Regularization

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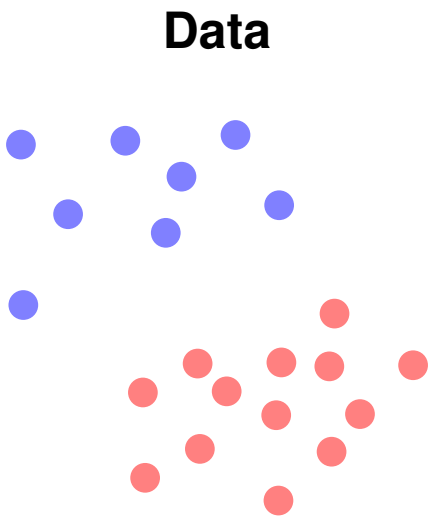
Pattern Recognition Lab, Friedrich-Alexander-Universität Erlangen-Nürnberg

November 13, 2018

Outline

Introduction to Regularization

Fitting appropriately



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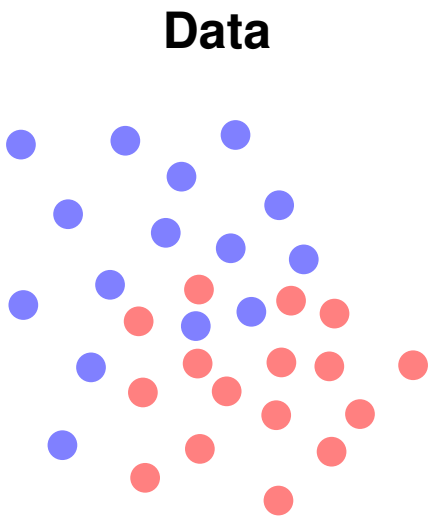

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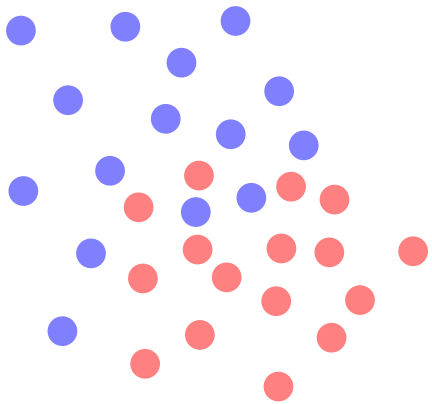
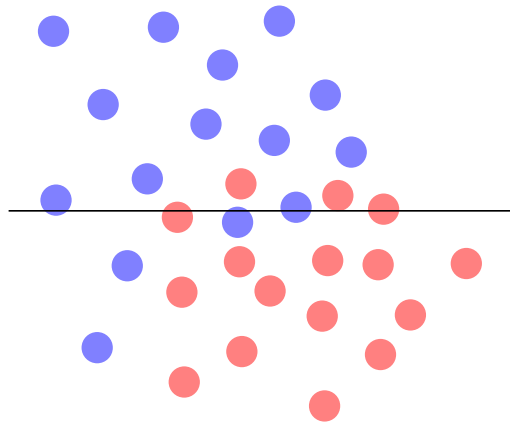
Data



Underfitting



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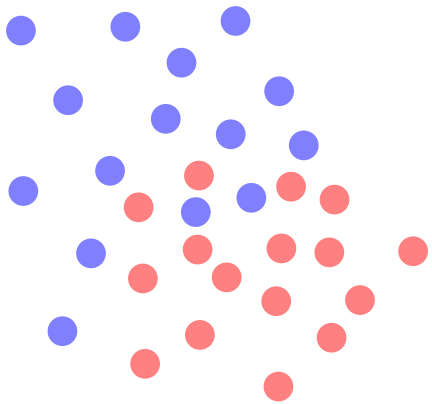

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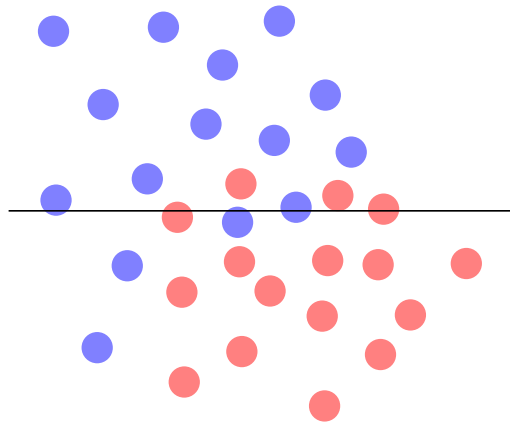

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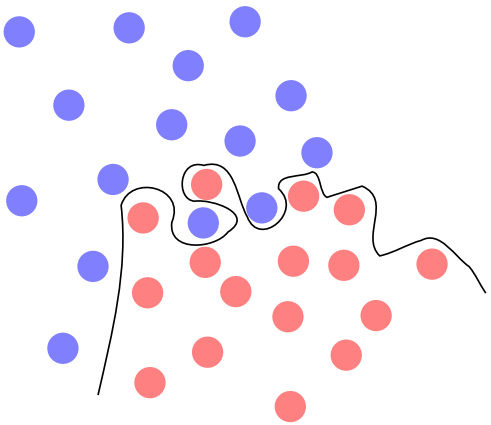

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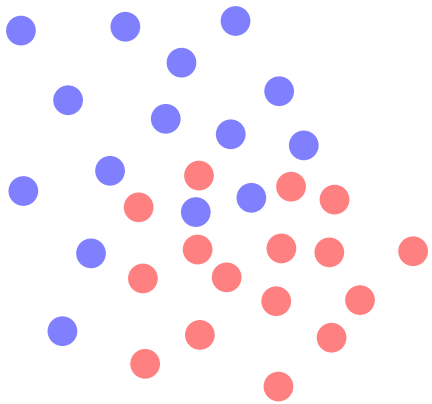

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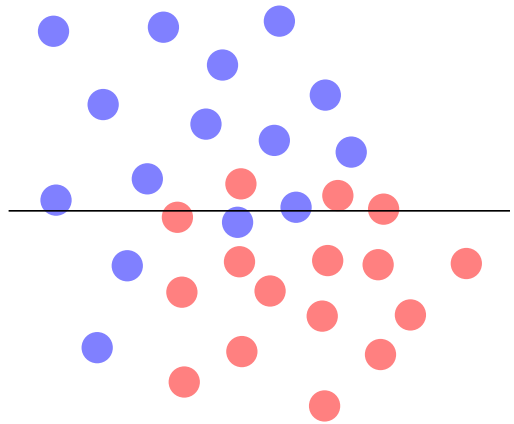

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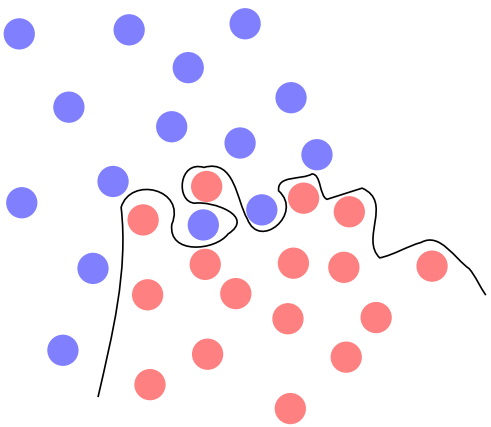
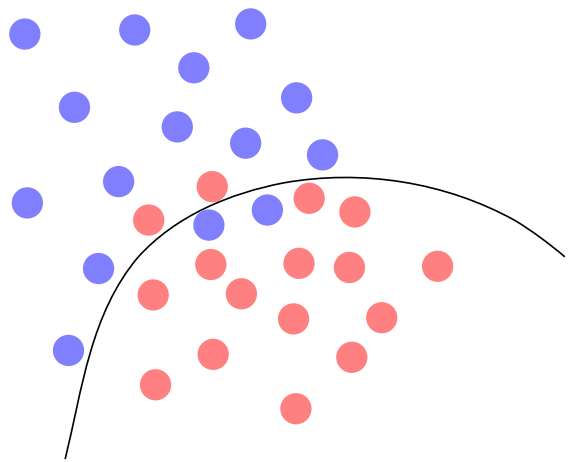


Sensible boundary



Bias Variance Decomposition

- Regression **problem** [5] with **h**, **ideal** value from the true distribution:

$$\mathbf{h} = h(\mathbf{x}) + \epsilon\,, \quad \epsilon = \mathcal{N}(0, \sigma_\epsilon)\,.$$

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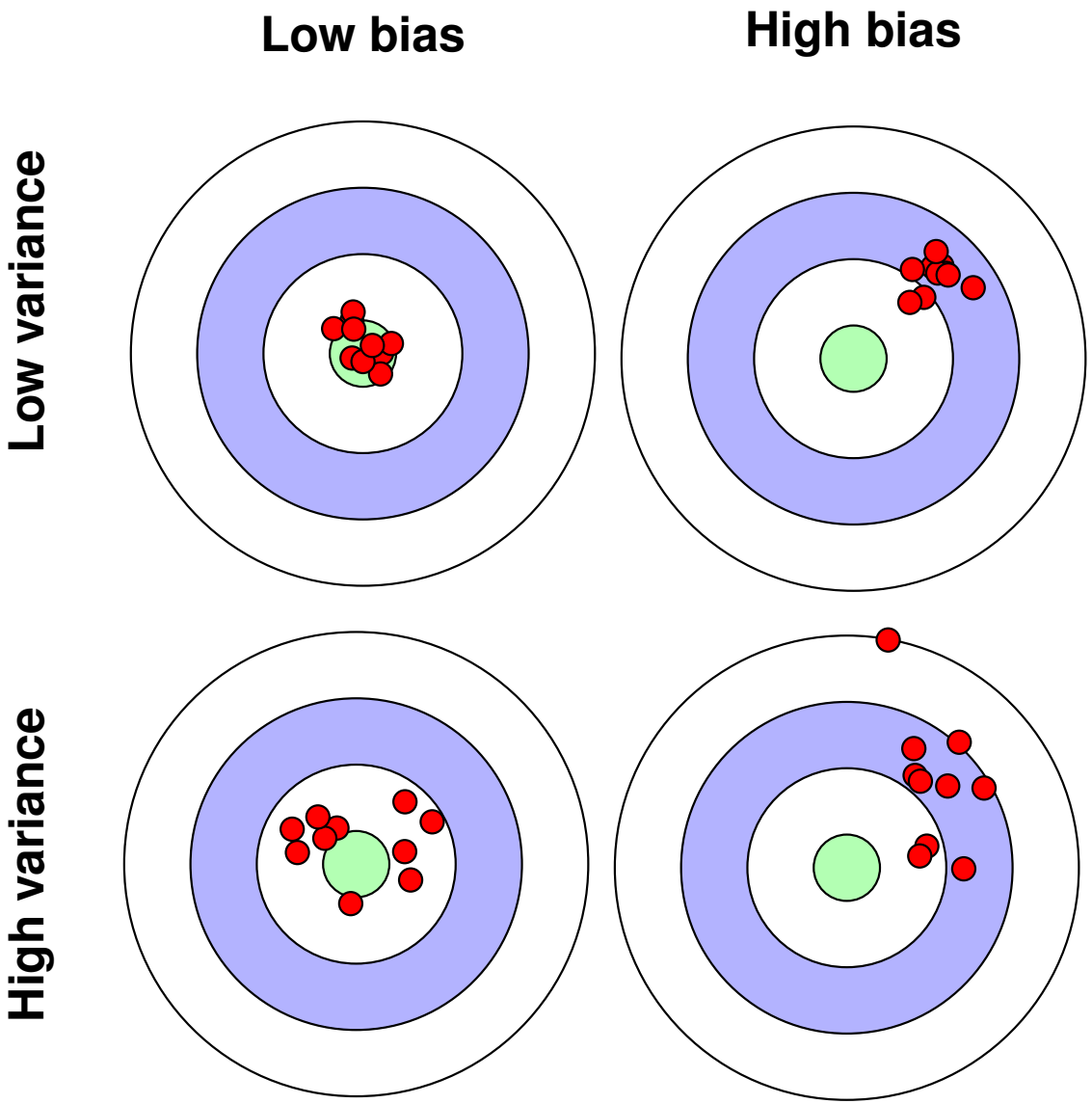
- Integrating this over every datapoint **x** we get $L_D(\mathbf{X})$ from the $\ell_D(\mathbf{x})$.
- A similar decomposition exists for classification using the zero-one loss [6, pp. 468-471].

Bias Variance Tradeoff



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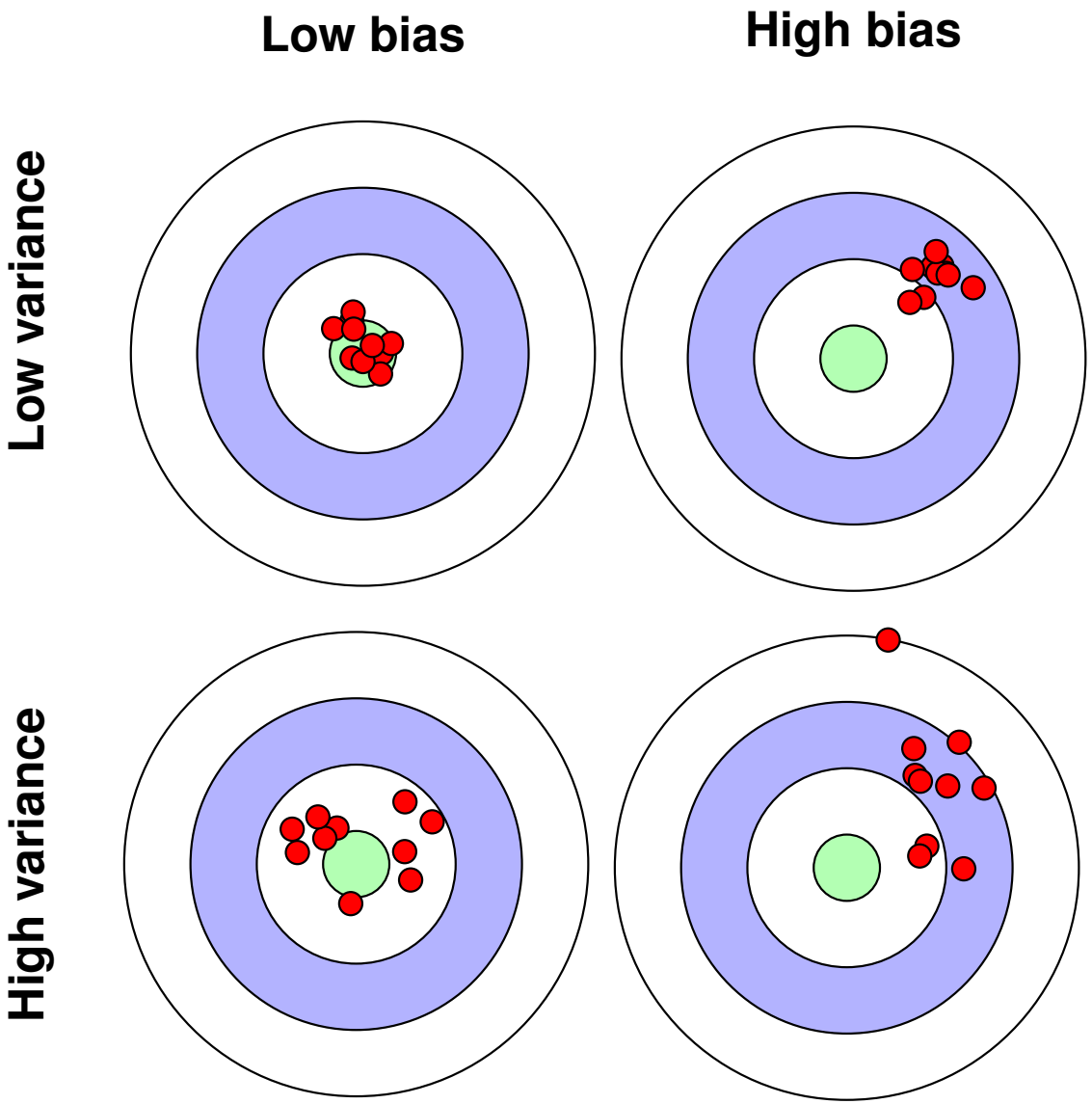

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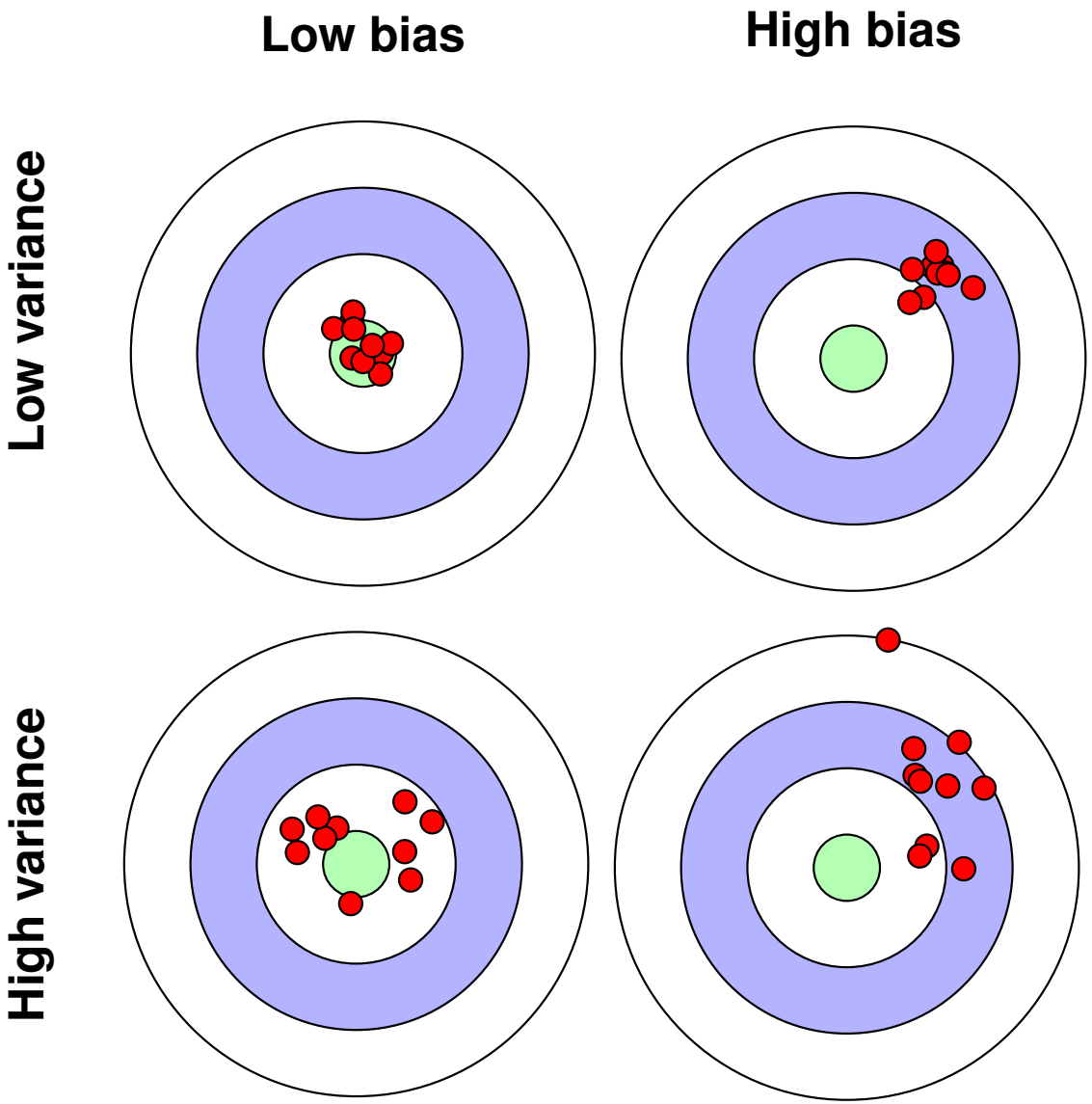

