Homework: 3, 5, 7, 13, 24, 34, 45, 49, 51, 56, 57, 59

3. If the 1 kg standard body has an acceleration of 2.00 m/s^2 at 20.0^0 to the positive direction of an x axis, what are (a) the x component and (b) the y component of the net force acting on the body, and (c) what is the net force in unit-vector notation?

$$F_x = m\,\underline{a_x} = m\,\underline{a\cos\theta}$$

$$F_y = m\,\underline{a_y} = m\,\underline{a\sin\theta}$$

$$\vec{F}_{net} = \underline{F_x}\,\hat{i} + \underline{F_y}\,\hat{j} = m\,a(\cos\theta\,\hat{i} + \sin\theta\,\hat{j})$$

5. Three astronauts propelled by jet backpacks, push and guide a 120 kg asteroid toward a processing dock, exerting the forces shown in the figure below, with F_1=32 N, F_2=55 N, F_3=41 N, $\theta_1=30^0$, and $\theta_3=60^0$. What is the asteroid's acceleration (a) in unit-vector notation and as (b) a magnitude and (c) a direction relative to the positive direction of the x axis?

$$F_x = m\,a_x$$

$$F_x = F_2 + F_1\cos\theta_1 + F_3\cos\theta_3$$

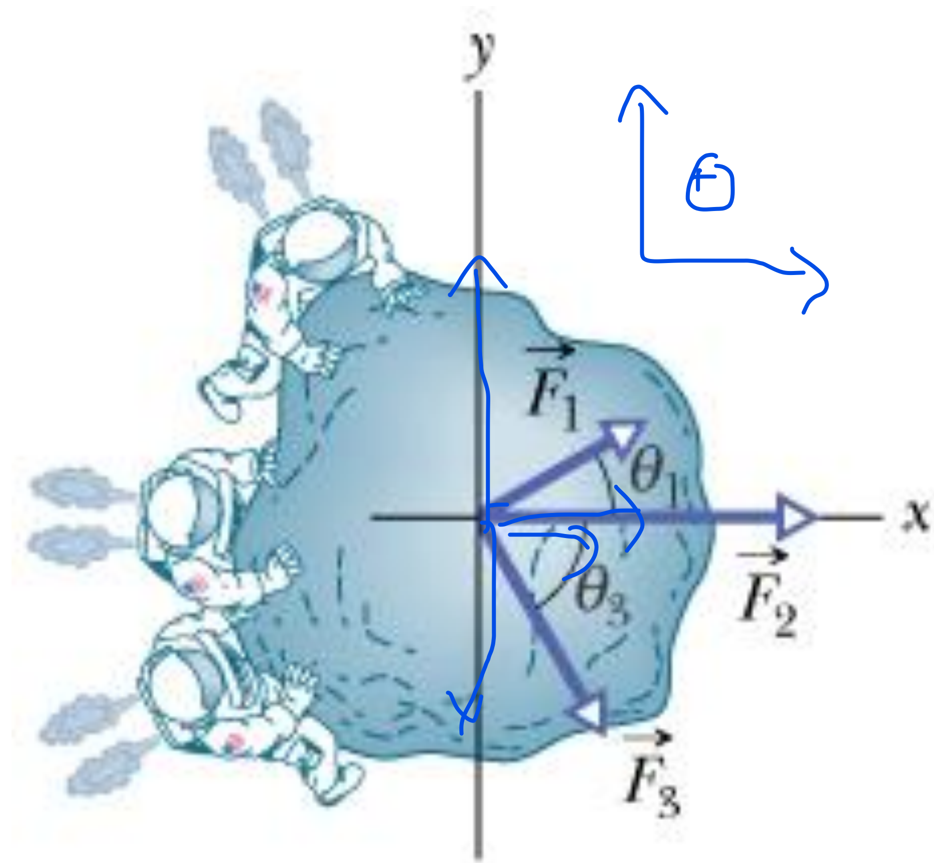
$$F_y = m\,a_y$$

$$F_y = F_1\sin\theta_1 - F_3\sin\theta_3$$

$$\Rightarrow \vec{a} = a_x\hat{i} + a_y\hat{j}$$

$$|\vec{a}| = \sqrt{a_x^2 + a_y^2}$$

$$\theta = \tan^{-1}\left(\frac{a_y}{a_x}\right)$$



13. The figure below shows an arrangement in which four disks are suspended by cords. The longer, top cord loops over a frictionless pulley and pulls with a force of magnitude 98 N on the wall to which it is attached. The tension in the shorter cords are T_1=58.8 N, T_2=49.0 N, and T_3=9.8 N. What are the masses of (a) disk A, (b) disk B, (c) disk C, and (d) disk D?

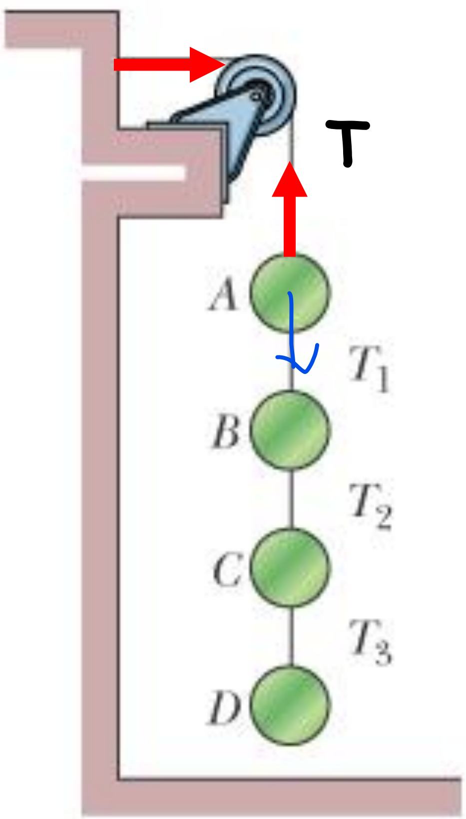
P

Disk A: $T=T_1+m_Ag \Rightarrow m_A=4.0$ (kg)

