

You should also practice calculating the heat and work for various processes (isothermal, adiabatic, constant volume, and constant pressure). Use the heat capacity or specific heat to compute the heat, use  $\int p dV$  to compute the work.

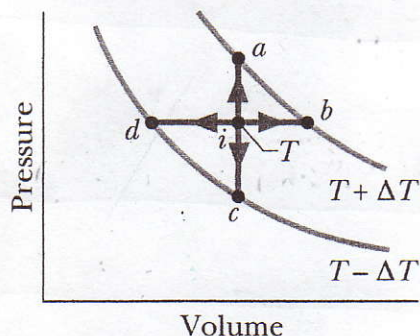
You should know how to calculate changes in entropy for various processes. Remember that  $\Delta S = \int (1/T) dQ$  for a reversible process between the initial and final state. To evaluate the integral you need to substitute for  $dQ$  and what you substitute depends on the process involved. In most cases you will use a heat capacity or specific heat to determine  $dQ$  in terms of the temperature change  $dT$ . In some cases you will need the first law of thermodynamics to determine  $dQ$  in terms of the work  $dW$  and the change  $dE_{\text{int}}$  in the internal energy. The change in the internal energy can always be written in terms of the heat capacity or specific heat at constant volume and the change in temperature.

Some problems deal with the microscopic view of entropy. You must calculate the number  $W$  of microstates for a given configuration and use  $S = -k \ln W$ .

## Questions and Example Problems from Chapter 20

### Question 1

Point  $i$  in the figure below represents the initial state of an ideal gas at temperature  $T$ . Taking algebraic signs into account, rank the entropy changes that the gas undergoes as it moves successively and reversibly, from point  $i$  to points  $a$ ,  $b$ ,  $c$ , and  $d$ , greatest first.



ideal gas:

$$\Delta S = n C_v \ln \left( \frac{T_f}{T_i} \right) + n R \ln \left( \frac{V_f}{V_i} \right)$$

$$a \leftarrow c \rightarrow V_f = V_i$$

$$c \leftarrow d \rightarrow \Delta T < 0$$

$$a \leftarrow b \rightarrow \Delta T > 0$$

**b, a, c, d**

### Question 2

A box contains 100 atoms in a configuration, with 50 atoms in each half of the box. Suppose that you could count the different microstates associated with this configuration at the rate of 100 billion states per second, using a supercomputer. Without written calculation, guess how much computing time you would need: a day, a year, or much more than a year.

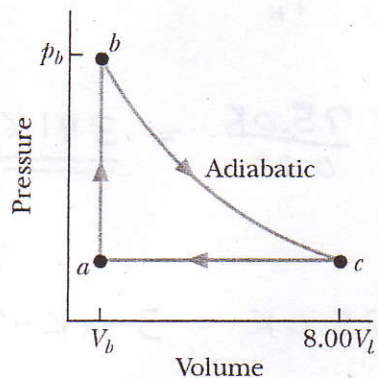
$$W = \frac{N!}{n_1! n_2!} = \frac{100!}{(50!)(50!)} = 1.01 \times 10^{29}$$

$$\text{time} = \frac{1.01 \times 10^{29}}{100 \times 10^9 / \text{s}} = 1.01 \times 10^{18} \text{ seconds} = \text{32 billion years}$$



### Problem 1

The figure below shows a reversible cycle through which one mole of a monatomic ideal gas is taken. Process bc is an adiabatic expansion, with  $p_b = 10.0 \text{ atm}$  and  $V_b = 1.00 \times 10^{-3} \text{ m}^3$ . Find (a) the energy added to the gas as heat, (b) the energy leaving the gas as heat, (c) the net work done by the gas, and (d) the efficiency of the cycle.



