9

Sinusoidal Steady State Analysis

Assessment Problems

AP 9.1 [a] $\mathbf{V} = 170\underline{/-40^\circ}\,\text{V}$

[b] $10\sin(1000t + 20^\circ) = 10\cos(1000t - 70^\circ)$

$$\therefore \qquad \mathbf{I} = 10\underline{/-70^\circ}\,\text{A}$$

[c] $\mathbf{I} = 5\underline{/36.87^\circ} + 10\underline{/-53.13^\circ}$

$$= 4 + j3 + 6 - j8 = 10 - j5 = 11.18\underline{/-26.57^\circ}\,\text{A}$$

[d] $\sin(20{,}000\pi t + 30^\circ) = \cos(20{,}000\pi t - 60^\circ)$

Thus,

$$\mathbf{V} = 300\underline{/45^\circ} - 100\underline{/-60^\circ} = 212.13 + j212.13 - (50 - j86.60)$$

$$= 162.13 + j298.73 = 339.90\underline{/61.51^\circ}\,\text{mV}$$

AP 9.2 [a] $v = 18.6\cos(\omega t - 54^\circ)\,\text{V}$

[b] $\mathbf{I} = 20\underline{/45^\circ} - 50\underline{/-30^\circ} = 14.14 + j14.14 - 43.3 + j25$

$$= -29.16 + j39.14 = 48.81\underline{/126.68^\circ}$$

Therefore $\quad i = 48.81\cos(\omega t + 126.68^\circ)\,\text{mA}$

[c] $\mathbf{V} = 20 + j80 - 30\underline{/15^\circ} = 20 + j80 - 28.98 - j7.76$

$$= -8.98 + j72.24 = 72.79\underline{/97.08^\circ}$$

$$v = 72.79\cos(\omega t + 97.08^\circ)\,\text{V}$$

AP 9.3 [a] $\omega L = (10^4)(20 \times 10^{-3}) = 200\,\Omega$

[b] $Z_L = j\omega L = j200\,\Omega$

[c] $\mathbf{V}_L = \mathbf{I}Z_L = (10\underline{/30^\circ})(200\underline{/90^\circ}) \times 10^{-3} = 2\underline{/120^\circ}\,\text{V}$

[d] $v_L = 2\cos(10{,}000t + 120^\circ)\,\text{V}$

AP 9.4 **[a]** $X_C = \dfrac{-1}{\omega C} = \dfrac{-1}{4000(5\times10^{-6})} = -50\,\Omega$

[b] $Z_C = jX_C = -j50\,\Omega$

[c] $\mathbf{I} = \dfrac{\mathbf{V}}{Z_C} = \dfrac{30\underline{/25^\circ}}{50\underline{/-90^\circ}} = 0.6\underline{/115^\circ}\,\text{A}$

[d] $i = 0.6\cos(4000t + 115^\circ)\,\text{A}$

AP 9.5 $\mathbf{I}_1 = 100\underline{/25^\circ} = 90.63 + j42.26$

$\mathbf{I}_2 = 100\underline{/145^\circ} = -81.92 + j57.36$

$\mathbf{I}_3 = 100\underline{/-95^\circ} = -8.72 - j99.62$

$\mathbf{I}_4 = -(\mathbf{I}_1 + \mathbf{I}_2 + \mathbf{I}_3) = (0 + j0)\,\text{A}, \qquad \text{therefore} \quad i_4 = 0\,\text{A}$

AP 9.6 **[a]** $\mathbf{I} = \dfrac{125\underline{/-60^\circ}}{|Z|\underline{/\theta_z}} = \dfrac{125}{|Z|}\underline{/(-60-\theta_Z)^\circ}$

But $-60 - \theta_Z = -105^\circ \qquad \therefore \;\; \theta_Z = 45^\circ$

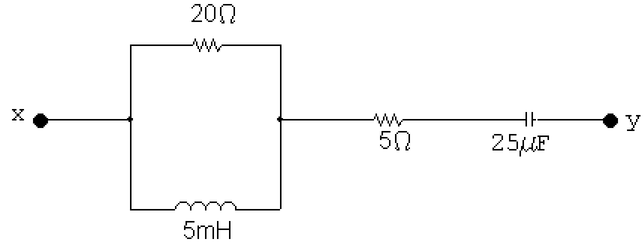
$Z = 90 + j160 + jX_C$

$\therefore \;\; X_C = -70\,\Omega; \qquad X_C = -\dfrac{1}{\omega C} = -70$

$\therefore \;\; C = \dfrac{1}{(70)(5000)} = 2.86\,\mu\text{F}$

[b] $\mathbf{I} = \dfrac{\mathbf{V}_s}{Z} = \dfrac{125\underline{/-60^\circ}}{(90 + j90)} = 0.982\underline{/-105^\circ}A; \qquad \therefore \;\; |\mathbf{I}| = 0.982\,\text{A}$

AP 9.7 **[a]**



$\omega = 2000\,\text{rad/s}$

$\omega L = 10\,\Omega, \qquad \dfrac{-1}{\omega C} = -20\,\Omega$

$$Z_{xy} = 20\|j10 + 5 + j20 = \frac{20(j10)}{(20 + j10)} + 5 - j20$$

$$= 4 + j8 + 5 - j20 = (9 - j12)\,\Omega$$

[b] $\omega L = 40\,\Omega, \qquad \dfrac{-1}{\omega C} = -5\,\Omega$

$$Z_{xy} = 5 - j5 + 20\|j40 = 5 - j5 + \left[\frac{(20)(j40)}{20 + j40}\right]$$

$$= 5 - j5 + 16 + j8 = (21 + j3)\,\Omega$$

[c] $Z_{xy} = \left[\dfrac{20(j\omega L)}{20 + j\omega L}\right] + \left(5 - \dfrac{j10^6}{25\omega}\right)$

$$= \frac{20\omega^2 L^2}{400 + \omega^2 L^2} + \frac{j400\omega L}{400 + \omega^2 L^2} + 5 - \frac{j10^6}{25\omega}$$

The impedance will be purely resistive when the j terms cancel, i.e.,

$$\frac{400\omega L}{400 + \omega^2 L^2} = \frac{10^6}{25\omega}$$

Solving for ω yields $\omega = 4000\,\text{rad/s}$.

[d] $Z_{xy} = \dfrac{20\omega^2 L^2}{400 + \omega^2 L^2} + 5 = 10 + 5 = 15\,\Omega$

AP 9.8 The frequency 4000 rad/s was found to give $Z_{\text{xy}} = 15\,\Omega$ in Assessment Problem 9.7. Thus,

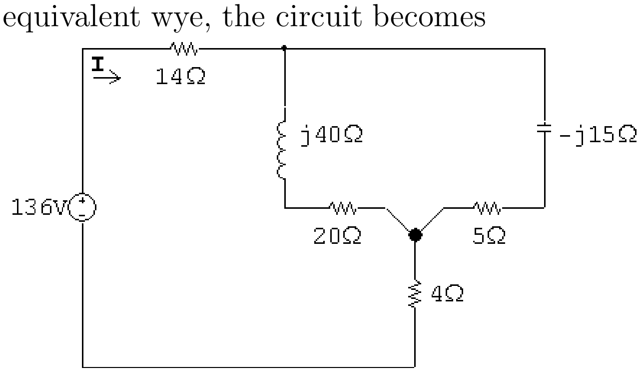
$$\mathbf{V} = 150\underline{/0^\circ}, \qquad \mathbf{I}_s = \frac{\mathbf{V}}{Z_{\text{xy}}} = \frac{150\underline{/0^\circ}}{15} = 10\underline{/0^\circ}\,\text{A}$$

Using current division,

$$\mathbf{I}_L = \frac{20}{20 + j20}(10) = 5 - j5 = 7.07\underline{/-45^\circ}\,\text{A}$$

$$i_L = 7.07\cos(4000t - 45^\circ)\,\text{A}, \qquad I_m = 7.07\,\text{A}$$

AP 9.9 After replacing the delta made up of the $50\,\Omega$, $40\,\Omega$, and $10\,\Omega$ resistors with its equivalent wye, the circuit becomes



The circuit is further simplified by combining the parallel branches,

$(20 + j40)\|(5 - j15) = (12 - j16)\,\Omega$

Therefore $\quad \mathbf{I} = \dfrac{136\angle 0^\circ}{14 + 12 - j16 + 4} = 4\angle 28.07^\circ\,\mathrm{A}$

AP 9.10

$\mathbf{V}_1 = 240\angle 53.13^\circ = 144 + j192\,\mathrm{V}$

$\mathbf{V}_2 = 96\angle -90^\circ = -j96\,\mathrm{V}$

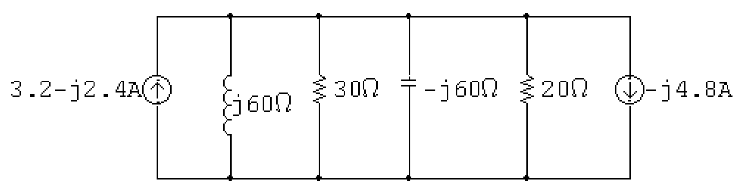
$j\omega L = j(4000)(15 \times 10^{-3}) = j60\,\Omega$

$\dfrac{1}{j\omega C} = -j\dfrac{6 \times 10^6}{(4000)(25)} = -j60\,\Omega$

Perform a source transformation:

$\dfrac{\mathbf{V}_1}{j60} = \dfrac{144 + j192}{j60} = 3.2 - j2.4\,\mathrm{A}$

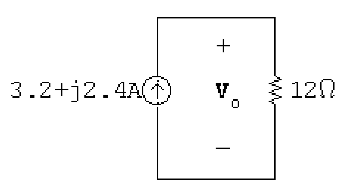
$\dfrac{\mathbf{V}_2}{20} = -j\dfrac{96}{20} = -j4.8\,\mathrm{A}$



Combine the parallel impedances:

$Y = \dfrac{1}{j60} + \dfrac{1}{30} + \dfrac{1}{-j60} + \dfrac{1}{20} = \dfrac{j5}{j60} = \dfrac{1}{12}$

$Z = \dfrac{1}{Y} = 12\,\Omega$



$\mathbf{V}_o = 12(3.2 + j2.4) = 38.4 + j28.8\,\mathrm{V} = 48\angle 36.87^\circ\,\mathrm{V}$

$v_o = 48\cos(4000t + 36.87^\circ)\,\mathrm{V}$

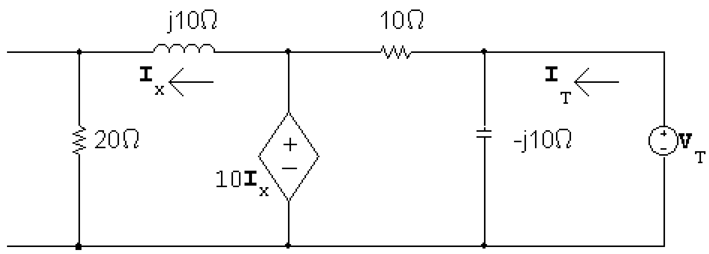
AP 9.11 Use the lower node as the reference node. Let $\mathbf{V}_1$ = node voltage across the $20\,\Omega$ resistor and $\mathbf{V}_{\text{Th}}$ = node voltage across the capacitor. Writing the node voltage equations gives us

$$\frac{\mathbf{V}_1}{20} - 2\angle 45^\circ + \frac{\mathbf{V}_1 - 10\mathbf{I}_x}{j10} = 0 \quad \text{and} \quad \mathbf{V}_{\text{Th}} = \frac{-j10}{10 - j10}(10\mathbf{I}_x)$$

We also have

$$\mathbf{I}_x = \frac{\mathbf{V}_1}{20}$$

Solving these equations for $\mathbf{V}_{\text{Th}}$ gives $\mathbf{V}_{\text{Th}} = 10\angle 45^\circ V$. To find the Thévenin impedance, we remove the independent current source and apply a test voltage source at the terminals a, b. Thus



It follows from the circuit that

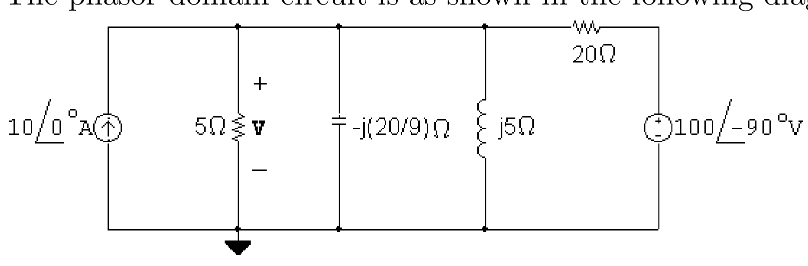
$$10\mathbf{I}_x = (20 + j10)\mathbf{I}_x$$

Therefore

$$\mathbf{I}_x = 0 \quad \text{and} \quad \mathbf{I}_T = \frac{\mathbf{V}_T}{-j10} + \frac{\mathbf{V}_T}{10}$$

$$Z_{\text{Th}} = \frac{\mathbf{V}_T}{\mathbf{I}_T}, \quad \text{therefore} \quad Z_{\text{Th}} = (5 - j5)\,\Omega$$

AP 9.12 The phasor domain circuit is as shown in the following diagram:



The node voltage equation is

$$-10 + \frac{\mathbf{V}}{5} + \frac{\mathbf{V}}{-j(20/9)} + \frac{\mathbf{V}}{j5} + \frac{\mathbf{V} - 100\angle -90^\circ}{20} = 0$$

Therefore $\mathbf{V} = 10 - j30 = 31.62\underline{/-71.57^\circ}$

Therefore $v = 31.62\cos(50{,}000t - 71.57^\circ)\,\text{V}$

AP 9.13 Let $\mathbf{I}_\text{a}$, $\mathbf{I}_\text{b}$, and $\mathbf{I}_\text{c}$ be the three clockwise mesh currents going from left to right. Summing the voltages around meshes a and b gives

$$33.8 = (1 + j2)\mathbf{I}_\text{a} + (3 - j5)(\mathbf{I}_\text{a} - \mathbf{I}_\text{b})$$

and

$$0 = (3 - j5)(\mathbf{I}_\text{b} - \mathbf{I}_\text{a}) + 2(\mathbf{I}_\text{b} - \mathbf{I}_\text{c}).$$

But

$$\mathbf{V}_x = -j5(\mathbf{I}_\text{a} - \mathbf{I}_\text{b}),$$

therefore

$$\mathbf{I}_\text{c} = -0.75[-j5(\mathbf{I}_\text{a} - \mathbf{I}_\text{b})].$$

Solving for $\mathbf{I} = \mathbf{I}_\text{a} = 29 + j2 = 29.07\underline{/3.95^\circ}\,\text{A}$.

AP 9.14 **[a]** $M = 0.4\sqrt{0.0625} = 0.1\,\text{H}, \qquad \omega M = 80\,\Omega$

$$Z_{22} = 40 + j800(0.125) + 360 + j800(0.25) = (400 + j300)\,\Omega$$

Therefore $|Z_{22}| = 500\,\Omega, \qquad Z_{22}^* = (400 - j300)\,\Omega$

$$Z_\tau = \left(\frac{80}{500}\right)^2 (400 - j300) = (10.24 - j7.68)\,\Omega$$

[b] $$\mathbf{I}_1 = \frac{245.20}{184 + 100 + j400 + Z_\tau} = 0.50\underline{/-53.13^\circ}\,\text{A}$$

$$i_1 = 0.5\cos(800t - 53.13^\circ)\,\text{A}$$

[c] $$\mathbf{I}_2 = \left(\frac{j\omega M}{Z_{22}}\right)\mathbf{I}_1 = \frac{j80}{500\underline{/36.87^\circ}}(0.5\underline{/-53.13^\circ}) = 0.08\underline{/0^\circ}\,\text{A}$$

$$i_2 = 80\cos 800t\,\text{mA}$$

AP 9.15

$$\mathbf{I}_1 = \frac{\mathbf{V}_s}{Z_1 + 2s^2 Z_2} = \frac{25 \times 10^3 \underline{/0^\circ}}{1500 + j6000 + (25)^2(4 - j14.4)}$$

$$= 4 + j3 = 5\underline{/36.87^\circ}\text{ A}$$

$$\mathbf{V}_1 = \mathbf{V}_s - Z_1\mathbf{I}_1 = 25{,}000\underline{/0^\circ} - (4 + j3)(1500 + j6000)$$

$$= 37{,}000 - j28{,}500$$

$$\mathbf{V}_2 = -\frac{1}{25}\mathbf{V}_1 = -1480 + j1140 = 1868.15\underline{/142.39^\circ}\text{ V}$$

$$\mathbf{I}_2 = \frac{\mathbf{V}_2}{Z_2} = \frac{1868.15\underline{/142.39^\circ}}{4 - j14.4} = 125\underline{/216.87^\circ}\text{ A}$$

Problems

P 9.1 **[a]** 80 V

[b] $2\pi f = 1000\pi; \qquad f = 500\,\text{Hz}$

[c] $\omega = 1000\pi = 3141.59 \text{ rad/s}$

[d] $\theta(\text{rad}) = \dfrac{-\pi}{6} = -0.5236 \text{ rad}$

[e] $\theta = -30^\circ$

[f] $T = \dfrac{1}{f} = \dfrac{1}{500} = 2\,\text{ms}$

[g] $1000\pi t - \dfrac{\pi}{6} = 0; \qquad \therefore \quad t = \dfrac{1}{6000} = 166.67\,\mu\text{s}$

[h]
$$v = 80\cos\left[1000\pi\left(t + \frac{0.002}{3}\right) - \frac{\pi}{6}\right]$$
$$= 80\cos[1000\pi t + (2\pi/3) - (\pi/6)]$$
$$= 80\cos[1000\pi t + (\pi/2)]$$
$$= -80\sin 1000\pi t\,\text{V}$$

[i] $1000\pi(t - t_o) - (\pi/6) = 1000\pi t - (\pi/2)$

$$\therefore \quad 1000\pi t_o = \frac{\pi}{3}; \qquad t_o = \frac{1}{3000} = 333.33\,\mu\text{s}$$

[j] $1000\pi(t + t_o) - (\pi/6) = 1000\pi t$

$$\therefore \quad 1000\pi t_o = \frac{\pi}{6}; \qquad t_o = \frac{1}{6000} = 166.67\,\mu\text{s}$$

P 9.2 **[a]** $\dfrac{T}{2} = 8 + 2 = 10\,\text{ms}; \qquad T = 20\,\text{ms}$

$$f = \frac{1}{T} = \frac{1}{20 \times 10^{-3}} = 50\,\text{Hz}$$

[b] $v = V_m \sin(\omega t + \theta)$

$$\omega = 2\pi f = 100\pi \text{ rad/s}$$

$$100\pi(-2 \times 10^{-3}) + \theta = 0; \qquad \therefore \quad \theta = \frac{\pi}{5} \text{ rad} = 36^\circ$$

$$v = V_m \sin[100\pi t + 36^\circ]$$

$$80.9 = V_m \sin 36^\circ; \qquad V_m = 137.64\,\text{V}$$

$$v = 137.64\sin[100\pi t + 36^\circ] = 137.64\cos[100\pi t - 54^\circ]\,\text{V}$$

P 9.3 **[a]** By hypothesis

$$i = 20\cos(\omega t + \theta)$$

$$\frac{di}{dt} = -20\omega\sin(\omega t + \theta)$$

$$\therefore \quad 20\omega = 8000\pi; \qquad \omega = 400\pi \text{ rad/s}$$

[b] $f = \dfrac{\omega}{2\pi} = 200$ Hz; $\qquad T = \dfrac{1}{f} = 5$ ms $= 5000\,\mu$s

$$\frac{625}{5000} = \frac{1}{8}, \qquad \therefore \quad \theta = -\frac{1}{8}(360) = -45^\circ$$

$$\therefore \quad i = 20\cos(400\pi t - 45^\circ)\,\text{A}$$

P 9.4 **[a]** $\omega = 2\pi f = 3769.91\,\text{rad/s}, \qquad f = \dfrac{\omega}{2\pi} = 600\,\text{Hz}$

[b] $T = 1/f = 1.67\,\text{ms}$

[c] $V_m = 10\,\text{V}$

[d] $v(0) = 10\cos(-53.13^\circ) = 6\,\text{V}$

[e] $\phi = -53.13^\circ; \qquad \phi = \dfrac{-53.13^\circ(2\pi)}{360^\circ} = -0.9273$ rad

[f] $V = 0$ when $3769.91t - 53.13^\circ = 90^\circ$. Now resolve the units:

$$(3769.91 \text{ rad/s})t = \frac{143.13^\circ}{57.3^\circ/\text{rad}} = 2.498\,\text{rad}, \qquad t = 662.64\,\mu\text{s}$$

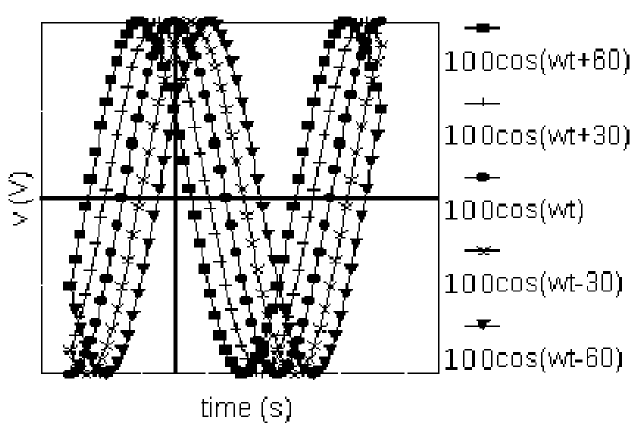
[g] $(dv/dt) = (-10)3769.91\sin(3769.91t - 53.13^\circ)$

$$(dv/dt) = 0 \quad \text{when} \quad 3769.91t - 53.13^\circ = 0^\circ$$

$$\text{or} \qquad 3769.91t = \frac{53.13^\circ}{57.3^\circ/\text{rad}} = 0.9273\,\text{rad}$$

Therefore $\quad t = 245.97\,\mu$s

P 9.5



[a] Left as ϕ becomes more positive

[b] Left

P 9.6
$$\int_{t_o}^{t_o+T} V_m^2 \cos^2(\omega t + \phi)\,dt = V_m^2 \int_{t_o}^{t_o+T} \frac{1}{2} + \frac{1}{2}\cos(2\omega t + 2\phi)\,dt$$
$$= \frac{V_m^2}{2}\left\{\int_{t_o}^{t_o+T} dt + \int_{t_o}^{t_o+T} \cos(2\omega t + 2\phi)\,dt\right\}$$
$$= \frac{V_m^2}{2}\left\{T + \frac{1}{2\omega}\left[\sin(2\omega t + 2\phi)\,\Big|_{t_o}^{t_o+T}\right]\right\}$$
$$= \frac{V_m^2}{2}\left\{T + \frac{1}{2\omega}\left[\sin(2\omega t_o + 4\pi + 2\phi) - \sin(2\omega t_o + 2\phi)\right]\right\}$$
$$= V_m^2\left(\frac{T}{2}\right) + \frac{1}{2\omega}(0) = V_m^2\left(\frac{T}{2}\right)$$

P 9.7 $V_m = \sqrt{2}V_{\text{rms}} = \sqrt{2}(240) = 339.41\,\text{V}$

P 9.8
$$V_{\text{rms}} = \sqrt{\frac{1}{T}\int_0^{T/2} V_m^2 \sin^2 \frac{2\pi}{T}t\,dt}$$

$$\int_0^{T/2} V_m^2 \sin^2\left(\frac{2\pi}{T}t\right)\,dt = \frac{V_m^2}{2}\int_0^{T/2}\left(1 - \cos\frac{4\pi}{T}t\right)\,dt = \frac{V_m^2 T}{4}$$

Therefore $V_{\text{rms}} = \sqrt{\dfrac{1}{T}\dfrac{V_m^2 T}{4}} = \dfrac{V_m}{2}$

P 9.9 **[a]** The numerical values of the terms in Eq. 9.8 are

$V_m = 20, \qquad R/L = 1066.67, \qquad \omega L = 60$

$\sqrt{R^2 + \omega^2 L^2} = 100$

$\phi = 25^\circ, \qquad \theta = \tan^{-1} 60/80, \qquad \theta = 36.87^\circ$

Substitute these values into Equation 9.9:

$$i = \left[-195.72e^{-1066.67t} + 200\cos(800t - 11.87^\circ)\right]\,\text{mA}, \qquad t \geq 0$$

[b] Transient component $= -195.72e^{-1066.67t}\,\text{mA}$
Steady-state component $= 200\cos(800t - 11.87^\circ)\,\text{mA}$

[c] By direct substitution into Eq 9.9 in part (a), $i(1.875\,\text{ms}) = 28.39\,\text{mA}$

[d] $200\,\text{mA}, \quad 800\,\text{rad/s}, \quad -11.87^\circ$

[e] The current lags the voltage by 36.87°.

P 9.10 [a] From Eq. 9.9 we have

$$L\frac{di}{dt} = \frac{V_m R\cos(\phi-\theta)}{\sqrt{R^2+\omega^2L^2}}e^{-(R/L)t} - \frac{\omega L V_m \sin(\omega t+\phi-\theta)}{\sqrt{R^2+\omega^2L^2}}$$

$$Ri = \frac{-V_m R\cos(\phi-\theta)e^{-(R/L)t}}{\sqrt{R^2+\omega^2L^2}} + \frac{V_m R\cos(\omega t+\phi-\theta)}{\sqrt{R^2+\omega^2L^2}}$$

$$L\frac{di}{dt} + Ri = V_m\left[\frac{R\cos(\omega t+\phi-\theta) - \omega L\sin(\omega t+\phi-\theta)}{\sqrt{R^2+\omega^2L^2}}\right]$$

But

$$\frac{R}{\sqrt{R^2+\omega^2L^2}} = \cos\theta \quad \text{and} \quad \frac{\omega L}{\sqrt{R^2+\omega^2L^2}} = \sin\theta$$

Therefore the right-hand side reduces to

$$V_m\cos(\omega t+\phi)$$

At $t = 0$, Eq. 9.9 reduces to

$$i(0) = \frac{-V_m\cos(\phi-\theta)}{\sqrt{R^2+\omega^2L^2}} + \frac{V_m\cos(\phi-\theta)}{\sqrt{R^2+\omega^2L^2}} = 0$$

[b] $$i_{ss} = \frac{V_m}{\sqrt{R^2+\omega^2L^2}}\cos(\omega t+\phi-\theta)$$

Therefore

$$L\frac{di_{ss}}{dt} = \frac{-\omega L V_m}{\sqrt{R^2+\omega^2L^2}}\sin(\omega t+\phi-\theta)$$

and

$$Ri_{ss} = \frac{V_m R}{\sqrt{R^2+\omega^2L^2}}\cos(\omega t+\phi-\theta)$$

$$L\frac{di_{ss}}{dt} + Ri_{ss} = V_m\left[\frac{R\cos(\omega t+\phi-\theta) - \omega L\sin(\omega t+\phi-\theta)}{\sqrt{R^2+\omega^2L^2}}\right]$$

$$= V_m\cos(\omega t+\phi)$$

P 9.11 [a] $\mathbf{Y} = 50\angle 60^\circ + 100\angle -30^\circ = 111.8\angle -3.43^\circ$

$$y = 111.8\cos(500t - 3.43^\circ)$$

[b] $\mathbf{Y} = 200\angle 50^\circ - 100\angle 60^\circ = 102.99\angle 40.29^\circ$

$$y = 102.99\cos(377t + 40.29^\circ)$$

[c] $\mathbf{Y} = 80\angle 30^\circ - 100\angle -225^\circ + 50\angle -90^\circ = 161.59\angle -29.96^\circ$

$$y = 161.59\cos(100t - 29.96^\circ)$$

[d] $\mathbf{Y} = 250\underline{/0^\circ} + 250\underline{/120^\circ} + 250\underline{/-120^\circ} = 0$

$y = 0$

P 9.12 [a] $\mathbf{V}_g = 300\underline{/78^\circ}$; $\mathbf{I}_g = 6\underline{/33^\circ}$

$$\therefore \quad Z = \frac{\mathbf{V}_g}{\mathbf{I}_g} = \frac{300\underline{/78^\circ}}{6\underline{/33^\circ}} = 50\underline{/45^\circ}\,\Omega$$

[b] i_g lags v_g by 45°:

$2\pi f = 5000\pi$; $f = 2500\,\text{Hz}$; $T = 1/f = 400\,\mu\text{s}$

$$\therefore \quad i_g \text{ lags } v_g \text{ by } \frac{45^\circ}{360^\circ}(400\,\mu\text{s}) = 50\,\mu\text{s}$$

P 9.13 [a] $\omega = 2\pi f = 160\pi \times 10^3 = 502.65\,\text{krad/s} = 502{,}654.82\,\text{rad/s}$

[b] $$\mathbf{I} = \frac{25 \times 10^{-3}\underline{/0^\circ}}{1/j\omega C} = j\omega C(25 \times 10^{-3})\underline{/0^\circ} = 25 \times 10^{-3}\omega C\underline{/90^\circ}$$

$\therefore \quad \theta_i = 90^\circ$

[c] $628.32 \times 10^{-6} = 25 \times 10^{-3}\,\omega C$

$$\frac{1}{\omega C} = \frac{25 \times 10^{-3}}{628.32 \times 10^{-6}} = 39.79\,\Omega, \quad \therefore \quad X_{\text{C}} = -39.79\,\Omega$$

[d] $$C = \frac{1}{39.79(\omega)} = \frac{1}{(39.79)(160\pi \times 10^3)}$$

$C = 0.05 \times 10^{-6} = 0.05\,\mu\text{F}$

[e] $$Z_c = j\left(\frac{-1}{\omega C}\right) = -j39.79\,\Omega$$

P 9.14 [a] 400 Hz

[b] $\theta_v = 0^\circ$

$$\mathbf{I} = \frac{100\underline{/0^\circ}}{j\omega L} = \frac{100}{\omega L}\underline{/-90^\circ}; \qquad \theta_i = -90^\circ$$

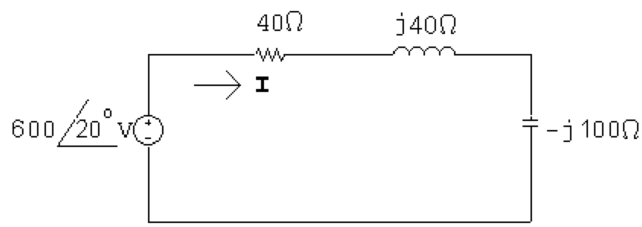
[c] $\dfrac{100}{\omega L} = 20$; $\omega L = 5\,\Omega$

[d] $$L = \frac{5}{800\pi} = 1.99\,\text{mH}$$

[e] $Z_L = j\omega L = j5\,\Omega$

P 9.15 **[a]** $Z_L = j(8000)(5\times10^{-3}) = j40\,\Omega$

$$Z_C = \frac{-j}{(8000)(1.25\times10^{-6})} = -j100\,\Omega$$

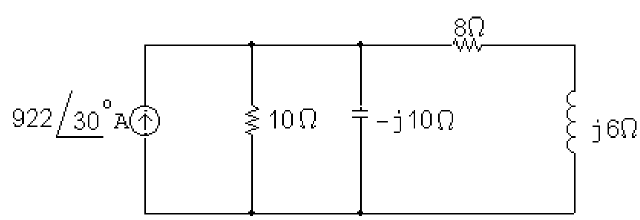


[b] $\mathbf{I} = \dfrac{600\underline{/20^\circ}}{40+j40-j100} = 8.32\underline{/76.31^\circ}\,\text{A}$

[c] $i = 8.32\cos(8000t + 76.31^\circ)\,\text{A}$

P 9.16 **[a]** $j\omega L = j(2\times10^4)(300\times10^{-6}) = j6\,\Omega$

$$\frac{1}{j\omega C} = -j\frac{1}{(2\times10^4)(5\times10^{-6})} = -j10\,\Omega; \qquad \mathbf{I}_g = 922\underline{/30^\circ}\,\text{A}$$



[b] $\mathbf{V}_o = 922\underline{/30^\circ}Z_e$

$$Z_e = \frac{1}{Y_e}; \qquad Y_e = \frac{1}{10} + j\frac{1}{10} + \frac{1}{8+j6}$$

$$Y_e = 0.18 + j0.04\,\text{S}$$

$$Z_e = \frac{1}{0.18+j0.04} = 5.42\underline{/-12.53^\circ}\,\Omega$$

$$\mathbf{V}_o = (922\underline{/30^\circ})(5.42\underline{/-12.53^\circ}) = 5000.25\underline{/17.47^\circ}\,\text{V}$$

[c] $v_o = 5000.25\cos(2\times10^4 t + 17.47^\circ)\,\text{V}$

P 9.17 **[a]** $Z_1 = R_1 + j\omega L_1$

$$Z_2 = \frac{R_2(j\omega L_2)}{R_2 + j\omega L_2} = \frac{\omega^2L_2^2R_2 + j\omega L_2R_2^2}{R_2^2+\omega^2L_2^2}$$

$$Z_1 = Z_2 \quad \text{when} \quad R_1 = \frac{\omega^2L_2^2R_2}{R_2^2+\omega^2L_2^2} \quad \text{and} \quad L_1 = \frac{R_2^2L_2}{R_2^2+\omega^2L_2^2}$$

[b] $R_1 = \dfrac{(4000)^2(1.25)^2(5000)}{5000^2 + 4000^2(1.25)^2} = 2500\,\Omega$

$$L_1 = \frac{(5000)^2(1.25)}{5000^2 + 4000^2(1.25)^2} = 625\,\text{mH}$$

P 9.18 [a] $Y_2 = \dfrac{1}{R_2} - \dfrac{j}{\omega L_2}$

$$Y_1 = \frac{1}{R_1 + j\omega L_1} = \frac{R_1 - j\omega L_1}{R_1^2 + \omega^2 L_1^2}$$

Therefore $\quad Y_2 = Y_1 \quad$ when

$$R_2 = \frac{R_1^2 + \omega^2 L_1^2}{R_1} \quad \text{and} \quad L_2 = \frac{R_1^2 + \omega^2 L_1^2}{\omega^2 L_1}$$

[b] $R_2 = \dfrac{8000^2 + 1000^2(4)^2}{8000} = 10\,\text{k}\Omega$

$$L_2 = \frac{8000^2 + 1000^2(4)^2}{1000^2(4)} = 20\,\text{H}$$

P 9.19 [a] $Z_1 = R_1 - j\dfrac{1}{\omega C_1}$

$$Z_2 = \frac{R_2/j\omega C_2}{R_2 + (1/j\omega C_2)} = \frac{R_2}{1 + j\omega R_2 C_2} = \frac{R_2 - j\omega R_2^2 C_2}{1 + \omega^2 R_2^2 C_2^2}$$

$$Z_1 = Z_2 \quad \text{when} \quad R_1 = \frac{R_2}{1 + \omega^2 R_2^2 C_2^2} \quad \text{and}$$

$$\frac{1}{\omega C_1} = \frac{\omega R_2^2 C_2}{1 + \omega^2 R_2^2 C_2^2} \quad \text{or} \quad C_1 = \frac{1 + \omega^2 R_2^2 C_2^2}{\omega^2 R_2^2 C_2}$$

[b] $R_1 = \dfrac{1000}{1 + (40 \times 10^3)^2(1000)^2(50 \times 10^{-4})^2} = 200\,\Omega$

$$C_1 = \frac{1 + (40 \times 10^3)^2(1000)^2(50 \times 10^{-9})^2}{(40 \times 10^3)^2(1000)^2(50 \times 10^{-9})} = 62.5\,\text{nF}$$

P 9.20 [a] $Y_2 = \dfrac{1}{R_2} + j\omega C_2$

$$Y_1 = \frac{1}{R_1 + (1/j\omega C_1)} = \frac{j\omega C_1}{1 + j\omega R_1 C_1} = \frac{\omega^2 R_1 C_1^2 + j\omega C_1}{1 + \omega^2 R_1^2 C_1^2}$$

Therefore $\quad Y_1 = Y_2 \quad$ when

$$R_2 = \frac{1 + \omega^2 R_1^2 C_1^2}{\omega^2 R_1 C_1^2} \quad \text{and} \quad C_2 = \frac{C_1}{1 + \omega^2 R_1^2 C_1^2}$$

[b] $$R_2 = \frac{1 + (50\times10^3)^2(1000)^2(40\times10^{-9})^2}{(50\times10^3)^2(1000)(40\times10^{-9})^2} = 1250\,\Omega$$

$$C_2 = \frac{40\times10^{-9}}{1 + (50\times10^3)^2(1000)^2(40\times10^{-9})^2} = 8\,\text{nF}$$

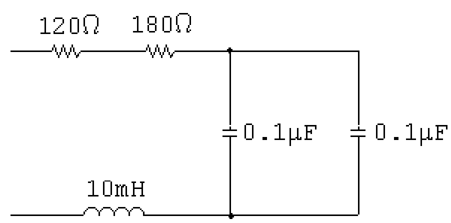
P 9.21 [a] $R = 300\,\Omega = 120\,\Omega + 180\,\Omega$

$$\omega L - \frac{1}{\omega C} = -400 \quad \text{so} \quad 10{,}000L - \frac{1}{10{,}000C} = -400$$

Choose L = 10 mH. Then,

$$\frac{1}{10{,}000C} = 100 + 400 \quad \text{so} \quad C = \frac{1}{10{,}000(500)} = 0.2\,\mu\text{F}$$

We can achieve the desired capacitance by combining two $0.1\,\mu$F capacitors in parallel. The final circuit is shown here:



[b] $$0.01\omega = \frac{1}{\omega(0.2\times10^{-6})} \quad \text{so} \quad \omega^2 = \frac{1}{0.01(0.2\times10^{-6})} = 5\times10^8$$

$$\therefore \quad \omega = 22{,}360.7 \text{ rad/s}$$

P 9.22 [a] Using the notation and results from Problem 9.18:

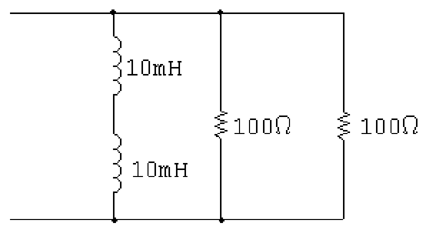
$$R\|L = 40 + j20 \quad \text{so} \quad R_1 = 40, \quad L_1 = \frac{20}{5000} = 4\,\text{mH}$$

$$R_2 = \frac{40^2 + 5000^2(0.004)^2}{40} = 50\,\Omega$$

$$L_2 = \frac{40^2 + 5000^2(0.004)^2}{5000^2(0.004)} = 20\,\text{mH}$$

$$R_2\|j\omega L_2 = 50\|j100 = 40 + j20\,\Omega \quad \text{(checks)}$$

The circuit, using combinations of components from Appendix H, is shown here:



[b] Using the notation and results from Problem 9.22:

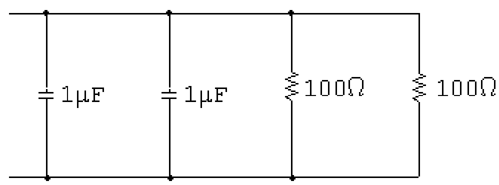
$$R\|C = 40 - j20 \quad \text{so} \quad R_1 = 40, \quad C_1 = 10\,\mu\text{F}$$

$$R_2 = \frac{1 + 5000^2(40)^2(10\,\mu)^2}{5000^2(40)(10\,\mu)^2} = 50\,\Omega$$

$$C_2 = \frac{10\,\mu}{1 + 5000^2(40)^2(10\,\mu)^2} = 2\,\mu\text{F}$$

$$R_2\|(-j/\omega C_2) = 50\|(-j100) = 40 - j20\,\Omega \quad \text{(checks)}$$

The circuit, using combinations of components from Appendix H, is shown here:



P 9.23 **[a]** $(40 + j20)\|(-j/\omega C) = 50\|j100\|(-j/\omega C)$

To cancel out the $j100\,\Omega$ impedance, the capacitive impedance must be $-j100\,\Omega$:

$$\frac{-j}{5000C} = -j100 \quad \text{so} \quad C = \frac{1}{(100)(5000)} = 2\,\mu\text{F}$$

Check:

$$R\|j\omega L\|(-j/\omega C) = 50\|j100\|(-j100) = 50\,\Omega$$

Create the equivalent of a $2\,\mu$F capacitor from components in Appendix H by combining two $1\,\mu$F capacitors in parallel.

