Esercizio 1. Let $p(x) \subseteq L(A)$ and let $\varphi(x;y) \in L(A)$ be a formula that defines, when restricted to $p(\mathcal{U})$, an equivalence relation with finitely many classes. Prove that there is a finite equivalence relation definable over A that coincides with $\varphi(x;y)$ on $p(\mathcal{U})$.

Esercizio 2. Let $A \subseteq \mathcal{U}$ and let $\mathcal{A}$ be a definable set with finite orbit over A. Without using the eq-expansion, prove that $\mathcal{A}$ is union of classes of a finite equivalence relation definable over A.

Esercizio 3. Let T be strongly minimal and let $\varphi(x\,;z) \in L(A)$ with $|x| = 1$. For arbitrary $b \in \mathcal{U}^{|z|}$, prove that if the orbit of $\varphi(\mathcal{U}\,;b)$ over A is finite, then $\varphi(\mathcal{U}\,;b)$ is definable over $\operatorname{acl} A$.

Esercizio 4. Assume that $\operatorname{acl}^{\mathrm{eq}} A = \operatorname{dcl}^{\mathrm{eq}}(\operatorname{acl} A)$ for every $A \subseteq \mathcal{U}$. Prove that, for every $A \subseteq \mathcal{U}$ and $a, b \in \mathcal{U}^{<\omega}$, the following are equivalent

1. $a \overset{\mathrm{Sh}}{\equiv}_A b$;
2. $a \equiv_{\operatorname{acl} A} b$.

Prove that the assumption above holds when T has weak elimination of imaginaries.