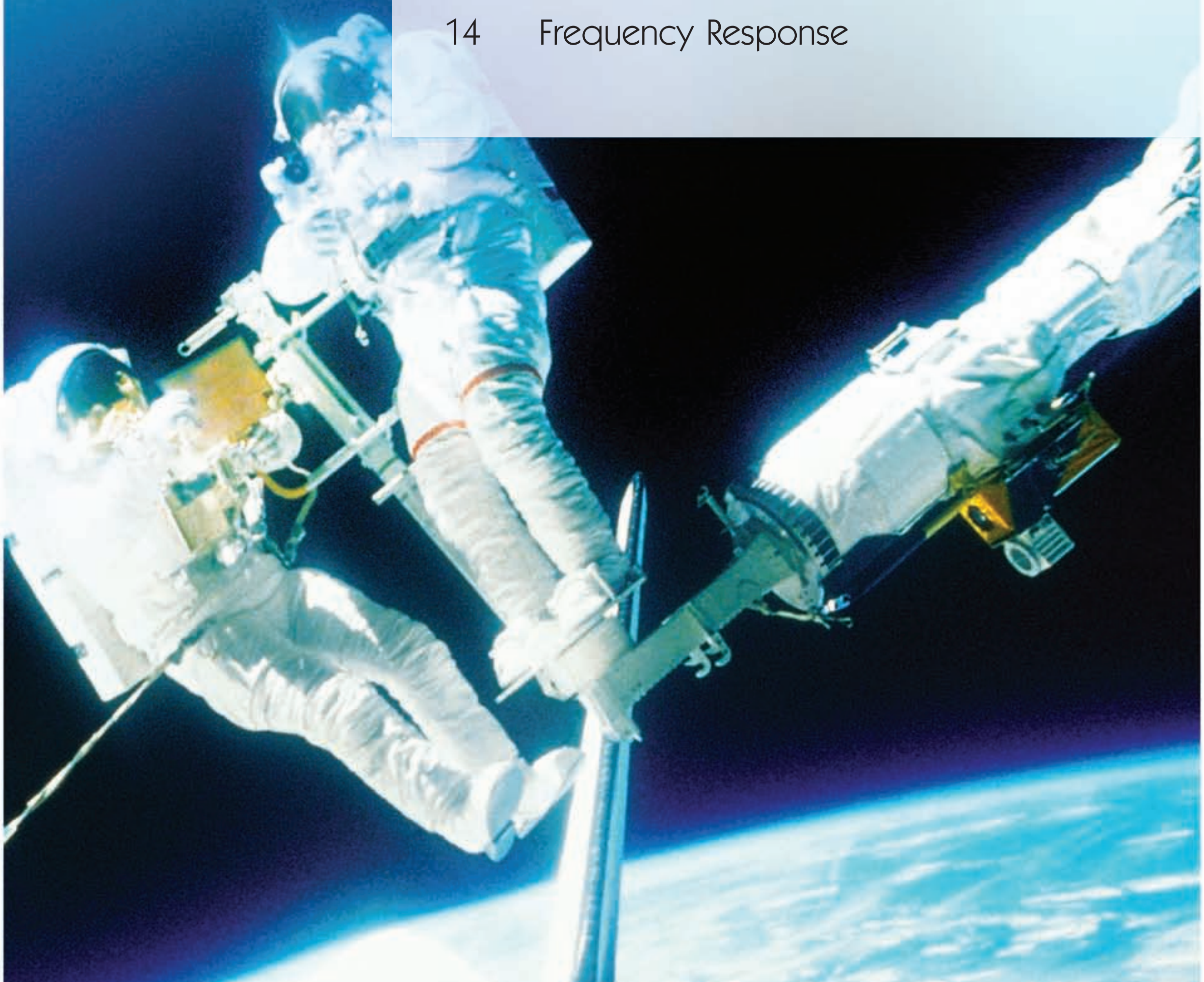
PART TWO

AC Circuits

OUTLINE



chapter

9

Sinusoids and Phasors

He who knows not, and knows not that he knows not, is a fool—shun him. He who knows not, and knows that he knows not, is a child—teach him. He who knows, and knows not that he knows, is asleep—wake him up. He who knows, and knows that he knows, is wise—follow him.

—Persian Proverb

Enhancing Your Skills and Your Career

ABET EC 2000 criteria (3.d), ***"an ability to function on multi-disciplinary teams."***

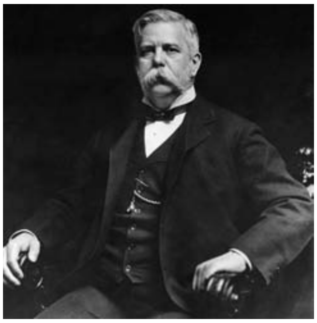
The "ability to function on multidisciplinary teams" is inherently critical for the working engineer. Engineers rarely, if ever, work by themselves. Engineers will always be part of some team. One of the things I like to remind students is that you do not have to like everyone on a team; you just have to be a successful part of that team.

Most frequently, these teams include individuals from of a variety of engineering disciplines, as well as individuals from nonengineering disciplines such as marketing and finance.

Students can easily develop and enhance this skill by working in study groups in every course they take. Clearly, working in study groups in nonengineering courses as well as engineering courses outside your discipline will also give you experience with multidisciplinary teams.

Photo by Charles Alexander

Historical



George Westinghouse. Photo

Nikola Tesla (1856–1943) and **George Westinghouse** (1846–1914) helped establish alternating current as the primary mode of electricity transmission and distribution.

Today it is obvious that ac generation is well established as the form of electric power that makes widespread distribution of electric power efficient and economical. However, at the end of the 19th century, which was the better—ac or dc—was hotly debated and had extremely outspoken supporters on both sides. The dc side was lead by Thomas Edison, who had earned a lot of respect for his many contributions. Power generation using ac really began to build after the successful contributions of Tesla. The real commercial success in ac came from George Westinghouse and the outstanding team, including Tesla, he assembled. In addition, two other big names were C. F. Scott and B. G. Lamme.

The most significant contribution to the early success of ac was the patenting of the polyphase ac motor by Tesla in 1888. The induction motor and polyphase generation and distribution systems doomed the use of dc as the prime energy source.

9.1 Introduction

Thus far our analysis has been limited for the most part to dc circuits: those circuits excited by constant or time-invariant sources. We have restricted the forcing function to dc sources for the sake of simplicity, for pedagogic reasons, and also for historic reasons. Historically, dc sources were the main means of providing electric power up until the late 1800s. At the end of that century, the battle of direct current versus alternating current began. Both had their advocates among the electrical engineers of the time. Because ac is more efficient and economical to transmit over long distances, ac systems ended up the winner. Thus, it is in keeping with the historical sequence of events that we considered dc sources first.

We now begin the analysis of circuits in which the source voltage or current is time-varying. In this chapter, we are particularly interested in sinusoidally time-varying excitation, or simply, excitation by a *sinusoid*.

> A **sinusoid** is a signal that has the form of the sine or cosine function.

A sinusoidal current is usually referred to as *alternating current* (*ac*). Such a current reverses at regular time intervals and has alternately positive and negative values. Circuits driven by sinusoidal current or voltage sources are called *ac circuits.*

We are interested in sinusoids for a number of reasons. First, nature itself is characteristically sinusoidal. We experience sinusoidal variation in the motion of a pendulum, the vibration of a string, the ripples on the ocean surface, and the natural response of underdamped second-order systems, to mention but a few. Second, a sinusoidal signal is easy to generate and transmit. It is the form of voltage generated throughout the world and supplied to homes, factories, laboratories, and so on. It is the dominant form of signal in the communications and electric power industries. Third, through Fourier analysis, any practical periodic signal can be represented by a sum of sinusoids. Sinusoids, therefore, play an important role in the analysis of periodic signals. Lastly, a sinusoid is easy to handle mathematically. The derivative and integral of a sinusoid are themselves sinusoids. For these and other reasons, the sinusoid is an extremely important function in circuit analysis.

A sinusoidal forcing function produces both a transient response and a steady-state response, much like the step function, which we studied in Chapters 7 and 8. The transient response dies out with time so that only the steady-state response remains. When the transient response has become negligibly small compared with the steady-state response, we say that the circuit is operating at sinusoidal steady state. It is this *sinusoidal steady-state response* that is of main interest to us in this chapter.

We begin with a basic discussion of sinusoids and phasors. We then introduce the concepts of impedance and admittance. The basic circuit laws, Kirchhoff's and Ohm's, introduced for dc circuits, will be applied to ac circuits. Finally, we consider applications of ac circuits in phase-shifters and bridges.

9.2 Sinusoids

Consider the sinusoidal voltage

$$v(t) = V_m \sin \omega t \tag{9.1}$$

where

V_m = the *amplitude* of the sinusoid
ω = the *angular frequency* in radians/s
ωt = the *argument* of the sinusoid

The sinusoid is shown in Fig. 9.1(a) as a function of its argument and in Fig. 9.1(b) as a function of time. It is evident that the sinusoid repeats itself every T seconds; thus, T is called the *period* of the sinusoid. From the two plots in Fig. 9.1, we observe that $\omega T = 2\pi$,

$$\boxed{T = \frac{2\pi}{\omega}} \tag{9.2}$$

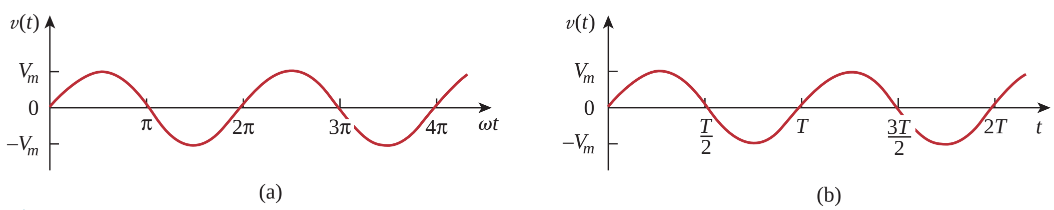


Figure 9.1
A sketch of $V_m \sin \omega t$: (a) as a function of ωt, (b) as a function of t.

Historical

The Burndy Library Collection at The Huntington Library, San Marino, California.

Heinrich Rudorf Hertz (1857–1894), a German experimental physicist, demonstrated that electromagnetic waves obey the same fundamental laws as light. His work confirmed James Clerk Maxwell's celebrated 1864 theory and prediction that such waves existed.

Hertz was born into a prosperous family in Hamburg, Germany. He attended the University of Berlin and did his doctorate under the prominent physicist Hermann von Helmholtz. He became a professor at Karlsruhe, where he began his quest for electromagnetic waves. Hertz successfully generated and detected electromagnetic waves; he was the first to show that light is electromagnetic energy. In 1887, Hertz noted for the first time the photoelectric effect of electrons in a molecular structure. Although Hertz only lived to the age of 37, his discovery of electromagnetic waves paved the way for the practical use of such waves in radio, television, and other communication systems. The unit of frequency, the hertz, bears his name.

The fact that $v(t)$ repeats itself every T seconds is shown by replacing t by $t + T$ in Eq. (9.1). We get

$$\begin{aligned} v(t+T) &= V_m \sin\omega(t+T) = V_m \sin\omega\left(t + \frac{2\pi}{\omega}\right) \\ &= V_m \sin(\omega t + 2\pi) = V_m \sin\omega t = v(t) \end{aligned} \tag{9.3}$$

Hence,

$$\boxed{v(t+T) = v(t)} \tag{9.4}$$

that is, v has the same value at $t + T$ as it does at t and $v(t)$ is said to be *periodic*. In general,

A **periodic function** is one that satisfies $f(t) = f(t + nT)$, for all t and for all integers n.

As mentioned, the *period* T of the periodic function is the time of one complete cycle or the number of seconds per cycle. The reciprocal of this quantity is the number of cycles per second, known as the *cyclic frequency* f of the sinusoid. Thus,

$$\boxed{f = \frac{1}{T}} \tag{9.5}$$

From Eqs. (9.2) and (9.5), it is clear that

$$\omega = 2\pi f \tag{9.6}$$

While ω is in radians per second (rad/s), f is in hertz (Hz).

The unit of f is named after the German physicist Heinrich R. Hertz (1857–1894).

Let us now consider a more general expression for the sinusoid,

$$v(t) = V_m \sin(\omega t + \phi) \tag{9.7}$$

where $(\omega t + \phi)$ is the argument and ϕ is the *phase*. Both argument and phase can be in radians or degrees.

Let us examine the two sinusoids

$$v_1(t) = V_m \sin \omega t \qquad \text{and} \qquad v_2(t) = V_m \sin(\omega t + \phi) \tag{9.8}$$

shown in Fig. 9.2. The starting point of v_2 in Fig. 9.2 occurs first in time. Therefore, we say that v_2 *leads* v_1 by ϕ or that v_1 *lags* v_2 by ϕ. If $\phi \neq 0$, we also say that v_1 and v_2 are *out of phase*. If $\phi = 0$, then v_1 and v_2 are said to be *in phase*; they reach their minima and maxima at exactly the same time. We can compare v_1 and v_2 in this manner because they operate at the same frequency; they do not need to have the same amplitude.

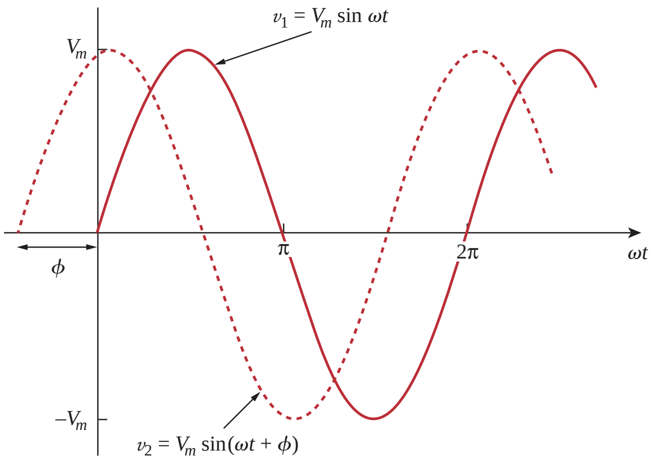


Figure 9.2
Two sinusoids with different phases.

A sinusoid can be expressed in either sine or cosine form. When comparing two sinusoids, it is expedient to express both as either sine or cosine with positive amplitudes. This is achieved by using the following trigonometric identities:

$$\begin{aligned} \sin(A \pm B) &= \sin A \cos B \pm \cos A \sin B \\ \cos(A \pm B) &= \cos A \cos B \mp \sin A \sin B \end{aligned} \tag{9.9}$$

With these identities, it is easy to show that

$$\begin{aligned} \sin(\omega t \pm 180^\circ) &= -\sin \omega t \\ \cos(\omega t \pm 180^\circ) &= -\cos \omega t \\ \sin(\omega t \pm 90^\circ) &= \pm \cos \omega t \\ \cos(\omega t \pm 90^\circ) &= \mp \sin \omega t \end{aligned} \tag{9.10}$$

Using these relationships, we can transform a sinusoid from sine form to cosine form or vice versa.

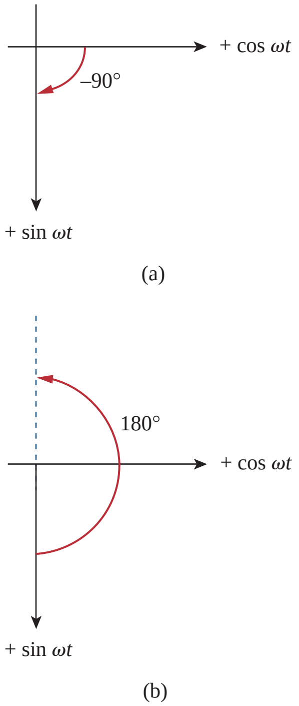


Figure 9.3
A graphical means of relating cosine and sine: (a) $\cos(\omega t - 90^\circ) = \sin \omega t$, (b) $\sin(\omega t + 180^\circ) = -\sin \omega t$.

A graphical approach may be used to relate or compare sinusoids as an alternative to using the trigonometric identities in Eqs. (9.9) and (9.10). Consider the set of axes shown in Fig. 9.3(a). The horizontal axis represents the magnitude of cosine, while the vertical axis (pointing down) denotes the magnitude of sine. Angles are measured positively counterclockwise from the horizontal, as usual in polar coordinates. This graphical technique can be used to relate two sinusoids. For example, we see in Fig. 9.3(a) that subtracting 90° from the argument of $\cos \omega t$ gives $\sin \omega t$, or $\cos(\omega t - 90^\circ) = \sin \omega t$. Similarly, adding 180° to the argument of $\sin \omega t$ gives $-\sin \omega t$, or $\sin(\omega t + 180^\circ) = -\sin \omega t$, as shown in Fig. 9.3(b).

The graphical technique can also be used to add two sinusoids of the same frequency when one is in sine form and the other is in cosine form. To add $A \cos \omega t$ and $B \sin \omega t$, we note that A is the magnitude of $\cos \omega t$ while B is the magnitude of $\sin \omega t$, as shown in Fig. 9.4(a). The magnitude and argument of the resultant sinusoid in cosine form is readily obtained from the triangle. Thus,

$$A \cos \omega t + B \sin \omega t = C \cos(\omega t - \theta) \tag{9.11}$$

where

$$C = \sqrt{A^2 + B^2}, \qquad \theta = \tan^{-1} \frac{B}{A} \tag{9.12}$$

For example, we may add $3 \cos \omega t$ and $-4 \sin \omega t$ as shown in Fig. 9.4(b) and obtain

$$3 \cos \omega t - 4 \sin \omega t = 5 \cos(\omega t + 53.1^\circ) \tag{9.13}$$

Compared with the trigonometric identities in Eqs. (9.9) and (9.10), the graphical approach eliminates memorization. However, we must not confuse the sine and cosine axes with the axes for complex numbers to be discussed in the next section. Something else to note in Figs. 9.3 and 9.4 is that although the natural tendency is to have the vertical axis point up, the positive direction of the sine function is down in the present case.

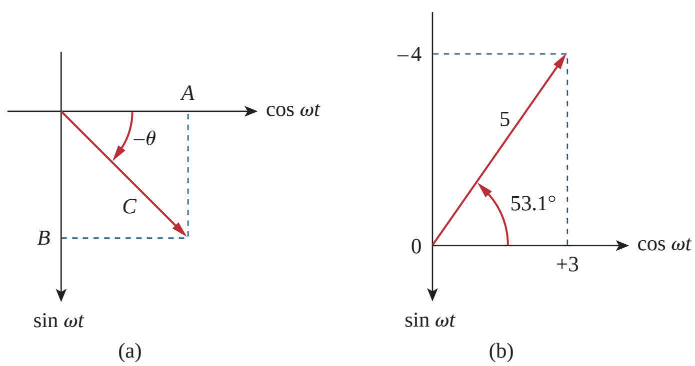


Figure 9.4
(a) Adding $A \cos \omega t$ and $B \sin \omega t$, (b) adding $3 \cos \omega t$ and $-4 \sin \omega t$.

Example 9.1

Find the amplitude, phase, period, and frequency of the sinusoid

$$v(t) = 12\cos(50t + 10^\circ)$$

Solution:

The amplitude is $V_m = 12$ V.
The phase is $\phi = 10^\circ$.
The angular frequency is $\omega = 50$ rad/s.
The period $T = \dfrac{2\pi}{\omega} = \dfrac{2\pi}{50} = 0.1257$ s.
The frequency is $f = \dfrac{1}{T} = 7.958$ Hz.

Practice Problem 9.1

Given the sinusoid $5\sin(4\pi t - 60^\circ)$, calculate its amplitude, phase, angular frequency, period, and frequency.

Answer: 5, -60°, 12.57 rad/s, 0.5 s, 2 Hz.

Example 9.2

Calculate the phase angle between $v_1 = -10\cos(\omega t + 50^\circ)$ and $v_2 = 12\sin(\omega t - 10^\circ)$. State which sinusoid is leading.

Solution:
Let us calculate the phase in three ways. The first two methods use trigonometric identities, while the third method uses the graphical approach.

METHOD 1 In order to compare v_1 and v_2, we must express them in the same form. If we express them in cosine form with positive amplitudes,

$$v_1 = -10\cos(\omega t + 50^\circ) = 10\cos(\omega t + 50^\circ - 180^\circ)$$
$$v_1 = 10\cos(\omega t - 130^\circ) \quad \text{or} \quad v_1 = 10\cos(\omega t + 230^\circ) \tag{9.2.1}$$

and

$$v_2 = 12\sin(\omega t - 10^\circ) = 12\cos(\omega t - 10^\circ - 90^\circ)$$
$$v_2 = 12\cos(\omega t - 100^\circ) \tag{9.2.2}$$

It can be deduced from Eqs. (9.2.1) and (9.2.2) that the phase difference between v_1 and v_2 is 30°. We can write v_2 as

$$v_2 = 12\cos(\omega t - 130^\circ + 30^\circ) \quad \text{or} \quad v_2 = 12\cos(\omega t + 260^\circ) \tag{9.2.3}$$

Comparing Eqs. (9.2.1) and (9.2.3) shows clearly that v_2 leads v_1 by 30°.

METHOD 2 Alternatively, we may express v_1 in sine form:

$$v_1 = -10\cos(\omega t + 50^\circ) = 10\sin(\omega t + 50^\circ - 90^\circ)$$
$$= 10\sin(\omega t - 40^\circ) = 10\sin(\omega t - 10^\circ - 30^\circ)$$

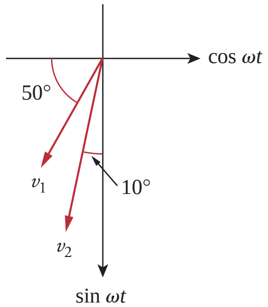


Figure 9.5
For Example 9.2.

But $v_2 = 12\sin(\omega t - 10^\circ)$. Comparing the two shows that v_1 lags v_2 by 30°. This is the same as saying that v_2 leads v_1 by 30°.

■ **METHOD 3** We may regard v_1 as simply $-10\cos\omega t$ with a phase shift of $+50^\circ$. Hence, v_1 is as shown in Fig. 9.5. Similarly, v_2 is $12\sin\omega t$ with a phase shift of -10°, as shown in Fig. 9.5. It is easy to see from Fig. 9.5 that v_2 leads v_1 by 30°, that is, $90^\circ - 50^\circ - 10^\circ$.

Practice Problem 9.2

Find the phase angle between

$$i_1 = -4\sin(377t + 25^\circ) \qquad \text{and} \qquad i_2 = 5\cos(377t - 40^\circ)$$

Does i_1 lead or lag i_2?

Answer: 155°, i_1 leads i_2.

9.3 Phasors

Sinusoids are easily expressed in terms of phasors, which are more convenient to work with than sine and cosine functions.

> A **phasor** is a complex number that represents the amplitude and phase of a sinusoid.

Charles Proteus Steinmetz (1865–1923) was a German-Austrian mathematician and electrical engineer.

Appendix B presents a short tutorial on complex numbers.

Phasors provide a simple means of analyzing linear circuits excited by sinusoidal sources; solutions of such circuits would be intractable otherwise. The notion of solving ac circuits using phasors was first introduced by Charles Steinmetz in 1893. Before we completely define phasors and apply them to circuit analysis, we need to be thoroughly familiar with complex numbers.

A complex number z can be written in rectangular form as

$$z = x + jy \tag{9.14a}$$

where $j = \sqrt{-1}$; x is the real part of z; y is the imaginary part of z. In this context, the variables x and y do not represent a location as in two-dimensional vector analysis but rather the real and imaginary parts of z in the complex plane. Nevertheless, we note that there are some resemblances between manipulating complex numbers and manipulating two-dimensional vectors.

The complex number z can also be written in polar or exponential form as

$$z = r\underline{/\phi} = re^{j\phi} \tag{9.14b}$$

Historical

Charles Proteus Steinmetz (1865–1923), a German-Austrian mathematician and engineer, introduced the phasor method (covered in this chapter) in ac circuit analysis. He is also noted for his work on the theory of hysteresis.

Steinmetz was born in Breslau, Germany, and lost his mother at the age of one. As a youth, he was forced to leave Germany because of his political activities just as he was about to complete his doctoral dissertation in mathematics at the University of Breslau. He migrated to Switzerland and later to the United States, where he was employed by General Electric in 1893. That same year, he published a paper in which complex numbers were used to analyze ac circuits for the first time. This led to one of his many textbooks, *Theory and Calculation of ac Phenomena*, published by McGraw-Hill in 1897. In 1901, he became the president of the American Institute of Electrical Engineers, which later became the IEEE.

where r is the magnitude of z, and ϕ is the phase of z. We notice that z can be represented in three ways:

$$\begin{aligned} z &= x + jy && \text{Rectangular form} \\ z &= r\underline{/\phi} && \text{Polar form} \\ z &= re^{j\phi} && \text{Exponential form} \end{aligned} \tag{9.15}$$

The relationship between the rectangular form and the polar form is shown in Fig. 9.6, where the x axis represents the real part and the y axis represents the imaginary part of a complex number. Given x and y, we can get r and ϕ as

$$r = \sqrt{x^2 + y^2}, \qquad \phi = \tan^{-1}\frac{y}{x} \tag{9.16a}$$

On the other hand, if we know r and ϕ, we can obtain x and y as

$$x = r\cos\phi, \qquad y = r\sin\phi \tag{9.16b}$$

Thus, z may be written as

$$\boxed{z = x + jy = r\underline{/\phi} = r(\cos\phi + j\sin\phi)} \tag{9.17}$$

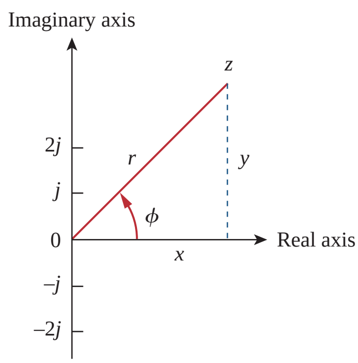


Figure 9.6
Representation of a complex number $z = x + jy = r\underline{/\phi}$.

Addition and subtraction of complex numbers are better performed in rectangular form; multiplication and division are better done in polar form. Given the complex numbers

$$z = x + jy = r\underline{/\phi}, \qquad z_1 = x_1 + jy_1 = r_1\underline{/\phi_1}$$
$$z_2 = x_2 + jy_2 = r_2\underline{/\phi_2}$$

the following operations are important.
Addition:

$$z_1 + z_2 = (x_1 + x_2) + j(y_1 + y_2) \tag{9.18a}$$

Subtraction:

$$z_1 - z_2 = (x_1 - x_2) + j(y_1 - y_2) \tag{9.18b}$$

Multiplication:

$$z_1 z_2 = r_1 r_2 \underline{/\phi_1 + \phi_2} \tag{9.18c}$$

Division:

$$\frac{z_1}{z_2} = \frac{r_1}{r_2} \underline{/\phi_1 - \phi_2} \tag{9.18d}$$

Reciprocal:

$$\frac{1}{z} = \frac{1}{r} \underline{/-\phi} \tag{9.18e}$$

Square Root:

$$\sqrt{z} = \sqrt{r} \underline{/\phi/2} \tag{9.18f}$$

Complex Conjugate:

$$z^* = x - jy = r\underline{/-\phi} = re^{-j\phi} \tag{9.18g}$$

Note that from Eq. (9.18e),

$$\frac{1}{j} = -j \tag{9.18h}$$

These are the basic properties of complex numbers we need. Other properties of complex numbers can be found in Appendix B.

The idea of phasor representation is based on Euler's identity. In general,

$$\boxed{e^{\pm j\phi} = \cos\phi \pm j\sin\phi} \tag{9.19}$$

which shows that we may regard $\cos\phi$ and $\sin\phi$ as the real and imaginary parts of $e^{j\phi}$; we may write

$$\cos\phi = \mathrm{Re}(e^{j\phi}) \tag{9.20a}$$

$$\sin\phi = \mathrm{Im}(e^{j\phi}) \tag{9.20b}$$

where Re and Im stand for the *real part of* and the *imaginary part of*. Given a sinusoid $v(t) = V_m \cos(\omega t + \phi)$, we use Eq. (9.20a) to express $v(t)$ as

$$v(t) = V_m \cos(\omega t + \phi) = \mathrm{Re}(V_m e^{j(\omega t + \phi)}) \tag{9.21}$$

or

$$v(t) = \mathrm{Re}(V_m e^{j\phi} e^{j\omega t}) \tag{9.22}$$

Thus,

$$\boxed{v(t) = \mathrm{Re}(\mathbf{V} e^{j\omega t})} \tag{9.23}$$

where

$$\mathbf{V} = V_m e^{j\phi} = V_m \underline{/\phi} \tag{9.24}$$

$\mathbf{V}$ is thus the *phasor representation* of the sinusoid $v(t)$, as we said earlier. In other words, a phasor is a complex representation of the magnitude and phase of a sinusoid. Either Eq. (9.20a) or Eq. (9.20b) can be used to develop the phasor, but the standard convention is to use Eq. (9.20a).

A phasor may be regarded as a mathematical equivalent of a sinusoid with the time dependence dropped.

One way of looking at Eqs. (9.23) and (9.24) is to consider the plot of the *sinor* $\mathbf{V}e^{j\omega t} = V_m e^{j(\omega t+\phi)}$ on the complex plane. As time increases, the sinor rotates on a circle of radius V_m at an angular velocity ω in the counterclockwise direction, as shown in Fig. 9.7(a). We may regard $v(t)$ as the projection of the sinor $\mathbf{V}e^{j\omega t}$ on the real axis, as shown in Fig. 9.7(b). The value of the sinor at time $t = 0$ is the phasor $\mathbf{V}$ of the sinusoid $v(t)$. The sinor may be regarded as a rotating phasor. Thus, whenever a sinusoid is expressed as a phasor, the term $e^{j\omega t}$ is implicitly present. It is therefore important, when dealing with phasors, to keep in mind the frequency ω of the phasor; otherwise we can make serious mistakes.

If we use sine for the phasor instead of cosine, then $v(t) = V_m \sin(\omega t + \phi) = \text{Im}(V_m e^{j(\omega t+\phi)})$ and the corresponding phasor is the same as that in Eq. (9.24).

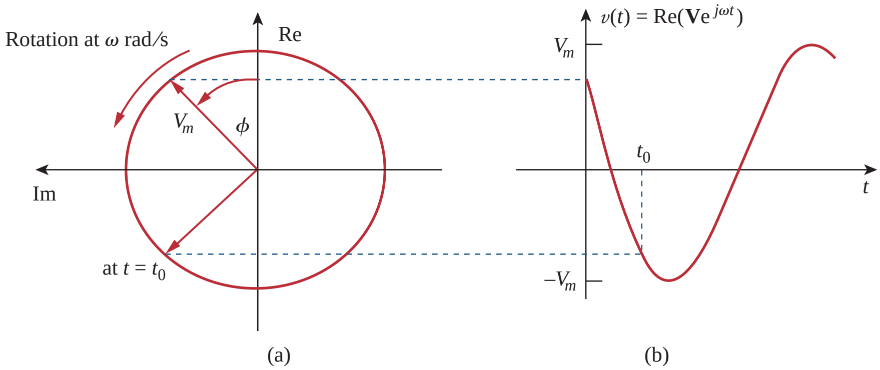


Figure 9.7
Representation of $\mathbf{V}e^{j\omega t}$: (a) sinor rotating counterclockwise, (b) its projection on the real axis, as a function of time.

Equation (9.23) states that to obtain the sinusoid corresponding to a given phasor $\mathbf{V}$, multiply the phasor by the time factor $e^{j\omega t}$ and take the real part. As a complex quantity, a phasor may be expressed in rectangular form, polar form, or exponential form. Since a phasor has magnitude and phase ("direction"), it behaves as a vector and is printed in boldface. For example, phasors $\mathbf{V} = V_m\underline{/\phi}$ and $\mathbf{I} = I_m\underline{/-\theta}$ are graphically represented in Fig. 9.8. Such a graphical representation of phasors is known as a *phasor diagram.*

We use lightface italic letters such as z to represent complex numbers but boldface letters such as $\mathbf{V}$ to represent phasors, because phasors are vector-like quantities.

Equations (9.21) through (9.23) reveal that to get the phasor corresponding to a sinusoid, we first express the sinusoid in the cosine form so that the sinusoid can be written as the real part of a complex number. Then we take out the time factor $e^{j\omega t}$, and whatever is left is the phasor corresponding to the sinusoid. By suppressing the time factor, we transform the sinusoid from the time domain to the phasor domain. This transformation is summarized as follows:

$$\underset{\text{(Time-domain representation)}}{v(t) = V_m \cos(\omega t + \phi)} \quad \Leftrightarrow \quad \underset{\text{(Phasor-domain representation)}}{\mathbf{V} = V_m\underline{/\phi}} \qquad \textbf{(9.25)}$$

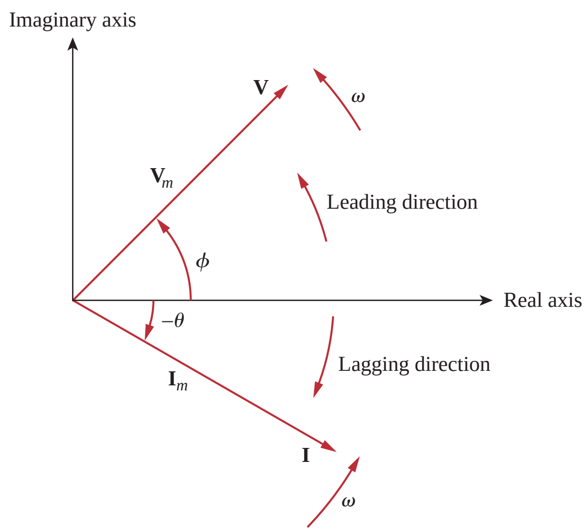


Figure 9.8
A phasor diagram showing $\mathbf{V} = V_m \angle \phi$ and $\mathbf{I} = I_m \angle -\theta$.

Given a sinusoid $v(t) = V_m \cos(\omega t + \phi)$, we obtain the corresponding phasor as $\mathbf{V} = V_m \angle \phi$. Equation (9.25) is also demonstrated in Table 9.1, where the sine function is considered in addition to the cosine function. From Eq. (9.25), we see that to get the phasor representation of a sinusoid, we express it in cosine form and take the magnitude and phase. Given a phasor, we obtain the time domain representation as the cosine function with the same magnitude as the phasor and the argument as ωt plus the phase of the phasor. The idea of expressing information in alternate domains is fundamental to all areas of engineering.

TABLE 9.1

Sinusoid-phasor transformation.

