VILNIAUS UNIVERSITETAS
MATEMATIKOS IR INFORMATIKOS FAKULTETAS
PROGRAMŲ SISTEMŲ STUDIJŲ PROGRAMA

Hibridinio genetinio paieškos algoritmo transporto maršrutų optimizavimo uždaviniams spręsti lygiagretinimas

Parallelization of Hybrid Genetic Search Algorithm for Solving Vehicle Routing Problem

Kursinis darbas

Atliko: 4 kurso 1 grupės studentas
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Darbo vadovas: Doc., Dr. Algirdas Lančinskas

Vilnius – 2025

Turinys

Terminai

Įvadas

paragrafas apie VRP relevance

Plačiau apie VRP constraints

Galimi constraints, kuriuos galima uždėti ant VRP problemų:

- kiekviena transporto priemonė gali turėti (skirtingą) maršruto pradžios laiką
- kiekviena transporto priemonė gali turėti (skirtingą) maksimalų atstumą, kurį gali nukeliauti
- kiekviena transporto priemonė gali turėti (skirtingą) maksimalią talpą
- kiekvienos transporto priemonės vairuotojas gali turėti (skirtingą) tvarkaraštį (skirtingas pamainos laikas, su arba be pertraukų)
- kiekvienas taškas gali turėti (skirtingas) veikimo valandas (galimai su pietų pertraukom, etc...)
- kiekvienas taškas gali turėti (skirtingas) service time
- kiekvienas taškui gali būti arba nebūti griežtas reikalavimas jį aplankyti
- apmokėjimo constraints:
  - per tašką
  - per atstumą
  - etc...

Metodai

- Exact methods / Mathematical models (google or-tools)
- Heuristic – A problem-specific rule or method to quickly find a good (not necessarily optimal) solution.
- Metaheuristic – A higher-level strategy/framework that guides heuristics to explore solutions more effectively.
  - Adaptive Large Neighborhood Search / Hybrid Adaptive Large Neighborhood Search
    🛠️ https://reinterpretcat.github.io/vrp/
  - Hybrid Genetic Search (HGS)
    🛠️ https://github.com/vidalt/HGS-CVRP
    🛠️ https://pyvrp.org/
  - Simulated Annealing Algorithm (SAA)
  - Ant colony optimization (ACO)

Konkursai

- DIMACS [DIM22]
- "2021 Amazon last mile routing research challenge: Data set" [Mer+24] (2024) TODO: properly cite the challenge, not just the dataset
- EURO meets NeurIPS 2022 vehicle routing competition

Įrankiai

- Matrix

  - OSMR
  - GraphHopper
  - Valhalla
  - BRouter
- Simulation https://roadsimulator3.fr/these/chapters/chapitre00.html

HGS

Hibridinis genetinis paieškos (HGS) algoritmas yra vienas iš efektyviausių būdų spręsti transporto maršrutų optimizavimo uždavinius. [Citation needed?]

Pirma aprašytas "A Hybrid Genetic Algorithm for Multidepot and Periodic Vehicle Routing Problems" [Vid+12] (2012) ir patobulintas [Vid+14], [Vid+16], [Vid17], [Vid+21].

- > We propose a metaheuristic that combines the exploration breadth of population-based evolutionary search, the aggressive-improvement capabilities of neighborhood-based metaheuristics, and advanced population-diversity management schemes. The method that we name *Hybrid Genetic Search with Adaptive Diversity Control* (HGSADC)
  >
  > — *Thibaut Vidal, Teodor Gabriel Crainic, Michel Gendreau, Nadia Lahrichi, Walter Rei [Vid+12]*
- > *HGSADC proves to be extremely competitive CVRP.*
  >
  > — *Thibaut Vidal, Teodor Gabriel Crainic, Michel Gendreau, Nadia Lahrichi, Walter Rei [Vid+12]*
- maintains diversity in search -> avoids local minima ir dar aukštesnės kokybės sprendimai ir reduced computational time.

