Inductance, Capacitance and Mutual Inductance

Textbook:

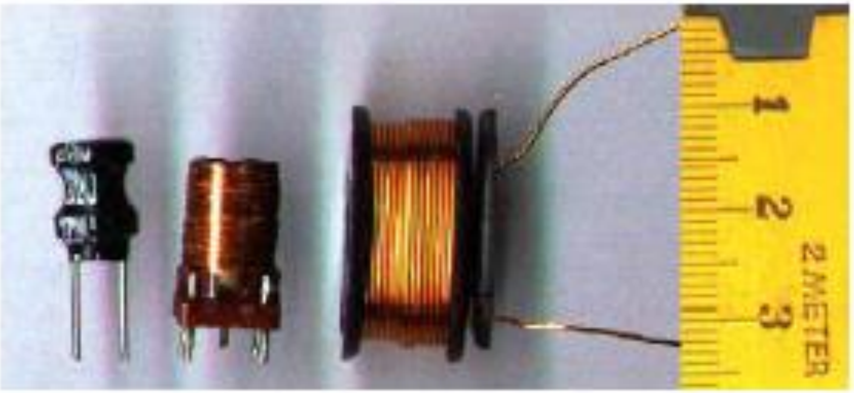
Electric Circuits

James W. Nilsson & Susan A. Riedel

10th Edition.

The inductor

- An **inductor** is a passive electrical device that stores energy in a magnetic field, typically by combining the effects of many loops of electric current.



- The inductance is measured in *Henrys* (H) (It is named after the American scientist *Joseph Henry*).

Inductor

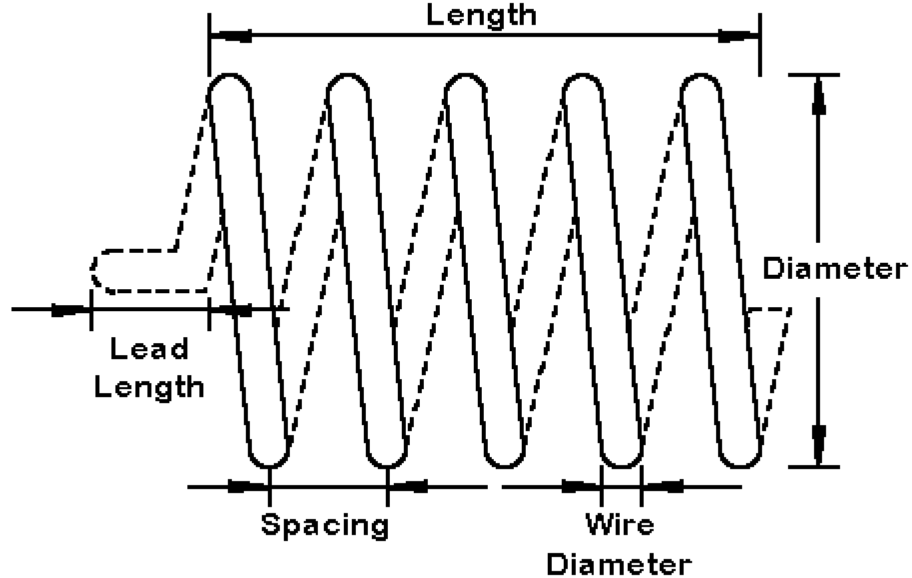
<u>Or other definitions:</u>

An inductor is a passive electrical device employed in electrical circuits for its property of inductance. An inductor can take many forms.

Or

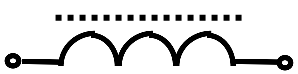
An inductor is a coil of wire through which a current is passed. The current can be either AC or DC.

Physical model of inductor



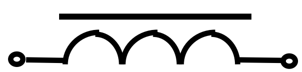
Symbol

L



Feritte core coil

L



Magnetic core coil

L

Air core coil

Inductor characteristics

Inductance (or **electric inductance**) is a measure of the amount of magnetic flux produced for a given electric current. The SI unit of inductance is the henry (H), in honor of Joseph Henry. The symbol *L* is used for inductance, possibly in honour of the physicist Heinrich Lenz.

The inductance (called *self-inductance*)

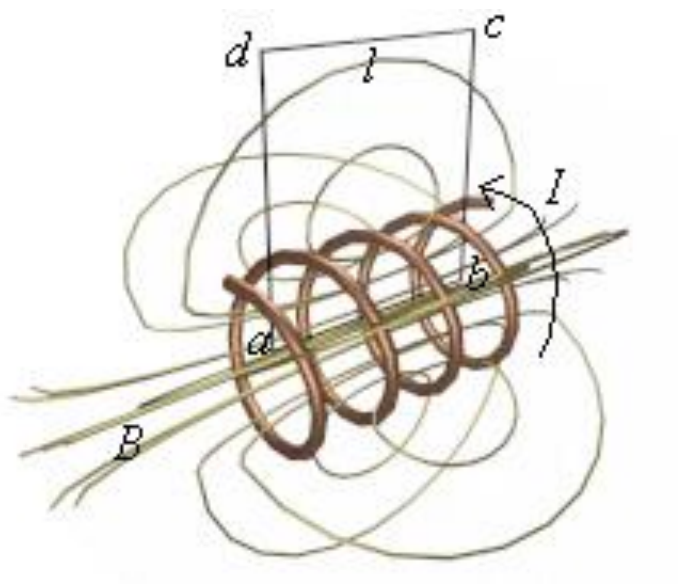
$$L = \frac{\Phi}{i}$$

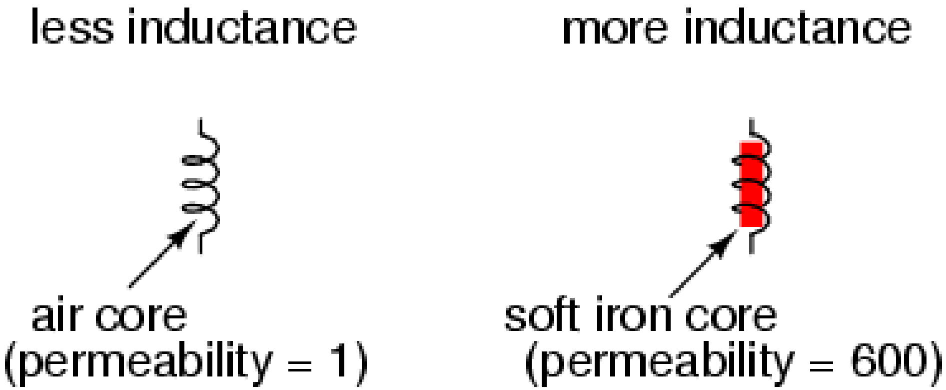
L is the inductance in H, mH, μH
i is the current in A,
Φ is the magnetic flux in Wb (webers)

The inductance (when a conductor is coiled-solenoid)

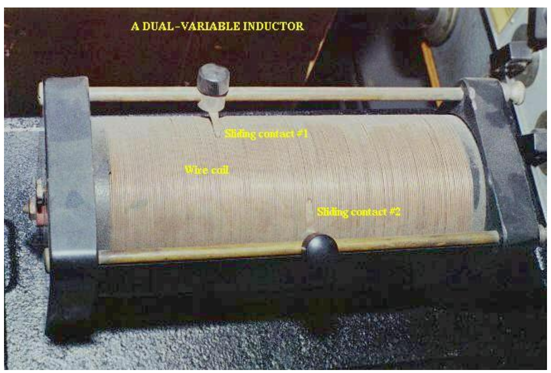
$$\boxed{L = \mu . N^2 . \frac{S}{l}}$$

S – coil's cross-sectional (m^2)
N – number of coil; l – length of coil (m)
μ- permeability of coil's core (H/ m). $\mu = \mu_r \times \mu_0$,
μ_0 - permeability of free space ($4\pi \times 10^{-7}$ H/m) ;
μ_r - relative permeability of the core (dimensionless)

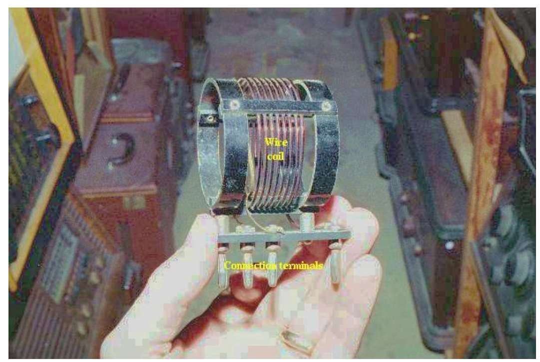




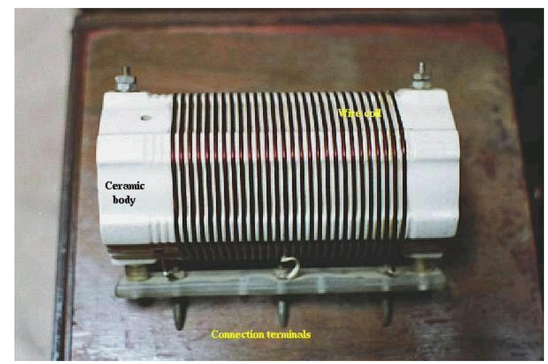
A core material with greater magnetic permeability results in greater magnetic field flux for any given amount of field force (amp-turns).



Variable inductors: providing a way to vary the number of wire turns in use at any given time, or by varying the core material (a sliding core that can be moved in and out of the coil).



Fixed-value inductor: another antique air-core unit built for radios. The connection terminals can be seen at the bottom, as well as the few turns of relatively thick wire

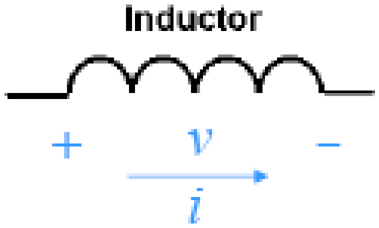


Inductor (of greater inductance value), also intended for radio applications. Its wire coil is wound around a white ceramic tube for greater rigidity

The inductor

- The voltage drop across the terminals is related to the current by

$$\boxed{v = L\frac{di}{dt}}$$



- The voltage across the terminals of an inductor is proportional to the time rate of change of the current in the inductor.
- If the current is constant (DC current) the inductor behaves as a short circuit.

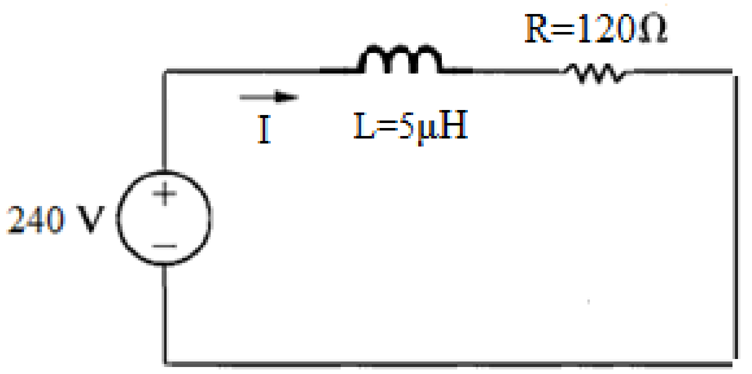
$$i = \text{constant} \qquad \frac{di}{dt} = 0 \qquad v = 0$$

- Current can not change instantaneously in an inductor (it cannot change by a finite amount in zero time)

$$\frac{di}{dt} \neq \infty \qquad v \neq \infty$$

Example

- Find the current I

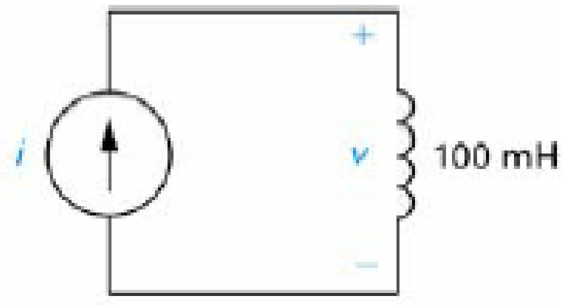


- Since the voltage course is a DC source, the inductor behaves as a short circuit.

