The Operational Amplifier

(Chapter 5)

Textbook:

Electric Circuits

James W. Nilsson & Susan A. Riedel

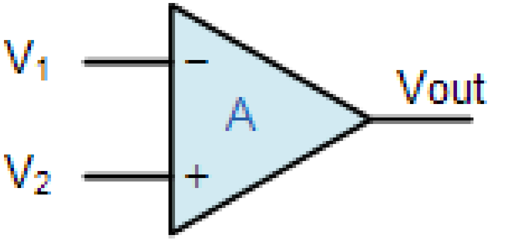
9th Edition.

Objectives

- Be able to name the five OP-AMP (Operational amplifier) terminals and describe and use the voltage and current constraints and the resulting simplifications they lead to in an ideal op amp.
- Be able to analyze simple circuits containing ideal OP-AMPs, and recognize the following op amp circuits: inverting amplifier, summing amplifier, non inverting amplifier, and difference amplifier.
- Understand the more realistic model for an op amp and be able to use this model to analyze simple circuits containing op amps.

Outline

- Operational amplifier terminals
- Terminal voltages and currents
- The inverting-amplifier circuit
- The summing-amplifier circuit
- The non inverting-amplifier circuit
- The difference-amplifier circuit



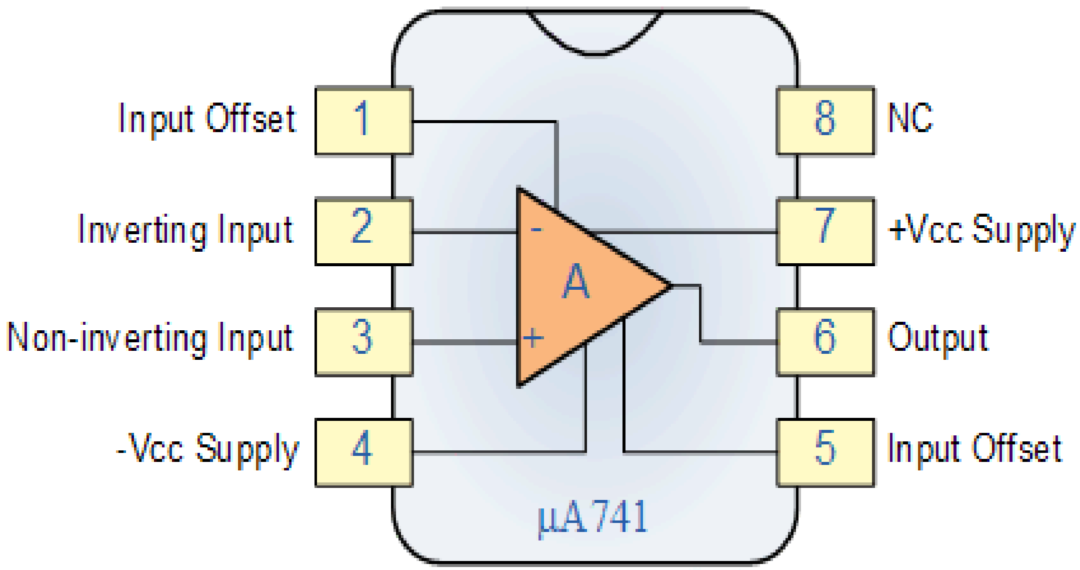
Op Amps

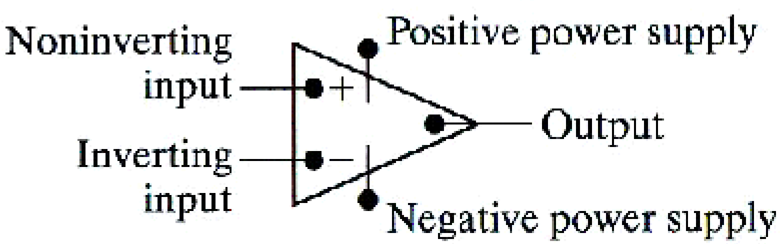
- Op Amp <==> operational amplifier.
- An operational amplifier has a very high input impedance and a very high gain.
- Op-amps can be configured in many different ways using resistors and other components.
- Most configurations use feedback.

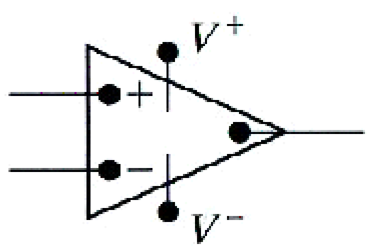
Applications of Op Amps

- Amplifiers provide gains in voltage or current.
- Op amps can convert current to voltage.
- Op amps can provide a buffer between two circuits.
- Op amps can be used to implement integrators and differentiators.
- Low-pass and band-pass filters.

Op Amp Terminals







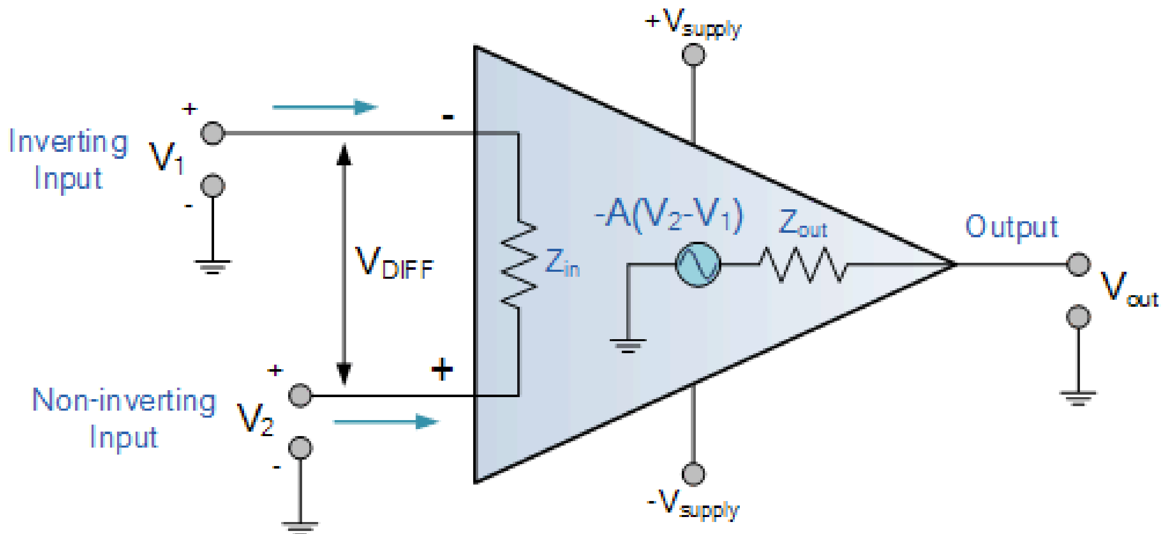
<u>Terminals of primary interest:</u>

- inverting input
- non-inverting input
- output
- positive power supply (+Vcc)
- negative power supply (-Vcc)

<u>Offset null terminals</u> may be used to compensate for a degradation in performance because of aging and imperfections.

The Op Amp Model

An operational amplifier is modeled as a voltage-controlled voltage source.



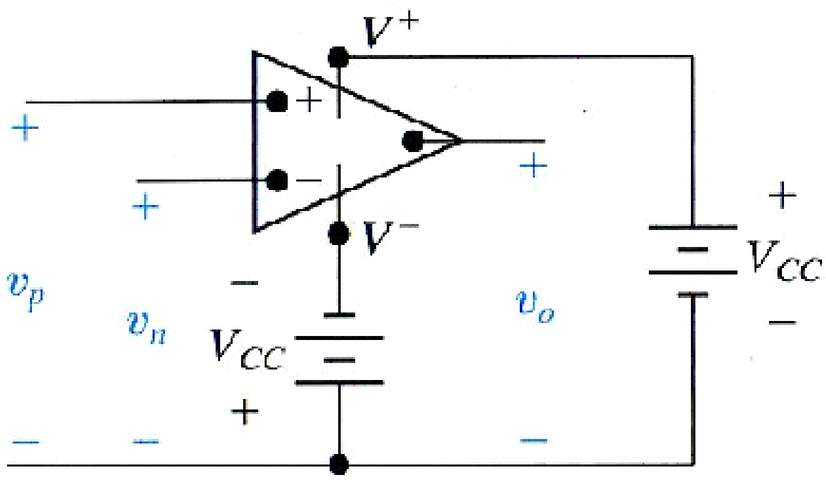
Ideal Op Amp:

- The input resistance is infinite: $Z_{in} = \infty$
- The gain is infinite: $Z_{in} = \infty$

Typical Op Amp:

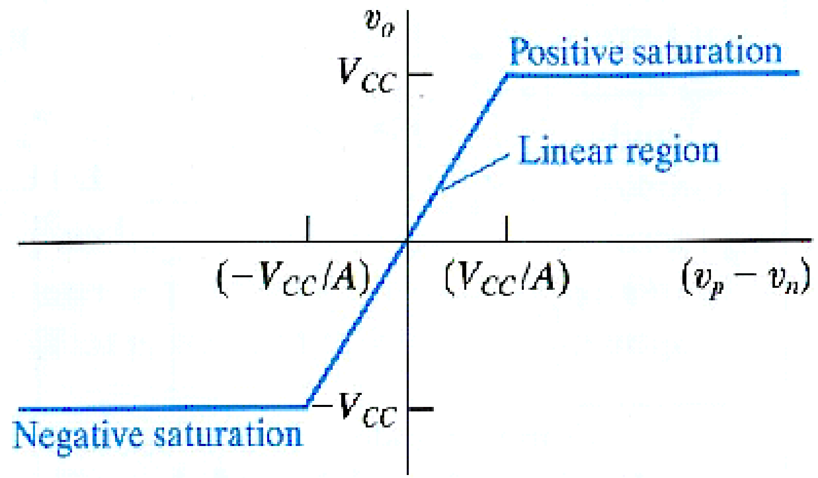
- The input resistance (impedance) Z_{in} is very large (practically infinite).
- The voltage gain A is very large (practically infinite).

