International University
School of Electrical Engineering

Introduction to Computers for Engineers

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Lecturely Topics

- Lecture 1 - Basics – variables, arrays, matrices
- Lecture 2 - Basics – matrices, operators, strings, cells
- Lecture 3 - Functions & Plotting
- Lecture 4 - User-defined Functions
- Lecture 5 - Relational & logical operators, if, switch statements
- Lecture 6 - For-loops, while-loops
- Lecture 7 - Review on Midterm Exam
- Lecture 8 - Solving Equations & Equation System (Matrix algebra)
- Lecture 9 - Data Fitting & Integral Computation
- Lecture 10 - Representing Signal and System
- Lecture 11 - Random variables & Wireless System
- Lecture 12 - Review on Final Exam

References: H. Moore, *MATLAB for Engineers*, 4/e, Prentice Hall, 2014
G. Recktenwald, *Numerical Methods with MATLAB*, Prentice Hall, 2000
A. Gilat, *MATLAB, An Introduction with Applications*, 4/e, Wiley, 2011

Matrix Algebra

- dot product
- matrix-vector multiplication
- matrix-matrix multiplication
- matrix inverse
- solving linear systems
- least-squares solutions
- determinant, rank, condition number
- vector & matrix norms
- iterative solutions of linear systems
- examples
- electric circuits
- temperature distributions

Operators and Expressions

operation	element-wise	matrix-wise
addition	`+`	`+`
subtraction	`-`	`-`
multiplication	`.*`	`*`
division	`./`	`/`
left division	`.\`	`\`
exponentiation	`.^`	`^`
transpose w/o complex conjugation	`.'`	
transpose with complex conjugation	`'`	

```
>> help /
>> help precedence
```

used in matrix algebra operations

```
>> A = [1 2; 3 4]
A =
     1     2
     3     4
```

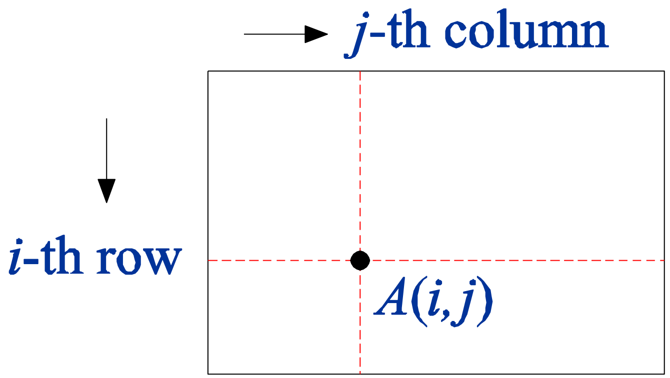
$$\begin{bmatrix}1 & 2\\3 & 4\end{bmatrix}\begin{bmatrix}1 & 2\\3 & 4\end{bmatrix} = \begin{bmatrix}7 & 10\\15 & 22\end{bmatrix}$$

```
>> [A, A.^2; A^2, A*A]          % form sub-blocks
ans =
      1      2      1      4
      3      4      9     16
      7     10      7     10    % note A^2 = A*A
     15     22     15     22
```

```
>> B = 10.^A;
>> [B, log10(B)]
ans =
          10          100            1            2
        1000        10000            3            4
```

$$B = \begin{bmatrix}10^1 & 10^2\\10^3 & 10^4\end{bmatrix}$$

$$A = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix}$$



matrix indexing convention

```
>> A = [1 2 3; 2 0 4; 0 8 5]
A =
     1     2     3
     2     0     4
     0     8     5

>> size(A)           % [N,M] = size(A), NxM matrix
ans =
     3     3
```

dot product

The dot product is the basic operation in matrix-vector and matrix-matrix multiplications

$$\mathbf{a} = \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

a, **b** must have the same dimension

$$\mathbf{a}^T \mathbf{b} = [a_1,\, a_2,\, a_3] \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} = a_1 b_1 + a_2 b_2 + a_3 b_3$$

$$\mathbf{a}^T \mathbf{b} = \mathbf{a}' \, \mathbf{b} = \mathbf{a} \cdot \mathbf{b} = \mathbf{a} \, .' * \mathbf{b}$$

math notations

MATLAB notation

dot product for complex-valued vectors

complex-conjugate transpose, or, hermitian conjugate of **a**

$$\mathbf{a}^\dagger \mathbf{b} = [a_1^*, a_2^*, a_3^*] \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} = a_1^* b_1 + a_2^* b_2 + a_3^* b_3$$

