Resistive Circuits

(Chapter 3)

Textbook:

Electric Circuits

James W. Nilsson & Susan A. Riedel 9th Edition.

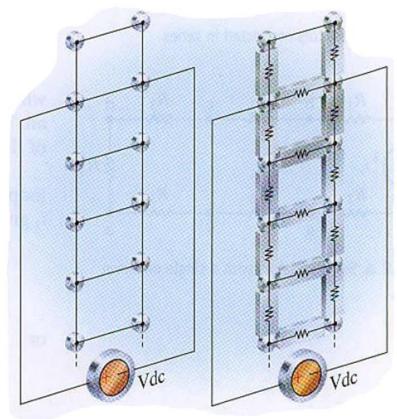
Link to download materials: Blackboard

Practical Perspective A Rear Window Defroster

The rear window defroster grid on an automobile is an example of a resistive circuit that performs a useful function.

How does this grid work to defrost the rear window?
How are the properties of the grid determined?

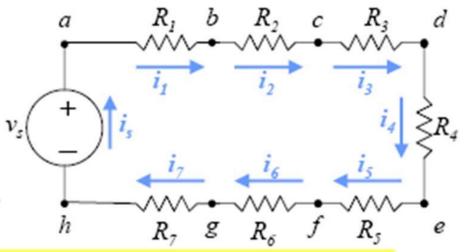




Resistors in series

- Just two elements connected at a single node are said to be in series.
- Applying KCL at all nodes.

$$i_s = i_1 = i_2 = i_3 = i_4 = i_5 = i_6 = i_7$$



Series-connected circuit elements carry the same current

Applying KVL

$$-v_{s} + i_{s}R_{1} + i_{s}R_{2} + i_{s}R_{3} + i_{s}R_{4} + i_{s}R_{5} + i_{s}R_{6} + i_{s}R_{7} = 0$$
or
$$v_{s} = i_{s} \left(R_{1} + R_{2} + R_{3} + R_{4} + R_{5} + R_{6} + R_{7} \right)$$

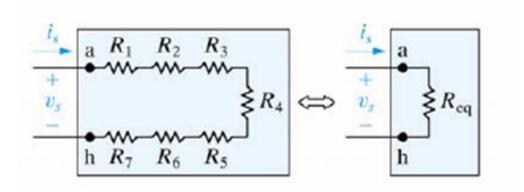
$$R_{eq}$$

$$R_{eq} = R_1 + R_2 + R_3 + R_4 + R_5 + R_6 + R_7 \longrightarrow v_s = i_s R_{eq}$$

Resistors in series (cont.)

 In general, if k resistors are connected in series, the equivalent single resistor has a resistance equal to the sum of the k resistances, or

$$R_{eq} = \sum_{i=1}^{k} R_i = R_1 + R_2 + \ldots + R_k$$



Resistors in parallel

- When two elements connect at a single <u>node</u> <u>pair</u>, they are said to be connected in parallel.
- Parallel-connected circuit elements have the same voltage

across their terminals.

Applying KCL

$$-- i_s = i_1 + i_2 + i_3 + i_4 - -$$

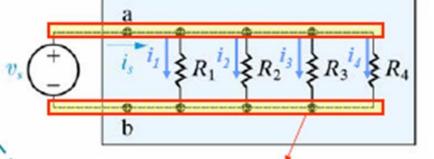
· From Ohm's law

$$i_1 R_1 = i_2 R_2 = i_3 R_3 = i_4 R_4 = v_{\varepsilon}$$

Therefore

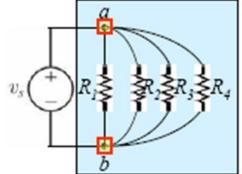
$$i_1 = \frac{v_s}{R_1}$$
, $i_2 = \frac{v_s}{R_2}$, $i_3 = \frac{v_s}{R_3} & i_4 = \frac{v_s}{R_4}$

$$- i_s = \frac{v_s}{R_1} + \frac{v_s}{R_2} + \frac{v_s}{R_3} + \frac{v_s}{R_4}$$



Same node "No elements connected

between nodes"

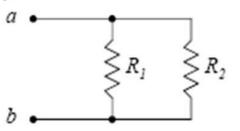


Resistors in parallel (cont.)

