PASSIVE FILTERS

<table>
<tr><th>Passive Filters</th><th>General</th><th>Types</th><th>Features</th></tr>
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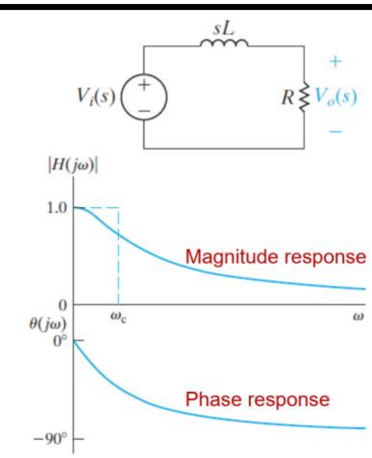
<td rowspan="2">Low-pass filters</td>
<td rowspan="2">$$H(s) = \frac{\omega_c}{s + \omega_c}$$</td>
<td>Series RL circuit</td>
<td>


Voltage transfer function:

$H(s) = \frac{R/L}{s + R/L}$ or $H(j\omega) = \frac{R/L}{j\omega + R/L}$

→ Transfer function magnitude:

$|H(j\omega)| = \frac{R/L}{\sqrt{\omega^2 + (R/L)^2}}$

→ Transfer function phase angle:

$\theta(j\omega) = -\tan^{-1}\left(\frac{\omega L}{R}\right)$

Cutoff frequency for RL filters:

$\boxed{\omega_c = R/L}$
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<td>Series RC circuit</td>
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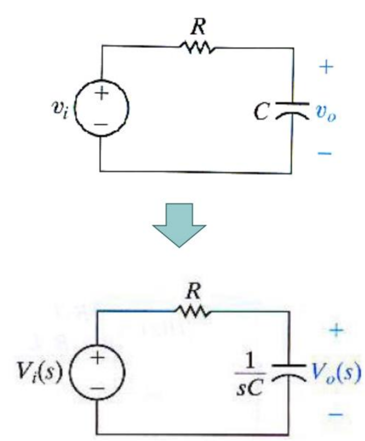

Transfer function:

$H(s) = \frac{1/RC}{s + 1/RC}$

$H(j\omega) = \frac{1/RC}{j\omega + 1/RC}$

Transfer function magnitude:

$|H(j\omega)| = \frac{1/RC}{\sqrt{\omega^2 + (1/RC)^2}}$

Cutoff frequency for RL filters:

$\omega_c = 1/RC$
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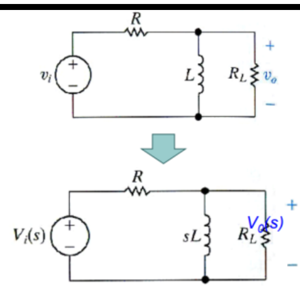
<tr>
<td rowspan="2">High-pass filters</td>
<td rowspan="2">$$H(s)=\frac{s}{s+\omega_c}$$</td>
<td>Series RL circuit</td>
<td>


<u>Examine</u> the effect of placing a load resistor in parallel with the inductor in the **RL high-pass filter**.

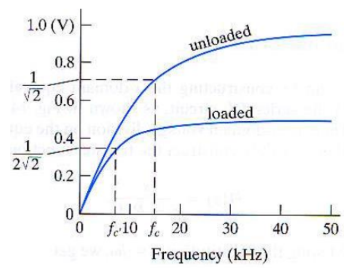
a) Determine the transfer function for the circuit.
b) Sketch the magnitude plot for the loaded R_L, high-pass filter, using the values for R and L from the circuit in previous example and letting R_L = R. On the same graph, sketch the magnitude plot for the unloaded **RL high-pass filter** of previous example.

<u>Sol:</u> a) From the S-domain, the transfer function:

$$H(s)=\frac{\frac{R_L sL}{R_L+sL}}{R+\frac{R_L sL}{R_L+sL}}=\frac{\left(\frac{R_L}{R+R_L}\right)s}{s+\left(\frac{R_L}{R+R_L}\right)\frac{R}{L}}=\frac{Ks}{s+\omega_C}$$

where $\omega_C = K.R/L$, $K=\frac{R_L}{R+R_L}$

ω_c is the cutoff frequency of the loaded filter



For the unloaded RL high-pass filter: K = 1

For loaded RL high-pass filter:

$$K=\frac{R_L}{R+R_L}$$

For K < 1, the effect of the load resistor is to reduce the passband magnitude by the factor K and to lower the cutoff frequency by the same factor.
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<tr>
<td>Series RC circuit</td>
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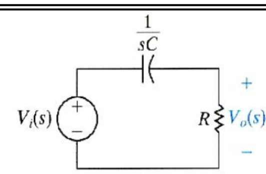