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- Bias and variance can be studied together as **model capacity**

Model Capacity

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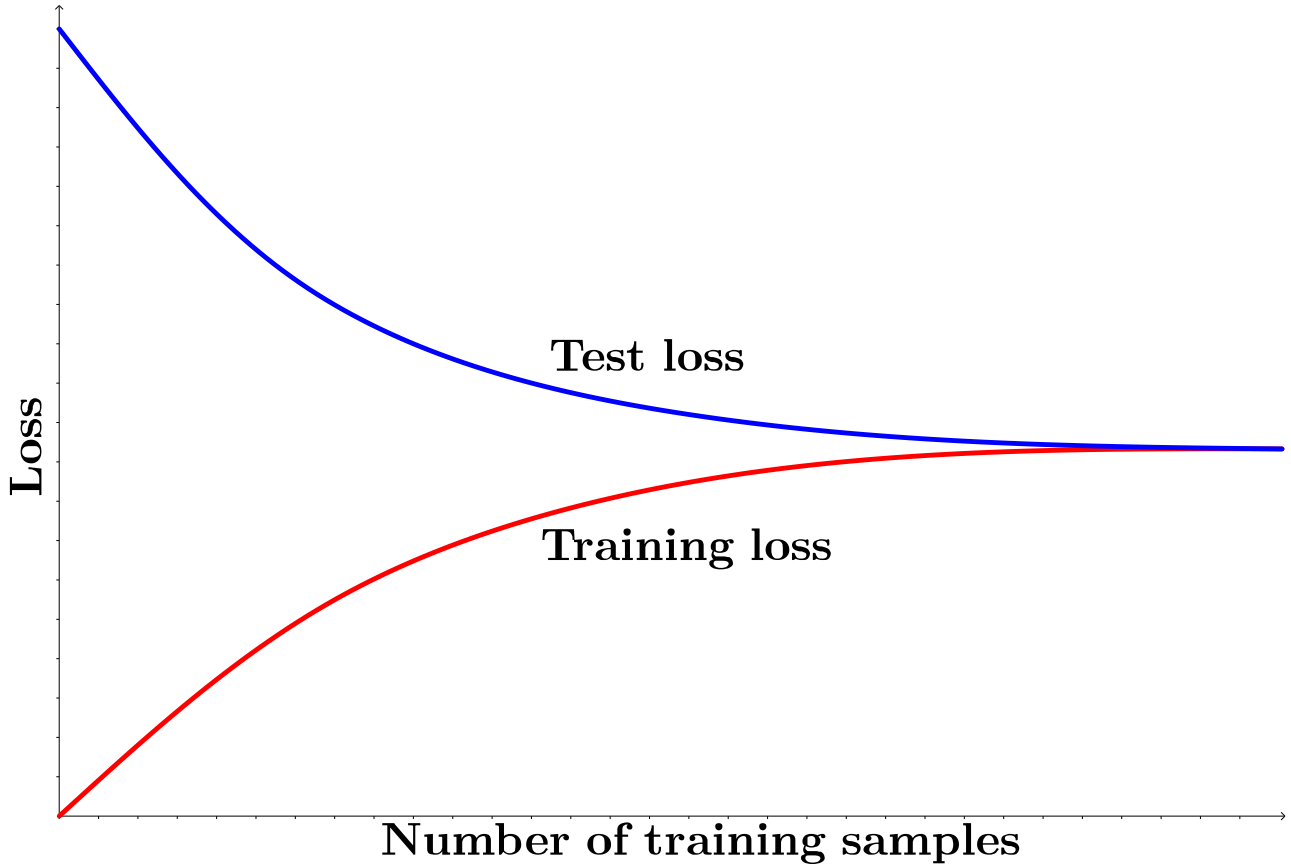
  - VC dimension of neural networks is usually **extremely high** compared to classical methods
  - Remember the paper learning ImageNet with random labels? [14]
  - ➔ VC dimension is **ineffective** in judging the **real capacity** of neural networks
- We can always reduce the **bias** by $\uparrow$ the **model capacity**

The Role of Data



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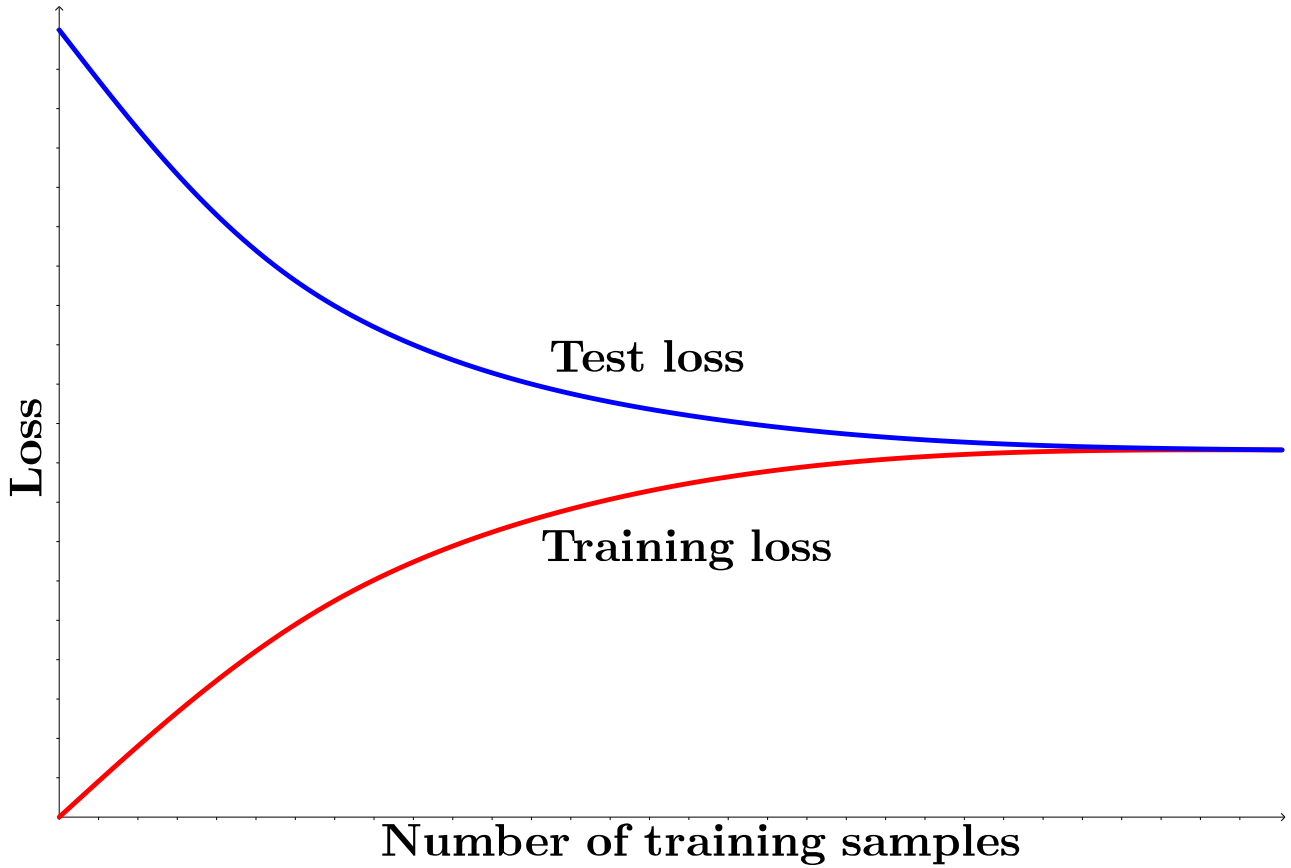

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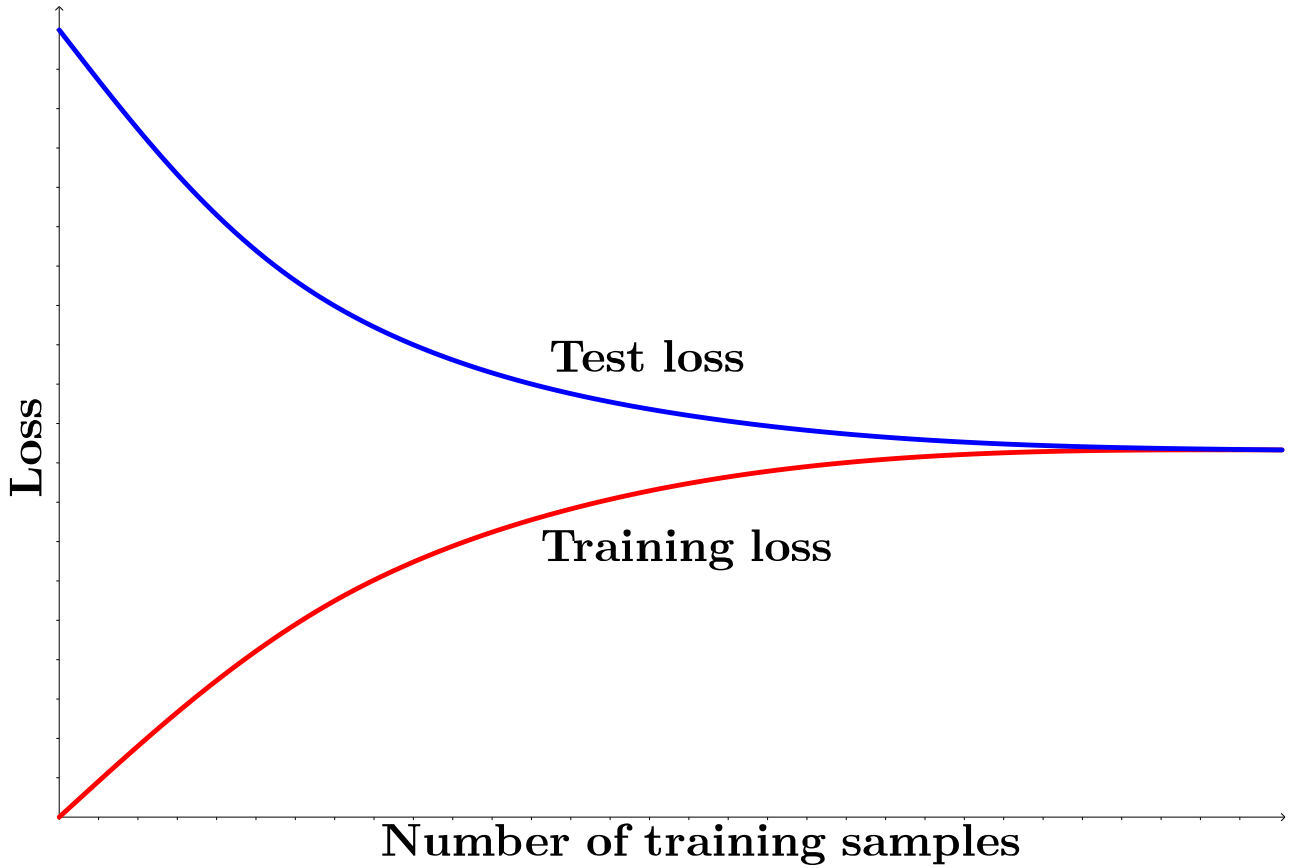

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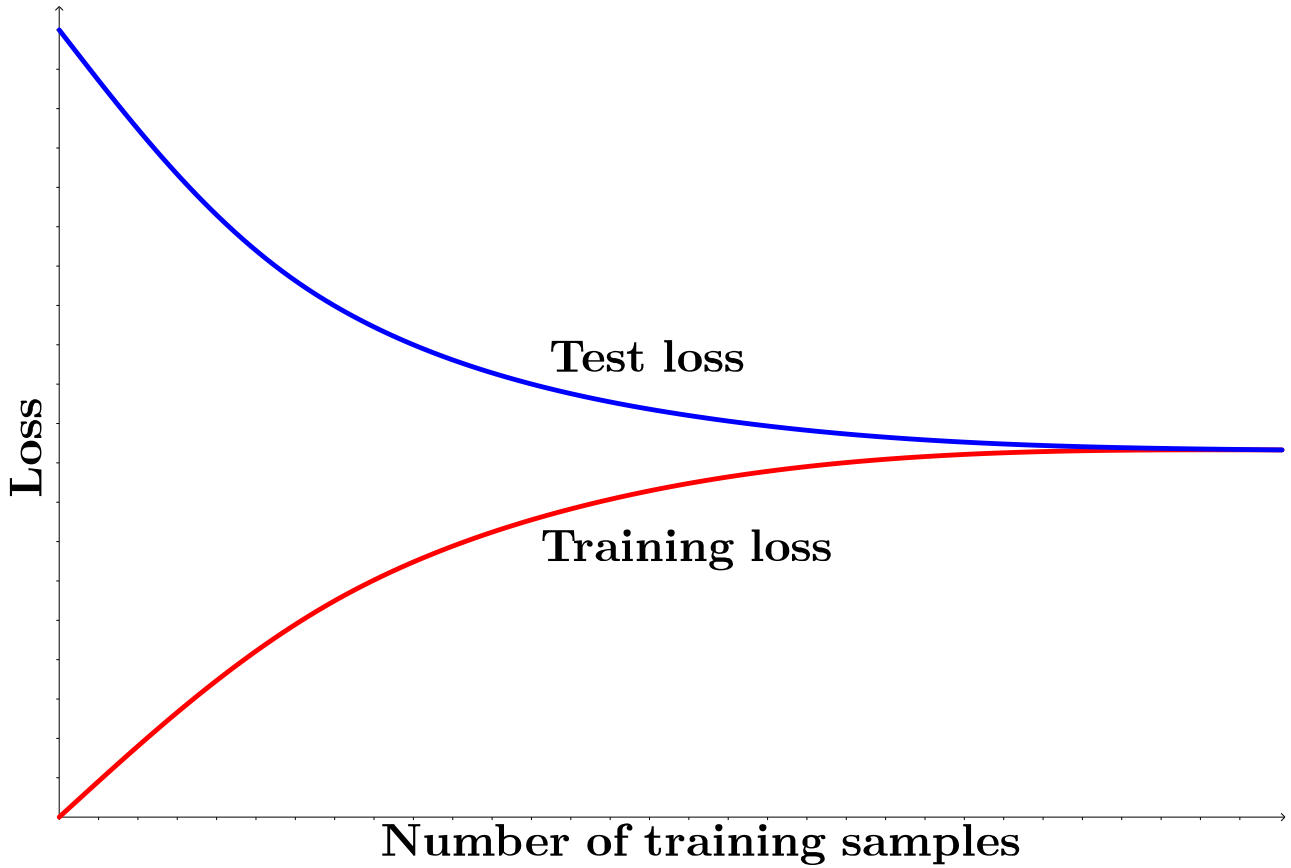

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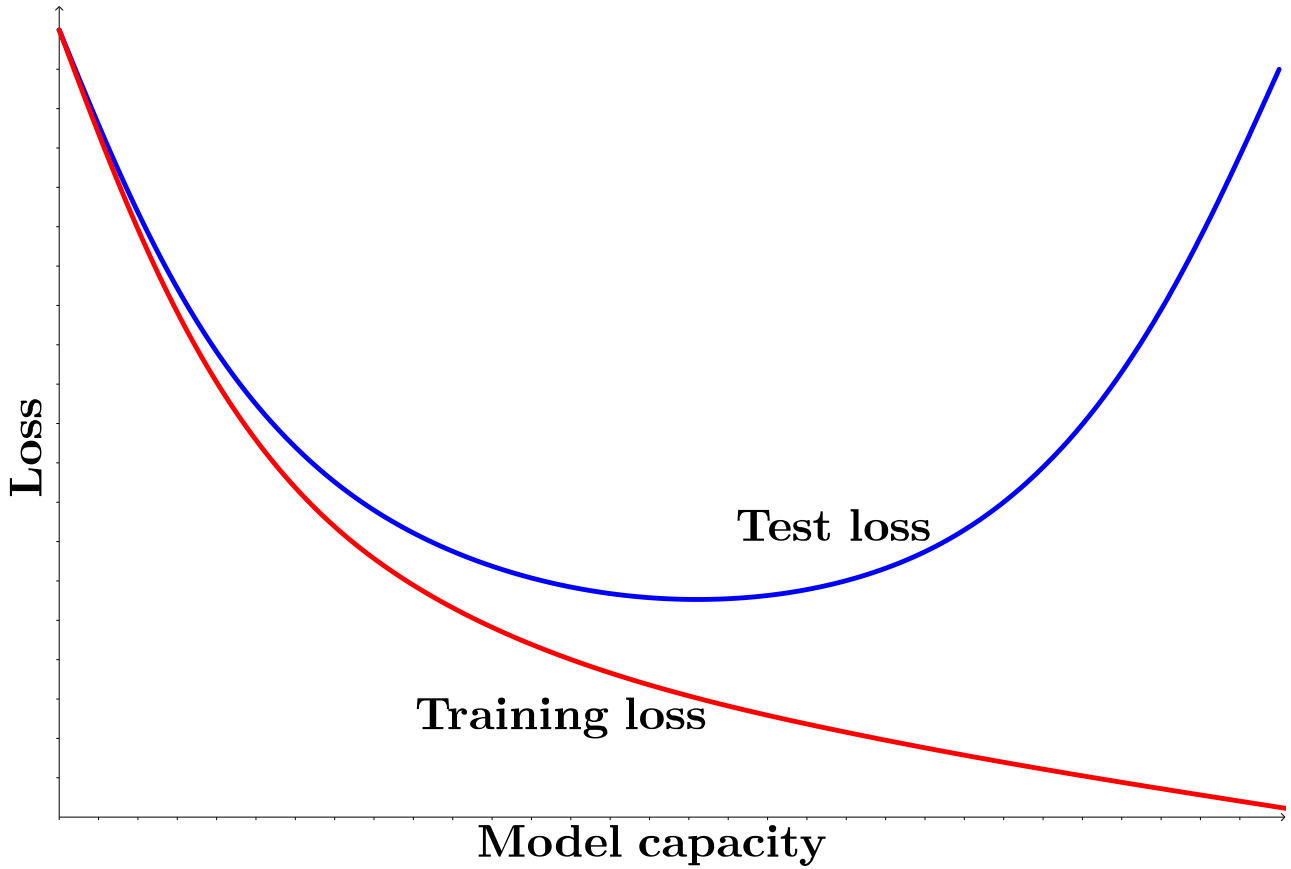
- What if we can't acquire more data?