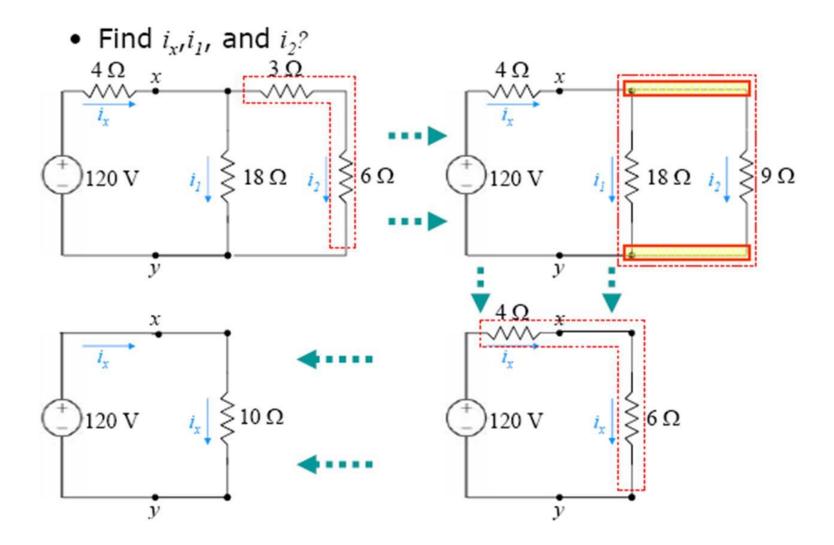
$$\begin{split} i_{z} &= v_{z} \left(\frac{1}{R_{1}} + \frac{1}{R_{2}} + \frac{1}{R_{3}} + \frac{1}{R_{4}} \right) \\ \frac{i_{z}}{v_{z}} &= \frac{1}{R_{eq}} = \left(\frac{1}{R_{1}} + \frac{1}{R_{2}} + \frac{1}{R_{3}} + \frac{1}{R_{4}} \right) \\ \frac{1}{R_{eq}} &= \sum_{i=1}^{k} \frac{1}{R_{i}} = \left(\frac{1}{R_{1}} + \frac{1}{R_{2}} + \dots + \frac{1}{R_{k}} \right) \\ G_{eq} &= \sum_{i=1}^{k} G_{i} = \left(G_{1} + G_{2} + \dots + G_{k} \right) \end{split}$$

Special Case (two resistors in parallel)

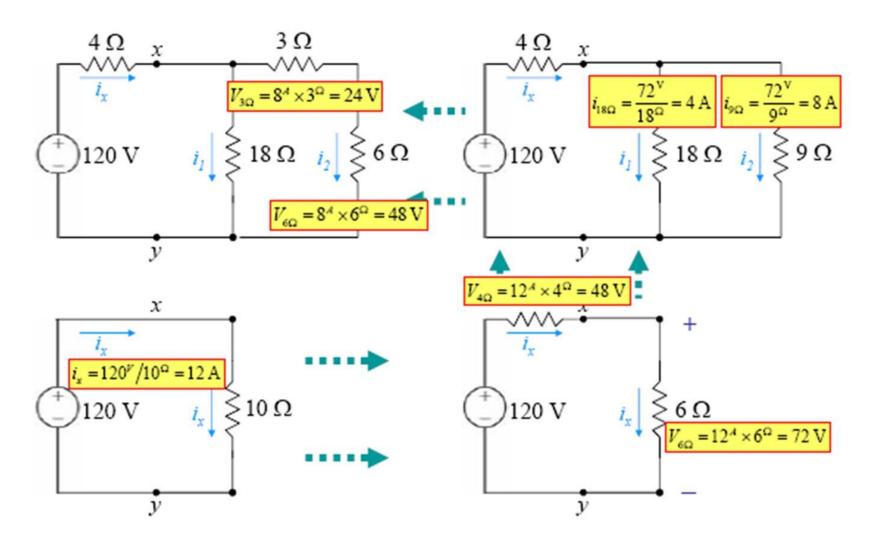
$$\begin{split} \frac{1}{R_{eq}} &= \frac{1}{R_1} + \frac{1}{R_2} = \frac{R_1 + R_2}{R_1 R_2} \\ R_{eq} &= \frac{R_1 R_2}{R_1 + R_2} \end{split}$$



Example 3.1



Example (Cont.)



Assessing Objective 1

Find (a) v, (b) power delivered to the circuit by the current source, and (c) the power dissipated in the 10 Ω resistor.

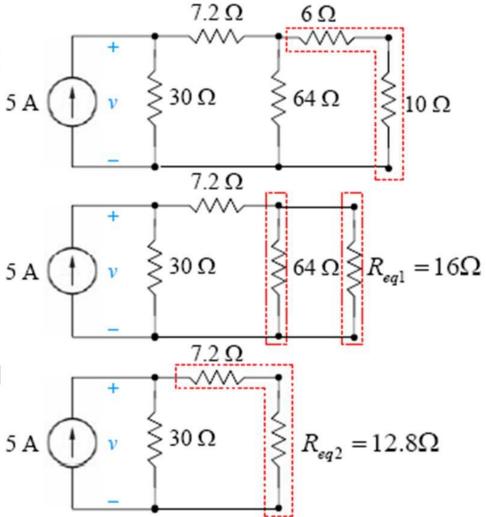


•The 6 Ω is in series with the 10 Ω ,

$$R_{eq1} = 6 + 10 = 16 \Omega$$

•The 16 Ω is in parallel with the 64 Ω ,

$$R_{eq2} = \frac{16 \times 64}{16 + 64} = 12.8 \,\Omega$$



Example (cont.)