=> $I = 240\ (V) / 120\Omega = 2\ (A)$

Example 1

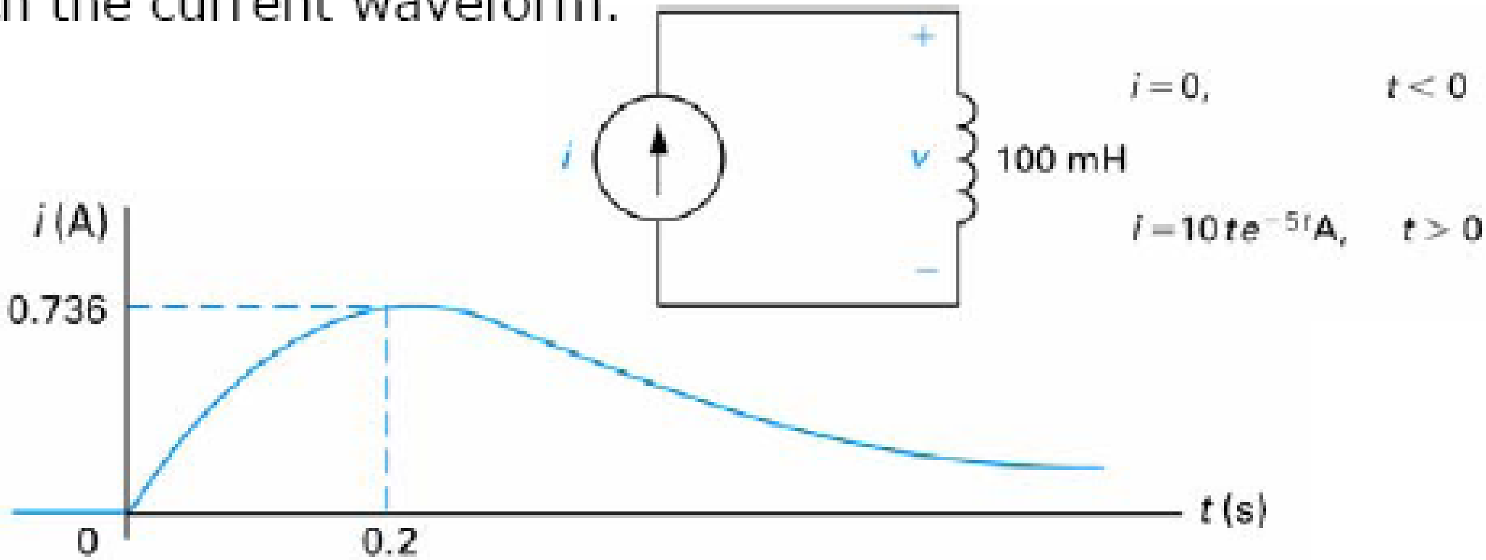
- If $\begin{array}{ll} i = 0 & \text{for } t \le 0 \\ i = 10te^{-5t} \text{ A} & \text{for } t \ge 0 \end{array}$



a) Sketch the current waveform.
b) At what instant of time is the current maximum.
c) Express the voltage across the terminals of the 100 mH inductor as a function of time.
d) Sketch the voltage waveform.
e) Are the voltage and the current at a maximum at the same time.
f) At what instant of time does the voltage change polarity?
g) Is there ever an instantaneous change in voltage across the inductor? If so, at what time?

Example (cont.)

a) Sketch the current waveform.



b) At what instant of time is the current maximum.

$$\frac{di}{dt} = 10e^{-5t} + 10(-5)te^{-5t}$$

$$\frac{di}{dt}\Big|_{max} = 0 \dashrightarrow 10e^{-5t} + 10(-5)te^{-5t} = 0 \dashrightarrow 10 - 50t = 0 \dashrightarrow \boxed{t = \frac{1}{5}\,s}$$

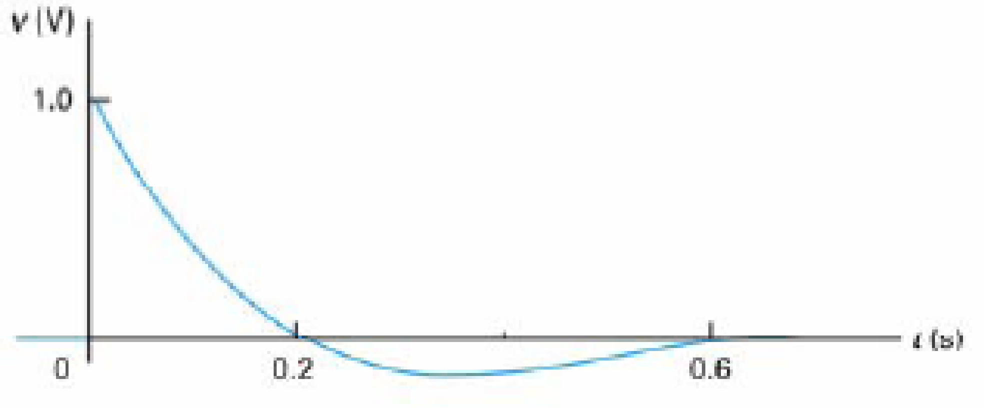
Example (cont.)

c) Express the voltage across the terminals of the 100 mH inductor as a function of time.

$$v = L\frac{di}{dt} = L(10-50t)e^{-5t} = 100\times 10^{-3}(10-50t)e^{-5t}\ \text{V}$$

$$\boxed{\begin{aligned} &v = 0 && \text{for } t \le 0 \\ &v = (1-5t)e^{-5t}\ \text{V} && \text{for } t \ge 0 \end{aligned}}$$

d) Sketch the voltage waveform.



Example (cont.)

e) Are the voltage and the current at a maximum at the same time.

- No v is proportional to di/dt and not i.

f) At what instant of time does the voltage change polarity?

- at t=0.2 s. (di/dt changes slope)

g) Is there ever an instantaneous change in voltage across the inductor? If so, at what time?

- Yes, at t=0.

Current in terms of voltage

- Integrate both sides in terms if dt of the equation

$$v = L\frac{di}{dt}$$

$$vdt = Ldi$$

$$L\int_{i(t_o)}^{i(t)} di = \int_{t_o}^{t} vdt$$

$$L[i(t) - i(t_o)] = \int_{t_o}^{t} vdt$$

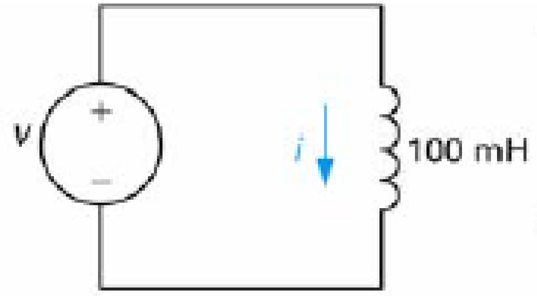
$$\boxed{i(t) = \frac{1}{L}\int_{t_o}^{t} vdt + i(t_o)}$$

- If t_o=0.

$$i(t) = \frac{1}{L}\int_{0}^{t} vdt + i(0)$$

Example 2

a) Sketch the voltage as a function of time.
b) Find the inductor current as a function of time.
c) Sketch the current as a function of time.

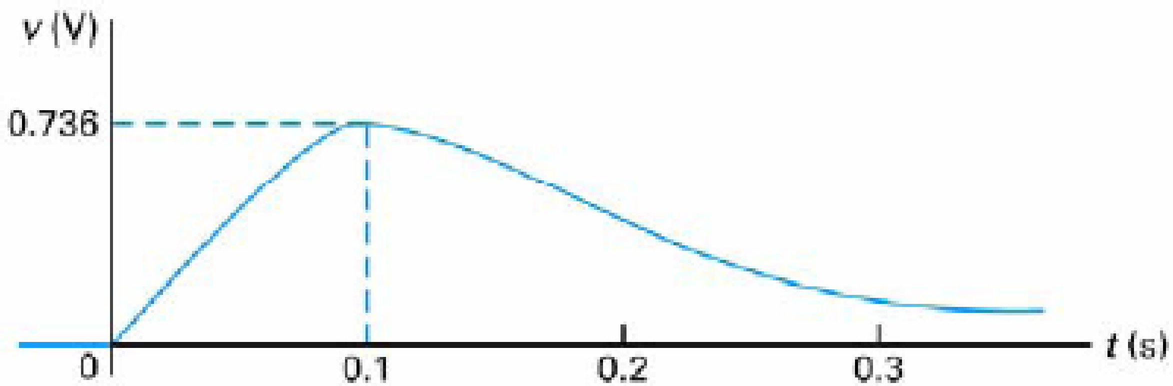


$v = 0, \quad t < 0$

$v = 20te^{-10t}\,\text{V}, \ t > 0$

Ans:-

a) Sketch the voltage as a function of time.



Example (cont.)

b) Find the inductor current as a function of time.

$$i(t) = \frac{1}{L}\int_0^t v\,dt + i(0)$$

$$i(t) = \frac{1}{L}\int_0^t 20te^{-10t}\,dt + i(0)$$

$$i(t) = \frac{20}{100\times 10^{-3}\times 100}\left(1 - e^{-10t} - 10te^{-10t}\right)$$

$$\boxed{i(t) = 2\left(1 - e^{-10t} - 10te^{-10t}\right)\mathrm{A}} \qquad t > 0$$

$$Int = \int_0^t te^{-10t}\,dt$$

$$Int = \int_0^t U\,dV$$

$$U = t \qquad dV = e^{-10t}dt$$

$$dU = dt \qquad V = \frac{e^{-10t}}{-10}$$

$$Int = UV - \int_0^t V\,dU$$

$$Int = \frac{te^{-10t}}{-10}\Big|_o^t - \int_0^t \frac{e^{-10t}}{-10}dt$$

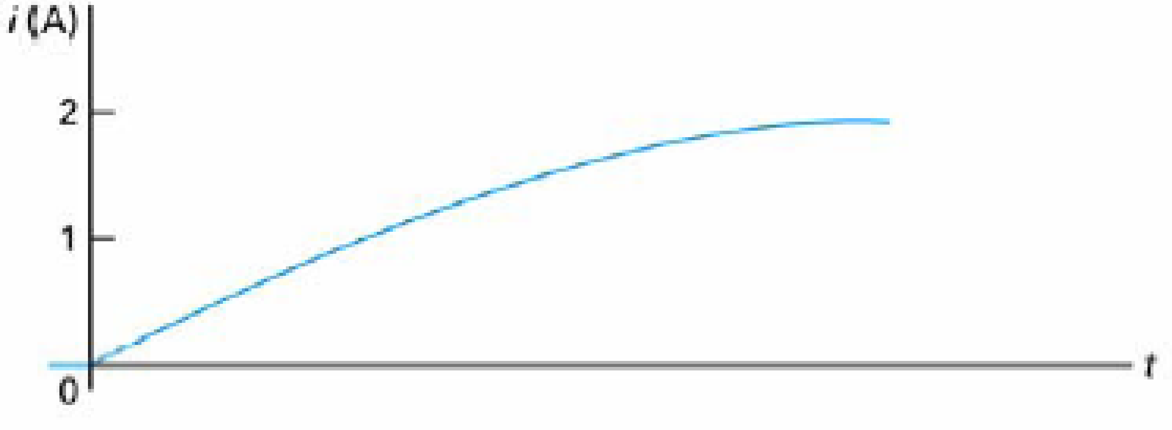
$$Int = \frac{te^{-10t}}{-10} - \frac{\left(e^{-10t} - 1\right)}{100}$$

$$Int = \frac{1}{100}\left(1 - e^{-10t} - 10te^{-10t}\right)$$

Example (cont.)

c) Sketch the current as a function of time.

$$\boxed{i(t) = 2\left(1 - e^{-10t} - 10te^{-10t}\right)\text{A}} \qquad t \geq 0$$



Power and Energy in the Inductor

Power :

$$p = vi$$

$$\boxed{P = iv = Li\frac{di}{dt}}$$ Power in an inductor

$$p = iv = v\left[\frac{1}{L}\int_{t_o}^{t} vdt + i(t_o)\right]$$

Energy :

$$p = \frac{dw}{dt} = Li\frac{di}{dt}$$

Multiplying both sides of Eq. 6.10 by a differential time

$$dw = Lidi$$

Both sides are integrated $\int_0^w dw = L\int_0^i idi$

$$\boxed{w = \frac{1}{2}Li^2}$$ Energy in an inductor

Example 3

a) For example 6.1 determine, plot i, v, p, and w versus time.

b) In what time interval is energy being stored on the inductor?

c) In what time interval is energy being extracted from the inductor?

