Rate 1 Quasi Orthogonal Universal Transmission and Combining for MIMO Systems Achieving Full Diversity

Barak Avraham, Elad Domanovitz, Uri Erez

***Abstract*—Recent work introduced the receiver-side dual of Alamouti modulation, enabling universal diversity combining for a 2×1 single-input multiple-output channel, where universality is in the sense that the combining weights do not depend on the channel realization, albeit the knowledge of the latter is required at the time of reconstruction. The present work addresses general multuple-input multiple-output systems and develops combined diversity transmission and combining schemes that are universal in the sense that the operations performed at both transmission ends are channel independent. Such schemes may be useful in a scenario where a multiple-antenna source node communicates with the cloud via a multiple-antenna "dumb relay" that forwards the received vector over a rate-constrained digital front-haul link or serves as relay performing an amplify-forward operation over the air. The proposed schemes are derived by establishing a certain operational equivalence relation between the true channel and an associated multiple-input single-output channel.**

***Index Terms*—Diversity methods, space-time codes, relays, MIMO systems, fronthaul, radio access network.**

I. Introduction

A well-known challenge for achieving the goal of ultra-reliable communication over a wireless link is overcoming channel fading. Caused by multiple scattering and moving objects, fading is a major impediment to achieving the goal in non line-of-sight environments, especially so when small payloads are to be transmitted and/or when operating over a narrow frequency band. One of the fundamental approaches to reduce the impact of fading is to use multiple transmit and/or receive antennas and thus reducing susceptibility to fading.

More specifically, recall that [1] the diversity order is defined as the slope of the error probability curve at (asymptotically) high signal-to-noise ratio (SNR) and that for i.i.d. Rayleigh fading, the maximal achievable diversity of a $N_t \times N_r$ multiple-input multiple-output (MIMO) channel is $N_r N_t$ [2]. Thus, multiple antennas allow to drastically enhance communication reliability.

Simple methods can be used to utilize the additional antennas. For instance, full diversity can be achieved via repetition (sending each symbol over the different antennas in sequence). However, repetition results in a drastic reduction in the effective symbol rate. Therefore, schemes that attain maximal diversity while retaining a high symbol rate are of interest.

The difficulty of meeting this goal greatly depends on two factors: the availability of channel state information (CSI) and the MIMO configurations. Clearly, the task is simple if CSI is available at both transmission ends. Another simple scenario is when only the receiver is equipped with multiple antennas (i.e., a SIMO setting) where it suffices for the receiver to have CSI in order to attain maximal diversity while supporting a single stream (symbol rate of 1) by employing maximal ratio combining (which effectively converts the SIMO channel to an equivalent single-input single output (SISO) channel thus enjoying also reduced detection complexity). Conversely, the same is true in the multiple-input single-output (MISO) scenario by employing transmit beamforming assuming the transmitter has access to CSI.

The MISO channel with CSI available only at the receiver, a scenario that is very common in practice, is more challenging. A well known approach to attain full diversity is to employ space-time codes, including space-time trellis codes and space-time block codes (STBC); see, e.g., [3].

Straightforward approaches for designing space-time codes achieving maximal diversity, while retaining a high symbol rate, require in general applying maximum likelihood (ML) detection. While performing ML for small number of antennas is tractable, as the decoding complexity of ML scales exponentially with the number of transmit and receive antennas, and as modern architectures strive to use high number of transmit and receive antennas, achieving maximal diversity while retaining a high symbol with reduced decoding complexity is desired.

A well-known attractive approach, when CSI is available at the receiver only, is to employ orthogonal space-time block-codes (OSTBC) [4]. As is well known, for the case of two transmit antennas the Alamouti OSTBC [5], supports a symbol rate of 1 with minimal possible block-length, i.e., two channel uses, and is thus "ideal". When the number of antennas grows however, the symbol rate of orthogonal OSTBCs decreases, ultimately down to $1/2$. Thus, full diversity while maintaining minimal detection complexity comes with a penalty in rate. Moreover, the blocklength of orthogonal STBCs grows rapidly with the number of antennas.

When the number of transmitter antennas is large, quasi-orthogonal space-time block codes (QOSTBC) [6]–[9], and even more so QOSTBCs with symbol rotations [10]–[12] are an attractive transmission scheme, since the latter variant achieves full diversity while maintaining a symbol rate of 1 and while further offering reduced detection complexity. Specifically, it was shown in [11] that the generalization of the code suggested in [6] achieves rate one, full diversity and a reduced detection complexity that amounts to separate detection for each half of the transmitted information symbols.

Consider now the single-input multiple-output (SIMO) channel. Recently, [13] and [14] addressed a problem that can be thought of as the dual to that of diversity transmission over MISO channels with CSI at the receiver only. Namely, the problem is that of diversity combining at the receiver in the absence of CSI. More specifically, for the case of a two-antenna SIMO channel, the latter works suggested a universal (channel independent) combining method that allows to achieve receiver diversity without CSI at the point of combining while offering full symbol rate, i.e., rate 1. While CSI is not required at the point of combining, it is assumed that at the point of detection, CSI is available. An example for such a receiver setting is the cloud radio access networks (C-RAN) architecture in which the transmitted symbols are received in a "dumb" relay and are then forwarded to the cloud for detection. As the motivation is to keep the relays as simple as possible, while reducing the throughput between the relays and the cloud, performing universal combining at the relays can be very beneficial. It was demonstrated that for a single transmit antenna and two receive antennas, maximal diversity and maximal rate (symbol rate one) along with SISO detection complexity can be achieved without CSI at the combining point.

In this paper, we show that rate 1 universal (i.e., both the transmitter and the combining unit have no access to CSI) transmission and combining achieving full-diversity is possible while at the same time supporting detection complexity which amounts to separate detection for each half of the transmitted information symbols can be achieved for transmission over any MIMO channel when. Starting with the simplest 2×2 channel, we first demonstrate that simple attempts at extending transmitter-only or receiver-only universal full-diversity schemes result in a reduced symbol rate (smaller than one). We present a combined transmitter and receiver universal scheme and show that while requiring no CSI either at the transmitter, nor at the combining unit, the suggested scheme achieves maximal diversity while offering rate one with reduced detection complexity.

II. System Model

We analyze a topology in which the receiver is composed of two blocks: a combining unit and a detector where these two blocks are not necessarily collocated. We begin by describing the channel between the transmitter and the combining unit.

The following channel notation is common in the space-time communication literature. The channel output is given by

$$\mathbf{r} = \mathbf{c}\mathbf{H} + \mathbf{z}, \tag{1}$$

where the input to the channel is a row vector $\mathbf{c} \in \mathbb{C}^{\mathbf{N_t}}$, i.e., the i'th element of $\mathbf{c}$ is the symbol transmitted through antenna i; $\mathbf{H} \in \mathbb{C}^{N_t \times N_r}$ is the channel matrix and we denote its entries, i.e., the channel coefficient from transmitting antenna i to receive antenna j by h_{ij}. We assume that the transmitted symbols are subjected to a sum power constraint of P. Finally, $\mathbf{z}$ is i.i.d. (over space and time) circularly-symmetric complex Gaussian noise with unit variance.

We further assume that the transmitter uses space-time block codes as defined in [15] which we briefly describe. The transmitter wishes to transmit K symbols $\tilde{x}_1, \ldots, \tilde{x}_K$ over T channels uses. The transmitted symbols may be described by a *space-time transmission matrix* denoted by $\mathbf{C}^{K,T} \in \mathbb{C}^{T \times N_t}$ where the entry $c_{t,i}$ is the transmitted symbol at time t from antenna i , $t \in [1, \ldots, T]$ and $i \in [1, \ldots, N_t]$. Recall that the minimum value of T required to achieve full diversity is $T = N_t$ [15]. Codes having the minimum value of T are called "delay optimal". The induced transmission symbol rate is denoted by

$$R_{\text{eff}} = \frac{K}{T}. \tag{2}$$

We assume that the entries of the transmission matrix may be one of the indeterminates $\pm x_1, \ldots, \pm x_K$ and their conjugates $\pm x_1^*, \ldots, \pm x_K^*$ (up to a power normalization factor). Those indeterminates are derived from the information symbols by (possibly) applying a certain transformation on them (e.g., symbols rotations as in [11]).

We set $\mathbb{E}\left[|x_l|^2\right] = P$ for $l \in [1, \ldots, K]$, so the power constraint is met for a normalization factor of N_t, i.e. $\mathbb{E}\left[|c_i^t|^2\right] = \frac{P}{N_t}$ for any i, t. It is assumed that the channel remains fixed during the transmission of the codeword $\mathbf{C}^{K,T}$. The output of the channel (1) can now be rewritten as

$$\mathbf{R} = \mathbf{C}^{K,T}\mathbf{H} + \mathbf{Z}, \tag{3}$$

where $\mathbf{R} \in \mathbb{C}^{T \times N_r}$ represents the received matrix (per receive antennas and over time). Similarly $\mathbf{Z} \in \mathbb{C}^{T \times N_r}$ is the space-time noise matrix. We denote by $r_{t,i}$ and $z_{t,i}$ the received symbols and the noise at receive antenna i at time t (see Figure 1).

We turn now to describe the link between the combining unit and the detector; we will refer to it as the fronthaul link in the sequel. The combining unit takes the matrix $\mathbf{R}$ as its input and returns as output a vector $\mathbf{s} \in \mathbb{C}^{\mathbf{T}}$ which represents the stream of symbols forwarded to the detector (and used to recover the K information symbols). Hence, the combining unit performs dimension reduction. Of course, in practice, the symbol stream after dimension reduction would also need to be quantized before transmission over the fronthaul.

We further constrain the dimension reduction operations to be any linear operation over the reals.[1] The combining operation applied to the matrix R is denoted by $\mathfrak{C}$, such that:

$$\begin{aligned} \mathbf{s} &= \mathfrak{C}(\mathbf{R}) \\ &= \mathfrak{C}(\mathbf{C}^{K,T}\mathbf{H} + \mathbf{Z}) \\ &\overset{(a)}{=} \mathfrak{C}(\mathbf{C}^{K,T}\mathbf{H}) + \mathfrak{C}(\mathbf{Z}) \end{aligned} \tag{4}$$

where (a) follows from constraining the combining operator to be widely linear. We note that the vector $\mathbf{s}$ is of size T so that one information symbol per channel use is conveyed over the fronthaul link, i.e. there is no bandwidth (BW) expansion or reduction.

We refer to the space-time code along with the combining operation as a *universal transmission-combining*

[1] Alternatively, the operations are assumed to be widely linear over the complex field.