Terminal Voltages



Terminal voltage variables

All voltages are considered as voltages rises from the common node.



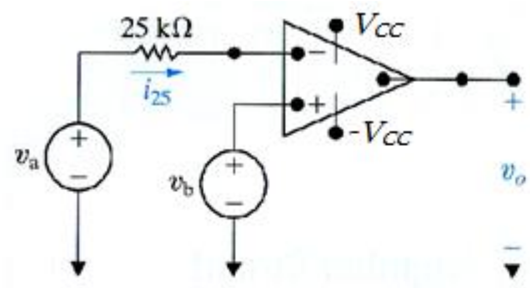
The voltage transfer characteristic of an op amp

$$v_0 = \begin{cases} -V_{CC} & A(v_p - v_n) < -V_{CC} \\ A(v_p - v_n) & -V_{CC} \leq A(v_p - v_n) \leq +V_{CC} \\ +V_{CC} & A(v_p - v_n) > +V_{CC} \end{cases}$$

When the magnitude of the input voltage difference ($|v_p - v_n|$) is small, the op amp behaves as a linear device, as the output voltage is a linear function of the input voltages (the output voltage is equal to the difference in its input voltages times the gain, A.

Configurations of Op-Amp

- Open-loop configuration

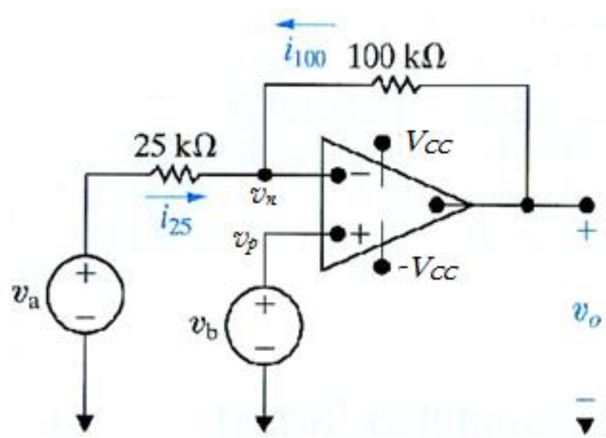


$$v_0 = A\left(v_p - v_n\right)$$

In the ideal case, A is infinite

$$v_0 = \begin{cases} -V_{CC} & v_p < v_n \\ +V_{CC} & v_p < v_n \end{cases}$$

- Negative feedback configuration



Input voltage constraint: $v_p = v_n$

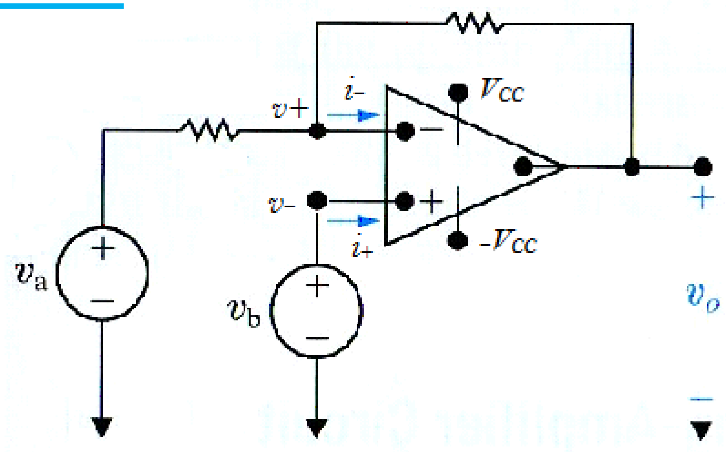
Input current constraint: $i_p = i_n = 0$

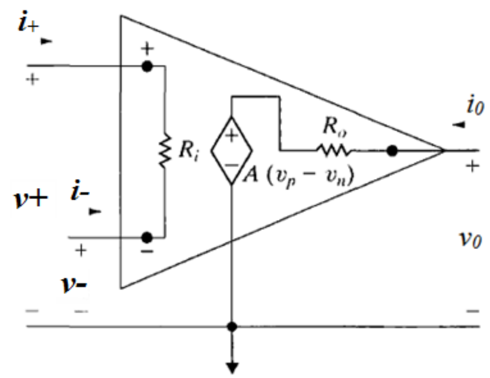
Applications

- Open-loop configuration. Applications
  - Comparator circuit
- Negative feedback. Applications
  - The inverting-amplifier circuit
  - The summing-amplifier circuit
  - The non inverting-amplifier circuit
  - The difference-amplifier circuit
- Negative feedback configuration. Applications
  - Oscillator

Operational Amplifier

Negative feedback



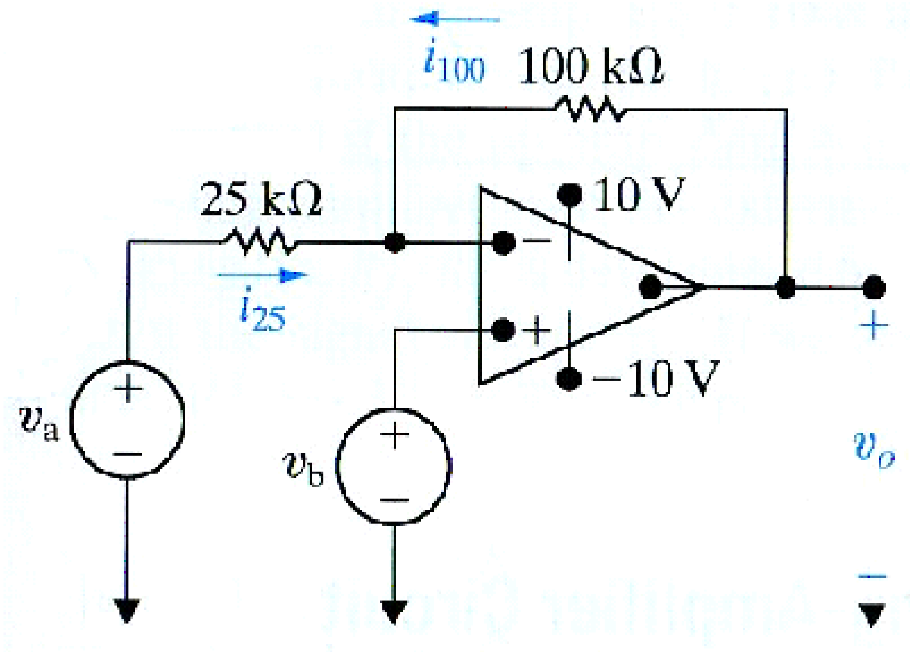


- In the ideal case, A is infinite, R_i is infinite (A→∞, Ri →∞) and R_0=0 then,

$$\left.\begin{array}{lll} v_p = v_n & \text{or} & v_+ = v_- \\ i_p = i_n = 0 & \text{or} & i_+ = i_- = 0 \end{array}\right\} \text{ when } -V_{cc} < V_{out} < +V_{cc}$$

$$\left.\begin{array}{lll} V_p \neq V_n & \text{or} & V_+ \neq V_- \\ i_p = i_n = 0 & \text{or} & i_+ = i_- = 0 \end{array}\right\} \text{ when } \begin{array}{l} V_{out} = -V_{cc} \\ V_{out} = V_{cc} \end{array}$$

Example 1



(a) Calculate v_0 if $v_a = 1$ V and $v_b = 0$ V.
(b) Calculate v_0 if $v_a = 1$ V and $v_b = 2$ V.
(c) If $v_a = 1.5$ V , specify the range of v_b that avoids amplifier saturation.

