Esercizio 1. Let T be a consistent theory. Suppose that all completions of T are of the form $T \cup S$ for some set S of quantifier-free sentences. Prove (in the most direct possible way) that, if all completion of T have elimination of quantifiers, so does T. Show that this fails when the completions of T have arbitrary complexity.

Esercizio 2. For every $a \in \mathcal{U}^{|u|}$ and $A \subseteq \mathcal{U}$, the following are equivalent

1. a is solution of some algebraic formula $\varphi(u) \in L(A)$;
2. $a = a_1, \dots, a_n$ for some $a_1, \dots, a_n \in \operatorname{acl}(A)$.

Esercizio 3. Let $\varphi(z) \in L(A)$ be a consistent formula. Prove that, if $a \in \mathrm{acl}(A, b)$ for every $b \models \varphi(z)$, then $a \in \mathrm{acl}(A)$. Prove the same claim with a type $p(z) \subseteq L(A)$ for $\varphi(z)$.

Esercizio 1. Dimostrare che se N è (elementarmente) saturo allora è (elementarmente) omogeneo.

La dimostrazione usa la tecnica dell'andirivieni (back-and-forth). Nelle note è esposta nel caso generale. Si provi a fare una dimostrazione nel caso specifico.

Esercizio 2. Let $\varphi(x;z) \in L$. Prove that if the set $\{\varphi(a;\mathcal{U}) \ : \ a \in \mathcal{U}^{|x|}\}$ is infinite then it has cardinality κ. Does the claim remains true with a type $p(x;z) \subseteq L$ for $\varphi(x;z)$?

Suggerimento per la seconda domanda: potrebbe esserci un controesempio in $\mathcal{U} \equiv \mathbb{N}$ nel linguaggio degli ordini.

Esercizio 3. prove that for every $a \in \mathcal{U}^{|u|}$ and $A \subseteq \mathcal{U}$, the following are equivalent

1. a is solution of some algebraic formula $\varphi(u) \in L(A)$;
2. $a = a_1, \ldots, a_n$ for some $a_1, \ldots, a_n \in \operatorname{acl}(A)$.