A, Derivatives: (Đạo hàm)

- $(\tan x)' = \sec^2 x = \frac{1}{\cos^2 x} = 1 + \tan^2 x$, $(\tan u)' = \frac{u'}{\cos^2 u}$
- $(\cot x)' = -\csc^2 x$, $(\cot u)' = -\frac{u'}{\sin^2 u}$
- $\left(\frac{1}{\cos x}\right)' = (\sec x)' = \sec x \tan x$
- $\left(\frac{1}{\sin x}\right)' = (\csc x)' = -\csc x \cot x$
- $\left(\frac{u}{v}\right)' = -\frac{v' u}{u^2}$... $\left(\cos^{-1}\frac{x}{a}\right)' = -\frac{1}{\sqrt{a^2-x^2}}$
- $\left(\sin^{-1}\frac{x}{a}\right)' = \frac{1}{\sqrt{a^2-x^2}}$ • $\left(\cot^{-1}\frac{x}{a}\right)' = \frac{-a}{a^2+x^2}$
- $\left(\tan^{-1}\frac{x}{a}\right)' = \frac{a}{a^2+x^2}$ • $(\csc^{-1}x)' = \frac{-1}{x\sqrt{x^2-1}}$
- $(\sec^{-1}x)' = \frac{1}{x\sqrt{x^2-1}}$ • $(e^u)' = u' e^u$
- $(a^x)' = a^x \ln a$ • $(a^u)' = a^u u' \ln a$
- $(\log_a x)' = \frac{1}{x \ln a}$ • $(\ln|u|)' = \frac{u'}{u}$ • $(\log_a |u|)' = \frac{u'}{u \ln a}$
- $(\sqrt[m]{u})' = \frac{u'}{m\sqrt[m]{u^{m-1}}}$ • $\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2}$
- $(\sin u)' = u' \cos u$, $(\cos u)' = u'(-\sin u)$

A, Antiderivatives:

