DC, AC

Sinusoidal Steady-State Analysis

We consider circuits energized by time-varying voltage or current sources.

Textbook:

Electric Circuits

James W. Nilsson & Susan A. Riedel

9th Edition.

Outline

- Complex Numbers Tutorial
- Sinusoids
- Phasors
- Techniques of Circuit Analysis
- Phasor Diagrams

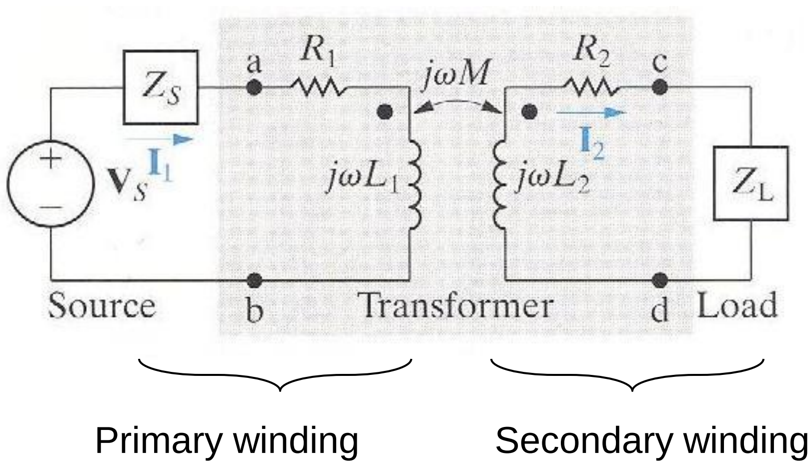
The transformer

- A transformer is a device that is based on magnetic coupling.
- Are used in both communication and power circuits.
- In communication circuits: transformer is used to matched impedance and eliminate dc signals from portions of the systems
- In power circuits: transformer is used to establish ac voltage levels that facilitate the transmission, distribution and consumption of electrical power.

only AC

Linear transformer

- **Primarily used in communication circuits.**
- **Is formed when two coils are wound on a single core to ensure magnetic coupling.**
- **Frequency domain circuit model of a transformer**

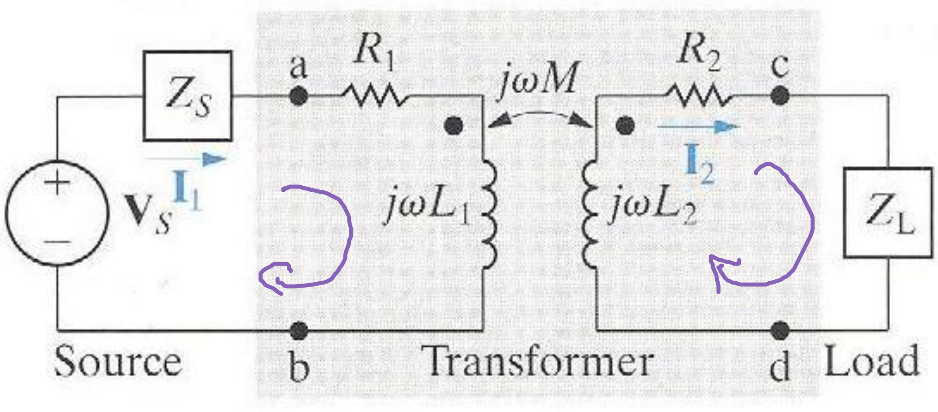


R_1 = the resistance of the primary winding
R_2 = the resistance of the secondary winding
L_1 = the self-inductance of the primary winding
L_2 = the self-inductance of the secondary winding
M = the mutual inductance
V_s = sinusoidal source
Z_s = internal impedance of the source
Z_L = the load
I_1 = primary current
I_2 = secondary current

New

$M = k\sqrt{L_1 L_2}$

Transformer circuit analysis



Mesh-current equations:

$$\boldsymbol{V_s} = \left(\boldsymbol{Z_s} + \boldsymbol{R_1} + \boldsymbol{j\omega L_1}\right)\boldsymbol{I}_1 - \boldsymbol{j\omega M I}_2$$
$$0 = -\boldsymbol{j\omega M I}_1 + \left(\boldsymbol{R_2} + \boldsymbol{j\omega L_2} + \boldsymbol{Z}_2\right)\boldsymbol{I}_2$$

