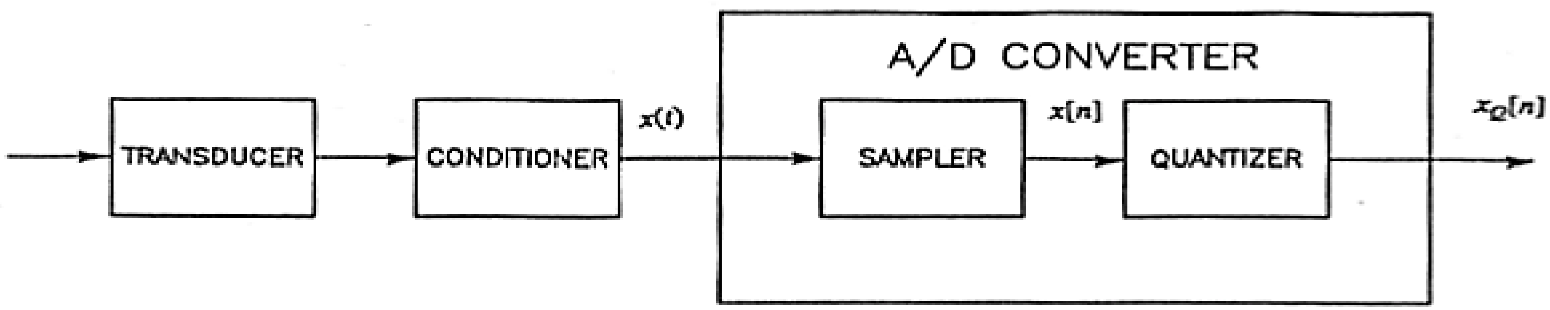
DATA ACQUISITION

- Data acquisition
- Continuous-time sinusoidal signals
- Sampling a continuous-time signal
- Sampling a sinusoid – aliasing
- The Nyquist sampling theorem
- Relations among frequency variables
- Quantization
- Analog to Digital (A/D) and Digital to Analog (D/A) conversion
- (Reconstructing continuous-time signals)

Data acquisition

- Data acquisition typically consists of three stages:
  - * Transduction (in general conversion of one form of energy to electrical energy which is suitable for encoding into a computer)
  - * Analog signal conditioning (amplifying and filtering the analog signal measured with a transducer to provide a good match between the typically low-amplitude, wide-bandwidth transducer signals and the analog-to-digital converter)
  - * Analog-to-digital converter (transforms a continuous-time signal into a digital signal: sampling – taking amplitudes of continuous-time signal at the discrete times, quantization – sample amplitudes can only take a finite set of values)



Continuous-time sinusoidal signals

- Cosine signal

$$x_a(t) = A\cos(\Omega t + \theta), \qquad -\infty < t < \infty$$

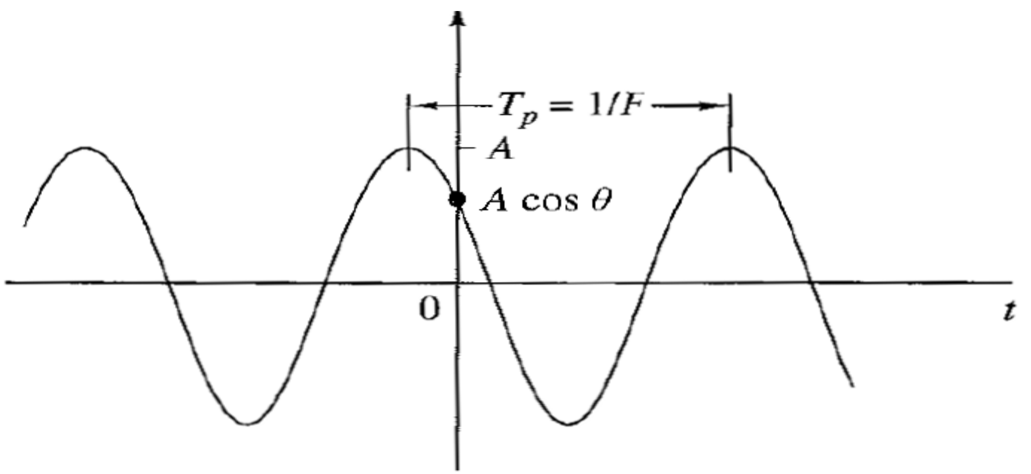
A is the amplitude

Ω is the frequency in radians per second [*rad/s*], *Ω = 2 π F*

Θ is the phase in radians [*rad*]

