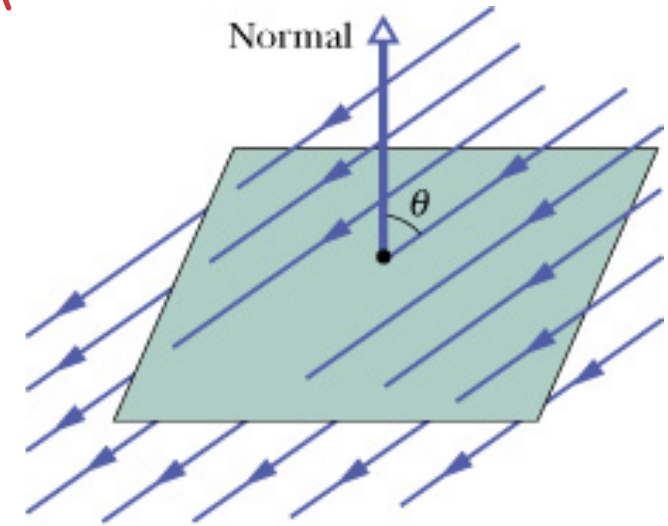
Homework (lecture 4):

1, 7, 12, 13, 14, 17, 21, 22, 24, 35, 36, 39, 43, 44, 51, 52

A = 3.2 mm

1. The square surface as shown measures 3.2 mm on each side. It is immersed in a uniform electric field with E = 1800 N/C and with field lines at an angle of θ = 35^0 with a normal to the surface. Calculate the electric flux through the surface.



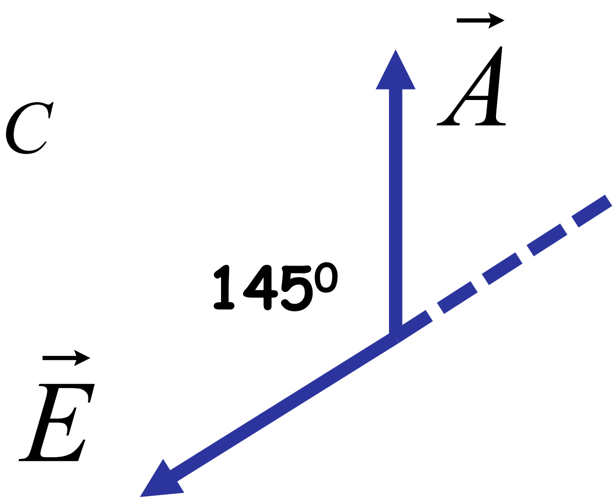
Φ

$$\Phi = \vec{E}\vec{A} = EA\cos\theta = (1800N/C)\times(3.2\times10^{-3}m)^{2}\cos(180^0 - 35^0)$$

$$\Phi = -1.51\times10^{-2}\,Nm^2/C$$

$\Phi = \vec{E}\vec{A}$

$= EA\cos\theta = Nm^2/C$



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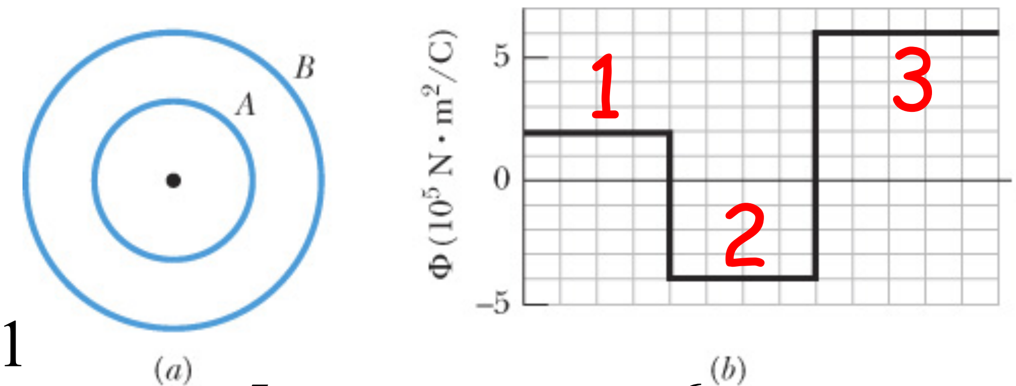
7. A point charge of 1.8 μC is at the center of a cubical Gaussian surface 55 cm on edge. What is the net electric flux through the surface?

Using Gauss's law:

$$\boxed{\varepsilon_0 \Phi = q_{\text{enclosed}}}$$

$$\Phi = \frac{q_{\text{enclosed}}}{\varepsilon_0} = \frac{1.8\times10^{-6}\,C}{8.85\times10^{-12}\,C^2/Nm^2} = 2.0\times10^{5}\,Nm^2/C$$

14. A charged particle is suspended at the center of two concentric spherical shells that are very thin and made of nonconducting material. Figure a shows a cross section. Figure b gives the net flux Φ through a Gaussian sphere centered on the particle, as a function of the radius r of the sphere. (a) What is the charge of the central particle? What are the net charges of (b) shell A and (c) shell B?



$$\varepsilon_0 \Phi = q_{\text{enclosed}}$$

(a) For r < r_A (region 1):

$$q_{\text{enclosed1}} = q_{\text{particle}} = \varepsilon_0 \Phi_1$$

$$q_{\text{particle}} = 8.85\times10^{-12}\times 2\times10^{5} = 1.77\times10^{-6}(C)$$

$$\approx 1.8\,(\mu C)$$

(b) For r_A < r < r_B (region 2): $q_{\text{enclosed2}} = q_{\text{particle}} + q_A = \varepsilon_0 \Phi_2$

$$\Phi_2 = -4\times10^{5}\ (Nm^2/C) \Rightarrow q_A = -5.3\times10^{-6}\ (C)\ \text{or}\ -5.3\mu C$$

(c) For r_B < r (region 3): $\Phi_3 = 6\times10^{5}\ (Nm^2/C) \Rightarrow q_B$

17. A uniformly charged conducting sphere of 1.2 m diameter has a surface charge density of 8.1 μC/m². (a) Find the net charge on the sphere. (b) What is the total electric flux leaving the surface of the sphere?

(a) charge = area x surface density

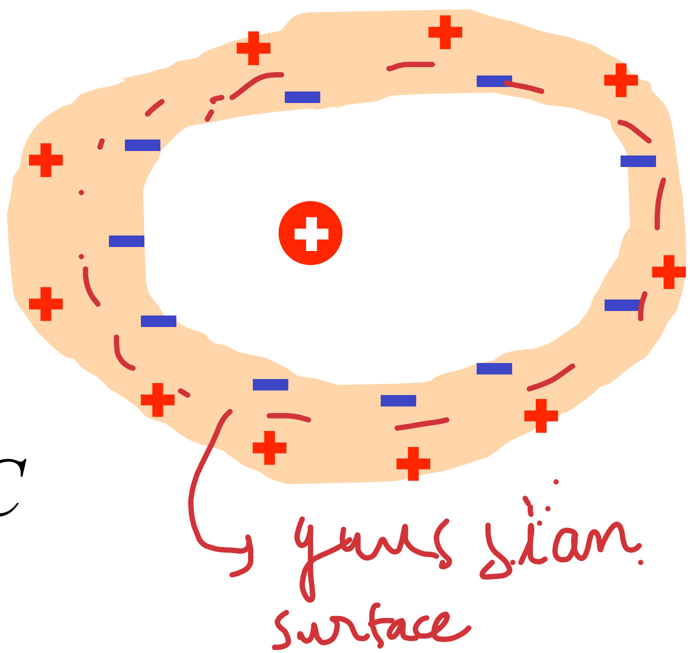
$$q = 4\pi r^2 \sigma = 4 \times 3.14 \times 0.6^2 \times 8.1 \times 10^{-6} = 3.7 \times 10^{-5} (C)$$

(b) We choose a Gaussian surface covers whole the sphere, using Gauss' law:

$$\Phi = \frac{q_{\text{enclosed}}}{\varepsilon_0} = \frac{3.7 \times 10^{-5}}{8.85 \times 10^{-12}} = 4.2 \times 10^{6}\, Nm^2 / C$$

21. An isolated conductor of arbitrary shape has a net charge of $+10\times10^{-6}$ C. Inside the conductor is a cavity within which is a point charge $q = +3.0\times10^{-6}$ C. What is the charge (a) on the cavity wall and (b) on the outer surface of the conductor?

