SOLUTION FOR PHYSICS 4

(Final)

January – 2011

2)

a) The nuclear reaction process:

$$^{14}_{6}C \rightarrow {}^{14}_{7}N + \beta^-$$

b) The mass effect:

$$\Delta m = M_C - M_N = 14.003241 - 14.003074 = 1.97 \text{ x } 10^{-4}(u)$$

The energy released by this reaction:

$$Q = \Delta mc^2 = 1.97 \text{ x } 10^{-4} \text{ x } 931.5 = 0.183 \ (MeV)$$

3)

The distance measured on Earth: $L_0 = 4.2 \text{ x } 10^6(m)$

The distance measured by the voyagers aboard the UFO:

$$L = L_0\sqrt{1 - \frac{u^2}{c^2}} = 4.2 \text{ x } 10^6\sqrt{1 - 0.7^2} \approx 3 \text{ x } 10^6(m)$$

4)

For the Balmer series:

$$\frac{1}{\lambda} = R\left(\frac{1}{n^2} - \frac{1}{m^2}\right) = R\left(\frac{1}{2^2} - \frac{1}{m^2}\right)$$

$$\rightarrow \lambda = \frac{1}{R\left(\frac{1}{2^2} - \frac{1}{m^2}\right)}$$

We have: For visible light, the wavelength is:

$$380 \ nm \leq \lambda \leq 760 \ nm$$

$$⇔380 \text{ x } 10^{-9} \leq \frac{1}{R\left(\frac{1}{2^2} - \frac{1}{m^2}\right)} \leq 760 \text{ x } 10^{-9} ⇔ 0.12 \leq \frac{1}{2^2} - \frac{1}{m^2} \leq 0.23$$

$$⇔0.02 \leq \frac{1}{m^2} \leq 0.13ό2.77 \leq m \leq 7.07ð\ m = \{3,4,5,6,7\}$$

Conclusion: All the lines which lie in the visible spectrum belong to Balmer series

5)

a) We have: The boundary condition:

$$\begin{cases} V = 0\ for\ -L \leq x \leq L \\ V = \infty\ otherwise \end{cases}$$

At $x = -L$: $\psi(-L) = 0 ⇨ \psi(-L) = Asin(-kL) + Bcos(-kL) = -Asin(kL) + Bcos(kL) = 0$

$$⇨ Bcos(kL) = Asin(kL)\ (1)$$

At $x = L$: $\psi(L) = 0 ⇨ \psi(L) = Asin(kL) + Bcos(kL) = 0$

$$⇨ Bcos(kL) = -Asin(kL)\ (2)$$

From (1) and (2): $Asin(kL) = -Asin(kL)$. This only makes sense if $A = 0$

Therefore: $\psi(x) = Bcos(kx)$

At $x = L$: $\psi(L) = Bcos(kL) = 0 \ \rightarrow kL = \frac{\pi}{2} + n\pi \ \rightarrow k = \frac{1}{L}\left(\frac{\pi}{2} + n\pi\right)$

b) The wave function:

$$\psi(x) = Bcos(kx)$$

The normalization condition:

$$P = \int_{-\infty}^{+\infty} |\psi^2| dx = \int_{-L}^{L} |\psi^2| dx = \int_{-L}^{L} (Bcos(kx)^2 dx = 1$$

Since: $\sin^2 x = \frac{1 - cos2x}{2}$; $\cos^2 x = \frac{1 + cos2x}{2}$; $\sin(2x) = 2sinxcosx$

$$⇨P = B^2 \int_{-L}^{L} \frac{1 + \cos(2kx)}{2} dx = \frac{B^2}{2} \int_{-L}^{L} (1 + \cos(2kx))\, dx = 1$$

$$= \frac{B^2}{2}\left(L + \frac{1}{2k}\sin(2kL) + L - \frac{1}{2k}\sin(2kL)\right) = B^2 L = 1 \rightarrow B = \sqrt{\frac{1}{L}}$$

Conclusion: The wave function of a particle in an infinite deep well one-dimensional well

$$\psi(x) = \sqrt{\frac{1}{L}}\cos(kx) = \sqrt{\frac{1}{L}}\cos\left(\frac{x\pi}{2L}(2n+1)\right)$$

c) For the value of k: $k = \frac{1}{L}\left(\frac{\pi}{2} + n\pi\right) = \frac{\pi}{2L}(2n+1)$

Since k is the wave number:

$$k = \frac{2\pi}{\lambda} \rightarrow \lambda = \frac{2\pi}{k} = \frac{4\pi L}{\pi(2n+1)} = \frac{4L}{2n+1}$$

The momentum of the particle:

$$p = \frac{h}{\lambda} = \frac{h(2n+1)}{4L}$$

The energy of the particle:

$$E = \frac{p^2}{2m} = \frac{h^2(2n+1)^2}{2m\,(4L)^2} = \frac{h^2(2n+1)^2}{32mL^2} = \frac{(2\pi\hbar)^2(2n+1)^2}{32mL^2} = \frac{\hbar^2\pi^2(2n+1)^2}{8mL^2}$$

