• PROGRAM OF "PHYSICS"

Lecturer: Dr. DO Xuan Hoi
Room A1 503
E-mail : dxhoi@hcmiu.edu.vn

PHYSICS 4
(Wave, Light, and Atoms)

02 credits (30 periods)

PHYSICS 4

Chapter 5
Relativity and Nuclear Physics

SPECIAL THEORY OF RELATIVITY

- Einstein's Postulates
- Relativity of Time Intervals and of Length
- Relativistic Dynamics
- General theory of relativity

NUCLEAR PHYSICS

- Properties of Nuclei
- Nuclear Binding and Nuclear Structure
- Nuclear Reactions
- Radioactivity
- Fundamental Particles - Quarks

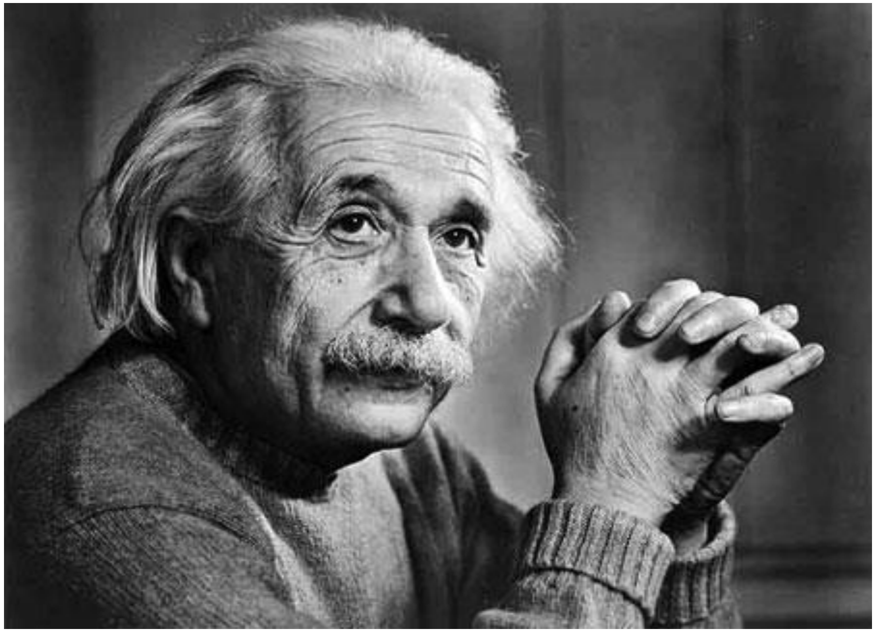
A. SPECIAL THEORY OF RELATIVITY

1 Einstein's Postulates

1.1 Einstein's First Postulate (the principle of relativity)

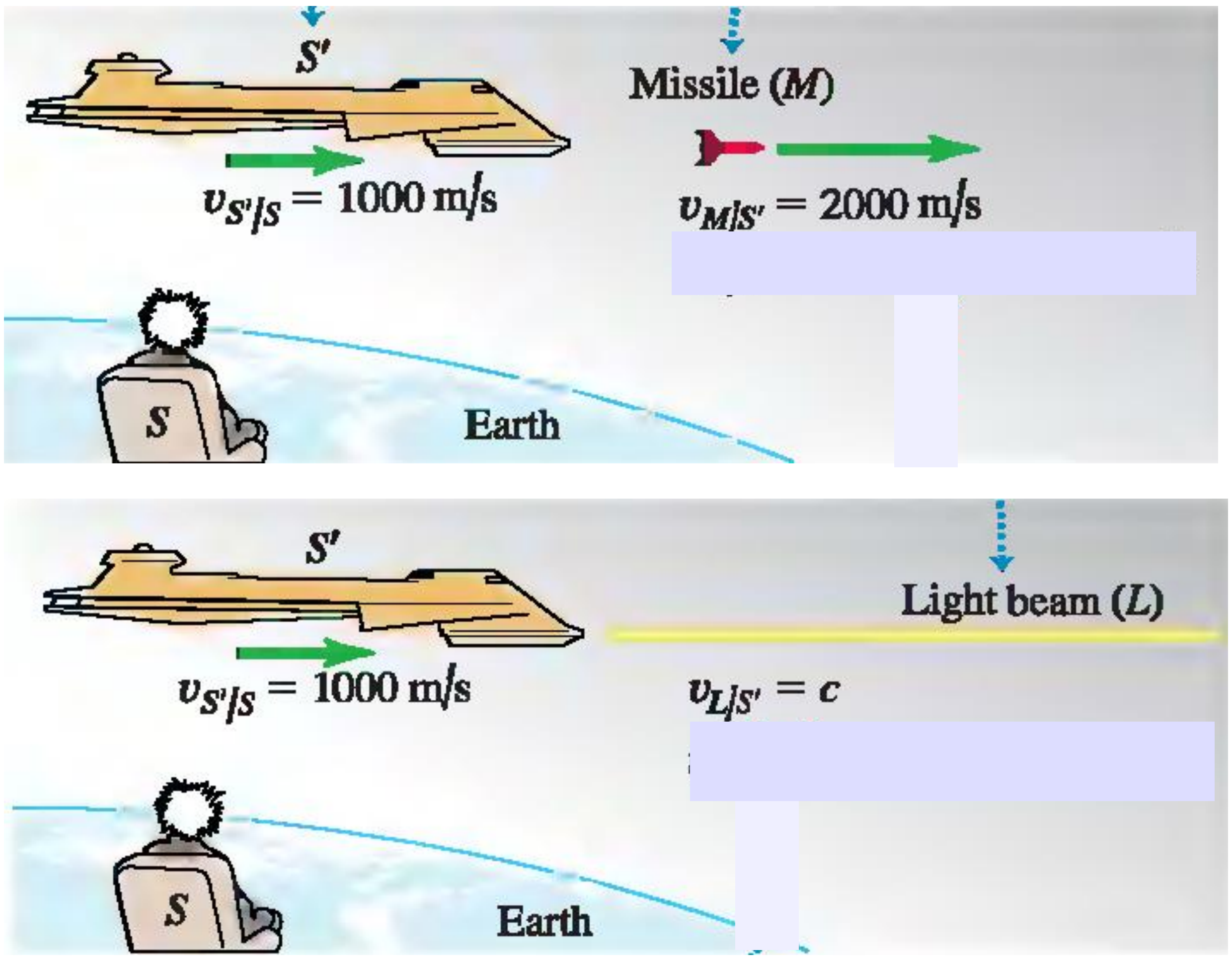
"The laws of physics are the same in every inertial frame of reference"

(inertial frame of reference : no acceleration)



1.2 Einstein's Second Postulate

"The speed of light in vacuum is **the same** in all inertial frames of reference and **is independent** of the motion of the source. "



2 Relativity of Time Intervals

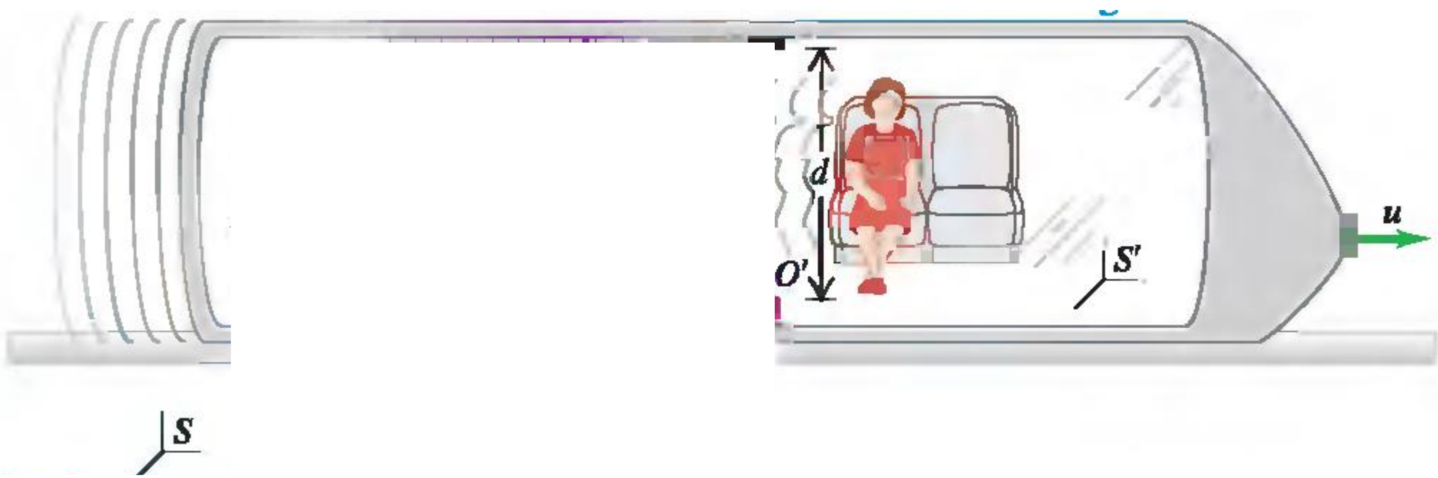
A frame of reference **S'** moves along the common x-x' -axis with constant speed ***u*** relative to a frame **S**

For S' : Event 1 is when a flash of light from a light source leaves 0'.