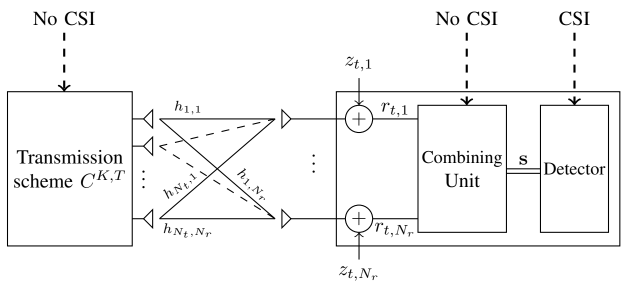


Figure 1: Combining scheme for $N_t \times N_r$ system where CSI is available only at the detector.

scheme. For a $N_t \times N_r$ channel, such a scheme is characterized by the pair $\{\mathbf{C}^{K,T}, \mathfrak{C}\}_{N_t,N_r}$, were $\mathbf{C}^{K,T}$ is the the STBC applied at the transmitter and where $\mathfrak{C}$ is the combining method operation applied at the receiver. Note that both units are independent of the channel matrix realization $\mathbf{H}$.

The vector $\mathbf{s}$ is used by the detector to recover the K information symbols where perfect CSI is assumed. For simplicity, in the sequel, we will consider the bit-error rate (BER) of different uncoded transmission schemes.

III. Main Results

We first introduce the concept of *scheme equivalency* which is the main tool we use in this paper. Consider a $N_t \cdot N_r \times 1$ MISO transmission scheme with the STBC transmission of $\hat{\mathbf{C}}^{K,\hat{T}} \in \mathbb{C}^{\hat{T} \times N_{\text{eff}}}$ where we denote

$$N_{\text{eff}} = N_t \cdot N_r. \tag{5}$$

The transmission scheme $\hat{\mathbf{C}}^{K,\hat{T}}$ is also comprised of the same indeterminants $\pm x_1, \ldots, \pm x_K$ and their conjugates $\pm x_1^*, \ldots, \pm x_K^*$ (up to a power normalization factor) so its rate is $\hat{R}_t = \frac{K}{\hat{T}}$. Noting that in the case of a MISO channel, no combining is needed (the combining unit forwards the received symbols as is), we can denote the above scheme by $\{\hat{\mathbf{C}}, \mathfrak{I}\}_{N_{\text{eff}},1}$ where $\mathfrak{I}$ stands for the operation of a combining unit that merely forwards its received symbols, namely,

$$\mathfrak{I}(\mathbf{R}) = \mathbf{R}. \tag{6}$$

For a MISO channel with channel vector $\hat{\mathbf{h}} \in \mathbb{C}^{N_{\text{eff}}}$, the received vector $\hat{\mathbf{r}} \in \mathbb{C}^{N_{\text{eff}}}$ is given by

$$\hat{\mathbf{r}} = \hat{\mathbf{C}}^{K,\hat{T}} \hat{\mathbf{h}} + \hat{\mathbf{z}}, \tag{7}$$

where the noise $\hat{\mathbf{z}} \in \mathbb{C}^{N_{\text{eff}}}$ is i.i.d circularly-symmetric complex Gaussian with unit variance. The output of the inactive combiner is $\hat{\mathbf{s}} = \hat{\mathbf{r}}$.

Consider now a $N_t \times N_r$ MIMO channel with a given channel matrix $\mathbf{H}$. We define a *conjugate-symmetric reordering transformation* $\mathcal{M}$ as a mapping from $\mathbf{H}$ to $\hat{\mathbf{h}}$, such that each entry of the matrix $\mathbf{H}$ is mapped to a distinct location in the vector $\hat{\mathbf{h}}$, up to conjugation and/or negation. Such a transformation $\mathcal{M}$ is associated with a one-to-one mapping m that maps each pair of indices $(i, j) : 1 \le i \le N_t, 1 \le j \le N_r$ to a distinct index $k : 1 \le k \le N_t \cdot N_r$, i.e we write $m(i, j) = k$ and hence $m^{-1}(k) = (i, j)$. Thus, for any k, we have

$$h_k \overset{\pm *}{=} \mathbf{H}_{m^{-1}(k)} \tag{8}$$

where the notation $\overset{\pm *}{=}$ stands for equality with possibly negation and/or conjugation. We denote

$$\hat{\mathbf{h}} = \mathcal{M}(\mathbf{H}). \tag{9}$$

As an example, we can define the following conjugate-symmetric reordering transformation from a 2×2 MIMO channel to a 4×1 MISO channel:

$$\begin{aligned} \mathcal{M}\left(\begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix}\right) &= \begin{bmatrix} h_{11} & -h_{12} & h_{21} & -h_{22}^* \end{bmatrix}^T \\ &= \begin{bmatrix} \hat{h}_1 & \hat{h}_2 & \hat{h}_3 & \hat{h}_4 \end{bmatrix}^T \end{aligned} \tag{10}$$

We are interested in such transformations as they do not change the distribution of the elements of $\mathbf{H}$ assuming they are conjugate-symmetrically distributed (e.g. circularly-symmetric complex Gaussian).

Definition 1 (Scheme equivalency)**.** *A universal MIMO transmission-combining scheme $\{\mathbf{C}^{K,T}, \mathfrak{C}\}_{N_t,N_r}$ is equivalent to the MISO transmission scheme $\{\hat{\mathbf{C}}^{K,\hat{T}}, \mathfrak{I}\}_{N_t \cdot N_r,1}$ if:*

1) *There exists a conjugate-symmetric reordering transformation $\hat{\mathbf{h}} = \mathcal{M}(\mathbf{H})$ such that for every realization of $\mathbf{H}$*

$$\mathfrak{C}(\mathbf{C}^{K,T} \mathbf{H}) = \hat{\mathbf{C}}^{K,\hat{T}} \hat{\mathbf{h}}. \tag{11}$$

2) *The noise matrix $\mathbf{Z}$ defined in (3) and noise vector $\hat{\mathbf{z}}$ defined in (10) satisfy that*

$$\mathfrak{C}(\mathbf{Z}) \text{ has the same distribution as } \hat{\mathbf{z}}. \tag{12}$$

In words, two transmission schemes are said to be equivalent if the "signal components" of the input to the detector are identical, and the noise parts have the same distribution. We denote such equivalency as

$$\{\mathbf{C}^{K,T}, \mathfrak{C}\}_{N_t,N_r} \longleftrightarrow \{\hat{\mathbf{C}}^{K,\hat{T}}, \mathfrak{I}\}_{N_{\text{eff}},1}. \tag{13}$$

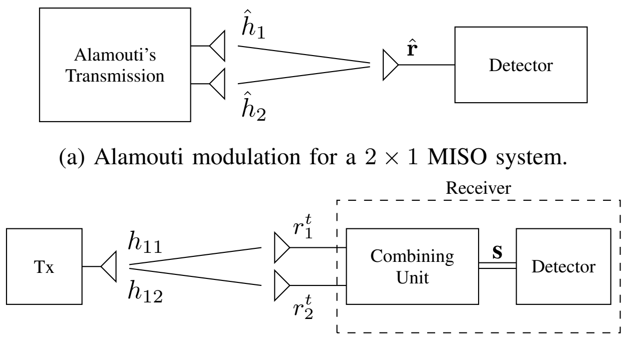
Remark 1. *This definition is, in fact, an extension of the concept presented in [13] in which equivalency between a 1×2 SIMO channel and a 2×1 MISO channel was presented. We recall this example in Section IV-A below.*

Remark 2. *We note that scheme equivalency does not necessarily mean that both transmission schemes have the same symbol rate. In fact, the rate of the STBC used over the MIMO channel is defined in (2) and our goal is to design equivalent schemes with high-rate STBC. More specifically, in the sequel our goal will be to obtain equal-rate scheme equivalency, i.e., we will require that $T = \hat{T}$. Accordingly, to simplify notations we henceforth omit the superscripts k and T from the description of a STBC scheme.*

Corollary 1. *For any quasi-static channel, two equivalent schemes have the same BER performance (and hence the same diversity order) when the same decoder is used.*

We will show that rate 1 and full diversity can be achieved for *any* MIMO channel while performing universal combining while maintaining reduced detection complexity. Specifically, our main result is as follows.

Proposition 1. *For any $N_t \times N_r$ MIMO channel, there exists a rate 1 STBC $\mathbf{C}$ supporting $K = 2^{\lceil \log_2 N_t \rceil} \cdot 2^{\lceil \log_2 N_r \rceil}$*



(a) Alamouti modulation for a 2×1 MISO system.

(b) Universal combining scheme for a 2×1 SIMO system.

Figure 2: Combining scheme and its equivalent multiple input single output (MISO). Combining scheme scheme scheme that is dual to Alamouti modulation.

information symbols with delay of $T = 2^{\lceil \log_2 N_t \rceil} \cdot 2^{\lceil \log_2 N_r \rceil}$ *and a universal (widely linear) combining operator* $\mathfrak{C}(\cdot)$ *such the universal transmission-combining scheme* $\{\mathbf{C}, \mathfrak{C}\}_{N_t,N_r}$ *achieves full diversity with detection complexity that is equal to the decoding complexity of the quasi-orthogonal STBC (QOSTBC) codes of [11] for an* $N_t \cdot N_t \times 1$ *MISO channel.*

IV. A Universal Transmission-Combining Scheme for 2×2IMO Channels

A. Known Results - Universal Transmission-Combining for 1×2IMO Systems

In [13] and [14] a universal transmission-combining scheme is derived for 1×2 SIMO systems which may be viewed as the dual to Alamouti modulation over 2×1 MISO systems.