Per daugelį iteracijų patobulintas aprašytas "Hybrid genetic search for the CVRP: Open-source implementation and SWAP* neighborhood" [Vid22] (2022).

- > *the generalization of this method into a unified algorithm for the vehicle routing problem (VRP) family (Vidal et al., 2014, 2016; Vidal, 2017; Vidal et al., 2021)*
  >
  > — *Thibaut Vidal [Vid22]*
- > *Beyond a simple reimplementation of the original algorithm, HGS- CVRP takes advantage of several lessons learned from the past decade of VRP studies: it relies on simple data structures to avoid move re- evaluations and uses the optimal linear-time Split algorithm of Vidal (2016). Moreover, its specialization to the CVRP permits significant methodological simplifications. In particular, it does not rely on the visit-pattern improvement (PI) operator (Vidal et al., 2012) originally designed for VRPs with multiple periods, and uses instead a new neigh- borhood called Swap*.*
  >
  > — *Thibaut Vidal [Vid22]*
- > *In HGS-CVRP, we rely on the efficient linear-time Split algorithm introduced by Vidal (2016) (autoriaus papildymas: [Vid16]) after each crossover operation.*
  >
  > — *Thibaut Vidal [Vid22]*

TODO: "Technical note: Split algorithm in O(n) for the capacitated vehicle routing problem" [Vid16] (2016)

- naudoja "New benchmark instances for the Capacitated Vehicle Routing Problem" [Uch+17a] (2017) metodiką rezultatų palyginimui
- 2000 eilučių C++ kodo

"A hybrid genetic search based approach for the generalized vehicle routing problem" [Lat25a] (2025)
grįstas HGS.
Pritaikytas *Generalized Vehicle Routing Problem* variantui
Nėra viešo source code.

> *We show that adapting the meta-heuristic strategies designed for the CVRP to the GVRP can be quite a straightforward process.*
>
> —

> *we report the numerical results on the well-known instances problems for both the GVRP and CluVRP.*
>
> —

Straipsnyje rezultatai palyginti tik su kitais CluVRP, GRVP-pritaikytais algoritmais.

"An application of a two-level genetic search for the soft-clustered vehicle routing problem" [Lat25b] (2025)
grįstas HGS.

> *we propose a tailored two-level HGS for the SoftCluVRP. Our approach integrates the efficient local search framework and data structures from [21] while restructuring HGS into a two-level algorithm.*
>
> —

pritaikytas SoftCluVRP/CluVRP VRP variantui
Straipsnyje rezultatai palyginti tik su kitais CluVRP-pritaikytais algoritmais.

"Exploring dynamic population Island genetic algorithm for solving the capacitated vehicle routing problem" [Rez+24] (2024)
grįstas HGS, pristato naujo algoritmą DPIGA-HGS

> *In the work herein, DPIGA-HGS is shown to outperform existing state-of-the-art algorithms from the literature*
>
> —

Kiti

- „Where to Split in Hybrid Genetic Search for the Capacitated Vehicle Routing Problem“

> *Results indicate that simple adjustments of the starting point for the splitting procedure can improve the performance of the genetic search, as measured by the average primal gaps of the final solutions obtained, by 3.9%.*
>
> —

- ACO-grįsti:
  - Multi-strategy ant colony optimization with k-means clustering algorithm for capacitated vehicle routing problem
- „Optimization of Heterogeneous Last-Mile Delivery of Fresh Products Considering Traffic Congestions and Other Real-World Parameters“
  The variants considered in this paper are: (CVRP), (VRPTW), (VRPSTW), (HVRP), (MTVRP), (SVRP), (SDVRP), (TDVRP),
- „A systematic literature review on the use of metaheuristics for the optimisation of multimodal transportation“

