● PROGRAM OF "PHYSICS"

<u>Lecturer</u>: **Dr. DO Xuan Hoi**

Room A1 503

E-mail : dxhoi@hcmiu.edu.vn

PHYSICS 4
(Wave, Light, and Atoms)

02 credits (30 periods)

Chapter 1 Vibration and Mechanical Wave

Chapter 2 Properties of Light

Chapter 3 Introduction to Quantum Physics

Chapter 4 Atomic Physics

Chapter 5 Relativity and Nuclear Physics

References :

Halliday D., Resnick R. and Walker, J. (2005), Fundamentals of Physics, Extended seventh edition. John Willey and Sons, Inc.

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Hecht, E. (2000). Physics. Calculus, Second Edition. Brooks/Cole.

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http://www.iop.org/index.html

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PHYSICS 4

Chapter 1 Vibration and Mechanical Wave

1. Vibration   *chuyển động tuần hoàn*

a. Simple Harmonic Motion (SHM)

When the restoring force is directly proportional to the displacement from equilibrium x (Hooke's law) :

$$F = -kx$$

the oscillation is called *simple harmonic motion*, abbreviated **SHM**.

Newton's second law :

$$a = \frac{d^2 x}{dt^2} \equiv x'' = \frac{F}{m} = -\frac{k}{m}x = -\omega^2 x$$

Solution : $\boxed{x = A\cos(\omega t + \varphi)}$ *(displacement in SHM)*

Solution : $\boxed{x = A\cos(\omega t + \varphi)}$ *(displacement in SHM)*

The period : $T = \dfrac{2\pi}{\omega}$

The frequency : $f = \dfrac{1}{T} = \dfrac{\omega}{2\pi}$
(Hz-Hertz)

$\begin{cases} x : \text{displacement at the instant } t \\ A\,(>0) : \text{amplitude} \\ \omega : \text{circular frequency} \\ \omega t + \varphi : \text{phase at the instant t } (\varphi : \text{initial phase}) \end{cases}$

State
T
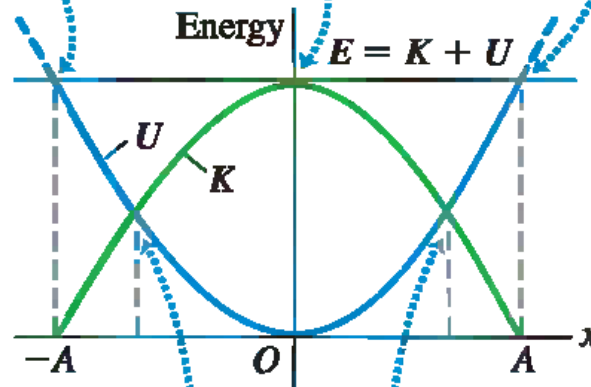
b. Energy in SHM

Total **mechanical energy** in SHM :

$$E = \frac{1}{2}mv^2 + \frac{1}{2}kx^2 = \frac{1}{2}kA^2 = const$$

Kinetic E

Energy

Potential

energy

At $x = \pm A$ the energy is all potential; the kinetic energy is zero.

At $x = 0$ the energy is all kinetic; the potential energy is zero.

Energy

$E = K + U$

U

K

−A   O   A   x

At these points the energy is half kinetic and half potential.

c. The Physical Pendulum → con lắc đơn

Consider a rigid body pivoted at a point O that is a distance d from the center of mass

The law of motion for a rigid body :

$$\sum \tau = I\alpha$$

τ : torque about an axis through O
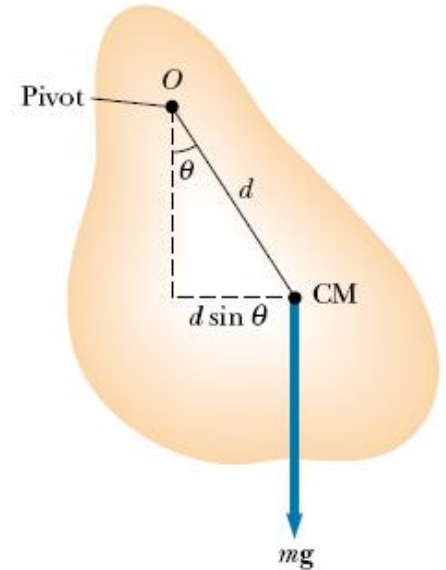
I : the moment of inertia about the axis through O

α : rotational acceleration

$$-mgd \sin\theta = I \frac{d^2\theta}{dt^2}$$

If θ small : $\sin\theta \approx \theta$ ⟶ $-mgd\theta = I \frac{d^2\theta}{dt^2}$; $\frac{d^2\theta}{dt^2} = -\left(\frac{mgd}{I}\right)\theta$

With : $\omega = \sqrt{\dfrac{mgd}{I}}$

⟶ $\dfrac{d^2\theta}{dt^2} \equiv \theta'' = -\omega^2\theta$; $\boxed{\theta = \theta_0 \cos(\omega t + \varphi)}$ **(SHM)**

The period : $T = \dfrac{2\pi}{\omega} = \dfrac{2\pi}{\sqrt{mgd/I}}$
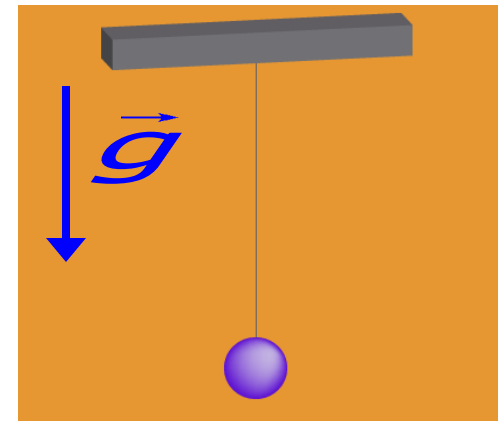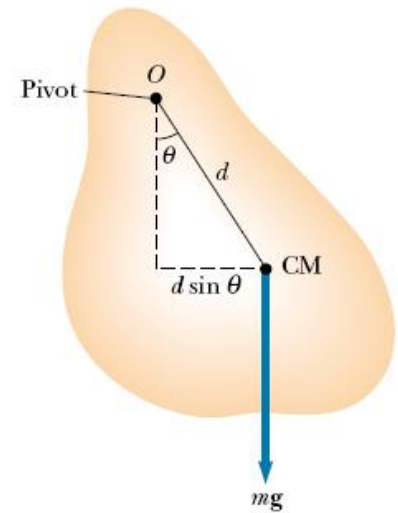
$$T = 2\pi \sqrt{\dfrac{I}{mgd}}$$

- A special case : **The simple pendulum**

(consisting of a particle-like bob of mass m suspended by a light string of length L that is fixed at the upper end)

$$I = mL^2 \; ; d = L$$

$$T = 2\pi \sqrt{\dfrac{mL^2}{mgL}} \; ; \quad T = 2\pi \sqrt{\dfrac{L}{g}}$$

PROBLEM 1 Calculate the period of a meter stick that is pivoted about one end and is oscillating in a vertical plane.
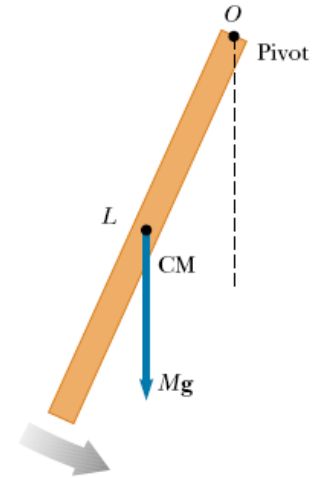
SOLUTION

$$T = 2\pi\sqrt{\frac{I}{mgd}}$$

The moment of inertia :

$$I = \frac{1}{3}mL^2 \; ; d = \frac{L}{2}$$

$$T = 2\pi\sqrt{\frac{\frac{1}{3}mL^2}{mgL/2}} \; ; \quad T = 2\pi\sqrt{\frac{2L}{3g}}$$

$$= 2\pi\sqrt{\frac{2\times 1\,m}{9.81\,m/s^2}} = 1.64\,s$$
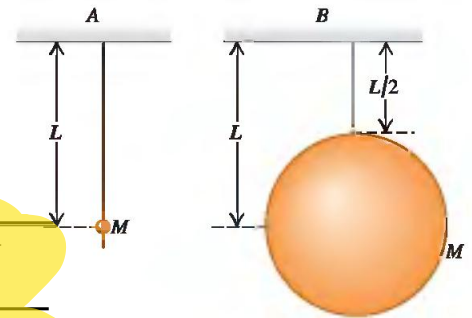
O
Pivot

L

CM

Mg

PROBLEM 2 The two pendulums shown in the figure each consist of a uniform solid ball of mass M supported by a massless string, but the ball for pendulum A is very tiny while the ball for pendulum B is much larger. Find the period of each pendulum for small displacements. Which ball takes longer to complete a swing?

SOLUTION

Pendulum A : simple pendulum: $T_A = 2\pi \sqrt{\dfrac{L}{g}}$

Pendulum B is a physical pendulum : $T_B = 2\pi \sqrt{\dfrac{I}{mgd}}$

For a solid sphere with an axis through its center :

$$I_{CM} = \frac{2}{5}mR^2 = \frac{2}{5}m(L/2)^2 = \frac{1}{10}mL^2$$

The parallel-axis theorem says : $I = I_{CM} + mL^2 = \dfrac{11}{10}mL^2$

PROBLEM 2 The two pendulums shown in the figure each consist of a uniform solid ball of mass M supported by a massless string, but the ball for pendulum A is very tiny while the ball for pendulum B is much larger. Find the period of each pendulum for small displacements. Which ball takes longer to complete a swing?

SOLUTION
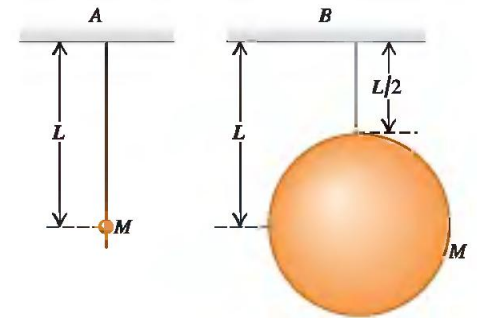
$$T_B = 2\pi\sqrt{\frac{I}{mgd}}$$

$$= 2\pi\sqrt{\frac{(11/10)mL^2}{mgL}} = \sqrt{\frac{11}{10}}\, 2\pi\sqrt{\frac{L}{g}} = 1.05 T_A$$

It takes pendulum *B* longer to complete a swing

d. Damped Oscillations.
Forced Oscillations and Resonance

Đao động

Tắt dần

● Damped Oscillations

In many *real systems*, dissipative forces, such as friction, retard the motion → the mechanical energy of the system *diminishes* in time, and the motion is said to be *damped*.