(a) Consider a Gaussian surface within the conductor that covers the cavity wall, in the conductor, E = 0:



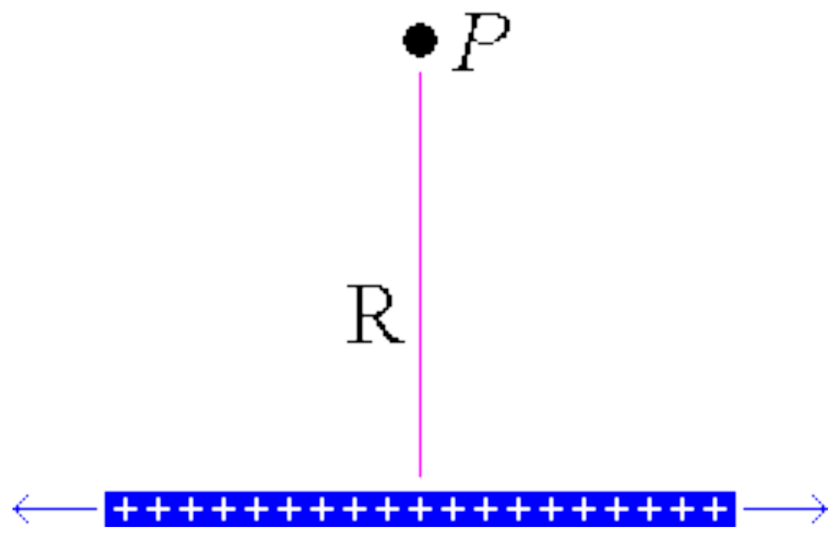
$$q_{\text{wall}} + q_{\text{point}} = 0$$

$$q_{\text{wall}} = -q_{\text{point}} = -3\times10^{-6}\,C \text{ or } -3\mu C$$

(b) the total charge of the conductor:

$$q_{\text{wall}} + q_{\text{outer}} = 10\times10^{-6} \Rightarrow q_{\text{outer}} = 13\times10^{-6}\,C \text{ or } 13\mu C$$

22. An electron is released from rest at a perpendicular distance of 9 cm from a line of charge on a very long nonconducting rod. That charge is uniformly distributed , with 4.5 μC per meter. What is the magnitude of the electron's initial acceleration?



Electric field at point P:

$$E = \frac{\lambda}{2\pi\varepsilon_0 R}$$

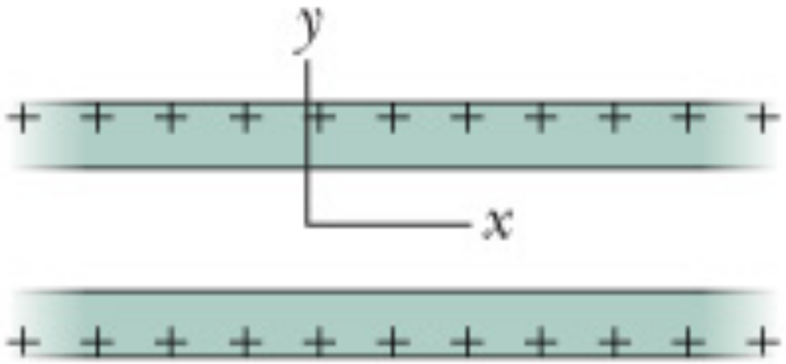
Force acting on the electron:

$$F = eE = \frac{e\lambda}{2\pi\varepsilon_0 R} = ma \Rightarrow a = \frac{e\lambda}{2\pi\varepsilon_0 mR}$$

R = 9 cm = 0.09 m

λ = 4.5 μC/m = 4.5×10^{-6} C/m

36. The figure shows cross sections through two large, parallel, nonconducting sheets with identical distributions of positive charge with surface charge density $\sigma = 2.31\times10^{-22}$ C/m^2. In unit-vector notation, what is $\vec{E}$ at points (a) above the sheets, (b) between them, and (c) below them?



For one non-conducting sheet:

$$E = \frac{\sigma}{2\varepsilon_0}$$

Using the superposition to calculate E due to two sheets:

(a)

$$E = 2\frac{\sigma}{2\varepsilon_0} = \frac{\sigma}{\varepsilon_0} = \frac{2.31\times10^{-22}}{8.85\times10^{-12}} = 2.61\times10^{-11}(N/C)$$

The net electric field direction is upward $\vec{E} = 2.61\times10^{-11}(N/C)\hat{j}$

(b) E = 0

(c) $\vec{E} = -2.61\times10^{-11}(N/C)\hat{j}$

The direction is downward

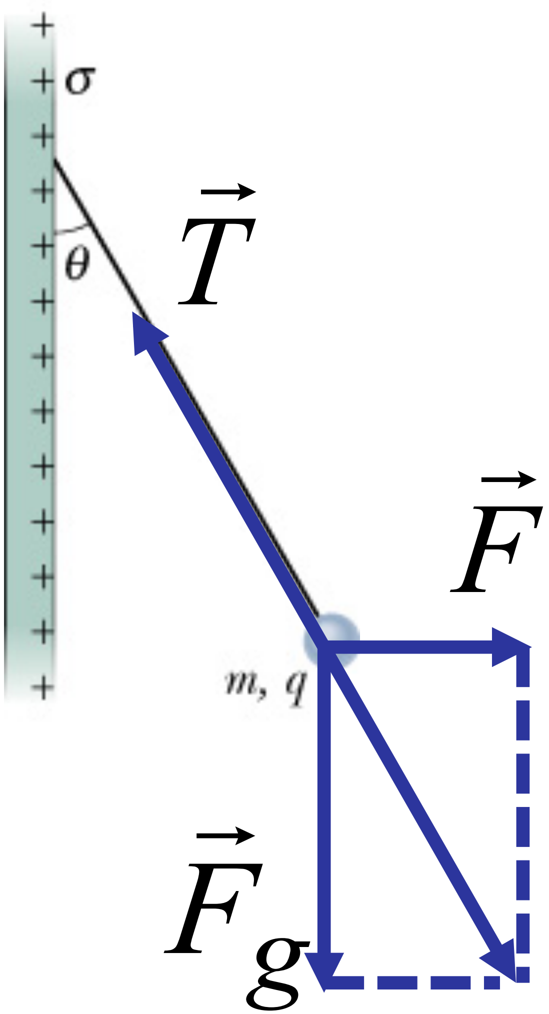
39. A small, nonconducting ball of mass m = 1 mg and charge q = 2×10^{-8} C hangs from an insulating thread that makes an angle θ = 30^0 with a vertical, uniformly charged nonconducting sheet. Considering the gravitational force on the ball and assuming the sheet extends far vertically and into and out of the page, calculate the surface charge density σ of the sheet.