Event 2 is when the flash returns to 0'

For S' : The time interval :

$$\Delta t_0 = \frac{2d}{c}$$



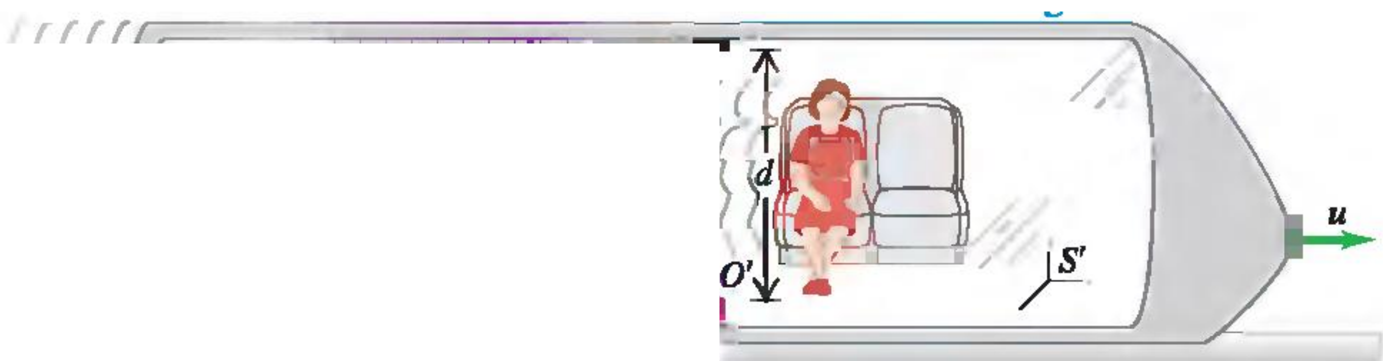
For S' : The time interval : $\Delta t_0 = \dfrac{2d}{c}$

For S : the round-trip distance is the longer distance 2l

$$l = \sqrt{d^2 + \left(\frac{u\Delta t}{2}\right)^2}$$

For S : The time interval

$$\Delta t = \frac{2l}{c} = \frac{2}{c}\sqrt{d^2 + \left(\frac{u\Delta t}{2}\right)^2} = \frac{2}{c}\sqrt{\left(\frac{c\Delta t_0}{2}\right)^2 + \left(\frac{u\Delta t}{2}\right)^2}$$

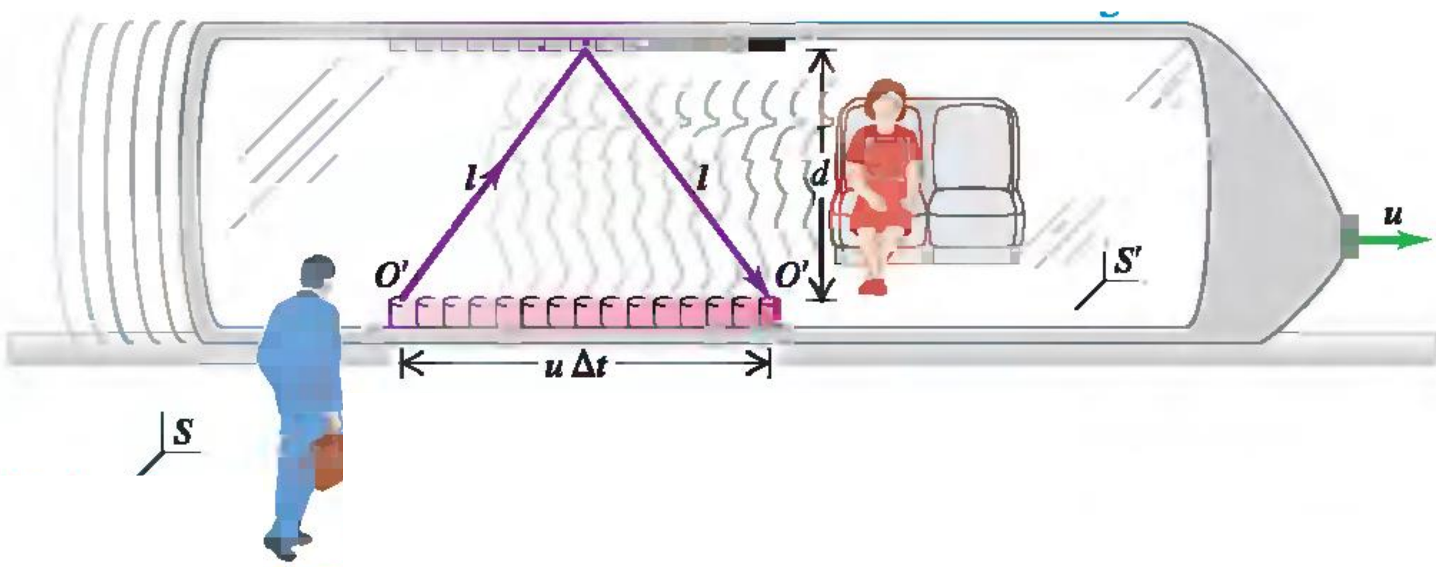


For S' : The time interval : Δt_0

For S : The time interval : Δt

$$\Delta t = \frac{2l}{c} = \frac{2}{c}\sqrt{d^2 + \left(\frac{u\Delta t}{2}\right)^2} = \frac{2}{c}\sqrt{\left(\frac{c\Delta t_0}{2}\right)^2 + \left(\frac{u\Delta t}{2}\right)^2}$$

$$\frac{c^2}{4}(\Delta t)^2 = \left(\frac{c\Delta t_0}{2}\right)^2 + \left(\frac{u\Delta t}{2}\right)^2 \longrightarrow \Delta t = \frac{\Delta t_0}{\sqrt{1 - u^2/c^2}}$$



For S' : The time interval : Δt_0

For S : The time interval : Δt

$$\Delta t = \frac{\Delta t_0}{\sqrt{1-u^2/c^2}}$$

In a particular frame of reference, suppose that two events occur at the same point in space. The time interval between these events, as measured by an observer at rest in this same frame is Δt_0 . Then an observer in a second frame moving with constant speed *u* relative to the rest frame will measure the time interval to be Δt , where

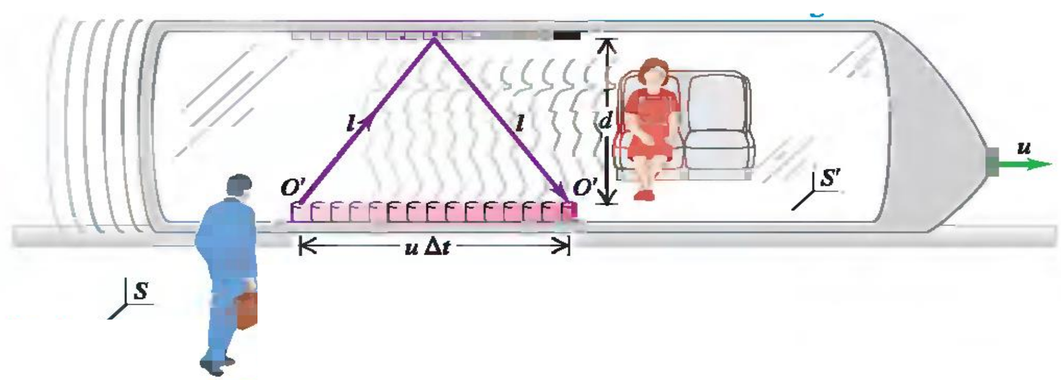
$$\Delta t = \frac{\Delta t_0}{\sqrt{1-u^2/c^2}}$$

$$\Delta t = \gamma \Delta t_0 \ ;$$

$$\gamma = \frac{1}{\sqrt{1-u^2/c^2}}$$

$\Delta t > \Delta t_0$: **time dilation**

Observers measure any clock to run slow if it moves relative to them



PROBLEM 1

A muon decays with a mean lifetime of 2.20×10^{-6} s as measured in a frame of reference in which it is at rest. If a muon is moving at 0.990c (about 2.97×10^{8} m/s) relative to the earth, what will you (an observer on earth) measure its mean lifetime to be?