*T*p is the duration of one cycle in seconds [*s*]

F = 1 / *T*p is the frequency in cycles per second or Hertz [*Hz*], *Hz* = 1/*s*



Sampling a continuous-time signal

- Discrete-time signals are obtained by sampling a continuous-time signal *x(t)* at regular intervals

$$x[n] \triangleq x(nT_s), \quad -\infty < n < \infty$$

$$F_s \triangleq \frac{1}{T_s}$$

- *Ts* is the *sampling interval* or *sampling period* in seconds [*sec*], [*s*]
- *Fs* is the *sampling frequency* or *sampling rate* in samples per second [*smp/s*] or in [*Hz*], [1/*s*]

Sampling a sinusoid - aliasing

- Sampling a continuous-time sinusoid:

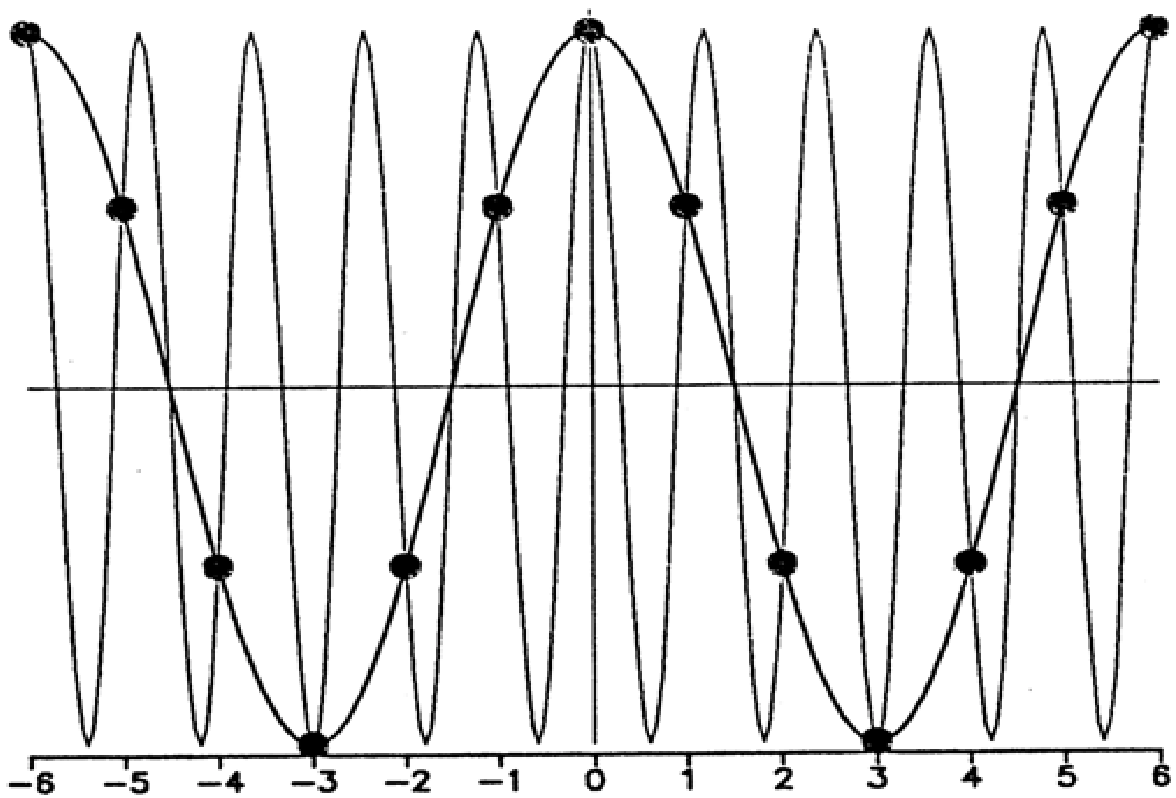
$$\frac{F}{F_S} = f$$

$$x[n] = x(nT_s) = a\cos(2\pi F n T_s + \phi) = a\cos(2\pi n F/F_s + \phi)$$

- $x[n]$ hides a difficulty arising from the ambiguity of frequency for discrete-time sinusoids:
  - It is not possible to know if the frequency of the original continuous-time signal *x(t)* was *F,* or *F + Fs*, or *F + 2Fs*, etc; or, *Fs – F* or *2Fs – F*, etc
- This phenomenon is known as aliasing because frequencies may not be what they appear to be once a continuous-time signal *x(t)* is sampled
- Aliasing – the error in a signal arising from limitations in the system that generates or processes the signal (Collins English Dictionary)