Sol. of Example 1

a) A negative feedback path exists from the op amp's output to its inverting input through the 100 kΩ resistor, → assume the op amp is working in linear operating region. ➔ write a node-voltage equation at the inverting input terminal. The voltage at the inverting input terminal is 0, as $v_p = v_b = 0$ from the connected voltage source, and $v_n = v_p$ from the voltage constraint. The node-voltage equation at v_n is: $i_{25} = i_{100} = i_n$

$i_{25} = (v_a - v_n)/25 = 1/25$ mA

$i_{100} = (v_o - v_n)/100 = v_o/100$ mA

The current constraint requires $i_n = 0$. Substituting the values for the three currents into the node-voltage equation, we obtain

$$\frac{1}{25} + \frac{v_o}{100} = 0. \Rightarrow v_o = -4 \text{ V}$$

b) $V_p = v_b = v_n = 2$ V ⇒

$$i_{25} = \frac{v_a - v_n}{25} = \frac{1-2}{25} = -\frac{1}{25} \text{ mA},$$

$$i_{100} = \frac{v_o - v_n}{100} = \frac{v_o - 2}{100} \text{ mA},$$

⇒ $i_{25} = - i_{100}$

$v_o = 6$ V

Sol. of Example 1

c) $v_n = v_p = v_b$, and $i_{25} = -i_{100}$

$v_a = 1.5$ V

$$\frac{1.5 - v_b}{25} = -\frac{v_o - v_b}{100}.$$

Solving for v_b as a function of *v_o gives*

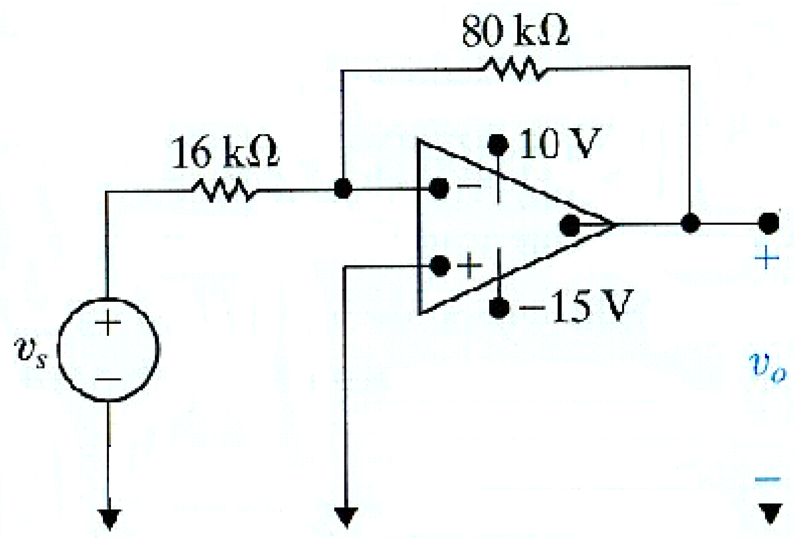
$$v_b = \frac{1}{5}(6 + v_o).$$

Now, if the amplifier is to be within the linear region of operation, $-10\text{ V} \le v_o \le 10\text{ V}.$

Substituting these limits on *vo into the expression* for *v_b, we see that v_b is limited to*

$$-0.8\text{ V} \le v_b \le 3.2\text{ V}.$$

Example 2



Assume that the op amp in the circuit shown is ideal.

- Calculate v_0 for the following values of if v_s : 0.4, 2.0, 3.5, -0.6, -1.6 and -2.4 V.
- Specify the range of v_s required to avoid amplifier saturation.

Sol. of example 2

[a] This is an inverting amplifier, so

$$v_o = (-R_f/R_i)v_s = (-80/16)v_s, \qquad \text{so} \qquad v_o = -5v_s$$

v_s(V)	0.4	2.0	3.5	-0.6	-1.6	-2.4
v_o(V)	-2.0	-10.0	-15.0	3.0	8.0	10.0

Two of the values, 3.5 V and -2.4 V, cause the op amp to saturate.

[b] Use the negative power supply value to determine the largest input voltage:

$$-15 = -5v_s, \quad v_s = 3 \text{ V}$$

Use the positive power supply value to determine the smallest input voltage:

$$10 = -5v_s, \qquad v_s = -2 \text{ V}$$

Therefore $\quad -2 \le v_s \le 3$ V

The Basic Inverting Amplifier

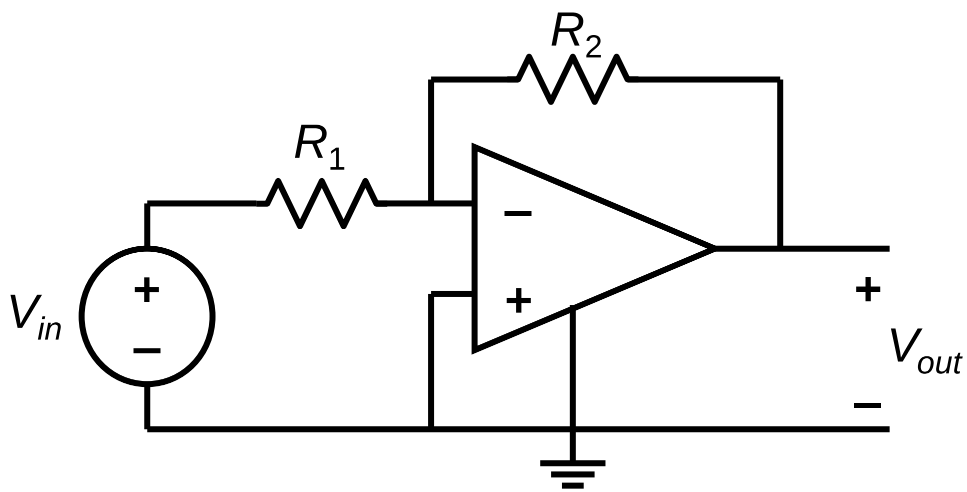
Consequences of the Ideal

- Infinite input resistance means the current into the inverting (–) input is zero:

$$i_- = 0$$

- Infinite gain means the difference between v_+ and v_- is zero:

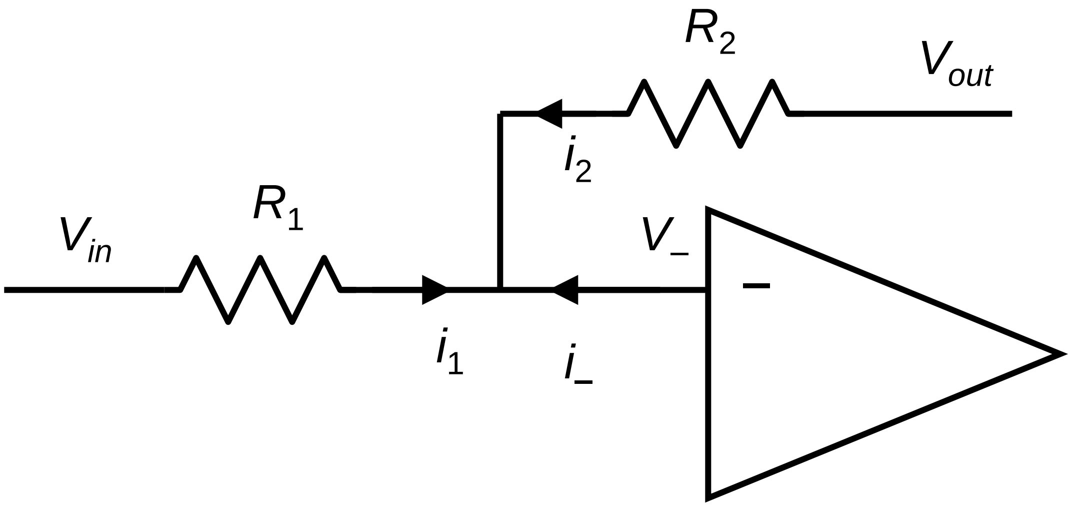
$$v_+ - v_- = 0$$



The Basic Inverting Amplifier

Solving the Amplifier Circuit

Apply KCL at the inverting (–) input:



$$i_- = 0$$

$$i_1 = \frac{V_{in} - V_-}{R_1} = \frac{V_{in}}{R_1}$$

$$i_2 = \frac{V_{out} - V_-}{R_2} = \frac{V_{out}}{R_2}$$

$$i_1 + i_2 + i_- = 0$$

The Basic Inverting Amplifier

Solve for V_{out}

- From KCL

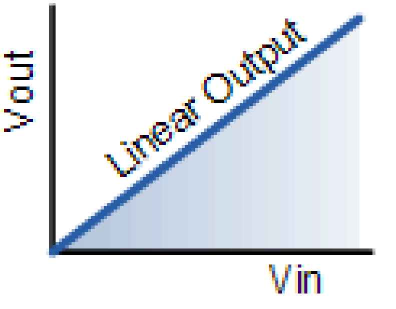
$$i_1 + i_2 + i_- = 0$$

$$\frac{V_{in}}{R_1} + \frac{V_{out}}{R_2} + 0 = 0$$

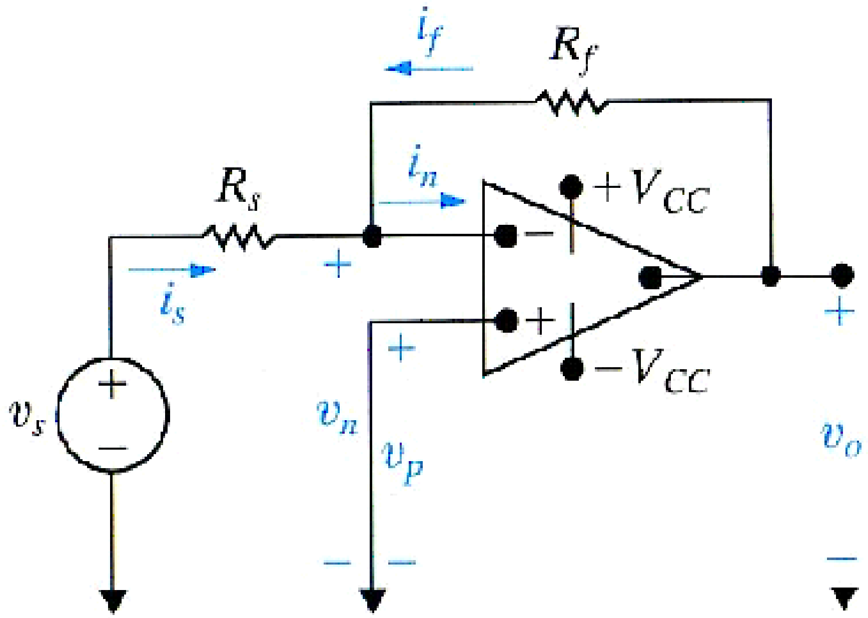
$$\frac{V_{in}}{R_1} = -\frac{V_{out}}{R_2}$$

