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1 Basic

```
1.1 vimrc
```

```
set rnu nu ai cin ts=4 sw=4 mouse=a
   map <F8> <ESC>:w<CR>:!clear && g++ "%" -o "%<" -std=c
       ++11 -Wall -Wextra -Wconversion -Wshadow -02 -DLITE
        -fsanitize=address -fsanitize=undefined && echo
       success < CR >
   map <F9> <ESC>:w<CR>:!clear && g++ "%" -o "%<" -std=c
       ++11 -Wall -Wextra -Wconversion -Wshadow -02 -DLITE
        && echo success<CR>
5 | map <F10> <ESC>:!./ "%<"<CR>
   1.2 Default Code
   #pragma GCC optimize ("02")
   #pragma GCC target("avx, tune=native")
   #include < bits / stdc++.h>
   #define ALL(a) begin(a), end(a)
   using namespace std;
   // deBuG function
   // example:
   // int a = 3; int b[] = {1,2}; string c = "abc";
   // BG(a, b[1], c); // stderr: int main():37: a = 3, b
[1] = 2, c = a
   #ifdef LITE
   void _BG(const char * s) {cerr<<s<<endl;}</pre>
   template < class T, class ... TT>
   void _BG(const char * s,T a, TT...b)
       for (size_t c = 0; *s && (c || *s != ','); cerr<<*s</pre>
           c += count(ALL("([{"), *s) - count(ALL(")]}"),
                *s); // Implementation defined. But that's
       ok since it only runs on local.
cerr<<" = "<<a;</pre>
       if (*s) {
           cerr<<", ";
           ++s;
       _BG(s,b...);
   #define BG(...) do { \
       cerr << __PRETTY_FUNCTION__ << ':' << __LINE__ << "
: "; \
        BG(#__VA_ARGS__,__VA_ARGS__); \
   } while(0)
   #define BG(...)
   #endif
   int main()
   1
```

1.3 IncreaseStackSize

cin.tie(0);
// lite !

ios::sync_with_stdio(0);

```
//stack resize
asm( "mov %0,%%esp\n" :: "g"(mem+10000000) );
//change esp to rsp if 64-bit system

//stack resize (linux)
#include <sys/resource.h>
void increase_stack_size() {
   const rlim_t ks = 64*1024*1024;
   struct rlimit rl;
   int res=getrlimit(RLIMIT_STACK, &rl);
   if(res==0) {
     if(rl.rlim_cur<ks) {</pre>
```

```
rl.rlim_cur=ks;
    res=setrlimit(RLIMIT_STACK, &rl);
    }
}
```

2 Data Structure

2.1 Bigint

```
struct Bigint{
  static const int LEN = 60;
  static const int BIGMOD = 10000;
  int vl, v[LEN];
  // vector<int> v;
  Bigint() : s(1) { vl = 0; }
  Bigint(long long a) {
  s = 1; vl = 0;
    if (a < 0) { s = -1; a = -a; }
    while (a) {
      push_back(a % BIGMOD);
      a /= BIGMOD;
    }
  Bigint(string str) {
   s = 1; vl = 0;
    int stPos = 0, num = 0;
    if (!str.empty() && str[0] == '-') {
      stPos = 1;
      s = -1:
    for (int i=SZ(str)-1, q=1; i>=stPos; i--) {
  num += (str[i] - '0') * q;
      if ((q *= 10) >= BIGMOD) {
        push_back(num);
        num = 0; q = 1;
      }
    if (num) push_back(num);
    n();
  int len() const {
   return vl;
          return SZ(v);
  bool empty() const { return len() == 0; }
  void push_back(int x) {
   v[vl++] = x;
        v.PB(x);
  void pop_back() {
   vl--;
         v.pop back();
  int back() const {
   return v[vl-1];
   // return v.back();
  void n() {
    while (!empty() && !back()) pop_back();
  void resize(int nl) {
   vl = nl;
    fill(v, v+vl, 0);
        v.resize(nl);
    //
          fill(ALL(v), 0);
  void print() const {
    if (empty()) { putchar('0'); return; }
    if (s == -1) putchar('-');
    printf("%d", back());
    for (int i=len()-2; i>=0; i--) printf("%.4d",v[i]);
  friend std::ostream& operator << (std::ostream& out,</pre>
      const Bigint &a) {
    if (a.empty()) { out << "0"; return out; }</pre>
    if (a.s == -1) out << "-";
    out << a.back();</pre>
```

```
for (int i=a.len()-2; i>=0; i--) {
    char str[10];
    snprintf(str, 5, "%.4d", a.v[i]);
    out << str;
  return out;
int cp3(const Bigint &b)const {
  if (s != b.s) return s - b.s;
  if (s == -1) return -(-*this).cp3(-b);
  if (len() != b.len()) return len()-b.len();//int
  for (int i=len()-1; i>=0; i--)
  if (v[i]!=b.v[i]) return v[i]-b.v[i];
  return 0;
bool operator < (const Bigint &b)const{ return cp3(b)</pre>
    <0; }
bool operator <= (const Bigint &b)const{ return cp3(b</pre>
    ) <=0; }
bool operator == (const Bigint &b)const{ return cp3(b
    )==0; }
bool operator != (const Bigint &b)const{ return cp3(b
    )!=0; }
bool operator > (const Bigint &b)const{ return cp3(b)
    >0; }
bool operator >= (const Bigint &b)const{ return cp3(b
    )>=0; }
Bigint operator - () const {
  Bigint r = (*this);
  r.s = -r.s;
  return r;
Bigint operator + (const Bigint &b) const {
  if (s == -1) return -(-(*this)+(-b));
  if (b.s == -1) return (*this)-(-b);
  Bigint r;
  int nl = max(len(), b.len());
  r.resize(nl + 1);
  for (int i=0; i<nl; i++) {</pre>
    if (i < len()) r.v[i] += v[i];</pre>
    if (i < b.len()) r.v[i] += b.v[i];</pre>
    if(r.v[i] >= BIGMOD) {
      r.v[i+1] += r.v[i] / BIGMOD;
      r.v[i] %= BIGMOD;
   }
  }
  r.n();
  return r;
Bigint operator - (const Bigint &b) const {
  if (s == -1) return -(-(*this)-(-b));
  if (b.s == -1) return (*this)+(-b);
  if ((*this) < b) return -(b-(*this));</pre>
  Bigint r;
  r.resize(len());
  for (int i=0; i<len(); i++) {</pre>
    r.v[i] += v[i];
    if (i < b.len()) r.v[i] -= b.v[i];</pre>
    if (r.v[i] < 0) {
  r.v[i] += BIGMOD;</pre>
      r.v[i+1]--;
   }
  }
  r.n();
  return r:
Bigint operator * (const Bigint &b) {
  Bigint r;
  r.resize(len() + b.len() + 1);
  r.s = s * b.s;
for (int i=0; i<len(); i++) {
    for (int j=0; j<b.len(); j++) {</pre>
      r.v[i+j] += v[i] * b.v[j];
      if(r.v[i+j] >= BIGMOD) {
        r.v[i+j+1] += r.v[i+j] / BIGMOD;
        r.v[i+j] = BIGMOD;
      }
   }
  }
  r.n();
  return r;
Bigint operator / (const Bigint &b) {
```

```
Bigint r
    r.resize(max(1, len()-b.len()+1));
    int oriS = s;
    Bigint b2 = b; // b2 = abs(b)
    s = b2.s = r.s = 1;
    for (int i=r.len()-1; i>=0; i--) {
      int d=0, u=BIGMOD-1;
      while(d<u) {</pre>
        int m = (d+u+1)>>1;
        r.v[i] = m;
        if((r*b2) > (*this)) u = m-1;
        else d = m;
      r.v[i] = d;
    }
    s = oriS;
    r.s = s * b.s;
    r.n();
    return r;
  Bigint operator % (const Bigint &b) {
    return (*this)-(*this)/b*b;
  }
};
```

2.2 unordered_map

```
struct Key {
  int first, second;
  Key () {}
  Key (int _x, int _y) : first(_x), second(_y) {}
bool operator == (const Key &b) const {
    return tie(F,S) == tie(b.F,b.S);
  }
};
struct KeyHasher {
 size_t operator()(const Key& k) const {
    return k.first + k.second*100000;
};
typedef unordered_map < Key, int, KeyHasher > map_t;
int main(int argc, char** argv){
  map_t mp;
  for (int i=0; i<10; i++)</pre>
    mp[Key(i,0)] = i+1;
  for (int i=0; i<10; i++)</pre>
    printf("%d\n", mp[Key(i,0)]);
  return 0;
}
```

2.3 extc_heap

```
#include <bits/extc++.h>
typedef __gnu_pbds::priority_queue<int> heap_t;
heap_t a,b;
int main() {
  a.clear();
  b.clear();
  a.push(1);
 a.push(3);
 b.push(2);
 b.push(4);
 assert(a.top() == 3);
  assert(b.top() == 4);
 // merge two heap
 a.ioin(b):
 assert(a.top() == 4);
  assert(b.empty());
  return 0;
}
```

2.4 extc_balance_tree

```
#include <ext/pb_ds/assoc_container.hpp>
using namespace std;
using namespace __gnu_pbds;
typedef tree<int,null_type,less<int>,rb_tree_tag,
    tree_order_statistics_node_update> set_t;
#include <ext/pb_ds/assoc_container.hpp>
using namespace __gnu_pbds;
typedef cc_hash_table<int,int> umap_t;
int main()
{
  // Insert some entries into s.
  set_t s;
  s.insert(12);
  s.insert(505);
  // The order of the keys should be: 12, 505.
  assert(*s.find_by_order(0) == 12);
  assert(s.find_by_order(2) == end(s));
  // The order of the keys should be: 12, 505.
  assert(s.order_of_key(12) == 0);
  assert(s.order_of_key(505) == 1);
  // Erase an entry.
  s.erase(12);
  // The order of the keys should be: 505.
  assert(*s.find_by_order(0) == 505);
  // The order of the keys should be: 505.
  assert(s.order_of_key(505) == 0);
```

2.5 Disjoint Set

```
struct DisjointSet {
  // save() is like recursive
  // undo() is like return
  int n, fa[MXN], sz[MXN];
  vector<pair<int*,int>> h;
  vector<int> sp;
  void init(int tn) {
    n=tn;
    for (int i=0; i<n; i++) {</pre>
      fa[i]=i:
      sz[i]=1;
    sp.clear(); h.clear();
  void assign(int *k, int v) {
    h.PB({k, *k});
  void save() { sp.PB(SZ(h)); }
  void undo() {
    assert(!sp.empty());
    int last=sp.back(); sp.pop_back();
    while (SZ(h)!=last) {
      auto x=h.back(); h.pop_back();
      *x.F=x.S;
    }
  int f(int x) {
    while (fa[x]!=x) x=fa[x];
    return x;
  void uni(int x, int y) {
    x=f(x); y=f(y);
    if (x==y) return ;
    if (sz[x]<sz[y]) swap(x, y);</pre>
    assign(&sz[x], sz[x]+sz[y]);
    assign(&fa[y], x);
}djs;
```

2.6 Treap

```
const int MEM = 16000004;
struct Treap {
   static Treap nil, mem[MEM], *pmem;
```

```
Treap *l, *r;
                                                                 return 0:
  char val:
  int size;
  Treap () : l(&nil), r(&nil), size(0) {}
  Treap (char _val) :
    l(&nil), r(&nil), val(_val), size(1) {}
                                                                    Heavy Light Decomposition
} Treap::nil, Treap::mem[MEM], *Treap::pmem = Treap::
    mem·
                                                               // only one segment tree / 0-base
int size(const Treap *t) { return t->size; }
                                                               // should call init after input N
void pull(Treap *t) {
  if (!size(t)) return;
                                                               // getPathSeg return the segment in order u->v
                                                               // fa[root] = root
  t \rightarrow size = size(t \rightarrow l) + size(t \rightarrow r) + 1;
                                                               typedef pair<int,int> pii;
Treap* merge(Treap *a, Treap *b) {
  if (!size(a)) return b;
                                                               int N,fa[MXN],belong[MXN],dep[MXN],sz[MXN],que[MXN];
  if (!size(b)) return a;
                                                               int step,line[MXN],stPt[MXN],edPt[MXN];
  Treap *t;
                                                               vector<int> E[MXN], chain[MXN];
  if (rand() % (size(a) + size(b)) < size(a)) {
    t = new (Treap::pmem++) Treap(*a);
                                                               void init() {
    t - r = merge(a - r, b);
                                                                 REP(i,N) {
  } else {
                                                                   E[i].clear();
    t = new (Treap::pmem++) Treap(*b);
                                                                   chain[i].clear();
    t \rightarrow l = merge(a, b \rightarrow l);
                                                                 }
  pull(t);
                                                               void DFS(int u){
  return t;
                                                                 vector<int> &c = chain[belong[u]];
                                                                 for (int i=c.size()-1; i>=0; i--){
void split(Treap *t, int k, Treap *&a, Treap *&b) {
                                                                   int v = c[i];
  if (!size(t)) a = b = &Treap::nil;
                                                                   stPt[v] = step;
  else if (size(t->l) + 1 <= k) {
                                                                   line[step++] = v;
    a = new (Treap::pmem++) Treap(*t);
    split(t->r, k - size(t->l) - 1, a->r, b);
                                                                 for (int i=0; i<(int)c.size(); i++){</pre>
    pull(a);
                                                                   u = c[i];
  } else {
                                                                   for (auto v : E[u]){
    b = new (Treap::pmem++) Treap(*t);
                                                                     if (fa[u] == v || (i && v == c[i-1])) continue;
    split(t->l, k, a, b->l);
                                                                     DFS(v);
    pull(b);
                                                                   edPt[u] = step-1;
}
                                                                 }
int nv:
                                                               void build_chain(int st){
Treap *rt[50005];
                                                                 int fr,bk;
                                                                 fr=bk=0; que[bk++]=st; fa[st]=st; dep[st]=0;
void print(const Treap *t) {
                                                                 while (fr < bk){</pre>
  if (!size(t)) return;
                                                                   int u=que[fr++];
  print(t->l):
                                                                   for (auto v : E[u]){
  cout << t->val:
                                                                     if (v == fa[u]) continue;
  print(t->r);
                                                                     que[bk++] = v;
                                                                     dep[v] = dep[u]+1;
                                                                     fa[v] = u;
int main(int argc, char** argv) {
                                                                   }
  IOS:
  rt[nv=0] = &Treap::nil;
                                                                 for (int i=bk-1,u,pos; i>=0; i--){
  Treap::pmem = Treap::mem;
                                                                   u = que[i]; sz[u] = 1; pos = -1;
for (auto v : E[u]){
  int Q, cmd, p, c, v;
  string s;
                                                                     if (v == fa[u]) continue;
  cin >> 0:
                                                                     sz[u] += sz[v];
  while (Q--) {
                                                                     if (pos==-1 || sz[v]>sz[pos]) pos=v;
    cin >> cmd;
    if (cmd == 1) {
                                                                   if (pos == -1) belong[u] = u;
      // insert string s after position p
                                                                   else belong[u] = belong[pos];
      cin >> p >> s;
Treap *tl, *tr;
                                                                   chain[belong[u]].PB(u);
      split(rt[nv], p, tl, tr);
for (int i=0; i<SZ(s); i++)</pre>
                                                                 step = 0:
                                                                 DFS(st);
        tl = merge(tl, new (Treap::pmem++) Treap(s[i]))
                                                               int getLCA(int u, int v){
      rt[++nv] = merge(tl, tr);
                                                                 while (belong[u] != belong[v]){
    } else if (cmd == 2) {
                                                                   int a = chain[belong[u]].back();
      // remove c characters starting at position
                                                                   int b = chain[belong[v]].back();
      Treap *tl, *tm, *tr;
                                                                   if (dep[a] > dep[b]) u = fa[a];
      cin >> p >> c;
                                                                   else v = fa[b];
      split(rt[nv], p-1, tl, tm);
      split(tm, c, tm, tr);
                                                                 return sz[u] >= sz[v] ? u : v;
      rt[++nv] = merge(tl, tr);
    } else if (cmd == 3) {
                                                               vector<pii> getPathSeg(int u, int v){
      // print c characters starting at position p, in
                                                                 vector<pii> ret1,ret2;
           version v
                                                                 while (belong[u] != belong[v]){
      Treap *tl, *tm, *tr;
                                                                   int a = chain[belong[u]].back();
int b = chain[belong[v]].back();
      cin >> v >> p >> c;
      split(rt[v], p-1, tl, tm);
                                                                   if (dep[a] > dep[b]){
      split(tm, c, tm, tr);
                                                                     ret1.PB({stPt[a],stPt[u]});
      print(tm);
                                                                     u = fa[a];
      cout << "\n";
                                                                   } else {
                                                                     ret2.PB({stPt[b],stPt[v]});
```

```
v = fa[b]:
   }
  if (dep[u] > dep[v]) swap(u,v);
  ret1.PB({stPt[u],stPt[v]});
  reverse(ret2.begin(), ret2.end());
  ret1.insert(ret1.end(),ret2.begin(),ret2.end());
  return ret1;
// Usage
void build(){
  build_chain(0); //change root
  init(0,step,0); //init segment tree
int get_answer(int u, int v){
  int ret = -2147483647;
  vector<pii> vec = getPathSeg(u,v);
  for (auto it : vec)
    ; // check answer with segment [it.F, it.S]
  return ret:
```

