Ch09 Solution Manual Material Science and Engineering 8th Edition

Material Science & Engineering (HITEC University)

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PHASE DIAGRAMS

PROBLEM SOLUTIONS

Solubility Limit

9.1 Consider the sugar–water phase diagram of Figure 9.1.

(a) How much sugar will dissolve in 1500 g water at 90°C (194°F)?

(b) If the saturated liquid solution in part (a) is cooled to 20°C (68°F), some of the sugar will precipitate out as a solid. What will be the composition of the saturated liquid solution (in wt% sugar) at 20°C?

(c) How much of the solid sugar will come out of solution upon cooling to 20°C?

<u>Solution</u>

(a) We are asked to determine how much sugar will dissolve in 1000 g of water at 90°C. From the solubility limit curve in Figure 9.1, at 90°C the maximum concentration of sugar in the syrup is about 77 wt%. It is now possible to calculate the mass of sugar using Equation 4.3 as

$$C_{sugar}(wt\%) = \frac{m_{sugar}}{m_{sugar} + m_{water}} \times 100$$

$$77 \text{ wt\%} = \frac{m_{sugar}}{m_{sugar} + 1500 \text{ g}} \times 100$$

Solving for m_{sugar} yields m_{sugar} = 5022 g

(b) Again using this same plot, at 20°C the solubility limit (or the concentration of the saturated solution) is about 64 wt% sugar.

(c) The mass of sugar in this saturated solution at 20°C (m'_{sugar}) may also be calculated using Equation 4.3 as follows:

$$64 \text{ wt\%} = \frac{m'_{sugar}}{m'_{sugar} + 1500 \text{ g}} \times 100$$

which yields a value for m'_{sugar} of 2667 g. Subtracting the latter from the former of these sugar concentrations yields the amount of sugar that precipitated out of the solution upon cooling m''_{sugar}; that is

$$m''_{sugar} = m_{sugar} - m'_{sugar} = 5022 \text{ g} - 2667 \text{ g} = 2355 \text{ g}$$

9.2 At 500°C (930°F), what is the maximum solubility (a) of Cu in Ag? (b) Of Ag in Cu?

<u>Solution</u>

(a) From Figure 9.7, the maximum solubility of Cu in Ag at 500°C corresponds to the position of the β–(α + β) phase boundary at this temperature, or to about 2 wt% Cu.

(b) From this same figure, the maximum solubility of Ag in Cu corresponds to the position of the α–(α + β) phase boundary at this temperature, or about 1.5 wt% Ag.

Microstructure

9.3 Cite three variables that determine the microstructure of an alloy.

<u>Solution</u>

Three variables that determine the microstructure of an alloy are (1) the alloying elements present, (2) the concentrations of these alloying elements, and (3) the heat treatment of the alloy.

Phase Equilibria

9.4 What thermodynamic condition must be met for a state of equilibrium to exist?

<u>Solution</u>

In order for a system to exist in a state of equilibrium the free energy must be a minimum for some specified combination of temperature, pressure, and composition.

One-Component (or Unary) Phase Diagrams

9.5 Consider a specimen of ice that is at ‒10°C and 1 atm pressure. Using Figure 9.2, the pressure–temperature phase diagram for H_2O, determine the pressure to which the specimen must be raised or lowered to cause it (a) to melt, and (b) to sublime.

Solution

The figure below shows the pressure-temperature phase diagram for H_2O. A vertical line has been constructed at -10°C, and the location on this line at 1 atm pressure (point B) is also noted.
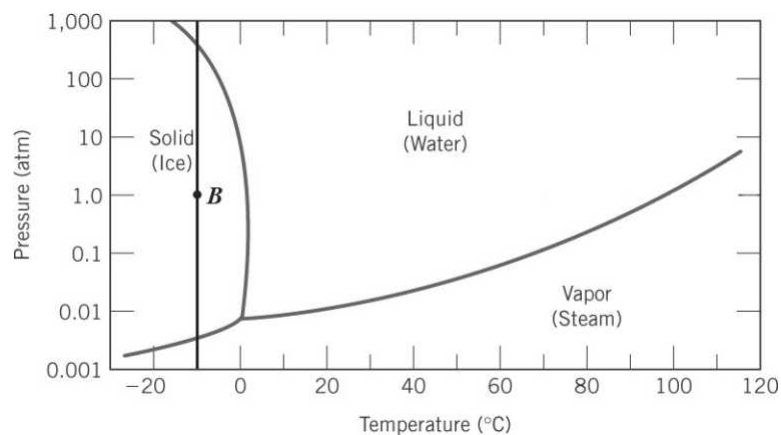

(a) Melting occurs, (by changing pressure) as, moving vertically (upward) at this temperature, we cross the Ice-Liquid phase boundary. This occurs at a pressure of approximately 570 atm; thus, the pressure of the specimen must be raised from 1 to 570 atm.

(b) In order to determine the pressure at which sublimation occurs at this temperature, we move vertically downward from 1 atm until we cross the Ice-Vapor phase boundary. This intersection occurs at approximately 0.0023 atm.

9.6   At a pressure of 0.01 atm, determine (a) the melting temperature for ice, and (b) the boiling temperature for water.

Solution

The melting temperature for ice and the boiling temperature for water at a pressure of 0.01 atm may be determined from the pressure-temperature diagram for this system (Figure 9.2), which is shown below. A horizontal line has been constructed across this diagram at a pressure of 0.01 atm.
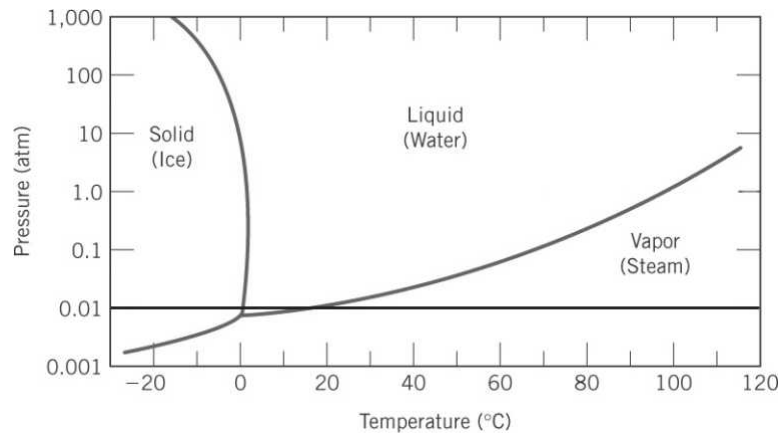
The melting and boiling temperatures for ice at a pressure of 0.01 atm may be determined by moving horizontally across the pressure-temperature diagram at this pressure. The temperature corresponding to the intersection of the Ice-Liquid phase boundary is the melting temperature, which is approximately 1°C. On the other hand, the boiling temperature is at the intersection of the horizontal line with the Liquid-Vapor phase boundary--approximately 16°C.

Binary Isomorphous Systems

9.7 Given here are the solidus and liquidus temperatures for the germanium-silicon system. Construct the phase diagram for this system and label each region.

Composition (wt% Si)	Solidus Temperature (°C)	Liquidus Temperature (°C)
0	938	938
10	1005	1147
20	1065	1226
30	1123	1278
40	1178	1315
50	1232	1346
60	1282	1367
70	1326	1385
80	1359	1397
90	1390	1408
100	1414	1414

Solution

The germanium-silicon phase diagram is constructed below.
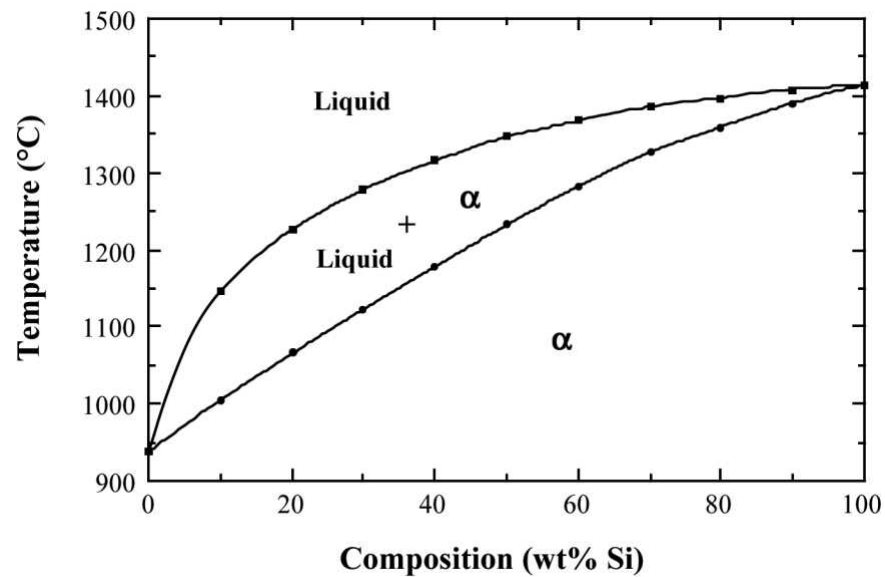
Interpretation of Phase Diagrams

9.8 Cite the phases that are present and the phase compositions for the following alloys:

(a) 90 wt% Zn-10 wt% Cu at 400°C (750°F)

(b) 75 wt% Sn-25 wt% Pb at 175°C (345°F)

(c) 55 wt% Ag-45 wt% Cu at 900°C (1650°F)

(d) 30 wt% Pb-70 wt% Mg at 425°C (795°F)

(e) 2.12 kg Zn and 1.88 kg Cu at 500°C (930°F)

(f) 37 lb_m Pb and 6.5 lb_m Mg at 400°C (750°F)

(g) 8.2 mol Ni and 4.3 mol Cu at 1250°C (2280°F)

(h) 4.5 mol Sn and 0.45 mol Pb at 200°C (390°F)

Solution

This problem asks that we cite the phase or phases present for several alloys at specified temperatures.

(a) That portion of the Cu-Zn phase diagram (Figure 9.19) that pertains to this problem is shown below; the point labeled "A" represents the 90 wt% Zn-10 wt% Cu composition at 400°C.
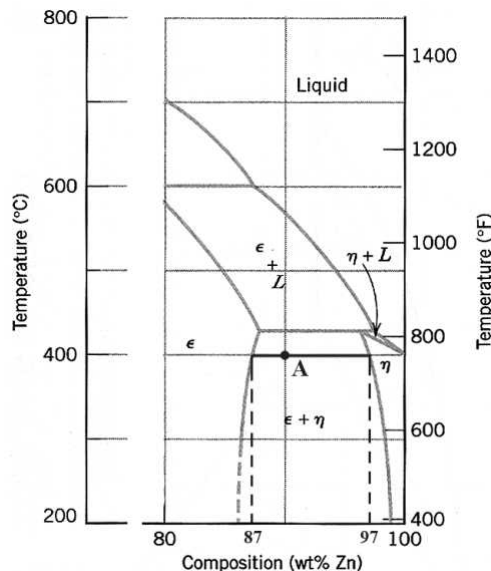
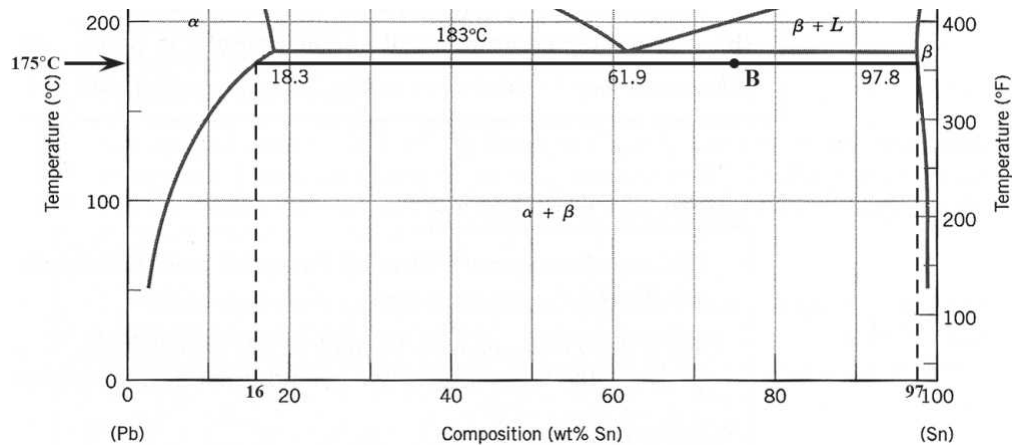As may be noted, point A lies within the ε and η phase field. A tie line has been constructed at 400°C; its intersection with the ε–ε + η phase boundary is at 87 wt% Zn, which corresponds to the composition of the ε phase. Similarly, the tie-line intersection with the ε + η–η phase boundary occurs at 97 wt% Zn, which is the composition of the η phase. Thus, the phase compositions are as follows:

$$C_\varepsilon = 87 \text{ wt\% Zn-13 wt\% Cu}$$
$$C_\eta = 97 \text{ wt\% Zn-3 wt\% Cu}$$

(b) That portion of the Pb-Sn phase diagram in Figure 9.8 that pertains to this problem is shown below; the point labeled "B" represents the 75 wt% Sn-25 wt% Pb composition at 175°C.
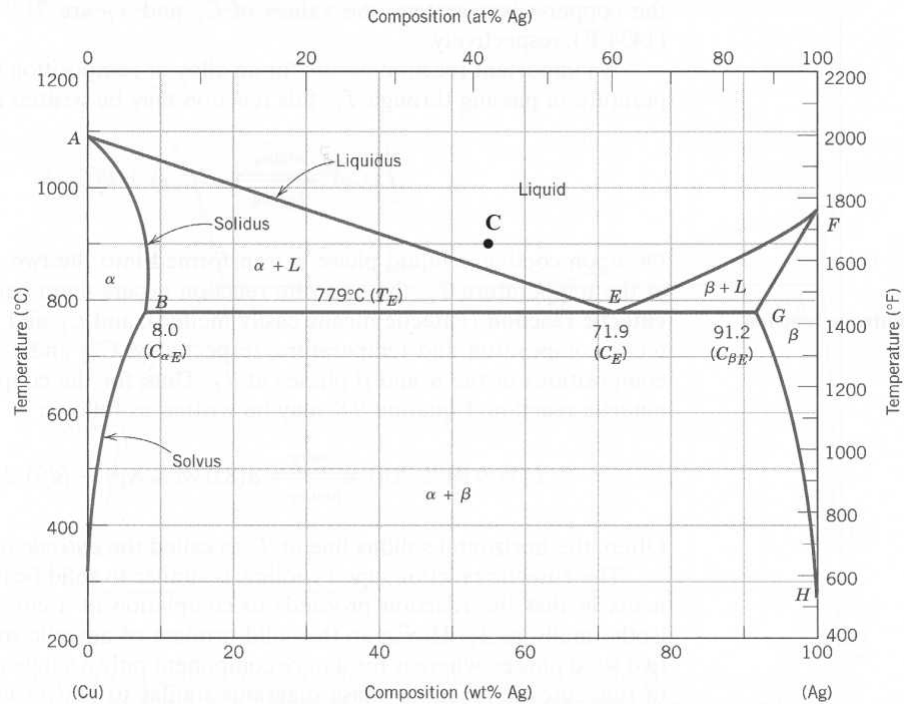

As may be noted, point B lies within the α + β phase field. A tie line has been constructed at 175°C; its intersection with the α–α + β phase boundary is at 16 wt% Sn, which corresponds to the composition of the α phase. Similarly, the tie-line intersection with the α + β–β phase boundary occurs at 97 wt% Sn, which is the composition of the β phase. Thus, the phase compositions are as follows:
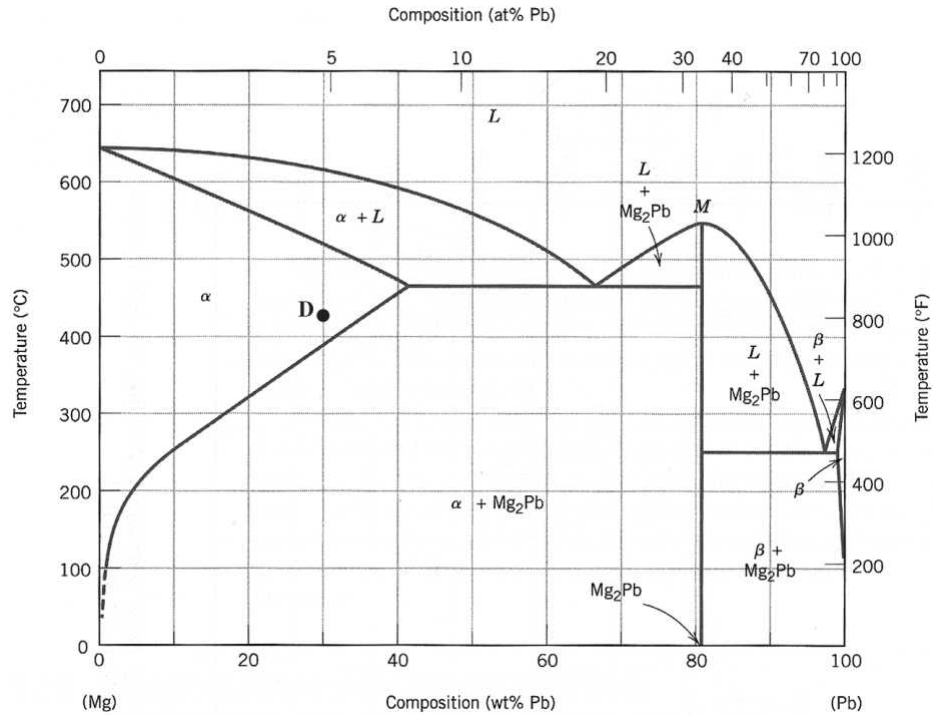
$$C_\alpha = 16 \text{ wt\% Sn-84 wt\% Pb}$$
$$C_\beta = 97 \text{ wt\% Sn-3 wt\% Pb}$$

(c) The Ag-Cu phase diagram in Figure 9.7 is shown below; the point labeled "C" represents the 55 wt% Ag-45 wt% Cu composition at 900°C.

As may be noted, point C lies within the Liquid phase field. Therefore, only the liquid phase is present; its composition is 55 wt% Ag-45 wt% Cu.

(d) The Mg-3E⎮SKDVH⎮GDJUDP⎮ᵇ⎮IJXUH ◀‼⎭ᵇᵘ⎮LV⎮VKRZQ⎮EHORZ⎮Ⅱ⎮WKH⎮SRLQW⎮ODEHODG⎮ᴮ‘‘⎮ᵘUᵇUHᵇQW⎮WKH⎮30 wt% Pb-70 wt% Mg composition at 425°C.

As may be noted, point D lies within the α phase field. Therefore, only the α phase is present; its composition is 30 wt% Pb-70 wt% Mg.

