

# Principles of Electrical Engineering 1 (EE051IU)

Textbook:

**Electric Circuits:** James W. Nilsson &  
Susan A. Riedel 9th Edition.

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- **Number of credits:** 3

# Grading

1. Midterm examination: 30%
2. Final examination : 40%
3. Attendance + Quiz + Homework + Project:  
30%

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# Course Policy

1. All assignments need to be submitted on the due date
2. Students are expected to do their own work at all times. Any evidence of plagiarism or cheating will be treated as grounds for failure in the class.
3. Attendance: at least 80%

# Course Syllabus

- Introduction to EE051IU: Circuit variables
- Simple resistive circuits
- Techniques of circuit analysis
- The operational amplifier
- Inductance, capacitance and mutual inductance
- Sinusoidal steady-state analysis
- Steady-state power calculations
- Two-port circuits.
- Balanced three-phase circuits: three-phase voltage sources, analysis of the wye-wye and wye-delta circuit, power calculation and measurements

# What is Electrical Engineering?

- Electrical engineering is a broad field that now subdivided into a wide range of subfields including **power**, **control systems**, **electronics**, **signal processing**, **telecommunications**, **instrumentation** and **microelectronics**.



Source Wikipedia (The free Encyclopedia)

[http://en.wikipedia.org/wiki/Electrical\\_engineering](http://en.wikipedia.org/wiki/Electrical_engineering)

# What is Electrical Engineering



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# Course Objectives

- Students who successfully fulfill the course requirements will have :
  - An ability to define and explain the fundamental principles of Ohm's law, Kirchhoff's Voltage Law (KCV) and Kirchhoff's current Law (KCL).
  - An ability to state and apply the techniques of superposition, source transformations, and Thevenin/Norton equivalent circuits to simplify the analysis of circuits and/or the computation of responses.
  - An ability to analyze op- amp circuits and design inverting, non-inverting, summing, and differential amplifier circuits using op- amps.
  - An in depth understanding of the behavior of inductances and capacitances, and differentiating and integrating op-amp circuits.
  - An ability to compute the steady state AC responses of basic circuits using the phasor method.
  - An ability to compute effective and average values of periodic signals and compute the instantaneous and average powers delivered to a circuit element.

# CHAPTER 1 – Circuit Variables



# What is an Electric Circuit?

“A **mathematical model** that approximates the behavior of an actual circuit system.”

## ■ Note:

- ❑ Electric circuits is a component that is common between all the electric systems.
- ❑ Circuit theory is a special case of electromagnetic field theory.

# International System of Units (SI)

*SI unit can be divided into two classes: **base units** and **derived units***

## Base units

Quantity	Name	Symbol
Length	Meter	m
Mass	Kilogram	kg
Time	Second	s
Electric Current	Ampere	A
Temperature	Degree Kelvin	K
Amount of Substance	Mole	mol
Luminous intensity	Candela	Cd

## Derived units

Quantity	Symbol	Formula
Frequency	Hertz (Hz)	$s^{-1}$
Force	Newton (N)	$kg.m/s^2$
Energy or Work	Joule (J)	$N.m$
Power	Watt (W)	$J/s$
Electric Charge	Coulomb (C)	$A.s$
Electric Potential	Volt (V)	$J/C$
Electric Resistance	Ohm ( $\Omega$ )	$V/A$
Electric Conductance	Siemens (S)	$A/V$
Electric Capacitance	Farad (F)	$C/V$
Magnetic Flux	Weber (Wb)	$V.s$
Inductance	Henry (H)	$Wb/A$

# Standardized prefixes to signify powers of 10

## Multiple Prefixes

Prefix	Symbol	Magnitude
yotta	Y	$10^{24}$
zetta	Z	$10^{21}$
exa	E	$10^{18}$
peta	P	$10^{15}$
tera	T	$10^{12}$
giga	G	$10^9$
mega	M	$10^6$
Kilo	k	$10^3$
hecto	h	$10^2$
deca	da	$10^1$

## Sub-multiple Prefixes

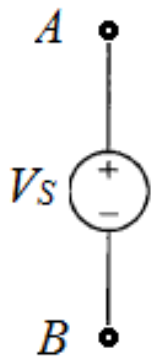
Prefix	Symbol	Magnitude
deci	d	$10^{-1}$
centi	c	$10^{-2}$
milli	m	$10^{-3}$
micro	$\mu$	$10^{-6}$
nano	n	$10^{-9}$
pico	p	$10^{-12}$
femto	f	$10^{-15}$
atto	a	$10^{-18}$
zepto	z	$10^{-21}$
yocto	y	$10^{-24}$

# Rules for using prefixes

- **No space** between the prefix symbol and the unit symbol.
  - 10 **cm** but not 10 **c m**
- The grouping formed by the prefix symbol attached to the unit symbol constitutes a new inseparable symbol which can be raised to a positive or negative power and combined with other unit symbols to form compound unit symbol.
  - $1 \text{ cm}^3 = (10^{-2} \text{ m})^3 = 10^{-6} \text{ m}^3$
  - $1 \text{ }\mu\text{s}^{-1} = (10^{-6} \text{ s})^{-1} = 10^6 \text{ s}^{-1}$
  - $1 \text{ V/cm} = (1 \text{ V}) / (10^{-2} \text{ m}) = 10^2 \text{ V/m}$
  - $1 \text{ cm}^{-1} = (10^{-2} \text{ m})^{-1} = 10^2 \text{ m}^{-1}$
- The position of two or more SI prefixes, are not used.
  - 1 **nm** but not 1 **mμm**
- A prefix is never used in isolation.
  - $10^6 / \text{m}^3$  but not **M/m**<sup>3</sup>

