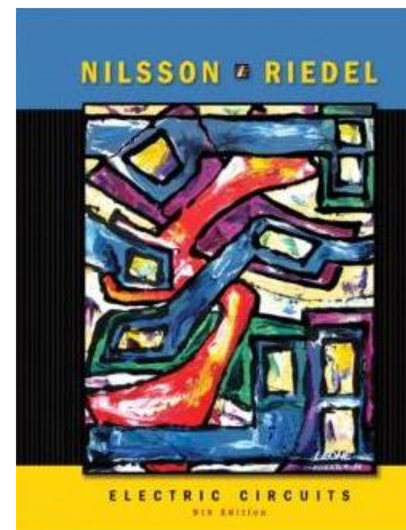
International University

School of Electrical Engineering

PRINCIPLES OF ELECTRICAL ENGINEERING 2

Lecture # 7: Introduction to frequency selective circuits

Chapter #14



Text book: Electric Circuits

James W. Nilsson & Susan A. Riedel

9th Edition.

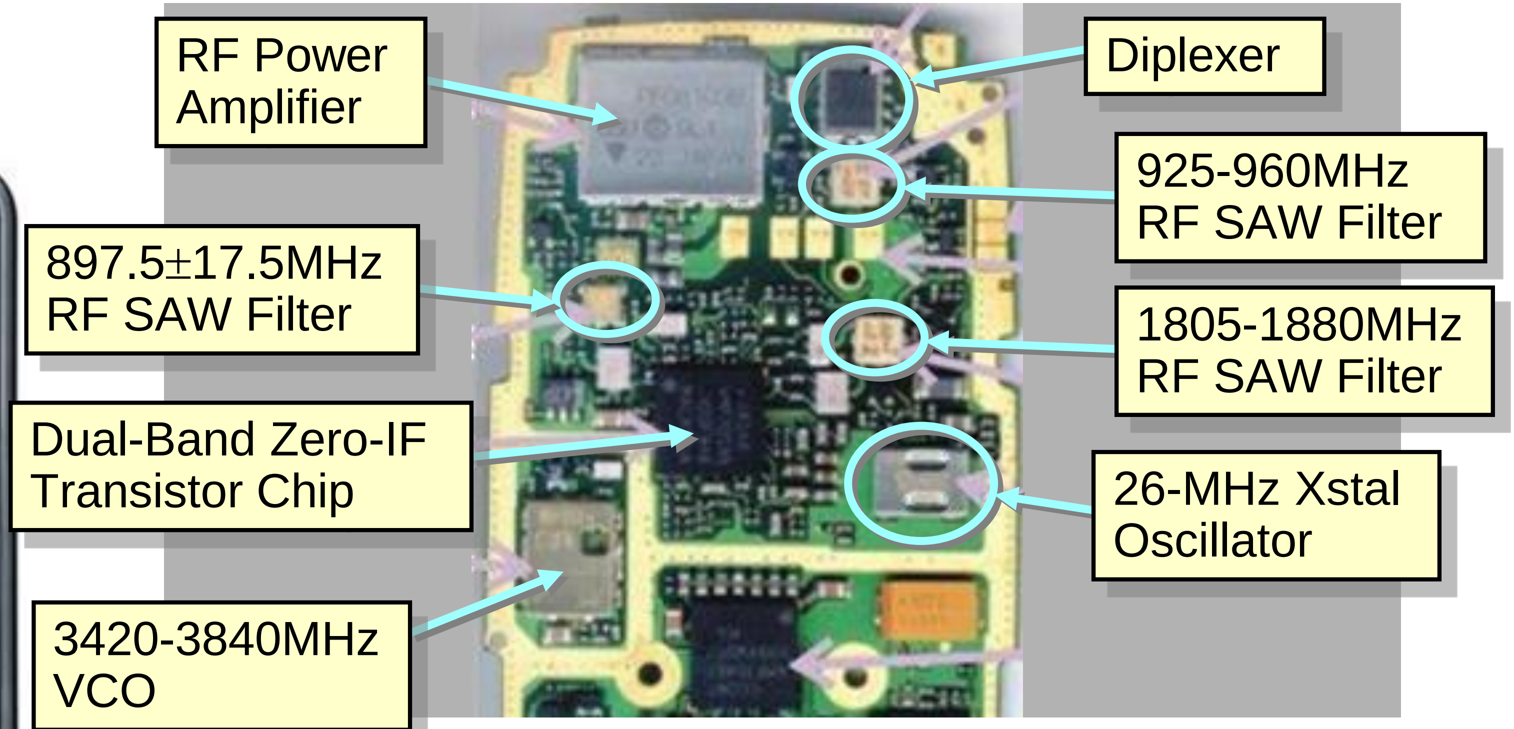
link: **http://blackboard.hcmiu.edu.vn/**

to download materials

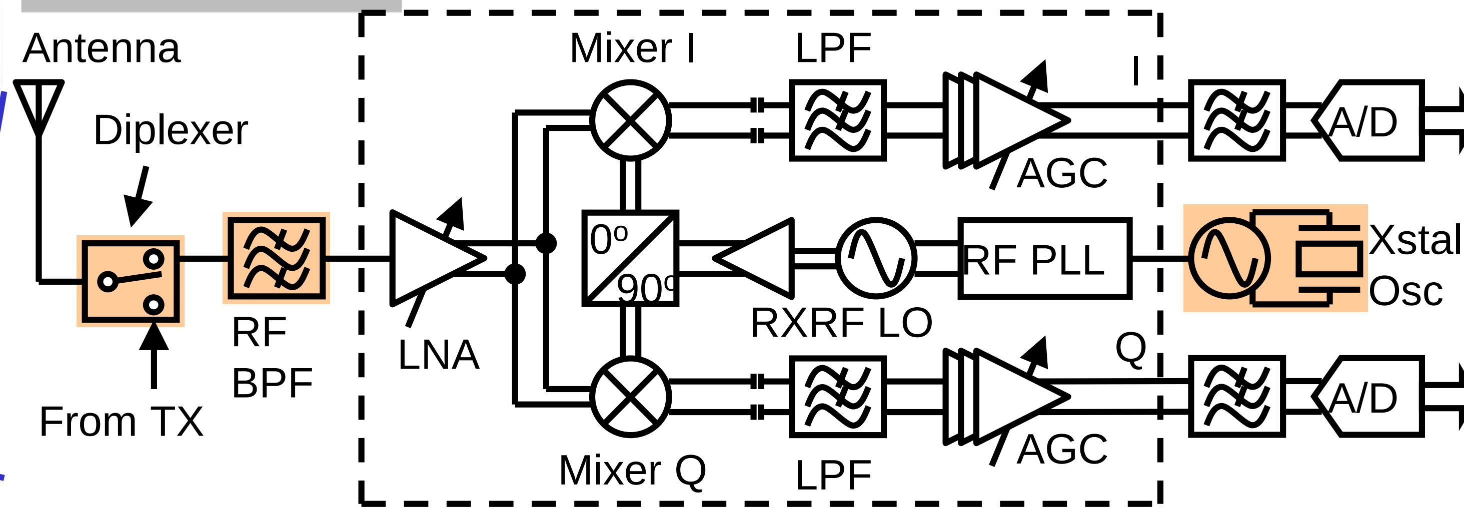
Overview

- We have seen that the response of a circuit depends on the types of elements, the way the elements are connected, and the impedance of the elements that varies with frequency.
- In this chapter, we analyze the effect of varying source frequency on circuit voltages and currents. In particular, the circuits made of passive elements (R, L, C) that pass only a finite range of input frequencies.

Cell phone's RF Front-Ends



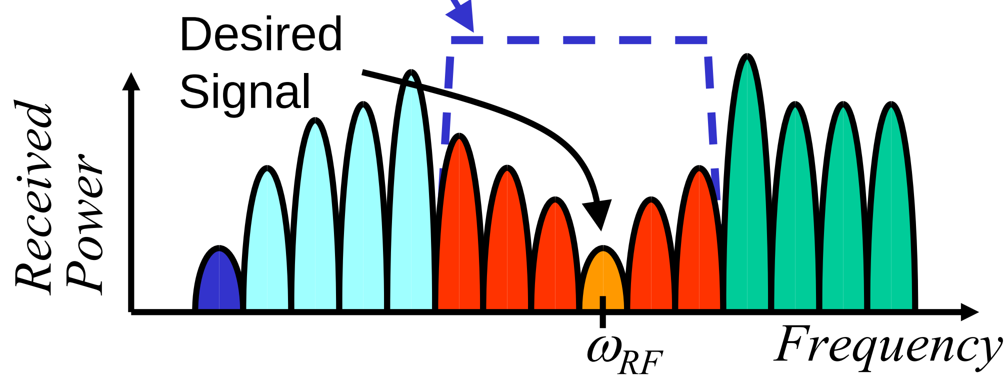
Wireless Phone



Frequency Selection

The higher the Q of the Pre-Select Filter ⇨ the simpler the demodulation electronics

