Esercizio 1. (Leonardo Centazzo) Let $A \subseteq \mathcal{U}$ and let $\mathcal{D}$ be a definable set with finite orbit over A. Without using the eq-expansion, prove that $\mathcal{D}$ is union of classes of a finite equivalence relation definable over A.

Esercizio 2. (Pietro Giura) Prove that the following are equivalent

1. T has weak elimination of imaginaries
2. T has geometric elimination of imaginaries and $\mathrm{acl}^{\mathrm{eq}} A = \mathrm{dcl}^{\mathrm{eq}}\big(\mathrm{acl} A\big)$ for every $A \subseteq \mathcal{U}$

Esercizio 3. (Costanza Furone) Let $A \subseteq N \models T_{\mathrm{rg}}$. Let $\varphi(x) \in L(A)$. Prove that if $\varphi(N)$ nonempty and disjoint from A then $\varphi(N)$ is a random graph.

Esercizio 4. (Francesco Sulpizi) Prove that if $\varphi(N)$ nonempty and disjoint from A then $\varphi(N)$ is a random graph. Let $p(x) \subseteq L(A)$ and let $\varphi(x\,;y) \in L(A)$ be a formula that defines, when restricted to $p(\mathcal{U}^x)$, an equivalence relation with finitely many classes. Prove that there is a finite equivalence relation definable over A that coincides with $\varphi(x\,;y)$ on $p(\mathcal{U}^x)$.

Esercizio 5. (Roberto Carnevale) The theory T has *uniform* elimination of imaginaries if for every formula $\varphi(x\,;u)$ there is a formula $\sigma(x,;z)$ such that

$$\forall u\, \exists^{=1} z\, \forall x\, \big[\varphi(x\,;u) \leftrightarrow \sigma(x\,;z)\big]$$

Claim. If $\mathcal{U}$ contains two definable elements (for simplicity, call them 0 and 1) then elimination of imaginaries implies uniform elimination of imaginaries.

Unfortunately, as stated the claim is false – if only for fatuous reasons. So, prove it assuming (in the definition of uniform elimination) that $\forall u\, \exists x\, \varphi(x\,;u)$.

Esercizio 6. (Alessandro Martina) Prove that the following are equivalent

1. T has weak elimination of imaginaries
2. for every $\mathcal{D} \in \mathcal{U}^{\mathrm{eq}}$ there exists the least algebraically closed set $A \subseteq \mathcal{U}$ such that $\mathcal{D}$ is definable over A.

Esercizio 7. (Davide Peccioli) Prove that following are equivalent for every $A \subseteq \mathcal{U}$

1. $\mathrm{acl}^{\mathrm{eq}} A = \mathrm{dcl}^{\mathrm{eq}}\big(\mathrm{acl} A\big)$ for every $A \subseteq \mathcal{U}$
2. $\mathrm{Aut}(\mathcal{U}/\mathrm{acl}^{\mathrm{eq}} A) = \mathrm{Aut}(\mathcal{U}/\mathrm{acl} A)$
3. $c \equiv_{\mathrm{acl} A} b \Leftrightarrow c \overset{\mathrm{Sh}}{\equiv}_A b$ for every $A \subseteq \mathcal{U}$ and $c, b \in \mathcal{U}^{<\omega}$.

Esercizio 8. (Matteo Bisi) Let T have elimination of imaginaries. Let $\varphi(x\,;z) \in L(A)$ and $c \in \mathcal{U}^z$ be given. Prove that if the orbit of $\varphi(\mathcal{U}^x; c)$ over A is finite, then $\varphi(\mathcal{U}^x; c)$ is definable over $\mathrm{acl} A$. Is the same true assuming only weak elemination?

Esercizio 9. (Angela Dosio) For which of the following theories every completion has elimination of imaginaries? (Some answers are trivial, others I do not know.)

1. PA
2. ZF+V=L
3. ZFC
4. ZF.