Laboratorinis darbas #1

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Užduotis #1

$$
\begin{aligned}
y(e^x+1)dx - e^x dy &= 0 \\
y(e^x+1)dx &= e^x dy \\
\frac{e^x+1}{e^x}dx &= \frac{1}{y}dy, \quad y \neq 0 \\
\frac{1}{y}dy &= \frac{e^x+1}{e^x}dx \\
\int \frac{1}{y}dy &= \int (1+e^{-x})dx \\
\ln|y| &= x - e^{-x} + C \\
y &= e^{x-e^{-x}+C} \\
y &= C_e e^{x-e^{-x}}, \quad C_e = \pm e^C
\end{aligned}
$$

$$y = 0 : 0 \equiv 0 - \text{tapatybė, todėl } y = 0 - \text{atskiras sprendinys}$$

`SymPy` pateiktas sprendimas:

$$y(x) = \left(C_1 e^{x-e^{-x}}\right)$$

Ativaizdu, kad čia tie patys sprendimai.

Užduotis #2

$$\equiv$$

$$
\begin{aligned}
xy' &= 2y + \frac{x^2}{y} \\
y' &= 2\frac{y}{x} + \frac{x}{y} \\
y' &= 2\frac{y}{x} + \left(\frac{y}{x}\right)^{-1} \\
u'x + u &= 2u + u^{-1} && \left[u = \frac{y}{x}, \quad u' = u'x + u\right] \\
\frac{du}{dx}x &= u + u^{-1} = \frac{u^2 + u}{u} \\
\frac{u}{u^2 + u}du &= \frac{1}{x}dx \\
\frac{1}{u+1}du &= \frac{1}{x}dx \\
\ln|u+1| &= \ln|x| + C \\
u + 1 &= C_e x \\
y &= C_e x^2 - x
\end{aligned}
$$

UŽDUOTIS #3

$$x^2y' + xy + 1 = 0, y(2) = 1$$
$$x^2y' + xy = -1$$
$$y' + \frac{y}{x} = -\frac{1}{x^2}$$

Naudojame konstantų variavimo metodą:

$$y' + \frac{y}{x} = 0$$
$$\frac{dy}{dx} = -\frac{y}{x}$$
$$\int \frac{1}{y} dy = -\int \frac{1}{x} dx, \qquad y \neq 0$$
$$\ln|y| = -\ln|x| + C$$
$$y = C\frac{1}{x}, \quad C \in \mathbb{R} \quad \pm e^C \to C$$
$$y' = \frac{C'}{x} - \frac{C}{x^2}$$

BLah blah blah:

$$x^2y' + xy + 1 = 0 \Rightarrow x^2\left(\frac{C'}{x} - \frac{C}{x^2}\right) + x\frac{C}{x} + 1 = 0$$
$$C'x - C + C + 1 = 0$$
$$C'x = -1$$
$$C' = -\frac{1}{x}, \quad x \neq 0$$
$$C = -\ln|x| + C_1$$

$$y = \frac{C}{x} \Rightarrow y = \frac{-\ln|x| + C_1}{x},$$

$$y(2) = 1 \Rightarrow \frac{-\ln|2| + C_1}{2} = 1$$
$$C_1 - \ln|2| = 2$$
$$C_1 = 2 + \ln|2|$$

$$\text{Ats.:} \quad y = \frac{-\ln|x| + \ln|2| + 2}{x}$$