Hands On 9

Algorithm Design

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1 Problem 1

After Diabolik's capture, Inspector Ginko is forced to free him because the judge, scared of Eva Kant's revenge, has acquitted him with an excuse. Outraged by the incident, the mayor of Clerville decides to introduce a new legislation to make judges personally liable for their mistakes. The new legislation allows the accused to sue the judge and have him punished in case of error. Consulted on the subject, Ginko is perplexed and decides to ask you to provide him with a formal demonstration of the correctness/incorrectness of this law.

1.1 Solution

We can model the game with three players: the judge, the accused, and the society. The society is a player with no actions but needed to decide whether if the law is correct or not. There exist multiple cases, let's start with the first one.

Case 1: the accused is guilty

		Accused	
		Sue	Don't sue
Judge	Guilty	0, 1 1	1, 0 2
(selfish)	Not guilty	0, 0 0	1, 3 0

If the judge is selfish, we have Nash Equilibrium in case (Not guilty, Don't sue). Therefore, the law is wrong since society's utility is low (it is 0).

		Accused	
		Sue	Don't sue
Judge	Guilty	2, 1 1	3, 0 2
(not selfish)	Not guilty	0, 0 0	1, 3 0

If the judge is not selfish, there is no need for the law. We can conclude that the law is not working properly if the accused is guilty.

Case 2: the accused is not guilty

Accused

		Sue	Don't sue
Judge (selfish)	Guilty	0,2 0	1,0 0
	Not guilty	0,0 1	1,3 2

If the judge is selfish, we have Nash Equilibrium in case (Not guilty, Don't sue). Therefore, the law is correct since society's utility is high (it is 2).

Accused

		Sue	Don't sue
Judge (not selfish)	Guilty	0,2 0	1,0 0
	Not guilty	2,0 1	3,3 2

If the judge is not selfish, we have Nash Equilibrium in case (Not guilty, Don't sue). So, in this case, the law is correct as well since society's utility is high (it is 2). We can conclude that the law is correct if the accused is not guilty.

2 Problem 2

An investment agency wants to collect a certain amount of money for a project. Aimed at convincing all the members of a group of N people to contribute to the fund, it proposes the following contract: each member can freely decide either to contribute with 100 euros or not to contribute (retaining money on its own wallet). Independently on this choice, after one year, the fund will be rewarded with an interest of 50% and uniformly redistributed among all the N members of the group. Describe the game and find the Nash equilibrium.

2.1 Solution

Let's say we have n players, $P_1, P_2, \dots, P_n$. Their actions are two: contribute or don't contribute. The following is the score function of a player if the number of players that opted to contribute is k:

$$score(contribute) = \frac{1.5 * (k + 1) * 100}{n}$$

$$score(not\_contribute) = \frac{1.5 * k * 100}{n} + 100$$

As we can see a player has a higher score function if it does not contribute. Because every player has the same score function, we can say that the players will prefer to don't contribute. We have Nash equilibrium.