(a) energy added as heat during  $a \rightarrow b$

$$Q = n C_v \Delta T \text{ constant volume}$$

$$C_v = \frac{3}{2} R \text{ monatomic gas}$$

$$Q = \frac{3}{2} n R \Delta T$$

$$pV = nRT \rightarrow \Delta(pV) = nR\Delta T$$

$$Q = \frac{3}{2} \Delta(pV) = \frac{3}{2} (p_b V_b - p_a V_a) = \frac{3}{2} (p_b - p_a) V$$

we need  $p_a \rightarrow p_a = p_c$  and  $p_c + p_b$  connected by adiabat

$$p_c V_c^\gamma = p_b V_b^\gamma \rightarrow p_c = p_b \left( \frac{V_b}{V_c} \right)^\gamma = (1.013 \times 10^6 \text{ Pa}) \left( \frac{1}{8} \right)^{5/3}$$

$$p_c = 3.167 \times 10^4 \text{ Pa} = p_a$$

$$Q = \frac{3}{2} (p_b - p_a) V_b = \frac{3}{2} (1.013 \times 10^6 \text{ Pa} - 3.167 \times 10^4 \text{ Pa}) (1.0 \times 10^{-3} \text{ m}^3) =$$

$$1.47 \times 10^3 \text{ J}$$

(b) energy leaves gas during  $c \rightarrow a$

$$Q = n C_p \Delta T \quad C_p = \frac{5}{2} R \text{ for monatomic} \rightarrow Q = \frac{5}{2} n R \Delta T$$

$$Q = \frac{5}{2} \Delta(pV) = \frac{5}{2} (p_a V_a - p_c V_c) = \frac{5}{2} p_a (V_a - V_c)$$

$$Q = \frac{5}{2} (3.167 \times 10^4 \text{ Pa}) (-7.00) (1.0 \times 10^{-3} \text{ m}^3) = \frac{5}{2} p_a (-7.0 V_b)$$

$$Q = -554 \text{ J}$$

(c)  $\Delta E_{\text{int}} = 0$  for cyclic path  $\rightarrow W = Q = Q_{ab} + Q_{bc} + Q_{ca}$

$$(d) \quad \epsilon = \frac{|W|}{|Q_H|} = \frac{|W|}{|Q_{ab}|} = \frac{916 \text{ J}}{1.47 \times 10^3 \text{ J}} \rightarrow \epsilon = 0.623$$



**Problem 2**

A Carnot engine has an efficiency of 22.0%. It operates between constant-temperature reservoirs differing in temperature by 75.0 °C. What are the temperatures of the two reservoirs?

$$\epsilon = 22.0\% \\ = 0.220$$

$$\epsilon_c = 1 - T_L/T_H = \frac{T_H - T_L}{T_H}$$

$$T_H - T_L = 75.0\text{C}^\circ \\ = 75.0\text{K}$$

$$T_H = \frac{T_H - T_L}{\epsilon_c} = \frac{75.0\text{K}}{0.220} = \underline{\underline{341\text{K}}}$$

$$T_H - T_L = 75.0\text{K} \rightarrow T_L = T_H - 75.0\text{K} = 341\text{K} - 75.0\text{K} \\ = \underline{\underline{266\text{K}}}$$

$$\boxed{\begin{array}{l} T_L = 266\text{K} \\ T_H = 341\text{K} \end{array}}$$

**Problem 3**

A Carnot engine operates between 235°C and 115°C, absorbing  $6.30 \times 10^4$  J per cycle at the higher temperature. (a) What is the efficiency of the engine? (b) How much work per cycle is this engine capable of performing?

$$T_H - T_L = 120\text{C}^\circ = 120\text{K}$$

$$Q_H = 6.30 \times 10^4 \text{ J}$$

$$\epsilon_c = 1 - T_L/T_H$$

$$T_H = 235\text{C}^\circ + 273.15 = 508.15\text{K}$$

$$\epsilon_c = \frac{T_H - T_L}{T_H}$$

$$(a) \epsilon_c = \frac{T_H - T_L}{T_H} = \frac{(120\text{K})}{508\text{K}} = \boxed{0.236 = 23.6\%}$$

$$(b) \epsilon = \frac{|W|}{|Q_H|} \rightarrow |W| = \epsilon |Q_H|$$

$$= (0.236)(6.3 \times 10^4 \text{ J})$$

$$\boxed{|W| = 1.49 \times 10^4 \text{ J}}$$



**Problem 4**

A Carnot air conditioner takes energy from the thermal energy of a room at  $70^\circ\text{F}$  and transfers it to the outdoors, which is at  $96^\circ\text{F}$ . For each joule of electric energy required to operate the air conditioner, how many joules are removed from the room?