- Thus, the amplifier gain is

$$\frac{V_{out}}{V_{in}} = -\frac{R_2}{R_1}$$



The Inverting Amplifier Circuit



$$v_0 = -\frac{R_f}{R_s} v_s$$

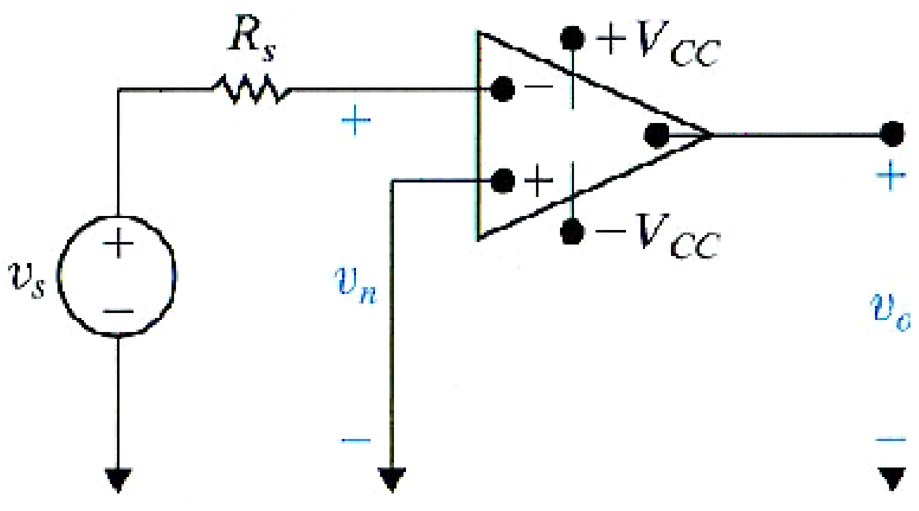
The output voltage is an inverted, scaled replica of the input.

R_f is *negative feedback* of the circuit.

Upper limit on the gain: $\frac{R_f}{R_s} < \left| \frac{V_{CC}}{v_s} \right|$

The Inverting Amplifier Circuit

When R_f is removed, the feedback path is opened and the amplifier is called *open loop*.



$$\boxed{v_0 = -Av_n}$$

A is called the open-loop gain of the op amp

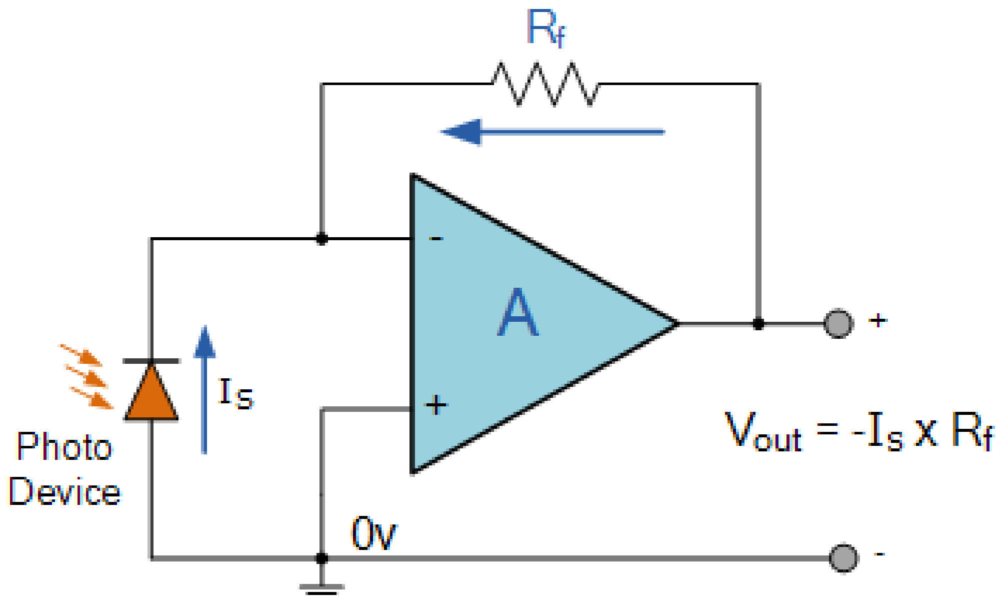
The negative sign in the equation indicates an inversion of the output signal with respect to the input as it is 180º out of phase.

The Inverting Amplifier Circuit

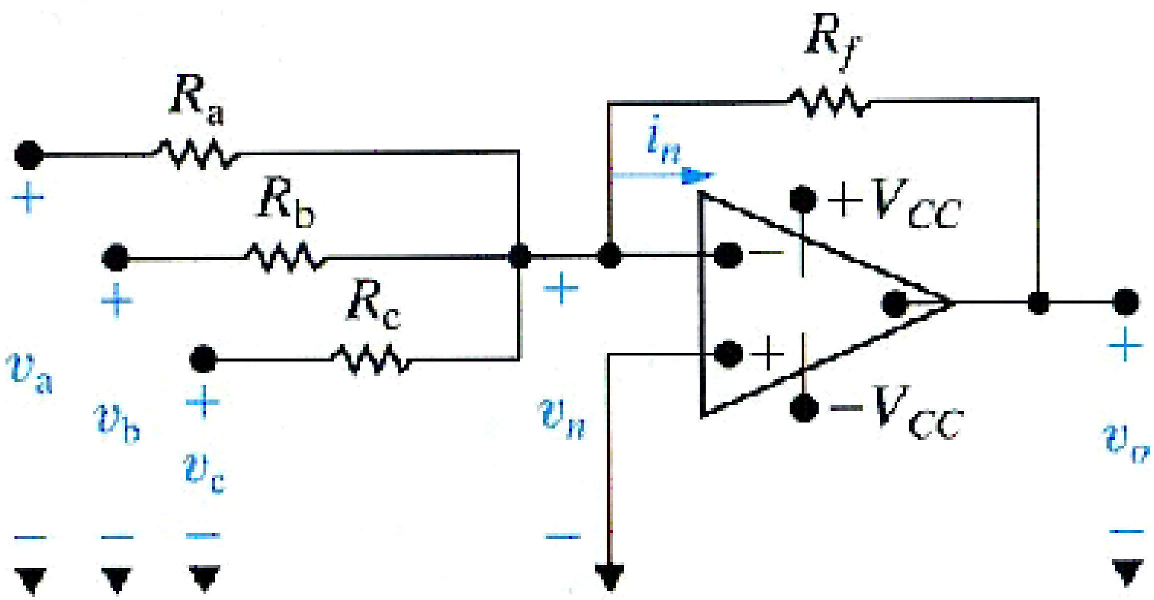
Transresistance Amplifier Circuit

Application of an inverting amplifier is that of a "transresistance amplifier" circuit. AKA. a "transimpedance amplifier", is basically a current-to-voltage converter (Current "in" and Voltage "out"). They can be used in low-power applications to convert a very small current generated by a photo-diode or photo-detecting device etc, into a usable output voltage which is proportional to the input current.

The output voltage is proportional to the amount of input current generated by the photo-diode.



The Summing Amplifier Circuit

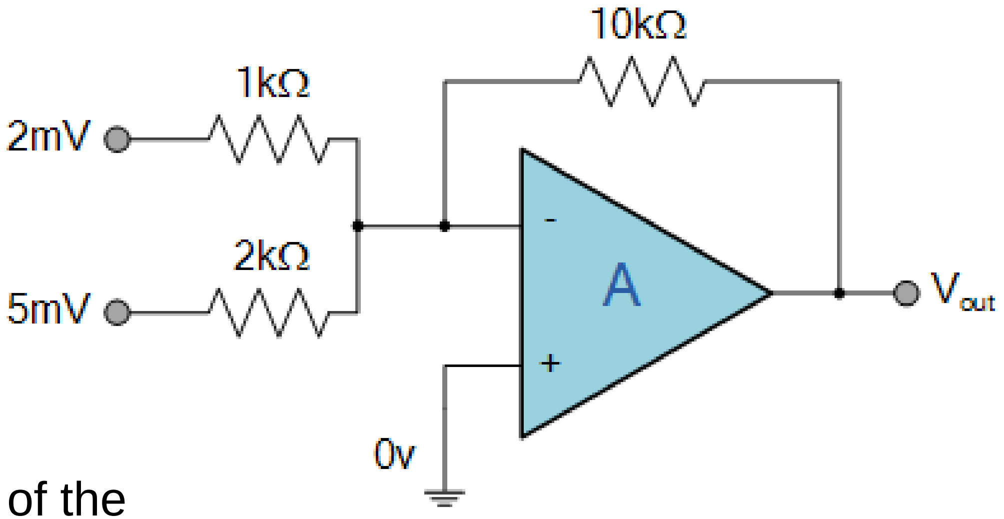


$$\boxed{v_0 = -\left(\frac{R_f}{R_a}v_a + \frac{R_f}{R_b}v_b + \frac{R_f}{R_c}v_c\right)}$$

The output voltage of a summing amplifier is an inverted, scaled sum of the voltages applied to the input of the amplifier.

