Ch04 Solution Manual Material Science and Engineering 8th Edition

Material Science & Engineering (HITEC University)

CHAPTER 4

IMPERFECTIONS IN SOLIDS

PROBLEM SOLUTIONS

Vacancies and Self-Interstitials

4.1 Calculate the fraction of atom sites that are vacant for lead at its melting temperature of 327°C (600 K). Assume an energy for vacancy formation of 0.55 eV/atom.

Solution

In order to compute the fraction of atom sites that are vacant in lead at 600 K, we must employ Equation 4.1. As stated in the problem, $Q_V = 0.55$ eV/atom. Thus,

$$\frac{N_V}{N} = \exp\left(-\frac{Q_V}{kT}\right) = \exp\left[-\frac{0.55 \text{ eV/atom}}{(8.62 \times 10^{-5} \text{ eV/atom-K})(600 \text{ K})}\right]$$

$$= 2.41 \times 10^{-5}$$

4.2 Calculate the number of vacancies per cubic meter in iron at 850°C. The energy for vacancy formation is 1.08 eV/atom. Furthermore, the density and atomic weight for Fe are 7.65 g/cm^3 and 55.85 g/mol, respectively.

Solution

Determination of the number of vacancies per cubic meter in iron at 850°C (1123 K) requires the utilization of Equations 4.1 and 4.2 as follows:

$$N_v = N \exp\left(-\frac{Q_v}{kT}\right) = \frac{N_A \rho_{Fe}}{A_{Fe}} \exp\left(-\frac{Q_v}{kT}\right)$$

And incorporation of values of the parameters provided in the problem statement into the above equation leads to

$$N_v = \frac{(6.022 \times 10^{23}\text{atoms/mol})(7.65\ \text{g/cm}^3)}{55.85\ \text{g/mol}} \exp\left[-\frac{1.08\ \text{eV/atom}}{(8.62 \times 10^{-5}\ \text{eV/atom-K})(850°\text{C} + 273\ \text{K})}\right]$$

$$= 1.18 \times 10^{18}\ \text{cm}^{-3} = 1.18 \times 10^{24}\ \text{m}^{-3}$$

4.3 Calculate the activation energy for vacancy formation in aluminum, given that the equilibrium number of vacancies at 500°C (773 K) is 7.57×10^{23} m^{-3}. The atomic weight and density (at 500°C) for aluminum are, respectively, 26.98 g/mol and 2.62 g/cm^3.

Solution

Upon examination of Equation 4.1, all parameters besides Q_v are given except N, the total number of atomic sites. However, N is related to the density, (ρ_{Al}), Avogadro's number (N_A), and the atomic weight (A_{Al}) according to Equation 4.2 as

$$N = \frac{N_A \rho_{Al}}{A_{Al}}$$

$$= \frac{(6.022 \times 10^{23} \text{ atoms/mol})(2.62 \text{ g/cm}^3)}{26.98 \text{ g/mol}}$$

$$= 5.85 \times 10^{22} \text{ atoms/cm}^3 = 5.85 \times 10^{28} \text{ atoms/m}^3$$

Now, taking natural logarithms of both sides of Equation 4.1,

$$\ln N_v = \ln N - \frac{Q_v}{kT}$$

and, after some algebraic manipulation

$$Q_v = -kT \ln\left(\frac{N_v}{N}\right)$$

$$= -(8.62 \times 10^{-5} \text{ eV/atom-K})(500\,^{\circ}\text{C} + 273 \text{ K}) \ln\left[\frac{7.57 \times 10^{23} \text{ m}^{-3}}{5.85 \times 10^{28} \text{ m}^{-3}}\right]$$

$$= 0.75 \text{ eV/atom}$$

Impurities in Solids

4.4 Below, atomic radius, crystal structure, electronegativity, and the most common valence are tabulated, for several elements; for those that are nonmetals, only atomic radii are indicated.

