

7

KINETIC ENERGY
AND WORK

7-1 WHAT IS PHYSICS?

One of the fundamental goals of physics is to investigate something that everyone talks about: energy. The topic is obviously important. Indeed, our civilization is based on acquiring and effectively using energy.

For example, everyone knows that any type of motion requires energy: Flying across the Pacific Ocean requires it. Lifting material to the top floor of an office building or to an orbiting space station requires it. Throwing a fastball requires it. We spend a tremendous amount of money to acquire and use energy. Wars have been started because of energy resources. Wars have been ended because of a sudden, overpowering use of energy by one side. Everyone knows many examples of energy and its use, but what does the term *energy* really mean?

7-2 What Is Energy?

The term *energy* is so broad that a clear definition is difficult to write. Technically, energy is a scalar quantity associated with the state (or condition) of one or more objects. However, this definition is too vague to be of help to us now.

A looser definition might at least get us started. Energy is a number that we associate with a system of one or more objects. If a force changes one of the objects by, say, making it move, then the energy number changes. After countless experiments, scientists and engineers realized that if the scheme by which we assign energy numbers is planned carefully, the numbers can be used to predict the outcomes of experiments and, even more important, to build machines, such as flying machines. This success is based on a wonderful property of our universe: Energy can be transformed from one type to another and transferred from one object to another, but the total amount is always the same (energy is *conserved*). No exception to this *principle of energy conservation* has ever been found.

Think of the many types of energy as being numbers representing money in many types of bank accounts. Rules have been made about what such money numbers mean and how they can be changed. You can transfer money numbers from one account to another or from one system to another, perhaps electronically with nothing material actually moving. However, the total amount (the total of all the money numbers) can always be accounted for: It is always conserved.

In this chapter we focus on only one type of energy (*kinetic energy*) and on only one way in which energy can be transferred (*work*). In the next chapter we examine a few other types of energy and how the principle of energy conservation can be written as equations to be solved.

7-3 Kinetic Energy

Kinetic energy K is energy associated with the *state of motion* of an object. The faster the object moves, the greater is its kinetic energy. When the object is stationary, its kinetic energy is zero.

For an object of mass m whose speed v is well below the speed of light,

$$K = \frac{1}{2}mv^2 \quad (\text{kinetic energy}). \quad (7-1)$$

For example, a 3.0 kg duck flying past us at 2.0 m/s has a kinetic energy of $6.0 \text{ kg} \cdot \text{m}^2/\text{s}^2$; that is, we associate that number with the duck's motion.

The SI unit of kinetic energy (and every other type of energy) is the **joule (J)**, named for James Prescott Joule, an English scientist of the 1800s. It is defined directly from Eq. 7-1 in terms of the units for mass and velocity:

$$1 \text{ joule} = 1 \text{ J} = 1 \text{ kg} \cdot \text{m}^2/\text{s}^2. \quad (7-2)$$

Thus, the flying duck has a kinetic energy of 6.0 J.

Sample Problem

Kinetic energy, train crash

In 1896 in Waco, Texas, William Crush parked two locomotives at opposite ends of a 6.4-km-long track, fired them up, tied their throttles open, and then allowed them to crash head-on at full speed (Fig. 7-1) in front of 30,000 spectators. Hundreds of people were hurt by flying debris; several were killed. Assuming each locomotive weighed $1.2 \times 10^6 \text{ N}$ and its acceleration was a constant 0.26 m/s^2 , what was the total kinetic energy of the two locomotives just before the collision?



Fig. 7-1 The aftermath of an 1896 crash of two locomotives. (Courtesy Library of Congress)

KEY IDEAS

(1) We need to find the kinetic energy of each locomotive with Eq. 7-1, but that means we need each locomotive's speed just before the collision and its mass. (2) Because we can assume each locomotive had constant acceleration, we can use the equations in Table 2-1 to find its speed v just before the collision.

Calculations: We choose Eq. 2-16 because we know values for all the variables except v :

$$v^2 = v_0^2 + 2a(x - x_0).$$

With $v_0 = 0$ and $x - x_0 = 3.2 \times 10^3 \text{ m}$ (half the initial separation), this yields

$$v^2 = 0 + 2(0.26 \text{ m/s}^2)(3.2 \times 10^3 \text{ m}),$$

$$\text{or} \quad v = 40.8 \text{ m/s}$$

(about 150 km/h).

We can find the mass of each locomotive by dividing its given weight by g :

$$m = \frac{1.2 \times 10^6 \text{ N}}{9.8 \text{ m/s}^2} = 1.22 \times 10^5 \text{ kg}.$$

Now, using Eq. 7-1, we find the total kinetic energy of the two locomotives just before the collision as

$$K = 2(\frac{1}{2}mv^2) = (1.22 \times 10^5 \text{ kg})(40.8 \text{ m/s})^2 = 2.0 \times 10^8 \text{ J}. \quad (\text{Answer})$$

This collision was like an exploding bomb.



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7-4 Work

If you accelerate an object to a greater speed by applying a force to the object, you increase the kinetic energy $K (= \frac{1}{2}mv^2)$ of the object. Similarly, if you decelerate the object to a lesser speed by applying a force, you decrease the kinetic energy of the object. We account for these changes in kinetic energy by saying that your force has transferred energy *to* the object from yourself or *from* the object to yourself. In such a transfer of energy via a force, **work** W is said to be *done on the object by the force*. More formally, we define work as follows:

Work W is energy transferred to or from an object by means of a force acting on the object. Energy transferred to the object is positive work, and energy transferred from the object is negative work.

“Work,” then, is transferred energy; “doing work” is the act of transferring the energy. Work has the same units as energy and is a scalar quantity.

The term *transfer* can be misleading. It does not mean that anything material flows into or out of the object; that is, the transfer is not like a flow of water. Rather, it is like the electronic transfer of money between two bank accounts: The number in one account goes up while the number in the other account goes down, with nothing material passing between the two accounts.

Note that we are not concerned here with the common meaning of the word “work,” which implies that *any* physical or mental labor is work. For example, if you push hard against a wall, you tire because of the continuously repeated muscle contractions that are required, and you are, in the common sense, working. However, such effort does not cause an energy transfer to or from the wall and thus is not work done on the wall as defined here.

To avoid confusion in this chapter, we shall use the symbol W only for work and shall represent a weight with its equivalent mg .

7-5 Work and Kinetic Energy

Finding an Expression for Work

Let us find an expression for work by considering a bead that can slide along a frictionless wire that is stretched along a horizontal x axis (Fig. 7-2). A constant force \vec{F} , directed at an angle ϕ to the wire, accelerates the bead along the wire. We can relate the force and the acceleration with Newton’s second law, written for components along the x axis:

$$F_x = ma_x, \quad (7-3)$$

where m is the bead’s mass. As the bead moves through a displacement \vec{d} , the force changes the bead’s velocity from an initial value \vec{v}_0 to some other value \vec{v} . Because the force is constant, we know that the acceleration is also constant. Thus, we can use Eq. 2-16 to write, for components along the x axis,

$$v^2 = v_0^2 + 2a_x d. \quad (7-4)$$

Solving this equation for a_x , substituting into Eq. 7-3, and rearranging then give us

$$\frac{1}{2}mv^2 - \frac{1}{2}mv_0^2 = F_x d. \quad (7-5)$$

The first term on the left side of the equation is the kinetic energy K_f of the bead at the end of the displacement d , and the second term is the kinetic energy K_i of the bead at the start of the displacement. Thus, the left side of Eq. 7-5 tells us the kinetic energy has been changed by the force, and the right side tells us the change is equal to $F_x d$. Therefore, the work W done on the bead by the force

(the energy transfer due to the force) is

$$W = F_x d. \quad (7-6)$$

If we know values for F_x and d , we can use this equation to calculate the work W done on the bead by the force.

To calculate the work a force does on an object as the object moves through some displacement, we use only the force component along the object's displacement. The force component perpendicular to the displacement does zero work.

From Fig. 7-2, we see that we can write F_x as $F \cos \phi$, where ϕ is the angle between the directions of the displacement \vec{d} and the force \vec{F} . Thus,

$$W = Fd \cos \phi \quad (\text{work done by a constant force}). \quad (7-7)$$

Because the right side of this equation is equivalent to the scalar (dot) product $\vec{F} \cdot \vec{d}$, we can also write

$$W = \vec{F} \cdot \vec{d} \quad (\text{work done by a constant force}), \quad (7-8)$$

where F is the magnitude of \vec{F} . (You may wish to review the discussion of scalar products in Section 3-8.) Equation 7-8 is especially useful for calculating the work when \vec{F} and \vec{d} are given in unit-vector notation.

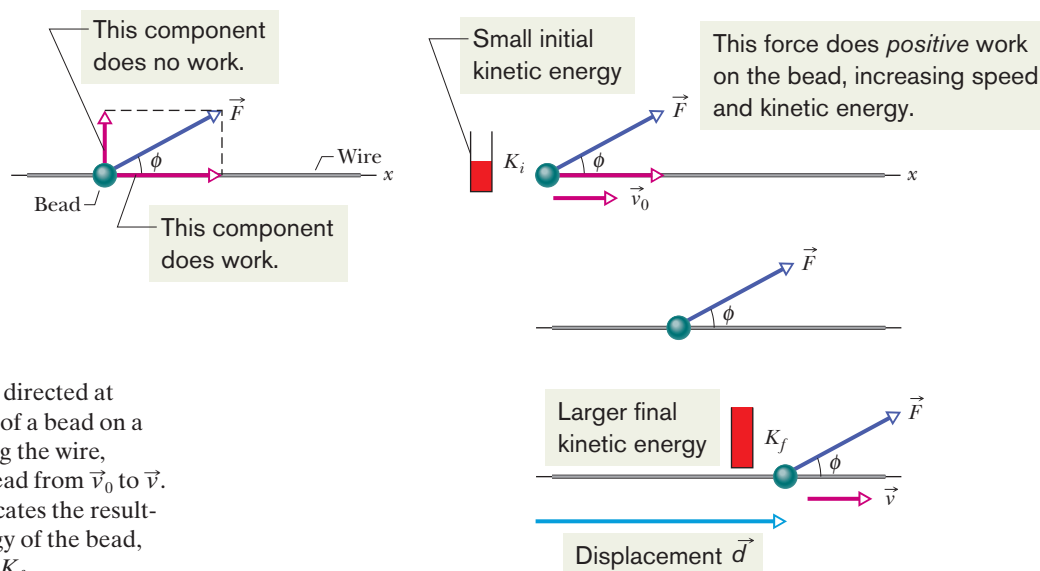


Fig. 7-2 A constant force \vec{F} directed at angle ϕ to the displacement \vec{d} of a bead on a wire accelerates the bead along the wire, changing the velocity of the bead from \vec{v}_0 to \vec{v} . A “kinetic energy gauge” indicates the resulting change in the kinetic energy of the bead, from the value K_i to the value K_f .

Cautions: There are two restrictions to using Eqs. 7-6 through 7-8 to calculate work done on an object by a force. First, the force must be a *constant force*; that is, it must not change in magnitude or direction as the object moves. (Later, we shall discuss what to do with a *variable force* that changes in magnitude.) Second, the object must be *particle-like*. This means that the object must be *rigid*; all parts of it must move together, in the same direction. In this chapter we consider only particle-like objects, such as the bed and its occupant being pushed in Fig. 7-3.

Signs for work. The work done on an object by a force can be either positive work or negative work. For example, if angle ϕ in Eq. 7-7 is less than 90° , then $\cos \phi$ is positive and thus so is the work. If ϕ is greater than 90° (up to 180°), then $\cos \phi$ is

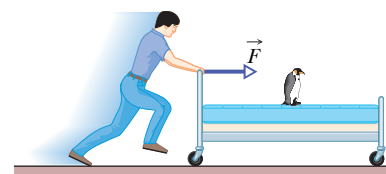


Fig. 7-3 A contestant in a bed race. We can approximate the bed and its occupant as being a particle for the purpose of calculating the work done on them by the force applied by the student.

negative and thus so is the work. (Can you see that the work is zero when $\phi = 90^\circ$?) These results lead to a simple rule. To find the sign of the work done by a force, consider the force vector component that is parallel to the displacement:

A force does positive work when it has a vector component in the same direction as the displacement, and it does negative work when it has a vector component in the opposite direction. It does zero work when it has no such vector component.

Units for work. Work has the SI unit of the joule, the same as kinetic energy. However, from Eqs. 7-6 and 7-7 we can see that an equivalent unit is the newton-meter ($\text{N} \cdot \text{m}$). The corresponding unit in the British system is the foot-pound ($\text{ft} \cdot \text{lb}$). Extending Eq. 7-2, we have

$$1 \text{ J} = 1 \text{ kg} \cdot \text{m}^2/\text{s}^2 = 1 \text{ N} \cdot \text{m} = 0.738 \text{ ft} \cdot \text{lb}. \quad (7-9)$$

Net work done by several forces. When two or more forces act on an object, the **net work** done on the object is the sum of the works done by the individual forces. We can calculate the net work in two ways. (1) We can find the work done by each force and then sum those works. (2) Alternatively, we can first find the net force \vec{F}_{net} of those forces. Then we can use Eq. 7-7, substituting the magnitude F_{net} for F and also the angle between the directions of \vec{F}_{net} and \vec{d} for ϕ . Similarly, we can use Eq. 7-8 with \vec{F}_{net} substituted for \vec{F} .

Work–Kinetic Energy Theorem

Equation 7-5 relates the change in kinetic energy of the bead (from an initial $K_i = \frac{1}{2}mv_0^2$ to a later $K_f = \frac{1}{2}mv^2$) to the work $W (= F_x d)$ done on the bead. For such particle-like objects, we can generalize that equation. Let ΔK be the change in the kinetic energy of the object, and let W be the net work done on it. Then

$$\Delta K = K_f - K_i = W, \quad (7-10)$$

which says that

$$\left(\begin{array}{c} \text{change in the kinetic} \\ \text{energy of a particle} \end{array} \right) = \left(\begin{array}{c} \text{net work done on} \\ \text{the particle} \end{array} \right).$$

We can also write

$$K_f = K_i + W, \quad (7-11)$$

which says that

$$\left(\begin{array}{c} \text{kinetic energy after} \\ \text{the net work is done} \end{array} \right) = \left(\begin{array}{c} \text{kinetic energy} \\ \text{before the net work} \end{array} \right) + \left(\begin{array}{c} \text{the net} \\ \text{work done} \end{array} \right).$$

These statements are known traditionally as the **work–kinetic energy theorem** for particles. They hold for both positive and negative work: If the net work done on a particle is positive, then the particle's kinetic energy increases by the amount of the work. If the net work done is negative, then the particle's kinetic energy decreases by the amount of the work.

For example, if the kinetic energy of a particle is initially 5 J and there is a net transfer of 2 J to the particle (positive net work), the final kinetic energy is 7 J. If, instead, there is a net transfer of 2 J from the particle (negative net work), the final kinetic energy is 3 J.

CHECKPOINT 1

A particle moves along an x axis. Does the kinetic energy of the particle increase, decrease, or remain the same if the particle's velocity changes (a) from -3 m/s to -2 m/s and (b) from -2 m/s to 2 m/s ? (c) In each situation, is the work done on the particle positive, negative, or zero?

Sample Problem

Work done by two constant forces, industrial spies

Figure 7-4a shows two industrial spies sliding an initially stationary 225 kg floor safe a displacement \vec{d} of magnitude 8.50 m, straight toward their truck. The push \vec{F}_1 of spy 001 is 12.0 N, directed at an angle of 30.0° downward from the horizontal; the pull \vec{F}_2 of spy 002 is 10.0 N, directed at 40.0° above the horizontal. The magnitudes and directions of these forces do not change as the safe moves, and the floor and safe make frictionless contact.

(a) What is the net work done on the safe by forces \vec{F}_1 and \vec{F}_2 during the displacement \vec{d} ?

KEY IDEAS

(1) The net work W done on the safe by the two forces is the sum of the works they do individually. (2) Because we can treat the safe as a particle and the forces are constant in both magnitude and direction, we can use either Eq. 7-7 ($W = Fd \cos \phi$) or Eq. 7-8 ($W = \vec{F} \cdot \vec{d}$) to calculate those works. Since we know the magnitudes and directions of the forces, we choose Eq. 7-7.

Calculations: From Eq. 7-7 and the free-body diagram for the safe in Fig. 7-4b, the work done by \vec{F}_1 is

$$W_1 = F_1 d \cos \phi_1 = (12.0 \text{ N})(8.50 \text{ m})(\cos 30.0^\circ) = 88.33 \text{ J},$$

and the work done by \vec{F}_2 is

$$W_2 = F_2 d \cos \phi_2 = (10.0 \text{ N})(8.50 \text{ m})(\cos 40.0^\circ) = 65.11 \text{ J}.$$

Thus, the net work W is

$$W = W_1 + W_2 = 88.33 \text{ J} + 65.11 \text{ J} = 153.4 \text{ J} \approx 153 \text{ J}. \quad (\text{Answer})$$

During the 8.50 m displacement, therefore, the spies transfer 153 J of energy to the kinetic energy of the safe.

(b) During the displacement, what is the work W_g done on the safe by the gravitational force \vec{F}_g and what is the work W_N done on the safe by the normal force \vec{F}_N from the floor?

KEY IDEA

Because these forces are constant in both magnitude and direction, we can find the work they do with Eq. 7-7.

Calculations: Thus, with mg as the magnitude of the gravitational force, we write

$$W_g = mgd \cos 90^\circ = mgd(0) = 0 \quad (\text{Answer})$$

and

$$W_N = F_N d \cos 90^\circ = F_N d(0) = 0. \quad (\text{Answer})$$

We should have known this result. Because these forces are perpendicular to the displacement of the safe, they do zero work on the safe and do not transfer any energy to or from it.

(c) The safe is initially stationary. What is its speed v_f at the end of the 8.50 m displacement?

KEY IDEA

The speed of the safe changes because its kinetic energy is changed when energy is transferred to it by \vec{F}_1 and \vec{F}_2 .

Calculations: We relate the speed to the work done by combining Eqs. 7-10 and 7-1:

$$W = K_f - K_i = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2.$$

The initial speed v_i is zero, and we now know that the work done is 153.4 J. Solving for v_f and then substituting known data, we find that

$$v_f = \sqrt{\frac{2W}{m}} = \sqrt{\frac{2(153.4 \text{ J})}{225 \text{ kg}}} = 1.17 \text{ m/s}. \quad (\text{Answer})$$

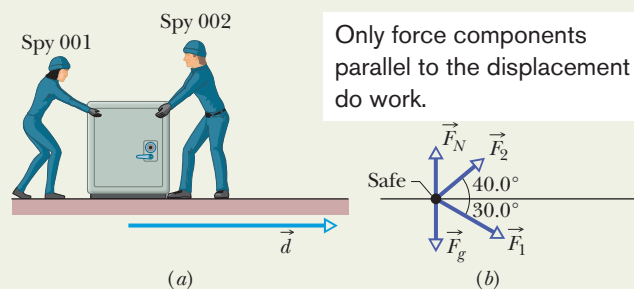


Fig. 7-4 (a) Two spies move a floor safe through a displacement \vec{d} . (b) A free-body diagram for the safe.

Sample Problem

Work done by a constant force in unit-vector notation

During a storm, a crate of crepe is sliding across a slick, oily parking lot through a displacement $\vec{d} = (-3.0 \text{ m})\hat{i}$ while a steady wind pushes against the crate with a force $\vec{F} = (2.0 \text{ N})\hat{i} + (-6.0 \text{ N})\hat{j}$. The situation and coordinate axes are shown in Fig. 7-5.

(a) How much work does this force do on the crate during the displacement?

KEY IDEA

Because we can treat the crate as a particle and because the wind force is constant (“steady”) in both magnitude and direction during the displacement, we can use either Eq. 7-7 ($W = Fd \cos \phi$) or Eq. 7-8 ($W = \vec{F} \cdot \vec{d}$) to calculate the work. Since we know \vec{F} and \vec{d} in unit-vector notation, we choose Eq. 7-8.

Calculations: We write

$$W = \vec{F} \cdot \vec{d} = [(2.0 \text{ N})\hat{i} + (-6.0 \text{ N})\hat{j}] \cdot [(-3.0 \text{ m})\hat{i}].$$

Of the possible unit-vector dot products, only $\hat{i} \cdot \hat{i}$, $\hat{j} \cdot \hat{j}$, and $\hat{k} \cdot \hat{k}$ are nonzero (see Appendix E). Here we obtain

$$\begin{aligned} W &= (2.0 \text{ N})(-3.0 \text{ m})\hat{i} \cdot \hat{i} + (-6.0 \text{ N})(-3.0 \text{ m})\hat{j} \cdot \hat{i} \\ &= (-6.0 \text{ J})(1) + 0 = -6.0 \text{ J}. \end{aligned} \quad (\text{Answer})$$

The parallel force component does *negative* work, slowing the crate.

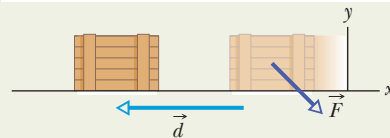


Fig. 7-5 Force \vec{F} slows a crate during displacement \vec{d} .

Thus, the force does a negative 6.0 J of work on the crate, transferring 6.0 J of energy from the kinetic energy of the crate.

(b) If the crate has a kinetic energy of 10 J at the beginning of displacement \vec{d} , what is its kinetic energy at the end of \vec{d} ?

KEY IDEA

Because the force does negative work on the crate, it reduces the crate’s kinetic energy.

Calculation: Using the work–kinetic energy theorem in the form of Eq. 7-11, we have

$$K_f = K_i + W = 10 \text{ J} + (-6.0 \text{ J}) = 4.0 \text{ J}. \quad (\text{Answer})$$

Less kinetic energy means that the crate has been slowed.



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7-6 Work Done by the Gravitational Force

We next examine the work done on an object by the gravitational force acting on it. Figure 7-6 shows a particle-like tomato of mass m that is thrown upward with initial speed v_0 and thus with initial kinetic energy $K_i = \frac{1}{2}mv_0^2$. As the tomato rises, it is slowed by a gravitational force \vec{F}_g ; that is, the tomato’s kinetic energy decreases because \vec{F}_g does work on the tomato as it rises. Because we can treat the tomato as a particle, we can use Eq. 7-7 ($W = Fd \cos \phi$) to express the work done during a displacement \vec{d} . For the force magnitude F , we use mg as the magnitude of \vec{F}_g . Thus, the work W_g done by the gravitational force \vec{F}_g is

$$W_g = mgd \cos \phi \quad (\text{work done by gravitational force}). \quad (7-12)$$

For a rising object, force \vec{F}_g is directed opposite the displacement \vec{d} , as indicated in Fig. 7-6. Thus, $\phi = 180^\circ$ and

$$W_g = mgd \cos 180^\circ = mgd(-1) = -mgd. \quad (7-13)$$

The minus sign tells us that during the object’s rise, the gravitational force acting on the object transfers energy in the amount mgd from the kinetic energy of the object. This is consistent with the slowing of the object as it rises.

After the object has reached its maximum height and is falling back down, the angle ϕ between force \vec{F}_g and displacement \vec{d} is zero. Thus,

$$W_g = mgd \cos 0^\circ = mgd(+1) = +mgd. \quad (7-14)$$

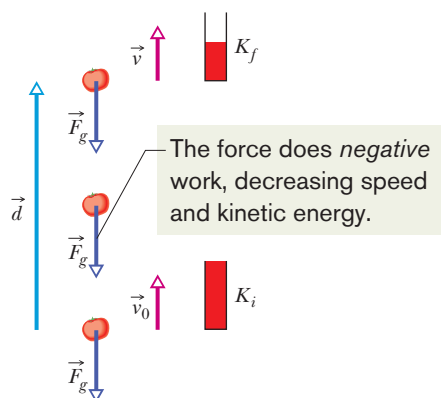


Fig. 7-6 Because the gravitational force \vec{F}_g acts on it, a particle-like tomato of mass m thrown upward slows from velocity \vec{v}_0 to velocity \vec{v} during displacement \vec{d} . A kinetic energy gauge indicates the resulting change in the kinetic energy of the tomato, from $K_i (= \frac{1}{2}mv_0^2)$ to $K_f (= \frac{1}{2}mv^2)$.

The plus sign tells us that the gravitational force now transfers energy in the amount mgd to the kinetic energy of the object. This is consistent with the speeding up of the object as it falls. (Actually, as we shall see in Chapter 8, energy transfers associated with lifting and lowering an object involve the full object–Earth system.)

Work Done in Lifting and Lowering an Object

Now suppose we lift a particle-like object by applying a vertical force \vec{F} to it. During the upward displacement, our applied force does positive work W_a on the object while the gravitational force does negative work W_g on it. Our applied force tends to transfer energy to the object while the gravitational force tends to transfer energy from it. By Eq. 7-10, the change ΔK in the kinetic energy of the object due to these two energy transfers is

$$\Delta K = K_f - K_i = W_a + W_g, \quad (7-15)$$

in which K_f is the kinetic energy at the end of the displacement and K_i is that at the start of the displacement. This equation also applies if we lower the object, but then the gravitational force tends to transfer energy *to* the object while our force tends to transfer energy *from* it.

In one common situation, the object is stationary before and after the lift—for example, when you lift a book from the floor to a shelf. Then K_f and K_i are both zero, and Eq. 7-15 reduces to

$$W_a + W_g = 0$$

or

$$W_a = -W_g. \quad (7-16)$$

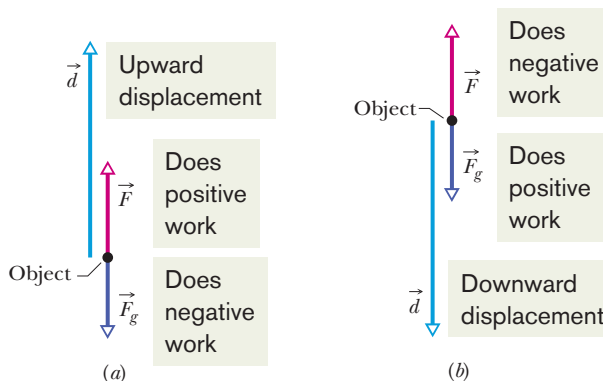
Note that we get the same result if K_f and K_i are not zero but are still equal. Either way, the result means that the work done by the applied force is the negative of the work done by the gravitational force; that is, the applied force transfers the same amount of energy to the object as the gravitational force transfers from the object. Using Eq. 7-12, we can rewrite Eq. 7-16 as

$$W_a = -mgd \cos \phi \quad (\text{work done in lifting and lowering; } K_f = K_i), \quad (7-17)$$

with ϕ being the angle between \vec{F}_g and \vec{d} . If the displacement is vertically upward (Fig. 7-7a), then $\phi = 180^\circ$ and the work done by the applied force equals mgd . If the displacement is vertically downward (Fig. 7-7b), then $\phi = 0^\circ$ and the work done by the applied force equals $-mgd$.

Equations 7-16 and 7-17 apply to any situation in which an object is lifted or lowered, with the object stationary before and after the lift. They are independent of the magnitude of the force used. For example, if you lift a mug from the floor to over your head, your force on the mug varies considerably during the lift. Still, because the mug is stationary before and after the lift, the work your force does on the mug is given by Eqs. 7-16 and 7-17, where, in Eq. 7-17, mg is the weight of the mug and d is the distance you lift it.

Fig. 7-7 (a) An applied force \vec{F} lifts an object. The object's displacement \vec{d} makes an angle $\phi = 180^\circ$ with the gravitational force \vec{F}_g on the object. The applied force does positive work on the object. (b) An applied force \vec{F} lowers an object. The displacement \vec{d} of the object makes an angle $\phi = 0^\circ$ with the gravitational force \vec{F}_g . The applied force does negative work on the object.



Sample Problem

Work done on an accelerating elevator cab

An elevator cab of mass $m = 500$ kg is descending with speed $v_i = 4.0$ m/s when its supporting cable begins to slip, allowing it to fall with constant acceleration $\vec{a} = \vec{g}/5$ (Fig. 7-8a).

(a) During the fall through a distance $d = 12$ m, what is the work W_g done on the cab by the gravitational force \vec{F}_g ?

KEY IDEA

We can treat the cab as a particle and thus use Eq. 7-12 ($W_g = mgd \cos \phi$) to find the work W_g .

Calculation: From Fig. 7-8b, we see that the angle between the directions of \vec{F}_g and the cab's displacement \vec{d} is 0° . Then, from Eq. 7-12, we find

$$\begin{aligned} W_g &= mgd \cos 0^\circ = (500 \text{ kg})(9.8 \text{ m/s}^2)(12 \text{ m})(1) \\ &= 5.88 \times 10^4 \text{ J} \approx 59 \text{ kJ.} \end{aligned} \quad (\text{Answer})$$

(b) During the 12 m fall, what is the work W_T done on the cab by the upward pull \vec{T} of the elevator cable?

KEY IDEAS

(1) We can calculate work W_T with Eq. 7-7 ($W = Fd \cos \phi$) if we first find an expression for the magnitude T of the cable's pull. (2) We can find that expression by writing Newton's second law for components along the y axis in Fig. 7-8b ($F_{\text{net},y} = ma_y$).

Calculations: We get

$$T - F_g = ma. \quad (7-18)$$

Solving for T , substituting mg for F_g , and then substituting the result in Eq. 7-7, we obtain

$$W_T = Td \cos \phi = m(a + g)d \cos \phi. \quad (7-19)$$

Next, substituting $-g/5$ for the (downward) acceleration a and then 180° for the angle ϕ between the directions of forces \vec{T} and $m\vec{g}$, we find

$$\begin{aligned} W_T &= m\left(-\frac{g}{5} + g\right)d \cos \phi = \frac{4}{5}mgd \cos \phi \\ &= \frac{4}{5}(500 \text{ kg})(9.8 \text{ m/s}^2)(12 \text{ m}) \cos 180^\circ \\ &= -4.70 \times 10^4 \text{ J} \approx -47 \text{ kJ.} \end{aligned} \quad (\text{Answer})$$

Caution: Note that W_T is not simply the negative of W_g . The reason is that, because the cab accelerates during the

fall, its speed changes during the fall, and thus its kinetic energy also changes. Therefore, Eq. 7-16 (which assumes that the initial and final kinetic energies are equal) does *not* apply here.

(c) What is the net work W done on the cab during the fall?

Calculation: The net work is the sum of the works done by the forces acting on the cab:

$$\begin{aligned} W &= W_g + W_T = 5.88 \times 10^4 \text{ J} - 4.70 \times 10^4 \text{ J} \\ &= 1.18 \times 10^4 \text{ J} \approx 12 \text{ kJ.} \end{aligned} \quad (\text{Answer})$$

(d) What is the cab's kinetic energy at the end of the 12 m fall?

KEY IDEA

The kinetic energy changes *because* of the net work done on the cab, according to Eq. 7-11 ($K_f = K_i + W$).

Calculation: From Eq. 7-1, we can write the kinetic energy at the start of the fall as $K_i = \frac{1}{2}mv_i^2$. We can then write Eq. 7-11 as

$$\begin{aligned} K_f &= K_i + W = \frac{1}{2}mv_i^2 + W \\ &= \frac{1}{2}(500 \text{ kg})(4.0 \text{ m/s})^2 + 1.18 \times 10^4 \text{ J} \\ &= 1.58 \times 10^4 \text{ J} \approx 16 \text{ kJ.} \end{aligned} \quad (\text{Answer})$$

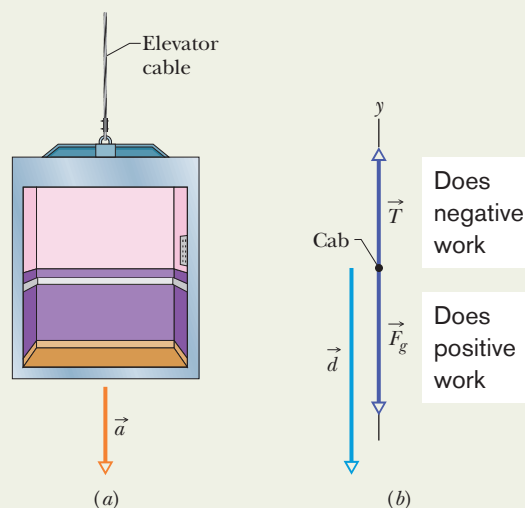


Fig. 7-8 An elevator cab, descending with speed v_i , suddenly begins to accelerate downward. (a) It moves through a displacement \vec{d} with constant acceleration $\vec{a} = \vec{g}/5$. (b) A free-body diagram for the cab, displacement included.

7-7 Work Done by a Spring Force

We next want to examine the work done on a particle-like object by a particular type of *variable force*—namely, a **spring force**, the force from a spring. Many forces in nature have the same mathematical form as the spring force. Thus, by examining this one force, you can gain an understanding of many others.

The Spring Force

Figure 7-9a shows a spring in its **relaxed state**—that is, neither compressed nor extended. One end is fixed, and a particle-like object—a block, say—is attached to the other, free end. If we stretch the spring by pulling the block to the right as in Fig. 7-9b, the spring pulls on the block toward the left. (Because a spring force acts to restore the relaxed state, it is sometimes said to be a *restoring force*.) If we compress the spring by pushing the block to the left as in Fig. 7-9c, the spring now pushes on the block toward the right.

