1. Thin lens

Positive lens — Convex

Negative lens — Concave

$$\frac{1}{f} = (n - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$ **(lens makers' equation)**

f is **positive** if the lens is **converging**.
f is **negative** if the lens is **diverging**.

R_2 is negative. (C_2 is on the opposite side from the outgoing light.)

R_1 is positive. (C_1 is on the same side as the outgoing light.)

Radius of curvature of second surface: R_2

Radius of curvature of first surface: R_1

Q

h

C_2

P

n

P'

h'

Q'

C_1

$\longleftarrow p \longrightarrow$ $\longleftarrow q \longrightarrow$
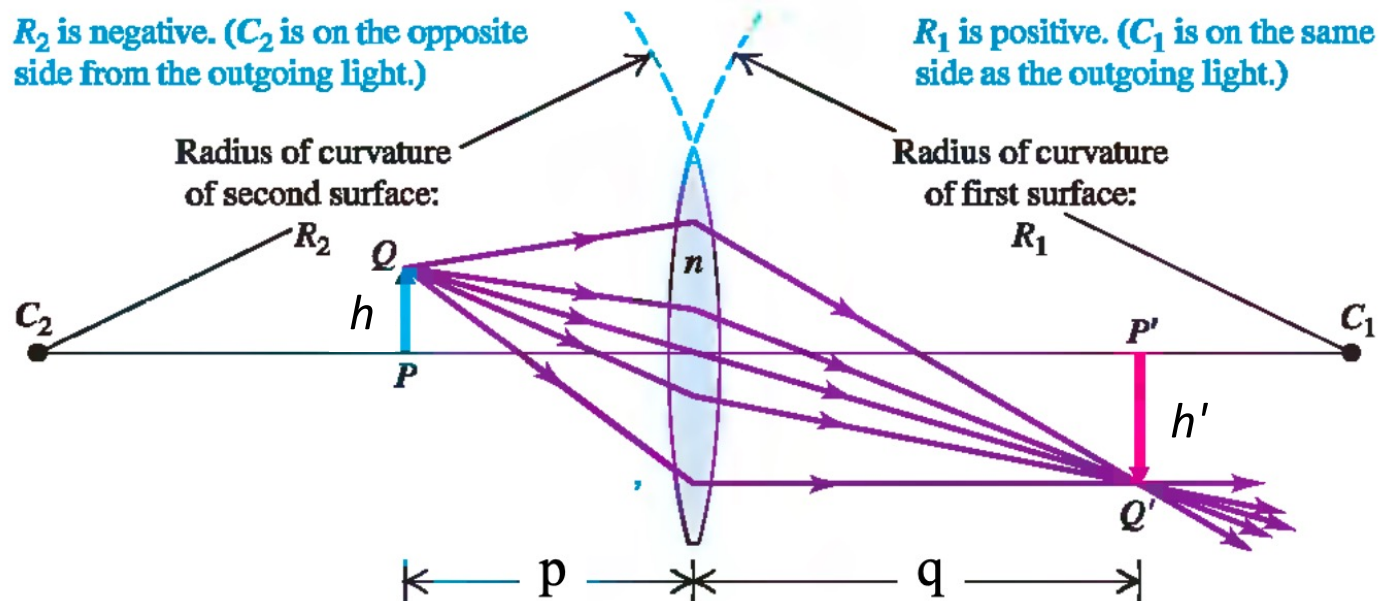
Thin-lens equation :