Element	Atomic Radius (nm)	Crystal Structure	Electronegativity	Valence
Cu	0.1278	FCC	1.9	+2
C	0.071			
H	0.046			
O	0.060			
Ag	0.1445	FCC	1.9	+1
Al	0.1431	FCC	1.5	+3
Co	0.1253	HCP	1.8	+2
Cr	0.1249	BCC	1.6	+3
Fe	0.1241	BCC	1.8	+2
Ni	0.1246	FCC	1.8	+2
Pd	0.1376	FCC	2.2	+2
Pt	0.1387	FCC	2.2	+2
Zn	0.1332	HCP	1.6	+2

Which of these elements would you expect to form the following with copper:

(a) A substitutional solid solution having complete solubility

(b) A substitutional solid solution of incomplete solubility

(c) An interstitial solid solution

Solution

In this problem we are asked to cite which of the elements listed form with Cu the three possible solid solution types. For complete substitutional solubility the following criteria must be met: 1) the difference in atomic radii between Cu and the other element ($\Delta R\%$) must be less than ±15%, 2) the crystal structures must be the same, 3) the electronegativities must be similar, and 4) the valences should be the same, or nearly the same. Below are tabulated, for the various elements, these criteria.

Element	ΔR%	Crystal Structure	ΔElectro-negativity	Valence
Cu		**FCC**		**2+**
C	–44			

H	–64			
O	–53			
Ag	+13	FCC	0	1+
Al	+12	FCC	-0.4	3+
Co	-2	HCP	-0.1	2+
Cr	-2	BCC	-0.3	3+
Fe	-3	BCC	-0.1	2+
Ni	-3	FCC	-0.1	2+
Pd	+8	FCC	+0.3	2+
Pt	+9	FCC	+0.3	2+
Zn	+4	HCP	-0.3	2+

(a) Ni, Pd, and Pt meet all of the criteria and thus form substitutional solid solutions having complete solubility. At elevated temperatures Co and Fe experience allotropic transformations to the FCC crystal structure, and thus display complete solid solubility at these temperatures.

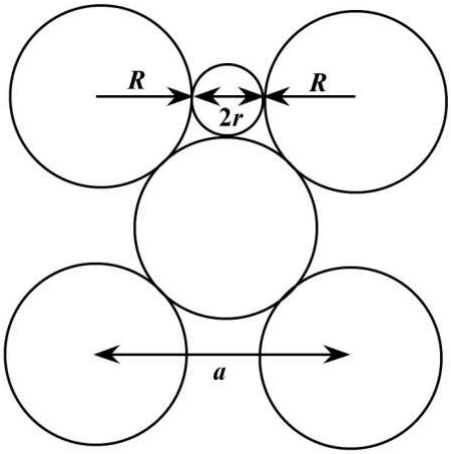
(b) Ag, Al, Co, Cr, Fe, and Zn form substitutional solid solutions of incomplete solubility. All these metals have either BCC or HCP crystal structures, and/or the difference between their atomic radii and that for Cu are greater than ±15%, and/or have a valence different than 2+.

(c) C, H, and O form interstitial solid solutions. These elements have atomic radii that are significantly smaller than the atomic radius of Cu.

4.5 For both FCC and BCC crystal structures, there are two different types of interstitial sites. In each case, one site is larger than the other, and is normally occupied by impurity atoms. For FCC, this larger one is located at the center of each edge of the unit cell; it is termed an octahedral interstitial site. On the other hand, with BCC the larger site type is found at $0\ \frac{1}{2}\ \frac{1}{4}$ positions—that is, lying on {100} faces, and situated midway between two unit cell edges on this face and one-quarter of the distance between the other two unit cell edges; it is termed a tetrahedral interstitial site. For both FCC and BCC crystal structures, compute the radius r of an impurity atom that will just fit into one of these sites in terms of the atomic radius R of the host atom.

Solution

In the drawing below is shown the [illegible] center of the edge.