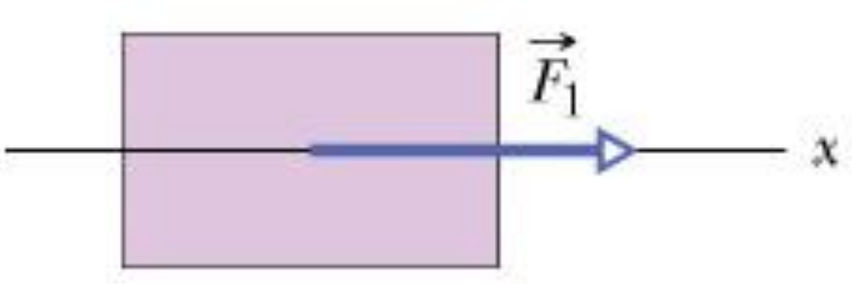
Disk B: $T_1=T_2+m_Bg \Rightarrow m_B=1.0$ (kg)

Disk C: $T_2=T_3+m_Cg \Rightarrow m_C=4.0$ (kg)

Disk D: $T_3=m_Dg \Rightarrow m_D=1.0$ (kg)



24. There are two horizontal forces on the 2.0 kg box in the overhead view of the figure below but only one (of magnitude F_1=30 N) is shown. The box moves along the x axis. For each of the following values for the acceleration a_x of the box, find the second force in unit-vector notation: (a) 10 m/s², (b) 20 m/s², (c) 0, (d) -10 m/s², and (e) -20 m/s².



$F_1+F_2=ma$

(a) $F_2=ma-F_1=2.0\times10-30=-10$ (N)

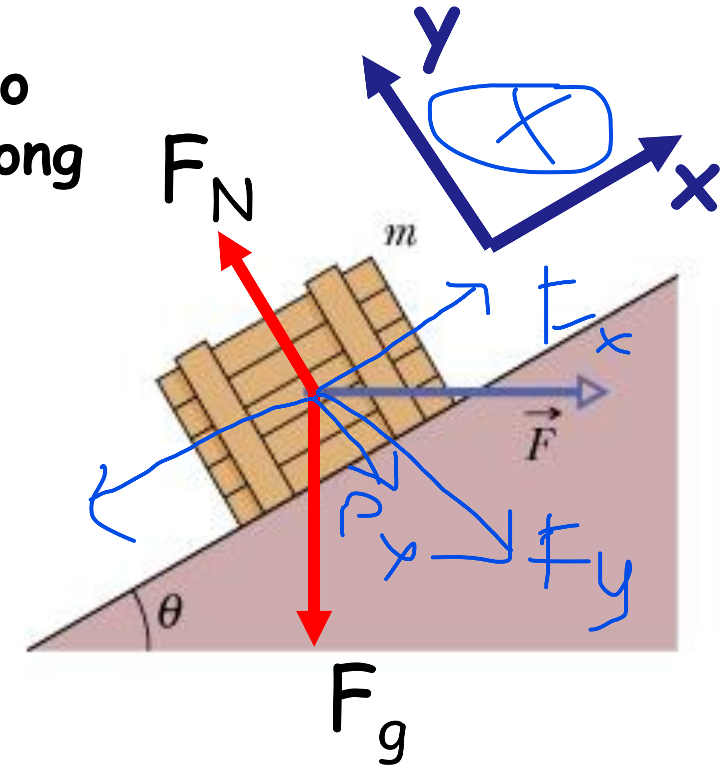
$$\vec{F}_2 = (-10\text{N})\hat{i}$$

(d) $F_2=ma-F_1=2.0\times(-10)-30=-50$ (N)

$$\vec{F}_2 = (-50\,\text{N})\hat{\text{i}}$$

$F_1+F_2=ma$

34. In the figure below, a crate of mass m=115 kg is pushed at constant speed up a frictionless ramp (θ=30.0^0) by a horizontal force $\vec{F}$. What are the magnitudes of (a) $\vec{F}$ and (b) the force on the crate from the ramp?



(a) The crate moves with a constant speed, so the net force acting on the crate is zero. Along the x axis:

$$F\cos\theta - mg\sin\theta = ma$$

$$F_x - P_x = ma$$

F_x

+ a = 0 (constant speed):

$$F\cos\theta = mg\sin\theta \rightarrow F = 651\ (N)$$

(b) Along the y axis:

$$F_N - P_y - F_y = 0$$

$$F_N - mg\cos\theta - F\sin\theta = 0$$

$$F_N = mg\cos\theta + F\sin\theta \rightarrow F_N = 1302\ (N)$$

45. An elevator cab that weighs 27.8 kN moves upward. What is the tension in the cable if the cab's speed is (a) increasing at a rate of 1.22 m/s^2 and (b) decreasing at a rate of 1.22 m/s^2?