In viscous fluid flow, additional force on the body due to friction :

$$-bv$$

v : velocity ; b :constant called the *damping coefficient*

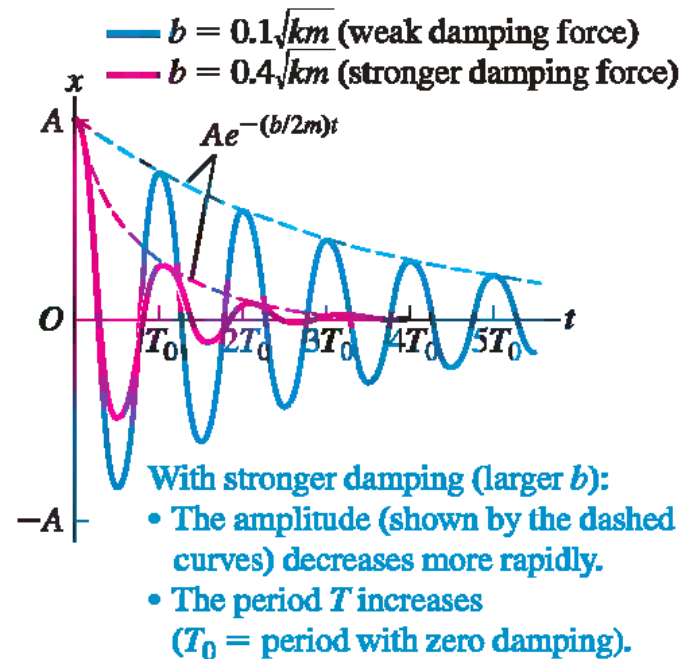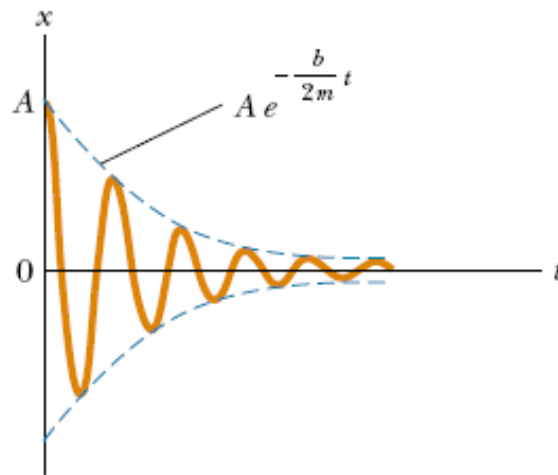The restoring force of the system : $-kx$

Newton's second law :

$$-kx - bv = ma \;; \; -kx - b\frac{dx}{dt} = m\frac{d^2x}{dt^2}$$

$$-kx - b\frac{dx}{dt} = m\frac{d^2x}{dt^2}$$

$\rightarrow$ a differential equation for x

Solution (b small) : $\boxed{x = Ae^{-(b/2m)t}\cos(\omega't + \varphi)\ ;}$ $\omega' = \sqrt{\dfrac{k}{m} - \dfrac{b^2}{4m^2}}$

$b = 0.1\sqrt{km}$ (weak damping force)
$b = 0.4\sqrt{km}$ (stronger damping force)

$Ae^{-(b/2m)t}$

With stronger damping (larger b):
• The amplitude (shown by the dashed curves) decreases more rapidly.
• The period T increases (T_0 = period with zero damping).

• **Forced Oscillations and Resonance** *Giao động cưỡng bức*

How to compensate for energy loss in a damped system ?

$\rightarrow$ Apply an external force that does positive work on the system (*driving force*)

Example : $F_{ext} = F_0 \cos \omega t$

Newton's second law : $F_0 \cos \omega t - kx - b\dfrac{dx}{dt} = m\dfrac{d^2 x}{dt^2}$

With t large enough : a *steady-state* condition is reached in which the oscillations proceed with *constant amplitude* (*forced oscillation*)

$$x = A\cos(\omega t + \varphi)$$

$$A = \frac{F_0 / m}{\sqrt{(\omega^2 - \omega_0^2)^2 + (b\omega / m)^2}} \; ; \; \omega_0 = \sqrt{\frac{k}{m}}$$
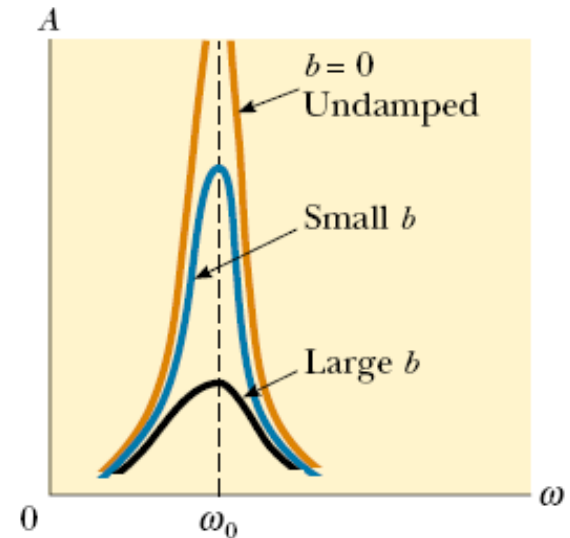
$$x = A\cos(\omega t + \varphi)$$

$$A = \frac{F_0 / m}{\sqrt{(\omega^2 - \omega_0^2)^2 + (b\omega / m)^2}} \; ; \; \omega_0 = \sqrt{\frac{k}{m}}$$

If $\omega \to \omega_0 : A$ becomes very large

$\to$ **resonance**

(the applied force is *in phase* with the velocity and that the power transferred to the oscillator is a maximum)

PROBLEM 3

A 2.20-kg mass oscillates on a spring of force constant 250.0 N/m with a period of 0.615 s.
Is this system damped or not? If it is damped, find the damping constant.

SOLUTION

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{250.0 \text{ N/m}}{2.20 \text{ kg}}} = 10.66 \text{ rad/s}$$

$\omega' < \omega$ so the system is damped.

$$\omega' = \sqrt{\frac{k}{m} - \frac{b^2}{4m^2}} \quad \rightarrow \quad b = 2m\sqrt{\frac{k}{m} - \omega'^2}$$

$$= 2(2.20 \text{ kg})\sqrt{\frac{250.0 \text{ N/m}}{2.20 \text{ kg}} - (10.22 \text{ rad/s})^2} = 13.3 \text{ kg/s}$$

PROBLEM 4 A 50.0-g object moves on the end of a spring with force constant $k = 25.0$ N/m. Its initial displacement is 0.300 m. A damping force $F = - bv$ acts on the object, and the amplitude of the motion decreases to 0.100 m in 5.00 s. Calculate the magnitude of the damping constant b.

SOLUTION

$$x = Ae^{-(b/2m)t} \cos(\omega't + \varphi); \quad \rightarrow \quad A_2 = A_1 \exp\left(-\frac{b}{2m}t\right)$$

$$b = \frac{2m}{t} \ln\left(\frac{A_1}{A_2}\right)$$

$$= \frac{2(0.050 \text{ kg})}{(5.00 \text{ s})} \ln\left(\frac{0.300 \text{ m}}{0.100 \text{ m}}\right) = 0.0220 \text{ kg/s.}$$

2. Mechanical Wave ▶

know everything

a. THE WAVE EQUATION

Wave: propagation of oscillation in space

Harmonic oscillation of A:

$$y_A = A \sin \omega t$$

$\neq \dfrac{\pi}{2}$

M begins to move after the time:
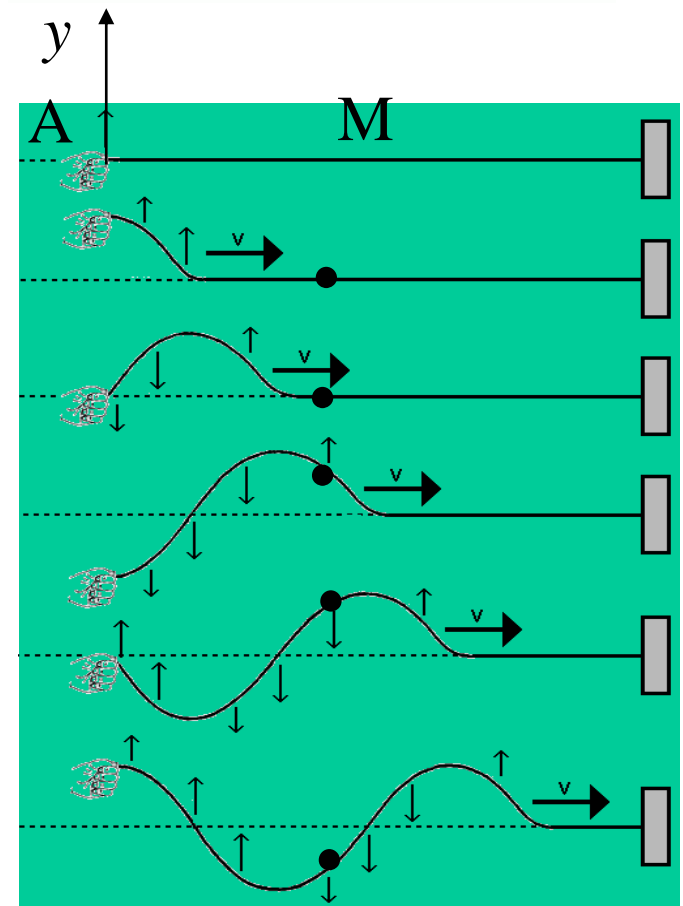
$$t = \frac{AM}{v} \equiv \frac{x}{v}$$

depend A, ω

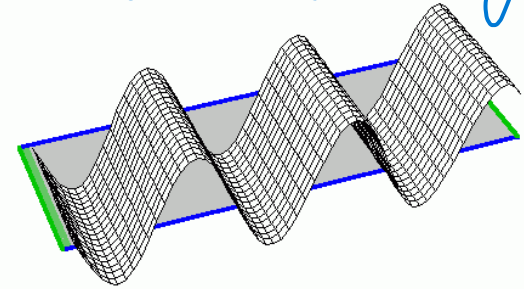*(v is the speed of the wave)*

Motion of M :

$$y = A \sin\left[\omega\left(t - \frac{x}{v}\right)\right]$$
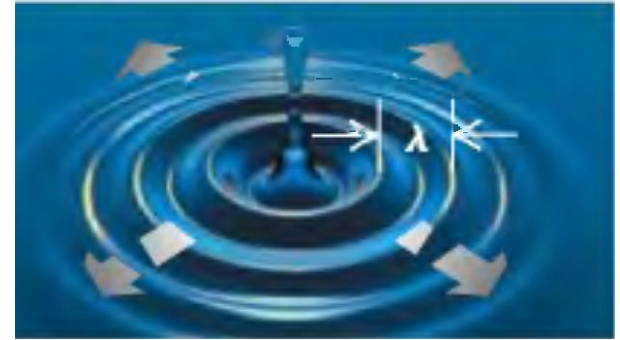
$$= A \sin\left(\omega t - \omega\frac{x}{v}\right)$$

Motion of M : $y = A\sin\left(\omega t - \omega\dfrac{x}{v}\right)$

$$\omega\frac{x}{v} = \frac{2\pi}{T}\frac{x}{v} = \frac{2\pi}{vT}x$$

The **wavelength :** $\lambda = vT$

The **wave number** : $K = \dfrac{2\pi}{\lambda}$

Wave Equation :

$$y = A\sin\left(\frac{2\pi}{T}t - \frac{2\pi}{\lambda}x\right) = A\sin\left(\omega t - Kx\right)$$

http://www.phy.hk/wiki/englishhtm/TwaveA.htm

b. TYPES OF WAVES

● A traveling wave that causes the particles of the disturbed medium to move perpendicular to the wave motion is called a **transverse wave**. Sóng ngang

● A traveling wave that causes the particles of the medium to move parallel to the direction of wave motion is called a **longitudinal wave**. ⟹ Sóng dọc
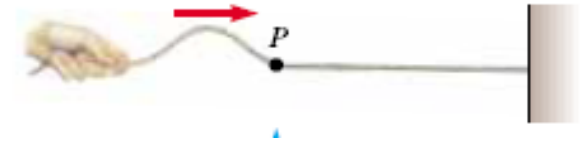