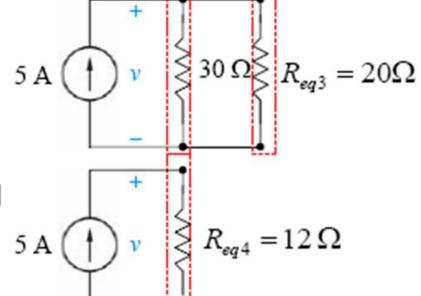
 The 12.8 Ω is in. series with the 7.2 Ω ,

$$R_{eq3} = 7.2 + 12.8 = 20 \Omega$$

•The 30 Ω is in parallel with the 20 Ω ,

$$R_{eq4} = \frac{30 \times 20}{30 + 20} = 12 \Omega$$



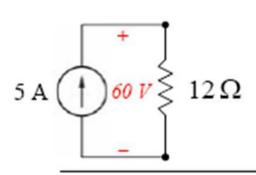


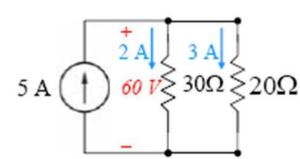
$$v = iR_{ea4} = 5^{A}12^{\Omega} = 60 \text{ V}$$

Power delivered by the current source

$$p = iv = 5^A 60^V = 300 \text{ W}$$

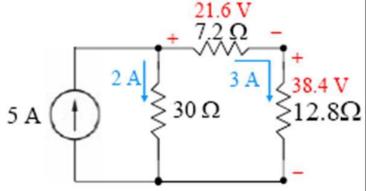
Example (cont.)





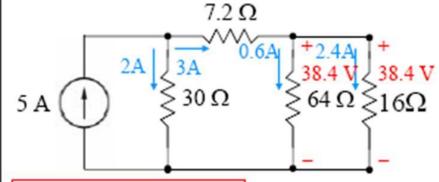
$$i_{30\Omega} = \frac{60^V}{30^\Omega} = 2 \text{ A}$$

$$i_{20\Omega} = \frac{60^{V}}{20^{\Omega}} = 3 \text{ A}$$



$$v_{7.2\Omega} = 3^A \times 7.2^{\Omega} = 21.6 \text{ V}$$

$$v_{12.8\Omega} = 3^A \times 12.8^{\Omega} = 38.4 \text{ V}$$



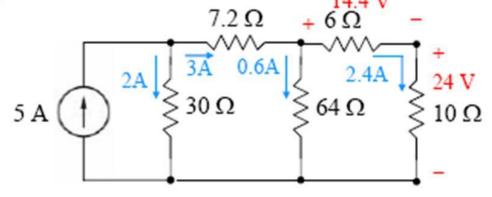
$$i_{64\Omega} = \frac{38.4^{V}}{64^{\Omega}} = 0.6 \text{ A}$$

$$i_{16\Omega} = \frac{38.4^{V}}{16^{\Omega}} = 2.4 \,\mathrm{A}$$

Example (cont.)

$$v_{6\Omega} = 2.4^A \times 6^{\Omega} = 14.4 \text{ V}$$

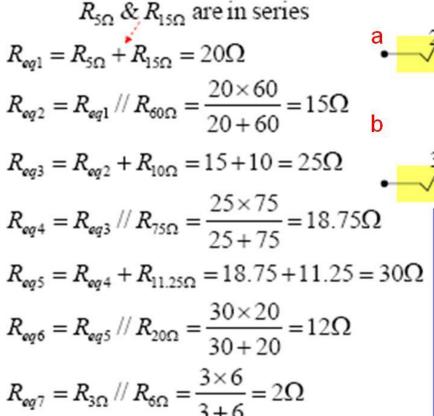
$$v_{10\Omega} = 2.4^A \times 10^\Omega = 24 \text{ V}$$

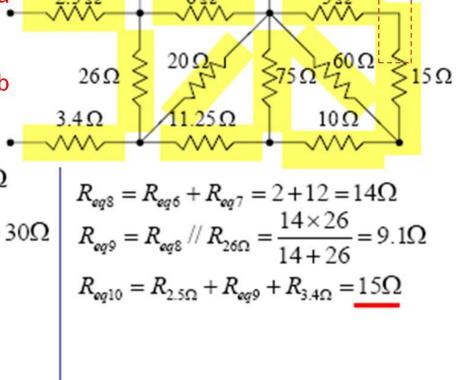


•The power dissipated in the 10 Ω resistor

$$p_{10\Omega} = 2.4^A \times 24^V = 57.6 \text{ W}$$

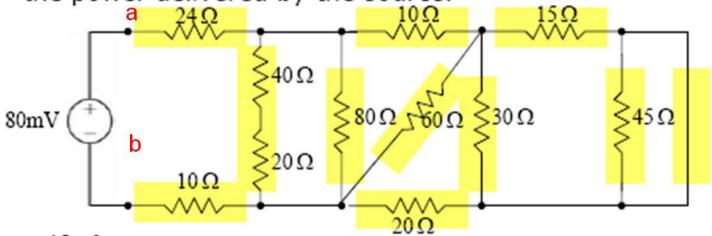
Find the equivalent resistance R_{ab} for the circuit in Figure.





