• PROGRAM OF "PHYSICS"

Lecturer: Dr. DO Xuan Hoi
Room A1 503
E-mail : dxhoi@hcmiu.edu.vn

PHYSICS 4

(Wave and Modern Physics)

02 credits (30 periods)

Chapter 1 Mechanical Wave

Chapter 2 Properties of Light

Chapter 3 Introduction to Quantum Physics

Chapter 4 Atomic Physics

Chapter 5 Relativity and Nuclear Physics

PHYSICS 4

Chapter 2 Properties of Light

A. WAVE OPTICS

The Nature of Light

Interference of Light Waves

Diffraction Patterns

Polarization

B. GEOMETRIC OPTICS

Light Rays

The Laws of Reflection and Refraction

Mirrors

Thin Lenses

1 The Nature of Light

1.1 Dual nature of light

- In some cases light behaves like a *wave* (classical E & M – light propagation)
- In some cases light behaves like a *particle* (photoelectric effect)
- Einstein formulated theory of light:

$$\boxed{\begin{array}{c} E = hf \\ h = 6.63\times10^{-34}\,J\cdot s \end{array}}$$

Planck's constant

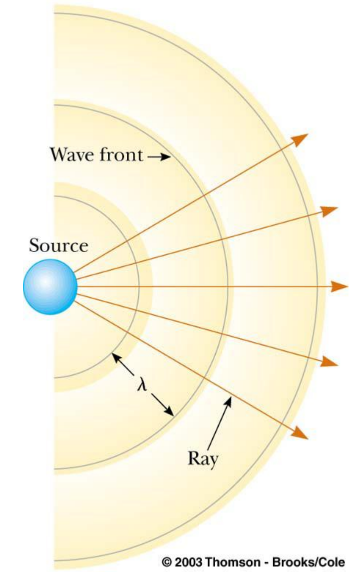
1.2 Huygens's Principle

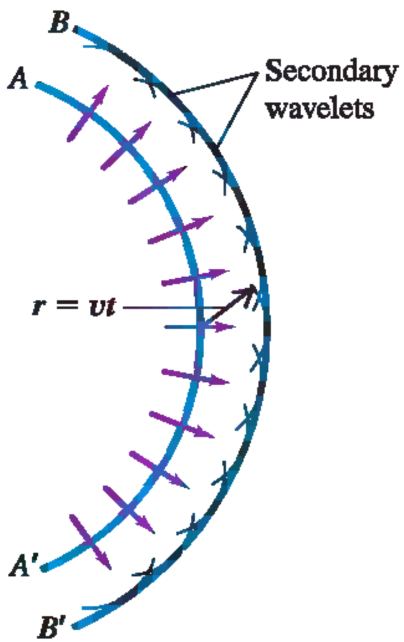
- Light travels at 3.00 x 10^8 m/s in vacuum (travels slower in liquids and solids)
- In order to describe propagation:

<u>Huygens method :</u>

$\rightarrow$ All points on given wave front taken as point sources for propagation of spherical waves

$\rightarrow$ Assume wave moves through medium in straight line in direction of rays





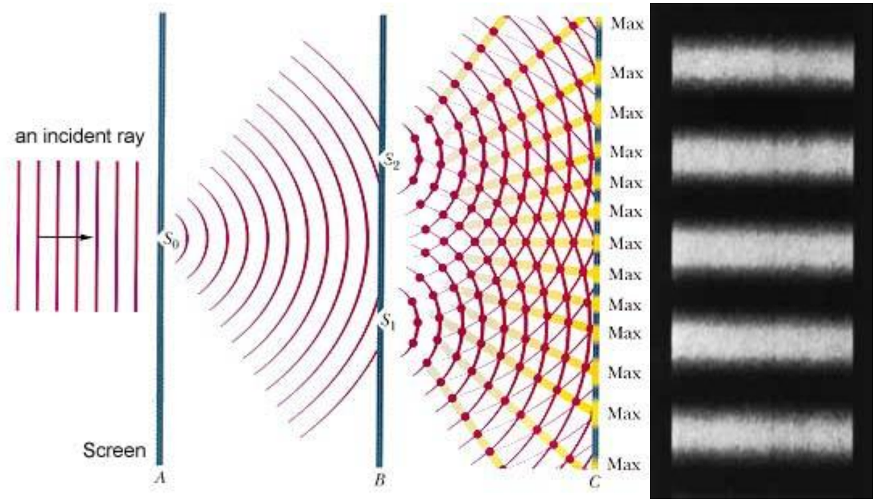
2 Interference of Light Waves

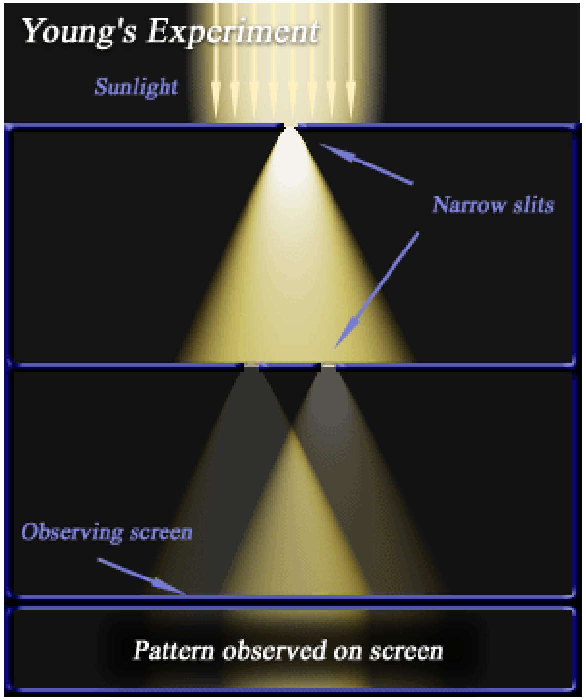
2.1 Conditions for interference

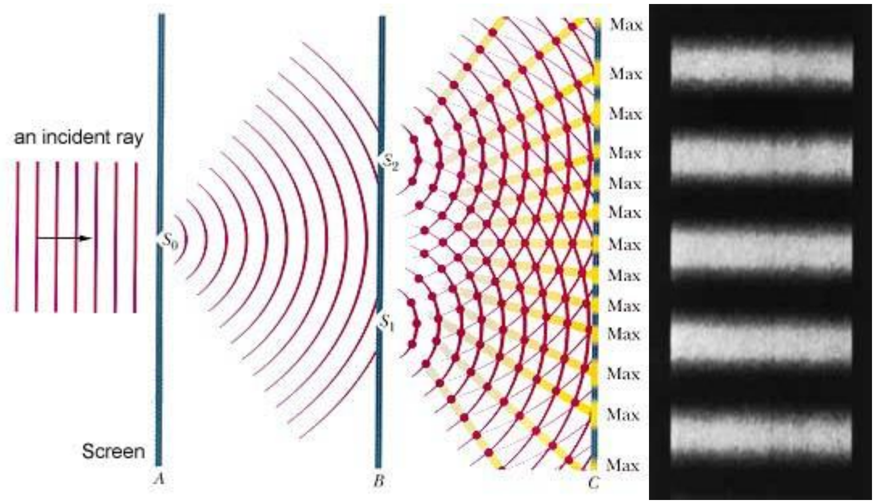
- light sources must be coherent (must maintain a constant phase with each other)
- sources must have identical wavelength
- superposition principle must apply

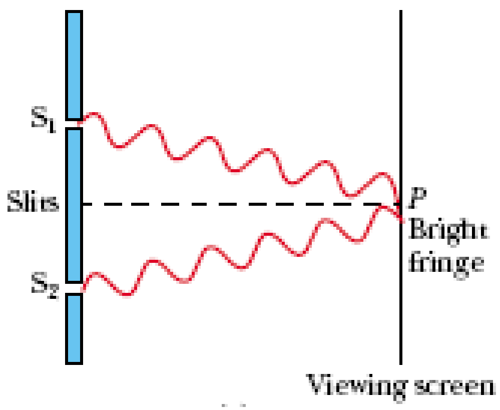
2.2 Young's Double-Slit Experiment

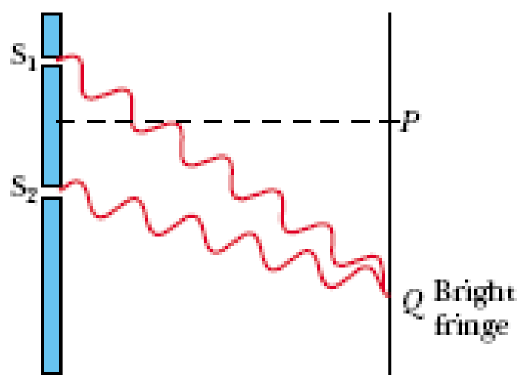
- Setup: light shines at the plane with two slits
- Result: a series of parallel dark and bright bands called fringes

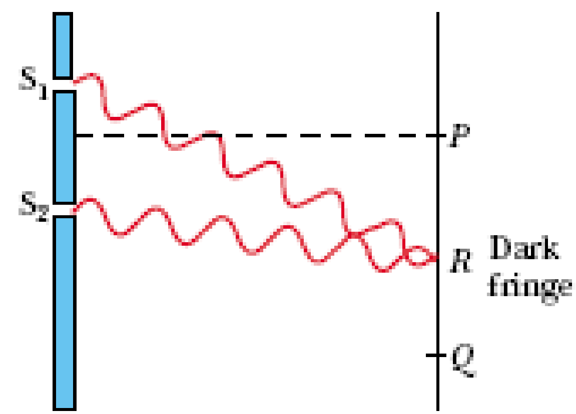












In phases

Opposite phases

- **Interference of Sinusoidal Waves**

$$y_1 = A\sin(\omega t - Kd_1)\ ; \quad y_2 = A\sin(\omega t - Kd_2)\ ; \quad K = \frac{2\pi}{\lambda}$$

$$y = y_1 + y_2 \quad = A\sin(\omega t - Kd_1) + A\sin(\omega t - Kd_2)$$

$$= A\left[\sin(\omega t - Kd_1) + \sin(\omega t - Kd_2)\right]$$

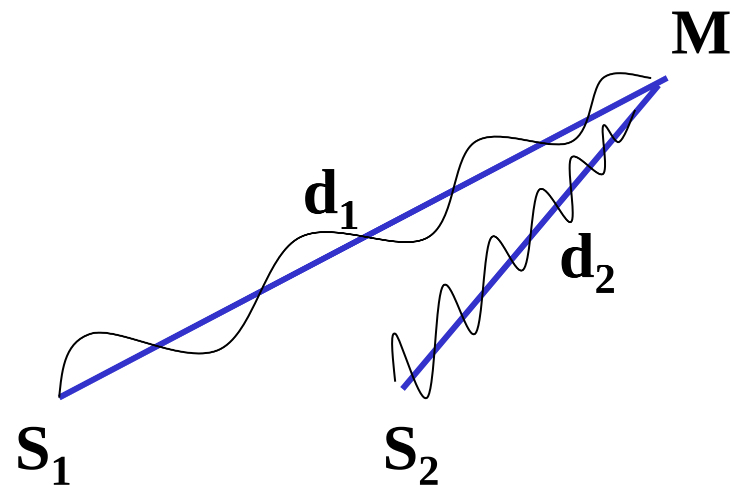
$$= 2A\cos\left(K\frac{d_2 - d_1}{2}\right)\sin(\omega t + K\frac{d_1 + d_2}{2})$$

Amplitude of y depends on the **path difference** : $\boldsymbol{\delta = d_2 - d_1}$

Amplitude of y maximum :

$$\cos\left(K\frac{d_2 - d_1}{2}\right) = \pm 1\ ; \ K\frac{d_2 - d_1}{2} = k\pi\ ;$$

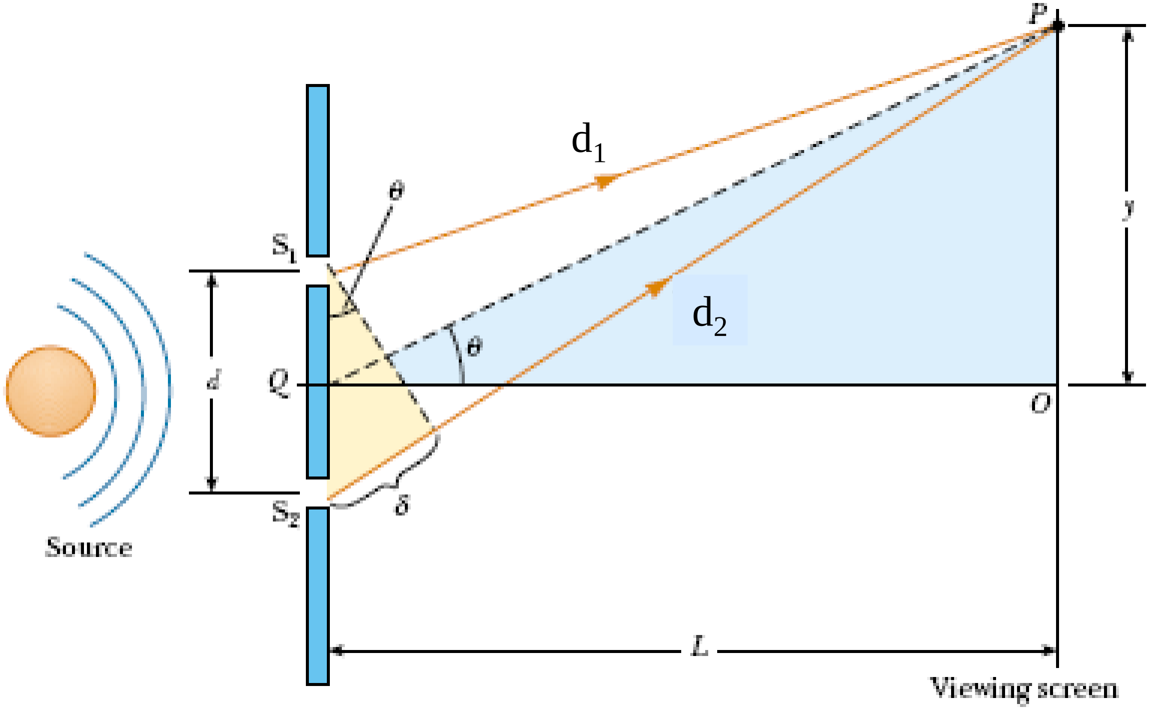
$$d_2 - d_1 = \frac{2k\pi}{2\pi/\lambda}\ ; \ \boxed{\delta = d_2 - d_1 = k\lambda}$$



Amplitude of y equal to 0 :

$$\boxed{\delta = d_2 - d_1 = (k + \frac{1}{2})\lambda}$$

- **The path difference in Young's Double-Slit Experiment**

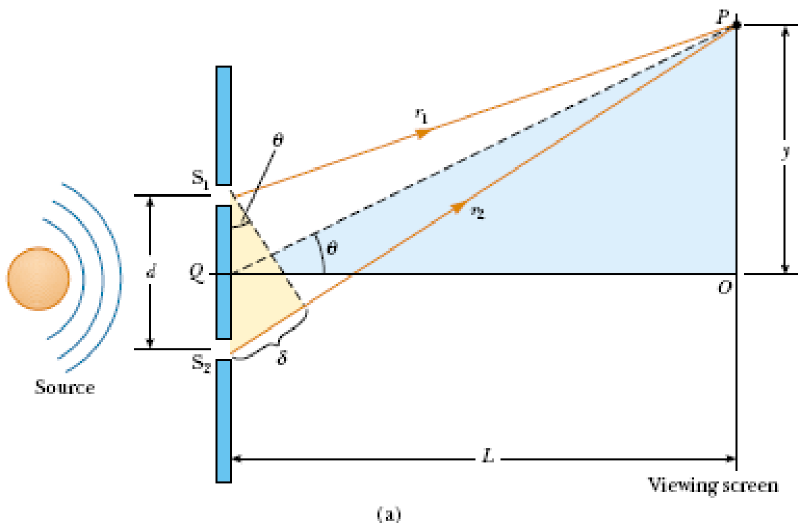


(a)

$$\delta = d_2 - d_1 \approx d\sin\theta \approx d\tan\theta$$

$$\boxed{\delta \approx d\frac{y}{L}}$$

- **Positions of fringes on the screen**



$$\boxed{\delta \approx d \sin\theta}$$

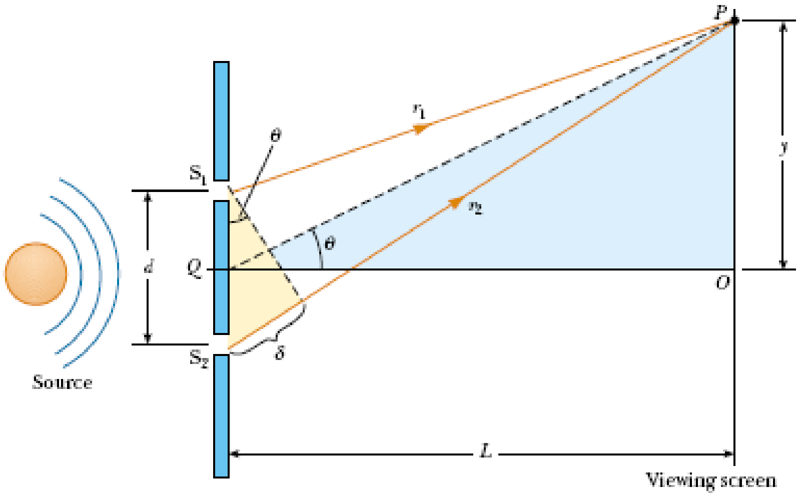
Bright fringes (constructive interference) :

$$\delta = \frac{d}{L} y_{BRIGHT} = k\lambda \; ; \quad \boxed{y_{BRIGHT} = k\frac{L}{d}\lambda = ki} \quad \boxed{i = \frac{L}{d}\lambda}$$

Dark fringes (destructive interference) :

$$\delta = \frac{d}{L} y_{DARK} = (k + \frac{1}{2})\lambda \; ; \quad \boxed{y_{DARK} = \left(k + \frac{1}{2}\right) i}$$

- **Angular Positions of Fringes**



(a)

$$\boxed{\delta \approx d\frac{y}{L}}$$

Bright regions (constructive interference) :

$$\boxed{\delta = d\sin\theta = k\lambda}$$

Dark regions (destructive interference) :

$$\boxed{\delta = d\sin\theta = (k + \frac{1}{2})\lambda}$$

PROBLEM 1 A viewing screen is separated from a double-lit source by 1.2 m. The distance between the two slits is 0.030 mm. The second-order bright fringe is 4.5 cm from the center line.

(a) Determine the wavelength of the light.

SOLUTION

(a) The position of second-order bright fringe :

$$y_2 = k\frac{L}{d}\lambda = 2\frac{L}{d}\lambda$$

$$\lambda = \frac{dy_2}{2L} = \frac{(3.0\times10^{-5}\ m)(4.5\times10^{-2}\ m)}{2(1.2\ m)}$$

$$= 5.6\times10^{-7}\ m = 560\ nm$$

PROBLEM 1 A viewing screen is separated from a double-lit source by 1.2 m. The distance between the two slits is 0.030 mm. The second-order bright fringe is 4.5 cm from the center line.

(b) Calculate the distance between adjacent bright fringes.

SOLUTION

(b) $$y_{k+1} - y_k = (k+1)\frac{L}{d}\lambda - k\frac{L}{d}\lambda = \frac{L}{d}\lambda = i$$

$$i = \frac{\lambda L}{d} = \frac{(5.6 \times 10^{-7}\ m)(1.2\ m)}{3.0 \times 10^{-5}\ m} = 2.2\ cm$$

PROBLEM 2 A light source emits visible light of two wavelengths: 430 nm and 510 nm. The source is used in a double-slit interference experiment in which L = 1.5 m and d = 0.025 mm. Find the separation distance between the third order bright fringes.

SOLUTION

$$y_3 = 3\frac{L}{d}\lambda = 3\frac{(1.5\ m)(430\times10^{-9}\ m)}{0.025\times10^{-3}\ m} = 7.74\times10^{-2}\ m$$

$$y'_3 = 3\frac{L}{d}\lambda' = 3\frac{(1.5\ m)(510\times10^{-9}\ m)}{0.025\times10^{-3}\ m} = 9.18\times10^{-2}\ m$$

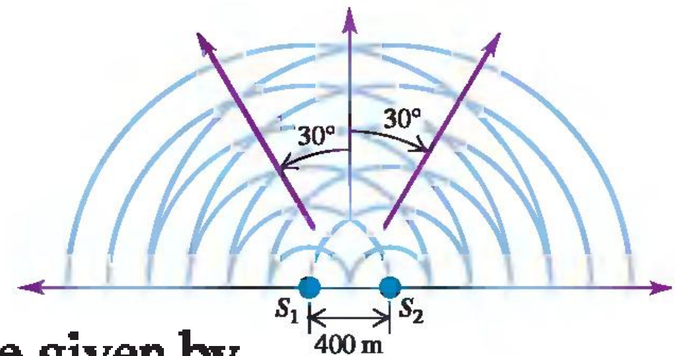
$$\Delta y = y'_3 - y_3 = 9.18\times10^{-2}\ m - 7.74\times10^{-2}\ m$$

$$= 1.40\times10^{-2}\ m = 1.40\ cm$$

PROBLEM 3

A radio station operating at a frequency of 1500 kHz = 1.5×10^6 Hz (near the top end of the AM broadcast band) has two identical vertical dipole antennas spaced 400 m apart, oscillating in phase. At distances much greater than 400 m, in what directions is the intensity greatest in the resulting radiation pattern?

SOLUTION



The wavelength is $\lambda = c/f = 200$ m.

