CHAPTER 3 THE KINETIC THEORY OF GASES

Homework:

Problems 13, 14, 20, 24, 32, 40 in Chapter 19 – Textbook

Problem 13. A sample of an ideal gas is taken through the cyclic process abca shown in the figure below; at point a, T=200 K. (a) How many moles of gas are in the sample? What are (b) the temperature of the gas at point b, (c) the temperature of the gas at point c, and (d) the net energy added to the gas as heat during the cycle?

(a) Applying the equation of state:

$$pV = nRT \Rightarrow n = \frac{pV}{RT}$$

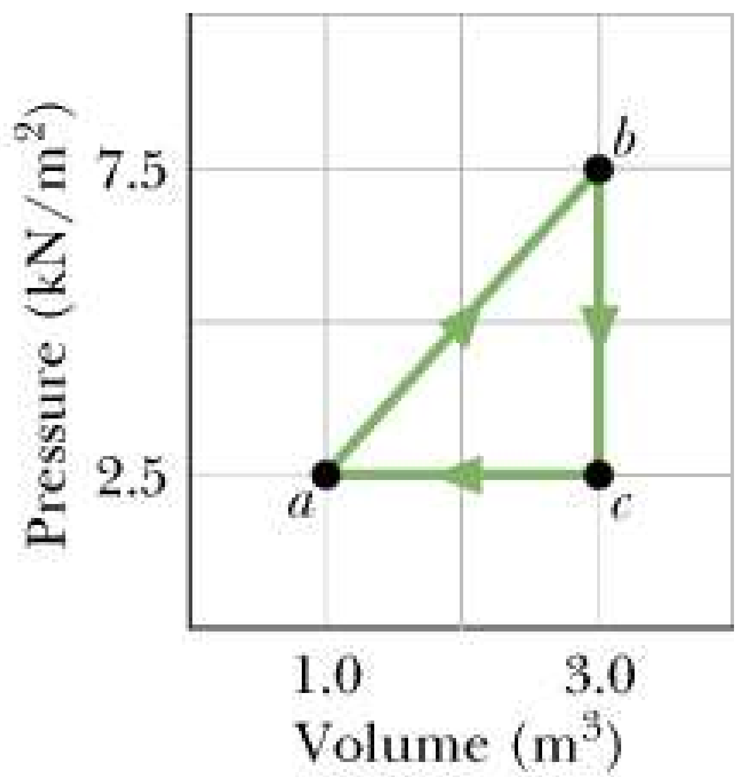
At point a, p=2.5 kN/m^2 or 2500 N/m^2; V=1 m^3; T = 200 K

$$n = \frac{2500 \times 1}{8.31 \times 200} = 1.5\,(\text{mol})$$

(b) $pV = nRT \Rightarrow p_b V_b = nRT_b$

At point b, p=7.5 kN/m^2 or 7500 N/m^2; V=3 m^3.

$$T_b = \frac{p_b V_b}{nR} = \frac{7500 \times 3}{1.5 \times 8.31} = 1800\,(\text{K})$$



$$pV = nRT \Rightarrow p_c V_c = nRT_c$$

(c) T_c=600 K;

(d) Applying the first law of thermodynamics:

$$\Delta E = Q - W$$

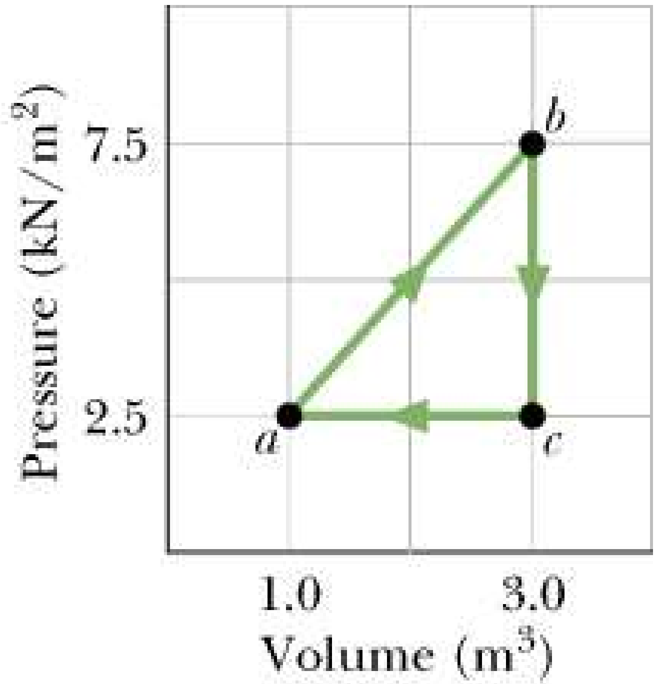
W: work done by the system.