2.8 Link-Cut Tree

```
const int MXN = 100005;
const int MEM = 100005:
struct Splay {
  static Splay nil, mem[MEM], *pmem;
  Splay *ch[2], *f;
 int val, rev, size;
Splay () : val(-1), rev(0), size(0) {
    f = ch[0] = ch[1] = &nil;
  Splay (int _val) : val(_val), rev(0), size(1) {
  f = ch[0] = ch[1] = &nil;
  bool isr() {
    return f->ch[0] != this && f->ch[1] != this;
  int dir() {
   return f->ch[0] == this ? 0 : 1;
  void setCh(Splay *c, int d) {
    ch[d] = c;
    if (c != &nil) c->f = this;
    pull();
  void push() {
    if (rev) {
      swap(ch[0], ch[1]);
if (ch[0] != &nil) ch[0]->rev ^= 1;
      if (ch[1] != &nil) ch[1]->rev ^= 1;
      rev=0;
   }
  void pull() {
    size = ch[0] -> size + ch[1] -> size + 1;
    if (ch[0] != &nil) ch[0]->f = this;
    if (ch[1] != &nil) ch[1]->f = this;
} Splay::nil, Splay::mem[MEM], *Splay::pmem = Splay::
    mem;
Splay *nil = &Splay::nil;
void rotate(Splay *x) {
 Splay *p = x - > f;
  int d = x->dir();
  if (!p->isr()) p->f->setCh(x, p->dir());
  else x->f = p->f;
 p->setCh(x->ch[!d], d);
 x->setCh(p, !d);
  p->pull(); x->pull();
vector<Splay*> splayVec;
void splay(Splay *x) {
  splayVec.clear();
  for (Splay *q=x;; q=q->f) {
    splayVec.push_back(q);
    if (q->isr()) break;
  reverse(begin(splayVec), end(splayVec));
  for (auto it : splayVec) it->push();
```

```
while (!x->isr()) {
    if (x->f->isr()) rotate(x);
     else if (x->dir()==x->f->dir()) rotate(x->f),rotate
        (x);
    else rotate(x),rotate(x);
  }
}
Splay* access(Splay *x) {
  Splay *q = nil;
  for (;x!=nil;x=x->f) {
    splay(x);
    x->setCh(q, 1);
    q = x;
  }
  return q;
}
void evert(Splay *x) {
  access(x);
  splay(x);
  x->rev ^= 1;
  x->push(); x->pull();
}
void link(Splay *x, Splay *y) {
// evert(x);
  access(x);
  splay(x);
  evert(y);
  x->setCh(y, 1);
void cut(Splay *x, Splay *y) {
// evert(x);
  access(y);
  splay(y);
  y->push();
  y - > ch[0] = y - > ch[0] - > f = nil;
int N, Q;
Splay *vt[MXN];
int ask(Splay *x, Splay *y) {
  access(x):
  access(y);
  splay(x);
  int res = x->f->val;
  if (res == -1) res=x->val;
  return res:
int main(int argc, char** argv) {
  scanf("%d%d", &N, &Q);
for (int i=1; i<=N; i++)
    vt[i] = new (Splay::pmem++) Splay(i);
  while (Q--) {
     char cmd[105];
    int u, v;
scanf("%s", cmd);
    if (cmd[1] == 'i') {
  scanf("%d%d", &u, &v);
       link(vt[v], vt[u]);
    } else if (cmd[0] == 'c') {
       scanf("%d", &v);
       cut(vt[1], vt[v]);
     } else {
       scanf("%d%d", &u, &v);
       int res=ask(vt[u], vt[v]);
       printf("%d\n", res);
  }
  return 0;
}
```

3 Graph

3.1 BCC Edge

```
struct BccEdge {
   static const int MXN = 100005;
   struct Edge { int v,eid; };
   int n,m,step,par[MXN],dfn[MXN],low[MXN];
   vector<Edge> E[MXN];
```

```
DisjointSet djs;
  void init(int _n) {
  n = _n;  m = 0;
  for (int i=0; i<n; i++) E[i].clear();</pre>
     djs.init(n);
  void add_edge(int u, int v) {
    E[u].PB({v, m});
     E[v].PB({u, m});
    m++;
  void DFS(int u, int f, int f_eid) {
     par[u] = f;
dfn[u] = low[u] = step++;
     for (auto it:E[u]) {
       if (it.eid == f_eid) continue;
       int v = it.v;
       if (dfn[v] == -1) {
         DFS(v, u, it.eid);
low[u] = min(low[u], low[v]);
       } else {
         low[u] = min(low[u], dfn[v]);
       }
    }
  void solve() {
     step = 0;
     memset(dfn, -1, sizeof(int)*n);
     for (int i=0; i<n; i++)</pre>
       if (dfn[i] == -1) DFS(i, i, -1);
     djs.init(n);
     for (int i=0; i<n; i++) {</pre>
       if (low[i] < dfn[i]) djs.uni(i, par[i]);</pre>
}graph;
```

3.2 BCC Vertex

```
struct BccVertex {
  int n,nBcc,step,root,dfn[MXN],low[MXN];
  vector<int> E[MXN], ap;
  vector<pii> bcc[MXN];
  int top;
  pii stk[MXN];
  void init(int _n) {
   n = _n;
    nBcc = step = 0;
    for (int i=0; i<n; i++) E[i].clear();</pre>
  void add_edge(int u, int v) {
    E[u].PB(v);
    E[v].PB(u);
  void DFS(int u, int f) {
    dfn[u] = low[u] = step++;
    int son = 0;
    for (auto v:E[u]) {
      if (v == f) continue;
      if (dfn[v] == -1) {
        son++:
        stk[top++] = \{u,v\};
        DFS(v,u);
        if (low[v] >= dfn[u]) {
          if(v != root) ap.PB(v);
          do {
            assert(top > 0);
            bcc[nBcc].PB(stk[--top]);
          } while (stk[top] != pii(u,v));
          nBcc++;
        low[u] = min(low[u], low[v]);
        if (dfn[v] < dfn[u]) stk[top++] = pii(u,v);</pre>
        low[u] = min(low[u],dfn[v]);
      }
    if (u == root \&\& son > 1) ap.PB(u);
  // return the edges of each bcc;
  vector<vector<pii>>> solve() {
    vector<vector<pii>>> res;
    for (int i=0; i<n; i++) {</pre>
```

```
dfn[i] = low[i] = -1;
}
ap.clear();
for (int i=0; i<n; i++) {
   if (dfn[i] == -1) {
      top = 0;
      root = i;
      DFS(i,i);
   }
} REP(i,nBcc) res.PB(bcc[i]);
return res;
}
}graph;</pre>
```

3.3 Strongly Connected Components

```
struct Scc{
   int n, nScc, vst[MXN], bln[MXN];
vector<int> E[MXN], rE[MXN], vec;
   void init(int _n){
     n = _n;
for (int i=0; i<n; i++){</pre>
       E[i].clear();
        rE[i].clear();
     }
   void add_edge(int u, int v){
      E[u].PB(v);
     rE[v].PB(u);
   void DFS(int u){
      vst[u]=1;
      for (auto v : E[u])
        if (!vst[v]) DFS(v);
      vec.PB(u);
   void rDFS(int u){
      vst[u] = 1;
      bln[u] = nScc;
      for (auto v : rE[u])
        if (!vst[v]) rDFS(v);
   void solve(){
     nScc = 0;
      vec.clear();
     for (int i=0; i<n; i++) vst[i] = 0;
for (int i=0; i<n; i++)</pre>
        if (!vst[i]) DFS(i);
      reverse(vec.begin(),vec.end());
     for (int i=0; i<n; i++) vst[i] = 0;
for (auto v : vec){</pre>
        if (!vst[v]){
          rDFS(v);
          nScc++;
        }
     }
   }
};
```

3.4 DMST with sol

```
const int INF = 1029384756;

struct edge_t{
   int u,v,w;
   set < pair < int, int > > add, sub;
   edge_t() : u(-1), v(-1), w(0) {}
   edge_t(int _u, int _v, int _w) {
      u = _u; v = _v; w = _w;
      add.insert({u, v});
}

edge_t& operator += (const edge_t& obj) {
   w += obj.w;
   FOR (it, obj.add) {
      if (!sub.count(*it)) add.insert(*it);
      else sub.erase(*it);
   }

FOR (it, obj.sub) {
      if (!add.count(*it)) sub.insert(*it);
      else add.erase(*it);
   }
}
```

```
return *this;
                                                              struct DominatorTree{
                                                                static const int MAXN = 200010;
  edge_t& operator -= (const edge_t& obj) {
                                                                int n,s;
                                                                vector<int> g[MAXN],pred[MAXN];
    w -= obj.w;
                                                                vector<int> cov[MAXN];
     FOR (it, obj.sub) {
                                                                int dfn[MAXN],nfd[MAXN],ts;
      if (!sub.count(*it)) add.insert(*it);
      else sub.erase(*it);
                                                                int par[MAXN];
                                                                int sdom[MAXN],idom[MAXN];
     for (auto it : obj.add) {
                                                                int mom[MAXN],mn[MAXN];
       if (!add.count(it)) sub.insert(it);
      else add.erase(it);
                                                                inline bool cmp(int u,int v) { return dfn[u] < dfn[v</pre>
                                                                     ]; }
    return *this;
                                                                int eval(int u) {
}eg[MXN*MXN],prv[MXN],EDGE_INF(-1,-1,INF);
                                                                  if(mom[u] == u) return u;
int N,M;
                                                                   int res = eval(mom[u]);
int cid,incyc[MXN],contracted[MXN];
                                                                  if(cmp(sdom[mn[mom[u]]],sdom[mn[u]]))
vector<int> E[MXN];
                                                                    mn[u] = mn[mom[u]];
                                                                   return mom[u] = res;
edge_t dmst(int rt){
  edge_t cost;
  for (int i=0; i<N; i++){
  contracted[i] = incyc[i] = 0;</pre>
                                                                void init(int _n, int _s) {
                                                                  n = _n;
    prv[i] = EDGE_INF;
                                                                   REP1(i,1,n) {
  cid = 0;
                                                                    g[i].clear();
                                                                    pred[i].clear();
  int u,v;
  while (true){
                                                                    idom[i] = 0;
    for (v=0; v<N; v++){</pre>
       if (v != rt && !contracted[v] && prv[v].w == INF)
                                                                void add_edge(int u, int v) {
                                                                  g[u].push_back(v);
    if (v >= N) break; // end
for (int i=0; i<M; i++){</pre>
                                                                  pred[v].push_back(u);
                                                                void DFS(int u) {
       if (eg[i].v == v && eg[i].w < prv[v].w)
         prv[v] = eg[i];
                                                                   ts++:
                                                                  dfn[u] = ts;
     if (prv[v].w == INF) // not connected
                                                                  nfd[ts] = u;
       return EDGE_INF;
                                                                   for(int v:g[u]) if(dfn[v] == 0) {
     cost += prv[v];
                                                                    par[v] = u;
     for (u=prv[v].u; u!=v && u!=-1; u=prv[u].u);
                                                                     DFS(v);
     if (u == -1) continue;
                                                                  }
     incyc[v] = ++cid;
                                                                }
     for (u=prv[v].u; u!=v; u=prv[u].u){
                                                                void build() {
       contracted[u] = 1;
                                                                  ts = 0;
       incyc[u] = cid;
                                                                   REP1(i,1,n) {
                                                                    dfn[i] = nfd[i] = 0;
     for (int i=0; i<M; i++){</pre>
                                                                    cov[i].clear();
      if (incyc[eg[i].u] != cid && incyc[eg[i].v] ==
                                                                    mom[i] = mn[i] = sdom[i] = i;
           cid){
         eg[i] -= prv[eg[i].v];
                                                                  DFS(s);
                                                                  for (int i=ts; i>=2; i--) {
                                                                    int u = nfd[i];
     for (int i=0; i<M; i++){</pre>
                                                                     if(u == 0) continue ;
       if (incyc[eg[i].u] == cid) eg[i].u = v;
                                                                     for(int v:pred[u]) if(dfn[v]) {
       if (incyc[eg[i].v] == cid) eg[i].v = v;
                                                                       eval(v);
                                                                       if(cmp(sdom[mn[v]],sdom[u])) sdom[u] = sdom[mn[
       if (eg[i].u == eg[i].v) eg[i--] = eg[--M];
                                                                           v11;
     for (int i=0; i<N; i++){</pre>
       if (contracted[i]) continue;
                                                                    cov[sdom[u]].push_back(u);
       if (prv[i].u>=0 && incyc[prv[i].u] == cid)
                                                                     mom[u] = par[u];
                                                                     for(int w:cov[par[u]]) {
         prv[i].u = v;
                                                                       eval(w);
    prv[v] = EDGE_INF;
                                                                       if(cmp(sdom[mn[w]],par[u])) idom[w] = mn[w];
  }
                                                                       else idom[w] = par[u];
  return cost;
                                                                    cov[par[u]].clear();
                                                                  REP1(i,2,ts) {
void solve(){
  edge_t cost = dmst(0);
                                                                    int u = nfd[i];
                                                                     if(u == 0) continue
  for (auto it : cost.add){ // find a solution
    E[it.F].PB(it.S);
                                                                     if(idom[u] != sdom[u]) idom[u] = idom[idom[u]];
    prv[it.S] = edge_t(it.F,it.S,0);
                                                                }
| }
                                                              }dom;
```