Compressed λ Compressed

Stretched λ Stretched
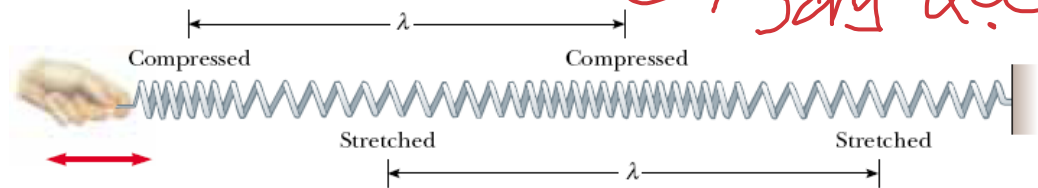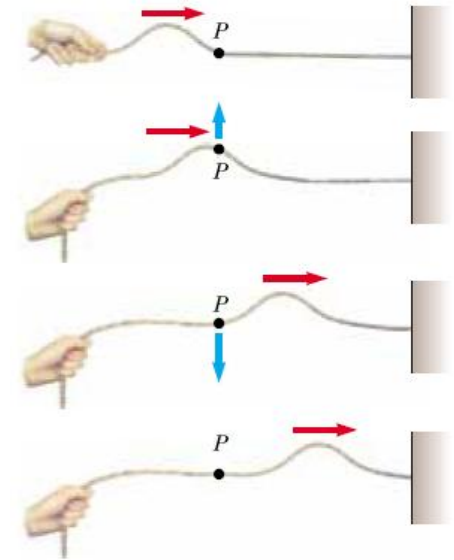
● Some waves in nature exhibit a combination of transverse and longitudinal displacements :
Surface water waves

Wave motion

Crest

Trough
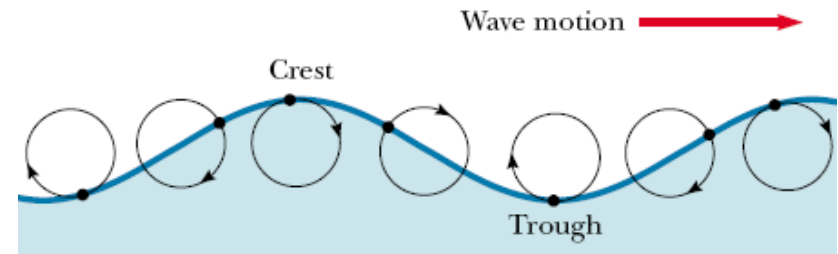
c. THE SPEED OF WAVES ON STRINGS

A transverse pulse traveling on a string
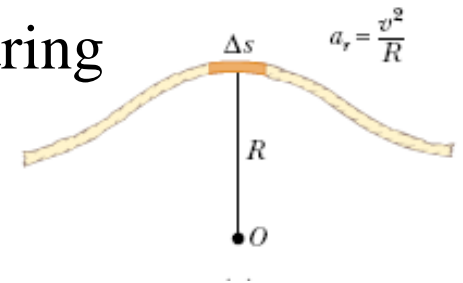with mass per unit length μ *(kg/m)*

A small segment of the string of length *s*
forms an approximate arc of a circle of radius *R*

Speed of the segment : v , T : the tension in the string

Total force on direction of R: $2T \sin\theta \approx 2T\theta$

Mass of the segment :

$$m = \mu\Delta s = \mu(R.2\theta)$$

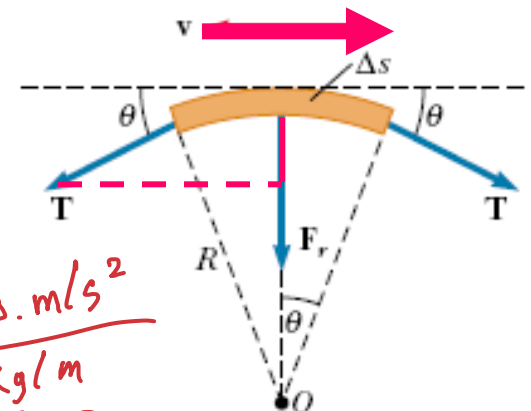Newton's second law (radial component):

$$2T\theta = ma = m\frac{v^2}{R}$$

→ circular motion

$$2T\theta = \frac{2\mu R\theta v^2}{R} \; ; \quad \boxed{V = \sqrt{\frac{T}{\mu}}}$$

→ $\dfrac{N}{Kg/m} = \dfrac{Kg.m/s^2}{Kg/m}$

$= m^2/s^2$

PROBLEM 5

A uniform cord has a mass of 0.300 kg and a length of 6.00 m. The cord passes over a pulley and supports a 2.00-kg object.
1. Find the speed of a pulse traveling along this cord
2. Find the time it takes the pulse to travel from the wall to the pulley.

SOLUTION

1. $T = mg = 0.300\ kg \times 9.80\ m/s^2 = 19.6\ N$
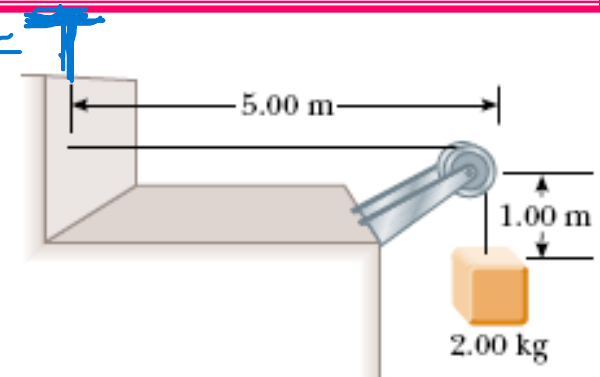
$\mu = \dfrac{m}{L} = \dfrac{0.300\ kg}{6.00\ m} = 0.00500\ kg/m$

$v = \sqrt{\dfrac{T}{\mu}} = \sqrt{\dfrac{19.6\ N}{0.00500\ kg/m}} = 19.8\ m/s$

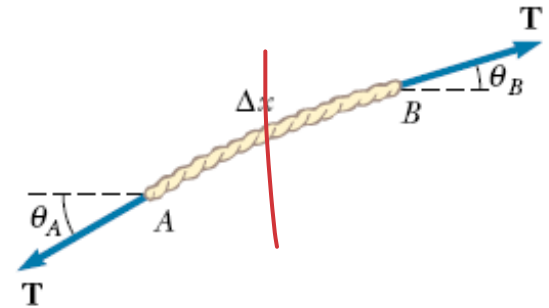2. $t = \dfrac{d}{v} = \dfrac{5.00\ m}{19.8\ m/s} = 0.253\ s$

d. CONSTRUCTION OF THE WAVE EQUATION

- A traveling wave is propagating along a string that is under a tension T.

- Δx : length of a small string segment

- Net force in the vertical direction :

$$T\sin\theta_B - T\sin\theta_A = T(\sin\theta_B - \sin\theta_A) \approx T(\tan\theta_B - \tan\theta_A)$$

$$\approx T\left[\left(\frac{\partial y}{\partial x}\right)_B - \left(\frac{\partial y}{\partial x}\right)_A\right]$$

- Newton's second law : $T\left[\left(\frac{\partial y}{\partial x}\right)_B - \left(\frac{\partial y}{\partial x}\right)_A\right] = ma_y = \mu\Delta x\left(\frac{\partial^2 y}{\partial t^2}\right)$

$$\frac{\mu}{T}\left(\frac{\partial^2 y}{\partial t^2}\right) = \frac{(\partial y/\partial x)_B - (\partial y/\partial x)_A}{\Delta x} \equiv \frac{\Delta(\partial y/\partial x)}{\Delta x} \longrightarrow = y''_x$$

$$\frac{\mu}{T}\left(\frac{\partial^2 y}{\partial t^2}\right) = \frac{\partial^2 y}{\partial x^2} \quad \longleftrightarrow \quad \boxed{\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2}\left(\frac{\partial^2 y}{\partial t^2}\right)}$$

The Partial Differential Equation of Wave

e. **General form of a traveling wave**

• A wave pulse traveling to the right with constant speed *v* on a long, taut string
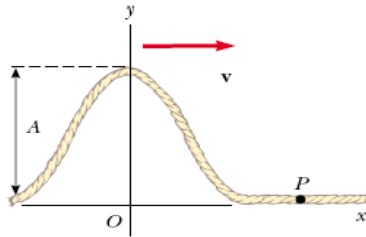
(a) Pulse at $t = 0$

• At $t = 0$ the shape of the pulse is given by $y = f(x)$
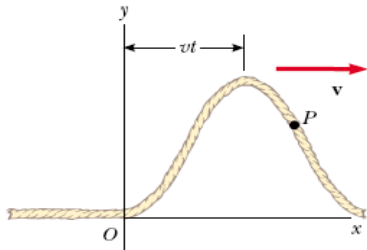
(b) Pulse at time *t*

• At some later time *t*, the shape remains unchanged and the vertical displacement of any point *P* of the medium is given by $y = f(x - vt)$.

The wave function measured in a stationary reference frame having its origin at O : $\boxed{y = f(x - vt)}$

For the wave traveling to the left : $\boxed{y = f(x + vt)}$

General form of a traveling wave : $\boxed{y = f(x \pm vt)}$

PROBLEM 6

A wave pulse moving to the right along the x axis is represented by the wave function $y(x,t) = \dfrac{2}{(x-3.0t)^2 + 1}$

where x and y are measured in centimeters and t is measured in seconds. Plot the wave function at $t = 0$ and $t = 1.0$ s

SOLUTION

- At $t = 0$: $y(x,t) = \dfrac{2}{x^2 + 1}$

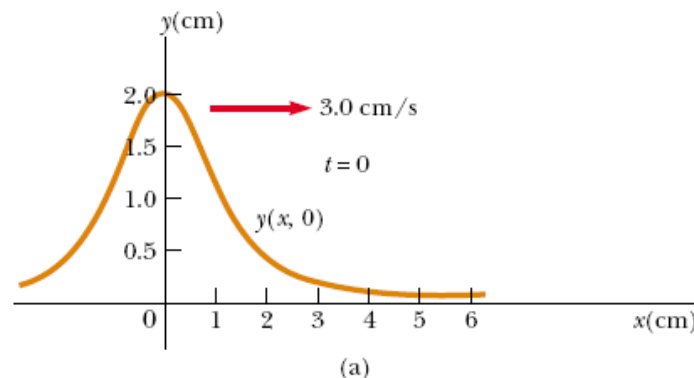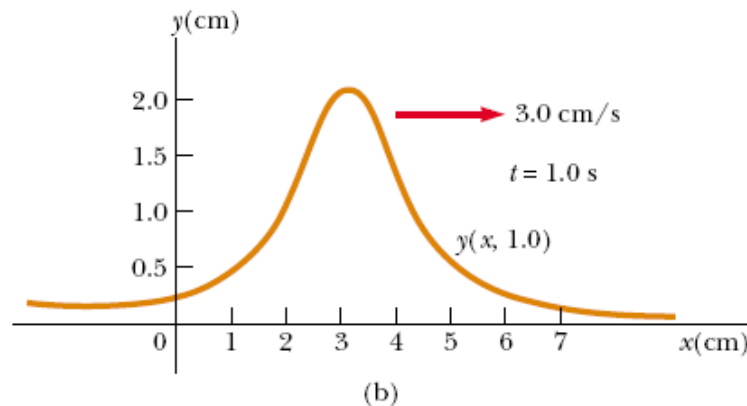$$x = x - 3\pi$$

(a)

PROBLEM 6

A wave pulse moving to the right along the x axis is represented by the wave function $y(x,t) = \dfrac{2}{(x-3.0t)^2 + 1}$

where x and y are measured in centimeters and t is measured in seconds. Plot the wave function at $t = 0$ and $t = 1.0$ s

