Overview

One goal of Physics is to provide the basic science for practical devices designed by engineers

In this part, we study such a practical device called **a capacitor** in which electrical energy can be stored
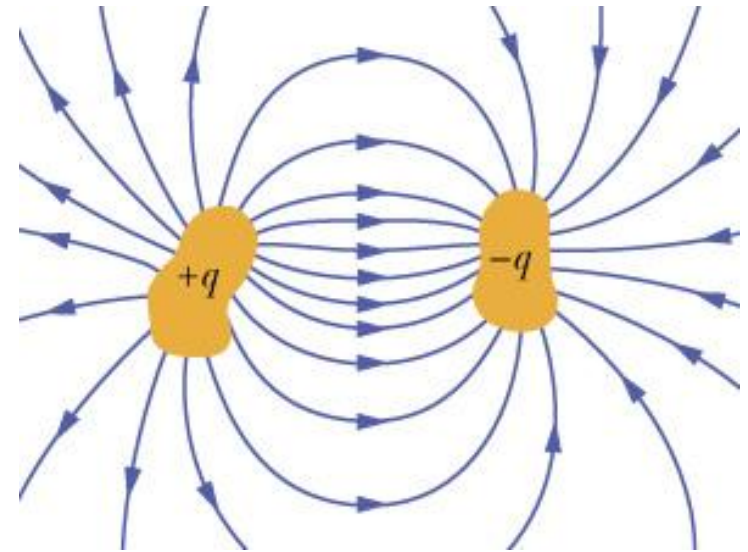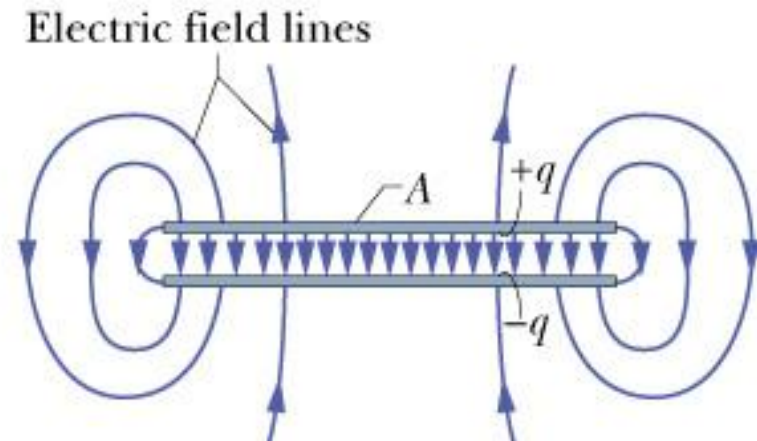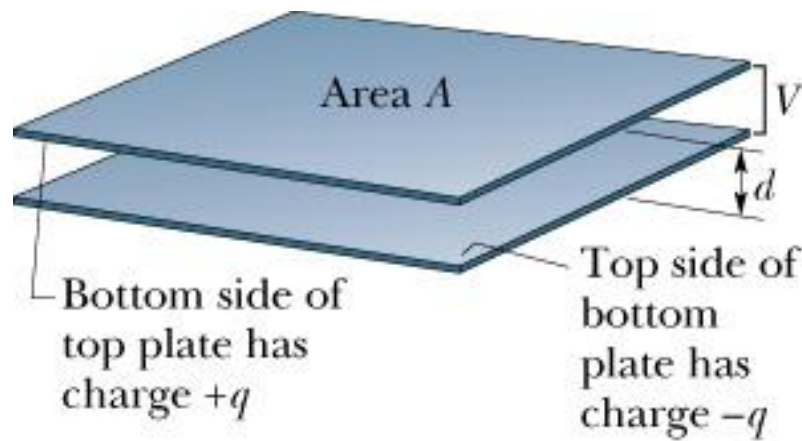
2.6. Capacitance. Capacitors in Parallel and in Series:

2.6.1. Capacitance:

Some concepts:

· A capacitor consists of two isolated conductors of any shape. These two conductors are called plates.

· A parallel-plate capacitor has two parallel conducting plates of area A separated by a distance d
(symbol: —||—)

· When a capacitor is charged, its plates have equal but opposite charges of +q and –q

Area A

V

d

Bottom side of top plate has charge $+q$

Top side of bottom plate has charge $-q$

Electric field lines

A

$+q$

$-q$

- All points on a plate have the same potential, **V** is the potential difference between the two plates
- *For electrical devices: symbol V (not ΔV) often represents a potential difference*
- When a capacitor is charged:

$$q = CV$$

- **C**: Capacitance of the capacitor
- The capacitance describes how much charge an arrangement of conductors can hold for a given voltage applied and its value depends on the geometry (dielectric, plate area, distance) of the plates
- Unit: **1 farad = 1 F = 1 Coulomb/Vol = 1 C/V**
- **1 μF = 10^{-6} F; 1 pF = 10^{-12} F**

- **Charging a capacitor:**
 - – Place the capacitor in an electric circuit with a battery which is a device that maintains a certain potential difference between its terminals

(a)

  - – In **schematic diagram** (b) of a **circuit**: a battery B; a capacitor C; a switch S; interconnecting wires
  - – <u>When S is closed</u>: the electric field drives electrons from capacitor plate h to the positive terminal of B, plate h loosing e- becomes positively charged; the field drives electrons from negative terminal of B to plate l, so it gains electrons and becomes negatively charged

(b)