# Circuit Elements

## ■ Five ideal basic circuit elements



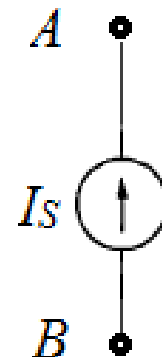
**DC Voltage source**

$$V_S = V_{AB} = -V_{BA}$$



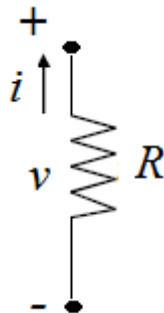
**AC Voltage source**

$$V_S = V_{AB} = -V_{BA}$$



**Current source**

$$I_S = I_{AB} = -I_{BA}$$



**Resistor R**

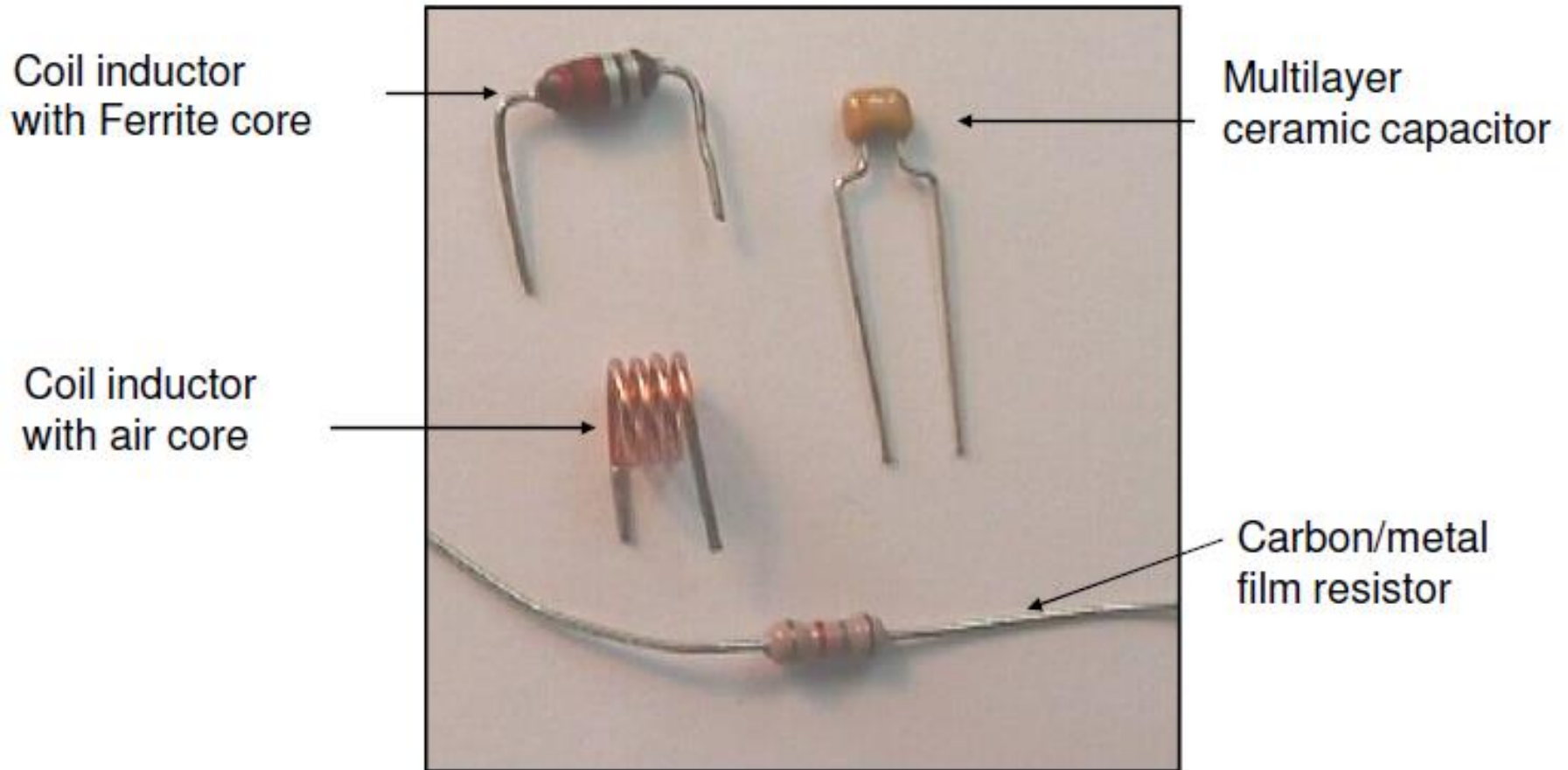


**Inductor L**

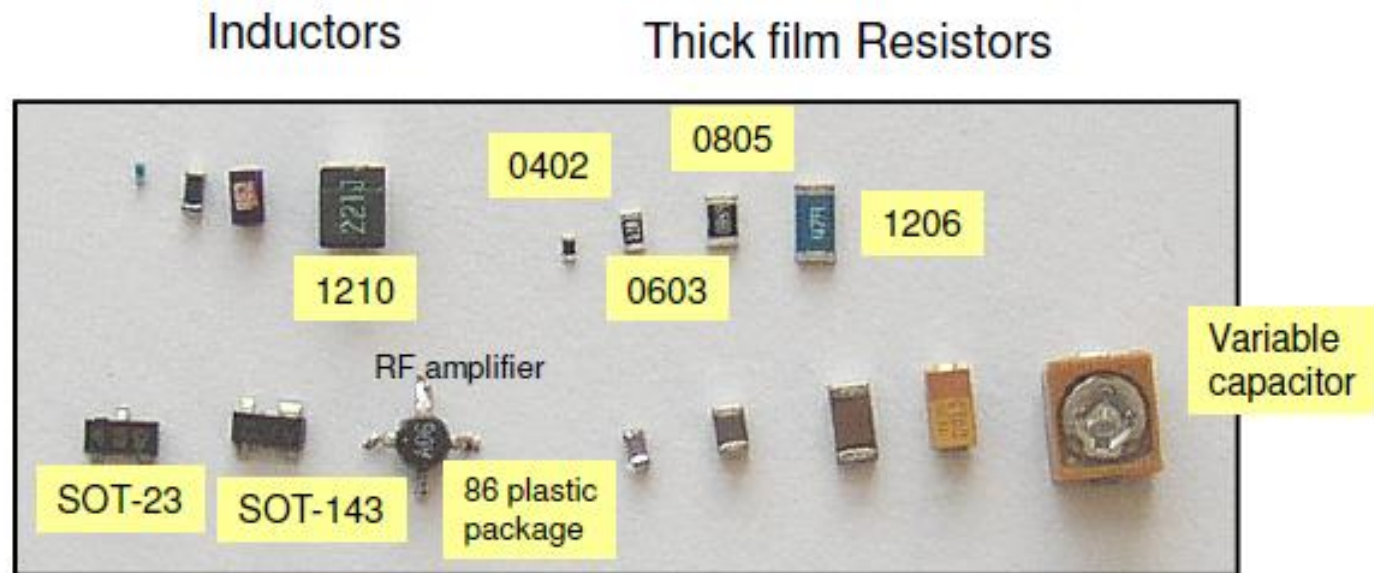


**Capacitor C**

# Passive Lumped Components for Medium Frequency (up to 300MHz)

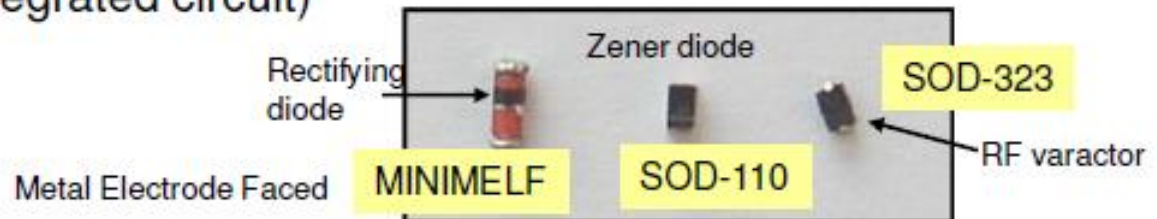


# Passive Lumped Components for Ultra High Frequencies (UHF) (>300MHz)



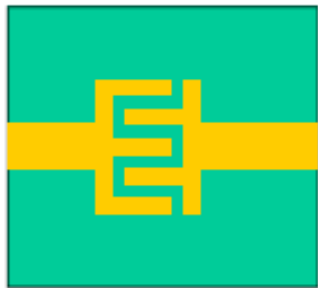
Transistors and  
MMIC (monolithic  
microwave integrated circuit)

Various types of capacitors

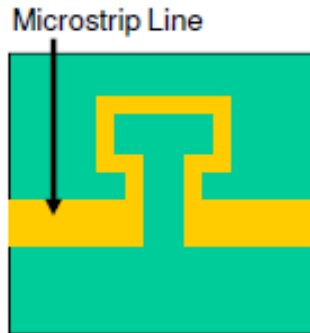


Various types of diodes

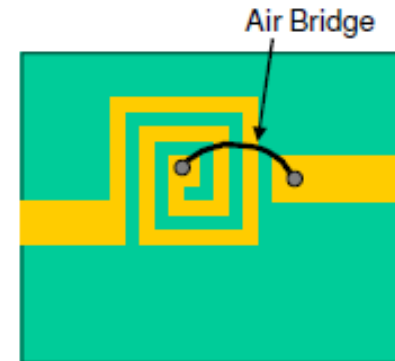
# Components for Microwave Frequency Microstrip Technology



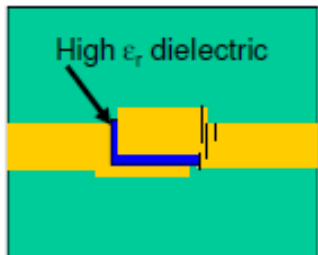
Inter-digital Capacitor



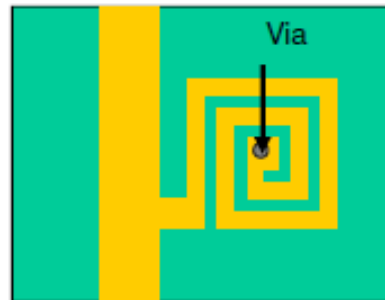
Series Single-Loop Spiral Inductor



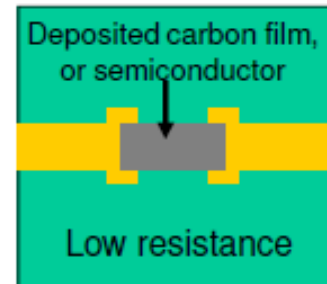
Series Multi-Loop Spiral Inductor



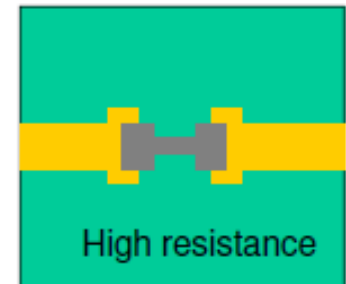
Metal-Insulator-Metal (MIM) Capacitor



Shunt Multi-Loop Spiral Inductor



Low resistance



High resistance

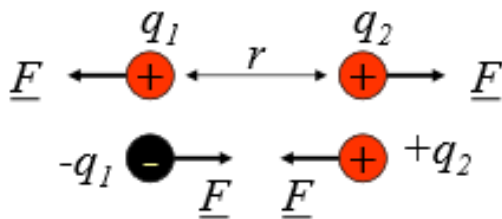
Resistors



# Voltage (Electric) “V”

The concept of electric charge is the basis for describing all electrical phenomena.

- The charge is bipolar, meaning that electrical effects are described in terms of positive and negative charges.
- The electric charge exists in discrete quantities, which are integral multiples of the electronic charge (proton, electron =  $1.6022 \times 10^{-19}$  C).
- The charges that occur in nature are integral multiples of the electronic charge



## Force

$$|F| = \frac{|q_1||q_2|}{4\pi\epsilon_0 r^2}$$



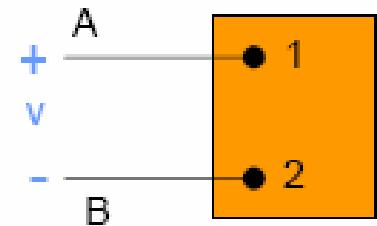
# Voltage (Electric) “V”

- Voltage is the amount of potential energy between two points on a circuit.
- One volt is the potential difference between two points when one joule of energy is used to move one coulomb of charge from one point to another
- The unit “volt” is named after the Italian physicist Alessandro Volta who invented what is considered the first chemical battery. Voltage is represented with the letter “V”.

The **voltage** (electric) is the energy per unit charge.

$$v = \frac{dw}{dq}$$

$v$  = the voltage in volts,  
 $w$  = the energy in joules,  
 $q$  = the charge in coulombs.



In circuit theory, the separation of charge creates an electric force (**voltage**), and the motion of charge creates an electric fluid (**current**).