[b] $(40 - j20)\|(j\omega L) = 50\|(-j100)\|(j\omega L)$

To cancel out the $-j100\,\Omega$ impedance, the inductive impedance must be $j100\,\Omega$:

$$j5000L = j100 \quad \text{so} \quad L = \frac{100}{5000} = 20\,\text{mH}$$

Check:

$$R\|j\omega L\|(-j/\omega C) = 50\|j100\|(-j100) = 50\,\Omega$$

Create the equivalent of a 20 mH inductor from components in Appendix H by combining two 10 mH inductors in series.

P 9.24 **[a]** $Y = \dfrac{1}{3 + j4} + \dfrac{1}{16 - j12} + \dfrac{1}{-j4}$

$$= 0.12 - j0.16 + 0.04 + j0.03 + j0.25$$

$$= 0.16 + j0.12 = 200\underline{/36.87^\circ}\,\text{mS}$$

[b] $G = 160\,\text{mS}$

[c] $B = 120\,\text{mS}$

[d] $\mathbf{I} = 8\underline{/0^\circ}\,\text{A}, \qquad \mathbf{V} = \frac{\mathbf{I}}{Y} = \frac{8}{0.2\underline{/36.87^\circ}} = 40\underline{/-36.87^\circ}\,\text{V}$

$$\mathbf{I}_C = \frac{\mathbf{V}}{Z_C} = \frac{40\underline{/-36.87^\circ}}{4\underline{/-90^\circ}} = 10\underline{/53.13^\circ}\,\text{A}$$

$$i_C = 10\cos(\omega t + 53.13^\circ)\,\text{A}, \qquad I_m = 10\,\text{A}$$

P 9.25 **[a]** $j\omega L = R\|(-j/\omega C) = j\omega L + \dfrac{-jR/\omega C}{R - j/\omega C}$

$$j\omega L + \frac{-jR}{\omega CR - j}$$

$$j\omega L + \frac{-jR(\omega CR + j)}{\omega^2 C^2 R^2 + 1}$$

$$\mathbf{Im}(Z_{\text{ab}}) = \omega L - \frac{\omega CR^2}{\omega^2 C^2 R^2 + 1} = 0$$

$$\therefore \quad L = \frac{CR^2}{\omega^2 C^2 R^2 + 1}$$

$$\therefore \quad \omega^2 C^2 R^2 + 1 = \frac{CR^2}{L}$$

$$\therefore \quad \omega^2 = \frac{(CR^2/L) - 1}{C^2 R^2} = \frac{\frac{(25\times 10^{-9})(100)^2}{160\times 10^{-6}} - 1}{(25 \times 10^{-9})^2 (100)^2} = 900 \times 10^8$$

$$\omega = 300\,\text{krad/s}$$

[b] $Z_{\text{ab}}(300 \times 10^3) = j48 + \dfrac{(100)(-j133.33)}{100 - j133.33} = 64\,\Omega$

P 9.26 First find the admittance of the parallel branches

$$Y_p = \frac{1}{6 - j2} + \frac{1}{4 + j12} + \frac{1}{5} + \frac{1}{j10} = 0.375 - j0.125\,\text{S}$$

$$Z_p = \frac{1}{Y_p} = \frac{1}{0.375 - j0.125} = 2.4 + j0.8\,\Omega$$

$$Z_{\text{ab}} = -j12.8 + 2.4 + j0.8 + 13.6 = 16 - j12\,\Omega$$

$$Y_{\text{ab}} = \frac{1}{Z_{\text{ab}}} = \frac{1}{16 - j12} = 0.04 + j0.03\,\text{S}$$

$$= 40 + j30\,\text{mS} = 50\underline{/36.87^\circ}\,\text{mS}$$

P 9.27 $Z_{\rm ab} = 1 - j8 + (2 + j4)\|(10 - j20) + (40\|j20)$

$$= 1 - j8 + 3 + j4 + 8 + j16 = 12 + j12\,\Omega = 16.97\underline{/45^\circ}\,\Omega$$

P 9.28 $\mathbf{V}_g = 40\underline{/-15^\circ}\,\text{V};\qquad \mathbf{I}_g = 40\underline{/-68.13^\circ}\,\text{mA}$

$$Z = \frac{\mathbf{V}_g}{\mathbf{I}_g} = 1000\underline{/53.13^\circ}\,\Omega = 600 + j800\,\Omega$$

$$Z = 600 + j\left(3.2\omega - \frac{0.4\times 10^6}{\omega}\right)$$

$$\therefore\quad 3.2\omega - \frac{0.4\times 10^6}{\omega} = 800$$

$$\therefore\quad \omega^2 - 250\omega - 125{,}000 = 0$$

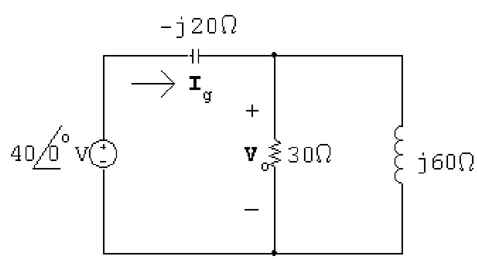
Solving,

$\omega = 500\,\text{rad/s}$

P 9.29 $\dfrac{1}{j\omega C} = \dfrac{1}{(1\times 10^{-6})(50\times 10^3)} = -j20\,\Omega$

$$j\omega L = j50\times 10^3(1.2\times 10^{-3}) = j60\,\Omega$$

$$\mathbf{V}_g = 40\underline{/0^\circ}\,\text{V}$$



$$Z_e = -j20 + 30\|j60 = 24 - j8\,\Omega$$

$$\mathbf{I}_g = \frac{40\underline{/0^\circ}}{24 - j8} = 1.5 + j0.5\,\text{mA}$$

$$\mathbf{V}_o = (30\|j60)\mathbf{I}_g = \frac{30(j60)}{30 + j60}(1.5 + j0.5) = 30 + j30 = 42.43\underline{/45^\circ}\,\text{V}$$

$$v_o = 42.43\cos(50{,}000t + 45^\circ)\,\text{V}$$

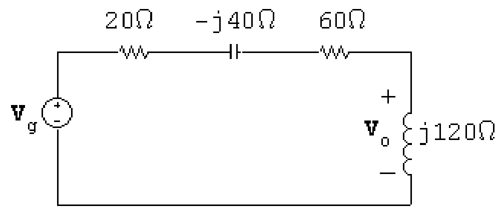
P 9.30 **[a]** $\dfrac{1}{j\omega C} = -j50\,\Omega$

$j\omega L = j120\,\Omega$

$Z_e = 100\| - j50 = 20 - j40\,\Omega$

$\mathbf{I}_g = 2\underline{/0^\circ}$

$\mathbf{V}_g = \mathbf{I}_g Z_e = 2(20 - j40) = 40 - j80\,\text{V}$



$$\mathbf{V}_o = \frac{j120}{80 + j80}(40 - j80) = 90 - j30 = 94.87\underline{/-18.43^\circ}\,\text{V}$$

$v_o = 94.87\cos(16 \times 10^5 t - 18.435^\circ)\,\text{V}$

[b] $\omega = 2\pi f = 16 \times 10^5; \qquad f = \dfrac{8 \times 10^5}{\pi}$

$$T = \frac{1}{f} = \frac{\pi}{8 \times 10^5} = 1.25\pi\,\mu\text{s}$$

$$\therefore \quad \frac{18.435}{360}(1.25\pi\,\mu\text{s}) = 201.09\,\text{ns}$$

$\therefore \quad v_o$ lags i_g by 201.09 ns.

P 9.31 $Z = 4 + j(50)(0.24) - j\dfrac{1}{(50)(0.0025)} = 5.66\underline{/45^\circ}\,\Omega$

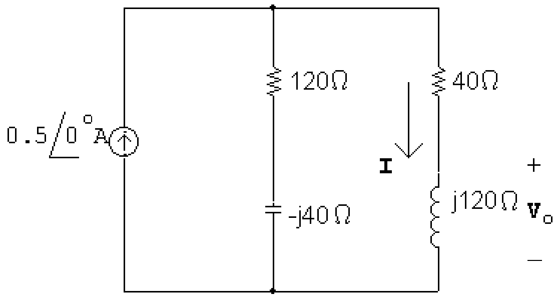
$$\mathbf{I}_o = \frac{\mathbf{V}}{Z} = \frac{0.1\underline{/-90^\circ}}{5.66\underline{/45^\circ}} = 17.67\underline{/-135^\circ}\,\text{mA}$$

$i_o(t) = 17.67\cos(50t - 135^\circ)\,\text{mA}$

P 9.32 $Z_L = j(2000)(60 \times 10^{-3}) = j120\,\Omega$

$$Z_C = \frac{-j}{(2000)(12.5 \times 10^{-6})} = -j40\,\Omega$$

Construct the phasor domain equivalent circuit:



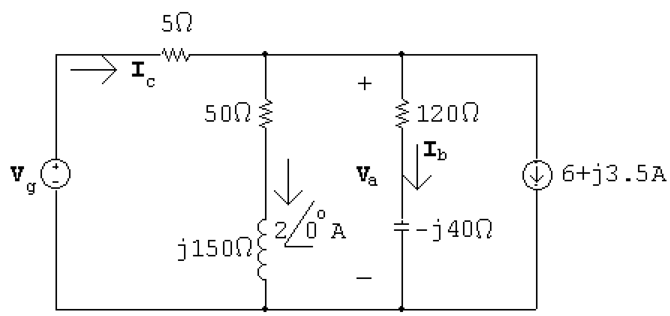
Using current division:

$$\mathbf{I} = \frac{(120 - j40)}{120 - j40 + 40 + j120}(0.5) = 0.25 - j0.25\,\text{A}$$

$$\mathbf{V}_o = j120\mathbf{I} = 30 + j30 = 42.43\underline{/45^\circ}$$

$$v_o = 42.43\cos(2000t + 45^\circ)\,\text{V}$$

P 9.33 [**a**]



$$\mathbf{V}_{\text{a}} = (50 + j150)(2\underline{/0^\circ}) = 100 + j300\,\text{V}$$

$$\mathbf{I}_{\text{b}} = \frac{100 + j300}{120 - j40} = j2.5\,\text{A}$$

$$\mathbf{I}_{\text{c}} = 2\underline{/0^\circ} + j2.5 + 6 + j3.5 = 8 + j6\,\text{A}$$

$$\mathbf{V}_g = 5\mathbf{I}_{\text{c}} + \mathbf{V}_{\text{a}} = 5(8 + j6) + 100 + j300 = 140 + j330\,\text{V}$$

[**b**] $i_{\text{b}} = 2.5\cos(800t + 90^\circ)\,\text{A}$

$$i_{\text{c}} = 10\cos(800t + 36.87^\circ)\,\text{A}$$

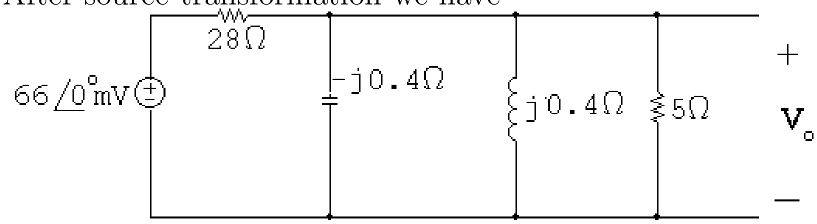
$$v_g = 358.47\cos(800t + 67.01^\circ)\,\text{V}$$

P 9.34 $\mathbf{I}_s = 3\underline{/0^\circ}\,\text{mA}$

$$\frac{1}{j\omega C} = -j0.4\,\Omega$$

$$j\omega L = j0.4\,\Omega$$

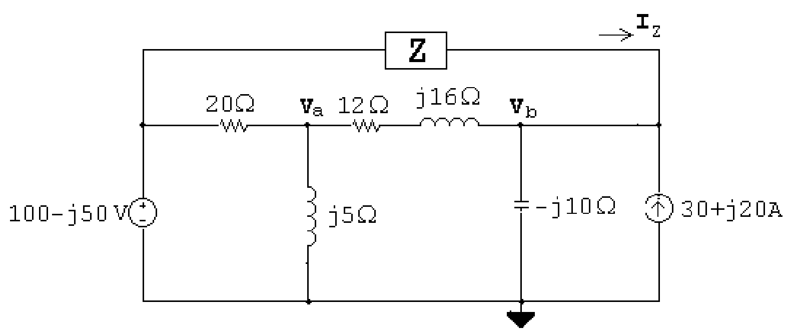
After source transformation we have



$$\mathbf{V}_o = \frac{-j0.4\|j0.4\|5}{28 + -j0.4\|j0.4\|5}(66\times 10^{-3}) = 10\,\text{mV}$$

$$v_o = 10\cos 200t\,\text{mV}$$

P 9.35



$$\frac{\mathbf{V}_\text{a} - (100 - j50)}{20} + \frac{\mathbf{V}_\text{a}}{j5} + \frac{\mathbf{V}_\text{a} - (140 + j30)}{12 + j16} = 0$$

Solving,

$$\mathbf{V}_\text{a} = 40 + j30\,\text{V}$$

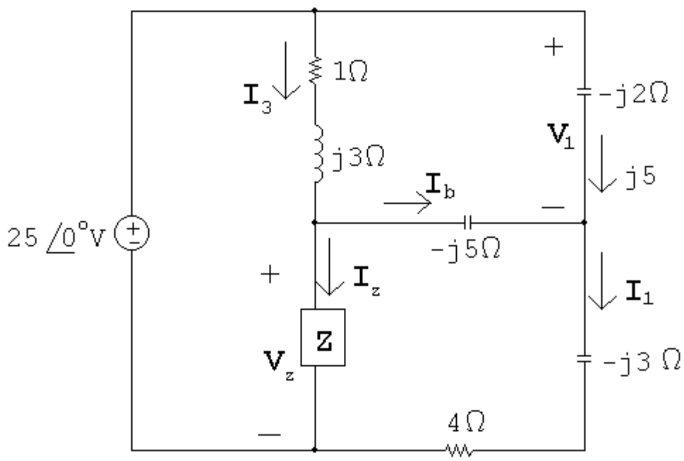
$$\mathbf{I}_Z + (30 + j20) - \frac{140 + j30}{-j10} + \frac{(40 + j30) - (140 + j30)}{12 + j16} = 0$$

Solving,

$$\mathbf{I}_Z = -30 - j10\,\text{A}$$

$$Z = \frac{(100 - j50) - (140 + j30)}{-30 - j10} = 2 + j2\,\Omega$$

P 9.36



$$\mathbf{V}_1 = j5(-j2) = 10\,\text{V}$$

$$-25 + 10 + (4 - j3)\mathbf{I}_1 = 0 \quad \therefore \quad \mathbf{I}_1 = \frac{15}{4 - j3} = 2.4 + j1.8\,\text{A}$$

$$\mathbf{I}_\text{b} = \mathbf{I}_1 - j5 = (2.4 + j1.8) - j5 = 2.4 - j3.2\,\text{A}$$

$$\mathbf{V}_Z = -j5\mathbf{I}_2 + (4 - j3)\mathbf{I}_1 = -j5(2.4 - j3.2) + (4 - j3)(2.4 + j1.8) = -1 - j12\,\text{V}$$

$$-25 + (1 + j3)\mathbf{I}_3 + (-1 - j12) = 0 \quad \therefore \quad \mathbf{I}_3 = 6.2 - j6.6\,\text{A}$$

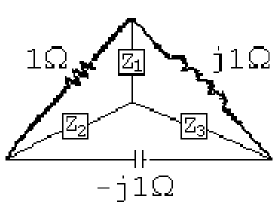
$$\mathbf{I}_Z = \mathbf{I}_3 - \mathbf{I}_2 = (6.2 - j6.6) - (2.4 - j3.2) = 3.8 - j3.4\,\text{A}$$

$$Z = \frac{\mathbf{V}_Z}{\mathbf{I}_Z} = \frac{-1 - j12}{3.8 - j3.4} = 1.42 - j1.88\,\Omega$$

P 9.37 Simplify the top triangle using series and parallel combinations:

$$(1 + j1)\|(1 - j1) = 1\,\Omega$$

Convert the lower left delta to a wye:

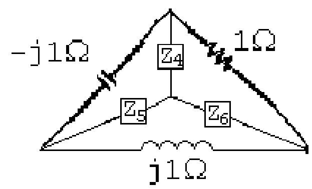


$$Z_1 = \frac{(j1)(1)}{1 + j1 - j1} = j1\,\Omega$$

$$Z_2 = \frac{(-j1)(1)}{1 + j1 - j1} = -j1\,\Omega$$

$$Z_3 = \frac{(j1)(-j1)}{1 + j1 - j1} = 1\,\Omega$$

Convert the lower right delta to a wye:

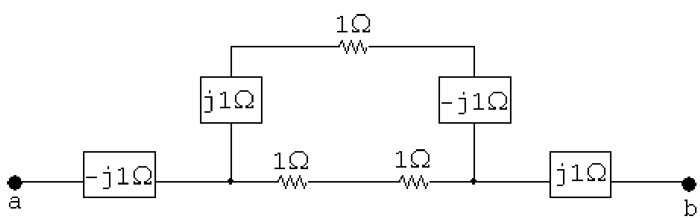


$$Z_4 = \frac{(-j1)(1)}{1 + j1 - j1} = -j1\,\Omega$$

$$Z_5 = \frac{(-j1)(j1)}{1 + j1 - j1} = 1\,\Omega$$

$$Z_6 = \frac{(j1)(1)}{1 + j1 - j1} = j1\,\Omega$$

The resulting circuit is shown below:



Simplify the middle portion of the circuit by making series and parallel combinations:

$$(1 + j1 - j1)\|(1 + 1) = 1\|2 = 2/3\,\Omega$$

$$Z_{\rm ab} = -j1 + 2/3 + j1 = 2/3\,\Omega$$

P 9.38 **[a]** $Z_g = 500 - j\dfrac{10^6}{\omega} + \dfrac{10^3(j0.5\omega)}{10^3 + j0.5\omega}$

$$= 500 - j\frac{10^6}{\omega} + \frac{500j\omega(1000 - j0.5\omega)}{10^6 + 0.25\omega^2}$$

$$= 500 - j\frac{10^6}{\omega} + \frac{250\omega^2}{10^6 + 0.25\omega^2} + j\frac{5 \times 10^5\omega}{10^6 + 0.25\omega^2}$$

$\therefore$ If Z_g is purely real, $\dfrac{10^6}{\omega} = \dfrac{5 \times 10^5\omega}{10^6 + 0.25\omega^2}$

$$2(10^6 + 0.25\omega^2) = \omega^2 \qquad \therefore \qquad 4\times 10^6 = \omega^2$$

$$\therefore \qquad \omega = 2000 \text{ rad/s}$$

[b] When $\omega = 2000\,\text{rad/s}$

$$Z_g = 500 - j500 + (j1000\|1000) = 1000\,\Omega$$

$$\therefore \quad \mathbf{I}_g = \frac{20/\underline{0^\circ}}{1000} = 20/\underline{0^\circ}\,\text{mA}$$

$$\mathbf{V}_o = \mathbf{V}_g - \mathbf{I}_g Z_1$$

$$Z_1 = 500 - j500\,\Omega$$

$$\mathbf{V}_o = 20/\underline{0^\circ} - (0.02/\underline{0^\circ})(500 - j500) = 10 + j10 = 14.14/\underline{45^\circ}\,\text{V}$$

$$v_o = 14.14\cos(2000t + 45^\circ)\,\text{V}$$

P 9.39 **[a]** $Z_{\text{eq}} = \dfrac{50{,}000}{3} + \dfrac{-j20\times 10^6}{\omega}\|(1200 + j0.2\omega)$

$$= \frac{50{,}000}{3} + \frac{-j20\times 10^6}{\omega}\,\frac{(1200 + j0.2\omega)}{1200 + j[0.2\omega - \frac{20\times10^6}{\omega}]}$$

$$= \frac{50{,}000}{3} + \frac{\frac{-j20\times10^6}{\omega}(1200 + j0.2\omega)\left[1200 - j\left(0.2\omega - \frac{20\times10^6}{\omega}\right)\right]}{1200^2 + \left(0.2\omega - \frac{20\times10^6}{\omega}\right)^2}$$

$$\mathbf{Im}(Z_{\text{eq}}) = -\frac{20\times10^6}{\omega}(1200)^2 - \frac{20\times10^6}{\omega}\left[0.2\omega\left(0.2\omega - \frac{20\times10^6}{\omega}\right)\right] = 0$$

$$-20\times10^6(1200)^2 - 20\times10^6\left[0.2\omega\left(0.2\omega - \frac{20\times10^6}{\omega}\right)\right] = 0$$

$$-(1200)^2 = 0.2\omega\left(0.2\omega - \frac{20\times10^6}{\omega}\right)$$

$$0.2^2\omega^2 - 0.2(20\times10^6) - 1200^2 = 0$$

$$\omega^2 = 64\times10^6 \qquad \therefore \qquad \omega = 8000\,\text{rad/s}$$

$$\therefore \qquad f = 1273.24 \text{ Hz}$$

[b] $Z_{\text{eq}} = \dfrac{50{,}000}{3} + -j2500\|(1200 + j1600)$

$$= \frac{50{,}000}{3} + \frac{(-j2500)(1200 + j1600)}{1200 - j900} = 20{,}000\,\Omega$$

$$\mathbf{I}_g = \frac{30/\underline{0^\circ}}{20{,}000} = 1.5/\underline{0^\circ}\,\text{mA}$$

$$i_g(t) = 1.5\cos 8000t\,\text{mA}$$

P 9.40 **[a]** $Z_p = \dfrac{\dfrac{R}{j\omega C}}{R + (1/j\omega C)} = \dfrac{R}{1 + j\omega RC}$

$$= \frac{10{,}000}{1 + j(5000)(10{,}000)C} = \frac{10{,}000}{1 + j50 \times 10^6 C}$$

$$= \frac{10{,}000(1 - j50 \times 10^6 C)}{1 + 25 \times 10^{14} C^2}$$

$$= \frac{10{,}000}{1 + 25 \times 10^{14} C^2} - j\frac{5 \times 10^{11} C}{1 + 25 \times 10^{14} C^2}$$

$j\omega L = j5000(0.8) = j4000$

$\therefore \quad 4000 = \dfrac{5 \times 10^{11} C}{1 + 25 \times 10^{14} C^2}$

$\therefore \quad 10^{14} C^2 - 125 \times 10^6 C + 1 = 0$

$\therefore \quad C^2 - 5 \times 10^{-8} C + 4 \times 10^{-16} = 0$

Solving,

$C_1 = 40\,\text{nF} \qquad\qquad C_2 = 10\,\text{nF}$

[b] $R_e = \dfrac{10{,}000}{1 + 25 \times 10^{14} C^2}$

When $C = 40\,\text{nF} \qquad R_e = 2000\,\Omega$;

$\mathbf{I}_g = \dfrac{80\underline{/0^\circ}}{2000} = 40\underline{/0^\circ}\,\text{mA}; \qquad i_g = 40\cos 5000t\,\text{mA}$

When $C = 10\,\text{nF} \qquad R_e = 8000\,\Omega$;

$\mathbf{I}_g = \dfrac{80\underline{/0^\circ}}{8000} = 10\underline{/0^\circ}\,\text{mA}; \qquad i_g = 10\cos 5000t\,\text{mA}$

P 9.41 **[a]** $Z_C = \dfrac{10^9}{j(50{,}000)(5)} = -j4000\,\Omega$

$$Z_1 = 10{,}000 \| j50{,}000L = \frac{10{,}000(j50{,}000L)}{10{,}000 + j50{,}000L} = \frac{250{,}000L^2 + j50{,}000L}{1 + 25L^2}$$

$$Z_T = Z_1 + Z_R + Z_C = \frac{250{,}000L^2 + j50{,}000L}{1 + 25L^2} - j4000 + 2000$$

Z_T is resistive when

$\dfrac{50{,}000L}{1 + 25L^2} = 4000 \qquad$ or

$L^2 - 0.5L + 0.04 = 0$

Solving, $L_1 = 0.4$ H and $L_2 = 0.1$ H.

[b] When $L = 0.4$ H:

$$Z_T = 2000 + \frac{250{,}000(0.16)}{1 + 25(0.16)} = 10{,}000\,\Omega$$

$$\mathbf{I}_g = \frac{50\underline{/0^\circ}}{10{,}000} = 5\underline{/0^\circ}\,\text{mA}$$

$$i_g = 5\cos 50{,}000t\,\text{mA}$$

When $L = 0.1$ H:

$$Z_T = 2000 + \frac{250{,}000(0.01)}{1 + 25(0.01)} = 4000\,\Omega$$

$$\mathbf{I}_g = \frac{50\underline{/0^\circ}}{4000} = 12.5\underline{/0^\circ}\,\text{mA}$$

$$i_g = 12.5\cos 50{,}000t\,\text{mA}$$

P 9.42 **[a]** $Y_1 = \dfrac{1}{5000} = 0.2 \times 10^{-3}\,\text{S}$

$$Y_2 = \frac{1}{1200 + j0.2\omega}$$

$$= \frac{1200}{1.44 \times 10^6 + 0.04\omega^2} - j\frac{0.2\omega}{1.44 \times 10^6 + 0.04\omega^2}$$

$$Y_3 = j\omega 50 \times 10^{-9}$$

$$Y_T = Y_1 + Y_2 + Y_3$$

For i_g and v_o to be in phase the j component of Y_T must be zero; thus,

$$\omega 50 \times 10^{-9} = \frac{0.2\omega}{1.44 \times 10^6 + 0.04\omega^2}$$

or

$$0.04\omega^2 + 1.44 \times 10^6 = \frac{0.2 \times 10^9}{50} = 4 \times 10^6$$

$$\therefore\quad 0.04\omega^2 = 2.56 \times 10^6 \qquad \therefore\quad \omega = 8000\,\text{rad/s} = 8\,\text{krad/s}$$

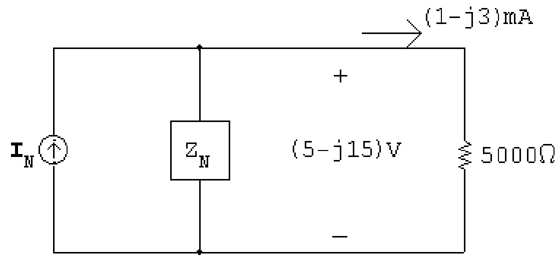
[b] $Y_T = 0.2 \times 10^{-3} + \dfrac{1200}{1.44 \times 10^6 + 0.04(64) \times 10^6} = 0.5 \times 10^{-3}\,\text{S}$

$$\therefore\quad Z_T = 2000\,\Omega$$

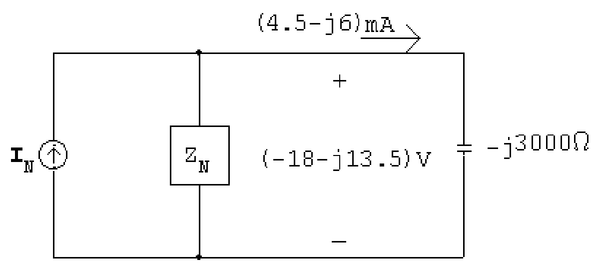
$$\mathbf{V}_o = (2.5 \times 10^{-3}\underline{/0^\circ})(2000) = 5\underline{/0^\circ}$$

$$v_o = 5\cos 8000t\,\text{V}$$

P 9.43



$$\mathbf{I}_N = \frac{5-j15}{Z_\mathrm{N}} + (1-j3)\,\mathrm{mA}, \quad Z_N \text{ in k}\Omega$$

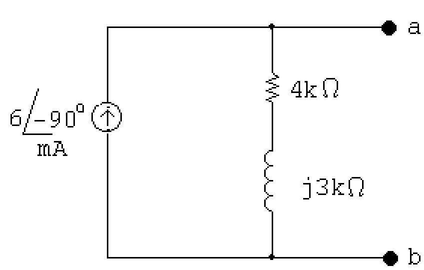


$$\mathbf{I}_N = \frac{-18-j13.5}{Z_N} + 4.5 - j6\,\mathrm{mA}, \quad Z_N \text{ in k}\Omega$$

$$\frac{5-j15}{Z_N} + 1 - j3 = \frac{-18-j13.5}{Z_N} + (4.5-j6)$$

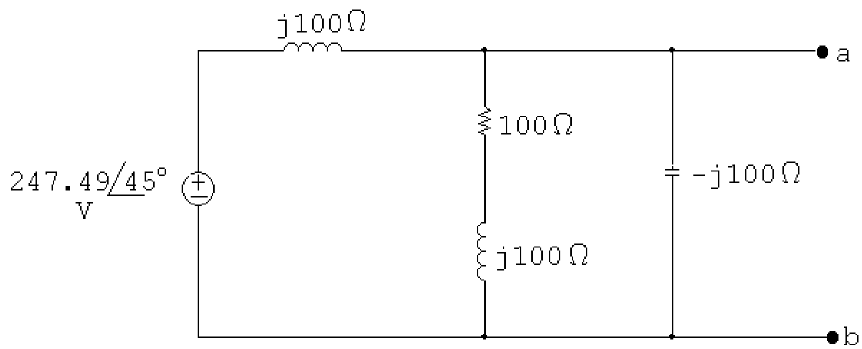
$$\frac{23-j15}{Z_N} = 3.5 - j3 \quad \therefore \qquad Z_N = 4 + j3\,\mathrm{k}\Omega$$

$$\mathbf{I}_N = \frac{5-j15}{4+j3} + 1 - j3 = -j6\,\mathrm{mA}$$



P 9.44 **[a]** $j\omega L = j(1000)(100)\times 10^{-3} = j100\,\Omega$

$$\frac{1}{j\omega C} = -j\frac{10^6}{(1000)(10)} = -j100\,\Omega$$



Using voltage division,

$$\mathbf{V}_{\rm ab} = \frac{(100+j100)\|(-j100)}{j100+(100+j100)\|(-j100)}(247.49\underline{/45^\circ}) = 350\underline{/0^\circ}$$

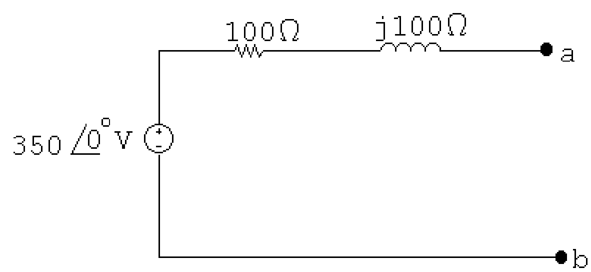
$$\mathbf{V}_{\rm Th} = \mathbf{V}_{\rm ab} = 350\underline{/0^\circ}\text{ V}$$

[**b**] Remove the voltage source and combine impedances in parallel to find $Z_{\rm Th} = Z_{\rm ab}$:

$$Y_{\rm ab} = \frac{1}{j100} + \frac{1}{100+j100} + \frac{1}{-j100} = 5 - j5\text{ mS}$$

$$Z_{\rm Th} = Z_{\rm ab} = \frac{1}{Y_{\rm ab}} = 100 + j100\,\Omega$$

[**c**]



P 9.45 Step 1 to Step 2:

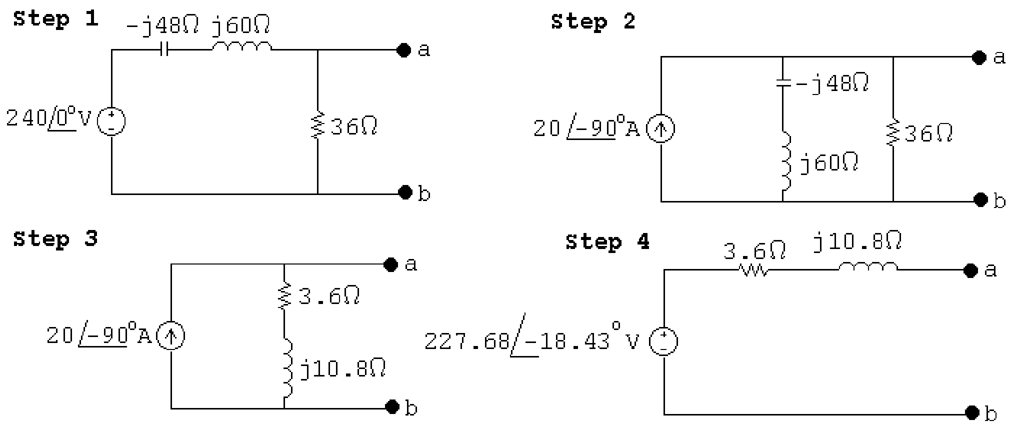
$$\frac{240\underline{/0^\circ}}{j12} = -j20 = 20\underline{/-90^\circ}\text{ A}$$

Step 2 to Step 3:

$$(j12)\|36 = 3.6 + j10.8\,\Omega$$

Step 3 to Step 4:

$$(20\underline{/-90^\circ})(3.6+j10.8) = 216 - j72 = 227.68\underline{/-18.43^\circ}\text{ V}$$



P 9.46 Step 1 to Step 2:

$$(4\angle 0^\circ)(50) = 200\angle 0^\circ\ \text{V}$$

Step 2 to Step 3:

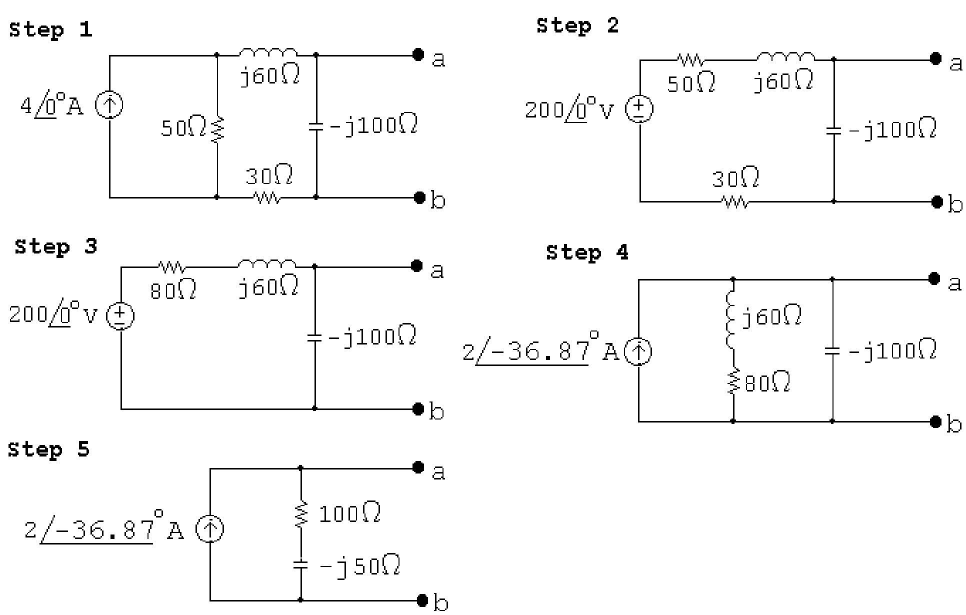
$$50 + 30 + j60 = (80 + j60)\,\Omega$$

Step 3 to Step 4:

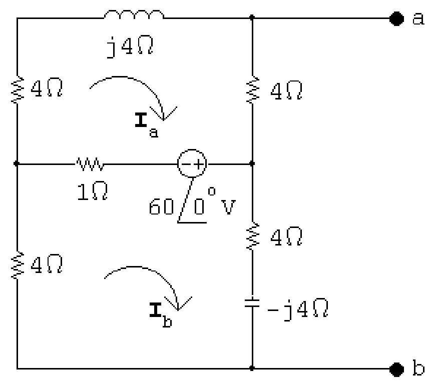
$$\frac{200\angle 0^\circ}{(80 + j60)} = 2\angle -36.87^\circ\ \text{A}$$

Step 4 to Step 5:

$$(80 + j60 \| - j100 = 100 - j50\,\Omega$$



P 9.47 Open circuit voltage:



$$(9 + j4)\mathbf{I}_a - \mathbf{I}_b = -60\underline{/0^\circ}$$

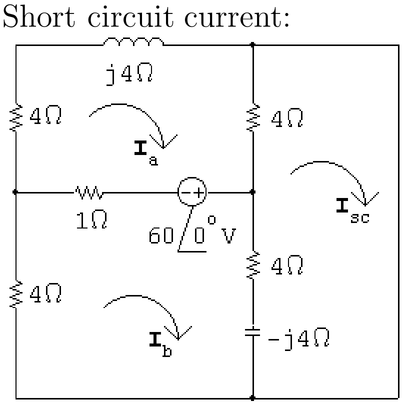
$$-\mathbf{I}_a + (9 - j4)\mathbf{I}_b = 60\underline{/0^\circ}$$

Solving,

$$\mathbf{I}_a = -5 + j2.5\,\text{A}; \qquad \mathbf{I}_b = 5 + j2.5\,\text{A}$$

$$\mathbf{V}_{\text{Th}} = 4\mathbf{I}_a + (4 - j4)\mathbf{I}_b = 10\underline{/0^\circ}\,\text{V}$$

Short circuit current:



$$(9 + j4)\mathbf{I}_a - 1\mathbf{I}_b - 4\mathbf{I}_{sc} = -60$$

$$-1\mathbf{I}_a + (9 - j4)\mathbf{I}_b - (4 - j4)\mathbf{I}_{sc} = 60$$

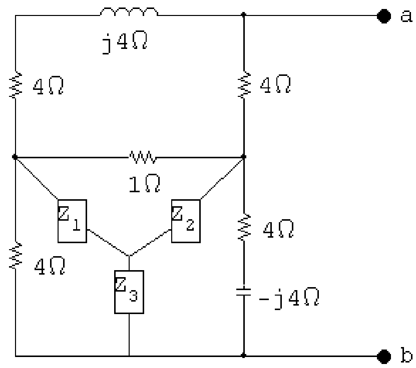
$$-4\mathbf{I}_a - (4 - j4)\mathbf{I}_b + (8 - j4)\mathbf{I}_{sc} = 0$$