Time domain representation	Phasor domain representation
$V_m \cos(\omega t + \phi)$	$V_m \angle \phi$
$V_m \sin(\omega t + \phi)$	$V_m \angle \phi - 90^\circ$
$I_m \cos(\omega t + \theta)$	$I_m \angle \theta$
$I_m \sin(\omega t + \theta)$	$I_m \angle \theta - 90^\circ$

Note that in Eq. (9.25) the frequency (or time) factor $e^{j\omega t}$ is suppressed, and the frequency is not explicitly shown in the phasor domain representation because ω is constant. However, the response depends on ω. For this reason, the phasor domain is also known as the *frequency domain.*

From Eqs. (9.23) and (9.24), $v(t) = \text{Re}(\mathbf{V}e^{j\omega t}) = V_m \cos(\omega t + \phi)$, so that

$$\begin{aligned} \frac{dv}{dt} &= -\omega V_m \sin(\omega t + \phi) = \omega V_m \cos(\omega t + \phi + 90^\circ) \\ &= \text{Re}(\omega V_m e^{j\omega t} e^{j\phi} e^{j90^\circ}) = \text{Re}(j\omega \mathbf{V} e^{j\omega t}) \end{aligned} \tag{9.26}$$

This shows that the derivative $v(t)$ is transformed to the phasor domain as $j\omega\mathbf{V}$

$$\underset{\textbf{(Time domain)}}{\frac{dv}{dt}} \quad \Leftrightarrow \quad \underset{\textbf{(Phasor domain)}}{j\omega\mathbf{V}} \tag{9.27}$$

Differentiating a sinusoid is equivalent to multiplying its corresponding phasor by $j\omega$.

Similarly, the integral of $v(t)$ is transformed to the phasor domain as $\mathbf{V}/j\omega$

$$\underset{\textbf{(Time domain)}}{\int v\,dt} \quad \Leftrightarrow \quad \underset{\textbf{(Phasor domain)}}{\frac{\mathbf{V}}{j\omega}} \tag{9.28}$$

Integrating a sinusoid is equivalent to dividing its corresponding phasor by $j\omega$.

Equation (9.27) allows the replacement of a derivative with respect to time with multiplication of $j\omega$ in the phasor domain, whereas Eq. (9.28) allows the replacement of an integral with respect to time with division by $j\omega$ in the phasor domain. Equations (9.27) and (9.28) are useful in finding the steady-state solution, which does not require knowing the initial values of the variable involved. This is one of the important applications of phasors.

Besides time differentiation and integration, another important use of phasors is found in summing sinusoids of the same frequency. This is best illustrated with an example, and Example 9.6 provides one.

Adding sinusoids of the same frequency is equivalent to adding their corresponding phasors.

The differences between $v(t)$ and $\mathbf{V}$ should be emphasized:

1. $v(t)$ is the *instantaneous or time domain* representation, while $\mathbf{V}$ is the *frequency or phasor domain* representation.
2. $v(t)$ is time dependent, while $\mathbf{V}$ is not. (This fact is often forgotten by students.)
3. $v(t)$ is always real with no complex term, while $\mathbf{V}$ is generally complex.

Finally, we should bear in mind that phasor analysis applies only when frequency is constant; it applies in manipulating two or more sinusoidal signals only if they are of the same frequency.

Example 9.3

Evaluate these complex numbers:

(a) $(40\underline{/50^\circ} + 20\underline{/-30^\circ})^{1/2}$

(b) $\dfrac{10\underline{/-30^\circ} + (3 - j4)}{(2 + j4)(3 - j5)^*}$

Solution:

(a) Using polar to rectangular transformation,

$$40\underline{/50^\circ} = 40(\cos 50^\circ + j\sin 50^\circ) = 25.71 + j30.64$$

$$20\underline{/-30^\circ} = 20[\cos(-30^\circ) + j\sin(-30^\circ)] = 17.32 - j10$$

Adding them up gives

$$40\underline{/50^\circ} + 20\underline{/-30^\circ} = 43.03 + j20.64 = 47.72\underline{/25.63^\circ}$$

Taking the square root of this,

$$(40\underline{/50^\circ} + 20\underline{/-30^\circ})^{1/2} = 6.91\underline{/12.81^\circ}$$

(b) Using polar-rectangular transformation, addition, multiplication, and division,

$$\begin{aligned}\frac{10\underline{/-30^\circ} + (3 - j4)}{(2 + j4)(3 - j5)^*} &= \frac{8.66 - j5 + (3 - j4)}{(2 + j4)(3 + j5)} \\ &= \frac{11.66 - j9}{-14 + j22} = \frac{14.73\underline{/-37.66^\circ}}{26.08\underline{/122.47^\circ}} \\ &= 0.565\underline{/-160.13^\circ}\end{aligned}$$

Practice Problem 9.3

Evaluate the following complex numbers:

(a) $[(5 + j2)(-1 + j4) - 5\underline{/60^\circ}]^*$

(b) $\dfrac{10 + j5 + 3\underline{/40^\circ}}{-3 + j4} + 10\underline{/30^\circ} + j5$

Answer: (a) $-15.5 - j13.67$, (b) $8.293 + j7.2$.

Example 9.4

Transform these sinusoids to phasors:

(a) $i = 6\cos(50t - 40^\circ)$ A
(b) $v = -4\sin(30t + 50^\circ)$ V

Solution:

(a) $i = 6\cos(50t - 40^\circ)$ has the phasor

$$\mathbf{I} = 6\underline{/-40^\circ}\text{ A}$$

(b) Since $-\sin A = \cos(A + 90^\circ)$,

$$\begin{aligned}v = -4\sin(30t + 50^\circ) &= 4\cos(30t + 50^\circ + 90^\circ) \\ &= 4\cos(30t + 140^\circ)\text{ V}\end{aligned}$$

The phasor form of v is

$$\mathbf{V} = 4\underline{/140^\circ}\text{ V}$$

Practice Problem 9.4

Express these sinusoids as phasors:

(a) $v = 7\cos(2t + 40^\circ)$ V
(b) $i = -4\sin(10t + 10^\circ)$ A

Answer: (a) $\mathbf{V} = 7\underline{/40^\circ}$ V, (b) $\mathbf{I} = 4\underline{/100^\circ}$ A.

Example 9.5

Find the sinusoids represented by these phasors:

(a) $\mathbf{I} = -3 + j4$ A
(b) $\mathbf{V} = j8e^{-j20^\circ}$ V

Solution:

(a) $\mathbf{I} = -3 + j4 = 5\underline{/126.87^\circ}$. Transforming this to the time domain gives

$$i(t) = 5\cos(\omega t + 126.87^\circ)\text{ A}$$

(b) Since $j = 1\underline{/90^\circ}$,

$$\begin{aligned}\mathbf{V} = j8\underline{/-20^\circ} &= (1\underline{/90^\circ})(8\underline{/-20^\circ})\\ &= 8\underline{/90^\circ - 20^\circ} = 8\underline{/70^\circ}\text{ V}\end{aligned}$$

Converting this to the time domain gives

$$v(t) = 8\cos(\omega t + 70^\circ)\text{ V}$$

Practice Problem 9.5

Find the sinusoids corresponding to these phasors:

(a) $\mathbf{V} = -10\underline{/30^\circ}$ V
(b) $\mathbf{I} = j(5 - j12)$ A

Answer: (a) $v(t) = 10\cos(\omega t + 210^\circ)$ V or $10\cos(\omega t - 150^\circ)$ V,
(b) $i(t) = 13\cos(\omega t + 22.62^\circ)$ A.

Example 9.6

Given $i_1(t) = 4\cos(\omega t + 30^\circ)$ A and $i_2(t) = 5\sin(\omega t - 20^\circ)$ A, find their sum.

Solution:
Here is an important use of phasors—for summing sinusoids of the same frequency. Current $i_1(t)$ is in the standard form. Its phasor is

$$\mathbf{I}_1 = 4\underline{/30^\circ}$$

We need to express $i_2(t)$ in cosine form. The rule for converting sine to cosine is to subtract 90°. Hence,

$$i_2 = 5\cos(\omega t - 20^\circ - 90^\circ) = 5\cos(\omega t - 110^\circ)$$

and its phasor is

$$\mathbf{I}_2 = 5\underline{/-110^\circ}$$

If we let $i = i_1 + i_2$, then

$$\begin{aligned}\mathbf{I} = \mathbf{I}_1 + \mathbf{I}_2 &= 4\underline{/30^\circ} + 5\underline{/-110^\circ}\\ &= 3.464 + j2 - 1.71 - j4.698 = 1.754 - j2.698\\ &= 3.218\underline{/-56.97^\circ}\text{ A}\end{aligned}$$

Transforming this to the time domain, we get

$$i(t) = 3.218\cos(\omega t - 56.97^\circ)\ \text{A}$$

Of course, we can find $i_1 + i_2$ using Eq. (9.9), but that is the hard way.

Practice Problem 9.6

If $v_1 = -10\sin(\omega t - 30^\circ)$ V and $v_2 = 20\cos(\omega t + 45^\circ)$ V, find $v = v_1 + v_2$.

Answer: $v(t) = 12.158\cos(\omega t + 55.95^\circ)$ V.

Example 9.7

Using the phasor approach, determine the current $i(t)$ in a circuit described by the integrodifferential equation

$$4i + 8\int i\,dt - 3\frac{di}{dt} = 50\cos(2t + 75^\circ)$$

Solution:

We transform each term in the equation from time domain to phasor domain. Keeping Eqs. (9.27) and (9.28) in mind, we obtain the phasor form of the given equation as

$$4\mathbf{I} + \frac{8\mathbf{I}}{j\omega} - 3j\omega\mathbf{I} = 50\underline{/75^\circ}$$

But $\omega = 2$, so

$$\mathbf{I}(4 - j4 - j6) = 50\underline{/75^\circ}$$

$$\mathbf{I} = \frac{50\underline{/75^\circ}}{4 - j10} = \frac{50\underline{/75^\circ}}{10.77\underline{/-68.2^\circ}} = 4.642\underline{/143.2^\circ}\ \text{A}$$

Converting this to the time domain,

$$i(t) = 4.642\cos(2t + 143.2^\circ)\ \text{A}$$

Keep in mind that this is only the steady-state solution, and it does not require knowing the initial values.

Practice Problem 9.7

Find the voltage $v(t)$ in a circuit described by the integrodifferential equation

$$2\frac{dv}{dt} + 5v + 10\int v\,dt = 50\cos(5t - 30^\circ)$$

using the phasor approach.

Answer: $v(t) = 5.3\cos(5t - 88^\circ)$ V.

9.4 Phasor Relationships for Circuit Elements

Now that we know how to represent a voltage or current in the phasor or frequency domain, one may legitimately ask how we apply this to circuits involving the passive elements R, L, and C. What we need to do is to transform the voltage-current relationship from the time domain to the frequency domain for each element. Again, we will assume the passive sign convention.

We begin with the resistor. If the current through a resistor R is $i = I_m \cos(\omega t + \phi)$, the voltage across it is given by Ohm's law as

$$v = iR = RI_m \cos(\omega t + \phi) \tag{9.29}$$

The phasor form of this voltage is

$$\mathbf{V} = RI_m \underline{/\phi} \tag{9.30}$$

But the phasor representation of the current is $\mathbf{I} = I_m \underline{/\phi}$. Hence,

$$\mathbf{V} = R\mathbf{I} \tag{9.31}$$

showing that the voltage-current relation for the resistor in the phasor domain continues to be Ohm's law, as in the time domain. Figure 9.9 illustrates the voltage-current relations of a resistor. We should note from Eq. (9.31) that voltage and current are in phase, as illustrated in the phasor diagram in Fig. 9.10.

For the inductor L, assume the current through it is $i = I_m \cos(\omega t + \phi)$. The voltage across the inductor is

$$v = L\frac{di}{dt} = -\omega L I_m \sin(\omega t + \phi) \tag{9.32}$$

Recall from Eq. (9.10) that $-\sin A = \cos(A + 90^\circ)$. We can write the voltage as

$$v = \omega L I_m \cos(\omega t + \phi + 90^\circ) \tag{9.33}$$

which transforms to the phasor

$$\mathbf{V} = \omega L I_m e^{j(\phi+90^\circ)} = \omega L I_m e^{j\phi} e^{j90^\circ} = \omega L I_m \underline{/\phi + 90^\circ} \tag{9.34}$$

But $I_m \underline{/\phi} = \mathbf{I}$, and from Eq. (9.19), $e^{j90^\circ} = j$. Thus,

$$\mathbf{V} = j\omega L\mathbf{I} \tag{9.35}$$

showing that the voltage has a magnitude of $\omega L I_m$ and a phase of $\phi + 90^\circ$. The voltage and current are 90° out of phase. Specifically, the current lags the voltage by 90°. Figure 9.11 shows the voltage-current relations for the inductor. Figure 9.12 shows the phasor diagram.

For the capacitor C, assume the voltage across it is $v = V_m \cos(\omega t + \phi)$. The current through the capacitor is

$$i = C\frac{dv}{dt} \tag{9.36}$$

By following the same steps as we took for the inductor or by applying Eq. (9.27) on Eq. (9.36), we obtain

$$\mathbf{I} = j\omega C\mathbf{V} \quad \Rightarrow \quad \mathbf{V} = \frac{\mathbf{I}}{j\omega C} \tag{9.37}$$

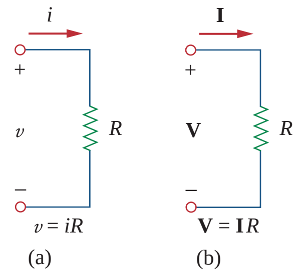


Figure 9.9
Voltage-current relations for a resistor in the: (a) time domain, (b) frequency domain.

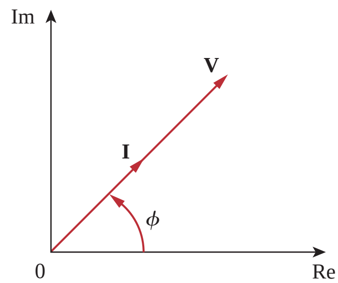


Figure 9.10
Phasor diagram for the resistor.

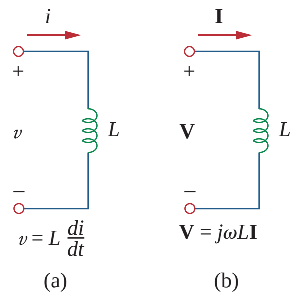


Figure 9.11
Voltage-current relations for an inductor in the: (a) time domain, (b) frequency domain.

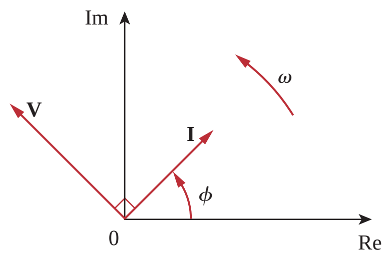


Figure 9.12
Phasor diagram for the inductor; **I** lags **V**.

Although it is equally correct to say that the inductor voltage leads the current by 90°, convention gives the current phase relative to the voltage.

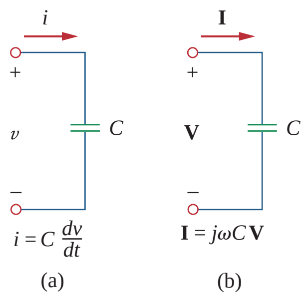


Figure 9.13
Voltage-current relations for a capacitor in the: (a) time domain, (b) frequency domain.

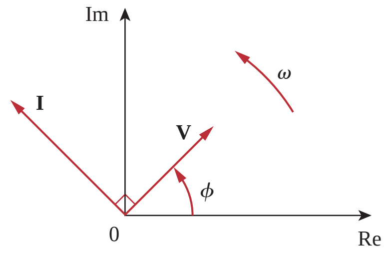


Figure 9.14
Phasor diagram for the capacitor; **I** leads **V**.

showing that the current and voltage are 90° out of phase. To be specific, the current leads the voltage by 90°. Figure 9.13 shows the voltage-current relations for the capacitor; Fig. 9.14 gives the phasor diagram. Table 9.2 summarizes the time domain and phasor domain representations of the circuit elements.

TABLE 9.2

Summary of voltage-current relationships.

Element	Time domain	Frequency domain
R	$v = Ri$	$\mathbf{V} = R\mathbf{I}$
L	$v = L\dfrac{di}{dt}$	$\mathbf{V} = j\omega L\mathbf{I}$
C	$i = C\dfrac{dv}{dt}$	$\mathbf{V} = \dfrac{\mathbf{I}}{j\omega C}$

Example 9.8

The voltage $v = 12\cos(60t + 45°)$ is applied to a 0.1-H inductor. Find the steady-state current through the inductor.

Solution:
For the inductor, $\mathbf{V} = j\omega L\mathbf{I}$, where $\omega = 60$ rad/s and $\mathbf{V} = 12\underline{/45°}$ V. Hence,

$$\mathbf{I} = \frac{\mathbf{V}}{j\omega L} = \frac{12\underline{/45°}}{j60 \times 0.1} = \frac{12\underline{/45°}}{6\underline{/90°}} = 2\underline{/-45°} \text{ A}$$

Converting this to the time domain,

$$i(t) = 2\cos(60t - 45°) \text{ A}$$

Practice Problem 9.8

If voltage $v = 10\cos(100t + 30°)$ is applied to a 50 μF capacitor, calculate the current through the capacitor.

Answer: $50\cos(100t + 120°)$ mA.

9.5 Impedance and Admittance

In the preceding section, we obtained the voltage-current relations for the three passive elements as

$$\mathbf{V} = R\mathbf{I}, \qquad \mathbf{V} = j\omega L\mathbf{I}, \qquad \mathbf{V} = \frac{\mathbf{I}}{j\omega C} \tag{9.38}$$

These equations may be written in terms of the ratio of the phasor voltage to the phasor current as

$$\frac{\mathbf{V}}{\mathbf{I}} = R, \qquad \frac{\mathbf{V}}{\mathbf{I}} = j\omega L, \qquad \frac{\mathbf{V}}{\mathbf{I}} = \frac{1}{j\omega C} \tag{9.39}$$

From these three expressions, we obtain Ohm's law in phasor form for any type of element as

$$\boxed{\mathbf{Z} = \frac{\mathbf{V}}{\mathbf{I}} \quad \text{or} \quad \mathbf{V} = \mathbf{Z}\mathbf{I}} \tag{9.40}$$

where $\mathbf{Z}$ is a frequency-dependent quantity known as *impedance*, measured in ohms.

> The **impedance** $\mathbf{Z}$ of a circuit is the ratio of the phasor voltage $\mathbf{V}$ to the phasor current $\mathbf{I}$, measured in ohms (Ω).

The impedance represents the opposition that the circuit exhibits to the flow of sinusoidal current. Although the impedance is the ratio of two phasors, it is not a phasor, because it does not correspond to a sinusoidally varying quantity.

The impedances of resistors, inductors, and capacitors can be readily obtained from Eq. (9.39). Table 9.3 summarizes their impedances. From the table we notice that $\mathbf{Z}_L = j\omega L$ and $\mathbf{Z}_C = -j/\omega C$. Consider two extreme cases of angular frequency. When $\omega = 0$ (i.e., for dc sources), $\mathbf{Z}_L = 0$ and $\mathbf{Z}_C \to \infty$, confirming what we already know—that the inductor acts like a short circuit, while the capacitor acts like an open circuit. When $\omega \to \infty$ (i.e., for high frequencies), $\mathbf{Z}_L \to \infty$ and $\mathbf{Z}_C = 0$, indicating that the inductor is an open circuit to high frequencies, while the capacitor is a short circuit. Figure 9.15 illustrates this.

As a complex quantity, the impedence may be expressed in rectangular form as

$$\mathbf{Z} = R + jX \tag{9.41}$$

where $R = \text{Re}\,\mathbf{Z}$ is the *resistance* and $X = \text{Im}\,\mathbf{Z}$ is the *reactance*. The reactance X may be positive or negative. We say that the impedance is inductive when X is positive or capacitive when X is negative. Thus, impedance $\mathbf{Z} = R + jX$ is said to be *inductive* or lagging since current lags voltage, while impedance $\mathbf{Z} = R - jX$ is capacitive or leading because current leads voltage. The impedance, resistance, and reactance are all measured in ohms. The impedance may also be expressed in polar form as

$$\mathbf{Z} = |\mathbf{Z}| \underline{/\theta} \tag{9.42}$$

TABLE 9.3

Impedances and admittances of passive elements.

Element	Impedance	Admittance
R	$\mathbf{Z} = R$	$\mathbf{Y} = \dfrac{1}{R}$
L	$\mathbf{Z} = j\omega L$	$\mathbf{Y} = \dfrac{1}{j\omega L}$
C	$\mathbf{Z} = \dfrac{1}{j\omega C}$	$\mathbf{Y} = j\omega C$

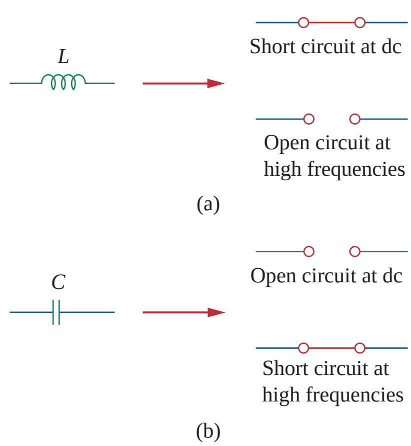


Figure 9.15
Equivalent circuits at dc and high frequencies: (a) inductor, (b) capacitor.

Comparing Eqs. (9.41) and (9.42), we infer that

$$\boxed{\mathbf{Z} = R + jX = |\mathbf{Z}|\underline{/\theta}} \qquad \textbf{(9.43)}$$

where

$$|\mathbf{Z}| = \sqrt{R^2 + X^2}, \qquad \theta = \tan^{-1}\frac{X}{R} \qquad \textbf{(9.44)}$$

and

$$R = |\mathbf{Z}|\cos\theta, \qquad X = |\mathbf{Z}|\sin\theta \qquad \textbf{(9.45)}$$

It is sometimes convenient to work with the reciprocal of impedance, known as *admittance*.

The **admittance Y** is the reciprocal of impedance, measured in siemens (S).

The admittance $\mathbf{Y}$ of an element (or a circuit) is the ratio of the phasor current through it to the phasor voltage across it, or

$$\boxed{\mathbf{Y} = \frac{1}{\mathbf{Z}} = \frac{\mathbf{I}}{\mathbf{V}}} \qquad \textbf{(9.46)}$$

The admittances of resistors, inductors, and capacitors can be obtained from Eq. (9.39). They are also summarized in Table 9.3.

As a complex quantity, we may write $\mathbf{Y}$ as

$$\boxed{\mathbf{Y} = G + jB} \qquad \textbf{(9.47)}$$

where $G = \text{Re}\,\mathbf{Y}$ is called the *conductance* and $B = \text{Im}\,\mathbf{Y}$ is called the *susceptance*. Admittance, conductance, and susceptance are all expressed in the unit of siemens (or mhos). From Eqs. (9.41) and (9.47),

$$G + jB = \frac{1}{R + jX} \qquad \textbf{(9.48)}$$

By rationalization,

$$G + jB = \frac{1}{R + jX} \cdot \frac{R - jX}{R - jX} = \frac{R - jX}{R^2 + X^2} \qquad \textbf{(9.49)}$$

Equating the real and imaginary parts gives

$$G = \frac{R}{R^2 + X^2}, \qquad B = -\frac{X}{R^2 + X^2} \qquad \textbf{(9.50)}$$

showing that $G \neq 1/R$ as it is in resistive circuits. Of course, if $X = 0$, then $G = 1/R$.

Example 9.9

Find $v(t)$ and $i(t)$ in the circuit shown in Fig. 9.16.

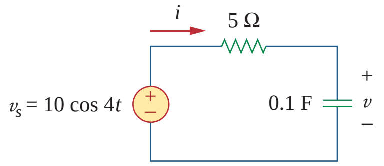


Figure 9.16
For Example 9.9.

Solution:
From the voltage source $10 \cos 4t$, $\omega = 4$,

$$\mathbf{V}_s = 10\underline{/0^\circ} \text{ V}$$

The impedance is

$$\mathbf{Z} = 5 + \frac{1}{j\omega C} = 5 + \frac{1}{j4 \times 0.1} = 5 - j2.5\ \Omega$$

Hence the current

$$\begin{aligned}\mathbf{I} &= \frac{\mathbf{V}_s}{\mathbf{Z}} = \frac{10\underline{/0^\circ}}{5 - j2.5} = \frac{10(5 + j2.5)}{5^2 + 2.5^2} \\ &= 1.6 + j0.8 = 1.789\underline{/26.57^\circ} \text{ A}\end{aligned} \tag{9.9.1}$$

The voltage across the capacitor is

$$\begin{aligned}\mathbf{V} = \mathbf{IZ}_C = \frac{\mathbf{I}}{j\omega C} &= \frac{1.789\underline{/26.57^\circ}}{j4 \times 0.1} \\ &= \frac{1.789\underline{/26.57^\circ}}{0.4\underline{/90^\circ}} = 4.47\underline{/-63.43^\circ} \text{ V}\end{aligned} \tag{9.9.2}$$

Converting **I** and **V** in Eqs. (9.9.1) and (9.9.2) to the time domain, we get

$$i(t) = 1.789 \cos(4t + 26.57^\circ) \text{ A}$$
$$v(t) = 4.47 \cos(4t - 63.43^\circ) \text{ V}$$

Notice that $i(t)$ leads $v(t)$ by 90° as expected.

Practice Problem 9.9

Refer to Fig. 9.17. Determine $v(t)$ and $i(t)$.

Answer: $8.944 \sin(10t + 93.43^\circ)$ V, $4.472 \sin(10t + 3.43^\circ)$ A.

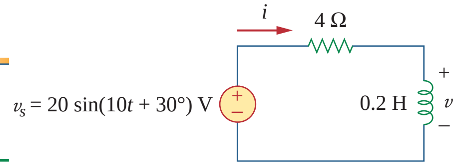


Figure 9.17
For Practice Prob. 9.9.

9.6 †Kirchhoff's Laws in the Frequency Domain

We cannot do circuit analysis in the frequency domain without Kirchhoff's current and voltage laws. Therefore, we need to express them in the frequency domain.

For KVL, let $v_1, v_2, \ldots, v_n$ be the voltages around a closed loop. Then

$$v_1 + v_2 + \cdots + v_n = 0 \tag{9.51}$$

In the sinusoidal steady state, each voltage may be written in cosine form, so that Eq. (9.51) becomes

$$\begin{aligned}V_{m1} \cos(\omega t + \theta_1) + V_{m2} \cos(\omega t + \theta_2) \\ + \cdots + V_{mn} \cos(\omega t + \theta_n) = 0\end{aligned} \tag{9.52}$$

This can be written as

$$\mathrm{Re}(V_{m1}e^{j\theta_1}e^{j\omega t}) + \mathrm{Re}(V_{m2}e^{j\theta_2}e^{j\omega t}) + \cdots + \mathrm{Re}(V_{mn}e^{j\theta_n}e^{j\omega t}) = 0$$

or

$$\mathrm{Re}[(V_{m1}e^{j\theta_1} + V_{m2}e^{j\theta_2} + \cdots + V_{mn}e^{j\theta_n})e^{j\omega t}] = 0 \tag{9.53}$$

If we let $\mathbf{V}_k = V_{mk}e^{j\theta_k}$, then

$$\mathrm{Re}[(\mathbf{V}_1 + \mathbf{V}_2 + \cdots + \mathbf{V}_n)e^{j\omega t}] = 0 \tag{9.54}$$

Since $e^{j\omega t} \neq 0$,

$$\mathbf{V}_1 + \mathbf{V}_2 + \cdots + \mathbf{V}_n = 0 \tag{9.55}$$

indicating that Kirchhoff's voltage law holds for phasors.

By following a similar procedure, we can show that Kirchhoff's current law holds for phasors. If we let $i_1, i_2, \ldots, i_n$ be the current leaving or entering a closed surface in a network at time t, then

$$i_1 + i_2 + \cdots + i_n = 0 \tag{9.56}$$

If $\mathbf{I}_1, \mathbf{I}_2, \ldots, \mathbf{I}_n$ are the phasor forms of the sinusoids $i_1, i_2, \ldots, i_n$, then

$$\mathbf{I}_1 + \mathbf{I}_2 + \cdots + \mathbf{I}_n = 0 \tag{9.57}$$

which is Kirchhoff's current law in the frequency domain.

Once we have shown that both KVL and KCL hold in the frequency domain, it is easy to do many things, such as impedance combination, nodal and mesh analyses, superposition, and source transformation.

9.7 Impedance Combinations

Consider the N series-connected impedances shown in Fig. 9.18. The same current $\mathbf{I}$ flows through the impedances. Applying KVL around the loop gives

$$\mathbf{V} = \mathbf{V}_1 + \mathbf{V}_2 + \cdots + \mathbf{V}_N = \mathbf{I}(\mathbf{Z}_1 + \mathbf{Z}_2 + \cdots + \mathbf{Z}_N) \tag{9.58}$$

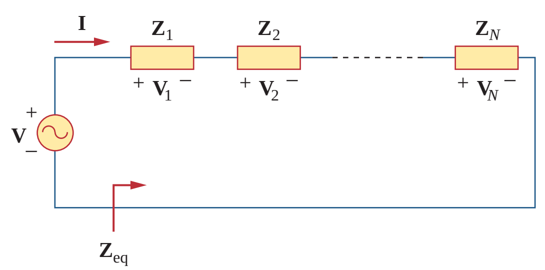


Figure 9.18
N impedances in series.

The equivalent impedance at the input terminals is

$$\mathbf{Z}_{\mathrm{eq}} = \frac{\mathbf{V}}{\mathbf{I}} = \mathbf{Z}_1 + \mathbf{Z}_2 + \cdots + \mathbf{Z}_N$$

or

$$\boxed{\mathbf{Z}_{\mathrm{eq}} = \mathbf{Z}_1 + \mathbf{Z}_2 + \cdots + \mathbf{Z}_N} \tag{9.59}$$

showing that the total or equivalent impedance of series-connected impedances is the sum of the individual impedances. This is similar to the series connection of resistances.

If $N = 2$, as shown in Fig. 9.19, the current through the impedances is

$$\mathbf{I} = \frac{\mathbf{V}}{\mathbf{Z}_1 + \mathbf{Z}_2} \tag{9.60}$$

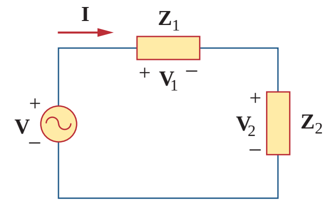


Figure 9.19
Voltage division.

Since $\mathbf{V}_1 = \mathbf{Z}_1\mathbf{I}$ and $\mathbf{V}_2 = \mathbf{Z}_2\mathbf{I}$, then

$$\boxed{\mathbf{V}_1 = \frac{\mathbf{Z}_1}{\mathbf{Z}_1 + \mathbf{Z}_2}\mathbf{V}, \qquad \mathbf{V}_2 = \frac{\mathbf{Z}_2}{\mathbf{Z}_1 + \mathbf{Z}_2}\mathbf{V}} \tag{9.61}$$

which is the *voltage-division* relationship.

In the same manner, we can obtain the equivalent impedance or admittance of the N parallel-connected impedances shown in Fig. 9.20. The voltage across each impedance is the same. Applying KCL at the top node,

$$\mathbf{I} = \mathbf{I}_1 + \mathbf{I}_2 + \cdots + \mathbf{I}_N = \mathbf{V}\left(\frac{1}{\mathbf{Z}_1} + \frac{1}{\mathbf{Z}_2} + \cdots + \frac{1}{\mathbf{Z}_N}\right) \tag{9.62}$$

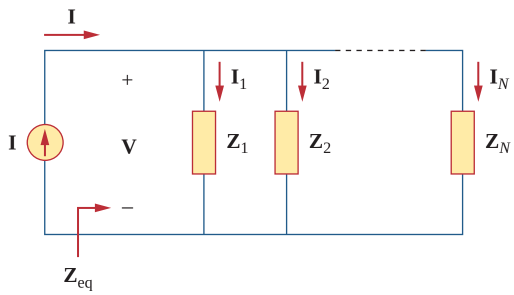


Figure 9.20
N impedances in parallel.

The equivalent impedance is

$$\frac{1}{\mathbf{Z}_{\text{eq}}} = \frac{\mathbf{I}}{\mathbf{V}} = \frac{1}{\mathbf{Z}_1} + \frac{1}{\mathbf{Z}_2} + \cdots + \frac{1}{\mathbf{Z}_N} \tag{9.63}$$

and the equivalent admittance is

$$\boxed{\mathbf{Y}_{\text{eq}} = \mathbf{Y}_1 + \mathbf{Y}_2 + \cdots + \mathbf{Y}_N} \tag{9.64}$$

This indicates that the equivalent admittance of a parallel connection of admittances is the sum of the individual admittances.

When $N = 2$, as shown in Fig. 9.21, the equivalent impedance becomes

$$\mathbf{Z}_{\text{eq}} = \frac{1}{\mathbf{Y}_{\text{eq}}} = \frac{1}{\mathbf{Y}_1 + \mathbf{Y}_2} = \frac{1}{1/\mathbf{Z}_1 + 1/\mathbf{Z}_2} = \frac{\mathbf{Z}_1\mathbf{Z}_2}{\mathbf{Z}_1 + \mathbf{Z}_2} \tag{9.65}$$

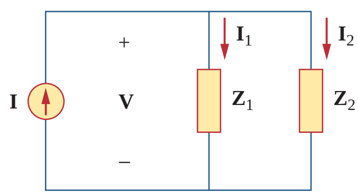


Figure 9.21
Current division.

Also, since

$$\mathbf{V} = \mathbf{I}\mathbf{Z}_{\text{eq}} = \mathbf{I}_1\mathbf{Z}_1 = \mathbf{I}_2\mathbf{Z}_2$$

the currents in the impedances are

$$\boxed{\mathbf{I}_1 = \frac{\mathbf{Z}_2}{\mathbf{Z}_1 + \mathbf{Z}_2}\mathbf{I}, \qquad \mathbf{I}_2 = \frac{\mathbf{Z}_1}{\mathbf{Z}_1 + \mathbf{Z}_2}\mathbf{I}} \tag{9.66}$$

which is the *current-division* principle.

The delta-to-wye and wye-to-delta transformations that we applied to resistive circuits are also valid for impedances. With reference to Fig. 9.22, the conversion formulas are as follows.

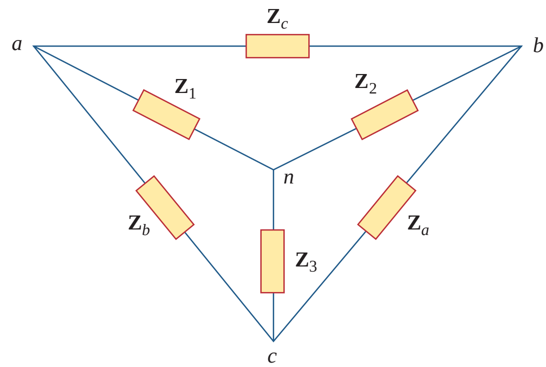


Figure 9.22
Superimposed Y and Δ networks.

