Activation Functions and Convolutional Neural Networks

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Outline

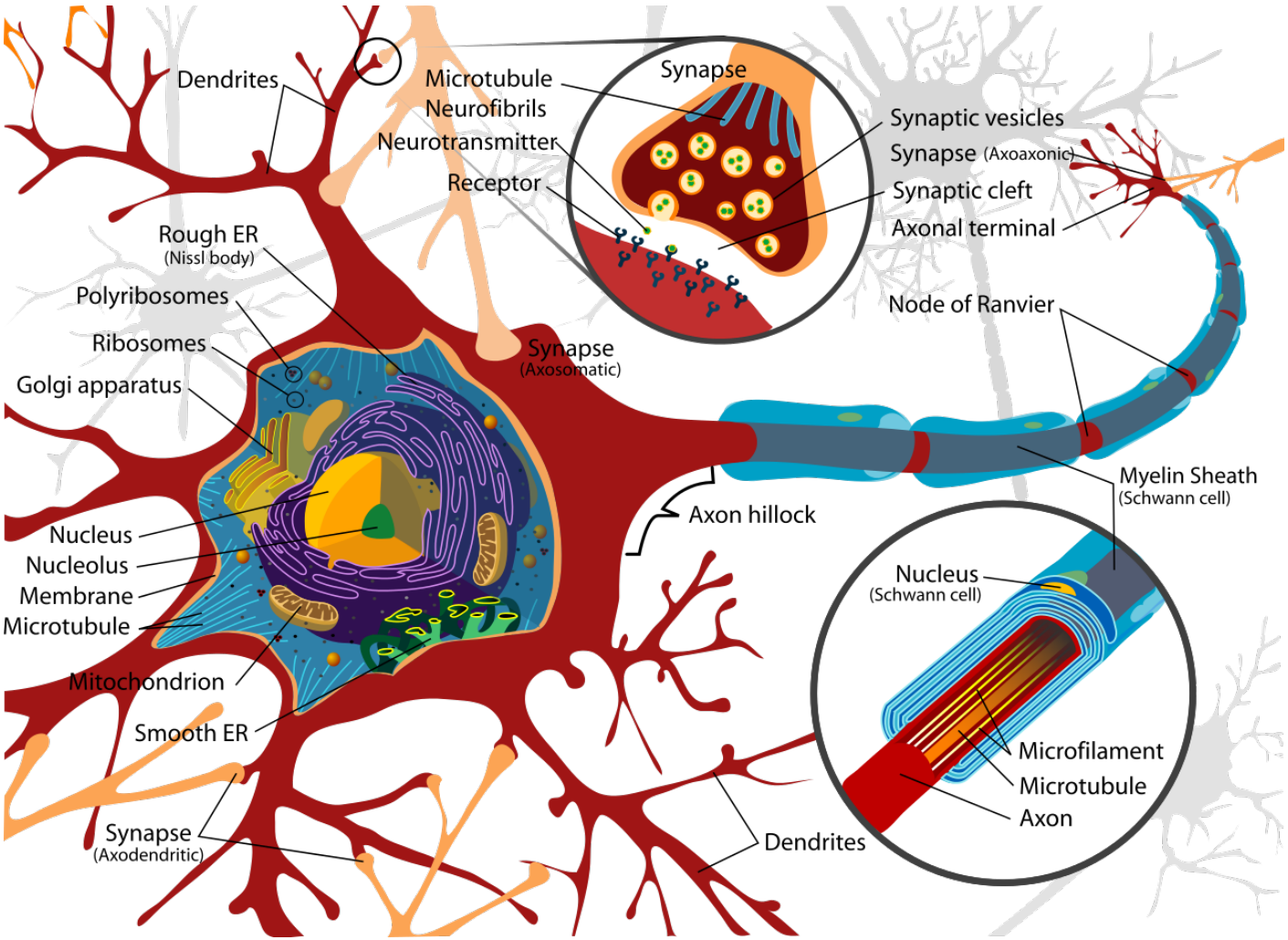
Activation Functions

Convolutional Neural Networks

- Convolutional Layers
- Pooling Layers

Activation Functions

Biological Activation



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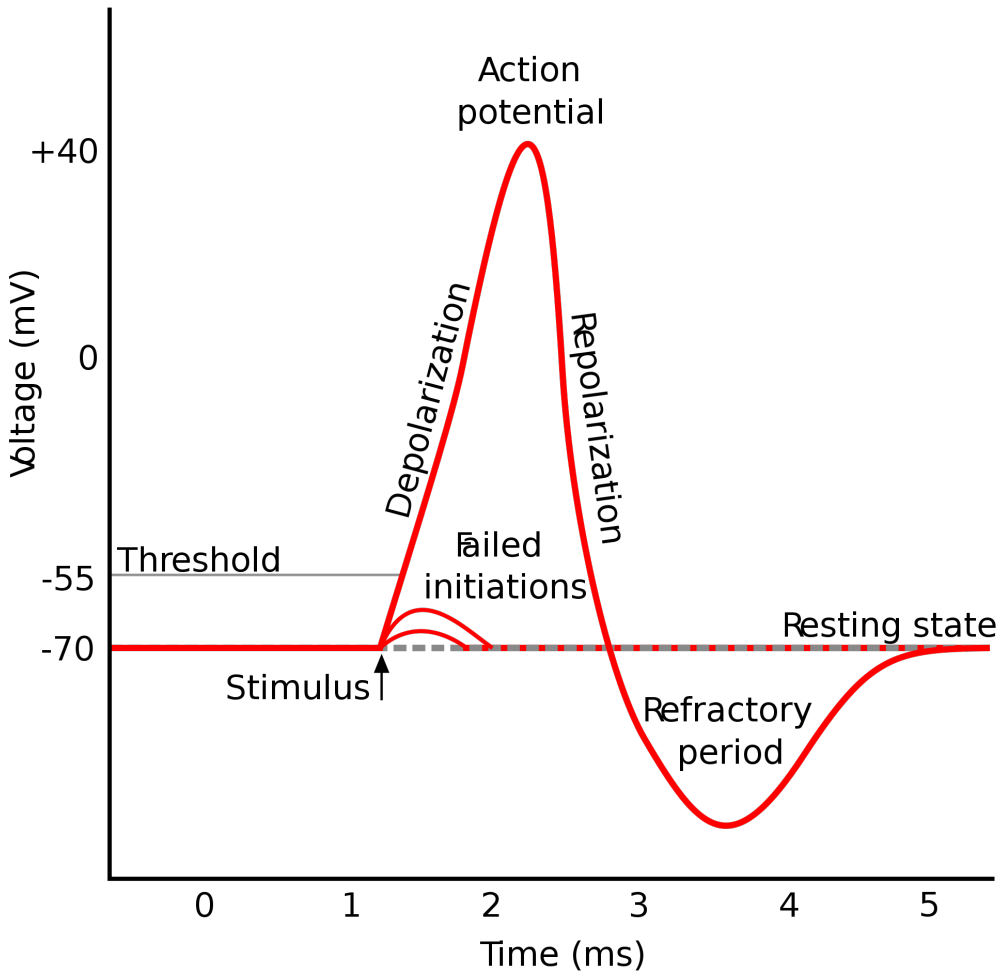

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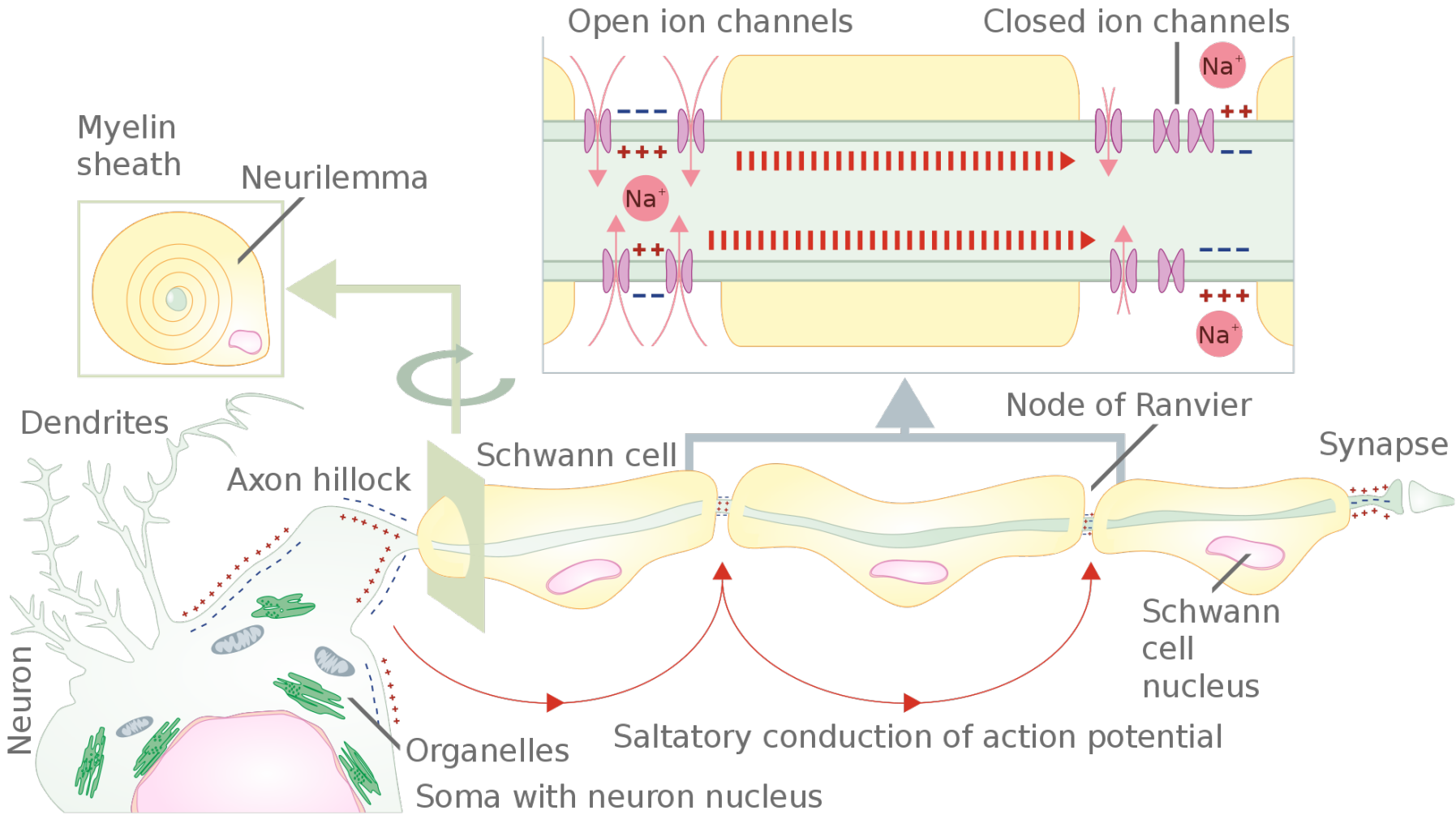

Summary - Activations in Biological Neurons

- The knowledge lies in the **connections**
- Both **inhibitory** and **excitatory** connections exist
- The synapses anatomically enforce **feed forward** processing
- However, the connections can be in any direction
- The **sum** of activations is crucial
- Activations are electric spikes
  - With **specified intensity**
  - Which also encode **information over time**

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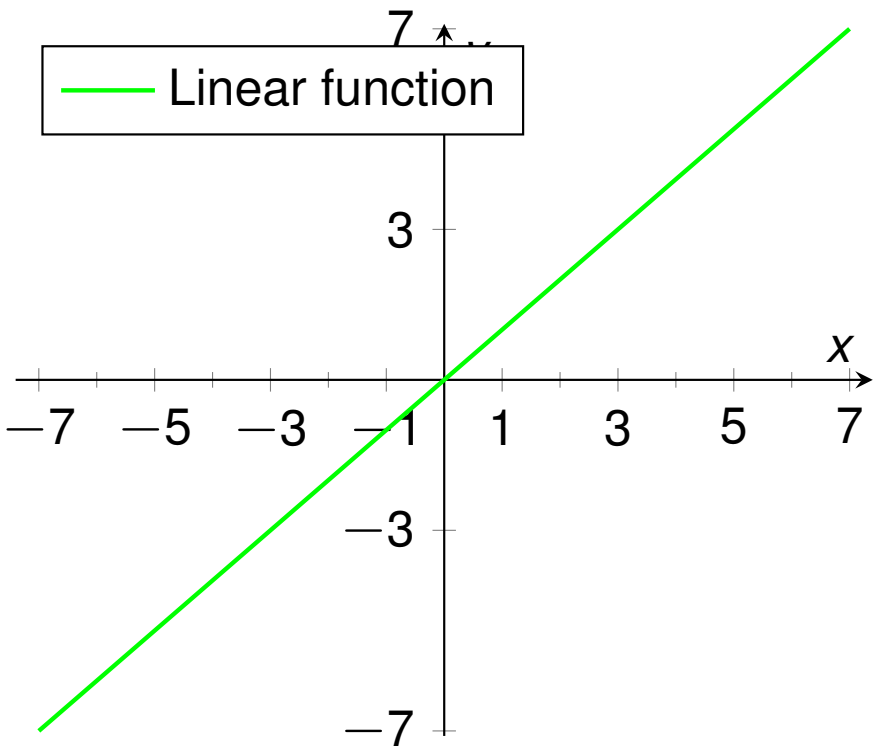
- Can we do better?