Finite Dataset



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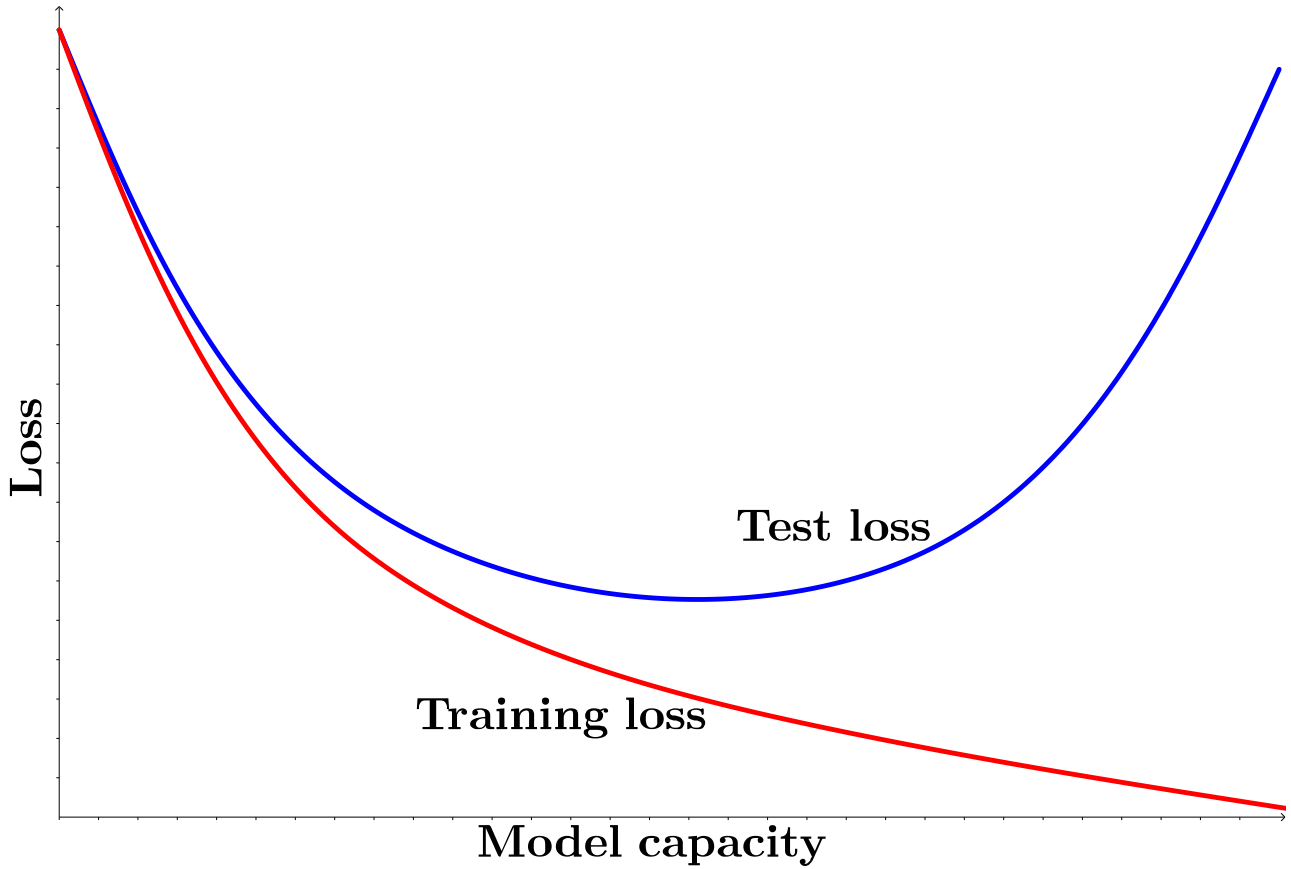

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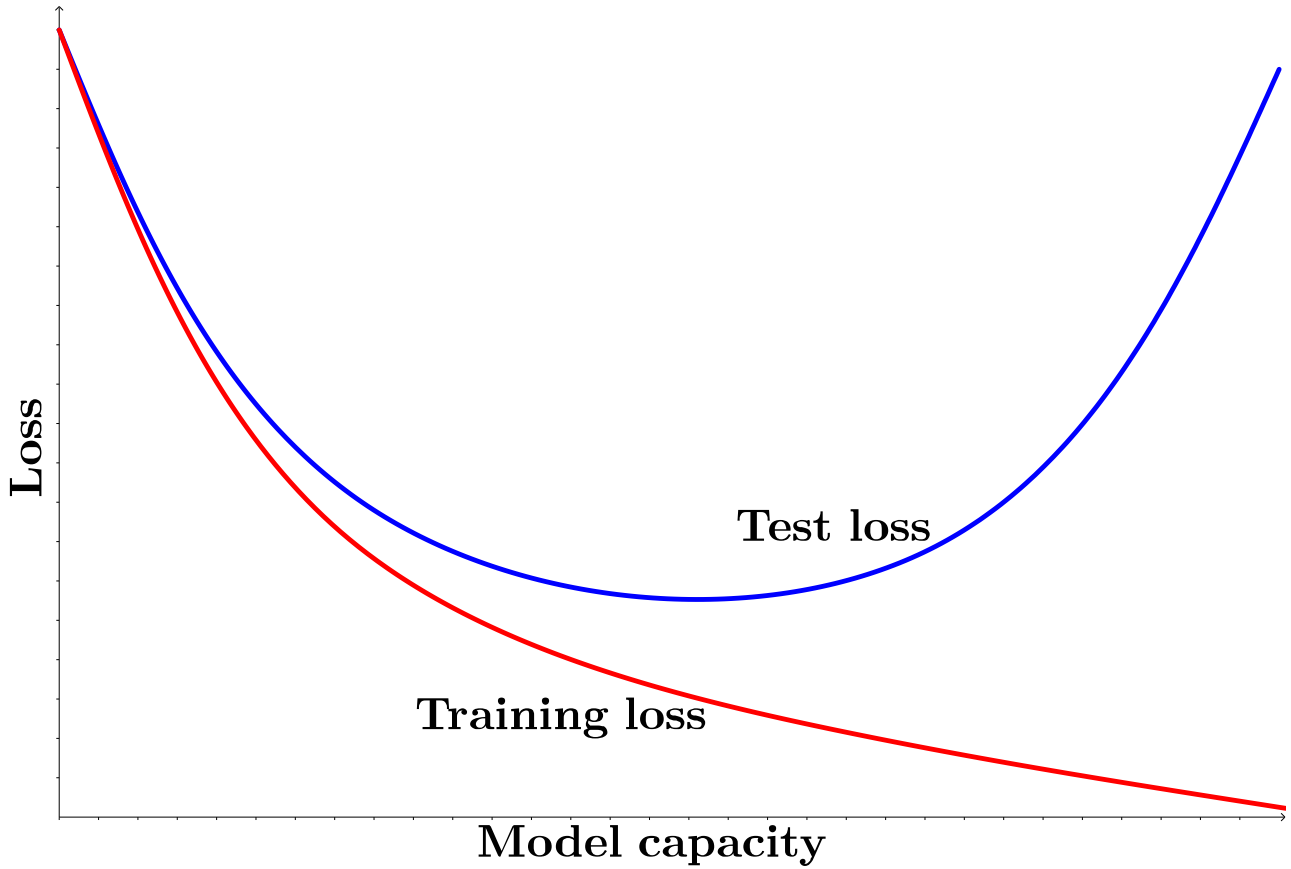

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➔ For a **specific problem** there might be favorable tradeoffs!

Regularization reduces Overfitting

How can we find such a favorable tradeoff?

➔ **By enforcing prior knowledge**

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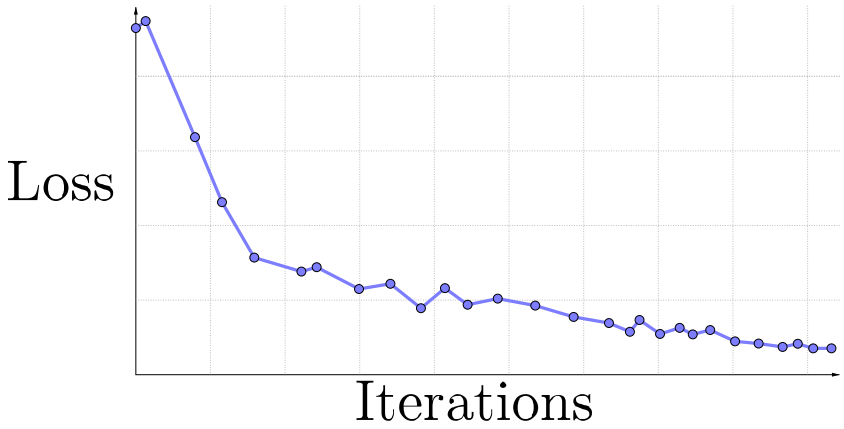
Actual regularizers

- Equality constraints
- Inequality constraints

Classical Techniques

Overfitting on Learning Curve

Training set



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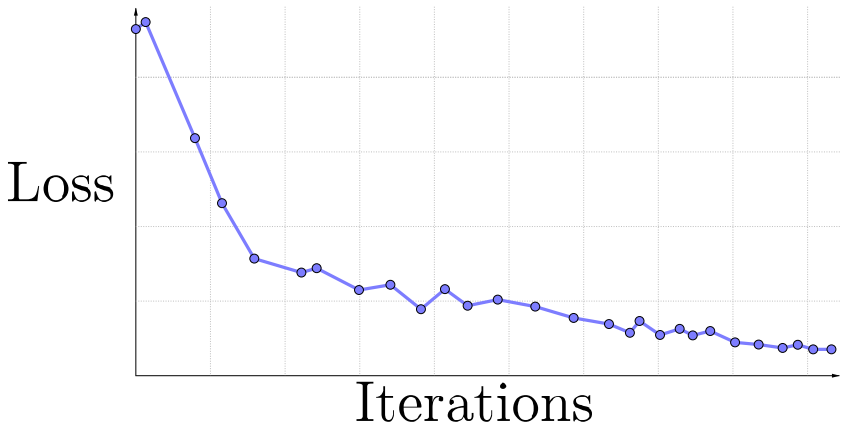

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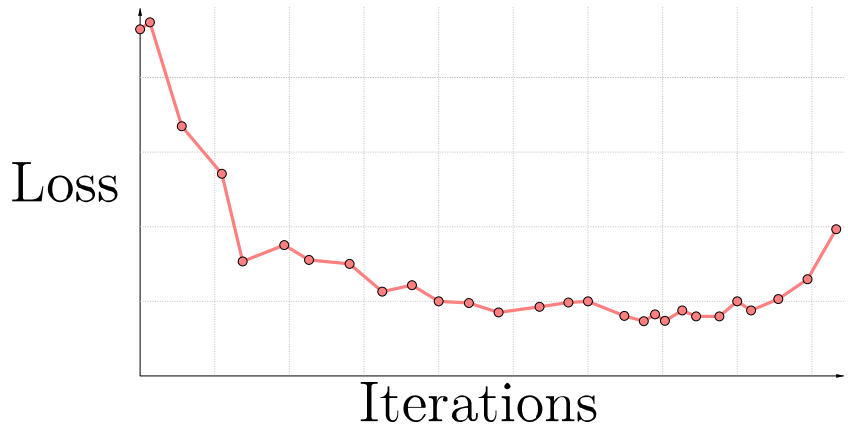

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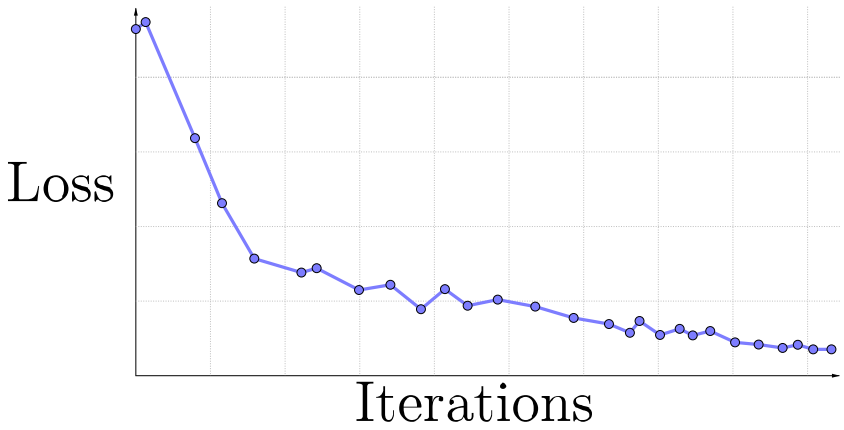

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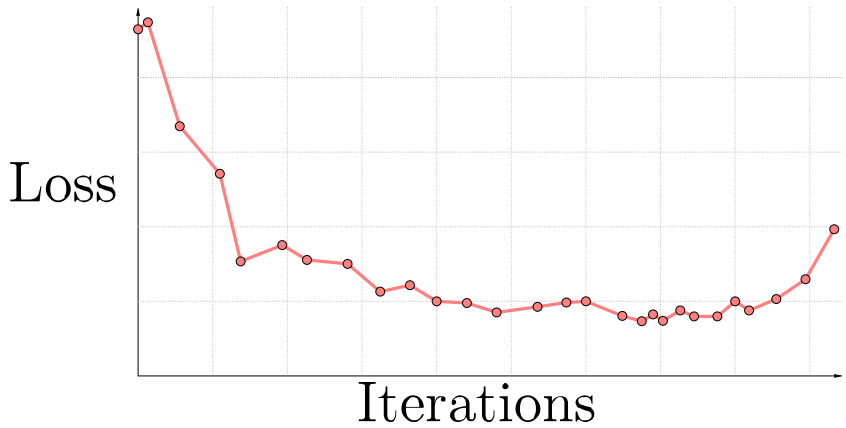

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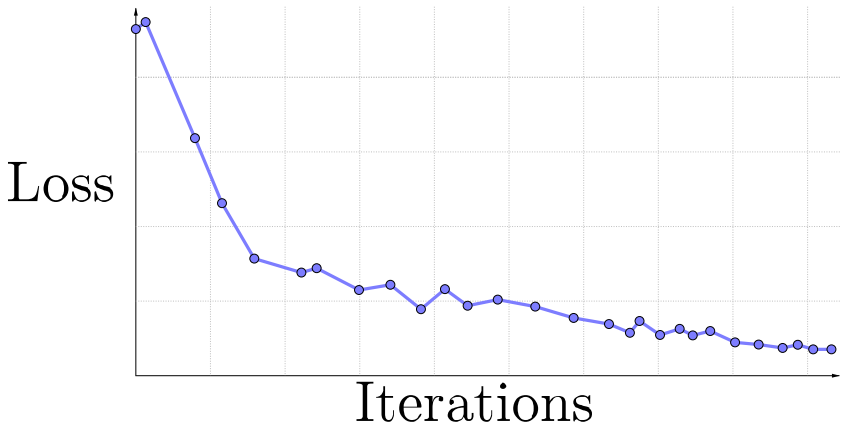

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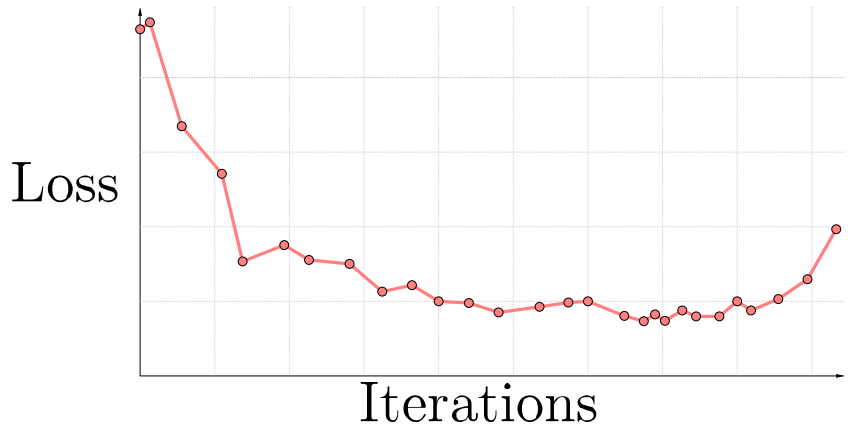