Presently use resonators with Q's ~ 400

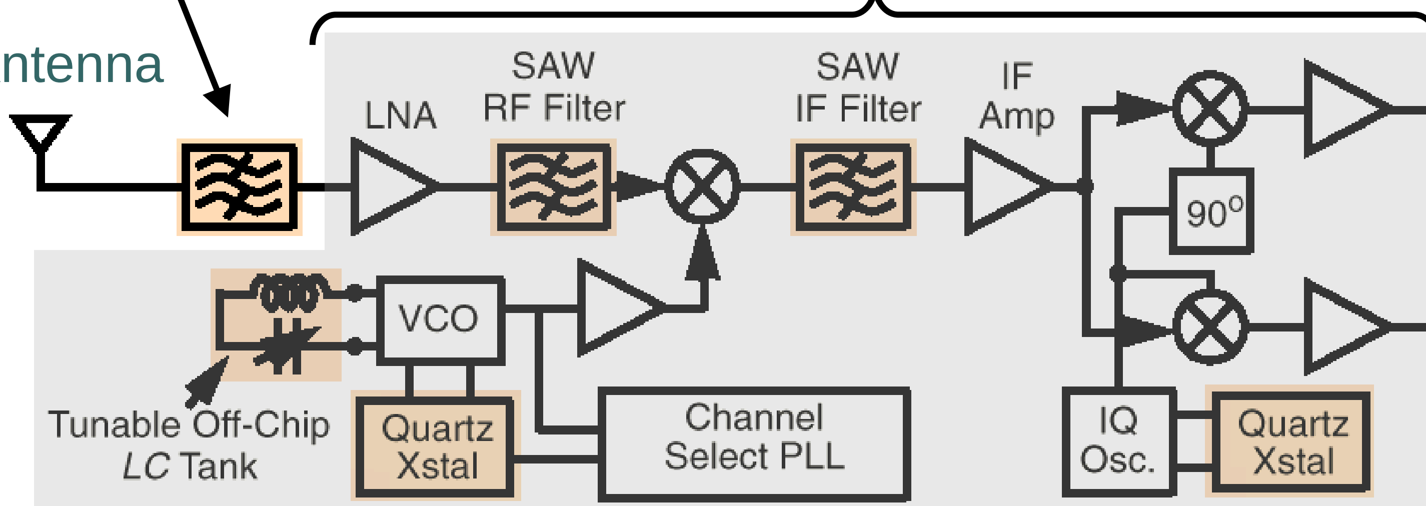


Pre-Select Filter in the GHz Range

Antenna

Wireless Phone

Demodulation Electronics



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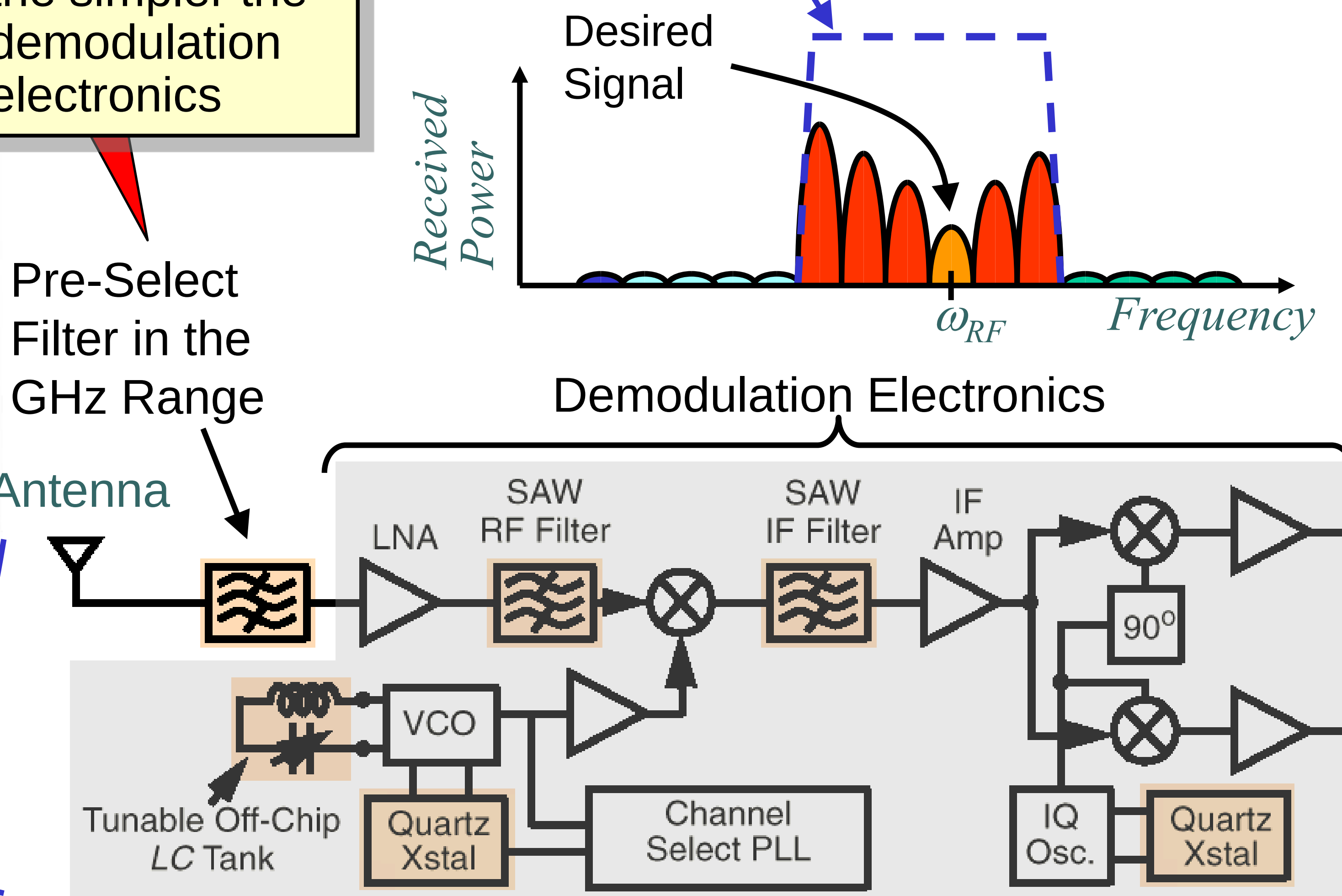

Frequency Selection

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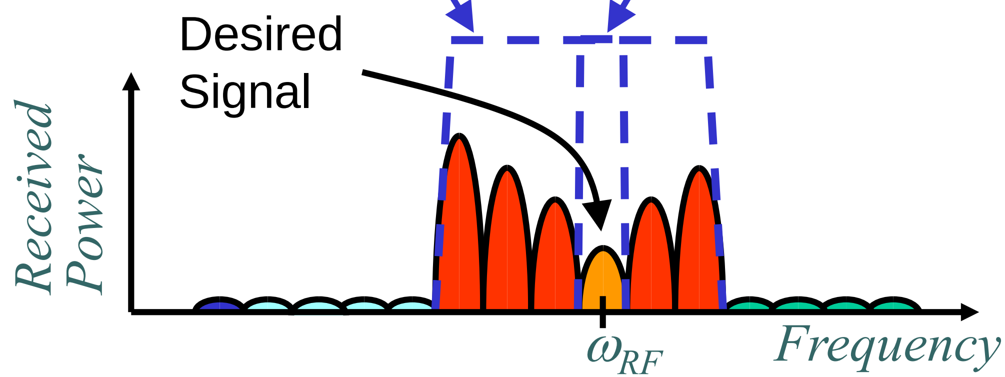
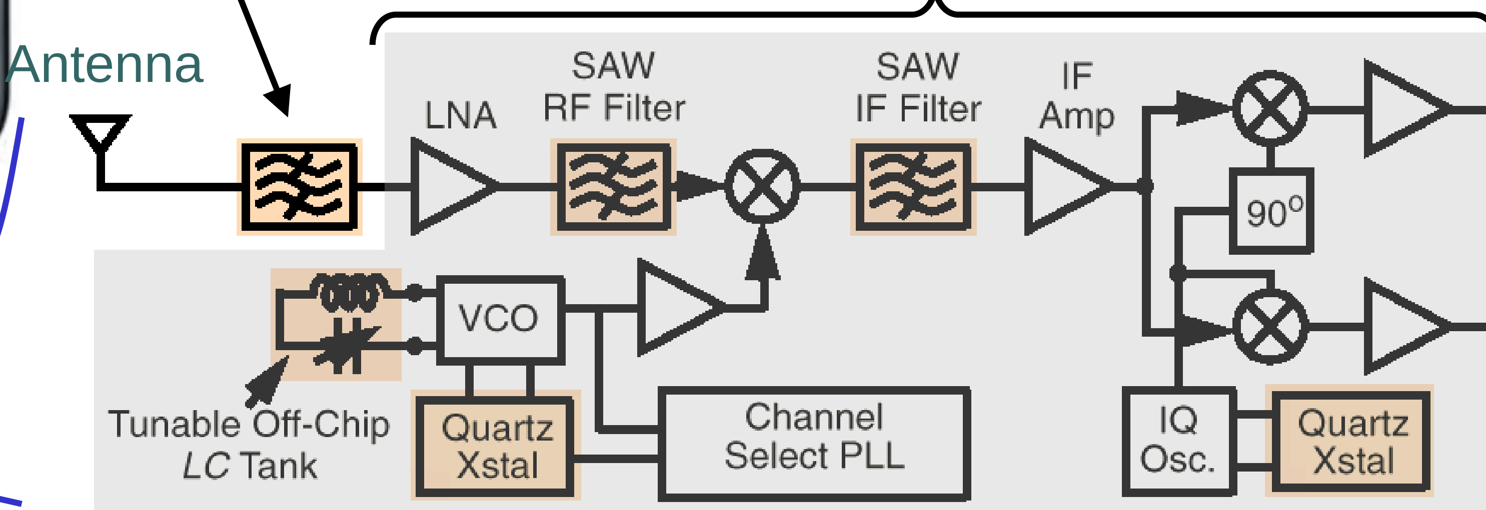
If can have resonator *Q*'s > 10,000



