Note

PHYSICS 1

BY

LE DIEP PHI

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Chapter I

KINEMATICS

1. Linear Motion

Considering a moving object on the Ox direction versus time t with x, s, v, and a are position, distance traveled, instantaneous velocity, and instantaneous acceleration, respectively. The relationships between these parameters are classified into 3 types as shown in the table below.

Table 1.1: Summary for linear motion.

Type of motion	**Fomulas**	
Uniform motion $a = 0$	$s = vt$	(m)
Uniform acceleration $a = \text{const}$	$a = \dfrac{\Delta v}{\Delta t}$	(m/s^2)
	$v = v_0 + at$	(m/s)
	$x = x_0 + v_0 t + \dfrac{1}{2}at^2$	(m)
	$v^2 - v_0^2 = 2a(\Delta x)$	
Varying with time $a = a(t)$	$a(t) = v'(t)$	(m/s^2)
	$\rightarrow v(t) = \int_0^t a(\tau)d\tau + v_0$	(m/s)
	$v(t) = x'(t)$	(m/s)
	$\rightarrow x(t) = \int_0^t v(\tau)d\tau + x_0$	(m)

Average speed

$$s_{ave} = \frac{\text{total distance}}{\Delta t} \tag{1.1}$$

Average velocity

$$v_{ave} = \frac{\Delta x}{\Delta t} \tag{1.2}$$

Average acceleration

$$a_{ave} = \frac{\Delta v}{\Delta t} \tag{1.3}$$

2. Motion Along a Curved Path

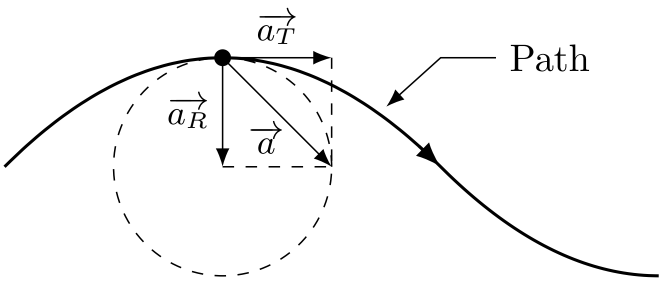


Figure 1.1: Object moving on a curved path.

At any point of a curved path, the acceleration of moving object on that path is a sum of two acceleration components

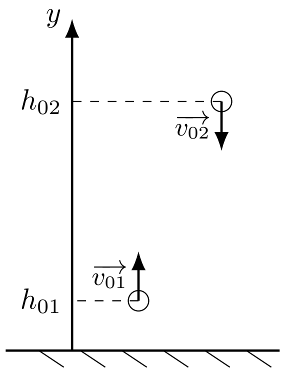
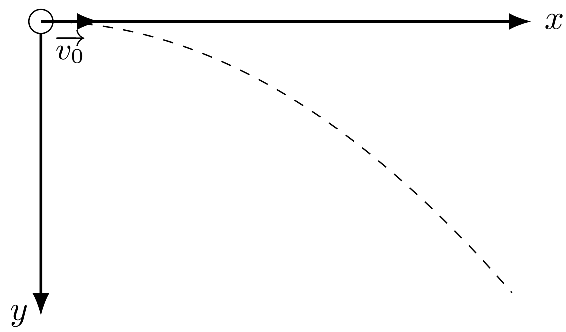
$$\overrightarrow{a} = \overrightarrow{a_R} + \overrightarrow{a_T} \tag{1.4}$$

where the magnitudes are

- $a = \sqrt{a_R^2 + a_T^2}$: Net acceleration.
- $a_R = \dfrac{v^2}{R}$: Radical acceleration.
- $a_T = \dfrac{d|\overrightarrow{v}|}{dt}$: Tangential acceleration.

3. Throwing Object Problems

Table 1.2: Summary for throwing object problems.

