International University

School of Electrical Engineering

PRINCIPLES OF ELECTRICAL ENGINEERING 2

Lecture #12: Two-port Circuits

Chapter #18

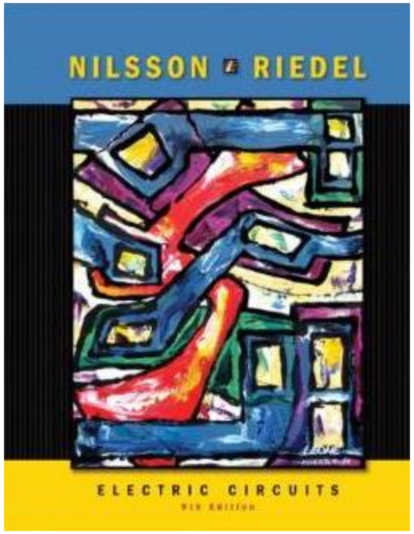
Text book: Electric Circuits

James W. Nilsson & Susan A. Riedel

9th Edition.

link: **http://blackboard.hcmiu.edu.vn/**

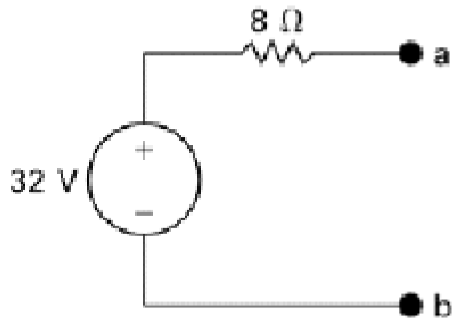
to download materials



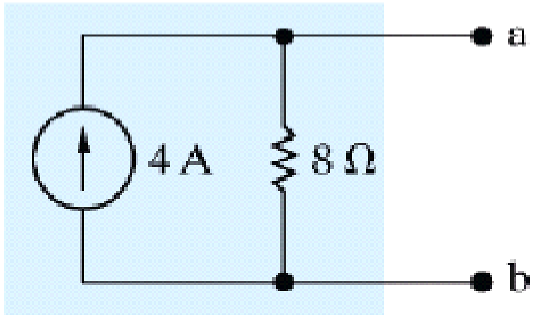
Lecture's Objectives

1. Be able to calculate any set of two-port parameters with any of the following methods:
   - Circuit analysis;
   - Measurements made on a circuit;
   - Converting from another set of two-port parameters using Table 18.1.
2. Be able to analyze a terminated two-port circuit to find currents, voltages, impedances, and ratios of interest using Table 18.2.
3. Know how to analyze a cascade interconnection of two-port circuits.

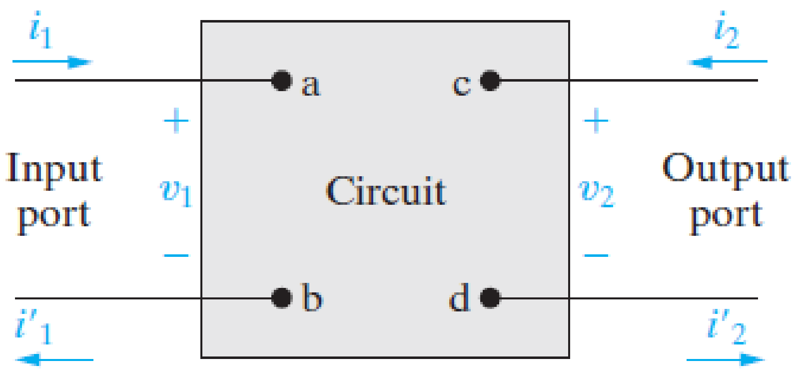
Introduction



Thevenin equivalent circuit



Norton equivalent circuit



Two-port circuit model

The **two-port model** is used to describe the performance of a circuit in terms of the voltage and current as its input and output ports.

The two-port circuit

A two-port network is an electrical network with two separate ports for input and output.

The model is limited to circuits in which:

- No independent sources are inside the circuit between the ports
- No energy is stored inside the circuit between the ports
- The current into the ports is equal to the current out of the port
- No external connections exist between the input and output ports.

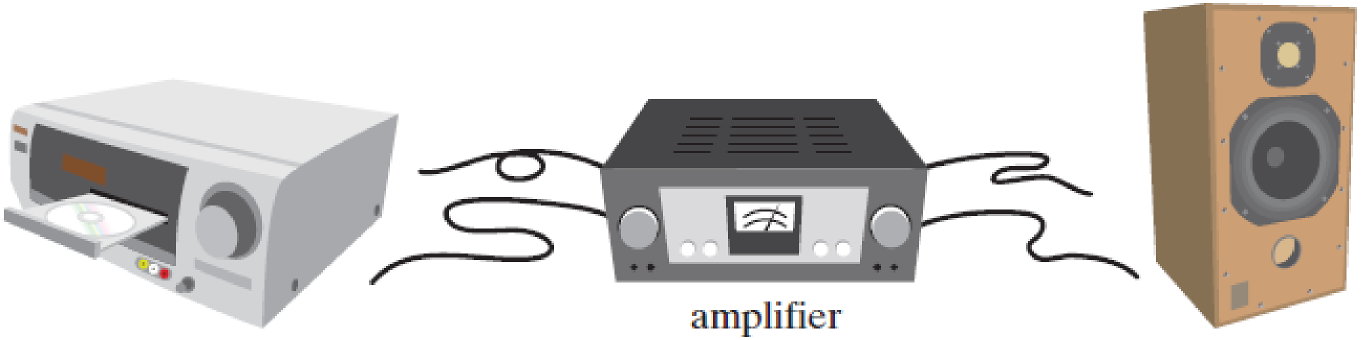
Characterizing an Unknown Circuit

To create a model of a circuit, we needed to know what types of components make up the circuit, the values of those components, and the interconnections among those components.

But what if we want to model a circuit that is inside a "black box", where the components, their values, and their interconnections are hidden?

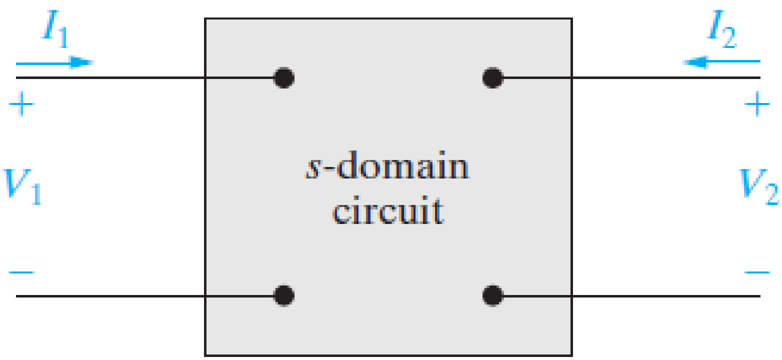
We can perform 2 simple experiments on such a black box to create a model that consists of just 4 values - the two-port parameter model for the circuit.

Use the two-port parameter model to predict the behavior of the circuit once we have attached a power source to one of its ports and a load to the other port.



The terminal equations

The most general description of the two-port network is carried out in the *s domain.*



Two of the four terminal variables (**V_1**, **I_1**, **V_2**, **I_2**) are independent; ➔ only two simultaneous equations involving the four variables are needed to describe the circuit.

