(9) Markov Chain

$$P^2 = \begin{bmatrix} 0.1 & 0.2 & 0.7 \\ 0.3 & 0.14 & 0.56 \\ 0.24 & 0.4 & 0.36 \end{bmatrix}$$

1. $$P = \begin{bmatrix} 0 & 1 & 0 \\ 0.1 & 0.2 & 0.7 \\ 0.4 & 0 & 0.6 \end{bmatrix}$$

$$P^3 = \begin{bmatrix} 0.3 & 0.14 & 0.56 \\ 0.238 & 0.328 & 0.434 \\ 0.184 & 0.32 & 0.496 \end{bmatrix}$$

a) $P(X_2=3, X_1=3 \mid X_0=1)$

$= P\{X_2=3 \mid X_1=3\} \cdot P\{X_1=3 \mid X_0=1\}$

$= P_{33} \cdot P_{13} = 0.6 \times 0 = 0$

b) $P(X_1=2, X_3=1 \mid X_0=2)$

$= P\{X_1=2 \mid X_3=1\} \cdot P\{X_3=1 \mid X_0=2\}$

$= P_{12} \cdot P^{(3)}_{21} = 1 \cdot 0.238 = 0.238$

c) $P(X_3=1 \mid X_0=3) = P^{(3)}_{31} = 0.184$

d) $P(X_5=1, X_2=0 \mid X_1=3)$

$= P\{X_5=1 \mid X_2=0\} \cdot P\{X_2=0 \mid X_1=3\}$

$= P^{(3)}_{01} \cdot P_{30}$

(2)

$$P = \begin{array}{c} \begin{matrix} 0 & 1 & 2 \end{matrix} \\ \begin{bmatrix} 1/2 & 1/3 & 1/6 \\ 0 & 1/3 & 2/3 \\ 1/2 & 0 & 1/2 \end{bmatrix} \end{array} \begin{matrix} 0 \\ 1 \\ 2 \end{matrix}$$

$$P^2 = \begin{bmatrix} 1/3 & 5/18 & 7/18 \\ 1/3 & 1/9 & 5/9 \\ 1/2 & 1/6 & 1/3 \end{bmatrix}$$

$$P^3 = \begin{bmatrix} 13/36 & 11/54 & 47/108 \\ 4/9 & 4/27 & 11/27 \\ 5/12 & 2/9 & 13/36 \end{bmatrix}$$

a) $P(X_3 = 2 \mid X_1 = 0) = P^{(2)}_{02} = 7/18$

b) $P(X_5 = 1, X_2 = 0 \mid X_0 = 2)$

$= P\{X_5 = 1 \mid X_2 = 0\} \cdot P\{X_2 = 0 \mid X_0 = 2\}$

$= P^{(3)}_{01} \cdot P^{(2)}_{20} = 11/54 \cdot 1/2 = 11/108$

c) initial distribution X_0 is

$\pi^{(0)} = (0.25 \quad 0.35 \quad 0.4)$

$$\pi^{(2)} = \pi^{(0)} \cdot P^{(2)} = (0.25 \quad 0.35 \quad 0.4) \begin{bmatrix} 1/3 & 5/18 & 7/18 \\ 1/3 & 1/9 & 5/9 \\ 1/2 & 1/6 & 1/3 \end{bmatrix}$$

$$= \left(2/5 \quad 7/10 \quad 17/40 \right)$$

$$E(X_2) = \frac{0.2}{5} + \frac{1.7}{40} + \frac{2.17}{40} = \frac{21}{40}$$