Physics 2 Quiz 2

Q1/ (50 pts) Fig. 1 shows the cross section of a wall made of three layers. The thermal conductivities are $k_1 = 0.03$ W/m.K, $k_2 = 0.01$ W/m.K and $k_3 = 0.02$ W/m.K. The thicknesses of the layers are $L_1 = 5$ cm, $L_2 = X$ cm and $L_3 = 10$ cm. The temperatures at the left and right sides of the wall are 20°C and -20°C, respectively.
(a) If the temperature difference across layer 1 is 3 C°, what is the temperature difference across layer 2?
(b) What is the thickness X of layer 2?

$\Delta T_1 = T_0 - T_1$

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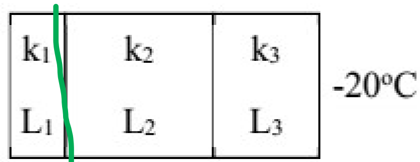

Fig. 1

$\Delta T_2 = T_1 - T_2$

$\Delta T_3 = T_2 - T_3$

Solution

a) The temperature at the end of the first layer: (30 pts)

$$T_1 = T_0 - \Delta T_1 = 20 - 3 = 17^oC$$

Assuming that the conduction rate P_{cond} throughout the wall is steady, we have:

$$P_{cond} = k_1 A\frac{\Delta T_1}{L_1} = k_3 A\frac{\Delta T_3}{L_3}$$

=> $0.03 \,.\frac{3}{0.05} = 0.02\,.\frac{T_2-(-20)}{0.1}$ => $T_2 = -11^oC$

=> The temperature difference across layer 2: $\Delta T_2 = T_1 - T_2 = 17 - (-11) = 28\ C^o$

Another way: $\Delta T_2 = \Delta T - \Delta T_1 - \Delta T_3$

b) We have: (20 pts)

$$P_{cond} = k_1 A\frac{\Delta T_1}{L_1} = k_2 A\frac{\Delta T_2}{L_2}$$

=> $0.03\,.\frac{3}{0.05} = 0.01\,.\frac{28}{X}$ => $X \approx 0.16\ (m)$

Q2/ (50 pts). A 100 g ice cube at -20°C is dropped into a thermally insulated container containing water at 40°C. The final temperature is 5°C at thermal equilibrium. The heat of fusion of water is 333 kJ/kg and the specific heats of ice and water are 2220 J/kg.K and 4187 J/kg.K, respectively.

(a) Determine the initial volume of water?

(b) If it takes 15 minutes for the equilibrium system, how long is the solid ice completely melted to liquid water? Assuming that the ice receives energy as heat at a constant rate

Solution

a) We have the heat equation for the water: (30 pts)

$$Q_w = c_{water} m_{water} \Delta T_{40\rightarrow 5} = 4187 \,.\, m_{water} \,.\, (-35) = -146545.\, m_{water}$$

We have the heat equation for the ice cube:

- From -20°C to 0°C: $Q_1 = c_{ice} m_{ice} \Delta T_{-20\rightarrow 0} = 2220 \,.\, 0.1 \,.\, 20 = 4440\ (J)$

- Phase change from ice to water: $Q_2 = L_F.\, m_{ice} = 333 \,.\, 10^3 .\, 0.1 = 33300\ (J)$

- From 0°C to 5°C: $Q_3 = c_{water} m_{ice} \Delta T_{0\rightarrow 5} = 4187 \,.\, 0.1 \,.\, 5 = 2093.5\ (J)$

When the temperature is at thermal equilibrium, we have:

=> $Q_w = -Q_i$

=> $-146545.\, m_{water} = -(Q_1 + Q_2 + Q_3) = -(4440 + 33300 + 2093.5)$

=> $m_{water} = 0.27\ (kg)$

The initial volume of water: $V_{water} = \frac{m_{water}}{\rho_{water}} = \frac{0.27}{1000} = 2.7 \,.\, 10^{-4} (m^3) = 271.82\ (cm^3)$

b) We have the rate of ice absorbing heat: (20 pts)

$R = \frac{Q_i}{t} = \frac{39833.5}{15} = 2655.57\ (\frac{J}{min})$

The amount of time needed for the ice to completely melted into liquid water is:

$$t_{melt} = \frac{Q_1 + Q_2}{R} = \frac{4440 + 33300}{2655.57} = 14.2\ (\text{min})$$