Homework: 1, 2, 8, 15, 24, 26, 29, 36, 43 (p. 159-163)

1. A proton (mass m=1.67 x 10^{-27} kg) is being accelerated along a straight line at 3.6 x 10^{15} m/s² in a machine. If the proton has an initial speed of 2.4 x 10^{7} m/s and travels 3.5 cm, what then is (a) its speed and (b) the increase in its kinetic energy?

m, a, v_i, x

(a)

$$v^2 - v_0^2 = 2a(x - x_0)$$

$$v = \sqrt{2a(x - x_0) + v_0^2}$$

$$v = \sqrt{2\times(3.6\times10^{15}\,\text{m/s}^2)\times(3.5\times10^{-2}\,\text{m}) + (2.4\times10^{7}\,\text{m/s})^2}$$

$$v = 2.9\times10^{7}\,\text{m/s}$$

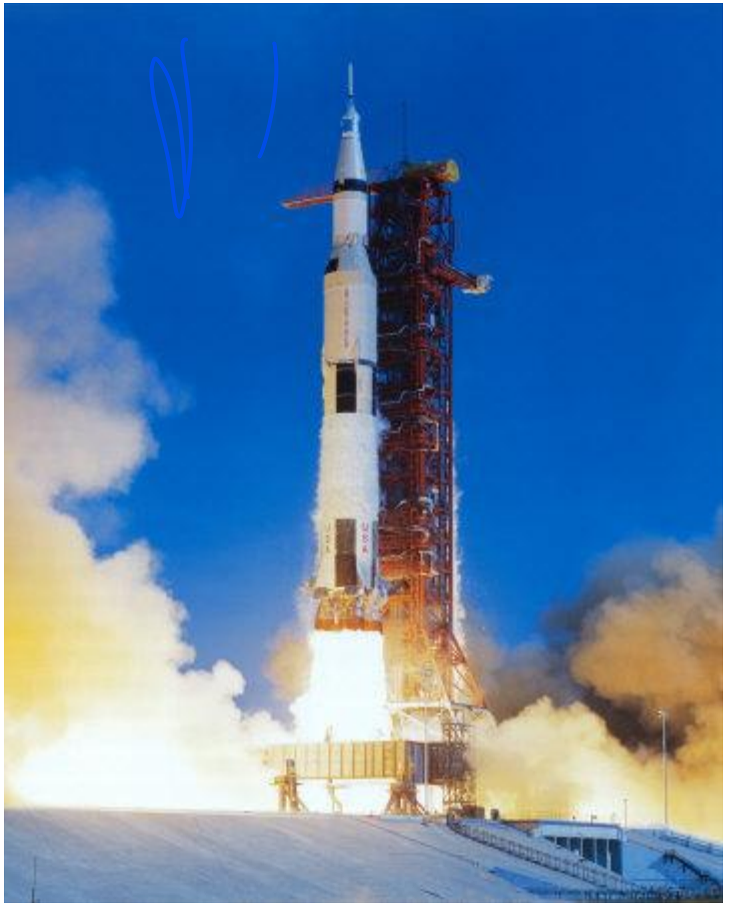
$\Delta K = E_f - E_i$

(b)

$$\Delta K = E_f - E_i = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 = \frac{1}{2}m(v_f^2 - v_i^2)$$

$$\Delta K = \frac{1}{2}\times1.67\times10^{-27}\left[(2.9\times10^{7})^2 - (2.4\times10^{7})^2\right] = 2.2\times10^{-13}\,(\text{J})$$

2. If a Saturn V rocket with an Apollo spacecraft attached had a combined mass of 2.9×10^5 kg and reached a speed of 11.2 km/s, how much kinetic energy would it then have?