 3Ω

 In the circuit shown, find the equivalent resistance R_{ab}, and the power delivered by the source.



$$R_{eq1} = \frac{45 \times 0}{45 + 0} = 0\Omega$$
 (Short Circuit)
 $R_{eq2} = \frac{15 \times 30}{15 + 30} = 10\Omega$
 $R_{eq3} = 30 + 20 = 50\Omega$
 $R_{eq4} = \frac{30 \times 60}{30 + 60} = 20\Omega$

$$R_{eq5} = 10 + 20 = 30\Omega$$

$$R_{eq6} = \frac{1}{\frac{1}{60} + \frac{1}{80} + \frac{1}{30}} = 16\Omega$$

$$R_{eq7} = 24 + 16 + 10 = 50\Omega$$

$$P_{80mV} = \frac{\left(80 \times 10^{-3}\right)^2}{50} = 128\mu\text{W}$$

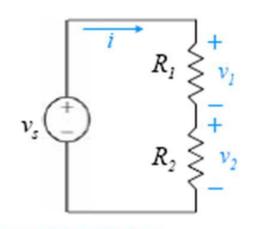
The voltage-divider circuit

Apply KVL

$$v_s = iR_1 + iR_2$$

$$i = \frac{V_z}{R_1 + R_2}$$

$$v_1 = iR_1 = v_s \frac{R_1}{R_1 + R_2}$$
 & $v_2 = iR_2 = v_s \frac{R_2}{R_1 + R_2}$

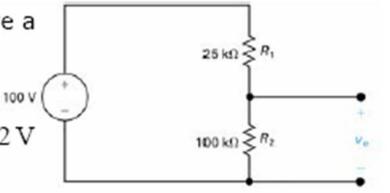


Example:

If the resistors used in the circuit have a tolerance of $\pm 10\%$. Find v_{omax} and v_{omin}

$$v_o(\text{max}) = 100 \frac{100 \times 1.1}{100 \times 1.1 + 25 \times 0.9} = 83.02 \text{ V}$$

$$v_o(\text{min}) = 100 \frac{100 \times 0.9}{100 \times 0.9 + 25 \times 1.1} = 76.60 \text{ V}$$



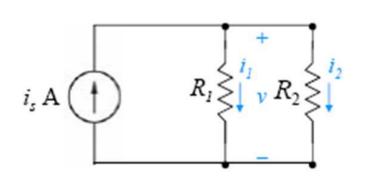
The current-divider circuit

$$R_1 // R_2 \Longrightarrow R_{eq} = \frac{R_1 R_2}{R_1 + R_2}$$

Ohm's Law

$$v = i_1 R_1 = i_2 R_2 = i_z R_{eq} = i_z \frac{R_1 R_2}{R_1 + R_2}$$

$$i_1 = \frac{R_2}{R_1 + R_2} i_s$$
 & $i_2 = \frac{R_1}{R_1 + R_2} i_s$



Example:

Find the power dissipated in the 6 Ω resistor

Ans.:-

$$6\Omega//4\Omega + 1.6\Omega$$

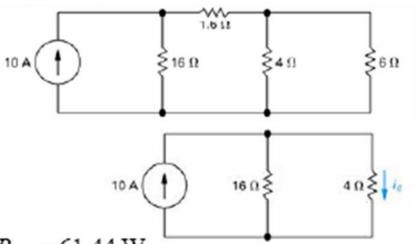
$$R_{eq} = \frac{4 \times 6}{4 + 6} + 1.6 = 4 \Omega$$

Using Current Divider

$$i_o = \frac{16}{16+4} 10 = 8 \text{ A}$$

Using Current Divider

$$i_6 = \frac{4}{4+6} 8 = 3.2 \text{ A}$$
 $P_6 = i_{6\Omega}^2 R_{6\Omega} = 61.44 \text{ W}$



Assessing Objective 2

Find (a) v_o at no load, (b) v_o when $R_L = 150 \text{k}\Omega$. (c) Power dissipated in 25 k Ω if the load is short circuited. (d) max. power in 75 k Ω .

Ans.:-

(a)
$$v_o = 200 \frac{75k}{75k + 25k} = 150 \text{ V}$$

(b)
$$R_{eq} = \frac{75k \times 150k}{75k + 150k} = 50k\Omega \implies v_o = 200 \frac{50k}{50k + 25k} = 133.33 \text{ V}$$

(c)
$$P_{25k\Omega} = \frac{V^2}{R_{25k\Omega}} = \frac{200^2}{25k} = 1.6 \text{ W}$$

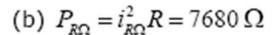
(d)
$$P_{75k\Omega}^{\text{max}} = \frac{V^2}{R_{75k\Omega}} = \frac{150^2}{75k} = 0.3 \text{ W}$$

Assessing Objective

Find (a) R so
$$i_{80\Omega}$$
=4A, (b) $P_{R\Omega}$, (c) P_{20A}

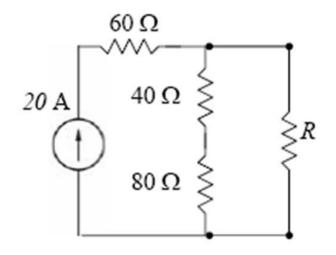
Ans.:-

(a)
$$i_{80\Omega} = 4 = \frac{R}{R + 80 + 40} 20$$
 $R = 30\Omega$



(c)
$$V_{20A} = 20^A \times 60^\Omega + 4^A \times 120^\Omega = 1680 \text{ V}$$

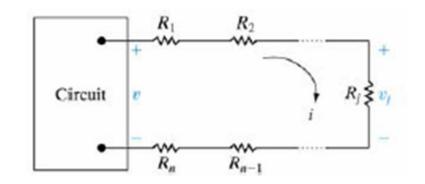
 $P_{20A} = 20^A \times 1680^V = 33600 \text{ W}$



