8.6 Exercises

1.) ND proof

$$\forall x(A(x) \to B(x)) \to (\forall x A(x) \to \forall x B(x))$$

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{\forall x(A(x)\to B(x))}\,1}{A(x)\to B(x)} \quad \dfrac{\dfrac{}{\forall x A(x)}\,2}{A(x)}}{B(x)}}{\forall x B(x)}}{\forall x A(x) \to \forall x B(x)}\,2}{\forall x(A(x)\to B(x)) \to (\forall x A(x) \to \forall x B(x))}\,1$$

2.) Give ND Proof of $\forall x B(x)$ from hypotheses: • $\forall x(A(x) \vee B(x))$
• $\forall y \neg A(y)$

$$\dfrac{\dfrac{\dfrac{\forall x(A(x)\vee B(x))}{A(x)\vee B(x)} \quad \dfrac{\dfrac{}{A(x)}\,2 \quad \dfrac{\forall y \neg A(y)}{\neg A(x)}}{\bot} \quad \dfrac{\dfrac{}{B(x)}\,2 \quad \dfrac{}{\neg B(x)}\,1}{\bot}\,2}{\bot}\,1}{B(x)}$$

$$\forall x B(x)$$

3.) From hypotheses:

- $\forall x(even(x) \vee odd(x))$
- $\forall x(odd(x) \rightarrow even(s(x)))$

... give a ND proof $\forall x(even(x) \vee even(s(x)))$

$$\dfrac{\dfrac{\forall x(even(x) \vee odd(x))}{even(x) \vee odd(x)} \quad \dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{odd(x)}\,1 \quad \forall x(odd(x) \rightarrow even(s(x)))}{odd(x) \rightarrow even(s(x))}}{even(s(x))} \quad \dfrac{}{even(x)}}{even(x) \vee even(s(x))}}{even(x) \vee even(s(x))}\,1$$

$$\dfrac{even(x) \vee even(s(x))}{\forall x(even(x) \vee even(s(x)))}$$