There are <u>six possible sets of equations</u> involving the four terminal variables:

z parameters

$$V_1 = z_{11}I_1 + z_{12}I_2$$
$$V_2 = z_{21}I_1 + z_{22}I_2$$

a parameters

$$V_1 = a_{11}V_2 - a_{12}I_2$$
$$I_1 = a_{21}V_2 - a_{22}I_2$$

h parameters

$$V_1 = h_{11}I_1 + h_{12}V_2$$
$$I_2 = h_{21}I_1 + h_{22}V_2$$

y parameters

$$I_1 = y_{11}V_1 + y_{12}V_2$$
$$I_2 = y_{21}V_1 + y_{22}V_2$$

b parameters

$$V_2 = b_{11}V_1 - b_{12}I_1$$
$$I_2 = b_{21}V_1 - b_{22}I_1$$

g parameters

$$I_1 = g_{11}V_1 + g_{12}I_2$$
$$V_2 = g_{21}V_1 + g_{22}I_2$$

The two-port parameters

z - parameters

The open circuit impedance (or **z-parameter**) characterization of two –port networks is based on exciting the network by I_1 and I_2

matrix form

The describing equations are: $V_1 = z_{11}I_1 + z_{12}I_2$

$$V_2 = z_{21}I_1 + z_{22}I_2$$

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = [z] \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$$z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0}$$

→ z_{11} is the impedance seen looking into port 1 when port 2 is open.

$$z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0}$$

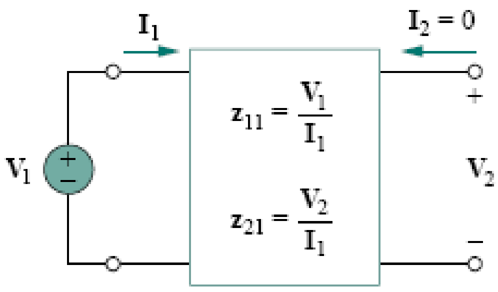
→ z_{21} is a transfer impedance. It is the ratio of the port 2 voltage to the port 1 current when port 2 is open.

$$z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0}$$

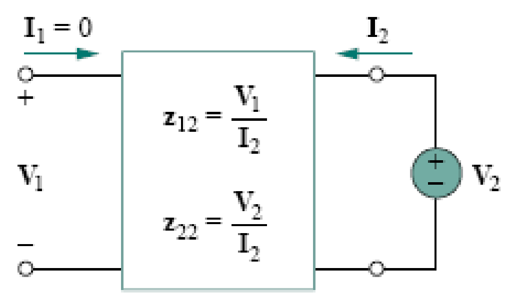
→ z_{12} is a transfer impedance. It is the ratio of the port 1 voltage to the port 2 current when port 1 is open.

$$z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0}$$

→ z_{22} is the impedance seen looking into port 2 when port 1 is open.



(a)



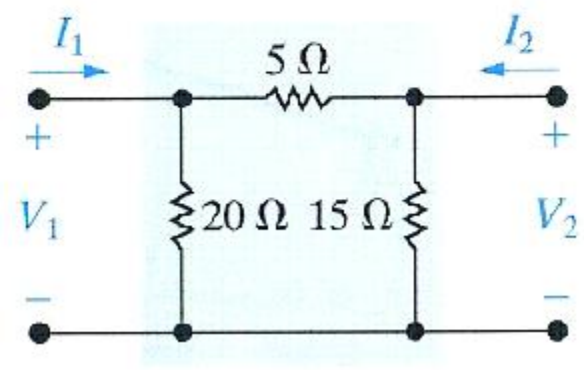
(b)

Each parameter is the ratio of a voltage to a current and therefore is an impedance with the dimension of ohms.

The two-port parameters

Example 1

Find the z parameters for the circuit below



Sol.

The circuit is purely resistive → the s-domain circuit is also purely resistive. With port 2 open, that is, $I_2 = 0$, the resistance seen looking into port 1 is the 20 Ω resistor in parallel with the series combination of the 5 and 15 Ω resistors. Therefore

$$z_{11} = \frac{V_1}{I_1}\bigg|_{I_2=0} = \frac{(20)(20)}{40} = 10\ \Omega.$$

When I_2 is zero, V_2 is $\quad V_2 = \frac{V_1}{15+5}(15) = 0.75V_1$

The two-port parameters

Example 1 **Sol.**

and therefore $$z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0} = \frac{0.75V_1}{V_1/10} = 7.5\ \Omega.$$

When I_1 is zero, the resistance seen looking into port 2 is the 15 Ω resistor in parallel with the series combination of the 5 and 20 Ω resistors. Therefore

$$z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0} = \frac{(15)(25)}{40} = 9.375\ \Omega.$$

When port 1 is open, I_1 is zero and the voltage V_1 $$V_1 = \frac{V_2}{5+20}(20) = 0.8V_2.$$

With port 1 open, the current into port 2 is $$I_2 = \frac{V_2}{9.375}.$$

$$\Rightarrow \quad z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0} = \frac{0.8V_2}{V_2/9.375} = 7.5\ \Omega.$$

Each parameter is the ratio of a voltage to a current and therefore is an impedance with the dimension of ohms.

The two-port parameters
y - parameters

The short circuit admittance (or **y-parameter**) characterization is based on exciting the network by V_1 and V_2 .

The describing equations are: $I_1 = y_{11}V_1 + y_{12}V_2$

$$I_2 = y_{21}V_1 + y_{22}V_2$$

matrix form

$$\begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \end{bmatrix} = \begin{bmatrix} \mathbf{y}_{11} & \mathbf{y}_{12} \\ \mathbf{y}_{21} & \mathbf{y}_{22} \end{bmatrix} \begin{bmatrix} \mathbf{V}_1 \\ \mathbf{V}_2 \end{bmatrix} = [\mathbf{y}] \begin{bmatrix} \mathbf{V}_1 \\ \mathbf{V}_2 \end{bmatrix}$$

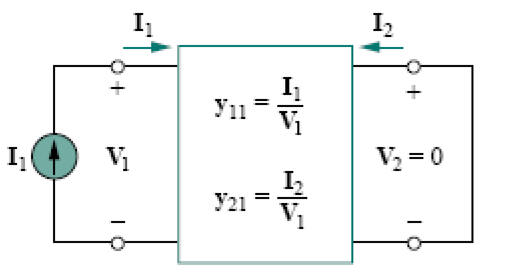
y-parameters can be calculated by following relations:

$$y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0}$$

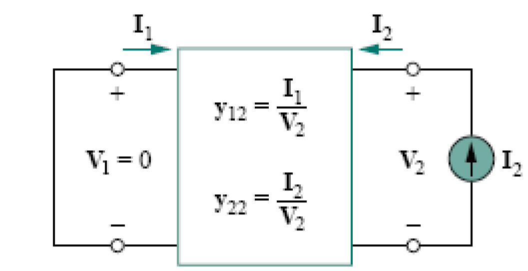
$$y_{12} = \left.\frac{I_1}{V_2}\right|_{V_1=0}$$

$$y_{21} = \left.\frac{I_2}{V_1}\right|_{V_2=0}$$

$$y_{22} = \left.\frac{I_2}{V_2}\right|_{V_1=0}$$



(a)



(b)

Each parameter is the ratio of a current to a voltage and therefore is an admittance with the dimension of siemens (**S**).

