SS 2018/11

Q1.

a) Given that: $x(t) = e^{-t}[u(t) - u(t-2)]$

$$E_x = \int_{-\infty}^{+\infty} |x(t)|^2 dt = \int_0^2 |e^{-t}|^2 dt = \frac{1}{2}(1 - e^{-4})$$

(Reader sketches the signal by yourself)

b)

$$P_x = \frac{1}{T}\int_{-T/4}^{3T/4} |x(t)|^2 dt = \frac{1}{T}\int_{-T/4}^{T/4} |A|^2 dt = \frac{A^2}{2}$$

Q2.

a) Given that: $y(t) = x^2(t)$

1. Check for linearity:

Let: $\begin{cases} x_1 \xrightarrow{s} y_1 = x_1^2(t) \\ x_2 \xrightarrow{s} y_2 = x_2^2(t) \end{cases}$

$\rightarrow a_1 y_1 + a_2 y_2 = a_1 x_1^2(t) + a_2 x_2^2(t)$ (1)

Let: $x = a_1 x_1 + a_2 x_2 \xrightarrow{s} y = \big(a_1 x_1(t) + a_2 x_2(t)\big)^2$ (2)

From (1) and (2), $a_1 y_1 + a_2 y_2 \neq \boldsymbol{S}\{a_1 x_1 + a_2 x_2\}$, the system is nonlinear.

2. Check for time invariant:

Let: $x(t) \xrightarrow{s} y = x^2(t)$

$\rightarrow y(t-T) = x^2(t-T)$ (1) (delay the ouput).

Let: $x_T(t) = x(t-T) \xrightarrow{s} y_T = x_T^2(t) = x^2(t-T)$ (2)

Since, (1) = (2), therefore, the system is time invariant.

b) Given that: $y[n] = 2x[2n-1] + 3$

1. Check for linearity:

Let: $\begin{cases} x_1 \xrightarrow{s} y_1 = 2x_1[2n-1] + 3 \\ x_2 \xrightarrow{s} y_2 = 2x_2[2n-1] + 3 \end{cases}$

$\rightarrow a_1 y_1 + a_2 y_2 = a_1(2x_1[2n-1] + 3) + a_2(2x_1[2n-1] + 3)$ (1)

Let: $x = a_1 x_1 + a_2 x_2 \xrightarrow{s} y = 2(a_1 x_1[2n-1] + a_2 x_2[2n-1]) + 3$ (2)

From (1) and (2), $a_1 y_1 + a_2 y_2 \neq \boldsymbol{S}\{a_1 x_1 + a_2 x_2\}$, the system is nonlinear.

2. Check for time invariant:

Let: $x[n] \xrightarrow{s} y[n] = 2x[2n-1] + 3$

$\rightarrow y[n-N] = 2x[2n-2N-1] + 3$ (1) (delay the ouput).

Let: $x_N[n] = x[n-N] \xrightarrow{s} y_N = 2x_N[2n-1] + 3 = 2x_N[2n-N-1] + 3$ (2)

Since, (1) $\neq$ (2), therefore, the system is time variant.

Q3.

Given that: $y[n] = 4x[n] - 2x[n-1] + 3x[n-2]$

a)

$$y[n] = x[n] * h[n] = \sum_{k=-\infty}^{+\infty} x[n-k]h[k] = \sum_{k=0}^{2} x[n-k]h[k]$$

$$= x[n]h[0] + x[n-1]h[1] + x[n-2]h[2] \ (1)$$

$$= 4x[n] - 2x[n-1] + 3x[n-2] \ (2)$$

Compare (1) and (2), we obtain: $h[n] = [4, -2, 3]$

b) Given that: $x[n] = [2, -1, 1, 3, 1]$

$$+\ y[0] = 4x[0] - 2x[-1] + 3x[-2] = 4 \times 2 - 2 \times 0 + 3 \times 0 = 8$$
$$+\ y[1] = 4x[1] - 2x[0] + 3x[-1] = 4 \times (-1) - 2 \times 2 + 3 \times 0 = -8$$
$$+\ y[2] = 4x[2] - 2x[1] + 3x[0] = 4 \times 1 - 2 \times (-1) + 3 \times 2 = 12$$
$$+\ y[3] = 4x[3] - 2x[2] + 3x[1] = 4 \times 3 - 2 \times 1 + 3 \times (-1) = 7$$
$$+\ y[4] = 4x[4] - 2x[3] + 3x[2] = 4 \times 1 - 2 \times 3 + 3 \times 1 = 1$$
$$+\ y[5] = 4x[5] - 2x[4] + 3x[3] = 4 \times 0 - 2 \times 1 + 3 \times 3 = 7$$
$$+\ y[6] = 4x[6] - 2x[5] + 3x[4] = 4 \times 0 - 2 \times 0 + 3 \times 1 = 3$$

