HO CHI MINH CITY INTERNATIONAL UNIVERSITY

FINAL EXAMINATION

Academic year 2023-2024, Semester 1

Duration: 120 minutes

SUBJECT: Differential Equations (MAFE202IU)

Head of the Department of Mathematics	Lecturer:
Professor Pham Huu Anh Ngoc	Pham Huu Anh Ngoc
	Signature:

Instructions:

- *Each student is allowed a scientific calculator and a maximum of two double-sided sheets of reference material (size A4 or similar), stapled together and marked with their name and ID. All other documents and electronic devices are forbidden..*

Question 1. (20 marks) Solve the following differential equation

$$y''' + 3y'' + 2y' = \frac{1}{e^x + 1}.$$

Question 2. (20 marks) Find the general solution of the following system of differential equations

$$\begin{cases} \dfrac{dx}{dt} &= x + 5y + e^{-3t} \\ \dfrac{dy}{dt} &= 5x + y - 2e^{-3t}. \end{cases}$$

Question 3. (20 marks) Let

$$X'(t) = \begin{pmatrix} \frac{1}{t-1} & -\frac{e^{-t}}{t-1} \\ \frac{e^t}{t+1} & \frac{1}{t+1} \end{pmatrix} X(t) + \begin{pmatrix} t^2 - 1 \\ t^2 - 1 \end{pmatrix}, \quad t \in (1, \infty).$$

Show that $X_1(t) = \begin{pmatrix} t \\ e^t \end{pmatrix}$, $X_2(t) = \begin{pmatrix} e^{-t} \\ t \end{pmatrix}$ are linearly independent solutions of the corresponding homogeneous system. Find the general solution of the non-homogeneous system.

Question 4. (20 marks) Find a particular solution of the following differential equation

$$y'''' - 4y''' + 6y'' - 4y' + y = x^2 + e^{-x}.$$

Question 5. (20 marks) (**Nutrient Flow in an Aquarium**) Consider a vessel of water containing a radioactive isotope, to be used as a tracer for the food chain, which consists of aquatic plankton varieties A and B. Plankton are aquatic organisms that drift with the currents, typically in an environment like Chesapeake Bay. Plankton can be divided into two groups, phytoplankton and zooplankton. The phytoplankton are plant-like drifters: diatoms and other alga. Zooplankton are animal-like drifters: copepods, larvae, and small crustaceans.

Let $x(t)$ = isotope concentration in the water, $y(t)$ = isotope concentration in A, $z(t)$ = isotope concentration in B. Typical differential equations are

$$\begin{aligned} x'(t) &= -3x(t) + 6y(t) + 5z(t) \\ y'(t) &= 2x(t) - 12y(t) \\ z'(t) &= x(t) + 6y(t) - 5z(t). \end{aligned}$$

Find $x(t), y(t), z(t)$.