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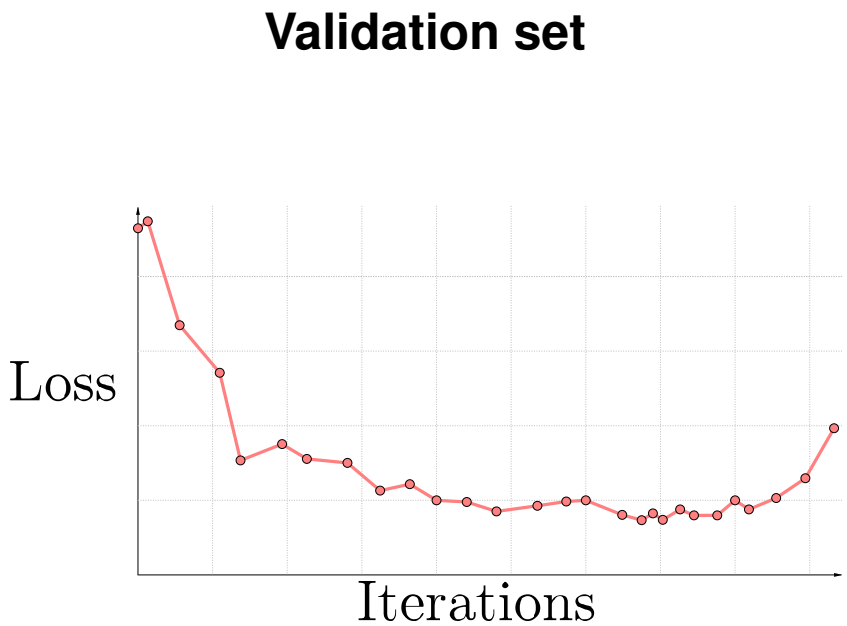
➔ Split of **validation set** from training data

Early Stopping



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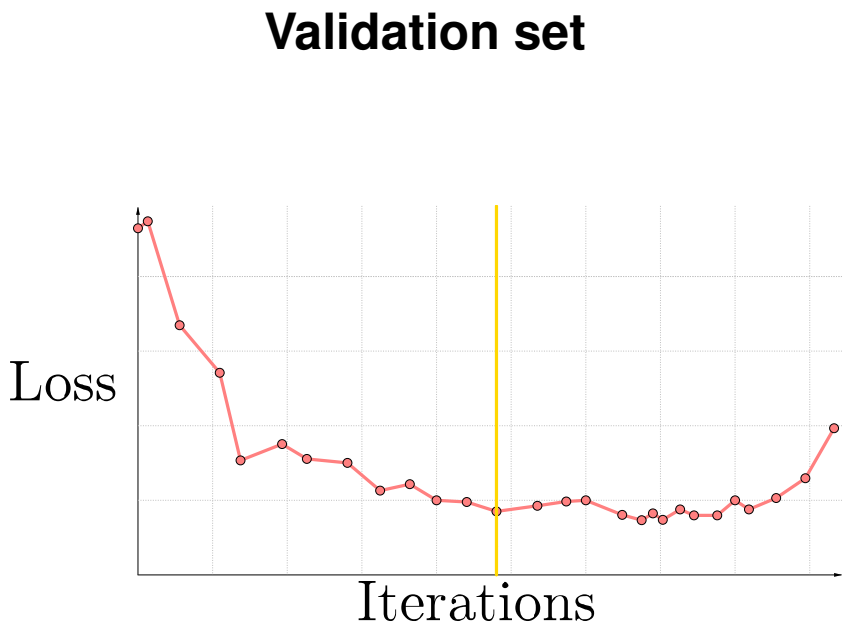

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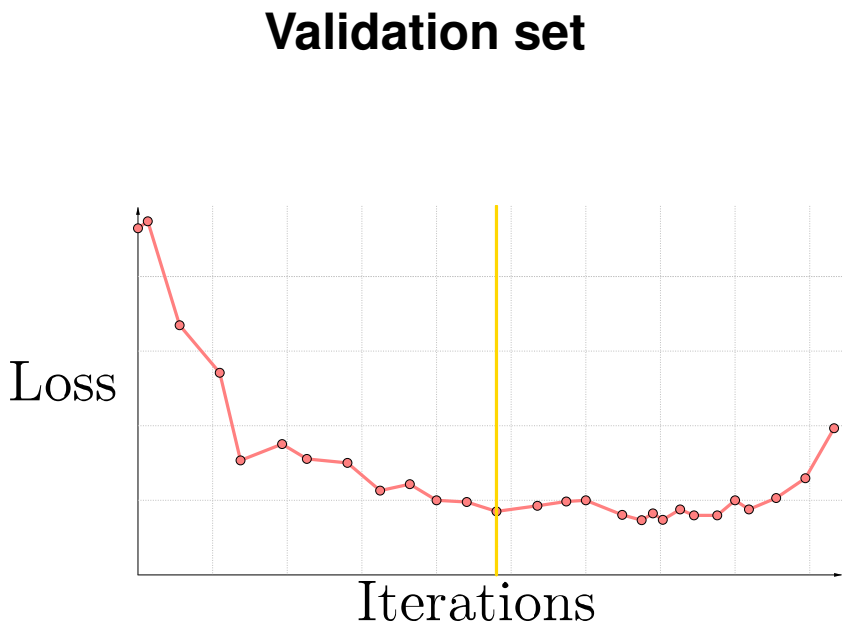

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$\rightarrow$ Use parameters with **minimum validation loss**

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Still you have to be careful!

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Data Augmentation

Common transformations:

1. random spatial transformations:
   - affine transformations
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2. pixel transformations
   - changing resolution
   - random noise
   - changing pixel distribution
   - ...

Regularization in the Loss Function

Maximum A Posteriori Estimation

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\end{aligned}
$$

- Because $p(\mathbf{Y}|\mathbf{X})$ does not depend on $\mathbf{w}$, and $\mathbf{X}$ and $\mathbf{w}$ are independent, we can obtain an estimate as:

$$
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$$

Maximum A Posteriori Estimation

- ➔ **Bayesian** approach considering the weights uncertain: $\mathbf{w} \mapsto p(\mathbf{w})$
- and we have a dataset $\mathbf{X}$ with associated labels $\mathbf{Y}$
- Now we can state:

$$
\begin{aligned}
p(\mathbf{w}, \mathbf{Y}, \mathbf{X}) &= p(\mathbf{w}|\mathbf{Y}, \mathbf{X})\, p(\mathbf{Y}, \mathbf{X}) \\
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$$\mathbf{w}^{(k+1)} = \underbrace{\left(1 - \eta\lambda\right)\mathbf{w}^{(k)}}_{\text{Shrinkage}} - \eta \frac{\partial L}{\partial \mathbf{w}^{(k)}}$$

- Often called “weight decay”

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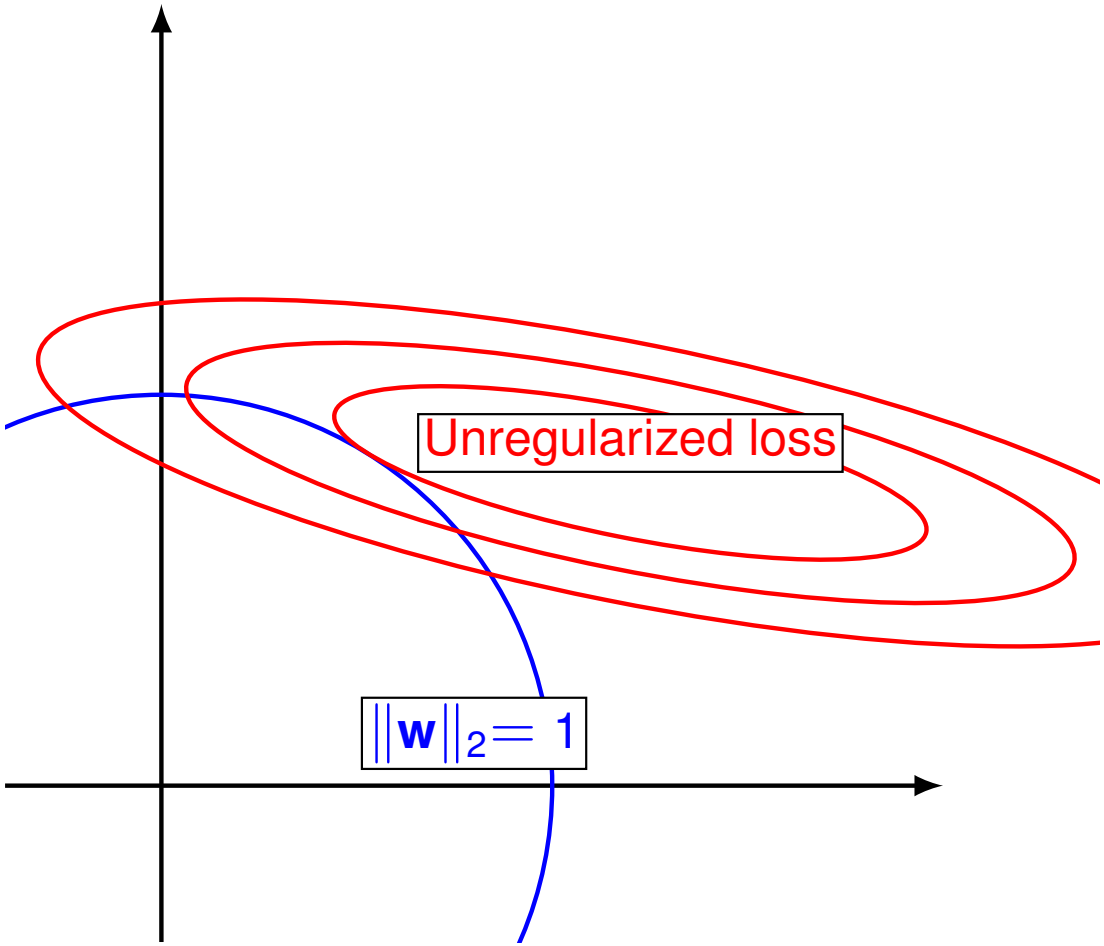
➔ We trade increased **bias** for reduced **variance**

Graphical Effect of L_2 Regularization



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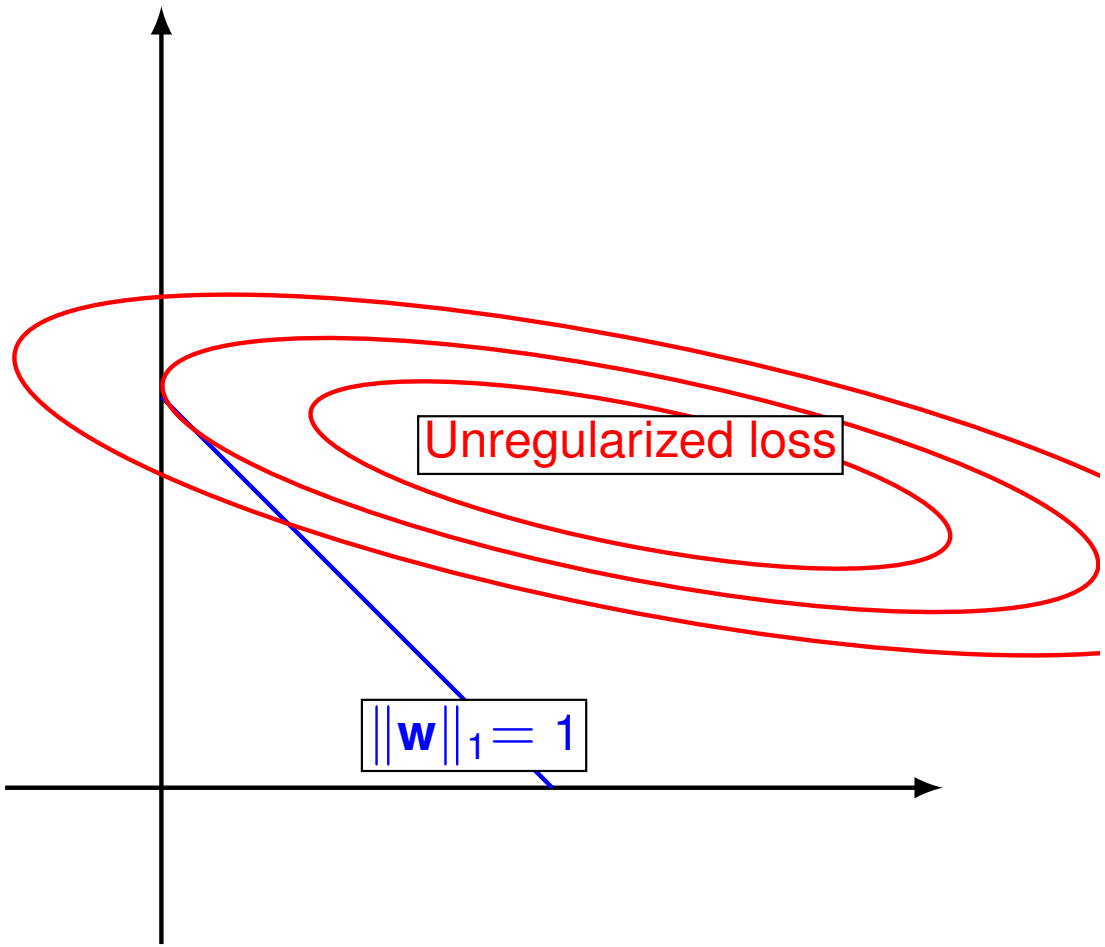
Backpropagation of the augmented loss with the corresponding **subgradient**:

$$\mathbf{w}^{(k+1)} = \underbrace{\mathbf{w}^{(k)} - \eta\lambda \text{sign}\left(\mathbf{w}^{(k)}\right)}_{\text{Other shrinkage}} - \eta \frac{\partial L}{\partial \mathbf{w}^{(k)}}$$

➔ Same as before

Graphical Effect (cont.)



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➔ Perform parameter update, then project to unit-"ball"

- Still a kind of shrinkage
- Prohibits **exploding gradients** but also hides it

Other Variants of Changing the Loss

- We can have a **constraint** and a λ **individual** for every layer

➔ Possible but **no gains** are reported

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- Instead of the weights, **activations** can be **constrained**
- Different variants have been used for **sparse autoencoders**
- We will cover this when we talk about unsupervised learning

Normalization

Internal Covariate Shift

- ReLU is not **zero-centered**
- Initialization and input distribution might not be normalized

➔ Input distribution **shifts** over time

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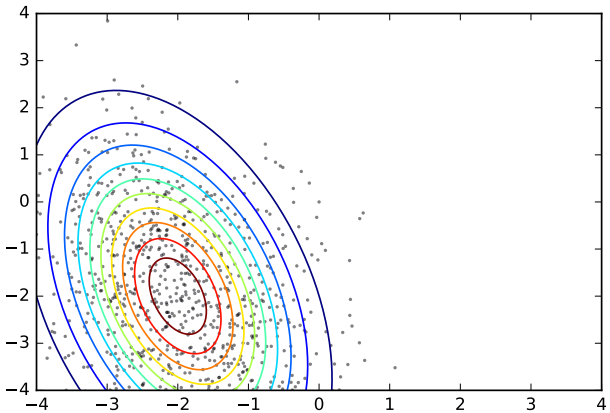
➔ **Slow learning**

Data Normalization

Original data



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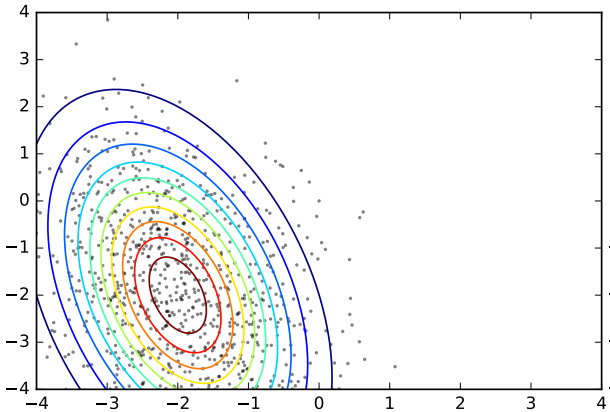
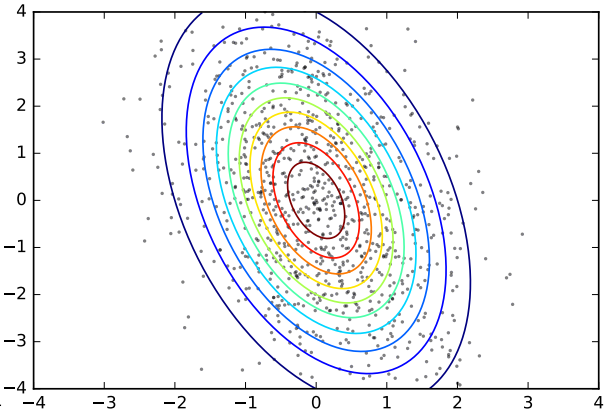


Mean subtracted



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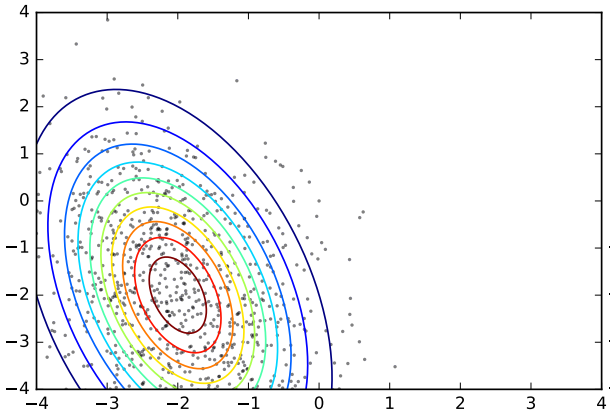

