Principles of EE2
Spring 2016
Midterm exam – SOLUTION

A. Fill out into the blacks:

Q1. Circuits that contain energy-storing elements, that is, ………………… and …………………, are represented by differential equations rather than algebraic equations. Analysis of these circuits requires the solutions of differential equations.

Ans.: Circuits that contain energy-storing elements, that is, capacitors and inductors, are represented by differential equations rather than algebraic equations. Analysis of these circuits requires the solutions of differential equations.

Q2. ………………… and ………………… can be used to encode, store, and process information. When a ………………… or ………………… is used to represent information, that ………………… or ………………… is called a signal. Electric circuits that process that information are called signal-processing circuits.

Ans.: Voltages and currents can be used to encode, store, and process information. When a voltage or current is used to represent information, that voltage or current is called a signal. Electric circuits that process that information are called signal-processing circuits.

Q3. Second-order circuits are circuits that are represented by a ……………… ……………… differential equation. The output of the circuit, also called the response of the circuit, can be the current or the voltage of any device in the circuit.

Ans.: Second-order circuits are circuits that are represented by a second-order differential equation. The output of the circuit, also called the response of the circuit, can be the current or the voltage of any device in the circuit.

Q4. First-order circuits contain one energy storage element and are represented by ………………………differential equations, which are reasonably easy to solve.

Ans.: First-order circuits contain one energy storage element and are represented by first-order differential equations, which are reasonably easy to solve.

Q5. The complete response can be separated into the transient response and the steady-state response. The transient response vanishes with ……………, leaving the steady-state response.

Ans.: The complete response can be separated into the transient response and the steady-state response. The transient response vanishes with time, leaving the steady-state response.

Q6. Pierre-Simon Laplace is credited with a transform that bears his name. The Laplace transform is defined as

$$L[f(t)] =$$

Ans.: Pierre-Simon Laplace is credited with a transform that bears his name. The Laplace transform is defined as

$$L[f(t)] = \int_{0^-}^{\infty} f(t)e^{-st}\,dt$$

Q7. The Laplace transform transforms the differential equation describing a circuit in the time domain into an algebraic equation in the ………………… ………………. After solving the algebraic equation, we use the inverse Laplace transform to obtain the circuit response in the time domain.

Ans.: The Laplace transform transforms the differential equation describing a circuit in the time domain into an algebraic equation in the frequency domain. After solving the algebraic equation, we use the inverse Laplace transform to obtain the circuit response in the time domain.

B. Select the correct answers:

Q8. For the circuit in Fig. Q8, the capacitor voltage at $t = 0^-$ (just before the switch is closed) is:

(a) 0 V (b) 4 V (c) 8 V (d) 12 V

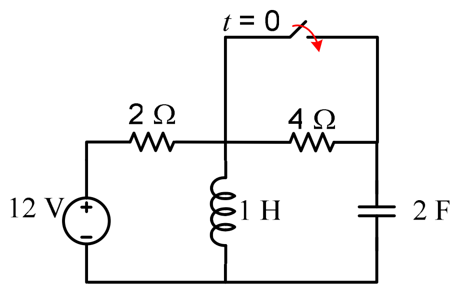


Fig. Q8

Ans.: (a)

Q9. For the circuit in Fig. Q8, the initial inductor current (at $t = 0$) is:

(a) 0 A (b) 2 A (c) 6 A (d) 12 A

Ans.: (c)

Q10. When a step input is applied to a second-order circuit, the final values of the circuit variables are found by:
(a) Replacing capacitors with closed circuits and inductors with open circuits.
(b) Replacing capacitors with open circuits and inductors with closed circuits.
(c) Doing neither of the above.

Ans.: (b)

Q11. If the roots of the characteristic equation of an RLC circuit are -2 and -3, the response is:

(a) $(A\cos 2t + B\sin 2t)e^{-3t}$
(b) $(A + 2Bt)e^{-3t}$
(c) $Ae^{-2t} + Bte^{-3t}$
(d) $Ae^{-2t} + Be^{-3t}$

Where A and B are constants.