Literatūros apžvalgos

- "A Detailed Review of the Capacitated Vehicle Routing Problem: Model, Computational Complexity, Solutions, and Practical Applications" [Ham+25] (2025)
  tl;dr: aprašto logistikos problemų kriterijus ir tipus, tada šias priskiria tam tikriem VRP tipams (e.g. VRPPD, VRPTW, etc...)
- "A review of recent advances in time-dependent vehicle routing" [Ada+24] (2024)
  tl;dr: pagrinde pristato ir aprašo CVRP. Išskiria metodų grupes (tikslūs; apytikslūs – heuristiniai ir metaheuristiniai). Iš metaheuristinių algoritmų grupių išskiria tris grupes:

> *•Evolutionary such as “Genetic Algorithm (GA)”;*
> *•Physic - Based such as “Simulated Annealing Algorithm (SAA)”; and*
> *•Swarm Intelligence like “Ant colony optimization (ACO)”.*
> *— Tommaso Adamo, Michel Gendreau, Gianpaolo Ghiani, Emanuela Guerriero [Ada+24]*

pasirinkti ACO grįsti algortimai ir palyginti tarpusavyje.

- "Operational Research: methods and applications" [Pet+23] (2023)
  tl;dr: apriebia visą *Operations Research* iš 200 psl. 2 skirta VRP. Pateikia įvairius naujus metaheuristinius algoritmus, išskiria HGS kaip vieną iš geresnių.

> *An up-to-date survey on recent trends can be found in Vidal et al. (2020) [[VLM20]]*
>
> —

> *Clear standards have been set by the CVRP com munity around which benchmark instances should be used for testing the performance of an algorithm, and which are ways of testing a computer code for a fair comparison with other previously proposed algorithms. Uchoa et al. (2017) discuss the most widely used instances and provides a link to the repository, in which the input data, as well as the best known solu tions, are provided and kept up-to-date by the authors. A more recent set of instances and best known solu tions is available in Queiroga et al. (2022), where the authors provide data enabling the use of machine learning approaches to solve the CVRP. Accorsi et al. (2022)*

> *present the standard practices to test CVRP algorithms: how to determine computing time (typic ally on a single thread), common ways of tuning parameters, and providing best and average solutions on a specified number of executions, among others.*
>
> —

- TODO: "A concise guide to existing and emerging vehicle routing problem variants" [VLM20] (2020)
- TODO: A hybrid genetic search and dynamic programming-based split algorithm for the multi-trip time-dependent vehicle routing problem
- TODO: Vehicle routing problem and related algorithms for logistics distribution: a literature review and classification (2020)

Lygiagretinimas

„Pathways to Efficient and Equitable Solutions for Large-Scale Routing Problems“ (2025)

Dar neišleista disertacija - PREVIEW
Pagreitina HGS veikimą naudojant deep learning (ir vėliau jį pritaiko last-mile gig-economy panaudojimui).

> *The third problem extends the classical Rural Postman Problem (RPP) to a mixed- fleet scenario involving multiple trucks and drones, with the objective of minimizing makespan*
>
> —

"Speeding up Local Optimization in Vehicle Routing with Tensor-based GPU Acceleration" [LHW25] (2025)

> *In this study, we explore a promising direction to address this challenge by introducing an original tensor-based GPU ac- celeration method designed to speed up the commonly used local search operators in vehicle routing.*
>
> —

> *[25] proposed a hybrid genetic algorithm integrating 2-opt local search to solve the capacitated VRP on GPU. The GPU was used to handle all algorith- mic components, including population initialization, reproduction, 2-opt local search, and refining processes. [26] developed a GPU-based multi-objective memetic algorithm for the VRP with route balancing. They proposed two schemes for the parallelism: solution-level parallelism, where multiple solutions were processed using parallel local search, and route-level parallelism, which provided a finer granularity by parallelizing route level evaluations. However, their method did not exploit the finer node-level parallelism commonly used in neighborhood evaluations. [27] explored GPU-based parallelization of 2-opt and 3-opt local search opera- tors for the CVRP, achieving significant speedups over CPU implementations. Similarly, [28] extended GPU-based local search for the CVRP by incorporat- ing additional operators such as or-opt, swap, and relocate, achieving compa- rable improvements in computational performance. However, their methods were limited to the basic travel distance evaluation. [29] addressed the single VRP with deliveries and selective pickups using a GPU-based variable neigh- borhood search, where the GPU was also tasked with parallel neighborhood*