  - – V between the plates is equal to that of battery B

<u>**Checkpoint 1:**</u> Does the capacitance C of a capacitor increase, decrease, or remain the same (a) when the charge q on it is doubled and (b) when the potential difference V across it is tripled? (a) the same; (b) the same
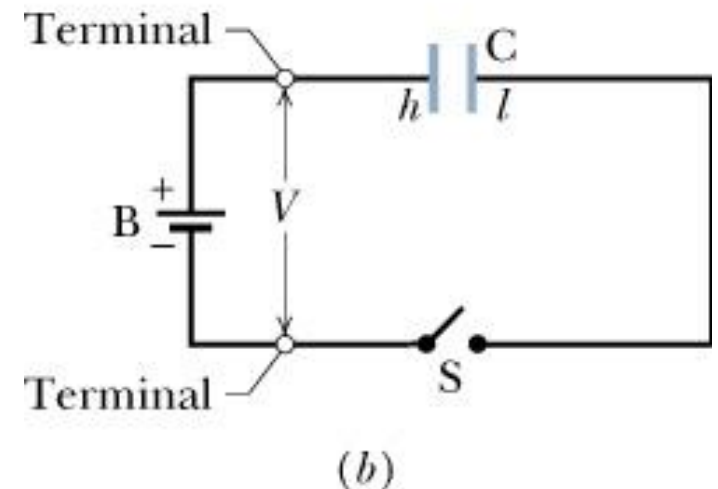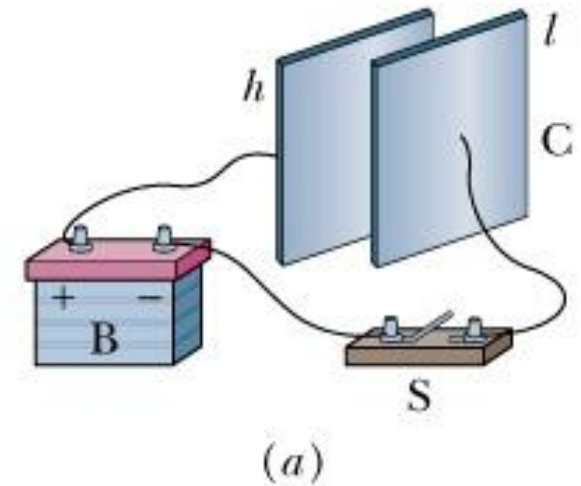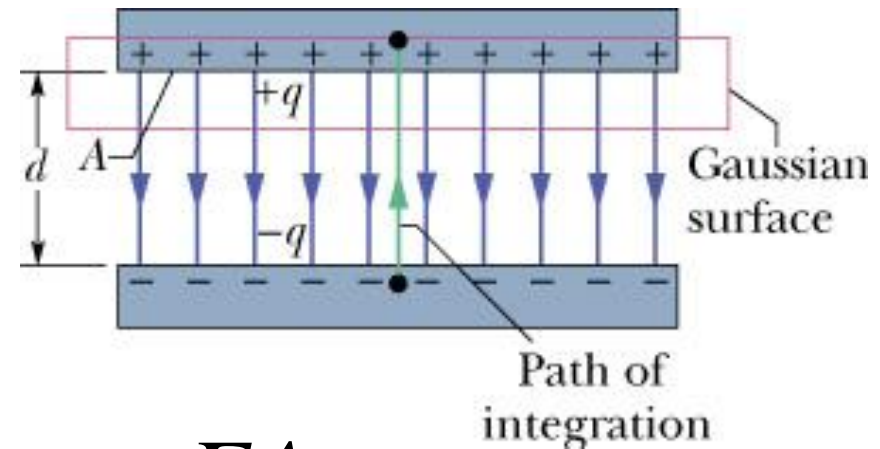
2.6.2. Calculating the Capacitance:

Goal: Calculate the capacitance of a capacitor once we know its geometry

$$q = CV$$

step 1: Calculating the electric field

Use Gauss' law for a Gaussian surface as indicated:

$$\varepsilon_0 \oint \vec{E} d\vec{A} = q \Rightarrow q = \varepsilon_0 EA$$

step 2: Calculating the potential difference

$$V = V_f - V_i = -\int_i^f \vec{E} d\vec{s}$$

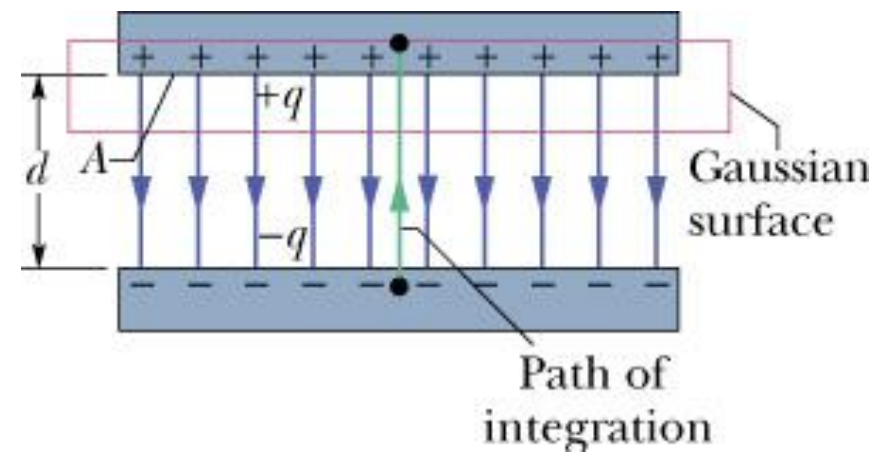If the integration path from the negative to the positive plates

$$V = \int_-^+ E ds$$

(a) A Parallel-Plate Capacitor:

$$q = \varepsilon_0 EA = CV$$

$$V = \int_{-}^{+} Eds = E \int_{0}^{d} ds = Ed$$

$$\Rightarrow \quad \boxed{C = \frac{\varepsilon_0 A}{d}}$$

Using the formula above, we can use:

$$\boxed{\varepsilon_0 = 8.85 \times 10^{-12} \text{ F/m} = 8.85 \text{ pF/m}}$$

instead of

$$\varepsilon_0 = 8.85 \times 10^{-12} \text{ } C^2/(N.m^2)$$

(b) A Cylindrical Capacitor:

- A cylindrical capacitor of length L includes two coaxial cylinders of radii a and b
- We choose a Gaussian surface as indicated:

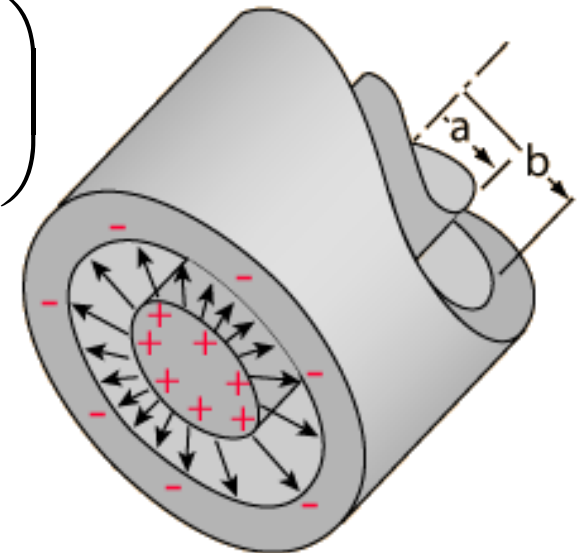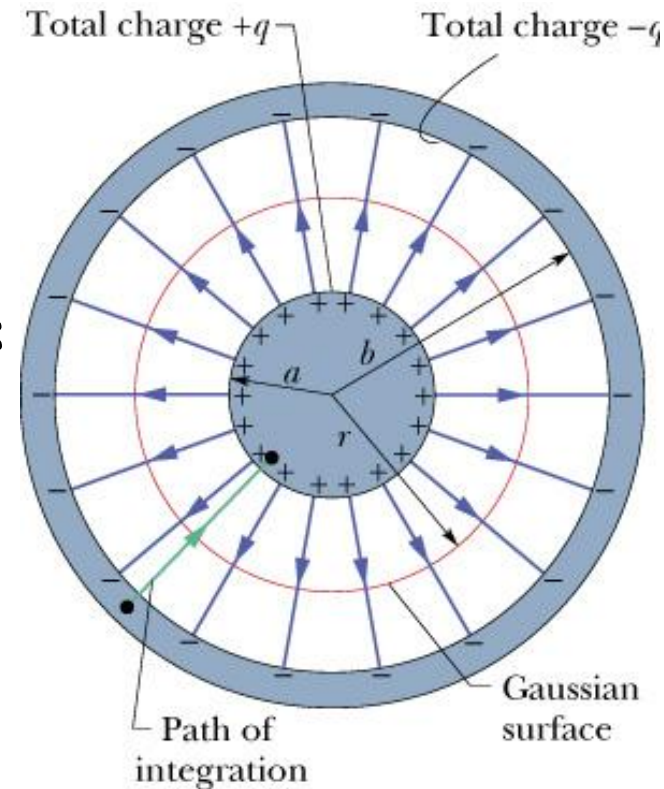$$q = \varepsilon_0 EA = \varepsilon_0 E(2\pi rL)$$