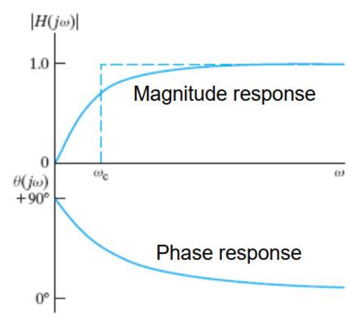

Transfer function:

$$H(s)=\frac{s}{s+1/RC} \quad \text{or} \quad H(j\omega)=\frac{j\omega}{j\omega+1/RC}$$

Transfer function magnitude:

$$|H(j\omega)|=\frac{\omega}{\sqrt{\omega^2+(1/RC)^2}}$$

Transfer function phase angle:

$$\theta(j\omega)=90-\tan^{-1}\omega RC$$

Cutoff frequency for RC filters:

$$\omega_c=1/RC$$
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<tr>
<td rowspan="2">Band-pass filters</td>
<td rowspan="2">$H(s)=\dfrac{\beta s}{s^2+\beta s+\omega_0^2}$</td>
<td>Series RLC circuit</td>
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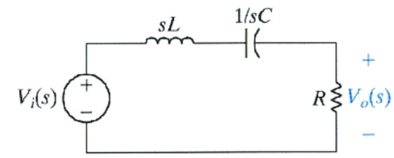

The transfer function

$$H(s)=\frac{V_0(s)}{V_i(s)}=\frac{(R/L)s}{s^2+(R/L)s+(1/LC)}$$

Substitute s = jω, we have:

Magnitude of the transfer function

$$|H(j\omega)|=\frac{\omega(R/L)}{\sqrt{[(1/LC)-\omega^2]^2+[\omega(R/L)]^2}}$$

Phase angle of the transfer function

$$\theta(j\omega)=90^\circ-\tan^{-1}\left[\frac{\omega(R/L)}{(1/LC)-\omega^2}\right]$$

Center frequency

$$\omega_0=\sqrt{\frac{1}{LC}}$$

<u>Cutoff frequencies</u> $\omega_{c1}=-\dfrac{R}{2L}+\sqrt{\left(\dfrac{R}{2L}\right)^2+\left(\dfrac{1}{LC}\right)}$ $\qquad \omega_{c2}=\dfrac{R}{2L}+\sqrt{\left(\dfrac{R}{2L}\right)^2+\left(\dfrac{1}{LC}\right)}$

Relationship between center frequency and cutoff frequencies:

$$\omega_0=\sqrt{\omega_{c1}\omega_{c2}}=\sqrt{\frac{1}{LC}}$$

Bandwidth: $\beta=\omega_{c2}-\omega_{c1}=\dfrac{R}{L}$

Quality factor: $Q=\dfrac{\omega_0}{\beta}=\dfrac{(1/RC)}{R/L}=\sqrt{\dfrac{L}{CR^2}}$

Alternative forms for these equations express the cutoff frequencies

$$\omega_{c1}=\omega_o\cdot\left[-\frac{1}{2Q}+\sqrt{1+\left(\frac{1}{2Q}\right)^2}\right],$$

$$\omega_{c2}=\omega_o\cdot\left[\frac{1}{2Q}+\sqrt{1+\left(\frac{1}{2Q}\right)^2}\right].$$

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<td>Parallel RLC circuit</td>
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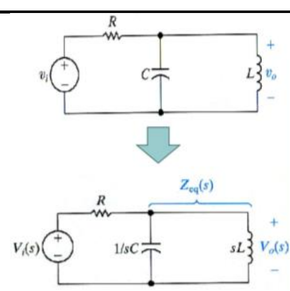

$$Z_{eq}(s)=\frac{L/C}{sL+1/sC}$$

The transfer function

$$H(s)=\frac{s/RC}{s^2+s/RC+1/LC}$$

Substitute s = jω, we have:

Magnitude of the transfer function

$$|H(j\omega)|=\frac{1}{\sqrt{1+\left(\omega RC-\dfrac{1}{\omega(L/R)}\right)^2}}$$