SOLUTION

$$\Delta t = \frac{\Delta t_0}{\sqrt{1 - u^2/c^2}} = \frac{2.20 \times 10^{-6}\ \mathrm{s}}{\sqrt{1 - (0.990)^2}} = 15.6 \times 10^{-6}\ \mathrm{s}$$

The mean lifetime of the muon in the earth frame (Δt) is about seven times longer than in the muon's frame (Δt_0).

PROBLEM 2

An airplane flies from San Francisco to New York (about 4800 km) at a steady speed of 300 m/s.
How much time does the trip take, as measured by an observer on the ground ? By an observer in the plane ?

SOLUTION

The time interval measured by ground observers **:**

$$\Delta t = \frac{4.80 \times 10^6 \text{ m}}{300 \text{ m/s}} = 1.60 \times 10^4 \text{ s}$$

The time interval in the airplane (proper time) :

$$\frac{u^2}{c^2} = \frac{(300 \text{ m/s})^2}{(3.00 \times 10^8 \text{ m/s})^2} = 1.00 \times 10^{-12}$$

$$\Delta t_0 = (1.60 \times 10^4 \text{ s})\sqrt{1 - 1.00 \times 10^{-12}}$$

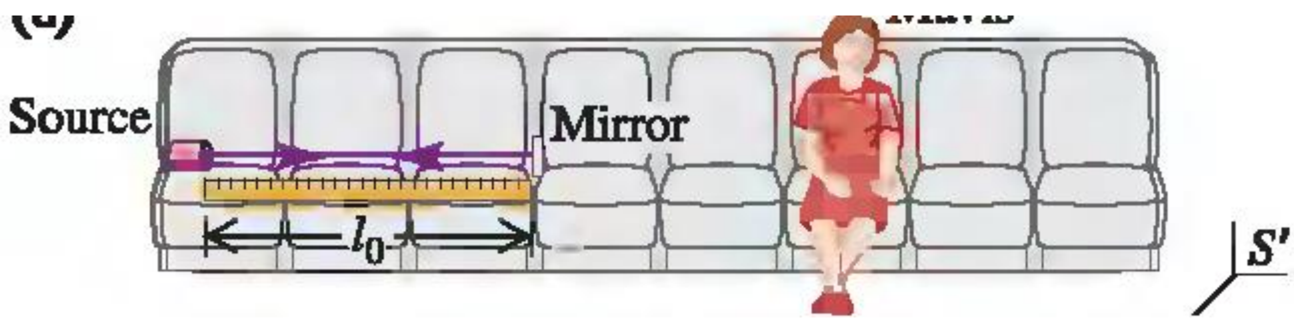
$$\Delta t_0 = (1.60 \times 10^4 \text{ s})(1 - 0.50 \times 10^{-12})$$

3 Relativity of Length

A frame of reference **S'** moves along the common x-x' -axis with constant speed ***u*** relative to a frame **S**

For S' : A light source to one end of a ruler and a mirror to the other end. The ruler is at rest in reference frame S', and its length in this frame is L_0 . Then the time required for a light pulse to make the round trip from source to mirror and back is

$$\Delta t_0 = \frac{2L_0}{c}$$



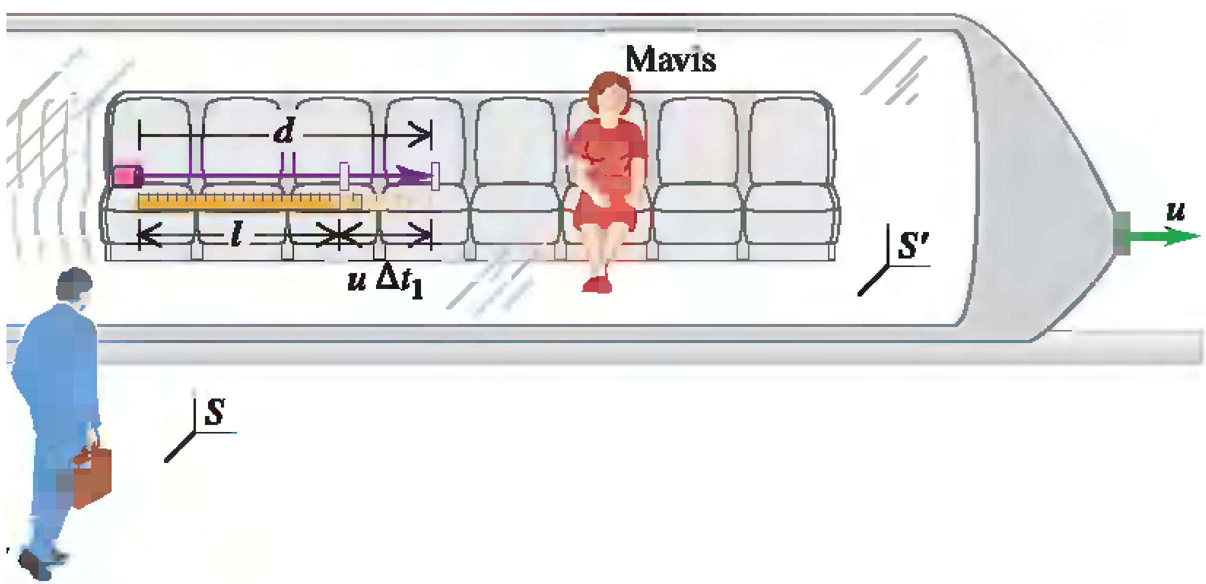
For S : the ruler is moving to the right with speed *u* during this travel of the light pulse

The length of the ruler in S is L, and the time of travel **from source to mirror**, as measured in S, is Δt_1.

The total length of path *d* from source to mirror is $d = L + u\Delta t_1$

The light pulse travels with speed *c*, so : $d = c\Delta t_1$

$$c\Delta t_1 = L + u\Delta t_1 \longrightarrow \Delta t_1 = \frac{L}{c-u}$$

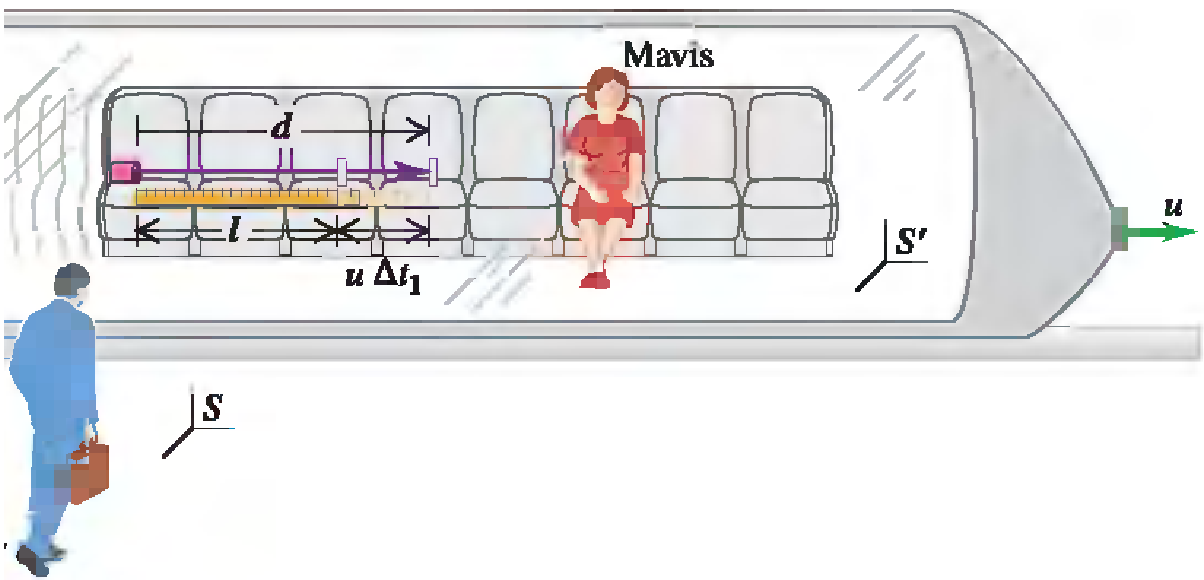


The length of the ruler in S is l, and the time of travel **from source to mirror**, as measured in S, is Δt_1 : $\Delta t_1 = \dfrac{L}{c-u}$

In the same way we can show that the time Δt_2 for the return trip **from mirror to source** : $\Delta t_2 = \dfrac{L}{c+u}$

The **total time** for the round trip, as measured in S :

$$\Delta t = \Delta t_1 + \Delta t_2 = \frac{L}{c-u} + \frac{L}{c+u} = \frac{2L}{c(1-u^2/c^2)}$$