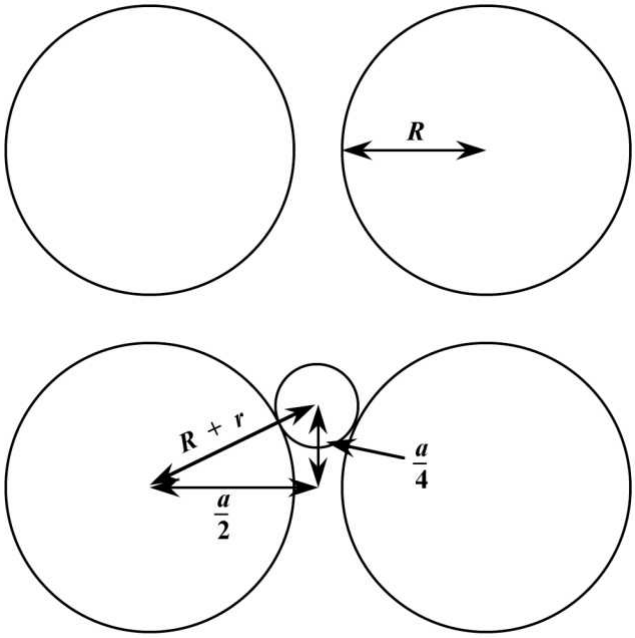
The diameter of an atom that will just fit into this site (2r) is just the difference between that unit cell edge length (a) and the radii of the two host atoms that are located on either side of the site (R); that is

$$2r = a - 2R$$

However, for FCC a is related to R according to Equation 3.1 as $a = 2R\sqrt{2}$; therefore, solving for r from the above equation gives

$$r = \frac{a - 2R}{2} = \frac{2R\sqrt{2} - 2R}{2} = 0.41R$$

A (100) face of a BCC unit cell is shown below.



The interstitial atom that just fits into this interstitial site is shown by the small circle. It is situated in the plane of this (100) face, midway between the two vertical unit cell edges, and one quarter of the distance between the bottom and top cell edges. From the right triangle that is defined by the three arrows we may write

$$\left(\frac{a}{2}\right)^2 + \left(\frac{a}{4}\right)^2 = (R + r)^2$$

However, from Equation 3.3, $a = \frac{4R}{\sqrt{3}}$, and, therefore, making this substitution, the above equation takes the form

$$\left(\frac{4R}{2\sqrt{3}}\right)^2 + \left(\frac{4R}{4\sqrt{3}}\right)^2 = R^2 + 2Rr + r^2$$

After rearrangement the following quadratic equation results:

$$r^2 + 2Rr - 0.667\,R^2 = 0$$

And upon solving for r:

$$r = \frac{-(2R) \pm \sqrt{(2R)^2 - (4)(1)(-0.667\,R^2)}}{2}$$

$$= \frac{-2R \pm 2.582\,R}{2}$$

And, finally

$$r(+) = \frac{-2R + 2.582\,R}{2} = 0.291\,R$$
$$r(-) = \frac{-2R - 2.582\,R}{2} = -2.291\,R$$

Of course, only the r(+) root is possible, and, therefore, r = 0.291R.

Thus, for a host atom of radius R, the size of an interstitial site for FCC is approximately 1.4 times that for BCC.

Specification of Composition

4.6 Derive the following equations:

(a) Equation 4.7a

(b) Equation 4.9a

(c) Equation 4.10a

(d) Equation 4.11b

Solution

(a) This problem asks that we derive Equation 4.7a. To begin, C_1 is defined according to Equation 4.3 as

$$C_1 = \frac{m_1}{m_1 + m_2} \times 100$$

or, equivalently

$$C_1 = \frac{m_1'}{m_1' + m_2'} \times 100$$

where the primed m's indicate masses in grams. From Equation 4.4 we may write

$$m_1' = n_{m1} A_1$$

$$m_2' = n_{m2} A_2$$

And, substitution into the C_1 expression above

$$C_1 = \frac{n_{m1} A_1}{n_{m1} A_1 + n_{m2} A_2} \times 100$$

From Equation 4.5 it is the case that

$$n_{m1} = \frac{C_1'(n_{m1} + n_{m2})}{100}$$

$$n_{m2} = \frac{C_2'(n_{m1} + n_{m2})}{100}$$

And substitution of these expressions into the above equation leads to

$$C_1 = \frac{C_1' A_1}{C_1' A_1 + C_2' A_2} \times 100$$

which is just Equation 4.7a.

