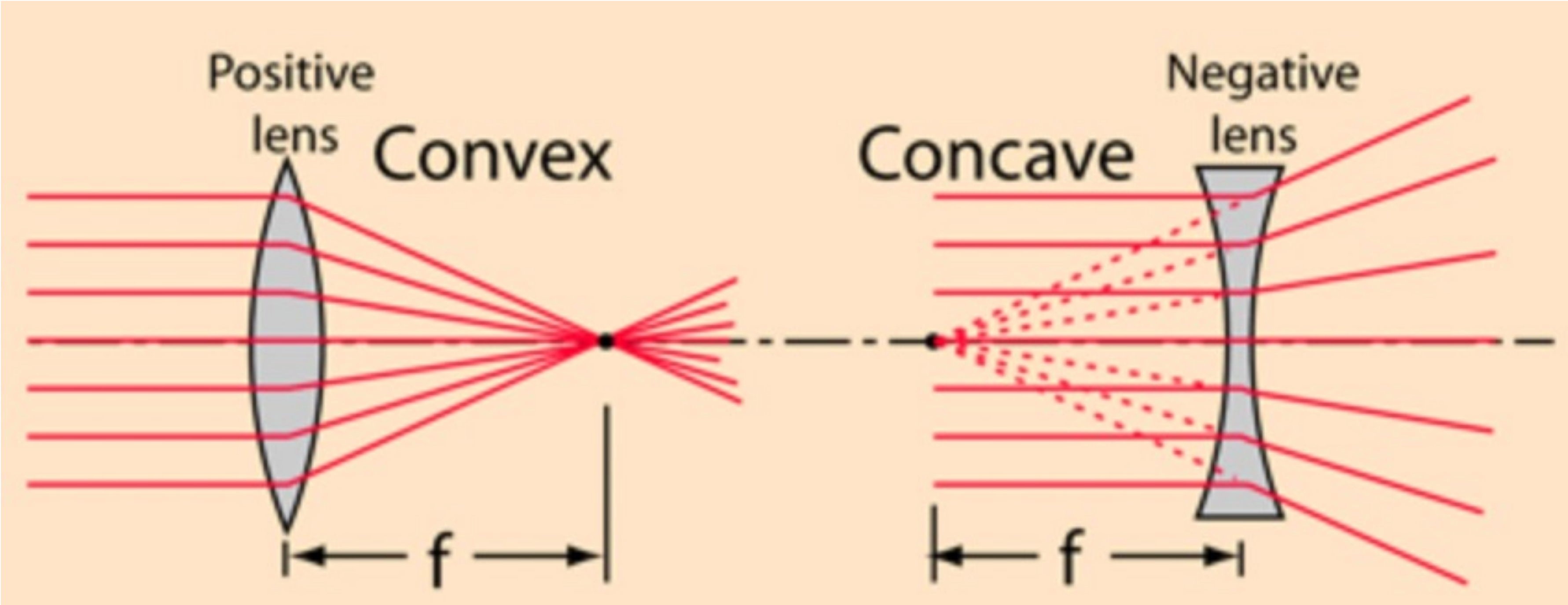
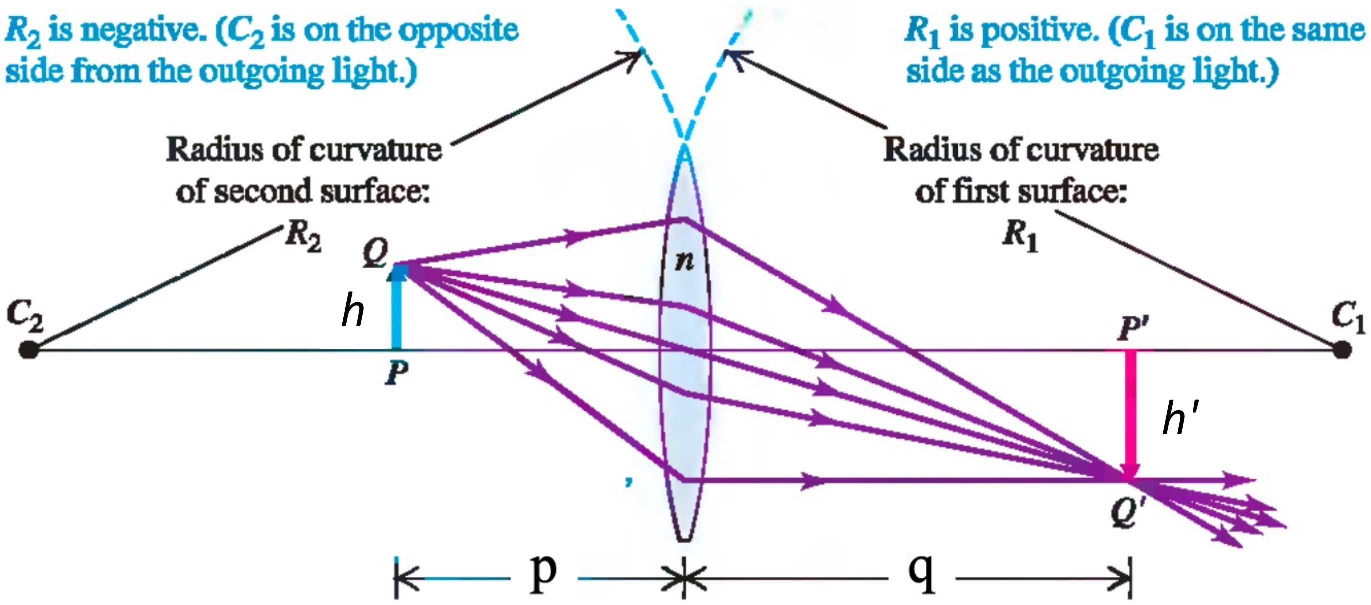
1. Thin lens