$$E = \frac{q}{2\pi\varepsilon_0 rL}$$

Total charge +q Total charge −q

Gaussian surface

Path of integration

$$ds = -dr : \text{integrated radially inward}$$

$$V = \int_{-}^{+} E\,ds = -\frac{q}{2\pi\varepsilon_0 L}\int_{b}^{a}\frac{dr}{r} = \frac{q}{2\pi\varepsilon_0 L}\ln\left(\frac{b}{a}\right)$$

$$C = 2\pi\varepsilon_0 \frac{L}{\ln(b/a)}$$
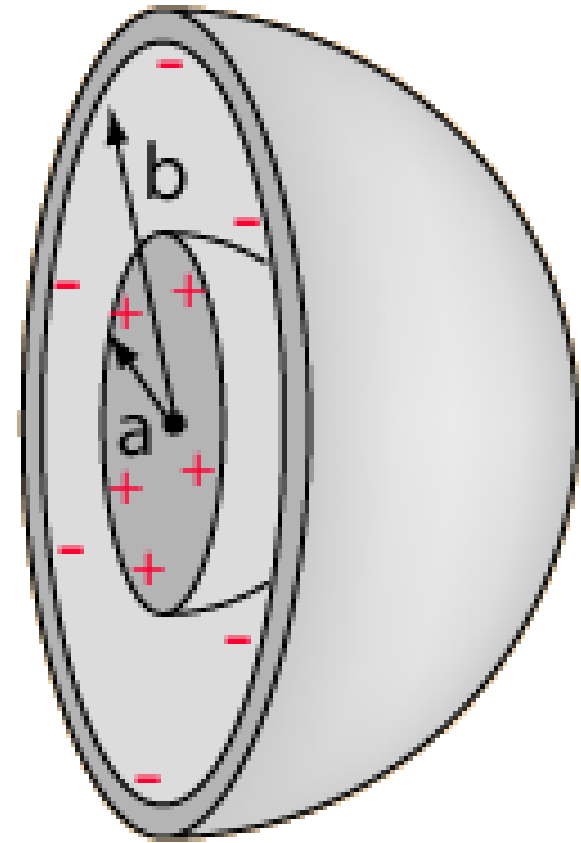
(c) A Spherical Capacitor:

- A capacitor consists of two concentric spherical shells of radii a and b

$$q = \varepsilon_0 EA = \varepsilon_0 E(4\pi r^2)$$

$$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}$$

$$V = \int_{-}^{+} Eds = -\frac{q}{4\pi\varepsilon_0}\int_{b}^{a}\frac{dr}{r^2} = \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{a} - \frac{1}{b}\right)$$

$$\boxed{C = 4\pi\varepsilon_0\frac{ab}{b-a}}$$

(d) An Isolated Sphere:

• A capacitor consists only a single isolated spherical conductor of radius R, assuming that the missing plate is a conducting sphere of infinite radius

$$C = 4\pi\varepsilon_0 \frac{ab}{b-a} = 4\pi\varepsilon_0 \frac{a}{1-\dfrac{a}{b}}$$

$$\boxed{C = 4\pi\varepsilon_0 R}$$

Checkpoint 2: For capacitors charged by the same battery, does the charge stored by the capacitor increase, decrease, or remain the same in each of the following situations? (a) The plate separation of a parallel-plate capacitor is increased. (b) The radius of the inner cylinder of a cylindrical capacitor is increased. (c) The radius of the outer spherical shell of a spherical capacitor is increased.

(a) decreases; (b) increases; (c) decreases

Capacitance Summary

- Unit: **1 farad = 1 F = 1 Coulomb/Vol = 1 C/V**
- $1 \ \mu F = 10^{-6} \ F$; $1 \ pF = 10^{-12} \ F$

Parallel-Plate Capacitor: $C = \dfrac{\varepsilon_0 A}{d}$

Cylindrical Capacitor: $C = 2\pi\varepsilon_0 \dfrac{L}{\ln(b/a)}$

Spherical Capacitor: $C = 4\pi\varepsilon_0 \dfrac{ab}{b-a}$

Isolated Sphere: $C = 4\pi\varepsilon_0 R$

$$\varepsilon_0 = 8.85 \times 10^{-12} \ \text{F/m} = 8.85 \ \text{pF/m}$$

2.6.3. Capacitors in Parallel and in Series:

If there is a combination of capacitors in a circuit, we can replace that with an equivalent capacitor, which has the same capacitance as the actual combination of capacitors
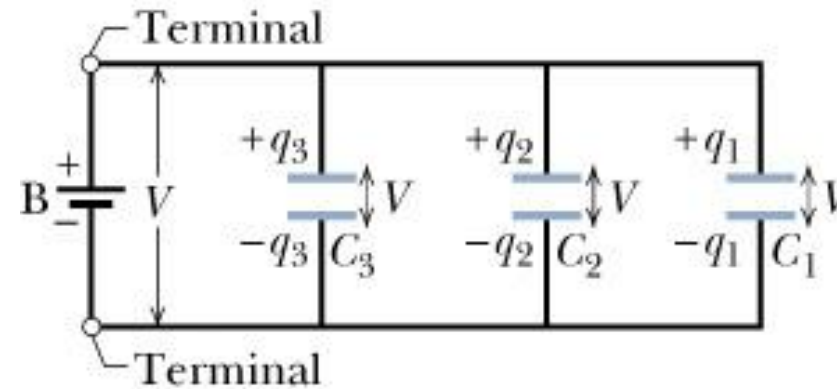
(a) Capacitors in Parallel:

