MATH 6620
Spring 2019

Homework 2, Due Thursday February 7 2019
Show all the work. Late homework will not be accepted.

Problem 1.
Given the interpolation data (points) $(0, 2), (0.5, 5), (1, 4)$:
a) Find the function $f(x) = c_0 + c_1 \cos(\pi x) + c_2 \sin(\pi x)$, which interpolates the given data
b) Find the quadratic polynomial interpolating this data
In each case, graph the interpolating function.
Problem 2.
Bound the error (in terms of h - a positive constant) of the quadratic interpolation to $f(x) = e^x$ on $[0, 1]$ with evenly spaced interpolation points $x_0, \quad x_1 = x_0 + h, \quad x_2 = x_0 + 2h$. Assume that x such that $x_0 < x < x_2$
Problem 3.
a) Suppose you are given symmetric data:

$$(x_i, y_i), \quad i = -n, -n+1, ..., n-1, n,$$

such that

$$x_{-i} = -x_i \text{ and } y_{-i} = -y_i, \quad i = 0, 1, ..., n.$$

What is the required degree of the interpolating polynomial p (x_is are distinct nodes)? Show that the interpolating polynomial is odd, i.e., $p(x) = -p(-x)$ for all real numbers x.
b) Let $l_i(x)$ are Lagrange basis functions with $x_0, x_1, ..., x_n$(as defined in class, $\{x_i\}$ are distinct nodes) with $n = 2019$. Prove that,

$$\sum_{i=0}^{2019} l_i(x) = 1$$

for all x.
Problem 4.
a) Consider finding a rational function $p(x) = \frac{a+bx}{1+dx}$ that satisfies

$$p(x_i) = y_i, \quad i = 1, 2, 3$$

with distinct x_1, x_2, x_3. Does such a function $p(x)$ exist, or are additional conditions needed to ensure existence and uniqueness of $p(x)$?
b) Let $x_0, ..., x_n$ be distinct real points, and consider the following interpolation problem. Choose a function

$$F_n(x) = \sum_{j=0}^{n} c_j e^{jx}$$

such that

$$F_n(x_i) = y_i \quad i = 0, 1, ..., n$$

with the $\{y_i\}$ given data. Show there is a unique choice of $c_0, ..., c_n$.

Problem 5. (Computational assignment. Please submit the codes by e-mail and include the printout of the results (and discussion of the results) with the theoretical part.)

Consider the function $f(x) = (x^2 + 1)^{-1}$ on the interval $-5 \le x \le 5$. For each $n \ge 1$, define $h = 10/n, \quad x_j = -5 + j \times h,$ for $j = 0, 1, ..., n$. Let $p_n(x)$ be the polynomial of degree n which interpolates f at the nodes $x_0, x_1, ..., x_n$. Compute p_n for $n = 1, 2, ..., 20$, plot $f(x)$ and $p_n(x)$ for each n, and estimate the maximum error $|f(x) - p_n(x)|$ for $x \in (-5, 5)$. Discuss what you find.