(b) This problem asks that we derive Equation 4.9a. To begin, C_1'' is defined as the mass of component 1 per unit volume of alloy, or

$$C_1'' = \frac{m_1}{V}$$

If we assume that the total alloy volume V is equal to the sum of the volumes of the two constituents--i.e., $V = V_1 + V_2$--then

$$C_1'' = \frac{m_1}{V_1 + V_2}$$

Furthermore, the volume of each constituent is related to its density and mass as

$$V_1 = \frac{m_1}{\rho_1}$$

$$V_2 = \frac{m_2}{\rho_2}$$

This leads to

$$C_1'' = \frac{m_1}{\frac{m_1}{\rho_1} + \frac{m_2}{\rho_2}}$$

From Equation 4.3, m_1 and m_2 may be expressed as follows:

$$m_1 = \frac{C_1(m_1 + m_2)}{100}$$

$$m_2 = \frac{C_2(m_1 + m_2)}{100}$$

Substitution of these equations into the preceding expression yields

$$C_1'' = \frac{\frac{C_1(m_1 + m_2)}{100}}{\frac{\frac{C_1(m_1 + m_2)}{100}}{\rho_1} + \frac{\frac{C_2(m_1 + m_2)}{100}}{\rho_2}}$$

$$= \frac{C_1}{\frac{C_1}{\rho_1} + \frac{C_2}{\rho_2}}$$

If the densities ρ_1 and ρ_2 are given in units of g/cm^3, then conversion to units of kg/m^3 requires that we multiply this equation by 10^3, inasmuch as

$$1\ g/cm^3 = 10^3\ kg/m^3$$

Therefore, the previous equation takes the form

$$C_1'' = \frac{C_1}{\frac{C_1}{\rho_1} + \frac{C_2}{\rho_2}} \times 10^3$$

which is the desired expression.

(c) Now we are asked to derive Equation 4.10a. The density of an alloy ρ_{ave} is just the total alloy mass M divided by its volume V

$$\rho_{ave} = \frac{M}{V}$$

Or, in terms of the component elements 1 and 2

$$\rho_{ave} = \frac{m_1 + m_2}{V_1 + V_2}$$

[Note: here it is assumed that the total alloy volume is equal to the separate volumes of the individual components, which is only an approximation; normally V will not be exactly equal to $(V_1 + V_2)$].

Each of V_1 and V_2 may be expressed in terms of its mass and density as,

$$V_1 = \frac{m_1}{\rho_1}$$

$$V_2 = \frac{m_2}{\rho_2}$$

When these expressions are substituted into the above equation, we get

$$\rho_{ave} = \frac{m_1 + m_2}{\frac{m_1}{\rho_1} + \frac{m_2}{\rho_2}}$$

Furthermore, from Equation 4.3

$$m_1 = \frac{C_1(m_1 + m_2)}{100}$$

$$m_2 = \frac{C_2(m_1 + m_2)}{100}$$

Which, when substituted into the above ρ_{ave} expression yields

$$\rho_{ave} = \frac{m_1 + m_2}{\frac{\frac{C_1(m_1 + m_2)}{100}}{\rho_1} + \frac{\frac{C_2(m_1 + m_2)}{100}}{\rho_2}}$$

And, finally, this equation reduces to

$$= \frac{100}{\frac{C_1}{\rho_1} + \frac{C_2}{\rho_2}}$$

(d) And, finally, the derivation of Equation 4.11b for A_{ave} is requested. The alloy average molecular weight is just the ratio of total alloy mass in grams M' and the total number of moles in the alloy N_m. That is

$$A_{ave} = \frac{M'}{N_m} = \frac{m_1' + m_2'}{n_{m1} + n_{m2}}$$

But using Equation 4.4 we may write

$$m_1' = n_{m1} A_1$$

$$m_2' = n_{m2} A_2$$

Which, when substituted into the above A_{ave} expression yields

$$A_{ave} = \frac{M'}{N_m} = \frac{n_{m1} A_1 + n_{m2} A_2}{n_{m1} + n_{m2}}$$

Furthermore, from Equation 4.5

$$n_{m1} = \frac{C_1' (n_{m1} + n_{m2})}{100}$$

$$n_{m2} = \frac{C_2' (n_{m1} + n_{m2})}{100}$$

Thus, substitution of these expressions into the above equation for A_{ave} yields

