

Homework

Chapter 1

Week 1

Recall that A system of linear equations is said to be **consistent** if it has either one solution or infinitely many solutions; a system is **inconsistent** if it has no solution.

1. Determine if the following system is consistent:

$$\begin{cases} x_1 - 4x_3 = 8 \\ 2x_1 - 3x_2 + 2x_3 = 1 \\ 4x_1 - 8x_2 + 12x_3 = 1 \end{cases}$$

2. Determine which matrices are in reduced echelon form and which others are only in echelon form.

a. $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}$ b. $\begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

c. $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ d. $\begin{bmatrix} 1 & 1 & 0 & 1 & 1 \\ 0 & 2 & 0 & 2 & 2 \\ 0 & 0 & 0 & 3 & 3 \\ 0 & 0 & 0 & 0 & 4 \end{bmatrix}$

3. Reduced the matrices to echelon form. Circle the pivot positions in the final matrix and in the original matrix, and list the pivot columns.

$$\text{a) } \begin{bmatrix} 1 & 2 & 3 & 4 \\ 4 & 5 & 6 & 7 \\ 6 & 7 & 8 & 9 \end{bmatrix}$$

$$\text{b) } \begin{bmatrix} 1 & 3 & 5 & 7 \\ 3 & 5 & 7 & 9 \\ 5 & 7 & 9 & 1 \end{bmatrix}$$

4. Find the general solutions of the systems whose augmented matrices

$$\text{a) } \begin{bmatrix} 1 & 3 & 4 & 7 \\ 3 & 9 & 7 & 6 \end{bmatrix}$$

$$\text{b) } \begin{bmatrix} 3 & -4 & 2 & 0 \\ -9 & 12 & -6 & 0 \\ -6 & 8 & -4 & 0 \end{bmatrix}$$

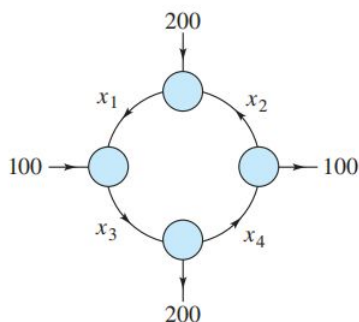
5 Solve the system

$$\text{a) } \begin{cases} 4x + 2y + z = 18 \\ 4x - 2y - 2z = 28 \\ 2x - 3y + 2z = -8 \end{cases}$$

$$\text{b) } \begin{cases} 2x_1 + x_2 + x_3 + 2x_4 = -1 \\ 5x_1 - 2x_2 + x_3 - 3x_4 = 0 \\ -x_1 + 3x_2 + 2x_3 + 2x_4 = 1 \\ 3x_1 + 2x_2 + 3x_3 - 5x_4 = 12 \end{cases}$$

Applications

6. (*Network Analysis*) The figure shows the flow of traffic (in vehicles per hour) through a network of streets.



- Solve this system for $x_i, i = 1, 2, 3, 4$
- Find the traffic flow when $x_4 = 0$.
- Find the traffic flow when $x_4 = 100$.

Week 2

1. Answer the following questions

- If a matrix A is 5×3 and the product AB is 5×7 , what is the size of B ?
- How many rows does B have if BC is a 3×4 matrix?

2. Let $A = \begin{pmatrix} 1 & 1 \\ -2 & -1 \\ 1 & 2 \end{pmatrix}$, $B = \begin{pmatrix} 1 & -1 & -2 \\ 2 & 1 & -2 \end{pmatrix}$, and $C = \begin{pmatrix} 1 & 1 & -3 \\ -1 & 2 & 1 \\ -3 & -1 & 0 \end{pmatrix}$.

Find the following if possible.

- $A + 20B$, $B - 5A^T$, and BA
 - $A + 4C^T$, AC and CA
3. Let $A = \begin{pmatrix} 2 & 5 \\ -3 & 1 \end{pmatrix}$ and $B = \begin{pmatrix} 4 & -5 \\ 3 & c \end{pmatrix}$. What is value of c such that $AB = BA$?