<table>
<tr><th>Throw vertically</th><th>Throw vertically</th></tr>
<tr><td>
</td><td>
</td></tr>
<tr><td>$$h_1 = h_{01} + v_{01}t - \frac{1}{2}gt^2$$ $$h_2 = h_{02} - v_{02}t - \frac{1}{2}gt^2$$</td><td>$$v_x = v_0, \qquad v_y = gt$$ $$x = v_0 t, \qquad y = \frac{1}{2}gt^2$$</td></tr>
</table>

4. Projectile Motion

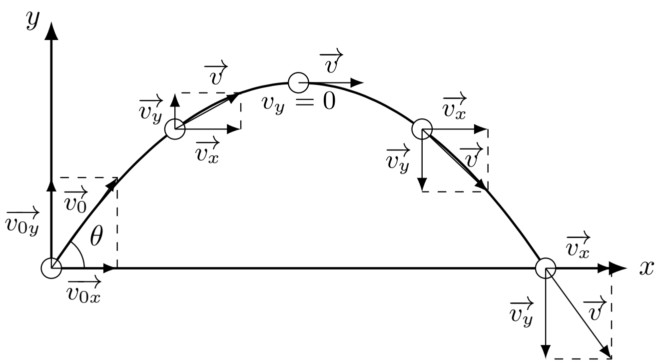


Figure 1.2: Path of projectile motion.

Equations of motion

$$\begin{aligned} v_x = v_{0x} = v_0 \cos\theta, &\qquad v_y = v_{0y} - gt = v_0 \sin\theta - gt \\ x = v_0 \cos\theta\ t, &\qquad y = v_{0y}t - \frac{1}{2}gt^2 = v_0 \sin\theta\ t - \frac{1}{2}gt^2 \end{aligned} \tag{1.5}$$

Horizontal range

$$R = \frac{v_0^2 \sin 2\theta}{g} \tag{1.6}$$

Maximum Height

$$H = \frac{v_0^2 \sin^2 \theta}{2g} \tag{1.7}$$

Time to reach the maximum height

$$t = \frac{v_0 \sin \theta}{g} \tag{1.8}$$

5. Relative Velocity

Considering two moving objects (1) and (2) with respect to a common stationary (3) as frame of reference. The following equation is hold

$$\overrightarrow{v_{12}} = \overrightarrow{v_{13}} + \overrightarrow{v_{32}} \tag{1.9}$$

Given that: $\overrightarrow{c} = \overrightarrow{a} + \overrightarrow{b}$, depending on each case, the following formulas can be helpful:

- If $\overrightarrow{a} \perp \overrightarrow{b}$, it leads to:

  $$c^2 = a^2 + b^2 \tag{1.10}$$

  The equation 1.10 is still holds for the case $\overrightarrow{c} = \overrightarrow{a} - \overrightarrow{b}$

- If $(\widehat{\overrightarrow{a}, \overrightarrow{b}}) = \theta$, it leads to

  $$c^2 = a^2 + b^2 + 2ab\cos\theta \tag{1.11}$$

  In the case of $\overrightarrow{c} = \overrightarrow{a} - \overrightarrow{b}$, the equation 1.11 becomes

  $$c^2 = a^2 + b^2 - 2ab\cos\theta \tag{1.12}$$

Chapter II

LAWS OF MOTION

1. Newton's Laws

1.1. Newton's First Law

The first law of Newton states that if no force or zero net force is applied to an object, the object at rest will stay at rest, and the object in motion will stay in motion.

$$\sum \overrightarrow{F} = \overrightarrow{0} \quad \Leftrightarrow \quad \overrightarrow{a} = \frac{d\overrightarrow{v}}{dt} = \overrightarrow{0} \tag{2.1}$$

1.2. Newton's Second Law

For a constant mass object, the net forced applied is directly proportional to the acceleration of the object; this acceleration is the same direction as net forced applied.

$$\sum \overrightarrow{F} = m\overrightarrow{a} \tag{2.2}$$

$$\Rightarrow \begin{cases} \sum F_x &= ma_x \\ \sum F_y &= ma_y \\ \sum F_z &= ma_z \end{cases}$$

1.3. Newton's Third Law

The third law states that all forces between two objects exist in equal magnitude and opposite direction. If an object (1) exerts a force $\overrightarrow{F_{12}}$ on an object (2), then simultaneously a force $\overrightarrow{F_{21}}$ exists from object (2) exerts on object (1).

$$\overrightarrow{F_{12}} = -\overrightarrow{F_{21}} \tag{2.3}$$

2. Sketch and Analyze a Mechanics Problem

Procedure for sketching and analyzing a mechanics problem:

1. Write the following sentence: "Let the positive direction and the system coordinate as the following figure" before sketching figure(s) for the problem.