Let us recall the latter scheme, depicted in Figure 2a. As before, we denote the scheme as $\{\hat{\mathbf{C}}, \mathfrak{I}\}_{2\times 1}$ where the transmission matrix is the well-known Alamouti matrix (see, e.g. [5]):

$$\hat{\mathbf{C}} = \frac{1}{\sqrt{2}} \begin{bmatrix} x_1 & x_2 \\ -x_2^* & x_1^* \end{bmatrix} \tag{14}$$

For a given channel vector $\hat{\mathbf{h}}$, the received vector as defined in (10) now becomes:

$$\begin{bmatrix} \hat{s}_1 \\ \hat{s}_2 \end{bmatrix} = \begin{bmatrix} \hat{r}_1 \\ \hat{r}_2 \end{bmatrix} = \underbrace{\frac{1}{\sqrt{2}} \begin{bmatrix} x_1 & x_2 \\ -x_2^* & x_1^* \end{bmatrix} \begin{bmatrix} \hat{h}_1 \\ \hat{h}_2 \end{bmatrix}}_{\hat{\mathbf{C}}\hat{\mathbf{h}}} + \underbrace{\begin{bmatrix} \hat{z}_1 \\ \hat{z}_2 \end{bmatrix}}_{\hat{\mathbf{z}}} \tag{15}$$

As for the universal transmission-combining scheme, we propose the following scheme, denoted by $\{\mathbf{C}, \mathfrak{C}\}_{1\times 2}$:

$$\mathbf{C} = \begin{bmatrix} x_1 \\ x_2^* \end{bmatrix} \tag{16}$$

i.e. the transmission consists of sending x_1 at time 1 and x_2 at time 2, and the combining operation is

$$\mathfrak{C}(\mathbf{R}) = \frac{1}{\sqrt{2}} \begin{bmatrix} r_{1,1} + r_{2,2}^* \\ -r_{2,1} + r_{1,2}^* \end{bmatrix}. \tag{17}$$

We note that the delay of the scheme is $T = 2$. For a given channel matrix (vector)

$$\mathbf{H} = \begin{bmatrix} h_{11} & h_{12} \end{bmatrix}, \tag{18}$$

we obtain the following received matrix (see Figure 2b):

$$\mathbf{R} = \begin{bmatrix} x_1 h_1 + z_{1,1} & x_1 h_2 + z_{1,2} \\ x_2^* h_1 + z_{2,1} & x_2^* h_2 + z_{2,2} \end{bmatrix}. \tag{19}$$

Thus, after applying the combining operation (17), the vector forwarded to the detector is:

$$\begin{bmatrix} s_1 \\ s_2 \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} (x_1 h_{11} + z_{1,1}) + (x_2^* h_{12} + z_{2,2})^* \\ -(x_2^* h_{11} + z_{2,1}) + (x_1 h_{12} + z_{1,2})^* \end{bmatrix}, \tag{20}$$

which can also be written as

$$\begin{bmatrix} s_1 \\ s_2 \end{bmatrix} = \underbrace{\frac{1}{\sqrt{2}} \begin{bmatrix} x_1 & x_2 \\ -x_2 & x_1 \end{bmatrix} \begin{bmatrix} h_{11} \\ h_{12}^* \end{bmatrix}}_{\mathfrak{C}(\mathbf{CH})} + \underbrace{\frac{1}{\sqrt{2}} \begin{bmatrix} z_{1,1} + z_{2,2}^* \\ -z_{2,1} + z_{1,2}^* \end{bmatrix}}_{\mathfrak{C}(\mathbf{Z})}. \tag{21}$$

By comparing (15) and (21), it is clear that under the following conjugate-symmetric reordering transformation

$$\begin{aligned} \mathcal{M}(\begin{bmatrix} h_{11} & h_{12} \end{bmatrix}) &= \begin{bmatrix} h_{11} \\ h_{12}^* \end{bmatrix} \\ &= \begin{bmatrix} \hat{h}_1 \\ \hat{h}_2 \end{bmatrix}, \end{aligned} \tag{22}$$

the two conditions of scheme equivalency (11) and (12) hold for any realization of $\mathbf{H}$.

Thus, we have established an equivalency between the suggested rate-1 transmission-combining scheme for the 1×2 SIMO channel and the well-known Alamouti modulation scheme for the 2×1 MISO channel. Following Corollary 1, we conclude that the suggested scheme has the same performance, and in particular the same diversity order, as Alamouti modulation.

B. Trivial Schemes for 2×2hannels

In order to motivate the approach we proposed, we first describe some naive attempts in the context of 2×2 channels, none of which achieves $R_t = 1$ and full diversity universally while meeting the "no BW expansion" constraint defined above on the link between the combiner and the detector (i.e., the received TN_r symbols are reduced to T symbols to be forwarded over the fronthaul link).

Specifically, simple schemes we will compare our results against are:

- **Repetition :** Each information symbol is transmitted four times (twice from each transmit antenna). The relay forwards to the detector at each time slot a symbol received from a different antenna (hence the dimension of the symbol stream sent to the detector equals the number of channel uses T). Thus, the detector gets four "copies" of the information symbol each received over different channel path (see Figure 3a). While full diversity is achieved, the resulting transmission rate is $R_t = \frac{1}{4}$.
- **Alamouti modulation:** We may use Alamouti modulation at the transmitter side while forwarding the symbols received from only one of the antennas in order to retain the no BW expansion constraint. While achieving $R_t = 1$, this scheme does not achieve full diversity (see Figure 3b), but rather its diversity order is 2. Alternatively one could transmit x_1 and x_2^* from

a single transmit antenna and apply the "Alamouti combining" described above, resulting with the same transmission rate and diversity order (see Figure 3c).

- **Alamouti modulation with repetition at the combining unit:** The relay forwards to the detector first the two symbols received at the first antenna, and then, the two symbols received from the second antenna (see Figure 3d). While full diversity is achieved, the resulting transmission rate is $R_t = \frac{1}{2}$.
- **Alamouti combining with repetition at the transmitter:** The dual to the previous scheme is to apply "block" repetition (i.e., transmit x_1, x_2^* from the first antenna and then from the second antenna), and apply Alamouti combining on each repetition (see Figure 3e). Again, while full diversity is achieved, the resulting transmission rate is $R_t = \frac{1}{2}$.
- **Doubling the BW:** As a benchmark, we also consider a scheme that violated the BW constraint. Thus, we may use Alamouti modulation while "combining unit" simply forwards the symbol streams received at both antennas, each of length $T = 2$. In other words, the combining unit forwards the symbols received from both antennas without any processing. Clearly, such a scheme obtains full diversity and $R_t = 1$ (see Figure 3f).

We note that all these methods enjoy low-complexity detection as the effective channel at the detector, after applying a unitary transformation, reduces to parallel scalar channels.

C. Example: New Transmission Scheme for 2×2hannels

We first recall the extended Alamouti (EA) QOSTBC for 4×1 MISO channels proposed in [6], [16]. In the sequel we will discuss QOSTBCs in greater generality and also recall the generalized EA-QOSTBC construction. The EA-QOSTBC for four transmit antennas is defined as

$$\breve{\mathbf{X}}^{(4)} = \frac{1}{\sqrt{4}} \begin{bmatrix} x_1 & x_2 & x_3 & x_4 \\ -x_2^* & x_1^* & -x_4^* & x_3^* \\ -x_3^* & -x_4^* & x_1^* & x_2^* \\ x_4 & -x_3 & -x_2 & x_1 \end{bmatrix}. \tag{23}$$

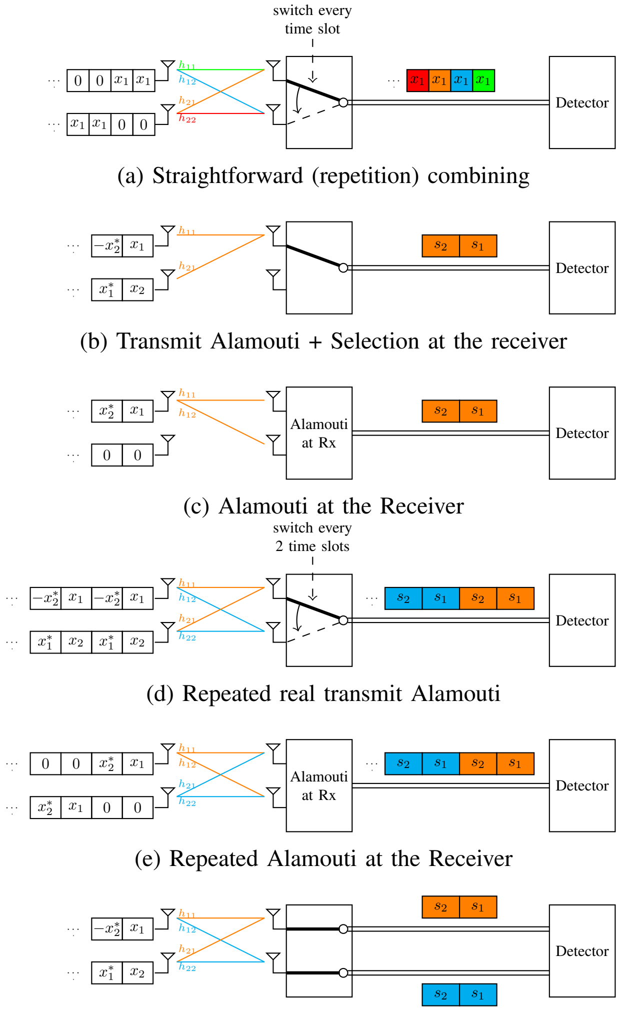
Such a scheme is attractive as it is rate 1 (whereas no rate 1 OSTBCs exist for four transmit antennas). The structure of the transmission matrix is comprised of two sets of orthogonal columns. Although it does not enjoy the low-complexity symbol-wise detection of OSTBCs, it does allow pairwise ML detection [16]. Further, when combined with symbol rotations applied to the information symbols [11], full diversity is achieved.

For the 2×2 channel, we suggest the following transmission-combining scheme. The following rate-1 space-time code at the transmit side:

$$\mathbf{C}^T = \frac{1}{\sqrt{2}} \begin{bmatrix} x_1 & -x_2^* & x_3^* & -x_4 \\ x_2 & x_1^* & x_4^* & x_3 \end{bmatrix} \tag{24}$$

The dimension reduction unit performs the following combining operations on the received vector:

$$\mathfrak{C}(\mathbf{R}) = \frac{1}{\sqrt{2}} \begin{bmatrix} r_{1,1} + r_{3,2}^* \\ r_{2,1} + r_{4,2}^* \\ -r_{3,1} + r_{1,2}^* \\ -r_{4,1} + r_{2,2}^* \end{bmatrix}. \tag{25}$$



(a) Straightforward (repetition) combining

(b) Transmit Alamouti + Selection at the receiver

(c) Alamouti at the Receiver

(d) Repeated real transmit Alamouti

(e) Repeated Alamouti at the Receiver

(f) Forward everything

Figure 3: s_1 and s_2 are the symbols acquired in a regular Alamouti's communication scheme. The two colors represent 2 versions of these symbols, i.e. influenced by different channel coefficients.

