2020 2021 DEqs S2 Midterm

Electrical Engineering (Trường Đại học Quốc tế, Đại học Quốc gia Thành phố Hồ Chí Minh)

HO CHI MINH CITY INTERNATIONAL UNIVERSITY

MIDTERM EXAMINATION

Academic year 2020-2021, Semester 2
Duration: 120 minutes

SUBJECT: Differential Equations (MAFE202IU)	
Head of Department of Mathematics	Lecturer:
Signature:	Signature:
Professor Pham Huu Anh Ngoc	Full name: Pham Huu Anh Ngoc

Instructions:

- *Each student is allowed a scientific calculator and a maximum of two double-sided sheets of reference material (size A4 or similar), stapled together and marked with their name and ID. All other documents and electronic devices are forbidden.*

Question 1. *(20 marks)* (Memorization) When forgetfulness is taken into account, the rate of memorization of a subject is given by

$$\frac{dA}{dt} = k_1(M - A) - k_2 A, \qquad A(0) = 0$$

where $k_1 > 0, k_2 > 0, A(t)$ is the amount memorized in time t, M is the total amount to be memorized and $M - A$ is the amount remaining to be memorized.

Find the limiting value of $A(t)$ as $t \to \infty$. Interpret the result.

Question 2. *(20 marks)* Prove that the differential equation

$$\big(\cos x - xy^2 + y\big)dx + \big(y(1 - x^2) + x\big)dy = 0,$$

is exact. Solve the differential equation.

Question 3. *(20 marks)* Find the solution to the initial value problem

$$xy' + (3x + 1)y = e^{-3x}, \qquad y(1) = 0.$$

Question 4. *(20 marks)* Find a particular solution of the following differential equation

$$y'' - 8y' + 16y = 6x^2 + 2020 + 2021e^{4x}.$$

Question 5. *(20 marks)* Find the general solution of the following differential equation

$$x^2y'' - 5xy' + 9y = x^2, \qquad x \in (0, \infty).$$

END.

SOLUTIONS:

Question 1. The limiting value of the population is $1,000,000$. The population will reach $500,000$ in 5.29 months.
Question 2. The given differential equation is rewritten as

$$(e^{2y}dx + xde^{2y}) - \cos(xy)(xdy + ydx) + dy^2 = 0.$$

Then, we get

$$d(e^{2y}x) - \cos(xy)d(xy) + dy^2 = d(e^{2y}x) + d(-\sin(xy)) + dy^2 = 0.$$

Therefore,

$$d(e^{2y}x - \sin(xy) + y^2) = 0.$$

Thus the general solution is given by

$$e^{2y}x - \sin(xy) + y^2 = C.$$

Question 3. Consider the differential equation

$$y' - (\sin x)y = 2\sin x.$$

The integrating factor is given by $I(x) = e^{\cos x}$. Thus, we get

$$e^{\cos x}y' - e^{\cos x}(\sin x)y = 2e^{\cos x}\sin x.$$

This gives

$$\frac{d}{dx}(e^{\cos x}y) = 2\int e^{\cos x}\sin x dx = -2e^{\cos x} + C.$$

Therefore, the general solution is

$$y(x) = -2 + \frac{C}{e^{\cos x}}.$$

Since $y(\frac{\pi}{2}) = 1$, the particular solution is $y(x) = -2 + \frac{3}{e^{\cos x}}$.
Question 4. a) The form of a particular solution of the differential equation

$$y'' - 4y' + 3y = e^{2x}(x^3 + 1) + e^x(x + 1)$$

is given by

$$y_p(x) = e^{2x}(Ax^3 + Bx^2 + Cx + D) + e^x(Ex^2 + Fx).$$

The general solution of the differential equation

$$y'' - 4y' + 3y = e^x(x + 1)$$

is given by

$$y(x) = c_1e^x + c_2e^{3x} - e^x(\frac{1}{4}x^2 + \frac{3}{4}x).$$

Question 5. a) $a = b = q, q \in \mathbb{R}$.
b) Note that $y_1(x) = x + 1$ is a particular solution of the differential equation

$$(1 - 2x - x^2)y'' + 2(x + 1)y' - 2y = 0.$$

By the Liouville formula, $y_2(x) = x^2 + x + 2$ is a solution of this equation such that y_1, y_2 are linearly independent. So, the general solution is given by

$$y(x) = c_1(x + 1) + c_2(x^2 + x + 2).$$