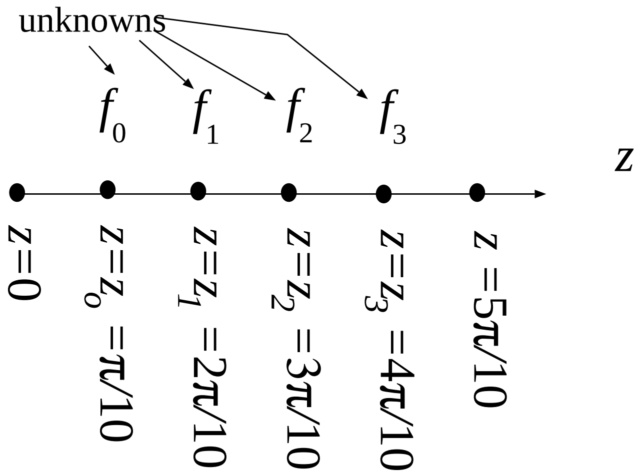
PHYS 2326
March 27, 2014

SOLUTION TO CORRECTED HOMEWORK SET 9




Impose the grid as shown. There are 6 relevant locations, $z = 0$, $z = \pi/2$ and $z_k$, with $k = 0, 1, 2, 3$. Let $f$ represent the unknown function $f(z)$. We are given the values of $f$ at $z = 0$ and $z = \pi/2$. Our task is to find $f_k$, with $k = 0, 1, 2, 3$, the values of $f$ at the points $z_k$.

With $h \equiv z_{k+1} - z_k = \pi/10$ the finite difference representation of the second deriviative is

$$\frac{f_{k+1} - 2f_k + f_{k-1}}{h^2} = f''(z_k) + \mathcal{O}(h^2) \tag{1}$$

where the $\mathcal{O}(h^2)$ term is proportional to the 4th derivative of $f$ somewhere on the interval $z_{k-1}, z_{k+1}$.

We can now write out 4 equations representing the finite difference approximation to the differential equation at the points $z_k$:

At $z_0$:

$$\frac{f(0) - 2f_0 + f_1}{h^2} + f_0 = z_0^2 + 2$$

At $z_1$:

$$\frac{f_0 - 2f_1 + f_2}{h^2} + f_1 = z_1^2 + 2$$

At $z_2$:

$$\frac{f_1 - 2f_2 + f_3}{h^2} + f_2 = z_2^2 + 2$$

At $z_2$:

$$\frac{f_2 - 2f_3 + f(\pi/2)}{h^2} + f_2 = z_3^2 + 2$$

These can be written in the following matrix form (in which the known values of $f(0)$ and $f(\pi/2)$ have been inserted.

$$
\begin{bmatrix}
1-\frac{2}{h^2} & \frac{1}{h^2} & 0 & 0 \\
\frac{1}{h^2} & 1-\frac{2}{h^2} & \frac{1}{h^2} & 0 \\
0 & \frac{1}{h^2} & 1-\frac{2}{h^2} & \frac{1}{h^2} \\
0 & 0 & \frac{1}{h^2} & 1-\frac{2}{h^2}
\end{bmatrix}
\begin{bmatrix}
f_0 \\ f_1 \\ f_2 \\ f_3
\end{bmatrix}
=
\begin{bmatrix}
2+h^2-\frac{1}{h^2} \\
2+4h^2 \\
2+9h^2 \\
2+16h^2-\frac{1}{h^2}\left(1+\left(\frac{\pi}{2}\right)^2\right)
\end{bmatrix}
$$

These are the numbers (with $h$ taken to be $\pi/10$ that are used in the input files `A.dat` and `B.dat`.