Y-Δ Conversion:

$$\boxed{\begin{aligned}
\mathbf{Z}_a &= \frac{\mathbf{Z}_1\mathbf{Z}_2 + \mathbf{Z}_2\mathbf{Z}_3 + \mathbf{Z}_3\mathbf{Z}_1}{\mathbf{Z}_1}\\
\mathbf{Z}_b &= \frac{\mathbf{Z}_1\mathbf{Z}_2 + \mathbf{Z}_2\mathbf{Z}_3 + \mathbf{Z}_3\mathbf{Z}_1}{\mathbf{Z}_2}\\
\mathbf{Z}_c &= \frac{\mathbf{Z}_1\mathbf{Z}_2 + \mathbf{Z}_2\mathbf{Z}_3 + \mathbf{Z}_3\mathbf{Z}_1}{\mathbf{Z}_3}
\end{aligned}} \tag{9.67}$$

Δ-Y Conversion:

$$\boxed{\begin{aligned}
\mathbf{Z}_1 &= \frac{\mathbf{Z}_b\mathbf{Z}_c}{\mathbf{Z}_a + \mathbf{Z}_b + \mathbf{Z}_c}\\
\mathbf{Z}_2 &= \frac{\mathbf{Z}_c\mathbf{Z}_a}{\mathbf{Z}_a + \mathbf{Z}_b + \mathbf{Z}_c}\\
\mathbf{Z}_3 &= \frac{\mathbf{Z}_a\mathbf{Z}_b}{\mathbf{Z}_a + \mathbf{Z}_b + \mathbf{Z}_c}
\end{aligned}} \tag{9.68}$$

A delta or wye circuit is said to be **balanced** if it has equal impedances in all three branches.

When a Δ-Y circuit is balanced, Eqs. (9.67) and (9.68) become

$$\mathbf{Z}_\Delta = 3\mathbf{Z}_Y \qquad \text{or} \qquad \mathbf{Z}_Y = \frac{1}{3}\mathbf{Z}_\Delta \tag{9.69}$$

where $\mathbf{Z}_Y = \mathbf{Z}_1 = \mathbf{Z}_2 = \mathbf{Z}_3$ and $\mathbf{Z}_\Delta = \mathbf{Z}_a = \mathbf{Z}_b = \mathbf{Z}_c$.

As you see in this section, the principles of voltage division, current division, circuit reduction, impedance equivalence, and Y-Δ transformation all apply to ac circuits. Chapter 10 will show that other circuit techniques—such as superposition, nodal analysis, mesh analysis, source transformation, the Thevenin theorem, and the Norton theorem—are all applied to ac circuits in a manner similar to their application in dc circuits.

Example 9.10

Find the input impedance of the circuit in Fig. 9.23. Assume that the circuit operates at $\omega = 50$ rad/s.

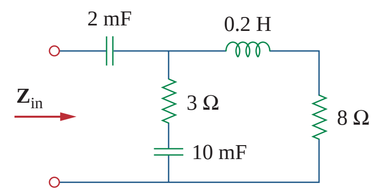


Figure 9.23
For Example 9.10.

Solution:
Let

$\mathbf{Z}_1$ = Impedance of the 2-mF capacitor

$\mathbf{Z}_2$ = Impedance of the 3-Ω resistor in series with the10-mF capacitor

$\mathbf{Z}_3$ = Impedance of the 0.2-H inductor in series with the 8-Ω resistor

Then

$$\mathbf{Z}_1 = \frac{1}{j\omega C} = \frac{1}{j50 \times 2 \times 10^{-3}} = -j10\ \Omega$$

$$\mathbf{Z}_2 = 3 + \frac{1}{j\omega C} = 3 + \frac{1}{j50 \times 10 \times 10^{-3}} = (3 - j2)\ \Omega$$

$$\mathbf{Z}_3 = 8 + j\omega L = 8 + j50 \times 0.2 = (8 + j10)\ \Omega$$

The input impedance is

$$\mathbf{Z}_{\text{in}} = \mathbf{Z}_1 + \mathbf{Z}_2 \parallel \mathbf{Z}_3 = -j10 + \frac{(3 - j2)(8 + j10)}{11 + j8}$$

$$= -j10 + \frac{(44 + j14)(11 - j8)}{11^2 + 8^2} = -j10 + 3.22 - j1.07\ \Omega$$

Thus,

$$\mathbf{Z}_{\text{in}} = 3.22 - j11.07\ \Omega$$

Practice Problem 9.10

Determine the input impedance of the circuit in Fig. 9.24 at $\omega = 10$ rad/s.

Answer: $(129.52 - j295)$

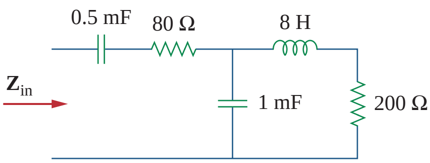


Figure 9.24
For Practice Prob. 9.10.

Example 9.11

Determine $v_o(t)$ in the circuit of Fig. 9.25.

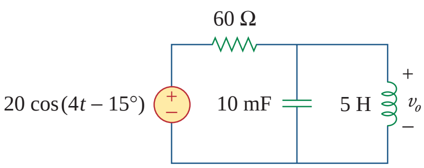


Figure 9.25
For Example 9.11.

Solution:
To do the analysis in the frequency domain, we must first transform the time domain circuit in Fig. 9.25 to the phasor domain equivalent in Fig. 9.26. The transformation produces

$$v_s = 20\cos(4t - 15^\circ) \quad \Rightarrow \quad \mathbf{V}_s = 20\underline{/-15^\circ}\text{ V}, \qquad \omega = 4$$

$$10\text{ mF} \quad \Rightarrow \quad \frac{1}{j\omega C} = \frac{1}{j4 \times 10 \times 10^{-3}}$$
$$= -j25\ \Omega$$

$$5\text{ H} \quad \Rightarrow \quad j\omega L = j4 \times 5 = j20\ \Omega$$

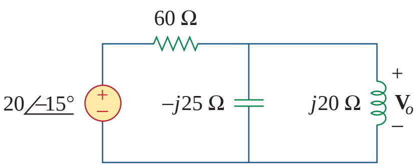


Figure 9.26
The frequency domain equivalent of the circuit in Fig. 9.25.

Let

$\mathbf{Z}_1$ = Impedance of the 60-Ω resistor

$\mathbf{Z}_2$ = Impedance of the parallel combination of the 10-mF capacitor and the 5-H inductor

Then $\mathbf{Z}_1 = 60\ \Omega$ and

$$\mathbf{Z}_2 = -j25 \parallel j20 = \frac{-j25 \times j20}{-j25 + j20} = j100\ \Omega$$

By the voltage-division principle,

$$\mathbf{V}_o = \frac{\mathbf{Z}_2}{\mathbf{Z}_1 + \mathbf{Z}_2}\mathbf{V}_s = \frac{j100}{60 + j100}(20\underline{/-15^\circ})$$
$$= (0.8575\underline{/30.96^\circ})(20\underline{/-15^\circ}) = 17.15\underline{/15.96^\circ}\text{ V}$$

We convert this to the time domain and obtain

$$v_o(t) = 17.15\cos(4t + 15.96^\circ)\text{ V}$$

Practice Problem 9.11

Calculate v_o in the circuit of Fig. 9.27.

Answer: $v_o(t) = 14.142\cos(10t - 35^\circ)$ V.

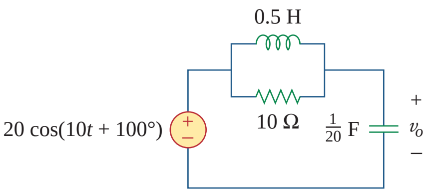


Figure 9.27
For Practice Prob. 9.11.

Example 9.12

Find current **I** in the circuit of Fig. 9.28.

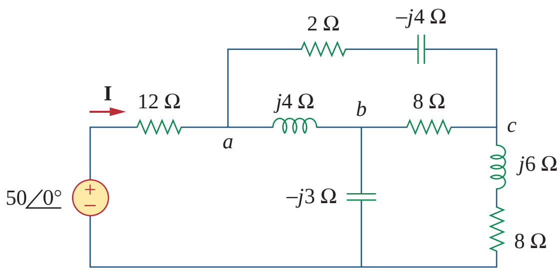


Figure 9.28
For Example 9.12.

Solution:
The delta network connected to nodes a, b, and c can be converted to the Y network of Fig. 9.29. We obtain the Y impedances as follows using Eq. (9.68):

$$\mathbf{Z}_{an} = \frac{j4(2 - j4)}{j4 + 2 - j4 + 8} = \frac{4(4 + j2)}{10} = (1.6 + j0.8)\ \Omega$$

$$\mathbf{Z}_{bn} = \frac{j4(8)}{10} = j3.2\ \Omega, \qquad \mathbf{Z}_{cn} = \frac{8(2 - j4)}{10} = (1.6 - j3.2)\ \Omega$$

The total impedance at the source terminals is

$$\begin{aligned}\mathbf{Z} &= 12 + \mathbf{Z}_{an} + (\mathbf{Z}_{bn} - j3) \parallel (\mathbf{Z}_{cn} + j6 + 8)\\ &= 12 + 1.6 + j0.8 + (j0.2) \parallel (9.6 + j2.8)\\ &= 13.6 + j0.8 + \frac{j0.2(9.6 + j2.8)}{9.6 + j3}\\ &= 13.6 + j1 = 13.64\underline{/4.204^\circ}\ \Omega\end{aligned}$$

The desired current is

$$\mathbf{I} = \frac{\mathbf{V}}{\mathbf{Z}} = \frac{50\underline{/0^\circ}}{13.64\underline{/4.204^\circ}} = 3.666\underline{/-4.204^\circ}\ \text{A}$$

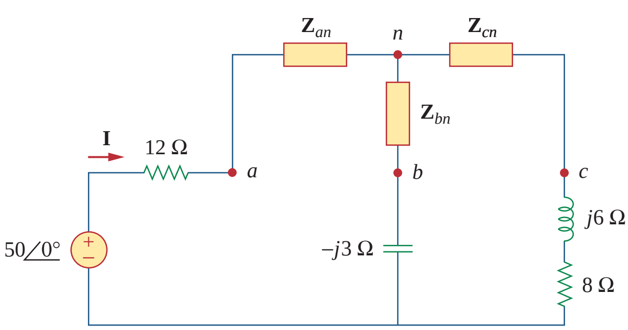


Figure 9.29
The circuit in Fig. 9.28 after delta-to-wye transformation.

Practice Problem 9.12

Find **I** in the circuit of Fig. 9.30.

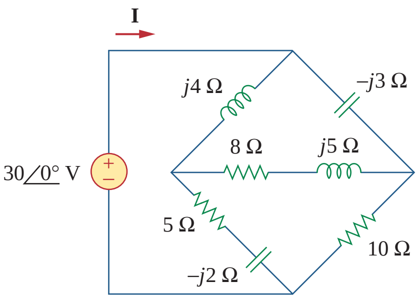


Figure 9.30
For Practice Prob. 9.12.

Answer: $6.364\underline{/3.8^\circ}$ A.

9.8 †Applications

In Chapters 7 and 8, we saw certain uses of *RC*, *RL*, and *RLC* circuits in dc applications. These circuits also have ac applications; among them are coupling circuits, phase-shifting circuits, filters, resonant circuits, ac bridge circuits, and transformers. This list of applications is inexhaustive. We will consider some of them later. It will suffice here to observe two simple ones: *RC* phase-shifting circuits, and ac bridge circuits.

9.8.1 Phase-Shifters

A phase-shifting circuit is often employed to correct an undesirable phase shift already present in a circuit or to produce special desired effects. An *RC* circuit is suitable for this purpose because its capacitor causes the circuit current to lead the applied voltage. Two commonly used *RC* circuits are shown in Fig. 9.31. (*RL* circuits or any reactive circuits could also serve the same purpose.)

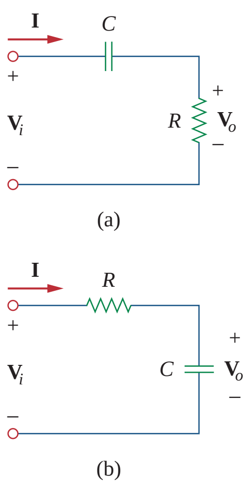


Figure 9.31
Series *RC* shift circuits: (a) leading output, (b) lagging output.

In Fig. 9.31(a), the circuit current **I** leads the applied voltage $\mathbf{V}_i$ by some phase angle θ, where $0 < \theta < 90^\circ$, depending on the values of *R* and *C*. If $X_C = -1/\omega C$, then the total impedance is $\mathbf{Z} = R + jX_C$, and the phase shift is given by

$$\theta = \tan^{-1}\frac{X_C}{R} \tag{9.70}$$

This shows that the amount of phase shift depends on the values of *R*, *C*, and the operating frequency. Since the output voltage $\mathbf{V}_o$ across the resistor is in phase with the current, $\mathbf{V}_o$ leads (positive phase shift) $\mathbf{V}_i$ as shown in Fig. 9.32(a).

In Fig. 9.31(b), the output is taken across the capacitor. The current **I** leads the input voltage $\mathbf{V}_i$ by θ, but the output voltage $v_o(t)$ across the capacitor lags (negative phase shift) the input voltage $v_i(t)$ as illustrated in Fig. 9.32(b).

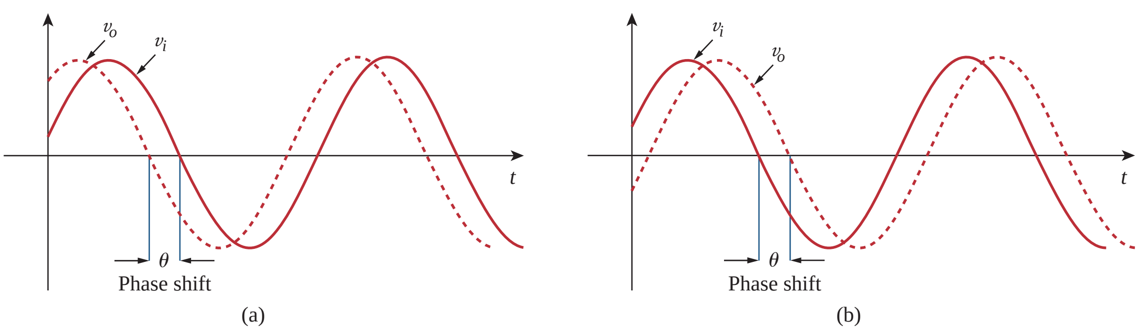


Figure 9.32
Phase shift in *RC* circuits: (a) leading output, (b) lagging output.

We should keep in mind that the simple RC circuits in Fig. 9.31 also act as voltage dividers. Therefore, as the phase shift θ approaches 90°, the output voltage $\mathbf{V}_o$ approaches zero. For this reason, these simple RC circuits are used only when small amounts of phase shift are required. If it is desired to have phase shifts greater than 60°, simple RC networks are cascaded, thereby providing a total phase shift equal to the sum of the individual phase shifts. In practice, the phase shifts due to the stages are not equal, because the succeeding stages load down the earlier stages unless op amps are used to separate the stages.

Example 9.13

Design an RC circuit to provide a phase of 90° leading.

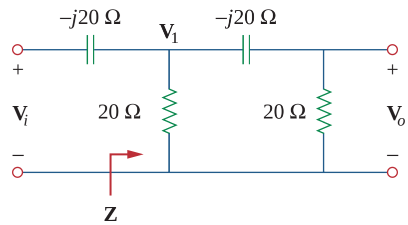


Figure 9.33
An RC phase shift circuit with 90° leading phase shift; for Example 9.13.

Solution:
If we select circuit components of equal ohmic value, say $R = |X_C| = 20\ \Omega$, at a particular frequency, according to Eq. (9.70), the phase shift is exactly 45°. By cascading two similar RC circuits in Fig. 9.31(a), we obtain the circuit in Fig. 9.33, providing a positive or leading phase shift of 90°, as we shall soon show. Using the series-parallel combination technique, $\mathbf{Z}$ in Fig. 9.33 is obtained as

$$\mathbf{Z} = 20 \parallel (20 - j20) = \frac{20(20 - j20)}{40 - j20} = 12 - j4\ \Omega \qquad \textbf{(9.13.1)}$$

Using voltage division,

$$\mathbf{V}_1 = \frac{\mathbf{Z}}{\mathbf{Z} - j20}\mathbf{V}_i = \frac{12 - j4}{12 - j24}\mathbf{V}_i = \frac{\sqrt{2}}{3}\underline{/45^\circ}\ \mathbf{V}_i \qquad \textbf{(9.13.2)}$$

and

$$\mathbf{V}_o = \frac{20}{20 - j20}\mathbf{V}_1 = \frac{\sqrt{2}}{2}\underline{/45^\circ}\ \mathbf{V}_1 \qquad \textbf{(9.13.3)}$$

Substituting Eq. (9.13.2) into Eq. (9.13.3) yields

$$\mathbf{V}_o = \left(\frac{\sqrt{2}}{2}\underline{/45^\circ}\right)\left(\frac{\sqrt{2}}{3}\underline{/45^\circ}\ \mathbf{V}_i\right) = \frac{1}{3}\underline{/90^\circ}\ \mathbf{V}_i$$

Thus, the output leads the input by 90° but its magnitude is only about 33 percent of the input.

Practice Problem 9.13

Design an RC circuit to provide a 90° lagging phase shift of the output voltage relative to the input voltage. If an ac voltage of 10 V rms is applied, what is the output voltage?

Answer: Figure 9.34 shows a typical design; 3.33 V rms.

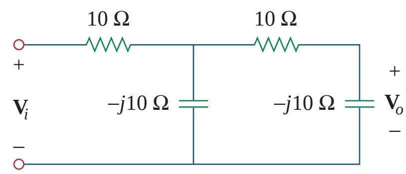


Figure 9.34
For Practice Prob. 9.13.

Example 9.14

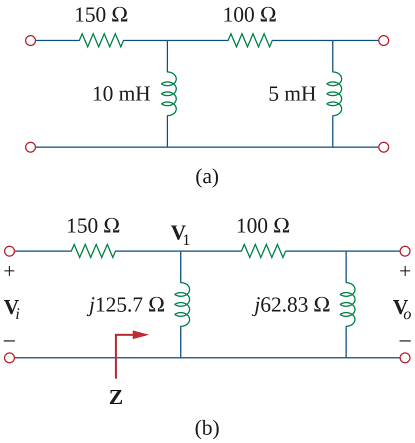


Figure 9.35
For Example 9.14.

For the *RL* circuit shown in Fig. 9.35(a), calculate the amount of phase shift produced at 2 kHz.

Solution:
At 2 kHz, we transform the 10-mH and 5-mH inductances to the corresponding impedances.

$$\begin{aligned} 10 \text{ mH} \quad &\Rightarrow \quad X_L = \omega L = 2\pi \times 2 \times 10^3 \times 10 \times 10^{-3} \\ &= 40\pi = 125.7\ \Omega \\ 5 \text{ mH} \quad &\Rightarrow \quad X_L = \omega L = 2\pi \times 2 \times 10^3 \times 5 \times 10^{-3} \\ &= 20\pi = 62.83\ \Omega \end{aligned}$$

Consider the circuit in Fig. 9.35(b). The impedance $\mathbf{Z}$ is the parallel combination of $j125.7\ \Omega$ and $100 + j62.83\ \Omega$. Hence,

$$\begin{aligned} \mathbf{Z} &= j125.7 \parallel (100 + j62.83) \\ &= \frac{j125.7(100 + j62.83)}{100 + j188.5} = 69.56\underline{/60.1^\circ}\ \Omega \end{aligned} \tag{9.14.1}$$

Using voltage division,

$$\begin{aligned} \mathbf{V}_1 &= \frac{\mathbf{Z}}{\mathbf{Z} + 150}\mathbf{V}_i = \frac{69.56\underline{/60.1^\circ}}{184.7 + j60.3}\mathbf{V}_i \\ &= 0.3582\underline{/42.02^\circ}\ \mathbf{V}_i \end{aligned} \tag{9.14.2}$$

and

$$\mathbf{V}_o = \frac{j62.832}{100 + j62.832}\mathbf{V}_1 = 0.532\underline{/57.86^\circ}\ \mathbf{V}_1 \tag{9.14.3}$$

Combining Eqs. (9.14.2) and (9.14.3),

$$\mathbf{V}_o = (0.532\underline{/57.86^\circ})(0.3582\underline{/42.02^\circ})\ \mathbf{V}_i = 0.1906\underline{/100^\circ}\ \mathbf{V}_i$$

showing that the output is about 19 percent of the input in magnitude but leading the input by 100°. If the circuit is terminated by a load, the load will affect the phase shift.

Practice Problem 9.14

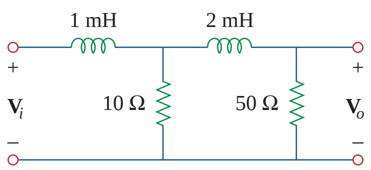


Figure 9.36
For Practice Prob. 9.14.

Refer to the *RL* circuit in Fig. 9.36. If 1 V is applied, find the magnitude and the phase shift produced at 5 kHz. Specify whether the phase shift is leading or lagging.

Answer: 0.172, 120.4°, lagging.

9.8.2 AC Bridges

An ac bridge circuit is used in measuring the inductance *L* of an inductor or the capacitance *C* of a capacitor. It is similar in form to the Wheatstone bridge for measuring an unknown resistance (discussed in Section 4.10) and follows the same principle. To measure *L* and *C*, however, an ac source is needed as well as an ac meter instead of the galvanometer. The ac meter may be a sensitive ac ammeter or voltmeter.

Consider the general ac bridge circuit displayed in Fig. 9.37. The bridge is *balanced* when no current flows through the meter. This means that $\mathbf{V}_1 = \mathbf{V}_2$. Applying the voltage division principle,

$$\mathbf{V}_1 = \frac{\mathbf{Z}_2}{\mathbf{Z}_1 + \mathbf{Z}_2}\mathbf{V}_s = \mathbf{V}_2 = \frac{\mathbf{Z}_x}{\mathbf{Z}_3 + \mathbf{Z}_x}\mathbf{V}_s \tag{9.71}$$

Thus,

$$\frac{\mathbf{Z}_2}{\mathbf{Z}_1 + \mathbf{Z}_2} = \frac{\mathbf{Z}_x}{\mathbf{Z}_3 + \mathbf{Z}_x} \quad \Rightarrow \quad \mathbf{Z}_2\mathbf{Z}_3 = \mathbf{Z}_1\mathbf{Z}_x \tag{9.72}$$

or

$$\boxed{\mathbf{Z}_x = \frac{\mathbf{Z}_3}{\mathbf{Z}_1}\mathbf{Z}_2} \tag{9.73}$$

This is the balanced equation for the ac bridge and is similar to Eq. (4.30) for the resistance bridge except that the R's are replaced by $\mathbf{Z}$'s.

Specific ac bridges for measuring L and C are shown in Fig. 9.38, where L_x and C_x are the unknown inductance and capacitance to be measured while L_s and C_s are a standard inductance and capacitance (the values of which are known to great precision). In each case, two resistors, R_1 and R_2, are varied until the ac meter reads zero. Then the bridge is balanced. From Eq. (9.73), we obtain

$$L_x = \frac{R_2}{R_1}L_s \tag{9.74}$$

and

$$C_x = \frac{R_1}{R_2}C_s \tag{9.75}$$

Notice that the balancing of the ac bridges in Fig. 9.38 does not depend on the frequency f of the ac source, since f does not appear in the relationships in Eqs. (9.74) and (9.75).

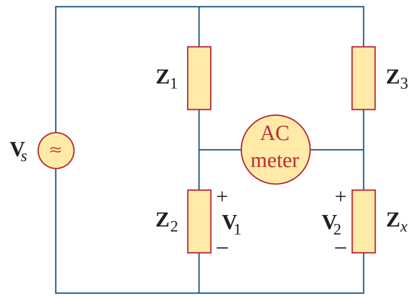


Figure 9.37
A general ac bridge.

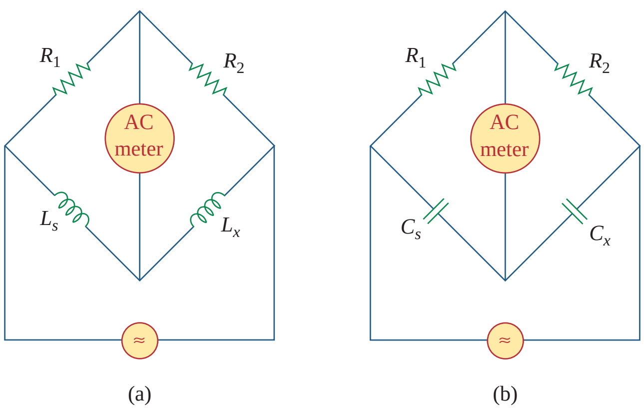


Figure 9.38
Specific ac bridges: (a) for measuring L, (b) for measuring C.

Example 9.15

The ac bridge circuit of Fig. 9.37 balances when $\mathbf{Z}_1$ is a 1-kΩ resistor, $\mathbf{Z}_2$ is a 4.2-kΩ resistor, $\mathbf{Z}_3$ is a parallel combination of a 1.5-MΩ resistor and a 12-pF capacitor, and $f = 2$ kHz. Find: (a) the series components that make up $\mathbf{Z}_x$, and (b) the parallel components that make up $\mathbf{Z}_x$.

Solution:

1. **Define.** The problem is clearly stated.
2. **Present.** We are to determine the unknown components subject to the fact that they balance the given quantities. Since a parallel and series equivalent exists for this circuit, we need to find both.
3. **Alternative.** Although there are alternative techniques that can be used to find the unknown values, a straightforward equality works best. Once we have answers, we can check them by using hand techniques such as nodal analysis or just using *PSpice*.
4. **Attempt.** From Eq. (9.73),

$$\mathbf{Z}_x = \frac{\mathbf{Z}_3}{\mathbf{Z}_1}\mathbf{Z}_2 \tag{9.15.1}$$

where $\mathbf{Z}_x = R_x + jX_x$,

$$\mathbf{Z}_1 = 1000\ \Omega, \qquad \mathbf{Z}_2 = 4200\ \Omega \tag{9.15.2}$$

and

$$\mathbf{Z}_3 = R_3 \parallel \frac{1}{j\omega C_3} = \frac{\dfrac{R_3}{j\omega C_3}}{R_3 + 1/j\omega C_3} = \frac{R_3}{1 + j\omega R_3 C_3}$$

Since $R_3 = 1.5$ MΩ and $C_3 = 12$ pF,

$$\mathbf{Z}_3 = \frac{1.5 \times 10^6}{1 + j2\pi \times 2 \times 10^3 \times 1.5 \times 10^6 \times 12 \times 10^{-12}} = \frac{1.5 \times 10^6}{1 + j0.2262}$$

or

$$\mathbf{Z}_3 = 1.427 - j0.3228\ \text{M}\Omega \tag{9.15.3}$$

(a) Assuming that $\mathbf{Z}_x$ is made up of series components, we substitute Eqs. (9.15.2) and (9.15.3) in Eq. (9.15.1) and obtain

$$\begin{aligned} R_x + jX_x &= \frac{4200}{1000}(1.427 - j0.3228) \times 10^6 \\ &= (5.993 - j1.356)\ \text{M}\Omega \end{aligned} \tag{9.15.4}$$

Equating the real and imaginary parts yields $R_x = 5.993$ MΩ and a capacitive reactance

$$X_x = \frac{1}{\omega C} = 1.356 \times 10^6$$

or

$$C = \frac{1}{\omega X_x} = \frac{1}{2\pi \times 2 \times 10^3 \times 1.356 \times 10^6} = 58.69\ \text{pF}$$

(b) Z_x remains the same as in Eq. (9.15.4) but R_x and X_x are in parallel. Assuming an *RC* parallel combination,

$$\begin{aligned}\mathbf{Z}_x &= (5.993 - j1.356)\ \text{M}\Omega \\ &= R_x \parallel \frac{1}{j\omega C_x} = \frac{R_x}{1 + j\omega R_x C_x}\end{aligned}$$

By equating the real and imaginary parts, we obtain

$$R_x = \frac{\text{Real}(\mathbf{Z}_x)^2 + \text{Imag}(\mathbf{Z}_x)^2}{\text{Real}(\mathbf{Z}_x)} = \frac{5.993^2 + 1.356^2}{5.993} = \mathbf{6.3\ M\Omega}$$

$$\begin{aligned}C_x &= -\frac{\text{Imag}(\mathbf{Z}_x)}{\omega[\text{Real}(\mathbf{Z}_x)^2 + \text{Imag}(\mathbf{Z}_x)^2]} \\ &= -\frac{-1.356}{2\pi(2000)(5.917^2 + 1.356^2)} = \mathbf{2.852\ \mu F}\end{aligned}$$

We have assumed a parallel *RC* combination which works in this case.

5. **Evaluate.** Let us now use *PSpice* to see if we indeed have the correct equalities. Running *PSpice* with the equivalent circuits, an open circuit between the "bridge" portion of the circuit, and a 10-volt input voltage yields the following voltages at the ends of the "bridge" relative to a reference at the bottom of the circuit:

```
FREQ       VM($N_0002)  VP($N_0002)
2.000E+03  9.993E+00  -8.634E-03
2.000E+03  9.993E+00  -8.637E-03
```

Since the voltages are essentially the same, then no measurable current can flow through the "bridge" portion of the circuit for any element that connects the two points together and we have a balanced bridge, which is to be expected. This indicates we have properly determined the unknowns.

There is a very important problem with what we have done! Do you know what that is? We have what can be called an ideal, "theoretical" answer, but one that really is not very good in the real world. The difference between the magnitudes of the upper impedances and the lower impedances is much too large and would never be accepted in a real bridge circuit. For greatest accuracy, the overall magnitude of the impedances must at least be within the same relative order. To increase the accuracy of the solution of this problem, I would recommend increasing the magnitude of the top impedances to be in the range of 500 kΩ to 1.5 MΩ. One additional real-world comment: the size of these impedances also creates serious problems in making actual measurements, so the appropriate instruments must be used in order to minimize their loading (which would change the actual voltage readings) on the circuit.

6. **Satisfactory?** Since we solved for the unknown terms and then tested to see if they woked, we validated the results. They can now be presented as a solution to the problem.

Practice Problem 9.15

In the ac bridge circuit of Fig. 9.37, suppose that balance is achieved when $\mathbf{Z}_1$ is a 4.8-kΩ resistor, $\mathbf{Z}_2$ is a 10-Ω resistor in series with a 0.25-μH inductor, $\mathbf{Z}_3$ is a 12-kΩ resistor, and $f = 6$ MHz. Determine the series components that make up $\mathbf{Z}_x$.

Answer: A 25-Ω resistor in series with a 0.625-μH inductor.

9.9 Summary

1. A sinusoid is a signal in the form of the sine or cosine function. It has the general form

$$v(t) = V_m \cos(\omega t + \phi)$$

where V_m is the amplitude, $\omega = 2\pi f$ is the angular frequency, $(\omega t + \phi)$ is the argument, and ϕ is the phase.
2. A phasor is a complex quantity that represents both the magnitude and the phase of a sinusoid. Given the sinusoid $v(t) = V_m \cos(\omega t + \phi)$, its phasor $\mathbf{V}$ is

$$\mathbf{V} = V_m \underline{/\phi}$$

3. In ac circuits, voltage and current phasors always have a fixed relation to one another at any moment of time. If $v(t) = V_m \cos(\omega t + \phi_v)$ represents the voltage through an element and $i(t) = I_m \cos(\omega t + \phi_i)$ represents the current through the element, then $\phi_i = \phi_v$ if the element is a resistor, ϕ_i leads ϕ_v by 90° if the element is a capacitor, and ϕ_i lags ϕ_v by 90° if the element is an inductor.
4. The impedance $\mathbf{Z}$ of a circuit is the ratio of the phasor voltage across it to the phasor current through it:

$$\mathbf{Z} = \frac{\mathbf{V}}{\mathbf{I}} = R(\omega) + jX(\omega)$$

The admittance $\mathbf{Y}$ is the reciprocal of impedance:

$$\mathbf{Y} = \frac{1}{\mathbf{Z}} = G(\omega) + jB(\omega)$$

Impedances are combined in series or in parallel the same way as resistances in series or parallel; that is, impedances in series add while admittances in parallel add.
5. For a resistor $\mathbf{Z} = R$, for an inductor $\mathbf{Z} = jX = j\omega L$, and for a capacitor $\mathbf{Z} = -jX = 1/j\omega C$.
6. Basic circuit laws (Ohm's and Kirchhoff's) apply to ac circuits in the same manner as they do for dc circuits; that is,

$$\mathbf{V} = \mathbf{Z}\mathbf{I}$$

$$\Sigma\, \mathbf{I}_k = 0 \quad \text{(KCL)}$$

$$\Sigma\, \mathbf{V}_k = 0 \quad \text{(KVL)}$$

7. The techniques of voltage/current division, series/parallel combination of impedance/admittance, circuit reduction, and Y-Δ transformation all apply to ac circuit analysis.
8. AC circuits are applied in phase-shifters and bridges.

Review Questions

9.1 Which of the following is *not* a right way to express the sinusoid $A \cos \omega t$?

(a) $A \cos 2\pi ft$ (b) $A \cos(2\pi t/T)$
(c) $A \cos \omega(t - T)$ (d) $A \sin(\omega t - 90°)$

9.2 A function that repeats itself after fixed intervals is said to be:

(a) a phasor (b) harmonic
(c) periodic (d) reactive

9.3 Which of these frequencies has the shorter period?

(a) 1 krad/s (b) 1 kHz

9.4 If $v_1 = 30 \sin(\omega t + 10°)$ and $v_2 = 20 \sin(\omega t + 50°)$, which of these statements are true?

(a) v_1 leads v_2 (b) v_2 leads v_1
(c) v_2 lags v_1 (d) v_1 lags v_2
(e) v_1 and v_2 are in phase

9.5 The voltage across an inductor leads the current through it by 90°.

(a) True (b) False

9.6 The imaginary part of impedance is called:

(a) resistance (b) admittance
(c) susceptance (d) conductance
(e) reactance

9.7 The impedance of a capacitor increases with increasing frequency.

(a) True (b) False

9.8 At what frequency will the output voltage $v_o(t)$ in Fig. 9.39 be equal to the input voltage $v(t)$?

(a) 0 rad/s (b) 1 rad/s (c) 4 rad/s
(d) ∞ rad/s (e) none of the above

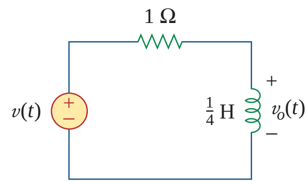


Figure 9.39
For Review Question 9.8.

9.9 A series RC circuit has $|V_R| = 12$ V and $|V_C| = 5$ V. The magnitude of the supply voltage is:

(a) −7 V (b) 7 V (c) 13 V (d) 17 V

9.10 A series RCL circuit has $R = 30\ \Omega$, $X_C = 50\ \Omega$, and $X_L = 90\ \Omega$. The impedance of the circuit is:

(a) $30 + j140\ \Omega$ (b) $30 + j40\ \Omega$
(c) $30 - j40\ \Omega$ (d) $-30 - j40\ \Omega$
(e) $-30 + j40\ \Omega$

Answers: 9.1d, 9.2c, 9.3b, 9.4b,d, 9.5a, 9.6e, 9.7b, 9.8d, 9.9c, 9.10b.

Problems

Section 9.2 Sinusoids

9.1 Given the sinusoidal voltage $v(t) = 50 \cos(30t + 10°)$ V, find: (a) the amplitude V_m, (b) the period T, (c) the frequency f, and (d) $v(t)$ at $t = 10$ ms.

9.2 A current source in a linear circuit has

$$i_s = 8 \cos(500\pi t - 25°) \text{ A}$$

(a) What is the amplitude of the current?
(b) What is the angular frequency?
(c) Find the frequency of the current.
(d) Calculate i_s at $t = 2$ ms.

