Đỗ Minh Duy - ITITSB22029.

4, Conditional probability - Multiplication rule.

5) ~~?~~ Let 2 components C_1 and C_2

$\Rightarrow P(\overline{C_1}) = 10\%$

$\Rightarrow P(\overline{C_2})$

- First component fails, the chance to fail second component $= 20\% \Rightarrow P(\overline{C_2}/\overline{C_1}) = 20\%$
- First component works, the chance to fail second component $= 5\% \Rightarrow P(\overline{C_2}/C_1) = 5\%$

a) $1 - P(\overline{C_1})P(\overline{C_2})$

$\Rightarrow 1 - (10\%)\times(20\%) = 98\%$

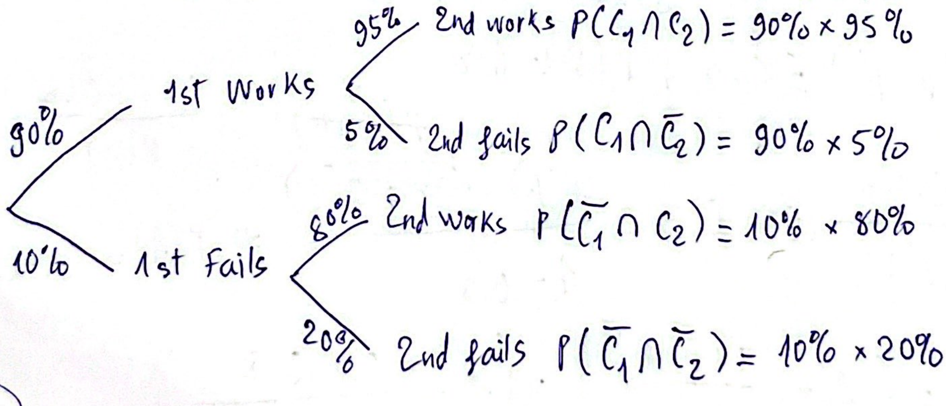
b) $\rightarrow P(C_1 \cap \overline{C_2}) + P(\overline{C_1} \cap C_2)$

$\Rightarrow P(C_1).P(\overline{C_2}/C_1) + P(\overline{C_1}).P(C_2/\overline{C_1})$

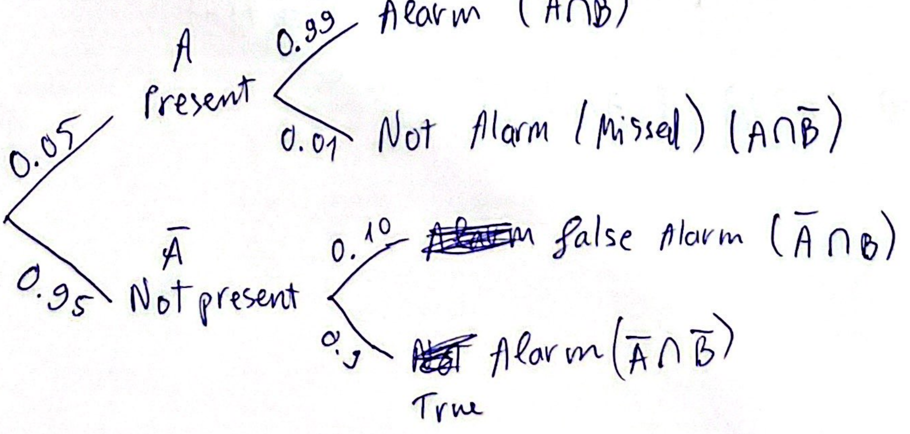
$\Rightarrow (1-10\%).(50\%) + (10\%).(\cancel{?}\,20\%) = 12.5\%$

c) $P(C_2) = P(\overline{C_1}).P(C_2/C_1) + P(C_1).P(C_2/\overline{C_1})$

$= 10\%.(1-20\%) + (1-10\%).(1-5\%) = 93.5\%$



6)



a) $P(\overline{A} \cap B) = 0.95 \times 0.1 = 0.095$

b) $P(A \cap \overline{B}) = 0.05 \times 0.01 = 0.0005$

5. Independence

1) $P(B) = \frac{5}{500}$; $P(B/A) = \frac{4}{499}$

$P(B) \neq P(B/A) \Rightarrow$ A and B are not independent events.