3.5 Dominator Tree

```
// idom[n] is the unique node that strictly dominates n
    but does
// not strictly dominate any other node that strictly
    dominates n.
// idom[n] = 0 if n is entry or the entry cannot reach
    n.
```

3.6 Maximum Clique

```
class MaxClique {
public:
    static const int MV = 210;
    int V;
    int el[MV][MV/30+1];
```

```
int dp[MV];
    int ans:
    int s[MV][MV/30+1];
    vector<int> sol;
    void init(int v) {
        V = v; ans = 0;
        FZ(el); FZ(dp);
     /* Zero Base */
    void addEdge(int u, int v) {
        if(u > v) swap(u, v);
        if(u == v) return;
         el[u][v/32] |= (1<<(v%32));
    }
    bool dfs(int v, int k) {
        int c = 0, d = 0;
         for(int i=0; i<(V+31)/32; i++) {</pre>
             s[k][i] = el[v][i];
             if(k != 1) s[k][i] &= s[k-1][i];
             c += __builtin_popcount(s[k][i]);
         if(c == 0) {
             if(k > ans) {
                 ans = k;
                 sol.clear();
                 sol.push_back(v);
                 return 1;
             return 0;
         for(int i=0; i<(V+31)/32; i++) {</pre>
             for(int a = s[k][i]; a ; d++) {
                 if(k + (c-d) <= ans) return 0;</pre>
                 int lb = a&(-a), lg = 0;
                 a ^= lb:
                 while(lb!=1) {
                      lb = (unsigned int)(lb) >> 1;
                 int u = i*32 + lg;
if(k + dp[u] <= ans) return 0;</pre>
                 if(dfs(u, k+1)) {
                      sol.push_back(v);
                      return 1:
                 }
             }
        return 0;
    }
    int solve() {
         for(int i=V-1; i>=0; i--) {
             dfs(i, 1);
             dp[i] = ans;
         return ans;
    }
};
```

3.7 MinimumMeanCycle

```
/* minimum mean cycle */
const int MAXE = 1805;
const int MAXN = 35;
const double inf = 1029384756;
const double eps = 1e-6;
struct Edge {
  int v,u;
  double c;
int n,m,prv[MAXN][MAXN], prve[MAXN][MAXN], vst[MAXN];
Edge e[MAXE];
vector<int> edgeID, cycle, rho;
double d[MAXN][MAXN];
inline void bellman_ford() {
   for(int i=0; i<n; i++) d[0][i]=0;</pre>
  for(int i=0; i<n; i++) {</pre>
     fill(d[i+1], d[i+1]+n, inf);
    for(int j=0; j<m; j++) {
  int v = e[j].v, u = e[j].u;
  if(d[i][v]<inf && d[i+1][u]>d[i][v]+e[j].c) {
```

```
d[i+1][u] = d[i][v]+e[j].c;
        prv[i+1][u] = v;
        prve[i+1][u] = j;
   }
 }
double karp_mmc() {
  // returns inf if no cycle, mmc otherwise
  double mmc=inf;
  int st = -1;
  bellman_ford();
  for(int i=0; i<n; i++) {</pre>
    double avg=-inf;
    for(int k=0; k<n; k++) {</pre>
      \dot{tf}(d[n][i] < inf-eps) avg=max(avg,(d[n][i]-d[k][i])
          /(n-k));
      else avg=max(avg,inf);
    if (avg < mmc) tie(mmc, st) = tie(avg, i);</pre>
  for(int i=0; i<n; i++) vst[i] = 0;</pre>
  edgeID.clear(); cycle.clear(); rho.clear();
  for (int i=n; !vst[st]; st=prv[i--][st]) {
    vst[st]++;
    edgeID.PB(prve[i][st]);
    rho.PB(st);
  while (vst[st] != 2) {
    int v = rho.back(); rho.pop_back();
    cycle.PB(v);
    vst[v]++;
  }
  reverse(ALL(edgeID));
  edgeID.resize(SZ(cycle));
  return mmc;
```

4 Flow

4.1 Dinic

struct Dinic{

```
static const int MXN = 10000;
struct Edge{ int v,f,re; };
int n,s,t,level[MXN];
vector < Edge > E[MXN];
void init(int _n, int _s, int _t){
    n = _n;    s = _s;    t = _t;
    for (int i=0; i<n; i++) E[i].clear();</pre>
void add_edge(int u, int v, int f){
  E[u].PB({v,f,SZ(E[v])});
  E[v].PB({u,0,SZ(E[u])-1});
bool BFS(){
  for (int i=0; i<n; i++) level[i] = -1;</pre>
  queue<int> que;
  que.push(s);
  level[s] = 0;
  while (!que.empty()){
     int u = que.front(); que.pop();
     for (auto it : E[u]){
       if (it.f > 0 && level[it.v] == -1){
         level[it.v] = level[u]+1;
         que.push(it.v);
    }
  return level[t] != -1;
int DFS(int u, int nf){
  if (u == t) return nf;
  int res = 0;
  for (auto &it : E[u]){
    if (it.f > 0 && level[it.v] == level[u]+1){
       int tf = DFS(it.v, min(nf,it.f));
       res += tf; nf -= tf; it.f -= tf;
       E[it.v][it.re].f += tf;
       if (nf == 0) return res;
    }
```

```
if (!res) level[u] = -1;
    return res;
                                                                    void init(int _n){
  int flow(int res=0){
                                                                     n = _n;
for (int i=0; i<n; i++)
    for (int j=0; j<n; j++)</pre>
    while ( BFS() )
      res += DFS(s,2147483647);
    return res;
                                                                          edge[i][j] = 0;
}flow;
                                                                      edge[x][y] = w;
4.2 Cost Flow
                                                                   bool DFS(int x){
                                                                      vx[x] = 1;
                                                                      for (int y=0; y<n; y++){</pre>
typedef pair<long long, long long> pll;
                                                                        if (vy[y]) continue;
struct CostFlow {
  static const int MXN = 205;
static const long long INF = 102938475610293847LL;
  struct Edge {
                                                                        } else {
    int v, r;
                                                                          vy[y] = 1;
    long long f, c;
  };
                                                                            match[y] = x;
  int n, s, t, prv[MXN], prvL[MXN], inq[MXN];
  long long dis[MXN], fl, cost;
                                                                             return true;
  vector<Edge> E[MXN];
                                                                       }
  void init(int _n, int _s, int _t) {
    n = _n; s = _s; t = _t;
for (int i=0; i<n; i++) E[i].clear();</pre>
                                                                      }
                                                                      return false;
    fl = cost = 0;
                                                                    int solve(){
                                                                      fill(match, match+n, -1);
  void add_edge(int u, int v, long long f, long long c)
                                                                      fill(lx,lx+n,-INF);
                                                                      fill(ly,ly+n,0);
    E[u].PB({v, SZ(E[v])
                             , f, c});
                                                                      for (int i=0; i<n; i++)</pre>
    E[v].PB({u, SZ(E[u])-1, 0, -c});
                                                                        for (int j=0; j<n; j++)</pre>
  pll flow() {
                                                                      for (int i=0; i<n; i++){</pre>
    while (true) {
      for (int i=0; i<n; i++) {
   dis[i] = INF;</pre>
                                                                        fill(slack,slack+n,INF);
                                                                        while (true){
                                                                          fill(vx,vx+n,0);
         inq[i] = 0;
                                                                          fill(vy,vy+n,0);
                                                                          if ( DFS(i) ) break;
       dis[s] = 0;
       queue<int> que;
       que.push(s);
       while (!que.empty()) {
         int u = que.front(); que.pop();
         inq[u] = 0;
         for (int i=0; i<SZ(E[u]); i++) {</pre>
                                                                            else slack[j] -= d;
           int v = E[u][i].v;
           long long w = E[u][i].c;
                                                                       }
           if (E[u][i].f > 0 && dis[v] > dis[u] + w) {
             prv[v] = u; prvL[v] = i;
             dis[v] = dis[u] + w;
                                                                      int res=0;
                                                                      for (int i=0; i<n; i++)</pre>
             if (!inq[v]) {
                                                                        res += edge[match[i]][i];
               inq[v] = 1;
                                                                      return res;
                que.push(v);
                                                                   }
             }
                                                                 }graph;
           }
         }
       if (dis[t] == INF) break;
                                                                 4.4 SW-Mincut
       long long tf = INF;
       for (int v=t, u, l; v!=s; v=u) {
         u=prv[v]; l=prvL[v];
                                                                 struct SW{ // O(V^3) 0-base
         tf = min(tf, E[u][l].f);
                                                                   static const int MXN = 514;
                                                                   int n,vst[MXN],del[MXN];
       for (int v=t, u, l; v!=s; v=u) {
                                                                    int edge[MXN][MXN],wei[MXN];
         u=prv[v]; l=prvL[v];
E[u][l].f -= tf;
                                                                   void init(int _n){
                                                                      n = _n;
         E[v][E[u][l].r].f += tf;
                                                                      for (int i=0; i<n; i++) {</pre>
                                                                        for (int j=0; j<n; j++)</pre>
       cost += tf * dis[t];
                                                                          edge[i][j] = 0;
      fl += tf;
                                                                        del[i] = 0;
    }
                                                                      }
    return {fl, cost};
}flow;
                                                                      edge[u][v] += w;
                                                                      edge[v][u] += w;
```

4.3 Kuhn Munkres

```
struct KM{
// Maximum Bipartite Weighted Matching (Perfect Match)
 static const int MXN = 650;
 static const int INF = 2147483647; // long long
```

```
int n,match[MXN],vx[MXN],vy[MXN];
int edge[MXN][MXN],lx[MXN],ly[MXN],slack[MXN];
// ^^^ long long
void add_edge(int x, int y, int w){ // long long
     if (lx[x]+ly[y] > edge[x][y]){
        slack[y] = min(slack[y], lx[x]+ly[y]-edge[x][y]
        if (match[y] == -1 || DFS(match[y])){
       lx[i] = max(lx[i], edge[i][j]);
        int d = INF; // long long
        for (int j=0; j<n; j++)</pre>
          if (!vy[j]) d = min(d, slack[j]);
        for (int j=0; j<n; j++){
  if (vx[j]) lx[j] -= d;</pre>
          if (vy[j]) ly[j] += d;
void add_edge(int u, int v, int w){
void search(int &s, int &t){
   for (int i=0; i<n; i++)</pre>
```

vst[i] = wei[i] = 0;

for (int i=0; i<n; i++)</pre>

int mx=-1, cur=0;

s = t = -1:

while (true){

```
if (!del[i] && !vst[i] && mx<wei[i])</pre>
           cur = i, mx = wei[i];
      if (mx == -1) break;
      vst[cur] = 1;
      s = t;
      t = cur;
      for (int i=0; i<n; i++)</pre>
        if (!vst[i] && !del[i]) wei[i] += edge[cur][i];
  int solve(){
    int res = 2147483647;
    for (int i=0,x,y; i<n-1; i++){</pre>
      search(x,y);
      res = min(res,wei[y]);
      del[y] = 1;
      for (int j=0; j<n; j++)</pre>
        edge[x][j] = (edge[j][x] += edge[y][j]);
    return res:
  }
}graph;
```

4.5 Maximum Simple Graph Matching

```
struct GenMatch { // 1-base
  static const int MAXN = 514;
  int V;
  bool el[MAXN][MAXN];
  int pr[MAXN];
  bool inq[MAXN],inp[MAXN],inb[MAXN];
  queue < int > qe;
  int st,ed;
  int nb;
  int bk[MAXN],djs[MAXN];
  int ans:
  void init(int _V) {
    V = V;
    for(int i = 0; i <= V; i++) {</pre>
      for(int j = 0; j <= V; j++) el[i][j] = 0;
pr[i] = bk[i] = djs[i] = 0;</pre>
      inq[i] = inp[i] = inb[i] = 0;
    }
    ans = 0;
  void add_edge(int u, int v) {
    el[u][v] = el[v][u] = 1;
  int lca(int u,int v) {
    for(int i = 0; i <= V; i++) inp[i] = 0;</pre>
    while(1) {
      u = djs[u];
      inp[u] = true;
      if(u == st) break;
      u = bk[pr[u]];
    while(1) {
      v = djs[v];
      if(inp[v]) return v;
      v = bk[pr[v]];
    }
    return v;
  }
  void upd(int u) {
    while(djs[u] != nb) {
      v = pr[u];
      inb[djs[u]] = inb[djs[v]] = true;
      u = bk[v];
      if(djs[u] != nb) bk[u] = v;
    }
  }
  void blo(int u,int v) {
    nb = lca(u,v);
    for (int i=0; i<=V; i++) inb[i] = 0;</pre>
    upd(u); upd(v);
    if(djs[u] != nb) bk[u] = v;
    if(djs[v] != nb) bk[v] = u;
    for(int tu = 1; tu <= V; tu++)</pre>
      if(inb[djs[tu]]) {
         djs[tu] = nb;
         if(!ing[tu]){
           qe.push(tu);
           inq[tu] = 1;
```

```
}
  void flow() {
    for(int i = 1; i <= V; i++) {</pre>
      inq[i] = 0;
      bk[i] = 0;
      djs[i] = i;
    while(qe.size()) qe.pop();
    qe.push(st);
     inq[st] = 1;
    ed = 0:
     while(qe.size()) {
      int u = qe.front(); qe.pop();
       for(int v = 1; v <= V; v++)</pre>
         if(el[u][v] && (djs[u] != djs[v]) && (pr[u] !=
             v)) {
           if((v == st) || ((pr[v] > 0) && bk[pr[v]] >
               0))
             blo(u,v);
           else if(bk[v] == 0) {
             bk[v] = u;
             if(pr[v] > 0) {
               if(!inq[pr[v]]) qe.push(pr[v]);
             } else {
               ed = v;
               return;
           }
         }
    }
  }
  void aug() {
    int u,v,w;
    u = ed;
    while(u > 0) {
      v = bk[u];
      w = pr[v];
      pr[v] = u;
      pr[u] = v;
      u = w;
    }
  int solve() {
    for(int i = 0; i <= V; i++) pr[i] = 0;</pre>
    for(int u = 1; u <= V; u++)</pre>
      if(pr[u] == 0) {
         st = u;
         flow();
         if(ed > 0) {
           aug();
           ans ++;
        }
    return ans;
  }
}G;
int main() {
  G.init(V);
  for(int i=0; i<E; i++) {</pre>
    int u, v;
    cin >> u >> v;
    G.add_edge(u, v);
  cout << G.solve() << endl;</pre>
}
      Minimum Weight Matching (Clique version)
```