$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

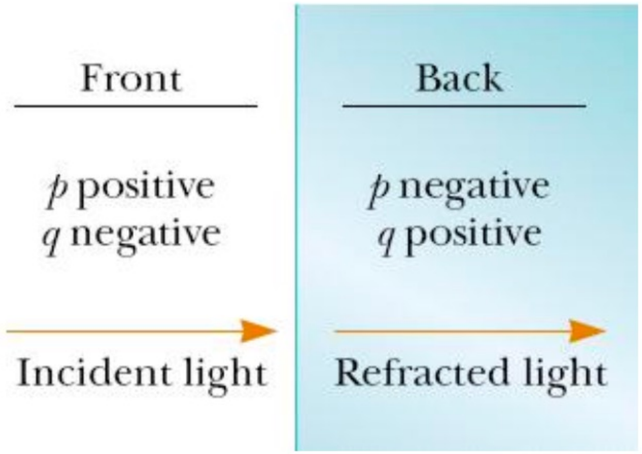
(lens makers' equation)

f is **positive** if the lens is **converging.**
f is **negative** if the lens is **diverging.**



Thin-lens equation :

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f}$$

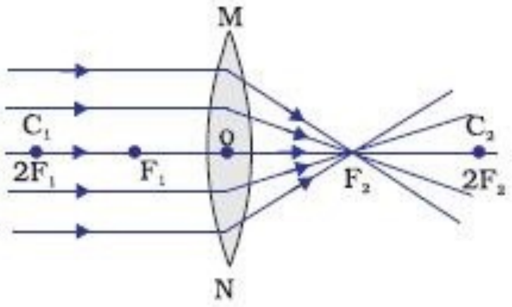


Lateral magnification :