Linear Activation Function

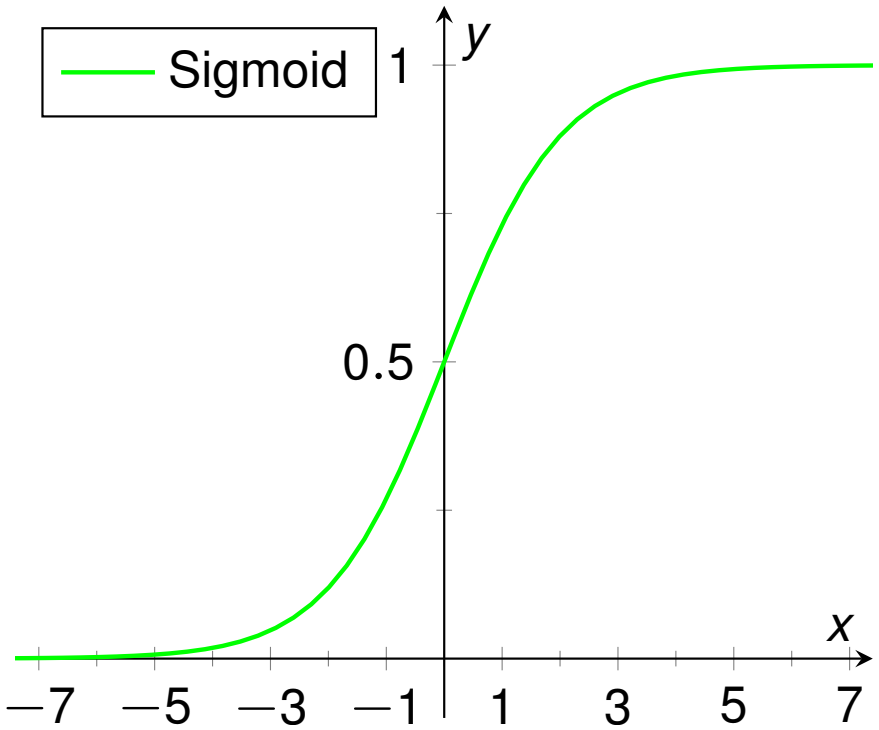


Linear function with parameter α:

$$f(x) = \alpha x$$
$$f'(x) = \alpha$$

- \+ Simple
- − Does not introduce non-linearity
- \+ Therefore, it renders the optimization problem convex
- Listed here mainly for completeness

Sigmoid Activation Function



Sigmoid (logistic function)

$$f(x) = \frac{1}{1 + exp(-x)}$$

$$f'(x) = f(x)(1 - f(x))$$

- Close to biological model, but differentiable
- \+ Probabilistic output
- − Saturates for $x \ll 0$ and $x \gg 0$
- − Not zero-centered

Zero-Centering

- Sigmoid: $f : \mathbb{R} \mapsto]0, 1[$

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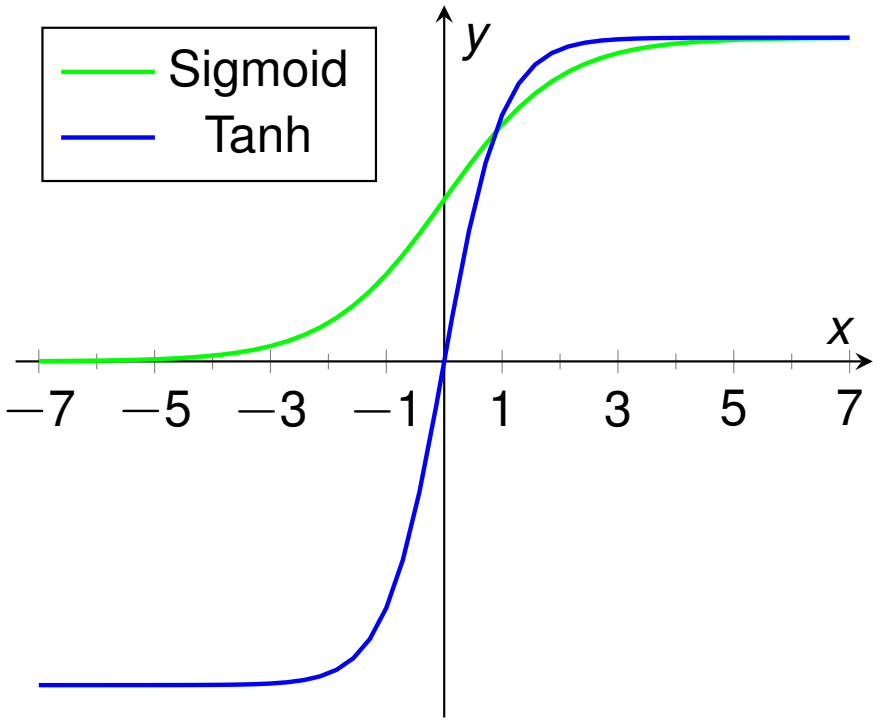
- A mean $\mu = 0$ of the input distribution will always be shifted to $\mu > 0$
  - ➔ **co-variate shift** of successive layers
  - ➔ layers **constantly** have to **adapt** to the shifting distribution
- Batch learning reduces the variance σ of the updates

Tanh Activation Function



Tanh

$$f(x) = tanh(x)$$
$$f'(x) = 1 - f(x)^2$$

- \+ Zero-centered (LeCun '91)
- − Shifted version of sigmoid σ: $\tanh(x) = 2\sigma(2x) - 1$
  - ➔ Still saturates
  - ➔ Still causes vanishing gradients

Vanishing and Exploding Gradients

- Essence of learning: How does **x** affect **y**?
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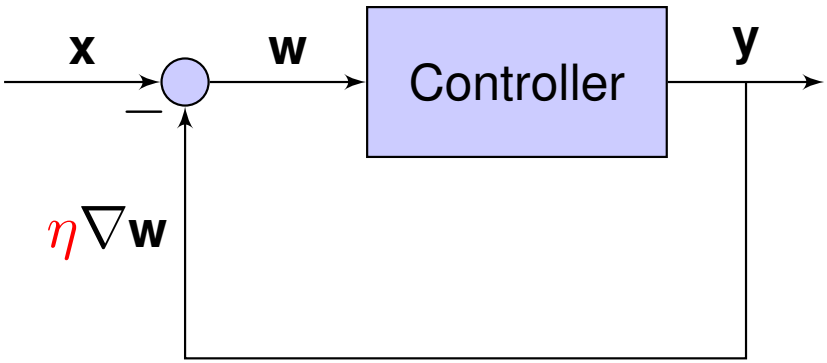
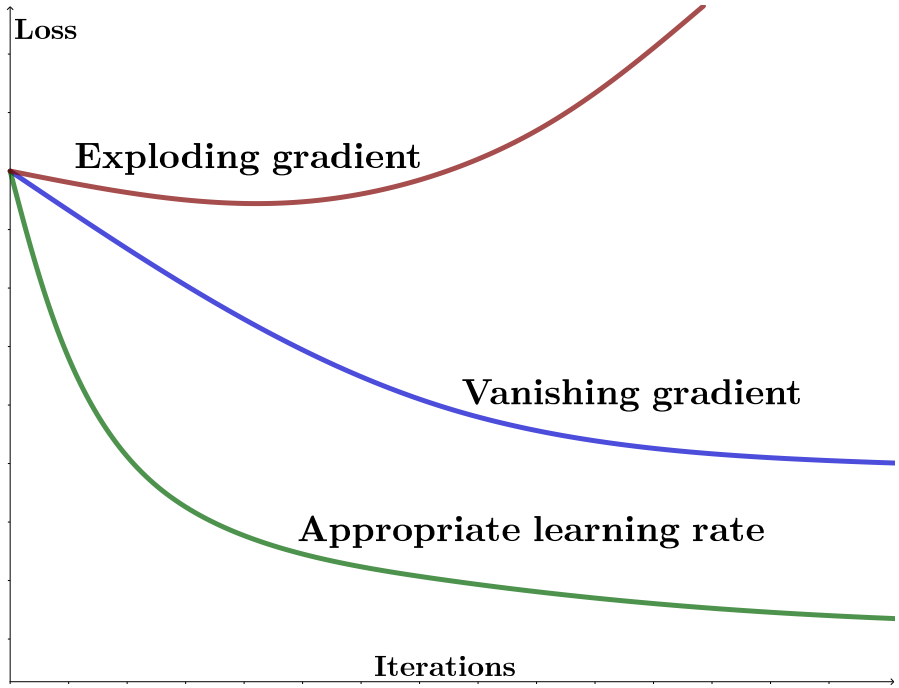
- Related problem: Exploding gradients

Recap: Feedback Loop - Vanishing and Exploding Gradients

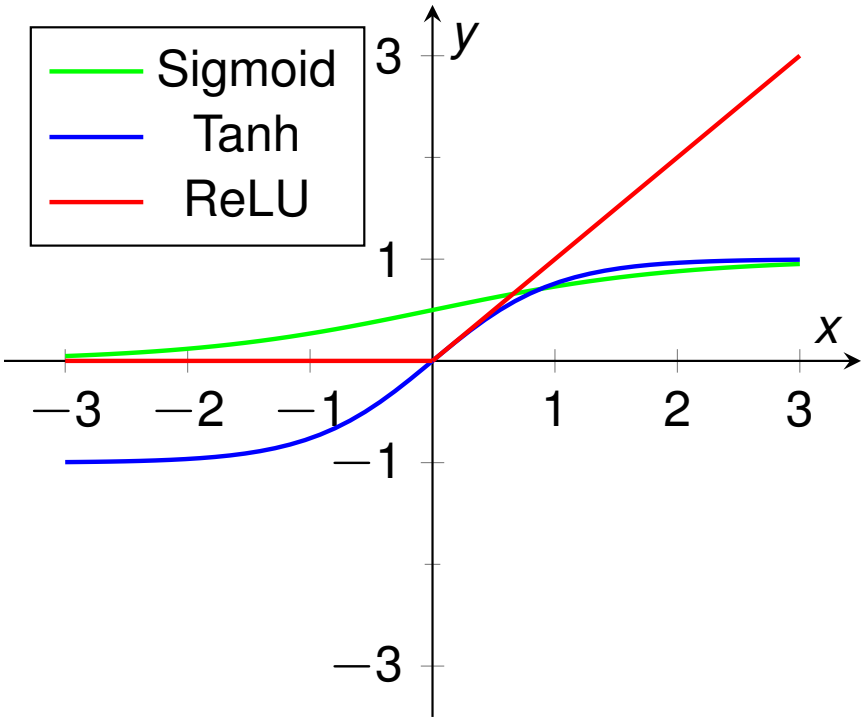


Analogy to control theory



- If η is to high $\mapsto$ **positive feedback** $\mapsto$ loss grows **without bounds**
- If η is to small $\mapsto$ **negative feedback** $\mapsto$ **gradient vanishes**
- Choice of η is **critical** for learning

Rectified Linear Units (ReLU)



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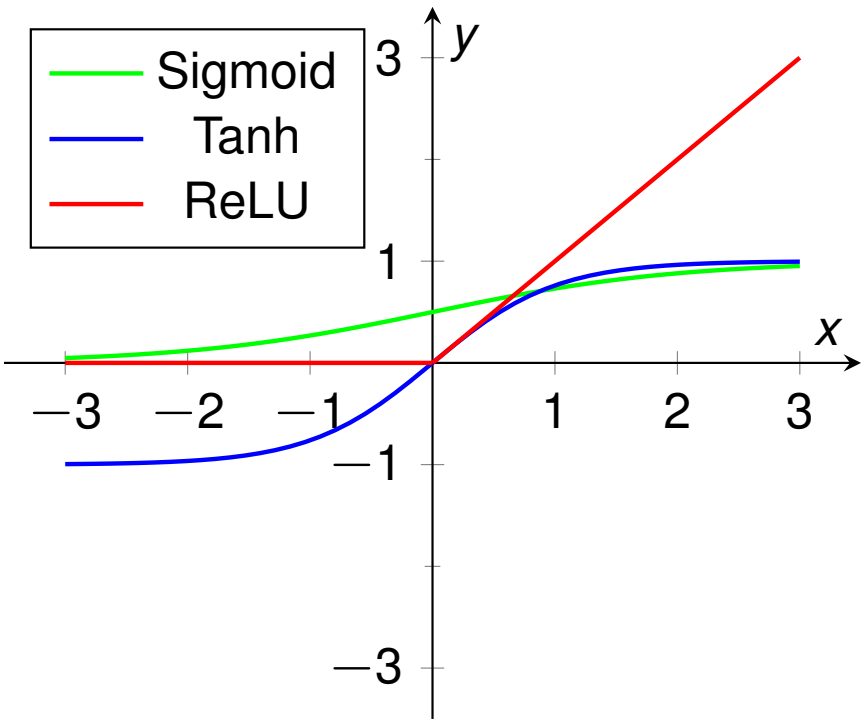

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+ Good generalization due to piece-wise linearity
+ Speed up during learning ($\approx$ 6x (Krizhevsky '12))
+ No vanishing gradient problem
− Not zero-centered

More on ReLUs

- ReLUs were a **big step forward**!
- ReLUs enable training of **deep** supervised neural networks **without unsupervised pre-training**
- First derivative is 1 if the unit is active, second derivative is 0 almost anywhere
  - ➔ No second-order effects

Dying ReLUs

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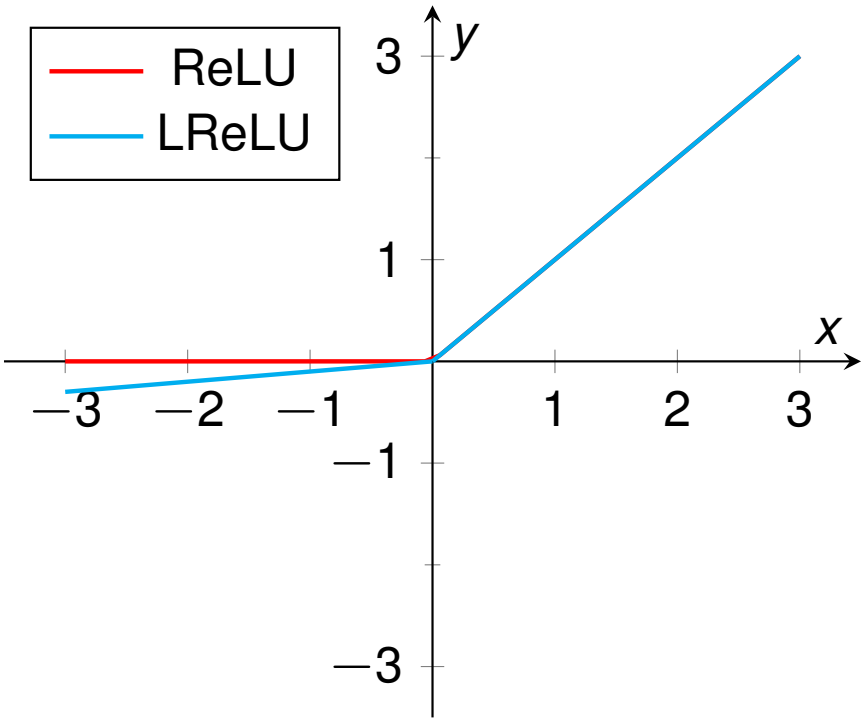
- Often related to a (too) high learning rate

Activation Function



Leaky ReLU / Parametric ReLU

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+ Fixes dying ReLU problem
- Leaky ReLU: $\alpha = 0.01$ [5]

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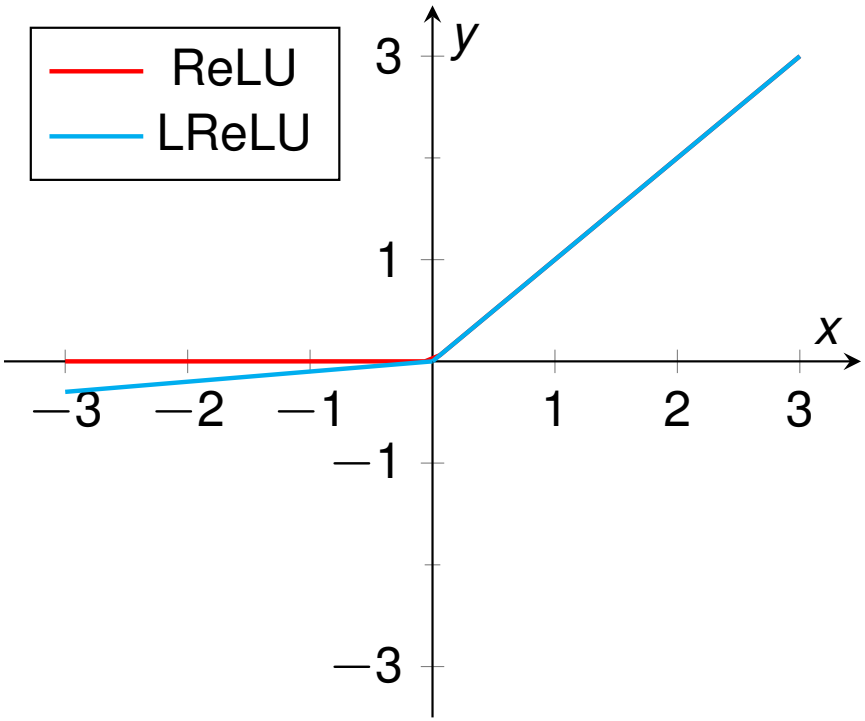

