

FINAL EXAMINATION

Academic year 2020-2021, Semester 1; Duration: 120 minutes

SUBJECT: Differential Equations (FERM)

Head of the Department of Mathematics

Professor Pham Huu Anh Ngoc

Lecturer:

Pham Huu Anh Ngoc

Signature:

Instructions:

- Each student is allowed a scientific calculator and a maximum of two double-sided sheets of reference material (size A4 or similar), stapled together and marked with their name and ID. All other documents and electronic devices are forbidden..

Question 1. (20 marks) Determine the form of a particular solution of the following differential equation:

$$y^{(5)} - 4y^{(4)} + y''' - 4y'' = x^3 + 1 - (x^2 + 1)e^{4x} + 12x \sin x.$$

Question 2. (i) (10 marks) Find $\alpha \in \mathbb{R}$ such that $y(x) = x^\alpha$ is a solution of the following differential equation

$$x^2 y''' + 10xy'' + 18y' = 0, \quad x \in (0, \infty).$$

(ii) (10 marks) Find the general solution of the following differential equation:

$$x^2 y''' + 10xy'' + 18y' = x^4 + x^2, \quad x \in (0, \infty).$$

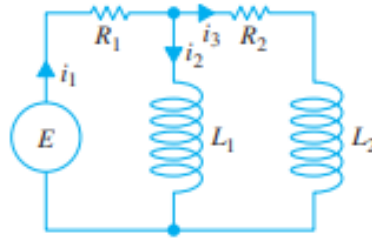
Question 3. (20 marks) Find the general solution of the linear system of differential equations

$$\frac{dx}{dt} = 4x + \frac{1}{3}y + e^t; \quad \frac{dy}{dt} = 9x + 6y - 2e^t.$$

Question 4. (20 marks) Find the general solution of the following differential equation

$$y^{(5)} - 3y^{(4)} + 2y''' = 2020 + 2021e^{2x}.$$

Question 5. (20 marks)



The system of differential equations for the currents $i_2(t)$ and $i_3(t)$ in the electrical network shown in the Figure is

$$\begin{cases} \frac{di_2}{dt} = -\frac{R_1}{L_1}i_2 - \frac{R_1}{L_1}i_3 + \frac{E}{L_1} \\ \frac{di_3}{dt} = -\frac{R_1}{L_2}i_2 - \frac{R_1 + R_2}{L_2}i_3 + \frac{E}{L_2} \end{cases}$$

(a) Use the method of undetermined coefficients to solve the system if $R_1 = 2 \Omega$, $R_2 = 3 \Omega$, $L_1 = 1 \text{ h}$, $L_2 = 1 \text{ h}$, $E = 60 \text{ V}$, $i_2(0) = 0$, and $i_3(0) = 0$.

(b) Determine the current $i_1(t)$.

The end.