Conclusion: The energy of the particle in state nth:

$$E_n = \frac{\hbar^2\pi^2 n^2}{8mL^2}\ (\mathrm{n} = 1,3,5,7,\dots)$$

June – 2012

1)

a) We have totally 10 different lines can be emitted by an hydrogen atoms: $(2 \rightarrow 1; 3 \rightarrow 1,; 4 \rightarrow 1,; 5 \rightarrow 1,; 3 \rightarrow 2; 4 \rightarrow 2,; 5 \rightarrow 2,; 4 \rightarrow 3; 5 \rightarrow 3,; 5 \rightarrow 4)$

Or we can calculate by using the formula:

$$N = \frac{n(n-1)}{2} = \frac{5(5-1)}{2} = 10$$

There are 3 spectral lines are found in the visible spectrum (in Balmer series): $3 \rightarrow 2\,;4 \rightarrow 2\,;5\ \rightarrow 2$

b) We have:

The energy state of excitation energy 10.19 eV: $E_f = -13.6 + 10.19 = -3.41\ (eV)$

$$\Delta E = hf = \frac{hc}{\lambda} = \frac{hc}{4890 \text{ x } 10^{-10}} = 2.54(eV)$$

$$\Delta E = E_i - E_f = E_i - (-3.41) = 2.54 \rightarrow E_i = -0.87\ (eV)$$

The principle quantum number of these state:

$$E_i = \frac{-13.6}{n_i^2} = -0.87 \rightarrow n_i = 4\,; E_f = \frac{-13.6}{n_f^2} = -3.41 \Rightarrow n_f = 2$$

2)

In Newtonian mechanics: $K_N = \frac{1}{2}m_0u^2$

In special relativity:

$$E = m_0c^2 + K_S \rightarrow K_S = E - m_0c^2 = \gamma m_0c^2 - m_0c^2 = (\gamma - 1)m_0c^2$$

With: $\gamma = \frac{1}{\sqrt{1-\frac{u^2}{c^2}}}$

Since the relativistic kinetic energy is 50% greater than the Newtonian value:

$$K_S = K_N + 50\%K_N = 1.5K_N \Leftrightarrow (\gamma - 1)m_0c^2$$

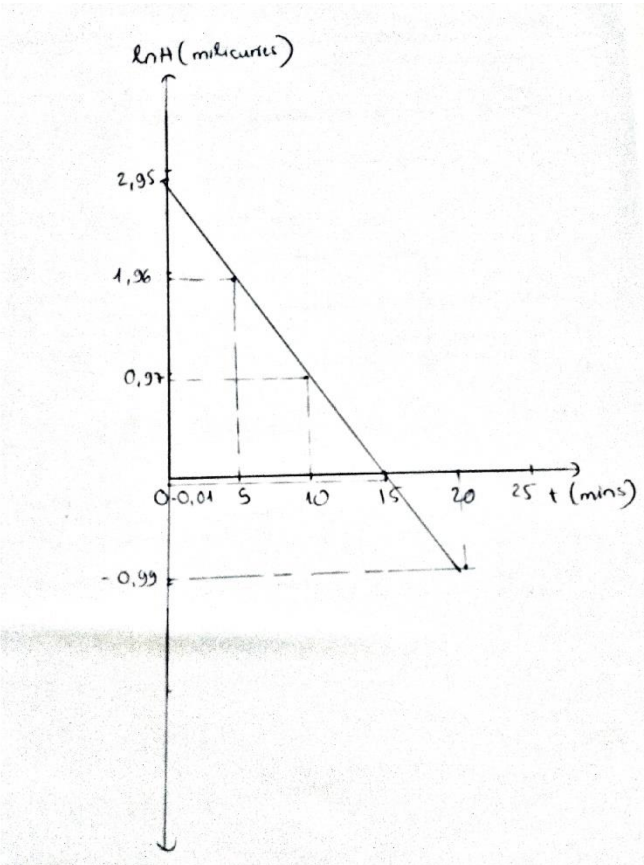
$$= 1.5 \text{ x} \frac{1}{2}m_0u^2 \Leftrightarrow \left(\frac{1}{\sqrt{1-\frac{u^2}{c^2}}} - 1\right)c^2 = \frac{3}{4}u^2$$

Let $\beta = \frac{u}{c}$

Therefore, the equation becomes:

$$\left(\frac{1}{\sqrt{1-\beta^2}} - 1\right) = \frac{3}{4}\beta^2 \Rightarrow \beta = 0.652\ ðu = 0.652c$$

3)



We have: The activity of the sample

$$H = \lambda N(t) = \lambda N_0 e^{-\lambda t}$$

$$\rightarrow lnH = \ln\left(\lambda N_0 e^{-\lambda t}\right) = ln\lambda N_0 + lne^{-\lambda t} = ln\lambda N_0 - \lambda t = a - bt$$

