Generalized eigenvalue problem

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Exam project

Abstract

Part of the exam project

The exam project part 1) did involve the task to prove analytically that the original $Ax = \lambda Nx$ can be represented as $By = \lambda y$, where $B = \sqrt{D}^{-1} V^T A V \sqrt{D}^{-1}$ and $y = \sqrt{D} V^T x$
For this, first transform $N = VDV^T$ and multiply both sides by $\sqrt{D}^{-1} V^T$ from the left-hand side. This gives (after using that $V^T V = identitymatrix$ and moving the scalar value λ)

$$\sqrt{D}^{-1} V^T Ax = \lambda \sqrt{D} V^T x$$

On the right-hand side, we already do have the outcome we wanted, $\lambda \sqrt{D} V^T x = \lambda y$
Taking a closer look at the left-hand side it is

$$\sqrt{D}^{-1} V^T AIx$$

Since I can freely introduce indentity matrices into the equation, that doesn't have any effect on it.
use $V^T V = V V^T = identitymatrix$ again, and introduce another I:

$$\sqrt{D}^{-1} V^T AVIV^T x$$

use $\sqrt{D}^{-1} \sqrt{D} = I$

$$\sqrt{D}^{-1} V^T AV \sqrt{D}^{-1} \sqrt{D} V^T x$$

This is exaclty By, so we did prove the equivalence of the two expressions.