Electromagnetic Theory

Chapter 1: Vector Algebra

1. Vector Calculation

Name	Operator
Dot product	$\boldsymbol{a}.\boldsymbol{b} = \lvert\boldsymbol{a}\rvert \cdot \lvert\boldsymbol{b}\rvert \cdot \cos\theta$
Cross product	$\lvert\boldsymbol{a} \times \boldsymbol{b}\rvert = \lvert\boldsymbol{a}\rvert \cdot \lvert\boldsymbol{b}\rvert \cdot \sin\theta$
Component of vector $\boldsymbol{a}$ along vector $\boldsymbol{b}$	$\text{comp}_{\boldsymbol{b}}\boldsymbol{a} = \dfrac{\boldsymbol{a}\cdot\boldsymbol{b}}{\lvert\boldsymbol{b}\rvert}$
Unit vector of vector $\boldsymbol{u}$	$\boldsymbol{a_u} = \dfrac{\boldsymbol{u}}{\lvert\boldsymbol{u}\rvert}$
Differential length vector	$d\boldsymbol{l} = dx\boldsymbol{x} + dy\boldsymbol{y} + dz\boldsymbol{z}$
Normal vector of a surface	$\boldsymbol{a_n} = \dfrac{d\boldsymbol{l_1} \times d\boldsymbol{l_2}}{\lvert d\boldsymbol{l_1} \times d\boldsymbol{l_2}\rvert}$
Differential suface vector	$d\boldsymbol{S} = \pm\boldsymbol{a_n}dS$
Del – Gradient vector	$\nabla = \dfrac{\partial}{\partial x}\boldsymbol{x} + \dfrac{\partial}{\partial y}\boldsymbol{y} + \dfrac{\partial}{\partial z}\boldsymbol{z}$

2. System Coordinates Conversion

2. 1. Cylindrical Coordinate Systems

Coordinate conversion	Vector conversion
$\begin{cases} r = \sqrt{x^2+y^2} \\ \phi = \arctan\dfrac{y}{x} \end{cases} \leftrightarrow \begin{cases} x = r\cos\phi \\ y = r\sin\phi \end{cases}$	$\begin{bmatrix}\boldsymbol{r}\\ \boldsymbol{\phi}\end{bmatrix} = \begin{bmatrix}\cos\phi & \sin\phi \\ -\sin\phi & \cos\phi\end{bmatrix}\begin{bmatrix}\boldsymbol{x}\\ \boldsymbol{y}\end{bmatrix}$

Differential vector:

- $d\boldsymbol{l} = dr\boldsymbol{r} + rd\phi\boldsymbol{\phi} + dz\boldsymbol{z}$
- $d\boldsymbol{S} = \pm rd\phi dz\boldsymbol{r}; \pm drdz\boldsymbol{\phi}; \pm rdrd\phi\boldsymbol{z}$
- $dv = rdrd\phi dz$

2. 2. Spherical Coordinate Systems

Coordinate conversion	Vector conversion
$\begin{cases} r = \sqrt{x^2+y^2+z^2} \\ \theta = \tan^{-1}\dfrac{\sqrt{x^2+y^2}}{z} \\ \phi = \arctan\dfrac{y}{x} \end{cases} \leftrightarrow \begin{cases} x = r\sin\theta\cos\phi \\ y = r\sin\theta\sin\phi \\ z = r\cos\theta \end{cases}$	$\begin{bmatrix}\boldsymbol{r}\\ \boldsymbol{\theta}\\ \boldsymbol{\phi}\end{bmatrix} = \begin{bmatrix}\sin\theta\cos\phi & \sin\theta\sin\phi & \cos\theta \\ \cos\theta\cos\phi & \cos\theta\sin\phi & -\sin\theta \\ -\sin\phi & \cos\phi & 0\end{bmatrix}\begin{bmatrix}\boldsymbol{x}\\ \boldsymbol{y}\\ \boldsymbol{z}\end{bmatrix}$

Differential vector:

- $d\boldsymbol{l} = dr\boldsymbol{r} + rd\theta\boldsymbol{\theta} + r\sin\theta\, d\phi\boldsymbol{\phi}$
- $d\boldsymbol{S} = \pm r^2\sin\theta\, d\theta d\phi\boldsymbol{r}; \pm r\sin\theta\, d\phi dr\boldsymbol{\theta}; \pm rdrd\theta\boldsymbol{\phi}$
- $dv = r^2\sin\theta\, drd\theta d\phi$

