Hands On 10

Algorithm Design

Ferraro Domenico
559813

1 Problem 1

Two players drive up to the same intersection at the same time. If both attempt to cross, the result is a fatal traffic accident. The game can be modeled by a payoff matrix where crossing successfully has a payoff of 1, not crossing pays 0, while an accident costs -100.

- Build the payoff matrix
- Find the Nash equilibria
- Find a mixed strategy Nash equilibrium
    - Compute the expected payoff for one player (the game is symmetric)

1.1 Solution

The game can be modelled by the following payoff matrix, where D_1 is the first driver and D_2 is the second one

		D_2	
		Cross	Stop
D_1	Cross	$-100, -100$	$1, 0$
	Stop	$0, 1$	$0, 0$

As we can see we have two Nash equilibria, (Cross, Stop) and (Stop, Cross).

To find the Mixed Strategy Nash Equilibrium, we first need to find the probability that each of the drivers assigns to each action. This will be done by calculating their Expected Payoffs. Let's define as p the probability to Cross for D_1. Then $1 - p$ is its probability to Stop. Similarly, we assume as q the probability that D_2 chooses to Cross and then $1 - q$ is its probability to Stop.

For D_1, the Expected Payoff of this game is the sum of the payoffs of the two possible actions, multiplied with the probability of D_2 choosing those actions:

$$EP_{D_1}[Cross] = -100q + 1 - q = 1 - 101q$$

$$EP_{D_1}[Stop] = 0q + 0 * 1 - q = 0$$

We do this similarly for D_2 but we multiply with the probabilities of D_1:

$$EP_{D_2}[Cross] = -100p + 1 - p = 1 - 101p$$

$$EP_{D_2}[Stop] = 0p + 0 * 1 - p = 0$$

So, we have that with $p = \frac{1}{101}$ and $q = \frac{1}{101}$ the two drivers are indifferent between the two options. Therefore, those probabilities are a mixed strategy Nash equilibrium.

2 Problem 2

Find the mixed strategy and expected payoff for the Back Stravinsky game.

2.1 Solution

We can apply the same reasoning as before. Let's define as p the probability to go to Bach for P_1. Then $1 - p$ is its probability to go to Stravinsky. Similarly, we assume as q the probability that P_2 chooses to Bach and then $1 - q$ is its probability to choose Stravinsky. For a player, the Expected Payoff of this game is the sum of the payoffs of the two possible actions, multiplied with the probability of the other player choosing those actions The game can be modelled like the following:

P_2

		Bach	Stravinsky	P_1's expected payoff
	Bach	$2, 1$	$0, 0$	$2q + 0(1 - q) = 2q$
P_1	Stravinsky	$0, 0$	$1, 2$	$0(q) + (1 - q) = 1 - q$
	P_2's expected payoff	$p + 0(1 - p) = p$	$0(p) + 2(1 - p) = 2 - 2p$	

As we can see we have two Nash equilibria, (Bach, Bach) and (Stravinsky, Stravinsky). Finally, the probabilities for a mixed strategy Nash equilibrium are:

$$2q = 1 - q \Rightarrow q = \frac{1}{3}$$

$$p = 2 - 2p \Rightarrow p = \frac{2}{3}$$

3 Problem 3

The Municipality of your city wants to implement an algorithm for the assignment of children to kindergartens that, on the one hand, takes into account the desiderata of families and, on the other hand, reduces city traffic caused by taking children to school. Every school has a maximum capacity limit that cannot be exceeded under any circumstances. As a form of welfare the Municipality has established the following two rules:

1. in case of a child already attending a certain school, the sibling is granted the same school;
2. families with only one parent have priority for schools close to the workplace.

Model the situation as a stable matching problem and describe the payoff functions of the players. Question: what happens to twin siblings?

3.1 Solution

The schools have preferences over the children based on the given criteria. The school may choose a child based on the child's distance from the school and on child's distance from its parent workplace. In case of a child already attending a certain school, the sibling will get a zero distance from the school, so it will be granted the same school.