> *evaluations. Despite incorporating multiple local search operators, their ap- proach primarily optimized the evaluation of travel distance and struggled to effectively manage complex constraints.*
>
> —

[25]: „M. F. Abdelatti, M. S. Sodhi, An improved gpu-accelerated heuristic technique applied to the capacitated vehicle routing problem, in: Proceedings of the 2020 Genetic and Evolutionary Computation Conference, 2020"

> *We present the first innovative tensor-based GPU acceleration method that can be embedded in local search algorithms for solving various VRPs.*
>
> —

> *Our tensor-based GPU acceleration (TGA) method is highly extensible and can be integrated into various local search based algorithms and frameworks.*
>
> —

> *we incorporated TGA into the MA-FIRD algorithm*
>
> —

- NĖRA SOURCE CODE
- PREPRINT
- ypatingas pagreitėjimas su ypač dideliais duomenų kiekiais
- pritaikytas šiem *local search operators* (Relocate, Swap, 2-opt*, and 2-opt)
- **IDEA: Pritaikyti HGS** (Swap* ir Swap nėra tas pats dalykas)
  Neaišku, kuriam VRP variantui, tikriausiai CVRP
  Galimai bus sunku pritaikyti HGS:

  > *the current design of the tensor representation of solutions doesn't*
  > *galima bandyti pritaikyti senesniems HGS variantams, kur naudojamas 2-opt/Swap.*
  > *support easy implementation of pruning strategies and neighborhood reduc- tion techniques that are often used in local search-based routing algorithms.*
  >
  > —

  vietoje pagreitinimo galima bandyti panaudoti GPU, kad atrasti visas galimybes, galimai gausis geresni rezultai:

  > *We therefore only evaluate Swap* moves between r and r' if the polar sectors (from the depot) associated with these routes intercept each other. As shown in our computational experiments, with this additional restriction, the computational effort needed to explore Swap* decreases*
  >
  > — *Thibaut Vidal [Vid22]*

- Gal galima pritaikyti [„Efficient Parallel Sparse Tensor Contraction"](), kad dar pagreitinti.

„A Parallel Hybrid Genetic Search for the Capacitated VRP with Pickup and Delivery" (2023)

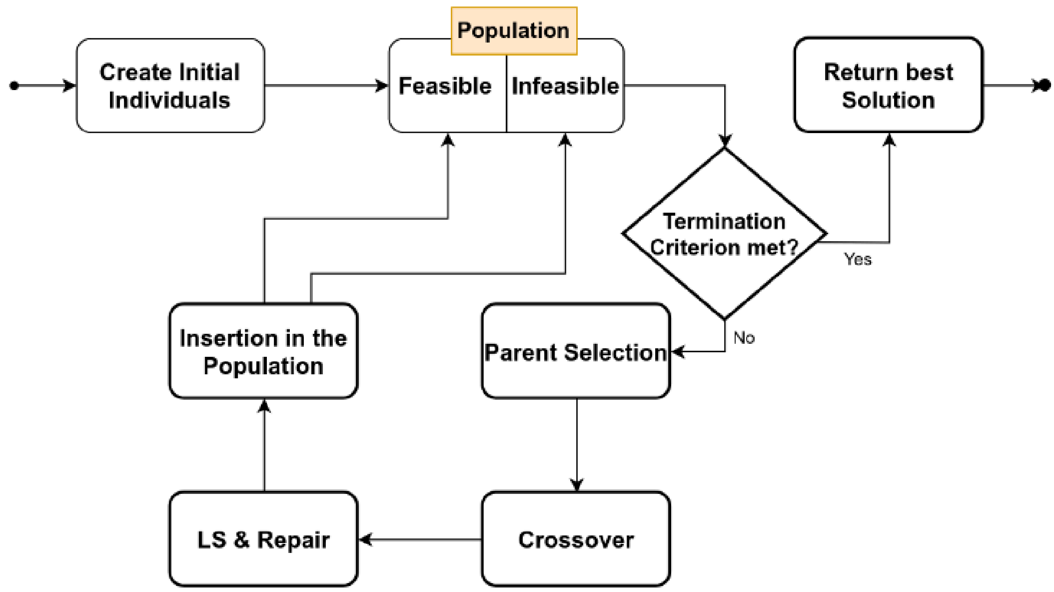
> *In our paper A Hybrid Genetic Algorithm for Solving the VRP with Pickup and Delivery in Rural Areas, we introduced an adapated gene transfer limiting the amount of possible mutations in each generation.*
>
> —

