Ng. Huy Quang - ITITWE22145

Đạo hàm:

1. $(u.v)' = u'v + u.v'$; $(C.u)' = C.u'$.
$\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2}$; $f'(u) = u' f'(u)$.
$(x^\alpha)' = \alpha x^{\alpha-1}$; $(u^\alpha)' = \alpha u^{\alpha-1} u'$.
$\left(\frac{1}{u}\right)' = -\frac{u'}{u^2}$; $(\sqrt{u})' = \frac{u'}{2\sqrt{u}}$ $(u>0)$
$(\sin x)' = \cos x$; $(\sin u)' = u' \cos u$.
$(\cos u)' = u'(-\sin u)$; $(\tan u)' = \frac{u'}{\cos^2 u}$
$(e^u)' = u' e^u$; $(a^u)' = u' a^u \ln a$.
$(\ln|u|)' = \frac{u'}{u}$; $(\cot u)' = -\frac{u'}{\sin^2 u}$

Nguyên hàm - Antiderivative:

$\int du = u + C$; $\int u^\alpha du = \frac{u^{\alpha+1}}{\alpha+1} + C$; $\int \frac{1}{u} du = \ln|u| + C$
$\int e^u du = e^u + C$; $\int a^u du = \frac{a^u}{\ln a} + C$; $\int \frac{du}{2\sqrt{u}} = \sqrt{u} + C$
$\int \cos u \, du = \sin u + C$; $\int \sin u \, du = -\cos u + C$
$\int e^{ax+b} dx = \frac{1}{a} e^{ax+b} + C$; $\int \frac{dx}{ax+b} = \frac{1}{a} \ln|ax+b| + C$
$\int \cos(ax+b) dx = \frac{1}{a} \sin(ax+b) + C$
$\int \sin(ax+b) dx = -\frac{1}{a} \cos(ax+b) + C$.

Sequences (Dãy Số).

$\ln(x) - \ln(y) = \ln\left(\frac{x}{y}\right)$
$\ln(x) + \ln(y) = \ln(xy)$

The Fibonacci Sequence:
$f_1 = 1$; $f_2 = 1$, $f_n = f_{n-1} + f_{n-2}$.

Limit. If and only if $\lim_{n\to\infty} a_n = L$, then the sequence converges. Otherwise, we say the sequence divergent.

Squeeze Theorem for sequences:
If $a_n \le b_n \le c_n$ for $n \ge n_0$ and $\lim_{n\to\infty} a_n = \lim_{n\to\infty} c_n = L$, then $\lim_{n\to\infty} b_n = L$.

Theorem: If $\lim_{n\to\infty} |a_n| = 0$, then $\lim_{n\to\infty} a_n = 0$.

Theorem: If $\lim_{n\to\infty} a_n = L$ and the f is cont at L, then, $\lim_{n\to\infty} f(a_n) = f(L)$

Theorem: The sequence $\{r^n\}$ is convergent if $-1 < r \le 1$ and divergent for all other values of r.

$$\lim_{n\to\infty} r^n = \begin{cases} 0 & \text{if } -1 < r < 1 \\ 1 & \text{if } r = 1 \end{cases}$$

A sequence is called monotonic If it either increasing or decreasing.

Series: Definition: Given a series $\sum_{n=1} a_n = a_1 + a_2 + \dots$, let S_n denote its nth partial sum: $S_n = \sum_{i=1}^{n} a_i = a_1 + a_2 + \dots$

Properties: (i) $\sum c a_n = c \sum a_n$
(ii) $\sum (a_n + b_n) = \sum a_n + \sum b_n$
(iii) $\sum (a_n - b_n) = \sum a_n - \sum b_n$

If $\sum_{n=1}^{\infty} a_n$ is convergent, then $\lim_{n\to\infty} a_n = 0$

Test For Series:

I. Test for Divergence:

divergent. (If you can see that $\lim_{n\to\infty} a_n$ maybe different from 0, then apply the Test for Divergence).

II. p-Series:

If the sequence is of the form $\sum \frac{1}{n^p}$, then it is a p-series, which we know to be convergent if $p > 1$ and divergent if $p \le 1$

III. Geometric Series:

If the series has a form $\sum a r^{n-1}$ or $\sum a r^n$, then it is a geometric series, which converges if $|r| < 1$ and diverges if $|r| \ge 1$. Some preliminary algebraic manipulation may be required to bring series into this form.

