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Week 7.

① Given $B = (b_1, b_2, b_3) = \{(8, 11, 0), (7, 0, 10), (1, 4, 6)\}$

$x = (3, 19, 2)$

Let $x = ab_1 + bb_2 + cb_3$

$$\Rightarrow a\begin{pmatrix} 8 \\ 11 \\ 0 \end{pmatrix} + b\begin{pmatrix} 7 \\ 0 \\ 10 \end{pmatrix} + c\begin{pmatrix} 1 \\ 4 \\ 6 \end{pmatrix} = \begin{pmatrix} 3 \\ 19 \\ 2 \end{pmatrix}$$

$$\Rightarrow \left(\begin{array}{ccc|c} 8 & 7 & 1 & 3 \\ 11 & 0 & 4 & 19 \\ 0 & 10 & 6 & 2 \end{array}\right) \xrightarrow{8R_2 - 11R_1} \left(\begin{array}{ccc|c} 8 & 7 & 1 & 3 \\ 0 & -77 & 21 & 119 \\ 0 & 5 & 3 & 1 \end{array}\right) \xrightarrow{5R_2 + 77R_3} \left(\begin{array}{ccc|c} 8 & 7 & 1 & 3 \\ 0 & -77 & 21 & 119 \\ 0 & 0 & 336 & 672 \end{array}\right)$$

$$\Rightarrow \begin{cases} 8a + 7b + c = 3 \\ -77b + 21c = 119 \\ 336c = 672 \end{cases} \rightarrow \begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix} \Rightarrow [x_B] = \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}$$

the cordinate of the vector $x = (4, 23, 8)$ relative to the basis B

is $[x]_B = \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}$

② Let $S = \{(1, -4), (3, -5)\}$ and $T = \{(-9, 1), (-5, 1)\}$

a) Denote $[w_1]_S = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$, $[w_2]_S = \begin{pmatrix} y_1 \\ y_2 \end{pmatrix}$

$$[w_1]_S = x_1 u_1 + x_2 u_2 = x_1\begin{pmatrix} 1 \\ -4 \end{pmatrix} + x_2\begin{pmatrix} 3 \\ -5 \end{pmatrix} = \begin{pmatrix} -9 \\ 1 \end{pmatrix}$$

$$[w_2]_u = y_1 u_1 + y_2 + u_2 = y_1\begin{pmatrix} 1 \\ -4 \end{pmatrix} + y_2\begin{pmatrix} 3 \\ -5 \end{pmatrix} = \begin{pmatrix} -5 \\ 1 \end{pmatrix}$$

- we have:

$$\begin{pmatrix} x_1 + 3x_2 \\ -4x_1 - 5x_2 \end{pmatrix} = \begin{pmatrix} -9 \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} y_1 + 3y_2 \\ -4y_1 - 5y_2 \end{pmatrix} = \begin{pmatrix} -5 \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 3 \\ -4 & -5 \end{pmatrix}\begin{pmatrix} x_1 & y_1 \\ x_2 & y_2 \end{pmatrix} = \begin{pmatrix} -9 & -5 \\ 1 & 1 \end{pmatrix}$$

$$\Leftrightarrow \left(\begin{array}{cc|cc} 1 & 3 & -9 & -5 \\ -4 & -5 & 1 & 1 \end{array}\right) \xrightarrow{4R_1 + R_2} \left(\begin{array}{cc|cc} 1 & 3 & -9 & -5 \\ 0 & 7 & -35 & -19 \end{array}\right)$$

$$\xrightarrow{7R_1 - 3R_2} \left(\begin{array}{cc|cc} 7 & 0 & 42 & 22 \\ 0 & 1 & -5 & -19/7 \end{array}\right) \rightarrow \left(\begin{array}{cc|cc} 1 & 0 & 6 & 22/7 \\ 0 & 1 & -5 & -19/7 \end{array}\right)$$

$\Rightarrow P_{T \to S} = \begin{pmatrix} 6 & 22/7 \\ -5 & -19/7 \end{pmatrix}$

b) Let vetor $v = (1, -2)$, Find $[v]_T$, $[v]$

$\left(\begin{array}{cc|c} -9 & -5 & 1 \\ 1 & -1 & -2 \end{array}\right)$

$\rightarrow \cancel{\left(\begin{array}{cc|c} 1 & -1 & -2 \end{array}\right)} \xrightarrow{5R_2 - R_1} \left(\begin{array}{cc|c} 14 & 0 & -11 \\ 0 & 14 & 17 \end{array}\right) \rightarrow \left(\begin{array}{cc|c} 1 & 0 & -11/14 \\ 0 & 1 & 17/14 \end{array}\right)$

$-9R_2 - R_1$

$\Rightarrow [v]_T = \begin{pmatrix} -11/14 \\ 17/14 \end{pmatrix}$

$[v]_S = P_{T \to S} [v]_T = \begin{pmatrix} 6 & 4 \\ -5 & -3 \end{pmatrix} \begin{pmatrix} -11/14 \\ 17/14 \end{pmatrix} = \begin{pmatrix} 1/7 \\ 2/7 \end{pmatrix}$

week 8:

(1) a) $A = \begin{pmatrix} 4 & -5 \\ 2 & -3 \end{pmatrix}$

$P(\lambda) = \det(A - \lambda I) = \begin{vmatrix} 4-\lambda & -5 \\ 2 & -3-\lambda \end{vmatrix}$