Based on the variation of lnH, we have:

The slope of the plot of lnH : $b = \lambda$

Therefore:

$$\lambda = \frac{(0.97 - 1.96) \text{ x } 3.7 \text{ x } 10^{10} \text{ x } 10^{-3}}{5 \text{ x } 60} = -122100\ (s^{-1})$$

The half-life of a sample ${}^{55}_{24}Cr$:

$$T_{half} = \frac{ln2}{\lambda} = \frac{ln2}{122100} \approx 5.67(\mu s)$$

4)

Conservation of energy:

$$E_U + E_\gamma = E_{Kr} + E_{Ba} + 3E_n$$

$$\Leftrightarrow K_U + M_U c^2 + K_\gamma = K_{Kr} + M_{Kr} c^2 + K_{Ba} + M_{Ba} c^2 + 3(K_n + M_n c^2)$$

Since $K_U = 0$; $K_\gamma = 6MeV$

$$\Leftrightarrow \sum K_f = M_U c^2 + K_\gamma - M_{Kr} c^2 - M_{Ba} c^2 - 3M_n c^2$$

$$\Leftrightarrow \sum K_f = (M_U - M_{Kr} - M_{Ba} - 3M_n)c^2 + K_\gamma$$

$$\Leftrightarrow \sum K_f = (235.043915 - 89.91972 - 141.91635 - 3 \text{ x } 1.008665) \text{x } 931.5 + 6 = 175.44\ (MeV)$$

5)

The wave function of a particle in a square well: (At the ground level: n =1)

$$\psi(x) = \sqrt{\frac{2}{L}} \sin\left(\frac{n\pi x}{L}\right) = \sqrt{\frac{2}{L}} \sin\left(\frac{\pi x}{L}\right)$$

a) The probability of finding particle in the region between 0 and L/4

$$P = \int_0^{L/4} |\psi^2| dx$$

$$= \frac{2}{L}\int_0^{L/4} \sin^2\left(\frac{\pi x}{L}\right) dx = \frac{2}{L}\int_0^{L/4} \frac{1 - \cos\left(\frac{2\pi x}{L}\right)}{2} dx$$

$$= \frac{1}{L}\int_0^{L/4} 1 - \cos\left(\frac{2\pi x}{L}\right) dx$$

$$= \frac{1}{L}\left(\frac{L}{4} - \frac{L}{2\pi} \sin\left(\frac{2\pi L}{4L}\right)\right) = \frac{1}{4} - \frac{1}{2\pi} \sin\left(\frac{\pi}{2}\right) \approx 9.1\%$$

b) The probability of finding particle in the region between L/4 and L/2

$$P = \int_{L/4}^{L/2} |\psi^2| dx$$

$$= \frac{2}{L}\int_{L/4}^{L/2} \sin^2\left(\frac{\pi x}{L}\right) dx = \frac{2}{L}\int_{L/4}^{L/2} \frac{1-\cos\left(\frac{2\pi x}{L}\right)}{2} dx$$

$$= \frac{1}{L}\int_{L/4}^{L/2} 1 - \cos\left(\frac{2\pi x}{L}\right) dx$$

$$= \frac{1}{L}(\frac{L}{2} - \frac{L}{2\pi}\sin(\pi) - \frac{L}{4} + \frac{L}{2\pi}\sin\left(\frac{\pi}{2}\right) = \frac{1}{4} + \frac{1}{2\pi}\left(\sin\left(\frac{\pi}{2}\right) - \sin(\pi)\right) \approx 40.9\%$$

c) From the results of part a/ and part b/ we have:

$P_a < P_b (9.1\% < 40.9\%)$ ⇨Particle is mostly found in the region between L/4 and L/2

d) The sum of two results calculated in part a/ and b/ (The probability of finding particle in the region between 0 and L/2

$P = P_a + P_b \approx 50\%$ ⇨The probability of finding particle in first-half of the width of the well is 50%

June – 2014

1)

Let the length of cube sides in a frame S is a_0

The volume of a metal cube: $V_0 = a_0^3$

The length measured by an observer in frame S' (move along x-axis)

$$a = a_0\sqrt{1 - \frac{u^2}{c^2}} = \frac{a_0}{\gamma}$$

The volume of a metal cube in frame S'

$$V = a \text{ x } a_0^2 = \frac{a_0}{\gamma} \text{ x } a_0^2 = \frac{a_0^3}{\gamma} = V_0\sqrt{1 - \frac{u^2}{c^2}}$$

3)

a) The nuclear reaction process:

$$^{230}_{90}Th \rightarrow X + \ \alpha\ (^{4}_{2}He)$$

Conservation of charge:

$Z_{Th} = Z_X + Z_\alpha \Leftrightarrow 90 = Z_X + 2 \Rightarrow Z_X = 88\ (protons)$

Conservation of nucleon number A:

$$A_{Th} = A_X + A_\alpha \Leftrightarrow 230 = A_X + 4 \ \rightarrow A_X = 226$$

