Matematica e BioStatistica con Applicazioni Informatiche
Esercitazione in aula del 9 gennaio 2018

Quesito 1. Della v.a. discreta X conosciamo la distribizione di probabilità

$$\Pr(X=4)=\frac{1}{2} \qquad \Pr(X=5)=\frac{1}{2}$$

Della v.a. discreta Y conosciamo la distribuzione condizionata a X

$$\Pr(Y=3 \mid X=4)=\frac{1}{2} \qquad \Pr(Y=3 \mid X=5)=\frac{1}{3}$$

$$\Pr(Y=2 \mid X=4)=\frac{1}{2} \qquad \Pr(Y=2 \mid X=5)=\frac{2}{3}$$

Calcolare la distribuzione di probablità di Y

Esprimere i numeri razionali come frazioni.

Quesito 2. Assume the null hypothesis is true and denote by P the random variable that gives the p-value you would get if you run a test.

1. What is the probability that $\Pr(P<0.05)$?
2. If we run the tests 8 times (independently), what is the probability of incorrectly rejecting at least once the null hypotheses with a significance $\alpha=5\%$?
3. If we run the tests 8 times (independently), how small do we have to make the cutoff (α above) to lower to 5% the probability of incorrectly rejecting at least once the null hypotheses?

Quesito 3. A manufacturer claims that the mean lifetime of a lightbulb is on average at least 10 thousand hours with a standard deviation of 0.3. In a sample of 9 lightbulbs, it was found that they only last 9.5 thousand hours on average. The sample standard deviation is 0.6 thousand hours. Can we reject the manufacturer's claim? Answer the following questions:

1. H_0? H_1?
2. What test is required?
3. What is the value of the statistic?
4. What is the p-value?

Formulario: se $X \sim B(\mathtt{n},\mathtt{p})$ allora $E(X)=np$
se $X \sim NB(\mathtt{n},\mathtt{p})$ allora $E(X)=n(1-p)/p$

$$T=\frac{\bar{X}-\bar{Y}}{S\cdot\sqrt{1/n_x+1/n_y}} \quad \text{dove } S^2 = \frac{n_x-1}{n_x+n_y-2}\cdot S_x^2+\frac{n_y-1}{n_x+n_y-2}\cdot S_y^2 \quad \text{ha distribuzione } t(n_x+n_y-2)$$

Si assuma noto il valore delle seguenti funzioni della libreria `scipy.stats` di `Python`
`binom.pmf(k, n, p)` $=\Pr\big(X=\mathtt{k}\big)$ dove $X\sim B(\mathtt{n},\mathtt{p})$
`binom.cdf(k, n, p)` $=\Pr\big(X\leq\mathtt{k}\big)$ dove $X\sim B(\mathtt{n},\mathtt{p})$
`bimom.ppf(q, n, p)` $=$ `k` dove `k` è tale che $\Pr\big(X\leq\mathtt{k}\big)\cong\mathtt{q}$ per $X\sim B(\mathtt{n},\mathtt{p})$
`nbinom.xxx(...)`, è l'analogo per $X\sim NB(\mathtt{n},\mathtt{p})$.
`norm.xxx(...)`, è l'analogo per $Z\sim N(0,1)$.
`t.xxx(...,` ν`)`, è l'analogo per $T\sim t(\nu)$.