$$\mathbf{a}^\dagger \mathbf{b} = \mathbf{a}^H \mathbf{b} = \mathbf{a}' * \mathbf{b}$$

math notations

MATLAB notation

for real-valued vectors, the operations `'` and `.'` are equivalent

$$\mathbf{a} = \begin{bmatrix} 1 \\ 2 \\ -3 \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} 4 \\ -5 \\ 2 \end{bmatrix}$$

$$[1,\ 2,\ -3]\begin{bmatrix} 4 \\ -5 \\ 2 \end{bmatrix} = 1 \times 4 + 2 \times (-5) + (-3) \times 2 = -12$$

```
>> a = [1; 2; -3];  b = [4; -5; 2];
>> a'*b
ans =
   -12
>> dot(a,b)                 % built-in function
ans =                       % same as sum(a.*b)
   -12
```

matrix-vector multiplication

$$[4,\ 1,\ 2]\begin{bmatrix} 5 \\ -4 \\ -7 \end{bmatrix} = 2$$

combine three dot product operations into a single matrix-vector multiplication

$$[1,\ -1,\ 1]\begin{bmatrix} 5 \\ -4 \\ -7 \end{bmatrix} = 2 \qquad \Rightarrow \qquad \begin{bmatrix} 4 & 1 & 2 \\ 1 & -1 & 1 \\ 2 & 1 & 1 \end{bmatrix}\begin{bmatrix} 5 \\ -4 \\ -7 \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \\ -1 \end{bmatrix}$$

$$[2,\ 1,\ 1]\begin{bmatrix} 5 \\ -4 \\ -7 \end{bmatrix} = -1$$

matrix-vector multiplication

combine three dot product operations into a single matrix-vector multiplication

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} a_{11}x_1 + a_{12}x_2 + a_{13}x_3 \\ a_{21}x_1 + a_{22}x_2 + a_{23}x_3 \\ a_{31}x_1 + a_{32}x_2 + a_{33}x_3 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

$$\mathbf{A}\,\mathbf{x} = \mathbf{b}$$

matrix-matrix multiplication

$$\begin{bmatrix} 4 & 1 & 2 \\ 1 & -1 & 1 \\ 2 & 1 & 1 \end{bmatrix} \begin{bmatrix} 5 \\ -4 \\ -7 \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \\ -1 \end{bmatrix}$$

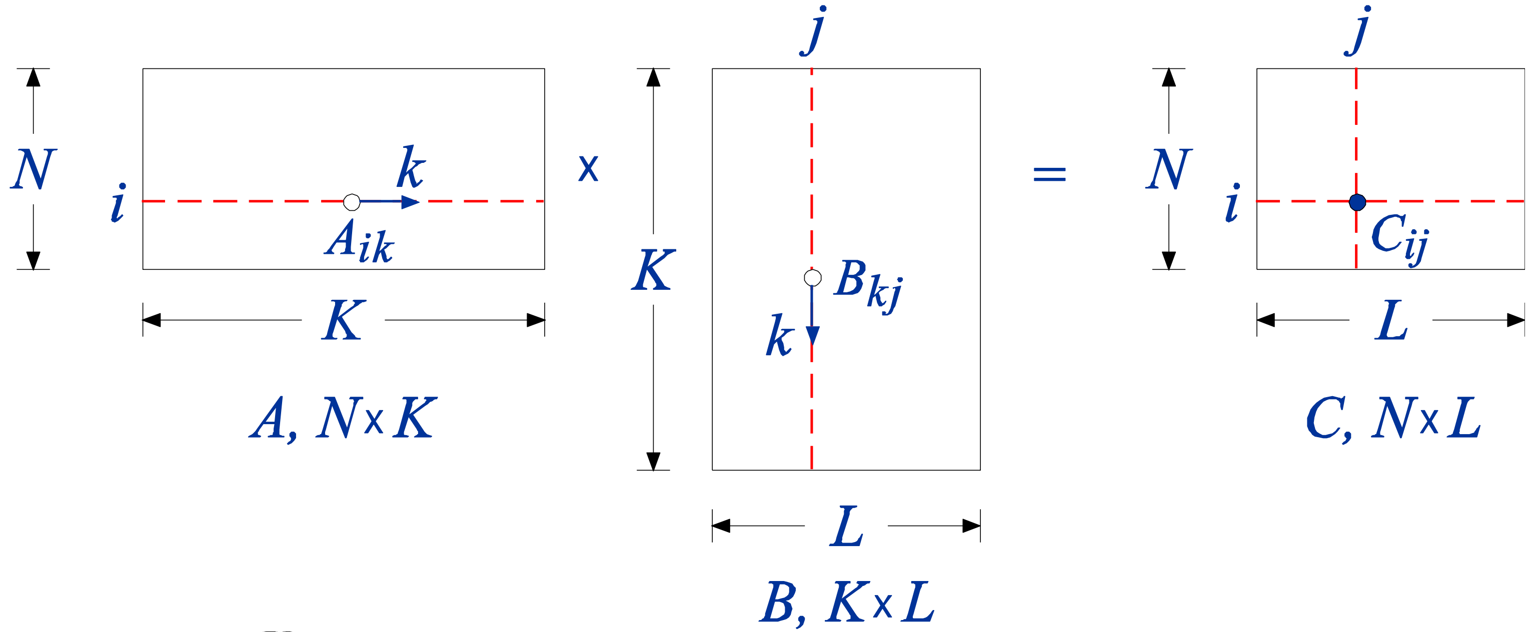
$$\begin{bmatrix} 4 & 1 & 2 \\ 1 & -1 & 1 \\ 2 & 1 & 1 \end{bmatrix} \begin{bmatrix} -1 \\ 3 \\ 2 \end{bmatrix} = \begin{bmatrix} 3 \\ -2 \\ 3 \end{bmatrix}$$