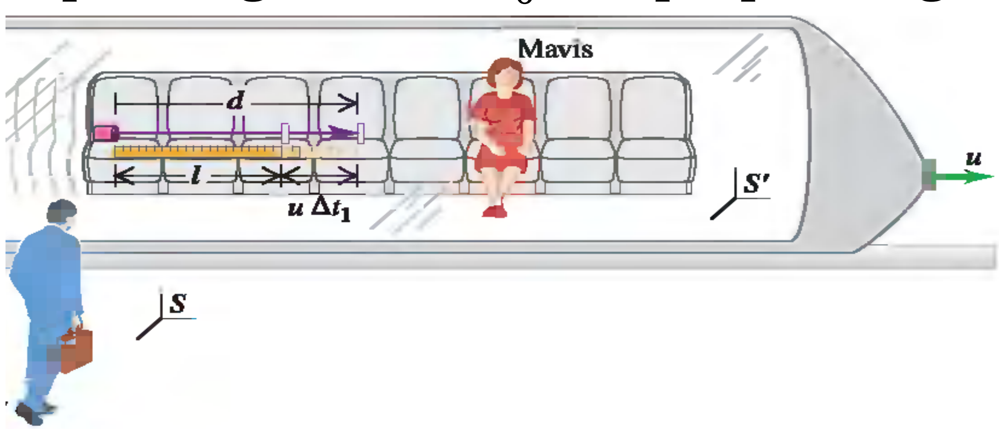
Because : $\Delta t = \dfrac{\Delta t_0}{\sqrt{1-u^2/c^2}} \rightarrow \dfrac{2L}{c(1-u^2/c^2)} = \dfrac{2L_0}{c}\dfrac{1}{\sqrt{1-u^2/c^2}}$

$$\rightarrow \boxed{L = L_0\sqrt{1-\frac{v^2}{c^2}}} \quad \boxed{L = \frac{L_0}{\gamma};} \quad \boxed{\gamma = \frac{1}{\sqrt{1-u^2/c^2}}}$$

$(L < L_0)$ **(Length contraction)**

The length L measured in S, in which the ruler is moving, is shorter than the length L_0 measured in its rest frame S'.

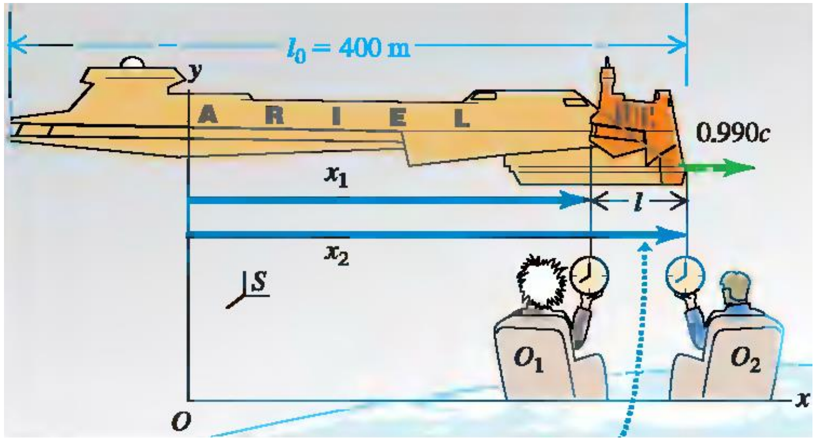
(A length measured in the frame in which the body is at rest is called a proper length; thus L_0 is a proper length in S')



PROBLEM 2

A spaceship flies past earth at a speed of 0.990c. A crew member on board the spaceship measures its length, obtaining the value 400 m. What length do observers measure on earth?

SOLUTION



$$l = l_0\sqrt{1 - \frac{u^2}{c^2}}$$

$$= (400\text{ m})\sqrt{1 - (0.990)^2}$$

$$= 56.4\text{ m}$$

4. Relativistic Dynamics

4.1 Relativistic Momentum and Relativistic Mass

Classical momentum : $p = mu = \dfrac{m\Delta x}{\Delta t}$

Relativistic Momentum : $p = \dfrac{m\Delta x}{\Delta t_0}$

(Δt_0 : time required to travel the distance Δx measured by an observer **moving with the particle**)

$$p = \frac{m\Delta x}{\Delta t_0} = \frac{m\Delta x}{\Delta t}\frac{\Delta t}{\Delta t_0}\,;\ \Delta t = \frac{\Delta t_0}{\sqrt{1-u^2/c^2}} \longrightarrow p = mu\frac{1}{\sqrt{1-u^2/c^2}}$$

With m : **rest mass** $\longrightarrow$ **Relativistic mass m_{rel} :**

$$\boxed{m_{rel} = \gamma m\,;} \quad \boxed{\gamma = \frac{1}{\sqrt{1-u^2/c^2}}}$$

Relativistic Momentum : $\boxed{\vec{p} = \gamma m\vec{u} = m_{rel}\vec{u}}$

4.2 Relativistic Energy

- **The relativistic generalization of Newton's second law**

$$F = \frac{dp}{dt} = \frac{d}{dt}\frac{mu}{\sqrt{1-u^2/c^2}} = \frac{m}{\left(1-u^2/c^2\right)^{3/2}}a$$

$$\boxed{\gamma = \frac{1}{\sqrt{1-u^2/c^2}}} \longrightarrow \boxed{F = \gamma^3 ma}$$

- **Relativistic Kinetic Energy**

$$K = W = \int_0^x F\,dx = \int_0^x m\gamma^3 a\,dx\;;\; a\,dx = \frac{du}{dt}dx = u\,du\;;$$

$$K = W = \int_0^v \frac{mu\,du}{\left(1-u^2/c^2\right)^{3/2}} = \frac{mc^2}{\sqrt{1-u^2/c^2}} - mc^2 = (\gamma - 1)mc^2$$

Rest energy : $\boxed{E_0 = mc^2}$

Total energy : $\boxed{E = E_0 + K = m_{rel}c^2}$

- **Total Energy and Relativistic Momentum**

$$\boxed{\gamma = \frac{1}{\sqrt{1-u^2/c^2}}} \qquad \boxed{\vec{p} = \gamma m\vec{u} = m_{rel}\vec{u}}$$

$$\left(\frac{p}{mc}\right)^2 = \frac{u^2/c^2}{\sqrt{1-u^2/c^2}} \quad (1)$$

$$\boxed{E_0 = mc^2} \qquad \boxed{E = E_0 + K = m\gamma c^2 = m_{rel}c^2}$$

$$\left(\frac{E}{mc^2}\right)^2 = \frac{1}{\sqrt{1-u^2/c^2}} \quad (2)$$

(1) and (2) $\rightarrow$ $\boxed{E^2 = (mc^2)^2 + (pc)^2}$

PROBLEM 3

A proton (rest mass 1.67×10^{-27} kg) has total energy that is 4.00 times its rest energy. What are (a) the kinetic energy of the proton ; (b) the magnitude of the momentum of the proton ; (c) the speed of the proton?

SOLUTION

(a) $E = mc^2 + K$, so $E = 4.00mc^2$ means $K = 3.00mc^2 = 4.50\times10^{-10}$ J

(b) $E^2 = (mc^2)^2 + (pc)^2$; $\quad E = 4.00mc^2$, so $15.0(mc^2)^2 = (pc)^2$

$p = \sqrt{15}mc = 1.94\times10^{-18}$ kg $\cdot$ m/s

(c) $E = mc^2 / \sqrt{1 - v^2/c^2}$

$E = 4.00mc^2$ gives $1 - v^2/c^2 = 1/16$ and $v = \sqrt{15/16}c = 0.968c$

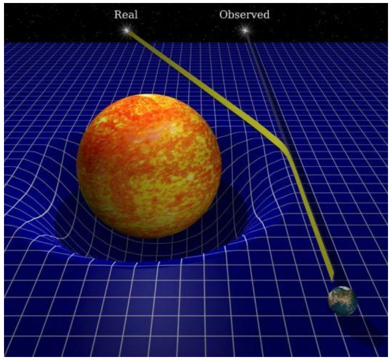
5. General theory of relativity

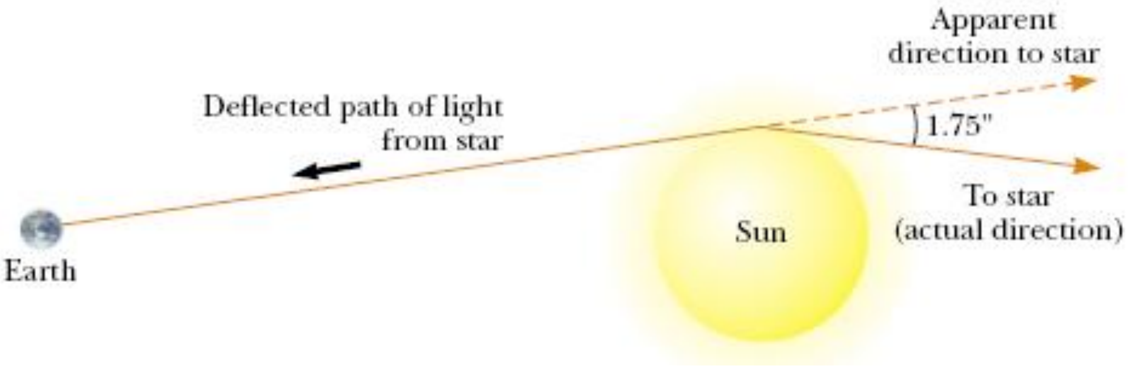
$$\left.\begin{array}{ll}\text{Gravitational property:} & W = m_g g \\ \text{Inertial property:} & F = m_i g\end{array}\right\} \quad m_i \equiv m_g \quad ?$$

Postulates of Einstein's general theory of relativity:

- All the laws of nature have the same form for observers in any frame of reference, whether accelerated or not.
- *Principle of equivalence:* In the vicinity of any point, a gravitational field is equivalent to an accelerated frame of reference in the absence of gravitational effects.

