Laboratorinis darbas #1

Domantas Keturakis

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Užduotis #1

$$
\begin{aligned}
y(e^x + 1)dx - e^x dy &= 0 \\
y(e^x + 1)dx &= e^x dy \\
\frac{e^x + 1}{e^x} dx &= \frac{1}{y} dy, \quad y \neq 0 \\
\frac{1}{y} dy &= \frac{e^x + 1}{e^x} dx \\
\int \frac{1}{y} dy &= \int (1 + e^{-x}) dx \\
\ln|y| &= x - e^{-x} + C \\
y &= e^{x - e^{-x} + C} \\
y &= C_e e^{x - e^{-x}}, \quad C_e = \pm e^C
\end{aligned}
$$

$$y = 0 : 0 \equiv 0 - \text{tapatybė, todėl } y = 0 - \text{atskiras sprendinys}$$

`SymPy` pateiktas sprendimas:

$$y(x) = \left(C_1 e^{x - e^{-x}}\right)$$

Ativaizdu, kad čia tie patys sprendimai.

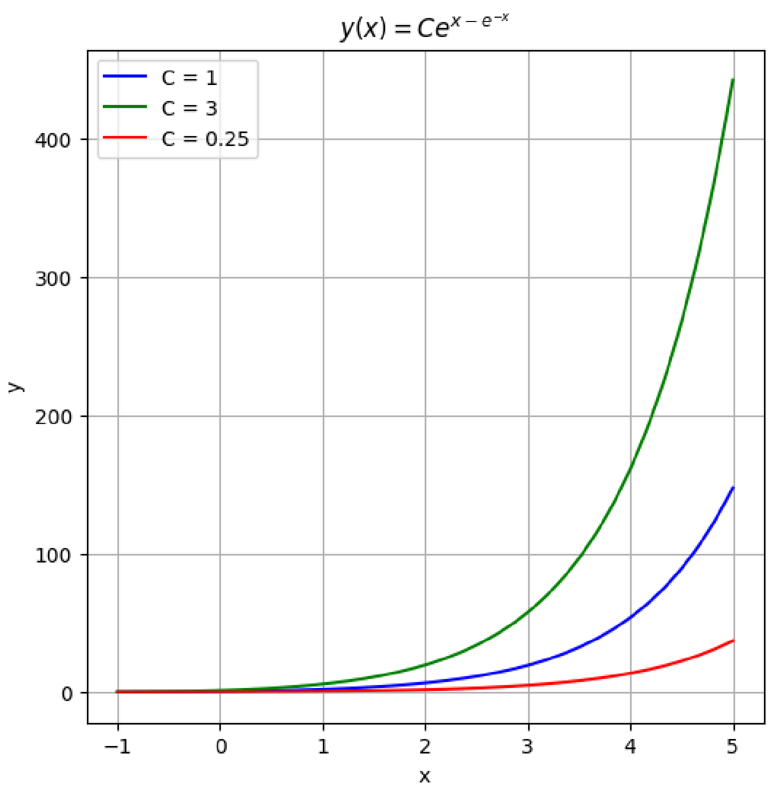

Fig. 1: Lygties $y(e^x + 1)dx - e^x dy = 0$ integralinės kreivės

Užduotis #2

$$
\begin{aligned}
xy' &= 2y + \frac{x^2}{y} \\
y' &= 2\frac{y}{x} + \frac{x}{y}, \quad x \neq 0 \\
y' &= 2\frac{y}{x} + \left(\frac{y}{x}\right)^{-1} \\
u'x + u &= 2u + u^{-1} \qquad\qquad \left[u = \frac{y}{x}, \quad y = ux \quad y' = u'x + u\right] \\
\frac{du}{dx}x &= u + u^{-1} = \frac{u^2+1}{u} \\
\frac{u}{u^2+1}du &= \frac{1}{x}dx \\
\frac{1}{2}\frac{1}{u^2+1}du^2 &= \frac{1}{x}dx \\
\frac{1}{2}\ln|u^2+1| &= \ln|x| + C \\
\ln|u^2+1| &= 2\ln|x| + C \\
u^2 + 1 &= e^{2\ln|x| + C} \\
u^2 + 1 &= e^{\ln|x^2| + C} \\
u^2 + 1 &= C_e x^2, \quad C_e = \pm e^C \\
y^2 &= C_e x^4 - x^2
\end{aligned}
$$