$$A_{ave} = \frac{\frac{C_1' A_1 (n_{m1} + n_{m2})}{100} + \frac{C_2' A_2 (n_{m1} + n_{m2})}{100}}{n_{m1} + n_{m2}}$$

$$= \frac{C_1' A_1 + C_2' A_2}{100}$$

which is the desired result.

4.7 What is the composition, in atom percent, of an alloy that consists of 30 wt% Zn and 70 wt% Cu?

Solution

In order to compute composition, in atom percent, of a 30 wt% Zn-70 wt% Cu alloy, we employ Equation 4.6 as

$$C'_{Zn} = \frac{C_{Zn} A_{Cu}}{C_{Zn} A_{Cu} + C_{Cu} A_{Zn}} \times 100$$

$$= \frac{(30)(63.55\ g/mol)}{(30)(63.55\ g/mol) + (70)(65.41\ g/mol)} \times 100$$

$$= 29.4\ at\%$$

$$C'_{Cu} = \frac{C_{Cu} A_{Zn}}{C_{Zn} A_{Cu} + C_{Cu} A_{Zn}} \times 100$$

$$= \frac{(70)(65.41\ g/mol)}{(30)(63.55\ g/mol) + (70)(65.41\ g/mol)} \times 100$$

$$= 70.6\ at\%$$

4.8 What is the composition, in weight percent, of an alloy that consists of 6 at% Pb and 94 at% Sn?

Solution

In order to compute composition, in weight percent, of a 6 at% Pb-94 at% Sn alloy, we employ Equation 4.7 as

$$C_{Pb} = \frac{C'_{Pb} A_{Pb}}{C'_{Pb} A_{Pb} + C'_{Sn} A_{Sn}} \times 100$$

$$= \frac{(6)(207.2\ g/mol)}{(6)(207.2\ g/mol) + (94)(118.71\ g/mol)} \times 100$$

$$= 10.0\ wt\%$$

$$C_{Sn} = \frac{C'_{Sn} A_{Sn}}{C'_{Pb} A_{Pb} + C'_{Sn} A_{Sn}} \times 100$$

$$= \frac{(94)(118.71\ g/mol)}{(6)(207.2\ g/mol) + (94)(118.71\ g/mol)} \times 100$$

$$= 90.0\ wt\%$$

4.9 Calculate the composition, in weight percent, of an alloy that contains 218.0 kg titanium, 14.6 kg of aluminum, and 9.7 kg of vanadium.

Solution

The concentration, in weight percent, of an element in an alloy may be computed using a modified form of Equation 4.3. For this alloy, the concentration of titanium (C_{Ti}) is just

$$C_{Ti} = \frac{m_{Ti}}{m_{Ti} + m_{Al} + m_V} \times 100$$

$$= \frac{218 \text{ kg}}{218 \text{ kg} + 14.6 \text{ kg} + 9.7 \text{ kg}} \times 100 = 89.97 \text{ wt\%}$$

Similarly, for aluminum

$$C_{Al} = \frac{14.6 \text{ kg}}{218 \text{ kg} + 14.6 \text{ kg} + 9.7 \text{ kg}} \times 100 = 6.03 \text{ wt\%}$$

And for vanadium

$$C_V = \frac{9.7 \text{ kg}}{218 \text{ kg} + 14.6 \text{ kg} + 9.7 \text{ kg}} \times 100 = 4.00 \text{ wt\%}$$

4.10 What is the composition, in atom percent, of an alloy that contains 98 g tin and 65 g of lead?

Solution

The concentration of an element in an alloy, in atom percent, may be computed using Equation 4.5. However, it first becomes necessary to compute the number of moles of both Sn and Pb, using Equation 4.4. Thus, the number of moles of Sn is just

$$n_{m_{Sn}} = \frac{m'_{Sn}}{A_{Sn}} = \frac{98\ g}{118.71\ g/mol} = 0.826\ mol$$

