Nome/i Cognome/i

**Esercizio 1.** Prove that the equivalence relation $a \stackrel{\text{L}}{\equiv}_A b$ is the transitive closure of the relation: there is a sequence $\langle c_i : i < \omega \rangle$ indiscernible over $A$ such that $c_0 = a$ and $c_1 = b$.

Si ragioni come nella proposizione 16.19 e nel teorema 16.8.

**Esercizio 2.** ($T$ stable) Prove that the following are equivalent for every $p(x) \in S(\mathfrak{U})$

1. $p(x)$ is finitely satisfiable in $M$;
2. $p(x)$ is invariant over $M$.

**Esercizio 3.** ($T$ stable) Prove that $a \perp_M b$ if and only if $b \perp_M a$.

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**Esercizio 4.** Let $\varphi(x, y) \in L$, where $|x| = |y| = 1$. Suppose there is an infinite set $A \subseteq \mathfrak{U}$ such that $\varphi(a, b) \nleftrightarrow \varphi(b, a)$ for every two distinct $a, b \in A$. Prove that $\varphi(x\,; y)$ is unstable.

**Esercizio 5.** Prove that the following are equivalent

1. $T$ is stable;
2. for every $\varphi(x\,; z) \in L$, every model $M$ and every $a \in \mathfrak{U}^{|x|}$ there is a formula $\psi(z) \in L(M)$ such that $\varphi(a\,; \mathfrak{U}) =_M \psi(\mathfrak{U})$.

**Esercizio 6.** Prove that if every formula $\varphi(x\,; z) \in L$ with $|x| = 1$ is stable then $T$ is stable.

Si dimostri che tutte le formule con parametri sono stabili, poi si usino gli indiscernibili (ma altre vie sono possibili).