· A potential difference V is applied across each capacitor. The total charge q stored on the capacitors is the sum of the charges stored on all capacitors:
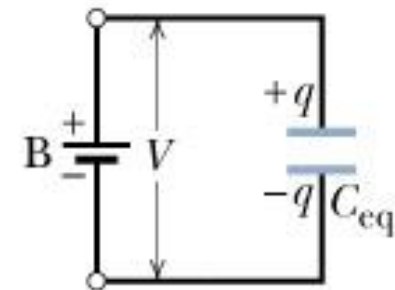
$$q = q_1 + q_2 + q_3 = (C_1 + C_2 + C_3)V$$

$$C_{eq} = \frac{q}{V} = C_1 + C_2 + C_3$$

If we have n capacitors in parallel:

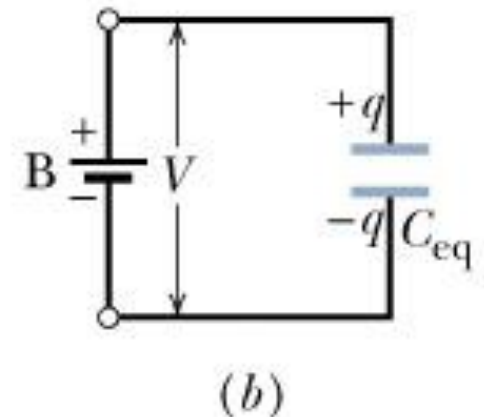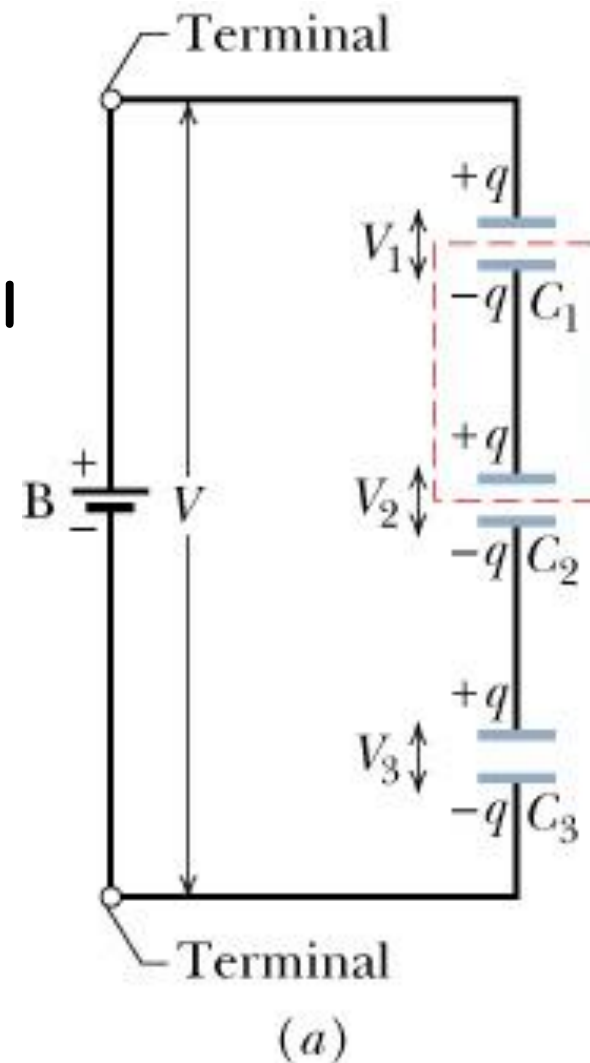$$C_{eq} = \sum_{i=1}^{n} C_i$$

(b) Capacitors in Series:

- All capacitors have identical charges q
- The sum of the potential differences across all the capacitors is equal to the applied potential difference V

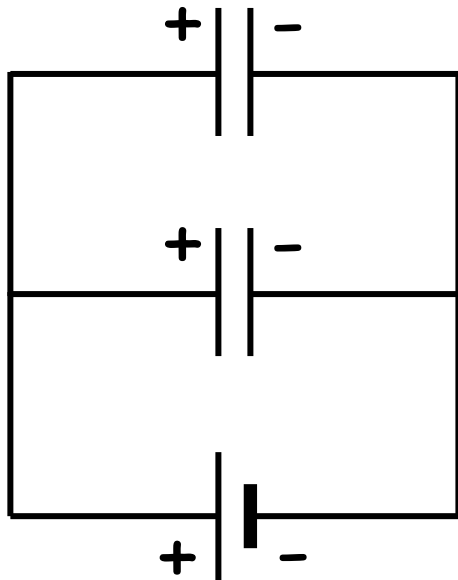$$V = V_1 + V_2 + V_3 = q\left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right)$$

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$
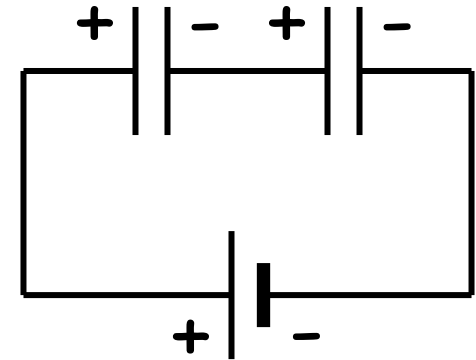
If we have n capacitors in series:

$$\frac{1}{C_{eq}} = \sum_{i=1}^{n} \frac{1}{C_i}$$

(a)

(b)

Checkpoint: A battery of potential V stores charge q on a combination of two identical capacitors. What are the potential difference across and the charge on either capacitor if the capacitors are (a) in parallel and (b) in series?