To a good approximation for many springs, the force \vec{F}_s from a spring is proportional to the displacement \vec{d} of the free end from its position when the spring is in the relaxed state. The *spring force* is given by

$$\vec{F}_s = -k\vec{d} \quad (\text{Hooke's law}), \quad (7-20)$$

which is known as **Hooke's law** after Robert Hooke, an English scientist of the late 1600s. The minus sign in Eq. 7-20 indicates that the direction of the spring force is always opposite the direction of the displacement of the spring's free end. The constant k is called the **spring constant** (or **force constant**) and is a measure of the stiffness of the spring. The larger k is, the stiffer the spring; that is, the larger k is, the stronger the spring's pull or push for a given displacement. The SI unit for k is the newton per meter.

In Fig. 7-9 an x axis has been placed parallel to the length of the spring, with the origin ($x = 0$) at the position of the free end when the spring is in its relaxed state. For this common arrangement, we can write Eq. 7-20 as

$$F_x = -kx \quad (\text{Hooke's law}), \quad (7-21)$$

where we have changed the subscript. If x is positive (the spring is stretched toward the right on the x axis), then F_x is negative (it is a pull toward the left). If x is negative (the spring is compressed toward the left), then F_x is positive (it is a push toward the right). Note that a spring force is a *variable force* because it is a function of x , the position of the free end. Thus F_x can be symbolized as $F(x)$. Also note that Hooke's law is a *linear* relationship between F_x and x .

The Work Done by a Spring Force

To find the work done by the spring force as the block in Fig. 7-9a moves, let us make two simplifying assumptions about the spring. (1) It is *massless*; that is, its mass is negligible relative to the block's mass. (2) It is an *ideal spring*; that is, it obeys Hooke's law exactly. Let us also assume that the contact between the block and the floor is frictionless and that the block is particle-like.

We give the block a rightward jerk to get it moving and then leave it alone. As the block moves rightward, the spring force F_x does work on the block, decreasing the kinetic energy and slowing the block. However, we *cannot* find this work by using Eq. 7-7 ($W = Fd \cos \phi$) because that equation assumes a constant force. The spring force is a variable force.

To find the work done by the spring, we use calculus. Let the block's initial position be x_i and its later position x_f . Then divide the distance between those two

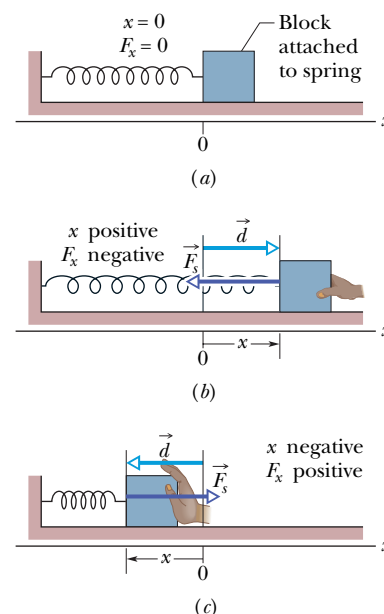


Fig. 7-9 (a) A spring in its relaxed state. The origin of an x axis has been placed at the end of the spring that is attached to a block. (b) The block is displaced by \vec{d} , and the spring is stretched by a positive amount x . Note the restoring force \vec{F}_s exerted by the spring. (c) The spring is compressed by a negative amount x . Again, note the restoring force.

positions into many segments, each of tiny length Δx . Label these segments, starting from x_i , as segments 1, 2, and so on. As the block moves through a segment, the spring force hardly varies because the segment is so short that x hardly varies. Thus, we can approximate the force magnitude as being constant within the segment. Label these magnitudes as F_{x1} in segment 1, F_{x2} in segment 2, and so on.

With the force now constant in each segment, we *can* find the work done within each segment by using Eq. 7-7. Here $\phi = 180^\circ$, and so $\cos \phi = -1$. Then the work done is $-F_{x1} \Delta x$ in segment 1, $-F_{x2} \Delta x$ in segment 2, and so on. The net work W_s done by the spring, from x_i to x_f , is the sum of all these works:

$$W_s = \sum -F_{xj} \Delta x, \quad (7-22)$$

where j labels the segments. In the limit as Δx goes to zero, Eq. 7-22 becomes

$$W_s = \int_{x_i}^{x_f} -F_x dx. \quad (7-23)$$

From Eq. 7-21, the force magnitude F_x is kx . Thus, substitution leads to

$$\begin{aligned} W_s &= \int_{x_i}^{x_f} -kx dx = -k \int_{x_i}^{x_f} x dx \\ &= \left(-\frac{1}{2}k\right)[x^2]_{x_i}^{x_f} = \left(-\frac{1}{2}k\right)(x_f^2 - x_i^2). \end{aligned} \quad (7-24)$$

Multiplied out, this yields

$$W_s = \frac{1}{2}kx_i^2 - \frac{1}{2}kx_f^2 \quad (\text{work by a spring force}). \quad (7-25)$$

This work W_s done by the spring force can have a positive or negative value, depending on whether the *net* transfer of energy is to or from the block as the block moves from x_i to x_f . *Caution:* The final position x_f appears in the *second* term on the right side of Eq. 7-25. Therefore, Eq. 7-25 tells us:

Work W_s is positive if the block ends up closer to the relaxed position ($x = 0$) than it was initially. It is negative if the block ends up farther away from $x = 0$. It is zero if the block ends up at the same distance from $x = 0$.

If $x_i = 0$ and if we call the final position x , then Eq. 7-25 becomes

$$W_s = -\frac{1}{2}kx^2 \quad (\text{work by a spring force}). \quad (7-26)$$

The Work Done by an Applied Force

Now suppose that we displace the block along the x axis while continuing to apply a force \vec{F}_a to it. During the displacement, our applied force does work W_a on the block while the spring force does work W_s . By Eq. 7-10, the change ΔK in the kinetic energy of the block due to these two energy transfers is

$$\Delta K = K_f - K_i = W_a + W_s, \quad (7-27)$$

in which K_f is the kinetic energy at the end of the displacement and K_i is that at the start of the displacement. If the block is stationary before and after the displacement, then K_f and K_i are both zero and Eq. 7-27 reduces to

$$W_a = -W_s. \quad (7-28)$$

If a block that is attached to a spring is stationary before and after a displacement, then the work done on it by the applied force displacing it is the negative of the work done on it by the spring force.

Caution: If the block is not stationary before and after the displacement, then this statement is *not* true.



CHECKPOINT 2

For three situations, the initial and final positions, respectively, along the x axis for the block in Fig. 7-9 are (a) -3 cm, 2 cm; (b) 2 cm, 3 cm; and (c) -2 cm, 2 cm. In each situation, is the work done by the spring force on the block positive, negative, or zero?

Sample Problem

Work done by spring to change kinetic energy

In Fig. 7-10, a cumin canister of mass $m = 0.40$ kg slides across a horizontal frictionless counter with speed $v = 0.50$ m/s. It then runs into and compresses a spring of spring constant $k = 750$ N/m. When the canister is momentarily stopped by the spring, by what distance d is the spring compressed?

KEY IDEAS

1. The work W_s done on the canister by the spring force is related to the requested distance d by Eq. 7-26 ($W_s = -\frac{1}{2}kx^2$), with d replacing x .
2. The work W_s is also related to the kinetic energy of the canister by Eq. 7-10 ($K_f - K_i = W$).
3. The canister's kinetic energy has an initial value of $K = \frac{1}{2}mv^2$ and a value of zero when the canister is momentarily at rest.

Calculations: Putting the first two of these ideas together, we write the work–kinetic energy theorem for the canister as

$$K_f - K_i = -\frac{1}{2}kd^2.$$

The spring force does *negative* work, decreasing speed and kinetic energy.

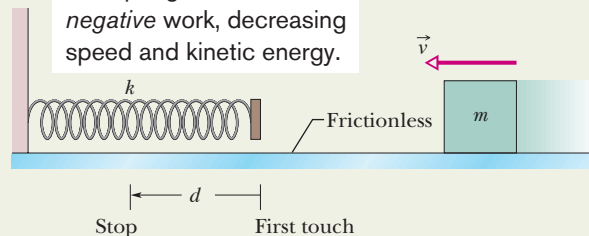


Fig. 7-10 A canister of mass m moves at velocity \vec{v} toward a spring that has spring constant k .

Substituting according to the third key idea gives us this expression

$$0 - \frac{1}{2}mv^2 = -\frac{1}{2}kd^2.$$

Simplifying, solving for d , and substituting known data then give us

$$\begin{aligned} d &= v \sqrt{\frac{m}{k}} = (0.50 \text{ m/s}) \sqrt{\frac{0.40 \text{ kg}}{750 \text{ N/m}}} \\ &= 1.2 \times 10^{-2} \text{ m} = 1.2 \text{ cm}. \end{aligned} \quad (\text{Answer})$$



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7-8 Work Done by a General Variable Force

One-Dimensional Analysis

Let us return to the situation of Fig. 7-2 but now consider the force to be in the positive direction of the x axis and the force magnitude to vary with position x . Thus, as the bead (particle) moves, the magnitude $F(x)$ of the force doing work on it changes. Only the magnitude of this variable force changes, not its direction, and the magnitude at any position does not change with time.

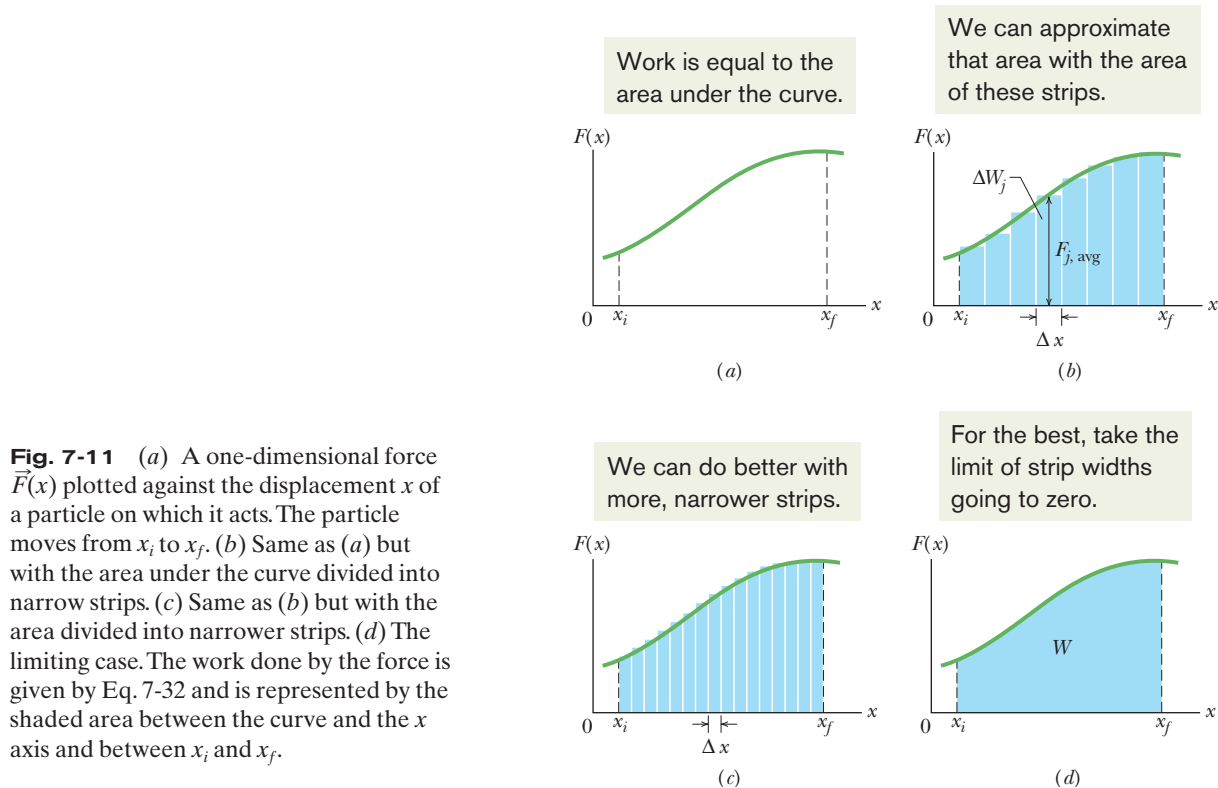


Fig. 7-11 (a) A one-dimensional force $\vec{F}(x)$ plotted against the displacement x of a particle on which it acts. The particle moves from x_i to x_f . (b) Same as (a) but with the area under the curve divided into narrow strips. (c) Same as (b) but with the area divided into narrower strips. (d) The limiting case. The work done by the force is given by Eq. 7-32 and is represented by the shaded area between the curve and the x axis and between x_i and x_f .

Figure 7-11a shows a plot of such a *one-dimensional variable force*. We want an expression for the work done on the particle by this force as the particle moves from an initial point x_i to a final point x_f . However, we *cannot* use Eq. 7-7 ($W = Fd \cos \phi$) because it applies only for a constant force \vec{F} . Here, again, we shall use calculus. We divide the area under the curve of Fig. 7-11a into a number of narrow strips of width Δx (Fig. 7-11b). We choose Δx small enough to permit us to take the force $F(x)$ as being reasonably constant over that interval. We let $F_{j,\text{avg}}$ be the average value of $F(x)$ within the j th interval. Then in Fig. 7-11b, $F_{j,\text{avg}}$ is the height of the j th strip.

With $F_{j,\text{avg}}$ considered constant, the increment (small amount) of work ΔW_j done by the force in the j th interval is now approximately given by Eq. 7-7 and is

$$\Delta W_j = F_{j,\text{avg}} \Delta x. \quad (7-29)$$

In Fig. 7-11b, ΔW_j is then equal to the area of the j th rectangular, shaded strip.

To approximate the total work W done by the force as the particle moves from x_i to x_f , we add the areas of all the strips between x_i and x_f in Fig. 7-11b:

$$W = \sum \Delta W_j = \sum F_{j,\text{avg}} \Delta x. \quad (7-30)$$

Equation 7-30 is an approximation because the broken “skyline” formed by the tops of the rectangular strips in Fig. 7-11b only approximates the actual curve of $F(x)$.

We can make the approximation better by reducing the strip width Δx and using more strips (Fig. 7-11c). In the limit, we let the strip width approach zero; the number of strips then becomes infinitely large and we have, as an exact result,

$$W = \lim_{\Delta x \rightarrow 0} \sum F_{j,\text{avg}} \Delta x. \quad (7-31)$$

This limit is exactly what we mean by the integral of the function $F(x)$ between the limits x_i and x_f . Thus, Eq. 7-31 becomes

$$W = \int_{x_i}^{x_f} F(x) dx \quad (\text{work: variable force}). \quad (7-32)$$

If we know the function $F(x)$, we can substitute it into Eq. 7-32, introduce the proper limits of integration, carry out the integration, and thus find the work.

(Appendix E contains a list of common integrals.) Geometrically, the work is equal to the area between the $F(x)$ curve and the x axis, between the limits x_i and x_f (shaded in Fig. 7-11*d*).

Three-Dimensional Analysis

Consider now a particle that is acted on by a three-dimensional force

$$\vec{F} = F_x \hat{i} + F_y \hat{j} + F_z \hat{k}, \quad (7-33)$$

in which the components F_x , F_y , and F_z can depend on the position of the particle; that is, they can be functions of that position. However, we make three simplifications: F_x may depend on x but not on y or z , F_y may depend on y but not on x or z , and F_z may depend on z but not on x or y . Now let the particle move through an incremental displacement

$$d\vec{r} = dx \hat{i} + dy \hat{j} + dz \hat{k}. \quad (7-34)$$

The increment of work dW done on the particle by \vec{F} during the displacement $d\vec{r}$ is, by Eq. 7-8,

$$dW = \vec{F} \cdot d\vec{r} = F_x dx + F_y dy + F_z dz. \quad (7-35)$$

The work W done by \vec{F} while the particle moves from an initial position r_i having coordinates (x_i, y_i, z_i) to a final position r_f having coordinates (x_f, y_f, z_f) is then

$$W = \int_{r_i}^{r_f} dW = \int_{x_i}^{x_f} F_x dx + \int_{y_i}^{y_f} F_y dy + \int_{z_i}^{z_f} F_z dz. \quad (7-36)$$

If \vec{F} has only an x component, then the y and z terms in Eq. 7-36 are zero and the equation reduces to Eq. 7-32.

Work–Kinetic Energy Theorem with a Variable Force

Equation 7-32 gives the work done by a variable force on a particle in a one-dimensional situation. Let us now make certain that the work is equal to the change in kinetic energy, as the work–kinetic energy theorem states.

Consider a particle of mass m , moving along an x axis and acted on by a net force $F(x)$ that is directed along that axis. The work done on the particle by this force as the particle moves from position x_i to position x_f is given by Eq. 7-32 as

$$W = \int_{x_i}^{x_f} F(x) dx = \int_{x_i}^{x_f} ma dx, \quad (7-37)$$

in which we use Newton's second law to replace $F(x)$ with ma . We can write the quantity $ma dx$ in Eq. 7-37 as

$$ma dx = m \frac{dv}{dt} dx. \quad (7-38)$$

From the chain rule of calculus, we have

$$\frac{dv}{dt} = \frac{dv}{dx} \frac{dx}{dt} = \frac{dv}{dx} v, \quad (7-39)$$

and Eq. 7-38 becomes

$$ma dx = m \frac{dv}{dx} v dx = mv dv. \quad (7-40)$$

Substituting Eq. 7-40 into Eq. 7-37 yields

$$\begin{aligned} W &= \int_{v_i}^{v_f} mv dv = m \int_{v_i}^{v_f} v dv \\ &= \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2. \end{aligned} \quad (7-41)$$

Note that when we change the variable from x to v we are required to express the limits on the integral in terms of the new variable. Note also that because the mass m is a constant, we are able to move it outside the integral.

Recognizing the terms on the right side of Eq. 7-41 as kinetic energies allows us to write this equation as

$$W = K_f - K_i = \Delta K,$$

which is the work–kinetic energy theorem.

Sample Problem

Work calculated by graphical integration

In an epidural procedure, as used in childbirth, a surgeon or an anesthetist must run a needle through the skin on the patient's back, through various tissue layers and into a narrow region called the epidural space that lies within the spinal canal surrounding the spinal cord. The needle is intended to deliver an anesthetic fluid. This tricky procedure requires much practice so that the doctor knows when the needle has reached the epidural space and not overshot it, a mistake that could result in serious complications.

The feel a doctor has for the needle's penetration is the variable force that must be applied to advance the needle through the tissues. Figure 7-12a is a graph of the force magnitude F versus displacement x of the needle tip in a typical epidural procedure. (The line segments have been straightened somewhat from the original data.) As x increases from 0, the skin resists the needle, but at $x = 8.0$ mm the force is finally great enough to pierce the skin, and then the required force decreases. Similarly, the needle finally pierces the interspinous ligament at $x = 18$ mm and the relatively tough ligamentum flavum at $x = 30$ mm. The needle then enters the epidural space (where it is to deliver the anesthetic fluid), and the force drops sharply. A new doctor must learn this pattern of force versus displacement to recognize when to stop pushing on the needle. (This is the pattern to be programmed into a virtual-reality simulation of an epidural procedure.) How much work W is done by the force exerted on the needle to get the needle to the epidural space at $x = 30$ mm?

KEY IDEAS

(1) We can calculate the work W done by a variable force $F(x)$ by integrating the force versus position x . Equation 7-32 tells us that

$$W = \int_{x_i}^{x_f} F(x) dx.$$

We want the work done by the force during the displacement from $x_i = 0$ to $x_f = 0.030$ m. (2) We can evaluate the integral by finding the area under the curve on the graph of Fig. 7-12a.

$$W = \left(\begin{array}{l} \text{area between force curve} \\ \text{and } x \text{ axis, from } x_i \text{ to } x_f \end{array} \right).$$

Calculations: Because our graph consists of straight-line segments, we can find the area by splitting the region below the curve into rectangular and triangular regions, as shown in Fig. 7-12b. For example, the area in triangular region A is

$$\text{area}_A = \frac{1}{2}(0.0080 \text{ m})(12 \text{ N}) = 0.048 \text{ N} \cdot \text{m} = 0.048 \text{ J}.$$

Once we've calculated the areas for all the labeled regions in Fig. 7-12b, we find that the total work is

$$\begin{aligned} W &= (\text{sum of the areas of regions A through K}) \\ &= 0.048 + 0.024 + 0.012 + 0.036 + 0.009 + 0.001 \\ &\quad + 0.016 + 0.048 + 0.016 + 0.004 + 0.024 \\ &= 0.238 \text{ J.} \end{aligned} \quad (\text{Answer})$$

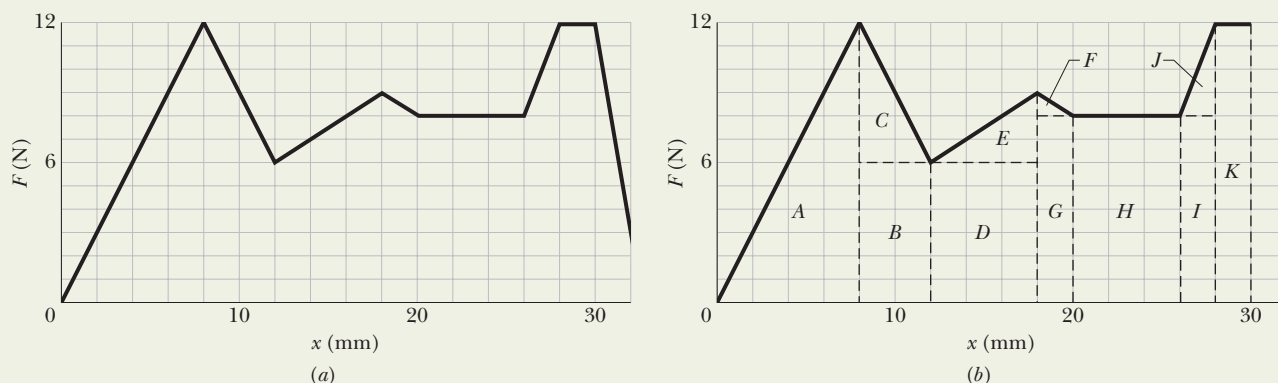


Fig. 7-12 (a) The force magnitude F versus the displacement x of the needle in an epidural procedure. (b) Breaking up the region between the plotted curve and the displacement axis to calculate the area.



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Sample Problem

Work, two-dimensional integration

Force $\vec{F} = (3x^2 \text{ N})\hat{i} + (4 \text{ N})\hat{j}$, with x in meters, acts on a particle, changing only the kinetic energy of the particle. How much work is done on the particle as it moves from coordinates (2 m, 3 m) to (3 m, 0 m)? Does the speed of the particle increase, decrease, or remain the same?

KEY IDEA

The force is a variable force because its x component depends on the value of x . Thus, we cannot use Eqs. 7-7 and 7-8 to find the work done. Instead, we must use Eq. 7-36 to integrate the force.

Calculation: We set up two integrals, one along each axis:

$$\begin{aligned} W &= \int_2^3 3x^2 dx + \int_3^0 4 dy = 3 \int_2^3 x^2 dx + 4 \int_3^0 dy \\ &= 3\left[\frac{1}{3}x^3\right]_2^3 + 4[y]_3^0 = [3^3 - 2^3] + 4[0 - 3] \\ &= 7.0 \text{ J.} \end{aligned} \quad (\text{Answer})$$

The positive result means that energy is transferred to the particle by force \vec{F} . Thus, the kinetic energy of the particle increases and, because $K = \frac{1}{2}mv^2$, its speed must also increase. If the work had come out negative, the kinetic energy and speed would have decreased.



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7-9 Power

The time rate at which work is done by a force is said to be the **power** due to the force. If a force does an amount of work W in an amount of time Δt , the **average power** due to the force during that time interval is

$$P_{\text{avg}} = \frac{W}{\Delta t} \quad (\text{average power}). \quad (7-42)$$

The **instantaneous power** P is the instantaneous time rate of doing work, which we can write as

$$P = \frac{dW}{dt} \quad (\text{instantaneous power}). \quad (7-43)$$

Suppose we know the work $W(t)$ done by a force as a function of time. Then to get the instantaneous power P at, say, time $t = 3.0 \text{ s}$ during the work, we would first take the time derivative of $W(t)$ and then evaluate the result for $t = 3.0 \text{ s}$.

The SI unit of power is the joule per second. This unit is used so often that it has a special name, the **watt** (W), after James Watt, who greatly improved the rate at which steam engines could do work. In the British system, the unit of power is the foot-pound per second. Often the horsepower is used. These are related by

$$1 \text{ watt} = 1 \text{ W} = 1 \text{ J/s} = 0.738 \text{ ft} \cdot \text{lb/s} \quad (7-44)$$

$$\text{and} \quad 1 \text{ horsepower} = 1 \text{ hp} = 550 \text{ ft} \cdot \text{lb/s} = 746 \text{ W}. \quad (7-45)$$

Inspection of Eq. 7-42 shows that work can be expressed as power multiplied by time, as in the common unit kilowatt-hour. Thus,

$$\begin{aligned} 1 \text{ kilowatt-hour} &= 1 \text{ kW} \cdot \text{h} = (10^3 \text{ W})(3600 \text{ s}) \\ &= 3.60 \times 10^6 \text{ J} = 3.60 \text{ MJ}. \end{aligned} \quad (7-46)$$

Perhaps because they appear on our utility bills, the watt and the kilowatt-hour have become identified as electrical units. They can be used equally well as units for other examples of power and energy. Thus, if you pick up a book from the floor and put it on a tabletop, you are free to report the work that you have done as, say, $4 \times 10^{-6} \text{ kW} \cdot \text{h}$ (or more conveniently as $4 \text{ mW} \cdot \text{h}$).



Fig. 7-13 The power due to the truck's applied force on the trailing load is the rate at which that force does work on the load. (REGLAIN FREDERIC/Gamma-Press, Inc.)

We can also express the rate at which a force does work on a particle (or particle-like object) in terms of that force and the particle's velocity. For a particle that is moving along a straight line (say, an x axis) and is acted on by a constant force \vec{F} directed at some angle ϕ to that line, Eq. 7-43 becomes

$$P = \frac{dW}{dt} = \frac{F \cos \phi \, dx}{dt} = F \cos \phi \left(\frac{dx}{dt} \right),$$

or

$$P = Fv \cos \phi. \quad (7-47)$$

Reorganizing the right side of Eq. 7-47 as the dot product $\vec{F} \cdot \vec{v}$, we may also write the equation as

$$P = \vec{F} \cdot \vec{v} \quad (\text{instantaneous power}). \quad (7-48)$$

For example, the truck in Fig. 7-13 exerts a force \vec{F} on the trailing load, which has velocity \vec{v} at some instant. The instantaneous power due to \vec{F} is the rate at which \vec{F} does work on the load at that instant and is given by Eqs. 7-47 and 7-48. Saying that this power is “the power of the truck” is often acceptable, but keep in mind what is meant: Power is the rate at which the applied *force* does work.

CHECKPOINT 3

A block moves with uniform circular motion because a cord tied to the block is anchored at the center of a circle. Is the power due to the force on the block from the cord positive, negative, or zero?

Sample Problem

Power, force, and velocity

Figure 7-14 shows constant forces \vec{F}_1 and \vec{F}_2 acting on a box as the box slides rightward across a frictionless floor. Force \vec{F}_1 is horizontal, with magnitude 2.0 N; force \vec{F}_2 is angled upward by 60° to the floor and has magnitude 4.0 N. The speed v of the box at a certain instant is 3.0 m/s. What is the power due to each force acting on the box at that instant, and what is the net power? Is the net power changing at that instant?

KEY IDEA

We want an instantaneous power, not an average power over a time period. Also, we know the box's velocity (rather than the work done on it).

Calculation: We use Eq. 7-47 for each force. For force \vec{F}_1 , at angle $\phi_1 = 180^\circ$ to velocity \vec{v} , we have

$$\begin{aligned} P_1 &= F_1 v \cos \phi_1 = (2.0 \text{ N})(3.0 \text{ m/s}) \cos 180^\circ \\ &= -6.0 \text{ W}. \end{aligned} \quad (\text{Answer})$$

This negative result tells us that force \vec{F}_1 is transferring energy *from* the box at the rate of 6.0 J/s.

For force \vec{F}_2 , at angle $\phi_2 = 60^\circ$ to velocity \vec{v} , we have

$$\begin{aligned} P_2 &= F_2 v \cos \phi_2 = (4.0 \text{ N})(3.0 \text{ m/s}) \cos 60^\circ \\ &= 6.0 \text{ W}. \end{aligned} \quad (\text{Answer})$$

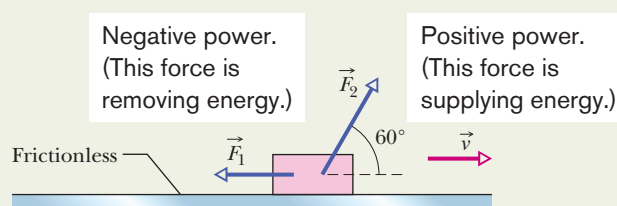


Fig. 7-14 Two forces \vec{F}_1 and \vec{F}_2 act on a box that slides rightward across a frictionless floor. The velocity of the box is \vec{v} .

This positive result tells us that force \vec{F}_2 is transferring energy *to* the box at the rate of 6.0 J/s.

The net power is the sum of the individual powers:

$$\begin{aligned} P_{\text{net}} &= P_1 + P_2 \\ &= -6.0 \text{ W} + 6.0 \text{ W} = 0, \end{aligned} \quad (\text{Answer})$$

which tells us that the net rate of transfer of energy to or from the box is zero. Thus, the kinetic energy ($K = \frac{1}{2}mv^2$) of the box is not changing, and so the speed of the box will remain at 3.0 m/s. With neither the forces \vec{F}_1 and \vec{F}_2 nor the velocity \vec{v} changing, we see from Eq. 7-48 that P_1 and P_2 are constant and thus so is P_{net} .



Additional examples, video, and practice available at WileyPLUS

REVIEW & SUMMARY

Kinetic Energy The **kinetic energy** K associated with the motion of a particle of mass m and speed v , where v is well below the speed of light, is

$$K = \frac{1}{2}mv^2 \quad (\text{kinetic energy}). \quad (7-1)$$

Work **Work** W is energy transferred to or from an object via a force acting on the object. Energy transferred to the object is positive work, and from the object, negative work.

Work Done by a Constant Force The work done on a particle by a constant force \vec{F} during displacement \vec{d} is

$$W = Fd \cos \phi = \vec{F} \cdot \vec{d} \quad (\text{work, constant force}), \quad (7-7, 7-8)$$

in which ϕ is the constant angle between the directions of \vec{F} and \vec{d} . Only the component of \vec{F} that is along the displacement \vec{d} can do work on the object. When two or more forces act on an object, their **net work** is the sum of the individual works done by the forces, which is also equal to the work that would be done on the object by the net force \vec{F}_{net} of those forces.

Work and Kinetic Energy For a particle, a change ΔK in the kinetic energy equals the net work W done on the particle:

$$\Delta K = K_f - K_i = W \quad (\text{work-kinetic energy theorem}), \quad (7-10)$$

in which K_i is the initial kinetic energy of the particle and K_f is the kinetic energy after the work is done. Equation 7-10 rearranged gives us

$$K_f = K_i + W. \quad (7-11)$$

Work Done by the Gravitational Force The work W_g done by the gravitational force \vec{F}_g on a particle-like object of mass m as the object moves through a displacement \vec{d} is given by

$$W_g = mgd \cos \phi, \quad (7-12)$$

in which ϕ is the angle between \vec{F}_g and \vec{d} .

Work Done in Lifting and Lowering an Object The work W_a done by an applied force as a particle-like object is either lifted or lowered is related to the work W_g done by the gravitational force and the change ΔK in the object's kinetic energy by

$$\Delta K = K_f - K_i = W_a + W_g. \quad (7-15)$$

If $K_f = K_i$, then Eq. 7-15 reduces to

$$W_a = -W_g, \quad (7-16)$$

which tells us that the applied force transfers as much energy to the object as the gravitational force transfers from it.

Spring Force The force \vec{F}_s from a spring is

$$\vec{F}_s = -k\vec{d} \quad (\text{Hooke's law}), \quad (7-20)$$

where \vec{d} is the displacement of the spring's free end from its position when the spring is in its **relaxed state** (neither compressed nor extended), and k is the **spring constant** (a measure of the spring's stiffness). If an x axis lies along the spring, with the origin at the location of the spring's free end when the spring is in its relaxed state, Eq. 7-20 can be written as

$$F_x = -kx \quad (\text{Hooke's law}). \quad (7-21)$$

A spring force is thus a variable force: It varies with the displacement of the spring's free end.