2. Sketch the gravitational force $\overrightarrow{P}$ ($P = mg$) downwards to the ground, and the normal force $\overrightarrow{N}$ perpendicular to the surface.

3. Sketch the force $\overrightarrow{F}$ as given in the problem.

4. Sketch the friction force $\overrightarrow{f}$ ($f = \mu N$), if it exists, opposites to the direction of motion.

5. Assign the system coordinate Oxy in which the direction Ox is the same as direction of motion to obtain $a_y = 0$.

6. Sketch the components of forces in which their directions are not parallel to Ox and Oy.

7. If the friction force exists, analyze the net force with respect to Oy first and then Ox. If not, there is no necessity to analyze the net force with respect to Oy.

Note: If a problem relates to two or more objects, the diagram for the problem should be split into smaller diagrams in which 1 object for 1 diagram, which are called Free Body Diagrams.

Chapter III

WORK - ENERGY

1. Work

1.1. Constant Force

Considering a constant force $\overrightarrow{F}$ exerting on an object to move a linear distance d, let $\overrightarrow{d}$ be the distance vector, the work done by the force $\overrightarrow{F}$ is given by

$$\begin{aligned} W &= \overrightarrow{F} \cdot \overrightarrow{d} \\ &= F \cdot d \cdot \cos\left(\overrightarrow{F}, \overrightarrow{d}\right) \end{aligned} \tag{3.1}$$

Consequence: For any force perpendiculars to the distance, the work done by that fore is zero.

1.2. Non-Constant Force

Considering a varying force $\overrightarrow{F}(t)$ exerting on an object with position vector $\overrightarrow{r}(t)$ to move from initial position r_i to final position r_f, the work done by the force is given by

$$dW = \overrightarrow{F} \cdot \overrightarrow{dr} \quad \Rightarrow \quad W = \int_{r_i}^{r_f} \overrightarrow{F} \cdot \overrightarrow{dr} \tag{3.2}$$

Consequence: For an object attached to an ideal spring with the spring constant k and the spring force of $F = -kx$, the work done by that spring if the object moves from position x_i to x_f on the Ox axis is

$$W = \int_{x_i}^{x_f} (-kx) \cdot dx \tag{3.3}$$

1.3. Average Power

Average power for a force $\overrightarrow{F}$ is given by

$$P_{ave} = \frac{W}{\Delta t} = Fv \tag{3.4}$$

2. Energy

Kinetic energy

$$K = \frac{1}{2}mv^2 \tag{3.5}$$

Gravitational potential energy

$$U_G = mgh \tag{3.6}$$

Elastic potential energy

$$U_E = \frac{1}{2}k(\Delta x)^2 \tag{3.7}$$

In general, the potential energy specifies the object's ability to do work

$$U = -\int_{r_i}^{r_f} \overrightarrow{F} \cdot \overrightarrow{dr} \tag{3.8}$$

Mechanical energy is the sum of kinetic energy and potential energy

$$E = K + U = K + U_G + U_E \tag{3.9}$$

3. Laws and Theory of Work – Energy

3.1. Conservation of Mechanical Energy

Mechanical energy of an isolated system is constant

$$E_i = E_f \tag{3.10}$$

3.2. Work – Kinetic Energy Theorem

The change in its kinetic energy equals to the net work done by the forces on an object

$$K_f - K_i = W_{net} \tag{3.11}$$

3.3. The Change in Mechanical Energy

The change in mechanical energy equals to the work done by the friction force

$$E_f - E_i = W_{fric} \tag{3.12}$$

3.4. Analyze a Mechanics Problem

The following procedure shows how to analyze a mechanics problem

1. Is there any friction?. If yes, go to step 2, if not, go to step 3.

2. List all the forces do the work, then apply equation 3.11.

3. List all the types of potential energy, then apply equation 3.10.

Note: The equation 3.10 can also be applied in the case of no friction force. The equation 3.12 is rarely used unless the problem asks for the work of friction force.