If the ball is in equilibrium:

$$\vec{F} + \vec{F}_g + \vec{T} = 0$$

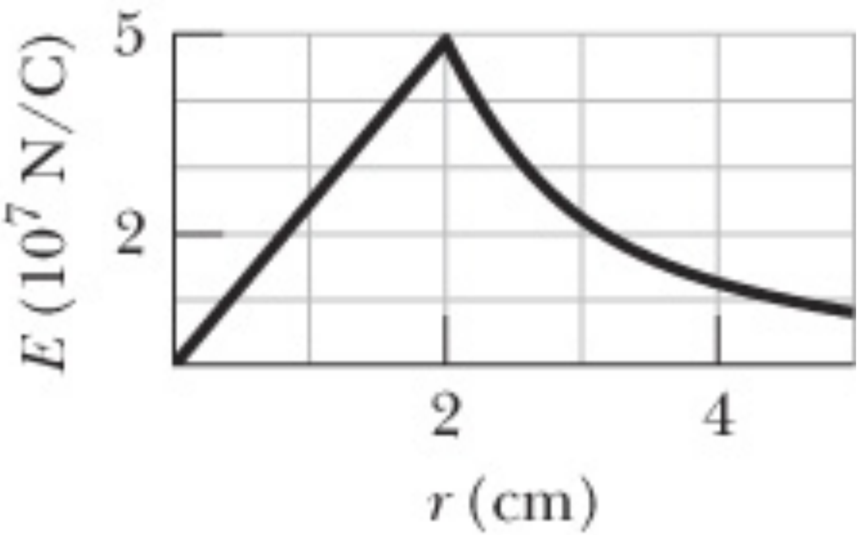
$$\tan\theta = \frac{F}{F_g} = \frac{qE}{mg} = \frac{q}{mg} \times \frac{\sigma}{2\varepsilon_0}$$

$$\sigma = \frac{2\varepsilon_0 mg \tan\theta}{q} = 5\times10^{-9}(C/m^2)$$



44. The figure gives the magnitude of the electric field inside and outside a sphere with a positive charge distributed uniformly throughout its volume. What is the charge on the sphere?

$$E = \left(\frac{q}{4\pi\varepsilon_0 R^3}\right)r \qquad (r \le R)$$

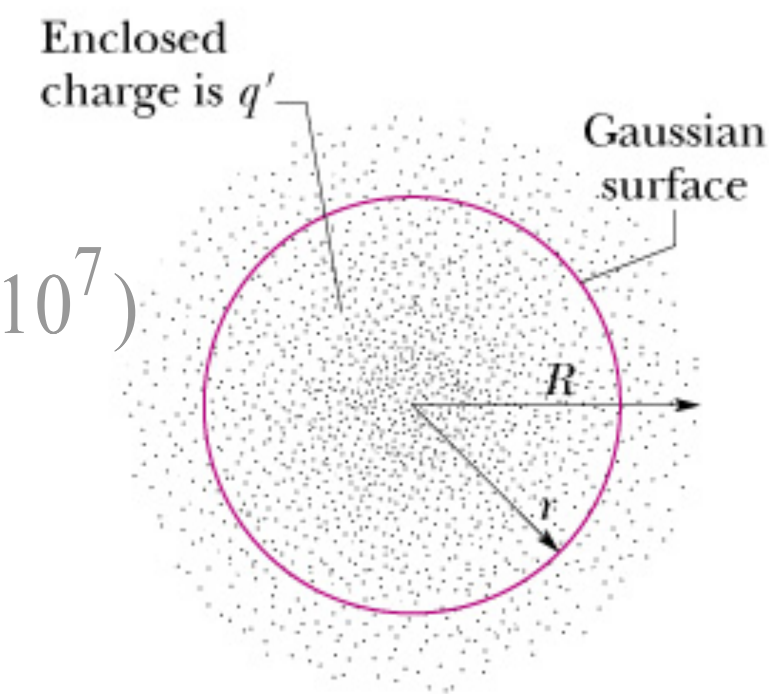


- At r = 2 cm, E is maximum, so R = 2 cm

$$E = \frac{q}{4\pi\varepsilon_0 R^2}$$

$$q = 4\times 3.14\times 8.85\times 10^{-12}\times(0.02)^2\times(5\times 10^7)$$

$$= 2.2\times 10^{-6}(C)$$



51. A nonconducting spherical shell of inner radius a = 2 cm and outer radius b = 2.4 cm has a positive volume charge density ρ = A/r, where A is a constant and r is the distance from the center of the shell. In addition, a small ball of charge q = 45 fC is located at that center. What value should A have if the electric field in the shell ($a \le r \le b$) is to be uniform?

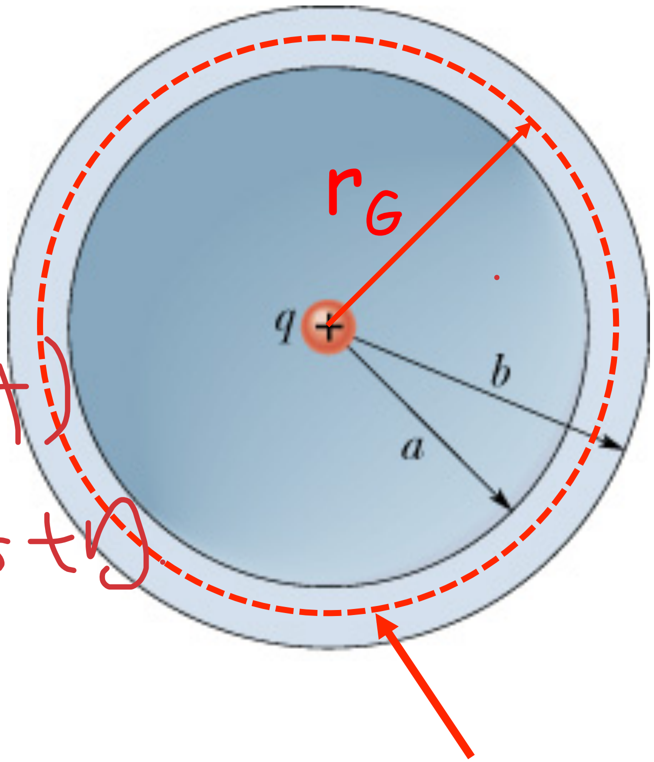
Key idea: First, we need to calculate E inside the shell, if the field is uniform, so E is independent of distance from the center