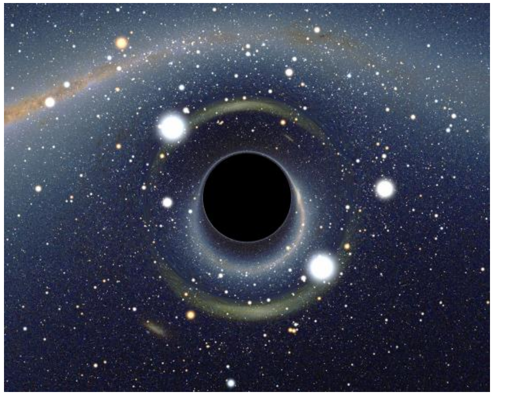
→ **The *curvature of space–time***: the presence of a mass causes a **curvature** of space–time in the vicinity of the mass, and this curvature dictates the space–time path that all freely moving objects must follow





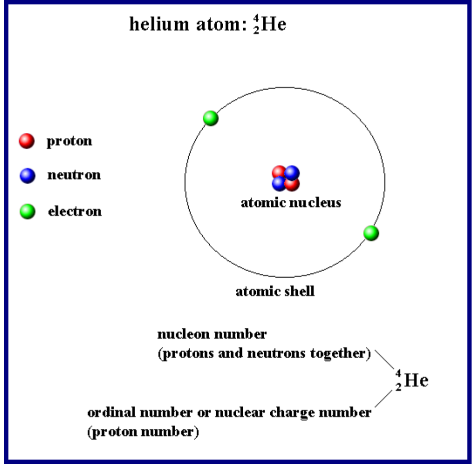
If the concentration of mass becomes very great (when a large star exhausts its nuclear fuel and collapses to a very small volume): **a black hole**.

The curvature of space–time is so extreme that, within a certain distance from the center of the black hole, all matter and light become trapped.



B. NUCLEAR PHYSICS

1 Properties of Nuclei $^{A}_{Z}X$



- Every atom contains at its center an dense, positively **charged nucleus**.
- **Model :** a nucleus is a **sphere** with a radius R that depends on the total number of **nucleons** (neutrons and protons): **Nucleon number A (mass number).**
- The proton mass and the neutron mass are both approximately $1u = 1.66053886 \times 10^{-27}$ kg = 931.5 MeV/c^2
- The number of protons in a nucleus is the **atomic number** Z. The number of neutrons is the neutron number N.

$$\boxed{A = Z + N}$$

- The radii of most nuclei : $\boxed{R = R_0 A^{1/3}}$

$$(R_0 = 1.2 \times 10^{-15}\,\text{m} = 1.2\ \text{fm})$$

- **A single nuclear species having specific values of both Z and N is called a nuclide.**
- **Some nuclides that have the same Z but different N: isotopes of that element**

EXAMPLE :

- **Chlorine (Cl, Z = 17). About 76% of chlorine nuclei have N = 18; the other 24% have N = 20.**
- **Two common isotopes of uranium with A = 235 and 238**

Nucleus	Mass Number (Total Number of Nucleons), A	Atomic Number (Number of Protons), Z	Neutron Number, $N = A - Z$
${}^{1}_{1}\mathrm{H}$	1	1	0
${}^{2}_{1}\mathrm{D}$	2	1	1
${}^{4}_{2}\mathrm{He}$	4	2	2
${}^{6}_{3}\mathrm{Li}$	6	3	3
${}^{7}_{3}\mathrm{Li}$	7	3	4
${}^{9}_{4}\mathrm{Be}$	9	4	5
${}^{10}_{5}\mathrm{B}$	10	5	5
${}^{11}_{5}\mathrm{B}$	11	5	6
${}^{12}_{6}\mathrm{C}$	12	6	6
${}^{13}_{6}\mathrm{C}$	13	6	7
${}^{14}_{7}\mathrm{N}$	14	7	7
${}^{16}_{8}\mathrm{O}$	16	8	8
${}^{23}_{11}\mathrm{Na}$	23	11	12
${}^{65}_{29}\mathrm{Cu}$	65	29	36
${}^{200}_{80}\mathrm{Hg}$	200	80	120
${}^{235}_{92}\mathrm{U}$	235	92	143
${}^{238}_{92}\mathrm{U}$	238	92	146

2 Nuclear Binding and Nuclear Structure

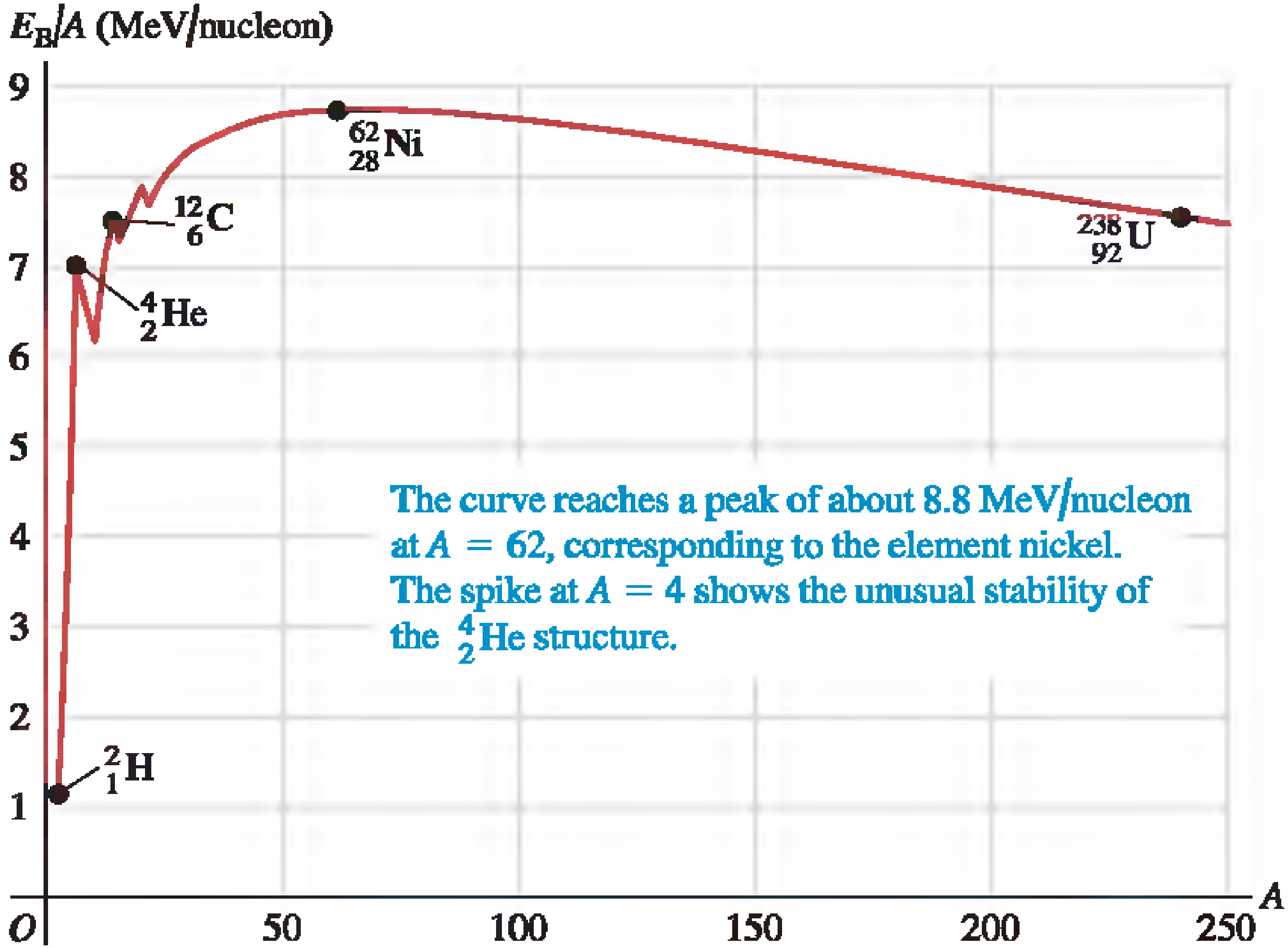
- **The binding energy E_B : the magnitude of the energy by which the nucleons are bound together.**

- **Total rest energy E_0 of the separated nucleons is greater than the rest energy of the nucleus.**

- **The rest energy of the nucleus :** $E_0 - E_B$

- **The binding energy for a nucleus containing Z protons and N neutrons :**

$$\boxed{E_B = (Zm_p + Nm_n - {}^{A}_{Z}M)c^2}$$


E_B/A (MeV/nucleon)
9
8
7
6
5
4
3
2
1
O
50
100
150
200
250
A
${}^{62}_{28}Ni$
${}^{12}_{6}C$
${}^{4}_{2}He$
${}^{2}_{1}H$
${}^{238}_{92}U$
The curve reaches a peak of about 8.8 MeV/nucleon at $A = 62$, corresponding to the element nickel.
The spike at $A = 4$ shows the unusual stability of the ${}^{4}_{2}He$ structure.

