Transpose Matrix

$A = [a_{ij}]$ then $A^T = [a_{ji}]$

$$A = \begin{bmatrix} 4 & -3 \\ 1 & 6 \\ -5 & 2 \end{bmatrix}$$

3 x 2

$$A^T_{2\times 3} = \begin{bmatrix} 4 & 1 & -5 \\ -3 & 6 & 2 \end{bmatrix}$$

$$B = \begin{bmatrix} 1 & 0 & -2 \\ 3 & 4 & 1 \\ -5 & -1 & 7 \end{bmatrix}$$

$$B^T = \begin{bmatrix} 1 & 3 & -5 \\ 0 & 4 & -1 \\ -2 & 1 & 7 \end{bmatrix}$$

Properties:

$(A+B)^T = A^T + B^T$

$(A^T)^T = A$

$(kA)^T = kA^T$

$(AB)^T = B^T A^T$

$(A^T)^{-1} = (A^{-1})^T$

$AA^T = (AA^T)^T$

$$\begin{bmatrix} 6 & 5 & 0 \\ 2 & 4 & -1 \end{bmatrix} \begin{bmatrix} 6 & 2 \\ 5 & 4 \\ 0 & -1 \end{bmatrix}$$

Given A → Find AA^T

$$A = \begin{matrix} a_1 \\ a_2 \\ a_3 \end{matrix} \begin{bmatrix} 1 & 6 \\ -2 & 2 \\ 0 & 3 \end{bmatrix} \quad R_1 \times C_1$$

2 x 3 · 3 x 2

$$\begin{bmatrix} 1 & -2 & 0 \\ 6 & 2 & 3 \end{bmatrix} \begin{bmatrix} 1 & 6 \\ -2 & 2 \\ 0 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 5 & 2 \\ 2 & 49 \end{bmatrix}$$

The identity matrix

If A is an $m \times n$ matrix, then

$$I_m A = A \quad \text{and} \quad A I_n = A$$

If $n \times n$

$$A I_n = I_n A = A$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 4 & 7 \\ -5 & -2 \end{bmatrix} = \begin{bmatrix} 4 & 7 \\ -5 & -2 \end{bmatrix} P_1$$

2×2 possible 2×2