$\sum_{n=1}^{\infty} a r^{n-1} = a + ar + ar^2 + \dots = \frac{a}{1-r}$ $|r| < 1$

If $|r| \ge 1$, The geometric series is divergent

$\boxed{S_n = \frac{a(1-r^n)}{1-r}}$ $a \ne 0$
r: common ratio

- $\sum_{n=0}^{\infty} x^n = 1 + x + x^2 + x^3 + \dots = \frac{1}{1-x}$ since $a = 1$ and $r = x$ ($|r| = |x| < 1$).

IV Integral Test:

If $a_n = f(n)$, where $\int_1^\infty f(x) dx$ is easily evaluated, then the integral test is effective (assuming the hypotheses of this test are satisfied).

Def: Suppose f is cont, pos, decreasing func.
(i) If $\int_1^\infty f(x) dx$ is conv, then $\sum_{n=1}^{\infty} a_n$ is conv
(ii) if $\int_1^\infty f(x) dx$ is diver, then $\sum_{n=1}^{\infty} a_n$ is diver.

V. Comparison Tests:

If the series has a form that is similar to a p-series or a geometric series, then one of the comparison tests should be considered.

Def: Suppose that $\sum a_n$ and $\sum b_n$ are series with pos terms.
(i) If $\sum b_n$ is convergent and $a_n \le b_n$, then a_n is also convergent
(ii) If $\sum b_n$ is divergent and $a_n \ge b_n$, then $\sum a_n$ also is divergent.

VI The limit comparison Tests:

Def: Suppose that $\sum a_n$ and $\sum b_n$ are series with positive terms. If $\lim_{n\to\infty} \frac{a_n}{b_n} = C$ where c is finite number and $c > 0$, then either both series converge or both diverge.

VII: Alternating Series Test:

If the series is of the form $\sum (-1)^{n-1} b_n$ or $\sum (-1)^n b_n$, then the alternating series test is an obvious possibility.

Def: $\sum_{n=1}^{\infty} (-1)^{n-1} b_n = b_1 - b_2 + b_3 - b_4 + \dots$ $(b_n > 0)$
Satisfy the conditions (i) $b_{n+1} \le b_n$ for all n
(ii) $\lim_{n\to\infty} b_n = 0$
then the series is convergent.

VIII The Ratio Test and Root test:

Ratio Test: $\lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right| = L$
Root Test: $\lim_{n\to\infty} \sqrt[n]{|a_n|} = L$.

If:
- $L < 1$: Absolute convergence.
- $L = 1$: No conclusion.
- $L > 1$: Divergence.

Absolute Convergence:

Def: $\sum a_n$ is absolutely convergent if $\sum |a_n|$ is convergent.
$\sum a_n$ is conditionally convergent if it is convergent but $\sum |a_n|$ is divergent.

Note: $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^p} = 1 - \frac{1}{2^p} + \frac{1}{3^p} - \frac{1}{4^p} + \dots$, $p > 0$
If $p > 1$, the series converges absolutely
$0 \le p \le 1$, the series converges but not absolutely

The approximation error using S_n as an estimate of S is:

$$\boxed{\begin{gathered} S - S_n \le a_{n+1} \\ |error| \le a_{n+1} \end{gathered}}$$

Power Series:

c: coefficient of series.

Form: $\sum_{n=0}^{\infty} c_n x^n = c_0 + c_1 x + c_2 x^2 + c_3 x^3 + \dots$

Generally, $\sum_{n=0}^{\infty} c_n (x-a)^n = c_0 + c_1(x-a) + c_2(x-a)^2 + \dots$

Interval of convergence:
For a power series $\sum_{n=0}^{\infty} c_n (x-a)^n$, there are only three possibilities:
(i) The series converges only when $x = a$
(ii) The series converges for all x
(iii) There is a positive number R such that the series converges if $|x-a| < R$ and diverges if $|x-a| > R$.

Note: $\frac{1}{1-x} = 1 + x + x^2 + x^3 + \dots = \sum_{n=0}^{\infty} x^n$, $|x| < 1$.