$$y : 0 \Rightarrow 2y = 0 - \text{ne tapatybė, todėl } y = 0 \text{ nėra užduoties sprendinys}$$

`SymPy` pateikti sprendimai:

$$
\begin{aligned}
y(x) &= \left(-x\sqrt{C_1x^2 - 1}\right) \\
y(x) &= \left(x\sqrt{C_1x^2 - 1}\right)
\end{aligned}
$$

Pertvarkius sprendimą:

$$
\begin{aligned}
y^2 &= C_e x^4 - x^2 \\
y^2 &= x^2(C_e x^2 - 1) \\
y &= \sqrt{x^2(C_e x^2 - 1)} \\
y &= \pm x\sqrt{(C_e x^2 - 1)}
\end{aligned}
$$

Ativaizdu, kad tai tie patys sprendimai.

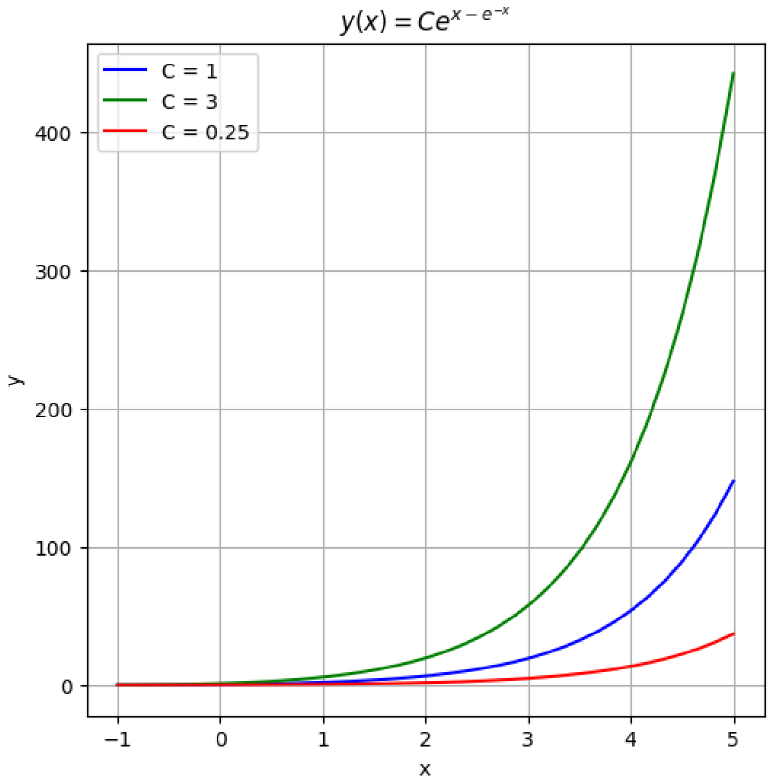

Fig. 2: Lygties $y' = 2y + \frac{x^2}{y}$ integralinės kreivės

Užduotis #3

$$x^2y' + xy + 1 = 0, y(2) = 1$$
$$x^2y' + xy = -1$$
$$y' + \frac{y}{x} = -\frac{1}{x^2}$$

Naudojant konstantų variavimo metodą:

$$y' + \frac{y}{x} = 0$$
$$\frac{dy}{dx} = -\frac{y}{x}$$
$$\int \frac{1}{y}dy = -\int \frac{1}{x}dx, \qquad y \neq 0$$
$$\ln|y| = -\ln|x| + C$$
$$y = C\frac{1}{x}, \quad C \in \mathbb{R} \quad \pm e^C \to C$$
$$y' = \frac{C'}{x} - \frac{C}{x^2}$$