Ans.: (d)

Q12. In a series RLC circuit, setting R = 0 will produce:
(a) an overdamped response
(b) a critically damped response
(c) an underdamped response
(d) an undamped response
(e) none of the above

Ans.: (d)

Q13. A parallel RLC circuit has L = 2 H and C = 0.25 F. The value of R that will produce unity damping factor is:
(a) 0.5 Ω (b) 1 Ω (c) 2 Ω (d) 4Ω

Ans.: (c)

Q14. Refer to the series RLC circuit in Fig. Q14. What kind of response will it produce?
(a) overdamped
(b) underdamped
(c) critically damped

(d) none of the above

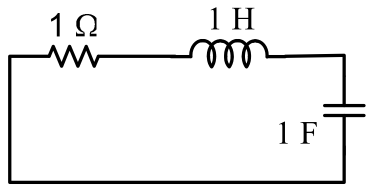


Fig. Q14

Ans.: (b)

Q15. Consider the parallel RLC circuit in Fig. Q15. What type of response will it produce?
(a) overdamped
(b) underdamped
(c) critically damped
(d) none of the above

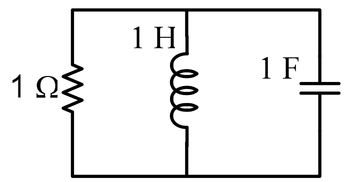


Fig. Q15

Ans.: (b)

Q16. An RC circuit has R = 2 Ω and C = 4 F. The time constant is:
(a) 0.5 s (b) 2 s (c) 4 s (d) 8 s (e) 15 s

Ans.: (d)

Q17. The time constant for an RL circuit with R = 2 Ω and L = 4 H is:
(a) 0.5 s (b) 2 s (c) 4 s (d) 8 s (e) 15 s

Ans.: (b)

Q18. A capacitor in an RC circuit with R = 2 Ω and C = 4 F is being charged. The time required for the capacitor voltage to reach 63.2 percent of its steady state value is:
(a) 2 s (b) 4 s (c) 8 s (d) 16 s (e) none of the above

Ans.: (c)

Q19. An RL circuit has R = 2 Ω and L = 4 H. The time needed for the inductor current to reach 40 percent of its steady-state value is:

(a) 0.5 s (b) 1 s (c) 2 s (d) 4 s (e) none of the above

Ans.: (b)

Q20. In the circuit of Fig. Q20, the capacitor voltage just before t = 0 is:
(a) 10 V (b) 7 V (c) 6 V (d) 4 V (e) 0 V

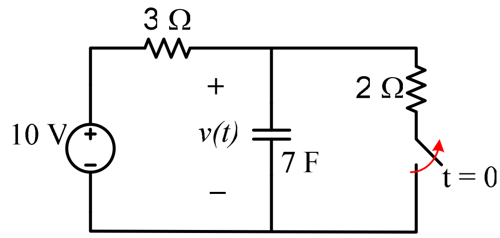


Fig. Q20

Ans.: (d)

Q21. In the circuit in Fig. Q20, v(∞) is:
(a) 10 V (b) 7 V (c) 6 V (d) 4 V (e) 0 V

Ans.: (a)

Q22. The variable s in the Laplace transform H(s) is called
(a) complex frequency (b) transfer function
(c) zero (d) pole

Ans.: (a)

Q23. The Laplace transform of u(t - 2) is:
(a) 1/(s + 2) (b) 1/(s - 2) (c) e^{2s}/s (d) e^{-2s}/s

Ans.: (d)

Q24. The zero of the function $F(s) = \dfrac{s+1}{(s+2)(s+3)(s+4)}$ is at
(a) - 4 (b) – 3 (c) – 2 (d) – 1

Ans.: (d)

Q25. The poles of the function $F(s) = \dfrac{s+1}{(s+2)(s+3)(s+4)}$ is at

(a) - 4 (b) – 3 (c) – 2 (d) – 1

Ans.: (a), (b), (c)

Q26. if F(s) = 1/(s + 2), then f(t) is
(a) $e^{2t}u(t)$ (b) $e^{-2t}u(t)$
(c) u(t - 2) (d) u(t + 2)

Ans. : (b)