> *Here, several heuristic methods are combined in an iterative process to find the most optimal solution to the problem [4].*
>
> —

> *Yelmewad and Talawar use a parallel version of the Local Search heuristic, for solving the Capacitated Vehicle Routing Problem (CVRP) [7].*
>
> —

> *In „A Multi-GPU Parallel Genetic Algorithm For Large-Scale Vehicle Rout- ing Problems“ Abdelatti et al. consider solving VRPs using GAs on high- performance computing (HPC) platforms with up to 8 GPUs. The authors focus on VRPs with up to 20, 000 nodes. To achieve the maximum degree of paral- lelism, each array of the algorithm is mapped to block threads to achieve high throughput and low latency [9].*
>
> —

- [4]: B. D. Backer, V. Furnon, P. Shaw, P. Kilby, and P. Prosser, „Solving vehicle routing problems using constraint programming and metaheuristics,“ vol. 6, no. 4, pp. 501–523.
- [7]: „Parallel Version of Local Search Heuristic Algorithm to Solve Capacitated Vehicle Routing Problem“ (2021)
- [9]: „A multi-gpu parallel genetic algorithm for large-scale vehicle routing problem“ (2022)

> 
>
>
> **Fig. 1.** Flow Chart for the sequential version of the HGS Algorithm
>
> —