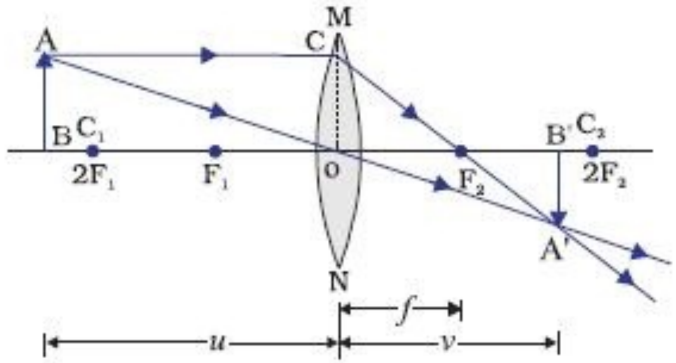
$$M = \frac{h'}{h} = -\frac{q}{p}$$

Real object $\leftrightarrow$ $p > 0$ Virtual object $\leftrightarrow$ $p < 0$

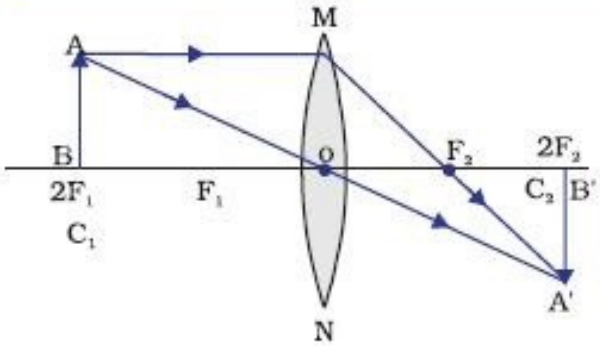
Real image $\leftrightarrow$ $q > 0$ Virtual image $\leftrightarrow$ $q < 0$



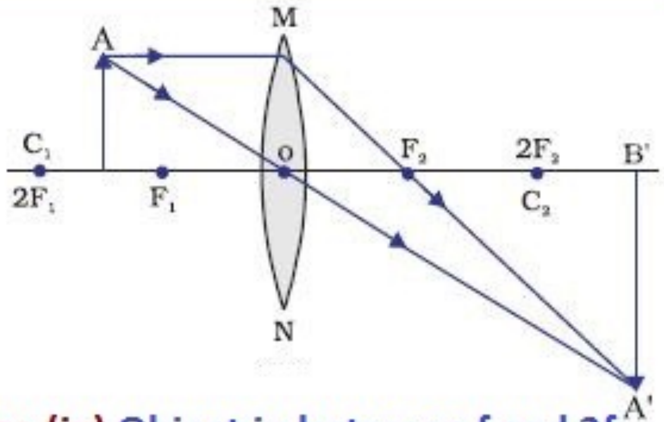
Case (i) Object at infinity



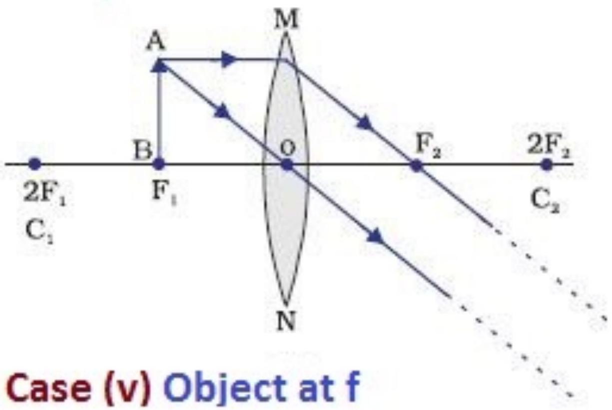
Case (ii) Object at beyond 2f



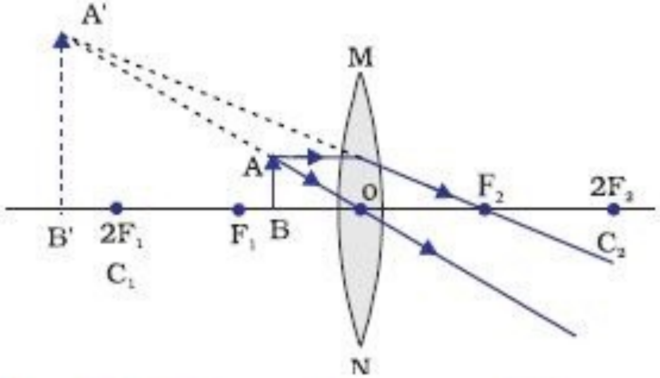
Case (iii) Object at 2f



Case (iv) Object in between f and 2f



Case (v) Object at f



Case (vi) Object distance < f

2. Matter-wave principle

De Broglie's relationships

We recall that
for a photon (E, p) associated to an electromagnetic wave (f, λ):