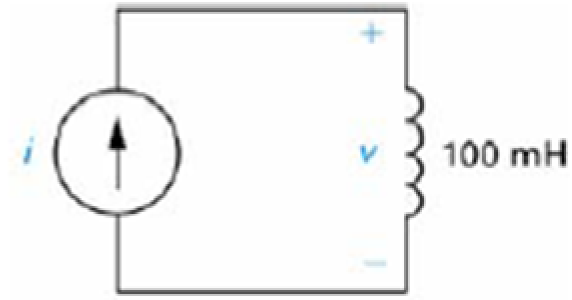
d) What is the maximum energy stored in the inductor?

e) Evaluate the integrals $\int_0^{0.2} p\,dt$ and $\int_{0.2}^{\infty} p\,dt$ and comment on their significance.

f) Sketch For Example 6.2, and comment why w is constant?

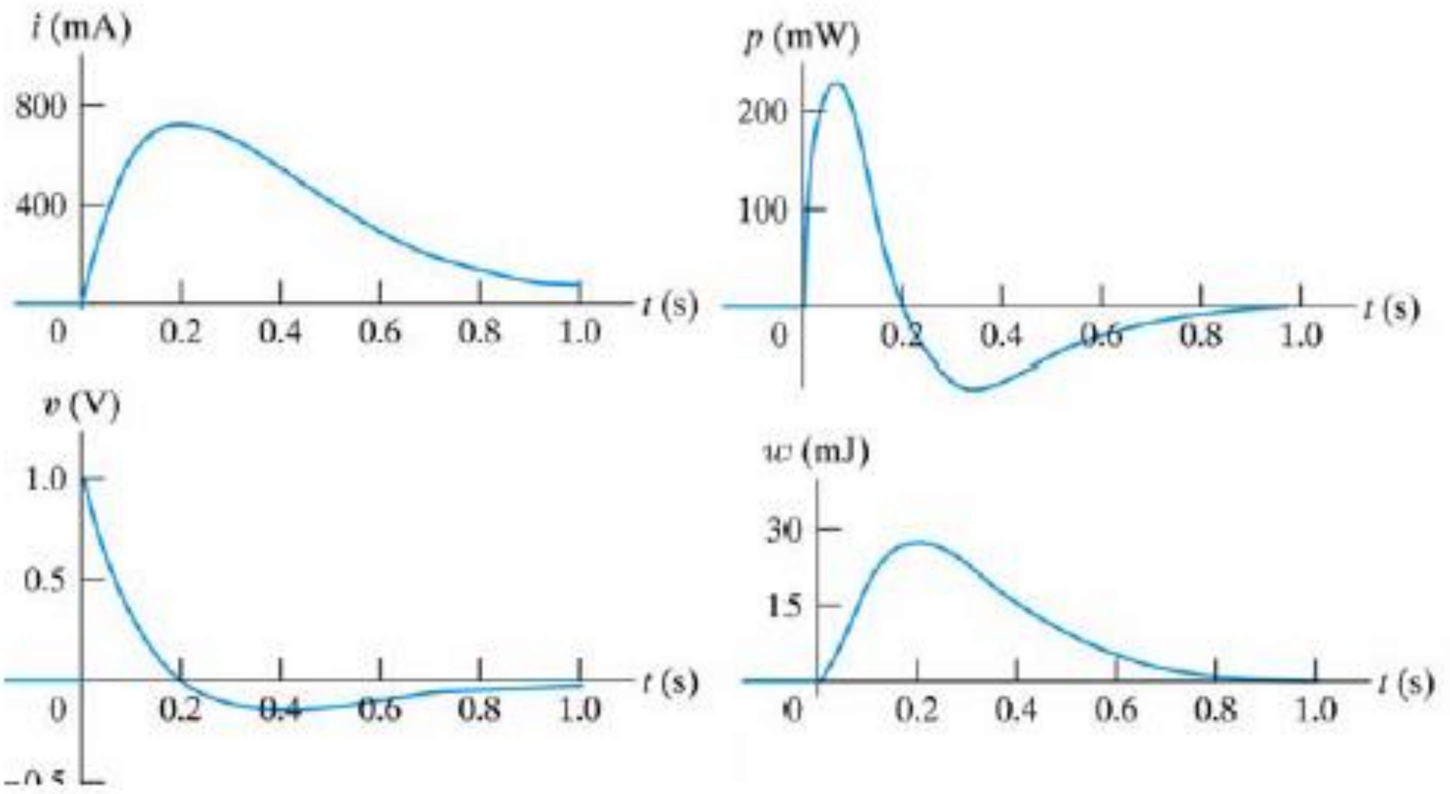
If $i = 0$ for $t \leq 0$

$i = 10te^{-5t}$ A for $t \geq 0$



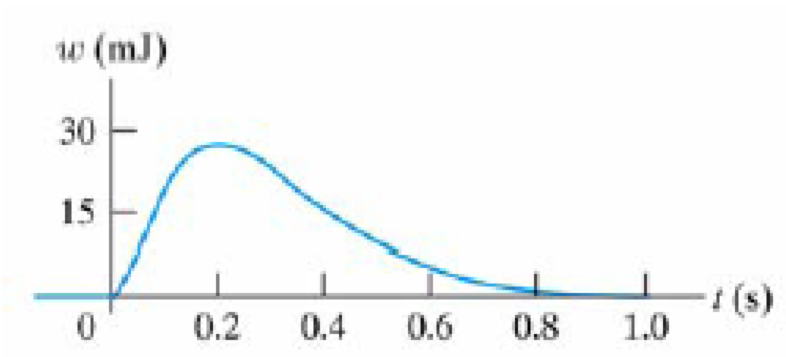
Example (cont.)

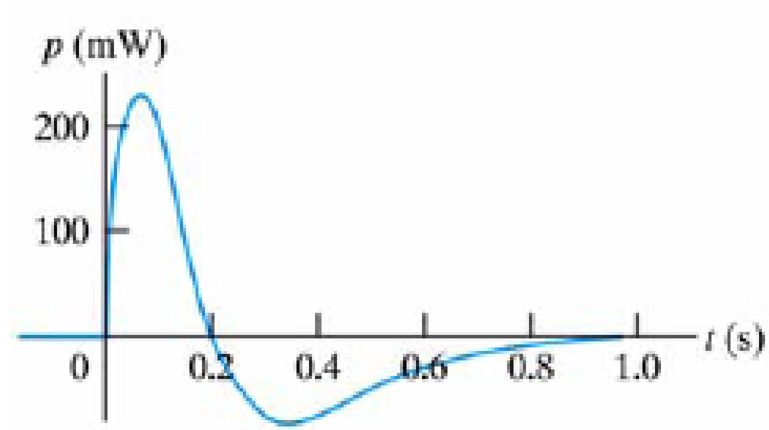
a) For the previous example determine, plot i, v, p, and w versus time.



Example (cont.)

b) In what time interval is energy being stored on the inductor? ans. (0<t<0.2 s)





c) In what time interval is energy being extracted from the inductor? ans. $(0.2 < t < \infty)$

d) What is the maximum energy stored in the inductor?

ans. (w_{max}= 27.07 mJ)

Example (cont.)

e) Evaluate the integrals $\int_0^{0.2} p\,dt$ and $\int_{0.2}^{\infty} p\,dt$ and comment on their significance.

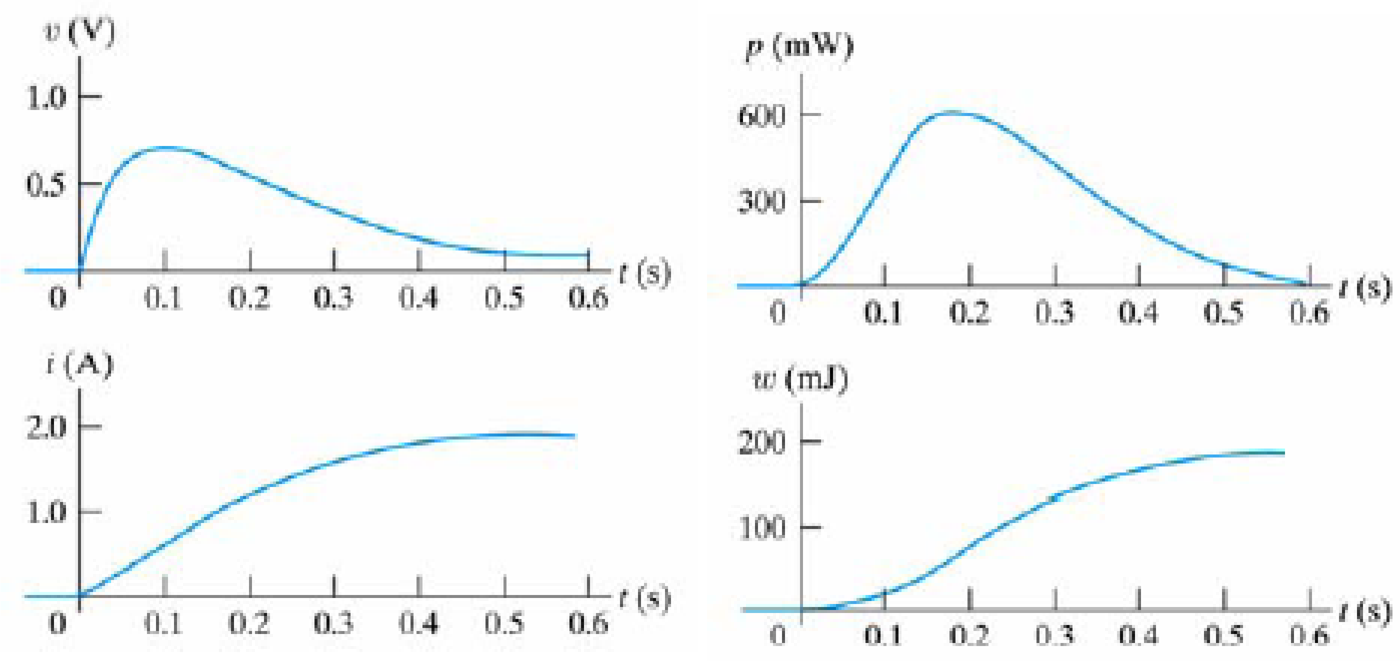
$$i = 10te^{-5t}\ \text{A} \qquad \& \qquad v = (1-5t)e^{-5t}\ \text{V}$$

$$p = vi = 10te^{-5t}(1-5t)e^{-5t} = 10t(1-5t)e^{-10t}\ \text{W}$$

$$\underbrace{\int_0^{0.2} p\,dt = 27.07\ mJ}_{\text{Energy Stored}} \qquad \underbrace{\int_{0.2}^{\infty} p\,dt = -27.07\ mJ}_{\text{Energy Extracted}}$$

Example (cont.)

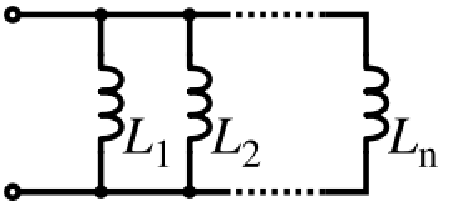
f) Sketch For Example 6.2, and comment why w is constant?



- Since both the source and the inductor is ideal, when the voltage returns to zero, the energy is trapped inside the inductor and there is no means of dissipating energy.

