PHYS 6352
March 30, 2014

PROJECT 3

In HW set 9 you solved an ordinary differential equation (ODE) using finite differences. This project extends a solution to a similar ODE and adds an investigation of error measures.

The ODE to be solved is

$$\frac{d^2 f}{dz^2} + f = \frac{1}{1+z} + \frac{2}{(1+z)^3}\,.$$

This is to be solved on the interval $0 \le z \le 5\pi/2$ with the endpoint conditions

$$f(0) = 2 \qquad\qquad f(5\pi/2) = 1 + \frac{1}{1+5\pi/2}\,.$$

The analytic solution to this problem is very simple, and will be needed for comparisons with the numerical solution.

You will solve this problem by using a finite difference scheme with N unknowns. In HW set 9, the value of N was 4; in this project it will be a parameter the user chooses, and can be much larger. (How large depends on the RAM and the settings on your computer.)

Part of this problem will involve a measure of the error, the difference between the numerical solution found and the true solution. A common family of measures is the set of "Ln" errors. For a set of M numerical values $num[k]$ with the analagous $true[k]$ exact values, the Ln error measure is

$$\mathrm{Ln} \equiv \left(\frac{\sum (num[k] - true[k])^n}{M} \right)^{1/n}\,,$$

where the sum is over all M values of k.

The most common error measure used is the L2 error:

$$\mathrm{L2} \equiv \sqrt{\frac{(\sum (num[k] - true[k])^2}{M}}\,.$$

The project:
You are to write two interacting programs to solve the ODE using N unknowns. One program will generate the matrix and column vector to define the linear algebra problem. The second, a version of `LUdecom` will solve that problem and write out useful results.

The first program, to be called `datagenerator.cpp` takes N as a user input and generates the output `A.dat` and `B.dat` to be used as input to the GSL LU solver. These output files will be analogous to the matrix data and right hand column data used in HW set 9 and elsewhere.

The second program is `LUdecomp.cpp`, a version of the program that we have used several times. For this project, your `LUdecomp.cpp` must have the following features:

- It must take N as a user input.
- It should output a file `numsoln.dat` with the value of z and the value of f, in two-column format, extending from $z = 0$ to $z = 5\pi/2$. (The number of values should be $N + 2$).
- It should output an analogous file `ansoln.dat` with the analytic solution in the same format over the same values of z.
- It should output the value of the L2 error for the chosen value of N.

In addition to these programs your project package should include a README file that addresses the following:

1. What is the L2 error for $N = 20$?
2. What is the smallest value of N for which the L2 error is less than 0.01?
3. Explain the trend (the dependence on N) of the L2 error as N varies from $N = 10$ to $N = 1000$ (or whatever maximum N your computer can reach).

The delivery:
You will deliver your project in the same "tarball" format as homework. Please put all materials in a directory named `PROJECT3` before tarring. The delivery should be prior to noon, Tuesday, April 8. We will arrange meetings starting on Tuesday, April 8, at which you explain your project to me.

You may work with other students, but any line of code you deliver must be written by you yourself. You must also be able to explain everything by yourself.