9.3 Express the following functions in cosine form:

(a) $4 \sin(\omega t - 30°)$ (b) $-2 \sin 6t$
(c) $-10 \sin(\omega t + 20°)$

9.4 Design a problem to help other students better understand sinusoids.

9.5 Given $v_1 = 20 \sin(\omega t + 60^\circ)$ and $v_2 = 60 \cos(\omega t - 10^\circ)$, determine the phase angle between the two sinusoids and which one lags the other.

9.6 For the following pairs of sinusoids, determine which one leads and by how much.

(a) $v(t) = 10 \cos(4t - 60^\circ)$ and $i(t) = 4 \sin(4t + 50^\circ)$

(b) $v_1(t) = 4 \cos(377t + 10^\circ)$ and $v_2(t) = -20 \cos 377t$

(c) $x(t) = 13 \cos 2t + 5 \sin 2t$ and $y(t) = 15 \cos(2t - 11.8^\circ)$

Section 9.3 Phasors

9.7 If $f(\phi) = \cos\phi + j \sin\phi$, show that $f(\phi) = e^{j\phi}$.

9.8 Calculate these complex numbers and express your results in rectangular form:

(a) $\dfrac{15\angle 45^\circ}{3 - j4} + j2$

(b) $\dfrac{8\angle -20^\circ}{(2 + j)(3 - j4)} + \dfrac{10}{-5 + j12}$

(c) $10 + (8\angle 50^\circ)(5 - j12)$

9.9 Evaluate the following complex numbers and leave your results in polar form:

(a) $5\angle 30^\circ\left(6 - j8 + \dfrac{3\angle 60^\circ}{2 + j}\right)$

(b) $\dfrac{(10\angle 60^\circ)(35\angle -50^\circ)}{(2 + j6) - (5 + j)}$

9.10 Design a problem to help other students better understand phasors.

9.11 Find the phasors corresponding to the following signals:

(a) $v(t) = 21 \cos(4t - 15^\circ)$ V

(b) $i(t) = -8 \sin(10t + 70^\circ)$ mA

(c) $v(t) = 120 \sin(10t - 50^\circ)$ V

(d) $i(t) = -60 \cos(30t + 10^\circ)$ mA

9.12 Let $\mathbf{X} = 8\angle 40^\circ$ and $\mathbf{Y} = 10\angle -30^\circ$. Evaluate the following quantities and express your results in polar form:

(a) $(\mathbf{X} + \mathbf{Y})\mathbf{X}^*$

(b) $(\mathbf{X} - \mathbf{Y})^*$

(c) $(\mathbf{X} + \mathbf{Y})/\mathbf{X}$

9.13 Evaluate the following complex numbers:

(a) $\dfrac{2 + j3}{1 - j6} + \dfrac{7 - j8}{-5 + j11}$

(b) $\dfrac{(5\angle 10^\circ)(10\angle -40^\circ)}{(4\angle -80^\circ)(-6\angle 50^\circ)}$

(c) $\begin{vmatrix} 2 + j3 & -j2 \\ -j2 & 8 - j5 \end{vmatrix}$

9.14 Simplify the following expressions:

(a) $\dfrac{(5 - j6) - (2 + j8)}{(-3 + j4)(5 - j) + (4 - j6)}$

(b) $\dfrac{(240\angle 75^\circ + 160\angle -30^\circ)(60 - j80)}{(67 + j84)(20\angle 32^\circ)}$

(c) $\left(\dfrac{10 + j20}{3 + j4}\right)^2 \sqrt{(10 + j5)(16 - j20)}$

9.15 Evaluate these determinants:

(a) $\begin{vmatrix} 10 + j6 & 2 - j3 \\ -5 & -1 + j \end{vmatrix}$

(b) $\begin{vmatrix} 20\angle -30^\circ & -4\angle -10^\circ \\ 16\angle 0^\circ & 3\angle 45^\circ \end{vmatrix}$

(c) $\begin{vmatrix} 1 - j & -j & 0 \\ j & 1 & -j \\ 1 & j & 1 + j \end{vmatrix}$

9.16 Transform the following sinusoids to phasors:

(a) $-10 \cos(4t + 75^\circ)$ (b) $5 \sin(20t - 10^\circ)$

(c) $4 \cos 2t + 3 \sin 2t$

9.17 Two voltages v_1 and v_2 appear in series so that their sum is $v = v_1 + v_2$. If $v_1 = 10 \cos(50t - \pi/3)$ V and $v_2 = 12 \cos(50t + 30^\circ)$ V, find v.

9.18 Obtain the sinusoids corresponding to each of the following phasors:

(a) $\mathbf{V}_1 = 60\angle 15^\circ$ V, $\omega = 1$

(b) $\mathbf{V}_2 = 6 + j8$ V, $\omega = 40$

(c) $\mathbf{I}_1 = 2.8e^{-j\pi/3}$ A, $\omega = 377$

(d) $\mathbf{I}_2 = -0.5 - j1.2$ A, $\omega = 10^3$

9.19 Using phasors, find:

(a) $3 \cos(20t + 10^\circ) - 5 \cos(20t - 30^\circ)$

(b) $40 \sin 50t + 30 \cos(50t - 45^\circ)$

(c) $20 \sin 400t + 10 \cos(400t + 60^\circ) - 5 \sin(400t - 20^\circ)$

9.20 A linear network has a current input $4 \cos(\omega t + 20^\circ)$ A and a voltage output $10 \cos(\omega t + 110^\circ)$ V. Determine the associated impedance.

9.21 Simplify the following:

(a) $f(t) = 5\cos(2t + 15^\circ) - 4\sin(2t - 30^\circ)$

(b) $g(t) = 8\sin t + 4\cos(t + 50^\circ)$

(c) $h(t) = \displaystyle\int_0^t (10\cos 40t + 50\sin 40t)\,dt$

9.22 An alternating voltage is given by $v(t) = 20\cos(5t - 30^\circ)$ V. Use phasors to find

$$10v(t) + 4\frac{dv}{dt} - 2\int_{-\infty}^{t} v(t)\,dt$$

Assume that the value of the integral is zero at $t = -\infty$.

9.23 Apply phasor analysis to evaluate the following.

(a) $v = 50\cos(\omega t + 30^\circ) + 30\cos(\omega t - 90^\circ)$ V

(b) $i = 15\cos(\omega t + 45^\circ) - 10\sin(\omega t + 45^\circ)$ A

9.24 Find $v(t)$ in the following integrodifferential equations using the phasor approach:

(a) $v(t) + \displaystyle\int v\,dt = 5\cos(t + 45^\circ)$ V

(b) $\dfrac{dv}{dt} + 5v(t) + 4\displaystyle\int v\,dt = 20\sin(4t + 10^\circ)$ V

9.25 Using phasors, determine $i(t)$ in the following equations:

(a) $2\dfrac{di}{dt} + 3i(t) = 4\cos(2t - 45^\circ)$

(b) $10\displaystyle\int i\,dt + \dfrac{di}{dt} + 6i(t) = 5\cos(5t + 22^\circ)$ A

9.26 The loop equation for a series *RLC* circuit gives

$$\frac{di}{dt} + 2i + \int_{-\infty}^{t} i\,dt = \cos 2t \text{ A}$$

Assuming that the value of the integral at $t = -\infty$ is zero, find $i(t)$ using the phasor method.

9.27 A parallel *RLC* circuit has the node equation

$$\frac{dv}{dt} + 50v + 100\int v\,dt = 110\cos(377t - 10^\circ) \text{ V}$$

Determine $v(t)$ using the phasor method. You may assume that the value of the integral at $t = -\infty$ is zero.

Section 9.4 Phasor Relationships for Circuit Elements

9.28 Determine the current that flows through an 8-Ω resistor connected to a voltage source $v_s = 110\cos 377t$ V.

9.29 What is the instantaneous voltage across a 2-μF capacitor when the current through it is $i = 4\sin(10^6 t + 25^\circ)$ A?

9.30 A voltage $v(t) = 100\cos(60t + 20^\circ)$ V is applied to a parallel combination of a 40-kΩ resistor and a 50-μF capacitor. Find the steady-state currents through the resistor and the capacitor.

9.31 A series *RLC* circuit has $R = 80\ \Omega$, $L = 240$ mH, and $C = 5$ mF. If the input voltage is $v(t) = 10\cos 2t$, find the currrent flowing through the circuit.

9.32 Using Fig. 9.40, design a problem to help other students better understand phasor relationships for circuit elements.

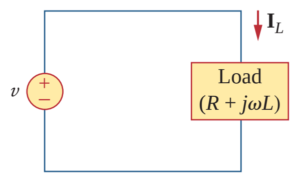


Figure 9.40
For Prob. 9.32.

9.33 A series *RL* circuit is connected to a 110-V ac source. If the voltage across the resistor is 85 V, find the voltage across the inductor.

9.34 What value of ω will cause the forced response v_o in Fig. 9.41 to be zero?

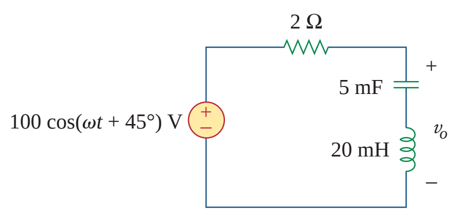


Figure 9.41
For Prob. 9.34.

Section 9.5 Impedance and Admittance

9.35 Find current i in the circuit of Fig. 9.42, when $v_s(t) = 50\cos 200t$ V.

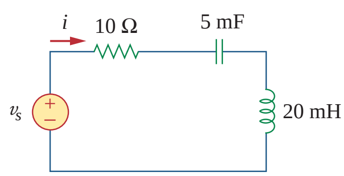


Figure 9.42
For Prob. 9.35.

9.36 Using Fig. 9.43, design a problem to help other students better understand impedance.

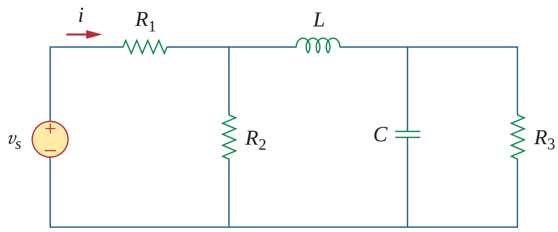


Figure 9.43
For Prob. 9.36.

9.37 Determine the admittance **Y** for the circuit in Fig. 9.44.

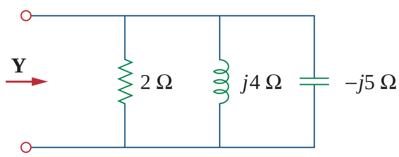


Figure 9.44
For Prob. 9.37.

9.38 Using Fig. 9.45, design a problem to help other students better understand admittance.

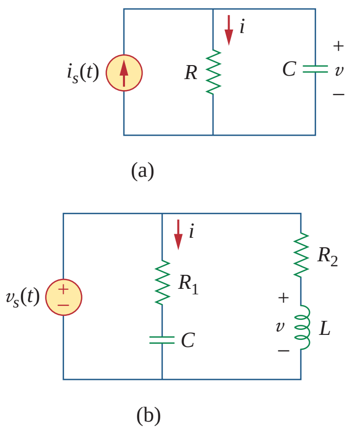


Figure 9.45
For Prob. 9.38.

9.39 For the circuit shown in Fig. 9.46, find Z_{eq} and use that to find current **I**. Let $\omega = 10$ rad/s.

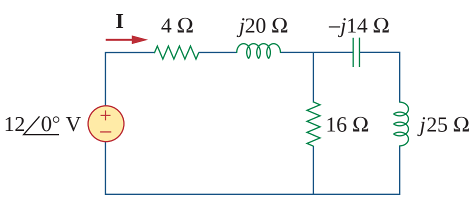


Figure 9.46
For Prob. 9.39.

9.40 In the circuit of Fig. 9.47, find i_o when:

(a) $\omega = 1$ rad/s (b) $\omega = 5$ rad/s
(c) $\omega = 10$ rad/s

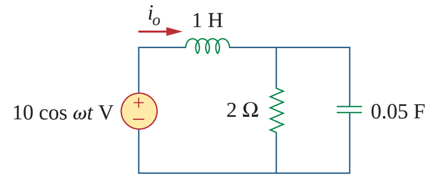


Figure 9.47
For Prob. 9.40.

9.41 Find $v(t)$ in the *RLC* circuit of Fig. 9.48.

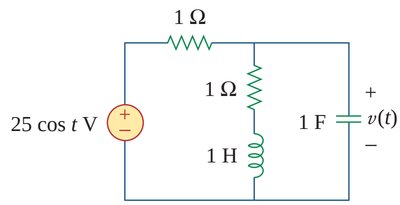


Figure 9.48
For Prob. 9.41.

9.42 Calculate $v_o(t)$ in the circuit of Fig. 9.49.

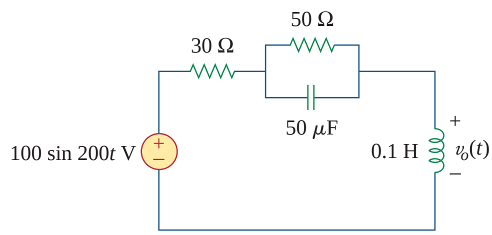


Figure 9.49
For Prob. 9.42.

9.43 Find current $\mathbf{I}_o$ in the circuit shown in Fig. 9.50.

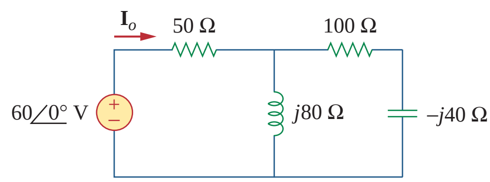


Figure 9.50
For Prob. 9.43.

9.44 Calculate $i(t)$ in the circuit of Fig. 9.51.

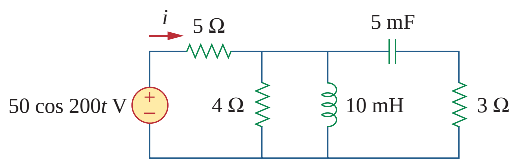


Figure 9.51
For prob. 9.44.

9.45 Find current $\mathbf{I}_o$ in the network of Fig. 9.52.

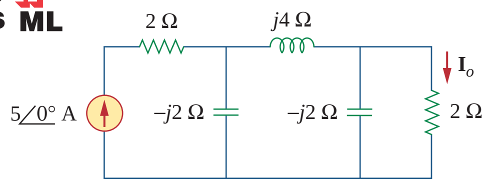


Figure 9.52
For Prob. 9.45.

9.46 If $i_s = 20 \cos(10t + 15°)$ A in the circuit of Fig. 9.53, find i_o.

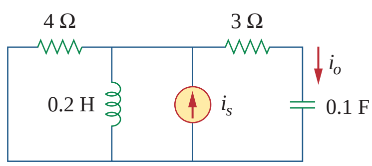


Figure 9.53
For Prob. 9.46.

9.47 In the circuit of Fig. 9.54, determine the value of $i_s(t)$.

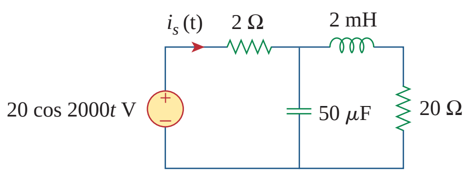


Figure 9.54
For Prob. 9.47.

9.48 Given that $v_s(t) = 20 \sin(100t - 40°)$ in Fig. 9.55, determine $i_x(t)$.

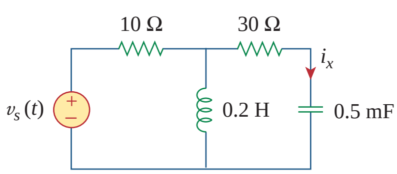


Figure 9.55
For Prob. 9.48.

9.49 Find $v_s(t)$ in the circuit of Fig. 9.56 if the current i_x through the 1-Ω resistor is $0.5 \sin 200t$ A.

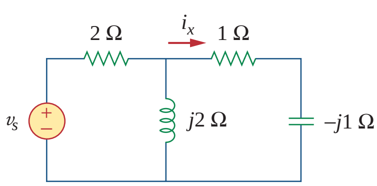


Figure 9.56
For Prob. 9.49.

9.50 Determine v_x in the circuit of Fig. 9.57. Let $i_s(t) = 5 \cos(100t + 40°)$ A.

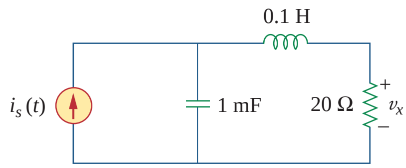


Figure 9.57
For Prob. 9.50.

9.51 If the voltage v_o across the 2-Ω resistor in the circuit of Fig. 9.58 is $-5 \cos 2t$ V, obtain i_s.

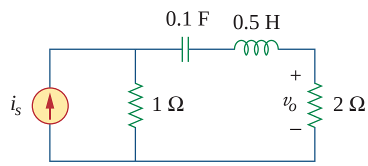


Figure 9.58
For Prob. 9.51.

9.52 If $\mathbf{V}_o = 20/45°$ V in the circuit of Fig. 9.59, find $\mathbf{I}_s$.

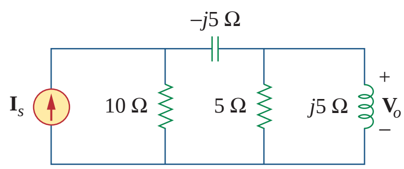


Figure 9.59
For Prob. 9.52.

9.53 Find $\mathbf{I}_o$ in the circuit of Fig. 9.60.

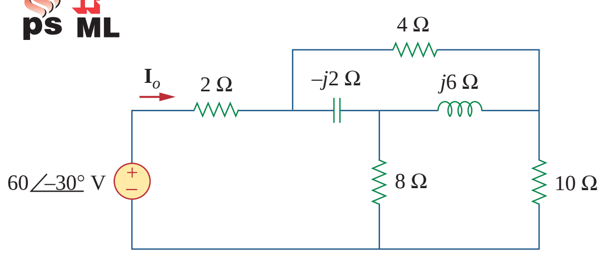


Figure 9.60
For Prob. 9.53.

9.54 In the circuit of Fig. 9.61, find $\mathbf{V}_s$ if $\mathbf{I}_o = 2/0°$ A.

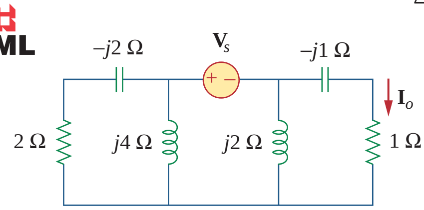


Figure 9.61
For Prob. 9.54.

***9.55** Find $\mathbf{Z}$ in the network of Fig. 9.62, given that $\mathbf{V}_o = 8\underline{/0^\circ}$ V.

ML

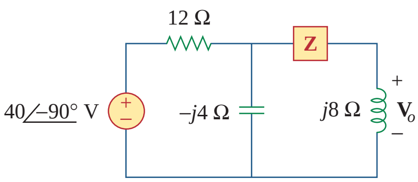


Figure 9.62
For Prob. 9.55.

Section 9.7 Impedance Combinations

9.56 At $\omega = 377$ rad/s, find the input impedance of the circuit shown in Fig. 9.63.

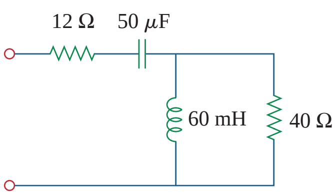


Figure 9.63
For Prob. 9.56.

9.57 At $\omega = 1$ rad/s, obtain the input admittance in the circuit of Fig. 9.64.

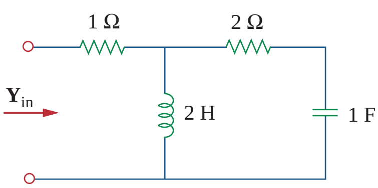


Figure 9.64
For Prob. 9.57.

9.58 Using Fig. 9.65, design a problem to help other students better understand impedance combinations.

e2d

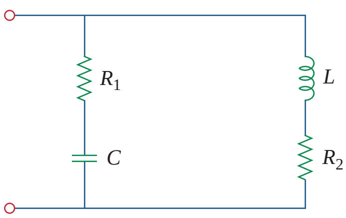


Figure 9.65
For Prob. 9.58.

* An asterisk indicates a challenging problem.

9.59 For the network in Fig. 9.66, find $\mathbf{Z}_{\text{in}}$. Let $\omega = 10$ rad/s.

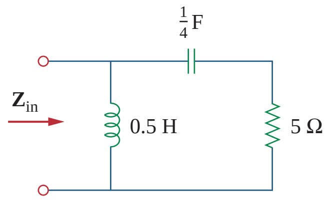


Figure 9.66
For Prob. 9.59.

9.60 Obtain $\mathbf{Z}_{\text{in}}$ for the circuit in Fig. 9.67.

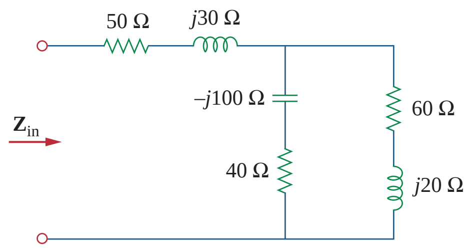


Figure 9.67
For Prob. 9.60.

9.61 Find $\mathbf{Z}_{\text{eq}}$ in the circuit of Fig. 9.68.

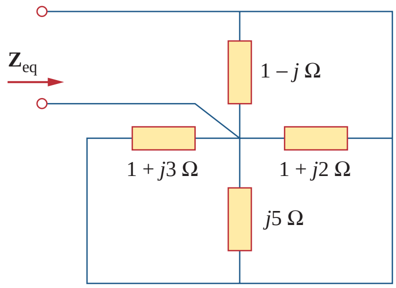


Figure 9.68
For Prob. 9.61.

9.62 For the circuit in Fig. 9.69, find the input impedance $\mathbf{Z}_{\text{in}}$ at 10 krad/s.

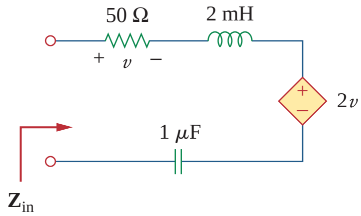


Figure 9.69
For Prob. 9.62.

9.63 For the circuit in Fig. 9.70, find the value of $\mathbf{Z}_T$.

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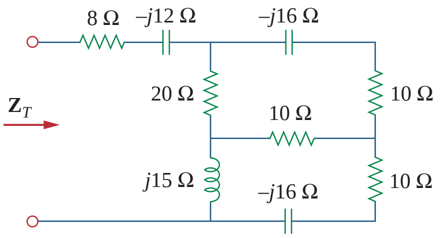

Figure 9.70
For Prob. 9.63.

9.64 Find $\mathbf{Z}_T$ and $\mathbf{I}$ in the circuit of Fig. 9.71.

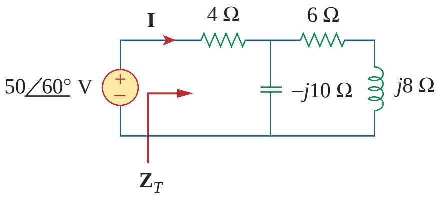

Figure 9.71
For Prob. 9.64.

9.65 Determine $\mathbf{Z}_T$ and $\mathbf{I}$ for the circuit in Fig. 9.72.

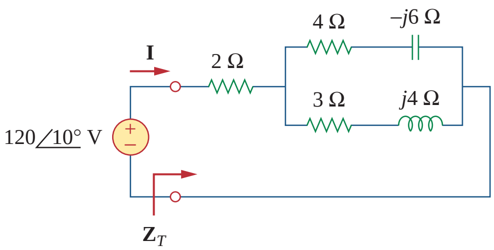

Figure 9.72
For Prob. 9.65.

9.66 For the circuit in Fig. 9.73, calculate $\mathbf{Z}_T$ and $\mathbf{V}_{ab}$.

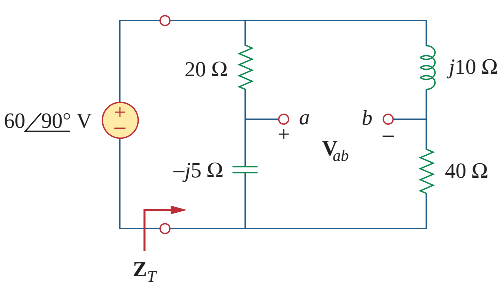

Figure 9.73
For Prob. 9.66.

9.67 At $\omega = 10^3$ rad/s, find the input admittance of each of the circuits in Fig. 9.74.

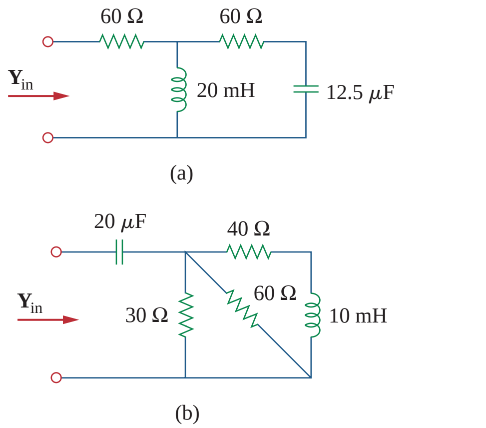

Figure 9.74
For Prob. 9.67.

9.68 Determine $\mathbf{Y}_{eq}$ for the circuit in Fig. 9.75.

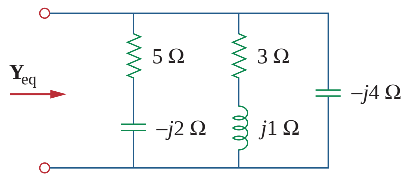

Figure 9.75
For Prob. 9.68.

9.69 Find the equivalent admittance $\mathbf{Y}_{eq}$ of the circuit in Fig. 9.76.

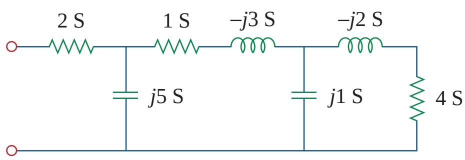

Figure 9.76
For Prob. 9.69.

9.70 Find the equivalent impedance of the circuit in Fig. 9.77.

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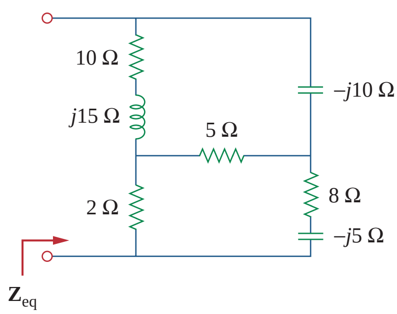


Figure 9.77
For Prob. 9.70.

9.71 Obtain the equivalent impedance of the circuit in Fig. 9.78.

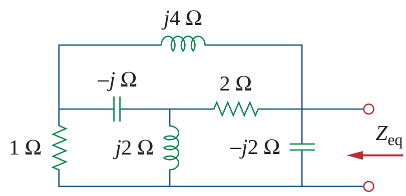


Figure 9.78
For Prob. 9.71.

9.72 Calculate the value of $\mathbf{Z}_{ab}$ in the network of Fig. 9.79.

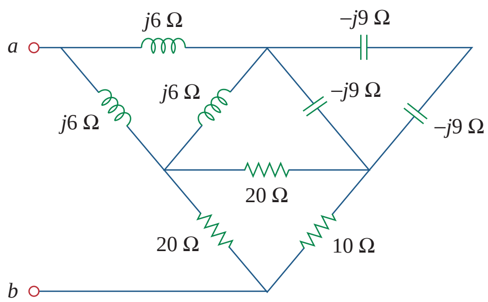


Figure 9.79
For Prob. 9.72.

9.73 Determine the equivalent impedance of the circuit in Fig. 9.80.

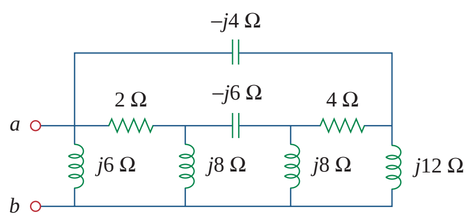


Figure 9.80
For Prob. 9.73.

Section 9.8 Applications

9.74 Design an *RL* circuit to provide a 90° leading phase shift.

9.75 Design a circuit that will transform a sinusoidal voltage input to a cosinusoidal voltage output.

9.76 For the following pairs of signals, determine if v_1 leads or lags v_2 and by how much.

(a) $v_1 = 10\cos(5t - 20°)$, $\quad v_2 = 8\sin 5t$
(b) $v_1 = 19\cos(2t + 90°)$, $\quad v_2 = 6\sin 2t$
(c) $v_1 = -4\cos 10t$, $\quad v_2 = 15\sin 10t$

9.77 Refer to the *RC* circuit in Fig. 9.81.

(a) Calculate the phase shift at 2 MHz.
(b) Find the frequency where the phase shift is 45°.

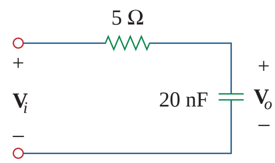


Figure 9.81
For Prob. 9.77.

9.78 A coil with impedance $8 + j6\ \Omega$ is connected in series with a capacitive reactance X. The series combination is connected in parallel with a resistor R. Given that the equivalent impedance of the resulting circuit is $5\angle 0°\ \Omega$, find the value of R and X.

9.79 (a) Calculate the phase shift of the circuit in Fig. 9.82.
(b) State whether the phase shift is leading or lagging (output with respect to input).
(c) Determine the magnitude of the output when the input is 120 V.

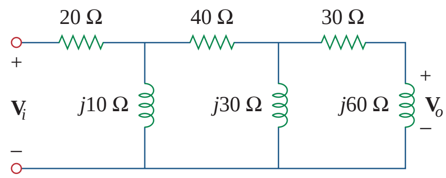


Figure 9.82
For Prob. 9.79.

9.80 Consider the phase-shifting circuit in Fig. 9.83. Let $\mathbf{V}_i = 120$ V operating at 60 Hz. Find:

(a) $\mathbf{V}_o$ when R is maximum
(b) $\mathbf{V}_o$ when R is minimum
(c) the value of R that will produce a phase shift of 45°

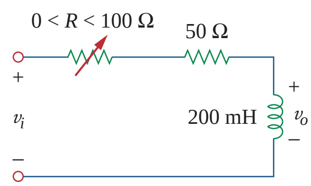


Figure 9.83
For Prob. 9.80.

9.81 The ac bridge in Fig. 9.37 is balanced when $R_1 = 400\ \Omega$, $R_2 = 600\ \Omega$, $R_3 = 1.2\ \text{k}\Omega$, and $C_2 = 0.3\ \mu\text{F}$. Find R_x and C_x. Assume R_2 and C_2 are in series.

9.82 A capacitance bridge balances when $R_1 = 100\ \Omega$, $R_2 = 2\ \text{k}\Omega$, and $C_s = 40\ \mu\text{F}$. What is C_x, the capacitance of the capacitor under test?

9.83 An inductive bridge balances when $R_1 = 1.2\ \text{k}\Omega$, $R_2 = 500\ \Omega$, and $L_s = 250$ mH. What is the value of L_x, the inductance of the inductor under test?

9.84 The ac bridge shown in Fig. 9.84 is known as a *Maxwell bridge* and is used for accurate measurement of inductance and resistance of a coil in terms of a standard capacitance C_s. Show that when the bridge is balanced,

$$L_x = R_2 R_3 C_s \qquad \text{and} \qquad R_x = \frac{R_2}{R_1} R_3$$

Find L_x and R_x for $R_1 = 40\ \text{k}\Omega$, $R_2 = 1.6\ \text{k}\Omega$, $R_3 = 4\ \text{k}\Omega$, and $C_s = 0.45\ \mu\text{F}$.

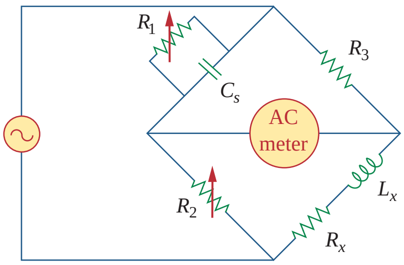


Figure 9.84
Maxwell bridge; For Prob. 9.84.

9.85 The ac bridge circuit of Fig. 9.85 is called a *Wien bridge*. It is used for measuring the frequency of a source. Show that when the bridge is balanced,

$$f = \frac{1}{2\pi\sqrt{R_2 R_4 C_2 C_4}}$$

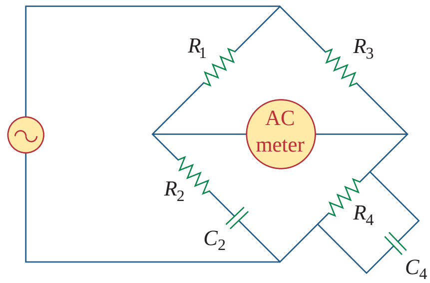


Figure 9.85
Wein bridge; For Prob. 9.85.

Comprehensive Problems

9.86 The circuit shown in Fig. 9.86 is used in a television receiver. What is the total impedance of this circuit?

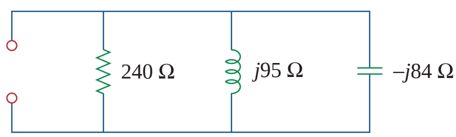


Figure 9.86
For Prob. 9.86.

9.87 The network in Fig. 9.87 is part of the schematic describing an industrial electronic sensing device. What is the total impedance of the circuit at 2 kHz?

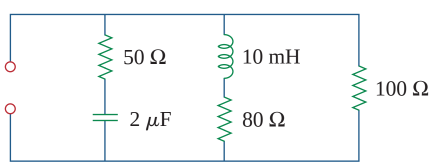


Figure 9.87
For Prob. 9.87.

9.88 A series audio circuit is shown in Fig. 9.88.

(a) What is the impedance of the circuit?

(b) If the frequency were halved, what would be the impedance of the circuit?

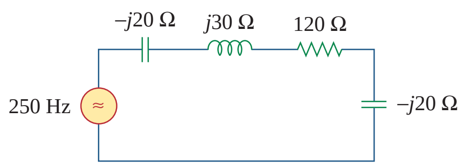


Figure 9.88
For Prob. 9.88.

9.89 An industrial load is modeled as a series combination of a capacitance and a resistance as shown in Fig. 9.89. Calculate the value of an inductance L across the series combination so that the net impedance is resistive at a frequency of 50 kHz.

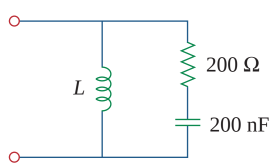


Figure 9.89
For Prob. 9.89.

9.90 An industrial coil is modeled as a series combination of an inductance L and resistance R, as shown in Fig. 9.90. Since an ac voltmeter measures only the magnitude of a sinusoid, the following measurements are taken at 60 Hz when the circuit operates in the steady state:

$$|\mathbf{V}_s| = 145 \text{ V}, \quad |\mathbf{V}_1| = 50 \text{ V}, \quad |\mathbf{V}_o| = 110 \text{ V}$$

Use these measurements to determine the values of L and R.