Inductors in a parallel each have the same potential difference (voltage). Total equivalent inductance (L_{eq}):

$$\frac{1}{L_{\mathrm{eq}}} = \frac{1}{L_1} + \frac{1}{L_2} + \cdots + \frac{1}{L_n}$$



The current through **inductors in series** stays the same, but the voltage across each inductor can be different. The sum of the potential differences (voltage) is equal to the total voltage. Total equivalent inductance (*L*eq):

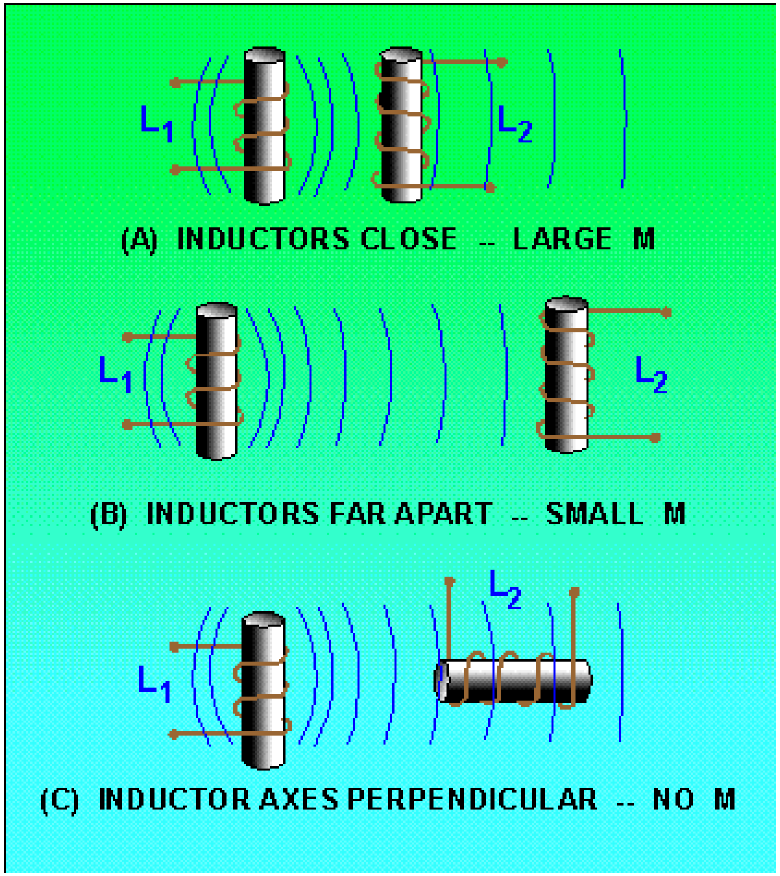
$$L_{\mathrm{eq}} = L_1 + L_2 + \cdots + L_n$$

Impedance:

Inductive reactance X_L , the impedance of an inductor to an AC signal, is found by the equation

$$\boxed{X_L = 2\pi f\, L}$$

X_L = inductive reactance, Ω;
f = frequency, Hz; and
L = inductance, H.



Mutual Inductance

Mutual inductance is the property that exists between two conductors carrying current when their magnetic lines of force link together.

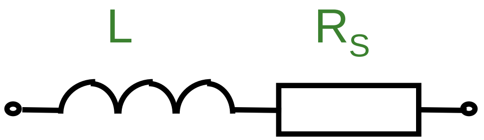
The mutual inductance of two coils with fields interacting can be determined by the equation

$$\boxed{M = \frac{L_A - L_B}{4}}$$

M = mutual inductance of L_A and L_B, H; L_A = total inductance, H, of coils L_1 and L_2 with fields aiding; and L_B = total inductance, H, of coils L_1 and L_2 with fields opposing.

Quality factor (Q) of Inductor

The quality factor of an inductor is the ratio of its inductive reactance to its resistance at a given frequency (ω), and is a measure of its efficiency.
The higher the Q factor of the inductor, the closer it approaches the behavior of an ideal, lossless, inductor.



The Q factor of an inductor can be found through the following formula, where R_S is its internal electrical resistance
X_L = inductive reactance of the coil (Ω)

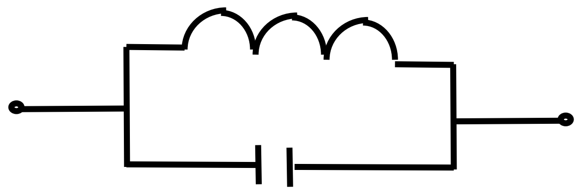
$$\boxed{Q = \frac{\omega L}{R_S} = \frac{2\pi f L}{R_S} = \frac{X_L}{R_S}}$$

Working frequency

When working frequency is small enough, the paracitic capacitors between coils of inductor is negligible. But at high frequency, these parasitic capacitors cannot be ignored.

At high frequency, the inductor become a parallel resonant circuit. The resonance of this circuit called self-resonance frequency $\mathbf{f_0}$.

At higher frequency, f > $\mathbf{f_0}$, the coil has more capacitive property. Thus, the maximum frequency of the coil should less than $\mathbf{f_0}$.



$$f_{\max} < f_0 = \frac{1}{2\pi\sqrt{LC}}$$

Inductor code guide

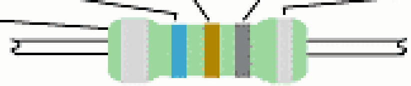
INDUCTOR COLOR GUIDE

Result Is In µH

4-BAND-CODE  270µH ± 5%

COLOR	1st BAND	2nd BAND	MULTIPLIER	TOLERANCE
BLACK	0	0	1	± 20%
BROWN	1	1	10	Military ± 1%
RED	2	2	100	Military ± 2%
ORANGE	3	3	1,000	Military ± 3%
YELLOW	4	4	10,000	Military ± 4%
GREEN	5	5		
BLUE	6	6		
VIOLET	7	7		
GREY	8	8		
WHITE	9	9		
NONE				Military ± 20%
GOLD			0.1 / Mil. Dec. Pt.	Both ± 5%
SILVER			0.01	Both ±10%

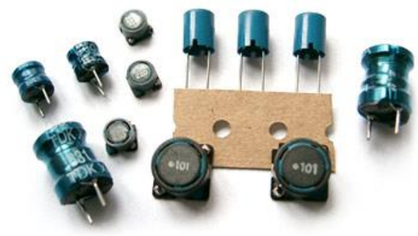
Military Identifier



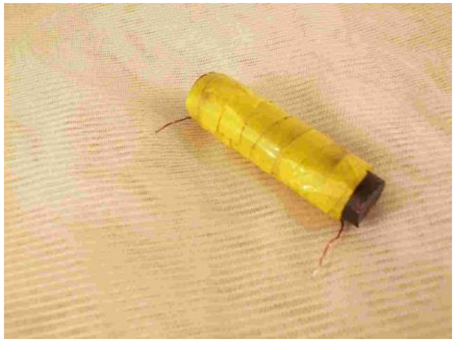
6.8µH ± 10%
MILITARY CODE

Electronix Express / RSR
http://www.elexp.com

1-800-972-2225
In NJ 732-381-8020

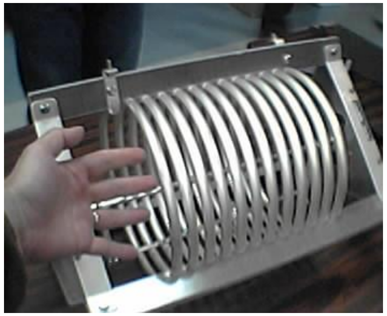


Power Inductor



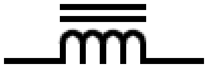
Ferrite Rod Inductor

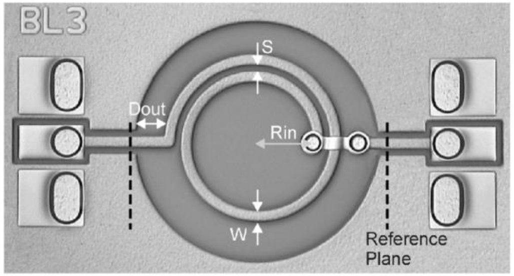
SMD Wound Chip Inductor



Roller inductor for FM diplexer

DC filter choke Inductor

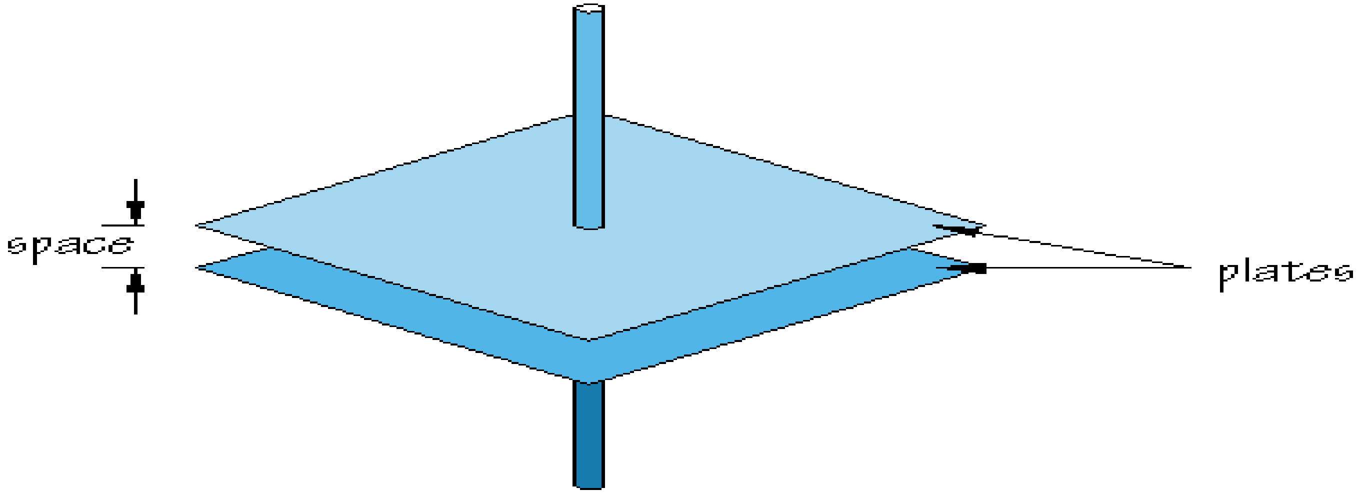




Spiral inductor with *N*=1.5 turns, *W*=20 μm, *S*=10 μm and R_{in}=100 μm (area=0.14 mm^2). (called On-chip inductor)

The capacitor

- A capacitor is a device that stores energy in the electric field created between a pair of conductors on which equal but opposite electric charges have been placed

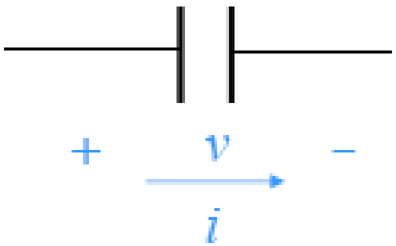


- The capacitance is measured in *Farads* (F) (It is named after the English chemist *Michael Faraday*).

