PHYS 6352
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The notes here will be for 2 nonlinear equations in 2 unknowns. The generalization is obvious.

We have two functions F_1 and F_2 of two variables x_1 and x_2. The system of eqs is therefore:

$$F_1(x_a) = 0 \quad F_2(x_a) = 0 \ .$$

We now assume that the actual root x_a is related to our guess x_a^{old} according to

$$x_a = x_a^{old} + \delta_a$$

and linearization gives us:

$$0 = F_1 + F_{1,1}\delta_1 + F_{1,2}\delta_2$$

$$0 = F_2 + F_{2,1}\delta_1 + F_{2,2}\delta_2$$

where the Fs and their partials are evaluated at $\{x_a^{old}\}$.

The equation for the delta's can then be written:

$$\begin{bmatrix} F_{1,1} & F_{1,2} \\ F_{2,1} & F_{2,2} \end{bmatrix} \begin{bmatrix} \delta_1 \\ \delta_2 \end{bmatrix} = - \begin{bmatrix} F_1 \\ F_2 \end{bmatrix} \ .$$

For generalization, it is useful to note that the square matrix on the left is the Jacobean.

The solution to these equations is:

$$\begin{bmatrix} \delta_1 \\ \delta_2 \end{bmatrix} = -\frac{1}{F_{1,1}F_{2,2} - F_{1,2}F_{2,1}} \begin{bmatrix} F_{2,2} & -F_{1,2} \\ -F_{2,1} & F_{1.1} \end{bmatrix} \begin{bmatrix} F_1 \\ F_2 \end{bmatrix} \ ,$$

which in greater generality would have the matrix form

$$\boldsymbol{\delta} = -\mathbf{J}^{-1}\mathbf{F}$$

where $\mathbf{J}^{-1}$ is the inverse of the Jacobean.

A good example: Let

$$F_1 = \sin(x+y) - e^{y/x} + 5$$

$$F_2 = x^4 + y^4 - 1 \ .$$

This *can* be converted to a single function of x or y and a solution found to be: $x, y = \{0.5453925, 0.977106\}$, but it can also be solved as described above.