$$\boxed{E = h\,f}$$

$$\boxed{p = h\,\frac{1}{\lambda}}$$

$$\underbrace{}_{\text{particle}} \qquad \underbrace{}_{\text{wave}}$$

Planck constant:
$h = 6.63 \times 10^{-34}\ m^2\ kg/s$

De Broglie's hypothesis:

To a **particle** (E, p) is associated a **matter wave**,
which has a frequency f and a wavelength λ

$$\boxed{f = \frac{E}{h}} \qquad \boxed{\lambda = \frac{h}{p}}$$

- From $\boxed{f = \frac{E}{h}}$ and $\boxed{\lambda = \frac{h}{p}}$

if we put: $\boxed{h = 2\pi \hbar}$ $\Rightarrow$ $\boxed{E = 2\pi f \hbar}$

$\Rightarrow$ $\boxed{p = \frac{2\pi}{\lambda} \hbar}$

$$\left.\begin{array}{l} \boxed{E = \hbar\omega} \\ \boxed{\vec{p} = \hbar \vec{K}} \end{array}\right\} \textbf{Planck-Einstein's relationship}$$

λ is called **de Broglie wavelength**

$\vec{K}$ is **the angular wave vector** (describing how many oscillations it completes per unit of distance)

$$\vec{K} = 2\pi/\lambda$$

The Schrödinger's equation

For the case of one-dimensional motion,
when a particle with the mass m has a potential energy $U(x)$
Schrödinger's equation is

$$\boxed{\frac{d^2\psi}{dx^2}+\frac{8\pi^2 m}{h^2}[E-U(x)]\psi=0}$$

Probability density:

Assume that the free particle travels only in the positive direction
Relabel the constant A as ψ_0: $\psi(x)=\psi_0 e^{iKx}$
The probability density is:

$$|\psi|^2=\left|\psi_0 e^{iKx}\right|^2=(\psi_0)^2\left|e^{iKx}\right|^2$$

Because:

$$\left|e^{iKx}\right|^2=(e^{iKx})(e^{iKx})^*=(e^{iKx})(e^{-iKx})=e^0=1$$

we have:

$$|\psi|^2=(\psi_0)^2=const$$

3. A particle in an infinite well

With n is an integer: $n = 1; 2; 3;...$

the energy can only have the <u>discrete values</u>: $E_n = \left(\frac{\pi^2\hbar^2}{2ma^2}\right)n^2$

$$E_n = \left(\frac{\pi^2(h/2\pi)^2}{2ma^2}\right)n^2 = \frac{h^2}{8ma^2}n^2 \longrightarrow \boxed{E_n = \frac{h^2}{8ma^2}n^2}$$

We say that the <u>energy is quantized</u>

these values of energy are called <u>energy levels</u>

$n = 1 \longrightarrow$ ground state (E_1)

$n = 2 \longrightarrow$ first excited state (E_2)

$n = 3 \longrightarrow$ second excited state (E_3)

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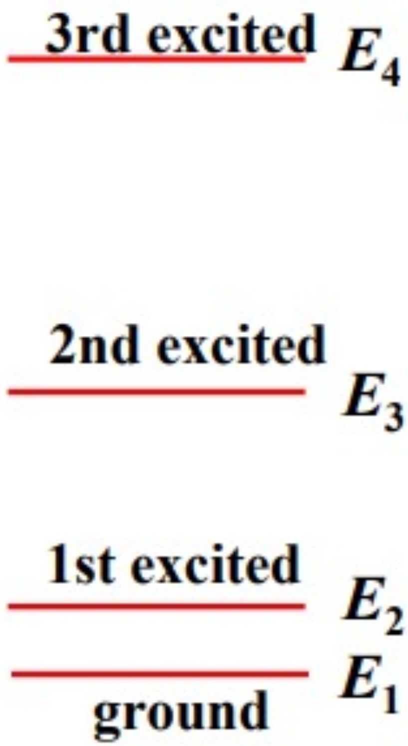
The integer n is called <u>the quantum number</u>