The capacitor

- The current drop across the terminals is related to the voltage by

$$\boxed{i = C\frac{dv}{dt}}$$



- If the voltage is constant (DC voltage) the capacitor behaves as an open circuit.

$$v = \text{constant} \qquad \frac{dv}{dt} = 0 \qquad i = 0$$

- The voltage cannot change instantaneously across the terminals of a capacitor.

$$\frac{dv}{dt} \neq \infty \qquad i \neq \infty$$

The capacitor power and energy

$$idt = Cdv \rightarrow dv = \frac{1}{C} idt \rightarrow \boxed{v(t) = \frac{1}{C}\int_{t_o}^{t} idt + v(t_o)}$$

$$\boxed{p = vi = Cv\frac{dv}{dt}}$$

Power in a capacitor

$$p = \frac{dw}{dt} = Cv\frac{dv}{dt}$$

$$dw = Cvdv$$

$$\int_o^w dw = \int_o^v Cvdv \qquad \boxed{w = \frac{1}{2}Cv^2}$$

Energy in a capacitor

Example 4

The voltage pulse is impressed across the terminals of a 0.5 μF capacitor:

$$v(t) = \begin{cases} 0 & , t \le 0 \text{ s} \\ 4t & , 0 \le t \le 1 \text{ s} \\ 4e^{-(t-1)} & , t \ge 1 \text{ s} \end{cases}$$

a) Derive the expressions for the capacitor current, power, and energy.
b) Sketch the voltage, current, power, and energy as functions of time.
c) Specify the interval of time when energy is being stored in the capacitor.
d) Specify the interval of time when energy is being delivered by the capacitor.
e) Evaluate the integrals $\int_0^1 p\,dt$ and $\int_1^\infty p\,dt$

Example (Cont.)

a) Derive the expressions for the capacitor current, power, and energy.

$$i = C\frac{dv}{dt}$$

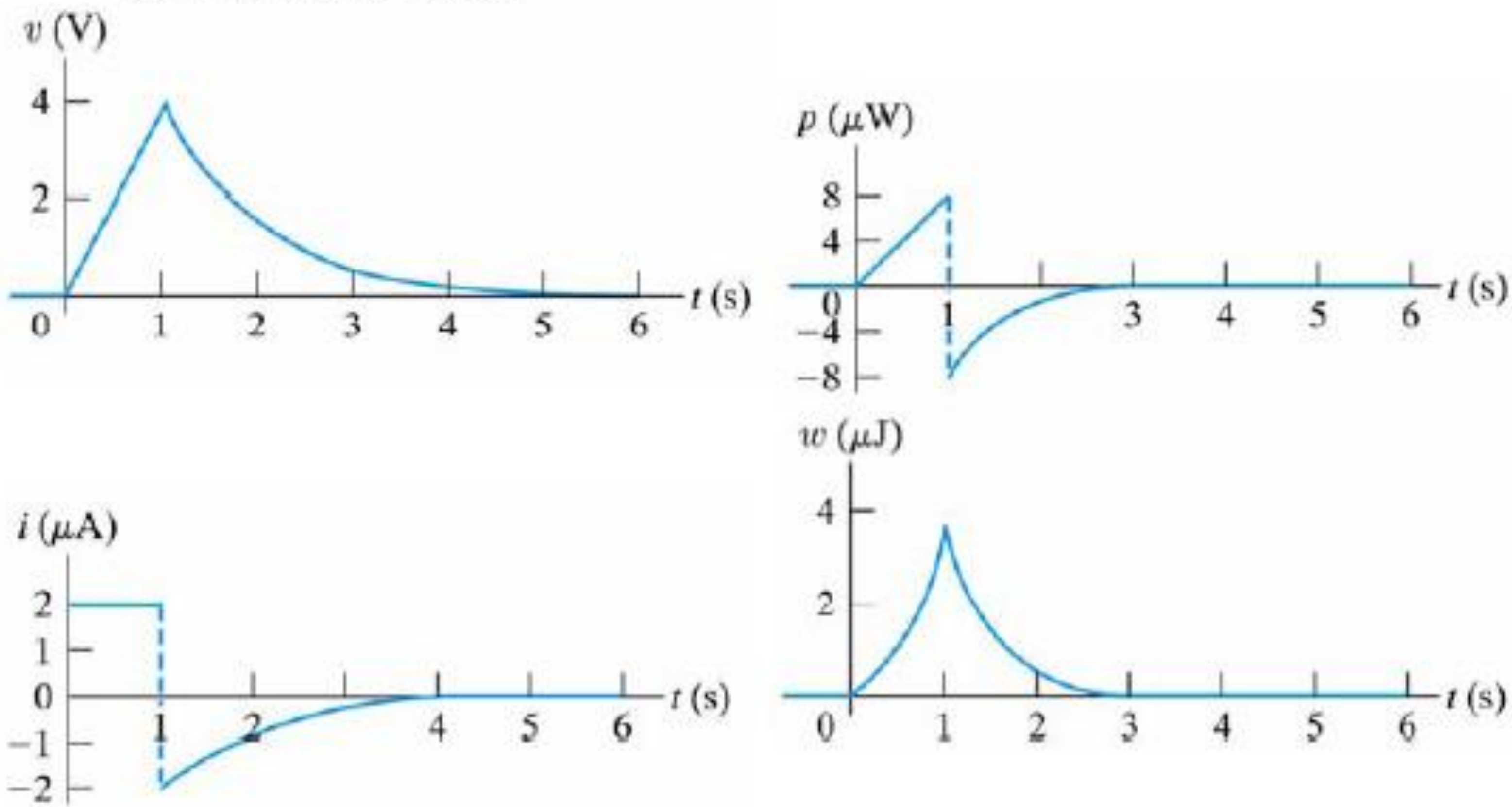
$$i(t) = C\frac{dv}{dt} = \begin{cases} 0 \times 0.5 \times 10^{-6} \\ 4 \times 0.5 \times 10^{-6} \\ -4e^{-(t-1)} \times 0.5 \times 10^{-6} \end{cases} = \boxed{\begin{cases} 0 & ,\ t \le 0\ \text{s} \\ 2\ \mu\text{A} & ,\ 0 \le t \le 1\ \text{s} \\ -2e^{-(t-1)}\ \mu\text{A} & ,\ t \ge 1\ \text{s} \end{cases}}$$

$$p(t) = iv = \boxed{\begin{cases} 0 & ,\ t \le 0\ \text{s} \\ 8t\ \mu\text{W} & ,\ 0 \le t \le 1\ \text{s} \\ -8e^{-2(t-1)}\ \mu\text{W} & ,\ t \ge 1\ \text{s} \end{cases}}$$

$$w = \frac{1}{2}Cv^2 = \begin{cases} 0 \\ 8Ct^2 \\ 8Ce^{-2(t-1)} \end{cases} = \boxed{\begin{cases} 0 & ,\ t \le 0\ \text{s} \\ 4t^2\ \mu\text{J} & ,\ 0 \le t \le 1\ \text{s} \\ 4e^{-2(t-1)}\ \mu\text{J} & ,\ t \ge 1\ \text{s} \end{cases}}$$

Example (Cont.)

b) Sketch the voltage, current, power, and energy as functions of time.



Example (Cont.)

c) Specify the interval of time when energy is being stored in the capacitor.

- ($0 < t < 1$ s)

d) Specify the interval of time when energy is being delivered by the capacitor.

- ($t > 1$ s)

e) Evaluate the integrals $\int_0^1 p\,dt$ and $\int_1^\infty p\,dt$

$$\int_0^1 p\,dt = \int_0^1 8t\,dt = 4\ \mu J \dashrightarrow \text{Energy Stored}$$

$$\int_1^\infty p\,dt = \int_1^\infty -8e^{-2(t-1)}dt = -4\ \mu\text{J} \dashrightarrow \text{Energy Extracted}$$

Example 5

An uncharged 0.2 μF capacitor is driven by a triangle angular current pulse. The current pulse is described by

$$i(t) = \begin{cases} 0 & , t \leq 0 \text{ s} \\ 5000t \text{ A} & , 0 \leq t \leq 20 \ \mu\text{s} \\ 0.2 - 5000t \text{ A} & , 20 \leq t \leq 40 \ \mu\text{s} \\ 0 & , t \geq 40 \ \mu\text{s} \end{cases}$$

a) Derive the expressions for the capacitor voltage, power, and energy.
b) Sketch the current, voltage, power, and energy as functions of time.
c) Why does a voltage remain on the capacitor after the current returns to zero.

Example (Cont.)

a) Derive the expressions for the capacitor voltage, power, and energy.

$\boxed{t \leq 0 \text{ s}}$ (All equal to zero)

$\boxed{0 \leq t \leq 20\ \mu\text{s}}$

$$v = \frac{1}{0.2\mu}\int_0^{20\,\mu s} i\,dt + 0 = \frac{1}{0.2\mu}\int_0^{20\,\mu s} 5000t\,dt = 12.5\times10^9 t^2 \text{ V}$$

$$p = vi = 62.5\times10^{12} t^3 \text{ W}$$

$$w = \frac{1}{2}Cv^2 = 15.625\times10^{12} t^4 \text{ J}$$

$\boxed{20 \leq t \leq 40\ \mu\text{s}}$

$$v = (10^6 t - 12.5\times10^9 t^2 - 10) \text{ V}$$

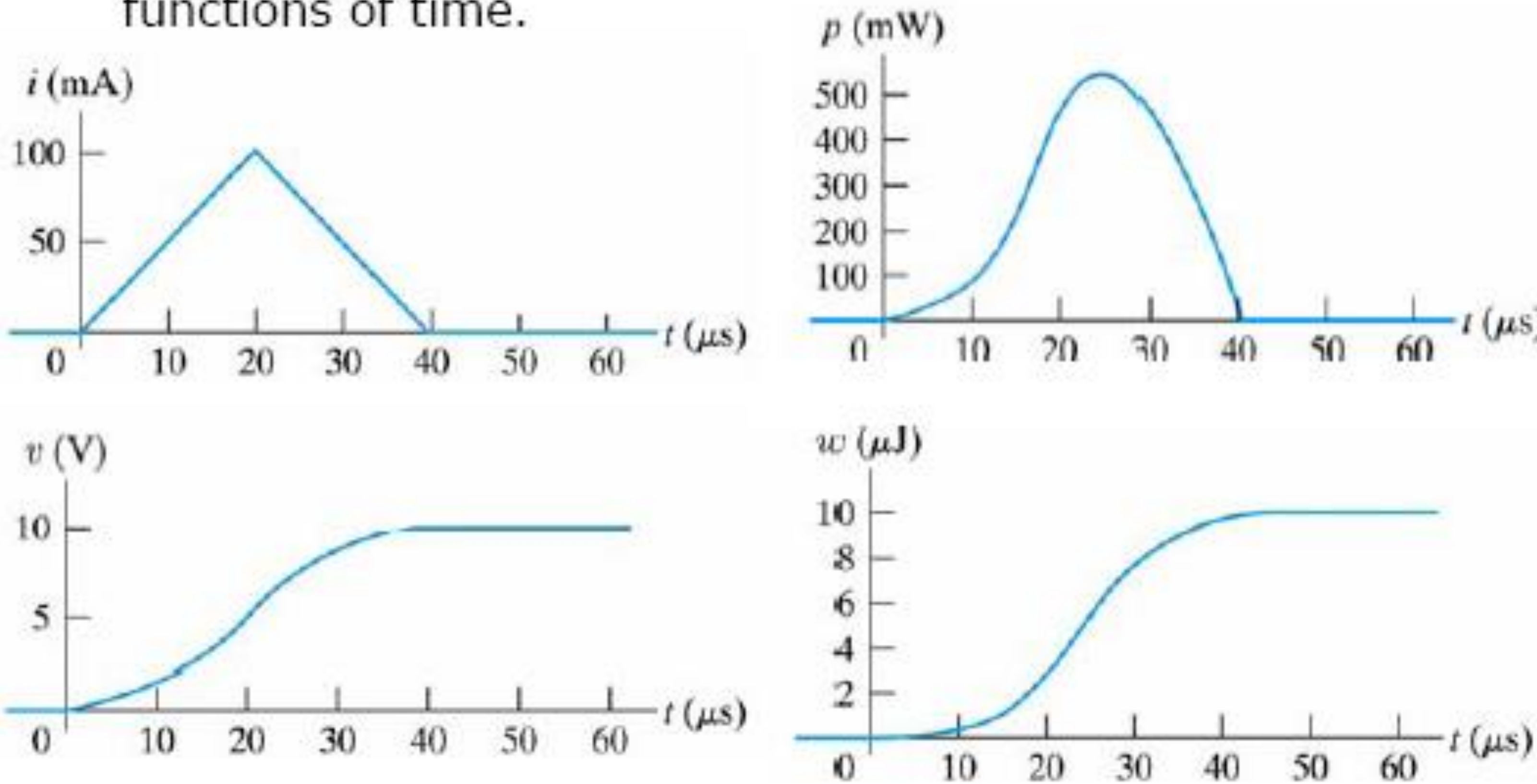
$$p = vi = (62.5\times10^{12} t^3 - 7.5\times10^9 t^2 + 2.5\times10^5 t - 2) \text{ W}$$

$$w = \frac{1}{2}Cv^2 = (15.625\times10^{12} t^4 - 2.5\times10^9 t^3 + 0.125\times10^6 t^2 - 2t + 10^{-5}) \text{ J}$$

Example (Cont.)