(a) Applying Newton's second law, $a=+1.22$ m/s^2:

$$T-\overset{P}{mg} = ma$$

$$m = (27.8\times1000)/9.8 = 2837 \text{ (kg)}$$

$$T = 2837\,(9.8+1.22) = 31.3 \times 10^3 \text{ (N)}$$

(b) $a=-1.22$ m/s^2: $T = 2837\,(9.8-1.22) = 24.3 \times 10^3$ (N)

$P = ma = 27.8 \text{ kN}$

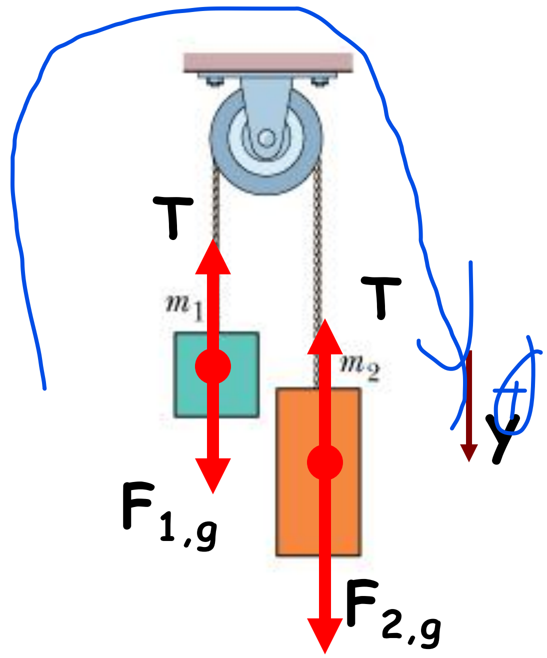
$\Rightarrow m\overline{a} = \dfrac{27.8 \times 1000}{9.8} = \ldots$

+

51. The figure below shows two blocks connected by a cord (of negligible mass) that passes over a frictionless pulley (also of negligible mass). The arrangement is known as Atwood's machine. One block has mass m_1=1.3 kg; the other has mass m_2=2.8 kg. What are (a) the magnitude of the block's acceleration and (b) the tension in the cord?

P

$$m_1 g - T = m_1 a_1$$
$$m_2 g - T = m_2 a_2$$



$a_1 = -a_2 = -a$:

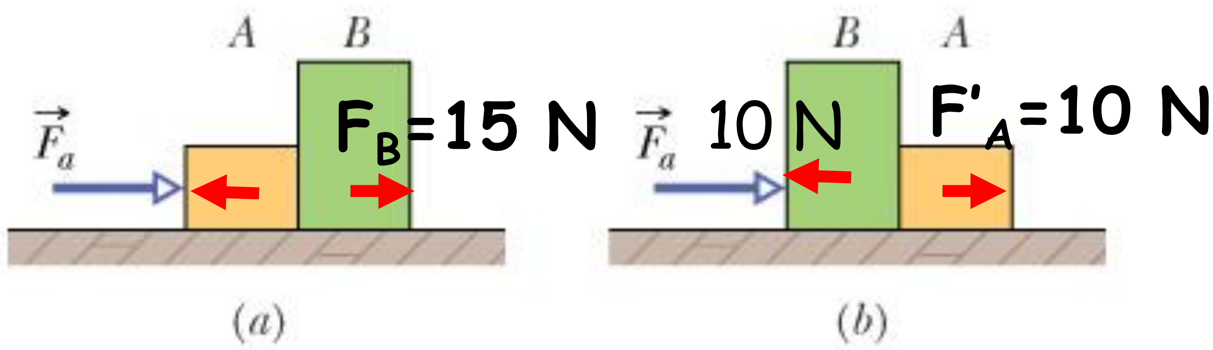
$$m_1 g - T = -m_1 a$$
$$m_2 g - T = m_2 a$$

a = 10

$$a = \frac{(m_2 - m_1)g}{m_1 + m_2} = 3.6(\mathrm{m/s^2})$$

$$T = m_1(g+a)$$
$$T = 17.4\ (\mathrm{N})$$

56. In Figure a, a constant horizontal force $\vec{F}_a$ is applied to block A, which pushes against block B with a 15.0 N force directed horizontally to the right. In Figure b, the same force $\vec{F}_a$ is applied to block B; now block A pushes on block B with a 10.0 N force directed horizontally to the left. The blocks have a combined mass of 12.0 kg. What are the magnitudes of (a) their acceleration in Figure a and (b) force $\vec{F}_a$?



(a) Figure a: $F_B = m_B a$

Figure b: $F'_A = m_A a$

$a = (F'_A + F_B)/(m_A + m_B)$

$F_B = 15$ N; $F'_A = 10$ N; $m_A + m_B = 12$ kg $\Rightarrow a = 2.08\ (m/s^2)$

(b) $F_a = (m_A + m_B)a = 25$ (N)

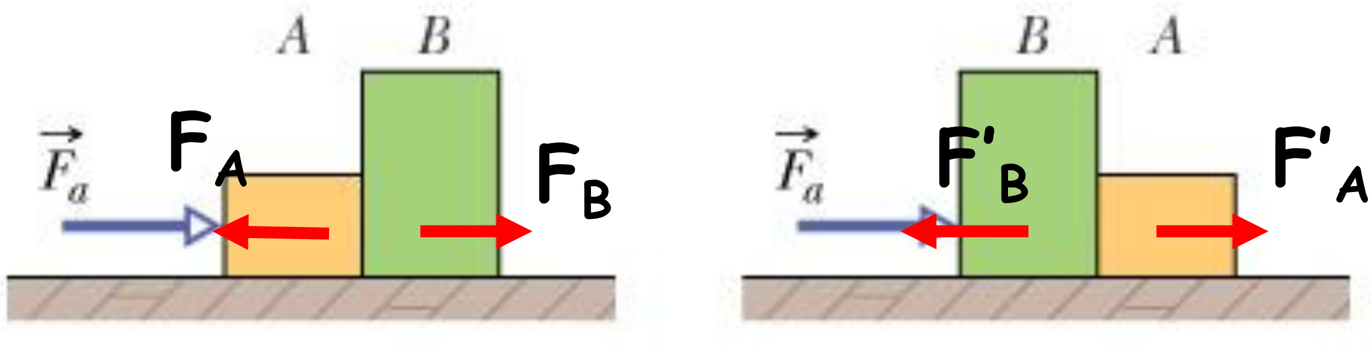
<u>Additional question:</u> what are masses m_A and m_B?

$$m_A = F'_A/a = 10/2.08 \approx 4.8 \text{ kg}$$

$$m_B = F_B/a = 15/2.08 \approx 7.2 \text{ kg}$$

Inverse problem:

If we know $F_a = 25$ N, $m_A = 4.8$ kg and $m_B = 7.2$ kg, Determine contact forces between the blocks in Figure a and b.