PROBLEM 4

Because it has the highest binding energy per nucleon of all nuclides, Ni (Z=28;A=62) may be described as the most strongly bound. Its neutral atomic mass is 61.928349 u. Find its mass defect. its total binding energy, and its binding energy per nucleon.

SOLUTION

We use $Z = 28$, $M_{\mathrm{H}} = 1.007825\ \mathrm{u}$, $N = A - Z = 62 - 28 = 34$, $m_{\mathrm{n}} = 1.008665\ \mathrm{u}$, and ${}^{A}_{Z}M = 61.928349\ \mathrm{u}$

$$\boxed{E_B = (Zm_p + Nm_n - {}^{A}_{Z}M)c^2}$$

$$E_B = (0.585361\ \mathrm{u})(931.5\ \mathrm{MeV/u}) = 545.3\ \mathrm{MeV}$$

It would require a minimum of 545.3 MeV to pull a ${}^{62}_{28}\mathrm{Ni}$ nucleus completely apart into 62 separate nucleons. The binding energy *per nucleon* is $\frac{1}{62}$ of this, or 8.795 MeV per nucleon.

3 Nuclear Reactions

• **Nuclear reactions : Rearrangements of nuclear components that result from a bombardment by a particle**

$$A + B \rightarrow C + D$$

• **Several conservation laws : The classical conservation principles for charge, momentum, angular momentum, and energy (including rest energies)**

EXAMPLE : $${}^{4}_{2}He + {}^{14}_{7}Na \rightarrow {}^{17}_{8}O + {}^{1}_{1}H$$

Conservation of charge : 2 + 7 = 8 + 1

Conservation of nucleon number A : 4 +14 = 17 + 1

• **Reaction energy :**

$$\boxed{Q = (\underbrace{M_A + M_B - M_C - M_C}_{\Delta m\,:\,\text{mass defect}})c^2}$$

PROBLEM 5

When lithium $^{7}_{3}Li$ is bombarded by a proton, two alpha particles ($^{4}_{2}He$) are produced. Find the reaction energy.

SOLUTION

$$^{1}_{1}H + ^{7}_{3}Li \rightarrow ^{4}_{2}He + ^{4}_{2}He$$

$$M_A + M_B - M_C - M_D = 0.018623\ \text{u}$$

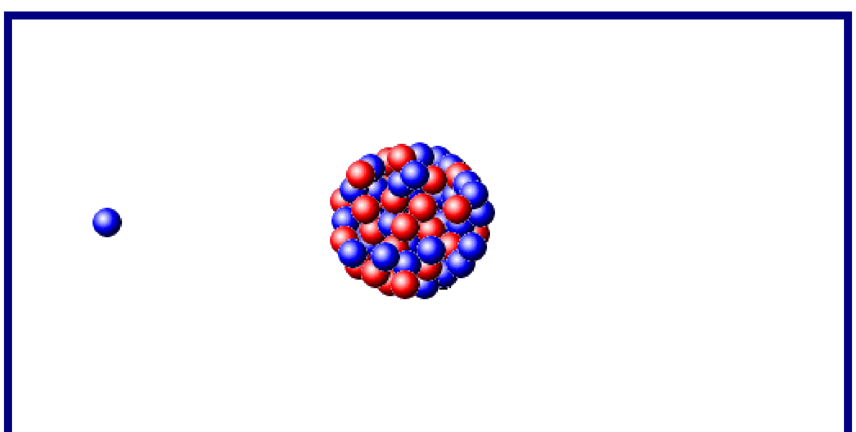
$$Q = (0.018623\ \text{u})(931.5\ \text{MeV/u}) = 17.35\ \text{MeV}$$

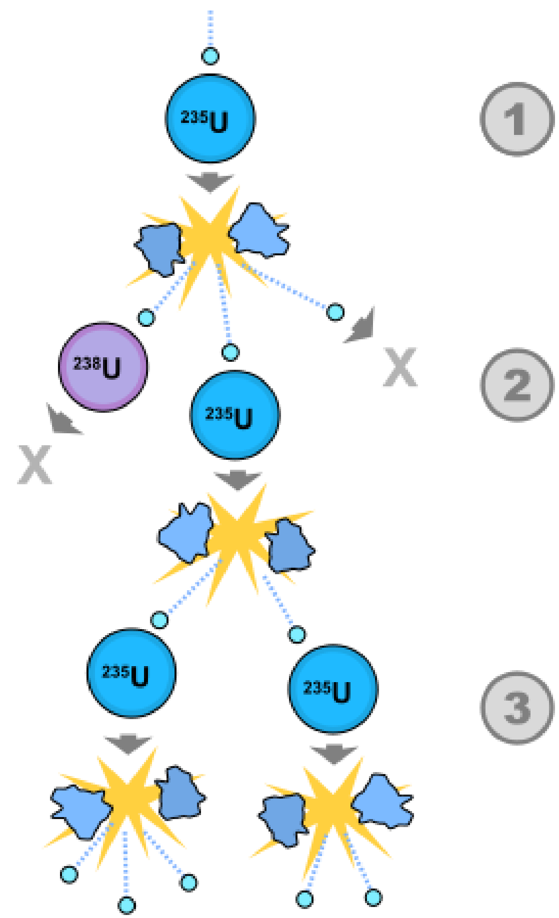
3.1 Fission Reactions

• **Nuclear fission is a decay process in which an unstable nucleus splits into two fragments of comparable mass.**

EXAMPLE : $${}^{235}_{92}\mathrm{U} + {}^{1}_{0}\mathrm{n} \rightarrow {}^{235}_{92}\mathrm{U} \rightarrow {}^{144}_{56}\mathrm{Ba} + {}^{89}_{36}\mathrm{Kr} + 3\,{}^{1}_{0}\mathrm{n}$$

• **Chain Reactions : Fission of a uranium nucleus, triggered by neutron bombardment, releases other neutrons that can trigger more fissions**





PROBLEM 6

Calculate the energy released in the fission reaction

$$^{235}_{92}\mathrm{U} + {}^{1}_{0}\mathrm{n} \rightarrow {}^{140}_{54}\mathrm{Xe} + {}^{94}_{38}\mathrm{Sr} + 2\,{}^{1}_{0}\mathrm{n}$$

You can ignore the initial kinetic energy of the absorbed neutron. The atomic masses are $^{235}_{92}\mathrm{U}$, 235.043923 u; $^{140}_{54}\mathrm{Xe}$, 139.921636 u; and $^{94}_{38}\mathrm{Sr}$, 93.915360 u.

