P2.1. $\mathbf{F} = y\mathbf{a}_x - z\mathbf{a}_y + x\mathbf{a}_z$

(a) $x = y = z; \quad dx = dy = dz$

$$d\mathbf{l} = dx\,\mathbf{a}_x + dx\,\mathbf{a}_y + dx\,\mathbf{a}_z$$

$$\mathbf{F} = x\mathbf{a}_x - x\mathbf{a}_y + x\mathbf{a}_z$$

$$\mathbf{F} \bullet d\mathbf{l} = x\,dx - x\,dx + x\,dx = x\,dx$$

$$\int_{(0,0,0)}^{(1,1,1)} \mathbf{F} \bullet d\mathbf{l} = \int_0^1 x\,dx = \left[\frac{x^2}{2}\right]_0^1 = \frac{1}{2}$$

(b) $x = y = z^3; \quad dx = dy = 3z^2\,dz$

$$d\mathbf{l} = 3z^2\,dz\,\mathbf{a}_x + 3z^2\,dz\,\mathbf{a}_y + dz\,\mathbf{a}_z$$

$$\mathbf{F} = z^3\mathbf{a}_x - z\mathbf{a}_y + z^3\mathbf{a}_z$$

$$\mathbf{F} \bullet d\mathbf{l} = (3z^5 - 3z^3 + z^3)\,dz = (3z^5 - 2z^3)\,dz$$

$$\int_{(0,0,0)}^{(1,1,1)} \mathbf{F} \bullet d\mathbf{l} = \int_0^1 (3z^5 - 2z^3)\,dz = \left[\frac{z^6}{2} - \frac{z^4}{2}\right]_0^1 = 0$$

P2.2. $\mathbf{F} \bullet d\mathbf{l} = (xy\mathbf{a}_x + yz\mathbf{a}_y + zx\mathbf{a}_z) \bullet (dx\,\mathbf{a}_x + dy\,\mathbf{a}_y + dz\,\mathbf{a}_z)$

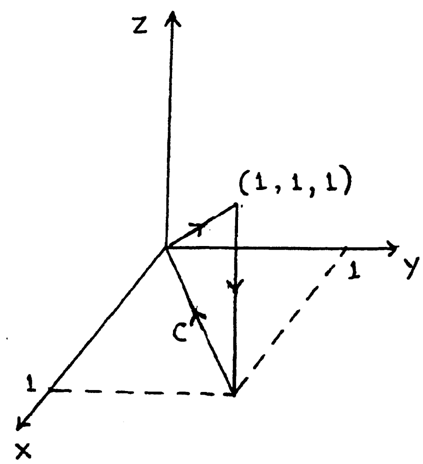
$= xy\,dx + yz\,dy + zx\,dz$

From (0, 0, 0) to (1, 1, 1),

$x = y = z,\ dx = dy = dz$

$\mathbf{F} \bullet d\mathbf{l} = x^2\,dx + x^2\,dx + x^2\,dx = 3x^2\,dx$

$$\int_{(0,0,0)}^{(1,1,1)} \mathbf{F} \bullet d\mathbf{l} = \int_0^1 3x^2\,dx = \left[x^3\right]_0^1 = 1$$



From (1, 1, 1) to (1, 1, 0),

$x = y = 1,\ dx = dy = 0$

$\mathbf{F} \bullet d\mathbf{l} = 0 + 0 + z\,dz = z\,dz$

$$\int_{(1,1,1)}^{(1,1,0)} \mathbf{F} \bullet d\mathbf{l} = \int_1^0 z\,dz = \left[\frac{z^2}{2}\right]_1^0 = -\frac{1}{2}$$

From (1, 1, 0) to (0, 0, 0),

$y = x,\ z = 0;\ dy = dx,\ dz = 0$

$\mathbf{F} \bullet d\mathbf{l} = x^2\,dx + 0 + 0 = x^2\,dx$

$$\int_{(1,1,0)}^{(0,0,0)} \mathbf{F} \bullet d\mathbf{l} = \int_1^0 x^2\,dx = \left[\frac{x^3}{3}\right]_1^0 = -\frac{1}{3}$$

$$\therefore \oint_C \mathbf{F} \bullet d\mathbf{l} = 1 - \frac{1}{2} - \frac{1}{3} = \frac{1}{6}$$

P2.3. $\mathbf{F} \bullet d\mathbf{l} = (\cos y\, \mathbf{a}_x - x \sin y\, \mathbf{a}_y) \bullet (dx\, \mathbf{a}_x + dy\, \mathbf{a}_y + dz\, \mathbf{a}_z)$

$= \cos y\, dx - x \sin y\, dy$

(a) Equation for the straight line path from $(0, 0, 0)$ to $(1, 2\pi, 1)$ is

$y = 2\pi x = 2\pi z$

$\therefore\ dy = 2\pi\, dx = 2\pi\, dz$

$\mathbf{F} \bullet d\mathbf{l} = \cos 2\pi x\, dx - 2\pi x \sin 2\pi x\, dx$

$$\int_{(0,0,0)}^{(1,2\pi,1)} \mathbf{F} \bullet d\mathbf{l} = \int_0^1 (\cos 2\pi x\, dx - 2\pi x \sin 2\pi x\, dx)$$

$$= [x \cos 2\pi x]_0^1$$

$$= 1$$

(b) For $x = z = \sin \frac{y}{4}$,

$dx = dz = \frac{1}{4} \cos \frac{y}{4}\, dy$

$\mathbf{F} \bullet d\mathbf{l} = \frac{1}{4} \cos y \cos \frac{y}{4}\, dy - \sin \frac{y}{4} \sin y\, dy$

$$\int_{(0,0,0)}^{(1,2\pi,1)} \mathbf{F} \bullet d\mathbf{l} = \int_0^{2\pi} \left(\frac{1}{4} \cos y \cos \frac{y}{4} - \sin \frac{y}{4} \sin y \right) dy$$

$$= \left[\cos y \sin \frac{y}{4} \right]_0^{2\pi}$$

$$= 1$$

(c) $\mathbf{F} \bullet d\mathbf{l} = \cos y\, dx - x \sin y\, dy = d(x \cos y)$

$$\int_{(0,0,0)}^{(1,2\pi,1)} \mathbf{F} \bullet d\mathbf{l} = \int_{(0,0,0)}^{(1,2\pi,1)} d(x \cos y)$$

$$= [x \cos y]_{(0,0,0)}^{(1,2\pi,1)}$$

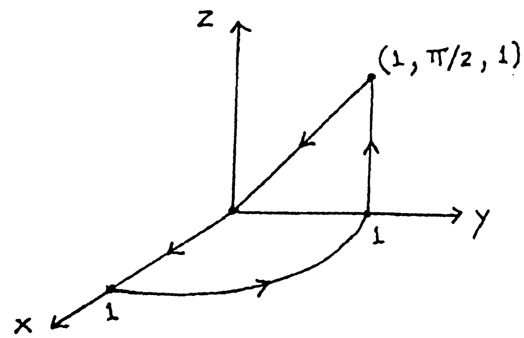
$$= (1)(\cos 2\pi) - (0)(\cos 0)$$

$$= 1 - 0$$

$$= 1$$

The vector field is conservative, since in view of (c), the line integral between any two points (x_1, y_1, z_1) and (x_2, y_2, z_2) is given by $[x \cos y]_{(x_1,y_1,z_1)}^{(x_2,y_2,z_2)} = x_2 \cos y_2 - x_1 \cos y_1$, and is independent of the path.

P2.4. $\mathbf{A} = 2r \sin \phi \, \mathbf{a}_r + r^2 \mathbf{a}_\phi + z\mathbf{a}_z$



From (0, 0, 0) to (1, 0, 0),

$$\phi = 0, z = 0; \; d\phi = dz = 0$$

$$d\mathbf{l} = dr \, \mathbf{a}_r, \mathbf{A} = r^2 \mathbf{a}_\phi$$

$$\mathbf{A} \cdot d\mathbf{l} = 0$$

$$\int \mathbf{A} \cdot d\mathbf{l} = 0$$

From (1, 0, 0) to (1, π/2, 0),

$$r = 1, z = 0; \; dr = dz = 0$$

$$d\mathbf{l} = 1 \, d\phi \, \mathbf{a}_\phi, \mathbf{A} = 2 \sin \phi \, \mathbf{a}_r + \mathbf{a}_\phi$$

$$\mathbf{A} \cdot d\mathbf{l} = d\phi$$

$$\int \mathbf{A} \cdot d\mathbf{l} = \int_0^{\pi/2} d\phi = \frac{\pi}{2}$$

From (1, π/2, 0) to (1, π/2, 1),

$$\phi = \pi/2, r = 1; \; d\phi = dr = 0$$

$$d\mathbf{l} = dz \, \mathbf{a}_z, \mathbf{A} = 2r \, \mathbf{a}_r + \mathbf{a}_\phi + z\mathbf{a}_z$$

$$\mathbf{A} \cdot d\mathbf{l} = z \, dz$$

$$\int \mathbf{A} \cdot d\mathbf{l} = \int_0^1 z \, dz = \frac{1}{2}$$

From (1, π/2, 1) to (0, 0, 0),

$$r = z, \phi = \pi/2, dr = dz, d\phi = 0$$

$$d\mathbf{l} = dr \, \mathbf{a}_r + dr \, \mathbf{a}_z, \mathbf{A} = 2r \, \mathbf{a}_r + r^2 \mathbf{a}_\phi + r\mathbf{a}_z$$

$$\mathbf{A} \cdot d\mathbf{l} = 2r \, dr + r \, dr = 3r \, dr$$

$$\int \mathbf{A} \cdot d\mathbf{l} = \int_1^0 3r \, dr = -\frac{3}{2}$$

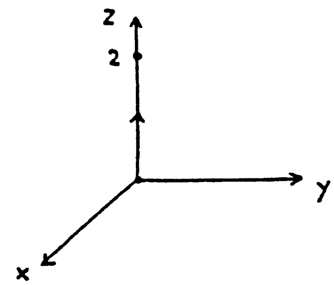
$$\therefore \oint_C \mathbf{A} \cdot d\mathbf{l} = 0 + \frac{\pi}{2} + \frac{1}{2} - \frac{3}{2}$$

$$= 0.5708$$

P2.5. $\mathbf{A} = e^{-r}(\cos\theta\,\mathbf{a}_r + \sin\theta\,\mathbf{a}_\theta) + r\sin\theta\,\mathbf{a}_\phi$

