REVIEW PHYSICS 1 MIDTERM EXAM

1/ A book slides off a horizontal tabletop with a speed of 1.10 m/s. It strikes the floor in 0.350 s. Ignore air resistance. Find (a) the height of the tabletop above the floor ; (b) the horizontal distance from the edge of the table to the point where the book strikes the floor ; (c) the magnitude and direction of its velocity, just before the book reaches the floor.

EXECUTE: **(a)** $y - y_0 = ?$

$y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2 = 0 + \frac{1}{2}(-9.80 \text{ m/s}^2)(0.350 \text{ s})^2 = -0.600 \text{ m}$. The table top is 0.600 m above the floor.

(b) $x - x_0 = ?$

$x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2 = (1.10 \text{ m/s})(0.350 \text{ s}) + 0 = 0.358 \text{ m}$.

(c) $v_x = v_{0x} + a_x t = 1.10 \text{ m/s}$ (The x-component of the velocity is constant, since $a_x = 0$.)

$v_y = v_{0y} + a_y t = 0 + (-9.80 \text{ m/s}^2)(0.350 \text{ s}) = -3.43 \text{ m/s}$

$$v = \sqrt{v_x^2 + v_y^2} = 3.60 \text{ m/s}$$

$$\tan\alpha = \frac{v_y}{v_x} = \frac{-3.43 \text{ m/s}}{1.10 \text{ m/s}} = -3.118$$

$$\alpha = -72.2°$$

Direction of $\vec{v}$ is $72.2°$ below the horizontal

2/ A woman is driving along a straight highway in her car. At time t = 0, when she is moving at 10 m/s in the positive x-direction, she passes a signpost at x = 50 m. Her acceleration is a function of time : $a = 2.0\, m/s^2 - (1.10\, m/s^2)t$.

(a) Find her velocity and position as functions of time.

(b) When is her velocity greatest ? What is the maximum velocity ?

$$v_x = 10 \text{ m/s} + \int_0^t [2.0 \text{ m/s}^2 - (0.10 \text{ m/s}^3)t]\, dt$$

$$= 10 \text{ m/s} + (2.0 \text{ m/s}^2)t - \frac{1}{2}(0.10 \text{ m/s}^3)t^2$$

$$x = 50 \text{ m} + \int_0^t \left[10 \text{ m/s} + (2.0 \text{ m/s}^2)t - \frac{1}{2}(0.10 \text{ m/s}^3)t^2\right] dt$$

$$= 50 \text{ m} + (10 \text{ m/s})t + \frac{1}{2}(2.0 \text{ m/s}^2)t^2 - \frac{1}{6}(0.10 \text{ m/s}^3)t^3$$

$$0 = 2.0 \text{ m/s}^2 - (0.10 \text{ m/s}^3)t$$

$$t = \frac{2.0 \text{ m/s}^2}{0.10 \text{ m/s}^3} = 20 \text{ s}$$

$$v_{\text{max-}x} = 10 \text{ m/s} + (2.0 \text{ m/s}^2)(20 \text{ s}) - \frac{1}{2}(0.10 \text{ m/s}^3)(20 \text{ s})^2$$

$$= 30 \text{ m/s}$$

3/ A baseball is hit so that it leaves the ground at speed $v_0 = 37.0$ m/s and at an angle $\alpha_0 = 53.1°$.

(a) Find the position of the ball, the magnitude and the direction of its velocity at t = 2.00 s.

(b) Find the time when the ball reaches the highest point of its flight and find its height at this point.

$$v_{0x} = v_0 \cos\alpha_0 = (37.0 \text{ m/s}) \cos 53.1° = 22.2 \text{ m/s}$$
$$v_{0y} = v_0 \sin\alpha_0 = (37.0 \text{ m/s}) \sin 53.1° = 29.6 \text{ m/s}$$

EXECUTE: (a) We want to find x, y, v_x, and v_y at time $t = 2.00$ s. From Eqs. (3.20) through (3.23),

$$x = v_{0x}t = (22.2 \text{ m/s})(2.00 \text{ s}) = 44.4 \text{ m}$$
$$y = v_{0y}t - \frac{1}{2}gt^2$$
$$= (29.6 \text{ m/s})(2.00 \text{ s}) - \frac{1}{2}(9.80 \text{ m/s}^2)(2.00 \text{ s})^2$$
$$= 39.6 \text{ m}$$
$$v_x = v_{0x} = 22.2 \text{ m/s}$$
$$v_y = v_{0y} - gt = 29.6 \text{ m/s} - (9.80 \text{ m/s}^2)(2.00 \text{ s})$$
$$= 10.0 \text{ m/s}$$