The **z** and **y** parameters are impedance and admittance parameters; therefore are grouped into the **immittance** parameters.

The two-port parameters
a and b - parameters

a & b parameters are called the **transmission** parameters because they describe the voltage and current and one end of the two-port network in terms of the voltage and current at the other end.

a - parameters

$$V_1 = a_{11}V_2 - a_{12}I_2$$
$$I_1 = a_{21}V_2 - a_{22}I_2$$

$$a_{11} = \left.\frac{V_1}{V_2}\right|_{I_2=0}$$

$$a_{12} = -\left.\frac{V_1}{I_2}\right|_{V_2=0}$$

$$a_{21} = \left.\frac{I_1}{V_2}\right|_{I_2=0}$$

$$a_{22} = -\left.\frac{I_1}{I_2}\right|_{V_2=0}$$

b - parameters

$$V_2 = b_{11}V_1 - b_{12}I_1$$
$$I_2 = b_{21}V_1 - b_{22}I_1$$

$$b_{11} = \left.\frac{V_2}{V_1}\right|_{I_1=0}$$

$$b_{12} = -\left.\frac{V_2}{I_1}\right|_{V_1=0}$$

$$b_{21} = \left.\frac{I_2}{V_1}\right|_{I_1=0}$$

$$b_{22} = -\left.\frac{I_2}{I_1}\right|_{V_1=0}$$

The two-port parameters
g and h - parameters

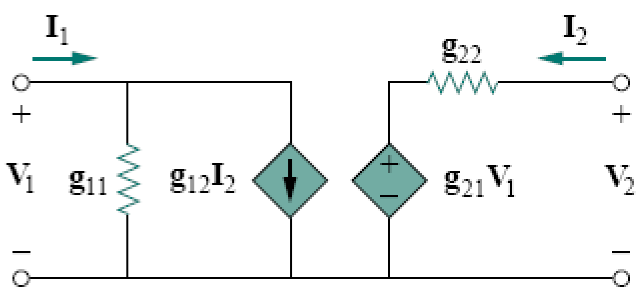
g and h parameters are called the **hybrid** parameters because they relate cross-variables, that is, an input voltage and output current to an output voltage and input current.

g - parameters

$$I_1 = g_{11}V_1 + g_{12}I_2$$
$$V_2 = g_{21}V_1 + g_{22}I_2$$

$$g_{11} = \left.\frac{I_1}{V_1}\right|_{I_2=0}$$
$$g_{12} = \left.\frac{I_1}{I_2}\right|_{V_1=0}$$
$$g_{21} = \left.\frac{V_2}{V_1}\right|_{I_2=0}$$
$$g_{22} = \left.\frac{V_2}{I_2}\right|_{V_1=0}$$

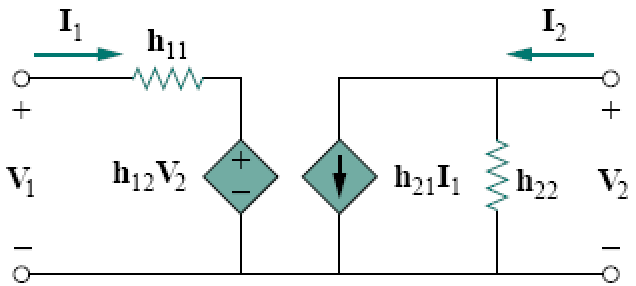
The *g-parameter model of a* two-port network.



h - parameters

$$V_1 = h_{11}I_1 + h_{12}V_2$$
$$I_2 = h_{21}I_1 + h_{22}V_2$$

$$h_{11} = \left.\frac{V_1}{I_1}\right|_{V_2=0}$$
$$h_{12} = \left.\frac{V_1}{V_2}\right|_{I_1=0}$$
$$h_{21} = \left.\frac{I_2}{I_1}\right|_{V_2=0}$$
$$h_{22} = \left.\frac{I_2}{V_2}\right|_{I_1=0}$$



The *h-parameter equivalent* network of a two-port network.

Example 2:

The following measurements pertain to a 2-port circuit operating in the sinusoidal steady state. With port 2 open, a voltage equal to 150cos4000t V is applied to port 1. The current into port 1 is 25cos(4000t - 45°) A, and the port 2 voltage is 100cos(4000t + 15°) V. With port 2 short-circuited, a voltage equal to 30cos4000t V is applied to port 1. The current into port 1 is 1.5cos(4000t + 30°) A, and the current into port 2 is 0.25cos(4000t + 150°) A. *Find the a parameters that can describe the sinusoidal steady-state behavior of the circuit.*

Solution:

The first set of measurements gives
$$\begin{cases} \mathbf{V}_1 = 150\angle 0^\circ \text{ V}, & \mathbf{I}_1 = 25\angle -45^\circ \text{ A}, \\ \mathbf{V}_2 = 100\angle 15^\circ \text{ V}, & \mathbf{I}_2 = 0 \text{ A}. \end{cases}$$

$$\Rightarrow a_{11} = \left.\frac{\mathbf{V}_1}{\mathbf{V}_2}\right|_{I_2=0} = \frac{150\angle 0^\circ}{100\angle 15^\circ} = 1.5\angle -15^\circ, \quad a_{21} = \left.\frac{\mathbf{I}_1}{\mathbf{V}_2}\right|_{I_2=0} = \frac{25\angle -45^\circ}{100\angle 15^\circ} = 0.25\angle -60^\circ \text{S}.$$

The second set of measurements gives
$$\begin{cases} \mathbf{V}_1 = 30\angle 0^\circ \text{ V}, & \mathbf{I}_1 = 1.5\angle 30^\circ \text{A}, \\ \mathbf{V}_2 = 0 \text{ V}, & \mathbf{I}_2 = 0.25\angle 150^\circ \text{ A}. \end{cases}$$

Therefore

$$a_{12} = -\left.\frac{\mathbf{V}_1}{\mathbf{I}_2}\right|_{V_2=0} = \frac{-30\angle 0^\circ}{0.25\angle 150^\circ} = 120\angle 30^\circ\ \Omega, \quad a_{21} = -\left.\frac{\mathbf{I}_1}{\mathbf{I}_2}\right|_{V_2=0} = \frac{-1.5\angle 30^\circ}{0.25\angle 150^\circ} = 6\angle 60^\circ.$$

Relationships among the two-port parameters

- The six sets of equations relate to the same variables, therefore, we can derive all the other sets from the known set.
- The relationships among the six sets of parameters are given in Table 18.1 page 736 from the textbook.