# Voltage (Electric) “V”

P.E=POTENTIAL ENERGY

$$P.E=m.g.h$$

m: mass (kg)

g: acceleration of gravity  
( $m/s^2$ )

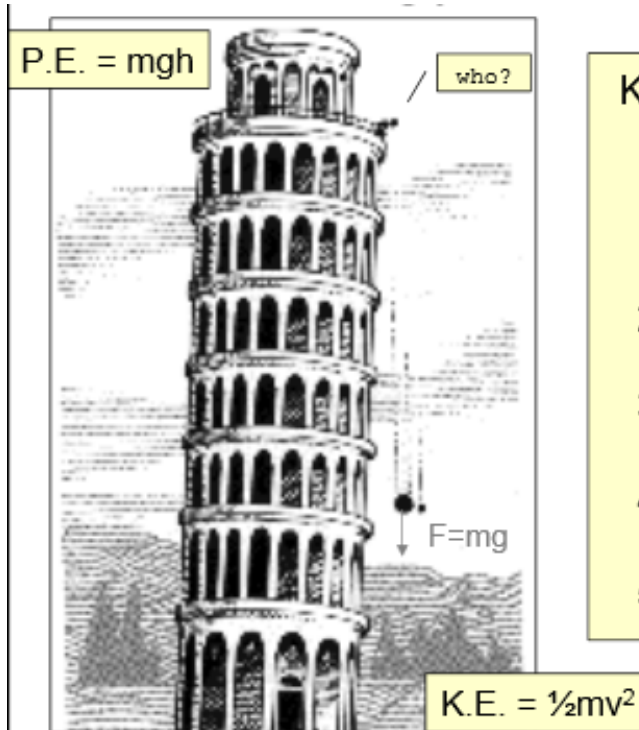
h: height (m)

K.E= KINETIC ENERGY

$$K.E=(1/2)m.v^2$$

m: mass (kg)

v: velocity or speed (m/s)



K.E.		P.E.	P.D.
0	++++	5 J	5 V
1 J	⊕ Q=1 C ↓ F=EQ	4 J	4 V
2 J		3 J	3 V
3 J		2 J	2 V
4 J		1 J	1 V
5 J	----- E ----- ----	0	0

A potential difference of 1 volt will give 1 joule of kinetic energy to a charge of 1 coulomb: **Energy=Q.V**

A charged particle moving in an electric field has exactly the same dynamics as a mass falling under gravity

# Electric Current “I”

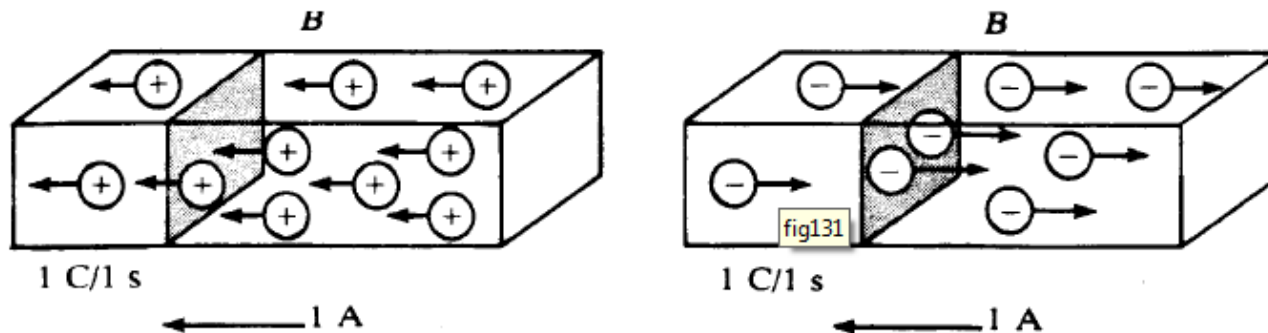
The electrical effects caused by charges in motion depend on the rate of charge flow. The rate of charge flow is known as the **electric current**, which is expressed as

$$i = \frac{dq}{dt}$$

C/s or (A)

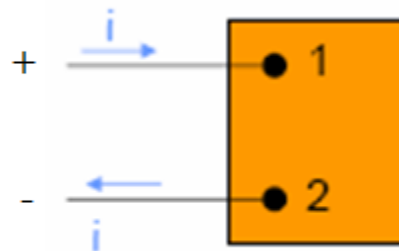
$i$  = the current in amperes,  
 $q$  = the charge in coulombs,  
 $t$  = the time in seconds.

Thus in a **current** of 1 ampere, charge is being transferred at a rate of 1 coulomb per second



# The ideal basic circuit element

- Attributes of the **ideal basic circuit** element:
  - It has **only two terminal**
  - It is **described mathematically in terms of current and/or voltage**
  - It **cannot be subdivided** into other elements.
- Note:
  - It is called **ideal** because it **does not exist as a realizable physical component**.



# Interpretation of Reference Directions

## Positive Value

$v$  voltage drop from terminal 1 to terminal 2

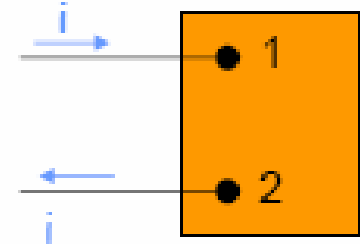
*or*

voltage rise from terminal 2 to terminal 1

$i$  positive charge flowing from terminal 1 to terminal 2

*or*

negative charge flowing from terminal 2 to terminal 1



## Negative Value

voltage rise from terminal 1 to terminal 2

*or*

voltage drop from terminal 2 to terminal 1

positive charge flowing from terminal 2 to terminal 1

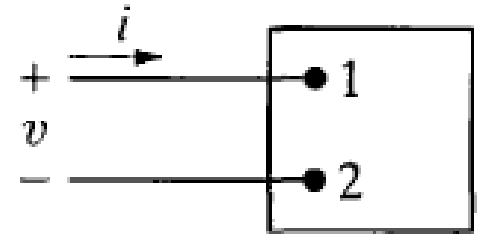
*or*

negative charge flowing from terminal 1 to terminal 2

*The passive sign convention*

## Example 1.1

No charge exists at the upper terminal of the element in Fig. for  $t < 0$ . At  $t = 0$ , a 5 A current begins to flow into the upper terminal.



- a)** Derive the expression for the charge accumulating at the upper terminal of the element for  $t > 0$ .
- b)** If the current is stopped after 10 seconds, how much charge has accumulated at the upper terminal?

## Solution

- a) From the definition of current, the expression for charge accumulation due to current flow is

$$q(t) = \int_0^t i(x) dx.$$

Therefore,

$$q(t) = \int_0^t 5 dx = 5x \Big|_0^t = 5t - 5(0) = 5t \text{ C for } t > 0.$$

- b) The total charge that accumulates at the upper terminal in 10 seconds due to a 5 A current is  $q(10) = 5(10) = 50 \text{ C}$ .

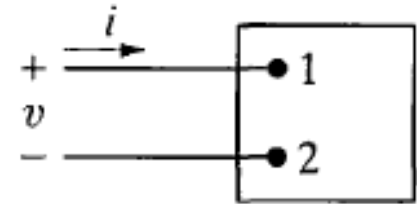


## Example 1.2

The current at the terminals of the element in Fig is:

$$i = 0, \quad t < 0;$$

$$i = 20e^{-5000t} \text{ A}, \quad t \geq 0.$$



Calculate the total charge (in microcoulombs) entering the element at its upper terminal.

## Solution

Current is the time rate of change of charge, or  $i = dq/dt$ . In this problem, we are given the current and asked to find the total charge. To do this, we must integrate the equation to find an expression for charge in terms of currents:

$$q(t) = \int_0^t i(x) dx$$

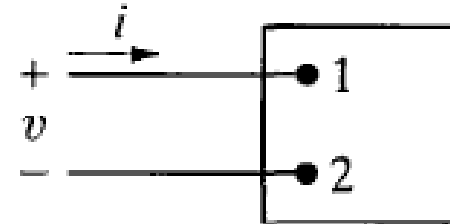
We are given the expression for current,  $i$ , which can be substituted into the above expression. To find the total charge, we let  $t \rightarrow \infty$  in the integral. Thus we have:

$$\begin{aligned} q_{\text{total}} &= \int_0^{\infty} 20e^{-5000x} dx = \left. \frac{20}{-5000} e^{-5000x} \right|_0^{\infty} = \frac{20}{-5000} (e^{-\infty} - e^0) \\ &= \frac{20}{-5000} (0 - 1) = \frac{20}{5000} = 0.004 \text{ C} = 4000 \mu\text{C} \end{aligned}$$

## Example 1.3

The expression for the charge entering the upper terminal of Fig is

$$q = \frac{1}{\alpha^2} - \left( \frac{t}{\alpha} + \frac{1}{\alpha^2} \right) e^{-\alpha t} \text{ C.}$$



Find the maximum value of the current entering the terminal if  $\alpha = 0.03679 \text{ s}^{-1}$ .