Center frequency

$$\omega_0=\sqrt{\frac{1}{LC}}$$

Cutoff frequencies

$$\omega_{c1}=-\frac{1}{2RC}+\sqrt{\left(\frac{1}{2RC}\right)^2+\left(\frac{1}{LC}\right)}$$

$$\omega_{c2}=\frac{1}{2RC}+\sqrt{\left(\frac{1}{2RC}\right)^2+\left(\frac{1}{LC}\right)}$$

Bandwidth:

$$\beta=\omega_{c2}-\omega_{c1}=\frac{1}{RC}$$

Quality factor:

$$Q=\frac{\omega_0}{\beta}=\sqrt{\frac{CR^2}{L}}$$

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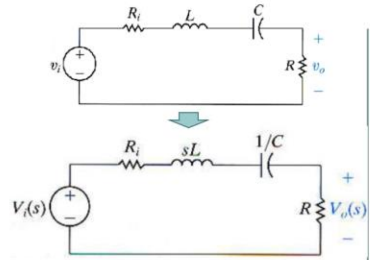
<td rowspan="2">Band-pass filters</td>
<td rowspan="2">$H(s)=\dfrac{\beta s}{s^2+\beta s+\omega_0^2}$</td>
<td>Effect of Nonideal Voltage Source</td>
<td>


The transfer function of the series RLC bandpass filter:

$$H(s)=\frac{\frac{R}{L}s}{s^2+\left(\frac{R+R_i}{L}\right)s+\frac{1}{LC}}$$

$$|H(j\omega)|=\frac{\frac{R}{L}\omega}{\sqrt{\left(\frac{1}{LC}-\omega^2\right)^2+\left(\omega\frac{R+R_i}{L}\right)^2}}$$

The center frequency, ω_o, is the frequency at which this transfer function magnitude is maximum,

$$\omega_o=\sqrt{\frac{1}{LC}}$$

At the center frequency the maximum magnitude is

$$H_{\max}=|H(j\omega_o)|=\frac{R}{R_i+R}$$

The cutoff frequencies can be computed by setting the transfer function magnitude equal to $(1/2^{0.5})H_{max}$

$$\omega_{c1}=-\frac{R+R_i}{2L}+\sqrt{\left(\frac{R+R_i}{2L}\right)^2+\frac{1}{LC}}$$

$$\omega_{c2}=\frac{R+R_i}{2L}+\sqrt{\left(\frac{R+R_i}{2L}\right)^2+\frac{1}{LC}}$$

The bandwidth is calculated from the cutoff frequencies: $\beta=\dfrac{R+R_i}{L}$

Quality factor is computed from the ω_o the β: $Q=\dfrac{\sqrt{L/C}}{R+R_i}$

Transfer function of the series RLC bandpass filter with nonzero source resistance as $H(s)=\dfrac{K\beta s}{s^2+\beta s+\omega_0^2}$

where

$$\beta=\frac{R+R_i}{L},\quad K=\frac{R}{R+R_i}\quad \text{and}\quad \omega_0=\sqrt{\frac{1}{LC}}$$

For ideal source (nonzero source resistance):

$R_i = 0$ then $K = 1$ ⇒ $H(s)=\dfrac{\beta s}{s^2+\beta s+\omega_o^2}$
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<tr>
<td>Summary</td>
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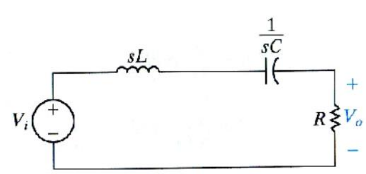
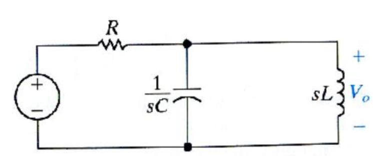
Transfer function: $\boxed{H(s)=\dfrac{\beta s}{s^2+\beta s+\omega_0^2}}$



$$H(s)=\frac{(R/L)s}{s^2+(R/L)s+(1/LC)}$$

$$\omega_0=\sqrt{\frac{1}{LC}}\qquad \beta=\frac{R}{L}$$



$$H(s)=\frac{s/RC}{s^2+s/RC+1/LC}$$

$$\omega_0=\sqrt{\frac{1}{LC}}\qquad \beta=\frac{1}{RC}$$
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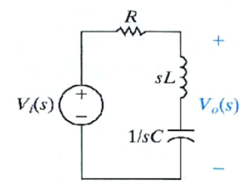
<td rowspan="2">Band-reject filters</td>
<td rowspan="2">$H(s)=\dfrac{s^2+\omega_0^2}{s^2+\beta s+\omega_0^2}$</td>
<td>Series RLC circuit</td>
<td>