<u>Ex.:</u> Given the *z parameters, let us obtain the y parameters.*

$$I_1 = y_{11}V_1 + y_{12}V_2$$
$$I_2 = y_{21}V_1 + y_{22}V_2$$

$$V_1 = \frac{\begin{vmatrix} I_1 & y_{12} \\ I_2 & y_{22} \end{vmatrix}}{\begin{vmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{vmatrix}} = \frac{y_{22}}{\Delta y} I_1 - \frac{y_{12}}{\Delta y} I_2,$$

$$V_2 = \frac{\begin{vmatrix} y_{11} & I_1 \\ y_{21} & I_2 \end{vmatrix}}{\Delta y} = -\frac{y_{21}}{\Delta y} I_1 + \frac{y_{11}}{\Delta y} I_2.$$

$$V_1 = z_{11}I_1 + z_{12}I_2$$
$$V_2 = z_{21}I_1 + z_{22}I_2$$

$$\mathbf{y}_{11} = \frac{\mathbf{z}_{22}}{\Delta_z}, \qquad \mathbf{y}_{12} = -\frac{\mathbf{z}_{12}}{\Delta_z},$$
$$\mathbf{y}_{21} = -\frac{\mathbf{z}_{21}}{\Delta_z}, \qquad \mathbf{y}_{22} = \frac{\mathbf{z}_{11}}{\Delta_z}$$

TABLE 18.1 Parameter Conversion Table

$$z_{11} = \frac{y_{22}}{\Delta y} = \frac{a_{11}}{a_{21}} = \frac{b_{22}}{b_{21}} = \frac{\Delta h}{h_{22}} = \frac{1}{g_{11}}$$

$$z_{12} = -\frac{y_{12}}{\Delta y} = \frac{\Delta a}{a_{21}} = \frac{1}{b_{21}} = \frac{h_{12}}{h_{22}} = -\frac{g_{12}}{g_{11}}$$

$$z_{21} = \frac{-y_{21}}{\Delta y} = \frac{1}{a_{21}} = \frac{\Delta b}{b_{21}} = -\frac{h_{21}}{h_{22}} = \frac{g_{21}}{g_{11}}$$

$$z_{22} = \frac{y_{11}}{\Delta y} = \frac{a_{22}}{a_{21}} = \frac{b_{11}}{b_{21}} = \frac{1}{h_{22}} = \frac{\Delta g}{g_{11}}$$

$$y_{11} = \frac{z_{22}}{\Delta z} = \frac{a_{22}}{a_{12}} = \frac{b_{11}}{b_{12}} = \frac{1}{h_{11}} = \frac{\Delta g}{g_{22}}$$

$$y_{12} = -\frac{z_{12}}{\Delta z} = -\frac{\Delta a}{a_{12}} = -\frac{1}{b_{12}} = -\frac{h_{12}}{h_{11}} = \frac{g_{12}}{g_{22}}$$

$$y_{21} = -\frac{z_{21}}{\Delta z} = -\frac{1}{a_{12}} = -\frac{\Delta b}{b_{12}} = \frac{h_{21}}{h_{11}} = -\frac{g_{21}}{g_{22}}$$

$$y_{22} = \frac{z_{11}}{\Delta z} = \frac{a_{11}}{a_{12}} = \frac{b_{22}}{b_{12}} = \frac{\Delta h}{h_{11}} = \frac{1}{g_{22}}$$

$$a_{11} = \frac{z_{11}}{z_{21}} = -\frac{y_{22}}{y_{21}} = \frac{b_{22}}{\Delta b} = -\frac{\Delta h}{h_{21}} = \frac{1}{g_{21}}$$

$$a_{12} = \frac{\Delta z}{z_{21}} = -\frac{1}{y_{21}} = \frac{b_{12}}{\Delta b} = -\frac{h_{11}}{h_{21}} = \frac{g_{22}}{g_{21}}$$

$$a_{21} = \frac{1}{z_{21}} = -\frac{\Delta y}{y_{21}} = \frac{b_{21}}{\Delta b} = -\frac{h_{22}}{h_{21}} = \frac{g_{11}}{g_{21}}$$

$$a_{22} = \frac{z_{22}}{z_{21}} = -\frac{y_{11}}{y_{21}} = \frac{b_{11}}{\Delta b} = -\frac{1}{h_{21}} = \frac{\Delta g}{g_{21}}$$

$$b_{11} = \frac{z_{22}}{z_{12}} = -\frac{y_{11}}{y_{12}} = \frac{a_{22}}{\Delta a} = \frac{1}{h_{12}} = -\frac{\Delta g}{g_{12}}$$

$$b_{12} = \frac{\Delta z}{z_{12}} = -\frac{1}{y_{12}} = \frac{a_{12}}{\Delta a} = \frac{h_{11}}{h_{12}} = -\frac{g_{22}}{g_{12}}$$

$$b_{21} = \frac{1}{z_{12}} = -\frac{\Delta y}{y_{12}} = \frac{a_{21}}{\Delta a} = \frac{h_{22}}{h_{12}} = -\frac{g_{11}}{g_{12}}$$

$$b_{22} = \frac{z_{11}}{z_{12}} = \frac{y_{22}}{y_{12}} = \frac{a_{11}}{\Delta a} = \frac{\Delta h}{h_{12}} = -\frac{1}{g_{12}}$$

$$h_{11} = \frac{\Delta z}{z_{22}} = \frac{1}{y_{11}} = \frac{a_{12}}{a_{22}} = \frac{b_{12}}{b_{11}} = \frac{g_{22}}{\Delta g}$$

$$h_{12} = \frac{z_{12}}{z_{22}} = -\frac{y_{12}}{y_{11}} = \frac{\Delta a}{a_{22}} = \frac{1}{b_{11}} = -\frac{g_{12}}{\Delta g}$$

$$h_{21} = -\frac{z_{21}}{z_{22}} = \frac{y_{21}}{y_{11}} = -\frac{1}{a_{22}} = -\frac{\Delta b}{b_{11}} = -\frac{g_{21}}{\Delta g}$$

$$h_{22} = \frac{1}{z_{22}} = \frac{\Delta y}{y_{11}} = \frac{a_{21}}{a_{22}} = \frac{b_{21}}{b_{11}} = \frac{g_{11}}{\Delta g}$$

$$g_{11} = \frac{1}{z_{11}} = \frac{\Delta y}{y_{22}} = \frac{a_{21}}{a_{11}} = \frac{b_{21}}{b_{22}} = \frac{h_{22}}{\Delta h}$$

$$g_{12} = -\frac{z_{12}}{z_{11}} = \frac{y_{12}}{y_{22}} = -\frac{\Delta a}{a_{11}} = -\frac{1}{b_{22}} = -\frac{h_{12}}{\Delta h}$$

$$g_{21} = \frac{z_{21}}{z_{11}} = -\frac{y_{21}}{y_{22}} = \frac{1}{a_{11}} = \frac{\Delta b}{b_{22}} = -\frac{h_{21}}{\Delta h}$$

$$g_{22} = \frac{\Delta z}{z_{11}} = \frac{1}{y_{22}} = \frac{a_{12}}{a_{11}} = \frac{b_{12}}{b_{22}} = \frac{h_{11}}{\Delta h}$$

$$\Delta z = z_{11}z_{22} - z_{12}z_{21}$$

$$\Delta y = y_{11}y_{22} - y_{12}y_{21}$$

$$\Delta a = a_{11}a_{22} - a_{12}a_{21}$$

$$\Delta b = b_{11}b_{22} - b_{12}b_{21}$$

$$\Delta h = h_{11}h_{22} - h_{12}h_{21}$$

$$\Delta g = g_{11}g_{22} - g_{12}g_{21}$$

Example 3:

Two sets of measurements are made on a two-port resistive circuit. The first set is made with port 2 open, and the second set is made with port 2 short-circuited. The results are as follows:

Port 2 Open	**Port 2 Short-Circuited**
$V_1 = 10\text{ mV}$	$V_1 = 24\text{ mV}$
$I_1 = 10\ \mu\text{A}$	$I_1 = 20\ \mu\text{A}$
$V_2 = -40\text{ V}$	$I_2 = 1\text{ mA}$

Find the h parameters of the circuit.