Taylor series and Maclaurin Series:

If f has a power series rep. (expand) at a,
$$\boxed{f(x) = \sum_{n=0}^{\infty} c_n (x-a)^n, \ |x-a| < R.}$$
then its coefficients: $c_n = \frac{f^{(n)}(a)}{n!}$

Taylor Series: centered at a (about a)
$$f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!} (x-a)^n = f(a) + \frac{f'(a)}{1!}(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \dots$$

Maclaurin Series: $a = 0$.
$$f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(0)}{n!} x^n = f(0) + \frac{f'(0)}{1!} + \frac{f''(0)}{2!} x^2 + \dots$$

Important Maclaurin Series and their Radius of convergence.

$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n = 1 + x + x^2 + x^3 + \dots$ $R = 1$
$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} = \sum_{n=0}^{\infty} \frac{e^a}{n!}(x-a)^n = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \dots$ $R = \infty$
$\sin x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!} = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \dots$
$\lim_{n\to\infty} \frac{x^n}{n!} = 0$ for every real number.
$\cos x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!} = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \dots$ $R = \infty$
$\tan^{-1} x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{2n+1} = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \dots$ $R = 1$
$\ln(1+x) = \sum_{n=0}^{\infty} (-1)^{n-1} \frac{x^n}{n} = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \dots$ $R = 1$
$\ln(1-x) = -\sum_{n=1}^{\infty} \frac{x^n}{n} = -x - \frac{x^2}{2} - \frac{x^3}{3} - \dots$, $|x| = 1$, $R = 1$

Vectors. 1. Def:

$\vec{u} = (x; y; z) \Leftrightarrow \vec{u} = x\vec{i} + y\vec{j} + z\vec{k}$.
The standard of basic vectors:
$\vec{i} = (1,0,0)$; $\vec{j} = (0,1,0)$; $\vec{z} = (0,0,1)$.

2. Properties: $\vec{a} = (a_1, a_2, a_3)$, $\vec{b} = (b_1, b_2, b_3)$
- $\vec{a} + \vec{b} = (a_1 + b_1, a_2 + b_2, a_3 + b_3)$
- $\vec{a} - \vec{b} = (a_1 - b_1, a_2 - b_2, a_3 - b_3)$
- $c\vec{a} = (ca_1, ca_2, ca_3)$, $c \in \mathbb{R}$.
- $\vec{a} = \vec{b}$ $(\vec{a} // \vec{b}) \Leftrightarrow \begin{cases} a_1 = b_1 \\ a_2 = b_2 \\ a_3 = b_3 \end{cases}$; $\vec{a}.\vec{b} = a_1 b_1 + a_2 b_2 + a_3 b_3$.
- $\vec{a} \perp \vec{b} \Leftrightarrow \vec{a}.\vec{b} = 0 \Leftrightarrow a_1 b_1 + a_2 b_2 + a_3 b_3 = 0$.
- $(\vec{a})^2 = a_1^2 + a_2^2 + a_3^2$; $|\vec{a}| = \sqrt{a_1^2 + a_2^2 + a_3^2}$

Properties of vectors: If $\vec{a}$, $\vec{b}$ and $\vec{c}$ are Vector and c and d are scalars then:
1. $\vec{a} + \vec{b} = \vec{b} + \vec{a}$
2. $\vec{a} + (\vec{b} + \vec{c}) = (\vec{a} + \vec{b}) + \vec{c}$
3. $\vec{a} + 0 = \vec{a}$
4. $\vec{a} + (-\vec{a}) = 0$
5. $c(\vec{a} + \vec{b}) = c\vec{a} + c\vec{b}$
6. $(c+d)\vec{a} = c\vec{a} + d\vec{a}$
7. $(cd)\vec{a} = c(d\vec{a})$
8. $1\vec{a} = \vec{a}$.

The length of $A(x_1, y_1, z_1)$ and $B(x_2, y_2, z_2)$
$|\overrightarrow{AB}| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$.

Note: Any vectors whose length is 1 is a unit vector. Ex: $\vec{i}, \vec{j}, \vec{k}$ are unit vectors
The direction of the vector:
$u = \frac{1}{|a|} a = \frac{a}{|a|}$ $a(x_1, x_2, x_3)$

The Dot Product: $\vec{a}(a_1, a_2, a_3)$, $\vec{b}(b_1, b_2, b_3)$
The dot Product of a and b is a number
$a \cdot b = a_1 b_1 + a_2 b_2 + a_3 b_3$

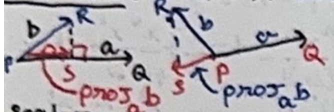
Properties:
1. $\vec{a} \cdot \vec{a} = |\vec{a}|^2 = a_1^2 + a_2^2 + a_3^2$
2. $\vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{a}$
3. $\vec{a} \cdot (\vec{b} + \vec{c}) = \vec{a} \cdot \vec{b} + \vec{a} \cdot \vec{c}$
4. $(d\vec{a}) \cdot \vec{b} = d(\vec{a} \cdot \vec{b}) = a \cdot (d\vec{b})$.
5. $0 \cdot \vec{a} = \vec{a} \cdot 0 = 0$.

