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1 Basic

1.1 vimrc

```
set rnu nu ai cin ts=4 sw=4 mouse=a

map <F9> <ESC>:w<CR>:!clear && g++ "%*" -o "%*" -std=c
++11 -Wall -Wextra -Wconversion -Wshadow -O2 &&
echo success<CR>
map <F10> <ESC>:!./*%*<CR>
```

1.2 IncreaseStackSize

```
//stack resize
asm( "mov %0,%esp\n" ::"g"(mem+10000000) );
//change esp to rsp if 64-bit system

//stack resize (linux)
#include <sys/resource.h>
void increase_stack_size() {
    const rlim_t ks = 64*1024*1024;
    struct rlimit rl;
    int res=getrlimit(RLIMIT_STACK, &rl);
    if(res==0){
        if(rl.rlim_cur<ks){
            rl.rlim_cur=ks;
            res=setrlimit(RLIMIT_STACK, &rl);
        }
    }
}
```

1.3 Default Code

```
#pragma GCC optimize ("O2")
#include<bits/stdc++.h>
#include<unistd.h>
using namespace std;
#define FZ(n) memset((n),0,sizeof(n))
#define FMO(n) memset((n),-1,sizeof(n))
#define F first
#define S second
#define PB push_back
#define ALL(x) begin(x),end(x)
#define SZ(x) ((int)(x).size())
#define IOS ios_base::sync_with_stdio(0); cin.tie(0)
#define REP(i,x) for (int i=0; i<(x); i++)
#define REPI(i,a,b) for (int i=(a); i<=(b); i++)
#ifdef ONLINE_JUDGE
#define FILEIO(name) \
    freopen(name".in", "r", stdin); \
    freopen(name".out", "w", stdout);
#else
#define FILEIO(name)
#endif
#define FILEIO(name)
#endif
template<typename A, typename B>
ostream& operator <<(ostream &s, const pair<A,B> &p) {
    return s<<"("<<p.first<<","<<p.second<<")";
}
template<typename T>
ostream& operator <<(ostream &s, const vector<T> &c) {
    s<<"[";
    for (auto it : c) s << it << " ";
    s<<"]";
    return s;
}
// Let's Fight!

int main() {
    IOS;

    return 0;
}
```

2 Data Structure

2.1 Bigint

```

struct Bigint{
    static const int LEN = 60;
    static const int BIGMOD = 100000;

    int s;
    int vl, v[LEN];
    // vector<int> v;
    Bigint() : s(1) { vl = 0; }
    Bigint(long long a) {
        s = 1; vl = 0;
        if (a < 0) { s = -1; a = -a; }
        while (a) {
            push_back(a % BIGMOD);
            a /= BIGMOD;
        }
    }
    Bigint(string str) {
        s = 1; vl = 0;
        int stPos = 0, num = 0;
        if (!str.empty() && str[0] == '-') {
            stPos = 1;
            s = -1;
        }
        for (int i=SZ(str)-1, q=1; i>=stPos; i--) {
            num += (str[i] - '0') * q;
            if ((q *= 10) >= BIGMOD) {
                push_back(num);
                num = 0; q = 1;
            }
        }
        if (num) push_back(num);
        n();
    }

    int len() const {
        return vl;
        // return SZ(v);
    }
    bool empty() const { return len() == 0; }
    void push_back(int x) {
        v[vl++] = x;
        // v.PB(x);
    }
    void pop_back() {
        vl--;
        // v.pop_back();
    }
    int back() const {
        return v[vl-1];
        // return v.back();
    }
    void n() {
        while (!empty() && !back()) pop_back();
    }
    void resize(int nl) {
        vl = nl;
        fill(v, v+vl, 0);
        // v.resize(nl);
        // fill(ALL(v), 0);
    }

    void print() const {
        if (empty()) { putchar('0'); return; }
        if (s == -1) putchar('-');
        printf("%d", back());
        for (int i=len()-2; i>=0; i--) printf("%.4d", v[i]);
    }
    friend ostream& operator << (ostream& out,
        const Bigint &a) {
        if (a.empty()) { out << "0"; return out; }
        if (a.s == -1) out << "-";
        out << a.back();
        for (int i=a.len()-2; i>=0; i--) {
            char str[10];
            snprintf(str, 5, "%.4d", a.v[i]);
            out << str;
        }
        return out;
    }

    int cp3(const Bigint &b) const {
        if (s != b.s) return s - b.s;
        if (s == -1) return -(*this).cp3(-b);
        if (len() != b.len()) return len()-b.len(); //int
        for (int i=len()-1; i>=0; i--)
            if (v[i] != b.v[i]) return v[i]-b.v[i];

```

```

        return 0;
    }

    bool operator < (const Bigint &b) const { return cp3(b) < 0; }
    bool operator <= (const Bigint &b) const { return cp3(b) <= 0; }
    bool operator == (const Bigint &b) const { return cp3(b) == 0; }
    bool operator != (const Bigint &b) const { return cp3(b) != 0; }
    bool operator > (const Bigint &b) const { return cp3(b) > 0; }
    bool operator >= (const Bigint &b) const { return cp3(b) >= 0; }

    Bigint operator - () const {
        Bigint r = (*this);
        r.s = -r.s;
        return r;
    }

    Bigint operator + (const Bigint &b) const {
        if (s == -1) return -(*this)+(-b);
        if (b.s == -1) return (*this)-(-b);
        Bigint r;
        int nl = max(len(), b.len());
        r.resize(nl + 1);
        for (int i=0; i<nl; i++) {
            if (i < len()) r.v[i] += v[i];
            if (i < b.len()) r.v[i] += b.v[i];
            if (r.v[i] >= BIGMOD) {
                r.v[i+1] += r.v[i] / BIGMOD;
                r.v[i] %= BIGMOD;
            }
        }
        r.n();
        return r;
    }

    Bigint operator - (const Bigint &b) const {
        if (s == -1) return -(*this)-(-b);
        if (b.s == -1) return (*this)+(-b);
        if ((*this) < b) return -(*this)-b;
        Bigint r;
        r.resize(len());
        for (int i=0; i<len(); i++) {
            r.v[i] += v[i];
            if (i < b.len()) r.v[i] -= b.v[i];
            if (r.v[i] < 0) {
                r.v[i] += BIGMOD;
                r.v[i+1]--;
            }
        }
        r.n();
        return r;
    }

    Bigint operator * (const Bigint &b) {
        Bigint r;
        r.resize(len() + b.len() + 1);
        r.s = s * b.s;
        for (int i=0; i<len(); i++) {
            for (int j=0; j<b.len(); j++) {
                r.v[i+j] += v[i] * b.v[j];
                if (r.v[i+j] >= BIGMOD) {
                    r.v[i+j+1] += r.v[i+j] / BIGMOD;
                    r.v[i+j] %= BIGMOD;
                }
            }
        }
        r.n();
        return r;
    }

    Bigint operator / (const Bigint &b) {
        Bigint r;
        r.resize(max(1, len()-b.len()+1));
        int oriS = s;
        Bigint b2 = b; // b2 = abs(b)
        s = b2.s = r.s = 1;
        for (int i=r.len()-1; i>=0; i--) {
            int d=0, u=BIGMOD-1;
            while(d<u) {
                int m = (d+u+1)>>1;
                r.v[i] = m;
                if ((r*b2) > (*this)) u = m-1;
                else d = m;
            }
            r.v[i] = d;

```

```

    }
    s = oriS;
    r.s = s * b.s;
    r.n();
    return r;
}
Bigint operator % (const Bigint &b) {
    return (*this)-(*this)/b*b;
}
};

```

2.2 unordered_map

```

struct Key {
    int first, second;
    Key () {}
    Key (int _x, int _y) : first(_x), second(_y) {}
    bool operator == (const Key &b) const {
        return tie(F,S) == tie(b.F,b.S);
    }
};

struct KeyHasher {
    size_t operator()(const Key& k) const {
        return k.first + k.second*100000;
    }
};

typedef unordered_map<Key,int,KeyHasher> map_t;

int main(int argc, char** argv){
    map_t mp;
    for (int i=0; i<10; i++)
        mp[Key(i,0)] = i+1;
    for (int i=0; i<10; i++)
        printf("%d\n", mp[Key(i,0)]);

    return 0;
}

```

2.3 extc_heap

```

#include <bits/extc++.h>
typedef __gnu_pbds::priority_queue<int> heap_t;
heap_t a,b;

int main() {
    a.clear();
    b.clear();
    a.push(1);
    a.push(3);
    b.push(2);
    b.push(4);
    assert(a.top() == 3);
    assert(b.top() == 4);
    // merge two heap
    a.join(b);
    assert(a.top() == 4);
    assert(b.empty());

    return 0;
}

```

2.4 extc_balance_tree

```

#include <ext/pb_ds/assoc_container.hpp>
using namespace std;
using namespace __gnu_pbds;
typedef tree<int,null_type,less<int>,rb_tree_tag,
    tree_order_statistics_node_update> set_t;

#include <ext/pb_ds/assoc_container.hpp>
using namespace __gnu_pbds;
typedef cc_hash_table<int,int> umap_t;

int main()
{
    // Insert some entries into s.
    set_t s;
    s.insert(12);

```

```

s.insert(505);

// The order of the keys should be: 12, 505.
assert(*s.find_by_order(0) == 12);
assert(s.find_by_order(2) == end(s));

// The order of the keys should be: 12, 505.
assert(s.order_of_key(12) == 0);
assert(s.order_of_key(505) == 1);

// Erase an entry.
s.erase(12);

// The order of the keys should be: 505.
assert(*s.find_by_order(0) == 505);

// The order of the keys should be: 505.
assert(s.order_of_key(505) == 0);
}

```

2.5 Disjoint Set

```

struct DisjointSet {
    // save() is like recursive
    // undo() is like return
    int n, fa[MXN], sz[MXN];
    vector<pair<int*,int*>> h;
    vector<int> sp;
    void init(int tn) {
        n=tn;
        for (int i=0; i<n; i++) {
            fa[i]=i;
            sz[i]=1;
        }
        sp.clear(); h.clear();
    }
    void assign(int *k, int v) {
        h.PB({k, *k});
        *k=v;
    }
    void save() { sp.PB(SZ(h)); }
    void undo() {
        assert(!sp.empty());
        int last=sp.back(); sp.pop_back();
        while (SZ(h)!=last) {
            auto x=h.back(); h.pop_back();
            *x.F=x.S;
        }
    }
    int f(int x) {
        while (fa[x]!=x) x=fa[x];
        return x;
    }
    void uni(int x, int y) {
        x=f(x); y=f(y);
        if (x==y) return;
        if (sz[x]<sz[y]) swap(x, y);
        assign(&sz[x], sz[x]+sz[y]);
        assign(&fa[y], x);
    }
}djs;

```

2.6 Treap

```

const int MEM = 160000004;
struct Treap {
    static Treap nil, mem[MEM], *pmem;
    Treap *l, *r;
    char val;
    int size;
    Treap () : l(&nil), r(&nil), size(0) {}
    Treap (char _val) :
        l(&nil), r(&nil), val(_val), size(1) {}
} Treap::nil, Treap::mem[MEM], *Treap::pmem = Treap::
    mem;

int size(const Treap *t) { return t->size; }
void pull(Treap *t) {
    if (!size(t)) return;
    t->size = size(t->l) + size(t->r) + 1;
}
Treap* merge(Treap *a, Treap *b) {

```

```

if (!size(a)) return b;
if (!size(b)) return a;
Treap *t;
if (rand() % (size(a) + size(b)) < size(a)) {
    t = new (Treap::pmem++) Treap(*a);
    t->r = merge(a->r, b);
} else {
    t = new (Treap::pmem++) Treap(*b);
    t->l = merge(a, b->l);
}
pull(t);
return t;
}

void split(Treap *t, int k, Treap *&a, Treap *&b) {
    if (!size(t)) a = b = &Treap::nil;
    else if (size(t->l) + 1 <= k) {
        a = new (Treap::pmem++) Treap(*t);
        split(t->r, k - size(t->l) - 1, a->r, b);
        pull(a);
    } else {
        b = new (Treap::pmem++) Treap(*t);
        split(t->l, k, a, b->l);
        pull(b);
    }
}

int nv;
Treap *rt[50005];

void print(const Treap *t) {
    if (!size(t)) return;
    print(t->l);
    cout << t->val;
    print(t->r);
}

int main(int argc, char** argv) {
    IOS;
    rt[nv=0] = &Treap::nil;
    Treap::pmem = Treap::mem;
    int Q, cmd, p, c, v;
    string s;
    cin >> Q;
    while (Q--) {
        cin >> cmd;
        if (cmd == 1) {
            // insert string s after position p
            cin >> p >> s;
            Treap *tl, *tr;
            split(rt[nv], p, tl, tr);
            for (int i=0; i<SZ(s); i++)
                tl = merge(tl, new (Treap::pmem++) Treap(s[i]));
            rt[++nv] = merge(tl, tr);
        } else if (cmd == 2) {
            // remove c characters starting at position
            Treap *tl, *tm, *tr;
            cin >> p >> c;
            split(rt[nv], p-1, tl, tm);
            split(tm, c, tm, tr);
            rt[++nv] = merge(tl, tr);
        } else if (cmd == 3) {
            // print c characters starting at position p, in
            // version v
            Treap *tl, *tm, *tr;
            cin >> v >> p >> c;
            split(rt[v], p-1, tl, tm);
            split(tm, c, tm, tr);
            print(tm);
            cout << "\n";
        }
    }
    return 0;
}

```

2.7 Heavy Light Decomposition

```

// only one segment tree / 0-base
// should call init after input N
// getPathSeg return the segment in order u->v
// fa[root] = root

typedef pair<int,int> pii;

