Nome/i Cognome/i

Esercizio 1. (Angela Dosio) Let $L(A)$ be countable. Let $P \subseteq S_x(A)$. Sketch (stress sketch) a proof of the following (if true). The following are equivalent

1. there is a model $M \supseteq A$ that omits all types in P
2. P is meager in the A-topology
3. there is a model M such that $\{p \in S_x(M) \ : \ p{\restriction}A \in P\}$ is meager in the M-topology.

Esercizio 2. (Costanza Furone) Let $p(x) \subseteq L(B)$ and $p_n(x) \subseteq L(A)$, for $n < \omega$, be such that

$$p(x) \ \rightarrow \ \bigvee_{i<\omega} p_i(x)$$

Prove that $p(x) \wedge \varphi(x) \ \rightarrow \ p_n(x)$ for some $n < \omega$ and some formula $\varphi(x) \in L(A)$ consistent with $p(x)$.

Esercizio 3. (Leonardo Centazzo) Let $A \subseteq B$ and $p(x) \subseteq L(A)$. Suppose that $\mathrm{tp}(a/B)$ is isolated over B for every $a \models p(x)$. Prove that $p(x)$ is isolated over A.

Esercizio 4. (Pietro Giura) Prove that the following are equivalent for every finite set A

1. T is ω-categorical
2. T is ω-categorical over A.

Esercizio 5. (Francesco Sulpizi) Assume L is countable and that T is complete. Suppose that for every finite tuple x there is a model M that realizes only finitely many types in $S_x(T)$. Prove that T is ω-categorical.

Esercizio 6. (Roberto Carnevale) The language L contains only a finite number of relational symbols. Let M be a countable structure that is *set-ultrahomogeneous*, that is, for every finite partial embedding $k : M \rightarrow M$ there is an $h \in \mathrm{Aut}(M)$ such that $h[\mathrm{dom}\, k] = \mathrm{im}\, k$. Prove that $\mathrm{Th}(M)$ is ω-categorical and model-complete.

Esercizio 7. (Alessandro Martina) Assume L is countable and let T be strongly minimal. Prove that the following are equivalent

1. T is ω-categorical
2. the algebraic closure of a finite set is finite.

Esercizio 8. (Davide Peccioli) Let $|x| = 1$. Prove that if $S_x(A)$ is countable for every finite set A, then T is small.

Esercizio 9. (Matteo Bisi) Let T be ω-categorical. Prove that if all algebraically closed sets are homogeneous, then T is not finitely axiomatizable.