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- Leaky ReLU: $\alpha = 0.01$ [5]
- Parametric ReLU (PReLU): learn α [2]

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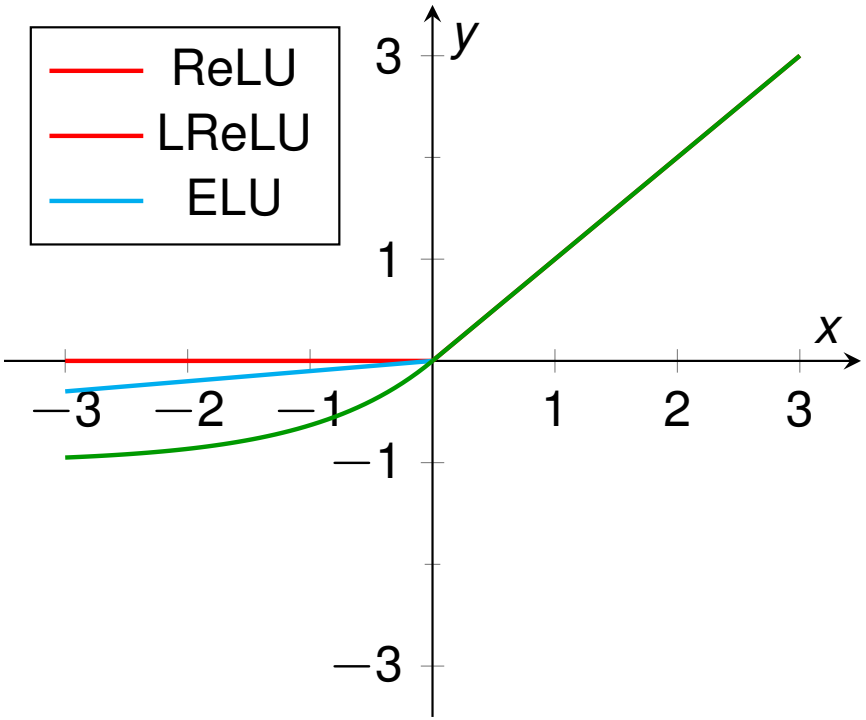

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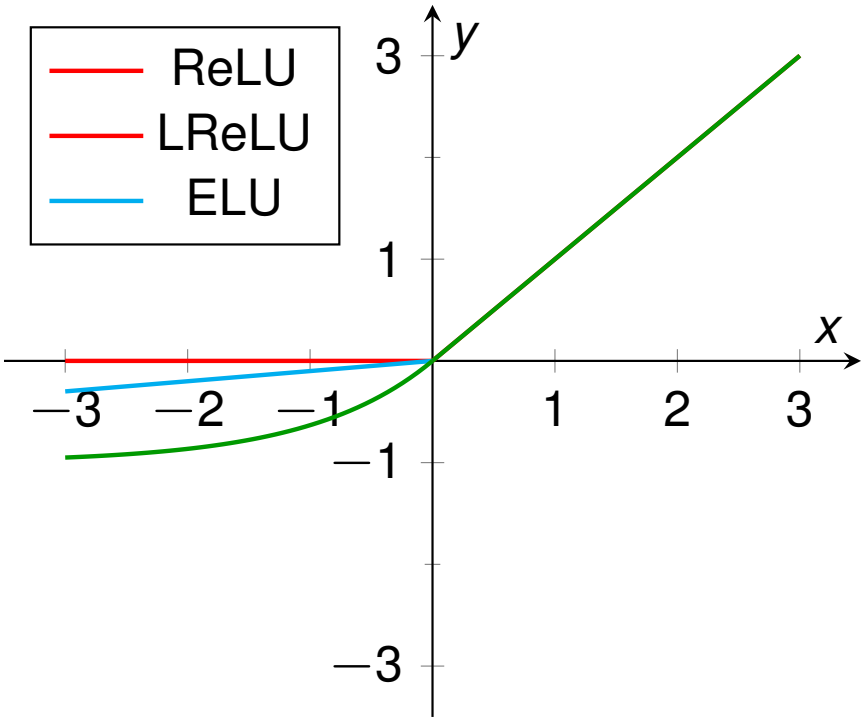

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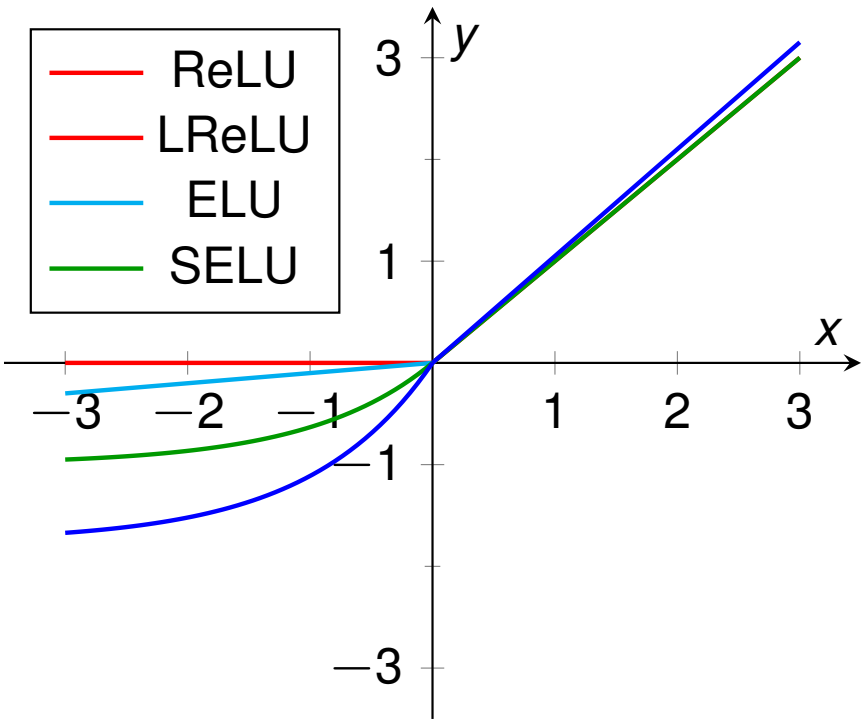
+ Vanishing gradient is not back
+ Can keep zero mean activations
- Gradient is in general not continuous!

Scaled ELU (SELU)



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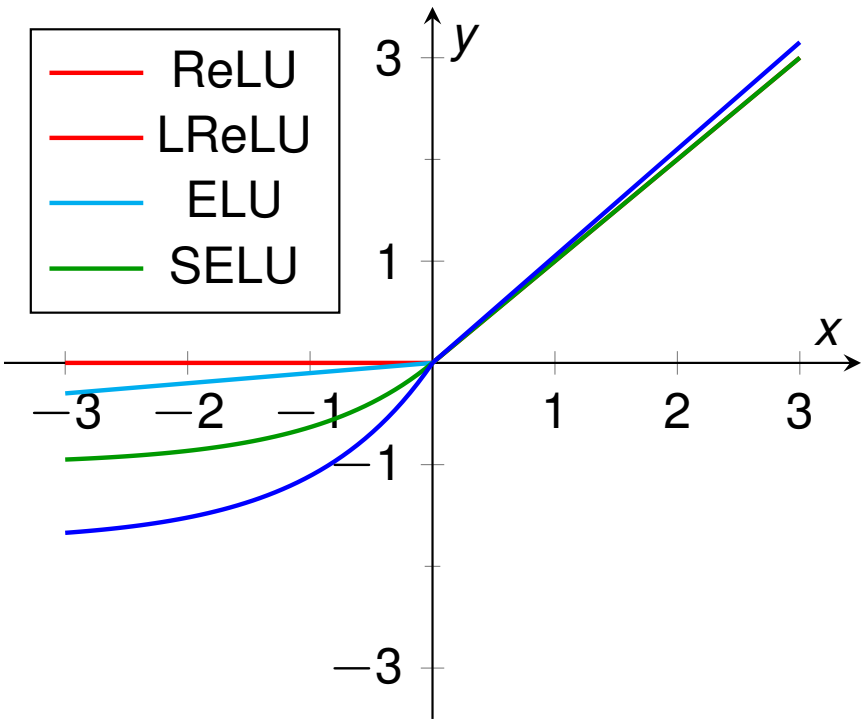

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- Brand new variant [3]
- Idea: Self-normalizing - $\mu = 0, \sigma = 1 \mapsto \lambda_{01}, \alpha_{01}$
- Alternative to Batch Normalization? (see next lecture)

Other Activation Functions

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This is getting ridiculous - what should we use?

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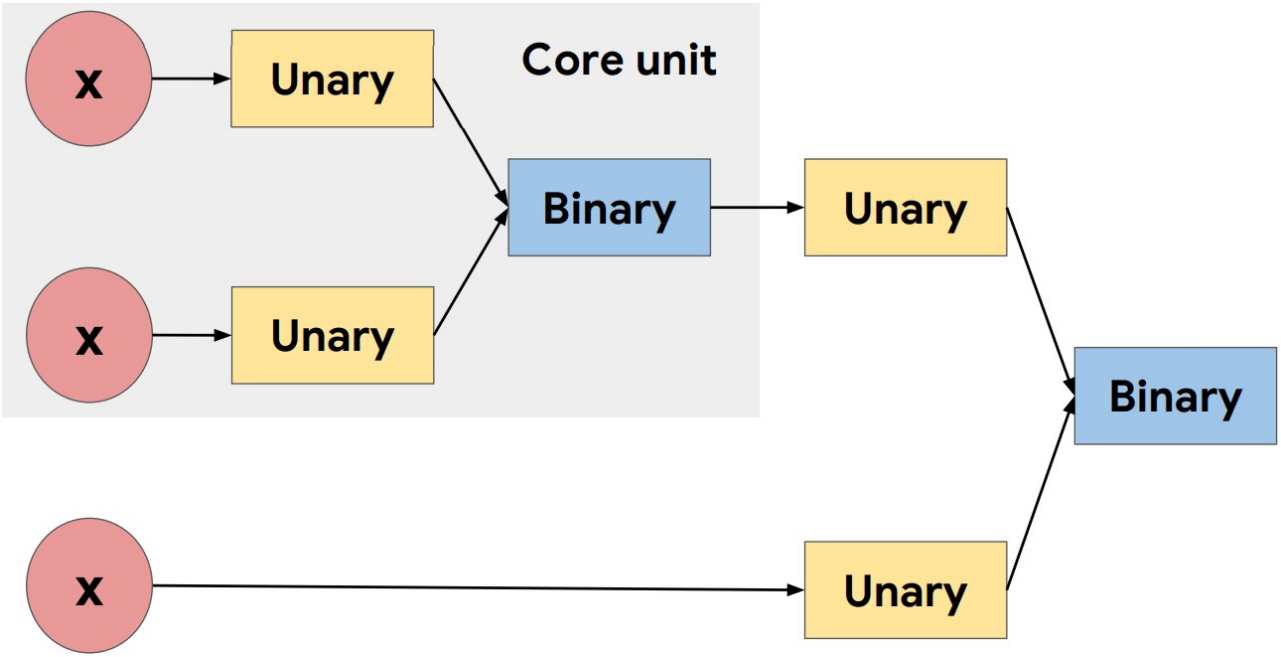
Strategy

1. Define a search-space
2. Perform the search using a RNN with reinforcement learning
3. Use the best result

Searching for Activation Functions [6]



Search space for activation functions

Source: https://arxiv.org/pdf/1710.05941.pdf

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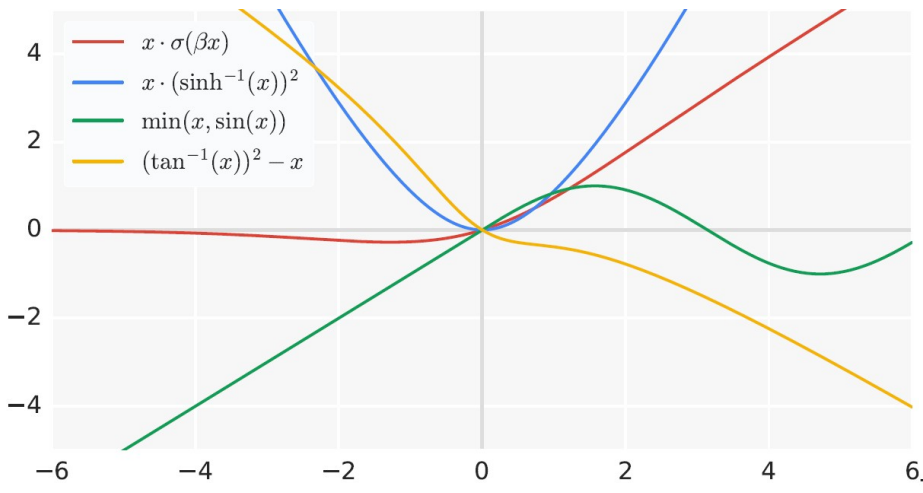

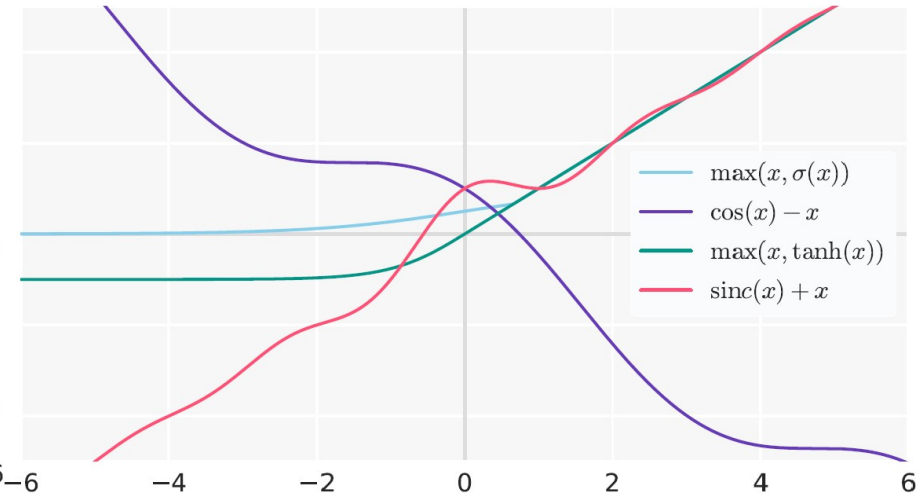

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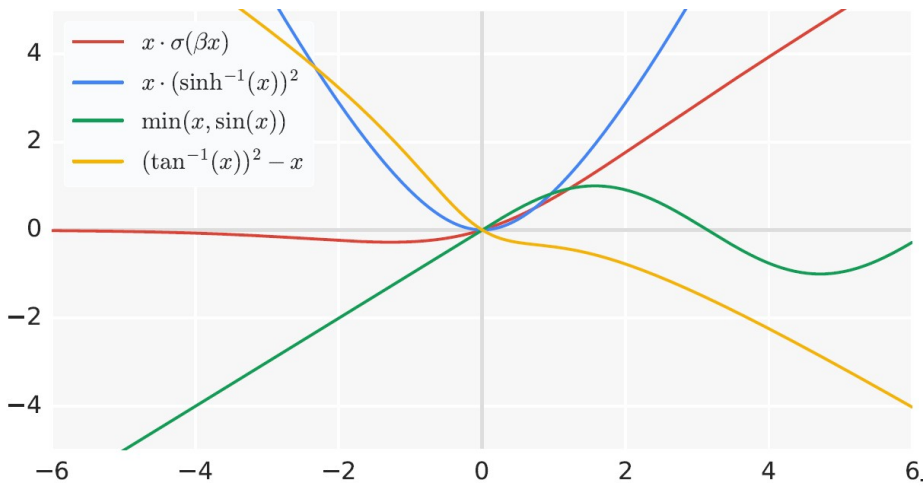