(e) For an alloy composed of 2.12 kg Zn and 1.88 kg Cu and at 500°C, we must first determine the Zn and Cu concentrations, as

$$C_{Zn} = \frac{2.12 \text{ kg}}{2.12 \text{ kg} + 1.88 \text{ kg}} \times 100 = 53 \text{ wt\%}$$

$$C_{Cu} = \frac{1.88 \text{ kg}}{2.12 \text{ kg} + 1.88 \text{ kg}} \times 100 = 47 \text{ wt\%}$$

That portion of the Cu-Zn phase diagram (Figure 9.19) that pertains to this problem is shown below; the point labeled "E" represents the 53 wt% Zn-47 wt% Cu composition at 500°C.
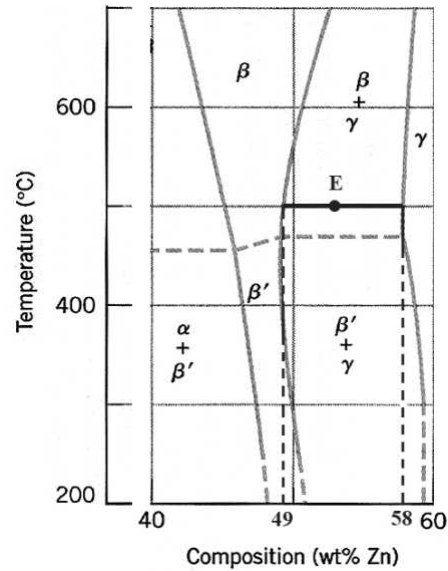
As may be noted, point E lies within the $\beta + \gamma$ phase field. A tie line has been constructed at 500°C; its intersection with the β–$\beta + \gamma$ phase boundary is at 49 wt% Zn, which corresponds to the composition of the β phase. Similarly, the tie-line intersection with the $\beta + \gamma$–γ phase boundary occurs at 58 wt% Zn, which is the composition of the γ phase. Thus, the phase compositions are as follows:

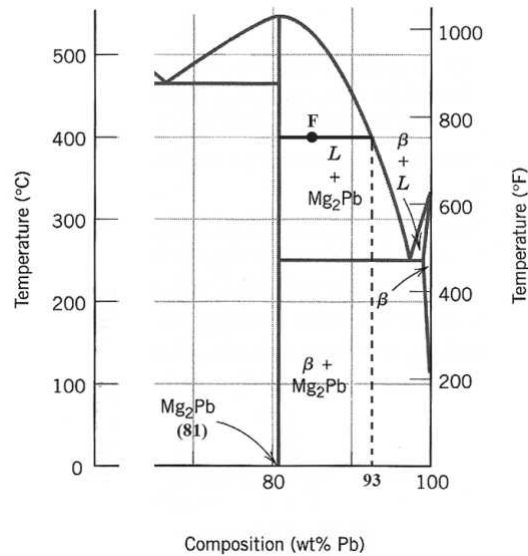$$C_\beta = 49 \text{ wt\% Zn-51 wt\% Cu}$$
$$C_\gamma = 58 \text{ wt\% Zn-42 wt\% Cu}$$

(f) For an alloy composed of 37 lb_m Pb and 6.5 lb_m Mg and at 400°C, we must first determine the Pb and Mg concentrations, as

$$C_{Pb} = \frac{37 \text{ lb}_m}{37 \text{ lb}_m + 6.5 \text{ lb}_m} \times 100 = 85 \text{ wt\%}$$

$$C_{Mg} = \frac{6.5 \text{ lb}_m}{37 \text{ lb}_m + 6.5 \text{ lb}_m} \times 100 = 15 \text{ wt\%}$$

That portion of the Mg-3E phase diagram (Figure 9.20) that pertains to this problem is shown below; the point labeled "F" represents the 85 wt% Pb-15 wt% Mg composition at 400°C.

Composition (wt% Pb)

As may be noted, point F lies within the L + Mg$_2$Pb phase field. A tie line has been constructed at 400°C; it intersects the vertical line at 81 wt% Pb, which corresponds to the composition of Mg$_2$Pb. Furthermore, the tie line intersection with the L + Mg$_2$Pb-L phase boundary is at 93 wt% Pb, which is the composition of the liquid phase. Thus, the phase compositions are as follows:

$$C_{Mg_2Pb} = 81 \text{ wt\% Pb-19 wt\% Mg}$$

$$C_L = 93 \text{ wt\% Pb-7 wt\% Mg}$$

(g) For an alloy composed of 8.2 mol Ni and 4.3 mol Cu and at 1250°C, it is first necessary to determine the Ni and Cu concentrations, which we will do in wt% as follows:
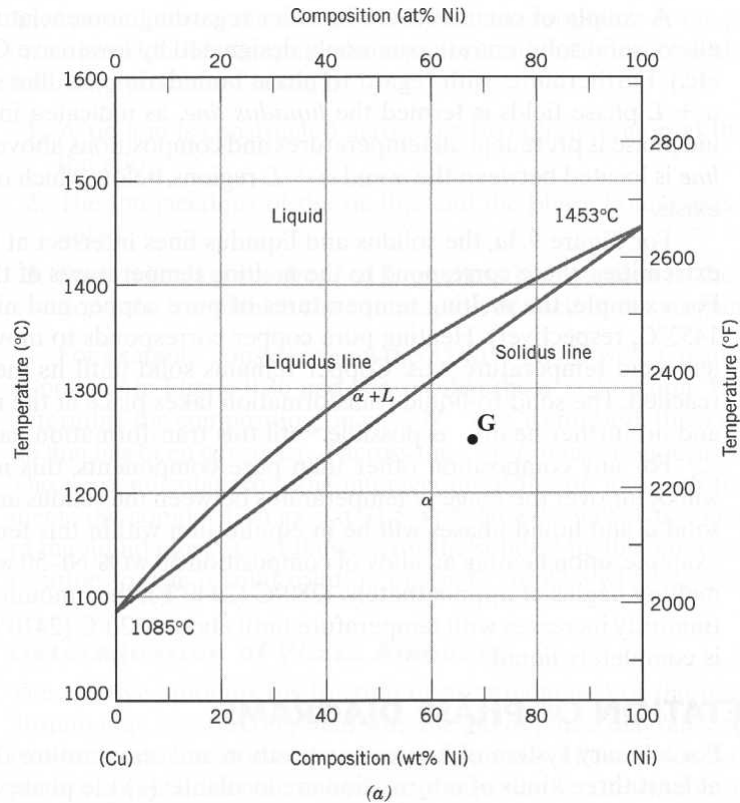
$$n'_{Ni} = n_{m_{Ni}} A_{Ni} = (8.2 \text{ mol})(58.69 \text{ g/mol}) = 481.3 \text{ g}$$

$$n'_{Cu} = n_{m_{Cu}} A_{Cu} = (4.3 \text{ mol})(63.55 \text{ g/mol}) = 273.3 \text{ g}$$

$$C_{Ni} = \frac{481.3 \text{ g}}{481.3 \text{ g} + 273.3 \text{ g}} \times 100 = 63.8 \text{ wt\%}$$

$$C_{Cu} = \frac{273.3 \text{ g}}{481.3 \text{ g} + 273.3 \text{ g}} \times 100 = 36.2 \text{ wt\%}$$

The Cu-Ni phase diagram (Figure 9.3a) is shown below; the point labeled "G" represents the 63.8 wt% Ni-36.2 wt% Cu composition at 1250°C.

As may be noted, point G lies within the α phase field. Therefore, only the α phase is present; its composition is 63.8 wt% Ni-36.2 wt% Cu.

(h) For an alloy composed of 4.5 mol Sn and 0.45 mol Pb and at 200°C, it is first necessary to determine the Sn and Pb concentrations, which we will do in weight percent as follows:
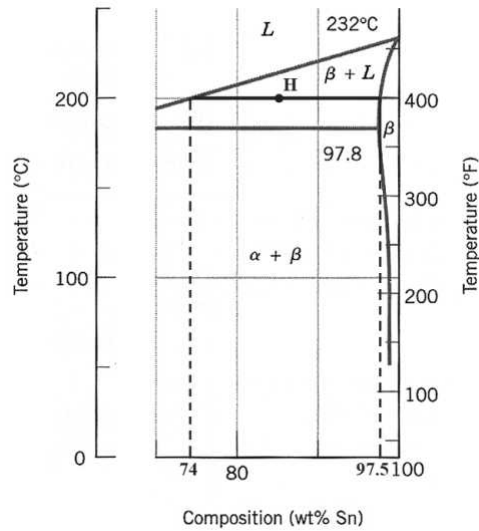
$$n'_{Sn} = n_{m_{Sn}} A_{Sn} = (4.5 \text{ mol})(118.71 \text{ g/mol}) = 534.2 \text{ g}$$

$$n'_{Pb} = n_{m_{Pb}} A_{Pb} = (0.45 \text{ mol})(207.2 \text{ g/mol}) = 93.2 \text{ g}$$

$$C_{Sn} = \frac{534.2 \text{ g}}{534.2 \text{ g} + 93.2 \text{ g}} \times 100 = 85.1 \text{ wt\%}$$

$$C_{Pb} = \frac{93.2 \text{ g}}{534.2 \text{ g} + 93.2 \text{ g}} \times 100 = 14.9 \text{ wt\%}$$

That portion of the Pb-Sn phase diagram (Figure 9.8) that pertains to this problem is shown below; the point labeled "H" represents the 85.1 wt% Sn-14.9 wt% Pb composition at 200°C.

As may be noted, point H lies within the β + L phase field. A tie line has been constructed at 200°C; its intersection with the L–β + L phase boundary is at 74 wt% Sn, which corresponds to the composition of the L phase. Similarly, the tie-line intersection with the β + L–β phase boundary occurs at 97.5 wt% Sn, which is the composition of the β phase. Thus, the phase compositions are as follows:

$$C_\beta = 97.5 \text{ wt\% Sn-2.5 wt\% Pb}$$
$$C_L = 74 \text{ wt\% Sn-26 wt\% Pb}$$

9.9   Is it possible to have a copper–nickel alloy that, at equilibrium, consists of a liquid phase of composition 20 wt% Ni–80 wt% Cu and also an α phase of composition 37 wt% Ni–63 wt% Cu? If so, what will be the approximate temperature of the alloy? If this is not possible, explain why.

Solution

It is not possible to have a Cu-Ni alloy, which at equilibrium, consists of a liquid phase of composition 20 wt% Ni-80 wt% Cu and an α phase of composition 37 wt% Ni-63 wt% Cu. From Figure 9.3a, a single tie line does not exist within the α + L region that intersects the phase boundaries at the given compositions. At 20 wt% Ni, the L-(α + L) phase boundary is at about 1200°C, whereas at 37 wt% Ni the (L + α)-α phase boundary is at about 1230°C.

9.10 Is it possible to have a copper-zinc alloy that, at equilibrium, consists of an ε phase of composition 80 wt% Zn-20 wt% Cu, and also a liquid phase of composition 95 wt% Zn-5 wt% Cu? If so, what will be the approximate temperature of the alloy? If this is not possible, explain why.

<u>Solution</u>

It is not possible to have a Cu-Zn alloy, which at equilibrium consists of an ε phase of composition 80 wt% Zn-20 wt% Cu and also a liquid phase of composition 95 wt% Zn-5 wt% Cu. From Figure 9.19 a single tie line does not exist within the ε + L region which intersects the phase boundaries at the given compositions. At 80 wt% Zn, the ε-(ε + L) phase boundary is at about 575°C, whereas at 95 wt% Zn the (ε + L)-L phase boundary is at about 490°C.

9.11 A copper-nickel alloy of composition 70 wt% Ni-30 wt% Cu is slowly heated from a temperature of 1300°C (2370°F).

(a) At what temperature does the first liquid phase form?

(b) What is the composition of this liquid phase?

(c) At what temperature does complete melting of the alloy occur?

(d) What is the composition of the last solid remaining prior to complete melting?

Solution

Shown below is the Cu-Ni phase diagram (Figure 9.3a) and a vertical line constructed at a composition of 70 wt% Ni-30 wt% Cu.

(a)
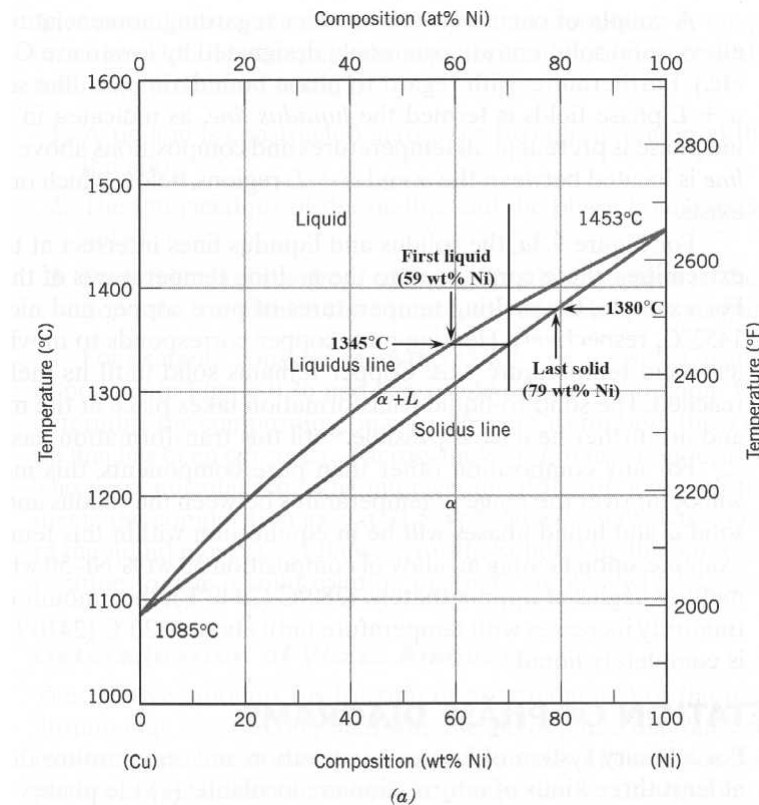
(a) Upon heating from 1300°C, the first liquid phase forms at the temperature at which this vertical line intersects the α-(α + L) phase boundary--i.e., about 1345°C.

(b) The composition of this liquid phase corresponds to the intersection with the (α + L)-L phase boundary, of a tie line constructed across the α + L phase region at 1345°C--I44K ZWL L

(c) Complete melting of the alloy occurs at the intersection of this same vertical line at 70 wt% Ni with the (α + L)-L phase boundary--i.e., about 1380°C;

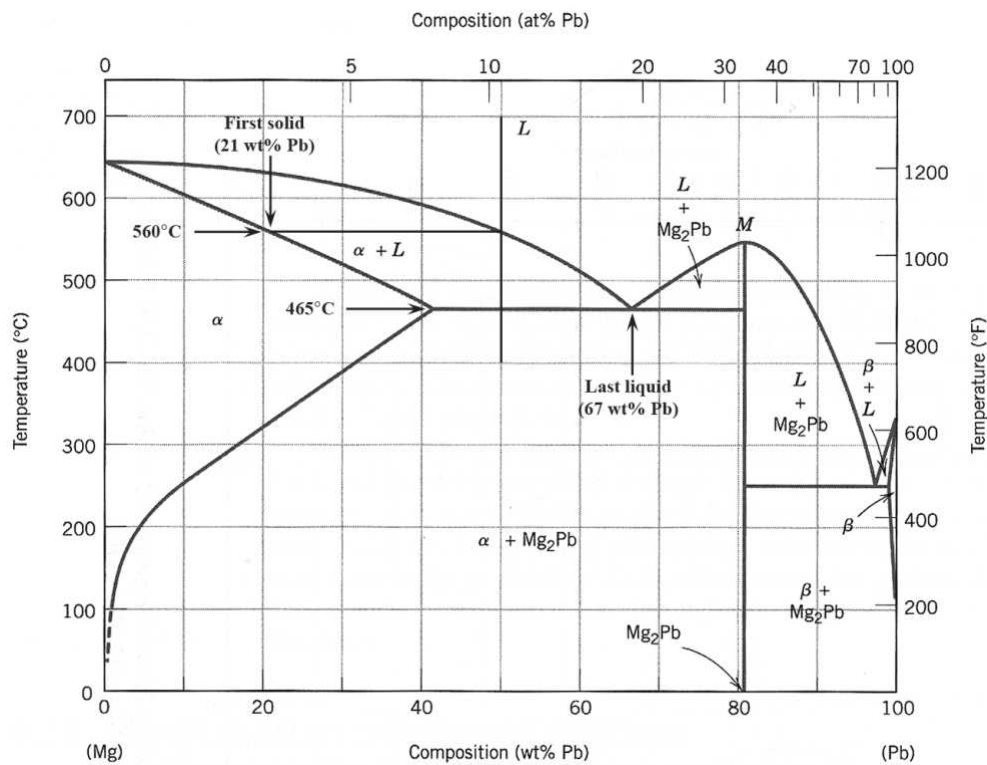(d) The composition of the last solid remaining prior to complete melting corresponds to the intersection with α-$(\alpha + L)$ phase boundary, of the tie line constructed across the $\alpha + L$ phase region at 1380°C--i.e., about 79 wt% Ni.

9.12 A 50 wt% Pb-50 wt% Mg alloy is slowly cooled from 700°C (1290°F) to 400°C (750°F).

(a) At what temperature does the first solid phase form?

(b) What is the composition of this solid phase?

(c) At what temperature does the liquid solidify?

(d) What is the composition of this last remaining liquid phase?

Solution

Shown below is the Mg-Pb phase diagram (Figure 9.20) and a vertical line constructed at a composition of 50 wt% Pb-50 wt% Mg.


(a) Upon cooling from 700°C, the first solid phase forms at the temperature at which a vertical line at this composition intersects the L-($\alpha + L$) phase boundary--i.e., about 560°C;

(b) The composition of this solid phase corresponds to the intersection with the α-($\alpha + L$) phase boundary, of a tie line constructed across the $\alpha + L$ phase region at 560°C--i.e., 21 wt% Pb-→ ‌Z‌ ‍ 0 J

(c) Complete solidification of the alloy occurs at the intersection of this same vertical line at 50 wt% Pb with the eutectic isotherm--i.e., about 465°C;

(d) The composition of the last liquid phase remaining prior to complete solidification corresponds to the eutectic composition--i.e., about 67 wt% Pb-33 wt% Mg.

9.13 For an alloy of composition 74 wt% Zn-26 wt% Cu, cite the phases present and their compositions at the following temperatures: 850°C, 750°C, 680°C, 600°C, and 500°C.

<u>Solution</u>

This problem asks us to determine the phases present and their concentrations at several temperatures, for an alloy of composition 74 wt% Zn-26 wt% Cu. From Figure 9.19 (the Cu-Zn phase diagram), which is shown below with a vertical line constructed at the specified composition:
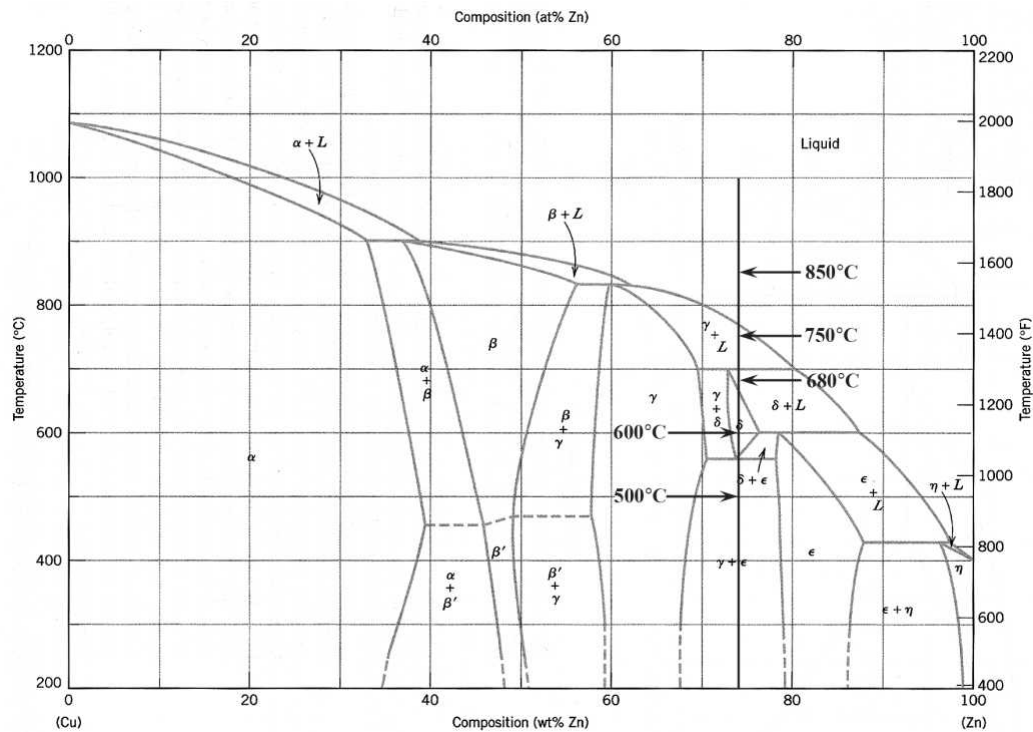
At 850°C, a liquid phase is present; C_L = 74 wt% Zn-26 wt% Cu

At 750°C, γ and liquid phases are present; C_γ = 67 wt% Zn-⊤⊤ Z ◐ &X C_L = 77 wt% Zn-23 wt% Cu

At 680°C, δ and liquid phases are present; C_δ = 73 wt% Zn-└→ Z ◐ &X C_L = 82 wt% Zn-18 wt% Cu

At 600°C, the δ phase is present; C_δ = 74 wt% Zn-26 wt% Cu

At 500°C, γ and ε phases are present; C_γ = 69 wt% Zn-⊤¶ Z ◐ &X C_ε = 78 wt% Zn-22 wt% Cu

9.14   Determine the relative amounts (in terms of mass fractions) of the phases for the alloys and temperatures given in Problem 9.8.