$$\begin{bmatrix} 4 & 1 & 2 \\ 1 & -1 & 1 \\ 2 & 1 & 1 \end{bmatrix} \begin{bmatrix} -3 \\ 1 \\ 6 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$$

combine three matrix-vector multiplications into a single matrix-matrix multiplication

$$\begin{bmatrix} 4 & 1 & 2 \\ 1 & -1 & 1 \\ 2 & 1 & 1 \end{bmatrix} \begin{bmatrix} 5 & -1 & -3 \\ -4 & 3 & 1 \\ -7 & 2 & 6 \end{bmatrix} = \begin{bmatrix} 2 & 3 & 1 \\ 2 & -2 & 2 \\ -1 & 3 & 1 \end{bmatrix}$$

```
>> A = [4 1 2; 1 -1 1; 2 1 1]
A =
     4     1     2
     1    -1     1
     2     1     1

>> B = [5 -1 -3; -4 3 1; -7 2 6]
B =
     5    -1    -3
    -4     3     1
    -7     2     6

>> C = A*B
C =
     2     3     1
     2    -2     2
    -1     3     1
```



$$C_{ij} = \sum_{k=1}^{K} A_{ik} B_{kj}, \qquad 1 \le i \le N, \quad 1 \le j \le L$$

$C(i,j)$ is the dot product of i-th row of A with j-th column of B

$$\begin{bmatrix} 4 & 1 & 2 \\ 1 & -1 & 1 \\ 2 & 1 & 1 \end{bmatrix} \begin{bmatrix} 5 & -1 & -3 \\ -4 & 3 & 1 \\ -7 & 2 & 6 \end{bmatrix} = \begin{bmatrix} 2 & 3 & 1 \\ 2 & -2 & 2 \\ -1 & 3 & 1 \end{bmatrix}$$

$$2 \times (-1) + 1 \times 3 + 1 \times 2 = 3$$

note:
A*B ≠ B*A

$$
\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}
\begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}
$$

$$
= \left[\begin{array}{c|c} a_{11}b_{11} + a_{12}b_{21} & a_{11}b_{12} + a_{12}b_{22} \\ \hline a_{21}b_{11} + a_{22}b_{21} & a_{21}b_{12} + a_{22}b_{22} \end{array}\right]
$$

Rule of thumb:

(NxK)x(KxM) --> NxM

A is NxK
B is KxM
then, A*B is NxM

vector-vector multiplication

$$[a_1, a_2, a_3]\begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} = a_1 b_1 + a_2 b_2 + a_3 b_3$$

(1x3)x(3x1) --> 1x1 = scalar

row * column = scalar

$$\begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix}[b_1, b_2, b_3] = \begin{bmatrix} a_1 b_1 & a_1 b_2 & a_1 b_3 \\ a_2 b_1 & a_2 b_2 & a_2 b_3 \\ a_3 b_1 & a_3 b_2 & a_3 b_3 \end{bmatrix}$$

