TABLE 8.4 In Determining the Step Response of a Second-Order Circuit, We Apply the Appropriate Equations Depending on the Damping

Damping	Step Response Equations[a]	Coefficient Equations
Overdamped	$x(t) = X_f + A_1' e^{s_1 t} + A_2' e^{s_2 t}$	$x(0) = X_f + A_1' + A_2'$; $dx/dt(0) = A_1' s_1 + A_2' s_2$
Underdamped	$x(t) = X_f + (B_1' \cos \omega_d t + B_2' \sin \omega_d t)e^{-\alpha t}$	$x(0) = X_f + B_1'$; $dx/dt(0) = -\alpha B_1' + \omega_d B_2'$
Critically damped	$x(t) = X_f + D_1' te^{-\alpha t} + D_2' e^{-\alpha t}$	$x(0) = X_f + D_2'$; $dx/dt(0) = D_1' - \alpha D_2'$

[a] where X_f is the final value of $x(t)$.

Problems

Sections 8.1–8.2

8.1 The resistance, inductance, and capacitance in a parallel *RLC* circuit are 2000 Ω, 250 mH, and 10 nF, respectively.

a) Calculate the roots of the characteristic equation that describe the voltage response of the circuit.

b) Will the response be over-, under-, or critically damped?

c) What value of R will yield a damped frequency of 12 krad/s?

d) What are the roots of the characteristic equation for the value of R found in (c)?

e) What value of R will result in a critically damped response?

8.2 PSPICE MULTISIM The circuit elements in the circuit in Fig. 8.1 are $R = 200\ \Omega$, $C = 200$ nF, and $L = 50$ mH. The initial inductor current is -45 mA, and the initial capacitor voltage is 15 V.

a) Calculate the initial current in each branch of the circuit.

b) Find $v(t)$ for $t \geq 0$.

c) Find $i_L(t)$ for $t \geq 0$.

8.3 PSPICE MULTISIM The resistance in Problem 8.2 is increased to 312.5 Ω. Find the expression for $v(t)$ for $t \geq 0$.

8.4 PSPICE MULTISIM The resistance in Problem 8.2 is increased to 250 Ω. Find the expression for $v(t)$ for $t \geq 0$.

8.5 a) Design a parallel *RLC* circuit (see Fig. 8.1) using component values from Appendix H, with a resonant radian frequency of 5000 rad/s. Choose a resistor or create a resistor network so that the response is critically damped. Draw your circuit.

b) Calculate the roots of the characteristic equation for the resistance in part (a).

8.6 a) Change the resistance for the circuit you designed in Problem 8.5(a) so that the response is underdamped. Continue to use components from Appendix H. Calculate the roots of the characteristic equation for this new resistance.

b) Change the resistance for the circuit you designed in Problem 8.5(a) so that the response is overdamped. Continue to use components from Appendix H. Calculate the roots of the characteristic equation for this new resistance.

8.7 The natural voltage response of the circuit in Fig. 8.1 is

$$v(t) = 75e^{-8000t}(\cos 6000t - 4 \sin 6000t)\text{V}, \quad t \geq 0,$$

when the inductor is 400 mH. Find (a) C; (b) R; (c) V_0; (d) I_0; and (e) $i_L(t)$.

8.8 Suppose the capacitor in the circuit shown in Fig. 8.1 has a value of 0.1 μF and an initial voltage of 24 V. The initial current in the inductor is zero. The resulting voltage response for $t \geq 0$ is

$$v(t) = -8e^{-250t} + 32e^{-1000t}\ \text{V}.$$

a) Determine the numerical values of R, L, α, and ω_0.

b) Calculate $i_R(t)$, $i_L(t)$, and $i_C(t)$ for $t \geq 0^+$.

8.9 The voltage response for the circuit in Fig. 8.1 is known to be

$$v(t) = D_1te^{-500t} + D_2e^{-500t}, \quad t \geq 0.$$

The initial current in the inductor (I_0) is −10 mA, and the initial voltage on the capacitor (V_0) is 8 V. The inductor has an inductance of 4 H.

a) Find the values of R, C, D_1, and D_2.

b) Find $i_C(t)$ for $t \geq 0^+$.

8.10 The natural response for the circuit shown in Fig. 8.1 is known to be

$$v(t) = -11e^{-100t} + 20e^{-400t}\ \text{V}, \quad t \geq 0.$$

If $C = 2\ \mu\text{F}$ and $L = 12.5$ H, find $i_L(0^+)$ in milliamperes.

