Work

$$W = \vec{F}\,\vec{d} = F d \cos(\theta)$$

Unit: J

Power

$$P = \frac{|W|}{\Delta t}$$

Unit: J/s = W

Kinetic energy

$$K = \frac{1}{2} m v^2$$

Unit: J

Gravitational potential energy

$$U = mgy \qquad \text{Unit: J}$$

Elastic potential energy

$$U = \frac{1}{2}kx^2 \qquad \text{Unit: J}$$

Conservation of mechanical energy

$$E_{mec} = K + U \qquad K_1 + U_1 = K_2 + U_2$$

Work done on a system (no friction involved)

$$W = \Delta E_{mec} = \Delta K + \Delta U$$ Unit: J

Work done on a system (friction involved)

$$W = \Delta E_{mec} + \Delta E_{thermal}$$ Unit: J

Work done by friction

$$|W_{friction}| = \Delta E_{thermal} = -\Delta E_{mec}$$ Unit: J

$$W_{friction} = \vec{F}\,\vec{d}$$

Center of mass

$$x_{com} = \frac{m_1x_1 + m_2x_2 + \cdots + m_nx_n}{m_1 + m_2 + \ \ldots + m_n} = \frac{A_1x_1 + A_2x_2}{A_1 + A_2}$$

Unit: m

$$m = \rho \text{ x thickness x area}$$

Linear momentum

$$\vec{p} = m\,\vec{v}$$

Unit: kg m/s

Conservation of linear momentum

$$p = p'$$

$$m_1v_1 + m_2v_2 = m_1v_1{}' + m_2v_2{}'$$

Impulse

$$\mathrm{J} = \mathrm{F}_{\mathrm{avg}} \mathrm{x}\, \Delta \mathrm{t} = \Delta \mathrm{p} \qquad \Delta\vec{\mathrm{p}} = \vec{\mathrm{J}}$$

Unit: N.s

Collision

- Elastic collision: p and K conserved
- Inelastic collision: only p conserved
  - + Perfectly inelastic collision
  - + Non-perfectly inelastic collision

→ Elastic or inelastic? $K_i = K_f \rightarrow$ elastic

$K_i \neq K_f \rightarrow$ inelastic

Angular position: θ Unit: rad (1 rev = 2π rad)

Angular velocity: ω Unit: rad/s

Angular acceleration: α Unit: rad/s^2

Period of revolution: T = $\frac{2\pi}{\omega}$ Unit: s

Tangential acceleration: $a_t = \alpha r$ Unit: m/s^2

Radial acceleration: $a_r = \frac{v^2}{r} = \omega^2 r$ Unit: m/s^2

Rotational inertia: I Unit: kg.m^2

Torque: $\vec{\tau} = \vec{F}\vec{r}$ Unit: N.m

Angular momentum: $\vec{l} = \vec{p}\vec{r}$ Unit: kg m^2 s^{-1}

Translation	Rotation
x	θ
v	ω
a	α
$\vec{F} = m\,\vec{a}$	$\vec{\tau} = \vec{F}\,\vec{r} = I\,\alpha$
m	I
$\vec{p} = m\,\vec{v}$	$\vec{l} = \vec{p}\,\vec{r} = I\,\omega$
$\vec{W} = \vec{F}\,\vec{d}$	$\vec{W} = \vec{\tau}\,\vec{\varphi}$
$K = \frac{1}{2}mv^2$	$K = \frac{1}{2}I\omega^2$