$$E_r = \frac{1}{4\pi\varepsilon_0}\frac{q_{total}}{r^2}$$

$$q_{total} = q + q_{shell}$$

$E = f(A)$

$= CA + D$



Gaussian surface

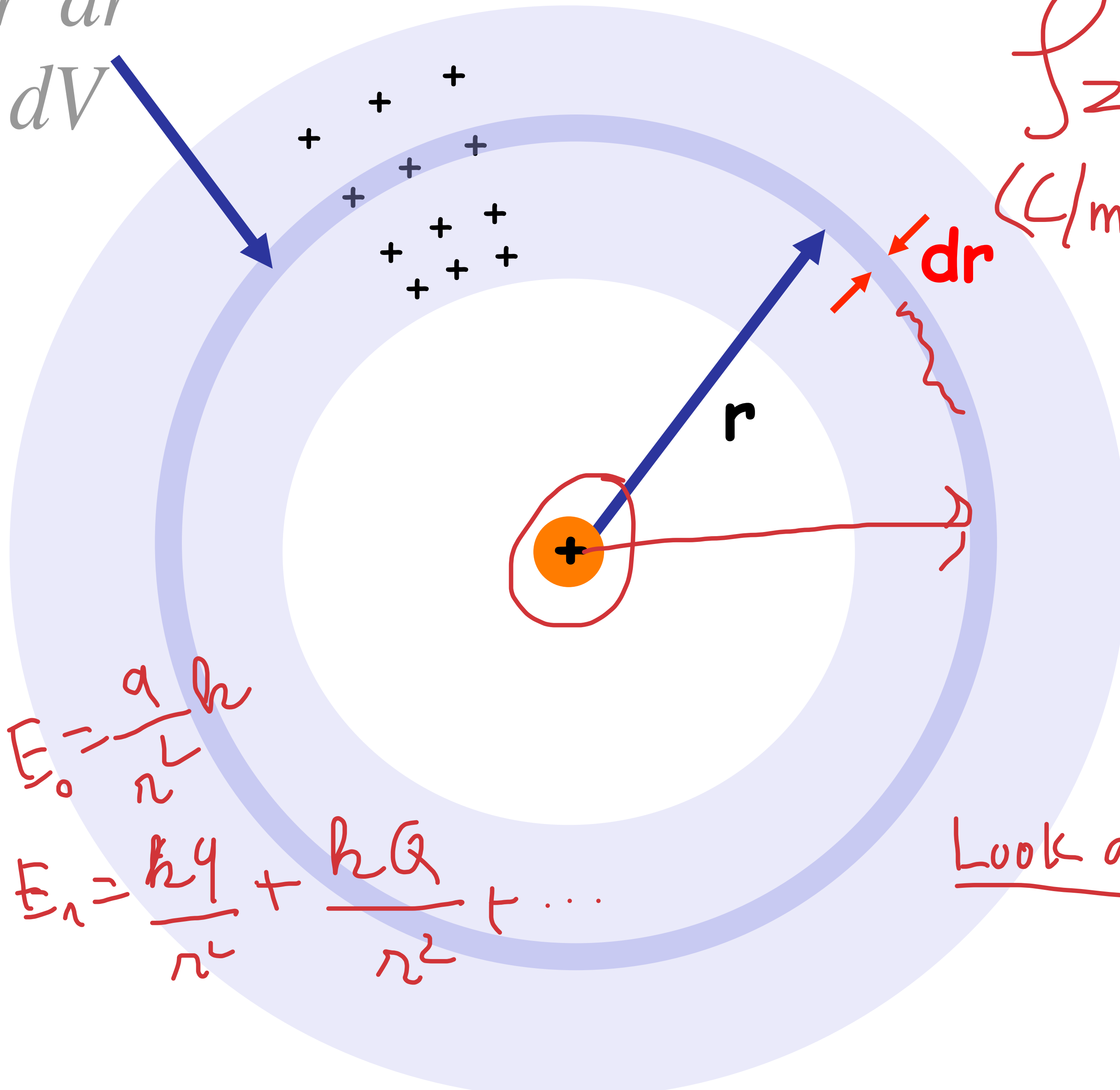
q_{shell} is the enclosed charge in the shell of thickness $r_G - a$: $dq_{shell} = \rho \times dV = \rho \times 4\pi r^2 dr$

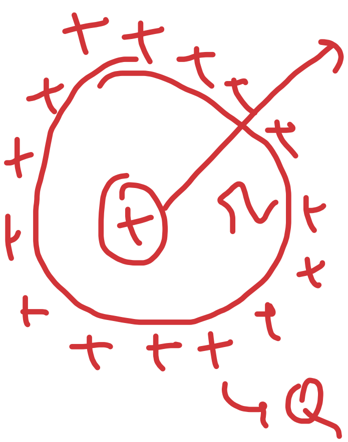
$$q_{shell} = 4\pi \int_a^{r_G} \frac{A}{r} r^2 dr = 2\pi A(r_G^2 - a^2)$$

$$dV = 4\pi r^2 dr$$

$$dq = \rho \times dV$$

$$\rho = \frac{A}{r} \quad (C/m)$$





$$E_0 = \frac{kq}{r^2}$$

$$E_r = \frac{kq}{r^2} + \frac{kQ}{r^2} + \cdots$$

Look at this one!

Using Gauss' law: $$\varepsilon_0 \Phi = q_{\text{total}}$$

$$\varepsilon_0 E 4\pi r_G^2 = q_{\text{total}}$$

$$E = \frac{q_{\text{total}}}{\varepsilon_0 4\pi r_G^2} = \frac{q + 2\pi A(r_G^2 - a^2)}{4\pi\varepsilon_0 r_G^2}$$

We rewrite:

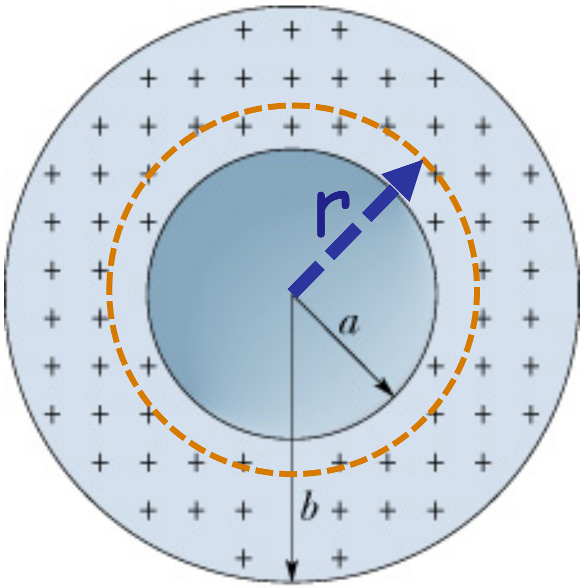
$$E = \frac{A}{2\varepsilon_0} + \frac{1}{2\varepsilon_0}\left(\frac{q}{2\pi} - Aa^2\right) \times \frac{1}{r_G^2}$$

If E is uniform in the shell:

$$\frac{q}{2\pi} - Aa^2 = 0 \Rightarrow A = \frac{q}{2\pi a^2}$$

$$A = \frac{45 \times 10^{-15} C}{2 \times 3.14 \times (0.02m)^2} = 1.79 \times 10^{-11} (C/m^2)$$

52. The figure below shows a spherical shell with uniform volume charge density ρ = 1.56 nC/m^3, inner radius a = 10 cm, and outer radius b = 2a. What is the magnitude of the electric field at radial distances (a) r = 0, (b) r = a/2, (c) r = a, (d) r = 1.5 a, (e) r = b, and (f) r = 3b?