SOLUTION

• At $t = 1.0$ s : $y(x,t) = \dfrac{2}{(x-3.0)^2 + 1}$

(b)

f. Representations of Waves

- **Wave fronts** are the concentric arcs
  - The distance between successive wave fronts is the wavelength
- **Rays** are the radial lines pointing out from the source and perpendicular to the wave fronts
- A **spherical wave** propagates radially outward from the oscillating sphere

  The energy propagates equally in all directions

- Far away from the source, the wave fronts are nearly **parallel planes**

  The rays are nearly parallel lines

  A small segment of the wave front is approximately a **plane wave**

Wave front →

Source

λ

Ray

Spherical wave front

r_1

r_2

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Rays

Wave fronts
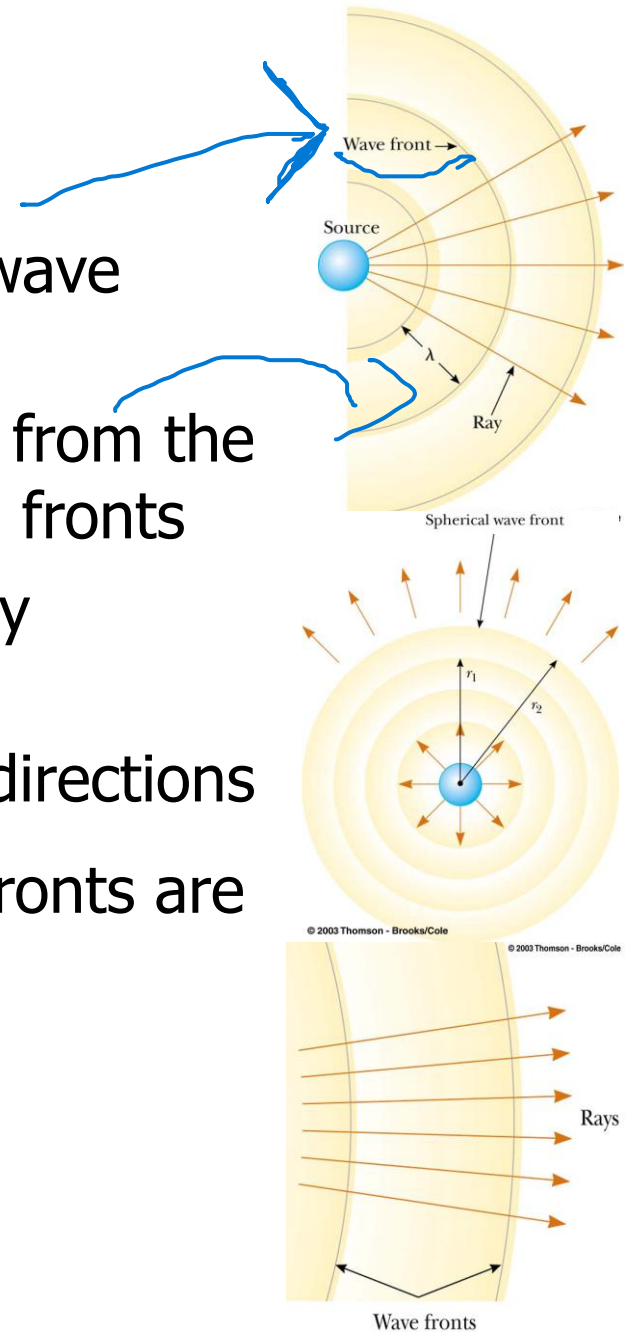
3 Superposition and Interference

a. Superposition principle for linear waves

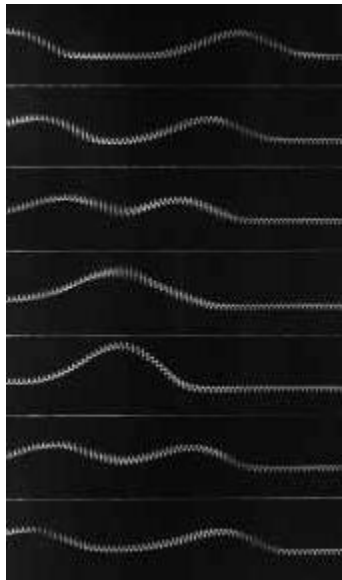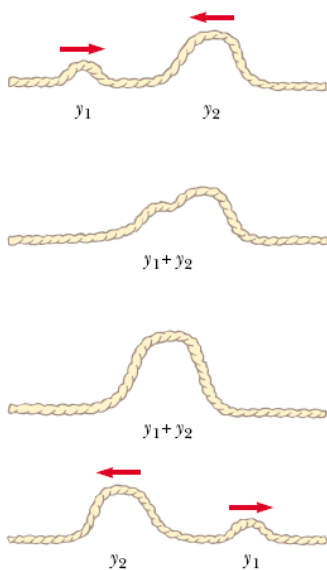"If two or more traveling waves are moving through a medium, the **resultant wave function** at any point is the **algebraic sum** of the wave functions of the individual waves."

$$y = y_1 + y_2$$

● *Constructive interference*   ● *Destructive interference*
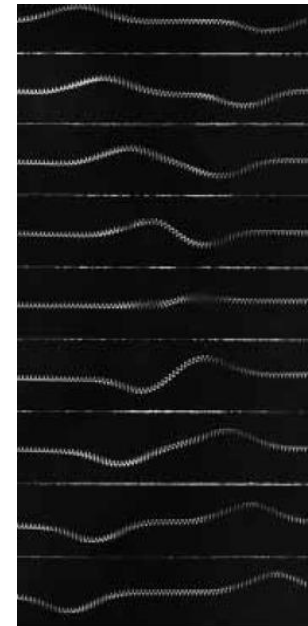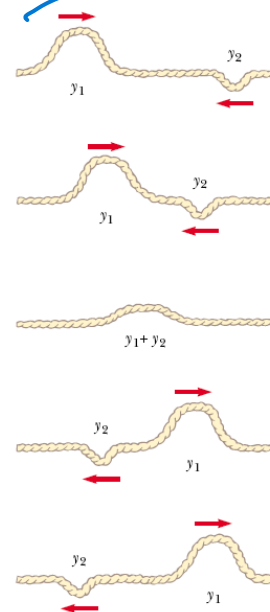
b. Superposition and Interference of Sinusoidal Waves

$$y_1 = A\sin(\omega t - Kx_1) \,;$$

$$y_2 = A\sin(\omega t - Kx_2)$$

$$y_2 = A\sin\left[\omega t - Kx_1 + (Kx_1 - Kx_2)\right]$$

$$y_2 = A\sin\left[\omega t - Kx_1 + \phi\right]$$

M

x_1

x_2

S_1 S_2
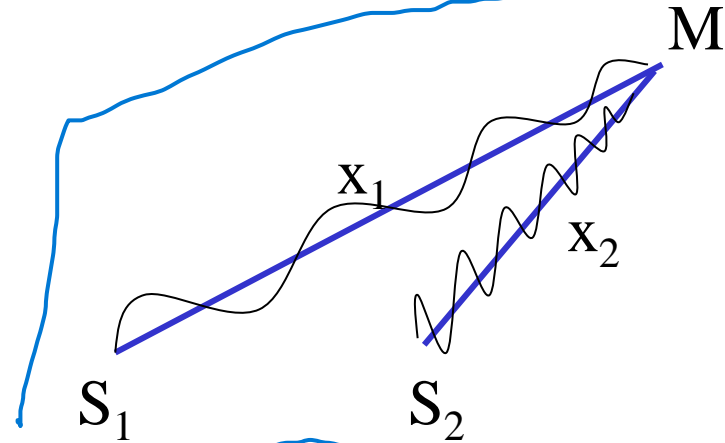
$$\phi = K(x_1 - x_2) = K\delta : \textbf{ Phase difference}$$

$$\delta = x_1 - x_2 : \quad \textbf{Path difference} \quad \Delta x$$

$$y = y_1 + y_2 \;\; = A\sin(\omega t - Kx_1) + A\sin(\omega t - Kx_1 + \phi)$$

$$= A\left[\sin(\omega t - Kx_1) + \sin(\omega t - Kx_1 + \phi)\right]$$

$$= 2A\cos\left(\frac{\phi}{2}\right)\sin(\omega t - Kx_1 + \frac{\phi}{2})$$

Conclusion : **The amplitude of the resultant wave is**
$2A\left|\cos(\phi/2)\right|$, and its phase is $\phi/2$ $(+\pi)$

$$y = y_1 + y_2 = 2A\cos\left(\frac{\phi}{2}\right)\sin\left(\omega t - Kx_1 + \frac{\phi}{2}\right)$$

<u>Conclusion :</u> The amplitude of the resultant wave is 2A cos(φ/2), and its phase is φ/2

$\phi = k2\pi$ $\longrightarrow$ **The amplitude of the resultant wave is 2A**
(constructive interference)

$\phi = (2k+1)\pi$ $\longrightarrow$ **The amplitude of the resultant wave is 0**
(destructive interference)

In terms of path difference : $\delta = x_1 - x_2$

Constructive interference : $\phi = K\delta = k2\pi$; $\dfrac{2\pi}{\lambda}\delta = k2\pi$; $\boxed{\delta = k\lambda}$

Destructive interference : $\phi = K\delta = (2k+1)\pi$; $\dfrac{2\pi}{\lambda}\delta = (2k+1)\pi$;

$$\boxed{\delta = \left(k + \frac{1}{2}\right)\lambda}$$

PROBLEM 7 A pair of speakers placed 3.00 m apart are driven by the same oscillator. A listener is originally at point O, which is located 8.00 m from the center of the line connecting the two speakers.

1. The listener then walks to point P, which is a perpendicular distance 0.350 m from O, before reaching the *first minimum* in sound intensity. What is the frequency of the oscillator?

SOLUTION

1.

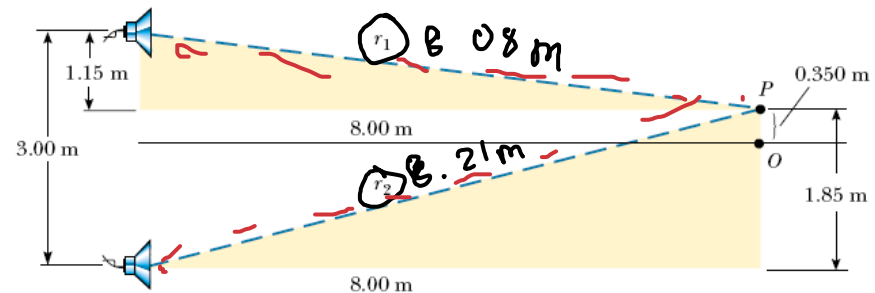$$r_1 = \sqrt{(8.00 \text{ m})^2 + (1.15 \text{ m})^2} = 8.08 \text{ m}$$

$$r_2 = \sqrt{(8.00 \text{ m})^2 + (1.85 \text{ m})^2} = 8.21 \text{ m}$$

the path difference is $r_2 - r_1 = 0.13$ m.

For the first minimum : $r_2 - r_1 = \lambda/2 \; ; \; \lambda = 0.26 \, m$

$$0.13 = \frac{\lambda}{2}$$

$$f = \frac{v}{\lambda} = \frac{343 \text{ m/s}}{0.26 \text{ m}} = \boxed{1.3 \text{ kHz}}$$

$$\lambda = \frac{v}{f}$$

PROBLEM 7
A pair of speakers placed 3.00 m apart are driven by the same oscillator. A listener is originally at point O, which is located 8.00 m from the center of the line connecting the two speakers.

2. If the oscillator frequency is adjusted such that the first location at which a listener hears no sound is at a distance of 0.75 m from O, what is the new frequency?