Note that the combining scheme converts the 8 symbols of $\mathbf{R}$ to a vector of size 4, which equals to the number of channel uses per transmission, hence the BW is retained over the fronthaul. The resultant combined vector is:

$$\begin{bmatrix} s_1 \\ s_2 \\ s_3 \\ s_4 \end{bmatrix} = \frac{1}{\sqrt{4}} \begin{bmatrix} x_1 & x_2 & x_3 & x_4 \\ -x_2^* & x_1^* & -x_4^* & x_3^* \\ -x_3^* & -x_4^* & x_1^* & x_2^* \\ x_4 & -x_3 & -x_2 & x_1 \end{bmatrix} \cdot \begin{bmatrix} h_{11} \\ h_{21} \\ h_{12}^* \\ h_{22}^* \end{bmatrix} + \frac{1}{\sqrt{2}} \begin{bmatrix} z_{1,1} + z_{3,2}^* \\ z_{2,1} + z_{4,2}^* \\ -z_{3,1} + z_{1,2}^* \\ -z_{4,1} + z_{2,2}^* \end{bmatrix}. \tag{26}$$

Now using the transformation

$$\mathcal{M}\left(\begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix}\right) = \begin{bmatrix} h_{11} & h_{21} & h_{12}^* & h_{22}^* \end{bmatrix}^T = \begin{bmatrix} \hat{h}_1 & \hat{h}_2 & \hat{h}_3 & \hat{h}_4 \end{bmatrix}^T, \tag{27}$$

it follows that the transmission-combining scheme $\{\mathbf{C}, \mathfrak{C}\}_{2,2}$ is equivalent to the transmission scheme $\{\breve{\mathbf{X}}^{(4)}, \mathfrak{I}\}_{4,1}$.

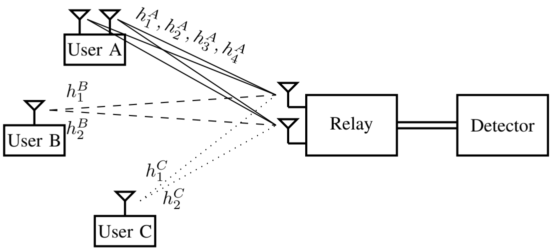


Figure 4: Dimension Reduction application for multiple users with different number of transmit antennas

Thus, by Corollary 1), we conclude that that we have obtained a rate-1 full-diversity scheme that meets the fronthaul BW constraint.

Remark 3. *We note that had we insisted on maintaining orthogonality (either aiming for an equivalent rate-$3/4$ orthogonal STBC (OSTBC) scheme or rate-$1/2$ OSTBC scheme), the resulting transmission-combining scheme turns out to support a symbol rate rate smaller than 1 (in fact, no greater than $1/2$).*

D. Different Variations

We opted to describe the proposed scheme based on the EA QOSTBC for reasons of simplicity of exposition. Nonetheless the approach may be applied also to other QOSTBCs schemes that also enjoy reduced-complexity detection. Such variations are discussed Appendix A.

The benefit for considering also other QOSTBCs may readily be understood in the context of 2×2 channel. To that end, consider the ABBA-QOSTBC presented in [8], [16]. It is easy to show that the following transmission-combining operations satisfy scheme equivalency with former.

At the transmit side, use Alamouti modulation applied in sequence to two pairs of information symbols: x_1, x_2 and then x_3, x_4. The combining operation is ???MISSING DESCRIPTION. IS IT THE SAME AS FOR ER???

The advantage of the ABBA variant over the EA one is that a transition from a 2×1 system with Alamouti transmission to a 2×2 system does not involve changing the transmitter, i.e, we can utilize an additional antenna at the receiver to improve the diversity order without modifying the transmitter side.

Another variation, again based on the ABBA QOSTBC, can be derived such that the same combining operation can be retained while supporting users equipped equipped with either a single antenna or two antennas. See Figure 4).

V. Universal Transmission-Combining for $N_t \times N_r$hannels

Thus far, we have limited the discussion to universal transmission combining schemes for 2×2 channels. By establishing equivalency of the proposed scheme, as described in (24) and (25), to the EA MISO scheme, we showed that the scheme achieves rate 1 transmission and full diversity, while allowing for reduced complexity detection and no BW expansion over the fraunthal link.

A. Generalized EA-QOSTBC

We now describe the generalization of the EA QOSTBC scheme to MISO channels with more than two antennas. This will serve as the basis for the generalization of the transmission-combining scheme we described in the previous section.

EA-QOSTBC transmits N information symbols in N channel uses. we split these information symbols to two groups denoted as $\mathbf{x} = \{x_1, \cdots, x_{\frac{N}{2}}\}$ and $\mathbf{y} = \{x_{\frac{N}{2}+1}, \cdots, x_N\}$. The transmission matrix $\check{\mathbf{X}}^{(N)} \in \mathbb{C}^{N \times N}$ can be obtained recursively by

$$\check{\mathbf{X}}^{(N)} = \frac{1}{\sqrt{2}} \left[\begin{array}{c|c} \check{\mathbf{X}}_{\{x\}}^{(N/2)} & \check{\mathbf{X}}_{\{y\}}^{(N/2)} \\ \hline -\check{\mathbf{X}}_{\{y\}}^{(N/2)*} & \check{\mathbf{X}}_{\{x\}}^{(N/2)*} \end{array} \right], \tag{28}$$

where $\check{\mathbf{X}}_{\{x\}}^{(N/2)}$ and $\check{\mathbf{X}}_{\{y\}}^{(N/2)}$ are the matrices obtained by placing the sets $\mathbf{x}$ and $\mathbf{y}$ correspondingly is the EA-QOSTBC transmission matrix for $N/2 \times 1$ MISO channel, where $X^{(1)}$ is a transmission containing a single information symbol x_1. We also note that $X^{(2)}$ is the Alamouti transmission matrix. Notice that N channel uses are utilized to transmit N information symbols. Hence, it is a delay optimal scheme with rate 1. Furthermore, the detection may be done over two separate sets of information symbols, each contains $\frac{N}{2}$, and thus is reduced by order of square root related to a full exhaustive search detection.

B. Schemes for integer power of 2 channels

We generalize the universal transmission-combining scheme described in Section V for 2×2 to a scheme suitable for a $N_t \times N_r$ channel where N_t and N_r are an integer power of 2. For convenience we denote $\mathbf{X}^{(p)} \triangleq \check{\mathbf{X}}^{(2^p)}$. The proposed scheme, denoted by $\{\mathbf{C}^{(p,q)}, \mathfrak{C}\}_{N_t,N_r}$, is as follows.

The transmission scheme is defined by the following recursion:

$$\begin{aligned} \mathbf{C}^{(p,q)} &= \mathbf{X}^{(p)} && \mathit{for}\ q = 0, p \geq 0 \\ \mathbf{C}^{(p,q)} &= \left[\begin{array}{c} \mathbf{C}_{\{x\}}^{(p,q-1)} \\ \hline \mathbf{C}_{\{y\}}^{(p,q-1)*} \end{array} \right] && \mathit{for}\ q > 0, p \geq 0. \end{aligned} \tag{29}$$

where $\mathbf{C}_{\{x\}}^{(p,q-1)}$ and $\mathbf{C}_{\{y\}}^{(p,q-1)}$ are the matrices obtained by placing in the transmission matrix for the $2^p \times 2^{q-1}$ MIMO channel the sets $\mathbf{x}$ and $\mathbf{y}$ correspondingly. The derived transmission rate is 1 for any $p, q > 0$.

The proposed combining scheme is defined over the matrix of received symbols $\mathbf{R}$, where $\mathbf{R}$ has dimensions of $T \times 2^q$. Matrix $\mathbf{R}$ can be split as follows

$$\mathbf{R} = \left[\begin{array}{c|c} \mathbf{A} & \mathbf{B} \\ \hline \mathbf{C} & \mathbf{D} \end{array} \right] \tag{30}$$

where each sub-matrix ($\mathbf{A}$, $\mathbf{B}$, $\mathbf{C}$, $\mathbf{D}$) is a $T/2 \times 2^{q-1}$ matrix. The combining scheme is defined recursively as

$$\begin{aligned} \mathfrak{C}(\mathbf{R}) &= \mathbf{R} && \mathit{for}\ q = 0, p > 0 \\ \mathfrak{C}(\mathbf{R}) &= \frac{1}{\sqrt{2}} \left[\begin{array}{c} \mathfrak{C}(\mathbf{A}) + \mathfrak{C}(\mathbf{D})^* \\ \hline -\mathfrak{C}(\mathbf{C}) + \mathfrak{C}(\mathbf{B})^* \end{array} \right] && \mathit{for}\ q > 0, p > 0. \end{aligned} \tag{31}$$

The recursion stops when the interim input is a column vector with dimensions $2^p \times 1$.

Proposition 2.

$$\{\mathbf{C}^{(p,q)}, \mathfrak{C}\}_{N_t,N_r} \longleftrightarrow \{\check{\mathbf{X}}^{(N_{\text{eff}})}, \mathfrak{I}\}_{N_t \cdot N_r \times 1}, \tag{32}$$

where $\mathbf{C}^{(p,q)}$ is defined in (29) and $\mathfrak{C}$ in (31).

Proof. Appendix B □

C. Schemes for general $N_t \times N_r$hannels

Next, we show how to generate a transmission scheme and combining scheme when N_t, N_r does not equal to two to the power of an integer.

We extend the $N_t \times N_r$ MIMO channel to $\tilde{N}_t \times \tilde{N}_r$ MIMO channel such that $\tilde{N}_t, \tilde{N}_r$ are the smallest integer power of 2 that are greater than N_t, N_r, i.e $\tilde{N}_t = 2^p$ and $\tilde{N}_r = 2^q$ where

$$\begin{aligned} p &= \lceil \log_2 N_t \rceil \\ q &= \lceil \log_2 N_r \rceil, \end{aligned} \tag{33}$$

From Proposition 2, there exists a universal transmission-combining scheme $\{\tilde{\mathbf{C}}^{(p,q)}, \tilde{\mathfrak{C}}\}_{\tilde{N}_t,\tilde{N}_r}$ such that

$$\{\tilde{\mathbf{C}}^{(p,q)}, \tilde{\mathfrak{C}}\}_{\tilde{N}_t,\tilde{N}_r} \longleftrightarrow \{\check{\mathbf{X}}^{(\tilde{N}_{\text{eff}})}, \mathfrak{I}\}_{\tilde{N}_{\text{eff}} \times 1}, \tag{34}$$

where

$$\tilde{N}_{\text{eff}} = \tilde{N}_t \cdot \tilde{N}_r. \tag{35}$$

Our scheme $\{\mathbf{C}, \mathfrak{C}\}_{N_t,N_r}$ will be constructed as follows: $\mathbf{C}$ is constructed by eliminating (puncturing) the last $\tilde{N}_t - N_t$ columns of $\tilde{\mathbf{C}}^{(p,q)}$ (up to power normalization) i.e. if we partition to two submatrices : $\tilde{\mathbf{C}}_A^{(p,q)} \in \mathbb{C}^{T \times N_t}$ and $\tilde{\mathbf{C}}_B^{(p,q)} \in \mathbb{C}^{T \times \tilde{N}_t - N_t}$ such that

$$\tilde{\mathbf{C}}^{(p,q)} = \left[\begin{array}{c|c} \tilde{\mathbf{C}}_A^{(p,q)} & \tilde{\mathbf{C}}_B^{(p,q)} \end{array} \right], \tag{36}$$

we define:

$$\mathbf{C} = \sqrt{\frac{\tilde{N}_t}{N_t}} \tilde{\mathbf{C}}_A^{(p,q)}. \tag{37}$$

The induced delay is T = $\tilde{N}_{\text{eff}}$.