Solution

We can find h_{11} and h_{21} directly from the short circuit test:

$$h_{11} = \left.\frac{V_1}{I_1}\right|_{V_2=0} = \frac{24 \times 10^{-3}}{20 \times 10^{-6}} = 1.2\text{ k}\Omega,$$

$$h_{21} = \left.\frac{I_2}{I_1}\right|_{V_2=0} = \frac{10^{-3}}{20 \times 10^{-6}} = 50.$$

The parameters h_{12} and h_{22} cannot be obtained directly from the open-circuit test. However, a check of Eqs. of z parameters & immittance indicates that the four a parameters can be derived from the test data. Therefore, h_{12} and h_{22} can be obtained through the conversion table. Specifically,

$$h_{12} = \frac{\Delta a}{a_{22}} \qquad h_{22} = \frac{a_{21}}{a_{22}}.$$

Example 3: Solution

The a parameters are

$$a_{11} = \left.\frac{V_1}{V_2}\right|_{I_2=0} = \frac{10 \times 10^{-3}}{-40} = -0.25 \times 10^{-3},$$

$$a_{21} = \left.\frac{I_1}{V_2}\right|_{I_2=0} = \frac{10 \times 10^{-6}}{-40} = -0.25 \times 10^{-6}\,\mathrm{S},$$

$$a_{12} = -\left.\frac{V_1}{I_2}\right|_{V_2=0} = -\frac{24 \times 10^{-3}}{10^{-3}} = -24\ \Omega,$$

$$a_{22} = -\left.\frac{I_1}{I_2}\right|_{V_2=0} = -\frac{20 \times 10^{-6}}{10^{-3}} = -20 \times 10^{-3}.$$

The numerical value of $\Delta a = a_{11}a_{22} - a_{12}a_{21} = 5{\times}10^{-6} - 6{\times}10^{-6} = -10^{-6}$

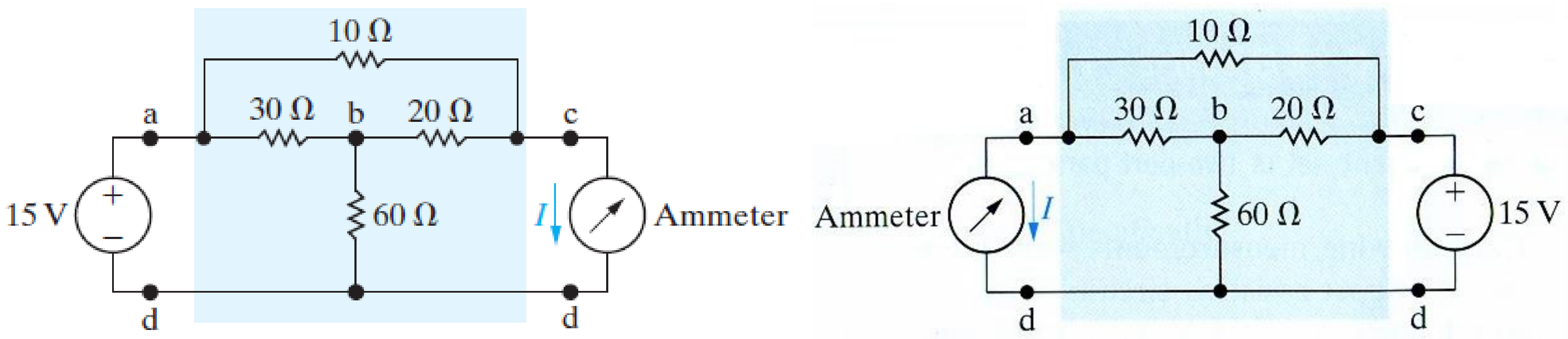
$$h_{12} = \frac{\Delta a}{a_{22}} = \frac{-10^{-6}}{-20 \times 10^{-3}} = 5 \times 10^{-5},$$

$$h_{22} = \frac{a_{21}}{a_{22}} = \frac{-0.25 \times 10^{-6}}{-20 \times 10^{-3}} = 12.5\ \mu\mathrm{S}.$$

Reciprocal two-port circuits

A two-port circuit is reciprocal if the interchange of an ideal voltage source at one port with an ideal ammeter at the other port produces the same ammeter reading.

Example of a reciprocal two-port network:



When a voltage source of 15 V is applied to port **ad**, it produces a current of 1.75 A in the ammeter at port **cd**. If the voltage source and ammeter are interchanged, the ammeter will still read 1.75 A.

Reciprocal two-port circuits

The effect of reciprocity on the two-port parameters is given by:

$$z_{12} = z_{21}$$

$$y_{12} = y_{21}$$

$$\Delta a = a_{11}a_{22} - a_{12}a_{21} = 1$$

$$\Delta b = b_{11}b_{22} - b_{12}b_{21} = 1$$

$$h_{12} = -h_{21}$$

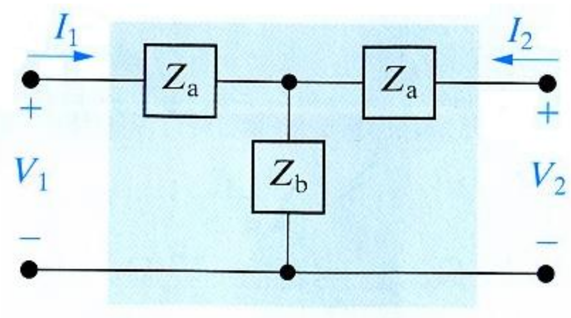
$$g_{12} = -g_{21}$$

For a reciprocal two-port circuit, only three calculations or measurements are needed to determine a set of parameters.

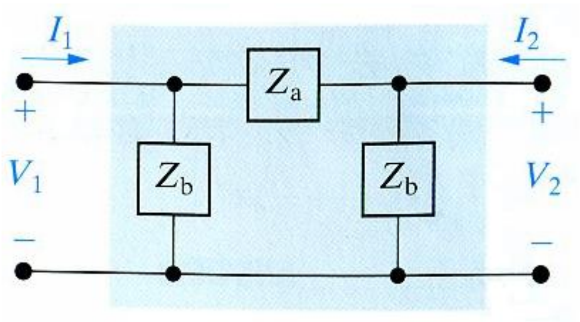
Symmetric reciprocal two-port circuits

A reciprocal two-port circuit is symmetric if its ports can be interchanged without disturbing the values of the terminal currents and voltages.