$m = 0, \pm 1$, and ± 2, the intensity maxima are given by

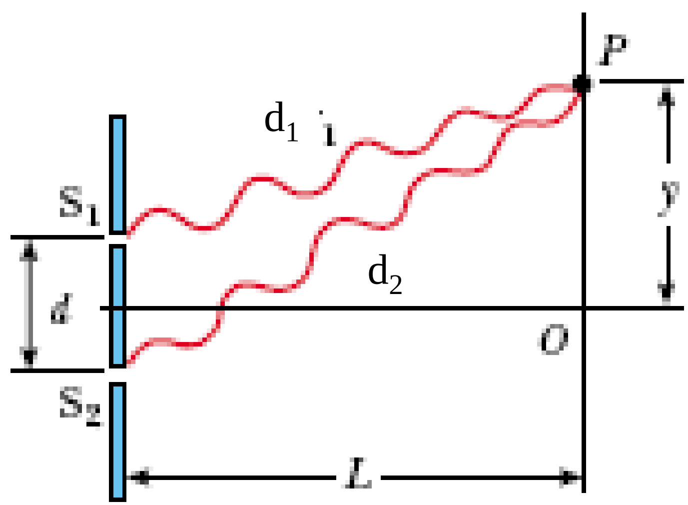
$$\sin\theta = \frac{m\lambda}{d} = \frac{m(200\text{ m})}{400\text{ m}} = \frac{m}{2} \qquad \theta = 0, \pm 30°, \pm 90°$$

minimum intensity (destructive interference)

$$\sin\theta = \frac{(m + \frac{1}{2})\lambda}{d} = \frac{m + \frac{1}{2}}{2} \qquad \theta = \pm 14.5°, \pm 48.6°$$

2.3 Intensity distribution of the double-slit. Interference pattern

- **light : electromagnetic wave**
- **suppose that the two slits represent coherent sources of sinusoidal waves : the two waves from the slits have the same angular frequency ω**
- **assume that the two waves have the same amplitude E_0 of the electric field at point P**

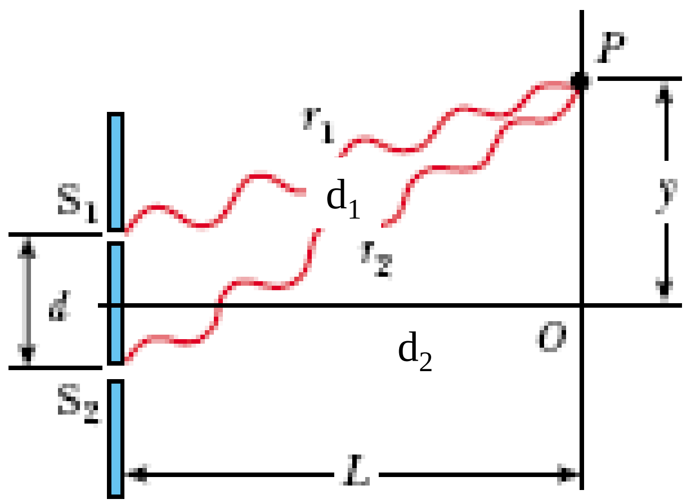


At point P:

$$E_1 = E_0 \sin(\omega t - Kd_1)\ ;\ E_2 = E_0 \sin(\omega t - Kd_2)\ ;\ K = \frac{2\pi}{\lambda}$$

$$E = E_1 + E_2 = 2E_0 \cos\left(K\frac{d_2 - d_1}{2}\right)\sin(\omega t + K\frac{d_1 + d_2}{2})$$

Amplitude of E: $2E_0\left|\cos\left(K\frac{d_2 - d_1}{2}\right)\right| = 2E_0\left|\cos\left(K\frac{\delta}{2}\right)\right|$



At point P:

Amplitude of E: $2E_0\left|\cos\left(K\dfrac{d_2-d_1}{2}\right)\right| = 2E_0\left|\cos\left(K\dfrac{\delta}{2}\right)\right|$

The intensity of a wave is proportional to the square of the electric field magnitude at that point:

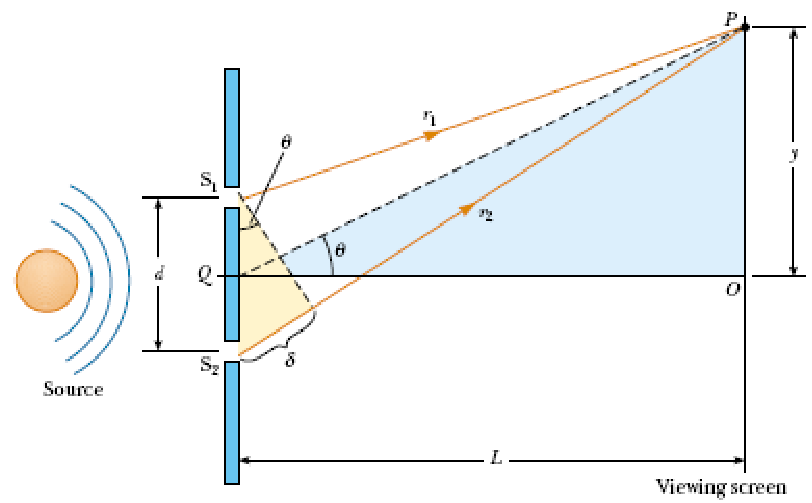
$$I = I_0\cos^2\left(K\frac{\delta}{2}\right)$$

$$\delta = d_2 - d_1 \approx d\sin\theta \longrightarrow I = I_0\cos^2\left(\frac{2\pi}{\lambda}\frac{d\sin\theta}{2}\right)$$

$$\boxed{I = I_0\cos^2\left(\frac{\pi d\sin\theta}{\lambda}\right)}$$

$$\sin\theta \approx \tan\theta = \frac{y}{L};$$

$$\boxed{I = I_0\cos^2\left(\frac{\pi d y}{L\lambda}\right)}$$

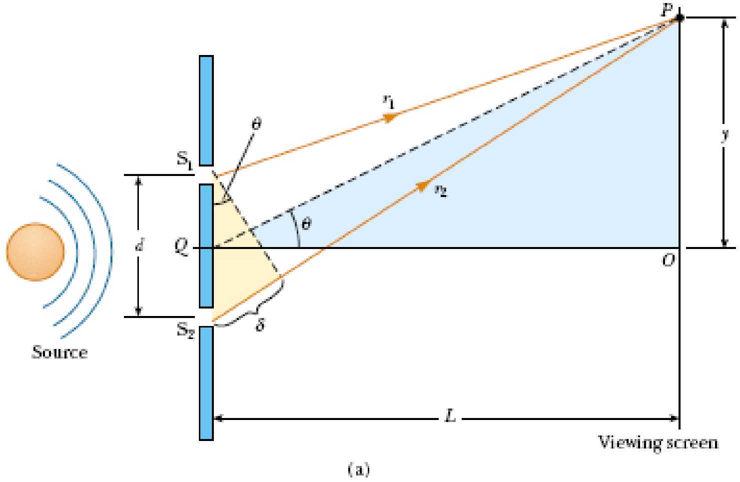


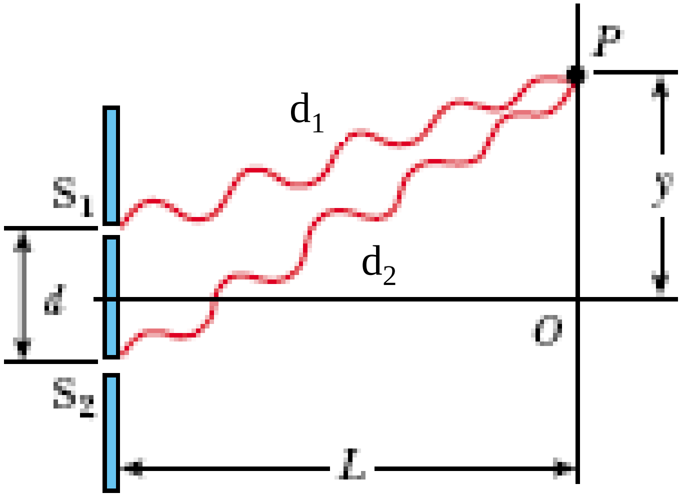
(a)

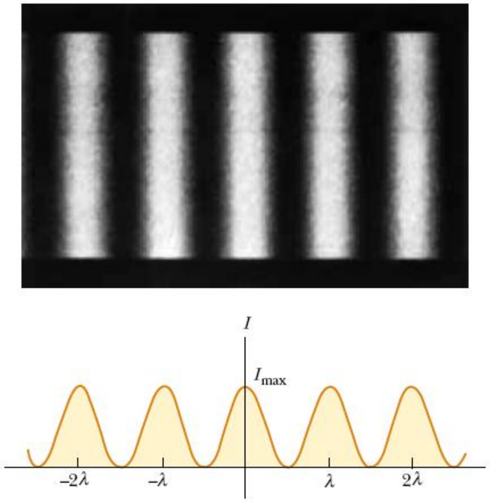
- The intensity *maximum* :

$$\frac{\pi d \sin\theta}{\lambda} = n\pi \; ; \quad \frac{\pi d y}{L\lambda} = n\pi$$

$$\sin\theta = n\frac{\lambda}{d} \; ; \quad y = n\frac{L\lambda}{d}$$

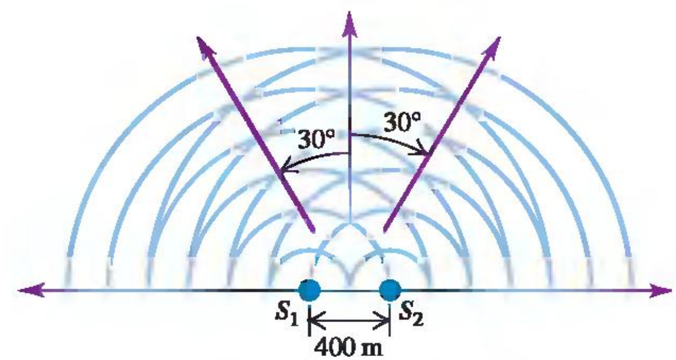






PROBLEM 4

Suppose the two identical radio antennas in Fig. 35.8 are moved to be only 10.0 m apart and the frequency of the radiated waves is increased to $f = 60.0$ MHz. The intensity at a distance of 700 m in the $+x$-direction (corresponding to $\theta = 0$ in Fig. 35.5) is $I_0 = 0.020\ \mathrm{W/m^2}$. (a) What is the intensity in the direction $\theta = 4.0°$? (b) In what direction near $\theta = 0$ is the intensity $I_0/2$? (c) In what directions is the intensity zero?



SOLUTION

$$\lambda = \frac{c}{f} = \frac{3.00 \times 10^8\ \mathrm{m/s}}{60.0 \times 10^6\ \mathrm{s^{-1}}} = 5.00\ \mathrm{m}$$

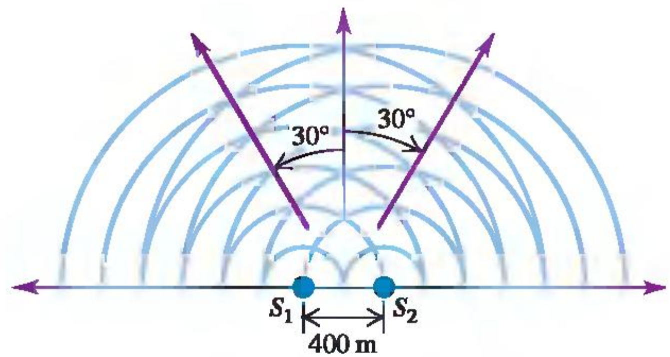
$$I = I_0 \cos^2\left(\frac{\pi d}{\lambda}\sin\theta\right)$$

$$= (0.020\ \mathrm{W/m^2})\cos^2\left[\frac{\pi(10.0\ \mathrm{m})}{5.00\ \mathrm{m}}\sin\theta\right]$$

$$= (0.020\ \mathrm{W/m^2})\cos^2[(2.00\pi\ \mathrm{rad})\sin\theta]$$

PROBLEM 4

Suppose the two identical radio antennas in Fig. 35.8 are moved to be only 10.0 m apart and the frequency of the radiated waves is increased to $f = 60.0$ MHz. The intensity at a distance of 700 m in the $+x$-direction (corresponding to $\theta = 0$ in Fig. 35.5) is $I_0 = 0.020\ \mathrm{W/m^2}$. (a) What is the intensity in the direction $\theta = 4.0°$? (b) In what direction near $\theta = 0$ is the intensity $I_0/2$? (c) In what directions is the intensity zero?



SOLUTION

$$I = (0.020\ \mathrm{W/m^2})\cos^2[(2.00\pi\ \mathrm{rad})\sin\theta]$$

(a) When $\theta = 4.0°$,

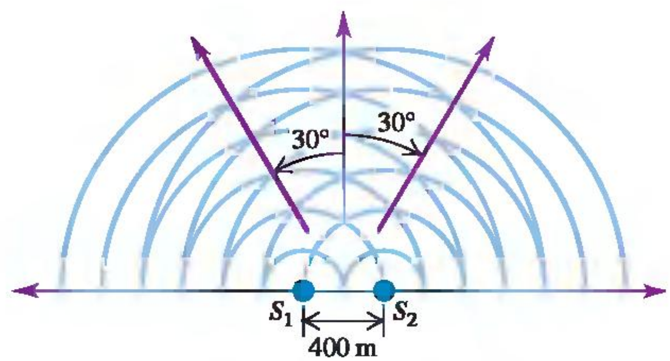
$$I = (0.020\ \mathrm{W/m^2})\cos^2[(2.00\pi\ \mathrm{rad})\sin 4.0°]$$
$$= 0.016\ \mathrm{W/m^2}$$

This is about 82% of the intensity at $\theta = 0$.

PROBLEM 4

Suppose the two identical radio antennas in Fig. 35.8 are moved to be only 10.0 m apart and the frequency of the radiated waves is increased to $f = 60.0$ MHz. The intensity at a distance of 700 m in the $+x$-direction (corresponding to $\theta = 0$ in Fig. 35.5) is $I_0 = 0.020\ \mathrm{W/m^2}$. (a) What is the intensity in the direction $\theta = 4.0°$? (b) In what direction near $\theta = 0$ is the intensity $I_0/2$? (c) In what directions is the intensity zero?

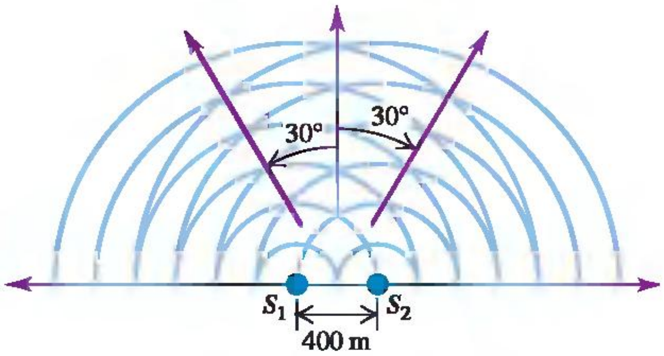
SOLUTION



(b) The intensity I equals $I_0/2$ when the cosine in Eq. (35.14) has the value $\pm 1/\sqrt{2}$. This occurs when $2.00\pi \sin\theta = \pm\pi/4$ rad, so that $\sin\theta = \pm(1/8.00) = \pm 0.125$ and $\theta = \pm 7.2°$.

PROBLEM 4

Suppose the two identical radio antennas in Fig. 35.8 are moved to be only 10.0 m apart and the frequency of the radiated waves is increased to $f = 60.0$ MHz. The intensity at a distance of 700 m in the $+x$-direction (corresponding to $\theta = 0$ in Fig. 35.5) is $I_0 = 0.020\ \mathrm{W/m^2}$. (a) What is the intensity in the direction $\theta = 4.0°$? (b) In what direction near $\theta = 0$ is the intensity $I_0/2$? (c) In what directions is the intensity zero?



SOLUTION

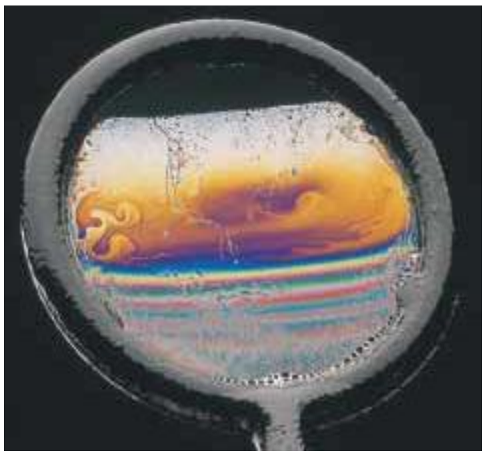
$$I = (0.020\ \mathrm{W/m^2})\cos^2[(2.00\pi\ \mathrm{rad})\sin\theta]$$

(c) The intensity is zero when $\cos[(2.00\pi\ \mathrm{rad})\sin\theta] = 0$. This occurs when $2.00\pi\sin\theta = \pm\pi/2, \pm 3\pi/2, \pm 5\pi/2, \ldots$, or $\sin\theta = \pm 0.250, \pm 0.750, \pm 1.25, \ldots$. Values of $\sin\theta$ greater than 1 have no meaning, and we find

$$\theta = \pm 14.5°, \pm 48.6°$$

2.4 INTERFERENCE IN THIN FILMS

- **Interference effects are commonly observed in thin films, such as thin layers of oil on water or the thin surface of a soap bubble**



- **The varied colors observed when white light is incident on such films result from the interference of waves reflected from the two surfaces of the film**

a. Change of phase due to reflection

- Interference pattern with *Lloyd's mirror*

P' is equidistant from points *S* and *S*:

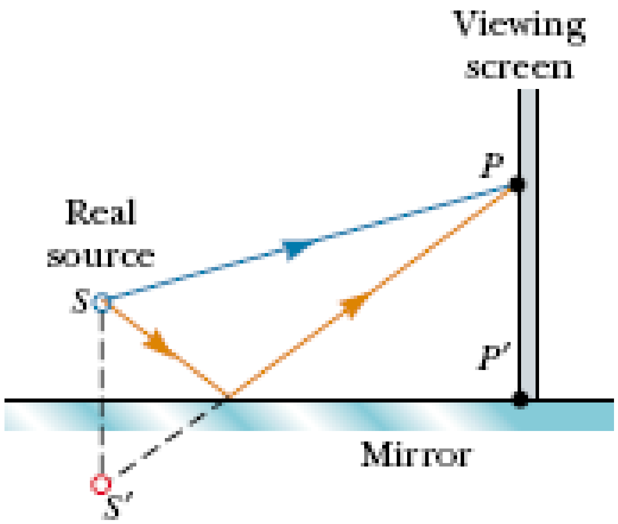
the path difference is zero

→ bright fringe at P' ?

We observe a dark fringe at *P'*

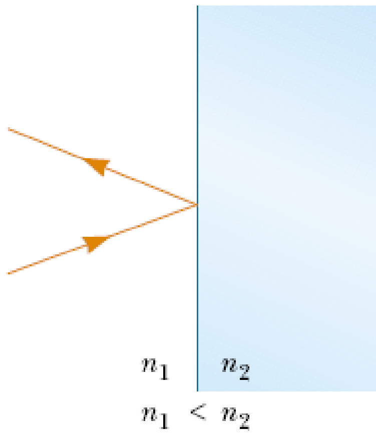
⇓

There a 180° phase change produced by reflection

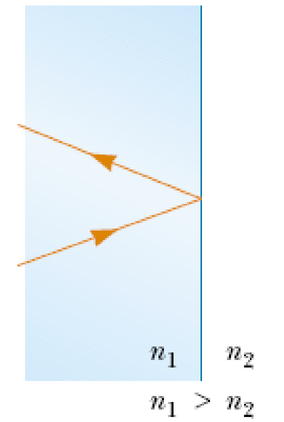


CONCLUSION : an electromagnetic wave undergoes a phase change of 180° upon reflection from a medium that has a higher index of refraction than the one in which the wave is traveling.

- **Change of phase due to reflection**



180° phase change



No phase change

- **If λ is the wavelength of the light in free space**

$$\lambda = \frac{c}{f}$$

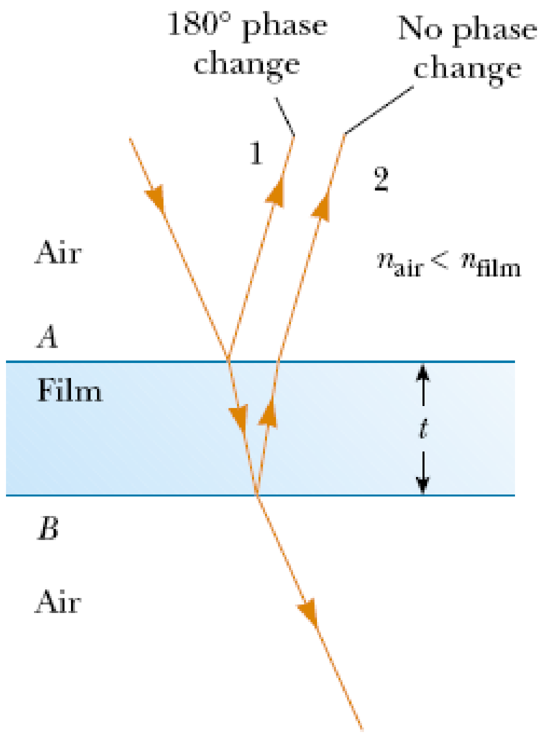
and if the medium has the refraction index *n* : $n = \frac{c}{v}$

The wavelength of light in this medium :

$$\lambda_n = \frac{v}{f} = \frac{c}{nf} \; ; \quad \boxed{\lambda_n = \frac{\lambda}{n}}$$

b. Interference in thin films

- Consider a film of uniform thickness t and index of refraction n. Assume that the light rays traveling in air are nearly normal to the two surfaces of the film.

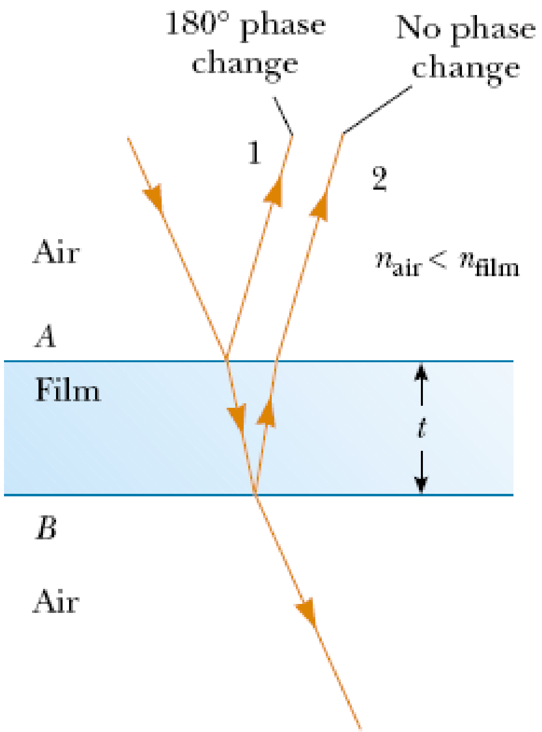


- Reflected ray 1 : phase change of 180° with respect to the incident wave.
- Reflected ray 2 : no phase change

- Ray 1 is 180° out of phase with ray 2 $\rightarrow$ a path difference of $\lambda_n/2$.

- ray 2 travels an extra distance $2t$ before the waves recombine in the air

- Ray 1 is 180° out of phase with ray 2 → a path difference of $\lambda_n/2$.
- ray 2 travels an extra distance $2t$ before the waves recombine in the air



Condition for constructive interference :

$$\boxed{2t = (m + \frac{1}{2})\lambda_n}$$

$$= (m + \frac{1}{2})\frac{\lambda}{n}$$

$$\boxed{2nt = (m + \frac{1}{2})\lambda}$$

Condition for destructive interference :

$$\boxed{2nt = m\lambda_n} \qquad m = 0; 1; 2; 3; \ldots$$

PROBLEM 5 Calculate the minimum thickness of a soap-bubble ($n = 1.33$) film that results in constructive interference in the reflected light if the film is illuminated with light whose wavelength in free space is $\lambda = 600$ nm.

SOLUTION

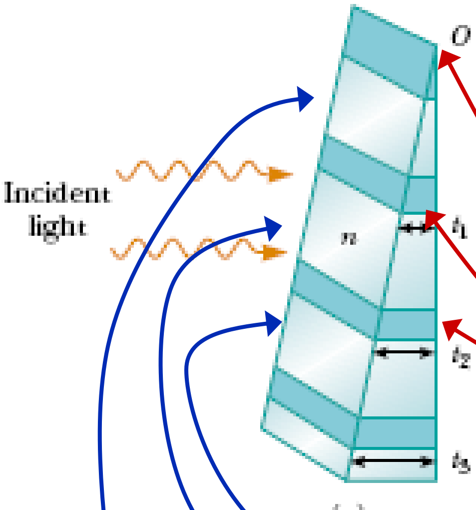
Constructive interference : $2nt = (m + \frac{1}{2})\lambda$

The minimum film thickness : $m = 0$

$$2nt = \frac{\lambda}{2}$$

$$t = \frac{\lambda}{4n} = \frac{600\ nm}{4(1.33)} = 113\ nm$$

PROBLEM 6 A thin, wedge-shaped film of refractive index n is illuminated with monochromatic light of wavelength , as illustrated in the figure. Describe the interference pattern observed for this case.



SOLUTION

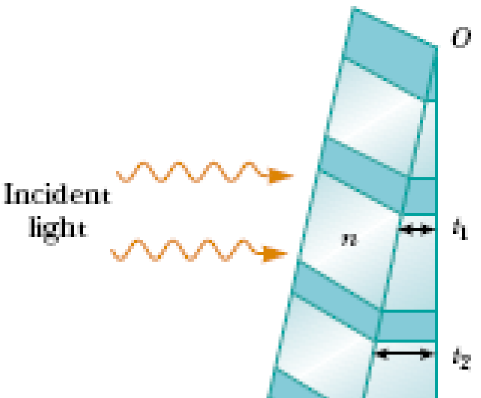
Dark fringes : $2nt_m = m\lambda \; ; \; t_m = \dfrac{m\lambda}{2n}$

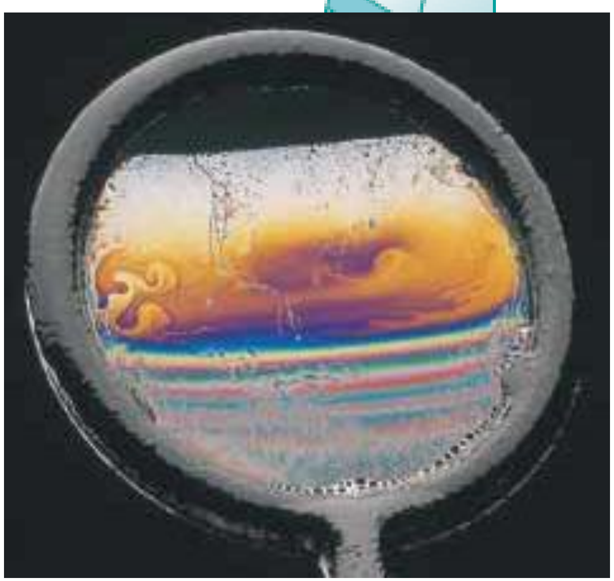
$$t_0 = 0 \; ; \; t_1 = \frac{1.\lambda}{2n} \; ; \; t_2 = \frac{2.\lambda}{2n} \; ; \ldots$$

Bright fringes : $2nt_m = (m + \dfrac{1}{2})\lambda$

$$t_0 = \frac{\lambda}{4n} \; ; \; t_1 = \frac{3.\lambda}{4n} \; ; \; t_2 = \frac{5.\lambda}{4n} \; ; \ldots$$

PROBLEM 6 A thin, wedge-shaped film of refractive index n is illuminated with monochromatic light of wavelength , as illustrated in the figure. Describe the interference pattern observed for this case.