Solving,

$$\mathbf{I}_{sc} = 2.07\underline{/0^\circ}$$

$$Z_{\text{Th}} = \frac{\mathbf{V}_{\text{Th}}}{\mathbf{I}_{\text{sc}}} = \frac{10/0^\circ}{2.07/0^\circ} = 4.83\,\Omega$$

Alternate calculation for Z_{Th}:

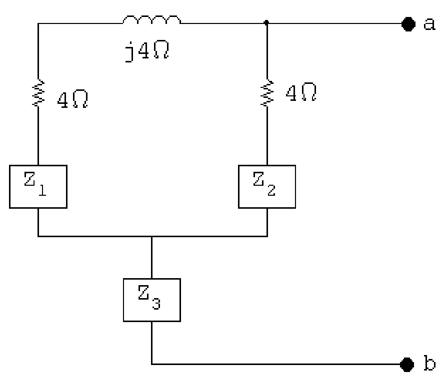


$$\sum Z = 4 + 1 + 4 - j4 = 9 - j4$$

$$Z_1 = \frac{4}{9 - j4}$$

$$Z_2 = \frac{4 - j4}{9 - j4}$$

$$Z_3 = \frac{16 - j16}{9 - j4}$$



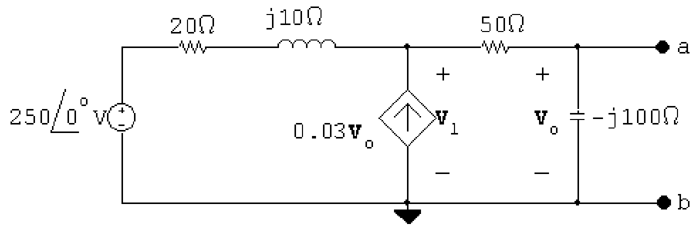
$$Z_{\text{a}} = 4 + j4 + \frac{4}{9 - j4} = \frac{56 + j20}{9 - j4}$$

$$Z_{\text{b}} = 4 + \frac{4 - j4}{9 - j4} = \frac{40 - j20}{9 - j4}$$

$$Z_{\text{a}} \| Z_{\text{b}} = \frac{2640 - j320}{884 - j384}$$

$$Z_3 + Z_{\text{a}} \| Z_{\text{b}} = \frac{16 - j16}{9 - j4} + \frac{2640 - j320}{884 - j384} = 4.83\,\Omega$$

P 9.48 Open circuit voltage:



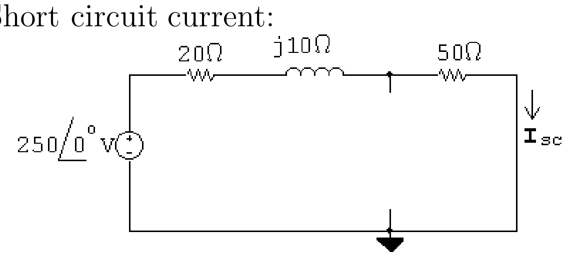
$$\frac{\mathbf{V}_1 - 250}{20 + j10} - 0.03\mathbf{V}_o + \frac{\mathbf{V}_1}{50 - j100} = 0$$

$$\therefore \quad \mathbf{V}_o = \frac{-j100}{50 - j100}\mathbf{V}_1$$

$$\frac{\mathbf{V}_1}{20 + j10} + \frac{j3\mathbf{V}_1}{50 - j100} + \frac{\mathbf{V}_1}{50 - j100} = \frac{250}{20 + j10}$$

$$\mathbf{V}_1 = 500 - j250\,\text{V}; \qquad \mathbf{V}_o = 300 - j400\,\text{V} = \mathbf{V}_{\text{Th}}$$

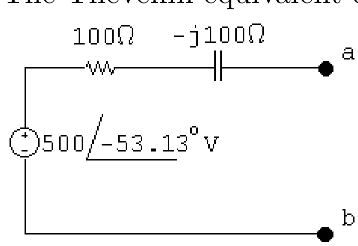
Short circuit current:



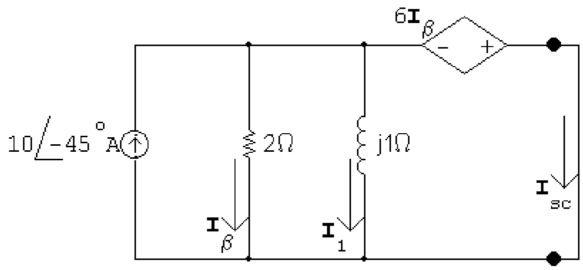
$$\mathbf{I}_{\text{sc}} = \frac{250\underline{/0^\circ}}{70 + j10} = 3.5 - j0.5\,\text{A}$$

$$Z_{\text{Th}} = \frac{\mathbf{V}_{\text{Th}}}{\mathbf{I}_{\text{sc}}} = \frac{300 - j400}{3.5 - j0.5} = 100 - j100\,\Omega$$

The Thévenin equivalent circuit:



P 9.49 Short circuit current

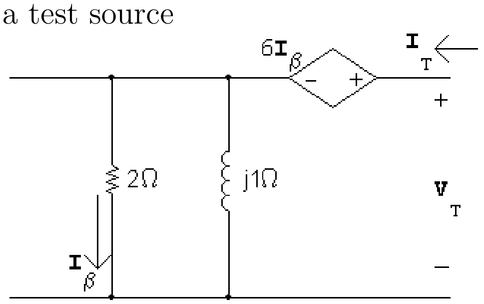


$$\mathbf{I}_\beta = \frac{-6\mathbf{I}_\beta}{2}$$

$$2\mathbf{I}_\beta = -6\mathbf{I}_\beta; \qquad \therefore \quad \mathbf{I}_\beta = 0$$

$$\mathbf{I}_1 = 0; \qquad \therefore \quad \mathbf{I}_{sc} = 10\underline{/-45^\circ} \text{ A} = \mathbf{I}_N$$

The Norton impedance is the same as the Thévenin impedance. Find it using a test source

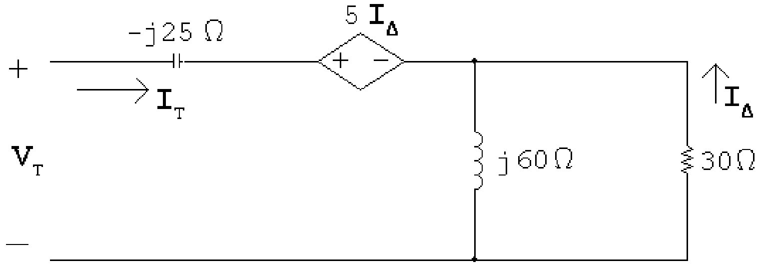


$$\mathbf{V}_T = 6\mathbf{I}_\beta + 2\mathbf{I}_\beta = 8\mathbf{I}_\beta, \qquad \mathbf{I}_\beta = \frac{j1}{2+j1}\mathbf{I}_T$$

$$Z_{\text{Th}} = \frac{\mathbf{V}_T}{\mathbf{I}_T} = \frac{8\mathbf{I}_\beta}{[(2+j1)/j1]\mathbf{I}_\beta} = \frac{j8}{2+j1} = 1.6 + j3.2\,\Omega$$

P 9.50 $j\omega L = j100 \times 10^3(0.6 \times 10^{-3}) = j60\,\Omega$

$$\frac{1}{j\omega C} = \frac{-j}{(100 \times 10^3)(0.4 \times 10^{-6})} = -j25\,\Omega$$



$$\mathbf{V}_T = -j25\mathbf{I}_T + 5\mathbf{I}_\Delta - 30\mathbf{I}_\Delta$$

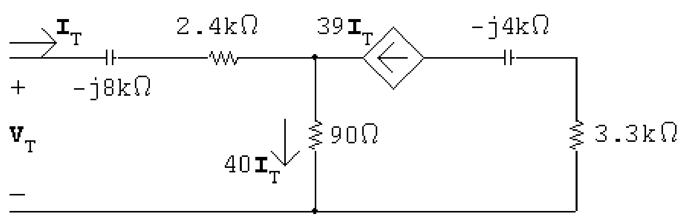
$$\mathbf{I}_\Delta = \frac{-j60}{30+j60}\mathbf{I}_T$$

$$\mathbf{V}_T = -j25\mathbf{I}_T + 25\frac{j60}{30+j60}\mathbf{I}_T$$

$$\frac{\mathbf{V}_T}{\mathbf{I}_T} = Z_{\text{ab}} = 20 - j15 = 25\underline{/-36.87^\circ}\,\Omega$$

P 9.51 $$\frac{1}{\omega C_1} = \frac{10^9}{50{,}000(2.5)} = 8\,\text{k}\Omega$$

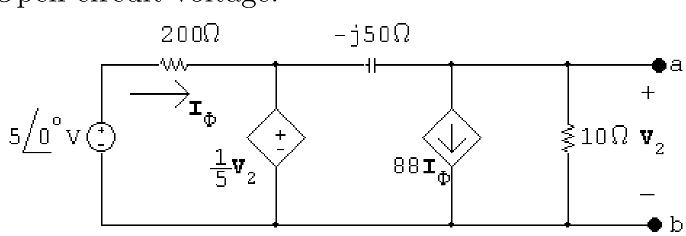
$$\frac{1}{\omega C_2} = \frac{10^9}{50{,}000(5)} = 4\,\text{k}\Omega$$



$$\mathbf{V}_T = (2400 - j8000)\mathbf{I}_T + 40\mathbf{I}_T(90)$$

$$Z_{\text{Th}} = \frac{\mathbf{V}_T}{\mathbf{I}_T} = 6000 - j8000\,\Omega$$

P 9.52 Open circuit voltage:



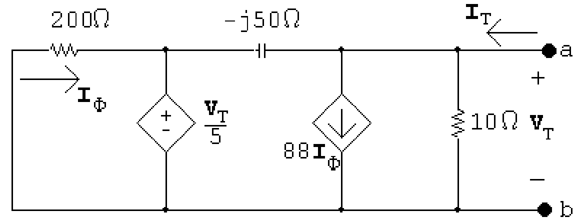
$$\frac{\mathbf{V}_2}{10} + 88\mathbf{I}_\phi + \frac{\mathbf{V}_2 - \frac{1}{5}\mathbf{V}_2}{-j50} = 0$$

$$\mathbf{I}_\phi = \frac{5 - (\mathbf{V}_2/5)}{200}$$

Solving,

$$\mathbf{V}_2 = -66 + j88 = 110\underline{/126.87^\circ}\,\text{V} = \mathbf{V}_{\text{Th}}$$

Find the Thévenin equivalent impedance using a test source:



$$\mathbf{I}_T = \frac{\mathbf{V}_T}{10} + 88\mathbf{I}_\phi + \frac{0.8\mathbf{V}_t}{-j50}$$

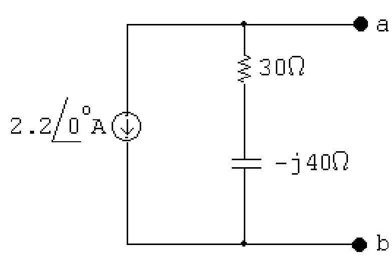
$$\mathbf{I}_\phi = \frac{-\mathbf{V}_T/5}{200}$$

$$\mathbf{I}_T = \mathbf{V}_T\left(\frac{1}{10} - 88\frac{\mathbf{V}_T/5}{200} + \frac{0.8}{-j50}\right)$$

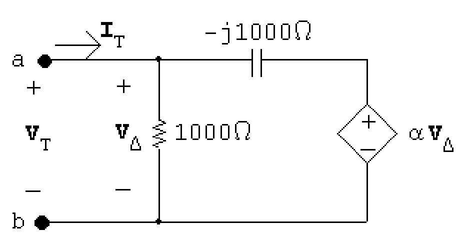
$$\therefore \qquad \frac{\mathbf{V}_T}{\mathbf{I}_T} = 30 - j40 = Z_{\text{Th}}$$

$$\mathbf{I}_{\text{N}} = \frac{\mathbf{V}_{\text{Th}}}{Z_{\text{Th}}} = \frac{-66 + j88}{30 - j40} = -2.2 + j0\,\text{A}$$

The Norton equivalent circuit:



P 9.53 [a]



$$\mathbf{I}_T = \frac{\mathbf{V}_T}{1000} + \frac{\mathbf{V}_T - \alpha\mathbf{V}_T}{-j1000}$$

$$\frac{\mathbf{I}_T}{\mathbf{V}_T} = \frac{1}{1000} - \frac{(1-\alpha)}{j1000} = \frac{j - 1 + \alpha}{j1000}$$

$$\therefore \quad Z_{\text{Th}} = \frac{\mathbf{V}_T}{\mathbf{I}_T} = \frac{j1000}{\alpha - 1 + j}$$

Z_{Th} is real when $\alpha = 1$.

[b] $Z_{\text{Th}} = 1000\,\Omega$

[c] $Z_{\text{Th}} = 500 - j500 = \dfrac{j1000}{\alpha - 1 + j}$

$$= \frac{1000}{(\alpha-1)^2 + 1} + j\frac{1000(\alpha - 1)}{(\alpha-1)^2 + 1}$$

Equate the real parts:

$$\frac{1000}{(\alpha-1)^2+1} = 500 \quad \therefore \quad (\alpha-1)^2 + 1 = 2$$

$$\therefore \quad (\alpha - 1)^2 = 1 \quad \text{so} \quad \alpha = 0$$

Check the imaginary parts:

$$\left.\frac{(\alpha-1)1000}{(\alpha-1)^2+1}\right|_{\alpha=1} = -500$$

Thus, $\alpha = 0$.

[d] $Z_{\text{Th}} = \dfrac{1000}{(\alpha-1)^2+1} + j\dfrac{1000(\alpha-1)}{(\alpha-1)^2+1}$

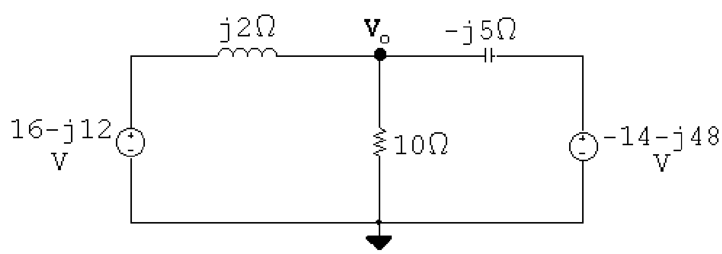
For $\mathbf{Im}(Z_{\text{Th}}) > 0$, α must be greater than 1. So Z_{Th} is inductive for $1 < \alpha \le 10$.

P 9.54 $j\omega L = j(2000)(1 \times 10^{-3}) = j2\,\Omega$

$$\frac{1}{j\omega C} = -j\frac{10^6}{(2000)(100)} = -j5\,\Omega$$

$$\mathbf{V}_{g1} = 20\underline{/-36.87^\circ} = 16 - j12\,\text{V}$$

$$\mathbf{V}_{g2} = 50\underline{/-106.26^\circ} = -14 - j48\,\text{V}$$



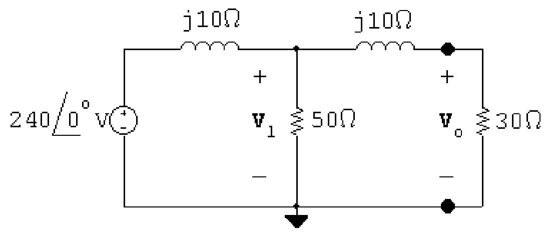
$$\frac{\mathbf{V}_o - (16 - j12)}{j2} + \frac{\mathbf{V}_o}{10} + \frac{\mathbf{V}_o - (-14 - j48)}{-j5} = 0$$

Solving,

$$\mathbf{V}_o = 36\underline{/0^\circ}$$

$$v_o(t) = 36\cos 2000t\,\text{V}$$

P 9.55



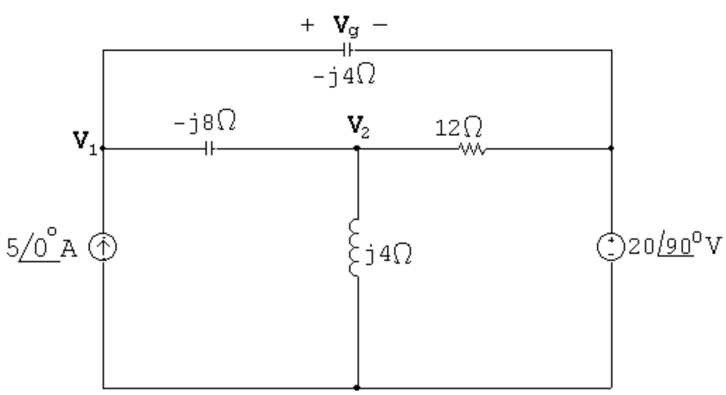
$$\frac{\mathbf{V}_1 - 240}{j10} + \frac{\mathbf{V}_1}{50} + \frac{\mathbf{V}_1}{30 + j10} = 0$$

Solving for $\mathbf{V}_1$ yields

$$\mathbf{V}_1 = 198.63\underline{/-24.44^\circ}\,\text{V}$$

$$\mathbf{V}_o = \frac{30}{30 + j10}(\mathbf{V}_1) = 188.43\underline{/-42.88^\circ}\,\text{V}$$

P 9.56 Set up the frequency domain circuit to use the node voltage method:



At $\mathbf{V}_1$: $$-5\underline{/0^\circ} + \frac{\mathbf{V}_1 - \mathbf{V}_2}{-j8} + \frac{\mathbf{V}_1 - 20\underline{/90^\circ}}{-j4} = 0$$

At $\mathbf{V}_2$: $$\frac{\mathbf{V}_2 - \mathbf{V}_1}{-j8} + \frac{\mathbf{V}_2}{j4} + \frac{\mathbf{V}_2 - 20\underline{/90^\circ}}{12} = 0$$

In standard form:

$$\mathbf{V}_1\left(\frac{1}{-j8}+\frac{1}{-j4}\right)+\mathbf{V}_2\left(-\frac{1}{-j8}\right)=5\underline{/0^\circ}+\frac{20\underline{/90^\circ}}{-j4}$$

$$\mathbf{V}_1\left(-\frac{1}{-j8}\right)+\mathbf{V}_2\left(\frac{1}{-j8}+\frac{1}{j4}+\frac{1}{12}\right)=\frac{20\underline{/90^\circ}}{12}$$

Solving on a calculator:

$$\mathbf{V}_1=-\frac{8}{3}+j\frac{4}{3} \qquad \mathbf{V}_2=-8+j4$$

Thus

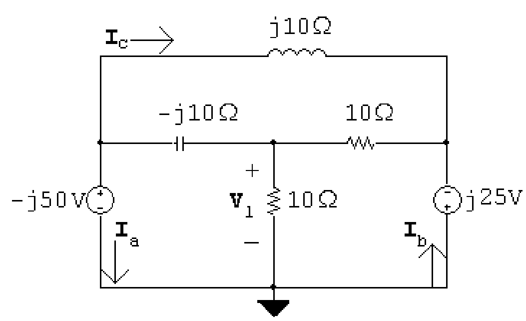
$$\mathbf{V}_g=\mathbf{V}_1-20\underline{/90^\circ}=-\frac{8}{3}-j\frac{56}{3}\,\mathrm{V}$$

P 9.57 $j\omega L=j10^6(10\times10^{-6})=j10\,\Omega$

$$\frac{1}{j\omega C}=\frac{-j}{10^6(100\times10^{-9})}=-j10\,\Omega$$

$$\mathbf{V}_\mathrm{a}=50\underline{/-90^\circ}=-j50\,\mathrm{V}$$

$$\mathbf{V}_\mathrm{b}=25\underline{/90^\circ}=j25\,\mathrm{V}$$



$$\frac{\mathbf{V}_1}{10}+\frac{\mathbf{V}_1+j25}{10}+\frac{\mathbf{V}_1+j50}{-j10}=0$$

Solving,

$$\mathbf{V}_1=25\underline{/-53.13^\circ}\,\mathrm{V}=15-j20\,\mathrm{V}$$

$$\mathbf{I}_a = \frac{\mathbf{V}_1 + j50}{-j10} + \frac{-j25 + j50}{j10}$$

$$= -0.5 + j1.5 = 1.58\angle 108.43^\circ \text{ A}$$

$$i_a = 1.58\cos(10^6 t + 108.43^\circ) \text{ A}$$

$$\mathbf{I}_b = \frac{-j25 - \mathbf{V}_1}{10} + \frac{-j25 + j50}{j10}$$

$$= 1 - j0.5 = 1.12\angle -26.57^\circ \text{ A}$$

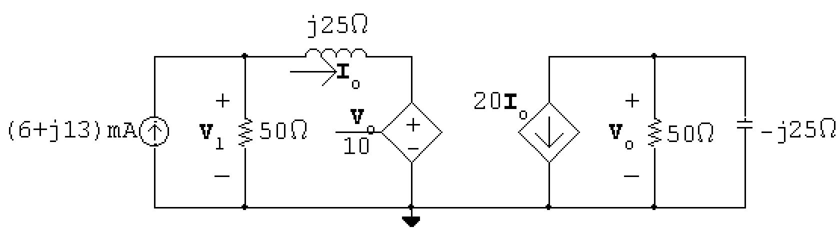
$$i_b = 1.12\cos(10^6 t - 26.57^\circ) \text{ A}$$

$$\mathbf{I}_c = \frac{-j50 + j25}{j10}$$

$$= -2.5 \text{ A}$$

$$i_c = 2.5\cos(10^6 t + 180^\circ) \text{ A}$$

P 9.58



$$\frac{\mathbf{V}_o}{50} + \frac{\mathbf{V}_o}{-j25} + 20\mathbf{I}_o = 0$$

$$(2 + j4)\mathbf{V}_o = -2000\mathbf{I}_o$$

$$\mathbf{V}_o = (-200 + j400)\mathbf{I}_o$$

$$\mathbf{I}_o = \frac{\mathbf{V}_1 - (\mathbf{V}_o/10)}{j25}$$

$$\therefore \quad \mathbf{V}_1 = (-20 + j65)\mathbf{I}_o$$

$$0.006 + j0.013 = \frac{\mathbf{V}_1}{50} + \mathbf{I}_o = (-0.4 + j1.3)\mathbf{I}_o + \mathbf{I}_o = (0.6 + j1.3)\mathbf{I}_o$$

$$\therefore \quad \mathbf{I}_o = \frac{0.6 + j1.3(10 \times 10^{-3})}{(0.6 + j1.3)} = 10\angle 0^\circ \text{ mA}$$

$$\mathbf{V}_o = (-200 + j400)\mathbf{I}_o = -2 + j4 = 4.47\angle 116.57^\circ \text{ V}$$

P 9.59 Write a KCL equation at the top node:

$$\frac{\mathbf{V}_o}{-j8} + \frac{\mathbf{V}_o - 2.4\mathbf{I}_\Delta}{j4} + \frac{\mathbf{V}_o}{5} - (10 + j10) = 0$$

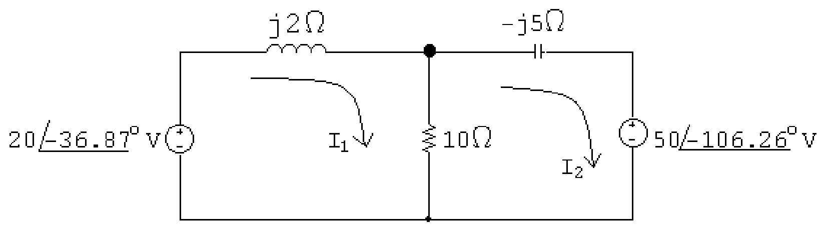
The constraint equation is:

$$\mathbf{I}_\Delta = \frac{\mathbf{V}_o}{-j8}$$

Solving,

$$\mathbf{V}_o = j80 = 80\underline{/90^\circ}\,\mathrm{V}$$

P 9.60 The circuit with the mesh currents identified is shown below:



The mesh current equations are:

$$-20\underline{/-36.87^\circ} + j2\mathbf{I}_1 + 10(\mathbf{I}_1 - \mathbf{I}_2) = 0$$

$$50\underline{/-106.26^\circ} + 10(\mathbf{I}_2 - \mathbf{I}_1) - j5\mathbf{I}_2 = 0$$

In standard form:

$$\mathbf{I}_1(10 + j2) + \mathbf{I}_2(-10) = 20\underline{/-36.87^\circ}$$

$$\mathbf{I}_1(-10) + \mathbf{I}_2(10 - j5) = 50\underline{/-106.26^\circ}$$

Solving on a calculator yields:

$$\mathbf{I}_1 = -6 + j10\,\mathrm{A}; \qquad \mathbf{I}_2 = -9.6 + j10\,\mathrm{A}$$

Thus,

$$\mathbf{V}_o = 10(\mathbf{I}_1 - \mathbf{I}_2) = 36\,\mathrm{V}$$

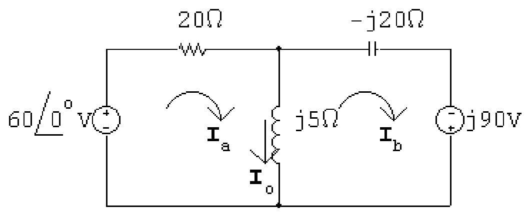
and

$$v_o(t) = 36\cos 2000t\,\mathrm{V}$$

P 9.61 $\mathbf{V}_\mathrm{a} = 60\angle 0^\circ\,\mathrm{V}; \qquad \mathbf{V}_\mathrm{b} = 90\angle 90^\circ\,\mathrm{V}$

$$j\omega L = j(4\times 10^4)(125\times 10^{-6}) = j5\Omega$$

$$\frac{-j}{\omega C} = \frac{-j10^6}{40{,}000(1.25)} = -j20\,\Omega$$



$$60 = (20 + j5)\mathbf{I}_\mathrm{a} - j5\mathbf{I}_\mathrm{b}$$

$$j90 = -j5\mathbf{I}_\mathrm{a} - j15\mathbf{I}_\mathrm{b}$$

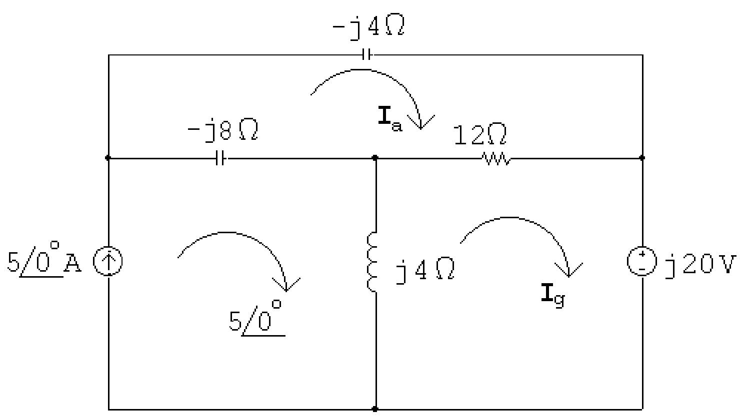
Solving,

$$\mathbf{I}_\mathrm{a} = 2.25 - j2.25\,\mathrm{A}; \qquad \mathbf{I}_\mathrm{b} = -6.75 + j0.75\,\mathrm{A}$$

$$\mathbf{I}_o = \mathbf{I}_\mathrm{a} - \mathbf{I}_\mathrm{b} = 9 - j3 = 9.49\angle -18.43^\circ\,\mathrm{A}$$

$$i_o(t) = 9.49\cos(40{,}000t - 18.43^\circ)\,\mathrm{A}$$

P 9.62



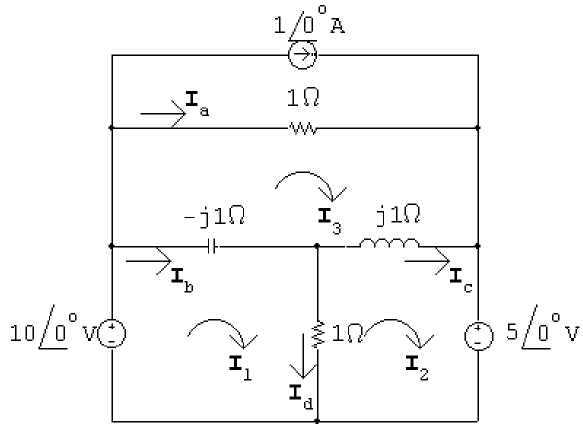
$$(12 - j12)\mathbf{I}_\mathrm{a} - 12\mathbf{I}_g - 5(-j8) = 0$$

$$-12\mathbf{I}_\mathrm{a} + (12 + j4)\mathbf{I}_g + j20 - 5(j4) = 0$$

Solving,

$$\mathbf{I}_g = 4 - j2 = 4.47\angle -26.57^\circ\,\mathrm{A}$$

P 9.63



$$10\underline{/0^\circ} = (1 - j1)\mathbf{I}_1 - 1\mathbf{I}_2 + j1\mathbf{I}_3$$

$$-5\underline{/0^\circ} = -1\mathbf{I}_1 + (1 + j1)\mathbf{I}_2 - j1\mathbf{I}_3$$

$$1 = j1\mathbf{I}_1 - j1\mathbf{I}_2 + \mathbf{I}_3$$

Solving,

$\mathbf{I}_1 = 11 + j10\,\mathrm{A}$; $\quad\mathbf{I}_2 = 11 + j5\,\mathrm{A}$; $\quad\mathbf{I}_3 = 6\,\mathrm{A}$

$$\mathbf{I}_\mathrm{a} = \mathbf{I}_3 - 1 = 5\,\mathrm{A}$$

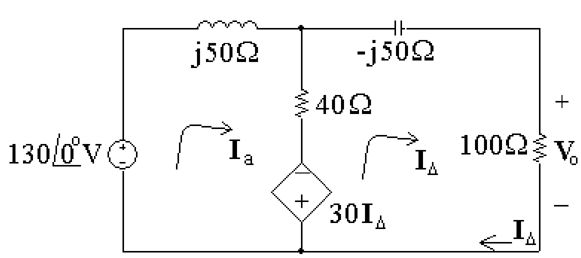
$$\mathbf{I}_\mathrm{b} = \mathbf{I}_1 - \mathbf{I}_3 = 5 + j10\,\mathrm{A}$$

$$\mathbf{I}_\mathrm{c} = \mathbf{I}_2 - \mathbf{I}_3 = 5 + j5\,\mathrm{A}$$

$$\mathbf{I}_\mathrm{d} = \mathbf{I}_1 - \mathbf{I}_2 = j5\,\mathrm{A}$$

P 9.64 $\quad j\omega L = j10{,}000(5 \times 10^{-3}) = j50\,\Omega$

$$\frac{1}{j\omega C} = \frac{-j}{(10{,}000)(2 \times 10^{-6})} = -j50\,\Omega$$



$$130\underline{/0^\circ} = (40 + j50)\mathbf{I}_\mathrm{a} - 40\mathbf{I}_\Delta + 30\mathbf{I}_\Delta$$

$$0 = -40\mathbf{I}_a + 30\mathbf{I}_\Delta + (140 - j50)\mathbf{I}_\Delta$$

Solving,

$$\mathbf{I}_\Delta = (400 - j400)\,\text{mA}$$

$$\mathbf{V}_o = 100\mathbf{I}_\Delta = 40 - j40 = 56.57\underline{/-45^\circ}$$

$$v_o = 56.57\cos(10{,}000t - 45^\circ)\,\text{V}$$

P 9.65 $$\frac{1}{j\omega C} = -j\frac{10^9}{(12{,}500)(800)} = -j100\,\Omega$$

$$j\omega L = j(12{,}500)(0.04) = j500\,\Omega$$

Let $Z_1 = 50 - j100\,\Omega$; $\qquad Z_2 = 250 + j500\,\Omega$

$$\mathbf{I}_g = 125\underline{/0^\circ}\,\text{mA}$$

$$\mathbf{I}_o = \frac{-\mathbf{I}_g Z_2}{Z_1 + Z_2} = \frac{-125\underline{/0^\circ}(250 + j500)}{(300 + j400)}$$

$$= -137.5 - j25\,\text{mA} = 139.75\underline{/-169.7^\circ}\,\text{mA}$$

$$i_o = 139.75\cos(12{,}500t - 169.7^\circ)\,\text{mA}$$

P 9.66 $$Z_o = 12{,}000 - j\frac{10^9}{(20{,}000)(3.125)} = 12{,}000 - j16{,}000\,\Omega$$

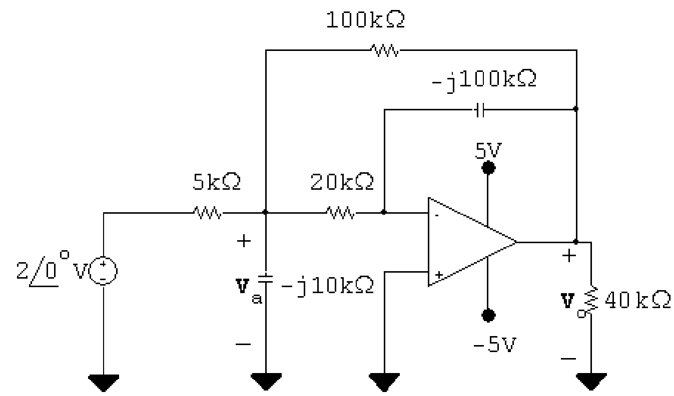
$$Z_T = 6000 + j40{,}000 + 12{,}000 - j16{,}000 = 18{,}000 + j24{,}000\,\Omega = 30{,}000\underline{/53.13^\circ}\,\Omega$$

$$\mathbf{V}_o = \mathbf{V}_g\frac{Z_o}{Z_T} = \frac{(75\underline{/0^\circ})(20{,}000\underline{/-53.13^\circ})}{30{,}000\underline{/53.13^\circ}} = 50\underline{/-106.26^\circ}\,\text{V}$$

$$v_o = 50\cos(20{,}000t - 106.26^\circ)\,\text{V}$$

P 9.67 $$\frac{1}{j\omega C_1} = -j10\,\text{k}\Omega$$

$$\frac{1}{j\omega C_2} = -j100\,\text{k}\Omega$$



$$\frac{\mathbf{V}_a - 2}{5000} + \frac{\mathbf{V}_a}{-j10{,}000} + \frac{\mathbf{V}_a}{20{,}000} + \frac{\mathbf{V}_a - \mathbf{V}_o}{100{,}000} = 0$$

$$20\mathbf{V}_a - 40 + j10\mathbf{V}_a + 5\mathbf{V}_a + \mathbf{V}_a - \mathbf{V}_o = 0$$

$$\therefore \qquad (26 + j10)\mathbf{V}_a - \mathbf{V}_o = 40$$

$$\frac{0 - \mathbf{V}_a}{20{,}000} + \frac{0 - \mathbf{V}_o}{-j100{,}000} = 0$$

$$j5\mathbf{V}_a - \mathbf{V}_o = 0$$

Solving,

$$\mathbf{V}_o = 1.43 + j7.42 = 7.56\underline{/79.09^\circ}\,\mathrm{V}$$

$$v_o(t) = 7.56\cos(10^6 t + 79.09^\circ)\,\mathrm{V}$$

P 9.68 **[a]** $\mathbf{V}_g = 25\underline{/0^\circ}\,\mathrm{V}$

$$\mathbf{V}_p = \frac{20}{100}\mathbf{V}_g = 5\underline{/0^\circ}; \qquad \mathbf{V}_n = \mathbf{V}_p = 5\underline{/0^\circ}\,\mathrm{V}$$

$$\frac{5}{80{,}000} + \frac{5 - \mathbf{V}_o}{Z_p} = 0$$

$$Z_p = -j80{,}000 \| 40{,}000 = 32{,}000 - j16{,}000\,\Omega$$

$$\mathbf{V}_o = \frac{5Z_p}{80{,}000} + 5 = 7 - j = 7.07\underline{/-8.13^\circ}$$

$$v_o = 7.07\cos(50{,}000t - 8.13^\circ)\,\mathrm{V}$$

[**b**] $\mathbf{V}_\mathrm{p} = 0.2V_m\underline{/0^\circ}; \qquad \mathbf{V}_\mathrm{n} = \mathbf{V}_\mathrm{p} = 0.2V_m\underline{/0^\circ}$

$$\frac{0.2V_m}{80{,}000} + \frac{0.2V_m - \mathbf{V}_o}{32{,}000 - j16{,}000} = 0$$

$$\therefore \quad \mathbf{V}_o = 0.2V_m + \frac{32{,}000 - j16{,}000}{80{,}000}V_m(0.2) = V_m(0.28 - j0.04)$$

$$\therefore \quad |V_m(0.28 - j0.04)| \leq 10$$

$$\therefore \quad V_m \leq 35.36\,\mathrm{V}$$

P 9.69 $\mathbf{V}_g = 4\underline{/0^\circ}\,\mathrm{V}; \qquad \dfrac{1}{j\omega C} = -j20\,\mathrm{k}\Omega$

Let $\mathbf{V}_\mathrm{a}$ = voltage across the capacitor, positive at upper terminal
Then:

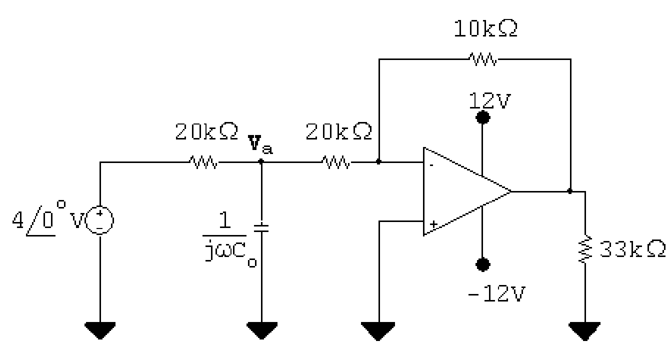
$$\frac{\mathbf{V}_\mathrm{a} - 4\underline{/0^\circ}}{20{,}000} + \frac{\mathbf{V}_\mathrm{a}}{-j20{,}000} + \frac{\mathbf{V}_\mathrm{a}}{20{,}000} = 0; \qquad \therefore \quad \mathbf{V}_\mathrm{a} = (1.6 - j0.8)\,\mathrm{V}$$

$$\frac{0 - \mathbf{V}_\mathrm{a}}{20{,}000} + \frac{0 - \mathbf{V}_o}{10{,}000} = 0; \qquad \mathbf{V}_o = -\frac{\mathbf{V}_\mathrm{a}}{2}$$

$$\therefore \quad \mathbf{V}_o = -0.8 + j0.4 = 0.89\underline{/153.43^\circ}\,\mathrm{V}$$

$$v_o = 0.89\cos(200t + 153.43^\circ)\,\mathrm{V}$$

P 9.70 [**a**]



$$\frac{\mathbf{V}_\mathrm{a} - 4\underline{/0^\circ}}{20{,}000} + j\omega C_o\mathbf{V}_\mathrm{a} + \frac{\mathbf{V}_\mathrm{a}}{20{,}000} = 0$$

$$\mathbf{V}_\mathrm{a} = \frac{4}{2 + j20{,}000\omega C_o}$$

$$\mathbf{V}_o = -\frac{\mathbf{V}_\mathrm{a}}{2}$$

$$\mathbf{V}_o = \frac{-2}{2 + j4\times 10^6 C_o} = \frac{2\underline{/180^\circ}}{2 + j4\times 10^6 C_o}$$