4. Let $A = \begin{pmatrix} 3 & -6 \\ -1 & 2 \end{pmatrix}$. Find matrix B such that $AB = 0$

5. Consider the following system of equation

$$\begin{cases} 3x_1 + x_2 + x_3 = 3 \\ x_1 - x_2 - x_3 = 1 \\ x_1 + 2x_2 + 2x_3 = 1 \end{cases}$$

Denote $x = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$ the vector solution of the equation. Express your solution in the form $x = tv + su$, where v and u are column vector in three dimensions, $t, s \in \mathbb{R}$.

Week 3

Inverse matrices

1. Suppose A , B , and X are $n \times n$ matrices with A , X , and $A - AX$ is invertible and and suppose

$$(A - AX)^{-1} = X^{-1}B \quad (*)$$

a) Explain why B is invertible

b) Solve $(*)$ for X . If you need to invert a matrix, explain why that matrix is invertible.

2. Find the inverses of the matrices, if they exist

a)

$$\begin{pmatrix} 1 & 0 & -2 \\ -3 & 1 & 4 \\ 2 & -3 & 4 \end{pmatrix}$$

b)

$$\begin{pmatrix} 1 & -2 & 1 \\ 4 & -7 & 3 \\ -2 & 6 & -4 \end{pmatrix}$$

c)

$$\begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{pmatrix}$$

3. If A , B , and C are $n \times n$ invertible matrices, does the equation $C^{-1}(A + X)B^{-1} = I_n$ have a solution, X ? If so, find it.

4. Use an inverse matrix to solve system of linear equations.

$$x_1 + x_2 - 2x_3 = -1$$

$$x_1 - 2x_2 + x_3 = 2$$

$$x_2 - x_2 - x_3 = 0$$

5. Prove that if $A^2 = A$ then $I - 2A = (I - 2A)^{-1}$.

Chapter 2

Week 4

Determinants

1. Find the determinants in the following problems by row reduction to echelon form.

a)

$$\begin{vmatrix} 1 & 3 & 0 & 2 \\ -2 & -5 & 7 & 4 \\ 3 & 5 & 2 & 1 \\ 1 & -1 & 2 & -3 \end{vmatrix}$$

b)

$$\begin{vmatrix} 1 & 3 & 2 & -4 \\ 0 & 1 & 2 & -5 \\ 2 & 7 & 6 & -3 \\ -3 & -10 & -7 & 2 \end{vmatrix}$$

2. We know that

$$\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = 7.$$

Find the determinant of the following matrix

$$\begin{vmatrix} a & b & c \\ d & e & f \\ 3g & 3h & 3i \end{vmatrix}$$

3. Compute $\det(B^4)$ where

$$B = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 2 \\ 1 & 2 & 1 \end{bmatrix}.$$

4. Using cofactors, find inverse of the following matrices

a)

$$A = \begin{pmatrix} 2 & 4 & -1 \\ 0 & 3 & 1 \\ 6 & -2 & 5 \end{pmatrix}$$

b)

$$B = \begin{pmatrix} 1 & 2 & 0 \\ -1 & 0 & 1 \\ 0 & 3 & 1 \end{pmatrix}$$

5. Use Gaussian elimination and Cramer's Rule to solve the systems.

a)

$$\begin{cases} 7x_1 + x_2 - 4x_3 = 3 \\ -6x_1 - 4x_2 + x_3 = 0 \\ 4x_1 - x_2 - 2x_3 = 6 \end{cases}$$

b)

$$\begin{cases} 2x_1 + 3x_2 - 5x_3 = 2 \\ 3x_1 - x_2 + 2x_3 = 1 \\ 5x_1 + 4x_2 - 6x_3 = 3 \end{cases}$$

Chapter 3. Vector Spaces

Week 5

1. Determine whether the set, together with the standard operations, is a vector space?

a) The set $S = \{(x, y) : x \geq 0, y \in \mathbb{R}\}$

b) The set $S = \{(x, x/2) : x \in \mathbb{R}\}$

2. Determine whether the set \mathbb{R}^2 with the operations

$$(x_1, y_1) + (x_2, y_2) = (x_1 y_1, x_2 y_2)$$

and $c(x_1, y_1) = (cx_1, cy_1)$ where $c \in \mathbb{R}$,

is a vector space. If it is, verify each vector space axiom; if it is not, state all vector space axioms that fail.

