

Assignment I - CALCULUS I - Group 5

Deadline: 8PM, May 27th 2023

Exercise 1. Let $A(1, 2)$ and $B(4, 3)$ be two points in \mathbb{R}^2 . Find

- (i) \overrightarrow{AB}
- (ii) Parametric and symmetric equations of the line passing through A and B .

Exercise 2. Find a formula for the inverse of the function

$$f(x) = x^2 + 2x, \quad x \geq -1.$$

Exercise 3. Let $f(x) = \sqrt{2017 - \sin(x + 1)}$

- (i) Find the domain and the range of f .
- (ii) Find functions g, h so that $f(x) = g(h(x))$, $x \in (-\infty, \infty)$.

Exercise 4. Let C be a circle with radius 2 centred at the point $(2, 0)$.

- (i) Write an equation for the circle C .
- (ii) Is curve C the graph of a function of x ? Explain your answer.
- (iii) Write parametric equations to traverse C once, in a clockwise direction, starting from the origin.

Exercise 5. Evaluate the limit

$$\lim_{x \rightarrow 2} \frac{\sqrt{6-x} - 2}{\sqrt{3-x} - 1}.$$

Exercise 6. Use Squeeze theorem to evaluate the limit

$$\lim_{x \rightarrow 0} x \cos(\ln |x|).$$

Exercise 7. Which of the following is true for the function $f(x)$ given by

$$f(x) = \begin{cases} 2x - 1 & \text{if } x < -1 \\ x^2 + 1 & \text{if } -1 \leq x \leq 1 \\ x + 1 & \text{if } x > 1. \end{cases}$$

- (i) f is continuous everywhere,
- (ii) f is continuous everywhere except at $x = -1$ and $x = 1$,
- (iii) f is continuous everywhere except at $x = -1$,
- (iv) f is continuous everywhere except at $x = 1$,
- (v) None of the above.

Explain your choice in details.

Exercise 8. Let

$$g(x) = \begin{cases} \cos x & \text{if } x < 0 \\ 0 & \text{if } x = 0 \\ 1 - x^2 & \text{if } x > 0. \end{cases}$$

- (i) Explain why $g(x)$ is discontinuous at $x = 0$.
- (ii) Sketch the graph of $g(x)$.

Exercise 9. Show that the equation

$$x^3 - x \sin x - 1 = x\sqrt{x+2}$$

has a real root in the interval $[0, 2]$.