Esercizio 1. The language contains only the binary relations $<$ and e. The theory T_0 says that $<$ is a strict linear order and that e is an equivalence relation. Let $\mathcal{M}$ consists of models of T_0 and partial isomorphisms. Do rich models exist? Can we axiomatize their theory? If so, does it have elimination of quantifiers? Is it λ-categorical for some λ?

Esercizio 2. Prove that every model of $T_{\rm dag}$ is ω-ultraomogeneous (indipendenlty of cardinality and rank). Does the same holds for models of $T_{\rm acf}$?

Esercizio 3. Let $\varphi(x) \in L$. Prove that the following are equivalent

1. $\varphi(x)$ is equivalent to some $\psi(x) \in L_{\rm qf}$;
2. $\varphi(a) \leftrightarrow \varphi(fa)$ for every partial isomorphism $f : \mathcal{U} \to \mathcal{U}$ defined in a.

Use the result to prove that if T complete and partial isomorphisms between models of T preserve the truth of all formulas then T has elimination of quantifiers.

Esercizio 4. Let $\varphi(x, y) \in L(\mathcal{U})$. Prove that if the set $\{\varphi(a, \mathcal{U}) \ : \ a \in \mathcal{U}^{|x|}\}$ is infinite then it has cardinality κ. Does the claim remain true with a type $p(x, y) \subseteq L(A)$ for $\varphi(x, y)$?