3. Determine whether the set W is a subspace of \mathbb{R}^3 with the standard operations. Justify your answer.

a) $W = \{(0, x_2, x_3) : x_2, x_3 \text{ are real numbers}\}$

b) $W = \{(x_1, x_2, 4) : x_1 \text{ and } x_2 \text{ are real numbers}\}$

4. Write each vector as a linear combination of the vectors in S (if possible).

$$S = \{(2, 0, 7), (2, 4, 5), (2, -12, 13)\}$$

a) $u = (-1, 5, -6)$

b) $v = (-3, 15, 18)$

5. Determine whether the set S spans \mathbb{R}^3 .

a) $S = \{(4, 7, 3), (-1, 2, 6), (2, -3, 5)\}$

b) $S = \{(5, 6, 5), (2, 1, -5), (0, -4, 1)\}$

6. Determine whether the set S is linearly independent or linearly dependent.

a) $S = \{(-2, 1, 3), (2, 9, -3), (2, 3, -3)\}$

b) $S = \{(-4, -3, 4), (1, -2, 3), (6, 0, 0)\}$

7. For which values of t is each set linearly independent?

$$S = \{(t, 1, 1), (1, t, 1), (1, 1, t)\}$$

Week 6

1. Find rank, nullity ($\dim \text{Null}$) of the following matrix

a)

$$A = \begin{pmatrix} 1 & -4 & 9 & -7 \\ -1 & 2 & -4 & 1 \\ 5 & -6 & 10 & 7 \end{pmatrix}$$

b)

$$B = \begin{pmatrix} 1 & 3 & -2 & 4 \\ 0 & 1 & -1 & 2 \\ -2 & -6 & 4 & -8 \end{pmatrix}$$

2. Finding a Basis for a Row Space and Rank of the following matrix

a)

$$A = \begin{pmatrix} 2 & -3 & 1 \\ 5 & 10 & 6 \\ 8 & -7 & 5 \end{pmatrix}$$

b)

$$B = \begin{pmatrix} -2 & -4 & 4 & -5 \\ 3 & 6 & -6 & -4 \\ -2 & -4 & 4 & 9 \end{pmatrix}$$

3. Determining whether a Set is a Basis. If it is, write $u = (8, 3, 8)$ as a linear combination of the vectors in S

a) $S = \{(4, 3, 2), (0, 3, 2), (0, 0, 2)\}$

b) $S = \{(1, 0, 0), (1, 1, 0), (1, 1, 1)\}$

c) $S = \{(0, 0, 0), (1, 3, 4), (6, 1, -2)\}$

4. Find the dimension and basis of the subspace

$$H = \left\{ \begin{pmatrix} a - 4b - 2c \\ 2a + 5b - 4c \\ -a + 2c \\ -3a + 7b + 6c \end{pmatrix}, a, b, c \in \mathbb{R} \right\}$$

Week 7

1. Let $B = \{(8, 11, 0), (7, 0, 10), (1, 4, 6)\}$ be a basis of \mathbb{R}^3 . Find the coordinate of vector $x = (3, 19, 2)$?

2. Let $S = \{(1, -4), (3, -5)\}$, and $T = \{(-9, 1), (-5, -1)\}$ be bases in \mathbb{R}^2 .

a. Find the change of coordinates matrix from T to S (transition matrix).

b. Let vector $v = (1, -2)$. Find the coordinate of v in the basis T then use the transition matrix at question a) to find the coordinate of v in the basis S .

Chapter 4. Eigenvalues and Eigenvectors

Week 8

1. Find eigenvalues and eigenvectors of the following matrices

a)

$$A = \begin{pmatrix} 4 & -5 \\ 2 & -3 \end{pmatrix}$$

b)

$$A = \begin{pmatrix} 6 & -3 \\ -2 & 1 \end{pmatrix}$$

2. Find eigenvalues and eigenvectors of the following matrix

$$A = \begin{pmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ 1 & -2 & 0 \end{pmatrix}$$