(a) (b)

$$F_a = (m_A+m_B)a \rightarrow a = F_a/(m_A+m_B) = 25/12 \approx 2.08 \text{ m/s}^2$$

Figure a: $F_B = m_B a \rightarrow F_B = 7.2 \times 2.08 \approx 15$ N

$$F_a - F_A = m_A a \rightarrow F_A = 25 - 4.8 \times 2.08 \approx 15 \text{ N}$$

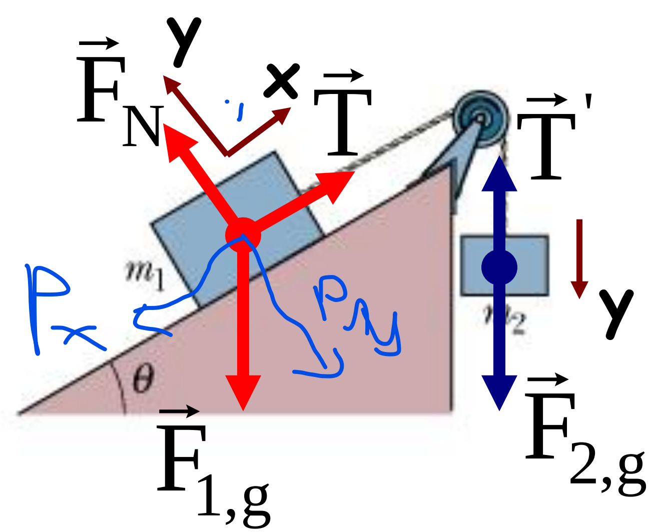
Figure b: $F'_A = m_A a \rightarrow F'_A = 4.8 \times 2.08 \approx 10$ N

$$F_a - F'_B = m_B a \rightarrow F'_B = 25 - 7.2 \times 2.08 \approx 10 \text{ N}$$

57. A block of mass m_1=3.7 kg on a frictionless plane inclined at angle θ=30.0^0 is connected by a cord over a massless, frictionless pulley to a second block of mass m_2=2.30 kg hanging vertically. What are (a) the magnitude of the acceleration of each block, (b) the direction of the acceleration of the hanging block, and (c) the tension in the cord?

1. Force analysis

2. Applying Newton's second law:



Block 1: $F_N - F_{1,g}\cos\theta = 0$ (0) y

$T - F_{1,g}\sin\theta = m_1 a$ (0) x

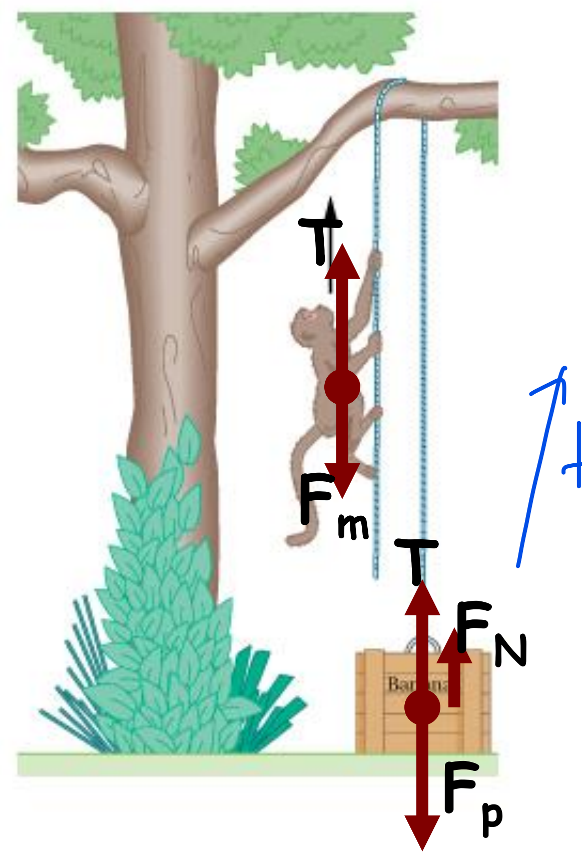
Block 2: $F_{2,g} - T = m_2 a$

$$\Rightarrow a = \frac{m_2 g - m_1 g \sin\theta}{m_1 + m_2} = 0.735\,(m/s^2)$$

a>0: the direction of the acceleration of block 2 is downward.

$$T = F_{2,g} - m_2 a = m_2(g - a) = 20.9\,(N)$$

59. A 10 kg monkey climbs up a massless rope that runs over a frictionless tree limb and back down to a 15 kg package on the ground (a) What is the magnitude of the least acceleration the monkey must have if it is to lift the package off the ground? If, after the package has been lifted, the monkey stops its climb and holds onto the rope, what are the (b) magnitude and (c) direction of the monkey's acceleration and (d) the tension in the rope?



(a) T: the force the rope pulls upward on the monkey: $T - mg = ma_m$

For the package:

$$T + F_N - Mg = Ma_p$$

To lift the package off the ground: $F_N = 0$, and the least acceleration a_m requires $a_p = 0$, so:

$$T = Mg$$

$$Mg - mg = ma_m$$

$$\rightarrow \quad a_m = 4.9\ (m/s^2)$$

(b) See Problem 51: $a = \frac{(M - m)g}{M + m} = 1.96\ (m/s^2)$

(c) See Problem 51: $T = m(g + a_m) \approx 118\ (N)$

Chapter 2 Force and Motion

2.1. Newton's First Law and Inertial Frames
2.2. Newton's Second Law
2.3. Some Particular Forces. The Gravitational Force and Weight
2.4. Newton's Third Law