}

```
for (int j=0; j<n; j++)
  edge[i][j] = 0;</pre>
                                                                        }
  void add_edge(int u, int v, int w) {
    edge[u][v] = edge[v][u] = w;
  bool SPFA(int u){
    if (onstk[u]) return true;
     stk.PB(u);
     onstk[u] = 1;
    for (int v=0; v<n; v++){
  if (u != v && match[u] != v && !onstk[v]){</pre>
         int m = match[v];
         if (dis[m] > dis[u] - edge[v][m] + edge[u][v]){
            dis[m] = dis[u] - edge[v][m] + edge[u][v];
           onstk[v] = 1;
           stk.PB(v):
           if (SPFA(m)) return true;
           stk.pop_back();
           onstk[v] = 0;
         }
      }
    }
    onstk[u] = 0;
    stk.pop_back();
    return false;
  int solve() {
     // find a match
    for (int i=0; i<n; i+=2){
  match[i] = i+1;</pre>
       match[i+1] = i;
    while (true){
  int found = 0;
       for (int i=0; i<n; i++)</pre>
         dis[i] = onstk[i] = 0;
       for (int i=0; i<n; i++){</pre>
         stk.clear();
         if (!onstk[i] && SPFA(i)){
            found = 1;
            while (SZ(stk)>=2){
              int u = stk.back(); stk.pop_back();
              int v = stk.back(); stk.pop_back();
              match[u] = v;
              match[v] = u;
           }
         }
       if (!found) break;
     int ret = 0;
     for (int i=0; i<n; i++)</pre>
       ret += edge[i][match[i]];
     ret /= 2;
    return ret;
}graph;
```

4.7 General Graph Matching (wangyenjen)

```
/// {{{ general graph matching template
#define MAXN 505
vector < int > g[MAXN];
int pa[MAXN] , match[MAXN] , st[MAXN] , S[MAXN] , vis[
    MAXN];
int n;
inline int lca(int u,int v){
    static int t=0;
    for(++t;;swap(u,v)){
        if(u==0)continue;
        if(vis[u]==t)return u;
        vis[u]=t;
        u=st[pa[match[u]]];
#define qpush(u) q.push(u),S[u]=0
inline void flower(int u,int v,int l,queue<int> &q){
    while(st[u]!=l){
        pa[u]=v
        v=match[u];
        if(S[v]==1)qpush(v);
        st[u]=st[v]=l;
```

```
inline bool agument(int u,int v){
    for(int lst;u;v=lst,u=pa[v]){
         lst=match[u];
         match[u]=v;
         match[v]=u;
inline bool bfs(int u){
    for(int i=1;i<=n;++i)st[i]=i;</pre>
    memset(S+1,-1,sizeof(int)*n);
    queue < int > q;
    qpush(u);
    while(q.size()){
         u=q.front(),q.pop();
         for(size_t i=0;i<g[u].size();++i){</pre>
             int v=g[u][i];
             if(S[v]==-1){
                 pa[v]=u;
                  S[v]=1:
                  if(!match[v]){
                      agument(u,v);
                      return true:
                 }
                  qpush(match[v]);
             }else if(!S[v]&&st[v]!=st[u]){
                 int l=lca(v,u);
                  flower(v,u,l,q);
                  flower(u,v,l,q);
         }
    return false;
inline int blossom(){
    memset(pa+1,0,sizeof(int)*n);
    memset(match+1,0,sizeof(int)*n);
    int ans=0;
    for(int i=1;i<=n;++i)</pre>
         if(!match[i]&&bfs(i))++ans;
    return ans;
int main() {
    int t;
    RI(t);
    while(t--) {
         int m;
         RI(n , m);
         REP1(i , 1 , n) g[i].clear();
REP(i , m) {
             int x , y;
             RI(x, y);
             x++ , y++;
             g[x].PB(y);
             g[y].PB(x);
         PL(blossom());
    return 0;
}
```

u=pa[v];

4.8 (+1) SW-mincut O(NM)

```
// {{{ StoerWagner
const int inf=1000000000;
// should be larger than max.possible mincut
class StoerWagner {
  public:
    int n,mc; // node id in [0,n-1]
vector<int> adj[MAXN];
    int cost[MAXN][MAXN];
    int cs[MAXN];
    bool merged[MAXN],sel[MAXN];
    // --8<-- include only if cut is explicitly needed
      DisjointSet djs;
    vector<int> cut;
    //--8<----
      StoerWagner(int _n):n(_n),mc(inf),djs(_n) {
         for(int i=0;i<n;i++)</pre>
           merged[i]=0;
         for(int i=0;i<n;i++)</pre>
           for(int j=0;j<n;j++)</pre>
```

```
cost[i][j]=cost[j][i]=0;
    void append(int v,int u,int c) {
      if(v==u) return;
      if(!cost[v][u]&&c) {
        adj[v].PB(u);
        adj[u].PB(v);
      }
      cost[v][u]+=c;
      cost[u][v]+=c;
    void merge(int v,int u) {
      merged[u]=1;
      for(int i=0;i<n;i++)</pre>
        append(v,i,cost[u][i]);
      // --8<-- include only if cut is explicitly
          needed
        djs.merge(v,u);
    void phase() {
      priority_queue < pii > pq;
      for(int v=0; v<n; v++) {</pre>
        if(merged[v]) continue;
        cs[v]=0;
        sel[v]=0;
        pq.push({0,v});
      int v,s,pv;
      while(pq.size()) {
        if(cs[pq.top().S]>pq.top().F) {
          pq.pop();
          continue;
        }
        pv=v;
        v=pq.top().S;
        s=pq.top().F;
        pq.pop();
        sel[v]=1;
        for(int i=0;i<adj[v].size();i++) {</pre>
          int u=adj[v][i];
          if(merged[u]||sel[u]) continue;
          cs[u]+=cost[v][u];
          pq.push({cs[u],u});
        }
      if(s<mc) {</pre>
        // --8<-- include only if cut is explicitly
        needed -----
          cut.clear():
        for(int i=0;i<n;i++)</pre>
          if(djs.getrep(i)==djs.getrep(v)) cut.PB(i);
      }
      merge(v,pv);
    int mincut() {
      if(mc==inf) {
        for(int t=0;t<n-1;t++)</pre>
          phase();
      return mc;
    // --8<-- include only if cut is explicitly needed
      vector<int> getcut() { // return one side of the
          cut
        mincut();
        return cut;
    //--8<-----
// }}}
```

5 Math

5.1 ax+by=gcd

```
typedef pair<int, int> pii;
```

```
pii gcd(int a, int b){
   if(b == 0) return make_pair(1, 0);
   else{
     int p = a / b;
     pii q = gcd(b, a % b);
     return make_pair(q.second, q.first - q.second * p);
   }
}
```

5.2 Fast Fourier Transform

```
// const int MAXN = 262144;
// (must be 2^k)
typedef long double ld;
typedef complex < ld > cplx;
const ld PI = acosl(-1);
const cplx I(0, 1);
cplx omega[MAXN+1];
void pre_fft()
  for(int i=0; i<=MAXN; i++)</pre>
    omega[i] = exp(i * 2 * PI / MAXN * I);
}
void fft(int n, cplx a[], bool inv=false)
  int basic = MAXN / n;
  int theta = basic;
  for (int m = n; m >= 2; m >>= 1) {
    int mh = m >> 1;
    for (int i = 0; i < mh; i++) {</pre>
      cplx w = omega[inv ? MAXN-(i*theta%MAXN) : i*
          theta%MAXN];
       for (int j = i; j < n; j += m) {</pre>
        int k = j + mh;
         cplx x = a[j] - a[k];
         a[j] += a[k];
         a[k] = w * x;
      }
    theta = (theta * 2) % MAXN;
  for (int j = 1; j < n - 1; j++) {
  for (int k = n >> 1; k > (i ^= k); k >>= 1);
    if (j < i) swap(a[i], a[j]);</pre>
  if (inv)
    for (i = 0; i < n; i++)</pre>
      a[i] /= n;
}
// wangyenjen
typedef complex < double > cplx;
const double PI = acos(-1.0);
const int MAX_N = 1<<20;
void fft(cplx *a , int n</pre>
                             , int dir) {
    static cplx tmp[MAX_N];
     if(n == 1) return;
    REP(i , n) tmp[i] = a[i];
    REP(i , n) a[(i\&1) ? (n>>1) + (i>>1) : (i>>1)] =
         tmp[i];
    cplx *a1 = a , *a2 = a + (n>>1);
fft(a1 , n>>1 , dir);
fft(a2 , n>>1 , dir);
     cplx w_base(cos(2.0 * PI / (double)n) , sin(2.0 *
         PI / (double)n));
     cplx w(1.0 , 0.0);
     if(dir < 0) w_base = conj(w_base);</pre>
     for(int i = 0; (i<<1) < n; i++ , w *= w_base) {</pre>
         tmp[i] = a1[i] + w * a2[i];
         tmp[(n>>1) + i] = a1[i] - w * a2[i];
    REP(i, n) a[i] = tmp[i];
inline int mult(cplx *a , int la , cplx *b , int lb ,
    cplx *c) {
  int n = 2;
  while(n < la + lb) n <<= 1;</pre>
  REP1(i , la , n - 1) a[i] = cplx(0.0 , 0.0);
  REP1(i , lb , n - 1) b[i] = cplx(0.0 , 0.0);
  fft(a , n , 1);
```

```
fft(b , n , 1);
REP(i , n) c[i] = a[i] * b[i];
  fft(b , n
  fft(c , n , -1);
  REP(i, n) c[i] /= n;
  return la + lb - 1;
cplx a[MAX_N] , b[MAX_N] , c[MAX_N];
int main() {
  int n , m;
  RI(n , m);
REP(i , n + 1) {
    int x;
    RI(x);
    a[i] = cplx((double)x , 0.0);
  REP(i , m + 1) \{
    int x;
    RI(x);
    b[i] = cplx((double)x , 0.0);
  int len = mult(a , n + 1 , b , m + 1 , c);
REP(i , len) printf("%d%c",(int)real(c[i] + 0.5)," \n
       "[i == len - 1]);
  return 0;
```

5.3 Fast Linear Recurrence

```
ll n,m,dp[N+N];
void pre dp(){
  dp[0] = 1;
  ll bdr = min(m+m,n);
  for(ll i=1; i<=bdr; i++)</pre>
    for(ll j=i-1; j>=max(0ll,i-m); j--)
      dp[i]= add(dp[i],dp[j]);
vector<ll> Mul(const vector<ll>& v1,const vector<ll>&
    v2){
  int sz1 = (int)v1.size();
  int sz2 = (int)v2.size();
  assert(sz1 == m and sz2 == m);
  vector<ll> _v(m+m);
  for(int i=0; i<m+m; i++) _v[i]= 0;</pre>
  // expand
  for(int i=0; i<sz1; i++)</pre>
    for(int j=0; j<sz2; j++)</pre>
       _v[i+j+1]= add(_v[i+j+1],mul(v1[i],v2[j]));
  // shrink
  for(int i=0; i<m; i++)</pre>
    for(int j=1; j<=m; j++)</pre>
       _v[i + j]= add(_v[i + j],_v[i]);
  for(int i=0; i<m; i++)
   _v[i]= _v[i + m];</pre>
  _v.resize(m);
  return v;
vector<ll> I,A;
ll solve(){
  pre_dp();
  if(n <= m+m)return dp[n];</pre>
  I.resize(m);
  A.resize(m);
  for(int i=0; i<m; i++) I[i]=A[i]=1;</pre>
  // dp[n]= /Sum_{i=0}^{m-1} A_i * dp[n - i - 1]
  ll dlt = (n - m) / m;
  ll rdlt = dlt * m;
  while(dlt){
    if(dlt & 1ll) I = Mul(I,A);
    A = Mul(A,A);
    dlt >>= 1;
  ll\ ans = 0;
  for(int i=0; i<m; i++)</pre>
    ans = add(ans,mul(I[i],dp[n-i-1-rdlt]));
  return ans:
```