Solution

This problem asks that we determine the phase mass fractions for the alloys and temperatures in Problem 9.8.

(a) From Problem 9.8a, ε and η phases are present for a 90 wt% Zn-10 wt% Cu alloy at 400°C, as represented in the portion of the Cu-Zn phase diagram shown below (at point A).
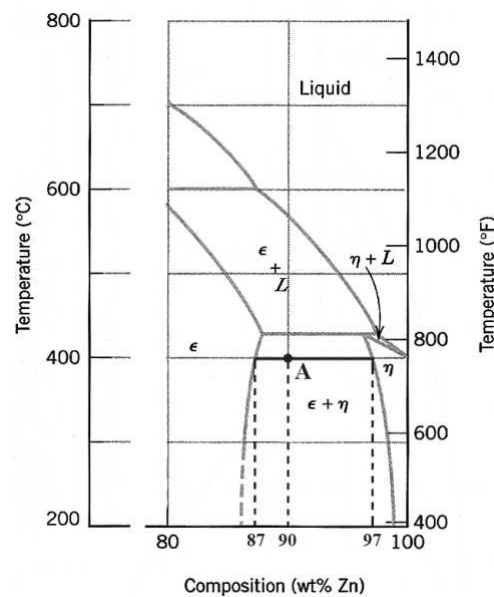
Furthermore, the compositions of the phases, as determined from the tie line are

C_ε = 87 wt% Zn-13 wt% Cu

C_η = 97 wt% Zn-3 wt% Cu

Inasmuch as the composition of the alloy C_0 = 90 wt% Zn, application of the appropriate lever rule expressions (for compositions in weight percent zinc) leads to

$$W_\varepsilon = \frac{C_\eta - C_0}{C_\eta - C_\varepsilon} = \frac{97 - 90}{97 - 87} = 0.70$$

$$W_\eta = \frac{C_0 - C_\varepsilon}{C_\eta - C_\varepsilon} = \frac{90 - 87}{97 - 87} = 0.30$$

(b) From Problem 9.8b, α and β phases are present for a 75 wt% Sn-25 wt% Pb alloy at 175°C, as represented in the portion of the Pb-Sn phase diagram shown below (at point B).

Furthermore, the compositions of the phases, as determined from the tie line are

C_α = 16 wt% Sn-84 wt% Pb
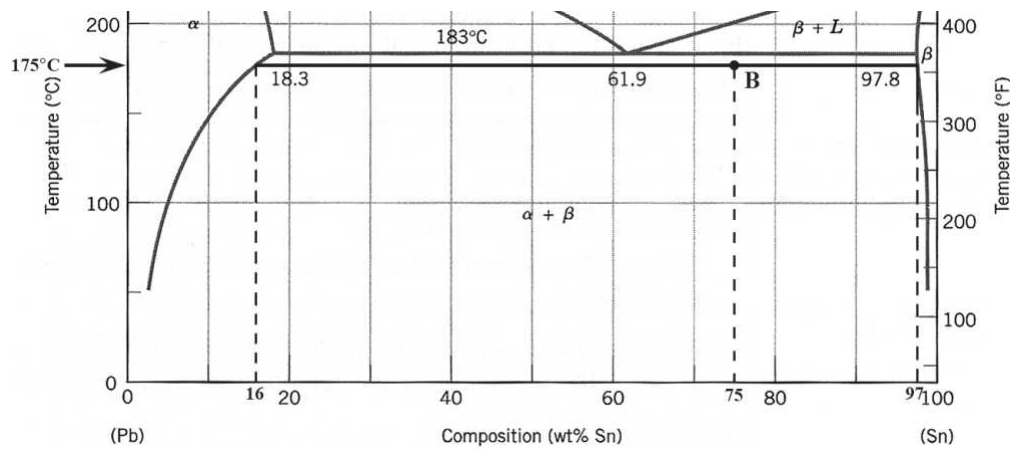
C_β = 97 wt% Sn-3 wt% Pb

Inasmuch as the composition of the alloy C_0 = 75 wt% Sn, application of the appropriate lever rule expressions (for compositions in weight percent tin) leads to

$$W_\alpha = \frac{C_\beta - C_0}{C_\beta - C_\alpha} = \frac{97 - 75}{97 - 16} = 0.27$$

$$W_\beta = \frac{C_0 - C_\alpha}{C_\beta - C_\alpha} = \frac{75 - 16}{97 - 16} = 0.73$$

(c) From Problem 9.8c, just the liquid phase is present for a 55 wt% Ag-45 wt% Cu alloy at 900°C, as may be noted in the Ag-Cu phase diagram shown below (at point C)—i.e., W_L = 1.0
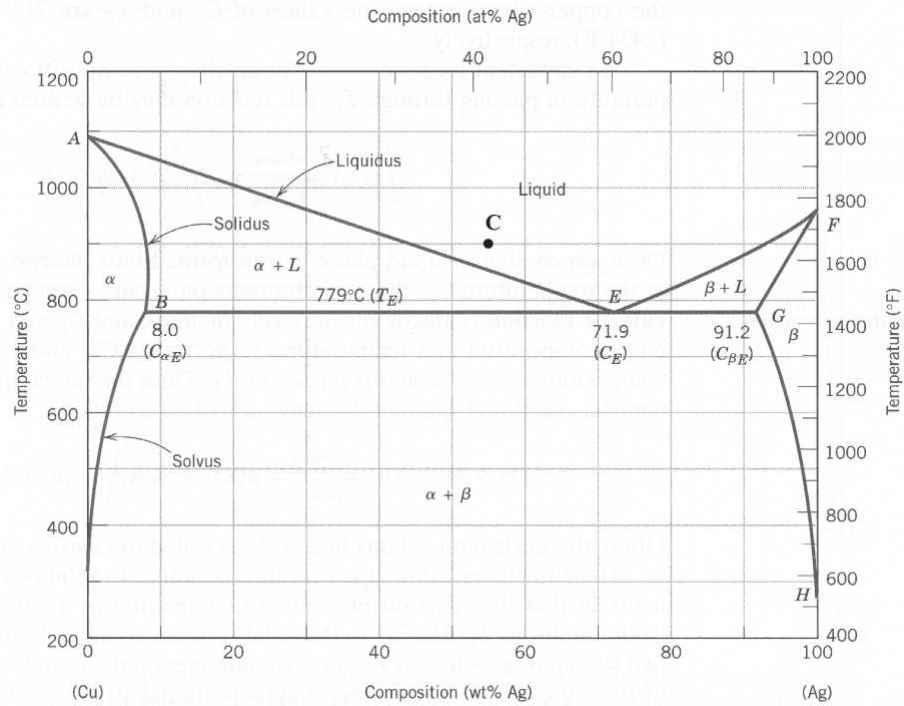
(d) From Problem 9.8d, just the α phase is present for a 30 wt% Pb-70 wt% Mg alloy at 425°C, as may be noted in the Mg-Pb phase diagram shown below (at point D)—i.e., $W_\alpha = 1.0$
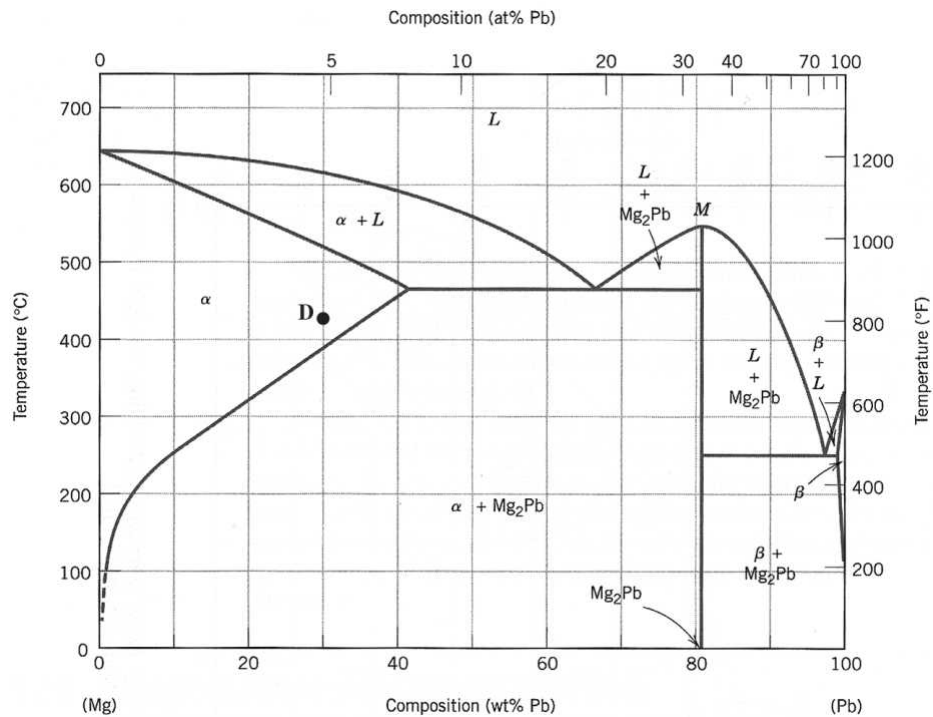
(e) From Problem 9.8e, β and γ phases are present for an alloy composed of 2.12 kg Zn and 1.88 kg Cu (i.e., of composition 53 wt% Zn-47 wt% Cu) at 500°C. This is represented in the portion of the Cu-Zn phase diagram shown below (at point E).

Furthermore, the compositions of the phases, as determined from the tie line are

C_β = 49 wt% Zn-51 wt% Cu

C_γ = 58 wt% Zn-42 wt% Cu

Inasmuch as the composition of the alloy C_0 = 53 wt% Zn and application of the appropriate lever rule expressions (for compositions in weight percent zinc) leads to
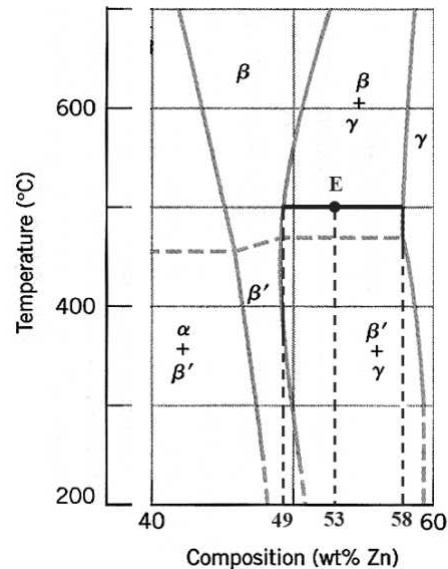
$$W_\beta = \frac{C_\gamma - C_0}{C_\gamma - C_\beta} = \frac{58 - 53}{58 - 49} = 0.56$$

$$W_\gamma = \frac{C_0 - C_\beta}{C_\gamma - C_\beta} = \frac{53 - 49}{58 - 49} = 0.44$$

(f) From Problem 9.8f, L and Mg_2Pb phases are present for an alloy composed of 37 lb$_m$ Pb and 6.5 lb$_m$ Mg (85 wt% Pb-15 wt% Mg) at 400°C. This is represented in the portion of the Pb-Mg phase diagram shown below (at point F).

Composition (wt% Pb)

Furthermore, the compositions of the phases, as determined from the tie line are
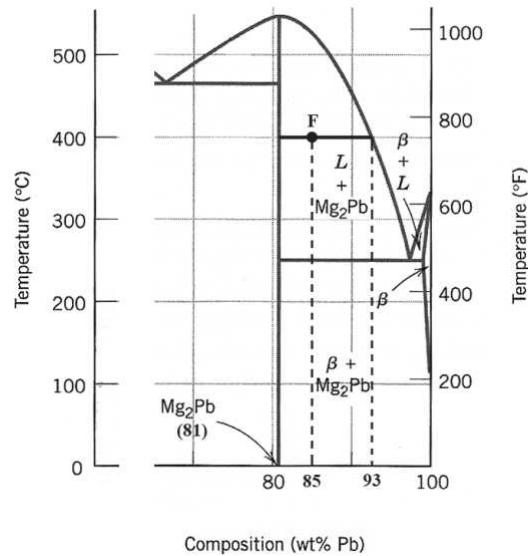
$$C_{Mg_2Pb} = 81 \text{ wt\% Pb-19 wt\% Mg}$$

$$C_L = 93 \text{ wt\% Pb-7 wt\% Mg}$$

Inasmuch as the composition of the alloy $C_0 = 85$ wt% Pb and application of the appropriate lever rule expressions (for compositions in weight percent lead) leads to

$$W_{Mg_2Pb} = \frac{C_L - C_0}{C_L - C_{Mg_2Pb}} = \frac{93 - 85}{93 - 81} = 0.67$$

$$W_L = \frac{C_0 - C_{Mg_2Pb}}{C_L - C_{Mg_2Pb}} = \frac{85 - 81}{93 - 81} = 0.33$$

(g) From Problem 9.8g, just the α phase is present (i.e., $W_\alpha = 1.0$) for an alloy composed of 8.2 mol Ni and 4.3 mol Cu (i.e., 63.8 wt% Ni-36.2 wt% Cu) at 1250°C; such may be noted (as point G) in the Cu-Ni phase diagram shown below.

Composition (at% Ni)

(a)
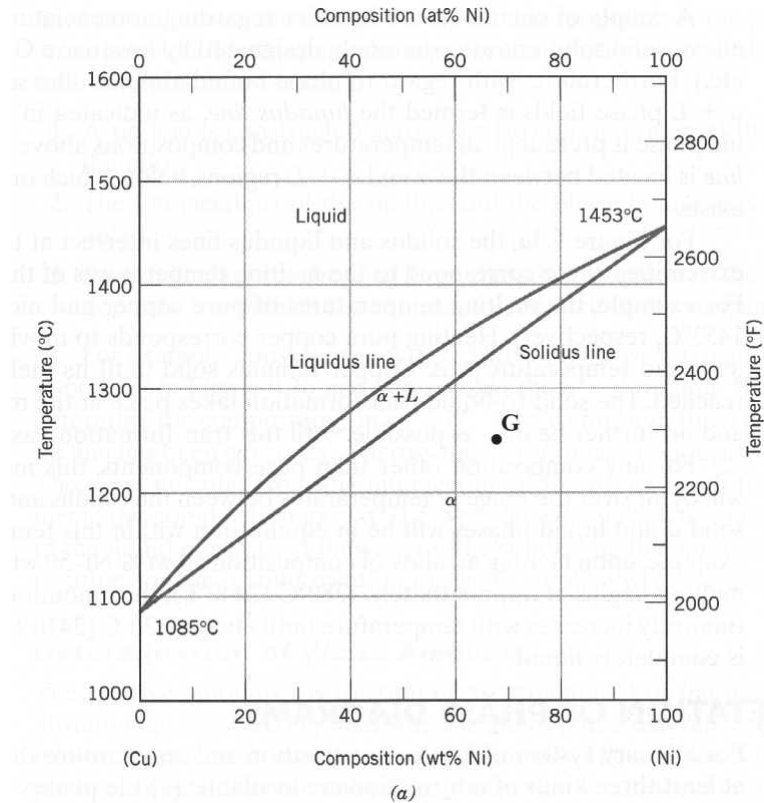
(h) From Problem 9.8h, β and L phases are present for an alloy composed of 4.5 mol Sn and 0.45 mol Pb (85.1 wt% Sn-14.9 wt% Pb) and at 200°C. This is represented in the portion of the Pb-Sn phase diagram shown below (at point H).

Composition (wt% Sn)

Furthermore, the compositions of the phases, as determined from the tie line are

$C_\beta = 97.5$ wt% Sn-2.5 wt% Pb

$C_L = 74$ wt% Sn-26 wt% Pb

Inasmuch as the composition of the alloy $C_0 = 85.1$ wt% Sn, application of the appropriate lever rule expressions (for compositions in weight percent lead) leads to

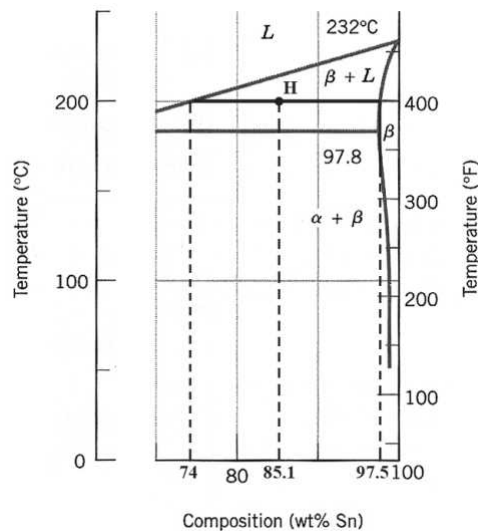$$W_\beta = \frac{C_0 - C_L}{C_\beta - C_L} = \frac{85.1 - 74}{97.5 - 74} = 0.47$$

$$W_L = \frac{C_\beta - C_0}{C_\beta - C_L} = \frac{97.5 - 85.1}{97.5 - 74} = 0.53$$

9.15 A 1.5-kg specimen of a 90 wt% Pb–10 wt% Sn alloy is heated to 250°C (480°F); at this temperature it is entirely an α-phase solid solution (Figure 9.8). The alloy is to be melted to the extent that 50% of the specimen is liquid, the remainder being the α phase. This may be accomplished either by heating the alloy or changing its composition while holding the temperature constant.

(a) To what temperature must the specimen be heated?

(b) How much tin must be added to the 1.5-kg specimen at 250°C to achieve this state?

Solution

(a) Probably the easiest way to solve this part of the problem is by trial and error--that is, on the Pb-Sn phase diagram (Figure 9.8), moving vertically at the given composition, through the α + L region until the tie-line lengths on both sides of the given composition are the same. This occurs at approximately 295°C (560°F).

(b) We can also produce a 50% liquid solution at 250°C, by adding Sn to the alloy. At 250°C and within the α + L phase region

$$C_\alpha = 14 \text{ wt\% Sn-86 wt\% Pb}$$
$$C_L = 34 \text{ wt\% Sn-66 wt\% Pb}$$

Let C_0 be the new alloy composition to give $W_\alpha = W_L = 0.5$. Then,

$$W_\alpha = 0.5 = \frac{C_L - C_0}{C_L - C_\alpha} = \frac{34 - C_0}{34 - 14}$$

And solving for C_0 gives 24 wt% Sn. Now, let m_{Sn} be the mass of Sn added to the alloy to achieve this new composition. The amount of Sn in the original alloy is

$$(0.10)(1.5 \text{ kg}) = 0.15 \text{ kg}$$

Then, using a modified form of Equation 4.3

$$\left[\frac{0.15 \text{ kg} + m_{Sn}}{1.5 \text{ kg} + m_{Sn}} \right] \times 100 = 24$$

And, solving for m_{Sn} (the mass of tin to be added), yields $m_{Sn} = 0.276$ kg.

