

Esercizio 1. (Leonardo Centazzo) Let $A \subseteq \mathcal{U}$ and let \mathcal{D} be a definable set with finite orbit over A . Without using the eq-expansion, prove that \mathcal{D} is union of classes of a finite equivalence relation definable over A .

Esercizio 2. (Pietro Giura) Prove that the following are equivalent

1. T has weak elimination of imaginaries
2. T has geometric elimination of imaginaries and $\text{acl}^{\text{eq}} A = \text{dcl}^{\text{eq}}(\text{acl} A)$ for every $A \subseteq \mathcal{U}$

Esercizio 3. (Costanza Furone) Let $A \subseteq N \models T_{\text{rg}}$. Let $\varphi(x) \in L(A)$. Prove that if $\varphi(N)$ nonempty and disjoint from A then $\varphi(N)$ is a random graph.

Esercizio 4. (Francesco Sulpizi) Prove that if $\varphi(N)$ nonempty and disjoint from A then $\varphi(N)$ is a random graph. Let $p(x) \subseteq L(A)$ and let $\varphi(x; y) \in L(A)$ be a formula that defines, when restricted to $p(\mathcal{U}^x)$, an equivalence relation with finitely many classes. Prove that there is a finite equivalence relation definable over A that coincides with $\varphi(x; y)$ on $p(\mathcal{U}^x)$.

Esercizio 5. (Roberto Carnevale) The theory T has *uniform* elimination of imaginaries if for every formula $\varphi(x; u)$ there is a formula $\sigma(x; z)$ such that

$$\forall u \exists^{=1} z \forall x [\varphi(x; u) \leftrightarrow \sigma(x; z)]$$

Claim. If \mathcal{U} contains two definable elements (for simplicity, call them 0 and 1) then elimination of imaginaries implies uniform elimination of imaginaries.

Unfortunately, as stated the claim is false – if only for fatuous reasons. So, prove it assuming (in the definition of uniform elimination) that $\forall u \exists x \varphi(x; u)$.

Esercizio 6. (Alessandro Martina) Prove that the following are equivalent

1. T has weak elimination of imaginaries
2. for every $\mathcal{D} \in \mathcal{U}^{\text{eq}}$ there exists the least algebraically closed set $A \subseteq \mathcal{U}$ such that \mathcal{D} is definable over A .

Esercizio 7. (Davide Peccioli) Prove that following are equivalent for every $A \subseteq \mathcal{U}$

1. $\text{acl}^{\text{eq}} A = \text{dcl}^{\text{eq}}(\text{acl} A)$ for every $A \subseteq \mathcal{U}$
2. $\text{Aut}(\mathcal{U}/\text{acl}^{\text{eq}} A) = \text{Aut}(\mathcal{U}/\text{acl} A)$
3. $c \equiv_{\text{acl} A} b \Leftrightarrow c \equiv_A^{\text{Sh}} b$ for every $A \subseteq \mathcal{U}$ and $c, b \in \mathcal{U}^{<\omega}$.

Esercizio 8. (Matteo Bisi) Let T have elimination of imaginaries. Let $\varphi(x; z) \in L(A)$ and $c \in \mathcal{U}^z$ be given. Prove that if the orbit of $\varphi(\mathcal{U}^x; c)$ over A is finite, then $\varphi(\mathcal{U}^x; c)$ is definable over $\text{acl} A$. Is the same true assuming only weak elimination?

Esercizio 9. (Angela Dosio) For which of the following theories every completion has elimination of imaginaries? (Some answers are trivial, for others, I do not know.)

1. PA
2. $ZF+V=L$
3. ZFC
4. ZF