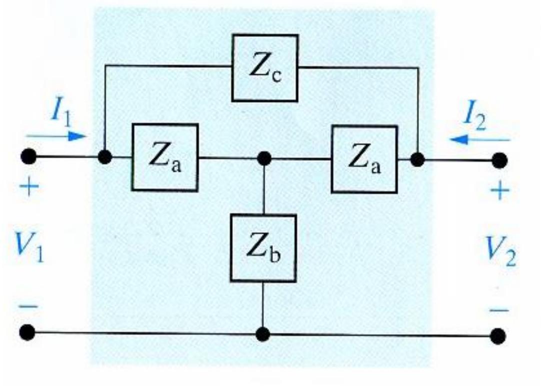
<u>Example of symmetric reciprocal two-port network:</u>



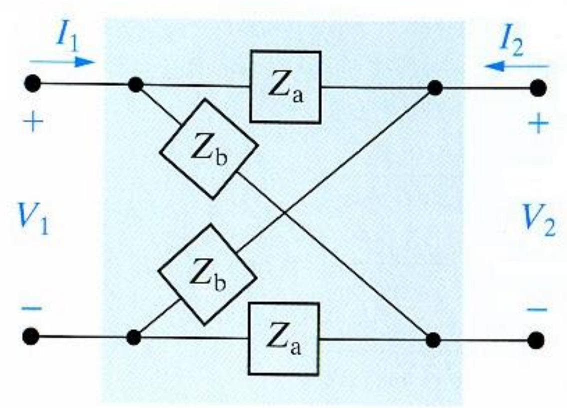
Symmetric tee (**T**)



Symmetric pi (π)



Symmetric bridged tee (**T**)



Symmetric lattice

Symmetric reciprocal two-port circuits

For symmetric reciprocal two-port circuits, the following additional relationships exist among the port parameters:

$$z_{11} = z_{22}$$

$$y_{11} = y_{22}$$

$$a_{11} = a_{22}$$

$$b_{11} = b_{22}$$

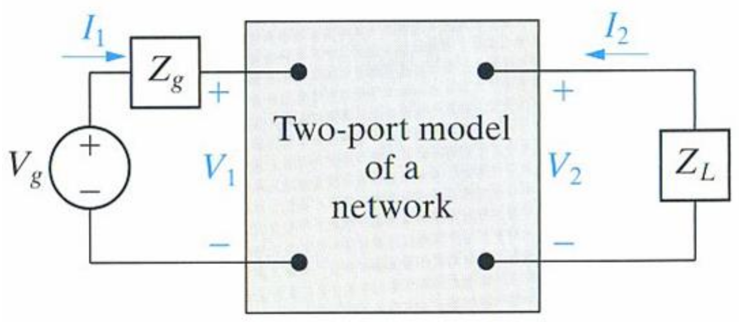
$$\Delta h = h_{11}h_{22} - h_{12}h_{21} = 1$$

$$\Delta g = g_{11}g_{22} - g_{12}g_{21} = 1$$

For symmetric reciprocal two-port circuits, only two calculations or measurements are necessary to determine all the two-port parameters.

Analysis of the terminated two-port circuit

In the typical application of a two-port model, the circuit is driven at port 1 and loaded at port 2.



V_g = internal voltage of the source

Z_g = internal impedance of the source

Z_L = load impedance

Six characteristics of the terminated two-port circuit define its terminal behavior:

1) The input impedance $Z_{in} = V_1/I_1$, or the admittance $Y_{in} = I_1/V_1$
2) The output current I_2
3) The Thevenin voltage and impedance (V_{th}, Z_{th}) with respect to port 2
4) The current gain I_2/I_1
5) The voltage gain V_2/V_1
6) The voltage gain V_2/V_g

The relationships of these parameters are given in Table 18.2, page 742 from textbook.

Terminated Two-Port Equations

TABLE 18.2 Terminated Two-Port Equations

***z* Parameters**

$$Z_{in} = z_{11} - \frac{z_{12}z_{21}}{z_{22} + Z_L}$$

$$I_2 = \frac{-z_{21}V_g}{(z_{11} + Z_g)(z_{22} + Z_L) - z_{12}z_{21}}$$

$$V_{Th} = \frac{z_{21}}{z_{11} + Z_g}V_g$$

$$Z_{Th} = z_{22} - \frac{z_{12}z_{21}}{z_{11} + Z_g}$$

$$\frac{I_2}{I_1} = \frac{-z_{21}}{z_{22} + Z_L}$$

$$\frac{V_2}{V_1} = \frac{z_{21}Z_L}{z_{11}Z_L + \Delta z}$$

$$\frac{V_2}{V_g} = \frac{z_{21}Z_L}{(z_{11} + Z_g)(z_{22} + Z_L) - z_{12}z_{21}}$$

***y* Parameters**

$$Y_{in} = y_{11} - \frac{y_{12}y_{21}Z_L}{1 + y_{22}Z_L}$$

$$I_2 = \frac{y_{21}V_g}{1 + y_{22}Z_L + y_{11}Z_g + \Delta y Z_g Z_L}$$

$$V_{Th} = \frac{-y_{21}V_g}{y_{22} + \Delta y Z_g}$$

$$Z_{Th} = \frac{1 + y_{11}Z_g}{y_{22} + \Delta y Z_g}$$

$$\frac{I_2}{I_1} = \frac{y_{21}}{y_{11} + \Delta y Z_L}$$

$$\frac{V_2}{V_1} = \frac{-y_{21}Z_L}{1 + y_{22}Z_L}$$

$$\frac{V_2}{V_g} = \frac{y_{21}Z_L}{y_{12}y_{21}Z_gZ_L - (1 + y_{11}Z_g)(1 + y_{22}Z_L)}$$

Terminated Two-Port Equations

a Parameters

$$Z_{in} = \frac{a_{11}Z_L + a_{12}}{a_{21}Z_L + a_{22}}$$

$$I_2 = \frac{-V_g}{a_{11}Z_L + a_{12} + a_{21}Z_g Z_L + a_{22}Z_g}$$

$$V_{Th} = \frac{V_g}{a_{11} + a_{21}Z_g}$$

$$Z_{Th} = \frac{a_{12} + a_{22}Z_g}{a_{11} + a_{21}Z_g}$$

$$\frac{I_2}{I_1} = \frac{-1}{a_{21}Z_L + a_{22}}$$

$$\frac{V_2}{V_1} = \frac{Z_L}{a_{11}Z_L + a_{12}}$$

$$\frac{V_2}{V_g} = \frac{Z_L}{(a_{11} + a_{21}Z_g)Z_L + a_{12} + a_{22}Z_g}$$

b Parameters

$$Z_{in} = \frac{b_{22}Z_L + b_{12}}{b_{21}Z_L + b_{11}}$$

$$I_2 = \frac{-V_g \Delta b}{b_{11}Z_g + b_{21}Z_g Z_L + b_{22}Z_L + b_{12}}$$

$$V_{Th} = \frac{V_g \Delta b}{b_{22} + b_{21}Z_g}$$

$$Z_{Th} = \frac{b_{11}Z_g + b_{12}}{b_{21}Z_g + b_{22}}$$

$$\frac{I_2}{I_1} = \frac{-\Delta b}{b_{11} + b_{21}Z_L}$$

$$\frac{V_2}{V_1} = \frac{\Delta b Z_L}{b_{12} + b_{22}Z_L}$$

$$\frac{V_2}{V_g} = \frac{\Delta b Z_L}{b_{12} + b_{11}Z_g + b_{22}Z_L + b_{21}Z_g Z_L}$$

