International University

School of Electrical Engineering

PRINCIPLES OF ELECTRICAL ENGINEERING 2

Lecture # 8b Supplement

FREQUENCY RESPONSE OF AC CIRCUIT

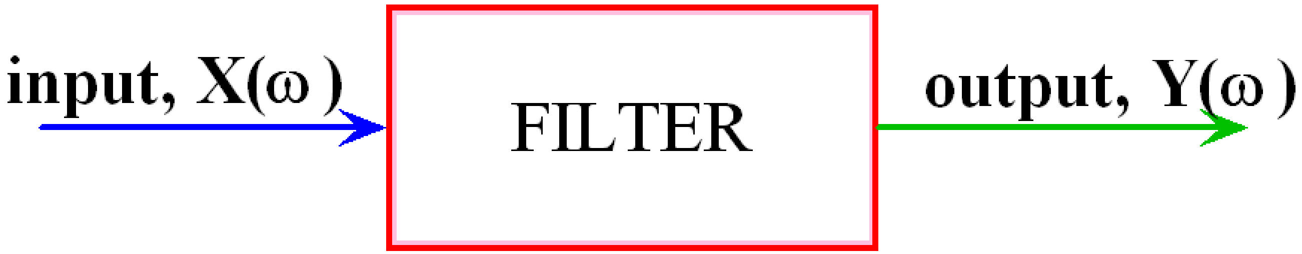
- Some Preliminaries

  ✓Analysis of a circuit with varying frequency of a sinusoidal sources is called the frequency response of a circuit.

  ✓Frequency selection in the circuits are called filters because of their ability to filter out certain input signals on the basis of frequency.

TRANSFER FUNCTION (TF or tf)

- Frequency response can be obtained by using transfer function.



- DEFINITION: Transfer function, H(ω) is a ratio between output & input signals (in s-domain or jω).

$$H(\omega) = \frac{Y(\omega)}{X(\omega)}$$

TRANSFER FUNCTION

$$H(\omega) = \frac{Y(\omega)}{X(\omega)}$$

Output signal

Input signal

$$\boxed{H(\omega) = \left|H(\omega)\right| \angle \phi} = He^{j\phi}$$

Using sinusoidal source, the transfer function will be the magnitude and phase of output voltage to the magnitude and phase of input voltage of a circuit .

4 conditions of TF:

$$H(\omega) = voltage \quad gain = \frac{V_o(\omega)}{V_i(\omega)}$$

$$H(\omega) = current \quad gain = \frac{I_o(\omega)}{I_i(\omega)}$$

Because there is no unit, they are called GAIN

$$H(\omega) = impedance = \frac{V(\omega)}{I(\omega)}$$

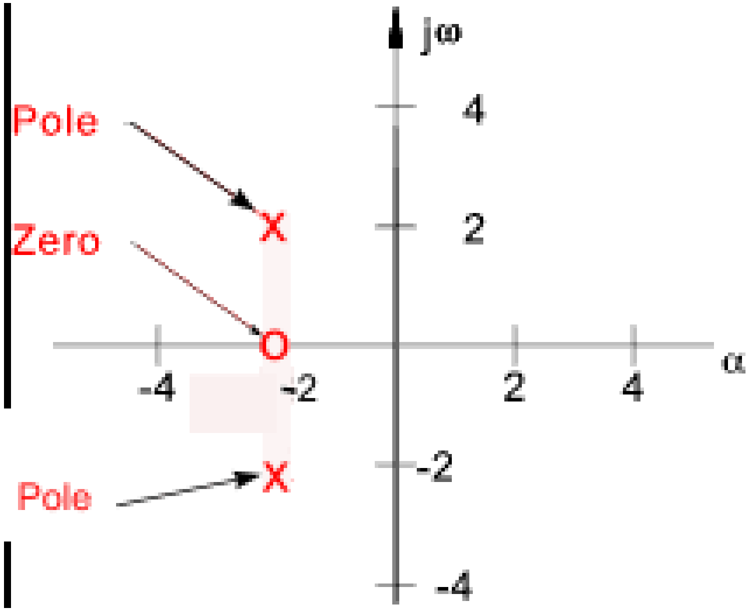
$$H(\omega) = admi\tan ce = \frac{I(\omega)}{V(\omega)}$$

POLES & ZEROS

- Transfer function is written in fraction
- The numerator and denominator can be existed as a polynomial

$$H(\omega) = \frac{Y(\omega)}{X(\omega)} \Rightarrow H(\omega) = \frac{N(\omega)}{D(\omega)}$$

- The roots of numerator also known as **ZEROS**. Zeros exist when N(ω)=0
- The roots of denominator also known as **POLES**. Poles exist when D(ω)=0



- The symbol for pole is x
- The symbol for zero is o
- Complex s-plane is used to plot poles and zeros.

POLES/ZEROS

Poles/zeros at the origin

real zero

quadratic zero

$$H(\omega)=\frac{K(j\omega)^{\pm 1}(1+j\omega/z_1)\left[1+j2\xi_1\omega/\omega_k+(j\omega/\omega_k)^2\right]\ldots}{(1+j\omega/p_1)\left[1+j2\xi_2\omega/\omega_n+(j\omega/\omega_n)^2\right]\ldots}$$

real pole

quadratic pole

LOCATION OF POLES/ZEROS

- Zeros/poles at the origin: Zeros/poles that are located at 0
- Real Zeros/poles: Zeros/poles that are located at real axis (-1, -2, 1, 2, 10, etc.)
- Quadratic Zeros/poles: Zeros/poles that are not located at imaginary or real axis (-1+j2, 2+j5, 3-j3, etc.)

EXAMPLE

$$H(\omega) = \frac{j\omega(j\omega+1)(j\omega+2)}{(j\omega+1)(j\omega+4)}$$

Simplified,

$$H(\omega) = \frac{j\omega(j\omega+2)}{(j\omega+4)}$$

Simplified,

$$H(\omega)=\frac{j\omega\left(j\omega+2\right)}{\left(j\omega+4\right)}$$

ZEROS

- Let numerator, N(ω)=0

$$j\omega\left(j\omega+2\right)=0$$

1st zero:

$$j\omega=0$$

2nd zero:

$$j\omega+2=0$$

$$\therefore\ j\omega=-2$$

POLE

- Let denominator, D(ω)=0

$$\left(j\omega+4\right)=0$$

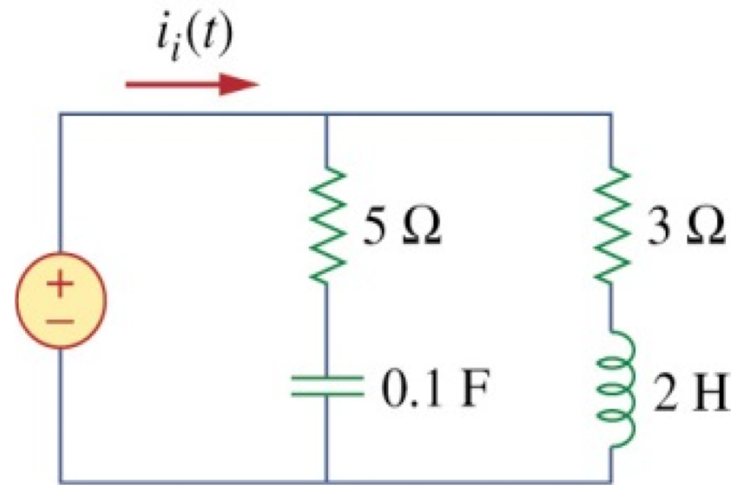
pole:

$$j\omega+4=0$$

$$\therefore\ j\omega=-4$$

Example

Find the input impedance $Z_i(\omega)$, poles and zeroes, ω_n and ζ.



$$\mathbf{Z}_i(s) = \frac{\mathbf{V}_o(s)}{\mathbf{I}_o(s)} = \left(5 + \frac{1}{s/10}\right) \| (3+2s)$$

$$\mathbf{Z}_i(s) = \frac{(5+10/s)(3+2s)}{5+10/s+3+2s} = \underline{\frac{\mathbf{5(s+2)(s+1.5)}}{\mathbf{s^2+4s+5}}}$$

$$Z_i(\omega) = \frac{5(j\omega+2)(j\omega+1.5)}{(j\omega)^2+4j\omega+5} = 3\frac{\left(1+j\frac{\omega}{2}\right)\left(1+j\frac{\omega}{1.5}\right)}{1+j2\left(\frac{2}{\sqrt{5}}\right)\left(\frac{\omega}{\sqrt{5}}\right)+\left(j\frac{\omega}{\sqrt{5}}\right)^2} = K\frac{\left(1+j\frac{\omega}{z_1}\right)\left(1+j\frac{\omega}{z_2}\right)}{1+j2\zeta\frac{\omega}{\omega_n}+\left(j\frac{\omega}{\omega_n}\right)^2}$$

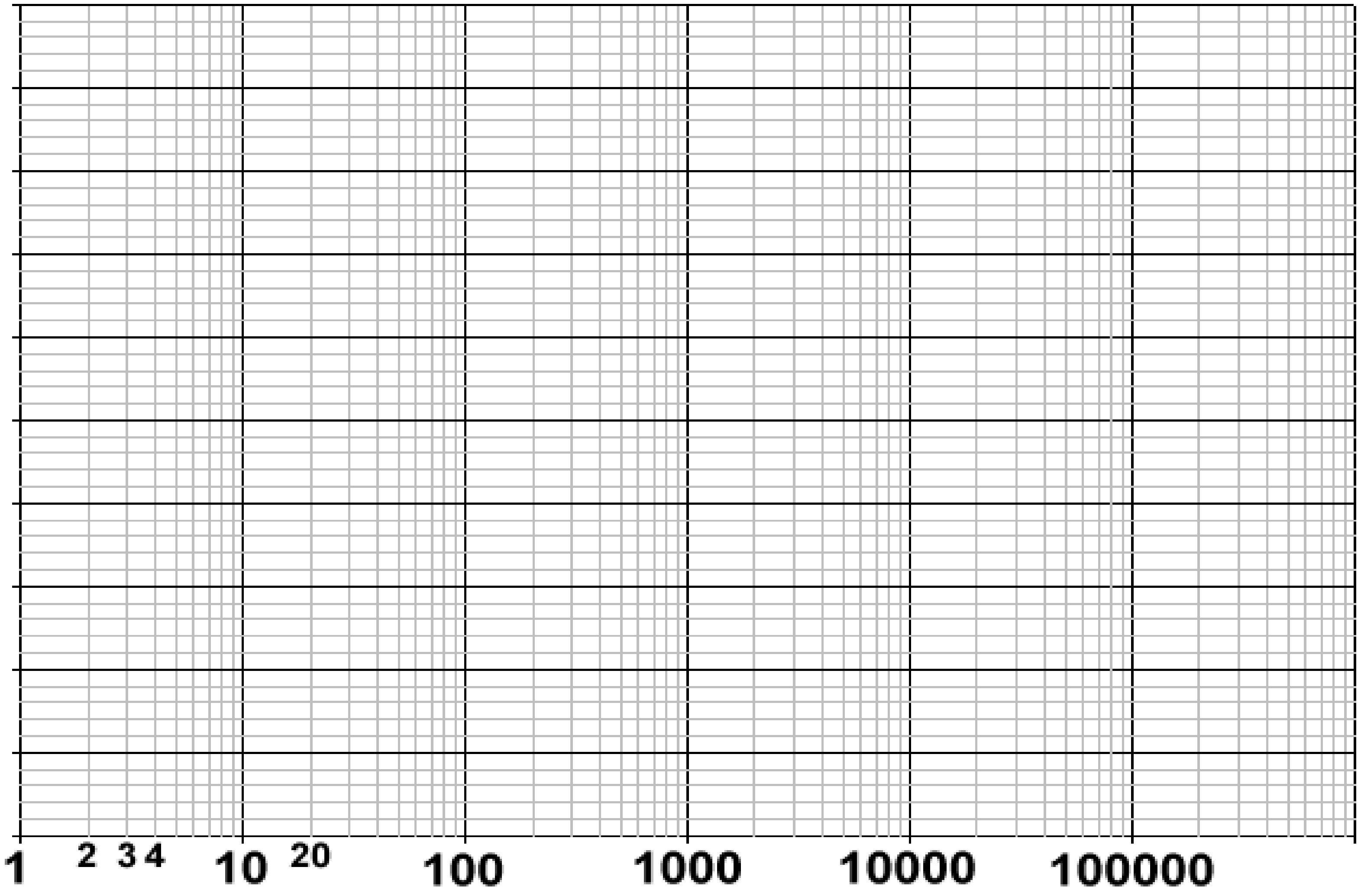
$K=3,\ z_1=2,\ z_2=1.5.$ Quadratic Pole: damping factor $\zeta = \frac{2}{\sqrt{5}},\ \omega_n = \sqrt{5}$

zeros at $s = j\omega = -2$ and -1.5

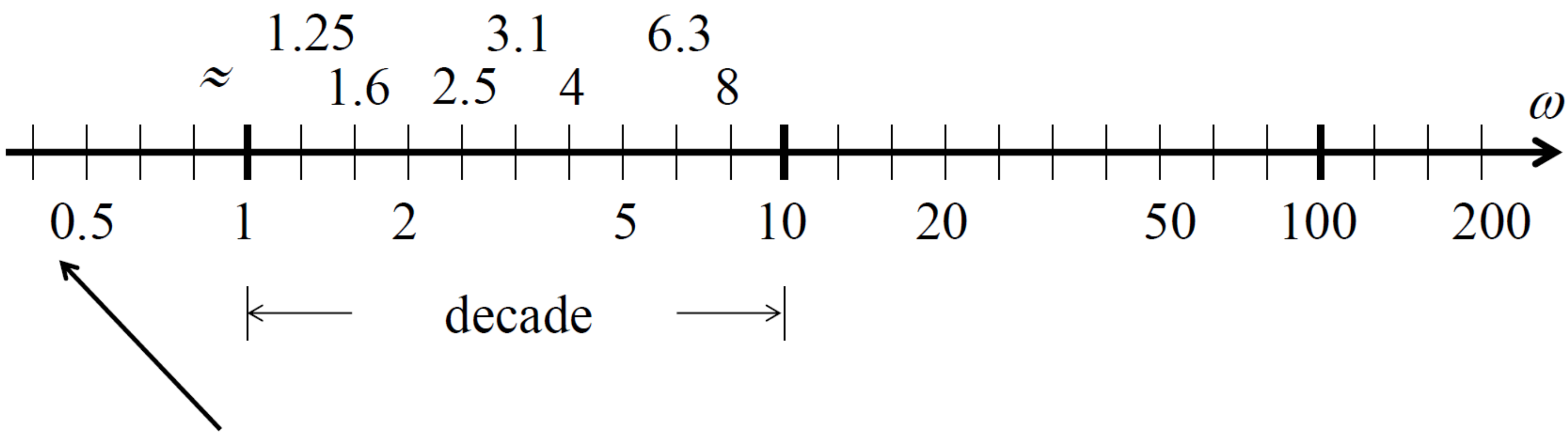
Note that, z_1 and z_2 in this example are based on the transfer function definition mentioned before.