The number of neutrons for the nuclide X:

$$N_X = A_X - Z_X = 226 - 88 = 138\ (neutrons)$$

b) Conservation of energy:

$$E_{Th} = E_X + E_\alpha$$

$$\Leftrightarrow K_{Th} + M_{\mathrm{Th}}c^2 = K_X + M_{\mathrm{X}}c^2 + K_\alpha + M_\alpha c^2 \Leftrightarrow K_X + K_\alpha - K_{Th} = (M_{\mathrm{Th}} - M_{\mathrm{X}} - M_\alpha)c^2$$

The thorium is at rest: $K_{Th} = 0$

The recoil effect of the daughter nucleus: $K_X = K_\alpha$

⇨The kinetic energy of the emitted α particle:

$$2K_\alpha = (M_{\mathrm{Th}} - M_{\mathrm{X}} - M_\alpha)c^2 = (230.033127\ - 226.025403\ - 4.002603)\text{x}\ 931.5\ \rightarrow K_\alpha = 2.385\ (MeV)$$

5)

a) We have: The photon energy arising from the transition between two energy levels (from level E_n to level E_m)

$$\Delta E = hf = \frac{hc}{\lambda} = -13.6\left(\frac{1}{m^2} - \frac{1}{n^2}\right)\ (eV) \Leftrightarrow \frac{1}{\lambda} = \frac{-13.6\ \text{x}\ 1.6\ \text{x}\ 10^{-19}}{hc}\left(\frac{1}{m^2} - \frac{1}{n^2}\right)$$

$$\Rightarrow \frac{1}{\lambda} = 1.094\ \text{x}\ 10^{-7}\left(\frac{1}{n^2} - \frac{1}{m^2}\right) \Rightarrow R = 1.094\ \text{x}\ 10^{7}(m^{-1})$$

b) The shortest wavelength in the Lyman series: From $n = \ \infty$ to $n = 1$

$$\Rightarrow \frac{1}{\lambda} = 1.094 \text{ x } 10^7 \left(\frac{1}{1^2} - \frac{1}{\infty^2}\right) = -1.094 \text{ x } 10^7 \Rightarrow \lambda = 91.4\ (nm) \rightarrow \text{Ultraviolet}$$

The longest wavelength in the Paschen series: From $n = 4$ to $n = 3$

$$\Rightarrow \frac{1}{\lambda} = 1.094 \text{ x } 10^7 \left(\frac{1}{3^2} - \frac{1}{4^2}\right) \Rightarrow \lambda = 1880\ (nm) \rightarrow \text{Infrared}$$

June – 2015

1)

a) The first line of Balmer series: $\lambda = 658\ nm$

$$\Delta E = hf = \frac{hc}{\lambda} = \frac{hc}{658 \text{ x } 10^{-9}} = 1.88\ (eV)$$

The potential difference for the electron to accelerate:

$$e\Delta V = \Delta E \rightarrow \Delta V = \frac{\Delta E}{e} = 1.88\ (V)$$

b) We have: H atoms are excited by UV radiation with $\lambda = 100nm$

$$\Delta E = \frac{hc}{\lambda} = \frac{hc}{100 \text{ x } 10^{-9}} = 12.42(eV)$$

$$\Delta E = E_i - E_f = -13.6\left(\frac{1}{n_i^2} - \frac{1}{n_f^2}\right) = -13.6\left(\frac{1}{\text{n}^2} - 1\right) = 12.42 \Rightarrow n = 3$$

Conclusion: That is Lyman spectral line from $n = 3$ to $n = 1$

2)

a) Conservation of charge:

$$Z_{Si} = Z_X + Z_{Mg} \Leftrightarrow 14 = Z_X + 12 \rightarrow Z_X = 2$$

Conservation of nucleon number A:

$$A_{Si} = A_X + A_{Mg} \Leftrightarrow 28 = A_X + 24 \rightarrow A_X = 4$$

The number of neutrons for the nuclide X:

$$N_X = A_X - Z_X = 4 - 2 = 2\ (neutrons)$$

b) The energy released in this reaction

$$Q = \left(M_{Si} - M_X - M_{Mg}\right)c^2 = (27.976927 \; - 4.002603 \; - 23.985042) \; x \; 931.5$$
$$= -9.98 \; (MeV)$$

This energy is transferred into gamma radiation

$$Q = hf = \frac{hc}{\lambda} = 9.98 \; (MeV) \Rightarrow \lambda = 0.124 \; (pm)$$

3)

a) The width of the aperture: Δy = d = 0.5mm

According to the Heisenberg's uncertainty principle:

$$\Delta p_y \Delta y \geq \hbar \rightarrow \Delta p_y \geq \frac{\hbar}{\Delta y} = \frac{\hbar}{0.5 \text{ x } 10^{-3}} = 2.11 \text{ x } 10^{-31} (kg.\frac{m}{s})$$