3. Direction Line

Coordinate		
Rectangular	**Cylindrical**	**Spherical**
$\frac{dx}{F_x} = \frac{dy}{F_y} = \frac{dz}{F_z}$	$\frac{dr}{F_r} = \frac{rd\phi}{F_\phi} = \frac{dz}{F_z}$	$\frac{dr}{F_r} = \frac{rd\theta}{F_\theta} = \frac{r\sin\theta\, d\phi}{F_\phi}$

4. Electromagnetic Field

Point charge	Current element
$\boldsymbol{E} = \frac{Q}{4\pi\varepsilon_0 R^2}\boldsymbol{r}$	$\boldsymbol{B} = \frac{\mu_0}{4\pi}\frac{Id\boldsymbol{l} \times \boldsymbol{r}}{R^2}$
Long line charge	**Long line current**
$\boldsymbol{E} = \frac{\rho}{2\pi\varepsilon_0 R}\boldsymbol{r}$	$\boldsymbol{B} = \frac{\mu_0 I}{2\pi R}\boldsymbol{\phi}$
Sheet of charge	**Sheet of current**
$\boldsymbol{E} = \frac{\rho}{2\varepsilon_0}\boldsymbol{a_n}$	$\boldsymbol{B} = \frac{\mu_0}{2}\mathbf{J_s} \times \boldsymbol{a_n}$

Where: $\varepsilon_0 = 10^{-9}/36\pi$ F/m, $\mu_0 = 4\pi \times 10^{-7}$ H/m.

5. Differential Force

$$d\boldsymbol{F_1} = I_1 d\boldsymbol{l_1} \times (\frac{\mu_0}{4\pi}\frac{I_2 d\boldsymbol{l_2} \times \boldsymbol{a_{21}}}{R^2})$$

6. Lorentz Force Equation

$$\boldsymbol{F} = \boldsymbol{F_E} + \boldsymbol{F_M} = q\boldsymbol{E} + q\boldsymbol{v} \times \boldsymbol{B}$$

*(Newton's second law **F** = m**a** may be useful in some cases)*

7. Curl and Divergence

i. Curl/Stoke's Theorem

$$\oint_C \boldsymbol{A}.d\boldsymbol{l} = \int_S (\nabla \times \boldsymbol{A}).d\boldsymbol{S}$$

ii. Gradient Vectors

	Rectangular	Cylindrical	Spherical
$\nabla \times \boldsymbol{A} =$	$\begin{vmatrix} \boldsymbol{x} & \boldsymbol{y} & \boldsymbol{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix}$	$\begin{vmatrix} \frac{\boldsymbol{r}}{r} & \boldsymbol{\phi} & \frac{\boldsymbol{z}}{r} \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \phi} & \frac{\partial}{\partial z} \\ A_r & rA_\phi & A_z \end{vmatrix}$	$\begin{vmatrix} \frac{\boldsymbol{r}}{r^2\sin\theta} & \frac{\boldsymbol{\phi}}{r\sin\theta} & \frac{\boldsymbol{\theta}}{r} \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial \phi} \\ A_r & rA_\theta & r\sin\theta\, A_\phi \end{vmatrix}$

7. 1. Divergence Theorem

$$\oint_S \boldsymbol{A}.d\boldsymbol{S} = \int_V (\nabla.\boldsymbol{A})dv$$

Chapter 2: Maxwell's Equations

Note that:

$$\boldsymbol{D} = \varepsilon_0 \boldsymbol{E}; \qquad \boldsymbol{H} = \frac{\boldsymbol{B}}{\mu_0}$$

1. Line, Surface Integral

Voltage – line integral	Magnetic flux – surface integral
$V_{AB} = \frac{W_{AB}}{q} = \int_A^B \boldsymbol{E}.d\boldsymbol{l}$	$\Psi = \int_S \boldsymbol{B}.d\boldsymbol{S}$

