Đỗ Minh Duy - ITITSB 22029.

Week 2:

① a) $(m \times n).(n \times k) = (m \times k)$

$\Rightarrow$ acc. to the question: $m = 5$; $n = 3$

$\Rightarrow (5 \times 3).(n \times k) = (5 \times 7)$

$\Rightarrow n = 3$; $k = 7$ $\Rightarrow$ Size of $B = (3 \times 7)$

b) $B.C = BC = 3 \times 4$

Therefore, $m = 3$; $k = 4$

$(m \times n).(n \times k) = (m \times k)$

$\Rightarrow m \to$ row; $k \to$ column

So B has 3 rows

③ $A = \begin{pmatrix} 2 & 5 \\ -3 & 1 \end{pmatrix}$; $B = \begin{pmatrix} 4 & -5 \\ 3 & c \end{pmatrix}$

$$AB = \begin{bmatrix} 2\times4 + 5\times3 & 2\times(-5) + 5c \\ (-3)(4) + 1\times3 & (-3)\times(-5) + c \end{bmatrix} = \begin{bmatrix} 23 & -10+5c \\ -9 & 15+c \end{bmatrix}$$

$$BA = \begin{bmatrix} 4\times2 + (-5)(-3) & 4\times5 + (-5)(1) \\ 3\times\cancel{4}\,2 + (-3)c & 3\times5 + c \end{bmatrix} = \begin{bmatrix} 23 & 15 \\ 6-3c & 15+c \end{bmatrix}$$

Then $\left.\begin{matrix} 6 - 3c = -9 \Rightarrow c = 5 \\ -10 + 5c = 15 \Rightarrow c = 5 \end{matrix}\right\} \Rightarrow c = 5$

④ $A = \begin{pmatrix} 3 & -6 \\ -1 & 2 \end{pmatrix}$; $B = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$

$$AB = \begin{pmatrix} 3a - 6c & 3b - 6d \\ -1a + 2c & -1b + 2d \end{pmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$\begin{cases} 3a - 6c = 0 \Rightarrow 6c - 6c = 0 \\ -a + 2c = 0 \Rightarrow a = 2c \end{cases}$

$\begin{cases} 3b - 6d = 0 \Rightarrow 6d - 6d = 0 \\ -b + 2d = 0 \Rightarrow b = 2d \end{cases}$

Therefore, $a = 2c$; $b = 2d$

Let $c = 1 \Rightarrow a = 2$; $d = 2 \Rightarrow b = 4$

$\Rightarrow \begin{bmatrix} 2 & 4 \\ 1 & 2 \end{bmatrix}$

5) $$\left[\begin{array}{ccc|c} 3 & 1 & 1 & 3 \\ 1 & -1 & -1 & 1 \\ 1 & 2 & 2 & 1 \end{array}\right] \xrightarrow[R_3 = R_1 - 3R_3]{R_2 = R_1 - 3R_2} \left[\begin{array}{ccc|c} 3 & 1 & 1 & 3 \\ 0 & 4 & 4 & 0 \\ 0 & -5 & -5 & 0 \end{array}\right] \xrightarrow{R_3 = 4R_3 + 5R_2} \left[\begin{array}{ccc|c} 3 & 1 & 1 & 3 \\ 0 & 4 & 4 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$$\xrightarrow{R_2 = 4R_3 - 0.R_2} \left[\begin{array}{ccc|c} 3 & 1 & 1 & 3 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$\Rightarrow \begin{cases} 3x_1 + x_2 + x_3 = 3 \\ \cancel{4x_2 + 4x_3 = 0}\ x_2 = x_2 = s \text{ (free var)} \\ x_3 = x_3 = t \text{ (free var)} \end{cases}$

$\Rightarrow \begin{cases} x_1 = \frac{3}{3} - \frac{s}{3} - \frac{t}{3} \\ x_2 = s \\ x_3 = t \end{cases} \Rightarrow \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} =$

② $A = \begin{pmatrix} 1 & 1 \\ -2 & -1 \\ 1 & 2 \end{pmatrix}$, $B = \begin{pmatrix} 1 & -1 & -2 \\ 2 & 1 & -2 \end{pmatrix}$, $C = \begin{pmatrix} 1 & 1 & -3 \\ -1 & 2 & 1 \\ -3 & -1 & 0 \end{pmatrix}$

a) $A + 20B \Rightarrow$ can not

$$B.A = \underset{2\times 3}{\begin{pmatrix} 1 & -1 & -2 \\ 2 & 1 & -2 \end{pmatrix}} \underset{3\times 2}{\begin{pmatrix} 1 & 1 \\ -2 & -1 \\ 1 & 2 \end{pmatrix}} = \begin{pmatrix} 1\times 1 + (-1)(-2) + (-2)(1) & 1 + (-1)(-1) + (-2)2 \\ 2\times 1 + 1(-2) + (-2)1 & 2\times 1 + 1(-1) + (-2)(2) \end{pmatrix}$$

$$= \begin{pmatrix} 1 & -2 \\ -2 & -3 \end{pmatrix}$$

$$B - 5A^T = \begin{pmatrix} 1 & -1 & -2 \\ 2 & 1 & -2 \end{pmatrix} - 5\begin{pmatrix} 1 & -2 & 1 \\ 1 & -1 & 2 \end{pmatrix} = \begin{pmatrix} 1 & -1 & -2 \\ 2 & 1 & -2 \end{pmatrix} - \begin{pmatrix} 5 & -10 & 5 \\ 5 & -5 & 10 \end{pmatrix}$$

$$= \begin{pmatrix} -4 & 9 & -7 \\ -3 & 6 & -12 \end{pmatrix}$$

b) $A + 4C^T \Rightarrow$ can not

$AC \Rightarrow$ can not

$$CA = \underset{3\times 3}{\begin{pmatrix} 1 & 1 & -3 \\ -1 & 2 & 1 \\ -3 & -1 & 0 \end{pmatrix}} \underset{3\times 2}{\begin{pmatrix} 1 & 1 \\ -2 & -1 \\ 1 & 2 \end{pmatrix}} = \begin{pmatrix} 1 + 1(-2) + (-3)1 & 1 + 1(-1) + (-3)2 \\ (-1) + 2(-2) + 1 & (-1) + (-2) + 2 \\ (-3) + (-1)(-2) + 0 & (-3) + (-1)(-1) + 0 \end{pmatrix}$$

$$= \begin{pmatrix} -4 & -6 \\ -4 & -1 \\ -1 & -2 \end{pmatrix}$$