The uncertainty in the component of the electron's velocity:

$$\Delta p_y = m \Delta v_y \rightarrow \Delta v_y = \frac{\Delta p_y}{m} = \frac{2.11 \text{ x } 10^{-31}}{9.11 \text{ x } 10^{-31}} = 0.231 \; (\frac{m}{s})$$

b) There is the uncertainty in momentum of the electron ⇨ Affecting on its direction when it strikes the screen ⇨ Electron diffraction

We have: $K = \frac{1}{2} mv^2 = \frac{p^2}{2m} = eV$

$$\rightarrow p = \sqrt{2meV} = \sqrt{2 \text{ x } 9.11 \text{ x } 10^{-31} \text{x } 1.6 \text{ x } 10^{-19} \text{ x } 15 \; x \; 10^3}$$
$$= 6.612 \text{ x } 10^{-23} (kg.\frac{m}{s})$$

The De Broglie's wavelength of the electron:

$$\lambda = \frac{h}{p} = \frac{6.62 \text{ x } 10^{-23}}{6.612 \text{ x } 10^{-23}} = 10.02 \; (pm)$$

The first minima (dark fringe) in electron diffraction:

$$dsin\theta = \lambda \rightarrow sin\theta = \frac{\lambda}{d} = \frac{10.2 \text{ x } 10^{-12}}{0.5 \text{ x } 10^{-3}} = 2.04 \text{ x } 10^{-8}$$

We have:

$$sin\theta \sim tan\theta = \frac{\Delta p_y}{\Delta p_x} = \frac{\Delta y_{screen}}{L} = 2.04 \text{ x } 10^{-8} \rightarrow \Delta y_{screen} = 6.12 \; (nm)$$

Conclusion: The position of the electron on the screen will spread over all direction.

4)

a) We have: The normalization condition:

$$P = \int_{-\infty}^{+\infty} \psi^2(x)\,dx = \int_{-\infty}^{0} \psi^2(x)dx + \int_{0}^{+\infty} \psi^2(x)dx = 1$$

$$\Leftrightarrow \int_{-\infty}^{0} A^2e^{2bx}dx + \int_{0}^{+\infty} A^2e^{-2bx}dx = \frac{A^2}{2b}(e^0 - e^{-2b\infty}) - \frac{A^2}{2b}(e^{-2b\infty} - e^0) = 1$$

$$\Leftrightarrow \frac{A^2}{2b}(1-0) - \frac{A^2}{2b}(0-1) = \frac{2A^2}{2b} = 1 \Rightarrow A = \sqrt{2}$$

b) The probability of finding this particle within 50cm of the origin:

$$P = \int_{-0.25}^{0.25} \psi^2(x)dx$$

$$= \int_{-0.25}^{0} \psi^2(x)dx + \int_{0}^{0.25} \psi^2(x)dx = \int_{-0.25}^{0} A^2e^{2bx}dx + \int_{0}^{0.25} A^2e^{-2bx}dx$$

$$= \int_{-0.25}^{0} 2e^{4x}dx + \int_{0}^{0.25} 2e^{-4x}dx = \frac{1}{2}(e^0 - e^{-1}) - \frac{1}{2}(e^{-1} - e^0) \approx 63.21\%$$

5)

a) The length of a spacecraft runway measured by an observer on the Earth: $L = 3600\ (m)$

⇨ The length of the runway measured by a pilot on a spacecraft:

$$L = L_0\sqrt{1 - \frac{u^2}{c^2}} \rightarrow L_0 = \frac{L}{\sqrt{1 - \frac{u^2}{c^2}}} = \frac{3600}{\sqrt{1 - \left(\frac{4 \text{ x } 10^7}{3 \text{ x } 10^8}\right)^2}} = 3632.433\ (m)$$

b) The time that takes him to travel from one end of the runway to the other end:

$$\Delta t_0 = \frac{L_0}{u} = \frac{3632.433}{4 \text{ x } 10^7} = 90.81\ (\mu m)$$

June – 2016

1)

When a hydrogen atom absorbs a photon, it jumps from the ground level to the higher level

$$E_n = E_1 + \varepsilon = -13.6 + 12.75 = -0.85(eV)$$

$$E_n = \frac{-13.6}{n^2} = -0.85 \ \rightarrow n = 4$$

a) The shortest wavelength of the photons: From $n = 4$ to $n = 1$

$$\Delta E = hf = \frac{hc}{\lambda} = E_i - E_f = -13.6\left(\frac{1}{n_i^2} - \frac{1}{n_f^2}\right) = -13.6\left(\frac{1}{4^2} - \frac{1}{1^2}\right)$$
$$= 12.75\ (eV) ⇨ \lambda_{min} = 97.4\ (nm)$$