FREQUENCY RESPONSE PLOT

- USING SEMILOG GRAPH



Sketching a Log Frequency Scale



Note: There is no point $\omega = 0$ on a logarithmic frequency scale !

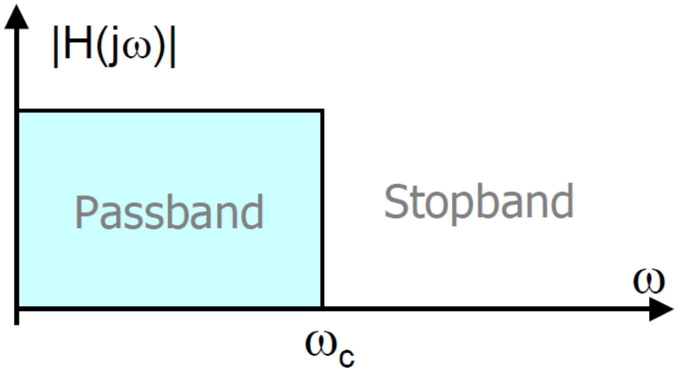
A decade is an interval between two frequencies with a ratio of 10;
e.g., between ω_0 and $10\omega_0$, or between 10 Hz and 100 Hz.

MAGNITUDE PLOT & PHASE PLOT

Using transfer function of circuit, we plot a frequency response of the circuit for both **amplitude** & **phase** with changing **source frequency**

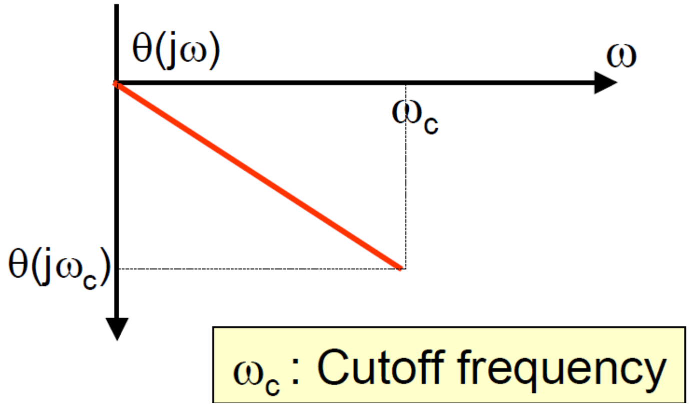
- Magnitude plot

$$\left|H(j\omega)\right| \;\; vs \;\; frequency(\omega)$$



- Phase angle plot

$$\theta(j\omega) \;\; vs \;\; frequency(\omega)$$



HOW TO DO MAGNITUDE AND PHASE PLOT

i. Transform the time domain circuit (t) into freq. domain circuit (ω)

ii. Determine the TF, H(ω)

iii. Plot the magnitude of that TF, $|H(\omega)|$ against ω.

iv. Plot the phase of that TF, $\phi(^{o})$ against ω.

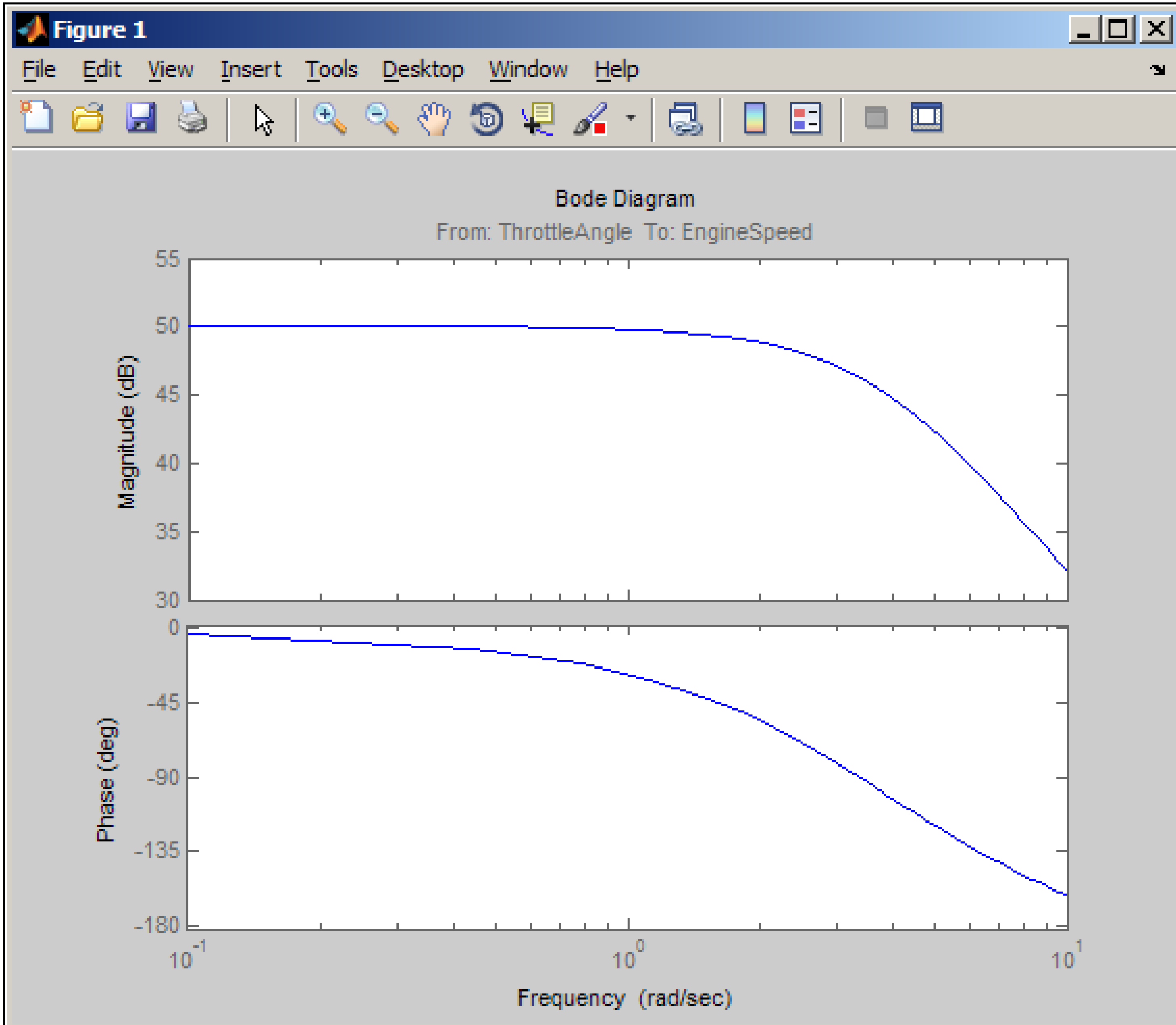
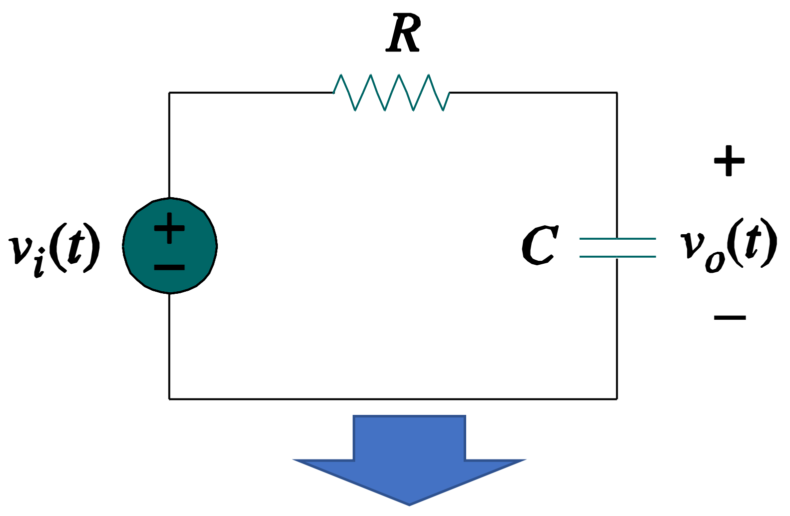


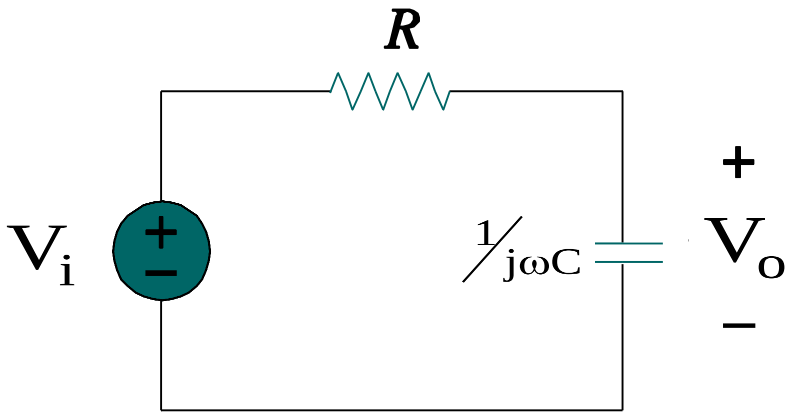
Figure 1
File
Edit
View
Insert
Tools
Desktop
Window
Help
Bode Diagram
From: ThrottleAngle To: EngineSpeed
Magnitude (dB)
55
50
45
40
35
30
Phase (deg)
0
-45
-90
-135
-180
10^{-1}
10^{0}
10^{1}
Frequency (rad/sec)

THE CONCEPT OF TF



CIRCUIT IN FREQUENCY DOMAIN

- Input is V_i & Output is V_o,



➔ OBTAINED THE TF:

$$H(\omega) = \frac{V_o}{V_i} = \frac{1/\,j\omega C}{R + 1/\,j\omega C}$$

$$= \frac{1}{1 + j\omega RC}$$

MAGNITUDE OF TF

$$H(\omega)=\frac{1}{1+j\omega RC}$$

Magnitude;

$$\therefore\left|H(\omega)\right|=\frac{1}{\sqrt{1+(\omega RC)^2}}$$

$$\text{If } \omega_c = 1/RC$$

$$\therefore\left|H(\omega)\right|=\frac{1}{\sqrt{1+\left(\omega/\omega_c\right)^2}}$$

PHASE OF TF

$$\left|H(\omega)\right| = \frac{1}{\sqrt{1+(\omega RC)^2}}$$

$$\phi = -Tan^{-1}\left(\frac{\omega RC}{1}\right) = -Tan^{-1}(\omega RC)$$

If $\omega_c = 1/RC$

$$\therefore \phi = -Tan^{-1}\left(\frac{\omega}{\omega_C}\right)$$

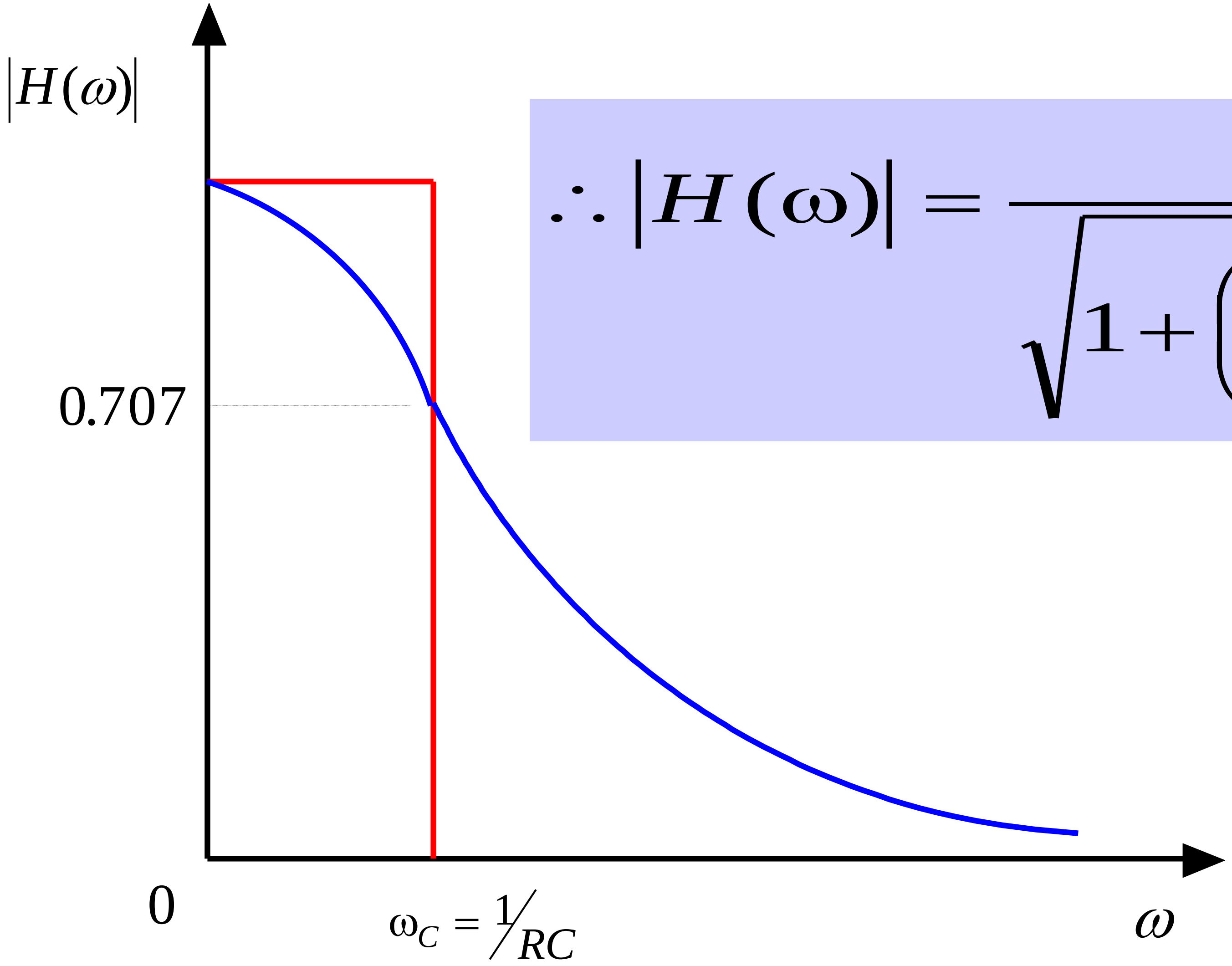
CUT-OFF FREQUENCY:

$$\omega_C = \frac{1}{RC}$$

THE VALUES OF MAGNITUDE AND PHASE

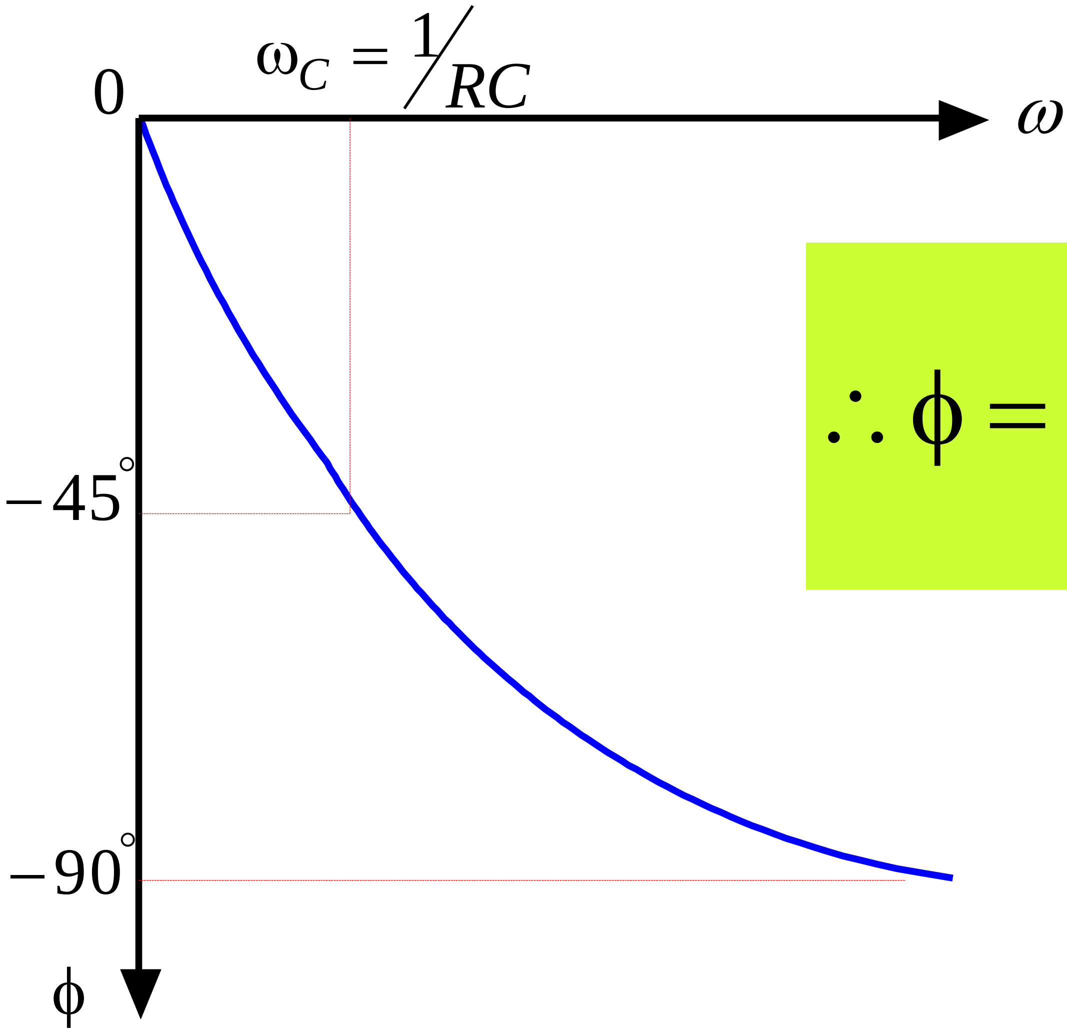
ω/ω_C	H	ϕ
0	1	0
1	0.71	-45°
2	0.45	-63°
3	0.32	-72°
10	0.1	-84°
20	0.05	-87°
100	0.01	-89°
∞	0	-90°

MAGNITUDE PLOT



$$\therefore |H(\omega)| = \frac{1}{\sqrt{1 + \left(\omega / \omega_c \right)^2}}$$

PHASE PLOT



$$\therefore \phi = -Tan^{-1}\left(\frac{\omega}{\omega_C}\right)$$

FREQUENCY RESPONSE PLOT

BODE PLOTS

Hendrik Wade Bode

From Wikipedia, the free encyclopedia

Hendrik Wade Bode (/ˈboʊdi/ *boh-dee*; Dutch: [ˈbodə])[1] (December 24, 1905 – June 21, 1982)[1] was an American engineer, researcher, inventor, author and scientist, of Dutch ancestry. As a pioneer of modern control theory and electronic telecommunications he revolutionized both the content and methodology of his chosen fields of research.

He made important contributions to the design, guidance and control of anti-aircraft systems during World War II and, continuing post-World War II during the Cold War, to the design and control of missiles and anti-ballistic missiles.[2]

He also made important contributions to control system theory and mathematical tools for the analysis of stability of linear systems, inventing Bode plots, gain margin and phase margin.

Bode was one of the great engineering philosophers of his era.[3] Long respected in academic circles worldwide,[4][5] he is also widely known to modern engineering students mainly for developing the asymptotic magnitude and phase plot that bears his name, the Bode plot.

His research contributions in particular were not only multidimensional but far

Hendrik Wade Bode

Hendrik Wade Bode

Born	December 24, 1905 Madison, Wisconsin
Died	June 21, 1982 (aged 76) Cambridge, Massachusetts
Residence	Cambridge, Massachusetts
Nationality	American

FREQUENCY RESPONSE PLOT

BODE PLOTS

Bode plots are semilog plots of magnitude (in decibels) and phase (in degrees) of a transfer function versus frequency

DECIBEL SCALE

Logarithm

$$\log P_1 P_2 = \log P_1 + \log P_2$$

$$\log P_1 / P_2 = \log P_1 - \log P_2$$

$$\log P^n = n \log P$$

$$\log 1 = 0$$

BODE PLOT CHARACTERISTIC FOR **POLES & ZEROS**

Logarithm of TF:

$$H(\omega) = \frac{N(\omega)}{D(\omega)}$$

$$\therefore \log H = \log[N] - \log[D]$$

Generally, the power gain is measured in *bels*

$$G = \text{Number of } bels = \log_{10}\frac{P_2}{P_1}; \ (P_1 \text{ and } P_2 \text{ are power.})$$

- Decibel (dB) is 1/10th of a *bel*, and is

$$G_{dB} = 10\log_{10}\frac{P_2}{P_1}$$

$$G_{dB} = 20\log_{10}\frac{V_2}{V_1}$$

TRANSFER FUNCTION

$$\mathbf{H} = H\angle\phi = He^{j\phi} \qquad H_{\text{dB}} = 20\log_{10} H$$

Magnitude H	**20** $\log_{10} H$ **(dB)**
0.001	-60
0.01	-40
0.1	-20
0.5	-6
$1/\sqrt{2}$	-3
1	0
$\sqrt{2}$	3
2	6
10	20
20	26
100	40

GENERAL EQUATION/STANDARD FORM OF TF

- Before draw, make sure the general equation of tf is obtained first:

$$H(\omega)=\frac{K(j\omega)^{\pm}(1+j\omega/z_1)\left[(j\omega)^2+2\xi_1\omega_n+\omega_n{}^2\right]}{(1+j\omega/p_1)[(j\omega)^2+2\xi_2\omega_n+\omega_n{}^2]}$$

COMPARE

Ex.

$$H(\omega)=\frac{2(j\omega)(j\omega+1)\left[(j\omega)^2+30\omega+100\right]}{(j\omega+2)[(j\omega)^2+50\omega+400]}$$

ξ is called the damping factor

$$H(\omega)=\frac{2(j\omega)(1+j\omega/1)\left[(j\omega)^2+30\omega+10^2\right]}{(1+j\omega/2)[(j\omega)^2+50\omega+20^2]}$$

Constant:

$K=2$

Zero at the origin:

$j\omega=0$

Real zero:

$j\omega=1$

Quadratic zero:

$\omega_n=10$

Real pole:

$j\omega=2$

Quadratic pole:

$\omega_n=20$

Bode Plots

Steps to construct a Bode plot

– Plot each factor separately.

– Additively combine all of them graphically because of the logarithms involved

- The mathematical convenience of the logarithm makes the Bode plots a powerful tool.
- Straight-line plots used instead of actual plots

BODE PLOT OF A CONSTANT, K

(1) (GAIN)

constant

$$H(\omega) = \frac{K(j\omega)^{\pm}(1+j\omega/z_1)\left[(j\omega)^2 + 2\xi_1\omega_n + \omega_n{}^2\right]}{(1+j\omega/p_1)[(j\omega)^2 + 2\xi_2\omega_n + \omega_n{}^2]}$$

CHARACTERISTICS

- Magnitude for constant is : $\mathrm{H} = 20\log|\mathrm{K}|$
- Phase angle for constant is: $\phi = 0^{\circ}$

BODE PLOT FOR CONSTANT

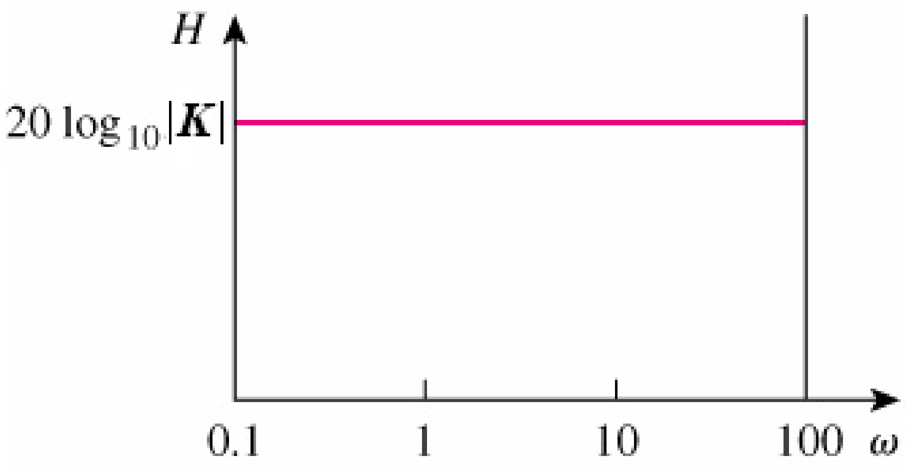
$$\mathbf{H}^{\text{const}}(\omega) = K$$

$$\Rightarrow H_{\text{dB}}^{\text{const}} = 20\log_{10}|K|$$

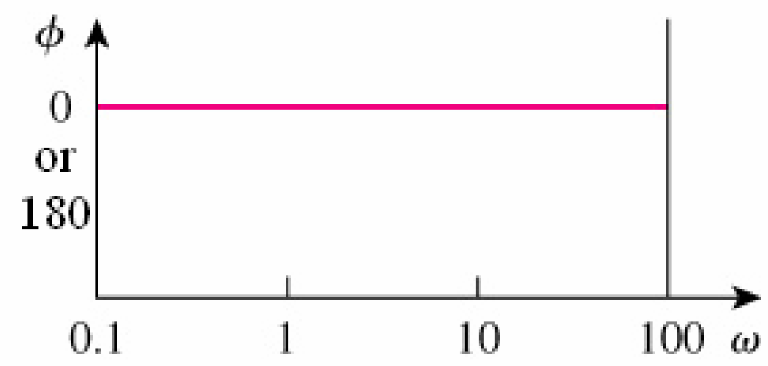
$$\Rightarrow \begin{cases} \phi = 0^\circ & \text{if } K > 0 \\ \phi = \pm 180^\circ & \text{if } K < 0 \end{cases}$$

For the gain K, the magnitude is 20 $\log_{10}(K)$ and the phase is 0°; both are constant with frequency. If K is negative, the magnitude is 20 $\log_{10}(|K|)$ but the phase is ±180°.

- magnitude plot



- phase plot



BODE PLOT FOR ZERO AT THE ORIGIN

(2) ZERO AT THE ORIGIN $(j\omega)^N$

$$H(\omega) = \frac{K\boxed{(j\omega)^{\pm}}(1 + j\omega/z_1)\left[(j\omega)^2 + 2\xi_1\omega_n + \omega_n{}^2\right]}{(1 + j\omega/p_1)[(j\omega)^2 + 2\xi_2\omega_n + \omega_n{}^2]}$$

$$\boxed{\begin{gathered}\mathbf{H}^{\text{origin\_zero}}(\omega) = j\omega = \omega\angle 90^\circ \\ \Rightarrow \begin{cases} H_{\text{dB}}^{\text{origin\_zero}} = 20\log_{10}\omega \\ \phi = 90^\circ \end{cases}\end{gathered}}$$

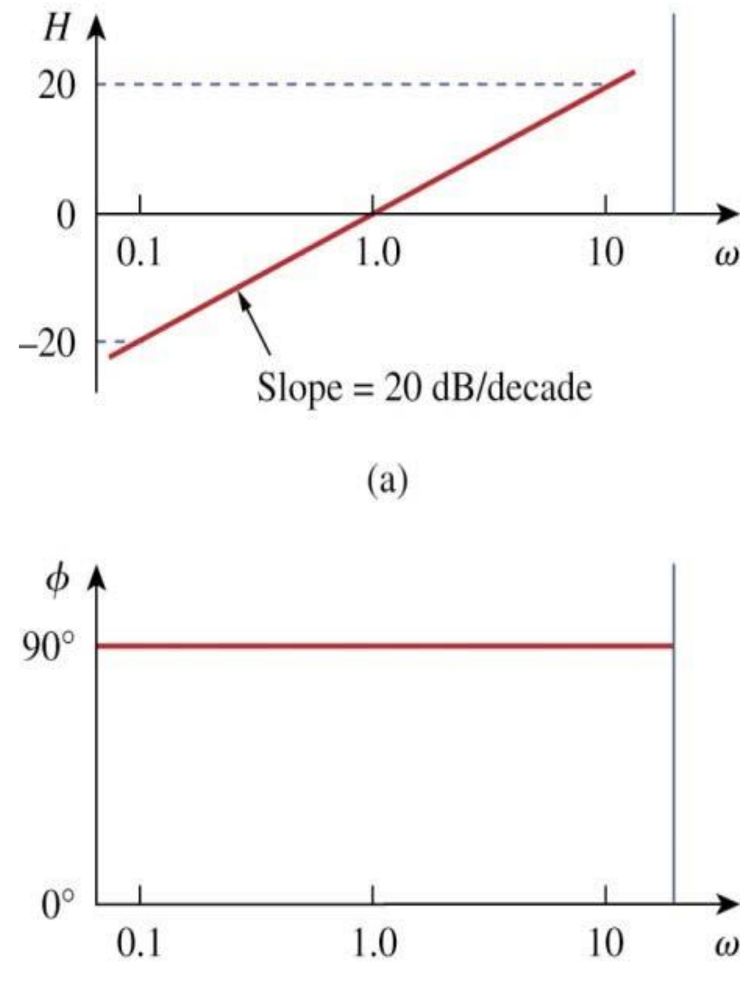
(2) ZERO AT THE ORIGIN $(j\omega)^N$

For the zero $(j\omega)$ at the origin, the magnitude is $20\log_{10}(\omega)$ and the phase is 90°.

The slope of the magnitude plot is 20 dB/decade, while the phase is 90° and constant with frequency.

The Bode plots for the pole $1/(j\omega)$ are similar except that the slope of the magnitude plot is -20 dB/decade while the phase is -90°.