v=11.2 km/s or v=11200 m/s

The kinetic energy:

$$K = \frac{1}{2}mv^2$$

$$K = \frac{1}{2}(2.9\times10^5)\times11200^2 = 1.82\times10^{13}\,(J)$$

8. A floating ice block is pushed through a displacement $\vec{d} = (20\text{m})\,\hat{i} - (16\text{m})\,\hat{j}$ along a straight embankment by rushing water, which exerts a force $\vec{F} = (210\text{N})\hat{i} - (150\text{N})\hat{j}$ on the block. How much work does the force do on the block during the displacement?

$$W = \vec{F}\vec{d} = \left[210\,\hat{i} - 150\,\hat{j}\right] \bullet \left[20\,\hat{i} - 16\,\hat{j}\right]$$

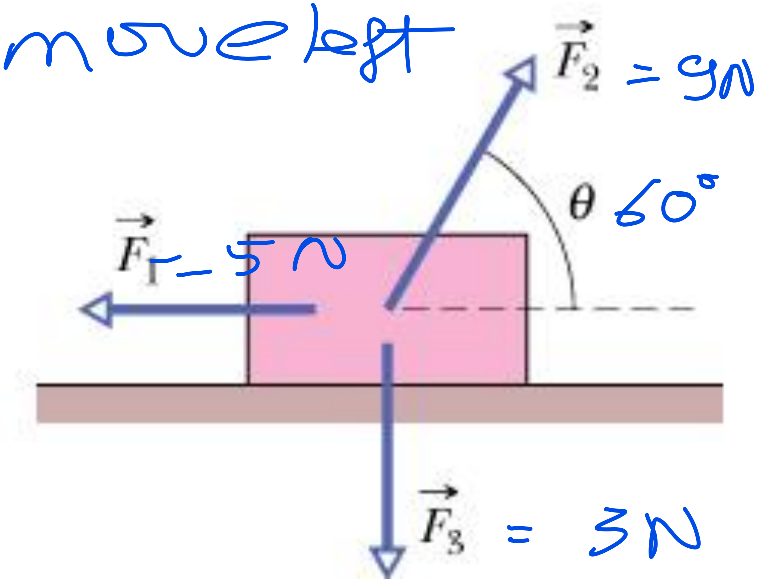
$$W = 210 \times 20 + (-150)(-16) = 6600\,(\text{J})$$

$W = \vec{d}\ \vec{F}$

15. The figure below shows three forces applied to a trunk that moves leftward by 3.00 m over a frictionless floor. The force magnitudes are F_1=5.00 N, F_2=9.00 N, and F_3=3.00 N, and the indicated angle is θ=60.0^0. During the displacement, (a) what is the net work done on the trunk by the three forces and (b) does the kinetic energy of the trunk increase or decrease?

(a) No friction

move left



$$W = \vec{F}\vec{d} = (\vec{F}_1 + \vec{F}_2 + \vec{F}_3)\vec{d}$$

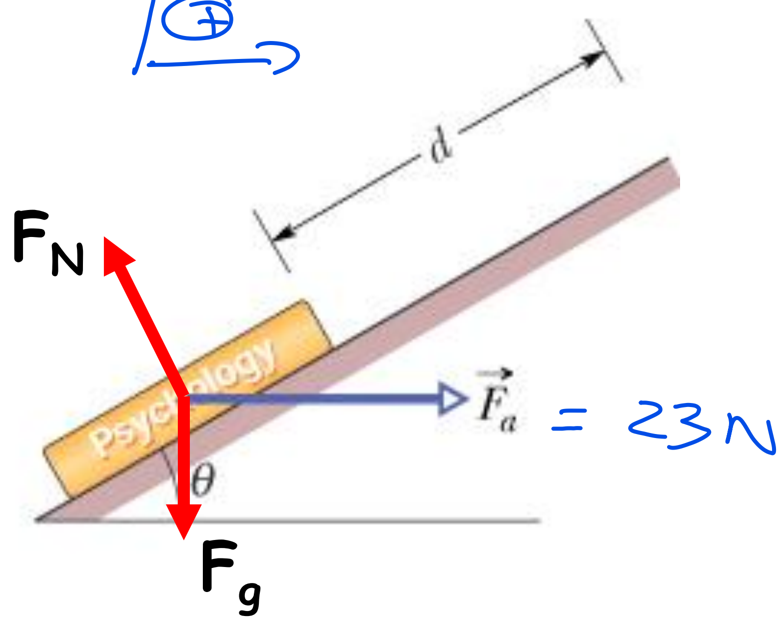
$$= F_1 d - F_2 d\cos\theta = 5\times3 - 9\times3\times0.5 = 1.5\,(J)$$