$= (4-\lambda)(-3-\lambda) + 10$

$= \lambda^2 - \lambda - 2 = 0 \Rightarrow \left[\begin{array}{l} \lambda = -1 \\ \lambda = 2 \end{array}\right.$

For $\lambda = -1$

$\begin{bmatrix} 5 & -5 \\ 2 & -2 \end{bmatrix} \begin{bmatrix} a_1 \\ b_1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \Rightarrow a_1 = b_1$

Setting $a_1 = 1$ ve get $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$, satisfied $\lambda = -1$

For $\lambda = 2$

$\begin{bmatrix} 2 & -5 \\ 2 & -5 \end{bmatrix} \begin{bmatrix} a_2 \\ b_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \Rightarrow 2a_2 = 5b_2$

Setting $a_2 = 5$, we get $\begin{bmatrix} 5 \\ 2 \end{bmatrix}$, satisfied $\lambda = 2$

b) $A = \begin{pmatrix} 6 & -3 \\ -2 & 1 \end{pmatrix}$

$P(\lambda) = \det(A - \lambda I) = \begin{vmatrix} 6-\lambda & -3 \\ -2 & 1-\lambda \end{vmatrix}$

$= (6-\lambda)(1-\lambda) - 6 = 0$

$\Leftrightarrow 6 - 6\lambda - \lambda + \lambda^2 - 6 = 0$

$\Leftrightarrow \lambda^2 - 7\lambda = 0$

$\Rightarrow \left[\begin{array}{l} \lambda = 0 \\ \lambda = 7 \end{array}\right.$

For $\lambda = 0$:

$\begin{bmatrix} 6 & -3 \\ -2 & 1 \end{bmatrix} \begin{bmatrix} a_1 \\ b_1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$\rightarrow \left(\begin{array}{cc|c} 6 & -3 & 0 \\ -2 & 1 & 0 \end{array}\right) \xrightarrow{R_2 + 3R_2} \left(\begin{array}{cc|c} 6 & -3 & 0 \\ 0 & 0 & 0 \end{array}\right)$

$6a_1 - 3b_1 = 0 \Leftrightarrow 2a_1 = b_1$

Setting $a_1 = 1$ we get $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$, satisfied $\lambda = 0$

For $\lambda = 7$

$$\begin{bmatrix} -1 & -3 \\ -2 & -6 \end{bmatrix}\begin{bmatrix} a_2 \\ b_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\Rightarrow \left(\begin{array}{cc|c} -1 & -3 & 0 \\ -2 & -6 & 0 \end{array}\right) \longrightarrow \left(\begin{array}{cc|c} -1 & -3 & 0 \\ 0 & 0 & 0 \end{array}\right) \Rightarrow -a_2 - 3b_2 = 0 \Rightarrow a_2 = -3b_2$$

setting $b_2 = 1$ we get $\begin{bmatrix} -3 \\ 1 \end{bmatrix}$, satisfied $\lambda = 7$

② $A = \begin{pmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ 1 & -2 & 0 \end{pmatrix}$

$$P(\lambda) = \det(A - \lambda I) = \begin{vmatrix} -2-\lambda & 2 & -3 \\ 2 & 1-\lambda & -6 \\ 1 & -2 & -\lambda \end{vmatrix}$$

$$= (-2-\lambda)\left[-\lambda(1-\lambda) - 12\right] - 2\left[-2\lambda + 6\right] - 3\left[-4 - (1-\lambda)\right]$$

$$= (-2-\lambda)(-\lambda + \lambda^2 - 12) + 4\lambda - \cancel{12} + \cancel{12} + 3 - 3\lambda$$

$$= \underline{2\lambda} - \underline{2\lambda^2} + 24 + \underline{\lambda^2} - \underline{\lambda^3} + \underline{12\lambda} + 3 + \underline{\lambda}$$

$$= -\lambda^3 + 15\lambda - \lambda^2 + 27 = 0 \Leftrightarrow \begin{cases} \lambda = -3 \\ \lambda = -\sqrt{10} + 1 \\ \lambda = \sqrt{10} + 1 \end{cases}$$

For $\lambda = -3$:

$$\Rightarrow \left[\begin{array}{ccc|c} 1 & 2 & -3 & 0 \\ 2 & 4 & -6 & 0 \\ 1 & -2 & 3 & 0 \end{array}\right]$$

$$\begin{array}{l} \\ \vec{R_2} - 2R_1 \\ R_1 - R_3 \end{array}\left[\begin{array}{ccc|c} 1 & 2 & -3 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 4 & -6 & 0 \end{array}\right]$$

$$\Rightarrow \left[\begin{array}{ccc|c} 1 & 2 & -3 & 0 \\ 0 & 4 & -6 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$\Rightarrow 4x_2 = 6x_3$ | let $x_3 = 1 \Rightarrow x_2 = \frac{3}{2}$

$x_1 + 2x_2 - 3x_3 = 0$

$\Rightarrow x_1 = -2x_2 + 3x_3 = 0$

$\vec{x} = \begin{bmatrix} 0 \\ 3/2 \\ 1 \end{bmatrix} \Rightarrow \lambda_1 = -3$

For $\lambda = -\sqrt{10} + 1$

$$\Rightarrow \left[\begin{array}{ccc|c} \sqrt{10} - 3 & 2 & -3 & 0 \\ 2 & \sqrt{10} & -6 & 0 \\ 1 & -2 & \sqrt{10} - 1 & 0 \end{array}\right]$$