2) A : ~~the~~ lab speimen contains high levels of contamination.

a) $P(A_1' \cap A_2' \cap A_3' \cap A_4' \cap A_5') = 0.9^5 = 0.5905 = P(A)$

b) $5 \times P(A_1' \cap A_2 \cap A_3' \cap A_4' \cap A_5') = 5 \times [0.9^4 \times 0.1] = 0.33 = P(B)$

c) $P(C) = 1 - P(A_1' \cap A_2' \cap A_3' \cap A_4' \cap A_5') = 1 - 0.5905 = 0.4095$

3) $P(BC) = 1 - (1-0.7).(1-0.8) = 0.94$

a) $P(W) = P(A) \times P(BC) \times P(D) = 0.95 \times 0.94 \times 0.9 = 0.8037$

b) $P(B'/W) = \frac{P(A)\,P(C)\,P(D)}{P(W)} = \frac{0.684}{0.8037} = 0.85$

6. Total probability - Bayes's formula.

1) $P(A/B) = 0.2$; $P(A/B') = 0.3$; $P(B) = 0.8$

Bayes's formula

$P(A/B') = \frac{P(B'/A) \times P(A)}{P(B')} \Leftrightarrow 0.3 = \frac{P(A) \times \left(1 - \frac{0.2 \times 0.8}{P(A)}\right)}{1-0.8} \Rightarrow P(A) = 0.22$

$P(B'/A) = (1 - P(B/A)) = \left(1 - \frac{P(A \cap B)}{P(A)}\right) = \left(1 - \frac{0.2 \times 0.8}{P(A)}\right)$

$P(A/B) = \frac{P(A \cap B)}{P(B)} \Rightarrow P(A \cap B) = P(A/B).P(B)$

(2) L: the event that costumer buy latex paint

S: the event that costumer buy semigloss paint

R: the event that costumer buy a roller.

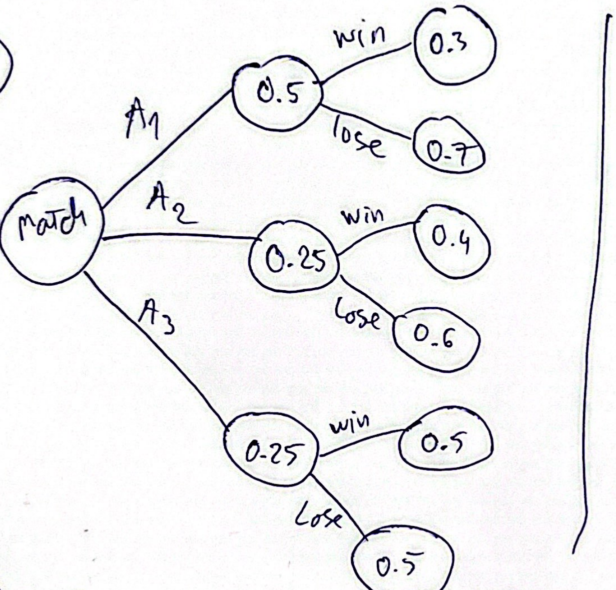
$P(L) = 0.75$; $P(S) = 1 - 0.75 = 0.25$

$P(R/L) = 0.6$

$P(R/S) = 0.3$

$$P(L/R) = \frac{P(R/L)\,P(L)}{P(R/L)\,P(L) + P(R/S)\,P(S)} = \frac{0.6 \times 0.75}{0.6 \times 0.75 + 0.3 \times 0.25} = 0.857$$

(3)



$P(\text{Winning}) = 0.5 \times 0.3 + 0.25 \times 0.4$

$+ 0.25 \times 0.5 = 0.375$

(4) A: Event that the odd box is chosen

B: Event that the even box is chosen

C: Event that ball 3 is drawn

$$P(C) = P(A) \times P(C/A) + P(B) \times P(C/B)$$

$$= \frac{1}{2} \times \frac{1}{3} + \frac{1}{2} \times 0 = \frac{1}{6}$$

(5) P: Event that test gives positive result.

N: Event that test gives Negative result.

D: Event that person has disease.

$\bar{D}$: Event that person has no disease.

$P(P/D) = 0.95$; $P(P/\bar{D}) = 0.02$

$P(D) = 0.01 \Rightarrow P(\bar{D}) = 1 - 0.01 = 0.99$

$$P(D/P) = \frac{P(P/D) \times P(D)}{P(P)} = \frac{0.95 \times 0.01}{0.0293}$$

$= 0.324$

$P(P) = 1\% \times 95\% + 99\% \times 2\% = 0.0293$