```

```

int N, fa[MXN], belong[MXN], dep[MXN], sz[MXN], que[MXN];
int step, line[MXN], stPt[MXN], edPt[MXN];
vector<int> E[MXN], chain[MXN];

void init() {
    REP(i, N) {
        E[i].clear();
        chain[i].clear();
    }
}

void DFS(int u) {
    vector<int> &c = chain[belong[u]];
    for (int i=c.size()-1; i>=0; i--) {
        int v = c[i];
        stPt[v] = step;
        line[step++] = v;
    }
    for (int i=0; i<(int)c.size(); i++) {
        u = c[i];
        for (auto v : E[u]) {
            if (fa[u] == v || (i && v == c[i-1])) continue;
            DFS(v);
        }
        edPt[u] = step-1;
    }
}

void build_chain(int st) {
    int fr, bk;
    fr=bk=0; que[bk++] = st; fa[st]=st; dep[st]=0;
    while (fr < bk) {
        int u=que[fr++];
        for (auto v : E[u]) {
            if (v == fa[u]) continue;
            que[bk++] = v;
            dep[v] = dep[u]+1;
            fa[v] = u;
        }
    }
    for (int i=bk-1, u, pos; i>=0; i--) {
        u = que[i]; sz[u] = 1; pos = -1;
        for (auto v : E[u]) {
            if (v == fa[u]) continue;
            sz[u] += sz[v];
            if (pos == -1 || sz[v] > sz[pos]) pos = v;
        }
        if (pos == -1) belong[u] = u;
        else belong[u] = belong[pos];
        chain[belong[u]].PB(u);
    }
    step = 0;
    DFS(st);
}

int getLCA(int u, int v) {
    while (belong[u] != belong[v]) {
        int a = chain[belong[u]].back();
        int b = chain[belong[v]].back();
        if (dep[a] > dep[b]) u = fa[a];
        else v = fa[b];
    }
    return sz[u] >= sz[v] ? u : v;
}

vector<pii> getPathSeg(int u, int v) {
    vector<pii> ret1, ret2;
    while (belong[u] != belong[v]) {
        int a = chain[belong[u]].back();
        int b = chain[belong[v]].back();
        if (dep[a] > dep[b]) {
            ret1.PB({stPt[a], stPt[u]});
            u = fa[a];
        } else {
            ret2.PB({stPt[b], stPt[v]});
            v = fa[b];
        }
    }
    if (dep[u] > dep[v]) swap(u, v);
    ret1.PB({stPt[u], stPt[v]});
    reverse(ret2.begin(), ret2.end());
    ret1.insert(ret1.end(), ret2.begin(), ret2.end());
    return ret1;
}

// Usage
void build() {
    build_chain(0); //change root
    init(0, step, 0); //init segment tree
}

int get_answer(int u, int v) {

```

```

int ret = -2147483647;
vector<pii> vec = getPathSeg(u,v);
for (auto it : vec)
    ; // check answer with segment [it.F, it.S]
return ret;
}

```

2.8 Link-Cut Tree

```

const int MXN = 100005;
const int MEM = 100005;

struct Splay {
    static Splay nil, mem[MEM], *pmem;
    Splay *ch[2], *f;
    int val, rev, size;
    Splay () : val(-1), rev(0), size(0) {
        f = ch[0] = ch[1] = &nil;
    }
    Splay (int _val) : val(_val), rev(0), size(1) {
        f = ch[0] = ch[1] = &nil;
    }
    bool isr() {
        return f->ch[0] != this && f->ch[1] != this;
    }
    int dir() {
        return f->ch[0] == this ? 0 : 1;
    }
    void setCh(Splay *c, int d) {
        ch[d] = c;
        if (c != &nil) c->f = this;
        pull();
    }
    void push() {
        if (rev) {
            swap(ch[0], ch[1]);
            if (ch[0] != &nil) ch[0]->rev ^= 1;
            if (ch[1] != &nil) ch[1]->rev ^= 1;
            rev=0;
        }
    }
    void pull() {
        size = ch[0]->size + ch[1]->size + 1;
        if (ch[0] != &nil) ch[0]->f = this;
        if (ch[1] != &nil) ch[1]->f = this;
    }
} Splay::nil, Splay::mem[MEM], *Splay::pmem = Splay::mem;
Splay *nil = &Splay::nil;

void rotate(Splay *x) {
    Splay *p = x->f;
    int d = x->dir();
    if (!p->isr()) p->f->setCh(x, p->dir());
    else x->f = p->f;
    p->setCh(x->ch[!d], d);
    x->setCh(p, !d);
    p->pull(); x->pull();
}

vector<Splay*> splayVec;
void splay(Splay *x) {
    splayVec.clear();
    for (Splay *q=x;; q=q->f) {
        splayVec.push_back(q);
        if (q->isr()) break;
    }
    reverse(begin(splayVec), end(splayVec));
    for (auto it : splayVec) it->push();
    while (!x->isr()) {
        if (x->f->isr()) rotate(x);
        else if (x->dir()==x->f->dir()) rotate(x->f), rotate(x);
        else rotate(x), rotate(x);
    }
}

Splay* access(Splay *x) {
    Splay *q = nil;
    for (;x!=nil;x=x->f) {
        splay(x);
        x->setCh(q, 1);
        q = x;
    }
}

```

```

return q;
}

void evert(Splay *x) {
    access(x);
    splay(x);
    x->rev ^= 1;
    x->push(); x->pull();
}

void link(Splay *x, Splay *y) {
    // evert(x);
    access(x);
    splay(x);
    evert(y);
    x->setCh(y, 1);
}

void cut(Splay *x, Splay *y) {
    // evert(x);
    access(y);
    splay(y);
    y->push();
    y->ch[0] = y->ch[0]->f = nil;
}

int N, Q;
Splay *vt[MXN];

int ask(Splay *x, Splay *y) {
    access(x);
    access(y);
    splay(x);
    int res = x->f->val;
    if (res == -1) res=x->val;
    return res;
}

int main(int argc, char** argv) {
    scanf("%d%d", &N, &Q);
    for (int i=1; i<=N; i++)
        vt[i] = new (Splay::pmem++) Splay(i);
    while (Q--) {
        char cmd[105];
        int u, v;
        scanf("%s", cmd);
        if (cmd[1] == 'i') {
            scanf("%d%d", &u, &v);
            link(vt[u], vt[v]);
        } else if (cmd[0] == 'c') {
            scanf("%d", &v);
            cut(vt[1], vt[v]);
        } else {
            scanf("%d%d", &u, &v);
            int res=ask(vt[u], vt[v]);
            printf("%d\n", res);
        }
    }

    return 0;
}

```

3 Graph

3.1 BCC Edge

```

struct BccEdge {
    static const int MXN = 100005;
    struct Edge { int v,eid; };
    int n,m,step,par[MXN],dfn[MXN],low[MXN];
    vector<Edge> E[MXN];
    DisjointSet djs;
    void init(int _n) {
        n = _n; m = 0;
        for (int i=0; i<n; i++) E[i].clear();
        djs.init(n);
    }
    void add_edge(int u, int v) {
        E[u].PB({v, m});
        E[v].PB({u, m});
        m++;
    }
    void DFS(int u, int f, int f_eid) {
        par[u] = f;
        dfn[u] = low[u] = step++;
        for (auto it:E[u]) {

```

```

    if (it.eid == f_eid) continue;
    int v = it.v;
    if (dfn[v] == -1) {
        DFS(v, u, it.eid);
        low[u] = min(low[u], low[v]);
    } else {
        low[u] = min(low[u], dfn[v]);
    }
}
}
void solve() {
    step = 0;
    memset(dfn, -1, sizeof(int)*n);
    for (int i=0; i<n; i++) {
        if (dfn[i] == -1) DFS(i, i, -1);
    }
    djs.init(n);
    for (int i=0; i<n; i++) {
        if (low[i] < dfn[i]) djs.uni(i, par[i]);
    }
}
}graph;

```

3.2 BCC Vertex

```

struct BccVertex {
    int n, nBcc, step, root, dfn[MXN], low[MXN];
    vector<int> E[MXN], ap;
    vector<pii> bcc[MXN];
    int top;
    pii stk[MXN];
    void init(int _n) {
        n = _n;
        nBcc = step = 0;
        for (int i=0; i<n; i++) E[i].clear();
    }
    void add_edge(int u, int v) {
        E[u].PB(v);
        E[v].PB(u);
    }
    void DFS(int u, int f) {
        dfn[u] = low[u] = step++;
        int son = 0;
        for (auto v:E[u]) {
            if (v == f) continue;
            if (dfn[v] == -1) {
                son++;
                stk[top++] = {u,v};
                DFS(v,u);
                if (low[v] >= dfn[u]) {
                    if (v != root) ap.PB(v);
                    do {
                        assert(top > 0);
                        bcc[nBcc].PB(stk[--top]);
                    } while (stk[top] != pii(u,v));
                    nBcc++;
                }
                low[u] = min(low[u], low[v]);
            } else {
                if (dfn[v] < dfn[u]) stk[top++] = pii(u,v);
                low[u] = min(low[u], dfn[v]);
            }
        }
        if (u == root && son > 1) ap.PB(u);
    }
    // return the edges of each bcc;
    vector<vector<pii>> solve() {
        vector<vector<pii>> res;
        for (int i=0; i<n; i++) {
            dfn[i] = low[i] = -1;
        }
        ap.clear();
        for (int i=0; i<n; i++) {
            if (dfn[i] == -1) {
                top = 0;
                root = i;
                DFS(i,i);
            }
        }
        REP(i, nBcc) res.PB(bcc[i]);
        return res;
    }
}graph;

```

3.3 Strongly Connected Components

```

struct Scc{
    int n, nScc, vst[MXN], bln[MXN];
    vector<int> E[MXN], rE[MXN], vec;
    void init(int _n){
        n = _n;
        for (int i=0; i<n; i++){
            E[i].clear();
            rE[i].clear();
        }
    }
    void add_edge(int u, int v){
        E[u].PB(v);
        rE[v].PB(u);
    }
    void DFS(int u){
        vst[u]=1;
        for (auto v : E[u])
            if (!vst[v]) DFS(v);
        vec.PB(u);
    }
    void rDFS(int u){
        vst[u] = 1;
        bln[u] = nScc;
        for (auto v : rE[u])
            if (!vst[v]) rDFS(v);
    }
    void solve(){
        nScc = 0;
        vec.clear();
        for (int i=0; i<n; i++) vst[i] = 0;
        for (int i=0; i<n; i++)
            if (!vst[i]) DFS(i);
        reverse(vec.begin(), vec.end());
        for (int i=0; i<n; i++) vst[i] = 0;
        for (auto v : vec){
            if (!vst[v]){
                rDFS(v);
                nScc++;
            }
        }
    }
};

```

3.4 DMST_with_sol

```

const int INF = 1029384756;

struct edge_t{
    int u,v,w;
    set< pair<int,int> > add, sub;
    edge_t() : u(-1), v(-1), w(0) {}
    edge_t(int _u, int _v, int _w) {
        u = _u; v = _v; w = _w;
        add.insert({u, v});
    }
    edge_t& operator += (const edge_t& obj) {
        w += obj.w;
        FOR (it, obj.add) {
            if (!sub.count(*it)) add.insert(*it);
            else sub.erase(*it);
        }
        FOR (it, obj.sub) {
            if (!add.count(*it)) sub.insert(*it);
            else add.erase(*it);
        }
        return *this;
    }
    edge_t& operator -= (const edge_t& obj) {
        w -= obj.w;
        FOR (it, obj.sub) {
            if (!sub.count(*it)) add.insert(*it);
            else sub.erase(*it);
        }
        for (auto it : obj.add) {
            if (!add.count(it)) sub.insert(it);
            else add.erase(it);
        }
        return *this;
    }
};
eg[MXN*MXN], prv[MXN], EDGE_INF(-1,-1,INF);
int N,M;

```

```

int cid, incyc[MXN], contracted[MXN];
vector<int> E[MXN];

edge_t dmst(int rt){
    edge_t cost;
    for (int i=0; i<N; i++){
        contracted[i] = incyc[i] = 0;
        prv[i] = EDGE_INF;
    }
    cid = 0;
    int u, v;
    while (true){
        for (v=0; v<N; v++){
            if (v != rt && !contracted[v] && prv[v].w == INF)
                break;
        }
        if (v >= N) break; // end
        for (int i=0; i<M; i++){
            if (eg[i].v == v && eg[i].w < prv[v].w)
                prv[v] = eg[i];
        }
        if (prv[v].w == INF) // not connected
            return EDGE_INF;
        cost += prv[v];
        for (u=prv[v].u; u!=v && u!=-1; u=prv[u].u);
        if (u == -1) continue;
        incyc[v] = ++cid;
        for (u=prv[v].u; u!=v; u=prv[u].u){
            contracted[u] = 1;
            incyc[u] = cid;
        }
        for (int i=0; i<M; i++){
            if (incyc[eg[i].u] != cid && incyc[eg[i].v] == cid){
                eg[i] -= prv[eg[i].v];
            }
        }
        for (int i=0; i<M; i++){
            if (incyc[eg[i].u] == cid) eg[i].u = v;
            if (incyc[eg[i].v] == cid) eg[i].v = v;
            if (eg[i].u == eg[i].v) eg[i--] = eg[--M];
        }
        for (int i=0; i<N; i++){
            if (contracted[i]) continue;
            if (prv[i].u >= 0 && incyc[prv[i].u] == cid)
                prv[i].u = v;
        }
        prv[v] = EDGE_INF;
    }
    return cost;
}

void solve(){
    edge_t cost = dmst(0);
    for (auto it : cost.add){ // find a solution
        E[it.F].PB(it.S);
        prv[it.S] = edge_t(it.F, it.S, 0);
    }
}