Define (some) matrix $\mathbf{W}$ as the input to the combining scheme $\mathfrak{C}()$. $\mathfrak{C}()$ is defined as applying the combining scheme $\tilde{\mathfrak{C}}()$ over an extended matrix

$$\tilde{\mathbf{W}} = \left[\begin{array}{c|ccc} \mathbf{W} & 0 & & 0 \\ & \vdots & \dots & \vdots \\ & 0 & & 0 \end{array} \right] \tag{38}$$

which is the outcome of concatenating $\tilde{N}_r - N_r$ zero columns. Hence

$$\mathfrak{C}(\mathbf{W}) \triangleq \sqrt{\frac{\tilde{N}_r}{N_r}} \tilde{\mathfrak{C}}(\tilde{\mathbf{W}}). \tag{39}$$

Proposition 3. *The universal transmission-combining scheme $\{\mathbf{C}, \mathfrak{C}\}_{N_t,N_r}$ for general N_t and N_r as defined in (37) and (39) is equivalent to the transmission scheme $\{\check{\mathbf{X}}_{\text{punc}}^{(N_{\text{eff}})}, \mathfrak{I}\}_{N_{\text{eff}} \times 1}$ where $\check{\mathbf{X}}_{\text{punc}}^{(N_{\text{eff}})}$ is obtained by puncturing $\tilde{N}_{\text{eff}} - N_{\text{eff}}$ of $\check{\mathbf{X}}^{(\tilde{N}_{\text{eff}})}$ columns.*

Proof. Assume that the equivalency in (34) is obtained using the conjugate-symmetric reordering transformation $\tilde{\mathcal{M}}$ i.e.:

1) For any realization of the channel matrix $\mathbf{H}' \in \mathbb{C}^{\tilde{N}_t \times \tilde{N}_r}$ we have:

$$\tilde{\mathfrak{C}}(\tilde{\mathbf{C}}^{(p,q)} \mathbf{H}') = \check{\mathbf{X}}^{(\tilde{N}_{\text{eff}})} \tilde{\mathcal{M}}(\mathbf{H}') \tag{40}$$

2) For a noise matrix $\mathbf{Z}'$, the vector $\mathfrak{C}(\mathbf{Z}')$ is made of i.i.d Gaussian variables with unit variance.

For any realization of channel matrix $\mathbf{H}$ of the channel $N_t \times N_r$ we define the extended channel matrix $\tilde{\mathbf{H}}$ by concatenating $\tilde{N}_r - N_r$ columns of zeros from the right and $\tilde{N}_t - N_t$ raws of zeros from the bottom:

$$\tilde{\mathbf{H}} = \sqrt{\frac{\tilde{N}_{\text{eff}}}{N_{\text{eff}}}} \begin{bmatrix} & & 0 & & 0 \\ & \mathbf{H} & \vdots & \dots & \vdots \\ & & 0 & & 0 \\ 0 & 0 & \cdots & & 0 \\ & & \vdots & & \\ 0 & 0 & \cdots & & 0 \end{bmatrix} \tag{41}$$

For the construction of the transmission scheme $\mathbf{C}$ in (37) we have:

$$\begin{aligned} \tilde{\mathbf{C}}^{(p,q)} \tilde{\mathbf{H}} &= \left[\begin{array}{c|c} \tilde{\mathbf{C}}_A^{(p,q)} & \tilde{\mathbf{C}}_B^{(p,q)} \end{array} \right] \left[\begin{array}{c|c} \mathbf{H} & 0 \\ \hline 0 & 0 \end{array} \right] \\ &= \left[\begin{array}{c|c} \mathbf{CH} & 0 \end{array} \right]. \end{aligned} \tag{42}$$

With the combining scheme in (39) we get:

$$\begin{aligned} \mathfrak{C}(\mathbf{CH}) &= \sqrt{\frac{\tilde{N}_{\text{eff}}}{N_{\text{eff}}}} \tilde{\mathfrak{C}}(\left[\begin{array}{c|c} \mathbf{CH} & 0 \end{array} \right]) \\ &= \sqrt{\frac{\tilde{N}_{\text{eff}}}{N_{\text{eff}}}} \tilde{\mathfrak{C}}(\tilde{\mathbf{C}}^{(p,q)} \tilde{\mathbf{H}}) \end{aligned} \tag{43}$$

Further assume that the one-to-one index mapping $\tilde{m}$ associated with the transformation $\tilde{\mathcal{M}}$ maps the pair of indices which involve an index from $\mathcal{I} = \{N_t{+}1, \dots, \tilde{N}_t\}$ as the first index or $\mathcal{J} = \{N_r + 1, \dots, \tilde{N}_r\}$ as the second index to the set of indices $\mathcal{K}$. More specifically:

$$\mathcal{K} = \tilde{m}(\mathcal{I} \times \{1, \dots, N_r\} \cup \{1, \dots, N_t\} \times \mathcal{J}), \tag{44}$$

where we get

$$|\mathcal{K}| = \tilde{N}_{\text{eff}} - N_{\text{eff}}. \tag{45}$$

By defining $\hat{\tilde{\mathbf{h}}} = \tilde{\mathcal{M}}(\tilde{\mathbf{H}})$ we get that the elements of $\hat{\tilde{\mathbf{h}}}$ that their origin are the zeros entries of the matrix $\tilde{\mathbf{H}}$ are indexed by indices from $\mathcal{K}$, namely:

$$\hat{\tilde{h}}_k = 0 \quad \text{for } k \in \mathcal{K}. \tag{46}$$

For the matrix $\check{\mathbf{X}}^{(\tilde{N}_{\text{eff}})} \in \mathbb{C}^{T \times \tilde{N}_{\text{eff}}}$, we define $\check{\mathbf{X}}_{\text{punc}}^{(N_{\text{eff}})} \in \mathbb{C}^{T \times N_{\text{eff}}}$ as the matrix obtained after puncturing all $\check{\mathbf{X}}^{(\tilde{N}_{\text{eff}})}$ columns indexed by indices from $\mathcal{K}$ (up to power normalization of $\sqrt{\frac{\tilde{N}_{\text{eff}}}{N_{\text{eff}}}}$). Hence we get:

$$\sqrt{\frac{\tilde{N}_{\text{eff}}}{N_{\text{eff}}}} \check{\mathbf{X}}^{(\tilde{N}_{\text{eff}})} \cdot \tilde{\mathcal{M}}(\tilde{\mathbf{H}}) = \check{\mathbf{X}}_{\text{punc}}^{(N_{\text{eff}})} \cdot \mathcal{M}(\mathbf{H}) \tag{47}$$

where $\mathcal{M}(\mathbf{H})$ is defined as the transformation obtained after expanding $\mathbf{H}$ to $\tilde{\mathbf{H}}$ and then puncturing all the elements with indices from $\mathcal{K}$. From (44), (47) and (40) we get:

$$\mathfrak{C}(\mathbf{CH}) = \check{\mathbf{X}}_{\text{punc}}^{(N_{\text{eff}})} \mathcal{M}(\mathbf{H}) \tag{48}$$

i.e. the first condition of the equivalency is met.

As for the second condition, we expand the noise matrix $\mathbf{Z}$ in the $N_t \times N_r$ channel by concatenating $\tilde{N}_r - N_r$ columns of zeros such that we get the matrix $\tilde{\mathbf{Z}}$:

$$\left[\mathbf{Z} \left| \begin{matrix} 0 & & 0 \\ \vdots & \dots & \vdots \\ 0 & & 0 \end{matrix} \right. \right] \tag{49}$$

From Remark 7 we get that $\tilde{\mathfrak{C}}(\tilde{\mathbf{Z}})$ is a Gaussian vector with unit variance i.i.d elements. Thus, from the definition of $\mathfrak{C}$ in (39) we get that also $\mathfrak{C}(\mathbf{Z})$ is a random vector with unit variance Gaussian and i.i.d elements (where the power normalization ensures the unit variance). □

Remark 4. *The proof showed that puncturing transmit and receive antennas from the last indices leads to an quasi-orthogonal equivalent MISO. The same is true for any antennas puncturing.*

Examples for the suggested scheme (and its equivalency to EA applied over the matching MISO channel) is given in Appendix C for 3×2 MIMO channel and 1×3 SIMO channel.

Proof of Theorem 1. From propositions **??** and 3 we have that for any $N_t \times N_r$ MIMO channel, there exists a transmission scheme (the STBC defined in (37) at rate one with delay $T = \tilde{N}_t \cdot \tilde{N}_r$ and the universal combining scheme defined in (39)) which is equivalent to EA-QOSTBC applied over $N_t \cdot N_r \times 1$ MISO channel. Following Corollary 1, this equivalency means that the transmission scheme has the same diversity as the EA-QOSTBC. Following Propositions 5 and 6 in [11] it follows that the suggested transmission scheme (the STBC defined in (**??**) at rate one with delay $T = \tilde{N}_t \cdot \tilde{N}_r$ and the universal combining scheme defined in (**??**)) achieve full diversity. □

Remark 5. *We recall that in Proposition 6 in [11] it was stated that full diversity is achieved by puncturing any column of the EA-QOSTBC.*

[] provides a Python script that generates the (symbolic) STBC $\mathbf{C}^{(p,q)}$ and the combining scheme $\mathfrak{C}()$.

VI. Simulations

In this section we present simulation results for the new schemes described above while comparing its performance to some known schemes. We assume complex uncorrelated Gaussian channel drawn independently for each transmission block, but remains constant in it (Rayleigh fading).

