

Homework week 5

Chapter 2: Determinants

Exercise 1

a)

$$\begin{bmatrix} 1 & 3 & 0 & 2 \\ -2 & -5 & 7 & 4 \\ 3 & 5 & 2 & 1 \\ 1 & -1 & 2 & -3 \end{bmatrix}$$

$$R_2 \leftarrow 2R_1 + R_2$$

$$R_3 \leftarrow -3 \cdot R_1 + R_3$$

$$R_4 \leftarrow R_1 - R_4$$

$$\begin{array}{cc|cc} 2 & 3 & 1 & 0 \\ 4 & 5 & 0 & 1 \end{array}$$

$$\begin{bmatrix} 1 & 3 & 0 & 2 \\ 0 & 1 & 7 & 8 \\ 0 & -4 & 2 & -5 \\ 0 & 4 & -2 & 5 \end{bmatrix}$$

$$R_3 \leftarrow 4R_2 + R_3$$

$$R_4 \leftarrow R_3 + R_4$$

$$\begin{bmatrix} 1 & 3 & 0 & 2 \\ 0 & 1 & 7 & 8 \\ 0 & 0 & 30 & 27 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Sum
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$$\Rightarrow \det = 1 \times 1 \times 30 \times 0 = 0$$

b)

$$\begin{bmatrix} 1 & 3 & -1 & 0 & -2 \\ 0 & 2 & -4 & -2 & -6 \\ -2 & -6 & 2 & 3 & 10 \\ 1 & 5 & -6 & 2 & -3 \\ 0 & 2 & -4 & 5 & 9 \end{bmatrix}$$

$$R_3 \leftarrow 2R_1 + R_3$$

$$R_4 \leftarrow R_1 - R_4$$

$$\begin{bmatrix} 1 & 3 & -1 & 0 & -2 \\ 0 & 2 & -4 & -2 & -6 \\ 0 & 0 & 0 & 3 & 6 \\ 0 & -2 & 5 & -2 & 1 \\ 0 & 2 & -4 & 5 & 9 \end{bmatrix}$$

$$R_4 \leftarrow R_2 + R_4$$

$$R_5 \leftarrow R_2 - R_5$$

$$\begin{bmatrix} 1 & 3 & -1 & 0 & -2 \\ 0 & 2 & -4 & -2 & -6 \\ 0 & 0 & 0 & 3 & 6 \\ 0 & 0 & 1 & -4 & -5 \\ 0 & 0 & 0 & -7 & -15 \end{bmatrix}$$

$$R_5 \leftarrow R_3 \times \frac{7}{3} + R_5$$

$$\begin{bmatrix} 1 & 3 & -1 & 0 & -2 \\ 0 & 2 & -4 & -2 & -6 \\ 0 & 0 & 0 & 3 & 6 \\ 0 & 0 & 1 & -4 & -5 \\ 0 & 0 & 0 & 0 & -1 \end{bmatrix}$$

$$R_3 \leftrightarrow R_4$$

$$\begin{bmatrix} 1 & 3 & -1 & 0 & -2 \\ 0 & 2 & -4 & -2 & -6 \\ 0 & 0 & 1 & -4 & -5 \\ 0 & 0 & 0 & 3 & 6 \\ 0 & 0 & 0 & 0 & -1 \end{bmatrix}$$

Answer
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$$\text{Det} = 1 \times 2 \times 1 \times 3 \times -1 \times (-1) = 6$$

$$c) \begin{bmatrix} 1 & 3 & 2 & -4 \\ 0 & 1 & 2 & -5 \\ 2 & 7 & 6 & -3 \\ -3 & -10 & -7 & 2 \end{bmatrix}$$

$$R_3 \leftarrow -2R_1 + R_3$$

$$R_4 \leftarrow 3R_1 + R_4$$

$$\begin{bmatrix} 1 & 3 & 2 & -4 \\ 0 & 1 & 2 & -5 \\ 0 & 1 & 2 & 5 \\ 0 & -1 & -1 & -10 \end{bmatrix}$$

$$R_3 \leftarrow R_2 - R_3$$

$$R_4 \leftarrow R_2 + R_4$$

$$\begin{bmatrix} 1 & 3 & 2 & -4 \\ 0 & 1 & 2 & -5 \\ 0 & 0 & 0 & -10 \\ 0 & 0 & 1 & -15 \end{bmatrix}$$

$$R_3 \leftrightarrow R_4$$

$$\begin{bmatrix} 1 & 3 & 2 & -4 \\ 0 & 1 & 2 & -5 \\ 0 & 0 & 1 & -15 \\ 0 & 0 & 0 & +10 \end{bmatrix}$$

line
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$$\det = [1 \times 1 \times 1 \times (+10)] \times (-1) \\ = -10$$

Exercise 2:

$$\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = 7$$

$$\begin{vmatrix} a & b & c & a & b \\ d & e & f & d & e \\ g & h & i & g & h \end{vmatrix}$$

$$\Leftrightarrow aei + bfg + cdh - ceg - afh - bdi = 7$$

$$\begin{vmatrix} a & b & c \\ d & e & f \\ 3g & 3h & 3i \end{vmatrix}$$

$$\begin{vmatrix} a & b & c & a & b \\ d & e & f & d & e \\ 3g & 3h & 3i & 3g & 3h \end{vmatrix}$$

$$\Rightarrow 3aei + 3bfg + 3cdh - 3ceg - 3afh - 3bdi$$

$$= 3(aei + bfg + cdh - ceg - afh - bdi)$$

$$= 3 \cdot 7 = 21$$

Excercise 3:

$$B = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 2 \\ 1 & 2 & 1 \end{bmatrix}$$

$$\begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix}$$

$$\begin{aligned} \Rightarrow \det(B) &= 1 \times 1 \times 1 + 0 \times 2 \times 1 + 1 \times 1 \times 2 \\ &\quad - 1 \times 1 \times 1 - 2 \times 2 \times 1 - 1 \times 1 \times 0 \\ &= 1 + 0 + 2 - 1 - 4 - 0 \\ &= -2 \end{aligned}$$

$$1 \times 4 - 3 \times 2$$

$$\Rightarrow \det(B^4) = [\det(B)]^4 = (-2)^4 = 16$$

2. We know that

$$\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = 7.$$

Find the determinant of the following matrix

$$\begin{vmatrix} a & b & c \\ d & e & f \\ 3g & 3h & 3i \end{vmatrix}$$

3. Compute $\det(B^4)$ where

$$B = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 2 \\ 1 & 2 & 1 \end{bmatrix}.$$

$$\det(B^4) = [\det(B)]^4$$

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