```

```

if(cmp(sdom[mn[mom[u]]], sdom[mn[u]]))
    mn[u] = mn[mom[u]];
return mom[u] = res;
}

void init(int _n, int _s) {
    n = _n;
    s = _s;
    REP1(i, 1, n) {
        g[i].clear();
        pred[i].clear();
        idom[i] = 0;
    }
}

void add_edge(int u, int v) {
    g[u].push_back(v);
    pred[v].push_back(u);
}

void DFS(int u) {
    ts++;
    dfn[u] = ts;
    nfd[ts] = u;
    for(int v:g[u]) if(dfn[v] == 0) {
        par[v] = u;
        DFS(v);
    }
}

void build() {
    ts = 0;
    REP1(i, 1, n) {
        dfn[i] = nfd[i] = 0;
        cov[i].clear();
        mom[i] = mn[i] = sdom[i] = i;
    }
    DFS(s);
    for (int i=ts; i>=2; i--) {
        int u = nfd[i];
        if(u == 0) continue;
        for(int v:pred[u]) if(dfn[v]) {
            eval(v);
            if(cmp(sdom[mn[v]], sdom[u])) sdom[u] = sdom[mn[v]];
        }
        cov[sdom[u]].push_back(u);
        mom[u] = par[u];
        for(int w:cov[par[u]]) {
            eval(w);
            if(cmp(sdom[mn[w]], par[u])) idom[w] = mn[w];
            else idom[w] = par[u];
        }
        cov[par[u]].clear();
    }
    REP1(i, 2, ts) {
        int u = nfd[i];
        if(u == 0) continue;
        if(idom[u] != sdom[u]) idom[u] = idom[idom[u]];
    }
}
}dom;

```

3.5 Dominator Tree

```

// idom[n] is the unique node that strictly dominates n
// but does
// not strictly dominate any other node that strictly
// dominates n.
// idom[n] = 0 if n is entry or the entry cannot reach
// n.
struct DominatorTree{
    static const int MAXN = 200010;
    int n, s;
    vector<int> g[MAXN], pred[MAXN];
    vector<int> cov[MAXN];
    int dfn[MAXN], nfd[MAXN], ts;
    int par[MAXN];
    int sdom[MAXN], idom[MAXN];
    int mom[MAXN], mn[MAXN];

    inline bool cmp(int u, int v) { return dfn[u] < dfn[v]; }

    int eval(int u) {
        if(mom[u] == u) return u;
        int res = eval(mom[u]);
    }
}

```

3.6 Maximum Clique

```

class MaxClique {
public:
    static const int MV = 210;

    int V;
    int el[MV][MV/30+1];
    int dp[MV];
    int ans;
    int s[MV][MV/30+1];
    vector<int> sol;

    void init(int v) {
        V = v; ans = 0;
        FZ(el); FZ(dp);
    }

    /* Zero Base */
    void addEdge(int u, int v) {
        if(u > v) swap(u, v);
        if(u == v) return;
        el[u][v/32] |= (1<<(v%32));
    }
}

```



```

bool dfs(int v, int k) {
    int c = 0, d = 0;
    for(int i=0; i<(V+31)/32; i++) {
        s[k][i] = eL[v][i];
        if(k != 1) s[k][i] &= s[k-1][i];
        c += __builtin_popcount(s[k][i]);
    }
    if(c == 0) {
        if(k > ans) {
            ans = k;
            sol.clear();
            sol.push_back(v);
            return 1;
        }
        return 0;
    }
    for(int i=0; i<(V+31)/32; i++) {
        for(int a = s[k][i]; a; a += d) {
            if(k + (c-d) <= ans) return 0;
            int lb = a & (-a), lg = 0;
            a ^= lb;
            while(lb!=1) {
                lb = (unsigned int)(lb) >> 1;
                lg++;
            }
            int u = i*32 + lg;
            if(k + dp[u] <= ans) return 0;
            if(dfs(u, k+1)) {
                sol.push_back(v);
                return 1;
            }
        }
    }
    return 0;
}

int solve() {
    for(int i=V-1; i>=0; i--) {
        dfs(i, 1);
        dp[i] = ans;
    }
    return ans;
}
};

```

3.7 MinimumMeanCycle

```

/* minimum mean cycle */
const int MAXE = 1805;
const int MAXN = 35;
const double inf = 1029384756;
const double eps = 1e-6;
struct Edge {
    int v,u;
    double c;
};
int n,m,prv[MAXN][MAXN], prve[MAXN][MAXN], vst[MAXN];
Edge e[MAXE];
vector<int> edgeID, cycle, rho;
double d[MAXN][MAXN];
inline void bellman_ford() {
    for(int i=0; i<n; i++) d[0][i]=0;
    for(int i=0; i<n; i++) {
        fill(d[i+1], d[i+1]+n, inf);
        for(int j=0; j<m; j++) {
            int v = e[j].v, u = e[j].u;
            if(d[i][v]<inf && d[i+1][u]>d[i][v]+e[j].c) {
                d[i+1][u] = d[i][v]+e[j].c;
                prv[i+1][u] = v;
                prve[i+1][u] = j;
            }
        }
    }
}

double karp_mmc() {
    // returns inf if no cycle, mmc otherwise
    double mmc=inf;
    int st = -1;
    bellman_ford();
    for(int i=0; i<n; i++) {
        double avg=-inf;
        for(int k=0; k<n; k++) {

```

```

            if(d[n][i]<inf-eps) avg=max(avg,(d[n][i]-d[k][i])/(n-k));
            else avg=max(avg,inf);
        }
        if (avg < mmc) tie(mmc, st) = tie(avg, i);
    }
    for(int i=0; i<n; i++) vst[i] = 0;
    edgeID.clear(); cycle.clear(); rho.clear();
    for (int i=n; !vst[st]; st=prv[i--][st]) {
        vst[st]++;
        edgeID.PB(prve[i][st]);
        rho.PB(st);
    }
    while (vst[st] != 2) {
        int v = rho.back(); rho.pop_back();
        cycle.PB(v);
        vst[v]++;
    }
    reverse(ALL(edgeID));
    edgeID.resize(SZ(cycle));
    return mmc;
}

```

4 Flow

4.1 Dinic

```

struct Dinic{
    static const int MXN = 10000;
    struct Edge{ int v,f,re; };
    int n,s,t,level[MXN];
    vector<Edge> E[MXN];
    void init(int _n, int _s, int _t){
        n = _n; s = _s; t = _t;
        for (int i=0; i<n; i++) E[i].clear();
    }
    void add_edge(int u, int v, int f){
        E[u].PB({v,f,SZ(E[v])});
        E[v].PB({u,0,SZ(E[u])-1});
    }
    bool BFS(){
        for (int i=0; i<n; i++) level[i] = -1;
        queue<int> que;
        que.push(s);
        level[s] = 0;
        while (!que.empty()){
            int u = que.front(); que.pop();
            for (auto it : E[u]){
                if (it.f > 0 && level[it.v] == -1){
                    level[it.v] = level[u]+1;
                    que.push(it.v);
                }
            }
        }
        return level[t] != -1;
    }
    int DFS(int u, int nf){
        if (u == t) return nf;
        int res = 0;
        for (auto &it : E[u]){
            if (it.f > 0 && level[it.v] == level[u]+1){
                int tf = DFS(it.v, min(nf,it.f));
                res += tf; nf -= tf; it.f -= tf;
                E[it.v][it.re].f += tf;
                if (nf == 0) return res;
            }
        }
        if (!res) level[u] = -1;
        return res;
    }
    int flow(int res=0){
        while (BFS())
            res += DFS(s,2147483647);
        return res;
    }
}flow;

```

4.2 Cost Flow


```

typedef pair<long long, long long> pll;
struct CostFlow {
    static const int MXN = 205;
    static const long long INF = 102938475610293847LL;
    struct Edge {
        int v, r;
        long long f, c;
    };
    int n, s, t, prv[MXN], prvl[MXN], inq[MXN];
    long long dis[MXN], fl, cost;
    vector<Edge> E[MXN];
    void init(int _n, int _s, int _t) {
        n = _n; s = _s; t = _t;
        for (int i=0; i<n; i++) E[i].clear();
        fl = cost = 0;
    }
    void add_edge(int u, int v, long long f, long long c) {
        E[u].PB({v, SZ(E[v])}, f, c);
        E[v].PB({u, SZ(E[u])-1, 0, -c});
    }
    pll flow() {
        while (true) {
            for (int i=0; i<n; i++) {
                dis[i] = INF;
                inq[i] = 0;
            }
            dis[s] = 0;
            queue<int> que;
            que.push(s);
            while (!que.empty()) {
                int u = que.front(); que.pop();
                inq[u] = 0;
                for (int i=0; i<SZ(E[u]); i++) {
                    int v = E[u][i].v;
                    long long w = E[u][i].c;
                    if (E[u][i].f > 0 && dis[v] > dis[u] + w) {
                        prv[v] = u; prvl[v] = i;
                        dis[v] = dis[u] + w;
                        if (!inq[v]) {
                            inq[v] = 1;
                            que.push(v);
                        }
                    }
                }
            }
            if (dis[t] == INF) break;
            long long tf = INF;
            for (int v=t, u, l; v!=s; v=u) {
                u=prv[v]; l=prvl[v];
                tf = min(tf, E[u][l].f);
            }
            for (int v=t, u, l; v!=s; v=u) {
                u=prv[v]; l=prvl[v];
                E[u][l].f -= tf;
                E[v][E[u][l].r].f += tf;
            }
            cost += tf * dis[t];
            fl += tf;
        }
        return {fl, cost};
    }
} flow;

```

4.3 Kuhn Munkres

```

struct KM{
    // Maximum Bipartite Weighted Matching (Perfect Match)
    static const int MXN = 650;
    static const int INF = 2147483647; // long long
    int n, match[MXN], vx[MXN], vy[MXN];
    int edge[MXN][MXN], lx[MXN], ly[MXN], slack[MXN];
    // ^^^^ long long
    void init(int _n){
        n = _n;
        for (int i=0; i<n; i++)
            for (int j=0; j<n; j++)
                edge[i][j] = 0;
    }
    void add_edge(int x, int y, int w){ // long long
        edge[x][y] = w;
    }
    bool DFS(int x){
        vx[x] = 1;

```

```

        for (int y=0; y<n; y++){
            if (vy[y]) continue;
            if (lx[x]+ly[y] > edge[x][y]){
                slack[y] = min(slack[y], lx[x]+ly[y]-edge[x][y]);
            }
            else {
                vy[y] = 1;
                if (match[y] == -1 || DFS(match[y])){
                    match[y] = x;
                    return true;
                }
            }
        }
        return false;
    }
    int solve(){
        fill(match, match+n, -1);
        fill(lx, lx+n, -INF);
        fill(ly, ly+n, 0);
        for (int i=0; i<n; i++){
            for (int j=0; j<n; j++)
                lx[i] = max(lx[i], edge[i][j]);
            for (int i=0; i<n; i++){
                fill(slack, slack+n, INF);
                while (true){
                    fill(vx, vx+n, 0);
                    fill(vy, vy+n, 0);
                    if (DFS(i)) break;
                    int d = INF; // long long
                    for (int j=0; j<n; j++)
                        if (!vy[j]) d = min(d, slack[j]);
                    for (int j=0; j<n; j++){
                        if (vx[j]) lx[j] -= d;
                        if (vy[j]) ly[j] += d;
                        else slack[j] -= d;
                    }
                }
            }
            int res=0;
            for (int i=0; i<n; i++)
                res += edge[match[i]][i];
            return res;
        }
    }
} graph;

```

4.4 SW-Mincut

```

struct SW{ // O(V^3) 0-base
    static const int MXN = 514;
    int n, vst[MXN], del[MXN];
    int edge[MXN][MXN], wei[MXN];
    void init(int _n){
        n = _n;
        for (int i=0; i<n; i++) {
            for (int j=0; j<n; j++)
                edge[i][j] = 0;
            del[i] = 0;
        }
    }
    void add_edge(int u, int v, int w){
        edge[u][v] += w;
        edge[v][u] += w;
    }
    void search(int &s, int &t){
        for (int i=0; i<n; i++)
            vst[i] = wei[i] = 0;
        s = t = -1;
        while (true){
            int mx=-1, cur=0;
            for (int i=0; i<n; i++)
                if (!del[i] && !vst[i] && mx<wei[i])
                    cur = i, mx = wei[i];
            if (mx == -1) break;
            vst[cur] = 1;
            s = t;
            t = cur;
            for (int i=0; i<n; i++)
                if (!vst[i] && !del[i]) wei[i] += edge[cur][i];
        }
    }
    int solve(){
        int res = 2147483647;
        for (int i=0, x, y; i<n-1; i++){
            search(x, y);

```

```

    res = min(res,wei[y]);
    del[y] = 1;
    for (int j=0; j<n; j++)
        edge[x][j] = (edge[j][x] += edge[y][j]);
    }
    return res;
}
}graph;

```

4.5 Maximum Simple Graph Matching

```

struct GenMatch { // 1-base
    static const int MAXN = 514;
    int V;
    bool el[MAXN][MAXN];
    int pr[MAXN];
    bool inq[MAXN],inp[MAXN],inb[MAXN];
    queue<int> qe;
    int st,ed;
    int nb;
    int bk[MAXN],djs[MAXN];
    int ans;
    void init(int _V) {
        V = _V;
        for(int i = 0; i <= V; i++) {
            for(int j = 0; j <= V; j++) el[i][j] = 0;
            pr[i] = bk[i] = djs[i] = 0;
            inq[i] = inp[i] = inb[i] = 0;
        }
        ans = 0;
    }
    void add_edge(int u, int v) {
        el[u][v] = el[v][u] = 1;
    }
    int lca(int u,int v) {
        for(int i = 0; i <= V; i++) inp[i] = 0;
        while(1) {
            u = djs[u];
            inp[u] = true;
            if(u == st) break;
            u = bk[pr[u]];
        }
        while(1) {
            v = djs[v];
            if(inp[v]) return v;
            v = bk[pr[v]];
        }
        return v;
    }
    void upd(int u) {
        int v;
        while(djs[u] != nb) {
            v = pr[u];
            inb[djs[u]] = inb[djs[v]] = true;
            u = bk[v];
            if(djs[u] != nb) bk[u] = v;
        }
    }
    void blo(int u,int v) {
        nb = lca(u,v);
        for (int i=0; i<=V; i++) inb[i] = 0;
        upd(u); upd(v);
        if(djs[u] != nb) bk[u] = v;
        if(djs[v] != nb) bk[v] = u;
        for(int tu = 1; tu <= V; tu++)
            if(inb[djs[tu]]) {
                djs[tu] = nb;
                if(!inq[tu]){
                    qe.push(tu);
                    inq[tu] = 1;
                }
            }
    }
    void flow() {
        for(int i = 1; i <= V; i++) {
            inq[i] = 0;
            bk[i] = 0;
            djs[i] = i;
        }

        while(qe.size()) qe.pop();
        qe.push(st);
        inq[st] = 1;
        ed = 0;
    }