5.4 (+1) ntt

```
int P=605028353,root=3,MAXNUM=262144;
// Remember coefficient are mod P
/*
```

```
p=a*2^n+1
    2^n
n
                                а
                                       root
5
    32
                 97
                                3
                                       5
6
    64
                 193
                                3
                                       5
                 257
    128
                                2
                                       3
8
    256
                 257
                                       3
9
                                       17
    512
                 7681
                                15
10
    1024
                 12289
                                12
                                       11
11
    2048
                 12289
                                6
                                       11
    4096
                 12289
                                3
                                       11
    8192
                 40961
                                5
13
                                       3
14
    16384
                 65537
                                4
                                       3
15
    32768
                 65537
                                2
                                       3
    65536
                 65537
16
                                1
                                       3
17
    131072
                 786433
                                6
                                       10
18
    262144
                 786433
                                       10 (605028353,
    2308, 3)
19
    524288
                 5767169
                                       3
                                11
20
    1048576
                 7340033
                                       3
    2097152
                 23068673
                                11
21
                                       3
22
    4194304
                 104857601
                                25
                                       3
    8388608
                 167772161
23
                                20
                                       3
    16777216
                 167772161
24
                                10
25
    33554432
                 167772161
                                 5
                                       3 (1107296257, 33,
    10)
26
    67108864
                 469762049
    134217728
27
                 2013265921
                                15
                                       31
*/
int bigmod(long long a,int b){
  if(b==0)return 1;
  return (bigmod((a*a)%P,b/2)*(b%2?a:1ll))%P;
int inv(int a,int b){
  if(a==1)return 1;
  return (((long long)(a-inv(b%a,a))*b+1)/a)%b;
std::vector<long long> ps(MAXNUM);
std::vector<int> rev(MAXNUM);
struct poly{
  std::vector<unsigned int> co;
  int n;//polynomial degree = n
  poly(int d){n=d;co.resize(n+1,0);}
  void trans2(int NN){
    int r=0,st,N;
    unsigned int a,b;
    while((1<<r)<(NN>>1))++r;
    for(N=2;N<=NN;N<<=1,--r){
      for(st=0;st<NN;st+=N){</pre>
         int i,ss=st+(N>>1);
         for(i=(N>>1)-1;i>=0;--i){
           a=co[st+i]; b=(ps[i<<r]*co[ss+i])%P;
           co[st+i]=a+b; if(co[st+i]>=P)co[st+i]-=P;
           co[ss+i]=a+P-b; if(co[ss+i]>=P)co[ss+i]-=P;
        }
      }
    }
  void trans1(int NN){
    int r=0,st,N;
    unsigned int a,b;
    for(N=NN;N>1;N>>=1,++r){
      for(st=0;st<NN;st+=N){</pre>
         int i,ss=st+(N>>1);
         for(i=(N>>1)-1;i>=0;--i){
           a=co[st+i]; b=co[ss+i];
           co[st+i]=a+b; if(co[st+i]>=P)co[st+i]-=P;
           co[ss+i]=((a+P-b)*ps[i<<r])%P;
        }
      }
    }
  }
  poly operator*(const poly& _b)const{
    poly a=*this,b=_b;
    int k=n+b.n,i,N=1;
    while(N<=k)N*=2;</pre>
    a.co.resize(N,0); b.co.resize(N,0);
    int r=bigmod(root,(P-1)/N),Ni=inv(N,P);
    ps[0]=1;
    for(i=1;i<N;++i)ps[i]=(ps[i-1]*r)%P;</pre>
    a.trans1(N);b.trans1(N);
    for(i=0;i<N;++i)a.co[i]=((long long)a.co[i]*b.co[i</pre>
         ])%P
    r=inv(r,P);
    for(i=1;i<N/2;++i)std::swap(ps[i],ps[N-i]);</pre>
    a.trans2(N);
```

```
for(i=0;i<N;++i)a.co[i]=((long long)a.co[i]*Ni)%P;
    a.n=n+_b.n; return a;
}

5.5 Mod

/// _fd(a,b) floor(a/b).</pre>
```

```
/// _rd(a,m) a-floor(a/m)*m.
/// _pv(a,m,r) largest x s.t x<=a && x%m == r.
/// _nx(a,m,r) smallest x s.t x>=a && x%m == r.
/// _{ct(a,b,m,r)} /A/ , A = { x : a<=x<=b && x%m == r }.
int _fd(int a,int b){ return a<0?(-~a/b-1):a/b; }</pre>
int _rd(int a,int m){ return a-_fd(a,m)*m; }
int _pv(int a,int m,int r)
{
    r = (r%m+m)%m:
    return _fd(a-r,m)*m+r;
int _nt(int a,int m,int r)
{
    m=abs(m):
    r=(r%m+m)%m;
    return _fd(a-r-1,m)*m+r+m;
int _ct(int a,int b,int m,int r)
    m=abs(m);
    a=_nt(a,m,r);
    b=_pv(b,m,r);
    return (a>b)?0:((b-a+m)/m);
}
```

5.6 (+1) Miller Rabin

```
// n < 4,759,123,141
                            3: 2, 7, 61
  n < 1,122,004,669,633
                            4 : 2, 13, 23, 1662803
// n < 3,474,749,660,383
                                   6 : pirmes <= 13
// n < 2^64
// 2, 325, 9375, 28178, 450775, 9780504, 1795265022
// Make sure testing integer is in range [2, n-2] if
// you want to use magic.
long long power(long long x,long long p,long long mod){
 long long s=1,m=x;
 while(p) {
   if(p&1) s=mult(s,m,mod);
   p>>=1;
   m=mult(m,m,mod);
 }
  return s:
bool witness(long long a,long long n,long long u,int t)
  long long x=power(a,u,n);
  for(int i=0;i<t;i++) {</pre>
    long long nx=mult(x,x,n);
    if(nx==1&&x!=1&&x!=n-1) return 1;
   x=nx;
 }
  return x!=1;
bool miller_rabin(long long n,int s=100) {
 // iterate s times of witness on n
  // return 1 if prime, 0 otherwise
  if(n<2) return 0;</pre>
  if(!(n&1)) return n==2;
  long long u=n-1;
 int t=0;
  // n-1 = u*2^t
  while(!(u&1)) {
   u>>=1;
    t++:
 while(s--) {
    long long a=randll()%(n-1)+1;
    if(witness(a,n,u,t)) return 0;
  return 1;
```

5.7 Pollard Rho

```
// does not work when n is prime
long long modit(long long x,long long mod) {
  if(x>=mod) x-=mod;
   //if(x<0) x+=mod;
   return x;
long long mult(long long x,long long y,long long mod) {
  long long s=0,m=x%mod;
   while(y) {
     if(y&1) s=modit(s+m,mod);
     v>>=1;
     m=modit(m+m,mod);
   return s;
long long f(long long x,long long mod) {
  return modit(mult(x,x,mod)+1,mod);
long long pollard_rho(long long n) {
   if(!(n&1)) return 2;
   while (true) {
     long long y=2, x=rand()%(n-1)+1, res=1;
for (int sz=2; res==1; sz*=2) {
       for (int i=0; i<sz && res<=1; i++) {</pre>
         x = f(x, n);
         res = \_gcd(abs(x-y), n);
       }
       y = x;
     if (res!=0 && res!=n) return res;
}
```

5.8 Algorithms about Primes

```
* 12721
 * 13331
 * 14341
* 75577
 * 123457
 * 222557
 * 556679
 * 999983
 * 1097774749
 * 1076767633
 * 100102021
 * 999997771
  1001010013
 * 1000512343
 * 987654361
 * 999991231
 * 999888733
 * 98789101
 * 987777733
 * 999991921
 * 1010101333
 * 1010102101
 * 1000000000039
 * 1000000000000037
 * 2305843009213693951
 * 4611686018427387847
 * 9223372036854775783
 * 18446744073709551557
int mu[MX],p_tbl[MX];
vector<int> primes;
void sieve() {
  mu[1] = p_tbl[1] = 1;
  for (int i=2; i<MX; i++) {</pre>
    if (!p_tbl[i]) {
      p_tbl[i] = i;
      primes.PB(i);
      mu[i] = -1;
    for (auto p : primes) {
      int x = i*p;
      if (x >= M) break;
      p_{tbl}[x] = p;
      mu[x] = -mu[i];
```

```
if (i%p==0) {
          mu[x] = 0;
         break:
     }
  }
}
 vector<int> factor(int x) {
   vector<int> fac{1};
   while (x > 1) {
   int fn=SZ(fac), p=p_tbl[x], pos=0;
     while (x\%p == 0) {
       x /= p;
       for (int i=0; i<fn; i++)</pre>
         fac.PB(fac[pos++]*p);
     }
   }
   return fac;
openssl prime -generate -bits 31
```

5.9 (+1) PolynomialGenerator

```
class PolynomialGenerator {
    ^{\prime}* for a nth-order polynomial f(x), *
   * given f(0), f(1), ..., f(n) *
    * express f(x) as sigma_i{c_i*C(x,i)} */
   public:
     int n;
     vector < long long > coef;
     // initialize and calculate f(x), vector f(x) should
     // filled with f(0) to f(n)
       PolynomialGenerator(int _n, vector < long long > _fx)
           :n(_n
           ),coef(_fx) {
         for(int i=0;i<n;i++)</pre>
           for(int j=n;j>i;j--)
             coef[j]-=coef[j-1];
     // evaluate f(x), runs in O(n)
     long long eval(int x) {
       long long m=1,ret=0;
       for(int i=0;i<=n;i++) {</pre>
         ret+=coef[i]*m;
         m=m*(x-i)/(i+1);
       return ret;
};
```

5.10 Pseudoinverse of Square matrix

```
Mat pinv(Mat m)
{
  Mat res = I;
  FZ(used);
  for(int i=0; i<W; i++)</pre>
    int piv = -1;
    for(int j=0; j<W; j++)</pre>
      if(used[j]) continue;
      if(abs(m.v[j][i]) > EPS)
        piv = j;
        break;
      }
    if(piv == -1)
      continue;
    used[i] = true;
    swap(m.v[piv], m.v[i]);
    swap(res.v[piv], res.v[i]);
    ld rat = m.v[i][i];
    for(int j=0; j<W; j++)</pre>
```

```
m.v[i][j] /= rat;
    res.v[i][j] /= rat;
}

for(int j=0; j<W; j++)
{
    if(j == i) continue;
    rat = m.v[j][i];
    for(int k=0; k<W; k++)
    {
        m.v[j][k] -= rat * m.v[i][k];
        res.v[j][k] -= rat * res.v[i][k];
    }
}

for(int i=0; i<W; i++)
{
    if(used[i]) continue;
    for(int j=0; j<W; j++)
        res.v[i][j] = 0;
}

return res;
}</pre>
```

5.11 Theorem

5.11.1 Lucas' Theorem

For non-negative integer n,m and prime p, $\binom{m}{n} \equiv \prod_{i=0}^k \binom{m_i}{n_i} \pmod{p}$ where m_i is the i-th digit of m in base p.

5.11.2 Sum of Two Squares Thm (Legendre)

For a given positive integer n, let $D_1=$ (# of positive integers d dividing N that $1\equiv d\pmod 4$) $D_3=$ (# of positive integers d dividing N that $3\equiv d\pmod 4$) then n can be written as a sum of two squares in exactly $R(n)=4(D_1-D_3)$ ways.

5.11.3 Difference of D1-D3 Thm

```
\begin{array}{l} \operatorname{let} n = 2^t \cdot (p_1^{e_1} \cdot \ldots \cdot p_r^{e_r}) \cdots (q_1^{f_1} \cdot \ldots \cdot q_s^{f_s}) \\ \operatorname{where} p_i, q_i \text{ are primes and } 1 \equiv p_i \pmod{4}, 3 \equiv q_i \pmod{4} \\ \operatorname{then} D_1 - D_3 = \begin{cases} (e_1 + 1)(e_2 + 1) \ldots (e_r + 1), & \text{if } f_i \text{ all even} \\ 0, & \text{if any } f_i \text{ is odd} \end{cases}
```

5.11.4 Krush-Kuhn-Tucker Conditions

Stationarity

```
For maximizing f(x): \nabla f(x^*) = \sum_{i=1}^m \mu_i \nabla g_i(x^*) + \sum_{j=1}^l \lambda_j \nabla h_j(x^*)
For minimizing f(x): -\nabla f(x^*) = \sum_{i=1}^m \mu_i \nabla g_i(x^*) + \sum_{j=1}^l \lambda_j \nabla h_j(x^*)
```

Primal feasibility

```
g_i(x^*) \le 0, for all i = 1, ..., m
h_j(x^*) = 0, for all j = 1, ..., l
```

Dual feasibility

 $\mu_i \geq 0$, for all $i = 1, \ldots, m$

Complementary slackness

 $\mu_i g_i(x^*) = 0$, for all i = 1, ..., m

5.11.5 Chinese remainder theorem

```
\begin{split} x &\equiv r_i \mod p_i \\ N &= \prod p_i \\ N_i &= N/p_i \\ x &\equiv \sum r_i N_i (N_i)_{p_i}^{-1} \mod N \end{split}
```

5.12 Simplex

```
const int maxn = 111;
const int maxm = 111;
const double eps = 1E-10;

double a[maxn][maxm], b[maxn], c[maxm], d[maxn][maxm];
double x[maxm];
int ix[maxn + maxm]; // !!! array all indexed from 0
// max{cx} subject to {Ax<=b,x>=0}
```

```
n: constraints, m: vars !!!
// x[] is the optimal solution vector
//
// usage :
// value = simplex(a, b, c, N, M);
double simplex(double a[maxn][maxm], double b[maxn],
    double c[maxm], int n, int m) {
    ++m;
    int r = n, s = m - 1;
    memset(d, 0, sizeof(d));
    for (int i = 0; i < n + m; ++i) ix[i] = i;
for (int i = 0; i < n; ++i) {</pre>
        for (int j = 0; j < m - 1; ++j)
             d[i][j] = -a[i][j];
        d[i][m - 1] = 1;
        d[i][m] = b[i];
        if (d[r][m] > d[i][m]) r = i;
    for (int j = 0; j < m - 1; ++j) d[n][j] = c[j];</pre>
    d[n + 1][m - 1] = -1;
    for (double dd;; ) {
        if (r < n) {
             int t = ix[s];
             ix[s] = ix[r + m]; ix[r + m] = t;
             d[r][s] = 1.0 / d[r][s];
             for (int j = 0; j <= m; ++j)
   if (j != s) d[r][j] *= -d[r][s];</pre>
             for (int i = 0; i <= n + 1; ++i)</pre>
                 if (i != r) {
                      for (int j = 0; j <= m; ++j)</pre>
                          if (j != s)
                              d[i][j] += d[r][j]*d[i][s];
                     d[i][s] *= d[r][s];
                 }
        r = -1; s = -1;
        for (int j = 0; j < m; ++j)</pre>
             if (s < 0 || ix[s] > ix[j]) {
                 if (d[n + 1][j] > eps || (d[n + 1][j] >
                       -eps && d[n][j] > eps)) s = j;
        if (s < 0) break;</pre>
        ix[r + m] > ix[i + m])) r = i;
         if (r < 0) return -1; // not bounded
    if (d[n + 1][m] < -eps) return -1; // not</pre>
         executable
    double ans = 0;
    for(int i=0; i<m; i++) x[i] = 0;</pre>
    for (int i = m; i < n + m; ++i) { // the missing</pre>
         enumerated x[i] = 0
         if (ix[i] < m - 1)</pre>
        {
             ans += d[i - m][m] * c[ix[i]];
             x[ix[i]] = d[i-m][m];
    return ans;
}
```

6 Geometry

6.1 Point operators

```
#define x first
#define y second

#define cpdd const pdd
struct pdd : pair < double, double > {
    using pair < double, double > :: pair;

    pdd operator + (cpdd &p) const {
        return {x+p.x, y+p.y};
    }

    pdd operator - () const {
        return {-x, -y};
    }
}
```

```
pdd operator - (cpdd &p) const {
    return (*this) + (-p);
}

pdd operator * (double f) const {
    return {f*x, f*y};
}

double operator * (cpdd &p) const {
    return x*p.x + y*p.y;
}
};

double abs(cpdd &p) { return hypot(p.x, p.y); }
double arg(cpdd &p) { return atan2(p.y, p.x); }
double cross(cpdd &p, cpdd &q) { return p.x*q.y - p.y*q.x; }
double cross(cpdd &p, cpdd &q, cpdd &o) { return cross(p-o, q-o); }
pdd operator * (double f, cpdd &p) { return p*f; } //
    !! Not f*p !!
```

