Chapter 2: Laplace Transform

1. Definition

The Laplace transform of a function $f(t)$, defined for all real numbers $t \geq 0$, is the function $F(s)$, which is a unilateral transform defined by

$$F(s) = \mathcal{L}\{f(t)\} = \int_0^\infty f(t)e^{-st}dt \qquad \text{(Eq 2.1)}$$

2. Properties

$\boldsymbol{f(t)}$	$\boldsymbol{\mathcal{L}\{f(t)u(t)\}}$	$\boldsymbol{f(t)}$	$\boldsymbol{\mathcal{L}\{f(t)u(t)\}}$
$f(at)$	$\frac{1}{\lvert a\rvert}F\left(\frac{s}{a}\right)$	$e^{-at}f(t)$	$F(s+a)$
$t^n f(t)$	$(-1)^n F^{(n)}(s)$	$f(t-a)u(t-a)$	$e^{-as}F(s)$
$f'(t)$	$sF(s) - f(0)$	$(f * g)(t)$	$F(s).G(s)$
$f''(t)$	$s^2F(s) - sf(0) - f'(0)$	$\frac{f(t)}{t}$	$\int_s^{+\infty} F(\tau)d\tau$
$f^{(n)}(t)$	$s^nF(s) - s^{n-1}f(0) - \cdots - f^{n-1}(0)$	$\int_0^t f(\tau)d\tau = u(t) * f(t)$	$\frac{1}{s}F(s)$

3. Formulas

$\boldsymbol{f(t)}$	$\boldsymbol{\mathcal{L}\{f(t)u(t)\}}$	$\boldsymbol{f(t)}$	$\boldsymbol{\mathcal{L}\{f(t)u(t)\}}$
1	$\frac{1}{s}$	$\delta(t-a)$	e^{-as}
t^n	$\frac{n!}{s^{n+1}}$	$e^{-at}\cos\omega t$	$\frac{s+a}{(s+a)^2+\omega^2}$
e^{-at}	$\frac{1}{s+a}$	$e^{-at}\sin\omega t$	$\frac{\omega}{(s+a)^2+\omega^2}$
$\cos\omega t$	$\frac{s}{s^2+\omega^2}$	$t\cos\omega t$	$\frac{s^2-\omega^2}{(s^2+\omega^2)^2}$
$\sin\omega t$	$\frac{\omega}{s^2+\omega^2}$	$t\sin\omega t$	$\frac{2\omega s}{(s^2+\omega^2)^2}$
$\cosh at$	$\frac{s}{s^2-a^2}$	$t^n e^{-at}$	$\frac{n!}{(s+a)^{n+1}}$
$\sinh at$	$\frac{a}{s^2-a^2}$	$tf(t)$	$-F'(s)$

4. Initial and Final Value Theorem

Initial-value theorem

$$\lim_{t\to 0^+} f(t) = \lim_{s\to\infty} sF(s) = f(0^+) \qquad \text{(Eq 2.2)}$$

Final-value theorem

$$\lim_{t\to\infty} f(t) = \lim_{s\to 0} sF(s) \qquad \text{(Eq 2.3)}$$