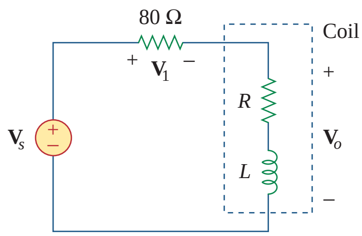


Figure 9.90
For Prob. 9.90.

9.91 Figure 9.91 shows a parallel combination of an inductance and a resistance. If it is desired to connect a capacitor in series with the parallel combination such that the net impedance is resistive at 10 MHz, what is the required value of C?

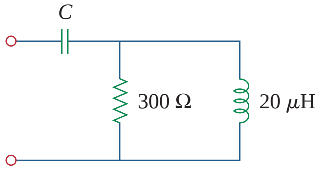


Figure 9.91
For Prob. 9.91.

9.92 A transmission line has a series impedance of $\mathbf{Z} = 100\angle 75^\circ\ \Omega$ and a shunt admittance of $\mathbf{Y} = 450\angle 48^\circ\ \mu$S. Find: (a) the characteristic impedance $\mathbf{Z}_o = \sqrt{\mathbf{Z}/\mathbf{Y}}$, (b) the propagation constant $\gamma = \sqrt{\mathbf{ZY}}$.

9.93 A power transmission system is modeled as shown in Fig. 9.92. Given the following;

Source voltage	$\mathbf{V}_s = 115\angle 0^\circ$ V,
Source impedance	$\mathbf{Z}_s = (2 + j)\ \Omega$,
Line impedance	$\mathbf{Z}_\ell = (0.8 + j0.6)\ \Omega$,
Load impedance	$\mathbf{Z}_L = (46.4 + j37.8)\ \Omega$,

Find the load current $\mathbf{I}_L$.

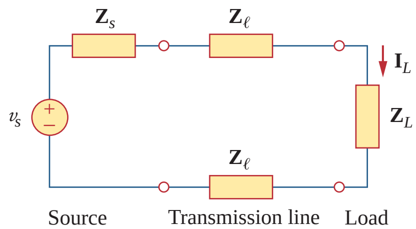


Figure 9.92
For Prob. 9.93.

chapter

10

Sinusoidal Steady-State Analysis

Three men are my friends—he that loves me, he that hates me, he that is indifferent to me. Who loves me, teaches me tenderness; who hates me, teaches me caution; who is indifferent to me, teaches me self-reliance.

—J. E. Dinger

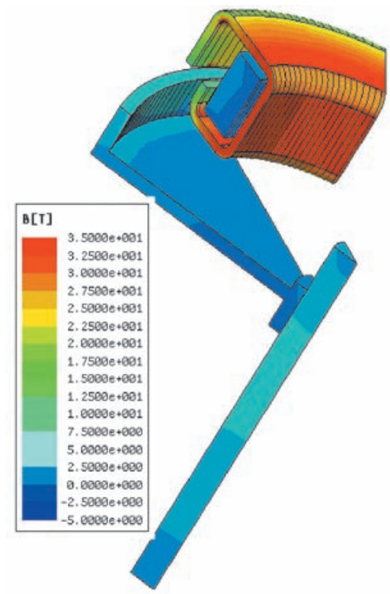
Enhancing Your Career

Career in Software Engineering

Software engineering is that aspect of engineering that deals with the practical application of scientific knowledge in the design, construction, and validation of computer programs and the associated documentation required to develop, operate, and maintain them. It is a branch of electrical engineering that is becoming increasingly important as more and more disciplines require one form of software package or another to perform routine tasks and as programmable microelectronic systems are used in more and more applications.

The role of a software engineer should not be confused with that of a computer scientist; the software engineer is a practitioner, not a theoretician. A software engineer should have good computer-programming skills and be familiar with programming languages, in particular C^{++}, which is becoming increasingly popular. Because hardware and software are closely interlinked, it is essential that a software engineer have a thorough understanding of hardware design. Most important, the software engineer should have some specialized knowledge of the area in which the software development skill is to be applied.

All in all, the field of software engineering offers a great career to those who enjoy programming and developing software packages. The higher rewards will go to those having the best preparation, with the most interesting and challenging opportunities going to those with graduate education.



Output of a modeling software.
Courtesy Ansoft

10.1 Introduction

In Chapter 9, we learned that the forced or steady-state response of circuits to sinusoidal inputs can be obtained by using phasors. We also know that Ohm's and Kirchhoff's laws are applicable to ac circuits. In this chapter, we want to see how nodal analysis, mesh analysis, Thevenin's theorem, Norton's theorem, superposition, and source transformations are applied in analyzing ac circuits. Since these techniques were already introduced for dc circuits, our major effort here will be to illustrate with examples.

Analyzing ac circuits usually requires three steps.

Steps to Analyze AC Circuits:

1. Transform the circuit to the phasor or frequency domain.
2. Solve the problem using circuit techniques (nodal analysis, mesh analysis, superposition, etc.).
3. Transform the resulting phasor to the time domain.

Step 1 is not necessary if the problem is specified in the frequency domain. In step 2, the analysis is performed in the same manner as dc circuit analysis except that complex numbers are involved. Having read Chapter 9, we are adept at handling step 3.

Frequency domain analysis of an ac circuit via phasors is much easier than analysis of the circuit in the time domain.

Toward the end of the chapter, we learn how to apply *PSpice* in solving ac circuit problems. We finally apply ac circuit analysis to two practical ac circuits: oscillators and ac transistor circuits.

10.2 Nodal Analysis

The basis of nodal analysis is Kirchhoff's current law. Since KCL is valid for phasors, as demonstrated in Section 9.6, we can analyze ac circuits by nodal analysis. The following examples illustrate this.

Example 10.1

Find i_x in the circuit of Fig. 10.1 using nodal analysis.

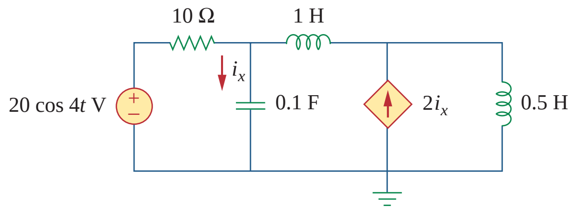


Figure 10.1
For Example 10.1.

Solution:

We first convert the circuit to the frequency domain:

$$\begin{aligned}
20 \cos 4t \quad &\Rightarrow \quad 20\underline{/0^\circ}, \qquad \omega = 4 \text{ rad/s} \\
1 \text{ H} \quad &\Rightarrow \quad j\omega L = j4 \\
0.5 \text{ H} \quad &\Rightarrow \quad j\omega L = j2 \\
0.1 \text{ F} \quad &\Rightarrow \quad \frac{1}{j\omega C} = -j2.5
\end{aligned}$$

Thus, the frequency domain equivalent circuit is as shown in Fig. 10.2.

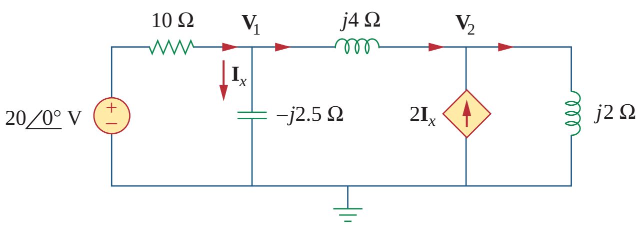


Figure 10.2
Frequency domain equivalent of the circuit in Fig. 10.1.

Applying KCL at node 1,

$$\frac{20 - \mathbf{V}_1}{10} = \frac{\mathbf{V}_1}{-j2.5} + \frac{\mathbf{V}_1 - \mathbf{V}_2}{j4}$$

or

$$(1 + j1.5)\mathbf{V}_1 + j2.5\mathbf{V}_2 = 20 \tag{10.1.1}$$

At node 2,

$$2\mathbf{I}_x + \frac{\mathbf{V}_1 - \mathbf{V}_2}{j4} = \frac{\mathbf{V}_2}{j2}$$

But $\mathbf{I}_x = \mathbf{V}_1/-j2.5$. Substituting this gives

$$\frac{2\mathbf{V}_1}{-j2.5} + \frac{\mathbf{V}_1 - \mathbf{V}_2}{j4} = \frac{\mathbf{V}_2}{j2}$$

By simplifying, we get

$$11\mathbf{V}_1 + 15\mathbf{V}_2 = 0 \tag{10.1.2}$$

Equations (10.1.1) and (10.1.2) can be put in matrix form as

$$\begin{bmatrix} 1 + j1.5 & j2.5 \\ 11 & 15 \end{bmatrix} \begin{bmatrix} \mathbf{V}_1 \\ \mathbf{V}_2 \end{bmatrix} = \begin{bmatrix} 20 \\ 0 \end{bmatrix}$$

We obtain the determinants as

$$\Delta = \begin{vmatrix} 1 + j1.5 & j2.5 \\ 11 & 15 \end{vmatrix} = 15 - j5$$

$$\Delta_1 = \begin{vmatrix} 20 & j2.5 \\ 0 & 15 \end{vmatrix} = 300, \qquad \Delta_2 = \begin{vmatrix} 1 + j1.5 & 20 \\ 11 & 0 \end{vmatrix} = -220$$

$$\mathbf{V}_1 = \frac{\Delta_1}{\Delta} = \frac{300}{15 - j5} = 18.97\underline{/18.43^\circ} \text{ V}$$

$$\mathbf{V}_2 = \frac{\Delta_2}{\Delta} = \frac{-220}{15 - j5} = 13.91\underline{/198.3^\circ} \text{ V}$$

The current $\mathbf{I}_x$ is given by

$$\mathbf{I}_x = \frac{\mathbf{V}_1}{-j2.5} = \frac{18.97\underline{/18.43^\circ}}{2.5\underline{/-90^\circ}} = 7.59\underline{/108.4^\circ}\text{ A}$$

Transforming this to the time domain,

$$i_x = 7.59\cos(4t + 108.4^\circ)\text{ A}$$

Practice Problem 10.1

Using nodal analysis, find v_1 and v_2 in the circuit of Fig. 10.3.

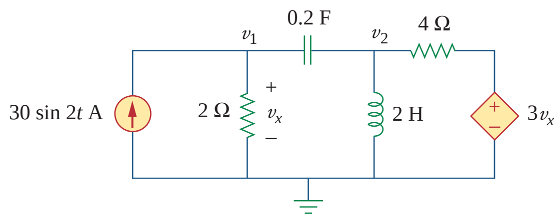


Figure 10.3
For Practice Prob. 10.1.

Answer: $v_1(t) = 33.96\sin(2t + 60.01^\circ)$ V,
$v_2(t) = 99.06\sin(2t + 57.12^\circ)$ V.

Example 10.2

Compute $\mathbf{V}_1$ and $\mathbf{V}_2$ in the circuit of Fig. 10.4.

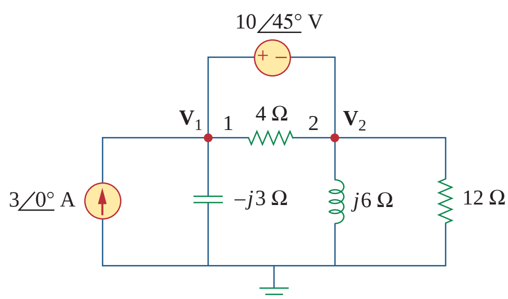


Figure 10.4
For Example 10.2.

Solution:
Nodes 1 and 2 form a supernode as shown in Fig. 10.5. Applying KCL at the supernode gives

$$3 = \frac{\mathbf{V}_1}{-j3} + \frac{\mathbf{V}_2}{j6} + \frac{\mathbf{V}_2}{12}$$

or

$$36 = j4\mathbf{V}_1 + (1 - j2)\mathbf{V}_2 \tag{10.2.1}$$

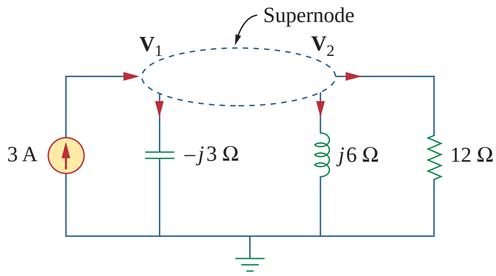


Figure 10.5
A supernode in the circuit of Fig. 10.4.

But a voltage source is connected between nodes 1 and 2, so that

$$\mathbf{V}_1 = \mathbf{V}_2 + 10\underline{/45^\circ} \qquad \textbf{(10.2.2)}$$

Substituting Eq. (10.2.2) in Eq. (10.2.1) results in

$$36 - 40\underline{/135^\circ} = (1 + j2)\mathbf{V}_2 \qquad \Rightarrow \qquad \mathbf{V}_2 = 31.41\underline{/-87.18^\circ}\text{ V}$$

From Eq. (10.2.2),

$$\mathbf{V}_1 = \mathbf{V}_2 + 10\underline{/45^\circ} = 25.78\underline{/-70.48^\circ}\text{ V}$$

Practice Problem 10.2

Calculate $\mathbf{V}_1$ and $\mathbf{V}_2$ in the circuit shown in Fig. 10.6.

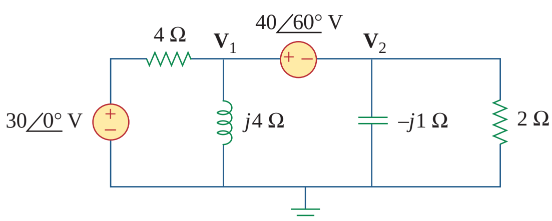


Figure 10.6
For Practice Prob. 10.2.

Answer: $\mathbf{V}_1 = 38.72\underline{/69.67^\circ}$ V, $\mathbf{V}_2 = 6.752\underline{/165.7^\circ}$ V.

10.3 Mesh Analysis

Kirchhoff's voltage law (KVL) forms the basis of mesh analysis. The validity of KVL for ac circuits was shown in Section 9.6 and is illustrated in the following examples. Keep in mind that the very nature of using mesh analysis is that it is to be applied to planar circuits.

Example 10.3

Determine current $\mathbf{I}_o$ in the circuit of Fig. 10.7 using mesh analysis.

Solution:
Applying KVL to mesh 1, we obtain

$$(8 + j10 - j2)\mathbf{I}_1 - (-j2)\mathbf{I}_2 - j10\mathbf{I}_3 = 0 \qquad \textbf{(10.3.1)}$$

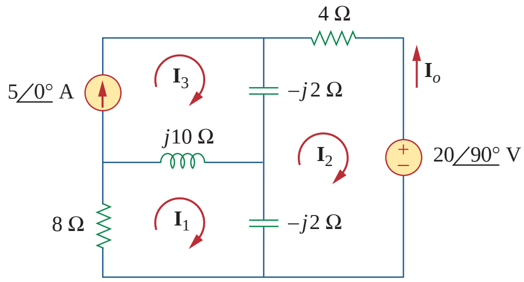


Figure 10.7
For Example 10.3.

For mesh 2,

$$(4 - j2 - j2)\mathbf{I}_2 - (-j2)\mathbf{I}_1 - (-j2)\mathbf{I}_3 + 20\underline{/90°} = 0 \qquad \textbf{(10.3.2)}$$

For mesh 3, $\mathbf{I}_3 = 5$. Substituting this in Eqs. (10.3.1) and (10.3.2), we get

$$(8 + j8)\mathbf{I}_1 + j2\mathbf{I}_2 = j50 \qquad \textbf{(10.3.3)}$$

$$j2\mathbf{I}_1 + (4 - j4)\mathbf{I}_2 = -j20 - j10 \qquad \textbf{(10.3.4)}$$

Equations (10.3.3) and (10.3.4) can be put in matrix form as

$$\begin{bmatrix} 8 + j8 & j2 \\ j2 & 4 - j4 \end{bmatrix} \begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \end{bmatrix} = \begin{bmatrix} j50 \\ -j30 \end{bmatrix}$$

from which we obtain the determinants

$$\Delta = \begin{vmatrix} 8 + j8 & j2 \\ j2 & 4 - j4 \end{vmatrix} = 32(1 + j)(1 - j) + 4 = 68$$

$$\Delta_2 = \begin{vmatrix} 8 + j8 & j50 \\ j2 & -j30 \end{vmatrix} = 340 - j240 = 416.17\underline{/-35.22°}$$

$$\mathbf{I}_2 = \frac{\Delta_2}{\Delta} = \frac{416.17\underline{/-35.22°}}{68} = 6.12\underline{/-35.22°} \text{ A}$$

The desired current is

$$\mathbf{I}_o = -\mathbf{I}_2 = 6.12\underline{/144.78°} \text{ A}$$

Practice Problem 10.3

Find $\mathbf{I}_o$ in Fig. 10.8 using mesh analysis.

Answer: $3.582\underline{/65.45°}$ A.

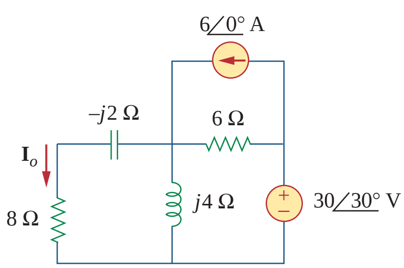


Figure 10.8
For Practice Prob. 10.3.

Example 10.4

Solve for $\mathbf{V}_o$ in the circuit of Fig. 10.9 using mesh analysis.

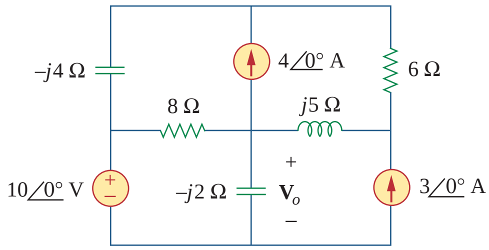


Figure 10.9
For Example 10.4.

Solution:

As shown in Fig. 10.10, meshes 3 and 4 form a supermesh due to the current source between the meshes. For mesh 1, KVL gives

$$-10 + (8 - j2)\mathbf{I}_1 - (-j2)\mathbf{I}_2 - 8\mathbf{I}_3 = 0$$

or

$$(8 - j2)\mathbf{I}_1 + j2\mathbf{I}_2 - 8\mathbf{I}_3 = 10 \tag{10.4.1}$$

For mesh 2,

$$\mathbf{I}_2 = -3 \tag{10.4.2}$$

For the supermesh,

$$(8 - j4)\mathbf{I}_3 - 8\mathbf{I}_1 + (6 + j5)\mathbf{I}_4 - j5\mathbf{I}_2 = 0 \tag{10.4.3}$$

Due to the current source between meshes 3 and 4, at node A,

$$\mathbf{I}_4 = \mathbf{I}_3 + 4 \tag{10.4.4}$$

■ **METHOD 1** Instead of solving the above four equations, we reduce them to two by elimination.

Combining Eqs. (10.4.1) and (10.4.2),

$$(8 - j2)\mathbf{I}_1 - 8\mathbf{I}_3 = 10 + j6 \tag{10.4.5}$$

Combining Eqs. (10.4.2) to (10.4.4),

$$-8\mathbf{I}_1 + (14 + j)\mathbf{I}_3 = -24 - j35 \tag{10.4.6}$$

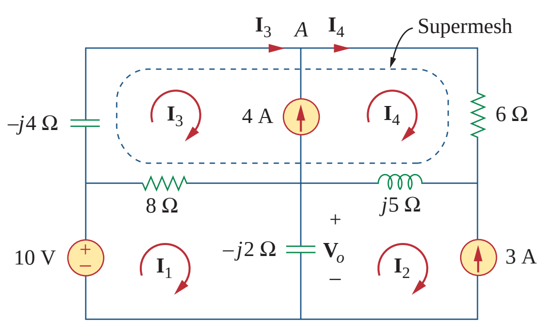


Figure 10.10
Analysis of the circuit in Fig. 10.9.

From Eqs. (10.4.5) and (10.4.6), we obtain the matrix equation

$$\begin{bmatrix} 8 - j2 & -8 \\ -8 & 14 + j \end{bmatrix} \begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_3 \end{bmatrix} = \begin{bmatrix} 10 + j6 \\ -24 - j35 \end{bmatrix}$$

We obtain the following determinants

$$\Delta = \begin{vmatrix} 8 - j2 & -8 \\ -8 & 14 + j \end{vmatrix} = 112 + j8 - j28 + 2 - 64 = 50 - j20$$

$$\Delta_1 = \begin{vmatrix} 10 + j6 & -8 \\ -24 - j35 & 14 + j \end{vmatrix} = 140 + j10 + j84 - 6 - 192 - j280$$
$$= -58 - j186$$

Current $\mathbf{I}_1$ is obtained as

$$\mathbf{I}_1 = \frac{\Delta_1}{\Delta} = \frac{-58 - j186}{50 - j20} = 3.618\underline{/274.5^\circ} \text{ A}$$

The required voltage $\mathbf{V}_0$ is

$$\mathbf{V}_o = -j2(\mathbf{I}_1 - \mathbf{I}_2) = -j2(3.618\underline{/274.5^\circ} + 3)$$
$$= -7.2134 - j6.568 = 9.756\underline{/222.32^\circ} \text{ V}$$

METHOD 2 We can use *MATLAB* to solve Eqs. (10.4.1) to (10.4.4). We first cast the equations as

$$\begin{bmatrix} 8 - j2 & j2 & -8 & 0 \\ 0 & 1 & 0 & 0 \\ -8 & -j5 & 8 - j4 & 6 + j5 \\ 0 & 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \\ \mathbf{I}_3 \\ \mathbf{I}_4 \end{bmatrix} = \begin{bmatrix} 10 \\ -3 \\ 0 \\ 4 \end{bmatrix} \qquad \textbf{(10.4.7a)}$$

or

$$\mathbf{AI} = \mathbf{B}$$

By inverting **A**, we can obtain **I** as

$$\mathbf{I} = \mathbf{A}^{-1}\mathbf{B} \qquad \textbf{(10.4.7b)}$$

We now apply *MATLAB* as follows:

```
>> A = [(8-j*2)  j*2   -8       0;
        0        1     0        0;
        -8       -j*5  (8-j*4)  (6+j*5);
        0        0     -1       1];
>> B = [10 -3 0 4]';
>> I = inv(A)*B

I =
  0.2828 - 3.6069i
  -3.0000
  -1.8690 - 4.4276i
   2.1310 - 4.4276i
>> Vo = -2*j*(I(1) - I(2))

Vo =
  -7.2138 - 6.5655i
```

as obtained previously.

Practice Problem 10.4

Calculate current $\mathbf{I}_o$ in the circuit of Fig. 10.11.

Answer: $2.538\underline{/5.943^\circ}$ A.

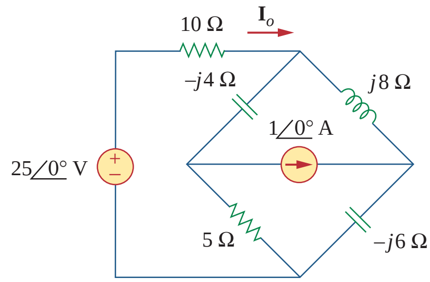


Figure 10.11
For Practice Prob. 10.4.

10.4 Superposition Theorem

Since ac circuits are linear, the superposition theorem applies to ac circuits the same way it applies to dc circuits. The theorem becomes important if the circuit has sources operating at *different* frequencies. In this case, since the impedances depend on frequency, we must have a different frequency domain circuit for each frequency. The total response must be obtained by adding the individual responses in the *time* domain. It is incorrect to try to add the responses in the phasor or frequency domain. Why? Because the exponential factor $e^{j\omega t}$ is implicit in sinusoidal analysis, and that factor would change for every angular frequency ω. It would therefore not make sense to add responses at different frequencies in the phasor domain. Thus, when a circuit has sources operating at different frequencies, one must add the responses due to the individual frequencies in the time domain.

Example 10.5

Use the superposition theorem to find $\mathbf{I}_o$ in the circuit in Fig. 10.7.

Solution:
Let

$$\mathbf{I}_o = \mathbf{I}'_o + \mathbf{I}''_o \tag{10.5.1}$$

where $\mathbf{I}'_o$ and $\mathbf{I}''_o$ are due to the voltage and current sources, respectively. To find $\mathbf{I}'_o$, consider the circuit in Fig. 10.12(a). If we let $\mathbf{Z}$ be the parallel combination of $-j2$ and $8 + j10$, then

$$\mathbf{Z} = \frac{-j2(8 + j10)}{-2j + 8 + j10} = 0.25 - j2.25$$

and current $\mathbf{I}'_o$ is

$$\mathbf{I}'_o = \frac{j20}{4 - j2 + \mathbf{Z}} = \frac{j20}{4.25 - j4.25}$$

or

$$\mathbf{I}'_o = -2.353 + j2.353 \tag{10.5.2}$$

To get $\mathbf{I}''_o$, consider the circuit in Fig. 10.12(b). For mesh 1,

$$(8 + j8)\mathbf{I}_1 - j10\mathbf{I}_3 + j2\mathbf{I}_2 = 0 \tag{10.5.3}$$

For mesh 2,

$$(4 - j4)\mathbf{I}_2 + j2\mathbf{I}_1 + j2\mathbf{I}_3 = 0 \tag{10.5.4}$$

For mesh 3,

$$\mathbf{I}_3 = 5 \tag{10.5.5}$$

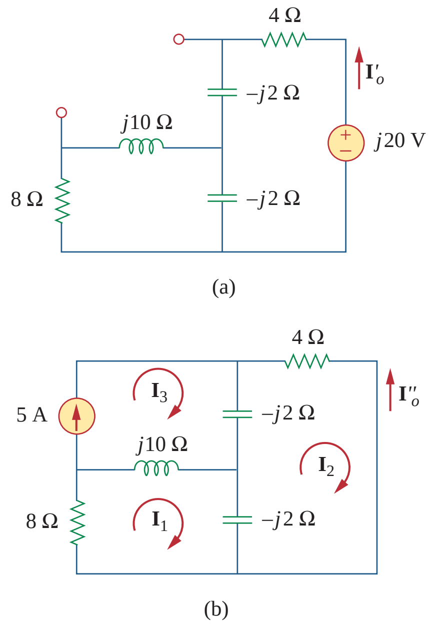


Figure 10.12
Solution of Example 10.5.

From Eqs. (10.5.4) and (10.5.5),

$$(4 - j4)\mathbf{I}_2 + j2\mathbf{I}_1 + j10 = 0$$

Expressing $\mathbf{I}_1$ in terms of $\mathbf{I}_2$ gives

$$\mathbf{I}_1 = (2 + j2)\mathbf{I}_2 - 5 \tag{10.5.6}$$

Substituting Eqs. (10.5.5) and (10.5.6) into Eq. (10.5.3), we get

$$(8 + j8)[(2 + j2)\mathbf{I}_2 - 5] - j50 + j2\mathbf{I}_2 = 0$$

or

$$\mathbf{I}_2 = \frac{90 - j40}{34} = 2.647 - j1.176$$

Current $\mathbf{I}_o''$ is obtained as

$$\mathbf{I}_o'' = -\mathbf{I}_2 = -2.647 + j1.176 \tag{10.5.7}$$

From Eqs. (10.5.2) and (10.5.7), we write

$$\mathbf{I}_o = \mathbf{I}_o' + \mathbf{I}_o'' = -5 + j3.529 = 6.12\underline{/144.78^\circ} \text{ A}$$

which agrees with what we got in Example 10.3. It should be noted that applying the superposition theorem is not the best way to solve this problem. It seems that we have made the problem twice as hard as the original one by using superposition. However, in Example 10.6, superposition is clearly the easiest approach.

Practice Problem 10.5

Find current $\mathbf{I}_o$ in the circuit of Fig. 10.8 using the superposition theorem.

Answer: $3.582\underline{/65.45^\circ}$ A.

Example 10.6

Find v_o of the circuit of Fig. 10.13 using the superposition theorem.

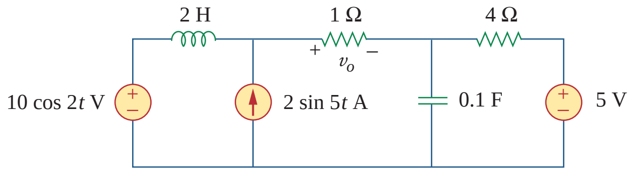


Figure 10.13
For Example 10.6.

Solution:
Since the circuit operates at three different frequencies ($\omega = 0$ for the dc voltage source), one way to obtain a solution is to use superposition, which breaks the problem into single-frequency problems. So we let

$$v_o = v_1 + v_2 + v_3 \tag{10.6.1}$$

where v_1 is due to the 5-V dc voltage source, v_2 is due to the $10 \cos 2t$ V voltage source, and v_3 is due to the $2 \sin 5t$ A current source.

To find v_1, we set to zero all sources except the 5-V dc source. We recall that at steady state, a capacitor is an open circuit to dc while an inductor is a short circuit to dc. There is an alternative way of looking at this. Since $\omega = 0$, $j\omega L = 0$, $1/j\omega C = \infty$. Either way, the equivalent circuit is as shown in Fig. 10.14(a). By voltage division,

$$-v_1 = \frac{1}{1+4}(5) = 1 \text{ V} \tag{10.6.2}$$

To find v_2, we set to zero both the 5-V source and the $2 \sin 5t$ current source and transform the circuit to the frequency domain.

$$\begin{aligned} 10 \cos 2t \quad &\Rightarrow \quad 10\angle 0^\circ, \qquad \omega = 2 \text{ rad/s} \\ 2 \text{ H} \quad &\Rightarrow \quad j\omega L = j4\ \Omega \\ 0.1 \text{ F} \quad &\Rightarrow \quad \frac{1}{j\omega C} = -j5\ \Omega \end{aligned}$$

The equivalent circuit is now as shown in Fig. 10.14(b). Let

$$\mathbf{Z} = -j5 \parallel 4 = \frac{-j5 \times 4}{4 - j5} = 2.439 - j1.951$$

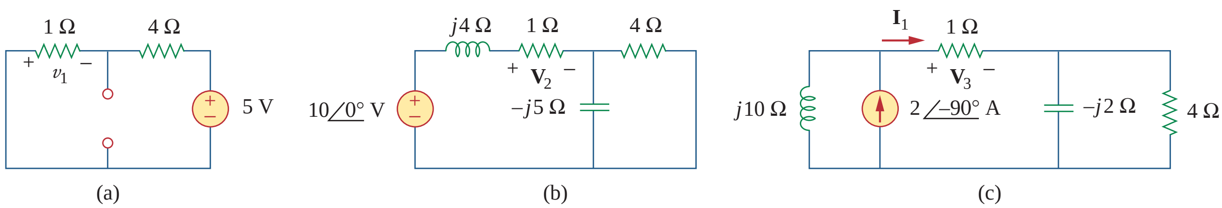


Figure 10.14
Solution of Example 10.6: (a) setting all sources to zero except the 5-V dc source, (b) setting all sources to zero except the ac voltage source, (c) setting all sources to zero except the ac current source.

By voltage division,

$$\mathbf{V}_2 = \frac{1}{1 + j4 + \mathbf{Z}}(10\angle 0^\circ) = \frac{10}{3.439 + j2.049} = 2.498\angle -30.79^\circ$$

In the time domain,

$$v_2 = 2.498 \cos(2t - 30.79^\circ) \tag{10.6.3}$$

To obtain v_3, we set the voltage sources to zero and transform what is left to the frequency domain.

$$\begin{aligned} 2 \sin 5t \quad &\Rightarrow \quad 2\angle -90^\circ, \qquad \omega = 5 \text{ rad/s} \\ 2 \text{ H} \quad &\Rightarrow \quad j\omega L = j10\ \Omega \\ 0.1 \text{ F} \quad &\Rightarrow \quad \frac{1}{j\omega C} = -j2\ \Omega \end{aligned}$$

The equivalent circuit is in Fig. 10.14(c). Let

$$\mathbf{Z}_1 = -j2 \parallel 4 = \frac{-j2 \times 4}{4 - j2} = 0.8 - j1.6\ \Omega$$

By current division,

$$\mathbf{I}_1 = \frac{j10}{j10 + 1 + \mathbf{Z}_1}(2\underline{/-90^\circ})\ \text{A}$$

$$\mathbf{V}_3 = \mathbf{I}_1 \times 1 = \frac{j10}{1.8 + j8.4}(-j2) = 2.328\underline{/-80^\circ}\ \text{V}$$

In the time domain,

$$v_3 = 2.33\cos(5t - 80^\circ) = 2.33\sin(5t + 10^\circ)\ \text{V} \tag{10.6.4}$$

Substituting Eqs. (10.6.2) to (10.6.4) into Eq. (10.6.1), we have

$$v_o(t) = -1 + 2.498\cos(2t - 30.79^\circ) + 2.33\sin(5t + 10^\circ)\ \text{V}$$

Practice Problem 10.6

Calculate v_o in the circuit of Fig. 10.15 using the superposition theorem.

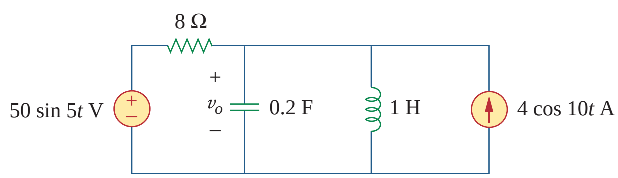


Figure 10.15
For Practice Prob. 10.6.

Answer: $7.718\sin(5t - 81.12^\circ) + 2.102\cos(10t - 86.24^\circ)$ V.

10.5 Source Transformation

As Fig. 10.16 shows, source transformation in the frequency domain involves transforming a voltage source in series with an impedance to a current source in parallel with an impedance, or vice versa. As we go from one source type to another, we must keep the following relationship in mind:

$$\boxed{\mathbf{V}_s = \mathbf{Z}_s\mathbf{I}_s \quad \Leftrightarrow \quad \mathbf{I}_s = \frac{\mathbf{V}_s}{\mathbf{Z}_s}} \tag{10.1}$$

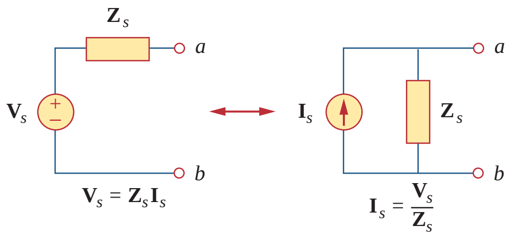


Figure 10.16
Source transformation.

Example 10.7

Calculate $\mathbf{V}_x$ in the circuit of Fig. 10.17 using the method of source transformation.

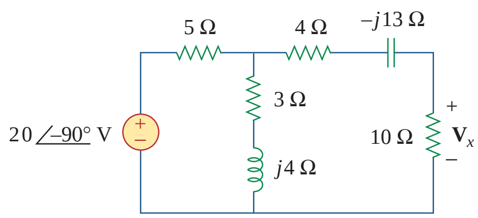


Figure 10.17
For Example 10.7.