Work Done by a Spring Force If an object is attached to the spring's free end, the work W_s done on the object by the spring force when the object is moved from an initial position x_i to a final position x_f is

$$W_s = \frac{1}{2}kx_i^2 - \frac{1}{2}kx_f^2. \quad (7-25)$$

If $x_i = 0$ and $x_f = x$, then Eq. 7-25 becomes

$$W_s = -\frac{1}{2}kx^2. \quad (7-26)$$

Work Done by a Variable Force When the force \vec{F} on a particle-like object depends on the position of the object, the work done by \vec{F} on the object while the object moves from an initial position r_i with coordinates (x_i, y_i, z_i) to a final position r_f with coordinates (x_f, y_f, z_f) must be found by integrating the force. If we assume that component F_x may depend on x but not on y or z , component F_y may depend on y but not on x or z , and component F_z may depend on z but not on x or y , then the work is

$$W = \int_{x_i}^{x_f} F_x dx + \int_{y_i}^{y_f} F_y dy + \int_{z_i}^{z_f} F_z dz. \quad (7-36)$$

If \vec{F} has only an x component, then Eq. 7-36 reduces to

$$W = \int_{x_i}^{x_f} F(x) dx. \quad (7-32)$$

Power The **power** due to a force is the *rate* at which that force does work on an object. If the force does work W during a time interval Δt , the *average power* due to the force over that time interval is

$$P_{\text{avg}} = \frac{W}{\Delta t}. \quad (7-42)$$

Instantaneous power is the instantaneous rate of doing work:

$$P = \frac{dW}{dt}. \quad (7-43)$$

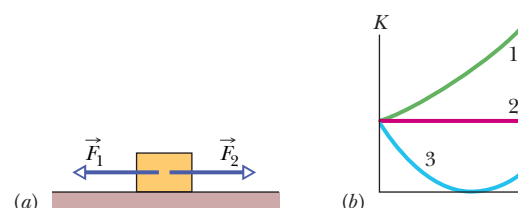
For a force \vec{F} at an angle ϕ to the direction of travel of the instantaneous velocity \vec{v} , the instantaneous power is

$$P = Fv \cos \phi = \vec{F} \cdot \vec{v}. \quad (7-47, 7-48)$$

1 Rank the following velocities according to the kinetic energy a particle will have with each velocity, greatest first: (a) $\vec{v} = 4\hat{i} + 3\hat{j}$, (b) $\vec{v} = -4\hat{i} + 3\hat{j}$, (c) $\vec{v} = -3\hat{i} + 4\hat{j}$, (d) $\vec{v} = 3\hat{i} - 4\hat{j}$, (e) $\vec{v} = 5\hat{i}$, and (f) $v = 5$ m/s at 30° to the horizontal.

2 Figure 7-15a shows two horizontal forces that act on a block that is sliding to the right across a frictionless floor. Figure 7-15b shows three plots of the block's kinetic energy K versus time t .

Fig. 7-15
Question 2.



Which of the plots best corresponds to the following three situations: (a) $F_1 = F_2$, (b) $F_1 > F_2$, (c) $F_1 < F_2$?

3 Is positive or negative work done by a constant force \vec{F} on a particle during a straight-line displacement \vec{d} if (a) the angle between \vec{F} and \vec{d} is 30° ; (b) the angle is 100° ; (c) $\vec{F} = 2\hat{i} - 3\hat{j}$ and $\vec{d} = -4\hat{i}$?

4 In three situations, a briefly applied horizontal force changes the velocity of a hockey puck that slides over frictionless ice. The overhead views of Fig. 7-16 indicate, for each situation, the puck's initial speed v_i , its final speed v_f , and the directions of the corresponding velocity vectors. Rank the situations according to the work done on the puck by the applied force, most positive first and most negative last.

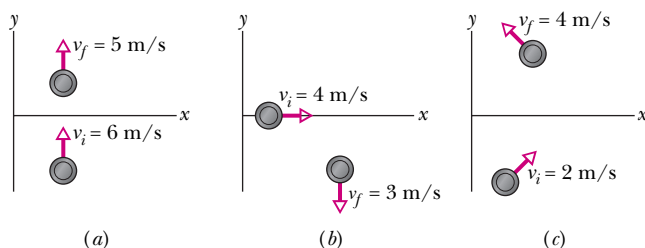


Fig. 7-16 Question 4.

5 Figure 7-17 shows four graphs (drawn to the same scale) of the x component F_x of a variable force (directed along an x axis) versus the position x of a particle on which the force acts. Rank the graphs according to the work done by the force on the particle from $x = 0$ to $x = x_1$, from most positive work first to most negative work last.

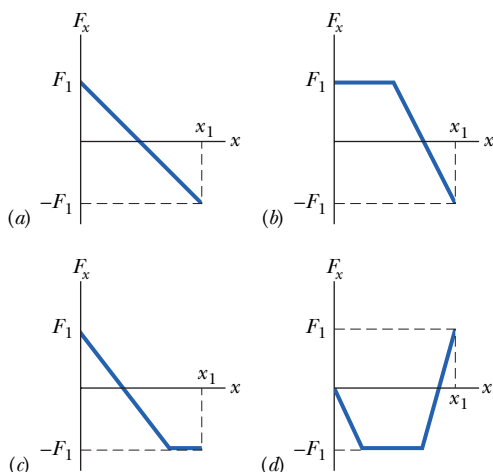


Fig. 7-17 Question 5.

6 Figure 7-18 gives the x component F_x of a force that can act on a particle. If the particle begins at rest at $x = 0$, what is its coordinate when it has (a) its greatest kinetic energy, (b) its greatest speed, and (c) zero speed? (d) What is the particle's direction of travel after it reaches $x = 6$ m?

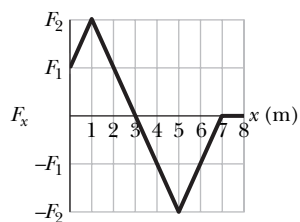


Fig. 7-18 Question 6.

7 In Fig. 7-19, a greased pig has a choice of three frictionless slides along which to slide to the ground. Rank the slides according to how much work the gravitational force does on the pig during the descent, greatest first.

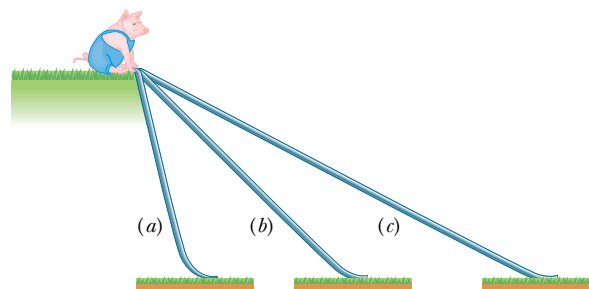


Fig. 7-19 Question 7.

8 Figure 7-20a shows four situations in which a horizontal force acts on the same block, which is initially at rest. The force magnitudes are $F_2 = F_4 = 2F_1 = 2F_3$. The horizontal component v_x of the block's velocity is shown in Fig. 7-20b for the four situations. (a) Which plot in Fig. 7-20b best corresponds to which force in Fig. 7-20a? (b) Which plot in Fig. 7-20c (for kinetic energy K versus time t) best corresponds to which plot in Fig. 7-20b?

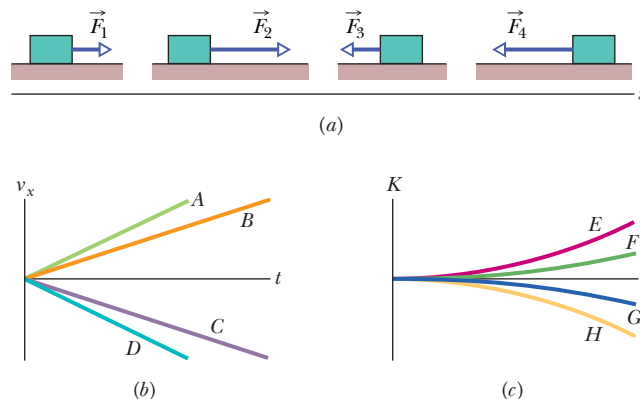


Fig. 7-20 Question 8.

9 Spring A is stiffer than spring B ($k_A > k_B$). The spring force of which spring does more work if the springs are compressed (a) the same distance and (b) by the same applied force?

10 A glob of slime is launched or dropped from the edge of a cliff. Which of the graphs in Fig. 7-21 could possibly show how the kinetic energy of the glob changes during its flight?

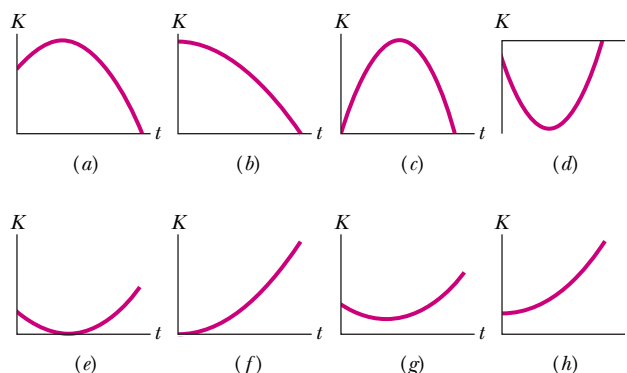


Fig. 7-21 Question 10.

PROBLEMS



Tutoring problem available (at instructor's discretion) in WileyPLUS and WebAssign



Worked-out solution available in Student Solutions Manual

WWW Worked-out solution is at



Number of dots indicates level of problem difficulty

ILW Interactive solution is at

<http://www.wiley.com/college/halliday>



Additional information available in *The Flying Circus of Physics* and at flyingcircusofphysics.com

sec. 7-3 Kinetic Energy

•1 **SSM** A proton (mass $m = 1.67 \times 10^{-27}$ kg) is being accelerated along a straight line at 3.6×10^{15} m/s² in a machine. If the proton has an initial speed of 2.4×10^7 m/s and travels 3.5 cm, what then is (a) its speed and (b) the increase in its kinetic energy?

•2 If a Saturn V rocket with an Apollo spacecraft attached had a combined mass of 2.9×10^5 kg and reached a speed of 11.2 km/s, how much kinetic energy would it then have?

•3 On August 10, 1972, a large meteorite skipped across the atmosphere above the western United States and western Canada, much like a stone skipped across water. The accompanying fireball was so bright that it could be seen in the daytime sky and was brighter than the usual meteorite trail. The meteorite's mass was about 4×10^6 kg; its speed was about 15 km/s. Had it entered the atmosphere vertically, it would have hit Earth's surface with about the same speed. (a) Calculate the meteorite's loss of kinetic energy (in joules) that would have been associated with the vertical impact. (b) Express the energy as a multiple of the explosive energy of 1 megaton of TNT, which is 4.2×10^{15} J. (c) The energy associated with the atomic bomb explosion over Hiroshima was equivalent to 13 kilotons of TNT. To how many Hiroshima bombs would the meteorite impact have been equivalent?

•4 A bead with mass 1.8×10^{-2} kg is moving along a wire in the positive direction of an x axis. Beginning at time $t = 0$, when the bead passes through $x = 0$ with speed 12 m/s, a constant force acts on the bead. Figure 7-22 indicates the bead's position at these four times: $t_0 = 0$, $t_1 = 1.0$ s, $t_2 = 2.0$ s, and $t_3 = 3.0$ s. The bead momentarily stops at $t = 3.0$ s. What is the kinetic energy of the bead at $t = 10$ s?

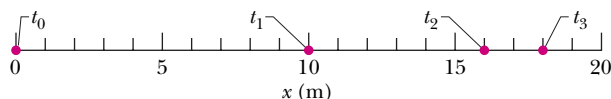


Fig. 7-22 Problem 4.

•5 A father racing his son has half the kinetic energy of the son, who has half the mass of the father. The father speeds up by 1.0 m/s and then has the same kinetic energy as the son. What are the original speeds of (a) the father and (b) the son?

•6 A force \vec{F}_a is applied to a bead as the bead is moved along a straight wire through displacement $+5.0$ cm. The magnitude of \vec{F}_a is set at a certain value, but the angle ϕ between \vec{F}_a and the bead's displacement can be chosen. Figure 7-23 gives the work W done by \vec{F}_a on the bead for a range of ϕ values; $W_0 = 25$ J. How much work is done by \vec{F}_a if ϕ is (a) 64° and (b) 147° ?

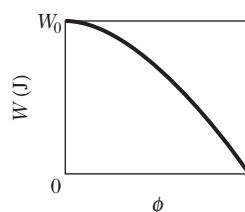


Fig. 7-23 Problem 6.

sec. 7-5 Work and Kinetic Energy

•7 A 3.0 kg body is at rest on a frictionless horizontal air track when a constant horizontal force \vec{F} acting in the positive direction of an x axis along the track is applied to the body. A stroboscopic graph of the position of the body as it slides to the right is shown in Fig. 7-24. The force \vec{F} is applied to the body at $t = 0$, and the graph records the position of the body at 0.50 s intervals. How much work is done on the body by the applied force \vec{F} between $t = 0$ and $t = 2.0$ s?

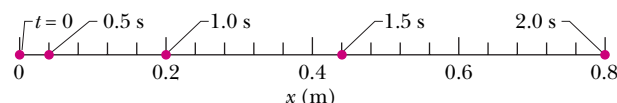


Fig. 7-24 Problem 7.

•8 A ice block floating in a river is pushed through a displacement $\vec{d} = (15 \text{ m})\hat{i} - (12 \text{ m})\hat{j}$ along a straight embankment by rushing water, which exerts a force $\vec{F} = (210 \text{ N})\hat{i} - (150 \text{ N})\hat{j}$ on the block. How much work does the force do on the block during the displacement?

•9 The only force acting on a 2.0 kg canister that is moving in an xy plane has a magnitude of 5.0 N. The canister initially has a velocity of 4.0 m/s in the positive x direction and some time later has a velocity of 6.0 m/s in the positive y direction. How much work is done on the canister by the 5.0 N force during this time?

•10 A coin slides over a frictionless plane and across an xy coordinate system from the origin to a point with xy coordinates (3.0 m, 4.0 m) while a constant force acts on it. The force has magnitude 2.0 N and is directed at a counterclockwise angle of 100° from the positive direction of the x axis. How much work is done by the force on the coin during the displacement?

•11 A 12.0 N force with a fixed orientation does work on a particle as the particle moves through the three-dimensional displacement $\vec{d} = (2.00\hat{i} - 4.00\hat{j} + 3.00\hat{k})$ m. What is the angle between the force and the displacement if the change in the particle's kinetic energy is (a) $+30.0$ J and (b) -30.0 J?

•12 A can of bolts and nuts is pushed 2.00 m along an x axis by a broom along the greasy (frictionless) floor of a car repair shop in a version of shuffleboard. Figure 7-25 gives the work W done on the

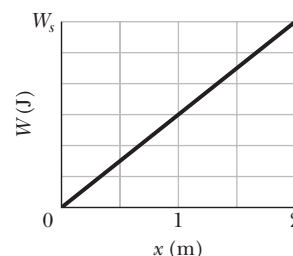



Fig. 7-25 Problem 12.

can by the constant horizontal force from the broom, versus the can's position x . The scale of the figure's vertical axis is set by $W_s = 6.0$ J. (a) What is the magnitude of that force? (b) If the can had an initial kinetic energy of 3.00 J, moving in the positive direction of the x axis, what is its kinetic energy at the end of the 2.00 m?

••13 A luge and its rider, with a total mass of 85 kg, emerge from a downhill track onto a horizontal straight track with an initial speed of 37 m/s. If a force slows them to a stop at a constant rate of 2.0 m/s², (a) what magnitude F is required for the force, (b) what distance d do they travel while slowing, and (c) what work W is done on them by the force? What are (d) F , (e) d , and (f) W if they, instead, slow at 4.0 m/s²?

••14  Figure 7-26 shows an overhead view of three horizontal forces acting on a cargo canister that was initially stationary but now moves across a frictionless floor. The force magnitudes are $F_1 = 3.00$ N, $F_2 = 4.00$ N, and $F_3 = 10.0$ N, and the indicated angles are $\theta_2 = 50.0^\circ$ and $\theta_3 = 35.0^\circ$. What is the net work done on the canister by the three forces during the first 4.00 m of displacement?

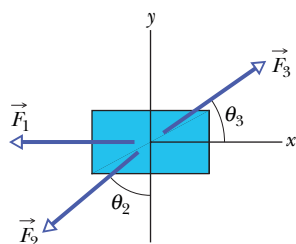



Fig. 7-26 Problem 14.

••15  Figure 7-27 shows three forces applied to a trunk that moves leftward by 3.00 m over a frictionless floor. The force magnitudes are $F_1 = 5.00$ N, $F_2 = 9.00$ N, and $F_3 = 3.00$ N, and the indicated angle is $\theta = 60.0^\circ$. During the displacement, (a) what is the net work done on the trunk by the three forces and (b) does the kinetic energy of the trunk increase or decrease?

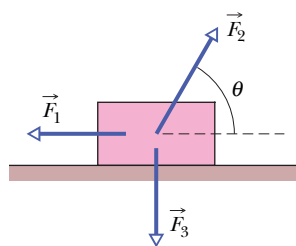



Fig. 7-27 Problem 15.

••16  An 8.0 kg object is moving in the positive direction of an x axis. When it passes through $x = 0$, a constant force directed along the axis begins to act on it. Figure 7-28 gives its kinetic en-

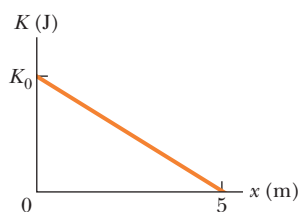






Fig. 7-28 Problem 16.

ergy K versus position x as it moves from $x = 0$ to $x = 5.0$ m; $K_0 = 30.0$ J. The force continues to act. What is v when the object moves back through $x = -3.0$ m?

sec. 7-6 Work Done by the Gravitational Force

•17   A helicopter lifts a 72 kg astronaut 15 m vertically from the ocean by means of a cable. The acceleration of the astronaut is $g/10$. How much work is done on the astronaut by (a) the force from the helicopter and (b) the gravitational force on her? Just before she reaches the helicopter, what are her (c) kinetic energy and (d) speed?

•18  (a) In 1975 the roof of Montreal's Velodrome, with a weight of 360 kN, was lifted by 10 cm so that it could be centered. How much work was done on the roof by the forces making the lift? (b) In 1960 a Tampa, Florida, mother reportedly raised one end of a car that had fallen onto her son when a jack failed. If her panic lift effectively raised 4000 N (about $\frac{1}{4}$ of the car's weight) by 5.0 cm, how much work did her force do on the car?

••19  In Fig. 7-29, a block of ice slides down a frictionless ramp at angle $\theta = 50^\circ$ while an ice worker pulls on the block (via a rope) with a force \vec{F}_r that has a magnitude of 50 N and is directed up the ramp. As the block slides through distance $d = 0.50$ m along the ramp, its kinetic energy increases by 80 J. How much greater would its kinetic energy have been if the rope had not been attached to the block?

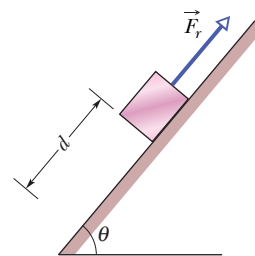


Fig. 7-29 Problem 19.

••20 A block is sent up a frictionless ramp along which an x axis extends upward. Figure 7-30 gives the kinetic energy of the block as a function of position x ; the scale of the figure's vertical axis is set by $K_s = 40.0$ J. If the block's initial speed is 4.00 m/s, what is the normal force on the block?

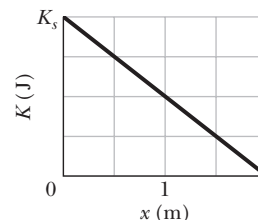



Fig. 7-30 Problem 20.

••21  A cord is used to vertically lower an initially stationary block of mass M at a constant downward acceleration of $g/4$. When the block has fallen a distance d , find (a) the work done by the cord's force on the block, (b) the work done by the gravitational force on the block, (c) the kinetic energy of the block, and (d) the speed of the block.

••22 A cave rescue team lifts an injured spelunker directly upward and out of a sinkhole by means of a motor-driven cable. The

lift is performed in three stages, each requiring a vertical distance of 10.0 m: (a) the initially stationary spelunker is accelerated to a speed of 5.00 m/s; (b) he is then lifted at the constant speed of 5.00 m/s; (c) finally he is decelerated to zero speed. How much work is done on the 80.0 kg rescuee by the force lifting him during each stage?

••23 In Fig. 7-31, a constant force \vec{F}_a of magnitude 82.0 N is applied to a 3.00 kg shoe box at angle $\phi = 53.0^\circ$, causing the box to move up a frictionless ramp at constant speed. How much work is done on the box by \vec{F}_a when the box has moved through vertical distance $h = 0.150$ m?

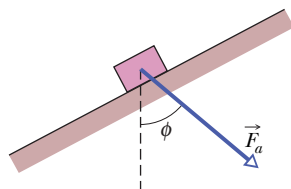


Fig. 7-31 Problem 23.

••24 GO In Fig. 7-32, a horizontal force \vec{F}_a of magnitude 20.0 N is applied to a 3.00 kg psychology book as the book slides a distance $d = 0.500$ m up a frictionless ramp at angle $\theta = 30.0^\circ$. (a) During the displacement, what is the net work done on the book by \vec{F}_a , the gravitational force on the book, and the normal force on the book? (b) If the book has zero kinetic energy at the start of the displacement, what is its speed at the end of the displacement?

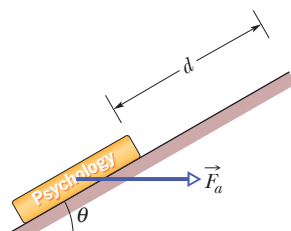


Fig. 7-32 Problem 24.

•••25 GO In Fig. 7-33, a 0.250 kg block of cheese lies on the floor of a 900 kg elevator cab that is being pulled upward by a cable through distance $d_1 = 2.40$ m and then through distance $d_2 = 10.5$ m. (a) Through d_1 , if the normal force on the block from the floor has constant magnitude $F_N = 3.00$ N, how much work is done on the cab by the force from the cable? (b) Through d_2 , if the work done on the cab by the (constant) force from the cable is 92.61 kJ, what is the magnitude of F_N ?

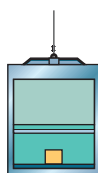


Fig. 7-33 Problem 25.

sec. 7-7 Work Done by a Spring Force

•26 In Fig. 7-9, we must apply a force of magnitude 80 N to hold the block stationary at $x = -2.0$ cm. From that position, we then slowly move the block so that our force does +4.0 J of work on the spring-block system; the block is then again stationary. What is the block's position? (*Hint:* There are two answers.)

•27 A spring and block are in the arrangement of Fig. 7-9. When the block is pulled out to $x = +4.0$ cm, we must apply a force of magnitude 360 N to hold it there. We pull the block to $x = 11$ cm and then release

it. How much work does the spring do on the block as the block moves from $x_i = +5.0$ cm to (a) $x = +3.0$ cm, (b) $x = -3.0$ cm, (c) $x = -5.0$ cm, and (d) $x = -9.0$ cm?

•28 During spring semester at MIT, residents of the parallel buildings of the East Campus dorms battle one another with large catapults that are made with surgical hose mounted on a window frame. A balloon filled with dyed water is placed in a pouch attached to the hose, which is then stretched through the width of the room. Assume that the stretching of the hose obeys Hooke's law with a spring constant of 100 N/m. If the hose is stretched by 5.00 m and then released, how much work does the force from the hose do on the balloon in the pouch by the time the hose reaches its relaxed length?

•29 In the arrangement of Fig. 7-9, we gradually pull the block from $x = 0$ to $x = +3.0$ cm, where it is stationary. Figure 7-34 gives the work that our force does on the block. The scale of the figure's vertical axis is set by $W_s = 1.0$ J. We then pull the block out to $x = +5.0$ cm and release it from rest. How much work does the spring do on the block when the block moves from $x_i = +5.0$ cm to (a) $x = +4.0$ cm, (b) $x = -2.0$ cm, and (c) $x = -5.0$ cm?

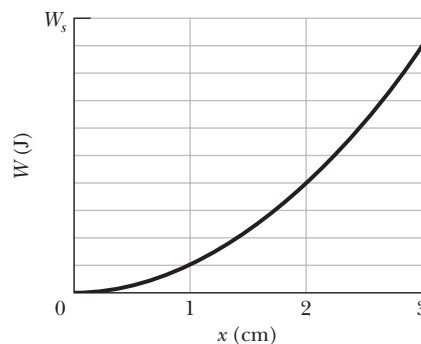


Fig. 7-34 Problem 29.

•30 In Fig. 7-9a, a block of mass m lies on a horizontal frictionless surface and is attached to one end of a horizontal spring (spring constant k) whose other end is fixed. The block is initially at rest at the position where the spring is unstretched ($x = 0$) when a constant horizontal force \vec{F} in the positive direction of the x axis is applied to it. A plot of the resulting kinetic energy of the block versus its position x is shown in Fig. 7-35. The scale of the figure's vertical axis is set by $K_s = 4.0$ J. (a) What is the magnitude of \vec{F} ? (b) What is the value of k ?

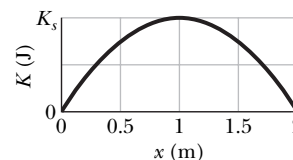


Fig. 7-35 Problem 30.

•31 SSM WWW The only force acting on a 2.0 kg body as it moves along a positive x axis has an x component $F_x = -6x$ N, with x in meters. The velocity at $x = 3.0$ m is 8.0 m/s. (a) What is the velocity of the body at $x = 4.0$ m? (b) At what positive value of x will the body have a velocity of 5.0 m/s?

••32 Figure 7-36 gives spring force F_x versus position x for the spring–block arrangement of Fig. 7-9. The scale is set by $F_s = 160.0$ N. We release the block at $x = 12$ cm. How much work does the spring do on the block when the block moves from $x_i = +8.0$ cm to (a) $x = +5.0$ cm, (b) $x = -5.0$ cm, (c) $x = -8.0$ cm, and (d) $x = -10.0$ cm?

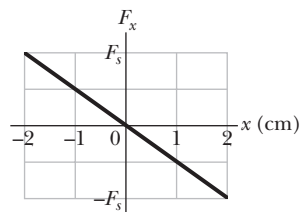


Fig. 7-36 Problem 32.

••33 The block in Fig. 7-9a lies on a horizontal frictionless surface, and the spring constant is 50 N/m. Initially, the spring is at its relaxed length and the block is stationary at position $x = 0$. Then an applied force with a constant magnitude of 3.0 N pulls the block in the positive direction of the x axis, stretching the spring until the block stops. When that stopping point is reached, what are (a) the position of the block, (b) the work that has been done on the block by the applied force, and (c) the work that has been done on the block by the spring force? During the block's displacement, what are (d) the block's position when its kinetic energy is maximum and (e) the value of that maximum kinetic energy?

sec. 7-8 Work Done by a General Variable Force

•34 ILW A 10 kg brick moves along an x axis. Its acceleration as a function of its position is shown in Fig. 7-37. The scale of the figure's vertical axis is set by $a_s = 20.0$ m/s². What is the net work performed on the brick by the force causing the acceleration as the brick moves from $x = 0$ to $x = 8.0$ m?

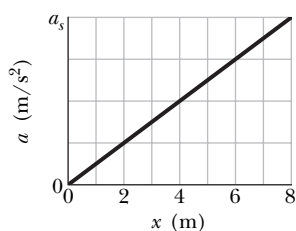


Fig. 7-37 Problem 34.

•35 SSM WWW The force on a particle is directed along an x axis and given by $F = F_0(x/x_0 - 1)$. Find the work done by the force in moving the particle from $x = 0$ to $x = 2x_0$ by (a) plotting $F(x)$ and measuring the work from the graph and (b) integrating $F(x)$.

•36 A 5.0 kg block moves in a straight line on a horizontal frictionless surface under the influence of a force that varies with position as shown in Fig. 7-38.

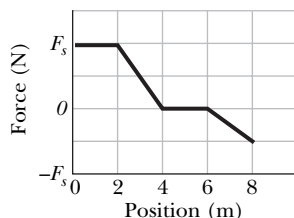


Fig. 7-38 Problem 36.

The scale of the figure's vertical axis is set by $F_s = 10.0$ N. How much work is done by the force as the block moves from the origin to $x = 8.0$ m?

••37 Figure 7-39 gives the acceleration of a 2.00 kg particle as an applied force \vec{F}_a moves it from rest along an x axis from $x = 0$ to $x = 9.0$ m. The scale of the figure's vertical axis is set by $a_s = 6.0$ m/s². How much work has the force done on the particle when the particle reaches (a) $x = 4.0$ m, (b) $x = 7.0$ m, and (c) $x = 9.0$ m? What is the particle's speed and direction of travel when it reaches (d) $x = 4.0$ m, (e) $x = 7.0$ m, and (f) $x = 9.0$ m?

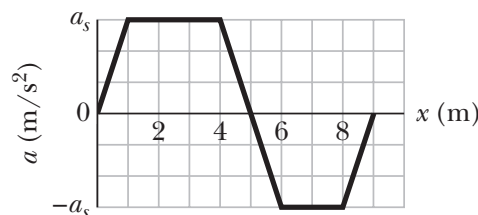


Fig. 7-39 Problem 37.

••38 A 1.5 kg block is initially at rest on a horizontal frictionless surface when a horizontal force along an x axis is applied to the block. The force is given by $\vec{F}(x) = (2.5 - x^2)\hat{i}$ N, where x is in meters and the initial position of the block is $x = 0$. (a) What is the kinetic energy of the block as it passes through $x = 2.0$ m? (b) What is the maximum kinetic energy of the block between $x = 0$ and $x = 2.0$ m?

••39 GO A force $\vec{F} = (cx - 3.00x^2)\hat{i}$ acts on a particle as the particle moves along an x axis, with \vec{F} in newtons, x in meters, and c a constant. At $x = 0$, the particle's kinetic energy is 20.0 J; at $x = 3.00$ m, it is 11.0 J. Find c .

••40 A can of sardines is made to move along an x axis from $x = 0.25$ m to $x = 1.25$ m by a force with a magnitude given by $F = \exp(-4x^2)$, with x in meters and F in newtons. (Here \exp is the exponential function.) How much work is done on the can by the force?

••41 A single force acts on a 3.0 kg particle-like object whose position is given by $x = 3.0t - 4.0t^2 + 1.0t^3$, with x in meters and t in seconds. Find the work done on the object by the force from $t = 0$ to $t = 4.0$ s.

••42 Figure 7-40 shows a cord attached to a cart that can slide along a frictionless horizontal rail aligned along an x axis. The left end of the cord is pulled over a pulley, of negligible mass and friction and at cord height $h = 1.20$ m, so the cart slides from $x_1 = 3.00$ m to $x_2 = 1.00$ m. During the move, the tension in the cord is a constant 25.0 N. What is the change in the kinetic energy of the cart during the move?

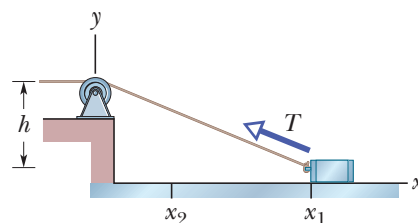


Fig. 7-40 Problem 42.

sec. 7-9 Power

•43 **SSM** A force of 5.0 N acts on a 15 kg body initially at rest. Compute the work done by the force in (a) the first, (b) the second, and (c) the third seconds and (d) the instantaneous power due to the force at the end of the third second.

•44 A skier is pulled by a towrope up a frictionless ski slope that makes an angle of 12° with the horizontal. The rope moves parallel to the slope with a constant speed of 1.0 m/s. The force of the rope does 900 J of work on the skier as the skier moves a distance of 8.0 m up the incline. (a) If the rope moved with a constant speed of 2.0 m/s, how much work would the force of the rope do on the skier as the skier moved a distance of 8.0 m up the incline? At what rate is the force of the rope doing work on the skier when the rope moves with a speed of (b) 1.0 m/s and (c) 2.0 m/s?

•45 **SSM ILW** A 100 kg block is pulled at a constant speed of 5.0 m/s across a horizontal floor by an applied force of 122 N directed 37° above the horizontal. What is the rate at which the force does work on the block?

•46 The loaded cab of an elevator has a mass of 3.0×10^3 kg and moves 210 m up the shaft in 23 s at constant speed. At what average rate does the force from the cable do work on the cab?

•47 A machine carries a 4.0 kg package from an initial position of $\vec{d}_i = (0.50 \text{ m})\hat{i} + (0.75 \text{ m})\hat{j} + (0.20 \text{ m})\hat{k}$ at $t = 0$ to a final position of $\vec{d}_f = (7.50 \text{ m})\hat{i} + (12.0 \text{ m})\hat{j} + (7.20 \text{ m})\hat{k}$ at $t = 12$ s. The constant force applied by the machine on the package is $\vec{F} = (2.00 \text{ N})\hat{i} + (4.00 \text{ N})\hat{j} + (6.00 \text{ N})\hat{k}$. For that displacement, find (a) the work done on the package by the machine's force and (b) the average power of the machine's force on the package.

•48 A 0.30 kg ladle sliding on a horizontal frictionless surface is attached to one end of a horizontal spring ($k = 500 \text{ N/m}$) whose other end is fixed. The ladle has a kinetic energy of 10 J as it passes through its equilibrium position (the point at which the spring force is zero). (a) At what rate is the spring doing work on the ladle as the ladle passes through its equilibrium position? (b) At what rate is the spring doing work on the ladle when the spring is compressed 0.10 m and the ladle is moving away from the equilibrium position?

•49 **SSM** A fully loaded, slow-moving freight elevator has a cab with a total mass of 1200 kg, which is required to travel upward 54 m in 3.0 min, starting and ending at rest. The elevator's counterweight has a mass of only 950 kg, and so the elevator motor must help. What average power is required of the force the motor exerts on the cab via the cable?