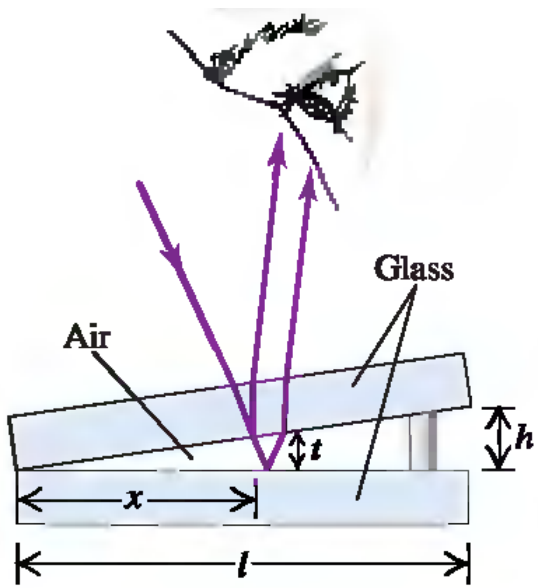
SOLUTION

White light :

Dark fringes : $2nt_m = m\lambda_\alpha$; $t_m = \dfrac{m\lambda_\alpha}{2n}$

Bright fringes : $2nt_m = (m + \dfrac{1}{2})\lambda_\alpha$

PROBLEM 7 Suppose the two glass plates in the figure are two microscope slides 10.0 cm long. At one end they are in contact; at the other end they are separated by a piece of paper 0.0200 mm thick. What is the spacing of the interference fringes seen by reflection? Is the fringe at the line of contact bright or dark? Assume monochromatic light with a wavelength in air of λ = 500nm.



SOLUTION

$$\text{Dark fringes}: 2nt = 2t = m\lambda$$

$$\frac{t}{x} = \frac{h}{l}; \quad \frac{m\lambda}{2x} = \frac{h}{l};$$

$$x = m\frac{\lambda l}{2h} = m\frac{(0.100\ m)(500\times 10^{-9}\ m)}{2(0.002\times 10^{-3}\ m)}$$

$$= 1.25m\ (mm) = 0; 1.25\ mm; 2.50\ mm; ...$$

PROBLEM 7 Suppose the two glass plates in the figure are two microscope slides 10.0 cm long. At one end they are in contact; at the other end they are separated by a piece of paper 0.0200 mm thick. What is the spacing of the interference fringes seen by reflection? Is the fringe at the line of contact bright or dark? Assume monochromatic light with a wavelength in air of λ = 500nm. Suppose the glass plates have n = 1.52 and the space between plates contains water (n = 1.33) instead of air. What happens now?

SOLUTION

In the film of water (n = 1.33), the wavelength :

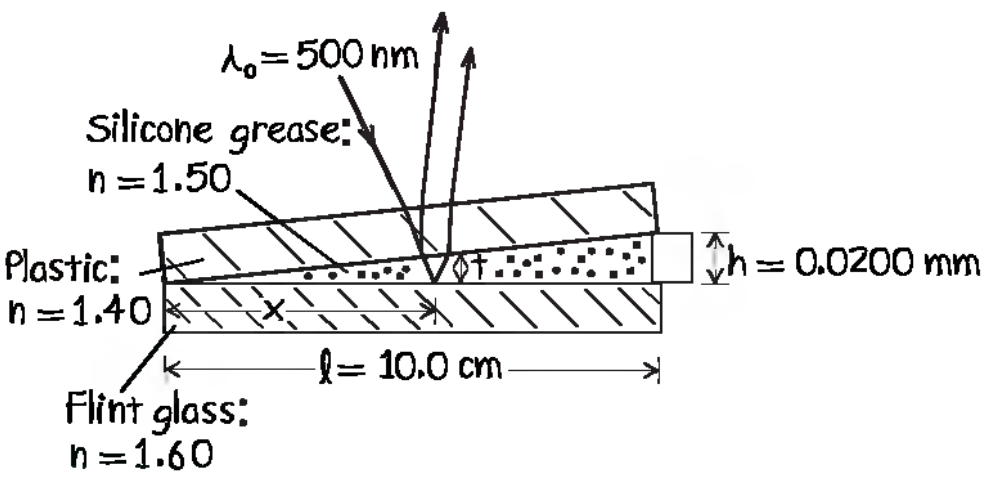
$$\lambda' = \frac{\lambda}{n} = \frac{500\ nm}{1.33} = 376\ nm$$

$$x' = m\frac{\lambda' l}{2h} = m\frac{\lambda l}{n2h} = m\frac{(0.100\ m)(500\times 10^{-9}\ m)}{2(0.002\times 10^{-3}\ m)(1.33)}$$

$$= 1.25m / 1.33\ (mm) = 0.0150m\ (mm)$$

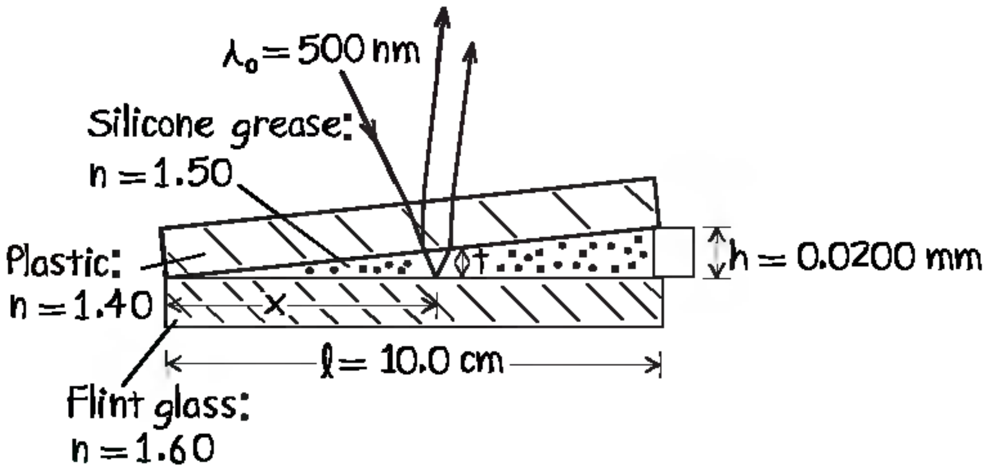
$$= 0; 0.015\ mm; 0.030\ mm; ...$$

PROBLEM 8 Suppose the two glass plates in the figure are two microscope slides 10.0 cm long. At one end they are in contact; at the other end they are separated by a piece of paper 0.0200 mm thick. What is the spacing of the interference fringes seen by reflection? Is the fringe at the line of contact bright or dark? Assume monochromatic light with a wavelength in air of λ = 500nm. Suppose the upper of the two plates is a plastic material with n = 1.40, the wedge is filled with a silicone grease having n = 1.50, and the bottom plate is a dense flint glass with n = 1.60. What happens now?



SOLUTION

SOLUTION



The wavelength in the silicone grease: $\lambda' = \dfrac{\lambda}{n} = \dfrac{500\ nm}{1.50} = 333\ nm$

The two reflected waves from the line of contact are in phase (they both undergo the same phase shift), so the line of contact is at a bright fringe.

$$x' = 0.833\ mm$$

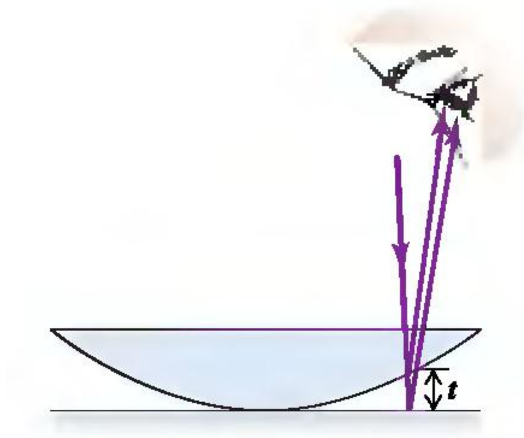
c. Newton’s Rings

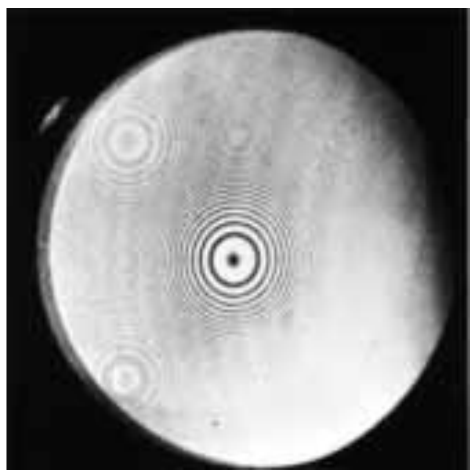
The convex surface of a lens in contact with a plane glass plate.

A thin film of air is formed between the two surfaces.

View the setup with monochromatic light :

Circular interference fringes





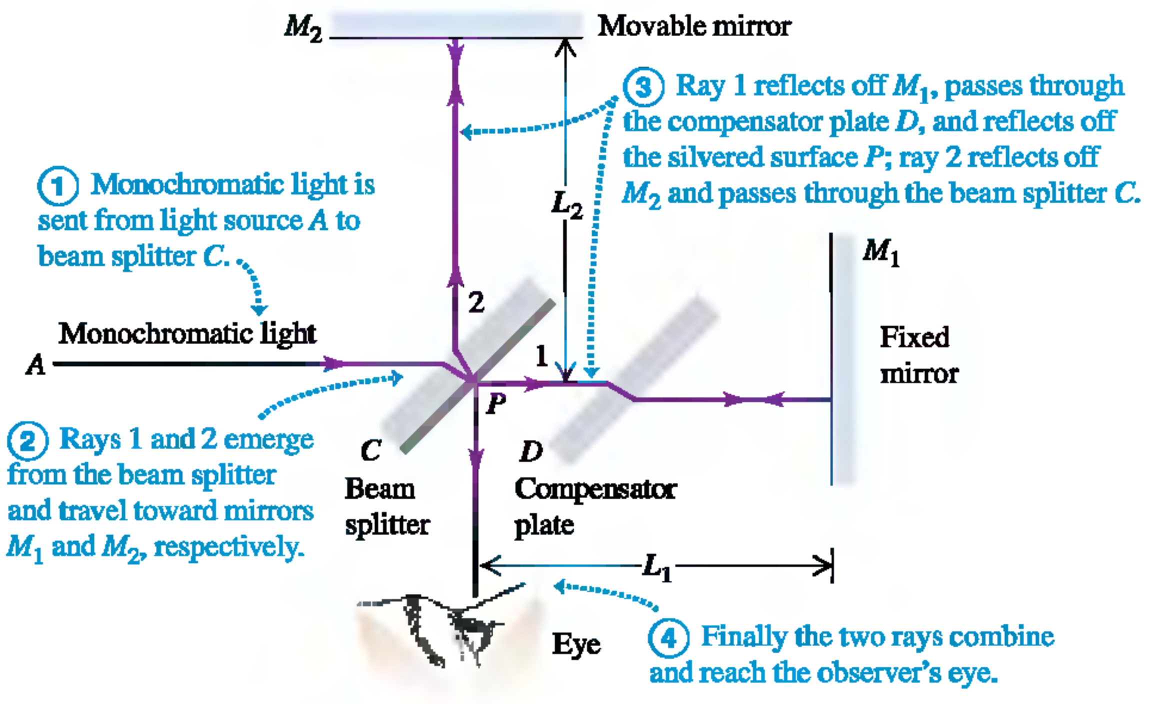
Ray 1 : a phase change of 180° upon reflection; ray 2 : no phase change

→ the conditions for **constructive and destructive interference**

2.5 THE MICHELSON INTERFEROMETER

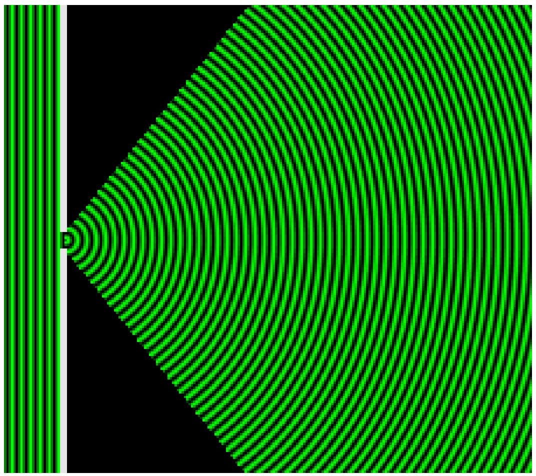
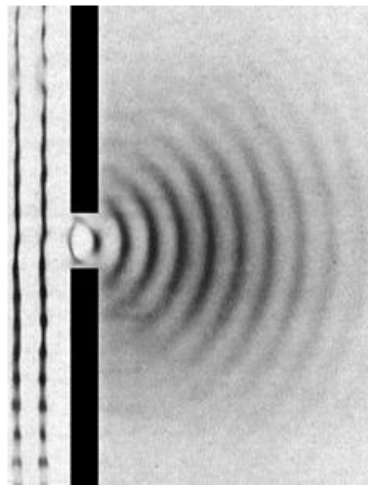
- **Principle : splits a light beam into two parts and then recombines the parts to form an interference pattern**
- **Use : To measure wavelengths or other lengths with great precision**

The observer sees an interference pattern that results from the difference in path lengths for rays 1 and 2.



3. Light Diffraction

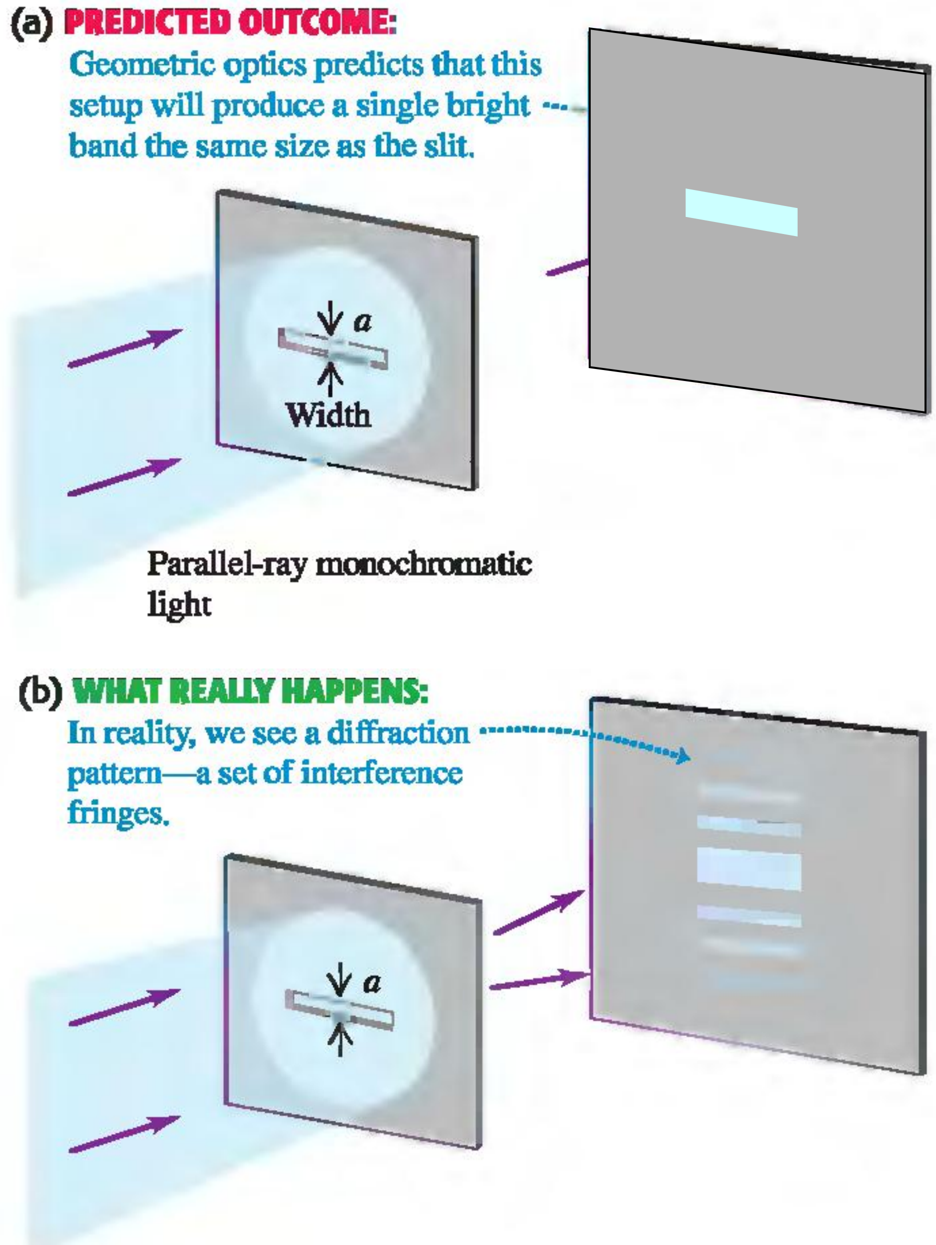
3.1 Introduction If a wave encounters a barrier that has an opening of dimensions similar to the wavelength, the part of the wave that passes through the opening will spread out (diffract) into the region beyond the barrier


That is another proof of wave nature

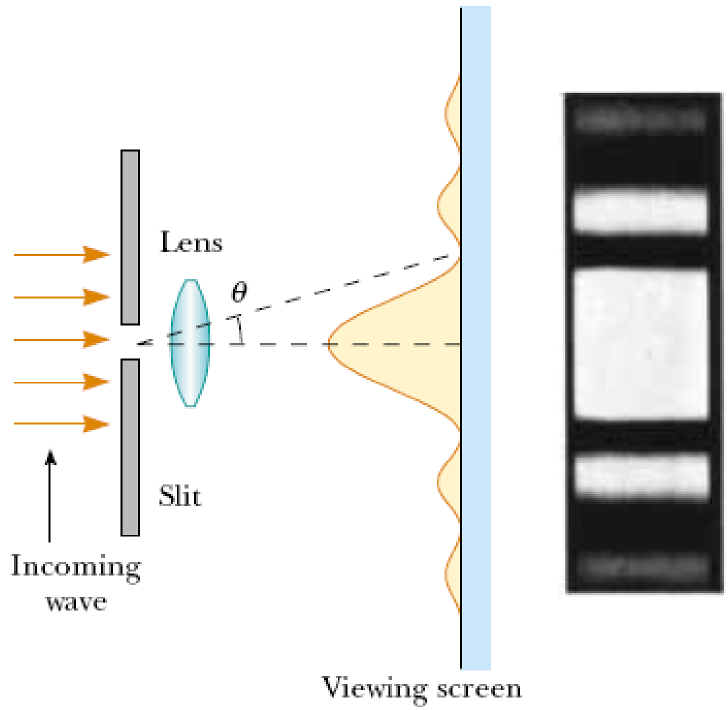
Diffraction Phenomenon $\Leftrightarrow$ WAVE

Light produces also the diffraction: a light (with appropriate wavelength) beam going through a hole gives a number of circular fringes

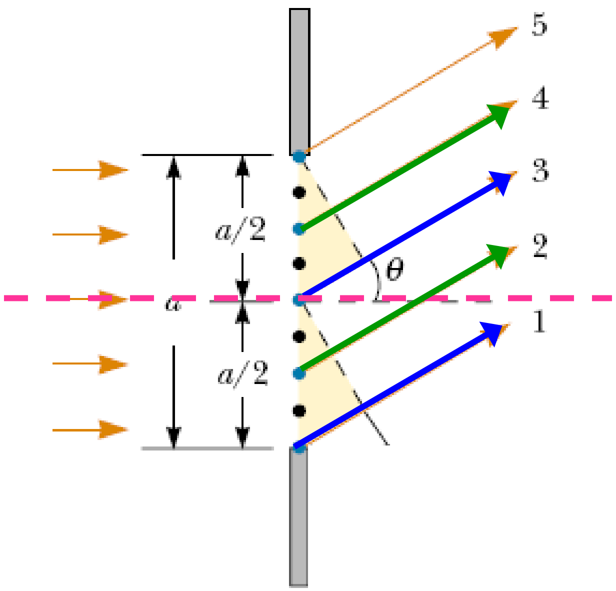

(a) PREDICTED OUTCOME:
Geometric optics predicts that this setup will produce a single bright band the same size as the slit.
a
Width
Parallel-ray monochromatic light
(b) WHAT REALLY HAPPENS:
In reality, we see a diffraction pattern—a set of interference fringes.
a

3.2 Fraunhofer diffraction

All the rays passing through a narrow slit are approximately parallel to one another.