V, q/2

V/2, q

2.7. Energy Stored in a Charged Capacitor:

• When we charge a capacitor, work must be done by an external agent and the work is stored in the form of electric potential energy U in the electric field between the plates

• At any instant, the capacitor has q' and V' = q'/C, if an extra charge dq' is then transferred:

$$dW = V'dq' = \frac{q'}{C}dq'$$

$$W = \int dW = \frac{1}{C}\int_0^q q'dq' = \frac{q^2}{2C}$$

The potential energy of the capacitor:

$$U = \frac{q^2}{2C}$$

$$U = \frac{1}{2}CV^2$$
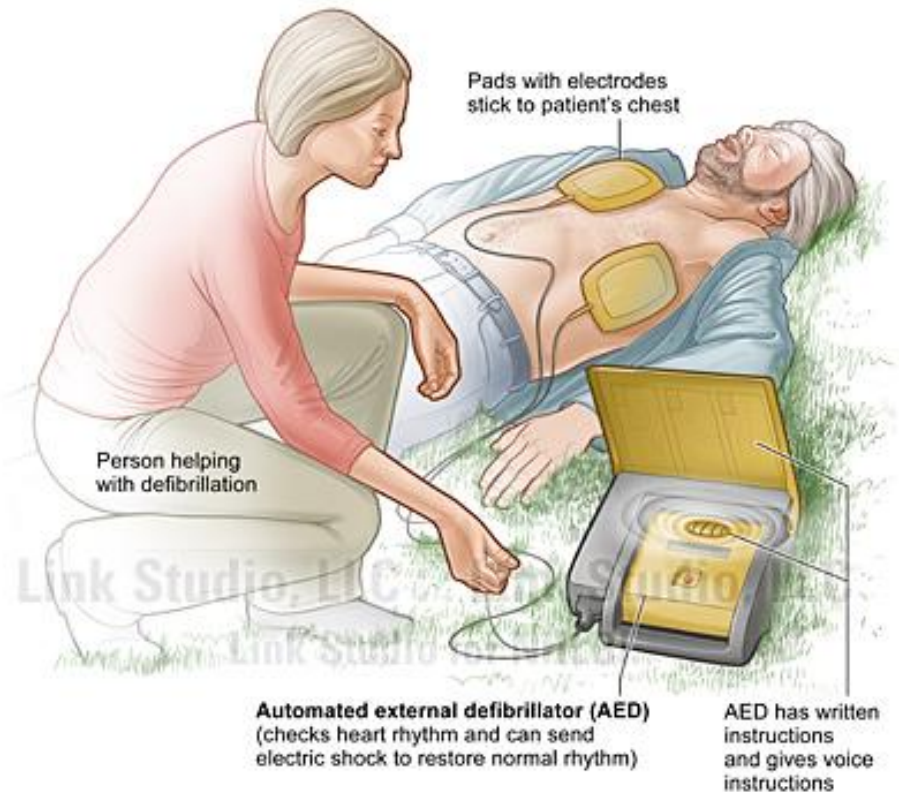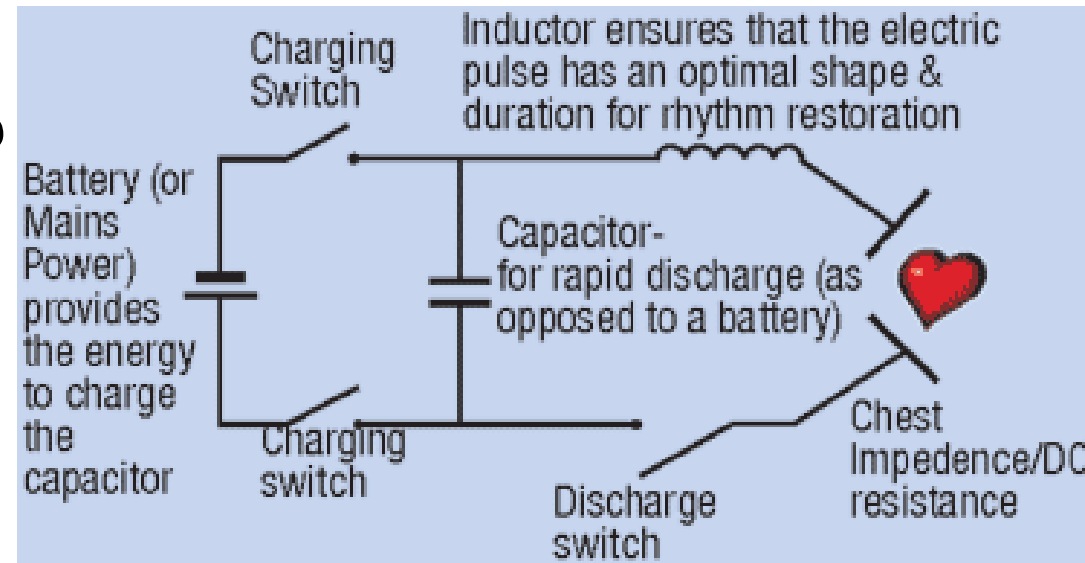
The medical defibrillator:

A medical device is used to stop the fibrillation of heart attack victims based on the ability of storing potential energy of a capacitor

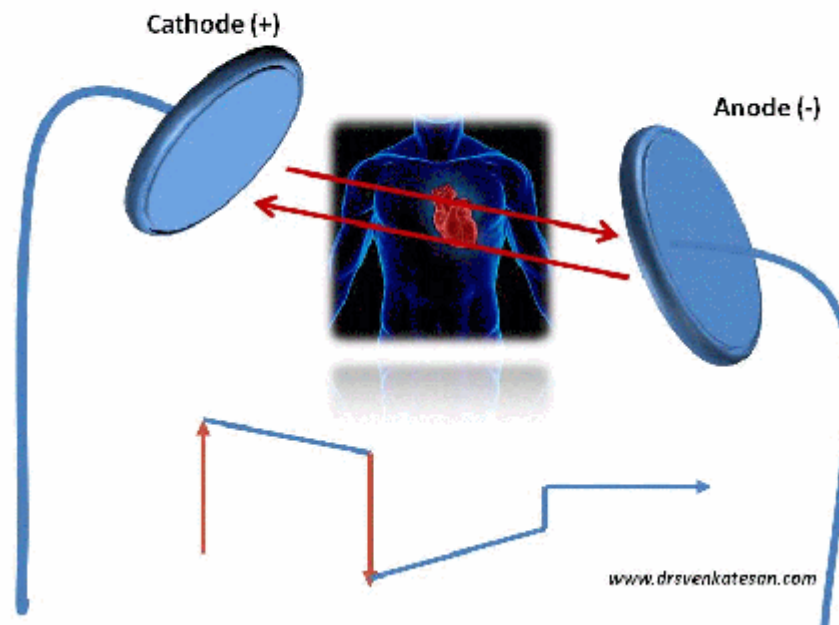*Example:* a 70 µF capacitor in a defibrillator is charged to 5000 V giving an amount of energy U:

$$U = \frac{1}{2}CV^2 = 875 \, (\text{J})$$

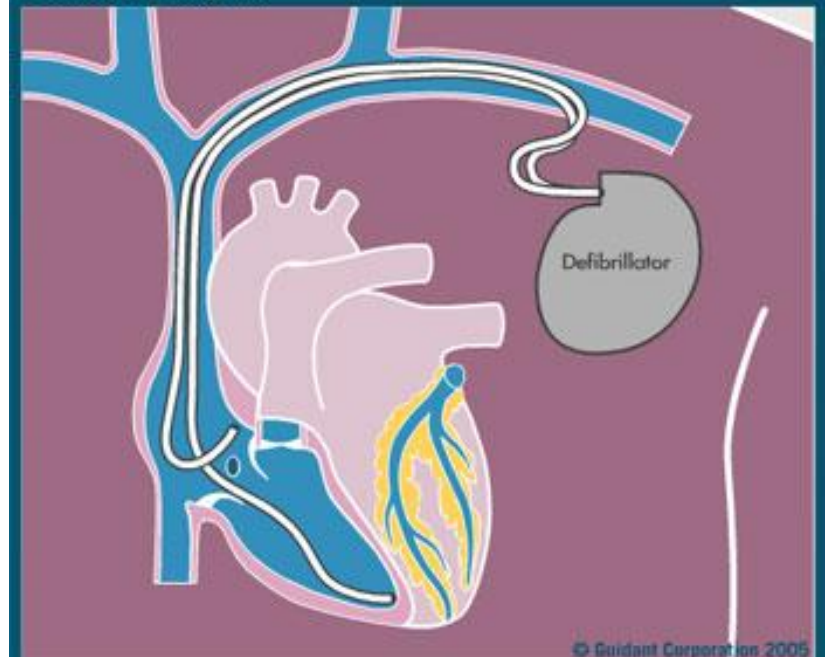About 200 J is sent through the victim in a pulse of 2 ms, so the power is:

$$P = \frac{U}{t} = 100 \, (\text{kW})$$

Bi-Phasic DC shock

Cathode (+)

Anode (-)

www.drsvenkatesan.com

Defibrillator Implant

Defibrillator

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Energy density:

Consider a parallel-plate capacitor, the uniform energy density u, the potential energy per unit volume, between the plates:

$$u = \frac{U}{Ad} = \frac{CV^2}{2Ad}$$

$$C = \frac{\varepsilon_0 A}{d} \Rightarrow u = \frac{1}{2}\varepsilon_0 \left(\frac{V}{d}\right)^2$$

The electric field is uniform between the plates (neglecting fringing):

$$E = -\frac{\Delta V}{\Delta s} = \frac{V}{d}$$

$$\Rightarrow \boxed{u = \frac{1}{2}\varepsilon_0 E^2}$$

2.8. Capacitor with a Dielectric:

Some concepts:

Dielectric: an insulating material such as mineral oil or plastic

. If we fill the space between the plates of a capacitor with a dielectric, **its capacitance increases by a factor κ** (kappa), which is called the dielectric constant (for a vacuum, $\kappa = 1$)

. A dielectric has a maximum potential difference V_{max} that can be applied between the plates, called **breakdown potential.**

. Every dielectric has a characteristic **dielectric strength** E_{max} corresponding to V_{max}

$$C' = \kappa C_{air}$$

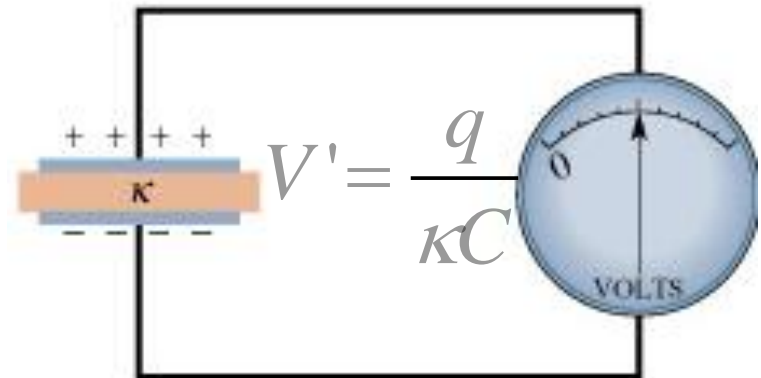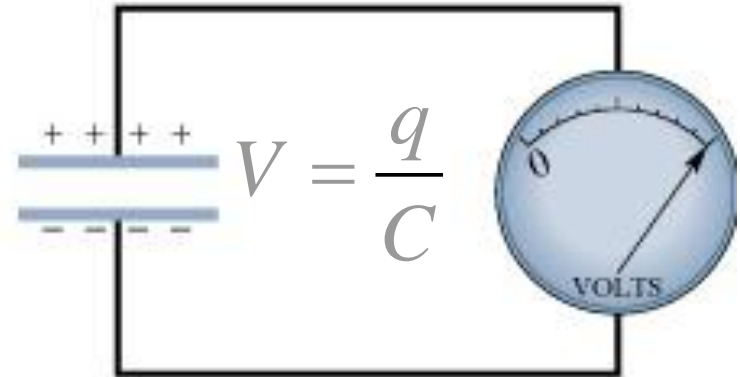. C': capacitance with a dielectric

. C_{air}: air between the plates

☐ **If V is maintained (e.g., by a battery), more charge flows to the capacitor with a dielectric**

$$q = CV$$

$$q' = \kappa CV$$
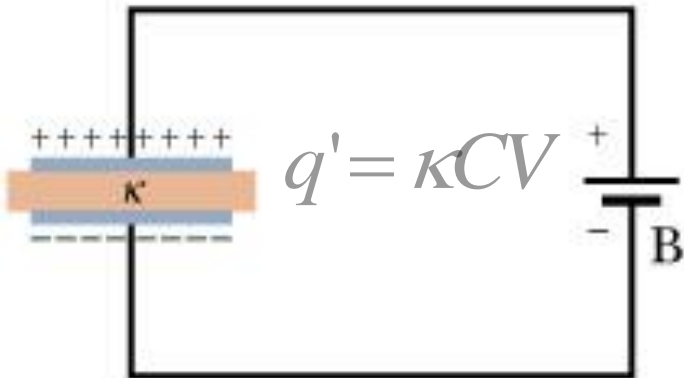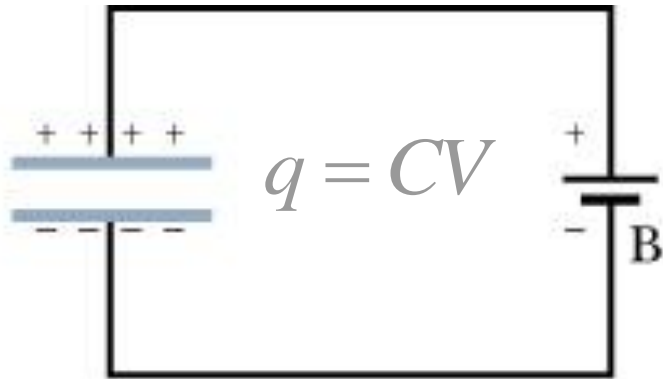
$$V = \text{a constant}$$

☐ **If q is maintained, voltage drops in the capacitor with a dielectric**

$$V = \frac{q}{C}$$

VOLTS

$$V' = \frac{q}{\kappa C}$$

VOLTS

$$q = \text{a constant}$$

In a region completely filled by a dielectric material, all electrostatic equations containing ε_0 are to be corrected by replacing ε_0 with $\kappa\varepsilon_0$

2.8.1. Dielectrics: An Atomic View

(a) What happens when we put a dielectric in an external electric field?