```

```

        while(qe.size()) {
            int u = qe.front(); qe.pop();
            for(int v = 1; v <= V; v++)
                if(el[u][v] && (djs[u] != djs[v]) && (pr[u] != v)) {
                    if((v == st) || ((pr[v] > 0) && bk[pr[v]] > 0))
                        blo(u,v);
                    else if(bk[v] == 0) {
                        bk[v] = u;
                        if(pr[v] > 0) {
                            if(!inq[pr[v]]) qe.push(pr[v]);
                        } else {
                            ed = v;
                            return;
                        }
                    }
                }
        }
    }
void aug() {
    int u,v,w;
    u = ed;
    while(u > 0) {
        v = bk[u];
        w = pr[v];
        pr[v] = u;
        pr[u] = v;
        u = w;
    }
}
int solve() {
    for(int i = 0; i <= V; i++) pr[i] = 0;
    for(int u = 1; u <= V; u++)
        if(pr[u] == 0) {
            st = u;
            flow();
            if(ed > 0) {
                aug();
                ans ++;
            }
        }
    return ans;
}
}G;
int main() {
    G.init(V);
    for(int i=0; i<E; i++) {
        int u, v;
        cin >> u >> v;
        G.add_edge(u, v);
    }
    cout << G.solve() << endl;
}

```

4.6 Minimum Weight Matching (Clique version)

```

struct Graph {
    // Minimum General Weighted Matching (Perfect Match)
    0-base
    static const int MXN = 105;

    int n, edge[MXN][MXN];
    int match[MXN],dis[MXN],onstk[MXN];
    vector<int> stk;

    void init(int _n) {
        n = _n;
        for (int i=0; i<n; i++)
            for (int j=0; j<n; j++)
                edge[i][j] = 0;
    }
    void add_edge(int u, int v, int w) {
        edge[u][v] = edge[v][u] = w;
    }
    bool SPFA(int u){
        if (onstk[u]) return true;
        stk.PB(u);
        onstk[u] = 1;
        for (int v=0; v<n; v++){
            if (u != v && match[u] != v && !onstk[v]){
                int m = match[v];
                if (dis[m] > dis[u] - edge[v][m] + edge[u][v]){

```

```

        dis[m] = dis[u] - edge[v][m] + edge[u][v];
        onstk[v] = 1;
        stk.PB(v);
        if (SPFA(m)) return true;
        stk.pop_back();
        onstk[v] = 0;
    }
}
onstk[u] = 0;
stk.pop_back();
return false;
}

int solve() {
    // find a match
    for (int i=0; i<n; i+=2){
        match[i] = i+1;
        match[i+1] = i;
    }
    while (true){
        int found = 0;
        for (int i=0; i<n; i++){
            dis[i] = onstk[i] = 0;
            for (int i=0; i<n; i++){
                stk.clear();
                if (!onstk[i] && SPFA(i)){
                    found = 1;
                    while (SZ(stk)>=2){
                        int u = stk.back(); stk.pop_back();
                        int v = stk.back(); stk.pop_back();
                        match[u] = v;
                        match[v] = u;
                    }
                }
            }
            if (!found) break;
        }
        int ret = 0;
        for (int i=0; i<n; i++)
            ret += edge[i][match[i]];
        ret /= 2;
        return ret;
    }
}graph;

```

4.7 (+1) SW-mincut $O(NM)$

```

// {{{ StoerWagner
const int inf=1000000000;
// should be larger than max.possible mincut
class StoerWagner {
public:
    int n,mc; // node id in [0,n-1]
    vector<int> adj[MAXN];
    int cost[MAXN][MAXN];
    int cs[MAXN];
    bool merged[MAXN],sel[MAXN];
    // --8<-- include only if cut is explicitly needed
    DisjointSet djs;
    vector<int> cut;
    //--8<-----
    StoerWagner(int _n):n(_n),mc(inf),djs(_n) {
        for (int i=0; i<n; i++)
            merged[i]=0;
        for (int i=0; i<n; i++)
            for (int j=0; j<n; j++)
                cost[i][j]=cost[j][i]=0;
    }
    void append(int v,int u,int c) {
        if (v==u) return;
        if (!cost[v][u]&&c) {
            adj[v].PB(u);
            adj[u].PB(v);
        }
        cost[v][u]+=c;
        cost[u][v]+=c;
    }
    void merge(int v,int u) {
        merged[u]=1;
        for (int i=0; i<n; i++)
            append(v,i,cost[u][i]);
        // --8<---include only if cut is explicitly
        needed
    }
}

```

```

        djs.merge(v,u);
        // --8<-----
    }
    void phase() {
        priority_queue<pii> pq;
        for (int v=0; v<n; v++) {
            if (merged[v]) continue;
            cs[v]=0;
            sel[v]=0;
            pq.push({0,v});
        }
        int v,s,pv;
        while (pq.size()) {
            if (cs[pq.top().S]>pq.top().F) {
                pq.pop();
                continue;
            }
            pv=v;
            v=pq.top().S;
            s=pq.top().F;
            pq.pop();
            sel[v]=1;
            for (int i=0; i<adj[v].size(); i++) {
                int u=adj[v][i];
                if (merged[u]||sel[u]) continue;
                cs[u]+=cost[v][u];
                pq.push({cs[u],u});
            }
            if (s<mc) {
                mc=s;
                // --8<-- include only if cut is explicitly
                needed -----
                cut.clear();
                for (int i=0; i<n; i++)
                    if (djs.getrep(i)==djs.getrep(v)) cut.PB(i);
                //--8<-----
            }
            merge(v,pv);
        }
        int mincut() {
            if (mc==inf) {
                for (int t=0; t<n-1; t++)
                    phase();
            }
            return mc;
        }
        // --8<-- include only if cut is explicitly needed
        -----
        vector<int> getcut() { // return one side of the
            cut
            mincut();
            return cut;
        }
        //--8<-----
    };
    // }}}

```

5 Math

5.1 $ax+by=gcd$

```

typedef pair<int, int> pii;

pii gcd(int a, int b){
    if (b == 0) return make_pair(1, 0);
    else{
        int p = a / b;
        pii q = gcd(b, a % b);
        return make_pair(q.second, q.first - q.second * p);
    }
}

```

5.2 Fast Fourier Transform

```

// const int MAXN = 262144;
// (must be 2^k)

```

```

typedef long double ld;
typedef complex<ld> cplx;
const ld PI = acos(-1);
const cplx I(0, 1);

cplx omega[MAXN+1];
void pre_fft()
{
    for(int i=0; i<=MAXN; i++)
        omega[i] = exp(i * 2 * PI / MAXN * I);
}

void fft(int n, cplx a[], bool inv=false)
{
    int basic = MAXN / n;
    int theta = basic;
    for (int m = n; m >= 2; m >>= 1) {
        int mh = m >> 1;
        for (int i = 0; i < mh; i++) {
            cplx w = omega[inv ? MAXN - (i*theta%MAXN) : i*theta%MAXN];
            for (int j = i; j < n; j += m) {
                int k = j + mh;
                cplx x = a[j] - a[k];
                a[j] += a[k];
                a[k] = w * x;
            }
        }
        theta = (theta * 2) % MAXN;
    }
    int i = 0;
    for (int j = 1; j < n - 1; j++) {
        for (int k = n >> 1; k > (i ^= k); k >>= 1);
        if (j < i) swap(a[i], a[j]);
    }
    if (inv)
        for (i = 0; i < n; i++)
            a[i] /= n;
}

```

5.3 Fast Linear Recurrence

```

ll n,m,dp[N+N];
void pre_dp(){
    dp[0]=1;
    ll bdr = min(m+m,n);
    for(ll i=1; i<=bdr; i++)
        for(ll j=i-1; j>=max(0, i-m); j--)
            dp[i]= add(dp[i], dp[j]);
}

vector<ll> Mul(const vector<ll>& v1, const vector<ll>& v2){
    int sz1 = (int)v1.size();
    int sz2 = (int)v2.size();
    assert(sz1 == m and sz2 == m);
    vector<ll> _v(m+m);
    for(int i=0; i<m+m; i++) _v[i]=0;
    // expand
    for(int i=0; i<sz1; i++)
        for(int j=0; j<sz2; j++)
            _v[i+j+1]= add(_v[i+j+1], mul(v1[i], v2[j]));
    // shrink
    for(int i=0; i<m; i++)
        for(int j=1; j<=m; j++)
            _v[i+j]= add(_v[i+j], _v[i]);
    for(int i=0; i<m; i++)
        _v[i]= _v[i+m];
    _v.resize(m);
    return _v;
}

vector<ll> I,A;
ll solve(){
    pre_dp();
    if(n <= m+m) return dp[n];
    I.resize(m);
    A.resize(m);
    for(int i=0; i<m; i++) I[i]=A[i]=1;
    // dp[n]= /Sum_{i=0}^{m-1} A_i * dp[n - i - 1]
    ll dlt = (n - m) / m;
    ll rdlt = dlt * m;
    while(dlt){
        if(dlt & 1ll) I = Mul(I,A);
        A = Mul(A,A);
        dlt >>= 1;
    }
}

```

```

ll ans = 0;
for(int i=0; i<m; i++)
    ans = add(ans, mul(I[i], dp[n-i-1-rdlt]));
return ans;
}

```

5.4 (+1) ntt

```

int P=605028353, root=3, MAXNUM=262144;
// Remember coefficient are mod P
/*
p=a*2^n+1
n  2^n      p      a      root
5   32      97      3       5
6   64     193      3       5
7  128     257      2       3
8  256     257      1       3
9  512     7681     15      17
10 1024    12289     12      11
11 2048    12289      6      11
12 4096    12289      3      11
13 8192    40961      5       3
14 16384   65537      4       3
15 32768   65537      2       3
16 65536   65537      1       3
17 131072  786433      6      10
18 262144  786433      3      10 (605028353,
    2308, 3)
19 524288  5767169     11       3
20 1048576 7340033      7       3
21 2097152 23068673     11       3
22 4194304 104857601    25       3
23 8388608 167772161    20       3
24 16777216 167772161   10       3
25 33554432 167772161    5      3 (1107296257, 33,
    10)
26 67108864 469762049    7       3
27 134217728 2013265921 15      31
*/
int bigmod(long long a, int b){
    if(b==0) return 1;
    return (bigmod((a*a)%P, b/2) * (b%2?a:1ll))%P;
}

int inv(int a, int b){
    if(a==1) return 1;
    return (((long long)(a - inv(b%a, a)) * b + 1) / a) % b;
}

std::vector<long long> ps(MAXNUM);
std::vector<int> rev(MAXNUM);
struct poly{
    std::vector<unsigned int> co;
    int n; // polynomial degree = n
    poly(int d){n=d; co.resize(n+1, 0);}
    void trans2(int NN){
        int r=0, st, N;
        unsigned int a, b;
        while((1<<r)<(NN>>1)) ++r;
        for(N=2; N<=NN; N<=<=1, --r){
            for(st=0; st<NN; st+=N){
                int i, ss=st+(N>>1);
                for(i=(N>>1)-1; i>=0; --i){
                    a=co[st+i]; b=(ps[i<<r]*co[ss+i])%P;
                    co[st+i]=a+b; if(co[st+i]>=P) co[st+i]-=P;
                    co[ss+i]=a-P-b; if(co[ss+i]>=P) co[ss+i]-=P;
                }
            }
        }
    }

    void trans1(int NN){
        int r=0, st, N;
        unsigned int a, b;
        for(N=NN; N>1; N>=>=1, ++r){
            for(st=0; st<NN; st+=N){
                int i, ss=st+(N>>1);
                for(i=(N>>1)-1; i>=0; --i){
                    a=co[st+i]; b=co[ss+i];
                    co[st+i]=a+b; if(co[st+i]>=P) co[st+i]-=P;
                    co[ss+i]=((a-P-b)*ps[i<<r])%P;
                }
            }
        }
    }

    poly operator*(const poly& _b) const{
        poly a=*this, b=_b;
    }
}

```

```

    int k=n+b.n,i,N=1;
    while(N<=k)N*=2;
    a.co.resize(N,0); b.co.resize(N,0);
    int r=bigmod(root,(P-1)/N),Ni=inv(N,P);
    ps[0]=1;
    for(i=1;i<N;++i)ps[i]=(ps[i-1]*r)%P;
    a.trans1(N);b.trans1(N);
    for(i=0;i<N;++i)a.co[i]=((long long)a.co[i]*b.co[i])%P;
    ;
    r=inv(r,P);
    for(i=1;i<N/2;++i)std::swap(ps[i],ps[N-i]);
    a.trans2(N);
    for(i=0;i<N;++i)a.co[i]=((long long)a.co[i]*Ni)%P;
    a.n=n+_b.n; return a;
}
};