2. Law Maxwell's Equations

Law	Integral form	Differential form
Faraday	$emf = \oint_C \boldsymbol{E}.d\boldsymbol{l} = -\frac{d}{dt}\int_S \boldsymbol{B}.d\boldsymbol{S}$	$\nabla \times \boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t}$
Ampère	$\oint_C \boldsymbol{H}.d\boldsymbol{l} = \int_S \mathbf{J}.d\boldsymbol{S} + \frac{d}{dt}\int_S \boldsymbol{D}.d\boldsymbol{S}$	$\nabla \times \boldsymbol{H} = \mathbf{J} + \frac{\partial \boldsymbol{D}}{\partial t}$
Gauss	**For the electric field**	
	$Q = \oint_S \boldsymbol{D}.d\boldsymbol{S} = \int_V \rho\, dv$	$\rho = \nabla.\boldsymbol{D}$
	For the magnetic field	
	$\oint_S \boldsymbol{B}.d\boldsymbol{S} = 0$	$\nabla.\boldsymbol{B} = 0$
Conservation of charge	$\oint_S \mathbf{J}.d\boldsymbol{S} = -\frac{d}{dt}\int_V \rho\, dv$	$\nabla.\mathbf{J} = -\frac{\partial \rho}{\partial t}$

Given that:

$$\nabla \times \boldsymbol{E} = \begin{vmatrix} \boldsymbol{x} & \boldsymbol{y} & \boldsymbol{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ E_x & E_y & E_z \end{vmatrix}; \qquad \nabla \times \boldsymbol{H} = \begin{vmatrix} \boldsymbol{x} & \boldsymbol{y} & \boldsymbol{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ H_x & H_y & H_z \end{vmatrix}$$

Chapter 3: Uniform Plane Waves in Free Space

Note:

$\boldsymbol{E} = E_x(z,t)\boldsymbol{x}$; $\boldsymbol{H} = H_y(z,t)\boldsymbol{y}$.

$\mathbf{J_S} = -\mathrm{J_S}(t)\boldsymbol{x}$, at $z = 0$; $\eta_0 = 120\pi \approx 377\Omega$.

1. Wave Equation

Wave equation	
$\dfrac{\partial^2 E_x}{\partial z^2} = \varepsilon_0\mu_0 \dfrac{\partial^2 E_x}{\partial t^2}$	$\dfrac{\partial^2 H_y}{\partial z^2} = \varepsilon_0\mu_0 \dfrac{\partial^2 H_y}{\partial t^2}$
Solution of wave equation	
$\boldsymbol{E}(z,t) = \dfrac{\eta_0}{2}\mathrm{J_S}\left(t \mp \dfrac{z}{v_p}\right)\boldsymbol{x}$	$\boldsymbol{H}(z,t) = \pm\dfrac{1}{2}\mathrm{J_S}\left(t \mp \dfrac{z}{v_p}\right)\boldsymbol{y}$

2. Sinusoidally Time-varying Uniform Plane Waves in Free Space

In the case of $\mathbf{J_S} = -\mathrm{J_{S0}}\cos(\omega t)\,\boldsymbol{x}$, at $z = 0$, the solution of wave equation becomes:

$$\boldsymbol{E}(z,t) = \frac{\eta_0 \mathrm{J_{S0}}}{2}\cos(\omega t \mp \beta z)\,\boldsymbol{x}; \qquad \boldsymbol{H}(z,t) = \pm\frac{\mathrm{J_{S0}}}{2}\cos(\omega t \mp \beta z)\,\boldsymbol{y}$$

Parameters

Phase constant	$\beta = \dfrac{\omega}{v_p} = \omega\sqrt{\varepsilon_0\mu_0} = \left\lvert\dfrac{\Delta\phi}{\Delta z}\right\rvert$	(m^{-1})
Frequency	$\omega = 2\pi f = \dfrac{2\pi}{T} = \left\lvert\dfrac{\Delta\phi}{\Delta t}\right\rvert$	(rad/s)
Wavelength	$\lambda = \dfrac{2\pi}{\beta} = v_p T = \dfrac{v_p}{f}$	(m)
Poynting vector	$\boldsymbol{P} = \boldsymbol{E}\times\boldsymbol{H} = \pm\dfrac{\eta_0 \mathrm{J_{S0}^2}}{4}\cos^2(\omega t \mp \beta z)\,\boldsymbol{z}$	(W/m)

3. Polarization Sinusoidally Time-varying Vector Field

Given that:

$$\boldsymbol{F} = F_1\cos(\omega t + \varphi_1)\,\boldsymbol{a} + F_2\cos(\omega t + \varphi_2)\,\boldsymbol{b}$$

1) $\boldsymbol{F}$ is called as linear polarization in either of two following cases:

- $F_2 = 0$ or
- $\Delta\varphi = \varphi_1 - \varphi_2 = 0°$ or $\pm 180°$.

