

**Esercizio 1.** (Alessandro Martina) Let  $\bar{a} = \langle a_i : i \in I \rangle$  be an  $A$ -indiscernible sequence and let  $J \supseteq I$  with  $|J| \leq \kappa$ . Then there is an  $A$ -indiscernible sequence  $\bar{c} = \langle c_i : i \in J \rangle$  such that  $c_{\upharpoonright I} = \bar{a}$ .

**Esercizio 2.** (Francesco Sulpizi) Let  $p(x) \in S(\mathcal{U})$  be a global type invariant over  $A$ . Let  $a, b \models p_{\upharpoonright A}(x)$ . Prove that there is a sequence  $\bar{c} = \langle c_i : i < \omega \rangle$  such that  $a, \bar{c}$  and  $b, \bar{c}$  are both sequences of  $A$ -indiscernibles.

**Esercizio 3.** (Matteo Bisi) Let  $M$  be an arbitrary model. Let  $\varphi(x, y) \in L(M)$ , where  $|x| = |y|$ , be such that  $M \models \varphi(\mathcal{U}, a)$  for some  $a \in \mathcal{U}$ . Prove that there is a sequence  $\langle a_i : i < \omega \rangle$  in  $M^x$  such that  $\varphi(a_i, a_j)$  holds for every  $i < j < \omega$ .

**Esercizio 4.** (Leonardo Centazzo) Assume that there is an elementary extension  $\mathcal{U} \leq \mathbb{N}$  that contains an element  $a$  such that  $a + a \equiv a$  (the signature is arbitrary but contains a symbol for  $+$ ). Prove that for every coloring of  $\mathbb{N}$  in finitely many colors and for every  $n < \omega$ , there is a monochromatic arithmetical progression of length  $n$ . An *arithmetical progression* of length  $n$  is a sequence of natural numbers  $\langle a_i : i \leq n \rangle$  such that  $a_{i+1} - a_i$  is constant for  $i < n$ .

**Esercizio 5.** (Roberto Carnevale) Let  $\langle c_i : i < \omega \rangle$  be an indiscernible sequence. Prove that there is an indiscernible sequence  $\langle d_i : i < \omega \rangle$  such that  $d_0, d_1 = c_1, c_0$ .

**Esercizio 6.** (Davide Peccioli) Let  $\langle \mathcal{D}_i : i < \omega \rangle$  be an  $A$ -indiscernible sequence in  $\mathcal{U}^{\text{eq}}$ . Prove that there is a formula  $\varphi(x; z) \in L$  and an  $A$ -indiscernible sequence  $\langle b_i : i < \omega \rangle$  in  $\mathcal{U}^z$  such that  $\mathcal{D}_i = \varphi(\mathcal{U}; b_i)$ .

**Esercizio 7.** (Pietro Giura) Let  $M$  be an arbitrary model. Let  $\varphi(x, y) \in L(M)$ , where  $|x| = |y|$ . Prove that there is a sequence  $\langle a_i : i < \omega \rangle$  in  $M^x$  such that

$$\varphi(a_i, a_j) \leftrightarrow \varphi(a_h, a_k)$$

for every  $i < j < \omega$  and  $h < k < \omega$ .

**Esercizio 8.** (Costanza Furone) Let  $M$  be a graph with the property that for every finite  $A \subseteq M$  there is a  $c \in M$  such that  $A \subseteq r(c, \mathcal{U})$ . A star in  $M$  is a subgraph whose edges all share a common vertex. We say that a coloring of the edges of  $M$  is locally finite if there is a  $k$  such that every star has at most  $k$  colors. Prove that for every locally finite coloring of the edges of  $M$ , there is an infinite monochromatic complete subgraph.

Nome/i Cognome/i

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**Esercizio 9.** (Angela Dosio)