SOLUTION

$$\Delta m = M\left({}^{235}_{92}\mathrm{U}\right) - M\left({}^{140}_{54}\mathrm{Xe}\right) - M\left({}^{94}_{38}\mathrm{Sr}\right) - m_{\mathrm{n}}$$

$$\Delta m = 235.043923\ \mathrm{u} - 139.921636\ \mathrm{u} - 93.915360\ \mathrm{u} - 1.008665\ \mathrm{u} = 0.1983\ \mathrm{u}$$

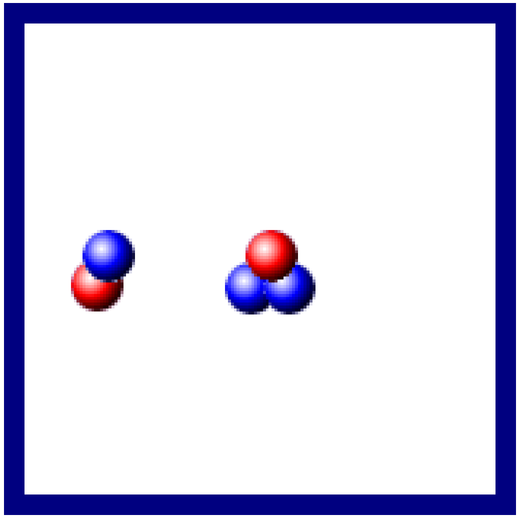
$$E = (\Delta m)c^2 = (0.1983\ \mathrm{u})(931.5\ \mathrm{MeV/u}) = 185\ \mathrm{MeV}.$$

3.2 Nuclear Fusion

- **In a nuclear fusion reaction, two or more small light nuclei come together, or fuse. to form a larger nucleus.**
- **Fusion reactions release energy for the same reason as fission reactions: The binding energy per nucleon after the reaction is greater than before.**

EXAMPLE : $^{2}_{1}H + ^{1}_{1}H \rightarrow ^{3}_{2}He + \gamma$

- **Atoms have this much energy only at extremely high temperatures : thermonuclear reactions.**



PROBLEM 7

Two deuterons fuse to form a triton (a nucleus of tritium, or $^{3}_{1}\mathrm{H}$) and a proton. How much energy is liberated?

SOLUTION

$$^{2}_{1}\mathrm{H} + ^{2}_{1}\mathrm{H} \rightarrow ^{3}_{2}\mathrm{He} + \mathrm{p}$$

$$\begin{aligned} Q &= [2(2.014102\ \mathrm{u}) - 3.016049\ \mathrm{u} - 1.007825\ \mathrm{u}] \\ &\quad \times (931.5\ \mathrm{MeV/u}) \\ &= 4.03\ \mathrm{MeV} \end{aligned}$$

4. Radioactivity

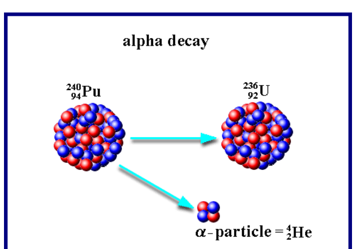
• Among about 2500 known nuclides, fewer than 300 are stable. The others are **unstable structures that decay to form other nuclides** by emitting particles and electromagnetic radiation, a process called r**adioactivity**.

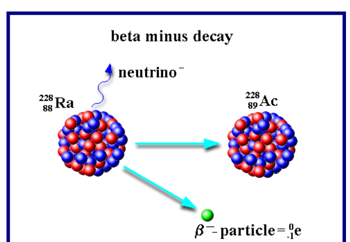
• The time scale of these decay processes ranges from a small fraction of a **microsecond to billions of years**.

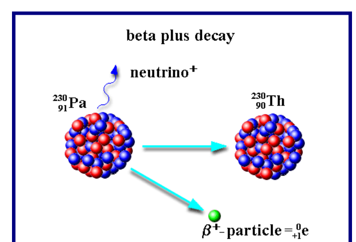
• When unstable nuclides decay into different nuclides, they usually emit **alpha (α)** or **beta (β)** particles:

Alpha particle is a **^{4}He nucleus**, a beta-minus particle (β^{-}) is an **electron**, beta-plus particle (β^{+}) is a **positron** (antiparticle of electron)

$$ {}_{Z}^{A}X \rightarrow {}_{2}^{4}He + {}_{Z-2}^{A-4}Y \; ; {}_{Z}^{A}X \rightarrow {}_{-1}^{0}e + {}_{Z+1}^{A}Y \; ; \; {}_{Z}^{A}X \rightarrow {}_{1}^{0}e + {}_{Z-1}^{A}Y $$







EXAMPLE :

What nuclide is produced in the following radioactive decays?

(a) α decay of $^{239}_{94}Pu$

(b) β^- decay of $^{24}_{11}Na$

(c) β^+ decay of $^{15}_{8}O$

SOLUTION

$$^{239}_{94}\mathrm{Pu} \longrightarrow {}^{4}_{2}\mathrm{He} + {}^{235}_{92}\mathrm{U}$$

$$^{24}_{11}\mathrm{Na} \longrightarrow {}^{0}_{-1}\mathrm{e} + {}^{24}_{12}\mathrm{Mg}$$

$$^{15}_{8}\mathrm{O} \longrightarrow {}^{0}_{+1}\mathrm{e} + {}^{15}_{7}\mathrm{N}$$

- **Radioactive Decay Rates**

- ***N*(*t*) : the (very large) number of radioactive nuclei in a sample at time *t*,**

dN*(*t*) : the (negative) change in that number during a short time interval *dt

H(t) =- *dN*(*t*)/*dt* : The decay rate or the activity of the specimen.

Activity H (t): becquerel (Bq) in SI or curie (Ci)

$$1\text{Ci} = 3.70\times10^{10}\,\text{Bq} = 3.70\times10^{10}\,\text{decays / s}$$

$$H(t) = -\frac{dN(t)}{dt} = \lambda N(t) \qquad \boxed{N(t) = N_0 e^{-\lambda t}}$$

λ : decay constant

- **The half-life $T_{1/2}$ is the time required for the number of radioactive nuclei to decrease to one-half the original number N_0**

$$\boxed{T_{1/2} = \frac{\ln 2}{\lambda} = \frac{0.693}{\lambda}}$$

- **The mean lifetime T_{mean} (generally called the lifetime):**

$$\boxed{T_{mean} = \frac{1}{\lambda} = \frac{T_{1/2}}{\ln 2}}$$

PROBLEM 8

The radioactive isotope ^{57}Co decays by electron capture with a half-life of 272 days. (a) Find the decay constant and the lifetime. (b) If you have a radiation source containing ^{57}Co, with activity 2.00 μCi, how many radioactive nuclei does it contain? (c) What will be the activity of your source after one year?

SOLUTION

$$T_{1/2} = (272\text{ days})(86{,}400\text{ s/day}) = 2.35 \times 10^{7}\text{ s.}$$

$$T_{\text{mean}} = \frac{T_{1/2}}{\ln 2} = \frac{2.35 \times 10^{7}\text{ s}}{0.693} = 3.39 \times 10^{7}\text{ s}$$

$$\lambda = \frac{1}{T_{\text{mean}}} = 2.95 \times 10^{-8}\text{ s}^{-1}$$

PROBLEM 8

The radioactive isotope ^{57}Co decays by electron capture with a half-life of 272 days. (a) Find the decay constant and the lifetime. (b) If you have a radiation source containing ^{57}Co, with activity 2.00 μCi, how many radioactive nuclei does it contain? (c) What will be the activity of your source after one year?

SOLUTION

$$-\frac{dN(t)}{dt} = 2.00\ \mu\text{Ci} = (2.00 \times 10^{-6})(3.70 \times 10^{10}\ \text{s}^{-1})$$

$$= 7.40 \times 10^{4}\ \text{decays/s}$$

$$N(t) = -\frac{dN(t)/dt}{\lambda} = \frac{7.40 \times 10^{4}\ \text{s}^{-1}}{2.95 \times 10^{-8}\ \text{s}^{-1}} = 2.51 \times 10^{12}\ \text{nuclei}$$

$$N(t) = N_0 e^{-\lambda t} = N_0 e^{-(2.95\times 10^{-8}\ \text{s}^{-1})(3.156\times 10^{7}\ \text{s})} = 0.394 N_0$$

$$(0.394)(2.00\ \mu\text{Ci}) = 0.788\ \mu\text{Ci}.$$

PROBLEM 9

The isotope ^{226}Ra undergoes a decay with a half-life of 1620 years.
What is the activity of 1.00 g of ^{226}Ra?
Express your answer in Bq and in Ci.

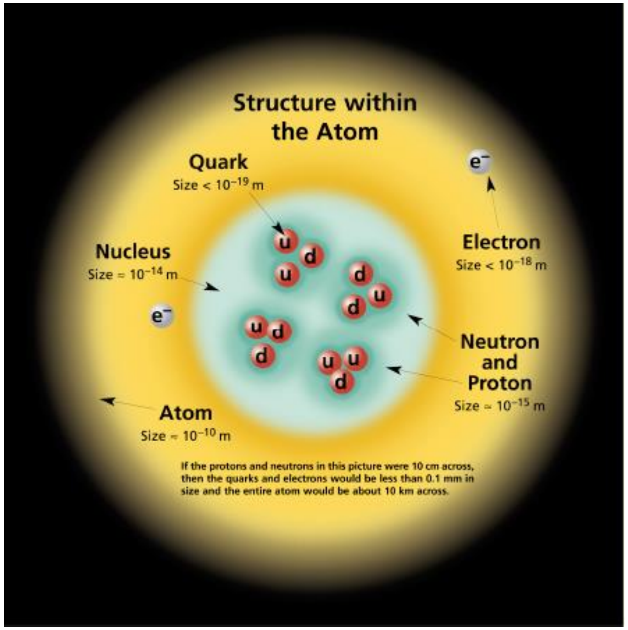
SOLUTION

$$\frac{dN}{dt} = \lambda N \; . \quad \lambda = \frac{\ln 2}{T_{1/2}} = \frac{\ln 2}{1620 \text{ yr}\left(3.15\times10^{7} \text{ s/yr}\right)} = 1.36\times10^{-11} \text{ s}^{-1} \, .$$

$$N = 1 \text{ g}\left(\frac{6.022\times10^{23} \text{atoms}}{226 \text{ g}}\right) = 2.665\times10^{25} \text{ atoms} \, .$$

$$\frac{dN}{dt} = \lambda N = (2.665\times10^{25})(1.36\times10^{-11}\text{s}^{-1}) = 3.62\times10^{10} \text{ decays/s} = 3.62\times10^{10}\,\text{Bq}$$

5. Fundamental Particles. Quarks



More than 30 long-lived particles and antiparticles have been detected experimentally.