As a baseline, we take "double BW" scheme as depicted in Figure 3f, in which the combining unit forwards the symbols received at the 2 antennas, by allowing a bandwidth expansion by a factor of 2 in the frauntha link. The scheme can be implemented easily by utilizing maximal ratio combining (MRC), which necessitates full channel state information (CSI), at the detector followed by the detection method itself. Such scheme enjoys full diversity and transmission symbol rate of 1, as it is similar to a scheme operating MRC at the combining unit, while allowing the knowledge of CSI in it, which is known to be a full diversity scheme. Hence no one of the other universal schemes, which are deprived of any CSI, can outperform the "double BW" scheme.

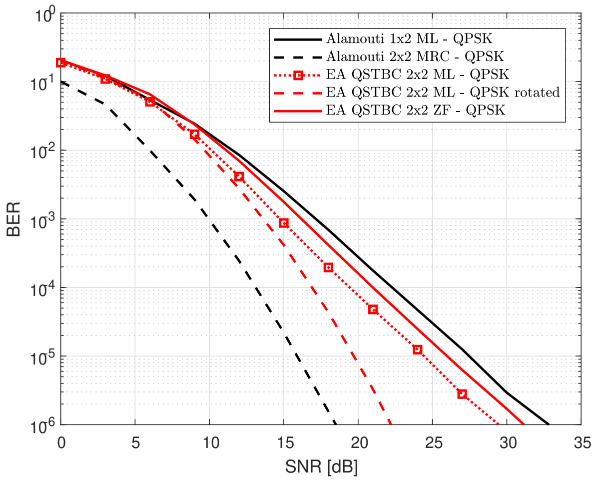


Figure 5: BER performance of Alamouti, EA QOSTBC with ML detector, rotated QOSTBC with ML detector, QOSTBC with ZF detector and MRC

In Figure 5, we present the bit error ratio (BER) curve of the "double BW" baseline using QPSK constellation. The "transmit Alamouti" scheme (Figure 3b) or "Alamouti at the receiver" (Figure 3c) complex versions obviously don't achieve full diversity.

The "repeated transmit Alamouti" or the "repeated Alamouti at the receiver" complex analogs do not appear in the graph. Instead the proposed scheme from Section IV-C is presented while using 3 variants: zero forcing (ZF) detection with QPSK constellation, maximum likelihood (ML) detection with QPSK constellation and ML detection with rotated QPSK constellation. The scheme with the ZF detection enjoys a simple low complexity symbol-wise detector and outperforms the "transmit Alamouti" scheme by approximately 1dB. Alternatively ML detection with QPSK constellation is slightly more complicated but results with a performance enhancement of approximately 2dB related to the "transmit Alamouti", but still not a full diversity scheme. As explained in Section V-A, a technique involved with rotating the QPSK constellation [10] leads to a full diversity scheme as its BER curve slope is equal to that of the "double BW" scheme, but with a constant degradation of approximately 3dB.

In Figure 6, we show the performance of the proposed combining scheme equivalent to generalized EA for more general multiple input multiple output (MIMO) systems. The symbol constellation is taken with suitable rotation such that full diversity is achieved.

Appendix A
Different Variations for 2×2ystem

We first present the variation which transmits the Alamouti's scheme applied on each of the pairs x_1, x_2 and

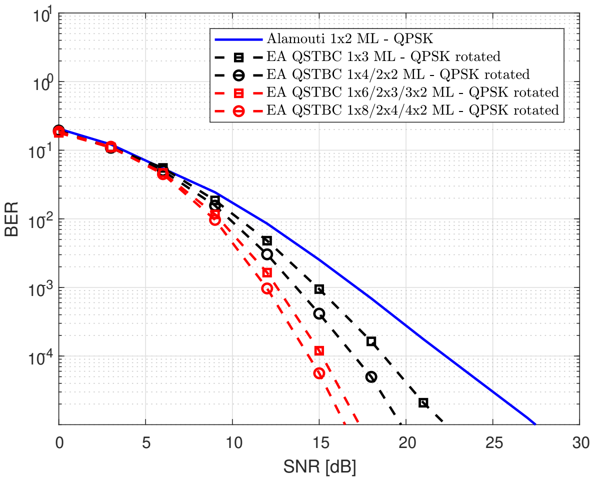


Figure 6: BER performance of Alamouti, EA QOSTBC equivalent for 1x3, 2x2 / 1x4, 2x3, 3x2, 2x4 / 4x2 / 1x8 using rotated QPSK constellation and ML detector

x_3, x_4:

$$\mathbf{C^T} = \frac{1}{\sqrt{2}} \begin{bmatrix} x_1 & -x_2^* & x_3 & -x_4^* \\ x_2 & x_1^* & x_4 & x_3^* \end{bmatrix} \tag{50}$$

We propose the following combining scheme:

$$\mathfrak{C}(\mathbf{R}) = \frac{1}{\sqrt{2}} \begin{bmatrix} r_{1,1} + r_{3,2} \\ r_{2,1} + r_{4,2} \\ r_{3,1} + r_{1,2} \\ r_{4,1} + r_{2,2} \end{bmatrix} \tag{51}$$

Thus we obtain the following combined vector:

$$\begin{bmatrix} s_1 \\ s_2 \\ s_3 \\ s_4 \end{bmatrix} = \frac{1}{\sqrt{4}} \begin{bmatrix} x_1 & x_2 & x_3 & x_4 \\ -x_2^* & x_1^* & -x_4^* & x_3^* \\ x_3 & x_4 & x_1 & x_2 \\ -x_4^* & x_3^* & -x_2^* & x_1^* \end{bmatrix} \cdot \begin{bmatrix} h_{11} \\ h_{21} \\ h_{12} \\ h_{22} \end{bmatrix} + \frac{1}{\sqrt{2}} \begin{bmatrix} z_{1,1} + z_{2,2} \\ z_{2,1} + z_{1,2} \\ z_{3,1} + z_{4,2} \\ z_{4,1} + z_{3,2} \end{bmatrix}. \tag{52}$$

which amounts for the 4×1 MISO channel with EA-QOSTBC transmission.

The second variation uses the "Alamouti at the receiver" combining applied on each of the received symbols at times $1, 2$ and $3, 4$, i.e.:

$$\mathfrak{C}(\mathbf{R}) = \frac{1}{\sqrt{2}} \begin{bmatrix} r_{1,1} + r_{2,2}^* \\ -r_{2,1} + r_{1,2}^* \\ r_{3,1} + r_{4,2}^* \\ -r_{4,1} + r_{3,2}^* \end{bmatrix}. \tag{53}$$

The proposed transmission scheme is:

$$\mathbf{C^T} = \frac{1}{\sqrt{2}} \begin{bmatrix} x_1 & x_2^* & x_3 & x_4^* \\ x_3 & x_4^* & x_1 & x_2^* \end{bmatrix}, \tag{54}$$

which leads to the following reduced signals vector

$$\begin{bmatrix} s_1 \\ s_2 \\ s_3 \\ s_4 \end{bmatrix} = \frac{1}{\sqrt{4}} \begin{bmatrix} x_1 & x_2 & x_3 & x_4 \\ -x_2^* & x_1^* & -x_4^* & x_3^* \\ x_3 & x_4 & x_1 & x_2 \\ -x_4^* & x_3^* & -x_2^* & x_1^* \end{bmatrix} \cdot \begin{bmatrix} h_{11} \\ h_{21} \\ h_{22} \\ h_{12} \end{bmatrix} + \frac{1}{\sqrt{2}} \begin{bmatrix} z_1^1 + z_{2,2}^* \\ -z_{2,1} + z_{1,2}^* \\ z_{3,1} + z_{4,2}^* \\ -z_{4,1} + z_{3,2}^* \end{bmatrix}, \tag{55}$$

i.e. is also equivalent to the 4×1 MISO channel with ABBA-QOSTBC transmission.

Appendix B
Generalization for Integer Power of 2

Proof. We prove this Proposition by induction. The initial step $(q = 0, p)$ is a MISO channel, thus, as suggested in (29), the transmission to be used is EA-QOSTBC and no combining is needed. Therefore the equivalency

$$\{\mathbf{C}^{(p,0)}, \mathfrak{C}\}_{2^p,1} \longleftrightarrow \{\mathbf{X}^{(p)}, \mathfrak{I}\}_{2^p \times 1}, \tag{56}$$

trivially holds.

For $(q > 0, p)$ we show recursively that the equivalency holds, while assuming that the equivalency for $(q-1, p)$ is true:

$$\{\mathbf{C}^{(p,q-1)}, \mathfrak{C}\}_{2^p,2^{q-1}} \longleftrightarrow \{\mathbf{X}^{(p+q-1)}, \mathfrak{I}\}_{2^{p+q-1} \times 1}. \tag{57}$$

We start with the first condition for equivalency as in (11): For any realization of channel matrix for MIMO channel $2^p \times 2^q$, denoted by $\mathbf{H}^{(p,q)} \in \mathbb{C}^{2^p \times 2^q}$, we define the following sub-matrices:

$$\mathbf{H^{(p,q)}} = \left[\ \mathbf{H_L^{(p,q-1)}} \ \middle| \ \mathbf{H_R^{(p,q-1)}} \ \right]. \tag{58}$$

Remark 6. *$\mathbf{H_L^{(p,q-1)}}$ is the channel resulting from taking the first 2^{q-1} receive antennas and $\mathbf{H_R^{(p,q-1)}}$ is the channel resulting from taking the last 2^{q-1} receive antennas.*

We use the following relations:

$$\mathbf{X^{(n)}_{\{x^*\}}} = \mathbf{X^{(n)*}_{\{x\}}} \tag{59}$$

$$\mathfrak{C}(\mathbf{R}^*) = \mathfrak{C}(\mathbf{R})^* \tag{60}$$

$$\mathfrak{C}(\mathbf{R_1} + \mathbf{R_2}) = \mathfrak{C}(\mathbf{R_1}) + \mathfrak{C}(\mathbf{R_1}) \tag{61}$$

