Esercizio 1. Assume L is countable and let $M \preceq N$ have arbitrary (large) cardinality. Let $A \subseteq N$ be countable. Prove there is a countable model K such that $A \subseteq K \preceq N$ and $K \cap M \preceq N$ (in particular, $K \cap M$ is a model). Hint: adapt the *construction* used to prove the downward Löwenheim-Skolem Theorem.

Esercizio 2. Prove either that 1a⇔2a, or that 1b⇔2b. (Choose whichever you prefer.)

1a. $X \subseteq \mathbb{R}$ is open;

2a. $b \approx a \in X \;\Rightarrow\; b \in {}^*X$ for every $b \in {}^*\mathbb{R}$.

1b. $X \subseteq \mathbb{R}$ is closed;

2b. $a \in {}^*X \;\Rightarrow\; \operatorname{st} a \in X$ for every finite $a \in {}^*\mathbb{R}$.

Open and closed are understood w.r.t. the usual tolopology on $\mathbb{R}$.

Esercizio 3. (Bonus question) For which sets $X \subseteq \mathbb{R}$ does the following hold?

2. $b \approx a \in {}^*X \;\Rightarrow\; b \in {}^*X$ for every $a, b \in {}^*\mathbb{R}$.