$d\mathbf{l} = dr\,\mathbf{a}_r + r\,d\theta\,\mathbf{a}_\theta + r\sin\theta\,d\phi\,\mathbf{a}_\phi$

(a) $\theta = 0,\ \phi = 0;\ d\theta = d\phi = 0$



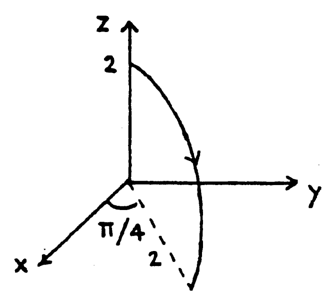
$\mathbf{A} = e^{-r}\,\mathbf{a}_r$

$d\mathbf{l} = dr\,\mathbf{a}_r$

$\mathbf{A}\cdot d\mathbf{l} = e^{-r}\,dr$

$$\int_{(0,0,0)}^{(2,0,0)} \mathbf{A}\cdot d\mathbf{l} = \int_0^2 e^{-r}\,dr = \left[-e^{-r}\right]_0^2 = 1 - e^{-2}$$

(b) $r = 2,\ \phi = \pi/4;\ dr = d\phi = 0$



$\mathbf{A} = e^{-2}(\cos\theta\,\mathbf{a}_r + \sin\theta\,\mathbf{a}_\theta) + 2\sin\theta\,\mathbf{a}_\phi$

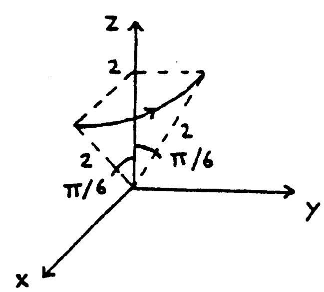
$d\mathbf{l} = 2\,d\theta\,\mathbf{a}_\theta$

$\mathbf{A}\cdot d\mathbf{l} = 2e^{-2}\sin\theta\,d\theta$

$$\int_{(2,0,\pi/4)}^{(2,\pi/2,\pi/4)} \mathbf{A}\cdot d\mathbf{l} = \int_0^{\pi/2} 2e^{-2}\sin\theta\,d\theta$$

$$= 2e^{-2}\left[-\cos\theta\right]_0^{\pi/2} = 2e^{-2}$$

(c) $r = 2,\ \theta = \pi/6;\ dr = d\theta = 0$



$$\mathbf{A} = e^{-2}\left(\frac{\sqrt{3}}{2}\,\mathbf{a}_r + \frac{1}{2}\,\mathbf{a}_\theta\right) + \mathbf{a}_\phi$$

$d\mathbf{l} = d\phi\,\mathbf{a}_\phi$

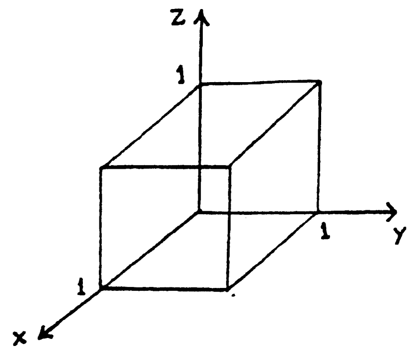
$\mathbf{A}\cdot d\mathbf{l} = d\phi$

$$\int_{(2,\pi/6,0)}^{(2,\pi/6,\pi/2)} \mathbf{A}\cdot d\mathbf{l} = \int_0^{\pi/2} d\phi = \frac{\pi}{2}$$

P2.6. $\mathbf{A} = x^2yz\mathbf{a}_x + y^2zx\mathbf{a}_y + z^2xy\mathbf{a}_z$

For $x = 0$, $y = 0$, $z = 0$, $\mathbf{A} = 0$

$$\int \mathbf{A} \cdot d\mathbf{S} = 0$$



For $x = 1$,

$$\mathbf{A} = yz\mathbf{a}_x + y^2z\mathbf{a}_y + z^2y\mathbf{a}_z$$

$$d\mathbf{S} = dy\, dz\, \mathbf{a}_x$$

$$\mathbf{A} \cdot d\mathbf{S} = yz\, dy\, dz$$

$$\int \mathbf{A} \cdot d\mathbf{S} = \int_{y=0}^{1}\int_{z=0}^{1} yz\, dy\, dz = \frac{1}{4}$$

For $y = 1$,

$$\mathbf{A} = x^2z\mathbf{a}_x + zx\mathbf{a}_y + z^2x\mathbf{a}_z$$

$$d\mathbf{S} = dz\, dx\, \mathbf{a}_y$$

$$\mathbf{A} \cdot d\mathbf{S} = zx\, dz\, dx$$

$$\int \mathbf{A} \cdot d\mathbf{S} = \int_{z=0}^{1}\int_{x=0}^{1} zx\, dz\, dx = \frac{1}{4}$$

For $z = 1$,

$$\mathbf{A} = x^2y\mathbf{a}_x + y^2x\mathbf{a}_y + xy\mathbf{a}_z$$

$$d\mathbf{S} = dx\, dy\, \mathbf{a}_z$$

$$\mathbf{A} \cdot d\mathbf{S} = xy\, dx\, dy$$

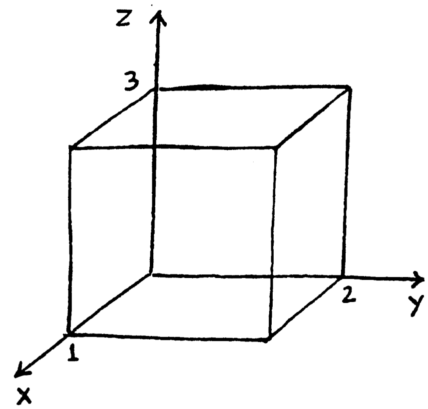
$$\int \mathbf{A} \cdot d\mathbf{S} = \int_{x=0}^{1}\int_{y=0}^{1} xy\, dx\, dy = \frac{1}{4}$$

$$\therefore \oint_S \mathbf{A} \cdot d\mathbf{S} = 0 + 0 + 0 + \frac{1}{4} + \frac{1}{4} + \frac{1}{4} = \frac{3}{4}$$

P2.7. $\mathbf{A} = (x^2y + 2)\mathbf{a}_x + 3\mathbf{a}_y - 2xyz\mathbf{a}_z$

For $x = 0$, $d\mathbf{S} = -dy\,dz\,\mathbf{a}_x$, $\mathbf{A} = 2\mathbf{a}_x + 3\mathbf{a}_y$

$$\mathbf{A} \bullet d\mathbf{S} = -2\,dy\,dz$$
$$\int \mathbf{A} \bullet d\mathbf{S} = \int_{z=0}^{3}\int_{y=0}^{2} (-2)\,dy\,dz = -12$$



For $y = 0$, $d\mathbf{S} = -dz\,dx\,\mathbf{a}_y$, $\mathbf{A} = 2\mathbf{a}_x + 3\mathbf{a}_y$

$$\mathbf{A} \bullet d\mathbf{S} = -3\,dz\,dx$$
$$\int \mathbf{A} \bullet d\mathbf{S} = \int_{z=0}^{3}\int_{x=0}^{1} (-3)\,dz\,dx = -9$$

For $z = 0$, $d\mathbf{S} = -dx\,dy\,\mathbf{a}_z$, $\mathbf{A} = (x^2y + 2)\mathbf{a}_x + 3\mathbf{a}_y$

$$\mathbf{A} \bullet d\mathbf{S} = 0$$
$$\int \mathbf{A} \bullet d\mathbf{S} = 0$$

For $x = 1$, $d\mathbf{S} = dy\,dz\,\mathbf{a}_x$, $\mathbf{A} = (\,y + 2)\mathbf{a}_x + 3\mathbf{a}_y - 2yz\mathbf{a}_z$

$$\mathbf{A} \bullet d\mathbf{S} = (\,y + 2)\,dy\,dz$$
$$\int \mathbf{A} \bullet d\mathbf{S} = \int_{z=0}^{3}\int_{y=0}^{2} (y+2)\,dy\,dz = 18$$

For $y = 2$, $d\mathbf{S} = dz\,dx\,\mathbf{a}_y$, $\mathbf{A} = (2x^2 + 2)\mathbf{a}_x + 3\mathbf{a}_y - 4xz\mathbf{a}_z$

$$\mathbf{A} \bullet d\mathbf{S} = 3\,dz\,dx$$
$$\int \mathbf{A} \bullet d\mathbf{S} = \int_{z=0}^{3}\int_{x=0}^{1} 3\,dz\,dx = 9$$

For $z = 3$, $d\mathbf{S} = dx\,dy\,\mathbf{a}_z$, $\mathbf{A} = (x^2y + 2)\mathbf{a}_x + 3\mathbf{a}_y - 6xy\mathbf{a}_z$

$$\mathbf{A} \bullet d\mathbf{S} = -6\,xy\,dx\,dy$$
$$\int \mathbf{A} \bullet d\mathbf{S} = \int_{y=0}^{2}\int_{x=0}^{1} (-6)\,xy\,dx\,dy = -6$$

$$\therefore \oint_S \mathbf{A} \bullet d\mathbf{S} = -12 - 9 + 0 + 18 + 9 - 6 = 0$$

P2.8. $\mathbf{A} = r\cos\phi\,\mathbf{a}_r - r\sin\phi\,\mathbf{a}_\phi$

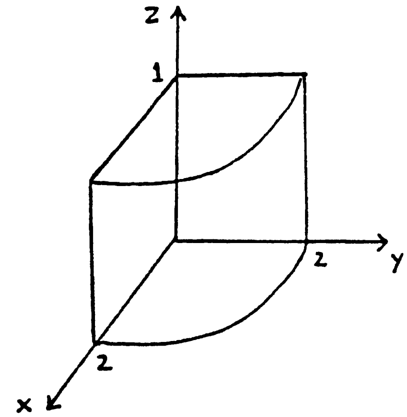
For $\phi = 0$,

$$\mathbf{A} = r\mathbf{a}_r$$

$$d\mathbf{S} = -dr\,dz\,\mathbf{a}_\phi$$

$$\mathbf{A}\cdot d\mathbf{S} = 0$$

$$\int \mathbf{A}\cdot d\mathbf{S} = 0$$



For $\phi = \pi/2$,

$$\mathbf{A} = -r\mathbf{a}_\phi$$

$$d\mathbf{S} = dr\,dz\,\mathbf{a}_\phi$$

$$\mathbf{A}\cdot d\mathbf{S} = -r\,dr\,dz$$

$$\int \mathbf{A}\cdot d\mathbf{S} = -\int_{r=0}^{2}\int_{z=0}^{1} r\,dr\,dz = -2$$