SOLUTION

2.

$$f = \frac{v}{\lambda} = 0.63 \, kHz$$
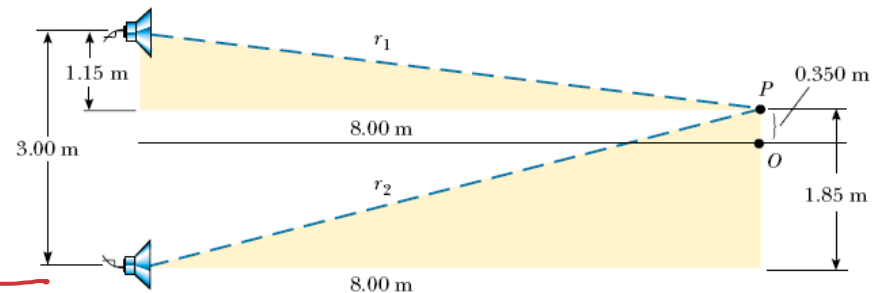
$$\approx \frac{343}{544.44}$$

$$\lambda = \frac{v}{8}$$

Figure labels: 1.15 m, 3.00 m, 8.00 m, r_1, r_2, 0.350 m, P, O, 1.85 m, 8.00 m

PROBLEM 8 Two small loudspeakers, A and B, are driven by the same amplifier and emit pure sinusoidal waves in phase. If the speed of sound is 350 m/s,

(a) for what frequencies does constructive interference occur at point P?

(b) For what frequencies does destructive interference occur at point P?

SOLUTION

The distance from speaker A to point P is $[(2.00\,\text{m})^2 + (4.00\,\text{m})^2]^{1/2} = 4.47\,\text{m}$, and the distance from speaker B to point P is $[(1.00\,\text{m})^2 + (4.00\,\text{m})^2]^{1/2} = 4.12\,\text{m}$. The path difference is $d = 4.47\,\text{m} - 4.12\,\text{m} = 0.35\,\text{m}$.

PROBLEM 8 Two small loudspeakers, A and B, are driven by the same amplifier and emit pure sinusoidal waves in phase. If the speed of sound is 350 m/s,
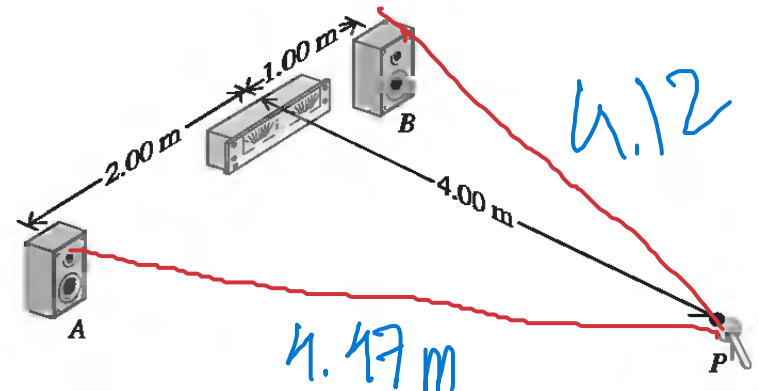
(a) for what frequencies does constructive interference occur at point P?

(b) For what frequencies does destructive interference occur at point P?

SOLUTION

(a) Constructive interference occurs when the path difference is $d = 0, \lambda, 2\lambda, \ldots$ or $d = 0, v/f, 2v/f, \ldots = nv/f$. So the possible frequencies are

$$f_n = \frac{nv}{d} = n\frac{350 \text{ m/s}}{0.35 \text{ m}} \qquad (n = 1, 2, 3, \ldots)$$

$$= 1000 \text{ Hz}, 2000 \text{ Hz}, 3000 \text{ Hz}, \ldots$$

PROBLEM 8 Two small loudspeakers, A and B, are driven by the same amplifier and emit pure sinusoidal waves in phase. If the speed of sound is 350 m/s,
(a) for what frequencies does constructive interference occur at point P?
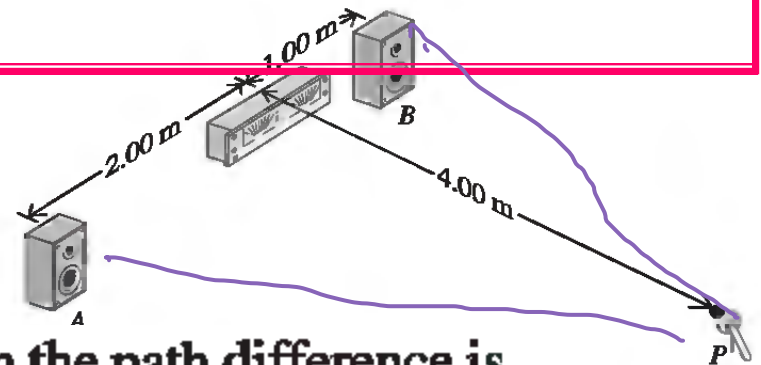**(b)** For what frequencies does destructive interference occur at point P?

SOLUTION
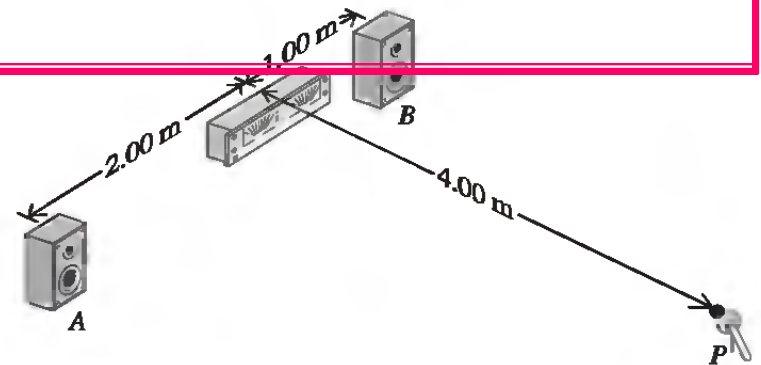
(b) Destructive interference occurs when the path difference is $d = \lambda/2,\ 3\lambda/2,\ 5\lambda/2,\ \ldots$ or $d = v/2f,\ 3v/2f,\ 5v/2f,\ \ldots$. The possible frequencies are

$$f_n = \frac{nv}{2d} = n\frac{350\ \text{m/s}}{2(0.35\ \text{m})} \qquad (n = 1, 2, 3, \ldots)$$

$$= 500\ \text{Hz},\ 1500\ \text{Hz},\ 2500\ \text{Hz},\ \ldots$$

PROBLEM 9 Two identical loudspeakers are located at points A and B, 2.00 m apart. The loudspeakers are driven by the same amplifier and produce sound waves with a frequency of 784 Hz. Take the speed of sound in air to be 344 m/s. A small microphone is moved out from point B along a line perpendicular to the line connecting A and B (line BC).

(a) At what distances from B will there be destructive interference?

SOLUTION

$$\sqrt{x^2 + h^2} - x = \beta\lambda.$$

$$x = \frac{h^2}{2\beta\lambda} - \frac{\beta}{2}\lambda.$$

Using $\lambda = 0.439$ m and $h = 2.00$ m yields 9.01 m

for $\beta = \frac{1}{2}$, 2.71 m for $\beta = \frac{3}{2}$, 1.27 m for $\beta = \frac{5}{2}$,

and 0.026 m for $\beta = \frac{9}{2}$. (These are the only allowable values of β that give positive solutions for x)

PROBLEM 9 Two identical loudspeakers are located at points A and B, 2.00 m apart. The loudspeakers are driven by the same amplifier and produce sound waves with a frequency of 784 Hz. Take the speed of sound in air to be 344 m/s. A small microphone is moved out from point B along a line perpendicular to the line connecting A and B (line BC).

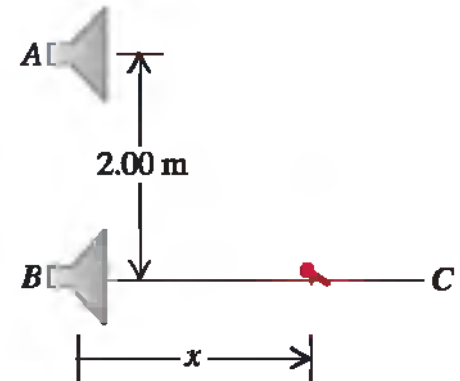**(b)** At what distances from B will there be constructive interference?
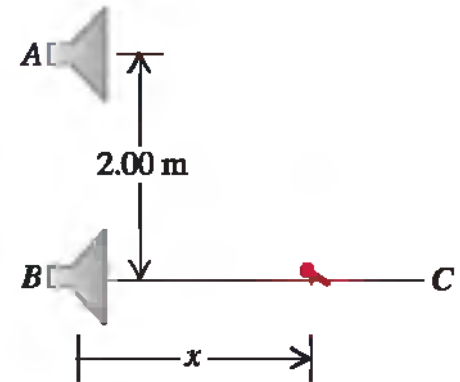
(b)

constructive interference occurs at 4.34 m, 1.84 m, 0.86 m, 0.26 m.

PROBLEM 9 Two identical loudspeakers are located at points A and B, 2.00 m apart. The loudspeakers are driven by the same amplifier and produce sound waves with a frequency of 784 Hz. Take the speed of sound in air to be 344 m/s. A small microphone is moved out from point B along a line perpendicular to the line connecting A and B (line BC).

(c) If the frequency is made low enough, there will be no positions along the line BC at which destructive interference occurs. How low must the frequency be for this to be the case?

(c) If $h = \lambda/2$, there will be destructive interference at speaker B. If $\lambda/2 > h$, the path difference can never be as large as $\lambda/2$. (This is also obtained from the above expression for x, with $x = 0$ and $\beta = 1/2$). The minimum frequency is then $v/2h = (344 \text{ m s})/(4.0 \text{ m}) = 86$ Hz.
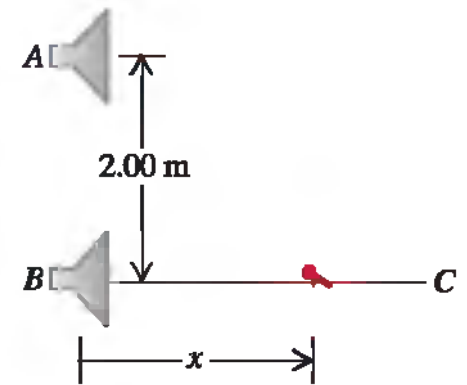
4 STANDING WAVES

4.1 General consideration

Consider these two waves traveling in two opposite directions:

$$y_1 = A\sin(Kx - \omega t) \; ; \; y_2 = A\sin(Kx + \omega t)$$

The combined wave is

$$y = y_1 + y_2 = A\sin(Kx - \omega t) + A\sin(Kx + \omega t)$$
$$= 2A\sin(Kx)\cos(\omega t)$$