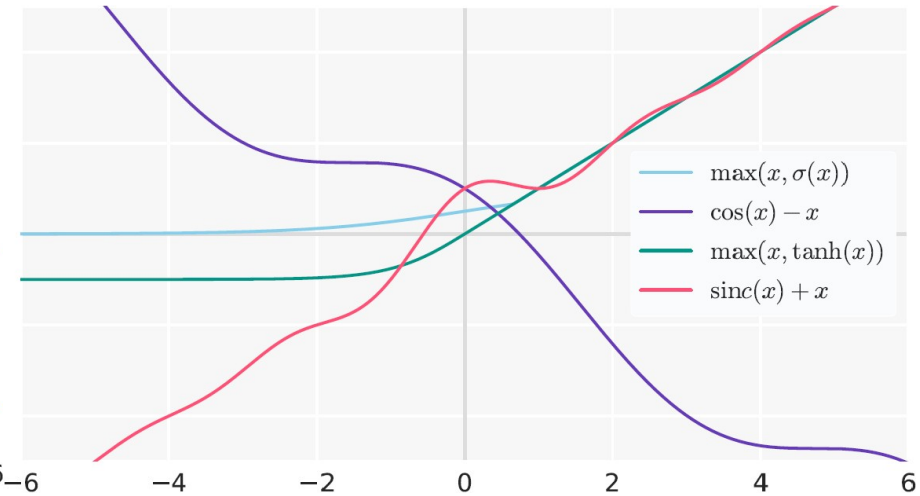

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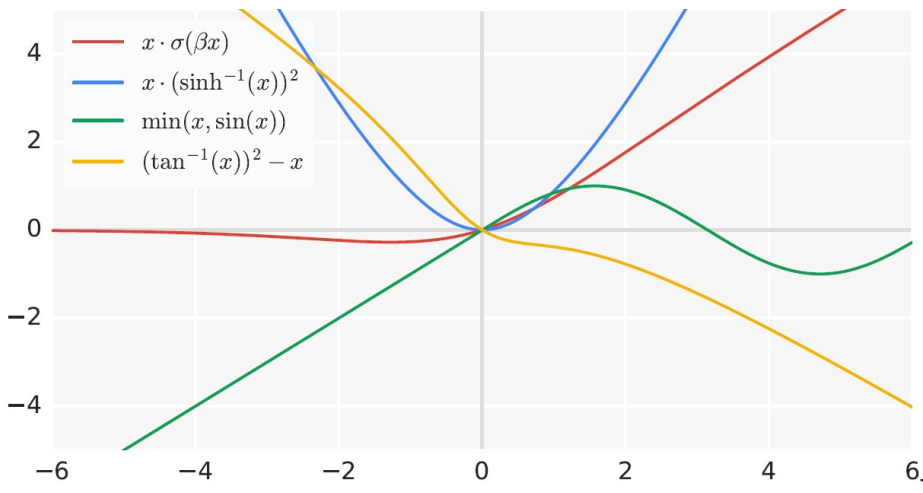

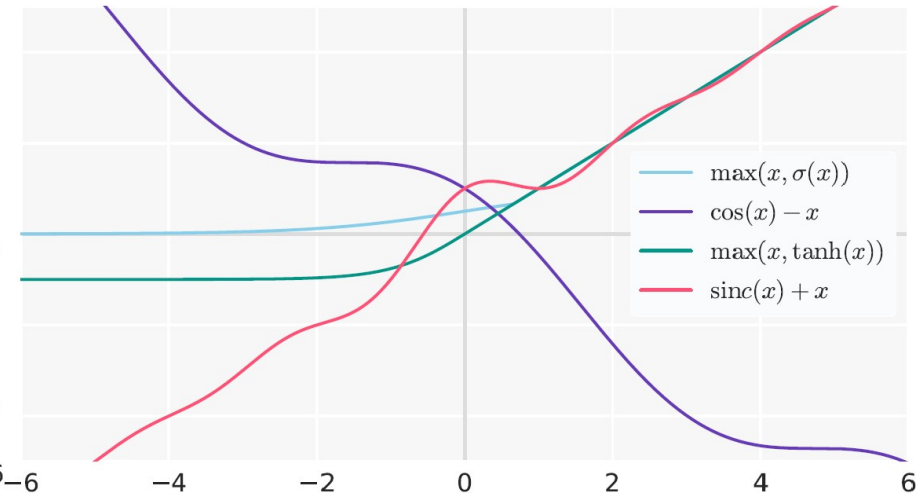

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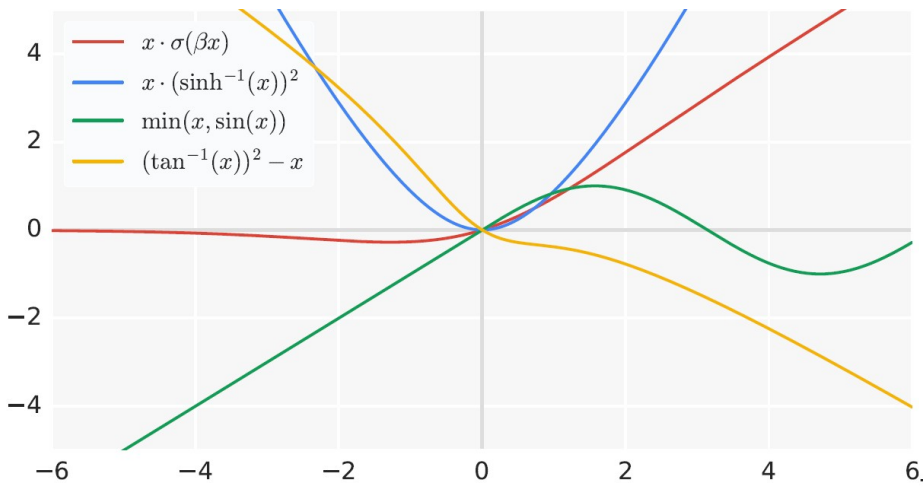

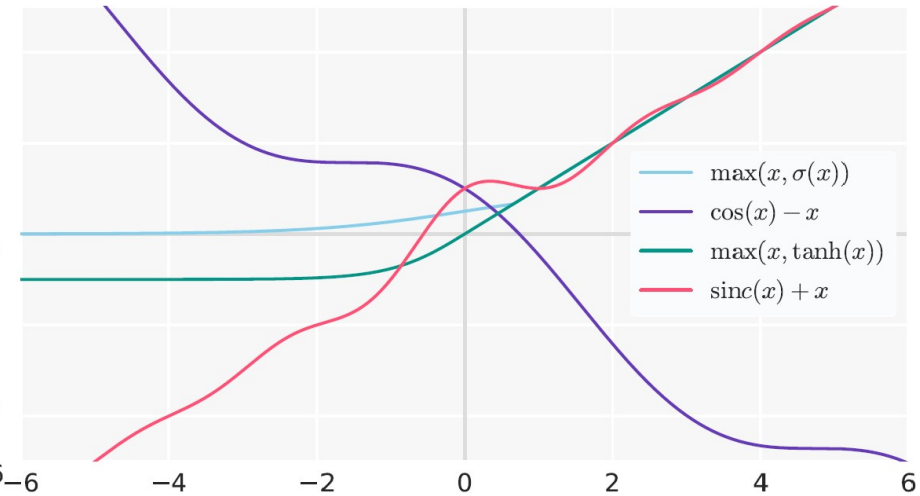

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- They now call $x \cdot \sigma(\beta x)$ the "Swish" function
- Has been proposed before as "Sigmoid-weighted Linear Unit" [7]

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Let's look into their results in detail

Disclaimer

Never show tables in you slides. Try hard to find a better representation.

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Model	Top-1 Acc. (%)		
LReLU	79.5	79.5	79.6
PReLU	79.7	79.8	80.1
Softplus	80.1	80.2	80.4
ELU	75.8	79.9	80.0
SELU	79.0	79.2	79.2
GELU	79.6	79.6	79.9
ReLU	79.5	79.6	79.8
Swish-1	80.2	80.3	80.4
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Inception-Resnet-V2 architecture trained on ImageNet.

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- Are any of these differences significant?

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ELU	75.8	79.9	80.0
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Summary Activation Functions

- Go-to solution: **ReLU**

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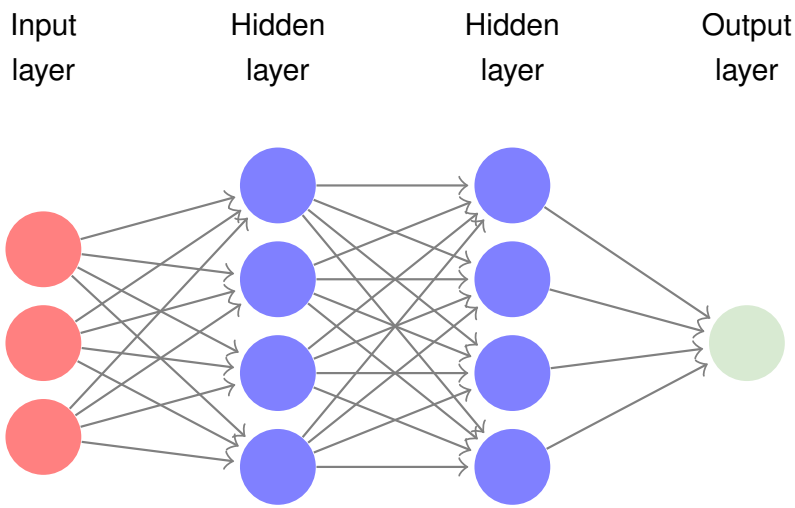
What we know about good activation functions:

- They have almost linear areas to prevent **vanishing gradients**.
- They have **saturating areas** to provide **non-linearity**.
- They should be **monotonic**.

Convolutional Neural Networks

Motivation

- So far: Fully connected layers - each input is connected to each node
- Very powerful: Can represent any kind of (linear) relationship between inputs



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- Large part of machine learning: images/videos/sounds
- Assume we have:
  - An image with size 512×512 pixels
  - One hidden layer with 8 neurons
    - ➔ $(512^2 + 1) \cdot 8 > 2$ million trainable weights!

Motivation (cont.)

So the size is a problem. Is there something else?

Source: https://news.nationalgeographic.com

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  - Scale dependent
  - Intensity variations
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- Example: Classify between cat and dog
- Pixels are **bad** features!
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  - ...
- Pixels are a bad representation from a machine learning point of view

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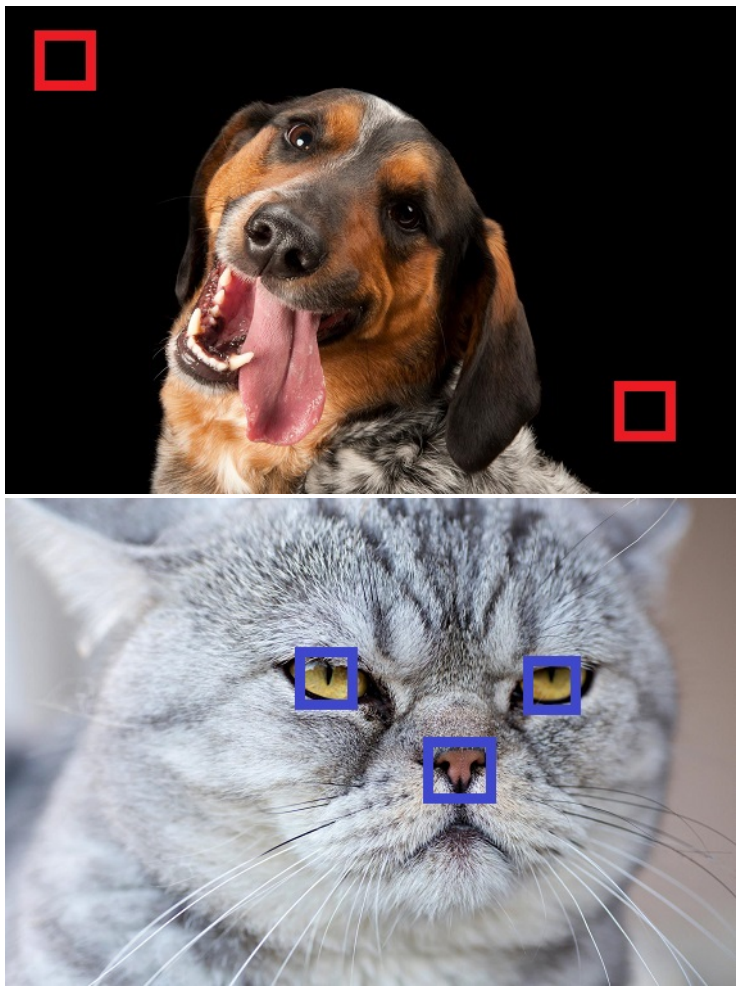
Motivation (cont.)

Can we find a better representation?