## Solution

Again, current is the time rate of change of charge, or  $i = dq/dt$ . In this problem we are given an expression for the charge, and asked to find the maximum current. First, we will find an expression for the current using the equation:

$$\begin{aligned} i &= \frac{dq}{dt} = \frac{d}{dt} \left[ \frac{1}{\alpha^2} - \left( \frac{t}{\alpha} + \frac{1}{\alpha^2} \right) e^{-\alpha t} \right] = \frac{d}{dt} \left( \frac{1}{\alpha^2} \right) - \frac{d}{dt} \left( \frac{t}{\alpha} e^{-\alpha t} \right) - \frac{d}{dt} \left( \frac{1}{\alpha^2} e^{-\alpha t} \right) \\ &= \frac{d}{dt} \left( \frac{1}{\alpha^2} \right) - \frac{d}{dt} \left( \frac{t}{\alpha} e^{-\alpha t} \right) - \frac{d}{dt} \left( \frac{1}{\alpha^2} e^{-\alpha t} \right) = \left( -\frac{1}{\alpha} + t + \frac{1}{\alpha} \right) e^{-\alpha t} \\ &= t e^{-\alpha t} \end{aligned}$$

Now that we have an expression for the current, we can find the maximum value of the current by setting the first derivative of the current to zero and solving for  $t$ :

$$\frac{di}{dt} = \frac{d}{dt} (t e^{-\alpha t}) = e^{-\alpha t} + t(-\alpha) e^{-\alpha t} = (1 - \alpha t) e^{-\alpha t} = 0$$

Since  $e^{-\alpha t}$  never equals 0 for a finite value of  $t$ , the expression equals 0 only when  $(1 - \alpha t) = 0$ . Thus,  $t = 1/\alpha$  will cause the current to be maximum. For this value of  $t$ , the current is:

$$i = \frac{1}{\alpha} e^{-\alpha/\alpha} = \frac{1}{\alpha} e^{-1} = \frac{1}{0.03679} e^{-1} \cong 10 \text{ A}$$

# Active and Passive Elements

- **Active element** is one that models a device that is **capable of generating** electric energy

*Example:* Sources

- **Passive element** is one that models a device that **cannot generate** electric energy

*Example:* Resistors, inductors and capacitors

# Power and Energy

Power and energy calculations also are important in circuit analysis.

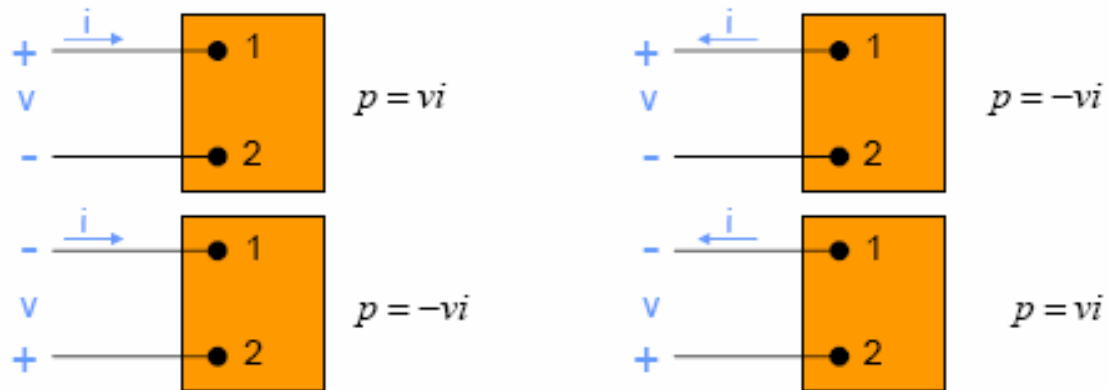
**Power** is the time rate of expanding or absorbing energy

$$p = \frac{dw}{dt} = \left( \frac{dw}{dq} \right) \left( \frac{dq}{dt} \right) = vi$$

$p$  = the power in watts,  $w$  = the energy in joules,  $t$  = the time in seconds.  $p$  = the power in watts,  $v$  = the voltage in volts,  $i$  = the current in amperes.

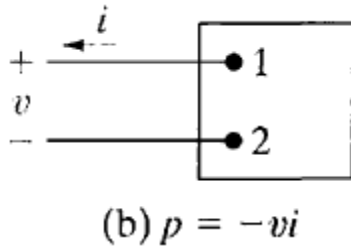
If the current reference is in the direction of a reference voltage rise across the terminals, the expression for the power is  $p = -vi$

- If the power is positive (that is, if  $p > 0$ ), power is being **delivered** to the circuit inside the box.
- If the power is negative (that is, if  $p < 0$ ), power is being **extracted** from the circuit inside the box.



If we use the passive sign convention, Eq. is correct if the reference direction for the current is in the direction of the reference voltage drop across the terminals

For example, suppose that we have selected the polarity references shown in Fig. (b). Assume further that our calculations for the current and voltage yield the following numerical results:

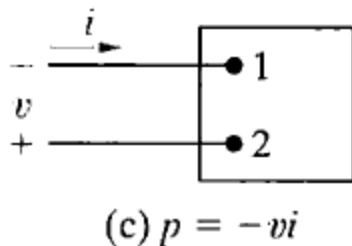


$$i = 4 \text{ A and } v = -10 \text{ V.}$$

Then the power associated with the terminal pair 1,2 is

$$p = -(-10)(4) = 40 \text{ W.}$$

Thus the circuit inside the box is absorbing 40 W.



**If** we choose the reference polarities shown in Fig. (c). The resulting numerical values are

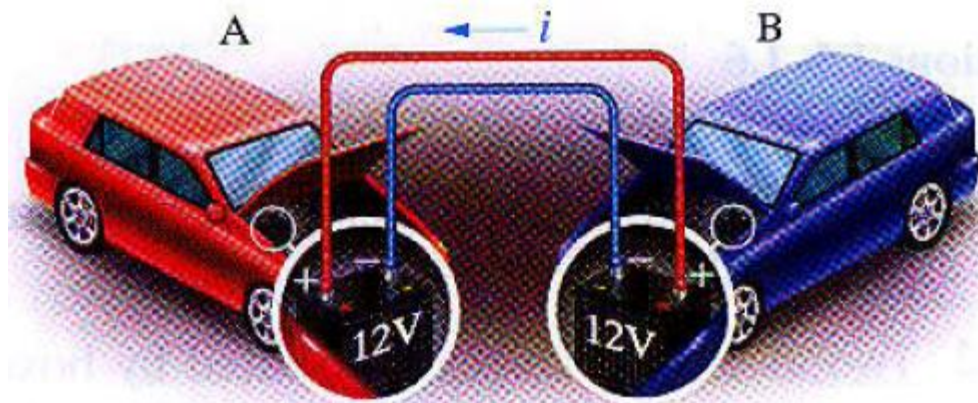
$$i = -4 \text{ A, } v = 10 \text{ V, and } p = 40 \text{ W.}$$

Note that interpreting these results in terms of this reference system gives the same conclusions that we previously obtained—namely, that the circuit inside the box is absorbing 40 W.

# Example 1.4

When a car has a dead battery, it can often be started by connecting the battery from another car across its terminals. The positive terminals are connected together as are the negative terminals. The connection is illustrated in the Figure. Assume the current  $i$  in the Figure is measured and found to be 30 A.

- a) Which car has the dead battery (power of battery)?
- b) If this connection is maintained for 1 min, how much energy is transferred to the dead battery?





## Solution

- a) In car A, the current  $i$  is in the direction of the voltage drop across the 12 V battery (the current  $i$  flows into the + terminal of the battery of car A). Thus using the passive sign convention,

$$p = vi = 30 \times 12 = 360 \text{ W}$$

since the power is positive, the battery in car A is absorbing power, so car A must have the “dead” battery.

b)  $w(t) = \int_0^t p \, dx; \quad 1 \text{ min} = 60 \text{ s}$

$$w(60) = \int_0^{60} 360 \, dx$$

$$w = 360(60 - 0) = 360(60) = 21,600 \text{ J} = 21.6 \text{ kJ}$$

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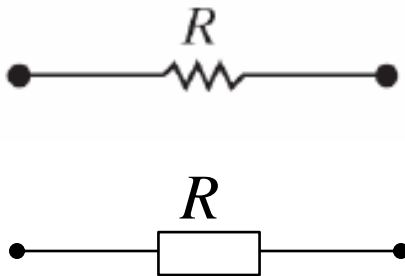
# CHAPTER 2 – Circuit Elements

**There are five ideal basic circuit elements:**

- resistors,
- voltage sources,
- current sources,
- inductors, and
- capacitors.

# Electrical Resistance

- **Resistance** is the capacity of materials to holdback the flow of **current** or, more specifically, the flow of **electric charge**.
- The circuit element used to model this behavior is the **resistor**.



# Voltage and current sources

- An **electrical source** is a device that is capable of converting **non-electric energy** to **electric energy** and vice versa.

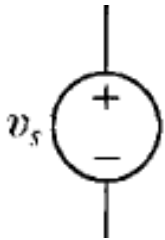
Example: battery, generator, etc.

- Ideal voltage and current sources

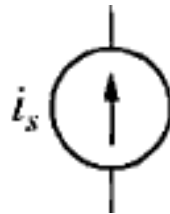
- An **ideal voltage source** is a circuit element that maintains a prescribed voltage across its terminal regardless of the current flowing in those terminals.
- An **ideal current source** is a circuit element that maintains a prescribed current across its terminal regardless of the voltage across those terminals.

# Independent and dependent sources

- An **independent source** establishes a **voltage** or **current** without relying on voltage or currents elsewhere in the circuit.
- A **dependent source** establishes a **voltage** or **current** whose value depends on the value of a voltage or current elsewhere in the circuit.