• **The amplitude is zero** $\sin(Kx) = 0 \rightarrow Kx = n\pi$

With the wave number : $K = 2\pi / \lambda$ we get the position of the **nodes**:

$$\boxed{x = n\lambda / 2} \; (n = 1, 2, 3, \ldots)$$

• **The amplitude is maximum** :

$$\sin(Kx) = \pm 1 \rightarrow Kx = \left(n + \frac{1}{2}\right)\pi$$

We get the position of the **antinodes** : $\boxed{x = \left(n + \frac{1}{2}\right)\lambda / 2}$
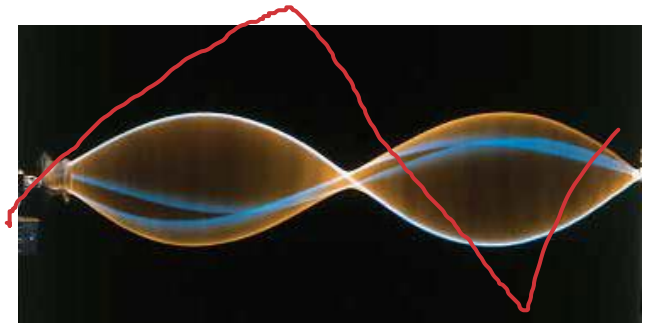
- A *node* occurs where the two traveling waves have the same magnitude of displacement, but the displacements are in opposite directions
  - Net displacement is zero at that point
  - The distance between two nodes is ½λ
- An *antinode* occurs where the standing wave vibrates at maximum amplitude
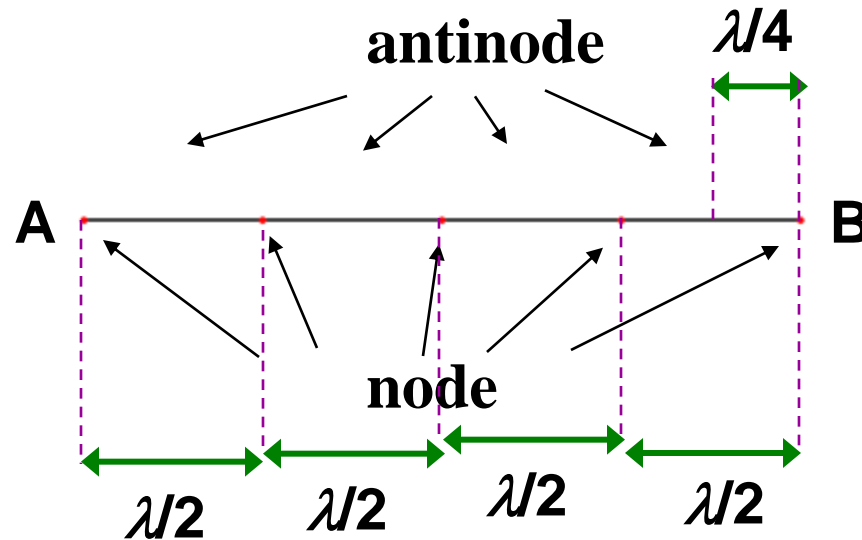
4.2 Standing waves in a string fixed at both ends

- The string has a number of natural patterns of oscillation, called normal modes with wavelengths :

$$\lambda_n = \frac{2L}{n} \quad n = 1, 2, 3, \ldots$$

- Frequencies of normal modes as functions of wave speed and length of string:

$$f_n = \frac{v}{\lambda_n} = n\frac{v}{2L} = \frac{n}{2L}\sqrt{\frac{F}{\mu}}$$

- The lowest frequency f_1, which corresponds to $n = 1$ is the fundamental :

$$f_1 = \frac{1}{2L}\sqrt{\frac{F}{\mu}} \; ; \quad f_n = nf_1 = \frac{n}{2L}\sqrt{\frac{F}{\mu}}$$

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L

f_2

$n = 2$ (a) (c) $L = \lambda_2$

N A N

f_1 f_3

$n = 1$ (b) $L = \frac{1}{2}\lambda_1$ $n = 3$ (d) $L = \frac{3}{2}\lambda_3$
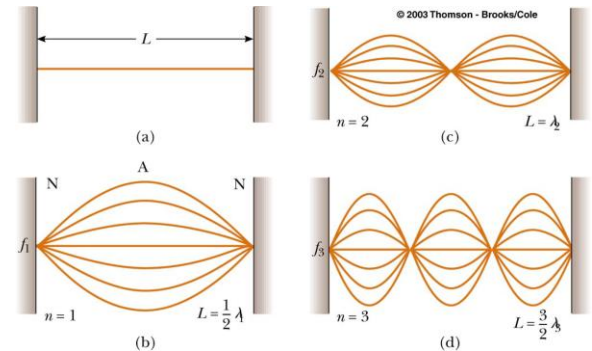
Fig 14.18, p. 443
Slide 25

- The normal modes are called harmonics. The fundamental frequency f_1 is the frequency of the first harmonic; the frequency $f_2 = 2f_1$ is the frequency of the second harmonic; and the frequency f_n is the frequency of the nth harmonic

PROBLEM 10

Middle C on a piano has a fundamental frequency of 262 Hz, and the first A above middle C has a fundamental frequency of 440 Hz.

(a) Calculate the frequencies of the next two harmonics of the C string.

Solution Knowing that the frequencies of higher harmonics are integer multiples of the fundamental frequency $f_1 = 262$ Hz, we find that

$$f_2 = 2f_1 = \boxed{524 \text{ Hz}}$$

$$f_3 = 3f_1 = \boxed{786 \text{ Hz}}$$

PROBLEM 10

Middle C on a piano has a fundamental frequency of 262 Hz, and the first A above middle C has a fundamental frequency of 440 Hz.
(b) If the A and C strings have the same linear mass density and length L, determine the ratio of tensions in the two strings.

SOLUTION

$$f_{1A} = \frac{1}{2L}\sqrt{\frac{T_A}{\mu}} \quad \text{and} \quad f_{1C} = \frac{1}{2L}\sqrt{\frac{T_C}{\mu}}$$

$$\frac{f_{1A}}{f_{1C}} = \sqrt{\frac{T_A}{T_C}}$$

$$\frac{T_A}{T_C} = \left(\frac{f_{1A}}{f_{1C}}\right)^2 = \left(\frac{440}{262}\right)^2 = \boxed{2.82}$$

PROBLEM 11

The high E string on a guitar measures 64.0 cm in length and has a fundamental frequency of 330 Hz. By pressing down on it at the first fret, the string is shortened so that it plays an F note that has a frequency of 350 Hz. How far is the fret from the neck end of the string?

SOLUTION

The speed of the wave on the string :

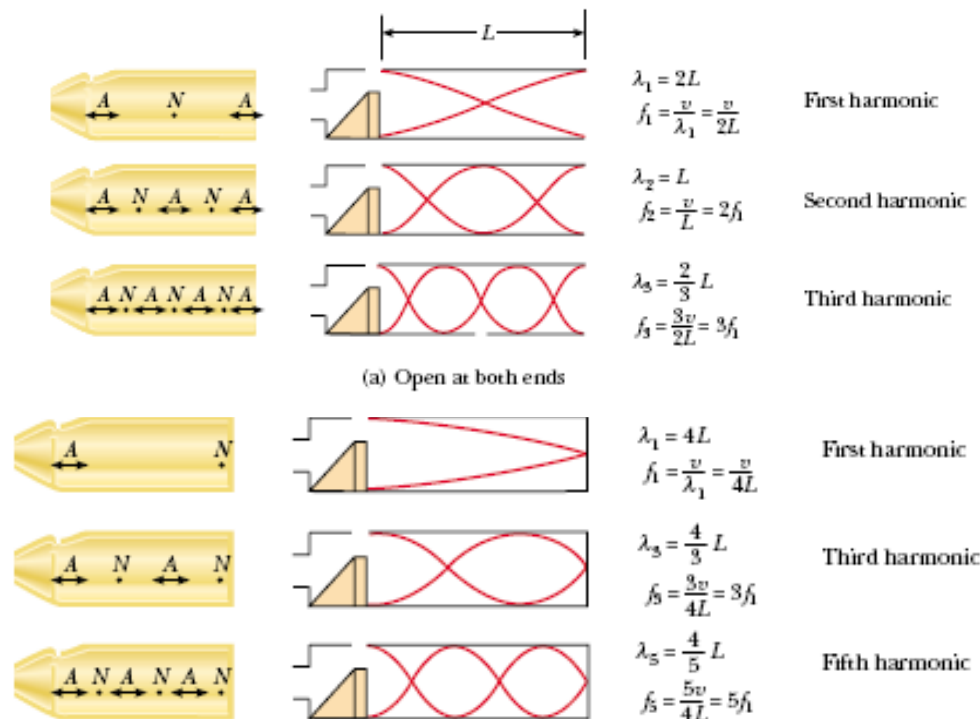$$v = \frac{2L}{n} f_n = \frac{2(0.640 \text{ m})}{1} (330 \text{ Hz}) = 422 \text{ m/s}$$

The shortened string length : $L = n\dfrac{v}{2f_n} = (1)\dfrac{422 \text{ m/s}}{2(350 \text{ Hz})} = 0.603 \text{ m}$

The difference between this length and the measured length of 64.0 cm is the distance from the fret to the neck end of the string, or   3.70 cm.

4.3 Standing waves in air columns

- In a pipe closed at one end, the **closed end** is a displacement **node** because the wall at this end does not allow longitudinal motion of the air molecules

- The **open end** of an air column is approximately a displacement **antinode** (only odd harmonics are present)

$\lambda_1 = 2L$

$f_1 = \dfrac{v}{\lambda_1} = \dfrac{v}{2L}$ First harmonic

$\lambda_2 = L$

$f_2 = \dfrac{v}{L} = 2f_1$ Second harmonic

$\lambda_3 = \dfrac{2}{3}L$

$f_3 = \dfrac{3v}{2L} = 3f_1$ Third harmonic

(a) Open at both ends

$\lambda_1 = 4L$

$f_1 = \dfrac{v}{\lambda_1} = \dfrac{v}{4L}$ First harmonic

$\lambda_3 = \dfrac{4}{3}L$

$f_3 = \dfrac{3v}{4L} = 3f_1$ Third harmonic

$\lambda_5 = \dfrac{4}{5}L$

$f_5 = \dfrac{5v}{4L} = 5f_1$ Fifth harmonic

(b) Closed at one end, open at the other

PROBLEM 12

A section of drainage culvert 1.23 m in length makes a howling noise when the wind blows.
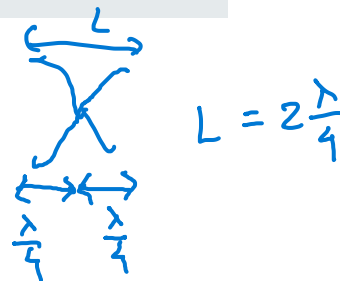**(a)** Determine the frequencies of the first three harmonics of the culvert if it is open at both ends. Take 343 m/s as the speed of sound in air.

SOLUTION

(a) The frequency of the first harmonic of a pipe open at both ends :

$$f_1 = \frac{v}{2L} = \frac{343 \text{ m/s}}{2(1.23 \text{ m})} = 139 \text{ Hz}$$

$$f_2 = 2f_1 = 278 \text{ Hz} \quad \text{and} \quad f_3 = 3f_1 = 417 \text{ Hz}.$$

$$L = 2\frac{\lambda}{4}$$

PROBLEM 12

A section of drainage culvert 1.23 m in length makes a howling noise when the wind blows.
(b) What are the three lowest natural frequencies of the culvert if it is blocked at one end?

SOLUTION

(b) The fundamental frequency of a pipe closed at one end :

$$f_1 = \frac{v}{4L} = \frac{343 \text{ m/s}}{4(1.23 \text{ m})} = \boxed{69.7 \text{ Hz}}$$

Only odd harmonics are present :

$$\longrightarrow \quad f_3 = 3f_1 = \boxed{209 \text{ Hz}} \quad f_5 = 5f_1 = \boxed{349 \text{ Hz.}}$$

PROBLEM 12

A section of drainage culvert 1.23 m in length makes a howling noise when the wind blows.
(c) For the culvert open at both ends, how many of the harmonics present fall within the normal human hearing range (20 to 17 000 Hz)?