$\therefore$ denominator angle $= 45^\circ$

so $4\times 10^6 C_o = 2 \quad \therefore \quad C = 0.5\,\mu\text{F}$

[**b**] $\mathbf{V}_o = \dfrac{2\underline{/180^\circ}}{2 + j2} = 0.707\underline{/135^\circ}\,\text{V}$

$v_o = 0.707\cos(200t + 135^\circ)\,\text{V}$

P 9.71 [**a**] $\dfrac{1}{j\omega C} = \dfrac{-j10^9}{(10^6)(10)} = -j100\,\Omega$

$\mathbf{V}_g = 30\underline{/0^\circ}\,\text{V}$

$$\mathbf{V}_\text{p} = \frac{\mathbf{V}_g(1/j\omega C_o)}{25 + (1/j\omega C_o)} = \frac{30\underline{/0^\circ}}{1 + j25\omega C_o} = \mathbf{V}_\text{n}$$

$$\frac{\mathbf{V}_\text{n}}{100} + \frac{\mathbf{V}_\text{n} - \mathbf{V}_o}{-j100} = 0$$

$$\mathbf{V}_o = \frac{1 + j1}{j}\mathbf{V}_\text{n} = (1 - j1)\mathbf{V}_\text{n} = \frac{30(1 - j1)}{1 + j25\omega C_o}$$

$$|\mathbf{V}_o| = \frac{30\sqrt{2}}{\sqrt{1 + 625\omega^2 C_o^2}} = 6$$

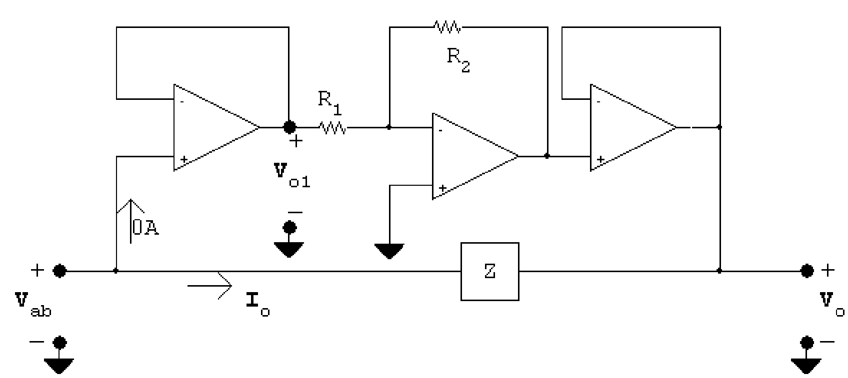
Solving,

$C_o = 280\,\text{nF}$

[**b**] $\mathbf{V}_o = \dfrac{30(1 - j1)}{1 + j7} = 6\underline{/-126.87^\circ}$

$v_o = 6\cos(10^6 t - 126.87^\circ)\,\text{V}$

P 9.72 [**a**]



Because the op-amps are ideal $\mathbf{I}_{\text{in}} = \mathbf{I}_o$, thus

$$Z_{\text{ab}} = \frac{\mathbf{V}_{\text{ab}}}{\mathbf{I}_{\text{in}}} = \frac{\mathbf{V}_{\text{ab}}}{\mathbf{I}_o}; \qquad \mathbf{I}_o = \frac{\mathbf{V}_{\text{ab}} - \mathbf{V}_o}{Z}$$

$$\mathbf{V}_{o1} = \mathbf{V}_{\text{ab}}; \qquad \mathbf{V}_{o2} = -\left(\frac{R_2}{R_1}\right)\mathbf{V}_{o1} = -K\mathbf{V}_{o1} = -K\mathbf{V}_{\text{ab}}$$

$$\mathbf{V}_o = \mathbf{V}_{o2} = -K\mathbf{V}_{\text{ab}}$$

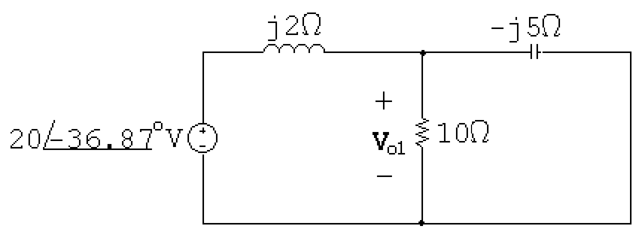
$$\therefore \quad \mathbf{I}_o = \frac{\mathbf{V}_{\text{ab}} - (-K\mathbf{V}_{\text{ab}})}{Z} = \frac{(1+K)\mathbf{V}_{\text{ab}}}{Z}$$

$$\therefore \quad Z_{\text{ab}} = \frac{\mathbf{V}_{\text{ab}}}{(1+K)\mathbf{V}_{\text{ab}}}Z = \frac{Z}{(1+K)}$$

[b] $Z = \dfrac{1}{j\omega C}; \qquad Z_{\text{ab}} = \dfrac{1}{j\omega C(1+K)}; \qquad \therefore \quad C_{\text{ab}} = C(1+K)$

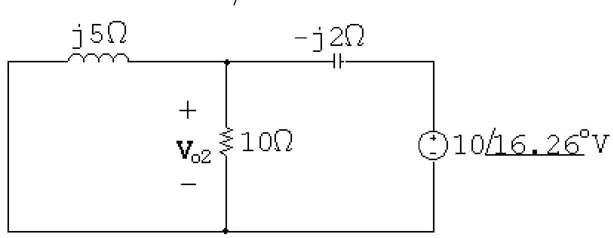
P 9.73 **[a]** Superposition must be used because the frequencies of the two sources are different.

[b] For $\omega = 2000$ rad/s:



$$10\| - j5 = 2 - j4\,\Omega \quad \text{so} \quad \mathbf{V}_{o1} = \frac{2 - j4}{2 - j4 + j2}(20\underline{/-36.87^\circ} = 31.62\underline{/-55.3^\circ}\text{ V}$$

For $\omega = 5000$ rad/s:



$$j5\|10 = 2 + j4\,\Omega$$

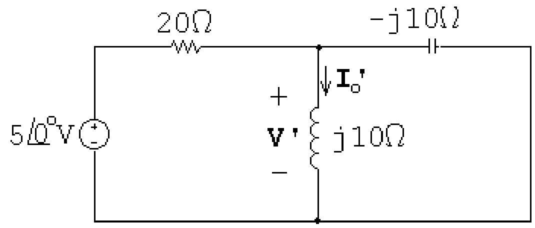
$$\mathbf{V}_{o2} = \frac{2 + j4}{2 + j4 - j2}(10\underline{/16.26^\circ}) = 15.81\underline{/34.69^\circ}\text{ V}$$

Thus,

$$v_o(t) = [31.62\cos(2000t - 55.3^\circ) + 15.81\cos(5000t + 34.69^\circ)]\text{ V}, \quad t \ge 0$$

P 9.74 **[a]** Superposition must be used because the frequencies of the two sources are different.

[b] For $\omega = 80{,}000$ rad/s:



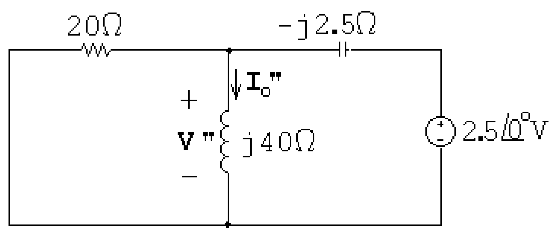
$$\frac{\mathbf{V}_o' - 5}{20} + \frac{\mathbf{V}_o'}{j10} + \frac{\mathbf{V}_o'}{-j10} = 0$$

$$\mathbf{V}_o'\left(\frac{1}{20} + \frac{1}{j10} + \frac{1}{-j10}\right) = \frac{5}{20}$$

$$\therefore \quad \mathbf{V}_o' = 5\underline{/0^\circ}\,\text{V}$$

$$\mathbf{I}_o' = \frac{\mathbf{V}_o'}{j10} = -j0.5 = 500\underline{/-90^\circ}\,\text{mA}$$

For $\omega = 320{,}000$ rad/s:



$$20\|j40 = 16 + j8\,\Omega$$

$$\mathbf{V}'' = \frac{16 + j8}{16 + j8 - j2.5}(2.5\underline{/0^\circ}) = 2.643\underline{/7.59^\circ}\,\text{V}$$

$$\therefore \quad \mathbf{I}_o'' = \frac{\mathbf{V}''}{j40} = 66.08\underline{/-82.4^\circ}\,\text{mA}$$

Thus,

$$i_o(t) = [500\sin 80{,}000t + 66.08\cos(320{,}000t - 82.4^\circ)]\,\text{mA}, \quad t \geq 0$$

P 9.75 **[a]** $j\omega L_L = j100\,\Omega$

$$j\omega L_2 = j500\,\Omega$$

$$Z_{22} = 300 + 500 + j100 + j500 = 800 + j600\,\Omega$$

$$Z_{22}^* = 800 - j600\,\Omega$$

$$\omega M = 270\,\Omega$$

$$Z_r = \left(\frac{270}{1000}\right)^2 [800 - j600] = 58.32 - j43.74\,\Omega$$

[b] $Z_{\text{ab}} = R_1 + j\omega L_1 + Z_r = 41.68 + j180 + 58.32 - j43.74 = 100 + j136.26\,\Omega$

P 9.76 **[a]** $j\omega L_1 = j(200 \times 10^3)(10^{-3}) = j200\,\Omega$

$$j\omega L_2 = j(200 \times 10^3)(4 \times 10^{-3}) = j800\,\Omega$$

$$\frac{1}{j\omega C} = \frac{-j}{(200 \times 10^3)(12.5 \times 10^{-9})} = -j400\,\Omega$$

$$\therefore \quad Z_{22} = 100 + 200 + j800 - j400 = 300 + j400\,\Omega$$

$$\therefore \quad Z_{22}^* = 300 - j400\,\Omega$$

$$M = k\sqrt{L_1 L_2} = 2k \times 10^{-3}$$

$$\omega M = (200 \times 10^3)(2k \times 10^{-3}) = 400k$$

$$Z_r = \left[\frac{400k}{500}\right]^2 (300 - j400) = k^2(192 - j256)\,\Omega$$

$$Z_{\text{in}} = 200 + j200 + 192k^2 - j256k^2$$

$$|Z_{\text{in}}| = [(200 + 192k)^2 + (200 - 256k)^2]^{\frac{1}{2}}$$

$$\frac{d|Z_{\text{in}}|}{dk} = \frac{1}{2}[(200 + 192k)^2 + (200 - 256k)^2]^{-\frac{1}{2}} \times$$

$$[2(200 + 192k^2)384k + 2(200 - 256k^2)(-512k)]$$

$$\frac{d|Z_{\text{in}}|}{dk} = 0 \text{ when}$$

$$768k(200 + 192k^2) - 1024k(200 - 256k^2) = 0$$

$$\therefore \quad k^2 = 0.125; \qquad \therefore \quad k = \sqrt{0.125} = 0.3536$$

[b]
$$Z_{\text{in}}\ (\text{min}) = 200 + 192(0.125) + j[200 - 0.125(256)]$$
$$= 224 + j168 = 280\underline{/36.87^\circ}\,\Omega$$

$$\mathbf{I}_1\ (\text{max}) = \frac{560\underline{/0^\circ}}{224 + j168} = 2\underline{/-36.87^\circ}\,\text{A}$$

$$\therefore \quad i_1\ (\text{peak}) = 2\,\text{A}$$

Note — You can test that the k value obtained from setting $d|Z_{\rm in}|/dt = 0$ leads to a minimum by noting $0 \le k \le 1$. If $k = 1$,

$$Z_{\rm in} = 392 - j56 = 395.98\underline{/-8.13^\circ}\,\Omega$$

Thus,

$$|Z_{\rm in}|_{k=1} > |Z_{\rm in}|_{k=\sqrt{0.125}}$$

If $k = 0$,

$$Z_{\rm in} = 200 + j200 = 282.84\underline{/45^\circ}\,\Omega$$

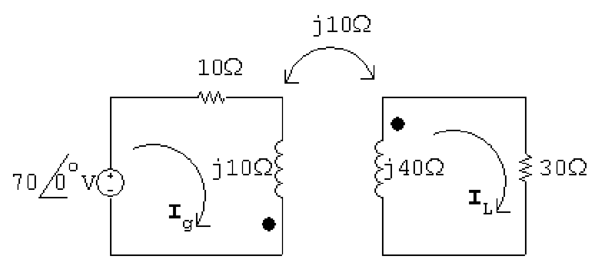
Thus,

$$|Z_{\rm in}|_{k=0} > |Z_{\rm in}|_{k=\sqrt{0.125}}$$

P 9.77 **[a]** $j\omega L_1 = j(5000)(2\times10^{-3}) = j10\,\Omega$

$$j\omega L_2 = j(5000)(8\times10^{-3}) = j40\,\Omega$$

$$j\omega M = j10\,\Omega$$



$$70 = (10 + j10)\mathbf{I}_g + j10\mathbf{I}_L$$

$$0 = j10\mathbf{I}_g + (30 + j40)\mathbf{I}_L$$

Solving,

$$\mathbf{I}_g = 4 - j3\,\text{A}; \qquad \mathbf{I}_L = -1\,\text{A}$$

$$i_g = 5\cos(5000t - 36.87^\circ)\,\text{A}$$

$$i_L = 1\cos(5000t - 180^\circ)\,\text{A}$$

[b] $k = \dfrac{M}{\sqrt{L_1L_2}} = \dfrac{2}{\sqrt{16}} = 0.5$

[c] When $t = 100\pi\ \mu$s,

$$5000t = (5000)(100\pi)\times10^{-6} = 0.5\pi = \pi/2\,\text{rad} = 90^\circ$$

$$i_g(100\pi\mu s) = 5\cos(53.13^\circ) = 3\,\text{A}$$

$$i_L(100\pi\mu s) = 1\cos(-90^\circ) = 0\,\text{A}$$

$$w = \frac{1}{2}L_1 i_1^2 + \frac{1}{2}L_2 i_2^2 + M i_1 i_2 = \frac{1}{2}(2 \times 10^{-3})(9) + 0 + 0 = 9\,\text{mJ}$$

When $t = 200\pi\ \mu\text{s}$,

$$5000t = \pi\,\text{rad} = 180^\circ$$

$$i_g(200\pi\mu s) = 5\cos(180 - 53.13) = -4\,\text{A}$$

$$i_L(200\pi\mu s) = 1\cos(180 - 180) = 1\,\text{A}$$

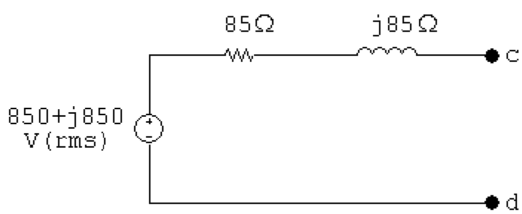
$$w = \frac{1}{2}(2 \times 10^{-3})(16) + \frac{1}{2}(8 \times 10^{-3})(1) + 2 \times 10^{-3}(-4)(1) = 12\,\text{mJ}$$

P 9.78 Remove the voltage source to find the equivalent impedance:

$$Z_{\text{Th}} = 45 + j125 + \left(\frac{20}{|5 + j5|}\right)^2 (5 + j5) = 85 + j85\,\Omega$$

Using voltage division:

$$\mathbf{V}_{\text{Th}} = \mathbf{V}_{\text{cd}} = j20\mathbf{I}_1 = j20\left(\frac{425}{5 + j5}\right) = 850 + j850\,\text{V}$$



P 9.79 $j\omega L_1 = j50\,\Omega$

$$j\omega L_2 = j32\,\Omega$$

$$\frac{1}{j\omega C} = -j20\,\Omega$$

$$j\omega M = j(4 \times 10^3)k\sqrt{(12.5)(8)} \times 10^{-3} = j40k\,\Omega$$

$$Z_{22} = 5 + j32 - j20 = 5 + j12\,\Omega$$

$$Z_{22}^* = 5 - j12\,\Omega$$

$$Z_r = \left[\frac{40k}{|5 + j12|}\right]^2 (5 - j12) = 47.337k^2 - j113.609k^2$$

$$Z_{\rm ab} = 20 + j50 + 47.337k^2 - j113.609k^2 = (20 + 47.337k^2) + j(50 - 113.609k^2)$$

$Z_{\rm ab}$ is resistive when

$$50 - 113.609k^2 = 0 \quad \text{or} \quad k^2 = 0.44 \quad \text{so} \quad k = 0.66$$

$$\therefore \quad Z_{\rm ab} = 20 + (47.337)(0.44) = 40.83\,\Omega$$

P 9.80 In Eq. 9.69 replace $\omega^2 M^2$ with $k^2\omega^2 L_1 L_2$ and then write $X_{\rm ab}$ as

$$X_{\rm ab} = \omega L_1 - \frac{k^2\omega^2 L_1 L_2(\omega L_2 + \omega L_L)}{R_{22}^2 + (\omega L_2 + \omega L_L)^2}$$

$$= \omega L_1 \left\{1 - \frac{k^2\omega L_2(\omega L_2 + \omega L_L)}{R_{22}^2 + (\omega L_2 + \omega L_L)^2}\right\}$$

For $X_{\rm ab}$ to be negative requires

$$R_{22}^2 + (\omega L_2 + \omega L_L)^2 < k^2\omega L_2(\omega L_2 + \omega L_L)$$

or

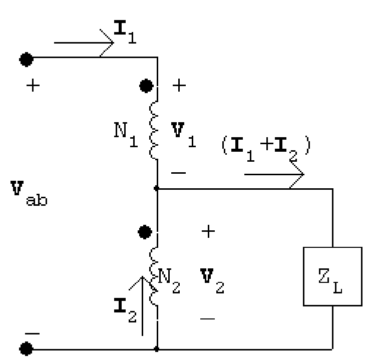
$$R_{22}^2 + (\omega L_2 + \omega L_L)^2 - k^2\omega L_2(\omega L_2 + \omega L_L) < 0$$

which reduces to

$$R_{22}^2 + \omega^2 L_2^2(1 - k^2) + \omega L_2 \omega L_L(2 - k^2) + \omega^2 L_L^2 < 0$$

But $k \leq 1$, so it is impossible to satisfy the inequality. Therefore $X_{\rm ab}$ can never be negative if X_L is an inductive reactance.

P 9.81 [a]



$$Z_{\rm ab} = \frac{\mathbf{V}_{\rm ab}}{\mathbf{I}_1} = \frac{\mathbf{V}_1 + \mathbf{V}_2}{\mathbf{I}_1}$$

$$\frac{\mathbf{V}_1}{N_1} = \frac{\mathbf{V}_2}{N_2}, \qquad \mathbf{V}_2 = \frac{N_2}{N_1}\mathbf{V}_1$$

$$N_1\mathbf{I}_1 = N_2\mathbf{I}_2, \qquad \mathbf{I}_2 = \frac{N_1}{N_2}\mathbf{I}_1$$

$$\mathbf{V}_2 = (\mathbf{I}_1 + \mathbf{I}_2)Z_L = \mathbf{I}_1\left(1 + \frac{N_1}{N_2}\right)Z_L$$

$$\mathbf{V}_1 + \mathbf{V}_2 = \left(\frac{N_1}{N_2} + 1\right)\mathbf{V}_2 = \left(1 + \frac{N_1}{N_2}\right)^2 Z_L\mathbf{I}_1$$

$$\therefore \quad Z_{\text{ab}} = \frac{(1 + N_1/N_2)^2 Z_L\mathbf{I}_1}{\mathbf{I}_1}$$

$$Z_{\text{ab}} = \left(1 + \frac{N_1}{N_2}\right)^2 Z_L \quad \text{Q.E.D.}$$

[b] Assume dot on N_2 is moved to the lower terminal, then

$$\frac{\mathbf{V}_1}{N_1} = \frac{-\mathbf{V}_2}{N_2}, \qquad \mathbf{V}_1 = \frac{-N_1}{N_2}\mathbf{V}_2$$

$$N_1\mathbf{I}_1 = -N_2\mathbf{I}_2, \qquad \mathbf{I}_2 = \frac{-N_1}{N_2}\mathbf{I}_1$$

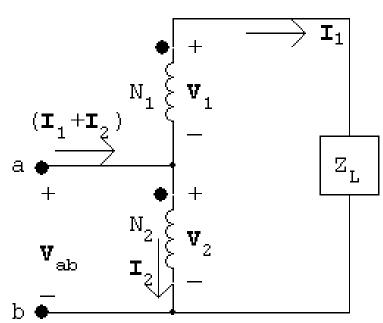
As in part [a]

$$\mathbf{V}_2 = (\mathbf{I}_2 + \mathbf{I}_1)Z_L \quad \text{and} \quad Z_{\text{ab}} = \frac{\mathbf{V}_1 + \mathbf{V}_2}{\mathbf{I}_1}$$

$$Z_{\text{ab}} = \frac{(1 - N_1/N_2)\mathbf{V}_2}{\mathbf{I}_1} = \frac{(1 - N_1/N_2)(1 - N_1/N_2)Z_L\mathbf{I}_1}{\mathbf{I}_1}$$

$$Z_{\text{ab}} = [1 - (N_1/N_2)]^2 Z_L \quad \text{Q.E.D.}$$

P 9.82 **[a]**



$$N_1\mathbf{I}_1 = N_2\mathbf{I}_2, \qquad \mathbf{I}_2 = \frac{N_1}{N_2}\mathbf{I}_1$$

$$Z_{\text{ab}} = \frac{\mathbf{V}_{\text{ab}}}{\mathbf{I}_1 + \mathbf{I}_2} = \frac{\mathbf{V}_2}{\mathbf{I}_1 + \mathbf{I}_2} = \frac{\mathbf{V}_2}{(1 + N_1/N_2)\mathbf{I}_1}$$

$$\frac{\mathbf{V}_1}{\mathbf{V}_2} = \frac{N_1}{N_2}, \qquad \mathbf{V}_1 = \frac{N_1}{N_2}\mathbf{V}_2$$

$$\mathbf{V}_1 + \mathbf{V}_2 = Z_L\mathbf{I}_1 = \left(\frac{N_1}{N_2} + 1\right)\mathbf{V}_2$$

$$Z_{\text{ab}} = \frac{\mathbf{I}_1 Z_L}{(N_1/N_2 + 1)(1 + N_1/N_2)\mathbf{I}_1}$$

$$\therefore \quad Z_{\text{ab}} = \frac{Z_L}{[1 + (N_1/N_2)]^2} \quad \text{Q.E.D.}$$

[b] Assume dot on the N_2 coil is moved to the lower terminal. Then

$$\mathbf{V}_1 = -\frac{N_1}{N_2}\mathbf{V}_2 \quad \text{and} \quad \mathbf{I}_2 = -\frac{N_1}{N_2}\mathbf{I}_1$$

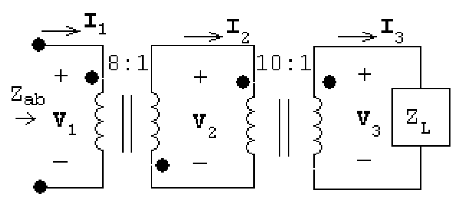
As before

$$Z_{\text{ab}} = \frac{\mathbf{V}_2}{\mathbf{I}_1 + \mathbf{I}_2} \quad \text{and} \quad \mathbf{V}_1 + \mathbf{V}_2 = Z_L\mathbf{I}_1$$

$$\therefore \quad Z_{\text{ab}} = \frac{\mathbf{V}_2}{(1 - N_1/N_2)\mathbf{I}_1} = \frac{Z_L\mathbf{I}_1}{[1 - (N_1/N_2)]^2\mathbf{I}_1}$$

$$Z_{\text{ab}} = \frac{Z_L}{[1 - (N_1/N_2)]^2} \quad \text{Q.E.D.}$$

P 9.83



$$Z_L = \frac{\mathbf{V}_3}{\mathbf{I}_3}$$

$$\frac{\mathbf{V}_2}{10} = \frac{\mathbf{V}_3}{1}; \qquad 10\mathbf{I}_2 = 1\mathbf{I}_3$$

$$\frac{\mathbf{V}_1}{8} = -\frac{\mathbf{V}_2}{1}; \qquad 8\mathbf{I}_1 = -1\mathbf{I}_2$$

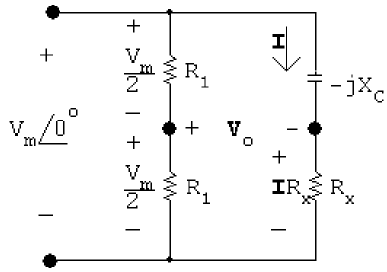
$$Z_{\text{ab}} = \frac{\mathbf{V}_1}{\mathbf{I}_1}$$

Substituting,

$$Z_{\text{ab}} = \frac{\mathbf{V}_1}{\mathbf{I}_1} = \frac{-8\mathbf{V}_2}{-\mathbf{I}_2/8} = \frac{8^2\mathbf{V}_2}{\mathbf{I}_2}$$

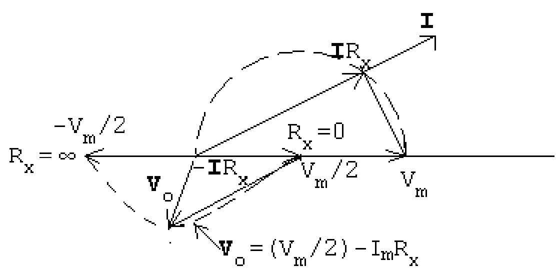
$$= \frac{8^2(10\mathbf{V}_3)}{\mathbf{I}_3/10} = \frac{(8)^2(10)^2\mathbf{V}_3}{\mathbf{I}_3} = (8)^2(10)^2 Z_L = (8)^2(10)^2(80\underline{/60^\circ}) = 512{,}000\underline{/60^\circ}\,\Omega$$

P 9.84 The phasor domain equivalent circuit is



$$V_o = \frac{V_m}{2} - \mathbf{I}R_x; \qquad \mathbf{I} = \frac{V_m}{R_x - jX_C}$$

As R_x varies from 0 to ∞, the amplitude of v_o remains constant and its phase angle increases from 0° to −180°, as shown in the following phasor diagram:



P 9.85 [a] $\mathbf{I} = \frac{240}{24} + \frac{240}{j32} = (10 - j7.5)\,\text{A}$

$$\mathbf{V}_s = 240\underline{/0^\circ} + (0.1 + j0.8)(10 - j7.5) = 247 + j7.25 = 247.11\underline{/1.68^\circ}\,\text{V}$$

[b] Use the capacitor to eliminate the j component of $\mathbf{I}$, therefore

$$\mathbf{I}_c = j7.5\,\text{A}, \qquad Z_c = \frac{240}{j7.5} = -j32\,\Omega$$

$$\mathbf{V}_s = 240 + (0.1 + j0.8)10 = 241 + j8 = 241.13\underline{/1.90^\circ}\,\text{V}$$

[c] Let I_c denote the magnitude of the current in the capacitor branch. Then

$$\mathbf{I} = (10 - j7.5 + jI_c) = 10 + j(I_c - 7.5)\,\text{A}$$

$$\mathbf{V}_s = 240\underline{/\alpha} = 240 + (0.1 + j0.8)[10 + j(I_c - 7.5)]$$
$$= (247 - 0.8I_c) + j(7.25 + 0.1I_c)$$

It follows that

$$240\cos\alpha = (247 - 0.8I_c) \quad \text{and} \quad 240\sin\alpha = (7.25 + 0.1I_c)$$

Now square each term and then add to generate the quadratic equation

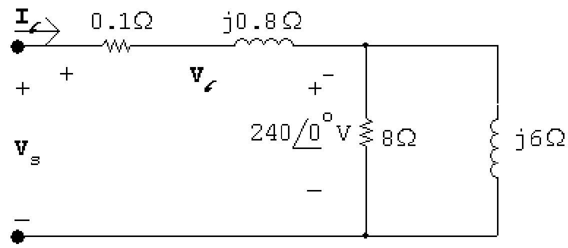
$I_c^2 - 605.77I_c + 5325.48 = 0; \qquad I_c = 302.88 \pm 293.96$

Therefore

$I_c = 8.92\,\text{A}$ (smallest value) and $Z_c = 240/j8.92 = -j26.90\,\Omega$.

Therefore, the capacitive reactance is $-26.90\,\Omega$.

P 9.86 [a]

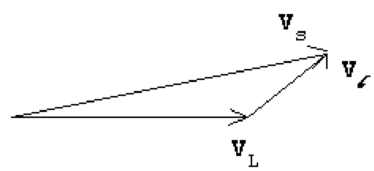


$$\mathbf{I}_\ell = \frac{240}{8} + \frac{240}{j6} = 30 - j40\,\text{A}$$

$$\mathbf{V}_\ell = (0.1 + j0.8)(30 - j40) = 35 + j20 = 40.31\underline{/29.74^\circ}\,\text{V}$$

$$\mathbf{V}_s = 240\underline{/0^\circ} + \mathbf{V}_\ell = 275 + j20 = 275.73\underline{/4.16^\circ}\,\text{V}$$

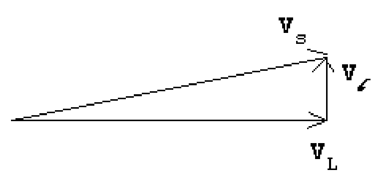
[b]



[c] $\mathbf{I}_\ell = 30 - j40 + \dfrac{240}{-j5} = 30 + j8\,\text{A}$

$$\mathbf{V}_\ell = (0.1 + j0.8)(30 + j8) = -3.4 + j24.8 = 25.03\underline{/97.81^\circ}$$

$$\mathbf{V}_s = 240\underline{/0^\circ} + \mathbf{V}_\ell = 236.6 + j24.8 = 237.9\underline{/5.98^\circ}$$



P 9.87 [a] $\mathbf{I}_1 = \dfrac{120}{24} + \dfrac{240}{8.4 + j6.3} = 23.29 - j13.71 = 27.02\underline{/-30.5^\circ}\,\text{A}$

$$\mathbf{I}_2 = \frac{120}{12} - \frac{120}{24} = 5\underline{/0^\circ}\,\text{A}$$

$$\mathbf{I}_3 = \frac{120}{12} + \frac{240}{8.4 + j6.3} = 28.29 - j13.71 = 31.44\underline{/-25.87^\circ}\,\text{A}$$

$$\mathbf{I}_4 = \frac{120}{24} = 5\underline{/0^\circ}\,\text{A}; \qquad \mathbf{I}_5 = \frac{120}{12} = 10\underline{/0^\circ}\,\text{A}$$

$$\mathbf{I}_6 = \frac{240}{8.4 + j6.3} = 18.29 - j13.71 = 22.86\underline{/-36.87^\circ}\,\text{A}$$

[**b**] When fuse A is interrupted,

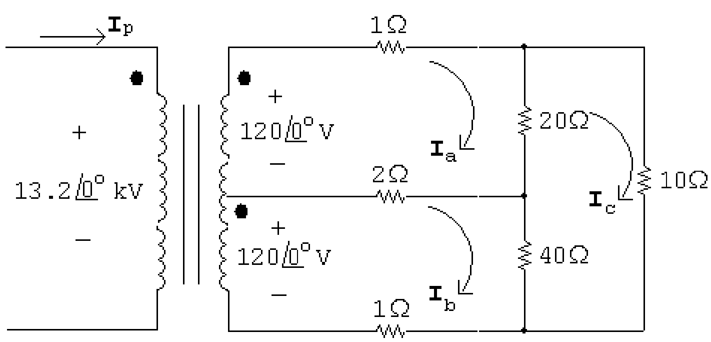
$\mathbf{I}_1 = 0$	$\mathbf{I}_3 = 15\,\text{A}$	$\mathbf{I}_5 = 10\,\text{A}$
$\mathbf{I}_2 = 10 + 5 = 15\,\text{A}$	$\mathbf{I}_4 = -5\,\text{A}$	$\mathbf{I}_6 = 5\,\text{A}$

[**c**] The clock and television set were fed from the uninterrupted side of the circuit, that is, the $12\,\Omega$ load includes the clock and the TV set.

[**d**] No, the motor current drops to 5 A, well below its normal running value of 22.86 A.

[**e**] After fuse A opens, the current in fuse B is only 15 A.

P 9.88 [a] The circuit is redrawn, with mesh currents identified:



The mesh current equations are:

$$120\underline{/0^\circ} = 23\mathbf{I}_a - 2\mathbf{I}_b - 20\mathbf{I}_c$$

$$120\underline{/0^\circ} = -2\mathbf{I}_a + 43\mathbf{I}_b - 40\mathbf{I}_c$$

$$0 = -20\mathbf{I}_a - 40\mathbf{I}_b + 70\mathbf{I}_c$$

Solving,

$$\mathbf{I}_a = 24\underline{/0^\circ}\,\text{A} \qquad \mathbf{I}_b = 21.96\underline{/0^\circ}\,\text{A} \qquad \mathbf{I}_c = 19.40\underline{/0^\circ}\,\text{A}$$

The branch currents are:

$$\mathbf{I}_1 = \mathbf{I}_a = 24\underline{/0^\circ}\,\text{A}$$

$$\mathbf{I}_2 = \mathbf{I}_a - \mathbf{I}_b = 2.04\underline{/0^\circ}\,\text{A}$$

$$\mathbf{I}_3 = \mathbf{I}_b = 21.96\underline{/0^\circ}\,\text{A}$$

$\mathbf{I}_4 = \mathbf{I}_c = 19.40\underline{/0^\circ}\,\text{A}$

$\mathbf{I}_5 = \mathbf{I}_a - \mathbf{I}_c = 4.6\underline{/0^\circ}\,\text{A}$

$\mathbf{I}_6 = \mathbf{I}_b - \mathbf{I}_c = 2.55\underline{/0^\circ}\,\text{A}$

[b] Let N_1 be the number of turns on the primary winding; because the secondary winding is center-tapped, let $2N_2$ be the total turns on the secondary. From Fig. 9.58,

$$\frac{13{,}200}{N_1} = \frac{240}{2N_2} \qquad \text{or} \qquad \frac{N_2}{N_1} = \frac{1}{110}$$

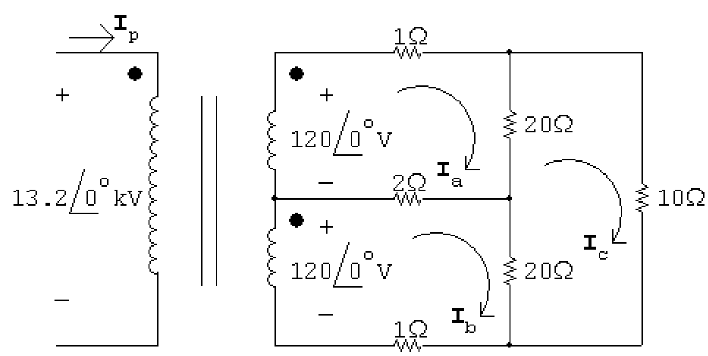
The ampere turn balance requires

$$N_1\mathbf{I}_p = N_2\mathbf{I}_1 + N_2\mathbf{I}_3$$

Therefore,

$$\mathbf{I}_p = \frac{N_2}{N_1}(\mathbf{I}_1 + \mathbf{I}_3) = \frac{1}{110}(24 + 21.96) = 0.42\underline{/0^\circ}\,\text{A}$$

P 9.89 **[a]**



The three mesh current equations are

$$120\underline{/0^\circ} = 23\mathbf{I}_\text{a} - 2\mathbf{I}_\text{b} - 20\mathbf{I}_\text{c}$$

$$120\underline{/0^\circ} = -2\mathbf{I}_\text{a} + 23\mathbf{I}_\text{b} - 20\mathbf{I}_\text{c}$$

$$0 = -20\mathbf{I}_\text{a} - 20\mathbf{I}_\text{b} + 50\mathbf{I}_\text{c}$$

Solving,

$$\mathbf{I}_\text{a} = 24\underline{/0^\circ}\,\text{A}; \qquad \mathbf{I}_\text{b} = 24\underline{/0^\circ}\,\text{A}; \qquad \mathbf{I}_\text{c} = 19.2\underline{/0^\circ}\,\text{A}$$

$$\therefore \quad \mathbf{I}_2 = \mathbf{I}_\text{a} - \mathbf{I}_\text{b} = 0\,\text{A}$$

[b] $$\mathbf{I}_\text{p} = \frac{N_2}{N_1}(\mathbf{I}_1 + \mathbf{I}_3) = \frac{N_2}{N_1}(\mathbf{I}_\text{a} + \mathbf{I}_\text{b})$$

$$= \frac{1}{110}(24 + 24) = 0.436\underline{/0^\circ}\,\text{A}$$

[c] Yes; when the two 120 V loads are equal, there is no current in the "neutral" line, so no power is lost to this line. Since you pay for power, the cost is lower when the loads are equal.

