Name:

ID :

PRINCIPLES OF EE1

Homework #4

IMPORTANT: You should write on **A4 paper** that contains a full and detailed description of all the work done on the homework. Then you must submit the test hand-written by scanning and uploading the file in **pdf** form on Blackboard (Assignment Session). Marks will be deducted if there are sign of violation of regulation and late submission (20% for each day).

Tip: You draw a bounding box or highlight for your final answer. Ex: Y = ABC + AC = $\boxed{ABC}$

Problem 1: (25 marks) Perform some operations below.

In rectangular form:

a. $(4.2 + j6.8) + (7.6 + j0.2)$

b. $(4 \times 10^{-6} + j76) + (7.2 \times 10^{-7} - j5)$

c. $42\angle 45^0 + 62\angle 60^0 - 70\angle 120^0$

In polar form:

d. $(400 - j200)(-0.01 - j0.5)(-1 + j3)$

e. $\frac{-4.5-j6}{0.1-j0.8}$

f. $\frac{42\angle 10^0}{7\angle 60^0}$

g. $\frac{8\angle 60^0}{(2\angle 0^0)+(100+j400)}$

h. $\frac{(6\angle 20^0)(120\angle -40^0)(3+j8)}{(2\angle -30^0)}$

Solution:

a. $11.8 + j7.0$

b. $4.72 \times 10^{-6} + j71$

c. $(29.698 + j29.698) + (31.0 + j53.69) - (-35 + j60.62) = 95.7 + j22.77$

d. $707.2 \angle -9.27^\circ$

e. $9.30 \angle -43.99^\circ$

f. $6.0 \angle -50^\circ$

g. $19.38 \times 10^{-3} \angle -15.69^{\circ}$

h. $3.07 \times 10^{3} \angle 79.44^{\circ}$

Problem 2: **(25 marks)**

The maximum amplitude of a sinusoidal current is $40A$. The current passes through one complete cycle in $0.5\ ms$. The magnitude of the initial current is $10\ A$. Finding the characteristics of a Sinusoidal Current via questions as below:

a. What is the frequency (in Hz) of the current?

b. What is the frequency in rad/s?

c. Write the expression for $i(t)$ using the cosine function. Express ϕ in degrees.

d. What is the rms value of the current?

Solution:

a. We have: $T = 0.5\ ms$. So $\boldsymbol{f = 1/T = 2000\ Hz.}$

b. $\boldsymbol{\omega = 2\pi f = 4000\pi\ rad/s.}$

c.

We have:

$$i(t) = I_m cos(\omega t + \phi) = 40\ cos(2000\pi t + \phi),$$

and $i(0) = 10\ A.$

$\Rightarrow 10 = 40 \cos\phi \quad \Rightarrow \quad \boldsymbol{\phi = 75.5^{\circ}}$

Thus, the expression for $i(t)$: $\boldsymbol{i(t) = 40cos(4000\pi t + 75.5^{\circ}).}$

d.

From $I_{rms} = \frac{I_m}{\sqrt{2}}$, we get the rms value of a sinusoidal current is $40/\sqrt{2}$.

Therefore, the rms value is: $\boldsymbol{I_{rms} = \frac{40}{\sqrt{2}} = 28.28\ A.}$

Problem 3: **(25 marks)**

Determine the total input impedance Z_{in} of the following circuit

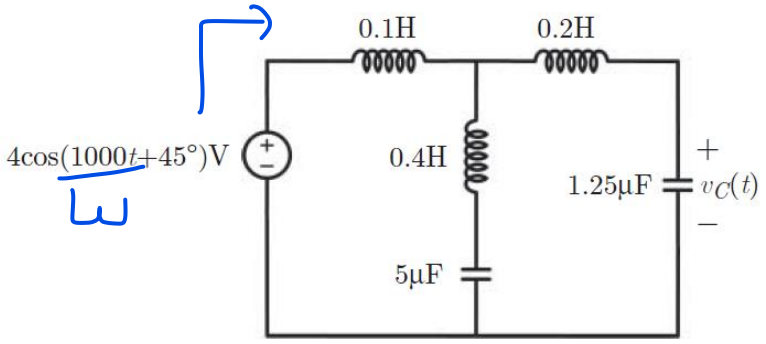


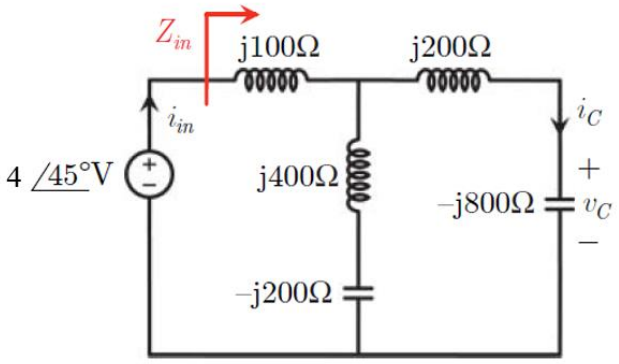
Figure 1

Solution:

We have: $\omega = 1000 \frac{\text{rad}}{\text{s}}$

The impedances of some circuit elements are calculated:

$$0.1\,H \longrightarrow j \times 1000 \times 0.1 = j100\,\Omega,$$
$$0.2\,H \longrightarrow j \times 1000 \times 0.2 = j200\,\Omega,$$
$$0.4\,H \longrightarrow j \times 1000 \times 0.4 = j400\,\Omega,$$
$$1.25\,\mu F \longrightarrow 1/(j \times 1000 \times 1.25 \times 10-6) = -j800\,\Omega,$$
$$5\,\mu F \longrightarrow 1/(j \times 1000 \times 5 \times 10-6) = -j200$$



The total impedance:

$$\boldsymbol{Z_{in}} = j100 + [(j400 - j200) \parallel (j200 - j800)]$$
$$= j100 + \frac{j200 \times (-j600)}{j200 - j600} = j100 + j300 = \boldsymbol{j400\,\Omega}.$$

Problem 4: **(25 marks)**

In the following circuit, the voltage source is $v_s(t) = 3\sqrt{2}\,cos(2000t + 45^0)V$

a/ Find $v_L(t)$ in steady state.

b/ Compare phase angle between $v_L(t)$ and voltage source, state which one is leading?

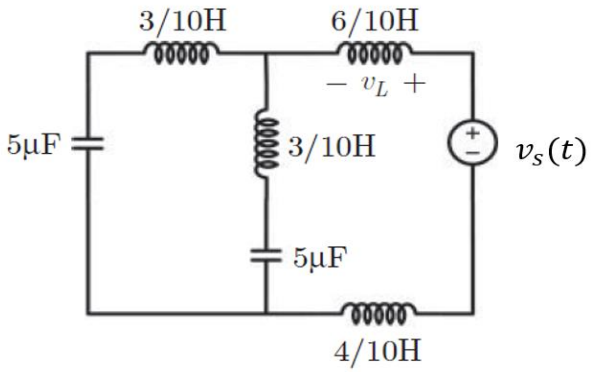


Figure 2

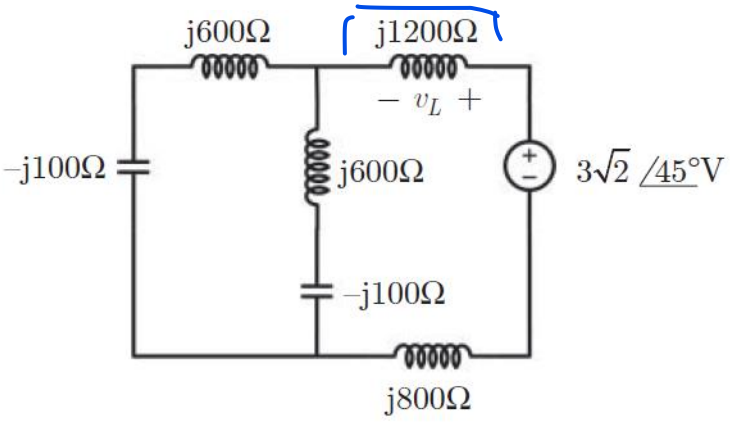
Solution:

a/

We have: $v_s(t) = 3\sqrt{2}\,cos(2000t + 45^0)V \longrightarrow 3\sqrt{2}\angle 45^0 = 3(1 + j)$ V,

The impedances of some circuit elements are calculated:

- $3/10\,H \longrightarrow j \times 2000 \times 3/10 = j600\,\Omega$,
- $4/10\,H \longrightarrow j \times 2000 \times 4/10 = j800\,\Omega$,
- $6/10\,H \longrightarrow j \times 2000 \times 6/10 = j1200\,\Omega$,
- $5\,\mu F \longrightarrow 1/(j \times 2000 \times 5 \times 10 - 6) = -j100\,\Omega$,



The total impedance:

$$Z_{in} = (j600 - j100) \parallel (j600 - j100) + j1200 + j800 = j2250\,\Omega.$$

The current through inductor $j1200$: $\boldsymbol{I}_L = \frac{3+j3}{j2250} = \frac{1+j}{750}\ (A)$

The voltage through inductor $j1200$: $\boldsymbol{V}_L = j1200 \times \boldsymbol{I}_L = \frac{8+j8}{5} = \frac{8\sqrt{2}}{5} \angle 45^0 (A)$

So, in time domain: $\boldsymbol{v_L(t) = 2.26\cos(2000t + 45^0)}$

b/

The voltage of the inductor $v_L(t)$ is in phase with the voltage source.