For a closed cycle, ΔE=0:

$$Q = W$$

$$W = \frac{1}{2}(p_b - p_c)(V_b - V_a)$$

$$W = \frac{1}{2} \times 5000.0 \times 2 = 5 \times 10^3 \text{ (J)}$$



Problem 14. In the temperature range 310 K to 330 K, the pressure p of a certain non-ideal gas is related to volume V and temperature T by:

$$p = (24.9\ J/K)\frac{T}{V} - (0.00662\ J/K^2)\frac{T^2}{V}$$

How much work is done by the gas if its temperature is raised from 315 K to 330 K while the pressure is held constant?

Work done by the gas is computed by the following formula:

$$W = \int_{V_i}^{V_f} p dV = p(V_f - V_i)$$ Due to the constant pressure process

$$W = pV_f - pV_i = 24.9(T_f - T_i) - 0.00662(T_f^2 - T_i^2)$$

$$T_f = 330K; T_i = 315K \Rightarrow W \approx 310\,(\text{J})$$

Problem 20. Calculate the rms speed of helium atoms at 1000 K, the molar mass of helium atoms is 4.0026 g/mol.

$$v_{rms} = \sqrt{\frac{3RT}{M}} = \sqrt{\frac{3\times 8.31\times 1000}{4.0026\times 10^{-3}}} = 2.5\times 10^{3}\ \text{(m/s)}$$

Problem 24. At 273 K and 1.0 x 10^{-2} atm, the density of a gas is 1.24 x 10^{-5} g/cm^3. (a) Find v_{rms} for the gas molecules. (b) Find the molar mass of the gas and (c) identify the gas (hint: see Table 19-1).

(a) Root-mean-square speed:

$$v_{rms} = \sqrt{\frac{3RT}{M}} \quad (1)$$

$$\rho = \frac{M_{gas}}{V} = \frac{nM}{V} \Rightarrow M = \frac{\rho V}{n} \quad (2)$$

(1) and (2): $v_{rms} = \sqrt{\frac{3nRT}{\rho V}} = \sqrt{\frac{3p}{\rho}}$ $\quad pV = nRT \quad (3)$

$$\rho = 1.24 \times 10^{-5}\,\text{g/cm}^3 = 1.24 \times 10^{-2}\,\text{kg/m}^3$$

$$p = 1.0 \times 10^{-2}\,\text{atm} = 1.01 \times 10^3\,\text{Pa}$$

$$v_{rms} \approx 494\ \text{m/s}$$

(b) $M = \frac{\rho V}{n}$ (2)

Equation of state:

$$pV = nRT \quad (3)$$

$$\Rightarrow M = \frac{\rho V}{n} = \frac{\rho RT}{p}$$

$$\Rightarrow M \approx 0.028 \text{ kg/mol} = 28 \text{ g/mol}$$

(c) From Table 19.1,
the gas is nitrogen (N_2)

Table 19-1 Some RMS Speeds at Room Temperature (T = 300 K)[a]

Gas	Molar Mass (10^{-3} kg/mol)	v_{rms} (m/s)
Hydrogen (H_2)	2.02	1920
Helium (He)	4.0	1370
Water vapor (H_2O)	18.0	645
Nitrogen (N_2)	28.0	517
Oxygen (O_2)	32.0	483
Carbon dioxide (CO_2)	44.0	412
Sulfur dioxide (SO_2)	64.1	342