- **Diffraction from narrow slit**



Huygens's principle : Each portion of the slit acts as a source of light waves

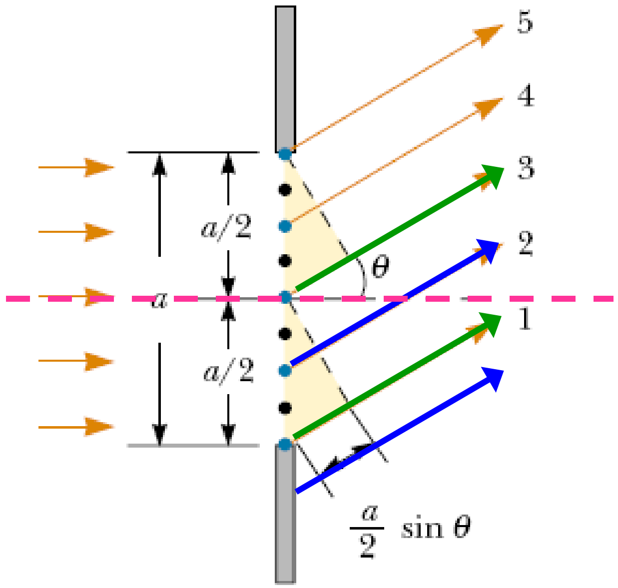
→ Divide the slit into **two halves**

Rays 1 and 3 : ray 1 travels farther than ray 3 by an amount equal to the path difference $(a/2)\sin\theta$

Rays 2 and 4 : ray 2 travels farther than ray 4 by an amount equal to the path difference $(a/2)\sin\theta$

Waves from the upper half of the slit interfere destructively with waves from the lower half when :

$$\frac{a}{2}\sin\theta = \frac{\lambda}{2}\,;\quad \sin\theta = \frac{\lambda}{a}$$



$\rightarrow$ Divide the slit into **two halves**

$$\frac{a}{2}\sin\theta = \frac{\lambda}{2}; \quad \sin\theta = \frac{\lambda}{a}$$

$\rightarrow$ Divide the slit into **four parts :**

Dark fringes :

$$\frac{a}{4}\sin\theta = \frac{\lambda}{2}; \quad \sin\theta = \frac{2\lambda}{a}$$

$\rightarrow$ Divide the slit into **six parts :**

Dark fringes :

$$\frac{a}{6}\sin\theta = \frac{\lambda}{2}; \quad \sin\theta = \frac{3\lambda}{a}$$

Divide the slit into **two halves :**

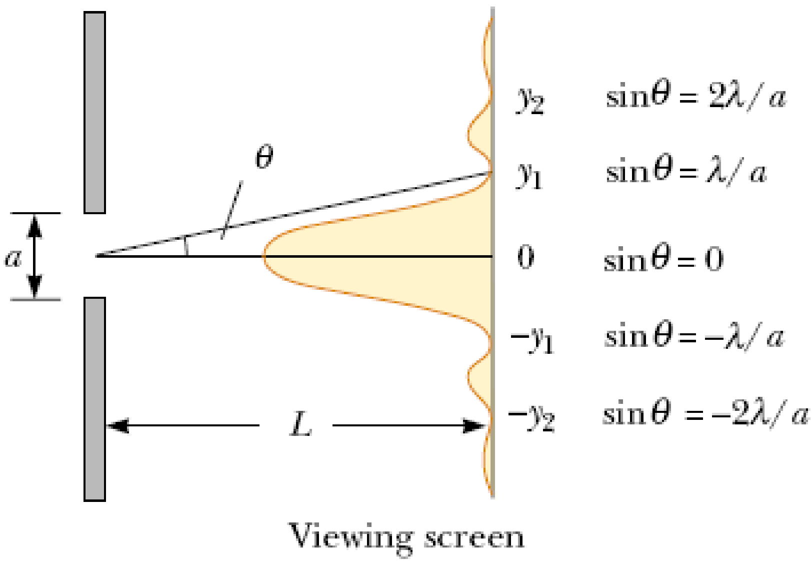
$$\sin\theta = \frac{\lambda}{a}$$

Divide the slit into **four parts :**

Dark fringes : $\sin\theta = \frac{2\lambda}{a}$

Divide the slit into **six parts :**

Dark fringes : $\sin\theta = \frac{3\lambda}{a}$



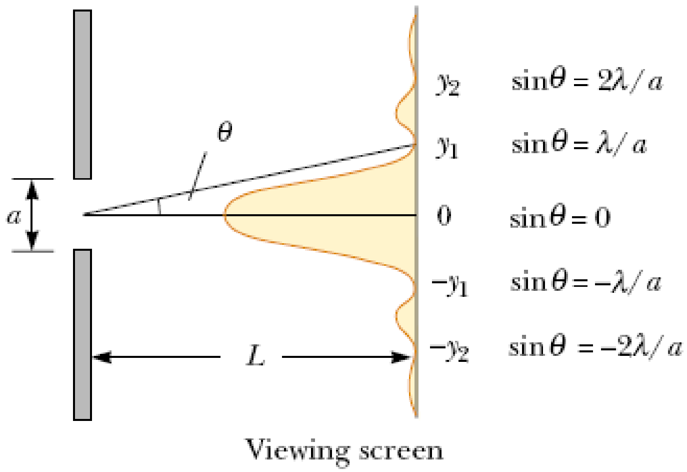
The general condition for destructive interference :

$$\boxed{\sin\theta = m\frac{\lambda}{a}; \quad m = \pm 1; \pm 2; \pm 3; \ldots}$$

$$\lambda << a \longrightarrow \sin\theta \approx \theta \approx \tan\theta = \frac{y_m}{L}$$

Positions of dark fringes : $\boxed{y_m = m\frac{L\lambda}{a}; \quad m = \pm 1; \pm 2; \pm 3; \ldots}$

PROBLEM 9 Light of wavelength 580 nm is incident on a slit having a width of 0.300 mm. The viewing screen is 2.00 m from the slit. Find the positions of the first dark fringes and the width of the central bright fringe.



SOLUTION

$$\boxed{\sin\theta = m\frac{\lambda}{a}\,; \quad m = \pm 1; \pm 2; \pm 3; \ldots}$$

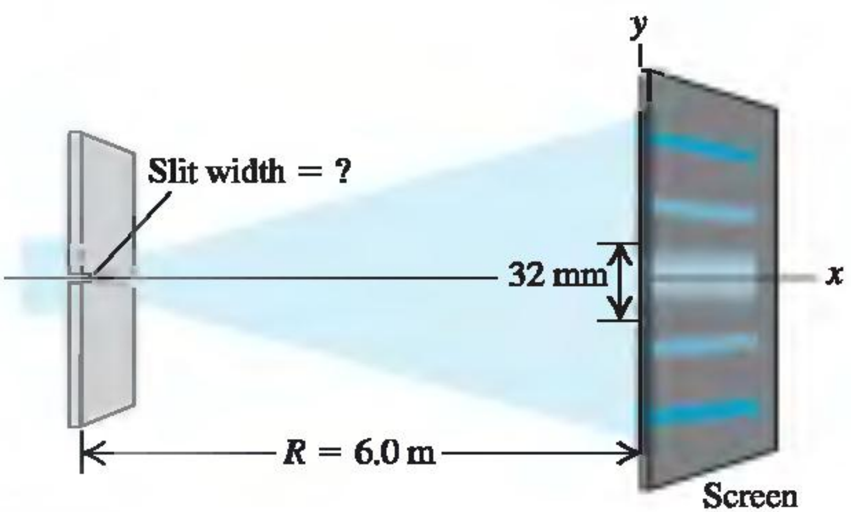
The two dark fringes that flank the central bright fringe : $m = \pm 1$

$$y_1 \approx L\sin\theta = \pm L\frac{\lambda}{a} = \pm 3.87 \times 10^{-3}\ \mathrm{m}$$

The width of the central bright fringe :

$$2|y_1| = 2(3.87\ mm) = 7.74\ mm$$

PROBLEM 10 You pass 633-nm laser light through a narrow slit and observe the diffraction pattern on a screen 6.0 m away. You find that the distance on the screen between the centers of the first minima outside the central bright fringe is 32 mm. How wide is the slit?



SOLUTION

Dark fringes :

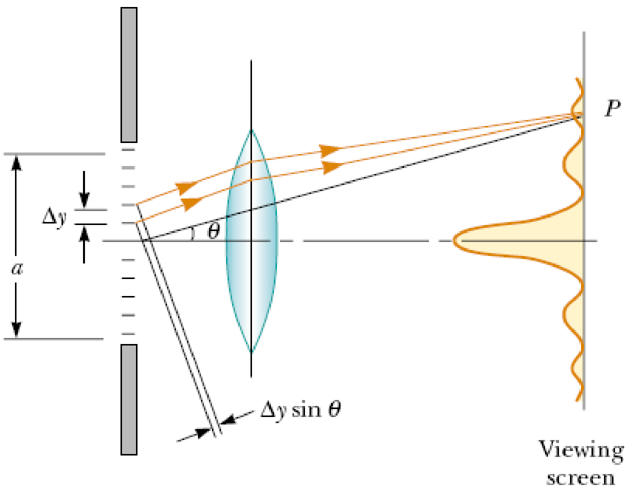
$$y_m = m\frac{L\lambda}{a}; \quad m = \pm 1; \pm 2; \pm 3; \ldots$$

The first minimum : $m = 1$

$$y_1 = 1\frac{L\lambda}{a} = \frac{32\ mm}{2}$$

$$a = \frac{L\lambda}{y_1} = \frac{(6\ m)(633\times 10^{-9}\ m)}{32\times 10^{-3}\ m/2} = 0.24\ mm$$

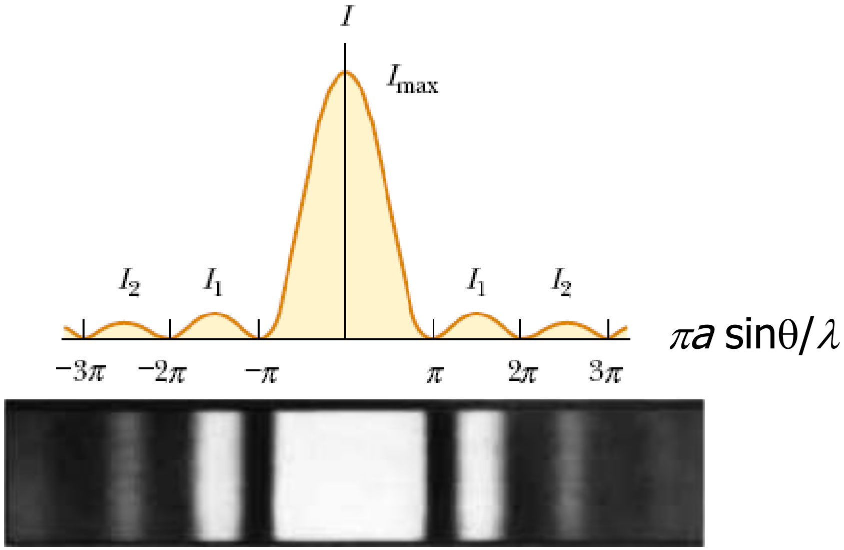
• Intensity of Single-Slit Diffraction Patterns



The intensity at each point on the screen :

$$\boxed{I = I_{\max}\left[\frac{\sin(\pi a \sin\theta / \lambda)}{\pi a \sin\theta / \lambda}\right]^2}$$

where $I_{\max}$ is the intensity at $\theta = 0$ (the central maximum).



We see that minima occur when :

$$\pi a \sin\theta / \lambda = m\pi$$

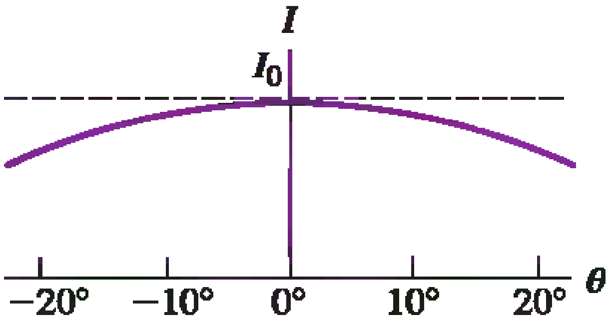
$$\sin\theta = m\,\frac{\lambda}{a}$$

(The general condition for destructive interference)

- **Width of the Single-Slit Pattern**

$a = \lambda$

If the slit width is equal to or narrower than the wavelength, only one broad maximum forms.



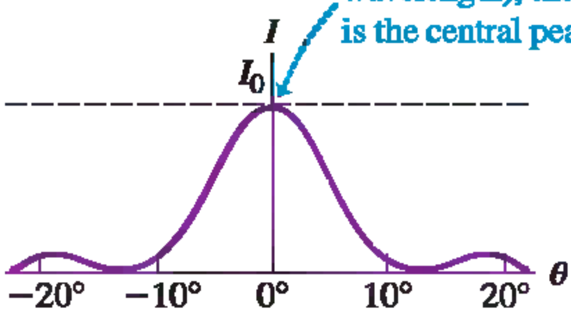
$$\boxed{I = I_{\max}\left[\frac{\sin(\pi a \sin\theta / \lambda)}{\pi a \sin\theta / \lambda}\right]^2}$$

First dark fringes : $\sin\theta = 1\,\dfrac{\lambda}{a}$

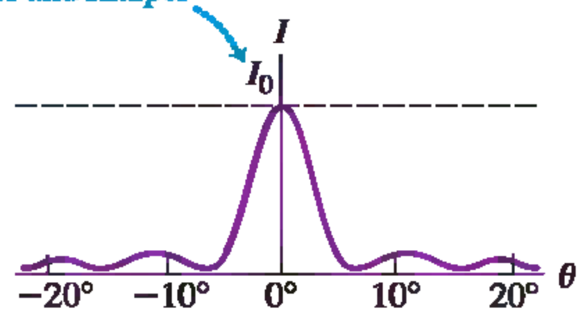
The single-slit diffraction pattern depends on the ratio : $\dfrac{\lambda}{a}$

$a = \lambda : \sin\theta = 1 \;\rightarrow\; \theta = \pi/2$

The wider the slit (or the shorter the wavelength), the narrower and sharper is the central peak.



$a = 5\lambda$



$a = 8\lambda$

Consequence :

• $a >> \lambda$: $\sin\theta << 1 \;\rightarrow\; \theta \approx 0$:

We can consider practically all the light to be concentrated at the **geometrical focus.**

• $a < \lambda$: $\theta \approx \pi/2$:

The central maximum spreads over 180° : the fringe pattern is **not seen at all**.

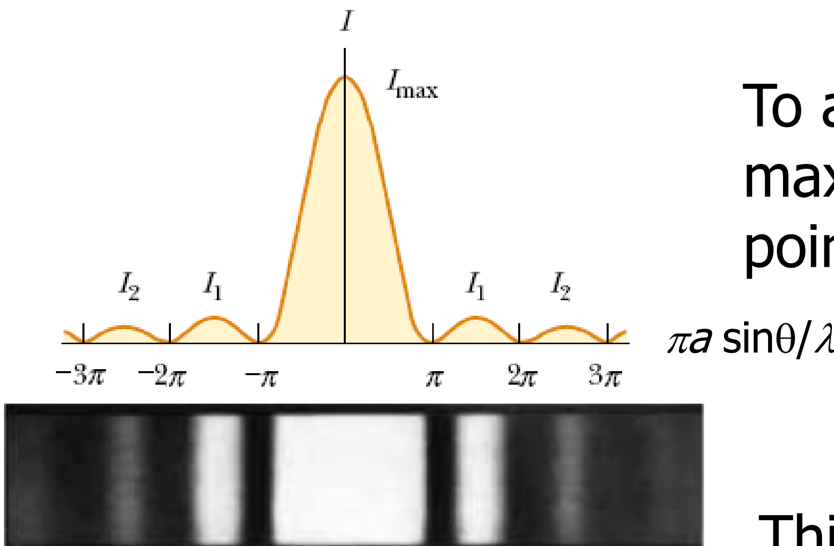
Example :

• The sound waves in speech : $\lambda \approx 1$ m. Doorway < 1 m : $a < \lambda$
$\rightarrow$ The central intensity maximum extends over 180°.
The sounds coming through an open doorway or can bend around the head of an instructor.

• Visible light ($\lambda \approx 5 \times 10^{-7}$ m), doorway (a $\approx$ 1 m) : $\lambda << a$
$\rightarrow$ No diffraction of light; you cannot see around corners

PROBLEM 11 Find the ratio of the intensities of the secondary maxima to the intensity of the central maximum for the single-slit Fraunhofer diffraction pattern.

SOLUTION



To a good approximation, the secondary maxima lie midway between the zero points

$$\boxed{I = I_{max}\left[\frac{\sin(\pi a \sin\theta/\lambda)}{\pi a \sin\theta/\lambda}\right]^2}$$

This corresponds to $\pi a \sin\theta/\lambda$ values of $3\pi/2$, $5\pi/2$, $7\pi/2$, . . .

$$\frac{I_1}{I_{max}} = \left[\frac{\sin(3\pi/2)}{(3\pi/2)}\right]^2 = 0.045$$

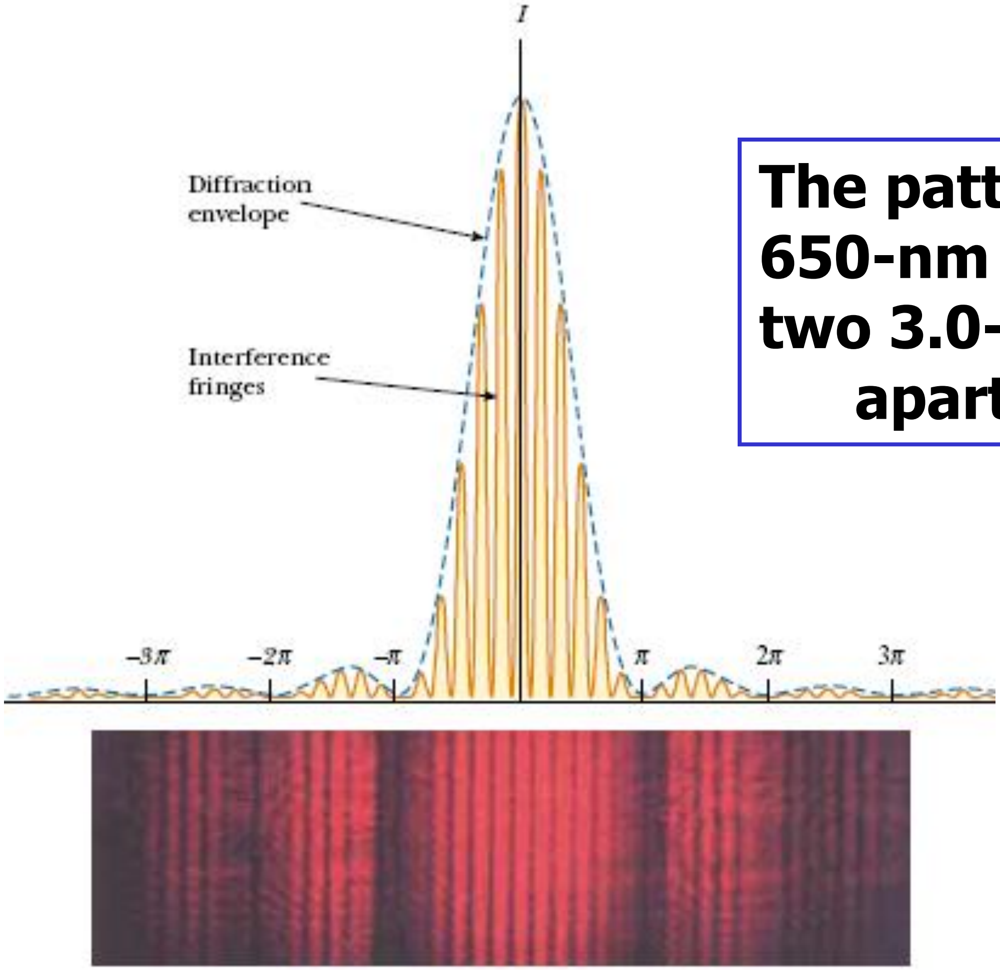
$$\frac{I_2}{I_{max}} = \left[\frac{\sin(5\pi/2)}{(5\pi/2)}\right]^2 = 0.016$$

The **first secondary maxima** (the ones adjacent to the central maximum) have an intensity of 4.5% that of the central maximum, and the **next secondary maxima** : 1.6%

• Intensity of Two-Slit Diffraction Patterns

Must consider not only diffraction due to the individual slits but also the **interference** of the waves coming from different slits.

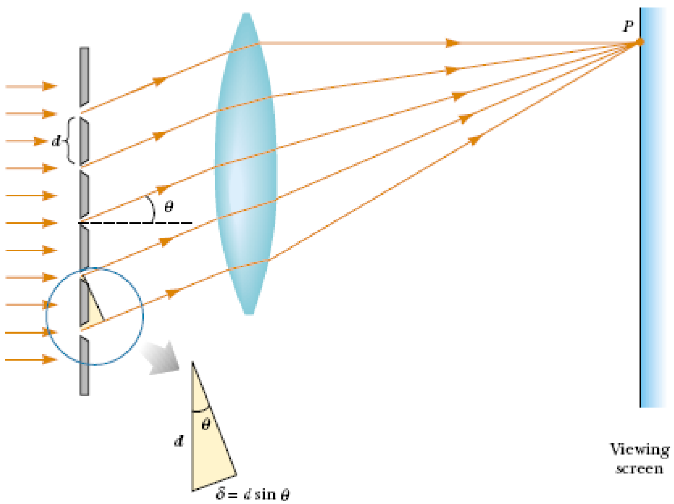
→ **The combined effects of diffraction and interference**

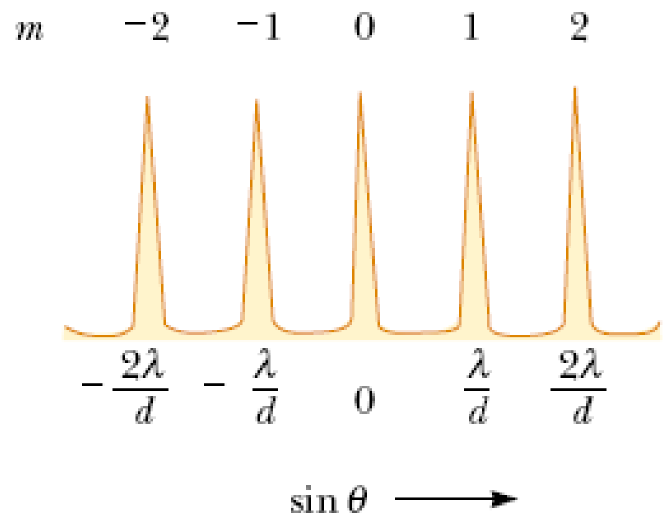


The pattern produced when 650-nm light waves pass through two 3.0-μm slits that are 18 μm apart

3.3 THE DIFFRACTION GRATING

An array of a large number of parallel slits, all with the same width a and spaced equal distances d between centers, is called a diffraction grating





The condition for maxima in the interference pattern at the angle θ:

$$\boxed{d\sin\theta = m\lambda \qquad m = 0;1;2;...}$$

Use this expression to calculate the wavelength if we know the grating spacing d and the angle θ

PROBLEM 12 The wavelengths of the visible spectrum are approximately 400 nm (violet) to 700 nm (red).
Find the angular width of the first-order visible spectrum produced by a plane grating with 600 slits per millimeter when white light falls normally on the grating.

SOLUTION

The grating spacing : $d = \dfrac{1}{600\ slits\ /\ mm} = 1.67 \times 10^{-6}\, m$

m = 1, the angular deviation of the violet light :

$$\sin\theta_V = 1.\frac{\lambda_V}{d} = \frac{400 \times 10^{-9}\, m}{1.67 \times 10^{-6}\, m}\ ;\ \theta_V = 13.9^0$$

The angular deviation of the red light :

$$\sin\theta_R = 1.\frac{\lambda_R}{d} = \frac{700 \times 10^{-9}\, m}{1.67 \times 10^{-6}\, m}\ ;\ \theta_R = 24.8^0$$

The angular width of the first-order visible spectrum :

$$\theta_R - \theta_V = 24.8^0 - 13.9^0 = 10.9^0$$

PROBLEM 13 Monochromatic light from a helium-neon laser (λ = 632.8 nm) is incident normally on a diffraction grating containing 6 000 lines per centimeter.
Find the angles at which the first order, second-order, and third-order maxima are observed.

SOLUTION

The slit separation : $d = \dfrac{1}{6000} cm = 1667\ nm$

For the first-order maximum (m = 1) we obtain :

$$\sin\theta_1 = 1.\frac{\lambda}{d} = \frac{632.8\ nm}{1667\ nm}\ ;\quad \theta_1 = 22.31^0$$

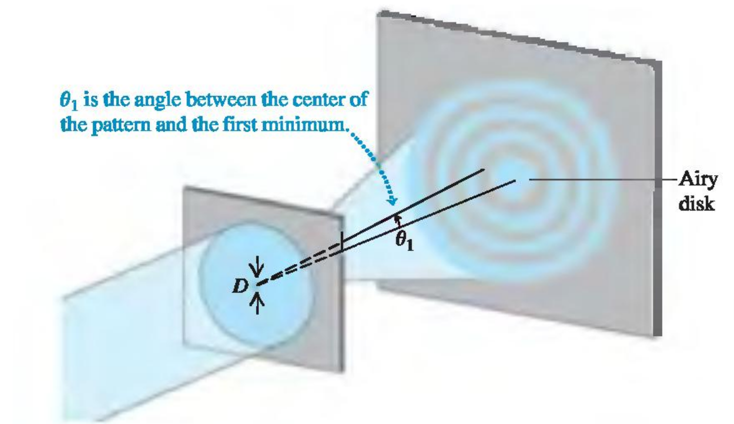
For the second-order maximum (m = 2) we find:

$$\sin\theta_2 = 2.\frac{\lambda}{d} = 2\frac{632.8\ nm}{1667\ nm}\ ;\ \theta_2 = 49.39^0$$

For m = 3 we find that $\sin\theta_3 > 1$: only zeroth-, first-, and second-order maxima are observed

3.4 Circular Apertures

The diffraction pattern formed by a circular aperture consists of a central bright spot surrounded by a series of bright and dark rings



If the aperture diameter is D and the wavelength is λ, the angular radius θ_1 of the **first , the second dark ring,...** are given by :

$$\sin\theta_1 = 1.22\frac{\lambda}{D}\ ;\ \sin\theta_2 = 2.23\frac{\lambda}{D}$$

Between these are bright rings with angular radii given by :

$$\sin\theta = 1.63\frac{\lambda}{D}\ ;2.68\frac{\lambda}{D}\ ;...$$

(The central bright spot is called **the Airy disk**)

PROBLEM 14 Monochromatic light with wavelength 620 nm passes through a circular aperture with diameter 7.4 μm. The resulting diffraction pattern is observed on a screen that is 4.5 m from the aperture. What is the diameter of the Airy disk on the screen?