(b) According to the work-kinetic energy theorem:

$$W = \Delta K = 1.5\,(J)$$

→The kinetic energy of the box increases by 1.5 J.

24. In the figure below, a horizontal force $\vec{F}_a$ of magnitude 23.0 N is applied to a 3.00 kg psychology book as the book slides a distance d=0.580 m up a frictionless ramp at angle θ=30.0°. (a) During the displacement, what is the net work done on the book by $\vec{F}_a$, the gravitational force on the book, and the normal force on the book? (b) If the book has zero kinetic energy at the start of the displacement, what is its speed at the end of the displacement?



(a) Work done by $\vec{F}_a$:

$$W_a = \vec{F}_a \vec{d} = F_a \cos\theta \times d$$
$$= 23.0 \times \cos(30) \times 0.58 = 11.6 \text{ (J)}$$

Work done by F_g:

$$W_g = \vec{F}_g \vec{d} = -F_g \cos(90-\theta) \times d$$
$$= -3.0 \times 9.8 \times \cos(60) \times 0.58 = -8.5 \text{ (J)}$$

Work done by F_N:

$$W_N = \vec{F}_N \vec{d} = 0 = 0$$

→ the net work done by the three forces:

$$W_{net} = W_a + W_g + W_N = 11.6 + (-8.5) + 0 = 3.1 \text{(J)}$$

(b)

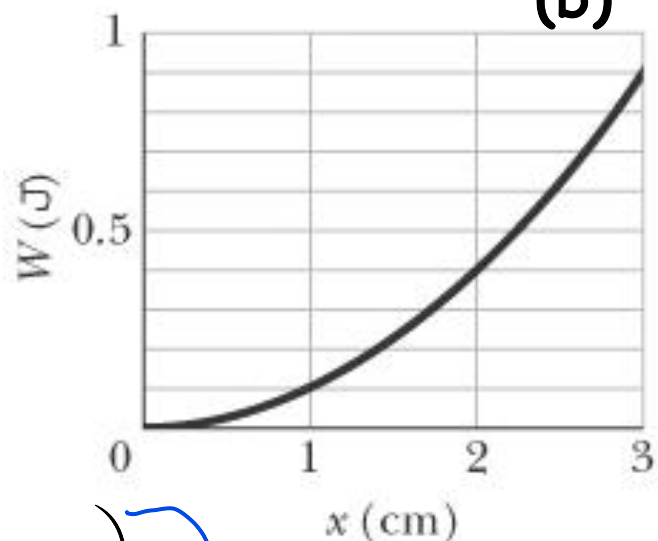
$$W = \Delta K = \frac{1}{2}mv^2 \Rightarrow v = \sqrt{\frac{2W}{m}}$$