$$T_H = 96^\circ\text{F} = 36^\circ\text{C} = 309\text{K}$$

$$T_L = 70^\circ\text{F} = 21^\circ\text{C} = 294\text{K}$$

$$K_c = \frac{|Q_L|}{|W|} = \frac{T_L}{T_H - T_L}$$

$$K_c = \frac{T_L}{T_H - T_L} = \frac{294\text{K}}{(309\text{K} - 294\text{K})} = \underline{\underline{19.6}}$$

$$K_c = \frac{|Q_L|}{|W|} \rightarrow |Q_L| = K_c |W| = (19.6)(1\text{J}) = \underline{\underline{19.6\text{J}}}$$

$$\boxed{|Q_L| = 20\text{J}}$$

**Problem 5**

The electric motor of a heat pump transfers energy as heat from the outdoors, which is at  $-5.0^\circ\text{C}$ , to a room, which is at  $17^\circ\text{C}$ . If the heat pump were a Carnot heat pump (a Carnot engine working in reverse), how many joules of heat would be transferred to the thermal energy of the room for each joule of electric energy consumed?

$$T_L = -5.0^\circ\text{C} = 268\text{K}$$

$$K_c = \frac{T_L}{T_H - T_L}$$

$$K = \frac{|Q_H|}{|W|}$$

$$T_H = 17^\circ\text{C} = 290\text{K}$$

$$|W| = 1\text{J}$$

$$|Q_L| + |W| = |Q_H| \rightarrow |Q_L| = |Q_H| - |W|$$

$$|Q_H| = ?$$

$$K = \frac{|Q_H|}{|W|} = \frac{|Q_H| - |W|}{|W|} = \frac{|Q_H|}{|W|} - 1$$

$$|Q_H| = (K+1)|W|$$

$$K_c = \frac{268\text{K}}{(290\text{K} - 268\text{K})} = 12.2$$

$$|Q_H| = (12.2 + 1)(1\text{J})$$

$$\boxed{|Q_H| = 13.2\text{J}}$$



### Problem 6

Suppose 4.00 mol of an ideal gas undergo a reversible isothermal expansion from volume  $V_1$  to volume  $V_2 = 2.00V_1$  at temperature  $T = 400$  K. Find (a) the work done by the gas and (b) the entropy change of the gas. (c) If the expansion is reversible and adiabatic instead of isothermal, what is the entropy change of the gas?

$$n = 4.00$$

$$V_2 = 2.00V_1$$

$$T = 400 \text{ K}$$

$$(a) W = \int p dv = nRT \ln \left( \frac{V_2}{V_1} \right) \text{ for isothermal process}$$

$$W = (4.00 \text{ mol})(8.31 \frac{\text{J}}{\text{mol} \cdot \text{K}})(400 \text{ K}) \ln 2 = \boxed{9.22 \times 10^3 \text{ J}}$$

$$(b) \Delta S = \int \frac{dQ}{T} = Q/T \text{ for isothermal process}$$

$$\Delta E_{\text{int}} = 0 \text{ for isotherm since } \Delta T = 0 \rightarrow Q = W \quad \Delta S = Q/T = W/T$$

$$\Delta S = \frac{(9.22 \times 10^3 \text{ J})}{400 \text{ K}} = \boxed{23.1 \text{ K}}$$

$$(c) \text{ for an adiabatic process}$$

$$Q = 0 \text{ so } \Delta S = 0$$

### Problem 7

In an experiment, 200 g of aluminum (with a specific heat of 900 J/kg·K) at 100°C is mixed with 50.0 g of water at 20.0°C, with the mixture thermally isolated. (a) What is the equilibrium temperature? What are the entropy changes of (b) the aluminum, (c) the water, and (d) the aluminum-water system?

