Esercizio 1. Let $\varphi(x\,;y) \in L(\mathfrak{U})$. Prove that the following are equivalent

1. there is a sequence $\langle a_i : i \in \omega \rangle$ such that $\varphi(\mathfrak{U}\,;a_i) \subset \varphi(\mathfrak{U}\,;a_{i+1})$ for every $i < \omega$;
2. there is a sequence $\langle a_i : i \in \omega \rangle$ such that $\varphi(\mathfrak{U}\,;a_{i+1}) \subset \varphi(\mathfrak{U}\,;a_i)$ for every $i < \omega$.

Esercizio 2. Let M and N be elementarily homogeneous structures of the same cardinality λ. Suppose that $M \models \exists x\, p(x) \Leftrightarrow N \models \exists x\, p(x)$ for every $p(x) \subseteq L$ such that $|x| < \lambda$. Prove that the two structures are isomorphic.

Esercizio 3. Let L be a language that extends that of strict linear orders with the constants $\{c_i : i \in \omega\}$. Let T be the theory that extends T_{dlo} with the axioms $c_i < c_{i+1}$ for every $i \in \omega$. Exhibit a countable saturated model and a countable model that is not homogeneous.