A Simple Receive Diversity Technique for Distributed Beamforming

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Abstract

A simple method is proposed for use in a scenario involving a single-antenna source node communicating with a destination node that is equipped with two antennas via multiple single-antenna relay nodes, where each relay is subject to an individual power constraint. Furthermore, ultra-reliable and low-latency communication are desired. The latter requirement translates to considering only schemes that make use of local channel state information. Whereas for a receiver equipped with a single antenna, distributed beamforming is a well known and adequate solution, no straightforward extension is known. In this paper, a scheme is proposed based on a space-time diversity transformation that is applied as a front-end operation at the destination node. This results in an effective unitary channel matrix replacing the scalar coefficient corresponding to each user. Each relay node then inverts its associated channel matrix, which is the generalization of undoing the channel phase in the classical case of distributed beamforming to a single-antenna receiver, and then repeats the message over the resulting “gain-only” channel. In comparison to a single-antenna destination node, the method doubles the diversity order without requiring any channel state information at the receiver while at the same time retaining the array gain offered by the relays.

I. Introduction

Cooperative diversity is a means to boost the reliability of communication over a wireless fading medium where adjacent devices collaborate and share their antennas to facilitate communication between a source and destination node. Different approaches and transmission protocols have been investigated over the years to address this goal.

The potential of using multiple single-antenna relay nodes as a means of forming a virtual antenna array has been recognized and studied in depth since the pioneering work of [1]–[4]. Depending on the assumptions made on the availability of channel state information (CSI) at the relays, the virtual antenna array can serve either to provide diversity alone or to obtain also array (power) gain; the former not strictly requiring (forward, channel from relay to destination) CSI at the relays whereas the latter requiring at least local CSI to be available at the relays.

When the goal is to attain diversity alone, one can employ distributed space-time coding as suggested in [4], by means of opportunistic relay selection as suggested in [5] and further studied in [6], or by using standard codes in a cooperative/distributed scenario as described in [7], [8]. For further basic results on cooperative diversity transmission techniques, we refer the reader to [9]–[11] and references therein. It is worth noting that all of these diversity-oriented schemes offer no array gain, which as exemplified in the sequel can be very substantial.

Recently, the need for communication protocols that can provide ultra reliability while maintaining low latency has become apparent; see, e.g. [12] and [13]. While it is obvious that increasing the number of antennas enables potentially to attain higher diversity as well as a power gain, utilizing these while meeting stringent latency constraints introduces substantial challenges, one of which is the need for acquiring channel state information rapidly.

While traditional distributed space-time coding schemes assume full CSI at the receiver (and thus do not meet stringent latency constraints), differential space-time coding can be used without any CSI. See, e.g., [14], [15] and [16]. While loosing approximately 3 dB compared to coherent detection, the same transmit diversity (compared to having full CSI at the receiver) can be attained. However, this loss comes in addition to losing all of the array gain offered by the relays. Furthermore, the scenario where only a sub-group of relays is active (based on the channel realizations) poses a major challenge to distributed space-time schemes as either the relay nodes must coordinate how to partition the code between them (at the expense of additional latency) or else, the active nodes will need to use a space-time code that is designed for the *total number of relay nodes*. The latter option will again induce high latency as the latency of space-time codes rapidly grows with the number of transmit antennas. Further, the symbol rate of orthogonal design space time codes fast reduces one half as the number of transmit antennas grows.

Interestingly, there are scenarios where acquiring transmitter-side CSI is easier than acquiring receiver-side CSI; the transmission phase from the relays to the destination node in the considered setting is among these. Namely, a reasonable approach is to have the relays acquire local CSI (on both source-to-relay and relay-to-destination links) via channel reciprocity, employing time-division duplex (TDD). See further discussion in [6]. Such an approach fits well a scenario where there is a large number of “potential” relays, but only a rather small subset will be active in a given communication round. In such a scenario, it would be highly inefficient for the source and destination nodes to try to acquire CSI and then feed the CSI back to the relays, due to the large pool of potential relays. We also note that similar considerations lead to TDD being advocated for use in massive (non-distributed) MIMO systems; see, e.g., [17], [18].

It is well known that in the case of a system where all nodes are equipped with a single antenna and each relay knows the channel gain between itself and the destination, distributed (phase-only) beamforming offers both diversity and array gain [1]. In fact, only a small power loss, with respect to the full (centralized) array gain, is incurred by the availability of only local CSI. Specifically, the loss is identical to that incurred in the dual (receiver side) scenario of performing equal-gain combining rather than maximal-ratio combining (MRC), which is a classic problem that has been explored in depth [19]. Further, it has been shown in [20] that given a per-relay power constraint, such phase-only beamforming is optimal (in the sense of maximizing the receive SNR).

In the present paper, we extend the latter insight (i.e., that phase-only beamforming loses little with respect to centralized beamforming) to a scenario where the destination node is equipped with two receive antennas. That is, we show that given local CSI at the relays, when universal space-time diversity combining transformation (recently introduced in [21]) is applied at the receiver, each relay can invert its equivalent channel and thus one can attain outage probabilities that are comparable to those attained when the relays perform centralized beamforming on the maximum singular vector of the joint channel, up to a moderate power loss. Since the resulting channel from each relay to the destination is *unitary*, the channel inversion operation can be seen as the analogue of undoing the phase, as performed in the case of a single-antenna receiver.

As mentioned above, the key element used in the proposed scheme is a recently introduced universal space-time diversity combining transformation [21] that is performed as a front-end operation at the destination node. This transformation may be viewed as the dual of Alamouti modulation [22]. The transformation is universal in the sense that it is channel independent and so the receiver is not required to acquire any CSI. Similarly, we do not require the transmitter node to have access to any CSI. Rather, we assume that perfect, yet local only, CSI is available at each relay, a scenario studied and discussed in depth in e.g., [5], [6].

The rest of the paper is organized as follows. Section II describes the system model in more detail and recalls known results for the scenario of a single-antenna destination node. Section III presents the proposed transmission protocol as well as describes the space-time diversity combining transformation that constitutes a key component of it. Section V provides a performance analysis and comparison to some useful benchmarks. Conclusions are given in Section VI.

II. System model and Review of Known Results

We assume transmission between a source node and a destination node via an array of M relays where the source and each of the relays are equipped with a single antenna whereas the receiver is equipped with N_r antennas as depicted in Figure 1.

For the most part, we will consider the case of $N_r = 2$. A discussion of extensions is carried out in Section IV. The extension of the proposed scheme to relays equipped with multiple antennas is rather straightforward and is addressed in Section IV.

We largely assume the system setup as described in [5], [6] which we therefore only briefly recall, highlighting mostly the difference in assumptions, and referring the reader to the latter works for more details on the general problem formulation. The main differences in the assumptions are:

- In the present work, perfect synchronization of all nodes is assumed and we strive to achieve array gain in addition to diversity gain.
- The number of transmit antennas per node is not limited to one and in particular the destination node is equipped with two antennas (or possibly more).

