# Quiz #2. Answer key below.

Determine the series is convergent or divergent?

**1.** $\displaystyle\sum_{n=1}^{\infty} \frac{\sqrt{n}+1}{n^2-\sqrt{n}+1}$

**2.** $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n \sin(\frac{n\pi}{4})}{n^2+1}$

**3.** $\displaystyle\sum_{n=2}^{\infty} \frac{n}{2^n}$. Hint: Can use Ratio Test.

**4.** $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n}\sin(\frac{1}{n})$. Hint: Limit comparison test with $b_n := \dfrac{1}{n^2}$ and use $\displaystyle\lim_{t\to 0} \frac{\sin t}{t} = 1$.

# Quiz #2. Answer key.

**1.** $\displaystyle\sum_{n=2}^{\infty} \frac{\sqrt{n}+1}{n^3-\sqrt{n}+1}$.

Use the limit comparison test (LCT).

Let $a_n = \dfrac{\sqrt{n}+1}{n^3-\sqrt{n}+1}$ and $b_n = \dfrac{1}{n^{5/2}}$.

Note that $\displaystyle\lim_{n\to\infty} \frac{a_n}{b_n} = 1$ and $\displaystyle\sum_{n=2}^{\infty} b_n$ is convergent (p-series with $p = 5/2 > 1$). Thus, $\displaystyle\sum_{n=2}^{\infty} a_n$ is also convergent.

# Quiz #2. Answer key.

**2.** $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n \sin(\frac{n\pi}{4})}{n^2+1}$

Use limit comparison test and comparison test.

Note that $\displaystyle\left|\frac{(-1)^n \sin(\frac{n\pi}{4})}{n^2+1}\right| \leq \frac{1}{n^2+1}$.

On the other hand, the series $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^2+1}$ is convergent by using the limit comparison test with the series $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^2}$ .

Therefore, the original series is absolutely convergent.

# Quiz #2. Answer key.

**3.** $\sum_{n=2}^{\infty} \frac{n}{2^n}$.

One can use the ratio test.

Let $a_n = \frac{n}{2^n}$. We have

$$\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty} \frac{(n+1)}{n}\frac{1}{2} = \frac{1}{2} < 1.$$

By the Ratio Test, the given series is absolutely convergent.

# Quiz #2. Answer key.

**4.** $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n} \sin(\frac{1}{n})$.

Use the limit comparison test.

Let $a_n = \dfrac{1}{n} \sin(\dfrac{1}{n})$ and $b_n = \dfrac{1}{n^2}$.

Note that $\displaystyle\lim_{x \to 0} \frac{\sin x}{x} = 1$. It implies that $\displaystyle\lim_{n \to \infty} \frac{a_n}{b_n} = \lim_{n \to \infty} \frac{\sin(\frac{1}{n})}{\frac{1}{n}} = 1$.

In addition, the series $\displaystyle\sum_{n=1}^{\infty} b_n$ is convergent (p-series with $p = 2$).

As a result, $\displaystyle\sum_{n=1}^{\infty} a_n$ is also convergent.