- $\int 0\,dx = C$
- $\int dx = x + C$
- $\int x^\alpha dx = \frac{1}{\alpha+1} x^{\alpha+1} + C$ $(\alpha \neq -1)$
- $\int \frac{1}{x^2} dx = -\frac{1}{x} + C$
- $\int \frac{1}{x} dx = \ln|x| + C$
- $\int e^x dx = e^x + C$
- $\int a^x dx = \frac{a^x}{\ln a} + C$
- $\int \frac{1}{x^n} dx = -\frac{1}{(n-1)x^{n-1}} + C$
- $\int \cos x\,dx = \sin x + C$
- $\int \sin x\,dx = -\cos x + C$
- $\int \tan x\,dx = -\ln|\cos x| + C$
- $\int \cot x\,dx = \ln|\sin x| + C$
- $\int \frac{1}{\cos^2 x} dx = \tan x + C$
- $\int \frac{1}{\sin^2 x} dx = -\cot x + C$
- $\int (1+\tan^2 x)dx = \tan x + C$
- $\int (1+\cot^2 x)dx = -\cot x + C$
- $\int (ax+b)^\alpha dx = \frac{1}{a}\frac{(ax+b)^{\alpha+1}}{\alpha+1} + C$ $(\alpha \neq -1)$
- $\int x\,dx = \frac{x^2}{2} + C$
- $\int \frac{dx}{ax+b} = \frac{1}{a}\ln|ax+b| + C$
- $\int e^{ax+b} dx = \frac{1}{a} e^{ax+b} + C$
- $\int a^{kx+b} dx = \frac{1}{k}\frac{a^{kx+b}}{\ln a} + C$
- $\int \cos(ax+b)dx = \frac{1}{a}\sin(ax+b) + C$
- $\int \tan(ax+b)dx = -\frac{1}{a}\ln|\cos(ax+b)| + C$
- $\int \cot(ax+b)dx = \frac{1}{a}\ln|\sin(ax+b)| + C$
- $\int \frac{1}{\cos^2(ax+b)}dx = \frac{1}{a}\tan(ax+b) + C$
- $\int \frac{1}{\sin^2(ax+b)}dx = -\frac{1}{a}\cot(ax+b) + C$
- $\int (1+\tan^2(ax+b))dx = \frac{1}{a}\tan(ax+b) + C$
- $\int (1+\cot^2(ax+b))dx = -\frac{1}{a}\cot(ax+b) + C$
- $\int \frac{1}{2\sqrt{x}} dx = \sqrt{x} + C$
- $\int \sin^{-1}\frac{x}{a} dx = x\sin^{-1}\frac{x}{a} + \sqrt{a^2-x^2} + C$
- $\int \cos^{-1}\frac{x}{a} dx = x\cos^{-1}\frac{x}{a} - \sqrt{a^2-x^2} + C$
- $\int \tan^{-1}\frac{x}{a} dx = x\tan^{-1}\frac{x}{a} - \frac{a}{2}\ln(a^2+x^2) + C$
- $\int \cot^{-1}\frac{x}{a} dx = x\cot^{-1}\frac{x}{a} + \frac{a}{2}\ln(a^2+x^2) + C$
- $\int \sqrt{a^2-x^2}dx = \frac{x\sqrt{a^2-x^2}}{2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a} + C$
- $\int \ln(ax+b)dx = \left(x+\frac{b}{a}\right)\ln(ax+b) - x + C$
- $\int \sec^2 x\,dx = \tan x + C$
- $\int a^{px+q} dx = \frac{1}{p\ln a} a^{px+q} + C$, $\int \frac{dx}{1+x^2} dx = \tan^{-1}x + C$
- $\int \frac{dx}{\sqrt{1-x^2}} = \sin^{-1}x + C$, $\int \frac{dx}{a^2+x^2} = \frac{1}{a}\tan^{-1}\frac{x}{a} + C$
- $\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}\frac{x}{a} + C$, $\int \frac{dx}{\sqrt{a^2+x^2}} = \ln\left(x+\sqrt{x^2+a^2}\right) + C$
- $\int \frac{dx}{\sin^2(ax+b)} = -\frac{1}{a}\cot(ax+b) + C$ $\boxed{\int \csc^2 x = -\cot x + C}$
- $\int \frac{dx}{\cos^2(ax+b)} = \frac{1}{a}\tan(ax+b) + C$ $\boxed{\int \sec x \tan x = \sec x + C}$
- $\int e^{ax}\sin bx\,dx = \frac{e^{ax}(a\sin bx - b\cos bx)}{a^2+b^2} + C$
- $\int e^{ax}\cos bx\,dx = \frac{e^{ax}(a\cos bx + b\sin bx)}{a^2+b^2} + C$
- $\int \sqrt{a^2+x^2}dx = \frac{x\sqrt{a^2+x^2}}{2} + \frac{a^2}{2}\ln\left(x+\sqrt{a^2+x^2}\right) + C$

Chapter 3

$\int \csc x \cot x\,dx = -\csc x + C$

$\int \tan x\,dx = \ln|\sec x| + C$

Related rate (how fast/what rate/increasing or decreasing/find the speed).

Ex: At a certain movement, triangle has base length 20 cm and perpendicular height 10 cm. The length is increasing at a rate of 3 cm/s, while the height is decreasing at 2 cm/s. How fast is the area changing? Is the area increasing or decreasing?

Let: l ___; P ___; A ___

We have: $\frac{dl}{dt} = 3$; $\frac{dP}{dt} = -2$

We want to find: $\frac{dA}{dt}$, when $l = 20$, $p = 10$

At all time, we have: $A = \frac{1}{2}pl$

Derivative both sides $\frac{dA}{dt} = \frac{1}{2}p\frac{dl}{dt} + \frac{1}{2}l\frac{dp}{dt}$

Hence: $\frac{dA}{dt} = -5 \Rightarrow$ decreasing at $5\,\mathrm{cm^2/s}$

Mean Value Theorem: If f is cont on $[a,b]$ and differ on (a,b), then exist $c \in (a,b)$ that