For (a), (b), (c) using Gauss's law, we find

E = 0

For (d), (e) $a \le r \le b$:

The enclosed charge:

$$q_{enc} = \rho \times V = \rho\left(\frac{4}{3}\pi r^3 - \frac{4}{3}\pi a^3\right)$$

The electric field:

$$E = \frac{1}{4\pi\varepsilon_0}\frac{q_{enc}}{r^2}$$

For (f):

$$E = \frac{1}{4\pi\varepsilon_0}\frac{q_{total}}{r^2} = \frac{1}{4\pi\varepsilon_0}\frac{\rho \times \frac{4}{3}\pi(b^3 - a^3)}{r^2}$$

$$\boxed{E = \frac{\rho}{3\varepsilon_0}\frac{b^3 - a^3}{r^2}}$$

Homework (lecture 5):

1, 6, 8, 11, 14, 18, 24, 28, 29, 35, 43, 59, 60, 64

1. A particular 12 V car battery can send a total charge of 84 A.h through a circuit, from one terminal to the other. (a) How many coulombs of charge does this represent? (b) If this entire charge undergoes a change in electric potential of 12 V, how much energy is involved?

(a) In the previous lecture, we mentioned that the coulomb unit is derived from ampere for electric current i:

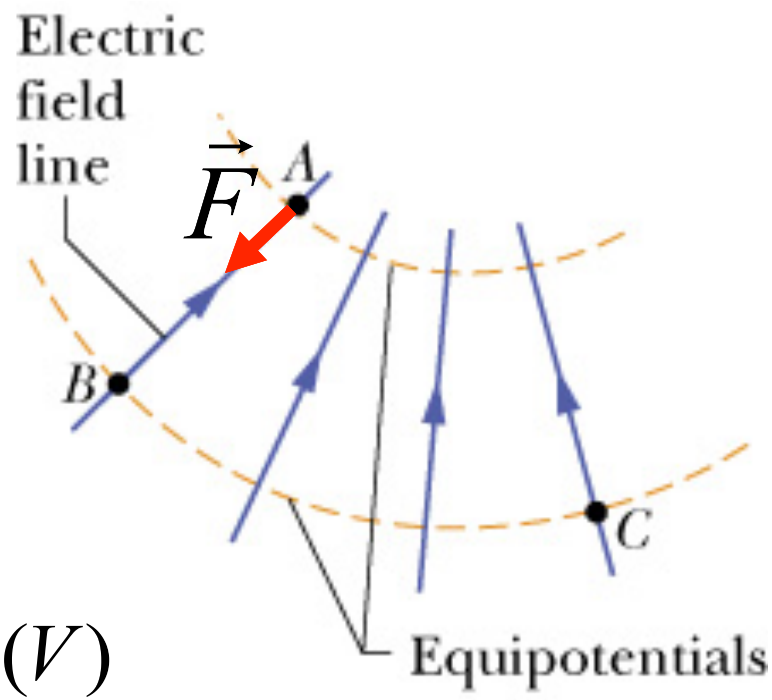
$$i = \frac{dq}{dt} \Rightarrow dq = idt$$

$$Q = 84(\mathrm{C/s}) \times 3600(\mathrm{s}) = 3 \times 10^5 (\mathrm{C})$$

(b) Energy is computed by:

$$\Delta U = \Delta V \times Q = 12 \times 3 \times 10^5 = 3.6 \times 10^6 (\mathrm{J})$$

6. When an electron moves from A to B along an electric field, see the figure. The electric field does 4.78×10^{-19} J of work on it. What are the electric potential differences (a) $V_B - V_A$, (b) $V_C - V_A$, and (c) $V_C - V_B$?



(a) We have work done by the electric field:

$$W = -q\Delta V$$

$$W = -(-e)(V_B - V_A)$$

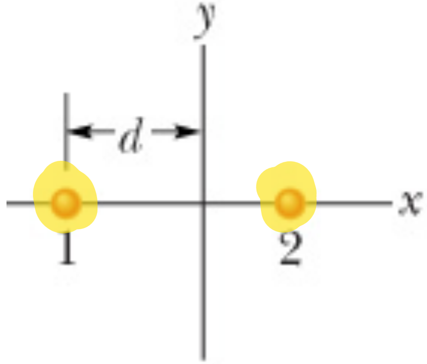
$$V_B - V_A = \frac{W}{e} = \frac{4.78\times10^{-19}}{1.6\times10^{-19}} = 3.0(V)$$

(b)

$$V_C - V_A = V_B - V_A = 3.0(V)$$

(c) $V_C - V_B = 0$: on the same equipotential

18. Two charged particles are shown in Figure a. Particle 1, with charge q_1, is fixed in place at distance d. Particle 2, with charge q_2, can be moved along the x axis. Figure b gives the net electric potential V at the origin due to the two particles as a function of the x coordinate of particle 2. The plot has an asymptote of V = 5.92×10^{-7} V as $x \to \infty$. What is q_2 in terms of e?



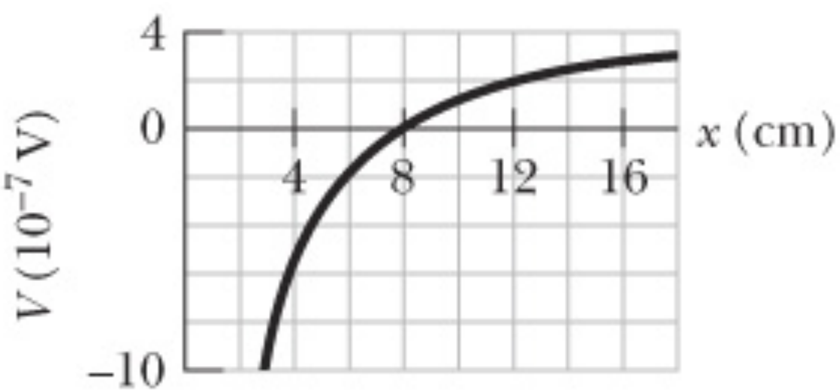
(a)

Potential due to a point charge: $V = k\dfrac{q}{r}$

Potential at the origin (O) due to q_1 and q_2:

$$V_O = k\frac{q_1}{d} + k\frac{q_2}{x}$$

$$V_{O,x=\infty} = k\frac{q_1}{d} = 5.92\times10^{-7}\,(V)$$



(b)

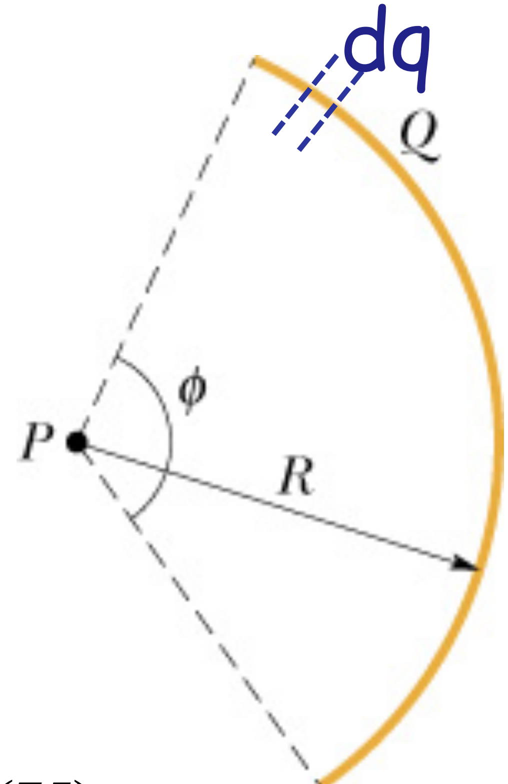
At x = 8 cm, V_O = 0:

$$V_{O,x=8} = V_{O,x=\infty} + k\frac{q_2}{x}$$

$$q_2 = -\frac{V_{O,x=\infty}x}{k} = -\frac{5.92\times10^{-7}\times0.08}{8.99\times10^{9}} = -5.27\times10^{-18}\,(C) \text{ or } -33e$$

24. The figure shows a plastic rod having a uniformly distributed charge Q = -28.9 pC has been bent into a circular arc of radius R = 3.71 cm and central angle Φ = 120^0. With V=0 at infinity, what is the electric potential at P, the center of curvature of the rod?

