Esercizio 1. Let M be a group. Given a 2-coloring of $M^{(2)}$ prove that there is a non-constant sequence $\langle a_i : i < \omega \rangle$ such that the set of pairs of the form $\{\prod a{\restriction}I,\ \prod a{\restriction}J\}$ for any $I < J$ subsets of ω is monocromatic. By $\prod\langle a_1, \ldots, a_n\rangle$ we denote the product $a_1 \cdot \cdots \cdot a_n$.

Esercizio 2. Let A be a coheir extension base. Prove that for every $b \in \mathcal{U}^z$ there is a structure $\mathcal{V} \preceq \mathcal{U}$ isomorphic to $\mathcal{U}$ over A such that $\mathcal{V} \mathrel{\raise0.2ex\hbox{$\smile$}\!\!\!\!\lower0.2ex\hbox{$\mid$}}_M b$ for every $A \subseteq M \preceq \mathcal{V}$.

Esercizio 3. Let $\varphi(x;z) \in L$. Let $p(x) \in S(\mathcal{U})$ be finitely satisfied in every $M \supseteq A$. Prove that if $\mathcal{D}_{p,\varphi}$ is saturated. We say that $\mathcal{D} \subseteq \mathcal{U}^x$ is saturated when the model $\mathcal{U}$ expanded with a predicate for $\mathcal{D}$ is saturated (in the expanded language).

Esercizio 4. Prove that the equivalence relation $a \overset{\mathrm{L}}{\equiv}_A b$ is the transitive closure of the relation: there is a sequence $\langle c_i : i < \omega\rangle$ indiscernible over A such that $c_0 = a$ and $c_1 = b$.

Esercizio 5. Let $G = \{f \in \mathrm{Aut}(\mathcal{U}/A) : f\mathcal{D} = \mathcal{D}$ for every $\mathcal{D}$ with $o(\mathcal{D}/A)$ bounded$\}$. Prove that $G = \mathrm{Aut}^{\mathrm{f}}(\mathcal{U}/A)$.