(An antiparticle has the same mass and spin as its associated particle, but the electromagnetic properties, such as charge and magnetic moment, are opposite in a particle and its antiparticle.
EX: electron and positron)

Hundreds of resonances particles have been observed.
In contrast with the relatively stable particles, a resonance particle is extremely short-lived (< 10^{-21} s).

Table 3. Intrinsic Properties of Elementary Particles. Mass in MeV/c^2, charge in units of e, spin in units of $\hbar$.

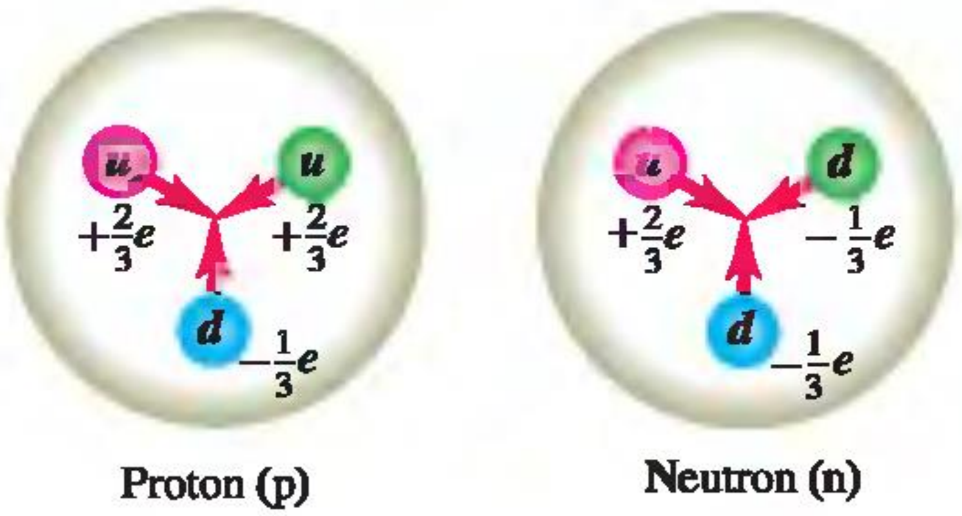
Family	Particle	Symbol	Mass	Charge	Spin
	photon	γ	0	0	1
LEPTONS	electron's neutrino	ν_e	0	0	1/2
	muon's neutrino	ν_μ	0	0	1/2
	tau's neutrino	ν_τ	0	0	1/2
	electron	e	0.511	−1	1/2
	muon	μ	105.66	−1	1/2
	tau	τ	1784.2	−1	1/2
HADRONS:					
mesons	pion	π^0	134.96	0	0
		π^+	139.57	+1	0
		π^-	139.57	−1	0
	Kaon	K^+	493.8	+1	0
		K^-	493.8	−1	0
		K^0	493.8	0	0
	eta	η	548.8	0	0
baryons	proton	p	938.26	+1	1/2
	neutron	n	939.55	0	1/2
	lambda	Λ^0	1115.6	0	1/2
	sigma	Σ^+	1189.4	+1	1/2
		Σ^0	1192.5	0	1/2
		Σ^-	1197.4	−1	1/2
	xi	Ξ^0	1315	0	1/2
		Ξ^-	1321.3	−1	1/2
	omega	Ω^-	1673	−1	3/2

truly elementary particle (not composed of smaller entities)

composed of more basic particles (quarks)

QUARKS MODEL (M. Gell-Mann and G. Zweig, 1963)

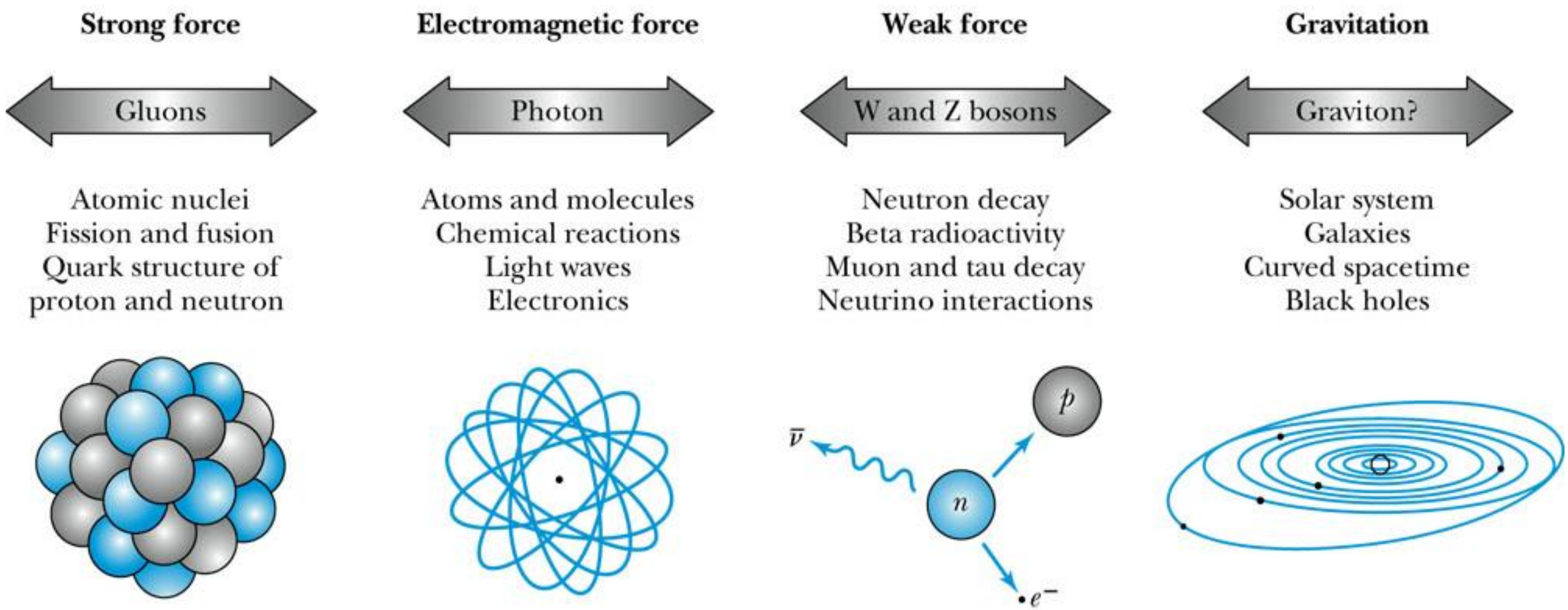
Hadrons are built from six quarks (u, d, s, c, b, t)
and their six antiquarks ($\bar{u}$, $\bar{d}$, $\bar{s}$, $\bar{c}$, $\bar{b}$, $\bar{t}$).



u : up,
d : down
s : strange
c : charm
b : bottom
t : top

proton quark content : (uud)
antiproton quark content : ($\bar{u}$, $\bar{u}$, $\bar{d}$)
neutron quark content : (udd)
antineutron quark content : ($\bar{u}$, $\bar{d}$, $\bar{d}$)

The Fundamental Interactions



The Standard Model

Three families of particles:

(1) the six leptons, which have no strong interactions;

(2) the six quarks, from which all hadrons are made;

(3) the particles that mediate the various interactions.

These mediators are :

Gluons for the strong interaction among quarks,

Photons for the electromagnetic interaction,

$W^{\pm}$ and Z^0 particles for the weak interaction

Graviton for the gravitational interaction.

The Standard Model

Three families of particles:

(1) the six leptons, which have no strong interactions;

(2) the six quarks, from which all hadrons are made;

(3) the particles that mediate the various interactions. These mediators are :

Gluons for the strong interaction among quarks, Photons for the electromagnetic interaction,

$W^{\pm}$ and Z^0 particles for the weak interaction

Graviton for the gravitational interaction.