Solution:
We transform the voltage source to a current source and obtain the circuit in Fig. 10.18(a), where

$$\mathbf{I}_s = \frac{20\underline{/-90^\circ}}{5} = 4\underline{/-90^\circ} = -j4 \text{ A}$$

The parallel combination of 5-Ω resistance and $(3 + j4)$ impedance gives

$$\mathbf{Z}_1 = \frac{5(3 + j4)}{8 + j4} = 2.5 + j1.25\ \Omega$$

Converting the current source to a voltage source yields the circuit in Fig. 10.18(b), where

$$\mathbf{V}_s = \mathbf{I}_s\mathbf{Z}_1 = -j4(2.5 + j1.25) = 5 - j10 \text{ V}$$

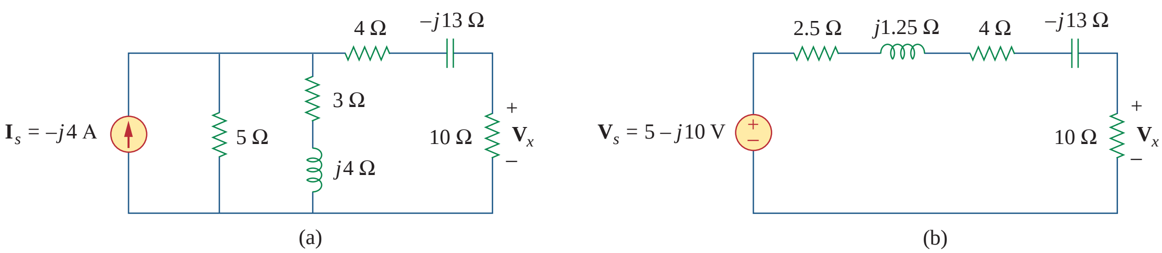


Figure 10.18
Solution of the circuit in Fig. 10.17.

By voltage division,

$$\mathbf{V}_x = \frac{10}{10 + 2.5 + j1.25 + 4 - j13}(5 - j10) = 5.519\underline{/-28^\circ} \text{ V}$$

Practice Problem 10.7

Find $\mathbf{I}_o$ in the circuit of Fig. 10.19 using the concept of source transformation.

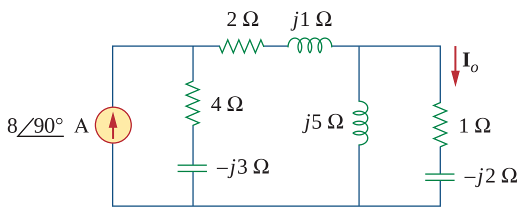


Figure 10.19
For Practice Prob. 10.7.

Answer: $6.576\underline{/99.46^\circ}$ A.

10.6 Thevenin and Norton Equivalent Circuits

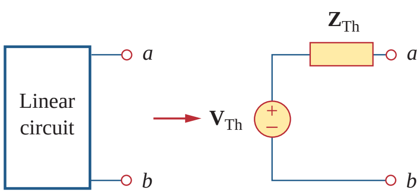


Figure 10.20
Thevenin equivalent.

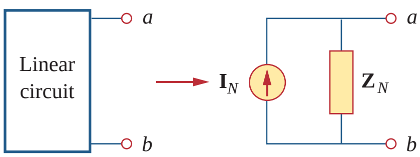


Figure 10.21
Norton equivalent.

Thevenin's and Norton's theorems are applied to ac circuits in the same way as they are to dc circuits. The only additional effort arises from the need to manipulate complex numbers. The frequency domain version of a Thevenin equivalent circuit is depicted in Fig. 10.20, where a linear circuit is replaced by a voltage source in series with an impedance. The Norton equivalent circuit is illustrated in Fig. 10.21, where a linear circuit is replaced by a current source in parallel with an impedance. Keep in mind that the two equivalent circuits are related as

$$\boxed{\mathbf{V}_{\text{Th}} = \mathbf{Z}_N \mathbf{I}_N, \qquad \mathbf{Z}_{\text{Th}} = \mathbf{Z}_N} \tag{10.2}$$

just as in source transformation. $\mathbf{V}_{\text{Th}}$ is the open-circuit voltage while $\mathbf{I}_N$ is the short-circuit current.

If the circuit has sources operating at different frequencies (see Example 10.6, for example), the Thevenin or Norton equivalent circuit must be determined at each frequency. This leads to entirely different equivalent circuits, one for each frequency, not one equivalent circuit with equivalent sources and equivalent impedances.

Example 10.8

Obtain the Thevenin equivalent at terminals *a*-*b* of the circuit in Fig. 10.22.

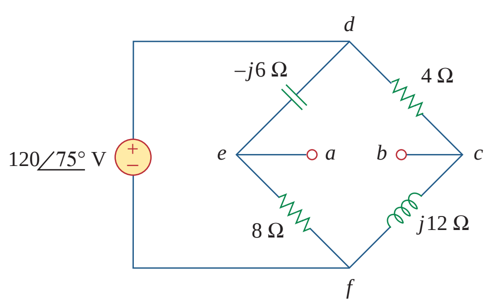


Figure 10.22
For Example 10.8.

Solution:

We find $\mathbf{Z}_{\mathrm{Th}}$ by setting the voltage source to zero. As shown in Fig. 10.23(a), the 8-Ω resistance is now in parallel with the $-j6$ reactance, so that their combination gives

$$\mathbf{Z}_1 = -j6 \parallel 8 = \frac{-j6 \times 8}{8 - j6} = 2.88 - j3.84\ \Omega$$

Similarly, the 4-Ω resistance is in parallel with the $j12$ reactance, and their combination gives

$$\mathbf{Z}_2 = 4 \parallel j12 = \frac{j12 \times 4}{4 + j12} = 3.6 + j1.2\ \Omega$$

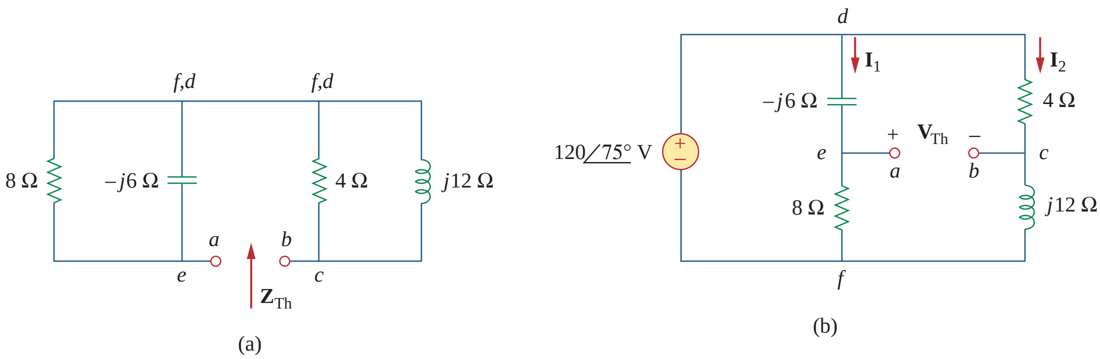


Figure 10.23
Solution of the circuit in Fig. 10.22: (a) finding $\mathbf{Z}_{\mathrm{Th}}$, (b) finding $\mathbf{V}_{\mathrm{Th}}$.

The Thevenin impedance is the series combination of $\mathbf{Z}_1$ and $\mathbf{Z}_2$; that is,

$$\mathbf{Z}_{\mathrm{Th}} = \mathbf{Z}_1 + \mathbf{Z}_2 = 6.48 - j2.64\ \Omega$$

To find $\mathbf{V}_{\mathrm{Th}}$, consider the circuit in Fig. 10.23(b). Currents $\mathbf{I}_1$ and $\mathbf{I}_2$ are obtained as

$$\mathbf{I}_1 = \frac{120\angle 75^\circ}{8 - j6}\ \text{A}, \qquad \mathbf{I}_2 = \frac{120\angle 75^\circ}{4 + j12}\ \text{A}$$

Applying KVL around loop *bcdeab* in Fig. 10.23(b) gives

$$\mathbf{V}_{\mathrm{Th}} - 4\mathbf{I}_2 + (-j6)\mathbf{I}_1 = 0$$

or

$$\begin{aligned}\mathbf{V}_{\mathrm{Th}} = 4\mathbf{I}_2 + j6\mathbf{I}_1 &= \frac{480\angle 75^\circ}{4 + j12} + \frac{720\angle 75^\circ + 90^\circ}{8 - j6} \\ &= 37.95\angle 3.43^\circ + 72\angle 201.87^\circ \\ &= -28.936 - j24.55 = 37.95\angle 220.31^\circ\ \text{V}\end{aligned}$$

Practice Problem 10.8

Find the Thevenin equivalent at terminals *a-b* of the circuit in Fig. 10.24.

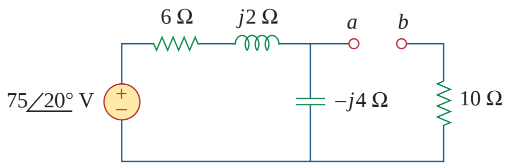


Figure 10.24
For Practice Prob. 10.8.

Answer: $\mathbf{Z}_{\text{Th}} = 12.4 - j3.2\ \Omega$, $\mathbf{V}_{\text{Th}} = 47.42\angle -51.57^\circ$ V.

Example 10.9

Find the Thevenin equivalent of the circuit in Fig. 10.25 as seen from terminals *a-b*.

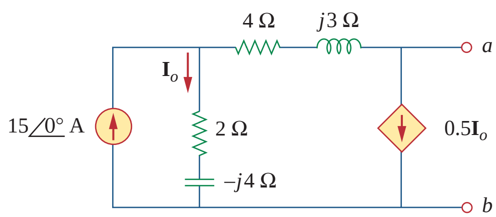


Figure 10.25
For Example 10.9.

Solution:
To find $\mathbf{V}_{\text{Th}}$, we apply KCL at node 1 in Fig. 10.26(a).

$$15 = \mathbf{I}_o + 0.5\mathbf{I}_o \qquad \Rightarrow \qquad \mathbf{I}_o = 10 \text{ A}$$

Applying KVL to the loop on the right-hand side in Fig. 10.26(a), we obtain

$$-\mathbf{I}_o(2 - j4) + 0.5\mathbf{I}_o(4 + j3) + \mathbf{V}_{\text{Th}} = 0$$

or

$$\mathbf{V}_{\text{Th}} = 10(2 - j4) - 5(4 + j3) = -j55$$

Thus, the Thevenin voltage is

$$\mathbf{V}_{\text{Th}} = 55\angle -90^\circ \text{ V}$$

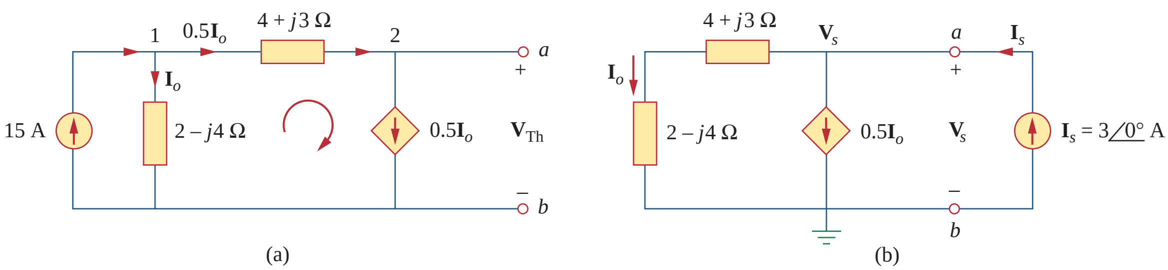


Figure 10.26
Solution of the problem in Fig. 10.25: (a) finding $\mathbf{V}_{\text{Th}}$, (b) finding $\mathbf{Z}_{\text{Th}}$.

To obtain $\mathbf{Z}_{\text{Th}}$, we remove the independent source. Due to the presence of the dependent current source, we connect a 3-A current source (3 is an arbitrary value chosen for convenience here, a number divisible by the sum of currents leaving the node) to terminals *a-b* as shown in Fig. 10.26(b). At the node, KCL gives

$$3 = \mathbf{I}_o + 0.5\mathbf{I}_o \qquad \Rightarrow \qquad \mathbf{I}_o = 2 \text{ A}$$

Applying KVL to the outer loop in Fig. 10.26(b) gives

$$\mathbf{V}_s = \mathbf{I}_o(4 + j3 + 2 - j4) = 2(6 - j)$$

The Thevenin impedance is

$$\mathbf{Z}_{\text{Th}} = \frac{\mathbf{V}_s}{\mathbf{I}_s} = \frac{2(6 - j)}{3} = 4 - j0.6667\ \Omega$$

Practice Problem 10.9

Determine the Thevenin equivalent of the circuit in Fig. 10.27 as seen from the terminals *a-b*.

Answer: $\mathbf{Z}_{\text{Th}} = 4.473\underline{/-7.64^\circ}\ \Omega$, $\mathbf{V}_{\text{Th}} = 29.4\underline{/72.9^\circ}$ V.

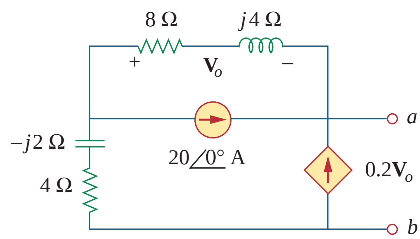


Figure 10.27
For Practice Prob. 10.9.

Example 10.10

Obtain current $\mathbf{I}_o$ in Fig. 10.28 using Norton's theorem.

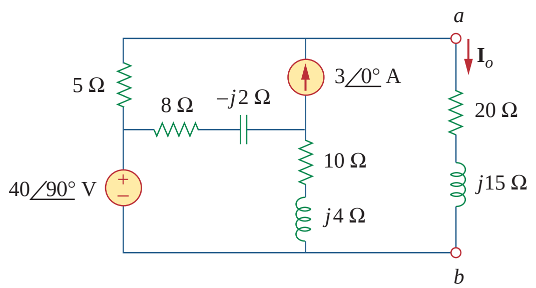


Figure 10.28
For Example 10.10.

Solution:
Our first objective is to find the Norton equivalent at terminals *a-b*. $\mathbf{Z}_N$ is found in the same way as $\mathbf{Z}_{\text{Th}}$. We set the sources to zero as shown in Fig. 10.29(a). As evident from the figure, the $(8 - j2)$ and $(10 + j4)$ impedances are short-circuited, so that

$$\mathbf{Z}_N = 5\ \Omega$$

To get $\mathbf{I}_N$, we short-circuit terminals *a-b* as in Fig. 10.29(b) and apply mesh analysis. Notice that meshes 2 and 3 form a supermesh because of the current source linking them. For mesh 1,

$$-j40 + (18 + j2)\mathbf{I}_1 - (8 - j2)\mathbf{I}_2 - (10 + j4)\mathbf{I}_3 = 0 \qquad \textbf{(10.10.1)}$$

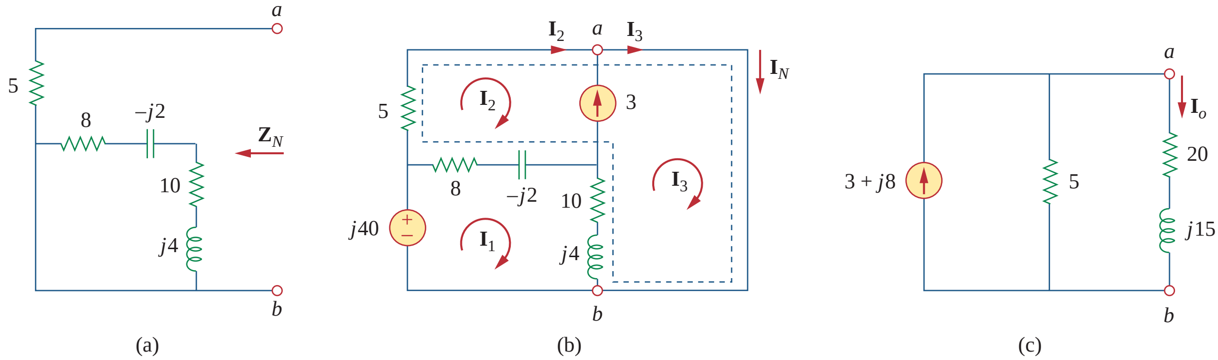


Figure 10.29
Solution of the circuit in Fig. 10.28: (a) finding $\mathbf{Z}_N$, (b) finding $\mathbf{V}_N$, (c) calculating $\mathbf{I}_o$.

For the supermesh,

$$(13 - j2)\mathbf{I}_2 + (10 + j4)\mathbf{I}_3 - (18 + j2)\mathbf{I}_1 = 0 \qquad \textbf{(10.10.2)}$$

At node a, due to the current source between meshes 2 and 3,

$$\mathbf{I}_3 = \mathbf{I}_2 + 3 \qquad \textbf{(10.10.3)}$$

Adding Eqs. (10.10.1) and (10.10.2) gives

$$-j40 + 5\mathbf{I}_2 = 0 \qquad \Rightarrow \qquad \mathbf{I}_2 = j8$$

From Eq. (10.10.3),

$$\mathbf{I}_3 = \mathbf{I}_2 + 3 = 3 + j8$$

The Norton current is

$$\mathbf{I}_N = \mathbf{I}_3 = (3 + j8) \text{ A}$$

Figure 10.29(c) shows the Norton equivalent circuit along with the impedance at terminals a-b. By current division,

$$\mathbf{I}_o = \frac{5}{5 + 20 + j15}\mathbf{I}_N = \frac{3 + j8}{5 + j3} = 1.465\underline{/38.48^\circ} \text{ A}$$

Practice Problem 10.10

Determine the Norton equivalent of the circuit in Fig. 10.30 as seen from terminals a-b. Use the equivalent to find $\mathbf{I}_o$.

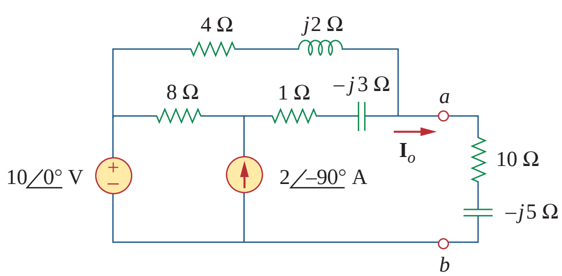


Figure 10.30
For Practice Prob. 10.10.

Answer: $\mathbf{Z}_N = 3.176 + j0.706\ \Omega$, $\mathbf{I}_N = 4.198\underline{/-32.68^\circ}$ A, $\mathbf{I}_o = 985.5\underline{/-2.101^\circ}$ mA.

10.7 Op Amp AC Circuits

The three steps stated in Section 10.1 also apply to op amp circuits, as long as the op amp is operating in the linear region. As usual, we will assume ideal op amps. (See Section 5.2.) As discussed in Chapter 5, the key to analyzing op amp circuits is to keep two important properties of an ideal op amp in mind:

1. No current enters either of its input terminals.
2. The voltage across its input terminals is zero.

The following examples will illustrate these ideas.

Example 10.11

Determine $v_o(t)$ for the op amp circuit in Fig. 10.31(a) if $v_s = 3\cos 1000t$ V.

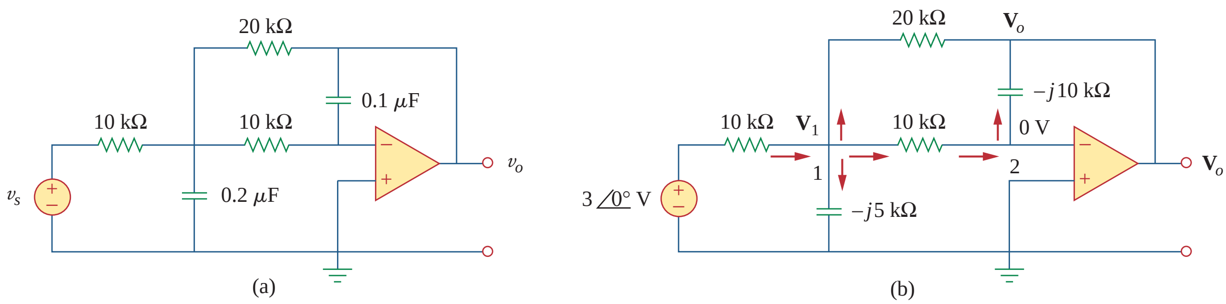


Figure 10.31
For Example 10.11: (a) the original circuit in the time domain, (b) its frequency domain equivalent.

Solution:
We first transform the circuit to the frequency domain, as shown in Fig. 10.31(b), where $\mathbf{V}_s = 3\underline{/0°}$, $\omega = 1000$ rad/s. Applying KCL at node 1, we obtain

$$\frac{3\underline{/0°} - \mathbf{V}_1}{10} = \frac{\mathbf{V}_1}{-j5} + \frac{\mathbf{V}_1 - 0}{10} + \frac{\mathbf{V}_1 - \mathbf{V}_o}{20}$$

or

$$6 = (5 + j4)\mathbf{V}_1 - \mathbf{V}_o \qquad \textbf{(10.11.1)}$$

At node 2, KCL gives

$$\frac{\mathbf{V}_1 - 0}{10} = \frac{0 - \mathbf{V}_o}{-j10}$$

which leads to

$$\mathbf{V}_1 = -j\mathbf{V}_o \qquad \textbf{(10.11.2)}$$

Substituting Eq. (10.11.2) into Eq. (10.11.1) yields

$$6 = -j(5 + j4)\mathbf{V}_o - \mathbf{V}_o = (3 - j5)\mathbf{V}_o$$

$$\mathbf{V}_o = \frac{6}{3 - j5} = 1.029\underline{/59.04°}$$

Hence,

$$v_o(t) = 1.029\cos(1000t + 59.04°)\text{ V}$$

Practice Problem 10.11

Find v_o and i_o in the op amp circuit of Fig. 10.32. Let $v_s = 4 \cos 5000t$ V.

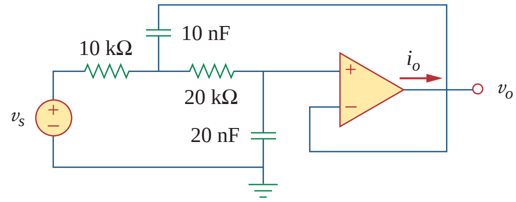


Figure 10.32
For Practice Prob. 10.11.

Answer: 1.3333 sin 5000t V, 133.33 sin 5000t μA.

Example 10.12

Compute the closed-loop gain and phase shift for the circuit in Fig. 10.33. Assume that $R_1 = R_2 = 10$ kΩ, $C_1 = 2$ μF, $C_2 = 1$ μF, and $\omega = 200$ rad/s.

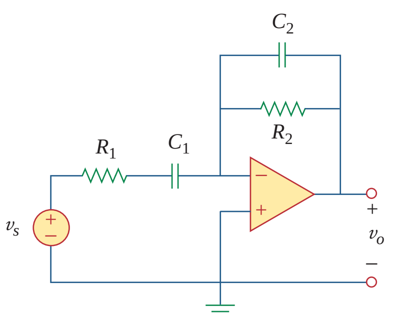


Figure 10.33
For Example 10.12.

Solution:
The feedback and input impedances are calculated as

$$\mathbf{Z}_f = R_2 \left\| \frac{1}{j\omega C_2} = \frac{R_2}{1 + j\omega R_2 C_2}\right.$$

$$\mathbf{Z}_i = R_1 + \frac{1}{j\omega C_1} = \frac{1 + j\omega R_1 C_1}{j\omega C_1}$$

Since the circuit in Fig. 10.33 is an inverting amplifier, the closed-loop gain is given by

$$\mathbf{G} = \frac{\mathbf{V}_o}{\mathbf{V}_s} = -\frac{\mathbf{Z}_f}{\mathbf{Z}_i} = \frac{-j\omega C_1 R_2}{(1 + j\omega R_1 C_1)(1 + j\omega R_2 C_2)}$$

Substituting the given values of R_1, R_2, C_1, C_2, and ω, we obtain

$$\mathbf{G} = \frac{-j4}{(1 + j4)(1 + j2)} = 0.434\underline{/130.6^\circ}$$

Thus, the closed-loop gain is 0.434 and the phase shift is 130.6°.

Practice Problem 10.12

Obtain the closed-loop gain and phase shift for the circuit in Fig. 10.34. Let $R = 10$ kΩ, $C = 1$ μF, and $\omega = 1000$ rad/s.

Answer: 1.015, −5.6°.

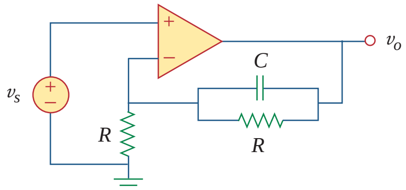


Figure 10.34
For Practice Prob. 10.12.

10.8 AC Analysis Using *PSpice*

PSpice affords a big relief from the tedious task of manipulating complex numbers in ac circuit analysis. The procedure for using *PSpice* for ac analysis is quite similar to that required for dc analysis. The reader should read Section D.5 in Appendix D for a review of *PSpice* concepts for ac analysis. AC circuit analysis is done in the phasor or frequency domain, and all sources must have the same frequency. Although ac analysis with *PSpice* involves using AC Sweep, our analysis in this chapter requires a single frequency $f = \omega/2\pi$. The output file of *PSpice* contains voltage and current phasors. If necessary, the impedances can be calculated using the voltages and currents in the output file.

Example 10.13

Obtain v_o and i_o in the circuit of Fig. 10.35 using *PSpice*.

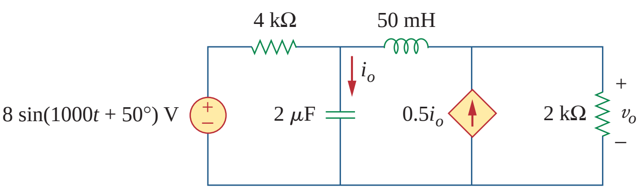


Figure 10.35
For Example 10.13.

Solution:
We first convert the sine function to cosine.

$$\begin{aligned} 8 \sin(1000t + 50°) &= 8 \cos(1000t + 50° - 90°) \\ &= 8 \cos(1000t - 40°) \end{aligned}$$

The frequency f is obtained from ω as

$$f = \frac{\omega}{2\pi} = \frac{1000}{2\pi} = 159.155 \text{ Hz}$$

The schematic for the circuit is shown in Fig. 10.36. Notice that the current-controlled current source F1 is connected such that its current flows from node 0 to node 3 in conformity with the original circuit in Fig. 10.35. Since we only want the magnitude and phase of v_o and i_o, we set the attributes of IPRINT and VPRINT1 each to *AC* = *yes*, *MAG* = *yes*, *PHASE* = *yes*. As a single-frequency analysis, we select **Analysis/Setup/AC Sweep** and enter *Total Pts* = 1, *Start Freq* = 159.155, and *Final Freq* = 159.155. After saving the schematic, we simulate it by selecting **Analysis/Simulate.** The output file includes the source frequency in addition to the attributes checked for the pseudocomponents IPRINT and VPRINT1,

```
FREQ         IM(V_PRINT3)   IP(V_PRINT3)
1.592E+02    3.264E-03      -3.743E+01

FREQ         VM(3)          VP(3)
1.592E+02    1.550E+00      -9.518E+01
```

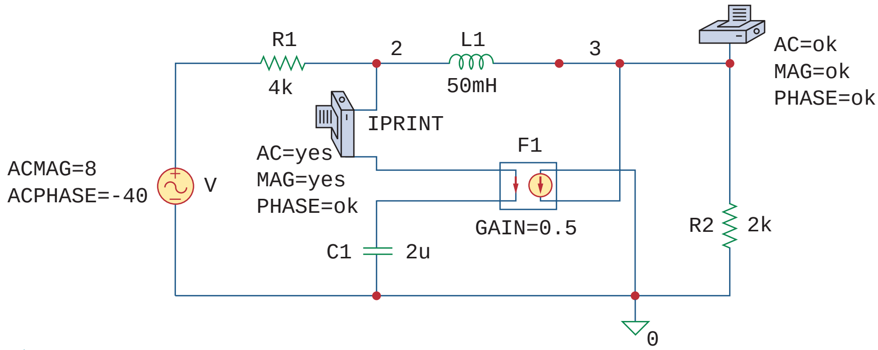


Figure 10.36
The schematic of the circuit in Fig. 10.35.

From this output file, we obtain

$$\mathbf{V}_o = 1.55\underline{/-95.18^\circ} \text{ V}, \qquad \mathbf{I}_o = 3.264\underline{/-37.43^\circ} \text{ mA}$$

which are the phasors for

$$v_o = 1.55 \cos(1000t - 95.18^\circ) = 1.55 \sin(1000t - 5.18^\circ) \text{ V}$$

and

$$i_o = 3.264 \cos(1000t - 37.43^\circ) \text{ mA}$$

Practice Problem 10.13

Use *PSpice* to obtain v_o and i_o in the circuit of Fig. 10.37.

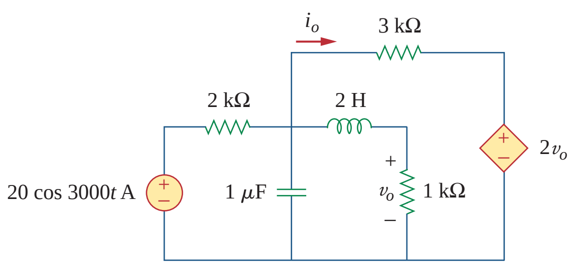


Figure 10.37
For Practice Prob. 10.13.

Answer: $536.4 \cos(3000t - 154.6^\circ)$ mV, $1.088 \cos(3000t - 55.12^\circ)$ mA.

Example 10.14

Find $\mathbf{V}_1$ and $\mathbf{V}_2$ in the circuit of Fig. 10.38.

Solution:

1. **Define.** In its present form, the problem is clearly stated. Again, we must emphasize that time spent here will save lots of time and expense later on! One thing that might have created a problem for you is that, if the reference was missing for this problem, you would then need to ask the individual assigning

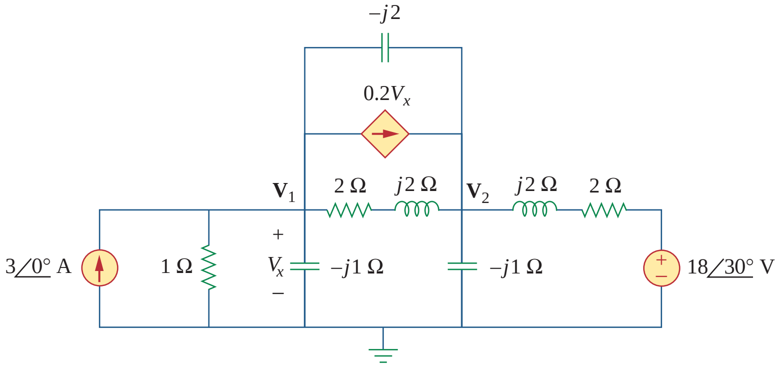


Figure 10.38
For Example 10.14.

the problem where it is to be located. If you could not do that, then you would need to assume where it should be and then clearly state what you did and why you did it.

2. **Present.** The given circuit is a frequency domain circuit and the unknown node voltages $\mathbf{V}_1$ and $\mathbf{V}_2$ are also frequency domain values. Clearly, we need a process to solve for these unknowns in the frequency domain.
3. **Alternative.** We have two direct alternative solution techniques that we can easily use. We can do a straightforward nodal analysis approach or use *PSpice*. Since this example is in a section dedicated to using *PSpice* to solve problems, we will use *PSpice* to find $\mathbf{V}_1$ and $\mathbf{V}_2$. We can then use nodal analysis to check the answer.
4. **Attempt.** The circuit in Fig. 10.35 is in the time domain, whereas the one in Fig. 10.38 is in the frequency domain. Since we are not given a particular frequency and *PSpice* requires one, we select any frequency consistent with the given impedances. For example, if we select $\omega = 1$ rad/s, the corresponding frequency is $f = \omega/2\pi =$ 0.15916 Hz. We obtain the values of the capacitance ($C = 1/\omega X_C$) and inductances ($L = X_L/\omega$). Making these changes results in the schematic in Fig. 10.39. To ease wiring, we have

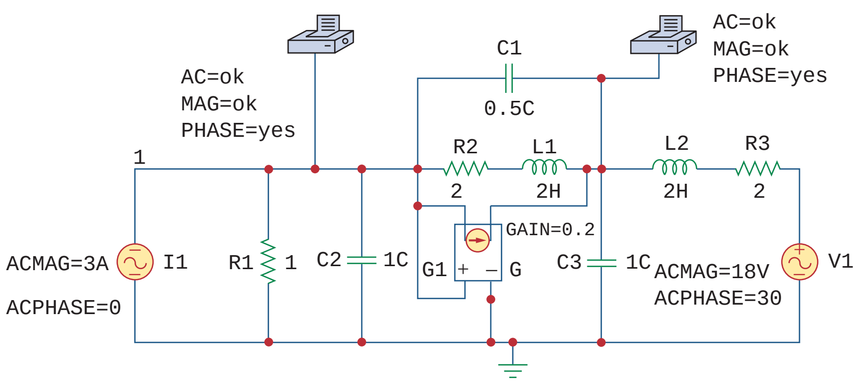


Figure 10.39
Schematic for circuit in the Fig. 10.38.

exchanged the positions of the voltage-controlled current source G1 and the $2 + j2\ \Omega$ impedance. Notice that the current of G1 flows from node 1 to node 3, while the controlling voltage is across the capacitor C2, as required in Fig. 10.38. The attributes of pseudocomponents VPRINT1 are set as shown. As a single-frequency analysis, we select **Analysis/Setup/AC Sweep** and enter *Total Pts* = 1, *Start Freq* = 0.15916, and *Final Freq* = 0.15916. After saving the schematic, we select **Analysis/Simulate** to simulate the circuit. When this is done, the output file includes

```
FREQ          VM(1)         VP(1)
1.592E-01     2.708E+00     -5.673E+01

FREQ          VM(3)         VP(3)
1.592E-01     4.468E+00     -1.026E+02
```

from which we obtain,

$$\mathbf{V}_1 = \mathbf{2.708}\underline{/\mathbf{-56.74^\circ}\ \mathbf{V}} \quad \text{and} \quad \mathbf{V}_2 = \mathbf{6.911}\underline{/\mathbf{-80.72^\circ}\ \mathbf{V}}$$

5. **Evaluate.** One of the most important lessons to be learned is that when using programs such as *PSpice* you still need to validate the answer. There are many opportunities for making a mistake, including coming across an unknown "bug" in *PSpice* that yields incorrect results.

   So, how can we validate this solution? Obviously, we can rework the entire problem with nodal analysis, and perhaps using *MATLAB*, to see if we obtain the same results. There is another way we will use here: write the nodal equations and substitute the answers obtained in the *PSpice* solution, and see if the nodal equations are satisfied.

   The nodal equations for this circuit are given below. Note we have substituted $\mathbf{V}_1 = \mathbf{V}_x$ into the dependent source.

$$-3 + \frac{\mathbf{V}_1 - 0}{1} + \frac{\mathbf{V}_1 - 0}{-j1} + \frac{\mathbf{V}_1 - \mathbf{V}_2}{2 + j2} + 0.2\mathbf{V}_1 + \frac{\mathbf{V}_1 - \mathbf{V}_2}{-j2} = 0$$

$$(1 + j + 0.25 - j0.25 + 0.2 + j0.5)\mathbf{V}_1$$

$$- (0.25 - j0.25 + j0.5)\mathbf{V}_2 = 3$$

$$(1.45 + j1.25)\mathbf{V}_1 - (0.25 + j0.25)\mathbf{V}_2 = 3$$

$$1.9144\underline{/40.76^\circ}\ \mathbf{V}_1 - 0.3536\underline{/45^\circ}\ \mathbf{V}_2 = 3$$

   Now, to check the answer, we substitute the *PSpice* answers into this.

$$\begin{aligned} 1.9144\underline{/40.76^\circ} \times 2.708\underline{/-56.74^\circ} - 0.3536\underline{/45^\circ} \times 6.911\underline{/-80.72^\circ} \\ = 5.184\underline{/-15.98^\circ} - 2.444\underline{/-35.72^\circ} \\ = 4.984 - j1.4272 - 1.9842 + j1.4269 \\ = 3 - j0.0003 \qquad \text{[Answer checks]} \end{aligned}$$

6. **Satisfactory?** Although we used only the equation from node 1 to check the answer, this is more than satisfactory to validate the answer from the *PSpice* solution. We can now present our work as a solution to the problem.