8.11 The initial value of the voltage v in the circuit in Fig. 8.1 is zero, and the initial value of the capacitor current, $i_c(0^+)$, is 45 mA. The expression for the capacitor current is known to be

$$i_c(t) = A_1e^{-200t} + A_2e^{-800t}, \quad t \geq 0^+,$$

when R is 250 Ω. Find

a) the values of α, ω_0, L, C, A_1, and A_2

$$\left(Hint: \frac{di_C(0^+)}{dt} = -\frac{di_L(0^+)}{dt} - \frac{di_R(0^+)}{dt} = \frac{-v(0)}{L} - \frac{1}{R}\frac{i_C(0^+)}{C}\right)$$

b) the expression for $v(t), t \geq 0$,

c) the expression for $i_R(t) \geq 0$,

d) the expression for $i_L(t) \geq 0$.

8.12 Assume the underdamped voltage response of the circuit in Fig. 8.1 is written as

$$v(t) = (A_1 + A_2)e^{-\alpha t}\cos\omega_d t + j(A_1 - A_2)e^{-\alpha t}\sin\omega_d t$$

The initial value of the inductor current is I_0, and the initial value of the capacitor voltage is V_0. Show that A_2 is the conjugate of A_1. (Hint: Use the same process as outlined in the text to find A_1 and A_2.)

8.13 Show that the results obtained from Problem 8.12—that is, the expressions for A_1 and A_2—are consistent with Eqs. 8.30 and 8.31 in the text.

8.14 PSPICE MULTISIM In the circuit in Fig. 8.1, $R = 5\ \text{k}\Omega$, $L = 8$ H, $C = 125$ nF, $V_0 = 30$ V, and $I_0 = 6$ mA.

a) Find $v(t)$ for $t \geq 0$.

b) Find the first three values of t for which dv/dt is zero. Let these values of t be denoted t_1, t_2, and t_3.

c) Show that $t_3 - t_1 = T_d$.

d) Show that $t_2 - t_1 = T_d/2$.

e) Calculate $v(t_1)$, $v(t_2)$, and $v(t_3)$.

f) Sketch $v(t)$ versus t for $0 \leq t \leq t_2$.

8.15 PSPICE MULTISIM a) Find $v(t)$ for $t \geq 0$ in the circuit in Problem 8.14 if the 5 kΩ resistor is removed from the circuit.

b) Calculate the frequency of $v(t)$ in hertz.

c) Calculate the maximum amplitude of $v(t)$ in volts.

8.16 PSPICE MULTISIM In the circuit shown in Fig. 8.1, a 2.5 H inductor is shunted by a 100 nF capacitor, the resistor R is adjusted for critical damping, $V_0 = -15$ V, and $I_0 = -5$ mA.

a) Calculate the numerical value of R.

b) Calculate $v(t)$ for $t \geq 0$.

c) Find $v(t)$ when $i_C(t) = 0$.

d) What percentage of the initially stored energy remains stored in the circuit at the instant $i_C(t)$ is 0?

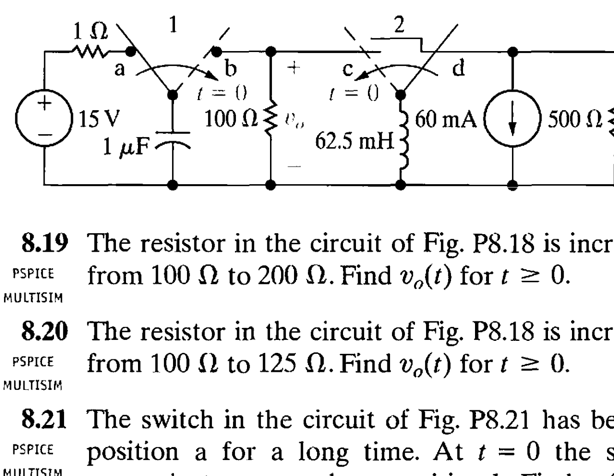
8.17 PSPICE MULTISIM The resistor in the circuit in Example 8.4 is changed to 3200 Ω.

a) Find the numerical expression for $v(t)$ when $t \geq 0$.

b) Plot $v(t)$ versus t for the time interval $0 \leq t \leq 7$ ms. Compare this response with the one in Example 8.4 ($R = 20\ \text{k}\Omega$) and Example 8.5 ($R = 4\ \text{k}\Omega$). In particular, compare peak values of $v(t)$ and the times when these peak values occur.