Įsistačius gautas y ir y' reikšmes gauname:

$$x^2y' + xy + 1 = 0 \Rightarrow x^2\left(\frac{C'}{x} - \frac{C}{x^2}\right) + x\frac{C}{x} + 1 = 0$$
$$C'x - C + C + 1 = 0$$
$$C'x = -1$$
$$C' = -\frac{1}{x}, \quad x \neq 0$$
$$C = -\ln|x| + C_1$$
$$y = \frac{C}{x} \Rightarrow y = \frac{C_1 - \ln|x|}{x},$$
$$x : 0 \Rightarrow 1 \neq 0 - x = 0 \text{ nėra atskiras sprendinys}$$

Koši sąlygos sprendimas:

$$y(2) = 1 \Rightarrow \frac{C_1 - \ln|2|}{2} = 1$$
$$C_1 - \ln|2| = 2$$
$$C_1 = 2 + \ln|2|$$
$$\text{Ats.:} \quad y = \frac{\ln|2| + 2 - \ln|x|}{x}$$

`SymPy` pateiktas sprendimas:

$$y(x) = \frac{C - \ln(x)}{x}$$

Ativaizdu, kad tai tie patys sprendimai.

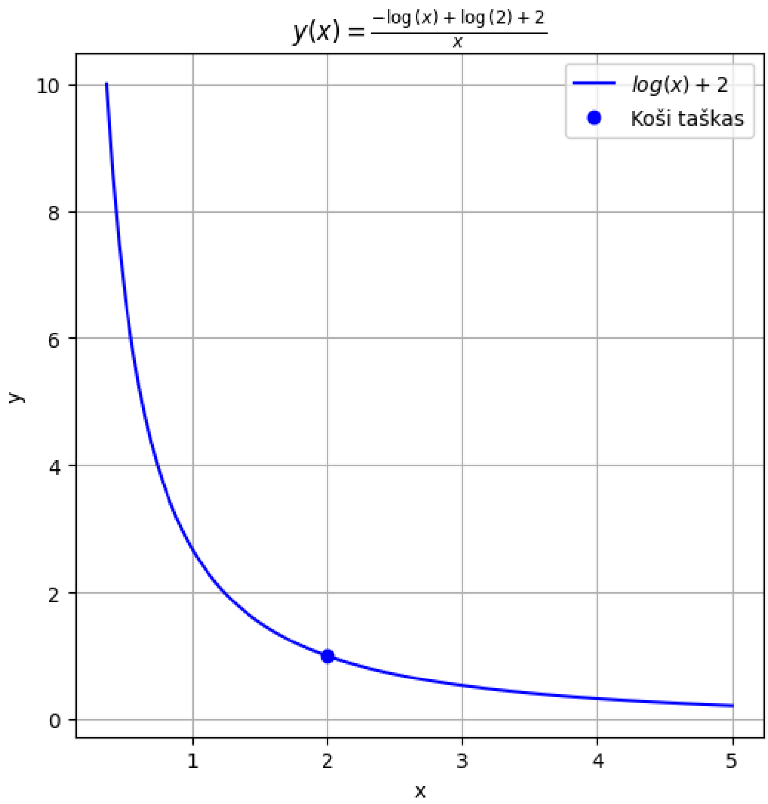

Fig. 3: Lygties $x^2y' + xy + 1 = 0, y(2) = 1$ integralinė krevė ir Koši taškas

Priedai

Visuose kodo pavyzdžiose naudojami šie įtraukimai:

```
from sympy import Function, dsolve, Derivative, E, Eq, lambdify, symbols
from sympy import print_python, print_maple_code, print_latex, latex
from sympy.abc import x
import sympy as sm
import numpy as np
import matplotlib.pyplot as plt
```