Theorem: If θ is the angle between $\vec{a}$ and $\vec{b}$, then: $\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}| \cos\theta$.
$\Rightarrow \cos\theta = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}||\vec{b}|}$

$$a \cdot b \begin{cases} > 0 : \theta \text{ acute} \\ = 0 : \theta = \frac{\pi}{2} \\ < 0 : \theta \text{ obtuse.} \end{cases}$$

Two vectors a and b are orthogonal if and If only $a \cdot b = 0$.

Projections (hình chiếu). $a = \vec{PQ}$; $b = \vec{PR}$



scalar projection of b onto a $\left\{ comp_a b = \frac{a \cdot b}{|a|} \right\}$

vector projection of b onto a:

$\left\{ proj_a b = \left(\frac{a\cdot b}{|a|}\right)\frac{a}{|a|} = \frac{a\cdot b}{|a|^2}(a) \right\}$

Work: The work done by a constant force $\vec{F}$ acting through a displacement $\vec{PQ}$

$\left\{ work = \vec{F}\cdot\vec{PQ} = |F||\vec{PQ}|\cos\theta \right\}$

The cross Product: $\vec{a} = (a_1, a_2, a_3)$, $\vec{b} = (b_1, b_2, b_3)$

$a\times b = (a_2b_3 - a_3b_2, a_3b_1 - a_1b_3, a_1b_2 - a_2b_1)$

Determinant of order 3:

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = a_1\begin{vmatrix} b_2 & b_3 \\ c_2 & c_3\end{vmatrix} - a_2\begin{vmatrix} b_1 & b_3 \\ c_1 & c_3\end{vmatrix} + a_3\begin{vmatrix} b_1 & b_2 \\ c_1 & c_2\end{vmatrix}$$

If: $a = a_1\vec{i} + a_2\vec{j} + a_3\vec{k}$

$b = b_1\vec{i} + b_2\vec{j} + b_3\vec{k}$.

$$a\times b = \begin{vmatrix} i & j & k \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3\end{vmatrix} = \left[\begin{vmatrix} a_2 & a_3 \\ b_2 & b_3\end{vmatrix} i - \begin{vmatrix} a_1 & a_3 \\ b_1 & b_3\end{vmatrix} j - \begin{vmatrix} a_1 & a_2 \\ b_1 & b_2\end{vmatrix}\vec{k}\right]$$

Theorem: The vector $a\times b$ is orthogonal to both a and b.

Theorem: θ is the angle btw a and b

$\left\{ |a\times b| = |a||b|\sin\theta \right\}$

Corollary: Two nonzero vector $a // b$ if:

$\left\{ a\times b = 0 \right\}$

Application: $\vec{a}$, $\vec{b}$ and $\vec{c}$ coplanar: $\boxed{(\vec{a}\times\vec{b})\cdot\vec{c} = 0}$

A, B, C, D uncoplaner $\left\{ (\vec{AB}\times\vec{AC})\cdot\vec{AP} \neq 0 \right\}$

Area of parallelogram $\left\{ A_{ABCD} = |\vec{AB}\times\vec{AD}| \right\}$

Area of triangle: $\left\{ A_{ABC} = \frac{1}{2}|\vec{AB}\times\vec{AC}| \right\}$

Volume of the box (parallelepiped):

$V = |a\cdot(b\times c)| = |(\vec{AB}\times\vec{AD})\cdot\vec{AA'}|$.

Theorems:

$i\times j = k$ $\quad j\times k = i$ $\quad k\times i = j$

$j\times i = -k$ $\quad k\times j = -i$ $\quad i\times k = -j$.

