THE INTERNATIONAL UNIVERSITY(IU) - VIETNAM NATIONAL UNIVERSITY HCMC

FINAL EXAMINATION

January, 2011

Duration: 120 minutes

SUBJECT: CALCULUS 3	
Head of Dept. Maths.	Lecturer:
Signature:	Signature:
Prof. DrSc. Phan Q. Khanh	Dr. N.N. Hai

INSTRUCTIONS: • Answer ALL questions in Section A and TWO questions in Section B.

- **Open-book examination. Computers and laptops prohibited.**
- **Exchanging documents strictly prohibited.**

PART A

Question A1 *[15 marks]* Let

$$f(t) = \begin{cases} \sin t, & 0 \le t < \frac{\pi}{4}, \\ \sin t + \cos(t - \pi/4), & t \ge \frac{\pi}{4}. \end{cases}$$

Find $\mathcal{L}\{f(t)\}$.

Question A2 *[15 marks]* Using $\mathcal{Z}$-transform methods, solve the difference equation

$$y_{k+2} - 5y_{k+1} + 6y_k = 5, \qquad y_0 = 0, \quad y_1 = 1.$$

Question A3 *[15 marks]* Obtain the Fourier series for the function defined by

$$f(t) = \begin{cases} 0, & -2 < t < -1 \\ 5, & -1 < t < 1, \\ 0, & 1 < t < 2. \end{cases}$$

Question A4 *[15 marks]* Find the Fourier transform of

$$f(x) = \begin{cases} 1 & 0 \le x \le a, \\ 0, & \text{otherwise}, \end{cases}$$

where a is a positive constant. Show that the Fourier transform vanishes as $\omega \to \pm\infty$.

PART B

Question B1 *[20 marks]* Solve the initial value problem

$$y'' + 4y = g(t), \qquad y(0) = 0, \quad y'(0) = 0,$$

where

$$g(t) = \begin{cases} 0, & 0 \leq t < 5, \\ \frac{1}{5}(t-5), & 5 \leq t < 10, \\ 1, & t \geq 0. \end{cases}$$

Question B2 *[20 marks]* (a) Solve the initial value problem

$$\frac{d^2y}{dt^2} + 3\frac{dy}{dt} + 2y = \delta(t-1) - \delta(t-2)$$

subject to $y = 0$ and $dy/dt = 0$ at $t = 0$.

(b) Find the inverse transform of

$$G(s) = \frac{1 - e^{-2s}}{s^2}.$$

Question B3 *[20 marks]* A periodic function $f(t)$, of period 2π, is defined within the period $-\pi < t < \pi$ by

$$f(t) = \begin{cases} 0, & -\pi < t < 0, \\ 1, & 0 \leq t < \pi. \end{cases}$$

(a) Find the Fourier series representation for $f(t)$.

(b) Use Parseval's theorem to show that

$$\sum_{k=1}^{\infty} \frac{1}{(2k-1)^2} = \frac{\pi^2}{8}.$$

END OF QUESTION PAPER