Esercizio 1. Let $\varphi(x\,;y) \in L(\mathcal{U})$. Prove that the following are equivalent

1. there is a sequence $\langle a_i : i \in \omega \rangle$ such that $\varphi(\mathcal{U}\,;a_i) \subset \varphi(\mathcal{U}\,;a_{i+1})$ for every $i < \omega$;
2. there is a sequence $\langle a_i : i \in \omega \rangle$ such that $\varphi(\mathcal{U}\,;a_{i+1}) \subset \varphi(\mathcal{U}\,;a_i)$ for every $i < \omega$.

Esercizio 2. Let $\varphi(x,y) \in L(\mathcal{U})$. Prove that if the set $\{\varphi(a,\mathcal{U}) \ : \ a \in \mathcal{U}^{|x|}\}$ is infinite then it has cardinality κ. Does the claim remain true with a type $p(x,y) \subseteq L(A)$ for $\varphi(x,y)$?

Suggerimento: si cerchi un controesempio con $\mathcal{U} \equiv \mathbb{N}$ nel linguaggio degli ordini.