Esercizio 1. Prove that the following are equivalent for every subset $X \subseteq \mathbb{R}$

1. X is an open set in the usual topology on $\mathbb{R}$;
2. $b \approx a \in X \quad \Rightarrow \quad b \in {}^*X$ for every $b \in {}^*\mathbb{R}$.

Esercizio 2. Prove that the following are equivalent for every subset $X \subseteq \mathbb{R}$

1. X is bounded and closed in the usual topology on $\mathbb{R}$;
2. for every $b \in {}^*X$ there is an $a \in X$ such that $a \approx b$.

Esercizio 3. For which sets $X \subseteq \mathbb{R}$ does the following hold?

2. $b \approx a \in {}^*X \;\Rightarrow\; b \in {}^*X$ for every $a, b \in {}^*\mathbb{R}$.

Esercizio 4. Assume L is countable and let $M \preceq N$ have arbitrary (large) cardinality. Let $A \subseteq N$ be countable. Prove there is a countable model K such that $A \subseteq K \preceq N$ and $K \cap M \preceq N$ (in particular, $K \cap M$ is a model).

Esercizio 5. Let L be the language of strict orders expanded with countably many constants $\{c_i : i \in \omega\}$. Let T be the theory that extends T_{dlo} by the axioms $c_i < c_{i+1}$ for all i. Prove (sketch) that T is complete. Describe three non isomorphic countable models of this theory.