9.16   A magnesium-lead alloy of mass 5.5 kg consists of a solid α phase that has a composition that is just slightly below the solubility limit at 200°C (390°F).

(a) What mass of lead is in the alloy?

(b) If the alloy is heated to 350°C (660°F), how much more lead may be dissolved in the α phase without exceeding the solubility limit of this phase?

Solution

(a) This portion of the problem asks that we calculate, for a Pb-Mg alloy, the mass of lead in 5.5 kg of the solid α phase at 200°C just below the solubility limit. From Figure 9.20, the solubility limit for the α phase at 200°C corresponds to the position (composition) of the α-α + Mg_2Pb phase boundary at this temperature, which is about 5 wt% Pb. Therefore, the mass of Pb in the alloy is just $(0.05)(5.5 \text{ kg}) = 0.28$ kg.

(b) At 350°C, the solubility limit of the α phase increases to approximately 25 wt% Pb. In order to determine the additional amount of Pb that may be added (m_{Pb}), we utilize a modified form of Equation 4.3 as

$$C_{Pb} = 25 \text{ wt\%} = \frac{0.28 \text{ kg} + m_{Pb}}{5.5 \text{ kg} + m_{Pb}} \times 100$$

Solving for m_{Pb} yields $m_{Pb} = 1.46$ kg.

9.17 A 90 wt% Ag-10 wt% Cu alloy is heated to a temperature within the β + liquid phase region. If the composition of the liquid phase is 85 wt% Ag, determine:

    (a) The temperature of the alloy

    (b) The composition of the β phase

    (c) The mass fractions of both phases

    Solution

(a) In order to determine the temperature of a 90 wt% Ag-10 wt% Cu alloy for which β and liquid phases are present with the liquid phase of composition 85 wt% Ag, we need to construct a tie line across the β + L phase region of Figure 9.7 that intersects the liquidus line at 85 wt% Ag; this point lies about 800°C.

(b) The composition of the β phase at this temperature is determined from the intersection of this same tie line with solidus line, which corresponds to about 95 wt% Ag.

(c) The mass fractions of the two phases are determined using the lever rule, Equations 9.1 and 9.2 with C_0 = 90 wt% Ag, C_L = 85 wt% Ag, and C_β = 95 wt% Ag, as

$$W_\beta = \frac{C_0 - C_L}{C_\beta - C_L} = \frac{90 - 85}{95 - 85} = 0.50$$

$$W_L = \frac{C_\beta - C_0}{C_\beta - C_L} = \frac{95 - 90}{95 - 85} = 0.50$$

9.18   A 30 wt% Sn-70 wt% Pb alloy is heated to a temperature within the α + liquid phase region. If the mass fraction of each phase is 0.5, estimate:

(a) The temperature of the alloy

(b) The compositions of the two phases

<u>Solution</u>

(a) We are given that the mass fractions of α and liquid phases are both 0.5 for a 30 wt% Sn-70 wt% Pb alloy and asked to estimate the temperature of the alloy. Using the appropriate phase diagram, Figure 9.8, by trial and error with a ruler, a tie line within the α + L phase region that is divided in half for an alloy of this composition exists at about 230°C.

(b) We are now asked to determine the compositions of the two phases. This is accomplished by noting the intersections of this tie line with both the solidus and liquidus lines. From these intersections, C_α = 15 wt% Sn, and C_L = 43 wt% Sn.

9.19 For alloys of two hypothetical metals A and B, there exist an α, A-rich phase and a β, B-rich phase. From the mass fractions of both phases for two different alloys provided in the table below, (which are at the same temperature), determine the composition of the phase boundary (or solubility limit) for both α and β phases at this temperature.

Alloy Composition	Fraction α Phase	Fraction β Phase
60 wt% A–40 wt% B	0.57	0.43
30 wt% A–70 wt% B	0.14	0.86

<u>Solution</u>

The problem is to solve for compositions at the phase boundaries for both α and β phases (i.e., C_α and C_β). We may set up two independent lever rule expressions, one for each composition, in terms of C_α and C_β as follows:

$$W_{\alpha 1} = 0.57 = \frac{C_\beta - C_{01}}{C_\beta - C_\alpha} = \frac{C_\beta - 60}{C_\beta - C_\alpha}$$

$$W_{\alpha 2} = 0.14 = \frac{C_\beta - C_{02}}{C_\beta - C_\alpha} = \frac{C_\beta - 30}{C_\beta - C_\alpha}$$

In these expressions, compositions are given in wt% of A. Solving for C_α and C_β from these equations, yield

$$C_\alpha = 90 \text{ (or 90 wt\% A-10 wt\% B)}$$

$$C_\beta = 20.2 \text{ (or 20.2 wt\% A-79.8 wt\% B)}$$

9.20 A hypothetical A–B alloy of composition 55 wt% B–45 wt% A at some temperature is found to consist of mass fractions of 0.5 for both α and β phases. If the composition of the β phase is 90 wt% B–10 wt% A, what is the composition of the α phase?

<u>Solution</u>

For this problem, we are asked to determine the composition of the β phase given that

$$C_0 = 55 \text{ (or 55 wt\% B-45 wt\% A)}$$

$$C_\beta = 90 \text{ (or 90 wt\% B-10 wt\% A)}$$

$$W_\alpha = W_\beta = 0.5$$

If we set up the lever rule for W_α

$$W_\alpha = 0.5 = \frac{C_\beta - C_0}{C_\beta - C_\alpha} = \frac{90 - 55}{90 - C_\alpha}$$

And solving for C_α

$$C_\alpha = 20 \text{ (or 20 wt\% B-80 wt\% A)}$$

9.21 Is it possible to have a copper-silver alloy of composition 50 wt% Ag-50 wt% Cu, which, at equilibrium, consists of α and β phases having mass fractions $W_\alpha = 0.60$ and $W_\beta = 0.40$? If so, what will be the approximate temperature of the alloy? If such an alloy is not possible, explain why.

Solution

It is not possible to have a Cu-Ag alloy of composition 50 wt% Ag-50 wt% Cu which consists of mass fractions $W_\alpha = 0.60$ and $W_\beta = 0.40$. Using the appropriate phase diagram, Figure 9.7, and, using Equations 9.1 and 9.2 let us determine W_α and W_β at just below the eutectic temperature and also at room temperature. At just below the eutectic, $C_\alpha = 8.0$ wt% Ag and $C_\beta = 91.2$ wt% Ag; thus

$$W_\alpha = \frac{C_\beta - C_0}{C_\beta - C_\alpha} = \frac{91.2 - 50}{91.2 - 8} = 0.50$$

$$W_\beta = 1.00 - W_\alpha = 1.00 - 0.50 = 0.50$$

Furthermore, at room temperature, $C_\alpha = 0$ wt% Ag and $C_\beta = 100$ wt% Ag; employment of Equations 9.1 and 9.2 yields

$$W_\alpha = \frac{C_\beta - C_0}{C_\beta - C_\alpha} = \frac{100 - 50}{100 - 0} = 0.50$$

And, $W_\beta = 0.50$. Thus, the mass fractions of the α and β phases, upon cooling through the α + β phase region will remain approximately constant at about 0.5, and will never have values of $W_\alpha = 0.60$ and $W_\beta = 0.40$ as called for in the problem.

9.22 For 11.20 kg of a magnesium-lead alloy of composition 30 wt% Pb-70 wt% Mg, is it possible, at equilibrium, to have α and Mg_2Pb phases having respective masses of 7.39 kg and 3.81 kg? If so, what will be the approximate temperature of the alloy? If such an alloy is not possible, explain why.

Solution

Yes, it is possible to have a 30 wt% Pb-70 wt% Mg alloy which has masses of 7.39 kg and 3.81 kg for the α and Mg_2Pb phases, respectively. In order to demonstrate this, it is first necessary to determine the mass fraction of each phase as follows:

$$W_\alpha = \frac{m_\alpha}{m_\alpha + m_{Mg_2Pb}} = \frac{7.39 \text{ kg}}{7.39 \text{ kg} + 3.81 \text{ kg}} = 0.66$$

$$W_{Mg_2Pb} = 1.00 - 0.66 = 0.34$$

Now, if we apply the lever rule expression for W_α

$$W_\alpha = \frac{C_{Mg_2Pb} - C_0}{C_{Mg_2Pb} - C_\alpha}$$

Since the Mg_2Pb phase exists only at 81 wt% Pb, and $C_0 = 30$ wt% Pb

$$W_\alpha = 0.66 = \frac{81 - 30}{81 - C_\alpha}$$

Solving for C_α from this expression yields $C_\alpha = 3.7$ wt% Pb. The position along the α–($\alpha + Mg_2Pb$) phase boundary of Figure 9.20 corresponding to this composition is approximately 190°C.

9.23 Derive Equations 9.6a and 9.7a, which may be used to convert mass fraction to volume fraction, and vice versa.

<u>Solution</u>

This portion of the problem asks that we derive Equation 9.6a, which is used to convert from phase weight fraction to phase volume fraction. Volume fraction of phase α, V_α, is defined by Equation 9.5 as

$$V_\alpha = \frac{v_\alpha}{v_\alpha + v_\beta} \tag{9.S1}$$

where v_α and v_β are the volumes of the respective phases in the alloy. Furthermore, the density of each phase is equal to the ratio of its mass and volume, or upon rearrangement

$$v_\alpha = \frac{m_\alpha}{\rho_\alpha} \tag{9.S2a}$$

$$v_\beta = \frac{m_\beta}{\rho_\beta} \tag{9.S2b}$$

Substitution of these expressions into Equation 9.S1 leads to

$$V_\alpha = \frac{\dfrac{m_\alpha}{\rho_\alpha}}{\dfrac{m_\alpha}{\rho_\alpha} + \dfrac{m_\beta}{\rho_\beta}} \tag{9.S3}$$

in which m's and ρ's denote masses and densities, respectively. Now, the mass fractions of the α and β phases (i.e., W_α and W_β) are defined in terms of the phase masses as

$$W_\alpha = \frac{m_\alpha}{m_\alpha + m_\beta} \tag{9.S4a}$$

$$W_\beta = \frac{m_\beta}{m_\alpha + m_\beta} \tag{9.S4b}$$

Which, upon rearrangement yield

$$m_\alpha = W_\alpha \left(m_\alpha + m_\beta \right) \tag{9.S5a}$$

$$m_\beta = W_\beta \left(m_\alpha + m_\beta \right) \tag{9.S5b}$$

Incorporation of these relationships into Equation 9.S3 leads to

$$V_\alpha = \frac{\dfrac{W_\alpha \left(m_\alpha + m_\beta \right)}{\rho_\alpha}}{\dfrac{W_\alpha \left(m_\alpha + m_\beta \right)}{\rho_\alpha} + \dfrac{W_\beta \left(m_\alpha + m_\beta \right)}{\rho_\beta}}$$

$$V_\alpha = \frac{\dfrac{W_\alpha}{\rho_\alpha}}{\dfrac{W_\alpha}{\rho_\alpha} + \dfrac{W_\beta}{\rho_\beta}} \tag{9.S6}$$

which is the desired equation.

For this portion of the problem we are asked to derive Equation 9.7a, which is used to convert from phase volume fraction to mass fraction. Mass fraction of the α phase is defined as

$$W_\alpha = \frac{m_\alpha}{m_\alpha + m_\beta} \tag{9.S7}$$

From Equations 9.S2a and 9.S2b

$$m_\alpha = v_\alpha \rho_\alpha \tag{9.S8a}$$

$$m_\beta = v_\beta \rho_\beta \tag{9.S8b}$$

Substitution of these expressions into Equation 9.S7 yields

$$W_\alpha = \frac{v_\alpha \rho_\alpha}{v_\alpha \rho_\alpha + v_\beta \rho_\beta} \tag{9.S9}$$

From Equation 9.5 and its equivalent for V_β the following may be written:

$$v_\alpha = V_\alpha(v_\alpha + v_\beta) \tag{9.S10a}$$

$$v_\beta = V_\beta(v_\alpha + v_\beta) \tag{9.S10b}$$

Substitution of Equations 9.S10a and 9.S10b into Equation 9.S9 yields

$$W_\alpha = \frac{V_\alpha(v_\alpha + v_\beta)\rho_\alpha}{V_\alpha(v_\alpha + v_\beta)\rho_\alpha + V_\beta(v_\alpha + v_\beta)\rho_\beta}$$

$$W_\alpha = \frac{V_\alpha\rho_\alpha}{V_\alpha\rho_\alpha + V_\beta\rho_\beta} \tag{9.S11}$$

which is the desired expression.

9.24 Determine the relative amounts (in terms of volume fractions) of the phases for the alloys and temperatures given in Problem 9.8a, b, and c. Below are given the approximate densities of the various metals at the alloy temperatures:

Metal	Temperature (°C)	Density (g/cm³)
Ag	900	9.97
Cu	400	8.77
Cu	900	8.56
Pb	175	11.20
Sn	175	7.22
Zn	400	6.83

Solution

This problem asks that we determine the phase volume fractions for the alloys and temperatures in Problems 9.8a, b, and c. This is accomplished by using the technique illustrated in Example Problem 9.3, and also the results of Problems 9.8 and 9.14.

(a) This is a Cu-Zn alloy at 400°C, wherein

$C_\varepsilon = 87$ wt% Zn-13 wt% Cu

$C_\eta = 97$ wt% Zn-3 wt% Cu

$W_\varepsilon = 0.70$

$W_\eta = 0.30$

$\rho_{Cu} = 8.77$ g/cm³

$\rho_{Zn} = 6.83$ g/cm³

Using these data it is first necessary to compute the densities of the ε and η phases using Equation 4.10a. Thus

$$\rho_\varepsilon = \frac{100}{\dfrac{C_{Zn(\varepsilon)}}{\rho_{Zn}} + \dfrac{C_{Cu(\varepsilon)}}{\rho_{Cu}}}$$

$$= \frac{100}{\dfrac{87}{6.83 \ \text{g/cm}^3} + \dfrac{13}{8.77 \ \text{g/cm}^3}} = 7.03 \ \text{g/cm}^3$$

$$\rho_\eta = \frac{100}{\dfrac{C_{Zn(\eta)}}{\rho_{Zn}} + \dfrac{C_{Cu(\eta)}}{\rho_{Cu}}}$$

$$= \frac{100}{\dfrac{97}{6.83 \ \text{g/cm}^3} + \dfrac{3}{8.77 \ \text{g/cm}^3}} = 6.88 \ \text{g/cm}^3$$

Now we may determine the V_ε and V_η values using Equation 9.6. Thus,

$$V_\varepsilon = \frac{\dfrac{W_\varepsilon}{\rho_\varepsilon}}{\dfrac{W_\varepsilon}{\rho_\varepsilon} + \dfrac{W_\eta}{\rho_\eta}}$$

$$= \frac{\dfrac{0.70}{7.03 \ \text{g/cm}^3}}{\dfrac{0.70}{7.03 \ \text{g/cm}^3} + \dfrac{0.30}{6.88 \ \text{g/cm}^3}} = 0.70$$

$$V_\eta = \frac{\dfrac{W_\eta}{\rho_\eta}}{\dfrac{W_\varepsilon}{\rho_\varepsilon} + \dfrac{W_\eta}{\rho_\eta}}$$

$$= \frac{\dfrac{0.30}{6.88 \ \text{g/cm}^3}}{\dfrac{0.70}{7.03 \ \text{g/cm}^3} + \dfrac{0.30}{6.88 \ \text{g/cm}^3}} = 0.30$$

(b) This is a Pb-Sn alloy at 175°C, wherein

$C_\alpha = 16$ wt% Sn-84 wt% Pb

$C_\beta = 97$ wt% Sn-3 wt% Pb

$W_\alpha = 0.27$

$W_\beta = 0.73$

$\rho_{Sn} = 7.22 \text{ g/cm}^3$

$\rho_{Pb} = 11.20 \text{ g/cm}^3$

Using this data it is first necessary to compute the densities of the α and β phases. Thus

$$\rho_\alpha = \frac{100}{\dfrac{C_{Sn(\alpha)}}{\rho_{Sn}} + \dfrac{C_{Pb(\alpha)}}{\rho_{Pb}}}$$

$$= \frac{100}{\dfrac{16}{7.22 \text{ g/cm}^3} + \dfrac{84}{11.20 \text{ g/cm}^3}} = 10.29 \text{ g/cm}^3$$

$$\rho_\beta = \frac{100}{\dfrac{C_{Sn(\beta)}}{\rho_{Sn}} + \dfrac{C_{Pb(\beta)}}{\rho_{Pb}}}$$

$$= \frac{100}{\dfrac{97}{7.22 \text{ g/cm}^3} + \dfrac{3}{11.20 \text{ g/cm}^3}} = 7.30 \text{ g/cm}^3$$

Now we may determine the V_α and V_β values using Equation 9.6. Thus,

$$V_\alpha = \frac{\dfrac{W_\alpha}{\rho_\alpha}}{\dfrac{W_\alpha}{\rho_\alpha} + \dfrac{W_\beta}{\rho_\beta}}$$

$$= \frac{\dfrac{0.27}{10.29 \text{ g/cm}^3}}{\dfrac{0.27}{10.29 \text{ g/cm}^3} + \dfrac{0.73}{7.30 \text{ g/cm}^3}} = 0.21$$

$$V_\beta = \frac{\dfrac{W_\beta}{\rho_\beta}}{\dfrac{W_\alpha}{\rho_\alpha} + \dfrac{W_\beta}{\rho_\beta}}$$

$$= \frac{\dfrac{0.73}{7.30 \ \text{g/cm}^3}}{\dfrac{0.27}{10.29 \ \text{g/cm}^3} + \dfrac{0.73}{7.30 \ \text{g/cm}^3}} = 0.79$$

(c) This is a Ag-Cu alloy at 900°C, wherein only the liquid phase is present. Therefore, $V_L = 1.0$.

Development of Microstructure in Isomorphous Alloys

9.25 (a) Briefly describe the phenomenon of coring and why it occurs.

(b) Cite one undesirable consequence of coring.

<u>Solution</u>

(a) Coring is the phenomenon whereby concentration gradients exist across grains in polycrystalline alloys, with higher concentrations of the component having the lower melting temperature at the grain boundaries. It occurs, during solidification, as a consequence of cooling rates that are too rapid to allow for the maintenance of the equilibrium composition of the solid phase.

(b) One undesirable consequence of a cored structure is that, upon heating, the grain boundary regions will melt first and at a temperature below the equilibrium phase boundary from the phase diagram; this melting results in a loss in mechanical integrity of the alloy.

Mechanical Properties of Isomorphous Alloys

9.26 It is desirable to produce a copper-nickel alloy that has a minimum noncold-worked tensile strength of 350 MPa (50,750 psi) and a ductility of at least 48%EL. Is such an alloy possible? If so, what must be its composition? If this is not possible, then explain why.

Solution

From Figure 9.6a, a tensile strength greater than 350 MPa (50,750 psi) is possible for compositions between about 22.5 and 98 wt% Ni. On the other hand, according to Figure 9.6b, ductilities greater than 48%EL exist for compositions less than about 8 wt% and greater than about 98 wt% Ni. Therefore, the stipulated criteria are met only at a composition of 98 wt% Ni.

Binary Eutectic Systems

9.27   A 45 wt% Pb–55 wt% Mg alloy is rapidly quenched to room temperature from an elevated temperature in such a way that the high-temperature microstructure is preserved. This microstructure is found to consist of the α phase and Mg_2Pb, having respective mass fractions of 0.65 and 0.35. Determine the approximate temperature from which the alloy was quenched.

