Esercizio 1. Assume L is countable and let $M \preceq N$ have arbitrary (large) cardinality. Let $A \subseteq N$ be countable. Prove there is a countable model K such that $A \subseteq K \preceq N$ and $K \cap M \preceq N$ (in particular, $K \cap M$ is a model).

Esercizio 2. Let L be the language of strict orders expanded with countably many constants $\{c_i : i \in \omega\}$. Let T be the theory that extends T_{dlo} by the axioms $c_i < c_{i+1}$ for all i. Prove (sketch) that T is complete. Describe three non isomorphic countable models of this theory.

(Basta descrivere senza troppi dettagli i tre modelli, senza dimostrazione)

Per quale di questi modelli vale un lemma di estensione simile a quelli dimostrati in classe?

Esercizio 3. Let N be free union of two random graphs N_1 and N_2. That is, $N = N_1 \sqcup N_2$ and $r^N = r^{N_1} \sqcup r^{N_2}$, where $\sqcup$ denotes the disjoint union. Show that N_1 is not definable without parameters. Write a first order formula $\psi(x, y)$ true if x and y belong to the same connected component of N. Axiomatize the class of graphs that are free union of two random graphs.

(Assiomatizzare in modo discorsivo, non scrivere lunghe formule.)