$\boxed{f'(c) = \frac{f(b)-f(a)}{b-a}}$ (C+ Nhớ "0 đc thiếu" cm f diff continuous)

L'Hopital Rule: $\lim_{x\to a}\frac{f(x)}{g(x)} = \lim_{x\to a}\frac{f'(x)}{g'(x)}$ $(g'(x) \neq 0)$ $\left[\frac{0}{0}\right], \left[\frac{\infty}{\infty}\right]$

• Type 1: $(\infty - \infty)$, ...

$\lim_{x\to 1}\left(\frac{\ln x}{\sin(\pi x)}\right)$ has form $\left[\frac{0}{0}\right]$, so by L'Hôpital R: $\lim_{x\to 1}\left(\frac{\ln x}{\sin(\pi x)}\right) = \lim_{x\to 1}\frac{1/x}{\pi\cos(\pi x)} = -\frac{1}{\pi}$

• Type 2: $(0\cdot\infty)$

$\lim_{x\to 0} x\cot x = \lim_{x\to 0}\frac{x}{\tan x}$ $\left(\frac{0}{0}\right)$

Using ... $\lim_{x\to 0}\frac{x}{\tan x} = \lim_{x\to 0}\frac{x'}{(\tan x)'} = \lim_{x\to 0}\frac{1}{1+\tan^2 x}$

Type 3: (∞^0), (0^0), (1^∞)

Let $y = a^x$, $\ln y = x\ln a$

$\lim y = \lim e^{\ln y} = e^{\lim \ln y}$

$\lim_{x\to 0}(1+x)^{\frac{1}{\sin x}}$. Let $y = (1+x)^{\frac{1}{\sin x}}$, then taking both sides $\ln$, $\ln y = \frac{\ln(1+x)}{\sin x}$. By ...

$\lim_{x\to 0}\frac{\ln(1+x)}{\sin x} = \lim_{x\to 0}\frac{\frac{1}{1+x}}{\cos x} = 1$

Hence, $\lim_{x\to 0}(1+x)^{\frac{1}{\sin x}} = e^1 = e$

Newton's Method: $\boxed{x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}}$

Ex: Use Newton's method to find the value of $\sqrt{2}$

Let $f(x) = x^2 - 2$. Then $f'(x) = 2x$ for Newton

$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} = \frac{1}{2}\left(x_n + \frac{2}{x_n}\right)$

Starting with $x_1 = 1$, one has $x_2 = 1.5$, $x_3 = 1.41666667$, $x_4 = 1.41421568$... $x_6 = $ ... so the approx

A, Note: decimal places: Sau dấu phẩy, significant figures: trước dấu phẩy

Max/Min Problem: Optimization problem.

Ex: $p = 50 - \frac{q}{200}$. All batches produced are sold, so the revenue function is $R = pq$. Find the daily production level which max the revenue.

The R.F $R(q) = p.q = 50q - \frac{q^2}{200}$

To find max the revenue, we have to find C.P of the equation. Thus, Derivative the ... respect to q: $R'(q) = \frac{dR}{dq} = 50 - \frac{q}{100}$

Let $R'(q) = 0$, then $50 - \frac{q}{100} = 0$ we have $q = 5000$

Take second deri... of the revenue function respect to q that $q = 5000$ is exactly local...

$R''(q) = -\frac{1}{100} < 0$ ($f'' < 0$ then q is local max)

Hence, ...