For $r = 2$,

$$\mathbf{A} = 2\cos\phi\,\mathbf{a}_r - 2\sin\phi\,\mathbf{a}_\phi$$

$$d\mathbf{S} = 2\,d\phi\,dz\,\mathbf{a}_r$$

$$\mathbf{A}\cdot d\mathbf{S} = 4\cos\phi\,d\phi\,dz$$

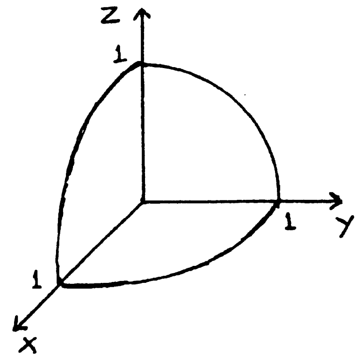
$$\int \mathbf{A}\cdot d\mathbf{S} = \int_{\phi=0}^{\pi/2}\int_{z=0}^{1} 4\cos\phi\,d\phi\,dz = 4$$

For $z = 0$, $d\mathbf{S} = -r\,dr\,d\phi\,\mathbf{a}_z$, $\mathbf{A}\cdot d\mathbf{S} = 0$, $\int \mathbf{A}\cdot d\mathbf{S} = 0$

For $z = 1$, $d\mathbf{S} = r\,dr\,d\phi\,\mathbf{a}_z$, $\mathbf{A}\cdot d\mathbf{S} = 0$, $\int \mathbf{A}\cdot d\mathbf{S} = 0$

$$\therefore \oint_S \mathbf{A}\cdot d\mathbf{S} = 0 - 2 + 4 + 0 + 0 = 2$$

P2.9. $\mathbf{A} = r^2\mathbf{a}_r + r\sin\theta\,\mathbf{a}_\theta$



For $\phi = 0$, $d\mathbf{S} = -r\,dr\,d\theta\,\mathbf{a}_\phi$

$\mathbf{A} = r^2\mathbf{a}_r + r\sin\theta\,\mathbf{a}_\theta$

$\mathbf{A} \cdot d\mathbf{S} = 0$

$\int \mathbf{A} \cdot d\mathbf{S} = 0$

For $\phi = \frac{\pi}{2}$, $d\mathbf{S} = r\,dr\,d\theta\,\mathbf{a}_\phi$, $\mathbf{A} = r^2\mathbf{a}_r + r\sin\theta\,\mathbf{a}_\theta$

$\mathbf{A} \cdot d\mathbf{S} = 0$

$\int \mathbf{A} \cdot d\mathbf{S} = 0$

For $\theta = \frac{\pi}{2}$, $d\mathbf{S} = r\sin\frac{\pi}{2}\,dr\,d\phi\,\mathbf{a}_\theta = r\,dr\,d\phi\,\mathbf{a}_\theta$

$\mathbf{A} = r^2\mathbf{a}_r + r\mathbf{a}_\theta$

$\mathbf{A} \cdot d\mathbf{S} = r^2\,dr\,d\phi$

$$\int \mathbf{A} \cdot d\mathbf{S} = \int_{r=0}^{1}\int_{\phi=0}^{\pi/2} r^2\,dr\,d\phi = \frac{\pi}{6}$$

For $r = 1$, $d\mathbf{S} = (1)^2\sin\theta\,d\theta\,d\phi\,\mathbf{a}_r = \sin\theta\,d\theta\,d\phi\,\mathbf{a}_r$

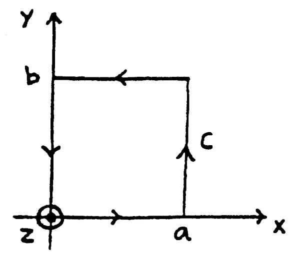
$\mathbf{A} = \mathbf{a}_r + \sin\theta\,\mathbf{a}_\theta$

$\mathbf{A} \cdot d\mathbf{S} = \sin\theta\,d\theta\,d\phi$

$$\int \mathbf{A} \cdot d\mathbf{S} = \int_{\theta=0}^{\pi/2}\int_{\phi=0}^{\pi/2} \sin\theta\,d\theta\,d\phi = \frac{\pi}{2}$$

$$\therefore \oint_S \mathbf{A} \cdot d\mathbf{S} = 0 + 0 + \frac{\pi}{6} + \frac{\pi}{2} = \frac{2\pi}{3}$$

P2.10. $\oint_C \mathbf{E} \cdot d\mathbf{l} = -\frac{d}{dt}\int_S \mathbf{B} \cdot d\mathbf{S}$



(a) $\mathbf{B} = \frac{B_0 a^2}{(x+a)^2} e^{-t}\mathbf{a}_z$

$d\mathbf{S} = dx\, dy\, \mathbf{a}_z$

$$\int_S \mathbf{B} \cdot d\mathbf{S} = \int_{x=0}^{a}\int_{y=0}^{b} \frac{B_0 a^2}{(x+a)^2} e^{-t}\, dx\, dy$$

$$= B_0 b a^2 e^{-t}\left[\frac{-1}{x+a}\right]_{x=0}^{a} = \frac{B_0 abe^{-t}}{2}$$

$$\text{emf} = -\frac{d}{dt}\left(\frac{B_0 abe^{-t}}{2}\right) = \frac{B_0 abe^{-t}}{2}$$

(b) $\mathbf{B} = B_0 \sin\frac{\pi x}{a} \cos \omega t\, \mathbf{a}_z$

$d\mathbf{S} = dx\, dy\, \mathbf{a}_z$

$$\int_S \mathbf{B} \cdot d\mathbf{S} = \int_{x=0}^{a}\int_{y=0}^{b} B_0 \sin\frac{\pi x}{a} \cos \omega t\, dx\, dy$$

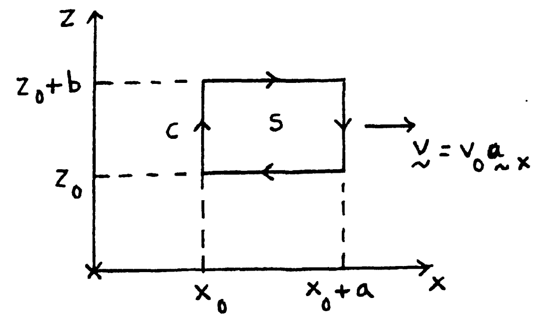
$$= B_0 b \cos \omega t \cdot \frac{a}{\pi}\left[-\cos\frac{\pi x}{a}\right]_{x=0}^{a}$$

$$= \frac{2B_0 ab}{\pi} \cos \omega t$$

$$\text{emf} = -\frac{d}{dt}\left(\frac{2B_0 ab}{\pi} \cos \omega t\right) = \frac{2B_0 ab\omega}{\pi} \sin \omega t$$

P2.11. $\int_S \mathbf{B} \cdot d\mathbf{S}$

$$= \int_{x=x_0}^{x_0+a} \int_{z=z_0}^{z_0+b} \frac{B_0}{x} \mathbf{a}_y \cdot dx\, dz\, \mathbf{a}_y$$



$$= \int_{x=x_0}^{x_0+a} \int_{z=z_0}^{z_0+b} \frac{B_0}{x}\, dx\, dz$$

$$= B_0 b[\ \ln x]_{x_0}^{x_0+a}$$

$$= B_0 b[\ \ln (x_0 + a) - \ln x_0]$$

$$\oint_C \mathbf{E} \cdot d\mathbf{l} = -\frac{d}{dt} \int_S \mathbf{B} \cdot d\mathbf{S}$$

$$= -\frac{d}{dt} \{B_0 b[\ \ln (x_0 + a) - \ln x_0]\}$$

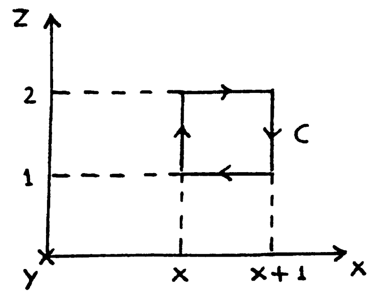
$$= -B_0 b \left(\frac{1}{x_0 + a} - \frac{1}{x_0} \right) \frac{dx_0}{dt}$$

$$= B_0 b v_0 \left(\frac{1}{x_0} - \frac{1}{x_0 + a} \right)$$

From the motional emf concept, the induced emf is $\left(v_0 \frac{B_0}{x_0} b - v_0 \frac{B_0}{x_0 + a} b \right)$, which agrees with the above result.

P2.12. $\mathbf{B} = B_0 \cos \pi(x - v_0 t)\, \mathbf{a}_y$

(a) $\int_S \mathbf{B} \cdot d\mathbf{S}$



$$= \int_{x=x}^{x+1} \int_{z=1}^{2} B_0 \cos \pi(x - v_0 t)\, \mathbf{a}_y \cdot dz\, dx\, \mathbf{a}_y$$

$$= \int_{x=x}^{x+1} \int_{z=1}^{2} B_0 \cos \pi(x - v_0 t)\, dz\, dx$$

$$= \frac{B_0}{\pi} [\sin \pi(x - v_0 t)]_x^{x+1}$$

$$= \frac{B_0}{\pi} [\sin \pi(x + 1 - v_0 t) - \sin \pi(x - v_0 t)]$$

$$= -\frac{2B_0}{\pi} \sin \pi(x - v_0 t)$$

$$\oint_C \mathbf{E} \cdot d\mathbf{l} = -\frac{d}{dt}\left[-\frac{2B_0}{\pi} \sin \pi(x - v_0 t)\right]$$

$$= -2B_0 v_0 \cos \pi(x - v_0 t)$$

(b) $x = x_0 + v_0 t$

$$\int_S \mathbf{B} \cdot d\mathbf{S} = -\frac{2B_0}{\pi} \sin \pi(x_0 + v_0 t - v_0 t)$$

$$= -\frac{2B_0}{\pi} \sin \pi x_0$$

$$\oint_C \mathbf{E} \cdot d\mathbf{l} = -\frac{d}{dt}\left(-\frac{2B_0}{\pi} \sin \pi x_0\right) = 0$$

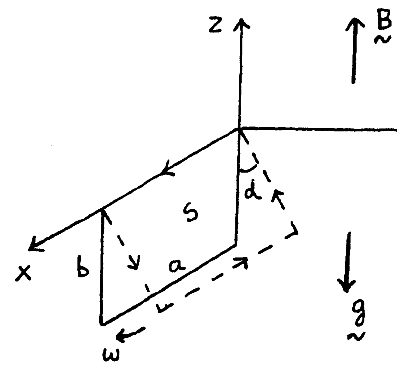
P2.13. $\oint_C \mathbf{E} \cdot d\mathbf{l} = -\frac{d}{dt}\int_S \mathbf{B} \cdot d\mathbf{S}$

$= -\frac{d}{dt}(B_0 ab \sin\alpha)$

$= -B_0 ab \cos\alpha \, \frac{d\alpha}{dt}$

$= -B_0 ab \cos\alpha \, (-\omega)$

$= B_0 ab\omega \cos\alpha$



For small α, $\cos\alpha \approx 1$.