SOLUTION

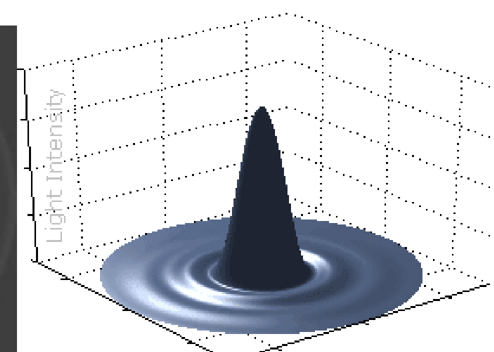
The angular size of the first dark ring :

$$\sin\theta_1 = 1.22\frac{\lambda}{d} = 1.22\,\frac{620\times10^{-9}\ m}{7.4\times10^{-6}\ m}\ ;\ \theta_1 = 0.1022\ rad$$

The radius of the Airy disk (central bright spot) :

$$r = (4.5\ m)\tan\theta_1 = 0.462\ m$$

The diameter : $2r$ = 0.92 m = 92 cm.



3.5 DIFFRACTION OF X-RAYS BY CRYSTALS

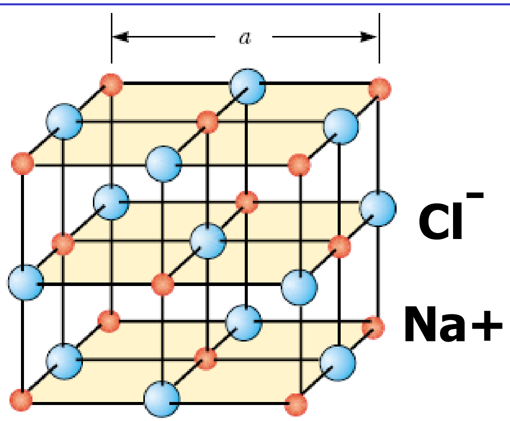
X-rays are electromagnetic waves of very short wavelength (of the order of **0.1 nm**)

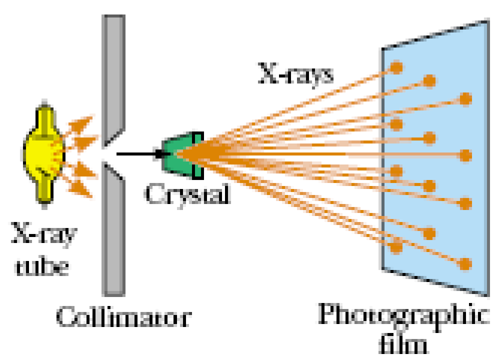
How to construct a grating having such a small spacing?

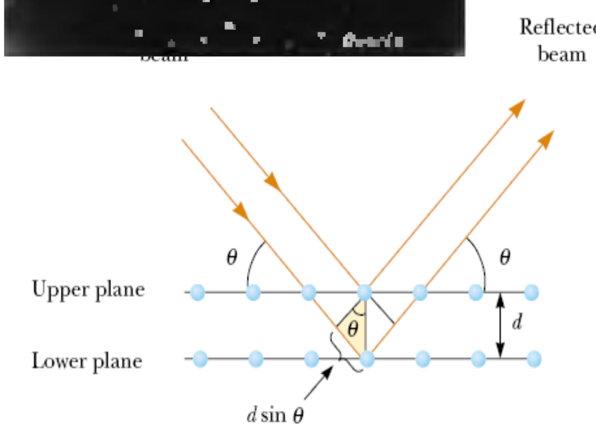
The beam reflected from the lower plane travels farther than the one reflected from the upper plane by a distance $2d\sin\theta$

The diffracted beams are very intense in certain directions
$\rightarrow$ constructive interference : ***Laue pattern***

Crystalline structure
of sodium chloride (NaCl)
$a \approx 0.56$ nm







PROBLEM 15 You direct a beam of x rays with wavelength 0.154 nm at certain planes of a silicon crystal. As you increase the angle of incidence from zero, you find the first strong interference maximum from these planes when the beam makes an angle of 34.5° with the planes.

(a) How far apart are the planes?

SOLUTION

(a) The Bragg equation : $2d\sin\theta = m\lambda$

The distance between adjacent planes :

$$d = \frac{m\lambda}{2\sin\theta} = \frac{(1)(154\ nm)}{2.\sin 34.5^0} = 0.136\ nm$$

PROBLEM 15 You direct a beam of x rays with wavelength 0.154 nm at certain planes of a silicon crystal. As you increase the angle of incidence from zero, you find the first strong interference maximum from these planes when the beam makes an angle of 34.5° with the planes.

(b) Other interference maxima from these planes at larger angles?

SOLUTION

(a) The Bragg equation : $2d\sin\theta = m\lambda$

The distance between adjacent planes :

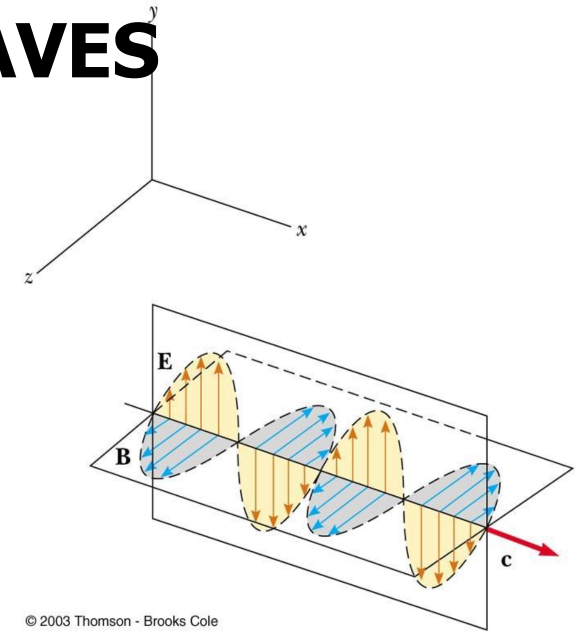
$$d = \frac{m\lambda}{2\sin\theta} = \frac{(1)(154\ nm)}{2.\sin 34.5^0} = 0.136\ nm$$

(b) $\sin\theta = \dfrac{m\lambda}{2d} = m\dfrac{154\ nm}{2(0.136\ nm)} = 0.566m$

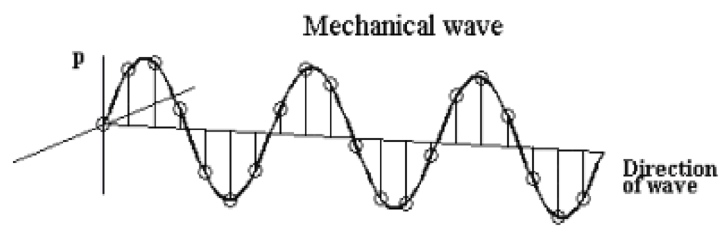
$m = 2$: $\sin\theta > 1$ $\rightarrow$ there are no other angles for interference maxima for this particular set of crystal planes.

4. POLARIZATION OF LIGHT WAVES

4.1 Introduction Light wave is electromagnetic (EM) wave traveling at the speed of light $\boxed{c = \dfrac{E}{B}}$



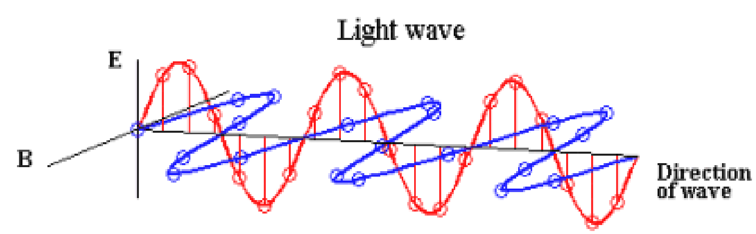
- The **E** and **B** fields are **perpendicular** to each other
- Both fields are perpendicular to the direction of motion



Therefore, em waves are transverse waves

The wavelength of light in the vacuum :

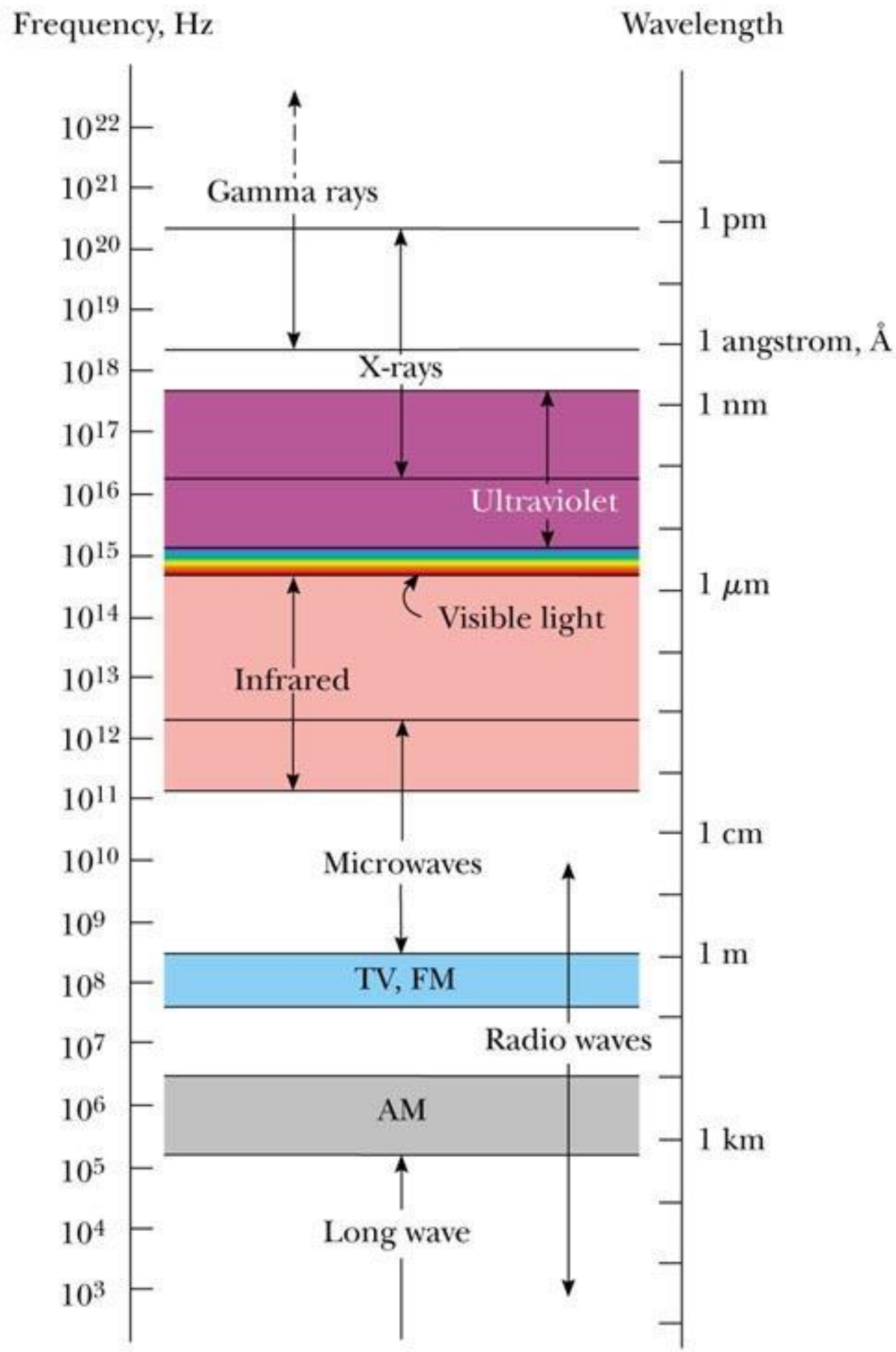
$$\boxed{\lambda = \frac{c}{f}}$$



isvr

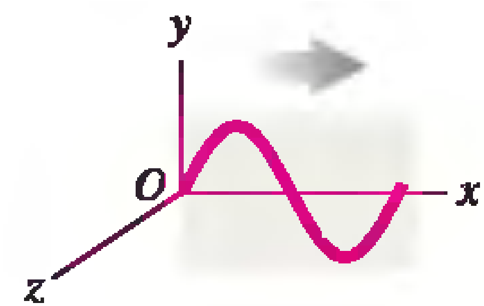
The EM Spectrum

- Note the overlap between types of waves
- Visible light is a small portion of the spectrum
- Types are distinguished by frequency or wavelength

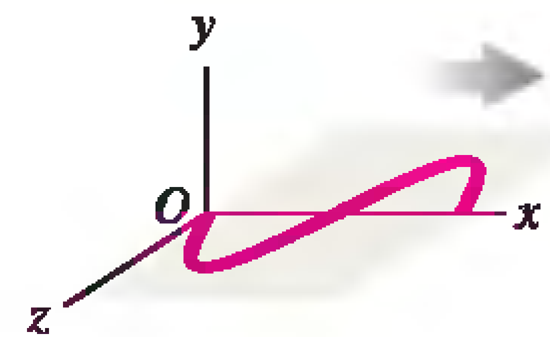


Polarized waves on a string.

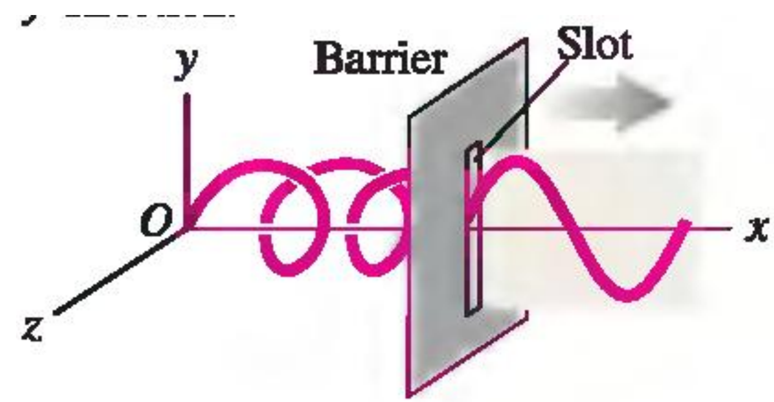
(a) Transverse wave linearly polarized in the *y*-direction



(b) Transverse wave linearly polarized in the *z*-direction



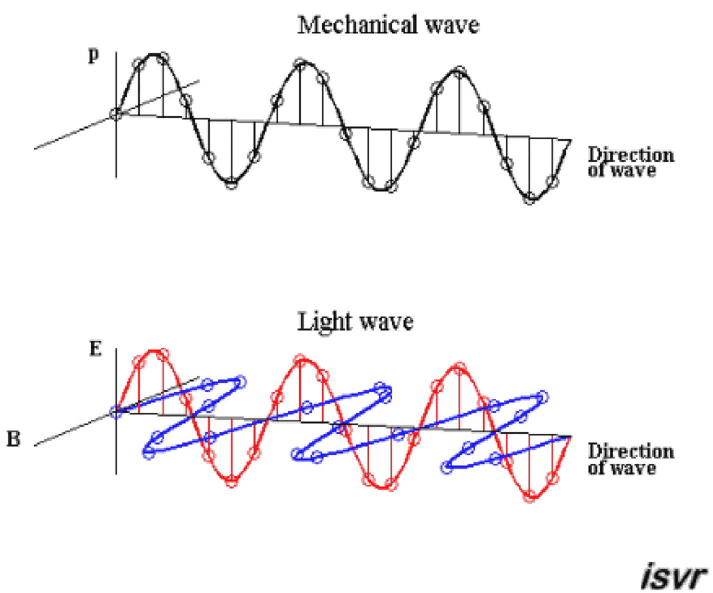
(c) The slot functions as a polarizing filter, passing only components polarized in the *y*-direction.

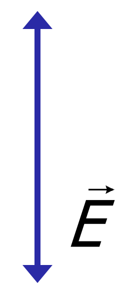


4.2 Polarization of an electromagnetic wave

We always define the direction of polarization of an electromagnetic wave to be the direction of the electric-field vector ***E***; not the magnetic field ***B***

The plane formed by ***E*** and the direction of propagation is called the *plane of polarization* of the wave.





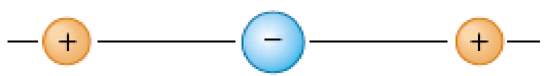
A linearly polarized light beam viewed along the direction of propagation (perpendicular to the screen)

CAUTION

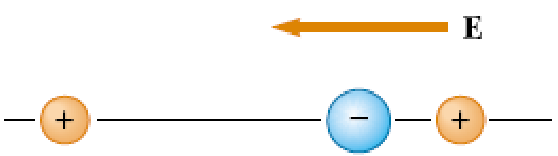
Polarization :

• **1st meaning :** to describe the the shifting of electric charge within a body, such as in response to a nearby charged body

• **2nd meaning :** to describe the direction of ***E*** in an electromagnetic wave



A symmetric molecule has no permanent polarization



An external electric field induces a polarization in the molecule.

These two concepts have the same name, they do not describe the same phenomenon

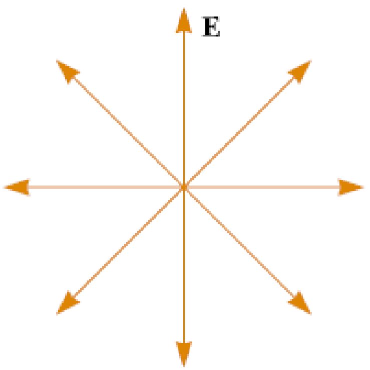
4.3 Polarizing Filters

- Waves emitted by a radio transmitter are usually linearly polarized.

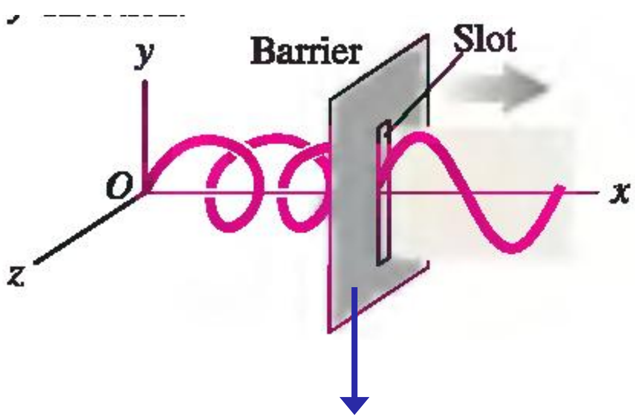
- An ordinary beam of light consists of a large number of waves emitted by the atoms of the light source. Each atom produces a particular direction of ***E***

All directions of vibration from a wave source are possible, the resultant electromagnetic wave is a superposition of waves vibrating in many different directions.

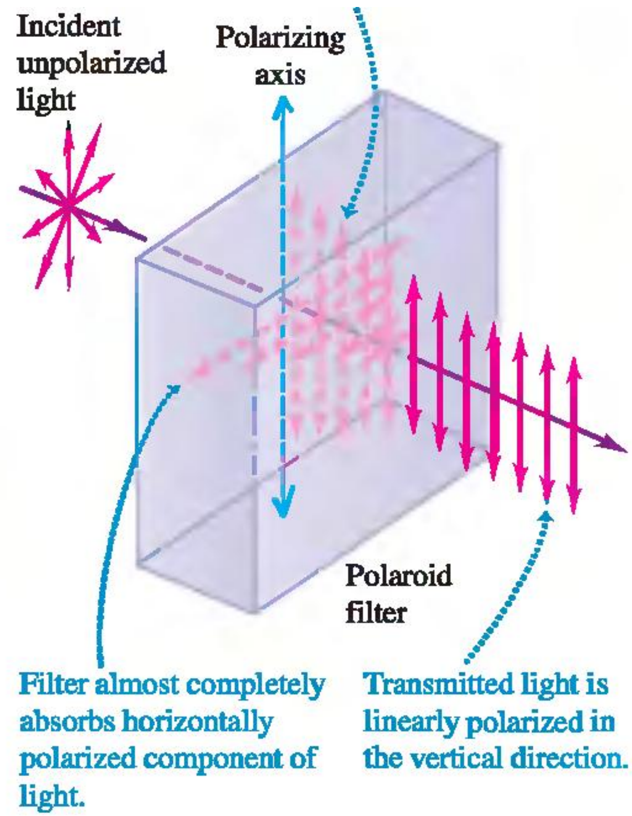
⟶ **An unpolarized light beam**



Polarizing Filters : To create **polarized light** from unpolarized natural light



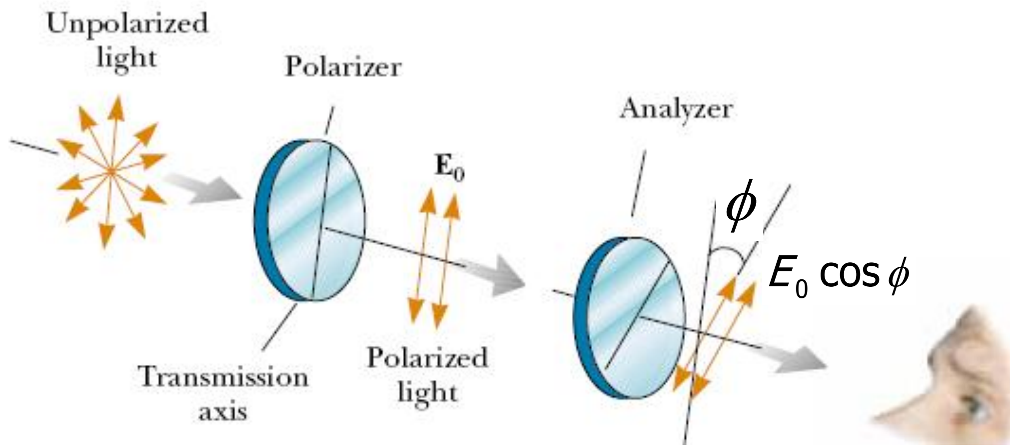
Polarizing Filter



The most common polarizing filter for visible light : called **Polaroid**

Polaroid : material is fabricated in thin sheets of long-chain hydrocarbons which transmits 80% or more of the intensity of a wave that is polarized parallel to a certain axis in the material, called the polarizing axis

What happens when the linearly polarized light emerging from a polarizer passes through a second polarizer (analyser)?



ϕ : angle between the polarizing axes of the polarizer and analizer

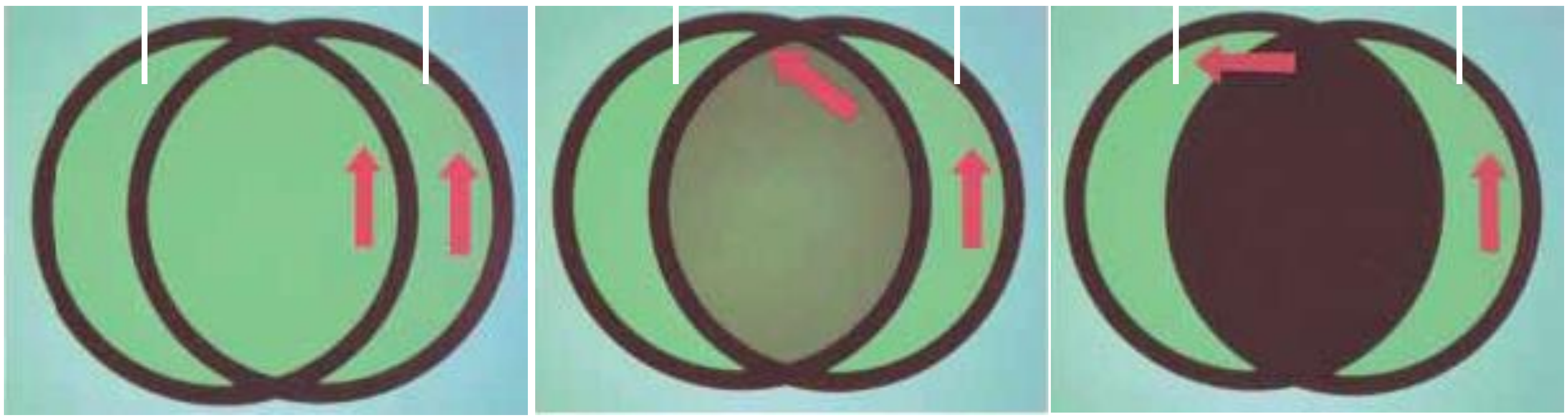
The **intensity of the light** transmitted through the analyzer :

$$\boxed{I = I_{MAX}\cos^2\phi}$$ **(Malus's law)**

($I_{\max}$ is the intensity of the polarized beam incident on the analyzer)

$\boxed{I = I_{MAX} \cos^2 \phi}$ **(Malus's law)**

polarizer analyser polarizer analyser polarizer analyser



$\phi = 0$; $I = I_{\mathrm{MAX}}$ $\phi = 45^0$; $I < I_{\mathrm{MAX}}$ $\phi = 90^0$; $I = 0$

PROBLEM 16 If the incident unpolarized light has intensity I_0 , find the intensities transmitted by the first and second polarizers if the angle between the axes of the two filters is 30°.