Absolute max/min: GTLN – GTNN

Local max/min: cực đại – tiểu

Critical point: $f'(x) = 0$ or $f'(x)$ does not exist

Inflection point: $f''(x) = 0$ { f'' ... }

Increasing & Decreasing Function:

$f'(x) > 0$: then f is increasing

$f'(x) < 0$: then f is decreasing

Concavity: độ lõi lõm [concave up $\Rightarrow f'' > 0$ ∪; concave down $\Rightarrow f'' < 0$ ∩]

If $f''(c) > 0$, then $f''(c)$ has local min at c

If $f''(c) < 0$, then $f''(c)$ has local max at c

+ Asymptotes (chú ý domain)

Vertical Asymptotes: $\lim_{x\to a^+} = +\infty$, $\lim_{x\to a^-} y = -\infty$

Horizontal Asymptotes: $\lim_{x\to\infty} y = a$

Rolle's Theorem: If f is continuous on $[a,b]$ and differ on (a,b) and if $f(a) = f(b)$, then exist a point $c \in (a,b)$ such that $f'(c) = 0$

Ex: $f(x) = 1 + x^5(x-1)^7$. Show that $f'(x) = 0$ has at least one real root on $(0,1)$

$\rightarrow$ We have $f(0) = f(1) = 0$

Apply RT, there exist a $c \in (0,1)$ s that $f'(c) = 0$

Hence, $f'(x) = 0$ has at least one real root (...)

Ex: S that $e^x < 1 + xe^x$, $\forall x > 0$ Let $f(x) = 1 + xe^x - e^x$, $x \geq 0$ F(x x>0. By MVT, there exist $c \in (0,x)$ s that $\frac{f(x)-f(0)}{x-0} = f'(c) = ce^c > 0$ This implies $f(x) - f(0) > 0$ for all $x > 0$. Therefore...	Ex: Use MVT $\sqrt{1+x} < 1 + \frac{1}{2}x$, $\forall x > 0$ Let $f(x) = 1 + \frac{1}{2}x - \sqrt{1+x}$, $x \geq 0$ We have $f(0) = 0$ and $f'(x) = \frac{1}{2} - \frac{1}{2\sqrt{1+x}} > 0$, $\forall x > 0$ By MVT, $f(x) - f(0) = f'(c)x$ Therefore, ...

CHAPTER 4 | Integration + Technique

Type 1) Substitution $\int f(g(x)).g'(x)dx = \int f(u)du$ $(u = g(x))$

Ex: $\int_0^1 2x\cos(x^2-1)dx$

Let $t = x^2 - 1$, then $dt = 2x\,dx$. When $x = 0$, $t = -1$; $x = 1$, $t = 0$

Hence, $I = \int_{-1}^{0}\cos u\,du = \sin u\Big|_{-1}^{0} = -\sin(-1) = \sin 1$

Ex2: $\int_0^2 x^2\sqrt{x^3+1}\,dx$

Let $t = \sqrt{x^3+1}$, $t^2 = x^3 + 1$. Then, $\frac{2}{3}t\,dt = x^2 dx$

When $x = 0$ correspond to $t = 1$ and $x = 2$, $t = 3$

Hence, $\int_1^3 \frac{2}{3}t^2 dt = \frac{2}{3}\frac{t^3}{3}\Big|_1^3 = \frac{52}{9}$

A, Note: • If $f(x)$ is odd $f(x) = -f(-x)$ $\rightarrow I = \int_{-a}^{a} f(x)dx = 0$

• If $f(x)$ is even $f(x) = f(-x)$ $\rightarrow I = \int_{-a}^{a} f(x)dx = 2\int_0^a f(x)dx$

Ex: $\int_{-1}^{1}(1 + |t^3|)dt$

Because the function is even,

$\int_{-1}^{1}(1+|t^3|)dt = 2\int_0^1(1+|t^3|)dt = 2\int_0^1(1+t^3)dt = \frac{5}{2}$

Type 2) Integration by parts. $\int u\,dv = uv - \int v\,du$

Suggestion: $\Rightarrow$ Nhất log, nhì đa, tam lượng, tứ mũ

Type 3: Trigonometric functions. $\int \sin^m x\cos^n x\,dx$

– If m or n is odd $(2k+1)$, then $\int \sin^m x\cos^{2k}x\cos x\,dx$

$= \int \sin^m(1-\sin^2 x)^k\cos x\,dx \Rightarrow$ Let $t = \sin x$ (ngược lại)