2.5. Friction and Properties of Friction. Motion in the Presence of Resistive Forces

2.6. Uniform Circular Motion and Non-uniform Circular Motion
2.5. Motion in Accelerated Frames

2.5. Friction and Properties of Friction. Motion in the Presence of Resistive Forces

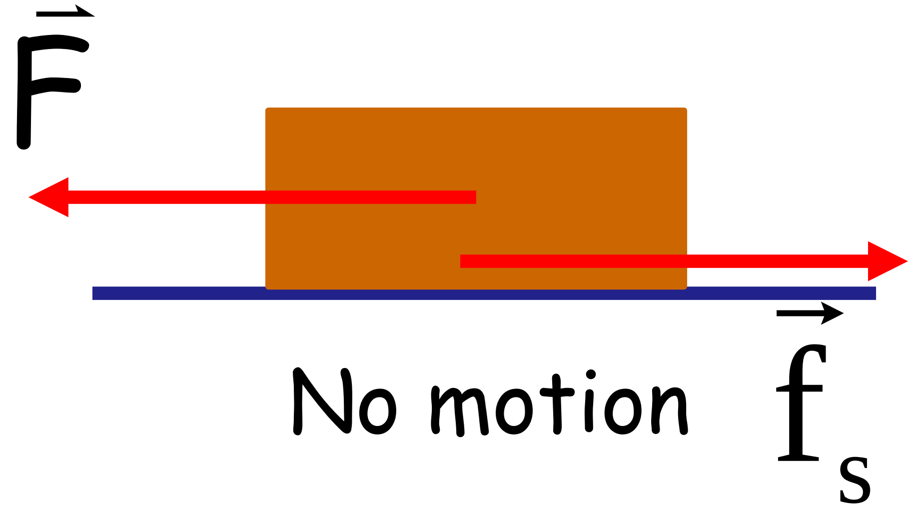
- **Friction:**
- No motion of the block:

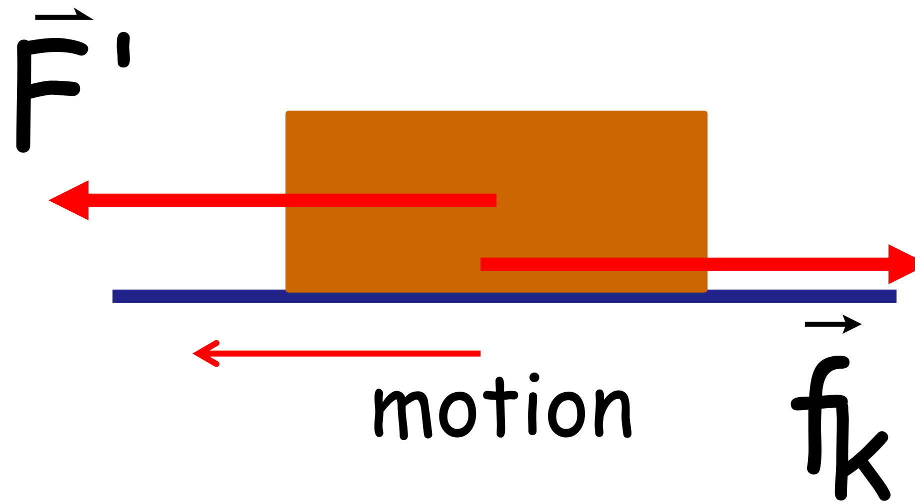
  $\vec{f}_s$: static frictional force

- Motion of the block:

  $\vec{f}_k$: kinetic frictional force

$$\left|\vec{f}_k\right| < \left|\vec{f}_{s,max}\right|$$





- **Properties of friction:**

Property 1: If the body does not move, $\vec{f}_s$ and the component of $\vec{F}$ that is parallel to the surface are equal in magnitude and opposite in direction.

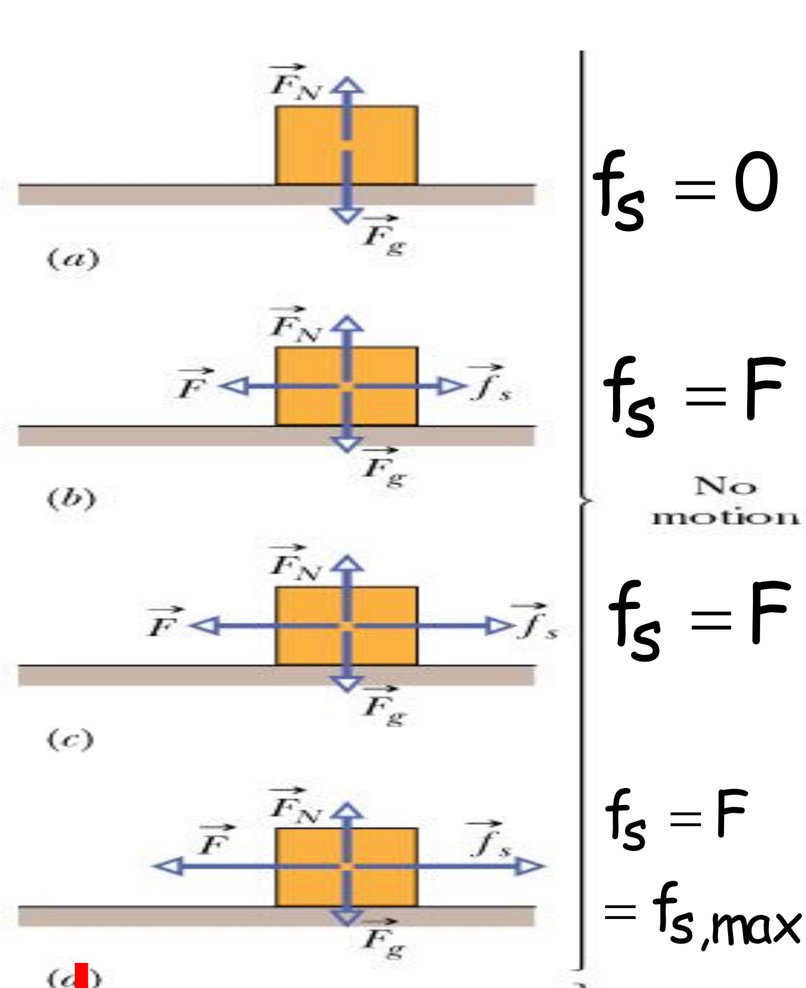
Property 2: The magnitude of $\vec{f}_s$ has a maximum value computed by:

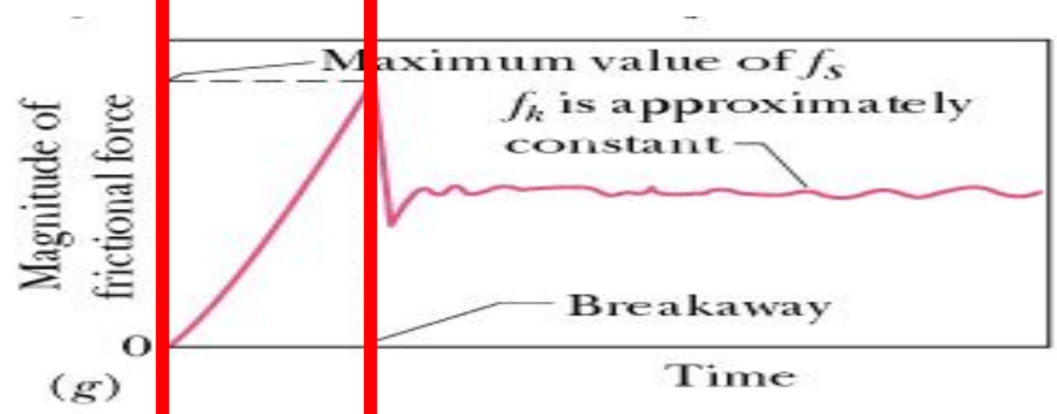
mg

$$\boxed{f_{s,max} = \mu_s F_N}$$

μ_s is the coefficient of static friction.

F_N is the magnitude of the normal force on the body from the surface.



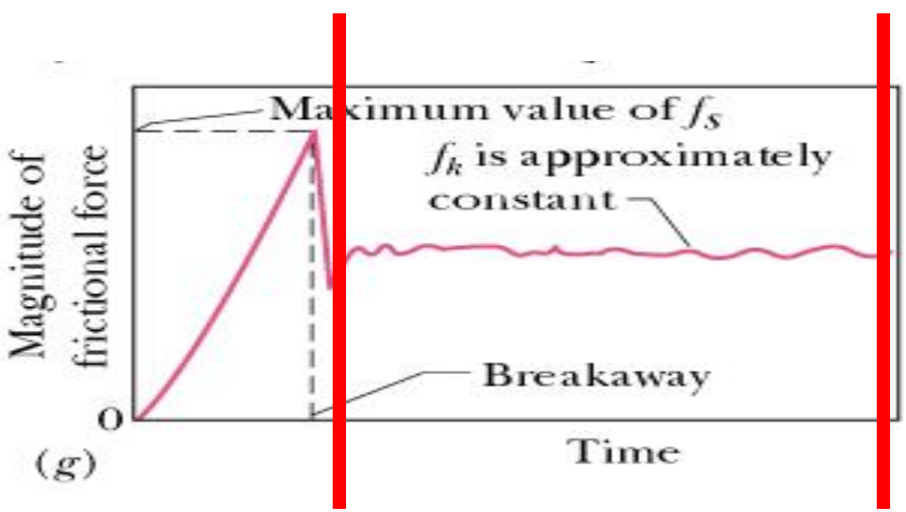


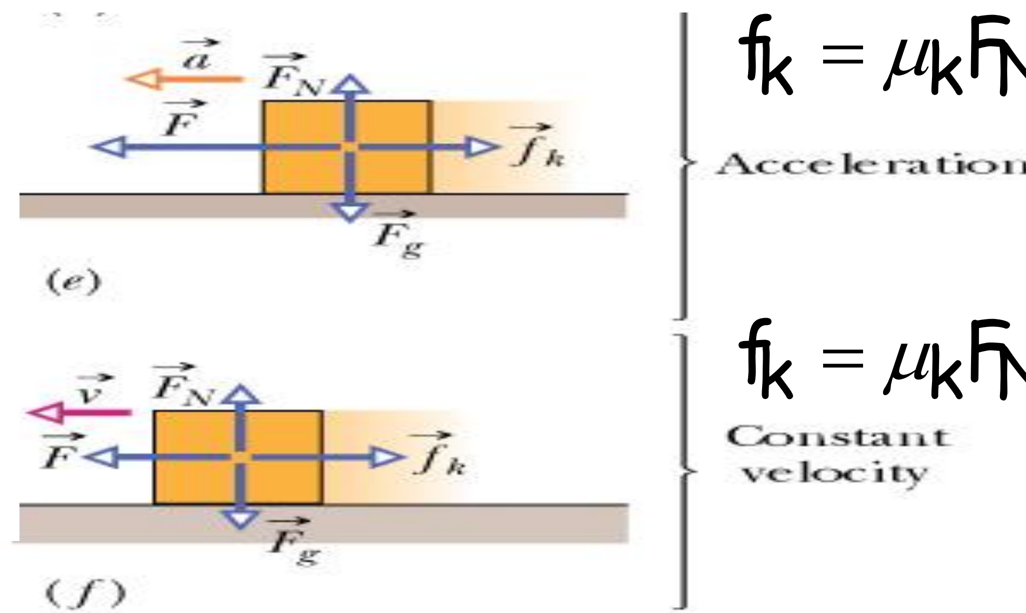
- **Properties of friction:**

Property 3: If the body moves, the magnitude of the frictional force decreases to a value f_k calculated by:

$$f_k = \mu_k F_N$$

μ_k is the coefficient of kinetic friction





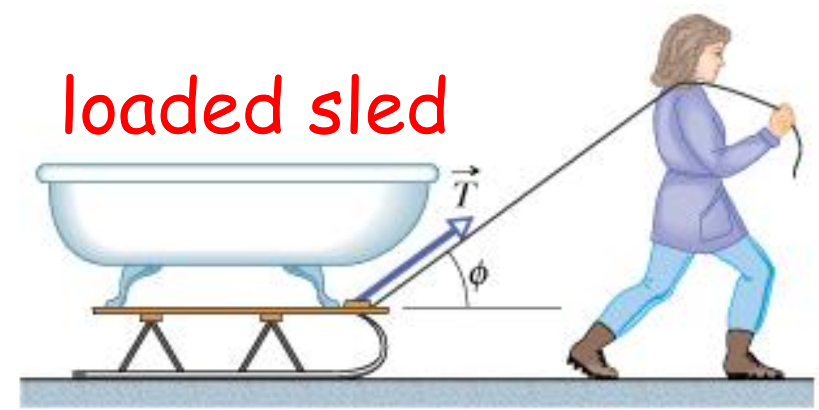
$f_k = \mu_k F_N$

$f_k = \mu_k F_N$

- **Sample Problem:**

A woman pulls a loaded sled of $m=75$ kg at constant speed; $\mu_k=0.10$; $\phi=42^0$; determine:

(a) $|\vec{T}|$ (b) T increases, how about f_k?



$$\vec{F}_{net} = m\vec{a}$$

Constant speed requires a = 0, so:

- For the x axis:

$$T\cos\Phi - f_k = 0; \; f_k = \mu_k F_N$$

$$T\cos\Phi - \mu_k F_N = 0 \; (1)$$

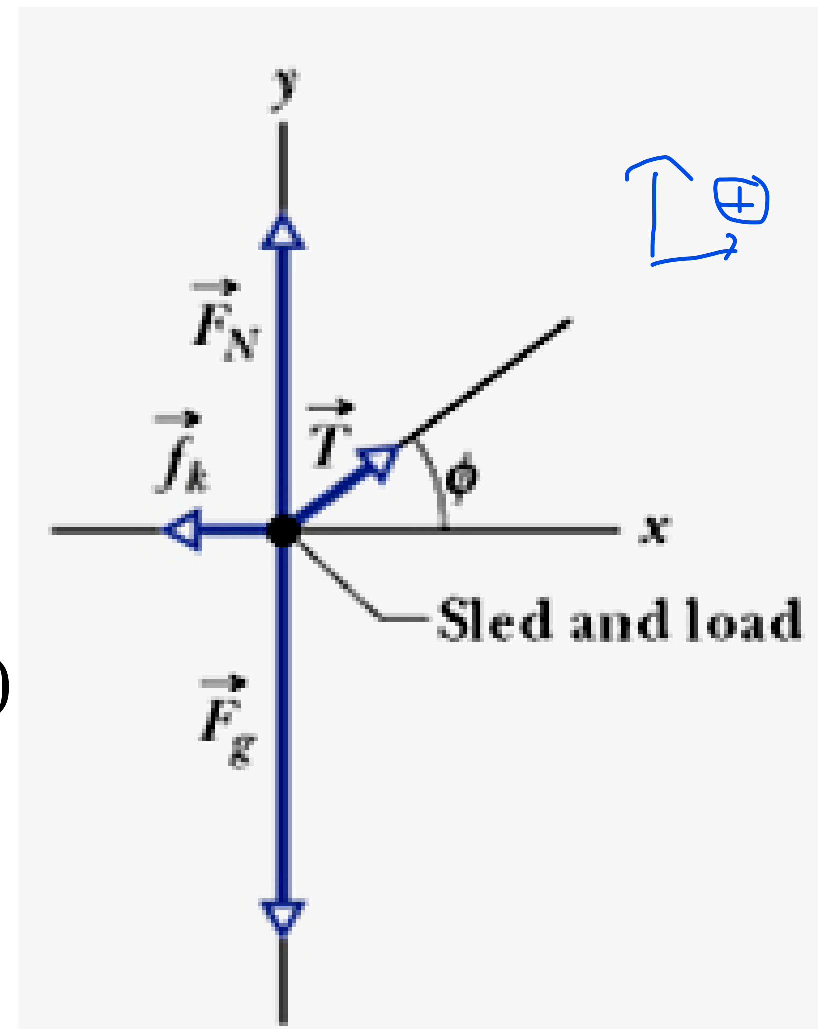
- For the y axis:

$$T\sin\Phi + F_N - mg = 0 \; (2)$$

$$(1) \,\&\, (2) \Rightarrow T = \frac{\mu_k mg}{\cos\Phi + \mu_k \sin\Phi} = 90.7 \text{ (N)}$$

$$F_N = mg - T\sin\Phi$$

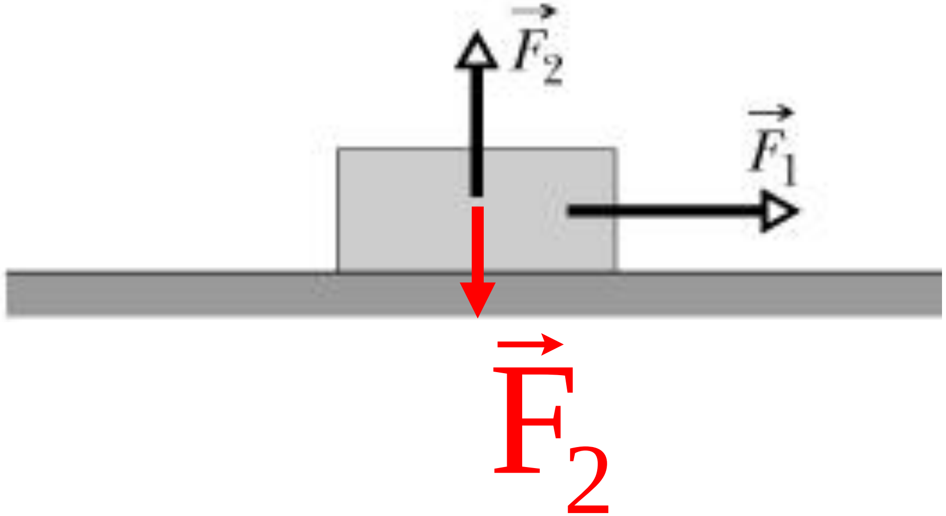
➔ If T increases, F_N will decrease ➔ f_k decreases