$$v = \sqrt{\frac{2\times 3.1}{3}} = 1.44\ (m/s)$$

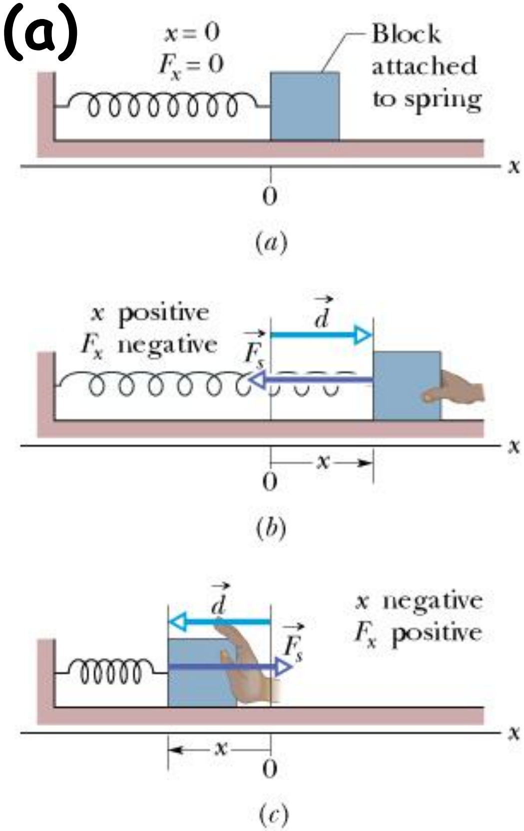
29. In the arrangement of figure a, we gradually pull the block from x=0 to x=+3.0 cm, where it is stationary. Figure b gives the work that our force does on the block. We then pull the block out to x=+5.0 cm and release it from rest. How much work does the spring do on the block when the block moves from x_i=+5.0 cm to (a) x=+4.0 cm, (b) x=-2.0 cm, and (c) x=-5.0 cm?

$$W_s = \frac{1}{2}kx_i^2 - \frac{1}{2}kx_f^2 \quad (1)$$

(b)



(a)



→ to calculate W_s, we need to compute k:

$$W = -W_s = -\left(\frac{1}{2}kx_i^2 - \frac{1}{2}kx_f^2\right)$$

from 0 to x_f=3 cm, W=0.9 J:

$$W = -\left(\frac{1}{2}kx_i^2 - \frac{1}{2}kx_f^2\right) = \frac{1}{2}kx_f^2$$

$$x_f = 3\text{cm} : k = \frac{2W}{x_f^2} = \frac{2\times(0.9\text{J})}{\left(3\times10^{-2}\,\text{m}\right)^2} = 2\times10^3(\text{N/m})$$

$K\,(N/m)$

(a) $$W_s = \frac{1}{2}kx_i^2 - \frac{1}{2}kx_f^2 = \frac{1}{2}\times 2\times10^3\left[(5\times10^{-2})^2 - (4\times10^{-2})^2\right]$$

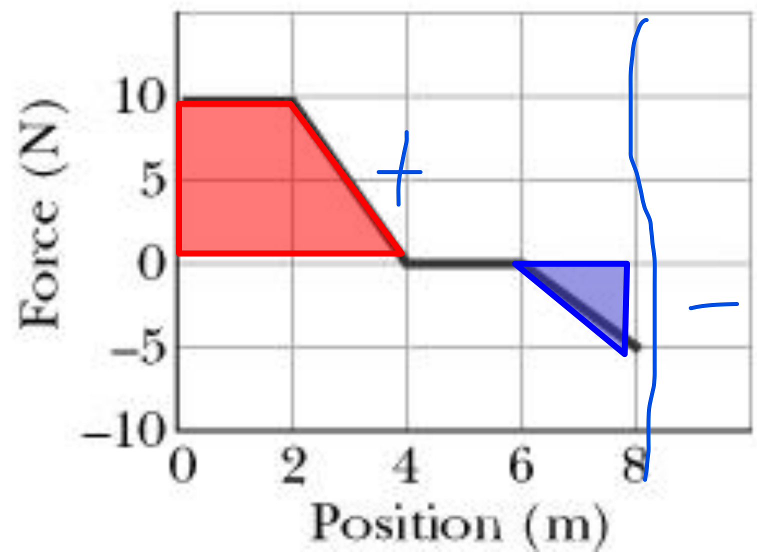
$$W_s = 0.9\,(\text{J})$$

(b) and (c): Use Equation (1)

$$W_s = U_{Ei} - U_{Ef}$$

36. A 2.5 kg block moves in a straight line on a horizontal frictionless surface under the influence of a force that varies with position as shown in the figure below. How much work is done by the force as the block moves from the origin to x=8.0 m?