Consider potential at P due to an element dq:



$$dV = k\frac{dq}{R}$$

$$V = \int k\frac{dq}{R} = k\frac{Q}{R}$$

$$V = \frac{8.99\times10^{9}\times(-28.9\times10^{-12})}{3.71\times10^{-2}} = -7.0(V)$$

35. The electric potential at points in an xy plane is given by $V = (2\,\mathrm{V/m^2})x^2 - (3\,\mathrm{V/m^2})y^2$. In unit vector notation, what is the electric field at the point (3.0 m, 2.0 m)?

We have:

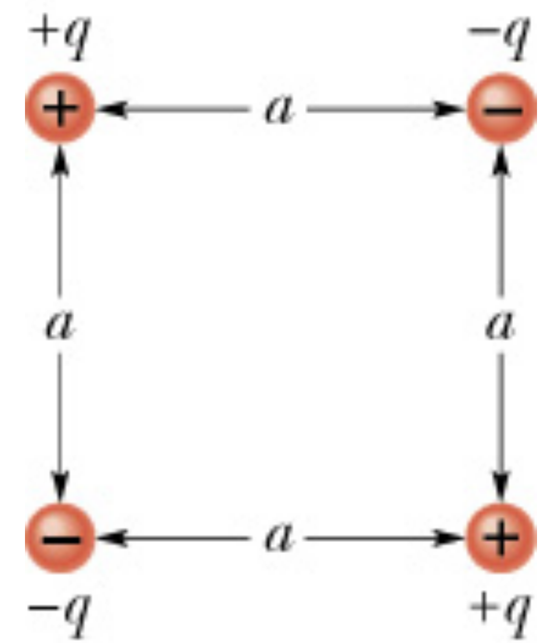
$$\boxed{\vec{E} = -\nabla V}$$

$$E_x = -\frac{\partial V}{\partial x}; E_y = -\frac{\partial V}{\partial y}$$

$$E_x = -4x = -12(V/m); E_y = 6y = 12(V/m)$$

$$\boxed{\vec{E} = -12(V/m)\hat{i} + 12(V/m)\hat{j}}$$

43. How much work is required to set up the arrangement of the figure below if q = 2.3 pC, a = 64 cm, and the particles are initially infinitely far apart and at rest?



We have 4 charges, so we have N = 6 pairs:

$$N = \frac{n(n-1)}{2}$$

$$W_{\text{applied}} = U_{\text{system}}$$

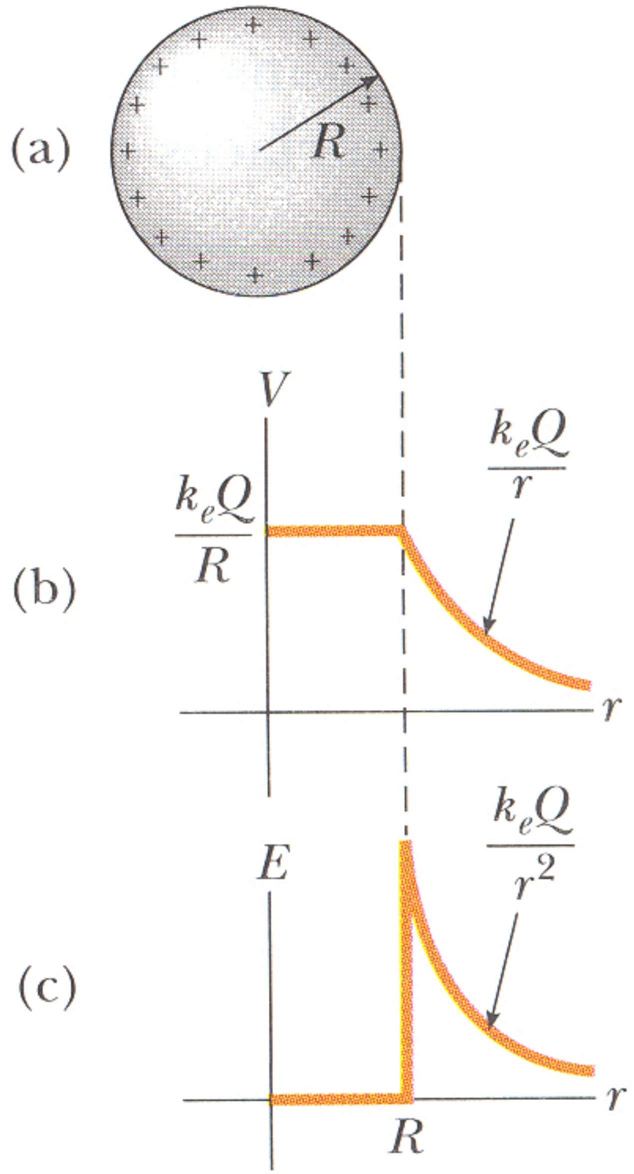
$$U_{\text{system}} = kq^2\left(-\frac{1}{a} - \frac{1}{a} + \frac{1}{a\sqrt{2}} - \frac{1}{a} + \frac{1}{a\sqrt{2}} - \frac{1}{a}\right)$$

$$U_{\text{system}} = \frac{2kq^2}{a}\left(\frac{1}{\sqrt{2}} - 2\right)$$

Note: q = 2.3 pC = 2.3×10^{-12} C; a = 64 cm = 0.64 m

64. A hollow metal sphere has a potential of +300 V with respect to ground (defined to be at V = 0) and a charge of 5.0×10^{-9} C. Find the electric potential at the center of the sphere.

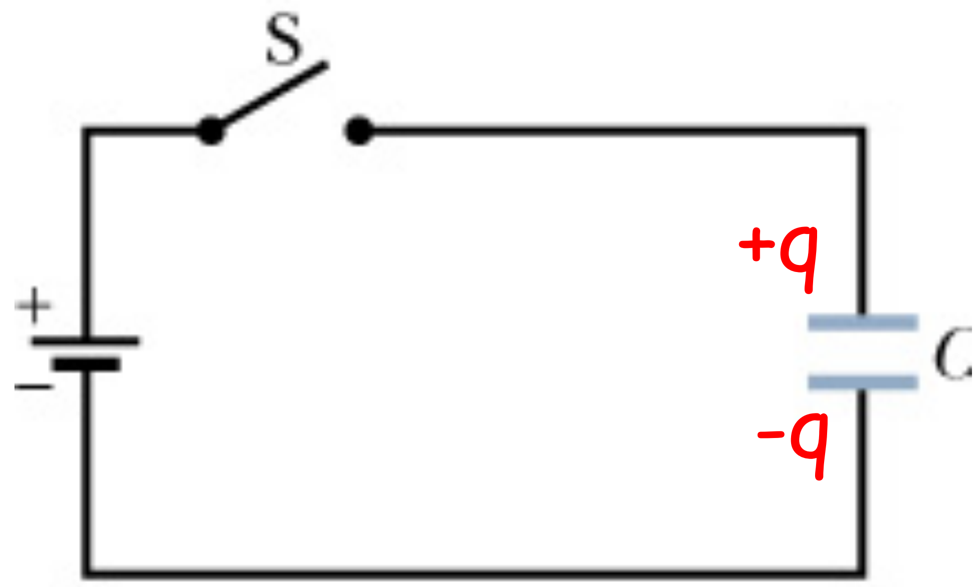
V = constant = +300 V throughout the entire conductor, this is valid for solid and hollow metal spheres.