Chapter IV

LINEAR MOMENTUM

1. Momentum and Impulse

For an object with constant mass m moving with velocity $\overrightarrow{v}$, the linear momentum for the object is given by

$$\overrightarrow{p} = m\overrightarrow{v} \tag{4.1}$$

Impulse is the change in momentum of the object

$$\overrightarrow{J} = \Delta\overrightarrow{p} = m\Delta\overrightarrow{v} \tag{4.2}$$

Average force

$$\overrightarrow{F} = \frac{\overrightarrow{J}}{\Delta t} = \frac{m\Delta\overrightarrow{v}}{\Delta t} \tag{4.3}$$

2. Conservation of Momentum

In a closed system, the total momentum of the system remains constant

$$\sum m\overrightarrow{v} = \sum m'\overrightarrow{v}' \tag{4.4}$$

Key ideas to solve a collision problem:

1. For an 1 dimension collision, choose the positive direction according to the direction of a particular object. Project all the velocities and then substitute into equation 4.4.

2. For a 2 dimension collision, choose the system coordinate in a way that there are highest number of vectors that parallel to Ox and Oy.

- Use projection method if the problem asks about angle of velocities in collision.
- Use unit vectors if most of the velocities are parallel to Ox and Oy.

3. Conservation of Kinetic Energy

In an elastic collision, the total kinetic energy of the system is conserved

$$\sum K_i = \sum K_f \tag{4.5}$$

The change in kinetic energy is given by

$$\Delta K = \sum K_f - \sum K_i \tag{4.6}$$

If $\Delta K = 0$, the kinetic energy is conserved, which means that the collision is elastic and vice versa.

Chapter V

ROTATIONAL MOTION

1. Linear Motion and Rotational Motion Conversion

Considering a rolling object with radius of R on a flat surface without slipping, let θ, ω, and α be angle of rotation, angular velocity, and angular acceleration of the object, respectively. The table below shows the conversion between linear motion and rotational motion of the object.

Table 5.1: Conversion table between linear motion and rotational motion.

Rotational motion		**Conversion**	**Linear motion**	
θ (rad)	Angle of rotation	$s = \theta R$	s (m)	Linear distance
ω (rad/s)	Angular velocity	$v = \omega R$	v (m/s)	Linear velocity
α (rad/s^2)	Angular acceleration	$a = \alpha R$	a (m/s^2)	Linear acceleration

2. Kinematics of Rotational Motion

Table 5.2: Summary for rotational motion.

Type of motion	**Fomulas**	
Uniform motion $\alpha = 0$	$\theta = \omega t$	(rad)
Uniform acceleration $\alpha = \text{const}$	$\alpha = \dfrac{\Delta\omega}{\Delta t}$	(rad/s^2)
	$\omega = \omega_0 + \alpha t$	(rad/s)
	$\theta = \theta_0 + \omega_0 t + \dfrac{1}{2}\alpha t^2$	(rad)
	$\omega^2 - \omega_0^2 = 2\alpha(\Delta\theta)$	

Table 5.2 (cont): Summary for rotational motion.

Type of motion	Fomulas
Varying with time $\alpha = \alpha(t)$	$\alpha(t) = \omega'(t)$ (rad/s^2) $\rightarrow \omega(t) = \int_0^t \alpha(\tau)d\tau + v_0$ (rad/s) $\omega(t) = \theta'(t)$ (rad/s) $\rightarrow \theta(t) = \int_0^t \omega(\tau)d\tau + \theta_0$ (rad)

3. Torque, Momentum, and Energy of Rotation

If forces F_i apply on a rigid body with inertia I at distances r_i from its axis and perpendicular to the radius, the torque of the rigid body is given by

$$\tau = I\alpha = \sum F_i r_i \tag{5.1}$$

Angular momentum of a rigid body

$$L = I\omega \tag{5.2}$$

The angular momentum is a conserved quantity or $L_f = L_i$.

For an object rotating on an orbit (circular, elliptic, ...) about a fixed origin, let r be the distance from the object to the origin and θ be the angle between velocity $\overrightarrow{v}$ and tangent line of the orbit at that point. The orbital angular momentum is given by

$$L = mrv\sin\theta \tag{5.3}$$

Angular kinetic energy of a rotating object

$$K = \frac{1}{2}I\omega^2 \tag{5.4}$$