The longest wavelength of the photon: From $n = 4$ to $n = 3$

$\Delta E = hf = \frac{hc}{\lambda} = E_i - E_f = -13.6\left(\frac{1}{n_i^2} - \frac{1}{n_f^2}\right) = -13.6\left(\frac{1}{4^2} - \frac{1}{3^2}\right) =$ $0.661\ (eV) ⇨ \lambda_{max} = 1879.2\ (nm)$

b) For the shortest wavelength: $\lambda_{min} = 97.4\ (nm)$ ⇨Lyman series

For the longest wavelength: $\lambda_{max} = 1879.2\ (nm)$ ⇨ Paschen series

2)

a) The number of nuclei in the sample at t =0:

$$N_0 = N_A \frac{m}{M} = N_A \frac{0.0035}{11} = 1.92 \text{ x } 10^{20} (nuclei)$$

b) After 8 hours, the number of nuclei:

$$N(t) = N_0 2^{-\frac{t}{T_{half}}} ⇔ 53.76 \text{ x } 10^6 = 1.92 \text{ x } 10^{20}\ 2^{-\frac{8 \text{ x } 3600}{T_{half}}}$$

$$⇨ T_{half} \approx 690.\ 65(s)$$

3)

a) We have: The normalization condition:

$$P = \int_{-\infty}^{+\infty} \psi^2(x)\, dx = \int_{-\infty}^{0} \psi^2(x) dx + \int_{0}^{+\infty} \psi^2(x) dx = 1$$

$$\Leftrightarrow \int_{-\infty}^{0} A^2 e^{2bx} dx + \int_{0}^{+\infty} A^2 e^{-2bx} dx = \frac{A^2}{2b}(e^0 - e^{-2b\infty}) - \frac{A^2}{2b}(e^{-2b\infty} - e^0) = 1$$

$$\Leftrightarrow \frac{A^2}{2b}(1-0) - \frac{A^2}{2b}(0-1) = \frac{2A^2}{2b} = 1 \Rightarrow A = \sqrt{2}$$

b) The probability of finding this particle within 50cm of the origin:

$$P = \int_{-0.25}^{0.25} \psi^2(x) dx$$

$$= \int_{-0.25}^{0} \psi^2(x) dx + \int_{0}^{0.25} \psi^2(x) dx = \int_{-0.25}^{0} A^2 e^{2bx} dx + \int_{0}^{0.25} A^2 e^{-2bx} dx$$

$$= \int_{-0.25}^{0} 2e^{4x} dx + \int_{0}^{0.25} 2e^{-4x} dx = \frac{1}{2}(e^0 - e^{-1}) - \frac{1}{2}(e^{-1} - e^0) \approx 63.21\%$$

4)

The length of an object when it moves: L $= 74\ (m)$

The length of an object when it is stationary:

$$L = L_0\sqrt{1 - \frac{u^2}{c^2}} \rightarrow L_0 = \frac{L}{\sqrt{1 - \frac{u^2}{c^2}}} = \frac{74}{\sqrt{1 - 0.6^2}} = 92.5\ (m)$$

5)

a) The wavelength of the emitted photon:

$$E = \frac{hc}{\lambda} \Rightarrow \lambda = \frac{hc}{E} = \frac{6.62 \text{ x } 10^{-34} \text{ x } 3 \text{ x } 10^{8}}{2.58 \text{ x } 1.6 \text{ x } 10^{-19}} = 4.81 \text{ x } 10^{-7}(m) = 0.418\ (\mu m)$$

b) According to the uncertainty principle of energy:

$$\Delta E_{\min}\Delta t = \hbar \Rightarrow \Delta E_{\min} = \frac{\hbar}{\Delta t} = \frac{\hbar}{1.64 \text{ x } 10^{-7}} = 6.43 \text{ x } 10^{-28}(J)$$

2018

1)

The longest wavelength of the Lyman series: From $n = 2$ to $n = 1$

$$\Delta E = hf = \frac{hc}{\lambda} = E_i - E_f = -13.6\left(\frac{1}{n_i^2} - \frac{1}{n_f^2}\right) = -13.6\left(\frac{1}{2^2} - \frac{1}{1^2}\right) = 10.2\ (eV)$$

$$\Rightarrow \lambda_{max} = 121.8\ (nm)$$

The shortest wavelength of the Lyman series: From $n = \infty$ to $n = 1$

$$\Delta E = hf = \frac{hc}{\lambda} = E_i - E_f = -13.6\left(\frac{1}{n_i^2} - \frac{1}{n_f^2}\right) = -13.6\left(\frac{1}{\infty^2} - \frac{1}{1^2}\right)$$
$$= 3.4\ (eV)$$

$$\Rightarrow \lambda_{min} = 91.3\ (nm)$$

Since $91.3 \leq \lambda \leq 121.8$ → Lyman series falls in ultraviolet region of the electromagnetic spectrum