– If m and n are both even. $\boxed{\sin^2 x = \frac{1}{2}(1-\cos 2x),\ \cos^2 x = \frac{1}{2}(1+\cos 2x)}$

Ex: $\int \sin^3 x\cos^8 x\,dx$

$= \int (1-\cos^2 x)\cos^8 x\sin x\,dx$, Let $t = \cos x$ =)

Các công thức lượng giác cơ bản:

- $\sin^2 x + \cos^2 x = 1$
- $\frac{1}{\sin^2 x} = 1 + \cot^2 x$ • $\frac{1}{\cos^2 x} = 1 + \tan^2 x$
- $\sin 2a = 2\sin a\cos a$ • $\tan x\cot x = 1$
- $\sin 3a = 3\sin a - 4\sin^3 a$ • $\cos 2a = 1 - 2\sin^2 a = \cos^2 a - \sin^2 a$
- $\cos 3a = 4\cos^3 a - 3\cos a$

Type 4: Partial Fraction Decomposition. $\left[\frac{P(x)}{Q(x)}\right]$

bậc $P(x) \geq Q(x)$ $\rightarrow$ chia đa thức

bậc $P(x) < Q(x)$ $\rightarrow$ xét mẫu số

- $\frac{P(x)}{Q(x)} = \frac{A_1}{x-a_1} + \frac{A_2}{x-a_2} + \dots + \frac{A_n}{x-a_n}$
- $\frac{P(x)}{Q(x)} = \frac{A}{x-a} + \frac{B}{x-b} + \frac{C}{(x-b)^2}$ (có n° đơn và n° kép)

Fundamental Theorem of Calculus $\boxed{(F(u))' = u'.f(u)}$

$F(x) = \int_a^x f(t)dt \rightarrow F'(x) = f(x)$ for $x \in [a,b]$

Ex: Let f be a cont f and $F(x) = \int_x^{x^2} f(t)dt$. If $F'(1) = 3$, find $f(1)$

By FTC, we can write $\int_x^{x^2} f(t)dt = \int_x^a f(t)dt + \int_a^{x^2} f(t)dt$

$= \int_a^{x^2} f(t)dt - \int_a^x f(t)dt = F(x^2) - F(x)$

Derivative ... $= 2xf(x^2) - f(x)$

Sub $x = 1$, $F'(1) = 2f(1^2) - f(1)$

$F'(1) = f(1) = 3$

Thus, ...

b) $H(x) = \int_{x^4}^{3x^2} e^{-t^2}dt$. Represent $H(x) = \int_1^{3x^2} e^{-t^2}dt - \int_1^{x^4} e^{-t^2}dt$

By FTC and chain rule

$H'(x) = H_1'(x) - H_2'(x)$

$= 6xe^{-9x^4} - 4x^3e^{-x^8}$

Application of Integrals:

Type 1: Length of curves (Arc Length)

+ $y=f(x)$: $S=\int_a^b \sqrt{1+[f'(x)]^2}\,dx$
+ $x=g(y)$: $S=\int_a^b \sqrt{1+[g'(y)]^2}\,dy$
+ $x=f(t)$; $y=g(t)$: $S=\int_a^b \sqrt{[x'(t)]^2+[y'(t)]^2}\,dt$ (parametric form)

• Type 2: Areas

The area A of the region bounded by the curves $y=f(x)$, $y=g(x)$ and the lines $x=a$, $x=b$ where f and g are cont and $f(x)\geq g(x)$ for all x in $[a,b]$ is: $\boxed{A=\int_a^b [f(x)-g(x)]dx}$

$\begin{cases} y=f(x) \\ x=a, x=b \\ y=x\text{-axis}=0 \end{cases} \rightarrow A=\int_a^b [f(x)]dx$