Pre-Select Filter in the GHz Range

Demodulation Electronics



Wireless Phone

Frequency Selection

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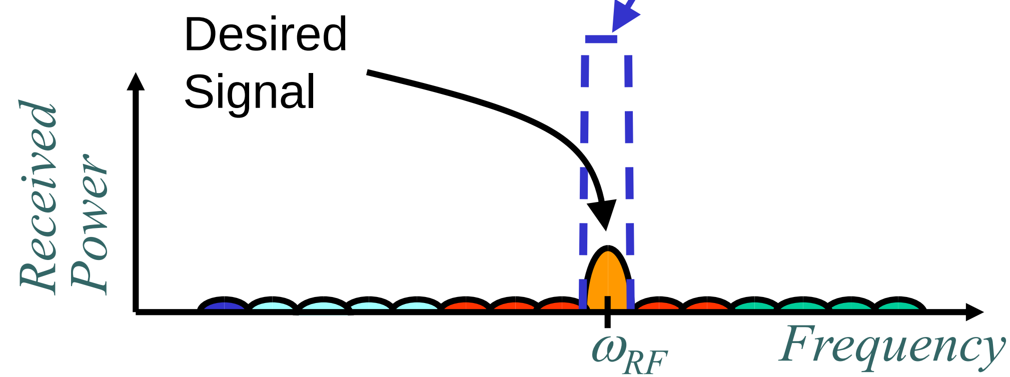
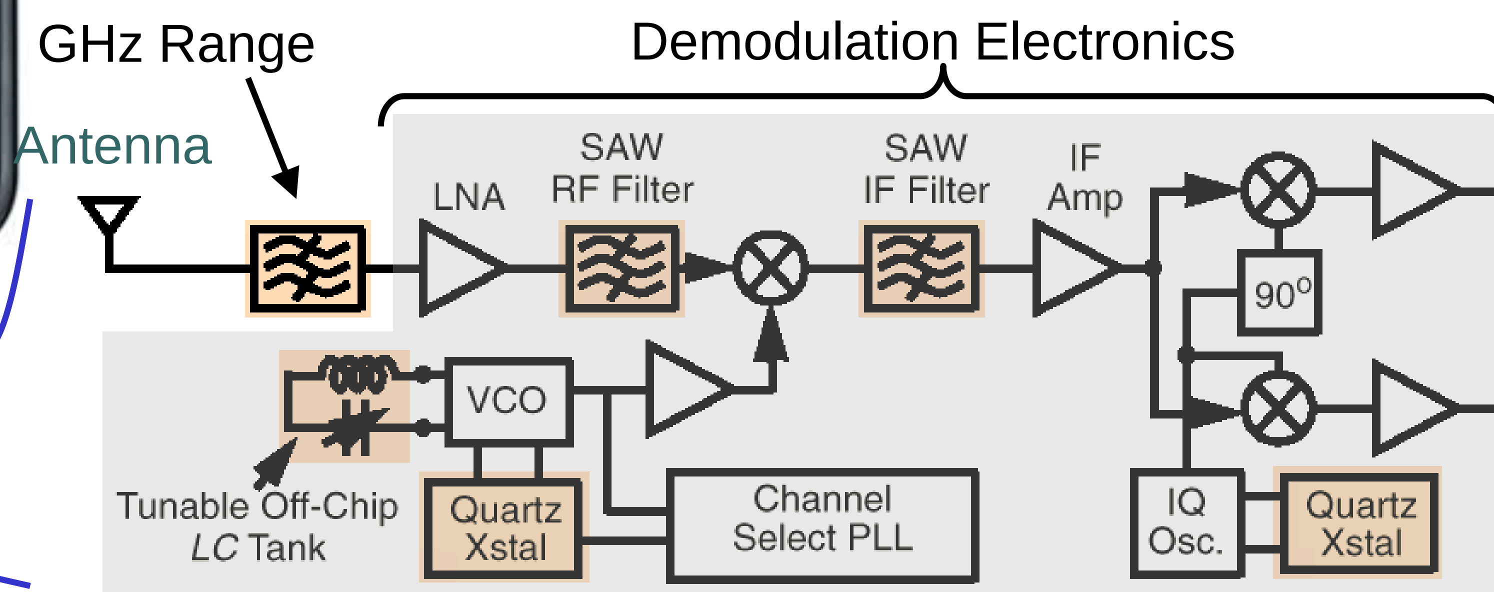


Wireless
Phone

Pre-Select
Filter in the
GHz Range



CONTENTS

1. Introductions
2. Low-Pass Filters
3. High-Pass Filters
4. Bandpass Filters
5. Bandreject Filters

Objectives

1. Know the RL & RC circuit configurations that act as low-pass filters/ high-pass filters and be able to design RL and RC circuit component values to meet a specified cutoff frequency.
2. Know the *RLC* circuit configurations that act as bandpass filters, understand the definition of and relationship among the center frequency, cutoff frequencies, bandwidth, and quality factor of a bandpass filter, and be able to design *RLC* circuit component values to meet design specifications.
3. Know the *RLC* circuit configurations that act as bandreject filters, understand the definition of and relationship among the center frequency, cutoff frequencies, bandwidth, and quality factor of a bandreject filter, and be able to design *RLC* circuit component values to meet design specifications.

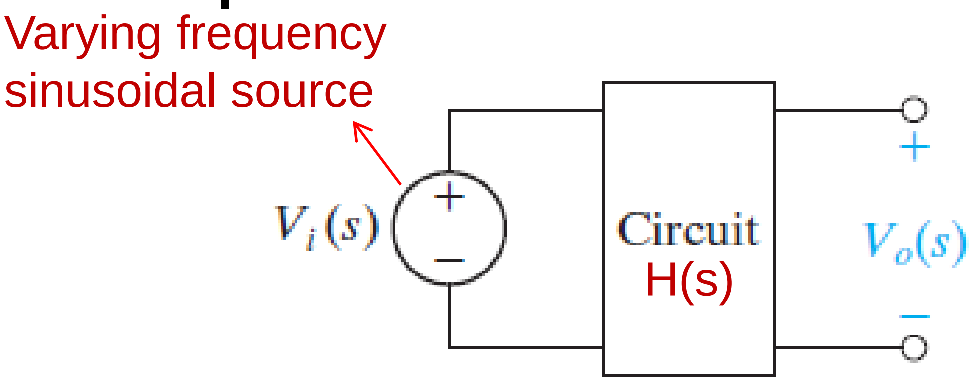
Introduction

Impedance of some elements (L and C) depends on the frequency of the source. Hence, we can construct a circuit that pass to the output only those input signal that reside in a desired range of frequencies. Such circuits are called frequency-selective circuits, or filters.

In this lecture, we will analyze the following filter types:

- Low-pass filters
- High-pass filters
- Band-pass filters
- Band-reject filters

Introduction



The transfer function

$H(s) = V_0(s)/V_i(s)$

- *Frequency response* is the results of the analysis the effect of varying source frequency on circuit voltages and currents.

The steady-state response due to a sinusoidal input $A\cos\omega t$ is determined by sampling the transfer function $H(s)$ along the imaginary axis, i.e. $H(j\omega)$.

Since $H(j\omega) \in C$, a frequency response plot consists of two parts: (1) magnitude plot $|H(j\omega)|$, (2) phase angle plot $\theta(j\omega)$.

Introduction

- A *frequency selective circuit*, or *filter*, enables signals at certain frequencies to reach the output, and it attenuates signals at other frequencies to prevent them from reaching the output.

➢ Filters are circuits that are capable of *passing signals within a band* of frequencies while *rejecting or blocking* signals of frequencies *outside this band*. This property of filters is also called "frequency selectivity".

➢ Filter can be passive or active filter.

Passive filters: The circuits built using RC, RL, or RLC circuits.

Active filters : The circuits that employ one or more op-amps in the design an addition to resistors and capacitors

Introduction

- *Frequency response plot* shows how a circuit's transfer function (both amplitude and phase) changes as the source frequency changes. A frequency response plot has two parts: *magnitude plot* ($|H(j\omega)|$) and *phase angle plot* ($\theta(j\omega)$).

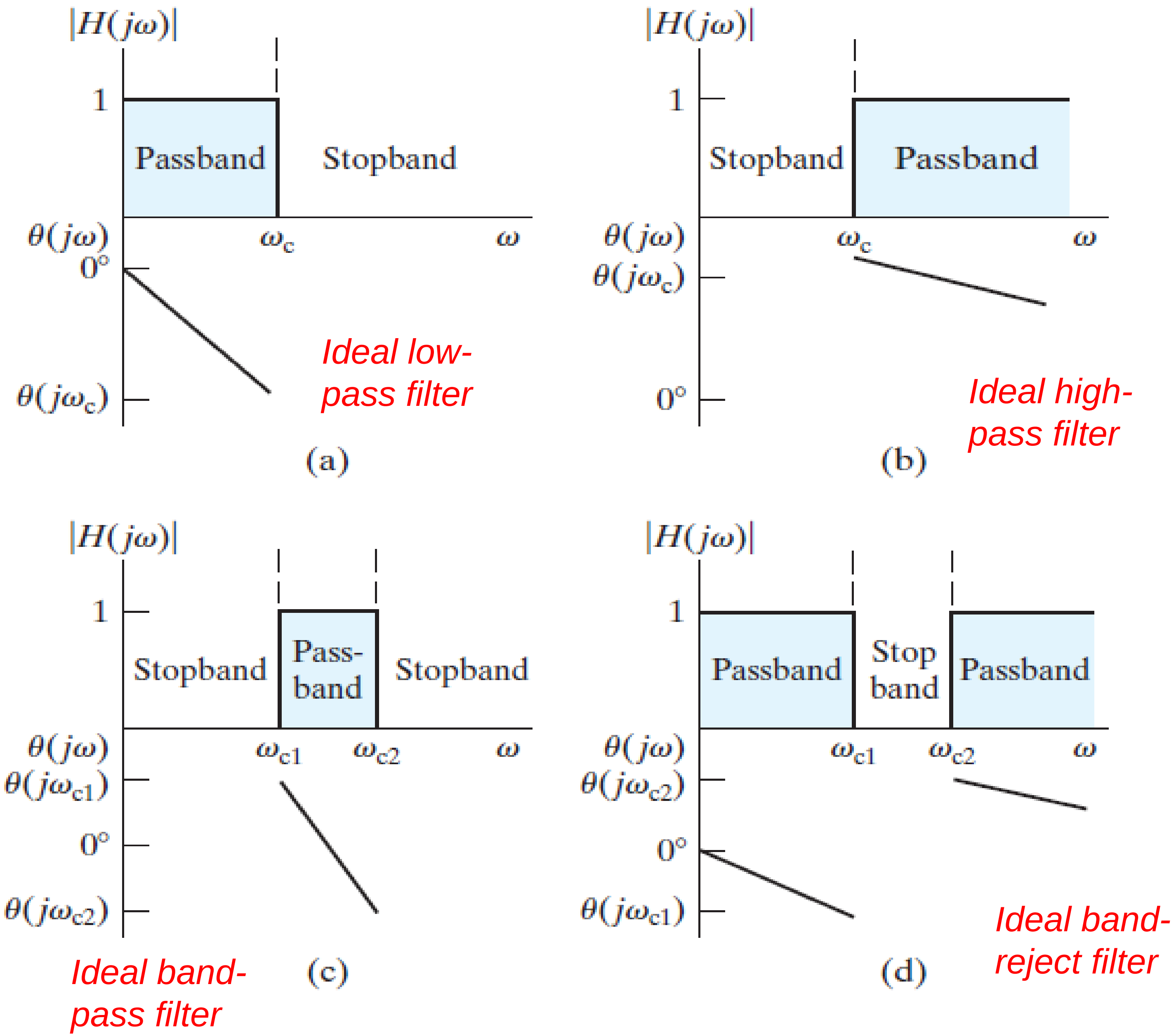
- The *cutoff frequency*, ω_c, identifies the location on the frequency axis that separates the stopband from the passband. At the cutoff frequency, the magnitude of the transfer function decreased by the factor $1/\sqrt{2}$

$$\left|H\left(j\omega_c\right)\right| = \frac{1}{\sqrt{2}} H_{\max}$$

- The average power delivered by the circuit is one half the maximum average power $P(j\omega_c) = P_{max}/2$ → ω_c is also called the half-power frequency

- There are 4 types of filter circuits: (1) low-pass filter, (2) high-pass filter, (3) band-pass filter, (4) band-reject filter.