SOLUTION

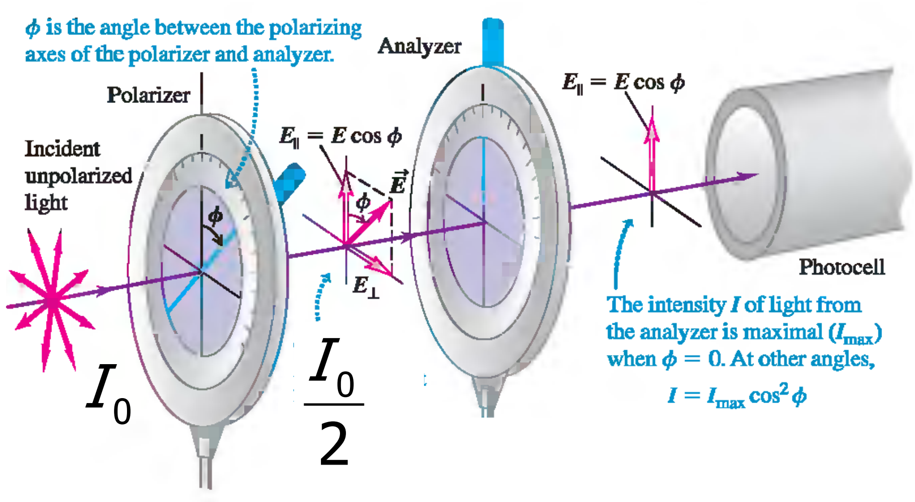
This problem involves a polarizer (a polarizing filter on which unpolarized light shines, producing polarized light) and an analyzer (a second polarizing filter on which the polarized light shines).

The angle between the polarizing axes : $\varphi = 30^0$

Malus's law : $\boxed{I = I_{MAX} \cos^2 \phi}$

$$= \frac{I_0}{2} \cos^2 30^0$$

$$I = \frac{3}{8} I_0$$



A. WAVE OPTICS

B. GEOMETRIC OPTICS

Light Rays

The Laws of Reflection and Refraction

Mirrors

Thin Lenses

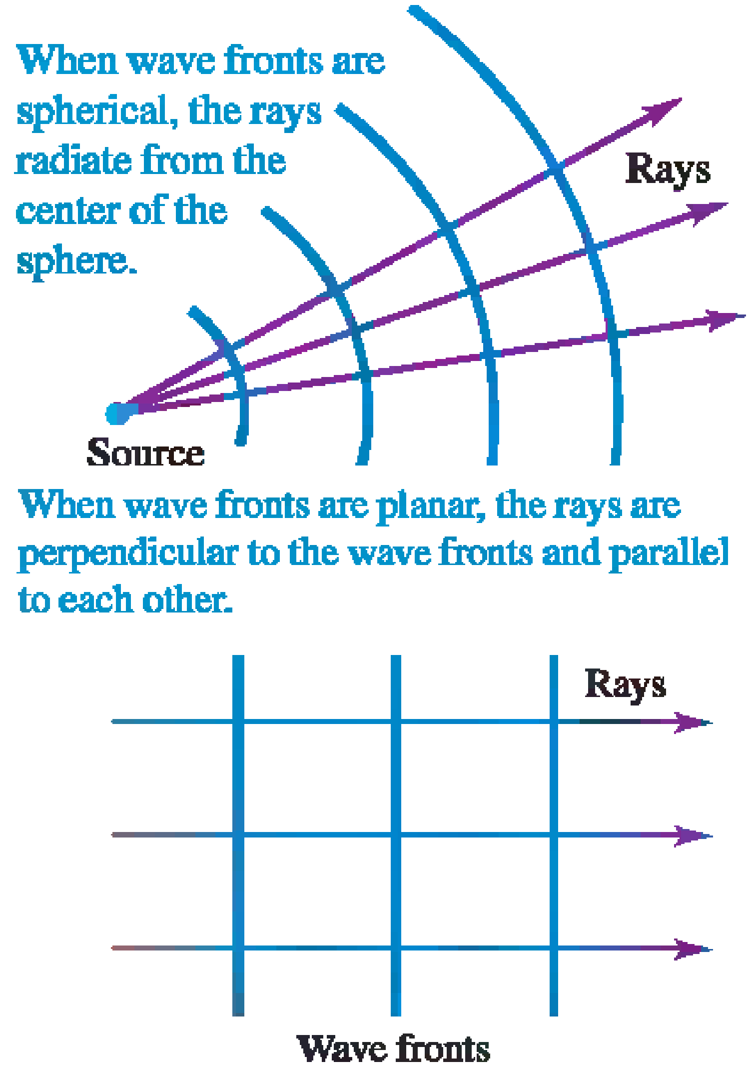
5. Light Rays

A wave front is a surface of constant phase;
wave fronts move with a speed equal to the propagation speed of the wave.

A ray is an imaginary line along the direction of travel of the wave

Spherical wave fronts :
the rays are the radii of the sphere

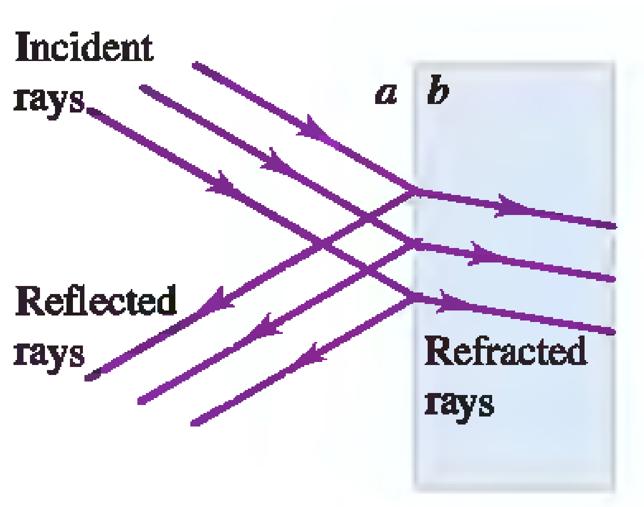
Planar wave fronts :
the rays are straight lines perpendicular to the wave fronts.



6. Reflection and Refraction

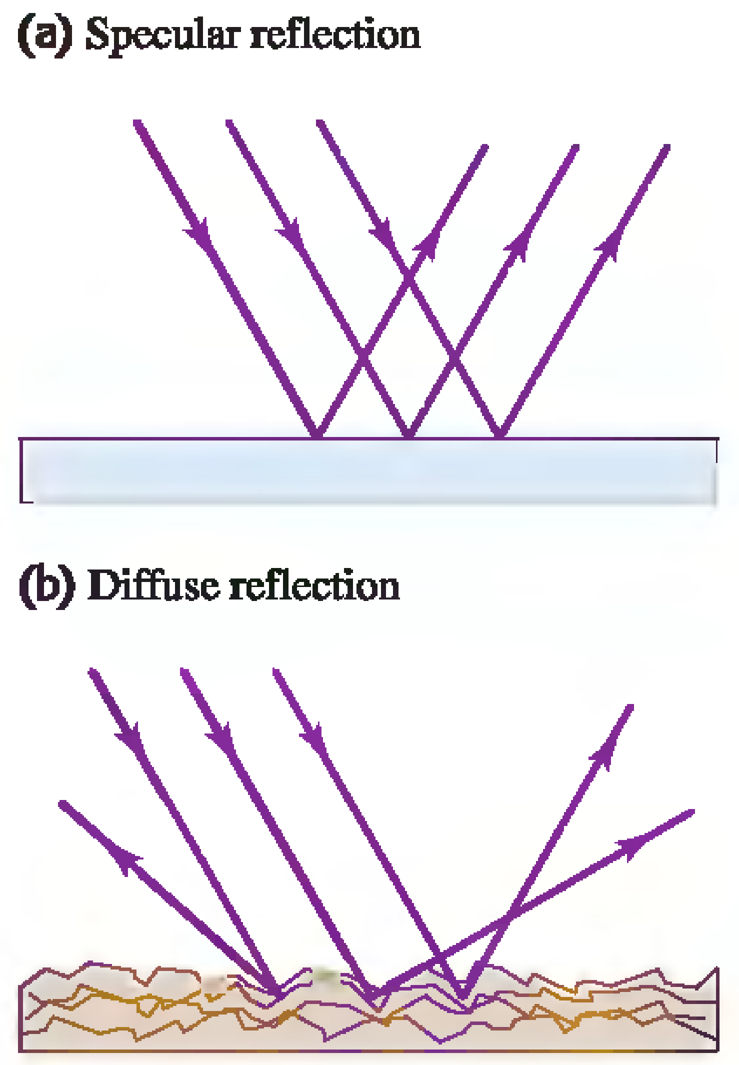
6.1 Basic notions

• When a light wave strikes a smooth interface separating two transparent materials (such as air and glass or water and glass), the wave is in general partly reflected and partly refracted (transmitted) into the second material



If the interface is rough, both the transmitted light and the reflected light are scattered in various directions

Reflection at a definite angle from a very smooth surface is called specular reflection (from the Latin word for "mirror"); scattered reflection from a rough surface is called diffuse reflection.

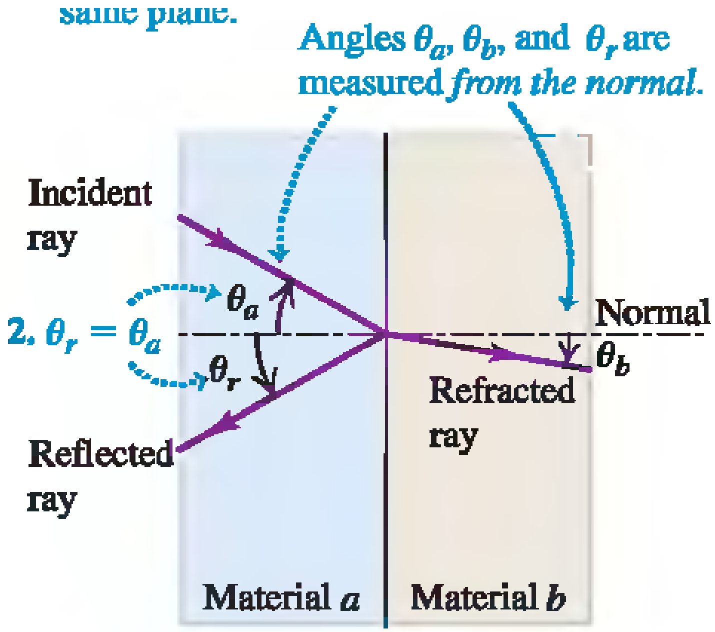


The directions of the incident, reflected, and refracted rays at a interface between two optical materials in terms of the angles they make with the normal (perpendicular) to the surface at the point of incidence

The index of refraction of an optical material (also called the refractive index), denoted by *n*.

$$\boxed{n = \frac{c}{v}}$$

(the ratio of the speed of light c in vacuum to the speed v in the material)



$c \geq v \longrightarrow n \geq 1$

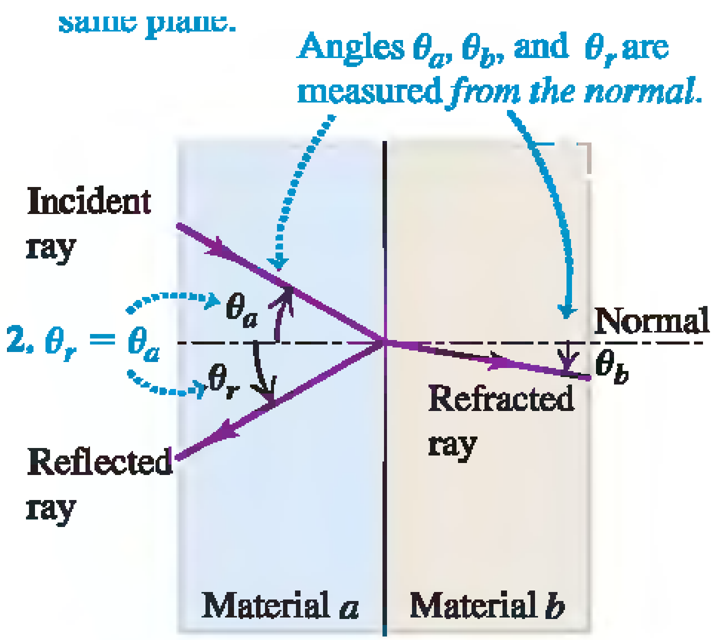
6.2 The Laws of Reflection and Refraction

• The incident, reflected, and refracted rays and the normal to the surface all lie in the same plane

• The angle of reflection is equal to the angle of incidence for all wavelengths and for any pair of materials.

$$\boxed{\theta_a = \theta_b}$$

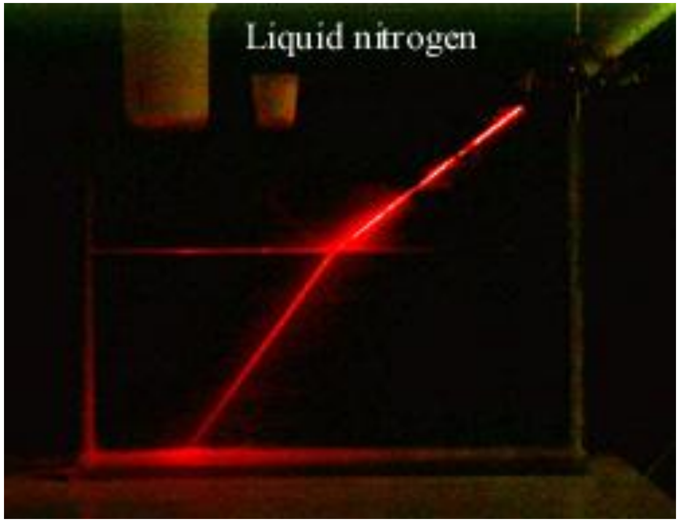
(law of reflection)

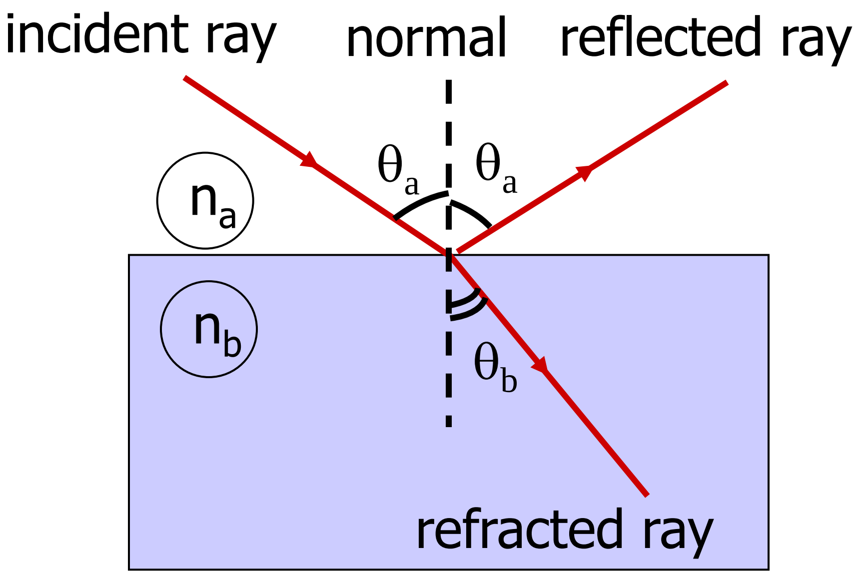


• For monochromatic light and for a given pair of materials, *a* and *b*, on opposite sides of the interface, the ratio of the sins of the angles θ_a and θ_b, is equal to the inverse ratio of the two indexes of refraction:

$$\boxed{n_a \sin\theta_a = n_b \sin\theta_b}$$

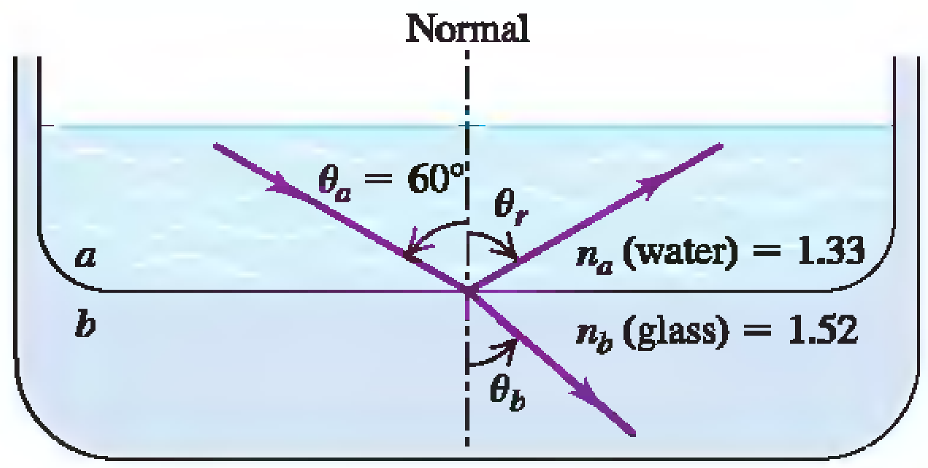
(law of refraction - Snell's law)





PROBLEM 17 In the figure, material *a* is water and material is a glass with index of refraction 1.52. If the incident ray makes an angle of 60^0 with the normal, find the directions of the reflected and refracted rays.

SOLUTION



$$\theta_r = \theta_a = 60.0°.$$

$$n_a \sin\theta_a = n_b \sin\theta_b$$

$$\sin\theta_b = \frac{n_a}{n_b}\sin\theta_a = \frac{1.33}{1.52}\sin 60.0° = 0.758$$

$$\theta_b = 49.3°$$

PROBLEM 18 The wavelength of the red light from a helium-neon laser is 633 μm in air but 474 μm in the aqueous humor inside your eye-ball. Calculate the index of refraction of the aqueous humor and the speed and frequency of the light in this substance.

SOLUTION

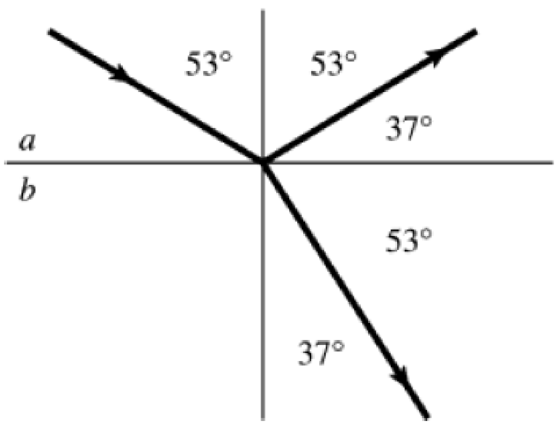
$$\lambda = \frac{\lambda_0}{n} \qquad n = \frac{\lambda_0}{\lambda} = \frac{633\ \mathrm{nm}}{474\ \mathrm{nm}} = 1.34$$

$$v = \frac{c}{n} = \frac{3.00 \times 10^8\ \mathrm{m/s}}{1.34} = 2.25 \times 10^8\ \mathrm{m/s}$$

$$f = \frac{v}{\lambda} = \frac{2.25 \times 10^8\ \mathrm{m/s}}{474 \times 10^{-9}\ \mathrm{m}} = 4.74 \times 10^{14}\ \mathrm{Hz}$$

PROBLEM 19 Light traveling in water strikes a glass plate at an angle of incidence of 53.0^0; part of the beam is reflected and part is refracted. If the reflected and refracted portions make an angle of 90.0^0 with each other, what is the index of refraction of the glass?

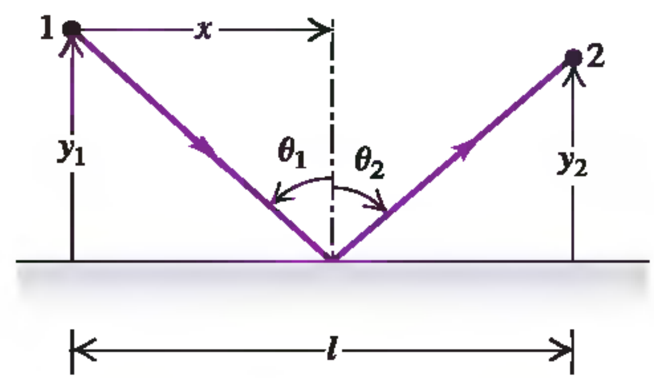
SOLUTION



From the figure, $\theta_b = 37.0°$ and $n_b = n_a \dfrac{\sin\theta_a}{\sin\theta_b} = 1.33\dfrac{\sin 53°}{\sin 37°} = 1.77.$

PROBLEM 20 A ray of light traveling with speed c leaves point 1 shown in the figure and is reflected to point 2. The ray strikes the reflecting surface a horizontal distance x from point 1. (a) What is the time t required for the light to travel from 1 to 2 ? (b) When does this time reaches its minimum value ?

SOLUTION



(a) The distance traveled by the light ray

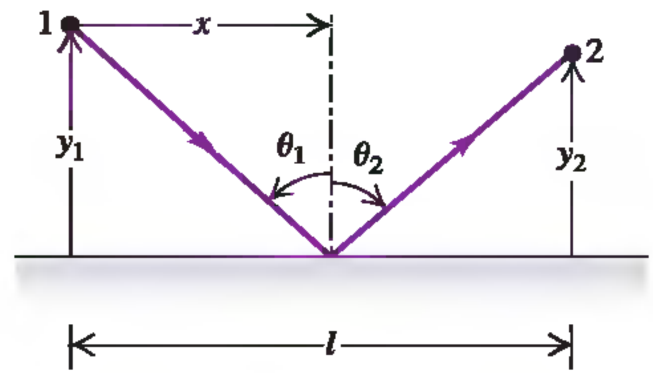
$$d = (x^2 + y_1^2)^{1/2} + ((l-x)^2 + y_2^2)^{1/2}$$

$$t = \frac{d}{c} = \frac{(x^2 + y_1^2)^{1/2} + ((l-x)^2 + y_2^2)^{1/2}}{c}$$

PROBLEM 20 A ray of light traveling with speed *c* leaves point 1 shown in the figure and is reflected to point 2. The ray strikes the reflecting surface a horizontal distance x from point 1. (a) What is the time *t* required for the light to travel from 1 to 2 ? (b) When does this time reaches its minimum value ?

