

Homework: 1, 2, 3, 5, 8 (page 556)

Final Exam: Chapter 18, 19, and 20 (not including "Engines")

1. Suppose 4.0 mol of an ideal gas undergoes a reversible isothermal expansion from volume  $V_1$  to volume  $V_2=2.0V_1$  at temperature  $T=400\text{K}$ . Find (a) the work done by the gas and (b) the entropy change of the gas. (c) If the expansion is reversible and adiabatic instead of isothermal, what is the entropy change of the gas?

(a) 
$$W = nRT \ln \frac{V_2}{V_1}$$
$$W = 4.0 \times 8.31 \times 400 \times \ln \frac{2.0V_1}{V_1} = 9216 \text{ (J)}$$

(b) 
$$\Delta S = \int_i^f \frac{dQ}{T}$$

For an isothermal process:  $T=\text{constant}$

$$\Delta S = \frac{Q}{T}; \Delta E_{\text{int}} = Q - W = 0 \Rightarrow Q = W$$
$$\Delta S = \frac{W}{T} = \frac{9216}{400} = 23.0 \text{ (J/K)}$$

(c)  $Q = 0 \Rightarrow \Delta S = 0$

2. An ideal gas undergoes a reversible isothermal expansion at 77.0°C, increasing its volume from 1.30 L to 3.90 L. The entropy change of the gas is 22.0 J/K. How many moles of gas are present?

$$\Delta S = S_f - S_i = nR \ln \frac{V_f}{V_i} + nC_V \ln \frac{T_f}{T_i}$$

$$\Delta S = nR \ln \frac{V_f}{V_i} \Rightarrow n = \frac{\Delta S}{R \ln \frac{V_f}{V_i}} = \frac{22.0}{8.31 \times \ln \frac{3.9}{1.3}}$$

$$n = 2.41 \text{ (mol)}$$

3. A 2.5 mol sample of an ideal gas expands reversibly and isothermally at 360 K until its volume is doubled. What is the increase in entropy of the gas?

$$\Delta S = S_f - S_i = nR \ln \frac{V_f}{V_i} + nC_V \ln \frac{T_f}{T_i}$$

For an isothermal process,  $T = \text{constant}$ :

$$\Delta S = S_f - S_i = nR \ln \frac{V_f}{V_i}$$

$$\Delta S = 2.5 \times 8.31 \times \ln 2 = 14.4 \text{ (J/K)}$$

5. Find (a) the energy absorbed as heat and (b) the change in entropy of a 2.0 kg block of copper whose temperature is increased reversibly from 25.0°C to 100°C. The specific heat of copper is 386 J.kg<sup>-1</sup>.K<sup>-1</sup>.

(a) Energy absorbed as heat to increase the copper temperature:

$$Q = cm\Delta T$$

$$Q = 386 \times 2 \times 75 = 57900 \text{ (J)}$$

(b) The change in entropy:

$$\Delta S = \int_{T_1}^{T_2} \frac{dQ}{T} = \int_{T_1}^{T_2} \frac{cm dT}{T} = cm \ln \frac{T_2}{T_1}$$

$$T_1 = 25 + 273.15 = 298.15^0 \text{K}; T_2 = 100 + 273.15 = 373.15^0 \text{K}$$

$$\Delta S = 386 \times 2 \times \ln \frac{373.15}{298.15} = 173.2 \text{ (J/K)}$$

8. At very low temperatures, the molar specific heat  $C_v$  of many solids is approximately  $C_v = AT^3$ , where  $A$  depends of the particular substance. For aluminum,  $A = 3.15 \times 10^{-5} \text{ J mol}^{-1} \text{ K}^{-4}$ . Find the entropy change for 4.0 mol of aluminum when its temperature is raised from 5.0 K to 10.0 K.