```

5.5 Mod

```

/// _fd(a,b) floor(a/b).
/// _rd(a,m) a-floor(a/m)*m.
/// _pv(a,m,r) largest x s.t x<=a && x%m == r.
/// _nx(a,m,r) smallest x s.t x>=a && x%m == r.
/// _ct(a,b,m,r) |A| , A = { x : a<=x<=b && x%m == r }.

int _fd(int a,int b){ return a<0?(-~a/b-1):a/b; }
int _rd(int a,int m){ return a-_fd(a,m)*m; }
int _pv(int a,int m,int r)
{
    r=(r%m+m)%m;
    return _fd(a-r,m)*m+r;
}
int _nt(int a,int m,int r)
{
    m=abs(m);
    r=(r%m+m)%m;
    return _fd(a-r-1,m)*m+r+m;
}
int _ct(int a,int b,int m,int r)
{
    m=abs(m);
    a=_nt(a,m,r);
    b=_pv(b,m,r);
    return (a>b)?0:((b-a+m)/m);
}

```

5.6 (+1) Miller Rabin

```

// n < 4,759,123,141      3 : 2, 7, 61
// n < 1,122,004,669,633  4 : 2, 13, 23, 1662803
// n < 3,474,749,660,383  6 : pimes <= 13
// n < 2^64              7 :
// 2, 325, 9375, 28178, 450775, 9780504, 1795265022
// Make sure testing integer is in range [2, n-2] if
// you want to use magic.
long long power(long long x,long long p,long long mod){
    long long s=1,m=x;
    while(p) {
        if(p&1) s=mult(s,m,mod);
        p>>=1;
        m=mult(m,m,mod);
    }
    return s;
}
bool witness(long long a,long long n,long long u,int t)
{
    long long x=power(a,u,n);
    for(int i=0;i<t;i++) {
        long long nx=mult(x,x,n);
        if(nx==1&&x!=1&&x!=n-1) return 1;
        x=nx;
    }
    return x!=1;
}
bool miller_rabin(long long n,int s=100) {
    // iterate s times of witness on n
    // return 1 if prime, 0 otherwise
    if(n<2) return 0;
    if(!(n&1)) return n==2;
    long long u=n-1;
    int t=0;

```

```

// n-1 = u*2^t
while(!(u&1)) {
    u>>=1;
    t++;
}
while(s--) {
    long long a=randll()%(n-1)+1;
    if(witness(a,n,u,t)) return 0;
}
return 1;
}

```

5.7 Pollard Rho

```

// does not work when n is prime
long long modit(long long x,long long mod) {
    if(x>=mod) x-=mod;
    //if(x<0) x+=mod;
    return x;
}
long long mult(long long x,long long y,long long mod) {
    long long s=0,m=x%mod;
    while(y) {
        if(y&1) s=modit(s+m,mod);
        y>>=1;
        m=modit(m+m,mod);
    }
    return s;
}
long long f(long long x,long long mod) {
    return modit(mult(x,x,mod)+1,mod);
}
long long pollard_rho(long long n) {
    if(!(n&1)) return 2;
    while (true) {
        long long y=2, x=rand()%(n-1)+1, res=1;
        for (int sz=2; res==1; sz*=2) {
            for (int i=0; i<sz && res<=1; i++) {
                x = f(x, n);
                res = __gcd(abs(x-y), n);
            }
            y = x;
        }
        if (res!=0 && res!=n) return res;
    }
}

```

5.8 Algorithms about Primes

```

/*
* 12721
* 13331
* 14341
* 75577
* 123457
* 222557
* 556679
* 999983
* 1097774749
* 1076767633
* 100102021
* 999997771
* 1001010013
* 1000512343
* 987654361
* 999991231
* 999888733
* 98789101
* 987777733
* 999991921
* 1010101333
* 1010102101
* 10000000000039
* 1000000000000037
* 2305843009213693951
* 4611686018427387847
* 9223372036854775783
* 18446744073709551557
*/

int mu[MX],p_tbl[MX];
vector<int> primes;

```

```

void sieve() {
    mu[1] = p_tbl[1] = 1;
    for (int i=2; i<MX; i++) {
        if (!p_tbl[i]) {
            p_tbl[i] = i;
            primes.PB(i);
            mu[i] = -1;
        }
        for (auto p : primes) {
            int x = i*p;
            if (x >= M) break;
            p_tbl[x] = p;
            mu[x] = -mu[i];
            if (i%p==0) {
                mu[x] = 0;
                break;
            }
        }
    }
}

vector<int> factor(int x) {
    vector<int> fac{1};
    while (x > 1) {
        int fn=SZ(fac), p=p_tbl[x], pos=0;
        while (x%p == 0) {
            x /= p;
            for (int i=0; i<fn; i++)
                fac.PB(fac[pos++]*p);
        }
    }
    return fac;
}

```

```

}
if(piv == -1)
    continue;
used[i] = true;
swap(m.v[piv], m.v[i]);
swap(res.v[piv], res.v[i]);

ld rat = m.v[i][i];
for(int j=0; j<W; j++)
{
    m.v[i][j] /= rat;
    res.v[i][j] /= rat;
}

for(int j=0; j<W; j++)
{
    if(j == i) continue;
    rat = m.v[j][i];
    for(int k=0; k<W; k++)
    {
        m.v[j][k] -= rat * m.v[i][k];
        res.v[j][k] -= rat * res.v[i][k];
    }
}

for(int i=0; i<W; i++)
{
    if(used[i]) continue;
    for(int j=0; j<W; j++)
        res.v[i][j] = 0;
}

return res;
}

```

5.9 (+1) PolynomialGenerator

```

class PolynomialGenerator {
    /* for a nth-order polynomial f(x), *
    * given f(0), f(1), ..., f(n) *
    * express f(x) as sigma_i{c_i*C(x,i)} */
public:
    int n;
    vector<long long> coef;
    // initialize and calculate f(x), vector _fx should
    // be
    // filled with f(0) to f(n)
    PolynomialGenerator(int _n,vector<long long> _fx)
        :n(_n)
        ,coef(_fx) {
        for(int i=0;i<n;i++)
            for(int j=n;j>i;j--)
                coef[j]-=coef[j-1];
    }
    // evaluate f(x), runs in O(n)
    long long eval(int x) {
        long long m=1,ret=0;
        for(int i=0;i<=n;i++) {
            ret+=coef[i]*m;
            m=m*(x-i)/(i+1);
        }
        return ret;
    }
};

```

5.10 Pseudoinverse of Square matrix

```

Mat pinv(Mat m)
{
    Mat res = I;

    FZ(used);
    for(int i=0; i<W; i++)
    {
        int piv = -1;
        for(int j=0; j<W; j++)
        {
            if(used[j]) continue;
            if(abs(m.v[j][i]) > EPS)
            {
                piv = j;
                break;
            }
        }
    }
}

```

5.11 Theorem

5.11.1 Lucas' Theorem

For non-negative integer n , m and prime p , $\binom{m}{n} \equiv \prod_{i=0}^k \binom{m_i}{n_i} \pmod{p}$ where m_i is the i -th digit of m in base p .

5.11.2 Sum of Two Squares Thm (Legendre)

For a given positive integer n , let $D_1 = (\# \text{ of positive integers } d \text{ dividing } N \text{ that } 1 \equiv d \pmod{4})$
 $D_3 = (\# \text{ of positive integers } d \text{ dividing } N \text{ that } 3 \equiv d \pmod{4})$
 then n can be written as a sum of two squares in exactly $R(n) = 4(D_1 - D_3)$ ways.

5.11.3 Difference of D1-D3 Thm

let $n = 2^t \cdot (p_1^{e_1} \cdot \dots \cdot p_r^{e_r}) \cdot \dots \cdot (q_1^{f_1} \cdot \dots \cdot q_s^{f_s})$
 where p_i, q_i are primes and $1 \equiv p_i \pmod{4}, 3 \equiv q_i \pmod{4}$
 then $D_1 - D_3 = \begin{cases} (e_1 + 1)(e_2 + 1) \dots (e_r + 1), & \text{if } f_i \text{ all even} \\ 0, & \text{if any } f_i \text{ is odd} \end{cases}$

5.11.4 Krush-Kuhn-Tucker Conditions

Stationarity

For maximizing $f(x)$: $\nabla f(x^*) = \sum_{i=1}^m \mu_i \nabla g_i(x^*) + \sum_{j=1}^l \lambda_j \nabla h_j(x^*)$
 For minimizing $f(x)$: $-\nabla f(x^*) = \sum_{i=1}^m \mu_i \nabla g_i(x^*) + \sum_{j=1}^l \lambda_j \nabla h_j(x^*)$

Primal feasibility

$g_i(x^*) \leq 0$, for all $i = 1, \dots, m$
 $h_j(x^*) = 0$, for all $j = 1, \dots, l$

Dual feasibility

$\mu_i \geq 0$, for all $i = 1, \dots, m$

Complementary slackness

$\mu_i g_i(x^*) = 0$, for all $i = 1, \dots, m$

5.11.5 Chinese remainder theorem

$x \equiv r_i \pmod{p_i}$
 $N = \prod p_i$
 $N_i = N/p_i$
 $x \equiv \sum r_i N_i (N_i)^{-1}_{p_i} \pmod{N}$

5.12 Simplex

```
const int maxn = 111;
const int maxm = 111;
const double eps = 1E-10;

double a[maxn][maxm], b[maxn], c[maxm], d[maxn][maxm];
double x[maxm];
int ix[maxn + maxm]; // !!! array all indexed from 0
// max{cx} subject to {Ax<=b, x>=0}
// n: constraints, m: vars !!!
// x[] is the optimal solution vector
//
// usage :
// value = simplex(a, b, c, N, M);
double simplex(double a[maxn][maxm], double b[maxn],
double c[maxm], int n, int m) {
    ++m;
    int r = n, s = m - 1;
    memset(d, 0, sizeof(d));
    for (int i = 0; i < n + m; ++i) ix[i] = i;
    for (int i = 0; i < n; ++i) {
        for (int j = 0; j < m - 1; ++j)
            d[i][j] = -a[i][j];
        d[i][m - 1] = 1;
        d[i][m] = b[i];
        if (d[r][m] > d[i][m]) r = i;
    }
    for (int j = 0; j < m - 1; ++j) d[n][j] = c[j];
    d[n + 1][m - 1] = -1;
    for (double dd;;) {
        if (r < n) {
            int t = ix[s];
            ix[s] = ix[r + m]; ix[r + m] = t;
            d[r][s] = 1.0 / d[r][s];
            for (int j = 0; j <= m; ++j)
                if (j != s) d[r][j] *= -d[r][s];
            for (int i = 0; i <= n + 1; ++i)
                if (i != r) {
                    for (int j = 0; j <= m; ++j)
                        if (j != s)
                            d[i][j] += d[r][j] * d[i][s];
                    d[i][s] *= d[r][s];
                }
        }
        r = -1; s = -1;
        for (int j = 0; j < m; ++j)
            if (s < 0 || ix[s] > ix[j]) {
                if (d[n + 1][j] > eps || (d[n + 1][j] >
                    -eps && d[n][j] > eps)) s = j;
            }
        if (s < 0) break;
        for (int i = 0; i < n; ++i) if (d[i][s] < -eps) {
            if (r < 0 || (dd = d[r][m] / d[r][s] - d[i][m] / d[i][s]) < -eps || (dd < eps &&
                ix[r + m] > ix[i + m])) r = i;
        }
        if (r < 0) return -1; // not bounded
    }
    if (d[n + 1][m] < -eps) return -1; // not
    // executable
    double ans = 0;
    for (int i = 0; i < m; i++) x[i] = 0;
    for (int i = m; i < n + m; ++i) { // the missing
        // enumerated x[i] = 0
        if (ix[i] < m - 1) {
            ans += d[i - m][m] * c[ix[i]];
            x[ix[i]] = d[i - m][m];
        }
    }
    return ans;
}
```

```
struct pdd : pair<double, double> {
    using pair<double, double>::pair;

    pdd operator + (cpdd &p) const {
        return {x+p.x, y+p.y};
    }

    pdd operator - () const {
        return {-x, -y};
    }

    pdd operator - (cpdd &p) const {
        return (*this) + (-p);
    }

    pdd operator * (double f) const {
        return {f*x, f*y};
    }

    double operator * (cpdd &p) const {
        return x*p.x + y*p.y;
    }
};

double abs(cpdd &p) { return hypot(p.x, p.y); }
double arg(cpdd &p) { return atan2(p.y, p.x); }
double cross(cpdd &p, cpdd &q) { return p.x*q.y - p.y*q.x; }
double cross(cpdd &p, cpdd &q, cpdd &o) { return cross(p-o, q-o); }
pdd operator * (double f, cpdd &p) { return p*f; } //
    !! Not f*p !!
```

6.2 Intersection of two circles

```
using ld = double;
vector<pdd> interCircle(pdd o1, double r1, pdd o2,
double r2) {
    ld d2 = (o1 - o2) * (o1 - o2);
    ld d = sqrt(d2);
    if (d < abs(r1-r2)) return {};
    if (d > r1+r2) return {};
    pdd u = 0.5*(o1+o2) + ((r2*r2-r1*r1)/(2*d2))*(o1-o2);
    double A = sqrt((r1+r2+d) * (r1-r2+d) * (r1+r2-d) *
        (-r1+r2+d));
    pdd v = A / (2*d2) * pdd(o1.S-o2.S, -o1.F+o2.F);
    return {u+v, u-v};
}
```

6.3 Intersection of two lines

```
const double EPS = 1e-9;

pdd interPnt(pdd p1, pdd p2, pdd q1, pdd q2, bool &res)
{
    double f1 = cross(p2, q1, p1);
    double f2 = -cross(p2, q2, p1);
    double f = (f1 + f2);

    if (fabs(f) < EPS) {
        res = false;
        return {};
    }

    res = true;
    return (f2 / f) * q1 + (f1 / f) * q2;
}
```

6.4 Half Plane Intersection

```
const double EPS = 1e-9;

pdd interPnt(Line l1, Line l2, bool &res){
    pdd p1, p2, q1, q2;
    tie(p1, p2) = l1;
    tie(q1, q2) = l2;
    double f1 = cross(p2, q1, p1);
    double f2 = -cross(p2, q2, p1);
    double f = (f1 + f2);
```

