FINAL EXAMINATION
Academic year 2022-2023, Semester 2
Duration: 120 minutes

SUBJECT: Differential Equations (MA024IU)	
Head of the Department of Mathematics	Lecturer:
Professor Pham Huu Anh Ngoc	Pham Huu Anh Ngoc
	Signature:

Instructions:

- *Each student is allowed a scientific calculator and a maximum of two double-sided sheets of reference material (size A4 or similar), stapled together and marked with their name and ID. All other documents and electronic devices are forbidden..*

Question 1. (20 marks) Find the general solution of the following differential equation

$$xy''' - 3y'' = x, \quad x \in (0, \infty).$$

Question 2. (20 marks) Find a particular solution of the following differential equation

$$y^{(4)} - 4y''' + 4y'' = 2022 + 2023e^{2x}.$$

Question 3. (20 marks) Solve the following system of differential equations

$$\begin{cases} \dfrac{dx}{dt} = x + 3y + 2e^{2t} \\ \\ \dfrac{dy}{dt} = 2x - e^{2t} \end{cases}$$

Question 4. (20 marks) Find a particular solution of the system of differential equations

$$\begin{cases} \dfrac{dx}{dt} = x + y + t \\ \\ \dfrac{dy}{dt} = 3x + y + 2t + 5. \end{cases}$$

Question 5. (20 marks) (**Forecasting Prices**) A cosmetics manufacturer has a marketing policy based upon the price x(t) of its salon shampoo. The price $x(t)$ (in dollars) and the inventory level $I(t)$ obey the following differential equations

$$x'(t) = I(t) - 50; \ I'(t) = \frac{13}{4}x(t) - 6I(t) + 289.$$

Assume that $x(0) = 10$ and $I(0) = 7$. Find $\lim_{t \to \infty} x(t)$ and $\lim_{t \to \infty} I(t)$. Interpret the obtained result.

The end.