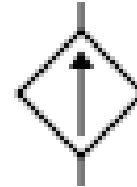
Voltage source



Current source



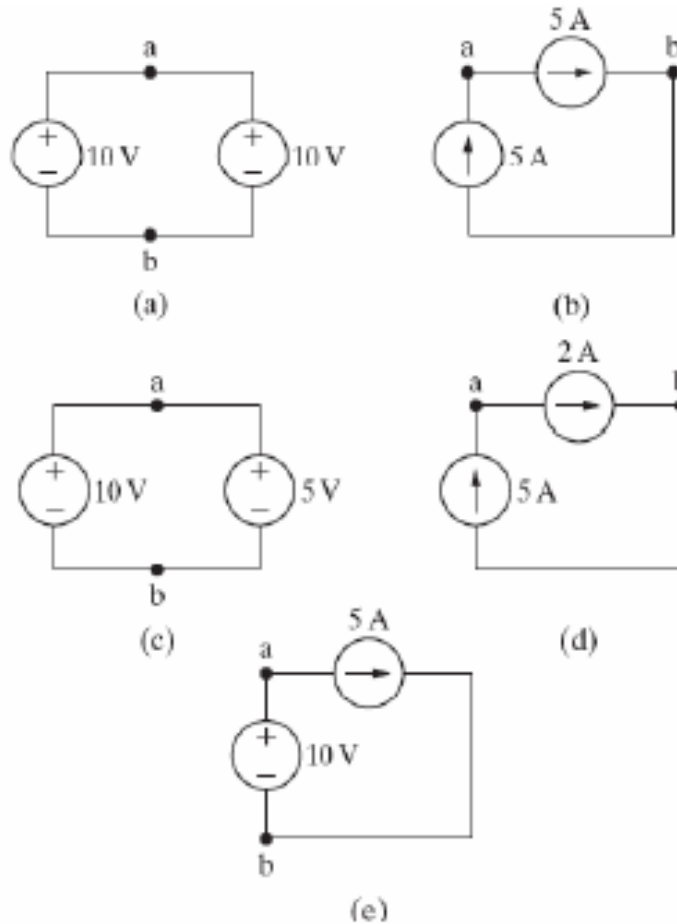
Controlled voltage  
source



Controlled current  
source

## Example 2.1

Which connection is valid and which is not?



#	Sol.
(a)	Valid
(b)	Valid
(c)	Not Valid
(d)	Not Valid
(e)	Valid

Note:

*Valid only theoretically.*

# Ohm's Law

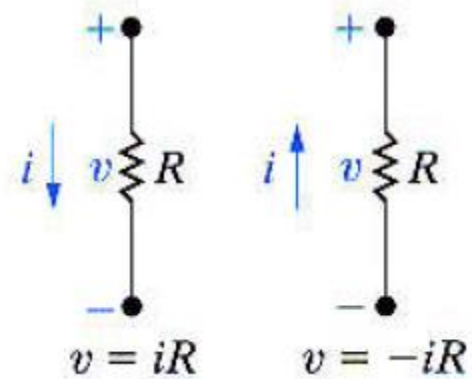
It is the algebraic relationship between voltage and current for a resistor

$$v = iR$$

$v$  = the voltage in volts

$i$  = the current in amperes

$R$  = the resistance in ohms ( $\Omega$ )



# Ohm's Law

- The **conductance** "G" is the reciprocal of the resistance.

$$i = \frac{v}{R} = Gv$$

- The **conductance** is measured in siemens (S) or mho ( $\mathfrak{U}$ )

- Examples:  $R = 8\ \Omega$    $G = 0.125\ \mathfrak{U}$

- Power:

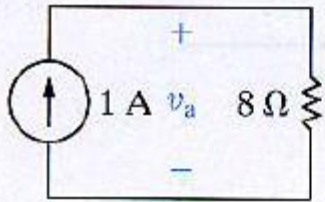
$$p = vi = (iR)i = i^2 R = \frac{i^2}{G}$$

$$p = vi = v\left(\frac{v}{R}\right) = \frac{v^2}{R} = v^2 G$$

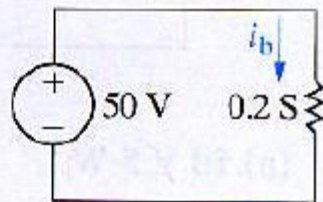


## Example 2.2

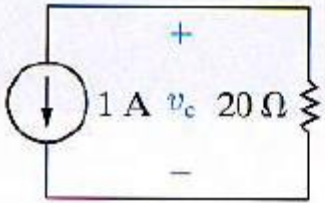
In each circuit in the Figure, either the value of  $v$  or  $i$  is not known.



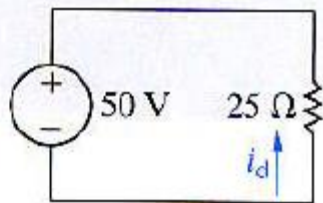
(a)



(b)



(c)

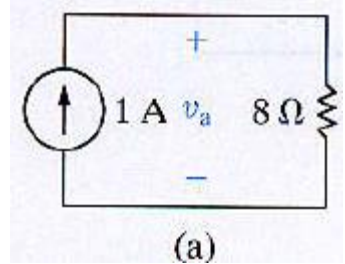


(d)

- a) Calculate the values of  $v$  and  $i$ .
- b) Determine the power dissipated in each resistor.

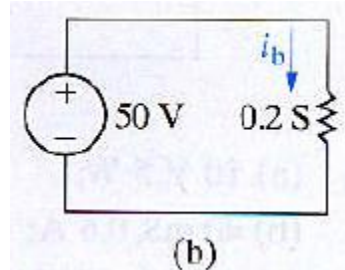
## Solution

a) The voltage  $v_a$  in Fig. (a) is a drop in the direction of the current in the resistor. Therefore,  $V_a = (1)(8) = 8$  V.

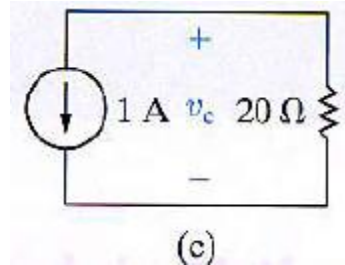


The current  $i_b$  in the resistor with a conductance of  $0.2$  S in Fig. (b) is in the direction of the voltage drop across the resistor. Thus

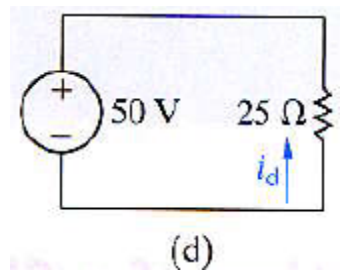
$$i_b = (50)(0.2) = 10\text{ A}.$$



The voltage  $v_c$  in Fig. (c) is a rise in the direction of the current in the resistor. Hence  $v_c = -(1)(20) = -20$  V.



The current  $i_d$  in the  $25\ \Omega$  resistor in Fig. (d) is in the direction of the voltage rise across the resistor. So  $i_d = -50/25 = -2$  A



b) The power dissipated in each of the four resistors is

$$p_{8\Omega} = \frac{(8)^2}{8} = (1)^2(8) = 8 \text{ W},$$

$$p_{0.2S} = (50)^2(0.2) = 500 \text{ W},$$

$$p_{20\Omega} = \frac{(-20)^2}{20} = (1)^2(20) = 20 \text{ W},$$

$$p_{25\Omega} = \frac{(50)^2}{25} = (-2)^2(25) = 100 \text{ W}.$$

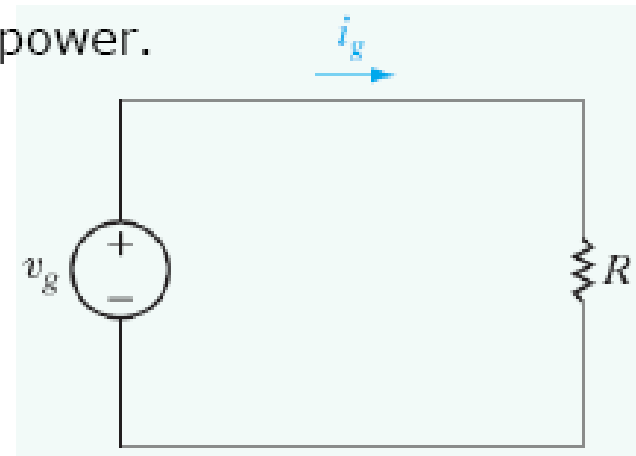
## Example 2.3

a) If  $v_g = 1$  kV and  $i_g = 5$  mA, find  $R$  & power.

ans.:

$$R = \frac{v_g}{i_g} = \frac{1 \times 10^3}{5 \times 10^{-3}} = \underline{200 \text{ k}\Omega}$$

$$p = v_g i_g = (1 \times 10^3) \times (5 \times 10^{-3}) = \underline{5 \text{ W}}$$



b) If  $R = 300 \Omega$  and the power absorbed by  $R$  is 480 mW. Find  $i_g$  and  $v_g$ .

ans.:

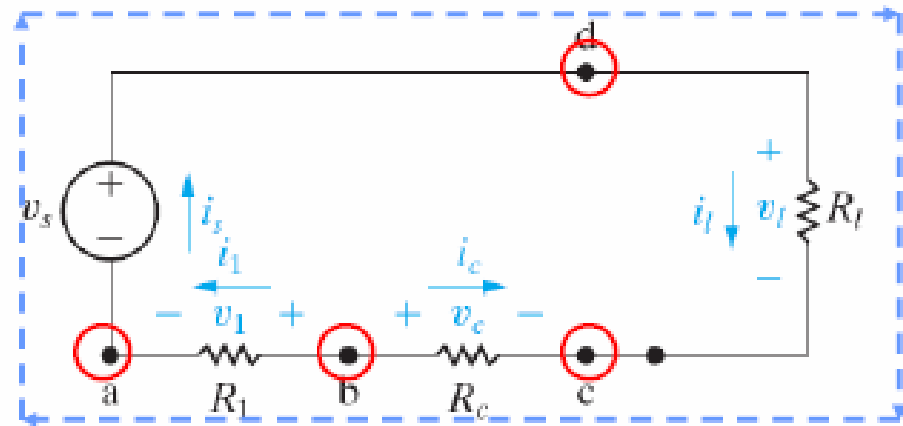
$$R = \frac{v_g}{i_g} = 300 \Omega \quad \& \quad p = v_g i_g = 480 \text{ mW}$$

$$\frac{v_g}{i_g} \times i_g v_g = v_g^2 = 300 \times 480 \times 10^{-3} = 144$$

$$v_g = \sqrt{144} = \underline{12 \text{ V}} \quad \longrightarrow \quad i_g = \frac{12}{300} = \underline{0.04 \text{ A}}$$

# Kirchhoff's Law

- A **node**
  - Point where **two** or **more** circuit elements meet.
  - Nodes **a**, **b**, **c** or **d**.