6 Geometry

6.1 Point operators

```
#define x first
#define y second
#define cpdd const pdd
```



```

    if(fabs(f) < EPS) {
        res = false;
        return {0, 0};
    }

    res = true;
    return (f2 / f) * q1 + (f1 / f) * q2;
}

bool isin(Line l0, Line l1, Line l2) {
    // Check inter(l1, l2) in l0
    bool res;
    pdd p = interPnt(l1, l2, res);
    return cross(l0.S, p, l0.F) > EPS;
}

/* If no solution, check: 1. ret.size() < 3
 * Or more precisely, 2. interPnt(ret[0], ret[1])
 * in all the lines. (use (l.S - l.F).cross(p - l.F) >
 * 0)
 */
vector<Line> halfPlaneInter(vector<Line> lines) {
    int sz = lines.size();
    vector<double> ata(sz), ord(sz);
    for (int i=0; i<sz; i++) {
        ord[i] = i;
        pdd d = lines[i].S - lines[i].F;
        ata[i] = atan2(d.y, d.x);
    }
    sort(ALL(ord), [&](int i, int j) {
        if (abs(ata[i] - ata[j]) < EPS) {
            return cross(lines[i].S, lines[j].S, lines[i].F) < 0;
        }
        return ata[i] < ata[j];
    });
    vector<Line> fin;
    for (int i=0; i<sz; i++) {
        if (!i or fabs(ata[ord[i]] - ata[ord[i-1]]) > EPS) {
            fin.PB(lines[ord[i]]);
        }
    }

    deque<Line> dq;
    for (int i=0; i<SZ(fin); i++) {
        while(SZ(dq) >= 2 and
            not isin(fin[i], dq[SZ(dq)-2], dq[SZ(dq)-1])) {
            dq.pop_back();
        }
        while(SZ(dq) >= 2 and
            not isin(fin[i], dq[0], dq[1])) {
            dq.pop_front();
        }
        dq.push_back(fin[i]);
    }

    while (SZ(dq) >= 3 and
        not isin(dq[0], dq[SZ(dq)-2], dq[SZ(dq)-1])) {
        dq.pop_back();
    }

    while (SZ(dq) >= 3 and
        not isin(dq[SZ(dq)-1], dq[0], dq[1])) {
        dq.pop_front();
    }
    vector<Line> res(ALL(dq));
    return res;
}

```

6.5 2D Convex Hull

```

vector<pdd> convex_hull(vector<pdd> pt){
    sort(pt.begin(), pt.end());
    int top=0;
    vector<pdd> stk(2*pt.size());
    for (int i=0; i<(int)pt.size(); i++){
        while (top >= 2 && cross(stk[top-1], pt[i], stk[top-2]) <= 0)
            top--;
        stk[top++] = pt[i];
    }
}

```

```

}
for (int i=pt.size()-2, t=top+1; i>=0; i--){
    while (top >= t && cross(stk[top-1], pt[i], stk[top-2]) <= 0)
        top--;
    stk[top++] = pt[i];
}
stk.resize(top-1);
return stk;
}

```

6.6 3D Convex Hull

```

// return the faces with pt indexes
int flag[MXN][MXN];
struct Point{
    ld x,y,z;
    Point operator - (const Point &b) const {
        return (Point){x-b.x, y-b.y, z-b.z};
    }
    Point operator * (const ld &b) const {
        return (Point){x*b, y*b, z*b};
    }
    ld len() const { return sqrtl(x*x+y*y+z*z); }
    ld dot(const Point &a) const {
        return x*a.x+y*a.y+z*a.z;
    }
    Point operator * (const Point &b) const {
        return (Point){y*b.z-b.y*z, z*b.x-b.z*x, x*b.y-b.x*y};
    }
};
Point ver(Point a, Point b, Point c) {
    return (b - a) * (c - a);
}
vector<Face> convex_hull_3D(const vector<Point> pt) {
    int n = SZ(pt);
    REP(i,n) REP(j,n)
        flag[i][j] = 0;

    vector<Face> now;
    now.push_back((Face){0,1,2});
    now.push_back((Face){2,1,0});
    int ftop = 0;
    for (int i=3; i<n; i++){
        ftop++;
        vector<Face> next;
        REP(j, SZ(now)) {
            Face& f=now[j];
            ld d=(pt[i]-pt[f.a]).dot(ver(pt[f.a], pt[f.b], pt[f.c]));
            if (d <= 0) next.push_back(f);
            int ff = 0;
            if (d > 0) ff=ftop;
            else if (d < 0) ff=-ftop;
            flag[f.a][f.b] = flag[f.b][f.c] = flag[f.c][f.a] = ff;
        }
        REP(j, SZ(now)) {
            Face& f=now[j];
            if (flag[f.a][f.b] > 0 and flag[f.a][f.b] != flag[f.b][f.a])
                next.push_back((Face){f.a, f.b, i});
            if (flag[f.b][f.c] > 0 and flag[f.b][f.c] != flag[f.c][f.b])
                next.push_back((Face){f.b, f.c, i});
            if (flag[f.c][f.a] > 0 and flag[f.c][f.a] != flag[f.a][f.c])
                next.push_back((Face){f.c, f.a, i});
        }
        now=next;
    }
    return now;
}

```

6.7 Minimum Covering Circle

```

struct Mcc{
    // return pair of center and r^2
    static const int MAXN = 1000100;
    int n;
    pdd p[MAXN], cen;
}

```

```

double r2;

void init(int _n, pdd _p[]){
    n = _n;
    memcpy(p,_p,sizeof(pdd)*n);
}

double sqr(double a){ return a*a; }
double abs2(pdd a){ return a*a; }
pdd center(pdd p0, pdd p1, pdd p2) {
    pdd a = p1-p0;
    pdd b = p2-p0;
    double c1=abs2(a)*0.5;
    double c2=abs2(b)*0.5;
    double d = a % b;
    double x = p0.x + (c1 * b.y - c2 * a.y) / d;
    double y = p0.y + (a.x * c2 - b.x * c1) / d;
    return pdd(x,y);
}

pair<pdd,double> solve(){
    random_shuffle(p,p+n);
    r2=0;
    for (int i=0; i<n; i++){
        if (abs2(cen-p[i]) <= r2) continue;
        cen = p[i];
        r2 = 0;
        for (int j=0; j<i; j++){
            if (abs2(cen-p[j]) <= r2) continue;
            cen = 0.5 * (p[i]+p[j]);
            r2 = abs2(cen-p[j]);
            for (int k=0; k<j; k++){
                if (abs2(cen-p[k]) <= r2) continue;
                cen = center(p[i],p[j],p[k]);
                r2 = abs2(cen-p[k]);
            }
        }
    }
    return {cen,r2};
}
}mcc;

```

6.8 KDTree (Nearest Point)

```

const int MXN = 100005;

struct KDTree {
    struct Node {
        int x,y,x1,y1,x2,y2;
        int id,f;
        Node *L, *R;
    }tree[MXN];
    int n;
    Node *root;

    long long dis2(int x1, int y1, int x2, int y2) {
        long long dx = x1-x2;
        long long dy = y1-y2;
        return dx*dx+dy*dy;
    }

    static bool cmpx(Node& a, Node& b){ return a.x<b.x; }
    static bool cmpy(Node& a, Node& b){ return a.y<b.y; }
    void init(vector<pair<int,int>> ip) {
        n = ip.size();
        for (int i=0; i<n; i++) {
            tree[i].id = i;
            tree[i].x = ip[i].first;
            tree[i].y = ip[i].second;
        }
        root = build_tree(0, n-1, 0);
    }

    Node* build_tree(int L, int R, int dep) {
        if (L>R) return nullptr;
        int M = (L+R)/2;
        tree[M].f = dep%2;
        nth_element(tree+L, tree+M, tree+R+1, tree[M].f ?
            cmpy : cmpx);
        tree[M].x1 = tree[M].x2 = tree[M].x;
        tree[M].y1 = tree[M].y2 = tree[M].y;

        tree[M].L = build_tree(L, M-1, dep+1);
        if (tree[M].L) {
            tree[M].x1 = min(tree[M].x1, tree[M].L->x1);
            tree[M].x2 = max(tree[M].x2, tree[M].L->x2);
            tree[M].y1 = min(tree[M].y1, tree[M].L->y1);

```

```

        tree[M].y2 = max(tree[M].y2, tree[M].L->y2);
    }

    tree[M].R = build_tree(M+1, R, dep+1);
    if (tree[M].R) {
        tree[M].x1 = min(tree[M].x1, tree[M].R->x1);
        tree[M].x2 = max(tree[M].x2, tree[M].R->x2);
        tree[M].y1 = min(tree[M].y1, tree[M].R->y1);
        tree[M].y2 = max(tree[M].y2, tree[M].R->y2);
    }

    return tree+M;
}

int touch(Node* r, int x, int y, long long d2){
    long long dis = sqrt(d2)+1;
    if (x<r->x1-dis || x>r->x2+dis || y<r->y1-dis || y>
        r->y2+dis)
        return 0;
    return 1;
}

void nearest(Node* r, int x, int y, int &mID, long
    long &md2) {
    if (!r || !touch(r, x, y, md2)) return;
    long long d2 = dis2(r->x, r->y, x, y);
    if (d2 < md2 || (d2 == md2 && mID < r->id)) {
        mID = r->id;
        md2 = d2;
    }
    // search order depends on split dim
    if ((r->f == 0 && x < r->x) ||
        (r->f == 1 && y < r->y)) {
        nearest(r->L, x, y, mID, md2);
        nearest(r->R, x, y, mID, md2);
    } else {
        nearest(r->R, x, y, mID, md2);
        nearest(r->L, x, y, mID, md2);
    }
}

int query(int x, int y) {
    int id = 1029384756;
    long long d2 = 102938475612345678LL;
    nearest(root, x, y, id, d2);
    return id;
}
}tree;

```

6.9 Triangulation

```

bool inCircle(pdd a, pdd b, pdd c, pdd d) {
    b = b - a;
    c = c - a;
    d = d - a;
    if (cross(b, c) < 0) swap(b, c);
    double m[3][3] = {
        {b.x, b.y, b*b},
        {c.x, c.y, c*c},
        {d.x, d.y, d*d}
    };

    double det = m[0][0] * (m[1][1]*m[2][2] - m[1][2]*m
        [2][1])
        + m[0][1] * (m[1][2]*m[2][0] - m[1][0]*m
        [2][2])
        + m[0][2] * (m[1][0]*m[2][1] - m[1][1]*m
        [2][0]);

    return det < 0;
}

bool intersect(pdd a, pdd b, pdd c, pdd d) {
    return cross(b, c, a) * cross(b, d, a) < 0 and
        cross(d, a, c) * cross(d, b, c) < 0;
}

const double EPS = 1e-12;
struct Triangulation {
    static const int MXN = 1e5+5;
    int N;
    vector<int> ord;
    vector<pdd> pts;
    set<int> E[MXN];
    vector<vector<int>> solve(vector<pdd> p) {
        N = SZ(p);
        ord.resize(N);

```

```

    for (int i=0; i<N; i++) {
        E[i].clear();
        ord[i] = i;
    }
    sort(ALL(ord), [&p](int i, int j) {
        return p[i] < p[j];
    });

    pts.resize(N);
    for (int i=0; i<N; i++) pts[i] = p[ord[i]];

    go(0, N);

    vector<vector<int>> res(N);
    for (int i=0; i<N; i++) {
        int o = ord[i];
        for (auto x: E[i]) {
            res[o].PB(ord[x]);
        }
    }
    return res;
}

void add_edge(int u, int v) {
    E[u].insert(v);
    E[v].insert(u);
}

void remove_edge(int u, int v) {
    E[u].erase(v);
    E[v].erase(u);
}

void go(int l, int r) {
    int n = r - l;

    if (n <= 3) {
        for (int i=l; i<r; i++)
            for (int j=i+1; j<r; j++) add_edge(i, j);
        return;
    }
    int md = (l+r)/2;

    go(l, md);
    go(md, r);

    int il = l, ir = r-1;
    while (1) {
        int nx = -1;
        for (auto i: E[il]) {
            double cs = cross(pts[il], pts[i], pts[ir]);
            if (cs > EPS ||
                (abs(cs) < EPS and abs(pts[i]-pts[ir]) < abs(pts[il]-pts[ir]))) {
                nx = i;
                break;
            }
        }
        if (nx != -1) {
            il = nx;
            continue;
        }
        for (auto i: E[ir]) {
            double cs = cross(pts[ir], pts[i], pts[il]);
            if (cs < -EPS ||
                (abs(cs) < EPS and abs(pts[i]-pts[il]) < abs(pts[ir]-pts[il]))) {
                nx = i;
                break;
            }
        }
        if (nx != -1) {
            ir = nx;
        } else break;
    }

    add_edge(il, ir);

    while (1) {
        int nx = -1;
        bool is2 = false;

```

```

        for (int i: E[il]) {
            if (cross(pts[il], pts[i], pts[ir]) < -EPS and
                (nx == -1 or inCircle(pts[il], pts[ir], pts[nx], pts[i]))) nx = i;
        }

        for (int i: E[ir]) {
            if (cross(pts[ir], pts[i], pts[il]) > EPS and
                (nx == -1 or inCircle(pts[il], pts[ir], pts[nx], pts[i]))) nx = i,
                is2 = 1;
        }

        if (nx == -1) break;

        int a = il, b = ir;
        if (is2) swap(a, b);

        for (auto i: E[a]) {
            if (intersect(pts[a], pts[i], pts[b], pts[nx])) {
                remove_edge(a, i);
            }
        }
        if (is2) {
            add_edge(il, nx);
            ir = nx;
        } else {
            add_edge(ir, nx);
            il = nx;
        }
    }
}
} tri;

