ASSIGNMENT PHYSICS 4

Group 11: Nhữ Thị Thu Ngân - EEACIU21163 (Leader)
Phan Nguyễn Xuân Phúc - ITITIU19042
Nguyễn Thế Hiền - EEACIU20113
Nguyễn Hùng Quốc - ITCSIU21101
Phan Thuỳ Đoan Trang - ITCSIU22257
Phạm Xuân Quý - ITITWE22047
Phùng Huy Quang - ITITWE22145
Vũ Minh Sơn - EEEEIU20044
Lê Tuyết Nhung - ITITDK22043

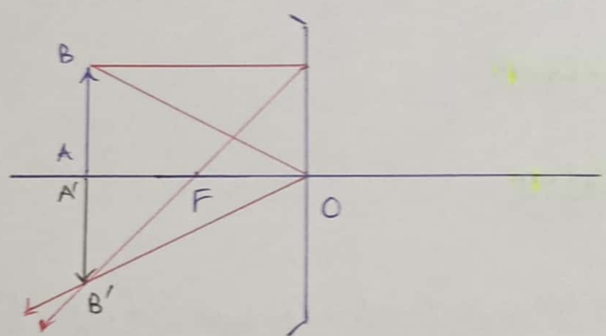
Sol.

Question 1:

(a) Since the concave mirror, so the focal length: $f = 20cm$ and $p = 40cm$

+ Height of object is: $h = 4cm$. Let h' be height of image
+ Mirror equation given that: $\frac{1}{f} = \frac{1}{p} + \frac{1}{q} \to \frac{1}{20} = \frac{1}{40} + \frac{1}{q} \to q = 40cm$ (real image)
+ The size of this image: $M = -\frac{q}{p} = -\frac{40}{40} = -1 = \frac{h}{h'} \to h' = -4cm$

(b)



Question 2:

- The wave fuction of the electron confined to an infinite potential well is given by: $\Psi(x) = \sqrt{\frac{2}{a}} \sin\left(\frac{n\pi}{a} x\right)$. For the ground state: $\Psi_1(x) = \sqrt{\frac{2}{a}} \sin\left(\frac{\pi x}{a}\right)$

(a): The probability that the electron can be detected in the middle one-third of the well:
$\left(x_1 = \frac{a}{3}, x_2 = \frac{2a}{3} \right)$

$$P_{mid} = P\left(\frac{a}{3} < x < \frac{2a}{3}\right) = \int_{a/3}^{2a/3} |\Psi_1(x)|^2 dx = \int_{a/3}^{2a/3} \frac{2}{a} \sin^2\left(\frac{\pi x}{a}\right) dx$$

$$P_{mid} = \frac{2}{a} \int_{a/3}^{2a/3} \frac{1}{2} \times \left(1 - \cos\left(\frac{2\pi x}{a}\right)\right) dx = \frac{1}{a} \int_{a/3}^{2a/3} 1 dx - \frac{1}{a} \int_{a/3}^{2a/3} \cos\left(\frac{2\pi x}{a}\right) dx \quad \left(u = \frac{2\pi x}{a},\ du = \frac{2\pi}{a} dx \right)$$

$$P_{mid} = \frac{1}{a} \times x \Big|_{a/3}^{2a/3} - \frac{1}{2\pi} \times \sin\left(\frac{2\pi x}{a}\right) \Big|_{a/3}^{2a/3} = \frac{1}{3} - \frac{1}{2\pi} \times \left(\sin\left(\frac{4\pi}{3}\right) - \sin\left(\frac{2\pi}{3}\right) \right)$$

$$P_{mid} = \frac{1}{3} - \frac{1}{2\pi}\left(-\frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{2}\right) \approx 0.60 \approx 60\%$$

(b): The probability that the electron can be detected in the left one-third of the well $(x_1 = 0, x_2 = a/3)$

$$P_{left} = P(0 < x < \frac{a}{3}) = \int_0^{a/3} \frac{2}{a}\sin^2(\frac{\pi x}{a})dx = \frac{1}{a}\int_0^{a/3} 1\,dx - \int_0^{a/3} \frac{1}{a}\cos(\frac{2\pi x}{a})dx$$

$$P_{left} = \frac{1}{3} - \frac{1}{2\pi}\sin(\frac{2\pi x}{a})\Big|_0^{a/3} = \frac{1}{3} - \frac{1}{2\pi} \times \frac{\sqrt{3}}{2} \approx 0.20 = 20\%$$