$\therefore$ Induced emf $\approx B_0 ab\omega$

The polarity of the induced emf is such that the current flows in the same sense as C, resulting in a force on the bottom side of the loop away from the vertical. Thus the loop swings slower than in the absence of the magnetic field.

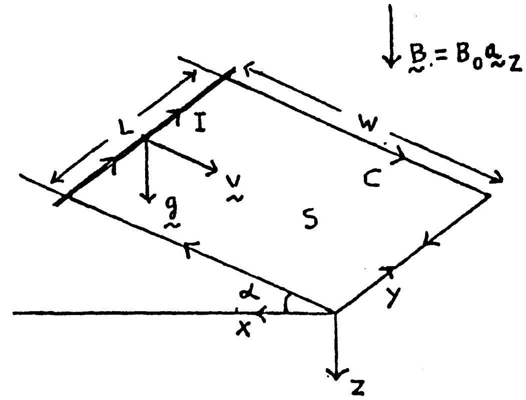
P2.14. $\oint_C \mathbf{E} \cdot d\mathbf{l} = -\frac{d}{dt}\int_S \mathbf{B} \cdot d\mathbf{S}$

$$= -\frac{d}{dt}\left[(B_0 \cos\alpha)\, Lw\right]$$

$$= -B_0 L \cos\alpha \frac{dw}{dt}$$

$$= -B_0 L \cos\alpha\,(-v)$$

$$= B_0 L v \cos\alpha$$



$$I = \frac{\text{emf}}{R} = \frac{B_0 L v \cos\alpha}{R}$$

Magnetic force on the bar

$$= -ILB_0\mathbf{a}_x = -\frac{B_0^2 L^2 v \cos\alpha}{R}\mathbf{a}_x$$

Equating the components of the magnetic force and the gravitational force acting on the bar, we have

$$Mg \sin\alpha = \frac{B_0^2 L^2 v \cos\alpha}{R} \cos\alpha$$

$$v = \frac{MgR}{B_0^2 L^2} \tan\alpha \sec\alpha$$

P2.15. **(a)** $\mathbf{B} = B_0\mathbf{a}_y$

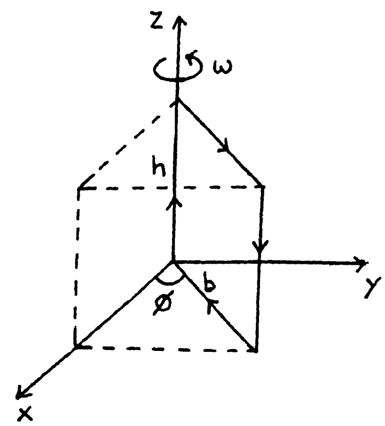
$$\int_S \mathbf{B} \cdot d\mathbf{S} = \int_{x=0}^{b\cos\phi} \int_{z=0}^{h} B_0\mathbf{a}_y \cdot dx\,dz\,\mathbf{a}_y$$

$$= B_0 hb \cos\phi$$

$$= B_0 hb \cos\omega t$$

$$\text{emf} = -\frac{d}{dt}(B_0 hb \cos\omega t)$$

$$= B_0 hb\omega \sin\omega t$$



(b) $\mathbf{B} = B_0(y\mathbf{a}_x - x\mathbf{a}_y)$

$$\int_S \mathbf{B} \cdot d\mathbf{S} = \int_{x=0}^{b\cos\phi} \int_{z=0}^{h} B_0(0\mathbf{a}_x - x\mathbf{a}_y) \cdot dx\,dz\,\mathbf{a}_y$$

$$+ \int_{y=0}^{b\sin\phi} \int_{z=0}^{h} B_0(y\mathbf{a}_x - b\cos\phi\,\mathbf{a}_y) \cdot (-\,dy\,dz\,\mathbf{a}_x)$$

$$= -B_0 h \frac{b^2\cos^2\phi}{2} - B_0 h \frac{b^2\sin^2\phi}{2}$$

$$= -B_0 h \frac{b^2}{2}$$

$$\text{emf} = -\frac{d}{dt}\left(-B_0 h \frac{b^2}{2}\right) = 0$$

P2.16. $\mathbf{B} = B_0(\sin \omega t\, \mathbf{a}_x + \cos \omega t\, \mathbf{a}_y)$

(a) $\int \mathbf{B} \bullet d\mathbf{S} = AB_0 \cos \omega t$

$$\text{emf} = -\frac{d}{dt}(AB_0 \cos \omega t)$$

$$= \omega AB_0 \sin \omega t$$

(b) $d\mathbf{S} = dS\, \mathbf{a}_\phi = dS\,(-\sin \phi\, \mathbf{a}_x + \cos \phi\, \mathbf{a}_y)$

$$\mathbf{B} \bullet d\mathbf{S} = B_0\, dS\,(-\sin \phi \sin \omega t + \cos \phi \cos \omega t)$$

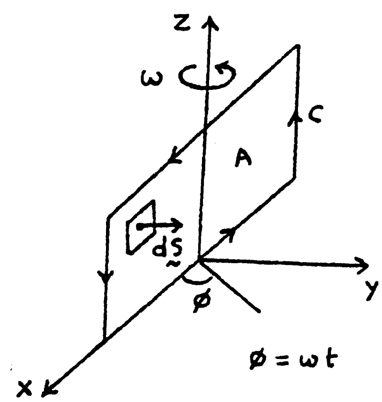
$$= B_0\, dS \cos(\omega t + \phi)$$

$$= B_0\, dS \cos 2\omega t$$

$$\int \mathbf{B} \bullet d\mathbf{S} = B_0 A \cos 2\omega t$$

$$\text{emf} = -\frac{d}{dt}(AB_0 \cos 2\omega t)$$

$$= 2\omega AB_0 \sin 2\omega t$$



(c) $\phi = -\omega t$

$$\mathbf{B} \bullet d\mathbf{S} = B_0\, dS\,(-\sin \phi \sin \omega t + \cos \phi \cos \omega t)$$

$$= B_0\, dS \cos(\omega t + \phi)$$

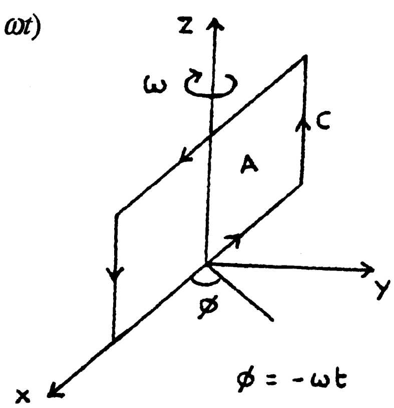
$$= B_0\, dS \cos(\omega t - \omega t)$$

$$= B_0\, dS$$

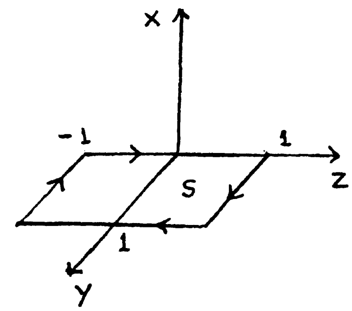
$$\int \mathbf{B} \bullet d\mathbf{S} = B_0 A$$

$$\text{emf} = -\frac{d}{dt}(B_0 A)$$

$$= 0$$



P2.17. From $\oint_C \mathbf{H} \bullet d\mathbf{l} = \int_S \mathbf{J} \bullet d\mathbf{S} + \frac{d}{dt} \int_S \mathbf{D} \bullet d\mathbf{S}$



$$\int_S \mathbf{J} \bullet d\mathbf{S} = \oint_C \mathbf{H} \bullet d\mathbf{l} - \frac{d}{dt} \int \mathbf{D} \bullet d\mathbf{S}$$

$$\oint_C \mathbf{H} \bullet d\mathbf{l} = H_0\left(t - \sqrt{\mu_0 \varepsilon_0}\right)^2 + H_0\left(t - \sqrt{\mu_0 \varepsilon_0}\right)^2$$

$$= 2H_0\left(t - \sqrt{\mu_0 \varepsilon_0}\right)^2$$

$$\int_S \mathbf{D} \bullet d\mathbf{S} = -\sqrt{\mu_0 \varepsilon_0}\; H_0 \left[\int_{-1}^{0} \left(t + \sqrt{\mu_0 \varepsilon_0}\, z\right)^2 dz\right.$$

$$\left. + \int_0^1 \left(t - \sqrt{\mu_0 \varepsilon_0}\, z\right)^2 dz\right]$$

$$= -\sqrt{\mu_0 \varepsilon_0}\; H_0 \left\{\left[\frac{\left(t + \sqrt{\mu_0 \varepsilon_0}\, z\right)^3}{3\sqrt{\mu_0 \varepsilon_0}}\right]_{z=-1}^{0}\right.$$

$$\left. - \left[\frac{\left(t - \sqrt{\mu_0 \varepsilon_0}\, z\right)^3}{3\sqrt{\mu_0 \varepsilon_0}}\right]_{z=0}^{1}\right\}$$

$$= -\frac{H_0}{3}\left[t^3 - \left(t - \sqrt{\mu_0 \varepsilon_0}\right)^3 - \left(t - \sqrt{\mu_0 \varepsilon_0}\right)^3 + t^3\right]$$

$$\frac{d}{dt} \int_S \mathbf{D} \bullet d\mathbf{S} = -\frac{H_0}{3}\left[6t^2 - 6\left(t - \sqrt{\mu_0 \varepsilon_0}\right)^2\right]$$

$$= -2H_0 t^2 + 2H_0 \left(t - \sqrt{\mu_0 \varepsilon_0}\right)^2$$

$$\int_S \mathbf{J} \bullet d\mathbf{S} = 2H_0\left(t - \sqrt{\mu_0 \varepsilon_0}\right)^2 + 2H_0 t^2 - 2H_0\left(t - \sqrt{\mu_0 \varepsilon_0}\right)^2$$

$$= 2H_0 t^2$$

P2.18. $\frac{d}{dt}\oint_S \mathbf{D} \cdot d\mathbf{S} = -\oint_S \mathbf{J} \cdot d\mathbf{S}$

