**Esercizio 1.** Let $T$ be a consistent theory. Suppose that all completions of $T$ are of the form $T \cup S$ for some set $S$ of quantifier-free sentences. Prove (in the most direct possible way) that, if all completion of $T$ have elimination of quantifiers, so does $T$. Show that this fails when the completions of $T$ have arbitrary complexity.

**Esercizio 2.** Let $L$ be the language of strict orders. Let $T$ be the theory of *discrete linear orders*. Namely, $T$ extends $T_{\text{dlo}}$ with the following two of axioms

dis↑. $\exists z\, \big[x < z \ \wedge\ \neg\exists y\ x < y < z\big]$;

dis↓. $\exists z\, \big[z < x \ \wedge\ \neg\exists y\ z < y < x\big]$.

Let $\Delta$ be the set of formulas that contains (all alphabetic variants of) the formulas

$$x <_n y \ := \ \exists^{\geq n} z\, (x{<}z{<}y)$$

and their negations (read $<_0$ as $<$).

1. Prove that the structure $\mathbb{Q} \times \mathbb{Z}$ ordered with the lexicographic order

   $$(a_1, a_2) < (b_1, b_2) \quad \Leftrightarrow \quad a_1 < b_1 \ \text{ or } \ (a_1 = b_1 \ \text{ e } \ a_2 < b_2)$$

   is a saturated model of $T$.
2. Prove that the theory of discrete linear orders has $\Delta$-elimination of quantifiers.
3. Prove that T is model-complete.

**Esercizio 3.** Let $\varphi(z) \in L(A)$ be a consistent formula. Prove that, if $a \in \mathrm{acl}(A, b)$ for every $b \models \varphi(z)$, then $a \in \mathrm{acl}(A)$. Prove the same claim with a type $p(z) \subseteq L(A)$ for $\varphi(z)$.

**Esercizio 1.** Dimostrare che se $N$ è (elementarmente) saturo allora è (elementarmente) omogeneo.

La dimostrazione usa la tecnica dell'andirivieni (back-and-forth). Nelle note è esposta nel caso generale. Si provi a fare una dimostrazione nel caso specifico.

**Esercizio 2.** Let $\varphi(x;z) \in L$. Prove that if the set $\{\varphi(a;\mathcal{U}) \,:\, a \in \mathcal{U}^{|x|}\}$ is infinite then it has cardinality $\kappa$. Does the claim remains true with a type $p(x;z) \subseteq L$ for $\varphi(x;z)$?

Suggerimento per la seconda domanda: potrebbe esserci un controesempio in $\mathcal{U} \equiv \mathbb{N}$ nel linguaggio degli ordini.

**Esercizio 3.** Si dia una dimostrazione sintattica di $\operatorname{acl}(\operatorname{acl} A) \subseteq \operatorname{acl} A$