Transfer function:
$H(s)=\dfrac{sL+1/sC}{R+sL+1/sC}=\dfrac{s^2+1/LC}{s^2+(R/L)s+(1/LC)}$
Substitute s = jω, we have:
Magnitude of the transfer function
$|H(j\omega)|=\dfrac{|(1/LC)-\omega^2|}{\sqrt{[(1/LC)-\omega^2]^2+\left[\dfrac{\omega R}{L}\right]^2}}$
Phase angle of the transfer function
$\theta(j\omega)=-\tan^{-1}\left[\dfrac{\omega R/L}{(1/LC)-\omega^2}\right]$
Center frequency $\omega_0=\sqrt{\dfrac{1}{LC}}$
Cutoff frequencies:
$\omega_{c1}=-\dfrac{R}{2L}+\sqrt{\left(\dfrac{R}{2L}\right)^2+\left(\dfrac{1}{LC}\right)}$ $\quad\omega_{c2}=\dfrac{R}{2L}+\sqrt{\left(\dfrac{R}{2L}\right)^2+\left(\dfrac{1}{LC}\right)}$
Relationship btw. ω_0 & cutoff freq. $\omega_0=\sqrt{\omega_{c1}\omega_{c2}}=\sqrt{\dfrac{1}{LC}}$
Bandwidth: $\beta=\omega_{c2}-\omega_{c1}=\dfrac{R}{L}$
Quality factor: $Q=\dfrac{\omega_0}{\beta}=\sqrt{\dfrac{L}{CR^2}}$
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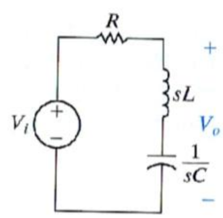
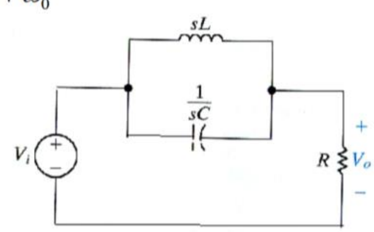
<tr>
<td>Summary</td>
<td>
Transfer function: $H(s)=\dfrac{s^2+\omega_0^2}{s^2+\beta s+\omega_0^2}$


$H(s)=\dfrac{s^2+(1/LC)}{s^2+(R/L)s+(1/LC)}$
$\omega_0=\sqrt{\dfrac{1}{LC}}$ $\quad\beta=\dfrac{R}{L}$