$$\omega = \omega_o + \alpha t$$

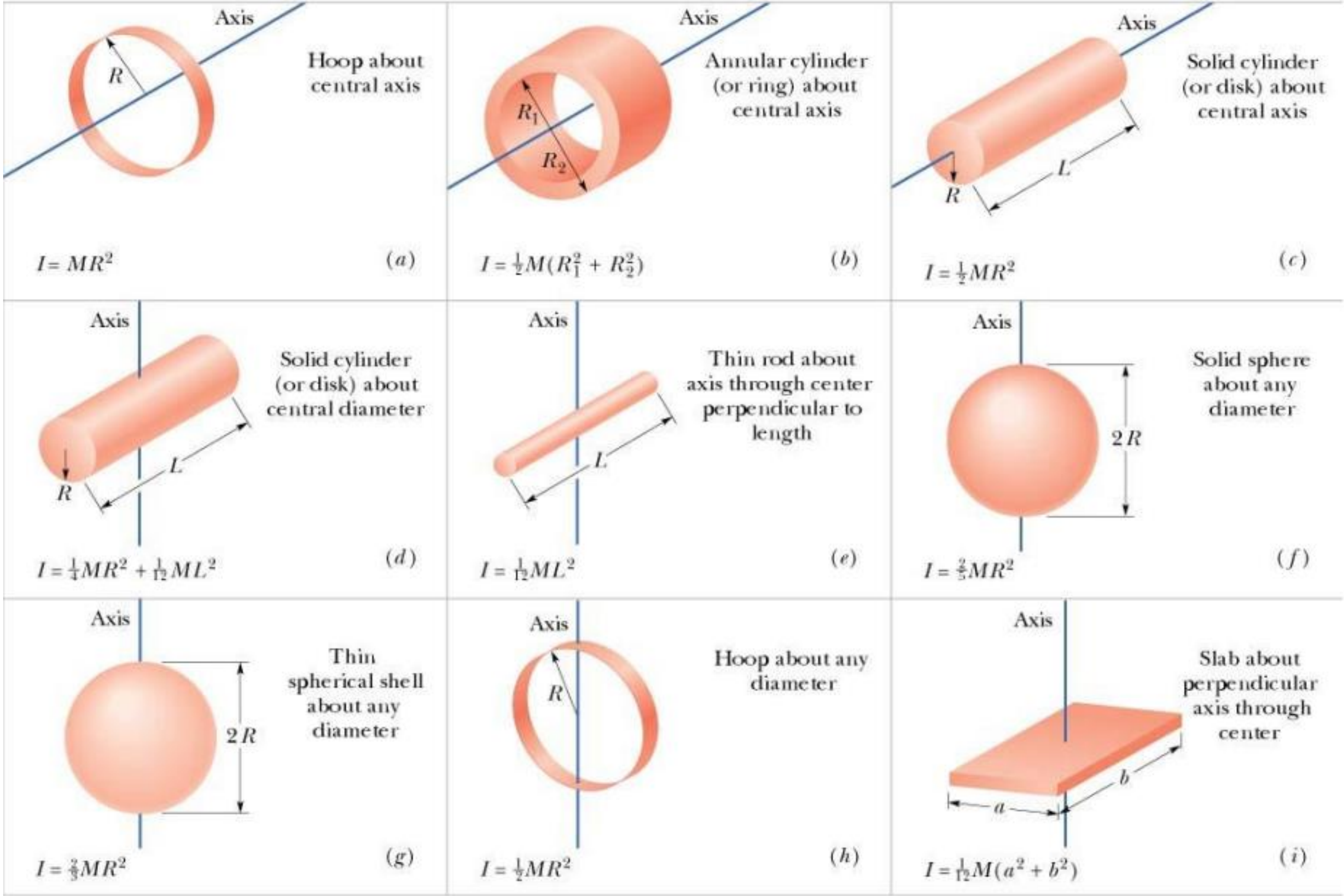
$$\theta = \theta_o + \omega_o t + \frac{1}{2}\alpha t^2$$

$$\omega^2 - \omega_o^2 = 2\alpha(\theta - \theta o)$$

$$\vec{\tau} = \vec{F}\,\vec{r} = I\,\alpha \qquad \vec{l} = \vec{p}\,\vec{r} = I\,\omega$$

Rotational inertia (I)

- For a point mass: $I = mr^2$
- For a rigid body: depends on the shape


Axis
R
Hoop about central axis
$I = MR^2$
(a)
Axis
R_1
R_2
Annular cylinder (or ring) about central axis
$I = \frac{1}{2}M(R_1^2 + R_2^2)$
(b)
Axis
R
L
Solid cylinder (or disk) about central axis
$I = \frac{1}{2}MR^2$
(c)
Axis
R
L
Solid cylinder (or disk) about central diameter
$I = \frac{1}{4}MR^2 + \frac{1}{12}ML^2$
(d)
Axis
L
Thin rod about axis through center perpendicular to length
$I = \frac{1}{12}ML^2$
(e)
Axis
2R
Solid sphere about any diameter
$I = \frac{2}{5}MR^2$
(f)
Axis
2R
Thin spherical shell about any diameter
$I = \frac{2}{3}MR^2$
(g)
Axis
R
Hoop about any diameter
$I = \frac{1}{2}MR^2$
(h)
Axis
a
b
Slab about perpendicular axis through center
$I = \frac{1}{12}M(a^2 + b^2)$
(i)