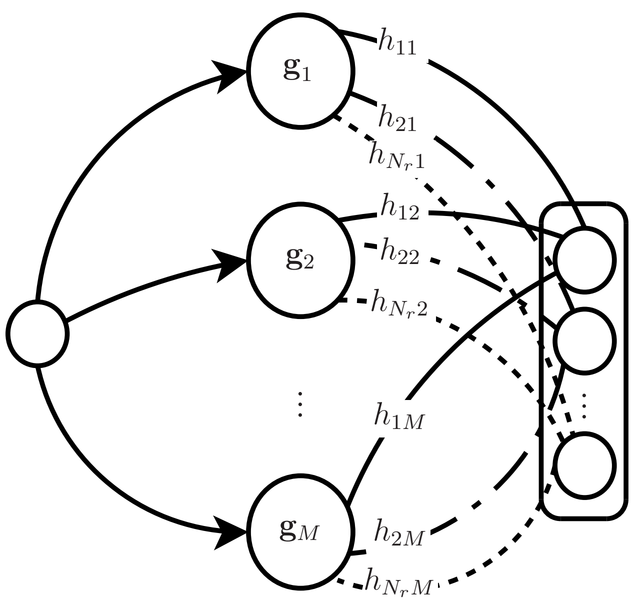

Fig. 1. Basic communication scenario between a source node and a destination node via a group of relays.

We consider a two-phase protocol. In the first phase, the source node transmits the coded message and all relays are in listening mode.

As for the second phase, the proposed scheme can equally fit a decode-and-forward (DaF) or an amplify-and-forward (AaF) mode of operation. For simplicity, we will describe a DaF protocol where all nodes that have successfully decoded the message participate in performing distributed beamforming as described next. We denote the (random) number of relays that successfully decode the message by M'. We note that in the terminology of [6], we are considering what is referred to as "reactive multiple-relay DaF".

We assume that the channel coefficients do not change over the entire transmission period of $2T$ symbols, where each phase occupies T symbols. All nodes are assumed to operate in half-duplex mode and for simplicity we assume there is no direct link between the source and destination.

As for CSI, we assume that before transmission begins, both source node and destination node send a beacon (clear-to-send, ready-to-receive) signal, from which the relays obtain *local* CSI (which is assumed to be perfect) by invoking channel reciprocity. Thus, we assume that transmission during both phases takes place over the same frequency band, i.e., we assume TDD.

The source node encodes the data to form the transmitted signal $x(t)$, $t = 1, 2, \ldots, T$, where T is the blocklength. The transmitted signal must satisfy the power constraint $\mathbb{E}\{|x(t)|^2\} \leq P_s$. The received signal at relay j is

$$r_j(t) = h_j^{s \to r} x(t) + n_j(t), \tag{1}$$

where $n_j(t)$ is circularly-symmetric complex normal $\mathcal{CN}(0, 1)$ and is i.i.d. over time and between relays. The channel coefficients are distributed in the same manner. Therefore, we may define the nominal SNR between the source and a relay node by

$$\mathsf{SNR}^{s \to r} \triangleq P_s. \tag{2}$$

Now each link from a relay to the destination is single-input multiple-output (SIMO) channel with coefficients

$$\mathbf{h}_j \triangleq \mathbf{h}_j^{r\to d} \triangleq \begin{bmatrix} h_{1j} & h_{2j} & \cdots & h_{N_r j} \end{bmatrix}^T, \tag{3}$$

for $j = 1, \ldots, M'$, where M' denotes the number of relays that have successfully decoded the message. Without loss of generality, we may assume that the relays with indices $1, \ldots, M'$, are the "successful" relays.

We denote the symbols sent from the relays by $x_j(t)$ and assume that each active relay must satisfy the (individual) power constraint $\mathbb{E}\{|x_j|^2\} = P_{r,i}$. For simplicity, we further assume that $P_{r,i} = P_r$ for all j. Thus, signal received at the destination is given by

$$\mathbf{s}(t) = \sum_{j=1}^{M'} \mathbf{h}_j x_j(t) + \mathbf{n}(t), \tag{4}$$

for $t = 1, \ldots, T$, and where $n_j(t)$ is i.i.d. $\mathcal{CN}(0,1)$ (over space and time). We note that with a slight abuse of notation we let t run form 1 to T in both phases of transmission. We define the nominal SNR between a relay node and the destination by

$$\mathsf{SNR}^{r\to d} \triangleq P_r. \tag{5}$$

We now describe the second phase of transmission. Each of the M' relays that have successfully decoded the message has access to the transmitted symbols $x(t)$, $t = 1, \ldots, T$. We will only consider relaying operations that amount to applying a linear transformation to the received codeword. We do assume that buffering of symbols is possible and hence linear space-time modulation can be applied at the relay.

Nonetheless, for simplicity, we first describe the simplest setting (without buffering), in which case the operation done at each relay amounts to multiplying each codeword symbol by some complex number which we take to be independent of t. We denote this scalar by g_j, $j =$

$1, \ldots, M'$. Thus, the output of each relay is simply

$$x_j(t) = g_j x(t), \quad t = 1, \ldots, T,$$

and hence the destination node receives

$$\mathbf{s}(t) = \sum_{j=1}^{M'} \mathbf{h}_j g_j x(t) + \mathbf{n}(t). \tag{6}$$

Let us define

$$\alpha \triangleq \sqrt{P_r/P_s}. \tag{7}$$

It follows that the gains g_j should be chosen such that $|g_j| = \alpha$. When considering more general space-time processing at the relays, (6) is replaced with a corresponding matrix variant as will be explicitly described in the sequel.

We will compare the outage probability attained by different schemes and take as a figure of merit, the receive SNR attained at the destination node. This can be directly translated to an outage probability for either uncoded transmission or coded transmission, depending on the stringency of the latency constraints. Both cases will be analyzed.

In particular, in order to provide simple performance bounds, we will analyze the mutual information attained by a scheme, defined by

$$I(\mathsf{SNR}_{\text{scheme}}) \triangleq \log(1 + \mathsf{SNR}_{\text{scheme}}). \tag{8}$$

Correspondingly, for coded transmission (with long blocklength), outage is defined as the event where the mutual information is below the target rate R_{tar}, i.e.

$$\Pr\left(I(\mathsf{SNR}_{\text{scheme}}) < R_{\text{tar}}\right). \tag{9}$$

A. Receiver Equipped with a Single Antenna: Known Results

In [20], it has been shown that given a per-relay power constraint (in the context of [20], this corresponds to a per antenna power constraint), the optimal beamforming vector which maximizes the received SNR is

$$g_j = \alpha \cdot \frac{h_j^*}{|h_j|}. \tag{10}$$

Note that such beamforming is dual to equal-gain combining. We further note that while in [20] full CSI at the transmitter is assumed, optimal beamforming subject to a per-antenna power constraint only makes use of local CSI (specifically, the phase of the forward channel) and hence can be employed in a distributed scenario.

When using (10), the received signal is

$$s(t) = \sum_{j=1}^{M'} \alpha |h_j| x(t) + n(t), \tag{11}$$

and hence, the resulting SNR is

$$\mathsf{SNR} = \left(\sum_{j=1}^{M'} |h_j| \right)^2 P_r. \tag{12}$$

As a benchmark, we consider the performance attained when employing the optimal "centralized" beamforming vector

$$g_j = \alpha \cdot \frac{h_j^*}{\|\mathbf{h}\|} \sqrt{M'}, \quad j = 1, \dots, M'. \tag{13}$$

Namely, this is the optimal beamforming vector subject to a global constraint on the total power transmitted by all the active relays, i.e., subject to the constraint $\sum_{j=1}^{M'} |x_j|^2 = M' P_r$. The corresponding SNR at the destination node is

$$\mathsf{SNR} = \|\mathbf{h}\|^2 M' P_r. \tag{14}$$

B. Receiver with Two Antennas: Performance Benchmark

We now return to the more general model where the destination node is equipped with $N_r > 1$ antennas where our focus is on the special case of $N_r = 2$.