SOLUTION

$$t = \frac{d}{c} = \frac{(x^2 + y_1^2)^{1/2} + ((l -}{c}$$



(b) Taking the derivative with respect t

$$\frac{dt}{dx} = \frac{1}{c}\frac{d}{dt}\left[(x^2 + y_1^2)^{1/2} + \left((l-x)^2 + y_2^2\right)\right]$$

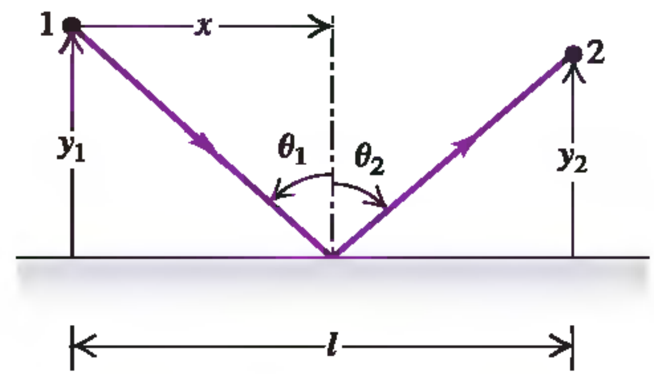
$$= \frac{1}{c}\left[x(x^2 + y_1^2)^{-1/2} - (l-x)\left((l-x)^2 + y_2^2\right)^{-1/2}\right] = 0$$

$$\frac{x}{\sqrt{x^2 + y_1^2}} = \frac{(l-x)}{\sqrt{(l-x)^2 + y_2^2}}, \ \sin\theta_1 = \sin\theta_2 \ \text{and} \ \theta_1 = \theta_2.$$

PROBLEM 20 A ray of light traveling with speed c leaves point 1 shown in the figure and is reflected to point 2. The ray strikes the reflecting surface a horizontal distance x from point 1. (a) What is the time t required for the light to travel from 1 to 2 ? (b) When does this time reaches its minimum value ?

SOLUTION

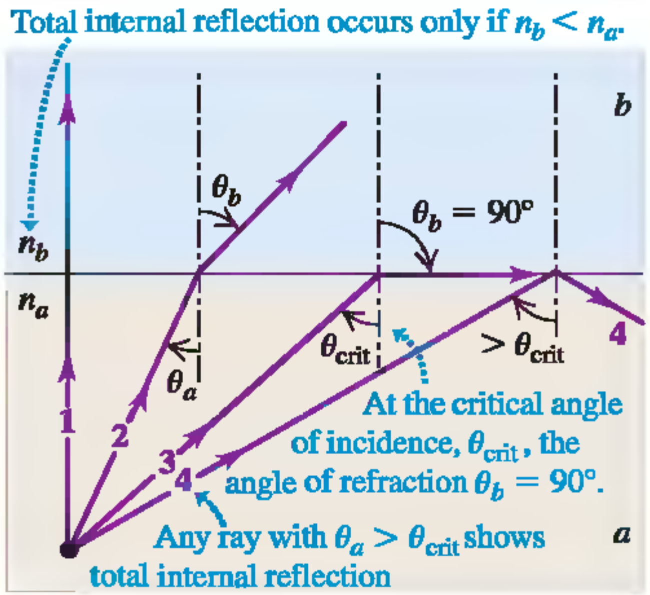
$$\theta_1 = \theta_2 .$$



(The law of reflection corresponding to the actual path taken by the light)

Fermat's Principle of Least Time : among all possible paths between two points, the one actually taken by a ray of light is that for which the time of travel is a minimum.

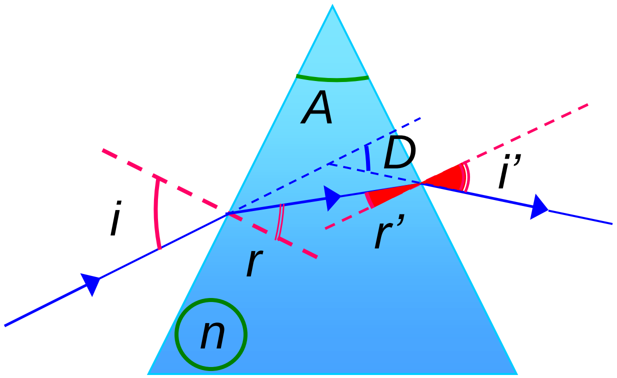
- **Total Internal Reflection**



$$\sin\theta_b = \frac{n_a}{n_b}\sin\theta_a$$

$$\sin\theta_{crit} = \frac{n_b}{n_a} \qquad \text{(critical angle for total internal reflection)}$$

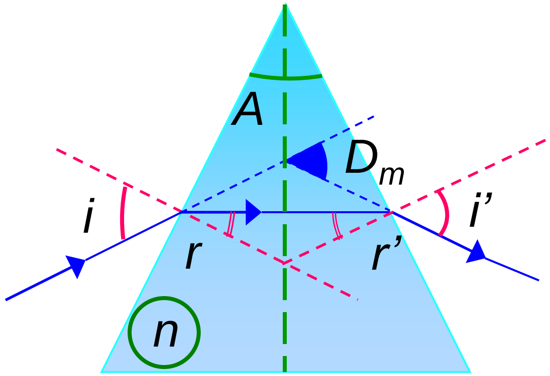
• Prism



$$\boxed{\sin i = n \sin r\ ;} \quad \boxed{\sin i' = n \sin r'\ ;}$$

$$\boxed{A = r + r'\ ;}$$

$$\boxed{D = i + i' - A}$$



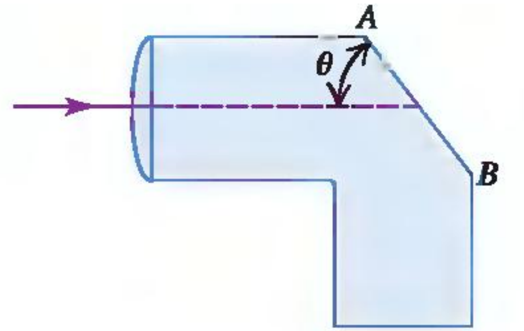
• Minimum deflection :

$$\boxed{i' = i\ ;} \quad \boxed{r = r' = \frac{A}{2}\ ;}$$

$$\boxed{D_m = 2i - A}$$

PROBLEM 21 Light enters a solid pipe made of plastic having an index of refraction of 1.60. The light travels parallel to the upper part of the pipe. You want to cut the face AB so that all the light will reflect back into the pipe after it first strikes that face. (a) What is the largest that θ can be if the pipe is in air? (b) If the pipe is immersed in water of refractive index 1.33, what is the largest that θ can be?

SOLUTION



(a) $$(1.60)\sin\theta_a = (1.00)\sin 90^\circ$$

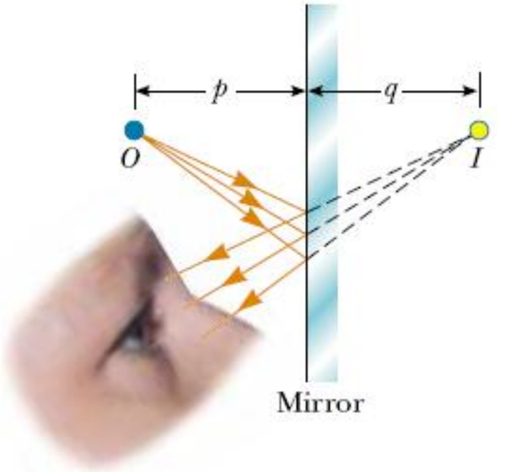
$$\theta_a = 38.7^\circ.\ \ \theta = 90^\circ - \theta_a = 51.3^\circ$$

(b) $$(1.60)\sin\theta_a = (1.333)\sin 90^\circ$$

$$\theta_a = 56.4^\circ.\ \ \theta = 90^\circ - \theta_a = 33.6^\circ$$

7. Mirrors

7.1 Flat mirrors

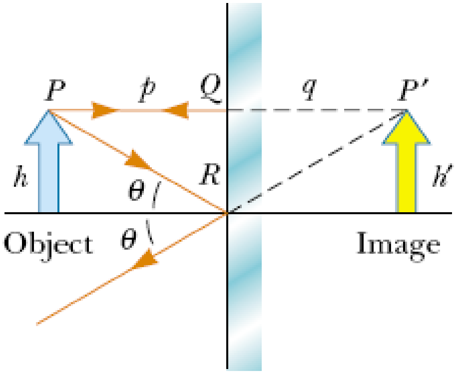


The distance p is called the object distance.

The distance q is called the image distance.

Real object $\rightarrow$ Virtual image

$$|p| = |q| \text{ and } h = h'.$$



Lateral magnification

$$M \equiv \frac{\text{Image height}}{\text{Object height}} = \frac{h'}{h}$$

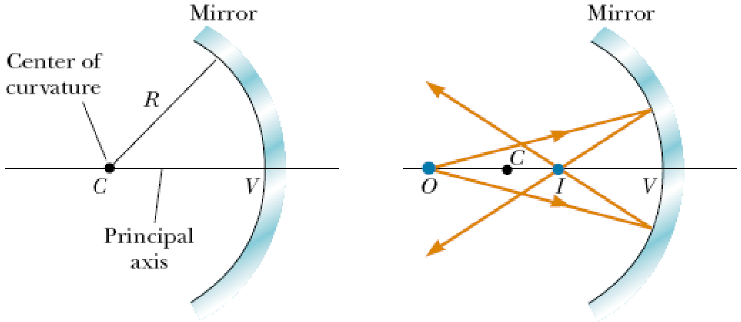
$$M = 1 \text{ because } h' = h.$$

7. Mirrors

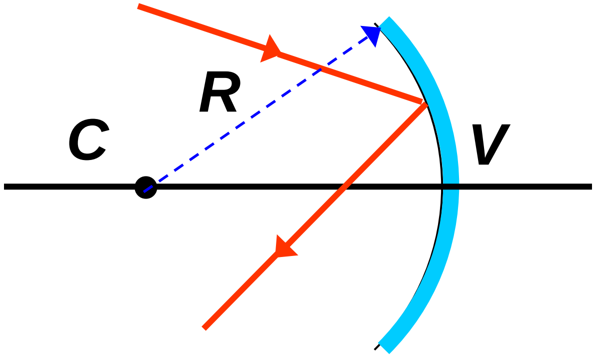
7.2 Spherical mirrors

A spherical mirror has the shape of a section of a sphere.

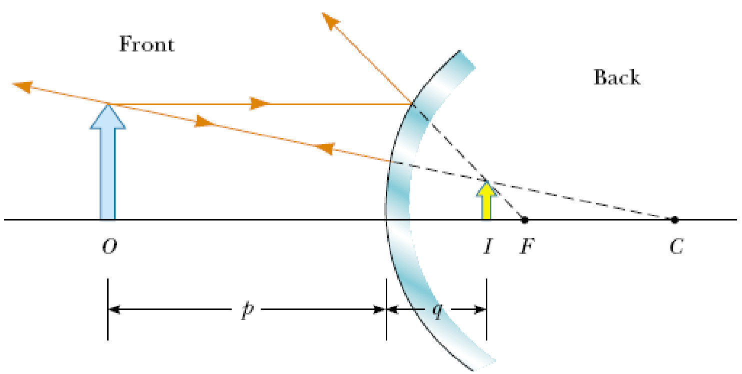
Light reflected from the inner (concave surface): concave mirror.

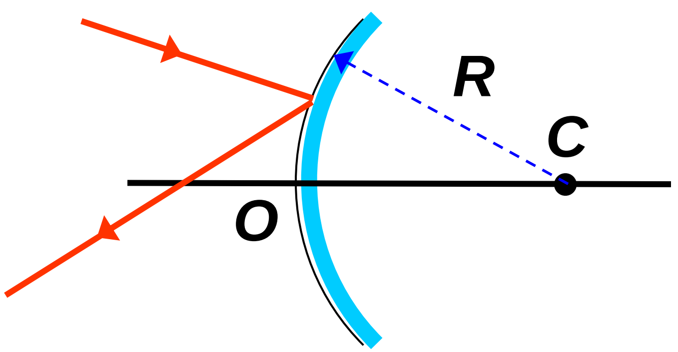


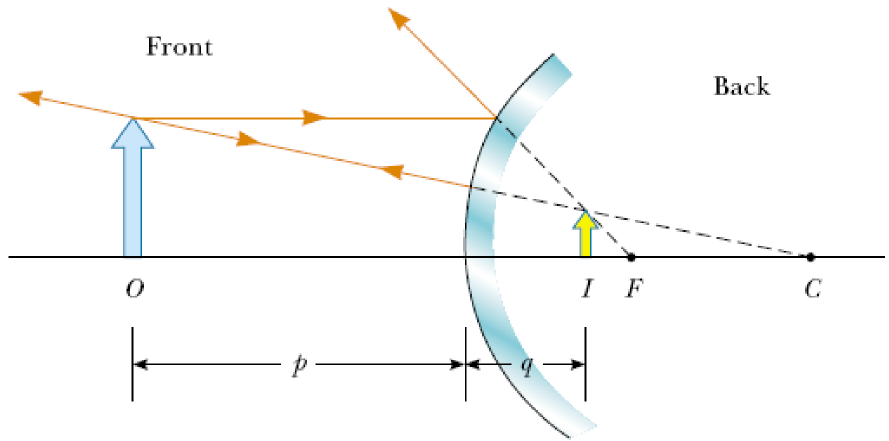
(a) (b)



Light reflected from the outer (convex surface) : convex mirror.

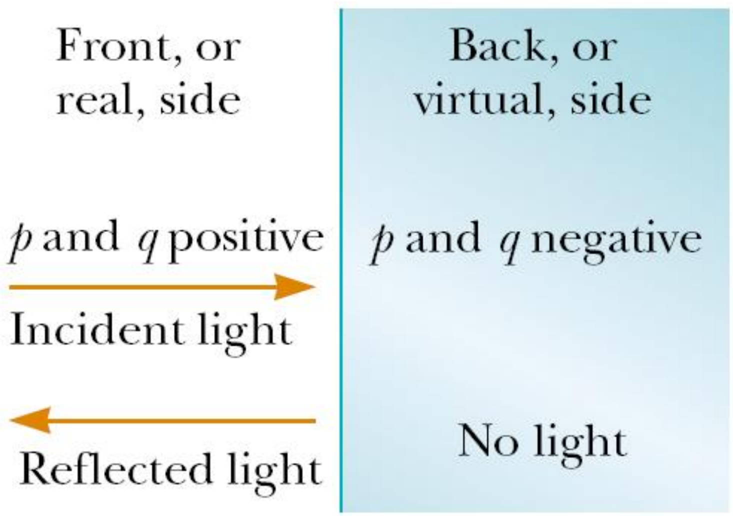






F : focal point ; *f* : focal length

Mirror equation : $$\boxed{\frac{1}{p}+\frac{1}{q}=\frac{1}{f}} \qquad f=\frac{R}{2}$$



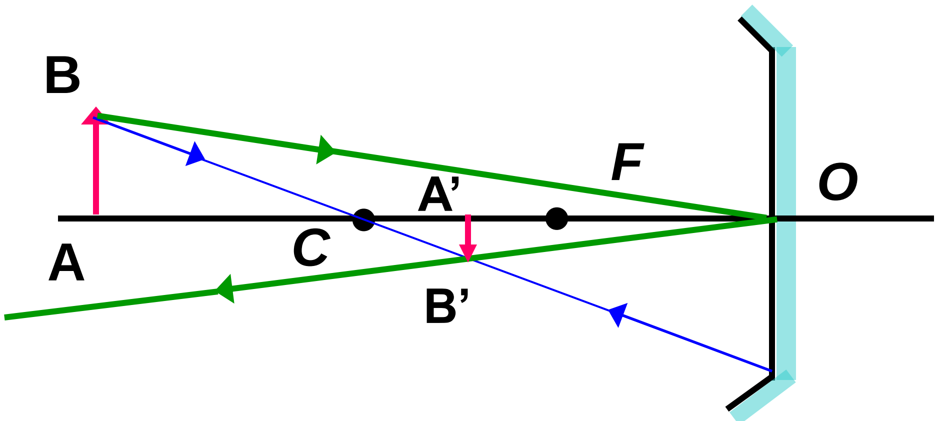
Real object $\leftrightarrow$ $p > 0$

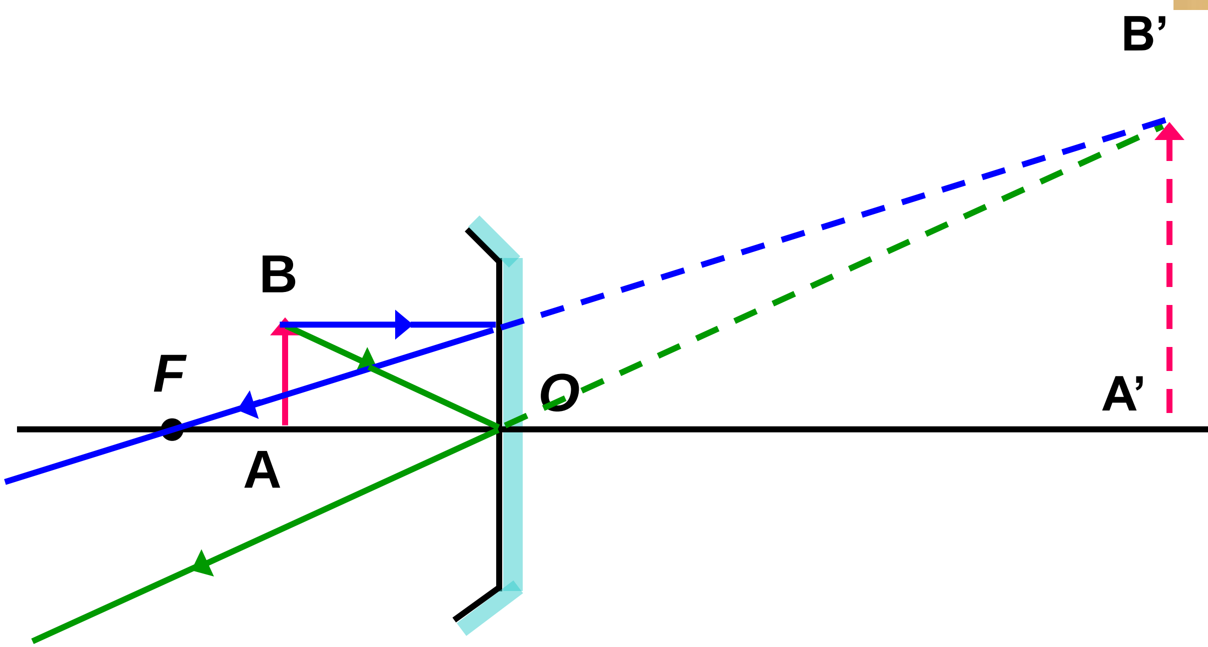
Virtual object $\leftrightarrow$ $p < 0$

Real image $\leftrightarrow$ $q > 0$

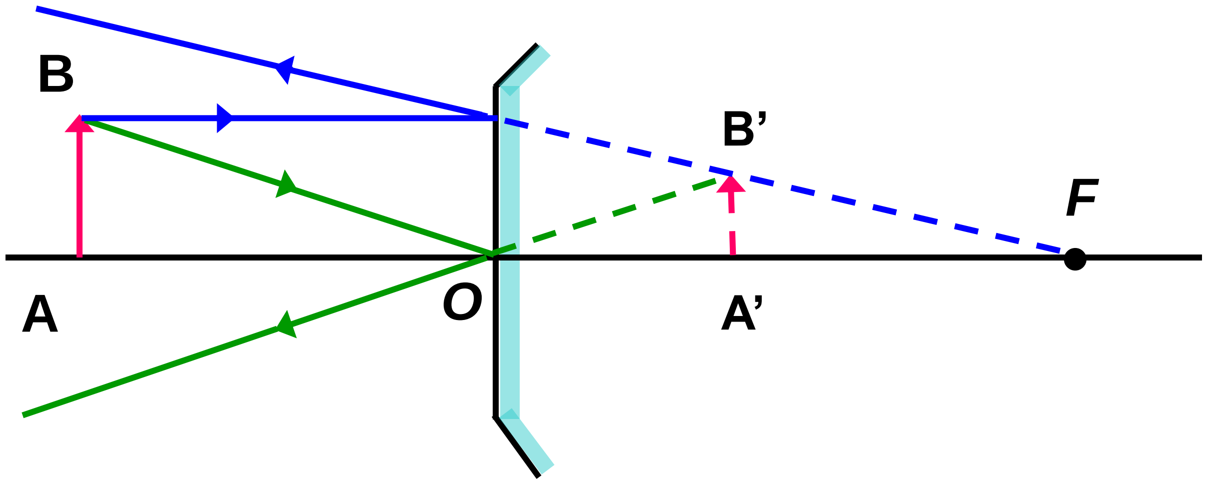
Virtual image $\leftrightarrow$ $q < 0$

Concave mirror :





Convex mirror :



PROBLEM 22 Assume that a certain spherical mirror has a focal length of 10.0 cm. Locate and describe the image for object distances of (a) 25.0 cm, (b) 10.0 cm, and (c) 5.00 cm.

SOLUTION

(a)

$$\frac{1}{p}+\frac{1}{q}=\frac{1}{f}$$

$$\frac{1}{25.0\text{ cm}}+\frac{1}{q}=\frac{1}{10.0\text{ cm}}$$

$$q=\boxed{16.7\text{ cm}}$$

$$M=-\frac{q}{p}=-\frac{16.7\text{ cm}}{25.0\text{ cm}}=-0.668$$

PROBLEM 22 Assume that a certain spherical mirror has a focal length of 10.0 cm. Locate and describe the image for object distances of (a) 25.0 cm, (b) 10.0 cm, and (c) 5.00 cm.

SOLUTION

(b)

$$\frac{1}{10.0\text{ cm}} + \frac{1}{q} = \frac{1}{10.0\text{ cm}} \qquad q = \boxed{\infty}$$

(c)

$$\frac{1}{5.00\text{ cm}} + \frac{1}{q} = \frac{1}{10.0\text{ cm}}$$

$$q = \boxed{-10.0\text{ cm}}$$

$$M = -\frac{q}{p} = -\left(\frac{-10.0\text{ cm}}{5.00\text{ cm}}\right) = 2.00$$

PROBLEM 23 A woman who is 1.5 m tall is located 3.0 m from an antishoplifting mirror. The focal length of the mirror is 0.25 m. Find the position of her image and the magnification.

SOLUTION

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f} = \frac{1}{-0.25\ \text{m}}$$

$$\frac{1}{q} = \frac{1}{-0.25\ \text{m}} - \frac{1}{3.0\ \text{m}}$$

$$q = \boxed{-0.23\ \text{m}}$$

The magnification is

$$M = -\frac{q}{p} = -\left(\frac{-0.23\ \text{m}}{3.0\ \text{m}}\right) = \boxed{0.077}$$

8. Thin Lenses

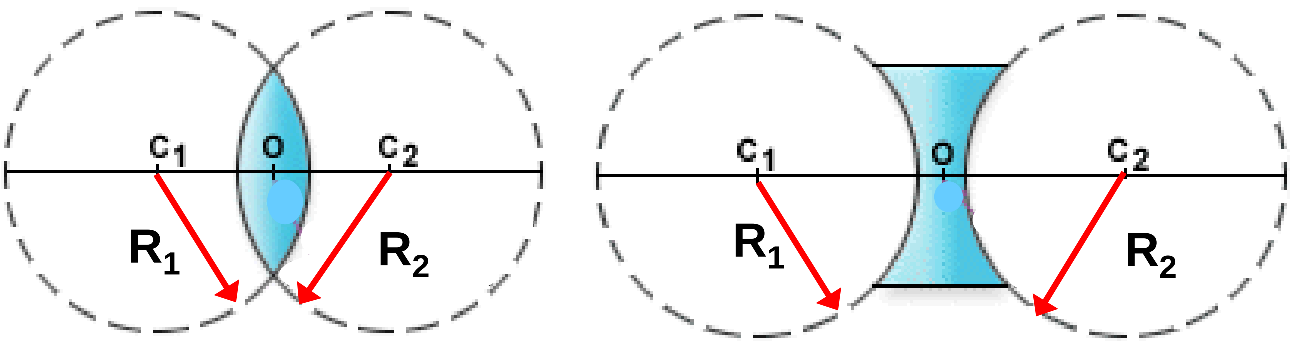
8.1 Notions

A lens is an optical system with two refracting surfaces. If two spherical surfaces close enough together that we can neglect the distance between them (the thickness of the lens) : **thin lens**.

