Classici t-test.

Quesito 1. A machine fills 500ml of milk into packages. It is suspected that the machine is not working correctly and that the amount filled is less than the setpoint. A sample of 16 packages filled by the machine are collected. The sample mean is 490ml and the sample standard deviation is 18ml. Can we reject the hypothesis that the machine is working correctly?

Answer the following questions: H_0? H_1? What test is required? What is the value of the statistic? What is the p-value?

Risposta

H_0 : $\mu = \mu_0 = 500$

H_1 : $\mu < \mu_0 = 500$

We use a one tail t-test (lower tail)

$n = 16$ sample size

$s = 18$ sample standard deviation

$\bar{x} = 490$ sample mean

$t = \dfrac{\bar{x} - \mu_0}{s/\sqrt{n}} = -2.2222$ value of the t-test statistic

$n - 1 = 15$ degrees of freedom

$\Pr\left(T_{n-1} < t\right) = \texttt{t.cdf}(-2.2222, 15) = 0.021$ p-value

Quesito 2. A machine fills 750ml of acetone into packages. It is suspected that the machine is not working correctly and that the amount filled is more than the setpoint. A sample of 4 packages filled by the machine are collected. The sample mean is 758ml and the sample standard deviation is 14ml. Can we reject the hypothesis that the machine is working correctly?

Answer the following questions: H_0? H_1? What test is required? What is the value of the statistic? What is the p-value?

Risposta

H_0 : $\mu = \mu_0 = 750$

H_1 : $\mu > \mu_0 = 750$

We use a one tail t-test (upper tail)

$n = 4$ sample size

$s = 14$ sample standard deviation

$\bar{x} = 758$ sample mean

$t = \dfrac{\bar{x} - \mu_0}{s/\sqrt{n}} = 1.1429$ value of the t-test statistic

$n - 1 = 3$ degrees of freedom

$\Pr\left(T_{n-1} > t\right) = 1 - \Pr\left(T_{n-1} < t\right)$ $1 - \texttt{t.cdf}(1.1429, 3) = 0.168$ p-value

Quesito 3. It is claimed that a new treatment is more effective than the standard treatment for prolonging the lives of terminal cancer patients. The standard treatment has been in use for a long time and from records in medical journals the mean survival period has been 4.1 years with a standard deviation of 1.1 years. The new treatment is administered to 20 patients and their average duration of survival is calculated to be 4.8 years with a standard deviation of 1.3. Can we reject the hypothesis that the new treatment is as effective as the old one?

Answer the following questions: H_0? H_1? What test is required? What is the value of the statistic? What is the p-value?

Risposta

$\mu_0 = 4.1$

H_0 : $\mu = \mu_0$

H_1 : $\mu > \mu_0$

We use a one tail t-test (upper tail)

$n = 20$ sample size

$s = 1.3$ sample standard deviation

$\bar{x} = 4.8$ sample mean

$t = \dfrac{\bar{x} - \mu_0}{s/\sqrt{n}} = 2.4081$ value of the t-test statistic

$n - 1 = 19$ degrees of freedom

$\Pr\left(T_{n-1} > t\right) = 1 - \Pr\left(T_{n-1} < t\right) = \texttt{t.cdf}(2.4081, 19) = 0.0132$ p-value

Quesito 4. A manufacturer claims that the mean lifetime of a lightbulb is on average at least 14 thousand hours with a standard deviation of 0.3. In a sample of 9 lightbulbs, it was found that they only last 13.3 thousand hours on average. The sample standard deviation is 0.5 thousand hours. Can we reject the manufacturer's claim?

Answer the following questions: H_0? H_1? What test is required? What is the value of the statistic? What is the p-value?

Risposta

$\mu_0 = 14$

$H_0 :$ $\mu = \mu_0$

$H_1 :$ $\mu < \mu_0$

We use a one tail t-test (lower tail)

$n = 9$ sample size

$s = 0.5$ sample standard deviation

$\bar{x} = 13.3$ sample mean

$t = \dfrac{\bar{x} - \mu_0}{s/\sqrt{n}} = 4.2$ value of the t-test statistic

$n - 1 = 8$ degrees of freedom

$\Pr\left(T_{n-1} < -t\right) =$ `t.cdf(4.2, 8)` $= 0.0015$ p-value