Introduction

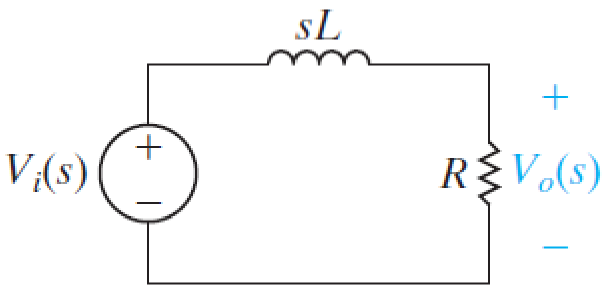


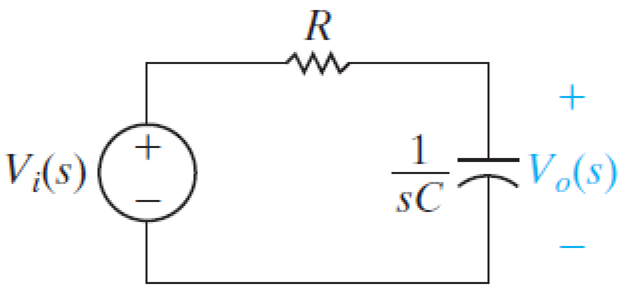
Low-pass filters

A *low-pass filter* passes voltages at frequencies below ω_c, and attenuates frequencies above ω_c . Any circuit with the transfer function:

$$\boxed{H(s) = \frac{\omega_c}{s + \omega_c}}$$

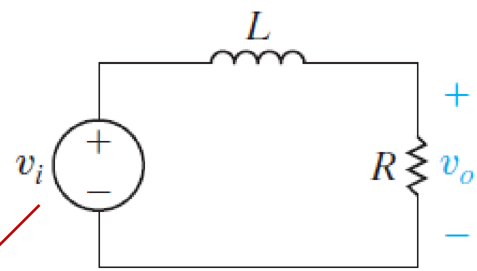
functions as a low-pass filter.





Low-pass filters

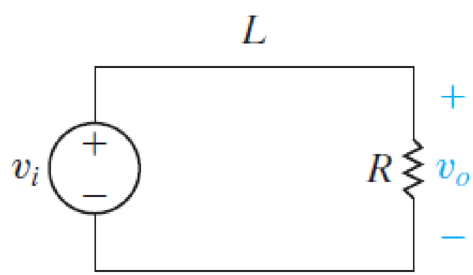
The series RL circuit



(a)

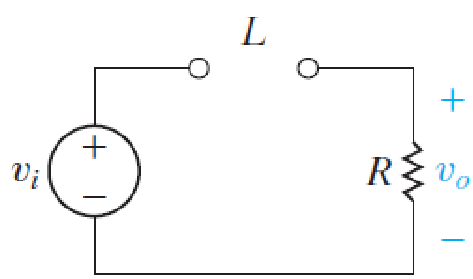
sinusoidal voltage source with varying frequency

As ω↑ the impedance of the inductor (jωL) ↑ relative to the impedance of the resistor, and the source voltage divides between the resistive impedance and the inductive impedance.



(b)

@ Zero frequency (ω = 0): the impedance of the *inductor (j*ωL*)* is *zero*, and the inductor acts as a short circuit. The input and output voltages are thus the same.

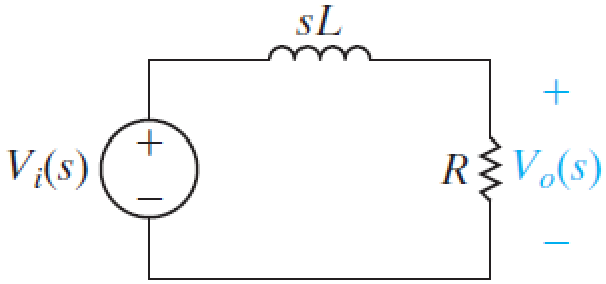


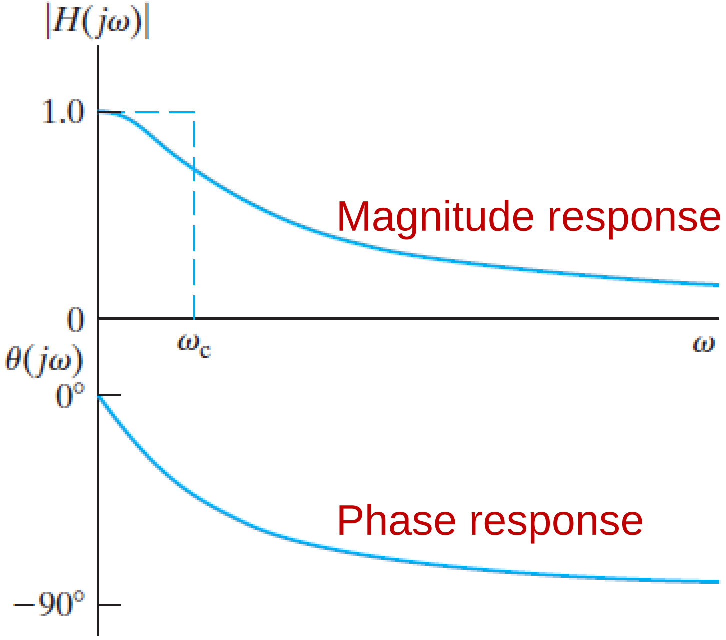
(c)

@ Infinite frequency (ω = ∞): the impedance of the *inductor* is *infinite*, and the inductor acts as a open circuit. The output voltage is thus zero.

Low-pass filters

The series RL circuit





Voltage transfer function:

$$H(s)=\frac{R/L}{s+R/L} \quad \text{or} \quad H(j\omega)=\frac{R/L}{j\omega+R/L}$$

→ Transfer function magnitude:

$$|H(j\omega)|=\frac{R/L}{\sqrt{\omega^2+(R/L)^2}}$$

→ Transfer function phase angle:

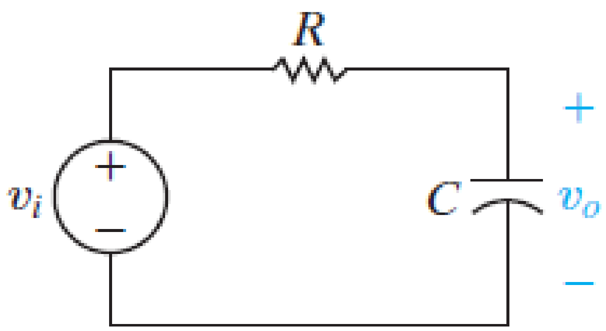
$$\theta(j\omega)=-\tan^{-1}\left(\frac{\omega L}{R}\right)$$

Cutoff frequency for RL filters:

$$\boxed{\omega_c=R/L}$$

Low-pass filters

The series RC circuit

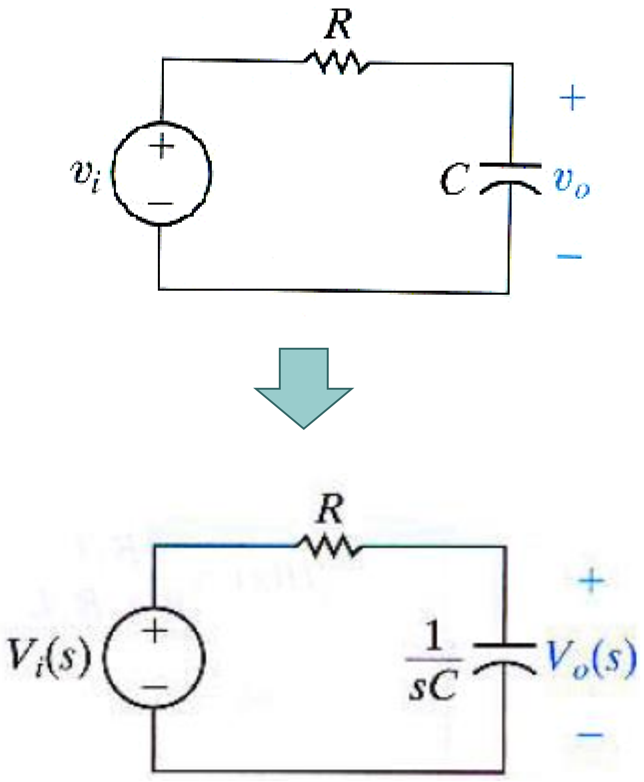


The impedance of the capacitor *decreases* relative to the impedance of the resistor, and the source voltage divides between the resistive impedance and the capacitive impedance. The output voltage is thus smaller than the source voltage.

Zero frequency ($\omega = 0$): the impedance of the *capacitor* is *infinite*, and the capacitor acts as a open circuit. The input and output voltages are thus the same.

Infinite frequency ($\omega = \infty$): the impedance of the *capacitor* is *zero*, and the capacitor acts as a short circuit. The output voltage is thus zero.

Low-pass filters
The series RC circuit



Transfer function:

$$H(s) = \frac{1/RC}{s + 1/RC}$$

$$H(j\omega) = \frac{1/RC}{j\omega + 1/RC}$$

Transfer function magnitude:

$$|H(j\omega)| = \frac{1/RC}{\sqrt{\omega^2 + (1/RC)^2}}$$

Cutoff frequency for RL filters:

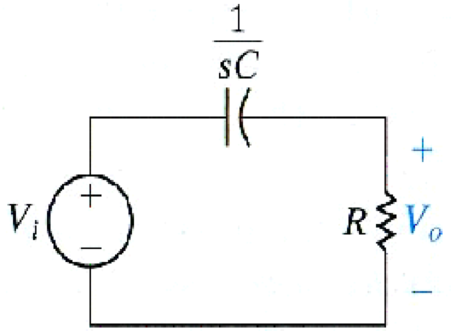
$$\omega_c = 1/RC$$

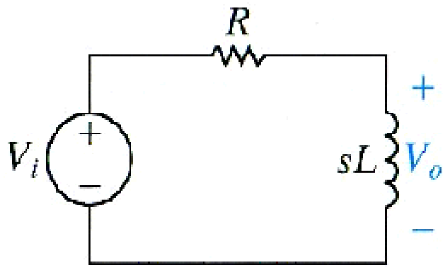
High-pass filters

A *high-pass filter* passes voltages at frequencies above ω_c, and attenuates frequencies below ω_c . Any circuit with the transfer function:

$$\boxed{H(s) = \frac{s}{s+\omega_c}}$$

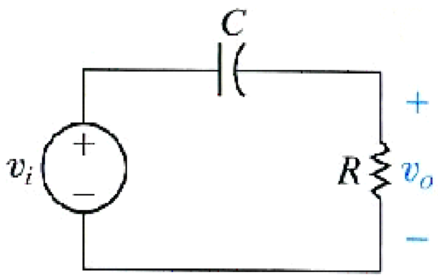
functions as a high-pass filter.