Voltage Division and Current Division

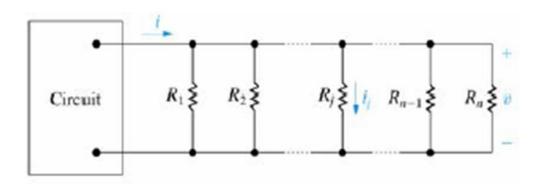
$$i = \frac{v}{R_1 + R_2 + \dots + R_n} = \frac{v}{R_{eq}}$$
$$v_j = iR_j$$

$$v_j = \frac{R_j}{R_{eq}} v$$



$$v = i(R_1 || R_2 || \dots || R_n) = iR_{eq}$$
$$v = i_j R_j$$

$$i_j = \frac{R_{eq}}{R_j}i$$



Example 3.4

Use current division to find i_o and voltage division to find v_o

Ans.:-

$$R_{eq} = \frac{1}{\frac{1}{80} + \frac{1}{10} + \frac{1}{80} + \frac{1}{24}} = 6 \Omega$$

Current Division

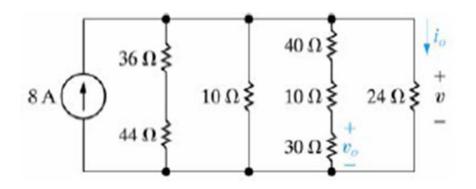
$$i_o = \frac{6}{24} 8^A = 2 \text{ A}$$

Ohm's Law

$$v_{24} = 2^{A}24^{\Omega} = 48 \text{ V}$$

Voltage Division

$$v_o = 48^{\rm V} \frac{30^{\rm \Omega}}{80^{\rm \Omega}} = 18 \,{\rm V}$$



Assessing Objective 3

Use voltage division & current division to find (a) v_{o} (b) $i_{40\Omega}$ &

 $i_{30\Omega}$, (c) $P_{50\Omega}$.

Ans.:-

(a)
$$R_{eq1} = \frac{1}{\frac{1}{20} + \frac{1}{30} + \frac{1}{60}} = 10 \,\Omega$$

 $v_o = 60^V \frac{40}{40 + 10 + 70} = 20 \,\text{V}$

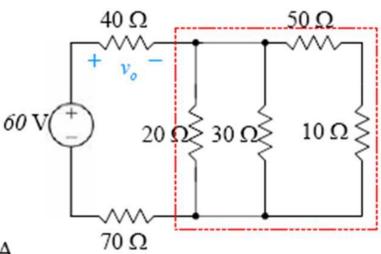
(b)
$$i_{40\Omega} = \frac{60^{V}}{120^{\Omega}} = 0.5 \text{ A}$$

$$i_{30\Omega} = i_{40\Omega} \frac{R_{eq1}}{R_{30\Omega}} = 0.5^{A} \frac{10}{30} = 0.1667 \text{ A}$$
(c) $v_{R_{eq1}} = 60^{V} \frac{10}{40 + 10 + 70} = 5 \text{ V}$

$$v_{50\Omega} = 5^{V} \frac{50}{50 + 10} = 4.1667 \text{ V}$$

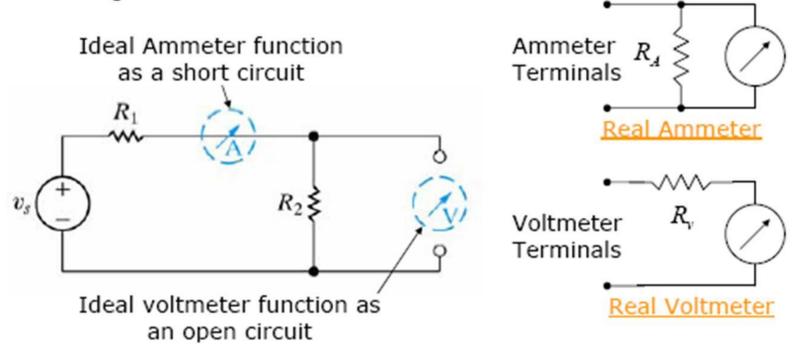
(c)
$$v_{R_{eq1}} = 60^{\nu} \frac{10}{40 + 10 + 70} = 5 \text{ V}$$
 $v_{50\Omega} = 5^{\nu} \frac{50}{50 + 10} = 4.1667 \text{ V}$

