Esercizio 1. (Alessandro Martina) Let $\bar{a} = \langle a_i : i \in I\rangle$ be an A-indiscernible sequence and let $J \supseteq I$ with $|J| \leq \kappa$. Then there is an A-indiscernible sequence $\bar{c} = \langle c_i : i \in J\rangle$ such that $c_{\restriction I} = \bar{a}$.

Esercizio 2. (Francesco Sulpizi) Let $p(x) \in S(\mathcal{U})$ be a global type invariant over A. Let $a, b \models p_{\restriction A}(x)$. Prove that there is a sequence $\bar{c} = \langle c_i : i < \omega\rangle$ such that $a, \bar{c}$ and $b, \bar{c}$ are both sequences of A-indiscernibles.

Esercizio 3. (Matteo Bisi) Let M be an arbitrary model. Let $\varphi(x, y) \in L(M)$, where $|x| = |y|$, be such that $M \subseteq \varphi(\mathcal{U}, a)$ for some $a \in \mathcal{U}$. Prove that there is a sequence $\langle a_i : i < \omega\rangle$ in M^x such that $\varphi(a_i, a_j)$ holds for every $i < j < \omega$.

Esercizio 4. (Leonardo Centazzo) Assume that there is an elementary extension $\mathcal{U} \preceq \mathbb{N}$ that contains an element a such that $a + a \equiv a$ (the signature is arbitrary but contains a symbol for $+$). Prove that for every coloring of $\mathbb{N}$ in finitely many colors and for every $n < \omega$, there is a monochromatic arithmetical progression of length n. An *arithmetical progression* of length n is a sequence of natural numbers $\langle a_i : i \leq n\rangle$ such that $a_i - a_{i+1}$ is constant for $i < n$.

Esercizio 5. (Roberto Carnevale) Let $\langle c_i : i < \omega\rangle$ be an indiscernible sequence. Prove that there is an indiscernible sequence $\langle d_i : i < \omega\rangle$ such that $d_0, d_1 = c_1, c_0$.

Esercizio 6. (Davide Peccioli) Let $\langle \mathcal{D}_i : i < \omega\rangle$ be an A-indiscernible sequence in $\mathcal{U}^{\mathrm{eq}}$. Prove that there is an A-indiscernible sequence $\langle b_i : i < \omega\rangle$ in $\mathcal{U}^z$ and a formula $\varphi(x; z) \in L$ such that $\mathcal{D}_i = \varphi(\mathcal{U}; b_i)$.

Esercizio 7. (Pietro Giura) Let M be an arbitrary model. Let $\varphi(x, y) \in L(M)$, where $|x| = |y|$. Prove that there is a sequence $\langle a_i : i < \omega\rangle$ in M^x such that

$$\varphi(a_i, a_j) \leftrightarrow \varphi(a_h, a_k)$$

for every $i < j < \omega$ and $h < k < \omega$.

Esercizio 8. (Costanza Furone) Let M be a graph with the property that for every finite $A \subseteq M$ there is a $c \in M$ such that $A \subseteq r(c, \mathcal{U})$. A star in M is a subgraph whose edges all share a common vertex. We say that a coloring of the edges of M is locally finite if there is a k such that every star has at most k colors. Prove that for every locally finite coloring of the edges of M, there is an infinite monochromatic complete subgraph.

Esercizio 9. (Angela Dosio)