

Esercizio 1. (Alessandro Martina) Let $\varphi(x, y) \in L$, where $|x| = |y| = 1$. Suppose there is an infinite set $A \subseteq \mathcal{U}$ such that $\varphi(a, b) \leftrightarrow \varphi(b, a)$ for every two distinct $a, b \in A$. Prove that $\varphi(x; y)$ is unstable.

Esercizio 2. (Francesco Sulpizi) Find some unstable $p(x; z) \subseteq L(A)$ that admits no ladder of infinite length. Hint: work in a dense linear order, and consider the set of formulas of the form $x < y \vee y \notin \{a_0, \dots, a_n\}$.

Esercizio 3. (Leonardo Centazzo) Prove that

$$\bigcup \{\mathcal{M} : \mathcal{M} \text{ a minimal left ideal}\} = \bigcup \{\mathcal{M} : \mathcal{M} \text{ a minimal right ideal}\}.$$

Esercizio 4. (Roberto Carnevale) For every finite coloring of \mathbb{N} and every $k < \omega$, there is an infinite sequence $\bar{a} = \langle a_i(x) : i < \omega \rangle$ of nonconstant univariate polynomials with coefficients in \mathbb{N} such that $\{a(h) : a(x) \in \text{fp}(\bar{a}), h < k\}$ is monochromatic. Can we require that the $a_i(x)$ have increasing degrees?

Esercizio 5. (Matteo Bisi) Let Σ be a finite set of homomorphisms $\sigma : S \rightarrow C$ between infinite semigroups such that the following intersection is nonempty for all $c \in C$

$$\Sigma^{-1}[c] = \bigcap_{\sigma \in \Sigma} \sigma^{-1}[c]$$

where $\sigma^{-1}[c] = \{s \in S : \sigma(s) = c\}$. Prove that, for every finite coloring of C , there is some $g \in S \setminus C$ such that the set $\{\sigma g : \sigma \in \Sigma\}$ is monochromatic.

Esercizio 6. (Davide Peccioli) The following claim is too strong to be true. For every finite coloring of $\mathbb{N}^{(2)}$ there is an infinite sequence $\bar{a} = \langle a_i : i < \omega \rangle$ in \mathbb{N} such that $\text{fp}(\bar{a})^{(2)}$ is monochromatic. Replace $\text{fp}(\bar{a})^{(2)}$ with a (slightly) smaller set that makes the claim true.

Esercizio 7. (Pietro Giura) Let \mathcal{M} and \mathcal{N} be type-definable minimal left ideals of \mathcal{C} . Let $u \in \mathcal{M}$ and $v \in \mathcal{N}$ be idempotents. Prove that $u * \mathcal{M}$ and $v * \mathcal{N}$ are isomorphic groups.

Esercizio 8. (Costanza Furone) Let \mathcal{M} be a type-definable minimal left ideal of \mathcal{C} . Let $u \in \mathcal{M}$ be an idempotent. Prove that $u * \mathcal{M}$ is a minimal right ideal of \mathcal{C} .

Esercizio 9. (Angela Dosio) For every finite coloring of \mathbb{N} and every $k < \omega$, there is an infinite sequence $\bar{a} = \langle a_i(x) : i < \omega \rangle$ of nonconstant univariate polynomials with coefficients in \mathbb{N} such that $\{a(h) : a(x) \in \text{fp}(\bar{a}), h < k\}$ is monochromatic. Can we require that the $a_i(x)$ have increasing degrees?