+ For polar dielectrics: the molecules already have electric dipole moments (e.g., water)

+ For nonpolar dielectrics: in a field, molecules acquire dipole moments by induction

So the molecules tend to line up with the field, Their alignment produces an electric field in the opposite direction to the external field and thus reduces the applied field


(a)

(b)

(a)	(b)	(c)
A nonpolar dielectric slab	An electric field is applied	The resultant field $\vec{E}$ inside the dielectric
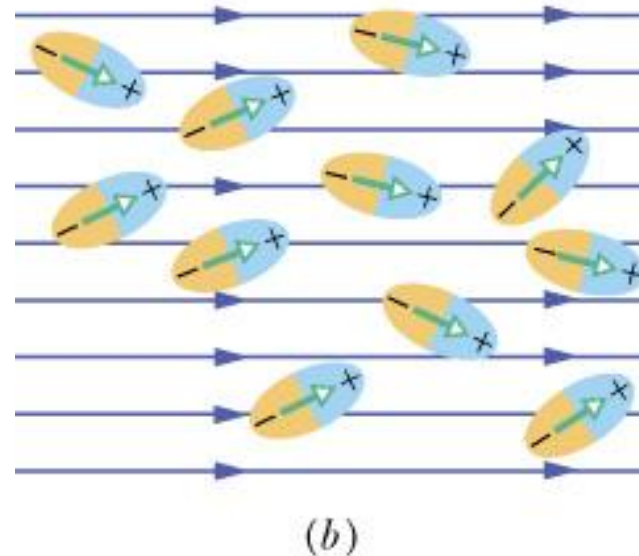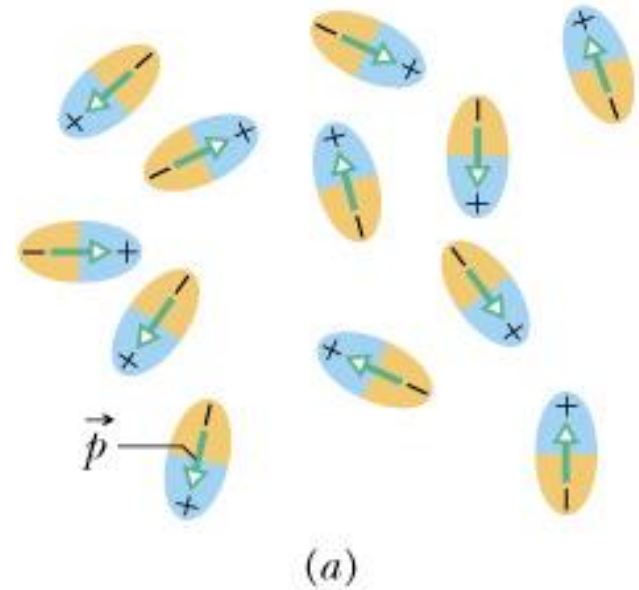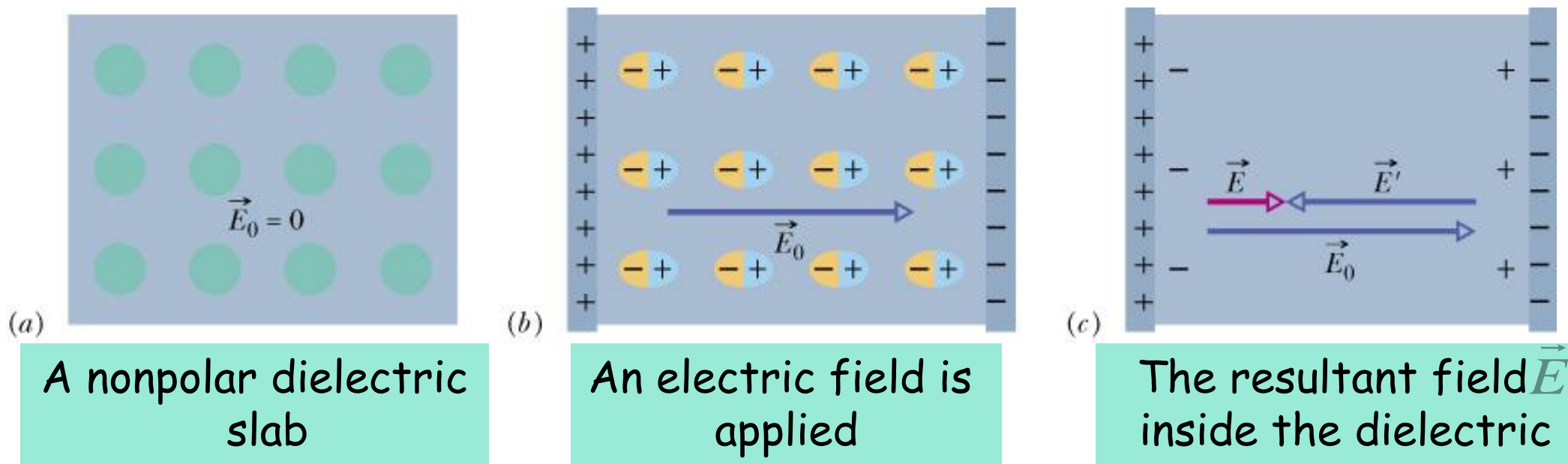
The presence of a dielectric will therefore reduce the difference potential for a given charge (q = constant) or will increase the charge on the plates.

2.8.2. Dielectrics and Gauss' Law:

Goal: Generalize Gauss's law if dielectrics are present
We consider a parallel-plate capacitor, without a dielectric:

$$\varepsilon_0 \oint \vec{E} d\vec{A} = \varepsilon_0 E_0 A = q$$
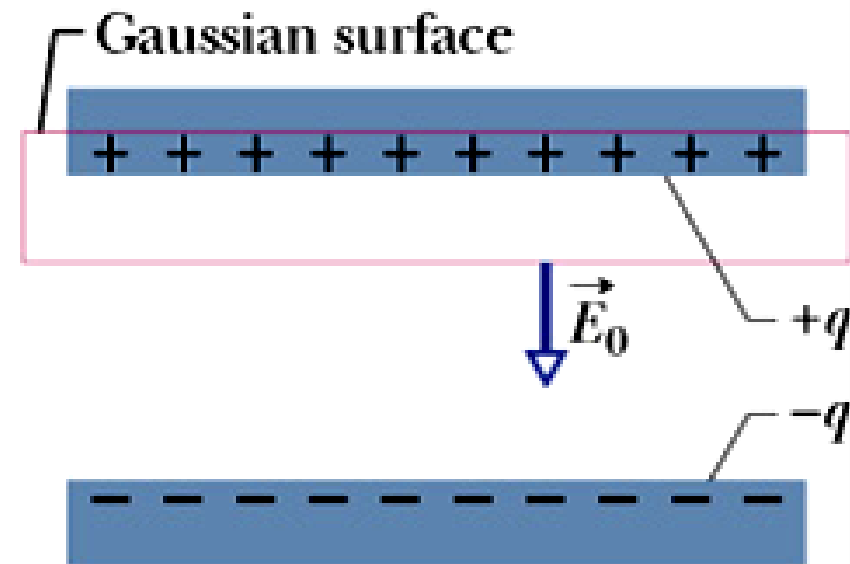
$$E_0 = \frac{q}{\varepsilon_0 A}$$

