**MID 22-NOV**

**(10 points) Flip a fair coin 4 times. Find the probability that the number of heads is greater than or equal to the number of tails.**

$$p(A) = \frac{n(A)}{n(\Omega)} = \frac{11}{16} = 0.6875$$

**(10 points) The percentages of people with each of the four blood types (O, A, B, and AB) in a region are as follows:**

| **Blood Type** | **A** | **B** | **AB** | **O** |
|---|---|---|---|---|
| **Percentage** | **30** | **12** | **3** | **55** |

**Select randomly a person in this region. Given that his/her blood type is either B or AB, find the (conditional) probability that his/her blood type is B.**

$$p(A|B) = \frac{n(A)}{n(B)} = \frac{12}{15} = 0.8$$

**(10 points) In a box, there is 2 blue balls and 18 green balls. Select randomly without replacement two balls from the box. What is the probability that the second ball selected is blue.**

$$p(A) = \frac{2}{20} \times {}^{1}\!/_{19} + {}^{18}\!/_{20} \times \frac{2}{19} = 0.1$$

**(10 points) A company has two stores of TV, one is located in Hanoi and another is in HCM city. At the store in Hanoi, 20% of TV are defective. The percentage of TV which are defective in HCM city is 15%. Choose randomly a store and from this store select randomly a TV. The selected TV is tested and found to be defective, what is the probability that it comes from the store in Hanoi.**

$$p(A|B) = \frac{p(B|A) \times p(A)}{p(B)} = \frac{0.2 \times 0.5}{0.2\ \times 0.5\ + 0.15\ \times 0.5} = \frac{4}{7} = 0.57$$

**(20 points) The probability function of a discrete random variable X has the form**

**Find c**

$$\Sigma p(x_i) = 33c = 1 \Rightarrow c = \frac{1}{33}$$

**Compute P(-2 ≤ X < 1).**

$$P(-2 \le x < 1) = P(x = -2) + P(x = -1) + P(x = 0) = \frac{1}{3} + \frac{5}{33} + \frac{1}{33} = \frac{17}{33}$$

**Evaluate E(X) and Var (X).**

$E(x) = \Sigma x_i p(x_i) = -22c + -5c + 0 + 5c + 22c = 0$

$$Var(x) = E(x^2) - [E(x)]^2 = 98c - 0^2 = 98c = \frac{98}{33} = 2.96$$

**6. (20 points) The borrowing period, in days, for a particular book at a university library can be regarded as random variable X which has normal distribution with mean = 8 and standard deviation = 2. A book need to be return within 10 days. Compute P(X > 10) - the probability that a new borrower returns the book after 10 days.**

$p(Z > 1) = 1 - P(Z < 1) = 1 - 0.8413 = 0.1587$

**For a late return, the borrower has to pay a penalty of $5. Otherwise, the borrower pays 0. Evaluate the average payment of a borrower.**

$E(x) = 0.1587 \times 5 + 0.8413 \times 0 = 0.7935$

**7. (10 points) Jack has invested $1000 in product A and $2000 in product B. He expects that if project A is success, he get a profit of $800 and lose his money that he invested in A if A is unsuccessful. For project B, a successfull investment yields a profit of $1000 and a uncessesfull of B makes him lose his money invested in B. He estimates the probability of success as following**

| | | **Project B** | |
|---|---|---|---|
| | | **successful** | **unsuccessful** |
| **Project A** | **successful** | **0.6** | **0.05** |
| | **unsuccessful** | **0.25** | **0.1** |

**Let X and Y be the his profit from project A and B respectively. Remark that X and Y can take negative value when he loses his money.**

**Determine the probability mass functions of X and Y.**

| | | |
|---|---|---|
| X | 800 | -1000 |
| P(X) | 0.65 | 0.35 |
| Y | 1000 | -2000 |

| | | |
|---|---|---|
| P(Y) | 0.85 | 0.15 |

**Calculate E(X) and E(Y) - the profits each project.**

$$E(X) = 800 \times 0.65 + 0.35 \times (-1000) = 170$$

$$E(Y) = 1000 \times 0.85 + 0.15 \times (-2000) = 550$$

**Compute E(X + Y) - the average of the overall profit from two projects.**

$$E(X + Y) = E(X) + E(Y) = 170 + 550 = 720$$

**8. (10 points) Consider a system of 4 components with structure as following**

**Suppose that all four components operate independently.**

**Compute the probability that the component 1 lasts more than 1 year.**

$$P_1 = P(T_1 > 1) = 1 - P(T_1 \leq 1) = 1 - \int_{-\infty}^{1} 0.1e^{-0.1x}\,dx = 0.9048$$

**Evaluate the probability that the system lasts more than 1 year.**

$$P(up) = p_1 \times [1 - (1 - p_2) \times (1 - p_3)] \times p_4 = 0.7917$$