Example 3.1:

Find the output voltage of the following Summing Amplifier circuit



Solution:

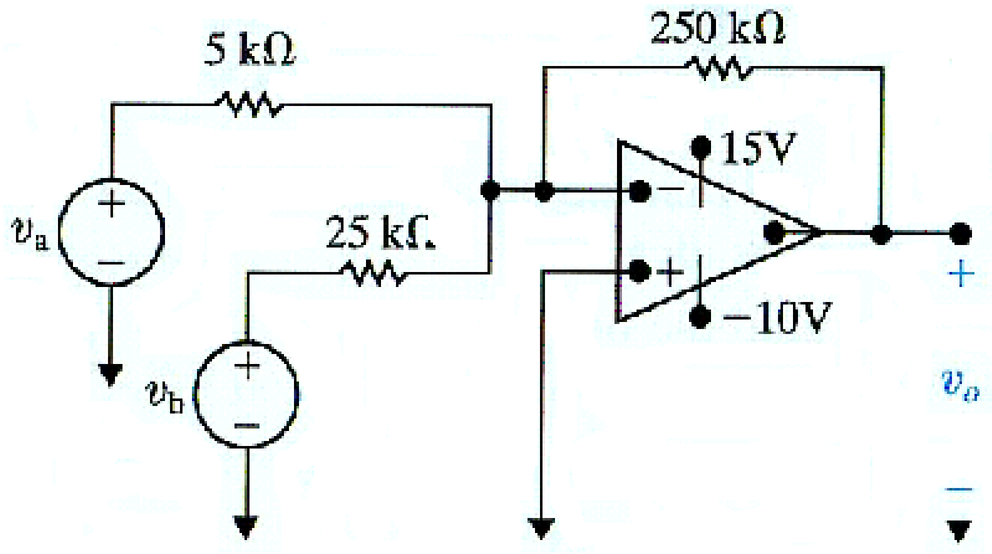
Using the previously found formula for the gain of the circuit

$$\text{Gain (Av)} = \frac{V_{out}}{V_{in}} = -\frac{R_f}{R_{in}}$$

we can now substitute the values of the resistors in the circuit as follows,

$$\begin{cases} A_1 = \dfrac{10k\Omega}{1k\Omega} = -10 \\ A_2 = \dfrac{10k\Omega}{2k\Omega} = -5 \end{cases} \Rightarrow \begin{cases} Vout = (A_1 \times V_1) + (A_2 \times V_2) \\ Vout = (-10(2mV)) + (-5(5mV)) = -45mV \end{cases}$$

Example 3.2



(a) Find v_0 with v_a =0.1 V and v_b = 0.25 V.
(b) If v_b = 0.25 V, how large can v_a be before the op amp saturates.
(c) If v_a = 0.1 V, how large can v_b be before the op amp saturates.
(d) Repeat (a), (b), and (c) with the polarity of v_b reversed.

Sol. of example 3

[a] This is an inverting summing amplifier so

$$v_o = (-R_f/R_\mathrm{a})v_\mathrm{a} + (-R_f/R_\mathrm{b})v_\mathrm{b} = -(250/5)v_\mathrm{a} - (250/25)v_\mathrm{b} = -50v_\mathrm{a} - 10v_\mathrm{b}$$

Substituting the values for v_a and v_b:

$$v_o = -50(0.1) - 10(0.25) = -5 - 2.5 = -7.5 \text{ V}$$

[b] Substitute the value for v_b into the equation for v_o from part (a) and use the negative power supply value:

$$v_o = -50v_\mathrm{a} - 10(0.25) = -50v_\mathrm{a} - 2.5 = -10 \text{ V}$$

$$\text{Therefore} \quad 50v_\mathrm{a} = 7.5, \quad \text{so} \quad v_\mathrm{a} = 0.15 \text{ V}$$

[c] Substitute the value for v_a into the equation for v_o from part (a) and use the negative power supply value:

$$v_o = -50(0.10) - 10v_\mathrm{b} = -5 - 10v_\mathrm{b} = -10 \text{ V};$$

$$\text{Therefore} \quad 10v_\mathrm{b} = 5, \quad \text{so} \quad v_\mathrm{b} = 0.5 \text{ V}$$

Sol. of example 3

[d] The effect of reversing polarity is to change the sign on the v_{b} term in each equation from negative to positive.

Repeat part (a):

$$v_o = -50v_{\mathrm{a}} + 10v_{\mathrm{b}} = -5 + 2.5 = -2.5 \ \mathrm{V}$$

Repeat part (b):

$$v_o = -50v_{\mathrm{a}} + 2.5 = -10 \ \mathrm{V}; \qquad 50v_{\mathrm{a}} = 12.5, \quad v_{\mathrm{a}} = 0.25 \ \mathrm{V}$$

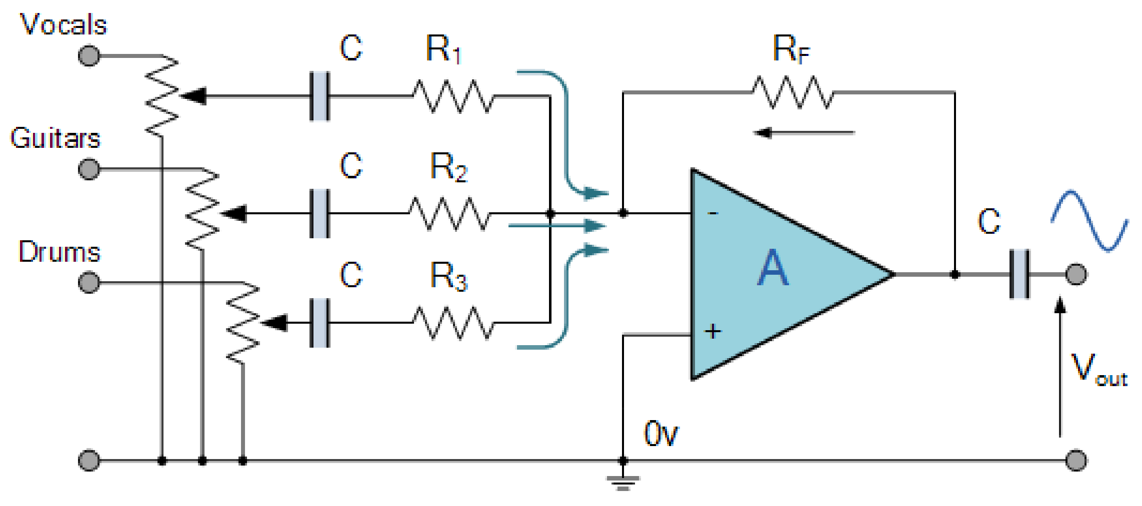
Repeat part (c), using the value of the positive power supply:

$$v_o = -5 + 10v_{\mathrm{b}} = 15 \ \mathrm{V}; \qquad 10v_{\mathrm{b}} = 20; \qquad v_{\mathrm{b}} = 2.0 \ \mathrm{V}$$

Summing Amplifier Applications

Summing Amplifier Audio Mixer

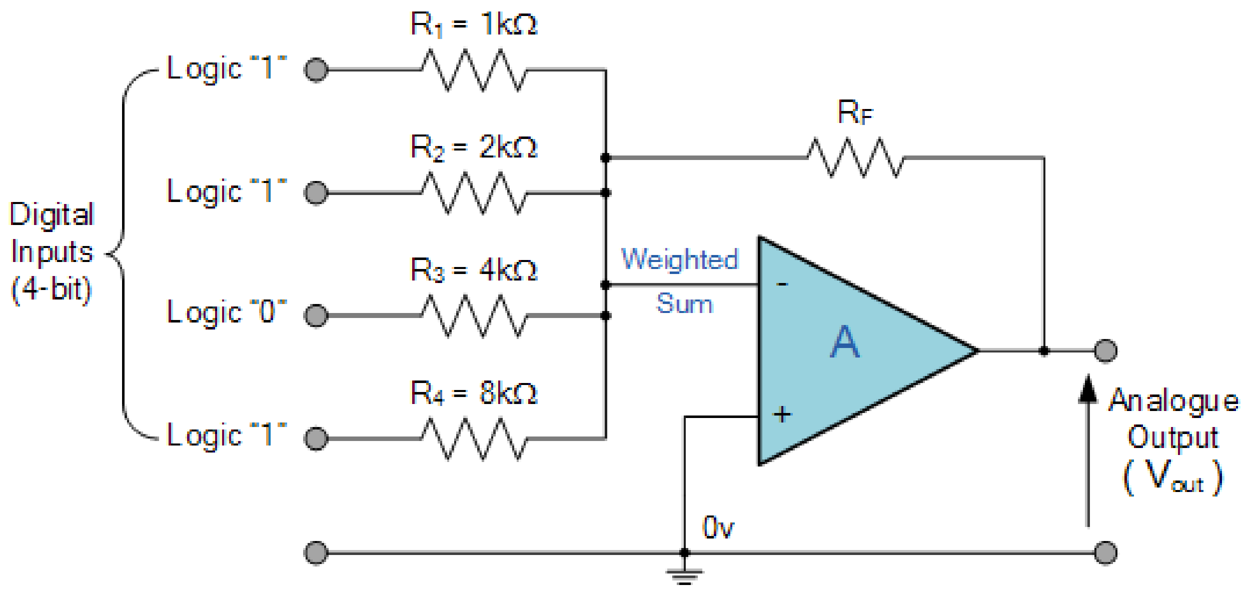
If the input resistances of a summing amplifier are connected to potentiometers the individual input signals can be mixed together by varying amounts. For example, measuring temperature, you could add a negative offset voltage to make the display read "0" at the freezing point or produce an audio mixer for adding or mixing together individual waveforms (sounds) from different source channels (vocals, instruments, etc) before sending them combined to an audio amplifier.