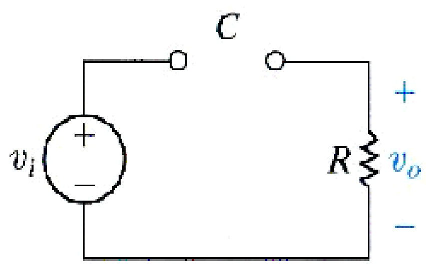


High-pass filters

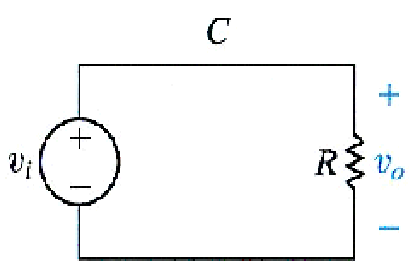
The series RC circuit



As the frequency (ω) of the voltage source ↑, the impedance of the capacitor ↓ relative to the impedance of the resistor, and the source voltage is divided between the capacitor and the resistor. The output voltage magnitude begins to increase

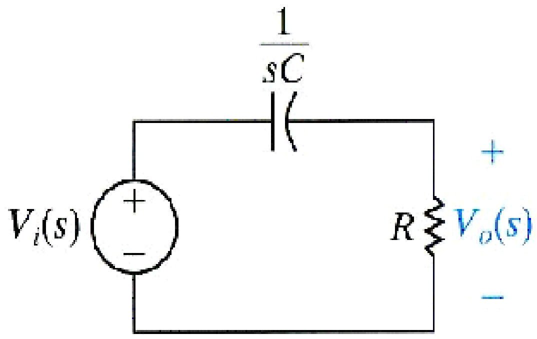


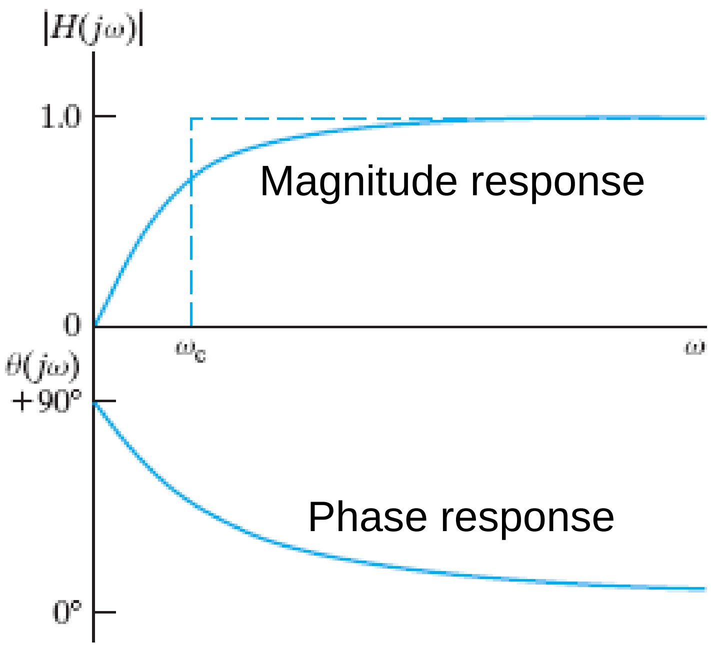
At ω = 0, the capacitor behaves as an *open circuit*. There is no voltage across the resistor, and the circuit filters out the low frequency source voltage before it reaches the circuit's output.



At ω = ∞, the capacitor behaves as an *short circuit*. There is no voltage across the capacitor. Therefore, the input and output voltages are the same.

High-pass filters
The series RC circuit





Transfer function:

$$H(s) = \frac{s}{s + 1/RC} \quad \text{or} \quad H(j\omega) = \frac{j\omega}{j\omega + 1/RC}$$

Transfer function magnitude:

$$|H(j\omega)| = \frac{\omega}{\sqrt{\omega^2 + (1/RC)^2}}$$

Transfer function phase angle:

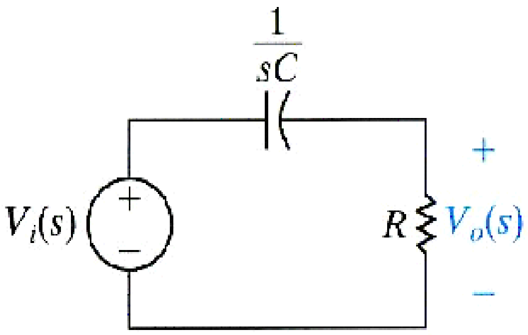
$$\theta(j\omega) = 90 - \tan^{-1}\omega RC$$

Cutoff frequency for RC filters:

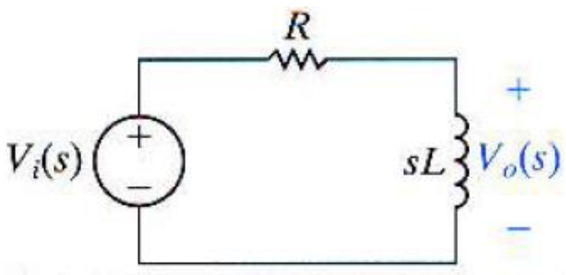
$$\omega_c = 1/RC$$

High-pass filters

The series RC circuit



Example: choose values for R and L that will yield a highpass filter with a cutoff frequency of 15 kHz.

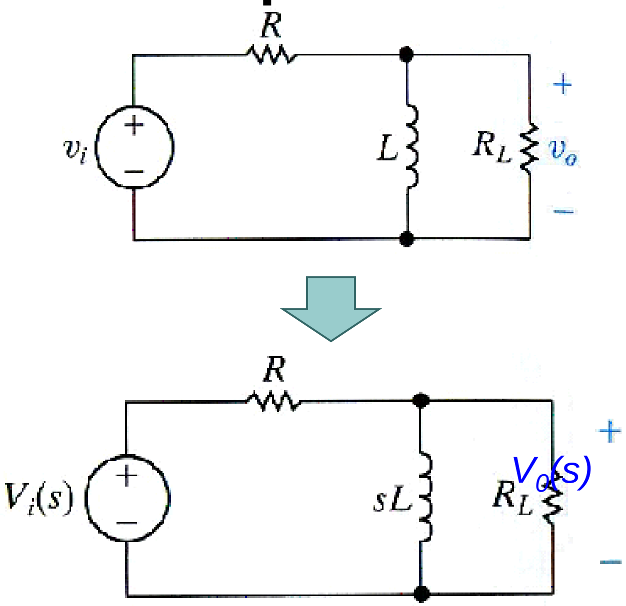


Sol: Let's arbitrarily select a value of 500 Ω for R. Remember to convert the cutoff frequency to radians per second

$$L = \frac{R}{\omega_c} = \frac{500}{(2\pi)(15{,}000)} = 5.31 \text{ mH}.$$

High-pass filters

The series RL circuit



<u>Examine</u> the effect of placing a load resistor in parallel with the inductor in the **RL high-pass filter**.

a) Determine the transfer function for the circuit.
b) Sketch the magnitude plot for the loaded R_L, high-pass filter, using the values for R and L from the circuit in previous example and letting R_L = R. On the same graph, sketch the magnitude plot for the unloaded **RL high-pass filter** of previous
example.

<u>**Sol:**</u> a) From the S-domain, the transfer function:

$$H(s)=\frac{\dfrac{R_L sL}{R_L+sL}}{R+\dfrac{R_L sL}{R_L+sL}}=\frac{\left(\dfrac{R_L}{R+R_L}\right)s}{s+\left(\dfrac{R_L}{R+R_L}\right)\dfrac{R}{L}}=\frac{Ks}{s+\omega_C}$$

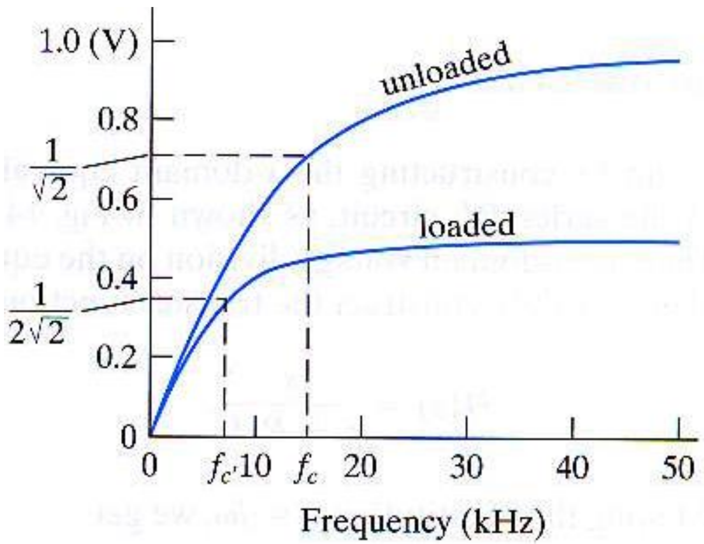
where
$$\begin{cases}\omega_C = K.R/L \\ K=\dfrac{R_L}{R+R_L}\end{cases}$$

ω_c is the cutoff frequency of the loaded filter

High-pass filters

The series RL circuit

b) For the unloaded RL high-pass filter from previous example the passband magnitude is 1, and the cutoff frequency is 15 kHz. For the loaded RL high-pass filter, R = R_L = 500Ω, so K = 1/2. Thus, for the loaded filter, the passband magnitudies (1)(1/2)= 1/2, and the cutoff frequency is 15,000×(1/2) = 7.5 kHz.



