Esercizio 1. Let C be a finite set. Prove that if $C \cap M \neq \varnothing$ for every model M containing A, then $C \cap \operatorname{acl}(A) \neq \varnothing$.

Esercizio 2. Prove that for every $A \subseteq N$ there is an M such that $\operatorname{acl} A = M \cap N$.

Esercizio 1. Let $\varphi(x) \in L(\mathfrak{U})$ and fix an arbitrary set A. Prove that the following are equivalent

1. there is some model M containing A and such that $M \cap \varphi(\mathfrak{U}) = \varnothing$;
2. there is no consistent formula $\psi(z_1,\ldots,z_n) \in L(A)$ such that
$$\psi(z_1,\ldots,z_n) \to \bigwedge_{i=1}^{n} \varphi(z_i).$$

Esercizio 2. Let $\varphi(z) \in L(A)$ be a consistent formula. Prove that, if $a \in \operatorname{acl}(A, b)$ for every $b \models \varphi(z)$, then $a \in \operatorname{acl}(A)$. Prove the same claim with a type $p(z) \subseteq L(A)$ for $\varphi(z)$.

Esercizio 3. Let $a \in \mathcal{U} \smallsetminus \operatorname{acl}\varnothing$. Prove that $\mathcal{U}$ is isomorphic to some $\mathcal{V} \preceq \mathcal{U}$ such that $a \notin \mathcal{V}$.

Esercizio 4. Let T be a complete theory without finite models. Prove that the following are equivalent

1. M is minimal;
2. $a \equiv_M b$ for every $a, b \in \mathfrak{U} \smallsetminus M$.