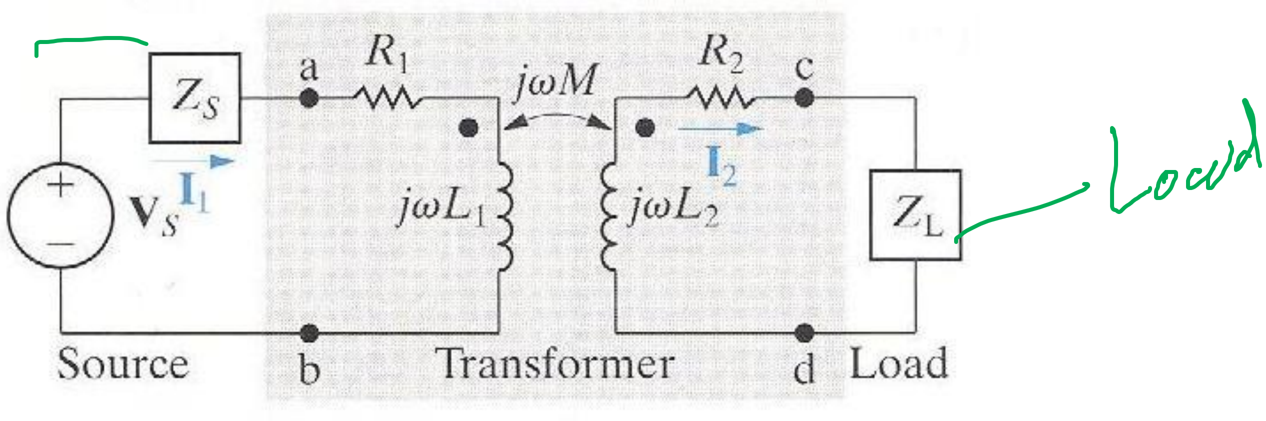
$$Z_{11} = Z_s + R_1 + j\omega L_1 \quad = \text{total self - impedance of the primary winding}$$
$$Z_{22} = R_2 + j\omega L_2 + Z_2 \quad = \text{total self - impedance of the secondary winding}$$

Yield:

$$\boldsymbol{I}_1 = \frac{\boldsymbol{Z}_{22}}{\boldsymbol{Z}_{11}\boldsymbol{Z}_{22} + \omega^2 \boldsymbol{M}^2}\boldsymbol{V_s}$$

$$\boldsymbol{I}_2 = \frac{\boldsymbol{j\omega M}}{\boldsymbol{Z}_{11}\boldsymbol{Z}_{22} + \omega^2 \boldsymbol{M}^2}\boldsymbol{V_s} = \frac{\boldsymbol{j\omega M}}{\boldsymbol{Z}_{22}}\boldsymbol{I}_1$$

Transformer circuit analysis



Impedance at the terminal of the source:

$$\boldsymbol{Z}_{ab} = \boldsymbol{R}_1 + \boldsymbol{j}\omega\boldsymbol{L}_1 + \frac{\omega^2 \boldsymbol{M}^2}{\left(\boldsymbol{R}_2 + \boldsymbol{j}\omega\boldsymbol{L}_2 + \boldsymbol{Z}_L\right)}$$

Z_{ab} is independent of the magnetic polarity of the transformer.

Z_{ab} shows how the transformer affects the impedance of the load as seen from the source

Reflected impedance

$$\Longrightarrow \quad Z_r = \left[\frac{\omega M}{|Z_{22}|}\right]^2 Z_{22}^*$$

$$Z_{ab} = R_1 + j\omega L_1 + \underbrace{\frac{\omega^2 M^2}{(R_2 + j\omega L_2 + Z_L)}}$$

Z_r = reflected impedance

= the impedance of the secondary circuit as seen from the terminals of the primary circuit or vice versa.