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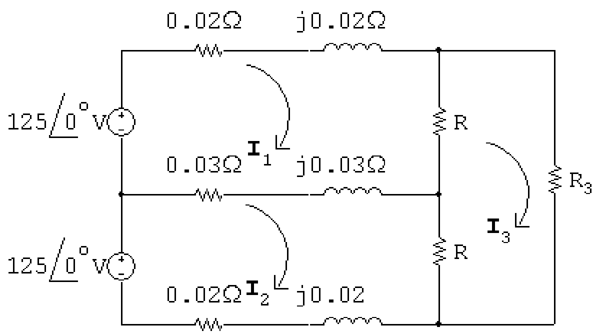
P 9.90 [a]



$$125 = (R + 0.05 + j0.05)\mathbf{I}_1 - (0.03 + j0.03)\mathbf{I}_2 - R\mathbf{I}_3$$

$$125 = -(0.03 + j0.03)\mathbf{I}_1 + (R + 0.05 + j0.05)\mathbf{I}_2 - R\mathbf{I}_3$$

Subtracting the above two equations gives

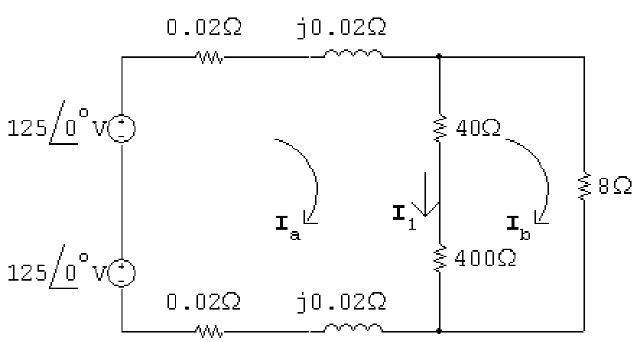
$$0 = (R + 0.08 + j0.08)\mathbf{I}_1 - (R + 0.08 + j0.08)\mathbf{I}_2$$

$$\therefore \quad \mathbf{I}_1 = \mathbf{I}_2 \quad \text{so} \quad \mathbf{I}_\text{n} = \mathbf{I}_1 - \mathbf{I}_2 = 0\,\text{A}$$

[b] $\mathbf{V}_1 = R(\mathbf{I}_1 - \mathbf{I}_3); \qquad \mathbf{V}_2 = R(\mathbf{I}_2 - \mathbf{I}_3)$

Since $\mathbf{I}_1 = \mathbf{I}_2$ (from part [a]) $\mathbf{V}_1 = \mathbf{V}_2$

[c]



$$250 = (440.04 + j0.04)\mathbf{I}_\text{a} - 440\mathbf{I}_\text{b}$$

$$0 = -440\mathbf{I}_\text{a} + 448\mathbf{I}_\text{b}$$

Solving,

$$\mathbf{I}_\text{a} = 31.656207 - j0.160343\,\text{A}$$

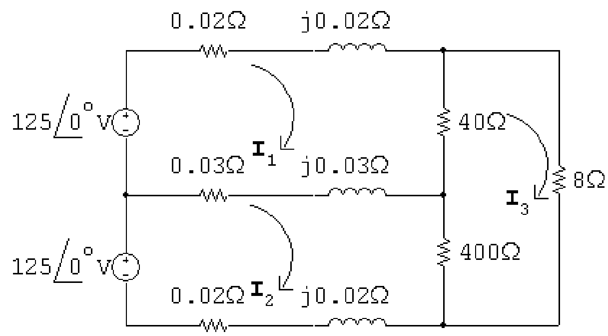
$\mathbf{I}_\mathrm{b} = 31.090917 - j0.157479\,\mathrm{A}$

$\mathbf{I}_1 = \mathbf{I}_\mathrm{a} - \mathbf{I}_\mathrm{b} = 0.56529 - j0.002864\,\mathrm{A}$

$\mathbf{V}_1 = 40\mathbf{I}_1 = 22.612 - j0.11456 = 22.612\underline{/-0.290282^\circ}\,\mathrm{V}$

$\mathbf{V}_2 = 400\mathbf{I}_1 = 226.116 - j1.1456 = 226.1189\underline{/-0.290282^\circ}\,\mathrm{V}$

[d]



$125 = (40.05 + j0.05)\mathbf{I}_1 - (0.03 + j0.03)\mathbf{I}_2 - 40\mathbf{I}_3$

$125 = -(0.03 + j0.03)\mathbf{I}_1 + (400.05 + j0.05)\mathbf{I}_2 - 400\mathbf{I}_3$

$0 = -40\mathbf{I}_1 - 400\mathbf{I}_2 + 448\mathbf{I}_3$

Solving,

$\mathbf{I}_1 = 34.19 - j0.182\,\mathrm{A}$

$\mathbf{I}_2 = 31.396 - j0.164\,\mathrm{A}$

$\mathbf{I}_3 = 31.085 - j0.163\,\mathrm{A}$

$\mathbf{V}_1 = 40(\mathbf{I}_1 - \mathbf{I}_3) = 124.2\underline{/-0.35^\circ}\,\mathrm{V}$

$\mathbf{V}_2 = 400(\mathbf{I}_2 - \mathbf{I}_3) = 124.4\underline{/-0.18^\circ}\,\mathrm{V}$

[e] Because an open neutral can result in severely unbalanced voltages across the 125 V loads.

P 9.91 **[a]** Let N_1 = primary winding turns and $2N_2$ = secondary winding turns. Then

$$\frac{14{,}000}{N_1} = \frac{250}{2N_2}; \qquad \therefore \quad \frac{N_2}{N_1} = \frac{1}{112} = a$$

In part c),

$\mathbf{I}_\mathrm{p} = 2a\mathbf{I}_\mathrm{a}$

$$\therefore \quad \mathbf{I}_\mathrm{p} = \frac{2N_2\mathbf{I}_\mathrm{a}}{N_1} = \frac{1}{56}\mathbf{I}_\mathrm{a}$$

$$= \frac{1}{56}(31.656 - j0.16)$$

$$\mathbf{I}_\mathrm{p} = 565.3 - j2.9\,\mathrm{mA}$$

In part d),

$$\mathbf{I}_\mathrm{p}N_1 = \mathbf{I}_1N_2 + \mathbf{I}_2N_2$$

$$\therefore \quad \mathbf{I}_\mathrm{p} = \frac{N_2}{N_1}(\mathbf{I}_1 + \mathbf{I}_2)$$

$$= \frac{1}{112}(34.19 - j0.182 + 31.396 - j0.164)$$

$$= \frac{1}{112}(65.586 - j0.346)$$

$$\mathbf{I}_\mathrm{p} = 585.6 - j3.1\,\mathrm{mA}$$

[b] Yes, because the neutral conductor carries non-zero current whenever the load is not balanced.

10

Sinusoidal Steady State Power Calculations

Assessment Problems

AP 10.1 [a] $\mathbf{V} = 100\underline{/-45^\circ}\text{ V}, \qquad \mathbf{I} = 20\underline{/15^\circ}\text{ A}$

Therefore

$$P = \frac{1}{2}(100)(20)\cos[-45 - (15)] = 500\,\text{W}, \qquad \text{A} \rightarrow \text{B}$$

$$Q = 1000\sin -60^\circ = -866.03\text{ VAR}, \qquad \text{B} \rightarrow \text{A}$$

[b] $\mathbf{V} = 100\underline{/-45^\circ}, \qquad \mathbf{I} = 20\underline{/165^\circ}$

$$P = 1000\cos(-210^\circ) = -866.03\,\text{W}, \qquad \text{B} \rightarrow \text{A}$$

$$Q = 1000\sin(-210^\circ) = 500\text{ VAR}, \qquad \text{A} \rightarrow \text{B}$$

[c] $\mathbf{V} = 100\underline{/-45^\circ}, \qquad \mathbf{I} = 20\underline{/-105^\circ}$

$$P = 1000\cos(60^\circ) = 500\,\text{W}, \qquad \text{A} \rightarrow \text{B}$$

$$Q = 1000\sin(60^\circ) = 866.03\text{ VAR}, \qquad \text{A} \rightarrow \text{B}$$

[d] $\mathbf{V} = 100\underline{/0^\circ}, \qquad \mathbf{I} = 20\underline{/120^\circ}$

$$P = 1000\cos(-120^\circ) = -500\,\text{W}, \qquad \text{B} \rightarrow \text{A}$$

$$Q = 1000\sin(-120^\circ) = -866.03\text{ VAR}, \qquad \text{B} \rightarrow \text{A}$$

AP 10.2

$$\text{pf} = \cos(\theta_v - \theta_i) = \cos[15 - (75)] = \cos(-60^\circ) = 0.5\text{ leading}$$

$$\text{rf} = \sin(\theta_v - \theta_i) = \sin(-60^\circ) = -0.866$$

AP 10.3

From Ex. 9.4 $\quad I_{\text{eff}} = \dfrac{I_\rho}{\sqrt{3}} = \dfrac{0.18}{\sqrt{3}}\,\text{A}$

$$P = I_{\text{eff}}^2 R = \left(\frac{0.0324}{3}\right)(5000) = 54\,\text{W}$$

AP 10.4 **[a]** $Z = (39 + j26)\|(-j52) = 48 - j20 = 52\underline{/-22.62^\circ}\,\Omega$

Therefore $\quad \mathbf{I}_\ell = \dfrac{250\underline{/0^\circ}}{48 - j20 + 1 + j4} = 4.85\underline{/18.08^\circ}\,\text{A (rms)}$

$$\mathbf{V}_\text{L} = Z\mathbf{I}_\ell = (52\underline{/-22.62^\circ})(4.85\underline{/18.08^\circ}) = 252.20\underline{/-4.54^\circ}\,\text{V (rms)}$$

$$\mathbf{I}_\text{L} = \frac{\mathbf{V}_\text{L}}{39 + j26} = 5.38\underline{/-38.23^\circ}\,\text{A (rms)}$$

[b] $S_\text{L} = \mathbf{V}_L\mathbf{I}_L^* = (252.20\underline{/-4.54^\circ})(5.38\underline{/+38.23^\circ}) = 1357\underline{/33.69^\circ}$

$= (1129.09 + j752.73)\,\text{VA}$

$P_\text{L} = 1129.09\,\text{W}; \qquad Q_\text{L} = 752.73\ \text{VAR}$

[c] $P_\ell = |\mathbf{I}_\ell|^2 1 = (4.85)^2 \cdot 1 = 23.52\,\text{W}; \qquad Q_\ell = |\mathbf{I}_\ell|^2 4 = 94.09\ \text{VAR}$

[d] $S_g(\text{delivering}) = 250\mathbf{I}_\ell^* = (1152.62 - j376.36)\,\text{VA}$

Therefore the source is delivering 1152.62 W and absorbing 376.36 magnetizing VAR.

[e] $Q_\text{cap} = \dfrac{|\mathbf{V}_\text{L}|^2}{-52} = \dfrac{(252.20)^2}{-52} = -1223.18\ \text{VAR}$

Therefore the capacitor is delivering 1223.18 magnetizing VAR.

Check: $\quad 94.09 + 752.73 + 376.36 = 1223.18\ \text{VAR}\quad$ and

$\quad 1129.09 + 23.52 = 1152.62\,\text{W}$

AP 10.5 Series circuit derivation:

$$S = 250\mathbf{I}^* = (40{,}000 - j30{,}000)$$

Therefore $\quad \mathbf{I}^* = 160 - j120 = 200\underline{/-36.87^\circ}\,\text{A (rms)}$

$$\mathbf{I} = 200\underline{/36.87^\circ}\,\text{A (rms)}$$

$$Z = \frac{\mathbf{V}}{\mathbf{I}} = \frac{250}{200\underline{/36.87^\circ}} = 1.25\underline{/-36.87^\circ} = (1 - j0.75)\,\Omega$$

Therefore $\quad R = 1\,\Omega, \qquad X_\text{C} = -0.75\,\Omega$

Parallel circuit derivation

$$P = \frac{(250)^2}{R}; \qquad \text{therefore} \quad R = \frac{(250)^2}{40{,}000} = 1.5625\,\Omega$$

$$Q = \frac{(250)^2}{X_C}; \qquad \text{therefore} \quad X_C = \frac{(250)^2}{-30{,}000} = -2.083\,\Omega$$

AP 10.6

$$S_1 = 15{,}000(0.6) + j15{,}000(0.8) = 9000 + j12{,}000\,\text{VA}$$

$$S_2 = 6000(0.8) - j6000(0.6) = 4800 - j3600\,\text{VA}$$

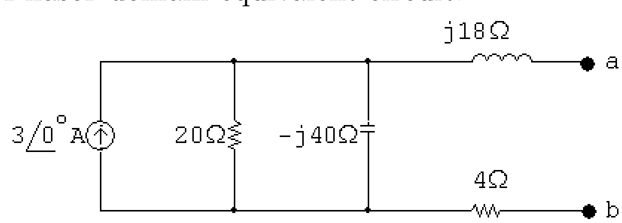
$$S_T = S_1 + S_2 = 13{,}800 + j8400\,\text{VA}$$

$$S_T = 200\mathbf{I}^*; \qquad \text{therefore} \quad \mathbf{I}^* = 69 + j42 \qquad \mathbf{I} = 69 - j42\,\text{A}$$

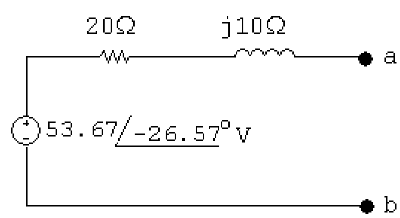
$$\mathbf{V}_s = 200 + j\mathbf{I} = 200 + j69 + 42 = 242 + j69 = 251.64\underline{/15.91^\circ}\,\text{V (rms)}$$

AP 10.7 [a] The phasor domain equivalent circuit and the Thévenin equivalent are shown below:

Phasor domain equivalent circuit:



Thévenin equivalent:



$$\mathbf{V}_{\text{Th}} = 3\frac{-j800}{20 - j40} = 48 - j24 = 53.67\underline{/-26.57^\circ}\,\text{V}$$

$$Z_{\text{Th}} = 4 + j18 + \frac{-j800}{20 - j40} = 20 + j10 = 22.36\underline{/26.57^\circ}\,\Omega$$

For maximum power transfer, $Z_L = (20 - j10)\,\Omega$

[b] $\mathbf{I} = \dfrac{53.67\angle -26.57^\circ}{40} = 1.34\angle -26.57^\circ\,\text{A}$

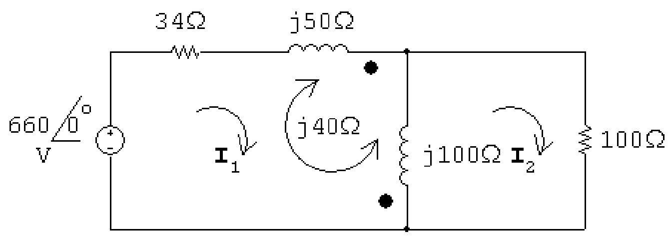
Therefore $P = \left(\dfrac{1.34}{\sqrt{2}}\right)^2 20 = 17.96\,\text{W}$

[c] $R_\text{L} = |Z_\text{Th}| = 22.36\,\Omega$

[d] $\mathbf{I} = \dfrac{53.67\angle -26.57^\circ}{42.36 + j10} = 1.23\angle -39.85^\circ\,\text{A}$

Therefore $P = \left(\dfrac{1.23}{\sqrt{2}}\right)^2 (22.36) = 17\,\text{W}$

AP 10.8



Mesh current equations:

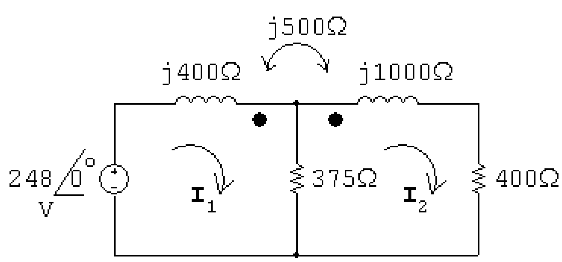
$$660 = (34 + j50)\mathbf{I}_1 + j100(\mathbf{I}_1 - \mathbf{I}_2) + j40\mathbf{I}_1 + j40(\mathbf{I}_1 - \mathbf{I}_2)$$

$$0 = j100(\mathbf{I}_2 - \mathbf{I}_1) - j40\mathbf{I}_1 + 100\mathbf{I}_2$$

Solving,

$$\mathbf{I}_2 = 3.5\angle 0^\circ\,\text{A}; \qquad \therefore \quad P = \frac{1}{2}(3.5)^2(100) = 612.50\,\text{W}$$

AP 10.9 [a]



$$248 = j400\mathbf{I}_1 - j500\mathbf{I}_2 + 375(\mathbf{I}_1 - \mathbf{I}_2)$$

$$0 = 375(\mathbf{I}_2 - \mathbf{I}_1) + j1000\mathbf{I}_2 - j500\mathbf{I}_1 + 400\mathbf{I}_2$$

Solving,

$$\mathbf{I}_1 = 0.80 - j0.62\,\text{A}; \qquad \mathbf{I}_2 = 0.4 - j0.3 = 0.5\angle -36.87^\circ$$

$$\therefore \quad P = \frac{1}{2}(0.25)(400) = 50\,\text{W}$$

[b] $\mathbf{I}_1 - \mathbf{I}_2 = 0.4 - j0.32\,\text{A}$

$$P_{375} = \frac{1}{2}|\mathbf{I}_1 - \mathbf{I}_2|^2(375) = 49.20\,\text{W}$$

[c] $P_g = \dfrac{1}{2}(248)(0.8) = 99.20\,\text{W}$

$$\sum P_{\text{abs}} = 50 + 49.2 = 99.20\,\text{W} \quad \text{(checks)}$$

AP 10.10 **[a]** $V_{\text{Th}} = 210\,\text{V}; \qquad \mathbf{V}_2 = \frac{1}{4}\mathbf{V}_1; \qquad \mathbf{I}_1 = \frac{1}{4}\mathbf{I}_2$

Short circuit equations:

$$840 = 80\mathbf{I}_1 - 20\mathbf{I}_2 + \mathbf{V}_1$$

$$0 = 20(\mathbf{I}_2 - \mathbf{I}_1) - \mathbf{V}_2$$

$$\therefore \quad \mathbf{I}_2 = 14\,\text{A}; \qquad R_{\text{Th}} = \frac{210}{14} = 15\,\Omega$$

[b] $P_{\max} = \left(\dfrac{210}{30}\right)^2 15 = 735\,\text{W}$

AP 10.11 **[a]** $\mathbf{V}_{\text{Th}} = -4(146\underline{/0^\circ}) = -584\underline{/0^\circ}\,\text{V (rms)}$

$$\mathbf{V}_2 = 4\mathbf{V}_1; \qquad \mathbf{I}_1 = -4\mathbf{I}_2$$

Short circuit equations:

$$146\underline{/0^\circ} = 80\mathbf{I}_1 - 20\mathbf{I}_2 + \mathbf{V}_1$$

$$0 = 20(\mathbf{I}_2 - \mathbf{I}_1) - \mathbf{V}_2$$

$$\therefore \quad \mathbf{I}_2 = -146/365 = -0.40\,\text{A}; \qquad R_{\text{Th}} = \frac{-584}{-0.4} = 1460\,\Omega$$

[b] $P = \left(\dfrac{-584}{2920}\right)^2 1460 = 58.40\,\text{W}$

Problems

P 10.1 **[a]** $P = \frac{1}{2}(100)(10)\cos(50 - 15) = 500\cos 35^\circ = 409.58\,\mathrm{W} \quad \text{(abs)}$

$Q = 500\sin 35^\circ = 286.79\ \mathrm{VAR} \quad \text{(abs)}$

[b] $P = \frac{1}{2}(40)(20)\cos(-15 - 60) = 400\cos(-75^\circ) = 103.53\,\mathrm{W} \quad \text{(abs)}$

$Q = 400\sin(-75^\circ) = -386.37\ \mathrm{VAR} \quad \text{(del)}$

[c] $P = \frac{1}{2}(400)(10)\cos(30 - 150) = 2000\cos(-120^\circ) = -1000\,\mathrm{W} \quad \text{(del)}$

$Q = 2000\sin(-120^\circ) = -1732.05\ \mathrm{VAR} \quad \text{(del)}$

[d] $P = \frac{1}{2}(200)(5)\cos(160 - 40) = 500\cos(120^\circ) = -250\,\mathrm{W} \quad \text{(del)}$

$Q = 500\sin(120^\circ) = 433.01\ \mathrm{VAR} \quad \text{(abs)}$

P 10.2 **[a]** hair dryer = 600 W   vacuum = 630 W

sun lamp = 279 W   air conditioner = 860 W

television = 240 W $\sum P = 2609\,\mathrm{W}$

Therefore $\quad I_{\text{eff}} = \frac{2609}{120} = 21.74\,\mathrm{A}$

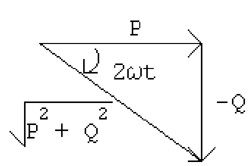
Yes, the breaker will trip.

[b] $\sum P = 2609 - 909 = 1700\,\mathrm{W}; \qquad I_{\text{eff}} = \frac{1700}{120} = 14.17\,\mathrm{A}$

Yes, the breaker will not trip if the current is reduced to 14.17 A.

P 10.3 $p = P + P\cos 2\omega t - Q\sin 2\omega t; \qquad \frac{dp}{dt} = -2\omega P\sin 2\omega t - 2\omega Q\cos 2\omega t$

$$\frac{dp}{dt} = 0 \quad \text{when} \quad -2\omega P\sin 2\omega t = 2\omega Q\cos 2\omega t \quad \text{or} \quad \tan 2\omega t = -\frac{Q}{P}$$



$$\cos 2\omega t = \frac{P}{\sqrt{P^2 + Q^2}}; \qquad \sin 2\omega t = -\frac{Q}{\sqrt{P^2 + Q^2}}$$

Let $\theta = \tan^{-1}(-Q/P)$, then p is maximum when $2\omega t = \theta$ and p is minimum when $2\omega t = (\theta + \pi)$.

Therefore $\quad p_{\max} = P + P \cdot \dfrac{P}{\sqrt{P^2+Q^2}} - \dfrac{Q(-Q)}{\sqrt{P^2+Q^2}} = P + \sqrt{P^2+Q^2}$

and $\quad p_{\min} = P - P \cdot \dfrac{P}{\sqrt{P^2+Q^2}} - Q \cdot \dfrac{Q}{\sqrt{P^2+Q^2}} = P - \sqrt{P^2+Q^2}$

P 10.4 **[a]** $P = \dfrac{1}{2}\dfrac{(240)^2}{480} = 60\,\text{W}$

$$-\frac{1}{\omega C} = \frac{-9 \times 10^6}{(5000)(5)} = -360\,\Omega$$

$$Q = \frac{1}{2}\frac{(240)^2}{(-360)} = -80\,\text{VAR}$$

$$p_{\max} = P + \sqrt{P^2+Q^2} = 60 + \sqrt{(60)^2 + (80)^2} = 160\,\text{W (del)}$$

[b] $p_{\min} = 60 - \sqrt{60^2 + 80^2} = -40\,\text{W (abs)}$

[c] $P = 60\,\text{W}$ from (a)

[d] $Q = -80\,\text{VAR}$ from (a)

[e] generates, because $Q < 0$

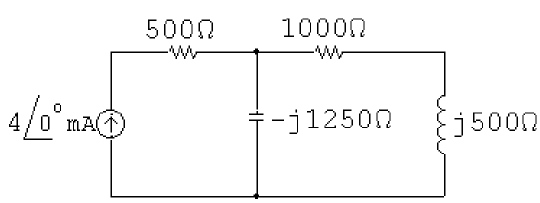
[f] pf $= \cos(\theta_v - \theta_i)$

$$\mathbf{I} = \frac{240}{480} + \frac{240}{-j360} = 0.5 + j0.67 = 0.83\underline{/53.13^\circ}\,\text{A}$$

$\therefore$ pf $= \cos(0 - 53.13^\circ) = 0.6$ leading

[g] rf $= \sin(-53.13^\circ) = -0.8$

P 10.5 $\mathbf{I}_g = 4\underline{/0^\circ}\,\text{mA}; \qquad \dfrac{1}{j\omega C} = -j1250\,\Omega; \qquad j\omega L = j500\,\Omega$

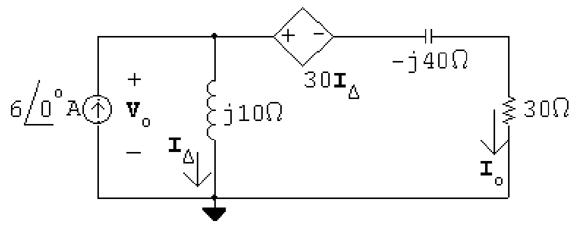


$$Z_{\text{eq}} = 500 + [-j1250 \| (1000 + j500)] = 1500 - j500\,\Omega$$

$$P_g = -\frac{1}{2}|I|^2 \text{Re}\{Z_{\text{eq}}\} = -\frac{1}{2}(0.004)^2(1500) = -12\,\text{mW}$$

The source delivers 12 mW of power to the circuit.

P 10.6 $j\omega L = j20{,}000(0.5 \times 10^{-3}) = j10\,\Omega; \qquad \frac{1}{j\omega C} = \frac{10^6}{j20{,}000(1.25)} = -j40\,\Omega$



$$-6 + \frac{\mathbf{V}_o}{j10} + \frac{\mathbf{V}_o - 30(\mathbf{V}_o/j10)}{30 - j40} = 0$$

$$\therefore \quad \mathbf{V}_o\left[\frac{1}{j10} + \frac{1 + j3}{30 - j40}\right] = 6$$

$$\therefore \quad \mathbf{V}_o = 100\underline{/126.87^\circ}\,\mathrm{V}$$

$$\therefore \quad \mathbf{I}_\Delta = \frac{\mathbf{V}_o}{j10} = 10\underline{/36.87^\circ}\,\mathrm{A}$$

$$\mathbf{I}_o = 6\underline{/0^\circ} - \mathbf{I}_\Delta = 6 - 8 - j6 = -2 - j6 = 6.32\underline{/-108.43^\circ}\,\mathrm{A}$$

$$P_{30\Omega} = \frac{1}{2}|\mathbf{I}_o|^2 30 = 600\,\mathrm{W}$$

P 10.7 $Z_\mathrm{f} = -j10{,}000 \| 20{,}000 = 4000 - j8000\,\Omega$

$$Z_\mathrm{i} = 2000 - j2000\,\Omega$$

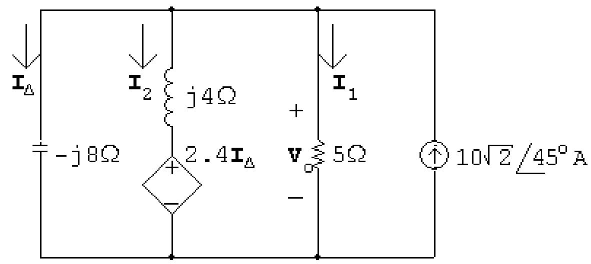
$$\therefore \quad \frac{Z_\mathrm{f}}{Z_\mathrm{i}} = \frac{4000 - j8000}{2000 - j2000} = 3 - j1$$

$$\mathbf{V}_o = -\frac{Z_\mathrm{f}}{Z_\mathrm{i}}\mathbf{V}_g; \qquad \mathbf{V}_g = 1\underline{/0^\circ}\,\mathrm{V}$$

$$\mathbf{V}_o = -(3 - j1)(1) = -3 + j1 = 3.16\underline{/161.57^\circ}\,\mathrm{V}$$

$$P = \frac{1}{2}\frac{V_m^2}{R} = \frac{1}{2}\frac{(10)}{1000} = 5 \times 10^{-3} = 5\,\mathrm{mW}$$

P 10.8 [a] From the solution to Problem 9.59 we have:



$$\mathbf{V}_o = j80 = 80\underline{/90^\circ}\,\text{V}$$

$$S_g = -\frac{1}{2}\mathbf{V}_o\mathbf{I}_g^* = -\frac{1}{2}(j80)(10 - j10) = -400 - j400\,\text{VA}$$

Therefore, the independent current source is delivering 400 W and 400 magnetizing vars.

$$\mathbf{I}_1 = \frac{\mathbf{V}_o}{5} = j16\,\text{A}$$

$$P_{5\Omega} = \frac{1}{2}(16)^2(5) = 640\,\text{W}$$

Therefore, the $8\,\Omega$ resistor is absorbing 640 W.

$$\mathbf{I}_\Delta = \frac{\mathbf{V}_o}{-j8} = -10\,\text{A}$$

$$Q_{\text{cap}} = \frac{1}{2}(10)^2(-8) = -400\,\text{VAR}$$

Therefore, the $-j8\,\Omega$ capacitor is developing 400 magnetizing vars.

$$2.4\mathbf{I}_\Delta = -24\,\text{V}$$

$$\mathbf{I}_2 = \frac{\mathbf{V}_o - 2.4\mathbf{I}_\Delta}{j4} = \frac{-j80 + 24}{j4}$$

$$= 20 - j6\,\text{A} = 20.88\underline{/-16.7^\circ}\,\text{A}$$

$$Q_{j4} = \frac{1}{2}|\mathbf{I}_2|^2(4) = 872\,\text{VAR}$$

Therefore, the $j4\,\Omega$ inductor is absorbing 872 magnetizing vars.

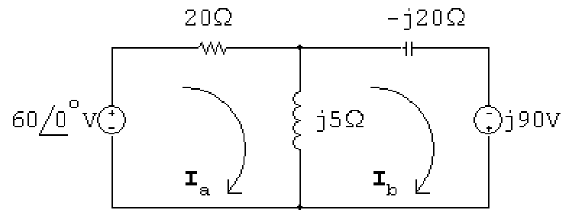
$$S_{\text{d.s.}} = \tfrac{1}{2}(2.4\mathbf{I}_\Delta)\mathbf{I}_2^* = \tfrac{1}{2}(-24)(20 + j6)$$
$$= -240 - j72\,\text{VA}$$

Thus the dependent source is delivering 240 W and 72 magnetizing vars.

[b] $\sum P_{\text{gen}} = 400 + 240 = 640\,\text{W} = \sum P_{\text{abs}}$

[c] $\sum Q_{\rm gen} = 400 + 400 + 72 = 872\,{\rm VAR} = \sum Q_{\rm abs}$

P 10.9 **[a]** From the solution to Problem 9.61 we have



$\mathbf{I}_{\rm a} = 2.25 - j2.25\,{\rm A}; \quad \mathbf{I}_{\rm b} = -6.75 + j0.75\,{\rm A}; \quad \mathbf{I}_o = 9 - j3\,{\rm A}$

$$S_{60V} = -\frac{1}{2}(60)\mathbf{I}_{\rm a}^* = -30(2.25 + j2.25) = -67.5 - j67.5\,{\rm VA}$$

Thus, the 60 V source is developing 67.5 W and 67.5 magnetizing vars.

$$S_{90V} = -\tfrac{1}{2}(j90)\mathbf{I}_{\rm b}^* = -j45(-6.75 - j0.75)$$
$$= -33.75 + j303.75\,{\rm VA}$$

Thus, the 90 V source is delivering 33.75 W and absorbing 303.75 magnetizing vars.

$$P_{20\Omega} = \frac{1}{2}|\mathbf{I}_{\rm a}|^2(20) = 101.25\,{\rm W}$$

Thus the $20\,\Omega$ resistor is absorbing 101.25 W.

$$Q_{-j20\Omega} = \frac{1}{2}|\mathbf{I}_{\rm b}|^2(-20) = -461.25\,{\rm VAR}$$

Thus the $-j20\,\Omega$ capacitor is developing 461.25 magnetizing vars.

$$Q_{j5\Omega} = \frac{1}{2}|\mathbf{I}_o|^2(5) = 225\,{\rm VAR}$$

Thus the $j5\,\Omega$ inductor is absorbing 225 magnetizing vars.

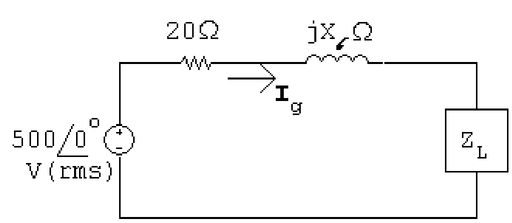
[b] $\sum P_{\rm dev} = 67.5 + 33.75 = 101.25\,{\rm W} = \sum P_{\rm abs}$

[c] $\sum Q_{\rm dev} = 67.5 + 461.25 = 528.75\,{\rm VAR}$

$$\sum Q_{\rm abs} = 225 + 303.75 = 528.75\,{\rm VAR} = \sum Q_{\rm dev}$$

P 10.10 **[a]** line loss $= 7500 - 2500 = 5\,{\rm kW}$

line loss $= |\mathbf{I}_g|^2 20 \qquad \therefore \quad |\mathbf{I}_g|^2 = 250$

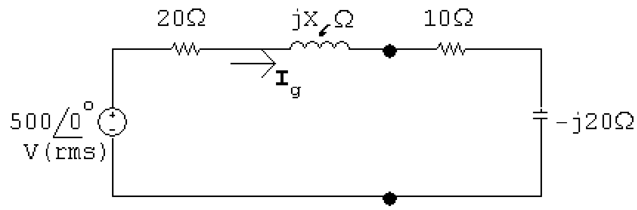


$|\mathbf{I}_g| = \sqrt{250}\,{\rm A}$

$|\mathbf{I}_g|^2 R_\mathrm{L} = 2500 \qquad \therefore \quad R_\mathrm{L} = 10\,\Omega$

$|\mathbf{I}_g|^2 X_\mathrm{L} = -5000 \qquad \therefore \quad X_\mathrm{L} = -20\,\Omega$

Thus,



$|Z| = \sqrt{(30)^2 + (X_\ell - 20)^2} \qquad |\mathbf{I}_g| = \dfrac{500}{\sqrt{900 + (X_\ell - 20)^2}}$

$$\therefore \quad 900 + (X_\ell - 20)^2 = \frac{25 \times 10^4}{250} = 1000$$

Solving, $\quad (X_\ell - 20) = \pm 10.$

Thus, $\quad X_\ell = 10\,\Omega \qquad$ or $\qquad X_\ell = 30\,\Omega$

[b] If $X_\ell = 30\,\Omega$:

$$\mathbf{I}_g = \frac{500}{30 + j10} = 15 - j5\,\mathrm{A}$$

$S_g = -500\mathbf{I}_g^* = -7500 - j2500\,\mathrm{VA}$

Thus, the voltage source is delivering 7500 W and 2500 magnetizing vars.

$Q_{j30} = |\mathbf{I}_g|^2 X_\ell = 250(30) = 7500\,\mathrm{VAR}$

Therefore the line reactance is absorbing 7500 magnetizing vars.

$Q_{-j20} = |\mathbf{I}_g|^2 X_\mathrm{L} = 250(-20) = -5000\,\mathrm{VAR}$

Therefore the load reactance is generating 5000 magnetizing vars.

$\sum Q_\mathrm{gen} = 7500\,\mathrm{VAR} = \sum Q_\mathrm{abs}$

If $X_\ell = 10\,\Omega$:

$$\mathbf{I}_g = \frac{500}{30 - j10} = 15 + j5\,\mathrm{A}$$

$S_g = -500\mathbf{I}_g^* = -7500 + j2500\,\mathrm{VA}$

Thus, the voltage source is delivering 7500 W and absorbing 2500 magnetizing vars.

$Q_{j10} = |\mathbf{I}_g|^2(10) = 250(10) = 2500\,\mathrm{VAR}$

Therefore the line reactance is absorbing 2500 magnetizing vars. The load continues to generate 5000 magnetizing vars.

$\sum Q_\mathrm{gen} = 5000\,\mathrm{VAR} = \sum Q_\mathrm{abs}$

P 10.11 **[a]** $I_{\text{eff}} = 40/115 \cong 0.35\,\text{A}$

[b] $I_{\text{eff}} = 130/115 \cong 1.13\,\text{A}$

P 10.12 $W_{\text{dc}} = \dfrac{V_{\text{dc}}^2}{R}T; \qquad W_s = \displaystyle\int_{t_o}^{t_o+T} \frac{v_s^2}{R}\,dt$

$$\therefore \quad \frac{V_{\text{dc}}^2}{R}T = \int_{t_o}^{t_o+T} \frac{v_s^2}{R}\,dt$$

$$V_{\text{dc}}^2 = \frac{1}{T}\int_{t_o}^{t_o+T} v_s^2\,dt$$

$$V_{\text{dc}} = \sqrt{\frac{1}{T}\int_{t_o}^{t_o+T} v_s^2\,dt} = V_{\text{rms}} = V_{\text{eff}}$$

P 10.13 $i(t) = 250t \qquad 0 \le t \le 80\,\text{ms}$

$$i(t) = 100 - 1000t \qquad 80\,\text{ms} \le t \le 100\,\text{ms}$$

$$\therefore \quad I_{\text{rms}} = \sqrt{\frac{1}{0.1}\left\{\int_0^{0.08}(250)^2t^2\,dt + \int_{0.08}^{0.1}(100-1000t)^2\,dt\right\}}$$

$$\int_0^{0.08}(250)^2t^2\,dt = (250)^2\frac{t^3}{3}\Big|_0^{0.08} = \frac{32}{3}$$

$$(100-1000t)^2 = 10^4 - 2\times10^5t + 10^6t^2$$

$$\int_{0.08}^{0.1} 10^4\,dt = 200$$

$$\int_{0.08}^{0.1} 2\times10^5t\,dt = 10^5t^2\Big|_{0.08}^{0.1} = 360$$

$$10^6\int_{0.08}^{0.1} t^2\,dt = \frac{10^6}{3}t^3\Big|_{0.08}^{0.1} = \frac{488}{3}$$

$$\therefore \quad I_{\text{rms}} = \sqrt{10\{(32/3) + 225 - 360 + (488/3)\}} = 11.55\,\text{A}$$

P 10.14 $P = I_{\text{rms}}^2R \qquad \therefore \quad R = \dfrac{1280}{(11.55)^2} = 9.6\,\Omega$

P 10.15 **[a]** Area under one cycle of v_g^2:

$$A = (100)(25\times10^{-6}) + 400(25\times10^{-6}) + 400(25\times10^{-6}) + 100(25\times10^{-6})$$
$$= 1000(25\times10^{-6})$$

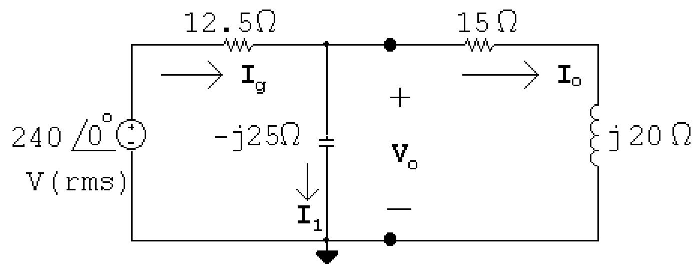
Mean value of v_g^2:

$$\text{M.V.} = \frac{A}{100\times10^{-6}} = \frac{1000(25\times10^{-6})}{100\times10^{-6}} = 250$$

$$\therefore \quad V_{\text{rms}} = \sqrt{250} = 15.81\,\text{V (rms)}$$

[b] $P = \dfrac{V_{\text{rms}}^2}{R} = \dfrac{250}{4} = 62.5\,\text{W}$

P 10.16 **[a]**



$$\frac{\mathbf{V}_o}{-j25} + \frac{\mathbf{V}_o - 240}{12.5} + \frac{\mathbf{V}_o}{15+j20} = 0$$

$$\therefore \quad \mathbf{V}_o = 183.53 - j14.12 = 184.07\underline{/-4.4^\circ}\,\text{V}$$

$$\mathbf{I}_g = \frac{240 - 183.53 + j14.12}{12.50} = 4.52 + j1.13\,\text{A}$$

$$S_g = -\mathbf{V}_g\mathbf{I}_g^* = -(240)(4.52 - j1.13)$$
$$= -1084.24 + j271.06\,\text{VA}$$

[b] Source is delivering 1084.24 W.

[c] Source is absorbing 271.06 magnetizing VAR.