For the unloaded RL high-pass filter: K = 1

For loaded RL high-pass filter:

$$K = \frac{R_L}{R + R_L}$$

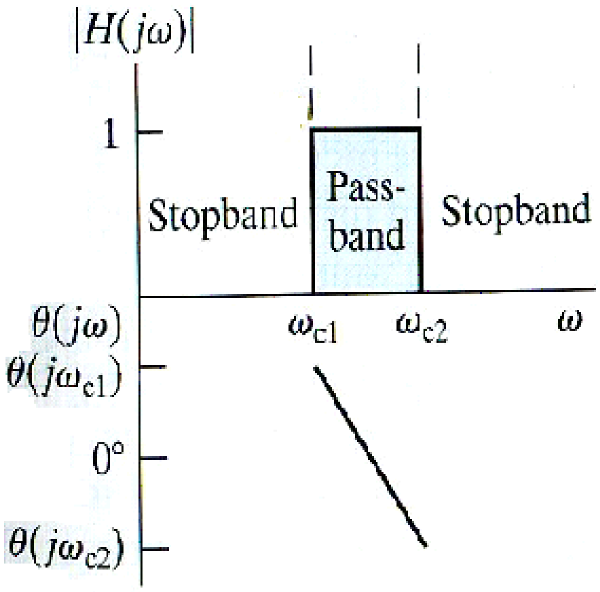
For K < 1, the effect of the load resistor is to reduce the passband magnitude by the factor K and to lower the cutoff frequency by the same factor.

Bandpass filters

A *bandpass filter* passes voltages at frequencies within the passband, which is between ω_{c1} and ω_{c2} . It attenuates frequencies outside of the passband.

Any circuit with the transfer function:

$$\boxed{H(s) = \frac{\beta s}{s^2 + \beta s + \omega_0^2}}$$



functions as a bandpass filter.

Bandpass filters

Center frequency , ω_0 , or *resonant frequency* , defined as the frequency for which a circuit's transfer function is purely real.

The center frequency is the geometric center of the passband

$$\omega_0 = \sqrt{\omega_{c1}\omega_{c2}}$$

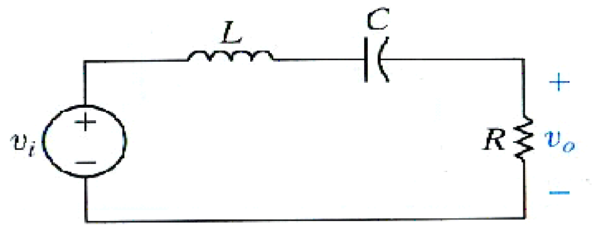
For bandpass filters, the magnitude of the transfer function is a maximum at the center frequency

$$H_{\max} = \left|H\left(j\omega_0\right)\right|$$

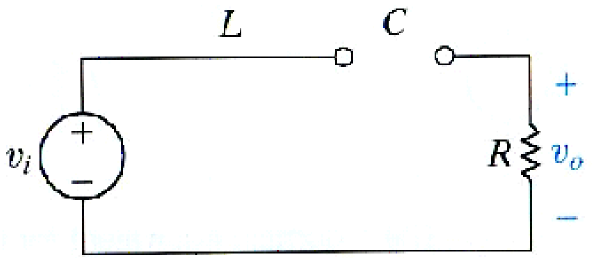
Bandwidth, *β*, is the width of the passband.

Quality factor, *Q*, is the ratio of the center frequency to the bandwidth. The quality factor gives a measure of the width of the passband, independent of its location on the frequency axis. It also describes the shape of the magnitude plot, independent of frequency.

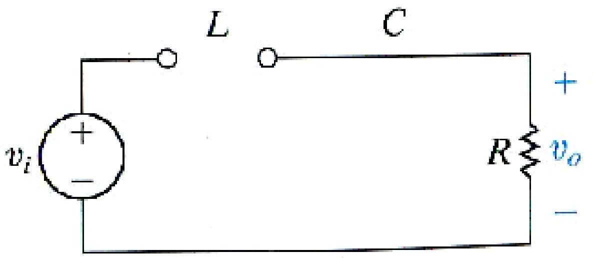
Bandpass filters
The series RLC circuit



At ω = 0, the capacitor behaves like an open circuit, and the inductor behaves like a short circuit. This results the zero output voltage.



At ω = ∞, the capacitor behaves like a short circuit, and the inductor behaves like an open circuit. This results the zero output voltage.

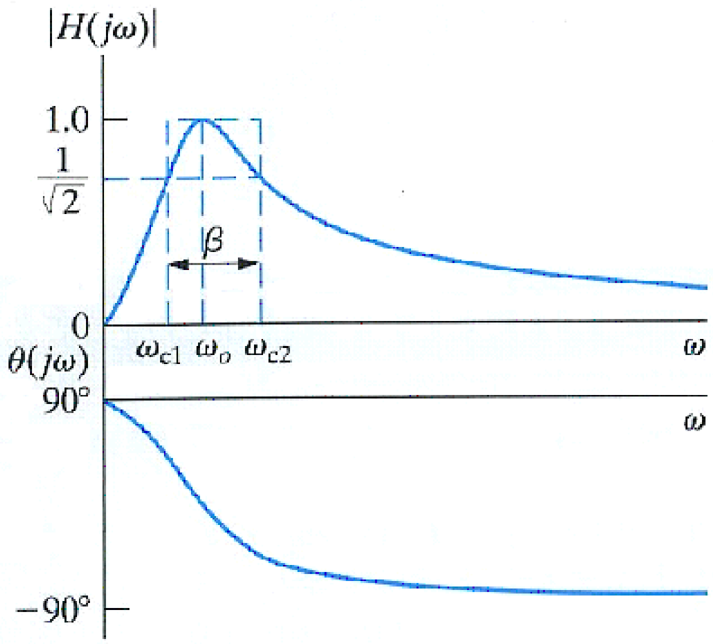


Between ω = 0 and ω = ∞, the output voltage changes depends on the changes of the impdedance of the capacitor and the inductor.

Bandpass filters

The series RLC circuit

Frequency response plot for the series RLC bandpass filter circuit



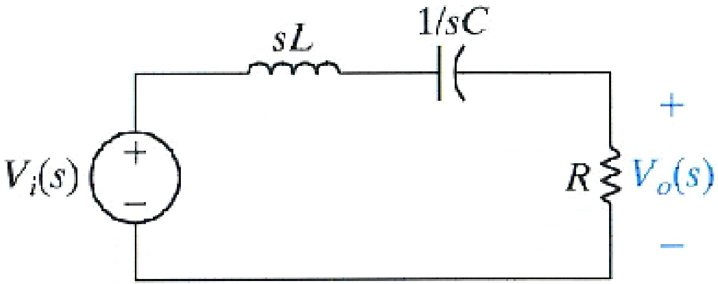
At center frequency, the output voltage is equal to the source voltage; the phase angles of the source and output voltage are also the same.

As the frequency ↓, the phase angle contribution from the capacitor is larger than that from the inductor. Because the capacitor contributes positive phase shift, the net phase angle at the output is positive. At very low frequencies, the phase angle at the output maximizes at $+90^0$.

Conversely, if the frequency ↑ from the frequency at which the source and the output voltage are in phase, the phase angle contribution from the inductor is larger than that from the capacitor. The inductor con tributes negative phase shift, so the net phase angle at the output is negative. At very high frequencies, the phase angle at the output reaches its negative maximum of -90^0.

Bandpass filters

The series RLC circuit



The transfer function

$$H(s) = \frac{V_0(s)}{V_i(s)} = \frac{(R/L)s}{s^2 + (R/L)s + (1/LC)}$$

Substitute s = jω, we have:

Magnitude of the transfer function

$$|H(j\omega)| = \frac{\omega(R/L)}{\sqrt{[(1/LC) - \omega^2]^2 + [\omega(R/L)]^2}}$$

Phase angle of the transfer function

$$\theta(j\omega) = 90^\circ - \tan^{-1}\left[\frac{\omega(R/L)}{(1/LC) - \omega^2}\right]$$

Center frequency

$$\omega_0 = \sqrt{\frac{1}{LC}}$$

Bandpass filters

The series RLC circuit

calculate the cutoff frequencies ω_1 and ω_2.

$$H_{max} = |H(j\omega_o)| = \frac{\omega_o(R/L)}{\sqrt{[(1/LC) - \omega_o^2]^2 + (\omega_o R/L)^2}}$$

$$= \frac{\sqrt{(1/LC)}(R/L)}{\sqrt{[(1/LC) - (1/LC)]^2 + \left[\sqrt{(1/LC)}(R/L)\right]^2}} = 1$$

$$\Rightarrow \quad \frac{1}{\sqrt{2}} = \frac{\omega_c(R/L)}{\sqrt{[(1/LC) - \omega_c^2]^2 + (\omega_c R/L)^2}} = \frac{1}{\sqrt{[(\omega_c L/R) - (1/\omega_c RC)]^2 + 1}}$$

We can equate the denominators of the two sides of Eq.

$$\Rightarrow \quad \pm 1 = \omega_c \frac{L}{R} - \frac{1}{\omega_c RC} \quad \Leftrightarrow \quad \omega_c^2 L \pm \omega_c R - 1/C = 0$$

Cutoff frequencies $\omega_{c1} = -\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \left(\frac{1}{LC}\right)}$ $\qquad \omega_{c2} = \frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \left(\frac{1}{LC}\right)}$

Bandpass filters

The series RLC circuit

Relationship between center frequency and cutoff frequencies:

$$\omega_0 = \sqrt{\omega_{c1}\omega_{c2}} = \sqrt{\frac{1}{LC}}$$

Bandwidth: $$\beta = \omega_{c2} - \omega_{c1} = \frac{R}{L}$$

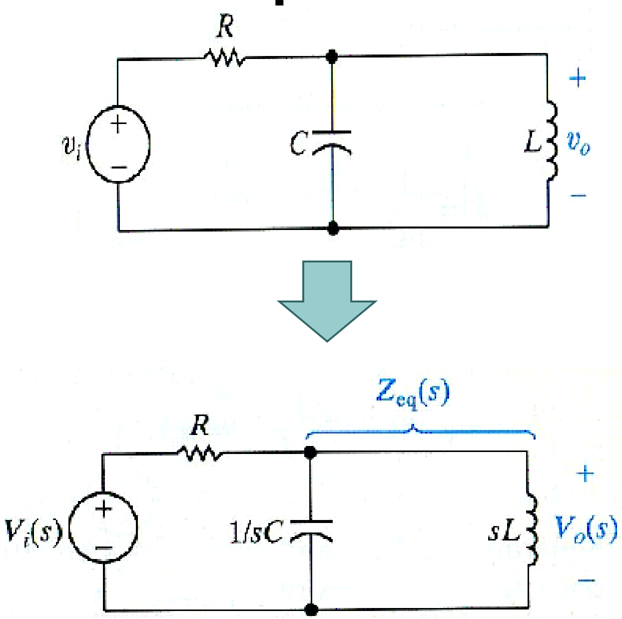
Quality factor: $$Q = \frac{\omega_0}{\beta} = \frac{(1/RC)}{R/L} = \sqrt{\frac{L}{CR^2}}$$

$$\omega_{c1} = \omega_o \cdot \left[-\frac{1}{2Q} + \sqrt{1 + \left(\frac{1}{2Q}\right)^2} \right],$$

$$\omega_{c2} = \omega_o \cdot \left[\frac{1}{2Q} + \sqrt{1 + \left(\frac{1}{2Q}\right)^2} \right].$$