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{(22.2 \text{ m/s})^2 + (10.0 \text{ m/s})^2}$$
$$= 24.3 \text{ m/s}$$
$$\alpha = \arctan\left(\frac{10.0 \text{ m/s}}{22.2 \text{ m/s}}\right) = \arctan 0.450 = 24.2°$$

$$v_y = v_{0y} - gt_1 = 0$$
$$t_1 = \frac{v_{0y}}{g} = \frac{29.6 \text{ m/s}}{9.80 \text{ m/s}^2} = 3.02 \text{ s}$$

$$h = v_{0y}t_1 - \frac{1}{2}gt_1^2$$
$$= (29.6 \text{ m/s})(3.02 \text{ s}) - \frac{1}{2}(9.80 \text{ m/s}^2)(3.02 \text{ s})^2$$
$$= 44.7 \text{ m}$$

4/ A river flows due south with a speed of 2.0 m/s. A man steers a motorboat across the river; his velocity relative to the water is 4.2 m/s due east. The river is 800 m wide.

(a) What is his velocity (magnitude and direction) relative to the earth?

(b) How much time is required to cross the river?

EXECUTE: $v_{\text{ME}} = \sqrt{v_{\text{MW}}^2 + v_{\text{WE}}^2} = \sqrt{(4.2 \text{ m/s})^2 + (2.0 \text{ m/s})^2} = 4.7 \text{ m/s}$ $\tan\theta = \frac{v_{\text{MW}}}{v_{\text{WE}}} = \frac{4.2 \text{ m/s}}{2.0 \text{ m/s}} = 2.10$; $\theta = 65°$; or $\phi = 90° - \theta = 25°$. The velocity of the man relative to the earth has magnitude 4.7 m/s and direction 25° S of E.

$$t = \frac{x - x_0}{v_x} = \frac{800 \text{ m}}{4.2 \text{ m/s}} = 190 \text{ s}.$$

5/ (a) Prove that a projectile launched at angle α_0 has the same horizontal range as one launched with the same speed at angle (90^o - α_0).

(b) A frog jumps at a speed of 2.2 m/s and lands 25 cm from its starting point. At which angles above the horizontal could it have jumped?.

$$R_1 = (v_0 \cos\alpha_0)\left(\frac{2v_0 \sin\alpha_0}{g}\right)$$

$$R_2 = (v_0 \cos(90° - \alpha_0))\left(\frac{2v_0 \sin(90° - \alpha_0)}{g}\right)$$

Thus $R_2 = (v_0 \sin\alpha_0)\left(\frac{2v_0 \cos\alpha_0}{g}\right) = (v_0 \cos\alpha_0)\left(\frac{2v_0 \sin\alpha_0}{g}\right) = R_1$.

(b) $R = \frac{v_0^2 \sin 2\alpha_0}{g}$ so $\sin 2\alpha_0 = \frac{Rg}{v_0^2} = \frac{(0.25 \text{ m})(9.80 \text{ m/s}^2)}{(2.2 \text{ m/s})^2}$.

This gives $\alpha = 15°$ or $75°$.

6/ On level ground an object is fired with an initial velocity of 80.0 m/s at 60.0° above the horizontal and feels no appreciable air resistance.

(a) Find its maximum height above the ground.

(b) How far from its firing point does this object land?

(c) At its highest point, find the horizontal and vertical components of its acceleration and velocity.

EXECUTE: **(a)** $v_{0x} = v_0 \cos\alpha_0 = (80.0 \text{ m/s})\cos 60.0° = 40.0 \text{ m/s}$, $v_{0y} = v_0 \sin\alpha_0 = (80.0 \text{ m/s})\sin 60.0° = 69.3 \text{ m/s}$.

(b) At the maximum height $v_y = 0$. $v_y = v_{0y} + a_y t$ gives $t = \frac{v_y - v_{0y}}{a_y} = \frac{0 - 69.3 \text{ m/s}}{-9.80 \text{ m/s}^2} = 7.07 \text{ s}$.

(c) $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ gives $y - y_0 = \frac{v_y^2 - v_{0y}^2}{2a_y} = \frac{0 - (69.3 \text{ m/s})^2}{2(-9.80 \text{ m/s}^2)} = 245 \text{ m}$.

(d) The total time in the air is twice the time to the maximum height, so
$x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2 = (40.0 \text{ m/s})(14.14 \text{ s}) = 566 \text{ m}$.

(e) At the maximum height, $v_x = v_{0x} = 40.0 \text{ m/s}$ and $v_y = 0$. At all points in the motion, $a_x = 0$ and $a_y = -9.80 \text{ m/s}^2$.

EVALUATE: The equation for the horizontal range R derived in Example 3.8 is $R = \frac{v_0^2 \sin 2\alpha_0}{g}$. This gives

$R = \frac{(80.0 \text{ m/s})^2 \sin(120.0°)}{9.80 \text{ m/s}^2} = 566 \text{ m}$, which agrees with our result in part (d).