Terminated Two-Port Equations

h Parameters

$$Z_{in} = h_{11} - \frac{h_{12}h_{21}Z_L}{1 + h_{22}Z_L}$$

$$I_2 = \frac{h_{21}V_g}{(1 + h_{22}Z_L)(h_{11} + Z_g) - h_{12}h_{21}Z_L}$$

$$V_{Th} = \frac{-h_{21}V_g}{h_{22}Z_g + \Delta h}$$

$$Z_{Th} = \frac{Z_g + h_{11}}{h_{22}Z_g + \Delta h}$$

$$\frac{I_2}{I_1} = \frac{h_{21}}{1 + h_{22}Z_L}$$

$$\frac{V_2}{V_1} = \frac{-h_{21}Z_L}{\Delta h Z_L + h_{11}}$$

$$\frac{V_2}{V_g} = \frac{-h_{21}Z_L}{(h_{11} + Z_g)(1 + h_{22}Z_L) - h_{12}h_{21}Z_L}$$

g Parameters

$$Y_{in} = g_{11} - \frac{g_{12}g_{21}}{g_{22} + Z_L}$$

$$I_2 = \frac{-g_{21}V_g}{(1 + g_{11}Z_g)(g_{22} + Z_L) - g_{12}g_{21}Z_g}$$

$$V_{Th} = \frac{g_{21}V_g}{1 + g_{11}Z_g}$$

$$Z_{Th} = g_{22} - \frac{g_{12}g_{21}Z_g}{1 + g_{11}Z_g}$$

$$\frac{I_2}{I_1} = \frac{-g_{21}}{g_{11}Z_L + \Delta g}$$

$$\frac{V_2}{V_1} = \frac{g_{21}Z_L}{g_{22} + Z_L}$$

$$\frac{V_2}{V_g} = \frac{g_{21}Z_L}{(1 + g_{11}Z_g)(g_{22} + Z_L) - g_{12}g_{21}Z_g}$$

Example 4

The two-port circuit is described in terms of its b parameters, the values of which are:

$b_{11} = -20$; $b_{12} = -3000\ \Omega$

$b_{21} = -2$ ms ; $b_{22} = -0.2$

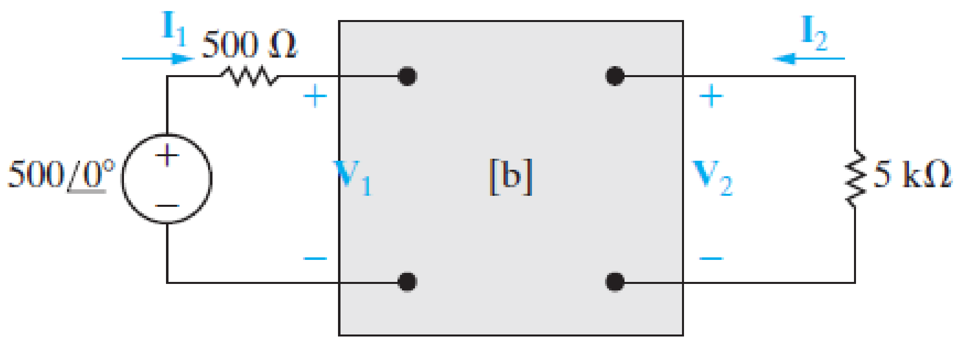
a) Find the phasor voltage V_2

b) Find the average power delivered to the 5 kΩ load

c) Find the average power delivered to the input port.

d) Find the load impedance for maximum average power transfer.

e) Find the maximum average power delivered to the load in (d)



Example 4 **Solution**

a) To find V_2, we have two choices from the entries in Table 18.2. We may choose to find I_2 and then find V_2 from the relationship $V_2 = -I_2Z_L$, *or we* may find the voltage gain V_2/V_g and calculate V_2 from the gain. Let's use the latter approach. For the 6-parameter values given, we have
$\Delta b = (-20)(-0.2) - (-3000)(-2 \times 10^{-3}) = 4 - 6 = -2$

From Table 18.2,

$$\frac{\mathbf{V}_2}{\mathbf{V}_g} = \frac{\Delta b Z_L}{b_{12} + b_{11}Z_g + b_{22}Z_L + b_{21}Z_gZ_L}$$

$$= \frac{(-2)(5000)}{-3000 + (-20)500 + (-0.2)5000 + [-2 \times 10^{-3}(500)(5000)]} = \frac{10}{19}.$$

$$\mathbf{V}_2 = \left(\frac{10}{19}\right)500 = 263.16\angle 0^\circ \text{ V}.$$

b) The average power delivered to the 5000 Ω load is $P_2 = \dfrac{263.16^2}{2(5000)} = 6.93 \text{ W}.$

Example 4 Solution

c) To find the average power delivered to the input port, we first find the input impedance Z_{in}. From Table 18.2,

$$Z_{in} = \frac{b_{22}Z_L + b_{12}}{b_{21}Z_L + b_{11}} = \frac{(-0.2)(5000) - 3000}{-2 \times 10^{-3}(5000) - 20} = \frac{400}{3} = 133.33\ \Omega.$$

Now I_1 follows directly: $\mathbf{I}_1 = \frac{500}{500 + 133.33} = 789.47\ \text{mA}.$

The average power delivered to the input port is

$$P_1 = \frac{0.78947^2}{2}(133.33) = 41.55\ \text{W}.$$

d) The load impedance for maximum power transfer equals the conjugate of the Thevenin impedance seen looking into port 2. From Table 18.2,

$$Z_{Th} = \frac{b_{11}Z_g + b_{12}}{b_{21}Z_g + b_{22}} = \frac{(-20)(500) - 3000}{(-2 \times 10^{-3})(500) - 0.2} = \frac{13{,}000}{1.2} = 10{,}833.33\ \Omega.$$

Therefore $Z_L = Z_{Th}^* = 10{,}833.33\ \Omega.$

Example 4 Solution

e) To find the maximum average power delivered to Z_L, we first find V_2 from the voltage-gain expression V_2/V_g. When Z_L is 10.833.33 Ω. This gain is

$$\frac{\mathbf{V}_2}{\mathbf{V}_g} = 0.8333.$$

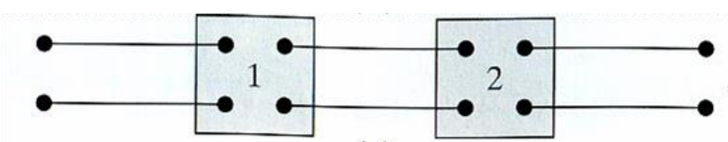
$$\mathbf{V}_2 = (0.8333)(500) = 416.67 \text{ V},$$

$$P_L(\text{maximum}) = \frac{1}{2}\frac{416.67^2}{10{,}833.33} = 8.01 \text{ W}.$$

Interconnected two port circuit

Synthesizing a large, complex system is simplified by first designing subsections of the systems. Then interconnecting these units to completes the systems.