Solution

We are asked to determine the approximate temperature from which a 45 wt% Pb-55 wt% Mg alloy was quenched, given the mass fractions of α and Mg_2Pb phases. We can write a lever-rule expression for the mass fraction of the α phase as

$$W_\alpha = 0.65 = \frac{C_{Mg_2Pb} - C_0}{C_{Mg_2Pb} - C_\alpha}$$

The value of C_0 is stated as 45 wt% Pb-55 wt% Mg, and C_{Mg_2Pb} is 81 wt% Pb-19 wt% Mg, which is independent of temperature. Therefore,

$$0.65 = \frac{81 - 45}{81 - C_\alpha}$$

which yields

$$C_\alpha = 25.6 \text{ wt\% Pb}$$

The temperature at which the α–(α + Mg_2Pb) phase boundary (Figure 9.20) has a value of 25.6 wt% Pb is about 360°C (680°F).

Development of Microstructure in Eutectic Alloys

9.28   Briefly explain why, upon solidification, an alloy of eutectic composition forms a microstructure consisting of alternating layers of the two solid phases.

<u>Solution</u>

Upon solidification, an alloy of eutectic composition forms a microstructure consisting of alternating layers of the two solid phases because during the solidification atomic diffusion must occur, and with this layered configuration the diffusion path length for the atoms is a minimum.

9.29 What is the difference between a phase and a microconstituent?

<u>Solution</u>

A "phase" is a homogeneous portion of the system having uniform physical and chemical characteristics, whereas a "microconstituent" is an identifiable element of the microstructure (that may consist of more than one phase).

9.30 Is it possible to have a copper-silver alloy in which the mass fractions of primary β and total β are 0.68 and 0.925, respectively, at 775°C (1425°F)? Why or why not?

Solution

This problem asks if it is possible to have a Cu-Ag alloy for which the mass fractions of primary β and total β are 0.68 and 0.925, respectively at 775°C. In order to make this determination we need to set up the appropriate lever rule expression for each of these quantities. From Figure 9.7 and at 775°C, $C_\alpha = 8.0$ wt% Ag, $C_\beta = 91.2$ wt% Ag, and $C_{eutectic} = 71.9$ wt% Ag.

For primary β

$$W_{\beta'} = \frac{C_0 - C_{eutectic}}{C_\beta - C_{eutectic}} = \frac{C_0 - 71.9}{91.2 - 71.9} = 0.68$$

Solving for C_0 gives $C_0 = 85$ wt% Ag.

Now the analogous expression for total β

$$W_\beta = \frac{C_0 - C_\alpha}{C_\beta - C_\alpha} = \frac{C_0 - 8.0}{91.2 - 8.0} = 0.925$$

And this value of C_0 is 85 wt% Ag. Therefore, since these two C_0 values are the same (85 wt% Ag), this alloy is possible.

9.31 For 6.70 kg of a magnesium-lead alloy, is it possible to have the masses of primary α and total α of 4.23 kg and 6.00 kg, respectively, at 460°C (860°F)? Why or why not?

Solution

This problem asks if it is possible to have a Mg-Pb alloy for which the masses of primary α and total α are 4.23 kg and 6.00 kg, respectively in 6.70 kg total of the alloy at 460°C. In order to make this determination we first need to convert these masses to mass fractions. Thus,

$$W_{\alpha'} = \frac{4.23 \text{ kg}}{6.70 \text{ kg}} = 0.631$$

$$W_{\alpha} = \frac{6.00 \text{ kg}}{6.70 \text{ kg}} = 0.896$$

Next it is necessary to set up the appropriate lever rule expression for each of these quantities. From Figure 9.20 and at 460°C, $C_{\alpha} = 41$ wt% Pb, $C_{Mg_2Pb} = 81$ wt% Pb, and $C_{eutectic} = 66$ wt% Pb

For primary α

$$W_{\alpha'} = \frac{C_{eutectic} - C_0}{C_{eutectic} - C_{\alpha}} = \frac{66 - C_0}{66 - 41} = 0.631$$

And solving for C_0 gives $C_0 = 50.2$ wt% Pb.

Now the analogous expression for total α

$$W_{\alpha} = \frac{C_{Mg_2Pb} - C_0}{C_{Mg_2Pb} - C_{\alpha}} = \frac{81 - C_0}{81 - 41} = 0.896$$

And this value of C_0 is 45.2 wt% Pb. Therefore, since these two C_0 values are different, this alloy is not possible.

9.32 For a copper-silver alloy of composition 25 wt% Ag-75 wt% Cu and at 775°C (1425°F) do the following:

(a) Determine the mass fractions of α and β phases.

(b) Determine the mass fractions of primary α and eutectic microconstituents.

(c) Determine the mass fraction of eutectic α.

<u>Solution</u>

(a) This portion of the problem asks that we determine the mass fractions of α and β phases for an 25 wt% Ag-75 wt% Cu alloy (at 775°C). In order to do this it is necessary to employ the lever rule using a tie line that extends entirely across the α + β phase field. From Figure 9.7 and at 775°C, C_α = 8.0 wt% Ag, C_β = 91.2 wt% Ag, and $C_{eutectic}$ = 71.9 wt% Sn. Therefore, the two lever-rule expressions are as follows:

$$W_\alpha = \frac{C_\beta - C_0}{C_\beta - C_\alpha} = \frac{91.2 - 25}{91.2 - 8.0} = 0.796$$

$$W_\beta = \frac{C_0 - C_\alpha}{C_\beta - C_\alpha} = \frac{25 - 8.0}{91.2 - 8.0} = 0.204$$

(b) Now it is necessary to determine the mass fractions of primary α and eutectic microconstituents for this same alloy. This requires us to utilize the lever rule and a tie line that extends from the maximum solubility of Ag in the α phase at 775°C (i.e., 8.0 wt% Ag) to the eutectic composition (71.9 wt% Ag). Thus

$$W_{\alpha'} = \frac{C_{eutectic} - C_0}{C_{eutectic} - C_\alpha} = \frac{71.9 - 25}{71.9 - 8.0} = 0.734$$

$$W_e = \frac{C_0 - C_\alpha}{C_{eutectic} - C_\alpha} = \frac{25 - 8.0}{71.9 - 8.0} = 0.266$$

(c) And, finally, we are asked to compute the mass fraction of eutectic α, $W_{e\alpha}$. This quantity is simply the difference between the mass fractions of total α and primary α as

$$W_{e\alpha} = W_\alpha - W_{\alpha'} = 0.796 - 0.734 = 0.062$$

9.33 The microstructure of a lead-tin alloy at 180°C (355°F) consists of primary β and eutectic structures. If the mass fractions of these two microconstituents are 0.57 and 0.43, respectively, determine the composition of the alloy.

<u>Solution</u>

Since there is a primary β microconstituent present, then we know that the alloy composition, C_0 is between 61.9 and 97.8 wt% Sn (Figure 9.8). Furthermore, this figure also indicates that C_β = 97.8 wt% Sn and $C_{eutectic}$ = 61.9 wt% Sn. Applying the appropriate lever rule expression for $W_{\beta'}$

$$W_{\beta'} = \frac{C_0 - C_{eutectic}}{C_\beta - C_{eutectic}} = \frac{C_0 - 61.9}{97.8 - 61.9} = 0.57$$

and solving for C_0 yields C_0 = 82.4 wt% Sn.

9.34 Consider the hypothetical eutectic phase diagram for metals A and B, which is similar to that for the lead-tin system, Figure 9.8. Assume that (1) α and β phases exist at the A and B extremities of the phase diagram, respectively; (2) the eutectic composition is 47 wt% B-53 wt% A; and (3) the composition of the β phase at the eutectic temperature is 92.6 wt% B-7.4 wt% A. Determine the composition of an alloy that will yield primary α and total α mass fractions of 0.356 and 0.693, respectively.

Solution

We are given a hypothetical eutectic phase diagram for which $C_{\text{eutectic}} = 47$ wt% B, $C_\beta = 92.6$ wt% B at the eutectic temperature, and also that $W_{\alpha'} = 0.356$ and $W_\alpha = 0.693$; from this we are asked to determine the composition of the alloy. Let us write lever rule expressions for $W_{\alpha'}$ and W_α

$$W_\alpha = \frac{C_\beta - C_0}{C_\beta - C_\alpha} = \frac{92.6 - C_0}{92.6 - C_\alpha} = 0.693$$

$$W_{\alpha'} = \frac{C_{\text{eutectic}} - C_0}{C_{\text{eutectic}} - C_\alpha} = \frac{47 - C_0}{47 - C_\alpha} = 0.356$$

Thus, we have two simultaneous equations with C_0 and C_α as unknowns. Solving them for C_0 gives $C_0 = 32.6$ wt% B.
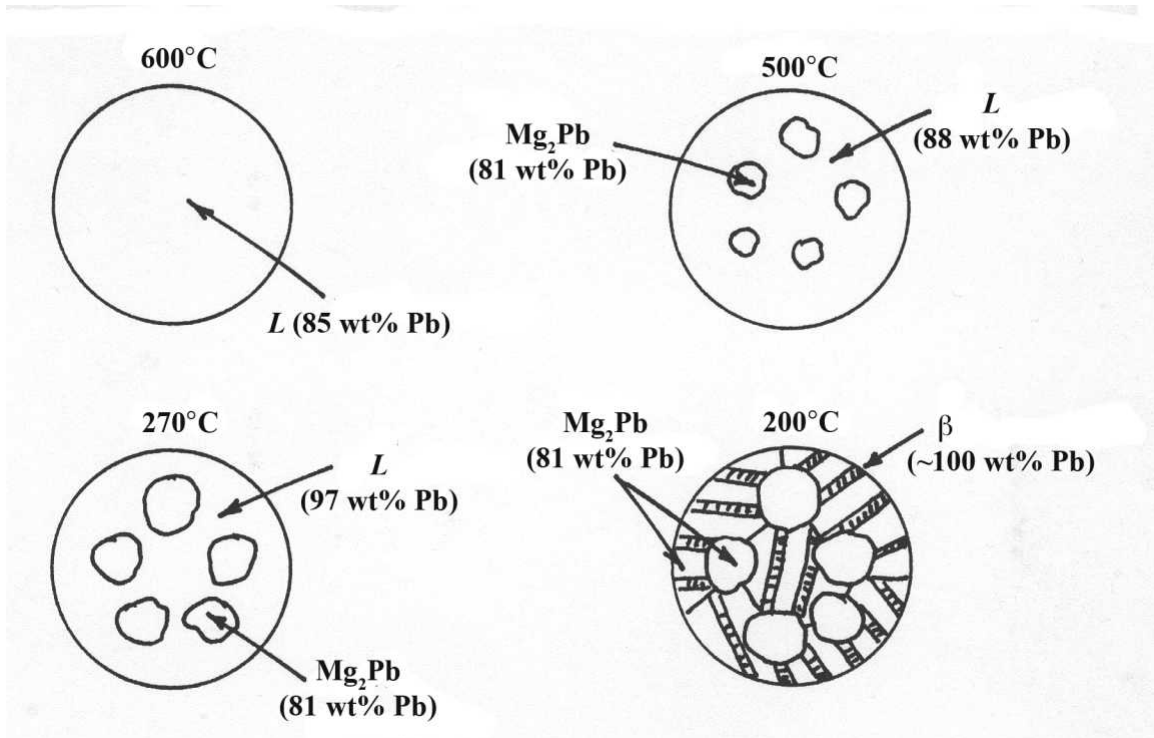
9.35 For an 85 wt% Pb-15 wt% Mg alloy, make schematic sketches of the microstructure that would be observed for conditions of very slow cooling at the following temperatures: 600°C (1110°F), 500°C (930°F), 270°C (520°F), and 200°C (390°F). Label all phases and indicate their approximate compositions.

<u>Solution</u>

The illustration below is the Mg-Pb phase diagram (Figure 9.20). A vertical line at a composition of 85 wt% Pb-15 wt% Mg has been drawn, and, in addition, horizontal arrows at the four temperatures called for in the problem statement (i.e., 600°C, 500°C, 270°C, and 200°C).
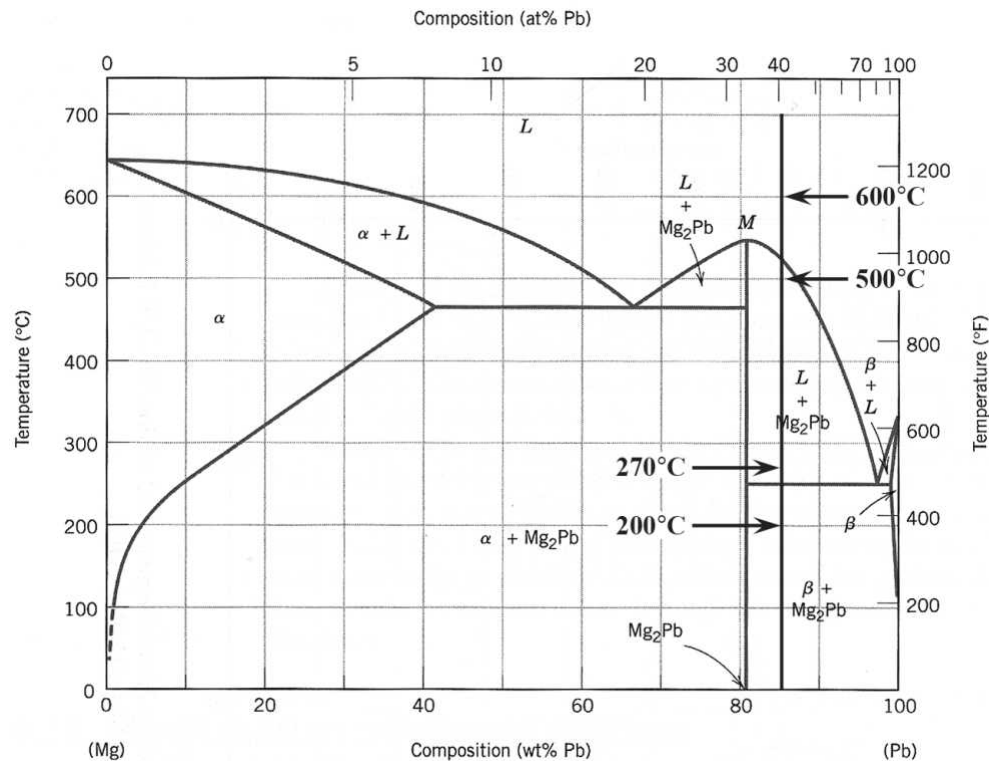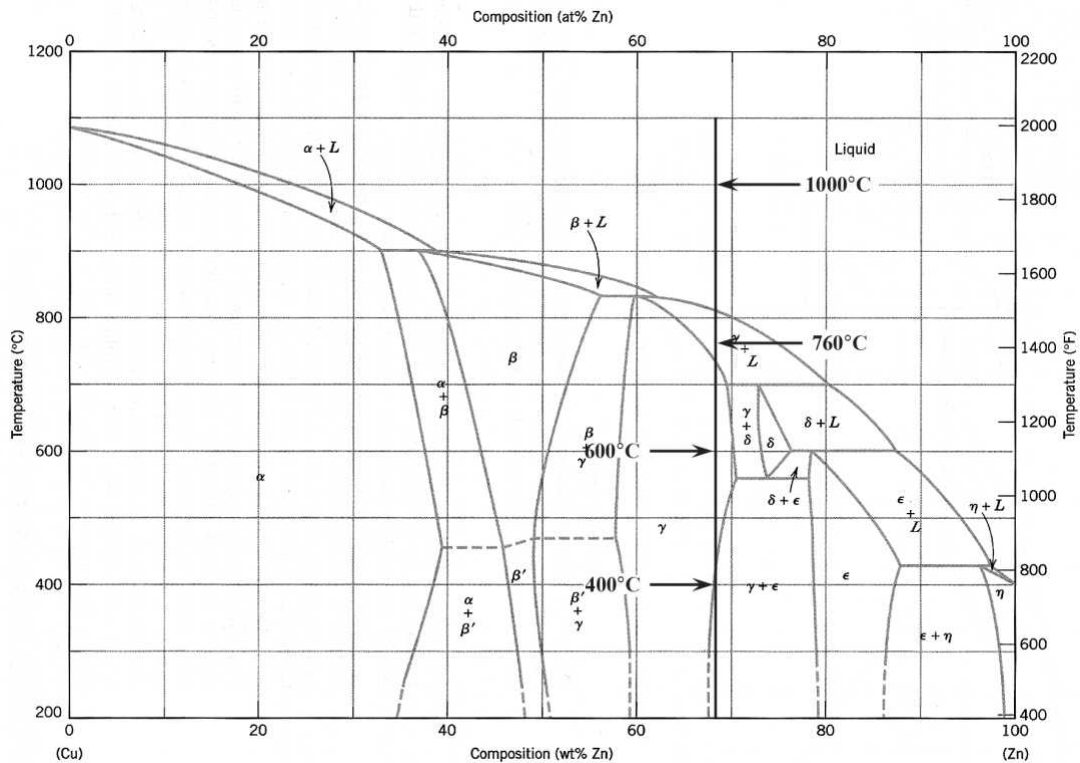

On the basis of the locations of the four temperature-composition points, schematic sketches of the four respective microstructures along with phase compositions are represented as follows:
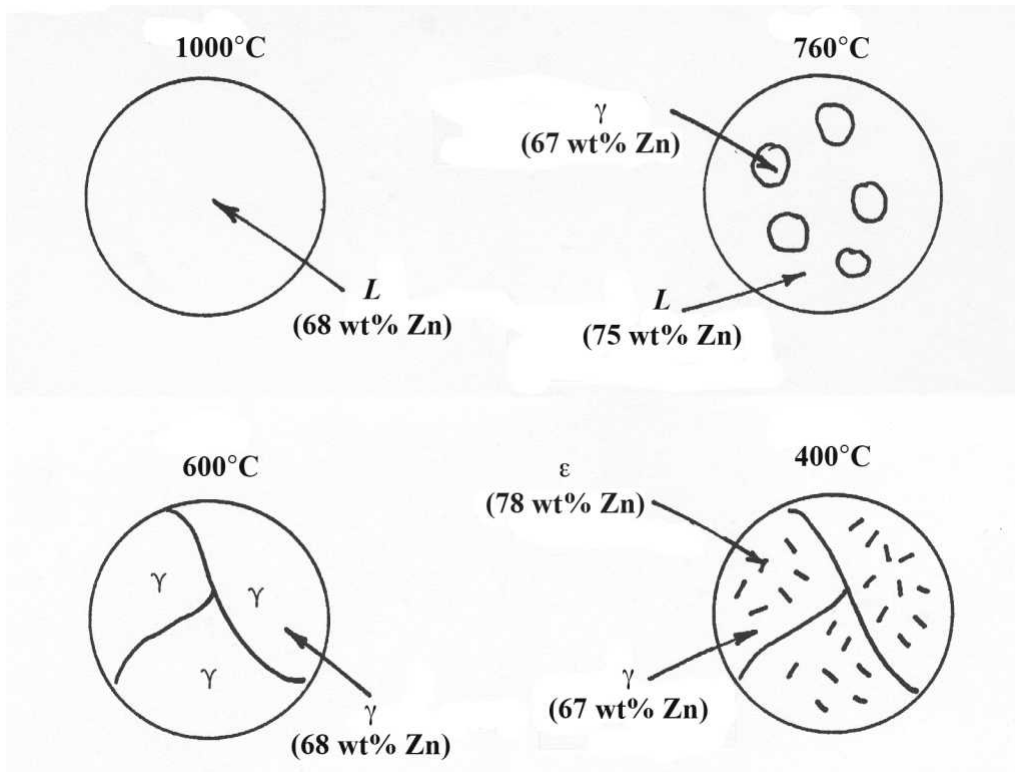
9.36 For a 68 wt% Zn-32 wt% Cu alloy, make schematic sketches of the microstructure that would be observed for conditions of very slow cooling at the following temperatures: 1000°C (1830°F), 760°C (1400°F), 600°C (1110°F), and 400°C (750°F). Label all phases and indicate their approximate compositions.

