Đo Minh Duy ITITSB22029 week 3

1) a) $(A - AX)^{-1} = X^{-1}B$

$\Rightarrow X(A - AX)^{-1} = X(X^{-1}B)$

$\Leftrightarrow X(A - AX)^{-1} = IB$

$\Leftrightarrow X(A - AX)^{-1} = B$

Therefore, B product of invertible matrices, so B is invertible

b) $(A - AX)^{-1} = X^{-1}B$

$\Rightarrow X(A - AX)^{-1}(A - AX) = XX^{-1}B(A - AX)$

$\Rightarrow X = BA - BAX$

$\Rightarrow X + BAX = BA$

$\Rightarrow (I + BA)X = BA$

$\Rightarrow X = BA(I + BA)^{-1}$

2) $A = \begin{pmatrix} 1 & 0 & -2 \\ -3 & 1 & 4 \\ 2 & -3 & 4 \end{pmatrix}$

$$A^{-1} = [A|I] = \left[\begin{array}{ccc|ccc} 1 & 0 & -2 & 1 & 0 & 0 \\ -3 & 1 & 4 & 0 & 1 & 0 \\ 2 & -3 & 4 & 0 & 0 & 1 \end{array}\right] \xrightarrow[2R_1 - R_3]{}$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & -2 & 1 & 0 & 0 \\ -3 & 1 & 4 & 0 & 1 & 0 \\ 0 & 3 & -8 & 2 & 0 & -1 \end{array}\right] \xrightarrow{3R_1 + R_2} \left[\begin{array}{ccc|ccc} 1 & 0 & -2 & 1 & 0 & 0 \\ 0 & 1 & -2 & 3 & 1 & 0 \\ 0 & 3 & -8 & 2 & 0 & -1 \end{array}\right]$$

$$\xrightarrow{3R_2 - R_3} \left[\begin{array}{ccc|ccc} 1 & 0 & -2 & 1 & 0 & 0 \\ 0 & 1 & -2 & 3 & 1 & 0 \\ 0 & 0 & 2 & 7 & 3 & 1 \end{array}\right] \xrightarrow{R_1 - R_2} \left[\begin{array}{ccc|ccc} 1 & -1 & 0 & -2 & -1 & 0 \\ 0 & 1 & -2 & 3 & 1 & 0 \\ 0 & 0 & 2 & 7 & 3 & 1 \end{array}\right]$$

$$\xrightarrow{R_2 + R_3} \left[\begin{array}{ccc|ccc} 1 & -1 & 0 & -2 & -1 & 0 \\ 0 & 1 & 0 & 10 & 4 & 1 \\ 0 & 0 & 2 & 7 & 3 & 1 \end{array}\right] \xrightarrow{R_1 + R_2} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 8 & 3 & 1 \\ 0 & 1 & 0 & 10 & 4 & 1 \\ 0 & 0 & 2 & 7 & 3 & 1 \end{array}\right]$$

$$\xrightarrow[\frac{1}{2}R_3]{} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 8 & 3 & 1 \\ 0 & 1 & 0 & 10 & 4 & 1 \\ 0 & 0 & 1 & 7/2 & 3/2 & 1/2 \end{array}\right]$$

$B = \begin{pmatrix} 1 & -2 & 1 \\ 4 & -7 & 3 \\ -2 & 6 & -4 \end{pmatrix}$

$$B^{-1} = [B|I] = \left[\begin{array}{ccc|ccc} 1 & -2 & 1 & 1 & 0 & 0 \\ 4 & -7 & 3 & 0 & 1 & 0 \\ -2 & 6 & -4 & 0 & 0 & 1 \end{array}\right]$$

$$\xrightarrow[1+R_3]{1-R_2} \left[\begin{array}{ccc|ccc} 1 & -2 & 1 & 1 & 0 & 0 \\ 0 & -1 & 1 & 4 & -1 & 0 \\ 0 & 2 & -2 & 2 & 0 & 1 \end{array}\right] \xrightarrow[2R_2 + R_3]{} \left[\begin{array}{ccc|ccc} 1 & -2 & 1 & 1 & 0 & 0 \\ 0 & -1 & 1 & 4 & -1 & 0 \\ 0 & 0 & 0 & 10 & -2 & 1 \end{array}\right]$$