$H(s)=\dfrac{s^2+1/LC}{s^2+s/RC+1/LC}$
$\omega_0=\sqrt{\dfrac{1}{LC}}$ $\quad\beta=\dfrac{1}{RC}$
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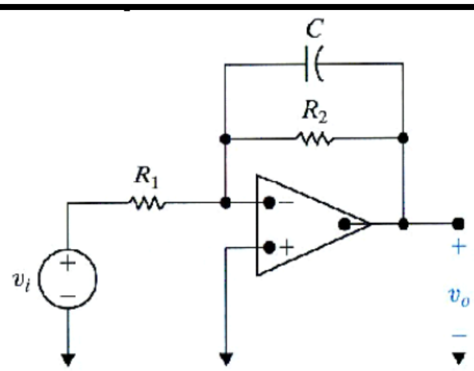
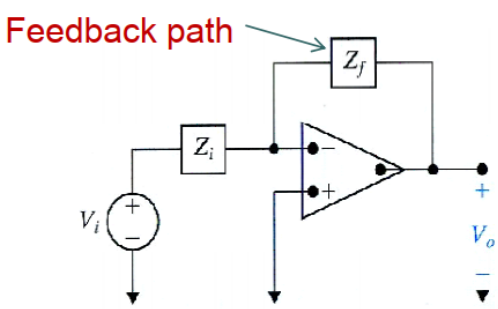
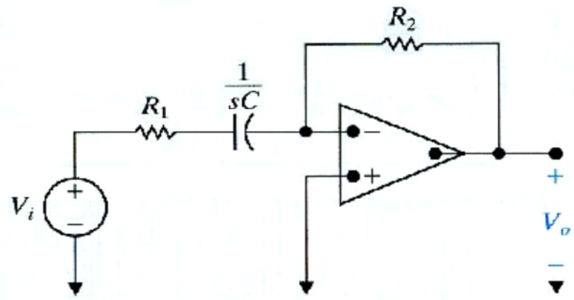
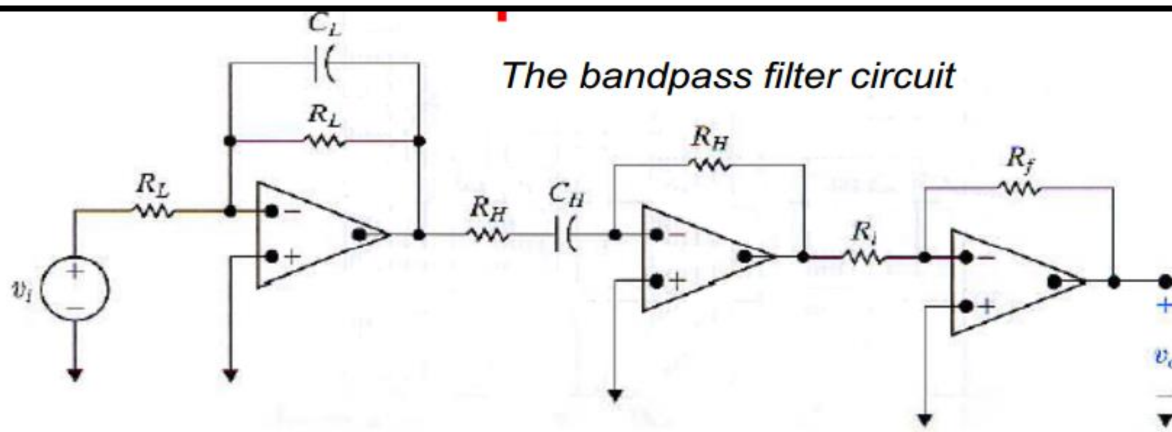
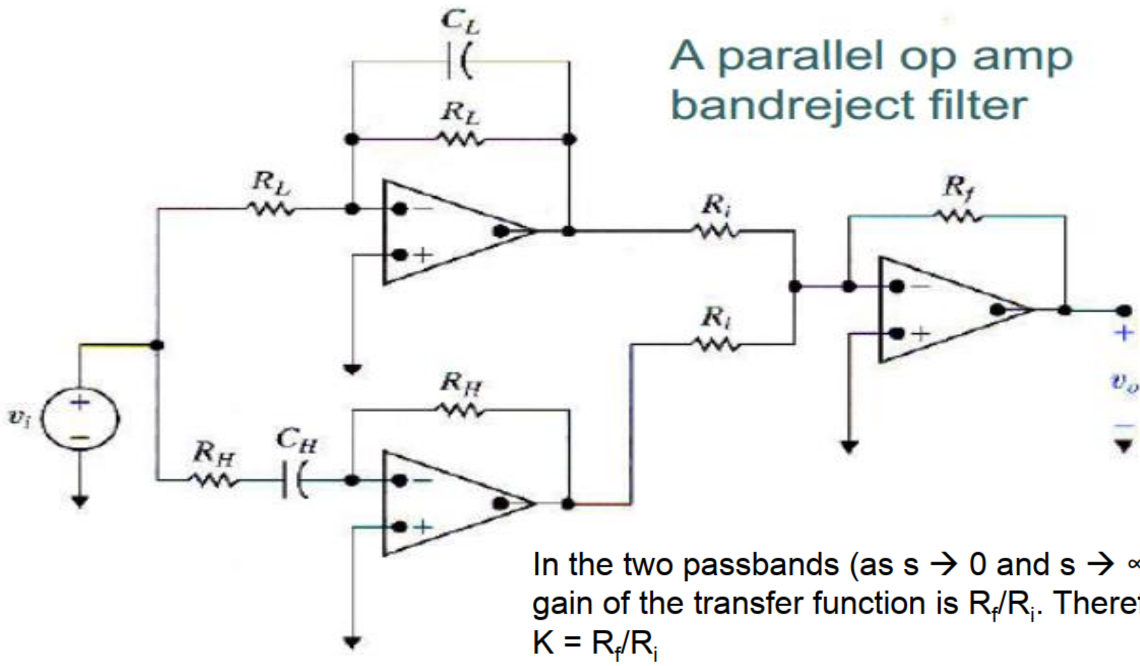
ACTIVE FILTERS