Q27. The initial value of f(t) with transform $F(s) = \frac{s+1}{(s+2)(s+3)}$ is:
(a) nonexistent (b) ∞ (c) 0
(d) 1 (e) 1/6

Ans.: (d)

Q28. The inverse Laplace transform of $\frac{s+2}{(s+2)^2+1}$ is
(a) $e^{-t}\cos 2t$ (b) $e^{-t}\sin 2t$ (c) $e^{-2t}\cos t$
(d) $e^{-2t}\sin 2t$ (e) none of the above

Ans.: (c)

Q29. The impedance of a 10-F capacitor is:
(a) 10/s (b) s/10 (c) 1/10s (d) 10s

Ans.: (c)

Q30. A transfer function is defined only when all initial conditions are zero.
(a) True (b) False

Ans.: (a)

C. Problems

Problem 1: Consider the circuit shown in Fig. P1. Assuming that the switch has been in position 1 for a long time, at time t = 0 the switch is moved to position 2.
(a) Calculate the $v(0^-)$.
(b) Write the differential equation to calculate the voltage $v(t)$ for $t > 0$.

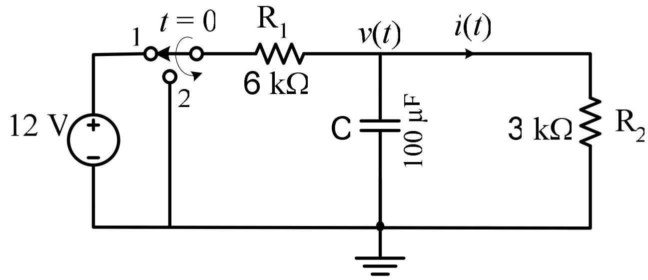
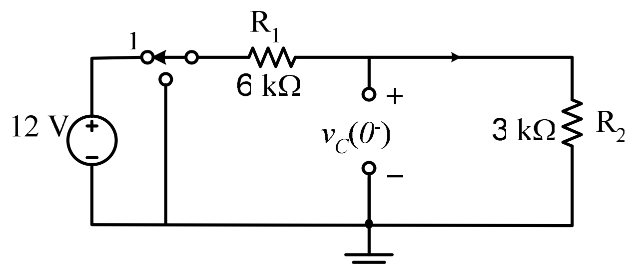


Fig. P1

Sol.:

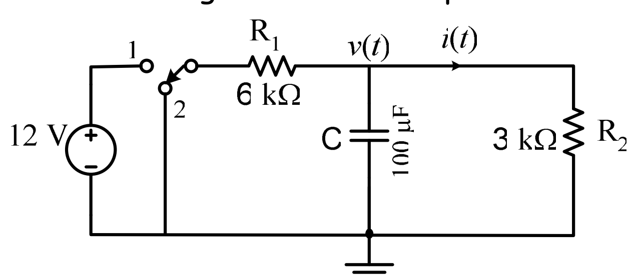
(a) At t = 0$^-$ the capacitor is fully charged and conducts no current since the capacitor acts like an open circuit to dc. The initial voltage across the capacitor can be found using voltage division.



$$v_C(0^-) = 12\left(\frac{3k}{6k+3k}\right) = 4 \quad (V)$$

(b) for t > 0:

Apply the KCL equation for the voltage across the capacitor is



$$\frac{v(t)}{R_1} + C\frac{dv(t)}{dt} + \frac{v(t)}{R_2} = 0$$

$$\Rightarrow \frac{dv(t)}{dt} + 5v(t) = 0$$

Problem 2: Write the second order differential equation for the circuit shown in Fig. P2. (Assume that energy may be initially stored in both the inductor and capacitor).

