Phys-5319 Midterm Exam July 19, 2016

Name_____

I. Choose (circle) one correct answer for each question: (*3 point each*)

1) On a 32-bit computer, a **single** precision floating-point number most likely has _____ significant digits (choose the closest one).
a) 3 b) 14 c) 7 d) 10 e) 32

2) On a 32-bit computer, a **double** precision floating-point number most likely has _____ significant digits (choose the closest one).
a) 3 b) 14 c) 7 d) 10 e) 32

3) A character is represented by 1 Byte (8-bit). What is the maximum number for different characters?
a) 8 b) 32 c) 128 d) 256 e) 1024

4) A good random number sequence should have _____ .
a) a period b) big numbers c) Gaussian distribution d) no correlations

5) A Fast Fourier Transform scales in order of _____ operations, where N is the number of uniform grid in the period.
a) O(N) b) O($N\log_2 N$) c) O(N^2) d) O(N^3)

6) For an one-dimensional integration, which method (rule) requires an odd number of points ?
a) Trapezoid b) Simpson's c) Monte Carlo d) Gaussian quadrature

7) For a one-dimensional integration, which method (rule) has a non-uniform interval **and** weight?
a) Trapezoid b) Simpson's c) Monte Carlo d) Gaussian quadrature

8) A classical Molecular Dynamics method is to integrate/solve _____ equation(s).
a) Poisson's b) Schrodinger c) Newton's d) Bernoulli's

9) The forward difference method for differentiation, $f'_{fd}(x) \approx \dfrac{f(x+h)-f(x)}{h}$, has error at the order of magnitude
a) O(h^5) b) O(h^3) c) O(h) d) O($\log_2 h$)

10) Before a computer can run it, your program code (Fortran, C or other Language) will be translated into the machine language by a
a) librarian b) compiler c) instructor d) editor e) printer

II. (a) Show the for (N+1) uniform points, the trapezoid rule is (10 points)

$$\int_a^b f(x)dx \approx h\left(\frac{1}{2}f_1 + f_2 + \ldots + f_{N-1} + f_N + \frac{1}{2}f_{N+1}\right)$$

where $h=(b-a)/N$.

(b) Derive the truncation error (in order of h). (5 points)

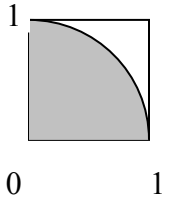
III. Consider an ordinary differential equation for a damped oscillation, $\frac{d^2x}{dt^2} + \omega^2 x + \alpha \frac{dx}{dt} = 0$.

(a) Express it into the standard form (i.e., a set of first-order differential eqs.) for numerical solution (*8 points*)

(b) Use Euler method to obtain the (i+1)-th values from i-th step's values (say, x_i, v_i) for a given ω, α and step-size h. (*7 points*)

IV. Give a formula and procedure (e.g. a flow chart) to use the Monte Carlo method to calculate the following integrals:

a) $\iint_{shaded} dxdy$ (for $x^2 + y^2 \le 1, x \ge 0, y \ge 0$) (*10 points*)



b) $\int_0^1 f(x)dx$ (*10 points*)

V. In calculation of a derivative by using a finite difference method, which one of the following two is more accurate? The forward difference method ($f'_{fd}(x) \approx \frac{f(x+h)-f(x)}{h}$) or the central difference method ($f'_{cd}(x) \approx \frac{f(x+h)-f(x-h)}{2h}$), where *h* is the step size. Please give your reason briefly. (10 points)

VI. The definition of a derivative is defined as $\frac{df}{dx} = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$ according to calculus. But when a finite difference approach to a derivative is used on any computer, does it mean that the smaller *h* (say, $h=10^{-32}$), the better accuracy? Please explain **why** in a few sentences. (*10 points*)