

8.6 Exercises

1.) ND proof

$$\forall x(A(x) \rightarrow B(x)) \rightarrow (\forall x A(x) \rightarrow \forall x B(x))$$

Hyp:
 $\forall x(A(x) \rightarrow B(x))$
 $\forall x A(x)$

$$\begin{array}{c} \overline{\forall x(A(x) \rightarrow B(x))}^1 \\ \hline A(x) \rightarrow B(x) \end{array} \quad \begin{array}{c} \overline{\forall x A(x)}^2 \\ \hline A(x) \end{array}$$

$$\begin{array}{c} B(x) \\ \hline \forall x B(x) \end{array} \quad \begin{array}{c} \hline \forall x A(x) \rightarrow \forall x B(x) \end{array}^2$$

$$\overline{\forall x(A(x) \rightarrow B(x)) \rightarrow (\forall x A(x) \rightarrow \forall x B(x))}^1$$

2.) Give ND Proof of $\forall x B(x)$ from hypotheses: $\bullet \forall x(A(x) \vee B(x))$
 $\bullet \forall y \neg A(y)$

Hyp

$$\begin{array}{c} \overline{\forall x(A(x) \vee B(x))}^1 \quad \overline{\forall y \neg A(y)}^2 \quad \overline{\forall x B(x)}^1 \\ \hline A(x) \vee B(x) \quad \neg A(x) \quad B(x) \end{array} \quad \begin{array}{c} \hline \vdash \end{array} \quad \begin{array}{c} \hline \vdash \end{array}^2$$

$$\begin{array}{c} \hline \vdash \end{array}^1 \quad \begin{array}{c} B(x) \\ \hline \forall x B(x) \end{array}$$

3.) From hypotheses:

- $\forall x (\text{even}(x) \vee \text{odd}(x))$
- $\forall x (\text{odd}(x) \rightarrow \text{even}(s(x)))$

... give a \Rightarrow proof $\forall x (\text{even}(x) \vee \text{even}(s(x)))$

$$\begin{array}{c}
 \frac{\forall x (\text{even}(x) \vee \text{odd}(x))}{\text{even}(x) \vee \text{odd}(x)} \quad \frac{\text{even}(x)}{\text{even}(x) \vee \text{even}(s(x))} \quad \frac{\frac{\frac{\frac{\text{odd}(x)}{\text{odd}(x) \rightarrow \text{even}(s(x))} \text{odd}(x) \rightarrow \text{even}(s(x))}{\text{even}(s(x))} \text{even}(x) \vee \text{even}(s(x))}{\text{even}(x) \vee \text{even}(s(x))}} \quad 1 \\
 \hline
 \forall x (\text{even}(x) \vee \text{even}(s(x)))
 \end{array}$$