$$M_{\text{Al}} = 0.200 \text{ Kg}$$

$$C_{\text{Al}} = 900 \text{ J/kg} \cdot \text{K}$$

$$T_{\text{Al}} = 373 \text{ K}$$

$$M_{\text{w}} = 0.050 \text{ Kg}$$

$$C_{\text{w}} = 4190 \text{ J/kg} \cdot \text{K}$$

$$T_{\text{w}} = 293 \text{ K}$$

$$(a) Q_{\text{Al}} + Q_{\text{w}} = 0 \rightarrow M_{\text{Al}} C_{\text{Al}} (T - T_{\text{Al}}) = M_{\text{w}} C_{\text{w}} (T - T_{\text{w}})$$

$$T = \frac{M_{\text{Al}} C_{\text{Al}} T_{\text{Al}} + M_{\text{w}} C_{\text{w}} T_{\text{w}}}{M_{\text{Al}} C_{\text{Al}} + M_{\text{w}} C_{\text{w}}}$$

$$T = \frac{(0.200 \text{ Kg})(900 \text{ J/kg} \cdot \text{K})(373 \text{ K}) + (0.050 \text{ Kg})(4190 \text{ J/kg} \cdot \text{K})(293 \text{ K})}{(0.200 \text{ Kg})(900 \text{ J/kg} \cdot \text{K}) + (0.050 \text{ Kg})(4190 \text{ J/kg} \cdot \text{K})}$$

$$\boxed{T = 330 \text{ K}}$$

$$T = ?$$

$$(b) \Delta S = \int \frac{dQ}{T} = \int \frac{mc dT}{T} = mc \int_{T_i}^{T_f} \frac{dT}{T}$$

$$\Delta S = mc \ln(T_f/T_i)$$

$$\text{for aluminum } \Delta S = (0.200 \text{ Kg})(900 \text{ J/kg} \cdot \text{K}) \ln \left( \frac{330 \text{ K}}{373 \text{ K}} \right) = \boxed{-22.0 \text{ J/K}}$$

$$(c) \text{ for water } \Delta S = (0.050 \text{ Kg})(4190 \text{ J/kg} \cdot \text{K}) \ln \left( \frac{330 \text{ K}}{293 \text{ K}} \right) = \boxed{24.9 \text{ J/K}}$$

$$(d) \text{ for aluminum-water } \Delta S = -22.0 \text{ J/K} + 24.9 \text{ J/K} = \boxed{2.9 \text{ J/K}}$$



### Problem 8

A 10 g ice cube at  $-10^\circ\text{C}$  is placed in a lake whose temperature is  $15^\circ\text{C}$ . Calculate the change in entropy of the cube-lake system as the ice cube comes to thermal equilibrium with the lake. The specific heat of ice is  $2220 \text{ J/kg}\cdot\text{K}$ . (Hint: Will the ice cube affect the temperature of the lake?)

$$\Delta S_{\text{cube-lake}} = \Delta S_{\text{ice}} + \Delta S_{\text{lake}}$$

$$m_{\text{ice}} = 0.010 \text{ kg}$$

$$T_{i,\text{ice}} = -10^\circ\text{C}$$

$$T_{i,\text{lake}} = 15^\circ\text{C}$$

$$T_{f,\text{ice}} = 15^\circ\text{C}$$

$$Q_{\text{ice}} + Q_{\text{lake}} = 0$$

$$Q_{\text{ice warms to } 0^\circ\text{C}} + Q_{\text{ice melts}} + Q_{\text{water warms to } 15^\circ\text{C}} + Q_{\text{lake}} = 0$$

material  
changes temp.  $\rightarrow$

$$dQ = mc dT$$

$$\Delta S = \int \frac{dQ}{T} = mc \int \frac{dT}{T} = mc \ln\left(\frac{T_f}{T_i}\right)$$

material changes phase  $\rightarrow Q = \pm mL$

$$\Delta S = \int \frac{dQ}{T} = \frac{Q}{T} = \pm \frac{mL}{T}$$

$$\Delta S_{\text{ice}} = m_i c_i \ln\left(\frac{T_f}{T_i}\right) + \frac{m_i L_f}{T} + m_i c_{\text{water}} \ln\left(\frac{T_f}{T_i}\right)$$