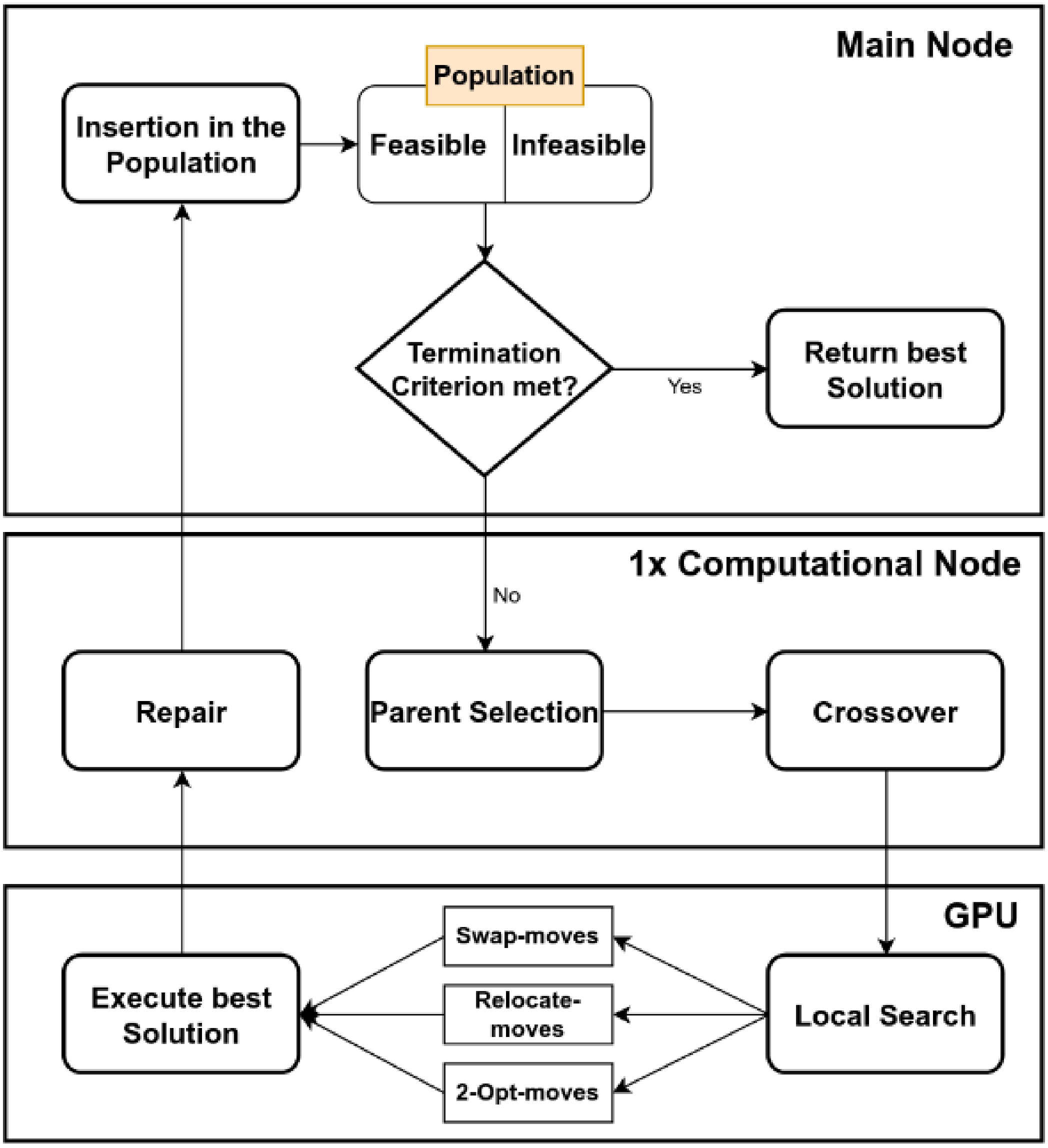


Fig. 4. Flowchart for the parallelized algorithm

- Grįstas HGS.
- padalina darbus per kelis įrenginius/GPUs? („nodes" straipsnyje) naudojant MPI. Naudoja CUDA, kad lygiagretinti LS.
  - Reikalauja kelių node'ų kiekvienas su GPU.
- nėra rezultatų palyginimų, su pvz.: BKS
- palyginimas su Tabu search grįstu algoritmu, ne HGS
- Nėra SOURCE CODE: autorius pametė jį

"Effective Parallelization of the Vehicle Routing Problem" [Mun+23] (2023)
- 1500 eilučių C++ kodo (https://github.com/mrprajesh/parMDS)

> *The state-of-the-art GPU implementations are due to Yelmewad and Talawar [35], and Abde- latti and Sodhi [1].*
>
> —

[1]: 2020. An improved GPU-accelerated heuristic technique applied to the capacitated vehicle routing problem.

[35]: 2021. Parallel Version of Local Search Heuristic Algorithm to Solve Capacitated Vehicle Routing Problem.

> work with the best-known-solution (BKS) for an instance. As a convention, the quality of a solution is measured as:
>
> $$\mathsf{Gap} = \frac{Z_S - Z_{\text{BKS}}}{Z_{\text{BKS}}} \times 100\ ,$$
>
> where Z_S is the cost of the solution reported by a solver S and Z_{BKS} is the BKS. Note that Gap is expressed as a percentage. The smaller
>
> —

[25]: 2018. A CPU-GPU Parallel Ant Colony Optimization Solver for the Vehicle Routing Problem

Naudojimas OpenMP pagreitinimui naudojant CPU (shared-memory).

