THE INTERNATIONAL UNIVERSITY(IU) - VIETNAM NATIONAL UNIVERSITY - HCMC

MIDTERM EXAMINATION
PROBABILITY, STATISTICS AND RANDOM PROCESS
Semester 2, 2020-21 ● April 2021 ● Total duration: 90 minutes

Chair of Mathematics Department	Lecturer
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INSTRUCTIONS: Each student is allowed calculators, statistical table for standard normal distribution and one double-sided sheet of reference material (size A4 or similar) marked with their name and ID. All other documents and electronic devices are forbidden.

1. *(20 points)* The following table summarizes the analysis of samples of galvanized steel for coating weight and surface roughness:

		coating weight	
		high	*low*
surface	*high*	12	16
roughness	*low*	88	34

   (a) If the coating weight of a sample is high, what is the probability that the surface roughness is high?

   (b) If the surface roughness of a sample is high, what is the probability that the coating weight is high?

2. *(20 points)* A spam filter is designed by looking at commonly occurring phrases in spam. Suppose that 80% of email is spam. In 10% of the spam emails, the phrase "free money" is used, whereas this phrase is only used in 1% of non-spam emails. A new email has just arrived, which does mention "free money". What is the probability that it is spam?

3. *(20 points)* **Random products mod 10.** Pick two successive digits from a table of random digits from $\{0, 1, ..., 9\}$. Multiply them together, and let X be the last digit of this random product. For example,

$$(3,9) \rightarrow 27 \rightarrow 7$$
$$(2,4) \rightarrow 8 \rightarrow 8$$

   Find the distribution of X, and calculate its mean $E(X)$.

4. *(20 points)* Motorola used the normal distribution to determine the probability of defects and the number of defects expected in a production process. Assume a production process produces items with a mean weight of 10 ounces.

   Calculate the probability of a defect and the expected number of defects for a 1000-unit production run in the following situations.

   (a) The process standard deviation is .15, and the process control is set at plus or minus one standard deviation. Units with weights less than 9.85 or greater than 10.15 ounces will be classified as defects.

   (b) Through process design improvements, the process standard deviation can be reduced to .05. Assume the process control remains the same, with weights less than 9.85 or greater than 10.15 ounces being classified as defects.

   (c) What is the advantage of reducing process variation, thereby causing process control limits to be at a greater number of standard deviations from the mean?

5. *(20 points)* A system consists of two independent components in series.
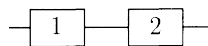
   The probability density functions of lifetime X_1 and X_2 (*in years*) of component 1 and component 2 are respectively given by

   $$f_1(x) = \begin{cases} \frac{1}{5}e^{-\frac{x}{5}} & \text{if } x > 0 \\ 0 & \text{otherwise} \end{cases}, \qquad f_2(x) = \begin{cases} \frac{1}{4}e^{-\frac{x}{4}} & \text{if } x > 0 \\ 0 & \text{otherwise} \end{cases}$$

   (a) Find the probability that component 1 breaks downs after 3 years.

   (b) What is the probability that the system lifetime is greater than 3 years?