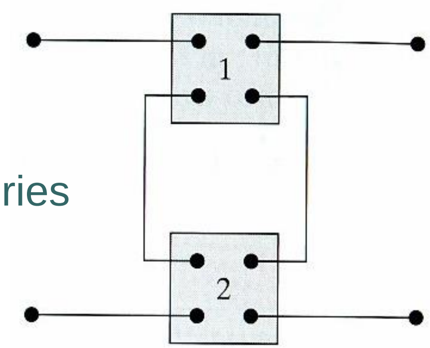
The subsection are modeled by two-port networks and can be interconnected by five ways:



(a)

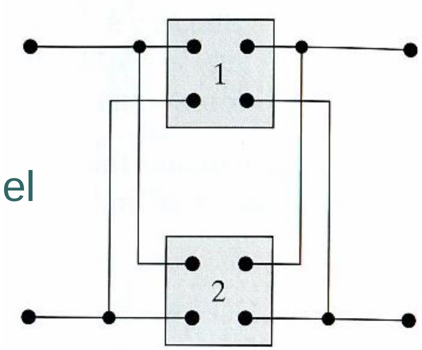
Cascade

Series



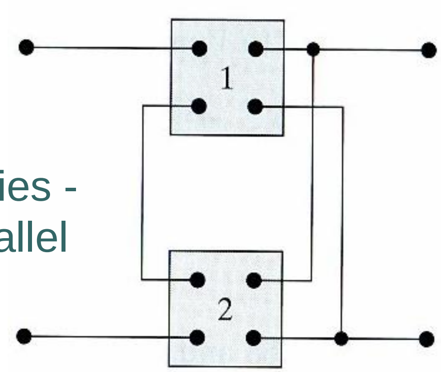
(b)

Parallel



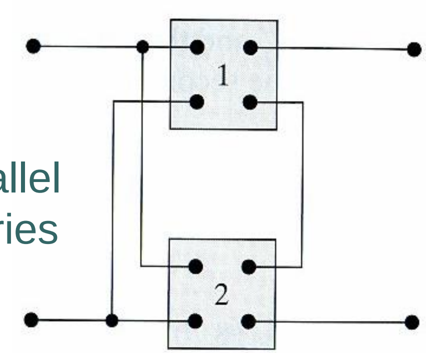
(c)

Series - parallel



(d)

Parallel - series

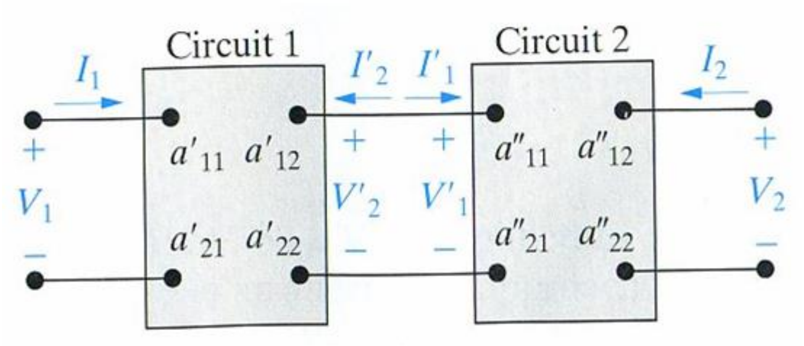


(e)

Cascaded two-port Circuits

The cascade connection is important because it occurs frequently in the large systems and there are no restrictions on using the parameters of the individual two-port circuits to obtain the parameters of the interconnected circuits.

The a parameters are best suited for describing the cascade connection.



$$V_1 = a_{11}V_2 - a_{12}I_2$$

$$I_1 = a_{21}V_2 - a_{22}I_2,$$

The describing equations are:

$$V_1 = (a'_{11}a''_{11} + a'_{12}a''_{21})V_2 - (a'_{11}a''_{12} + a'_{12}a''_{22})I_2$$

$$I_1 = (a'_{21}a''_{11} + a'_{22}a''_{21})V_2 - (a'_{21}a''_{12} + a'_{22}a''_{22})I_2$$

Cascaded two-port Circuits

The desired expressions for the a parameters of the interconnected networks, namely,

$$a_{11} = a'_{11}a''_{11} + a'_{12}a''_{21},$$

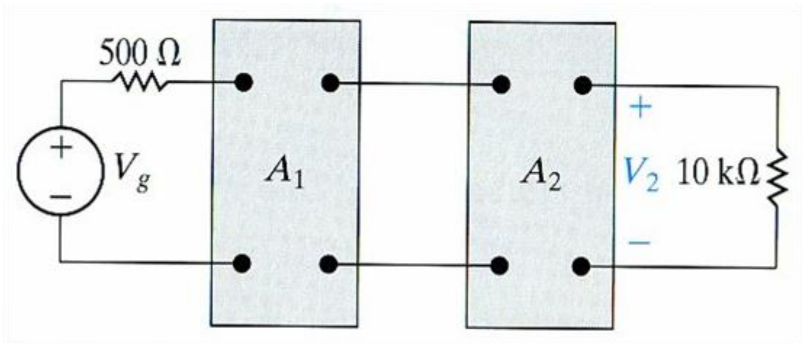
$$a_{12} = a'_{11}a''_{12} + a'_{12}a''_{22},$$

$$a_{21} = a'_{21}a''_{11} + a'_{22}a''_{21},$$

$$a_{22} = a'_{21}a''_{12} + a'_{22}a''_{22}.$$

If more than two units are connected in cascade, the a parameters of the equivalent two-port circuit can be found by successively reducing the original set of two-port circuits one pair at a time.

Example 5



Two identical amplifiers are connected in cascade as shown in Figure above.

Each amplifier is described in terms of its h parameters. The values are:

$h_{11} = 1000\ \Omega$ $\qquad$ $h_{12} = 0.0015$

$h_{21} = 100$ $\qquad$ $h_{22} = 100\ \mu s$

Find the voltage gain V_2/V_g.

Example 5 **Solution**

The first step in finding V_2/V_g is to convert from h parameters to a parameters. The amplifiers are identical, so one set of a parameters describes the amplifiers:

$$a'_{11} = \frac{-\Delta h}{h_{21}} = \frac{+0.05}{100} = 5 \times 10^{-4},$$

$$a'_{12} = \frac{-h_{11}}{h_{21}} = \frac{-1000}{100} = -10\ \Omega,$$

$$a'_{21} = \frac{-h_{22}}{h_{21}} = \frac{-100 \times 10^{-6}}{100} = -10^{-6}\,\mathrm{S},$$

$$a'_{22} = \frac{-1}{h_{21}} = \frac{-1}{100} = -10^{-2}.$$

To compute the a parameters of the cascaded amplifiers:

$$a_{11} = a'_{11}a'_{11} + a'_{12}a'_{21} \quad = 25 \times 10^{-8} + (-10)(-10^{-6}) \quad = 10.25 \times 10^{-6},$$

Example 5 Solution

$$a_{12} = a'_{11}a'_{12} + a'_{12}a'_{22} = (5 \times 10^{-4})(-10) + (-10)(-10^{-2}) = 0.095\ \Omega,$$

$$a_{21} = a'_{21}a'_{11} + a'_{22}a'_{21} = (-10^{-6})(5 \times 10^{-4}) + (-0.01)(-10^{-6}) = 9.5 \times 10^{-9}\,\text{S},$$

$$a_{22} = a'_{21}a'_{12} + a'_{22}a'_{22} = (-10^{-6})(-10) + (-10^{-2})^2 = 1.1 \times 10^{-4}.$$

From Table 18.2,

$$\frac{V_2}{V_g} = \frac{Z_L}{(a_{11} + a_{21}Z_g)Z_L + a_{12} + a_{22}Z_g}$$

$$= \frac{10^4}{[10.25 \times 10^{-6} + 9.5 \times 10^{-9}(500)]10^4 + 0.095 + 1.1 \times 10^{-4}(500)}$$

$$= \frac{10^4}{0.15 + 0.095 + 0.055} = \frac{10^5}{3} = 33{,}333.33.$$

Thus an input signal of 150 μV is amplified to an output signal of 5 V.