Esercizio 1. Assume L is countable and let $M \preceq N$ have arbitrary (large) cardinality. Let $A \subseteq N$ be countable. Prove there is a countable model K such that $A \subseteq K \preceq N$ and $K \cap M \preceq N$ (in particular, $K \cap M$ is a model). Hint: adapt the *construction* used to prove the downward Löwenheim-Skolem Theorem.

Esercizio 2. Prove (for the pair of statements of your choice) that that the following are equivalent for every $X \subseteq \mathbb{R}$.

1a. X is open;

1b. X is closed;

2a. $b \approx a \in X \Rightarrow b \in {}^*X$ for every $b \in {}^*\mathbb{R}$.

2b. $a \in {}^*X \Rightarrow \operatorname{st} a \in X$ for every finite $a \in {}^*\mathbb{R}$.

Open and closed are understood w.r.t. the usual tolopology on $\mathbb{R}$.

Esercizio 3. (Bonus question) For what sets $X \subseteq \mathbb{R}$ the following holds?

2. $b \approx a \in {}^*X \;\Rightarrow\; b \in {}^*X$ for every $a, b \in {}^*\mathbb{R}$.