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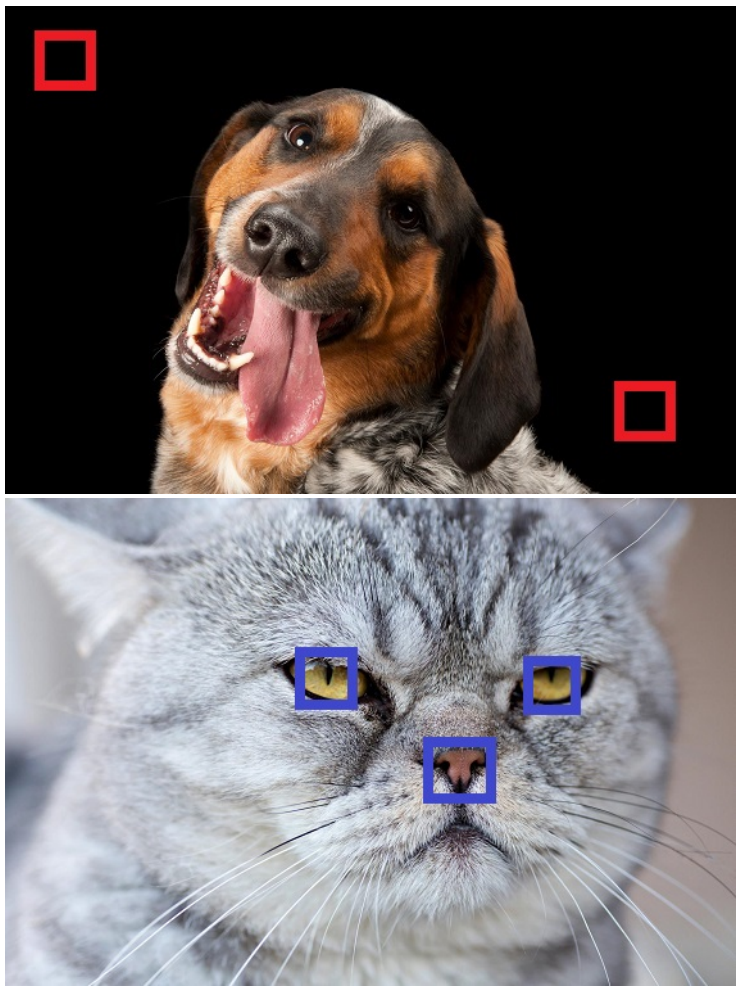

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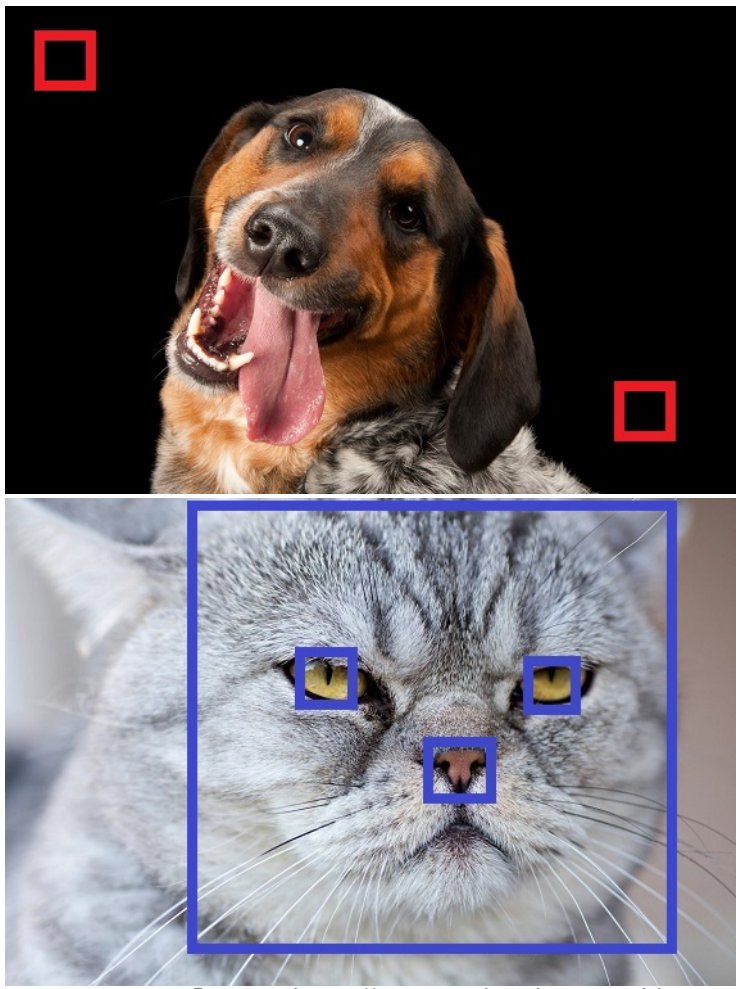

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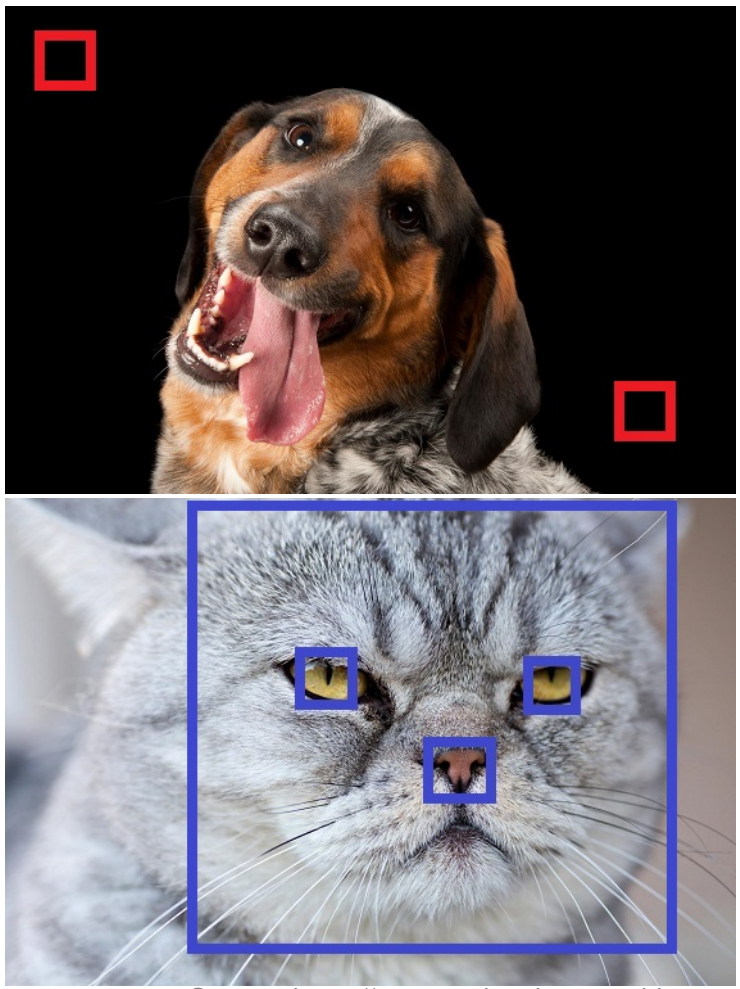

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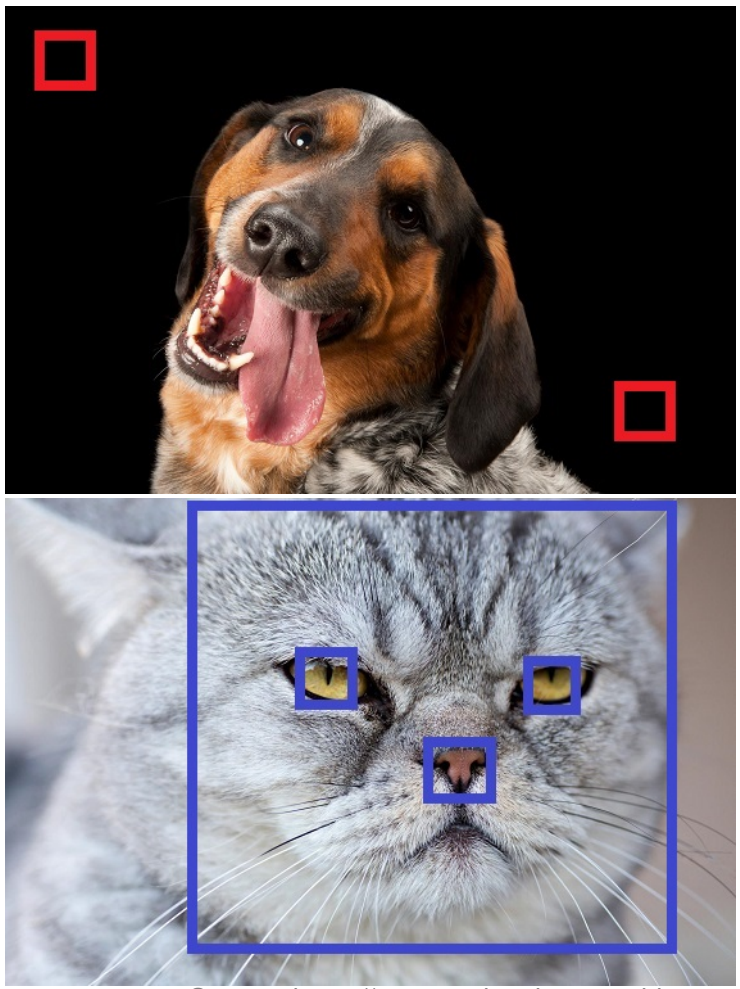

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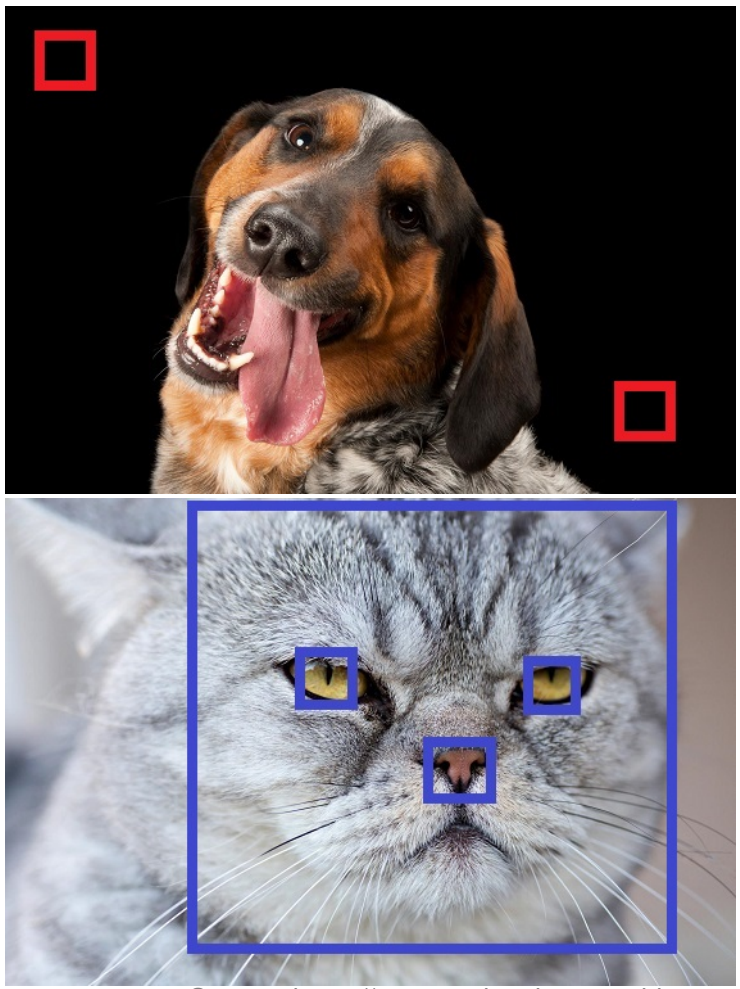
$\rightarrow$ Learn better representation, then classify!



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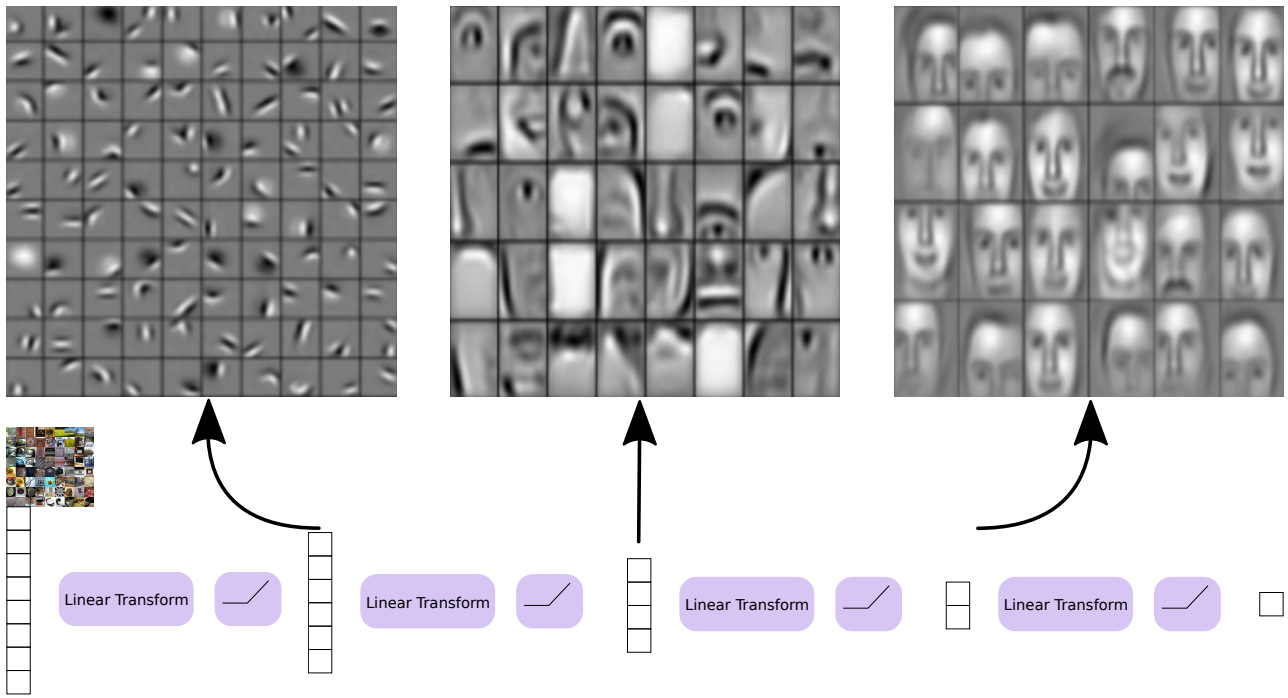
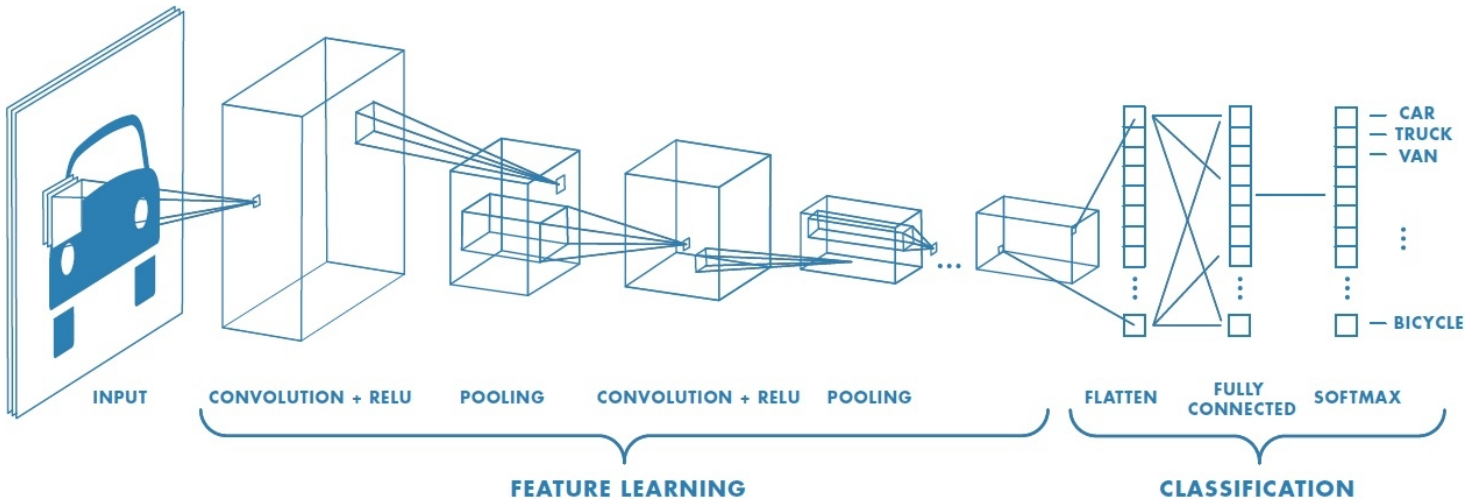
Convolutional Neural Networks