8.18 PSPICE MULTISIM The two switches in the circuit seen in Fig. P8.18 operate synchronously. When switch 1 is in position a, switch 2 is in position d. When switch 1 moves to position b, switch 2 moves to position c. Switch 1 has been in position a for a long time. At $t = 0$, the switches move to their alternate positions. Find $v_o(t)$ for $t \geq 0$.

Figure P8.18

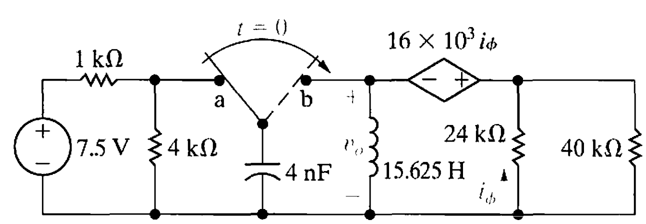


8.19 PSPICE MULTISIM The resistor in the circuit of Fig. P8.18 is increased from 100 Ω to 200 Ω. Find $v_o(t)$ for $t \geq 0$.

8.20 PSPICE MULTISIM The resistor in the circuit of Fig. P8.18 is increased from 100 Ω to 125 Ω. Find $v_o(t)$ for $t \geq 0$.

8.21 PSPICE MULTISIM The switch in the circuit of Fig. P8.21 has been in position a for a long time. At $t = 0$ the switch moves instantaneously to position b. Find $v_o(t)$ for $t \geq 0$.

Figure P8.21



8.22 The inductor in the circuit of Fig. P8.21 is decreased to 10 H. Find $v_o(t)$ for $t \geq 0$.

8.23 The inductor in the circuit of Fig. P8.21 is decreased to 6.4 H. Find $v_o(t)$ for $t \geq 0$.

Section 8.3

8.24 PSPICE MULTISIM For the circuit in Example 8.6, find, for $t \geq 0$, (a) $v(t)$; (b) $i_R(t)$; and (c) $i_C(t)$.

8.25 PSPICE MULTISIM For the circuit in Example 8.7, find, for $t \geq 0$, (a) $v(t)$ and (b) $i_C(t)$.

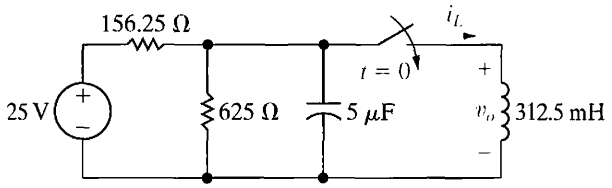
8.26 PSPICE MULTISIM For the circuit in Example 8.8, find $v(t)$ for $t \geq 0$.

8.27 PSPICE MULTISIM The switch in the circuit in Fig. P8.27 has been open a long time before closing at $t = 0$. Find

a) $v_o(t)$ for $t \geq 0^+$,

b) $i_L(t)$ for $t \geq 0$.

Figure P8.27



8.28 PSPICE MULTISIM Use the circuit in Fig. P8.27

a) Find the total energy delivered to the inductor.

b) Find the total energy delivered to the equivalent resistor.

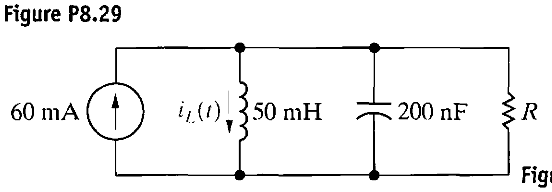
c) Find the total energy delivered to the capacitor.

d) Find the total energy delivered by the equivalent current source.

e) Check the results of parts (a) through (d) against the conservation of energy principle.

8.29 PSPICE MULTISIM Assume that at the instant the 60 mA dc current source is applied to the circuit in Fig. P8.29, the initial current in the 50 mH inductor is −45 mA, and the initial voltage on the capacitor is 15 V (positive at the upper terminal). Find the expression for $i_L(t)$ for $t \geq 0$ if R equals 200 Ω.