Note that: $i\times j \neq j\times i \Rightarrow (a\times b)\times c \neq a\times(b\times c)$

Properties:

1. $a\times b = -b\times a$ 2. $(da)\times b = d(a\times b) = a\times(db)$
3. $a\times(b+c) = a\times b + a\times c$ 3. $(a+b)\times c = a\times c + b\times c$
5. $a\cdot(b\times c) = (a\times b)\cdot c$ 6. $a\times(b\times c) = (a\cdot c)b - (a\cdot b)c$
7. $a\times a = 0$ $\forall a$ in V_3

Equation of Line: $\vec{u} // d$ or $\vec{u} \equiv d$, thus: $\vec{u}$ is directional vector of the line d: $\vec{u} // d$

The line through $M(x_0, y_0, z_0)$ and $\vec{u} = (a,b,c)$

Parametric equations (PT tham số)

$\begin{cases} x = x_0 + at \\ y = y_0 + bt \quad (t\in\mathbb{R}) \\ z = z_0 + ct \end{cases}$

Symmetric equations (PT chính tắc)

$\frac{x-x_0}{a} = \frac{y-y_0}{b} = \frac{z-z_0}{c} = t$.

Equation of line segments

$r = r_0 + t(r_1 - r_0) = (1-t)r_0 + tr_1$

Skew lines: they do not intersect and are not parallel (and therefore do not lie in the same plane)

Dạng:

1. d qua $M(x_0,y_0,z_0)$ có $\vec{u}$: d $\begin{cases} M(x_0,y_0,z_0) \\ \vec{u}_d = (a,b,c). \end{cases}$
2. d qua A và B: d $\begin{cases} \text{Qua A (hay B)} \\ \vec{u}_d = \vec{AB}. \end{cases}$
3. d qua M và // Δ: d $\begin{cases} M(x_0,y_0,z_0) \\ \vec{u}_d = \vec{u}_\Delta \end{cases}$ $\begin{pmatrix} // Ox \Rightarrow \vec{u}_d = \vec{i} \\ // Oy \Rightarrow \vec{u}_d = \vec{j} \\ // Oz \Rightarrow \vec{u}_d = \vec{k} \end{pmatrix}$
4. d qua M và ⊥(P); d $\begin{cases} M(x_0,y_0,z_0) \\ \vec{u}_d = \vec{n}_{(P)}. \end{cases}$
5. d // (P) và (Q): d $\begin{cases} M(x_0,y_0,z_0) \\ \vec{u}_d = (\vec{n}_P\times\vec{n}_Q) \end{cases}$
6. d // (P) và ⊥ Δ: d $\begin{cases} M(x_0,y_0,z_0) \\ \vec{u}_d = (\vec{n}_P\times\vec{u}_\Delta) \end{cases}$

Equation of planes:

orthogonal vector $\vec{n} = (a,b,c)$ is called normal vector. If $\vec{a}, \vec{b} // (P) \Rightarrow \vec{n} = (\vec{a}\times\vec{b})$

The plane pass through $M(x_0,y_0,z_0)$ and $\vec{n}(A,B,C)$, thus:

$A(x-x_0) + B(y-y_0) + C(z-z_0) = 0$

Rearrange: $Ax + By + Cz + D = 0$

($\vec{n}(A,B,C)$ is normal vector).

Special Planes:

$(Oxy): z = 0$; $(Oyz): x = 0$, $(Oxz): y = 0$

Dạng Bài

1. (P) // (Q) và qua M: (P) $\begin{cases} M(x_0,y_0,z_0) \\ \vec{n}_P = \vec{n}_Q = (A,B,C) \end{cases}$
2. (P) qua A, B, C: (P) $\begin{cases} \text{qua A (hay B hay C)} \\ \vec{n}_{(P)} = (\vec{AB}\times\vec{AC}). \end{cases}$
3. (P) qua M và ⊥ (α) và (β)

(P) $\begin{cases} M(x_0,y_0,z_0) \\ \vec{n}_{(P)} = (\vec{n}_{(\alpha)}\times\vec{n}_{(\beta)}). \end{cases}$

$(\alpha): \boxed{\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1}$ $\begin{pmatrix} \text{cắt Ox tại } A(a,0,0) \\ \text{Oy tại } B(0,b,0) \\ \text{Oz tại } C(0,0,c) \end{pmatrix}$

Sphere equation:

center $C(a,b,c)$ and radius R.

