Esercizio 1. Let $p(x) \subseteq L(A)$ and let $\varphi(x;y) \in L(A)$ be a formula that defines, when restricted to $p(\mathcal{U}^x)$, an equivalence relation with finitely many classes. Prove that there is a finite equivalence relation definable over A that coincides with $\varphi(x;y)$ on $p(\mathcal{U}^x)$.

Esercizio 2. Let T be strongly minimal and let $\varphi(x;z) \in L(A)$ with $|x| = 1$. For arbitrary $b \in \mathcal{U}^z$, prove that if the orbit of $\varphi(\mathcal{U}^x;b)$ over A is finite, then $\varphi(\mathcal{U}^x;b)$ is definable over $\operatorname{acl} A$.

Esercizio 3. Let T have elimination of imaginaries and $\varphi(x;z) \in L(A)$. For arbitrary $c \in \mathcal{U}^z$, prove that if the orbit of $\varphi(\mathcal{U}^x;c)$ over A is finite, then $\varphi(\mathcal{U}^x;c)$ is definable over $\operatorname{acl} A$.

Esercizio 4. Let $\bar{a}$ be a sequence such that $a_{\restriction I_0} \equiv a_{\restriction I_1}$ for every I_0, I_1 such that $I_0 < I_1$. Prove that $\bar{a}$ is a sequence of indiscernibles.

Esercizio 5. Let $\bar{\mathcal{D}} = \langle \mathcal{D}_n : n < \omega \rangle$ be a sequence of definable sets of sort $\sigma(x;z)$. Prove that if $\bar{\mathcal{D}}$ is a sequence of indiscernibles over A (in $\mathcal{U}^{\mathrm{eq}}$), then there is a sequence of A-indiscernibles $\bar{b} = \langle b_n : n < \omega \rangle$ such that $\bar{\mathcal{D}}_n = \sigma(\mathcal{U}^x; b_n)$.

Esercizio 6. Prove that following are equivalent for every $A \subseteq \mathcal{U}$

1. $\operatorname{acl}^{\mathrm{eq}} A = \operatorname{dcl}^{\mathrm{eq}}(\operatorname{acl} A)$ for every $A \subseteq \mathcal{U}$
2. $\operatorname{Aut}(\mathcal{U}/\operatorname{acl}^{\mathrm{eq}} A) = \operatorname{Aut}(\mathcal{U}/\operatorname{acl} A)$
3. $c \equiv_{\operatorname{acl} A} b \Leftrightarrow c \overset{\mathrm{Sh}}{\equiv}_A b$ for every $A \subseteq \mathcal{U}$ and $c, b \in \mathcal{U}^{<\omega}$.