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f}$$

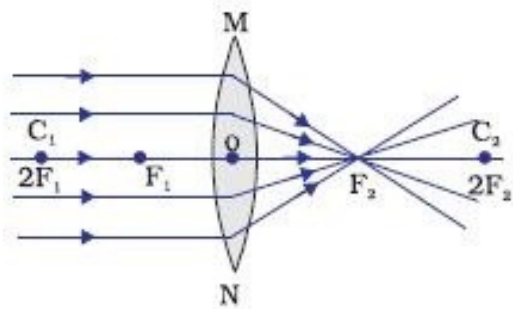| Front	Back
p positive	p negative
q negative	q positive
Incident light	Refracted light
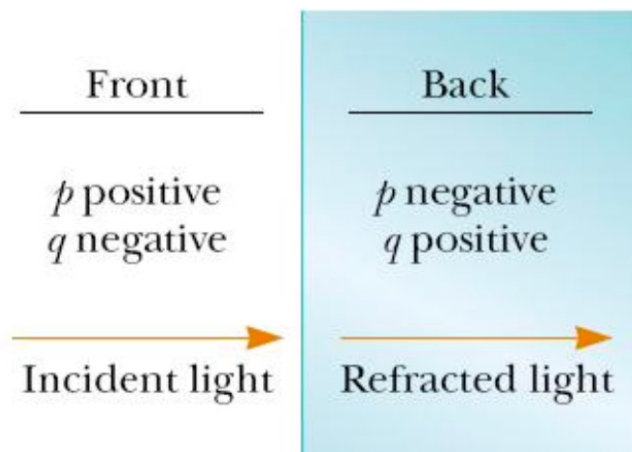
Lateral magnification :
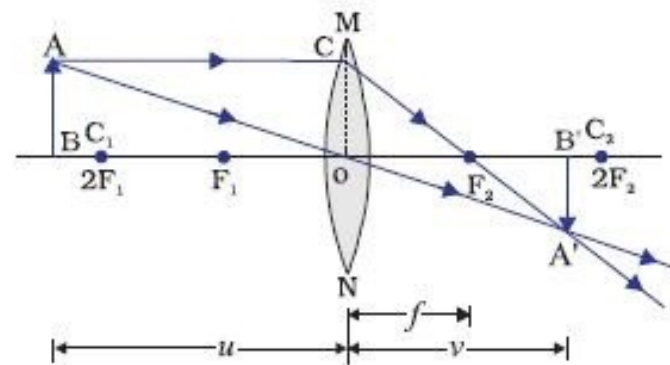
$$M = \frac{h'}{h} = -\frac{q}{p}$$

Real object $\leftrightarrow$ $p > 0$ Virtual object $\leftrightarrow$ $p < 0$
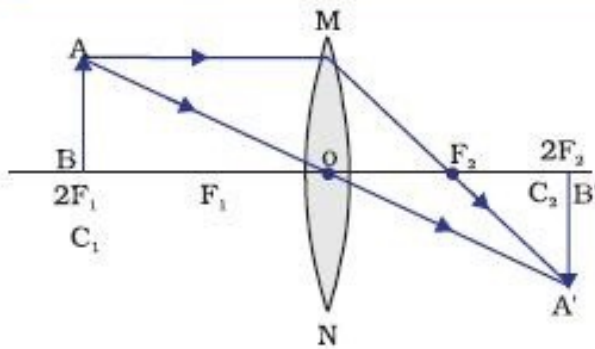
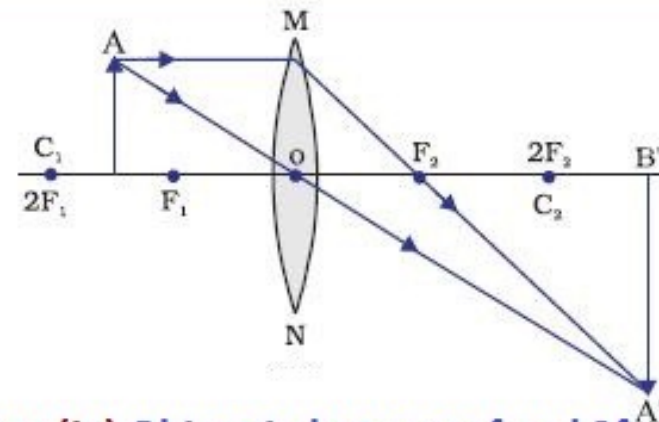Real image $\leftrightarrow$ $q > 0$ Virtual image $\leftrightarrow$ $q < 0$