Figure P8.29

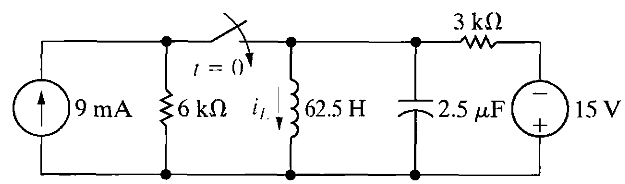


8.30 PSPICE MULTISIM The resistance in the circuit in Fig. P8.29 is changed to 312.5 Ω. Find $i_L(t)$ for $t \geq 0$.

8.31 PSPICE MULTISIM The resistance in the circuit in Fig. P8.29 is changed to 250 Ω. Find $i_L(t)$ for $t \geq 0$.

8.32 PSPICE MULTISIM The switch in the circuit in Fig. P8.32 has been open a long time before closing at $t = 0$. Find $i_L(t)$ for $t \geq 0$.

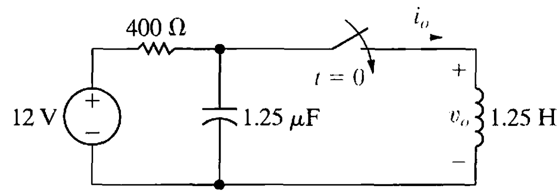
Figure P8.32



8.33 PSPICE MULTISIM Switches 1 and 2 in the circuit in Fig. P8.33 are synchronized. When switch 1 is opened, switch 2 closes and vice versa. Switch 1 has been open a long time before closing at $t = 0$. Find $i_L(t)$ for $t \geq 0$.

8.34 PSPICE MULTISIM The switch in the circuit in Fig. P8.34 has been open for a long time before closing at $t = 0$. Find $v_o(t)$ for $t \geq 0$.

Figure P8.34



8.35 PSPICE MULTISIM a) For the circuit in Fig. P8.34, find i_o for $t \geq 0$.

b) Show that your solution for i_o is consistent with the solution for v_o in Problem 8.34.

8.36 PSPICE MULTISIM The switch in the circuit in Fig. P8.36 has been open a long time before closing at $t = 0$. At the time the switch closes, the capacitor has no stored energy. Find v_o for $t \geq 0$.

Figure P8.36

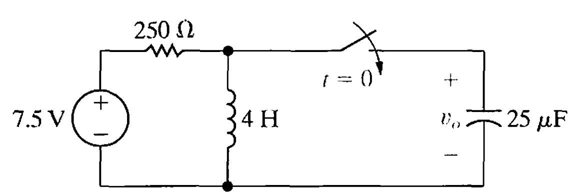
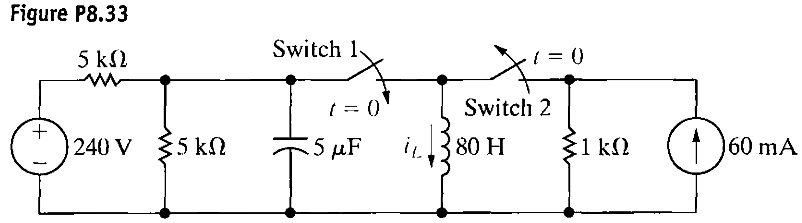
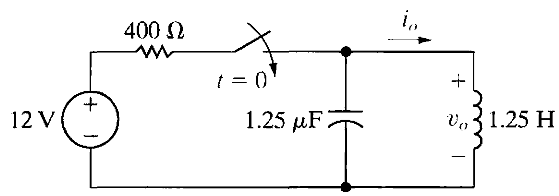


Figure P8.33



8.37 There is no energy stored in the circuit in Fig. P8.37 when the switch is closed at $t = 0$. Find $v_o(t)$ for $t \geq 0$.
PSPICE MULTISIM

Figure P8.37



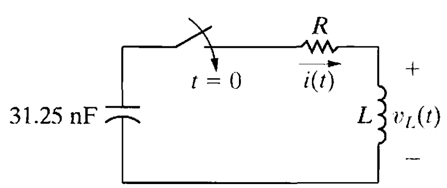
8.38 a) For the circuit in Fig. P8.37, find i_o for $t \geq 0$.
PSPICE MULTISIM
b) Show that your solution for i_o is consistent with the solution for v_o in Problem 8.37.

Section 8.4

8.39 The initial energy stored in the 31.25 nF capacitor in the circuit in Fig. P8.39 is 9 μJ. The initial energy stored in the inductor is zero. The roots of the characteristic equation that describes the natural behavior of the current i are $-4000\ \text{s}^{-1}$ and $-16{,}000\ \text{s}^{-1}$

a) Find the numerical values of R and L.
b) Find the numerical values of $i(0)$ and $di(0)/dt$ immediately after the switch has been closed.
c) Find $i(t)$ for
d) How many microseconds after the switch closes does the current reach its maximum value?
e) What is the maximum value of i in milliamperes?
f) Find $v_L(t)$ for $t \geq 0$.