Ganėtinai paprastas algoritmas, pagrinde naudojamas Local Search, iš esmės lygiagretinima ties kiekvienu bandymu ieškoti sprendimo (i.e. parelelizuojamas `for` ciklas):

> **Algorithm 1:** ParMDS: The proposed method
>
> ```
> Input: G = (V, E), Demands D := ⋃_{i=1}^{n} d_i, Capacity Q
> Output: R, a collection of routes as a valid CVRP solution
>         C_R, the cost of R
> T ← PRIMS_MST (G)                                  /* Step 1 */
> C_R ← ∞
> for i ← 1 to ρ do        /* Superloop */ /* Parallel */
>   | T_i ← RANDOMIZE (T) /* Shuffle Adjacency List */
>   | π_i ← DFS_VISIT (T_i, Depot)                   /* Step 2 */
>   | R_i ← CONVERT_TO_ROUTES (π_i, Q, D)            /* Step 3 */
>   | C_{R_i} ← CALCULATE_COST (R_i)               /* Parallel */
>   | if C_{R_i} < C_R then
>   |   | C_R ← C_{R_i}                      /* Current Min Cost */
>   |   | R′ ← R_i                     /* Current Min Cost Route */
>   | end
> end
> R ← REFINE_ROUTES (R′)                             /* Step 4 */
> return R, C_R
> ```
>
> —

> *We plan to develop a GPU-parallel version of the proposed method to further enhance performance. On the algorithmic front, we plan to build direction- awareness into the current scheme, and add inter-route refinement strategies to better the solution quality of ParMDS.*
>
> —

Autoriai yra parašę seriją straipsnių, kaip pagreitinti/lygiagretinti grafų operacijas. pvz.:

- https://scholar.google.com/citations?hl=fr&user=kfUNJb8AAAAJ&view_op=list_works&sortby=pubdate

- https://scholar.google.com/citations?hl=fr&user=nGUg9VUAAAAJ&view_op=list_works&sortby=pubdate

IDEA: galima bandyti pritaikyti HGS

"Standardized validation of vehicle routing algorithms" [Jas+24] (2024) [„1.1 Related work“ sekcija]

> *Finally, parallel techniques play an important role in solving different VRPs, as they can not only accelerate the computations [15, 54], but also allow to elaborate higher-quality routing schedules, e.g., through efficient cooperation of parallel solvers [6, 24, 51, 53, 59].*
>
> —

[15]: (2019) Solving generalized vehicle routing problem with occa- sional drivers via evolutionary multitaskin

[6]: (2023) Parallel cooperative memetic co-evolution for VRPTW

[24]: (2023) Path planning algorithm for the multiple depot vehicle routing problem based on parallel clustering.

[51]: (2023) Effective parallelization of the vehicle routing problem

[53]: (2015) Co-operation in the parallel memetic algorithm

[54]: 2015 A parallel algorithm with the search space partition for the pickup and delivery with time windows

[59]: (2013) New selection schemes in a memetic algorithm for the vehicle routing problem with time windows

Rezultatų palyginimas

Tikslas ir uždaviniai

Tikslas – Išlygiagretinti hibridinio genetinio paieškos algoritmą, skirto transporto maršrutų optimizavimo uždaviniams spręsti.

Uždavinai:

1. Išsirinkti duomenų rinkinį pagal, kurį galima būtų testuoti/analizuoti sprendimus, pvz.:
   - tikriausiai CVRPLIB repository (repository of BKSs - Best Known Solutions) (https://vrp.galgos.inf.puc-rio.br/index.php/en/)
   - Solomon
   - Neural Combinatorial Optimization for Real-World Routing (2025)
   - Test-data generation and integration for long-distance e-vehicle routing (2023)
   - "New benchmark instances for the Capacitated Vehicle Routing Problem" [Uch+17b] (2017)
   - > *For the CVRP and VRPTW, the BKS values are obtained*
     > - *[Jas+24] [3/1337 psl.]*