Case (i) Object at infinity
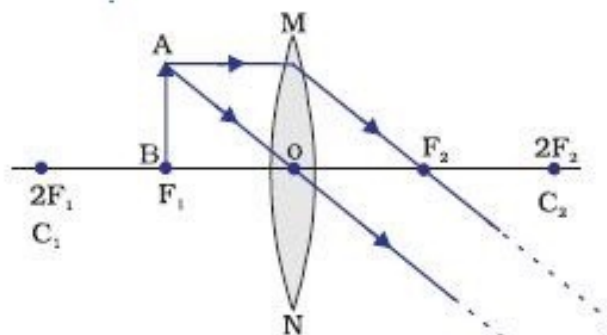
Case (ii) Object at beyond 2f
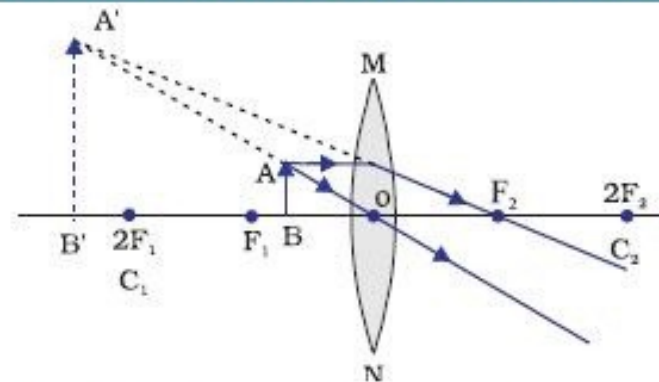
Case (iii) Object at 2f

Case (iv) Object in between f and 2f

Case (v) Object at f

Case (vi) Object distance < f

2. Matter-wave principle

De Broglie's relationships

We recall that
for a photon (E, p) associated to an electromagnetic wave (f, λ):

$$E = h f$$

$$p = h \frac{1}{\lambda}$$

particle *wave*

Planck constant:
$h = 6.63 \times 10^{-34}$ m^2 kg/s

De Broglie's hypothesis:

To a **particle** (E, p) is associated a **matter wave**,

which has a frequency f and a wavelength λ

$$f = \frac{E}{h}$$

$$\lambda = \frac{h}{p}$$

• From $f = \dfrac{E}{h}$ and $\lambda = \dfrac{h}{p}$

if we put: $\boxed{h = 2\pi\,\hbar}$ $\Rightarrow$ $\boxed{E = 2\pi f\,\hbar}$

$$\Rightarrow \boxed{p = \dfrac{2\pi}{\lambda}\hbar}$$

$\left.\begin{array}{c} \boxed{E = \hbar\omega} \\[2mm] \boxed{\vec{p} = \hbar\,\vec{K}} \end{array}\right\}$ **Planck-Einstein's relationship**

λ is called **de Broglie wavelength**

$\vec{K}$ is **the angular wave vector** (describing how many oscillations it completes per unit of distance)

$$\vec{K} = 2\pi/\lambda$$

The Schrödinger's equation

For the case of one-dimensional motion,
when a particle with the mass m has a <u>potential energy</u> $U(x)$
Schrödinger's equation is

$$\frac{d^2\psi}{dx^2} + \frac{8\pi^2 m}{h^2}[E - U(x)]\psi = 0$$

<u>Probability density:</u>

Assume that the free particle travels only in the positive direction
Relabel the constant A as ψ_0: $\quad \psi(x) = \psi_0 e^{iKx}$
The probability density is:

$$|\psi|^2 = |\psi_0 e^{iKx}|^2 = (\psi_0)^2 |e^{iKx}|^2$$

Because:

$$|e^{iKx}|^2 = (e^{iKx})(e^{iKx})^* = (e^{iKx})(e^{-iKx}) = e^0 = 1$$

we have:

$$|\psi|^2 = (\psi_0)^2 = const$$

3. A particle in an infinite well

With n is an integer: $n = 1; 2; 3;...$

the energy can only have the discrete values: $E_n = \left(\dfrac{\pi^2 \hbar^2}{2ma^2}\right) n^2$

$$E_n = \left(\dfrac{\pi^2 (h/2\pi)^2}{2ma^2}\right) n^2 = \dfrac{h^2}{8ma^2} n^2 \longrightarrow \boxed{E_n = \dfrac{h^2}{8ma^2} n^2}$$

We say that the <u>energy is quantized</u>

these values of energy are called <u>energy levels</u>

$n = 1 \longrightarrow$ ground state (E_1)

$n = 2 \longrightarrow$ first excited state (E_2)

$n = 3 \longrightarrow$ second excited state (E_3)

.

