INTERNATIONAL UNIVERSITY-VNUHCM

FINAL EXAMINATION

Semester 2, Academic Year 2020-2021
Duration: 90 minutes (online)

SUBJECT: **Calculus 2**	
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- **Students have to follow the IU regulations for online exams.**
- **Each question carries 20 marks.**

Question 1. Let $f(x, y) = xe^{x+y}$

a) Find the tangent plane to the graph of $f(x, y)$ at the point $(1, -1, 1)$.

b) Find the directional derivative $D_{\mathbf{u}}f(1, -1)$, where $\mathbf{u} = (1/\sqrt{2})\mathbf{i} + (1/\sqrt{2})\mathbf{j}$.

Question 2. Use Lagrange multipliers method to find the maximum and minimum values of the function $f(x, y, z) = x + y - z$ subject to the constraint $x^2 + y^2 + z^2 = 1$.

Question 3. a) Estimate the volume of the solid that lies below the surface $z = xe^{y^2/10}$ and above the rectangle $R = [0, 6] \times [0, 4]$ by using a Riemann sum with $m = 3, n = 2$ and the sample point to be the upper right corner of each square.

b) Find the volume of the solid under the surface $z = 2x^2y$ and above the region D in the xy-plane enclosed by $x = 0$ and $x = \sqrt{1 - y^2}$.

Question 4. Let $\mathbf{F}(x, y) = (x + y^2)\mathbf{i} + (2xy)\mathbf{j}$.

a) Show that $\mathbf{F}(x, y)$ is a conservative vector field;

b) Find a potential function f such that $\mathbf{F} = \nabla f$;

c) Use the part b) to evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$ along the curve C, where C is the arc of the curve $y = x^2$ from $(0, 0)$ to $(1, 1)$

Question 5. a) Find $\operatorname{curl}\mathbf{F}$ and $\operatorname{div}\mathbf{F}$ if $\mathbf{F}(x, y, z) = e^{x+2y}\mathbf{i} + (x - y)\mathbf{j} + (y + 3z)\mathbf{k}$.

b) Evaluate the surface integral of the vector field $\iint_S \mathbf{F} \cdot d\mathbf{S}$, where $\mathbf{F}(x, y, z) = 2x\mathbf{i} + \mathbf{j} + z\mathbf{k}$, and S is the surface $z = (x + 1)e^y, 0 \le x \le 1, 0 \le y \le 1$ with upward orientation.

———END OF QUESTIONS———