

The problem: Estimate the integral

$$I \equiv \int_{-1}^1 e^{-x} \sin \pi x \, dx .$$

A fairly precise value for this integral is

$$I = -0.6793261834020956 .$$

You are to do this in three different ways:

PROBLEM 1: (20 points) Use Gauss-Legendre (also known as plain “Gaussian”) integration to evaluate the integral with 5 nodes. Write a program `mygauss.cpp` that evaluates the 5 node estimate for the integral. Your program should ask for no input, and should output the estimate for the integral and the error.

Hint: Get the needed node locations and weights from Abramowitz & Stegun.

PROBLEM 2: (20 points) Use Gauss-Chebyshev integration. Write a program `mycheb.cpp` that asks the user to specify the number of Chebyshev nodes to use, then outputs both the answer for the estimate of that integral, and outputs the error. You are to deliver only the program `mycheb.cpp`.

PROBLEM 3: (30 points) Consider the (nonclassical) orthogonal polynomials –call them $B_n(x)$ – based on the interval $[-1, +1]$ and the weight function e^{-x} . Use a two-point Gaussian method based on this set of orthogonal polynomials to estimate the integral I . You are not to hand in a program, but rather a write up including: The values of the nodes x_1, x_2 ; the values of the weights w_1, w_2 ; the formula you use to find the estimate; the numerical value of the estimate. You are to hand in your solution and explanation in text, *.doc, pdf, or any reasonable format. (If it is not reasonable, I will send it back to you.) Hint: If you choose the polynomial to be “monic,” (i.e., to have the coefficient of its highest power be one) then the first two polynomials of this set, to sufficient accuracy are:

$$B_0(x) = 1$$

$$B_1(x) = x + 0.313035 \dots$$

PROBLEM 4: (20 points) You will be given a program, `struct4homework.cpp` that uses a struct type called “`stardata`” and prints out physical properties of the Sun. Modify the program so that it prints out the physical properties of the Sun followed by the physical properties of Sirius, the brightest star in the sky. Do this by adding another struct of the type `stardata`.