The longest wavelength of the Paschen series: From $n = 4$ to $n = 3$

$$\Delta E = hf = \frac{hc}{\lambda} = E_i - E_f = -13.6\left(\frac{1}{n_i^2} - \frac{1}{n_f^2}\right) = -13.6\left(\frac{1}{4^2} - \frac{1}{3^2}\right) = 0.66\ (eV)$$

$$\Rightarrow \lambda_{max} = 1879.2\ (nm)$$

The shortest wavelength of the Lyman series: From $n = \infty$ to $n = 3$

$$\Delta E = hf = \frac{hc}{\lambda} = E_i - E_f = -13.6\left(\frac{1}{n_i^2} - \frac{1}{n_f^2}\right) = -13.6\left(\frac{1}{\infty^2} - \frac{1}{3^2}\right)$$
$$= 1.51\ (eV)$$

$$\Rightarrow \lambda_{min} = 822.1\ (nm)$$

Since $822.1 \leq \lambda \leq 1879.2$ → Paschen series falls in infrared region of the electromagnetic spectrum

2)

a) We have: The normalization condition:

$$P = \int_{-\infty}^{+\infty} \psi^2(x)\,dx = \int_{-\infty}^{0} \psi^2(x)dx + \int_{0}^{+\infty} \psi^2(x)dx = 1$$

$$\Leftrightarrow \int_{-\infty}^{0} A^2 e^{2bx} dx + \int_{0}^{+\infty} A^2 e^{-2bx} dx = \frac{A^2}{2b}(e^0 - e^{-2b\infty}) - \frac{A^2}{2b}(e^{-2b\infty} - e^0) = 1$$

$$\Leftrightarrow \frac{A^2}{2b}(1-0) - \frac{A^2}{2b}(0-1) = \frac{2A^2}{2b} = 1 \Rightarrow A = \sqrt{2}$$

b) The probability of finding this particle within 50cm of the origin:

$$P = \int_{-0.25}^{0.25} \psi^2(x)dx$$

$$= \int_{-0.25}^{0} \psi^2(x)dx + \int_{0}^{0.25} \psi^2(x)dx = \int_{-0.25}^{0} A^2 e^{2bx} dx + \int_{0}^{0.25} A^2 e^{-2bx} dx$$

$$= \int_{-0.25}^{0} 2e^{4x} dx + \int_{0}^{0.25} 2e^{-4x} dx = \frac{1}{2}(e^0 - e^{-1}) - \frac{1}{2}(e^{-1} - e^0) \approx 63.21\%$$

3)

a) Since n is an integer (n = 1,2,3,..) ⇨ Only certain value of energy is evaluated ⇨The energy of this electron is quantized.

b) We have

$$\Delta E = E_4 - E_1 = \frac{16\pi^2\hbar^2}{2mL^2} - \frac{\pi^2\hbar^2}{2mL^2} = \frac{15\pi^2\hbar^2}{2mL^2}$$

$$= \frac{15\pi^2\hbar^2}{2 \text{ x } 9.1 \text{ x } 10^{-31} \text{ x } (0.125 \text{ x } 10^{-9})^2} = 361.4(eV)$$

c) We have:

$\Delta E = hf = \frac{hc}{\lambda} = 361.4(eV) \Rightarrow \lambda = 3.43\ (nm)$ ⇨ X-rays

4)

The Lorentz factor: $\gamma = \frac{1}{\sqrt{1-\frac{u^2}{c^2}}} = \frac{1}{\sqrt{1-0.99^2}} = 7.08$

a) The kinetic energy of the electron:

$$E = m_0c^2 + K_S \rightarrow K_S = E - m_0c^2 = \gamma m_0c^2 - m_0c^2 = (\gamma - 1)m_0c^2$$

$$\rightarrow K_S = (7.08 - 1) \text{x } 9.11 \text{ x } 10^{-31} \text{ x } (3 \text{ x } 10^8)^2 = 4.992 \text{ x } 10^{-13}(J)$$
$$= 3.12\ (MeV)$$

b) The kinetic energy of the electron (in classical mechanics)

$$K_N = \frac{1}{2}m_0u^2 = \frac{1}{2} \text{ x } 9.11 \text{ x } 10^{-31} \text{x } (0.99 \text{ x } 3 \text{ x } 10^8)^2 = 4.018\ x\ 10^{-14}(J)$$
$$= 0.25\ (MeV)$$

The difference between the classical value and relativistic value:

$$\%K = \frac{K_S - K_N}{K_S} = 91.98\ \%$$

Conclusion: There is an error when we use classical mechanics to calculate the kinetic energy of the electron when its velocity reaches to the light speed.