<u>Solution</u>

The illustration below is the Cu-Zn phase diagram (Figure 9.19). A vertical line at a composition of 68 wt% Zn-32 wt% Cu has been drawn, and, in addition, horizontal arrows at the four temperatures called for in the problem statement (i.e., 1000°C, 760°C, 600°C, and 400°C).
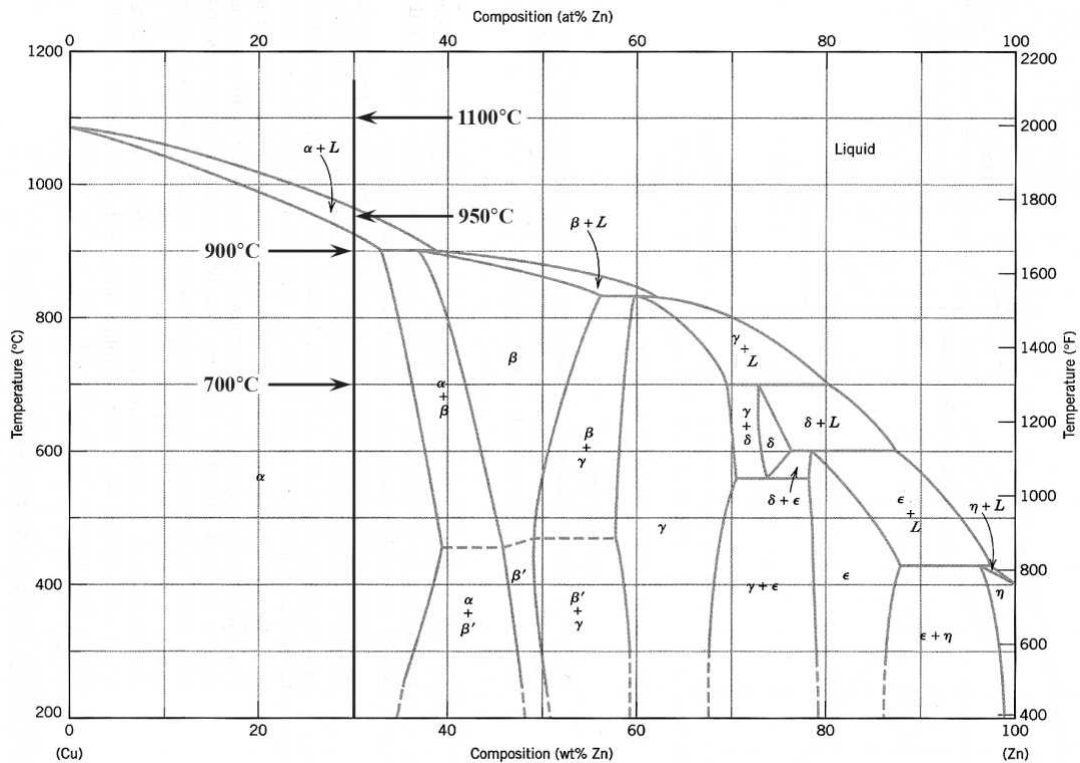

On the basis of the locations of the four temperature-composition points, schematic sketches of the four respective microstructures along with phase compositions are represented as follows:

1000°C

L
(68 wt% Zn)

760°C

γ
(67 wt% Zn)

L
(75 wt% Zn)

600°C

γ

γ

γ

γ
(68 wt% Zn)

400°C

ε
(78 wt% Zn)

γ
(67 wt% Zn)

9.37   For a 30 wt% Zn-70 wt% Cu alloy, make schematic sketches of the microstructure that would be observed for conditions of very slow cooling at the following temperatures: 1100°C (2010°F), 950°C (1740°F), 900°C (1650°F), and 700°C (1290°F). Label all phases and indicate their approximate compositions.

<u>Solution</u>

The illustration below is the Cu-Zn phase diagram (Figure 9.19). A vertical line at a composition of 30 wt% Zn-70 wt% Cu has been drawn, and, in addition, horizontal arrows at the four temperatures called for in the problem statement (i.e., 1100°C, 950°C, 900°C, and 700°C).
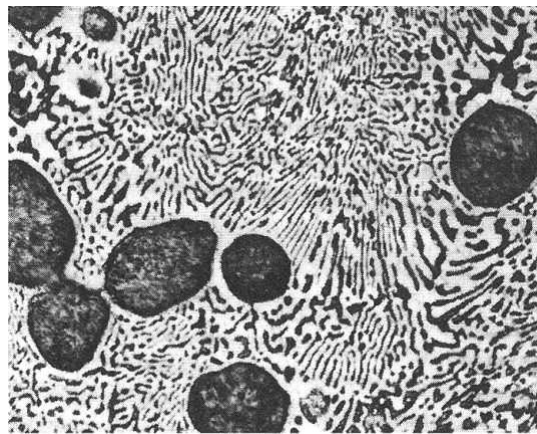

On the basis of the locations of the four temperature-composition points, schematic sketches of the four respective microstructures along with phase compositions are represented as follows:

1100°C

L
(30 wt% Zn)

950°C

L
(32 wt% Zn)

α
(27 wt% Zn)

900°C

α

α

α

α
(30 wt% Zn)

700°C

α
(30 wt% Zn)

α

α

α
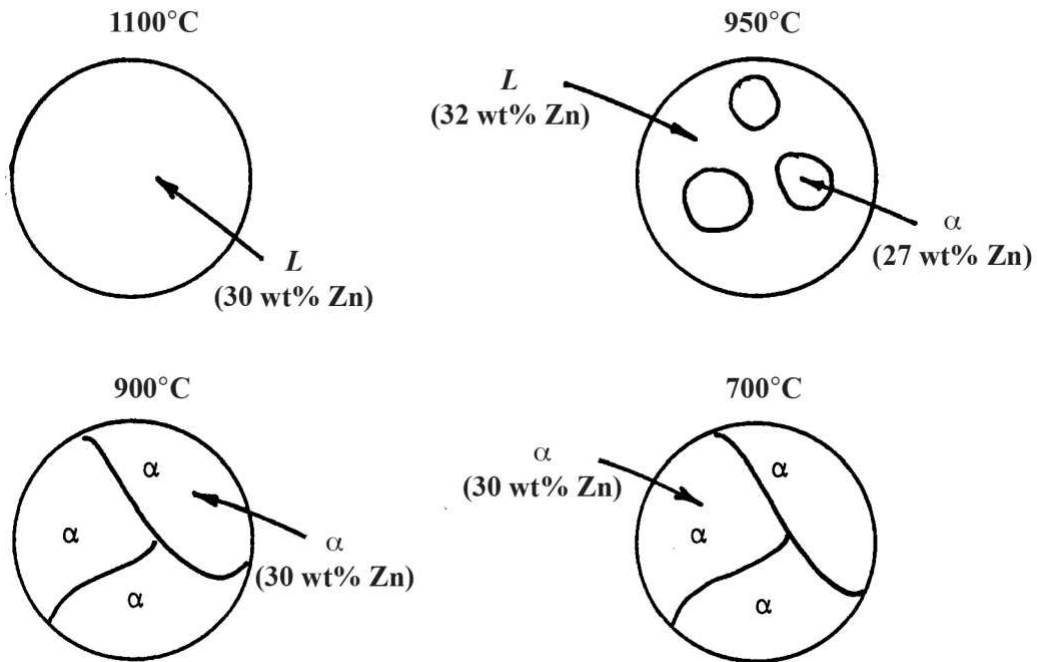
9.38   On the basis of the photomicrograph (i.e., the relative amounts of the microconstituents) for the lead–tin alloy shown in Figure 9.17 and the Pb–Sn phase diagram (Figure 9.8), estimate the composition of the alloy, and then compare this estimate with the composition given in the figure legend of Figure 9.17. Make the following assumptions: (1) the area fraction of each phase and microconstituent in the photomicrograph is equal to its volume fraction; (2) the densities of the α and β phases as well as the eutectic structure are 11.2 and 8.7 g/cm³, respectively; and (3) this photomicrograph represents the equilibrium microstructure at T = 180°C (356°F).

Solution

Below is shown the micrograph of the Pb-Sn alloy, Figure 9.17:

Primary α and eutectic microconstituents are present in the photomicrograph, and it is given that their densities are 11.2 and 8.7 g/cm³, respectively. Below is shown a square grid network onto which is superimposed outlines of the primary α phase areas.
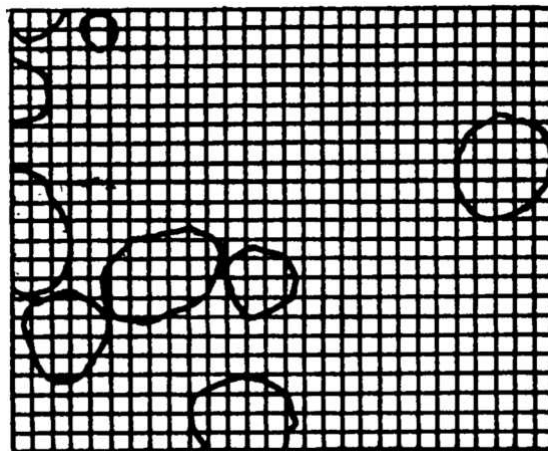
The area fraction of this primary α phase may be determined by counting squares. There are a total of 644 squares, and of these, approximately 104 lie within the primary α phase particles. Thus, the area fraction of primary α is 104/644 = 0.16, which is also assumed to be the volume fraction.

We now want to convert the volume fractions into mass fractions in order to employ the lever rule to the Pb-Sn phase diagram. To do this, it is necessary to utilize Equations 9.7a and 9.7b as follows:

$$W_{\alpha'} = \frac{V_{\alpha'}\,\rho_{\alpha'}}{V_{\alpha'}\rho_{\alpha'} + V_{eutectic}\,\rho_{eutectic}}$$

$$= \frac{(0.16)(11.2 \text{ g/cm}^3)}{(0.16)(11.2 \text{ g/cm}^3) + (0.84)(8.7 \text{ g/cm}^3)} = 0.197$$

$$W_{eutectic} = \frac{V_{eutectic}\,\rho_{eutectic}}{V_{\alpha'}\rho_{\alpha'} + V_{eutectic}\,\rho_{eutectic}}$$

$$= \frac{(0.84)(8.7 \text{ g/cm}^3)}{(0.16)(11.2 \text{ g/cm}^3) + (0.84)(8.7 \text{ g/cm}^3)} = 0.803$$

From Figure 9.8, we want to use the lever rule and a tie-line that extends from the eutectic composition (61.9 wt% Sn) to the α–($\alpha + \beta$) phase boundary at 180°C (about 18.3 wt% Sn). Accordingly

$$W_{\alpha'} = 0.197 = \frac{61.9 - C_0}{61.9 - 18.3}$$

wherein C_0 is the alloy composition (in wt% Sn). Solving for C_0 yields C_0 = 53.3 wt% Sn. This value is in good agreement with the actual composition—viz. 50 wt% Sn.

9.39 The room-temperature tensile strengths of pure lead and pure tin are 16.8 MPa and 14.5 MPa, respectively.

(a) Make a schematic graph of the room-temperature tensile strength versus composition for all compositions between pure lead and pure tin. (Hint: you may want to consult Sections 9.10 and 9.11, as well as Equation 9.24 in Problem 9.64.)

(b) On this same graph schematically plot tensile strength versus composition at 150°C.

(c) Explain the shapes of these two curves, as well as any differences between them.

Solution

The (a) and (b) portions of the problem ask that we make schematic plots on the same graph for the tensile strength versus composition for lead-tin alloys at both room temperature and 150°C; such a graph is shown below.
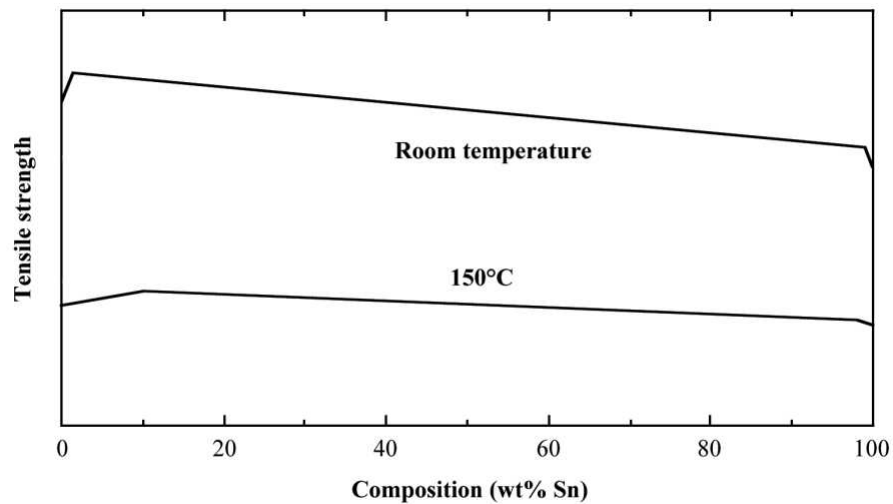
(c) Upon consultation of the Pb-Sn phase diagram (Figure 9.8) we note that, at room temperature (20°C), about 1.5 wt% of Sn is soluble in Pb (within the α-phase region at the left extremity of the phase diagram). Similarly, only about 1 wt% of Pb is soluble in Sn (within the β-phase region at the left extremity). Thus, there will a solid-solution strengthening effect on both ends of the phase diagram—strength increases slightly with additions of Sn to Pb [in the α phase region (left-hand side)] and with additions of Pb to Sn [in the β phase region (right-hand side)]; these effects are noted in the above figure. This figure also shows that the tensile strength of pure lead is greater than pure tin, which is in agreement with tensile strength values provided in the problem statement.

In addition, at room temperature, for compositions between about 1.5 wt% Sn and 99 wt% Sn, both α and β phase will coexist, (Figure 9.8), Furthermore, for compositions within this range, tensile strength will depend (approximately) on the tensile strengths of each of the α and β phases as well as their phase fractions in a manner described by Equation 9.24 for the elastic modulus (Problem 9.64). That is, for this problem

$$(TS)_{alloy} \cong (TS)_{\alpha}V_{\alpha} + (TS)_{\beta}V_{\beta}$$

in which TS and V denote tensile strength and volume fraction, respectively, and the subscripts represent the alloy/phases. Also, mass fractions of the α and β phases change linearly with changing composition (according to the lever rule). Furthermore, although there is some disparity between the densities of Pb and Sn (11.35 versus 7.27 g/cm^3), weight and volume fractions of the α and β phases will also be similar (see Equation 9.6).

At 150°C, the curve will be shifted to significantly lower tensile strengths inasmuch as tensile strength diminishes with increasing temperature (Section 6.6, Figure 6.14). In addition, according to Figure 9.8, solubility limits for both α and β phases increase—for the α phase from 1.5 to 10 wt% Sn, and for the β phase from 1 to about 2 wt% Pb. Thus, the compositional ranges over which solid-solution strengthening occurs increase somewhat from the room-temperature range ⌐the effect also not done the ¶⇑‼°C curve above. Furthermore, at 150°C, it would be expected that the tensile strength of lead will be greater than that of tin; and for compositions over which both α and β phases coexist, strength will decrease approximately linearly with increasing Sn content.

Equilibrium Diagrams Having Intermediate Phases or Compounds

9.40 Two intermetallic compounds, AB and AB$_2$, exist for elements A and B. If the compositions for AB and AB$_2$ are 34.3 wt% A–65.7 wt% B and 20.7 wt% A–79.3 wt% B, respectively, and element A is potassium, identify element B.

Solution

This problem gives us the compositions in weight percent for the two intermetallic compounds AB and AB$_2$, and then asks us to identify element B if element A is potassium. Probably the easiest way to solve this problem is to first compute the ratio of the atomic weights of these two elements using Equation 4.6a. Then, since we know the atomic weight of potassium (39.10 g/mol, per inside the front cover), it is possible to determine the atomic weight of element B, from which an identification may be made.

First of all, consider the AB intermetallic compound; inasmuch as it contains the same numbers of A and B atoms, its composition in atomic percent is 50 at% A-50 at% B. Equation 4.6a may be written in the form:

$$C_B' = \frac{C_B A_A}{C_A A_B + C_B A_A} \times 100$$

where A$_A$ and A$_B$ are the atomic weights for elements A and B, and C$_A$ and C$_B$ are their compositions in weight percent. For this AB compound, and making the appropriate substitutions in the above equation leads to

$$50 \text{ at\% B} = \frac{(65.7 \text{ wt\% B})(A_A)}{(34.3 \text{ wt\% A})(A_B) + (65.7 \text{ wt\% B})(A_A)} \times 100$$

Now, solving this expression yields,

$$A_B = 1.916 \, A_A$$

Since potassium is element A and it has an atomic weight of 39.10 g/mol, the atomic weight of element B is just

$$A_B = (1.916)(39.10 \text{ g/mol}) = 74.92 \text{ g/mol}$$

Upon consultation of the period table of the elements (Figure 2.6) we note the element that has an atomic weight closest to this value is arsenic (74.92 g/mol). Therefore, element B is arsenic, and the two intermetallic compounds are KAs and KAs$_2$.

Congruent Phase Transformations
Eutectoid and Peritectic Reactions

9.41 What is the principal difference between congruent and incongruent phase transformations?

<u>Solution</u>

The principal difference between congruent and incongruent phase transformations is that for congruent no compositional changes occur with any of the phases that are involved in the transformation. For incongruent there will be compositional alterations of the phases.

9.42 Figure 9.36 is the aluminum-neodymium phase diagram, for which only single-phase regions are labeled. Specify temperature-composition points at which all eutectics, eutectoids, peritectics, and congruent phase transformations occur. Also, for each, write the reaction upon cooling.

Solution

Below is shown the aluminum-neodymium phase diagram (Figure 9.36).
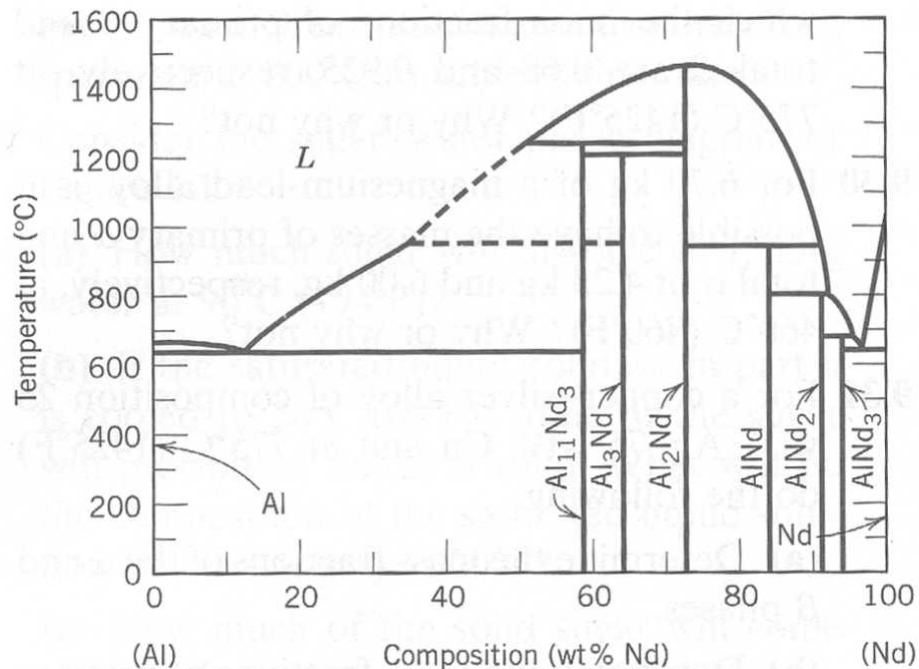
There are two eutectics on this phase diagram. One exists at 12 wt% Nd-88 wt% Al and 632°C. The reaction upon cooling is
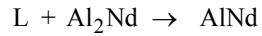
$$L \rightarrow Al + Al_{11}Nd_3$$
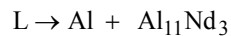
The other eutectic exists at about 97 wt% Nd-3 wt% Al and 635°C. This reaction upon cooling is

$$L \rightarrow AlNd_3 + Nd$$
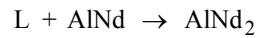
There are four peritectics. One exists at 59 wt% Nd-41 wt% Al and 1235°C. Its reaction upon cooling is as follows:

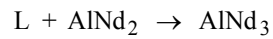$$L \; + \; Al_2Nd \; \rightarrow \; Al_{11}Nd_3$$
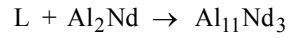
The second peritectic exists at 84 wt% Nd-16 wt% Al and 940°C. This reaction upon cooling is

$$L \; + \; Al_2Nd \; \rightarrow \; AlNd$$

The third peritectic exists at 91 wt% Nd-9 wt% Al and 795°C. This reaction upon cooling is

$$L \; + \; AlNd \; \rightarrow \; AlNd_2$$

The fourth peritectic exists at 94 wt% Nd-6 wt% Al and 675°C. This reaction upon cooling is

$$L \; + \; AlNd_2 \; \rightarrow \; AlNd_3$$

There is one congruent melting point at about 73 wt% Nd-27 wt% Al and 1460°C. Its reaction upon cooling is

$$L \; \rightarrow \; Al_2Nd$$
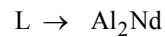
No eutectoids are present.

