

Homework

Chapter 1

Week 1

Recall that A system of linear equations is said to be **consistent** if it has either one solution or infinitely many solutions; a system is **inconsistent** if it has no solution.

1. Determine if the following system is consistent:

$$\begin{cases} x_1 - 4x_3 = 8 \\ 2x_1 - 3x_2 + 2x_3 = 1 \\ 4x_1 - 8x_2 + 12x_3 = 1 \end{cases}$$

2. Determine which matrices are in reduced echelon form and which others are only in echelon form.

a. $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}$ b. $\begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

c. $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ d. $\begin{bmatrix} 1 & 1 & 0 & 1 & 1 \\ 0 & 2 & 0 & 2 & 2 \\ 0 & 0 & 0 & 3 & 3 \\ 0 & 0 & 0 & 0 & 4 \end{bmatrix}$

3. Reduced the matrices to echelon form. Circle the pivot positions in the final matrix and in the original matrix, and list the pivot columns.

$$\text{a) } \begin{bmatrix} 1 & 2 & 3 & 4 \\ 4 & 5 & 6 & 7 \\ 6 & 7 & 8 & 9 \end{bmatrix}$$

$$\text{b) } \begin{bmatrix} 1 & 3 & 5 & 7 \\ 3 & 5 & 7 & 9 \\ 5 & 7 & 9 & 1 \end{bmatrix}$$

4. Find the general solutions of the systems whose augmented matrices

$$\text{a) } \begin{bmatrix} 1 & 3 & 4 & 7 \\ 3 & 9 & 7 & 6 \end{bmatrix}$$

$$\text{b) } \begin{bmatrix} 3 & -4 & 2 & 0 \\ -9 & 12 & -6 & 0 \\ -6 & 8 & -4 & 0 \end{bmatrix}$$

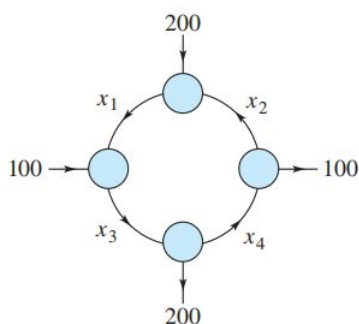
5 Solve the system

$$\text{a) } \begin{cases} 4x + 2y + z = 18 \\ 4x - 2y - 2z = 28 \\ 2x - 3y + 2z = -8 \end{cases}$$

$$\text{b) } \begin{cases} 2x_1 + x_2 + x_3 + 2x_4 = -1 \\ 5x_1 - 2x_2 + x_3 - 3x_4 = 0 \\ -x_1 + 3x_2 + 2x_3 + 2x_4 = 1 \\ 3x_1 + 2x_2 + 3x_3 - 5x_4 = 12 \end{cases}$$

Applications

6. (*Network Analysis*) The figure shows the flow of traffic (in vehicles per hour) through a network of streets.



- Solve this system for $x_i, i = 1, 2, 3, 4$
- Find the traffic flow when $x_4 = 0$.
- Find the traffic flow when $x_4 = 100$.

Week 2

1. Answer the following questions

- If a matrix A is 5×3 and the product AB is 5×7 , what is the size of B ?
- How many rows does B have if BC is a 3×4 matrix?

2. Let $A = \begin{pmatrix} 1 & 1 \\ -2 & -1 \\ 1 & 2 \end{pmatrix}$, $B = \begin{pmatrix} 1 & -1 & -2 \\ 2 & 1 & -2 \end{pmatrix}$, and $C = \begin{pmatrix} 1 & 1 & -3 \\ -1 & 2 & 1 \\ -3 & -1 & 0 \end{pmatrix}$.

Find the following if possible.

- $A + 20B$, $B - 5A^T$, and BA
 - $A + 4C^T$, AC and CA
3. Let $A = \begin{pmatrix} 2 & 5 \\ -3 & 1 \end{pmatrix}$ and $B = \begin{pmatrix} 4 & -5 \\ 3 & c \end{pmatrix}$. What is value of c such that $AB = BA$?

4. Let $A = \begin{pmatrix} 3 & -6 \\ -1 & 2 \end{pmatrix}$. Find matrix B such that $AB = 0$

5. Consider the following system of equation

$$\begin{cases} 3x_1 + x_2 + x_3 = 3 \\ x_1 - x_2 - x_3 = 1 \\ x_1 + 2x_2 + 2x_3 = 1 \end{cases}$$

Denote $x = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$ the vector solution of the equation. Express your solution in the form $x = tv + su$, where v and u are column vector in three dimensions, $t, s \in \mathbb{R}$.