```

7 Stringology

7.1 Suffix Array

```

const int N = 1.12e5, M = 11;
const char nil = 'a' - 1;
char s[N];
int sa[N], ct[N], w[2][N], rk[N], ht[N];
void suffix_array(int n, int m)
{
    int i, j, p = 0, *x = w[0], *y = w[1], h;
    memset(ct, 0, m * sizeof(int));
    for (i = 0; i < n; ++i) ++ct[x[i] = s[i] - ('a' - 1)];
    for (i = 1; i < m; ++i) ct[i] += ct[i - 1];
    for (i = n - 1; i >= 0; --i) sa[--ct[x[i]]] = i;
    for (j = p = 1; p < n; j *= 2, m = p) {
        for (i = n - j, p = 0; i < n; ++i) y[p++] = i;
        for (i = 0; i < n; ++i) if (sa[i] >= j) y[p++] = sa[i] - j;
        memset(ct, 0, m * sizeof(int));
        for (i = 0; i < n; ++i) ++ct[x[y[i]]];
        for (i = 1; i < m; ++i) ct[i] += ct[i - 1];
        for (i = n - 1; i >= 0; --i) sa[--ct[x[y[i]]]] = y[i];
        swap(x, y);
        x[sa[0]] = 0;
        p = 1;
        for (i = 1; i < n; ++i)
            x[sa[i]] = y[sa[i]] == y[sa[i-1]] && y[sa[i] + j]
                == y[sa[i-1] + j] ? p - 1 : p++;
    }
    for (i = 0; i < n; ++i) rk[sa[i]] = i;
    ht[0] = 0;
    h = 0;
    for (i = 0; rk[i]; ++i) {
        while (s[i + h] == s[sa[rk[i] - 1] + h]) ++h;
        ht[rk[i]] = h;
        if (h) --h;
    }
}

```

7.2 Suffix Array (SAIS TWT514)

```

struct SA{
#define REP(i,n) for ( int i=0; i<int(n); i++ )
#define REPl(i,a,b) for ( int i=(a); i<=int(b); i++ )
    static const int MXN = 300010;
    bool _t[MXN*2];
    int _s[MXN*2], _sa[MXN*2], _c[MXN*2], x[MXN], _p[MXN], _q[MXN*2], hei[MXN], r[MXN];
    int operator [] (int i){ return _sa[i]; }
    void build(int *s, int n, int m){
        memcpy(_s, s, sizeof(int) * n);
        sais(_s, _sa, _p, _q, _t, _c, n, m);
        mkhei(n);
    }
    void mkhei(int n){
        REP(i,n) r[_sa[i]] = i;
        hei[0] = 0;
        REP(i,n) if(r[i]) {
            int ans = i>0 ? max(hei[r[i-1]] - 1, 0) : 0;
            while(_s[i+ans] == _s[_sa[r[i]-1]+ans]) ans++;
            hei[r[i]] = ans;
        }
    }
    void sais(int *s, int *sa, int *p, int *q, bool *t, int *c, int n, int z){
        bool uniq = t[n-1] = true, neq;
        int nn = 0, nmzx = -1, *nsa = sa + n, *ns = s + n, lst = -1;
#define MS0(x,n) memset((x),0,n*sizeof(*(x)))
#define MAGIC(XD) MS0(sa, n); \
        memcpy(x, c, sizeof(int) * z); \
        XD; \
        memcpy(x + 1, c, sizeof(int) * (z - 1)); \
        REP(i,n) if(sa[i] && !t[sa[i]-1]) sa[x[s[sa[i]-1]]++] = sa[i]-1; \
        memcpy(x, c, sizeof(int) * z); \
        for(int i = n - 1; i >= 0; i--) if(sa[i] && t[sa[i]-1]) sa[--x[s[sa[i]-1]]] = sa[i]-1;
        MS0(c, z);
        REP(i,n) uniq &= ++c[s[i]] < 2;
        REP(i,z-1) c[i+1] += c[i];
        if (uniq) { REP(i,n) sa[--c[s[i]]] = i; return; }
        for(int i = n - 2; i >= 0; i--) t[i] = (s[i]==s[i+1] ? t[i+1] : s[i]<s[i+1]);
        MAGIC(REPl(i,1,n-1) if(t[i] && !t[i-1]) sa[--x[s[i]]]=p[q[i]=nn++]=i);
        REP(i, n) if (sa[i] && t[sa[i]] && !t[sa[i]-1]) {
            neq=lst<0||memcmp(s+sa[i],s+lst,(p[q[sa[i]]+1]-sa[i])*sizeof(int));
            ns[q[lst=sa[i]]]=nmzx+=neq;
        }
        sais(ns, nsa, p + nn, q + n, t + n, c + z, nn, nmzx + 1);
        MAGIC(for(int i = nn - 1; i >= 0; i--) sa[--x[p[nsa[i]]]] = p[nsa[i]]);
    }
}sa;

void suffix_array(int* ip, int len) {
    // should padding a zero in the back
    // s is int array, n is array length
    // s[0..n-1] != 0, and s[n] = 0
    // resulting SA will be length n+1
    ip[len++] = 0;
    sa.build(ip, len, 128);
    // original 1-base
    for (int i=0; i<l; i++) {
        hei[i] = sa.hei[i + 1];
        sa[i] = sa._sa[i + 1];
    }
}

```

7.3 Aho-Corasick Algorithm

```

struct ACautomata{
    struct Node{
        int cnt,dp;
        Node *go[26], *fail;
    }
}

```

```

Node (){
    cnt = 0;
    dp = -1;
    memset(go,0,sizeof(go));
    fail = 0;
}

Node *root, pool[1048576];
int nMem;

Node* new_Node(){
    pool[nMem] = Node();
    return &pool[nMem++];
}

void init(){
    nMem = 0;
    root = new_Node();
}

void add(const string &str){
    insert(root,str,0);
}

void insert(Node *cur, const string &str, int pos){
    if (pos >= (int)str.size()){
        cur->cnt++;
        return;
    }
    int c = str[pos]-'a';
    if (cur->go[c] == 0){
        cur->go[c] = new_Node();
        insert(cur->go[c],str,pos+1);
    }
}

void make_fail(){
    queue<Node*> que;
    que.push(root);
    while (!que.empty()){
        Node* fr=que.front();
        que.pop();
        for (int i=0; i<26; i++){
            if (fr->go[i]){
                Node *ptr = fr->fail;
                while (ptr && !ptr->go[i]) ptr = ptr->fail;
                if (!ptr) fr->go[i]->fail = root;
                else fr->go[i]->fail = ptr->go[i];
                que.push(fr->go[i]);
            }
        }
    }
}

```

7.4 KMP

```

#include<bits/stdc++.h>
using namespace std;

void build_fail_function(string B, int *fail) {
    int len = B.length(), pos;
    pos = fail[0] = -1;
    for (int i = 1; i < len; i++) {
        while (pos != -1 and B[pos + 1] != B[i])
            pos = fail[pos];
        if (B[pos + 1] == B[i]) pos++;
        fail[i] = pos;
    }
}

void match(string A, string B, int *fail) {
    int lenA = A.length(), lenB = B.length();
    int pos = -1;
    for (int i = 0; i < lenA; i++) {
        while (pos != -1 and B[pos + 1] != A[i])
            pos = fail[pos];

        if (B[pos + 1] == A[i]) pos++;

        if (pos == lenB - 1) {
            // Match ! A[i - lenB + 1, i] = B
            pos = fail[pos];
        }
    }
}

```

7.5 Z value

```
void Zval(const char *s, int len, int *z) {
    z[0] = 0;
    for (int b=0, i=1; i<len; i++) {
        z[i] = max(min(z[i-b], z[b] + b - i), 0);
        while (s[i + z[i]] == s[z[i]]) z[i] ++;
        if (i+z[i] > b+z[b]) b=i;
    }
}
```

7.6 Z value (palindrome ver.)

```
void Zpal(const char *s, int len, int *z) {
    // Only odd palindrome len is considered
    // z[i] means that the longest odd palindrom
    // centered at
    // i is [i-z[i] .. i+z[i]]
    z[0] = 0;
    for (int b=0, i=1; i<len; i++) {
        if (z[b] + b >= i) z[i] = min(z[2*b-i], b+z[b]-i);
        else z[i] = 0;
        while (i+z[i]+1 < len and i-z[i]-1 >= 0 and
            s[i+z[i]+1] == s[i-z[i]-1]) z[i] ++;
        if (z[i] + i > z[b] + b) b = i;
    }
}
```

7.7 palindromic tree

```
//bcw0x1bd2 {{{
#include<bits/stdc++.h>
#include<unistd.h>
using namespace std;
#define F first
#define S second
#define MP make_pair
#define PB push_back
#define IOS ios_base::sync_with_stdio(0); cin.tie(0);
#define SZ(x) ((int)((x).size()))
#define ALL(x) begin(x),end(x)
#define REP(i,x) for (int i=0; i<(x); i++)
#define REP1(i,a,b) for (int i=(a); i<=(b); i++)

typedef long long ll;
typedef pair<int,int> pii;
typedef pair<ll,ll> pll;
typedef long double ld;

#ifdef DARKHH
#define FILEIO(name)
#else
#define FILEIO(name) \
    freopen(name".in", "r", stdin); \
    freopen(name".out", "w", stdout);
#endif

#ifdef DARKHH
template<typename T>
void _dump( const char* s, T&& head ) { cerr<<s<<"="<<
    head<<endl; }

template<typename T, typename... Args>
void _dump( const char* s, T&& head, Args&&... tail ) {
    int c=0;
    while ( *s!=',' || c!=0 ) {
        if ( *s=='(' || *s=='[' || *s=='{' ) c++;
        if ( *s==')' || *s==']' || *s=='}' ) c--;
        cerr<<*s++;
    }
    cerr<<"="<<head<<" ";
    _dump(s+1,tail...);
}

#define dump(...) do { \
    fprintf(stderr, "%s:%d - ", __PRETTY_FUNCTION__, \
    __LINE__); \
    _dump(#__VA_ARGS__, __VA_ARGS__); \
} while (0)
```

```
template<typename Iter>
ostream& _out( ostream &s, Iter b, Iter e ) {
    s<<"[";
    for ( auto it=b; it!=e; it++ ) s<<(it==b?"":" ")<<*it
        ;
    s<<"]";
    return s;
}

template<typename A, typename B>
ostream& operator <<( ostream &s, const pair<A,B> &p )
    { return s<<"("<<p.first<<","<<p.second<<")"; }
template<typename T>
ostream& operator <<( ostream &s, const vector<T> &c )
    { return _out(s,ALL(c)); }
template<typename T, size_t N>
ostream& operator <<( ostream &s, const array<T,N> &c )
    { return _out(s,ALL(c)); }
template<typename T>
ostream& operator <<( ostream &s, const set<T> &c ) {
    return _out(s,ALL(c)); }
template<typename A, typename B>
ostream& operator <<( ostream &s, const map<A,B> &c ) {
    return _out(s,ALL(c)); }
#else
#define dump(...)
#endif
}}}

struct palindromic_tree{
    struct node{
        int next[26],fail,len;
        int cnt,num,st,ed;
        node(int l=0):fail(0),len(l),cnt(0),num(0){
            for(int i=0;i<26;++i)next[i]=0;
        }
    };
    vector<node> state;
    vector<char> s;
    int last,n;

    void init(){
        state.clear();
        s.clear();
        last=1;
        n=0;
        state.push_back(0);
        state.push_back(-1);
        state[0].fail=1;
        s.push_back(-1);
    }
    int get_fail(int x){
        while(s[n-state[x].len-1]!=s[n])x=state[x].fail;
        return x;
    }
    void add(int c){
        s.push_back(c-'a');
        ++n;
        int cur=get_fail(last);
        if(!state[cur].next[c]){
            int now=state.size();
            state.push_back(state[cur].len+2);
            state[now].fail=get_fail(state[cur].fail).next[c];
            state[cur].next[c]=now;
            state[now].num=state[state[now].fail].num+1;
        }
        last=state[cur].next[c];
        ++state[last].cnt;
    }
    int size(){
        return state.size()-2;
    }
}pt;

int main() {
    string s;
    cin >> s;
    pt.init();
    for (int i=0; i<SZ(s); i++) {
        int prvsz = pt.size();
        pt.add(s[i]);
        if (prvsz != pt.size()) {
            int r = i;
            int l = r - pt.state[pt.last].len + 1;

```

```

    cout << "Find pal @ [" << l << " " << r << "]" : "
        << s.substr(l,r-l+1) << endl;
}
}
return 0;
}

```

7.8 Lexicographically Smallest Rotation

```

string mcp(string s){
    int n = s.length();
    s += s;
    int i=0, j=1;
    while (i<n && j<n){
        int k = 0;
        while (k < n && s[i+k] == s[j+k]) k++;
        if (s[i+k] <= s[j+k]) j += k+1;
        else i += k+1;
        if (i == j) j++;
    }
    int ans = i < n ? i : j;
    return s.substr(ans, n);
}

```

7.9 Suffix Automaton

```

// par : fail link
// val : a topological order ( useful for DP )
// go[x] : automata edge ( x is integer in [0,26) )

struct SAM{
    struct State{
        int par, go[26], val;
        State () : par(0), val(0){ FZ(go); }
        State (int _val) : par(0), val(_val){ FZ(go); }
    };
    vector<State> vec;
    int root, tail;

    void init(int arr[], int len){
        vec.resize(2);
        vec[0] = vec[1] = State(0);
        root = tail = 1;
        for (int i=0; i<len; i++)
            extend(arr[i]);
    }
    void extend(int w){
        int p = tail, np = vec.size();
        vec.PB(State(vec[p].val+1));
        for ( ; p && vec[p].go[w]==0; p=vec[p].par)
            vec[p].go[w] = np;
        if (p == 0){
            vec[np].par = root;
        } else {
            if (vec[vec[p].go[w]].val == vec[p].val+1){
                vec[np].par = vec[p].go[w];
            } else {
                int q = vec[p].go[w], r = vec.size();
                vec.PB(vec[q]);
                vec[r].val = vec[p].val+1;
                vec[q].par = vec[np].par = r;
                for ( ; p && vec[p].go[w] == q; p=vec[p].par)
                    vec[p].go[w] = r;
            }
        }
        tail = np;
    }
};

