Đỗ Minh Duy ITITSB22029.

① Geometric distribution

1. $f(x) = 0.98^{x-1} \times 0.02$

a) $P(X=10) = 0.01667$

b) $P(X>5) = 1 - P(X=1) - P(X=2) - P(X=3) - P(X=4) - P(X=5) = 0.9039$

c) $E(X) = \frac{1}{0.02} = 50$

2. $p = 100\% - 80\% = 20\% = 0.2$

a) $P(X=k) = q^{k-1} p = (1-p)^{k-1} p$

$p = 0.8$:

$\Rightarrow P(X=k) = q^{k-1} p = (1-0.2)^{k-1} . 0.2 = 0.8^{k-1} 0.2$

b) $P(X=1) = 0.2$

$P(X=2) = 0.16$

$P(X \geq 2) = 1 - P(X<2) = 1 - 0.2 = 0.8$

c) $\mu = \frac{1}{p} = \frac{1}{0.2} = 5$

② Binomial distribution.

1, $P(X=k) = \cancel{C} \binom{n}{k} 0.2^k \times 0.8^{n-k}$

a) $n=5$

$P(X=1) = \binom{5}{1} 0.2^1 \times 0.8^4 = 0.4096$

b) and c) $n = 20$

$P(X=4) = \binom{20}{4} 0.2^4 \times 0.8^{16} = 0.218$

$P(X>4) = 1 - P(X \leq 4) = 1 - P(X=0) - P(X=1) - P(X=2) - P(X=3) - P(X=4)$
$= 0.37$

2. $n = 20$
$p = 13\%$

$P(X=k) = \frac{n!}{k!(n-k)!} \cdot p^k (1-p)^{n-k}$

a) $P(X=3) = \frac{20!}{\ldots} \cdot 0.13^3 (1-0.13)^{20-3} = 0.2347$

b) $P(x \geq 3) = 1 - P(X < 3) = 1 - P(X=0) - P(X=1) - P(X=2)$

$= 1 - 0.0617 - 0.1844 - 0.2618 = 0.4921$

c) $\mu = np = 20 \times 0.13 = 2.6$

$\sigma = \sqrt{npq} = \sqrt{np(1-p)} = \sqrt{20(0.13)(1-0.13)} = 1.5040$

(3) $p = 0.6$ and $n = 4$

$P(x \geq 2) = \binom{n}{k} p^2 (1-p)^{n-2} + \binom{n}{k} p^3 (1-p)^{n-3} + \binom{n}{k} p^4 (1-p)^{n-4}$

$= 4C2 (0.6)^2 (0.4)^2 + 4C3 (0.6)^3 (0.4)^1 + 4C4 (0.6)^4 (0.4)^0$

$= 0.8208$

(4) Normal distribution

(1) $\mu = 18 \; ; \; \sigma = 2.5$

a) $P(X < 15) = P(Z < -1.2) = 0.1151$

$Z = \frac{x - \mu}{\sigma} = \frac{15 - 18}{2.5} = -1.2$

b) $P\left(Z < \frac{k - \mu}{\sigma}\right) = 0.2236$

$P(Z < -0.76) = 0.02236$

$\Rightarrow \frac{k - \mu}{\sigma} = -0.76 \Rightarrow k = 16.1$

c) $P(X > k) = 0.1814$

$1 - P\left(Z \leq \frac{k-\mu}{\sigma}\right) = 0.1814$

$P\left(Z \leq \frac{k-\mu}{\sigma}\right) = 0.8186$

$P(Z \leq 0.91) = 0.8186$

$\Rightarrow \frac{k - \mu}{\sigma} = 0.91 \Rightarrow k = 20.275$

d) $Z_1 = \frac{x_1 - \mu}{\sigma} = \frac{17 - 18}{2.5} = -0.4$

$Z_2 = \frac{x_2 - \mu}{\sigma} = \frac{21 - 18}{2.5} = 1.2$

$P(17 < X < 21)$

$= P(-0.4 < Z < 1.2)$

$= P(Z < 1.2) - P(Z < -0.4)$

$= 0.8849 - 0.3446 = 0.5403$

$$2.\ z = \frac{x-\mu}{\sigma} = \frac{10.075-10}{0.03} = 2.5$$

a) $P(X > 10.075) = P(Z > 2.5)$

$= 1 - P(Z \leq 2.5)$

$= 1 - 0.9938 = 0.0062$

b) $P(9.97 < X < 10.03)$

$$\left.\begin{array}{l} z_1 = \dfrac{x_1-\mu}{\sigma} = \dfrac{9.97-10}{0.03} = -1 \\ z_2 = \dfrac{x_2-\mu}{\sigma} = \dfrac{10.03-10}{0.03} = 1 \end{array}\right\} \Rightarrow \begin{array}{l} P(-1 < Z < 1) \\ = P(Z<1) - P(Z<-1) \\ = 0.8413 - 0.1587 = 0.6826 \end{array}$$

c) $P(X < x) = 0.15$

$$P\left(Z < \frac{x-\mu}{\sigma}\right) = 0.15$$

$$P\left(Z < \frac{x-10}{0.03}\right) = 0.15$$

$$P(Z < -1.036) = 0.15$$

$$\Rightarrow \frac{x-10}{0.03} = -1.036 \Rightarrow x = 9.9689$$

$$3.\ z = \frac{x-\mu}{\sigma} = \frac{10-8}{2} = 1$$

a) $P(X>10) = \cancel{1 - P(X<10)}\ P(Z>1)$

$= 1 - P(Z \leq 1)$

$= 1 - 0.8413 = 0.1587$

b) $E(X) = \$5 \times \cancel{0.8413}\ 0.1587 = 0.7935$

4. $P(|X-9| > 1.5) = P(|Z| > 1.5) = 1 - P(|Z| < 1.5)$

$\boxed{Z = \dfrac{X-\mu}{\sigma} = X - 9}$ $= 1 - P(-1.5 < Z < 1.5)$

$= 1 - [P(Z<1.5) - P(Z<-1.5)]$

$= 1 - [P(Z<1.5) - (1 - P(Z-1.5))]$

$= 2 - 2P(Z<1.5) = 2 - 2 \times 0.9332 = 0.1336$

b) $$Z = \frac{X-\mu}{\sigma} = \frac{X-9}{\sigma}$$

$$P(9-1.5 < X < 9+1.5) = 0.99$$

$$\cancel{\Rightarrow 0.99 =}$$

$$= P\left(\frac{7.5-9}{\sigma} < \frac{X-9}{\sigma} < \frac{10.5-9}{\sigma}\right)$$

$$= P\left(-\frac{1.5}{\sigma} < Z < \frac{1.5}{\sigma}\right)$$

$$= P\left(Z < \frac{1.5}{\sigma}\right) - P\left(Z < -\frac{1.5}{\sigma}\right)$$

$$= P\left(Z < \frac{1.5}{\sigma}\right) - \left[1 - P\left(Z < \frac{1.5}{\sigma}\right)\right]$$

$$= 2P\left(Z < \frac{1.5}{\sigma}\right) - 1$$

$$\Rightarrow P\left(Z < \frac{1.5}{\sigma}\right) = \frac{1.99}{2} = 0.995 \Rightarrow \frac{1.5}{\sigma} = 2.575$$

$$\Rightarrow \sigma = 0.5825$$