Alternative forms for these equations express the cutoff frequencies

Bandpass filters
The parallel RLC circuit



$$Z_{eq}(s) = \frac{L/C}{sL + 1/sC}$$

The transfer function

$$H(s) = \frac{s/RC}{s^2 + s/RC + 1/LC}$$

Substitute s = jω, we have:

Magnitude of the transfer function

$$|H(j\omega)| = \frac{1}{\sqrt{1 + \left(\omega RC - \frac{1}{\omega(L/R)}\right)^2}}$$

Center frequency

$$\omega_0 = \sqrt{\frac{1}{LC}}$$

Cutoff frequencies

$$\omega_{c1} = -\frac{1}{2RC} + \sqrt{\left(\frac{1}{2RC}\right)^2 + \left(\frac{1}{LC}\right)}$$

$$\omega_{c2} = \frac{1}{2RC} + \sqrt{\left(\frac{1}{2RC}\right)^2 + \left(\frac{1}{LC}\right)}$$

Bandwidth:

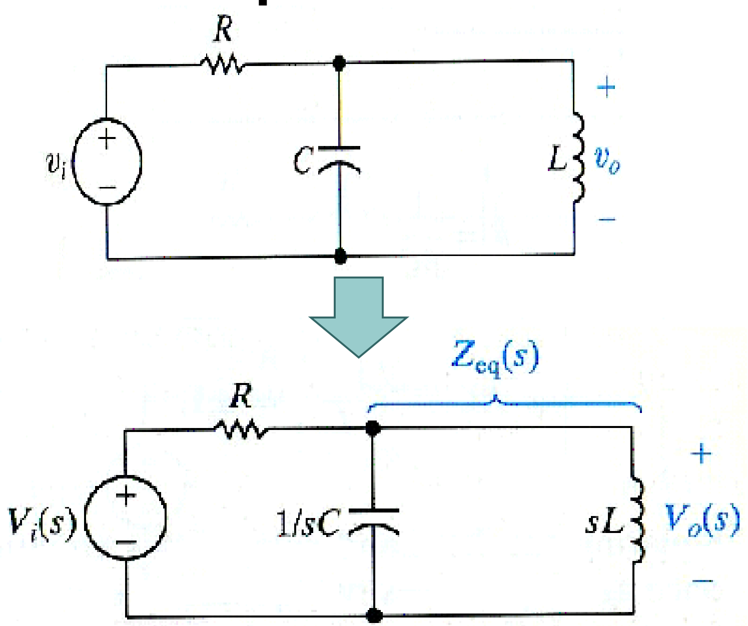
$$\beta = \omega_{c2} - \omega_{c1} = \frac{1}{RC}$$

Quality factor:

$$Q = \frac{\omega_0}{\beta} = \sqrt{\frac{CR^2}{L}}$$

Bandpass filters

The parallel RLC circuit



Example: Compute values for R & L to yield a bandpass filter (like the circuit in the figure) with a center frequency of 5 kHz and a bandwidth of 200 Hz, using a 5 μF capacitor.

<u>Sol:</u> Use the equation for bandwidth to compute a value for R with given 5 μF capacitor

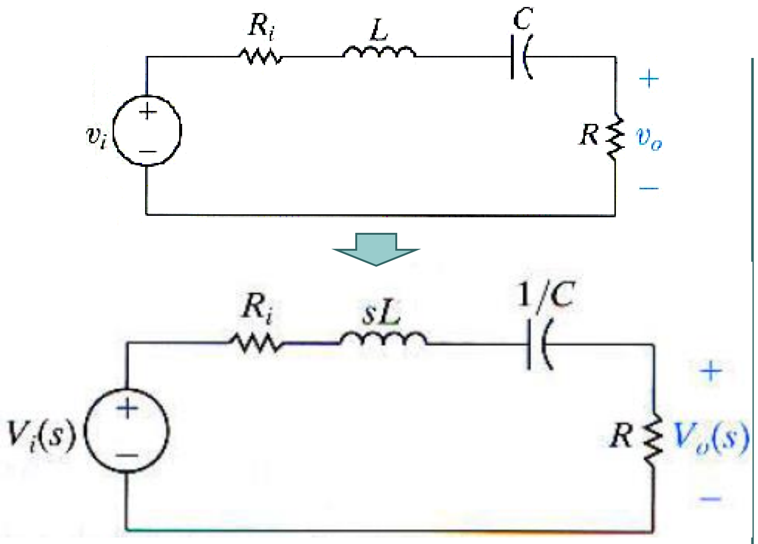
$$R = \frac{1}{\beta C} = \frac{1}{(2\pi)(200)(5 \times 10^{-6})} = 159.15\ \Omega$$

Using the value of capacitance and the equation for center frequency, calculate the inductance value:

$$L = \frac{1}{\omega_o^2 C} = \frac{1}{[2\pi(5000)]^2(5 \times 10^{-6})} = 202.64\ \mu\text{H}$$

Bandpass filters

Effect of Nonideal Voltage Source



The transfer function of the series RLC bandpass filter:

$$H(s) = \frac{\frac{R}{L}s}{s^2 + \left(\frac{R+R_i}{L}\right)s + \frac{1}{LC}}$$

$$|H(j\omega)| = \frac{\frac{R}{L}\omega}{\sqrt{\left(\frac{1}{LC} - \omega^2\right)^2 + \left(\omega\frac{R+R_i}{L}\right)^2}}$$

The center frequency, ω_o, is the frequency at which this transfer function magnitude is maximum,

$$\omega_o = \sqrt{\frac{1}{LC}}$$

At the center frequency the maximum magnitude is

$$H_{\max} = |H(j\omega_o)| = \frac{R}{R_i + R}$$

The cutoff frequencies can be computed by setting the transfer function magnitude equal to $(1/2^{0.5})H_{max}$

Bandpass filters

Effect of Nonideal Voltage Source

$$\omega_{c1} = -\frac{R + R_i}{2L} + \sqrt{\left(\frac{R + R_i}{2L}\right)^2 + \frac{1}{LC}},$$

$$\omega_{c2} = \frac{R + R_i}{2L} + \sqrt{\left(\frac{R + R_i}{2L}\right)^2 + \frac{1}{LC}}.$$

The bandwidth is calculated from the cutoff frequencies: $\beta = \frac{R + R_i}{L}$

Quality factor is computed from the ω_o the β: $Q = \frac{\sqrt{L/C}}{R + R_i}$

Transfer function of the series RLC bandpass filter with nonzero source resistance as $H(s) = \frac{K\beta s}{s^2 + \beta s + \omega_0^2}$

where

$\beta = \frac{R + R_i}{L}$, $K = \frac{R}{R + R_i}$ and $\omega_0 = \sqrt{\frac{1}{LC}}$

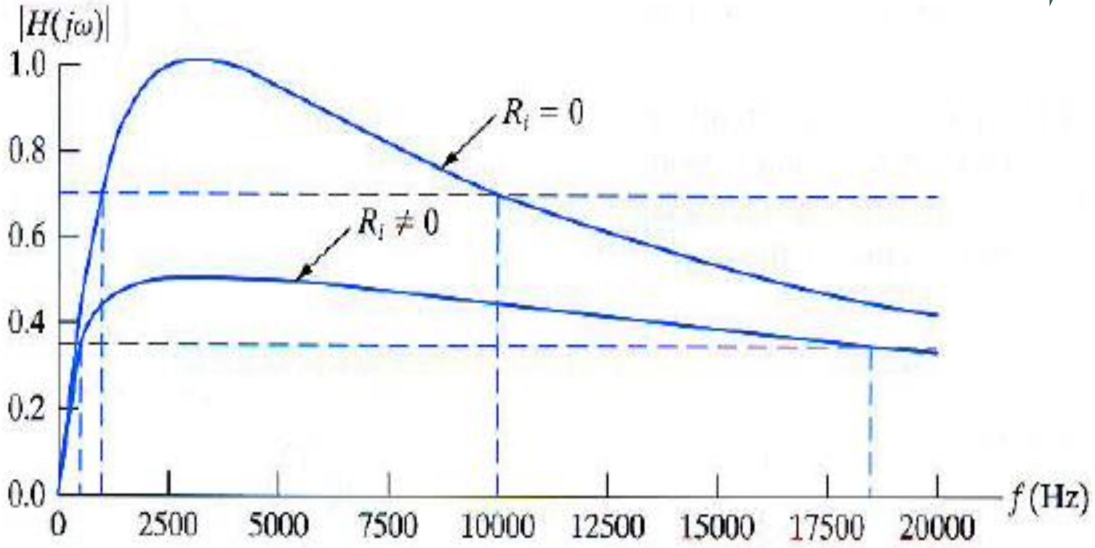
Bandpass filters

Effect of Nonideal Voltage Source

For ideal source (nonzero source resistance):

$R_i = 0$ then $K = 1$

$$H(s) = \frac{\beta s}{s^2 + \beta s + \omega_o^2}$$



The magnitude plots for a series RLC bandpass filter with a zero source resistance and a nonzero source resistance.

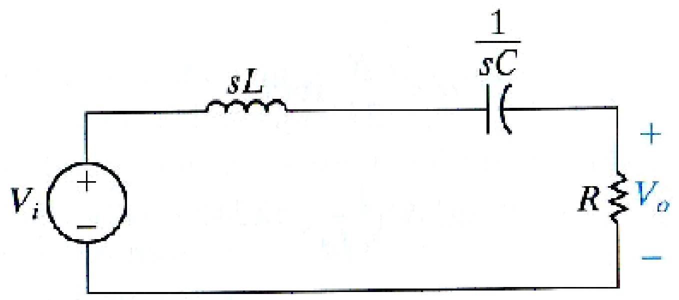
The addition of a nonzero source resistance to a series RLC bandpass filter leaves the center frequency unchanged but widens the bassband and reduces the bassband magnitude.