$\Rightarrow$ can not inverse

$C = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{pmatrix}$

$$C^{-1} = [C|I] = \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 \end{array}\right]$$

$$\xrightarrow[R_1 - R_3]{R_1 - R_2} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & -1 & 0 & 1 & -1 & 0 \\ 0 & -1 & -1 & 1 & 0 & -1 \end{array}\right]$$

$$\xrightarrow[R_2 - R_3]{} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & -1 & 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & 0 & -1 & 1 \end{array}\right]$$

$$\xrightarrow{-R_2} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & -1 & 1 & 0 \\ 0 & 0 & 1 & 0 & -1 & 1 \end{array}\right]$$

⑤ $I - 2A = (I - 2A)^{-1}$

$\Rightarrow (I - 2A)(I - 2A) = I$

$\Rightarrow I - 2A - 2A + 4A^2 = I$

$\Rightarrow I - 4A + 4A^2 = I$

$\Rightarrow I - 4A + 4A = I$

$\Rightarrow I = I$

④
$$\underbrace{\begin{bmatrix} 1 & 1 & -2 \\ 1 & -2 & 1 \\ 1 & -1 & -1 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}}_{X} = \underbrace{\begin{bmatrix} -1 \\ 2 \\ 0 \end{bmatrix}}_{B}$$

$$A^{-1} = [A \mid I] = \left[\begin{array}{ccc|ccc} 1 & 1 & -2 & 1 & 0 & 0 \\ 1 & -2 & 1 & 0 & 1 & 0 \\ 1 & -1 & -1 & 0 & 0 & 1 \end{array}\right] \xrightarrow{R_1 - R_3} \left[\begin{array}{ccc|ccc} 1 & 1 & -2 & 1 & 0 & 0 \\ 1 & -2 & 1 & 0 & 1 & 0 \\ 0 & 2 & -1 & 1 & 0 & -1 \end{array}\right] \xrightarrow{R_1 - R_2} \left[\begin{array}{ccc|ccc} 1 & 1 & -2 & 1 & 0 & 0 \\ 0 & 3 & -3 & 1 & -1 & 0 \\ 0 & 2 & -1 & 1 & 0 & -1 \end{array}\right]$$

$$\xrightarrow{{}_2 - 3R_3} \left[\begin{array}{ccc|ccc} 1 & 1 & -2 & 1 & 0 & 0 \\ 0 & 3 & -3 & 1 & -1 & 0 \\ 0 & 0 & -3 & -1 & -2 & 3 \end{array}\right] \xrightarrow{3R_1 - 2R_2} \left[\begin{array}{ccc|ccc} 3 & -3 & 0 & 1 & 2 & 0 \\ 0 & 3 & -3 & 1 & -1 & 0 \\ 0 & 0 & -3 & -1 & -2 & 3 \end{array}\right] \xrightarrow{R_2 - R_3} \left[\begin{array}{ccc|ccc} 3 & -3 & 0 & 1 & 2 & 0 \\ 0 & 3 & 0 & 2 & 1 & -3 \\ 0 & 0 & -3 & -1 & -2 & 3 \end{array}\right] \xrightarrow{R_1 + R_2} \left[\begin{array}{ccc|ccc} 3 & 0 & 0 & 3 & 3 & - \\ 0 & 3 & 0 & 2 & 1 & - \\ 0 & 0 & -3 & -1 & -2 & \end{array}\right]$$

$$\xrightarrow[\frac{1}{3}R_1,\ \frac{1}{3}R_2,\ -\frac{1}{3}R_3]{} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 1 & -1 \\ 0 & 1 & 0 & \frac{2}{3} & \frac{1}{3} & -1 \\ 0 & 0 & 1 & 1/3 & 2/3 & -1 \end{array}\right]$$
$\quad I \qquad A^{-1}$

$$X = \begin{bmatrix} -1 \\ 2 \\ 0 \end{bmatrix} \begin{bmatrix} 1 & 1 & -1 \\ 2/3 & 1/3 & -1 \\ 1/3 & 2/3 & -1 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$$

③ $C^{-1}(A + X)B^{-1} = I_n$

$\Rightarrow CC^{-1}(A + X)B^{-1} = CI_n$

$\Rightarrow I(A + X)B^{-1} = CI_n$

$\Rightarrow (A + X)B^{-1} = C$

$\Rightarrow (A + X) = CB$

$\Rightarrow CB - A = X \quad (*)$

Replace $(*)$ to equation:

$C^{-1}(A + CB - A)B^{-1} = \cancel{I_n}\ C^{-1}(CB)B^{-1} = I$