Figure P8.39



8.40 a) Design a series RLC circuit (see Fig. 8.3) using component values from Appendix H, with a resonant radian frequency of 20 krad/s. Choose a resistor or create a resistor network so that the response is critically damped. Draw your circuit.
b) Calculate the roots of the characteristic equation for the resistance in part (a).

8.41 a) Change the resistance for the circuit you designed in Problem 8.40(a) so that the response is underdamped. Continue to use components from Appendix H. Calculate the roots of the characteristic equation for this new resistance.
b) Change the resistance for the circuit you designed in Problem 8.40(a) so that the response is overdamped. Continue to use components from Appendix H. Calculate the roots of the characteristic equation for this new resistance.

8.42 The current in the circuit in Fig. 8.3 is known to be

$$i = B_1e^{-2000t}\cos 1500t + B_2e^{-2000t}\sin 1500t, \quad t \geq 0.$$

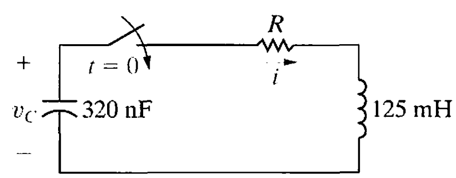
The capacitor has a value of 80 nF; the initial value of the current is 7.5 mA; and the initial voltage on the capacitor is –30 V. Find the values of R, L, B_1, and B_2.

8.43 Find the voltage across the 80 nF capacitor for the circuit described in Problem 8.42. Assume the reference polarity for the capacitor voltage is positive at the upper terminal.

8.44 In the circuit in Fig. P8.44, the resistor is adjusted for critical damping. The initial capacitor voltage is 15 V, and the initial inductor current is 6 mA.
PSPICE MULTISIM

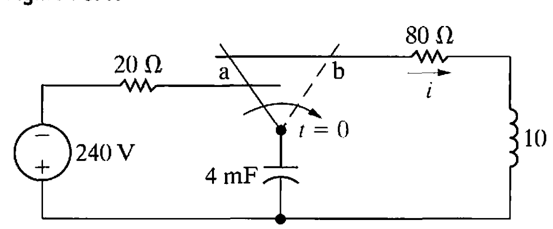
a) Find the numerical value of R.
b) Find the numerical values of i and di/dt immediately after the switch is closed.
c) Find $v_C(t)$ for $t \geq 0$.

Figure P8.44



8.45 The switch in the circuit shown in Fig. P8.45 has been in position a for a long time. At $t = 0$, the switch is moved instantaneously to position b. Find $i(t)$ for $t \geq 0$.
PSPICE MULTISIM

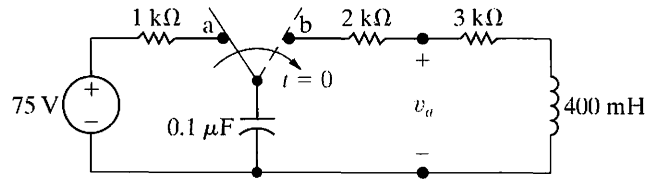
Figure P8.45



8.46 The switch in the circuit in Fig. P8.46 on the next page has been in position a for a long time. At $t = 0$, the switch moves instantaneously to position b.
PSPICE MULTISIM

a) What is the initial value of v_a?
b) What is the initial value of dv_a/dt?
c) What is the numerical expression for $v_a(t)$ for $t \geq 0$?

Figure P8.46

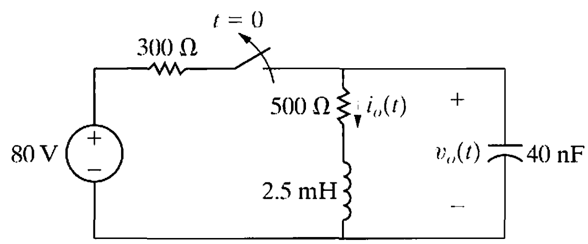


8.47 The switch in the circuit shown in Fig. P8.47 has been closed for a long time. The switch opens at $t = 0$. Find
PSPICE MULTISIM

a) $i_o(t)$ for $t \geq 0$,

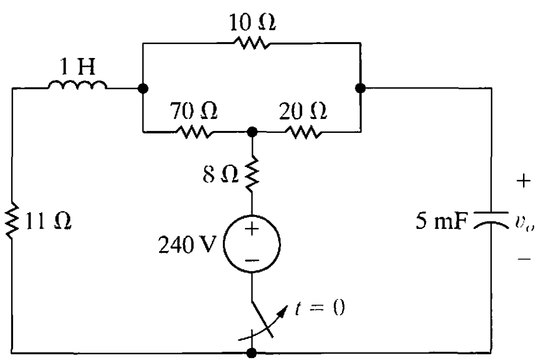
b) $v_o(t)$ for $t \geq 0$.