In general, for $(j\omega)^N$, where N is an integer, the magnitude plot will have a slope of $20N$ dB/decade, while the phase is $90N$ degrees.



(a)

(b)

CHARACTERISTIC OF $(j\omega)^N$

- Magnitude:
  - Straight line with 20dB/decade of slope that has a value of 0 dB at ω=1

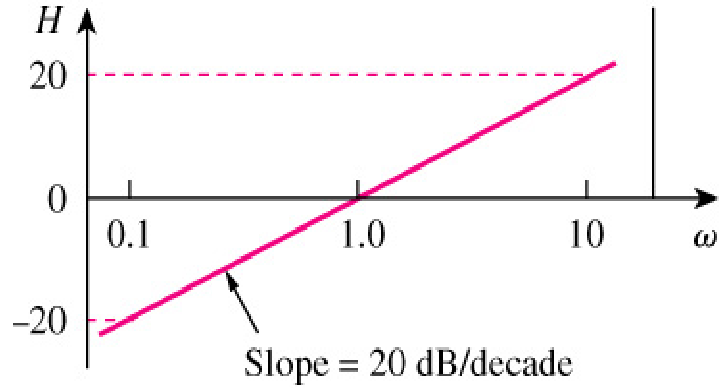
$$H = 20N \ \ (\text{dB/dec})$$

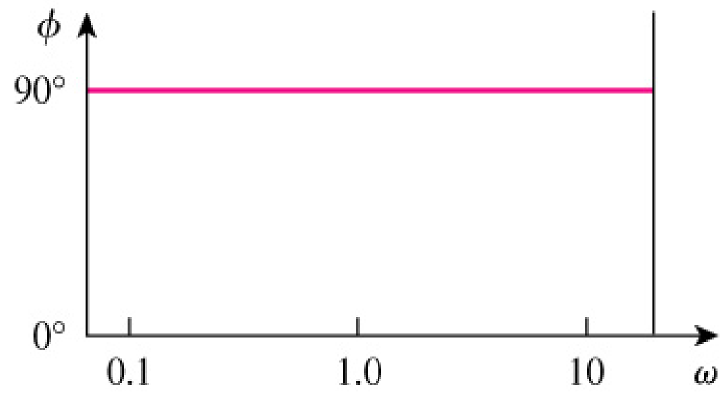
Thus, ±20 dB/decade means that the magnitude changes ± 20 dB whenever the frequency changes tenfold or one decade.

- Phase:

$$\phi = 90N^{\circ}$$

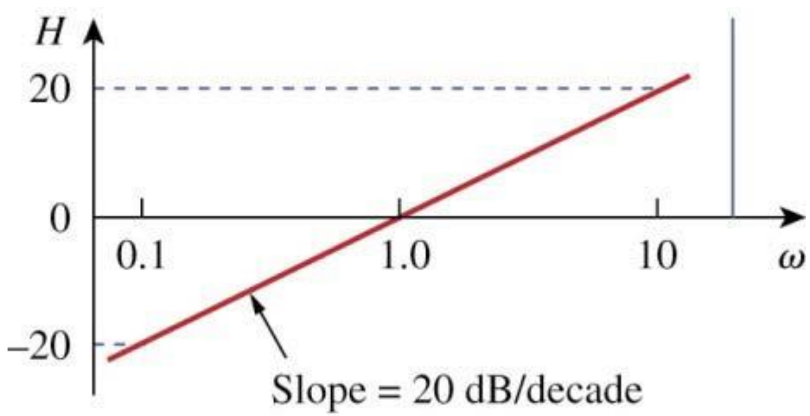
MAGNITUDE PLOT





PHASE PLOT

CHARACTERISTIC OF $(j\omega)^N$



$$H(\omega) = K \cdot j\omega, \ \text{slope} = +20\,\text{dB/decade}$$

$$K = ? \quad |H(\omega = 1)| = K$$

$$H_{dB} = 20\log_{10}(K) = 0\,\text{dB} \Rightarrow \text{K=1}$$

$$\text{If} \quad H(\omega) = K \cdot (j\omega)^2, \ \text{slope} = +40\,\text{dB/decade}$$

$$\text{If} \quad H(\omega) = \frac{K}{j\omega} \qquad \Rightarrow \text{slope} = -20\,\text{dB/decade}$$

BODE PLOT OF POLE AT THE ORIGIN

(3) POLE AT THE ORIGIN $1/(j\omega)^N$ @ $(j\omega)^{-N}$

$$H(\omega)=\frac{K(1+j\omega/z_1)\left[(j\omega)^2+2\xi_1\omega_n+\omega_n{}^2\right]}{\boxed{(j\omega)^{\pm}}(1+j\omega/p_1)[(j\omega)^2+2\xi_2\omega_n+\omega_n{}^2]}$$

$$\mathbf{H}^{\text{origin\_pole}}(\omega)=(j\omega)^{-1}=\omega^{-1}\angle-90^\circ$$

$$\Rightarrow\begin{cases} H_{\text{dB}}^{\text{origin\_pole}}=-20\log_{10}\omega \\ \phi=-90^\circ \end{cases}$$

CHARACTERISTIC OF $(j\omega)^{-N}$

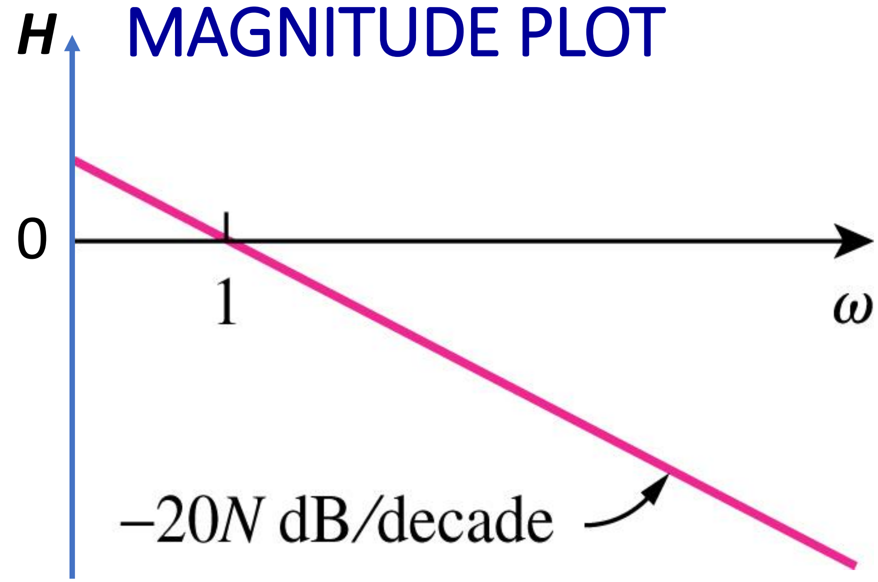
- Magnitude:
  - Straight line with -20dB/dec of slope that has a value of 0 dB at ω=1

$$H = -20N \ (dB/dec)$$

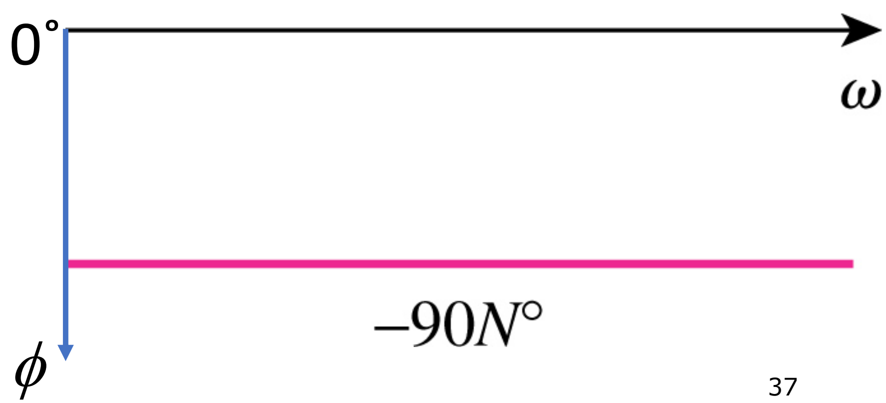
- Phase:

$$\phi = -90N^{\circ}$$

MAGNITUDE PLOT



PHASE PLOT



BODE PLOT OF REAL ZERO or SIMPLE ZERO

(4) REAL ZERO

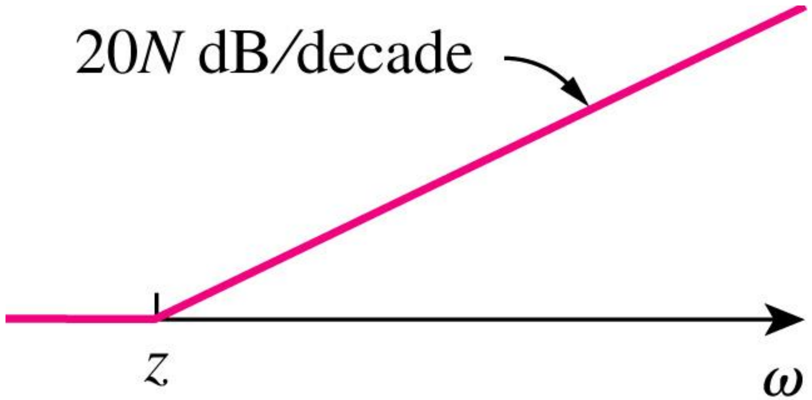
$$H(\omega)=\frac{K(j\omega)^{\pm}\boxed{(1+j\omega/z_1)}\left[(j\omega)^2+2\xi_1\omega_n+\omega_n{}^2\right]}{(1+j\omega/p_1)[(j\omega)^2+2\xi_2\omega_n+\omega_n{}^2]}$$

CHARACTERISTIC OF $(1+j\omega/z_1)^N$

- Magnitude:

$$H = \begin{cases} 0 & \omega < z_1 \\ 20\text{N (dB/dec)} & \omega \geq z_1 \end{cases}$$

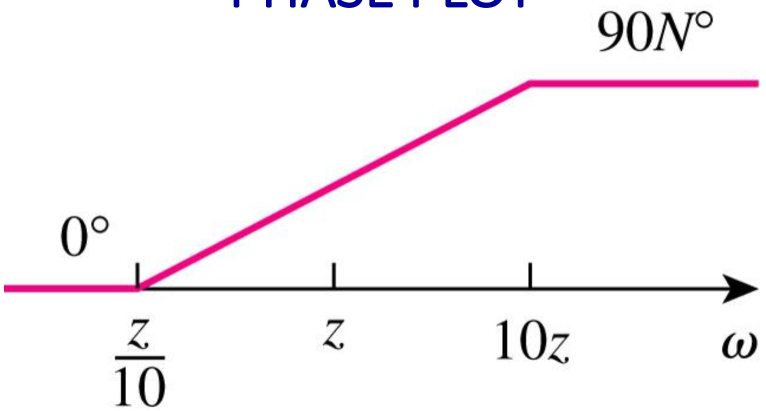
MAGNITUDE PLOT



- Phase:

$$\phi = \begin{cases} 0 & \omega = 0 \\ 45^o & \omega = z_1 \\ 90^o & \omega \to \infty \end{cases}$$

PHASE PLOT

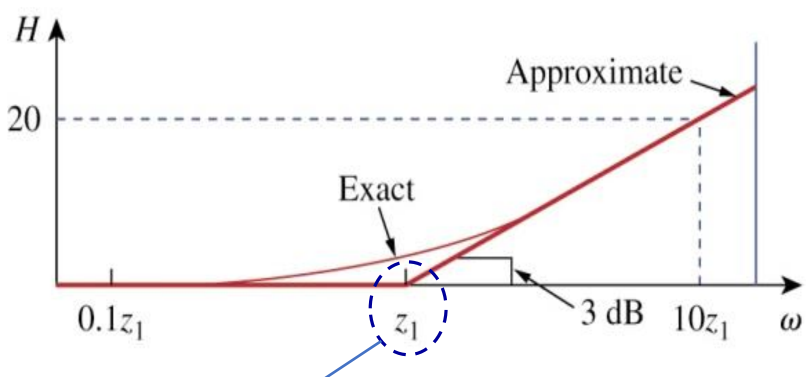


$$\mathbf{H}^{\text{simple\_zero}}(\omega) = \left(1 + j\omega/z_1\right)$$

$$\Rightarrow \begin{cases} H_{\text{dB}}^{\text{simple\_zero}} = 20\log_{10}\left|1 + j\omega/z_1\right| \\ \phi = \tan^{-1}\dfrac{\omega}{z_1} \end{cases}$$

$$H_{\text{dB}}^{\text{simple\_zero}} = \begin{cases} 20\log_{10} 1 = 0, & \omega \to 0 \\ 20\log_{10}\dfrac{\omega}{z_1}, & \omega \to \infty \end{cases}; \quad \phi = \tan^{-1}\frac{\omega}{z_1} = \begin{cases} 0°, & \omega \to 0 \\ 45°, & \omega = z_1 \\ 90°, & \omega \to \infty \end{cases}$$

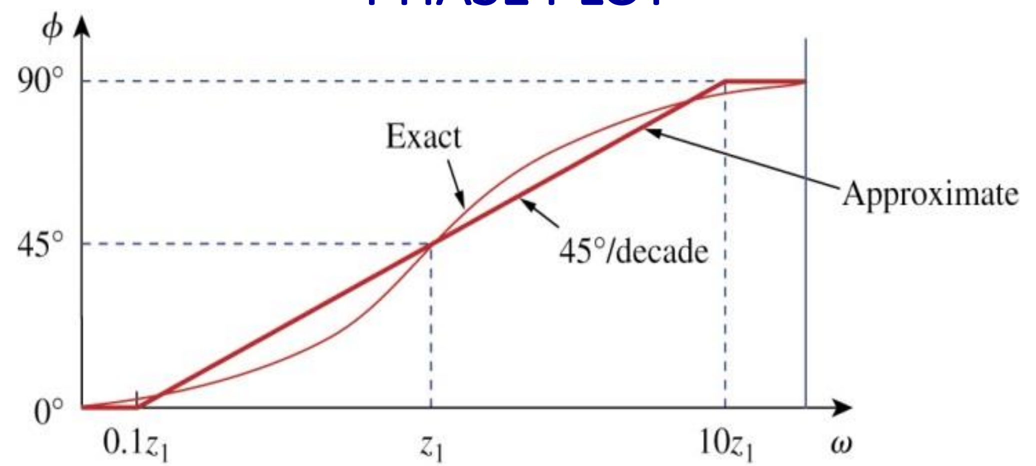
MAGNITUDE PLOT



(a)

Corner frequency/break frequency

PHASE PLOT



$\tan^{-1}(0.1)=5.7°$

(b)

$\tan^{-1}(10)=84.3°$

BODE PLOT OF REAL POLE or SIMPLE POLE

(5) REAL POLE

$$H(\omega)=\frac{K(j\omega)^{\pm}(1+j\omega/z_1)\left[(j\omega)^2+2\xi_1\omega_n+\omega_n{}^2\right]}{\boxed{(1+j\omega/p_1)}[(j\omega)^2+2\xi_2\omega_n+\omega_n{}^2]}$$

Simple Pole

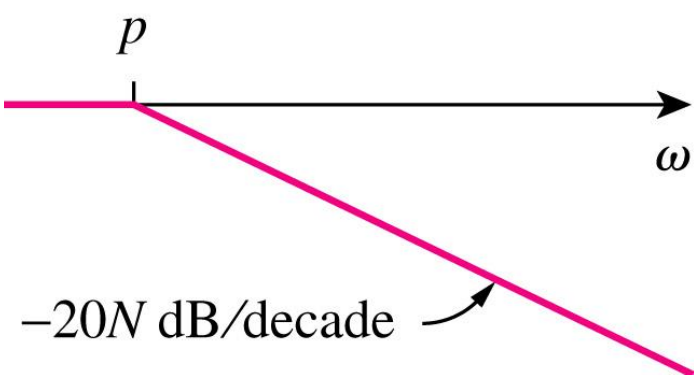
The Bode plots for the pole $1/(1+j\omega/p_1)$ are similar to those of the zero $(1+j\omega/z_1)$, except that the corner frequency is at $\omega = p_1$, the magnitude has a slope of -20 dB/decade, and the phase has a slope of -45° per decade.