6.2 Intersection of two circles

6.3 Intersection of two lines

```
const double EPS = 1e-9;
pdd interPnt(pdd p1, pdd p2, pdd q1, pdd q2, bool &res)
     {
          double f1 = cross(p2, q1, p1);
          double f2 = -cross(p2, q2, p1);
          double f = (f1 + f2);

          if(fabs(f) < EPS) {
                res = false;
                return {};
           }

          res = true;
          return (f2 / f) * q1 + (f1 / f) * q2;
}</pre>
```

6.4 Half Plane Intersection

```
const double EPS = 1e-9;

pdd interPnt(Line l1, Line l2, bool &res){
    pdd p1, p2, q1, q2;
    tie(p1, p2) = l1;
    tie(q1, q2) = l2;

    double f1 = cross(p2, q1, p1);
    double f2 = -cross(p2, q2, p1);

    double f = (f1 + f2);

    if(fabs(f) < EPS) {
        res = false;
        return {0, 0};
    }

    res = true;
    return (f2 / f) * q1 + (f1 / f) * q2;
}</pre>
```

```
bool isin(Line l0, Line l1, Line l2) {
    // Check inter(l1, l2) in l0
                                                               inline Vector operator - (Point a , Point b) {
    bool res;
                                                                   return Vector(a.x - b.x , a.y - b.y);
    pdd p = interPnt(l1, l2, res);
    return cross(l0.S, p, l0.F) > EPS;
                                                               inline Vector operator * (Vector v , double p) {
                                                                   return Vector(v.x * p , v.y * p);
/* If no solution, check: 1. ret.size() < 3</pre>
                                                               }
 * Or more precisely, 2. interPnt(ret[0], ret[1])
 * in all the lines. (use (l.S - l.F).cross(p - l.F) >
                                                               inline Vector operator / (Vector v , double p) {
                                                                   return Vector(v.x / p , v.y / p);
vector<Line> halfPlaneInter(vector<Line> lines) {
    int sz = lines.size();
                                                               inline double cross(Vector a , Vector b) {
    vector<double> ata(sz), ord(sz);
                                                                   return a.x * b.y - a.y * b.x;
    for (int i=0; i<sz; i++) {</pre>
                                                               }
        ord[i] = i;
        pdd d = lines[i].S - lines[i].F;
                                                               inline double dot(Vector a , Vector b) {
    return a.x * b.x + a.y * b.y;
        ata[i] = atan2(d.y, d.x);
    sort(ALL(ord), [&](int i, int j) {
        if (abs(ata[i] - ata[j]) < EPS) {</pre>
                                                               inline int dcmp(double x) {
             return cross(lines[i].S, lines[j].S, lines[
                                                                   return fabs(x) < EPS ? 0 : x > 0 ? 1 : -1;
                 i].F) < 0;
        return ata[i] < ata[j];</pre>
                                                               struct Line {
    });
                                                                   Point p;
    vector<Line> fin;
                                                                   Vector v;
    for (int i=0; i<sz; i++) {</pre>
                                                                   double ang;
        if (!i or fabs(ata[ord[i]] - ata[ord[i-1]]) >
             EPS) {
                                                                   Line() {}
             fin.PB(lines[ord[i]]);
                                                                   Line(Point _p , Vector _v) : p(_p) , v(_v) {
        }
                                                                       ang = atan2(_v.y , _v.x);
    }
    deque<Line> dq;
                                                                   bool operator < (const Line &rhs) const {</pre>
    for (int i=0; i<SZ(fin); i++) {</pre>
                                                                       return ang < rhs.ang;</pre>
        while(SZ(dq) >= 2 and
               not isin(fin[i], dq[SZ(dq)-2], dq[SZ(dq)
                                                               };
                   -1])) {
                                                               inline bool on_left(Line l , Point p) {
    return cross(l.v , p - l.p) > 0;
             dq.pop_back();
        while(SZ(dq) >= 2 and
                                                               }
               not isin(fin[i], dq[0], dq[1])) {
             dq.pop_front();
                                                               inline Point get_line_intersection(Line a , Line b) {
                                                                   Vector u = a.p - b.p;
                                                                   double t = cross(b.v , u) / cross(a.v , b.v);
        dq.push_back(fin[i]);
                                                                   return a.p + a.v * t;
    }
    while (SZ(dq) >= 3 and
           not isin(dq[0], dq[SZ(dq)-2], dq[SZ(dq)-1]))
                                                               vector<Point> half_plane_intersection(vector<Line> ls)
        dq.pop_back();
                                                                   int n = (int)ls.size();
                                                                   sort(ls.begin(), ls.end());
                                                                   int f , r;
                                                                   vector<Point> p(n) , ans;
    while (SZ(dq) >= 3 and
           not isin(dq[SZ(dq)-1], dq[0], dq[1])) {
                                                                   vector<Line> q(n);
                                                                   q[f = r = 0] = ls[0];
        dq.pop_front();
                                                                   for (int i = 1; i <= n - 1; i++) {</pre>
    vector<Line> res(ALL(dq));
                                                                        while(f < r && !on_left(ls[i] , p[r - 1])) r--;</pre>
    return res;
                                                                        while(f < r && !on_left(ls[i] , p[f])) f++;</pre>
                                                                        q[++r] = ls[i];
                                                                        if(dcmp(cross(q[r].v , q[r - 1].v)) == 0) {
#include <bits/stdc++.h>
                                                                            if(on_left(q[r] , ls[i].p)) q[r] = ls[i];
using namespace std;
                                                                        if(f < r) p[r - 1] = get_line_intersection(q[r</pre>
                                                                            - 1] , q[r]);
const double EPS = 1e-10;
                                                                   while(f < r && !on_left(q[f] , p[r - 1])) r--;</pre>
struct Point {
                                                                   if(r - f <= 1) return ans;</pre>
    double x , y;
                                                                   p[r] = get_line_intersection(q[r] , q[f]);
                                                                   for (int i = f; i <= r; i++) ans.push_back(p[i]);</pre>
    Point(double _x = 0.0, double _y = 0.0) : x(_x),
                                                                   return ans;
        y(_y) {}
                                                               }
    bool operator < (const Point &rhs) const {</pre>
         if(x != rhs.x) return x < rhs.x;</pre>
        else return y < rhs.y;</pre>
                                                               6.5 2D Convex Hull
    }
};
typedef Point Vector;
                                                               vector<pdd> convex_hull(vector<pdd> pt){
                                                                 sort(pt.begin(),pt.end());
inline Point operator + (Point p , Vector v) {
                                                                 int top=0;
    return Point(p.x + v.x , p.y + v.y);
                                                                 vector<pdd> stk(2*pt.size());
                                                                 for (int i=0; i<(int)pt.size(); i++){</pre>
```

6.6 3D Convex Hull

```
// return the faces with pt indexes
int flag[MXN][MXN];
struct Point{
  ld x,y,z;
  Point operator - (const Point &b) const {
    return (Point){x-b.x,y-b.y,z-b.z};
  Point operator * (const ld &b) const {
    return (Point){x*b,y*b,z*b};
  ld len() const { return sqrtl(x*x+y*y+z*z); }
  ld dot(const Point &a) const {
   return x*a.x+y*a.y+z*a.z;
  Point operator * (const Point &b) const {
    return (Point){y*b.z-b.y*z,z*b.x-b.z*x,x*b.y-b.x*y
        };
 }
Point ver(Point a, Point b, Point c) {
 return (b - a) * (c - a);
vector<Face> convex_hull_3D(const vector<Point> pt) {
  int n = SZ(pt);
  REP(i,n) REP(j,n)
    flag[i][j] = 0;
  vector < Face > now;
  now.push_back((Face){0,1,2});
  now.push_back((Face){2,1,0});
  int ftop = 0;
  for (int i=3; i<n; i++){</pre>
    ftop++;
    vector < Face > next;
    REP(j, SZ(now)) {
      Face& f=now[j];
      ld d=(pt[i]-pt[f.a]).dot(ver(pt[f.a], pt[f.b], pt
          [f.c]));
      if (d <= 0) next.push_back(f);</pre>
      int ff = 0;
      if (d > 0) ff=ftop;
else if (d < 0) ff=-ftop;</pre>
      flag[f.a][f.b] = flag[f.b][f.c] = flag[f.c][f.a]
          = ff;
    REP(j, SZ(now)) {
      Face& f=now[j];
      if (flag[f.a][f.b] > 0 and flag[f.a][f.b] != flag
          [f.b][f.a])
        next.push_back((Face){f.a,f.b,i});
      if (flag[f.b][f.c] > 0 and flag[f.b][f.c] != flag
          [f.c][f.b])
        next.push_back((Face){f.b,f.c,i});
      if (flag[f.c][f.a] > 0 and flag[f.c][f.a] != flag
          [f.a][f.c])
        next.push_back((Face){f.c,f.a,i});
    }
    now=next;
  return now;
```

```
struct Mcc{
   // return pair of center and r^2
   static const int MAXN = 1000100;
   int n;
   pdd p[MAXN],cen;
   double r2;
   void init(int _n, pdd _p[]){
     memcpy(p,_p,sizeof(pdd)*n);
   double sqr(double a){ return a*a; }
   double abs2(pdd a){ return a*a;
   pdd center(pdd p0, pdd p1, pdd p2) {
     pdd a = p1-p0;
     pdd b = p2 - p0;
     double c1=abs2(a)*0.5;
     double c2=abs2(b)*0.5;
     double d = a % b;
     double x = p0.x + (c1 * b.y - c2 * a.y) / d;
     double y = p0.y + (a.x * c2 - b.x * c1) / d;
     return pdd(x,y);
   pair<pdd,double> solve(){
     random_shuffle(p,p+n);
     r2 = 0;
     for (int i=0; i<n; i++){</pre>
       if (abs2(cen-p[i]) <= r2) continue;</pre>
       cen = p[i];
       r2 = 0;
       for (int j=0; j<i; j++){</pre>
         if (abs2(cen-p[j]) <= r2) continue;</pre>
         cen = 0.5 * (p[i]+p[j]);
         r2 = abs2(cen-p[j]);
         for (int k=0; k<j; k++){</pre>
           if (abs2(cen-p[k]) <= r2) continue;</pre>
           cen = center(p[i],p[j],p[k]);
           r2 = abs2(cen-p[k]);
      }
     }
     return {cen,r2};
  }
}mcc;
```

6.8 KDTree (Nearest Point)

```
const int MXN = 100005;
struct KDTree {
  struct Node {
    int x,y,x1,y1,x2,y2;
    int id,f;
    Node *L, *R;
  }tree[MXN];
  int n;
  Node *root:
  long long dis2(int x1, int y1, int x2, int y2) {
    long long dx = x1-x2;
    long long dy = y1-y2;
    return dx*dx+dy*dy;
  static bool cmpx(Node& a, Node& b){ return a.x<b.x; }</pre>
  static bool cmpy(Node& a, Node& b){ return a.y<b.y; }</pre>
  void init(vector<pair<int,int>> ip) {
    n = ip.size();
    for (int i=0; i<n; i++) {</pre>
      tree[i].id = i;
      tree[i].x = ip[i].first;
      tree[i].y = ip[i].second;
    root = build_tree(0, n-1, 0);
  Node* build_tree(int L, int R, int dep) {
    if (L>R) return nullptr;
    int M = (L+R)/2;
    tree[M].f = dep%2;
    nth_element(tree+L, tree+M, tree+R+1, tree[M].f ?
        cmpy : cmpx);
    tree[M].x1 = tree[M].x2 = tree[M].x;
    tree[M].y1 = tree[M].y2 = tree[M].y;
```

```
tree[M].L = build_tree(L, M-1, dep+1);
    if (tree[M].L) {
      tree[M].x1 = min(tree[M].x1, tree[M].L->x1);
      tree[M].x2 = max(tree[M].x2, tree[M].L->x2);
      tree[M].y1 = min(tree[M].y1, tree[M].L->y1);
      tree[M].y2 = max(tree[M].y2, tree[M].L->y2);
    tree[M].R = build_tree(M+1, R, dep+1);
    if (tree[M].R) {
      tree[M].x1 = min(tree[M].x1, tree[M].R->x1);
      tree[M].x2 = max(tree[M].x2, tree[M].R->x2);
      tree[M].y1 = min(tree[M].y1, tree[M].R->y1);
      tree[M].y2 = max(tree[M].y2, tree[M].R->y2);
    return tree+M:
  int touch(Node* r, int x, int y, long long d2){
    long long dis = sqrt(d2)+1;
    if (x<r->x1-dis || x>r->x2+dis || y<r->y1-dis || y>
        r->y2+dis)
      return 0;
    return 1;
  void nearest(Node* r, int x, int y, int &mID, long
      long &md2) {
    if (!r || !touch(r, x, y, md2)) return;
    long long d2 = dis2(r->x, r->y, x, y);
    if (d2 < md2 \mid | (d2 == md2 \&\& mID < r->id)) {
      mID = r -> id;
      md2 = d2;
    }
    // search order depends on split dim
    if ((r->f == 0 && x < r->x) ||
        (r->f == 1 && y < r->y)) {
      nearest(r->L, x, y, mID, md2);
      nearest(r->R, x, y, mID, md2);
     else {
      nearest(r->R, x, y, mID, md2);
      nearest(r->L, x, y, mID, md2);
   }
  int query(int x, int y) {
    int id = 1029384756;
    long long d2 = 102938475612345678LL;
    nearest(root, x, y, id, d2);
    return id:
 }
}tree;
```