(3x1)x(1x3) --> 3x3

column * row = matrix

vector-vector multiplication

```
>> [1, 2, 3] * [2 -3 -1]'

ans =

    -7

>> [1, 2, 3]' * [2 -3 -1]

ans =

     2    -3    -1
     4    -6    -2
     6    -9    -3
```

row x column
= scalar

column x row
= matrix

solving linear systems

$\mathbf{A\,x = b}$

Linear equations have a very large number of applications in engineering, science, social sciences, and economics

Linear Programming – Management Science

Computer Aided Design – aerodynamics of cars, planes

Signal Processing, Communications, Control, Radar, Sonar, Electromagnetics, Oil Exploration, Computer Vision, Pattern & Face Recognition

Chip Design – millions of transistors on a chip

Economic Models, Finance, Statistical Models, Data Mining, Social Models, Financial Engineering

Markov Models – Speech, Biology, Google Pagerank

Scientific Computing – solving very large problems

the only practical way to solve very large systems is iteratively

solving linear systems

$$
\begin{aligned}
2x_1 + x_2 &= 4 \\
x_1 + 5x_2 - x_3 &= 8 \\
x_1 - 2x_2 + 4x_3 &= 9
\end{aligned}
\quad \Rightarrow \quad
\begin{bmatrix} 2 & 1 & 0 \\ 1 & 5 & -1 \\ 1 & -2 & 4 \end{bmatrix}
\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}
=
\begin{bmatrix} 4 \\ 8 \\ 9 \end{bmatrix}
$$

$$A\mathbf{x} = \mathbf{b}$$

matrix inverse

$$\boxed{A\mathbf{x} = \mathbf{b} \quad \Rightarrow \quad \mathbf{x} = A^{-1}\mathbf{b} = A \backslash \mathbf{b}}$$

always use the **`backslash`** operator to solve a linear system, instead of **`inv(A)`**

solving linear systems (using backslash)

$$\begin{aligned} 2x_1 + x_2 &= 4 \\ x_1 + 5x_2 - x_3 &= 8 \\ x_1 - 2x_2 + 4x_3 &= 9 \end{aligned} \quad \Rightarrow \quad \begin{bmatrix} 2 & 1 & 0 \\ 1 & 5 & -1 \\ 1 & -2 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 4 \\ 8 \\ 9 \end{bmatrix}$$

```
>> A = [2 1 0; 1 5 -1; 1 -2 4];
>> b = [4 8 9]';
>> x = A\b              % solution of A*x = b
x =                     % x = A^-1 * b
    1                   % x = inv(A) * b
    2
    3

>> norm(A*x-b)          % test - should be zero
ans =                   % or, of the order of eps
     0
```

solving linear systems (using inv)

```
>> A = [2 1 0; 1 5 -1; 1 -2 4];
>> b = [4 8 9]';
>> inv(A)                     % same as A^(-1)
ans =
    0.5806   -0.1290   -0.0323
   -0.1613    0.2581    0.0645
   -0.2258    0.1613    0.2903

>> x = inv(A) * b             % but prefer backslash
x =                           % same as x = A^-1 * b
    1.0000
    2.0000
    3.0000

>> norm(A*x-b)
ans =
  1.8310e-015
```

```
>> inv(sym(A))
ans =
[ 18/31, -4/31, -1/31]
[ -5/31,  8/31,  2/31]
[ -7/31,  5/31,  9/31]
```

solving linear systems – back-slash and forward-slash

`A` of size **`NxN`** and invertible

`X` of size **`NxK`**

`B` of size **`NxK`**

equivalent

```
AX = B   -->   X = A\B = inv(A)*B
```

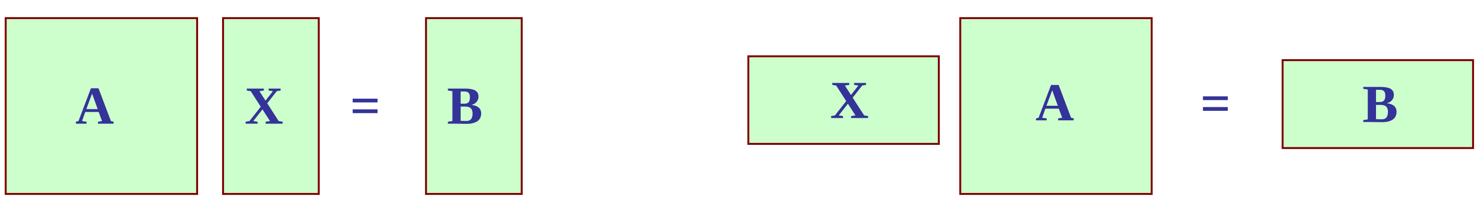
`A` of size **`NxN`** and invertible

`X` of size **`KxN`**

`B` of size **`KxN`**

equivalent

```
XA = B   -->   X = B/A = B*inv(A)
```



solving linear systems – least-squares solutions

A of size **NxM**
x of size **Mx1** column
b of size **Nx1** column

```
x = A\b
```

