

Esercizio 1. Let N be free union of two random graphs N_1 and N_2 . That is, $N = N_1 \sqcup N_2$ and $r^N = r^{N_1} \sqcup r^{N_2}$, where by \sqcup we denote the disjoint union.

1. Prove that N is *not* a random graph.
2. Assuming $|N_1| = |N_2| = \omega$ prove that N_1 is not definable without parameters.
Hint: definable sets are fixed by automorphisms.
3. Show that the assumption $|N_1| = |N_2| = \omega$ is unnecessary
Hint: use downward Löwenheim-Skolem.
4. Write a first order formula $\psi(x, y)$ true if x and y belong to the same connected component of N .
5. Conclude that N_1 is definable (with parameters).
6. Axiomatize (in words) the class of graphs that are free union of two random graphs.