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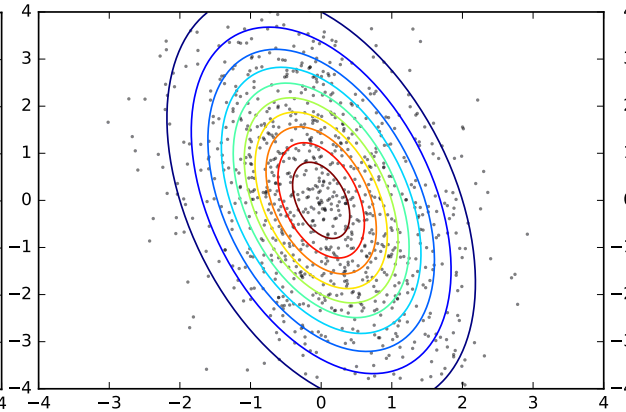
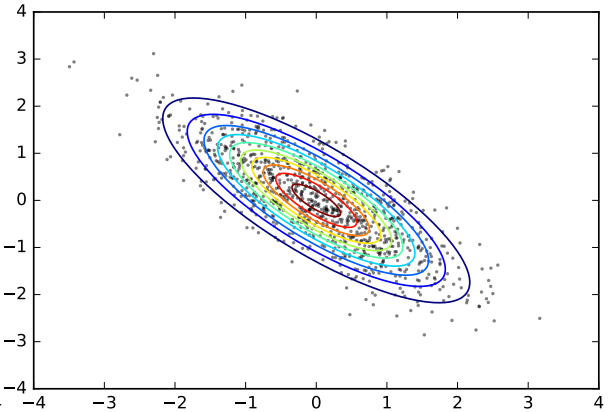
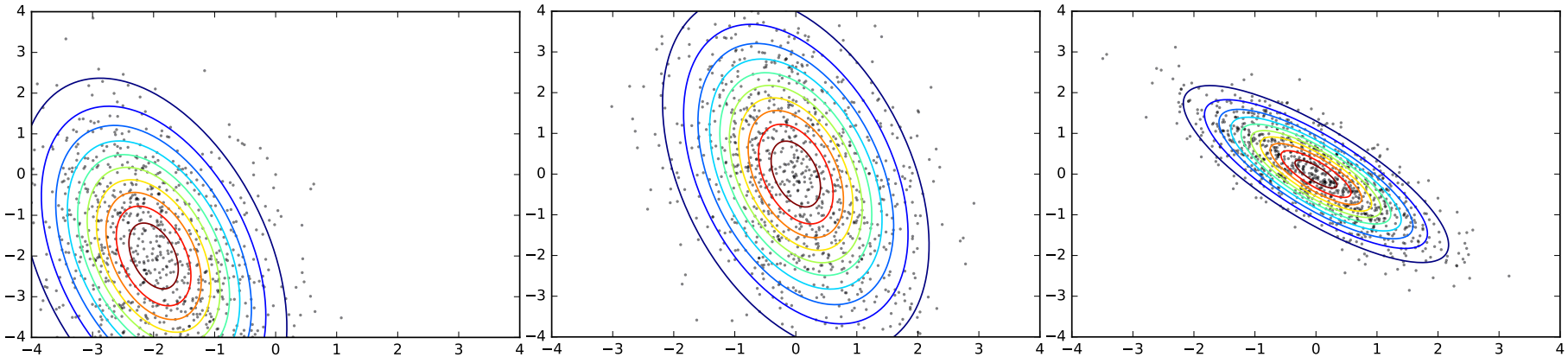


Normalized variance



Data Normalization

Original data | Mean subtracted | Normalized variance



- Normalization: min/max or **variance**

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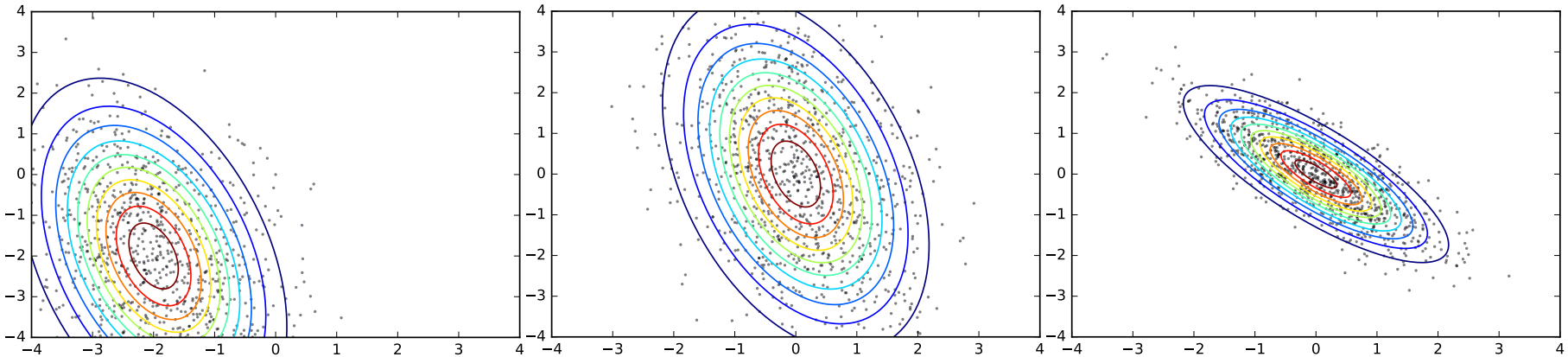

- Normalization: min/max or **variance**
- Only use **training data** to calculate normalization!

Batch normalization

- Normalization as a new layer with 2 parameters, γ and β

Batch normalization

➔ Normalization as a new layer with 2 parameters, γ and β

$$\tilde{x}_i = \frac{x_i - \mu_{B,i}}{\sqrt{\sigma_{B,i}^2 + \epsilon}}$$

μ_B and σ_B from **mini-batch**

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- So the μ and σ are **vectors, of the same dimension** as the **activation** vector.

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- So the μ and σ are **vectors, of the same dimension** as the **activation** vector.
- Paired with **convolutional** layers batch normalization is **different** by computing a **scalar** μ and σ for every **channel**.

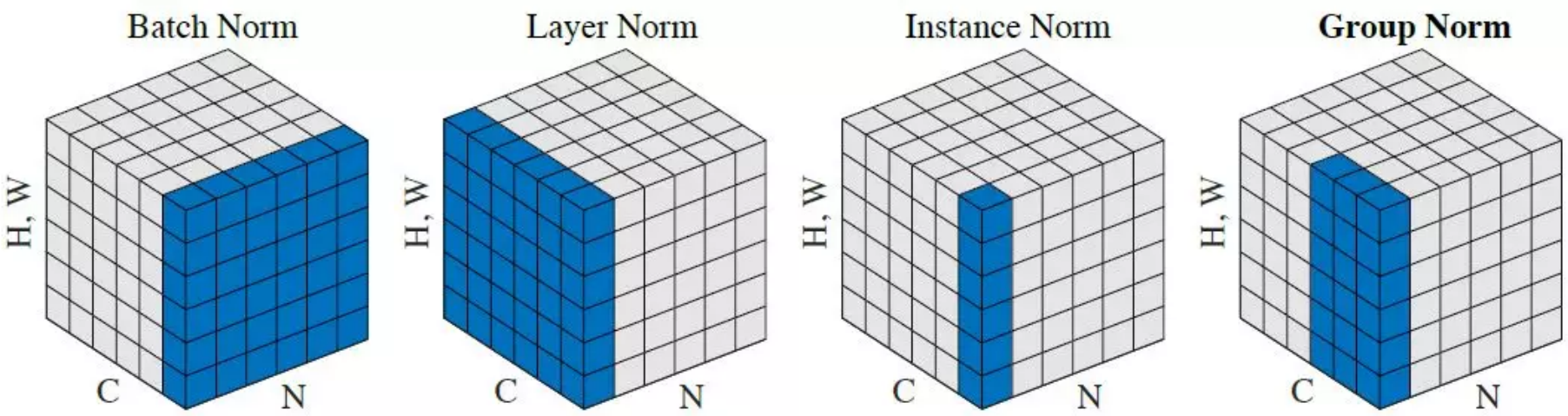
Generalizations

$$\hat{y} = f\left(g\frac{x-\mu}{\sigma} + b\right)$$

y: output
f: activation function
x: input
g,b: adaptive gain and bias
μ,σ: mean and std dev.

Calculating μ, σ over

- ... activations of a batch ➔ [13], a layer ➔ [3], spatial dims ➔ [1], a group ➔ [2]
- ... weights of a layer ➔ [11]



Source: Wu et al. [2]

Self Normalizing Neural Networks

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$$f(x) = \lambda \begin{cases} x & \text{if } x > 0 \\ \alpha(\exp(x) - 1) & \text{else} \end{cases}$$

  With $\lambda_{01} = 1.0507$ and $\alpha_{01} = 1.6733$ for $\mu = 0$ and $\sigma = 1$
- Guarantee that μ and the σ of the activations stays near 0 and 1 through the network

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- Additionally they use an **affine transform** to restore variance and mean
- The **SNN** concept resembles **batch normalization**


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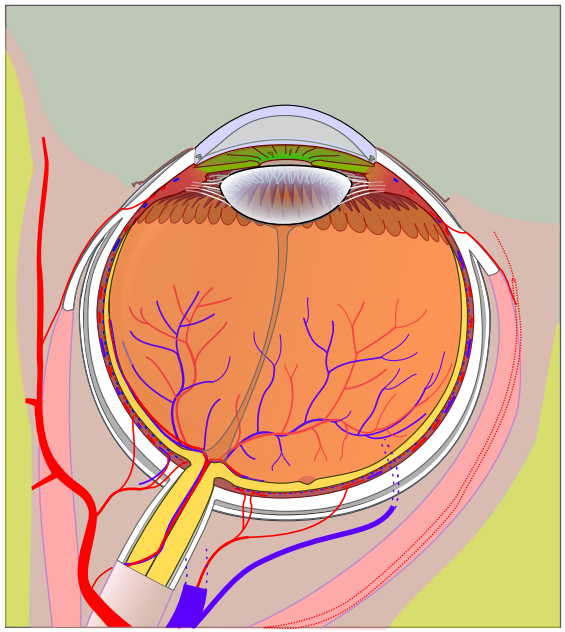
Dropout

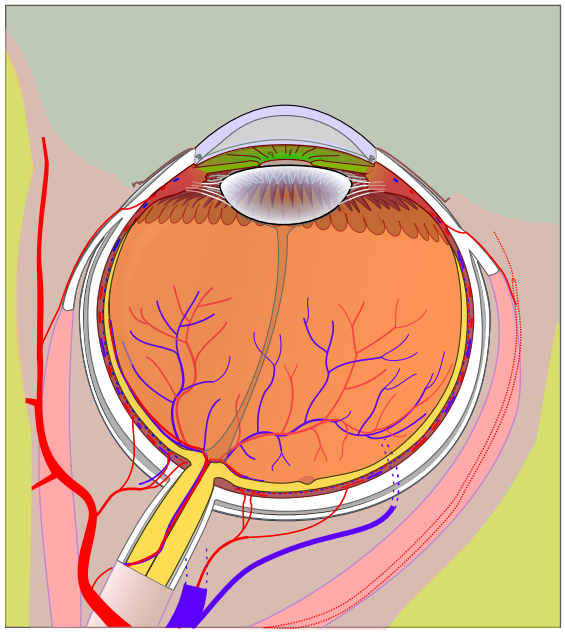

SIGILLVM ACADEMIAE FRIDERICO ALEXANDRINAE

Co-adaptation



All elements **depend** on each other

Co-adaptation



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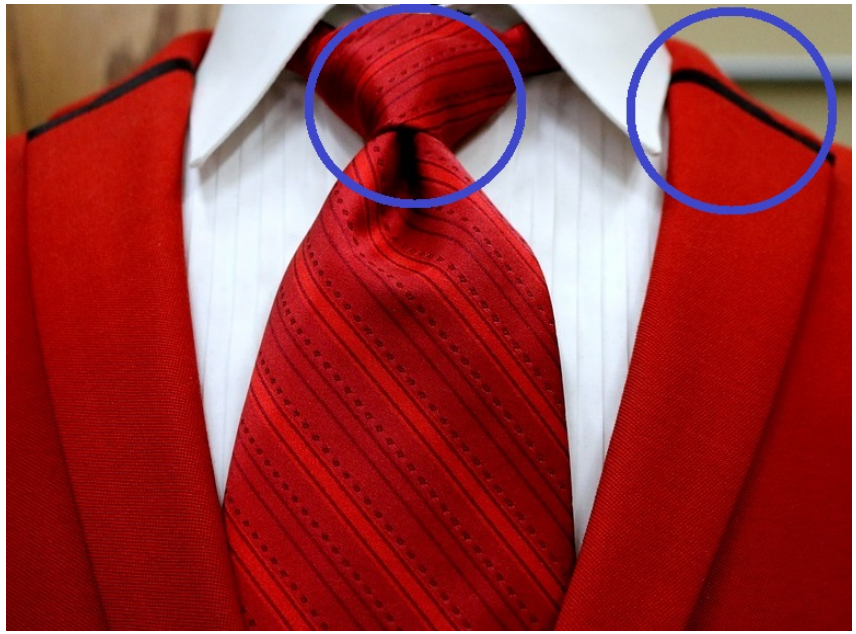
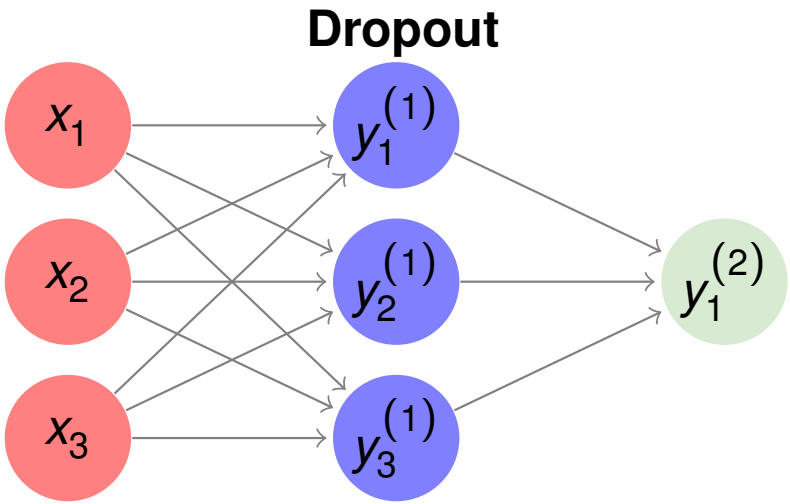


➔Contrary to **independent** features

Source: Jordi March / CC-BY-SA-3.0

Dropout and Dropconnect



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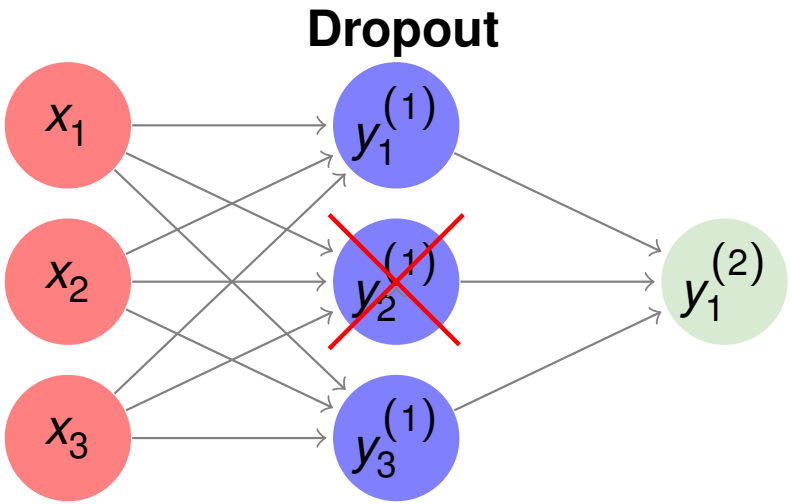
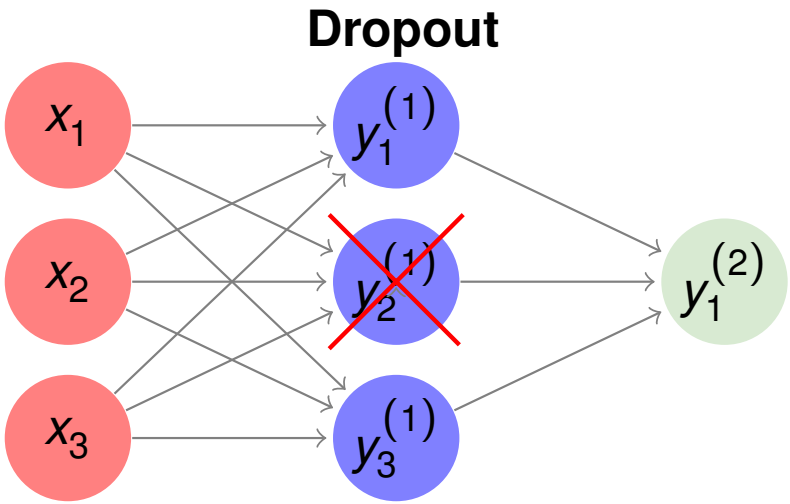


- Randomly set activations to zero with probability $1 - p$

Dropout and Dropconnect



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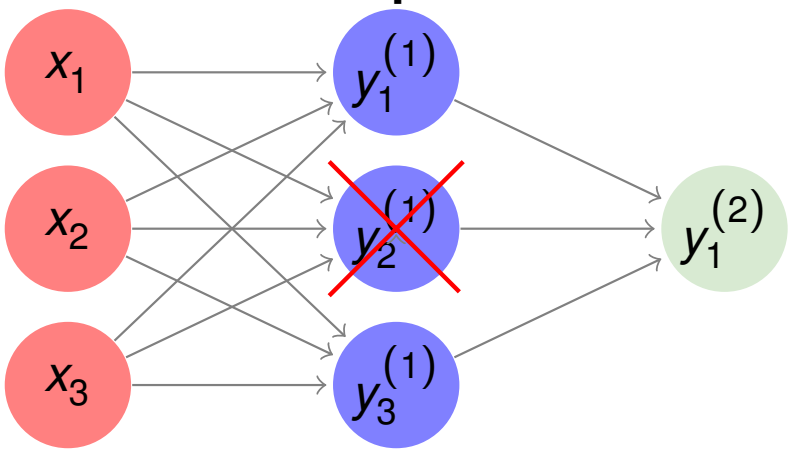

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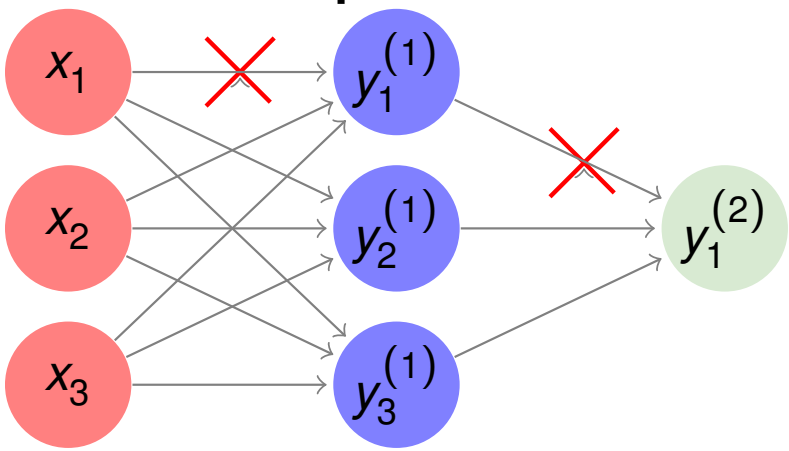

- Generalizes Dropout
- Less efficient implementation (masking)

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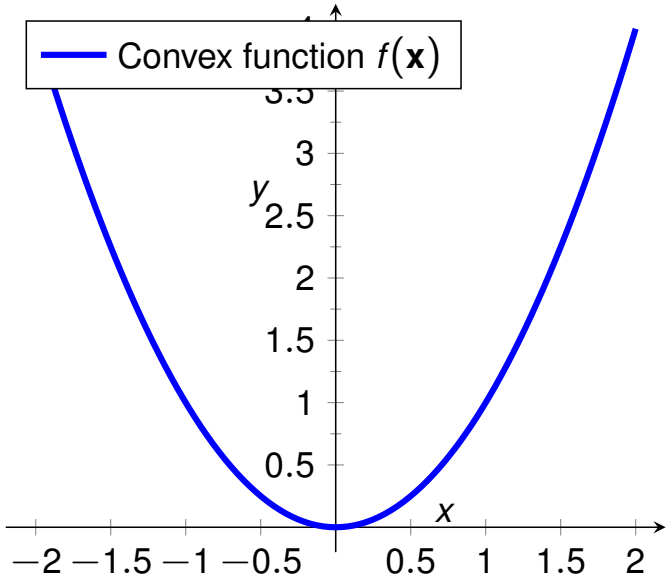
Initialization

Does initialization matter?



