

CALCULUS 2 - 90 minutes

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Instruction: Students must explain your answers in detail. No credit will be given for the answers alone. 1- Write your answers on A4 papers in 90 minutes. 2 - Right after 90 minutes, you must take photos or scan your works, put the photos in a file (Word for example). 3 - Then export a pdf file (name of the file should contain your names followed by your student numbers). 4 - Finally, you must submit your pdf file in blackboard (**ONLY** in the case you can not submit your answers in blackboard due to technical problems, you can send your file to my email address ptduong01@gmail.com). **You only have additional 30 minutes to submit your pdf files.**

Question 1 (10 marks). *Determine if the following series are convergent or divergent:*

$$(a) \sum_{n=1}^{\infty} (-1)^n \frac{1}{2^n}$$

$$(b) \sum_{n=1}^{\infty} \frac{e^n}{n^2}$$

Question 2 (10 marks). *Determine if the following series are convergent or divergent:*

$$(a) \sum_{n=1}^{\infty} \frac{2^n}{(1 + \frac{1}{n})^n}$$

$$(b) \sum_{n=1}^{\infty} \frac{\ln n}{n^2}$$

Question 3 (10 marks). *Find the radius of convergence and interval of convergence of the series*

$$\sum_{n=1}^{\infty} \sqrt{n+1} (x-2)^n$$

Question 4 (10 marks). *Find a power series representation for the function and determine the interval of convergence*

$$f(x) = \frac{3x}{1-2x}.$$

Question 5 (10 marks). (i) Find an equation of the sphere S that passes through the point $P(3, 2, 1)$ and has center $O(0, 0, 0)$.

(ii) Find a point on the sphere S that is closest to the point $Q(5, 5, 5)$.

Question 6 (10 marks). Let $f(x, y) = \begin{cases} \frac{x^4 y^4}{x^4 + y^4} & \text{if } (x, y) \neq (0, 0) \\ 0 & \text{if } (x, y) = (0, 0). \end{cases}$

(a) Prove that f is continuous at $(0, 0)$.

(b) Evaluate $f_x(0, 0)$ and $f_y(0, 0)$ if they exist.

Question 7 (10 marks). Find all the second partial derivatives of

(a) $u = \sqrt{x^2 + y^2 + z^2}$

(b) $v = e^{\sqrt[3]{x^2 + y^2}}$

Question 8 (10 marks). Find the equation of the tangent plane to the surface $(S) : z = x^2 - 3y^2$ at the point $P(1, 1, -2)$.

Question 9 (10 marks). Find the linear approximation of the function $f(x, y) = \sqrt{10 - 2x^3 - y^4}$ at the point $(1, 1)$ and use it to approximate $f(1.02, 0.97)$.

Question 10 (10 marks). Let $z = \sqrt{x^4 + y^4}$, $x = r \cos \theta$ and $y = r \sin \theta$. Use the chain rule to find $\partial z / \partial r$ and $\partial z / \partial \theta$.

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