will be used in
in weeks 11 & 12

x=A\b is a solution of **Ax=b**
in a least-squares sense,
i.e., **x** minimizes the norm squared
of the error **e = b - A*x**:
(b-Ax)'*(b-Ax) = min

x may or may not be unique
depending on whether the linear
system **Ax=b** is over-determined,
under-determined, or whether **A** has
full rank or not

pseudo-inverse

```
x = pinv(A)*b;
```

```
>> help \
>> help pinv
```

least-squares solutions - summary

Fundamental Theorem of Linear Algebra – what is it?

`A = NxM` matrix `A' = MxN` matrix
`x = Mx1` column `A'*A = MxM` matrix
`b = Nx1` column `A'*b = Mx1` column

Assuming full rank for **A**, we have the following cases:

1. `N>M`, overdetermined case, (most common in practice)
   `x = A\b =` unique least-squares solution, same as
   `x = pinv(A)*b,` and
   `x = (A'*A)^(-1) * (A'*b)`

   `x = A\b` is numerically the most accurate method

2. `N<M`, underdetermined case, (there are many solutions)
   `x=A\b, x=pinv(A)*b,` are two possible solutions

3. `N=M`, square invertible case, **x** is unique
   `x = A\b` is equivalent to `x = A^(-1)*b`

least-squares solutions - example

```
% overdetermined
% full-rank example

A = [1 2; 3 4; 5 6]
b = [4, 3, 8]';

x = A\b
% x = pinv(A)*b
% x = (A'*A)\(A'*b)

x =
    -1
     2
```

overdetermined system of

3 equations in 2 unknowns

$$x_1 + 2x_2 = 4$$
$$3x_1 + 4x_2 = 3$$
$$5x_1 + 6x_2 = 8$$

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 4 \\ 3 \\ 8 \end{bmatrix}$$

$$e = b - Ax = \begin{bmatrix} 4 \\ 3 \\ 8 \end{bmatrix} - \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 4 - x_1 - 2x_2 \\ 3 - 3x_1 - 4x_2 \\ 8 - 5x_1 - 6x_2 \end{bmatrix} = \text{error}$$

least-squares solutions - example

$$J = e^T e = (b - Ax)^T (b - Ax) = x^T (A^T A) x - 2x^T (A^T b) + b^T b = \min$$

$$\frac{\partial J}{\partial x} = 2A^T (Ax - b) = 0 \quad \Rightarrow \quad \boxed{x_{\text{opt}} = (A^T A)^{-1} A^T b}$$

inverse exists because ***A*** was assumed to have full rank

$$J_{\min} = J|_{x = x_{\text{opt}}} = \boxed{b^T b - b^T A (A^T A)^{-1} A^T b}$$

minimized value of ***J*** achieved at ***x* = *x*_opt**

$$\boxed{J = (x - x_{\text{opt}})^T (A^T A)(x - x_{\text{opt}}) + J_{\min} \geq J_{\min}}$$

strictly positive-definite quadratic form because of full rank of ***A***, vanishes only at ***x* = *x*_opt**

least-squares solutions - example

```
A = [1 2; 3 4; 5 6]
b = [4, 3, 8]';

x_opt = (A'*A)\(A'*b)

J_min = b'*b - ...
  b'*A*inv(A'*A)*A'*b


x_opt =
        -1
         2
J_min =
         6
```

$$A^T A = \begin{bmatrix} 1 & 3 & 5 \\ 2 & 4 & 6 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix} = \begin{bmatrix} 35 & 44 \\ 44 & 56 \end{bmatrix}$$

$$A^T b = \begin{bmatrix} 1 & 3 & 5 \\ 2 & 4 & 6 \end{bmatrix} \begin{bmatrix} 4 \\ 3 \\ 8 \end{bmatrix} = \begin{bmatrix} 53 \\ 68 \end{bmatrix}$$

