Special random variables

1 Geometric distribution

1. Assume that each of your calls to a popular radio station has a probability of 0.02 of connecting, that is, of not obtaining a busy signal. Assume that your calls are independent.
   (a) What is the probability that your first call that connects is your tenth call?
   (b) What is the probability that it requires more than five calls for you to connect?
   (c) What is the mean number of calls needed to connect?
2. A player of a video game is confronted with a series of opponents and has an 80% probability of defeating each one. Success with any opponent is independent of previous encounters. The player continues to contest opponents until defeated.
   (a) What is the probability mass function of the number of opponents contested in a game?
   (b) What is the probability that a player defeats at least two opponents in a game?
   (c) What is the expected number of opponents contested in a game?

2 Binomial distribution

1. A particularly long traffic light on your morning commute is green 20% of the time that you approach it. Assume that each morning represents an independent trial.
   (a) Over five mornings, what is the probability that the light is green on exactly one day?
   (b) Over 20 mornings, what is the probability that the light is green on exactly four days?
   (c) Over 20 mornings, what is the probability that the light is green on more than four days?
2. Heart failure is due to either natural occurrences (87%) or outside factors (13%). Outside factors are related to induced substances or foreign objects. Natural occurrences are caused by arterial blockage, disease, and infection. Suppose that 20 patients will visit an emergency room with heart failure. Assume that causes of heart failure between individuals are independent.
   (a) What is the probability that three individuals have conditions caused by outside factors?
   (b) What is the probability that three or more individuals have conditions caused by outside factors?
   (c) What are the mean and standard deviation of the number of individuals with conditions caused by outside factors?
3. A satellite system consists of 4 components and can function adequately if at least 2 of the 4 components are in working condition. If each component is, independently, in working condition with probability .6, what is the probability that the system functions adequately?

3 Poisson distribution

1. The number of telephone calls that arrive at a phone exchange is often modeled as a Poisson random variable. Assume that on the average there are 10 calls per hour.
   (a) What is the probability that there are exactly five calls in one hour?
   (b) What is the probability that there are three or fewer calls in one hour?
   (c) What is the probability that there are exactly 15 calls in two hours?

2. The number of flaws in bolts of cloth in textile manufacturing is assumed to be Poisson distributed with a mean of 0.1 flaw per square meter.

   (a) What is the probability that there are two flaws in 1 square meter of cloth?

   (b) What is the probability that there is one flaw in 10 square meters of cloth?

   (c) What is the probability that there are no flaws in 20 square meters of cloth?

   (d) What is the probability that there are at least two flaws in 10 square meters of cloth?

4 Normal distribution

1. Given the normally distributed variable X with mean 18 and standard deviation 2.5, find

   (a) $P(X < 15)$;

   (b) the value of k such that $P(X < k) = 0.2236$;

   (c) the value of k such that $P(X > k) = 0.1814$;

   (d) $P(17 < X < 21)$.

2. The finished inside diameter of a piston ring is normally distributed with a mean of 10 centimeters and a standard deviation of 0.03 centimeter.

   (a) What proportion of rings will have inside diameters exceeding 10.075 centimeters?

   (b) What is the probability that a piston ring will have an inside diameter between 9.97 and 10.03 centimeters?

   (c) Below what value of inside diameter will 15% of the piston rings fall?

3. The borrowing period, in days, for a particular book at a University library can be regarded as random variable X which has normal distribution with mean $\mu = 8$ and standard deviation $\sigma = 2$. A book need to be return within 10 days.

   (a) Compute $P(X > 10)$ - the probability that a new borrower returns the book after 10 days.

   (b) For a late return, the borrower has to pay a penalty of \$5. Otherwise, the borrower pays \$0. Evaluate the average payment of a borrower.

4. Suppose a filling machine is used to fill cartons with a liquid product. The specification that is strictly enforced for the filling machine is 9 ± 1.5 oz. If any carton is produced with weight outside these bounds, it is considered by the supplier to be defective. It is hoped that at least 99% of cartons will meet these specifications. Assume that the weight is normally distributed with mean μ and standard deviation σ.

   (a) With the conditions $\mu = 9$ and $\sigma = 1$, what proportion of cartons from the process are defective?

   (b) If changes are made to reduce variability, what must σ be reduced to in order to meet specifications with probability 0.99?