Denoting the channels from relay j to the receiver as

$$\mathbf{h}_j = \begin{bmatrix} h_{1j} & h_{2j} \end{bmatrix}^T, \quad j = 1, \ldots, M, \tag{15}$$

the received signal is

$$\mathbf{s}(t) = \sum_{j=1}^{M'} \mathbf{h}_j g_j x(t) + \mathbf{n}(t). \tag{16}$$

We describe several potential transmission protocols for this scenario which will serve as benchmarks for comparison. The main attributes of all considered methods, as well as those of the scheme proposed in the present work, are summarized in Table I.

1) Arbitrary antenna selection: The simplest scheme is a receiver which arbitrarily a priori chooses to use only one antenna. Clearly, the performance is then identical to the case of having a receiver with a single antenna as given above.

2) Optimal antenna selection: At the expense of increased latency, after the destination node sends a beacon signal from each antenna, the relays could perform distributed beamforming of a pilot sequence, first to receive antenna 1 and then to receive antenna 2. Then, the destination node would choose the receive antenna with the higher SNR and notify the relays to which of the antennas they should perform beamforming, in a similar spirit to the 1-bit feedback scheme proposed in [23]. Specifically, in case receive antenna 1 is chosen, the beamforming coefficient at the j'th relay is

$$g_j = \alpha \cdot \frac{h_{1j}^*}{|h_{1j}|} \tag{17}$$

where α is defined in (7). In case receive antenna 2 is chosen, the beamforming coefficient at the j'th relay should be

$$g_j = \alpha \cdot \frac{h_{2j}^*}{|h_{2j}|}. \tag{18}$$

The SNR attained by such a selection protocol is

$$\mathsf{SNR} = \max_{i=1,2} \mathsf{SNR}_i, \tag{19}$$

where

$$\mathsf{SNR}_i = \left(\sum_{j=1}^{M'} |h_{ij}| \right)^2 P_r, \quad i = 1, 2. \tag{20}$$

The main drawback of this protocol is the significant latency it entails and further, the process where the destination informs of its choice of antenna is susceptible to errors. Nonetheless, we take it as a benchmark for sake of performance comparisons where we neglect possible feedback errors (w.r.t. the antenna chosen by the destination node).

3) Opportunistic relaying: As another benchmark, we also consider opportunistic relaying as proposed in [5] and which we now briefly recall.

As soon as each relay receives the clear-to-send packet, it starts a timer for a time that is proportional to the channel gain from the relay to the receiver. The timer of the relay with the best channel conditions will expire first. That relay transmits a short duration flag packet, signaling its presence. All relays, while waiting for their timer to expire are in listening mode. As soon as they hear another relay flagging its presence or forward information (the best relay), they back off. Assuming the receiver uses MRC, the SNR attained by this protocol is

$$\mathsf{SNR} = \max_j \|\mathbf{h}_j\|^2 P_r \tag{21}$$

where $\mathbf{h}_j$ is defined in (15).

While the array gain is lost, full diversity is nonetheless achieved [5]. An advantage of the scheme over the method that we propose is that the time synchronization requirements between relays may be somewhat less stringent.

We also consider as a benchmark a variant of opportunistic relaying with a sum-power constraint. That is, each relay attempts to transmit with power $M'P_r$. The resulting SNR in this case is

$$\mathsf{SNR} = \max_j \|\mathbf{h}_j\|^2 M' P_r. \tag{22}$$

4) "Ideal" distributed space-time coding: As another benchmark we consider the performance attained by distributed space-time coding. We note that when orthogonal-design space time codes are used, there is an inherent symbol rate penalty whenever the number of relays exceeds two [24]. Nonetheless, in the performance comparison we carry out, we ignore the rate penalty, i.e., allowing for "ideal" space time coding. Accordingly, we assume the effective SNR attained is

$$\mathsf{SNR} = \sum_{j=1}^{M'} \|\mathbf{h}_j\|^2 P_r \tag{23}$$

where $\mathbf{h}_j$ is defined in (15).

Note that this method does not attain the array gain offered by the relays. Indeed, it cannot, as no CSI concerning the channel from relay to destination is assumed to be known. This of course has its merits but comes at a significant price in the form of a power penalty.

Further, as mentioned above, in order to use orthogonal space-time codes with minimal latency, a space time code designed for the number of *active* relays must be used. But in such a scenario, it is necessary to have some synchronization between these relays as to how they partition the code matrix between them, resulting in additional overhead and latency. Another option is to use a code designed for the total number of relays M. However, as the latency of orthogonal space-time codes grows rapidly with M [25]–[27], this option is only practical when M is quite small. Furthermore, distributed space-time coding assumes perfect CSI at the receiver and thus an a additional overhead is required to attain this CSI.

As mentioned above, differential distributed space-time coding can be used, having the advantage of not requiring any channel knowledge at the receiver (nor at the relays). However, this method suffers from an additional loss on top of losing all of the array gain, as for ideal space-time coding. Namely, there is an additional inherent loss due to non-coherent detection amounting to roughly 3 dB.

5) Centralized beamforming: The optimal solution for transmission of a single stream over a MIMO channel subject to an aggregate power constraint is to transmit in the direction of the singular vector corresponding to the largest singular value [28]. Denote the singular value decomposition (SVD) of the channel between the relays and the receiver by

$$\mathbf{H} = \mathbf{U}\mathbf{D}\mathbf{V}^H, \tag{24}$$

and the singular values by d_j. Thus, the beamforming vector to be used is the one which corresponds to the stronger of the two non-zero singular values, i.e.,

$$g_j = \mathbf{V}_{j,m}\alpha\sqrt{M'}, \quad j = 1, \dots, M', \tag{25}$$

where $m = \arg\max_{j=1,2} d_m$ and the scaling factor α is defined in (7). Denoting the maximal singular value by $d_{\max}$, the attained SNR is

$$\mathsf{SNR} = d_{\max}^2 M' P_r. \tag{26}$$

We note that this scheme requires an even higher level of coordination than the antenna selection protocol as each relay needs to know the channels coefficients between *a*ll relays and the receiver (or alternatively, a much greater amount of feedback from the receiver as it needs to update each relay on its beamforming coefficient). Furthermore, the scheme does not satisfy a per-relay power constraint.

III. New distributed beamforming protocol

The proposed method utilizes a recently developed universal diversity combining scheme that we employ as a front-end operation at the destination node. We therefore begin by briefly recalling the scheme presented in [21].

A. Universal Diversity Combining Transformation

Consider a 2×1 single-input multiple-output (SIMO) channel, with channel coefficients h_1 and h_2. The signal received at antenna $j = 1, 2$, at discrete time t, is

$$s_j(t) = h_j x(t) + n_j(t). \tag{27}$$

We assume that the noise $n_j(t)$ is i.i.d. over space and time with samples that are circularly-symmetric complex Gaussian random variables with unit variance. We further assume the transmitted symbols are subject to the power constraint $\mathbb{E}\{|x|^2\} = P$.