•50 (a) At a certain instant, a particle-like object is acted on by a force $\vec{F} = (4.0 \text{ N})\hat{i} - (2.0 \text{ N})\hat{j} + (9.0 \text{ N})\hat{k}$ while the object's velocity is $\vec{v} = -(2.0 \text{ m/s})\hat{i} + (4.0 \text{ m/s})\hat{k}$. What is the instantaneous rate at which the force does work on the object? (b) At some other time, the velocity consists of only a y component. If the force is unchanged and the instantaneous power is -12 W , what is the velocity of the object?

•51 A force $\vec{F} = (3.00 \text{ N})\hat{i} + (7.00 \text{ N})\hat{j} + (7.00 \text{ N})\hat{k}$ acts on a 2.00 kg mobile object that moves from an initial position of $\vec{d}_i = (3.00 \text{ m})\hat{i} - (2.00 \text{ m})\hat{j} + (5.00 \text{ m})\hat{k}$ to a final position of $\vec{d}_f = -(5.00 \text{ m})\hat{i} + (4.00 \text{ m})\hat{j} + (7.00 \text{ m})\hat{k}$ in 4.00 s. Find (a) the work done on the object by the force in the 4.00 s interval, (b) the average power due to the force during that interval, and (c) the angle between vectors \vec{d}_i and \vec{d}_f .

•52 A funny car accelerates from rest through a measured track distance in time T with the engine operating at a constant power P . If the track crew can increase the engine power by a differential amount dP , what is the change in the time required for the run?

Additional Problems

53 Figure 7-41 shows a cold package of hot dogs sliding rightward across a frictionless floor through a distance $d = 20.0 \text{ cm}$ while three forces act on the package. Two of them are horizontal and have the magnitudes $F_1 = 5.00 \text{ N}$ and $F_2 = 1.00 \text{ N}$; the third is angled down by $\theta = 60.0^\circ$ and has the magnitude $F_3 = 4.00 \text{ N}$. (a) For the 20.0 cm displacement, what is the *net* work done on the package by the three applied forces, the gravitational force on the package, and the normal force on the package? (b) If the package has a mass of 2.0 kg and an initial kinetic energy of 0, what is its speed at the end of the displacement?

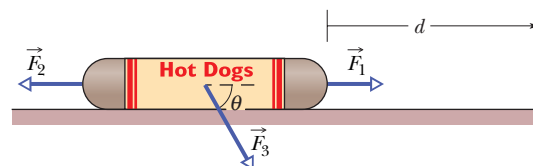


Fig. 7-41 Problem 53.

54 The only force acting on a 2.0 kg body as the body moves along an x axis varies as shown in Fig. 7-42. The scale of the figure's vertical axis is set by $F_s = 4.0 \text{ N}$. The velocity of the body at $x = 0$ is 4.0 m/s. (a) What is the kinetic energy of the body at $x = 3.0 \text{ m}$? (b) At what value of x will the body have a kinetic energy of 8.0 J? (c) What is the maximum kinetic energy of the body between $x = 0$ and $x = 5.0 \text{ m}$?

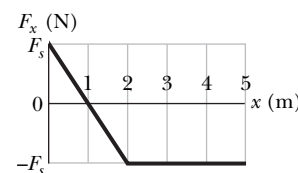


Fig. 7-42 Problem 54.

55 **SSM** A horse pulls a cart with a force of 40 lb at an angle of 30° above the horizontal and moves along at a speed of 6.0 mi/h. (a) How much work does the force do in 10 min? (b) What is the average power (in horsepower) of the force?

56 An initially stationary 2.0 kg object accelerates horizontally and uniformly to a speed of 10 m/s in 3.0 s. (a) In that 3.0 s interval, how much work is done on the object by the force accelerating it? What is the instantaneous power due to that force (b) at the end of the interval and (c) at the end of the first half of the interval?

57 A 230 kg crate hangs from the end of a rope of length $L = 12.0 \text{ m}$. You push horizontally on the crate with a varying force \vec{F} to move it distance $d = 4.00 \text{ m}$ to the side (Fig. 7-43). (a) What is the magnitude of \vec{F} when the crate is in this final position? During the crate's displacement, what are (b) the total work done on it, (c) the work done by the gravitational force on the crate, and (d) the work done by the pull on the crate from the rope? (e) Knowing that the crate is motionless before and after its displacement, use the answers to (b), (c), and (d) to find the work your

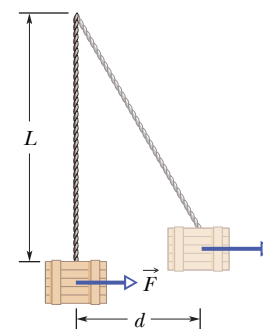


Fig. 7-43 Problem 57.

force \vec{F} does on the crate. (f) Why is the work of your force not equal to the product of the horizontal displacement and the answer to (a)?

58 To pull a 50 kg crate across a horizontal frictionless floor, a worker applies a force of 210 N, directed 20° above the horizontal. As the crate moves 3.0 m, what work is done on the crate by (a) the worker's force, (b) the gravitational force on the crate, and (c) the normal force on the crate from the floor? (d) What is the total work done on the crate?

59 An explosion at ground level leaves a crater with a diameter that is proportional to the energy of the explosion raised to the $\frac{1}{3}$ power; an explosion of 1 megaton of TNT leaves a crater with a 1 km diameter. Below Lake Huron in Michigan there appears to be an ancient impact crater with a 50 km diameter. What was the kinetic energy associated with that impact, in terms of (a) megatons of TNT (1 megaton yields 4.2×10^{15} J) and (b) Hiroshima bomb equivalents (13 kilotons of TNT each)? (Ancient meteorite or comet impacts may have significantly altered Earth's climate and contributed to the extinction of the dinosaurs and other life-forms.)

60 A frightened child is restrained by her mother as the child slides down a frictionless playground slide. If the force on the child from the mother is 100 N up the slide, the child's kinetic energy increases by 30 J as she moves down the slide a distance of 1.8 m. (a) How much work is done on the child by the gravitational force during the 1.8 m descent? (b) If the child is not restrained by her mother, how much will the child's kinetic energy increase as she comes down the slide that same distance of 1.8 m?

61 How much work is done by a force $\vec{F} = (2x \text{ N})\hat{i} + (3 \text{ N})\hat{j}$, with x in meters, that moves a particle from a position $\vec{r}_i = (2 \text{ m})\hat{i} + (3 \text{ m})\hat{j}$ to a position $\vec{r}_f = -(4 \text{ m})\hat{i} - (3 \text{ m})\hat{j}$?

62 A 250 g block is dropped onto a relaxed vertical spring that has a spring constant of $k = 2.5$ N/cm (Fig. 7-44). The block becomes attached to the spring and compresses the spring 12 cm before momentarily stopping. While the spring is being compressed, what work is done on the block by (a) the gravitational force on it and (b) the spring force? (c) What is the speed of the block just before it hits the spring? (Assume that friction is negligible.) (d) If the speed at impact is doubled, what is the maximum compression of the spring?

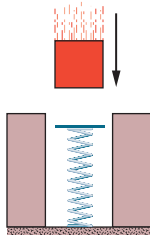


Fig. 7-44 Problem 62.

63 SSM To push a 25.0 kg crate up a frictionless incline, angled at 25.0° to the horizontal, a worker exerts a force of 209 N parallel to the incline. As the crate slides 1.50 m, how much work is done on the crate by (a) the worker's applied force, (b) the gravitational force on the crate, and (c) the normal force exerted by the incline on the crate? (d) What is the total work done on the crate?

64 Boxes are transported from one location to another in a warehouse by means of a conveyor belt that moves with a constant speed of 0.50 m/s. At a certain location the conveyor belt moves for 2.0 m up an incline that makes an angle of 10° with the horizontal, then for 2.0 m horizontally, and finally for 2.0 m down an incline that makes an angle of 10° with the horizontal. Assume that a 2.0 kg box rides on the belt without slipping. At what rate is the force of the conveyor belt doing work on the box as the box moves (a) up the 10° incline, (b) horizontally, and (c) down the 10° incline?

65 In Fig. 7-45, a cord runs around two massless, frictionless pulleys. A canister with mass $m = 20$ kg hangs from one pulley, and you exert a force \vec{F} on the free end of the cord. (a) What must be the magnitude of \vec{F} if you are to lift the canister at a constant speed? (b) To lift the canister by 2.0 cm, how far must you pull the free end of the cord? During that lift, what is the work done on the canister by (c) your force (via the cord) and (d) the gravitational force? (Hint: When a cord loops around a pulley as shown, it pulls on the pulley with a net force that is twice the tension in the cord.)

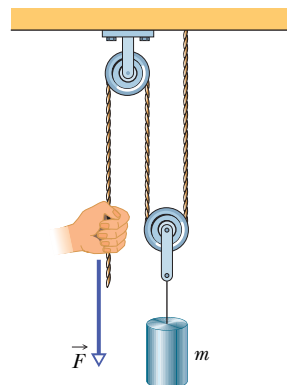


Fig. 7-45 Problem 65.

66 If a car of mass 1200 kg is moving along a highway at 120 km/h, what is the car's kinetic energy as determined by someone standing alongside the highway?

67 SSM A spring with a pointer attached is hanging next to a scale marked in millimeters. Three different packages are hung from the spring, in turn, as shown in Fig. 7-46. (a) Which mark on the scale will the pointer indicate when no package is hung from the spring? (b) What is the weight W of the third package?

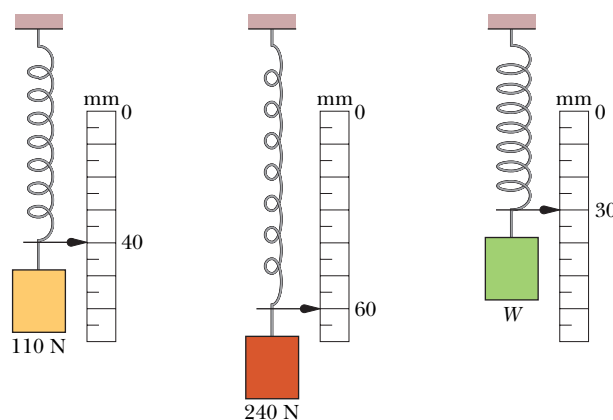


Fig. 7-46 Problem 67.

68 An iceboat is at rest on a frictionless frozen lake when a sudden wind exerts a constant force of 200 N, toward the east, on the boat. Due to the angle of the sail, the wind causes the boat to slide in a straight line for a distance of 8.0 m in a direction 20° north of east. What is the kinetic energy of the iceboat at the end of that 8.0 m?

69 If a ski lift raises 100 passengers averaging 660 N in weight to a height of 150 m in 60.0 s, at constant speed, what average power is required of the force making the lift?

VIEW ALL

CHAPTER 7: WORK, ENERGY, AND POWER

70 A force $\vec{F} = (4.0 \text{ N})\hat{i} + c\hat{j}$ acts on a particle as the particle goes through displacement $\vec{d} = (3.0 \text{ m})\hat{i} - (2.0 \text{ m})\hat{j}$. (Other forces also act on the particle.) What is c if the work done on the particle by force \vec{F} is (a) 0, (b) 17 J, and (c) -18 J?

71 A constant force of magnitude 10 N makes an angle of 150° (measured counterclockwise) with the positive x direction as it acts on a 2.0 kg object moving in an xy plane. How much work is done on the object by the force as the object moves from the origin to the point having position vector $(2.0 \text{ m})\hat{i} - (4.0 \text{ m})\hat{j}$?

72 In Fig. 7-47a, a 2.0 N force is applied to a 4.0 kg block at a downward angle θ as the block moves rightward through 1.0 m across a frictionless floor. Find an expression for the speed v_f of the block at the end of that distance if the block's initial velocity is (a) 0 and (b) 1.0 m/s to the right. (c) The situation in Fig. 7-47b is similar in that the block is initially moving at 1.0 m/s to the right, but now the 2.0 N force is directed downward to the left. Find an expression for the speed v_f of the block at the end of the 1.0 m distance. (d) Graph all three expressions for v_f versus downward angle θ for $\theta = 0^\circ$ to $\theta = 90^\circ$. Interpret the graphs.

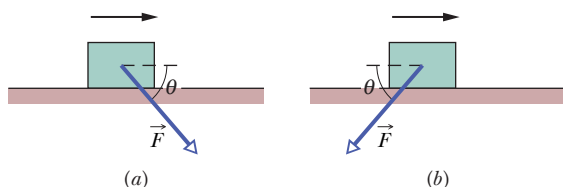


Fig. 7-47 Problem 72.

73 A force \vec{F} in the positive direction of an x axis acts on an object moving along the axis. If the magnitude of the force is $F = 10e^{-x/2.0} \text{ N}$, with x in meters, find the work done by \vec{F} as the object moves from $x = 0$ to $x = 2.0 \text{ m}$ by (a) plotting $F(x)$ and estimating the area under the curve and (b) integrating to find the work analytically.

74 A particle moves along a straight path through displacement $\vec{d} = (8 \text{ m})\hat{i} + c\hat{j}$ while force $\vec{F} = (2 \text{ N})\hat{i} - (4 \text{ N})\hat{j}$ acts on it. (Other forces also act on the particle.) What is the value of c if the work done by \vec{F} on the particle is (a) zero, (b) positive, and (c) negative?

75 SSM An elevator cab has a mass of 4500 kg and can carry a maximum load of 1800 kg. If the cab is moving upward at full load at 3.80 m/s, what power is required of the force moving the cab to maintain that speed?

76 A 45 kg block of ice slides down a frictionless incline 1.5 m long and 0.91 m high. A worker pushes up against the ice, parallel to the incline, so that the block slides down at constant speed. (a) Find the magnitude of the worker's force. How much work is done on the block by (b) the worker's force, (c) the gravitational force on the block, (d) the normal force on the block from the surface of the incline, and (e) the net force on the block?

77 As a particle moves along an x axis, a force in the positive direction of the axis acts on it. Figure 7-48 shows the magnitude F of the

force versus position x of the particle. The curve is given by $F = a/x^2$, with $a = 9.0 \text{ N} \cdot \text{m}^2$. Find the work done on the particle by the force as the particle moves from $x = 1.0 \text{ m}$ to $x = 3.0 \text{ m}$ by (a) estimating the work from the graph and (b) integrating the force function.

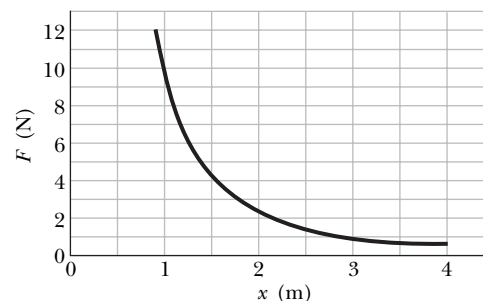


Fig. 7-48 Problem 77.

78 A CD case slides along a floor in the positive direction of an x axis while an applied force \vec{F}_a acts on the case. The force is directed along the x axis and has the x component $F_{ax} = 9x - 3x^2$, with x in meters and F_{ax} in newtons. The case starts at rest at the position $x = 0$, and it moves until it is again at rest. (a) Plot the work \vec{F}_a does on the case as a function of x . (b) At what position is the work maximum, and (c) what is that maximum value? (d) At what position has the work decreased to zero? (e) At what position is the case again at rest?

79 SSM A 2.0 kg lunchbox is sent sliding over a frictionless surface, in the positive direction of an x axis along the surface. Beginning at time $t = 0$, a steady wind pushes on the lunchbox in the negative direction of the x axis. Figure 7-49 shows the position x of the lunchbox as a function of time t as the wind pushes on the lunchbox. From the graph, estimate the kinetic energy of the lunchbox at (a) $t = 1.0 \text{ s}$ and (b) $t = 5.0 \text{ s}$. (c) How much work does the force from the wind do on the lunchbox from $t = 1.0 \text{ s}$ to $t = 5.0 \text{ s}$?

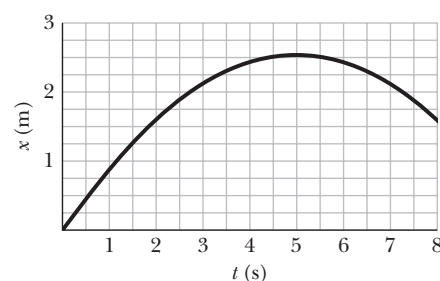


Fig. 7-49 Problem 79.

80 Numerical integration. A breadbox is made to move along an x axis from $x = 0.15 \text{ m}$ to $x = 1.20 \text{ m}$ by a force with a magnitude given by $F = \exp(-2x^2)$, with x in meters and F in newtons. (Here \exp is the exponential function.) How much work is done on the breadbox by the force?

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ASK QUESTIONS GET ANSWERS,

how many times you want.

POTENTIAL ENERGY AND CONSERVATION OF ENERGY

8-1 WHAT IS PHYSICS?

One job of physics is to identify the different types of energy in the world, especially those that are of common importance. One general type of energy is **potential energy** U . Technically, potential energy is energy that can be associated with the configuration (arrangement) of a system of objects that exert forces on one another.

This is a pretty formal definition of something that is actually familiar to you. An example might help better than the definition: A bungee-cord jumper plunges from a staging platform (Fig. 8-1). The system of objects consists of Earth and the jumper. The force between the objects is the gravitational force. The configuration of the system changes (the separation between the jumper and Earth decreases—that is, of course, the thrill of the jump). We can account for the jumper's motion and increase in kinetic energy by defining a **gravitational potential energy** U . This

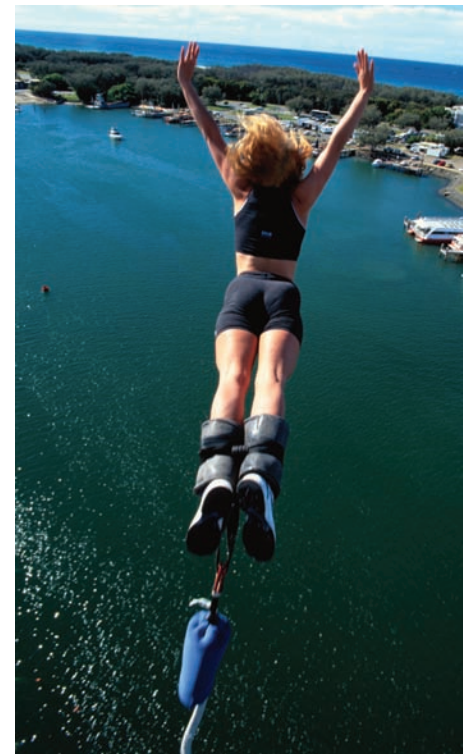


Fig. 8-1 The kinetic energy of a bungee-cord jumper increases during the free fall, and then the cord begins to stretch, slowing the jumper. (KOFUJIWARA/amana images/Getty Images News and Sport Services)

is the energy associated with the state of separation between two objects that attract each other by the gravitational force, here the jumper and Earth.

When the jumper begins to stretch the bungee cord near the end of the plunge, the system of objects consists of the cord and the jumper. The force between the objects is an elastic (spring-like) force. The configuration of the system changes (the cord stretches). We can account for the jumper's decrease in kinetic energy and the cord's increase in length by defining an **elastic potential energy** U . This is the energy associated with the state of compression or extension of an elastic object, here the bungee cord.

Physics determines how the potential energy of a system can be calculated so that energy might be stored or put to use. For example, before any particular bungee-cord jumper takes the plunge, someone (probably a mechanical engineer) must determine the correct cord to be used by calculating the gravitational and elastic potential energies that can be expected. Then the jump is only thrilling and not fatal.

8-2 Work and Potential Energy

In Chapter 7 we discussed the relation between work and a change in kinetic energy. Here we discuss the relation between work and a change in potential energy.

Let us throw a tomato upward (Fig. 8-2). We already know that as the tomato rises, the work W_g done on the tomato by the gravitational force is negative because the force transfers energy *from* the kinetic energy of the tomato. We can now finish the story by saying that this energy is transferred by the gravitational force *to* the gravitational potential energy of the tomato–Earth system.

The tomato slows, stops, and then begins to fall back down because of the gravitational force. During the fall, the transfer is reversed: The work W_g done on the tomato by the gravitational force is now positive—that force transfers energy *from* the gravitational potential energy of the tomato–Earth system *to* the kinetic energy of the tomato.

For either rise or fall, the change ΔU in gravitational potential energy is defined as being equal to the negative of the work done on the tomato by the gravitational force. Using the general symbol W for work, we write this as

$$\Delta U = -W. \quad (8-1)$$

This equation also applies to a block–spring system, as in Fig. 8-3. If we abruptly shove the block to send it moving rightward, the spring force acts leftward and thus does negative work on the block, transferring energy from the kinetic energy of the block to the elastic potential energy of the spring–block system. The block slows and eventually stops, and then begins to move leftward because the spring force is still leftward. The transfer of energy is then reversed—it is from potential energy of the spring–block system to kinetic energy of the block.

Conservative and Nonconservative Forces

Let us list the key elements of the two situations we just discussed:

1. The *system* consists of two or more objects.
2. A *force* acts between a particle-like object (tomato or block) in the system and the rest of the system.
3. When the system configuration changes, the force does *work* (call it W_1) on the particle-like object, transferring energy between the kinetic energy K of the object and some other type of energy of the system.

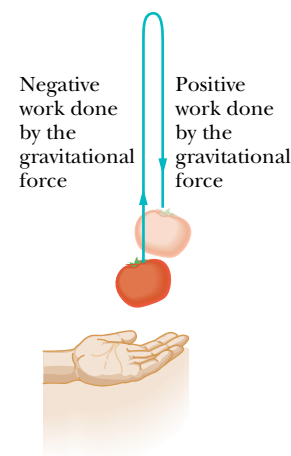


Fig. 8-2 A tomato is thrown upward. As it rises, the gravitational force does negative work on it, decreasing its kinetic energy. As the tomato descends, the gravitational force does positive work on it, increasing its kinetic energy.

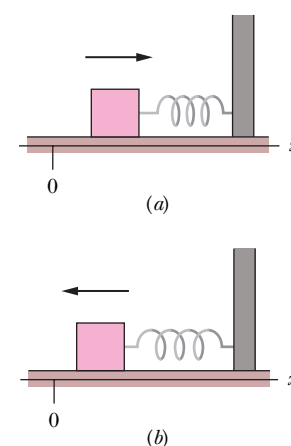


Fig. 8-3 A block, attached to a spring and initially at rest at $x = 0$, is set in motion toward the right. (a) As the block moves rightward (as indicated by the arrow), the spring force does negative work on it. (b) Then, as the block moves back toward $x = 0$, the spring force does positive work on it.

4. When the configuration change is reversed, the force reverses the energy transfer, doing work W_2 in the process.

In a situation in which $W_1 = -W_2$ is always true, the other type of energy is a potential energy and the force is said to be a **conservative force**. As you might suspect, the gravitational force and the spring force are both conservative (since otherwise we could not have spoken of gravitational potential energy and elastic potential energy, as we did previously).

A force that is not conservative is called a **nonconservative force**. The kinetic frictional force and drag force are nonconservative. For an example, let us send a block sliding across a floor that is not frictionless. During the sliding, a kinetic frictional force from the floor slows the block by transferring energy from its kinetic energy to a type of energy called *thermal energy* (which has to do with the random motions of atoms and molecules). We know from experiment that this energy transfer cannot be reversed (thermal energy cannot be transferred back to kinetic energy of the block by the kinetic frictional force). Thus, although we have a system (made up of the block and the floor), a force that acts between parts of the system, and a transfer of energy by the force, the force is not conservative. Therefore, thermal energy is not a potential energy.

When only conservative forces act on a particle-like object, we can greatly simplify otherwise difficult problems involving motion of the object. The next section, in which we develop a test for identifying conservative forces, provides one means for simplifying such problems.

8-3 Path Independence of Conservative Forces

The primary test for determining whether a force is conservative or nonconservative is this: Let the force act on a particle that moves along any *closed path*, beginning at some initial position and eventually returning to that position (so that the particle makes a *round trip* beginning and ending at the initial position). The force is conservative only if the total energy it transfers to and from the particle during the round trip along this and any other closed path is zero. In other words:

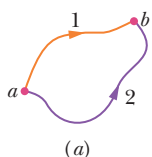
The net work done by a conservative force on a particle moving around any closed path is zero.

We know from experiment that the gravitational force passes this *closed-path test*. An example is the tossed tomato of Fig. 8-2. The tomato leaves the launch point with speed v_0 and kinetic energy $\frac{1}{2}mv_0^2$. The gravitational force acting on the tomato slows it, stops it, and then causes it to fall back down. When the tomato returns to the launch point, it again has speed v_0 and kinetic energy $\frac{1}{2}mv_0^2$. Thus, the gravitational force transfers as much energy *from* the tomato during the ascent as it transfers *to* the tomato during the descent back to the launch point. The net work done on the tomato by the gravitational force during the round trip is zero.

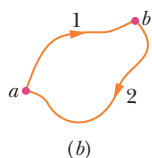
An important result of the closed-path test is that:

The work done by a conservative force on a particle moving between two points does not depend on the path taken by the particle.

For example, suppose that a particle moves from point a to point b in Fig. 8-4a along either path 1 or path 2. If only a conservative force acts on the particle, then the work done on the particle is the same along the two paths. In symbols, we can



The force is conservative.
Any choice of path between
the points gives the same
amount of work.



And a round trip gives
a total work of zero.

Fig. 8-4 (a) As a conservative force acts on it, a particle can move from point a to point b along either path 1 or path 2. (b) The particle moves in a round trip, from point a to point b along path 1 and then back to point a along path 2.

write this result as

$$W_{ab,1} = W_{ab,2}, \quad (8-2)$$

where the subscript ab indicates the initial and final points, respectively, and the subscripts 1 and 2 indicate the path.

This result is powerful because it allows us to simplify difficult problems when only a conservative force is involved. Suppose you need to calculate the work done by a conservative force along a given path between two points, and the calculation is difficult or even impossible without additional information. You can find the work by substituting some other path between those two points for which the calculation is easier and possible.

Proof of Equation 8-2

Figure 8-4b shows an arbitrary round trip for a particle that is acted upon by a single force. The particle moves from an initial point a to point b along path 1 and then back to point a along path 2. The force does work on the particle as the particle moves along each path. Without worrying about where positive work is done and where negative work is done, let us just represent the work done from a to b along path 1 as $W_{ab,1}$ and the work done from b back to a along path 2 as $W_{ba,2}$. If the force is conservative, then the net work done during the round trip must be zero:

$$W_{ab,1} + W_{ba,2} = 0,$$

and thus

$$W_{ab,1} = -W_{ba,2}. \quad (8-3)$$

In words, the work done along the outward path must be the negative of the work done along the path back.

Let us now consider the work $W_{ab,2}$ done on the particle by the force when the particle moves from a to b along path 2, as indicated in Fig. 8-4a. If the force is conservative, that work is the negative of $W_{ba,2}$:

$$W_{ab,2} = -W_{ba,2}. \quad (8-4)$$

Substituting $W_{ab,2}$ for $-W_{ba,2}$ in Eq. 8-3, we obtain

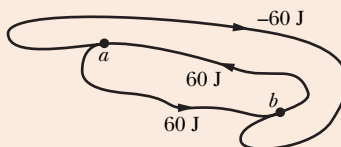
$$W_{ab,1} = W_{ab,2},$$

which is what we set out to prove.



CHECKPOINT 1

The figure shows three paths connecting points a and b . A single force \vec{F} does the indicated work on a particle moving along each path in the indicated direction. On the basis of this information, is force \vec{F} conservative?



Sample Problem

Equivalent paths for calculating work, slippery cheese

Figure 8-5a shows a 2.0 kg block of slippery cheese that slides along a frictionless track from point a to point b . The cheese travels through a total distance of 2.0 m along the track, and a net vertical distance of 0.80 m. How much work is done on the cheese by the gravitational force during the slide?

KEY IDEAS

(1) We *cannot* calculate the work by using Eq. 7-12 ($W_g = mgd \cos \phi$). The reason is that the angle ϕ between the direc-

The gravitational force is conservative. Any choice of path between the points gives the same amount of work.

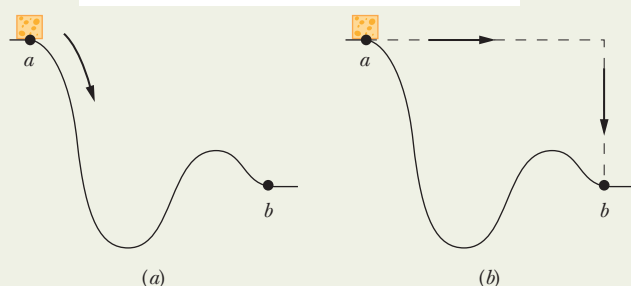


Fig. 8-5 (a) A block of cheese slides along a frictionless track from point a to point b . (b) Finding the work done on the cheese by the gravitational force is easier along the dashed path than along the actual path taken by the cheese; the result is the same for both paths.



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tions of the gravitational force \vec{F}_g and the displacement \vec{d} varies along the track in an unknown way. (Even if we did know the shape of the track and could calculate ϕ along it, the calculation could be very difficult.) (2) Because \vec{F}_g is a conservative force, we can find the work by choosing some other path between a and b —one that makes the calculation easy.

Calculations: Let us choose the dashed path in Fig. 8-5b; it consists of two straight segments. Along the horizontal segment, the angle ϕ is a constant 90° . Even though we do not know the displacement along that horizontal segment, Eq. 7-12 tells us that the work W_h done there is

$$W_h = mgd \cos 90^\circ = 0.$$

Along the vertical segment, the displacement d is 0.80 m and, with \vec{F}_g and \vec{d} both downward, the angle ϕ is a constant 0° . Thus, Eq. 7-12 gives us, for the work W_v done along the vertical part of the dashed path,

$$\begin{aligned} W_v &= mgd \cos 0^\circ \\ &= (2.0 \text{ kg})(9.8 \text{ m/s}^2)(0.80 \text{ m})(1) = 15.7 \text{ J}. \end{aligned}$$

The total work done on the cheese by \vec{F}_g as the cheese moves from point a to point b along the dashed path is then

$$W = W_h + W_v = 0 + 15.7 \text{ J} \approx 16 \text{ J}. \quad (\text{Answer})$$

This is also the work done as the cheese slides along the track from a to b .

8-4 Determining Potential Energy Values

Here we find equations that give the value of the two types of potential energy discussed in this chapter: gravitational potential energy and elastic potential energy. However, first we must find a general relation between a conservative force and the associated potential energy.

Consider a particle-like object that is part of a system in which a conservative force \vec{F} acts. When that force does work W on the object, the change ΔU in the potential energy associated with the system is the negative of the work done. We wrote this fact as Eq. 8-1 ($\Delta U = -W$). For the most general case, in which the force may vary with position, we may write the work W as in Eq. 7-32:

$$W = \int_{x_i}^{x_f} F(x) dx. \quad (8-5)$$

This equation gives the work done by the force when the object moves from point x_i to point x_f , changing the configuration of the system. (Because the force is conservative, the work is the same for all paths between those two points.)

Substituting Eq. 8-5 into Eq. 8-1, we find that the change in potential energy due to the change in configuration is, in general notation,

$$\Delta U = - \int_{x_i}^{x_f} F(x) dx. \quad (8-6)$$

Gravitational Potential Energy

We first consider a particle with mass m moving vertically along a y axis (the positive direction is upward). As the particle moves from point y_i to point y_f , the gravitational force \vec{F}_g does work on it. To find the corresponding change in the gravitational potential energy of the particle–Earth system, we use Eq. 8-6 with two changes: (1) We integrate along the y axis instead of the x axis, because the gravitational force acts vertically. (2) We substitute $-mg$ for the force symbol F , because \vec{F}_g has the magnitude mg and is directed down the y axis. We then have

$$\Delta U = - \int_{y_i}^{y_f} (-mg) dy = mg \int_{y_i}^{y_f} dy = mg \left[y \right]_{y_i}^{y_f},$$

which yields

$$\Delta U = mg(y_f - y_i) = mg \Delta y. \quad (8-7)$$

Only *changes* ΔU in gravitational potential energy (or any other type of potential energy) are physically meaningful. However, to simplify a calculation or a discussion, we sometimes would like to say that a certain gravitational potential value U is associated with a certain particle–Earth system when the particle is at a certain height y . To do so, we rewrite Eq. 8-7 as

$$U - U_i = mg(y - y_i). \quad (8-8)$$

Then we take U_i to be the gravitational potential energy of the system when it is in a **reference configuration** in which the particle is at a **reference point** y_i . Usually we take $U_i = 0$ and $y_i = 0$. Doing this changes Eq. 8-8 to

$$U(y) = mgy \quad (\text{gravitational potential energy}). \quad (8-9)$$

This equation tells us:

The gravitational potential energy associated with a particle–Earth system depends only on the vertical position y (or height) of the particle relative to the reference position $y = 0$, not on the horizontal position.