- *F* – *continuous-time frequency* in cycles per second [*cyc/s*], [*Hz*]
- *f* - *discrete-time frequency* in cycles per sample [*cycles/sample*], [*cyc/smp*]

Sampling a sinusoid - aliasing



The Nyquist sampling theorem

- How to avoid aliasing?

- Regarding the previous example, what is the minimum number of samples per sinusoid, N, that would still approximate a sinusoid? $N = ?$

- Regarding the previous example, what is the highest frequency F (expressed with Fs) of a sinusoid that would still be approximated, if using sampling frequency Fs?

Since: $Fs / F = N$ and $N \geq 2$, follows: $F = Fs / 2$

- If $Fs = 2.F$, the Fs is said to be Nyquist frequency

The Nyquist sampling theorem

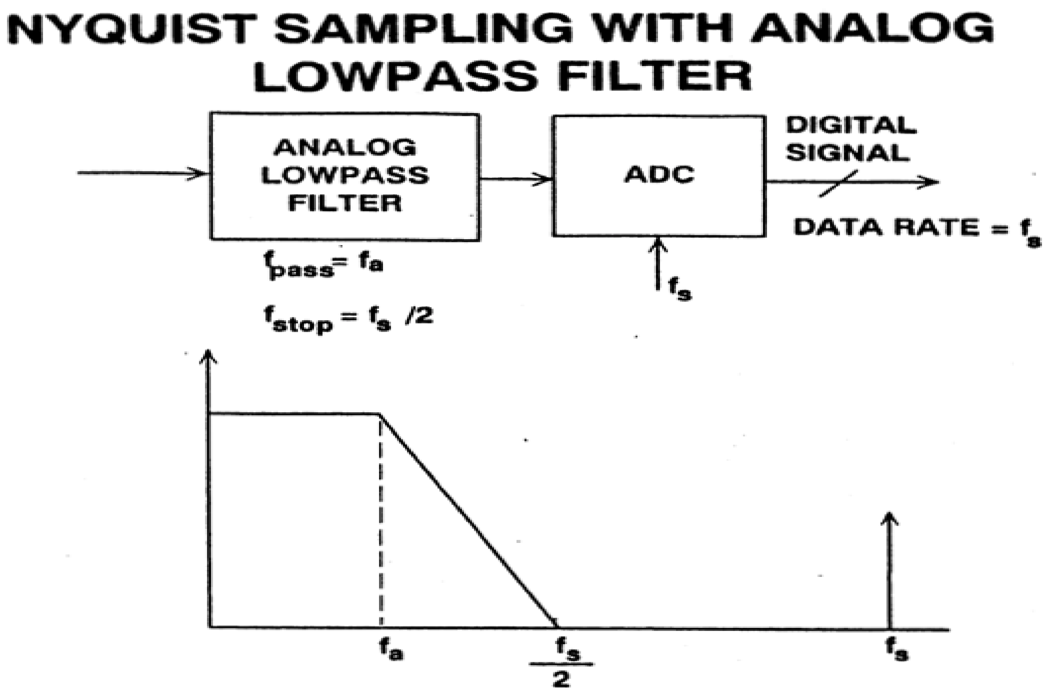
- Sampled analog signal *x(t)* should not contain frequencies higher than *Fs / 2*;

- Sampling frequency *Fs* should be higher than twice the highest frequency *F* present in the analog signal, $Fs \geq 2.F$

=> Principal value of discrete-time frequency $$0 \leq f \leq \frac{1}{2}$$

The Nyquist sampling theorem

- In practice always avoid aliasing by low-pass filtering the continuous-time signal *x(t)* before sampling
- In practice sample signals at about *Fs = (3-4).F*