$(S): (x-a)^2 + (y-b)^2 + (z-c)^2 = R^2$

$x^2 + y^2 + z^2 - 2ax - 2by - 2cz + d = 0$.

Radius of sphere: $R = \sqrt{a^2+b^2+c^2-d}$.

$(S): x^2+y^2+z^2 = R^2$: $\begin{cases} O(0,0,0) \\ \text{bán kính } R. \end{cases}$

Def: $x^2+y^2+z^2 \leq 4$: The solid ball bounded by the sphere $x^2+y^2+z^2 = 4$.

$x^2+y^2+z^2 = 4$ $(z\leq 0)$: The lower hemisphere cut from the sphere $x^2+y^2+z^2 = 4$ by (Oxy)

Angle:

Giữa hai mp(P) và (Q): $\begin{cases} \vec{n}_P = (A_1, A_2, A_3) \\ \vec{n}_Q = (B_1, B_2, B_3) \end{cases}$

$\cos(P,Q) = \frac{|A_1B_1 + A_2B_2 + A_3B_3|}{\sqrt{A_1^2+A_2^2+A_3^2}\sqrt{B_1^2+B_2^2+B_3^2}}$

Giữa 2 đg d và d' có $\vec{u}(a,b,c)$ và $\vec{u'} = (a',b',c')$

$\cos\theta = \frac{|aa' + bb' + cc'|}{\sqrt{a^2+b^2+c^2}\sqrt{(a')^2+(b')^2+(c')^2}}$

Giữa đg với mp: $\vec{u}(a,b,c)$, $\vec{n}(A,B,C)$

$\sin\theta = \frac{|Aa + Bb + Cc|}{\sqrt{A^2+B^2+C^2}\sqrt{a^2+b^2+c^2}}$

Distance: 1. Point to plane: $M(x_0,y_0,z_0)$

$d(M,(P)) = \frac{|Ax_0 + By_0 + Cz_0 + D|}{\sqrt{A^2+B^2+C^2}}$

2. Point to line (with $M_0 \in d$)

$d(M_1, d) = \frac{|(\vec{M_1M_0}\times\vec{u}_d)|}{|\vec{u}_d|}$

Cylinders and Quadric Surface:

Eq of cylinder: $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$; $\frac{y^2}{a^2} + \frac{z^2}{b^2} = 1$; $\frac{x^2}{a^2} + \frac{z^2}{b^2} = 1$

Vector function:

$r(t) = \langle f(t), g(t), h(t)\rangle = f(t)i + g(t)j + h(t)k$

Limits

$\lim_{t\to a} r(t) = (\lim_{t\to a} f(t), \lim_{t\to a} g(t), \lim_{t\to a} h(t))$.

Continuity: A vector $r(t)$ is continuous at a if $\lim_{t\to a} r(t) = r(a)$

Derivatives of vector functions

$r'(t) = \langle f'(t), g'(t), h'(t)\rangle$

$= f'(t)i + g'(t)j + h'(t)k$.

If this limit exists; The tangent vector is: $\left\{ T(t) = \frac{r'(t)}{|r(t)|} \right\}$

Differentiation Rules: u and v are diff, c is a scalar

1. $\frac{d}{dt}[u(t) + v(t)] = u'(t) + v'(t)$
2. $\frac{d}{dt}[cu(t)] = cu'(t)$
3. $\frac{d}{dt}[f(t)u(t)] = f'(t)u(t) + f(t)u'(t)$
4. $\frac{d}{dt}[u(t)\cdot v(t)] = u'(t)\cdot v(t) + u(t)\cdot v'(t)$
5. $\frac{d}{dt}[u(t)\times v(t)] = u'(t)\times v(t) + u(t)\times v'(t)$
6. $\frac{d}{dt}[u(f(t))] = f'(t)u'(f(t))$
7. $\frac{d}{dt}[u(t)\cdot u(t)] = 2r'(t)\cdot r(t)$.

If $|r(t)| = C$ (const), then $r'(t)$ is orthogonal to $r(t)$ for all t.