Elastic Potential Energy

We next consider the block–spring system shown in Fig. 8-3, with the block moving on the end of a spring of spring constant k . As the block moves from point x_i to point x_f , the spring force $F_x = -kx$ does work on the block. To find the corresponding change in the elastic potential energy of the block–spring system, we substitute $-kx$ for $F(x)$ in Eq. 8-6. We then have

$$\Delta U = - \int_{x_i}^{x_f} (-kx) dx = k \int_{x_i}^{x_f} x dx = \frac{1}{2}k \left[x^2 \right]_{x_i}^{x_f},$$

or

$$\Delta U = \frac{1}{2}kx_f^2 - \frac{1}{2}kx_i^2. \quad (8-10)$$

To associate a potential energy value U with the block at position x , we choose the reference configuration to be when the spring is at its relaxed length and the block is at $x_i = 0$. Then the elastic potential energy U_i is 0, and Eq. 8-10 becomes

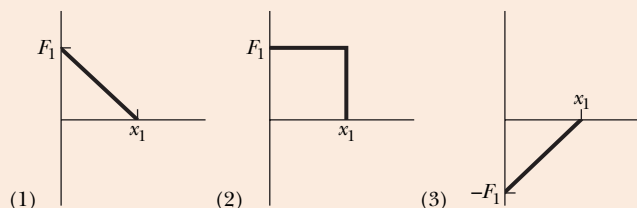
$$U - 0 = \frac{1}{2}kx^2 - 0,$$

which gives us

$$U(x) = \frac{1}{2}kx^2 \quad (\text{elastic potential energy}). \quad (8-11)$$

CHECKPOINT 2

A particle is to move along an x axis from $x = 0$ to x_1 while a conservative force, directed along the x axis, acts on the particle. The figure shows three situations in which the x component of that force varies with x . The force has the same maximum magnitude F_1 in all three situations. Rank the situations according to the change in the associated potential energy during the particle's motion, most positive first.



Sample Problem

Choosing reference level for gravitational potential energy, sloth

A 2.0 kg sloth hangs 5.0 m above the ground (Fig. 8-6).

(a) What is the gravitational potential energy U of the sloth–Earth system if we take the reference point $y = 0$ to be (1) at the ground, (2) at a balcony floor that is 3.0 m above the ground, (3) at the limb, and (4) 1.0 m above the limb? Take the gravitational potential energy to be zero at $y = 0$.

KEY IDEA

Once we have chosen the reference point for $y = 0$, we can calculate the gravitational potential energy U of the system *relative to that reference point* with Eq. 8-9.

Calculations: For choice (1) the sloth is at $y = 5.0$ m, and

$$U = mgy = (2.0 \text{ kg})(9.8 \text{ m/s}^2)(5.0 \text{ m}) = 98 \text{ J.} \quad (\text{Answer})$$

For the other choices, the values of U are

$$\begin{aligned} (2) \quad U &= mgy = mg(2.0 \text{ m}) = 39 \text{ J,} \\ (3) \quad U &= mgy = mg(0) = 0 \text{ J,} \\ (4) \quad U &= mgy = mg(-1.0 \text{ m}) \\ &= -19.6 \text{ J} \approx -20 \text{ J.} \end{aligned} \quad (\text{Answer})$$

(b) The sloth drops to the ground. For each choice of reference point, what is the change ΔU in the potential energy of the sloth–Earth system due to the fall?

KEY IDEA

The *change* in potential energy does not depend on the choice of the reference point for $y = 0$; instead, it depends on the change in height Δy .

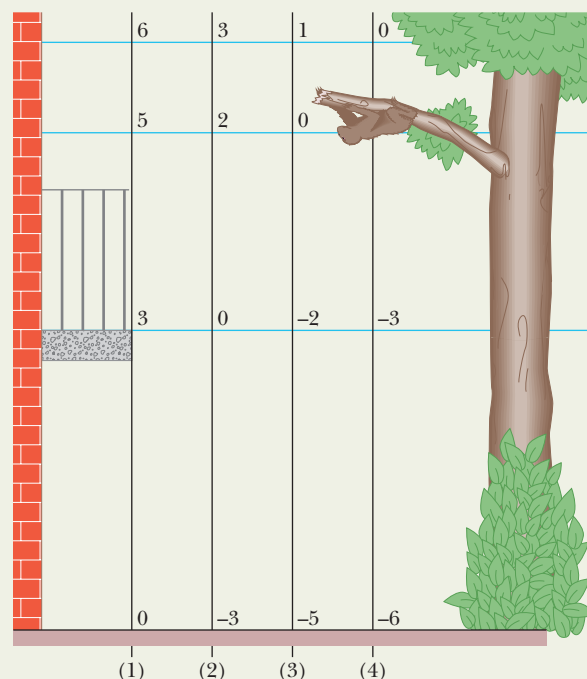


Fig. 8-6 Four choices of reference point $y = 0$. Each y axis is marked in units of meters. The choice affects the value of the potential energy U of the sloth–Earth system. However, it does not affect the change ΔU in potential energy of the system if the sloth moves by, say, falling.

Calculation: For all four situations, we have the same $\Delta y = -5.0$ m. Thus, for (1) to (4), Eq. 8-7 tells us that

$$\begin{aligned} \Delta U &= mg \Delta y = (2.0 \text{ kg})(9.8 \text{ m/s}^2)(-5.0 \text{ m}) \\ &= -98 \text{ J.} \end{aligned} \quad (\text{Answer})$$



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8-5 Conservation of Mechanical Energy

The **mechanical energy** E_{mec} of a system is the sum of its potential energy U and the kinetic energy K of the objects within it:

$$E_{\text{mec}} = K + U \quad (\text{mechanical energy}). \quad (8-12)$$

In this section, we examine what happens to this mechanical energy when only conservative forces cause energy transfers within the system—that is, when frictional and drag forces do not act on the objects in the system. Also, we shall assume that the system is *isolated* from its environment; that is, no *external force* from an object outside the system causes energy changes inside the system.

When a conservative force does work W on an object within the system, that force transfers energy between kinetic energy K of the object and potential energy U of the system. From Eq. 7-10, the change ΔK in kinetic energy is

$$\Delta K = W \quad (8-13)$$

and from Eq. 8-1, the change ΔU in potential energy is

$$\Delta U = -W. \quad (8-14)$$

Combining Eqs. 8-13 and 8-14, we find that

$$\Delta K = -\Delta U. \quad (8-15)$$

In words, one of these energies increases exactly as much as the other decreases.

We can rewrite Eq. 8-15 as

$$K_2 - K_1 = -(U_2 - U_1), \quad (8-16)$$

where the subscripts refer to two different instants and thus to two different arrangements of the objects in the system. Rearranging Eq. 8-16 yields

$$K_2 + U_2 = K_1 + U_1 \quad (\text{conservation of mechanical energy}). \quad (8-17)$$

In words, this equation says:

$$\left(\begin{array}{c} \text{the sum of } K \text{ and } U \text{ for} \\ \text{any state of a system} \end{array} \right) = \left(\begin{array}{c} \text{the sum of } K \text{ and } U \text{ for} \\ \text{any other state of the system} \end{array} \right),$$

when the system is isolated and only conservative forces act on the objects in the system. In other words:

In an isolated system where only conservative forces cause energy changes, the kinetic energy and potential energy can change, but their sum, the mechanical energy E_{mec} of the system, cannot change.

This result is called the **principle of conservation of mechanical energy**. (Now you can see where *conservative* forces got their name.) With the aid of Eq. 8-15, we can write this principle in one more form, as

$$\Delta E_{\text{mec}} = \Delta K + \Delta U = 0. \quad (8-18)$$

The principle of conservation of mechanical energy allows us to solve problems that would be quite difficult to solve using only Newton's laws:

When the mechanical energy of a system is conserved, we can relate the sum of kinetic energy and potential energy at one instant to that at another instant *without considering the intermediate motion and without finding the work done by the forces involved*.



In olden days, a person would be tossed via a blanket to be able to see farther over the flat terrain. Nowadays, it is done just for fun. During the ascent of the person in the photograph, energy is transferred from kinetic energy to gravitational potential energy. The maximum height is reached when that transfer is complete. Then the transfer is reversed during the fall. (©AP/Wide World Photos)

Fig. 8-7 A pendulum, with its mass concentrated in a bob at the lower end, swings back and forth. One full cycle of the motion is shown. During the cycle the values of the potential and kinetic energies of the pendulum–Earth system vary as the bob rises and falls, but the mechanical energy E_{mec} of the system remains constant. The energy E_{mec} can be described as continuously shifting between the kinetic and potential forms. In stages (a) and (e), all the energy is kinetic energy. The bob then has its greatest speed and is at its lowest point. In stages (c) and (g), all the energy is potential energy. The bob then has zero speed and is at its highest point. In stages (b), (d), (f), and (h), half the energy is kinetic energy and half is potential energy. If the swinging involved a frictional force at the point where the pendulum is attached to the ceiling, or a drag force due to the air, then E_{mec} would not be conserved, and eventually the pendulum would stop.

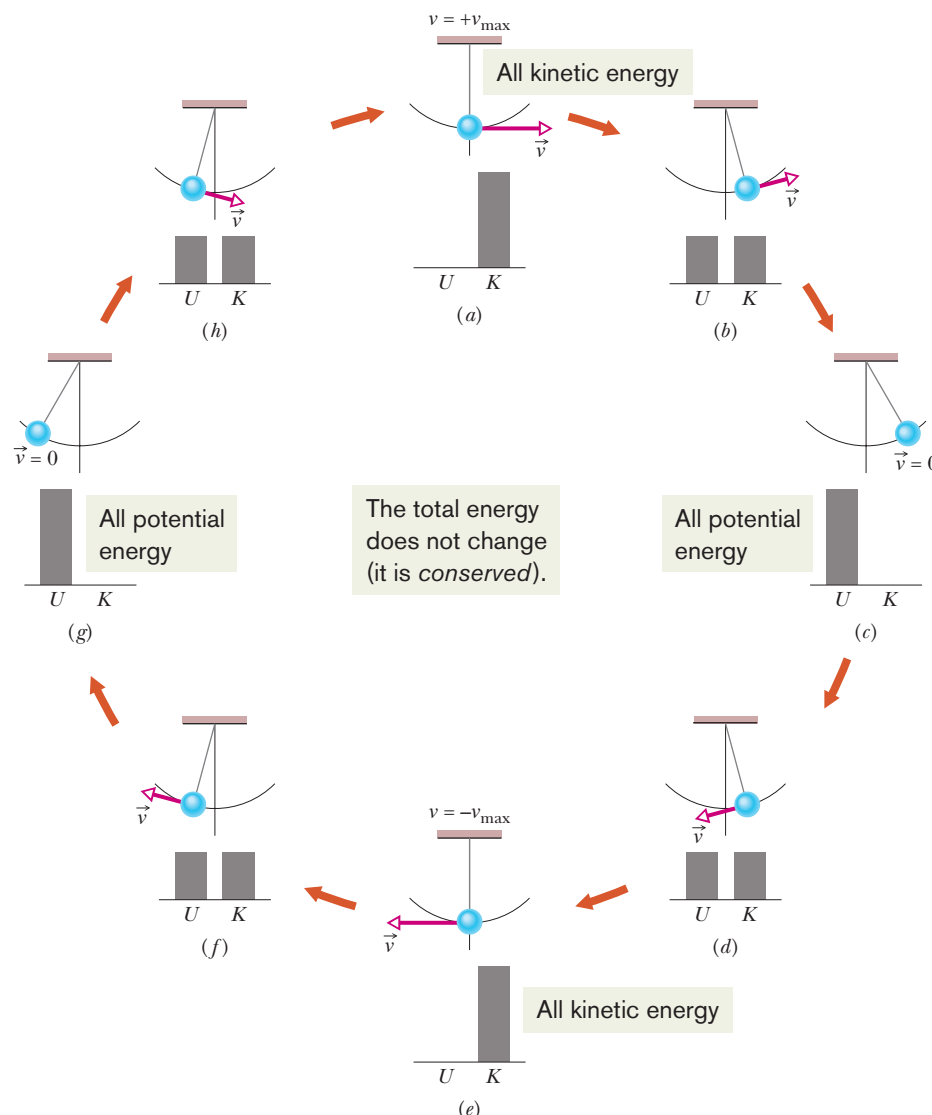


Figure 8-7 shows an example in which the principle of conservation of mechanical energy can be applied: As a pendulum swings, the energy of the pendulum–Earth system is transferred back and forth between kinetic energy K and gravitational potential energy U , with the sum $K + U$ being constant. If we know the gravitational potential energy when the pendulum bob is at its highest point (Fig. 8-7c), Eq. 8-17 gives us the kinetic energy of the bob at the lowest point (Fig. 8-7e).

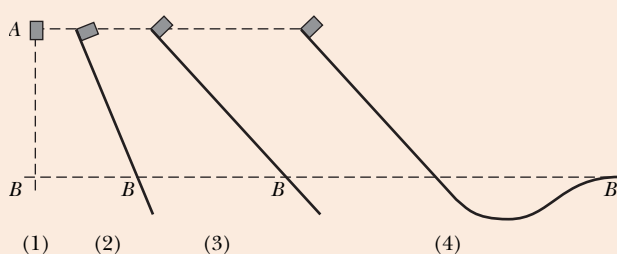
For example, let us choose the lowest point as the reference point, with the gravitational potential energy $U_2 = 0$. Suppose then that the potential energy at the highest point is $U_1 = 20 \text{ J}$ relative to the reference point. Because the bob momentarily stops at its highest point, the kinetic energy there is $K_1 = 0$. Putting these values into Eq. 8-17 gives us the kinetic energy K_2 at the lowest point:

$$K_2 + 0 = 0 + 20 \text{ J} \quad \text{or} \quad K_2 = 20 \text{ J}.$$

Note that we get this result without considering the motion between the highest and lowest points (such as in Fig. 8-7d) and without finding the work done by any forces involved in the motion.

CHECKPOINT 3

The figure shows four situations—one in which an initially stationary block is dropped and three in which the block is allowed to slide down frictionless ramps. (a) Rank the situations according to the kinetic energy of the block at point B, greatest first. (b) Rank them according to the speed of the block at point B, greatest first.



Sample Problem

Conservation of mechanical energy, water slide

In Fig. 8-8, a child of mass m is released from rest at the top of a water slide, at height $h = 8.5$ m above the bottom of the slide. Assuming that the slide is frictionless because of the water on it, find the child's speed at the bottom of the slide.

KEY IDEAS

(1) We cannot find her speed at the bottom by using her acceleration along the slide as we might have in earlier chapters because we do not know the slope (angle) of the slide. However, because that speed is related to her kinetic energy, perhaps we can use the principle of conservation of mechanical energy to get the speed. Then we would not need to know the slope. (2) Mechanical energy is conserved in a system *if* the system is isolated and *if* only conservative forces cause energy transfers within it. Let's check.

Forces: Two forces act on the child. The *gravitational force*, a conservative force, does work on her. The *normal force* on her from the slide does no work because its direction at any point during the descent is always perpendicular to the direction in which the child moves.

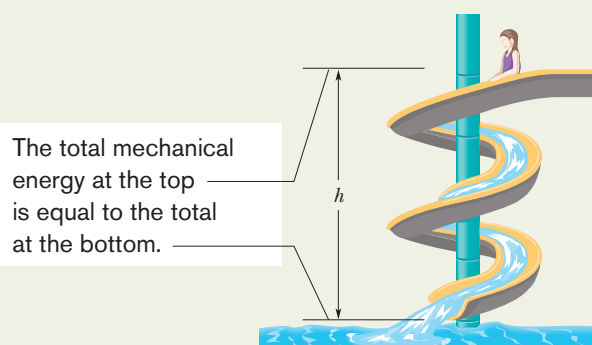


Fig. 8-8 A child slides down a water slide as she descends a height h .

System: Because the only force doing work on the child is the gravitational force, we choose the child–Earth system as our system, which we can take to be isolated.

Thus, we have only a conservative force doing work in an isolated system, so we *can* use the principle of conservation of mechanical energy.

Calculations: Let the mechanical energy be $E_{\text{mec},t}$ when the child is at the top of the slide and $E_{\text{mec},b}$ when she is at the bottom. Then the conservation principle tells us

$$E_{\text{mec},b} = E_{\text{mec},t}. \quad (8-19)$$

To show both kinds of mechanical energy, we have

$$K_b + U_b = K_t + U_t, \quad (8-20)$$

$$\text{or} \quad \frac{1}{2}mv_b^2 + mgy_b = \frac{1}{2}mv_t^2 + mgy_t.$$

Dividing by m and rearranging yield

$$v_b^2 = v_t^2 + 2g(y_t - y_b).$$

Putting $v_t = 0$ and $y_t - y_b = h$ leads to

$$v_b = \sqrt{2gh} = \sqrt{(2)(9.8 \text{ m/s}^2)(8.5 \text{ m})} = 13 \text{ m/s}. \quad (\text{Answer})$$

This is the same speed that the child would reach if she fell 8.5 m vertically. On an actual slide, some frictional forces would act and the child would not be moving quite so fast.

Comments: Although this problem is hard to solve directly with Newton's laws, using conservation of mechanical energy makes the solution much easier. However, if we were asked to find the time taken for the child to reach the bottom of the slide, energy methods would be of no use; we would need to know the shape of the slide, and we would have a difficult problem.



Additional examples, video, and practice available at WileyPLUS

8-6 Reading a Potential Energy Curve

Once again we consider a particle that is part of a system in which a conservative force acts. This time suppose that the particle is constrained to move along an x axis while the conservative force does work on it. We can learn a lot about the motion of the particle from a plot of the system's potential energy $U(x)$. However, before we discuss such plots, we need one more relationship.

Finding the Force Analytically

Equation 8-6 tells us how to find the change ΔU in potential energy between two points in a one-dimensional situation if we know the force $F(x)$. Now we want to go the other way; that is, we know the potential energy function $U(x)$ and want to find the force.

For one-dimensional motion, the work W done by a force that acts on a particle as the particle moves through a distance Δx is $F(x) \Delta x$. We can then write Eq. 8-1 as

$$\Delta U(x) = -W = -F(x) \Delta x. \quad (8-21)$$

Solving for $F(x)$ and passing to the differential limit yield

$$F(x) = -\frac{dU(x)}{dx} \quad (\text{one-dimensional motion}), \quad (8-22)$$

which is the relation we sought.

We can check this result by putting $U(x) = \frac{1}{2}kx^2$, which is the elastic potential energy function for a spring force. Equation 8-22 then yields, as expected, $F(x) = -kx$, which is Hooke's law. Similarly, we can substitute $U(x) = mgx$, which is the gravitational potential energy function for a particle–Earth system, with a particle of mass m at height x above Earth's surface. Equation 8-22 then yields $F = -mg$, which is the gravitational force on the particle.

The Potential Energy Curve

Figure 8-9a is a plot of a potential energy function $U(x)$ for a system in which a particle is in one-dimensional motion while a conservative force $F(x)$ does work on it. We can easily find $F(x)$ by (graphically) taking the slope of the $U(x)$ curve at various points. (Equation 8-22 tells us that $F(x)$ is the negative of the slope of the $U(x)$ curve.) Figure 8-9b is a plot of $F(x)$ found in this way.

Turning Points

In the absence of a nonconservative force, the mechanical energy E of a system has a constant value given by

$$U(x) + K(x) = E_{\text{mec}}. \quad (8-23)$$

Here $K(x)$ is the *kinetic energy function* of a particle in the system (this $K(x)$ gives the kinetic energy as a function of the particle's location x). We may rewrite Eq. 8-23 as

$$K(x) = E_{\text{mec}} - U(x). \quad (8-24)$$

Suppose that E_{mec} (which has a constant value, remember) happens to be 5.0 J. It would be represented in Fig. 8-9c by a horizontal line that runs through the value 5.0 J on the energy axis. (It is, in fact, shown there.)

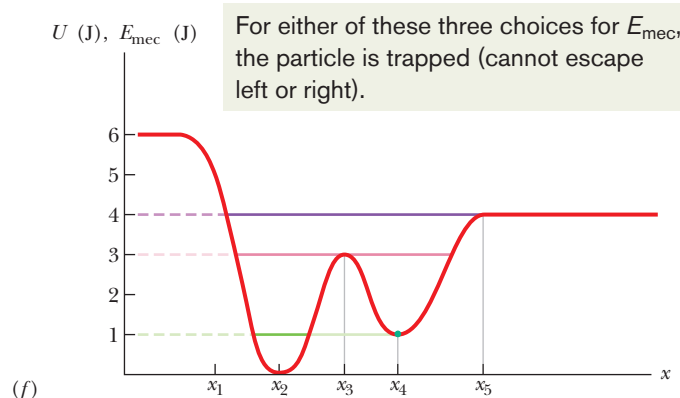
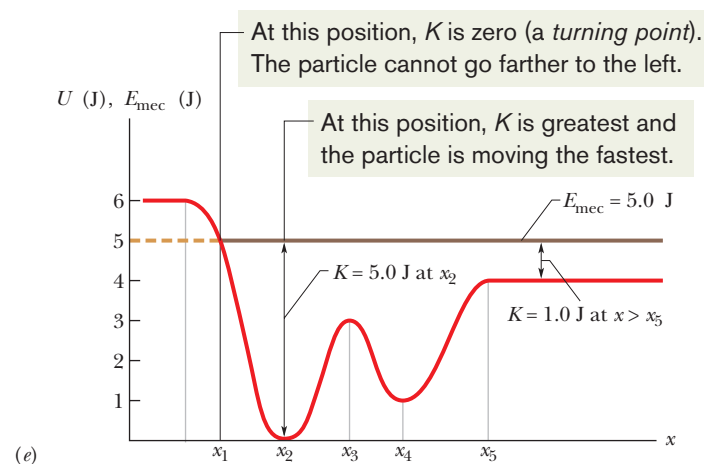
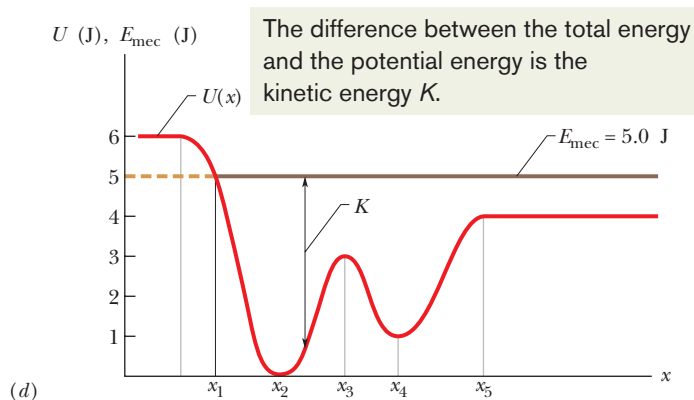
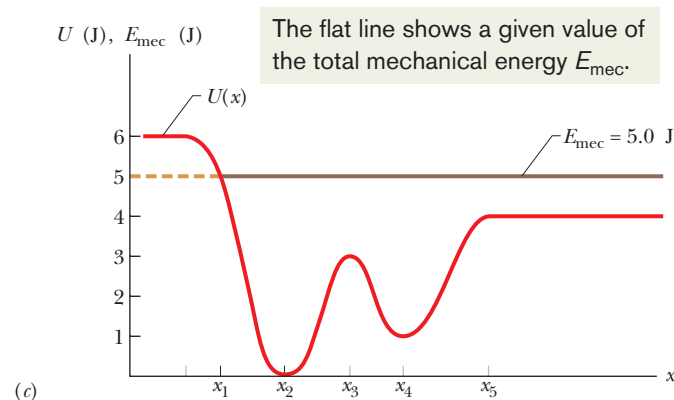
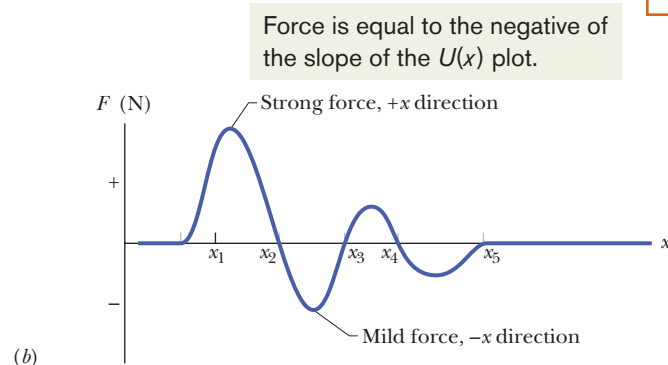
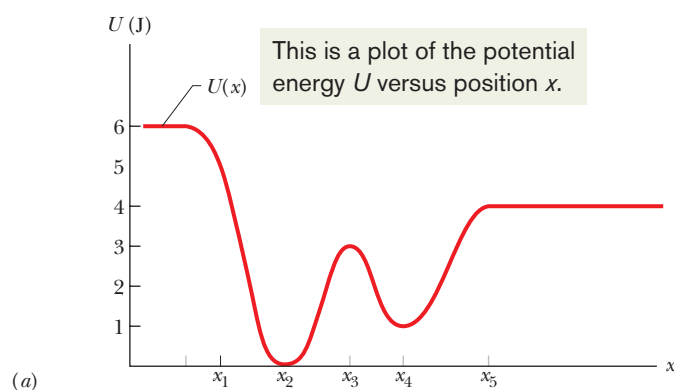


Fig. 8-9 (a) A plot of $U(x)$, the potential energy function of a system containing a particle confined to move along an x axis. There is no friction, so mechanical energy is conserved. (b) A plot of the force $F(x)$ acting on the particle, derived from the potential energy plot by taking its slope at various points. (c)–(e) How to determine the kinetic energy. (f) The $U(x)$ plot of (a) with three possible values of E_{mec} shown.

Equation 8-24 and Fig. 8-9*d* tell us how to determine the kinetic energy K for any location x of the particle: On the $U(x)$ curve, find U for that location x and then subtract U from E_{mec} . In Fig. 8-9*e* for example, if the particle is at any point to the right of x_5 , then $K = 1.0$ J. The value of K is greatest (5.0 J) when the particle is at x_2 and least (0 J) when the particle is at x_1 .

Since K can never be negative (because v^2 is always positive), the particle can never move to the left of x_1 , where $E_{\text{mec}} - U$ is negative. Instead, as the particle moves toward x_1 from x_2 , K decreases (the particle slows) until $K = 0$ at x_1 (the particle stops there).

Note that when the particle reaches x_1 , the force on the particle, given by Eq. 8-22, is positive (because the slope dU/dx is negative). This means that the particle does not remain at x_1 but instead begins to move to the right, opposite its earlier motion. Hence x_1 is a **turning point**, a place where $K = 0$ (because $U = E$) and the particle changes direction. There is no turning point (where $K = 0$) on the right side of the graph. When the particle heads to the right, it will continue indefinitely.

Equilibrium Points

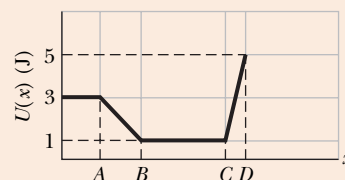
Figure 8-9*f* shows three different values for E_{mec} superposed on the plot of the potential energy function $U(x)$ of Fig. 8-9*a*. Let us see how they change the situation. If $E_{\text{mec}} = 4.0$ J (purple line), the turning point shifts from x_1 to a point between x_1 and x_2 . Also, at any point to the right of x_5 , the system's mechanical energy is equal to its potential energy; thus, the particle has no kinetic energy and (by Eq. 8-22) no force acts on it, and so it must be stationary. A particle at such a position is said to be in **neutral equilibrium**. (A marble placed on a horizontal tabletop is in that state.)

If $E_{\text{mec}} = 3.0$ J (pink line), there are two turning points: One is between x_1 and x_2 , and the other is between x_4 and x_5 . In addition, x_3 is a point at which $K = 0$. If the particle is located exactly there, the force on it is also zero, and the particle remains stationary. However, if it is displaced even slightly in either direction, a nonzero force pushes it farther in the same direction, and the particle continues to move. A particle at such a position is said to be in **unstable equilibrium**. (A marble balanced on top of a bowling ball is an example.)

Next consider the particle's behavior if $E_{\text{mec}} = 1.0$ J (green line). If we place it at x_4 , it is stuck there. It cannot move left or right on its own because to do so would require a negative kinetic energy. If we push it slightly left or right, a restoring force appears that moves it back to x_4 . A particle at such a position is said to be in **stable equilibrium**. (A marble placed at the bottom of a hemispherical bowl is an example.) If we place the particle in the cup-like *potential well* centered at x_2 , it is between two turning points. It can still move somewhat, but only partway to x_1 or x_3 .

CHECKPOINT 4

The figure gives the potential energy function $U(x)$ for a system in which a particle is in one-dimensional motion. (a) Rank regions AB , BC , and CD according to the magnitude of the force on the particle, greatest first. (b) What is the direction of the force when the particle is in region AB ?



Sample Problem

Reading a potential energy graph

A 2.00 kg particle moves along an x axis in one-dimensional motion while a conservative force along that axis acts on it. The potential energy $U(x)$ associated with the force is plotted in Fig. 8-10a. That is, if the particle were placed at any position between $x = 0$ and $x = 7.00$ m, it would have the plotted value of U . At $x = 6.5$ m, the particle has velocity $v_0 = (-4.00 \text{ m/s})\hat{i}$.

(a) From Fig. 8-10a, determine the particle's speed at $x_1 = 4.5$ m.

KEY IDEAS

(1) The particle's kinetic energy is given by Eq. 7-1 ($K = \frac{1}{2}mv^2$). (2) Because only a conservative force acts on the particle, the mechanical energy $E_{\text{mec}} (= K + U)$ is conserved as the particle moves. (3) Therefore, on a plot of $U(x)$ such as Fig. 8-10a, the kinetic energy is equal to the difference between E_{mec} and U .

Calculations: At $x = 6.5$ m, the particle has kinetic energy

$$\begin{aligned} K_0 &= \frac{1}{2}mv_0^2 = \frac{1}{2}(2.00 \text{ kg})(4.00 \text{ m/s})^2 \\ &= 16.0 \text{ J.} \end{aligned}$$

Because the potential energy there is $U = 0$, the mechanical energy is

$$E_{\text{mec}} = K_0 + U_0 = 16.0 \text{ J} + 0 = 16.0 \text{ J.}$$

This value for E_{mec} is plotted as a horizontal line in Fig. 8-10a. From that figure we see that at $x = 4.5$ m, the potential energy is $U_1 = 7.0 \text{ J}$. The kinetic energy K_1 is the difference between E_{mec} and U_1 :

$$K_1 = E_{\text{mec}} - U_1 = 16.0 \text{ J} - 7.0 \text{ J} = 9.0 \text{ J.}$$

Because $K_1 = \frac{1}{2}mv_1^2$, we find

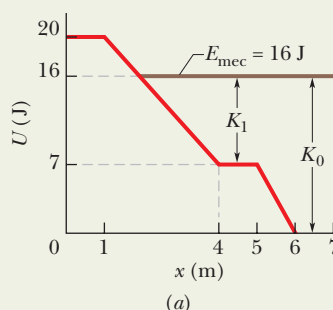
$$v_1 = 3.0 \text{ m/s.} \quad (\text{Answer})$$

(b) Where is the particle's turning point located?

KEY IDEA

The turning point is where the force momentarily stops and then reverses the particle's motion. That is, it is where the particle momentarily has $v = 0$ and thus $K = 0$.

Calculations: Because K is the difference between E_{mec} and U , we want the point in Fig. 8-10a where the plot of U rises to meet the horizontal line of E_{mec} , as shown in Fig. 8-10b. Because the plot of U is a straight line in Fig. 8-10b, we can draw nested right triangles as shown and then write



Kinetic energy is the difference between the total energy and the potential energy.

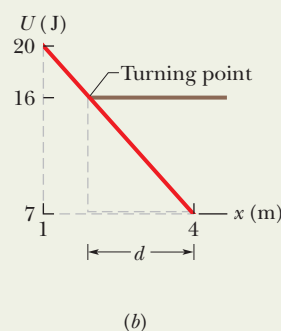


Fig. 8-10 (a) A plot of potential energy U versus position x . (b) A section of the plot used to find where the particle turns around.

the proportionality of distances

$$\frac{16 - 7.0}{d} = \frac{20 - 7.0}{4.0 - 1.0},$$

which gives us $d = 2.08$ m. Thus, the turning point is at

$$x = 4.0 \text{ m} - d = 1.9 \text{ m.} \quad (\text{Answer})$$

(c) Evaluate the force acting on the particle when it is in the region $1.9 \text{ m} < x < 4.0 \text{ m}$.

KEY IDEA

The force is given by Eq. 8-22 ($F(x) = -dU(x)/dx$). The equation states that the force is equal to the negative of the slope on a graph of $U(x)$.

Calculations: For the graph of Fig. 8-10b, we see that for the range $1.0 \text{ m} < x < 4.0 \text{ m}$ the force is

$$F = -\frac{20 \text{ J} - 7.0 \text{ J}}{1.0 \text{ m} - 4.0 \text{ m}} = 4.3 \text{ N.} \quad (\text{Answer})$$

Thus, the force has magnitude 4.3 N and is in the positive direction of the x axis. This result is consistent with the fact that the initially leftward-moving particle is stopped by the force and then sent rightward.



Additional examples, video, and practice available at WileyPLUS

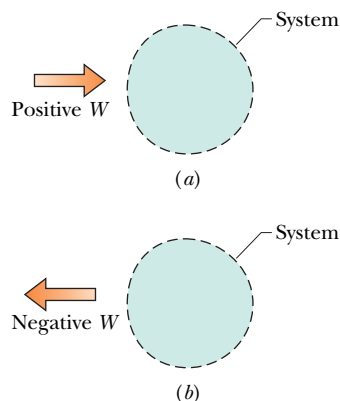


Fig. 8-11 (a) Positive work W done on an arbitrary system means a transfer of energy to the system. (b) Negative work W means a transfer of energy from the system.

