Esercizio 1. Let $\varepsilon(x, y) \subseteq L(A)$ be a type-definable equivalence relation with $< \lambda$ classes. Prove that every λ-saturated model containing A intersects every class of $\varepsilon(x, y)$.

Esercizio 2. Let $L = \{<\}$ and let N be a ω_1-saturated extension of $\mathbb{Q}$. Prove that there is an embedding $f : \mathbb{R} \to N$. Is it elementary? Is it an isomorphism?

Esercizio 3. Let $L = \{<\}$ and let N be a saturated extension of $\mathbb{Q}$. Prove that there $2^{|N|}$ Dedekind cuts of N.

Esercizio 4. Let M be an arbitrary structure of cardinality larger than $|L|$. Prove that M has an ω-homogeneous elementary extension of the same cardinality.

Esercizio 5. Let M and N be elementarily homogeneous structures of the same cardinality λ. Suppose that $M \models \exists x\, p(x) \Leftrightarrow N \models \exists x\, p(x)$ for every $p(x) \subseteq L$ such that $|x| < \lambda$ Prove that the two structures are isomorphic.

Esercizio 6. Let L be a countable language containing L_{gr} and assume $\mathcal{U}$ is a group. Prove that the following are equivalent

1. every model M is a normal subgroup of $\mathcal{U}$
2. some ω-saturated model M is a normal subgroup of $\mathcal{U}$
3. $\mathcal{U}$ is a BFC group.

A group G is BFC (has boundedly finite conjugacy classes) if $\{g\, a\, g^{-1} : g \in G\}$, as a ranges over G, have at most n elements, for some fixed n.

Esercizio 7. Prove that for every regular $\lambda \geq |L|$ every theory T with an infinite model has a model that is λ-saturated and λ-homogeneous.

Esercizio 8. Let λ be such that every first-order theory T with an infinite model has a saturated model of cardinality λ. Prove that $\lambda = \lambda^{<\lambda}$.