

$$\textcircled{1} a) \begin{vmatrix} 1 & 3 & 0 & 2 \\ -2 & -5 & 7 & 4 \\ 3 & 5 & 2 & 1 \\ 1 & -1 & 2 & -3 \end{vmatrix} \xrightarrow[\substack{2R_1+R_2 \\ -3R_1+R_3 \\ R_1-R_4}]{=} \begin{vmatrix} 1 & 3 & 0 & 2 \\ 0 & 1 & 7 & 8 \\ 0 & -4 & 2 & -5 \\ 0 & 4 & -2 & 5 \end{vmatrix} \xrightarrow[\substack{R_3+4R_2 \\ R_3+R_4}]{=} \begin{vmatrix} 1 & 3 & 0 & 2 \\ 0 & 1 & 7 & 8 \\ 0 & 0 & 30 & 27 \\ 0 & 0 & 0 & 0 \end{vmatrix} = 0$$

$$\Rightarrow \det = 1 \times 1 \times 30 \times 0 = 0$$

$$b) \begin{vmatrix} 1 & 3 & 2 & -4 \\ 0 & 1 & 2 & -5 \\ 2 & 7 & 6 & -3 \\ -3 & -10 & -7 & 2 \end{vmatrix} \xrightarrow[\substack{-2R_1+R_3 \\ 3R_1+R_4}]{=} \begin{vmatrix} 1 & 3 & 2 & -4 \\ 0 & 1 & 2 & -5 \\ 0 & 1 & 2 & -5 \\ 0 & -1 & -1 & -10 \end{vmatrix} \xrightarrow[\substack{R_2-R_3 \\ R_3+R_4}]{=} \begin{vmatrix} 1 & 3 & 2 & -4 \\ 0 & 1 & 2 & -5 \\ 0 & 0 & 0 & -10 \\ 0 & 0 & 1 & -15 \end{vmatrix}$$

$$= \begin{vmatrix} 1 & 3 & 2 & -4 \\ 0 & 1 & 2 & -5 \\ 0 & 0 & 1 & -15 \\ 0 & 0 & 0 & -10 \end{vmatrix}$$

$$\det = [1 \times 1 \times 1 \times 10] \times (-1) = -10$$

$$\textcircled{2} \begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = 7$$

$$\Rightarrow aei + bfg + cdh - ceg - afh - bdi = 7$$

$$\begin{vmatrix} a & b & c \\ d & e & f \\ 3g & 3h & 3i \end{vmatrix}$$

$$\Rightarrow 3aei + 3bfg + 3cdh - 3ceg - 3afh - 3bdi = 3(aei + bfg + cdh - ceg - afh - bdi) = 3 \times 7 = 21$$

$$\textcircled{3} B = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 2 \\ 1 & 2 & 1 \end{bmatrix}$$

$$\Rightarrow \det(B) = 1 + 0 + 2 - 1 - 4 - 0 = -2$$

$$\Rightarrow \det(B^4) = [\det(B)]^4 = (-2)^4 = 16$$

$$(4) A = \begin{pmatrix} 2 & 4 & -1 \\ 0 & 3 & 1 \\ 6 & -2 & 5 \end{pmatrix}$$

$$A^{-1} = \frac{[\text{cof}(A)]^T}{\det(A)}$$

$$\begin{vmatrix} 2 & 4 & -1 & 2 & 4 \\ 0 & 3 & 1 & 0 & 3 \\ 6 & -2 & 5 & 6 & -2 \end{vmatrix} \Rightarrow \det(A) = 2 \times 3 \times 5 + 4 \times 1 \times 6 + 0 - 6 \times 3 \times (-1) - 2 \times 1 \times (-2) - 0 = 76$$

$$\begin{aligned} a_{11} &= (-1)^{1+1} (3 \times 5 - (-2) \times 1) = 17 & a_{21} &= (-1)^{1+2} (20 - 2) = -18 & a_{31} &= (-1)^{1+3} (4 + 3) = 7 \\ a_{12} &= (-1)^{1+2} (0 - 6) = 6 & a_{22} &= (-1)^{1+3} (10 + 6) = 16 & a_{32} &= (-1)^{1+4} (2) = -2 \\ a_{13} &= (-1)^{1+3} (0 - 18) = -18 & a_{23} &= (-1)^{1+5} (-4 - 24) = 28 & a_{33} &= (-1)^{1+6} (6) = 6 \end{aligned}$$

$$\Rightarrow \text{cof}(A) = \begin{pmatrix} 17 & -18 & 7 \\ 6 & 16 & -2 \\ -18 & 28 & 6 \end{pmatrix} \Rightarrow [\text{cof}(A)]^T = \begin{pmatrix} 17 & 6 & -18 \\ -18 & 16 & 28 \\ 7 & -2 & 6 \end{pmatrix}$$

$$A^{-1} = \frac{[\text{cof}(A)]^T}{\det(A)} = \frac{1}{76} \begin{pmatrix} 17 & 6 & -18 \\ -18 & 16 & 28 \\ 7 & -2 & 6 \end{pmatrix}$$

$$b) B = \begin{pmatrix} 1 & 2 & 0 \\ -1 & 0 & 1 \\ 0 & 3 & 1 \end{pmatrix}$$

$$\begin{vmatrix} 1 & 2 & 0 & 1 & 2 \\ -1 & 0 & 1 & -1 & 0 \\ 0 & 3 & 1 & 0 & 3 \end{vmatrix} \Rightarrow \det(B) = 0 + 0 + 0 - 0 - 3 - (-2) = -1$$

$$\begin{aligned} b_{11} &= (-1)^{1+1} (0 - 3) = -3 & b_{21} &= (-1)^{1+2} (2) = -2 & b_{31} &= 2 \\ b_{12} &= (-1)^{1+2} (-1) = 1 & b_{22} &= (-1)^{1+3} (1) = 1 & b_{32} &= -1 \\ b_{13} &= (-1)^{1+3} (3) = 3 & b_{23} &= (-1)^{1+5} (3) = -3 & b_{33} &= 2 \end{aligned}$$

$$\Rightarrow \text{cof}(B) = \begin{pmatrix} -3 & 1 & 3 \\ -2 & 1 & -3 \\ 2 & -1 & 2 \end{pmatrix} \Rightarrow [\text{cof}(B)]^T = \begin{pmatrix} -3 & -2 & 2 \\ 1 & 1 & -1 \\ -3 & -3 & 2 \end{pmatrix}$$

$$B^{-1} = \frac{[\text{cof}(B)]^T}{\det(B)} = \frac{1}{-1} \begin{pmatrix} -3 & -2 & 2 \\ 1 & 1 & -1 \\ -3 & -3 & 2 \end{pmatrix}$$