```

8 Problems

8.1 Painter

```

#include<bits/stdc++.h>
using namespace std;
#define F first
#define S second

```

```

#define PB push_back
#define IOS ios_base::sync_with_stdio(0); cin.tie(0);
#define SZ(x) ((int)((x).size()))
#define ALL(x) begin(x),end(x)
#define REP(i,x) for (int i=0; i<(x); i++)
#define REP1(i,a,b) for (int i=(a); i<=(b); i++)

typedef long long ll;
typedef pair<ll,ll> pll;

typedef pll Point;
const int MXN = 100005;

Point operator + (const Point &a, const Point &b) {
    return Point(a.F+b.F, a.S+b.S); }
Point operator - (const Point &a, const Point &b) {
    return Point(a.F-b.F, a.S-b.S); }
ll operator * (const Point &a, const Point &b) { return
    a.F*b.F + a.S*b.S; }
ll operator % (const Point &a, const Point &b) { return
    a.F*b.S - a.S*b.F; }

struct Segment {
    int v,id;
    Point p,q;
    Segment () {}
    Segment (int _v, int _id, Point _p, Point _q) :
        v(_v), id(_id), p(_p), q(_q) {}
};

bool operator < (const Segment &a, const Segment &b) {
    if (a.p == b.q) return false;
    if (a.q == b.p) return true;
    if (a.p == b.p) return (a.q-a.p) % (b.q-a.p) > 0;
    if (a.q == b.q) return (a.p-a.q) % (b.p-a.q) < 0;
    if (a.p.F == b.p.F) return a.p.S < b.p.S;
    if (a.q.F == b.q.F) return a.q.S < b.q.S;
    if (a.p.F < b.p.F) return (a.q-a.p) % (b.p-a.p) > 0;
    else return (b.q-b.p) % (a.p-b.p) < 0;
}

bool operator == (const Segment &a, const Segment &b) {
    return tie(a.v,a.id,a.p,a.q) == tie(b.v,b.id,b.p,b.q)
        ;
}

struct Triangle {
    Point pt[3];
}ip[MXN];

const int MEM = 350004;
struct Treap {
    static Treap nil, mem[MEM], *pmem;
    Treap *l, *r;
    int sum,presum,size;
    Segment seg;
    Treap () : l(&nil), r(&nil), sum(0), presum(0), size
        (0), seg() {}
    Treap (Segment _val) :
        l(&nil), r(&nil), sum(_val.v), presum(max(_val.v,0)
        ), size(1), seg(_val) {}
} Treap:nil, Treap::mem[MEM], *Treap::pmem = Treap::
    mem;

int size(const Treap *t) { return t->size; }
void pull(Treap *t) {
    if (!size(t)) return;
    t->size = size(t->l) + size(t->r) + 1;
    t->sum = t->l->sum + t->seg.v + t->r->sum;
    t->presum = max(t->l->presum, t->l->sum + t->seg.v);
    t->presum = max(t->presum, t->l->sum + t->seg.v + t->
        r->presum);
}

Treap* merge(Treap *a, Treap *b) {
    if (!size(a)) return b;
    if (!size(b)) return a;
    Treap *t;
    if (rand() % (size(a) + size(b)) < size(a)) {
        t = a;
        t->r = merge(a->r, b);
    } else {
        t = b;
        t->l = merge(a, b->l);
    }
    pull(t);
    return t;
}

void split(Treap *t, int k, Treap *&a, Treap *&b) {
    if (!size(t)) a = b = &Treap::nil;
}

```



```

else if (size(t->l) + 1 <= k) {
    a = t;
    split(t->r, k - size(t->l) - 1, a->r, b);
    pull(a);
} else {
    b = t;
    split(t->l, k, a, b->l);
    pull(b);
}
}
int get_rank(Treap *t, Segment x) {
    if (!size(t)) return 0;
    if (x < t->seg) return get_rank(t->l, x);
    return get_rank(t->r, x) + size(t->l) + 1;
}
Treap* find_leftist(Treap *t) {
    while (size(t->l)) t = t->l;
    return t;
}
Treap* find_rightist(Treap *t) {
    while (size(t->r)) t = t->r;
    return t;
}

int N;
vector<int> allx;
vector<Segment> _seg[3*MXN];
#define seg(x) _seg[(x)+100000]

inline void add_seg(Segment s) {
    seg(s.p.F).PB(s);
    if (s.q.F != s.p.F) seg(s.q.F).PB(s);
}
void predo() {
    allx.clear();
    REP(i,N) REP(j,3) {
        seg(ip[i].pt[j].F).clear();
        allx.PB(ip[i].pt[j].F);
    }
    sort(ALL(allx));
    allx.resize(unique(ALL(allx))-begin(allx));
    REP(i,N) {
        sort(ip[i].pt, ip[i].pt+3);
        Point *pt = ip[i].pt;
        Segment seg1 = Segment(1,i,pt[0],pt[1]);
        Segment seg2 = Segment(1,i,pt[0],pt[2]);
        Segment seg3 = Segment(1,i,pt[1],pt[2]);
        if (seg2 < seg1) seg1.v = -1;
        else seg2.v = -1;
        seg3.v = seg1.v;
        add_seg(seg1);
        add_seg(seg2);
        add_seg(seg3);
    }
}
inline int sgn(ll x) { return x < 0 ? -1 : x > 0; }
bool interPnt(Point p1, Point p2, Point q1, Point q2){
    ll c1 = (p2-p1)%(q1-p1), c2 = (p2-p1)%(q2-p1);
    ll c3 = (q2-q1)%(p1-q1), c4 = (q2-q1)%(p2-q1);
    return sgn(c1) * sgn(c2) <= 0 and sgn(c3) * sgn(c4)
        <= 0;
}
bool check_error(Segment a, Segment b) {
    if (a.id == b.id) return false;
    return interPnt(a.p,a.q,b.p,b.q);
}
int solve() {
    Treap::pmem = Treap::mem;
    Treap *rt = &Treap::nil;
    int res = 0;
    for (auto i:allx) {
        for (auto l:seg(i)) {
            int k = get_rank(rt, l);
            Treap *t,*tl,*tm,*tr;
            split(rt,k,tl,tr);
            t = find_rightist(tl);
            if (size(t) and check_error(t->seg,l)) return -1;
            t = find_leftist(tr);
            if (size(t) and check_error(t->seg,l)) return -1;
            rt = merge(tl,tr);
            if (l.p.F == i and l.p.F != l.q.F) {
                k = get_rank(rt, l);
                split(rt,k,tl,tr);
                tm = new (Treap::pmem++) Treap(l);
                rt = merge(merge(tl,tm),tr);
            }
        }
    }
}

```

```

}
for (auto l:seg(i)) {
    if (l.q.F == i and l.p.F != l.q.F) {
        Treap *tl,*tm,*tr;
        int k = get_rank(rt, l);
        split(rt,k-1,tl,tm);
        split(tm,1,tm,tr);
        Treap *t1=find_rightist(tl),*t2=find_leftist(tr);
        if (size(t1) and size(t2) and check_error(t1->
            seg,t2->seg)) return -1;
        rt = merge(tl,tr);
    }
}
res = max(res, rt->presum);
}
res++;
return res;
}
int main() {
    IOS;
    int cas = 0;
    while (cin >> N) {
        if (N == -1) break;
        REP(i,N) {
            REP(j,3) cin >> ip[i].pt[j].F >> ip[i].pt[j].S;
        }
        predo();
        int ans = solve();
        cas++;
        cout << "Case " << cas << ": ";
        if (ans == -1) cout << "ERROR\n";
        else cout << ans << " shades\n";
    }
}
return 0;
}

```

8.2 Mo-Algorithm on Tree

```

#include<bits/stdc++.h>
using namespace std;
#define IOS ios_base::sync_with_stdio(0); cin.tie(0);
#define SZ(x) ((int)((x).size()))

const int MX = 500005;
const int SQ = 1400;
const int LOG = 17;

struct BIT {
    int bit[MX];
    int lb(int x) { return x & -x; }
    void add(int p, int v) {
        p++;
        for (int i=p; i<MX; i+=lb(i)) bit[i] += v;
    }
    int qry() {
        int v = 0;
        for (int i=1;<LOG; i>0; i>=1) {
            if ((v|i) < MX and bit[v|i]==i) v |= i;
        }
        return v;
    }
}bit;

struct Query {
    int l,r,qid;
}qry[MX];
struct Edge {
    int v,x;
};

int N,Q,timestamp[MX],ans[MX];
int in[MX],cnt[MX];
vector<Edge> E[MX];
vector<Edge> seq;

void DFS(int u, int f) {
    timestamp[u] = SZ(seq);
    for (auto it:E[u]) {
        if (it.v == f) continue;
        seq.push_back(it);
        DFS(it.v,u);
        seq.push_back(it);
    }
}

```



```

}
}
void poke(int id) {
    int v = seq[id].v;
    int x = seq[id].x;
    in[v] ^= 1;
    cnt[x] += in[v] ? 1 : -1;
    if (in[v] and cnt[x] == 1) bit.add(x, 1);
    if (!in[v] and cnt[x] == 0) bit.add(x, -1);
}
int main() {
    IOS;
    cin >> N >> Q;
    for (int i=0; i<N-1; i++) {
        int u,v,x;
        cin >> u >> v >> x;
        x = min(x,N);
        E[u].push_back({v,x});
        E[v].push_back({u,x});
    }
    DFS(1,1);
    for (int i=1; i<=Q; i++) {
        int u,v;
        cin >> u >> v;
        int l = timestamp[u], r = timestamp[v];
        if (l > r) swap(l,r);
        r--;
        qry[i] = {l,r,i};
    }
    sort(qry+1,qry+1+Q, [](Query a, Query b) {
        return make_pair(a.l/SQ,a.r) < make_pair(b.l/SQ,b.r);
    });

    int curL = 1, curR = 0;
    for (int i=1; i<=Q; i++) {
        int ql=qry[i].l,qr=qry[i].r;
        while (curL > ql) poke(--curL);
        while (curR < qr) poke(++curR);
        while (curL < ql) poke(curL++);
        while (curR > qr) poke(curR--);
        ans[qry[i].qid] = bit.qry();
    }

    for (int i=1; i<=Q; i++) cout << ans[i] << "\n";

    return 0;
}

```

8.3 Manhattan MST

```

#include<bits/stdc++.h>
#define REP(i,n) for(int i=0;i<n;i++)
using namespace std;
typedef long long LL;
const int N=200100;
int n,m;
struct PT {int x,y,z,w,id;}p[N];
inline int dis(const PT &a,const PT &b){return abs(a.x-b.x)+abs(a.y-b.y);}
inline bool cpx(const PT &a,const PT &b){return a.x!=b.x? a.x>b.x:a.y>b.y;}
inline bool cpz(const PT &a,const PT &b){return a.z<b.z;}
struct E{int a,b,c;}e[8*N];
bool operator<(const E&a,const E&b){return a.c<b.c;}
struct Node{
    int L,R,key;
}node[4*N];
int s[N];
int F(int x){return s[x]==x?s[x]:F(s[x]);}
void U(int a,int b){s[F(b)]=F(a);}
void init(int id,int L,int R) {
    node[id]=(Node){L,R,-1};
    if(L==R)return;
    init(id*2,L,(L+R)/2);
    init(id*2+1,(L+R)/2+1,R);
}
void ins(int id,int x) {
    if(node[id].key==-1 || p[node[id].key].w>p[x].w)node[id].key=x;
    if(node[id].L==node[id].R)return;
    if(p[x].z<=(node[id].L+node[id].R)/2)ins(id*2,x);
    else ins(id*2+1,x);
}

```

```

}
int Q(int id,int L,int R){
    if(R<node[id].L || L>node[id].R)return -1;
    if(L<=node[id].L && node[id].R<=R)return node[id].key;
    ;
    int a=Q(id*2,L,R),b=Q(id*2+1,L,R);
    if(b==-1 || (a!=-1 && p[a].w<p[b].w)) return a;
    else return b;
}
void calc() {
    REP(i,n) {
        p[i].z=p[i].y-p[i].x;
        p[i].w=p[i].x+p[i].y;
    }
    sort(p,p+n,cpz);
    int cnt=0,j,k;
    for(int i=0;i<n;i=j){
        for(j=i+1;p[j].z==p[i].z && j<n;j++){
            for(k=i,cnt++;k<j;k++)p[k].z=cnt;
        }
        init(1,1,cnt);
        sort(p,p+n,cpx);
        REP(i,n) {
            j=Q(1,p[i].z,cnt);
            if(j!=-1)e[m++]=(E){p[i].id,p[j].id,dis(p[i],p[j])};
        }
        ins(1,i);
    }
}
LL MST() {
    LL r=0;
    sort(e,e+m);
    REP(i,m) {
        if(F(e[i].a)==F(e[i].b))continue;
        U(e[i].a,e[i].b);
        r+=e[i].c;
    }
    return r;
}
int main(){
    int ts;
    scanf("%d",&ts);
    while (ts--) {
        m = 0;
        scanf("%d",&n);
        REP(i,n) {
            scanf("%d%d",&p[i].x,&p[i].y);
            p[i].id=s[i]=i;
        }
        calc();
        REP(i,n)p[i].y= -p[i].y;
        calc();
        REP(i,n)swap(p[i].x,p[i].y);
        calc();
        REP(i,n)p[i].x=-p[i].x;
        calc();
        printf("%lld\n",MST()*2);
    }
    return 0;
}

```