energy-level diagram

4. Spectral emission lines in Hydrogen atom

When the electron jumps down from an energy level E_m to a lower one E_n , the hydrogen atom emits a photon of energy:

$$\boxed{\varepsilon = hf_{mn} = \frac{hc}{\lambda_{mn}} = E_m - E_n}$$

with $\boxed{E_n = -\frac{13.6eV}{n^2}}$

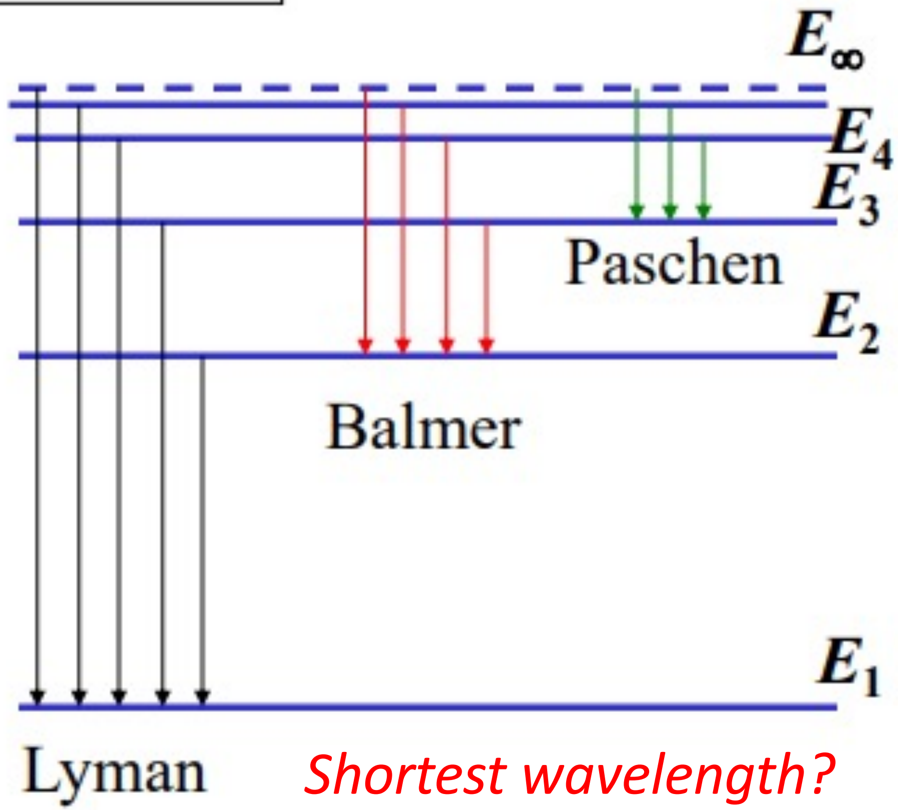
- If $E_n \equiv E_1$: Lyman series
- If $E_n \equiv E_2$: Balmer series
- If $E_n \equiv E_3$: Paschen series
- If $E_n \equiv E_4$: Brackett series

$$\boxed{\frac{1}{\lambda} = R_H\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)}$$

n_i, n_f are like m, n above

$$R_H = \frac{mk^2e^4}{4\pi c\hbar^3} = 1.097\times10^7 m^{-1}$$

(Rydberg constant)



Shortest wavelength?

Longest wavelength?

