INTERNATIONAL UNIVERSITY-VNUHCM

MIDTERM EXAMINATION

Semester 2, Academic Year 2021-2022

Duration: 90 minutes

SUBJECT: Calculus 2 (Groups 5 & 6)	
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INSTRUCTIONS:

- You can bring 2 A4 sheets of notes and a calculator.
- Each question has two parts, each part carries 10 points.

1. (a) Find

$$\lim_{n\to\infty} \frac{n-\sqrt{n^2-2}}{n^2-\sqrt{n^4+n}}.$$

(b) Let the sequence a_n be defined by $a_1 = 2$, $a_{n+1} = \dfrac{3}{4-a_n}$, $n \geq 1$. Show that the sequence $\{a_n\}$ converges and find its limit.

2. (a) Evaluate the following series

$$\sum_{n=3}^{\infty} \frac{1}{(n-1)(n+2)}.$$

(b) Determine if the following series converges

$$\sum_{n=1}^{\infty} \sqrt{n}\tan(1/n^2).$$

3. (a) Find the interval of convergence of

$$\sum_{n=2}^{\infty} \frac{(2n)!(x+2)^n}{(n!)^2}.$$

(b) Find the Maclaurin series of

$$f(x) = \sin(2x + \frac{\pi}{2})$$

4. Let $P(1,2,0)$, $Q(2,-3,2)$, $R(3,0,-1)$, $S(4,-1,2)$.

(a) Do these 4 points lie on a same plane?

(b) Find the distance from the midpoint of QS to the line PR.

5. (a) Suppose that the velocity of a particle is $\mathbf{v}(t) = \langle 2t^2+1, (t+1)e^{t-1}, -\sin(\pi t)\rangle$, and it position at time $t=1$ is $\langle 2,1,0\rangle$. Find its position function.

(b) Find the length of the curve

$$\mathbf{r}(t) = \left\langle t, t^2, \tfrac{2}{3}t^3 \right\rangle, \quad 0 \le t \le 3.$$

*** END OF QUESTIONS***