

Esercizio 1. (T strongly minimal.) Prove that every infinite algebraically closed set is a model.

Esercizio 2. Let $p(x) \subseteq L(A)$ and let $\varphi(x; y) \in L(A)$ be a formula that defines, when restricted to $p(\mathcal{U})$, an equivalence relation with finitely many classes. Prove that there is a finite equivalence relation definable over A that coincides with $\varphi(x; y)$ on $p(\mathcal{U})$.

Esercizio 3. Let M be a graph with the property that for every finite $A \subseteq M$ there is a $c \in M$ such that $A \subseteq r(c, \mathcal{U})$. (This holds in particular when M is a random graph.) A star in M is a subgraph whose edges all share a common vertex. We say that a coloring of the edges of M is locally finite if there is a k such that every star has at most k colors. Prove that for every locally finite coloring of the edges of M , there is an infinite monochromatic complete subgraph.

Esercizio 4. Let T have elimination of imaginaries and $\varphi(x; z) \in L(A)$. For arbitrary $c \in \mathcal{U}^{|z|}$, prove that if the orbit of $\varphi(\mathcal{U}; c)$ over A is finite, then $\varphi(\mathcal{U}; c)$ is definable over $\text{acl}A$.