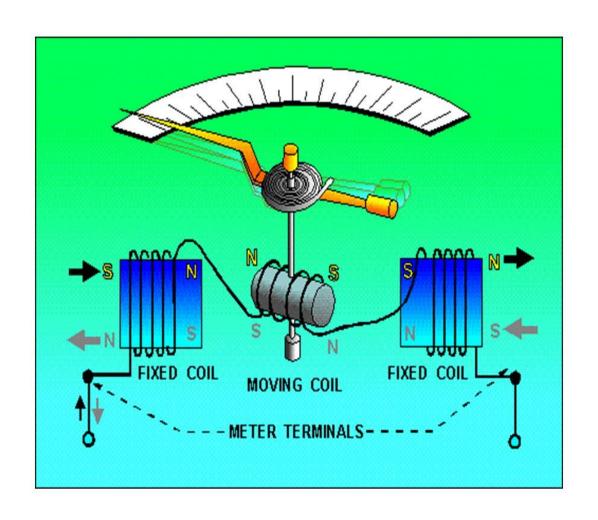
$$P_{50\Omega} = \frac{V_{50\Omega}^2}{R_{50\Omega}} = 0.3472 \text{ W}$$



Measuring Voltage and Current

- An ammeter is an instrument designed to measure current; it is placed in series with the circuit element whose current is being measured.
- A voltmeter is an instrument designed to measure voltage; it is placed in parallel with the element whose voltage is being measured.





Example 3.5 & 3.6

- (a) A 50 mV, 1 mA ammeter with a full scale of 10 mA. Determine R_A.
- (b) How much resistance is added to the circuit when the 10 mA meter is inserted to measure current?
 - (a) Meaning: When 10 mA is to be measured 1 mA will be moving in the coil; accordingly 9 mA will be moving in the R_A.

(b)
$$R_m = \frac{50 \, mV}{10 \, mA} = 5 \, \Omega$$

- (a) A 50 mV, 1 mA ammeter with a full scale of 150 V. Determine R_v.
- (b) How much resistance is added to the circuit when the 150 V meter is inserted to measure current?

ans.:

(a)
$$R_{movement} = \frac{50 \, mV}{1 \, mA} = 50 \, \Omega$$
, $50 \times 10^{-3} = \frac{50}{R_v + 50}$ 150 $R_v = 149.950 \, \Omega$

$$R_v = 149,950 \,\Omega$$

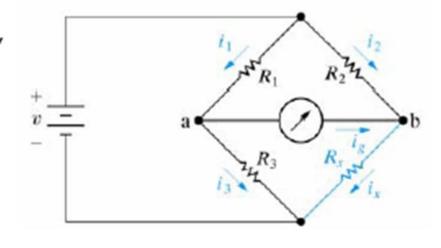
(b) $R_m = \frac{150^v}{10^{-3}} = 150,000 \,\Omega$

Measuring Resistance-The Wheatstone Bridge

 To find R_x, the value of R₃ is adjusted until there is no current in the meter.

$$i_g = 0$$
 Means balanced bridge.

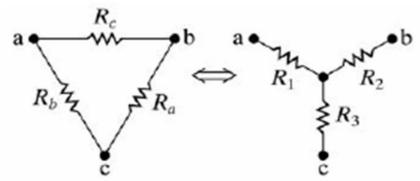
& V_{ab} =0 "same potential at a as b"



$$\frac{i_1}{i_2} = \frac{R_2}{R_1} = \frac{R_x}{R_3}$$

$$R_x = \frac{R_2}{R_1} R_3$$

The ∆-to-Y transformation (Delta-to-Wye/Pi-to-Tee)



 The resistance between terminals a and b must be the same whether we use Δ-connected set or the Y-connected circuit.

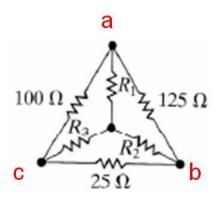
$$\begin{split} R_{ab} &= \frac{R_c \left(R_a + R_b \right)}{R_a + R_b + R_c} = R_1 + R_2 \\ R_{bc} &= \frac{R_a \left(R_b + R_c \right)}{R_a + R_b + R_c} = R_2 + R_3 \\ R_{ca} &= \frac{R_b \left(R_c + R_a \right)}{R_a + R_b + R_c} = R_1 + R_3 \\ \end{split} \qquad \begin{aligned} R_1 &= \frac{R_b R_c}{R_a + R_b + R_c} \\ R_2 &= \frac{R_c R_a}{R_a + R_b + R_c} \end{aligned} \qquad R_a = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_1} \\ R_2 &= \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_2} \end{aligned} \qquad R_b = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_2} \\ R_2 &= \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_2} \end{aligned} \qquad R_b = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_2} \end{aligned}$$

Example 3.7

Find the current and power supplied by the 40 V source.