.

.

The integer n is called <u>the quantum number</u>

___**3rd excited** E_4

2nd excited
_____ E_3

1st excited
_____ E_2
_____ E_1
ground

| energy-level diagram |

4. Spectral emission lines in Hydrogen atom

When the electron jumps down from an energy level E_m to a lower one E_n, the hydrogen atom emits a photon of energy:
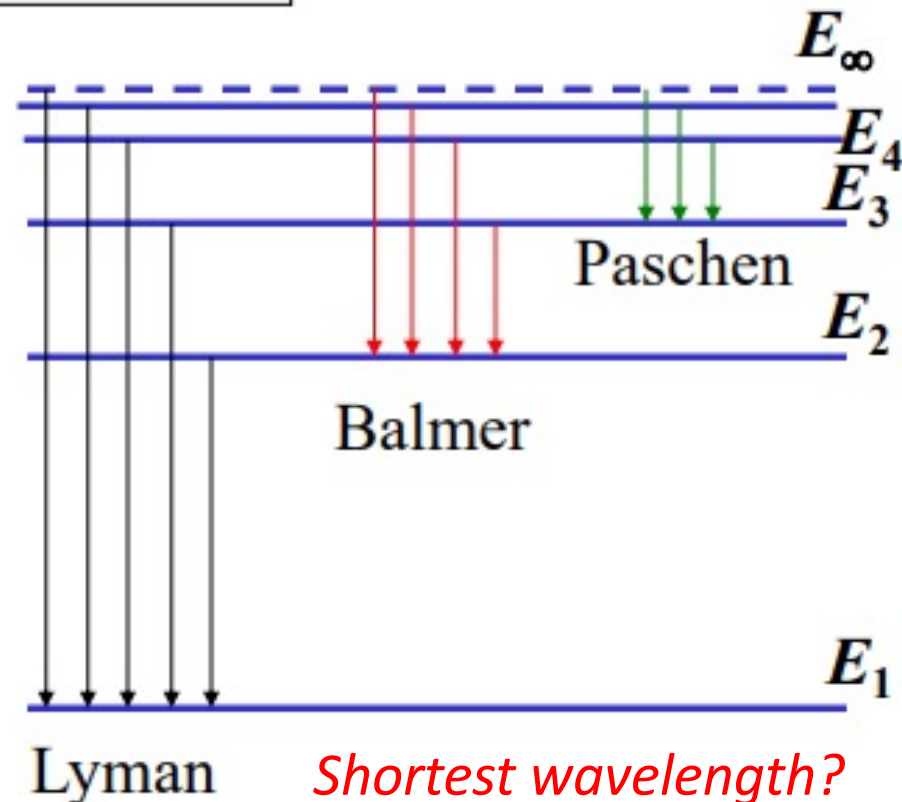
$$\varepsilon = hf_{mn} = \frac{hc}{\lambda_{mn}} = E_m - E_n$$

with

$$E_n = -\frac{13.6 eV}{n^2}$$

- If $E_n \equiv E_1$: Lyman series
- If $E_n \equiv E_2$: Balmer series
- If $E_n \equiv E_3$: Paschen series
- If $E_n \equiv E_4$: Brackett series

$$\frac{1}{\lambda} = R_H \left(\frac{1}{n_f^2} - \frac{1}{n_i^2} \right)$$

n_i, n_f are like m, n above

$$R_H = \frac{mk^2 e^4}{4\pi c \hbar^3} = 1.097 \times 10^7 m^{-1}$$

(Rydberg constant)

E_∞

E_4
E_3

Paschen

E_2

Balmer

E_1

Lyman

Shortest wavelength?
Longest wavelength?

