Esercizio 1. Show that for every $b \in \mathcal{U}^z$ there is a type $p(x;z) \subseteq L(M)$ such that for every $a \in \mathcal{U}^x$ and every $b' \in \mathcal{U}^z$

$$a \mathop{\downarrow}_M b' \equiv_M b \quad \Leftrightarrow \quad a, b' \models p(x;z).$$

Esercizio 2. Let T be strongly minimal. Let $a \in \mathcal{U}$ and $b \in \mathcal{U}^z$. Prove that $a \mathop{\downarrow}_M b$ if and only if $a \in M$, $b \in M^z$ or $a \notin \operatorname{acl}(M, b)$.

Esercizio 3. Let $a \mathop{\downarrow}_M b$ then there is $\mathcal{V} \preceq \mathcal{U}$ that is isomorphic to $\mathcal{U}$ over M, a and such that $\mathcal{V} \mathop{\downarrow}_M b$.

Esercizio 4. Let $a \mathop{\downarrow}_M b$. Prove that for every c there is $b' \equiv_{M,a} b$ such that $a, c \mathop{\downarrow}_M b'$.

Esercizio 5. Let $p(x) \in S(\mathcal{U})$ be a global type invariant over A. Let $a, b \models p_{\restriction A}(x)$. Prove that there is a sequence $\bar{c} = \langle c_i : i < \omega \rangle$ such that $a, \bar{c}$ and $b, \bar{c}$ are both sequences of A-indiscernibles.

Esercizio 6. Let $\langle c_i : i < \omega \rangle$ be an indiscernible sequence. Prove that there is an indiscernible sequence $\langle d_i : i < \omega \rangle$ such that $d_0, d_1 = c_1, c_0$.