5)

a) The decay constant:

$$T_{half} = \frac{ln2}{\lambda} \rightarrow \lambda = \frac{ln2}{T_{half}} = 3.75 \text{ x } 10^{-4}$$

The initial activity:

$$\frac{dN(0)}{dt} = -\lambda N(0) \Leftrightarrow -7.56 \text{ x } 10^{11} = -3.75 \text{ x } 10^{-4}N(0) \Rightarrow N(0)$$
$$= 2.016 \text{ x } 10^{15}(nuclei)$$

b) After 30.8 minutes, the number of nuclei:

$$N(t) = N_0 2^{-\frac{t}{T_{half}}} = 2.016 \text{ x } 10^{15} 2^{-\frac{t}{T_{half}}} = 2.016 \text{ x } 10^{15} \text{ x } 2^{-1}$$
$$= 1.008 \text{ x } 10^{15}(\text{nuclei})$$

The activity at this time:

$$\frac{dN(t)}{dt} = -\lambda N(t) = -3.75 \text{ x } 10^{-4} \text{x } 1.008 \text{ x } 10^{15} = -3.78 \text{ x } 10^{11}(Bq)$$

January - 2019

1)

The longest wavelength of the Balmer series: From $n = 3$ to $n = 2$

$$\Delta E = hf = \frac{hc}{\lambda} = E_i - E_f = -13.6\left(\frac{1}{n_i^2} - \frac{1}{n_f^2}\right) = -13.6\left(\frac{1}{3^2} - \frac{1}{2^2}\right) = 1.88\ (eV)$$

$$\Rightarrow \lambda_{max} = 660.8\ (nm)$$

The shortest wavelength of the Balmer series: From $n = \infty$ to $n = 2$

$$\Delta E = hf = \frac{hc}{\lambda} = E_i - E_f = -13.6\left(\frac{1}{n_i^2} - \frac{1}{n_f^2}\right) = -13.6\left(\frac{1}{\infty^2} - \frac{1}{2^2}\right)$$
$$= 3.4\ (eV)$$

$$\Rightarrow \lambda_{min} = 365.4\ (nm)$$

Since $365.4 \le \lambda \le 660.8 \rightarrow$ Balmer series falls in visible and ultraviolet region of the electromagnetic spectrum

2)

a) We have: The probability of finding particle from $0 \le x \le \frac{L}{a}$

$$P = \int_0^{L/a} |\psi^2| dx = \int_0^{L/a} \frac{2}{L}\sin^2\left(\frac{n\pi x}{L}\right)dx = \frac{1}{L}\int_0^{L/a}\left(1 - \cos\left(\frac{2n\pi x}{L}\right)\right)dx$$

$$= \frac{1}{L}\left(\frac{L}{a} - \frac{L}{2n\pi}\left(\sin\left(\frac{2n\pi}{a}\right) + sin0\right)\right) = \frac{1}{a} - \frac{1}{2n\pi}\sin\left(\frac{2n\pi}{a}\right)$$

b) When $n \rightarrow \infty$:

$$P = \lim_{n\to\infty}\frac{1}{a} - \frac{1}{2n\pi}\sin\left(\frac{2n\pi}{a}\right) = \lim_{n\to\infty}\frac{1}{a} - \lim_{n\to\infty}\frac{1}{2n\pi}\sin\left(\frac{2n\pi}{a}\right) = \frac{1}{a}$$

The probability is the same as the interval of finding particle (1/a) ⇨Fitting with the classical result

3)

a) We have:

$$\Delta E = E_2 - E_1 = \frac{4h^2}{8mL^2} - \frac{h^2}{8mL^2} = \frac{3h^2}{8mL^2} = \frac{3 \text{ x } (6.62 \text{ x } 10^{-34})^2}{8 \text{ x } 1.67 \text{ x } 10^{-27} \text{ x } (10 \text{ x } 10^{-15})^2}$$
$$= 9.84 \text{ x } 10^{-13} (J)$$

Also: The wavelength of the emitted photon

$$\Delta E = hf = \frac{hc}{\lambda} = 9.84 \text{ x } 10^{-13} (J) \Rightarrow \lambda = 20.2 (pm)$$

b) Since this wavelength is 20.2 pm ⇨ This photon is in the gamma region of the electromagnetic spectrum

4)

In classical physics:

$$s = u\Delta t_0 = 0.99 \text{ x } 3 \text{ x } 10^8 \text{ x } 0.1237 \text{ x } 10^{-6} = 36.74 \ (m)$$

In special relativity:

$$\Delta t = \frac{\Delta t_0}{\sqrt{1 - \frac{u^2}{c^2}}} = \frac{0.1237}{\sqrt{1 - 0.99^2}} = 0.876 \ (\mu m)$$

$$s = u\Delta t = 0.99 \text{ x } 3 \text{ x } 10^8 \text{ x } 0.\ 876 \text{ x } 10^{-6} = 260.43 \ (m)$$

5)

a) The number of ^{40}K nuclei (and thus atom) in this banana:

$$N = 0.0117\% \ N_A \frac{m}{M} = 0.0117\% N_A \frac{0.6}{40} = 1.057 \text{ x } 10^{18} (nuclei)$$

b) The decay constant:

$$T_{half} = \frac{ln2}{\lambda} \rightarrow \lambda = \frac{ln2}{T_{half}} = 1.76 \text{ x } 10^{-17}$$

The activity R of this banana

$$R = \lambda N = 1.76 \text{ x } 10^{-17} \text{ x } 1.057 \text{ x } 10^{18} = 18.58 \ (Bq)$$