PHYS 6352
April, 2014

PROJECT 4

For the ODE

$$x^2 f'' + f = (x^2+1)e^x \qquad (1)$$

the extra conditions at $f(0) = 1$ and $f(1) = e^1 = 2.718\cdots$. (It is easy to confirm that the exact solution to this equation is simply $f(x) = e^x$.) You are to solve this equation on the interval $[-1, 1]$ using the spectal method based on Legendre polynomials. You are to find the spectral solution for a spectral expansion up to x^3 (i.e., up to $P_3(x)$) and for a spectral expansion up to x^5.

You are to hand in a write up (not a program) giving the matrix equation you are solving (to each order 3 and 5) and the polynomial approximation to each order. For both order 3 and order 5 give the $L2$ error. The write-up you hand in should be a PDF.

This is not an excercise in writing code, but you may want to use your LUdecomposition program to solve the matrix equations. The matrix multiplication needed for the derivation of those matrix equations are feasible by hand, since the matrices are very sparse, but you are allowe/encouraged to use *Mathematica* or *Matlab* to do the matrix multiplications. You may use those tools for the matrix solutions also.

My advice: If you already have some familiarity with *Mathematica* or *Matlab*, use the to do all the matrix work. If you are not, do the matrix multiplications by hand (carefully) and find the solution using LUdecomp.cpp and find the $L2$ error with a simple program from earlier in the course.

Hint: For the order 5 computation the $L2$ error should be very impressive.

For those of you doing the computation mostly by hand, the following results might be of use:

$$(x^2+1)e^x = b_0P_0 + b_1P_1 + b_2P_2 + b_3P_3 + b_4P_4 + b_5P_5 + \cdots \qquad (2)$$

where

$$b_0 = 1.614643504944718 \qquad (3)$$
$$b_1 = 1.7778994262518077 \qquad (4)$$
$$b_2 = 1.3306064973487786 \qquad (5)$$
$$b_3 = 0.5481437588941276 \qquad (6)$$
$$b_4 = 0.13771247133081488 \qquad (7)$$
$$b_5 = 0.01965484432002995 \qquad (8)$$

You will email your results to me on or before midnight on Monday, April 21, and we will meet for individual discussions of your results on Tuesday afternoon, April 22.