Esercizio 1. Let M be an L-structure and let $\psi(x), \varphi(x,y) \in L$. For each of the following conditions, write a sentence true in M exactly when

a. $\psi(M) \in \{\varphi(a,M) : a \in M\}$;

b. $\{\varphi(a,M) : a \in M\}$ contains at least two sets;

c. $\{\varphi(a,M) : a \in M\}$ contains only sets that are pairwise disjoint.

Risposta secca, nessuna giustificazione.

Esercizio 2. Let $M \preceq N$ and let $\varphi(x) \in L(M)$. Prove that $\varphi(M)$ is finite if and only if $\varphi(N)$ is finite and in this case $\varphi(N) = \varphi(M)$.

Esercizio 3. Let $M \preceq N$ and let $\varphi(x,z) \in L$. Suppose there are finitely many sets of the form $\varphi(a,N)$ for some $a \in N^{|x|}$. Prove that all these sets are definable over M.

Esercizio 4. Assume L is countable and let $M \preceq N$ have arbitrary (large) cardinality. Let $A \subseteq N$ be countable. Prove there is a countable model K such that $A \subseteq K \preceq N$ and $K \cap M \preceq N$ (in particular, $K \cap M$ is a model).