Relations among frequency variables

Lowpass filtering and sampling

$$x_a(t) = A\sin(2\pi F t + \theta) \longrightarrow x_a[nT_s] = A\sin(2\pi F/F_s\, n + \theta) = x[n]$$

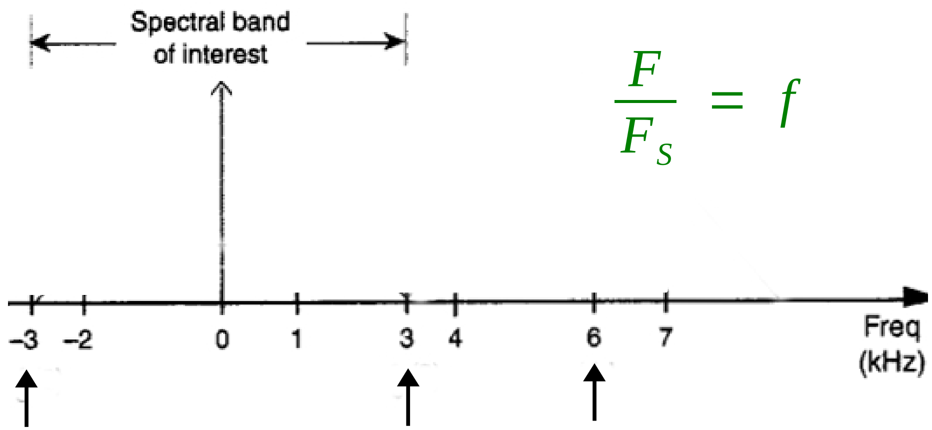
$$= A\sin(\Omega t + \theta) \qquad\qquad = A\sin(\omega n + \theta) = x[n]$$

$$\Omega = 2\pi F \longrightarrow \omega = 2\pi F/F_s \rightarrow \omega = 2\pi f$$

$(F_s = 6\text{ kHz})$

$\Omega\,, \quad -\infty < \Omega < \infty, \quad \Omega = 2\pi F,$

the frequency in radians per sec [*rad/s*]



$$\frac{F}{F_s} = f$$

$-F_s/2$	$F_s/2$	F_s	$\longrightarrow$	F
$-1/2$	$1/2$	1	$\longrightarrow$	f
$-\pi$	π	2π	$\longrightarrow$	ω

$F\,, \quad -\infty < F < \infty,$

the frequency in cycles per sec or Hertz [*Hz*]

$f\,, \quad -1/2 \le f \le 1/2,$

the frequency in cycles per sample [*cyc/smp*]

$\omega\,, \quad -\pi \le \omega \le \pi, \quad \omega = 2\pi f,$

the frequency in radians per sample [*rad/smp*]

Quantization

- A quantizer takes *x*[*n*] and produces a signal *xq*[*n*] that can only take a finite number of values
- The quantizer output *xq*[*n*] is equal to *kQ*, where *Q* is the quantization step, and *k* is the integer closest to *x*[*n*]/*Q*
- The number of quantization steps is a power of two
- The quantizer encodes signals whose values lie in the range:

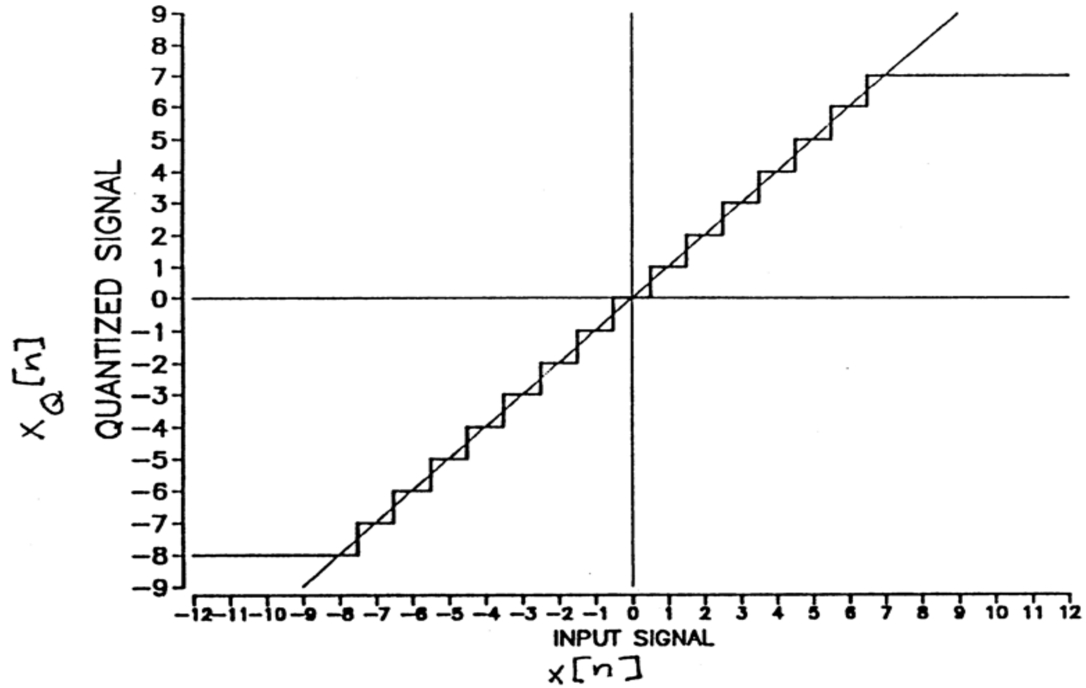
$$-V_{max} \leq x[n] < V_{max}$$

- Where *Vmax* is related to the quantization step by: $V_{max} = 2^{B-1}Q$

and *B* is the number of bits of the quantizer

Quantization

- $xq[n]$ as a function of $x[n]$ for $B = 4$ and $Q = 1$, corresponding to $Vmax = 8$



Analog to Digital (A/D) and Digital to Analog (D/A) conversion

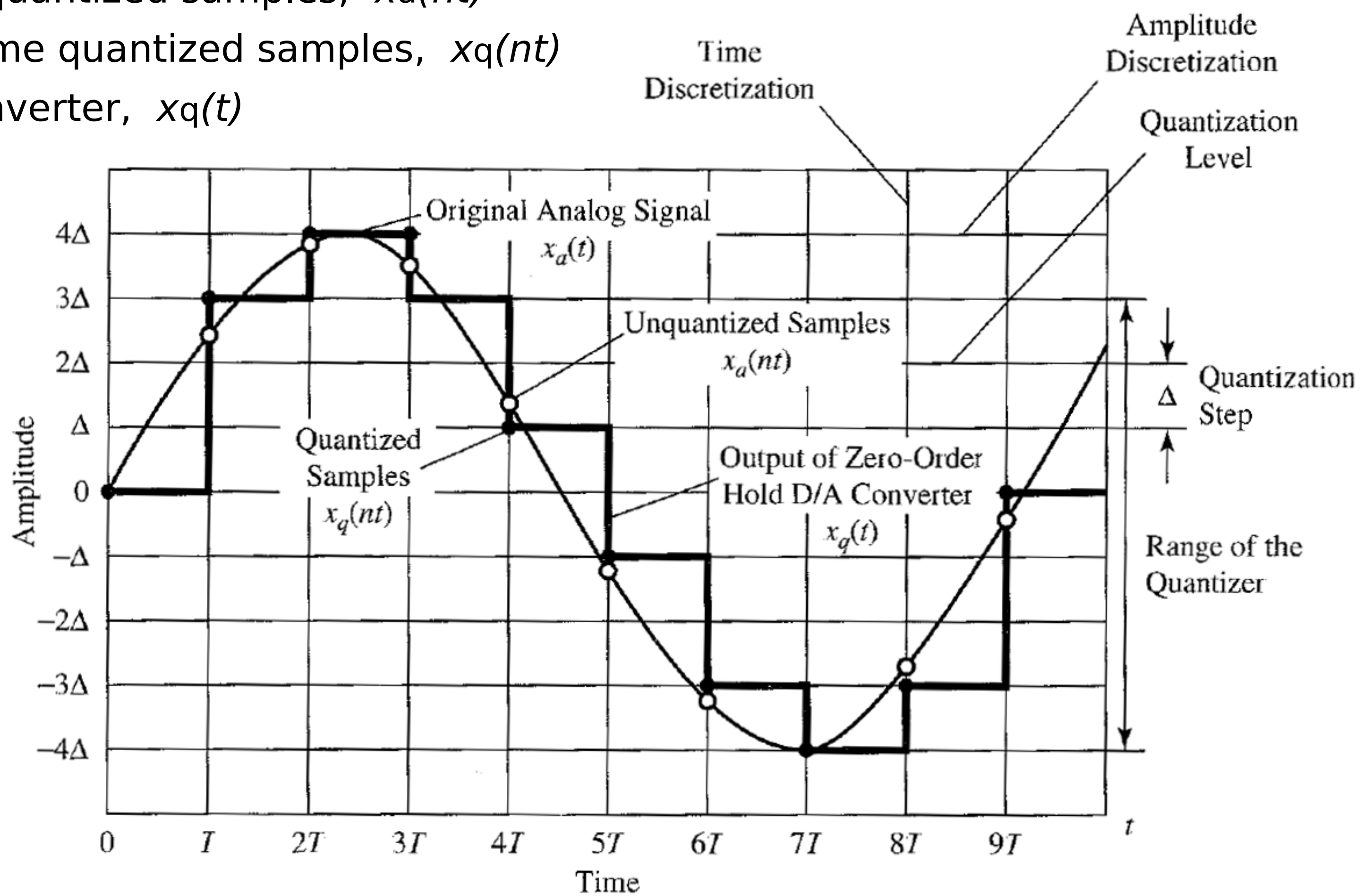
- Continuous-time signal, $x_a(t)$
- Discrete-time unquantized samples, $x_a(nt)$
- A/D → Discrete-time quantized samples, $x_q(nt)$
- Output of D/A converter, $x_q(t)$