5. Schrodinger equation for hydrogen atom

After solving Schrodinger equation for the radial part we have:

$$E_n = -\left(\frac{me^4}{8\varepsilon_0^2 h^2}\right)\frac{1}{n^2}$$

- *Wave function for the ground state*

The wave function for the ground state of the hydrogen atom:

$$\boxed{\psi(r) = \frac{1}{a^{3/2}\sqrt{\pi}} e^{-r/a}}$$

Where a is the Bohr radius: $a = 0.529 \times 10^{-10} m = 52.9 nm$

- *Wave function for the first excited state*

$$\boxed{\psi(r) = Ce^{-ar/2}(2 - ar)}$$

- *The radial probability density*

$$\boxed{P(r) = \frac{4}{a^3} e^{-2r/a} r^2}$$

6. Special relativity

$$\Delta t = \frac{\Delta t_0}{\sqrt{1-u^2/c^2}}$$

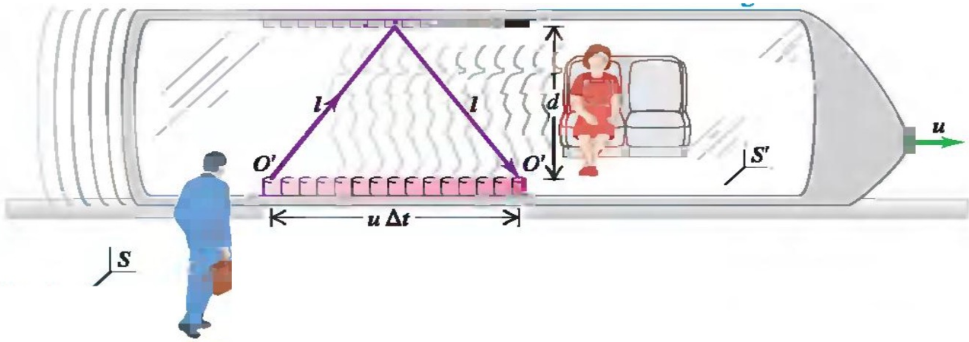
$$\Delta t = \gamma \Delta t_0 \;;$$

$$\gamma = \frac{1}{\sqrt{1-u^2/c^2}}$$

Lorentz constant

$\Delta t > \Delta t_0$: **time dilation**

Observers measure any clock to run slow if it moves relative to them



$$L = L_0\sqrt{1-\frac{v^2}{c^2}}$$

$$L = \frac{L_0}{\gamma}\;;$$

$$\gamma = \frac{1}{\sqrt{1-u^2/c^2}}$$

$(L < L_0)$ **(Length contraction)**

The length L measured in S, in which the ruler is moving, is shorter than the length L_0 measured in its rest frame S'.

7. Nuclear Radioactivity

- When unstable nuclides decay into different nuclides, they usually emit **alpha** (α) or **beta** (β) particles:

Alpha particle is a ^{4}He nucleus, a beta-minus particle (β^-) is an electron, beta-plus particle (β^+) is a positron (antiparticle of electron)

$${}^{A}_{Z}X \rightarrow {}^{4}_{2}He + {}^{A-4}_{Z-2}Y \;;\; {}^{A}_{Z}X \rightarrow {}^{0}_{-1}e + {}^{A}_{Z+1}Y \;;\; {}^{A}_{Z}X \rightarrow {}^{0}_{1}e + {}^{A}_{Z-1}Y$$

- $dN(t)/dt$ is called the **decay rate or the activity** of the specimen.

Activity: becquerel (Bq) in SI or curie (Ci)

$$1\text{Ci} = 3.70\times10^{10}\,\text{Bq} = 3.70\times10^{10}\,\text{decays/s}$$

$$-\frac{dN(t)}{dt} = \lambda N(t) \qquad \boxed{N(t) = N_0 e^{-\lambda t}}$$

λ **: decay constant**

$N(t)$: the (very large) number of radioactive nuclei in a sample at time t.

- **The half-life $T_{1/2}$** is the time required for the number of radioactive nuclei to decrease to one-half the original number N_0

$$\boxed{T_{1/2} = \frac{\ln 2}{\lambda} = \frac{0.693}{\lambda}}$$

- **The mean lifetime T_{mean}** (generally called the lifetime)**:**

$$\boxed{T_{mean} = \frac{1}{\lambda} = \frac{T_{1/2}}{\ln 2}}$$