Conservation of angular momentum: $I_i\omega_i = I_f\omega_f$

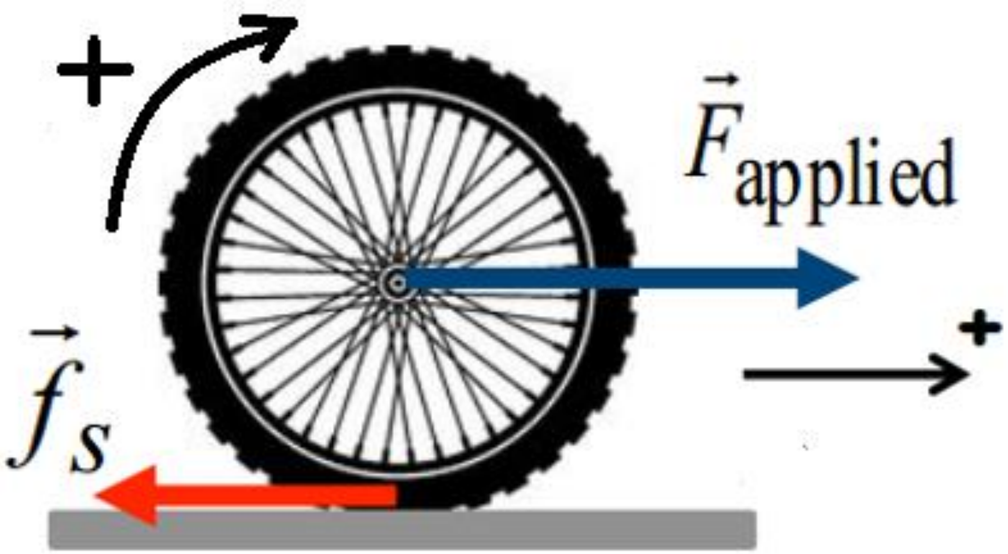
Rotational work

$$W = \Delta K = \frac{1}{2}I\omega_f{}^2 - \frac{1}{2}I\omega_i{}^2 = \tau\varphi$$

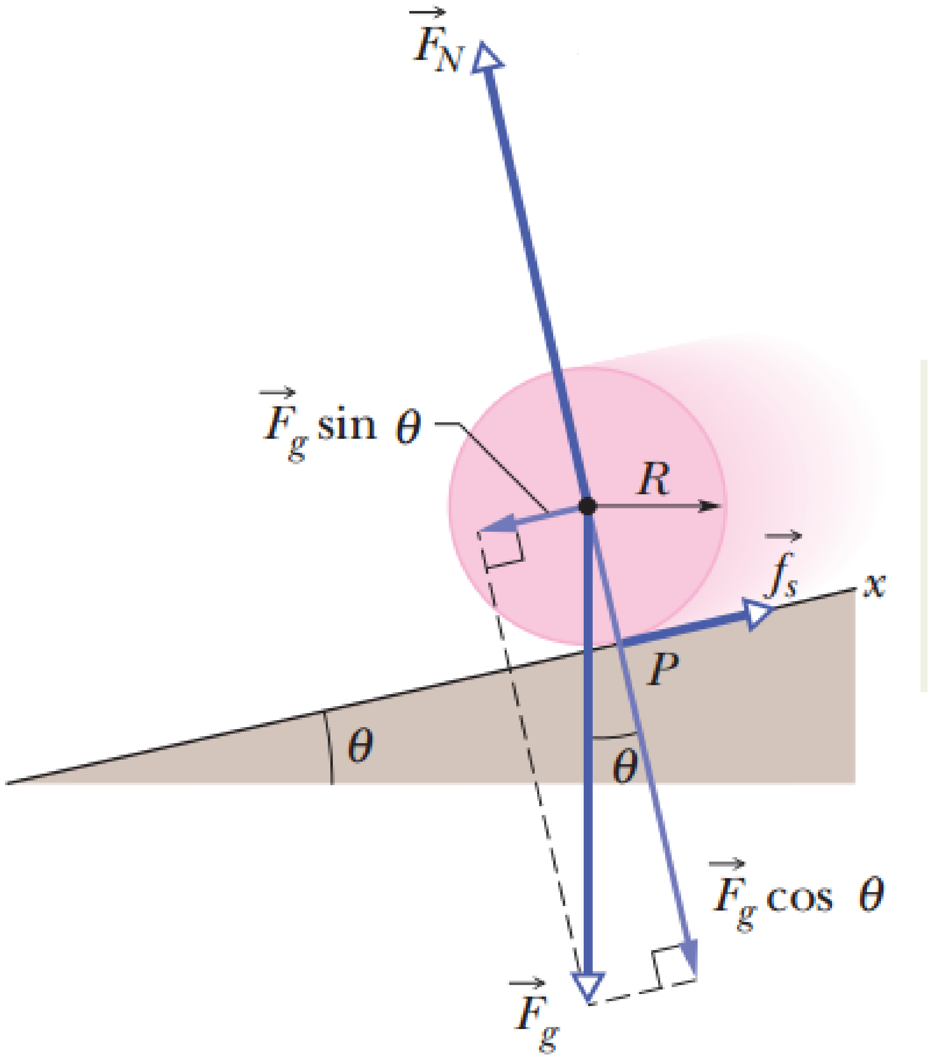
Rolling motion: Translational + Rotational