- **Checkpoint:**

F_1=10 N, F_2 increases from 0. Before the box begins to slide, do the following quantities increase, decrease or stay the same:
(a) f_s; (b) F_N; (c) $f_{s,max}$

(a) the same;
(b) $F_N+F_2=mg$ → F_N decreases;
(c) $f_{s,max} = \mu_s F_N$, so $f_{s.max}$ decreases



Summary

Steps for solving problems using Newton's laws

1. Draw a free-body diagram for each object of the system:
   - draw all possible forces: gravitational, normal, tension, friction (static or kinetic), any applied forces, third-law force pairs.
   - choose a coordinate system for each moveable object.
   - indicate the acceleration direction of each object, if unknown you can make an assumption.

2. Write Newton's second law: $\underline{\vec{F}_{net} = m\vec{a}}$

   - Write the equation above for each axis

$$F_{net,x} = ma_x; F_{net,y} = ma_y;$$

   - If the system is stationary or moving with a constant speed, then a = 0.

3. Put constraints on the accelerations of the objects

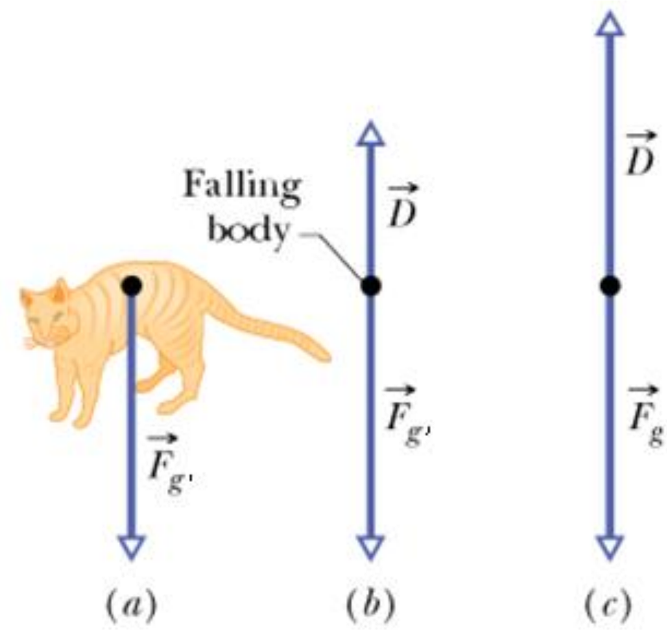
- **Motion in the Presence of Resistive Forces:**

If a body moves through a fluid (gas or liquid), the body will experience a drag force $\vec{D}$ (due to air or viscous resistance) that opposes the relative motion.

- **Drag at high velocity:**

$$\boxed{D = \frac{1}{2} C\rho A v^2}$$

ρ is the density of the fluid
v is the speed of the body relative to the fluid
A is the effective cross-sectional area
C is the drag coefficient



- For a body falling through air: $F_{g'} - D = ma$

$$F_{g'} = mg - F_{buoyant}$$

$$D \sim v^2$$

D increases until $D = F_{g'}$, and the body falls at a constant speed, called the terminal speed V_t:

$$F_{g'} - \frac{1}{2} C\rho A v_t^2 = 0$$

$$\Rightarrow v_t = \sqrt{\frac{2F_{g'}}{C\rho A}}$$

+

- **Drag at low velocity:**

$$\boxed{D = bv}$$

b is a constant, depending on the properties of the fluid and the dimension of the body
v is the speed of the body

$$mg - F_{buoyant} - bv = ma \qquad (1)$$

- D increases until the acceleration a=0: $mg - F_{buoyant} = bv_t \qquad (2)$

$$(1) \text{ and } (2) \Rightarrow b(v_t - v) = ma \text{ or } b(v_t - v) = m\frac{dv}{dt}$$

$$\frac{dv}{v - v_t} = -\frac{b}{m}dt \Rightarrow \int_0^v \frac{dv}{v - v_t} = -\frac{b}{m}\int_0^t dt$$

$$\ln\frac{v_t - v}{v_t} = -\frac{b}{m}t \Rightarrow v = v_t(1 - e^{-\frac{b}{m}t}) \qquad (3)$$

$$(1) \quad \Rightarrow \quad v_t = \frac{mg - F_{buoyant}}{b} = \frac{mg'}{b} \Rightarrow \boxed{v = \frac{mg'}{b}(1 - e^{-\frac{b}{m}t});} \qquad \boxed{a = g' e^{-\frac{b}{m}t}}$$

$$(3) \rightarrow \boxed{y = v_t t + v_t\frac{m}{b}(e^{-\frac{b}{m}t} - 1)}$$

$\tau = \frac{m}{b}$: the characteristic time

$$\boxed{v = v_t(1 - e^{-\frac{t}{\tau}})}$$

$$\boxed{a = g' e^{-\frac{t}{\tau}}}$$

$$\boxed{y = v_t t + v_t \tau (e^{-\frac{t}{\tau}} - 1)}$$

Homework: Read Sample Problem (p 123)
5, 9, 19, 25, 31, 34, 39 (p 130-134)