Practice Problem 10.14

Obtain $\mathbf{V}_x$ and $\mathbf{I}_x$ in the circuit depicted in Fig. 10.40.

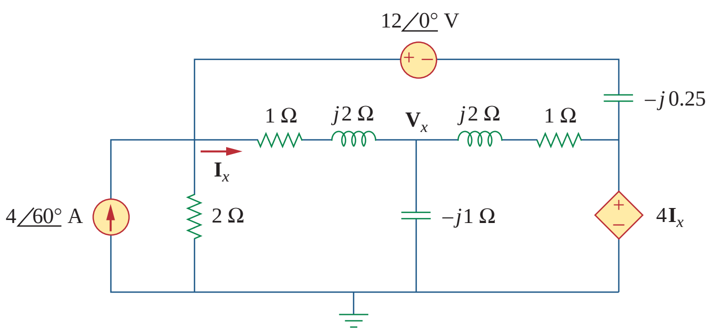


Figure 10.40
For Practice Prob. 10.14.

Answer: $9.842\angle 44.78^\circ$ V, $2.584\angle 158^\circ$ A.

10.9 †Applications

The concepts learned in this chapter will be applied in later chapters to calculate electric power and determine frequency response. The concepts are also used in analyzing coupled circuits, three-phase circuits, ac transistor circuits, filters, oscillators, and other ac circuits. In this section, we apply the concepts to develop two practical ac circuits: the capacitance multiplier and the sine wave oscillators.

10.9.1 Capacitance Multiplier

The op amp circuit in Fig. 10.41 is known as a *capacitance multiplier*, for reasons that will become obvious. Such a circuit is used in integrated-circuit technology to produce a multiple of a small physical capacitance C when a large capacitance is needed. The circuit in Fig. 10.41 can be used to multiply capacitance values by a factor up to 1000. For example, a 10-pF capacitor can be made to behave like a 100-nF capacitor.

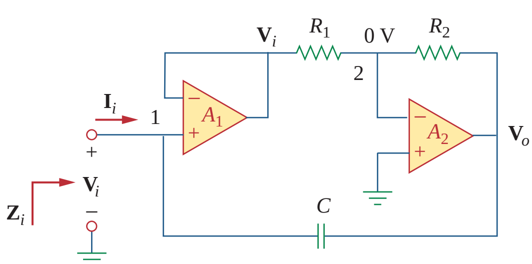


Figure 10.41
Capacitance multiplier.

In Fig. 10.41, the first op amp operates as a voltage follower, while the second one is an inverting amplifier. The voltage follower isolates the capacitance formed by the circuit from the loading imposed by the inverting amplifier. Since no current enters the input terminals of the op amp, the input current $\mathbf{I}_i$ flows through the feedback capacitor. Hence, at node 1,

$$\mathbf{I}_i = \frac{\mathbf{V}_i - \mathbf{V}_o}{1/j\omega C} = j\omega C(\mathbf{V}_i - \mathbf{V}_o) \tag{10.3}$$

Applying KCL at node 2 gives

$$\frac{\mathbf{V}_i - 0}{R_1} = \frac{0 - \mathbf{V}_o}{R_2}$$

or

$$\mathbf{V}_o = -\frac{R_2}{R_1}\mathbf{V}_i \tag{10.4}$$

Substituting Eq. (10.4) into (10.3) gives

$$\mathbf{I}_i = j\omega C\left(1 + \frac{R_2}{R_1}\right)\mathbf{V}_i$$

or

$$\frac{\mathbf{I}_i}{\mathbf{V}_i} = j\omega\left(1 + \frac{R_2}{R_1}\right)C \tag{10.5}$$

The input impedance is

$$\mathbf{Z}_i = \frac{\mathbf{V}_i}{\mathbf{I}_i} = \frac{1}{j\omega C_{\text{eq}}} \tag{10.6}$$

where

$$C_{\text{eq}} = \left(1 + \frac{R_2}{R_1}\right)C \tag{10.7}$$

Thus, by a proper selection of the values of R_1 and R_2, the op amp circuit in Fig. 10.41 can be made to produce an effective capacitance between the input terminal and ground, which is a multiple of the physical capacitance C. The size of the effective capacitance is practically limited by the inverted output voltage limitation. Thus, the larger the capacitance multiplication, the smaller is the allowable input voltage to prevent the op amps from reaching saturation.

A similar op amp circuit can be designed to simulate inductance. (See Prob. 10.89.) There is also an op amp circuit configuration to create a resistance multiplier.

Example 10.15

Calculate C_{eq} in Fig. 10.41 when $R_1 = 10\ \text{k}\Omega$, $R_2 = 1\ \text{M}\Omega$, and $C = 1$ nF.

Solution:

From Eq. (10.7)

$$C_{\text{eq}} = \left(1 + \frac{R_2}{R_1}\right)C = \left(1 + \frac{1 \times 10^6}{10 \times 10^3}\right)1\ \text{nF} = 101\ \text{nF}$$

Practice Problem 10.15

Determine the equivalent capacitance of the op amp circuit in Fig. 10.41 if $R_1 = 10\text{ k}\Omega$, $R_2 = 10\text{ M}\Omega$, and $C = 10\text{ nF}$.

Answer: $10\ \mu$F.

10.9.2 Oscillators

We know that dc is produced by batteries. But how do we produce ac? One way is using *oscillators*, which are circuits that convert dc to ac.

An **oscillator** is a circuit that produces an ac waveform as output when powered by a dc input.

The only external source an oscillator needs is the dc power supply. Ironically, the dc power supply is usually obtained by converting the ac supplied by the electric utility company to dc. Having gone through the trouble of conversion, one may wonder why we need to use the oscillator to convert the dc to ac again. The problem is that the ac supplied by the utility company operates at a preset frequency of 60 Hz in the United States (50 Hz in some other nations), whereas many applications such as electronic circuits, communication systems, and microwave devices require internally generated frequencies that range from 0 to 10 GHz or higher. Oscillators are used for generating these frequencies.

This corresponds to $\omega = 2\pi f = 377$ rad/s.

In order for sine wave oscillators to sustain oscillations, they must meet the *Barkhausen criteria:*

1. The overall gain of the oscillator must be unity or greater. Therefore, losses must be compensated for by an amplifying device.
2. The overall phase shift (from input to output and back to the input) must be zero.

Three common types of sine wave oscillators are phase-shift, twin T, and Wien-bridge oscillators. Here we consider only the Wien-bridge oscillator.

The *Wien-bridge oscillator* is widely used for generating sinusoids in the frequency range below 1 MHz. It is an RC op amp circuit with only a few components, easily tunable and easy to design. As shown in Fig. 10.42, the oscillator essentially consists of a noninverting amplifier with two feedback paths: the positive feedback path to the noninverting input creates oscillations, while the negative feedback path to the inverting input controls the gain. If we define the impedances of the RC series and parallel combinations as $\mathbf{Z}_s$ and $\mathbf{Z}_p$, then

$$\mathbf{Z}_s = R_1 + \frac{1}{j\omega C_1} = R_1 - \frac{j}{\omega C_1} \tag{10.8}$$

$$\mathbf{Z}_p = R_2 \parallel \frac{1}{j\omega C_2} = \frac{R_2}{1 + j\omega R_2 C_2} \tag{10.9}$$

The feedback ratio is

$$\frac{\mathbf{V}_2}{\mathbf{V}_o} = \frac{\mathbf{Z}_p}{\mathbf{Z}_s + \mathbf{Z}_p} \tag{10.10}$$

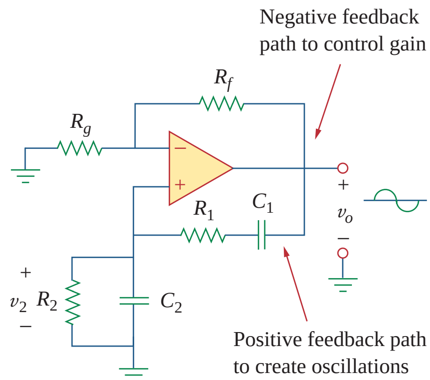


Figure 10.42 Wien-bridge oscillator.

Substituting Eqs. (10.8) and (10.9) into Eq. (10.10) gives

$$\begin{aligned}\frac{\mathbf{V}_2}{\mathbf{V}_o} &= \frac{R_2}{R_2 + \left(R_1 - \dfrac{j}{\omega C_1}\right)(1 + j\omega R_2 C_2)} \\ &= \frac{\omega R_2 C_1}{\omega(R_2C_1 + R_1C_1 + R_2C_2) + j(\omega^2 R_1C_1R_2C_2 - 1)}\end{aligned} \tag{10.11}$$

To satisfy the second Barkhausen criterion, $\mathbf{V}_2$ must be in phase with $\mathbf{V}_o$, which implies that the ratio in Eq. (10.11) must be purely real. Hence, the imaginary part must be zero. Setting the imaginary part equal to zero gives the oscillation frequency ω_o as

$$\omega_o^2 R_1C_1R_2C_2 - 1 = 0$$

or

$$\omega_o = \frac{1}{\sqrt{R_1R_2C_1C_2}} \tag{10.12}$$

In most practical applications, $R_1 = R_2 = R$ and $C_1 = C_2 = C$, so that

$$\omega_o = \frac{1}{RC} = 2\pi f_o \tag{10.13}$$

or

$$\boxed{f_o = \frac{1}{2\pi RC}} \tag{10.14}$$

Substituting Eq. (10.13) and $R_1 = R_2 = R$, $C_1 = C_2 = C$ into Eq. (10.11) yields

$$\frac{\mathbf{V}_2}{\mathbf{V}_o} = \frac{1}{3} \tag{10.15}$$

Thus, in order to satisfy the first Barkhausen criterion, the op amp must compensate by providing a gain of 3 or greater so that the overall gain is at least 1 or unity. We recall that for a noninverting amplifier,

$$\frac{\mathbf{V}_o}{\mathbf{V}_2} = 1 + \frac{R_f}{R_g} = 3 \tag{10.16}$$

or

$$R_f = 2R_g \tag{10.17}$$

Due to the inherent delay caused by the op amp, Wien-bridge oscillators are limited to operating in the frequency range of 1 MHz or less.

Example 10.16

Design a Wien-bridge circuit to oscillate at 100 kHz.

Solution:

Using Eq. (10.14), we obtain the time constant of the circuit as

$$RC = \frac{1}{2\pi f_o} = \frac{1}{2\pi \times 100 \times 10^3} = 1.59 \times 10^{-6} \tag{10.16.1}$$

If we select $R = 10\ \text{k}\Omega$, then we can select $C = 159$ pF to satisfy Eq. (10.16.1). Since the gain must be 3, $R_f/R_g = 2$. We could select $R_f = 20\ \text{k}\Omega$ while $R_g = 10\ \text{k}\Omega$.

Practice Problem 10.16

In the Wien-bridge oscillator circuit in Fig. 10.42, let $R_1 = R_2 = 2.5\ \text{k}\Omega$, $C_1 = C_2 = 1$ nF. Determine the frequency f_o of the oscillator.

Answer: 63.66 kHz.

10.10 Summary

1. We apply nodal and mesh analysis to ac circuits by applying KCL and KVL to the phasor form of the circuits.
2. In solving for the steady-state response of a circuit that has independent sources with different frequencies, each independent source *must* be considered separately. The most natural approach to analyzing such circuits is to apply the superposition theorem. A separate phasor circuit for each frequency *must* be solved independently, and the corresponding response should be obtained in the time domain. The overall response is the sum of the time domain responses of all the individual phasor circuits.
3. The concept of source transformation is also applicable in the frequency domain.
4. The Thevenin equivalent of an ac circuit consists of a voltage source $\mathbf{V}_{\text{Th}}$ in series with the Thevenin impedance $\mathbf{Z}_{\text{Th}}$.
5. The Norton equivalent of an ac circuit consists of a current source $\mathbf{I}_N$ in parallel with the Norton impedance $\mathbf{Z}_N$ $(=\mathbf{Z}_{\text{Th}})$.
6. *PSpice* is a simple and powerful tool for solving ac circuit problems. It relieves us of the tedious task of working with the complex numbers involved in steady-state analysis.
7. The capacitance multiplier and the ac oscillator provide two typical applications for the concepts presented in this chapter. A capacitance multiplier is an op amp circuit used in producing a multiple of a physical capacitance. An oscillator is a device that uses a dc input to generate an ac output.

Review Questions

10.1 The voltage $\mathbf{V}_o$ across the capacitor in Fig. 10.43 is:

(a) $5\angle 0^\circ$ V (b) $7.071\angle 45^\circ$ V
(c) $7.071\angle -45^\circ$ V (d) $5\angle -45^\circ$ V

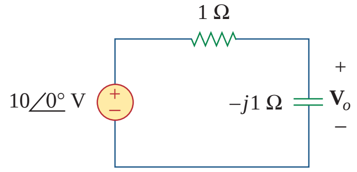


Figure 10.43
For Review Question 10.1.

10.2 The value of the current $\mathbf{I}_o$ in the circuit of Fig. 10.44 is:

(a) $4\angle 0^\circ$ A (b) $2.4\angle -90^\circ$ A
(c) $0.6\angle 0^\circ$ A (d) -1 A

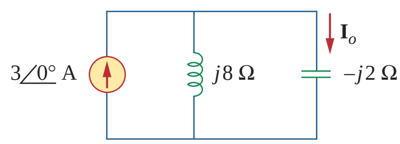


Figure 10.44
For Review Question 10.2.

10.3 Using nodal analysis, the value of $\mathbf{V}_o$ in the circuit of Fig. 10.45 is:

(a) -24 V (b) -8 V
(c) 8 V (d) 24 V

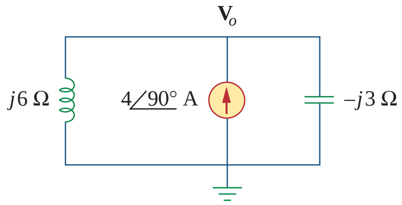


Figure 10.45
For Review Question 10.3.

10.4 In the circuit of Fig. 10.46, current $i(t)$ is:

(a) $10 \cos t$ A (b) $10 \sin t$ A (c) $5 \cos t$ A
(d) $5 \sin t$ A (e) $4.472 \cos(t - 63.43^\circ)$ A

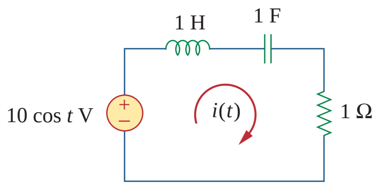


Figure 10.46
For Review Question 10.4.

10.5 Refer to the circuit in Fig. 10.47 and observe that the two sources do not have the same frequency. The current $i_x(t)$ can be obtained by:

(a) source transformation
(b) the superposition theorem
(c) *PSpice*

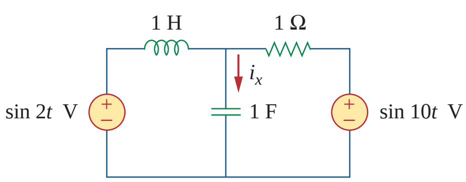


Figure 10.47
For Review Question 10.5.

10.6 For the circuit in Fig. 10.48, the Thevenin impedance at terminals *a-b* is:

(a) $1\ \Omega$ (b) $0.5 - j0.5\ \Omega$
(c) $0.5 + j0.5\ \Omega$ (d) $1 + j2\ \Omega$
(e) $1 - j2\ \Omega$

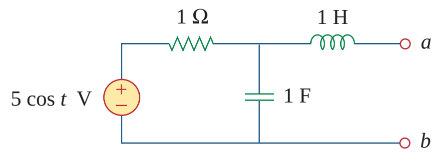


Figure 10.48
For Review Questions 10.6 and 10.7.

10.7 In the circuit of Fig. 10.48, the Thevenin voltage at terminals *a-b* is:

(a) $3.535\underline{/-45^\circ}$ V (b) $3.535\underline{/45^\circ}$ V
(c) $7.071\underline{/-45^\circ}$ V (d) $7.071\underline{/45^\circ}$ V

10.8 Refer to the circuit in Fig. 10.49. The Norton equivalent impedance at terminals *a-b* is:

(a) $-j4\ \Omega$ (b) $-j2\ \Omega$
(c) $j2\ \Omega$ (d) $j4\ \Omega$

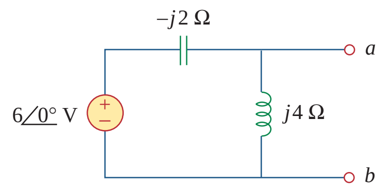


Figure 10.49
For Review Questions 10.8 and 10.9.

10.9 The Norton current at terminals *a-b* in the circuit of Fig. 10.49 is:

(a) $1\underline{/0^\circ}$ A (b) $1.5\underline{/-90^\circ}$ A
(c) $1.5\underline{/90^\circ}$ A (d) $3\underline{/90^\circ}$ A

10.10 *PSpice* can handle a circuit with two independent sources of different frequencies.

(a) True (b) False

Answers: 10.1c, 10.2a, 10.3d, 10.4a, 10.5b, 10.6c, 10.7a, 10.8a, 10.9d, 10.10b.

Problems

Section 10.2 Nodal Analysis

10.1 Determine i in the circuit of Fig. 10.50.

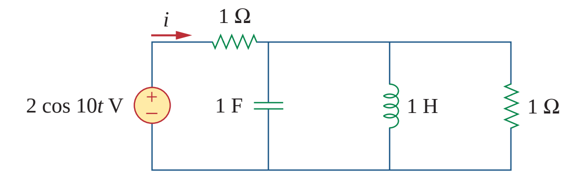


Figure 10.50
For Prob. 10.1.

10.2 Using Fig. 10.51, design a problem to help other students better understand nodal analysis.

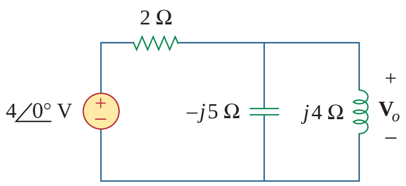


Figure 10.51
For Prob. 10.2.

10.3 Determine v_o in the circuit of Fig. 10.52.

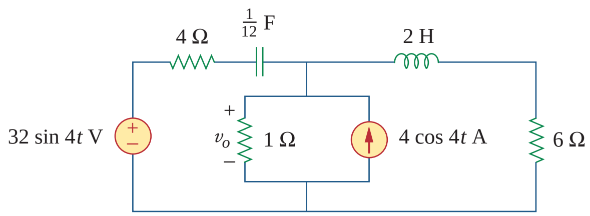


Figure 10.52
For Prob. 10.3.

10.4 Determine i_1 in the circuit of Fig. 10.53.

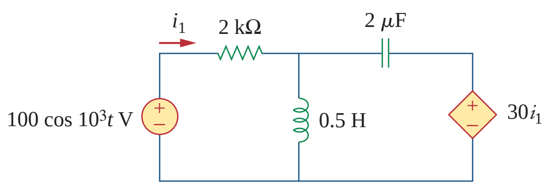


Figure 10.53
For Prob. 10.4.

10.5 Find i_o in the circuit of Fig. 10.54.

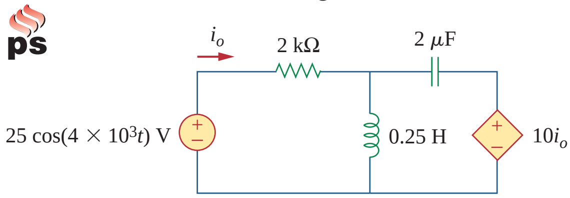


Figure 10.54
For Prob. 10.5.

10.6 Determine $\mathbf{V}_x$ in Fig. 10.55.

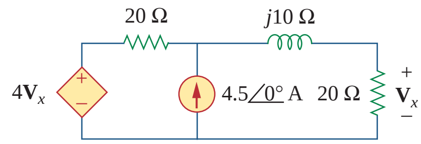


Figure 10.55
For Prob. 10.6.

10.7 Use nodal analysis to find $\mathbf{V}$ in the circuit of Fig. 10.56.

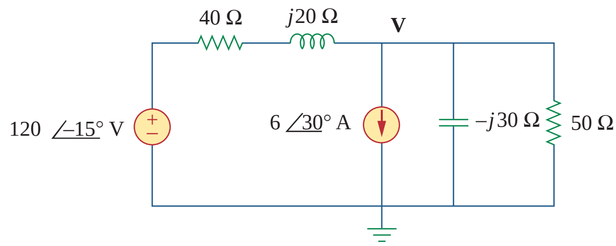


Figure 10.56
For Prob. 10.7.

10.8 Use nodal analysis to find current i_o in the circuit of Fig. 10.57. Let $i_s = 6\cos(200t + 15°)$ A.

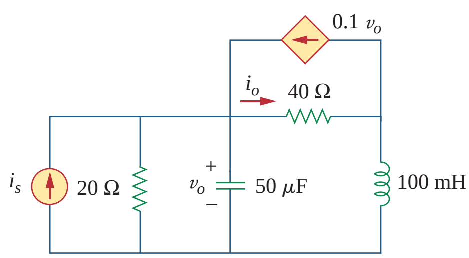


Figure 10.57
For Prob. 10.8.

10.9 Use nodal analysis to find v_o in the circuit of Fig. 10.58.

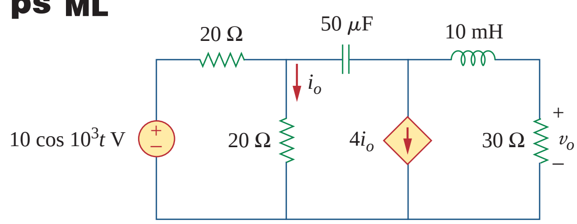


Figure 10.58
For Prob. 10.9.

10.10 Use nodal analysis to find v_o in the circuit of Fig. 10.59. Let $\omega = 2$ krad/s.

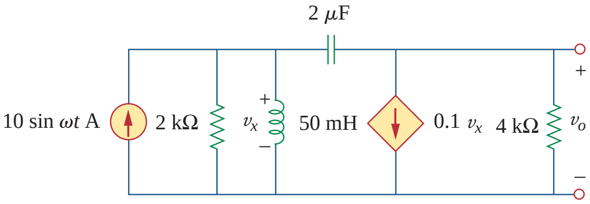


Figure 10.59
For Prob. 10.10.

10.11 Apply nodal analysis to the circuit in Fig. 10.60 and determine $\mathbf{I}_o$.

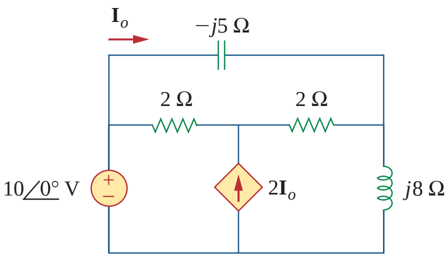


Figure 10.60
For Prob. 10.11.

10.12 Using Fig. 10.61, design a problem to help other students better understand nodal analysis.

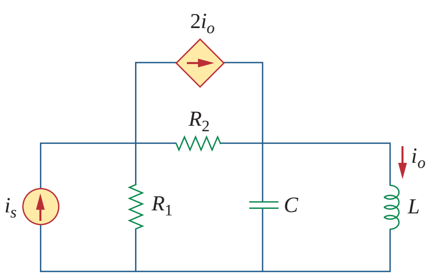


Figure 10.61
For Prob. 10.12.

10.13 Determine $\mathbf{V}_x$ in the circuit of Fig. 10.62 using any method of your choice.

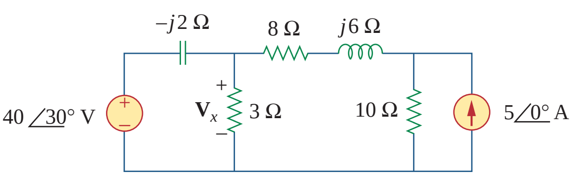


Figure 10.62
For Prob. 10.13.

10.14 Calculate the voltage at nodes 1 and 2 in the circuit of Fig. 10.63 using nodal analysis.

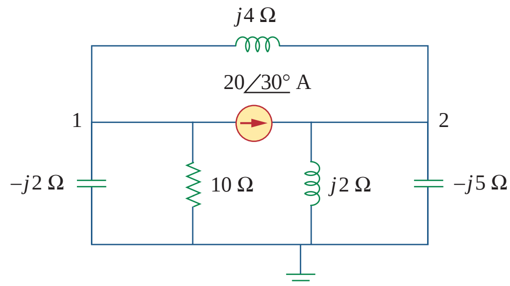


Figure 10.63
For Prob. 10.14.

10.15 Solve for the current $\mathbf{I}$ in the circuit of Fig. 10.64 using nodal analysis.

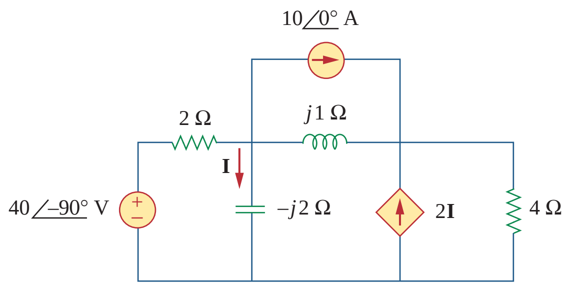


Figure 10.64
For Prob. 10.15.

10.16 Use nodal analysis to find $\mathbf{V}_x$ in the circuit shown in Fig. 10.65.

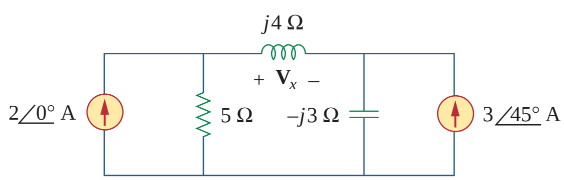


Figure 10.65
For Prob. 10.16.

10.17 By nodal analysis, obtain current $\mathbf{I}_o$ in the circuit of Fig. 10.66.

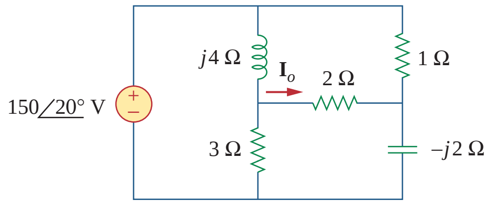


Figure 10.66
For Prob. 10.17.

10.18 Use nodal analysis to obtain $\mathbf{V}_o$ in the circuit of Fig. 10.67 below.

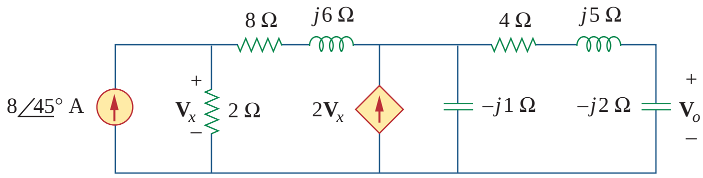


Figure 10.67
For Prob. 10.18.

10.19 Obtain $\mathbf{V}_o$ in Fig. 10.68 using nodal analysis.

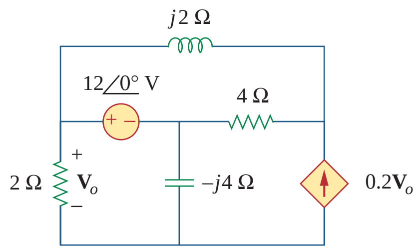


Figure 10.68
For Prob. 10.19.

10.20 Refer to Fig. 10.69. If $v_s(t) = V_m \sin \omega t$ and $v_o(t) = A \sin(\omega t + \phi)$, derive the expressions for A and ϕ.

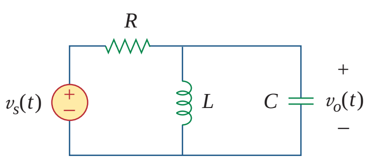


Figure 10.69
For Prob. 10.20.

10.21 For each of the circuits in Fig. 10.70, find $\mathbf{V}_o/\mathbf{V}_i$ for $\omega = 0$, $\omega \to \infty$, and $\omega^2 = 1/LC$.

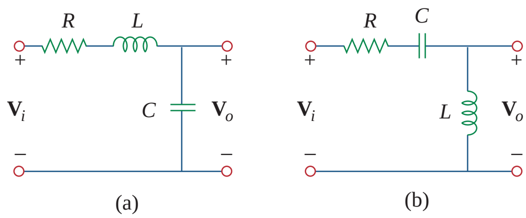


Figure 10.70
For Prob. 10.21.

10.22 For the circuit in Fig. 10.71, determine $\mathbf{V}_o/\mathbf{V}_s$.

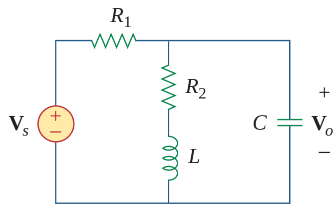


Figure 10.71
For Prob. 10.22.

10.23 Using nodal analysis obtain $\mathbf{V}$ in the circuit of Fig. 10.72.

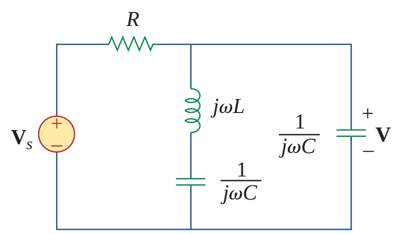


Figure 10.72
For Prob. 10.23.

Section 10.3 Mesh Analysis

10.24 Design a problem to help other students better understand mesh analysis.

10.25 Solve for i_o in Fig. 10.73 using mesh analysis.

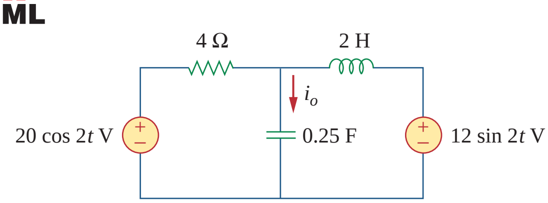


Figure 10.73
For Prob. 10.25.

10.26 Use mesh analysis to find current i_o in the circuit of Fig. 10.74.

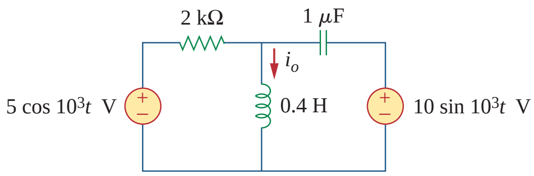


Figure 10.74
For Prob. 10.26.

10.27 Using mesh analysis, find $\mathbf{I}_1$ and $\mathbf{I}_2$ in the circuit of Fig. 10.75.

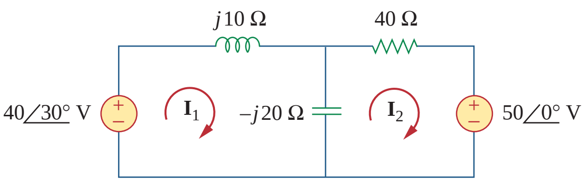


Figure 10.75
For Prob. 10.27.

10.28 In the circuit of Fig. 10.76, determine the mesh currents i_1 and i_2. Let $v_1 = 10 \cos 4t$ **V** and $v_2 = 20 \cos(4t - 30^\circ)$ V.

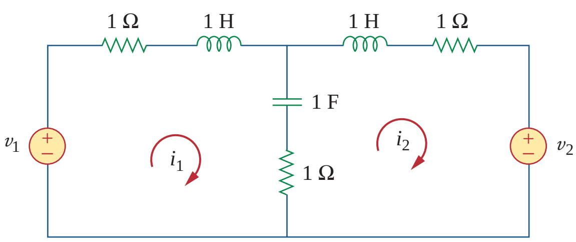


Figure 10.76
For Prob. 10.28.

10.29 Using Fig. 10.77, design a problem to help other students better understand mesh analysis.

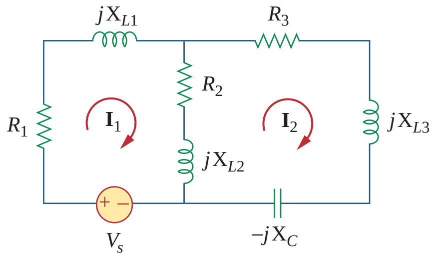


Figure 10.77
For Prob. 10.29.

10.30 Use mesh analysis to find v_o in the circuit of Fig. 10.78. Let $v_{s1} = 240 \cos(100t + 90^\circ)$ V, $v_{s2} = 160 \cos 100t$ V.

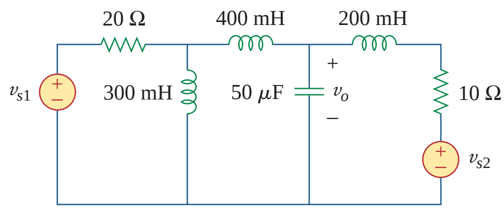


Figure 10.78
For Prob. 10.30.

10.31 Use mesh analysis to determine current $\mathbf{I}_o$ in the circuit of Fig. 10.79 below.

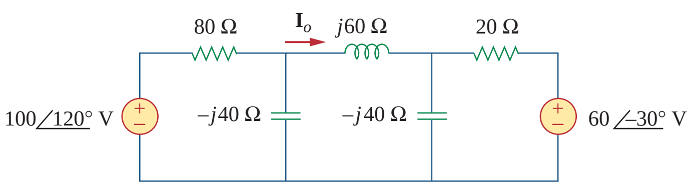


Figure 10.79
For Prob. 10.31.

10.32 Determine $\mathbf{V}_o$ and $\mathbf{I}_o$ in the circuit of Fig. 10.80 using mesh analysis.

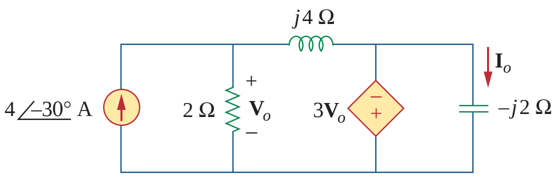


Figure 10.80
For Prob. 10.32.

10.33 Compute **I** in Prob. 10.15 using mesh analysis.

10.34 Use mesh analysis to find $\mathbf{I}_o$ in Fig. 10.28 (for Example 10.10).

10.35 Calculate $\mathbf{I}_o$ in Fig. 10.30 (for Practice Prob. 10.10) using mesh analysis.

10.36 Compute $\mathbf{V}_o$ in the circuit of Fig. 10.81 using mesh analysis.

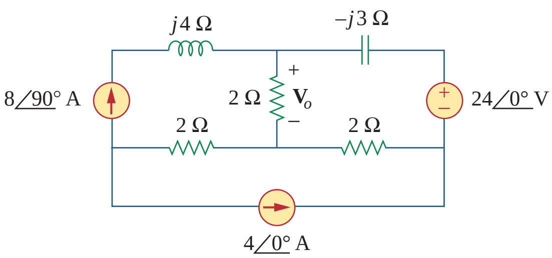


Figure 10.81
For Prob. 10.36.

