CALCULUS 1 (MA001IU) – FINAL EXAMINATION

Semester 3, 2021-22 • Duration: 120 minutes • Date: August 23, 2022

SUBJECT: CALCULUS 1	
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INSTRUCTIONS:

- Use of calculator is allowed. Each student is allowed two double-sided sheets of notes (size A4 or similar). All other documents and electronic devices are forbidden.
- Write the steps you use to arrive at the answers to each question. No marks will be given for the answer alone.
- There are a total of 10 (ten) questions. Each one carries 10 points.

1. Air is being pumped into balloon A with the rate of 2 cm^3/s. Concurrently, balloon B is being inflated in such a way that its radius is always 2 cm bigger than that of balloon A. What is the rate of change of the volume of balloon B at the moment its radius is 4 cm?
2. Find the absolute maximum and minimum values of $f(x) = (x^2+2x-1)e^{-2x}$ on $[-4,2]$.
3. Find the following limit if it exists, or show that the limit does not exist
$$\lim_{x\to 0^+} x^{\sqrt{x}}.$$
4. Using Newton's method starting with $x_1 = 0$, find the root of $x^4 - x^2 = 1$ correct to six decimal places.
5. Let f be a differentiable function with $f(0) = 1$, $f(1) = 2$, $f(2) = 3$. By considering $g(x) = f(x+1) - f(x)$, show that there exits $c \in (0,1)$ so that $f'(c+1) = f'(c)$.
6. Find the derivative of the function
$$H(x) = \int_1^{2x+1} \frac{t}{t^4+1}\,dt.$$
7. Evaluate $\displaystyle\int_0^1 (2x+1)e^{-x}\,dx$.
8. Determine whether the improper integral $\displaystyle\int_2^{\infty} \frac{2x^3+x}{x^4+x^2-2}\,dx$ is convergent or divergent. Explain.
9. The table below presents the dependence of the temperature T of a liquid on time t (in minutes). Use the Trapezoidal Rule to approximate the average temperature of this liquid during $0 \le t \le 5$.

Time t (in minutes)	0	0.5	1	1.5	2	2.5	3	3.5	4	4.5	5
Temperature T (in ^{o}C)	95	94.3	93.5	92.8	92.1	91.3	90.6	89.9	89.2	88.5	87.9

10. Find the arc length of the curve $y = 2(x-1)^{\frac{3}{2}}$ between $x = 1$ and $x = 3$.

—END OF THE QUESTION PAPER. GOOD LUCK!—