

### **Propositional Logic**

Symbolizing parts of arguments so that one can analyze whole arguments for validity

The method for analyzing arguments for validity involves assigning variables to the different parts of the argument, just like in algebra

© 2019 McGraw-Hill Companies. All Rights Reserved.

)-2

### **Conjunction**

When two simple statements are conjoined with an "and," we call it a "conjunction"

We represent each statement as a simple letter, and represent the "and" with an "&"

 Example: "Tina is tall, and Sarah is tall" gets symbolized as "p & q"

### Get it?

- p = Tina is tall
- q = Sarah is tall

© 2019 McGraw-Hill Companies. All Rights Reserved.

10-3

### **Truth Tables, 1**

Each of the variables has two possible truth values

· It could either be true or it could be false

Truth tables allow you to evaluate statements and arguments without knowing truth values by representing all possible truth value combinations

© 2019 McGraw-Hill Companies. All Rights Reserved.

### **Truth Tables, 2**

Recall that we symbolized "Tina is tall, and Sarah is tall" as "p & q"  $\,$ 

• We don't know if they are or not, but we can represent all possibilities this way:

р	q	p & q
Т	Т	Т
Т	F	F
F	Т	F
F	F	F

© 2019 McGraw-Hill Companies. All Rights Reserved.

10

### **Truth Tables, 3**

Notice that "p & q" means "both p and q are true"

- Unless both p and q are true, "p & q" will not be true
- That is why, p & q has a T only on the row on which both p and q both have a T as well

p & q is a propositional form that can stand for an infinite number of compound statements

• Example: "The train is late, and the bus is on time"

© 2019 McGraw-Hill Companies. All Rights Reserved.

### **Conjunction: A Word of Caution**

Not every use of the word "and" indicates a compound statement that can be represented by p & q

- Sometimes "and" joins two things within the same simple statement
  - Example: The Knicks and the Bulls are playing each other tonight
    - This can be symbolized as "e"
  - Note: What letter you give it really doesn't matter, just as long as you
    are consistent (use the same letter for that statement every time)
    and don't use the same letter for two different statements in the
    same argument

© 2019 McGraw-Hill Companies. All Rights Reserved.

10-

### Other Words for "And," 1

If you see any of the following words, treat them like "and" and symbolize the statement with an "&"

"But," "yet," "while," "whereas," "although," "though,"
 and "however"

The following compound statements are all correctly symbolized as p & q:

- Tony had steak, and Theresa had chicken
- Tony had steak, but Theresa had chicken

© 2019 McGraw-Hill Companies. All Rights Reserved.

.0-8

### Other Words for "And," 2

- Tony had steak, yet Theresa had chicken
- Tony had steak, while Theresa had chicken
- Tony had steak, whereas Theresa had chicken
- Tony had steak, although Theresa had chicken
- Tony had steak, though Theresa had chicken
- Tony had steak; however Theresa had chicken

© 2019 McGraw-Hill Companies. All Rights Reserved.

10-

# Using Truth Tables to Examine Validity, 1

Truth tables can be used to evaluate validity because:

- An argument is invalid only when it is possible for its premises to be true and the conclusion false
- Truth tables show us all possible truth values

© 2019 McGraw-Hill Companies. All Rights Reserved.

)-10

# Using Truth Tables to Examine Validity, 2

We use truth tables to determine all the possible truth values, and then look for a row where all the premises are true but the conclusion is false

- · If we find one, the argument is invalid
- · If there is no such row, the argument is valid

### Example

- Tina is tall
- · Sarah is tall
- · So, Tina is tall, and Sarah is tall

© 2019 McGraw-Hill Companies. All Rights Reserved.

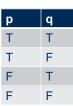
10-1

# Using Truth Tables to Examine Validity, 3

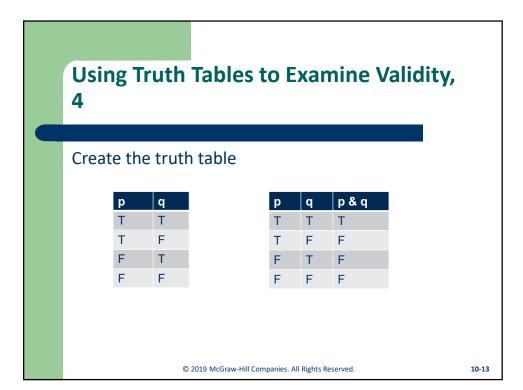
Argument can be symbolized as:

- •
- (
- ∴ p&q

First, represent all the statement letters and their truth values



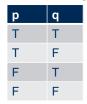
© 2019 McGraw-Hill Companies. All Rights Reserved.

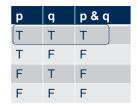


# Using Truth Tables to Examine Validity, 5

Look for rows where the premises are all true, and see if the conclusion is false in those rows

- · If there is such a row, then the argument is invalid
- In this case, the only row with all true premises is the one in which the conclusion is also true
  - · Thus, the argument is valid





© 2019 McGraw-Hill Companies. All Rights Reserved.