$\downarrow$  ice warms to  $0^\circ\text{C}$        $\downarrow$  ice melts       $\downarrow$  water warms to  $15^\circ\text{C}$

$$\Delta S_{\text{ice}} = (0.010 \text{ kg})(2220 \text{ J/kg}\cdot\text{K}) \ln\left(\frac{273 \text{ K}}{263 \text{ K}}\right) + \frac{(0.010 \text{ kg})(333 \times 10^3 \text{ J/kg})}{(273 \text{ K})} + (0.010 \text{ kg})(4190 \text{ J/kg}\cdot\text{K}) \ln\left(\frac{288 \text{ K}}{273 \text{ K}}\right)$$

$$\Delta S_{\text{ice}} = 0.828 \text{ J/K} + 12.2 \text{ J/K} + 2.24 \text{ J/K} \rightarrow \boxed{\Delta S_{\text{ice}} = 15.3 \text{ J/K}}$$

temperature of lake doesn't change  $\rightarrow \Delta S_{\text{lake}} = \frac{Q_{\text{lake}}}{T}$

$$Q_{\text{lake}} = -Q_{\text{ice}} = -(m_i c_i \Delta T_i + m_i L_f + m_i c_w \Delta T_w) = -4.18 \times 10^3 \text{ J}$$

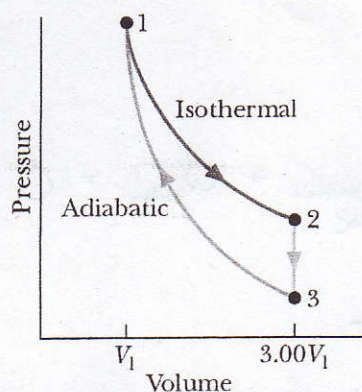
$$\Delta S_{\text{lake}} = \frac{-4.18 \times 10^3 \text{ J}}{288 \text{ K}} = \boxed{-14.5 \text{ J/K}}$$

$$\Delta S_{\text{cube-lake}} = 15.3 \text{ J/K} - 14.5 \text{ J/K} = \boxed{+0.80 \text{ J/K}}$$



### Problem 9

An ideal diatomic gas, whose molecules are rotating but not oscillating, is taken through the cycle in the figure below. Determine for all three processes, in terms of  $p_1$ ,  $V_1$ ,  $T_1$ , and  $R$ : (a)  $p_2$ ,  $p_3$ , and  $T_3$  and (b)  $W$ ,  $Q$ ,  $\Delta E_{\text{int}}$ , and  $\Delta S$  per mole.



diatomic gas  $\begin{cases} C_v = 5/2 R \\ C_p = 7/2 R \end{cases} \quad \gamma = 7/5$

(a) for isotherm:  $p_1 V_1 = p_2 V_2$

$$p_2 = p_1 \left( \frac{V_1}{V_2} \right) = p_1 \left( \frac{V_1}{3.00 V_1} \right) \rightarrow \boxed{p_2 = p_1 / 3}$$

adiabatic  $\rightarrow p_1 V_1^\gamma = p_3 V_3^\gamma \rightarrow p_3 = p_1 \left( \frac{V_1}{V_3} \right)^\gamma = p_1 \left( \frac{1}{3} \right)^{7/5} \rightarrow \boxed{p_3 = \frac{p_1}{3^{1.4}}}$

$$\rightarrow T_1 V_1^{\gamma-1} = T_3 V_3^{\gamma-1} \rightarrow T_3 = T_1 \left( \frac{V_1}{V_3} \right)^{\gamma-1} = T_1 \left( \frac{1}{3} \right)^{2/5} \rightarrow \boxed{T_3 = \frac{T_1}{3^{0.4}}}$$

