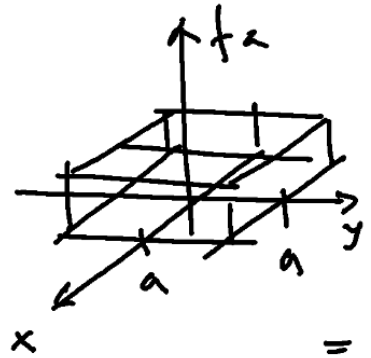
EXAMEN DE IMÁGENES

B1 $f_a(x,y) = \begin{cases} 1 & |x|<a, |y|<a \\ 0 & \text{sino} \end{cases}$



$$F_a(u,v) = \int_{-a}^{a}\int_{-a}^{a} e^{-j2\pi(ux+vy)}\,dx\,dy$$

$$= \int_{-a}^{a} e^{-j2\pi ux}\,dx \int_{-a}^{a} e^{-j2\pi vy}\,dy$$

$$= \frac{1}{-j2\pi u} e^{-j2\pi ux}\Big|_{-a}^{a} \cdot \frac{1}{-j2\pi v} e^{-j2\pi vy}\Big|_{-a}^{a}$$

$$= \frac{1}{-j2\pi u}\left(e^{-j2\pi ua} - e^{j2\pi ua}\right)\cdot\frac{1}{-j2\pi v}\left(e^{-j2\pi va} - e^{j2\pi va}\right)$$

$$= \frac{1}{-j2\pi u}\left(-2j\sin(2\pi ua)\right)\cdot\frac{1}{-j2\pi v}\left(-2j\sin(2\pi va)\right)$$

$$= \frac{\sin(2\pi ua)}{\pi u}\cdot\frac{\sin(2\pi va)}{\pi v} \qquad \operatorname{sinc}(x) = \frac{\sin(\pi x)}{\pi x}$$

$$= 2a\,\frac{\sin(2\pi ua)}{2\pi ua}\cdot 2a\,\frac{\sin(2\pi va)}{2\pi va}$$

$$= 4a^2 \operatorname{sinc}(2ua)\operatorname{sinc}(2va)$$

$f_a(x,y) \circ\!\!-\!\!\bullet\; F_a(u,v)$

$$F_a(u,v) = 4a^2 \operatorname{sinc}(2ua)\operatorname{sinc}(2va)$$

1) $f_1(x,y) \circ\!\!-\!\!\bullet\; 4\operatorname{sinc}(2u)\operatorname{sinc}(2v)$

2) $f_2(x,y) \circ\!\!-\!\!\bullet\; 16\operatorname{sinc}(4u)\operatorname{sinc}(4v)$

3) $f_1 + f_2 \circ\!\!-\!\!\bullet\; 4\operatorname{sinc}(2u)\operatorname{sinc}(2v) +$
$16\operatorname{sinc}(4u)\operatorname{sinc}(4v).$

(B2) $\|Wf\| \to \min$ sujeto a $\|H\hat{f} - g\| = 0$

$$Wf = f^+ - g = Pf - Hg$$

$$\Rightarrow W = P - H$$

$$f^+ = \begin{bmatrix} f_2 \\ f_3 \\ \vdots \\ f_{96} \\ f_{97} \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & 1 & & & \\ & 0 & 1 & & \\ & & 0 & 1 & \\ & & & \ddots & \ddots \\ & & & & 1 \end{bmatrix}}_{P \quad 96 \times 100} \begin{bmatrix} f_1 \\ f_2 \\ f_3 \\ \vdots \\ f_{100} \end{bmatrix}$$

$$V(f) = \lambda \|Hf - g\|^2 + \|Wf\|^2 \to \min$$

$$\frac{\partial V}{\partial f} = 2\lambda H^T(Hf - g) + 2W^T W f = 0$$

$$\lambda H^T H f + W^T W f = \lambda H^T g$$

$$[\lambda H^T H + W^T W] f = \lambda H^T g$$

$$\hat{f} = \lambda [\lambda H^T H + W^T W]^{-1} H^T g$$

B3

$X =$

0	0	4	4	4
0	0	4	4	4
0	0	4	4	4
0	0	4	4	4
0	0	4	4	4

Pasa Bajos

$X_a =$

0	0	0	0	0
0	4	8	12	0
0	4	8	12	0
0	4	8	12	0
0	0	0	0	0

$\cdot \frac{1}{3}$

Pasa Altos

$X_b^* =$

0	0	0	0	0
0	-4	4	0	0
0	-4	4	0	0
0	-4	4	0	0
0	0	0	0	0

$\cdot \frac{1}{3}$

Mediana

$X_c =$

0	0	0	0	0
0	0	4	4	0
0	0	4	4	0
0	0	4	4	0
0	0	0	0	0

$[-1\ 0\ 1]$

$X_d^* =$

0	4	4	0	0
0	4	4	0	0
0	4	4	0	0
0	4	4	0	0
0	4	4	0	0

$[-1\ 0\ 1]^T$

$X_e =$

0	0	0	0	0
0	0	0	0	0
0	0	0	0	0
0	0	0	0	0
0	0	0	0	0

*tomar versiones con cambio de signo también como válidas

f) Ecualización: a millones de soluciones

$X_f =$

0	1	2	3	4
0	1	2	3	4
0	1	2	3	4
0	1	2	3	4
0	1	2	3	4

Debe asegurarse que haya cinco de cada número y que los bajos estén a la izquierda y los altos a la derecha

B4

$X =$

0	0	0	0	0
0	0	0	0	0
0	1	1	1	0
0	0	0	0	0
0	0	0	0	0

Erosion

$X_a =$

0	0	0	0	0
0	0	0	0	0
0	0	0	0	0
0	0	0	0	0
0	0	0	0	0

Dilatacion

$X_b =$

0	0	0	0	0
0	1	1	1	0
0	1	1	1	0
0	1	1	1	0
0	0	0	0	0

Cierre = dilata, erosion

$X_c =$

0	0	0	0	0
0	0	0	0	0
0	0	1	0	0
0	0	0	0	0
0	0	0	0	0

Apertura = erosion y dilata

$X_d =$

0	0	0	0	0
0	0	0	0	0
0	0	0	0	0
0	0	0	0	0
0	0	0	0	0