

Example 2: $F(s) = \frac{12}{(s+2)^2(s+4)}$

① Factor denominator

$$\frac{12}{(s+2)^2(s+4)} = \frac{A}{(s+2)^2} + \frac{B}{s+2} + \frac{C}{s+4}$$

②

$$\frac{A(s+4) + B(s+2)(s+4) + C(s+2)^2}{(s+2)^2(s+4)}$$

$$\frac{A(s+4) + B(s^2+6s+8) + C(s^2+4s+4)}{(s+2)^2(s+4)}$$

③

$$\frac{As+4A+Bs^2+6Bs+8B+Cs^2+4Cs+4C}{(s+2)^2(s+4)} = \frac{12}{(s+2)^2(s+4)}$$

④ $s^2(B+C) = 0$
 $s(A+6B+4C) = 0$
 $4A+8B+4C = 12$

⑤

$$\begin{aligned} B+C &= 0 \\ -C & \\ B &= -C \\ 4A+4C+8B &= 12 \\ 4A &= 12 - 4C - 8B \\ \frac{4A}{4} &= \frac{12}{4} - \frac{4C}{4} - \frac{8B}{4} \\ A &= 3 - C - 2B \end{aligned}$$

⑥ $A+6B+4C=0$

$$\begin{aligned} 3 - C - 2B - 6C + 4C &= 0 \\ 3 - C + 2C - 6C + 4C &= 0 \\ 3 - C &= 0 \end{aligned}$$

$$\boxed{C=3}$$

$$\begin{aligned} A &= 3 - C - 2B \\ &= 3 - 3 + 6 \end{aligned}$$

$$\boxed{A=6}$$

$$B = -C$$

$$\boxed{B=-3}$$

⑦

$$\frac{A}{(s+2)^2} + \frac{B}{s+2} + \frac{C}{s+4} = \frac{6}{(s+2)^2} - \frac{3}{s+2} + \frac{3}{s+4}$$

⑧

$$\begin{aligned} &6 \cdot \mathcal{L}^{-1}\left\{\frac{1}{(s+2)^2}\right\} - 3 \mathcal{L}^{-1}\left\{\frac{1}{s+2}\right\} + 3 \cdot \mathcal{L}^{-1}\left\{\frac{1}{s+4}\right\} \\ &= \boxed{6te^{-2t}u(t) - 3e^{-2t}u(t) + 3e^{-4t}u(t)} \end{aligned}$$