The scheme works on batches of two time instances and for our purposes, it will suffice to describe it for time instances $t = 1, 2$. Let us stack the four complex samples received over $T = 2$ time instances, two over each antenna, into an 8×1 real vector:

$$\mathbf{s} = [s_{1R}(1)s_{1I}(1)s_{2R}(1)s_{2I}(1)s_{1R}(2)s_{1I}(2)s_{2R}(2)s_{2I}(2)]^T, \tag{28}$$

where x_R and x_I denote the real and imaginary parts of a complex number x. We similarly define the stacked noise vector $\mathbf{n}$. Likewise, we define

$$\mathbf{x} = [x_R(1)\, x_I(1)\, x_R(2)\, x_I(2)]^T. \tag{29}$$

Next, we form a real vector with 4 elements $\mathbf{y}$ by applying to the vector $\mathbf{s}$ the transformation

$$\mathbf{y} = \mathbf{G}\mathbf{s} \tag{30}$$

where

$$\mathbf{G} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \end{bmatrix}. \tag{31}$$

Note that unlike conventional diversity-combining schemes, here the combining matrix $\mathbf{G}$ is *universal*, i.e., it does not depend on the channel coefficients.

It is readily shown that the following holds

$$\begin{aligned} \mathbf{y} &= \frac{\|\mathbf{h}\|}{\sqrt{2}} \mathbf{O}(h_1, h_2)\mathbf{x} + \mathbf{G}\mathbf{n} \\ &= \frac{\|\mathbf{h}\|}{\sqrt{2}} \mathbf{O}(h_1, h_2)\mathbf{x} + \mathbf{n}', \end{aligned} \tag{32}$$

where

$$\mathbf{O}(h_1, h_2) = \frac{1}{\|\mathbf{h}\|} \begin{bmatrix} h_{1R} & -h_{1I} & h_{2R} & -h_{2I} \\ h_{1I} & h_{1R} & -h_{2I} & -h_{2R} \\ h_{2R} & -h_{2I} & -h_{1R} & h_{1I} \\ h_{2I} & h_{2R} & h_{1I} & h_{R1} \end{bmatrix}, \tag{33}$$

is an orthonormal matrix for any h_1, h_2 and where $\mathbf{n}'$ is i.i.d. and Gaussian with variance $1/2$.[1]

[1] The variance is $1/2$ as we chose above to normalize the complex noise to have unit power.

The receiver may reconstruct (up to additive noise) the original samples by forming

$$\begin{aligned}\hat{\mathbf{x}} &= \mathbf{O}^T(h_1, h_2) \cdot \mathbf{y} \\ &= \frac{\|\mathbf{h}\|}{\sqrt{2}}\mathbf{x} + \mathbf{n}''\end{aligned} \tag{34}$$

where $\mathbf{n}''$ is also i.i.d. Gaussian with variance $1/2$. We note that to perform this reconstruction step, CSI is needed at the receiver.[2]

Since the dimension (over the reals) of $\mathbf{y}$ is four rather than eight, as is the dimension of the received signal $\mathbf{s}$, we obtained a universal dimension-reducing linear combining scheme.

B. Universal Diversity Combining in a Multiple Access Scenario

We consider now the scenario of a $2 \times K$ MIMO multiple-access channel (MIMO-MAC) where K users, each equipped with a single antenna, transmit to a common receiver that is equipped with two antennas. The input/output relation of this MIMO-MAC can be expressed as

$$\mathbf{s}(t) = \sum_{k=1}^{K} \mathbf{h}_k x_k(t) + \mathbf{n}(t), \tag{35}$$

where $\mathbf{h}_k$ is the 2×1 channel matrix between user k and the receiver. We assume isotropic (“white”) transmission by each user and that all users are subject to the same power constraint P. The noise the noise $\mathbf{n}(t)$ is i.i.d. over space and time with samples that are circularly-symmetric complex Gaussian random variables with unit variance.

Now assume that the receiver applies as a front end the universal diversity combining transformation (30), applied over two consecutive time instances. Then by (32) and (35), in a multi-user scenario, the receiver output is given by

$$\mathbf{y} = \sum_{k=1}^{K} \frac{\|\mathbf{h}_k\|}{\sqrt{2}} \mathbf{O}(h_{1k}, h_{2k}) \mathbf{x}_k + \mathbf{n}', \tag{36}$$

[2] In the present paper, this operation will actually take place at the transmit side, at each relay, as detailed in the sequel.

where $\mathbf{h}_k$ is defined in (15) and $\mathbf{O}(h_{1j}, h_{2j})$ is given by (33).

C. Proposed Distributed Beamforming Scheme

In the proposed scheme, the destination node applies the universal space-time diversity transformation (30) as a front end-operation. As described above, this is done by buffering the symbols received during two consecutive time instances and applying to these the matrix $\mathbf{G}$ given in (31). Hence, (36) becomes

$$\mathbf{y} = \sum_{j=1}^{M'} \frac{\|\mathbf{h}_j\|}{\sqrt{2}} \mathbf{O}(h_{1j}, h_{2j}) \mathbf{x}_j + \mathbf{n}'. \tag{37}$$

Note that no CSI is needed at the destination.

Since we assume that each relay has perfect local CSI, at the expense of adding an additional delay of one symbol, each relay then simply "undoes" its channel matrix. Specifically, each relay transmits

$$\begin{aligned}\mathbf{x}_j &= \alpha \mathbf{O}(h_{1j}, h_{2j})^{-1} \mathbf{x} \\ &= \alpha \mathbf{O}(h_{1j}, h_{2j})^T \mathbf{x} \end{aligned} \tag{38}$$

where $\mathbf{x}$ is defined in (29) and should be interpreted as two symbols transmitted from the source node and correctly decoded at the M' relays participating in transmission, $\mathbf{O}$ is defined in (33), and α is defined in (7). Note that the per-antenna power constraint is satisfied due to the scaling factor α which is defined in (7).

Thus, the destination sees the effective channel

$$\begin{aligned}\mathbf{y} &= \sum_{j=1}^{M'} \alpha \frac{\|\mathbf{h}_j\|}{\sqrt{2}} \mathbf{O}(h_{1j}, h_{2j}) \mathbf{x}_j + \mathbf{n}' \\ &= \sum_{j=1}^{M'} \alpha \frac{\|\mathbf{h}_j\|}{\sqrt{2}} \mathbf{O}(h_{1j}, h_{2j}) \mathbf{O}(h_{1j}, h_{2j})^T \mathbf{x} + \mathbf{n}'\end{aligned}$$

$$= \sum_{j=1}^{M'} \alpha \frac{\|\mathbf{h}_j\|}{\sqrt{2}} \mathbf{x} + \mathbf{n}'. \tag{39}$$

The latter is a set of parallel (independent) scalar channels with SNR

$$\mathsf{SNR} = \frac{\left(\sum_{j=1}^{M'} \|\mathbf{h}_j\|\right)^2}{2} P_r. \tag{40}$$

The attained SNR is quite pleasing as we obtain both the maximal diversity gain while also enjoying transmit-side array gain (but not receive-side MRC gain). In fact, the attained performance is not far from that of the optimal selection benchmark (21) as we formalize in the form of a theorem.