Active Filters	Description
First-order low-pass filter	 *First order Low-passfilter*  *A general op amp circuit.* Transfer function of the low-pass filter: $$H(s) = -\frac{Z_f}{Z_i} = -\frac{R_2 \parallel (1/sC)}{R_1}$$ $$\boxed{H(s) = -K\frac{\omega_c}{s+\omega_c}}$$ $$K = \frac{R_2}{R_1} \qquad \omega_c = \frac{1}{R_2 C}$$ The transfer function has the same form as for passive low-pass filter <u>except the gain K in the pass-pand</u>. $$\text{Number of decibels} = 10\log_{10}\frac{p_{out}}{p_{in}}$$ $$\text{Number of decibels} = 20\log_{10}\frac{v_{out}}{v_{in}} = 20\log_{10}\frac{i_{out}}{i_{in}}$$

First-order high-pass filter	 *It also has the same form as passive high-pass filter, except for the gain.* Transfer function of the low-pass filter: $H(s) = -\frac{Z_f}{Z_i} = -\frac{R_2}{R_1 + (1/sC)}$ $\boxed{H(s) = -K\frac{s}{s+\omega_c}}$ $K = \frac{R_2}{R_1}$ $\qquad \omega_c = \frac{1}{R_1 C}$
Scaling	Scaling factor: $\boxed{k_m = \frac{R'}{R}; \qquad k_f = \frac{\omega}{\omega_0}}$ Thông thường thì $R = 1$ và $\omega_0 = 1$, Khi đó: $C' = \frac{C}{k_f k_m}; \qquad L' = \frac{L}{k_f k_m}$
First-order band-pass filter	*The bandpass filter circuit*  $\boxed{H(s) = \frac{-K\omega_{c2}s}{(s+\omega_{c1})(s+\omega_{c2})}}$ $K = \frac{R_f}{R_i} \qquad \omega_{c2} = \frac{1}{R_L C_L} \qquad \omega_{c1} = \frac{1}{R_H C_H}$

First-order band-reject filter



In the two passbands (as s → 0 and s → ∞), the gain of the transfer function is R_f/R_i. Therefore $K = R_f/R_i$

$$H(s)=\frac{R_f}{R_i}\left(\frac{s^2+2\omega_{c1}s+\omega_{c1}\omega_{c2}}{(s+\omega_{c1})(s+\omega_{c2})}\right)$$

the cutoff frequencies are

$$\omega_{c1}=\frac{1}{R_L C_L} \qquad \omega_{c2}=\frac{1}{R_H C_H}$$

Butterworth Filters

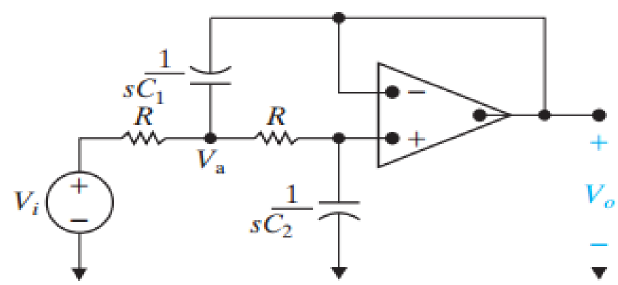


Figure 15.21 ▲ A circuit that provides the second-order transfer function for the Butterworth filter cascade.

n	*n*th-Order Butterworth Polynomial
1	$(s + 1)$
2	$(s^2 + \sqrt{2}s + 1)$
3	$(s + 1)(s^2 + s + 1)$
4	$(s^2 + 0.765s + 1)(s^2 + 1.848s + 1)$
5	$(s + 1)(s^2 + 0.618s + 1)(s^2 + 1.618s + 1)$
6	$(s^2 + 0.518s + 1)(s^2 + \sqrt{2} + 1)(s^2 + 1.932s + 1)$
7	$(s + 1)(s^2 + 0.445s + 1)(s^2 + 1.247s + 1)(s^2 + 1.802s + 1)$
8	$(s^2 + 0.390s + 1)(s^2 + 1.111s + 1)(s^2 + 1.6663s + 1)(s^2 + 1.962s + 1)$