- Local connectivity ➔ filters
- Use same filters over the whole image ➔ translational equivariance
- Hierarchy of filters working on different scales
- + learning = **Convolutional Neural Networks**

Convolutional Neural Networks - Architecture



Four essential building blocks:

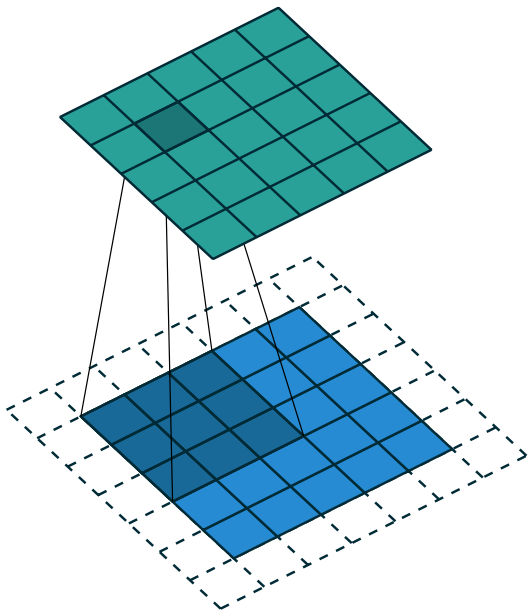
- Convolutional layer: Feature extraction
- Activation function: Nonlinearity
- Pooling layer: Compress information, save parameters
- Last layer: Fully-connected for classification

Source: https://de.mathworks.com/discovery/convolutional-neural-network.html

Convolutional Layers

Convolutional Layer - Local Connectivity

- Exploit spacial structure by only connecting pixels in a neighborhood
- Can be expressed as fully connected layer:
  Except for local connections, each entry in **W** is 0



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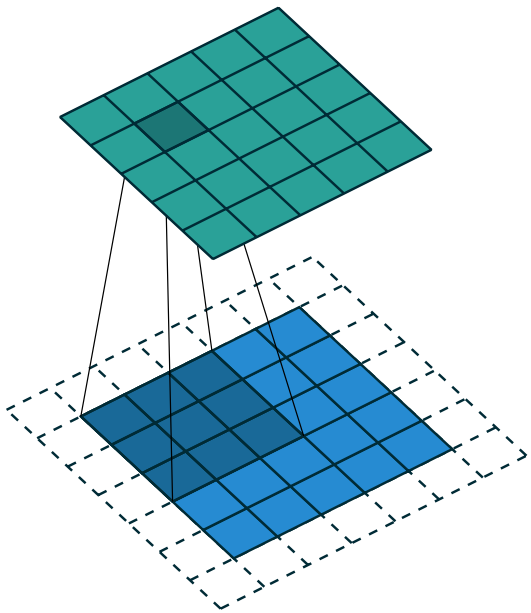

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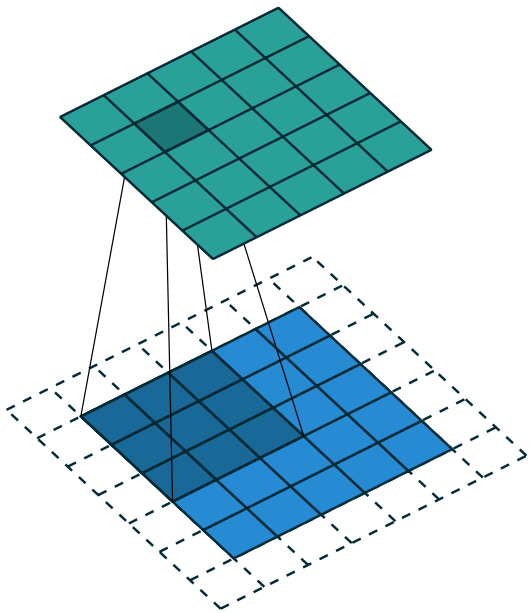

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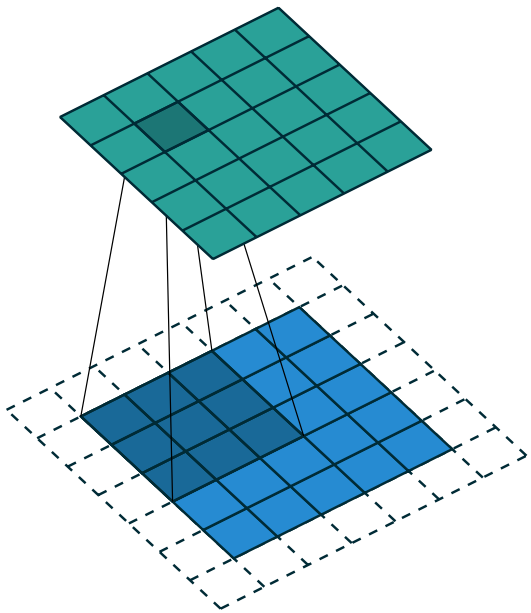
➔ **Convolution with trainable filters**



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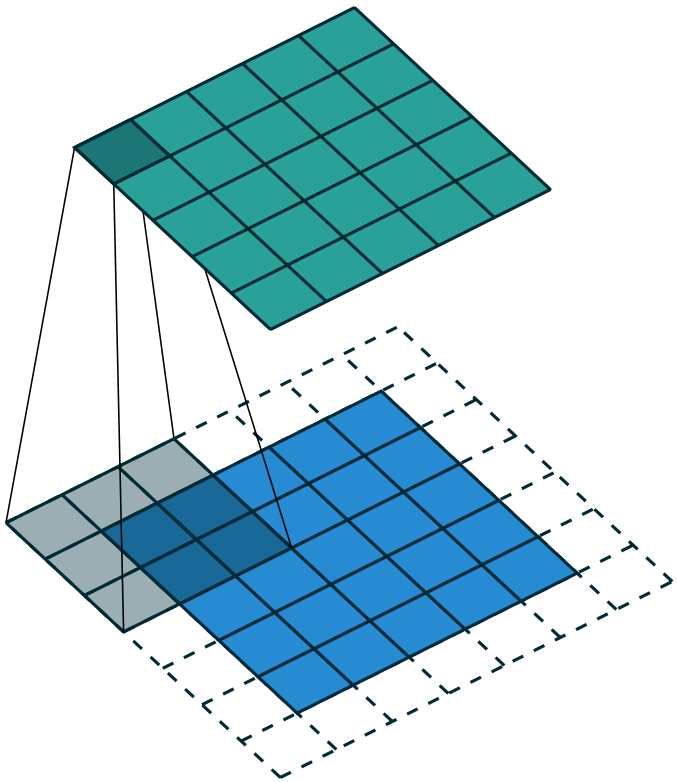

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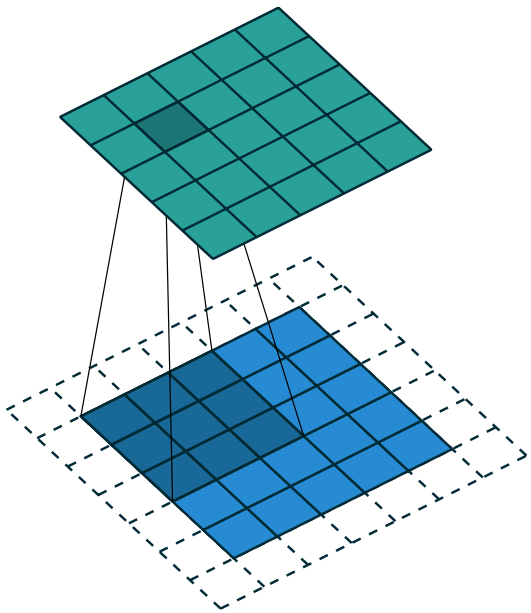
- Cross-correlation is convolution with a flipped kernel g – and vice versa!
- Implementation: Cross-correlation is frequently used in the forward pass - the weights are initialized randomly anyway

Padding

- Convolution reduces image size by $2 \cdot \lfloor \frac{n}{2} \rfloor$ pixels (n: kernel size).
- Necessary to pay attention to the borders:
- 'Same' padding (usually zero padding):
  - ➔ Input and output have the same size
- 'Valid'/no padding:
  - ➔ The output is smaller than the input



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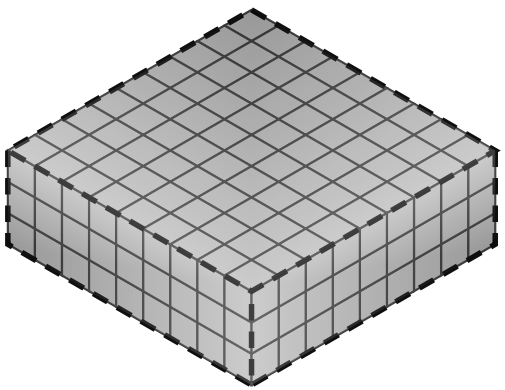
Forward Pass: Multi-channel convolution

- Input of size $X \times Y \times S$, where S is the number of input channels



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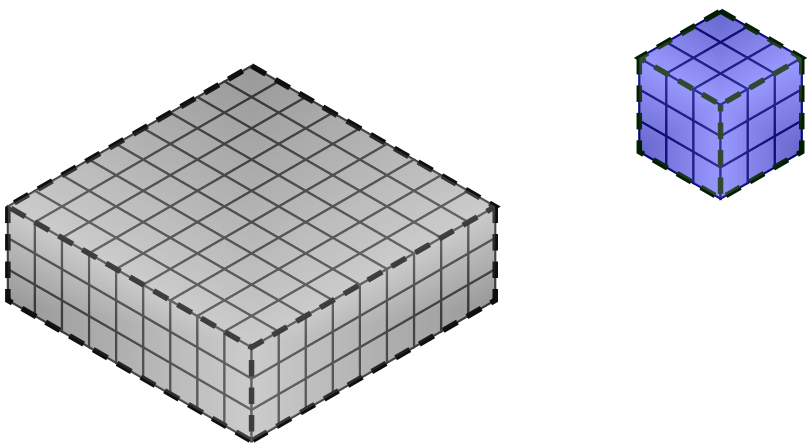

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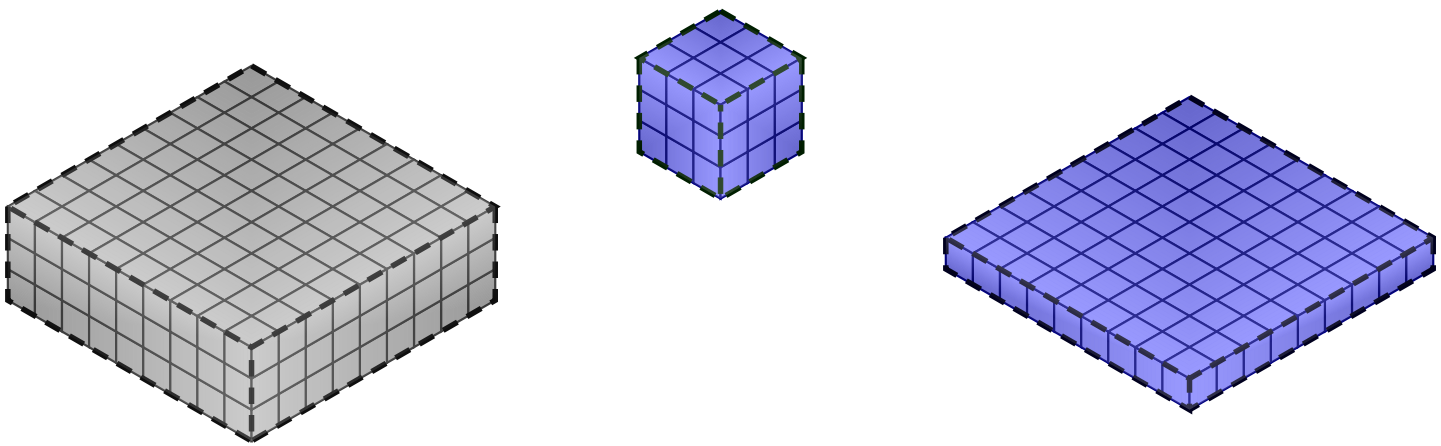

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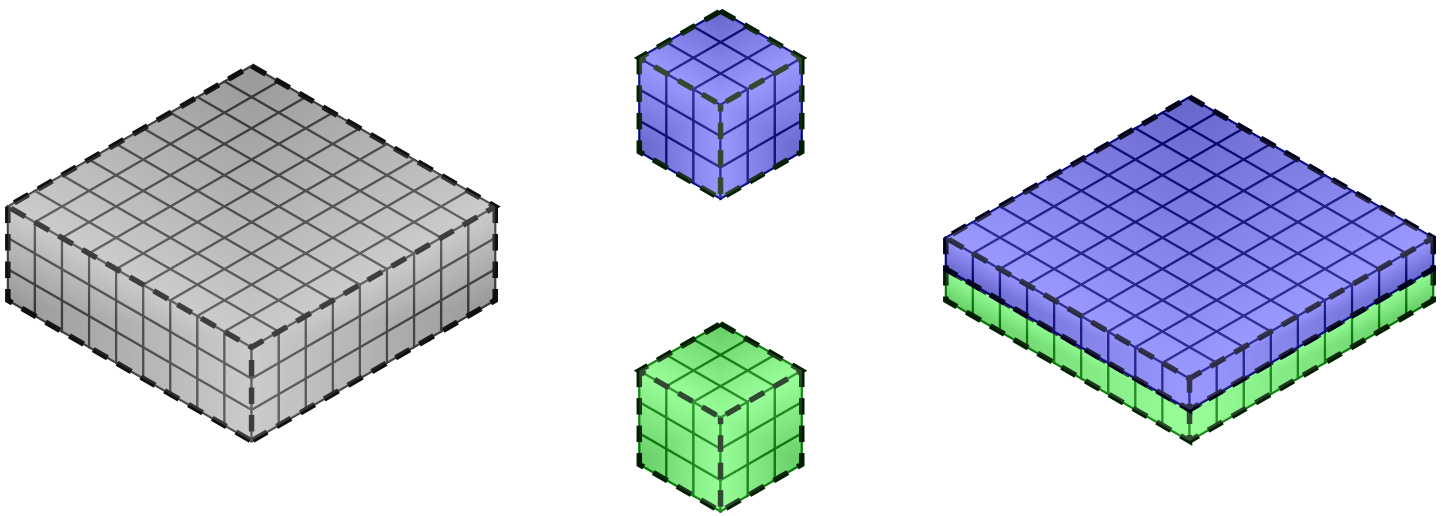

Backward pass: Multi-channel convolution

- Convolution can be expressed as matrix multiplication with matrix **W**: using a **Toeplitz matrix**
- We can use the **same formulas** as for the fully connected layer!

$$\mathbf{E}_{l-1} = \mathbf{W}^T \mathbf{E}_l$$
$$\nabla \mathbf{W} = \mathbf{E}_l \mathbf{X}^T$$

where

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- These can be expressed as convolutions $\mapsto$ exercise

Convolutional Layer - What have we gained?