Theorem 1. *For a given number of active relays M', each with power constraint P_r, the SNR of the proposed diversity-enhanced distributed beamforming scheme* (40) *suffers a power penalty factor no greater than 2 with respect to the optimal antenna selection benchmark* (21).

Proof: We may rewrite (40) as

$$\mathsf{SNR} = \left(\sum_{j=1}^{M'} \sqrt{\frac{|h_{1j}|^2 + |h_{2j}|^2}{2}}\right)^2 P_r. \tag{41}$$

Now, without loss of generality, let us assume that $\mathsf{SNR}_1 \geq \mathsf{SNR}_2$ where SNR_i is defined in (20). Thus, the selection schemes chooses SNR_1 and hence it attains

$$\mathsf{SNR} = \left(\sum_{j=1}^{M'} |h_{1j}|\right)^2 P_r. \tag{42}$$

Comparing (41) and (42), it is evident that for every element in the summation, the largest possible gap between the terms (in favor of selection) occurs in case where $|h_{2i}| = 0$. In this case, the gap is a factor of 2 for this summand. We therefore conclude that the maximal gap between (41) and (42) occurs when $|h_{2i}| = 0$ for all j and hence the proposed scheme suffers a power penalty that is no greater than 2 with respect to the optimal selection benchmark (21). ■

TABLE I
COMPARISON OF DIVERSITY COMBINING SCHEMES FOR A RECEIVER WITH TWO ANTENNAS

Scheme	**Distributed**	**Power constraint**	**CSI at relays**	**Low latency**	**SNR**
Arbitrary antenna selection	Yes	Per antenna	Local	Yes	$\left(\sum_{j=1}^{M'} \lvert h_{1j}\rvert\right)^2 P_r$
Optimal antenna selection	Partial	Per antenna	Full (or local+1-bit feedback)	No	$\max_j \left(\sum_{j=1}^{M'} \lvert h_{ij}\rvert\right)^2 P_r$
Opportunistic relaying	Yes	Per antenna	Local	Yes	$\max_j \lVert\mathbf{h}_j\rVert^2 P_r$
Opportunistic relaying, sum power	No	Sum power	Local	Yes	$\max_j \lVert\mathbf{h}_j\rVert^2 M' P_r$
Ideal distributed space-time coding	No	Per antenna	CSI at the receiver	No (function of M)	$\sum_{j=1}^{M'} \lVert\mathbf{h}_j\rVert^2 P_r$
Centralized beamforming	No	Sum power	Full (or receiver feedback)	No	$d_{\max}^2 M' P_r$
Unitary orthogonal combining	Yes	Per Antenna	Local	Yes (function of N_r)	$\frac{\left(\sum_{j=1}^{M'} \lVert\mathbf{h}_j\rVert\right)^2}{2} P_r$

IV. EXTENSIONS TO MORE ANTENNAS

The scheme described in the previous section can be extended (albeit not without some loss) to support more than a single antenna per node. In this section we present possible extensions for cases where the relays or the destination have more than a single antenna. Nevertheless, we still assume that the source is equipped with a single antenna and thus only a single stream is transmitted.

A. Relays with a Single Antenna, Destination with More than Two Antennas

Several extensions of the dimension reduction transformation to the case of a receiver with more than two antennas are detailed in [21]. As detailed in [21], all of the methods result in a non-orthogonal effective channel and thus suffer from some additional loss in performance with respect to MRC or antenna selection. Nevertheless, employing such transformations is still valuable as no CSI is needed.

Assuming the destination node has N_r antennas, and denoting by $\mathcal{F}$ the effective channel resulting from applying the universal transformation, the received signal is

$$\mathbf{y} = \sum_{j=1}^{M'} \frac{\|\mathbf{h}_j\|}{c} \mathcal{F}(h_{1j} \cdots , h_{N_r j}) \mathbf{x}_j + \mathbf{n}', \tag{43}$$

where c is a power normalization constant that depends (only) on the chosen transformation. See [21] for a detailed example for the case of four receive antennas.

Similar to the operation (38) applied in the case of a relay with two antennas, each relay now sends

$$\mathbf{x}_j = \alpha \beta_j \mathcal{F}(h_{1j} \cdots , h_{N_r j})^{-1} \mathbf{x}, \tag{44}$$

where β_j is a power normalization constant which depends on the chosen transformation as well as the actual channel (ensuring that the power constraint is satisfied). We note that this operation requires the ability to store $\mathbf{x}$ in the relays and results in an additional delay of T symbols at the relays, where T is the blocklength of the universal transformation employed and will grow with N_r, the number of antennas at the destination. This delay is added to the delay of T symbols which is required at the receiver to apply the transformation), resulting in a delay of $2T$ symbols.

B. MIMO Case - $N_r \times N_t$ Links Between Each Relay and Destination

Before describing the most general case, we discuss several intermediate cases. Further, for simplicity, we assume that all relays have the same number, N_t, of antennas.

1) Relays with N_t antennas, destination with a single antenna: In this case, since we assume that local CSI is available at the relays, each relay can apply local beamforming subject to the per-relay power constraint.

With a slight abuse of notation, denoting the channel from relay j to the destination by

$$\mathbf{h_j} \triangleq \begin{bmatrix} h_{1j} & \cdots & h_{N_tj}, \end{bmatrix} \tag{45}$$

the signal $x(t)$ will be multiplied at the i'th relay by the beamforming vector

$$\mathbf{g}_i = \alpha \frac{\mathbf{h_j^*}}{||\mathbf{h_j}||}. \tag{46}$$

Clearly, this scheme enjoys the maximal diversity possible in the considered scenario. Further, it achieves close to the maximal (centralized beamforming) array gain.

2) Relays with N_t antennas, destination with 2 antennas: First, we note that in case the destination has a single antenna, using the SVD of the channel between the j'th relay and the destination

$$\mathbf{h}_j = u_j \begin{bmatrix} d_j & 0 & \cdots & 0 \end{bmatrix}_{1 \times N_t} \mathbf{V}_j^H, \tag{47}$$

we may rewrite (40) alternatively as

$$\mathsf{SNR} = \frac{\left(\sum_{j=1}^{M'} d_j\right)^2}{2} P_r. \tag{48}$$

Next, we show that any beamforming vector (meeting the per-relay power constraint) can be applied in conjunction with the universal dimension reduction. To that end, we denote by $\mathbf{H}_j$ the $2 \times N_t$ channel from the j'th relay to the destination. The SVD of this channel takes the form

$$\mathbf{H}_j = \mathbf{U}_j \begin{bmatrix} d_{i1} & 0 & \cdots & 0 \\ 0 & d_{i2} & \cdots & 0 \end{bmatrix}_{2\times N_t} \mathbf{V}_j^H \tag{49}$$