- A **closed path or loop**
  - Starting at an arbitrarily selected node, we trace a closed path in a circuit through selected basic circuit elements and return to the original node **without passing through any intermediate node more than once**.
  - Loop  $a \rightarrow d \rightarrow c \rightarrow b \rightarrow a$

# Kirchhoff's Law



- Kirchhoff's Current Law (KCL)

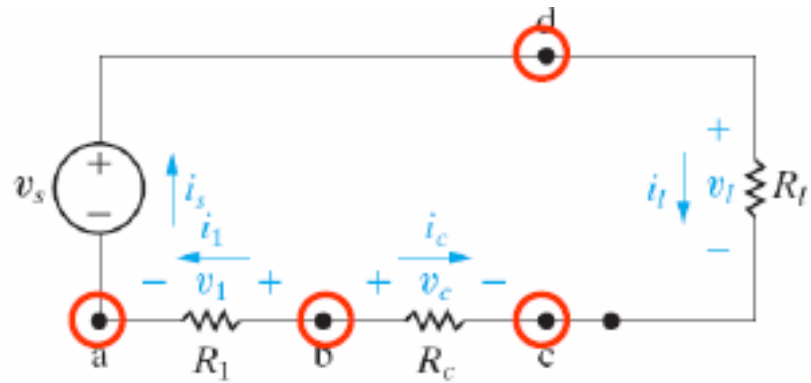
- The algebraic sum of all the **currents** at any node in the circuit equals zero
- The sum of the currents entering any node must equal the sum of the currents leaving that node

- Kirchhoff's Voltage Law (KVL)

- The algebraic sum of all the **voltages** around any close path equals a zero
- The sum of the potential difference across all elements around any closed loop must be zero

# Kirchhoff's Current Law

- Assign an **algebraic sign** corresponding to a reference direction.
  - Positive sign** to a current leaving.
  - Negative sign** to current entering the node.



At node **a**  $\longrightarrow i_s - i_1 = 0$

At node **b**  $\longrightarrow i_1 + i_c = 0$

At node **c**  $\longrightarrow -i_c - i_l = 0$

At node **d**  $\longrightarrow i_l - i_s = 0$

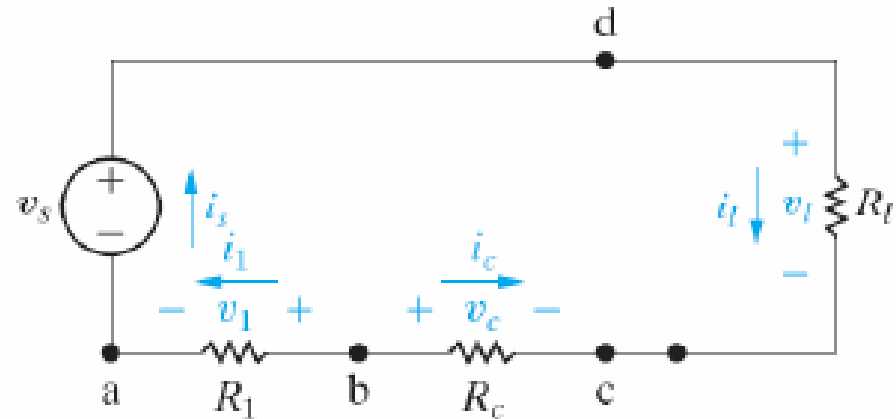
$$i_1 = -i_c = i_s = i_l$$

## Note:

In any circuit with  $n$  nodes,  $n - 1$  independent current equations can be derived from Kirchhoff's current law.

# Kirchhoff's Voltage Law

- Assign an **algebraic sign** (reference direction) to each voltage in the **loop**.
  - Positive** sign to a voltage rise requires assigning a **negative** sign to a voltage drop.



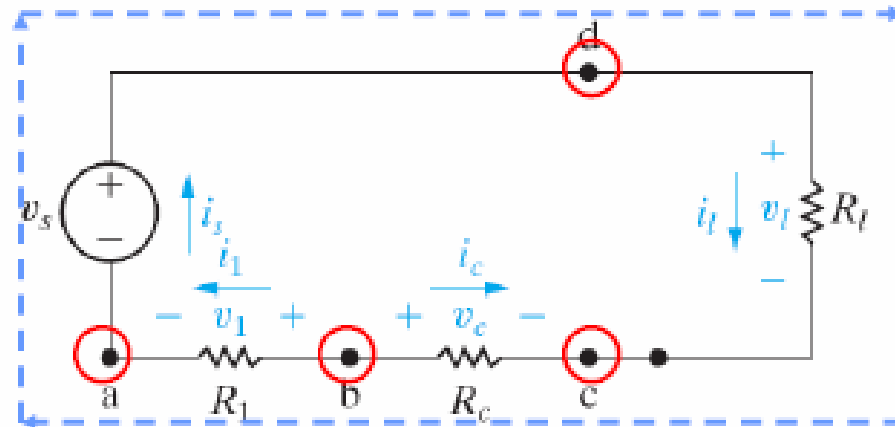
$$-v_s + v_l - v_c + v_1 = 0 \quad (a \rightarrow d \rightarrow c \rightarrow b \rightarrow a)$$

- Finally, applying **Ohm's law**

$$v_1 = i_1 R_1 \quad v_c = i_c R_c \quad v_l = i_l R_l$$



# Kirchhoff's Law



$$i_s - i_1 = 0$$

$$i_1 + i_c = 0$$

$$i_l - i_s = 0$$

KCL

$$-v_s + v_l - v_c + v_1 = 0$$

KVL

$$v_1 = i_1 R_1$$

$$v_c = i_c R_c$$

$$v_l = i_l R_l$$

Ohm's Law

$$-v_s + i_1 R_l + i_1 R_c + i_1 R_1 = 0$$

## Example 2.4

Sum the currents at each node in the circuit shown in Figure. Note that there is no connection dot (•) in the center of the diagram, where the  $4\ \Omega$  branch crosses the branch containing the ideal current source  $i_a$ .

- Apply KCL to the circuit?

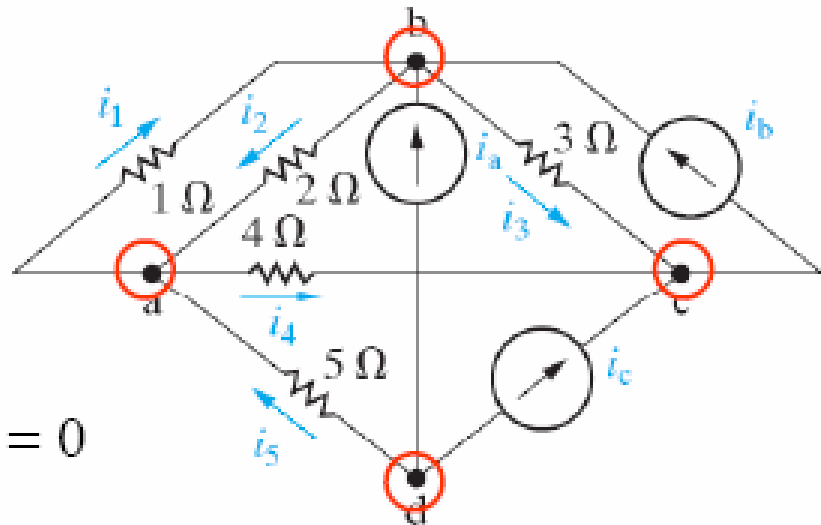
In writing the equations, we use a positive sign for a current leaving a node. The four equations are

At node **a**       $i_1 + i_4 - i_2 - i_5 = 0$

At node **b**       $i_2 + i_3 - i_1 - i_b - i_a = 0$

At node **c**       $i_b - i_3 - i_c - i_4 = 0$

At node **d**       $i_5 + i_c + i_a = 0$



## Example 2.5

Sum the voltages around each designated path in the circuit shown in Figure

- Apply KVL to the circuit?