2) $\boldsymbol{F}$ is called as circular polarization if it satisfies all 3 below conditions:

- $\Delta\varphi = \varphi_1 - \varphi_2 = \pm 90°$. (lệch pha 90°)
- $\boldsymbol{a}.\boldsymbol{b} = 0$. ($\boldsymbol{a}, \boldsymbol{b}$ vuông góc)
- $|F_1\boldsymbol{a}| = |F_2\boldsymbol{b}|$. (biên độ thành phần bằng nhau)

3) Elliptical polarization

- If 1) and 2) are not satisfy, therefore, the polarization must be elliptical

(Nếu xét 1) và 2) không thõa mãn vậy kết luận là "Elliptical polarization")

Chapter 4: Fields and Waves in Material Media

1. Material Media

Conductor – Semiconductor: (vật liệu dẫn điện – bán dẫn) $\boldsymbol{J_c} = \sigma \boldsymbol{E}$ $\sigma = \begin{cases} \mu_e N_e \lvert e \rvert, & \text{Conductor} \\ \mu_h N_h \lvert e \rvert + \mu_e N_e \lvert e \rvert, & \text{Semiconductor} \end{cases}$ Where: μ: Mobility. $N_{h,e}$: density holes(h). $V = El; \quad Il = VA\sigma;$ $I = J_c A; \quad R\sigma A = l$	Dielectric: $\boldsymbol{D} = \varepsilon \boldsymbol{E} = \varepsilon_0 \boldsymbol{E} + \boldsymbol{P}$ Where: ε: permittivity $\boldsymbol{P}$: polarization vector Magnetic material: $\boldsymbol{H} = \frac{\boldsymbol{B}}{\mu} = \frac{\boldsymbol{B}}{\mu_0} - \boldsymbol{M}$ Where: μ: permeability. $\boldsymbol{M}$: Magnetization vector

2. Waves in Material Media

Propagation constant

$$\bar{\gamma} = \sqrt{j\omega\mu(\sigma + j\omega\varepsilon)} = \alpha + j\beta \qquad (\mathrm{m}^{-1})$$

Intrinsic impedance

$$\bar{\eta} = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\varepsilon}} = r\angle\theta = \frac{\bar{\boldsymbol{E}}}{\bar{\boldsymbol{H}}} \qquad (\Omega)$$

Where:

- α: Attenuation constant. (Np/m)
- β: Phase constant. (rad/s)
- r: Ratio between E_0 and H_0. (Ω)
- θ: Phase different between electric field and magnetic field. (rad/s)

Relationship

$$\bar{\gamma}\,\bar{\eta} = j\omega\mu; \qquad \sigma = \mathrm{Re}\left(\frac{\bar{\gamma}}{\bar{\eta}}\right); \qquad \varepsilon = \mathrm{Im}\left(\frac{\bar{\gamma}}{\bar{\eta}}\right)$$

Note that: $\sqrt{z} = \sqrt{|z|}\angle(arg(z)/2)$.

Wave equation

$$\boldsymbol{E} = \begin{cases} E_0 e^{-\alpha z}\cos(\omega t - \beta z + \theta_1)\,\boldsymbol{x}, & z > 0 \\ E_0 e^{\alpha z}\cos(\omega t + \beta z + \theta_1)\,\boldsymbol{x}, & z < 0 \end{cases}$$

$$\leftrightarrow \boldsymbol{H} = \begin{cases} H_0 e^{-\alpha z}\cos(\omega t - \beta z + \theta_2)\,\boldsymbol{y}, & z > 0 \\ -H_0 e^{\alpha z}\cos(\omega t + \beta z + \theta_2)\,\boldsymbol{y}, & z < 0 \end{cases}$$

$$(\bar{\eta} = r\angle\theta;\ E_0 = rH_0;\ \theta_1 - \theta_2 = \theta;\ \boldsymbol{a_P} = \boldsymbol{a_E} \times \boldsymbol{a_H})$$

Poyting vector: $\boldsymbol{P} = \boldsymbol{E} \times \boldsymbol{H}$ → Average power: $P = \frac{1}{2}Re(\bar{\boldsymbol{E}} \times \bar{\boldsymbol{H}}^*).\Delta S$

Special cases:

Material media			
Perfect dielectric ($\sigma = 0$)	Imperfect dielectric ($\sigma \ll \omega$)	Good conductor ($\sigma \gg \omega\varepsilon$)	Perfect conductor ($\sigma \to \infty$)
$\bar{\gamma} = j\omega\sqrt{\mu\varepsilon}$ $\bar{\eta} = \sqrt{\frac{\mu}{\varepsilon}}$	$\bar{\gamma} = \frac{\sigma}{2}\sqrt{\frac{\mu}{\varepsilon}} + j\omega\sqrt{\mu\varepsilon}$ $\bar{\eta} = \sqrt{\frac{\mu}{\varepsilon}}\left(1 + \frac{j\sigma}{2\omega\varepsilon}\right)$	$\bar{\gamma} = \sqrt{\pi f\mu\sigma}(1 + j)$ $\bar{\eta} = \sqrt{\frac{2\pi f\mu}{\sigma}}\angle 45°$	$\bar{\gamma} = \alpha + j\beta$ $\alpha \to \infty$ $\bar{\eta} \to 0$ (No field inside)

3. Boundary Condition

Given that: there are two medium (1) and (2) which have its identities $\sigma_1, \mu_1, \varepsilon_1$ and $\sigma_2, \mu_2, \varepsilon_2$ respectively and normal vector points from medium (2) to (1). The boundary condition is given by:

$$\boldsymbol{a_n} \times (\boldsymbol{E_1} - \boldsymbol{E_2}) = \boldsymbol{0} \quad \text{or} \quad E_{t1} - E_{t2} = 0$$
$$\boldsymbol{a_n} \times (\boldsymbol{H_1} - \boldsymbol{H_2}) = \boldsymbol{J_S} \quad \text{or} \quad H_{t1} - H_{t2} = J_s$$
$$\boldsymbol{a_n} \cdot (\boldsymbol{D_1} - \boldsymbol{D_2}) = \rho_s \quad \text{or} \quad D_{n1} - D_{n2} = \rho_s$$
$$\boldsymbol{a_n} \cdot (\boldsymbol{B_1} - \boldsymbol{B_2}) = 0 \quad \text{or} \quad B_{n1} - B_{n2} = 0$$

Special cases:

Medium (2) is perfect conductor: ($\boldsymbol{E_2}, \boldsymbol{H_2} = 0$)	Medium (1) and (2) are dielectric: ((1) can be free space, $\rho_s = 0$, $\boldsymbol{J_S} = 0$)
$\boldsymbol{a_n} \times \boldsymbol{E_1} = \boldsymbol{0}$ or $E_{t1} = 0$	$\boldsymbol{a_n} \times (\boldsymbol{E_1} - \boldsymbol{E_2}) = \boldsymbol{0}$ or $E_{t1} = E_{t2}$
$\boldsymbol{a_n} \times \boldsymbol{H_1} = \boldsymbol{J_S}$ or $H_{t1} = J_s$	$\boldsymbol{a_n} \times (\boldsymbol{H_1} - \boldsymbol{H_2}) = \boldsymbol{0}$ or $H_{t1} = H_{t2}$
$\boldsymbol{a_n} \cdot \boldsymbol{D_1} = \rho_s$ or $D_{n1} = \rho_s$	$\boldsymbol{a_n} \cdot (\boldsymbol{D_1} - \boldsymbol{D_2}) = 0$ or $D_{n1} = D_{n2}$
$\boldsymbol{a_n} \cdot \boldsymbol{B_1} = 0$ or $B_{n1} = 0$	$\boldsymbol{a_n} \cdot (\boldsymbol{B_1} - \boldsymbol{B_2}) = 0$ or $B_{n1} = B_{n2}$

4. Reflection and transmission of uniform plane waves

Given that: there are two medium (1) and (2) which has its identities $\sigma_1, \mu_1, \varepsilon_1$ and $\sigma_2, \mu_2, \varepsilon_2$ respectively.

Reflection coefficient	
$\overline{\Gamma_E} = \frac{\overline{E_1^-}}{\overline{E_1^+}} = \frac{\overline{\eta_2} - \overline{\eta_1}}{\overline{\eta_2} + \overline{\eta_1}}$	$\overline{\Gamma_H} = \frac{\overline{H_1^-}}{\overline{H_1^+}} = -\overline{\Gamma_E}$
Transmission coefficient	
$\overline{\tau_E} = \frac{\overline{E_2^+}}{\overline{E_1^+}} = 1 + \overline{\Gamma_E} = \frac{2\overline{\eta_2}}{\overline{\eta_2} + \overline{\eta_1}}$	$\overline{\tau_H} = \frac{\overline{H_2^+}}{\overline{H_1^+}} = 1 - \overline{\Gamma_E} = \frac{2\overline{\eta_1}}{\overline{\eta_2} + \overline{\eta_1}}$

