Bacheca

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Abstract Poche idee ben confuse.

1 Introduction

2 Preliminaries

3 Global samples

Fix a tuple of variables x and let $\bar{x} = \langle x_i : i < \omega \rangle$, where $|x_i| = |x|$. For $p(\bar{x}) \in S(\mathfrak{U})$ and $q(x) \subseteq L(\mathfrak{U})$ we define

$$\operatorname{Av}_{p\restriction n} q(x) \;=\; \frac{1}{n}\Big|\big\{i < n \ :\ p(\bar{x}) \vdash q(x_i)\big\}\Big|$$

If $p(\bar{x})$ is the type containing $\bar{x} = \bar{a}$ for some $\bar{a} \in \mathfrak{U}^{|x|\cdot\omega}$, we write $\operatorname{Av}_{\bar{a}\restriction n} q(x)$.

3.1 Definition *We say that $p(\bar{x}) \in S(\mathfrak{U})$ is a global sample if the limit below exists for every formula $\varphi(x) \in L(\mathfrak{U})$.*

$$\operatorname{Av}_p \varphi(x) \;=\; \lim_{n\to\infty} \operatorname{Av}_{p\restriction n} \varphi(x). \qquad \square$$

Qualche esempio in cui il limite non esiste?

3.2 Proposition *If $p(\bar{x})$ is a global sample, then $\lim\limits_{n\to\infty} \operatorname{Av}_{p\restriction n} q(x)$ exists for any $q(x) \subseteq L(\mathfrak{U})$.*

Proof ??? $\square$

3.3 Definition *Let $\bar{x} = \langle x_i : i < \omega \rangle$ and $|x_i| = |x|$. We say that the type $p(\bar{x}) \in S(\mathfrak{U})$ is a uniform global sample if it is a global sample and for every $\varphi(x,z) \in L$, every $b \in \mathfrak{U}^{|z|}$, and every $\varepsilon > 0$, there is an n such that $\operatorname{Av}_{p\restriction n} \varphi(x,b)$ is within ε from $\operatorname{Av}_p \varphi(x,b)$ for all $b \in \mathfrak{U}^{|z|}$.* $\square$

Qualche esempio in cui l'uniformità è rilevante?

3.4 Proposition *Let $\bar{x} = \langle x_i : i < \omega \rangle$ and $|x_i| = |x|$ and let $p(\bar{x}) \in S(\mathfrak{U})$. Then the following are equivalent*

1. *$p(\bar{x})$ is a uniform global sample;*
2. *for every $\varepsilon > 0$, there is an n and a formula $\vartheta(\bar{x}) \in p$ such that $\operatorname{Av}_n\big(\bar{a}; \varphi(x,b)\big)$ is within ε from $\operatorname{Av}\big(p; \varphi(x,b)\big)$ for all $b \in \mathfrak{U}^{|z|}$ and all $\bar{a} \vDash \vartheta(\bar{x})$.*

Proof ??? $\square$

Note that, if $p(\bar{x})$ is definable, say over M, then there is a formula $\psi_{m/n}(z) \in L(M)$ such that

$$\psi_{m/n}(z) \quad \Leftrightarrow \quad \mathrm{Av}_n\big(p(\bar{x});\varphi(x,z)\big) = \frac{m}{n}.$$

Assume that $p(\bar{x})$ is a global sample finitely satisfied in M. Then, under the assumptions of the proposition above, for every $\varepsilon > 0$ there is an n and a tuple $\bar{a} \in M^{|x|\cdot n}$ such that $\mathrm{Av}_n\big(\bar{a};\varphi(x,b)\big)$ is within ε from $\mathrm{Av}\big(p;\varphi(x,b)\big)$ for all $b \in \mathcal{U}^{|z|}$.

Let $p(\bar{x})$ be as above. Let $q'(x) = q(x) \cup \{\varphi(x)\}$ for some $q(x) \in S(M)$ and $\varphi(x) \in L(\mathcal{U})$. Then $\mathrm{Av}\big(p;q'(x)\big)$ exists and is either 0 or 1.

4 References

[1] Pandelis Dodos and Vassilis Kanellopoulos, *Ramsey theory for product spaces*, Mathematical Surveys and Monographs, vol. 212, American Mathematical Society, 2016.

[2] Domenico Zambella, *A crèche course in model theory*, AMS Open Math Notes, 2018. (The link points to the github version).