Integrals

$\int_a^b r(t)dt = (\int_a^b f(t)dt)i + (\int_a^b g(t)dt)j + (\int_a^b h(t)dt)k$

$= R(t)|_a^b = R(b) - R(a)$

Motion in Space

$a(t) = v'(t) = r''(t)$

$\Rightarrow r(t) = \int v(t)dt$; $r'(t) = v(t)$

Note: $\boxed{\lim_{x\to 0}\frac{\sin x}{x} = 1}$; $\boxed{\lim_{x\to 0}\frac{\cos x}{x} = 0}$

If limit has form $\frac{0}{0}$: use L'Hopital's Rule

$\tan x \to \infty$ as $x \to (\frac{\pi}{2})^-$ $\Rightarrow \lim_{x\to\infty}\tan^{-1}x = \frac{\pi}{2}$.

$\lim_{n\to\infty}\frac{\cos(n)}{2^n} = 0$

Chapter 11: True-False:

1. If $\lim_{n\to\infty} a_n = 0$, then $\sum a_n$ is convergent (F)

Note that: $\lim_{n\to\infty} a_n = \lim_{n\to\infty}\frac{1}{n} = 0$. But $\sum\frac{1}{n}$ is the harmonic series is divergent.

2. The series $\sum_{n=1}^{\infty} n^{-\sin 1}$ is convergent (F)

By p-series, $\sum_{n=1}^{\infty}\frac{1}{n^{\sin 1}}$ with $\sin 1 < 1$. Hence, diver

3. If $\lim_{n\to\infty} a_n = L$, then $\lim_{n\to\infty} a_{2n+1} = L$ (T)

4. If $\sum c_n 6^n$ is convergent, then $\sum c_n(-2)^n$ is conv (T)

5. If $\sum c_n 6^n$ is convergent, then $\sum c_n(-6)^n$ is conv (F)

Let $c_n = \frac{(-1)^n}{n6^n}$; $\sum c_n 6^n = \sum\frac{(-1)^n}{n}$ (conv by AST)

$\sum c_n(-6)^n = \sum\frac{1}{n}$ (div by p-series)

6. If $\sum c_n x^n$ diverges when $x = 6$, then it div when $x = 10$. (T) (Let $a_n = c_n 10^n$ and $b_n = c_n 6^n$)

7. The Ratio Test can be used to determine whether $\sum\frac{1}{n^3}$ converges (F)

8. The ratio test can be used to determine whether $\sum\frac{1}{n!}$ converges (T)

9. If $0 \leq a_n \leq b_n$ and $\sum b_n$ div, then $\sum a_n$ div (F)

Consider $a_n = 0$, $b_n = 1$; $\sum b_n$ is div by n-term test

10. $\sum_{n=0}^{\infty}\frac{(-1)^n}{n!} = \frac{1}{e}$ (T) $\left(e^x = \sum_{n=0}^{\infty}\frac{x^n}{n!}; \frac{1}{e} = e^{-1}\right)$

11. If $-1 < \alpha < 1$, then $\lim_{n\to\infty}\alpha^n = 0$ (T)

12. If $\sum a_n$ is divergent, then $\sum|a_n|$ is div (T)

$|a_n| \geq a_n$. Therefore, $\sum|a_n| \geq \sum a_n \Rightarrow \sum|a_n| \geq \infty$

13. If $f(x) = 2x - x^2 + \frac{1}{3}x^3 - \dots$ convergent for all x, then $f'''(0) = 2$. (T) $\left(a_n = \frac{f^{(n)}(0)}{n!}x^n\right)$

14. If $\{a_n\}$ and $\{b_n\}$ are div, then $\{a_n + b_n\}$ is div (F) ($a_n = -n$, $b_n = n$, $a_n + b_n = 0$, conv)

15. If $\{a_n\}$ and $\{b_n\}$ are div, then $\{a_nb_n\}$ is div. (F) ($a_n = (-1)^n$, $b_n = (-1)^{n+1}$, $a_nb_n = -1$, conv)

16. If $\{a_n\}$ is decreasing and $a_n > 0$ for all n, then $\{a_n\}$ is conv. (T) (a_n decrease, so it is monotonic, $a_n > 0$ for all n, it bounded below $\Rightarrow$ conv)

17. If $a_n > 0$ and $\sum a_n$ converges, then $\sum(-1)^n a_n$ conv. (T) ($\sum a_n$ conv $\Rightarrow \lim_{n\to\infty} a_n = 0$)

18. If $a_n > 0$ and $\lim_{n\to\infty}\left(\frac{a_{n+1}}{a_n}\right) < 1$, then $\lim_{n\to\infty} a_n = 0$ (T) ($a_{n+1} < a_n$, $a_n > 0$)