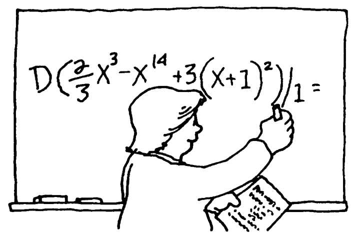
CLASSROOM CAPSULES



A Classroom Capsule is a short article that contains a new insight on a topic taught in the earlier years of undergraduate mathematics. Please submit manuscripts prepared according to the guidelines on the inside front cover to Nazanin Azarnia, Department of Mathematics, Santa Fe Community College Gainesville, FL 32606-6200

How Does the NFL Rate the Passing Ability of Quarterbacks?

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A list [**1**], [**2**], [**3**] of the NFL's "All-Time Leading Passers" is given in Table 1 below. In this list, which is restricted to players with a minimum of 1,500 pass attempts, players are ranked by a single numeric score. This rating system has been widely published in newspapers during recent NFL seasons to gauge the relative performance of quarterbacks for the current season. While it is known that this

Table 1
All-Time Leading Passers (through 1989–1990 season)

Player	Att	Comp	Yards	TD	Int	Rating
Joe Montana	4059	2593	31054	216	107	94.0
Dan Marino	3650	2174	27853	220	125	89.3
Boomer Esiason	2285	1296	18350	126	76	87.3
Dave Krieg	2842	1644	20858	169	116	83.7
Robert Staubach	2958	1685	22700	153	109	83.416
Bernie Kosar	1940	1134	13888	75	47	83.415
Ken O'Brien	2467	1471	17589	96	68	83.0
Jim Kelly	1742	1032	12901	81	63	82.74
Neil Lomax	3153	1817	22771	136	90	82.68
Sonny Jurgensen	4262	2433	32224	255	189	82.6
Len Dawson	3741	2136	28711	239	183	82.6
Ken Anderson	4475	2654	32838	197	160	81.9
Danny White	2950	1761	21959	155	132	81.7
Bart Starr	3149	1808	24718	152	138	80.5
Fran Tarkenton	6467	3686	47003	342	266	80.4
Tony Eason	1536	898	10987	61	50	80.3
Dan Fouts	5604	3297	43040	254	242	80.2
Jim McMahon	1831	1050	13335	77	66	79.2
Bert Jones	2551	1430	18190	124	101	78.2
Johnny Unitas	5186	2830	40239	290	253	78.2

rating is computed as a "percentage of completions, percentage of touchdown passes, percentage of interceptions, and average gain per pass attempt" [3], there seems to be no published account of exactly how it is computed.

As we will see, the NFL apparently computes its ratings of quarterbacks by taking a linear combination of these four statistics (see Table 2), including a nonzero constant term. That is, rating seems to be computed using

$$\text{Rating} = a + b(\%\ \text{Comp}) + c(\%\ \text{TD}) + d(\%\ \text{INT}) + e(\text{YDS}/\text{ATT}) \qquad (1)$$

Table 2
Statistics the Rating is Based Upon

Player	% Comp	% TD's	% Int's	Yds/Att
Joe Montana	63.8827	5.32151	2.63612	7.65065
Dan Marino	59.5616	6.02740	3.42466	7.63096
Boomer Esiason	56.7177	5.51422	3.32604	8.03063
Dave Krieg	57.8466	5.94652	4.08163	7.33920
Roger Staubach	56.9642	5.17241	3.68492	7.67410
Bernie Kosar	58.4536	3.86598	2.42268	7.15876
Ken O'Brien	59.6271	3.89137	2.75638	7.12971
Jim Kelly	59.2422	4.64983	3.61653	7.40586
Neil Lomax	57.6277	4.31335	2.85442	7.22201
Sonny Jurgensen	57.0859	5.98311	4.43454	7.56077
Len Dawson	57.0970	6.38867	4.89174	7.67469
Ken Anderson	59.3073	4.40223	3.57542	7.33810
Danny White	59.6949	5.25424	4.47458	7.44373
Bart Starr	57.4151	4.82693	4.38234	7.84948
Fran Tarkenton	56.9971	5.28839	4.11319	7.26813
Tony Eason	58.4635	3.97135	3.25521	7.15299
Dan Fouts	58.8330	4.53248	4.31834	7.68023
Jim McMahon	57.3457	4.20535	3.60459	7.28291
Bert Jones	56.0564	4.86084	3.95923	7.13054
Johnny Unitas	54.5700	5.59198	4.87852	7.75916

