Esercizi 1,2,3 per gli studenti della LM, esercizio 4 per gli studenti della LT.

Esercizio 1. Let $A \subseteq B$ and $p(x) \subseteq L(A)$. Suppose that $\mathrm{tp}(a/B)$ is isolated (over B) for every $a \vDash p(x)$. Prove that $p(x)$ is isolated (over A).

Esercizio 2. Let $|x| = 1$. Prove that if $S_x(A)$ is countable for every finite set A, then T is small.

Esercizio 3. Assumimamo L numerabile. Sia $p(x) \subseteq L$ un tipo consistente non isolato. Esiste sempre un modello omogeneo che non realizza $p(x)$?

Esercizio 4. Let $p(x) \subseteq L(B)$ and $p_n(x) \subseteq L(A)$, for $n < \omega$, be consistent types such that

$$p(x) \rightarrow \bigvee_{n<\omega} p_n(x)$$

Prove that there is an $n < \omega$ and a formula $\varphi(x) \in L(A)$ consistent with $p(x)$ such that

$$p(x) \wedge \varphi(x) \rightarrow p_n(x).$$

Give an example that proves that the claim does not hold when ω is replaced with un uncountable cardinal.