Esercizio 1. Let $\varphi(x, y) \in L$, where $|x| = |y| = 1$. Suppose there is an infinite set $A \subseteq \mathcal{U}$ such that $\varphi(a, b) \leftrightarrow \varphi(b, a)$ for every two distinct $a, b \in A$. Prove that $\varphi(x; y)$ is unstable.

Esercizio 2. Prove that the following are equivalent for every $p(x; z) \subseteq L(A)$

1. $p(x; z)$ is stable

2. for some finite n, there is no ladder of length n for $p(x; z)$

3. there is $q(x; z) \subseteq L(A)$ equivalent to $p(x; z)$ containing only stable formulas.

Esercizio 3. Prove that if $p(x; z)$ admits ladder sequences of arbitrary finite length, then it admits a ladder sequence of infinite length.

Esercizio 4. Prove that the following are equivalent

1. $\varphi(x; z)$ is a stable formula

2. $\left| S_\varphi(A^z) \right| \leq |A|$ for all countable sets A

3. $\left| S_\varphi(A^z) \right| < 2^{|A|}$ for all countable sets A.

Hint: the proof in the lecture notes is incomplete.