PHYS 6352
April 23, 2014

CHALLENGE 4 (40 pts)

If we limit ourselves to polynomials of order 5, then there are two polynomials that can be taken to represent the exponential function e^x. One is the Taylor expansion

$$e^x \approx 1 + \frac{1}{1!}x + \frac{1}{2!}x^2 + \frac{1}{3!}x^3 + \frac{1}{4!}x^4 + \frac{1}{5!}x^5 \,.$$

The other is the expansion of e^x in a Legendre series $\sum_{n=0}^{4} a_n P_n(x)$ where

$$a_n = \frac{2n+1}{2} \int_{-1}^{1} P_n(x) e^x dx \,.$$

In Project 4 you found a 5th order polynomial approximation to the solution of a differential equation. The exact solution of that differential equation is e^x. Is the solution found in Project 4 the Taylor expansion of e^x? Is it the Legendre expansion of e^x? Is it neither? Most important: EXPLAIN!!