$$W = \int_{x_i}^{x_f} F(x)dx$$



The red area = the work done by the force – its positive part

The blue area = the work done by the force - its negative part

1 box = 5 x 2 = 10 J

The net work done = the red area – the blue area = 3 boxes – 0.5 box = 2.5 x 10 = 25 J

|+| |-|

43. A force of 5.0 N acts on a 15 kg body initially at rest. Compute the work done by the force in (a) the first, (b) the second, and (c) the third seconds and (d) the instantaneous power due to the force at the end of the third second.

Wok done from t_i to t_f: $W = \vec{F}\vec{d} = F \times \left(\frac{1}{2} a t_f^2 - \frac{1}{2} a t_i^2 \right)$

Recall: $v_0 = 0 \Rightarrow d = \frac{1}{2} a t^2 : \; W = F \times \frac{1}{2} a t^2$

Newton's second law: $a = \frac{F}{m} \Rightarrow W = \frac{1}{2} \frac{F^2 t^2}{m}$

(a) $W = \frac{1}{2} \frac{5^2 \times 1^2}{15} = 0.83\,(J)$

(b) Work done from 1s to 2s: $W = \frac{1}{2} \frac{5^2 \times (2^2 - 1^2)}{15} = 2.5\,(J)$

(c) $W = 4.2\,(J)$

(d) Instantaneous power: $P = \frac{dW}{dt} = \frac{F^2 t}{m}; \text{at } t = 3\,s : P = \frac{5^2 \times 3}{15} = 5\,(W)$