Because the probability distribution is symmetical: $P_{left} = P_{right} = 20\%$

Thus, Normalization condition: $\int_{-\infty}^{+\infty} |\Psi(x)|^2 dx = 1 = P_{mid} + P_{left} + P_{right} = 0.6 + 0.2 + 0.2 = 1$

Question 3: Energy level of hydrogen atom: $E_n = -\frac{13.6}{n^2}$ (eV)

(a): Paschen series: $n = 3 \rightarrow E_3$

- The shortest wavelength: $\Delta E_{\infty 3} = E_\infty - E_3 = 0 - (-\frac{13.6}{9}) = 1.51$

$$\lambda_{shortest} = \frac{hc}{\Delta E_{\infty 3}} = \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{1.51 \times 1.6 \times 10^{-19}} = 823\,nm$$

- The longest wavelength: $\Delta E_{43} = E_4 - E_3 = -\frac{13.6}{16} + \frac{13.6}{9} = 0.66$

$$\lambda_{longest} = \frac{hc}{\Delta E_{43}} = \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{0.66 \times 1.6 \times 10^{-19}} \approx 1880\,nm$$

Thus, all the spectral lines of the Paschen series are in the infrared region ($750nm \rightarrow 1mm$)

(b): The three lagest wavelength of Paschen series: $E_4 \rightarrow E_3$, $E_5 \rightarrow E_3$, $E_6 \rightarrow E_3$

$\lambda_1 = 1880\,nm$ (from question a)

$$\lambda_2 = \frac{hc}{\Delta E_{53}} = \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{(-\frac{13.6}{25} + \frac{13.6}{9}) \times 1.6 \times 10^{-19}} = 1285\,nm$$

$$\lambda_3 = \frac{hc}{\Delta E_{63}} = \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{(-\frac{13.6}{36} + \frac{13.6}{9}) \times 1.6 \times 10^{-19}} = 1097\,nm$$

Question 4: Ground state wave function for a partical in a box: $\Psi(x) = \sqrt{\frac{2}{L}}\sin(\frac{\pi x}{L})$

(a): Probability of finding the particle in the region 0 and $\frac{L}{4}$

$$P(0 < x < \frac{L}{4}) = \int_0^{L/4} |\Psi(x)|^2 dx = \frac{2}{L}\int_0^{L/4} [(1 - \cos(\frac{2\pi x}{a})) \times \frac{1}{2}]dx = \frac{1}{4} - \frac{1}{2\pi} = 0.0908$$

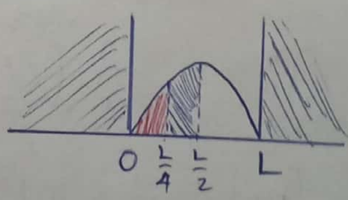
(b): that in region $\frac{L}{4}$ and $\frac{L}{2}$

$$P(\frac{L}{4} < x < \frac{L}{2}) = \int_{L/4}^{L/2} |\Psi(x)|^2 dx = \frac{2}{L}\int_{L/4}^{L/2} \frac{1}{2} \times (1 - \cos(\frac{2\pi x}{L}))\,dx = \frac{1}{4} + \frac{1}{2\pi} = 0.40915$$

(c): $P(0 < x < \frac{L}{4}) < P(\frac{L}{4} < x < \frac{L}{2})$ due to the term's effect

(d):

$$P(0 < x < \frac{L}{4}) + P(\frac{L}{4} < x < \frac{L}{2}) = \frac{1}{2}$$



Because particle is confined within the box, the total probability of finding it inside the box is 1. As a result, the likelihood of locating the particle between $x = 0$ and $x = \frac{L}{2}$ or $x = \frac{L}{2}$ and $x = L$ is equal $= \frac{1}{2}$ of probability