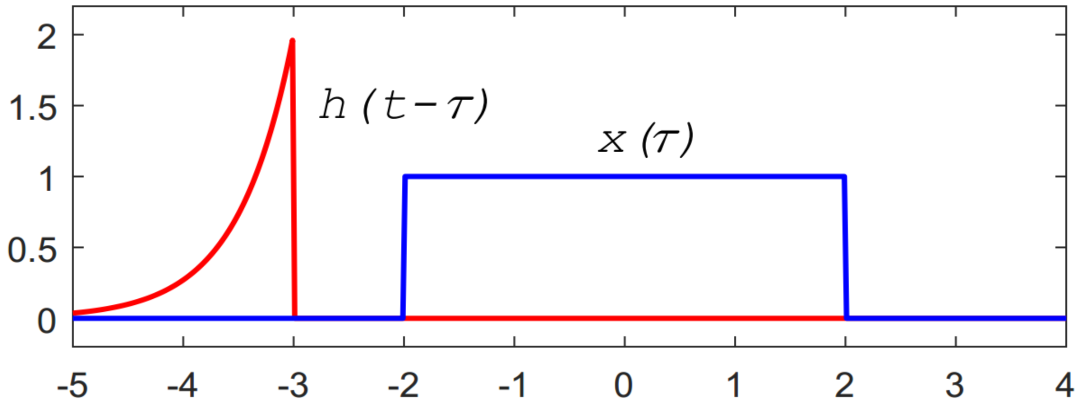
Therefore, $y[n] = [8, -8, 12, 7, 1, 7, 3]$

Q4.

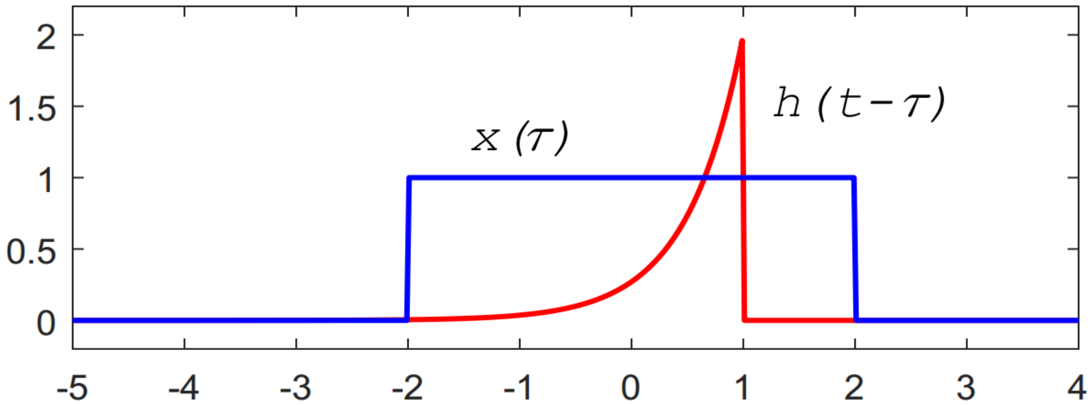
Given that: $h(t) = 2e^{-2t}u(t),\ x(t) = u(t+2) - u(t-2)$

a)

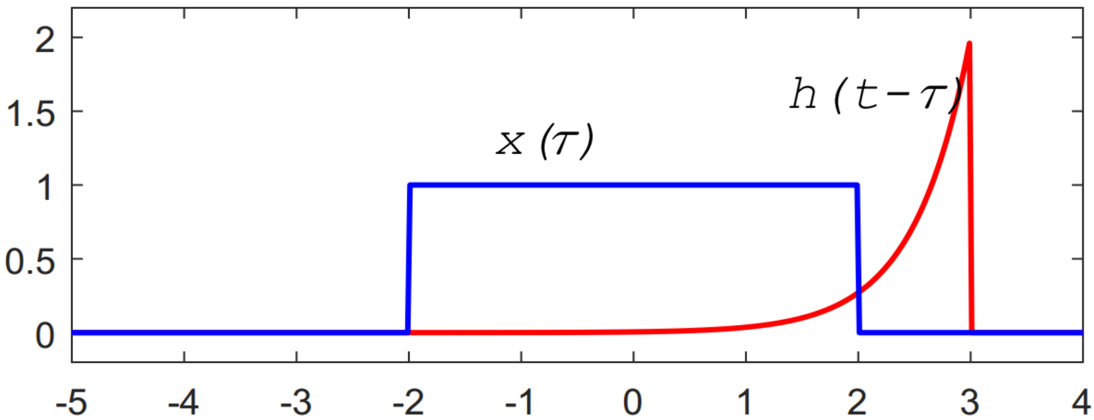
For $t < -2$:



For $-2 \le t < 2$:



For $t \ge 2$:



b)

For $t \leq -2$, $x(\tau)$ and $h(t-\tau)$ does not overlap, therefore $y(t) = 0$.

For $-2 \leq t < 2$:

$$y(t) = \int_{-\infty}^{+\infty} x(\tau)h(t-\tau)d\tau = \int_{-2}^{t} 1.\, e^{-2(t-\tau)}d\tau = e^{-2t}\int_{-2}^{t} e^{2\tau}d\tau = \frac{1}{2}e^{-2t}(e^{2t} - e^{-4})$$

For $t \geq 2$:

$$y(t) = \int_{-\infty}^{+\infty} x(\tau)h(t-\tau)d\tau = \int_{-2}^{2} 1.\, e^{-2(t-\tau)}d\tau = e^{-2t}\int_{-2}^{2} e^{2\tau}d\tau = \frac{1}{2}e^{-2t}(e^{4} - e^{-4})$$

Thus,

$$y(t) = \begin{cases} 0 & , \quad t \leq -2 \\ \frac{1}{2}e^{-2t}(e^{2t} - e^{-4}), & -2 < t < 2 \\ \frac{1}{2}e^{-2t}(e^{4} - e^{-4}), & t \geq 2 \end{cases}$$