Part B Laws of Conservation

Chapter 3 Work and Mechanical Energy

3.1. Kinetic Energy and Work. Power

3.2. Work-Kinetic Energy Theorem

3.3. Work and Potential Energy

3.4. Conservative and Non-conservative Forces. Conservative Forces and Potential Energy

3.5. Conservation of Mechanical Energy

3.6. Work Done on a System by an External Force. Conservation of Energy

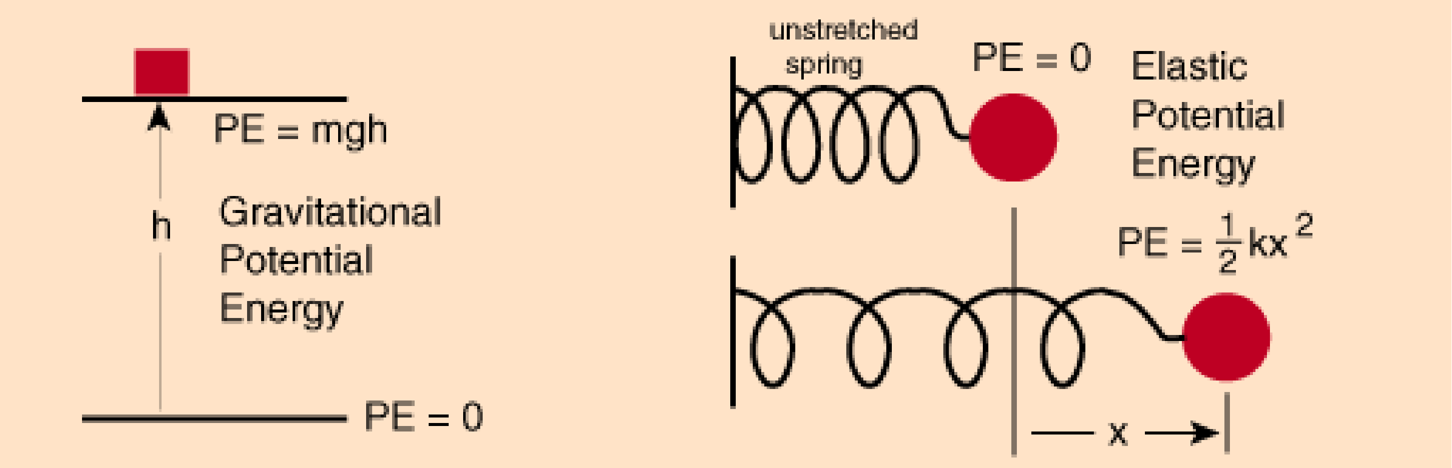
3.3. Work and Potential Energy

Potential energy

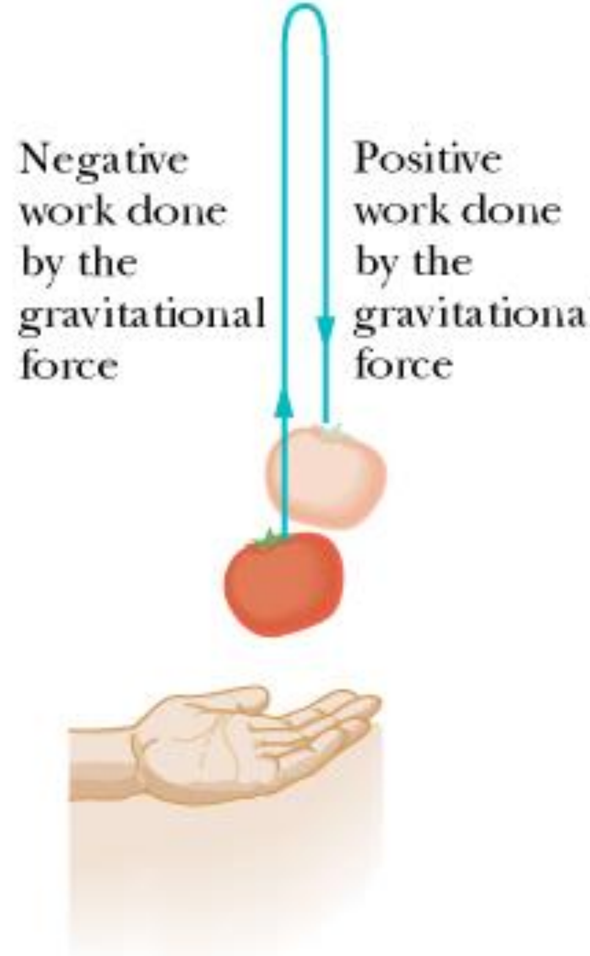
Potential energy U is energy associated with the configuration of a system of objects that exert forces on one another.

We study here two kinds of potential energy:

1. Gravitational potential energy: The energy is associated with the state of separation between two objects that attract each other, e.g., a jumper and the Earth.
2. Elastic potential energy: The energy is associated with the state of compression or extension of an elastic object, e.g., the bungee cord.



Example: throw a tomato upward.



When the tomato goes upward, the gravitational force does negative work, decreasing its kinetic energy: the kinetic energy is transferred by the gravitational force to the gravitational potential energy of the tomato-Earth system.

When it goes downward, the gravitational force does positive work, increasing its kinetic energy: the gravitational PE is transferred by the gravitational force to the KE of the tomato.

We define that the change ΔU in gravitational PE is equal to the negative of the work done on the object (e.g., the tomato) by the gravitational force:

$$\boxed{\Delta U = -W}$$

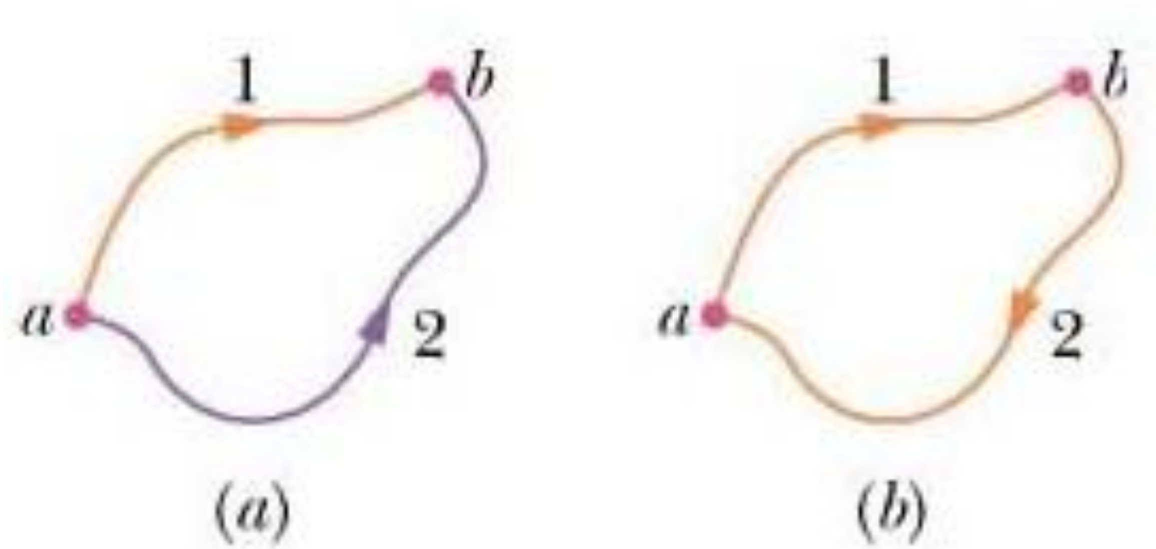
Note: This equation also applies to block-spring systems.

3.4. Conservative and Non-conservative Forces. Conservative Forces and Potential Energy

3.4.1. Conservative and Non-conservative Forces

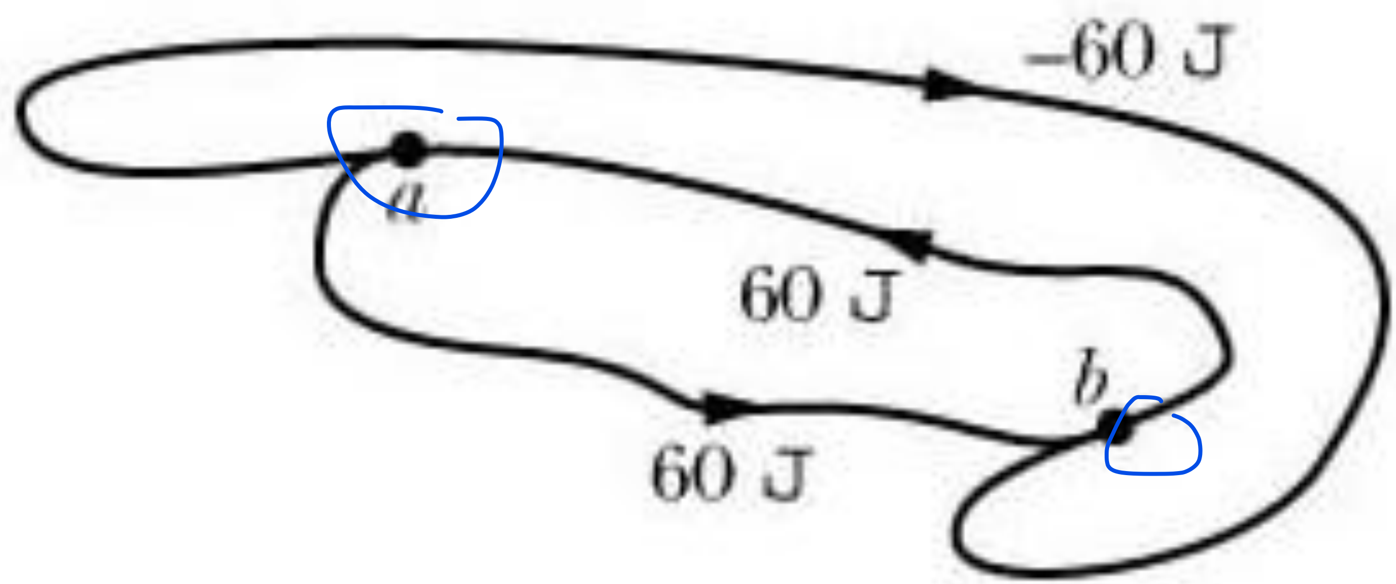
• A force is conservative if the work done by the force on an object moving between two points is independent of the path taken between the two points, e.g., gravitational force, spring force.