Homework (lecture 6):

2, 4, 6, 11, 14, 16, 26, 31, 33, 42, 48, 51

2. The capacitor in the figure below has a capacitance of 30 μF and is initially uncharged. The battery provides a potential difference of 120 V. After switch S is closed, how much charge will pass through it?

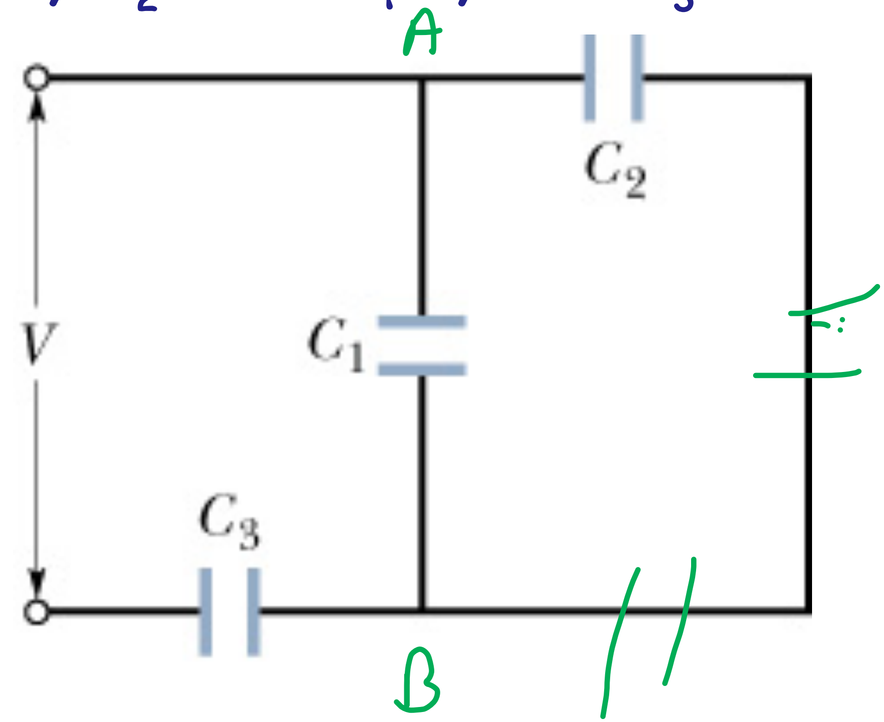


S is closed, the charge on the capacitor plates is:

$$\boxed{q = CV}$$

$$q = 30\times10^{-6}\times120 = 3.6\times10^{-3}(C)$$

11. In the figure below, find the equivalent capacitance of the combination. Assume that C_1 = 10.0 μF, C_2 = 5.0 μF, and C_3 = 4.0 μF



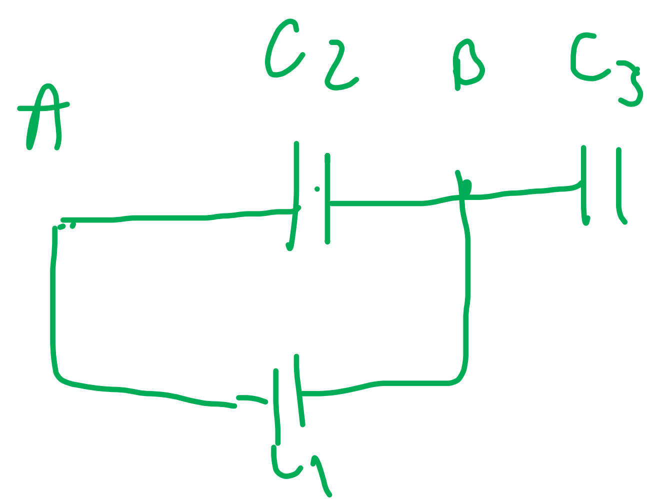
C_1 and C_2 are in parallel, the equivalent capacitance :

$$C_{12} = C_1 + C_2 = 15(\mu F)$$

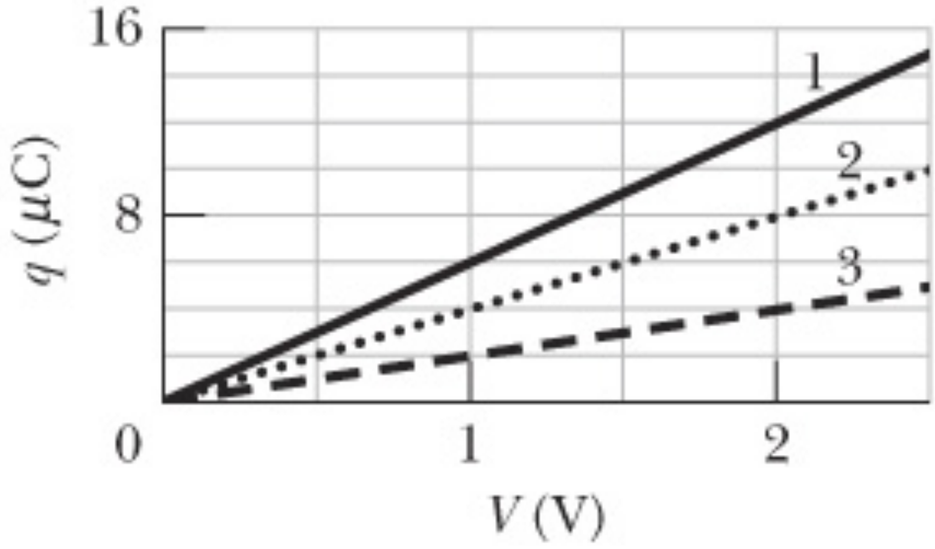
C_{12} and C_3 in series:

$$C_{123} = \frac{C_{12}C_3}{C_{12}+C_3} = \frac{15\times 4}{15+4} = 3.16(\mu F)$$

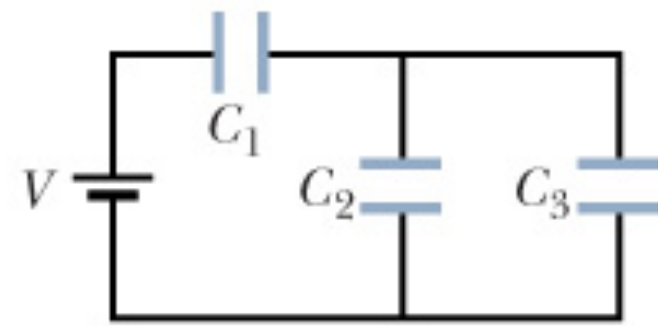
$$\frac{1}{C_{123}} = \frac{1}{C_{12}} + \frac{1}{C_3}$$



16. Plot 1 in Figure a gives the charge q that can be stored on capacitor 1 versus the electric potential V set up across it. Plots 2 and 3 are similar plots for capacitors 2 and 3, respectively. Figure b shows a circuit with those three capacitors and a 10.0 V battery. What is the charge stored on capacitor 2 in that circuit?



