**MIDTERM EXAMINATION - Answerkey**
**PROBABILITY, STATISTICS AND RANDOM PROCESS**
Semester 1, 2023-24 • November 2023 • Total duration: 90 minutes

1. *(10 points)* If the probability that student A will fail a certain examination is 0.4, the probability that student B will fail the examination is 0.2, and the probability that both student A and student B will fail the examination is 0.1, what is the probability that at least one of these two students will fail the examination? 0.4+0.2-0.1

2. *(20 points)* A box contains 4 reds ball and 3 blue balls. One ball is selected at random at its color observed. The ball is then returned to the box and 2 additional balls of the same color are also put into the box. A second ball is then selected at random, its color is observed, and it is returned to the box together with 2 additional balls of the same color.

   (a) Evaluate the conditional probability that the second ball is blue given that the first ball is red. $\frac{3}{9}$

   (b) Find the probability that two selected balls are blue. $\frac{3}{7} \times \frac{5}{9} = \frac{5}{21}$

3. *(10 points)* Consider a system of two components in parallel. The components work independently. It is known that the probability that each component fails is 0.95. Find the probability that the system works. $1-(0.95)^2$

4. *(10 points)* In a class of Calculus 1, 30% of students are from CS school, 20% from EE school, 40% from BME school and 10% from other schools. It is known that the fraction of students passing the course in CS school, EE school, BME school and other schools are 80%, 78%, 85% and 75% respectively. Take randomly a student from this class. Evaluate the probability that this student comes from CS school given that he/she passes the course. $\frac{30\%*80\%}{30\%*80\%+20\%*78\%+40\%*85\%+10\%*75\%}$

5. *(20 points)* Let $X$ be a discrete random variable with probability mass function is given by
$$p(x) = cx \quad \text{for} \quad x = 1,2,3,4,5,6$$

   (a) Find the value of $c$. $c = \frac{1}{1+2+3+4+5+6} = \frac{1}{21}$

   (b) Compute $P(2 < X < 6) = p(3) + p(4) + p(5) = \frac{12}{21}$.

   (c) Determine the mean and variance of $X$ $E(X) = \frac{91}{21} = \frac{13}{3}, Var(X) = \frac{441}{21} - \left(\frac{13}{3}\right)^2$.

6. *(20 points)* Suppose that $X$ is a continuous random variable with probability density function
$$f(x) = \begin{cases} cx^2 & \text{if } 1 \le x \le 2 \\ 0 & \text{elsewhere} \end{cases}$$

   (a) Find $c = \frac{3}{7}$.

   (b) Evaluate $E(X) = \int_1^2 \frac{3}{7}x^3 dx = \frac{45}{28}$ and $Var(X) = \int_1^2 \frac{3}{7}x^4 dx - \left(\frac{45}{28}\right)^2$.

   (c) Compute $P(X > 1.5) = \int_{1.5}^2 \frac{3}{7}x^2 dx$.

7. *(10 points)* The temperature in degrees Celsius at a certain location is normally distributed with a mean of 28 degrees and a standard deviation of 4 degrees. Find the probability that the temperature is between 26 and 30 degrees. $\phi\left(\frac{30-28}{4}\right) - \phi\left(\frac{26-28}{4}\right) \approx 0.6915 - 0.3085 = 0.3830$