Chapter 5: Transmission Line Essentials for Digital Electronics

1. Transmission Line

$V = E_x d$ $I = H_y w$ $\frac{\partial V}{\partial z} = -L\frac{\partial I}{\partial t}$ $\frac{\partial I}{\partial z} = -GV - C\frac{\partial V}{\partial t}$	$L = \frac{\mu d}{w} \quad (H/m)$ $G = \frac{\sigma w}{d} \quad (S/m)$ $C = \frac{\varepsilon w}{d} \quad (F/m)$

For lossless line ($G = 0$):

$$\frac{\partial^2 V}{\partial z^2} = LC\frac{\partial^2 V}{\partial t^2} \quad \rightarrow \begin{cases} V = Af\left(t - \frac{z}{v_p}\right) + Bg\left(t + \frac{z}{v_p}\right) \\ I = \frac{1}{Z_0}\left[f\left(t - \frac{z}{v_p}\right) - Bg\left(t + \frac{z}{v_p}\right)\right] \end{cases}$$

Where:

$$v_p = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{\mu\varepsilon}}; Z_0 = \sqrt{L/C} = \eta p; L = \mu p; C = \frac{\varepsilon}{p}$$

Special case:

1. Parallel-Plate Line: $p = d/w$
2. Coaxial Cable: $p = \frac{1}{2\pi}\ln\frac{b}{a}$
3. Parallel wire line: $p = \frac{1}{\pi}\cosh^{-1}\frac{d}{a}$

2. Terminated by Resistor

$$V = V^+ + V^-; \qquad I = I^+ + I^-; \qquad V^+ = I^+ Z_0; \qquad V^- = -I^- Z_0.$$

For Line with/without load resistor:

$$V^+ = V_g \frac{Z_0}{R_g + Z_0} \qquad I^+ = \frac{V^+}{Z_0} = \frac{V_g}{R_g + Z_0}$$

Reflection coefficients:

	Voltage	**Current**
At load	$\Gamma = \frac{V^-}{V^+} = \frac{R_L - Z_0}{R_L + Z_0}$	$\Gamma_I = \frac{I^-}{I^+} = -\Gamma$
At source	$\Gamma = \frac{V^{-+}}{V^-} = \frac{R_G - Z_0}{R_G + Z_0}$	$\Gamma_I = \frac{I^{-+}}{I^-} = -\Gamma$

Steady state of transmission:

$$V_{ss} = V_g \frac{R_L}{R_L + R_g} = V_{ss}^+ + V_{ss}^-; \qquad I_{ss} = \frac{V_g}{R_L + R_g} = I_{ss}^+ + I_{ss}^-$$

$$\rightarrow \begin{cases} V_{ss}^+ + V_{ss}^- = V_g - R_g(I_{ss}^+ + I_{ss}^-), & \text{At source} \\ V_{ss}^+ + V_{ss}^- = R_L(I_{ss}^+ + I_{ss}^-), & \text{At load} \end{cases}$$

3. Transmission Line Discontinuity

$$\begin{cases} V^+ + V^- = V^{++} \\ I^+ + I^- = I^{++} \end{cases}; \quad I^+ = \frac{V^+}{Z_{01}}; \quad I^- = -\frac{V^-}{Z_{01}}; \quad I^{++} = \frac{V^{++}}{Z_{02}}; \quad I^{+-} = -\frac{V^{+-}}{Z_{02}}$$

In this case reflection coefficient becomes

$$\Gamma = \frac{V^-}{V^+} = \frac{Z_{02} - Z_{01}}{Z_{02} + Z_{01}}$$

Define Transmission Coefficient:

Voltage	Current
$\tau_V = \frac{V^{++}}{V^+} = 1 + \Gamma$	$\tau_C = \frac{I^{++}}{I^+} = 1 - \Gamma$

Power transfer

$$P^{++} = (1 - \Gamma^2)P^+$$

4. Terminated by Reactive Components

Inductor	Capacitor
$V^-(l,t) = -\frac{V_0}{2} + V_0 e^{-\frac{Z_0}{L}(t-T)}, t > T$ $I^-(l,t) = \frac{V_0}{2Z_0} - \frac{V_0}{Z_0} e^{-\frac{Z_0}{L}(t-T)}, t > T$	$V^-(l,t) = \frac{V_0}{2} - V_0 e^{-\frac{1}{CZ_0}(t-T)}, t > T$ $I^-(l,t) = -\frac{V_0}{2Z_0} + \frac{V_0}{Z_0} e^{-\frac{1}{CZ_0}(t-T)}, t > T$