

Quesito 1. La v.a. discreta X ha distribuzione di probabilità

$$\Pr(X = -2) = \frac{1}{2}, \quad \Pr(X = 1) = \frac{1}{3}, \quad \Pr(X = 2) = \frac{1}{6}.$$

1. Calcolare la distribuzione di probabilità di X^2
2. Calcolare $\text{Var}(X)$.

Esprimere i numeri razionali come frazioni.

$$\Pr(X^2 = 4) = \frac{2}{3} \quad \text{e} \quad \Pr(X^2 = 1) = \frac{1}{3} \quad \text{Risposta 1}$$

$$E(X) = -2 \cdot \Pr(X = -2) + 1 \cdot \Pr(X = 1) + 2 \cdot \Pr(X = 2) = -1 + \frac{1}{3} + \frac{1}{3} = -\frac{1}{3}$$

$$E(X^2) = 4 \cdot \Pr(X^2 = 4) + 1 \cdot \Pr(X^2 = 1) = \frac{8}{3} + \frac{1}{3} = 3$$

$$\text{Var}(X) = E(X^2) - E(X)^2 = 3 - \frac{1}{9} = \frac{26}{9} \quad \text{Risposta 2}$$

Quesito 2. Si consideri la funzione $f(x) = \cos\left(\left|\frac{3x-4}{5x+2}\right|\right)$.

1. Determinare dominio e immagine della funzione.
2. Determinare il punto di massimo assoluto per $x \geq 0$.

$$\text{dom} f = \mathbb{R} \setminus \left\{-\frac{2}{5}\right\} \quad \text{e} \quad \text{im} f = [-1, 1] \quad \text{Risposta 1}$$

$$x = \frac{4}{3} \quad \text{Risposta 2}$$

Quesito 3. Marie is getting married tomorrow at an outdoor ceremony in the desert. In recent years it has rained only 7 days each year. But the weatherman has predicted rain for tomorrow. When it actually rains, the weatherman correctly forecasts rain 95% of the time. When it doesn't rain, he incorrectly forecasts rain 5% of the times. What is the probability that it will rain on the day of Marie's wedding?

R event: it rains on Marie's wedding

T_+ event: the weatherman predicts rain

$\Pr(R) = 7/365 = 1.9\%$ it rains 7 days out of 365

$\Pr(T_+|R) = 95\%$ when it rains, rain is predicted

$\Pr(T_+|\neg R) = 5\%$ when it does not rain, rain is predicted

$\Pr(T_+) = \Pr(T_+|R) \cdot \Pr(R) + \Pr(T_+|\neg R) \cdot \Pr(\neg R) = 6.7\%$

$$\Pr(R|T_+) = \frac{\Pr(R) \cdot \Pr(T_+|R)}{\Pr(T_+)} = 27.1\%$$

Risposta

Quesito 4. Si considerino le funzioni $f(x) = \frac{1}{2x}$ e $g(x) = \log(x)$.

1. Scrivere esplicitamente le funzioni $f \circ g$ e $g \circ f$.
2. Determinare dominio di $f \circ g$ e $g \circ f$.

$$(f \circ g)(x) = \frac{1}{2 \log(x)} \quad \text{e} \quad (g \circ f)(x) = \log\left(\frac{1}{2x}\right)$$

Risposta 1

$$\text{dom}(f \circ g) = (0, +\infty) \quad \text{e} \quad \text{dom}(g \circ f) = (0, 1) \cup (1, +\infty)$$

Risposta 2

Quesito 5. If 20% of the bolts produced by a machine are defective.

1. Determine the probability that out of 4 bolts chosen at random less than 2, bolts will be defective.
2. Out of 2000 bolts how many would you expect to be defective.

$$p = 0.2 \quad n = 2000 \quad X \sim B(n, p)$$

$$\begin{aligned} \Pr(X < 2) &= \Pr(X = 0) + \Pr(X = 1) \\ &= \binom{4}{0} p^0 (1-p)^4 + \binom{4}{1} p^1 (1-p)^3 \\ &= (1-p)^4 + 4p(1-p)^3 = 0.8192 \end{aligned}$$

Risposta 2

$$E(X) = 400$$

Risposta 2

Quesito 6. La v.a. discreta X ha valore atteso $E(X) = 5$ e varianza $\text{Var}(X) = 2$. Qual è il valore atteso di $X(X-3)$?

$$E(X(X-3)) = E(X^2) - 3 \cdot E(X) = \text{Var}(X) + E(X)^2 - 3 \cdot E(X) = 12$$

Risposta