Let $\mathcal{D}$ be a definable set. Let n be the minimal integer such that $\mathcal{D} = \varphi(\mathcal{U}; a)$ for some $\varphi(x; z) \in L$ and $a \in \mathcal{U}^n$. We claim that, if c is any other tuple such that $\mathcal{D} = \varphi(\mathcal{U}; c)$, then $\operatorname{rng} a = \operatorname{rng} c$.

Given a and c as above, assume for a contradiction that $\operatorname{rng} a \neq \operatorname{rng} c$.

Let $c = \langle c_0, \ldots, c_{n-1} \rangle$ and $a = \langle a_0, \ldots, a_{n-1} \rangle$.

Let $I \subseteq n$ be the set of those $i < n$ such that $a_i \in \operatorname{rng} c$.

Let $J \subseteq n$ be the set of those $j < n$ such that $c_j \in \operatorname{rng} a$.

Clearly $k := |J| = |I| < n$ and $b := a_{\restriction I} = c_{\restriction J}$.

Let u and v be two distinct tuples of variables of length k. Write (au) and (cv) for tuples of length n comprising parameters and variables such that $u = (au)_{\restriction I}$ and $v = (cv)_{\restriction J}$ and $a_{\restriction n \smallsetminus I} = (au)_{\restriction n \smallsetminus I}$ and $c_{\restriction n \smallsetminus J} = (cv)_{\restriction n \smallsetminus J}$.

$\forall u, v \; \forall x \; [\varphi(x; au) \leftrightarrow \varphi(x; cv)]$

Let y be a tuple of length k