Esercizio 1. Let M be a random graph. Prove that if $M_1, M_2 \subseteq M$ are such that $M_1 \sqcup M_2 = M$, then M_1 or M_2 is a random graph.

Esercizio 2. Let N be free union of two random graphs N_1 and N_2. That is, $N = N_1 \sqcup N_2$ and $r^N = r^{N_1} \sqcup r^{N_2}$, where by $\sqcup$ we denote the disjoint union.

1. Prove that N_1 is not definable without parameters.
2. Prove that N_1 is definable (with parameters).
3. Axiomatize (in words) $\mathrm{Th}(N)$.

Esercizio 3. A (countable) back-and-forth system between to models M and N is a set $\mathcal{P}$ of functions k such that

0. $k : M \to N$ is a finite partial embedding;
1a. for every $b \in M$ there is $h \in \mathcal{P}$ such that $k \subseteq h$ and $b \in \mathrm{dom}\, h$;
1b. for every $c \in N$ there is $h \in \mathcal{P}$ such that $k \subseteq h$ and $c \in \mathrm{rang}\, h$.

Prove (briefly) that if L countable and there is a back-and-forth system between M and N then $M \equiv N$.