

Esercizio 1. Let $L = \{<\}$ and let N be a ω_1 -saturated extension of \mathbb{Q} . Prove that there is an embedding $f: \mathbb{R} \rightarrow N$. Is it elementary? Can it be an isomorphism?

Esercizio 2. Prove that every model of T_{acf} is ω -ultrahomogeneous (independently of cardinality and transcendence degree).

Esercizio 3. Let M and N be elementarily homogeneous structures of the same cardinality λ . Suppose that $M \models \exists x p(x) \Leftrightarrow N \models \exists x p(x)$ for every $p(x) \subseteq L$ such that $|x| < \lambda$. Prove that the two structures are isomorphic.

Esercizio 4. Let $\varphi(x) \in L$. Prove that the following are equivalent

1. $\varphi(x)$ is equivalent to some $\psi(x) \in L_{\text{qf}}$;
2. $\varphi(a) \leftrightarrow \varphi(fa)$ for every a and every partial isomorphism $f: \mathcal{U} \rightarrow \mathcal{U}$ defined in a .

Use the result to prove Theorem 7.14 for T complete.