### Conjunction and Validity: Example 2, 1

### Example

- Grass is green
- Therefore, grass is green, and the sky is blue
  - Symbolized as:
    - p
    - ∴ p&q
- Truth table format:

р	q	p & q
Т	Т	Т
Т	F	F
F	Т	F
F	F	F

© 2019 McGraw-Hill Companies. All Rights Reserved.

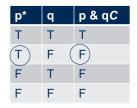
10-1

### Conjunction and Validity: Example 2, 2

### Note the following changes in the truth table:

- Premises are marked with an asterisk
- · Conclusion is marked with a capital C
- Line across that ultimately allows one to determine validity or invalidity will be circled

The second line allows us to determine that the argument form is invalid



© 2019 McGraw-Hill Companies. All Rights Reserved.

LO-16

### **Negation**

Negated statements can easily be represented with a "~"

- If "Sarah is tall" is "p," then "Sarah is not tall" is "~p"
- On a truth table, when p is true, ~p is false and vice versa

© 2019 McGraw-Hill Companies. All Rights Reserved.

10-1

### **Negation: Example A**

Argument: Tina is not tall, but Sarah is tall. So, Tina is not tall

- Symbolic form
  - ~p & q
  - · ~p
- The truth table for the argument
  - There are no cases in which all the premises are true and the conclusion is false, so we know that the argument form is valid

р	q	~p	~p & q*	~pC
Т	Т	F	F	F
Т	F	F	F	F
F	Т	Т	Т	T
F	F	Т	F	Т

© 2019 McGraw-Hill Companies. All Rights Reserved.

### Negation: Example B, 1

Argument: Frank does not drive a truck. So, Frank does not drive a truck, and Vinny does not drive a minivan.

- · Symbolic form
  - · ~p
  - ∴ ~p & ~q
- Truth table

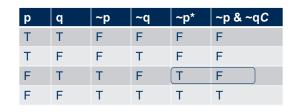
р	q	~p	~q	~p*	~p & ~qC
Т	Т	F	F	F	F
Т	F	F	Т	F	F
F	Т	Т	F	Т	F
F	F	Т	Т	Т	Т

© 2019 McGraw-Hill Companies. All Rights Reserved.

10-19

### **Negation: Example B, 2**

- In the highlighted line, the premise is true and the conclusion is false
  - · Therefore, the argument form is invalid



© 2019 McGraw-Hill Companies. All Rights Reserved.

### **Whole Statement Negations**

### Compound statements can be negated too

 Example: "It is not the case that Lisa drives a Jeep and Jennifer drives a Jeep" can be symbolized as ~(p & q)

Since "p & q" means "Both p and q are true," "~(p & q)" means "It is false that both p and q are true"

### But don't distribute (like in math)

- ~(p & q) is not the same as (~p & ~q)
  - ~(p & q) means they are not both true (at least one is false)
  - (~p & ~q) means they are both false

© 2019 McGraw-Hill Companies. All Rights Reserved.

10-2

### **Proof:** ~(p & q) ≠ (~p & ~q) ~(p & q) p&q F Т F Т F Т F Т ~(p& q) | ~p & ~q р q p&q ~p ~q Т Т F F F Т Т F Т

© 2019 McGraw-Hill Companies. All Rights Reserved.

11

# Truth Tables with Three Variables: Example A, 1

### Argument

- Tina is tall
- Sarah is not tall, but Missy is tall
- So, Tina is tall, and Missy is tall

### Symbolic form

- p
- ~q&r
- ∴ p&r

© 2019 McGraw-Hill Companies. All Rights Reserved.

10-23

# Truth Tables with Three Variables: Example A, 2

### Guide columns for the variables

р	q	r
Т	Т	T
Т	Т	F
Т	F	Т
Т	F	F
F	Т	Т
F	Т	F
F	F	Т
F	F	F

© 2019 McGraw-Hill Companies. All Rights Reserved.

# Truth Tables with Three Variables: Example A, 3

### Truth table for the argument

р	q	r	~q	p*	~q & r*	p & rC
Т	Т	Т	F	Т	F	Т
Т	Т	F	F	Т	F	F
Т	F	Т	Т	Т	Т	T
Т	F	F	Т	Т	F	F
F	Т	Т	F	F	F	F
F	Т	F	F	F	F	F
F	F	Т	Т	F	Т	F
F	F	F	Т	F	F	F

© 2019 McGraw-Hill Companies. All Rights Reserved.

10-25

# Truth Tables with Three Variables: Example A, 4

### The argument is valid because:

- The highlighted row contains true premises and a true conclusion
- There are no cases in which both premises are true and the conclusion is false

© 2019 McGraw-Hill Companies. All Rights Reserved.