(b) to figure out  $W$ ,  $Q$ ,  $\Delta E_{\text{int}}$ , +  $\Delta S$  per mole we let  $n=1$

process 1  $\rightarrow$  2 for isotherm  $\boxed{\Delta E = 0}$

$$Q = W = n R T \ln \left( \frac{V_f}{V_i} \right) = (1) R T_1 \ln 3$$

$$\boxed{Q = W = R T_1 \ln 3}$$

for isotherm  $\Delta S = Q/T = \frac{R T_1 \ln 3}{T_1} \rightarrow \boxed{\Delta S = R \ln 3 = 1.10 R}$

process 2  $\rightarrow$  3 for constant volume  $\boxed{W = 0}$   $\Delta E_{\text{int}} = Q = n C_v \Delta T$

$$\Delta E_{\text{int}} = Q = (1) \left( \frac{5}{2} R \right) (T_3 - T_2) = \frac{5}{2} R \left( \frac{T_1}{3^{0.4}} - T_1 \right) = \frac{5}{2} R T_1 \left( \frac{1}{3^{0.4}} - 1 \right)$$

$$\boxed{\Delta E_{\text{int}} = Q = -0.889 R T_1}$$

$$\Delta S = n R \ln \left( \frac{V_f}{V_i} \right) + n C_v \ln \left( \frac{T_f}{T_i} \right) = (1) \left( \frac{5}{2} R \right) \ln \left[ \frac{T_1 / 3^{0.4}}{T_1} \right]$$

$$\Delta S = \frac{5}{2} R \ln \left( \frac{1}{3^{0.4}} \right) = \boxed{-1.10 R}$$

process 3  $\rightarrow$  1 for adiabatic  $\boxed{Q = 0}$   
 $\boxed{\Delta S = 0}$

$$\Delta E_{\text{int}} = 0 = \Delta E_{\text{int}} + \Delta E_{\text{int}} + \Delta E_{\text{int}} \rightarrow \Delta E_{\text{int}} = -\Delta E_{\text{int}} - \Delta E_{\text{int}} = 0 - (-0.889 R T_1)$$

$$\boxed{\Delta E_{\text{int}} = 0.889 R T_1}$$

$$W = -\Delta E_{\text{int}} = -0.889 R T_1$$



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**Problem 10**

A box contains  $N$  gas molecules, equally divided between its two halves. For  $N = 50$ : (a) What is the multiplicity of this central configuration? (b) What is the total number of microstates for the system? (c) What percentage of the time does the system spend in its central configuration? (d) Repeat (a) through (c) for  $N = 100$ . (e) Repeat (a) through (c) for  $N = 200$ . (f) As  $N$  increases, you will find that the system spends less time (not more) in its central configuration. Explain why this is so.

$$W = \frac{N!}{n_1! n_2!}$$

(a)  $N = 50$   
 $n_1 = 25$   
 $n_2 = 25$

$$W = \frac{50!}{25! 25!} \rightarrow W = 1.26 \times 10^{14}$$

(b) total # of microstates  $\rightarrow$  2 choices for each molecule;  $N$  independent molecules

$$N_{\text{total}} = 2 \times 2 \times \dots \times 2 = 2^N = 2^{50} = 1.13 \times 10^{15}$$

(c) % of time = probability the system is in central configuration

$$= \frac{W_{\text{central configuration}}}{N_{\text{total}}} = \frac{1.26 \times 10^{14}}{1.13 \times 10^{15}} = 0.111 = 11.1\%$$

(d) for  $N = 100 \rightarrow$

$$W = 1.01 \times 10^{29}$$
$$N_{\text{total}} = 1.27 \times 10^{30}$$
$$\% \text{ time} = 8.0\%$$

(e) for  $N = 200 \rightarrow$

$$W = 9.05 \times 10^{58}$$
$$N_{\text{total}} = 1.61 \times 10^{60}$$
$$\% \text{ time} = 5.7\%$$

(f) As  $N$  increases, the available # of microstates increases, so there are more states to be occupied, leaving the probability less for the system to remain in the central configuration.