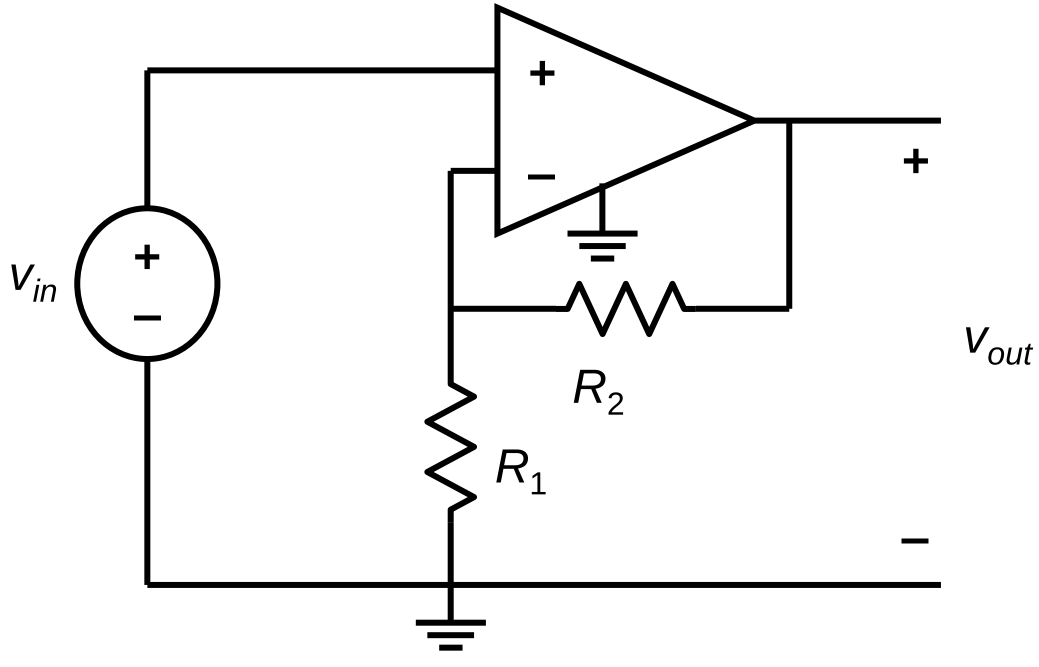
Summing Amplifier Applications

<u>Digital to Analogue Converter</u> (DAC summing amplifier circuit**)**

Another useful application of a **Summing Amplifier** is as a weighted sum digital-to-analogue converter. If the input resistors, Rin of the summing amplifier double in value for each input, for example, 1kΩ, 2kΩ, 4kΩ, 8kΩ, 16kΩ, etc, then a digital logical voltage, either a logic level "0" or a logic level "1" on these inputs will produce an output which is the weighted sum of the digital inputs. Consider the circuit below.

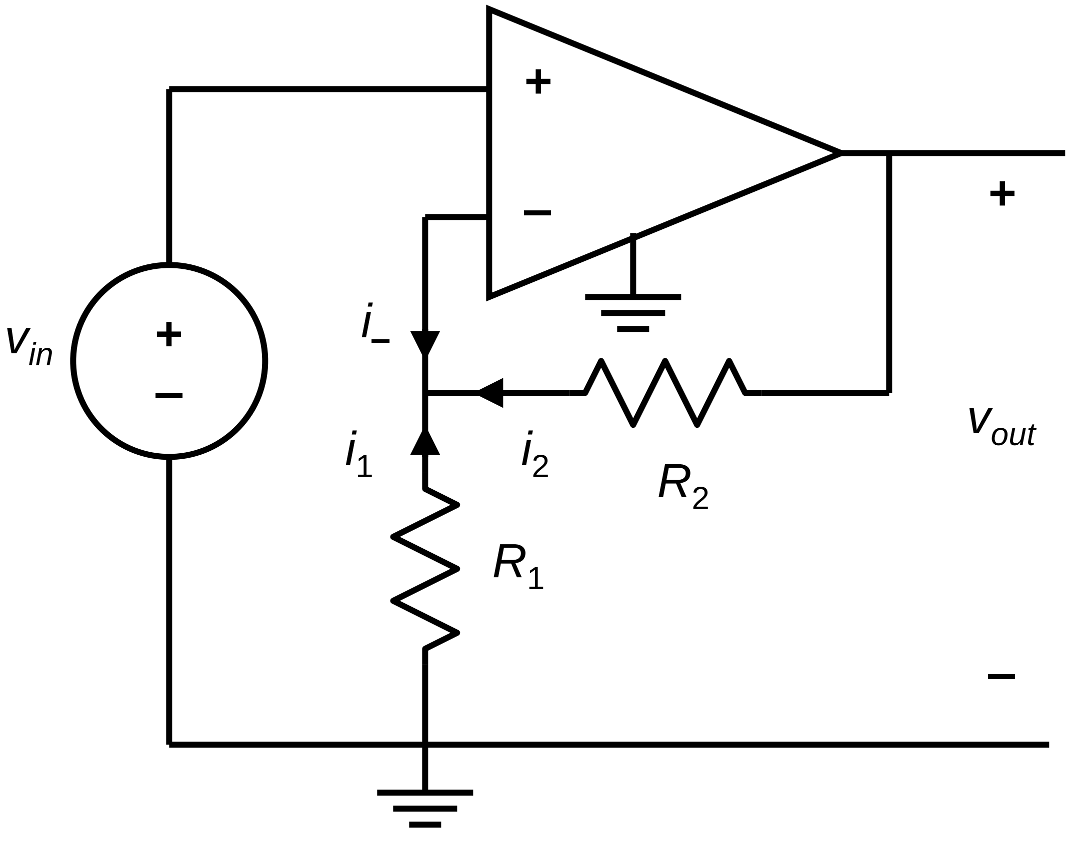


The Non-Inverting Amplifier



The Non-Inverting Amplifier

KCL at the Inverting Input



$$i_- = 0$$

$$i_1 = \frac{-v_-}{R_1} = \frac{-v_{in}}{R_1}$$

$$i_2 = \frac{v_{out} - v_-}{R_2}$$

$$= \frac{v_{out} - v_{in}}{R_2}$$

The Non-Inverting Amplifier

Solve for v_{out}

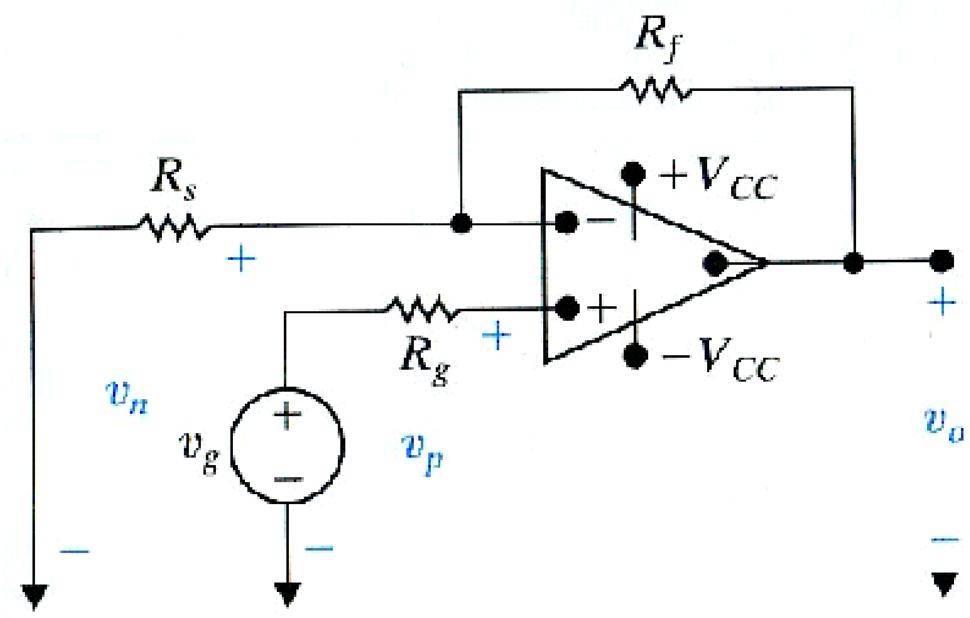
$$i_1 + i_2 + i_- = 0$$

$$\frac{-v_{in}}{R_1} + \frac{v_{out} - v_{in}}{R_2} = 0$$

$$v_{out} = v_{in}\left(1 + \frac{R_2}{R_1}\right)$$

- Hence, the non-inverting amplifier has a gained output (> unity) relative to the resistance ratio