10.37 Use mesh analysis to find currents $\mathbf{I}_1$, $\mathbf{I}_2$, and $\mathbf{I}_3$ in the circuit of Fig. 10.82.

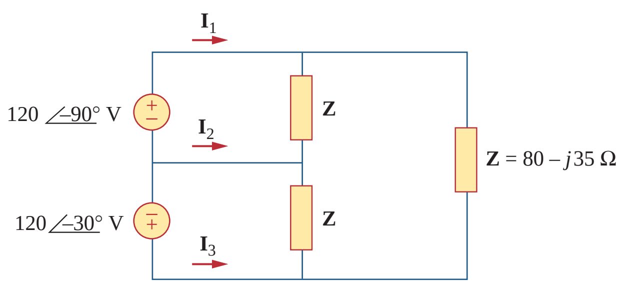


Figure 10.82
For Prob. 10.37.

10.38 Using mesh analysis, obtain $\mathbf{I}_o$ in the circuit shown in Fig. 10.83.

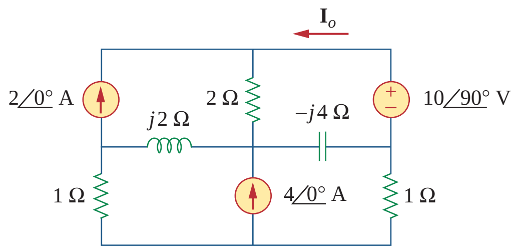


Figure 10.83
For Prob. 10.38.

10.39 Find $\mathbf{I}_1$, $\mathbf{I}_2$, $\mathbf{I}_3$, and $\mathbf{I}_x$ in the circuit of Fig. 10.84.

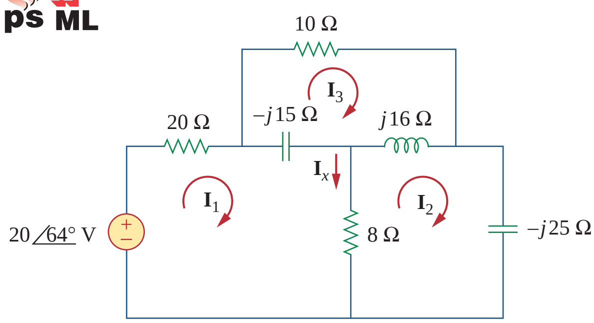


Figure 10.84
For Prob. 10.39.

Section 10.4 Superposition Theorem

10.40 Find i_o in the circuit shown in Fig. 10.85 using superposition.

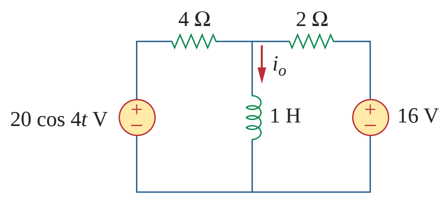


Figure 10.85
For Prob. 10.40.

10.41 Find v_o for the circuit in Fig. 10.86, assuming that $v_s = 3 \cos 2t + 8 \sin 4t$ V.

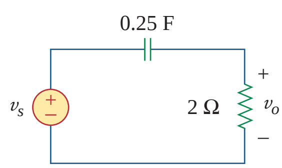


Figure 10.86
For Prob. 10.41.

10.42 Using Fig. 10.87, design a problem to help other students better understand the superposition theorem.

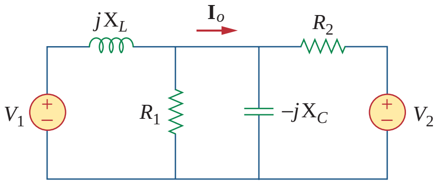


Figure 10.87
For Prob. 10.42.

10.43 Using the superposition principle, find i_x in the circuit of Fig. 10.88.

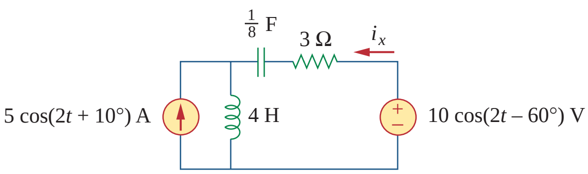


Figure 10.88
For Prob. 10.43.

10.44 Use the superposition principle to obtain v_x in the circuit of Fig. 10.89. Let $v_s = 25 \sin 2t$ V and $i_s = 6 \cos(6t + 10°)$ A.

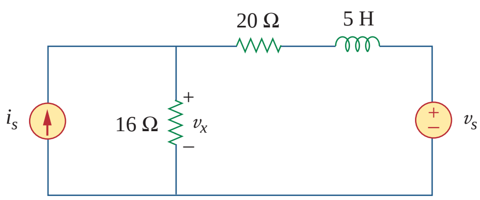


Figure 10.89
For Prob. 10.44.

10.45 Use superposition to find $i(t)$ in the circuit of Fig. 10.90.

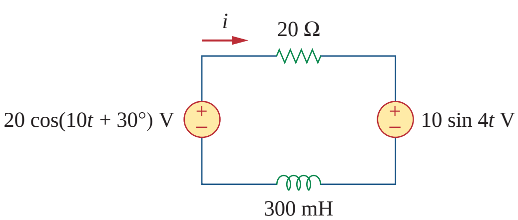


Figure 10.90
For Prob. 10.45.

10.46 Solve for $v_o(t)$ in the circuit of Fig. 10.91 using the superposition principle.

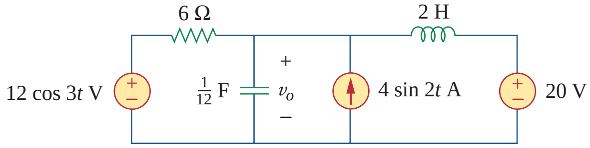


Figure 10.91
For Prob. 10.46.

10.47 Determine i_o in the circuit of Fig. 10.92, using the superposition principle.

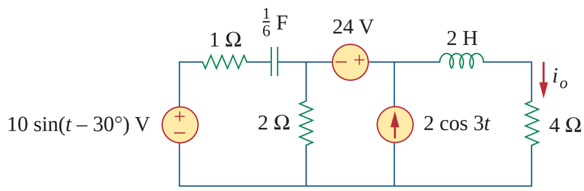


Figure 10.92
For Prob. 10.47.

10.48 Find i_o in the circuit of Fig. 10.93 using superposition.

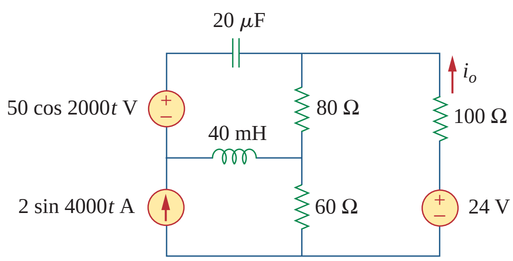


Figure 10.93
For Prob. 10.48.

Section 10.5 Source Transformation

10.49 Using source transformation, find i in the circuit of Fig. 10.94.

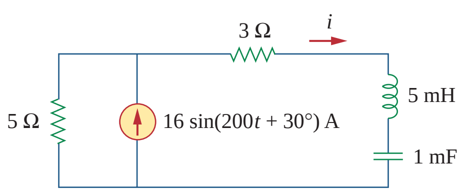


Figure 10.94
For Prob. 10.49.

10.50 Using Fig. 10.95, design a problem to help other students understand source transformation.

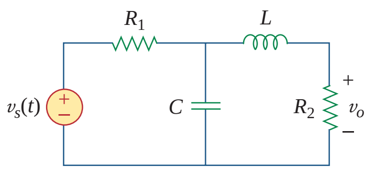


Figure 10.95
For Prob. 10.50.

10.51 Use source transformation to find $\mathbf{I}_o$ in the circuit of Prob. 10.42.

10.52 Use the method of source transformation to find $\mathbf{I}_x$ in the circuit of Fig. 10.96.

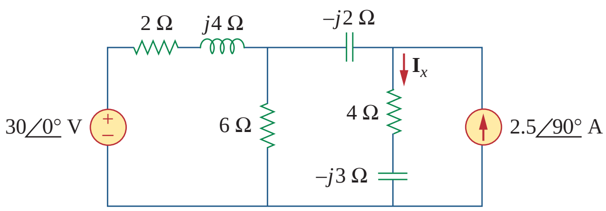


Figure 10.96
For Prob. 10.52.

10.53 Use the concept of source transformation to find $\mathbf{V}_o$ in the circuit of Fig. 10.97.

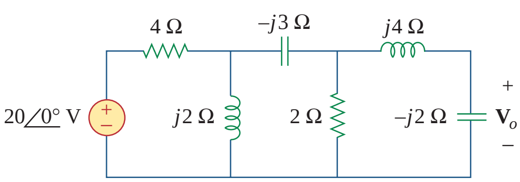
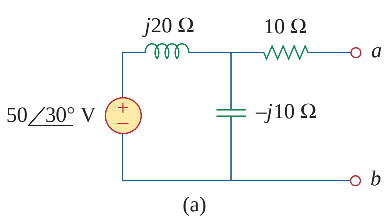


Figure 10.97
For Prob. 10.53.

10.54 Rework Prob. 10.7 using source transformation.

Section 10.6 Thevenin and Norton Equivalent Circuits

10.55 Find the Thevenin and Norton equivalent circuits at terminals *a-b* for each of the circuits in Fig. 10.98.



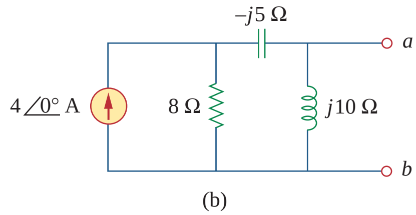


Figure 10.98
For Prob. 10.55.

10.56 For each of the circuits in Fig. 10.99, obtain Thevenin and Norton equivalent circuits at terminals *a-b*.

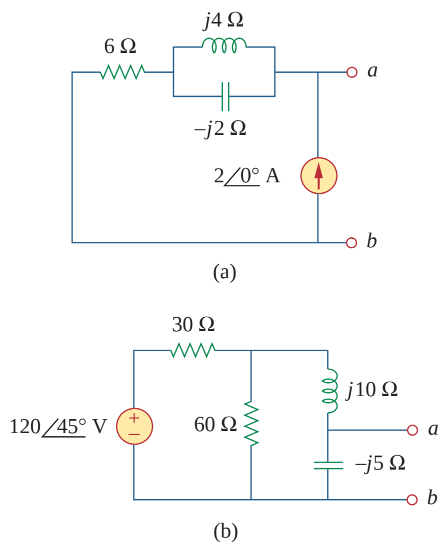


Figure 10.99
For Prob. 10.56.

10.57 Using Fig. 10.100, design a problem to help other students better understand Thevenin and Norton equivalent circuits.

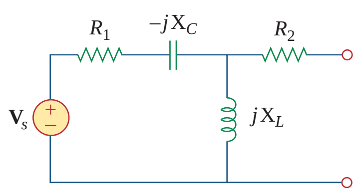


Figure 10.100
For Prob. 10.57.

10.58 For the circuit depicted in Fig. 10.101, find the Thevenin equivalent circuit at terminals *a-b*.

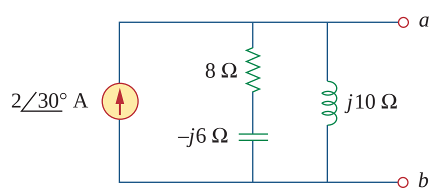


Figure 10.101
For Prob. 10.58.

10.59 Calculate the output impedance of the circuit shown in Fig. 10.102.

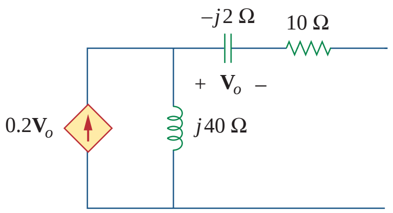


Figure 10.102
For Prob. 10.59.

10.60 Find the Thevenin equivalent of the circuit in Fig. 10.103 as seen from:

(a) terminals *a-b* (b) terminals *c-d*

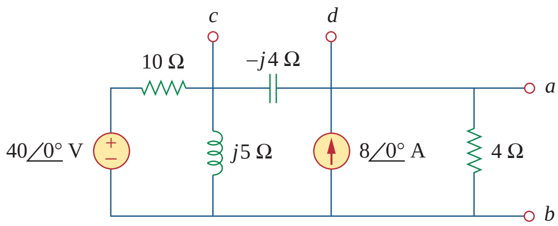


Figure 10.103
For Prob. 10.60.

10.61 Find the Thevenin equivalent at terminals *a-b* of the circuit in Fig. 10.104.

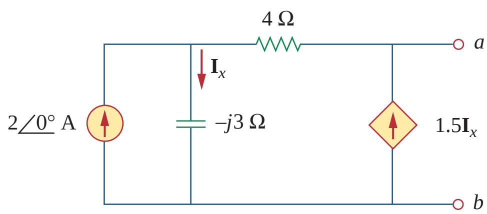


Figure 10.104
For Prob. 10.61.

10.62 Using Thevenin's theorem, find v_o in the circuit of Fig. 10.105.

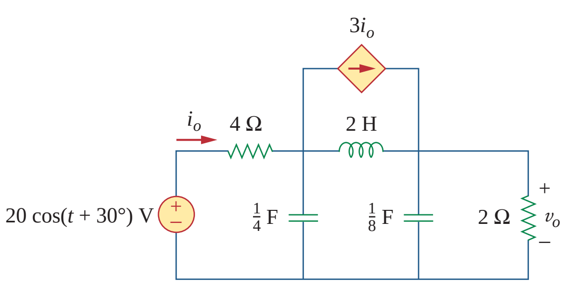


Figure 10.105
For Prob. 10.62.

10.63 Obtain the Norton equivalent of the circuit depicted in Fig. 10.106 at terminals *a-b*.

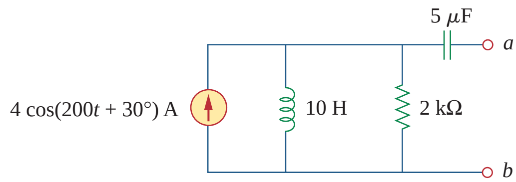


Figure 10.106
For Prob. 10.63.

10.64 For the circuit shown in Fig. 10.107, find the Norton equivalent circuit at terminals *a-b*.

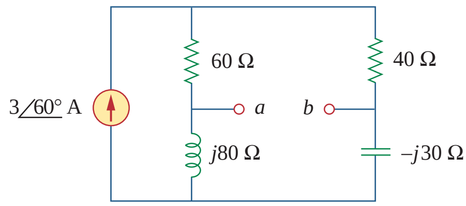


Figure 10.107
For Prob. 10.64.

10.65 Using Fig. 10.108, design a problem to help other students better understand Norton's theorem.

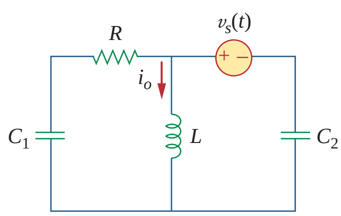


Figure 10.108
For Prob. 10.65.

10.66 At terminals *a-b*, obtain Thevenin and Norton equivalent circuits for the network depicted in Fig. 10.109. Take $\omega = 10$ rad/s.

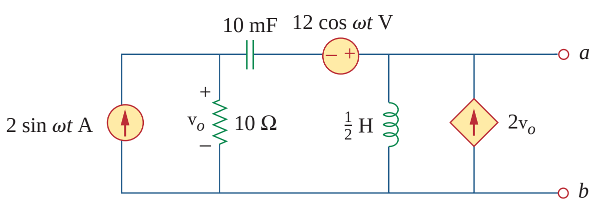


Figure 10.109
For Prob. 10.66.

10.67 Find the Thevenin and Norton equivalent circuits at terminals *a-b* in the circuit of Fig. 10.110.

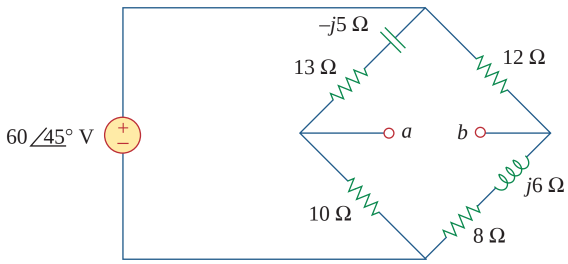


Figure 10.110
For Prob. 10.67.

10.68 Find the Thevenin equivalent at terminals *a-b* in the circuit of Fig. 10.111.

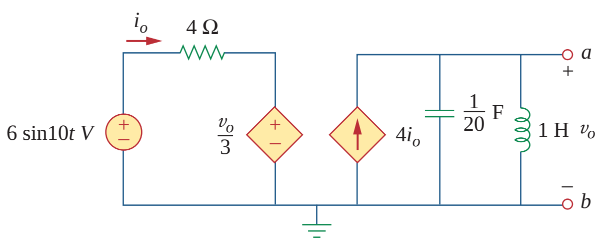


Figure 10.111
For Prob. 10.68.

Section 10.7 Op Amp AC Circuits

10.69 For the differentiator shown in Fig. 10.112, obtain $\mathbf{V}_o/\mathbf{V}_s$. Find $v_o(t)$ when $v_s(t) = \mathbf{V}_m \sin \omega t$ and $\omega = 1/RC$.

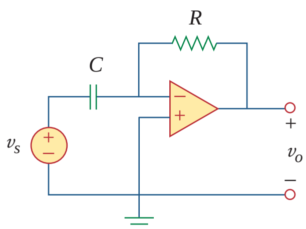


Figure 10.112
For Prob. 10.69.

10.70 Using Fig. 10.113, design a problem to help other students better understand op amps in AC circuits.

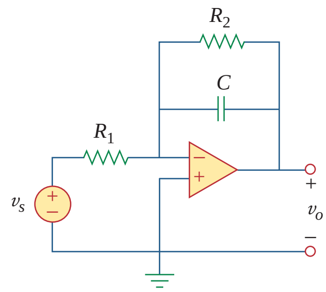


Figure 10.113
For Prob. 10.70.

10.71 Find v_o in the op amp circuit of Fig. 10.114.

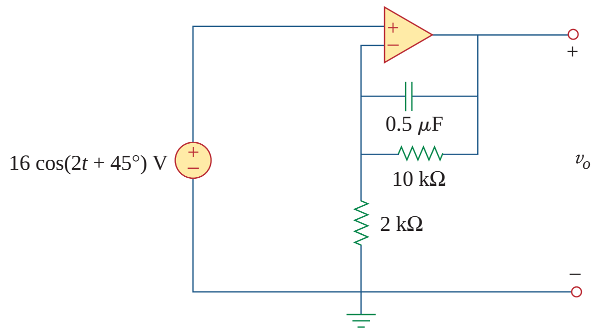


Figure 10.114
For Prob. 10.71.

10.72 Compute $i_o(t)$ in the op amp circuit in Fig. 10.115 if $v_s = 10 \cos(10^4 t + 30°)$ V.

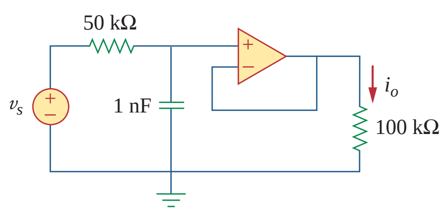


Figure 10.115
For Prob. 10.72.

10.73 If the input impedance is defined as $\mathbf{Z}_{in} = \mathbf{V}_s/\mathbf{I}_s$, find the input impedance of the op amp circuit in Fig. 10.116 when $R_1 = 10$ kΩ, $R_2 = 20$ kΩ, $C_1 = 10$ nF, $C_2 = 20$ nF, and $\omega = 5000$ rad/s.

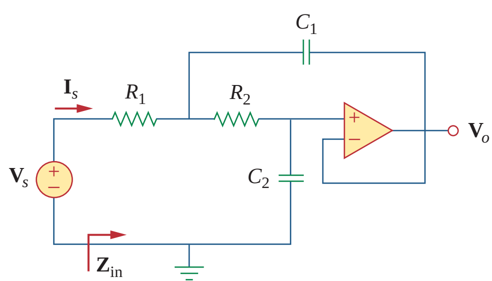


Figure 10.116
For Prob. 10.73.

10.74 Evaluate the voltage gain $\mathbf{A}_v = \mathbf{V}_o/\mathbf{V}_s$ in the op amp circuit of Fig. 10.117. Find $\mathbf{A}_v$ at $\omega = 0$, $\omega \to \infty$, $\omega = 1/R_1C_1$, and $\omega = 1/R_2C_2$.

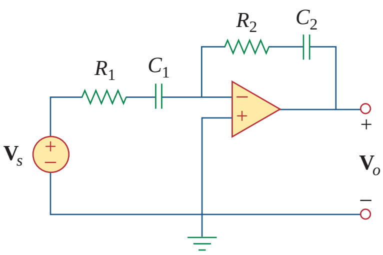


Figure 10.117
For Prob. 10.74.

10.75 In the op amp circuit of Fig. 10.118, find the closed-loop gain and phase shift of the output voltage with respect to the input voltage if $C_1 = C_2 = 1$ nF, $R_1 = R_2 = 100$ kΩ, $R_3 = 20$ kΩ, $R_4 = 40$ kΩ, and $\omega = 2000$ rad/s.

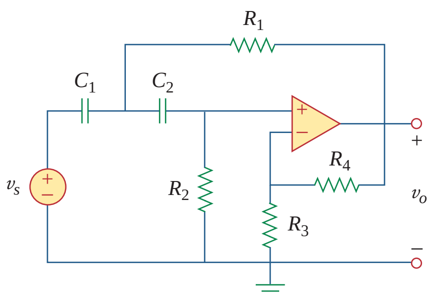


Figure 10.118
For Prob. 10.75.

10.76 Determine $\mathbf{V}_o$ and $\mathbf{I}_o$ in the op amp circuit of Fig. 10.119.

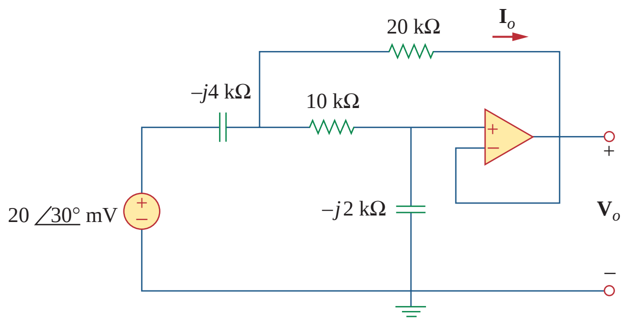


Figure 10.119
For Prob. 10.76.

10.77 Compute the closed-loop gain $\mathbf{V}_o/\mathbf{V}_s$ for the op amp circuit of Fig. 10.120.

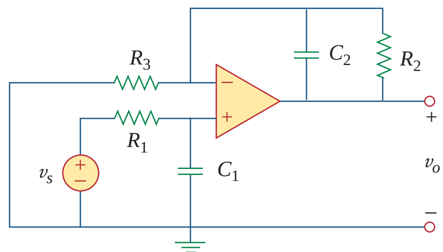


Figure 10.120
For Prob. 10.77.

10.78 Determine $v_o(t)$ in the op amp circuit in Fig. 10.121 below.

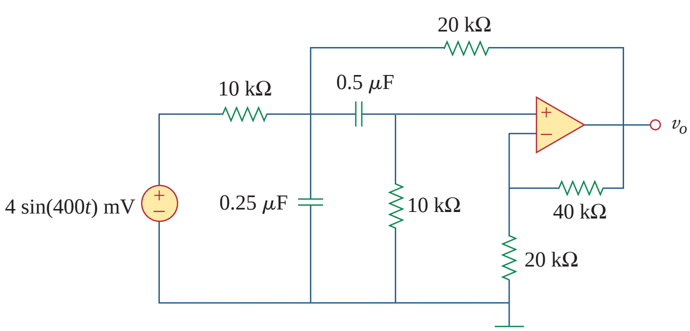


Figure 10.121
For Prob. 10.78.

10.79 For the op amp circuit in Fig. 10.122, obtain $v_o(t)$.

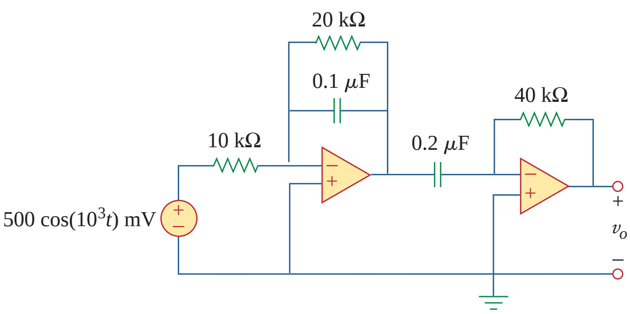


Figure 10.122
For Prob. 10.79.

10.80 Obtain $v_o(t)$ for the op amp circuit in Fig. 10.123 if $v_s = 4\cos(1000t - 60°)$ V.

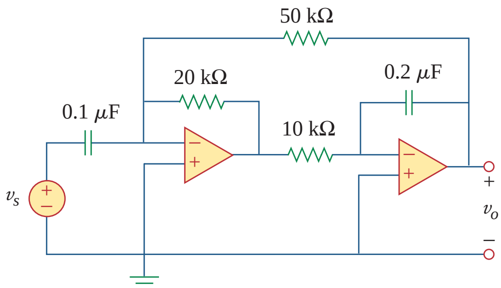


Figure 10.123
For Prob. 10.80.

Section 10.8 AC Analysis Using *PSpice*

10.81 Use *PSpice* to determine $\mathbf{V}_o$ in the circuit of Fig. 10.124. Assume $\omega = 1$ rad/s.

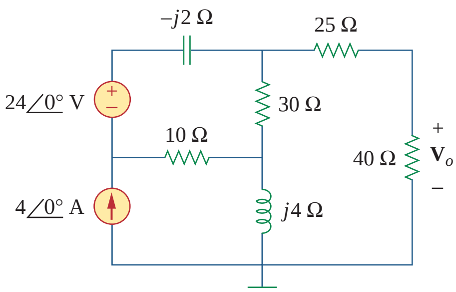


Figure 10.124
For Prob. 10.81.

10.82 Solve Prob. 10.19 using *PSpice*.

10.83 Use *PSpice* to find $v_o(t)$ in the circuit of Fig. 10.125. Let $i_s = 2\cos(10^3 t)$ A.

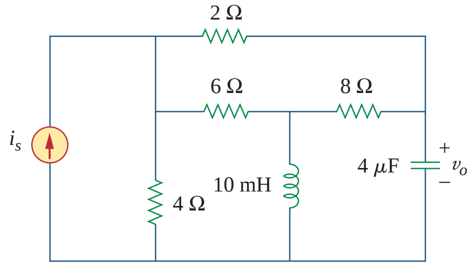


Figure 10.125
For Prob. 10.83.

10.84 Obtain $\mathbf{V}_o$ in the circuit of Fig. 10.126 using *PSpice*.

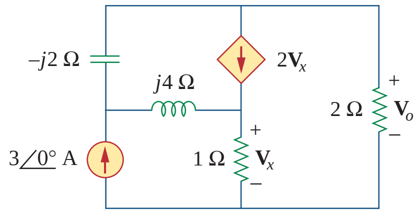


Figure 10.126
For Prob. 10.84.

10.85 Using Fig. 10.127, design a problem to help other students better understand performing AC analysis with *PSpice*.

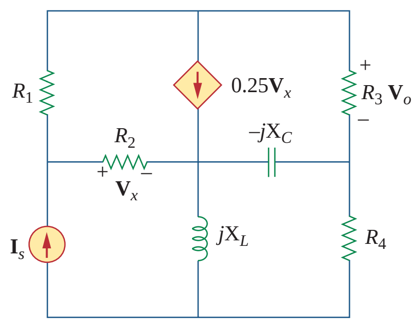


Figure 10.127
For Prob. 10.85.

10.86 Use *PSpice* to find $\mathbf{V}_1$, $\mathbf{V}_2$, and $\mathbf{V}_3$ in the network of Fig. 10.128.

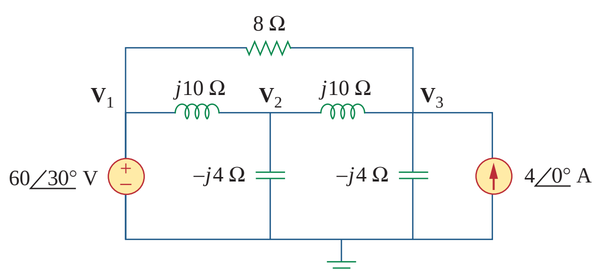


Figure 10.128
For Prob. 10.86.

10.87 Determine $\mathbf{V}_1$, $\mathbf{V}_2$, and $\mathbf{V}_3$ in the circuit of Fig. 10.129 using *PSpice*.

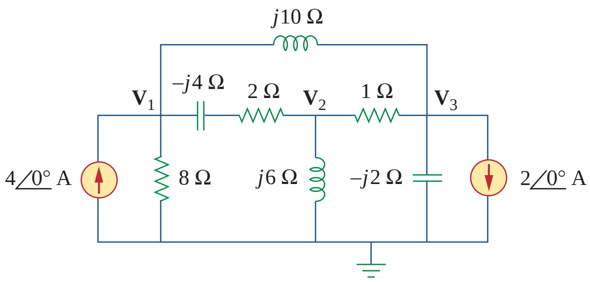


Figure 10.129
For Prob. 10.87.

10.88 Use *PSpice* to find v_o and i_o in the circuit of Fig. 10.130 below.

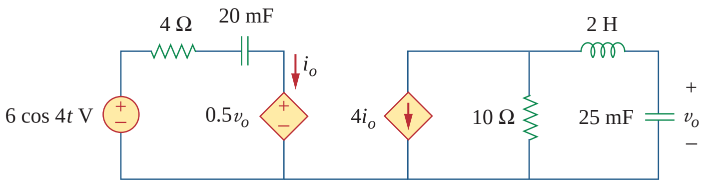


Figure 10.130
For Prob. 10.88.

Section 10.9 Applications

10.89 The op amp circuit in Fig. 10.131 is called an *inductance simulator*. Show that the input impedance is given by

$$\mathbf{Z}_{\text{in}} = \frac{\mathbf{V}_{\text{in}}}{\mathbf{I}_{\text{in}}} = j\omega L_{\text{eq}}$$

where

$$L_{\text{eq}} = \frac{R_1 R_3 R_4}{R_2} C$$

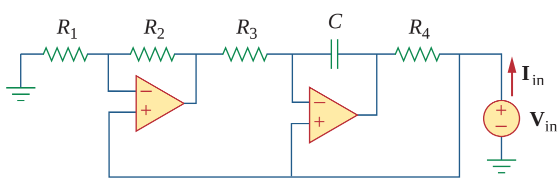


Figure 10.131
For Prob. 10.89.

10.90 Figure 10.132 shows a Wien-bridge network. Show that the frequency at which the phase shift between the input and output signals is zero is $f = \frac{1}{2}\pi RC$, and that the necessary gain is $\mathbf{A}_v = \mathbf{V}_o/\mathbf{V}_i = 3$ at that frequency.

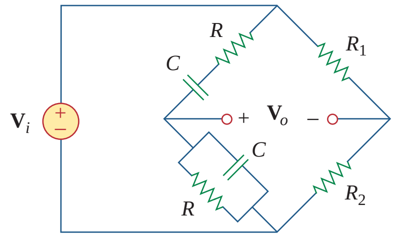


Figure 10.132
For Prob. 10.90.

10.91 Consider the oscillator in Fig. 10.133.

(a) Determine the oscillation frequency.

(b) Obtain the minimum value of R for which oscillation takes place.

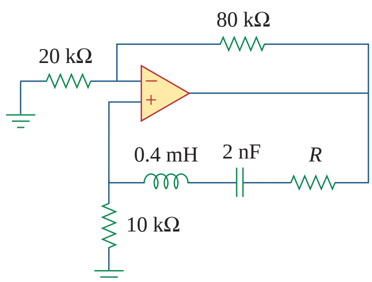


Figure 10.133
For Prob. 10.91.

10.92 The oscillator circuit in Fig. 10.134 uses an ideal op amp.

(a) Calculate the minimum value of R_o that will cause oscillation to occur.

(b) Find the frequency of oscillation.

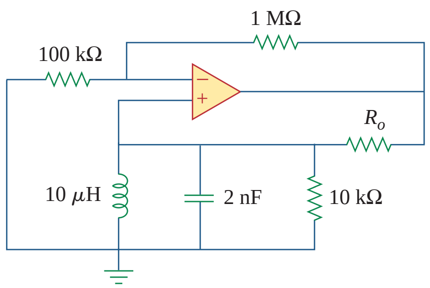


Figure 10.134
For Prob. 10.92.

10.93 Figure 10.135 shows a *Colpitts oscillator*. Show that the oscillation frequency is

$$f_o = \frac{1}{2\pi\sqrt{LC_T}}$$

where $C_T = C_1C_2/(C_1 + C_2)$. Assume $R_i \gg X_{C_2}$.

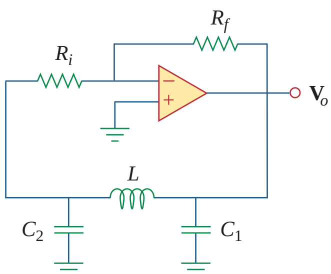


Figure 10.135
A Colpitts oscillator; for Prob. 10.93.

(*Hint:* Set the imaginary part of the impedance in the feedback circuit equal to zero.)

10.94 Design a Colpitts oscillator that will operate at 50 kHz.

10.95 Figure 10.136 shows a *Hartley oscillator*. Show that the frequency of oscillation is

$$f_o = \frac{1}{2\pi\sqrt{C(L_1 + L_2)}}$$

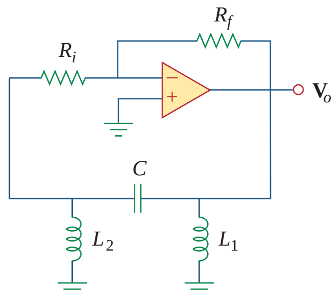


Figure 10.136
A Hartley oscillator; for Prob. 10.95.

10.96 Refer to the oscillator in Fig. 10.137.

(a) Show that

$$\frac{\mathbf{V}_2}{\mathbf{V}_o} = \frac{1}{3 + j(\omega L/R - R/\omega L)}$$

(b) Determine the oscillation frequency f_o.

(c) Obtain the relationship between R_1 and R_2 in order for oscillation to occur.

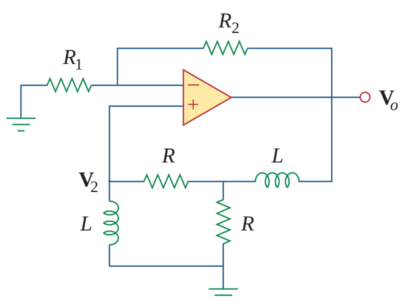


Figure 10.137
For Prob. 10.96.