[d] $Q_{\text{cap}} = \dfrac{(184.07)^2}{-25} = -1355.29$ VAR

$$P_{12.5\Omega} = |\mathbf{I}_g|^2(12.5) = 271.06\,\text{W}$$

$$|\mathbf{I}_o| = \frac{184.07}{25} = 7.36\,\text{A}$$

$$P_{15\Omega} = |\mathbf{I}_o|^2(15) = 813.18\,\text{W}$$

$$Q_{\text{ind}} = |\mathbf{I}_o|^2(20) = 1084.24\ \text{VAR}$$

[e] $\sum P_{\rm del} = 1084.24\,\text{W}$

$\sum P_{\rm diss} = 271.06 + 813.18 = 1084.24\,\text{W}$

$\therefore \quad \sum P_{\rm del} = \sum P_{\rm diss} = 1084.24\,\text{W}$

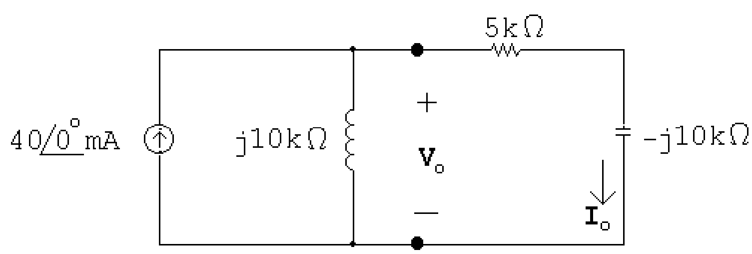
[f] $\sum Q_{\rm abs} = 271.06 + 1084.24 = 1355.29\,\text{VAR}$

$\sum Q_{\rm dev} = 1355.29\,\text{VAR}$

$\therefore \quad \sum \text{ mag VAR dev } = \sum \text{ mag VAR abs } = 1355.29\,\text{VAR}$

P 10.17 $\mathbf{I}_g = 40\underline{/0^\circ}\,\text{mA}$

$$j\omega L = j10{,}000\,\Omega; \qquad \frac{1}{j\omega C} = -j10{,}000\,\Omega$$



$$\mathbf{I}_o = \frac{j10{,}000}{5000}(40\underline{/0^\circ}) = 80\underline{/90^\circ}\,\text{mA}$$

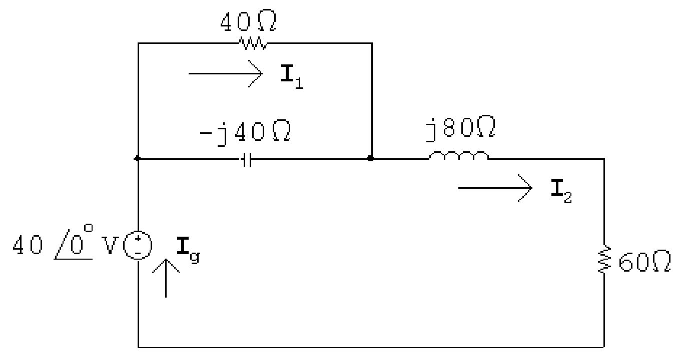
$$P = \frac{1}{2}|\mathbf{I}_o|^2(5000) = \frac{1}{2}(0.08)^2(5000) = 16\,\text{W}$$

$$Q = \frac{1}{2}|\mathbf{I}_o|^2(-10{,}000) = -32\,\text{VAR}$$

$$S = P + jQ = 16 - j32\,\text{VA}$$

$$|S| = 35.78\,\text{VA}$$

P 10.18 [a] $\dfrac{1}{j\omega C} = -j40\,\Omega; \qquad j\omega L = j80\,\Omega$



$$Z_{\text{eq}} = 40 \| -j40 + j80 + 60 = 80 + j60\,\Omega$$

$$\mathbf{I}_g = \frac{40\underline{/0^\circ}}{80 + j60} = 0.32 - j0.24\,\text{A}$$

$$S_g = -\frac{1}{2}\mathbf{V}_g\mathbf{I}_g^* = -\frac{1}{2}40(0.32 + j0.24) = -6.4 - j4.8\,\text{VA}$$

$$P = 6.4\,\text{W (del)}; \qquad Q = 4.8\,\text{VAR (del)}$$

$$|S| = |S_g| = 8\,\text{VA}$$

[b] $$\mathbf{I}_1 = \frac{-j40}{40 - j40}\mathbf{I}_g = 0.04 - j0.28\,\text{A}$$

$$P_{40\Omega} = \frac{1}{2}|\mathbf{I}_1|^2(40) = 1.6\,\text{W}$$

$$P_{60\Omega} = \frac{1}{2}|\mathbf{I}_g|^2(60) = 4.8\,\text{W}$$

$$\sum P_{\text{diss}} = 1.6 + 4.8 = 6.4\,\text{W} = \sum P_{\text{dev}}$$

[c] $$\mathbf{I}_{-j40\Omega} = \mathbf{I}_g - \mathbf{I}_1 = 0.28 + j0.04\,\text{A}$$

$$Q_{-j40\Omega} = \frac{1}{2}|\mathbf{I}_{-j40\Omega}|^2(-40) = -1.6\,\text{VAR (del)}$$

$$Q_{j80\Omega} = \frac{1}{2}|\mathbf{I}_g|^2(80) = 6.4\,\text{VAR (abs)}$$

$$\sum Q_{\text{abs}} = 6.4 - 1.6 = 4.8\,\text{VAR} = \sum Q_{\text{dev}}$$

P 10.19 $$S_{\text{T}} = 40{,}800 + j30{,}600\,\text{VA}$$

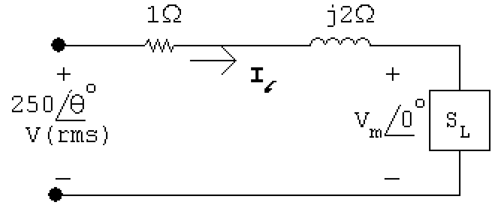
$$S_1 = 20{,}000(0.96 - j0.28) = 19{,}200 - j5600\,\text{VA}$$

$$S_2 = S_{\text{T}} - S_1 = 21{,}600 + j36{,}200 = 42{,}154.48\underline{/59.176^\circ}\,\text{VA}$$

$$\text{rf} = \sin(59.176^\circ) = 0.8587$$

$$\text{pf} = \cos(59.176^\circ) = 0.5124 \text{ lagging}$$

P 10.20 [a] Let $\mathbf{V}_\mathrm{L} = V_m\underline{/0^\circ}$:



$$S_\mathrm{L} = 2500(0.8 + j0.6) = 2000 + j1500\,\mathrm{VA}$$

$$\mathbf{I}_\ell^* = \frac{2000}{V_m} + j\frac{1500}{V_m}; \qquad \mathbf{I}_\ell = \frac{2000}{V_m} - j\frac{1500}{V_m}$$

$$250\underline{/\theta} = V_m + \left(\frac{2000}{V_m} - j\frac{1500}{V_m}\right)(1 + j2)$$

$$250V_m\underline{/\theta} = V_m^2 + (2000 - j1500)(1 + j2) = V_m^2 + 5000 + j2500$$

$$250V_m\cos\theta = V_m^2 + 5000; \qquad 250V_m\sin\theta = 2500$$

$$(250)^2V_m^2 = (V_m^2 + 5000)^2 + 2500^2$$

$$62{,}500V_m^2 = V_m^4 + 10{,}000V_m^2 + 31.25\times10^6$$

or

$$V_m^4 - 52{,}500V_m^2 + 31.25\times10^6 = 0$$

Solving,

$$V_m^2 = 26{,}250 \pm 25{,}647.86; \qquad V_m = 227.81\,\mathrm{V} \text{ and } V_m = 24.54\,\mathrm{V}$$

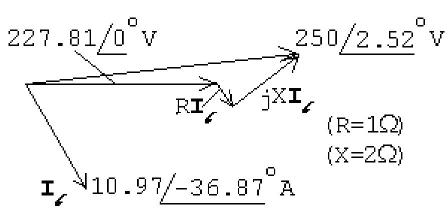
If $V_m = 227.81$ V:

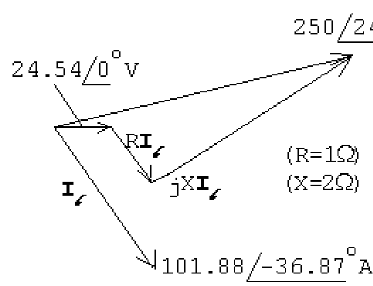
$$\sin\theta = \frac{2500}{(227.81)(250)} = 0.044; \qquad \therefore\quad \theta = 2.52^\circ$$

If $V_m = 24.54$ V:

$$\sin\theta = \frac{2500}{(24.54)(250)} = 0.4075; \qquad \therefore\quad \theta = 24.05^\circ$$

[b]





P 10.21 **[a]** $S_1 = 60{,}000 - j70{,}000\,\text{VA}$

$$S_2 = \frac{|\mathbf{V}_\text{L}|^2}{Z_2^*} = \frac{(2500)^2}{24 - j7} = 240{,}000 - j70{,}000\,\text{VA}$$

$$S_1 + S_2 = 300{,}000\,\text{VA}$$

$$2500\mathbf{I}_\text{L}^* = 300{,}000; \qquad \therefore \quad \mathbf{I}_\text{L} = 120\,\text{A(rms)}$$

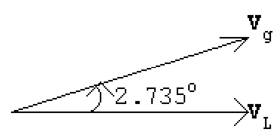
$$\mathbf{V}_g = \mathbf{V}_\text{L} + \mathbf{I}_\text{L}(0.1 + j1) = 2500 + (120)(0.1 + j1)$$

$$= 2512 + j120 = 2514.86\underline{/2.735^\circ}\,\text{Vrms}$$

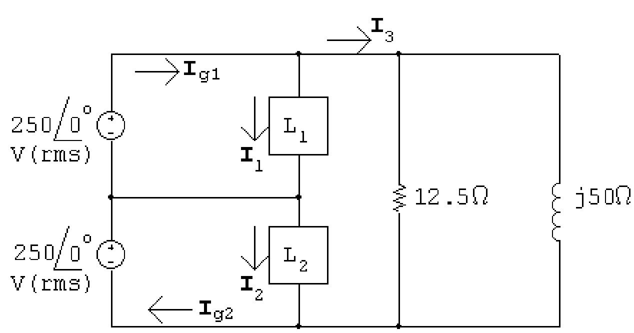
[b] $T = \dfrac{1}{f} = \dfrac{1}{60} = 16.67\,\text{ms}$

$$\frac{2.735^\circ}{360^\circ} = \frac{t}{16.67\text{ ms}}; \qquad \therefore \quad t = 126.62\,\mu\text{s}$$

[c] $\mathbf{V}_\text{L}$ lags $\mathbf{V}_g$ by 2.735° or $126.62\,\mu\text{s}$



P 10.22 **[a]**



$$250\mathbf{I}_1^* = 7500 + j2500; \qquad \therefore \quad \mathbf{I}_1 = 30 - j10\,\text{A(rms)}$$

$$250\mathbf{I}_2^* = 2800 - j9600; \qquad \therefore \quad \mathbf{I}_2 = 11.2 + j38.4\,\text{A(rms)}$$

$$\mathbf{I}_3 = \frac{500}{12.5} + \frac{500}{j50} = 40 - j10\,\text{A(rms)}$$

$$\mathbf{I}_{g1} = \mathbf{I}_1 + \mathbf{I}_3 = 70 - j20\,\text{A}$$

$$S_{g1} = 250(70 + j20) = 17{,}500 + j5000\,\text{VA}$$

Thus the $\mathbf{V}_{g1}$ source is delivering 17.5 kW and 5000 magnetizing vars.

$$\mathbf{I}_{g2} = \mathbf{I}_2 + \mathbf{I}_3 = 51.2 + j28.4\,\text{A(rms)}$$

$$S_{g2} = 250(51.2 - j28.4) = 12{,}800 - j7100\,\text{VA}$$

Thus the $\mathbf{V}_{g2}$ source is delivering 12.8 kW and absorbing 7100 magnetizing vars.

[b] $\sum P_{\text{gen}} = 17.5 + 12.8 = 30.3\,\text{kW}$

$$\sum P_{\text{abs}} = 7500 + 2800 + \frac{(500)^2}{12.5} = 30.3\,\text{kW} = \sum P_{\text{gen}}$$

$$\sum Q_{\text{del}} = 9600 + 5000 = 14.6\,\text{kVAR}$$

$$\sum Q_{\text{abs}} = 2500 + 7100 + \frac{(500)^2}{50} = 14.6\,\text{kVAR} = \sum Q_{\text{del}}$$

P 10.23 $S_1 = 1200 + 1196 = 2396 + j0\,\text{VA}$

$$\therefore\quad \mathbf{I}_1 = \frac{2396}{120} = 19.967\,\text{A}$$

$$S_2 = 860 + 600 + 240 = 1700 + j0\,\text{VA}$$

$$\therefore\quad \mathbf{I}_2 = \frac{1700}{120} = 14.167\,\text{A}$$

$$S_3 = 4474 + 12{,}200 = 16{,}674 + j0\,\text{VA}$$

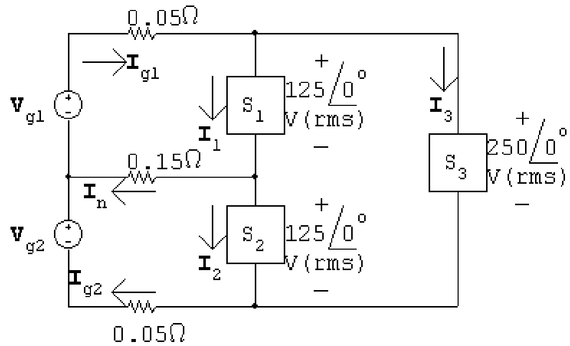
$$\therefore\quad \mathbf{I}_3 = \frac{16{,}674}{240} = 69.475\,\text{A}$$

$$\mathbf{I}_{g1} = \mathbf{I}_1 + \mathbf{I}_3 = 89.44\,\text{A}$$

$$\mathbf{I}_{g2} = \mathbf{I}_2 + \mathbf{I}_3 = 83.64\,\text{A}$$

Breakers will not trip since both feeder currents are less than 100 A.

P 10.24 [**a**]



$$\mathbf{I}_1 = \frac{5000 - j1250}{125} = 40 - j10\,\text{A (rms)}$$

$$\mathbf{I}_2 = \frac{6250 - j2500}{125} = 50 - j20\,\text{A (rms)}$$

$$\mathbf{I}_3 = \frac{8000 + j0}{250} = 32 + j0\,\text{A (rms)}$$

$$\therefore\quad \mathbf{I}_{g1} = 72 - j10\,\text{A (rms)}$$

$$\mathbf{I}_n = \mathbf{I}_1 - \mathbf{I}_2 = -10 + j10\,\text{A (rms)}$$

$$\mathbf{I}_{g2} = 82 - j20\,\text{A}$$

$$\mathbf{V}_{g1} = 0.05\mathbf{I}_{g1} + 125 + j0 + 0.15\mathbf{I}_n = 127.1 - j1\,\text{V(rms)}$$

$$\mathbf{V}_{g2} = -0.15\mathbf{I}_n + 125 + j0 + 0.05\mathbf{I}_{g2} = 130.6 - j2.5\,\text{V(rms)}$$

$$S_{g1} = -[(127.1 - j1)(72 + j10)] = -[9141.2 + j1343]\,\text{VA}$$

$$S_{g2} = -[(130.6 - j2.5)(82 + j20)] = -[10{,}759.2 + j2407]\,\text{VA}$$

Note: Both sources are delivering average power and magnetizing VAR to the circuit.

[**b**] $P_{0.05} = |\mathbf{I}_{g1}|^2(0.05) = 264.2\,\text{W}$

$$P_{0.15} = |\mathbf{I}_n|^2(0.15) = 30\,\text{W}$$

$$P_{0.05} = |\mathbf{I}_{g2}|^2(0.05) = 356.2\,\text{W}$$

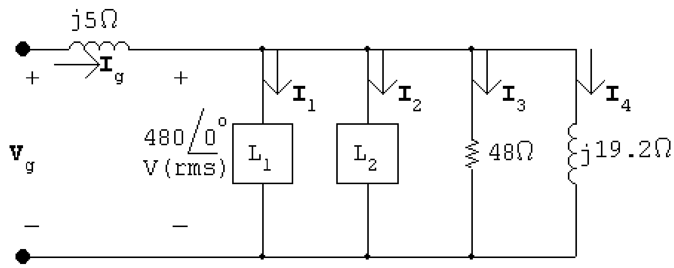
$$\sum P_{\text{dis}} = 264.2 + 30 + 356.2 + 5000 + 8000 + 6250 = 19{,}900.4\,\text{W}$$

$$\sum P_{\text{dev}} = 9141.2 + 10{,}759.2 = 19{,}900.4\,\text{W} = \sum P_{\text{dis}}$$

$$\sum Q_{\text{abs}} = 1250 + 2500 = 3750\,\text{VAR}$$

$$\sum Q_{\text{del}} = 1343 + 2407 = 3750\,\text{VAR} = \sum Q_{\text{abs}}$$

P 10.25



$$480\mathbf{I}_1^* = 7500 + j9000$$

$$\mathbf{I}_1^* = 15.625 + j18.75; \qquad \therefore \quad \mathbf{I}_1 = 15.625 - j18.75\,\text{A(rms)}$$

$$480\mathbf{I}_2^* = 2100 - j1800$$

$$\mathbf{I}_2^* = 4.375 - j3.75; \qquad \therefore \quad \mathbf{I}_2 = 4.375 + j3.75\,\text{A(rms)}$$

$$\mathbf{I}_3 = \frac{480/\underline{0^\circ}}{48} = 10 + j0\,\text{A}; \qquad \mathbf{I}_4 = \frac{480/\underline{0^\circ}}{j19.2} = 0 - j25\,\text{A}$$

$$\mathbf{I}_g = \mathbf{I}_1 + \mathbf{I}_2 + \mathbf{I}_3 + \mathbf{I}_4 = 30 - j40\,\text{A}$$

$$\mathbf{V}_g = 480 + (30 - j40)(j0.5) = 500 + j15 = 500.22/\underline{1.72^\circ}\,\text{V (rms)}$$

P 10.26 **[a]** $Z_1 = 240 + j70 = 250/\underline{16.26^\circ}\,\Omega$

$$\text{pf } = \cos(16.26^\circ) = 0.96 \text{ lagging}$$

$$\text{rf } = \sin(16.26^\circ) = 0.28$$

$$Z_2 = 160 - j120 = 200/\underline{-36.87^\circ}\,\Omega$$

$$\text{pf } = \cos(-36.87^\circ) = 0.8 \text{ leading}$$

$$\text{rf } = \sin(-36.87^\circ) = -0.6$$

$$Z_3 = 30 - j40 = 50/\underline{-53.13^\circ}\,\Omega$$

$$\text{pf } = \cos(-53.13^\circ) = 0.6 \text{ leading}$$

$$\text{rf } = \sin(-53.13^\circ) = -0.8$$

[b] $Y = Y_1 + Y_2 + Y_3$

$$Y_1 = \frac{1}{250\underline{/16.26^\circ}}; \qquad Y_2 = \frac{1}{200\underline{/-36.87^\circ}}; \qquad Y_3 = \frac{1}{50\underline{/-53.13^\circ}}$$

$$Y = 19.84 + j17.88\,\text{mS}$$

$$Z = \frac{1}{Y} = 37.44\underline{/-42.03^\circ}\,\Omega$$

$$\text{pf} = \cos(-42.03^\circ) = 0.74 \text{ leading}$$

$$\text{rf} = \sin(-42.03^\circ) = -0.67$$

P 10.27 [a] $S_1 = 16 + j18\,\text{kVA}; \qquad S_2 = 6 - j8\,\text{kVA}; \qquad S_3 = 8 + j0\,\text{kVA}$

$$S_\text{T} = S_1 + S_2 + S_3 = 30 + j10\,\text{kVA}$$

$$250\mathbf{I}^* = (30 + j10) \times 10^3; \qquad \therefore \quad \mathbf{I} = 120 - j40\,\text{A}$$

$$Z = \frac{250}{120 - j40} = 1.875 + j0.625\,\Omega = 1.98\underline{/18.43^\circ}\,\Omega$$

[b] $\text{pf} = \cos(18.43^\circ) = 0.9487 \text{ lagging}$

P 10.28 [a] From the solution to Problem 10.26 we have

$$\mathbf{I}_\text{L} = 120 - j40\,\text{A (rms)}$$

$$\therefore \quad \mathbf{V}_s = 250\underline{/0^\circ} + (120 - j40)(0.01 + j0.08) = 254.4 + j9.2$$

$$= 254.57\underline{/2.07^\circ}\,\text{V (rms)}$$

[b] $|\mathbf{I}_\text{L}| = \sqrt{16{,}000}$

$$P_\ell = (16{,}000)(0.01) = 160\,\text{W} \qquad Q_\ell = (16{,}000)(0.08) = 1280\,\text{VAR}$$

[c] $P_s = 30{,}000 + 160 = 30.16\,\text{kW} \qquad Q_s = 10{,}000 + 1280 = 11.28\,\text{kVAR}$

[d] $\eta = \dfrac{30}{30.16}(100) = 99.47\%$

P 10.29 [a] $\mathbf{I} = \dfrac{465\underline{/0^\circ}}{124 + j93} = 2.4 - j1.8 = 3\underline{/-36.87^\circ}\,\text{A(rms)}$

$$P = (3)^2(4) = 36\,\text{W}$$

[b] $Y_\text{L} = \dfrac{1}{120 + j90} = 5.33 - j4 \text{ mS}$

$$\therefore \quad X_\text{C} = \frac{1}{-4 \times 10^{-3}} = -250\,\Omega$$

[c] $Z_{\rm L} = \dfrac{1}{5.33 \times 10^{-3}} = 187.5\,\Omega$

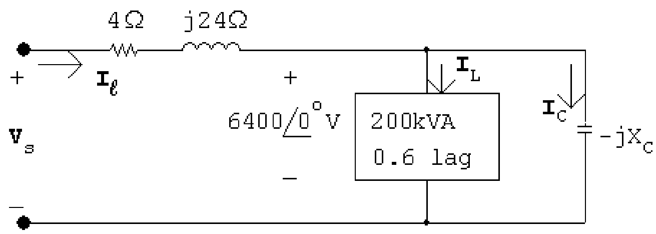
[d] $\mathbf{I} = \dfrac{465/\underline{0^\circ}}{191.5 + j3} = 2.4279/\underline{-0.9^\circ}\,\text{A}$

$$P = (2.4279)^2(4) = 23.58\,\text{W}$$

[e] $\% = \dfrac{23.58}{36}(100) = 65.5\%$

Thus the power loss after the capacitor is added is 65.5% of the power loss before the capacitor is added.

P 10.30



$$\mathbf{I}_{\rm L} = \frac{120{,}000 - j160{,}000}{6400} = 18.75 - j25\,\text{A (rms)}$$

$$\mathbf{I}_{\rm C} = \frac{6400}{-jX_{\rm C}} = j\frac{6400}{X_{\rm C}} = jI_{\rm C}$$

$$\mathbf{I}_\ell = 18.75 - j25 + jI_{\rm C} = 18.75 + j(I_{\rm C} - 25)$$

$$\begin{aligned}\mathbf{V}_s &= 6400 + (4 + j24)[18.75 + j(I_{\rm C} - 25)]\\ &= (7075 - 24I_{\rm C}) + j(350 + 4I_{\rm C})\end{aligned}$$

$$|\mathbf{V}_s|^2 = (7075 - 24I_{\rm C})^2 + (350 + 4I_{\rm C})^2 = (6400)^2$$

$$\therefore\quad 592I_{\rm C}^2 - 336{,}800I_{\rm C} + 9{,}218{,}125 = 0$$

$$I_{\rm C} = 284.46 \pm 255.63 = 28.33\,\text{A(rms)}^*$$

*Select the smaller value of $I_{\rm C}$ to minimize the magnitude of I_ℓ.

$$\therefore\quad X_{\rm C} = -\frac{6400}{28.33} = -221.99$$

$$\therefore\quad C = \frac{1}{(221.99)(120\pi)} = 11.95\,\mu\text{F}$$

P 10.31 **[a]** From Problem 9.75,

$$Z_{\rm ab} = 100 + j136.26 \qquad \text{so}$$

$$\mathbf{I}_1 = \frac{50}{100 + j13.74 + 100 + 136.26} = \frac{50}{200 + j150} = 160 - j120\,\text{mA}$$

$$\mathbf{I}_2 = \frac{j\omega M}{Z_{22}}\mathbf{I}_1 = \frac{j270}{800 + j600}(0.16 - j0.12) = 51.84 + j15.12\,\text{mA}$$

$$\mathbf{V}_L = (300 + j100)(0.05184 + j0.01512) = 14.04 + j9.72$$

$$|\mathbf{V}_L| = 17.08\,\text{V}$$

[b] $P_g(\text{ideal}) = 50(0.16) = 8\,\text{W}$

$$P_g(\text{practical}) = 8 - |\mathbf{I}_1|^2(100) = 4\,\text{W}$$

$$P_{\rm L} = |\mathbf{I}_2|^2(300) = 0.8748\,\text{W}$$

$$\%\text{ delivered } = \frac{0.8748}{4}(100) = 21.87\%$$

P 10.32 **[a]** $S_o = \text{ original load } = 1600 + j\dfrac{1600}{0.8}(0.6) = 1600 + j1200\,\text{kVA}$

$$S_f = \text{ final load } = 1920 + j\frac{1920}{0.96}(0.28) = 1920 + j560\,\text{kVA}$$

$$\therefore\quad Q_{\rm added} = 560 - 1200 = -640\,\text{kVAR}$$

[b] deliver

[c] $S_{\rm a} = \text{ added load } = 320 - j640 = 715.54\underline{/-63.43^\circ}\,\text{kVA}$

$$\text{pf } = \cos(-63.43) = 0.447 \text{ leading}$$

[d] $\mathbf{I}_{\rm L}^* = \dfrac{(1600 + j1200) \times 10^3}{2400} = 666.67 + j500\,\text{A}$

$$\mathbf{I}_{\rm L} = 666.67 - j500 = 833.33\underline{/-36.87^\circ}\,\text{A(rms)}$$

$$|\mathbf{I}_{\rm L}| = 833.33\,\text{A(rms)}$$

[e] $\mathbf{I}_{\rm L}^* = \dfrac{(1920 + j560) \times 10^3}{2400} = 800 + j233.33$

$$\mathbf{I}_{\rm L} = 800 - j233.33 = 833.33\underline{/-16.26^\circ}\,\text{A(rms)}$$

$$|\mathbf{I}_{\rm L}| = 833.33\,\text{A(rms)}$$

P 10.33 **[a]** $P_{\rm before} = P_{\rm after} = (833.33)^2(0.05) = 34{,}722.22\,\text{W}$

[b] $\mathbf{V}_s(\text{before}) = 2400 + (666.67 - j500)(0.05 + j0.4)$

$$= 2633.33 + j241.67 = 2644.4\underline{/5.24^\circ}\,\text{V(rms)}$$

$|\mathbf{V}_s(\text{before})| = 2644.4\,\text{V(rms)}$

$\mathbf{V}_s(\text{after}) = 2400 + (800 - j233.33)(0.05 + j0.4)$

$$= 2533.33 + j308.33 = 2552.028\underline{/6.94^\circ}\,\text{V(rms)}$$

$|\mathbf{V}_s(\text{after})| = 2552.028\,\text{V(rms)}$

P 10.34 **[a]** $S_\text{L} = 20{,}000(0.85 + j0.53) = 17{,}000 + j10{,}535.65\,\text{VA}$

$125\mathbf{I}_\text{L}^* = (17{,}000 + j10{,}535.65); \quad \mathbf{I}_\text{L}^* = 136 + j84.29\,\text{A(rms)}$

$\therefore\quad \mathbf{I}_\text{L} = 136 - j84.29\,\text{A(rms)}$

$$\mathbf{V}_s = 125 + (136 - j84.29)(0.01 + j0.08) = 133.10 + j10.04$$

$$= 133.48\underline{/4.31^\circ}\,\text{V(rms)}$$

$|\mathbf{V}_s| = 133.48\,\text{V(rms)}$

[b] $P_\ell = |\mathbf{I}_\ell|^2(0.01) = (160)^2(0.01) = 256\,\text{W}$

[c] $\dfrac{(125)^2}{X_\text{C}} = -10{,}535.65; \qquad X_\text{C} = -1.48306\,\Omega$

$$-\frac{1}{\omega C} = -1.48306; \qquad C = \frac{1}{(1.48306)(120\pi)} = 1788.59\,\mu\text{F}$$

[d] $\mathbf{I}_\ell = 136 + j0\,\text{A(rms)}$

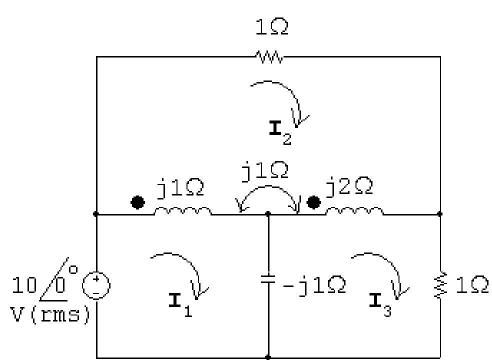
$$\mathbf{V}_s = 125 + 136(0.01 + j0.08) = 126.36 + j10.88$$

$$= 126.83\underline{/4.92^\circ}\,\text{V(rms)}$$

$|\mathbf{V}_s| = 126.83\,\text{V(rms)}$

[e] $P_\ell = (136)^2(0.01) = 184.96\,\text{W}$

P 10.35 **[a]**



$$10 = j1(\mathbf{I}_1 - \mathbf{I}_2) + j1(\mathbf{I}_3 - \mathbf{I}_2) - j1(\mathbf{I}_1 - \mathbf{I}_3)$$

$$0 = 1\mathbf{I}_2 + j2(\mathbf{I}_2 - \mathbf{I}_3) + j1(\mathbf{I}_2 - \mathbf{I}_1) + j1(\mathbf{I}_2 - \mathbf{I}_1) + j1(\mathbf{I}_2 - \mathbf{I}_3)$$

$$0 = \mathbf{I}_3 - j1(\mathbf{I}_3 - \mathbf{I}_1) + j2(\mathbf{I}_3 - \mathbf{I}_2) + j1(\mathbf{I}_1 - \mathbf{I}_2)$$

Solving,

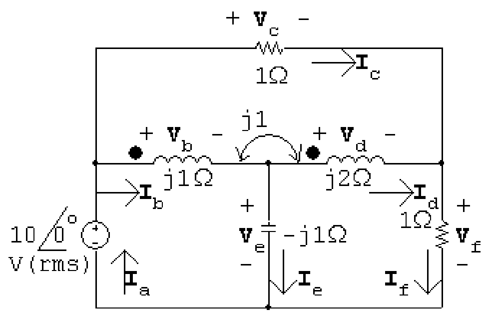
$\mathbf{I}_1 = 6.25 + j7.5\,\mathrm{A(rms)}$; $\quad \mathbf{I}_2 = 5 + j2.5\,\mathrm{A(rms)}$; $\quad \mathbf{I}_3 = 5 - j2.5\,\mathrm{A(rms)}$

$\mathbf{I}_\mathrm{a} = \mathbf{I}_1 = 6.25 + j7.5\,\mathrm{A}$ $\qquad \mathbf{I}_\mathrm{b} = \mathbf{I}_1 - \mathbf{I}_2 = 1.25 + j5\,\mathrm{A}$

$\mathbf{I}_\mathrm{c} = \mathbf{I}_2 = 5 + j2.5\,\mathrm{A}$ $\qquad \mathbf{I}_\mathrm{d} = \mathbf{I}_3 - \mathbf{I}_2 = -j5\,\mathrm{A}$

$\mathbf{I}_\mathrm{e} = \mathbf{I}_1 - \mathbf{I}_3 = 1.25 + j10\,\mathrm{A}$ $\qquad \mathbf{I}_\mathrm{f} = \mathbf{I}_3 = 5 - j2.5\,\mathrm{A}$

[**b**]



$\mathbf{V}_\mathrm{a} = 10\,\mathrm{V}$ $\qquad \mathbf{V}_\mathrm{b} = j1\mathbf{I}_\mathrm{b} + j1\mathbf{I}_\mathrm{d} = j1.25\,\mathrm{V}$

$\mathbf{V}_\mathrm{c} = 1\mathbf{I}_\mathrm{c} = 5 + j2.5\,\mathrm{V}$ $\qquad \mathbf{V}_\mathrm{d} = j2\mathbf{I}_\mathrm{d} - j1\mathbf{I}_\mathrm{b} = 5 + j1.25\,\mathrm{V}$

$\mathbf{V}_\mathrm{e} = -j1\mathbf{I}_\mathrm{e} = 10 - j1.25\,\mathrm{V}$ $\qquad \mathbf{V}_\mathrm{f} = 1\mathbf{I}_\mathrm{f} = 5 - j2.5\,\mathrm{V}$

$$S_\mathrm{a} = -10\mathbf{I}_\mathrm{a}^* = -62.5 + j75\,\mathrm{VA}$$

$$S_\mathrm{b} = \mathbf{V}_\mathrm{b}\mathbf{I}_\mathrm{b}^* = 6.25 + j1.5625\,\mathrm{VA}$$

$$S_\mathrm{c} = \mathbf{V}_\mathrm{c}\mathbf{I}_\mathrm{c}^* = 31.25 + j0\,\mathrm{VA}$$

$$S_\mathrm{d} = \mathbf{V}_\mathrm{d}\mathbf{I}_\mathrm{d}^* = -6.25 + j25\,\mathrm{VA}$$

$$S_\mathrm{e} = \mathbf{V}_\mathrm{e}\mathbf{I}_\mathrm{e}^* = 0 - j101.5625\,\mathrm{VA}$$

$$S_\mathrm{f} = \mathbf{V}_\mathrm{f}\mathbf{I}_\mathrm{f}^* = 31.25\,\mathrm{VA}$$

[**c**] $\sum P_\mathrm{dev} = 62.5\,\mathrm{W}$

$$\sum P_\mathrm{abs} = 31.25 + 31.25 = 62.5\,\mathrm{W}$$

Note that the total power absorbed by the coupled coils is zero:
$6.25 - 6.25 = 0 = P_\mathrm{b} + P_\mathrm{d}$

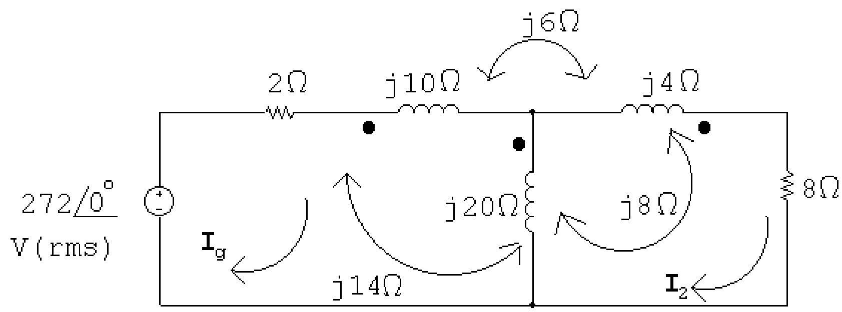
[d] $\sum Q_{\text{dev}} = 101.5625\,\text{VAR}$

Both the source and the capacitor are developing magnetizing vars.

$\sum Q_{\text{abs}} = 75 + 1.5625 + 25 = 101.5625\,\text{VAR}$

$\sum Q$ absorbed by the coupled coils is $Q_{\text{b}} + Q_{\text{d}} = 26.5625$

P 10.36 **[a]**



$$272\underline{/0^\circ} = 2\mathbf{I}_g + j10\mathbf{I}_g + j14(\mathbf{I}_g - \mathbf{I}_2) - j6\mathbf{I}_2$$
$$+j14\mathbf{I}_g - j8\mathbf{I}_2 + j20(\mathbf{I}_g - \mathbf{I}_2)$$
$$0 = j20(\mathbf{I}_2 - \mathbf{I}_g) - j14\mathbf{I}_g + j8\mathbf{I}_2 + j4\mathbf{I}_2$$
$$+j8(\mathbf{I}_2 - \mathbf{I}_g) - j6\mathbf{I}_g + 8\mathbf{I}_2$$

Solving,

$I_g = 20 - j4\,\text{A(rms)};\qquad \mathbf{I}_2 = 24\underline{/0^\circ}\,\text{A(rms)}$

$P_{8\Omega} = (24)^2(8) = 4608\,\text{W}$

[b] $P_g(\text{developed}) = (272)(20) = 5440\,\text{W}$

[c] $Z_{\text{ab}} = \dfrac{\mathbf{V}_g}{\mathbf{I}_g} - 2 = \dfrac{272}{20 - j4} - 2 = 11.08 + j2.62 = 11.38\underline{/13.28^\circ}\,\Omega$

[d] $P_{2\Omega} = |I_g|^2(2) = 832\,\text{W}$

$\sum P_{\text{diss}} = 832 + 4608 = 5440\,\text{W} = \sum P_{\text{dev}}$

P 10.37 **[a]** $Z_{\text{ab}} = \left(1 + \dfrac{N_1}{N_2}\right)^2(1 - j2) = 25 - j50\,\Omega$

$$\therefore\quad \mathbf{I}_1 = \frac{100\underline{/0^\circ}}{15 + j50 + 25 - j50} = 2.5\underline{/0^\circ}\,\text{A}$$

$$\mathbf{I}_2 = \frac{N_1}{N_2}\mathbf{I}_1 = 10\underline{/0^\circ}\,\text{A}$$

$\therefore\quad \mathbf{I}_{\text{L}} = \mathbf{I}_1 + \mathbf{I}_2 = 12.5\underline{/0^\circ}\,\text{A(rms)}$

$P_{1\Omega} = (12.5)^2(1) = 156.25\,\text{W}$

$P_{15\Omega} = (2.5)^2(15) = 93.75\,\text{W}$

[b] $P_g = -100(2.5\underline{/0^\circ}) = -250\,\text{W}$

$\sum P_{\text{abs}} = 156.25 + 93.75 = 250\,\text{W} = \sum P_{\text{dev}}$

P 10.38 [a] $25a_1^2 + 4a_2^2 = 500$

$\mathbf{I}_{25} = a_1\mathbf{I}; \qquad P_{25} = a_1^2\mathbf{I}^2(25)$

$\mathbf{I}_4 = a_2\mathbf{I}; \qquad P_4 = a_2^2\mathbf{I}^2(4)$

$P_4 = 4P_{25}; \qquad a_2^2\mathbf{I}^2 4 = 100a_1^2\mathbf{I}^2$

$\therefore \qquad 100a_1^2 = 4a_2^2$

$25a_1^2 + 100a_1^2 = 500; \qquad a_1 = 2$

$25(4) + 4a_2^2 = 500; \qquad a_2 = 10$

[b] $\mathbf{I} = \dfrac{2000\underline{/0^\circ}}{500 + 500} = 2\underline{/0^\circ}\,\text{A(rms)}$

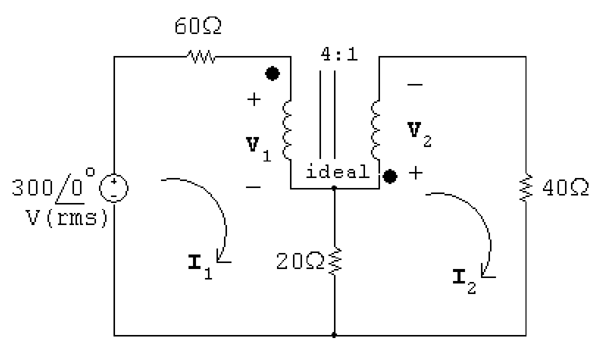
$\mathbf{I}_{25} = a_1\mathbf{I} = 4\,\text{A}$

$P_{25\Omega} = (16)(25) = 400\,\text{W}$

[c] $\mathbf{I}_4 = a_2\mathbf{I} = 10(2) = 20\,\text{A(rms)}$

$\mathbf{V}_4 = (20)(4) = 80\underline{/0^\circ}\,\text{V(rms)}$

P 10.39 [a]



$300 = 60\mathbf{I}_1 + \mathbf{V}_1 + 20(\mathbf{I}_1 - \mathbf{I}_2)$

$0 = 20(\mathbf{I}_2 - \mathbf{I}_1) + \mathbf{V}_2 + 40\mathbf{I}_2$

$\mathbf{V}_2 = \dfrac{1}{4}\mathbf{V}_1; \qquad \mathbf{I}_2 = -4\mathbf{I}_1$

Solving,

$\mathbf{V}_1 = 260\,\text{V (rms)}; \qquad \mathbf{V}_2 = 65\,\text{V (rms)}$

$\mathbf{I}_1 = 0.25\,\text{A (rms)}; \qquad \mathbf{I}_2 = -1.0\,\text{A (rms)}$

$\mathbf{V}_{5\text{A}} = \mathbf{V}_1 + 20(\mathbf{I}_1 - \mathbf{I}_2) = 285\,\text{V (rms)}$

$\therefore \quad P = -(285)(5) = -1425\,\text{W}$

Thus 1425 W is delivered by the current source to the circuit.