Ans.:

We can convert $\Delta(100, 125, 25 \Omega)$ or $\Delta(25,40,37.5 \Omega)$



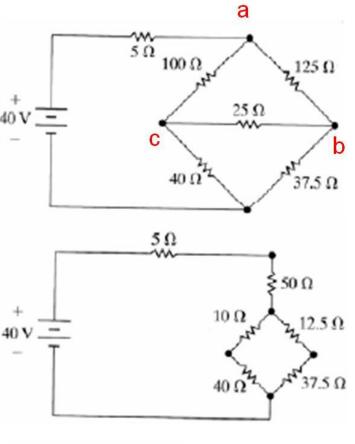
$$R_1 = \frac{100 \times 125}{100 + 125 + 25} = 50 \,\Omega$$

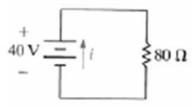
$$R_2 = \frac{125 \times 25}{100 + 125 + 25} = 12.5 \,\Omega$$

$$R_2 = \frac{100 \times 25}{100 + 125 + 25} = 10 \,\Omega$$

$$R_{eq} = 5 + 50 + \frac{(10 + 40) \times (12.5 + 37.5)}{(10 + 40) + (12.5 + 37.5)} = 80 \Omega$$

$$i = \frac{40^{\nu}}{80^{\Omega}} = \underline{0.5 \text{ A}}$$
 $P = 0.5^{\text{A}} \times 40 \text{v} = 20 \text{W}$





Assessing Objective 6

Use Y-to- Δ transformation to find ν .

Ans.:

$$R_a = \frac{20 \times 10 + 10 \times 5 + 5 \times 20}{5} = 70 \,\Omega$$

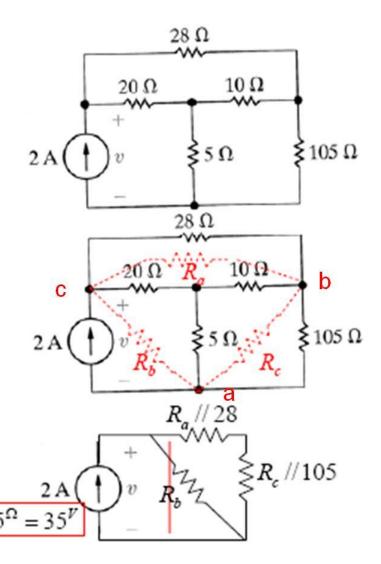
$$R_b = \frac{20 \times 10 + 10 \times 5 + 5 \times 20}{10} = 35 \,\Omega$$

$$R_c = \frac{20 \times 10 + 10 \times 5 + 5 \times 20}{20} = 17.5 \,\Omega$$

$$R_{eq1} = \frac{28 \times 70}{28 + 70} = 20 \,\Omega$$

$$R_{eq2} = \frac{17.5 \times 105}{17.5 + 105} = 15 \Omega$$

$$R_{eq3} = \frac{35 \times (20 + 15)}{35 + (20 + 15)} = 17.5 \,\Omega$$
 $v = 2^A \times 17.5^\Omega = 35^V$



Select R_1 , $R_2 \otimes R_3$ in the circuit to meet the following design requirements:

- a) The total power supplied is 36 W.
- b) $v_1 = 12 \text{ V}, v_2 = 6 \text{ V}, \text{ and } v_3 = -12 \text{ V}.$

Ans.:-

(a)
$$P_{24^{V}} = \frac{V^2}{R_{eq}} = \frac{(24)^2}{R_1 + R_2 + R_3} = 36$$

 $R_1 + R_2 + R_3 = 16 \Omega$



$$v_{1} = 24^{V} \frac{R_{1} + R_{2}}{R_{1} + R_{2} + R_{3}} = 12^{V}, R_{1} + R_{2} = \frac{12}{24} \times (R_{1} + R_{2} + R_{3}) = 8\Omega, R_{1} + R_{2} = 8\Omega$$

$$v_{2} = 24^{V} \frac{R_{2}}{R_{1} + R_{2} + R_{3}} = 6^{V}, R_{2} = \frac{6}{24} \times (R_{1} + R_{2} + R_{3}) = 4\Omega$$

$$R_{1} + R_{2} = 8\Omega$$

$$R_{2} = 4\Omega$$

$$R_{1} = 4\Omega$$

$$v_3 = -24^{\nu} \frac{R_3}{R_1 + R_2 + R_3} = -12^{\nu}$$
 $R_3 = 8 \Omega$

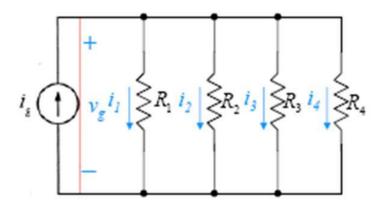
Common

Specify the value of the resistors in the circuit to meet the following design criteria: $i_g=8$ mA; $v_g=4$ V, $i_1=2i_2$; $i_2=10i_3$; and $i_2 = i_4$

Ans.:-
$$v_g = i_g R_{eq}$$
, $\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4}$

$$4^V = 8 \times 10^{-3} \left(\frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4}} \right)$$

$$\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4} = 2 \times 10^{-3}$$



$$i_j = \frac{v}{R_j} = \frac{R_{eq}}{R_j}i$$
 $i_1 = 2i_2 = \frac{R_{eq}}{R_1}i_g = 2\frac{R_{eq}}{R_2}i_g$, $R_2 = 2R_1$

$$i_2 = 10i_3 \longrightarrow R_3 = 10R_2$$

 $i_3 = i_4 \longrightarrow R_3 = R_4$
 $R_1 = 800 \Omega$

$$R_2 = 1.6 \,\mathrm{k}\Omega$$

$$i_2 = 10i_3 \longrightarrow R_3 = 10R_2$$

 $i_3 = i_4 \longrightarrow R_3 = R_4$
 $\frac{1}{R_1} + \frac{1}{2R_1} + \frac{1}{20R_1} + \frac{1}{20R_1} = \frac{32}{20R_1} = 2 \times 10^{-3}$

$$R_2 = 1.6 \,\mathrm{k}\Omega$$
 $R_3 = R_4 = 1.6 \,\mathrm{k}\Omega$

Find v_o ?