Ex: Find the area enclosed by the line $y=x-1$ and $y^2=2x+6$

Solving the system of x: $x=\frac{y^2}{2}-3$; $x=y+1$

we get 2 intersections on points $(-1;-2)$; $(5;4)$

So the area given by: $A=\int_{-2}^{4}\left(-\frac{y^2}{2}+y+4\right)dy$

• Type 3: Volume of revolution

+ By slicing: $V=\pi\int_a^b [f(x)]^2dx$
+ By washers: quanh trục Ox

S: $\begin{cases} (C): y=f(x)\geq 0\ \forall x\in[a,b] \\ Ox: y=0 \text{ (quay quanh trục Ox)} \\ \Delta_1, \Delta_2: x=a; x=b\ (a<b) \end{cases}$

$\rightarrow \boxed{V=\pi\int_a^b [f(x)]^2dx}$

S: $\begin{cases} (C_1): y=f(x)\ \forall x\in[a,b] \\ (C_2): y=g(x)\ \forall x\in[a,b] \\ 0\leq g(x)\leq f(x) \\ \Delta_1, \Delta_2: x=a; x=b\ (a<b) \end{cases}$

$\rightarrow \boxed{V=\pi\int_a^b [f^2(x)-g^2(x)]dx}$

Quanh trục Oy:

S: $\begin{cases} x=f(y) \\ x=0\ (Oy) \\ y=a; y=b\ (a<b) \end{cases} \rightarrow \boxed{V=\pi\int_a^b [f(y)]^2dy}$

S: $\begin{cases} x=f(y) \\ x=g(y) \\ y=a; y=b\ (a<b) \end{cases} \rightarrow \boxed{V=\pi\int_a^b [f(y)]^2-[g(y)]^2dy}$

+ By cylindrical shells: $V=2\pi\int_a^b xf(x)dx$ (rotate about y-axis)

Ex: rotating abt the y-axis the region bounded by $y=2x^2-x^3$ and $y=0$

From the sketch, by the shell method:

$\rightarrow V=2\pi\int_0^2(2x^3-x^4)dx=\frac{16}{5}\pi$

• Type 4: Average Value:

$\boxed{AVG(f,a,b)=f_{av}=\frac{1}{b-a}\int_a^b f(x)dx}$

Ex: A typical 24h period (°C)

$T(t)=3-\frac{2}{3}(t-13)^2$, $0\leq t\leq 24$.

What is the avg between 6am and 4pm?

By Average value,:

$\overline{T}=\frac{1}{16-6}\int_6^{16}\left[3-\frac{2}{3}(t-13)^2\right]dt=-\frac{47}{9}$

• Type 5: Improper Integral

1) The intervals is infinite
2) f has an infinite discontinuity on $[a,b]$

• $\int_a^{\infty} f(x)dx=\lim_{t\to\infty}\int_a^t f(x)dx$ $\begin{cases} \text{a finite number} \rightarrow \text{conv} \\ \text{infinite} \rightarrow \text{diver} \end{cases}$

• $\int_{-\infty}^{b} f(x)dx=\lim_{t\to-\infty}\int_t^b f(x)dx$ $\begin{cases} \text{exist} \rightarrow \text{conv} \\ \text{d. not exist} \rightarrow \text{dis} \end{cases}$

• $\int_{-\infty}^{\infty} f(x)dx=\int_{-\infty}^{a} f(x)dx+\int_a^{\infty} f(x)dx$

• $\int_a^{\infty}\frac{1}{x^p}dx$ $\begin{cases} \text{convergent if } p>1 \\ \text{divergent if } p\leq 1 \end{cases}$

• $\int_0^{a}\frac{1}{x^p}dx$ $\begin{cases} \text{convergent if } p<1 \\ \text{divergent if } p\geq 1 \end{cases}$

• $\int_a^b f(x)dx=\lim_{t\to b^-}\int_a^t f(x)dx$ (If $t\in[a,b)$)

• $\int_a^b f(x)dx=\lim_{t\to a^+}\int_t^b f(x)dx$ (If $t\in(a,b]$)

• $\int_a^b f(x)dx=\int_a^c f(x)dx+\int_c^b f(x)dx$.