6.9 Triangulation

```
bool inCircle(pdd a, pdd b, pdd c, pdd d) {
   b = b - a;
   c = c - a;
   d = d - a;
    if (cross(b, c) < 0) swap(b, c);
    double m[3][3] = {
       {b.x, b.y, b*b},
       {c.x, c.y, c*c},
{d.x, d.y, d*d}
   };
    double det = m[0][0] * (m[1][1]*m[2][2] - m[1][2]*m
        [2][1])
               + m[0][1] * (m[1][2]*m[2][0] - m[1][0]*m
              [2][2])
+ m[0][2] * (m[1][0]*m[2][1] - m[1][1]*m
                  [2][0]);
   return det < 0;</pre>
}
bool intersect(pdd a, pdd b, pdd c, pdd d) {
   }
const double EPS = 1e-12;
struct Triangulation {
   static const int MXN = 1e5+5;
   int N:
   vector<int> ord;
```

```
vector<pdd> pts:
set<int> E[MXN]:
vector<vector<int>> solve(vector<pdd>> p) {
    N = SZ(p);
    ord.resize(N);
    for (int i=0; i<N; i++) {</pre>
        E[i].clear();
        ord[i] = i;
    sort(ALL(ord), [&p](int i, int j) {
        return p[i] < p[j];</pre>
    pts.resize(N);
    for (int i=0; i<N; i++) pts[i] = p[ord[i]];</pre>
    qo(0. N):
    vector<vector<int>> res(N);
    for (int i=0; i<N; i++) {</pre>
        int o = ord[i];
        for (auto x: E[i]) {
            res[o].PB(ord[x]);
    return res;
void add_edge(int u, int v) {
    E[u].insert(v);
    E[v].insert(u);
void remove_edge(int u, int v) {
    E[u].erase(v);
    E[v].erase(u);
void go(int l, int r) {
    int n = r - l;
    if (n <= 3) {
        for (int i=1; i<r; i++)</pre>
             for (int j=i+1; j<r; j++) add_edge(i, j</pre>
        return:
    int md = (l+r)/2;
    go(l, md);
    go(md, r);
    int il = l, ir = r-1;
    while (1) {
        int nx = -1;
        for (auto i: E[il]) {
            double cs = cross(pts[il], pts[i], pts[
                 ir]);
             if (cs > EPS ||
                 (abs(cs) < EPS and abs(pts[i]-pts[</pre>
                    ir]) < abs(pts[il]-pts[ir]))) {</pre>
                 nx = i;
                 break;
            }
        if (nx != -1) {
            il = nx;
            continue:
        for (auto i: E[ir]) {
             double cs = cross(pts[ir], pts[i], pts[
                 il]);
             if (cs < -EPS ||
                 (abs(cs) < EPS and abs(pts[i]-pts[</pre>
                     il]) < abs(pts[ir]-pts[il]))) {
                 nx = i;
                 break;
            }
        if (nx != -1) {
             ir = nx;
        } else break;
    add_edge(il, ir);
```

```
while (1) {
             int nx = -1;
             bool is2 = false;
             for (int i: E[il]) {
                 if (cross(pts[il], pts[i], pts[ir]) < -</pre>
                     EPS and
                     (nx == -1 or inCircle(pts[il], pts[
                         ir], pts[nx], pts[i]))) nx = i;
             }
             for (int i: E[ir]) {
                 if (cross(pts[ir], pts[i], pts[il]) >
                     EPS and
                     (nx == -1 or inCircle(pts[il], pts[
                         ir], pts[nx], pts[i]))) nx = i,
                          is2 = 1;
             }
             if (nx == -1) break:
             int a = il, b = ir;
             if (is2) swap(a, b);
             for (auto i: E[a]) {
                 if (intersect(pts[a], pts[i], pts[b],
                     pts[nx])) {
                     remove_edge(a, i);
             if (is2) {
                 add_edge(il, nx);
                 ir = nx;
             } else {
                 add_edge(ir, nx);
                 il = nx;
             }
        }
} tri;
```

7 Stringology

7.1 Suffix Array

```
const int N = 1.12e5, M = 11;
const char nil = 'a' - 1;
char s[N];
int sa[N], ct[N], w[2][N], rk[N], ht[N];
void suffix_array(int n, int m)
 int i, j, p = 0, * x = w[0], * y = w[1], h;
memset(ct, 0, m * sizeof(int));
  for (i = 0; i < n; ++i) ++ct[x[i] = s[i] - ('a' - 1)
  for (i = 1; i < m; ++i) ct[i] += ct[i - 1];</pre>
  for (i = n - 1; i >= 0; --i) sa[--ct[x[i]]] = i;
  for (j = p = 1; p < n; j *= 2, m = p) {
  for (i = n - j, p = 0; i < n; ++i) y[p++] = i;</pre>
    for (i = 0; i < n; ++i) if (sa[i] >= j) y[p++] = sa
         [i] - j;
    memset(ct, 0, m * sizeof(int));
    for (i = 0; i < n; ++i) ++ct[x[y[i]]];</pre>
    for (i = 1; i < m; ++i) ct[i] += ct[i - 1];</pre>
    for (i = n - 1; i >= 0; --i) sa[--ct[x[y[i]]]] = y[
         i];
    swap(x, y);
x[sa[0]] = 0;
    p = 1;
    for (i = 1; i < n; ++i)</pre>
      x[sa[i]] = y[sa[i]] == y[sa[i-1]] && y[sa[i] + j]
            == y[sa[i-1] + j] ? p - 1 : p++;
  for (i = 0; i < n; ++i) rk[sa[i]] = i;
  ht[0] = 0;
  h = 0;
  for (i = 0; rk[i]; ++i) {
    while (s[i + h] == s[sa[rk[i] - 1] + h]) ++h;
    ht[rk[i]] = h;
    if (h) --h;
```

7.2 Suffix Array (SAIS TWT514)

#define REP(i,n) for (int i=0; i<int(n); i++)</pre>

#define REP1(i,a,b) **for** (**int** i=(a); i<=**int**(b); i++)

}

struct SA{

```
static const int MXN = 300010;
    bool _t[MXN*2];
    int operator [] (int i){ return _sa[i]; }
void build(int *s, int n, int m){
         memcpy(_s, s, sizeof(int) * n);
         sais(_s, _sa, _p, _q, _t, _c, n, m);
         mkhei(n);
     void mkhei(int n){
         REP(i,n) r[_sa[i]] = i;
         hei[0] = 0;
         REP(i,n) if(r[i]) {
              int ans = i>0 ? max(hei[r[i-1]] - 1, 0) :
              while(_s[i+ans] == _s[_sa[r[i]-1]+ans]) ans
              hei[r[i]] = ans;
         }
     void sais(int *s, int *sa, int *p, int *q, bool *t,
         int *c, int n, int z){
bool uniq = t[n-1] = true, neq;
         int nn = 0, nmxz = -1, *nsa = sa + n, *ns = s +
               n, lst = -1;
#define MSO(x,n) memset((x),0,n*sizeof(*(x)))
#define MAGIC(XD) MSO(sa, n); \
         memcpy(x, c, sizeof(int) * z); \
         XD; \
         memcpy(x + 1, c, sizeof(int) * (z - 1)); 
         REP(i,n) if(sa[i] && !t[sa[i]-1]) sa[x[s[sa[i
              ]-1]]++] = sa[i]-1; \
         memcpy(x, c, sizeof(int) * z); \
for(int i = n - 1; i >= 0; i--) if(sa[i] && t[
              sa[i]-1]) sa[--x[s[sa[i]-1]]] = sa[i]-1;
         MS0(c, z);
         REP(i,n) uniq \&= ++c[s[i]] < 2;
         REP(i,z-1) c[i+1] += c[i];
         if (uniq) { REP(i,n) sa[--c[s[i]]] = i; return;
         for(int i = n - 2; i >= 0; i--) t[i] = (s[i]==s
              [i+1] ? t[i+1] : s[i] < s[i+1]);
         \label{eq:magic_rep_1} \text{MAGIC}(\text{REP1}(\texttt{i}, 1, \texttt{n}-1) \ \textbf{if}(\texttt{t}[\texttt{i}] \ \&\& \ \texttt{!t}[\texttt{i}-1]) \ \texttt{sa}[--\texttt{x}[
              s[i]]=p[q[i]=nn++]=i);
         REP(i, n) if (sa[i] && t[sa[i]] && t[sa[i]-1])
              neq=lst<0||memcmp(s+sa[i],s+lst,(p[q[sa[i</pre>
                   ]]+1]-sa[i])*sizeof(int));
              ns[q[lst=sa[i]]]=nmxz+=neq;
         sais(ns, nsa, p + nn, q + n, t + n, c + z, nn,
              nmxz + 1);
         MAGIC(for(int i = nn - 1; i >= 0; i--) sa[--x[s
              [p[nsa[i]]]] = p[nsa[i]]);
}sa;
void suffix_array(int* ip, int len) {
    // should padding a zero in the back
// s is int array, n is array length
    // s[0..n-1] != 0, and s[n] = 0
     // resulting SA will be length n+1
    ip[len++] = 0;
    sa.build(ip, len, 128);
// original 1-base
     for (int i=0; i<1; i++) {</pre>
         hei[i] = sa.hei[i + 1];
         sa[i] = sa._sa[i + 1];
    }
}
```

7.3 Aho-Corasick Algorithm

```
struct ACautomata{
  struct Node{
    int cnt,dp;
    Node *go[26], *fail;
    Node (){
      cnt = 0;
      dp = -1:
      memset(go,0,sizeof(go));
      fail = 0;
    }
  };
  Node *root, pool[1048576];
  int nMem;
  Node* new_Node(){
    pool[nMem] = Node();
    return &pool[nMem++];
  void init(){
    nMem = 0;
    root = new_Node();
  void add(const string &str){
    insert(root,str,0);
  void insert(Node *cur, const string &str, int pos){
    if (pos >= (int)str.size()){
      cur->cnt++;
      return:
    int c = str[pos]-'a';
    if (cur->go[c] == 0){
      cur->go[c] = new_Node();
    insert(cur->go[c],str,pos+1);
  }
  void make_fail(){
    queue < Node *> que;
    que.push(root);
    while (!que.empty()){
      Node* fr=que.front();
      que.pop();
      for (int i=0; i<26; i++){</pre>
        if (fr->go[i]){
          Node *ptr = fr->fail;
          while (ptr && !ptr->go[i]) ptr = ptr->fail;
          if (!ptr) fr->go[i]->fail = root;
          else fr->go[i]->fail = ptr->go[i];
          que.push(fr->go[i]);
      }
    }
 }
};
```

7.4 KMP

```
#include < bits / stdc++.h>
using namespace std;
void build_fail_function(string B, int *fail) {
    int len = B.length(), pos;
    pos = fail[0] = -1;
    for (int i = 1; i < len; i ++) {</pre>
        while (pos != -1 and B[pos + 1] != B[i])
            pos = fail[pos];
        if (B[pos + 1] == B[i]) pos ++;
        fail[i] = pos;
    }
}
void match(string A, string B, int *fail) {
    int lenA = A.length(), lenB = B.length();
    int pos = -1;
    for (int i = 0; i < lenA; i ++) {</pre>
        while (pos != -1 and B[pos + 1] != A[i])
            pos = fail[pos];
        if (B[pos + 1] == A[i]) pos ++;
        if (pos == lenB - 1) {
            // Match ! A[i - lenB + 1, i] = B
            pos = fail[pos];
```

```
}
```

7.5 Z value

```
void Zval(const char *s, int len, int *z) {
   z[0] = 0;
   for (int b=0, i=1; i<len; i++) {
        z[i] = max(min(z[i-b], z[b] + b - i), 0);
        while (s[i + z[i]] == s[z[i]]) z[i] ++;
        if (i+z[i] > b+z[b]) b=i;
   }
}
```

7.6 Z value (palindrome ver.)

7.7 palindromic tree

```
//bcw0x1bd2 {{{
#include < bits / stdc++.h>
#include < unistd.h>
using namespace std;
#define F first
#define S second
#define MP make_pair
#define PB push_back
#define IOS ios_base::sync_with_stdio(0); cin.tie(0);
#define SZ(x) ((int)((x).size()))
#define ALL(x) begin(x),end(x)
#define REP(i,x) for (int i=0; i<(x); i++)</pre>
#define REP1(i,a,b) for (int i=(a); i<=(b); i++)</pre>
typedef long long ll;
typedef pair<int,int> pii;
typedef pair<ll,ll> pll;
typedef long double ld;
#ifdef DARKHH
#define FILEIO(name)
#else
#define FILEIO(name) \
  freopen(name".in", "r", stdin); \
  freopen(name".out", "w", stdout);
#endif
#ifdef DARKHH
template < typename T>
void _dump( const char* s, T&& head ) { cerr<<s<<"="<<</pre>
     head << endl; }
template < typename T, typename... Args >
void _dump( const char* s, T&& head, Args&&... tail ) {
  int c=0;
  while ( *s!=',' || c!=0 ) {
  if ( *s=='(' || *s=='[' || *s=='[' ) c++;
     if ( *s==')' || *s==']' || *s=='}' ) c--;
    cerr<<*s++;
  cerr<<"="<<head<<", ";
  dump(s+1,tail...);
```

```
#define dump(...) do { \
   fprintf(stderr, "%s:%d - ", __PRETTY_FUNCTION__
        _LINE__); \
   dump(#__VA_ARGS__, __VA_ARGS__); \
} while (0)
template < typename Iter >
ostream& _out( ostream &s, Iter b, Iter e ) {
  for ( auto it=b; it!=e; it++ ) s<<(it==b?"":" ")<<*it</pre>
  s<<"]";
  return s;
template < typename A, typename B>
ostream& operator <<( ostream &s, const pair<A,B> &p )
{ return s<<"("<<p.first<<","<<p.second<<")"; }
template < typename T>
ostream& operator <<( ostream &s, const vector<T> &c )
    { return _out(s,ALL(c)); }
template < typename T, size_t N>
ostream& operator <<( ostream &s, const array<T,N> &c )
     { return _out(s,ALL(c)); }
template < typename T>
ostream& operator <<( ostream &s, const set<T> &c ) {
    return _out(s,ALL(c)); }
template < typename A, typename B>
ostream& operator <<( ostream &s, const map < A, B > &c ) {
     return _out(s,ALL(c)); }
#define dump(...)
#endif
// }}}
struct palindromic_tree{
  struct node{
    int next[26],fail,len;
    int cnt,num,st,ed;
    node(int l=0):fail(0),len(l),cnt(0),num(0){
      for(int i=0;i<26;++i)next[i]=0;</pre>
  }:
  vector<node> state;
  vector<char> s;
  int last,n;
  void init(){
    state.clear();
    s.clear();
    last=1:
    n=0:
    state.push_back(0);
    state.push_back(-1);
    state[0].fail=1;
    s.push_back(-1);
  int get_fail(int x){
    while(s[n-state[x].len-1]!=s[n])x=state[x].fail;
  void add(int c){
    s.push_back(c-= 'a');
    int cur=get_fail(last);
    if(!state[cur].next[c]){
      int now=state.size();
      state.push_back(state[cur].len+2);
      state[now].fail=state[get_fail(state[cur].fail)].
           next[c];
       state[cur].next[c]=now;
      state[now].num=state[state[now].fail].num+1;
    last=state[cur].next[c];
    ++state[last].cnt;
  int size(){
    return state.size()-2;
}pt;
int main() {
  string s;
  cin >> s;
  pt.init();
  for (int i=0; i<SZ(s); i++) {</pre>
```

7.8 Lexicographically Smallest Rotation

```
string mcp(string s){
  int n = s.length();
  s += s;
  int i=0, j=1;
  while (i<n && j<n){
    int k = 0;
    while (k < n && s[i+k] == s[j+k]) k++;
    if (s[i+k] <= s[j+k]) j += k+1;
    else i += k+1;
    if (i == j) j++;
}
int ans = i < n ? i : j;
return s.substr(ans, n);
}</pre>
```