# Truth Tables with Three Variables: Example B, 1

### Argument

- ~(p & q)
- ~q&r
- ∴~p

			p & q	~(p & q)*	~q & r*	~p <i>C</i>
Т	Т	F	Т	F	F	F
Т	F	F	Т	F	F	F
F	Т	Т	F	Т	Т	F
F	F	Т	F	T	F	F
Т	Т	F	F	Т	F	Т
Т	F	F	F	Т	F	Т
F	Т	Т	F	Т	Т	Т
F	F	Т	F	Т	F	Т
	T F T T T	T F F T T T T F F T	T F F F T T T F F T T F T T T	T F F T F F T F F F F F F F F F F F F F	T F F T F  T T F T  T T F T  T T F T  T T F F T	T F F T F F  T T F T F  T T F T F  T T F T F

# Truth Tables with Three Variables: Example B, 2

This argument is invalid because both of the premises are true and the conclusion is false in the highlighted row

• It does not matter that in row 8 both of the premises are true and the conclusion is true

© 2019 McGraw-Hill Companies. All Rights Reserved.

### Disjunctions, 1

Two or more statements set apart, usually by the word "or"

- Example: "Frank is angry or Hank is tired"
  - Symbolized as p v q
    - To make things easier, don't ever use the letter "v" to symbolize a simple statement

© 2019 McGraw-Hill Companies. All Rights Reserved.

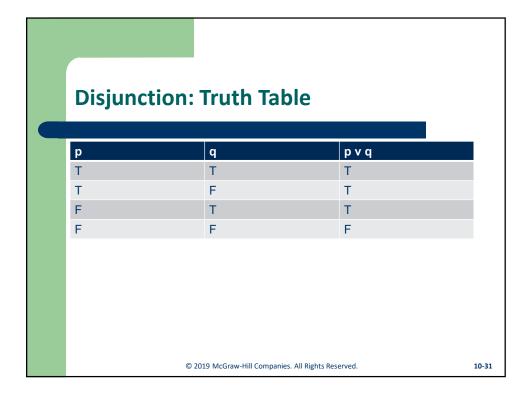
10-29

### Disjunctions, 2

"Or" can have two possible senses

- Exclusive sense: "A or b" means "a or b, but not both"
- Nonexclusive sense: "A or b" means "at least a or b, but maybe both"
  - The convention is to use "or" in its nonexclusive sense

© 2019 McGraw-Hill Companies. All Rights Reserved.



# Argument • Frank is angry or Hank is tired • So, Frank is angry Symbolic form • p v q • ∴ p The highlighted instance of all true premises and a false conclusion establishes that the argument is

© 2019 McGraw-Hill Companies. All Rights Reserved.

invalid

# Be Careful with Negations of Disjunctions, 1

Frank is not angry or Hank is tired

~avt

Frank is not angry or Hank is not tired

~a v ~t

It's not the case that Frank is angry or Hank is tired

• ~(a v t)

© 2019 McGraw-Hill Companies. All Rights Reserved.

10-33

# Be Careful with Negations of Disjunctions, 2

Neither is Frank angry nor is Hank tired

~(a v t)

Frank is not angry and Hank is not tired

(~a & ~t)

© 2019 McGraw-Hill Companies. All Rights Reserved.

)-34

# **Argument That Involves Disjunction, Negation, and Conjunction**

### Argument

- It's not the case that Frank is angry or Hank is tired
- So, Frank is not angry and Hank is not tired

### Symbolic form

- ~(p v q)
- ∴ ~p & ~q

р	q	~p	~q	pvq	~ (p v q)*	~p & ~qC
Т	Т	F	F	Т	F	F
Т	F	F	Т	Т	F	F
F	Т	Т	F	Т	F	F
F	F	Т	Т	F	Т	Т

### The argument is valid

© 2019 McGraw-Hill Companies. All Rights Reserved.

10-3

### Conditional (if, then) statements

Example: "If it rained, then the ground is wet"

- Where:
  - r = it rained (antecedent)
  - w = the ground is wet (consequent)
- Symbolic form: r → w

© 2019 McGraw-Hill Companies. All Rights Reserved.

### **Conditional Statements: Truth Table**

"p→q" means "every time p is true, q is true" or "when p is true, q is true"

So, only when the antecedent is true and the consequent is false, is it the case that the conditional is false?

р	q	p <del>→</del> q
Т	Т	Т
Т	F	F
F	Т	Т
F	F	Т

© 2019 McGraw-Hill Companies. All Rights Reserved.

10-3

# Be Careful with Negation and Conditionals, 1

The following arguments have different meanings:

- If it did not rain, then the game was played
  - ~p→q
- If it did not rain, then the game was not played
  - · ~p→~c
- It is not the case that if it rained then the game was played
  - · ~(p→q)

© 2019 McGraw-Hill Companies. All Rights Reserved.

