**Esercizio 1.** Let $p(x) \subseteq L(B)$ and $p_n(x) \subseteq L(A)$, for $n < \omega$, be consistent types such that

$$p(x) \to \bigvee_{n<\omega} p_n(x)$$

Prove that there is an $n < \omega$ and a formula $\varphi(x) \in L(A)$ consistent with $p(x)$ such that

$$p(x) \wedge \varphi(x) \to p_n(x).$$

**Esercizio 2.** Prove that a strongly minimal theory has always a prime model.

**Esercizio 3.** Assume $L$ is countable and let $T$ be strongly minimal. Prove that the following are equivalent

1. $T$ is $\omega$-categorical;
2. the algebraic closure of a finite set is finite.

**Esercizio 1.** Let $M$ be a second countable topological space (i.e. the topology has a countable base). We say that $A \subseteq M$ is meager if it is the countable union of nowhere dense sets.

Use Lemma 12.1 to prove the Kuratowski-Ulam Theorem, i.e. that for $A \subseteq M^2$ the following are equivalent

1. $A$ is meager in $M^2$ with the product topology;
2. $\left\{x \in M : A \cap \{x\} \times M \text{ is not meager}\right\}$ is meager in $M$.

**Esercizio 2.** Prove that the following are equivalent

1. $T$ is $\omega$-categorical;
2. there is countable model that is both saturated and atomic.

**Esercizio 3.** Assume $L$ is countable and that $T$ is complete. Suppose that for every finite tuple $x$ there is a model $M$ that realizes only finitely many types in $S_x(T)$. Prove that $T$ is $\omega$-categorical.

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**Esercizio 2.** Let $|x| = 1$. Prove that if $S_x(A)$ is countable for every finite set $A$, then $T$ is small.

**Esercizio 3.** Assume $L$ is countable and let $T$ be strongly minimal. Prove that the following are equivalent

1. $T$ is $\omega$-categorical;
2. the algebraic closure of a finite set is finite.

**Esercizio 1.** Suppose $L(A)$ is countable. Prove that if $T$ is small over $A$, then there exists a countable saturated model containing $A$

Si cerchi di trovare una dimostrazione diretta.

**Esercizio 2.** Assume $L$ is countable and that $T$ is complete. Suppose that for every finite tuple $x$ there is a model $M$ that realizes only finitely many types in $S_x(T)$. Prove that $T$ is $\omega$-categorical.

**Esercizio 3.** Prove that the following are equivalent

1. $T$ is $\omega$-categorical;
2. there is countable model that is both saturated and atomic.

**Esercizio 1.** Prove that no theory is $\omega$-categorical over an infinite set $A$.

**Esercizio 2.** Prove that the following are equivalent for every finite set $A$

1. $T$ is $\omega$-categorical;
2. $T$ is $\omega$-categorical over $A$.

**Esercizio 3.** Prove that a strongly minimal theory that has a model of finite dimension is not $\omega$-categorical.