SOLUTION

(c) All harmonics are present
$\rightarrow$ the frequency of the highest harmonic heard :

$$f_n = nf_1 ; \quad n = 1,2,3,...$$

$$f_1 = 139 \ Hz \ ; f_n = 17000 \ Hz$$
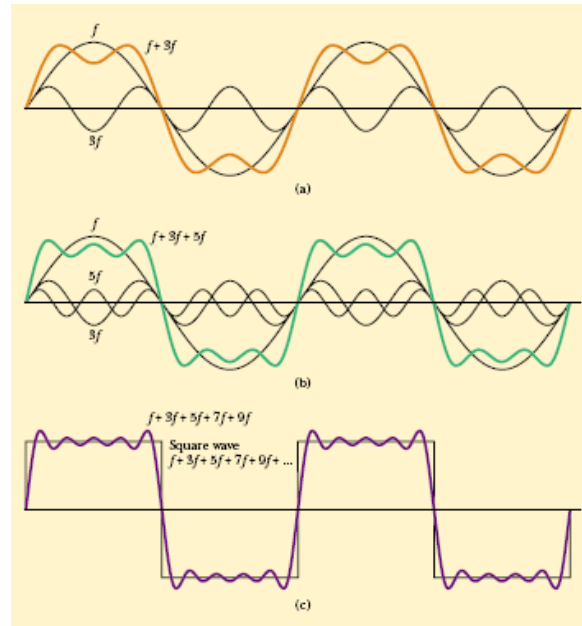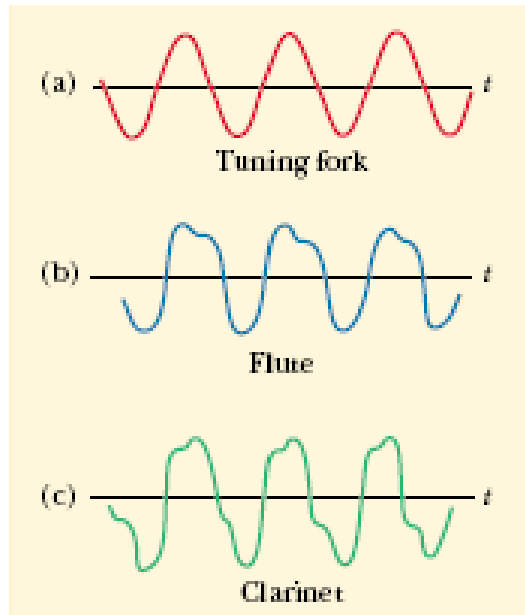
The number of harmonics present in the audible range :

$$n = \frac{f_n}{f_1} = \frac{17000 \ Hz}{139 \ Hz} = 122$$

(Only the first few harmonics are of sufficient amplitude to be heard)
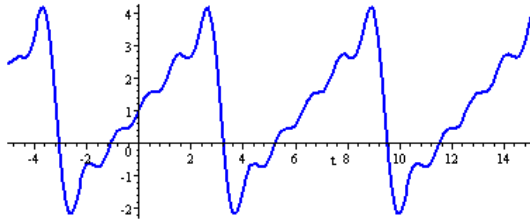
5 NON-SINUSOIDAL WAVE PATTERNS

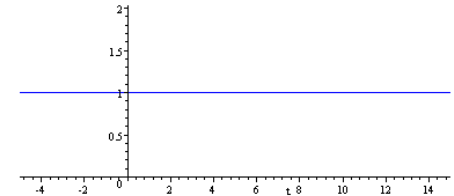The sound-wave patterns produced by the majority of musical instruments are **non-sinusoidal**

Fourier's theorem : any periodic function can be represented as a series of sine and cosine terms

Fourier series : $y(t) = \sum_{n} (A_n \sin 2\pi f_n t + A_n \cos 2\pi f_n t)$

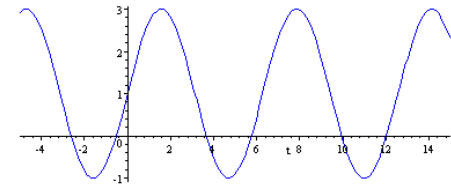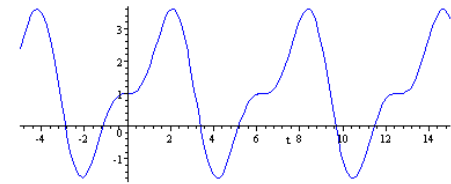# **Example** Function approximates a saw-tooth signal:

$$f(t) = 1$$

Take one extra
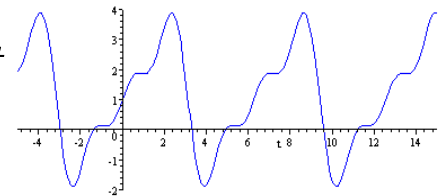term in the
series each
time and
drawing
separate graphs

$$f(t) = 1+2\sin t$$

$$f(t) = 1 + 2\sin t - \sin 2t$$

$$f(t) = 1 + 2\sin t - \sin 2t + 2/3\,\sin 3t$$

6 Energy Transfer by Waves

Don't have

Consider a sinusoidal wave traveling on a string.

A segment of the string of length Δx and mass $\Delta m = \mu \Delta x$

- The potential energy of this segment :

$$\Delta U = \frac{1}{2}(\Delta m)\omega^2 y^2 = \frac{1}{2}(\mu \Delta x)\omega^2 y^2$$

With $\Delta x \rightarrow dx$: $dU = \frac{1}{2}\mu\omega^2 A^2 y^2 dx$
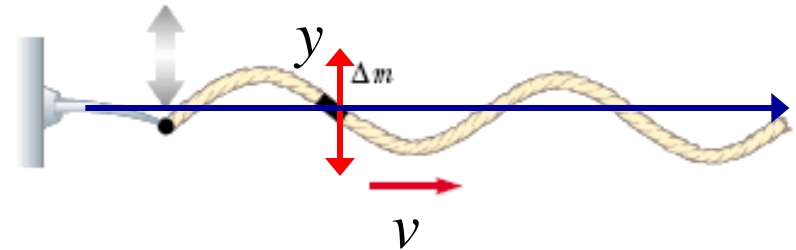
Choose $t = 0$ so that : $y = \sin(kx - \omega t) = \sin(kx)$

$\rightarrow$ $dU = \frac{1}{2}\mu\omega^2 A^2 \sin^2(kx)dx$

The total potential energy in one wavelength :

$$U_\lambda = \int_0^\lambda dU = \int_0^\lambda \frac{1}{2}\mu\omega^2 A^2 \sin^2(kx)dx = \frac{1}{2}\mu\omega^2 A^2 \int_0^\lambda \sin^2(kx)dx$$

$$U_\lambda = \frac{1}{4}\mu\omega^2 A^2 \lambda$$

- The total kinetic energy in one wavelength: $K_\lambda = \dfrac{1}{4} \mu \omega^2 A^2 \lambda$

- The total energy in one wavelength of the wave:

$$E_\lambda = U_\lambda + K_\lambda = \frac{1}{2} \mu \omega^2 A^2 \lambda$$
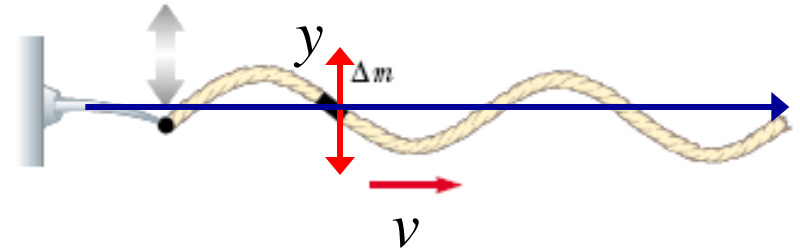
- The **power**, or **rate of energy transfer**, associated with the wave :

$$P = \frac{E_\lambda}{T} = \frac{1}{2} \mu \omega^2 A^2 \frac{\lambda}{T} \quad \rightarrow \quad P = \frac{1}{2} \mu \omega^2 A^2 v$$

- The rate of energy transfer in any sinusoidal wave is proportional to the **square of the angular frequency** and **to the square of the amplitude**

7 Sound Waves

- Sound waves are longitudinal waves traveling through a medium

- A tuning fork : an example of producing a sound wave
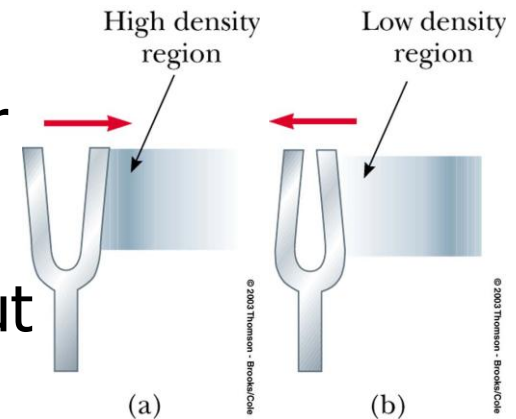
- A tuning fork will produce a pure musical note

- As the tines vibrate, they disturb the air near them

- As the tine swings to the right, it forces the air molecules near it closer together

- This produces a high density area in the air

- As the tine moves toward the left, the air molecules to the right of the tine spread out

- This produces an area of low density

  (a *rarefaction*)

High density region   Low density region

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(a)   (b)

(a)
(b)
© 2003 Thomson - Brooks/Cole
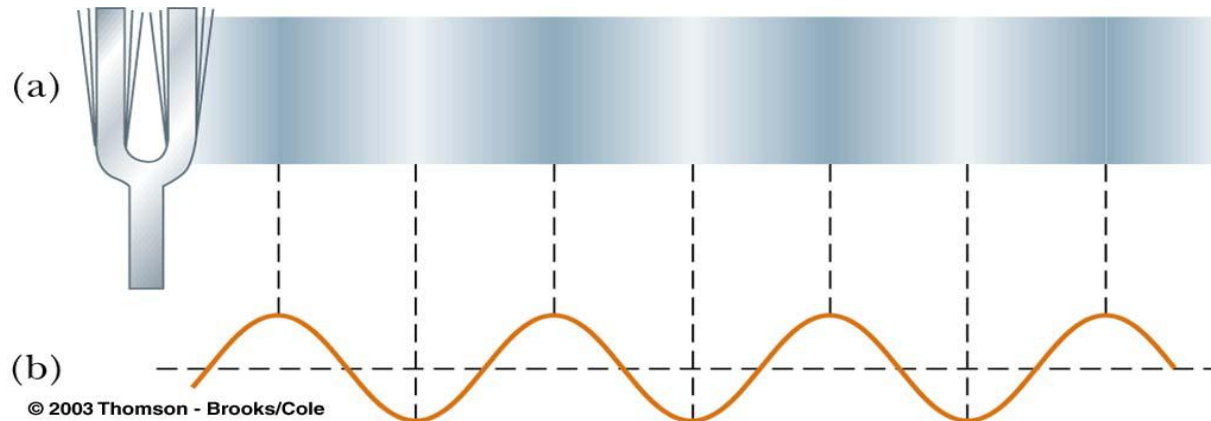
- As the tuning fork continues to vibrate, a **succession of compressions and rarefactions** spread out from the fork
- A **sinusoidal curve** can be used to represent the longitudinal wave
  - **Crests** correspond to compressions and troughs to rarefactions

Categories of Sound Waves

- **Audible waves**
  - Lay within the normal range of hearing of the human ear
  - Normally between **20 Hz to 20,000 Hz**
- **Infrasonic waves:** Frequencies are **below** the audible range
- **Ultrasonic waves:** Frequencies are **above** the audible range

Intensity of Sound Waves

- The ***intensity*** of a wave is the rate at which the energy flows through a unit area, A, oriented perpendicular to the direction of travel of the wave

$$I = \frac{\Delta E}{A\,\Delta t} = \frac{P}{A}$$

- ***P*** is the power, the rate of energy transfer
- Units are W/m²

Handwritten annotations: <20, >20000 → Siêu âm, Hạ âm, $I\,(W/m^2)$, $P = \dfrac{\Delta E}{\Delta t}$

- **Threshold of hearing** *(hg năng)*
  - **Faintest sound most humans can hear about $I_o = 1 \times 10^{-12}$ W/m²** *Const*
- **Threshold of pain**
  - **Loudest sound most humans can tolerate about 1 W/m²**
- **The sensation of loudness is logarithmic in the human hear**
- **β is the intensity sound level or the decibel level of the sound**

có số 10

$$\beta = 10 \log \frac{I}{I_o}$$

Normal conversation's intensity level is about 50 dB.