Notes:

1) The reflected impedance is due solely to the existence of mutual inductance

2) The linear transformer reflects the conjugate of the self-impedance of the secondary circuit (Z^*_{22}) into the primary winding by a scalar multiplier

$$Z_r = \frac{\omega^2 M^2}{|Z_{22}|^2}\underbrace{[(R_2 + R_L) - j(\omega L_2 + X_L)]}_{Z^*_{22}}$$

$$Z_r = \left[\frac{\omega M}{|Z_{22}|}\right]^2 Z_{22}^*$$

Example

$M = k\sqrt{L_1 L_2}$

The parameters of a certain linear transformer are R_1 = 200 Ω, R_2 = *100* Ω, L_1 = *9H*, L_2 = *4H, and k = 0.5. The transformer couples an impedance* consisting of an 800 Ω resistor in series with a 1 μF capacitor to a sinusoidal voltage source. The 300 V (rms) source has an internal impedance of 500 + j100 Ω and a frequency of 400 rad/s. ω Z_S

a) Construct a frequency-domain equivalent circuit of the system.

b) Calculate the self-impedance of the primary circuit. → Z_{11}

c) Calculate the self-impedance of the secondary circuit. → Z_{22}

d) Calculate the impedance reflected into the primary winding.

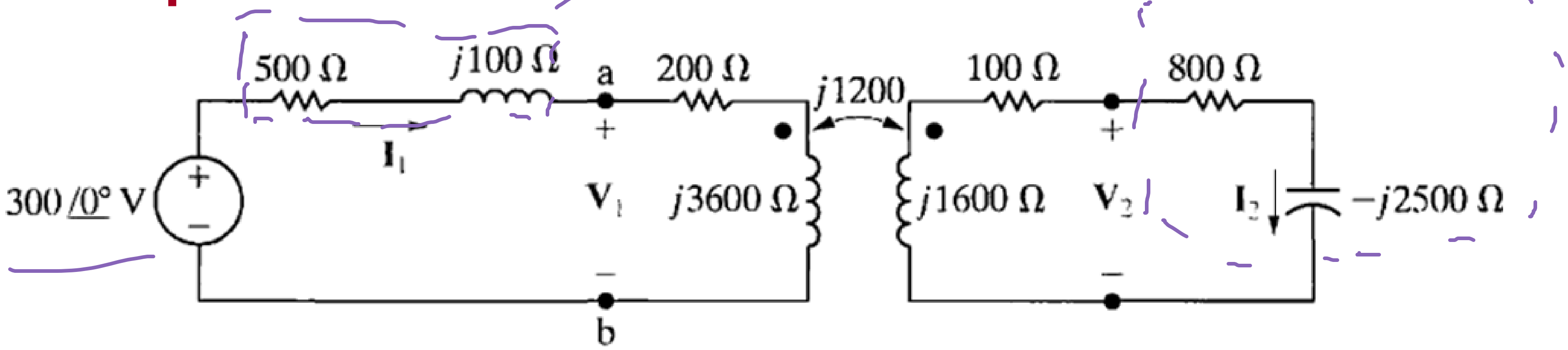
e) Calculate the scaling factor for the reflected impedance.

f) Calculate the impedance seen looking into the primary terminals of the transformer.

g) Calculate the Thevenin equivalent with respect to the terminals c, d.

$\omega = 400$ rad/s

Example – Sol.



The figure shows the frequency-domain equivalent circuit. Note that the internal voltage of the source serves as the reference phasor, and that **V**$_1$ and **V**$_2$ represent the terminal voltages of the transformer. In the circuit of the figure, we made the following calculations:

$$j\omega L_1 = j(400)(9) = j3600\ \Omega, \quad L1$$

$$j\omega L_2 = j(400)(4) = j1600\ \Omega, \quad L2$$

$$K\sqrt{L_1L_2} = M = 0.5\sqrt{(9)(4)} = 3\ \text{H},$$

$$j\omega M = j(400)(3) = j1200\ \Omega,$$

$$\frac{1}{j\omega C} = \frac{10^6}{j400} = -j2500\ \Omega. \quad C$$

Example – Sol.

