Esercizio 1. Prove that the following are equivalent

1. T has weak elimination of imaginaries;
2. for $\mathcal{A} \in \mathcal{U}^{\rm eq}$, there is a smallest algebraically closed set $A \subseteq \mathcal{U}$ such that $\mathcal{A} \in {\rm dcl}^{\rm eq} A$;
3. for $a, b \in \mathcal{U}$, if $\mathcal{A} \in {\rm dcl}^{\rm eq}(a) \cap {\rm dcl}^{\rm eq}(b)$ then $\mathcal{A} \in {\rm dcl}^{\rm eq}\big({\rm acl}\, a \cap {\rm acl}\, b\big)$.

Esercizio 2. Let M be a graph. A star in M is a subgraph whose edges all share a common vertex. We say that a coloring of the edges of M is locally finite if there is a k such that every star has at most k colors. Assum M has the property that for every finite $A \subseteq M$ there is a $c \in M$ such that $A \subseteq r(c, \mathcal{U})$. (This holds in particular when M is a random graph.) Prove that for every locally finite coloring of the edges M has an infinite monochromatic complete subgraph.