8-7 Work Done on a System by an External Force

In Chapter 7, we defined work as being energy transferred to or from an object by means of a force acting on the object. We can now extend that definition to an external force acting on a system of objects.

Work is energy transferred to or from a system by means of an external force acting on that system.

Figure 8-11a represents positive work (a transfer of energy *to* a system), and Fig. 8-11b represents negative work (a transfer of energy *from* a system). When more than one force acts on a system, their *net work* is the energy transferred to or from the system.

These transfers are like transfers of money to and from a bank account. If a system consists of a single particle or particle-like object, as in Chapter 7, the work done on the system by a force can change only the kinetic energy of the system. The energy statement for such transfers is the work–kinetic energy theorem of Eq. 7-10 ($\Delta K = W$); that is, a single particle has only one energy account, called kinetic energy. External forces can transfer energy into or out of that account. If a system is more complicated, however, an external force can change other forms of energy (such as potential energy); that is, a more complicated system can have multiple energy accounts.

Let us find energy statements for such systems by examining two basic situations, one that does not involve friction and one that does.

No Friction Involved

To compete in a bowling-ball-hurling contest, you first squat and cup your hands under the ball on the floor. Then you rapidly straighten up while also pulling your hands up sharply, launching the ball upward at about face level. During your upward motion, your applied force on the ball obviously does work; that is, it is an external force that transfers energy, but to what system?

To answer, we check to see which energies change. There is a change ΔK in the ball's kinetic energy and, because the ball and Earth become more separated, there is a change ΔU in the gravitational potential energy of the ball–Earth system. To include both changes, we need to consider the ball–Earth system. Then your force is an external force doing work on that system, and the work is

$$W = \Delta K + \Delta U, \quad (8-25)$$

$$\text{or} \quad W = \Delta E_{\text{mec}} \quad (\text{work done on system, no friction involved}), \quad (8-26)$$

where ΔE_{mec} is the change in the mechanical energy of the system. These two equations, which are represented in Fig. 8-12, are equivalent energy statements for work done on a system by an external force when friction is not involved.

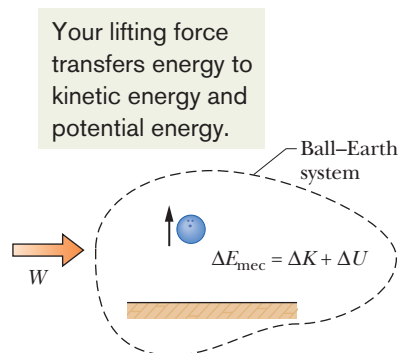


Fig. 8-12 Positive work W is done on a system of a bowling ball and Earth, causing a change ΔE_{mec} in the mechanical energy of the system, a change ΔK in the ball's kinetic energy, and a change ΔU in the system's gravitational potential energy.

Friction Involved

We next consider the example in Fig. 8-13a. A constant horizontal force \vec{F} pulls a block along an x axis and through a displacement of magnitude d , increasing the block's velocity from \vec{v}_0 to \vec{v} . During the motion, a constant kinetic frictional force \vec{f}_k from the floor acts on the block. Let us first choose the block as our

The applied force supplies energy. The frictional force transfers some of it to thermal energy.

So, the work done by the applied force goes into kinetic energy and also thermal energy.

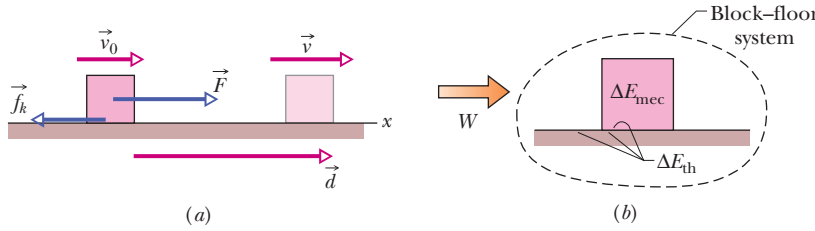


Fig. 8-13 (a) A block is pulled across a floor by force \vec{F} while a kinetic frictional force \vec{f}_k opposes the motion. The block has velocity \vec{v}_0 at the start of a displacement \vec{d} and velocity \vec{v} at the end of the displacement. (b) Positive work W is done on the block-floor system by force \vec{F} , resulting in a change ΔE_{mec} in the block's mechanical energy and a change ΔE_{th} in the thermal energy of the block and floor.

system and apply Newton's second law to it. We can write that law for components along the x axis ($F_{\text{net},x} = ma_x$) as

$$F - f_k = ma. \quad (8-27)$$

Because the forces are constant, the acceleration \vec{a} is also constant. Thus, we can use Eq. 2-16 to write

$$v^2 = v_0^2 + 2ad.$$

Solving this equation for a , substituting the result into Eq. 8-27, and rearranging then give us

$$Fd = \frac{1}{2}mv^2 - \frac{1}{2}mv_0^2 + f_k d \quad (8-28)$$

or, because $\frac{1}{2}mv^2 - \frac{1}{2}mv_0^2 = \Delta K$ for the block,

$$Fd = \Delta K + f_k d. \quad (8-29)$$

In a more general situation (say, one in which the block is moving up a ramp), there can be a change in potential energy. To include such a possible change, we generalize Eq. 8-29 by writing

$$Fd = \Delta E_{\text{mec}} + f_k d. \quad (8-30)$$

By experiment we find that the block and the portion of the floor along which it slides become warmer as the block slides. As we shall discuss in Chapter 18, the temperature of an object is related to the object's thermal energy E_{th} (the energy associated with the random motion of the atoms and molecules in the object). Here, the thermal energy of the block and floor increases because (1) there is friction between them and (2) there is sliding. Recall that friction is due to the cold-welding between two surfaces. As the block slides over the floor, the sliding causes repeated tearing and re-forming of the welds between the block and the floor, which makes the block and floor warmer. Thus, the sliding increases their thermal energy E_{th} .

Through experiment, we find that the increase ΔE_{th} in thermal energy is equal to the product of the magnitudes f_k and d :

$$\Delta E_{\text{th}} = f_k d \quad (\text{increase in thermal energy by sliding}). \quad (8-31)$$

Thus, we can rewrite Eq. 8-30 as

$$Fd = \Delta E_{\text{mec}} + \Delta E_{\text{th}}. \quad (8-32)$$

Fd is the work W done by the external force \vec{F} (the energy transferred by the force), but on which system is the work done (where are the energy transfers made)?

To answer, we check to see which energies change. The block's mechanical energy changes, and the thermal energies of the block and floor also change. Therefore, the work done by force \vec{F} is done on the block–floor system. That work is

$$W = \Delta E_{\text{mec}} + \Delta E_{\text{th}} \quad (\text{work done on system, friction involved}). \quad (8-33)$$

This equation, which is represented in Fig. 8-13*b*, is the energy statement for the work done on a system by an external force when friction is involved.



CHECKPOINT 5

In three trials, a block is pushed by a horizontal applied force across a floor that is not frictionless, as in Fig. 8-13*a*. The magnitudes F of the applied force and the results of the pushing on the block's speed are given in the table. In all three trials, the block is pushed through the same distance d . Rank the three trials according to the change in the thermal energy of the block and floor that occurs in that distance d , greatest first.

Trial	F	Result on Block's Speed
a	5.0 N	decreases
b	7.0 N	remains constant
c	8.0 N	increases

Sample Problem

Work, friction, change in thermal energy, cabbage heads

A food shipper pushes a wood crate of cabbage heads (total mass $m = 14 \text{ kg}$) across a concrete floor with a constant horizontal force \vec{F} of magnitude 40 N. In a straight-line displacement of magnitude $d = 0.50 \text{ m}$, the speed of the crate decreases from $v_0 = 0.60 \text{ m/s}$ to $v = 0.20 \text{ m/s}$.

(a) How much work is done by force \vec{F} , and on what system does it do the work?

KEY IDEA

Because the applied force \vec{F} is constant, we can calculate the work it does by using Eq. 7-7 ($W = Fd \cos \phi$).

Calculation: Substituting given data, including the fact that force \vec{F} and displacement \vec{d} are in the same direction, we find

$$\begin{aligned} W &= Fd \cos \phi = (40 \text{ N})(0.50 \text{ m}) \cos 0^\circ \\ &= 20 \text{ J}. \end{aligned} \quad (\text{Answer})$$

Reasoning: We can determine the system on which the work is done to see which energies change. Because the crate's speed changes, there is certainly a change ΔK in the crate's kinetic energy. Is there friction between the floor and the crate, and thus a change in thermal energy? Note that \vec{F} and the crate's velocity have the same direction.

Thus, if there is no friction, then \vec{F} should be accelerating the crate to a *greater* speed. However, the crate is *slowing*, so there must be friction and a change ΔE_{th} in thermal energy of the crate and the floor. Therefore, the system on which the work is done is the crate–floor system, because both energy changes occur in that system.

(b) What is the increase ΔE_{th} in the thermal energy of the crate and floor?

KEY IDEA

We can relate ΔE_{th} to the work W done by \vec{F} with the energy statement of Eq. 8-33 for a system that involves friction:

$$W = \Delta E_{\text{mec}} + \Delta E_{\text{th}}. \quad (8-34)$$

Calculations: We know the value of W from (a). The change ΔE_{mec} in the crate's mechanical energy is just the change in its kinetic energy because no potential energy changes occur, so we have

$$\Delta E_{\text{mec}} = \Delta K = \frac{1}{2}mv^2 - \frac{1}{2}mv_0^2.$$

Substituting this into Eq. 8-34 and solving for ΔE_{th} , we find

$$\begin{aligned} \Delta E_{\text{th}} &= W - \left(\frac{1}{2}mv^2 - \frac{1}{2}mv_0^2 \right) = W - \frac{1}{2}m(v^2 - v_0^2) \\ &= 20 \text{ J} - \frac{1}{2}(14 \text{ kg})[(0.20 \text{ m/s})^2 - (0.60 \text{ m/s})^2] \\ &= 22.2 \text{ J} \approx 22 \text{ J}. \end{aligned} \quad (\text{Answer})$$



8-8 Conservation of Energy

We now have discussed several situations in which energy is transferred to or from objects and systems, much like money is transferred between accounts. In each situation we assume that the energy that was involved could always be accounted for; that is, energy could not magically appear or disappear. In more formal language, we assumed (correctly) that energy obeys a law called the **law of conservation of energy**, which is concerned with the **total energy** E of a system. That total is the sum of the system's mechanical energy, thermal energy, and any type of *internal energy* in addition to thermal energy. (We have not yet discussed other types of internal energy.) The law states that

The total energy E of a system can change only by amounts of energy that are transferred to or from the system.

The only type of energy transfer that we have considered is work W done on a system. Thus, for us at this point, this law states that

$$W = \Delta E = \Delta E_{\text{mec}} + \Delta E_{\text{th}} + \Delta E_{\text{int}}, \quad (8-35)$$

where ΔE_{mec} is any change in the mechanical energy of the system, ΔE_{th} is any change in the thermal energy of the system, and ΔE_{int} is any change in any other type of internal energy of the system. Included in ΔE_{mec} are changes ΔK in kinetic energy and changes ΔU in potential energy (elastic, gravitational, or any other type we might find).

This law of conservation of energy is *not* something we have derived from basic physics principles. Rather, it is a law based on countless experiments. Scientists and engineers have never found an exception to it.

Isolated System

If a system is isolated from its environment, there can be no energy transfers to or from it. For that case, the law of conservation of energy states:

The total energy E of an isolated system cannot change.

Many energy transfers may be going on *within* an isolated system—between, say, kinetic energy and a potential energy or between kinetic energy and thermal energy. However, the total of all the types of energy in the system cannot change.

We can use the rock climber in Fig. 8-14 as an example, approximating him, his gear, and Earth as an isolated system. As he rappels down the rock face, changing the configuration of the system, he needs to control the transfer of energy from the gravitational potential energy of the system. (That energy cannot just disappear.) Some of it is transferred to his kinetic energy. However, he obviously does not want very much transferred to that type or he will be moving too quickly, so he has wrapped the rope around metal rings to produce friction between the rope and the rings as he moves down. The sliding of the rings on the rope then transfers the gravitational potential energy of the system to thermal energy of the rings and rope in a way that he can control. The total energy of the climber–gear–Earth system (the total of its gravitational potential energy, kinetic energy, and thermal energy) does not change during his descent.



Fig. 8-14 To descend, the rock climber must transfer energy from the gravitational potential energy of a system consisting of him, his gear, and Earth. He has wrapped the rope around metal rings so that the rope rubs against the rings. This allows most of the transferred energy to go to the thermal energy of the rope and rings rather than to his kinetic energy. (Tyler Stableford/The Image Bank/Getty Images)

For an isolated system, the law of conservation of energy can be written in two ways. First, by setting $W = 0$ in Eq. 8-35, we get

$$\Delta E_{\text{mec}} + \Delta E_{\text{th}} + \Delta E_{\text{int}} = 0 \quad (\text{isolated system}). \quad (8-36)$$

We can also let $\Delta E_{\text{mec}} = E_{\text{mec},2} - E_{\text{mec},1}$, where the subscripts 1 and 2 refer to two different instants—say, before and after a certain process has occurred. Then Eq. 8-36 becomes

$$E_{\text{mec},2} = E_{\text{mec},1} - \Delta E_{\text{th}} - \Delta E_{\text{int}}. \quad (8-37)$$

Equation 8-37 tells us:

In an isolated system, we can relate the total energy at one instant to the total energy at another instant *without considering the energies at intermediate times*.

This fact can be a very powerful tool in solving problems about isolated systems when you need to relate energies of a system before and after a certain process occurs in the system.

In Section 8-5, we discussed a special situation for isolated systems—namely, the situation in which nonconservative forces (such as a kinetic frictional force) do not act within them. In that special situation, ΔE_{th} and ΔE_{int} are both zero, and so Eq. 8-37 reduces to Eq. 8-18. In other words, the mechanical energy of an isolated system is conserved when nonconservative forces do not act in it.

External Forces and Internal Energy Transfers

An external force can change the kinetic energy or potential energy of an object without doing work on the object—that is, without transferring energy to the object. Instead, the force is responsible for transfers of energy from one type to another inside the object.

Figure 8-15 shows an example. An initially stationary ice-skater pushes away from a railing and then slides over the ice (Figs. 8-15*a* and *b*). Her kinetic energy increases because of an external force \vec{F} on her from the rail. However, that force does not transfer energy from the rail to her. Thus, the force does no work on

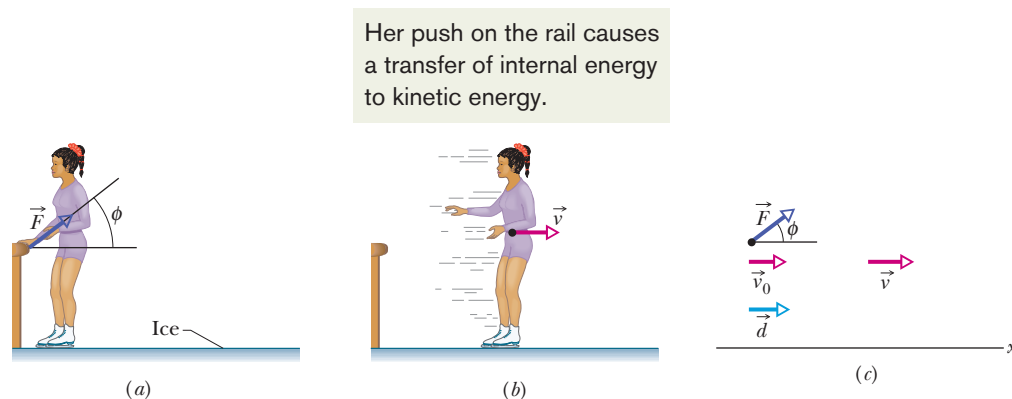
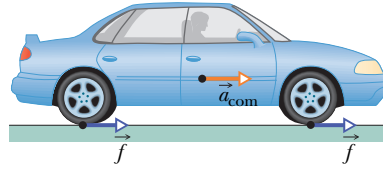


Fig. 8-15 (a) As a skater pushes herself away from a railing, the force on her from the railing is \vec{F} . (b) After the skater leaves the railing, she has velocity \vec{v} . (c) External force \vec{F} acts on the skater, at angle ϕ with a horizontal x axis. When the skater goes through displacement \vec{d} , her velocity is changed from $\vec{v}_0 (= 0)$ to \vec{v} by the horizontal component of \vec{F} .

Fig. 8-16 A vehicle accelerates to the right using four-wheel drive. The road exerts four frictional forces (two of them shown) on the bottom surfaces of the tires. Taken together, these four forces make up the net external force \vec{F} acting on the car.



her. Rather, her kinetic energy increases as a result of internal transfers from the biochemical energy in her muscles.

Figure 8-16 shows another example. An engine increases the speed of a car with four-wheel drive (all four wheels are made to turn by the engine). During the acceleration, the engine causes the tires to push backward on the road surface. This push produces frictional forces \vec{f} that act on each tire in the forward direction. The net external force \vec{F} from the road, which is the sum of these frictional forces, accelerates the car, increasing its kinetic energy. However, \vec{F} does not transfer energy from the road to the car and so does no work on the car. Rather, the car's kinetic energy increases as a result of internal transfers from the energy stored in the fuel.

In situations like these two, we can sometimes relate the external force \vec{F} on an object to the change in the object's mechanical energy if we can simplify the situation. Consider the ice-skater example. During her push through distance d in Fig. 8-15c, we can simplify by assuming that the acceleration is constant, her speed changing from $v_0 = 0$ to v . (That is, we assume \vec{F} has constant magnitude F and angle ϕ .) After the push, we can simplify the skater as being a particle and neglect the fact that the exertions of her muscles have increased the thermal energy in her muscles and changed other physiological features. Then we can apply Eq. 7-5 ($\frac{1}{2}mv^2 - \frac{1}{2}mv_0^2 = F_x d$) to write

$$K - K_0 = (F \cos \phi)d,$$

or

$$\Delta K = Fd \cos \phi. \quad (8-38)$$

If the situation also involves a change in the elevation of an object, we can include the change ΔU in gravitational potential energy by writing

$$\Delta U + \Delta K = Fd \cos \phi. \quad (8-39)$$

The force on the right side of this equation does no work on the object but is still responsible for the changes in energy shown on the left side.

Power

Now that you have seen how energy can be transferred from one type to another, we can expand the definition of power given in Section 7-9. There power is defined as the rate at which work is done by a force. In a more general sense, power P is the rate at which energy is transferred by a force from one type to another. If an amount of energy ΔE is transferred in an amount of time Δt , the **average power** due to the force is

$$P_{\text{avg}} = \frac{\Delta E}{\Delta t}. \quad (8-40)$$

Similarly, the **instantaneous power** due to the force is

$$P = \frac{dE}{dt}. \quad (8-41)$$

Sample Problem

Energy, friction, spring, and tamales

In Fig. 8-17, a 2.0 kg package of tamales slides along a floor with speed $v_1 = 4.0$ m/s. It then runs into and compresses a spring, until the package momentarily stops. Its path to the initially relaxed spring is frictionless, but as it compresses the spring, a kinetic frictional force from the floor, of magnitude 15 N, acts on the package. If $k = 10\,000$ N/m, by what distance d is the spring compressed when the package stops?

KEY IDEAS

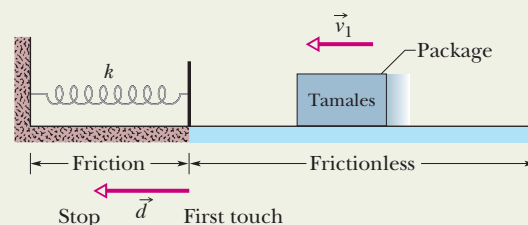
We need to examine all the forces and then to determine whether we have an isolated system or a system on which an external force is doing work.

Forces: The normal force on the package from the floor does no work on the package because the direction of this force is always perpendicular to the direction of the package's displacement. For the same reason, the gravitational force on the package does no work. As the spring is compressed, however, a spring force does work on the package, transferring energy to elastic potential energy of the spring. The spring force also pushes against a rigid wall. Because there is friction between the package and the floor, the sliding of the package across the floor increases their thermal energies.

System: The package–spring–floor–wall system includes all these forces and energy transfers in one isolated system. Therefore, because the system is isolated, its total energy cannot change. We can then apply the law of conservation of energy in the form of Eq. 8-37 to the system:

$$E_{\text{mec},2} = E_{\text{mec},1} - \Delta E_{\text{th}} \quad (8-42)$$

Calculations: In Eq. 8-42, let subscript 1 correspond to the initial state of the sliding package and subscript 2 correspond to the state in which the package is momentarily stopped and the spring is compressed by distance d . For both states the mechanical energy of the system is the sum



During the rubbing, kinetic energy is transferred to potential energy and thermal energy.

Fig. 8-17 A package slides across a frictionless floor with velocity \vec{v}_1 toward a spring of spring constant k . When the package reaches the spring, a frictional force from the floor acts on the package.

of the package's kinetic energy ($K = \frac{1}{2}mv^2$) and the spring's potential energy ($U = \frac{1}{2}kx^2$). For state 1, $U = 0$ (because the spring is not compressed), and the package's speed is v_1 . Thus, we have

$$E_{\text{mec},1} = K_1 + U_1 = \frac{1}{2}mv_1^2 + 0.$$

For state 2, $K = 0$ (because the package is stopped), and the compression distance is d . Therefore, we have

$$E_{\text{mec},2} = K_2 + U_2 = 0 + \frac{1}{2}kd^2.$$

Finally, by Eq. 8-31, we can substitute $f_k d$ for the change ΔE_{th} in the thermal energy of the package and the floor. We can now rewrite Eq. 8-42 as

$$\frac{1}{2}kd^2 = \frac{1}{2}mv_1^2 - f_k d.$$

Rearranging and substituting known data give us

$$5000d^2 + 15d - 16 = 0.$$

Solving this quadratic equation yields

$$d = 0.055 \text{ m} = 5.5 \text{ cm.} \quad (\text{Answer})$$



Additional examples, video, and practice available at WileyPLUS

REVIEW & SUMMARY

Conservative Forces A force is a **conservative force** if the net work it does on a particle moving around any closed path, from an initial point and then back to that point, is zero. Equivalently, a force is conservative if the net work it does on a particle moving between two points does not depend on the path taken by the particle. The gravitational force and the spring force are conservative forces; the kinetic frictional force is a **nonconservative force**.

Potential Energy A **potential energy** is energy that is associated with the configuration of a system in which a conservative force acts. When the conservative force does work W on a particle within the system, the change ΔU in the potential energy of the system is

$$\Delta U = -W. \quad (8-1)$$

If the particle moves from point x_i to point x_f , the change in the potential energy of the system is

$$\Delta U = - \int_{x_i}^{x_f} F(x) dx. \quad (8-6)$$

Gravitational Potential Energy The potential energy associated with a system consisting of Earth and a nearby particle is **gravitational potential energy**. If the particle moves from height y_i

to height y_f , the change in the gravitational potential energy of the particle–Earth system is

$$\Delta U = mg(y_f - y_i) = mg \Delta y. \quad (8-7)$$

If the **reference point** of the particle is set as $y_i = 0$ and the corresponding gravitational potential energy of the system is set as $U_i = 0$, then the gravitational potential energy U when the particle is at any height y is

$$U(y) = mgy. \quad (8-9)$$

Elastic Potential Energy Elastic potential energy is the energy associated with the state of compression or extension of an elastic object. For a spring that exerts a spring force $F = -kx$ when its free end has displacement x , the elastic potential energy is

$$U(x) = \frac{1}{2}kx^2. \quad (8-11)$$

The **reference configuration** has the spring at its relaxed length, at which $x = 0$ and $U = 0$.

Mechanical Energy The **mechanical energy** E_{mec} of a system is the sum of its kinetic energy K and potential energy U :

$$E_{\text{mec}} = K + U. \quad (8-12)$$

An *isolated system* is one in which no *external force* causes energy changes. If only conservative forces do work within an isolated system, then the mechanical energy E_{mec} of the system cannot change. This **principle of conservation of mechanical energy** is written as

$$K_2 + U_2 = K_1 + U_1, \quad (8-17)$$

in which the subscripts refer to different instants during an energy transfer process. This conservation principle can also be written as

$$\Delta E_{\text{mec}} = \Delta K + \Delta U = 0. \quad (8-18)$$

Potential Energy Curves If we know the potential energy function $U(x)$ for a system in which a one-dimensional force $F(x)$ acts on a particle, we can find the force as

$$F(x) = -\frac{dU(x)}{dx}. \quad (8-22)$$

If $U(x)$ is given on a graph, then at any value of x , the force $F(x)$ is the negative of the slope of the curve there and the kinetic energy of the particle is given by

$$K(x) = E_{\text{mec}} - U(x), \quad (8-24)$$

where E_{mec} is the mechanical energy of the system. A **turning point**

is a point x at which the particle reverses its motion (there, $K = 0$). The particle is in **equilibrium** at points where the slope of the $U(x)$ curve is zero (there, $F(x) = 0$).

Work Done on a System by an External Force Work W is energy transferred to or from a system by means of an external force acting on the system. When more than one force acts on a system, their *net work* is the transferred energy. When friction is not involved, the work done on the system and the change ΔE_{mec} in the mechanical energy of the system are equal:

$$W = \Delta E_{\text{mec}} = \Delta K + \Delta U. \quad (8-26, 8-25)$$

When a kinetic frictional force acts within the system, then the thermal energy E_{th} of the system changes. (This energy is associated with the random motion of atoms and molecules in the system.) The work done on the system is then

$$W = \Delta E_{\text{mec}} + \Delta E_{\text{th}}. \quad (8-33)$$

The change ΔE_{th} is related to the magnitude f_k of the frictional force and the magnitude d of the displacement caused by the external force by

$$\Delta E_{\text{th}} = f_k d. \quad (8-31)$$

Conservation of Energy The **total energy** E of a system (the sum of its mechanical energy and its internal energies, including thermal energy) can change only by amounts of energy that are transferred to or from the system. This experimental fact is known as the **law of conservation of energy**. If work W is done on the system, then

$$W = \Delta E = \Delta E_{\text{mec}} + \Delta E_{\text{th}} + \Delta E_{\text{int}}. \quad (8-35)$$

If the system is isolated ($W = 0$), this gives

$$\Delta E_{\text{mec}} + \Delta E_{\text{th}} + \Delta E_{\text{int}} = 0 \quad (8-36)$$

and

$$E_{\text{mec},2} = E_{\text{mec},1} - \Delta E_{\text{th}} - \Delta E_{\text{int}}, \quad (8-37)$$

where the subscripts 1 and 2 refer to two different instants.

Power The **power** due to a force is the *rate* at which that force transfers energy. If an amount of energy ΔE is transferred by a force in an amount of time Δt , the **average power** of the force is

$$P_{\text{avg}} = \frac{\Delta E}{\Delta t}. \quad (8-40)$$

The **instantaneous power** due to a force is

$$P = \frac{dE}{dt}. \quad (8-41)$$

1 In Fig. 8-18, a horizontally moving block can take three frictionless routes, differing only in elevation, to reach the dashed finish line.

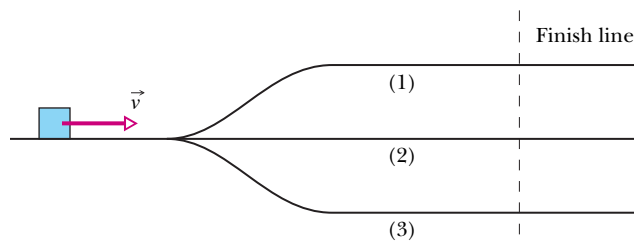


Fig. 8-18 Question 1.

Rank the routes according to (a) the speed of the block at the finish line and (b) the travel time of the block to the finish line, greatest first.

2 Figure 8-19 gives the potential energy function of a particle.

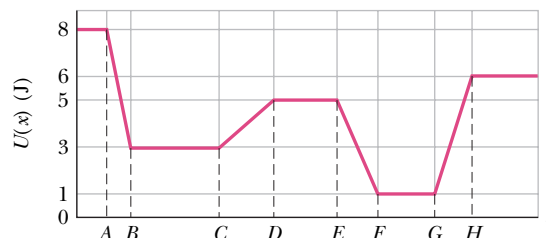


Fig. 8-19 Question 2.

(a) Rank regions AB , BC , CD , and DE according to the magnitude of the force on the particle, greatest first. What value must the mechanical energy E_{mec} of the particle not exceed if the particle is to be (b) trapped in the potential well at the left, (c) trapped in the potential well at the right, and (d) able to move between the two potential wells but not to the right of point H ? For the situation of (d), in which of regions BC , DE , and FG will the particle have (e) the greatest kinetic energy and (f) the least speed?

3 Figure 8-20 shows one direct path and four indirect paths from point i to point f . Along the direct path and three of the indirect paths, only a conservative force F_c acts on a certain object. Along the fourth indirect path, both F_c and a nonconservative force F_{nc} act on the object. The change ΔE_{mec} in the object's mechanical energy (in joules) in going from i to f is indicated along each straight-line segment of the indirect paths. What is ΔE_{mec} (a) from i to f along the direct path and (b) due to F_{nc} along the one path where it acts?

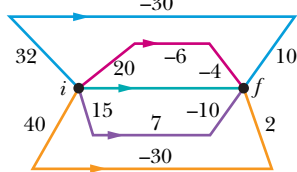


Fig. 8-20 Question 3.

4 In Fig. 8-21, a small, initially stationary block is released on a frictionless ramp at a height of 3.0 m. Hill heights along the ramp are as shown. The hills have identical circular tops, and the block does not fly off any hill. (a) Which hill is the first the block cannot cross? (b) What does the block do after failing to cross that hill? On which hilltop is (c) the centripetal acceleration of the block greatest and (d) the normal force on the block least?

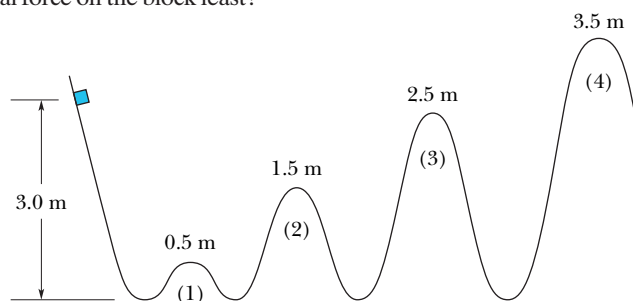


Fig. 8-21 Question 4.

5 In Fig. 8-22, a block slides from A to C along a frictionless ramp, and then it passes through horizontal region CD , where a frictional force acts on it. Is the block's kinetic energy increasing, decreasing, or constant in (a) region AB , (b) region BC , and (c) region CD ? (d) Is the block's mechanical energy increasing, decreasing, or constant in those regions?

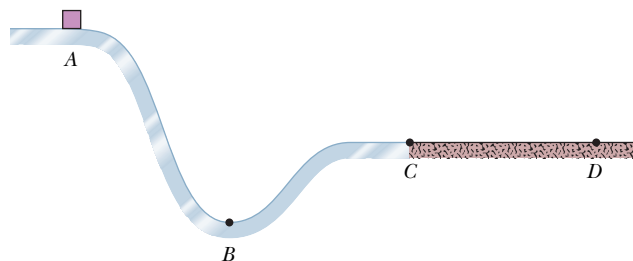


Fig. 8-22 Question 5.

6 In Fig. 8-23a, you pull upward on a rope that is attached to a cylinder on a vertical rod. Because the cylinder fits tightly on the rod, the cylinder slides along the rod with considerable friction. Your force

does work $W = +100 \text{ J}$ on the cylinder-rod-Earth system (Fig. 8-23b). An "energy statement" for the system is shown in Fig. 8-23c: the kinetic energy K increases by 50 J, and the gravitational potential energy U_g increases by 20 J. The only other change in energy within the system is for the thermal energy E_{th} . What is the change ΔE_{th} ?

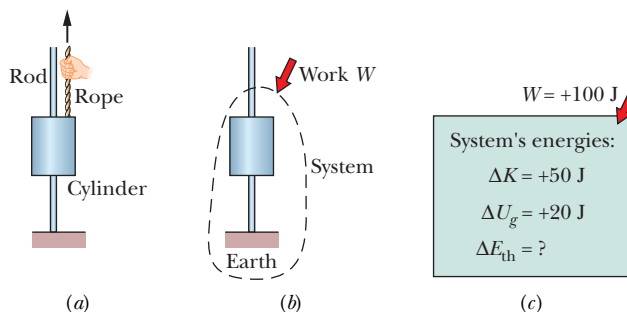


Fig. 8-23 Question 6.

7 The arrangement shown in Fig. 8-24 is similar to that in Question 6. Here you pull downward on the rope that is attached to the cylinder, which fits tightly on the rod. Also, as the cylinder descends, it pulls on a block via a second rope, and the block slides over a lab table. Again consider the cylinder-rod-Earth system, similar to that shown in Fig. 8-23b. Your work on the system is 200 J. The system does work of 60 J on the block. Within the system, the kinetic energy increases by 130 J and the gravitational potential energy decreases by 20 J. (a) Draw an "energy statement" for the system, as in Fig. 8-23c. (b) What is the change in the thermal energy within the system?