b) The self-impedance of the primary circuit is Z_{11} ①

$$Z_{11} = 500 + j100 + 200 + j3600 = 700 + j3700\ \Omega.$$

c) The self-impedance of the secondary circuit is Z_{22} ②

$$Z_{22} = 100 + j1600 + 800 - j2500 = 900 - j900\ \Omega.$$

$$Z_r = \left(\frac{\omega M}{|Z_{22}|}\right)^2 Z^*_{22}$$

d) The impedance reflected into the primary winding is

$$Z_r = \left(\frac{1200}{|900 - j900|}\right)^2 (900 + j900)$$

$$= \frac{8}{9}(900 + j900) = 800 + j800\ \Omega.$$

e) The scaling factor by which Z^*_{22} *is reflected is 8/9.* X Not Important.

f) The impedance seen looking into the primary terminals of the transformer is the impedance of the primary winding plus the reflected impedance; thus

$$Z_{ab} = 200 + j3600 + 800 + j800 = 1000 + j4400\ \Omega.$$

Z_R

Example – Sol.

g) The Thevenin voltage will equal the open circuit value of $\mathbf{V}_{cd}$. The open circuit value of $\boldsymbol{V}_{cd}$ *will* equal j1200 times the open circuit value of $\mathbf{I}_1$. The open circuit value of $\mathbf{I}_1$ is

$$\mathbf{I}_1 = \frac{300 \angle 0^\circ}{700 + j3700} = 79.67 \angle -79.29^\circ \text{ mA.}$$

Therefore $$\mathbf{V}_{\mathrm{Th}} = j1200(79.67 \angle -79.29^\circ) \times 10^{-3}$$

$$= 95.60 \angle 10.71^\circ \text{ V.}$$

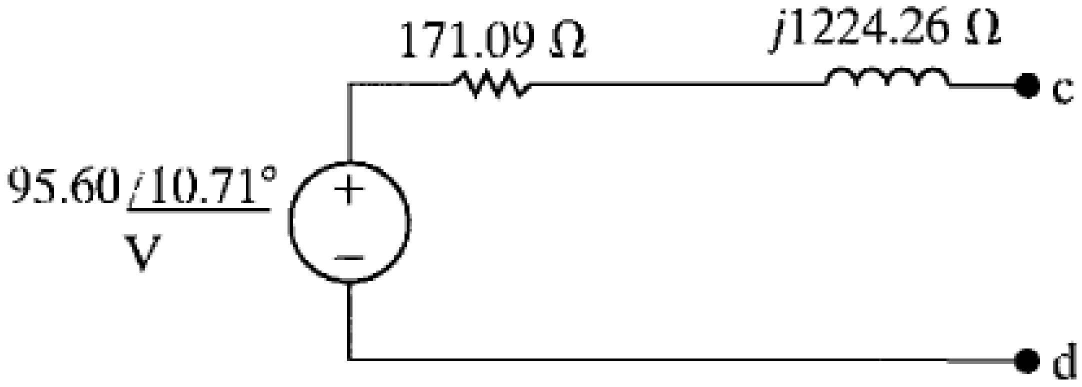
The Thevenin impedance will be equal to the impedance of the secondary winding plus the impedance reflected from the primary when the voltage source is replaced by a short-circuit. Thus

$$Z_{\mathrm{Th}} = 100 + j1600 + \left(\frac{1200}{|700 + j3700|}\right)^2 (700 - j3700)$$

$$= 171.09 + j1224.26\ \Omega.$$

Example – Sol.

The Thevenin equivalent is shown in the figure bellow



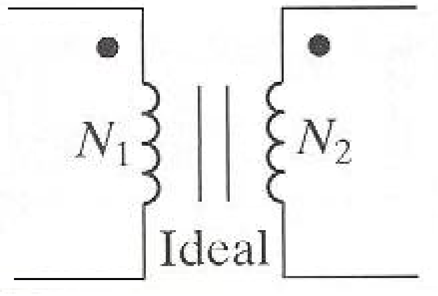
Ideal transformer

- Usually used to model the ferromagnetic transformer in power systems.
- An ideal transformer consists of two magnetically coupled coils having N1 and N2 turns, respectively, and exhibiting these three properties:

    1) The coefficient of coupling is unity ($k = 1$)

    2) The self-inductance of each coil is infinite ($L_1 = L_2 = \infty$)