(a)



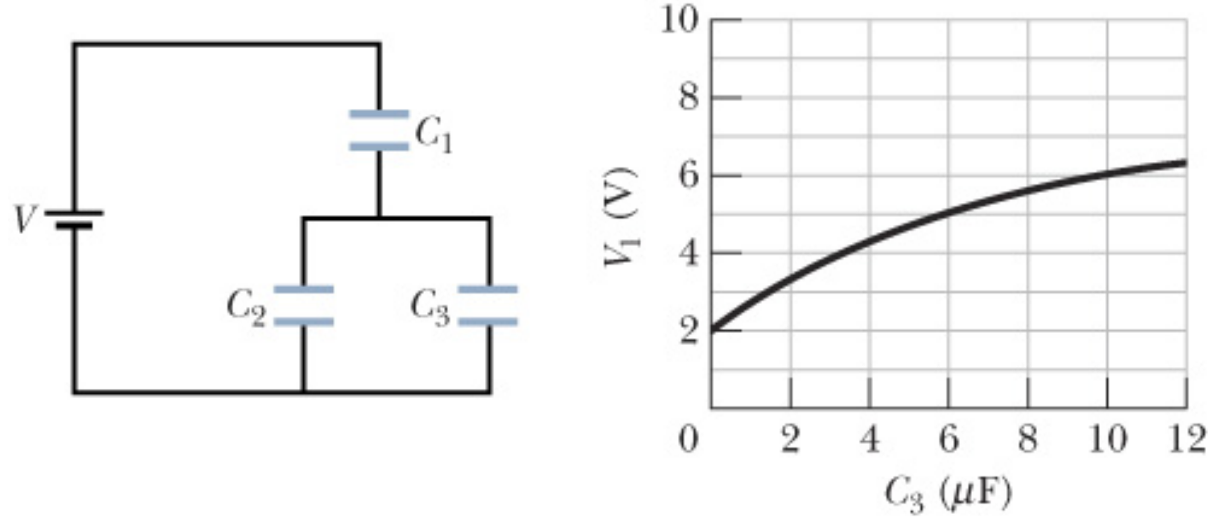
(b)

$$C_1 = \frac{q_1}{V_1} = \frac{12(\mu C)}{2(V)} = 6\mu F; C_2 = \frac{q_2}{V_2} = \frac{8(\mu C)}{2(V)} = 4\mu F; C_3 = \frac{q_3}{V_3} = \frac{4(\mu C)}{2(V)} = 2\mu F$$

$$C_{123} = 3(\mu F)$$

$$V_1 = \frac{q}{C_1} = \frac{C_{123}V}{C_1} = \frac{1}{2}10 = 5(\mathrm{V}) \Rightarrow q_2 = C_2V_2 = 4\mathrm{\mu F} \times 5\mathrm{V} = 20\mathrm{\mu C}$$

26. Capacitor 3 in Figure a is a variable capacitor (its capacitance C_3 can be varied). Figure b gives the electric potential V_1 across capacitor 1 versus C_3. Electric potential V_1 approaches an asymptote of 8 V as $C_3 \rightarrow \infty$. What are (a) the electric potential V across the battery, (b) C_1, and (c) C_2?



(a) (b)

(a)When $C_3 \rightarrow \infty$, $C_{123} = C_1$; so, $V = V_1 = 8$ V

(b)

$$C_{123} = \frac{C_1 C_{23}}{C_1 + C_{23}} = \frac{C_1(C_2 + C_3)}{C_1 + C_2 + C_3};$$

$$V_1 = \frac{q}{C_1} = \frac{C_{123}V}{C_1} = \frac{C_2 + C_3}{C_1 + C_2 + C_3}V$$

- **At C_3 = 0, V_1 = 2 V:**

$$C_1 = 3C_2$$

- **At C_3 = 6 μF, V_1 = 5 V:**

$$V_1 = \frac{C_2 + 6}{3C_2 + C_2 + 6}8 = 5$$

$$C_2 = 1.5\mu F; C_1 = 4.5\mu F$$

33. A charged isolated metal sphere of diameter 10 cm has a potential of 8000 V relative to V = 0 at infinity. Calculate the energy density in the electric field near the surface of the sphere.

In a general case, the energy density is computed by:

$$\boxed{u = \frac{1}{2}\varepsilon_0 E^2}$$

For a charged isolated metal sphere:

$$u = \frac{1}{2}\varepsilon_0\left(\frac{V}{R}\right)^2 = \frac{1}{2}8.85\times10^{-12}\left(\frac{8000}{0.05}\right)^2 = 0.113(\mathrm{J/m^3})$$

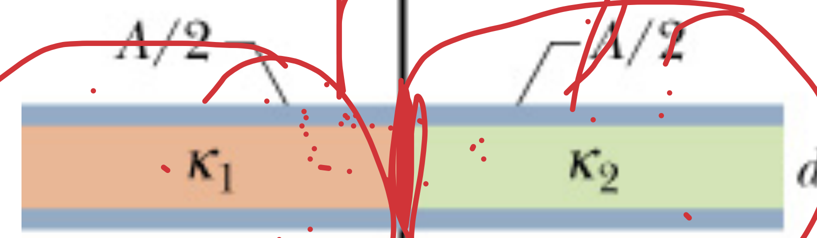
42. A parallel-plate air-filled capacitor has a capacitance of 50 pF: (a) If each of its plates has an area of 0.30 m², what is the separation? (b) If the region between the plates is now filled with material having κ = 5.6, what is the capacitance?

(a) For parallel-plate capacitors: $C = \frac{\varepsilon_0 A}{d}$

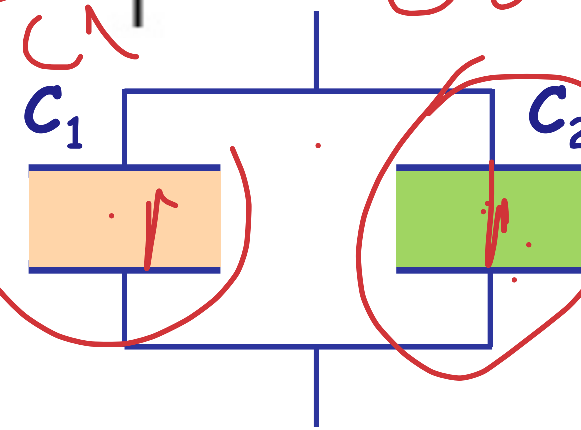
$$d = \frac{\varepsilon_0 A}{C} = \frac{8.85\times 10^{-12}\times 0.30}{50\times 10^{-12}} = 5.3\times 10^{-2}(m) = 5.3(cm)$$

(b) With a dielectric: $C' = \kappa C = 5.6\times 50 = 280(pF)$

48. The figure below shows a parallel-plate capacitor with a plate area $A = 5.56\ cm^2$ and separation $d = 5.56$ mm. The left half of the gap is filled with material of dielectric constant $\kappa_1 = 7.00$; the right half is filled with material of dielectric constant $\kappa_2 = 12.0$. What is the capacitance?



Their configuration is equivalent to a combination of **two capacitors in parallel** with dielectrics κ_1 and κ_2, respectively



equivalent capacitor

$$C_0 = \frac{\varepsilon_0 (A/2)}{d} =$$

$$= \frac{8.85\times10^{-12}\times5.56\times10^{-4}}{2\times5.56\times10^{-3}} = 4.43\times10^{-13}\,(F)$$

$$C_0 = 0.443\,\text{pF}$$

$$C_{\text{equivalent}} = C_1 + C_2 = (\kappa_1 + \kappa_2)C_0 = 8.42(\text{pF})$$