Nome/i Cognome/i

Esercizio 1. Prove that the equivalence relation $a \overset{\text{L}}{\equiv}_A b$ is the transitive closure of the relation: there is a sequence $\langle c_i : i < \omega \rangle$ indiscernible over A such that $c_0 = a$ and $c_1 = b$.

Si ragioni come nella proposizione 16.19 e nel teorema 16.8.

Esercizio 2. (T stable) Prove that the following are equivalent for every $p(x) \in S(\mathcal{U})$

1. $p(x)$ is finitely satisfiable in M;
2. $p(x)$ is invariant over M.

Esercizio 3. (T stable) Prove that $a \perp_M b$ if and only if $b \perp_M a$.

Esercizio 4. Let $\varphi(x, y) \in L$, where $|x| = |y| = 1$. Suppose there is an infinite set $A \subseteq \mathcal{U}$ such that $\varphi(a, b) \leftrightarrow \varphi(b, a)$ for every two distinct $a, b \in A$. Prove that $\varphi(x; y)$ is unstable.

Esercizio 5. (T stable) Prove that the following are equivalent

1. $\varphi(x; z)$ is stable;
2. for every M and $a \in \mathcal{U}^{|x|}$ there is a formula $\psi(z) \in L(M)$ such that $\varphi(a; \mathcal{U}) =_M \psi(\mathcal{U})$.

Esercizio 6. Prove that if every formula $\varphi(x; z) \in L$ with $|x| = 1$ is stable then T is stable.

Si dimostri che tutte le formule con parametri sono stabili, poi si usino gli indiscernibili (ma altre vie sono possibili).