

Project I

Due Thursday, July 14, 2016

Develop a program using Monte Carlo method to simulate the Ising model on a square lattice (2-dimension: $L \times L$) at the **no** external magnetic field case ($H=0$). Choose the lattice edge **$L=20$** . Use the periodic boundary condition.

Required format of the project:

1. Introduction

- Briefly describe the phases (ferromagnetic and paramagnetic) and the phase transition; Microstates and its probability; Statistical average;
- Ising model: spin and its possible values; the energy of the system; how many microstates for N spins ($N = L \times L$).
- Monte Carlo method on a Ising model; Important sampling; How to calculate E_{flip} .

2. Pseudocode of ‘flow chart’ of your design of the Monte Carlo simulation.

3. Write **your own code** (in FORTRAN, C/C++ or python). Attach your code (better use ‘script’ on omega).

4. Calculate the magnetization M ($=\langle s \rangle$) and the total energy $\langle E \rangle$ for 50 different temperature steps from $T=0.0$ to $T=5.0$ (or a range you can observe a phase transition). Plot them against T 's. Discuss the physics from your results. Use the unit $J/k_B=1$ (so your T is in J/k_B and $\langle E \rangle$ is in J). Estimate the Curie temperature T_c .

The total point is 100, distributed as 20/20/30/30 points for Section 1/2/3/4.

Hint on the structure of the code

Use (declare) a two dimension array ($L \times L$) (e.g. $L=20$)

isp(L,L), one dimension for columns and another for rows

Initialize the value to all “1”, i.e. all up.

At a given T (temperature):

Do a few hundred steps sweeps for initial transient (bring the system in equilibrium with the heat bath);

Start the real N sweeps to collect the average $\langle M \rangle = \sum M_\alpha / N$, $\langle E \rangle = \sum E_\alpha / N$; where α is a microstate or a sweep.

Output T, M, E

Repeat for a new T. (outer loop)

One Monte Carlo sweep with the periodic boundary:

```
    loop i=1, L
      i1=i-1
      if(i.eq.1) i1=L
      i2=i+1
      if(i.eq.L) i2=1
      loop j=1, L
        j1=j-1
        if(j.eq.1) j1=L
        j2=j+1
        if(j.eq.L) j2=1
        isum=isp(i1,j)+isp(i2,j)+isp(i,j1)+isp(i,j2)
c eflip is the flip energy
        eflip=2*isp(i,j)*isum
        if(eflip.lt.0) then
          isp(i,j)=-isp(i,j)
        elseif(rand().le.dexp(-eflip/T)) then
          isp(i,j)= -isp(i,j)
        endif
      end loop j
    end loop i
```