

Esercizio 1. Let M and N be elementarily homogeneous structures of the same cardinality λ . Suppose that $M \models \exists x p(x) \Leftrightarrow N \models \exists x p(x)$ for every $p(x) \subseteq L$ such that $|x| < \lambda$. Prove that the two structures are isomorphic.

Esercizio 2. Let $\varphi(x) \in L$. Prove that the following are equivalent

1. $\varphi(x)$ is equivalent to some $\psi(x) \in L_{\text{qf}}$;
2. $\varphi(a) \leftrightarrow \varphi(fa)$ for every partial isomorphism $f : \mathcal{U} \rightarrow \mathcal{U}$ defined in a .

Esercizio 3. Let $p(x) \subseteq L(A)$, with $|x| < \omega$. Prove that if $p(\mathcal{U})$ is infinite then it has cardinality κ . Show that this may not be true if x is an infinite tuple.