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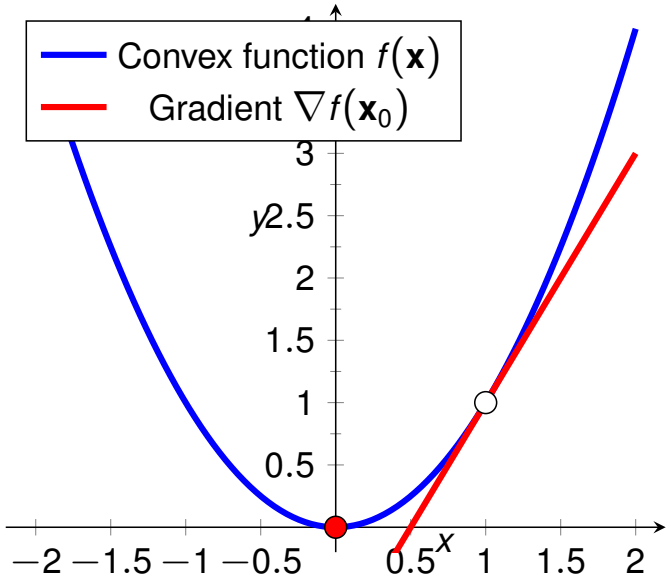

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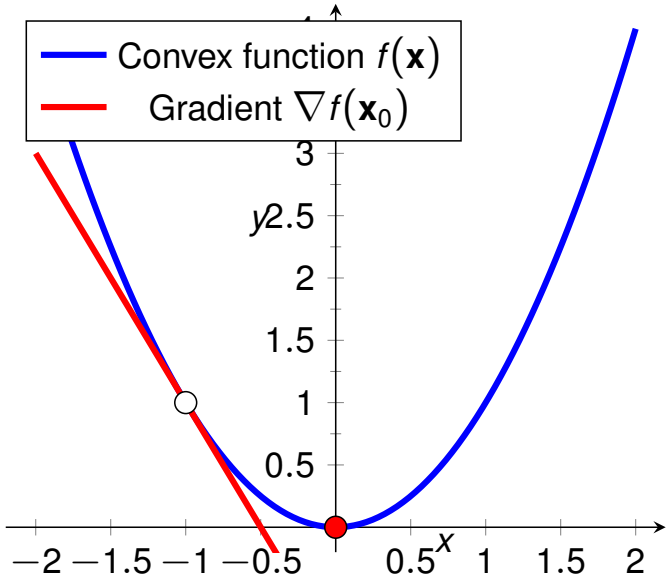

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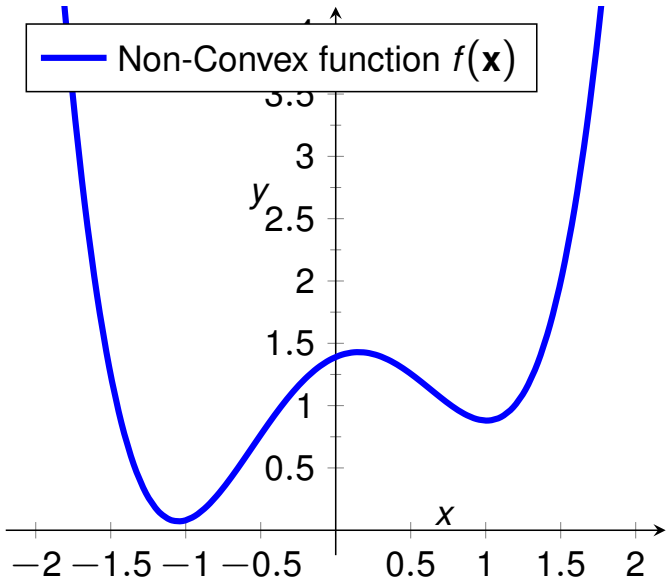

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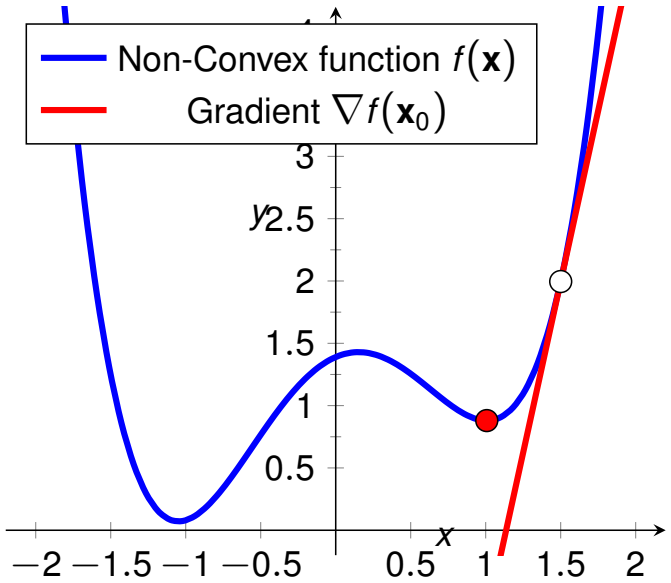

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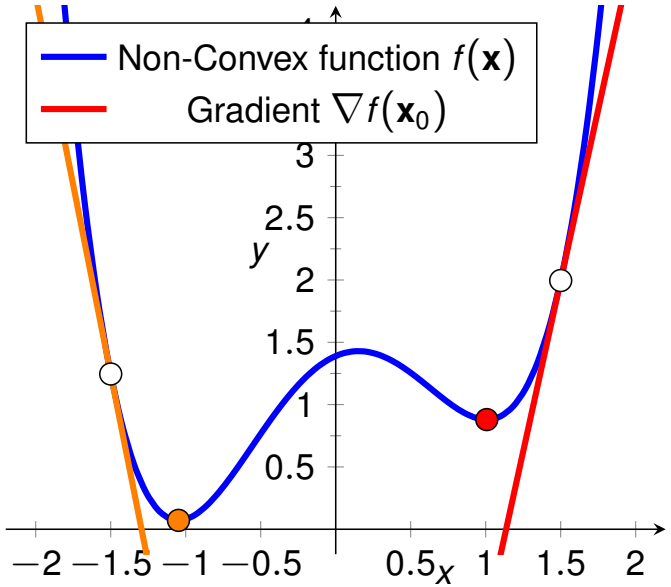

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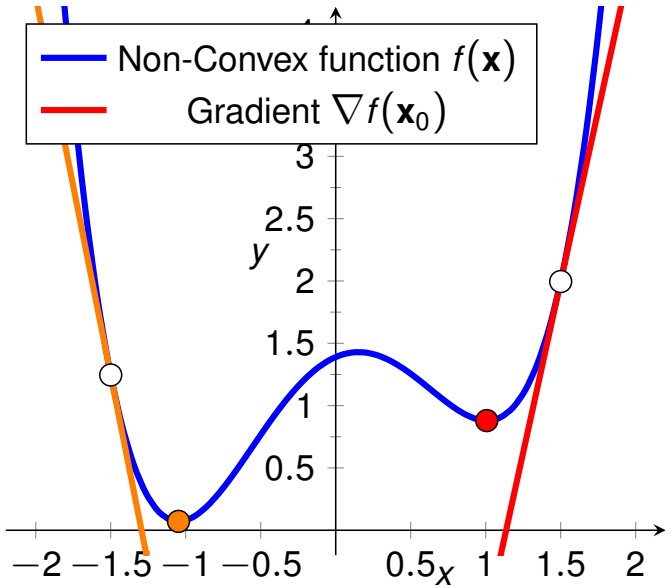

- No it doesn't for **convex** optimization problems
- But it **does** for every **non-convex** problem
- Neural Networks with a non-linearity are in general **non-convex**

Simple Initialization

Bias

- **Bias** units can simply be **initialized to zero**

Weights

Simple Initialization

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- Similar to the learning rate their **variance** influences the **stability** of learning
- **Small** uniform/Gaussian values work

Calibrating the variances

- Suppose we have a single **linear** neuron with weights **W** and an input **X**
- Capital letters here mark them as **random variables**!

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- If W_n and X_n have zero-mean: $\mathrm{Var}(W_n X_n) = \mathrm{Var}(W_n)\mathrm{Var}(X_n)$
- Now we assume the X_n and W_n to be **I**ndependent and **I**dentically **D**istributed:

$$\mathrm{Var}(\hat{Y}) = \underbrace{N\,\mathrm{Var}(W_n)}_{scales Var}\,\mathrm{Var}(X_n)$$

Xavier initialization

- We can “calibrate” the variances for the forward-pass

  by initializing with a zero-mean Gaussian: $\mathcal{N}(0, \sigma)$

  with $\sigma^2 = \frac{1}{\text{fan\_in}}$

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- So we average those two:

  $$\sigma = \sqrt{\frac{2}{\text{fan\_out}+\text{fan\_in}}}$$

- Named “**Xavier**” **initialization** after the first author [17]

He initialization

- ➔ the assumption of **linear** neurons is a problem
- He et al. [18] showed, for ReLUs it's better to use:

$$\sigma = \sqrt{\frac{2}{\text{fan\_in}}}$$

Conventional Initial Choices

- L_2 regularization
- Dropout using $p = 0.5$ for FCN, use selectively in CNNs
- Mean subtraction
- Batch normalization
- He initialization

Transfer Learning

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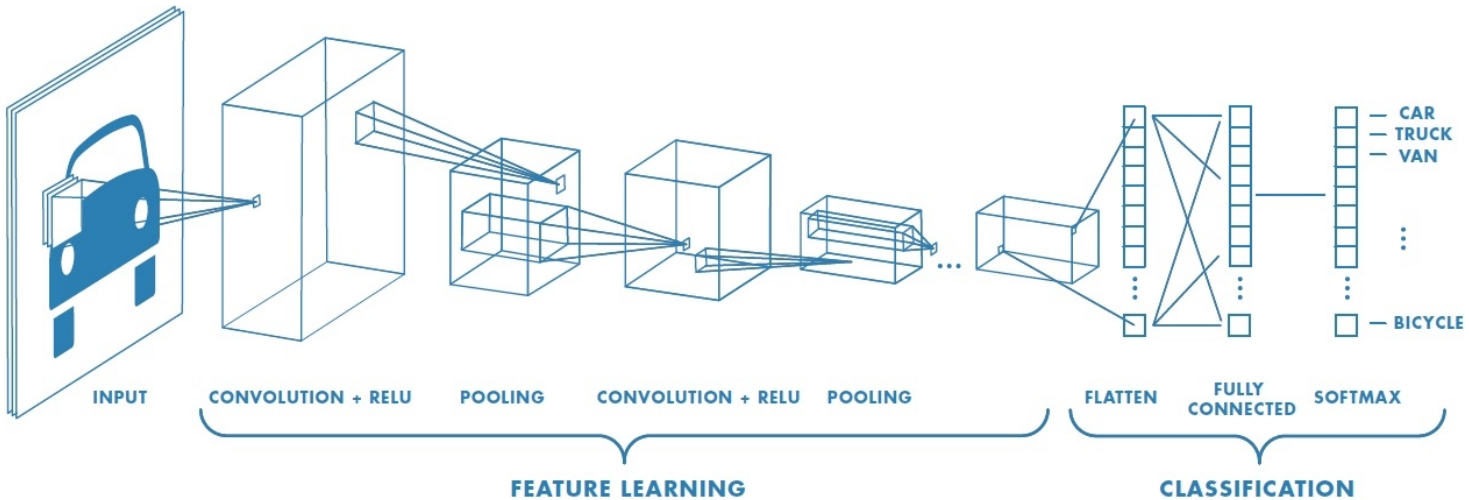
Transfer Learning

- Most **medical datasets** are prohibitively **small**
- **Reuse models** trained on e.g. ImageNet
  - ➔ for a **different task** on the **same data**
  - ➔ on **different data** for the **same task**
  - ➔ on **different data** for a **different task**

Weight Transfer



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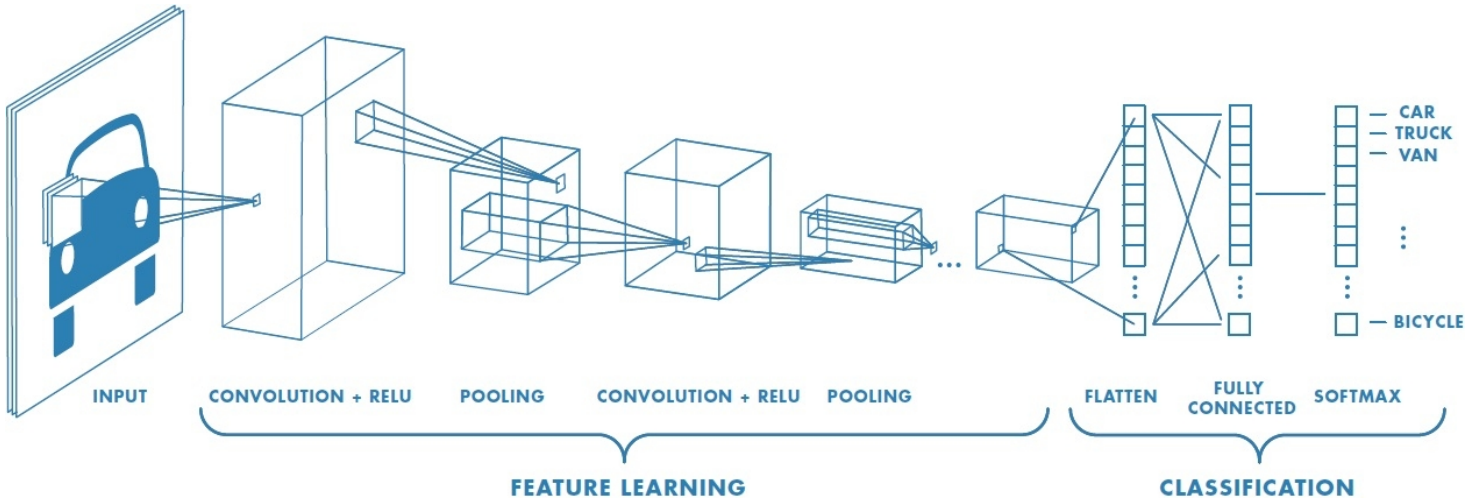

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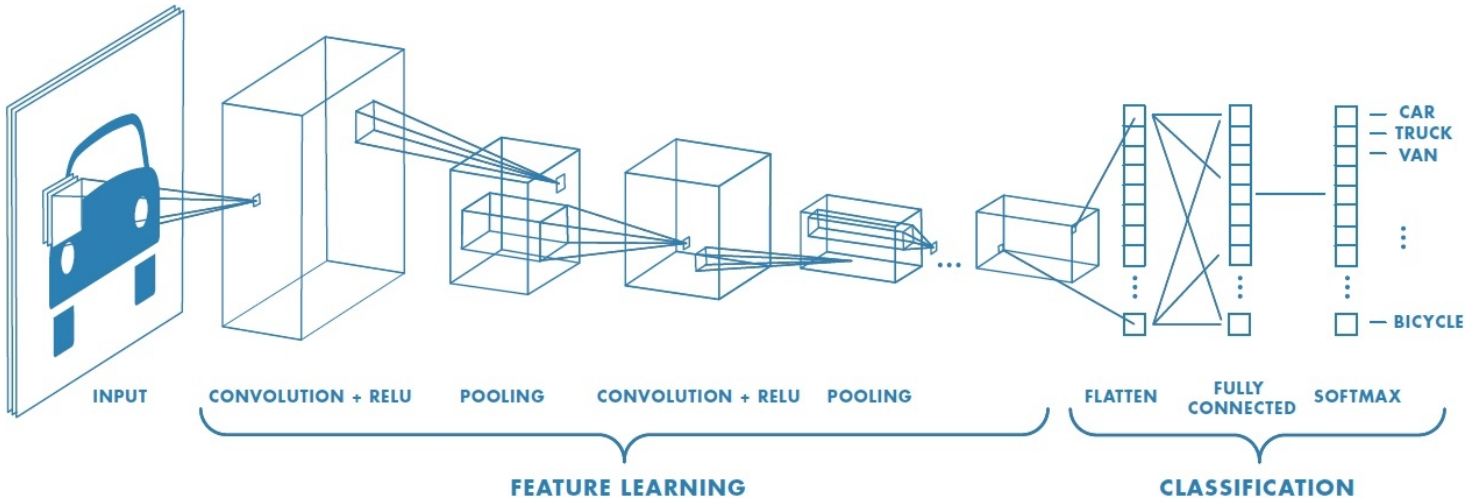

