BST 235 Project Proposal

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1 Background

We are thinking of building on the idea of uniformly randomized concentration inequalities introduced in [RM23].

The basic idea advanced in [RM23] is that standard concentration inequalities (e.g. Markov, Chebyshev, Chernoff) have "randomized" generalizations which are stronger than the original statements. For example, the standard Markov Inequality is as follows:

Theorem 1.1 (Markov Inequality)**.** *Let X be a non-negative random variable. Then for any $a > 0$ we know that*

$$\mathbb{P}(X \geqslant a) \leqslant \frac{\mathbb{E}\, X}{a}.$$

The Uniformly-Randomized Markov Inequality (UMI) from [RM23] is the following similar, but stronger, statement:

Theorem 1.2 (Uniformly-Randomized Markov Inequality)**.** *Let X be a non-negative random variable and U a random variable which stochastically dominates* Unif$(0, 1)$. Then for any $a > 0$ we know that

$$\mathbb{P}(X \geqslant aU) \leqslant \frac{\mathbb{E}\, X}{a}.$$

[RM23] also introduces uniformly-randomized generalizations of Chebyshev and three different styles of Chernoff bounds (Hoeffding, Bernstein, and empirical Bernstein) and shows how these improved bounds can be used in a variety of applications, including safe anytime-valid inference, randomized tests with e-values, and universal inference.

Our goal is to state and prove a uniformly-randomized version of another class of Chernoff bound (exact version TBD, but possibilities matrix concentration, concentration of self-normalized estimators, etc.), develop a possible application area, and show its improvement over the non-randomized version.

References

[RM23] Aaditya Ramdas and Tudor Manole. Randomized and exchangeable improvements of markov's, chebyshev's and chernoff's inequalities. *arXiv preprint arXiv:2304.02611*, 2023.