FINAL EXAMINATION
PROBABILITY, STATISTICS AND RANDOM PROCESS
Semester 1, 2021-22 • January 2022 • Total duration: 90 minutes

Chair of Mathematics Department	Lecturer
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INSTRUCTIONS: Each student is allowed calculators, statistical tables and one double-sided sheet of reference material (size A4 or similar) marked with their name and ID. All other documents and electronic devices are forbidden.

Exam Number: 1

1. *(10 points)*

   List below is speech record (in km/h) of random selected car

   57, 61, 57, 57, 58, 57, 61, 54, 68, 51, 49, 64, 50, 48, 65, 52, 56, 46, 54, 49, 50

   Compute the sample mean, sample median and sample standard deviation for car speech.

2. *(10 points)*

   The diameter of holes for a cable harness is known to have a normal distribution with standard deviation $\sigma = 0.01$ inch. We wanted the error in estimating the mean diameter with an error that is less than 0.005 inch at 99% confidence. What sample size is required?

3. *(10 points)*

   In a recent survey on 100 adults shows that 70% of adults use Internet. Contruct a 95% confidence interval for proportion of adults using Internet.

4. *(10 points)*

   In a sample of 20 weights of M&M candies, they have a standard deviation of 0.0518 g. Suppose that weight of M&M candy is normally distributed. Construct a 90% confidence interval estimate of the variance of weights of all M&M.

5. *(10 points)*

   List below are measured amount of lead (in micrograms per cubic meter $\mu g/m^3$) in the air.

   $$5.40, \quad 1.10, \quad 0.42, \quad 0.73, \quad 0.48, \quad 3.65$$

   Use a 0.05 significance level to test the claim that the sample is from a population with a mean great than the EPA standard of $1.5\mu g/m^3$

6. *(20 points)*

   The Consumer Price Index (CPI) (in %) and the cost of a slide of pizza (in dollars) each year were recorded as follow

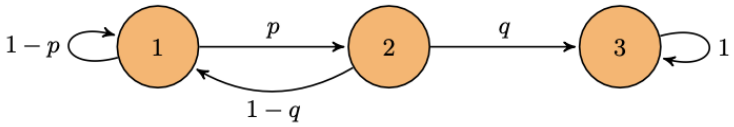
CPI	30.2	48.3	112.3	162.2	191.9	197.8
Cost of Pizza	0.15	0.35	1.00	1.25	1.75	2.00

   (a) Find equation of regression line.

   (b) In 2020, CPI is 181. Estimate the cost of a slide of pizza this year.

7. *(20 points)*

The double-heralding protocol for entanglement generation in quantum cryptography involves two rounds of photon transfer, the failure of either of which will cause the process to be restarted. The protocol is modeled using a Markov chain $(X_n)_{n\geq 0}$ described by the following graph



in which $p \in (0,1)$ is the probability of passing the first round, and $q \in (0,1)$ is the probability of passing the second round, conditional on passing the first.
Assume that $p = 0.9$ and $q = 0.95$ then the transition matrix of the Markov chain $(X_n)_{n\geq 0}$ is given by

$$P = \begin{bmatrix} 0.1 & 0.9 & 0 \\ 0.05 & 0 & 0.95 \\ 0 & 0 & 1 \end{bmatrix}$$

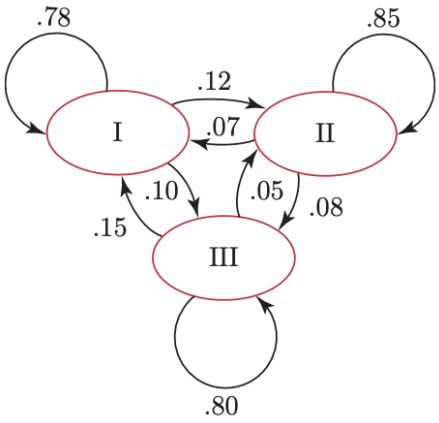
(a) Which state is recurrent? transient?

(b) Determine the recurrent class(es)?

(c) Compute $P(X_4 = 3|X_0 = 1)$.

8. *(10 points)*

The following figure describes the migration pattern of a species of bird from year to year among three habitats: I, II, and III - which is modeled by a Markov chain



In long run, what fraction of the birds will be located at each habitat? Use stationary distribution.