Denoting by $\hat{\mathbf{h}}_L^{(p+q-1)} = \mathcal{M}_{p+q-1}(\mathbf{H_L}^{\mathbf{(p,q-1)}})$ and $\hat{\mathbf{h}}_R^{(p+q-1)*} = \mathcal{M}_{p+q-1}(\mathbf{H_R}^{\mathbf{(p,q-1)*}})$ the conjugate-symmetrix reordering tranformations induced from the equivalency in (57), we get:

$$
\begin{aligned}
&\mathfrak{C}(\mathbf{C}^{(\mathbf{p},\mathbf{q})} \cdot \mathbf{H}^{(\mathbf{p},\mathbf{q})}) \\
&= \mathfrak{C}\left(\left[\begin{array}{c} \mathbf{C}_{\{\mathbf{x}\}}^{(\mathbf{p},\mathbf{q}-\mathbf{1})} \\ \hline \mathbf{C}_{\{\mathbf{y}\}}^{(\mathbf{p},\mathbf{q}-\mathbf{1})*} \end{array}\right] \left[\begin{array}{c|c} \mathbf{H}_{\mathbf{L}}^{(\mathbf{p},\mathbf{q}-\mathbf{1})} & \mathbf{H}_{\mathbf{R}}^{(\mathbf{p},\mathbf{q}-\mathbf{1})} \end{array}\right]\right) \\
&= \frac{1}{\sqrt{2}} \left[\begin{array}{c} \mathfrak{C}\left(\overbrace{\mathbf{C}_{\{\mathbf{x}\}}{}^{(\mathbf{p},\mathbf{q}-\mathbf{1})} \cdot \mathbf{H}_{\mathbf{L}}^{(\mathbf{p},\mathbf{q}-\mathbf{1})}}^{\mathbf{A}}\right) + \mathfrak{C}\left(\overbrace{\mathbf{C}_{\{\mathbf{y}\}}^{(\mathbf{p},\mathbf{q}-\mathbf{1})*} \cdot \mathbf{H}_{\mathbf{R}}^{(\mathbf{p},\mathbf{q}-\mathbf{1})}}^{\mathbf{D}}\right)^{*} \\ \hline -\mathfrak{C}\left(\underbrace{\mathbf{C}_{\{\mathbf{y}\}}^{(\mathbf{p},\mathbf{q}-\mathbf{1})*} \cdot \mathbf{H}_{\mathbf{L}}^{(\mathbf{p},\mathbf{q}-\mathbf{1})}}_{\mathbf{C}}\right) + \mathfrak{C}\left(\underbrace{\mathbf{C}_{\{\mathbf{x}\}}^{(\mathbf{p},\mathbf{q}-\mathbf{1})} \cdot \mathbf{H}_{\mathbf{R}}^{(\mathbf{p},\mathbf{q}-\mathbf{1})}}_{\mathbf{B}}\right)^{*} \end{array}\right] \\
&= \frac{1}{\sqrt{2}} \left[\begin{array}{c} \mathfrak{C}\left(\mathbf{C}_{\{\mathbf{x}\}}{}^{(\mathbf{p},\mathbf{q}-\mathbf{1})} \cdot \mathbf{H}_{\mathbf{L}}^{(\mathbf{p},\mathbf{q}-\mathbf{1})}\right) + \mathfrak{C}\left(\mathbf{C}_{\{\mathbf{y}\}}^{(\mathbf{p},\mathbf{q}-\mathbf{1})} \cdot \mathbf{H}_{\mathbf{R}}^{(\mathbf{p},\mathbf{q}-\mathbf{1})*}\right) \\ \hline -\mathfrak{C}\left(\mathbf{C}_{\{\mathbf{y}^*\}}^{(\mathbf{p},\mathbf{q}-\mathbf{1})} \cdot \mathbf{H}_{\mathbf{L}}^{(\mathbf{p},\mathbf{q}-\mathbf{1})}\right) + \mathfrak{C}\left(\mathbf{C}_{\{\mathbf{x}^*\}}^{(\mathbf{p},\mathbf{q}-\mathbf{1})} \cdot \mathbf{H}_{\mathbf{R}}^{(\mathbf{p},\mathbf{q}-\mathbf{1})*}\right) \end{array}\right] \\
&\overset{(a)}{=} \frac{1}{\sqrt{2}} \left[\begin{array}{c} \mathbf{X}_{\{\mathbf{x}\}}^{(\mathbf{p}+\mathbf{q}-\mathbf{1})} \cdot \hat{\mathfrak{h}}_{\mathbf{L}}^{(\mathbf{p}+\mathbf{q}-\mathbf{1})} + \mathbf{X}_{\{\mathbf{y}\}}^{(\mathbf{p}+\mathbf{q}-\mathbf{1})} \cdot \hat{\mathfrak{h}}_{\mathbf{R}}^{(\mathbf{p}+\mathbf{q}-\mathbf{1})*} \\ \hline -\mathbf{X}_{\{\mathbf{y}^*\}}^{(\mathbf{p}+\mathbf{q}-\mathbf{1})} \cdot \hat{\mathfrak{h}}_{\mathbf{L}}^{(\mathbf{p}+\mathbf{q}-\mathbf{1})} + \mathbf{X}_{\{\mathbf{x}^*\}}^{(\mathbf{p}+\mathbf{q}-\mathbf{1})} \cdot \hat{\mathfrak{h}}_{\mathbf{R}}^{(\mathbf{p}+\mathbf{q}-\mathbf{1})*} \end{array}\right] \\
&= \frac{1}{\sqrt{2}} \left[\begin{array}{c|c} \mathbf{X}_{\{\mathbf{x}\}}^{(\mathbf{p}+\mathbf{q}-\mathbf{1})} & \mathbf{X}_{\{\mathbf{y}\}}^{(\mathbf{p}+\mathbf{q}-\mathbf{1})} \\ \hline -\mathbf{X}_{\{\mathbf{y}\}}^{(\mathbf{p}+\mathbf{q}-\mathbf{1})*} & \mathbf{X}_{\{\mathbf{x}\}}^{(\mathbf{p}+\mathbf{q}-\mathbf{1})*} \end{array}\right] \left[\begin{array}{c} \hat{\mathfrak{h}}_{\mathbf{L}}^{(\mathbf{p}+\mathbf{q}-\mathbf{1})} \\ \hline \hat{\mathfrak{h}}_{\mathbf{R}}^{(\mathbf{p}+\mathbf{q}-\mathbf{1})*} \end{array}\right] \\
&\overset{(b)}{=} \mathbf{X}^{(\mathbf{p}+\mathbf{q})} \cdot \hat{\mathfrak{h}}^{(\mathbf{p}+\mathbf{q})},
\end{aligned} \tag{62}
$$

where equality (a) follows from the induction assumption in (57). Equality (b) follows from the definition of generalize EA-QOSTBC (28), and by denoting

$$
\hat{\mathfrak{h}}^{(p+q)} = \begin{bmatrix} \hat{\mathfrak{h}}_L^{(p+q-1)} \\ \hat{\mathfrak{h}}_R^{(p+q-1)*} \end{bmatrix}. \tag{63}
$$

we get that $\hat{\mathfrak{h}}^{(p+q)} = \mathcal{M}(\mathbf{H}^{(p,q)})$ is a conjugate-symmetric reordering transformation as it concatenates two such transformations applied on distinct elements of the matrix $H^{(p,q)}$. In order to prove the second condition of the equivalency as in (12), we partition $\mathbf{Z}^{(\mathbf{p},\mathbf{q})}$ similarly to (30):

$$
\mathbf{Z}^{(\mathbf{p},\mathbf{q})} = \left[\begin{array}{c|c} \mathbf{Z}_{\mathbf{A}}^{(\mathbf{p},\mathbf{q}-\mathbf{1})} & \mathbf{Z}_{\mathbf{B}}^{(\mathbf{p},\mathbf{q}-\mathbf{1})} \\ \hline \mathbf{Z}_{\mathbf{C}}^{(\mathbf{p},\mathbf{q}-\mathbf{1})} & \mathbf{Z}_{\mathbf{D}}^{(\mathbf{p},\mathbf{q}-\mathbf{1})} \end{array}\right]. \tag{64}
$$

from the equivalency in (57) we get that all the entries compriing $\mathfrak{C}(\mathbf{Z}_{\mathbf{A}}^{(\mathbf{p},\mathbf{q}-\mathbf{1})}), \mathfrak{C}(\mathbf{Z}_{\mathbf{B}}^{(\mathbf{p},\mathbf{q}-\mathbf{1})}), \mathfrak{C}(\mathbf{Z}_{\mathbf{C}}^{(\mathbf{p},\mathbf{q}-\mathbf{1})})$ and $\mathfrak{C}(\mathbf{Z}_{\mathbf{C}}^{(\mathbf{p},\mathbf{q}-\mathbf{1})})$ i.i.d complex Gaussian with unit variance. Hence we get that

$$
\mathfrak{C}(\mathbf{Z}^{(\mathbf{p},\mathbf{q})}) = \left[\begin{array}{c} \mathfrak{C}(\mathbf{Z}_{\mathbf{A}}^{(\mathbf{p},\mathbf{q}-\mathbf{1})}) + \mathfrak{C}(\mathbf{Z}_{\mathbf{D}}^{(\mathbf{p},\mathbf{q}-\mathbf{1})})^* \\ \hline -\mathfrak{C}(\mathbf{Z}_{\mathbf{C}}^{(\mathbf{p},\mathbf{q}-\mathbf{1})}) + \mathfrak{C}(\mathbf{Z}_{\mathbf{B}}^{(\mathbf{p},\mathbf{q}-\mathbf{1})})^* \end{array}\right] \tag{65}
$$

also has the same property. □

Remark 7. *We could also derive a stronger property; each element of* $\mathbf{Z}$ *is combined only once whereas equal weights are given to each element. The proof is derived by assuming the same on* $\mathfrak{C}(\mathbf{Z}_{\mathbf{A}}^{(\mathbf{p},\mathbf{q}-\mathbf{1})}), \mathfrak{C}(\mathbf{Z}_{\mathbf{B}}^{(\mathbf{p},\mathbf{q}-\mathbf{1})}), \mathfrak{C}(\mathbf{Z}_{\mathbf{C}}^{(\mathbf{p},\mathbf{q}-\mathbf{1})})$ *and* $\mathfrak{C}(\mathbf{Z}_{\mathbf{C}}^{(\mathbf{p},\mathbf{q}-\mathbf{1})})$.

