Esercizio 1. Let $p(x) \subseteq L(A)$ and let $\varphi(x;y) \in L(A)$ be a formula that defines, when restricted to $p(\mathfrak{U})$, an equivalence relation with finitely many classes. Prove that there is a finite equivalence relation definable over A that coincides with $\varphi(x;y)$ on $p(\mathfrak{U})$.

Esercizio 2. Let $A \subseteq \mathcal{U}$ and let $\mathcal{A}$ be a definable set with finite orbit over A. Without using the eq-expansion, prove that $\mathcal{A}$ is union of classes of a finite equivalence relation definable over A.

Esercizio 3. Let T be strongly minimal and let $\varphi(x\,;z) \in L(A)$ with $|x| = 1$. For arbitrary $b \in \mathcal{U}^{|z|}$, prove that if the orbit of $\varphi(\mathcal{U}\,;b)$ over A is finite, then $\varphi(\mathcal{U}\,;b)$ is definable over $\operatorname{acl} A$.

Esercizio 4. Let T have elimination of imaginaries and $\varphi(x;z) \in L(A)$. For arbitrary $c \in \mathcal{U}^{|z|}$, prove that if the orbit of $\varphi(\mathcal{U};c)$ over A is finite, then $\varphi(\mathcal{U};c)$ is definable over $\operatorname{acl} A$.