— Comparison Test for Improper Integral

$0\leq f(x)\leq g(x)$ for $x\geq a$ (f, g cont)

• $\int_a^{\infty} f(x)dx$ diver $\rightarrow \int_a^{\infty} g(x)dx$ diver

• $\int_a^{\infty} g(x)dx$ conver $\rightarrow \int_a^{\infty} f(x)dx$ conver

Ex: S that $\int_0^{\infty} e^{-x^2}dx$ convergent

$\rightarrow \int_0^{\infty} e^{-x^2}dx=\int_0^1 e^{-x^2}dx+\int_1^{\infty} e^{-x^2}dx$

$e^{-x^2}\leq e^{-x}$ for $x\geq 1$,

$\int_1^{\infty} e^{-x}dx$ conv then $\int_1^{\infty} e^{-x^2}dx$ conv.

Distance/Displacement

— Displacement during $[t_1,t_2]$ $\int_{t_1}^{t_2} v(t)dt$.

• Total distance travelled during $[t_1,t_2]$ $=\int_{t_1}^{t_2}|v(t)|dt$ $=S(2)-S(1)$

Consumer Surplus: $\int_0^x [P_d(x)-P]dx$

Producer Surplus: $\int_0^x [P-P_s(x)]dx$

Work done: $W=F(x)$ = force × distance

$W=\int_a^b dW=\int_a^b f(x)dx$

Ex: Rope 20m long, with linear d 0,5kg/m hang over the building. How much work?

we have $F(x)=0{,}5x$, $x\in[0,20]$

$W=\int_0^{20} 0{,}5x\,dx = 100\ m/kg\ (J)$

Net Change Theorem:

The integral of a rate of change is the net change:

$\int_a^b F'(x)dx=F(b)-F(a)$

Ex: $v(t)=t^2-t-6$ (m/s)

a) Find the displacement of particle during the time period:

By Net change Theorem, the dis is

$S(4)-S(1)=\int_1^4 v(t)dt=-\frac{9}{2}$ (m)

This means that the particle moved 4,5 m toward the left

b) Find the distance traveled during this time period.

Note that $v(t)=t^2-t-6=(t-3)(t+2)$ and so $v(t)\leq 0$ on $[1,3]$ and $v(t)\geq 0$ on $[3,4]$

Thus, $\int_1^4|v(t)|dt=\int_1^3[-v(t)]dt+\int_3^4 v(t)dt$

$=\frac{61}{6}\approx 10{,}17$ m

***CT tính diện tích, thể tích:**

$V_{chóp}=\frac{1}{3}B.h$ (B: S đáy), $S_{circle}=\pi R^2$

$\cdot V_{k.cầu}=\frac{4}{3}\pi R^3$, $S_{m.cầu}=4\pi R^2$ (Sphere)

$\cdot V_{trụ}=\pi R^2h$ (Cylinder), $S_{xq}=2\pi Rh$

$\cdot V_{nón}=\frac{1}{3}\pi R^2h$, $S_{xq}=\pi Rl$ $\quad S_{tp}=2\pi Rh+2\pi R^2$

$\cdot V_{hộp\,cn}=a.b.c$, $V_{l.phương}=a^3$ (Cube)

• By the Pythagorean Theorem, the radius of cylinder is: $r^2=R^2-\left(\frac{h}{2}\right)^2$

$\cdot S_{cube}=6a^2$; $V_{chóp\,cụt}=\frac{1}{3}\pi h(R^2+r^2+Rr)$

$\cdot C=2\pi R$ (circumference)

Trapezoidal Rule $\boxed{\Delta x=\frac{b-a}{n}}$ $x_0=a$, $x_n=b$

$T_n=\frac{\Delta x}{2}[f(x_0)+2(f(x_1)+f(x_2)+\dots+f(x_{n-1}))+f(x_n)]$ with $\boxed{x_i=a+i\Delta x}$