Bandpass filters

Summary

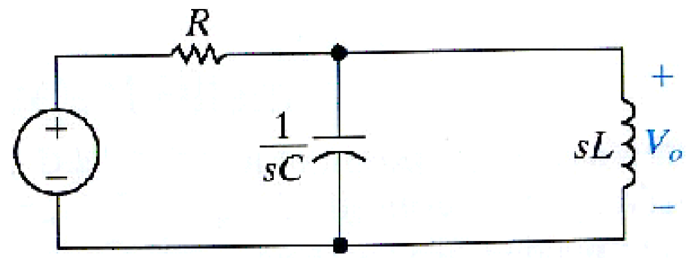
Transfer function:

$$\boxed{H(s) = \frac{\beta s}{s^2 + \beta s + \omega_0^2}}$$



$$H(s) = \frac{(R/L)s}{s^2 + (R/L)s + (1/LC)}$$

$$\omega_0 = \sqrt{\frac{1}{LC}} \qquad \beta = \frac{R}{L}$$



$$H(s) = \frac{s/RC}{s^2 + s/RC + 1/LC}$$

$$\omega_0 = \sqrt{\frac{1}{LC}} \qquad \beta = \frac{1}{RC}$$

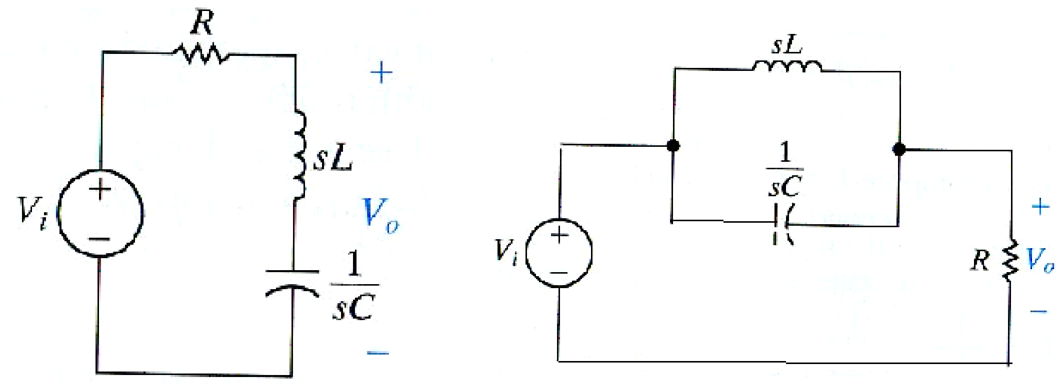
Band-reject filters

A ***band-reject filter*** attenuates voltages at frequencies within the stopband, which is between $\boldsymbol{\omega_{c1}}$ and $\boldsymbol{\omega_{c2}}$. It passes frequencies outside of the stopband.

Any circuit with the transfer function:

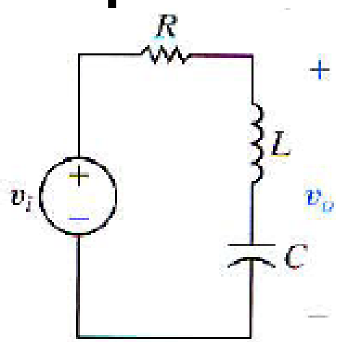
$$\boxed{H(s) = \frac{s^2 + \omega_0^2}{s^2 + \beta s + \omega_0^2}}$$

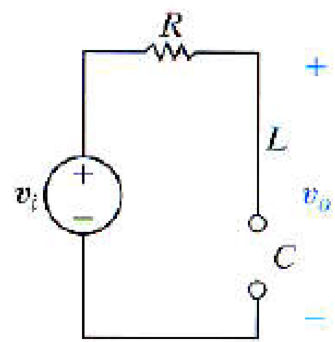
functions as a band-reject filter.



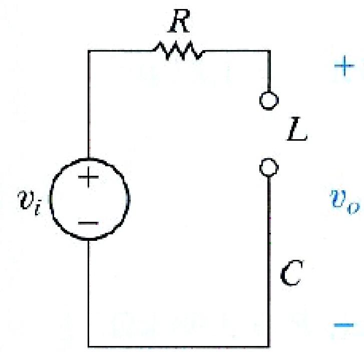
Two types of bandreject filters in s-domain

Bandreject filters
The series RLC circuit



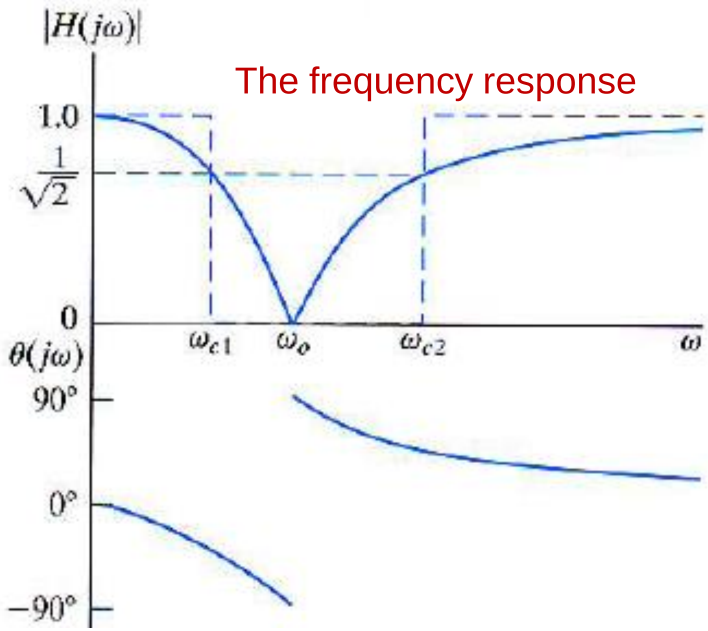


ω = 0



ω = ∞

The output voltage of the band-reject filter is defined across the inductor-capacitor pair.



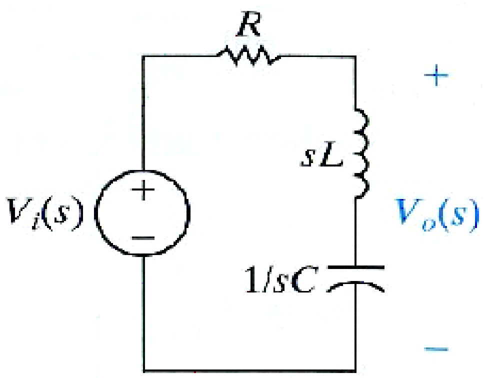
At ω = 0 and ω = ∞, the output voltage has the same magnitude as input voltage.

At center frequency ω_0, the magnitude of the output voltage is zero.

As the frequency is increased from zero, the impedance of the inductor increases and that of the capacitor decreases. The phase shift between the input and the output approaches - 90° . As soon as ω_L exceeds 1/ωC, the phase shift jumps to + 90° .and then approaches zero as frequency continues to increase.

Band-reject filters

The series RLC circuit



Transfer function:

$$H(s) = \frac{sL + 1/sC}{R + sL + 1/sC} = \frac{s^2 + 1/LC}{s^2 + (R/L)s + (1/LC)}$$

Substitute s = jω, we have:

Magnitude of the transfer function

$$|H(j\omega)| = \frac{|(1/LC) - \omega^2|}{\sqrt{[(1/LC) - \omega^2]^2 + \left[\frac{\omega R}{L}\right]^2}}$$

Phase angle of the transfer function

$$\theta(j\omega) = -\tan^{-1}\left[\frac{\omega R/L}{(1/LC) - \omega^2}\right]$$

Center frequency $\omega_0 = \sqrt{\frac{1}{LC}}$

Cutoff frequencies:

$$\omega_{c1} = -\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \left(\frac{1}{LC}\right)} \qquad \omega_{c2} = \frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \left(\frac{1}{LC}\right)}$$

Relationship btw. ω_0 & cutoff freq.

$$\omega_0 = \sqrt{\omega_{c1}\omega_{c2}} = \sqrt{\frac{1}{LC}}$$

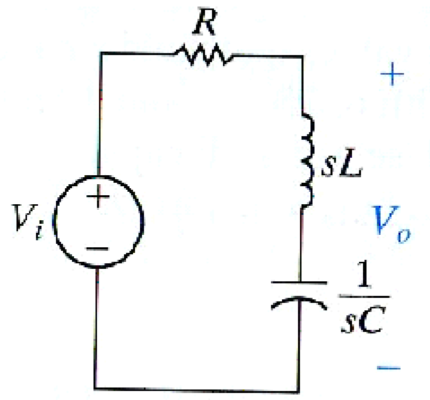
Bandwidth: $\beta = \omega_{c2} - \omega_{c1} = \frac{R}{L}$

Quality factor: $Q = \frac{\omega_0}{\beta} = \sqrt{\frac{L}{CR^2}}$

Band-reject filters

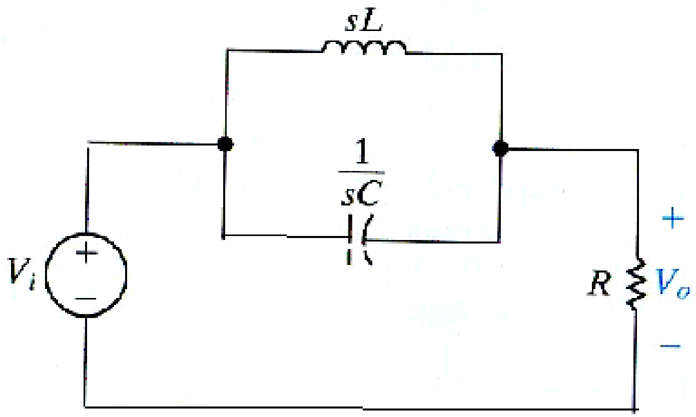
Summary

Transfer function: $$H(s)=\frac{s^2+\omega_0^2}{s^2+\beta s+\omega_0^2}$$



$$H(s)=\frac{s^2+(1/LC)}{s^2+(R/L)s+(1/LC)}$$

$$\omega_0=\sqrt{\frac{1}{LC}} \qquad \beta=\frac{R}{L}$$



$$H(s)=\frac{s^2+1/LC}{s^2+s/RC+1/LC}$$

$$\omega_0=\sqrt{\frac{1}{LC}} \qquad \beta=\frac{1}{RC}$$