9.43 Figure 9.37 is a portion of the titanium-copper phase diagram for which only single-phase regions are labeled. Specify all temperature-composition points at which eutectics, eutectoids, peritectics, and congruent phase transformations occur. Also, for each, write the reaction upon cooling.
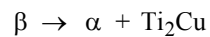
Solution

Below is shown the titanium-copper phase diagram (Figure 9.37).
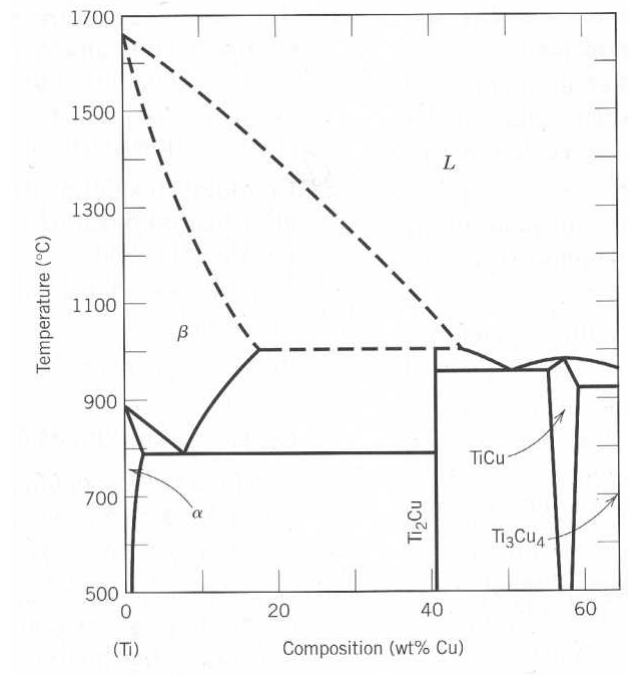
There is one eutectic on this phase diagram, which exists at about 51 wt% Cu-49 wt% Ti and 960°C. Its reaction upon cooling is

$$L \rightarrow Ti_2Cu + TiCu$$
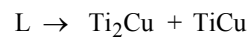
There is one eutectoid for this system. It exists at about 7.5 wt% Cu-92.5 wt% Ti and 790°C. This reaction upon cooling is

$$\beta \rightarrow \alpha + Ti_2Cu$$

There is one peritectic on this phase diagram. It exists at about 40 wt% Cu-60 wt% Ti and 1005°C. The reaction upon cooling is

$$\beta + L \rightarrow Ti_2Cu$$
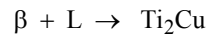
There is a single congruent melting point that exists at about 57.5 wt% Cu-42.5 wt% Ti and 982°C. The reaction upon cooling is

$$L \rightarrow TiCu$$
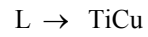
9.44   Construct the hypothetical phase diagram for metals A and B between temperatures of 600°C and 1000°C given the following information:

● The melting temperature of metal A is 940°C.

● The solubility of B in A is negligible at all temperatures.

● The melting temperature of metal B is 830°C.

● The maximum solubility of A in B is 12 wt% A, which occurs at 700°C.

● At 600°C, the solubility of A in B is 8 wt% A.

● One eutectic occurs at 700°C and 75 wt% B–25 wt% A.

● A second eutectic occurs at 730°C and 60 wt% B–40 wt% A.

● A third eutectic occurs at 755°C and 40 wt% B–60 wt% A.

● One congruent melting point occurs at 780°C and 51 wt% B–49 wt% A.

● A second congruent melting point occurs at 755°C and 67 wt% B–33 wt% A.

● The intermetallic compound AB exists at 51 wt% B–49 wt% A.

● The intermetallic compound AB_2 exists at 67 wt% B–33 wt% A.

Solution

Below is shown the phase diagram for these two A and B metals.
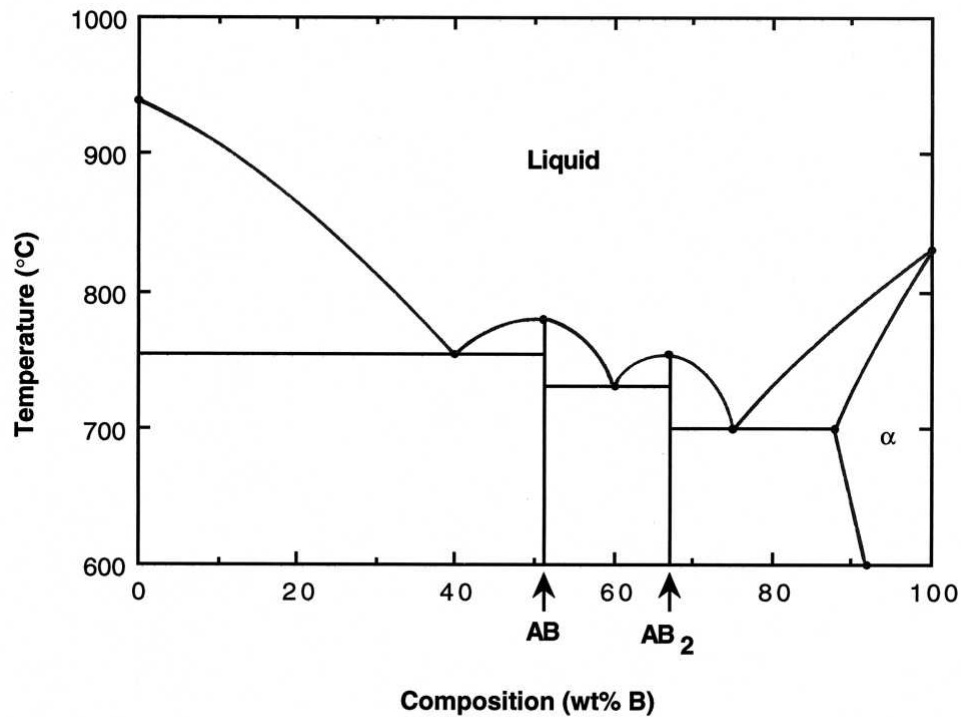
The Gibbs Phase Rule

9.45   In Figure 9.38 is shown the pressure–temperature phase diagram for H_2O. Apply the Gibbs phase rule at points A, B, and C; that is, specify the number of degrees of freedom at each of the points—that is, the number of externally controllable variables that need be specified to completely define the system.

Solution

We are asked to specify the value of F for Gibbs phase rule at points A, B, and C on the pressure-temperature diagram for H_2O, Figure 9.38, which is shown below.
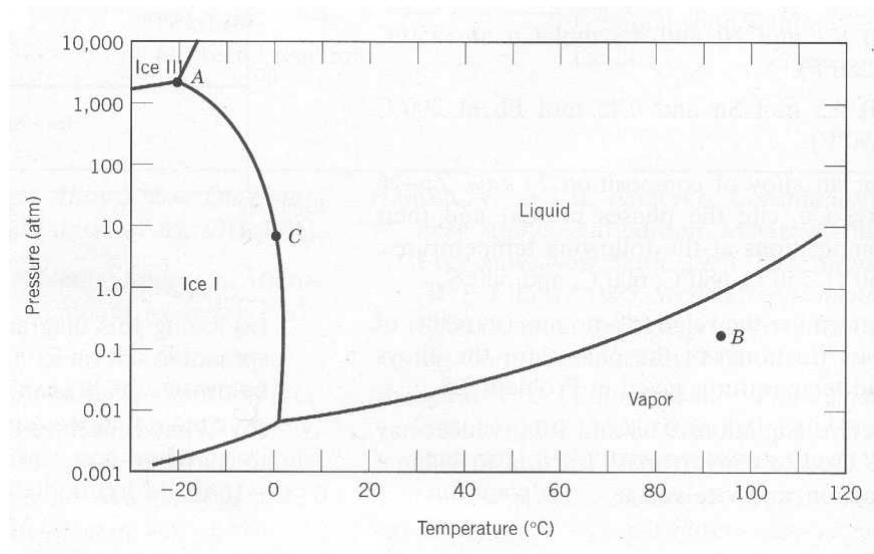
Gibbs phase rule in general form is

$$P + F = C + N$$

For this system, the number of components, C, is 1, whereas N, the number of noncompositional variables, is 2--viz. temperature and pressure. Thus, the phase rule now becomes

$$P + F = 1 + 2 = 3$$

Or

$$F = 3 - P$$

where P is the number of phases present at equilibrium.

At point A, three phases are present (viz. ice I, ice III, and liquid) and P ⊥ = ᴸκὺᵗ ὺκὐ QXP EHὐ RI ᵈGHJ UHὐ RI ᴸ freedom is zero since

$$F = 3 - P = 3 - 3 = 0$$

Thus, point A is an invariant point (in this case a triple point), and we have no choice in the selection of externally controllable variables in order to define the system.

At point B on the figure, only a single (vapor) phase is present (i.e., P = 1), or

$$F = 3 - P = 3 - 1 = 2$$

which means that specification of both temperature and pressure are necessary to define the system.

And, finally, at point C which is on the phase boundary between liquid and ice I phases, two phases are in equilibrium (P ⊥ ⊥ᵒ ⊥ KHQFH

$$F = 3 - P = 3 - 2 = 1$$

Or that we need to specify the value of either temperature or pressure, which determines the value of the other parameter (pressure or temperature).

The Iron-Iron Carbide (Fe-Fe$_3$C) Phase Diagram

Development of Microstructure in Iron-Carbon Alloys

9.46 Compute the mass fractions of α ferrite and cementite in pearlite.

<u>Solution</u>

This problem asks that we compute the mass fractions of α ferrite and cementite in pearlite. The lever-rule expression for ferrite is

$$W_\alpha = \frac{C_{Fe_3C} - C_0}{C_{Fe_3C} - C_\alpha}$$

and, since $C_{Fe_3C} = 6.70$ wt% C, $C_0 = 0.76$ wt% C, and $C_\alpha = 0.022$ wt% C

$$W_\alpha = \frac{6.70 - 0.76}{6.70 - 0.022} = 0.89$$

Similarly, for cementite

$$W_{Fe_3C} = \frac{C_0 - C_\alpha}{C_{Fe_3C} - C_\alpha} = \frac{0.76 - 0.022}{6.70 - 0.022} = 0.11$$

9.47 (a) What is the distinction between hypoeutectoid and hypereutectoid steels?

(b) In a hypoeutectoid steel, both eutectoid and proeutectoid ferrite exist. Explain the difference between them. What will be the carbon concentration in each?

<u>Solution</u>

(a) A "hypoeutectoid" steel has a carbon concentration less than the eutectoid; on the other hand, a "hypereutectoid" steel has a carbon content greater than the eutectoid.

(b) For a hypoeutectoid steel, the proeutectoid ferrite is a microconstituent that formed above the eutectoid temperature. The eutectoid ferrite is one of the constituents of pearlite that formed at a temperature below the eutectoid. The carbon concentration for both ferrites is 0.022 wt% C.

9.48   What is the carbon concentration of an iron–carbon alloy for which the fraction of total ferrite is 0.94?

<u>Solution</u>

This problem asks that we compute the carbon concentration of an iron-carbon alloy for which the fraction of total ferrite is 0.94. Application of the lever rule (of the form of Equation 9.12) yields

$$W_\alpha = 0.94 = \frac{C_{Fe_3C} - C_0'}{C_{Fe_3C} - C_\alpha} = \frac{6.70 - C_0'}{6.70 - 0.022}$$

and solving for C_0'

$$C_0' = 0.42 \text{ wt\% C}$$

9.49 What is the proeutectoid phase for an iron–carbon alloy in which the mass fractions of total ferrite and total cementite are 0.92 and 0.08, respectively? Why?

<u>Solution</u>

In this problem we are given values of W_α and W_{Fe_3C} (0.92 and 0.08, respectively) for an iron-carbon alloy and then are asked to specify the proeutectoid phase. Employment of the lever rule for total α leads to

$$W_\alpha = 0.92 = \frac{C_{Fe_3C} - C_0}{C_{Fe_3C} - C_\alpha} = \frac{6.70 - C_0}{6.70 - 0.022}$$

Now, solving for C_0, the alloy composition, leads to $C_0 = 0.56$ wt% C. Therefore, the proeutectoid phase is α-ferrite since C_0 is less than 0.76 wt% C.

9.50 Consider 1.0 kg of austenite containing 1.15 wt% C, cooled to below 727°C (1341°F).

(a) What is the proeutectoid phase?

(b) How many kilograms each of total ferrite and cementite form?

(c) How many kilograms each of pearlite and the proeutectoid phase form?

(d) Schematically sketch and label the resulting microstructure.

<u>Solution</u>

(a) The proeutectoid phase will be Fe_3C since 1.15 wt% C is greater than the eutectoid composition (0.76 wt% C).

(b) For this portion of the problem, we are asked to determine how much total ferrite and cementite form. Application of the appropriate lever rule expression yields

$$W_\alpha = \frac{C_{Fe_3C} - C_0}{C_{Fe_3C} - C_\alpha} = \frac{6.70 - 1.15}{6.70 - 0.022} = 0.83$$

which, when multiplied by the total mass of the alloy (1.0 kg), gives 0.83 kg of total ferrite.

Similarly, for total cementite,

$$W_{Fe_3C} = \frac{C_0 - C_\alpha}{C_{Fe_3C} - C_\alpha} = \frac{1.15 - 0.022}{6.70 - 0.022} = 0.17$$

And the mass of total cementite that forms is (0.17)(1.0 kg) = 0.17 kg.

(c) Now we are asked to calculate how much pearlite and the proeutectoid phase (cementite) form. Applying Equation 9.22, in which $C_1^{'} = 1.15$ wt% C

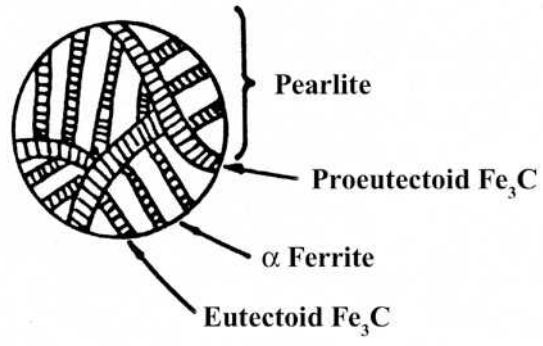$$W_p = \frac{6.70 - C_1^{'}}{6.70 - 0.76} = \frac{6.70 - 1.15}{6.70 - 0.76} = 0.93$$

which corresponds to a mass of 0.93 kg. Likewise, from Equation 9.23

$$W_{Fe_3C'} = \frac{C_1^{'} - 0.76}{5.94} = \frac{1.15 - 0.76}{5.94} = 0.07$$

which is equivalent to 0.07 kg of the total 1.0 kg mass.

(d) Schematically, the microstructure would appear as:

Pearlite

Proeutectoid Fe$_3$C

α Ferrite

Eutectoid Fe$_3$C

9.51 Consider 2.5 kg of austenite containing 0.65 wt% C, cooled to below 727°C (1341°F).

(a) What is the proeutectoid phase?

(b) How many kilograms each of total ferrite and cementite form?

(c) How many kilograms each of pearlite and the proeutectoid phase form?

(d) Schematically sketch and label the resulting microstructure.

Solution

(a) Ferrite is the proeutectoid phase since 0.65 wt% C is less than 0.76 wt% C.

(b) For this portion of the problem, we are asked to determine how much total ferrite and cementite form. For ferrite, application of the appropriate lever rule expression yields

$$W_\alpha = \frac{C_{Fe_3C} - C_0}{C_{Fe_3C} - C_\alpha} = \frac{6.70 - 0.65}{6.70 - 0.022} = 0.91$$

which corresponds to (0.91)(2.5 kg) = 2.27 kg of total ferrite.

Similarly, for total cementite,

$$W_{Fe_3C} = \frac{C_0 - C_\alpha}{C_{Fe_3C} - C_\alpha} = \frac{0.65 - 0.022}{6.70 - 0.022} = 0.09$$

Or (0.09)(2.5 kg) = 0.23 kg of total cementite form.

(c) Now consider the amounts of pearlite and proeutectoid ferrite. Using Equation 9.20

$$W_p = \frac{C'_0 - 0.022}{0.74} = \frac{0.65 - 0.022}{0.74} = 0.85$$

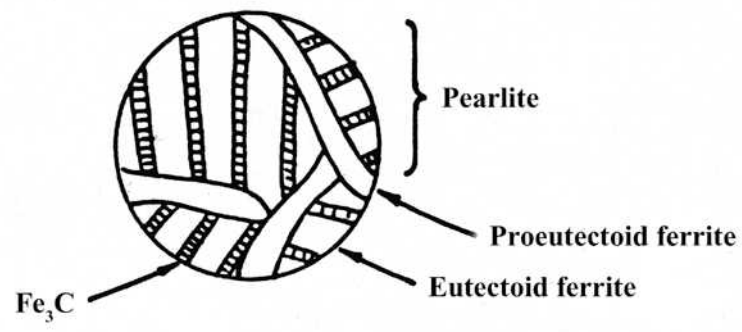This corresponds to (0.85)(2.5 kg) = 2.12 kg of pearlite.

Also, from Equation 9.21,

$$W_{\alpha'} = \frac{0.76 - 0.65}{0.74} = 0.15$$

Or, there are (0.15)(2.5 kg) = 0.38 kg of proeutectoid ferrite.

(d) Schematically, the microstructure would appear as:

9.52 Compute the mass fractions of proeutectoid ferrite and pearlite that form in an iron–carbon alloy containing 0.25 wt% C.

<u>Solution</u>

The mass fractions of proeutectoid ferrite and pearlite that form in a 0.25 wt% C iron-carbon alloy are considered in this problem. From Equation 9.20

$$W_p = \frac{C_0^{'} - 0.022}{0.74} = \frac{0.25 - 0.022}{0.74} = 0.31$$

And, from Equation 9.21 (for proeutectoid ferrite)

$$W_{\alpha'} = \frac{0.76 - C_0^{'}}{0.74} = \frac{0.76 - 0.25}{0.74} = 0.69$$

9.53 The microstructure of an iron–carbon alloy consists of proeutectoid ferrite and pearlite; the mass fractions of these two microconstituents are 0.286 and 0.714, respectively. Determine the concentration of carbon in this alloy.