$$\Delta S = \int_i^f \frac{dQ}{T}$$

We assume that the volume change is negligible:

$$dQ = nC_v dT \rightarrow AT^3$$

$$\Delta S = \int_i^f \frac{nC_v dT}{T} = nA \int_{5.0}^{10.0} T^2 dT =$$

$$= \frac{1}{3} nA T^3 \Big|_{5.0}^{10.0} = \frac{1}{3} \times 4.0 \times 3.15 \times 10^{-5} (10^3 - 5^3) = 0.037 \text{ (J/K)}$$

# Review

• Work done by the gas:

$$W = \int_{V_i}^{V_f} p dV$$

+3 special cases:

p = constant (isobaric):

$$W = p\Delta V = p(V_f - V_i) = nR(T_f - T_i)$$

V = constant (isochoric):

$$W = 0$$

T = constant (isothermal):

$$W = nRT \ln \frac{V_f}{V_i}$$

•Equation of State:

$$pV = nRT$$

$$\frac{p_i V_i}{T_i} = \frac{p_f V_f}{T_f}$$

•The First Law of Thermodynamics:

$$\Delta E_{\text{int}} = Q - W$$

Four Special Cases

Process	Restriction	Consequence
Adiabatic	$Q = 0$	$\Delta E_{\text{int}} = -W$
Constant volume	$W = 0$	$\Delta E_{\text{int}} = Q$
Closed cycle	$\Delta E_{\text{int}} = 0$	$Q = W$
Free expansion	$Q = W = 0$	$\Delta E_{\text{int}} = 0$



•For a closed cycle:  $\Delta E_{\text{int}} = 0$

•Work done by the gas:

-Expansion:  $W > 0$

-Compression:  $W < 0$

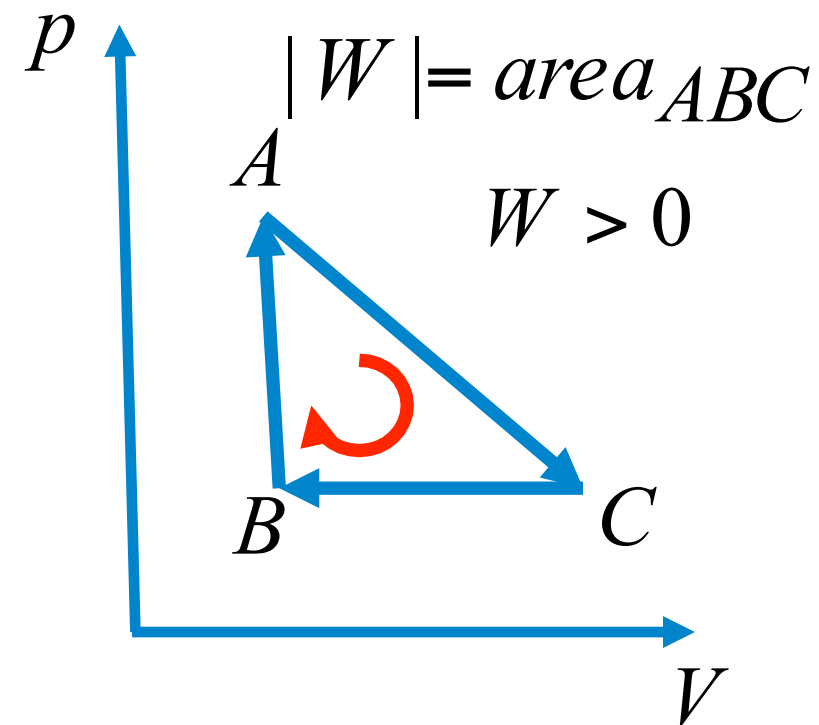
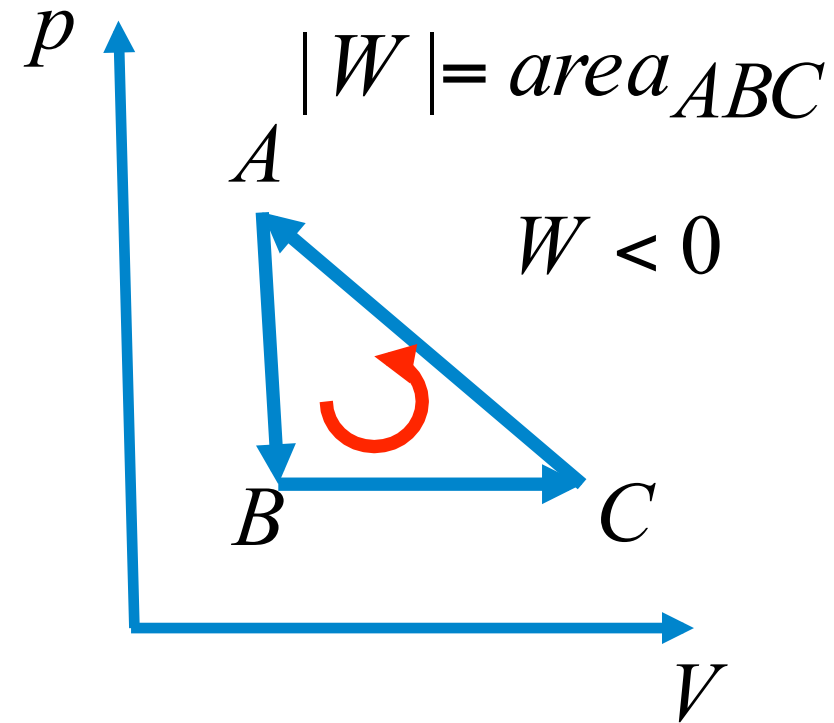
•Energy transferred as heat  $Q$ :