APPENDIX C
EXAMPLES FOR NON-INTEGER POWER OF 2 SYSTEMS

For N_t and N_r which are not an integer power of two, transmission and combining schemes (which achieve full diversity while transmitting at rate one, perform universal combining and meet the constraint defined above on the rate between the combining unit and the detector), can be obtained by truncating the transmission or the combining scheme of the closest greater power of two. For example, the transmission and combining scheme for 4×2 are:

$$
\mathbf{C} = \frac{1}{\sqrt{4}} \begin{bmatrix} x_1 & x_2 & x_3 & x_4 \\ -x_2^* & x_1^* & -x_4^* & x_3^* \\ -x_3^* & -x_4^* & x_1^* & x_2^* \\ x_4 & -x_3 & -x_2 & x_1 \\ x_5^* & x_6^* & x_7^* & x_8^* \\ -x_6 & x_5 & -x_8 & x_7 \\ -x_7 & -x_8 & x_5 & x_6 \\ x_8^* & -x_7^* & -x_6^* & x_5^* \end{bmatrix}, \tag{66}
$$

$$
\mathfrak{C}(\mathbf{R}) = \frac{1}{\sqrt{2}} \begin{bmatrix} r_{1,1} + r_{5,2}^* \\ r_{2,1} + r_{6,2}^* \\ r_{3,1} + r_{7,2}^* \\ r_{4,1} + r_{8,2}^* \\ -r_{5,1} + r_{1,2}^* \\ -r_{6,1} + r_{2,2}^* \\ -r_{7,1} + r_{3,2}^* \\ -r_{8,1} + r_{4,2}^* \end{bmatrix}, \tag{67}
$$

which results in the following system

$$
\begin{bmatrix} s_1 \\ s_2 \\ s_3 \\ s_4 \\ s_5 \\ s_6 \\ s_7 \\ s_8 \end{bmatrix} = \frac{1}{\sqrt{8}} \begin{bmatrix} x_1 & x_2 & x_3 & x_4 & x_5 & x_6 & x_7 & x_8 \\ -x_2^* & x_1^* & -x_4^* & x_3^* & -x_6^* & x_5^* & -x_8^* & x_7^* \\ -x_3^* & -x_4^* & x_1^* & x_2^* & -x_7^* & -x_8^* & x_5^* & x_6^* \\ x_4 & -x_3 & -x_2 & x_1 & x_8 & -x_7 & -x_6 & x_5 \\ -x_5^* & -x_6^* & -x_7^* & -x_8^* & x_1^* & x_2^* & x_3^* & x_4^* \\ x_6 & -x_5 & x_8 & -x_7 & -x_2 & x_1 & -x_4 & x_3 \\ x_7 & x_8 & -x_5 & -x_6 & -x_3 & -x_4 & x_1 & x_2 \\ -x_8^* & x_7^* & x_6^* & -x_5^* & x_4^* & -x_3^* & -x_2^* & x_1^* \end{bmatrix} \begin{bmatrix} h_{11} \\ h_{21} \\ h_{31} \\ h_{41} \\ h_{12}^* \\ h_{22}^* \\ h_{32}^* \\ h_{42}^* \end{bmatrix} + \frac{1}{\sqrt{2}} \begin{bmatrix} \hat{z}_1 & \hat{z}_2 & \hat{z}_3 & \hat{z}_4 & \hat{z}_5 & \hat{z}_6 & \hat{z}_7 & \hat{z}_8 \end{bmatrix}^T. \tag{68}
$$

This scheme is equivalent to the 8×1 MISO with EA-QOSTBC transmission denoted as $\{\breve{\mathbf{X}}^{(\tilde{N}_{\text{eff}})}, \mathfrak{I}\}_{\tilde{N}_{\text{eff}} \times 1}$. For 3×2 channel, the same transmission scheme can be used while eliminating one of the transmit antennas. For example, eliminating the 4'th transmit antenna results in the following transmission scheme

$$
\tilde{\mathbf{C}} = \frac{1}{\sqrt{3}} \begin{bmatrix} x_1 & x_2 & x_3 \\ -x_2^* & x_1^* & -x_4^* \\ -x_3^* & -x_4^* & x_1^* \\ x_4 & -x_3 & -x_2 \\ x_5^* & x_6^* & x_7^* \\ -x_6 & x_5 & -x_8 \\ -x_7 & -x_8 & x_5 \\ x_8^* & -x_7^* & -x_6^* \end{bmatrix}, \tag{69}
$$

where the normalization factor is adjusted to the new number of transmit antennas. Choosing the same combining operation

$$
\tilde{\mathfrak{C}}(\mathbf{R}) = \mathfrak{C}(\mathbf{R}) \tag{70}
$$

We get that the resulting received sybols in the system $\{\mathbf{C}^{(p,q)}, \mathfrak{C}\}_{N_t,N_r}$ (defined in (69) and (70)) is equal to those presented in (68) while substituting $h_{14} = h_{24} = 0$. This is equivalent to transmitting $\breve{\mathbf{X}}^{(8)}$ with the 4'th and 8'th columns punctured (with adjusted normalization factor).

Another example dealing with number of receive antennas which is not an integer power of two is demonstrated for the 1×3 SIMO channel. In case of 1×4 SIMO channel, the transmission and combining schemes are

$$
\tilde{\mathbf{C}}^T = \begin{bmatrix} x_1 & x_2^* & x_3^* & x_4 \end{bmatrix}, \tag{71}
$$

$$\tilde{\mathfrak{C}}(\mathbf{R}) = \frac{1}{\sqrt{4}} \begin{bmatrix} r_{1,1} + r_{2,2}^* + r_{3,3}^* + r_{4,4} \\ -r_{2,1} + r_{1,2}^* - r_{4,3}^* + r_{3,4} \\ -r_{3,1} - r_{4,2}^* + r_{1,3}^* + r_{2,4} \\ r_{4,1} - r_{3,2}^* - r_{2,3}^* + r_{1,4} \end{bmatrix}. \tag{72}$$

The resulting received vector are:

$$\begin{bmatrix} s_1 \\ s_2 \\ s_3 \\ s_4 \end{bmatrix} = \frac{1}{\sqrt{4}} \begin{bmatrix} x_1 & x_2 & x_3 & x_4 \\ -x_2^* & x_1^* & -x_4^* & x_3^* \\ -x_3^* & -x_4^* & x_1^* & x_2^* \\ x_4 & -x_3 & -x_2 & x_1 \end{bmatrix} \cdot \begin{bmatrix} h_{11} \\ h_{12}^* \\ h_{13}^* \\ h_{14} \end{bmatrix} + \begin{bmatrix} \hat{z}_1 \\ \hat{z}_2 \\ \hat{z}_3 \\ \hat{z}_4 \end{bmatrix}. \tag{73}$$

For a 1×3 SIMO channel, we use the transmission:

$$\mathbf{C} = \tilde{\mathbf{C}} \tag{74}$$

similar combining scheme can be used when nulling the 4'th receive antenna and ignoring its received symbols. Thus we get

$$\mathfrak{C}(\mathbf{R}) = \frac{1}{\sqrt{3}} \begin{bmatrix} r_{1,1} + r_{2,2}^* + r_{3,3}^* \\ -r_{2,1} + r_{1,2}^* - r_{4,3}^* \\ -r_{3,1} - r_{4,2}^* + r_{1,3}^* \\ r_{4,1} - r_{3,2}^* - r_{2,3}^* \end{bmatrix}, \tag{75}$$

where the normalization factor is adjusted to the new number of receive antennas. The resulting system is the one presented in (73) when substituting $h_{14} = 0$. This is similar to nulling the 4'th column of $\check{\mathbf{X}}^{(4)}$ (with adjusted normalization factor).

References

[1] D. Tse and P. Viswanath, *Fundamentals of wireless communication*. Cambridge university press, 2005.

[2] A. Hottinen, O. Tirkkonen, and R. Wichman, *Multi-antenna transceiver techniques for 3G and beyond*. John Wiley & Sons, 2004.

[13] E. Domanovitz and U. Erez, "Diversity combining via universal dimension-reducing space–time transformations," *IEEE Transactions on Communications*, vol. 67, no. 3, pp. 2464–2475, 2018.

[3] H. Jafarkhani, *Space-time coding: theory and practice*. Cambridge university press, 2005.

[4] V. Tarokh, H. Jafarkhani, and A. R. Calderbank, "Space-time block codes from orthogonal designs," *IEEE Transactions on Information theory*, vol. 45, no. 5, pp. 1456–1467, 1999.

[5] S. Alamouti, "A simple transmit diversity technique for wireless communications," *IEEE Journal on Selected Areas in Communications*, vol. 16, no. 8, pp. 1451–1458, 1998.

[6] H. Jafarkhani, "A quasi-orthogonal space-time block code," *IEEE Transactions on Communications*, vol. 49, no. 1, pp. 1–4, Jan 2001.

[7] C. B. Papadias and G. J. Foschini, "A space-time coding approach for systems employing four transmit antennas," in *2001 IEEE International Conference on Acoustics, Speech, and Signal Processing. Proceedings (Cat. No. 01CH37221)*, vol. 4. IEEE, 2001, pp. 2481–2484.

[8] O. Tirkkonen, A. Boariu, and A. Hottinen, "Minimal non-orthogonality rate 1 space-time block code for 3+ Tx antennas," *2000 IEEE Sixth International Symposium on Spread Spectrum Techniques and Applications. ISSTA 2000. Proceedings (Cat. No.00TH8536)*, vol. 2, pp. 429–432 vol.2, 2000.

[9] A. Sezgin and T. J. Oechtering, "Complete characterization of the equivalent mimo channel for quasi-orthogonal space–time codes," *IEEE Transactions on Information Theory*, vol. 54, no. 7, pp. 3315–3327, 2008.

[10] W. Su and X.-G. Xia, "Signal constellations for quasi-orthogonal space-time block codes with full diversity," *IEEE Transactions on Information Theory*, vol. 50, no. 10, pp. 2331–2347, 2004.

[11] N. Sharma and C. Papadias, "Full-rate full-diversity linear quasi-orthogonal space-time codes for any number of transmit antennas," *EURASIP Journal on Advances in Signal Processing*, vol. 2004, 08 2004.

[12] O. Tirkkonen, "Optimizing space-time block codes by constellation rotations," in *Proc. Finnish Wireless Commun. Workshop*, 2001, pp. 59–60.

[14] J. Joung, "Space–time line code," *IEEE Access*, vol. 6, pp. 1023–1041, 2017.

[15] Lizhong Zheng and D. N. C. Tse, "Diversity and multiplexing: a fundamental tradeoff in multiple-antenna channels," *IEEE Transactions on Information Theory*, vol. 49, no. 5, pp. 1073–1096, 2003.

[16] B. Badic, M. Rupp, and H. Weinrichter, "Quasi-orthogonal space-time block codes: approaching optimality," in *2005 13th European Signal Processing Conference*. IEEE, 2005, pp. 1–8.