$\boxed{t \geq 40\ \mu s}$ $v = 10\ \text{V}$ $p = vi = 0$ $w = \frac{1}{2}Cv^2 = 10\ \mu J$

b) Sketch the current, voltage, power, and energy as functions of time.

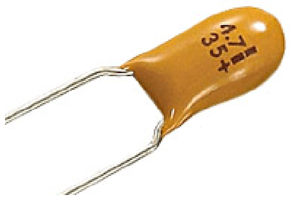


Example (Cont.)

c) Why does a voltage remain on the capacitor after the current returns to zero.

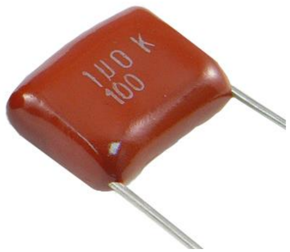
- Since both the source and the capacitor is ideal, when the current returns to zero, the energy is trapped inside the capacitor and there is no means of dissipating energy.

Various types of capacitors.



tantalum capacitor

Polypropylene Capacitor



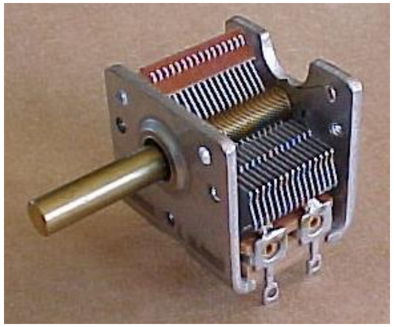
Polyester capacitor

High Voltage/power Capacitors

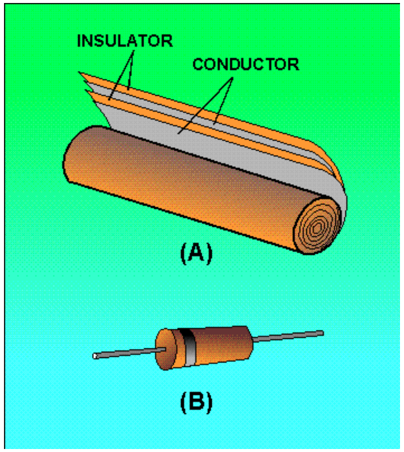
Multilayer Chip Ceramic Capacitor

Motor Running & Start Capacitors

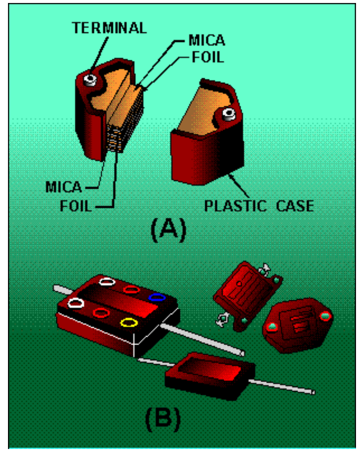
Variable Capacitor



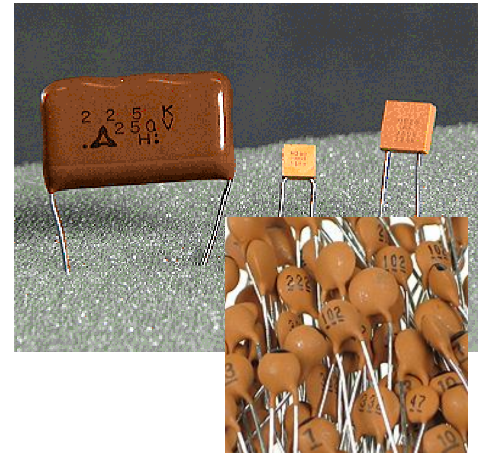
Tuning/Air Variable Capacitor



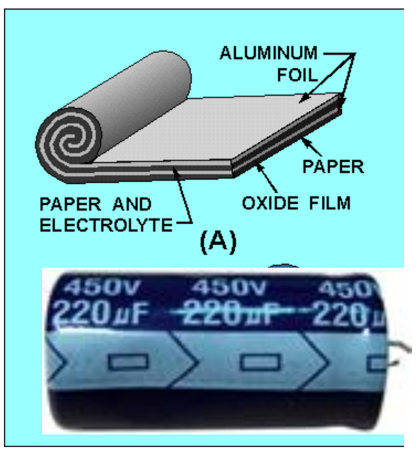
Paper capacitor (300pF - 4µF); max 600Volts



Mica capacitor (50pF -0.02µF)

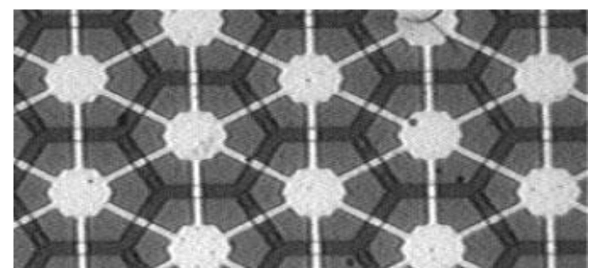


Ceramic Capacitor (1pF - 0.01µF); max 30kVolts



(Aluminum) Electrolytic Capacitor; (4µF ~ sevaral thousand F); max 500Volts

Oil capacitor (nF –sevaral hundred µF) several ten Kvolts



Top view of MEMS capacitor built at Stanford. The resonant frequency is 1.64 MHz with a Q of 18

Capacitor Code Guide

The 3 numbers: It is somewhat similar to the resistor code. The first two are the 1st and 2nd significant digits and the third is a multiplier code. Most of the time the last digit tells you how many zeros to write after the first two digits, but the standard (EIA standard) has a couple of curves that you probably will never see. But just to be complete here it is in a table.

Table 1 Digit multipliers

Third digit	Multiplier (this times the first two digits gives you the value in Pico-Farads)
0	1
1	10
2	100
3	1,000
4	10,000
5	100,000
6 not used	
7 not used	
8	.01
9	.1

Ex: A capacitor marked 104 is 10 with 4 more zeros or 100,000pF which is otherwise referred to as a .1 µF capacitor.

EIA = Electronic Industrial Association

http://xtronics.com/kits/ccode.htm

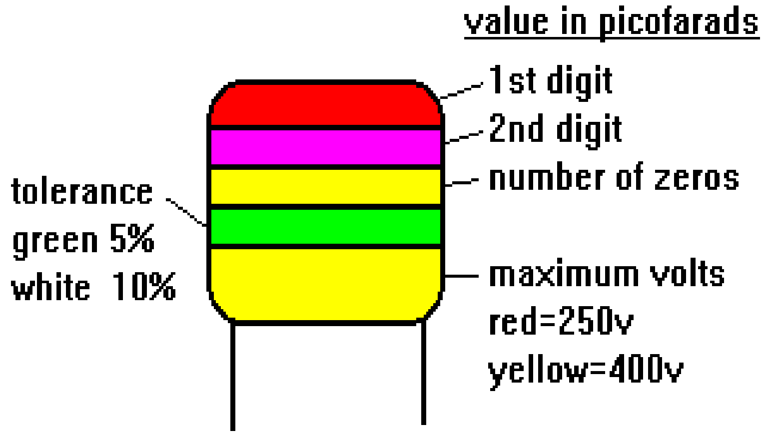
Capacitor Code Guide - EIA Capacitance Code

Table 2 Letter tolerance code	
Letter symbol	**Tolerance of capacitor**
B	+/- 0.10%
C	+/- 0.25%
D	+/- 0.5%
E	+/- 0.5%
F	+/- 1%
G	+/- 2%
H	+/- 3%
J	+/- 5%
K	+/- 10%
M	+/- 20%
N	+/- 0.05%
P	+100% ,-0%
Z	+80%, -20%

So a 103J is a 10,000 pF with +/-5% tolerance

Capacitor Code Guide – Color code

Some values are indicated with a colour code similar to resistors. There can be some confusion.
A 2200pf capacitor would have three red bands. These merge into one wide red band.



Some values are marked in picofarads using three digit numbers. The first two digits are the base number and the third digit is a multiplier.
For example, 102 is 1000 pF and 104 is 100,000 pF = 100 nF = 0.1 uF.

Impedance of capacitor:

The ratio of the phasor voltage across a circuit element to the phasor current through that element is called the impedance Z. For a capacitor, the impedance is given by

$$Z_C = \frac{V_C}{I_C} = \frac{-j}{2\pi f C} = -jX_C,$$

$X_C = \frac{1}{\omega C}$ is the capacitive reactance

$\omega = 2\pi f$ is the angular frequency

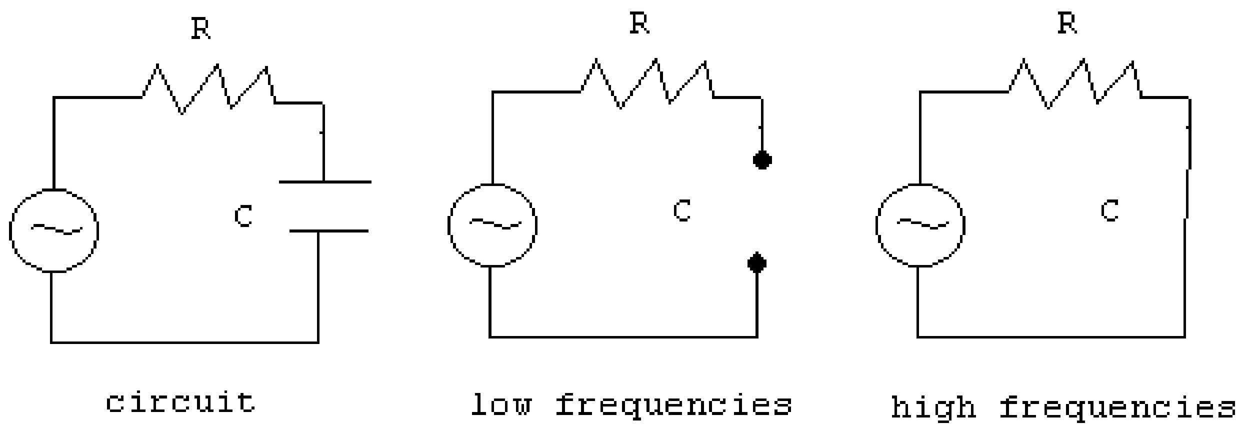
f is the frequency

C is the capacitance F

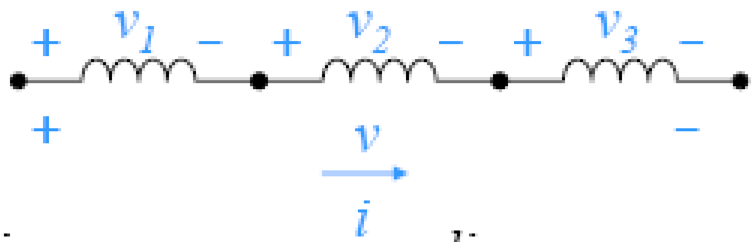
j is the imaginary unit

Capacitor Impedance

frequency	frequency approaches	impedance approaches	looks like	called
low	-->0	-->infinity	---● ●----	open circuit
high	-->infinity	-->0	--------------	short



