Thứ

Ngày No.

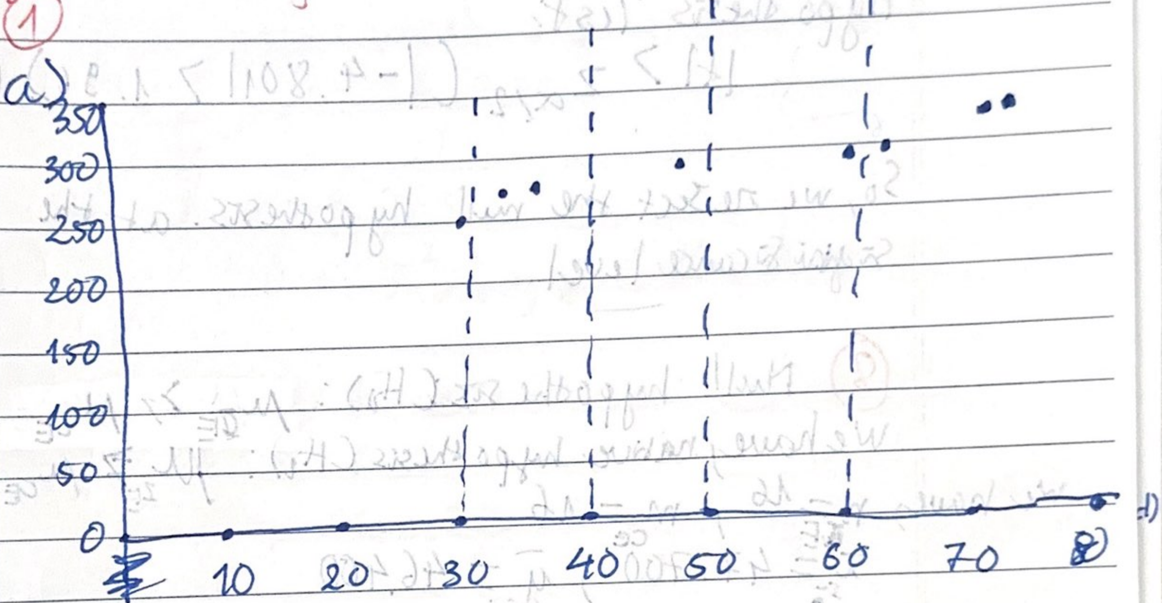
Phùng Huy Quang - ITITWE22145

HW 14

Linear Regression

1

a)



b)

Assume that we have a linear relationship btw (x) and (y), namely

$$y = \beta_0 + \beta_1 x$$

From the given data, we have

$$n = 8$$

$$\bar{X} = \frac{\Sigma x_i}{n} = 50.125 \; ; \; \bar{Y} = \frac{\Sigma y_i}{n} = 287.625$$

$$S_{xy} = \Sigma X_i Y_i - n\bar{X}\bar{Y} = 3314.375$$

$$S_{xx} = \Sigma X_i^2 - n\bar{X}^2 = 2894.875$$

$$S_{yy} = \Sigma Y_i^2 - n\bar{Y}^2 = 4683.875$$

$$SS = \frac{S_{xx} S_{yy} - S_{xy}^2}{S_{xx}} = 96.9627$$

Regression coefficients:

$$\beta_1 = \frac{S_{xy}}{S_{xx}} = \frac{\cancel{2394.875}\ 3314.375}{2394.875} = 1.384$$

$$\beta_0 = \bar{Y} - \beta_1 \bar{x} = 218.255$$

thus, $y = \beta_0 + \beta_1 x = 218.255 + 1.384x$

c)

$x = 65°F$

$y = 218.285 + 1.384(65)$
$= 308.215$

So, the predicted power consumption for $65°F$ is ≈ 308.215 BTU

2

Assume that we have relationship btw x and y

$$y = \beta_0 + \beta_1 x$$

From the given data, we have

$n = 9$

$\bar{X} = \frac{\Sigma x_i}{n} = 5$; $\bar{Y} = \frac{\Sigma y_i}{n} = 121.5556$

$S_{xy} = \Sigma X_i Y_i - n\bar{X}\bar{Y} = -121.8$

$S_{xx} = \Sigma X_i^2 - n\bar{X}^2 = 19.26$

$S_{yy} = \Sigma Y_i^2 - n\bar{Y}^2 = 804.222$

$SS = \frac{S_{xx} S_{yy} - S_{xy}^2}{S_{xx}} = 33.9605$

Thứ
Ngày
No.

Regression coefficient

$\beta_1 = \frac{S_{xy}}{S_{xx}} = -6.324$

$\beta_0 = \bar{Y} - \beta_1 \bar{X} = 153.176$

Thus,

$y = 153.176 - 6.324x.$

b)

The amount of particulate removed when the daily rainfall $x = 4.8$ is

$y = 153.176 - 6.324(4.8)$

$= 122.8208.$

3

a)

$y = \beta_0 + \beta_1 x$

$n = 19$

$\bar{X} = \frac{\Sigma x_i}{n} = 1.2473 \quad \bar{Y} = \frac{\Sigma y_i}{n} = 0.0196$

$S_{xy} = \Sigma X_i Y_i - n\bar{X}\bar{Y} = 0.0396$

$S_{xx} = \Sigma X_i^2 - n\bar{X}^2 = 6.125$

$S_{yy} = \Sigma Y_i^2 - n\bar{Y}^2 = 0.0003$

$S^2 = \frac{S_{xx}S_{yy} - S_{xy}^2}{S_{xx}} = 0.000044.$

$S = 0.00663.$

$\beta_1 = \frac{S_{xy}}{S_{xx}} = 0.0065$

$\beta_0 = \bar{Y} - \beta_1 \bar{X} = 0.0116$

Thus, Fitted line equation
$$y = 0.0116 + 0.0065x$$

b)

We have
$1 - \alpha = 95$
$t_{n-2,\alpha/2} = t_{17,0.025} = 2.110$

$ME = t_{n-2,\alpha/2} \dfrac{s}{\sqrt{S_{xx}}} = (2.110)\dfrac{0.00663}{\sqrt{6.125}} = 0.0056$

Thus,
$LB = \beta_1 - ME = 0.0009$
$UB = \beta_1 + ME = 0.0121$
Hence, 95% CI for β_1
$$0.0009 < \beta_1 < 0.0121$$

c) $\alpha = 1\% = 0.01$
$t_{obs} = \dfrac{\beta_1 - \beta_{10}}{s/\sqrt{S_{xx}}} = \dfrac{0.0065 - 0}{0.00663/\sqrt{6.125}} = 2.426$
$t_{0.005,17} = 2.898$

Hypothesis Test:
$$|t| < t_{\alpha/2,n-2} \quad (2.426 < 2.898)$$
So, we fail to reject the null hypothesis.

Thứ
Ngày . . No.

e)

$SSE = \sum_{i=1}^{n} (y_i - \hat{y}_i)^2 = (n-2)s^2$

$= (17)(0.000044)^2$

$= 0.032912 \times 10^{-6}$

$SSR = \sum_{i=1}^{n} (y_i - \bar{y})^2 = S_{YY} = 0.0003$

$R^2 = 1 - \frac{SSE}{SSR} = 0.9999 = 99.9\%$

∴ The linear model is a good ~~fit~~ model

$(R^2 \approx 1)$