$v = \omega R \qquad a = \alpha R \qquad K = \frac{1}{2}I\omega^2 + \frac{1}{2}mv^2$

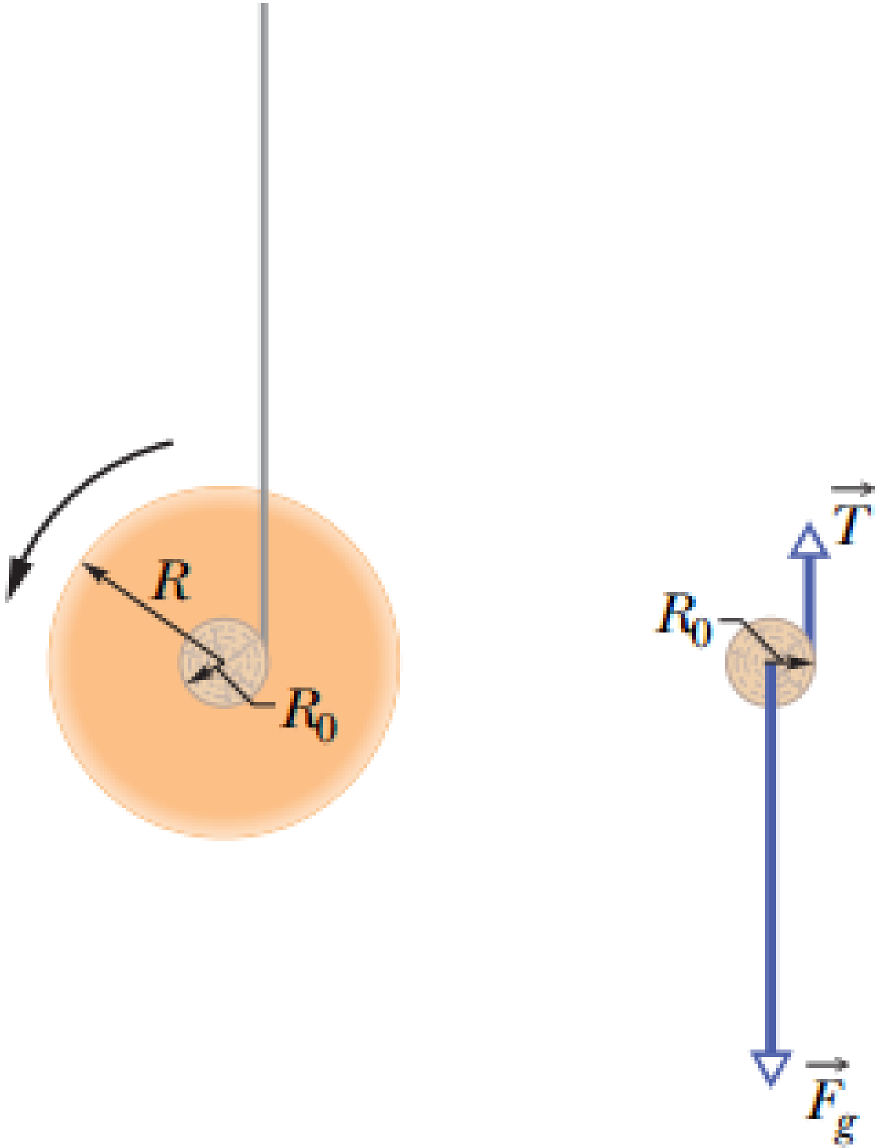
Rolling on horizontal surface



Rolling on an incline



The yo-yo



The pulley