> *from the CVRPLIB repository (http://vrp.galgos.inf.puc-rio.br/) as of April 30, 2025. For the CVRP, we use results from HGS-2012 [38] and HGS- CVRP [14]. For the VRPTW, with the objective of minimizing the total travel distance, we reference results from the DIMACS competition, including both the official DIMACS reference results and the champion team's algorithm, HGS-DIMACS [39]. For the VRPSPDTW, we report the best results from the state-of-the-art MA-FIRD method [32].*
>
> — *Zhenyu Lei, Jin-Kao Hao, Qinghua Wu [LHW25]*

2. Išanalizuoti, kaip veikia HGS algoritmas
3. Atrinkti paralelizuojamas dalis, ar dalis, kurias galima galima pakeisti paralelizuojamomis
4. Palyginti rezultatus su kitais state-of-the-art algoritmais
   1. Parinkti tinkamus (hyper-) parametrus (see [Jas+24] [3/1337 psl.])

Santrumpos

- VRP – *angl. Vechicle Routing Problem*.
- CVRP – *angl. Capacitated Vehicle Routing Problem*.

> *the CVRPPD divides stops into pickup and delivery points for passengers. Passengers are not arbitrary goods delivered to interchangeable destinations from a common depot, but they have individual starting points and destinations. Therefore, the pickup and delivery constraint has multiple implications. On the one hand, the order in which a person is picked up and dropped o by a vehicle must be in the correct order. In addition, the delivery must be performed by the same vehicle as the pickups*
>
> — *[SND23]*

- VRPTW – *angl. VRP with Time Windows*.
- CVRPPD – *angl. CVRP Pickup and Delivery*.
- MVRP – *angl. Multidepot VRP*.
- PVRP – *angl. Periodic VRP*.

> *In classical VRPs, typically the planning period is a single day. In the case of the Period Vehicle Routing Problem (PVRP), the classical VRP is generalized by extending the planning period to M days.*
>
> — https://neo.lcc.uma.es/vrp/vrp-flavors/periodic-vrp/

- MDPVRP – *angl. Multidepot Periodic VRP*.
- CVRP with Backhauls
- GVRP – *angl. Generalized VRP* –

> *In this problem each vertex belongs to a cluster, and only one vertex per cluster must be visited, satisfying the associated cluster demands.*
>
> — *Vittorio Latorre [Lat25a]*

- CluVRP – *angl. Clustered VRP* –

> *In the CluVRP, vehicles must visit all the nodes within a cluster before progressing to the next cluster, instead of visiting just one node per cluster as in the GVRP.*
>
> — *Vittorio Latorre [Lat25a]*

- VRPSPDTW – *angl. VRP with Simultaneous Pickup and Delivery and Time Windows*

Matematinis formulavimas

TODO

Notes

- VRPTW ∈ CVRP
- Specializuota optimizacija specializuotam uždaviniui
  "The vehicle routing problem with service level constraints" [Bul+18] (2018)
- depots and periods. A second general observation is that most methodological developments target a particular problem variant, the capacitated VRP (*CVRP*) or the VRP with time windows (*VRPTW*), for example, very few contributions aiming to address a broader set of problem settings.

Šaltiniai

[LHW25] Z. Lei, J.-K. Hao, ir Q. Wu, „Speeding up Local Optimization in Vehicle Routing with Tensor-based GPU Acceleration“. [Interaktyvus]. Adresas: https://arxiv.org/abs/2506.17357

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[Vid+16] T. Vidal, N. Maculan, L. S. Ochi, ir P. H. Vaz Penna, „Large Neighborhoods with Implicit Customer Selection for Vehicle Routing Problems with Profits“, *Transportation Science*, t. 50, nr. 2, p. 720–734, geg. 2016, doi: 10.1287/trsc.2015.0584.

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