Example 3: $F(s) = \frac{2s+26}{s^3+4s^2+13s}$

① Factor denominator

$$\frac{2s+26}{s(s^2+4s+13)} \rightarrow$$

② Perform Residue Method

$$2s+26 = \frac{A}{s} + \frac{Bs+C}{s^2+4s+13}$$

③ Multiply denominators

$$\frac{A(s^2+4s+13) + Bs+C(s)}{s(s^2+4s+13)} \rightarrow$$

④ Simplify

$$\frac{As^2+4As+13A+Bs^2+Cs}{s(s^2+4s+13)} = 2s+26$$

⑤ Gather Like Terms

$$s^2(A+B) = 0$$
$$s(4A+C) = 2s$$
$$13A = 26$$

⑥ Solve for A, B, C

$13A = 26$ → $A = \frac{26}{13}$ → $A = 2$

$4(2)+C = 2$ → $C = -6$

$2+B = 0$ → $B = -2$

⑦ Plug A, B, C in for Original equation

$$\frac{A}{s} + \frac{Bs+C}{s^2+4s+13} \rightarrow \frac{2}{s} + \frac{-2s-6}{s^2+4s+13}$$

⑧ Inverse laplace

$$\mathcal{L}^{-1}\left\{\frac{2}{s}\right\} + \mathcal{L}^{-1}\left\{\frac{-2s-6}{s^2+4s+13}\right\} \rightarrow$$

⑨ Inverse Laplace Simplify

$$\frac{2}{s} - \frac{2s}{s^2+4s+13} - \frac{6}{s^2+4s+13}$$

$$\frac{2}{s} - 2\left(\frac{s}{s^2+4s+13} + \frac{3}{s^2+4s+13}\right)$$

⑩ Factor denominator

$$\frac{2}{s} - 2\left(\frac{s+2-2}{(s^2+4s+4)+(13-4)} + \frac{3}{(s^2+4s+4)+(13-4)}\right)$$

$$\frac{2}{s} - 2\left(\frac{s+2}{(s+2)^2+9} + \frac{-2+3}{(s+2)^2+9}\right)$$

$$\frac{2}{s} - 2\left(\frac{s+2}{(s+2)^2+3^2}\right) - \frac{2}{(s+2)^2+3^2}$$

⑪ Simplify

$$\frac{2}{s} - 2\left(\frac{s+2}{(s+2)^2+3^2}\right) - \frac{2\cdot\frac{3}{3}}{(s+2)^2+3^2}$$

$$= \frac{2}{s} - 2\left(\frac{s+2}{(s+2)^2+3^2}\right) - \frac{2}{3}\cdot\frac{3}{(s+2)^2+3^2}$$

(12) Inverse laplace Transform

$$2\cdot\mathcal{L}^{-1}\left\{\frac{1}{s}\right\} - 2\cdot\mathcal{L}^{-1}\left\{\frac{s+2}{(s+2)^2+3^2}\right\} - \frac{2}{3}\cdot\mathcal{L}^{-1}\left\{\frac{3}{(s+2)^2+3^2}\right\}$$

$$= \boxed{2u(t) - 2e^{-2t}\cos(3t)u(t) - \frac{2}{3}e^{-2t}\sin(3t)u(t)}$$