CHARACTERISTIC OF $(1+j\omega/p1)^{-N}$

- **Magnitude:**

$$H = \begin{cases} 0 & \omega < p_1 \\ -20N\ \text{dB/dec} & \omega \geq p_1 \end{cases}$$

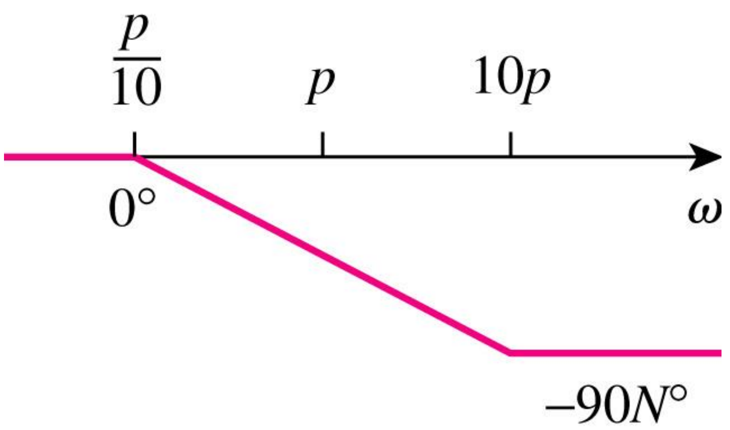
MAGNITUDE PLOT



- **Phase:**

$$\phi = \begin{cases} 0 & \omega = 0 \\ -45^o & \omega = p_1 \\ -90^o & \omega \to \infty \end{cases}$$

PHASE PLOT



(5) REAL POLE

Magnitude

$$H_{dB} = 20\log_{10}\left|\frac{1}{1+j\omega/p_1}\right| = -20\log_{10}\left|1+j\omega/p_1\right|$$

$$\omega \to 0 \;\Rightarrow\; H_{dB} = -20\log_{10} 1 = 0$$

$$\omega \to \infty \;\Rightarrow\; H_{dB} = -20\log_{10} \omega/p_1$$

$$\omega = p_1 \;\Rightarrow\; H_{dB} = -20\log_{10}\left|1+j1\right| = -20\log_{10}\sqrt{2}$$
$$= -3$$

Phase

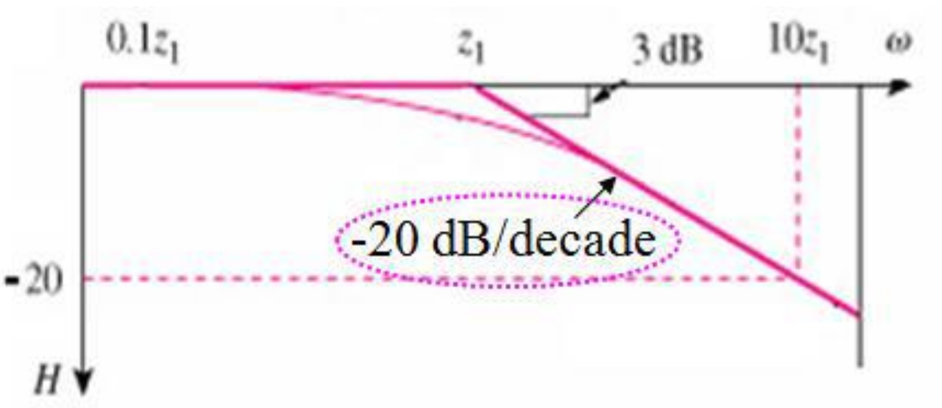
$$H(\omega) = \frac{1}{1+j\omega/p_1}$$

$$\Rightarrow \phi(\omega) = -\tan^{-1}(\omega/p_1)$$

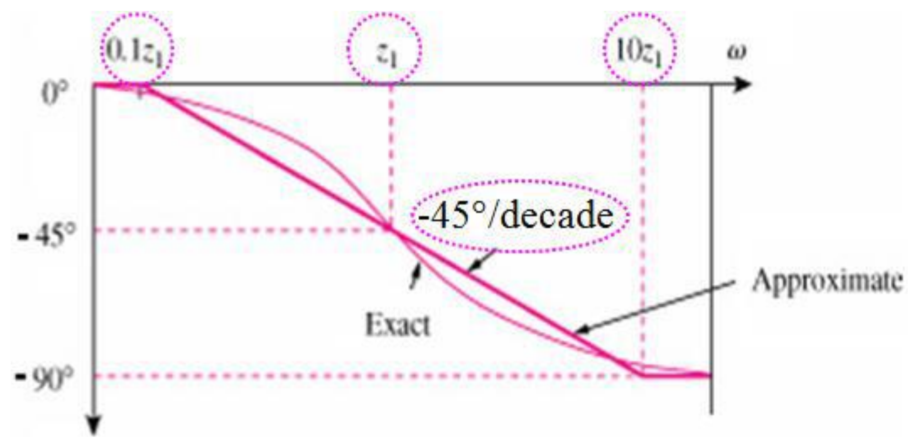
$$\phi(\omega \to 0) = 0$$

$$\phi(\omega \to \infty) = -90^0$$

$$\phi(\omega = z_1) = -45^0$$



MAGNITUDE PLOT



PHASE PLOT

BODE PLOT OF QUADRATIC ZERO

(6) QUADRATIC ZERO

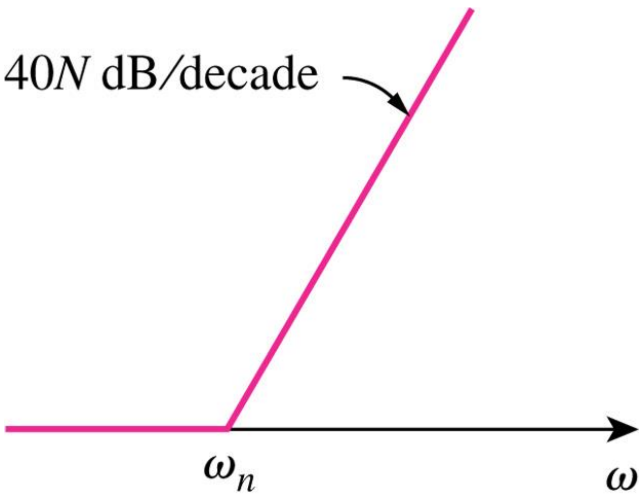
$$H(\omega) = \frac{K(j\omega)^{\pm}(1+ j\omega/z_1)\boxed{\left[(j\omega)^2 + 2\xi_1\omega_n + \omega_n{}^2\right]}}{(1+ j\omega/ p_1)[(j\omega)^2 + 2\xi_2\omega_n + \omega_n{}^2]}$$

CHARACTERISTIC OF $(j\omega^2+2\xi\omega_n+\omega_n^2)^N$

- Magnitude:

$$H = \begin{cases} 0 & \omega < \omega_n \\ 40N\,\mathrm{dB/dec} & \omega \geq \omega_n \end{cases}$$

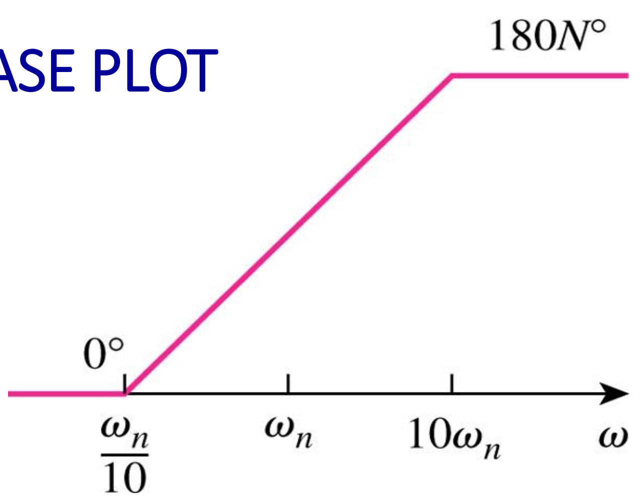
MAGNITUDE PLOT



- Phase:

$$\phi = \begin{cases} 0 & \omega = 0 \\ 90^o & \omega = \omega_n \\ 180^o & \omega \to \infty \end{cases}$$

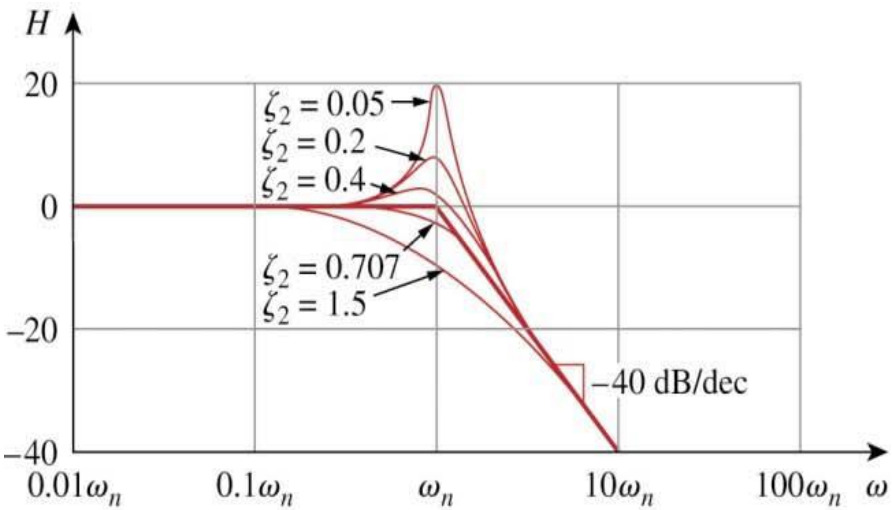
PHASE PLOT



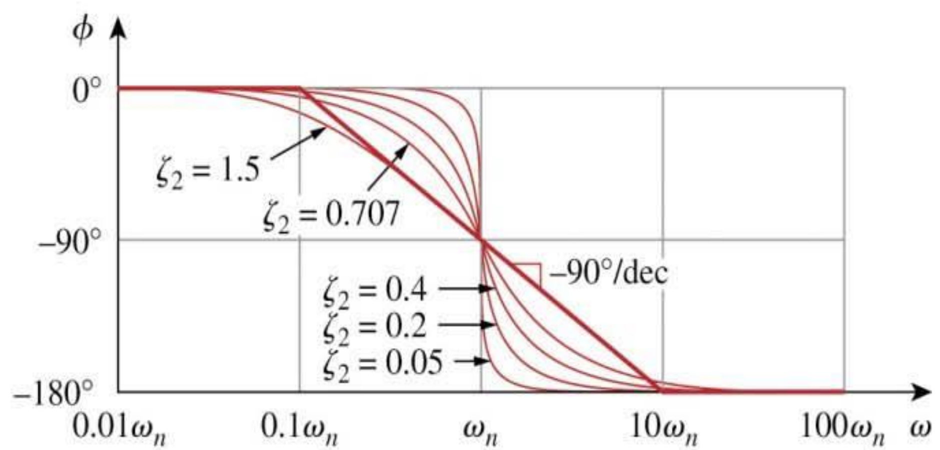
$$\mathbf{H}^{\text{quad\_zero}}(\omega) = 1 + j2\zeta_1\omega/\omega_k + (j\omega/\omega_k)^2 \qquad \text{(complex poles for } \zeta_2 < 1)$$

$$\Rightarrow \begin{cases} H_{\text{dB}}^{\text{quad\_zero}} = 20\log_{10}\left|1 + \dfrac{j2\zeta_1\omega}{\omega_k} + \left(\dfrac{j\omega}{\omega_k}\right)^2\right| \\ \phi = \tan^{-1}\dfrac{2\zeta_1\omega/\omega_k}{1-\omega^2/\omega_k^2} \end{cases}$$

$$\Rightarrow H_{\text{dB}}^{\text{quad\_zero}} = \begin{cases} 0\,, & \omega \to 0 \\ 40\log_{10}\dfrac{\omega}{\omega_k}\,, & \omega \to \infty \end{cases} ; \phi = \tan^{-1}\dfrac{2\zeta_1\omega/\omega_k}{1-\omega^2/\omega_k^2} = \begin{cases} 0\,, & \omega = 0 \\ 90°, & \omega = \omega_k \\ 180°, & \omega \to \infty \end{cases}$$



(a)



(b)

BODE PLOT OF QUADRATIC POLE

(7) QUADRATIC POLE

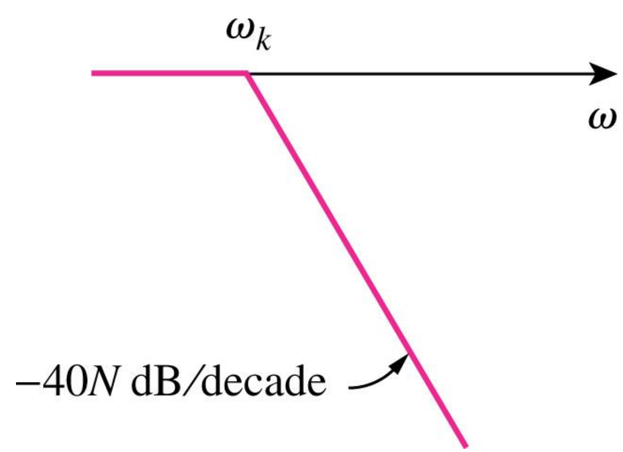
$$H(\omega)=\frac{K(j\omega)^{\pm}(1+j\omega/z_1)\left[(j\omega)^2+2\xi_1\omega_n+\omega_n{}^2\right]}{(1+j\omega/p_1)\boxed{\left[(j\omega)^2+2\xi_2\omega_n+\omega_n{}^2\right]}}$$

