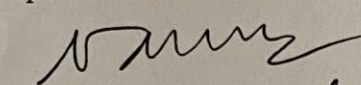
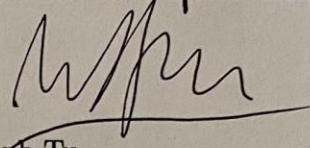


MIDTERM EXAMINATION
Semester 2, Academic Year 2021-2022
Duration: 90 minutes

SUBJECT: Calculus 2 (Groups 5 & 6)	
Department of Mathematics  Nguyễn Minh Quân	Lecturer  Nguyễn Anh Tu

INSTRUCTIONS:

- You can bring 2 A4 sheets of notes and a calculator.
- Each question has two parts, each part carries 10 points.

1. (a) Find

$$\lim_{n \rightarrow \infty} \frac{n - \sqrt{n^2 - 2}}{n^2 - \sqrt{n^4 + n}}.$$

(b) Let the sequence a_n be defined by $a_1 = 2$, $a_{n+1} = \frac{3}{4 - a_n}$, $n \geq 1$. Show that the sequence $\{a_n\}$ converges and find its limit.

2. (a) Evaluate the following series

$$\sum_{n=3}^{\infty} \frac{1}{(n-1)(n+2)}.$$

(b) Determine if the following series converges

$$\sum_{n=1}^{\infty} \sqrt{n} \tan(1/n^2).$$

3. (a) Find the interval of convergence of

$$\sum_{n=2}^{\infty} \frac{(2n)!(x+2)^n}{(n!)^2}.$$

(b) Find the Maclaurin series of

$$f(x) = \sin\left(2x + \frac{\pi}{2}\right)$$

4. Let $P(1, 2, 0)$, $Q(2, -3, 2)$, $R(3, 0, -1)$, $S(4, -1, 2)$.

(a) Do these 4 points lie on a same plane?

(b) Find the distance from the midpoint of QS to the line PR .

5. (a) Suppose that the velocity of a particle is $\mathbf{v}(t) = \langle 2t^2 + 1, (t + 1)e^{t-1}, -\sin(\pi t) \rangle$, and its position at time $t = 1$ is $\langle 2, 1, 0 \rangle$. Find its position function.

(b) Find the length of the curve

$$\mathbf{r}(t) = \langle t, t^2, \frac{2}{3}t^3 \rangle, \quad 0 \leq t \leq 3.$$

*** END OF QUESTIONS***