The Non-inverting Amplifier Circuit

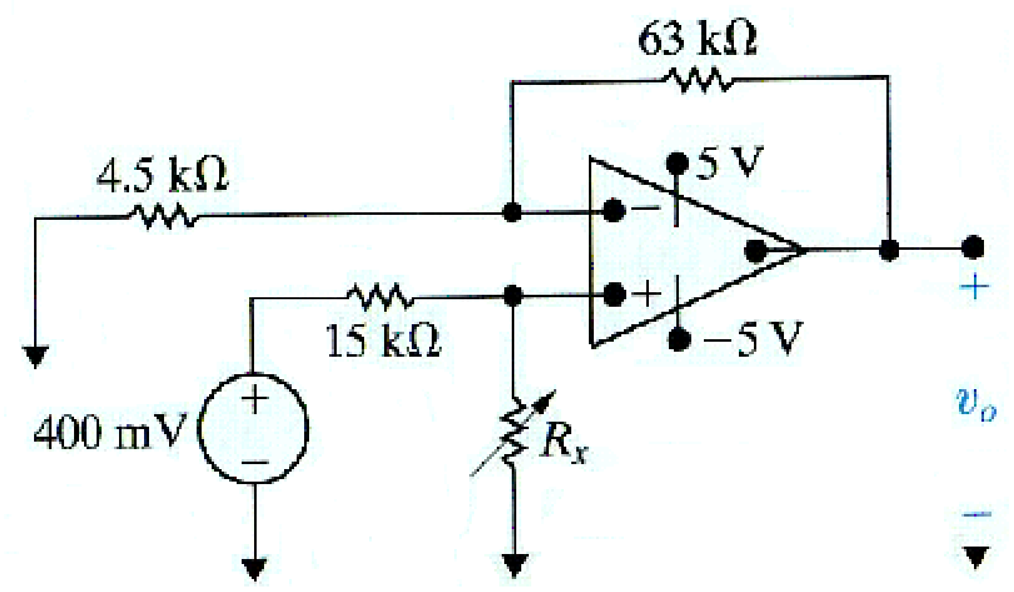


$$v_0 = \frac{R_s + R_f}{R_s} v_g$$

Requirement for operation in the linear region:

$$\frac{R_s + R_f}{R_s} < \left| \frac{V_{CC}}{V_g} \right|$$

Example 4



(a) Find the output voltage when R_x = 60 kΩ
(b) How large can R_x be before the amplifier saturates.

Sol. of example 4

[a] Write a node voltage equation at v_n; remember that for an ideal op amp, the current into the op amp at the inputs is zero:

$$\frac{v_n}{4500} + \frac{v_n - v_o}{63{,}000} = 0$$

Solve for v_o in terms of v_n by multiplying both sides by 63,000 and collecting terms:

$$14v_n + v_n - v_o = 0 \qquad \text{so} \qquad v_o = 15v_n$$

Now use voltage division to calculate v_p. We can use voltage division because the op amp is ideal, so no current flows into the non-inverting input terminal and the 400 mV divides between the 15 kΩ resistor and the R_x resistor:

$$v_p = \frac{R_x}{15{,}000 + R_x}(0.400)$$

Now substitute the value $R_x = 60$ kΩ:

Sol. of example 4

$$v_p = \frac{60{,}000}{15{,}000 + 60{,}000}(0.400) = 0.32\ \text{V}$$

Finally, remember that for an ideal op amp, $v_n = v_p$, so substitute the value of v_p into the equation for v_0

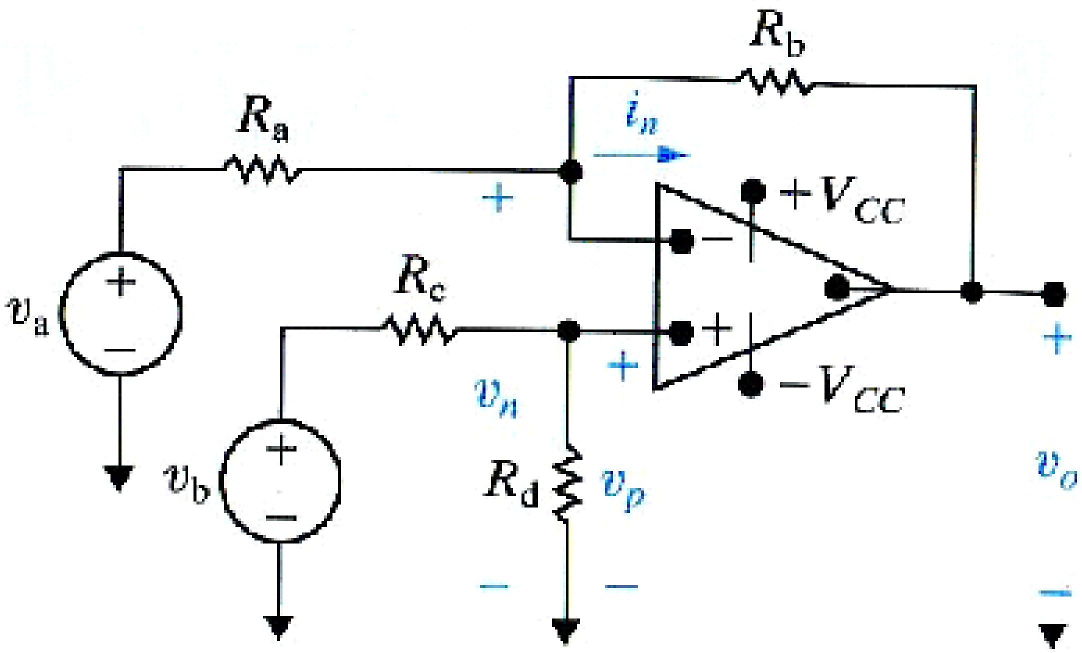
$$v_o = 15v_n = 15v_p = 15(0.32) = 4.8\ \text{V}$$

[b] Substitute the expression for v_p into the equation for v_o and set the resulting equation equal to the positive power supply value:

$$v_o = 15\left(\frac{0.4R_x}{15{,}000 + R_x}\right) = 5$$

$$15(0.4R_x) = 5(15{,}000 + R_x) \quad \text{so} \quad R_x = 75\,\text{k}\Omega$$

The Difference Amplifier Circuit



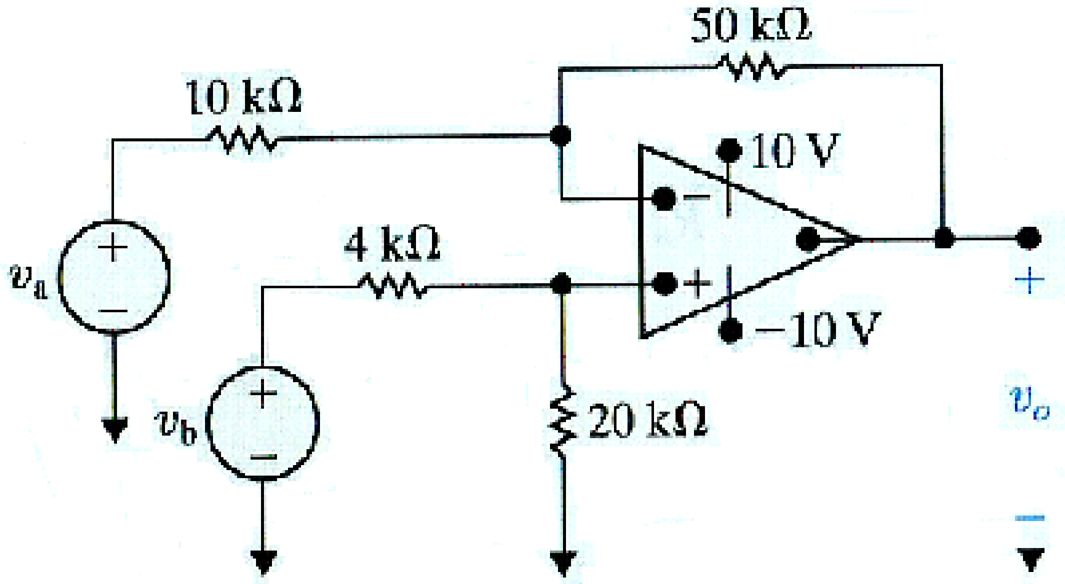
$$v_0 = \frac{R_d\left(R_a + R_b\right)}{R_a\left(R_c + R_d\right)} v_b - \frac{R_b}{R_a} v_a$$

(*)

If set $\frac{R_a}{R_b} = \frac{R_c}{R_d}$ then $$v_0 = \frac{R_b}{R_a}\left(v_b - v_a\right)$$

The output voltage of a difference amplifier is a scaled replica of the difference between the two input voltages. The scaling is controlled by the external resistors.