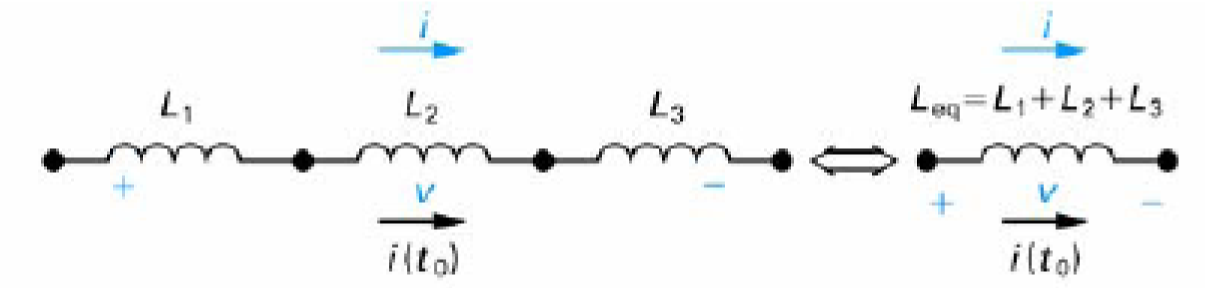
Series-parallel Combinations



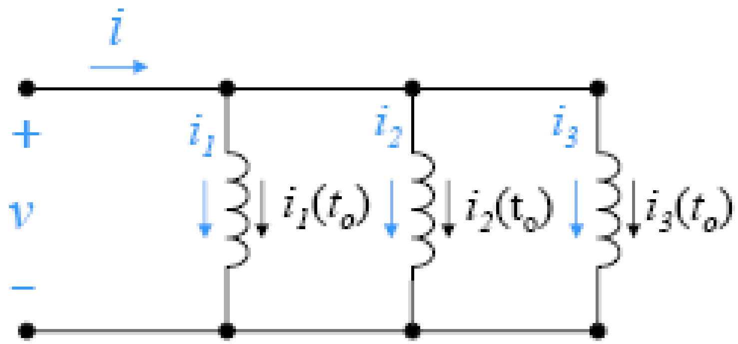
$$v_1 = L_1 \frac{di}{dt} \qquad v_2 = L_2 \frac{di}{dt} \qquad v_3 = L_3 \frac{di}{dt}$$

$$v = v_1 + v_2 + v_3 = (L_1 + L_2 + L_3)\frac{di}{dt}$$

$$L_{eq} = L_1 + L_2 + L_3 + \cdots + L_n$$



Series-parallel Combinations



$$i_1(t) = \frac{1}{L_1}\int_{t_o}^{t} v\,dt + i_1(t_o)$$

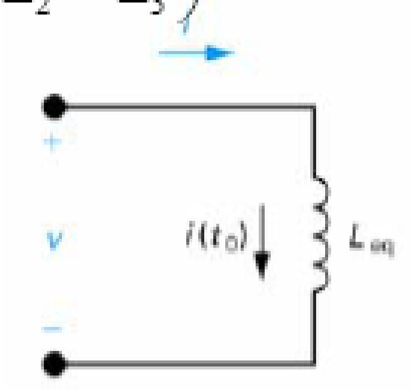
$$i_2(t) = \frac{1}{L_2}\int_{t_o}^{t} v\,dt + i_2(t_o)$$

$$i_3(t) = \frac{1}{L_3}\int_{t_o}^{t} v\,dt + i_3(t_o)$$

$$i = i_1 + i_2 + i_3 = \left(\frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3}\right)\int_{t_o}^{t} v\,dt + i_1(t_o) + i_2(t_o) + i_3(t_o)$$

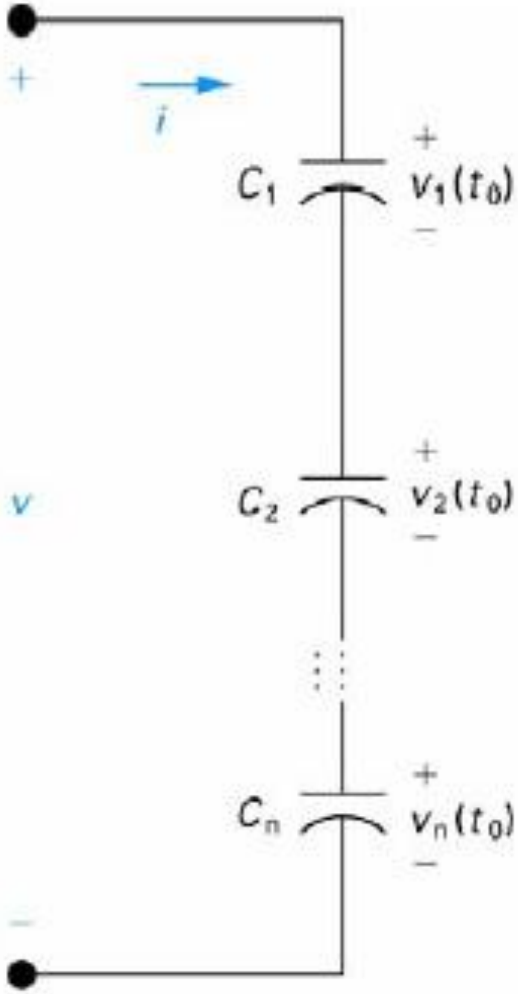
$$i = \frac{1}{L_{eq}}\int_{t_o}^{t} v\,dt + i(t_o)$$

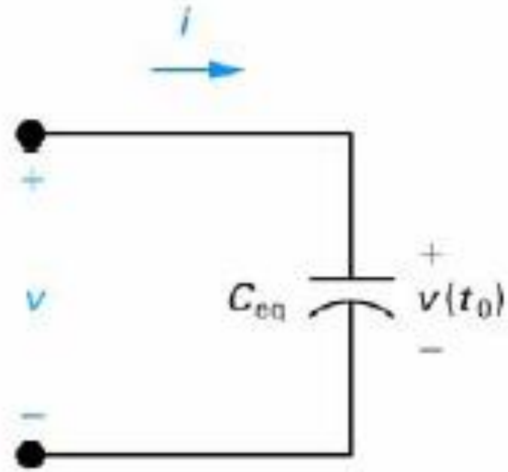
$$\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3}$$



$$\boxed{\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3} + \cdots + \frac{1}{L_n}}$$

Series-parallel Combinations

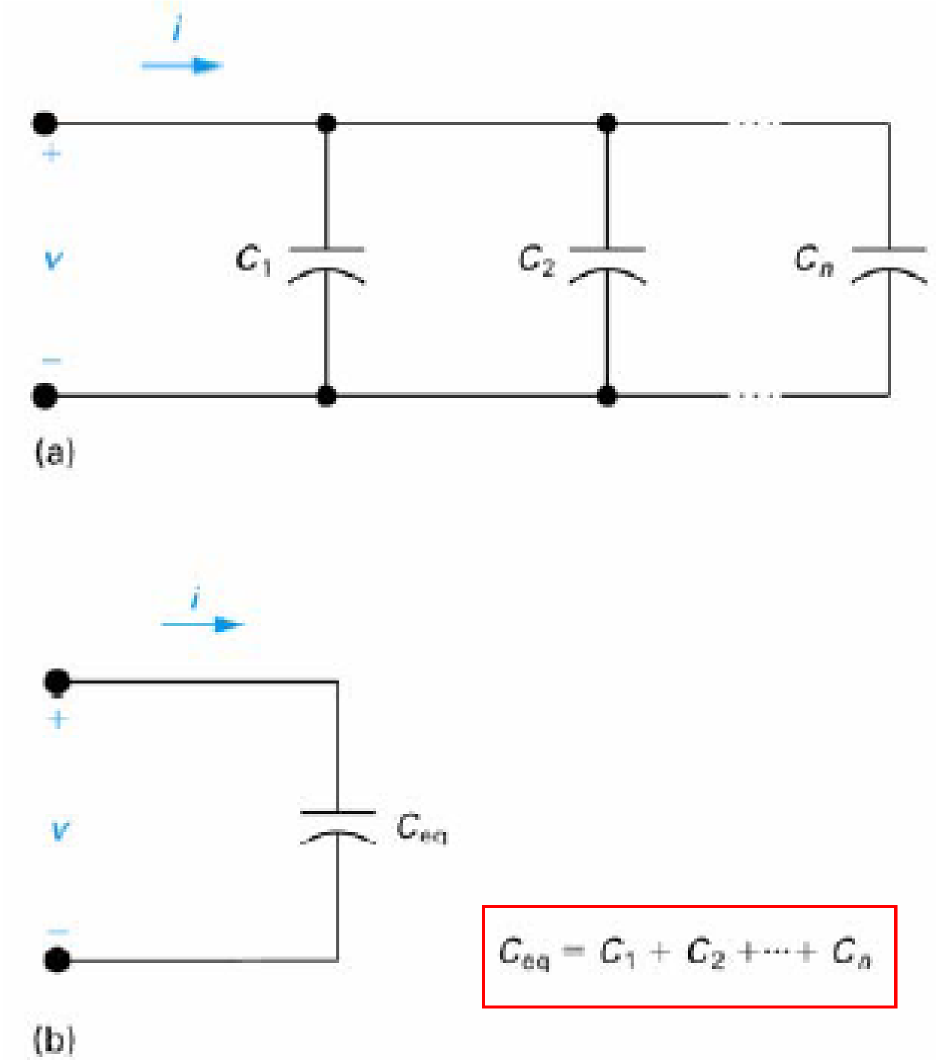




$$\boxed{\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \cdots + \frac{1}{C_n},}$$

$$v(t_0) = v_1(t_0) + v_2(t_0) + \cdots + v_n(t_0)$$

Series-parallel Combinations



$$\boxed{C_{eq} = C_1 + C_2 + \cdots + C_n}$$

Problem 1

Find L_{ab}?

Ans. :

$$L_{eq1} = 20 // 30 = 12\text{ H}$$

$$L_{eq2} = 12 + 8 = 20\text{ H}$$

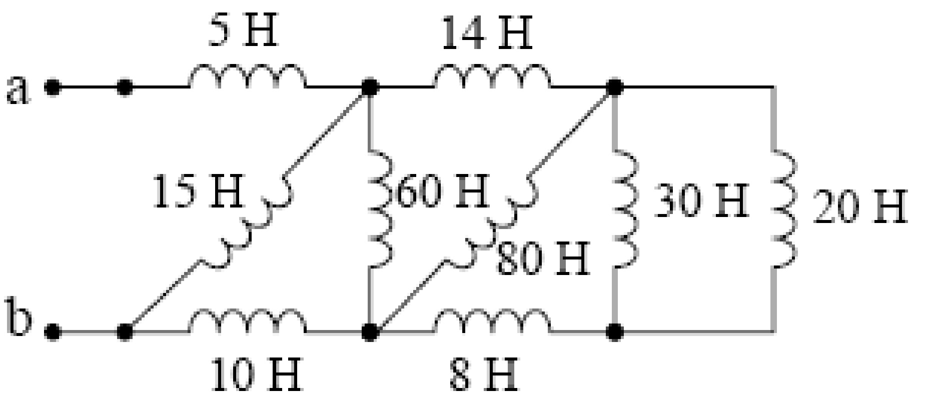
$$L_{eq3} = 20 // 80 = 16\text{ H}$$

$$L_{eq4} = 14 + 16 = 30\text{ H}$$

$$L_{eq5} = 30 // 60 = 20\text{ H}$$

$$L_{eq6} = 10 + 20 = 30\text{ H}$$

$$L_{eq7} = 30 // 15 = 10\text{ H}$$



$$\boxed{L_{ab} = 10 + 5 = 15\text{ H}}$$

Problem 2

Find C_{ab}?