<u>Solution</u>

This problem asks that we determine the carbon concentration in an iron-carbon alloy, given the mass fractions of proeutectoid ferrite and pearlite. From Equation 9.20

$$W_p = 0.714 \ = \frac{C_0^{'} \ - \ 0.022}{0.74}$$

which yields $C_0^{'} = 0.55$ wt% C.

9.54 The mass fractions of total ferrite and total cementite in an iron-carbon alloy are 0.88 and 0.12, respectively. Is this a hypoeutectoid or hypereutectoid alloy? Why?

Solution

In this problem we are given values of W_α and W_{Fe_3C} for an iron-carbon alloy (0.88 and 0.12, respectively), and then are asked to specify whether the alloy is hypoeutectoid or hypereutectoid. Employment of the lever rule for total α leads to

$$W_\alpha = 0.88 = \frac{C_{Fe_3C} - C_0}{C_{Fe_3C} - C_\alpha} = \frac{6.70 - C_0}{6.70 - 0.022}$$

Now, solving for C_0, the alloy composition, leads to $C_0 = 0.82$ wt% C. Therefore, the alloy is hypereutectoid since C_0 is greater than 0.76 wt% C.

9.55 The microstructure of an iron-carbon alloy consists of proeutectoid ferrite and pearlite; the mass fractions of these microconstituents are 0.20 and 0.80, respectively. Determine the concentration of carbon in this alloy.

<u>Solution</u>

We are asked in this problem to determine the concentration of carbon in an alloy for which $W_{\alpha'} = 0.20$ and $W_p = 0.80$. If we let C_0' equal the carbon concentration in the alloy, employment of the appropriate lever rule expression, Equation 9.20, leads to

$$W_p = \frac{C_0' - 0.022}{0.74} = 0.80$$

Solving for C_0' yields $C_0' = 0.61$ wt% C.

9.56   Consider 2.0 kg of a 99.6 wt% Fe–0.4 wt% C alloy that is cooled to a temperature just below the eutectoid.

(a) How many kilograms of proeutectoid ferrite form?

(b) How many kilograms of eutectoid ferrite form?

(c) How many kilograms of cementite form?

Solution

In this problem we are asked to consider 2.0 kg of a 99.6 wt% Fe-0.4 wt% C alloy that is cooled to a temperature below the eutectoid.

(a) Equation 9.21 must be used in computing the amount of proeutectoid ferrite that forms. Thus,

$$W_{\alpha'} = \frac{0.76 - C_0'}{0.74} = \frac{0.76 - 0.40}{0.74} = 0.49$$

Or, (0.49)(2.0 kg) = 0.98 kg of proeutectoid ferrite forms.

(b)   In order to determine the amount of eutectoid ferrite, it first becomes necessary to compute the amount of total ferrite using the lever rule applied entirely across the $\alpha + Fe_3C$ phase field, as

$$W_{\alpha} = \frac{C_{Fe_3C} - C_0'}{C_{Fe_3C} - C_{\alpha}} = \frac{6.70 - 0.40}{6.70 - 0.022} = 0.94$$

which corresponds to (0.94)(2.0 kg) = 1.88 kg. Now, the amount of eutectoid ferrite is just the difference between total and proeutectoid ferrites, or

$$1.88 \text{ kg} - 0.98 \text{ kg} = 0.90 \text{ kg}$$

(c)   With regard to the amount of cementite that forms, again application of the lever rule across the entirety of the $\alpha + Fe_3C$ phase field, leads to

$$W_{Fe_3C} = \frac{C_0' - C_{\alpha}}{C_{Fe_3C} - C_{\alpha}} = \frac{0.40 - 0.022}{6.70 - 0.022} = 0.057$$

which amounts to (0.057)(2.0 kg) = 0.114 kg cementite in the alloy.

9.57 Compute the maximum mass fraction of proeutectoid cementite possible for a hypereutectoid iron–carbon alloy.

<u>Solution</u>

This problem asks that we compute the maximum mass fraction of proeutectoid cementite possible for a hypereutectoid iron-carbon alloy. This requires that we utilize Equation 9.23 with $C_1' = 2.14$ wt% C, the maximum solubility of carbon in austenite. Thus,

$$W_{Fe_3C'} = \frac{C_1' - 0.76}{5.94} = \frac{2.14 - 0.76}{5.94} = 0.232$$

9.58   Is it possible to have an iron-carbon alloy for which the mass fractions of total ferrite and proeutectoid cementite are 0.846 and 0.049, respectively? Why or why not?

Solution

This problem asks if it is possible to have an iron-carbon alloy for which $W_\alpha = 0.846$ and $W_{Fe_3C'} = 0.049$. In order to make this determination, it is necessary to set up lever rule expressions for these two mass fractions in terms of the alloy composition, then to solve for the alloy composition of each; if both alloy composition values are equal, then such an alloy is possible. The expression for the mass fraction of total ferrite is

$$W_\alpha = \frac{C_{Fe_3C} - C_0}{C_{Fe_3C} - C_\alpha} = \frac{6.70 - C_0}{6.70 - 0.022} = 0.846$$

Solving for this C_0 yields $C_0 = 1.05$ wt% C. Now for $W_{Fe_3C'}$ we utilize Equation 9.23 as

$$W_{Fe_3C'} = \frac{C_1' - 0.76}{5.94} = 0.049$$

This expression leads to $C_1' = 1.05$ wt% C. And, since $C_0 = C_1'$, this alloy is possible.

9.59 Is it possible to have an iron-carbon alloy for which the mass fractions of total cementite and pearlite are 0.039 and 0.417, respectively? Why or why not?

<u>Solution</u>

This problem asks if it is possible to have an iron-carbon alloy for which $W_{Fe_3C} = 0.039$ and $W_p = 0.417$. In order to make this determination, it is necessary to set up lever rule expressions for these two mass fractions in terms of the alloy composition, then to solve for the alloy composition of each; if both alloy composition values are equal, then such an alloy is possible. The expression for the mass fraction of total cementite is

$$W_{Fe_3C} = \frac{C_0 - C_\alpha}{C_{Fe_3C} - C_\alpha} = \frac{C_0 - 0.022}{6.70 - 0.022} = 0.039$$

Solving for this C_0 yields $C_0 = 0.28$ wt% C. Therefore, this alloy is hypoeutectoid since C_0 is less than the eutectoid composition (0.76 wt%). Thus, it is necessary to use Equation 9.20 for W_p as

$$W_p = \frac{C_0' - 0.022}{0.74} = 0.417$$

This expression leads to $C_0' = 0.33$ wt% C. Since C_0 and C_0' are different, this alloy is not possible.

9.60 Compute the mass fraction of eutectoid ferrite in an iron-carbon alloy that contains 0.43 wt% C.

Solution

In order to solve this problem it is necessary to compute mass fractions of total and proeutectoid ferrites, and then to subtract the latter from the former. To calculate the mass fraction of total ferrite, it is necessary to use the lever rule and a tie line that extends across the entire $\alpha + Fe_3C$ phase field as

$$W_\alpha = \frac{C_{Fe_3C} - C_0}{C_{Fe_3C} - C_\alpha} = \frac{6.70 - 0.43}{6.70 - 0.022} = 0.939$$

Now, for the mass fraction of proeutectoid ferrite we use Equation 9.21 as

$$W_{\alpha'} = \frac{0.76 - C_0' -}{0.74} = \frac{0.76 - 0.43}{0.74} = 0.446$$

And, finally, the mass fraction of eutectoid ferrite $W_{\alpha''}$ is just

$$W_{\alpha''} = W_\alpha - W_{\alpha'} = 0.939 - 0.446 = 0.493$$

9.61 The mass fraction of eutectoid cementite in an iron-carbon alloy is 0.104. On the basis of this information, is it possible to determine the composition of the alloy? If so, what is its composition? If this is not possible, explain why.

<u>Solution</u>

This problem asks whether or not it is possible to determine the composition of an iron-carbon alloy for which the mass fraction of eutectoid cementite is 0.104. And, yes, it is possible to determine the alloy composition; and, in fact, there are two possible answers. For the first, the eutectoid cementite exists in addition to proeutectoid cementite. For this case the mass fraction of eutectoid cementite ($W_{Fe_3C''}$) is just the difference between total cementite and proeutectoid cementite mass fractions; that is

$$W_{Fe_3C''} = W_{Fe_3C} - W_{Fe_3C'}$$

Now, it is possible to write expressions for W_{Fe_3C} (of the form of Equation 9.12) and $W_{Fe_3C'}$ (Equation 9.23) in terms of C_0, the alloy composition. Thus,

$$W_{Fe_3C''} = \frac{C_0 - C_\alpha}{C_{Fe_3C} - C_\alpha} - \frac{C_0 - 0.76}{5.94}$$

$$= \frac{C_0 - 0.022}{6.70 - 0.022} - \frac{C_0 - 0.76}{5.94} = 0.104$$

And, solving for C_0 yields $C_0 = 1.11$ wt% C.

For the second possibility, we have a hypoeutectoid alloy wherein all of the cementite is eutectoid cementite. Thus, it is necessary to set up a lever rule expression wherein the mass fraction of total cementite is 0.104. Therefore,

$$W_{Fe_3C} = \frac{C_0 - C_\alpha}{C_{Fe_3C} - C_\alpha} = \frac{C_0 - 0.022}{6.70 - 0.022} = 0.104$$

And, solving for C_0 yields $C_0 = 0.72$ wt% C.

9.62 The mass fraction of eutectoid ferrite in an iron-carbon alloy is 0.82. On the basis of this information, is it possible to determine the composition of the alloy? If so, what is its composition? If this is not possible, explain why.

<u>Solution</u>

This problem asks whether or not it is possible to determine the composition of an iron-carbon alloy for which the mass fraction of eutectoid ferrite is 0.82. And if so, to calculate the composition. Yes, it is possible to determine the alloy composition; and, in fact, there are two possible answers. For the first, the eutectoid ferrite exists in addition to proeutectoid ferrite. For this case the mass fraction of eutectoid ferrite ($W_{\alpha''}$) is just the difference between total ferrite and proeutectoid ferrite mass fractions; that is

$$W_{\alpha''} = W_{\alpha} - W_{\alpha'}$$

Now, it is possible to write expressions for W_{α} (of the form of Equation 9.12) and $W_{\alpha'}$ (Equation 9.21) in terms of C_0, the alloy composition. Thus,

$$W_{\alpha''} = \frac{C_{Fe_3C} - C_0}{C_{Fe_3C} - C_{\alpha}} - \frac{0.76 - C_0}{0.74}$$

$$= \frac{6.70 - C_0}{6.70 - 0.022} - \frac{0.76 - C_0}{0.74} = 0.82$$

And, solving for C_0 yields $C_0 = 0.70$ wt% C.

For the second possibility, we have a hypereutectoid alloy wherein all of the ferrite is eutectoid ferrite. Thus, it is necessary to set up a lever rule expression wherein the mass fraction of total ferrite is 0.82. Therefore,

$$W_{\alpha} = \frac{C_{Fe_3C} - C_0}{C_{Fe_3C} - C_{\alpha}} = \frac{6.70 - C_0}{6.70 - 0.022} = 0.82$$

And, solving for C_0 yields $C_0 = 1.22$ wt% C.

9.63   For an iron-carbon alloy of composition 5 wt% C-95 wt% Fe, make schematic sketches of the microstructure that would be observed for conditions of very slow cooling at the following temperatures: 1175°C (2150°F), 1145°C (2095°F), and 700°C (1290°F).   Label the phases and indicate their compositions (approximate).

<u>Solution</u>

Below is shown the Fe-Fe$_3$C phase diagram (Figure 9.24). A vertical line at a composition of 5 wt% C-95 wt% Fe has been drawn, and, in addition, horizontal arrows at the three temperatures called for in the problem statement (i.e., 1175°C, 1145°C, and 700°C).
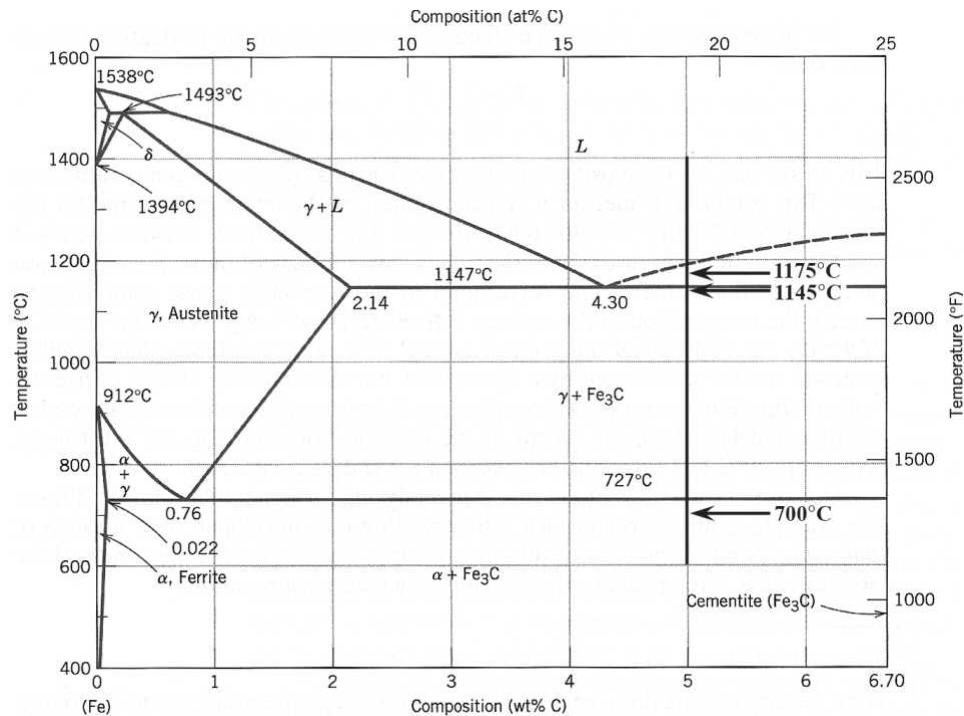
On the basis of the locations of the three temperature-composition points, schematic sketches of the respective microstructures along with phase compositions are represented as follows:
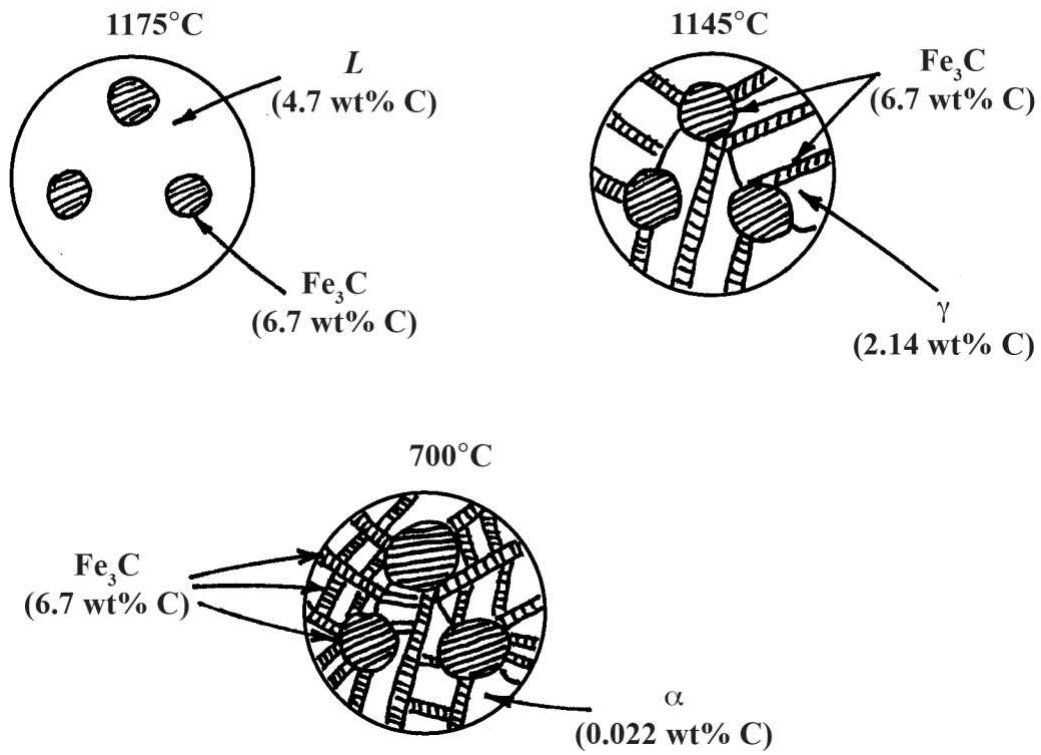
9.64 Often, the properties of multiphase alloys may be approximated by the relationship

$$E \text{ (alloy)} = E_\alpha V_\alpha + E_\beta V_\beta \qquad (9.24)$$

where E represents a specific property (modulus of elasticity, hardness, etc.), and V is the volume fraction. The subscripts α and β denote the existing phases or microconstituents. Employ the relationship above to determine the approximate Brinell hardness of a 99.80 wt% Fe–0.20 wt% C alloy. Assume Brinell hardnesses of 80 and 280 for ferrite and pearlite, respectively, and that volume fractions may be approximated by mass fractions.

<u>Solution</u>

This problem asks that we determine the approximate Brinell hardness of a 99.80 wt% Fe-0.20 wt% C alloy, using a relationship similar to Equation 9.24. First, we compute the mass fractions of pearlite and proeutectoid ferrite using Equations 9.20 and 9.21, as

$$W_p = \frac{C'_0 - 0.022}{0.74} = \frac{0.20 - 0.022}{0.74} = 0.24$$

$$W_{\alpha'} = \frac{0.76 - C'_0}{0.74} = \frac{0.76 - 0.20}{0.74} = 0.76$$

Now, we compute the Brinell hardness of the alloy using a modified form of Equation 9.24 as

$$HB_{alloy} = HB_{\alpha'} W_{\alpha'} + HB_p W_p$$

$$= (80)(0.76) + (280)(0.24) = 128$$

The Influence of Other Alloying Elements

9.65 A steel alloy contains 97.5 wt% Fe, 2.0 wt% Mo, and 0.5 wt% C.

(a) What is the eutectoid temperature of this alloy?

(b) What is the eutectoid composition?

(c) What is the proeutectoid phase?

Assume that there are no changes in the positions of other phase boundaries with the addition of Mo.

Solution

(a) From Figure 9.34, the eutectoid temperature for 2.0 wt% Mo is approximately 850°C.

(b) From Figure 9.35, the eutectoid composition is approximately 0.22 wt% C.

(c) Since the carbon concentration of the alloy (0.5 wt%) is greater than the eutectoid (0.22 wt% C), cementite is the proeutectoid phase.

9.66 A steel alloy is known to contain 93.8 wt% Fe, 6.0 wt% Ni, and 0.2 wt% C.

(a) What is the approximate eutectoid temperature of this alloy?

(b) What is the proeutectoid phase when this alloy is cooled to a temperature just below the eutectoid?

(c) Compute the relative amounts of the proeutectoid phase and pearlite.

Assume that there are no alterations in the positions of other phase boundaries with the addition of Ni.

<u>Solution</u>

(a) From Figure 9.34, the eutectoid temperature for 6.0 wt% Ni is approximately 650°C (1200°F).

(b) From Figure 9.35, the eutectoid composition is approximately 0.62 wt% C. Since the carbon concentration in the alloy (0.2 wt%) is less than the eutectoid (0.62 wt% C), the proeutectoid phase is ferrite.

(c) Assume that the α–(α + Fe_3C) phase boundary is at a negligible carbon concentration. Modifying Equation 9.21 leads to

$$W_{\alpha'} = \frac{0.62 - C_0'}{0.62 - 0} = \frac{0.62 - 0.20}{0.62} = 0.68$$

Likewise, using a modified Equation 9.20

$$W_p = \frac{C_0' - 0}{0.62 - 0} = \frac{0.20}{0.62} = 0.32$$