Example 5



(a) In the difference amplifier shown, v_b = 4 V. What range of values for v_a will result in linear operation.
(b) Repeat (a) with 20 kΩ resistor decreased to 8 kΩ

Sol. of example 5

[a] Since this is a difference amplifier, we can use the expression for the output voltage in terms of the input voltages and the resistor values given in Eq. **(*)**

$$v_o = \frac{20(60)}{10(24)} v_{\mathrm{b}} - \frac{50}{10} v_{\mathrm{a}}$$

Simplify this expression and subsitute in the value for v_{b}:

$$v_o = 5(v_{\mathrm{b}} - v_{\mathrm{a}}) = 20 - 5v_{\mathrm{a}}$$

Set this expression for v_o to the positive power supply value:

$$20 - 5v_{\mathrm{a}} = 10 \ \mathrm{V} \quad \text{so} \quad v_{\mathrm{a}} = 2 \ \mathrm{V}$$

Now set the expression for v_o to the negative power supply value:

$$20 - 5v_{\mathrm{a}} = -10 \ \mathrm{V} \quad \text{so} \quad v_{\mathrm{a}} = 6 \ \mathrm{V}$$

Therefore $\quad 2 \leq v_{\mathrm{a}} \leq 6 \ \mathrm{V}$

Sol. of example 5

[b] Begin as before by substituting the appropriate values into Eq. **(*)**

$$v_o = \frac{8(60)}{10(12)} v_\mathrm{b} - 5v_\mathrm{a} = 4v_\mathrm{b} - 5v_\mathrm{a}$$

Now substitute the value for v_b:

$$v_o = 4(4) - 5v_\mathrm{a} = 16 - 5v_\mathrm{a}$$

Set this expression for v_o to the positive power supply value:

$$16 - 5v_\mathrm{a} = 10 \ \mathrm{V} \quad \text{so} \quad v_\mathrm{a} = 1.2 \ \mathrm{V}$$

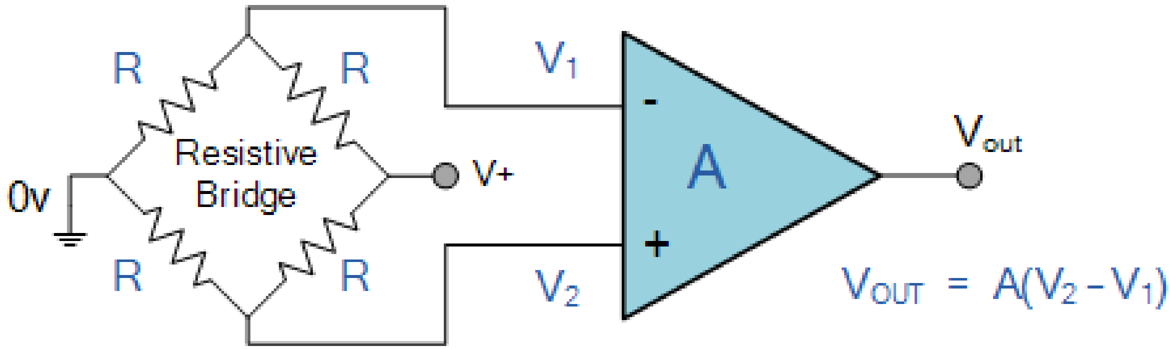
Now set the expression for v_o to the negative power supply value:

$$16 - 5v_\mathrm{a} = -10 \ \mathrm{V} \quad \text{so} \quad v_\mathrm{a} = 5.2 \ \mathrm{V}$$

Therefore $\quad 1.2 \leq v_\mathrm{a} \leq 5.2 \ \mathrm{V}$

The Difference Amplifier Circuit: Applications

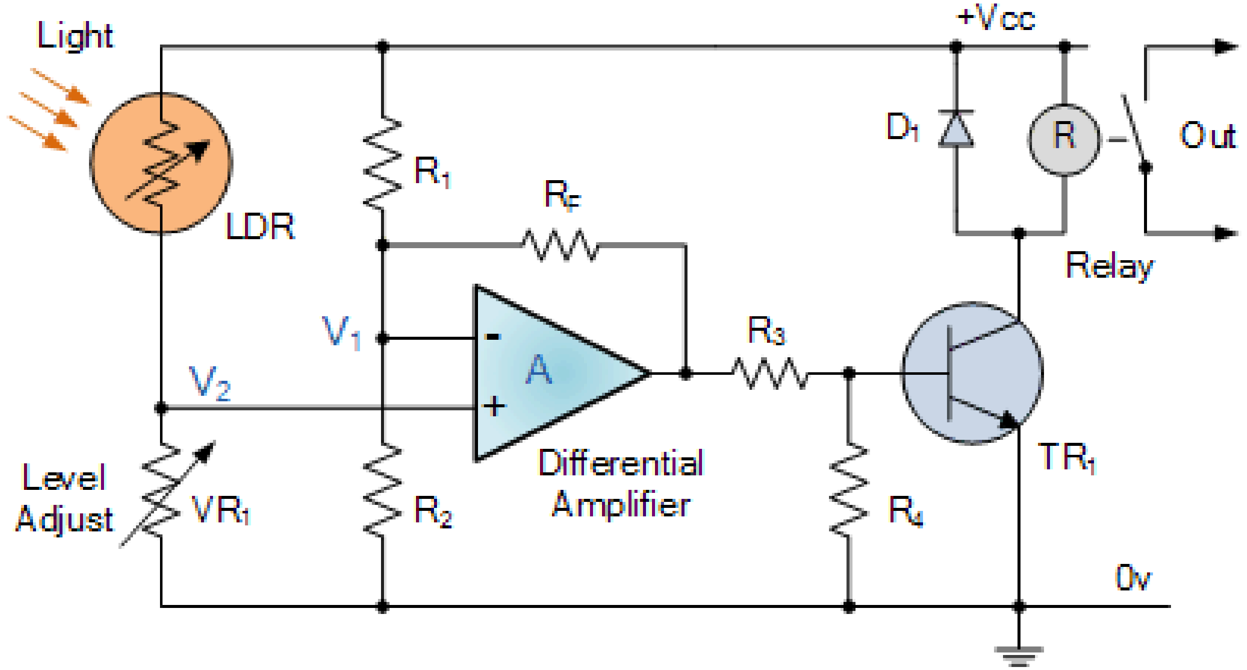
Bridge Amplifier



The standard Differential Amplifier circuit now becomes a differential voltage comparator by "Comparing" one input voltage to the other.
By connecting one input to a fixed voltage reference set up on one leg of the resistive bridge network and the other to either a "Thermistor" or a "Light Dependant Resistor" the amplifier circuit can be used to detect either low or high levels of temperature or light as the output voltage becomes a linear function of the changes in the active leg of the resistive bridge.

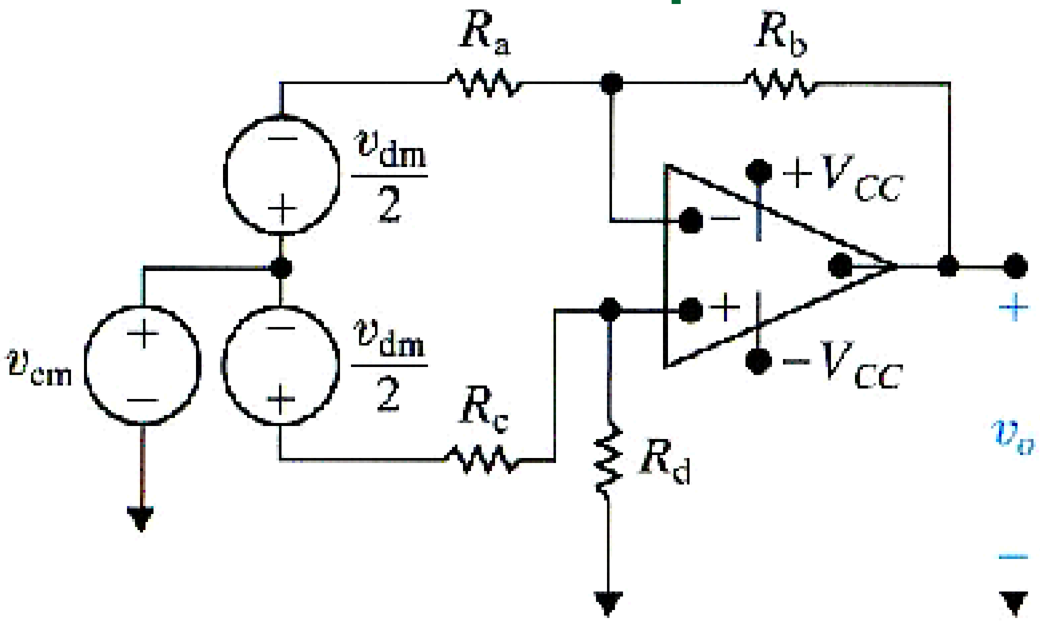
The Difference Amplifier Circuit: Applications

The circuit acts as a light-activated switch → turns the output relay either "ON" or "OFF" as the light level detected by the LDR resistor exceeds or falls below a pre-set value at V2 determined by the position of VR1. A fixed voltage reference is applied to the inverting input terminal V1 via the R1 - R2 voltage divider network and the variable voltage (proportional to the light level) applied to the non-inverting input terminal V2. It is also possible to detect temperature using this type of circuit by simply replacing the Light Dependant Resistor (LDR) with a thermistor. By interchanging the positions of VR1and the LDR, the circuit can be used to detect either light or dark, or heat or cold using a thermistor.



Light Activated Switch:

The difference amplifier – Another perspective



$$v_{dm} = v_b - v_a$$

$$v_{cm} = (v_a + v_b)/2$$

$$v_a = v_{cm} - \frac{1}{2}v_{dm},$$

$$v_b = v_{cm} + \frac{1}{2}v_{dm}.$$

$$v_o = \left[\frac{R_aR_d - R_bR_c}{R_a(R_c + R_d)}\right]v_{cm} + \left[\frac{R_d(R_a + R_b) + R_b(R_c + R_d)}{2R_a(R_c + R_d)}\right]v_{dm}, \Rightarrow v_0 = A_{cm}v_{cm} + A_{dm}v_{dm}$$

An ideal differential amplifier has zero common mode gain and non-zero (usually large) differential mode gain.

In practical applications, the differential mode signal contains information of interest, whereas the common mode signal is the noise found in all electric signals.