Esercizio 1. (Pietro Giura) Let $a \mathop{\smile\!\!\!\!\!\!\;\perp}_M b$ then there is $\mathcal{V} \preceq \mathfrak{U}$ that is isomorphic to $\mathfrak{U}$ over M, a and such that $\mathcal{V} \mathop{\smile\!\!\!\!\!\!\;\perp}_M b$.

Esercizio 2. (Costanza Furone) Let $\bar{c} = \langle c_i : i \in I\rangle$ be an M-indiscernible sequence. Let $a \mathop{\smile\!\!\!\!\!\!\;\perp}_M \bar{c}$. Prove that $\bar{c}$ is indiscernible over M, b.

Esercizio 3. (Francesco Sulpizi) Let $a \mathop{\smile\!\!\!\!\!\!\;\perp}_M b$. Prove that for every c there is $b' \equiv_{M,a} b$ such that $a, c \mathop{\smile\!\!\!\!\!\!\;\perp}_M b'$.

Esercizio 4. (Leonardo Centazzo) Show that for every $b \in \mathfrak{U}^z$ there is a type $p(x;z) \subseteq L(M)$ such that for every $a \in \mathfrak{U}^x$ and $b' \in \mathfrak{U}^z$

$$a, b' \models p(x;z) \;\;\Leftrightarrow\;\; a \mathop{\smile\!\!\!\!\!\!\;\perp}_M b' \equiv_M b.$$

Esercizio 5. (Roberto Carnevale) Let T be strongly minimal. Let $a \in \mathfrak{U}^x$ and $b \in \mathfrak{U}^z$. Characterize $a \mathop{\smile\!\!\!\!\!\!\;\perp}_M b$ using the notion of algebraic closure (or possibly, dimension). Show that it is equivalent to $b \mathop{\smile\!\!\!\!\!\!\;\perp}_M a$.

Esercizio 6. (Alessandro Martina) Prove Remark 14.7

Esercizio 7. (Davide Peccioli) Dei sequenti risultati dire quali (pochi) non valgono se sostituiamo M con un generico insieme A: Lemma 14.8, Proposizione 14.9, Poposizione 14.11.

Esercizio 8. (Matteo Bisi) Let $N \succeq M$ be saturated and of cardinality $> |M|$. Let $a_i \mathop{\smile\!\!\!\!\!\!\;\perp}_M N, b$, for $i = 1, 2$. Prove that if $a_1 \equiv_N a_2$ then $a_1 \equiv_{N,b} a_2$.

Esercizio 9. (Angela Dosio) Show that in general there is no type $p(x;z) \subseteq L(M)$ such that for every $a \in \mathfrak{U}^x$ and $b \in \mathfrak{U}^z$

$$a; b \models p(x;z) \;\;\Leftrightarrow\;\; a \mathop{\smile\!\!\!\!\!\!\;\perp}_M b.$$

Hint: consider the theory of dense linear orders without endpoints.