for particular choices of constants a, b, c, d, e. In our case, as rating is not given to us precisely, but generally only to the nearest 0.1, we cannot simply take 5 of our 20 quarterbacks and solve a system of 5 equations in the 5 unknowns a, b, c, d, e. It is well known, however, how to chose a, b, c, d, e to minimize the sum of squared deviations between the given (rounded) ratings and the right-hand side of (1). Let r denote the 20 by 1 vector of rounded ratings, and let A be the 20 by 5 matrix whose first column contains ones and whose second through fifth columns are those given in Table 2. From [4], for example, the solution $x = (a, b, c, d, e)$ minimizing the sum of squared deviations $(r - Ax)^t(r - Ax)$ satisfies

$$(A^tA)x = A^tr.$$

In our case, the above expression is nearly

$$\begin{bmatrix} 20.00 & 1162.79 & 100.01 & 74.69 & 149.38 \\ 1162.79 & 67672.34 & 5809.64 & 4328.66 & 8684.06 \\ 100.01 & 5809.64 & 511.25 & 379.69 & 749.24 \\ 74.69 & 4328.66 & 379.69 & 288.91 & 559.34 \\ 149.38 & 8684.06 & 749.24 & 559.34 & 1117.10 \end{bmatrix} x = \begin{bmatrix} 1655.4 \\ 96336.2 \\ 8294.2 \\ 6155.6 \\ 12370.3 \end{bmatrix}$$

which, solving for x, approximately yields

$$x = (1.91, 0.84, 3.33, -4.16, 4.14).$$

Using the solution for x as a starting point we find that the expression

$$\text{Rating} = \frac{50 + 20(\%\ \text{Comp}) + 80(\%\ \text{TD}) - 100(\%\ \text{INT}) + 100(\text{YDS}/\text{ATT})}{24}$$

gives the ratings of Table 1 up to rounding. Note, for example, that touchdown percentage is deemed to be four times as important as completion percentage with this rating scheme.

Rather than find the solution vector x which minimizes the sum of squared deviations (the "L_2" solution) one could find x to minimize the sum of absolute deviations (the "L_1" solution) or to minimize the maximum absolute deviation (the "L_∞" solution) by solving the appropriate linear programming problem (in our case these three solutions are nearly identical). For those familiar with linear programming the details are given, for example, in [**5**].

References

1. *The 1991 Information Please Almanac*, Houghton Mifflin, Boston, 1990 p. 889.
2. John W. Wright, (Ed.), *The Universal Almanac 1991*, Andrews & McMeel, Kansas City, 1990, p. 602.
3. *The World Almanac and Book of Facts 1991*, Pharos Books, New York, 1990 p. 910.
4. Ben Noble and James Daniel, *Applied Linear Algebra*, 2nd Edition, Prentice Hall, New Jersey, 1977, p. 60.
5. Vašek Chvátal, *Linear Programming*, W. H. Freeman, New York, 1980, pp. 222–226.

Stacking Ellipses—Revisited

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I recently read the September 1991 issue of the *CMJ* and the article *Stacking Ellipses*. The point of the article wasn't expressly stated, but I surmised that it was to determine the area of an ellipse without resorting to calculus, given Cavalieri's principle and the area of a circle. If this is the case, a much simpler proof in the same vein exists, which also does not need to distinguish between rational and irrational values for the semiaxes. Merely inscribe the ellipse $x^2/a^2 + y^2/b^2 = 1$ in its circumscribing circle $x^2/a^2 + y^2/a^2 = 1$ and compare y values. Since they are in the ratio b/a, the areas must be likewise. Hence the area of the ellipse is πab.

Quadratic Confidence Intervals

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In an introductory statistics course, students learn from the central limit theorem that for binomial data with sample proportion $\hat{p}$ and large enough sample size n, $(\hat{p} - p)/\sqrt{p(1-p)/n}$ is approximately distributed as a standard normal variable