-Heat transferred to the gas  
(receiving energy as heat):

$$Q > 0$$

-Heat transferred from the gas  
(releasing energy as heat):

$$Q < 0$$



•Isothermal process:

$$\Delta E_{\text{int}} = 0$$

$$W = Q = nRT \ln \frac{V_f}{V_i}$$

•RMS Speed:

$$v_{rms} = \sqrt{\frac{3RT}{M}} \quad \bar{v} = v_{avg} = \sqrt{\frac{8RT}{\pi M}}$$

•Translational Kinetic Energy per Molecule:

$$\bar{K} = \frac{3}{2} kT$$

•Total Translational Kinetic Energy (n moles):

$$K_{total} = \frac{3}{2} nRT$$

• Mean Free Path:

$$\lambda = \frac{1}{\sqrt{2}\pi d^2 N/V} = \frac{kT}{\sqrt{2}\pi d^2 p}$$

• The Change in Internal Energy:  $\Delta E_{\text{int}} = nC_V \Delta T$

• Molar Specific Heats of an Ideal Gas:

V = constant:  $Q = nC_V \Delta T$

p = constant:  $Q = nC_p \Delta T$

with  $C_p = C_V + R$

• Adiabatic Process ( $Q = 0$ ):  $pV^\gamma = \text{constant}; TV^{\gamma-1} = \text{constant}$

$$\gamma = \frac{C_p}{C_V}; C_V = \frac{f}{2} R; C_p = C_V + R$$

monatomic:  $f=3$ ; diatomic:  $f=5$ ; polyatomic:  $f=6$

• **Change in entropy:**

$$\Delta S = \int_i^f \frac{dQ}{T}$$

**1) Ideal gas:**  $\Delta S = S_f - S_i = nR \ln \frac{V_f}{V_i} + nC_V \ln \frac{T_f}{T_i}$

**+Some special cases:**

**+ T = constant:**  $\Delta S = nR \ln \frac{V_f}{V_i}$

**or**  $\Delta S = \frac{Q}{T} = \frac{W}{T}$

**+ V = constant:**  $\Delta S = nC_V \ln \frac{T_f}{T_i}$

**2) Liquid, solid:**

**+Cooling or heating:**  $\Delta S = \int_{T_1}^{T_2} \frac{dQ}{T} = \int_{T_1}^{T_2} \frac{cmdT}{T} = cm \ln \frac{T_2}{T_1}$

**+Phase change:**  $\Delta S = \frac{Lm}{T}$

**L is heat of vaporization or heat of fusion**