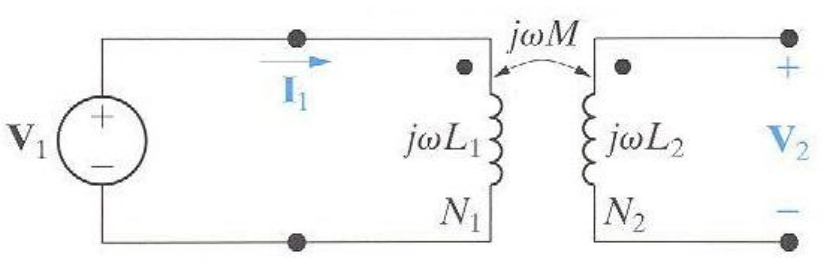
    3) The coil losses, due to parasitic resistance, are negligible ($R_1 = R_2 = 0$)



Ideal transformer

The circuit behavior is governed by the turns ratio a = N_2/N_1

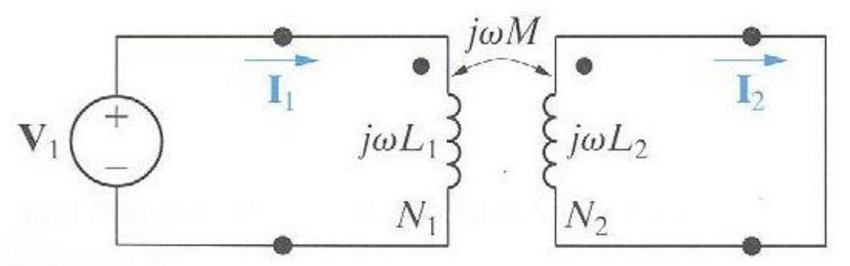
Volts per turns is the same for each winding



$$\left|\frac{V_1}{N_1}\right| = \left|\frac{V_2}{N_2}\right|$$

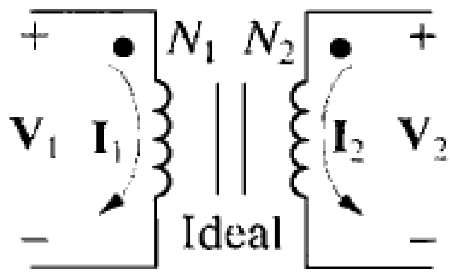
If the coil voltages V1 and V2 are both positive or negative at the dot-marked terminal, use a plus (+) sign. Otherwise, use a negative (-) sign.

Ampere turns are the same for each winding



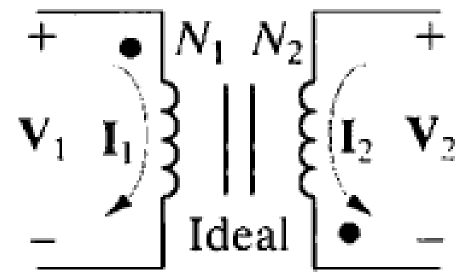
$$|I_1 N_1| = |I_2 N_2|$$

If the coil current I1 and I2 are both directed into or out of the dot-marked terminal, use a minus (-) sign. Otherwise, use a plus (+) sign.



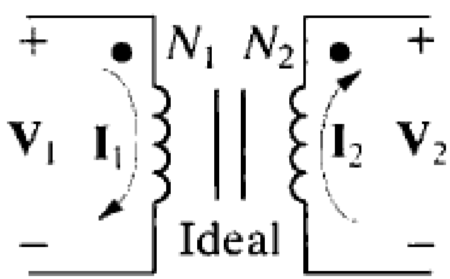
$$\frac{\mathbf{V}_1}{N_1} = \frac{\mathbf{V}_2}{N_2},$$
$$N_1\mathbf{I}_1 = -N_2\mathbf{I}_2$$

(a)



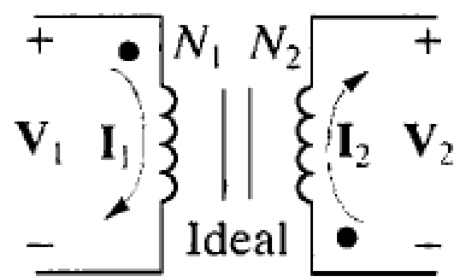
$$\frac{\mathbf{V}_1}{N_1} = -\frac{\mathbf{V}_2}{N_2},$$
$$N_1\mathbf{I}_1 = N_2\mathbf{I}_2$$

