FINAL EXAMINATION
PROBABILITY, STATISTICS AND RANDOM PROCESS
Semester 2, 2021-22 • June 2022 • Total duration: 90 minutes

Chair of Mathematics Department	Lecturer
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INSTRUCTIONS: Each student is allowed calculators, statistical tables and one double-sided sheet of reference material (size A4 or similar) marked with their name and ID. All other documents and electronic devices are forbidden.

1. *(20 points)*
Consider a Markov chain $(X_n)_{n\geq 0}$ with the transition matrix

$$P = \begin{array}{c} \\ \text{From} \end{array} \begin{array}{c} \begin{array}{cccc} \text{To} & & & \\ 0 & 1 & 2 & 3 \end{array} \\ \begin{array}{c} 0 \\ 1 \\ 2 \\ 3 \end{array} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0.6 & 0 & 0.4 & 0 \\ 0 & 0.6 & 0 & 0.4 \\ 0 & 0 & 0 & 1 \end{bmatrix} \end{array}.$$

   (a) Draw the diagram of this Markov chain.
   (b) Compute $P(X_3 = 2|X_0 = 1)$.
   (c) Which state(s) is recurrent? Which state(s) is transient?
   (d) List all recurrent class.

2. *(20 point)*
Determine the stationary distribution of the Markov chain with transition matrix

$$P = \begin{array}{c} \\ \text{From} \end{array} \begin{array}{c} \begin{array}{ccc} \text{To} & & \\ 1 & 2 & 3 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \end{array} \begin{bmatrix} 0.2 & 0.4 & 0.4 \\ 0.3 & 0.2 & 0.5 \\ 0.2 & 0.7 & 0.1 \end{bmatrix} \end{array}.$$

3. *(10 points)*
The net worth (in billions of dollars) of a sample of the richest people in the United States is shown.

59 52 28 26 19 19 18 17 17 17

Find the mean, median, mode, variance, and standard deviation for the data.

4. *(10 points)*

   The numbers of faculty at 32 randomly selected state-controlled colleges and universities with enrollment under 12,000 students are shown below.

$$\begin{bmatrix} 211 & 384 & 396 & 211 & 224 & 337 & 395 & 121 \\ 356 & 621 & 367 & 408 & 515 & 280 & 289 & 180 \\ 431 & 176 & 318 & 836 & 203 & 374 & 224 & 121 \\ 412 & 134 & 539 & 471 & 638 & 425 & 159 & 324 \end{bmatrix}$$

   Estimate the mean number of faculty at all state-controlled colleges and universities with enrollment under 12,000 with 95% confidence. Assume $\sigma = 165.1$.

5. *(10 points)*

   A pizza shop owner wishes to find the 99% confidence interval of the true mean cost of a large plain pizza. How large should the sample be if she wishes to be accurate to within \$0.12? A previous study showed that the standard deviation of the price was 0.26.

6. *(10 points)*

   A random sample of 250 adults in a medium-size college town were surveyed, and it was found that 110 were regular voters. Estimate the true proportion of regular voters with 95% confidence.

7. *(10 points)*

   A state executive claims that the average number of acres in western Pennsylvania state parks is less than 2000 acres. A random sample of five parks is selected, and the number of acres is shown.

$$959 \quad 1187 \quad 493 \quad 6249 \quad 541$$

   At a level of significant 0.01, is there enough evidence to support the claim?

8. *(10 points)*

   Daily weather observations for southwestern Pennsylvania for the first three weeks of January show daily high temperatures as follows: 55, 44, 51, 59, 62, 60, 46, 51, 37, 30, 46, 51, 53, 57, 57, 39, 28, 37, 35, and 28 degrees Fahrenheit. The normal standard deviation in high temperatures for this time period is usually no more than 8 degrees. A meteorologist believes that with the unusual trend in emperatures the standard deviation is greater. At $\alpha = 0.05$, can we conclude that the standard deviation is greater than 8 degrees?

9. *(10 points)*

   A product developer is interested in reducing the drying time of a primer paint. Two formulations of the paint are tested; formulation 1 is the standard chemistry, and formulation 2 has a new drying ingredient that should reduce the drying time. From experience, it is known that the standard deviation of drying time is 8 minutes, and this inherent variability should be unaffected by the addition of the new ingredient. Ten specimens are painted with formulation 1, and another 10 specimens are painted with formulation 2; the 20 specimens are painted in random order. The two sample average drying times are $\bar{x}_1 = 121$ minutes and $\bar{x}_2 = 112$ minutes respectively. What conclusions can the product developer draw about the effectiveness of the new ingredient (whether the new ingredient reduces the drying time) , using $\alpha = 0.05$?

THE END