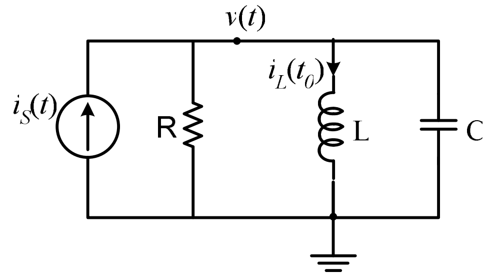


Fig. P2

Sol.: The node equation for the parallel RLC circuit is

$$\frac{v}{R}+\frac{1}{L}\int_{t_0}^{t} v(x)dx+i_L(t_0)+C\frac{dv}{dt}=i_s(t)$$

If the equation is differentiated with respect to time, we obtain

$$C\frac{d^2v}{dt^2}+\frac{1}{R}\frac{dv}{dt}+\frac{v}{L}=\frac{di_s}{dt}$$

Problem 3: Find $v_o(t)$ in the circuit of Fig. P3. Assume $v_o(0)$ = 5 V. (***Use the Laplace Transform in Circuit Analysis to solve this***)

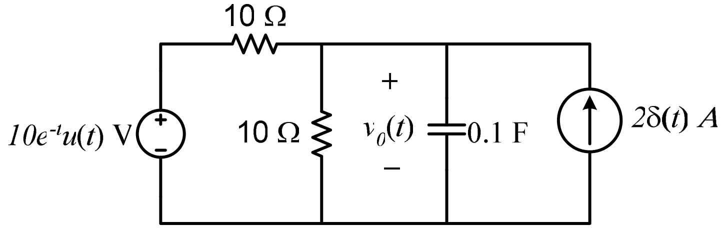


Fig. P3

Sol.:

We transform the circuit to the s-domain as shown in Fig. P3-Sol. The initial condition is included in the form of the current source $Cv_o(0)$ = 0.1(5) = 0.5 A. We apply nodal analysis. At the top node,

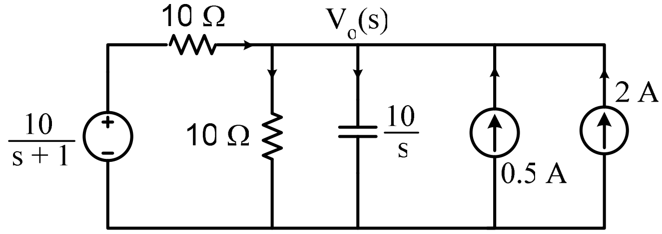


Fig. P3-Sol.

$$\frac{10/(s+1)-V_0}{10}+2+0.5=\frac{V_0}{10}+\frac{V_0}{10/s}$$

Or

$$\frac{1}{s+1}+2.5=\frac{2V_0}{10}+\frac{sV_0}{10}=\frac{1}{10}V_0(s+2)$$

$$\frac{10}{s+1}+25=V_0(s+2)$$

$$V_0=\frac{25s+35}{(s+1)(s+2)}=\frac{A}{s+1}+\frac{B}{s+2}$$

Where

$$A=(s+1)V_0(s)|_{s=-1}=\left.\frac{25s+35}{s+2}\right|_{s=-1}=\frac{10}{1}=10$$

$$B=(s+2)V_0(s)|_{s=-2}=\left.\frac{25s+35}{s+1}\right|_{s=-2}=\frac{-15}{-1}=15$$

Thus

$$V_0(s)=\frac{10}{s+1}+\frac{15}{s+2}$$

Taking the inverse Laplace transform, we obtain

$$v_o(t) = (10e^{-t} + 15e^{-2t})u(t) \qquad \text{(V)}$$

Problem 4: Determine the transfer function H(s) = $V_o(s)/I_o(s)$ of the circuit in Fig. P4.

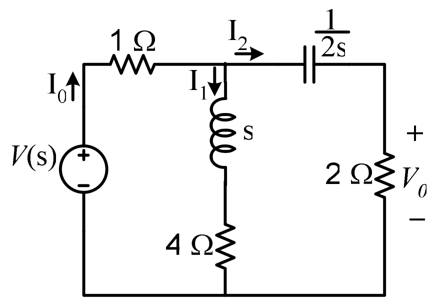


Fig. P4

<u>Sol.:</u>

By current division,

$$I_2=\frac{(s+4)I_0}{s+4+2+1/2s}$$

And

$$V_0=2I_2=\frac{2(s+4)I_0}{s+6+1/2s}$$

Hence

$$H(s)=\frac{V_0(s)}{I_0(s)}=\frac{4s(s+4)}{2s^2+12s+1}$$