

Quesito 1. Della v.a. discreta X conosciamo la distribuzione di probabilità

$$\Pr(X = 4) = \frac{1}{2} \qquad \Pr(X = 5) = \frac{1}{2}$$

Della v.a. discreta Y conosciamo la distribuzione condizionata a X

$$\begin{aligned} \Pr(Y = 3 \mid X = 4) &= \frac{1}{2} & \Pr(Y = 3 \mid X = 5) &= \frac{1}{3} \\ \Pr(Y = 2 \mid X = 4) &= \frac{1}{2} & \Pr(Y = 2 \mid X = 5) &= \frac{2}{3} \end{aligned}$$

Calcolare la distribuzione di probabilità di Y

Esprimere i numeri razionali come frazioni.

Quesito 2. Assume the null hypothesis is true and denote by P the random variable that gives the p-value you would get if you run a test.

1. What is the probability that $\Pr(P < 0.05)$?
2. If we run the tests 8 times (independently), what is the probability of incorrectly rejecting at least once the null hypotheses with a significance $\alpha = 5\%$?
3. If we run the tests 8 times (independently), how small do we have to make the cutoff (α above) to lower to 5% the probability of incorrectly rejecting at least once the null hypotheses?

Quesito 3. A manufacturer claims that the mean lifetime of a lightbulb is on average at least 10 thousand hours with a standard deviation of 0.3. In a sample of 9 lightbulbs, it was found that they only last 9.5 thousand hours on average. The sample standard deviation is 0.6 thousand hours. Can we reject the manufacturer's claim? Answer the following questions:

1. H_0 ? H_1 ?
2. What test is required?
3. What is the value of the statistic?
4. What is the p-value?

Formulario: se $X \sim B(n, p)$ allora $E(X) = np$
se $X \sim NB(n, p)$ allora $E(X) = n(1 - p)/p$

$$T = \frac{\bar{X} - \bar{Y}}{S \cdot \sqrt{1/n_x + 1/n_y}} \quad \text{dove } S^2 = \frac{n_x - 1}{n_x + n_y - 2} \cdot S_x^2 + \frac{n_y - 1}{n_x + n_y - 2} \cdot S_y^2 \quad \text{ha distribuzione } t(n_x + n_y - 2)$$

Si assuma noto il valore delle seguenti funzioni della libreria `scipy.stats` di Python

`binom.pmf(k, n, p)` = $\Pr(X = k)$ dove $X \sim B(n, p)$

`binom.cdf(k, n, p)` = $\Pr(X \leq k)$ dove $X \sim B(n, p)$

`bimom.ppf(q, n, p)` = k dove k è tale che $\Pr(X \leq k) \cong q$ per $X \sim B(n, p)$

`nbinom.xxx(...)`, è l'analogo per $X \sim NB(n, p)$.

`norm.xxx(...)`, è l'analogo per $Z \sim N(0, 1)$.

`t.xxx(..., ν)`, è l'analogo per $T \sim t(\nu)$.