Applying beamforming vector $\mathbf{p} = \begin{bmatrix} p_1 & \cdots & p_{N_t} \end{bmatrix}^{\mathbf{T}}$ (where $\sum |p_j|^2 = 1$), we get

$$\begin{aligned} \mathbf{H}_j\mathbf{p} &= \mathbf{U}_j \begin{bmatrix} d_{j1} & 0 & \cdots & 0 \\ 0 & d_{j2} & \cdots & 0 \end{bmatrix} \mathbf{V}_j^H \begin{bmatrix} p_1 & \cdots & p_{N_t} \end{bmatrix}^T \\ &= \mathbf{U}_j \begin{bmatrix} d_{j1} & 0 & \cdots & 0 \\ 0 & d_{j2} & \cdots & 0 \end{bmatrix} \begin{bmatrix} \tilde{p}_1 & \cdots & \tilde{p}_{N_t} \end{bmatrix}^T \\ &= \mathbf{U}_j \begin{bmatrix} d_{j1}\tilde{p}_1 \\ d_{j2}\tilde{p}_2 \end{bmatrix} \end{aligned} \tag{50}$$

where $|\tilde{p}_1|^2 + |\tilde{p}_2|^2 \leq 1$ since these are (only) two entries of a vector of unit norm (the result of a unitary transformation of the unit norm vector $\mathbf{p}$). The application of the beamforming vector transforms the $2 \times N_t$ MIMO channel to a 2×1 SIMO channel to which the universal dimension reduction transformation can be applied. Furthermore, it is readily seen that the singular value of this channel is $\tilde{d}_j = \sqrt{(d_{j1}\tilde{p}_1)^2 + (d_{j2}\tilde{p}_2)^2}$.

It can be easily shown that the optimal beamforming vector is the right singular vector of $\mathbf{H}$ corresponding to the largest singular value. Another simple, though suboptimal, beamforming vector corresponds to choosing the strongest the best of the N_t antennas at each relay.

3) Relays with N_t antennas, destination with $N_r > 2$ antennas: We may combine the method just described (in Section IV-B2) to convert the MIMO channel from each relay to the destination to a SIMO one with a universal combining transformation as described in Section IV-A. Specifically, given a chosen universal space-time transformation, each relay will choose as a beamforming vector the singular vector corresponding to the maximal singular value of the resulting *effective channel*.

V. Numerical Performance Evaluation

We compare the performance of different schemes when operating in a Rayleigh fading environment. As mentioned above, we consider the outage probability of the mutual information as well as the outage probability for uncoded transmission.

A. Evaluation of performance over second transmission phase

We consider a scenario in which exactly $M' = 4$ relays participate in the second phase of transmission. Figure 2 depicts the outage probability of the mutual information for different schemes for a receiver having a single antenna. The performance of both distributed (phase-only) and the centralized beamforming benchmark is shown. We also plot the performance of opportunistic relaying. As can be seen, the outage probability achieved by distributed beamforming is very close to that achieved by centralized beamforming. As expected, opportunistic beamforming has the same slope as the other methods but it does not benefit from an array gain. Figure 3 shows similar behavior when plotting the outage probabilities corresponding to uncoded QPSK transmission.

Figure 4 depicts the outage probability of the mutual information, corresponding to different relaying schemes, for a receiver equipped with two antennas. As simple benchmarks that are compatible with low latency constraints, we consider arbitrary selection and the opportunistic relaying.[3] Both methods suffer a significant penalty in terms of transmit power required to meet a given outage probability as compared with centralized beamforming.

Figure 4 also depicts methods *that do not meet* the low latency requirement or per-relay power constraint. Namely, optimal selection (which does not meet the former requirement) and

[3]Note that for the latter, in order to benefit from the MRC gain of the two antennas at the receiver, additional training is now required, once the best relay is chosen.

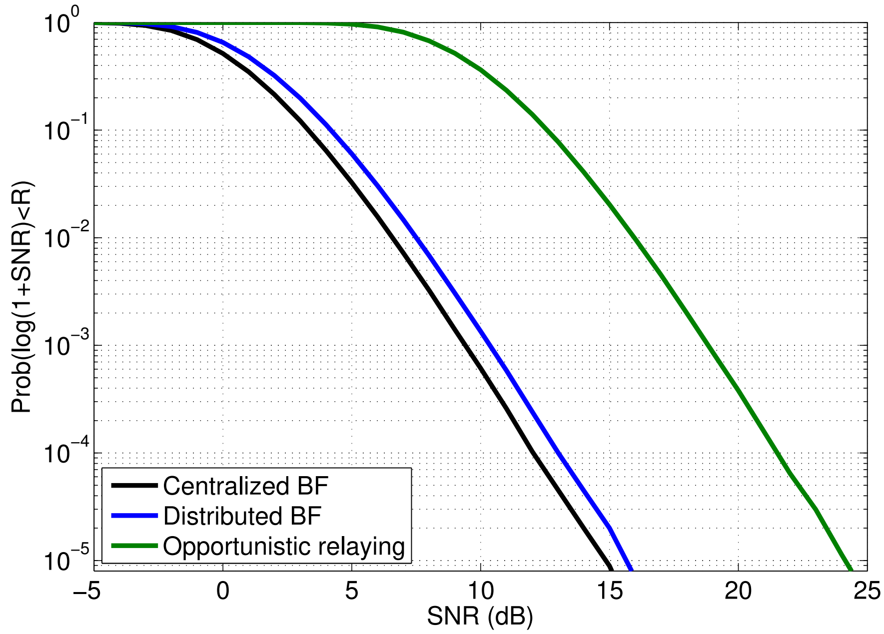

Fig. 2. Outage probability of mutual information for i.i.d. Rayleigh fading with $M' = 4$ relays, $R_{\mathrm{tar}} = 4$, as a function of the transmit power at the relays P_r for a receiver equipped with a single antenna.

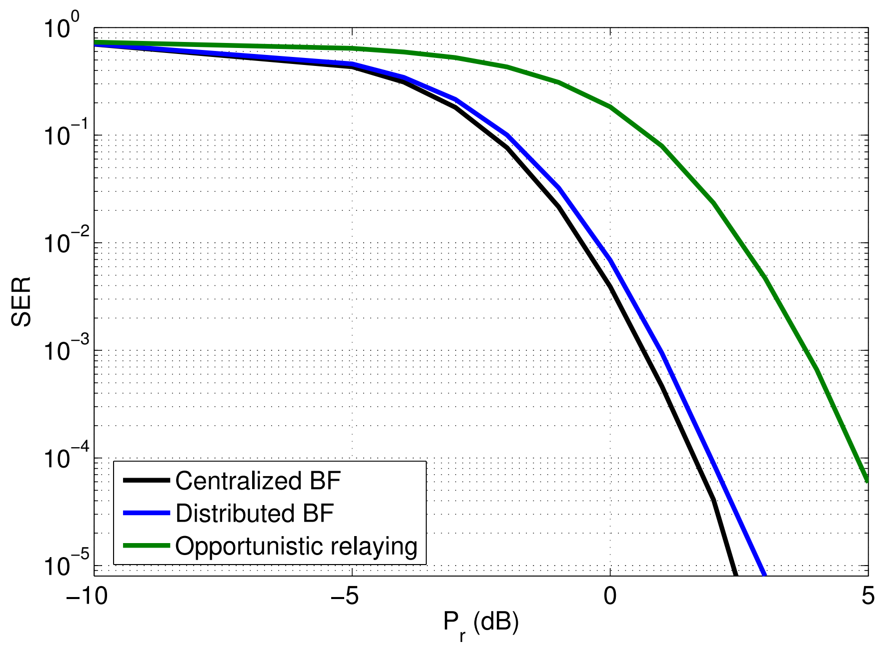

Fig. 3. Outage probability of uncoded QPSK for i.i.d. Rayleigh fading with $M' = 4$ relays as a function of the transmit power at the relays P_r for a receiver equipped with a single antenna.

optimal opportunistic relaying subject only to a sum power constraint (which does not meet the latter requirement) are depicted. These result in a much smaller gap with respect to centralized beamforming. It can be seen that the proposed method achieves similar performance while meeting both these requirements. The performance of ideal distributed space-time coding is also depicted in Figure 4. Even without taking into account additional penalties, the performance of this scheme is significantly worse than the proposed method (and the gap will be larger if compared against differential distributed space-time coding). Figure 5 demonstrates a similar behavior for uncoded QPSK transmission.