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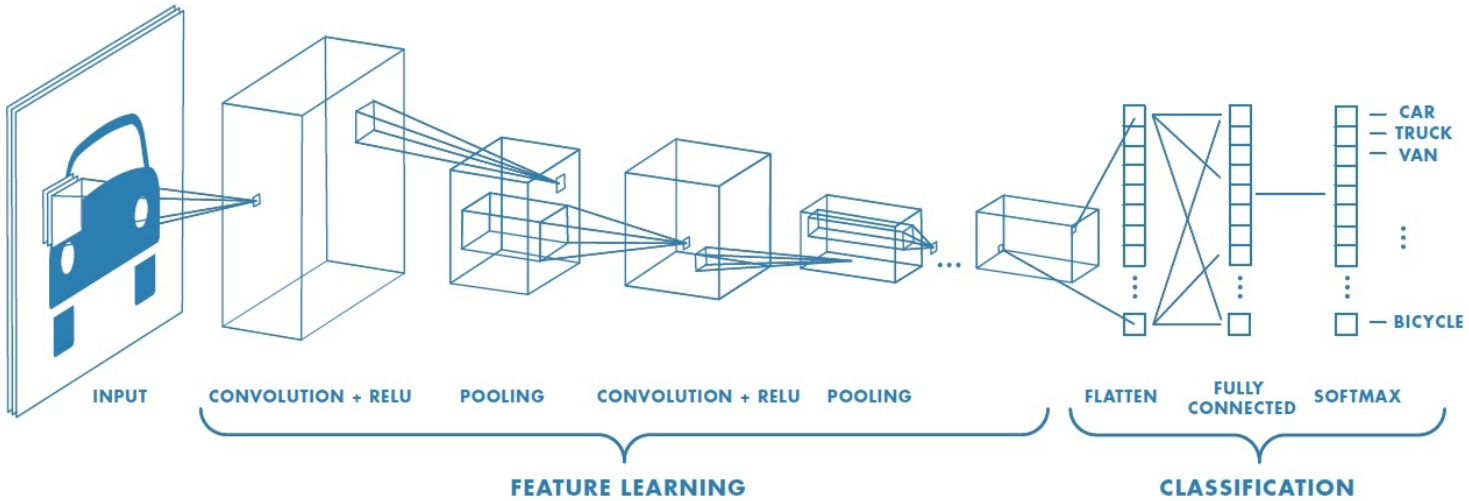

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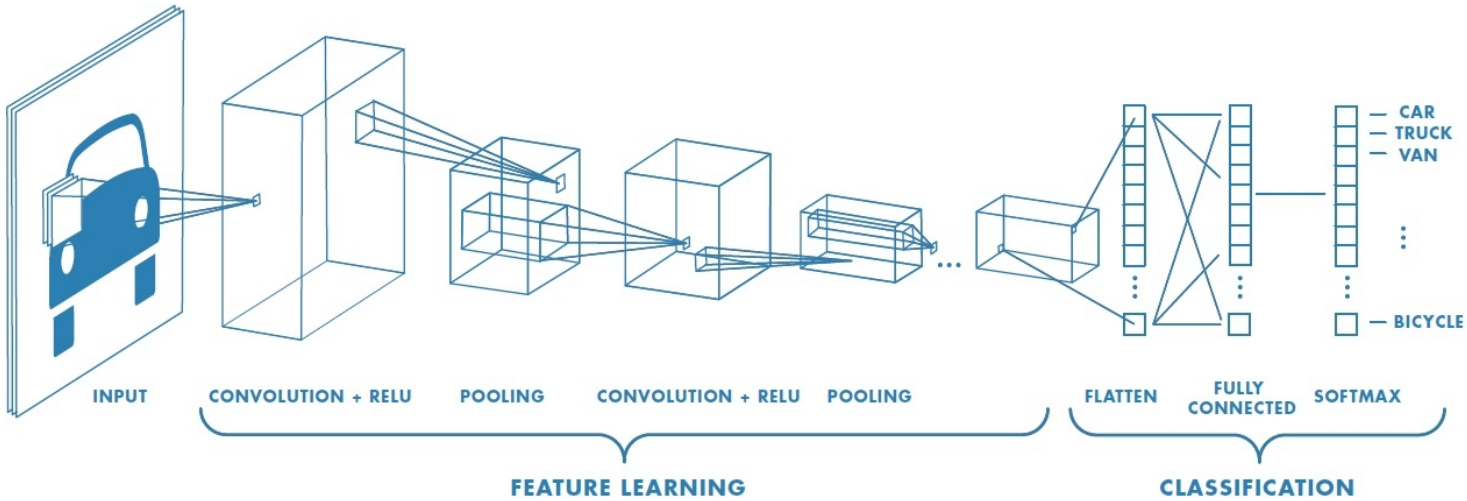
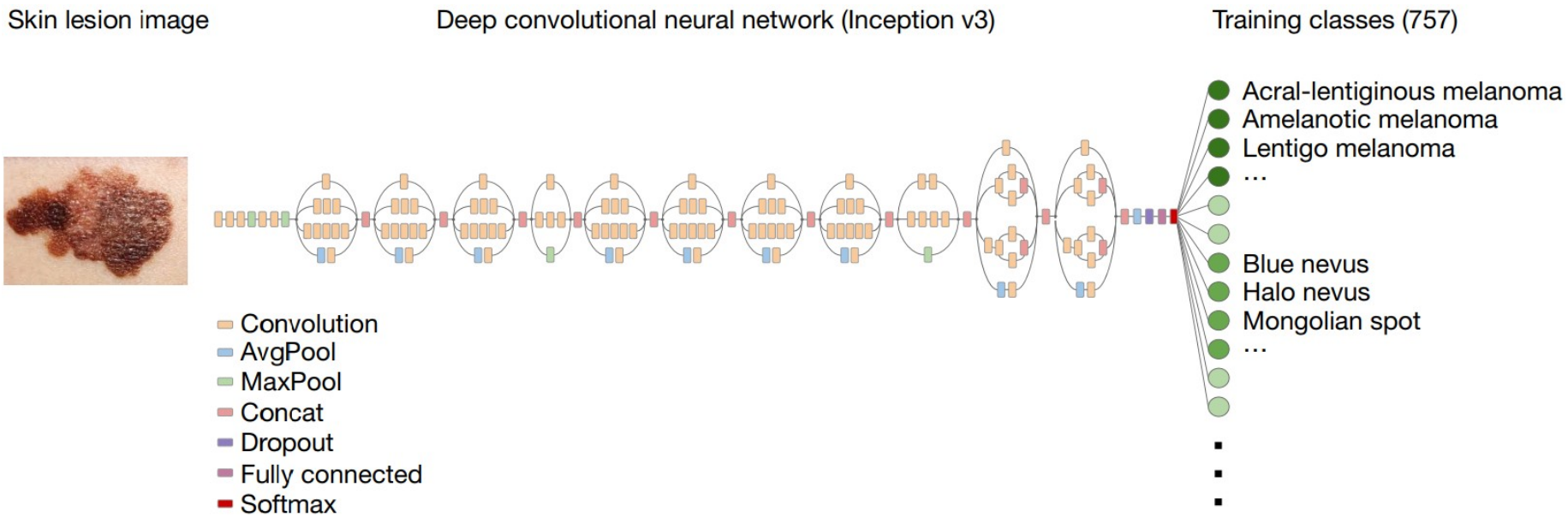


- What should we transfer?
  - ➔ Convolutional layers extract features
  - ➔ Expectation: Less task-specific in earlier layers
- We cut the network at some depth in the feature extraction part
- The extracted parts can be
  1. fixed by setting $\eta = 0$
  2. **fine-tuned**

Example: Skin Cancer classification



- State-of-the-art architecture, pre-trained on ImageNet
- Fine-tuned on skin cancer data [21]

Source: https://www.nature.com/articles/nature21056

Transfer between modalities

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➔ Always use transfer learning!

Multi-Task Learning (MTL)

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- So far: One network for one task
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- Can we do more?

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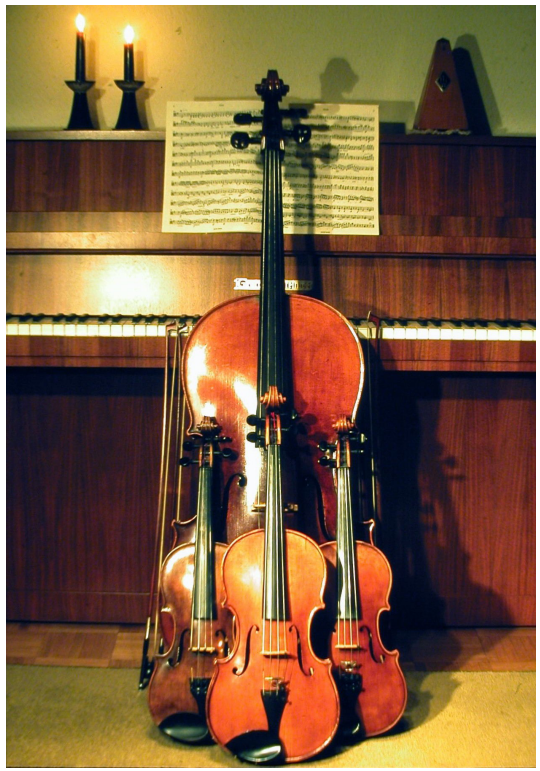

Source: https://commons.wikimedia.org/wiki/File:Klavierquintett.JPG

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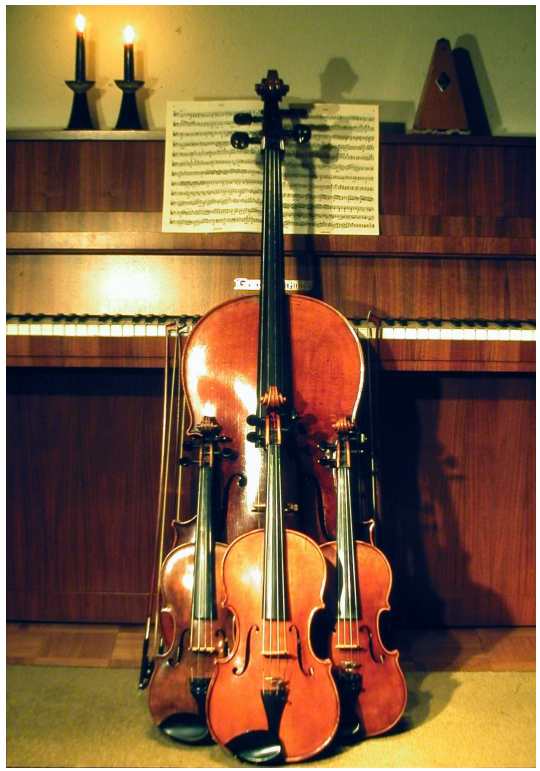

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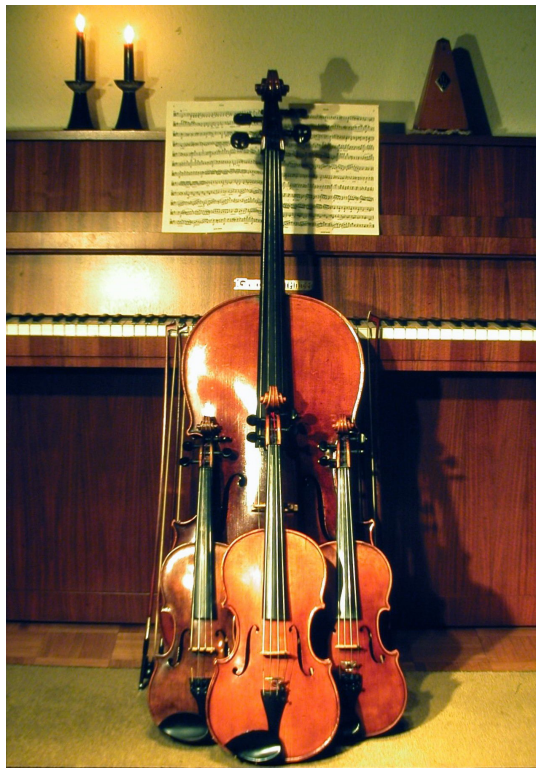

Even better than reusing: Learning them **simultaneously** can provide a better understanding of the **shared underlying concepts**.

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Concept (cont.)

- Idea: Train a network **simultaneously** on **multiple related** tasks

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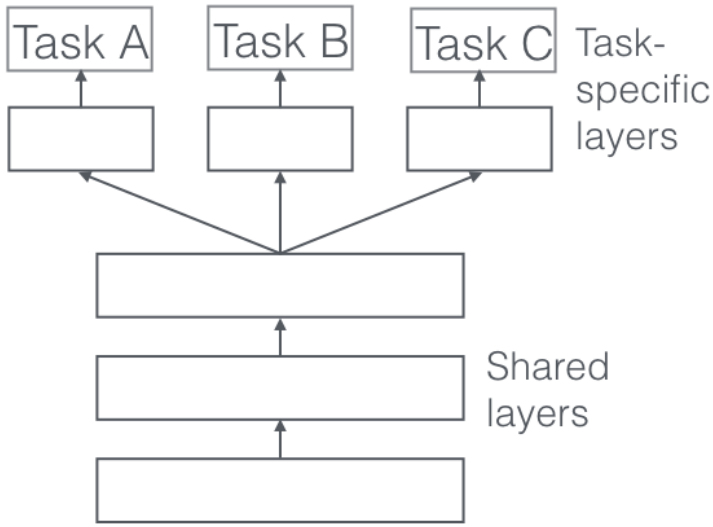
- Idea: Train a network **simultaneously** on **multiple related** tasks
- Adapt loss function to assess performance for multiple tasks
- Multi task learning introduces an **inductive bias**: We prefer a model that can explain more than one task
- Reduces risk of **overfitting** on one particular task [4]
  ➔ model generalizes better

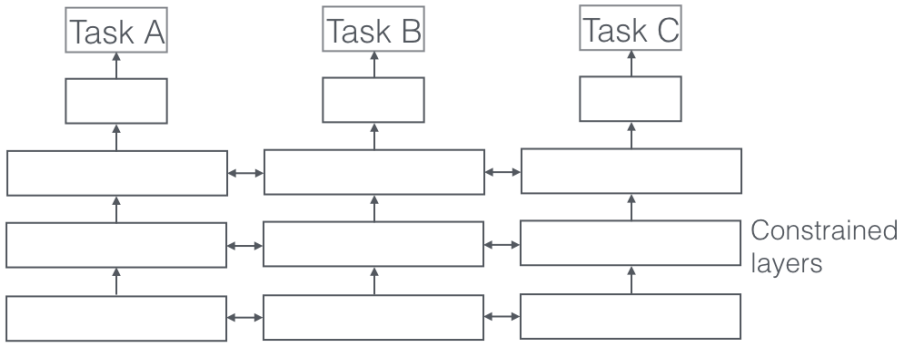
Hard parameter sharing



- Several hidden layers shared between all tasks
- Additional task-specific output layers
- Baxter ’97 [4]: Multi-task learning of N tasks reduces chance of overfitting by an order of N

Source: http://ruder.io/multi-task/

Soft parameter sharing



- Each model has its own parameters
- Instead of forcing equality, distance between parameters is regularized as part of the loss function
- Options e.g. l_2-norm, trace-norm, ...