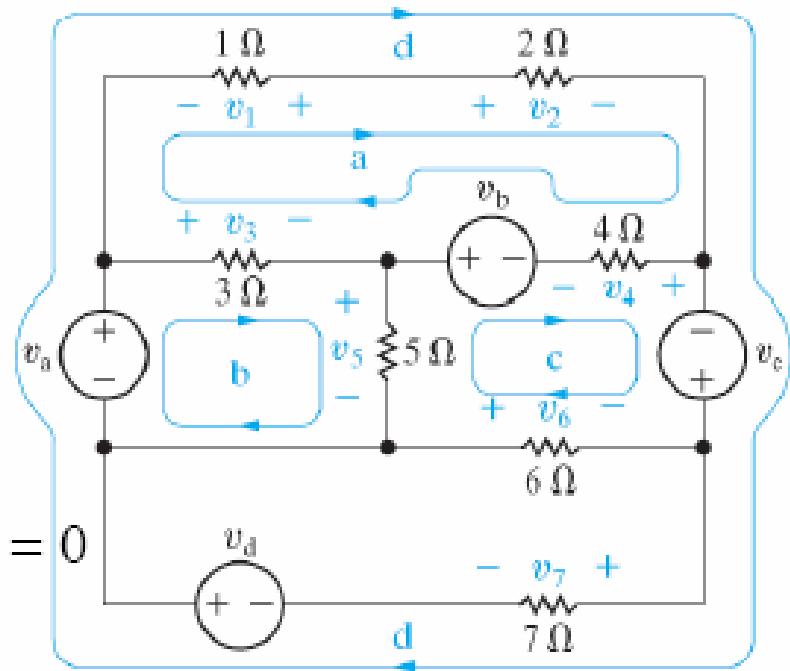
In writing the equations, we use a positive sign for a voltage drop.  
The four equations are

$$\text{Loop a} \quad -v_3 - v_1 + v_2 + v_4 - v_b = 0$$

$$\text{Loop b} \quad -v_a + v_3 + v_5 = 0$$

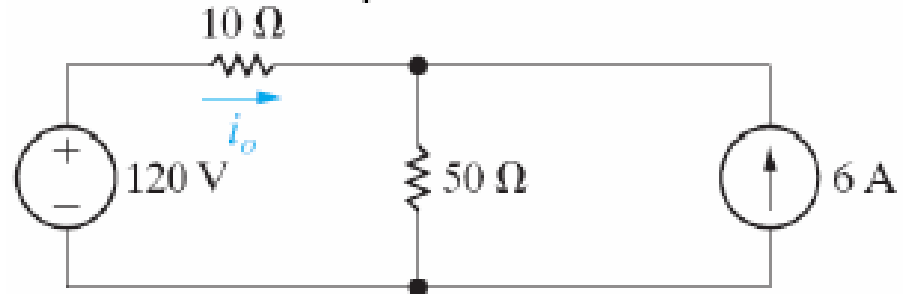
$$\text{Loop c} \quad v_b - v_4 - v_c - v_6 - v_5 = 0$$

$$\text{Loop d} \quad -v_a - v_1 + v_2 - v_c + v_7 - v_d = 0$$



## Example 2.6

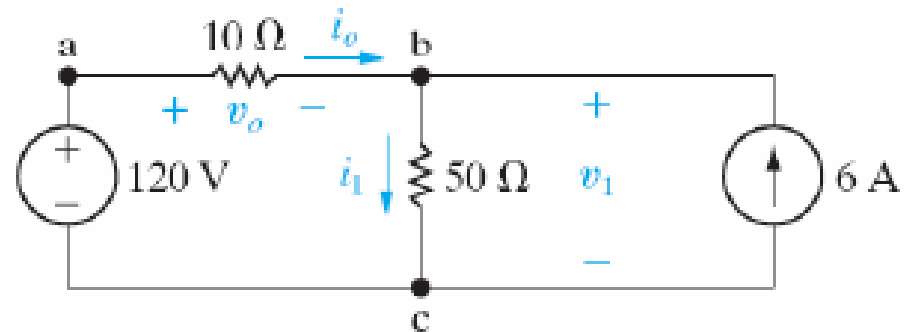
- Use *KCL*, *KVL* & *Ohm's* law to find  $i_o$ , verify that the total power generated equals the total dissipated.



Ans.:

- Redraw the circuit and assign unknown currents.
- Label the nodes.

- "2" nodes  $\rightarrow$  "1" KCL



At node "b"  $i_1 - 6 - i_o = 0$

## Example 2.6 (cont.)

- Apply KCL

$$v_o + v_1 - 120 = 0$$

- Ohm's Law

$$v_o = i_o 10$$

$$v_1 = i_1 50$$

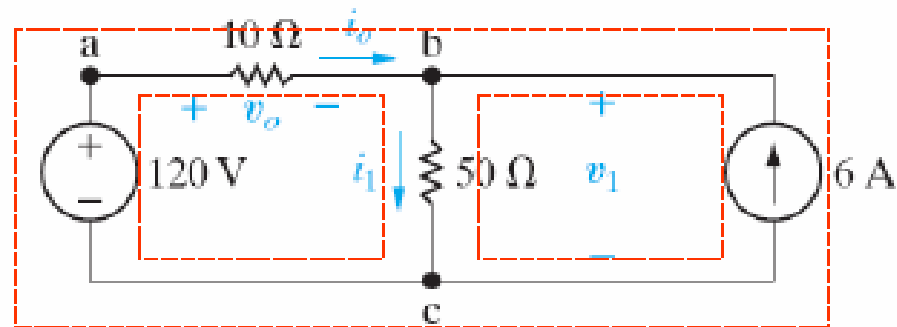
$$10i_o + 50i_1 - 120 = 0$$

$$i_1 - 6 - i_o = 0$$

&

$$10i_o + 50i_1 - 120 = 0$$

- Solving  $\rightarrow i_o = \underline{-3 \text{ A}} \quad i_1 = \underline{3 \text{ A}}$



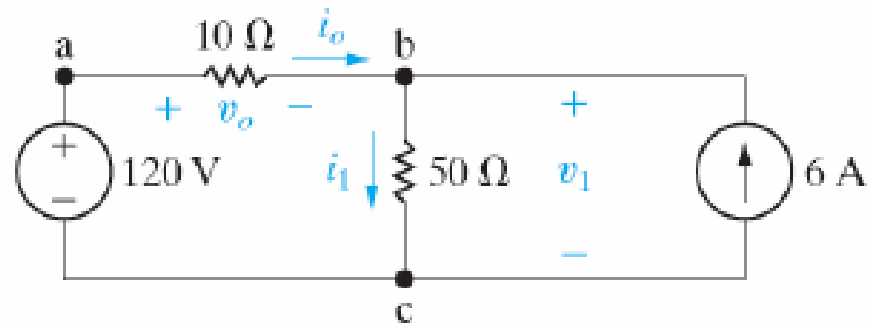
## Example 2.6 (cont.)

- Power dissipated in the resistors

$$p_R = i^2 R$$

$$p_{50\Omega} = (3)^2 (50) = \underline{450 \text{ W}}$$

$$p_{10\Omega} = (3)^2 (10) = \underline{90 \text{ W}}$$



- Power delivered to the 120 V supply

$$p_{120V} = iv = 3 \times 120 = \underline{360 \text{ W}}$$

- Power delivered by the 6 A supply

$$p_{6A} = iv = 6 \times 150 = \underline{900 \text{ W}}$$

$$p_{50\Omega} + p_{10\Omega} + p_{120V} = p_{6A} = 900 \text{ W}$$

# Circuit containing dependent sources

Find  $v_o$  &  $i_o$ ?

Ans.:

- KCL at b

$$i_o - i_{\Delta} - 5i_{\Delta} = 0$$

$$i_o = 6i_{\Delta}$$

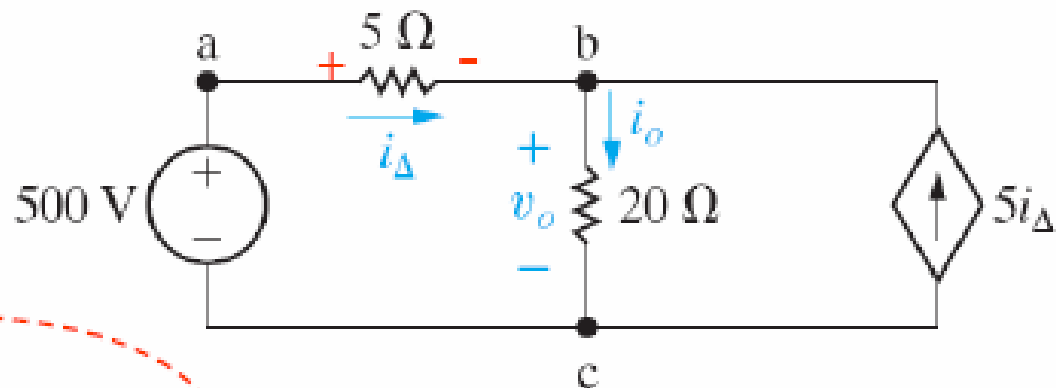
- KVL at (a  $\rightarrow$  b  $\rightarrow$  c)

$$i_{\Delta} 5 + i_o 20 - 500 = 0$$

$$i_{\Delta} 5 + (i_{\Delta} 6) \times 20 = 500 \quad \longrightarrow \quad i_{\Delta} = \underline{4 \text{ A}}$$

$$i_o = \underline{24 \text{ A}}$$

$$v_o = 20i_o = \underline{480 \text{ V}}$$



## Example 2.7

Use KL & Ohms law to find  $v_o$ , and show that total power developed equals the total power dissipated.