- **I_o is the threshold of hearing**

  Threshold of hearing is 0 dB. Threshold of pain is 120 dB

  Jet airplanes are about 150 dB. Normal conversation's intensity level is about 50 dB.

70–80dB → loa vừa...

PROBLEM 13

A point source emits sound waves with an average power output of 80.0 W. $P = 80W$

(a) Find the intensity 3.00 m from the source.

(b) Find the distance at which the sound level is 40 dB.

SOLUTION

$A = 4\pi R^2$

(a)
$$I = \frac{\mathscr{P}_{\text{av}}}{4\pi r^2} = \frac{80.0 \text{ W}}{4\pi(3.00 \text{ m})^2} = \boxed{0.707 \text{ W/m}^2}$$

(b)
$$I_0 = 1.00 \times 10^{-12} \text{ W/m}^2$$

$r \rightarrow \text{distance}$

$$10 \log\left(\frac{I}{I_0}\right) = 40 \text{ dB}$$

$$I = 1.00 \times 10^{-8} \text{ W/m}^2 \rightarrow I$$

$$r = \sqrt{\frac{\mathscr{P}_{\text{av}}}{4\pi I}} = \sqrt{\frac{80.0 \text{ W}}{4\pi \times 1.00 \times 10^{-8} \text{ W/m}^2}}$$

$$= \boxed{2.52 \times 10^4 \text{ m}}$$

PROBLEM 14

Consider an idealized model with a bird (treated as a point source) emitting constant sound power, with intensity inversely proportional to the square of the distance from the bird. By how many decibels does the sound intensity level drop when you move twice as far away from the bird?

SOLUTION

$$\beta_2 - \beta_1 = (10\,\text{dB})\left(\log\frac{I_2}{I_0} - \log\frac{I_1}{I_0}\right)$$

$$= (10\,\text{dB})[(\log I_2 - \log I_0) - (\log I_1 - \log I_0)]$$

$$= (10\,\text{dB})\log\frac{I_2}{I_1}$$

$$\beta_2 - \beta_1 = (10\,\text{dB})\log\frac{r_1^2}{r_2^2} = (10\,\text{dB})\log\frac{r_1^2}{(2r_1)^2}$$

$$= (10\,\text{dB})\log\frac{1}{4} = -6.0\,\text{dB}$$

$r_2 = 2r_1$

⇒ cự ra xa nghe nhỏ hơn

PROBLEM 15

Two identical machines are positioned the same distance from a worker. The intensity of sound delivered by each machine at the location of the worker is 2.0×10^{-7} W/m². I
   Find the sound level heard by the worker
(a) when one machine is operating and **(b)** when both machines are operating

SOLUTION

(a) $\beta_1 = 10 \log \left(\dfrac{2.0 \times 10^{-7} \ W/m^2}{1.0 \times 10^{-12} \ W/m^2} \right) = 53 \ dB$

(b) $\beta_2 = 10 \log \left(\dfrac{2 \times 2.0 \times 10^{-7} \ W/m^2}{1.0 \times 10^{-12} \ W/m^2} \right) = 56 \ dB$

mean

2 machine working at the same time.

If there are three machines ?

7.1 The Doppler Effect → why people know space expansion

- A **Doppler effect** is experienced whenever there is **relative motion** between a source of waves and an observer.

- Although the Doppler effect is commonly experienced with sound waves, it is a phenomena common to all waves

- An observer is moving toward a stationary source :

v : velocity of the sound. The frequency :

$$f = \frac{v}{\lambda} \longrightarrow f' = \frac{v + v_O}{\lambda} = \frac{v + v_O}{v/f}$$

$$f' = \frac{v + v_O}{v} f$$

$v_O \to v_S$

$\Rightarrow v_O > 0$

Listener

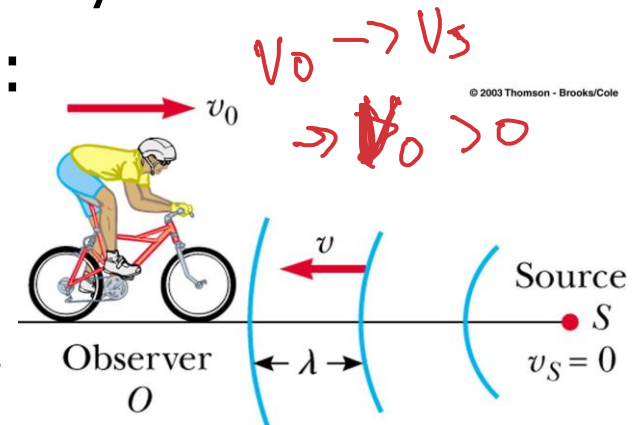© 2003 Thomson - Brooks/Cole

Source
S
$v_S = 0$

Observer
O

Fig 14.8, p. 435
Slide 12

v_O is positive if the observer is moving toward the source and negative if the observer is moving away from the source
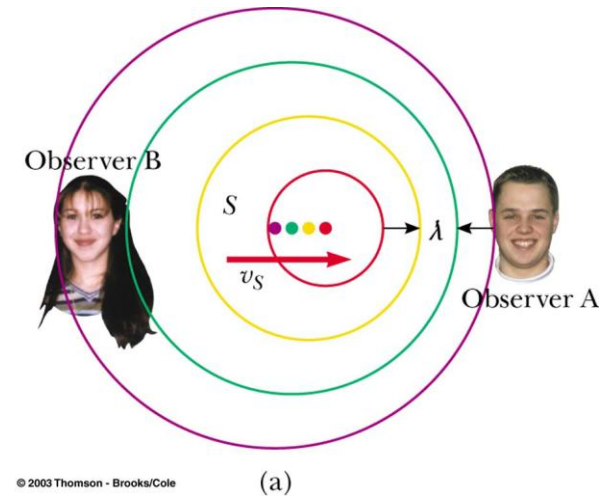
- Source in Motion

The distance between to front waves :

$$(v - v_S)T = \lambda'$$

$$(v - v_S)\frac{1}{f} = \frac{v}{f'}$$

$$f' = \frac{v}{v - v_S} f$$

Observer B

S

λ

v_S

Observer A

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(a)

v_S is positive when the source is moving toward the observer and negative when the source moves away from the observer

In general :

toward → +
away

- Both the source and the observer could be moving

$$f' = \frac{v + v_O}{v - v_S} f$$

→ 340 m/s

- Use positive values of v_o and v_s if the motion is toward
  - Frequency appears higher
- Use negative values of v_o and v_s if the motion is away
  - Frequency appears lower

PROBLEM 16

As an ambulance travels east down a highway at a speed of 33.5 m/s, its siren emits sound at a frequency of 400 Hz. What frequency is heard by a person in a car traveling west at 24.6 m/s?

(a) as the car approaches the ambulance and
(b) as the car moves away from the ambulance?

SOLUTION

(a)

$$f' = \left(\frac{v + v_O}{v - v_S}\right)f = \left(\frac{343 \text{ m/s} + 24.6 \text{ m/s}}{343 \text{ m/s} - 33.5 \text{ m/s}}\right)(400 \text{ Hz}) \quad \text{(approach)}$$

$$= \quad 475 \text{ Hz}$$

(b)

$$f' = \left(\frac{v - v_O}{v + v_S}\right)f = \left(\frac{343 \text{ m/s} - 24.6 \text{ m/s}}{343 \text{ m/s} + 33.5 \text{ m/s}}\right)(400 \text{ Hz}) \quad \text{(move away)}$$
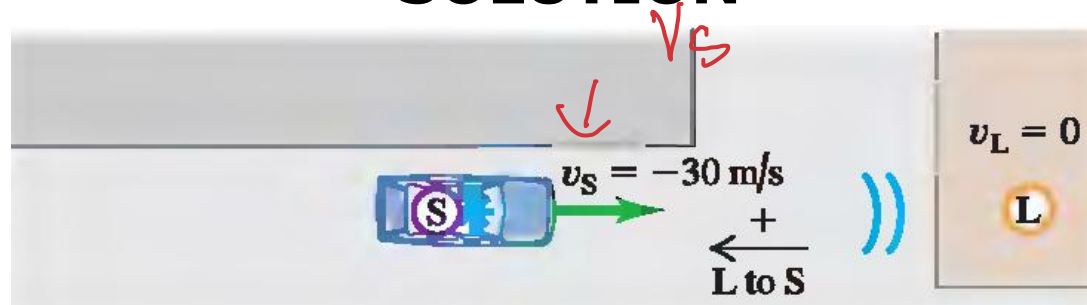
$$= \quad 338 \text{ Hz}$$

PROBLEM 17

The police car with its 300-Hz siren is moving toward a warehouse at 30 m/s, intending to crash through the door. What frequency does the driver of the police car hear reflected from the warehouse?

second stage we are not S anymore

SOLUTION

$v_S = -30$ m/s

$v_L = 0$

L to S

Two stages of the sound wave's motion :

1 : Sound travels from police car's siren (source S) to warehouse

The frequency reaching the warehouse :

$$f_W = \frac{v}{v + v_S} f_S = \frac{340\ \text{m/s}}{340\ \text{m/s} + (-30\ \text{m/s})}(300\ \text{Hz}) = 329\ \text{Hz}$$
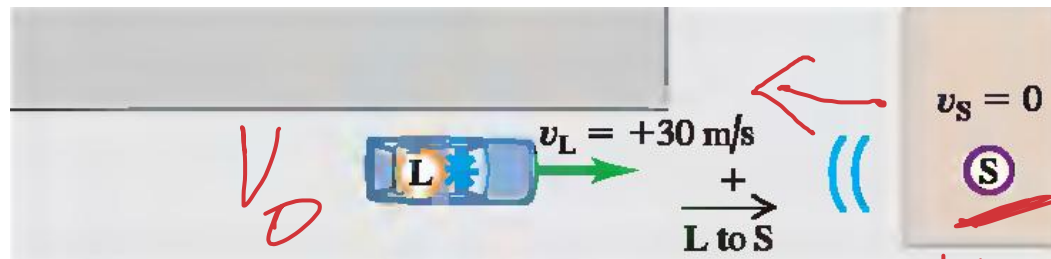
$v_0 = 0 \ / \ v_S = +30$

$$f = \frac{v + v_0}{v - v_S} f'$$

PROBLEM 15

The police car with its 300-Hz siren is moving toward a warehouse at 30 m/s, intending to crash through the door. What frequency does the driver of the police car hear reflected front the warehouse?

SOLUTION

2nd stage :

$v_L = +30$ m/s

$v_S = 0$

S

+
$\xrightarrow{\text{L to S}}$

$\dfrac{v_S}{f} = \dfrac{v + v_o}{v - v_S}$

Two stages of the sound wave's motion :

2 : Reflected sound travels from warehouse (source) to police car

The frequency heard by the driver :

→ from not reflected

$$f_L = \frac{v + v_L}{v} f_w = \frac{340 \text{ m/s} + 30 \text{ m/s}}{340 \text{ m/s}} (329 \text{ Hz}) = \mathbf{358 \text{ Hz}}$$