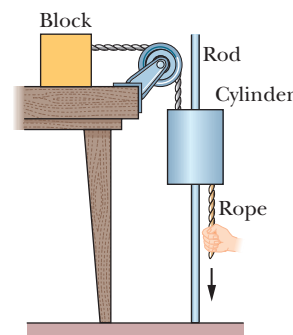


Fig. 8-24 Question 7.

8 In Fig. 8-25, a block slides along a track that descends through distance h . The track is frictionless except for the lower section. There the block slides to a stop in a certain distance D because of friction. (a) If we decrease h , will the block now slide to a stop in a distance that is greater than, less than, or equal to D ? (b) If, instead, we increase the mass of the block, will the stopping distance now be greater than, less than, or equal to D ?

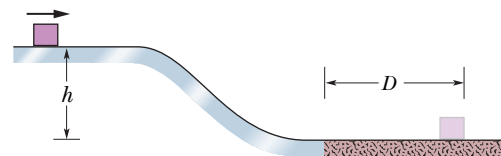


Fig. 8-25 Question 8.

9 Figure 8-26 shows three situations involving a plane that is not frictionless and a block sliding along the plane. The block begins with the same speed in all three situations and slides until the kinetic frictional force has stopped it. Rank the situations according to the increase in thermal energy due to the sliding, greatest first.

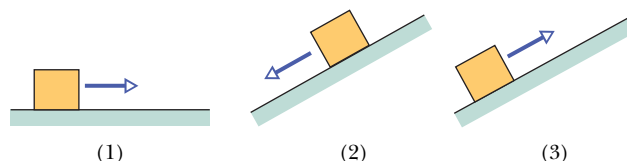


Fig. 8-26 Question 9.



Tutoring problem available (at instructor's discretion) in WileyPLUS and WebAssign



Worked-out solution available in Student Solutions Manual



Worked-out solution is at

<http://www.wiley.com/college/halliday>

Number of dots indicates level of problem difficulty

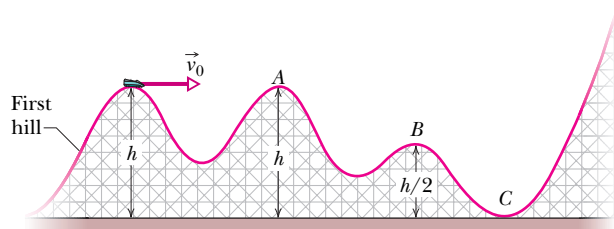


Interactive solution is at

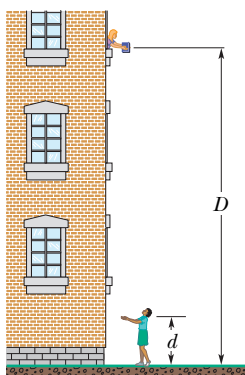
Additional information available in *The Flying Circus of Physics* and at flyingcircusofphysics.com**sec. 8-4 Determining Potential Energy Values**

•1 **SSM** What is the spring constant of a spring that stores 25 J of elastic potential energy when compressed by 7.5 cm?

•2 In Fig. 8-27, a single frictionless roller-coaster car of mass $m = 825$ kg tops the first hill with speed $v_0 = 17.0$ m/s at height $h = 42.0$ m. How much work does the gravitational force do on the car from that point to (a) point A, (b) point B, and (c) point C? If the gravitational potential energy of the car–Earth system is taken to be zero at C, what is its value when the car is at (d) B and (e) A? (f) If mass m were doubled, would the change in the gravitational potential energy of the system between points A and B increase, decrease, or remain the same?

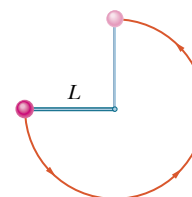
**Fig. 8-27** Problems 2 and 9.

•3 You drop a 2.00 kg book to a friend who stands on the ground at distance $D = 10.0$ m below. If your friend's outstretched hands are at distance $d = 1.50$ m above the ground (Fig. 8-28), (a) how much work W_g does the gravitational force do on the book as it drops to her hands? (b) What is the change ΔU in the gravitational potential energy of the book–Earth system during the drop? If the gravitational potential energy U of that system is taken to be zero at ground level, what is U (c) when the book is released and (d) when it reaches her hands? Now take U to be 100 J at ground level and again find (e) W_g , (f) ΔU , (g) U at the release point, and (h) U at her hands.

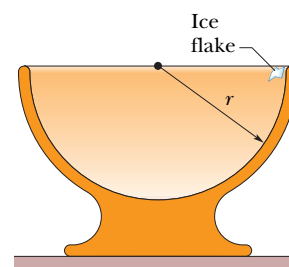
**Fig. 8-28** Problems 3 and 10.

•4 Figure 8-29 shows a ball with mass $m = 0.341$ kg attached to the end of a thin rod with length $L = 0.452$ m and negligible mass. The other end of the rod is pivoted so that the ball can move in a vertical circle. The rod is held horizontally as shown and then given enough of a downward push to cause the ball to swing down and around and just reach the vertically up position, with zero speed there. How much work is done on the ball by the gravitational force from the initial point to (a) the lowest point, (b) the highest point, and (c) the point on the right level with the initial point? If the gravitational potential energy of the ball–Earth system is

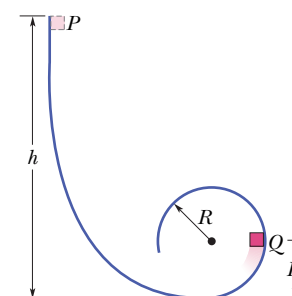
taken to be zero at the initial point, what is it when the ball reaches (d) the lowest point, (e) the highest point, and (f) the point on the right level with the initial point? (g) Suppose the rod were pushed harder so that the ball passed through the highest point with a nonzero speed. Would ΔU_g from the lowest point to the highest point then be greater than, less than, or the same as it was when the ball stopped at the highest point?

**Fig. 8-29** Problems 4 and 14.

•5 **SSM** In Fig. 8-30, a 2.00 g ice flake is released from the edge of a hemispherical bowl whose radius r is 22.0 cm. The flake–bowl contact is frictionless. (a) How much work is done on the flake by the gravitational force during the flake's descent to the bottom of the bowl? (b) What is the change in the potential energy of the flake–Earth system during that descent? (c) If that potential energy is taken to be zero at the bottom of the bowl, what is its value when the flake is released? (d) If, instead, the potential energy is taken to be zero at the release point, what is its value when the flake reaches the bottom of the bowl? (e) If the mass of the flake were doubled, would the magnitudes of the answers to (a) through (d) increase, decrease, or remain the same?

**Fig. 8-30** Problems 5 and 11.

••6 In Fig. 8-31, a small block of mass $m = 0.032$ kg can slide along the frictionless loop-the-loop, with loop radius $R = 12$ cm. The block is released from rest at point P, at height $h = 5.0R$ above the bottom of the loop. How much work does the gravitational force do on the block as the block travels from point P to (a) point Q and (b) the top of the loop? If the gravitational potential energy of the block–Earth system is taken to be zero at the bottom of the loop, what is that potential energy when the block is (c) at point P, (d) at point Q, and (e) at the top of the loop? (f) If, instead of merely being released, the block is given some initial speed downward along the track, do the answers to (a) through (e) increase, decrease, or remain the same?

**Fig. 8-31** Problems 6 and 17.

••7 Figure 8-32 shows a thin rod, of length $L = 2.00$ m and negligible mass, that can pivot about one end to rotate in a vertical circle. A ball of mass $m = 5.00$ kg is attached to the other end. The rod is pulled aside to angle $\theta_0 = 30.0^\circ$ and released with initial velocity $\vec{v}_0 = 0$. As the ball descends to its lowest point, (a) how much work does the gravitational force do on it and (b) what is the change in the gravitational potential energy of the ball–Earth system? (c) If the gravitational potential energy is taken to be zero at the lowest point, what is its value just as the ball is released? (d) Do the magnitudes of the answers to (a) through (c) increase, decrease, or remain the same if angle θ_0 is increased?

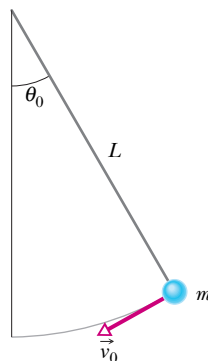


Fig. 8-32
Problems 7, 18,
and 21.

••8 A 1.50 kg snowball is fired from a cliff 12.5 m high. The snowball's initial velocity is 14.0 m/s, directed 41.0° above the horizontal. (a) How much work is done on the snowball by the gravitational force during its flight to the flat ground below the cliff? (b) What is the change in the gravitational potential energy of the snowball–Earth system during the flight? (c) If that gravitational potential energy is taken to be zero at the height of the cliff, what is its value when the snowball reaches the ground?

sec. 8-5 Conservation of Mechanical Energy

•9 **GO** In Problem 2, what is the speed of the car at (a) point A, (b) point B, and (c) point C? (d) How high will the car go on the last hill, which is too high for it to cross? (e) If we substitute a second car with twice the mass, what then are the answers to (a) through (d)?

•10 (a) In Problem 3, what is the speed of the book when it reaches the hands? (b) If we substituted a second book with twice the mass, what would its speed be? (c) If, instead, the book were thrown down, would the answer to (a) increase, decrease, or remain the same?

•11 **SSM WWW** (a) In Problem 5, what is the speed of the flake when it reaches the bottom of the bowl? (b) If we substituted a second flake with twice the mass, what would its speed be? (c) If, instead, we gave the flake an initial downward speed along the bowl, would the answer to (a) increase, decrease, or remain the same?

•12 (a) In Problem 8, using energy techniques rather than the techniques of Chapter 4, find the speed of the snowball as it reaches the ground below the cliff. What is that speed (b) if the launch angle is changed to 41.0° below the horizontal and (c) if the mass is changed to 2.50 kg?

•13 **SSM** A 5.0 g marble is fired vertically upward using a spring gun. The spring must be compressed 8.0 cm if the marble is to just reach a target 20 m above the marble's position on the compressed spring. (a) What is the change ΔU_g in the gravitational potential energy of the marble–Earth system during the 20 m ascent? (b) What is the change ΔU_s in the elastic potential energy of the spring during its launch of the marble? (c) What is the spring constant of the spring?

•14 (a) In Problem 4, what initial speed must be given the ball so that it reaches the vertically upward position with zero speed? What then is its speed at (b) the lowest point and (c) the point on the right at which the ball is level with the initial point? (d) If the ball's mass

were doubled, would the answers to (a) through (c) increase, decrease, or remain the same?

•15 **SSM** In Fig. 8-33, a runaway truck with failed brakes is moving downgrade at 130 km/h just before the driver steers the truck up a frictionless emergency escape ramp with an inclination of $\theta = 15^\circ$. The truck's mass is 1.2×10^4 kg. (a) What minimum length L must the ramp have if the truck is to stop (momentarily) along it? (Assume the truck is a particle, and justify that assumption.) Does the minimum length L increase, decrease, or remain the same if (b) the truck's mass is decreased and (c) its speed is decreased?

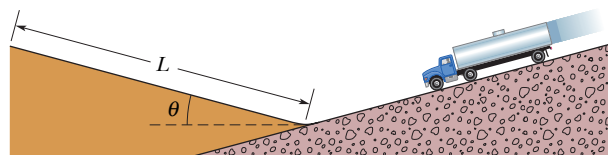


Fig. 8-33 Problem 15.

••16 A 700 g block is released from rest at height h_0 above a vertical spring with spring constant $k = 400$ N/m and negligible mass. The block sticks to the spring and momentarily stops after compressing the spring 19.0 cm. How much work is done (a) by the block on the spring and (b) by the spring on the block? (c) What is the value of h_0 ? (d) If the block were released from height $2.00h_0$ above the spring, what would be the maximum compression of the spring?

••17 In Problem 6, what are the magnitudes of (a) the horizontal component and (b) the vertical component of the *net* force acting on the block at point Q? (c) At what height h should the block be released from rest so that it is on the verge of losing contact with the track at the top of the loop? (*On the verge of losing contact* means that the normal force on the block from the track has just then become zero.) (d) Graph the magnitude of the normal force on the block at the top of the loop versus initial height h , for the range $h = 0$ to $h = 6R$.

••18 (a) In Problem 7, what is the speed of the ball at the lowest point? (b) Does the speed increase, decrease, or remain the same if the mass is increased?

••19 **GO** Figure 8-34 shows an 8.00 kg stone at rest on a spring. The spring is compressed 10.0 cm by the stone. (a) What is the spring constant? (b) The stone is pushed down an additional 30.0 cm and released. What is the elastic potential energy of the compressed spring just before that release? (c) What is the change in the gravitational potential energy of the stone–Earth system when the stone moves from the release point to its maximum height? (d) What is that maximum height, measured from the release point?

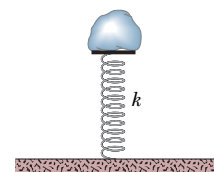



Fig. 8-34
Problem 19.

••20 **GO** A pendulum consists of a 2.0 kg stone swinging on a 4.0 m string of negligible mass. The stone has a speed of 8.0 m/s when it passes its lowest point. (a) What is the speed when the string is at 60° to the vertical? (b) What is the greatest angle with the vertical that the string will reach during the stone's motion? (c) If the potential energy of the pendulum–Earth system is taken to be zero at the stone's lowest point, what is the total mechanical energy of the system?

•21 Figure 8-32 shows a pendulum of length $L = 1.25$ m. Its bob (which effectively has all the mass) has speed v_0 when the cord makes an angle $\theta_0 = 40.0^\circ$ with the vertical. (a) What is the speed of the bob when it is in its lowest position if $v_0 = 8.00$ m/s? What is the least value that v_0 can have if the pendulum is to swing down and then up (b) to a horizontal position, and (c) to a vertical position with the cord remaining straight? (d) Do the answers to (b) and (c) increase, decrease, or remain the same if θ_0 is increased by a few degrees?

•22  A 60 kg skier starts from rest at height $H = 20$ m above the end of a ski-jump ramp (Fig. 8-35) and leaves the ramp at angle $\theta = 28^\circ$. Neglect the effects of air resistance and assume the ramp is frictionless. (a) What is the maximum height h of his jump above the end of the ramp? (b) If he increased his weight by putting on a backpack, would h then be greater, less, or the same?

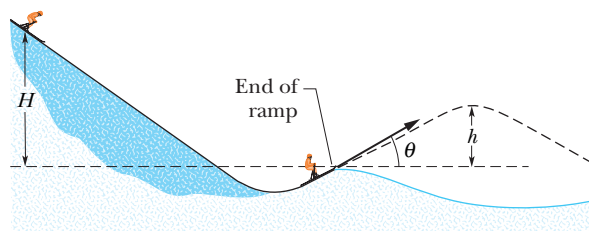




Fig. 8-35 Problem 22.

•23   The string in Fig. 8-36 is $L = 120$ cm long, has a ball attached to one end, and is fixed at its other end. The distance d from the fixed end to a fixed peg at point P is 75.0 cm. When the initially stationary ball is released with the string horizontal as shown, it will swing along the dashed arc. What is its speed when it reaches (a) its lowest point and (b) its highest point after the string catches on the peg?

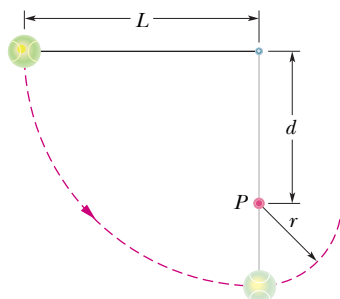


Fig. 8-36 Problems 23 and 70.

•24 A block of mass $m = 2.0$ kg is dropped from height $h = 40$ cm onto a spring of spring constant $k = 1960$ N/m (Fig. 8-37). Find the maximum distance the spring is compressed.

•25 At $t = 0$ a 1.0 kg ball is thrown from a tall tower with $\vec{v} = (18 \text{ m/s})\hat{i} + (24 \text{ m/s})\hat{j}$. What is ΔU of the ball–Earth system between $t = 0$ and $t = 6.0$ s (still free fall)?

•26 A conservative force $\vec{F} = (6.0x - 12)\hat{i}$ N, where x is in meters, acts on a particle moving along an x axis. The potential energy U associated with this force is assigned a value of 27 J at $x = 0$. (a) Write an expression for U as a function of x ,

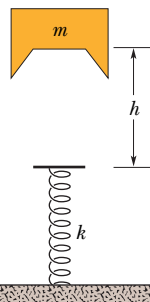


Fig. 8-37 Problem 24.

with U in joules and x in meters. (b) What is the maximum positive potential energy? At what (c) negative value and (d) positive value of x is the potential energy equal to zero?

•27 Tarzan, who weighs 688 N, swings from a cliff at the end of a vine 18 m long (Fig. 8-38). From the top of the cliff to the bottom of the swing, he descends by 3.2 m. The vine will break if the force on it exceeds 950 N. (a) Does the vine break? (b) If no, what is the greatest force on it during the swing? If yes, at what angle with the vertical does it break?

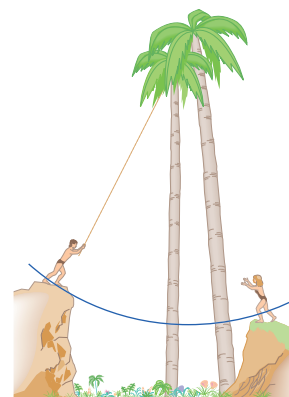


Fig. 8-38 Problem 27.

•28 Figure 8-39a applies to the spring in a cork gun (Fig. 8-39b); it shows the spring force as a function of the stretch or compression of the spring. The spring is compressed by 5.5 cm and used to propel a 3.8 g cork from the gun. (a) What is the speed of the cork if it is released as the spring passes through its relaxed position? (b) Suppose, instead, that the cork sticks to the spring and stretches it 1.5 cm before separation occurs. What now is the speed of the cork at the time of release?

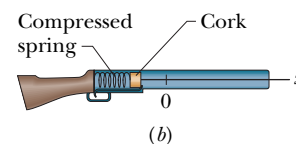
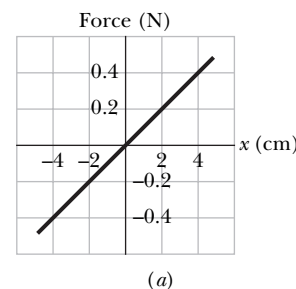




Fig. 8-39 Problem 28.

•29   In Fig. 8-40, a block of mass $m = 12$ kg is released from rest on a frictionless incline of angle $\theta = 30^\circ$. Below the block is a spring that can be compressed 2.0 cm by a force of 270 N. The block momentarily stops when it compresses the spring by 5.5 cm. (a) How far does the block move down the incline from

its rest position to this stopping point? (b) What is the speed of the block just as it touches the spring?

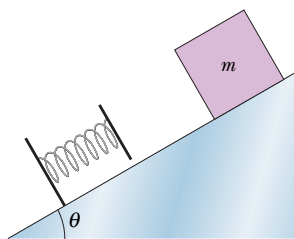


Fig. 8-40 Problems 29 and 35.

••30 GO A 2.0 kg breadbox on a frictionless incline of angle $\theta = 40^\circ$ is connected, by a cord that runs over a pulley, to a light spring of spring constant $k = 120 \text{ N/m}$, as shown in Fig. 8-41. The box is released from rest when the spring is unstretched. Assume that the pulley is massless and frictionless. (a) What is the speed of the box when it has moved 10 cm down the incline? (b) How far down the incline from its point of release does the box slide before momentarily stopping, and what are the (c) magnitude and (d) direction (up or down the incline) of the box's acceleration at the instant the box momentarily stops?

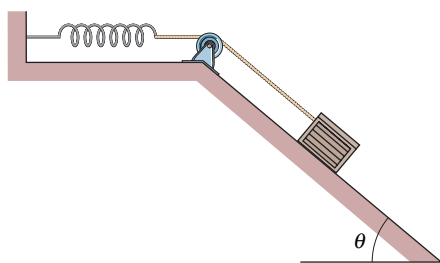


Fig. 8-41 Problem 30.

••31 ILW A block with mass $m = 2.00 \text{ kg}$ is placed against a spring on a frictionless incline with angle $\theta = 30.0^\circ$ (Fig. 8-42). (The block is not attached to the spring.) The spring, with spring constant $k = 19.6 \text{ N/cm}$, is compressed 20.0 cm and then released. (a) What is the elastic potential energy of the compressed spring? (b) What is the change in the gravitational potential energy of the block–Earth system as the block moves from the release point to its highest point on the incline? (c) How far along the incline is the highest point from the release point?

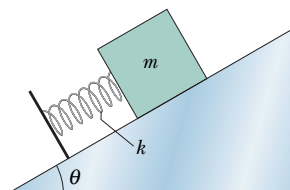


Fig. 8-42 Problem 31.

••32 In Fig. 8-43, a chain is held on a frictionless table with one-fourth of its length hanging over the edge. If the chain has length

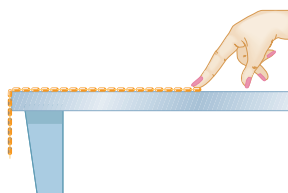


Fig. 8-43 Problem 32.

$L = 28 \text{ cm}$ and mass $m = 0.012 \text{ kg}$, how much work is required to pull the hanging part back onto the table?

••33 GO In Fig. 8-44, a spring with $k = 170 \text{ N/m}$ is at the top of a frictionless incline of angle $\theta = 37.0^\circ$. The lower end of the incline is distance $D = 1.00 \text{ m}$ from the end of the spring, which is at its relaxed length. A 2.00 kg canister is pushed against the spring until the spring is compressed 0.200 m and released from rest. (a) What is the speed of the canister at the instant the spring returns to its relaxed length (which is when the canister loses contact with the spring)? (b) What is the speed of the canister when it reaches the lower end of the incline?

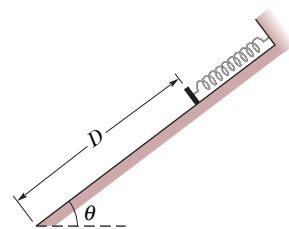


Fig. 8-44 Problem 33.

••34 GO A boy is initially seated on the top of a hemispherical ice mound of radius $R = 13.8 \text{ m}$. He begins to slide down the ice, with a negligible initial speed (Fig. 8-45). Approximate the ice as being frictionless. At what height does the boy lose contact with the ice?

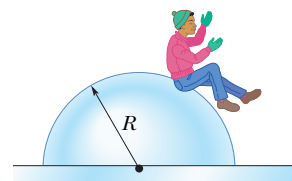


Fig. 8-45 Problem 34.

••35 In Fig. 8-40, a block of mass $m = 3.20 \text{ kg}$ slides from rest a distance d down a frictionless incline at angle $\theta = 30.0^\circ$ where it runs into a spring of spring constant 431 N/m . When the block momentarily stops, it has compressed the spring by 21.0 cm. What are (a) distance d and (b) the distance between the point of the first block–spring contact and the point where the block's speed is greatest?

••36 GO Two children are playing a game in which they try to hit a small box on the floor with a marble fired from a spring-loaded gun that is mounted on a table. The target box is horizontal distance $D = 2.20 \text{ m}$ from the edge of the table; see Fig. 8-46. Bobby

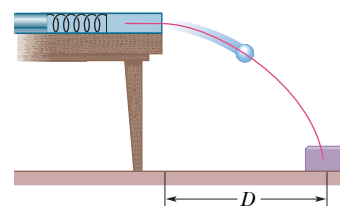


Fig. 8-46 Problem 36.

compresses the spring 1.10 cm, but the center of the marble falls 27.0 cm short of the center of the box. How far should Rhoda compress the spring to score a direct hit? Assume that neither the spring nor the ball encounters friction in the gun.

••37 A uniform cord of length 25 cm and mass 15 g is initially stuck to a ceiling. Later, it hangs vertically from the ceiling with only one end still stuck. What is the change in the gravitational potential energy of the cord with this change in orientation? (*Hint*: Consider a differential slice of the cord and then use integral calculus.)

sec. 8-6 Reading a Potential Energy Curve

••38 Figure 8-47 shows a plot of potential energy U versus position x of a 0.200 kg particle that can travel only along an x axis under the influence of a conservative force. The graph has these values: $U_A = 9.00$ J, $U_C = 20.00$ J, and $U_D = 24.00$ J. The particle is released at the point where U forms a “potential hill” of “height” $U_B = 12.00$ J, with kinetic energy 4.00 J. What is the speed of the particle at (a) $x = 3.5$ m and (b) $x = 6.5$ m? What is the position of the turning point on (c) the right side and (d) the left side?

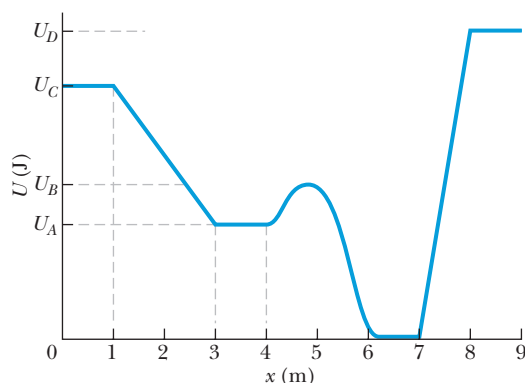


Fig. 8-47 Problem 38.

••39 Figure 8-48 shows a plot of potential energy U versus position x of a 0.90 kg particle that can travel only along an x axis. (Nonconservative forces are not involved.) Three values are $U_A = 15.0$ J, $U_B = 35.0$ J, and $U_C = 45.0$ J. The particle is released at $x = 4.5$ m with an initial speed of 7.0 m/s, headed in the negative x direction. (a) If the particle can reach $x = 1.0$ m, what is its speed there, and if it cannot, what is its turning point? What are the (b) magnitude and (c) direction of the force on the particle as it begins to move to the left of $x = 4.0$ m? Suppose, instead, the particle is headed in the positive x direction when it is released at $x = 4.5$ m at speed 7.0 m/s. (d) If the particle can reach $x = 7.0$ m, what is its speed there, and if it cannot, what is its turning point? What are the (e) magnitude and (f) direction of the force on the particle as it begins to move to the right of $x = 5.0$ m?

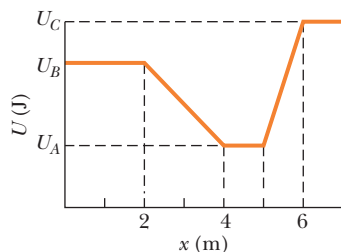


Fig. 8-48 Problem 39.

••40 The potential energy of a diatomic molecule (a two-atom system like H_2 or O_2) is given by

$$U = \frac{A}{r^{12}} - \frac{B}{r^6},$$

where r is the separation of the two atoms of the molecule and A and B are positive constants. This potential energy is associated with the force that binds the two atoms together. (a) Find the *equilibrium separation*—that is, the distance between the atoms at which the force on each atom is zero. Is the force repulsive (the atoms are pushed apart) or attractive (they are pulled together) if their separation is (b) smaller and (c) larger than the equilibrium separation?

••41 A single conservative force $F(x)$ acts on a 1.0 kg particle that moves along an x axis. The potential energy $U(x)$ associated with $F(x)$ is given by

$$U(x) = -4x e^{-x/4} \text{ J},$$

where x is in meters. At $x = 5.0$ m the particle has a kinetic energy of 2.0 J. (a) What is the mechanical energy of the system? (b) Make a plot of $U(x)$ as a function of x for $0 \leq x \leq 10$ m, and on the same graph draw the line that represents the mechanical energy of the system. Use part (b) to determine (c) the least value of x the particle can reach and (d) the greatest value of x the particle can reach. Use part (b) to determine (e) the maximum kinetic energy of the particle and (f) the value of x at which it occurs. (g) Determine an expression in newtons and meters for $F(x)$ as a function of x . (h) For what (finite) value of x does $F(x) = 0$?

sec. 8-7 Work Done on a System by an External Force

•42 A worker pushed a 27 kg block 9.2 m along a level floor at constant speed with a force directed 32° below the horizontal. If the coefficient of kinetic friction between block and floor was 0.20, what were (a) the work done by the worker's force and (b) the increase in thermal energy of the block–floor system?

•43 A collie drags its bed box across a floor by applying a horizontal force of 8.0 N. The kinetic frictional force acting on the box has magnitude 5.0 N. As the box is dragged through 0.70 m along the way, what are (a) the work done by the collie's applied force and (b) the increase in thermal energy of the bed and floor?

••44 A horizontal force of magnitude 35.0 N pushes a block of mass 4.00 kg across a floor where the coefficient of kinetic friction is 0.600. (a) How much work is done by that applied force on the block–floor system when the block slides through a displacement of 3.00 m across the floor? (b) During that displacement, the thermal energy of the block increases by 40.0 J. What is the increase in thermal energy of the floor? (c) What is the increase in the kinetic energy of the block?

••45 A rope is used to pull a 3.57 kg block at constant speed 4.06 m along a horizontal floor. The force on the block from the rope is 7.68 N and directed 15.0° above the horizontal. What are (a) the work done by the rope's force, (b) the increase in thermal energy of the block–floor system, and (c) the coefficient of kinetic friction between the block and floor?

sec. 8-8 Conservation of Energy

•46 An outfielder throws a baseball with an initial speed of 81.8 mi/h. Just before an infielder catches the ball at the same level, the ball's speed is 110 ft/s. In foot-pounds, by how much is the mechanical energy of the ball–Earth system reduced because of air drag? (The weight of a baseball is 9.0 oz.)

•47 A 75 g Frisbee is thrown from a point 1.1 m above the ground with a speed of 12 m/s. When it has reached a height of 2.1 m, its speed is 10.5 m/s. What was the reduction in E_{mec} of the Frisbee–Earth system because of air drag?

•48 In Fig. 8-49, a block slides down an incline. As it moves from point A to point B , which are 5.0 m apart, force \vec{F} acts on the block, with magnitude 2.0 N and directed down the incline. The magnitude of the frictional force acting on the block is 10 N. If the kinetic energy of the block increases by 35 J between A and B , how much work is done on the block by the gravitational force as the block moves from A to B ?

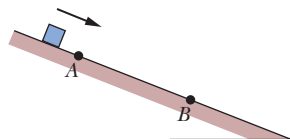


Fig. 8-49 Problems 48 and 71.

•49 SSM ILW A 25 kg bear slides, from rest, 12 m down a lodgepole pine tree, moving with a speed of 5.6 m/s just before hitting the ground. (a) What change occurs in the gravitational potential energy of the bear–Earth system during the slide? (b) What is the kinetic energy of the bear just before hitting the ground? (c) What is the average frictional force that acts on the sliding bear?

•50 A 60 kg skier leaves the end of a ski-jump ramp with a velocity of 24 m/s directed 25° above the horizontal. Suppose that as a result of air drag the skier returns to the ground with a speed of 22 m/s, landing 14 m vertically below the end of the ramp. From the launch to the return to the ground, by how much is the mechanical energy of the skier–Earth system reduced because of air drag?

•51 During a rockslide, a 520 kg rock slides from rest down a hillside that is 500 m long and 300 m high. The coefficient of kinetic friction between the rock and the hill surface is 0.25. (a) If the gravitational potential energy U of the rock–Earth system is zero at the bottom of the hill, what is the value of U just before the slide? (b) How much energy is transferred to thermal energy during the slide? (c) What is the kinetic energy of the rock as it reaches the bottom of the hill? (d) What is its speed then?

•52 A large fake cookie sliding on a horizontal surface is attached to one end of a horizontal spring with spring constant $k = 400 \text{ N/m}$; the other end of the spring is fixed in place. The cookie has a kinetic energy of 20.0 J as it passes through the spring's equilibrium position. As the cookie slides, a frictional force of magnitude 10.0 N acts on it. (a) How far will the cookie slide from the equilibrium position before coming momentarily to rest? (b) What will be the kinetic energy of the cookie as it slides back through the equilibrium position?

•53 GO In Fig. 8-50, a 3.5 kg block is accelerated from rest by a compressed spring of spring constant 640 N/m. The block leaves

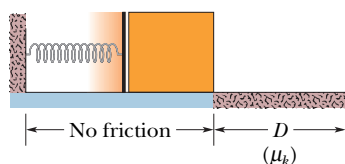


Fig. 8-50 Problem 53.

the spring at the spring's relaxed length and then travels over a horizontal floor with a coefficient of kinetic friction $\mu_k = 0.25$. The frictional force stops the block in distance $D = 7.8 \text{ m}$. What are (a) the increase in the thermal energy of the block–floor system, (b) the maximum kinetic energy of the block, and (c) the original compression distance of the spring?

•54 A child whose weight is 267 N slides down a 6.1 m playground slide that makes an angle of 20° with the horizontal. The coefficient of kinetic friction between slide and child is 0.10. (a) How much energy is transferred to thermal energy? (b) If she starts at the top with a speed of 0.457 m/s, what is her speed at the bottom?

•55 ILW In Fig. 8-51, a block of mass $m = 2.5 \text{ kg}$ slides head on into a spring of spring constant $k = 320 \text{ N/m}$. When the block stops, it has compressed the spring by 7.5 cm. The coefficient of kinetic friction between block and floor is 0.25. While the block is in contact with the spring and being brought to rest, what are (a) the work done by the spring force and (b) the increase in thermal energy of the block–floor system? (c) What is the block's speed just as it reaches the spring?