7.9 Suffix Automaton

```
par : fail link
// val : a topological order ( useful for DP )
 // go[x] : automata edge ( x is integer in [0,26) )
 struct SAM{
   struct State{
    int par, go[26], val;
State () : par(0), val(0){ FZ(go); }
     State (int _val) : par(0), val(_val){ FZ(go); }
   vector < State > vec;
   int root, tail;
   void init(int arr[], int len){
     vec.resize(2);
     vec[0] = vec[1] = State(0);
     root = tail = 1;
     for (int i=0; i<len; i++)</pre>
       extend(arr[i]);
   void extend(int w){
     int p = tail, np = vec.size();
     vec.PB(State(vec[p].val+1));
     for ( ; p && vec[p].go[w]==0; p=vec[p].par)
       vec[p].go[w] = np;
     if (p == 0){
       vec[np].par = root;
     } else {
       if (vec[vec[p].go[w]].val == vec[p].val+1){
         vec[np].par = vec[p].go[w];
       } else {
         int q = vec[p].go[w], r = vec.size();
         vec.PB(vec[q]);
         vec[r].val = vec[p].val+1;
         vec[q].par = vec[np].par = r;
         for ( ; p && vec[p].go[w] == q; p=vec[p].par)
           vec[p].go[w] = r;
       }
     tail = np;
   }
};
```

8 Problems

8.1 Painter

```
#include < bits / stdc++.h>
                                                                    return t:
using namespace std;
#define F first
#define S second
#define PB push_back
#define IOS ios_base::sync_with_stdio(0); cin.tie(0);
#define SZ(x) ((int)((x).size()))
                                                                      a = t;
#define ALL(x) begin(x),end(x)
#define REP(i,x) for (int i=0; i<(x); i++)</pre>
                                                                      else {
#define REP1(i,a,b) for (int i=(a); i<=(b); i++)</pre>
                                                                       b = t;
typedef long long ll;
typedef pair<ll,ll> pll;
                                                                    }
typedef pll Point;
const int MXN = 100005;
Point operator + (const Point &a, const Point &b) {
    return Point(a.F+b.F, a.S+b.S); }
Point operator - (const Point &a, const Point &b) {
    return Point(a.F-b.F, a.S-b.S); }
ll operator * (const Point &a, const Point &b) { return
                                                                    return t:
     a.F*b.F + a.S*b.S; }
ll operator % (const Point &a, const Point &b) { return
     a.F*b.S - a.S*b.F; }
                                                                    return t;
struct Segment {
  int v,id;
  Point p,q;
                                                                  int N;
  Segment () {}
  Segment (int _v, int _id, Point _p, Point _q) :
   v(_v), id(_id), p(_p), q(_q) {}
bool operator < (const Segment &a, const Segment &b) {</pre>
  if (a.p == b.q) return false;
  if (a.q == b.p) return true;
  if (a.p == b.p) return (a.q-a.p) % (b.q-a.p) > 0;
  if (a.q == b.q) return (a.p-a.q) % (b.p-a.q) < 0;</pre>
  if (a.p.F == b.p.F) return a.p.S < b.p.S;</pre>
  if (a.q.F == b.q.F) return a.q.S < b.q.S;</pre>
  if (a.p.F < b.p.F) return (a.q-a.p) % (b.p-a.p) > 0;
  else return (b.q-b.p) % (a.p-b.p) < 0;</pre>
bool operator == (const Segment &a, const Segment &b) {
  return tie(a.v,a.id,a.p,a.q) == tie(b.v,b.id,b.p,b.q)
                                                                    REP(i.N)
struct Triangle {
 Point pt[3];
}ip[MXN];
const int MEM = 350004;
struct Treap {
  static Treap nil, mem[MEM], *pmem;
  Treap *l, *r;
  int sum,presum,size;
  Segment seg;
  Treap () : l(&nil), r(&nil), sum(0), presum(0), size
       (0), seg() {}
  Treap (Segment _val) :
    l(&nil), r(&nil), sum(_val.v), presum(max(_val.v,0)
         ), size(1), seg(_val) {}
} Treap::nil, Treap::mem[MEM], *Treap::pmem = Treap::
    mem:
                                                                         <= 0;
int size(const Treap *t) { return t->size; }
void pull(Treap *t) {
  if (!size(t)) return;
 t \rightarrow size = size(t \rightarrow l) + size(t \rightarrow r) + 1;
  t \rightarrow sum = t \rightarrow l \rightarrow sum + t \rightarrow seg.v + t \rightarrow r \rightarrow sum;
  t - presum = max(t - > l - > presum, t - > l - > sum + t - > seg.v);
  t - presum = max(t - presum, t - > l - > sum + t - > seg.v + t - >
      r->presum);
Treap* merge(Treap *a, Treap *b) {
  if (!size(a)) return b;
  if (!size(b)) return a;
  Treap *t:
  if (rand() % (size(a) + size(b)) < size(a)) {
    t = a;
    t - r = merge(a - r, b);
  } else {
    t = b;
    t \rightarrow l = merge(a, b \rightarrow l);
  pull(t);
```

```
void split(Treap *t, int k, Treap *&a, Treap *&b) {
  if (!size(t)) a = b = &Treap::nil;
  else if (size(t->l) + 1 <= k) {
    split(t->r, k - size(t->l) - 1, a->r, b);
    pull(a);
    split(t->l, k, a, b->l);
    pull(b);
int get_rank(Treap *t, Segment x) {
  if (!size(t)) return 0;
  if (x < t->seg) return get_rank(t->l, x);
  return get_rank(t->r,x) + size(t->l) + 1;
Treap* find_leftist(Treap *t) {
  while (size(t->l)) t = t->l;
Treap* find_rightist(Treap *t) {
  while (size(t->r)) t = t->r;
vector<int> allx;
vector < Segment > _seg[3*MXN];
#define seg(x) _seg[(x)+100000]
inline void add_seg(Segment s) {
  seg(s.p.F).PB(s);
  if (s.q.F != s.p.F) seg(s.q.F).PB(s);
void predo() {
  allx.clear()
  REP(i,N) REP(j,3) {
    seg(ip[i].pt[j].F).clear();
    allx.PB(ip[i].pt[j].F);
  sort(ALL(allx));
  allx.resize(unique(ALL(allx))-begin(allx));
    sort(ip[i].pt, ip[i].pt+3);
    Point *pt = ip[i].pt;
    Segment seg1 = Segment(1,i,pt[0],pt[1]);
    Segment seg2 = Segment(1,i,pt[0],pt[2]);
    Segment seg3 = Segment(1,i,pt[1],pt[2]);
    if (seg2 < seg1) seg1.v = -1;
    else seg2.v = -1;
    seg3.v = seg1.v;
    add_seg(seg1);
    add_seg(seg2);
    add seq(seq3);
inline int sgn(ll x) { return x < 0 ? -1 : x > 0; }
bool interPnt(Point p1, Point p2, Point q1, Point q2){
  ll c1 = (p2-p1)\%(q1-p1), c2 = (p2-p1)\%(q2-p1);
  ll c3 = (q2-q1)%(p1-q1), c4 = (q2-q1)%(p2-q1);
return sgn(c1) * sgn(c2) <= 0 and sgn(c3) * sgn(c4)
bool check_error(Segment a, Segment b) {
  if (a.id == b.id) return false;
  return interPnt(a.p,a.q,b.p,b.q);
int solve() {
  Treap::pmem = Treap::mem;
  Treap *rt = &Treap::nil;
  int res = 0;
  for (auto i:allx) {
    for (auto l:seg(i)) {
      int k = get_rank(rt, l);
      Treap *t,*tl,*tm,*tr;
      split(rt,k,tl,tr);
      t = find_rightist(tl);
      if (size(t) and check_error(t->seg,l)) return -1;
      t = find_leftist(tr);
      if (size(t) and check_error(t->seg,l)) return -1;
      rt = merge(tl,tr);
      if (l.p.F == i and l.p.F != l.q.F) {
        k = get_rank(rt, l);
```

```
split(rt,k,tl,tr);
        tm = new (Treap::pmem++) Treap(l);
        rt = merge(merge(tl,tm),tr);
    for (auto l:seg(i)) {
  if (l.q.F == i and l.p.F != l.q.F) {
        Treap *tl,*tm,*tr;
         int k = get_rank(rt, l);
        split(rt,k-1,tl,tm);
         split(tm,1,tm,tr);
        Treap *t1=find_rightist(tl),*t2=find_leftist(tr
        if (size(t1) and size(t2) and check_error(t1->
             seg,t2->seg)) return -1;
        rt = merge(tl,tr);
      }
    res = max(res, rt->presum);
  res++;
  return res;
int main() {
  IOS:
  int cas = 0;
  while (cin >> N) {
    if (N == -1) break;
    REP(i,N) {
      REP(j,3) cin >> ip[i].pt[j].F >> ip[i].pt[j].S;
    predo();
    int ans = solve();
    cout << "Case " << cas << ": ";
if (ans == -1) cout << "ERROR\n";
    else cout << ans << " shades\n";</pre>
  return 0;
```

8.2 Mo-Algorithm on Tree

```
#include < bits / stdc++.h>
using namespace std;
#define IOS ios_base::sync_with_stdio(0); cin.tie(0);
#define SZ(x) ((int)((x).size()))
const int MX = 500005;
const int SQ = 1400;
const int LOG = 17;
struct BIT {
  int bit[MX];
  int lb(int x) { return x & -x; }
  void add(int p, int v) {
    for (int i=p; i<MX; i+=lb(i)) bit[i] += v;</pre>
  int qry() {
    int v = 0:
    for (int i=1<<LOG; i>0; i>>=1) {
      if ((v|i) < MX and bit[v|i]==i) v |= i;</pre>
    return v;
}bit;
struct Query {
  int l,r,qid;
}qry[MX];
struct Edge {
 int v,x;
int N,Q,timestamp[MX],ans[MX];
int in[MX],cnt[MX];
vector < Edge > E[MX];
vector < Edge > seq;
void DFS(int u, int f) {
  timestamp[u] = SZ(seq);
  for (auto it:E[u]) {
```

```
if (it.v == f) continue:
     seq.push_back(it);
     DFS(it.v,u);
     seq.push_back(it);
}
 void poke(int id) {
   int v = seq[id].v;
   int x = seq[id].x;
   in[v] ^= 1;
   cnt[x] += in[v] ? 1 : -1;
   if (in[v] \text{ and } cnt[x] == 1) bit.add(x, 1);
   if (!in[v] \text{ and } cnt[x] == 0) \text{ bit.add}(x, -1);
int main() {
  IOS;
   cin >> N >> Q;
   for (int i=0; i<N-1; i++) {</pre>
     int u,v,x;
     cin >> u >> v >> x;
     x = min(x,N);
     E[u].push_back({v,x});
     E[v].push_back({u,x});
   DFS(1,1);
   for (int i=1; i<=Q; i++) {</pre>
     int u,v;
     cin >> u >> v;
     int l = timestamp[u], r = timestamp[v];
     if (l > r) swap(l,r);
     qry[i] = {l,r,i};
   }
   sort(qry+1,qry+1+Q, [](Query a, Query b) {
       return make_pair(a.l/SQ,a.r) < make_pair(b.l/SQ,b</pre>
            .r);
   int curL = 1, curR = 0;
   for (int i=1; i<=Q; i++) {</pre>
     int ql=qry[i].l,qr=qry[i].r;
     while (curL > ql) poke(--curL);
     while (curR < qr) poke(++curR);</pre>
     while (curL < ql) poke(curL++);</pre>
     while (curR > qr) poke(curR--);
     ans[qry[i].qid] = bit.qry();
   for (int i=1; i<=Q; i++) cout << ans[i] << "\n";</pre>
   return 0;
}
```

8.3 Manhattan MST

```
#include < bits / stdc++.h>
#define REP(i,n) for(int i=0;i<n;i++)</pre>
using namespace std;
typedef long long LL;
const int N=200100;
int n.m:
struct PT {int x,y,z,w,id;}p[N];
inline int dis(const PT &a,const PT &b){return abs(a.x-
    b.x)+abs(a.y-b.y);}
inline bool cpx(const PT &a,const PT &b){return a.x!=b.
    x? a.x>b.x:a.y>b.y;}
inline bool cpz(const PT &a,const PT &b){return a.z<b.z</pre>
struct E{int a,b,c;}e[8*N];
bool operator<(const E&a,const E&b){return a.c<b.c;}</pre>
struct Node{
  int L,R,key;
}node[4*N];
int s[N];
int F(int x){return s[x]==x?x:s[x]=F(s[x]);}
void U(int a,int b){s[F(b)]=F(a);}
void init(int id,int L,int R) {
  node[id]=(Node){L,R,-1};
  if(L==R)return;
  init(id*2,L,(L+R)/2);
  init(id*2+1,(L+R)/2+1,R);
void ins(int id,int x) {
```

```
if(node[id].key==-1 || p[node[id].key].w>p[x].w)node[
      id].key=x;
  if(node[id].L==node[id].R)return;
  if(p[x].z<=(node[id].L+node[id].R)/2)ins(id*2,x);</pre>
  else ins(id*2+1,x);
int Q(int id,int L,int R){
  if(R<node[id].L || L>node[id].R)return -1;
  if(L<=node[id].L && node[id].R<=R)return node[id].key</pre>
  int a=Q(id*2,L,R),b=Q(id*2+1,L,R);
  if(b==-1 || (a!=-1 && p[a].w<p[b].w)) return a;</pre>
  else return b;
void calc() {
  REP(i,n) {
    p[i].z=p[i].y-p[i].x;
    p[i].w=p[i].x+p[i].y;
  sort(p,p+n,cpz);
  int cnt=0,j,k;
  for(int i=0;i<n;i=j){</pre>
    for(j=i+1;p[j].z==p[i].z && j<n;j++);</pre>
    for(k=i,cnt++;k<j;k++)p[k].z=cnt;</pre>
  init(1,1,cnt);
  sort(p,p+n,cpx);
  REP(i,n) {
    j=Q(1,p[i].z,cnt);
    if(j!=-1)e[m++]=(E){p[i].id,p[j].id,dis(p[i],p[j])
    ins(1,i);
  }
LL MST() {
  LL r=0;
  sort(e,e+m);
  REP(i,m) {
    if(F(e[i].a)==F(e[i].b))continue;
    U(e[i].a,e[i].b);
    r+=e[i].c;
  }
  return r;
int main(){
  int ts;
scanf("%d", &ts);
  while (ts--) {
m = 0;
    scanf("%d",&n);
    REP(i,n) {
   scanf("%d%d",&p[i].x,&p[i].y);
      p[i].id=s[i]=i;
    calc();
    REP(i,n)p[i].y= -p[i].y;
    calc();
    REP(i,n)swap(p[i].x,p[i].y);
    calc();
    REP(i,n)p[i].x=-p[i].x;
    calc();
    printf("%lld\n",MST()*2);
  return 0;
}
```

9 Graph paper

