Physics 2 Quiz 1

Q1/ (50 pts). Water is moving with a speed of 5.0 m/s through a pipe with a cross-sectional area of 4.0 cm^2. The water gradually descends 10 m as the pipe cross-sectional area increases to 8.0 cm^2.

(a) What is the speed at the lower level?

(b) If the pressure at the upper level is 1.5×10^5 Pa, what is the pressure at the lower level?

Equation of continuity:

$A_1 v_1 = A_2 v_2 = \text{constant}$

Bernoulli's equation:

$$p_1 + \frac{1}{2}\rho v_1^2 + \rho g y_1 = p_2 + \frac{1}{2}\rho v_2^2 + \rho g y_2 = \text{constant}$$

Solution

a) Because the water flowing rate $R_v = Av = \text{const}$, so applying the equation of continuity:

$A_1 v_1 = A_2 v_2$ (10 pts)

$\Rightarrow v_2 = \frac{A1\ v1}{A2}$

From the problem we have: $A_1 = 4.0 \times 10^{-4}$ m^2, $A_2 = 8.0 \times 10^{-4}$ m^2, $v_1 = 5.0$ m/s

$\Rightarrow v_2 = \frac{A1\ v1}{A2} = \frac{4.0 \times 10^{-4} \times 5.0}{8.0 \times 10^{-4}} = 2.5$ (m/s) (10 pts)

=> The speed at the lower level is 2.5 m/s

b) We use the Bernoulli's Equation:

$$p_1 + \frac{1}{2}\rho v_1^2 + \rho g h_1 = p_2 + \frac{1}{2}\rho v_2^2 + \rho g h_2 \quad \text{(10 pts)}$$

$$\Rightarrow p_2 = p_1 + \frac{1}{2}\rho(v_1^2 - v_2^2) + \rho g(h_1 - h_2)$$

We know that $p_1 = 1.5 \times 10^5$ Pa, $\rho = 1000 kg/m^3$, $v_1 = 5.0$ m/s, $v_2 = 2.5$ m/s, $h_1 - h_2 = 10m$ (10 pts)

$$\Rightarrow p_2 = 1.5 . 10^5 + \frac{1}{2}.1000(5^2 - 2.5^2) + 1000 . 9.8 . 10 = 257375\ (Pa) \quad \text{(10 pts)}$$

=> The pressure at the lower level is 257375 Pa

Q2/ (50 pts). An aluminum cup of 100 cm^3 capacity is completely filled with glycerin at 22°C. How much glycerin, if any, will spill out of the cup if the temperature of both the cup and the glycerin is increased to 28°C? (The coefficient of linear expansion of aluminum is $23 \times 10^{-6}/C°$ and the coefficient of volume expansion of glycerin is $5.1 \times 10^{-4}/C°$)

Solution

First, we calculate the volume expansion of the aluminum cup:

$\Delta V_{Al} = V_{Al}\beta_{Al}\Delta T$ (10 pts)

From the question, we have:

$V_{Al} = 100cm^3$

$\Delta T = 28°C - 22°C = 6C°$

The coefficient of linear expansion of Al: $\alpha_{Al} = 23 \times 10^{-6}/C°$

=> The coefficient of volume expansion of Al: $\beta_{Al} = 3\alpha_{Al} = 69 \times 10^{-6}/C°$ (10 pts)

=> $\Delta V_{Al} = V_{Al}\beta_{Al}\Delta T = 100 \,.\, 69 \times 10^{-6} \,.\, 6 = 0.0414(cm^3)$ (5 pts)

Secondly, we calculate the volume expansion of glycerin:

$\Delta V_{Gly} = V_{Gly}\beta_{Gly}\Delta T$ (10 pts)

From the question, we have:

$V_{Gly} = 100\ cm^3$

$\Delta T = 6C°$

The coefficient of volume expansion of Glycerin: $\beta_{Al} = 5.1 \times 10^{-4}/C°$

=> $\Delta V_{Gly} = V_{Gly}\beta_{Gly}\Delta T = 100 \,.\, 5.1 \times 10^{-4} \,.\, 6 = 0.306(cm^3)$ (5 pts)

Since $\Delta V_{Gly} > \Delta V_{Al}$ => The Glycerin will spill out of the cup with the volume of:

$\Delta V = \Delta V_{Gly} - \Delta V_{Al} = 0.306 - 0.0414 = 0.2646\ (cm^3)$ (10 pts)