DATE: April 17, 2019 – Duration: 90 minutes

Question 1 (20 marks)

a) Consider the signal $x(t) = |6\sin(100\pi t)|$. Sketch the signal $x(t)$ for $0 \le t \le 0.06$ seconds. Determine the period T_0 and calculate the power of the signal $x(t)$.

b) The analog signal $x(t) = 5\cos(10\pi t)$ is sampled at the sampling frequency $f_s = 20$ Hz. Determine the discrete-time signal $x[n]$. Sketch the signal $x[n]$ for n=0,1,2...20. Calculate the power of the signal $x[n]$.

Question 2 (20 marks)

A continuous-time system is described the following equation

$$y(t) = x(t-2) + x(2-t)$$

where $x(t)$ is the system input and $y(t)$ is the system output.

a) Discuss the linearity and time-invariance properties of the system.

b) If a system is bounded-input, bounded-output (BIBO) stable, then the output will be bounded for every input to the system that is bounded. Discuss the BIBO stability of the system.

Question 3 (20 marks)

Consider the discrete-time LTI system which has the unit impulse response

$$h[n] = [2, -1, 0, 2]$$

Let $x[n]$ be the input signal and $y[n]$ be the output signal.

a) Write the equation to describe the input-output relationship.

b) Find the output $y[n]$ for the input $x[n] = [2, 1, 0, 1, 3]$.

Question 4 (20 marks)

An LTI analog system has the following unit impulse response

$$h(t) = e^{-t/2}u(t)$$

a) Using convolution, determine and sketch the response $y_1(t)$ of the system for the input $x_1(t) = u(t)$.

b) Determine and sketch the response $y_2(t)$ of the system for the input $x_2(t) = 2[u(t) - u(t-3)]$.

Question 5 (20 marks)

Consider an analog system whose input-output relationship is defined by

$$\frac{d^2y(t)}{dt^2} + 5\frac{dy(t)}{dt} + 6y(t) = 6x(t)$$

where $x(t)$ is the input and $y(t)$ is the output.

a) Find the natural response $y^{(h)}(t)$ which is the solution to the homogeneous equation.

b) Assuming that $x(t) = u(t)$, find the forced response $y^{(p)}(t)$ for $t \ge 0$. Then, find the total response $y(t)$ for $t \ge 0$ if the initial conditions $y(0) = 10$ and $y'(0) = -2$ (i.e., $\left.\frac{dy(t)}{dt}\right|_{t=0} = -2$).