Uždieties #1 sprendimo kodas

```
y = Function('y')

eq = Eq(y(x) * (E**x + 1) - E**x * Derivative(y(x), x), 0)

solution = dsolve(eq)
display(solution)
```

Uždieties #1 vizualicazijos kodas

```
import numpy as np
import matplotlib.pyplot as plt

C = symbols('C')
solution = solution.subs('C1', C)

sol = solution.rhs

sol1 = lambdify(x, sol.subs('C', 1))
sol2 = lambdify(x, sol.subs('C', 3))
sol3 = lambdify(x, sol.subs('C', 0.25))

x_vals = np.linspace(-1, 5, 100)
y_vals1 = sol1(x_vals)
y_vals2 = sol2(x_vals)
y_vals3 = sol3(x_vals)

plt.figure(figsize=(6, 6))

plt.plot(x_vals, y_vals1, label='C = 1', color='blue')
plt.plot(x_vals, y_vals2, label='C = 3', color='green')
plt.plot(x_vals, y_vals3, label='C = 0.25', color='red')

plt.title(f"${latex(solution)}$")
plt.xlabel("x")
plt.ylabel("y")
plt.legend()
plt.grid(True)

plt.show()
```

Uždieties #2 sprendimo kodas

```
y = Function('y')

eq = Eq(
    x * Derivative(y(x), x),
    2*y(x) + x**2/y(x)
  )

solutions = dsolve(eq)

display(solutions[0])
display(solutions[1])
```

Uždieties #2 vizualicazijos kodas

```
sol1 = solutions[0].rhs
sol2 = solutions[1].rhs

sol11 = lambdify(x, sol1.subs('C1', 1))
sol12 = lambdify(x, sol1.subs('C1', 3))
sol13 = lambdify(x, sol1.subs('C1', 0.25))

sol21 = lambdify(x, sol2.subs('C1', 1))
sol22 = lambdify(x, sol2.subs('C1', 3))
sol23 = lambdify(x, sol2.subs('C1', 0.25))

x2_vals = np.linspace(-10, 10, 500)
x1_vals = np.linspace(-10, 10, 500)
y_vals1 = sol11(x1_vals)
y_vals2 = sol12(x1_vals)
y_vals3 = sol13(x1_vals)
y_vals4 = sol21(x2_vals)
y_vals5 = sol22(x2_vals)
y_vals6 = sol23(x2_vals)

plt.figure(figsize=(8, 8))

plt.plot(x1_vals, y_vals1, label='C = 1', color='blue')
plt.plot(x1_vals, y_vals2, label='C = 3', color='green')
plt.plot(x1_vals, y_vals3, label='C = 0.25', color='red')
plt.plot(x2_vals, y_vals4, color='blue')
plt.plot(x2_vals, y_vals5, color='green')
plt.plot(x2_vals, y_vals6, color='red')

plt.title(f"${latex(solutions)}$")
plt.xlabel("x")
plt.ylabel("y")
plt.legend()
plt.grid(True)

plt.show()
```

Uždieties #3 sprendimo kodas

```
y = Function('y')

eq = Eq(
    x**2 * Derivative(y(x), x) + x * y(x) + 1,
    0
    )

solution = dsolve(eq)
display(solution)
```

Uždieties #3 vizualicazijos kodas

```
sol = solution

sol = sol.subs('C1', sm.log(2) + 2)
sol1 = lambdify(x, sol.rhs)
eq = (sol.rhs)
x_limit = float(solve(Eq(sol.rhs, 10))[0])

x_vals = np.linspace(x_limit, 5, 100)
y_vals1 = sol1(x_vals)

plt.figure(figsize=(6, 6))

plt.plot(x_vals, y_vals1, label='$log(x) + 2$', color='blue')
plt.plot(2, 1, 'bo', label="Koši taškas")

plt.title(f"${latex(sol)}$")
plt.xlabel("x")
plt.ylabel("y")
plt.legend()
plt.grid(True)

plt.show()
```