(a) $\oint_S \mathbf{J} \cdot d\mathbf{S} = \int_{y=-2}^{2}\int_{z=-2}^{2} (-2)\, dy\, dz$

$+ \int_{y=-2}^{2}\int_{z=-2}^{2} 2(-\, dy\, dz)$

$+ \int_{x=-2}^{2}\int_{z=-2}^{2} (-2)\, dz\, dx$

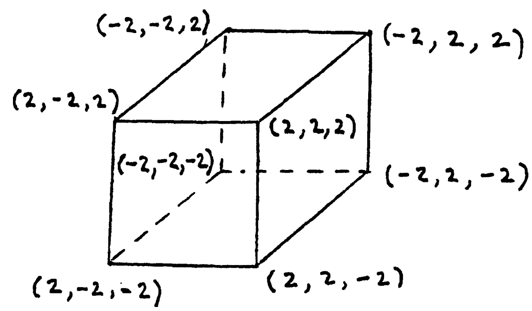
$+ \int_{x=-2}^{2}\int_{z=-2}^{2} 2\,(-dz\, dx)$

$+ \int_{x=-2}^{2}\int_{y=-2}^{2} (-\,4)\, dx\, dy$

$+ \int_{x=-2}^{2}\int_{y=-2}^{2} (-\,4)\,(-dx\, dy)$

$= -32 - 32 - 32 - 32 - 64 + 64$

$= -128$



$\therefore$ Displacement current emanating from box = 128 A

(b) $\mathbf{J} = -(x\mathbf{a}_x + y\mathbf{a}_y + z^2\mathbf{a}_z)$

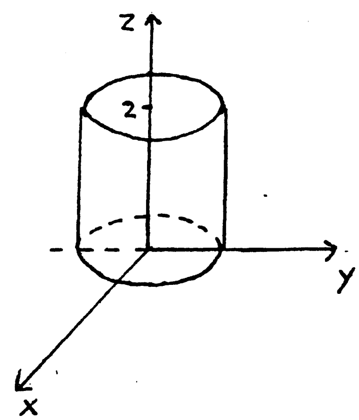
$= -r\mathbf{a}_r - z^2\mathbf{a}_z$

$\oint_S \mathbf{J} \cdot d\mathbf{S} = \int_{z=0}^{2}\int_{\phi=0}^{2\pi} (-1)\, d\phi\, dz$

$+ \int_{r=0}^{1}\int_{\phi=0}^{2\pi} (-\,4)\, r\, dr\, d\phi$

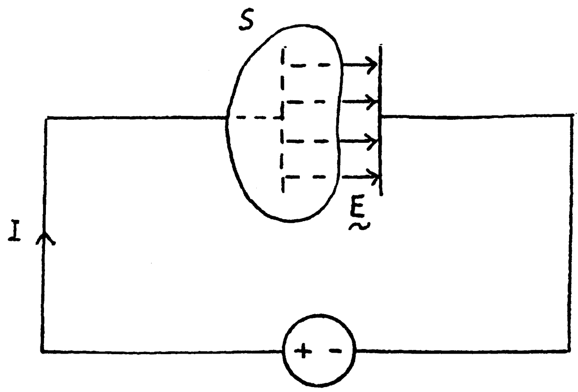
$= -\,4\pi - 4\pi$

$= -8\pi$



$\therefore$ Displacement current emanating from box = 8π A

P2.19.



$$I = \frac{d}{dt}\int_S \mathbf{D} \cdot d\mathbf{S}$$

$$= \frac{d}{dt} \int_{\substack{\text{area parallel}\\ \text{to the plates}}} \varepsilon_0 \, \mathbf{E} \cdot d\mathbf{S}$$

$$= \frac{d}{dt}\,[\varepsilon_0(180 \sin 2\pi \times 10^6 t \sin 4\pi \times 10^6 t) \times 0.1]$$

$$= \frac{10^{-9}}{4\pi}\frac{d}{dt}[\cos 2\pi \times 10^6 t - \cos 6\pi \times 10^6 t]$$

$$= \frac{10^{-3}}{4\pi}\,(-2\pi \sin 2\pi \times 10^6 t + 6\pi \sin 6\pi \times 10^6 t)$$

$$= \frac{10^{-3}}{2}\,(-\sin 2\pi \times 10^6 t + 3 \sin 6\pi \times 10^6 t)$$

Root-mean-square value of $I(t)$

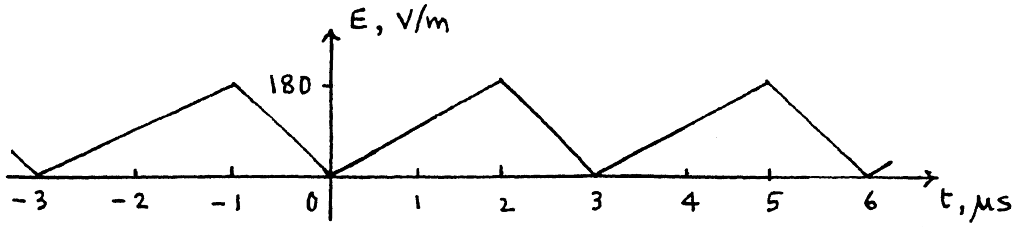
$$= \frac{10^{-3}}{2}\sqrt{\left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{3}{\sqrt{2}}\right)^2}$$

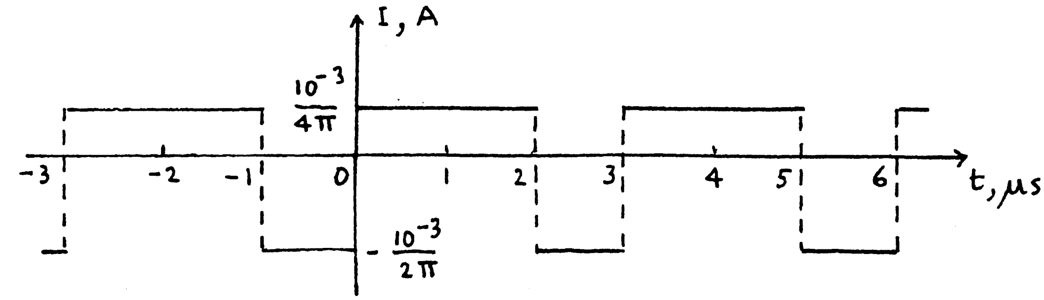
$$= 1.118 \times 10^{-3} \text{ A}$$

$$= 1.118 \text{ mA}$$

P2.20. $I(t) = \dfrac{d}{dt}(DA) = \dfrac{d}{dt}(0.1\varepsilon_0 E)$

$$= 0.1\varepsilon_0 \frac{dE}{dt}$$





$$\text{RMS value of } I(t) = \sqrt{\frac{1}{3\times 10^{-6}}\left[\left(\frac{10^{-3}}{4\pi}\right)^2 \times 2\times 10^{-6} + \left(\frac{10^{-3}}{2\pi}\right)^2 \times 10^{-6}\right]}$$

$$= \sqrt{\frac{1}{3\times 10^{-6}}\left(\frac{3\times 10^{-6}}{8\pi^2}\right)\times 10^{-6}}$$

$$= 0.1125 \times 10^{-3}\,\text{A}$$

$$= 0.1125\ \text{mA}$$

P2.21. (a) $\oint_S \mathbf{D} \bullet d\mathbf{S} = \int_V \rho \, dv$

$$= \int_{x=-1}^{1}\int_{y=-1}^{1}\int_{z=-1}^{1} \rho_0(3 - x^2 - y^2 - z^2)\, dx\, dy\, dz$$

$$= 8\rho_0 \int_{x=0}^{1}\int_{y=0}^{1}\int_{z=0}^{1} (3 - x^2 - y^2 - z^2)\, dx\, dy\, dz$$

$$= 8\rho_0\left(3 - \frac{1}{3} - \frac{1}{3} - \frac{1}{3}\right)$$

$$= 16\rho_0$$

(b) $\oint_S \mathbf{D} \bullet d\mathbf{S} = \int_V \rho \, dv$

$$= \int_{x=0}^{1}\int_{y=0}^{\sqrt{1-x^2}}\int_{z=0}^{\sqrt{1-x^2-y^2}} \rho_0\, xyz\, dx\, dy\, dz$$

$$= \frac{\rho_0}{2}\int_{x=0}^{1}\int_{y=0}^{\sqrt{1-x^2}} xy(1 - x^2 - y^2)\, dx\, dy$$

$$= \frac{\rho_0}{2}\int_{x=0}^{1}\left[\frac{xy^2}{2} - \frac{x^3y^2}{2} - \frac{xy^4}{4}\right]_{y=0}^{\sqrt{1-x^2}} dx$$

$$= \frac{\rho_0}{4}\int_{x=0}^{1}\left[x - x^3 - x^3 + x^5 - \frac{1}{2}(x - 2x^3 + x^5)\right] dx$$

$$= \frac{\rho_0}{4}\int_{x=0}^{1}\left(\frac{x}{2} - x^3 + \frac{1}{2}x^5\right) dx$$

$$= \frac{\rho_0}{4}\left[\frac{x^2}{4} - \frac{x^4}{4} + \frac{x^6}{12}\right]_0^1$$

$$= \frac{\rho_0}{48}$$

P2.22. **(a)** $\oint_S \mathbf{D} \bullet d\mathbf{S} = \int_V \rho\, dv$

$$= \int_{r=0}^{1}\int_{\phi=0}^{2\pi}\int_{z=0}^{1} (\rho_0 e^{-r^2})\, r\, dr\, d\phi\, dz$$