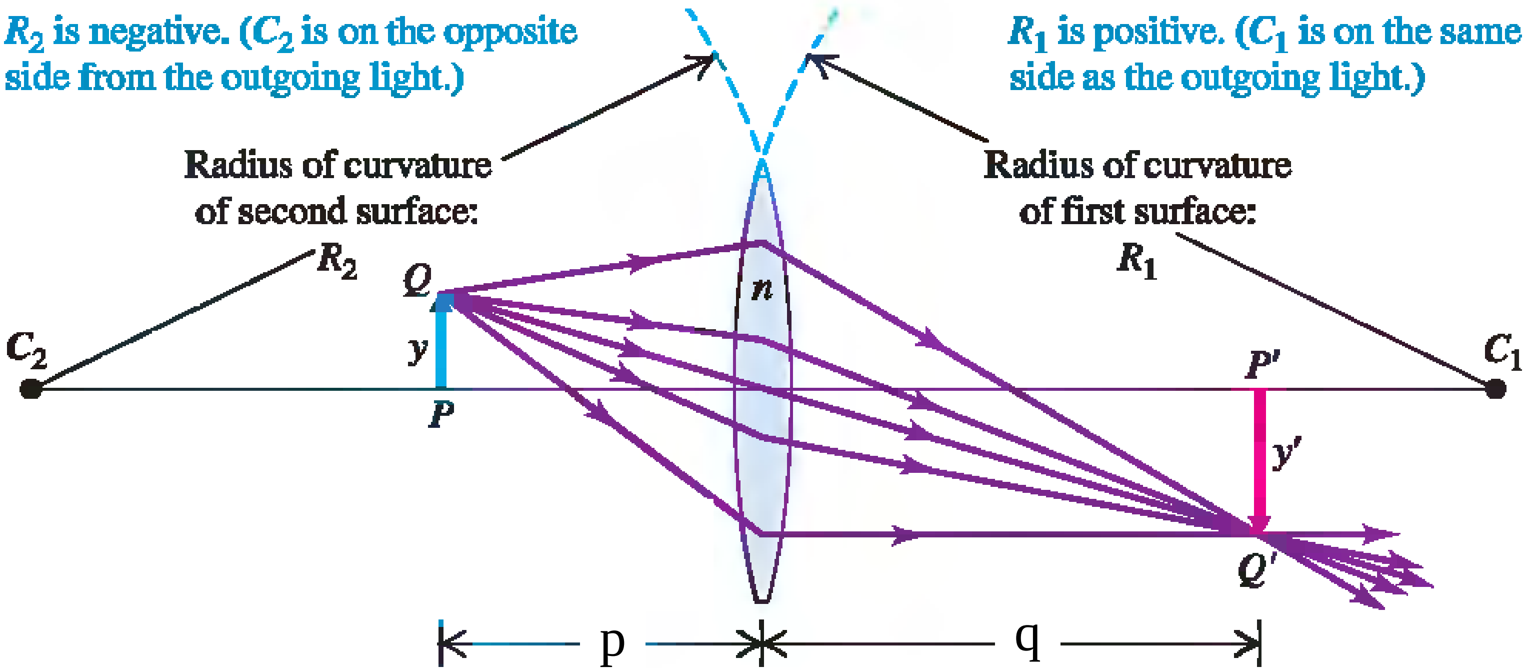
The focal length *f* :

$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

(lens makers' equation)

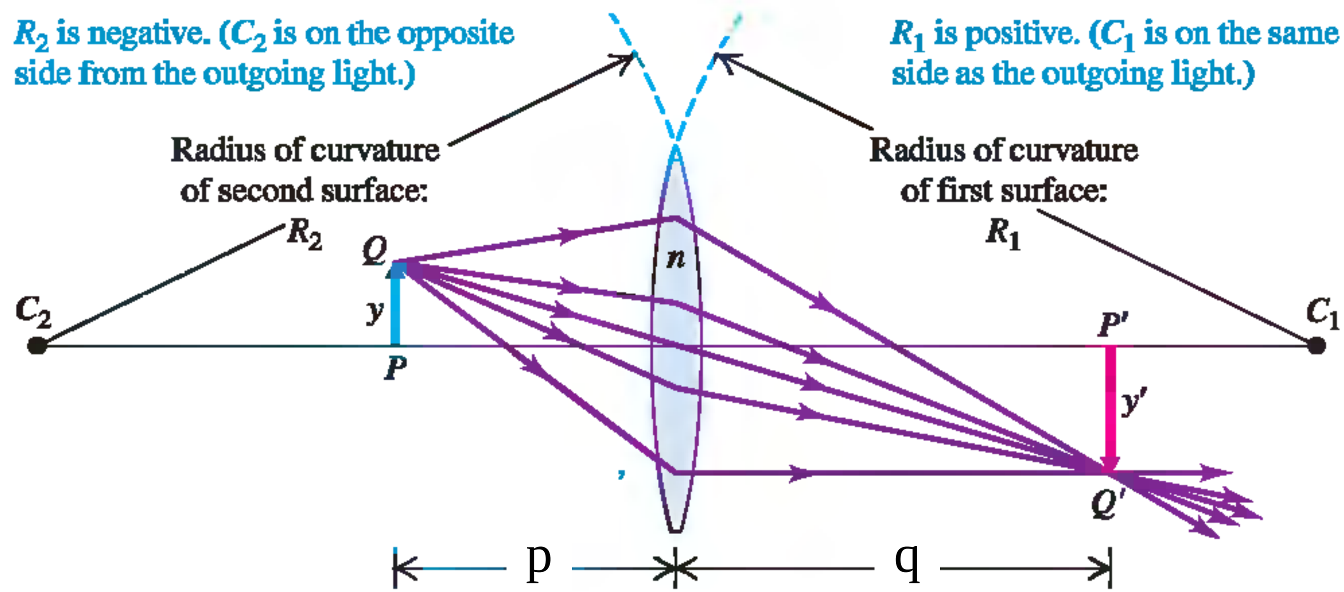


$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$ **(lens makers' equation)**



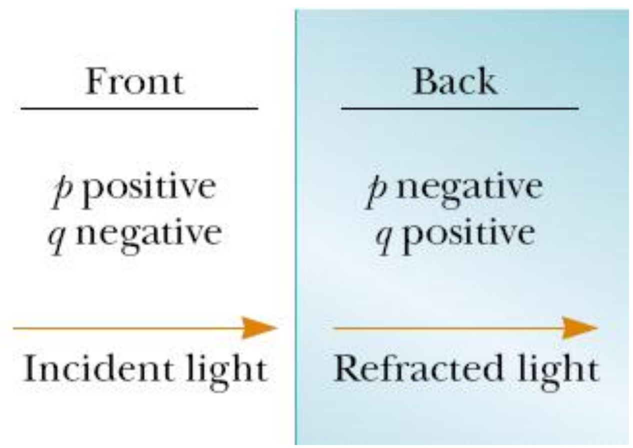
f is **positive** if the lens is **converging.**

f is **negative** if the lens is **diverging.**



Thin-lens equation :

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f}$$



Lateral magnification :

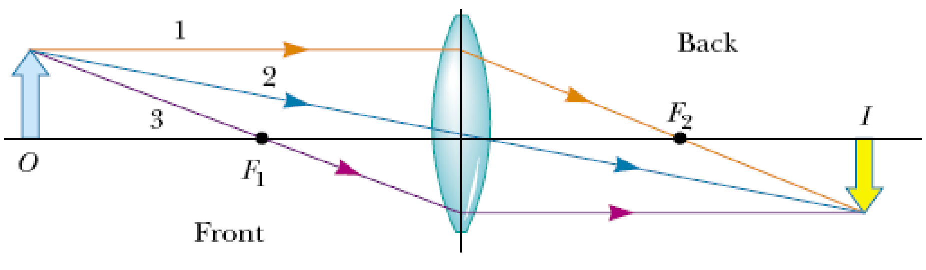
$$M = \frac{h'}{h} = -\frac{q}{p}$$

Real object $\leftrightarrow$ $p > 0$ Virtual object $\leftrightarrow$ $p < 0$

Real image $\leftrightarrow$ $q > 0$ Virtual image $\leftrightarrow$ $q < 0$

PROBLEM 24 A converging lens of focal length 10.0 cm forms an image of each of three objects placed
(a) 30.0 cm, (b) 10.0 cm, and (c) 5.00 cm in front of the lens.
In each case, find the image distance and describe the image.

SOLUTION (a)



(a)

$$\frac{1}{p}+\frac{1}{q}=\frac{1}{f}$$

$$\frac{1}{30.0\text{ cm}}+\frac{1}{q}=\frac{1}{10.0\text{ cm}}$$

$$q=\boxed{15.0\text{ cm}}$$

The positive sign indicates that the image is in back of the lens and real. The magnification is

$$M=-\frac{q}{p}=-\frac{15.0\text{ cm}}{30.0\text{ cm}}=\boxed{-0.500}$$

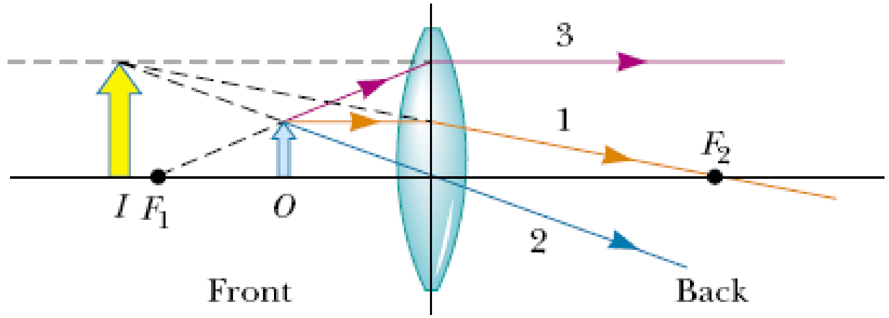
PROBLEM 24 A converging lens of focal length 10.0 cm forms an image of each of three objects placed (a) 30.0 cm, (b) 10.0 cm, and (c) 5.00 cm in front of the lens. In each case, find the image distance and describe the image.

SOLUTION (b)

When the object is placed at the focal point, the image is formed at infinity.

(c)

$$\frac{1}{5.00\text{ cm}} + \frac{1}{q} = \frac{1}{10.0\text{ cm}}$$



(b)

$$q = \boxed{-10.0\text{ cm}}$$

$$M = -\frac{q}{p} = -\left(\frac{-10.0\text{ cm}}{5.00\text{ cm}}\right) = \boxed{2.00}$$

PROBLEM 25 A diverging lens has a focal length of 20.0 cm. An object 2.00 cm tall is placed 30.0 cm in front of the lens. Locate the image. Determine both the magnification and the height of the image.

SOLUTION

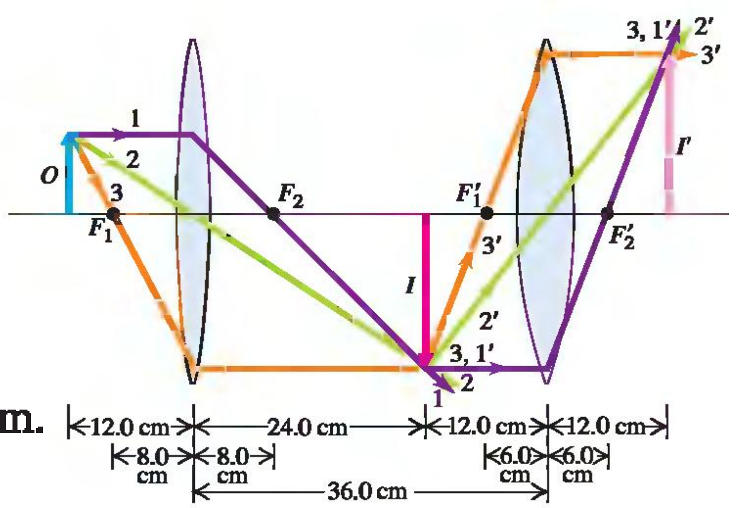
$$\frac{1}{30.0\ \text{cm}} + \frac{1}{q} = \frac{1}{-20.0\ \text{cm}}$$

$$q = \boxed{-12.0\ \text{cm}}$$

$M = 0.400$; $h' = 0.800$ cm.

PROBLEM 25 An object 8.0 cm high is placed 12.0 cm to the left of a converging lens of focal length 8.0 cm. A second converging lens of focal length 6.0 cm is placed 36.0 cm to the right of the first lens. Both lenses have the same optic axis. Find the position, size, and orientation of the image produced by the two lenses in combination.

SOLUTION



$$\frac{1}{12.0\text{ cm}} + \frac{1}{s_1'} = \frac{1}{8.0\text{ cm}} \qquad s_1' = +24.0\text{ cm}$$

$$m_1 = -(24.0\text{ cm})/(12.0\text{ cm}) = -2.00,$$

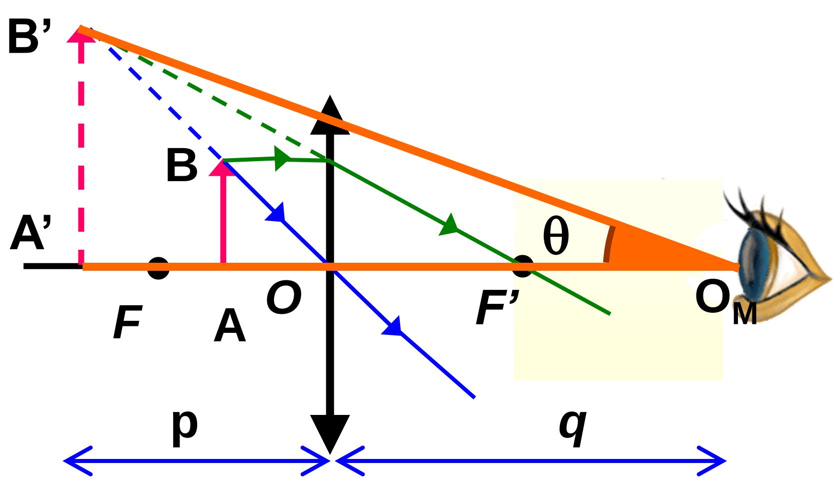
height of the first image is $(-2.0)(8.0\text{ cm}) = -16.0\text{ cm}.$

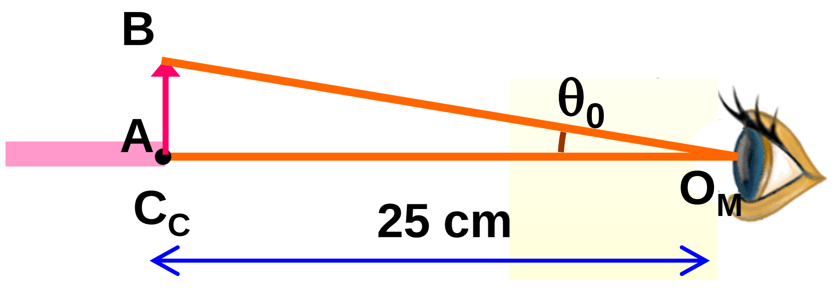
$$\frac{1}{12.0\text{ cm}} + \frac{1}{s_2'} = \frac{1}{6.0\text{ cm}} \qquad s_2' = +12.0\text{ cm}$$

$$m_2 = -(12.0\text{ cm})/(12.0\text{ cm}) = -1.0.$$

8. Thin Lenses

8.2 Simple magnifier





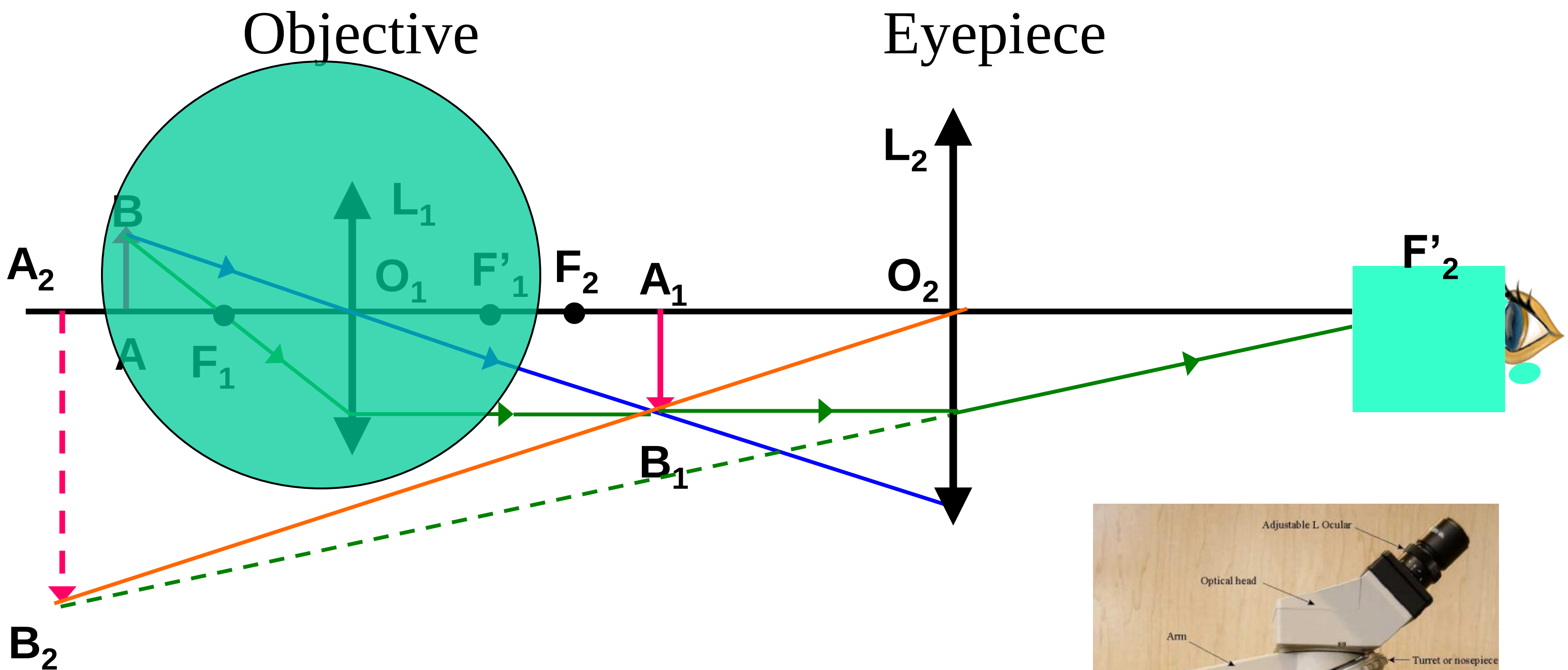
The simple magnifier consists of a single converging lens : this device increases the apparent size of an object.

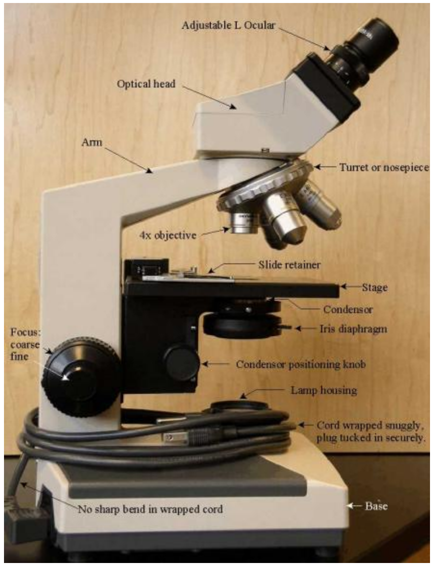
Angular magnification:

$$m \equiv \frac{\theta}{\theta_0}$$

8. Thin Lenses

8.3 The compound microscope



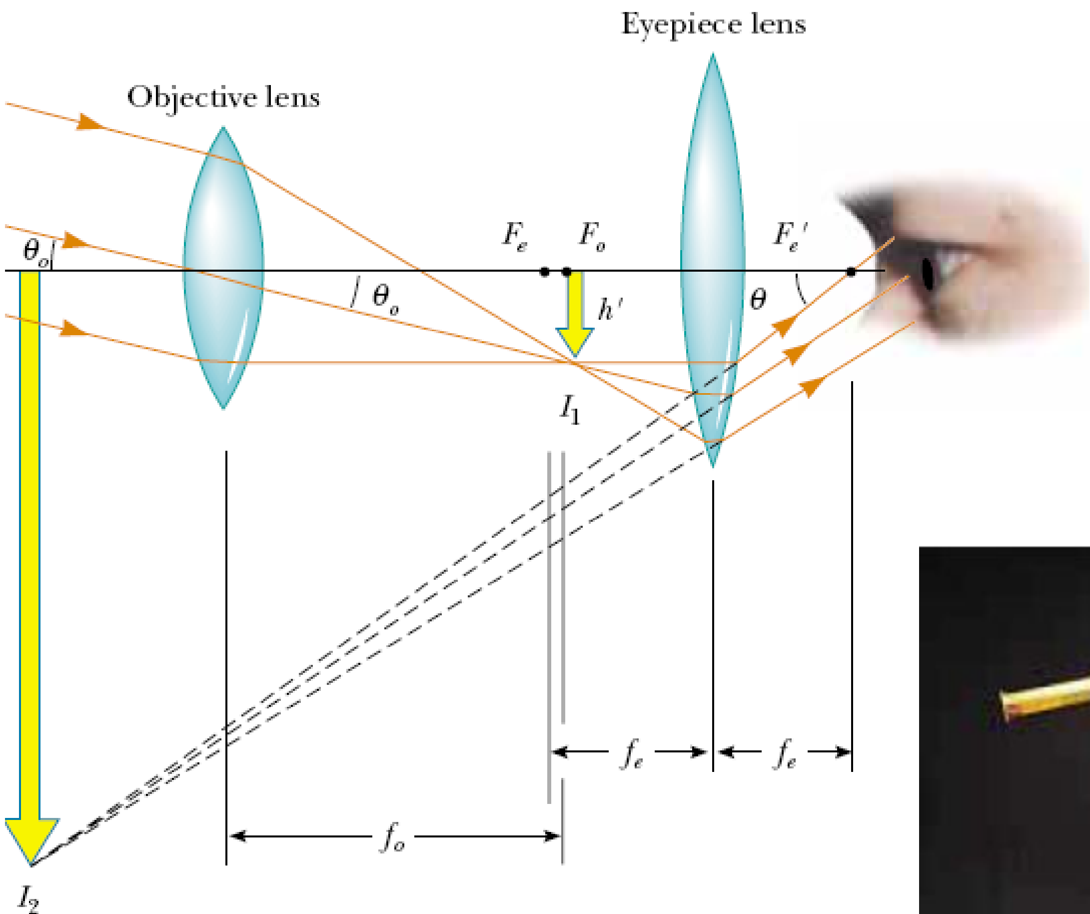


The *objective* has a very short focal length

The *eyepiece* has a focal length of a few centimeters.

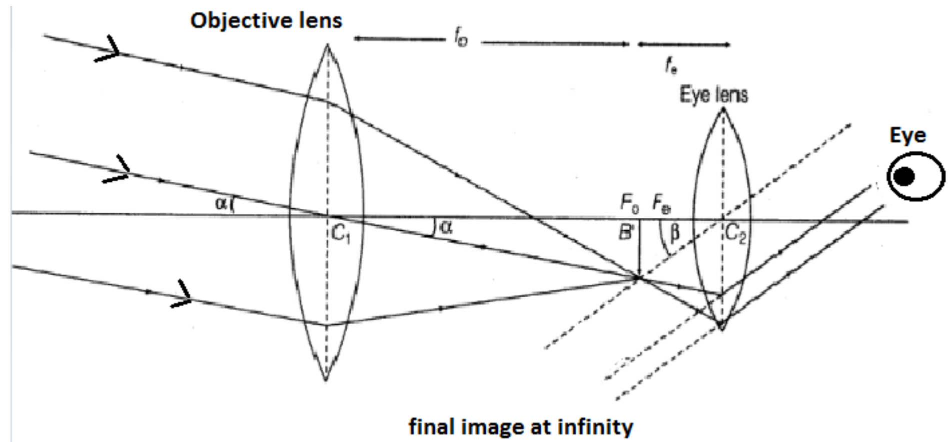
8. Thin Lenses

8.4 The Telescope



PROBLEM 26 The eyepiece of a refracting telescope has a focal length of 9.00 cm. The distance between objective and eyepiece is 1.80 m, and the final image is at infinity.
What is the angular magnification of the telescope?

SOLUTION



$$f_2 = 9.00\ cm\ ; \quad f_2 = ?$$

$$f_1 + f_2 = 1.80\ m\ ;$$

$$f_2 = 1.80 - 0.09 = 1.71\ m$$

$$M = \frac{\beta}{\alpha} = -\frac{A'B'/f_2}{A'B'/f_1} = -\frac{f_1}{f_2} = -\frac{171}{9} = -19.0$$