notations are used to denote the bound states of shallow impurities. The energies of these bound states are given by the **Rydberg series**:

$$E - E_c(\mathbf{0}) = -R/N^2 \quad (N = 1, 2, 3, \ldots). \tag{4.23}$$

R is the **Rydberg constant** for the donor electron and is related to the Rydberg constant for the hydrogen atom $[e^4 m_0/(2\hbar^2)]$ by

$$R = \left(\frac{m^*}{m_0}\right)\left(\frac{1}{\varepsilon_0^2}\right)\left(\frac{e^4 m_0}{2\hbar^2}\right)\frac{1}{(4\pi\varepsilon_0)^2}, \tag{4.24}$$

m_0 being the free electron mass. A schematic diagram of some of the bound states of a donor atom near a simple parabolic conduction band is shown in Fig. 4.1.

- The extent of the bound-state electron wave functions in real space is measured in terms of a *donor Bohr radius* a^*. It is related to the Bohr radius in the hydrogen atom $[\hbar^2/(m_0 e^2)]$ by

$$a^* = \left(\frac{\varepsilon_0 m_0}{m^*}\right)\left(\frac{\hbar^2}{m_0 e^2}\right)(4\pi\varepsilon_0). \tag{4.25}$$

In particular, the wave function of the 1s state is given by

$$C_{1s}(\boldsymbol{R}) = \left(\frac{1}{\pi}\right)^{1/2}\left(\frac{1}{a^*}\right)^{3/2}\exp\left(\frac{-R}{a^*}\right). \tag{4.26}$$

In order that $\boldsymbol{R}$ can be considered continuous rather than discrete, we require $a^* \gg a_0$. This condition also ensures that it is meaningful to approximate the entire conduction band structure by an effective mass m^*. The reason is that the extent in $\boldsymbol{R}$ of an envelope function $C(\boldsymbol{R})$ corresponding to the electron wave function $\Psi(\boldsymbol{r})$ scales as a^*. On the other hand, the extent in $\boldsymbol{k}$-space of Bloch functions (which are indexed by $\boldsymbol{k}$) to be summed over in the reciprocal space to construct $\Psi(\boldsymbol{r})$ can be small. This is because of an "uncertainty principle" for two variables that are related by Fourier

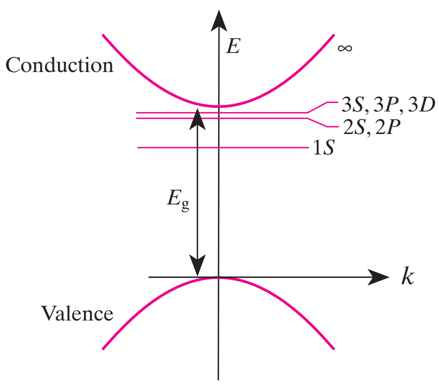

Fig. 4.1. Schematic diagram of the n=1, 2, and 3 bound states of a shallow donor electron near a nondegenerate and parabolic conduction band (corresponding to $n = \infty$). E_g is the bandgap