(b)



$$\frac{\mathbf{V}_1}{N_1} = \frac{\mathbf{V}_2}{N_2},$$
$$N_1\mathbf{I}_1 = N_2\mathbf{I}_2$$

(c)



$$\frac{\mathbf{V}_1}{N_1} = -\frac{\mathbf{V}_2}{N_2},$$
$$N_1\mathbf{I}_1 = -N_2\mathbf{I}_2$$

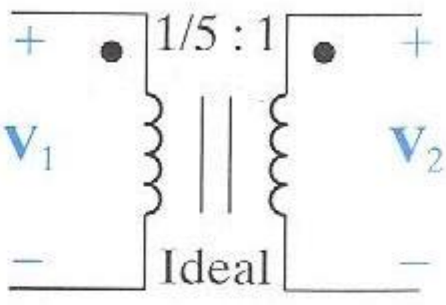
(d)

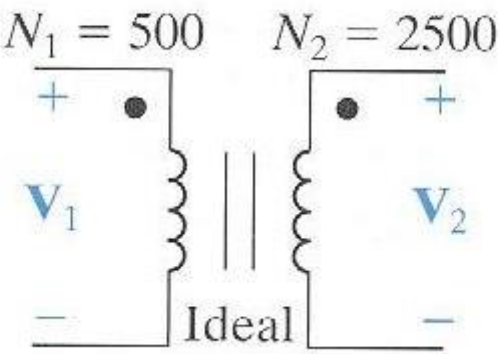
Ideal transformer

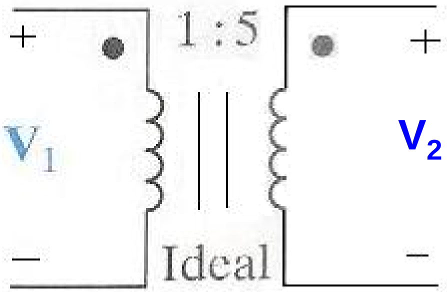
Turns ratio:

$$\boxed{a = \frac{N_2}{N_1}}$$

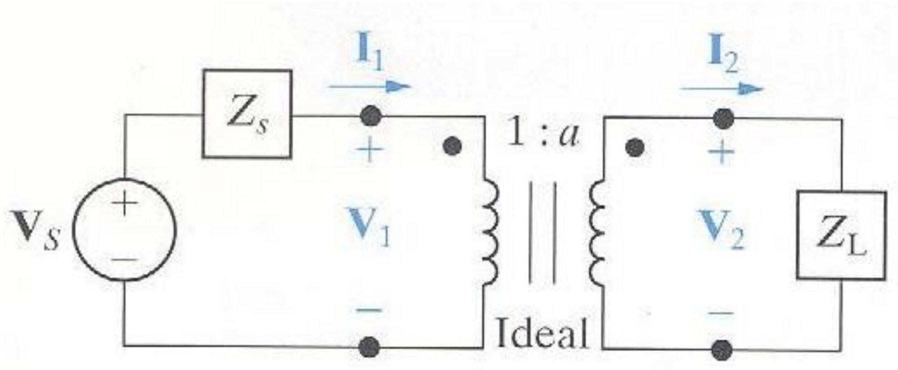
<u>Three ways to show the turns ratio of an ideal transformer</u>







Impedance matching by using ideal transformer



Relation of V_1 and I_1 by the transformer turns ratio:

$$V_1 = \frac{V_2}{a} \quad \text{and} \quad I_1 = aI_2$$

Impedance seen by the source and load respectively:

$$Z_{IN} = \frac{V_1}{I_1} \quad \text{and} \quad Z_L = \frac{V_2}{I_2}$$

Yield:

$$\boxed{Z_{IN} = \frac{1}{a^2} Z_L}$$

The ideal transformer's secondary coil reflects the load impedance back to the primary coil with the scaling factor $1/a^2$.