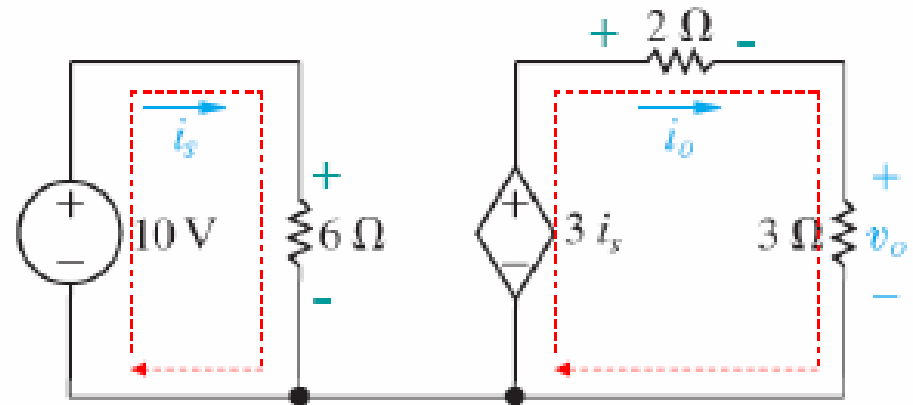
Ans.:

- Two closed paths  
(KVL & Ohm's Law)

$$i_s 6 - 10 = 0 \quad \rightarrow \quad i_s = \underline{5/3 \text{ A}}$$

$$i_o 2 + i_o 3 - i_s 3 = 0 \quad \rightarrow \quad i_o = \underline{1 \text{ A}}$$

$$v_o = i_o 3 = \underline{3 \text{ V}}$$



- Power **Dissipated**

$$P = P_{6\Omega} + P_{2\Omega} + P_{3\Omega} = i_s^2 6 + i_o^2 2 + i_o^2 3$$

$$P = 16.7 + 2 + 3 = 21.7 \text{ W}$$

- Power **Developed**

$$P = P_{10\text{V}} + P_{3i_s} = i_s 10 + i_o \times 3i_s$$

$$P = 16.7 + 5 = 21.7 \text{ W}$$



## Example 2.8

Find  $i_1$ ,  $v$ , and power generated and absorbed?

Ans.:

**KCL**  $i_{6k\Omega} - i_1 - 30i_1 = 0$

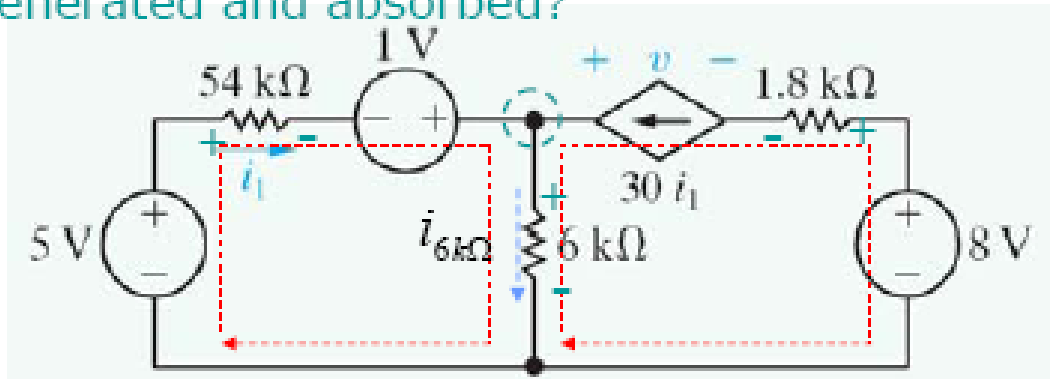
$$i_{6k\Omega} = \underline{31i_1}$$

**KVL**  $i_1(54 \times 10^3) - 1 + i_{6k\Omega}(6 \times 10^3) - 5 = 0 \quad \text{---} \rightarrow \quad i_1 = \underline{2.5 \times 10^{-5} \text{ A}}$

**KVL**  $-31i_1(6 \times 10^3) + v - 30i_1(1.8 \times 10^3) + 8 = 0 \quad \text{---} \rightarrow \quad \underline{v = -2 \text{ V}}$

Power **Dissipated**  $P = P_{54k\Omega} + P_{6k\Omega} + P_{1.8k\Omega} + P_{31i_1} = 0.00615 \text{ W} = \underline{6150 \mu\text{W}}$

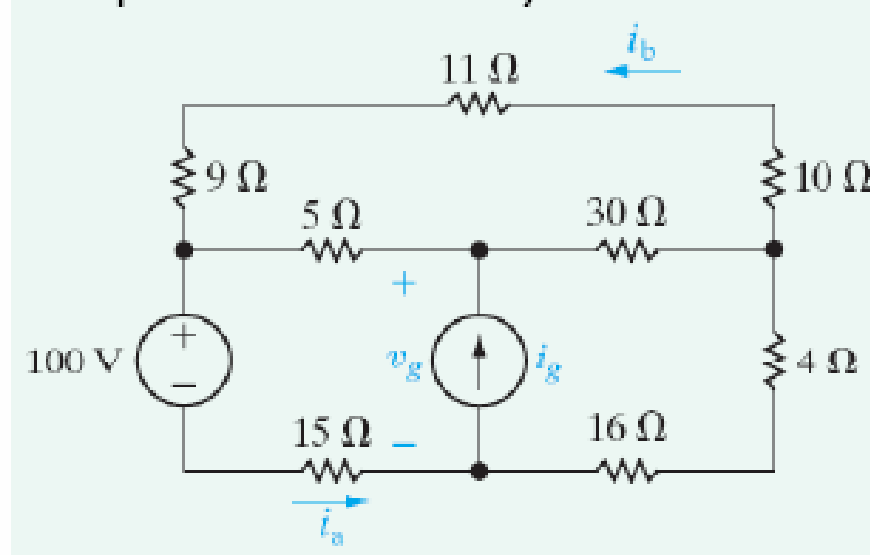
Power **Delivered**  $P = P_{5V} + P_{1V} + P_{8V} = 0.00615 \text{ W} = \underline{6150 \mu\text{W}}$



## Example 2.9

If  $i_a = 4$  A and  $i_b = -2$  A, respectively: Find

- Find  $i_g$ .
- Find the power dissipated in each resistor.
- Find  $v_g$ .
- Show that the power delivered by the current source is equal to the power absorbed by all other elements.



## Solution

Ohm's law

$$v_{15\Omega} = i_a \times 15 = 4 \times 15 = 60V$$

$$v_{9\Omega} = 2 \times 9 = 18V$$

$$v_{11\Omega} = 2 \times 11 = 22V$$

$$v_{10\Omega} = 2 \times 10 = 20V$$

KVL on outside loop

$$-60 - 100 + 18 + 22 + 20 + i_c 4 + i_c 16 = 0$$

$$i_c = 5A$$

KCL at a

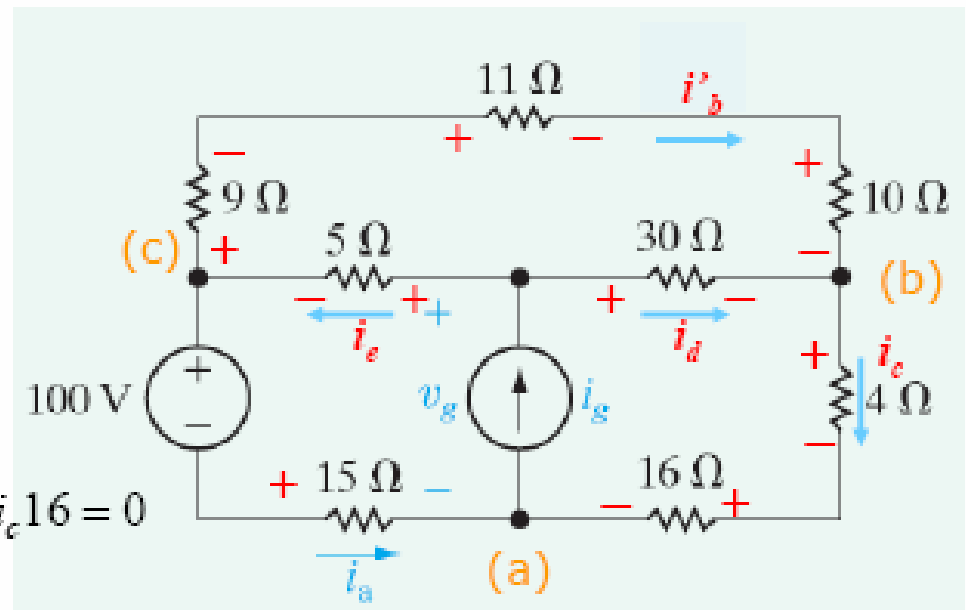
$$i_g = i_c + i_a = 5 + 4 = 9A$$

KCL at b

$$i_d = i_c - i'_b = 5 - 2 = 3A$$

KCL at c

$$i_e = i'_b + i_a = 2 + 4 = 6A$$



KVL on the bottom left loop

$$-60 - 100 - 5 \times 6 + v_g = 0$$

$$v_g = 190V$$

Calculate power using the formula  $p = R^2$ :

$$p_{9\Omega} = (9)(2)^2 = 36 \text{ W}; \quad p_{11\Omega} = (11)(2)^2 = 44 \text{ W}$$

$$p_{10\Omega} = (10)(2)^2 = 40 \text{ W}; \quad p_{30\Omega} = (30)(3)^2 = 270 \text{ W}$$

$$p_{5\Omega} = (5)(6)^2 = 180 \text{ W}; \quad p_{4\Omega} = (4)(5)^2 = 100 \text{ W}$$

$$p_{16\Omega} = (16)(5)^2 = 400 \text{ W}; \quad p_{15\Omega} = (15)(4)^2 = 240 \text{ W}$$

Sum the power dissipated by the resistors:

$$\sum p_{\text{diss}} = 36 + 44 + 40 + 270 + 180 + 100 + 400 + 240 = 1310 \text{ W}$$

The power associated with the sources is

$$p_{\text{voltage-source}} = (100 \text{ V})(4 \text{ A}) = 400 \text{ W}$$

$$p_{\text{current-source}} = -v_g i_g = -(190 \text{ V})(9 \text{ A}) = -1710 \text{ W}$$

Thus the total power dissipated is  $1310 + 400 = 1710 \text{ W}$  and the total power developed is  $1710 \text{ W}$ , so the power balances.