Scratch paper

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Abstract Poche idee, ben confuse.

1 Introduction

Al posto di $\mathbb{R}$ si potrebbe usare $\mathbb{Q}$ o anche meglio ω.

Per campioni si potrebbero prendere anche sequenze.

Per semplicità di notazione conviene usare misure con segno (e quindi campioni con segno). Ma non so quale sia il modo giusto. Per il momento faccio come se il problema non esistesse.

Soprattutto manca un'operazione tra campioni. Cioe se M e un semigruppo, vorrei avere un concetto di moltiplicazione tra campioni.

2 Abstract samples

Let M be a structure. The finite (fractional) sample expansion of M is a therefore-sorted expansion $\bar{M} = \langle M, \mathbb{R}, M^{\mathrm{s}} \rangle$ that has a domain for M, which we call the home-sort, a domain for $\mathbb{R}$, as an ordered field, and a domain M^{s}, which we call sample-sort, which is an $\mathbb{R}$-vector space that we describe below. Variables and elements of sample-sort (or tuple thereof) are denoted with symbols $x^{\mathrm{s}}, a^{\mathrm{s}}$ and variations thereof. The variable x is reserved for the home-sort. Otherwise we the context will clarify the sort.

An element $a^{\mathrm{s}} \in M^{\mathrm{s}}$ is a map $a^{\mathrm{s}} : M \to \mathbb{R}$ which is, non negative and almost always zero. These functions, which we call samples, are interpreted as (weighted) samples or as (signed) finite measures (up to normalization) that concentrated on the support of a^s.

The language of the expansion $\bar{M}$ is denoted by L. It expands the natural language of each sort: the language of M, the language ordered rings, and the language of $\mathbb{R}$-vector spaces. Moreover, for every formula $\varphi(x\,;y) \in L$, where y is a tuple a mixed sort, there is a symbol $\overline{\varphi}(x^{\mathrm{s}}\,;y)$ for a function $(M^{\mathrm{s}})^x \times M^y \to \mathbb{R}$. For $\varphi(x) \in L(\bar{M})$, we interpret $\overline{\varphi}(x^{\mathrm{s}})$ as the function that maps $a^{\mathrm{s}} \in M^{\mathrm{s}}$ to

$$\overline{\varphi}(a^{\mathrm{s}}) \;=\; \sum_{x \in \varphi(M)} a^s(x) \Big/ \sum_{x \in M} a^s(x)$$

Below, ε always ranges over the positive standard reals. If a and b are (hyper)reals, we write $a \approx_\varepsilon b$ for $|a-b| < \varepsilon$. We write $a \approx b$ if $a \approx_\varepsilon b$ holds for every ε.

2.1 Definition *Let $\mathcal{U}$ be a monster model of cardinality $\kappa > |L|$. Let $\bar{\mathcal{U}} = \langle \mathcal{U}, \mathbb{R}, \mathcal{U}^{\mathrm{s}} \rangle$ be as above and let $\bar{\mathcal{U}}^*$ be a saturated elementary extension of $\mathcal{U}^{\mathrm{s}}$ of cardinality κ. As all saturated models of cardinality κ are isomorphic, we can assume that $\mathcal{U}$ is the domain of the home-sort of $\bar{\mathcal{U}}^*$, hence we write $\bar{\mathcal{U}}^* = \langle \mathcal{U}, \mathbb{R}^*, \mathcal{U}^{\mathrm{s}*} \rangle$. Elements of $\mathcal{U}^{\mathrm{s}}$ are called finite samples, elements of $\mathcal{U}^{\mathrm{s}*}$ are called internal samples.*

Finally, an external sample is a maximally consistent set of formulas $p(x^{\mathrm{s}}) \subseteq L(\mathcal{U}, \mathbb{R}, \mathcal{U}^{\mathrm{s}*})$. (Note these are not types over the full of $\bar{\mathcal{U}}$.) □

2.2 Notation Let y and z be tuples of mixed sort. For any type $p(y)$ and formula $\varphi(y,z) \in L(\bar{\mathcal{U}}^*)$ we write

$$\varphi(p\,;\bar{\mathcal{U}}^*) \;=\; \Big\{ a \in (\bar{\mathcal{U}}^*)^z \;:\; p(y) \vdash \varphi(y\,;a) \Big\}$$

This notation intentionally confuses $p(y)$ with any of its realizations in some elementary extension of $\bar{\mathcal{U}}$. □

2.3 Definition *Let M be given. Let y be a tuple of the home-sort. A external sample $p(x^{\mathrm{s}})$ is*

1. *invariant if $\varphi(p\,;\mathcal{U}, \mathcal{U}^{\mathrm{s}*})$ is invariant over $\bar{M}$, for every $\varphi(x\,;y,z^{\mathrm{s}}) \in L$.*
2. *finitely satisfiable if every formula in $p(x^{\mathrm{s}})$ is satisfied by some element of M^{s}.*
3. *definable if $\varphi(p\,;\mathcal{U}, \mathcal{U}^{\mathrm{s}*})$ is definable over $\bar{M}$, for every $\varphi(x\,;y,z^{\mathrm{s}}) \in L$.* □

An external type is generically stable if it is definable and finitely satisfiable in some $\bar{M}$. □

Clearly every finitely satisfiable external sample is invariant.

We will use the symbol $\mathrel{\raise0.2ex\hbox{$\perp$}}_{\bar{M}}$ with the usual meaning.