- **If a dielectric is present:**

$$\varepsilon_0 \oint \vec{E} d\vec{A} = \varepsilon_0 E A = q - q'$$

$$E = \frac{q - q'}{\varepsilon_0 A}$$
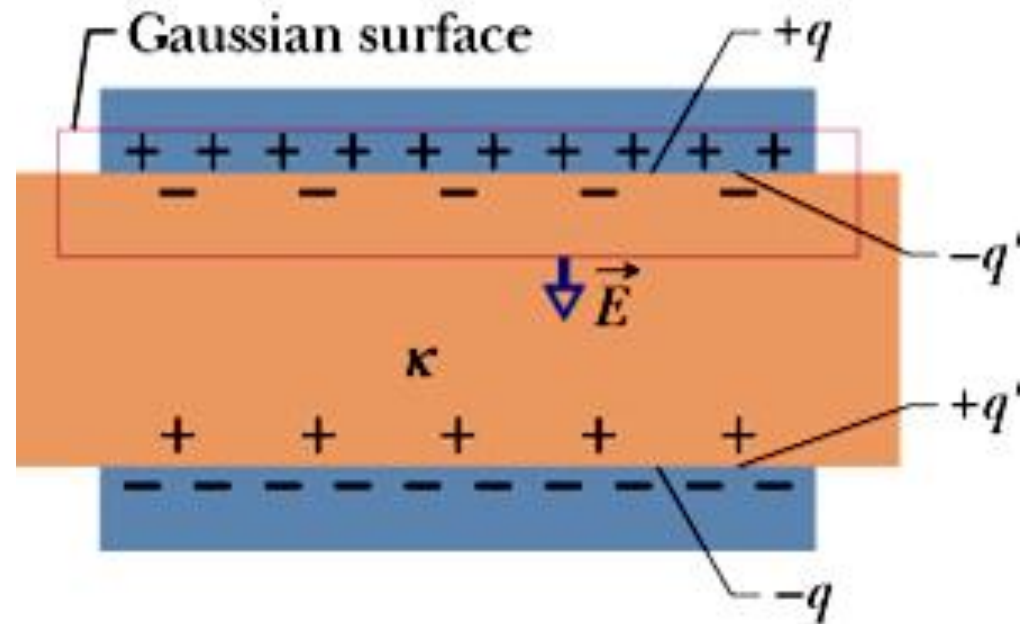

Gaussian surface

- **The effect of the dielectric is to reduce the applied field E$_0$:**

$$E = \frac{E_0}{\kappa} = \frac{q}{\kappa \varepsilon_0 A} \qquad \Rightarrow q - q' = \frac{q}{\kappa}$$

$$\boxed{\varepsilon_0 \oint \kappa \vec{E} d\vec{A} = q}$$

or

$$\boxed{\oint \vec{D} d\vec{A} = q; \vec{D} = \varepsilon_0 \kappa \vec{E} : electric\ displacement}$$

Checkpoint: Two identical parallel-plate capacitors are connected in series to a battery as shown below. If a dielectric is inserted in the lower capacitor, which of the following increase for that capacitor?

(a) Capacitance of the capacitor

(b) Charge on the capacitor

(c) Voltage across the capacitor

(d) Energy stored in the capacitor

$$C = \kappa C_0 \qquad q = CV \qquad U = \frac{1}{2}CV^2$$

1

V=constant

2

$$C = \frac{C_1 C_2}{C_1 + C_2}$$
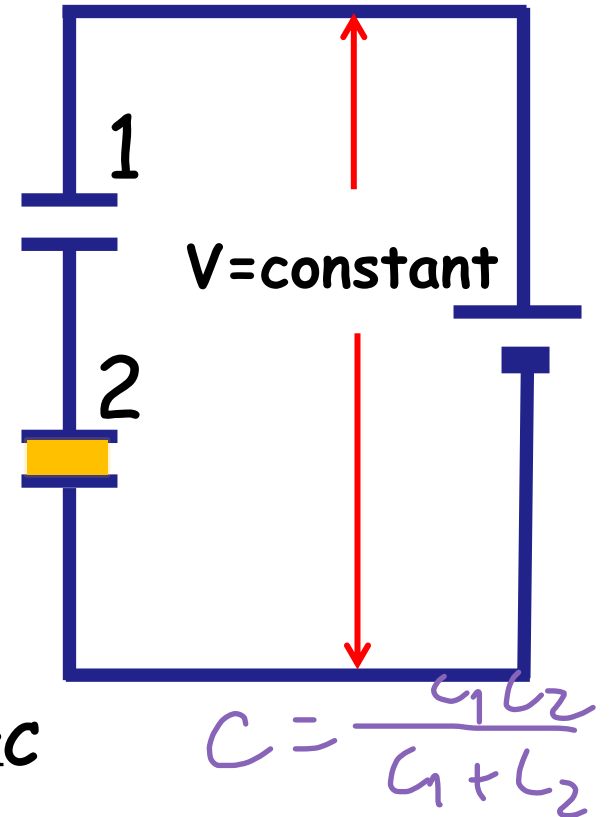
(a) the capacitance C_2 increases by κ, $C_2 = \kappa C$

(b) more charge flows to C_2 as $C_{equivalent}$ increases; q_2 increases

$q_1 = q_2$ also increases, $q' = 2\kappa q/(\kappa+1)$

(c) q_1 increases, so V_1 increases; $V_1+V_2 = V$ = constant, V_2 ↘

(d) before: $U = \dfrac{q^2}{2C}$ after: $U = \dfrac{q^2}{2C} \times \dfrac{4\kappa}{(\kappa+1)^2} < \dfrac{q^2}{2C}$ so, U ↘

Homework: 2, 4, 6, 11, 14, 16, 26, 31, 33, 42, 48, 51
(page 676-680)