PHYS 6352
April 17, 2014

CHALLENGE 2 (80 pts)

In class, on Wednesday February 12, we looked at an eigenvalue problem for the Schrödinger equation. In particular, we looked at the time-indepentent Schrödinger equation

$$\frac{d^2\psi}{dx^2} + 10(x^2 - 1)\psi = -E\psi\,, \tag{1}$$

with the conditions that $\psi = 0$ at $x = -1$ and at $x = +1$.

To find the eigenvalues, the allowed values of the (de-dimensionalized) energy E, we used "shooting." That is, we started at $x = -1$, with $\psi = 0$, with some nonzero choice for $d\psi/dx$ at $x = 0$, and with a guess for E. We then used a finite difference method to propagate the solution to $x = 1$, and we would note the value of ψ at $x = +1$. Next, we adjusted the guess for E and did another shooting, and another, and another until the value of ψ at $x = +1$ was acceptably near zero. In this way we found the "spectrum" of values $E = 11.029, 16.929, 29.135, \ldots.$

During the last two weeks of March, we found another way of solving differential equations. In Homework Set 9, we used a finite difference method to solve an ODE with fixed values at the end points. In Project 3, we solved a problem with fixed auxiliary data by a spectral method. Using either finite differences of spectral methods, therefore, we can solve an ODE, with fixed bounary conditions, without shooting.

So why can't we do this for the Schrödinger equation (1)? Or can we? Why not simply take Eq. (1) with *any* value of E, put in the boundary conditions $\psi = 0$ at $x = -1$ and $x = +1$, and solve for ψ? But to do this implies that we can solve the Schrödinger equation for any E and that there are no energy eigenvalues. This contradicts not only our physical intuition, but also the result of our shooting computations.

Explain.