

**Esercizio 1.** Prove that every type  $p(x) \subseteq L(\mathcal{U})$  that is strongly quasi-invariant over  $M$  extends to a global type that is quasi-invariant over  $M$ .

**Esercizio 2.** Let  $p(x) \in S(\mathcal{U})$  be a global type invariant over  $A$ . Let  $a, b \models p|_A(x)$ . Prove that there is a sequence  $\bar{c} = \langle c_i : i < \omega \rangle$  such that  $a, \bar{c}$  and  $b, \bar{c}$  are both sequences of  $A$ -indiscernibles.

**Esercizio 3.** For every  $\mathcal{D} \subseteq \mathcal{U}^{|z|}$  the following are equivalent

1.  $\mathcal{D}$  is approximated by  $\varphi(x; z)$ ;
2.  $\mathcal{D}$  is externally definable by  $\varphi(x; z)$ .

**Esercizio 4.** Prove that the equivalence relation  $a \stackrel{\perp}{\equiv}_A b$  is the transitive closure of the relation: there is a sequence  $\langle c_i : i < \omega \rangle$  indiscernible over  $A$  such that  $c_0 = a$  and  $c_1 = b$ .