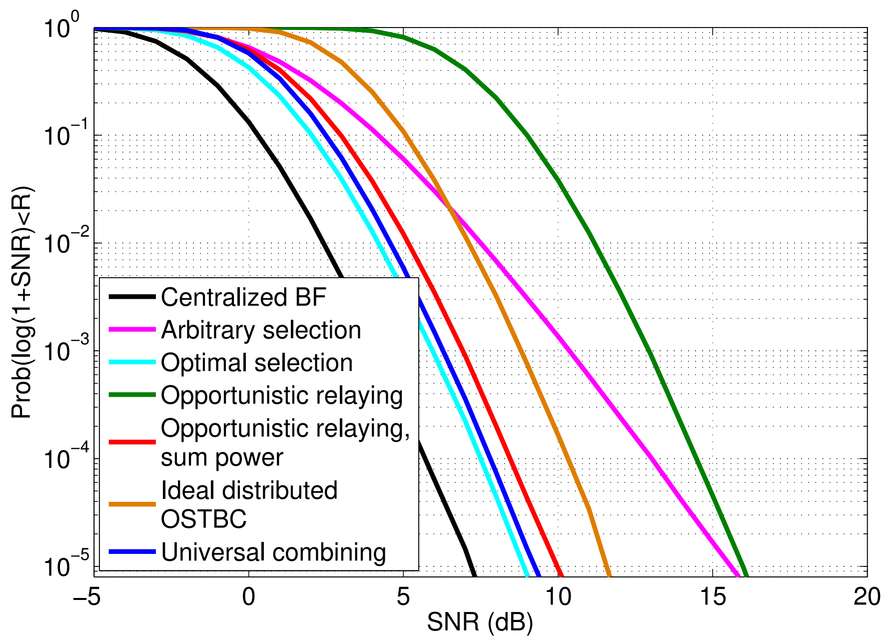

Fig. 4. Outage probability of mutual information for i.i.d. Rayleigh fading with $M' = 4$ relays, $R_{\mathrm{tar}} = 4$, as a function of the transmit power at the relays P_r for a receiver equipped with two antennas.

Another comparison of interest is to study the number of relays required to support a required outage probability and target rate. Figure 6 depicts the outage probability of the mutual information as a function of the number of active relays for the different schemes where we set $P_r = 0$ dB, a target rate $R_{\mathrm{tar}} = 4$ and the receiver is equipped with two antennas. As can be seen, the

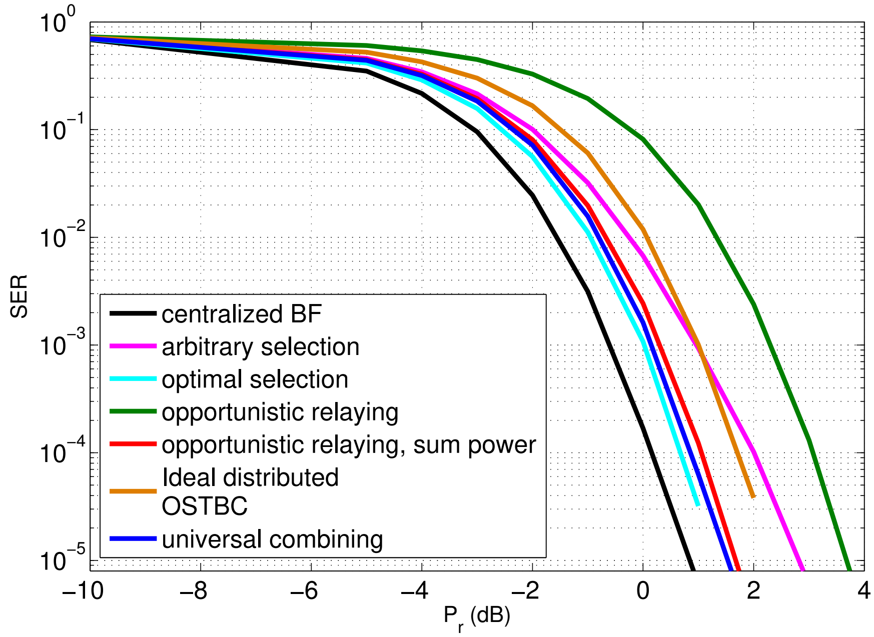

Fig. 5. Outage probability of uncoded QPSK for i.i.d. Rayleigh fading with $M' = 4$ relays as a function of the transmit power at the relays P_r for a receiver equipped with two antennas.

new scheme requires almost the same number of relays as optimal selection while meeting the low latency requirement. This is also reflected in Figure 7 for uncoded QPSK transmission.

B. End-to-End Simulation

We now simulate the end-to-end performance when both phases of transmission are in operation. That is, we now include the first hop in the simulation. Figures 8 and 9 show the behaviour of the different schemes as a function of the maximal possible relays M, where we set $P_s = 20$ dB and $P_r = 0$ dB. In these figures, the number of active relays M' is a random variable that depends on the SNR of the links between the source node and the relays.

VI. Conclusions and Outlook

We have introduced a novel distributed beamforming scheme with enhanced diversity for systems where the receiver is equipped with two antennas. The key ingredient is having the

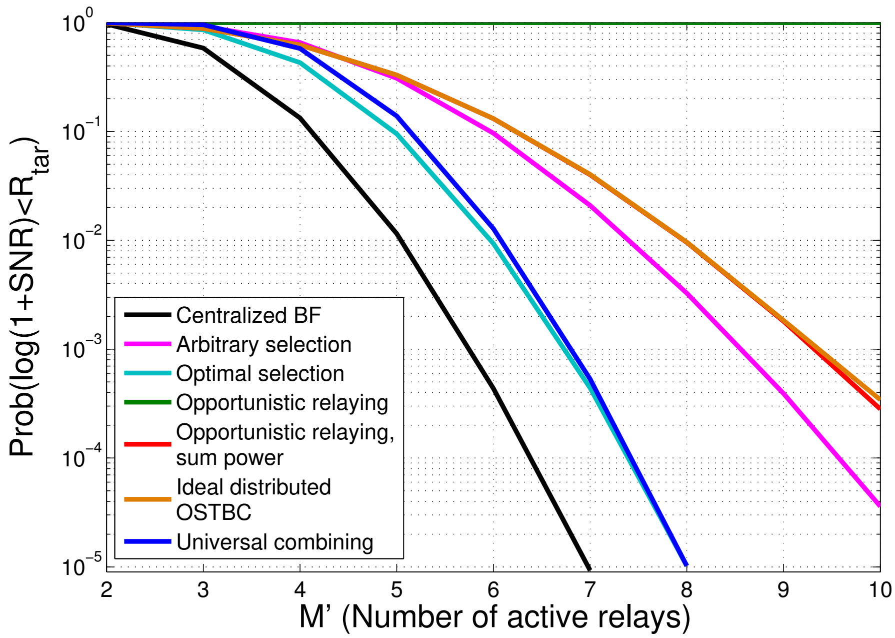

Fig. 6. Outage probability of mutual information for i.i.d. Rayleigh fading with $P_r = 0$ dB, $R_{\text{tar}} = 4$, as a function of the number of active relays M' for a receiver equipped with two antennas.

