

Esercizio 1. Let M be a group. Given a 2-coloring of $M^{(2)}$ prove that there is a non-constant sequence $\langle a_i : i < \omega \rangle$ such that the set of pairs of the form $\{\prod a \restriction I, \prod a \restriction J\}$ for any $I < J$ subsets of ω is monochromatic. By $\prod \langle a_1, \dots, a_n \rangle$ we denote the product $a_1 \cdots a_n$.

Esercizio 2. Let A be a coheir extension base. Prove that for every $b \in \mathcal{U}^z$ there is a structure $\mathcal{V} \preceq \mathcal{U}$ isomorphic to \mathcal{U} over A such that $\mathcal{V} \perp_M b$ for every $A \subseteq M \preceq \mathcal{V}$.

Esercizio 3. Let $\varphi(x; z) \in L$. Let $p(x) \in S(\mathcal{U})$ be finitely satisfied in every $M \supseteq A$. Prove that if $\mathcal{D}_{p,\varphi}$ is saturated then it is definable in $\text{acl}^{\text{eq}} A$. We say that $\mathcal{D} \subseteq \mathcal{U}^x$ is saturated when the model \mathcal{U} expanded with a predicate for \mathcal{D} is saturated (in the expanded language).

Esercizio 4. Prove that the equivalence relation $a \stackrel{L}{\equiv}_A b$ is the transitive closure of the relation: there is a sequence $\langle c_i : i < \omega \rangle$ indiscernible over A such that $c_0 = a$ and $c_1 = b$.

Esercizio 5. Let $G = \{f \in \text{Aut}(\mathcal{U}/A) : f\mathcal{D} = \mathcal{D} \text{ for every } \mathcal{D} \text{ with } o(\mathcal{D}/A) \text{ bounded}\}$. Prove that $G = \text{Aut}^f(\mathcal{U}/A)$.