Source: http://ruder.io/multi-task/

Auxiliary tasks

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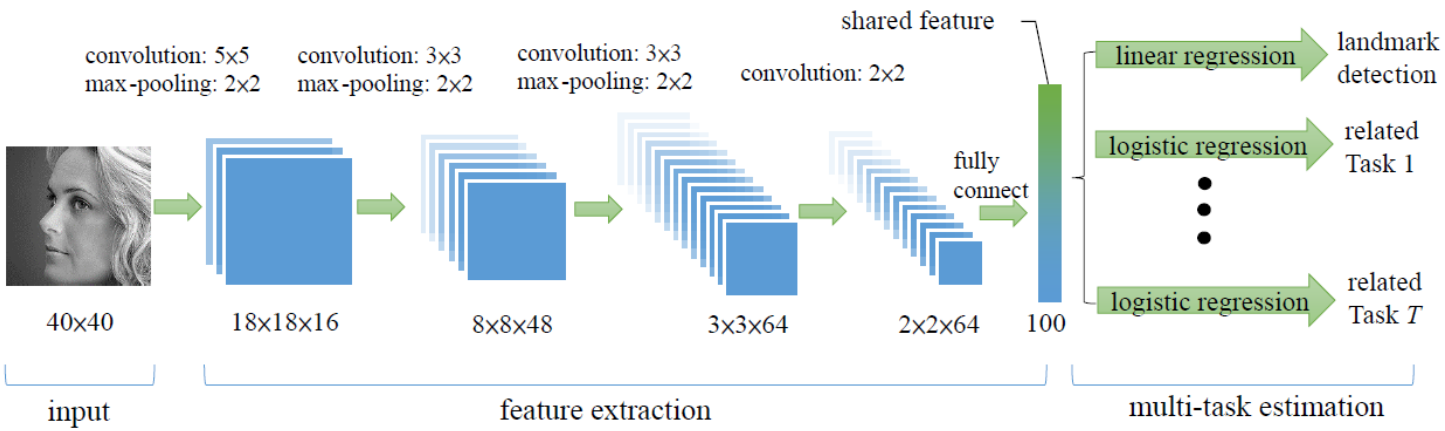
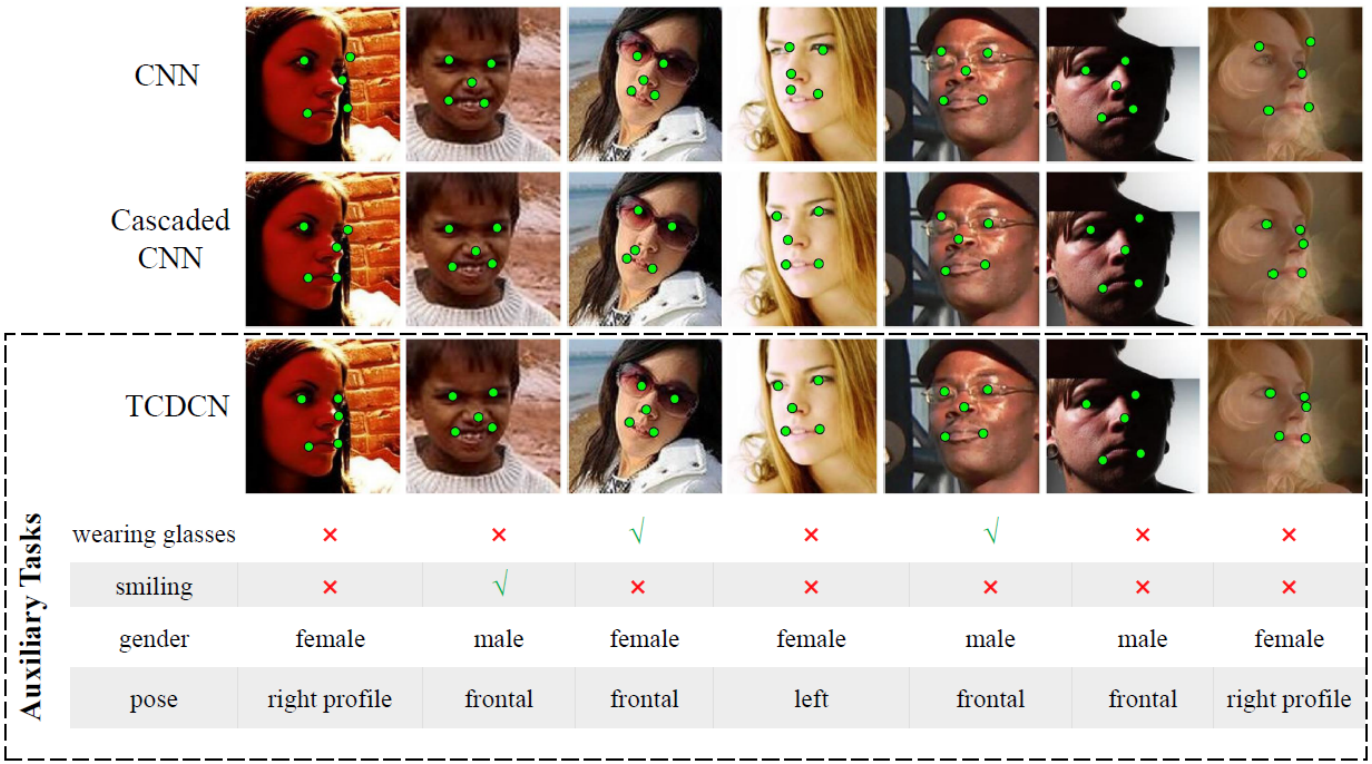
- Example: **Facial Landmark Detection** by Zhang et al. 2014 [20]
- Facial landmark detection impeded by occlusion and pose variances
- Simultaneously learn to estimate landmarks **and** “subtly” related task:
  - Face pose
  - Smiling/not smiling
  - Glasses/no glasses (occlusion)
  - Gender



Network architecture

Source: [20]

Auxiliary tasks (cont.)

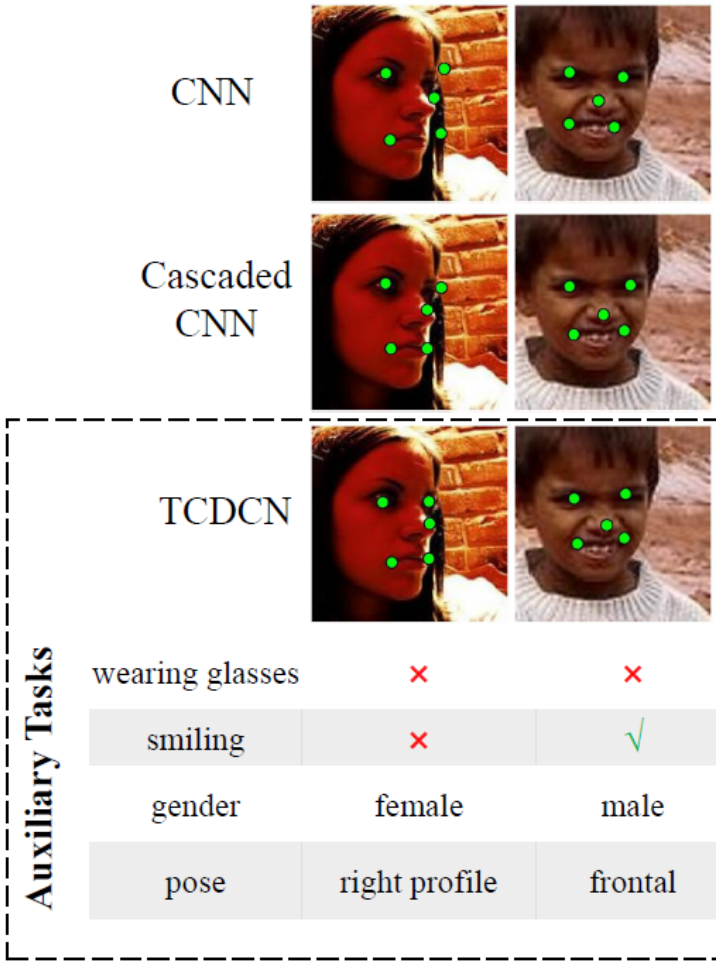


Landmark detection

Source: [20]

Auxiliary tasks (cont.)

- Certain features difficult to learn for one task but easy for a related one [7]



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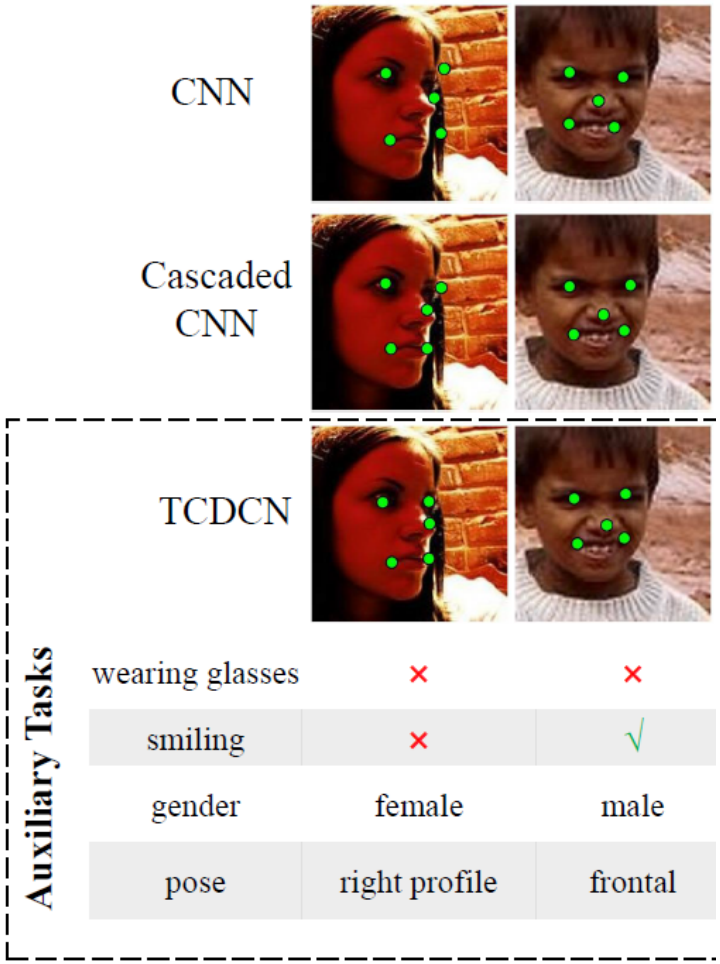

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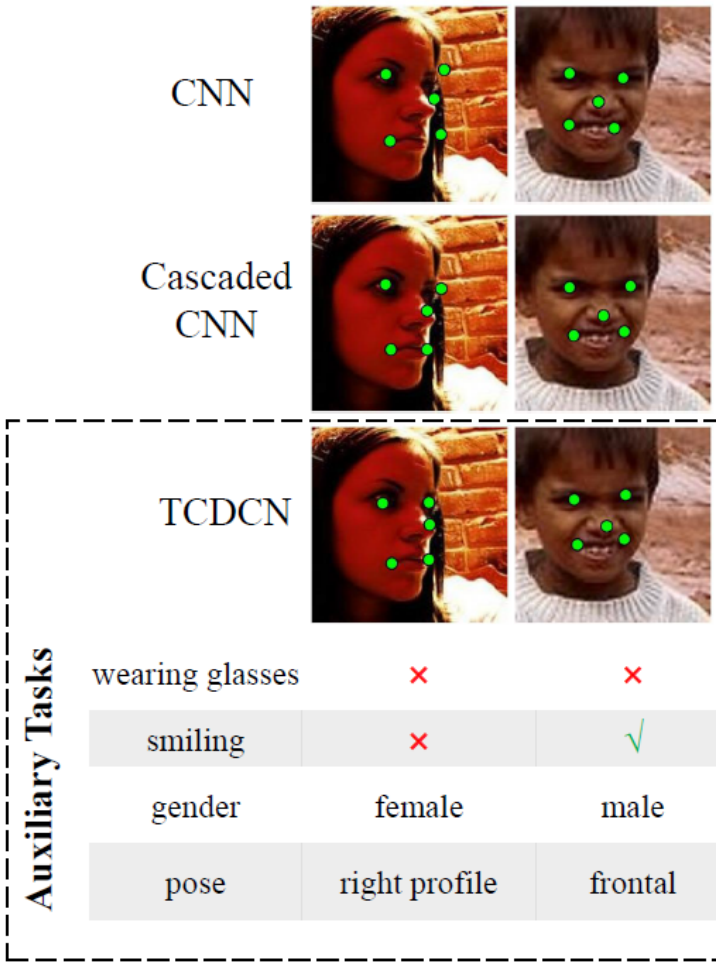

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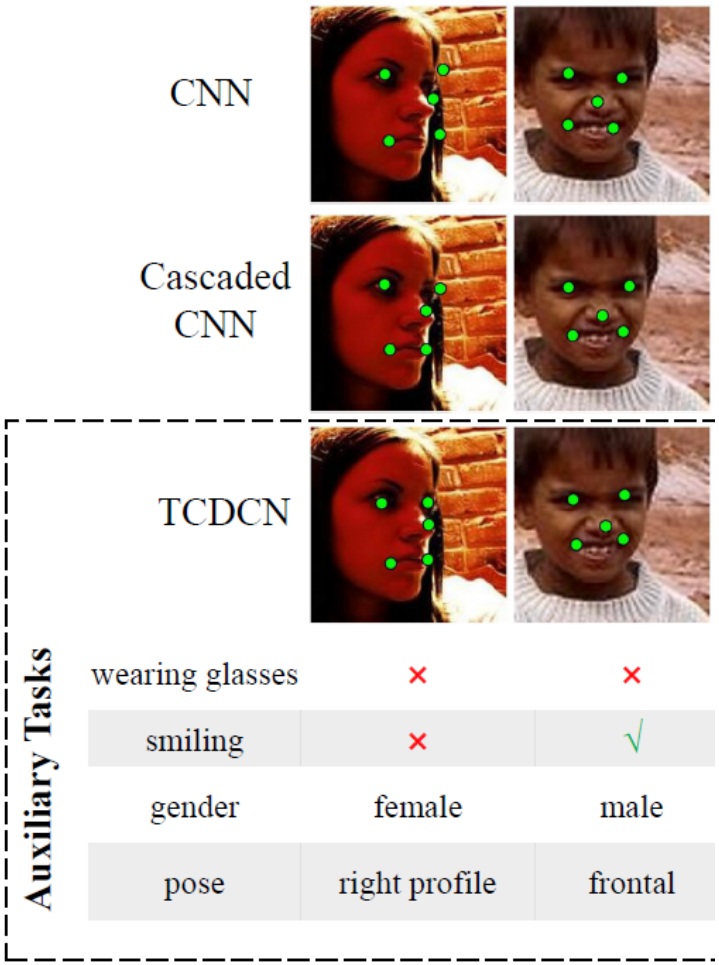
- Include prior knowledge by choosing appropriate auxiliary tasks
- Tasks can have different convergence rates
  ➔ Task-based early-stopping
- Open research question: **What are appropriate auxiliary tasks?**



Source: [20]

NEXT TIME

ON DEEP LEARNING

Coming Up

- Practical recommendations to make training work
- Evaluation of performance
- Methods to deal with common problems
- Concrete case studies using all the pieces you now learned

Comprehensive Questions

- What is the bias-variance tradeoff?
- What is model capacity?
- Describe three techniques to address overfitting in a neural network.
- Why do we often need a validation set?
- Can we optimize the hyperparameter λ by gradient descend on the training set?
- What connects the covariate shift problem and the ReLU?
- How can we address the covariate shift problem?
- Which problem do current initialization schemes try to address?
- What is transfer learning?

Further Reading

- Link - for details on Maximum A Posteriori estimation and the bias-variance decomposition
- Link - for a comprehensive text about practical recommendations for regularization
- Link - the paper about calibrating the variances

References

References I

[1] D Ulyanov, A Vedaldi, and VS Lempitsky.
Instance normalization: the missing ingredient for fast stylization. CoRR abs/16

[2] Yuxin Wu and Kaiming He. “Group normalization”. In: arXiv preprint arXiv:1803.08494 (2018).

[3] Jimmy Lei Ba, Jamie Ryan Kiros, and Geoffrey E Hinton. “Layer normalization”. In: arXiv preprint arXiv:1607.06450 (2016).

[4] Jonathan Baxter. “A Bayesian/Information Theoretic Model of Learning to Learn via Multiple Task Sampling”. In: Machine Learning 28.1 (July 1997), pp. 7–39.

[5] Christopher M. Bishop.
Pattern Recognition and Machine Learning (Information Science and Statistics)
Secaucus, NJ, USA: Springer-Verlag New York, Inc., 2006.

References II

[6] R. O. Duda, P. E. Hart, and D. G. Stork. Pattern Classification. John Wiley and Sons, inc., 2000.

[7] Richard Caruana. “Multitask Learning: A Knowledge-Based Source of Inductive Bias”. In: Proceedings of the Tenth International Conference on Machine Learning. Morgan Kaufmann, 1993, pp. 41–48.

[8] C. Ding, C. Xu, and D. Tao. “Multi-Task Pose-Invariant Face Recognition”. In: IEEE Transactions on Image Processing 24.3 (Mar. 2015), pp. 980–993.

[9] Li Wan, Matthew Zeiler, Sixin Zhang, et al. “Regularization of neural networks using dropconnect”. In: Proceedings of the 30th International Conference on Machine Learning (ICML-2013, pp. 1058–1066.

References III

[10] Nitish Srivastava, Geoffrey E Hinton, Alex Krizhevsky, et al. “Dropout: a simple way to prevent neural networks from overfitting.”. In: Journal of Machine Learning Research 15.1 (2014), pp. 1929–1958.

[11] Tim Salimans and Diederik P Kingma. “Weight Normalization: A Simple Reparameterization to Accelerate Training of Deep Neural Networks”. In: Advances in Neural Information Processing Systems 29. Curran Associates, Inc., 2016, pp. 901–909.

[12] Ian Goodfellow, Yoshua Bengio, and Aaron Courville. Deep Learning. http://www.deeplearningbook.org. MIT Press, 2016.

[13] Sergey Ioffe and Christian Szegedy. “Batch Normalization: Accelerating Deep Network Training by Reducing Internal Covariate Shift”. In: Proceedings of The 32nd International Conference on Machine Learning. 2015, pp. 448–456.

References IV

[14] Chiyuan Zhang, Samy Bengio, Moritz Hardt, et al. “Understanding deep learning requires rethinking generalization”. In: arXiv preprint arXiv:1611.03530 (2016).

[15] Yoshua Bengio. “Practical recommendations for gradient-based training of deep architectures”. In: Neural networks: Tricks of the trade. Springer, 2012, pp. 437–478.

[16] Nima Tajbakhsh, Jae Y Shin, Suryakanth R Gurudu, et al. “Convolutional neural networks for medical image analysis: Full training or fine tuning?” In: IEEE transactions on medical imaging 35.5 (2016), pp. 1299–1312.

[17] Xavier Glorot and Yoshua Bengio. “Understanding the difficulty of training deep feedforward neural networks”. In: Proceedings of the Thirteenth International Conference on Artificial Intelligence 2010, pp. 249–256.

References V

[18] Kaiming He, Xiangyu Zhang, Shaoqing Ren, et al. “Delving deep into rectifiers: Surpassing human-level performance on imagenet classification”. In: Proceedings of the IEEE international conference on computer vision. 2015, pp. 1026–1034.

[19] Günter Klambauer, Thomas Unterthiner, Andreas Mayr, et al. “Self-Normalizing Neural Networks”. In: Advances in Neural Information Processing Systems (NIPS). Vol. abs/1706.02515. 2017. arXiv: 1706.02515.

[20] Zhanpeng Zhang, Ping Luo, Chen Change Loy, et al. “Facial Landmark Detection by Deep Multi-task Learning”. In: Computer Vision – ECCV 2014: 13th European Conference, Zurich, Switzerlan Cham: Springer International Publishing, 2014, pp. 94–108.

References VI

[21] Andre Esteva, Brett Kuprel, Roberto A Novoa, et al. “Dermatologist-level classification of skin cancer with deep neural networks”. In: Nature 542.7639 (2017), pp. 115–118.