

Solving one-dimensional by the shooting method

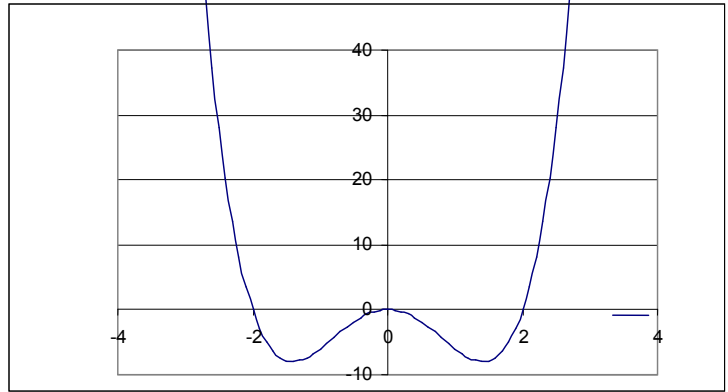
Use Runge-Kutta (4th order) method to solve a one-dimensional double-well problem

$$\left[-\frac{1}{2} \frac{d^2}{dx^2} + V(x)\right]\psi(x) = \varepsilon\psi(x)$$

where the atomic unit (a.u.) is used. The potential V is a double-well (as shown on the right)

$$V(x) = ax^4 - bx^2$$

with $a=2$ and $b=8$.



- Find all the energy eigenvalues below zero ($\varepsilon < 0$). (at least 4 significant figures)
- plot the lowest two states (orthonormal wave functions) together with $V(x)$ in $(-3 < x < 3)$.
- Discuss that if $V(x)$ is approximated as a simple harmonic oscillator around x_0 , where x_0 is one of the two bottoms, what kind of structures for the energy eigenvalues and wavefunctions will be. Compared (qualitatively) with your results from a) and b).

Please use the general format we used in other projects for the report.