[redacted] Minh Duy - ITITSB22029. week 5.

① a) $S = \{(x,y) : x \geq 0, y \in R\}$

Vector addition $(x_1, y_1), (x_2, y_2) \in S$

$\Rightarrow x_1, x_2 \geq 0$; $y_1, y_2 \in R$

$\Rightarrow (x_1, y_1) + (x_2, y_2) = (x_1 + x_2, y_1 + y_2) \in S$

Because of $x_1, x_2 \geq 0$; $x_1 + x_2 \geq 0$ and $y_1 + y_2 \in R$

• Scalar multiplication:

Let $(x,y) \in S \rightarrow x \geq 0$, $y \in R$

• $\alpha = -1$

$\Rightarrow \alpha(x,y) = -1(x,y) = (-x, y) \notin S$

$\Rightarrow$ S is not a vector space

b) $S = \{(x, \frac{x}{2}) : x \in R\}$

Vector addition $(x_1, y_1), (x_2, y_2) \in S$

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- Scalar multiplication:

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② Is not a vector space $(x_1, y_1) + (x_2, y_2)$

$= (x_1 y_1, x_2 y_2)$

$\neq$

$(x_2, y_2) + (x_1, y_1) = (x_2 y_2, x_1 y_1)$

③

a) $W = \{(0, x_2, x_3) : x_2, x_3$ are real numbers$\}$

Let $u, v \in w$

$\Rightarrow u = (0, x_2, x_3)$ and $v = (0, y_2, y_3)$ with

x_2, x_3, y_2, y_3 are real numbers

$\Rightarrow u + v = (0, x_2, x_3) + (0, y_2, y_3)$

$= (0, x_2 + y_2, x_3 + y_3) \in w$

$\Rightarrow u + v \in w$ (1)

$\alpha \in R$

$\Rightarrow \alpha u = \alpha(0, x_2, x_3)$

$= (0, \alpha x_2, \alpha x_3) \in w$

$\Rightarrow \alpha u \in w$ (2)

From (1)(2) $\Rightarrow$, w is subspace of R^3

b) $W = \{(x_1, x_2, 4) : x_1$ and x_2 are real numbers$\}$

Let $u, v \in w$

$u = (x_1, x_2, 4)$ and $v = (y_1, y_2, 4)$;

x_1, x_2, y_1, y_2 are real numbers

$\Rightarrow u + v = (x_1 + y_1, x_2 + y_2, 8) \notin w$

$\Rightarrow u + v \notin w$ (1)

$\alpha \in R$

$\Rightarrow \alpha u = \alpha(x_1, x_2, 4)$

$= (\alpha x_1, \alpha x_2, \alpha 4) \notin w$

$\Rightarrow \alpha u \notin w$ (2)

From (1)(2) $\Rightarrow$ w is not subspace of R^3

④ $S = \{(2,0,7), (2,4,5), (2,-12,13)\}$

a) $u = (-1, 5, -6)$

Let $u = C_1(2,0,7) + C_2(2,4,5) + C_3(2,-12,13)$

$\Rightarrow (-1, 5, -6) = (2C_1 + 2C_2 + 2C_3, 4C_2 - 12C_3, 7C_1 + 5C_2 + 13C_3)$

$$\begin{cases} 2C_1 + 2C_2 + 2C_3 = -1 \\ 4C_2 - 12C_3 = 5 \\ 7C_1 + 5C_2 + 13C_3 = -6 \end{cases}$$

$$\left[\begin{array}{ccc|c} 2 & 2 & 2 & -1 \\ 0 & 4 & -12 & 5 \\ 7 & 5 & 13 & -6 \end{array}\right] \xrightarrow[R_2 = R_2/4]{R_1 = \frac{1}{2}R_1} \left[\begin{array}{ccc|c} 1 & 1 & 1 & -\frac{1}{2} \\ 0 & 1 & -3 & 5/4 \\ 7 & 5 & 13 & -6 \end{array}\right]$$

$$\xrightarrow[R_3 = R_3 - 7R_1]{} \left[\begin{array}{ccc|c} 1 & 1 & 1 & -\frac{1}{2} \\ 0 & 1 & -3 & 5/4 \\ 0 & -2 & 6 & -5/2 \end{array}\right] \xrightarrow{R_3 + 2R_2} \left[\begin{array}{ccc|c} 1 & 1 & 1 & -1/2 \\ 0 & 1 & -3 & 5/4 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$$\Rightarrow \begin{cases} C_1 + C_2 + C_3 = -1/2 \\ C_2 - 3C_3 = 5/4 \end{cases}$$

Let $C_3 = 0$ $\Rightarrow \begin{cases} C_1 = -7/4 \\ C_2 = 5/4 \\ C_3 = 0 \end{cases}$

b) $v = (-3, 15, 18)$

$$\Rightarrow \begin{cases} 2c_1 + 2c_2 + 2c_3 = -3 \\ 4c_2 - 12c_3 = 15 \\ 7c_1 + 5c_2 + 13c_3 = 18 \end{cases}$$

$$\left[\begin{array}{ccc|c} 2 & 2 & 2 & -3 \\ 0 & 4 & -12 & 15 \\ 7 & 5 & 13 & 18 \end{array}\right] \rightarrow \left[\begin{array}{ccc|c} 1 & 1 & 1 & -1/2 \\ 0 & 4 & -12 & 15 \\ 0 & 0 & 0 & 36 \end{array}\right]$$

$\rightarrow$ No solution.

5) a) $S = \{(4,7,3), (-1,2,6), (2,-3,5)\}$

$$\begin{cases} 4a - b + 2c = 0 \\ 7a + 2b - 3c = 0 \\ 3a + 6b + 5c = 0 \end{cases} \Rightarrow a = b = c = 0$$

$\Rightarrow$ they are linearly independent $\rightarrow$ they span R^3

b) $S = \{(5,6,5), (2,1,-5), (0,-4,1)\}$

$$\begin{cases} 5a + 2b = 0 \\ 6a + b - 4c = 0 \\ 5a - 5b + c = 0 \end{cases} \Rightarrow a = b = c = 0$$

$\Rightarrow$ they are linearly independent $\Rightarrow$ they span R^3

6) a) $S = \{(-2,1,3), (2,9,-3), (2,3,-3)\}$

$$|D| = \begin{bmatrix} -2 & 1 & 3 \\ 2 & 9 & -3 \\ 2 & 3 & -3 \end{bmatrix} = -2(-27+9) - 1(-6+6) + 3(6-18) = 0$$

Since $|D| = 0$, the vectors A, B, C are linearly dependent

b) $S = \{(-4,-3,4), (1,-2,3), (6,0,0)\}$

$$|D| = \begin{bmatrix} -4 & -3 & 4 \\ 1 & -2 & 3 \\ 6 & 0 & 0 \end{bmatrix} = -6 \neq 0$$

Since $|D| = -6 \neq 0 \Rightarrow$ A, B, C are Linear independent.

7)

$$S = \left\{ \begin{bmatrix} x \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ x \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ x \end{bmatrix} \right\}$$

$$|D| = \begin{bmatrix} x & 1 & 1 \\ 1 & x & 1 \\ 1 & 1 & x \end{bmatrix} = x(x^2-1) - 1(x-1) + 1(1-x) = x^3 - 3x + 2$$

Set linearly dependent when $D = 0$

$\Rightarrow x^3 - 3x + 2 = 0$

$\Leftrightarrow \left[\begin{array}{l} x = 1 \\ x = -2 \end{array}\right.$

The set linearly independent when $x \neq -2$ and $x \neq 1$