Esercizio 1. Let L be a countable language containing L_{gr} and assume $\mathcal{U}$ is a group. Let $p(x) \subseteq L$ define a subgroup of $\mathcal{U}$. Prove that there are some formulas $\varphi_n(x) \in L$ such that

1. $p(x) \leftrightarrow \{\varphi_n(x) : n < \omega\}$
2. $\varphi_{n+1}(x) \wedge \varphi_{n+1}(y) \rightarrow \varphi_n(x \cdot y)$.

Esercizio 2. Let $\varphi(x; z) \in L$ and $a \in \mathcal{U}^z$. Prove that the following are equivalent

1. the type $\{\varphi(x; f\,a) : f \in \mathrm{Aut}(\mathcal{U})\}$ is realized (in $\mathcal{U}$)
2. there is a formula $\psi(z) \in L$ such that $\psi(a)$ and $\exists x \forall z\, [\psi(z) \rightarrow \varphi(x; z)]$.

Esercizio 3. Let $\varphi(x, y) \in L(\mathcal{U})$. Prove that if the set $\{\varphi(a, \mathcal{U}) : a \in \mathcal{U}^x\}$ is infinite then it has cardinality κ. Does the claim remain true with a type $p(x, y) \subseteq L(A)$ for $\varphi(x, y)$?

Esercizio 4. Let $\varphi(z) \in L(A)$ be a consistent formula. Prove that, if $a \in \mathrm{acl}(A, b)$ for every $b \models \varphi(z)$, then $a \in \mathrm{acl}A$. Prove the same claim with a type $p(z) \subseteq L(A)$ for $\varphi(z)$.

Esercizio 5. Let C be a finite set. Prove that if $C \cap M \neq \varnothing$ for every model M containing A, then $C \cap \mathrm{acl}A \neq \varnothing$.

Esercizio 6. Prove that for every $A \subseteq N$ there is an M such that $\mathrm{acl}A = M \cap N$.