$$= 2\pi\rho_0 \left[\frac{e^{-r^2}}{-2}\right]_0^1$$

$$= \pi\rho_0(1-e^{-1})$$

$$= 1.986\rho_0$$

(b) $\oint_S \mathbf{D} \bullet d\mathbf{S} = \int_V \rho\, dv$

$$= \int_{r=0}^{1}\int_{\theta=0}^{\pi/2}\int_{\phi=0}^{2\pi} \left(\frac{\rho_0}{r}\sin^2\theta\right) r^2 \sin\theta\, dr\, d\theta\, d\phi$$

$$= 2\pi\rho_0 \int_{r=0}^{1}\int_{\theta=0}^{\pi/2} r\sin^3\theta\, dr\, d\theta$$

$$= \frac{4}{3}\pi\rho_0 \int_{r=0}^{1} r\, dr$$

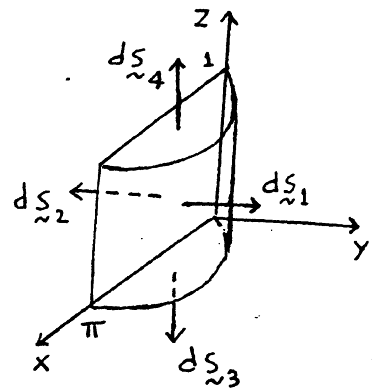
$$= \frac{2}{3}\pi\rho_0$$

P2.23. $\oint_S \mathbf{B} \cdot d\mathbf{S}$

$$= \int_{S_1} \mathbf{B} \cdot d\mathbf{S}_1 + \int_{S_2} \mathbf{B} \cdot d\mathbf{S}_2$$

$$+ \int_{S_3} \mathbf{B} \cdot d\mathbf{S}_3 + \int_{S_4} \mathbf{B} \cdot d\mathbf{S}_4$$

$$= 0$$



$$\therefore \int_{S_1} \mathbf{B} \cdot d\mathbf{S}_1$$

$$= -\int_{S_2} \mathbf{B} \cdot d\mathbf{S}_2 - \int_{S_3} \mathbf{B} \cdot d\mathbf{S}_3 - \int_{S_4} \mathbf{B} \cdot d\mathbf{S}_4$$

$$= -\int_{z=0}^{1} \int_{x=0}^{\pi} B_0 [\, y\mathbf{a}_x - x\mathbf{a}_y]_{y=0} \cdot (-dx\, dz\, \mathbf{a}_y) - 0 - 0$$

$$= -\int_{z=0}^{1} \int_{x=0}^{\pi} B_0\, x\, dx\, dz$$

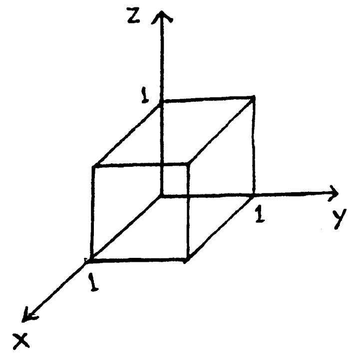
$$= -\frac{B_0 \pi^2}{2}$$

$\therefore$ Absolute value of the magnetic flux $= \dfrac{B_0 \pi^2}{2}$ Wb

P2.24. $\mathbf{J} = x\mathbf{a}_x + y\mathbf{a}_y + z\mathbf{a}_z$

$$\oint_S \mathbf{J} \cdot d\mathbf{S} + \frac{d}{dt}\int_V \rho \, dv = 0$$

$$-\frac{d}{dt}\int_V \rho \, dv = \oint_S \mathbf{J} \cdot d\mathbf{S}$$

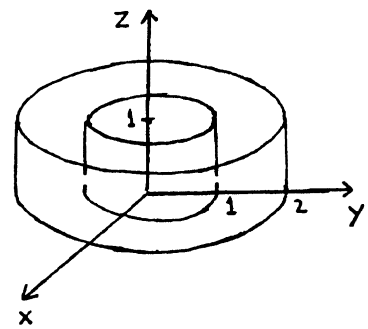


(a) $\oint_S \mathbf{J} \cdot d\mathbf{S} = 0 + 0 + 0 + 1 + 1 + 1 = 3$

$$-\frac{d}{dt}\int_V \rho \, dv = 3 \text{ A}$$

(b) $\mathbf{J} = x\mathbf{a}_x + y\mathbf{a}_y + z\mathbf{a}_z$

$= r_c\mathbf{a}_{rc} + z\mathbf{a}_z$

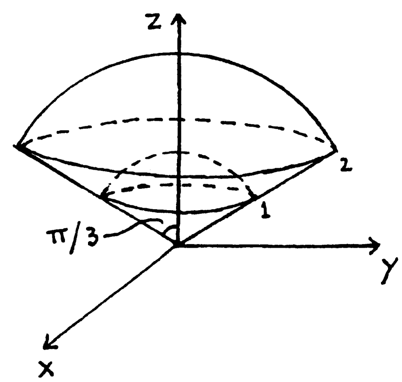


$$\begin{aligned}\oint_S \mathbf{J} \cdot d\mathbf{S} &= 4\pi \times 2 - 2\pi \times 1 \\ &\quad + 0 + (4\pi - \pi) \times 1 \\ &= 8\pi - 2\pi + 3\pi \\ &= 9\pi\end{aligned}$$

$$-\frac{d}{dt}\int_V \rho \, dv = 9\pi \text{ A}$$

(c) $\mathbf{J} = x\mathbf{a}_x + y\mathbf{a}_y + z\mathbf{a}_z$

$= r_s\mathbf{a}_{rs}$

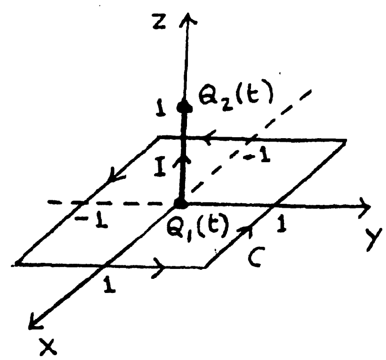


$$\begin{aligned}\oint_S \mathbf{J} \cdot d\mathbf{S} &= \int_{\theta=0}^{\pi/3}\int_{\phi=0}^{2\pi} 8 \sin\theta \, d\theta \, d\phi \\ &\quad - \int_{\theta=0}^{\pi/3}\int_{\phi=0}^{2\pi} \sin\theta \, d\theta \, d\phi + 0 \\ &= 14\pi\,[-\cos\theta]_0^{\pi/3} = 7\pi\end{aligned}$$

$$-\frac{d}{dt}\int_V \rho \, dv = 7\pi \text{ A}$$

P2.25. $\oint_C \mathbf{H} \cdot d\mathbf{l} = \int_S \mathbf{J} \cdot d\mathbf{S} + \frac{d}{dt} \int_S \mathbf{D} \cdot d\mathbf{S}$

Considering the plane surface S bounded by the closed path C, except for a slight upward bulge at the origin to avoid $Q_1(t)$, we have



$$\int_S \mathbf{J} \cdot d\mathbf{S} = I$$

$$\frac{d}{dt} \int_S \mathbf{D} \cdot d\mathbf{S} = \frac{d}{dt}\left(\frac{Q_1}{2} - \frac{Q_2}{6}\right)$$

$$= \frac{1}{2}\frac{dQ_1}{dt} - \frac{1}{6}\frac{dQ_2}{dt}$$

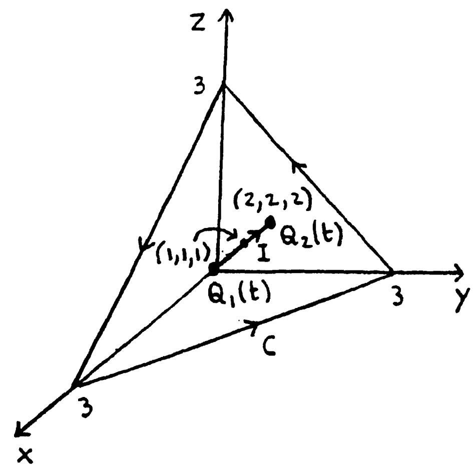
$$= \frac{1}{2}(-I) - \frac{1}{6}(I)$$

$$= -\frac{2}{3} I$$

$$\oint_C \mathbf{H} \cdot d\mathbf{l} = I - \frac{2}{3} I$$

$$= \frac{1}{3} I$$

P2.26. $\oint_C \mathbf{H} \cdot d\mathbf{l} = \int_S \mathbf{J} \cdot d\mathbf{S} + \frac{d}{dt} \int_S \mathbf{D} \cdot d\mathbf{S}$



Considering the plane surface S bounded by the closed path C and noting that the point (1, 1, 1) is at the center point of that surface, so that the points (0, 0, 0) and (2, 2, 2) are symmetrically situated on either side of S, we have

$$\int_S \mathbf{J} \cdot d\mathbf{S} = I$$

$$\frac{d}{dt} \int_S \mathbf{D} \cdot d\mathbf{S} = \frac{d}{dt}\left(\frac{Q_1}{8} - \frac{Q_2}{8}\right)$$

$$= \frac{1}{8}\frac{dQ_1}{dt} - \frac{1}{8}\frac{dQ_2}{dt}$$

$$= \frac{1}{8}(-I) - \frac{1}{8}(I)$$

$$= -\frac{1}{4} I$$

$$\oint_C \mathbf{H} \cdot d\mathbf{l} = I - \frac{1}{4} I$$

$$= \frac{3}{4} I$$

P2.27. From symmetry considerations and Gauss' law for the electric field, displacement flux emanating from one side of the box

$$= \frac{1}{6}\oint_S \mathbf{D} \bullet d\mathbf{S} = \frac{1}{6}\int_V \rho \, dv$$

$$= \frac{1}{6} \times 8 \times \int_{x=0}^{1} \int_{y=0}^{1} \int_{z=0}^{1} \rho(x, y, z)\, dx\, dy\, dz$$

(a) $\rho(x, y, z) = 3 - x^2 - y^2 - z^2$

$$\text{Required flux} = \frac{4}{3} \int_{x=0}^{1} \int_{y=0}^{1} \int_{z=0}^{1} (3 - x^2 - y^2 - z^2)\, dx\, dy\, dz$$

$$= \frac{4}{3}\left(3 - \frac{1}{3} - \frac{1}{3} - \frac{1}{3}\right) = \frac{8}{3} \text{ C}$$