$$b^T b = [4,\ 3,\ 8] \begin{bmatrix} 4 \\ 3 \\ 8 \end{bmatrix} = 89$$

$$x_{\mathrm{opt}} = (A^T A)^{-1} A^T b = \begin{bmatrix} -1 \\ 2 \end{bmatrix}, \quad J_{\min} = b^T b - b^T A^T (A^T A)^{-1} A b = 6$$

least-squares solutions - example

$$e = b - Ax = \begin{bmatrix} 4 \\ 3 \\ 8 \end{bmatrix} - \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 4 - x_1 - 2x_2 \\ 3 - 3x_1 - 4x_2 \\ 8 - 5x_1 - 6x_2 \end{bmatrix} = \text{error}$$

$$J = (4 - x_1 - 2x_2)^2 + (3 - 3x_1 - 4x_2)^2 + (8 - 5x_1 - 6x_2)^2$$

$$= [x_1, x_2] \begin{bmatrix} 35 & 44 \\ 44 & 56 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} - 2 \cdot [53,\ 68] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + 89$$

$$= 35x_1^2 + 88x_1x_2 + 56x_2^2 - 106x_1 - 136x_2 + 89$$

$$= 35(x_1 + 1)^2 + 88(x_1 + 1)(x_2 - 2) + 56(x_2 - 2)^2 + 6$$

$$= [x_1 + 1,\ x_2 - 2] \begin{bmatrix} 35 & 44 \\ 44 & 56 \end{bmatrix} \begin{bmatrix} x_1 + 1 \\ x_2 - 2 \end{bmatrix} + 6\,, \quad x - x_{\text{opt}} = \begin{bmatrix} x_1 + 1 \\ x_2 - 2 \end{bmatrix}$$

least-squares solutions - example

$$J = 35(x_1+1)^2 + 88(x_1+1)(x_2-2) + 56(x_2-2)^2 + 6 \geq 6$$

J is minimized at $x_1 = -1$, $x_2 = 2$, with minimum value, $J = 6$

```
% we can also minimize J with fminsearch,
% i.e., the multivariable version of fminbnd

J = @(x) 35*(x(1)+1).^2 +...
         88*(x(1)+1).*(x(2)-2)+...
         56*(x(2)-2).^2 + 6;

x0 = [0,0]';    % arbitrary initial search point

[xmin,Jmin] = fminsearch(J,x0)

% xmin =              % Jmin = 6
%      -1.0000
%       2.0000
```

Invertibility, rank, determinants, condition number

The inverse **`inv(A)`** of an **`NxN`** square matrix **`A`** exists if its **determinant** is non-zero, or, equivalently if it has **full rank**, i.e., when its **rank** is equal to the row or column dimension **`N`**

```
>> doc inv
>> doc det
>> doc rank
>> doc cond
```

```
a = [1 2 3]'; b = [4 5 6]';
A = [a, a+b, b]

A =
      1      5      4
      2      7      5
      3      9      6
```

`det(A) = 0`

```
>> det(A)

ans =
     0


>> rank(A)

ans =
     2
```

Invertibility, rank, determinants, condition number

The larger the **cond(A)** the more ill-conditioned the linear system, and the less reliable the solution.

```
A = [1,  5,           4
     2,  7 + 1e-8,    5
     3,  9,           6];
```

```
>> cond(A)
ans =
   3.3227e+009
```

```
A\[1; 2; 3]

ans =
     1
     0
     0
```

```
A\[1.001; 2.0002; 3.000003]

ans =
      30150.999185
     -30150.000183
      30150.000683
```

```
det(A) = -6.0000e-008
```

Determinant and inverse of a 2x2 matrix

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$A^{-1} = \frac{1}{ad - bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

$$\det(A) = ad - bc$$

Example:

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}^{-1} = \frac{1}{4-6} \begin{bmatrix} 4 & -2 \\ -3 & 1 \end{bmatrix} = \begin{bmatrix} -2 & 1 \\ 1.5 & -0.5 \end{bmatrix}$$

Matrix Exponential

Used widely in solving linear dynamic systems

$$\exp(A) = \sum_{n=0}^{\infty} \frac{A^n}{n!} = 1 + A + \frac{A^2}{2!} + \frac{A^3}{3!} + \cdots$$

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

```
>> A = [1 2;3 4];

>> expm(A)        % matrix exponential
ans =
   51.9690   74.7366
  112.1048  164.0738

>> exp(A)         % element-wise exponential
ans =
    2.7183    7.3891
   20.0855   54.5982
```

```
>> doc expm
>> doc exp
```