CHARATERISTIC OF $(j\omega^2+2\xi\omega_n+\omega_n^2)^{-N}$

- **Magnitude:**

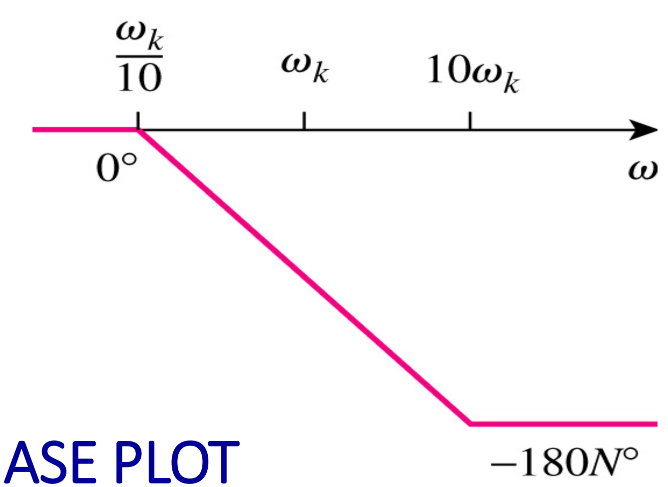
$$H=\begin{cases}0 & \omega<\omega_n\\ -40N\,\mathrm{dB/dec} & \omega\ge\omega_n\end{cases}$$

MAGNITUDE PLOT



- **Phase:**

$$\phi=\begin{cases}0 & \omega=0\\ -90^o & \omega=\omega_n\\ -180^o & \omega\to\infty\end{cases}$$

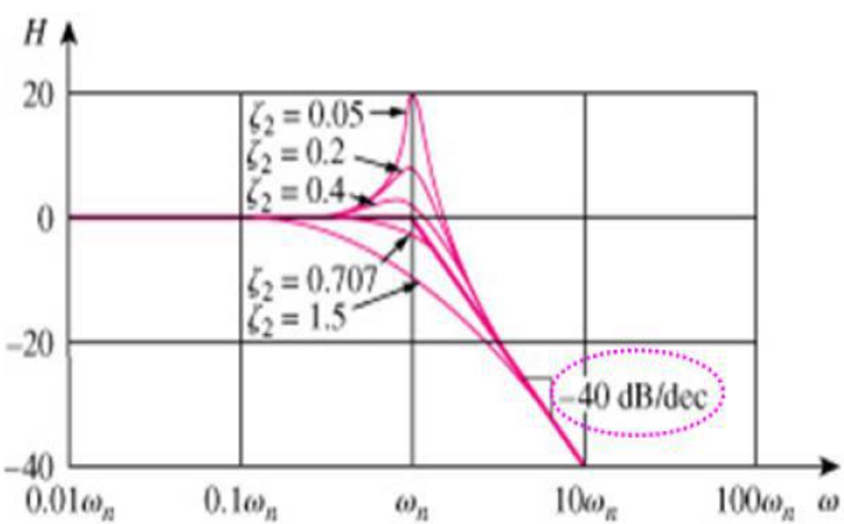


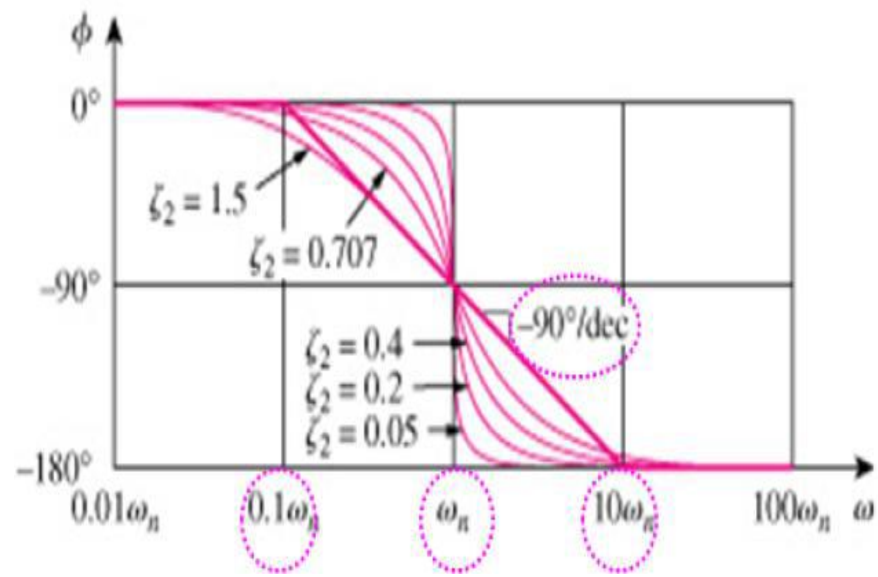
PHASE PLOT

$$\mathbf{H}^{\text{quad\_zero}}(\omega) = 1\Big/\Big(1 + j2\zeta_2\omega/\omega_n + (j\omega/\omega_n)^2\Big) \quad \text{(complex poles for } \zeta_2 < 1\text{)}$$

$$\Rightarrow \begin{cases} H_{\text{dB}}^{\text{quad\_zero}} = -20\log_{10}\left|1 + \dfrac{j2\zeta_2\omega}{\omega_n} + \left(\dfrac{j\omega}{\omega_n}\right)^2\right| \\[2ex] \phi = -\tan^{-1}\dfrac{2\zeta_2\omega/\omega_n}{1-\omega^2/\omega_n^2} \end{cases}$$

$$\Rightarrow H_{\text{dB}}^{\text{quad\_zero}} = \begin{cases} 0\,, & \omega \to 0 \\ -40\log_{10}\dfrac{\omega}{\omega_n}\,, & \omega \to \infty \end{cases} ; \quad \phi = -\tan^{-1}\dfrac{2\zeta_2\omega/\omega_n}{1-\omega^2/\omega_n^2} = \begin{cases} 0\,, & \omega = 0 \\ -90^\circ, & \omega = \omega_n \\ -180^\circ, & \omega \to \infty \end{cases}$$





HOW TO DRAW A BODE PLOT

- While drawing the bode plot, every factor (i.e zeros/poles) were drawn separately on the semilog graph.
- Finally, all of the factor are combined to form the answer.

EX.1

Draw the Bode plot for the given TF below:

$$H(\omega) = \frac{200\, j\omega}{(j\omega + 2)(j\omega + 10)}$$

SOLUTION

General equation:

$$H(\omega) = \frac{200\, j\omega}{(j\omega + 2)(j\omega + 10)}$$

$$= \frac{200\, j\omega}{(2)(1 + j\omega/2)(10)(1 + j\omega/10)}$$

$$= \frac{10\, j\omega}{(1 + j\omega/2)(1 + j\omega/10)}$$

Magnitude of TF:

$$H_{dB} = 20\log_{10} 10 + 20\log_{10} |j\omega| - 20\log_{10}\left|1 + \frac{j\omega}{2}\right| - 20\log_{10}\left|1 + \frac{j\omega}{10}\right|$$

$$20 \log 10 \; = 20\text{dB: straight line}$$

- Phase of TF:

$$\phi = 90^{o} - \tan^{-1}\frac{\omega}{2} - \tan^{-1}\frac{\omega}{10}$$

Zero at the origin

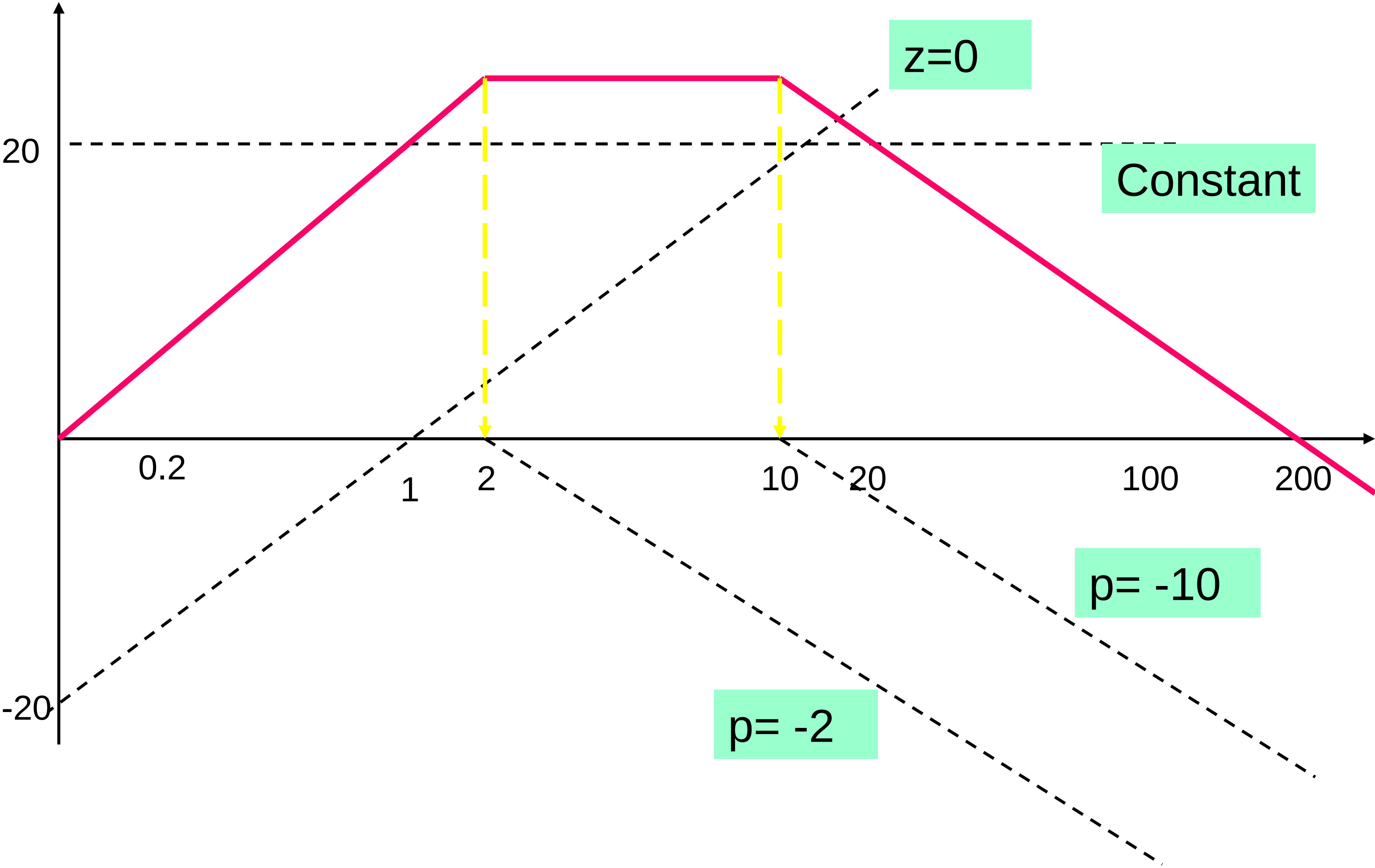
Pole at 2

Pole at 10

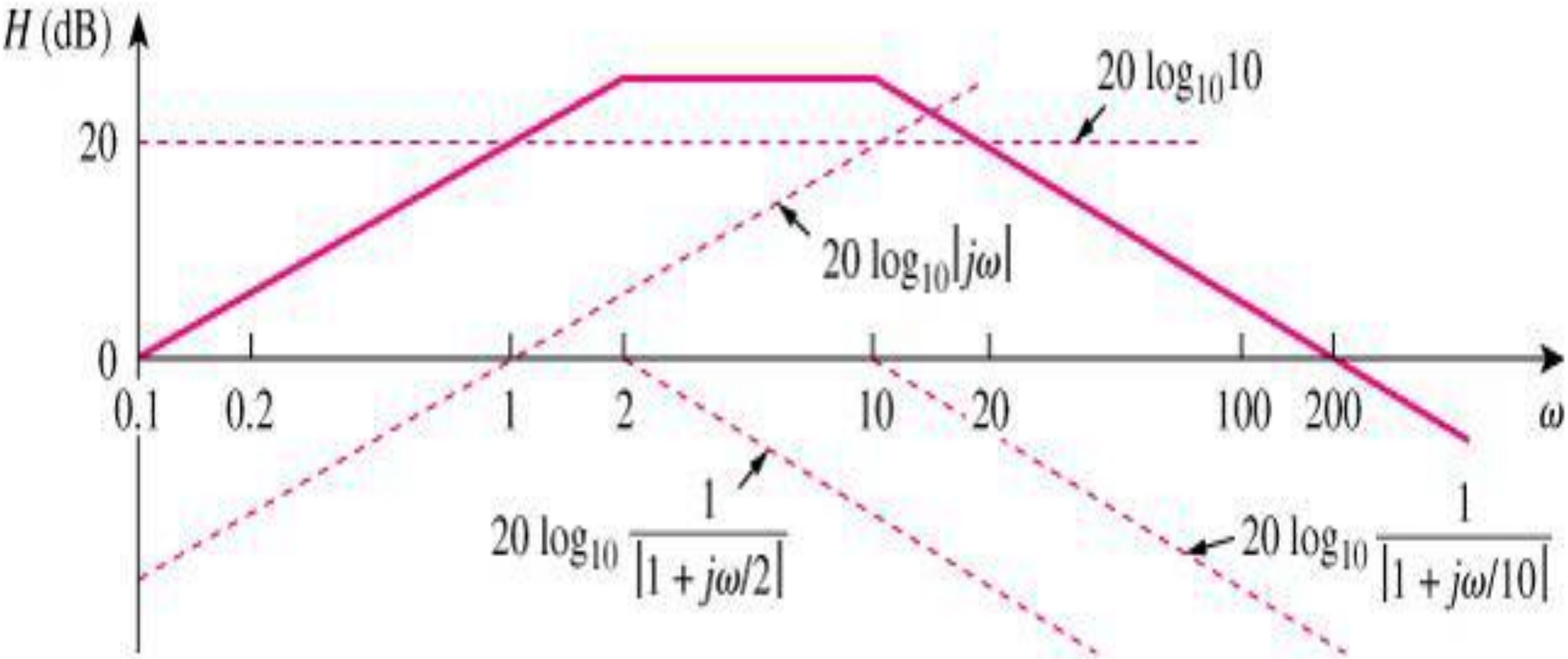
MAGNITUDE PLOT GUIDANCE

	ω=0.1	ω=2	ω=10	ω=100
z=0	20dB/dec	20dB/dec	20dB/dec	20dB/dec
p=2	0dB/dec	-20dB/dec	-20dB/dec	-20dB/dec
p=10	0dB/dec	0dB/dec	-20dB/dec	-20dB/dec
Resultant	=20dB/dec	=0dB/dec	=-20dB/dec	=-20dB/dec

Magnitude plot



$$H_{dB} = 20\log_{10} 10 + 20\log_{10} |j\omega| - 20\log_{10} \left|1+\frac{j\omega}{2}\right| - 20\log_{10} \left|1+\frac{j\omega}{10}\right|$$