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- Layer with 8 filters (nodes) with 5×5 neighborhood
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  - ➔ $5^2 \cdot 8 = \underline{200}$ weights
- Convolution: **Independent** of image size!
- Much more training data for one weight!

Strided Convolutions

- Instead of multiplying the filter at each pixel position, we can **skip** some **positions**
- **Stride** *s* describes the offset

Source: https://en.wikipedia.org, Monty Python's Flying Circus episode, Dinsdale

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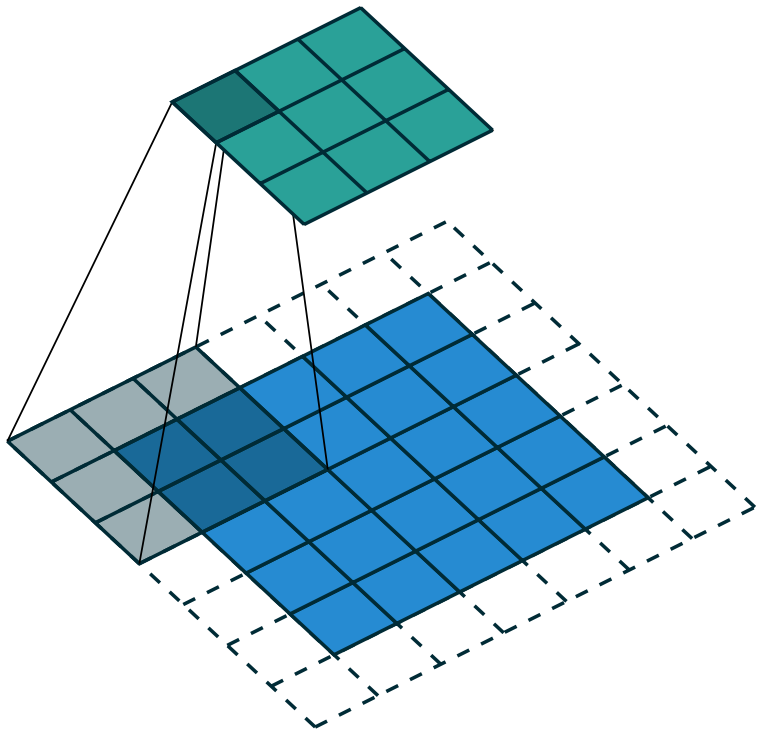
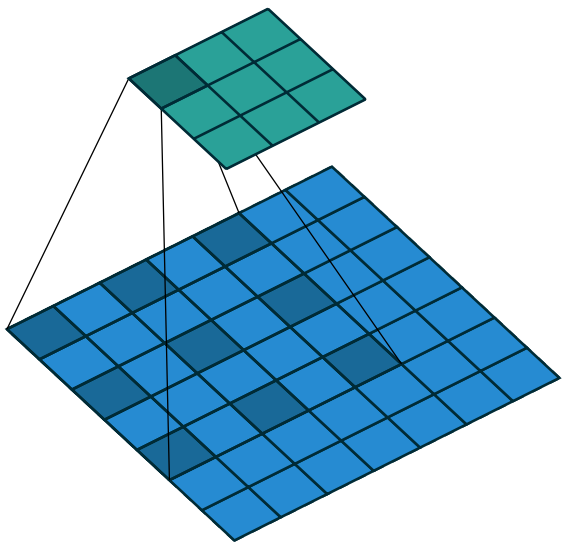


Strided Convolution with stride $s = 2$

Source: https://github.com/vdumoulin/conv_arithmetic

Dilated/Atrous Convolutions

- Additional variant of convolution in neural networks
- Dilate convolution kernel: Skip certain pixels
- Goal: Wider receptive field with less parameters/weights



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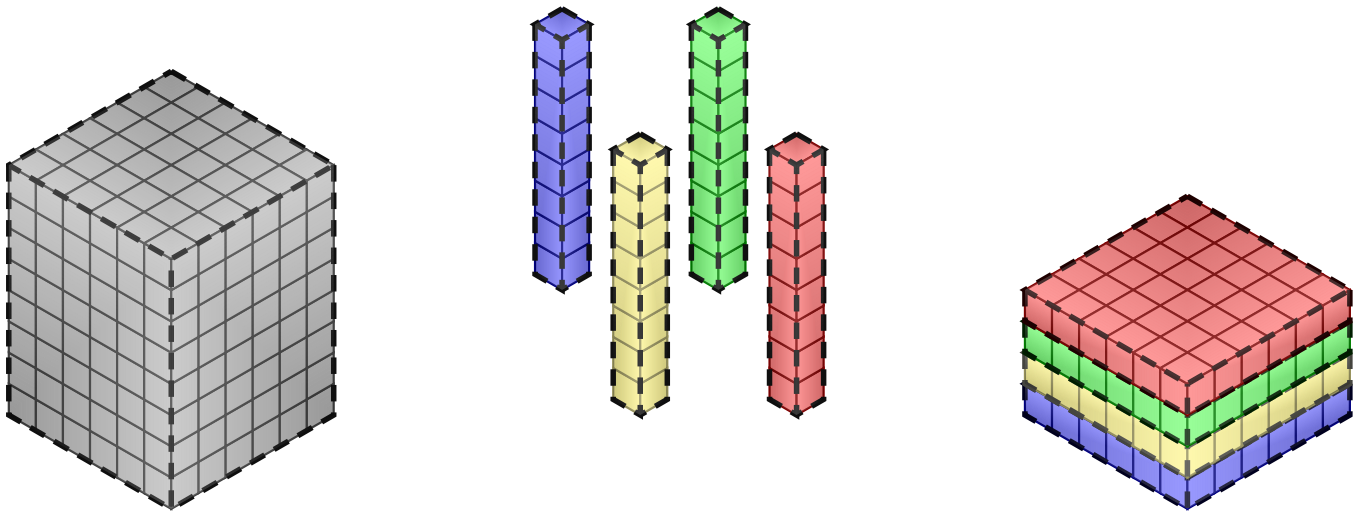

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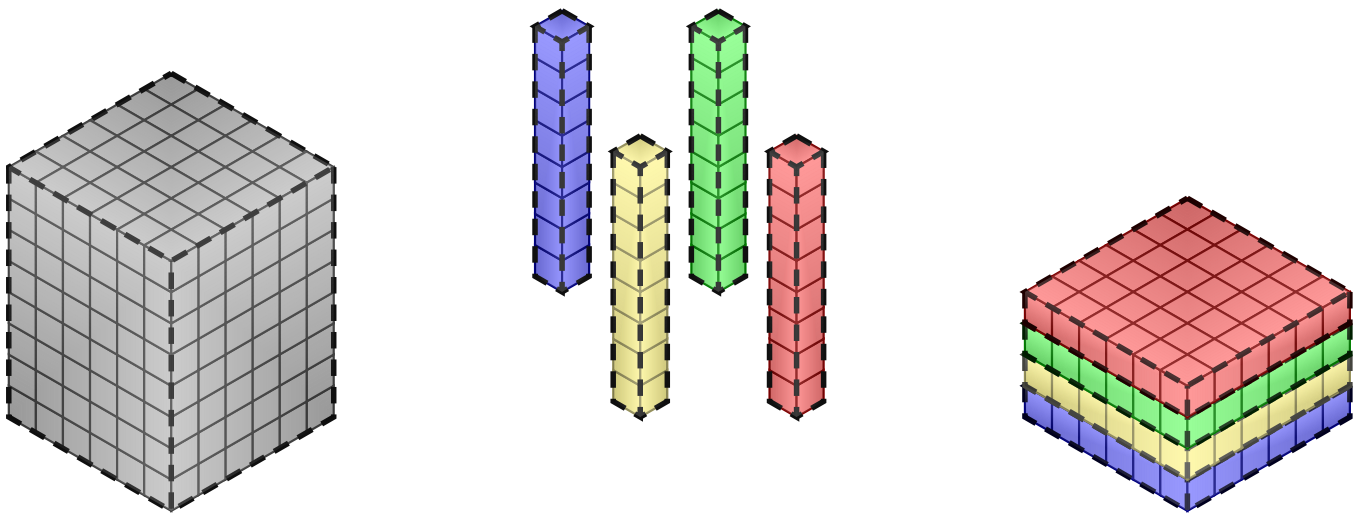
- We can decrease the neighborhood to 1×1
- And **just** use the fully connected property in the depth dimension



➔ Dimensionality reduction/expansion from S channels to H channels

➔ **If we flatten the input, 1×1 convolutions are fully connected layer!**

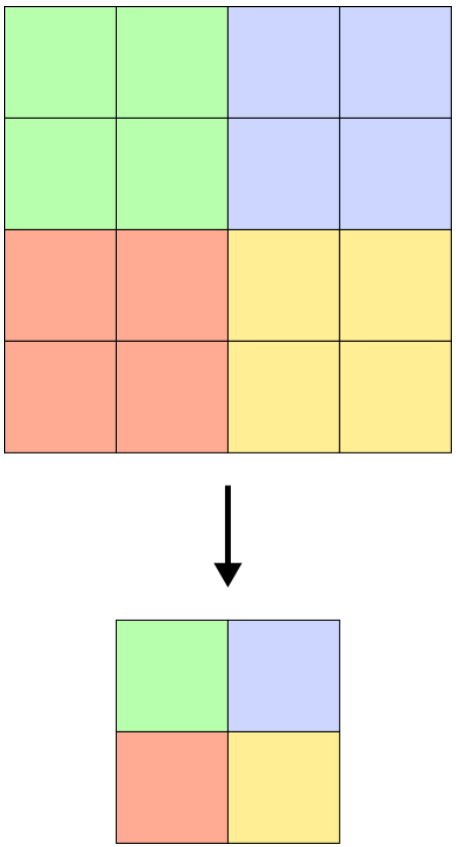
1×1 Convolution Concept (cont.)

- First described in “Network in Network” by Lin et al. [4]
- 1×1 convolutions simply calculate **inner products** at each position
- Simple and efficient method to **decrease** the **size** of a network
- Learns dimensionality reduction, e.g., can reduce redundancy in your feature maps

Pooling Layers

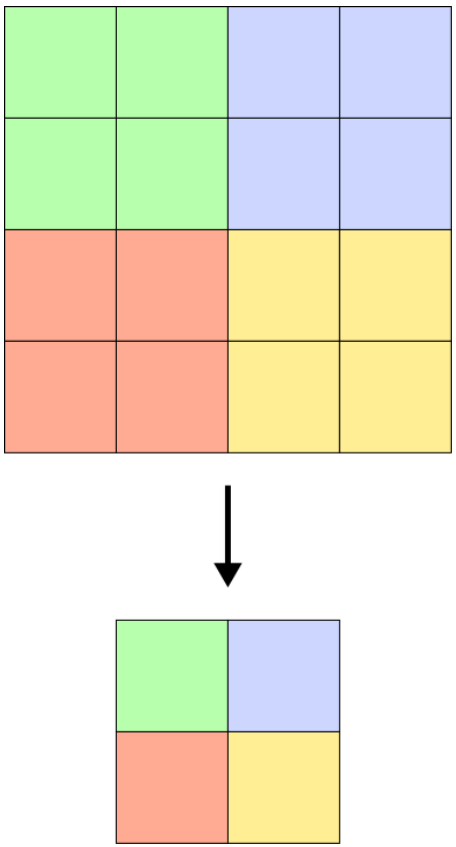
Idea behind Pooling Layers

- Fuses information of input across spatial locations
- Decreases number of parameters
- Reduces computational costs and overfitting



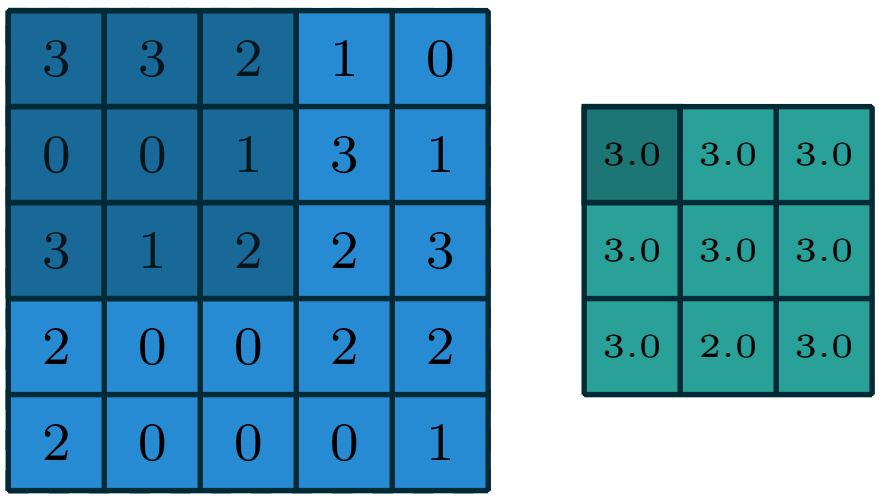
Idea behind Pooling Layers

- Fuses information of input across spatial locations
- Decreases number of parameters
- Reduces computational costs and overfitting
- Assumptions:
  - Features are hierarchically structured
  - Creates “summaries” of regions
  - Provides translational invariance
  - Exact location of a feature is not important



Max Pooling – Forward Pass

- Propagate maximum value in a neighborhood to next layer
- Typical choices: 2×2 or 3×3 neighborhood
- "Stride" of pooling usually equals the neighborhood size
- Maximum propagation adds additional non-linearity



Max pooling concept. Note that usually a stride > 1 is used for pooling.