[b] $\mathbf{I}_{20\Omega} = \mathbf{I}_1 - \mathbf{I}_2 = 1.25\,\text{A(rms)}$

$\therefore \quad P_{20\Omega} = (1.25)^2(20) = 31.25\,\text{W}$

P 10.40 $Z_\text{L} = |Z_\text{L}|\underline{/\theta^\circ} = |Z_\text{L}|\cos\theta^\circ + j|Z_\text{L}|\sin\theta^\circ$

$$\text{Thus} \quad |\mathbf{I}| = \frac{|\mathbf{V}_\text{Th}|}{\sqrt{(R_\text{Th} + |Z_\text{L}|\cos\theta)^2 + (X_\text{Th} + |Z_\text{L}|\sin\theta)^2}}$$

$$\text{Therefore} \quad P = \frac{0.5|\mathbf{V}_\text{Th}|^2|Z_\text{L}|\cos\theta}{(R_\text{Th} + |Z_\text{L}|\cos\theta)^2 + (X_\text{Th} + |Z_\text{L}|\sin\theta)^2}$$

Let D = demoninator in the expression for P, then

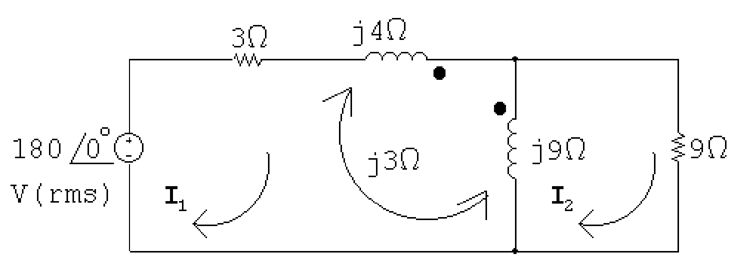
$$\frac{dP}{d|Z_\text{L}|} = \frac{(0.5|\mathbf{V}_\text{Th}|^2\cos\theta)(D\cdot 1 - |Z_\text{L}|dD/d|Z_\text{L}|)}{D^2}$$

$$\frac{dD}{d|Z_\text{L}|} = 2(R_\text{Th} + |Z_\text{L}|\cos\theta)\cos\theta + 2(X_\text{Th} + |Z_\text{L}|\sin\theta)\sin\theta$$

$$\frac{dP}{d|Z_\text{L}|} = 0 \quad \text{when} \quad D = |Z_\text{L}|\left(\frac{dD}{d|Z_\text{L}|}\right)$$

Substituting the expressions for D and $(dD/d|Z_\text{L}|)$ into this equation gives us the relationship $R_\text{Th}^2 + X_\text{Th}^2 = |Z_\text{L}|^2$ or $|Z_\text{Th}| = |Z_\text{L}|$.

P 10.41 [a]



$180 = 3\mathbf{I}_1 + j4\mathbf{I}_1 + j3(\mathbf{I}_2 - \mathbf{I}_1) + j9(\mathbf{I}_1 - \mathbf{I}_2) - j3\mathbf{I}_1$

$0 = 9\mathbf{I}_2 + j9(\mathbf{I}_2 - \mathbf{I}_1) + j3\mathbf{I}_1$

Solving,

$\mathbf{I}_1 = 18 - j18\,\text{A(rms)}; \qquad \mathbf{I}_2 = 12\underline{/0^\circ}\,\text{A(rms)}$

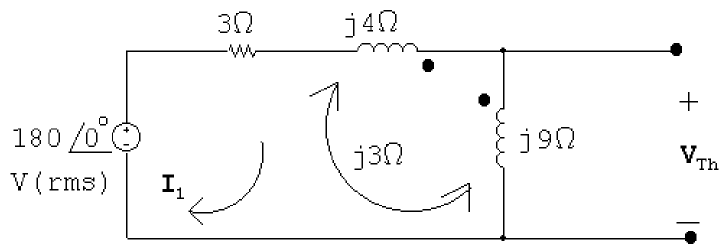
$\therefore \quad \mathbf{V}_o = (12)(9) = 108\,\text{V(rms)}$

[b] $P = (12)^2(9) = 1296\,\text{W}$

[c] $S_g = -(180)(18 + j18) = -3240 - j3240\,\text{VA} \quad \therefore \quad P_g = -3240\,\text{W}$

$$\%\text{ delivered } = \frac{1296}{3240}(100) = 40\%$$

P 10.42 [a] Open circuit voltage:

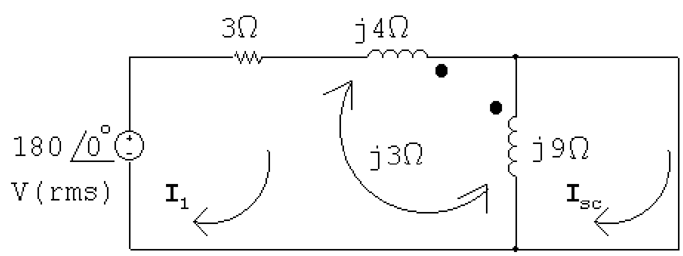


$$180 = 3\mathbf{I}_1 + j4\mathbf{I}_1 - j3\mathbf{I}_1 + j9\mathbf{I}_1 - j3\mathbf{I}_1$$

$$\therefore \quad \mathbf{I}_1 = \frac{180}{3 + j7} = 9.31 - j21.72\,\text{A(rms)}$$

$$\mathbf{V}_{\text{Th}} = j9\mathbf{I}_1 - j3\mathbf{I}_1 = j6\mathbf{I}_1 = 130.34 + j55.86\,\text{V}$$

Short circuit current:



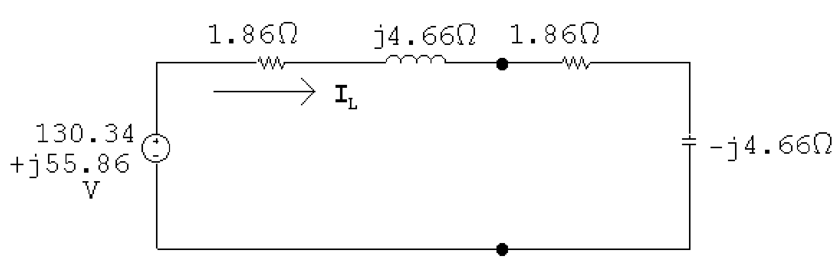
$$180 = 3\mathbf{I}_1 + j4\mathbf{I}_1 + j3(\mathbf{I}_{\text{sc}} - \mathbf{I}_1) + j9(\mathbf{I}_1 - \mathbf{I}_{\text{sc}}) - j3\mathbf{I}_1$$

$$0 = -j9(\mathbf{I}_{\text{sc}} - \mathbf{I}_1) + j3\mathbf{I}_1$$

Solving,

$$\mathbf{I}_{\text{sc}} = 20 - j20\,\text{A}$$

$$Z_{\text{Th}} = \frac{\mathbf{V}_{\text{Th}}}{\mathbf{I}_{\text{sc}}} = \frac{130.34 + j55.86}{20 - j20} = 1.86 + j4.66\,\Omega$$



$$\mathbf{I}_{\text{L}} = \frac{130.34 + j55.86}{3.72} = 35 + j15 = 38.08\underline{/23.20^\circ}\,\text{A}$$

$$P_L = (38.08)^2(1.86) = 2700\,\text{W}$$

[b] $$\mathbf{I}_1 = \frac{Z_o + j9}{j6}\mathbf{I}_2 = \frac{1.86 - j4.66 + j9}{j6}(35 + j15) = 30\underline{/0^\circ}\,\text{A(rms)}$$

$$P_{\text{dev}} = (180)(30) = 5400\,\text{W}$$

[c] Begin by choosing the capacitor value from Appendix H that is closest to the required reactive impedance, assuming the frequency of the source is 60 Hz:

$$4.66 = \frac{1}{2\pi(60)C} \qquad \text{so} \qquad C = \frac{1}{2\pi(60)(4.66)} = 569.22\,\mu\text{F}$$

Choose the capacitor value closest to this capacitance from Appendix H, which is 470 μF. Then,

$$X_L = -\frac{1}{2\pi(60)(470 \times 10^{-6})} = -5.6438\,\Omega$$

Now set R_L as close as possible to $\sqrt{R_{\text{Th}}^2 + (X_L + X_{\text{Th}})^2}$:

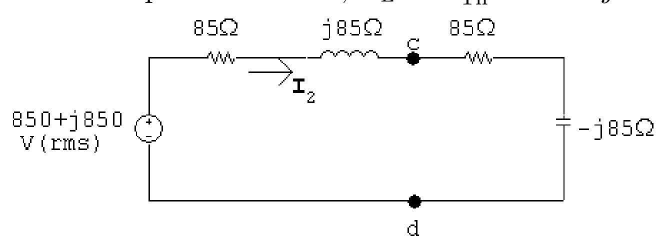
$$R_L = \sqrt{1.856^2 + (4.66 - 5.6438)^2} = 2.11\,\Omega$$

The closest single resistor value from Appendix H is 10 Ω. The resulting real power developed by the source is calculated below, using the Thévenin equivalent circuit:

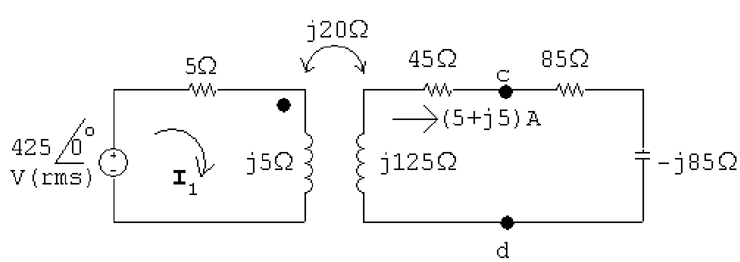
$$\mathbf{I} = \frac{130.34 + j55.86}{1.86 + j4.66 + 10 - j5.6438} = 11.9157\underline{/27.94^\circ}$$

$$P = |130.34 + j55.86|(11.9157) = 1689.7\,\text{W} \qquad \text{(instead of 5400 W)}$$

P 10.43 [a] From Problem 9.78, $Z_{\text{Th}} = 85 + j85\,\Omega$ and $\mathbf{V}_{\text{Th}} = 850 + j850\,\text{V}$. Thus, for maximum power transfer, $Z_L = Z_{\text{Th}}^* = 85 - j85\,\Omega$:



$$\mathbf{I}_2 = \frac{850 + j850}{170} = 5 + j5\,\text{A}$$



$$425\underline{/0^\circ} = (5 + j5)\mathbf{I}_1 - j20(5 + j5)$$

$$\therefore \quad \mathbf{I}_1 = \frac{325 + j100}{5 + j5} = 42.5 - j22.5\,\text{A}$$

$$S_g(\text{del}) = 425(42.5 + j22.5) = 18{,}062.5 + j9562.5\,\text{VA}$$

$$P_g = 18{,}062.5\,\text{W}$$

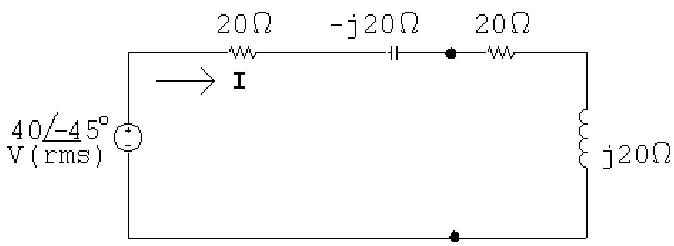
[b] $P_{\text{loss}} = |\mathbf{I}_1|^2(5) + |\mathbf{I}_2|^2(45) = 11{,}562.5 + 2250 = 13{,}812.5\,\text{W}$

$$\%\ \text{loss in transformer} = \frac{13{,}812.5}{18{,}062.5}(100) = 76.47\%$$

P 10.44 [a] $Z_{\text{Th}} = -j40 + \dfrac{(40)(j40)}{40 + j40} = 20 - j20\,\Omega$

$$\therefore \quad Z_{\text{L}} = Z_{\text{Th}}^* = 20 + j20\,\Omega$$

[b] $\mathbf{V}_{\text{Th}} = \dfrac{80\underline{/0^\circ}(40)}{40 + j40} = 40(1 - j1) = 40\sqrt{2}\underline{/-45^\circ}\,\text{V}$



$$\mathbf{I} = \frac{40\sqrt{2}\underline{/-45^\circ}}{40} = \sqrt{2}\underline{/-45^\circ}\,\text{A}$$

$$|\mathbf{I}_{\text{rms}}| = 1\,\text{A}$$

$$P_{\text{load}} = (1)^2(20 \times 10^3) = 20\,\text{W}$$

[c] The closest resistor value from Appendix H is $22\,\Omega$. Find the inductor value:

$$(5000)L = 20 \qquad \text{so} \qquad L = 4\,\text{mH}$$

The closest inductor value is 1 mH.

$$\mathbf{I} = \frac{40\underline{/-45^\circ}}{20 - j20 + 22 + j5} = \frac{40\underline{/-45^\circ}}{42 - j15} = 0.8969\underline{/-25.35^\circ}\ \text{A(rms)}$$

$$P_{\text{load}} = (0.8969)^2(22) = 17.70\,\text{W} \qquad \text{(instead of 20 W)}$$

P 10.45 [a] $\dfrac{115.2 - j86.4 - 240}{Z_{\text{Th}}} + \dfrac{115.2 - j80}{90 - j30} = 0$

$$\therefore \quad Z_{\text{Th}} = \frac{240 - 115.2 + j86.4}{1.44 - j0.48} = 60 + j80\,\Omega$$

$$\therefore \quad Z_{\text{L}} = 60 - j80\,\Omega$$

[b] $\mathbf{I} = \dfrac{240\underline{/0^\circ}}{120\underline{/0^\circ}} = 2\underline{/0^\circ}\,\mathrm{A(rms)}$

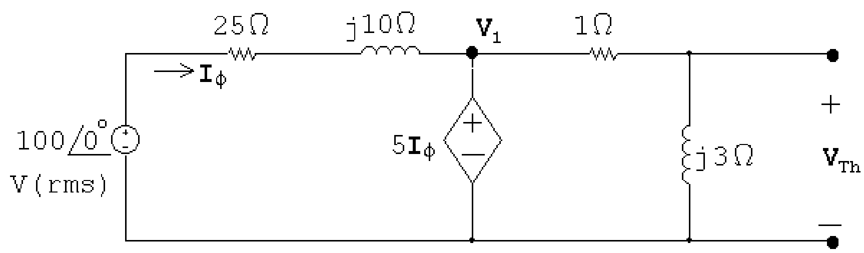
$P = (2)^2(60) = 240\,\mathrm{W}$

[c] Let $R = 15\,\Omega + 15\,\Omega + 15\,\Omega + 15\,\Omega = 60\,\Omega$

$$\frac{1}{2\pi(60)C} = 80 \qquad \text{so} \qquad C = \frac{1}{2\pi(60)(80)} = 33.16\,\mu\mathrm{F}$$

Let $C = 22\,\mu\mathrm{F}\|10\,\mu\mathrm{F}\|1\,\mu\mathrm{F} = 33\,\mu\mathrm{F}$

P 10.46 **[a]** Open circuit voltage:



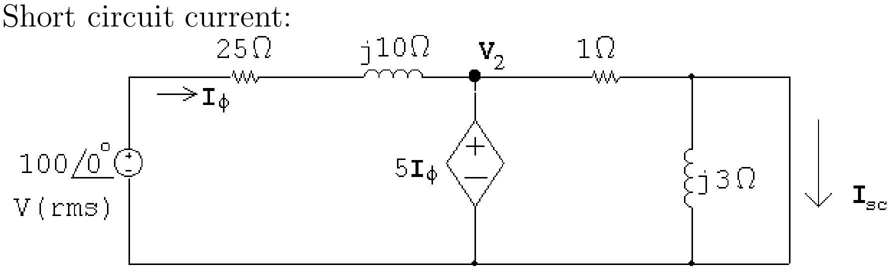
$$\mathbf{V}_1 = 5\mathbf{I}\phi = 5\frac{100 - 5\mathbf{I}_\phi}{25 + j10}$$

$$(25 + j10)\mathbf{I}_\phi = 100 - 5\mathbf{I}\phi$$

$$\mathbf{I}_\phi = \frac{100}{30 + j10} = 3 - j\,\mathrm{A}$$

$$\mathbf{V}_{\mathrm{Th}} = \frac{j3}{1 + j3}(5\mathbf{I}_\phi) = 15\,\mathrm{V}$$

Short circuit current:



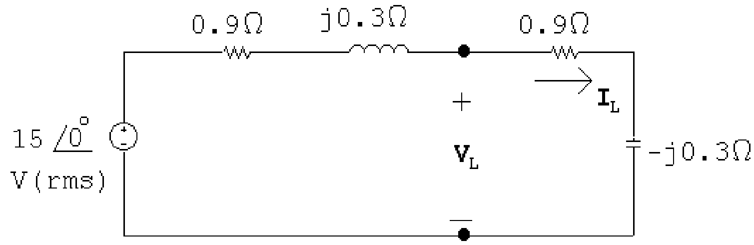
$$\mathbf{V}_2 = 5\mathbf{I}_\phi = \frac{100 - 5\mathbf{I}_\phi}{25 + j10}$$

$$\mathbf{I}_\phi = 3 - j1\,\mathrm{A}$$

$$\mathbf{I}_{\mathrm{sc}} = \frac{5\mathbf{I}_\phi}{1} = 15 - j5\,\mathrm{A}$$

$$Z_{\mathrm{Th}} = \frac{15}{15 - j5} = 0.9 + j0.3\,\Omega$$

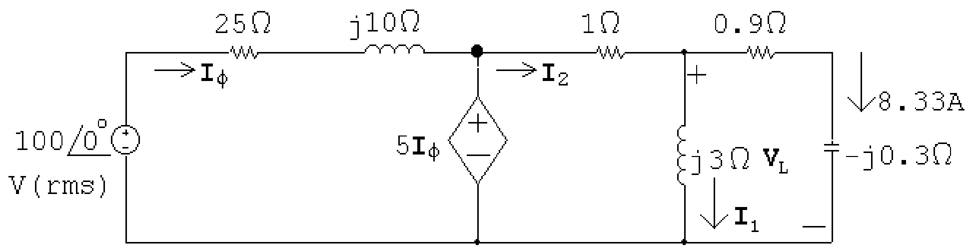
$Z_L = Z_{\text{Th}}^* = 0.9 - j0.3\,\Omega$



$$\mathbf{I}_{\mathrm{L}} = \frac{0.3}{1.8} = 8.33\,\mathrm{A(rms)}$$

$$P = |\mathbf{I}_L|^2(0.9) = 62.5\,\mathrm{W}$$

[b] $\mathbf{V}_{\mathrm{L}} = (0.9 - j0.3)(8.33) = 7.5 - j2.5\,\mathrm{V(rms)}$



$$\mathbf{I}_1 = \frac{\mathbf{V}_{\mathrm{L}}}{j3} = -0.833 - j2.5\,\mathrm{A(rms)}$$

$$\mathbf{I}_2 = \mathbf{I}_1 + \mathbf{I}_{\mathrm{L}} = 7.5 - j2.5\,\mathrm{A(rms)}$$

$$5\mathbf{I}_\phi = \mathbf{I}_2 + \mathbf{V}_L \quad \therefore \quad \mathbf{I}_\phi = 3 - j1\,\mathrm{A}$$

$$\mathbf{I}_{\mathrm{d.s.}} = \mathbf{I}_\phi - \mathbf{I}_2 = -4.5 + j1.5\,\mathrm{A}$$

$$S_g = -100(3 + j1) = -300 - j100\,\mathrm{VA}$$

$$S_{\mathrm{d.s.}} = 5(3 - j1)(-4.5 - j1.5) = -75 + j0\,\mathrm{VA}$$

$$P_{\mathrm{dev}} = 300 + 75 = 375\,\mathrm{W}$$

$$\%\ \text{developed} = \frac{62.5}{375}(100) = 16.67\%$$

Checks:

$$P_{25\Omega} = (10)(25) = 250\,\mathrm{W}$$

$$P_{1\Omega} = (67.5)(1) = 67.5\,\mathrm{W}$$

$$P_{0.9\Omega} = 62.5\,\mathrm{W}$$

$$\sum P_{\mathrm{abs}} = 230 + 62.5 + 67.5 = 375 = \sum P_{\mathrm{dev}}$$

$$Q_{j10} = (10)(10) = 100\,\text{VAR}$$

$$Q_{j3} = (6.94)(3) = 20.82\,\text{VAR}$$

$$Q_{-j0.3} = (69.4)(-0.3) = -20.82\,\text{VAR}$$

$$Q_{\text{source}} = -100\,\text{VAR}$$

$$\sum Q = 100 + 20.82 - 20.82 - 100 = 0$$

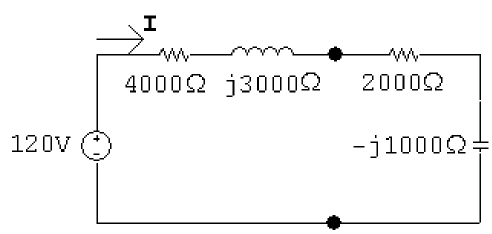
P 10.47 [a] First find the Thévenin equivalent:

$$j\omega L = j3000\,\Omega$$

$$Z_{\text{Th}} = 6000 \| 12{,}000 + j3000 = 4000 + j3000\,\Omega$$

$$\mathbf{V}_{\text{Th}} = \frac{12{,}000}{6000 + 12{,}000}(180) = 120\,\text{V}$$

$$\frac{-j}{\omega C} = -j1000\,\Omega$$



$$\mathbf{I} = \frac{120}{6000 + j2000} = 18 - j6\,\text{mA}$$

$$P = \frac{1}{2}|\mathbf{I}|^2(2000) = 360\,\text{mW}$$

[b] Set $C_o = 0.1\,\mu\text{F}$ so $-j/\omega C = -j2000\,\Omega$
Set R_o as close as possible to

$$R_o = \sqrt{4000^2 + (3000 - 2000)^2} = 4123.1\,\Omega$$

$$\therefore \quad R_o = 4000\,\Omega$$

[c] $\mathbf{I} = \dfrac{120}{8000 + j1000} = 14.77 - j1.85\,\text{mA}$

$$P = \frac{1}{2}|\mathbf{I}|^2(4000) = 443.1\,\text{mW}$$

Yes; $443.1\,\text{mW} > 360\,\text{mW}$

[d] $\mathbf{I} = \dfrac{120}{8000} = 15\,\text{mA}$

$$P = \frac{1}{2}(0.015)^2(4000) = 450\,\text{mW}$$

[e] $R_o = 4000\,\Omega$; $\quad C_o = 66.67\,\text{nF}$

[f] Yes; $\quad 450\,\text{mW} > 443.1\,\text{mW}$

P 10.48 [a] Set $C_o = 0.1\,\mu\text{F}$, so $-j/\omega C = -j2000\,\Omega$; also set $R_o = 4123.1\,\Omega$

$$\mathbf{I} = \frac{120}{8123.1 + j1000} = 14.55 - j1.79\,\text{mA}$$

$$P = \frac{1}{2}|\mathbf{I}|^2(4123.1) = 443.18\,\text{mW}$$

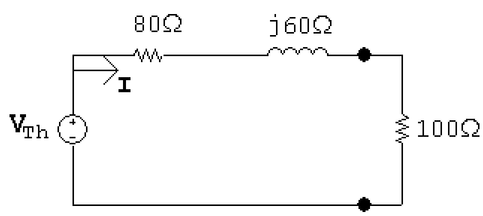
[b] Yes; $\quad 443.18\,\text{mW} > 360\,\text{mW}$

[c] Yes; $\quad 443.18\,\text{mW} < 450\,\text{mW}$

P 10.49 [a] $Z_{\text{Th}} = 20 + j60 + \dfrac{(j20)(6 - j18)}{6 + j2} = 80 + j60 = 100\underline{/36.87^\circ}\,\Omega$

$$\therefore \quad R = |Z_{\text{Th}}| = 100\,\Omega$$

[b] $\mathbf{V}_{\text{Th}} = \dfrac{j20}{6 - j18 + j20}(480\underline{/0^\circ}) = 480 + j1440\,\text{V(rms)}$

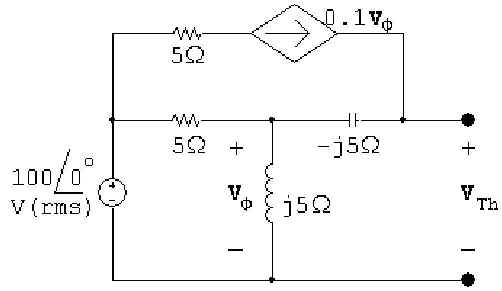


$$\mathbf{I} = \frac{480 + j1440}{180 + j60} = 4.8 + j6.4 = 8\underline{/53.13^\circ}\ \text{A(rms)}$$

$$P = 8^2(100) = 6400\,\text{W}$$

[c] Pick the $100\,\Omega$ resistor from Appendix H to match exactly.

P 10.50 [a] Open circuit voltage:

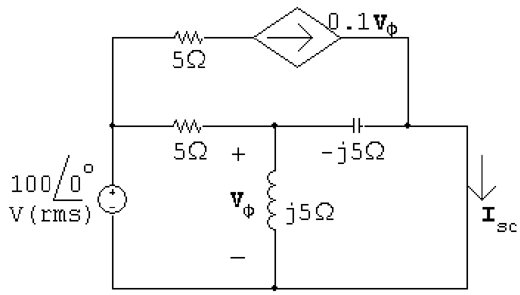


$$\frac{\mathbf{V}_\phi - 100}{5} + \frac{\mathbf{V}_\phi}{j5} - 0.1\mathbf{V}_\phi = 0$$

$$\therefore \quad \mathbf{V}_\phi = 40 + j80\,\mathrm{V(rms)}$$

$$\mathbf{V}_{\mathrm{Th}} = \mathbf{V}_\phi + 0.1\mathbf{V}_\phi(-j5) = \mathbf{V}_\phi(1 - j0.5) = 80 + j60\,\mathrm{V(rms)}$$

Short circuit current:



$$\mathbf{I}_{\mathrm{sc}} = 0.1\mathbf{V}_\phi + \frac{\mathbf{V}_\phi}{-j5} = (0.1 + j0.2)\mathbf{V}_\phi$$

$$\frac{\mathbf{V}_\phi - 100}{5} + \frac{\mathbf{V}_\phi}{j5} + \frac{\mathbf{V}_\phi}{-j5} = 0$$

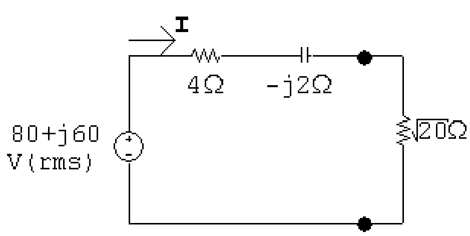
$$\therefore \quad \mathbf{V}_\phi = 100\,\mathrm{V(rms)}$$

$$\mathbf{I}_{\mathrm{sc}} = (0.1 + j0.2)(100) = 10 + j20\,\mathrm{A(rms)}$$

$$Z_{\mathrm{Th}} = \frac{\mathbf{V}_{\mathrm{Th}}}{\mathbf{I}_{\mathrm{sc}}} = \frac{80 + j60}{10 + j20} = 4 - j2\,\Omega$$

$$\therefore \quad R_o = |Z_{\mathrm{Th}}| = 4.47\,\Omega$$

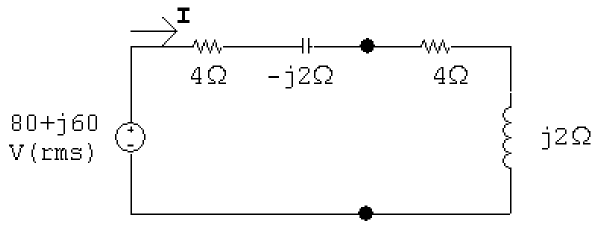
[b]



$$\mathbf{I} = \frac{80 + j60}{4 + \sqrt{20} - j2} = 7.36 + j8.82\,\mathrm{A\ (rms)}$$

$$P = (11.49)^2(\sqrt{20}) = 590.17\,\mathrm{W}$$

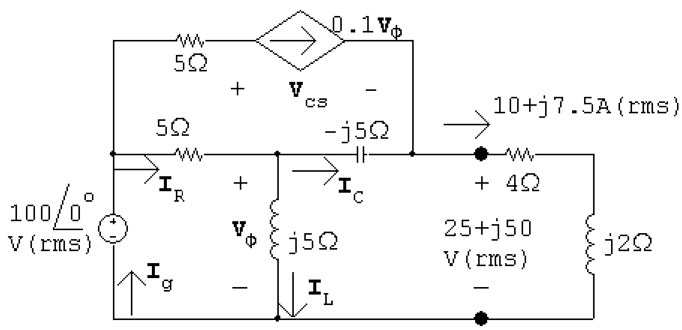
[c]



$$\mathbf{I} = \frac{80 + j60}{8} = 10 + j7.5\,\text{A (rms)}$$

$$P = (10^2 + 7.5^2)(4) = 625\,\text{W}$$

[d]



$$\frac{\mathbf{V}_\phi - 100}{5} + \frac{\mathbf{V}_\phi}{j5} + \frac{\mathbf{V}_o - (25 + j50)}{-j5} = 0$$

$$\mathbf{V}_\phi = 50 + j25\,\text{V (rms)}$$

$$0.1\mathbf{V}_\phi = 5 + j2.5\,\text{V (rms)}$$

$$5 + j2.5 + \mathbf{I}_C = 10 + j7.5$$

$$\mathbf{I}_C = 5 + j5\,\text{A (rms)}$$

$$\mathbf{I}_L = \frac{\mathbf{V}_\phi}{j5} = 5 - j10\,\text{A (rms)}$$

$$\mathbf{I}_R = \mathbf{I}_C + \mathbf{I}_L = 10 - j5\,\text{A (rms)}$$

$$\mathbf{I}_g = \mathbf{I}_R + 0.1\mathbf{V}_\phi = 15 - j2.5\,\text{A (rms)}$$

$$S_g = -100\mathbf{I}_g^* = -1500 - j250\,\text{VA}$$

$$100 = 5(5 + j2.5) + \mathbf{V}_{\text{cs}} + 25 + j50 \quad \therefore \quad \mathbf{V}_{\text{cs}} = 50 - j62.5\,\text{V (rms)}$$

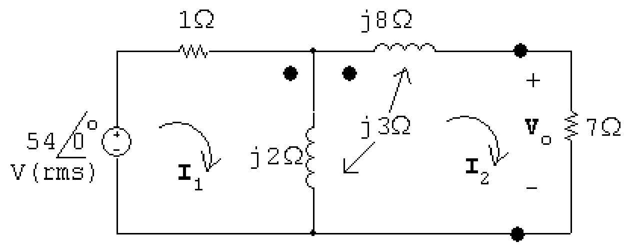
$$S_{\text{cs}} = (50 - j62.5)(5 - j2.5) = 93.75 - j437.5\,\text{VA}$$

Thus,

$$\sum P_{\text{dev}} = 1500$$

$$\%\text{ delivered to } R_o = \frac{625}{1500}(100) = 41.67\%$$

P 10.51 [**a**]



$$54 = \mathbf{I}_1 + j2(\mathbf{I}_1 - \mathbf{I}_2) + j3\mathbf{I}_2$$

$$0 = 7\mathbf{I}_2 + j2(\mathbf{I}_2 - \mathbf{I}_1) - j3\mathbf{I}_2 + j8\mathbf{I}_2 + j3(\mathbf{I}_1 - \mathbf{I}_2)$$

Solving,

$\mathbf{I}_1 = 12 - j21\,\text{A (rms)}; \qquad \mathbf{I}_2 = -3\,\text{A (rms)}$

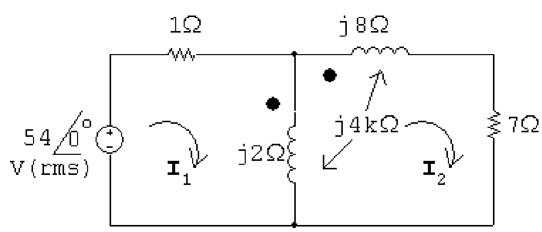
$\mathbf{V}_o = 7\mathbf{I}_2 = -21\underline{/0^\circ}\,\text{V(rms)}$

[**b**] $P = |\mathbf{I}_2|^2(7) = 63\,\text{W}$

[**c**] $P_g = (54)(12) = 648\,\text{W}$

$$\%\text{ delivered } = \frac{63}{648}(100) = 9.72\%$$

P 10.52 [**a**]



$$54 = \mathbf{I}_1 + j2(\mathbf{I}_1 - \mathbf{I}_2) + j4k\mathbf{I}_2$$

$$0 = 7\mathbf{I}_2 + j2(\mathbf{I}_2 - \mathbf{I}_1) - j4k\mathbf{I}_2 + j8\mathbf{I}_2 + j4k(\mathbf{I}_1 - \mathbf{I}_2)$$

Place the equations in standard form:

$$54 = (1 + j2)\mathbf{I}_1 + j(4k - 2)\mathbf{I}_2$$

$$0 = j(4k - 2)\mathbf{I}_1 + [7 + j(10 - 8k)]\mathbf{I}_2$$

$$\mathbf{I}_1 = \frac{54 - \mathbf{I}_2 j(4k - 2)}{(1 + j2)}$$

Substituting,

$$\mathbf{I}_2 = \frac{j54(4k - 2)}{[7 + j(10 - 8k)](1 + j2) - (4k - 2)}$$

For $\mathbf{V}_o = 0, \mathbf{I}_2 = 0$, so if $4k - 2 = 0$, then $k = 0.5$.