$x_a(nT) \rightarrow x[n]$

Discrete signal

$x_q(nT) \rightarrow x[n]$

Digital signal



[Proakis, Manolakis]

(Reconstructing continuous-time signals)

- If a continuous-time signal *x(t)* contains no frequency components higher than *F*, it can be exactly reconstructed from samples taken at a frequency *Fs > 2F*
- The Nyquist theorem gives an explicit *interpolation formula* for reconstructing *x(t)* from the discrete-time signal *x*[*n*]:

$$x(t) = \sum_{n=-\infty}^{\infty} x[n]\phi(t - nT_s)$$

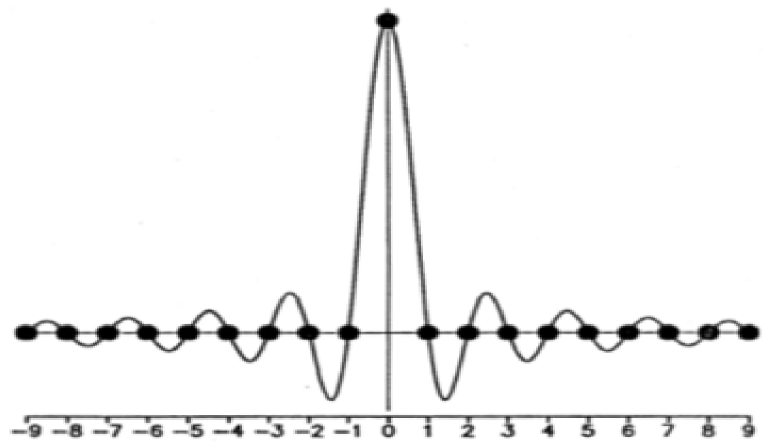
with a basic function *Φ(t)*:

$$\phi(t) = \frac{\sin(\pi F_s t)}{\pi F_s t}$$

- Time-dependent weights *Φ(t - nTs)* are obtained by delaying the basic function *Φ(t)*

(Reconstructing continuous-time signals)

- The basic function *Φ(t)*:



- This function verifies the property:

$$\phi(nT_s) = \begin{cases} 1 & \text{if } n = 0 \\ 0 & \text{otherwise} \end{cases}$$

- This property implies that $x(t) = x[n]$ for $t = nTs$
- The signal is said to be sampled at Nyquist frequency if $Fs = 2F$