Figure P8.47



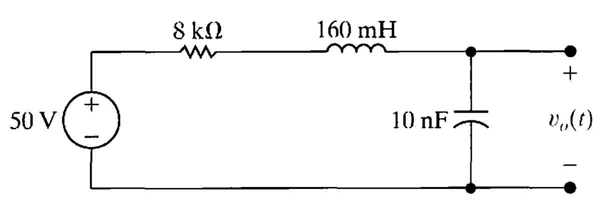
8.48 The switch in the circuit shown in Fig. P8.48 has been closed for a long time. The switch opens at $t = 0$. Find $v_o(t)$ for $t \geq 0$.

Figure P8.48



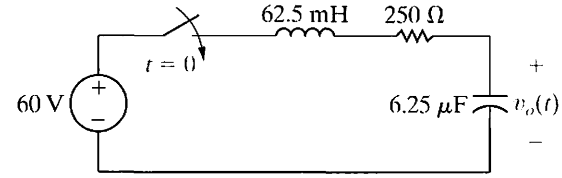
8.49 The circuit shown in Fig. P8.49 has been in operation for a long time. At $t = 0$, the source voltage suddenly jumps to 250 V. Find $v_o(t)$ for $t \geq 0$.
PSPICE MULTISIM

Figure P8.49



8.50 The initial energy stored in the circuit in Fig. P8.50 is zero. Find $v_o(t)$ for $t \geq 0$.
PSPICE MULTISIM

Figure P8.50



8.51 The capacitor in the circuit shown in Fig. P8.50 is changed to 4 μF. The initial energy stored is still zero. Find $v_o(t)$ for $t \geq 0$.

8.52 The capacitor in the circuit shown in Fig. P8.50 is changed to 2.56 μF. The initial energy stored is still zero. Find $v_o(t)$ for $t \geq 0$.

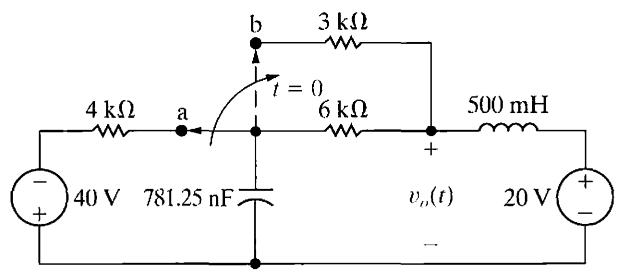
8.53 The switch in the circuit of Fig. P8.53 has been in position a for a long time. At $t = 0$ the switch moves instantaneously to position b. Find
PSPICE MULTISIM

a) $v_o(0^+)$

b) $dv_o(0^+)/dt$

c) $v_o(t)$ for $t \geq 0$.

Figure P8.53

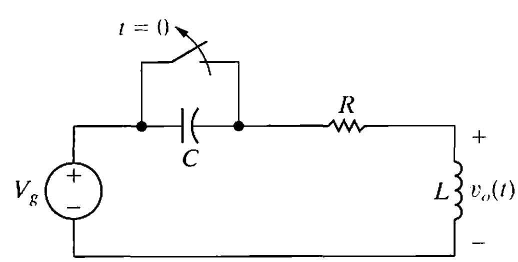


8.54 The switch in the circuit shown in Fig. P8.54 has been closed for a long time before it is opened at $t = 0$. Assume that the circuit parameters are such that the response is underdamped.

a) Derive the expression for $v_o(t)$ as a function of V_g, α, ω_d, C, and R for $t \geq 0$.

b) Derive the expression for the value of t when the magnitude of v_o is maximum.

Figure P8.54



8.55 The circuit parameters in the circuit of Fig. P8.54 are $R = 4800\ \Omega$, $L = 64$ mH, $C = 4$ nF, and $v_g = -72$ V.
PSPICE MULTISIM

a) Express $v_o(t)$ numerically for $t \geq 0$.

b) How many microseconds after the switch opens is the inductor voltage maximum?