

**MIDTERM EXAMINATION PRACTICE**

Semester 2, 2021-2022 • Date: March 30, 2022 • Duration: 90 minutes

<b>SUBJECT: Differential Equations for ISE</b>	
Department of Mathematics	Lecturer
Chair:	
	Associate Prof. Tran Vu Khanh, PhD

**INSTRUCTIONS:**

*Each student is allowed a scientific calculator and a maximum of ONE double-sided sheet of reference material (size A4 or similar) marked with their name and ID. All other documents and electronic devices are forbidden.*

**Arguments and computations must be detailed so that they are easy to follow.**

**Please indicate precisely which problem and question you are solving, e.g. Problem 1, question a.**

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**Question 1.** Show that  $x = e^{4t}$  is a solution to  $x''' - 12x'' + 48x' - 64x = 0$ .

**Question 2.** Let  $xy'' - y' = 0$ . Try a solution of the form  $y = x^r$ . Is this a solution for some  $r$ ? If so, find all such  $r$ .

**Question 3.** Classify the following equations. Are they ODE or PDE? Is it an equation or a system? What is the order? Is it linear or nonlinear, and if it is linear, is it homogeneous, constant coefficient? If it is an ODE, is it autonomous?

(a)  $\sin(t) \frac{d^2x}{dt^2} + \cos(t)x = t^2$

(b)  $\frac{\partial u}{\partial x} + 3 \frac{\partial u}{\partial y} = xy$

(c)  $y'' + 3y + 5x = 0, \quad x'' + x - y = 0$

(d)  $\frac{\partial^2 u}{\partial t^2} + u \frac{\partial^2 u}{\partial s^2} = 0$

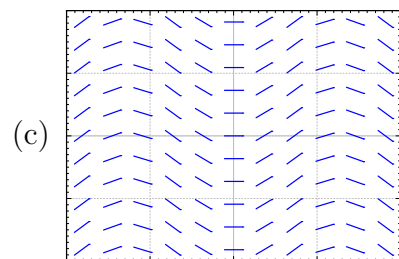
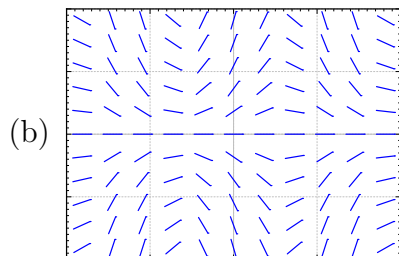
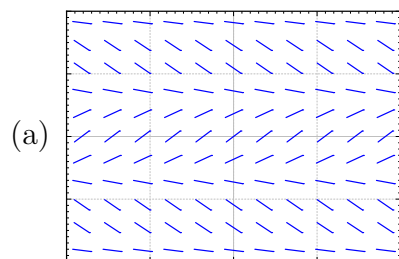
(e)  $x'' + tx^2 = t$

**Question 4.** Find the general solution to  $y'''' = 0$ . How many distinct constants do you need?

**Question 5.** Solve:

- (a)  $\frac{dx}{dt} = -4x, \quad x(0) = 9$
- (b)  $\frac{d^2T}{dx^2} = 4T, \quad T(0) = 0, \quad T'(0) = 6$
- (c)  $\frac{dy}{dx} = \frac{1}{x^2-1}$  for  $y(0) = 0$ .
- (d)  $y' = y^3$  for  $y(0) = 1$ .
- (e)  $y' = (y-1)(y+1)$  for  $y(0) = 3$ .

**Question 6.** Match equations  $y' = \sin x$ ,  $y' = \cos y$ ,  $y' = y \cos(x)$  to slope fields. Justify.



**Question 7.** Find an implicit solution for  $\frac{dy}{dx} = \frac{x^2 + 1}{y^2 + 1}$ , for  $y(0) = 1$ .

**Question 8.** Find an explicit solution for  $y' = xe^{-y}$ ,  $y(0) = 1$ .

**Question 4.** Solve

- (a)  $y' + 6y = e^x$ .
- (b) Solve  $y' + xy = y^4$ , with  $y(0) = 1$ .

**Question 9.** Solve the exact equations,  $(2xy + x^2) dx + (x^2 + y^2 + 1) dy = 0$  implicit general solutions will suffice.

**Question 10.** Take  $(1 - x^2)y'' - xy' + y = 0$ .

- (a) Show that  $y = x$  is a solution.
- (b) Use reduction of order to find a second linearly independent solution.
- (c) Write down the general solution.

**Question 11.**

- (a) Find the general solution of  $y'' + 9y' - 10y = 0$ .
- (b) Find the general solution of  $y'' + 6y' + 13y = 0$ .
- (c) Find the general solution for  $y''' + 2y'' + 2y' = 0$ .

**Question 12.** Construct an equation such that  $y = C_1e^{3x} + C_2e^{-2x}$  is the general solution.

—END OF QUESTION PAPER—