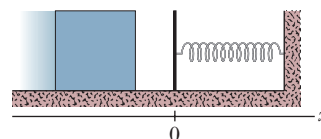


Fig. 8-51 Problem 55.

•56 You push a 2.0 kg block against a horizontal spring, compressing the spring by 15 cm. Then you release the block, and the spring sends it sliding across a tabletop. It stops 75 cm from where you released it. The spring constant is 200 N/m. What is the block–table coefficient of kinetic friction?

•57 GO In Fig. 8-52, a block slides along a track from one level to a higher level after passing through an intermediate valley. The track is frictionless until the block reaches the higher level. There a frictional force stops the block in a distance d . The block's initial speed v_0 is 6.0 m/s, the height difference h is 1.1 m, and μ_k is 0.60. Find d .

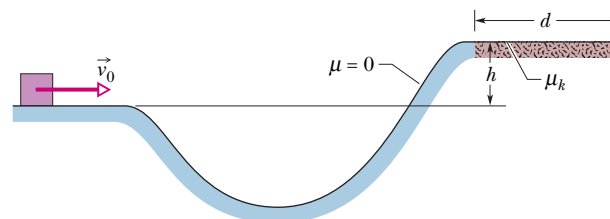


Fig. 8-52 Problem 57.

•58 A cookie jar is moving up a 40° incline. At a point 55 cm from the bottom of the incline (measured along the incline), the jar has a speed of 1.4 m/s. The coefficient of kinetic friction between jar and incline is 0.15. (a) How much farther up the incline will the jar move? (b) How fast will it be going when it has slid back to the bottom of the incline? (c) Do the answers to (a) and (b) increase, decrease, or remain the same if we decrease the coefficient of kinetic friction (but do not change the given speed or location)?

•59 A stone with a weight of 5.29 N is launched vertically from ground level with an initial speed of 20.0 m/s, and the air drag on it

is 0.265 N throughout the flight. What are (a) the maximum height reached by the stone and (b) its speed just before it hits the ground?

••60 A 4.0 kg bundle starts up a 30° incline with 128 J of kinetic energy. How far will it slide up the incline if the coefficient of kinetic friction between bundle and incline is 0.30?

••61 When a click beetle is upside down on its back, it jumps upward by suddenly arching its back, transferring energy stored in a muscle to mechanical energy. This launching mechanism produces an audible click, giving the beetle its name. Videotape of a certain click-beetle jump shows that a beetle of mass $m = 4.0 \times 10^{-6}$ kg moved directly upward by 0.77 mm during the launch and then to a maximum height of $h = 0.30$ m. During the launch, what are the average magnitudes of (a) the external force on the beetle's back from the floor and (b) the acceleration of the beetle in terms of g ?

••62 GO In Fig. 8-53, a block slides along a path that is without friction until the block reaches the section of length $L = 0.75$ m, which begins at height $h = 2.0$ m on a ramp of angle $\theta = 30^\circ$. In that section, the coefficient of kinetic friction is 0.40. The block passes through point A with a speed of 8.0 m/s. If the block can reach point B (where the friction ends), what is its speed there, and if it cannot, what is its greatest height above A?

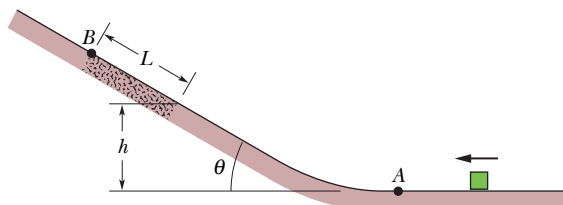


Fig. 8-53 Problem 62.

••63 The cable of the 1800 kg elevator cab in Fig. 8-54 snaps when the cab is at rest at the first floor, where the cab bottom is a distance $d = 3.7$ m above a spring of spring constant $k = 0.15$ MN/m. A safety device clamps the cab against guide rails so that a constant frictional force of 4.4 kN opposes the cab's motion. (a) Find the speed of the cab just before it hits the spring. (b) Find the maximum distance x that the spring is compressed (the frictional force still acts during this compression). (c) Find the distance that the cab will bounce back up the shaft. (d) Using conservation of energy, find the approximate total distance that the cab will move before coming to rest. (Assume that the frictional force on the cab is negligible when the cab is stationary.)

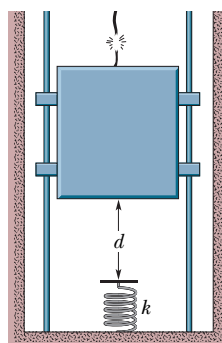


Fig. 8-54 Problem 63.

••64 In Fig. 8-55, a block is released from rest at height $d = 40$ cm and slides down a frictionless ramp and onto a first plateau, which has length d and where the coefficient of kinetic friction is 0.50. If the block is still moving, it then slides down a second frictionless ramp through height $d/2$ and onto a lower plateau, which has length $d/2$ and where the coefficient of kinetic friction is again 0.50. If the block is still moving, it then slides up a frictionless ramp until it (momentarily) stops. Where does the block stop? If its final

stop is on a plateau, state which one and give the distance L from the left edge of that plateau. If the block reaches the ramp, give the height H above the lower plateau where it momentarily stops.

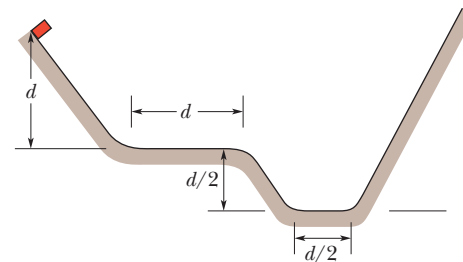


Fig. 8-55 Problem 64.

••65 A particle can slide along a track with elevated ends and a flat central part, as shown in Fig. 8-56. The flat part has length $L = 40$ cm. The curved portions of the track are frictionless, but for the flat part the coefficient of kinetic friction is $\mu_k = 0.20$. The particle is released from rest at point A, which is at height $h = L/2$. How far from the left edge of the flat part does the particle finally stop?

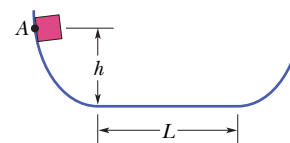


Fig. 8-56 Problem 65.

Additional Problems

66 A 3.2 kg sloth hangs 3.0 m above the ground. (a) What is the gravitational potential energy of the sloth–Earth system if we take the reference point $y = 0$ to be at the ground? If the sloth drops to the ground and air drag on it is assumed to be negligible, what are the (b) kinetic energy and (c) speed of the sloth just before it reaches the ground?

67 SSM A spring ($k = 200$ N/m) is fixed at the top of a frictionless plane inclined at angle $\theta = 40^\circ$ (Fig. 8-57). A 1.0 kg block is projected up the plane, from an initial position that is distance $d = 0.60$ m from the end of the relaxed spring, with an initial kinetic energy of 16 J. (a) What is the kinetic energy of the block at the instant it has compressed the spring 0.20 m? (b) With what kinetic energy must the block be projected up the plane if it is to stop momentarily when it has compressed the spring by 0.40 m?

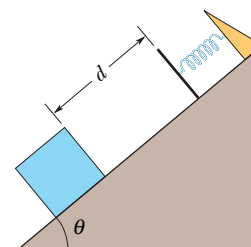


Fig. 8-57 Problem 67.

68 From the edge of a cliff, a 0.55 kg projectile is launched with an initial kinetic energy of 1550 J. The projectile's maximum upward displacement from the launch point is +140 m. What are the

(a) horizontal and (b) vertical components of its launch velocity? (c) At the instant the vertical component of its velocity is 65 m/s, what is its vertical displacement from the launch point?

69 SSM In Fig. 8-58, the pulley has negligible mass, and both it and the inclined plane are frictionless. Block *A* has a mass of 1.0 kg, block *B* has a mass of 2.0 kg, and angle θ is 30° . If the blocks are released from rest with the connecting cord taut, what is their total kinetic energy when block *B* has fallen 25 cm?

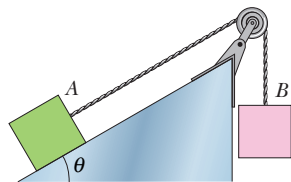


Fig. 8-58 Problem 69.

70 In Fig. 8-36, the string is $L = 120$ cm long, has a ball attached to one end, and is fixed at its other end. A fixed peg is at point *P*. Released from rest, the ball swings down until the string catches on the peg; then the ball swings up, around the peg. If the ball is to swing completely around the peg, what value must distance d exceed? (*Hint*: The ball must still be moving at the top of its swing. Do you see why?)

71 SSM In Fig. 8-49, a block is sent sliding down a frictionless ramp. Its speeds at points *A* and *B* are 2.00 m/s and 2.60 m/s, respectively. Next, it is again sent sliding down the ramp, but this time its speed at point *A* is 4.00 m/s. What then is its speed at point *B*?

72 Two snowy peaks are at heights $H = 850$ m and $h = 750$ m above the valley between them. A ski run extends between the peaks, with a total length of 3.2 km and an average slope of $\theta = 30^\circ$ (Fig. 8-59). (a) A skier starts from rest at the top of the higher peak. At what speed will he arrive at the top of the lower peak if he coasts without using ski poles? Ignore friction. (b) Approximately what coefficient of kinetic friction between snow and skis would make him stop just at the top of the lower peak?

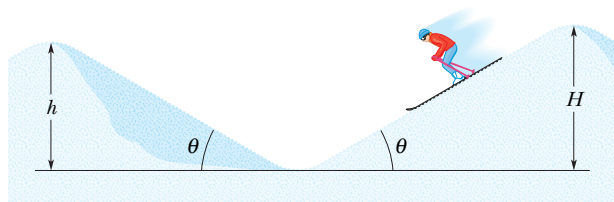


Fig. 8-59 Problem 72.

73 SSM The temperature of a plastic cube is monitored while the cube is pushed 3.0 m across a floor at constant speed by a horizontal force of 15 N. The thermal energy of the cube increases by 20 J. What is the increase in the thermal energy of the floor along which the cube slides?

74 A skier weighing 600 N goes over a frictionless circular hill of radius $R = 20$ m (Fig. 8-60). Assume

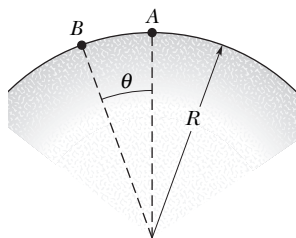


Fig. 8-60 Problem 74.

that the effects of air resistance on the skier are negligible. As she comes up the hill, her speed is 8.0 m/s at point *B*, at angle $\theta = 20^\circ$. (a) What is her speed at the hilltop (point *A*) if she coasts without using her poles? (b) What minimum speed can she have at *B* and still coast to the hilltop? (c) Do the answers to these two questions increase, decrease, or remain the same if the skier weighs 700 N instead of 600 N?

75 SSM To form a pendulum, a 0.092 kg ball is attached to one end of a rod of length 0.62 m and negligible mass, and the other end of the rod is mounted on a pivot. The rod is rotated until it is straight up, and then it is released from rest so that it swings down around the pivot. When the ball reaches its lowest point, what are (a) its speed and (b) the tension in the rod? Next, the rod is rotated until it is horizontal, and then it is again released from rest. (c) At what angle from the vertical does the tension in the rod equal the weight of the ball? (d) If the mass of the ball is increased, does the answer to (c) increase, decrease, or remain the same?

76 We move a particle along an x axis, first outward from $x = 1.0$ m to $x = 4.0$ m and then back to $x = 1.0$ m, while an external force acts on it. That force is directed along the x axis, and its x component can have different values for the outward trip and for the return trip. Here are the values (in newtons) for four situations, where x is in meters:

Outward	Inward
(a) +3.0	−3.0
(b) +5.0	+5.0
(c) +2.0 <i>x</i>	−2.0 <i>x</i>
(d) +3.0 <i>x</i> ²	+3.0 <i>x</i> ²

Find the net work done on the particle by the external force *for the round trip* for each of the four situations. (e) For which, if any, is the external force conservative?

77 SSM A conservative force $F(x)$ acts on a 2.0 kg particle that moves along an x axis. The potential energy $U(x)$ associated with $F(x)$ is graphed in Fig. 8-61. When the particle is at $x = 2.0$ m, its velocity is -1.5 m/s. What are the (a) magnitude and (b) direction of $F(x)$ at this position? Between what positions on the (c) left and (d) right does the particle move? (e) What is the particle's speed at $x = 7.0$ m?

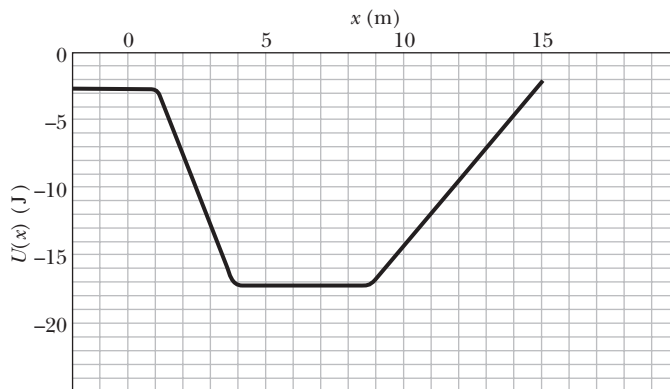


Fig. 8-61 Problem 77.

78 At a certain factory, 300 kg crates are dropped vertically from a packing machine onto a conveyor belt moving at 1.20 m/s (Fig. 8-62). (A motor maintains the belt's constant speed.) The coefficient of kinetic friction between the belt and each crate is 0.400. After a short time, slipping between the belt and the crate ceases, and the crate then moves along with the belt. For the period of time during which the crate is being brought to rest relative to the belt, calculate, for a coordinate system at rest in the factory, (a) the kinetic energy supplied to the crate, (b) the magnitude of the kinetic frictional force acting on the crate, and (c) the energy supplied by the motor. (d) Explain why answers (a) and (c) differ.

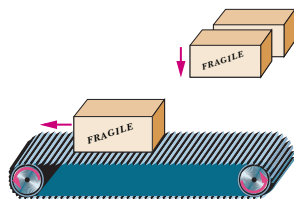


Fig. 8-62 Problem 78.

79 SSM A 1500 kg car begins sliding down a 5.0° inclined road with a speed of 30 km/h. The engine is turned off, and the only forces acting on the car are a net frictional force from the road and the gravitational force. After the car has traveled 50 m along the road, its speed is 40 km/h. (a) How much is the mechanical energy of the car reduced because of the net frictional force? (b) What is the magnitude of that net frictional force?

80 In Fig. 8-63, a 1400 kg block of granite is pulled up an incline at a constant speed of 1.34 m/s by a cable and winch. The indicated distances are $d_1 = 40$ m and $d_2 = 30$ m. The coefficient of kinetic friction between the block and the incline is 0.40. What is the power due to the force applied to the block by the cable?

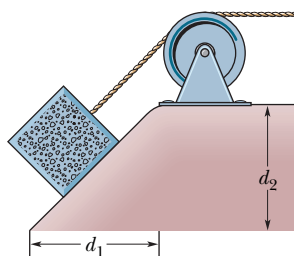


Fig. 8-63 Problem 80.

81 A particle can move along only an x axis, where conservative forces act on it (Fig. 8-64 and the following table). The particle is released at $x = 5.00$ m with a kinetic energy of $K = 14.0$ J and a potential energy of $U = 0$. If its motion is in the negative direction of the x axis, what are its (a) K and (b) U at $x = 2.00$ m and its (c) K and (d) U at $x = 0$? If its motion is in the positive direction of the x axis, what are its (e) K and (f) U at $x = 11.0$ m, its (g) K and (h) U at $x = 12.0$ m, and its (i) K and (j) U at $x = 13.0$ m? (k) Plot $U(x)$ versus x for the range $x = 0$ to $x = 13.0$ m.

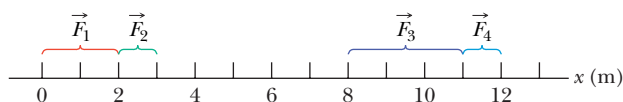


Fig. 8-64 Problems 81 and 82.

Next, the particle is released from rest at $x = 0$. What are (l) its kinetic energy at $x = 5.0$ m and (m) the maximum positive position x_{\max} it reaches? (n) What does the particle do after it reaches x_{\max} ?

Range	Force
0 to 2.00 m	$\vec{F}_1 = +(3.00 \text{ N})\hat{i}$
2.00 m to 3.00 m	$\vec{F}_2 = +(5.00 \text{ N})\hat{i}$
3.00 m to 8.00 m	$F = 0$
8.00 m to 11.0 m	$\vec{F}_3 = -(4.00 \text{ N})\hat{i}$
11.0 m to 12.0 m	$\vec{F}_4 = -(1.00 \text{ N})\hat{i}$
12.0 m to 15.0 m	$F = 0$

82 For the arrangement of forces in Problem 81, a 2.00 kg particle is released at $x = 5.00$ m with an initial velocity of 3.45 m/s in the negative direction of the x axis. (a) If the particle can reach $x = 0$ m, what is its speed there, and if it cannot, what is its turning point? Suppose, instead, the particle is headed in the positive x direction when it is released at $x = 5.00$ m at speed 3.45 m/s. (b) If the particle can reach $x = 13.0$ m, what is its speed there, and if it cannot, what is its turning point?

83 SSM A 15 kg block is accelerated at 2.0 m/s^2 along a horizontal frictionless surface, with the speed increasing from 10 m/s to 30 m/s. What are (a) the change in the block's mechanical energy and (b) the average rate at which energy is transferred to the block? What is the instantaneous rate of that transfer when the block's speed is (c) 10 m/s and (d) 30 m/s?

84 A certain spring is found *not* to conform to Hooke's law. The force (in newtons) it exerts when stretched a distance x (in meters) is found to have magnitude $52.8x + 38.4x^2$ in the direction opposing the stretch. (a) Compute the work required to stretch the spring from $x = 0.500$ m to $x = 1.00$ m. (b) With one end of the spring fixed, a particle of mass 2.17 kg is attached to the other end of the spring when it is stretched by an amount $x = 1.00$ m. If the particle is then released from rest, what is its speed at the instant the stretch in the spring is $x = 0.500$ m? (c) Is the force exerted by the spring conservative or nonconservative? Explain.

85 SSM Each second, 1200 m^3 of water passes over a waterfall 100 m high. Three-fourths of the kinetic energy gained by the water in falling is transferred to electrical energy by a hydroelectric generator. At what rate does the generator produce electrical energy? (The mass of 1 m^3 of water is 1000 kg.)

86 GO In Fig. 8-65, a small block is sent through point A with a speed of 7.0 m/s. Its path is without friction until it reaches the section of length $L = 12$ m, where the coefficient of kinetic friction is 0.70. The indicated heights are $h_1 = 6.0$ m and $h_2 = 2.0$ m. What are the speeds of the block at (a) point B and (b) point C? (c) Does

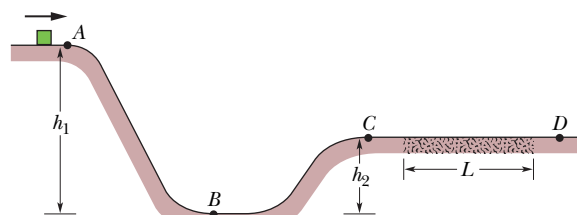


Fig. 8-65 Problem 86.

the block reach point D ? If so, what is its speed there; if not, how far through the section of friction does it travel?

87 SSM A massless rigid rod of length L has a ball of mass m attached to one end (Fig. 8-66). The other end is pivoted in such a way that the ball will move in a vertical circle. First, assume that there is no friction at the pivot. The system is launched downward from the horizontal position A with initial speed v_0 . The ball just barely reaches point D and then stops. (a) Derive an expression for v_0 in terms of L , m , and g . (b) What is the tension in the rod when the ball passes through B ? (c) A little grit is placed on the pivot to increase the friction there. Then the ball just barely reaches C when launched from A with the same speed as before. What is the decrease in the mechanical energy during this motion? (d) What is the decrease in the mechanical energy by the time the ball finally comes to rest at B after several oscillations?

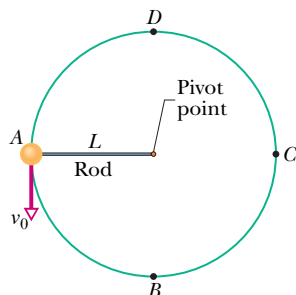


Fig. 8-66 Problem 87.

88 A 1.50 kg water balloon is shot straight up with an initial speed of 3.00 m/s. (a) What is the kinetic energy of the balloon just as it is launched? (b) How much work does the gravitational force do on the balloon during the balloon's full ascent? (c) What is the change in the gravitational potential energy of the balloon–Earth system during the full ascent? (d) If the gravitational potential energy is taken to be zero at the launch point, what is its value when the balloon reaches its maximum height? (e) If, instead, the gravitational potential energy is taken to be zero at the maximum height, what is its value at the launch point? (f) What is the maximum height?

89 A 2.50 kg beverage can is thrown directly downward from a height of 4.00 m, with an initial speed of 3.00 m/s. The air drag on the can is negligible. What is the kinetic energy of the can (a) as it reaches the ground at the end of its fall and (b) when it is halfway to the ground? What are (c) the kinetic energy of the can and (d) the gravitational potential energy of the can–Earth system 0.200 s before the can reaches the ground? For the latter, take the reference point $y = 0$ to be at the ground.

90 A constant horizontal force moves a 50 kg trunk 6.0 m up a 30° incline at constant speed. The coefficient of kinetic friction between the trunk and the incline is 0.20. What are (a) the work done by the applied force and (b) the increase in the thermal energy of the trunk and incline?

91 Two blocks, of masses $M = 2.0$ kg and $2M$, are connected to a spring of spring constant $k = 200$ N/m that has one end fixed, as shown in Fig. 8-67. The horizontal surface and the pulley are frictionless, and the pulley has negligible mass. The blocks are released from rest with the spring relaxed. (a) What is the combined kinetic energy of the two blocks when the hanging block has fallen 0.090 m? (b) What is the kinetic energy of the hanging block when it has

fallen that 0.090 m? (c) What maximum distance does the hanging block fall before momentarily stopping?

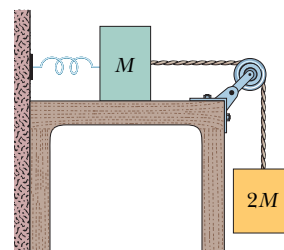


Fig. 8-67 Problem 91.

92 A volcanic ash flow is moving across horizontal ground when it encounters a 10° upslope. The front of the flow then travels 920 m up the slope before stopping. Assume that the gases entrapped in the flow lift the flow and thus make the frictional force from the ground negligible; assume also that the mechanical energy of the front of the flow is conserved. What was the initial speed of the front of the flow?

93 A playground slide is in the form of an arc of a circle that has a radius of 12 m. The maximum height of the slide is $h = 4.0$ m, and the ground is tangent to the circle (Fig. 8-68). A 25 kg child starts from rest at the top of the slide and has a speed of 6.2 m/s at the bottom. (a) What is the length of the slide? (b) What average frictional force acts on the child over this distance? If, instead of the ground, a vertical line through the *top of the slide* is tangent to the circle, what are (c) the length of the slide and (d) the average frictional force on the child?

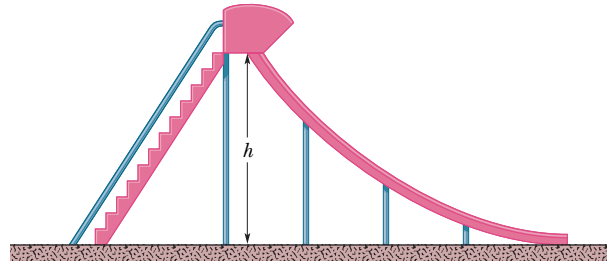


Fig. 8-68 Problem 93.

94 The luxury liner *Queen Elizabeth 2* has a diesel-electric power plant with a maximum power of 92 MW at a cruising speed of 32.5 knots. What forward force is exerted on the ship at this speed? (1 knot = 1.852 km/h.)

95 A factory worker accidentally releases a 180 kg crate that was being held at rest at the top of a ramp that is 3.7 m long and inclined at 39° to the horizontal. The coefficient of kinetic friction between the crate and the ramp, and between the crate and the horizontal factory floor, is 0.28. (a) How fast is the crate moving as it reaches the bottom of the ramp? (b) How far will it subsequently slide across the floor? (Assume that the crate's kinetic energy does not change as it moves from the ramp onto the floor.) (c) Do the answers to (a) and (b) increase, decrease, or remain the same if we halve the mass of the crate?

96 If a 70 kg baseball player steals home by sliding into the plate with an initial speed of 10 m/s just as he hits the ground, (a) what is the decrease in the player's kinetic energy and (b) what is the increase in the thermal energy of his body and the ground along which he slides?

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97 A 0.50 kg banana is thrown directly upward with an initial speed of 4.00 m/s and reaches a maximum height of 0.80 m. What change does air drag cause in the mechanical energy of the banana–Earth system during the ascent?

98 A metal tool is sharpened by being held against the rim of a wheel on a grinding machine by a force of 180 N. The frictional forces between the rim and the tool grind off small pieces of the tool. The wheel has a radius of 20.0 cm and rotates at 2.50 rev/s. The coefficient of kinetic friction between the wheel and the tool is 0.320. At what rate is energy being transferred from the motor driving the wheel to the thermal energy of the wheel and tool and to the kinetic energy of the material thrown from the tool?

99 A swimmer moves through the water at an average speed of 0.22 m/s. The average drag force is 110 N. What average power is required of the swimmer?

100 An automobile with passengers has weight 16 400 N and is moving at 113 km/h when the driver brakes, sliding to a stop. The frictional force on the wheels from the road has a magnitude of 8230 N. Find the stopping distance.

101 A 0.63 kg ball thrown directly upward with an initial speed of 14 m/s reaches a maximum height of 8.1 m. What is the change in the mechanical energy of the ball–Earth system during the ascent of the ball to that maximum height?

102 The summit of Mount Everest is 8850 m above sea level. (a) How much energy would a 90 kg climber expend against the gravitational force on him in climbing to the summit from sea level? (b) How many candy bars, at 1.25 MJ per bar, would supply an energy equivalent to this? Your answer should suggest that work done against the gravitational force is a very small part of the energy expended in climbing a mountain.

103 A sprinter who weighs 670 N runs the first 7.0 m of a race in 1.6 s, starting from rest and accelerating uniformly. What are the sprinter's (a) speed and (b) kinetic energy at the end of the 1.6 s? (c) What average power does the sprinter generate during the 1.6 s interval?

104 A 20 kg object is acted on by a conservative force given by $F = -3.0x - 5.0x^2$, with F in newtons and x in meters. Take the potential energy associated with the force to be zero when the object is at $x = 0$. (a) What is the potential energy of the system associated with the force when the object is at $x = 2.0$ m? (b) If the object has a velocity of 4.0 m/s in the negative direction of the x axis when it is at $x = 5.0$ m, what is its speed when it passes through the origin? (c) What are the answers to (a) and (b) if the potential energy of the system is taken to be -8.0 J when the object is at $x = 0$?

105 A machine pulls a 40 kg trunk 2.0 m up a 40° ramp at constant velocity, with the machine's force on the trunk directed parallel to the ramp. The coefficient of kinetic friction between the trunk and the ramp is 0.40. What are (a) the work done on the trunk by the machine's force and (b) the increase in thermal energy of the trunk and the ramp?

106 The spring in the muzzle of a child's spring gun has a spring constant of 700 N/m. To shoot a ball from the gun, first the spring is compressed and then the ball is placed on it. The gun's trigger then releases the spring, which pushes the ball through the muzzle. The ball leaves the spring just as it leaves the outer end of the muzzle. When the gun is inclined upward by 30° to the horizontal, a 57 g ball is shot to a maximum height of 1.83 m above the gun's muzzle.

Assume air drag on the ball is negligible. (a) At what speed does the spring launch the ball? (b) Assuming that friction on the ball within the gun can be neglected, find the spring's initial compression distance.

107 The only force acting on a particle is conservative force \vec{F} . If the particle is at point A , the potential energy of the system associated with \vec{F} and the particle is 40 J. If the particle moves from point A to point B , the work done on the particle by \vec{F} is $+25$ J. What is the potential energy of the system with the particle at B ?

108 In 1981, Daniel Goodwin climbed 443 m up the *exterior* of the Sears Building in Chicago using suction cups and metal clips. (a) Approximate his mass and then compute how much energy he had to transfer from biomechanical (internal) energy to the gravitational potential energy of the Earth–Goodwin system to lift himself to that height. (b) How much energy would he have had to transfer if he had, instead, taken the stairs inside the building (to the same height)?

109 A 60.0 kg circus performer slides 4.00 m down a pole to the circus floor, starting from rest. What is the kinetic energy of the performer as she reaches the floor if the frictional force on her from the pole (a) is negligible (she will be hurt) and (b) has a magnitude of 500 N?

110 A 5.0 kg block is projected at 5.0 m/s up a plane that is inclined at 30° with the horizontal. How far up along the plane does the block go (a) if the plane is frictionless and (b) if the coefficient of kinetic friction between the block and the plane is 0.40? (c) In the latter case, what is the increase in thermal energy of block and plane during the block's ascent? (d) If the block then slides back down against the frictional force, what is the block's speed when it reaches the original projection point?

111 A 9.40 kg projectile is fired vertically upward. Air drag decreases the mechanical energy of the projectile–Earth system by 68.0 kJ during the projectile's ascent. How much higher would the projectile have gone were air drag negligible?

112 A 70.0 kg man jumping from a window lands in an elevated fire rescue net 11.0 m below the window. He momentarily stops when he has stretched the net by 1.50 m. Assuming that mechanical energy is conserved during this process and that the net functions like an ideal spring, find the elastic potential energy of the net when it is stretched by 1.50 m.

113 A 30 g bullet moving a horizontal velocity of 500 m/s comes to a stop 12 cm within a solid wall. (a) What is the change in the bullet's mechanical energy? (b) What is the magnitude of the average force from the wall stopping it?

114 A 1500 kg car starts from rest on a horizontal road and gains a speed of 72 km/h in 30 s. (a) What is its kinetic energy at the end of the 30 s? (b) What is the average power required of the car during the 30 s interval? (c) What is the instantaneous power at the end of the 30 s interval, assuming that the acceleration is constant?

115 A 1.50 kg snowball is shot upward at an angle of 34.0° to the horizontal with an initial speed of 20.0 m/s. (a) What is its initial kinetic energy? (b) By how much does the gravitational potential energy of the snowball–Earth system change as the snowball moves from the launch point to the point of maximum height? (c) What is that maximum height?

116 A 68 kg sky diver falls at a constant terminal speed of 59 m/s. (a) At what rate is the gravitational potential energy of the

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Earth–sky diver system being reduced? (b) At what rate is the system’s mechanical energy being reduced?

117 A 20 kg block on a horizontal surface is attached to a horizontal spring of spring constant $k = 4.0 \text{ kN/m}$. The block is pulled to the right so that the spring is stretched 10 cm beyond its relaxed length, and the block is then released from rest. The frictional force between the sliding block and the surface has a magnitude of 80 N. (a) What is the kinetic energy of the block when it has moved 2.0 cm from its point of release? (b) What is the kinetic energy of the block when it first slides back through the point at which the spring is relaxed? (c) What is the maximum kinetic energy attained by the block as it slides from its point of release to the point at which the spring is relaxed?

118 Resistance to the motion of an automobile consists of road friction, which is almost independent of speed, and air drag, which is proportional to speed-squared. For a certain car with a weight of 12 000 N, the total resistant force F is given by $F = 300 + 1.8v^2$, with F in newtons and v in meters per second. Calculate the power (in horsepower) required to accelerate the car at 0.92 m/s^2 when the speed is 80 km/h.

119 SSM A 50 g ball is thrown from a window with an initial velocity of 8.0 m/s at an angle of 30° above the horizontal. Using energy methods, determine (a) the kinetic energy of the ball at the

top of its flight and (b) its speed when it is 3.0 m below the window. Does the answer to (b) depend on either (c) the mass of the ball or (d) the initial angle?

120 A spring with a spring constant of 3200 N/m is initially stretched until the elastic potential energy of the spring is 1.44 J. ($U = 0$ for the relaxed spring.) What is ΔU if the initial stretch is changed to (a) a stretch of 2.0 cm, (b) a compression of 2.0 cm, and (c) a compression of 4.0 cm?

121 A locomotive with a power capability of 1.5 MW can accelerate a train from a speed of 10 m/s to 25 m/s in 6.0 min. (a) Calculate the mass of the train. Find (b) the speed of the train and (c) the force accelerating the train as functions of time (in seconds) during the 6.0 min interval. (d) Find the distance moved by the train during the interval.

122 SSM A 0.42 kg shuffleboard disk is initially at rest when a player uses a cue to increase its speed to 4.2 m/s at constant acceleration. The acceleration takes place over a 2.0 m distance, at the end of which the cue loses contact with the disk. Then the disk slides an additional 12 m before stopping. Assume that the shuffleboard court is level and that the force of friction on the disk is constant. What is the increase in the thermal energy of the disk–court system (a) for that additional 12 m and (b) for the entire 14 m distance? (c) How much work is done on the disk by the cue?

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