Esercizio 1. (Angela Dosio) Let M and N be elementarily homogeneous structures of the same cardinality λ. Suppose that $M \models \exists x\, p(x) \Leftrightarrow N \models \exists x\, p(x)$ for every $p(x) \subseteq L$ such that $|x| < \lambda$ Prove that the two structures are isomorphic.

Esercizio 2. (Davide Peccioli) Let $|L| \leq \lambda$. Prove that the following are equivalent

1. N is λ-saturated
2. N is weakly λ-saturated and weakly λ-homogeneous.

Esercizio 3. (Costanza Furone) Let $\varphi(x) \in L$. Prove that the following are equivalent

1. $\varphi(x)$ is equivalent (in $\mathfrak{U}$) to some $\psi(x) \in L_{\rm qf}$
2. $\varphi(a) \leftrightarrow \varphi(fa)$ for every partial embedding $f : \mathfrak{U} \to \mathfrak{U}$ and $a \in (\mathrm{dom} f)^x$.

Can we use this result to infer that if all partial embeddings between models of T (any T, non necessarily complete) are elementary then T has elimination of quantifiers?

Esercizio 4. (Alessandro Martina) Let M be ω-saturated and let $k : M \to N$ be a finite Δ-morphism. Prove that the following are equivalent

1. $k : M \to N$ is a $\{\forall\vee\}\Delta$-morphism
2. for every $c \in N^{<\omega}$ there is $b \in M^{<\omega}$ such that $k \cup \{\langle b, c\rangle\} : M \to N$ is a Δ-morphism.

Esercizio 5. (Matteo Bisi) Let T be a complete theory without finite models in a language that consists only of unary predicates. Prove that T has elimination of quantifiers. Is the claim true if T is not complete?

Esercizio 6. (Roberto Carnevale) Let T be the theory of *discrete linear orders*, that is, T extends the theory of linear orders $T_{\rm lo}$ with the following two of axioms

dis↑. $\exists z \left[x < z \wedge \neg\exists y\; x < y < z\right]$
dis↓. $\exists z \left[z < x \wedge \neg\exists y\; z < y < x\right]$.

Let Δ be the set of formulas that contains (all alphabetic variants of) the formulas

$$x <_n y \;:=\; \exists^{\geq n} z\,(x<z<y)$$

and their negations, for every $n > 0$.

Prove that the theory of discrete linear orders has Δ-elimination of quantifiers. Prove that the structure $\mathbb{Q} \times \mathbb{Z}$ ordered with the lexicographic order is a saturated model of T.

Esercizio 7. (Leonardo Centazzo) Work in a monster model $\mathfrak{U}$. Suppose that every formula $\varphi(x) \in L$ is equivalent to a some quantifier-free formula $\psi(x) \in L(\mathfrak{U})$. Prove that T has positive Δ-elimination of quantifiers for Δ the set of formulas of the form $\exists y\, \forall z\, \theta(x, y, z)$ with $\theta(x, y, z) \in L_{\rm qf}$.

Esercizio 8. (Francesco Sulpizi) Let T be a consistent theory without finite models. Suppose that all completions of T are of the form $T \cup S$ for some set S of quantifier-free sentences. Apply Corollary 10.12 to prove that if all completions of T have elimination of quantifiers, so does T. Show that this fails when the completions of T have arbitrary complexity.

Esercizio 9. (Pietro Giura) Let T be a consistent theory without finite models. Suppose that all completions of T are of the form $T \cup S$ for some set S of quantifier-free sentences. Apply the compactness theorem prove that if all completions of T have elimination of quantifiers, so does T.