Vector & Matrix Norms

>> doc norm

L_1, L_2, and L_∞ norms of a vector

$$\mathbf{x} = [x_1, x_2, \ldots, x_N]$$

$$\|\mathbf{x}\|_1 = \sum_{n=1}^{N} |x_n|$$

L_1 norm

$$\|\mathbf{x}\|_2 = \sqrt{\sum_{n=1}^{N} |x_n|^2}$$

Euclidean, L_2 norm

$$\|\mathbf{x}\|_\infty = \max(|x_1|, |x_2|, \ldots, |x_N|)$$

L_∞ norm

$$d(\mathbf{x}, \mathbf{y}) = \|\mathbf{x} - \mathbf{y}\|$$

used as distance measure between two vectors or matrices

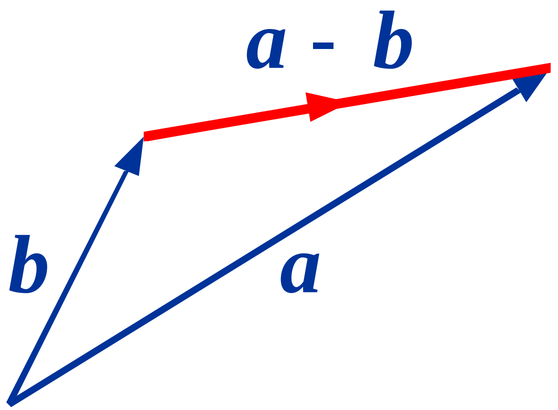
```
x = [1, -4, 5, 3]; p = inf;

switch p
    case 1
        N = sum(abs(x));                % N = norm(x,1);
    case 2
        N = sqrt(sum(abs(x).^2));       % N = norm(x,2);
    case inf
        N = max(abs(x));                % N = norm(x,inf);
    otherwise
        N = sqrt(sum(abs(x).^2));       % N = norm(x,2);
end
```

equivalent calculation using the built-in function **norm** :

useful for comparing two vectors or matrices

```
>> norm(a-b)      % a,b vectors of same size
>> norm(A-B)      % A,B matrices of same size
```

$$\mathbf{a} = \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$



$$\|\mathbf{a}-\mathbf{b}\|_2 = \mathrm{norm}(\mathbf{a}-\mathbf{b})$$

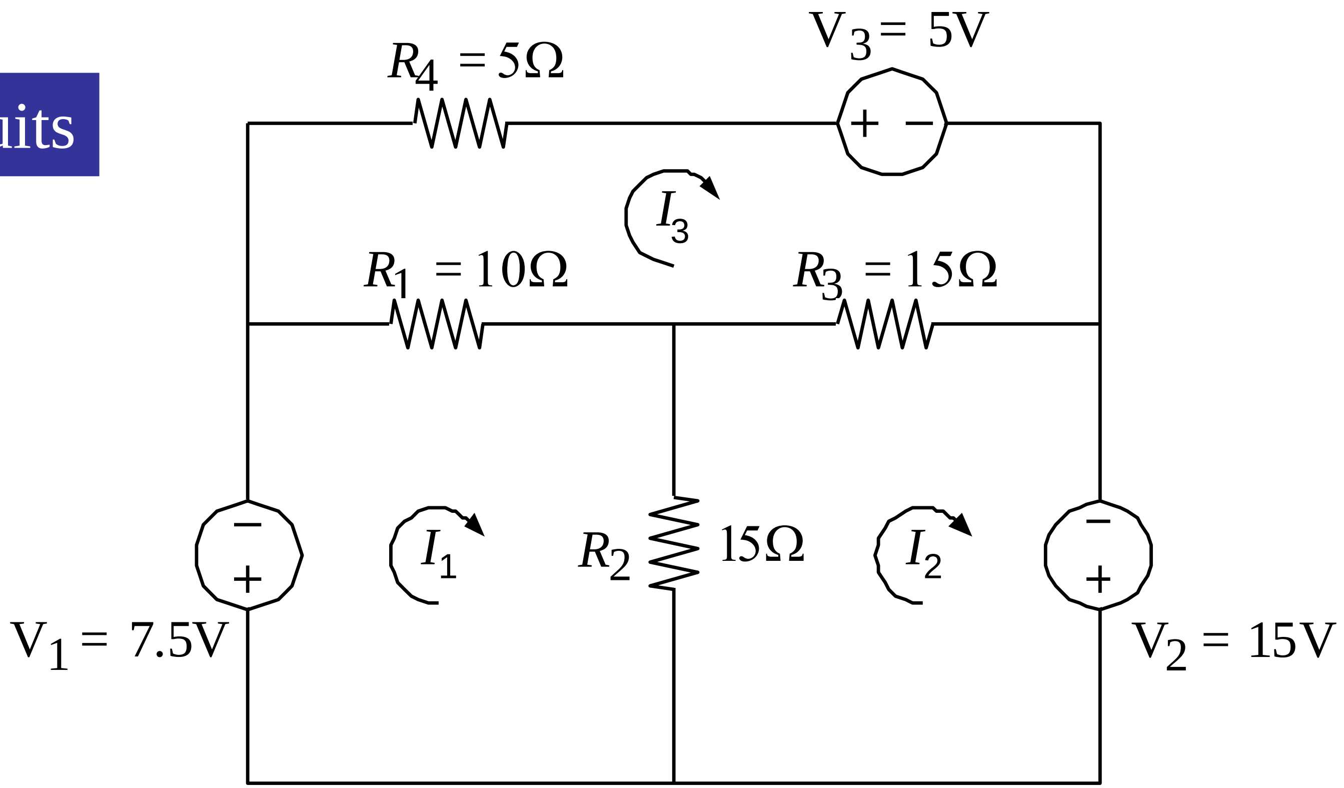
$$= \sqrt{(a_1-b_1)^2+(a_2-b_2)^2+(a_3-b_3)^2}$$