[**b**] When $\mathbf{I}_2 = 0$

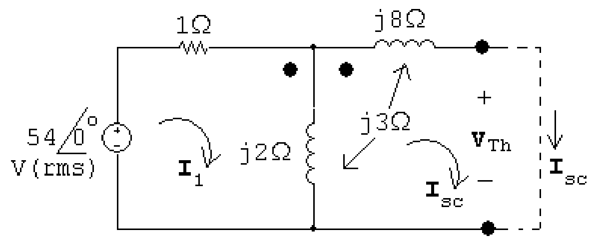
$$\mathbf{I}_1 = \frac{54}{1 + j2} = 10.8 - j21.6\,\text{A(rms)}$$

$$P_g = (54)(10.8) = 583.2\,\text{W}$$

Check:

$$P_{\text{loss}} = |\mathbf{I}_1|^2(1) = 583.2\,\text{W}$$

P 10.53 [**a**]



Open circuit:

$$\mathbf{V}_{\text{Th}} = -j3\mathbf{I}_1 + j2\mathbf{I}_1 = -j\mathbf{I}_1$$

$$\mathbf{I}_1 = \frac{54}{1 + j2} = 10.8 - j21.6$$

$$\mathbf{V}_{\text{Th}} = -21.6 - j10.8\,\text{V}$$

Short circuit:

$$54 = \mathbf{I}_1 + j2(\mathbf{I}_1 - \mathbf{I}_{\text{sc}}) + j3\mathbf{I}_{\text{sc}}$$

$$0 = j2(\mathbf{I}_{\text{sc}} - \mathbf{I}_1) - j3\mathbf{I}_{\text{sc}} + j8\mathbf{I}_{\text{sc}} + j3(\mathbf{I}_1 - \mathbf{I}_{\text{sc}})$$

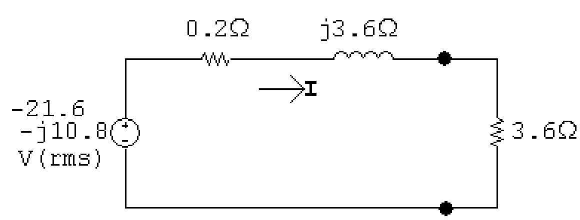
Solving,

$$\mathbf{I}_{\text{sc}} = -3.32 + j5.82$$

$$Z_{\text{Th}} = \frac{\mathbf{V}_{\text{Th}}}{\mathbf{I}_{\text{sc}}} = \frac{-21.6 - j10.8}{-3.32 + j5.82} = 0.2 + j3.6 = 3.6\underline{/86.86^\circ}\,\Omega$$

$$\therefore \quad R_{\text{L}} = |Z_{\text{Th}}| = 3.6\,\Omega$$

[**b**]



$$\mathbf{I} = \frac{-21.6 - j10.8}{3.8 + j3.6} = 4.614\underline{/163.1^\circ}$$

$$P = |\mathbf{I}|^2(3.6) = 76.6\,\text{W}, \quad \text{which is greater than when } R_L = 7\,\Omega$$

P 10.54 [a] $\dfrac{1}{\omega C} = 100\,\Omega; \qquad C = \dfrac{1}{(60)(200\pi)} = 26.53\,\mu\text{F}$

[b] $\mathbf{V}_{\text{swo}} = 4000 + (40)(1.25 + j10) = 4050 + j400$

$= 4069.71\underline{/5.64^\circ}\,\text{V(rms)}$

$\mathbf{V}_{\text{sw}} = 4000 + (40 - j40)(1.25 + j10) = 4450 + j350 = 4463.73\underline{/4.50^\circ}\,\text{V(rms)}$

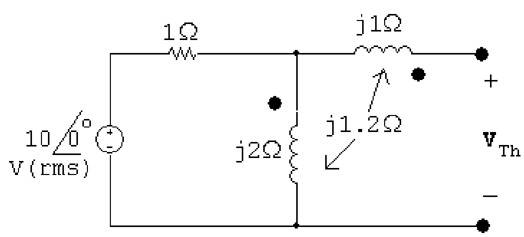
$\%\text{ increase } = \left(\dfrac{4463.73}{4069.71} - 1\right)(100) = 9.68\%$

[c] $P_{\ell\text{wo}} = (40\sqrt{2})^2(1.25) = 4000\,\text{W}$

$P_{\ell\text{w}} = 40^2(1.25) = 2000\,\text{W}$

$\%\text{ increase } = \left(\dfrac{4000}{2000} - 1\right)(100) = 100\%$

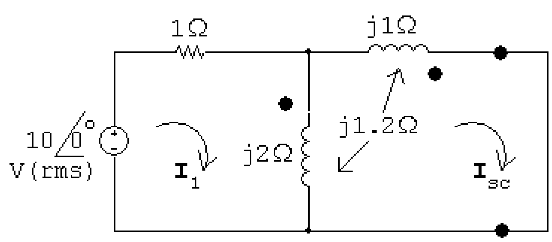
P 10.55 Open circuit voltage:



$\mathbf{I}_1 = \dfrac{10\underline{/0^\circ}}{1 + j2} = 2 - j4\,\text{A}$

$\mathbf{V}_{\text{Th}} = j2\mathbf{I}_1 + j1.2\mathbf{I}_1 = j3.2\mathbf{I}_1 = 12.8 + j6.4 = 14.31\underline{/26.57^\circ}$

Short circuit current:



$10\underline{/0^\circ} = (1 + j2)\mathbf{I}_1 - j3.2\mathbf{I}_{\text{sc}}$

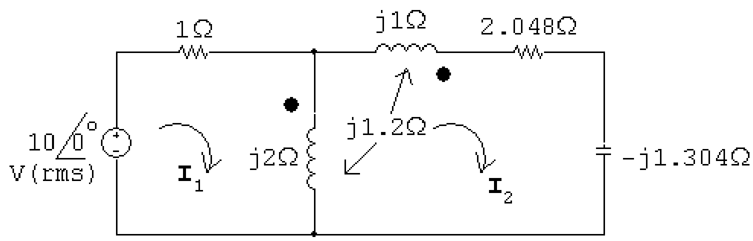
$0 = -j3.2\mathbf{I}_1 + j5.4\mathbf{I}_{\text{sc}}$

Solving,

$\mathbf{I}_{\text{sc}} = 5.89\underline{/-5.92^\circ}\,\text{A}$

$$Z_{\rm Th} = \frac{14.31\underline{/26.57^\circ}}{5.89\underline{/-5.92^\circ}} = 2.43\underline{/32.49^\circ} = 2.048 + j1.304\,\Omega$$

$$\therefore \quad \mathbf{I}_2 = \frac{14.31\underline{/26.57^\circ}}{4.096} = 3.49\underline{/26.57^\circ}\,\text{A}$$

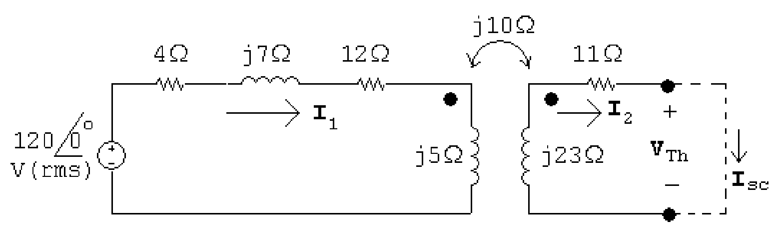


$$10\underline{/0^\circ} = (1 + j2)\mathbf{I}_1 - j3.2\mathbf{I}_2$$

$$\therefore \qquad \mathbf{I}_1 = \frac{10 + j3.2\mathbf{I}_2}{1 + j2} = \frac{10 + j3.2(3.49\underline{/26.57^\circ})}{1 + j2} = 5\,\text{A}$$

$$Z_g = \frac{10\underline{/0^\circ}}{5} = 2 + j0 = 2\underline{/0^\circ}\,\Omega$$

P 10.56 [**a**]



Open circuit:

$$\mathbf{V}_{\rm Th} = \frac{120}{16 + j12}(j10) = 36 + j48\,\text{V}$$

Short circuit:

$$(16 + j12)\mathbf{I}_1 - j10\mathbf{I}_{\rm sc} = 120$$

$$-j10\mathbf{I}_1 + (11 + j23)\mathbf{I}_{\rm sc} = 0$$

Solving,

$$\mathbf{I}_{\rm sc} = 2.4\,\text{A}$$

$$Z_{\rm Th} = \frac{36 + j48}{2.4} = 15 + j20\,\Omega$$

$$\therefore \quad Z_{\rm L} = Z^*_{\rm Th} = 15 - j20\,\Omega$$

$$\mathbf{I}_{\rm L} = \frac{\mathbf{V}_{\rm Th}}{Z_{\rm Th} + Z_L} = \frac{36 + j48}{30} = 1.2 + j1.6\,\text{A(rms)}$$

$$P_{\rm L} = |\mathbf{I}_{\rm L}|^2(15) = 60\,\text{W}$$

[b] $$\mathbf{I}_1 = \frac{Z_{22}\mathbf{I}_2}{j\omega M} = \frac{26 + j3}{j10}(1.2 + j1.6) = 5.23\underline{/-30.29^\circ}\,\text{A (rms)}$$

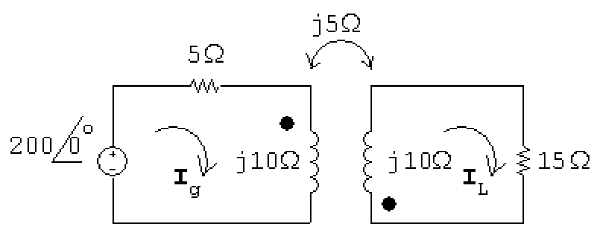
$$P_{\rm transformer} = (120)(5.23)\cos(-30.29^\circ) - (5.23)^2(4) = 432.8\,\text{W}$$

$$\%\ \text{delivered} = \frac{60}{432.8}(100) = 13.86\%$$

P 10.57 [a] $j\omega L_1 = j(10{,}000)(1 \times 10^{-3}) = j10\,\Omega$

$$j\omega L_2 = j(10{,}000)(1 \times 10^{-3}) = j10\,\Omega$$

$$j\omega M = j10\,\Omega$$



$$200 = (5 + j10)\mathbf{I}_g + j5\mathbf{I}_{\rm L}$$

$$0 = j5\mathbf{I}_g + (15 + j10)\mathbf{I}_{\rm L}$$

Solving,

$\mathbf{I}_g = 10 - j15\,\text{A}; \qquad \mathbf{I}_{\rm L} = -5\,\text{A}$

Thus,

$$i_g = 18.03\cos(10{,}000t - 56.31^\circ)\,\text{A}$$

$$i_{\rm L} = 5\cos(10{,}000t - 180^\circ)\,\text{A}$$

[b] $$k = \frac{M}{\sqrt{L_1 L_2}} = \frac{0.5}{\sqrt{1}} = 0.5$$

[c] When $t = 50\pi\ \mu$s:

$$10{,}000t = (10{,}000)(50\pi) \times 10^{-6} = 0.5\pi\ \text{rad} = 90^\circ$$

$$i_g(50\pi\ \mu\text{s}) = 18.03\cos(90^\circ - 56.31^\circ) = 15\,\text{A}$$

$$i_{\rm L}(50\pi\ \mu\text{s}) = 5\cos(90^\circ + 180^\circ) = 0\,\text{A}$$

$$w = \frac{1}{2}L_1 i_1^2 + \frac{1}{2}L_2 i_2^2 + M i_1 i_2 = \frac{1}{2}(10^{-3})(15)^2 + 0 + 0 = 112.5\,\text{mJ}$$

When $t = 100\pi\ \mu\text{s}$:

$10{,}000t = (10^4)(100\pi) \times 10^{-6} = \pi = 180^\circ$

$i_g(100\pi\ \mu\text{s}) = 18.03\cos(180 - 56.31^\circ) = -10\,\text{A}$

$i_\text{L}(100\pi\ \mu\text{s}) = 5\cos(180 - 180^\circ) = 5\,\text{A}$

$$w = \frac{1}{2}(10^{-3})(10)^2 + \frac{1}{2}(10^{-3})(5)^2 + 0.5 \times 10^{-3}(-10)(5) = 37.5\,\text{mJ}$$

[d] From (a), $I_m = 5$ A,

$$\therefore \quad P = \frac{1}{2}(5)^2(15) = 187.5\,\text{W}$$

[e] Open circuit:

$$\mathbf{V}_\text{Th} = \frac{200}{5 + j10}(-j5) = -80 - j40\,\text{V}$$

Short circuit:

$200 = (5 + j10)\mathbf{I}_1 + j5\mathbf{I}_\text{sc}$

$0 = j5\mathbf{I}_1 + j10\mathbf{I}_\text{sc}$

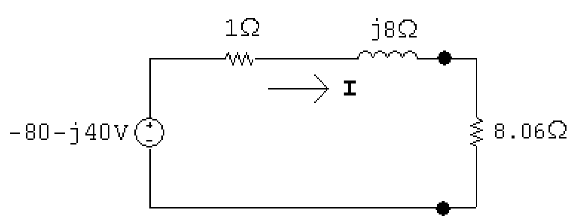
Solving,

$$\mathbf{I}_\text{sc} = -\frac{80}{13} + j\frac{120}{13}$$

$$Z_\text{Th} = \frac{\mathbf{V}_\text{Th}}{\mathbf{I}_\text{sc}} = \frac{-80 - j40}{-(80/13) + j(120/13)} = 1 + j8\,\Omega$$

$$\therefore \quad R_\text{L} = 8.06\,\Omega$$

[f]



$$\mathbf{I} = \frac{-80 - j40}{1 + j8 + 8.06} = 7.40\underline{/165.12^\circ}\,\text{A}$$

$$P = \frac{1}{2}(7.40)^2(8.06) = 223.42\,\text{W}$$

[g] $Z_\text{L} = Z^*_\text{Th} = 1 - j8\,\Omega$

[h] $\mathbf{I} = \dfrac{-80 - j40}{2} = 44.72\underline{/-153.43^\circ}$

$$P = \frac{1}{2}(44.72)^2(1) = 1000\,\text{W}$$

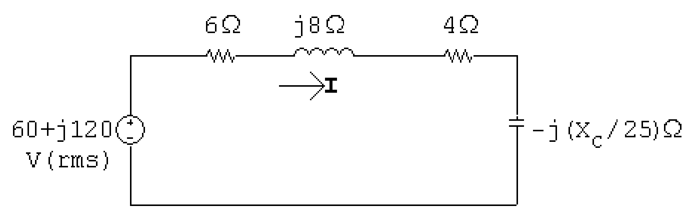
P 10.58 [a] Replace the circuit to the left of the primary winding with a Thévenin equivalent:

$$\mathbf{V}_{\text{Th}} = (15)(20\|j10) = 60 + j120\,\text{V}$$

$$Z_{\text{Th}} = 2 + 20\|j10 = 6 + j8\,\Omega$$

Transfer the secondary impedance to the primary side:

$$Z_p = \frac{1}{25}(100 - jX_{\text{C}}) = 4 - j\frac{X_{\text{C}}}{25}\,\Omega$$



Now maximize $\mathbf{I}$ by setting $(X_{\text{C}}/25) = 8\,\Omega$:

$$\therefore\quad C = \frac{1}{200(20\times 10^3)} = 0.25\,\mu\text{F}$$

[b] $\mathbf{I} = \dfrac{60 + j120}{10} = 6 + j12\,\text{A}$

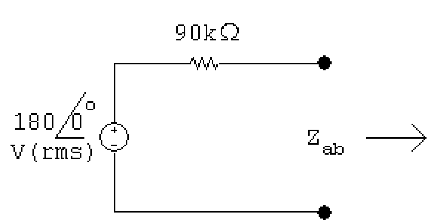
$$P = |\mathbf{I}|^2(4) = 720\,\text{W}$$

[c] $\dfrac{R_o}{25} = 6\,\Omega; \qquad \therefore\quad R_o = 150\,\Omega$

[d] $\mathbf{I} = \dfrac{60 + j120}{12} = 5 + j10\,\text{A}$

$$P = |\mathbf{I}|^2(6) = 750\,\text{W}$$

P 10.59 [a]



For maximum power transfer, $Z_{\text{ab}} = 90\,\text{k}\Omega$

$$Z_{\text{ab}} = \left(1 - \frac{N_1}{N_2}\right)^2 Z_{\text{L}}$$

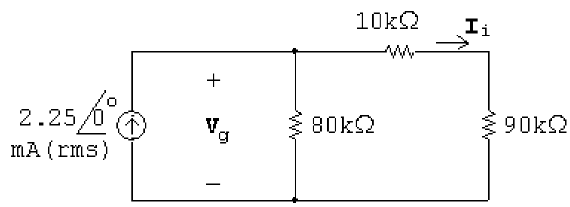
$$\therefore \quad \left(1 - \frac{N_1}{N_2}\right)^2 = \frac{90{,}000}{400} = 225$$

$$1 - \frac{N_1}{N_2} = \pm 15; \qquad \frac{N_1}{N_2} = 15 + 1 = 16$$

[b] $P = |\mathbf{I}_i|^2(90{,}000) = \left(\dfrac{180}{180{,}000}\right)^2 (90{,}000) = 90\,\text{mW}$

[c] $\mathbf{V}_1 = R_i\mathbf{I}_i = (90{,}000)\left(\dfrac{180}{180{,}000}\right) = 90\,\text{V}$

[d]



$$\mathbf{V}_g = (2.25 \times 10^{-3})(100{,}000 \| 80{,}000) = 100\,\text{V}$$

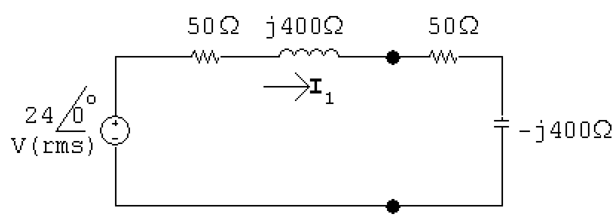
$$P_g(\text{del}) = (2.25 \times 10^{-3})(100) = 225\,\text{mW}$$

$$\%\ \text{delivered} = \frac{90}{225}(100) = 40\%$$

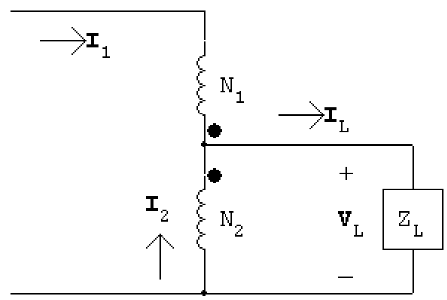
P 10.60 [a] $Z_{\text{ab}} = 50 - j400 = \left(1 - \dfrac{N_1}{N_2}\right)^2 Z_{\text{L}}$

$$\therefore \quad Z_{\text{L}} = \frac{1}{(1-6)^2}(50 - j400) = 2 - j16\,\Omega$$

[b]



$$\mathbf{I}_1 = \frac{24}{100} = 240\underline{/0^\circ}\,\text{mA}$$



$$N_1\mathbf{I}_1 = -N_2\mathbf{I}_2$$

$$\mathbf{I}_2 = -6\mathbf{I}_1 = -1.44\underline{/0^\circ}\,\text{A}$$

$$\mathbf{I}_\text{L} = \mathbf{I}_1 + \mathbf{I}_2 = -1.68\underline{/0^\circ}\,\text{A}$$

$$\mathbf{V}_\text{L} = (2 - j16)\mathbf{I}_\text{L} = -3.36 + j26.88 = 27.1\underline{/97.13^\circ}\,\text{V(rms)}$$

P 10.61 [a] $Z_\text{Th} = 720 + j1500 + \left(\frac{200}{50}\right)^2 (40 - j30) = 1360 + j1020 = 1700\underline{/36.87^\circ}\,\Omega$

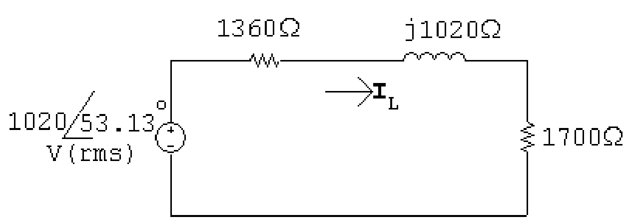
$$\therefore \quad Z_\text{ab} = 1700\,\Omega$$

$$Z_\text{ab} = \frac{Z_\text{L}}{(1 + N_1/N_2)^2}$$

$$(1 + N_1/N_2)^2 = 6800/1700 = 4$$

$$\therefore \quad N_1/N_2 = 1 \quad \text{or} \quad N_2 = N_1 = 1000 \text{ turns}$$

[b] $\mathbf{V}_\text{Th} = \frac{255\underline{/0^\circ}}{40 + j30}(j200) = 1020\underline{/53.13^\circ}\,\text{V}$

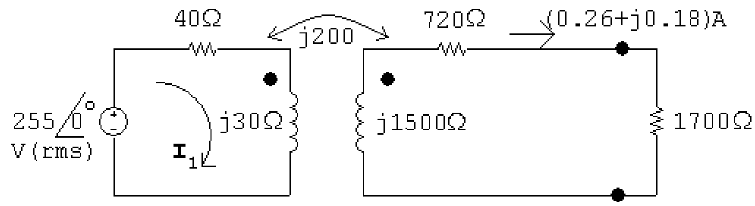


$$\mathbf{I}_L = \frac{1020\underline{/53.13^\circ}}{3060 + j1020} = 0.316\underline{/34.7^\circ}\,\text{A(rms)}$$

Since the transformer is ideal, $P_{6800} = P_{1700}$.

$$P = |\mathbf{I}|^2(1700) = 170\,\text{W}$$

[**c**]



$$255\underline{/0^\circ} = (40 + j30)\mathbf{I}_1 - j200(0.26 + j0.18)$$

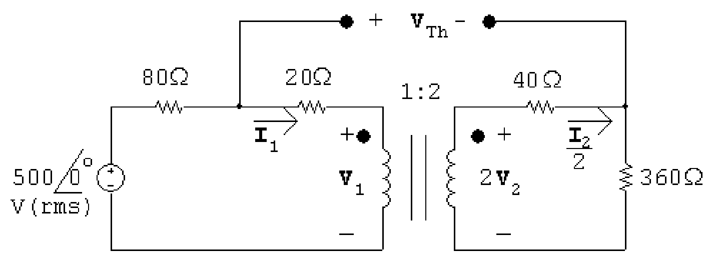
$$\therefore \quad \mathbf{I}_1 = 4.13 - j1.80\,\text{A(rms)}$$

$$P_{\text{gen}} = (255)(4.13) = 1053\,\text{W}$$

$$P_{\text{diss}} = 1053 - 170 = 883\,\text{W}$$

$$\%\ \text{dissipated} = \frac{883}{1053}(100) = 83.85\%$$

P 10.62 [**a**] Open circuit voltage:



$$500 = 100\mathbf{I}_1 + \mathbf{V}_1$$

$$\mathbf{V}_2 = 400\mathbf{I}_2$$

$$\frac{\mathbf{V}_1}{1} = \frac{\mathbf{V}_2}{2} \qquad \therefore \qquad \mathbf{V}_2 = 2\mathbf{V}_1$$

$$\mathbf{I}_1 = 2\mathbf{I}_2$$

Substitute and solve:

$$2\mathbf{V}_1 = 400\mathbf{I}_1/2 = 200\mathbf{I}_1 \qquad \therefore \qquad \mathbf{V}_1 = 100\mathbf{I}_1$$

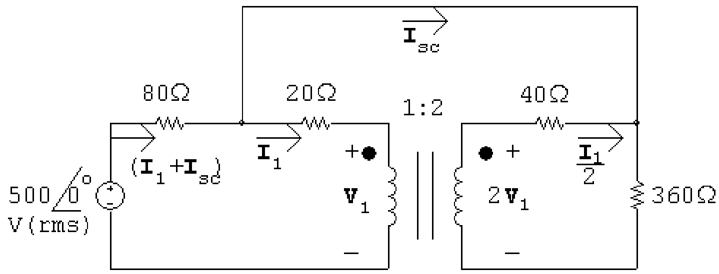
$$500 = 100\mathbf{I}_1 + 100\mathbf{I}_1 \qquad \therefore \qquad \mathbf{I}_1 = 500/200 = 2.5\,\text{A}$$

$$\therefore \qquad \mathbf{I}_2 = \frac{1}{2}\mathbf{I}_1 = 1.25\,\text{A}$$

$$\mathbf{V}_1 = 100(2.5) = 250\,\text{V}; \qquad \mathbf{V}_2 = 2\mathbf{V}_1 = 500\,\text{V}$$

$$V_{\text{Th}} = 20\mathbf{I}_1 + \mathbf{V}_1 - \mathbf{V}_2 + 40\mathbf{I}_2 = -150\,\text{V(rms)}$$

Short circuit current:



$$500 = 80(\mathbf{I}_{\text{sc}} + \mathbf{I}_1) + 360(\mathbf{I}_{\text{sc}} + 0.5\mathbf{I}_1)$$

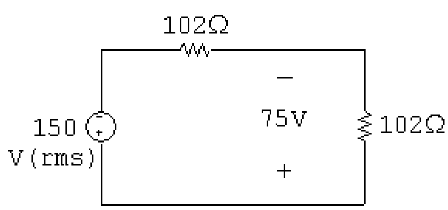
$$2\mathbf{V}_1 = 40\frac{\mathbf{I}_1}{2} + 360(\mathbf{I}_{\text{sc}} + 0.5\mathbf{I}_1)$$

$$500 = 80(\mathbf{I}_1 + \mathbf{I}_{\text{sc}}) + 20\mathbf{I}_1 + \mathbf{V}_1$$

Solving,

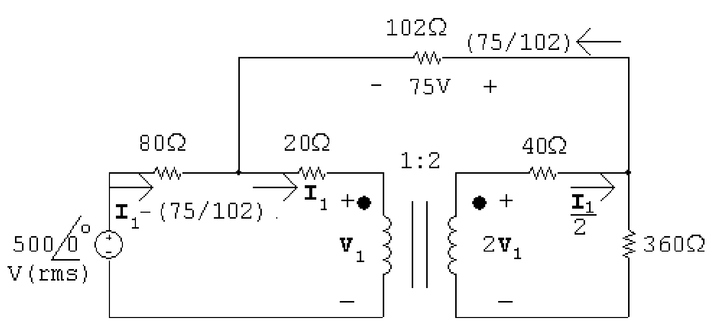
$$\mathbf{I}_{\text{sc}} = -1.47\,\text{A}$$

$$R_{\text{Th}} = \frac{\mathbf{V}_{\text{Th}}}{\mathbf{I}_{\text{sc}}} = \frac{-150}{-1.47} = 102\,\Omega$$



$$P = \frac{75^2}{102} = 55.15\,\text{W}$$

[b]



$$500 = 80[\mathbf{I}_1 - (75/102)] - 75 + 360[\mathbf{I}_2 - (75/102)]$$

$$575 + \frac{6000}{102} + \frac{27{,}000}{102} = 80\mathbf{I}_1 + 180\mathbf{I}_2$$

$$\therefore \quad \mathbf{I}_1 = 3.456\,\text{A}$$

$$P_{\text{source}} = (500)[3.456 - (75/102)] = 1360.35\,\text{W}$$

$$\%\ \text{delivered} = \frac{55.15}{1360.35}(100) = 4.05\%$$

[c] $P_{80\Omega} = 80(\mathbf{I}_1 + \mathbf{I}_L)^2 = 592.13\,\text{W}$

$$P_{20\Omega} = 20\mathbf{I}_1^2 = 238.86\,\text{W}$$

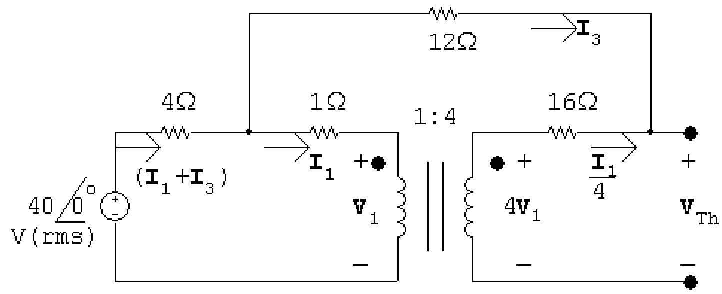
$$P_{40\Omega} = 40\mathbf{I}_2^2 = 119.43\,\text{W}$$

$$P_{102\Omega} = 102\mathbf{I}_L^2 = 55.15\,\text{W}$$

$$P_{360\Omega} = 360(\mathbf{I}_2 + \mathbf{I}_L)^2 = 354.73\,\text{W}$$

$$\sum P_{\text{abs}} = 592.13 + 238.86 + 119.43 + 55.15 + 354.73 = 1360.3\,\text{W} = \sum P_{\text{dev}}$$

P 10.63 [a] Open circuit voltage:



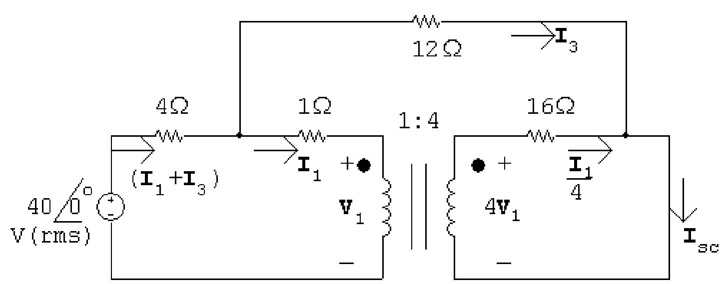
$$40\underline{/0^\circ} = 4(\mathbf{I}_1 + \mathbf{I}_3) + 12\mathbf{I}_3 + \mathbf{V}_{\text{Th}}$$

$$\frac{\mathbf{I}_1}{4} = -\mathbf{I}_3; \qquad \mathbf{I}_1 = -4\mathbf{I}_3$$

Solving,

$$\mathbf{V}_{\text{Th}} = 40\underline{/0^\circ}\,\text{V}$$

Short circuit current:



$$40\underline{/0^\circ} = 4\mathbf{I}_1 + 4\mathbf{I}_3 + \mathbf{I}_1 + \mathbf{V}_1$$

$$4\mathbf{V}_1 = 16(\mathbf{I}_1/4) = 4\mathbf{I}_1; \qquad \therefore\ \ \mathbf{V}_1 = \mathbf{I}_1$$

$$\therefore\ \ 40\underline{/0^\circ} = 6\mathbf{I}_1 + 4\mathbf{I}_3$$

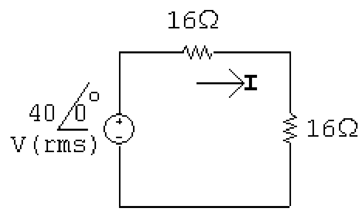
Also,

$40\underline{/0^\circ} = 4(\mathbf{I}_1 + \mathbf{I}_3) + 12\mathbf{I}_3$

Solving,

$\mathbf{I}_1 = 6\,\text{A}; \qquad \mathbf{I}_3 = 1\,\text{A}; \qquad \mathbf{I}_{\text{sc}} = \mathbf{I}_1/4 + \mathbf{I}_3 = 2.5\,\text{A}$

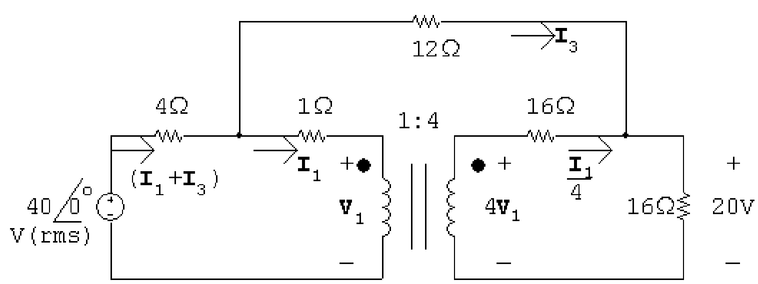
$$R_{\text{Th}} = \frac{\mathbf{V}_{\text{Th}}}{\mathbf{I}_{\text{sc}}} = \frac{40}{2.5} = 16\,\Omega$$



$$\mathbf{I} = \frac{40\underline{/0^\circ}}{32} = 1.25\underline{/0^\circ}\,\text{A(rms)}$$

$P = (1.25)^2(16) = 25\,\text{W}$

[**b**]



$40 = 4(\mathbf{I}_1 + \mathbf{I}_3) + 12\mathbf{I}_3 + 20$

$4\mathbf{V}_1 = 4\mathbf{I}_1 + 16(\mathbf{I}_1/4 + \mathbf{I}_3); \qquad \therefore\ \mathbf{V}_1 = 2\mathbf{I}_1 + 4\mathbf{I}_3$

$40 = 4\mathbf{I}_1 + 4\mathbf{I}_3 + \mathbf{I}_1 + \mathbf{V}_1$

$\therefore\ \mathbf{I}_1 = 6\,\text{A}; \qquad \mathbf{I}_3 = -0.25\,\text{A}; \qquad \mathbf{I}_1 + \mathbf{I}_3 = 5.75\underline{/0^\circ}\,\text{A}$

$P_{40V}(\text{developed}) = 40(5.75) = 230\,\text{W}$

$$\therefore\ \%\ \text{delivered} = \frac{25}{230}(100) = 10.87\%$$

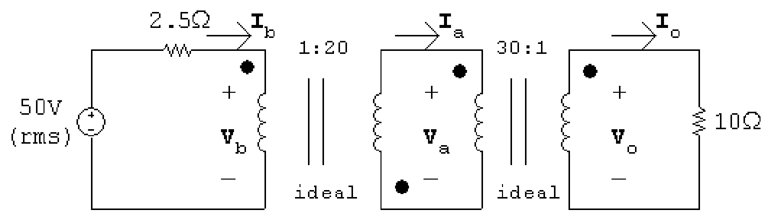
[**c**] $P_{R_L} = 25\,\text{W}; \qquad P_{16\Omega} = (1.5)^2(16) = 36\,\text{W}$

$P_{4\Omega} = (5.75)^2(4) = 132.25\,\text{W}; \qquad P_{1\Omega} = (6)^2(1) = 36\,\text{W}$

$P_{12\Omega} = (-0.25)^2(12) = 0.75\,\text{W}$

$\sum P_{\text{abs}} = 25 + 36 + 132.25 + 36 + 0.75 = 230\,\text{W} = \sum P_{\text{dev}}$

P 10.64



$$30\mathbf{V}_o = \mathbf{V}_a; \qquad \frac{\mathbf{I}_o}{30} = \mathbf{I}_a; \qquad \text{therefore} \quad \frac{\mathbf{V}_a}{\mathbf{I}_a} = 9\,\text{k}\Omega$$

$$\frac{\mathbf{V}_b}{1} = \frac{-\mathbf{V}_a}{20}; \qquad \mathbf{I}_b = -20\mathbf{I}_a; \qquad \text{therefore} \quad \frac{\mathbf{V}_b}{\mathbf{I}_b} = \frac{9000}{400} = 22.5\,\Omega$$

Therefore $\mathbf{I}_b = [50/(2.5 + 22.5)] = 2$ A (rms); since the ideal transformers are lossless, $P_{10\Omega} = P_{22.5\Omega}$, and the power delivered to the 22.5 Ω resistor is $2^2(22.5)$ or 90 W.

P 10.65 **[a]** $\dfrac{\mathbf{V}_b}{\mathbf{I}_b} = \dfrac{a^2 10}{400} = 2.5\,\Omega; \qquad \text{therefore} \quad a^2 = 100, \qquad a = 10$

[b] $\mathbf{I}_b = \dfrac{50}{5} = 10\,\text{A}; \qquad P = (100)(2.5) = 250\,\text{W}$

P 10.66 **[a]** Begin with the MEDIUM setting, as shown in Fig. 10.31, as it involves only the resistor R_2. Then,

$$P_{\text{med}} = 500\,\text{W} = \frac{V^2}{R_2} = \frac{120^2}{R_2}$$

Thus,

$$R_2 = \frac{120^2}{500} = 28.8\,\Omega$$

[b] Now move to the LOW setting, as shown in Fig. 10.30, which involves the resistors R_1 and R_2 connected in series:

$$P_{\text{low}} = \frac{V^2}{R_1 + R_2} = \frac{V^2}{R_1 + 28.8} = 250\,\text{W}$$

Thus,

$$R_1 = \frac{120^2}{250} - 28.8 = 28.8\,\Omega$$

[c] Note that the HIGH setting has R_1 and R_2 in parallel:

$$P_{\text{high}} = \frac{V^2}{R_1 \| R_2} = \frac{120^2}{28.8 \| 28.8} = 1000\,\text{W}$$

If the HIGH setting has required power other than 1000 W, this problem could not have been solved. In other words, the HIGH power setting was chosen in such a way that it would be satisfied once the two resistor values were calculated to satisfy the LOW and MEDIUM power settings.

P 10.67 **[a]** $P_{\rm L} = \dfrac{V^2}{R_1 + R_2}; \qquad R_1 + R_2 = \dfrac{V^2}{P_{\rm L}}$

$$P_{\rm M} = \frac{V^2}{R_2}; \qquad R_2 = \frac{V^2}{P_{\rm M}}$$

$$P_{\rm H} = \frac{V^2(R_1 + R_2)}{R_1 R_2}$$

$$R_1 + R_2 = \frac{V^2}{P_{\rm L}}; \qquad R_1 = \frac{V^2}{P_{\rm L}} - \frac{V^2}{P_{\rm M}}$$

$$P_{\rm H} = \frac{V^2 V^2/P_{\rm L}}{\left(\frac{V^2}{P_{\rm L}} - \frac{V^2}{P_{\rm M}}\right)\left(\frac{V^2}{P_{\rm M}}\right)} = \frac{P_{\rm M} P_{\rm L} P_{\rm M}}{P_{\rm L}(P_{\rm M} - P_{\rm L})}$$

$$P_{\rm H} = \frac{P_{\rm M}^2}{P_{\rm M} - P_{\rm L}}$$

[b] $P_{\rm H} = \dfrac{(750)^2}{(750 - 250)} = 1125\,\text{W}$

P 10.68 First solve the expression derived in P10.67 for $P_{\rm M}$ as a function of $P_{\rm L}$ and $P_{\rm H}$. Thus

$$P_{\rm M} - P_{\rm L} = \frac{P_{\rm M}^2}{P_{\rm H}} \quad \text{or} \quad \frac{P_{\rm M}^2}{P_{\rm H}} - P_{\rm M} + P_{\rm L} = 0$$

$$P_{\rm M}^2 - P_{\rm M} P_{\rm H} + P_{\rm L} P_{\rm H} = 0$$

$$\therefore \quad P_{\rm M} = \frac{P_{\rm H}}{2} \pm \sqrt{\left(\frac{P_{\rm H}}{2}\right)^2 - P_{\rm L} P_{\rm H}}$$

$$= \frac{P_{\rm H}}{2} \pm P_{\rm H}\sqrt{\frac{1}{4} - \left(\frac{P_{\rm L}}{P_{\rm H}}\right)}$$

For the specified values of $P_{\rm L}$ and $P_{\rm H}$

$$P_{\rm M} = 500 \pm 1000\sqrt{0.25 - 0.24} = 500 \pm 100$$

$$\therefore \quad P_{M1} = 600\,\text{W}; \qquad P_{M2} = 400\,\text{W}$$

Note in this case we design for two medium power ratings
If $P_{M1} = 600\,\text{W}$

$$R_2 = \frac{(120)^2}{600} = 24\,\Omega$$

$$R_1 + R_2 = \frac{(120)^2}{240} = 60\,\Omega$$

$$R_1 = 60 - 24 = 36\,\Omega$$

$$\text{CHECK: } P_{\text{H}} = \frac{(120)^2(60)}{(36)(24)} = 1000\,\text{W}$$

If $P_{M2} = 400\,\text{W}$

$$R_2 = \frac{(120)^2}{400} = 36\,\Omega$$

$$R_1 + R_2 = 60\,\Omega \quad \text{(as before)}$$

$$R_1 = 24\,\Omega$$

$$\text{CHECK: } P_{\text{H}} = 1000\,\text{W}$$

P 10.69 $$R_1 + R_2 + R_3 = \frac{(120)^2}{600} = 24\,\Omega$$

$$R_2 + R_3 = \frac{(120)^2}{900} = 16\,\Omega$$

$$\therefore \quad R_1 = 24 - 16 = 8\,\Omega$$

$$R_3 + R_1 \| R_2 = \frac{(120)^2}{1200} = 12\,\Omega$$

$$\therefore \quad 16 - R_2 + \frac{8R_2}{8 + R_2} = 12$$

$$R_2 - \frac{8R_2}{8 + R_2} = 4$$

$$8R_2 + R_2^2 - 8R_2 = 32 + 4R_2$$

$$R_2^2 - 4R_2 - 32 = 0$$

$$R_2 = 2 \pm \sqrt{4 + 32} = 2 \pm 6$$

$$\therefore \quad R_2 = 8\,\Omega; \qquad \therefore \quad R_3 = 8\,\Omega$$

P 10.70 $R_2 = \frac{(220)^2}{500} = 96.8\,\Omega$

$$R_1 + R_2 = \frac{(220)^2}{250} = 193.6\,\Omega$$

$$\therefore \quad R_1 = 96.8\,\Omega$$

CHECK: $R_1 \| R_2 = 48.4\,\Omega$

$$P_{\mathrm{H}} = \frac{(220)^2}{48.4} = 1000\,\mathrm{W}$$

P 10.71 Choose $R_1 = 22\,\Omega$ and $R_2 = 33\,\Omega$:

$$P_L = \frac{120^2}{22+33} = 262\,\mathrm{W} \qquad \text{(instead of 240 W)}$$

$$P_M = \frac{120^2}{33} = 436\,\mathrm{W} \qquad \text{(instead of 400 W)}$$

$$P_H = \frac{120^2(55)}{(22)(33)} = 1091\,\mathrm{W} \qquad \text{(instead of 1000 W)}$$

P 10.72 Choose $R_1 = R_2 = 100\,\Omega$:

$$P_L = \frac{220^2}{100+100} = 242\,\mathrm{W} \qquad \text{(instead of 250 W)}$$

$$P_M = \frac{220^2}{100} = 484\,\mathrm{W} \qquad \text{(instead of 500 W)}$$

$$P_H = \frac{220^2(200)}{(100)(100)} = 968\,\mathrm{W} \qquad \text{(instead of 1000 W)}$$