Esercizio 1. Let $p(x) \subseteq L(B)$ and $p_n(x) \subseteq L(A)$, for $n < \omega$, be consistent types such that

$$p(x) \to \bigvee_{n<\omega} p_n(x)$$

Prove that there is an $n < \omega$ and a formula $\varphi(x) \in L(A)$ consistent with $p(x)$ such that

$$p(x) \wedge \varphi(x) \to p_n(x).$$

Esercizio 2. Prove that a strongly minimal theory has always a prime model.

Esercizio 3. Assume L is countable and let T be strongly minimal. Prove that the following are equivalent

1. T is ω-categorical;
2. the algebraic closure of a finite set is finite.

Esercizio 1. Let M be a second countable topological space (i.e. the topology has a countable base). We say that $A \subseteq M$ is meager if it is the countable union of nowhere dense sets.

Use Lemma 12.1 to prove the Kuratowski-Ulam Theorem, i.e. that for $A \subseteq M^2$ the following are equivalent

1. A is meager in M^2 with the product topology;
2. $\left\{x \in M : A \cap \{x\} \times M \text{ is not meager}\right\}$ is meager in M.

Esercizio 2. Prove that the following are equivalent

1. T is ω-categorical;
2. there is countable model that is both saturated and atomic.

Esercizio 3. Assume L is countable and that T is complete. Suppose that for every finite tuple x there is a model M that realizes only finitely many types in $S_x(T)$. Prove that T is ω-categorical.

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Esercizio 2. Let $|x| = 1$. Prove that if $S_x(A)$ is countable for every finite set A, then T is small.

Esercizio 3. Assume L is countable and let T be strongly minimal. Prove that the following are equivalent

1. T is ω-categorical;
2. the algebraic closure of a finite set is finite.

Esercizio 1. Suppose $L(A)$ is countable. Prove that if T is small over A, then there exists a countable saturated model containing A

Si cerchi di trovare una dimostrazione diretta.

Esercizio 2. Assume L is countable and that T is complete. Suppose that for every finite tuple x there is a model M that realizes only finitely many types in $S_x(T)$. Prove that T is ω-categorical.

Esercizio 3. Prove that the following are equivalent

1. T is ω-categorical;
2. there is countable model that is both saturated and atomic.

Esercizio 1. Prove that no theory is ω-categorical over an infinite set A.

Esercizio 2. Prove that a strongly minimal theory that has a model of finite dimension is not ω-categorical.

Esercizio 3. Prove that in a strongly minimal theory the following are equivalent for every type $p(x) \in S(A)$ and every infinite set A

1. $p(x)$ is isolated;
2. $p(x)$ is algebraic.