$$= \sqrt{(\mathbf{a}-\mathbf{b})'(\mathbf{a}-\mathbf{b})}$$

Euclidean distance

dot product

Electric Circuits



Kirchhoff's Voltage Law

$$R_1(I_1 - I_3) + R_2(I_1 - I_2) + V_1 = 0$$

$$R_2(I_2 - I_1) + R_3(I_2 - I_3) - V_2 = 0$$

$$R_4 I_3 + R_3(I_3 - I_2) + R_1(I_3 - I_1) + V_3 = 0$$

Electric Circuits

$$(R_1 + R_2)I_1 - R_2 I_2 - R_1 I_3 = -V_1$$

$$-R_2 I_1 + (R_2 + R_3)I_2 - R_3 I_3 = V_2$$

$$-R_1 I_1 - R_3 I_2 + (R_1 + R_3 + R_4)I_3 = -V_3$$

$$\begin{bmatrix} R_1 + R_2 & -R_2 & -R_1 \\ -R_2 & R_2 + R_3 & -R_3 \\ -R_1 & -R_3 & R_1 + R_3 + R_4 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} -V_1 \\ V_2 \\ -V_3 \end{bmatrix}$$

$$R_1 = 10, \quad R_2 = 15, \quad R_3 = 15, \quad R_4 = 5$$

$$V_1 = 7.5, \quad V_2 = 15, \quad V_3 = 10$$

$$\begin{bmatrix} R_1 + R_2 & -R_2 & -R_1 \\ -R_2 & R_2 + R_3 & -R_3 \\ -R_1 & -R_3 & R_1 + R_3 + R_4 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} -V_1 \\ V_2 \\ -V_3 \end{bmatrix}$$

$$\begin{bmatrix} 25 & -15 & -10 \\ -15 & 30 & -15 \\ -10 & -15 & 30 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} -7.5 \\ 15 \\ -5 \end{bmatrix}$$

$$\mathbf{A}\,\mathbf{x} = \mathbf{b}$$

```
A = [25, -15, -10; -15, 30, -15; -10, -15, 30]
b = [-7.5; 15; -5]

A =
     25   -15   -10
    -15    30   -15
    -10   -15    30

b =
    -7.5000
    15.0000
    -5.0000

x = A\b

x =
     0.5000
     1.0000
     0.5000
```

$$\mathbf{x} = \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 0.5 \\ 1.0 \\ 0.5 \end{bmatrix}$$

```
inv(A)

ans =
     0.2571     0.2286     0.2000
     0.2286     0.2476     0.2000
     0.2000     0.2000     0.2000

inv(sym(A)) --> (1/105) * [27    24    21
                           24    26    21
                           21    21    21]
```

$$\mathbf{x} = A^{-1}\mathbf{b} = \frac{1}{105}\begin{bmatrix} 27 & 24 & 21 \\ 24 & 26 & 21 \\ 21 & 21 & 21 \end{bmatrix}\begin{bmatrix} -7.5 \\ 15 \\ -5 \end{bmatrix} = \begin{bmatrix} 0.5 \\ 1.0 \\ 0.5 \end{bmatrix}$$