5. Schrodinger equation for hydrogen atom

After solving Schrodinger equation for the radial part we have:

$$E_n = -\left(\frac{me^4}{8\varepsilon_0^2 h^2}\right)\frac{1}{n^2}$$

- *Wave function for the ground state*

The wave function for the ground state of the hydrogen atom:

$$\psi(r) = \frac{1}{a^{3/2}\sqrt{\pi}}e^{-r/a}$$

Where a is the Bohr radius: $a = 0.529 \times 10^{-10} m = 52.9nm$

- *Wave function for the first excited state*

$$\psi(r) = Ce^{-ar/2}(2-ar)$$

- *The radial probability density*

$$P(r) = \frac{4}{a^3}e^{-2r/a}r^2$$

6. Special relativity

$$\Delta t = \frac{\Delta t_0}{\sqrt{1 - u^2/c^2}}$$

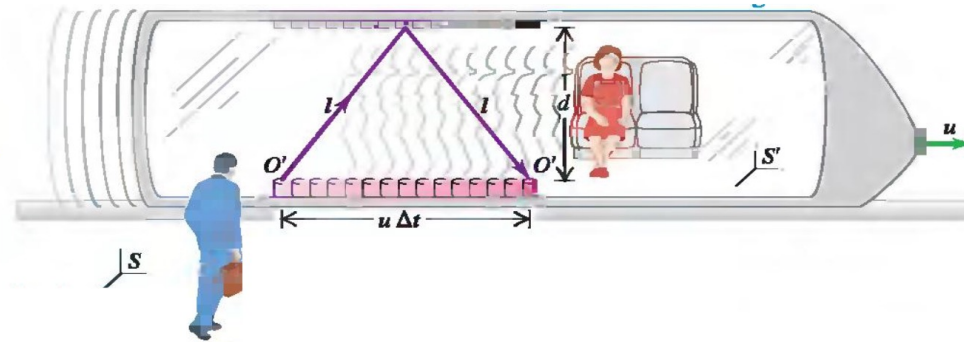$$\Delta t = \gamma \Delta t_0 \; ;$$

$$\gamma = \frac{1}{\sqrt{1 - u^2/c^2}}$$

Lorentz constant

$\Delta t > \Delta t_0$: **time dilation**

Observers measure any clock to run slow if it moves relative to them

$$L = L_0 \sqrt{1 - \frac{v^2}{c^2}}$$

$$L = \frac{L_0}{\gamma} \; ;$$
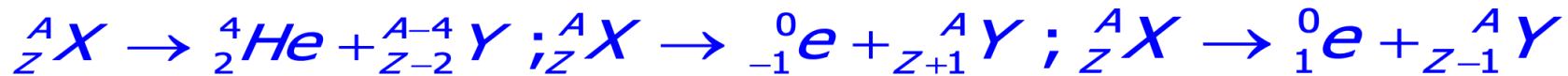
$$\gamma = \frac{1}{\sqrt{1 - u^2/c^2}}$$

$(L < L_0)$ **(Length contraction)**

The length L measured in S, in which the ruler is moving, is shorter than the length L_0 measured in its rest frame S'.

7. Nuclear Radioactivity

• When unstable nuclides decay into different nuclides, they usually emit **alpha** (α) or **beta** (β) particles:

Alpha particle is a ^{4}He nucleus, a beta-minus particle (β^-) is an electron, beta-plus particle (β^+) is a positron (antiparticle of electron)

$$_Z^A X \rightarrow {}_2^4 He + {}_{Z-2}^{A-4} Y \; ; {}_Z^A X \rightarrow {}_{-1}^{0} e + {}_{Z+1}^{A} Y \; ; {}_Z^A X \rightarrow {}_1^0 e + {}_{Z-1}^{A} Y$$

• $dN(t)/dt$ is called the **decay rate or the activity** of the specimen.

Activity: becquerel (Bq) in SI or curie (Ci)

$$1Ci = 3.70 \times 10^{10} Bq = 3.70 \times 10^{10} \text{ decays / s}$$

$$-\frac{dN(t)}{dt} = \lambda N(t) \qquad \boxed{N(t) = N_0 e^{-\lambda t}} \qquad \lambda : \textbf{decay constant}$$

$N(t)$: the (very large) number of radioactive nuclei in a sample at time t.

- **The half-life $T_{1/2}$** is the time required for the number of radioactive nuclei to decrease to one-half the original number N_0

$$T_{1/2} = \frac{\ln 2}{\lambda} = \frac{0.693}{\lambda}$$

- **The mean lifetime T_{mean}** (generally called the lifetime):

$$T_{mean} = \frac{1}{\lambda} = \frac{T_{1/2}}{\ln 2}$$