• The term "conservative" force comes from the fact that the conservative forces may cause the energy transfers within an isolated system but the mechanical of the system is conserved.



• A net work done by a conservative force on a particle moving around <u>any</u> closed path is zero.

Checkpoint 1: The figure shows three paths connecting points a and b. A single force F does the indicated work on a particle moving along each path in the indicated direction. On the basis information, is the force F conservative?

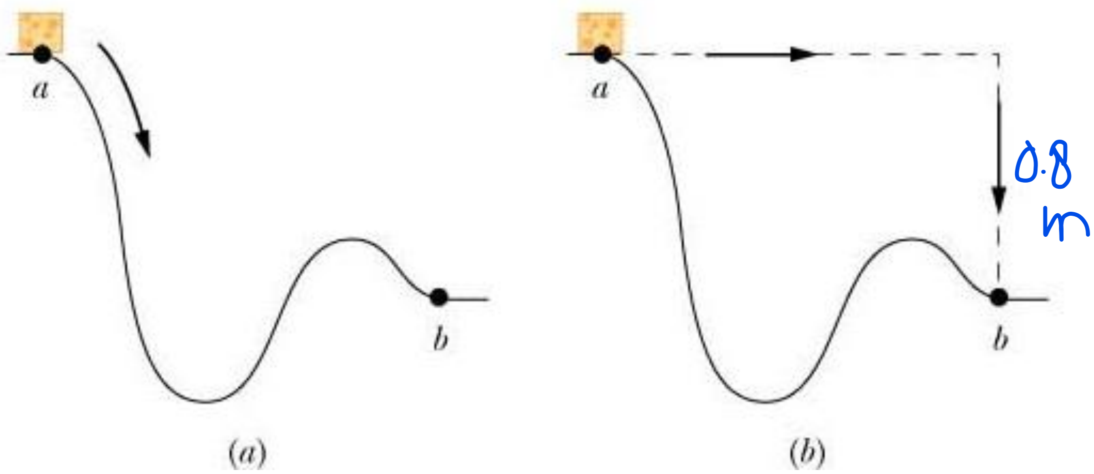


• **Nonconservative force:** A force that is not conservative is called a nonconservative force, e.g., frictional force: the work done by a frictional force depends on the path taken.

Sample Problem (p. 170): Figure 1 shows a 2 kg block of slippery cheese that slides along a frictionless track from point a to point b. The cheese travels through a total distance of 2.0 m along the track, and a net vertical distance of 0.8 m. How much work is done on the cheese by the gravitational force during the slide?

$d = 2$ m

The gravitational force F_g is a conservative force → choose a simple path to calculate the work done by F_g.



$$W = W_{horizontal} + W_{vertical}$$

$$W_{horizontal} = F_g \Delta x \cos\theta = 0 \text{ because } \theta = (F, \Delta x) = 90^0$$

$$W_{vertical} = F_g \Delta y \cos\theta = mg\Delta y \;\; (\theta = (F, \Delta y) = 0^0)$$

$$W_{vertical} = 2 \times 9.8 \times 0.8 \approx 15.7 \;\; (J)$$

$$W = W_{horizontal} + W_{vertical} = 15.7 \;\; (J)$$