Ans. :

8 μF // 16 μF ⇨ 24 μF

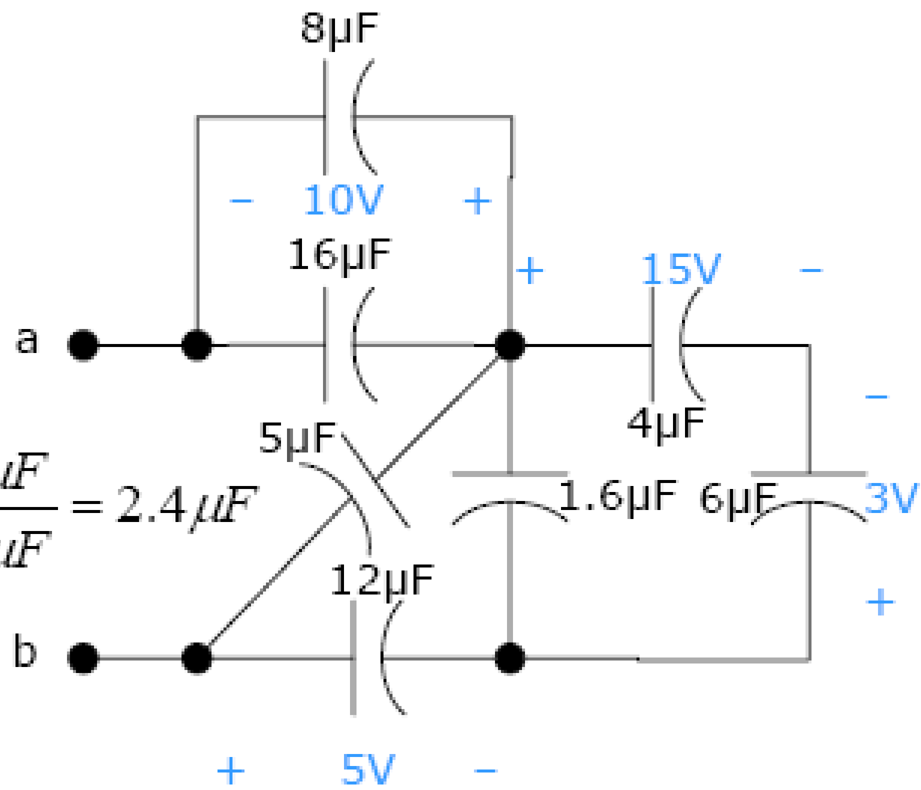
6 μF series 4 μF ⇨ $\dfrac{6\,\mu F \times 4\,\mu F}{4\,\mu F + 6\,\mu F} = 2.4\,\mu F$

2.4 μF // 1.6 μF ⇨ 4 μF

12 μF series 4 μF ⇨ 3 μF

5 μF // 3 μF ⇨ 8 μF

24 μF series 8 μF ⇨ $\boxed{6\ \mu F}$

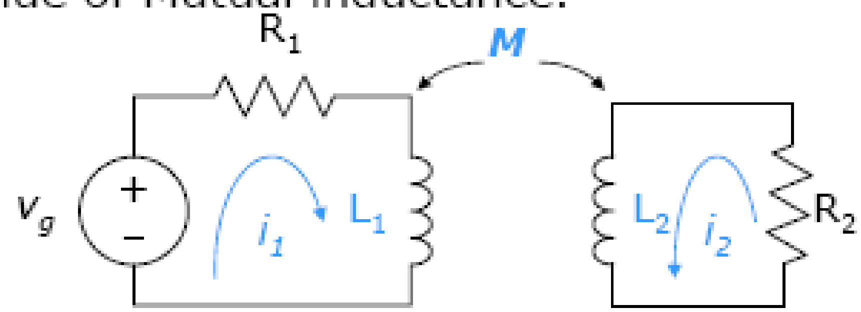


Mutual Inductance

The voltage induced in the second circuit can be related to the time-varying current in the first circuit by a parameter known as mutual inductance.

The inductance introduced earlier is known as the self inductance (L_1 & L_2).

M is the value of Mutual inductance.



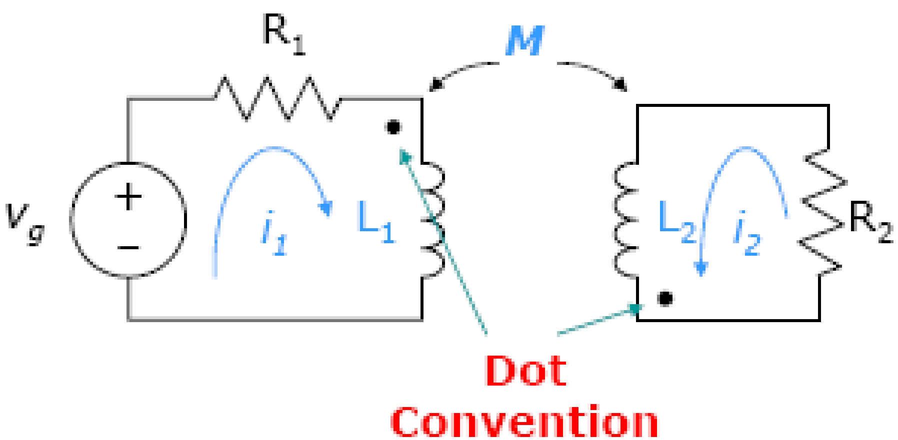
Consider coil 1 (left side)

$L_1 \frac{di_1}{dt}$ (Self Inductance) ? $M \frac{di_2}{dt}$ (Mutual Inductance)

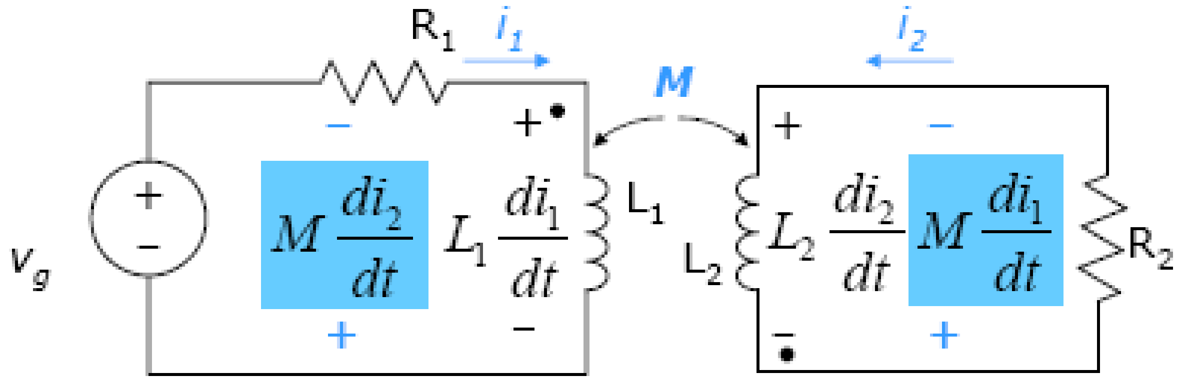
$$-v_g + i_1 R_1 + L_1 \frac{di_1}{dt} \boxed{\pm} M \frac{di_2}{dt} = 0$$

Mutual Inductance

- The sign of the mutual induced voltage depends on the way the coils are wound in relation to the reference direction of coil currents.



Mutual Inductance



When the reference direction for a current enters the dotted terminal of a coil, the **reference polarity** of the voltage that it induces in the other coil is positive at its dotted terminal.

When the reference direction for a current leaves terminal of a coil, the **reference polarity** of the voltage that it induces in the other coil is negative at its dotted terminal.

$$-v_g + i_1 R_1 + L_1\frac{di_1}{dt} - M\frac{di_2}{dt} = 0 \qquad i_2 R_2 + L_2\frac{di_2}{dt} - M\frac{di_1}{dt} = 0$$

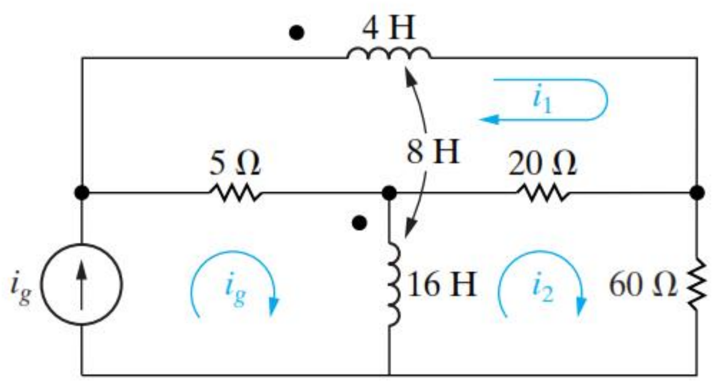
Example 6

a) Write a set of mesh-current equations that describe the circuit in Fig. 6.25 in terms of the currents i_1 and i_2.

b) Verify that if there is no energy stored in the circuit at $t = 0$ and if $i_g = 16 - 16e^{-5t}$ A, the solutions for i_1 and i_2 are

$$i_1 = 4 + 64e^{-5t} - 68e^{-4t} \text{ A},$$

$$i_2 = 1 - 52e^{-5t} + 51e^{-4t} \text{ A}.$$



a) Summing the voltages around the i_1 mesh yields

$$4\frac{di_1}{dt} + 8\frac{d}{dt}(i_g - i_2) + 20(i_1 - i_2) + 5(i_1 - i_g) = 0.$$

The i_2 mesh equation is

$$20(i_2 - i_1) + 60i_2 + 16\frac{d}{dt}(i_2 - i_g) - 8\frac{di_1}{dt} = 0.$$

Note that the voltage across the 4 H coil due to the current $(i_g - i_2)$, that is, $8d(i_g - i_2)/dt$, is a voltage drop in the direction of i_1. The voltage induced in the 16 H coil by the current i_1, that is, $8di_1/dt$, is a voltage rise in the direction of i_2.

b) To check the validity of i_1 and i_2, we begin by testing the initial and final values of i_1 and i_2. We know by hypothesis that $i_1(0) = i_2(0) = 0$. From the given solutions we have

$$i_1(0) = 4 + 64 - 68 = 0,$$

$$i_2(0) = 1 - 52 + 51 = 0.$$

Now we observe that as t approaches infinity the source current (i_g) approaches a constant value of 16 A, and therefore the magnetically coupled coils behave as short circuits. Hence at $t = \infty$ the circuit reduces to that shown in Fig. 6.26. From Fig. 6.26 we see that at $t = \infty$ the three resistors are in parallel across the 16 A source. The equivalent resistance is 3.75 Ω and thus the voltage across the 16 A current source is 60 V. It follows that

$$i_1(\infty) = \frac{60}{20} + \frac{60}{60} = 4 \text{ A},$$

$$i_2(\infty) = \frac{60}{60} = 1 \text{ A}.$$

These values agree with the final values predicted by the solutions for i_1 and i_2.

Finally we check the solutions by seeing if they satisfy the differential equations derived in (a). We will leave this final check to the reader via Problem 6.37.

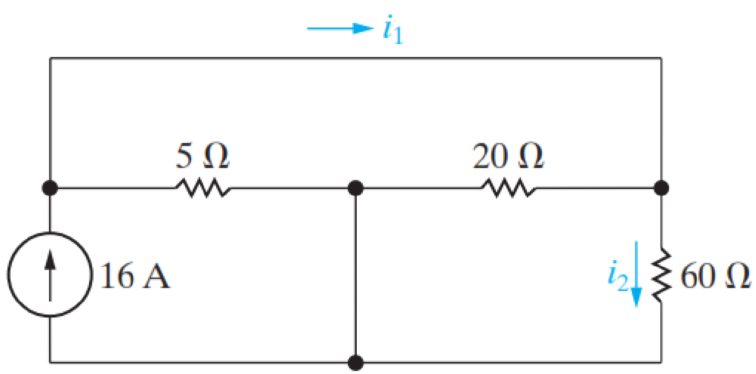


TABLE 6.1 Terminal Equations for Ideal Inductors and Capacitors

Inductors	
$v = L\frac{di}{dt}$	(V)
$i = \frac{1}{L}\int_{t_0}^{t} v\,d\tau + i(t_0)$	(A)
$p = vi = Li\frac{di}{dt}$	(W)
$w = \frac{1}{2}Li^2$	(J)
Capacitors	
$v = \frac{1}{C}\int_{t_0}^{t} i\,d\tau + v(t_0)$	(V)
$i = C\frac{dv}{dt}$	(A)
$p = vi = Cv\frac{dv}{dt}$	(W)
$w = \frac{1}{2}Cv^2$	(J)

TABLE 6.2 Equations for Series- and Parallel-Connected Inductors and Capacitors

Series-Connected
$L_{eq} = L_1 + L_2 + \cdots + L_n$
$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \cdots + \frac{1}{C_n}$
Parallel-Connected
$\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2} + \cdots + \frac{1}{L_n}$
$C_{eq} = C_1 + C_2 + \cdots + C_n$