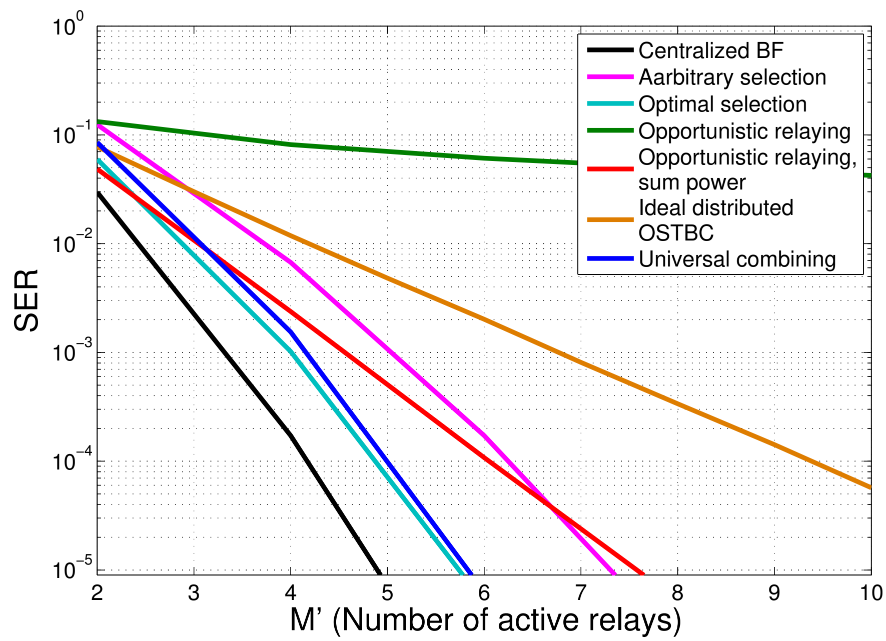

Fig. 7. Outage probability for uncoded QPSK transmission, for i.i.d. Rayleigh fading with $P_r = 0$ dB, and a varying number of active relays M' for a receiver equipped with two antennas.

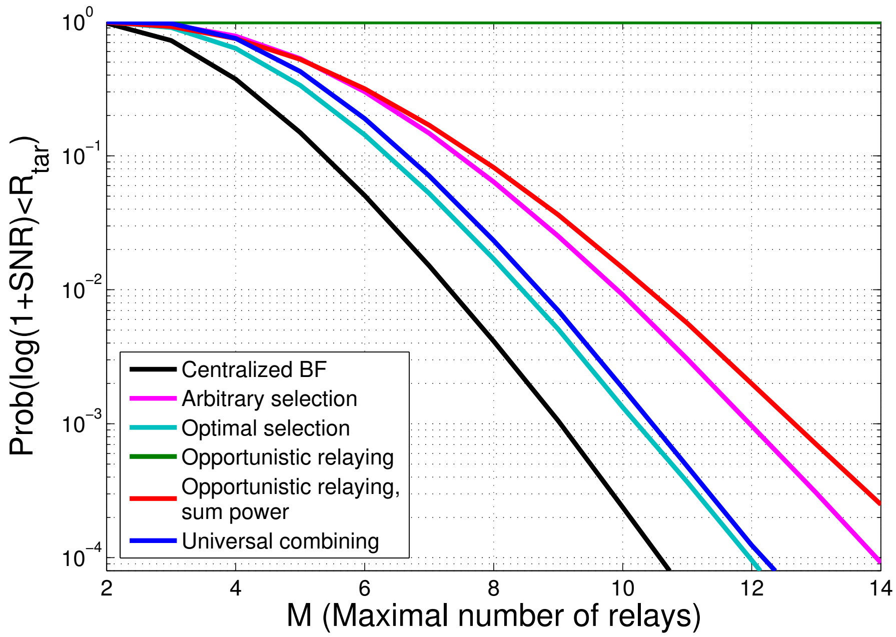

Fig. 8. Outage probability of mutual information for i.i.d. Rayleigh fading with $R_{\mathrm{tar}} = 4$, $P_s = 20$ dB, $P_r = 0$ dB, as a function of the total number of relays (the number M' of active ones being random) for a receiver equipped with two antennas.

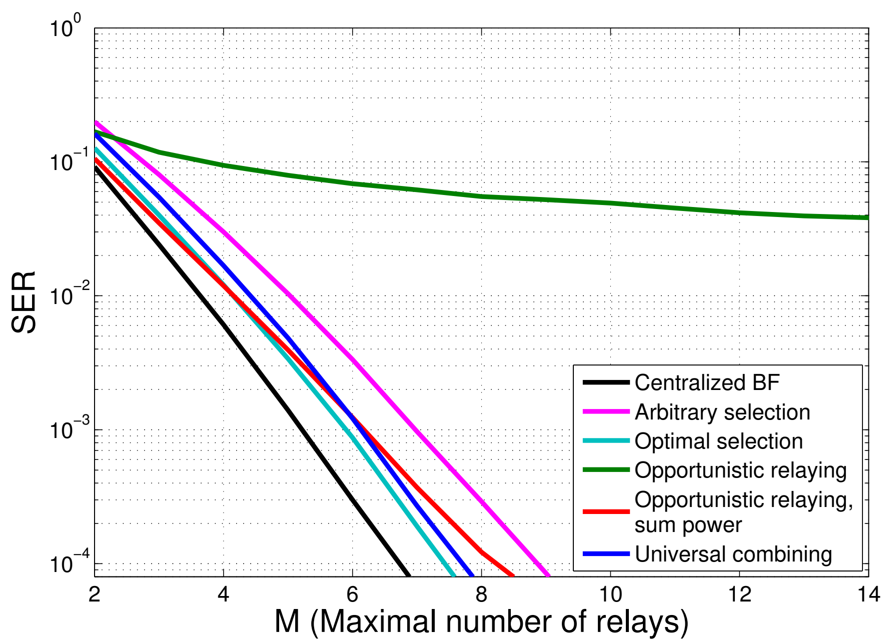

Fig. 9. Outage probability for uncoded QPSK transmission for i.i.d. Rayleigh fading channel with $P_s = 20$ dB, $P_r = 0$ dB, as a function of the total number of relays M (the number of active ones M' being random) doe a receiver equipped with two antennas.

receiver employ a universal space-time diversity combining transformation as a front-end operation, along with simple unitary precoding at the relays, utilizing only local channel state information. The scheme allows to enjoy both full diversity as well as substantial array gain. An interesting area for further research is extending the results to receivers equipped with more than two antennas. A possible avenue for such an extension is the use of more general orthogonal space-time block codes [24] or quasi-orthogonal codes [29]–[31] as the space-time diversity combining transformations. A preliminary study in this direction appears in [21].

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