(b) $\rho(x, y, z) = \sqrt{|xyz|} = \sqrt{xyz}$ for $0 < x < 1, 0 < y < 1, 0 < z < 1$

$$\text{Required flux} = \frac{4}{3} \int_{x=0}^{1} \int_{y=0}^{1} \int_{z=0}^{1} x^{1/2}y^{1/2}z^{1/2}\, dx\, dy\, dz$$

$$= \frac{4}{3} \times \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} = \frac{32}{81} \text{ C}$$

P2.28. From considerations of symmetry and application of

$$\oint_S \mathbf{D} \cdot d\mathbf{S} = \int_V \rho \, dv$$

to a cylindrical surface of radius r having the z-axis as its axis and lying between $z = 0$ and $z = l$, we have

$$2\pi r l D_r = \begin{cases} \int_{r=0}^{r}\int_{\phi=0}^{2\pi}\int_{z=0}^{l} \rho_0 e^{-r^2} r \, dr \, d\phi \, dz & \text{for } r \le 1 \\ \int_{r=0}^{1}\int_{\phi=0}^{2\pi}\int_{z=0}^{l} \rho_0 e^{-r^2} r \, dr \, d\phi \, dz & \text{for } r \ge 1 \end{cases}$$

$$= \begin{cases} 2\pi\rho_0 l \left[\dfrac{e^{-r^2}}{-2}\right]_0^r & \text{for } r \le 1 \\ 2\pi\rho_0 l \left[\dfrac{e^{-r^2}}{-2}\right]_0^1 & \text{for } r \ge 1 \end{cases}$$

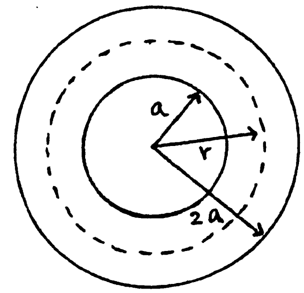
$$= \begin{cases} \pi\rho_0 l \left(1 - e^{-r^2}\right) & \text{for } r \le 1 \\ \pi\rho_0 l \left(1 - e^{-1}\right) & \text{for } r \ge 1 \end{cases}$$

$$\mathbf{D} = \begin{cases} \dfrac{\rho_0\left(1 - e^{-r^2}\right)}{2r} \mathbf{a}_r & \text{for } r \le 1 \\ \dfrac{\rho_0\left(1 - e^{-1}\right)}{2r} \mathbf{a}_r & \text{for } r \ge 1 \end{cases}$$

P2.29. From considerations of spherical symmetry and application of

$$\oint_S \mathbf{D} \cdot d\mathbf{S} = \int_V \rho \, dv$$

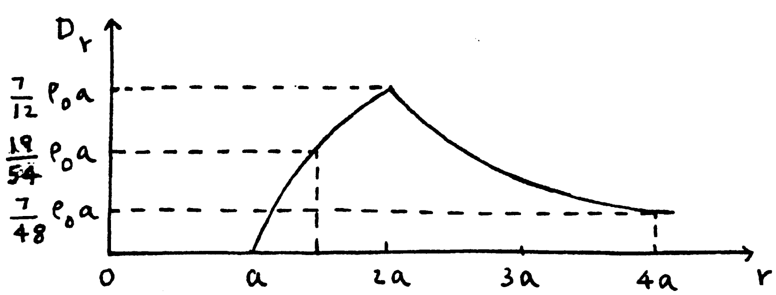
to a spherical surface of radius r centered at the origin, we obtain



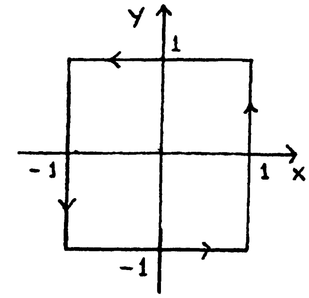
$$4\pi r^2 D_r = \begin{cases} 0 & \text{for } r < a \\ \left(\frac{4}{3}\pi r^3 - \frac{4}{3}\pi a^3\right)\rho_0 & \text{for } a < r < 2a \\ \left[\frac{4}{3}\pi (2a)^3 - \frac{4}{3}\pi a^3\right]\rho_0 & \text{for } r > 2a \end{cases}$$

$$= \begin{cases} 0 & \text{for } r < a \\ \frac{4}{3}\pi (r^3 - a^3)\rho_0 & \text{for } a < r < 2a \\ \frac{4}{3}\pi (7a^3)\rho_0 & \text{for } r > 2a \end{cases}$$

$$\mathbf{D} = \begin{cases} \mathbf{0} & \text{for } r < a \\ \frac{\rho_0 (r^3 - a^3)}{3r^2}\mathbf{a}_r & \text{for } a < r < 2a \\ \frac{7a^3 \rho_0}{3r^2}\mathbf{a}_r & \text{for } r > 2a \end{cases}$$



P2.30. From symmetry considerations of the square cross section and the given current densities,



$$\underset{\substack{\text{one side}\\\text{of square}}}{\int} \mathbf{H} \cdot d\mathbf{l} = \frac{1}{4} \underset{\substack{\text{around}\\\text{the square}}}{\oint} \mathbf{H} \cdot d\mathbf{l}$$

$$= \frac{1}{4} \int_S \mathbf{J} \cdot d\mathbf{S} = \frac{1}{4} \times 4 \int_{x=0}^{1} \int_{y=0}^{1} J_z\,(x, y)\, dx\, dy$$

(a) $J_z\,(x, y) = |x| + |y| = x + y$ for $0 < x < 1,\ 0 < y < 1$

$$\underset{\text{one side}}{\int} \mathbf{H} \cdot d\mathbf{l} = \int_{x=0}^{1} \int_{y=0}^{1} (x + y)\, dx\, dy$$

$$= \frac{1}{2} + \frac{1}{2} = 1 \text{ A}$$

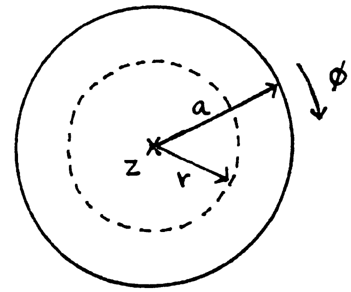
(b) $J_z\,(x, y) = x^2 y^2$

$$\underset{\text{one side}}{\int} \mathbf{H} \cdot d\mathbf{l} = \int_{x=0}^{1} \int_{y=0}^{1} x^2 y^2\, dx\, dy$$

$$= \frac{1}{3} \times \frac{1}{3} = \frac{1}{9} \text{ A}$$

P2.31. From considerations of symmetry and application of

$$\oint_C \mathbf{H} \bullet d\mathbf{l} = \int_S \mathbf{J} \bullet d\mathbf{S}$$



to a circular path of radius r centered on the axis of the wire and lying in the cross sectional plane of the wire, we obtain

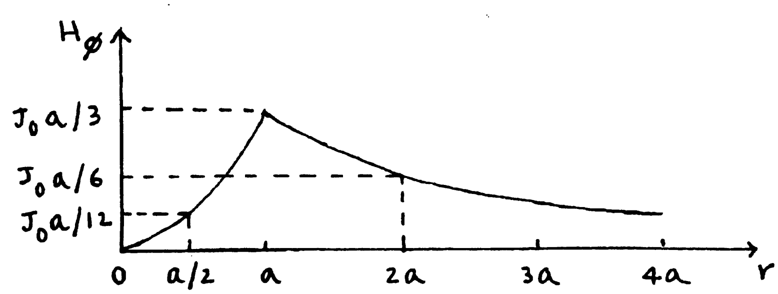
$$2\pi r H_\phi = \begin{cases} \int_{r=0}^{r}\int_{\phi=0}^{2\pi} J_0 \frac{r}{a}\mathbf{a}_z \bullet r\,dr\,d\phi\,\mathbf{a}_z & \text{for } r < a \\ \int_{r=0}^{a}\int_{\phi=0}^{2\pi} J_0 \frac{r}{a}\mathbf{a}_z \bullet r\,dr\,d\phi\,\mathbf{a}_z & \text{for } r > a \end{cases}$$

$$= \begin{cases} \frac{2\pi J_0}{a}\int_0^r r^2\,dr & \text{for } r < a \\ \frac{2\pi J_0}{a}\int_0^a r^2\,dr & \text{for } r > a \end{cases}$$

$$= \begin{cases} \frac{2\pi J_0 r^3}{3a} & \text{for } r < a \\ \frac{2\pi J_0 a^3}{3} & \text{for } r > a \end{cases}$$

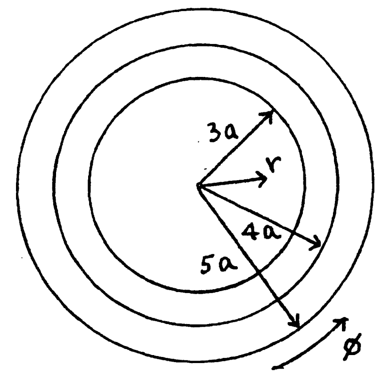
Thus

$$\mathbf{H} = \begin{cases} \frac{J_0 r^2}{3a}\mathbf{a}_\phi & \text{for } r < a \\ \frac{J_0 a^2}{3r}\mathbf{a}_\phi & \text{for } r > a \end{cases}$$



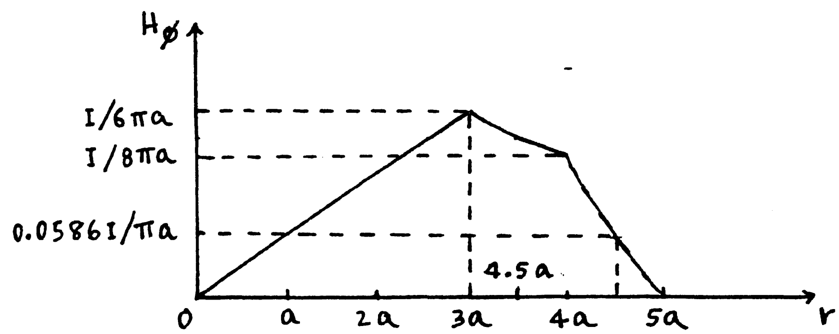
P2.32. From $\oint_C \mathbf{H} \cdot d\mathbf{l} = \int_S \mathbf{J} \cdot d\mathbf{S}$,

we have



$$2\pi r H_\phi = \begin{cases} I\dfrac{\pi r^2}{9\pi a^2} & \text{for } r < 3a \\ I & \text{for } 3a < r < 4a \\ I - I\dfrac{\pi(r^2 - 16a^2)}{\pi(25a^2 - 16a^2)} & \text{for } 4a < r < 5a \\ 0 & \text{for } r > 5a \end{cases}$$

$$\mathbf{H} = \begin{cases} \dfrac{Ir}{18\pi a^2}\,\mathbf{a}_\phi & \text{for } r < 3a \\ \dfrac{I}{2\pi r}\,\mathbf{a}_\phi & \text{for } 3a < r < 4a \\ \dfrac{I}{2\pi r}\left(\dfrac{25a^2 - r^2}{9a^2}\right)\mathbf{a}_\phi & \text{for } 4a < r < 5a \\ \mathbf{0} & \text{for } r > 5a \end{cases}$$



