Esercizio 1. Let $p(x) \subseteq L(A)$ and let $\varphi(x;y) \in L(A)$ be a formula that defines, when restricted to $p(\mathcal{U})$, an equivalence relation with finitely many classes. Prove that there is a finite equivalence relation definable over A that coincides with $\varphi(x;y)$ on $p(\mathcal{U})$.

Esercizio 2. Let $A \subseteq \mathcal{U}$ and let \mathcal{A} be a definable set with finite orbit over A. Without using the eq-expansion, prove that \mathcal{A} is union of classes of a finite equivalence relation definable over A.

Esercizio 3. Let T be strongly minimal and let $\varphi(x;z) \in L(A)$ with |x| = 1. For arbitrary $b \in \mathcal{U}^{|z|}$, prove that if the orbit of $\varphi(\mathcal{U};b)$ over A is finite, then $\varphi(\mathcal{U};b)$ is definable over $\mathrm{acl} A$.

Esercizio 4. Let T have weak elimination of imaginaries. Prove that for every $A \subseteq \mathcal{U}$ the following are equivalent

- 1. $a \stackrel{\text{Sh}}{\equiv}_A b$;
- 2. $a \equiv_{\operatorname{acl} A} b$.