ELECTROMAGNETIC THEORY

2021

Homework #2

Deadline:

Problem 1:

Given $\boldsymbol{A} = xyz(x\boldsymbol{i_x} + y\boldsymbol{i_y} + z\boldsymbol{i_z})$, evaluate $\oint_S \boldsymbol{A} \cdot d\boldsymbol{S}$, where S is the surface of the cubical box bounded by the planes $x = 0,\ x = 2,\ y = 0,\ y = 2,\ z = 0,\ z = 2$.

Problem 2:

A rigid rectangular loop of area *A* is situated in the xz-plane and symmetrically about the z-axis in a region of magnetic field $\boldsymbol{B} = B_0[\cos(2\omega t)\ \boldsymbol{a_x} + \sin(2\omega t)\ \boldsymbol{a_y}]\ Wb/m^2$. Find the induced emf around the closed path C of the loop for the cases where:

a) The loop is stationary
b) The loop revolves around the z-axis in the sense of decreasing ϕ with uniform angular velocity of $\omega\ rad/s$

The position of the loop at $t = 0$ is described in Fig. 1

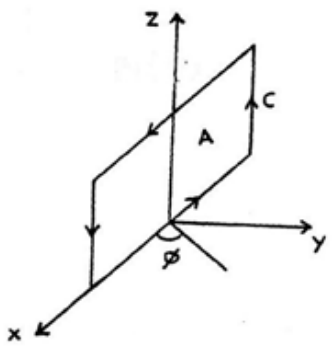

Figure 1

Problem 3:

A current density due to flow of charges is given by $\mathbf{J} = -(x\boldsymbol{a}_x + y\boldsymbol{a}_y + z^2\boldsymbol{a}_z)\ A/m^2$. Find the displacement current emanating from the closed surface of the cylindrical box bounded by the surfaces r = 1, z =0 and z =2 as the figure 1.

Hint: *Using the property:* $r\boldsymbol{a}_r = x\boldsymbol{a}_x + y\boldsymbol{a}_y$

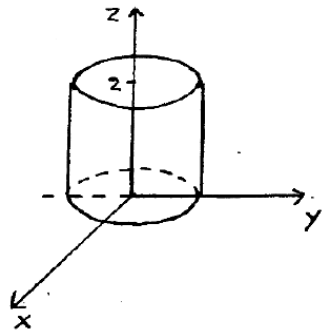

Figure 2

Problem 4:

Find the absolute value of the magnetic flux crossing that portion of the surface $y=sinx$ bounded by $x=0$, $x=\pi$, $z=0$ and $z=1$ for $\boldsymbol{B}= B0(y\boldsymbol{a}_x - x\boldsymbol{a}\boldsymbol{y})$ Wb/m^2.

Hint: Using the property that $\oint \boldsymbol{B}\cdot\boldsymbol{dS}=\boldsymbol{0}$, $\boldsymbol{S}$ find $\int_{S1} \boldsymbol{B}\cdot\boldsymbol{dS1}$ in the figure 3.

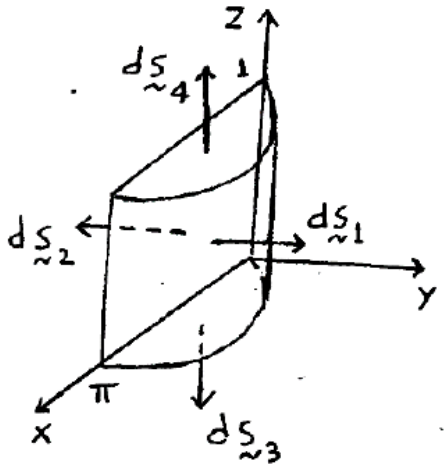

Figure 3