Ans.:-

Using current dividers

$$i_1 = \frac{200 + 1000}{300 + 300 + 200 + 1000} 15 \text{mA}$$

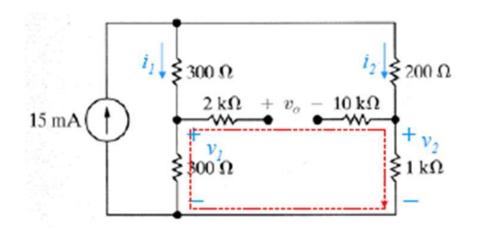
$$i_1 = 10 \text{mA}$$

$$i_2 = \frac{300 + 300}{300 + 300 + 200 + 1000} 15 \text{mA}$$

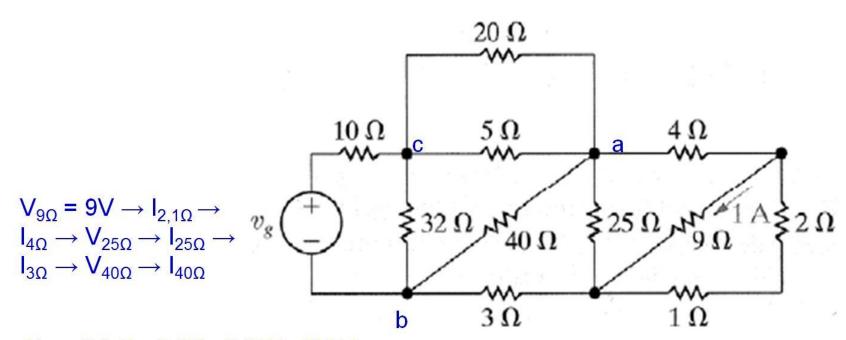
$$i_2 = 5 \text{mA}$$

$$v_1 = 10 \times 10^{-3} \times 300 = 3 \text{ V}$$
, $v_2 = 5 \times 10^{-3} \times 1000 = 5 \text{ V}$

Applying KVL
$$v_o + v_2 - v_1 = 0$$
 $v_o = v_1 - v_2 = 3 - 5$
 $v_o = -2 \text{ V}$



Find (a) v_g , (b) power dissipated in 20 Ω .



$$\begin{split} Z_{ab} = & [(\{[\ (2+1)//9] + 4\}//25) + 3]//40 \\ V_{ab} = & V_{40\Omega} = V_{32\Omega} \ Z_{ab}/(Z_{ab} + 20//5) \to V_{32\Omega} \to I_{32\Omega} \ \to I_g = I_{40\Omega} \ + I_{3\Omega} \ + I_{32\Omega} \\ Z_{cb} = & (Z_{ab} + (20//5))//32 \to V_g = I_g(10 + Z_{cb}) \to P_{20\Omega} \end{split}$$

Find v_x when the device in (b) is connected to the circuit.

Ans.:

$$R_{eq_1} = \frac{40 \times 10}{40 + 10} = 8 \Omega$$

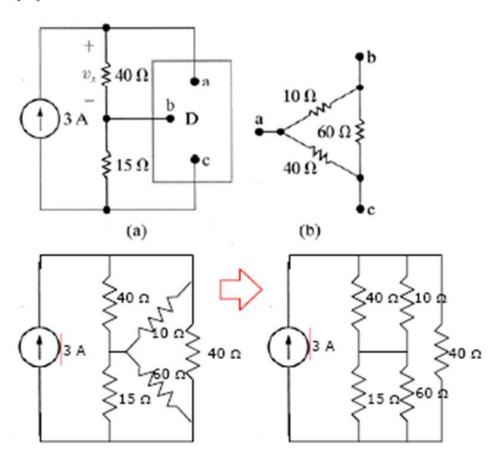
$$R_{eq2} = \frac{60 \times 15}{60 + 15} = 12 \,\Omega$$

$$R_{eq3} = 8 + 12 = 20 \Omega$$

$$i_{\text{Re}q3} = 3\frac{40}{20+40} = 2 \text{ A}$$

$$i_{40\Omega} = 2\frac{10}{10 + 40} = 0.4 \text{ A}$$

$$v_x = 0.4^A 40^\Omega = 16 \text{ V}$$



- (a) Find the resistance seen by the ideal voltage source in the circuit.
- (b) If v_{ab} equals 400 V, how much power is dissipated in the 31 Ω resistor.

Ans.:

$$R_{eq_{ab}} = 80 \Omega$$

$$P_{31\Omega} = 279 \text{ W}$$

Convert upper and lower delta circuits to Y cicuits, we have a simpler circuit

