Mid-term

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I multiple choices

1. c
2. b
3. c
4. d
5. b
6. b
7. d
8. c
9. c
10. b

II

- a)

$\int_a^b f(x)dx$
$\simeq (\frac{1}{2}(f_1 + f_2) + \frac{1}{2}(f_2 + f_3) + \ldots + \frac{1}{2}(f_N + f_{N+1})) * \frac{b-a}{N}$
$= \frac{h}{2}\Sigma_{k=1}^{N+1}(f(x_{k+1}) + f(x_k))$
$= h(\frac{1}{2}f_1 + f_2 + \ldots + f_N + \frac{1}{2}f_{N+1})$

where $h = (b-a)/N$

- b)

$\int_{x_i}^{x_{i+1}} f(x)dx \simeq \int_{x_i}^{x_{i+1}} [f_i + (x - x_i)f_i' + \frac{1}{2}(x - x_i)^2 f_i'']dx$

I get stuck for the expansion and forget the trick. It should be derived by Tyler expansion minus the trapezoid equation. The result I remember is about $\frac{1}{12}h^3 f''$

III

- a)

$$\frac{dx}{dt} = v$$

$$\frac{dv}{dt} = \frac{d^2x}{xt^2} = -\omega^2 x - \alpha v$$

- b)

$x_{i+1} - x_i = hv_i$ ->> $x_{i+1} = x_i + hv_i$

$v_{i+1} - v_i = h(-\omega^2 x_i - \alpha v_i)$ ->> $v_{i+1} = v_i - h\omega^2 x_i - h\alpha v_i$

IV

- a)

$$\frac{\text{area of shadow}}{\text{area of square}} = \frac{1}{4}\pi r^2 = \frac{\text{points in shadow}}{\text{total points}}$$

$$\iint dxdy = \frac{\text{points in shadow}}{\text{total points}}$$

procedure

```
do i = 1,max
    x=drand48()
    y=drand48()
    if (x*x+y*y.le.1 ) pts = pts + 1
enddo
print *, 'integral is ', pts/max
```

- b)

```
do i = 1,max
    x=drand48()
    value = value + f(x)
enddo
print *, 'integral is ', value/max
```

V

The central method is more accurate.

- the forward method: $f(x+h) \simeq f(x) + hf'(x) + \frac{1}{2}h^2 f''(x) + O(h^3 f''')+\ldots$ (1) thus $f'_{fd}(x) \simeq \frac{f(x+h)-f(x)}{h} - \frac{1}{2}hf''(x) \sim O(hf'')$
- the central method $f(x-h) \simeq f(x) - hf'(x) + \frac{1}{2}h^2 f''(x) - O(h^3 f''')+\ldots$ (2)
  (1)-(2): $f(x+h) - f(x-h) = 2hf'(x) + 2O(h^3 f''')$
  $f'_{cd}(x) \simeq \frac{f(x+h)-f(x-h)}{2h} - O(h^2 f''') \sim O(h^2 f''')$

VI

No. Computer has finite accuracy and limited computing power. For double precision float, it use 53bit for decimal points, which means the minimum it can represent is $2^{-53} \simeq 10^{-16}$. Below that, the truncation error will domain and accuracy won't increase anymore.