R2.1. $\mathbf{F} = \cos\theta \sin\phi\, \mathbf{a}_r - \sin\theta \sin\phi\, \mathbf{a}_\theta + \cot\theta \cos\phi\, \mathbf{a}_\phi$

$$d\mathbf{l} = dr\, \mathbf{a}_r + r\, d\theta\, \mathbf{a}_\theta + r \sin\theta\, d\phi\, \mathbf{a}_\phi$$

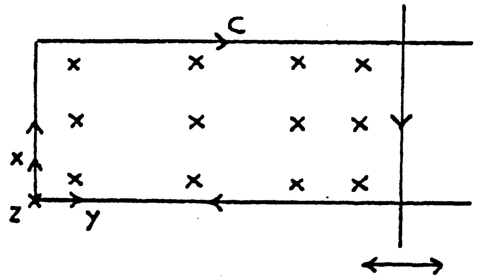
$$\begin{aligned} \mathbf{F} \bullet d\mathbf{l} &= \cos\theta \sin\phi\, dr - r \sin\theta \sin\phi\, d\theta + r \cos\theta \cos\phi\, d\phi \\ &= d(r \cos\theta \sin\phi) \end{aligned}$$

$$\begin{aligned} \int_{(r_1, \theta_1, \phi_1)}^{(r_2, \theta_2, \phi_2)} \mathbf{F} \bullet d\mathbf{l} &= \int_{(r_1, \theta_1, \phi_1)}^{(r_2, \theta_2, \phi_2)} d(r \cos\theta \sin\phi) \\ &= \left[r \cos\theta \sin\phi\right]_{(r_1, \theta_1, \phi_1)}^{(r_2, \theta_2, \phi_2)} \\ &= r_2 \cos\theta_2 \sin\phi_2 - r_1 \cos\theta_1 \sin\phi_1 \end{aligned}$$

is independent of the path from (r_1, θ_1, ϕ_1) to (r_2, θ_2, ϕ_2). Therefore, **F** is a conservative field.

$$\begin{aligned} \int_{(1, \pi/6, \pi/3)}^{(4, \pi/3, \pi/6)} \mathbf{F} \bullet d\mathbf{l} &= 4 \cos\frac{\pi}{3} \sin\frac{\pi}{6} - 1 \cos\frac{\pi}{6} \sin\frac{\pi}{3} \\ &= 4 \times \frac{1}{2} \times \frac{1}{2} - 1 \times \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2} \\ &= 1 - 0.75 \\ &= 0.25 \end{aligned}$$

R2.2.



$$\int_S \mathbf{B} \cdot d\mathbf{S} = \int_{x=0}^{l} \int_{y=0}^{y} B_0 y \mathbf{a}_z \cdot dx\, dy\, \mathbf{a}_z$$

$$= \int_{x=0}^{l} \int_{y=0}^{y} B_0 y\, dx\, dy$$

$$= B_0 l \frac{y^2}{2}$$

$$\text{emf} = -\frac{d}{dt} \int_S \mathbf{B} \cdot d\mathbf{S}$$

$$= -\frac{d}{dt}\left(B_0 l \frac{y^2}{2} \right)$$

$$= -B_0 l y \frac{dy}{dt}$$

$$= B_0 l\, (y_0 + a \cos \omega t)\, (\omega a \sin \omega t)$$

For $0 < \omega t < \pi$, y is decreasing, flux enclosed by C is decreasing, and emf is positive opposing the change in the flux. For $\pi < \omega t < 2\pi$, y is increasing, flux enclosed by C is increasing, and emf is negative opposing the change in the flux. Thus, Lenz' law is verified. Also,

$$\text{emf} = B_0 l y_0 a \omega \sin \omega t + B_0 l a^2 \omega \cos \omega t \sin \omega t$$

$$= B_0 l y_0 a \omega \sin \omega t + \frac{1}{2} B_0 l a^2 \omega \sin 2\omega t$$

Thus the induced emf has two frequency components, ω and 2ω.

R2.3. $I(t) = \frac{d}{dt}\int_S \mathbf{D} \bullet d\mathbf{S}$

$$= \frac{d}{dt}\int_{r=0}^{a}\int_{\phi=0}^{2\pi} \varepsilon_0 E_0 \sin\frac{\pi r}{2a}\cos\omega t\,\mathbf{a}_z \bullet r\,dr\,d\phi\,\mathbf{a}_z$$

$$= -\omega\varepsilon_0 E_0 \sin\omega t \int_{r=0}^{a}\int_{\phi=0}^{2\pi} r\sin\frac{\pi r}{2a}\,dr\,d\phi$$

$$= -2\pi\omega\varepsilon_0 E_0 \sin\omega t\left[\frac{1}{(\pi/2a)^2}\sin\frac{\pi r}{2a} - \frac{1}{(\pi/2a)}r\cos\frac{\pi r}{2a}\right]_0^a$$

$$= -2\pi\omega\varepsilon_0 E_0 \sin\omega t\left[\frac{4a^2}{\pi^2}\right]$$

$$= -\frac{8\omega\varepsilon_0 E_0 a^2}{\pi}\sin\omega t$$

Amplitude of the current $= \frac{8\omega\varepsilon_0 E_0 a^2}{\pi}$

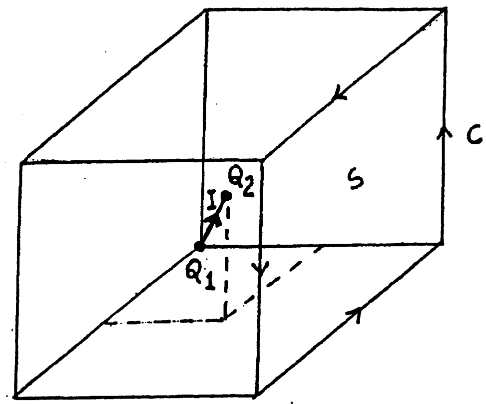
R2.4. $$\oint_C \mathbf{H} \cdot d\mathbf{l} = \int_S \mathbf{J} \cdot d\mathbf{S} + \frac{d}{dt}\int_S \mathbf{D} \cdot d\mathbf{S}$$

$$= 0 + \frac{d}{dt}\left(\frac{Q_1}{24} + \frac{Q_2}{6}\right)$$

$$= \frac{1}{24}\frac{dQ_1}{dt} + \frac{1}{6}\frac{dQ_2}{dt}$$

$$= \frac{1}{24}(-I) + \frac{1}{6}(I)$$

$$= \frac{1}{8}I$$



R2.5. From considerations of symmetry and application of

$$\oint_S \mathbf{D} \cdot d\mathbf{S} = \int_V \rho \, dv$$

to a spherical surface of radius r having the origin as its center, we have

$$4\pi r^2 D_r = \begin{cases} \displaystyle\int_{r=0}^{r} \int_{\theta=0}^{\pi} \int_{\phi=0}^{2\pi} \rho_0 \left(\frac{r}{a}\right)^2 r^2 \sin\theta \, dr \, d\theta \, d\phi & \text{for } r \le a \\ \displaystyle\int_{r=0}^{a} \int_{\theta=0}^{\pi} \int_{\phi=0}^{2\pi} \rho_0 \left(\frac{r}{a}\right)^2 r^2 \sin\theta \, dr \, d\theta \, d\phi & \text{for } r \ge a \end{cases}$$

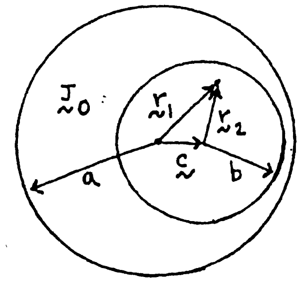
$$= \begin{cases} \dfrac{4\pi\rho_0}{a^2} \left[\dfrac{r^5}{5}\right]_0^r & \text{for } r \le a \\ \dfrac{4\pi\rho_0}{a^2} \left[\dfrac{r^5}{5}\right]_0^a & \text{for } r \ge a \end{cases}$$

$$= \begin{cases} \dfrac{4\pi\rho_0 r^5}{5a^2} & \text{for } r \le a \\ \dfrac{4\pi\rho_0 a^3}{5} & \text{for } r \ge a \end{cases}$$

$$D_r = \begin{cases} \dfrac{\rho_0 r^3}{5a^2} & \text{for } r \le a \\ \dfrac{\rho_0 a^3}{5r^2} & \text{for } r \ge a \end{cases}$$

$$\mathbf{D} = \begin{cases} \dfrac{\rho_0 r^3}{5a^2} \mathbf{a}_r & \text{for } r \le a \\ \dfrac{\rho_0 a^3}{5r^2} \mathbf{a}_r & \text{for } r \ge a \end{cases}$$

R2.6. We can consider the situation as the superposition of a current distribution $\mathbf{J}_0$ within the cylindrical region of radius a and a current distribution $-\mathbf{J}_0$ within the cylindrical region of radius b. Then expressing the result of $\mathbf{H}$ in Ex. 3.8 for the region $r < a$ in the manner



$$\mathbf{H} = \frac{J_0 r}{2}\mathbf{a}_\phi = \frac{J_0 \mathbf{a}_z \times r\mathbf{a}_r}{2} = \frac{1}{2}\mathbf{J}_0 \times \mathbf{r}$$

and applying it for the current-free region inside the cylindrical surface of radius b, we can write

$$\mathbf{H} = \frac{1}{2}\mathbf{J}_0 \times \mathbf{r}_1 + \frac{1}{2}(-\mathbf{J}_0) \times \mathbf{r}_2$$

$$= \frac{1}{2}\mathbf{J}_0 \times (\mathbf{r} - \mathbf{r}_2)$$

$$= \frac{1}{2}\mathbf{J}_0 \times \mathbf{c}$$