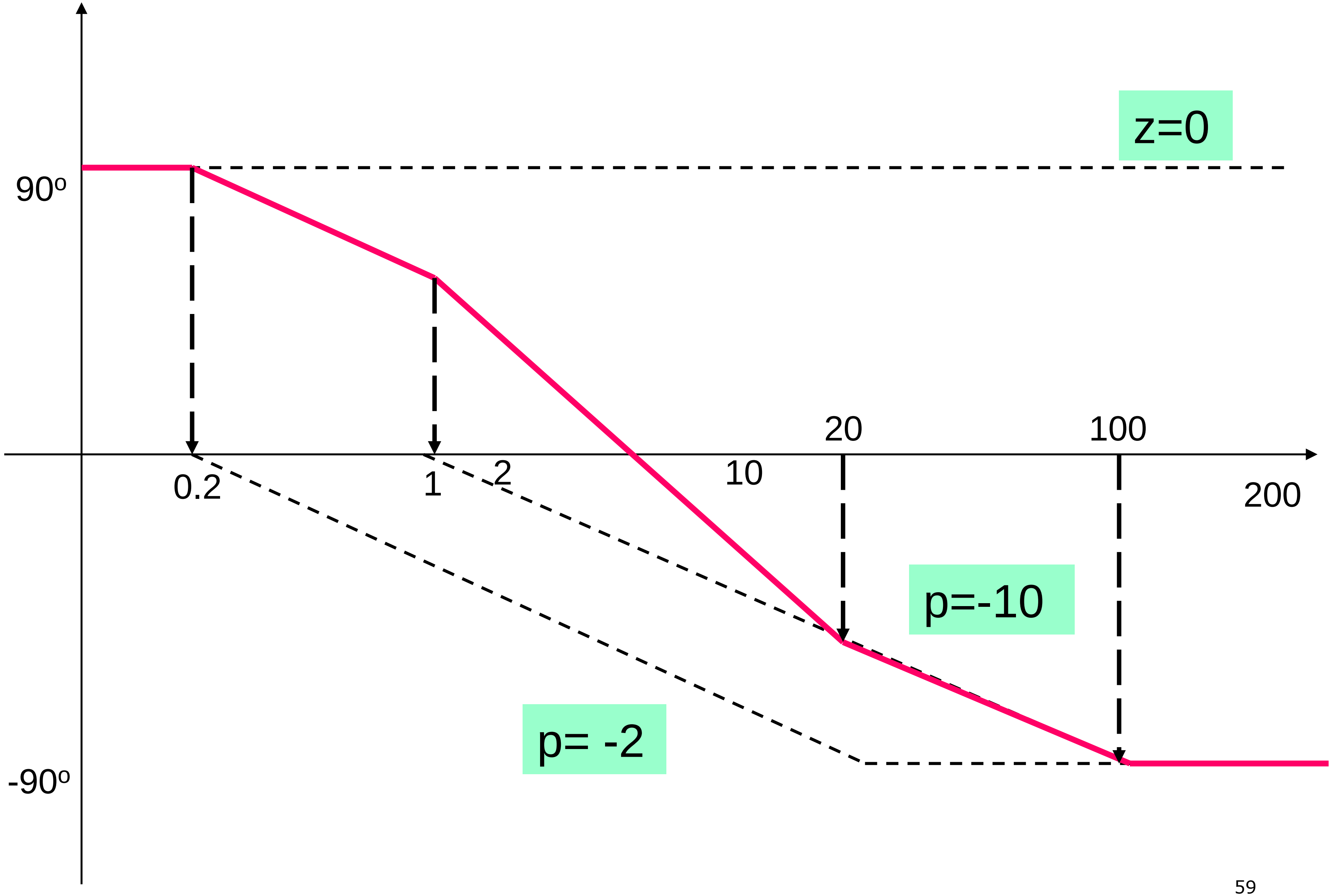


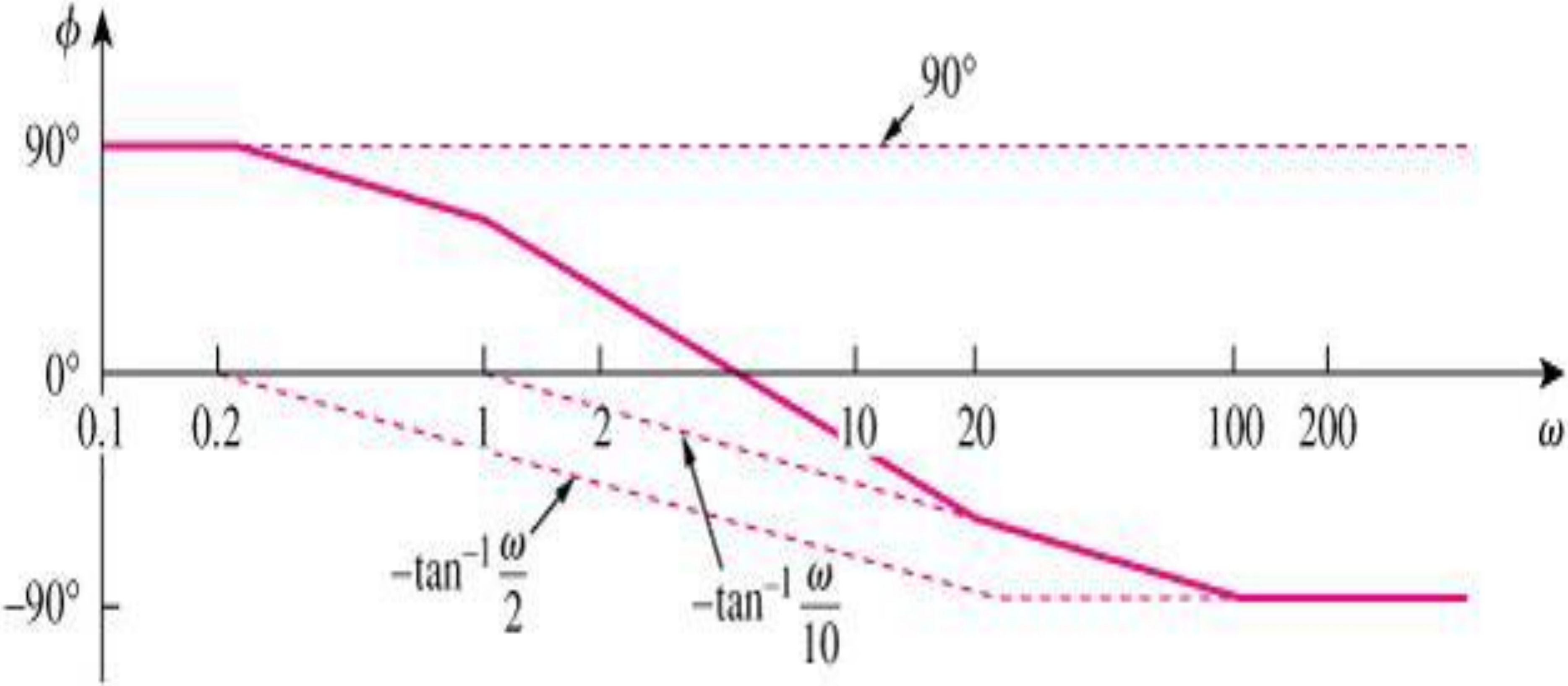
PHASE PLOT GUIDANCE

	$\omega=0$	$\omega=0.2$	$\omega=1$	$\omega=20$	$\omega=100$
z=0	90°	90°	90°	90°	90°
p=2	0°/dec	-45°/dec	-45°/dec	-90 °	-90 °
p=10	0°/dec	0°/dec	-45°/dec	-45°/dec	-90°
Resultant	90°	-45°/dec	-90°/dec	-45°/dec	-90°

Add all of the lines that having a slope only

Phase plot




ϕ
90°
90°
0°
−90°
0.1
0.2
1
2
10
20
100
200
ω
$-\tan^{-1}\frac{\omega}{2}$
$-\tan^{-1}\frac{\omega}{10}$

EX.2

Draw the Bode plot for the given TF below:

$$H(\omega) = \frac{(j\omega + 10)}{j\omega(j\omega + 2)}$$

SOLUTION

- General equation:

$$\mathrm{H}(\omega) = \frac{(j\omega + 10)}{j\omega(j\omega + 2)}$$

$$= \frac{5(1 + j\omega/10)}{j\omega(1 + j\omega/2)}$$

Magnitude of TF :

$$H_{dB} = 20\log_{10} 5 + 20\log_{10}\left|1 + \frac{j\omega}{10}\right|$$

$$- 20\log_{10}|j\omega| - 20\log_{10}\left|1 + \frac{j\omega}{2}\right|$$

$20 \log 5 = 14\text{dB}$: straight line

- Phase of TF:

$$\phi = -90^o + \tan^{-1}\frac{\omega}{10} - \tan^{-1}\frac{\omega}{2}$$

$p_1=0$

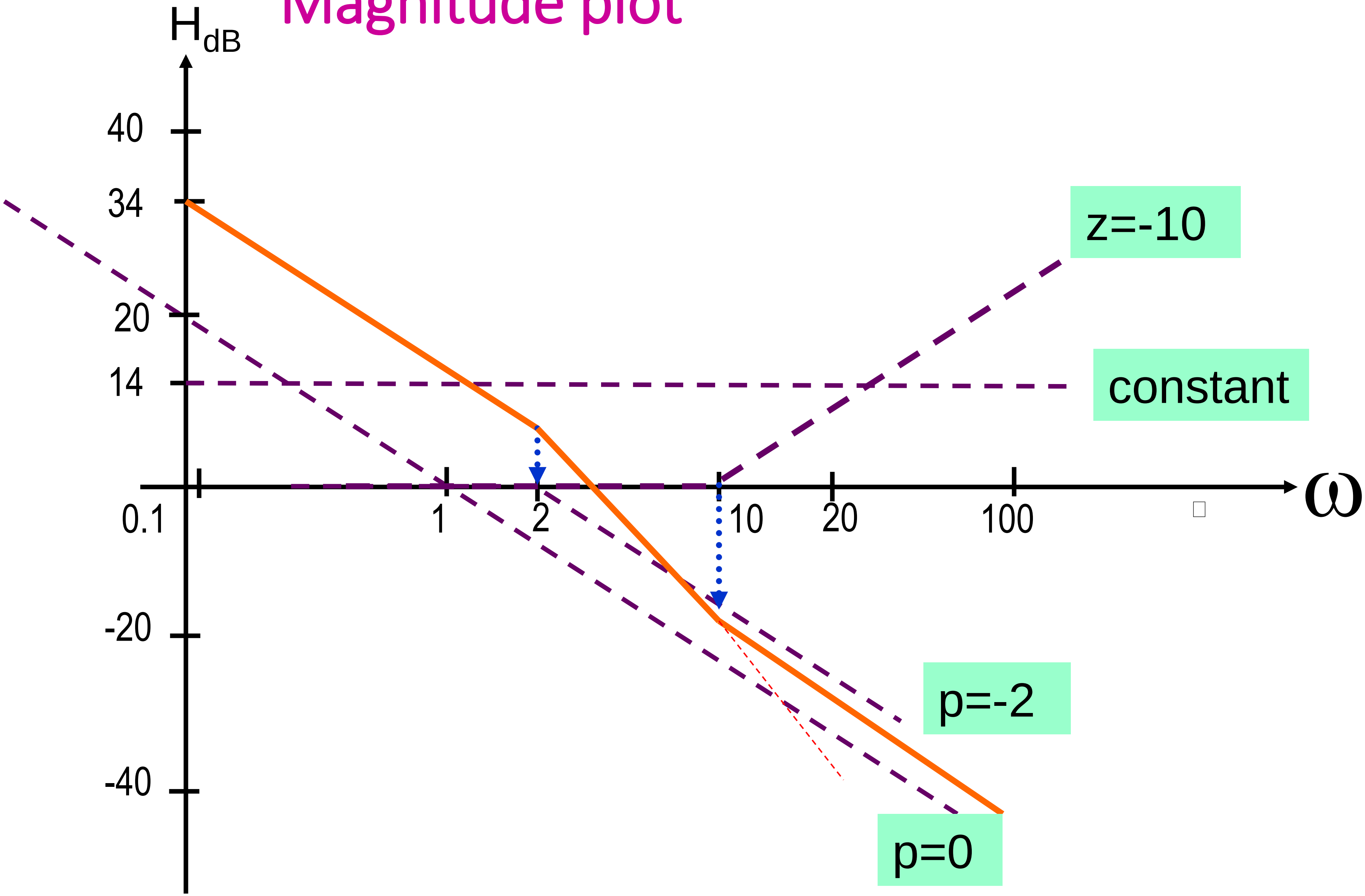
$z_1 = -10$

$p_2= -2$

MAGNITUDE PLOT GUIDANCE

	ω=0.1	ω=2	ω=10	ω=100
p=0	-20dB/dec	-20dB/dec	-20dB/dec	-20dB/dec
p=2	0dB/dec	-20dB/dec	-20dB/dec	-20dB/dec
z=10	0dB/dec	0dB/dec	20dB/dec	20dB/dec
Resultant	-20dB/dec	-40dB/dec	-20dB/dec	- 20dB/dec

