Esercizio 1. Let L be a countable language containing $L_{\rm gr}$ and assume $\mathcal U$ is a group. Let $p(x)\subseteq L$ define a subgroup of $\mathcal U$. Prove that there are some formulas $\varphi_n(x)\in L$ such that

- 1. $p(x) \leftrightarrow \{\varphi_n(x) : n < \omega\}$
- 2. $\varphi_{n+1}(x) \wedge \varphi_{n+1}(y) \rightarrow \varphi_n(x \cdot y)$.

Esercizio 2. Let $\varphi(x;z) \in L$ and $a \in \mathcal{U}^z$. Prove that the following are equivalent

- 1. the type $\{\varphi(x; fa) : f \in Aut(\mathcal{U})\}\$ is realized (in \mathcal{U})
- 2. there is a formula $\psi(z) \in L$ such that $\psi(a)$ and $\exists x \forall z [\psi(z) \rightarrow \varphi(x; z)]$.

Esercizio 3. Let $\varphi(x, y) \in L(\mathcal{U})$. Prove that if the set $\{\varphi(a, \mathcal{U}) : a \in \mathcal{U}^x\}$ is infinite then it has cardinality κ . Does the claim remain true with a type $p(x, y) \subseteq L(A)$ for $\varphi(x, y)$?

Esercizio 4. Let $\varphi(z) \in L(A)$ be a consistent formula. Prove that, if $a \in \operatorname{acl}(A, b)$ for every $b \models \varphi(z)$, then $a \in \operatorname{acl} A$. Prove the same claim with a type $p(z) \subseteq L(A)$ for $\varphi(z)$.

Esercizio 5. Let *C* be a finite set. Prove that if $C \cap M \neq \emptyset$ for every model *M* containing *A*, then $C \cap \operatorname{acl} A \neq \emptyset$.

Esercizio 6. Prove that for every $A \subseteq N$ there is an M such that $acl A = M \cap N$.