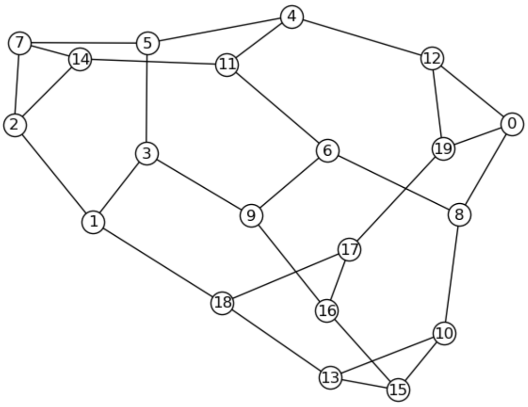
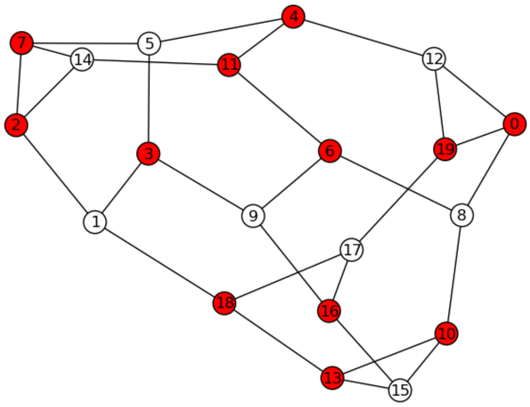
AGT 2025/26 Project

Consider a network, such as the one represented in the picture below (left). Your task is that of securing the network ensuring that the traffic on every edge is controlled by at least one node. That is, you have to select a set of nodes on which security measures (cameras, security checkpoints, etc…) have to be implemented. Let us call this set as **network security set** and let us define it as follows: a node is either in the network security set or all of its neighbors are in the security set.

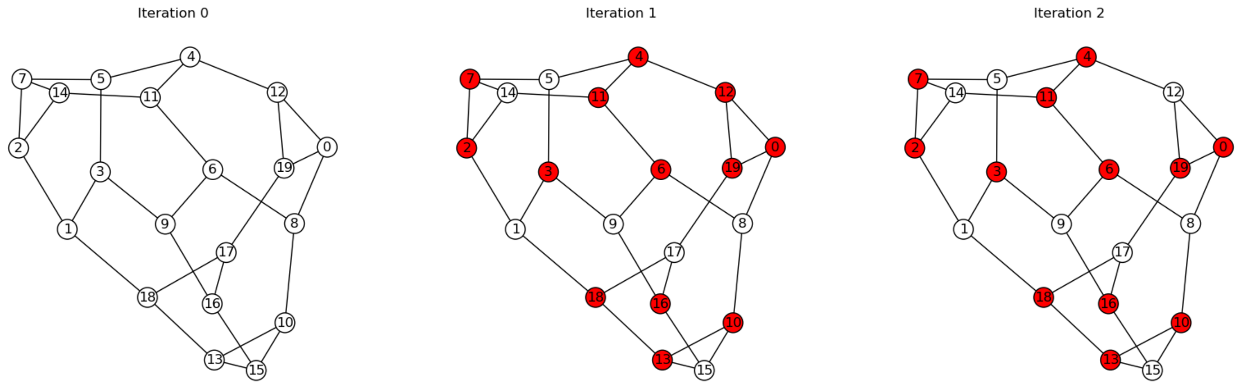




You want to select a **minimal set** of such nodes – i.e. a set S such that S is a network security set and $S \setminus \{i\}$ is not a network security set, for all nodes $i \in S$. See the picture above (right).

Your task is to use game theoretic tools to find **minimal network security sets** on graphs. In particular:

1. Use tools from strategic (non-cooperative) game theory to define a game where nodes are players and the Pure Nash Equilibria of the game corresponds to minimal network security sets. You need to define a payoff structure that incentivizes players to self-organize into a minimal network security set. Once you have formalized the game, find a Pure Nash Equilibrium. If modeled correctly, the game always has one Pure Nash Equilibrium and iterative algorithms such as Fictitious play, Regret Matching, and Best Response Dynamics should converge to a pure equilibrium. Consider comparing all these three algorithms and their outputs. (In the picture below, an example computed with the *Best Response Dynamics* algorithm.



2. Use tools from coalitional game theory to define a coalitional game that encodes the network security set problem. Specifically, define an appropriate characteristic function that given a coalition

returns a value which informs us whether the coalition represents a network security set or how far it is from a security set. You can also try to measure the portion of a network secured by a given coalition and penalize a coalition if it is not minimal. Explore different characteristic functions and remember that Shapley values are additive. Once you have a function which scores coalitions, approximate the Shapley values of the resulting game with a Monte Carlo sampling approach. Then, build a minimal network security set induced by the Shapley values in descending order. Compare the approaches and solutions provided by the strategic game modeling with those provided by the coalitional modeling approach.

3. Now that you have found at least one minimal network security set, imagine you are a planner and you have to assign the players in the minimal network security set to vendors of security services. Suppose every player has a budget (generate random integers between 1 and 100) and suppose every vendor offers only one type of security services for a given price (generate random integers between 1 and 100) and a given security level (generate random integers between 1 and 10). Define a utility which takes into account both the prices with respect to players' budgets (in particular, security services that have costs higher than a player's budget are incompatible with said player) and the security levels of the services offered. You want to maximize the social welfare (sum of utilities). Imagine two cases: 1) in which each vendor has an infinite number of items and 2) in which vendors have limited number of items. It's fine if some agents remain unmatched because either their budgets are incompatible with prices, or because the limited capacity of vendors.
4. Now, consider a situation where you want to buy a secure path on the network. Your utility depends on both the cost of the path, as well as the number of unsecure nodes on said path. Each node in the network is an agent which sells edges. Each agent has a price (you can generate random integers) and you want to **create a truthful auction mechanism** where agents on the network have no incentive to lie on their prices. Define your utility, the mechanism, and a payment schema which keeps the mechanism truthful.

Remarks

You should prepare a report where you explain your design and modeling choices, as well as the solution that you have implemented.

In addition, prepare a 10 minute presentation for the exam where you present the problem, your modeling choices, your solutions along with your experiments, and a discussion/reflection on how to extend/push forward what you have done.

You can work in groups of maximum 3 people.

Write your code in a way that it can run on *any* input network. I don't expect industrial strength code which scales to million of nodes, of course. But do not write your code in a way that it works only on the network in the example.

Try your solutions on different classes of networks: the one in the figures is a regular graph, but try it also on Erdős-Rényi graphs and power law networks (Barabasi-Albert graphs).