Max Pooling – Backward Pass

Max Pooling – Backward Pass

- ➔ Only one value contributes to error
- ➔ Error is propagated only along the path of the maximum value

Average Pooling

- Propagate average of the neighborhood
- Does not consistently perform better than max pooling
- Backward pass: Error is shared to equal parts

Additional Pooling Strategies

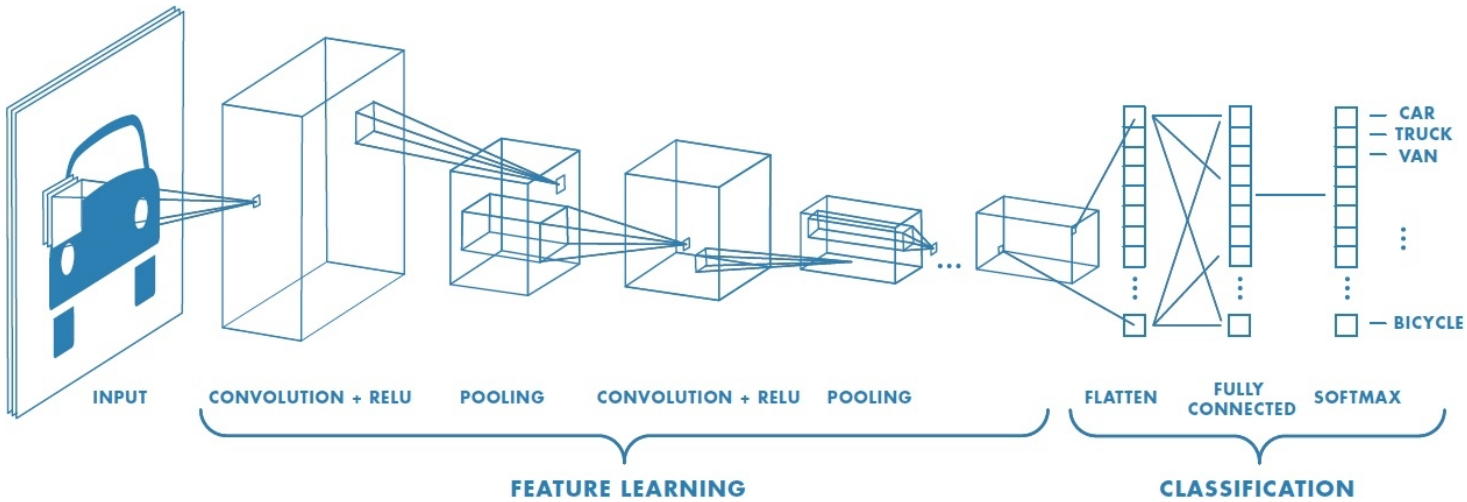
- Fractional max pooling
- L_p pooling
- Stochastic pooling
- Spacial pyramid pooling
- Generalized pooling
- ...

Alternative: Strided Convolution

- Historically, max pooling was the most frequently used pooling strategies due to additional non-linearity
- More recently, convolution with stride $s > 1$ has become more common

➔ Allows for trainable downsampling strategy

Recap: Convolutional Neural Networks - Architecture

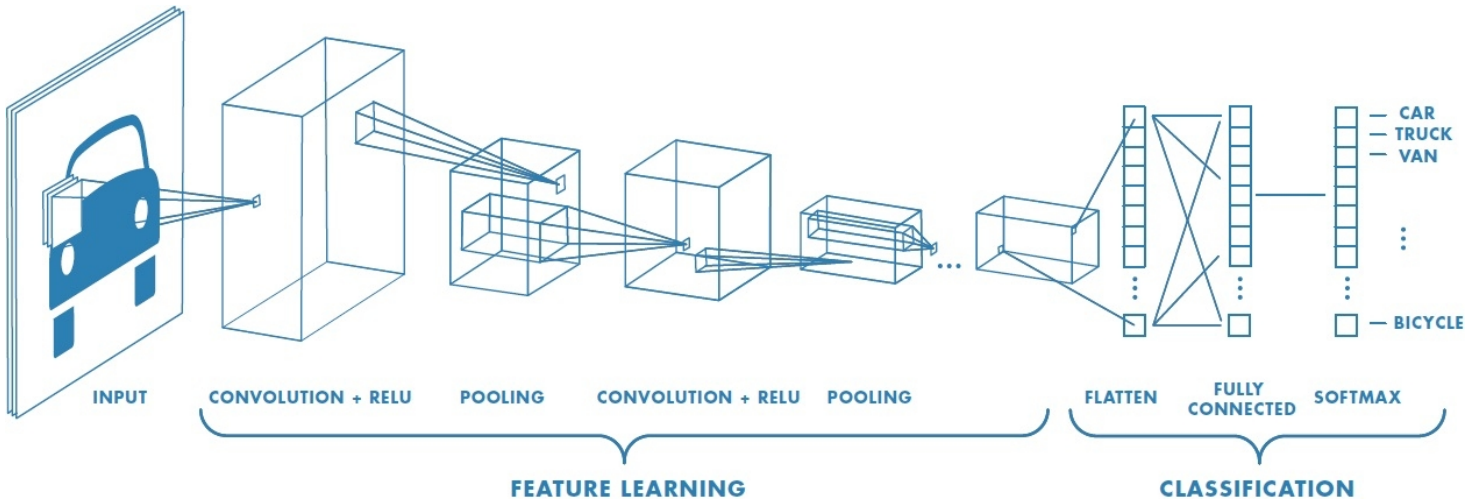


Four essential building blocks:

- Convolutional layer: Feature extraction
- Activation function: Nonlinearity
- Pooling layer: Compress information, save parameters
- Last layer: Fully-connected for classification

Source: https://de.mathworks.com/discovery/convolutional-neural-network.html

Recap: Convolutional Neural Networks - Architecture

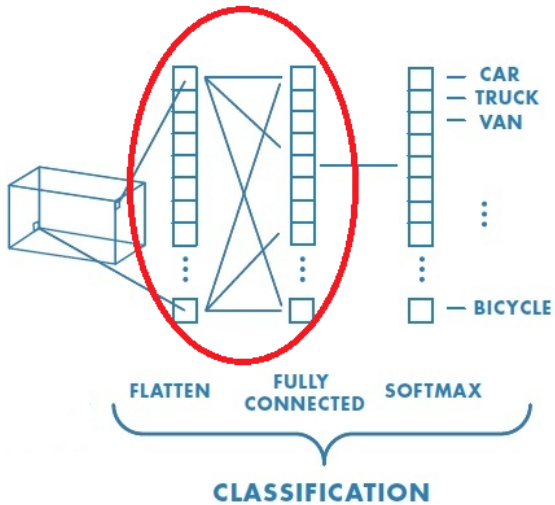


~~Four~~Three essential building blocks:

- Convolutional layer: Feature extraction
- Activation function: Nonlinearity
- Pooling layer: Compress information, save parameters
- ~~Last layer: Fully-connected for classification~~**We can replace this layer!**

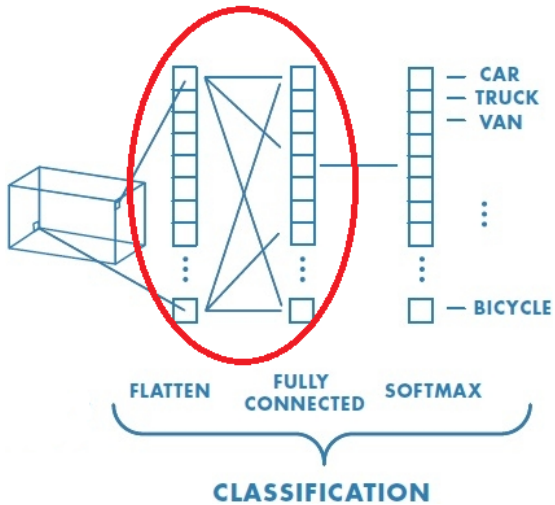
Source: https://de.mathworks.com/discovery/convolutional-neural-network.html

Replacing the Fully Connected Layer



- Conv and pooling layers generate better representation ➔ better features
- Fully connected layers for classification

Replacing the Fully Connected Layer



- Conv and pooling layers generate better representation ➔ better features
- Fully connected layers for classification
- **Alternatively and equivalently**: Use flatten & 1×1 convolution or $N \times N$ convolution
- Enables **arbitrary input sizes** in combination with global average pooling!

Inception model

- Szegedy et al. (2014): Going Deeper With Convolutions [8]
  - Very influencial publication: > 5000 citations (Nov. ’17)
  - Won ImageNet Large-Scale Visual Recognition Challenge 2014
  - GoogLeNet as one incarnation
  - Inspired by Network in Network [4]

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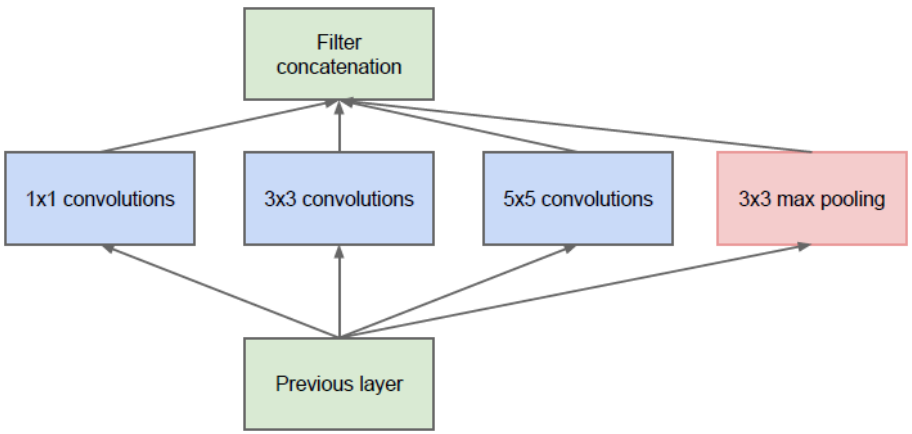
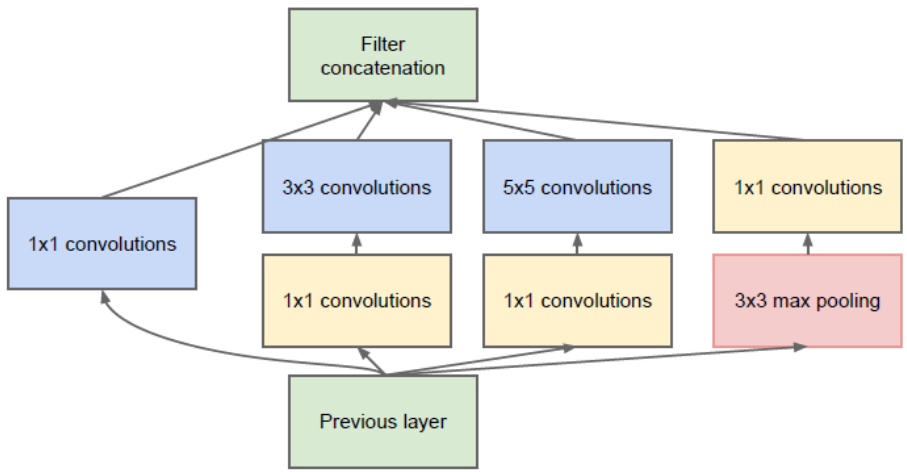
Source: http://knowyourmeme.com/photos/531557-we-need-to-go-deeper

Inception model

- **Idea**: Why use only one type of filter in one layer?
  Why not combine different neighborhoods/pooling/etc.?
- Construction of ‘inception modules’ that are stacked to form a large network.



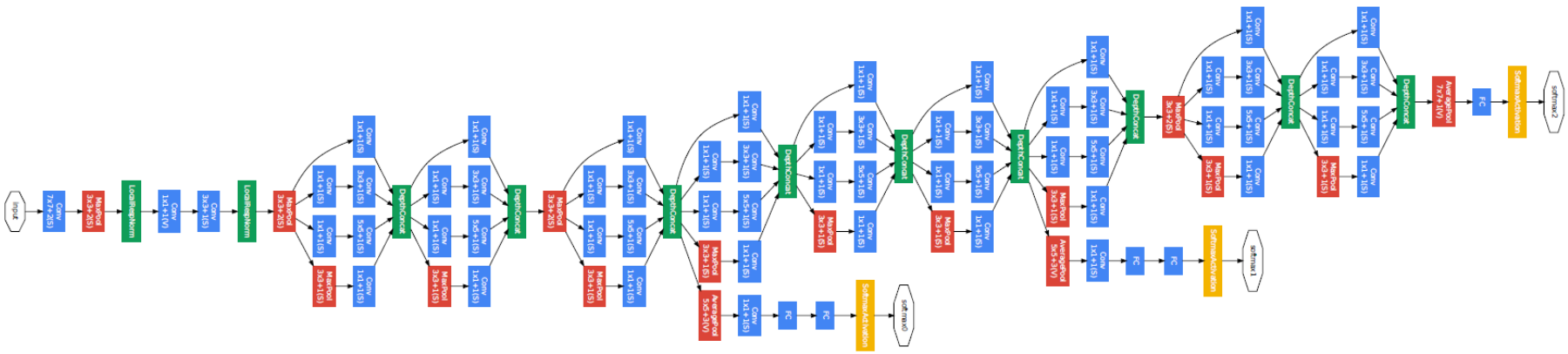
Naïve inception module



Inception module with dimensionality reduction

Source: [8]

Inception model – GoogLeNet



GoogLeNet architecture with inception modules.

Source: [8]

NEXT TIME

ON DEEP LEARNING

Coming up

- How to prevent networks from just memorizing the training data?
- Is there a way to force features to be independent?
- How can we make sure our network also recognizes cats in different poses?
- Can we fix the covariate shift problem?

Comprehensive Questions

- Name five activation functions.
- Discuss those 5 activation functions.
- What is the zero-centering problem?
- Why does ReLU as activation function perform much better than sigmoid/tanh in a large number of tasks?
- Why are convolutional networks well suited for image and audio processing?
- Write down a mathematical description of strided convolution.
- What is the connection between 1×1 convolutions and fully connected layers?
- How would you implement a classifier which operates on image patches?
- What is a pooling layer?
- Why do we use pooling layers?
- On what data would CNNs probably perform bad?

Further Reading

- Link - [3] for a paper about Self Normalizing Networks
- Link - [4] for a creative Network in Network paper
- Link - [6] for details on learned activation functions
- Link - [8] if everything so far was not deep enough for you

Questions?

References

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[2] Kaiming He, Xiangyu Zhang, Shaoqing Ren, et al. “Delving Deep into Rectifiers: Surpassing Human-Level Performance on ImageNet Classification”. In: CoRR abs/1502.01852 (2015). arXiv: 1502.01852.

[3] Günter Klambauer, Thomas Unterthiner, Andreas Mayr, et al. “Self-Normalizing Neural Networks”. In: Advances in Neural Information Processing Systems (NIPS). Vol. abs/1706.02515. 2017. arXiv: 1706.02515.

[4] Min Lin, Qiang Chen, and Shuicheng Yan. “Network In Network”. In: CoRR abs/1312.4400 (2013). arXiv: 1312.4400.

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[5] Andrew L. Maas, Awni Y. Hannun, and Andrew Y. Ng. “Rectifier Nonlinearities Improve Neural Network Acoustic Models”. In: Proc. ICML. Vol. 30. 1. 2013.

[6] Prajit Ramachandran, Barret Zoph, and Quoc V. Le. “Searching for Activation Functions”. In: CoRR abs/1710.05941 (2017). arXiv: 1710.05941.

[7] Stefan Elfwing, Eiji Uchibe, and Kenji Doya. “Sigmoid-weighted linear units for neural network function approximation in reinforcement learning”. In: arXiv preprint arXiv:1702.03118 (2017).

[8] Christian Szegedy, Wei Liu, Yangqing Jia, et al. “Going Deeper with Convolutions”. In: CoRR abs/1409.4842 (2014). arXiv: 1409.4842.