Esercizio 1. Suppose that every formula $\varphi(x) \in L$ is equivalent to a some quantifier-free formula $\psi(x) \in L(\mathcal{U})$. Prove that T has positive Δ-elimination of quantifiers for Δ the set of formulas of the form $\exists y \forall z \theta(x, y, z)$ with $\theta(x, y, z) \in L_{\rm qf}$.

Esercizio 2. Let $A \subseteq B$ and $p(x) \subseteq L(A)$. Suppose that ${\rm tp}(a/B)$ is isolated over B for every $a \models p(x)$. Prove that $p(x)$ is isolated over A.

Esercizio 3. A countable structure M is *set-ultrahomogeneous* if for every finite partial embedding $k : M \to M$ there is an $h \in {\rm Aut}(M)$ such that ${\rm dom} k = {\rm dom} h$ and ${\rm im} k = {\rm im} h$. Prove that if L contains only a finite number of relational symbols, then set-homogeneous models have an ω-categorical theory.

Esercizio 4. Let M be set-homogeneous set-homogeneous in a language L that contains only a finite number of relational symbols. Prove that ${\rm Th}(M)$ is model-complete[1].

Esercizio 5. Assume L is countable and that T is complete. Suppose that for every finite tuple x there is a model M that realizes only finitely many types in $S_x(T)$. Prove that T is ω-categorical.

Esercizio 6. Let $|x| = 1$. Prove that if $S_x(A)$ is countable for every finite set A, then T is small.

Esercizio 7. Let T be a complete theory without finite models in a language that consists only of unary predicates. Prove that T has elimination of quantifiers.

Esercizio 8. Let T be a complete theory without finite models. Prove that the following are equivalent

1. M is minimal
2. $a \equiv_M b$ for every $a, b \in \mathcal{U} \smallsetminus M$.

Esercizio 9. Let $\varphi(x) \in L_{\rm qf}(\mathcal{U})$. Find a simple theory that disproves the equivalence of the following

1. $\varphi(x)$ is equivalent to some formula $\psi(x) \in L_{\rm qf}(A)$
2. $\varphi(x)$ is invariant over A.

[1] Potrebbe essere meno semplice.