Midpoint Rule: $\overline{x}_i=\frac{1}{2}(x_{i-1}+x_i)$

$x_i=x_{i-1}+\Delta x$

$M_n=\Delta x[f(\overline{x}_1)+f(\overline{x}_2)+\dots+f(\overline{x}_n)]$

• Simpson's Rule: $x_i=a+i\Delta x$

$\int_a^b f(x)dx\approx\frac{\Delta x}{3}(f(x_0)+4f(x_1)+2f(x_2)+\dots+2f(x_{n-2})+4f(x_{n-1})+f(x_n)]$

Where $\Delta x=\frac{b-a}{n}$; $x_0=a$, $x_n=b$ and n is even.

Ex: Find an approx $\int_1^2\frac{1}{x}dx$ using $n=5$

a) Midpoint Rule.

We have $n=5$, $a=1$ and $b=2$, so $\Delta x=(2-1)/5=\frac{1}{5}$

a) The midpoint of subintervals are: 1,1; 1,3; 1,5; 1,7; 1,9.

So the midpoint Rule gives

$M_5=\Delta x[f(1{,}1)+f(1{,}3)+f(1{,}5)+f(1{,}7)+f(1{,}9)]$

$\approx 0{,}691908$.

b) The trapezoidal rule gives:

$\int_1^2\frac{1}{x}dx\approx T_5=\frac{\Delta x}{2}[f(1)+2[f(1{,}2)+\dots+f(1{,}8)]+f(2)]=\frac{1}{10}\left[\frac{1}{1}+\frac{2}{1{,}2}+\dots+\frac{2}{1{,}8}+\frac{1}{2}\right]\approx 0{,}696$

Các dạng đổi biến thường gặp:

$\int f(ax+b)dx$: → Đặt $t=ax+b$

$\int f(\sin x)\cos x\,dx$: → Đặt $t=\sin x$

$\int f(\cos x)\sin x\,dx$: → Đặt $t=\cos x$

$\int f(\ln x)\frac{1}{x}dx$ → Đặt $t=\ln x$

$\int f(\tan x)\frac{1}{\cos^2x}dx$ → Đặt $t=\tan x$

$\int f(e^x)e^x dx$ → Đặt $t=e^x$

• Có căn ⇒ đặt căn bằng t

• Có $[f(x)]^n$ đặt $f(x)=t$

• $\int\frac{u(x)}{v(x)+c}dx \Rightarrow t=v(x)+C$

• mẫu số $=t$ (phức tạp nhất $=t$)

. Khi gặp $I=\int_a^b|f(x)|dx$ → xét dấu hàm

Ex:

x	a		x_1		x_2		b
$f(x)$		+	0	−	0	+	

$I=\int_a^{x_1}f(x)dx-\int_{x_1}^{x_2}f(x)dx+\int_{x_2}^b f(x)dx$

$\left(\frac{1}{u}\right)'=-\frac{u'}{u^2}$ $\quad\int\ln x=x\ln x-x+C$ (đặt $u=\ln x$, $dv=dx$).

t	0	5	10	15	20	...	50
s	0	12	18	20	21	...	0
	x_0	x_1	x_2	x_3	x_4	x_n	x_{10}

Evaluate: $\int_1^{\infty}\frac{dx}{(2x-1)(x+2)}$

By partial fraction, $\frac{1}{(2x-1)(x+2)}=\frac{A}{(\ )}+\frac{B}{(\ )}$

where $1=A(x+2)+B(2x-1)\ \forall x$, $A=\frac{2}{5}$, $B=-\frac{1}{5}$

$\int_1^{\infty}\frac{dx}{(2x-1)(x+2)}=\lim_{t\to\infty}\int_1^t\left[\frac{\frac{2}{5}}{(\ )}-\frac{\frac{1}{5}}{(\ )}\right]dx$

$=\lim_{t\to\infty}\left[\frac{1}{5}\ln\left(\frac{2t-1}{t+2}\right)-\ln\frac{1}{3}\right]=\frac{\ln 6}{5}$