Magnitude plot

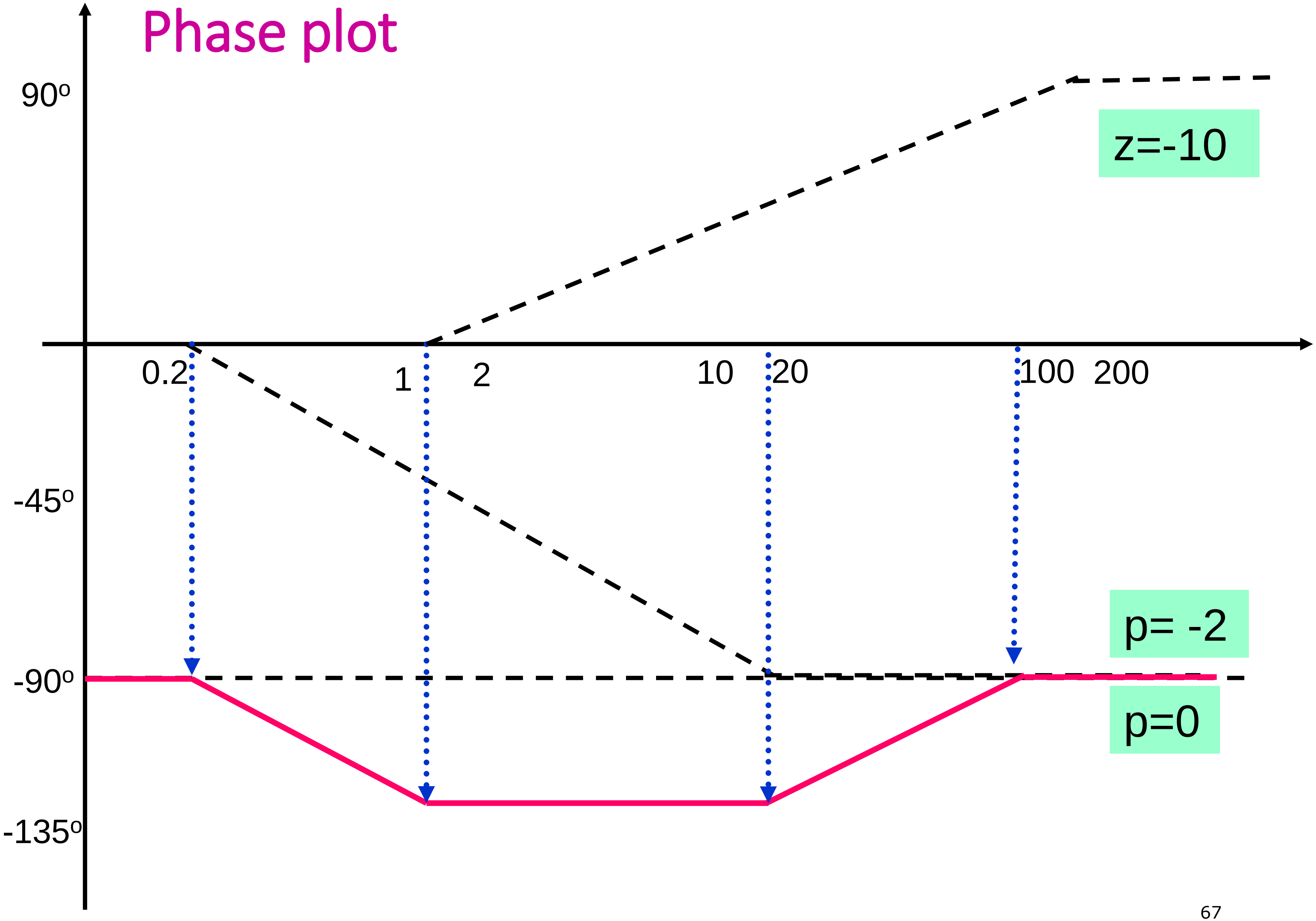


Phase plot Guidance

	$\omega=0$	$\omega=0.2$	$\omega=1$	$\omega=20$	$\omega=100$
p=0	-90°	-90°	-90°	-90°	-90°
p=2	0°/dec	-45°/dec	-45°/dec	-90 °	-90 °
z=10	0°/dec	0°/dec	45°/dec	45°/dec	90°
Resultant	-90°	-45°/dec	0°/dec	45°/dec	-90°

Add all the lines that having a slope only

Phase plot



EXAMPLE 3

- Draw the Bode plot for the given TF below:

$$H(s) = \frac{s}{s^2 + 10s + 100}$$

SOLUTION

- Standard equation: $$H(s) = \frac{s}{s^2 + 10s + 100}$$

Replace s = jω and divide it with 100;

$$\mathbf{H}(\omega) = \frac{j\omega}{100\,(1 + j\omega/10 - \omega^2/100)}$$

- Magnitude of TF:

$$H_{dB} = 20\log_{10}|j\omega| - 20\log_{10}|100| - 20\log_{10}\left|1 + j\omega/10 - \omega^2/100\right|$$

$$-20 \log 100 = -40\text{dB} : \text{straight line}$$

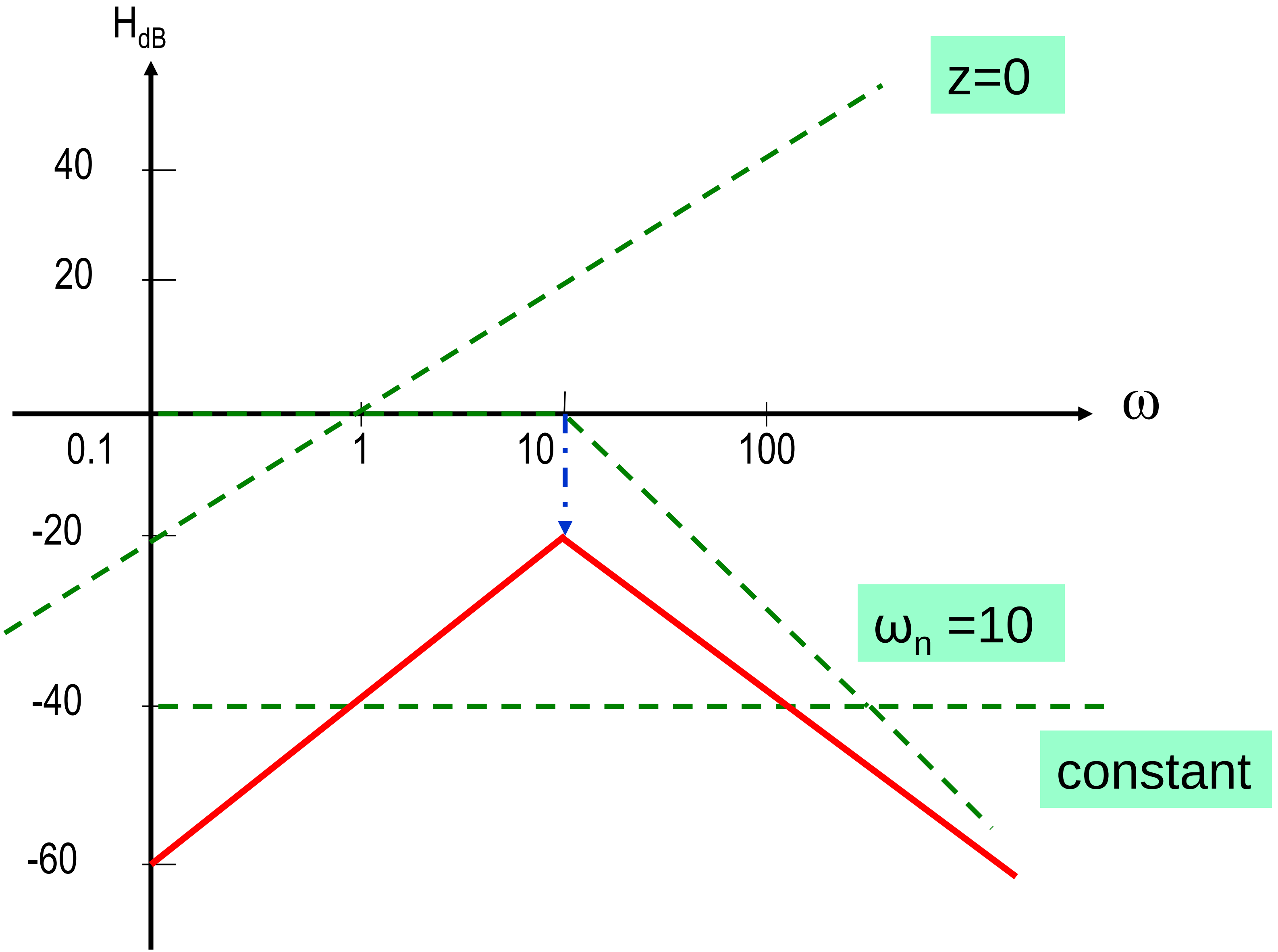
- Phase of TF:

$$\phi = 90^\circ - \tan^{-1}\left(\frac{\omega/10}{1-\omega^2/100}\right)$$

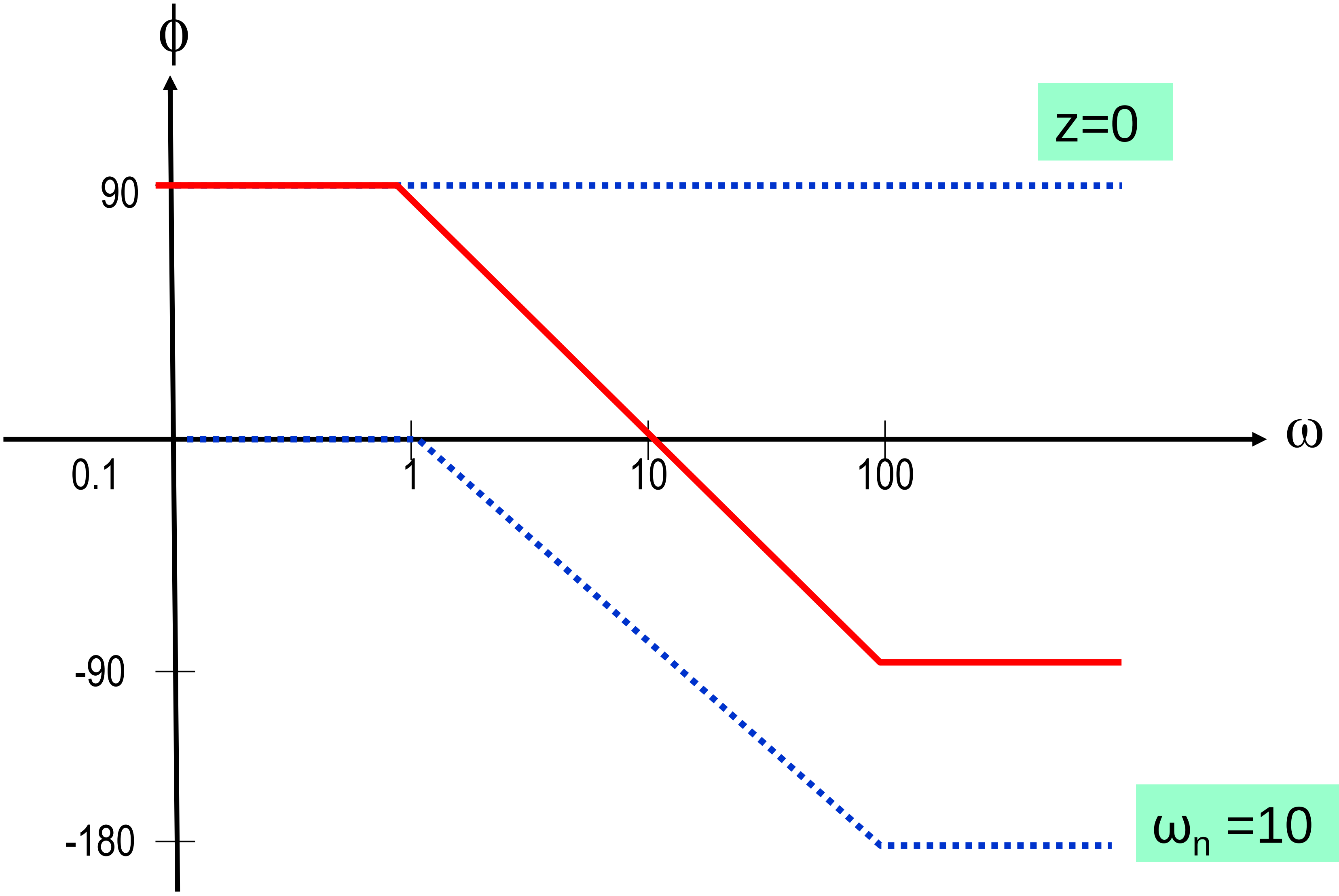
$z_1=0$

$\omega_n= 10$

Magnitude plot

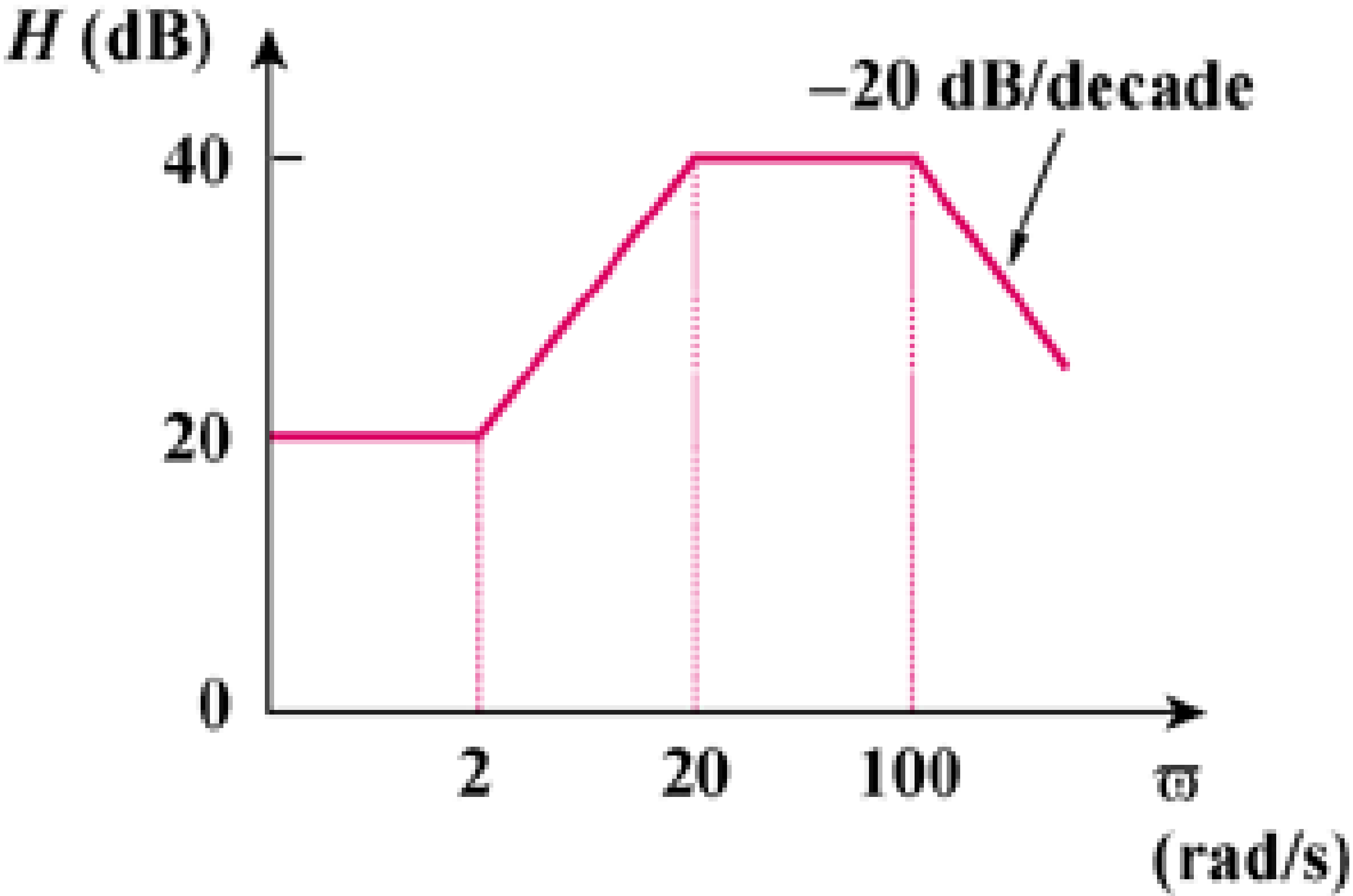


Phase plot



EXAMPLE 4

- Determine the TF ?



ANSWER

$$\mathbf{H}(\omega) = \frac{\mathbf{10^4\,(2 + j\omega)}}{\mathbf{(20 + j\omega)(100 + j\omega)}}$$