Likewise, for Pb

$$n_{m_{Pb}} = \frac{65\ g}{207.2\ g/mol} = 0.314\ mol$$

Now, use of Equation 4.5 yields

$$C'_{Sn} = \frac{n_{m_{Sn}}}{n_{m_{Sn}} + n_{m_{Pb}}} \times 100$$

$$= \frac{0.826\ mol}{0.826\ mol + 0.314\ mol} \times 100 = 72.5\ at\%$$

Also,

$$C'_{Pb} = = \frac{0.314\ mol}{0.826\ mol + 0.314\ mol} \times 100 = 27.5\ at\%$$

4.11 What is the composition, in atom percent, of an alloy that contains 99.7 lb_m copper, 102 lb_m zinc, and 2.1 lb_m lead?

Solution

In this problem we are asked to determine the concentrations, in atom percent, of the Cu-Zn-Pb alloy. It is first necessary to convert the amounts of Cu, Zn, and Pb into grams.

$$m'_{Cu} = (99.7\ lb_m)(453.6\ g/lb_m) = 45{,}224\ g$$

$$m'_{Zn} = (102\ lb_m)(453.6\ g/lb_m) = 46{,}267\ g$$

$$m'_{Pb} = (2.1\ lb_m)(453.6\ g/lb_m) = 953\ g$$

These masses must next be converted into moles (Equation 4.4), as

$$n_{m_{Cu}} = \frac{m'_{Cu}}{A_{Cu}} = \frac{45{,}224\ g}{63.55\ g/mol} = 711.6\ mol$$

$$n_{m_{Zn}} = \frac{46{,}267\ g}{65.41\ g/mol} = 707.3\ mol$$

$$n_{m_{Pb}} = \frac{953\ g}{207.2\ g/mol} = 4.6\ mol$$

Now, employment of a modified form of Equation 4.5, gives

$$C'_{Cu} = \frac{n_{m_{Cu}}}{n_{m_{Cu}} + n_{m_{Zn}} + n_{m_{Pb}}} \times 100$$

$$= \frac{711.6\ mol}{711.6\ mol + 707.3\ mol + 4.6\ mol} \times 100 = 50.0\ at\%$$

$$C'_{Zn} = \frac{707.3\ mol}{711.6\ mol + 707.3\ mol + 4.6\ mol} \times 100 = 49.7\ at\%$$

$$C'_{Pb} = \frac{4.6 \text{ mol}}{711.6 \text{ mol} + 707.3 \text{ mol} + 4.6 \text{ mol}} \times 100 = 0.3 \text{ at\%}$$

4.12 What is the composition, in atom percent, of an alloy that consists of 97 wt% Fe and 3 wt% Si?

Solution

We are asked to compute the composition of an Fe-Si alloy in atom percent. Employment of Equation 4.6 leads to

$$C'_{Fe} = \frac{C_{Fe} A_{Si}}{C_{Fe} A_{Si} + C_{Si} A_{Fe}} \times 100$$

$$= \frac{97\,(28.09\ \text{g/mol})}{97\,(28.09\ \text{g/mol}) + 3\,(55.85\ \text{g/mol})} \times 100$$

$$= 94.2\ \text{at\%}$$

$$C'_{Si} = \frac{C_{Si} A_{Fe}}{C_{Si} A_{Fe} + C_{Fe} A_{Si}} \times 100$$

$$= \frac{3\,(55.85\ \text{g/mol})}{3\,(55.85\ \text{g/mol}) + 97\,(28.09\ \text{g/mol})} \times 100$$

$$= 5.8\ \text{at\%}$$

4.13 Convert the atom percent composition in Problem 4.11 to weight percent.

Solution

The composition in atom percent for Problem 4.11 is 50.0 at% Cu, 49.7 at% Zn, and 0.3 at% Pb. Modification of Equation 4.7 to take into account a three-component alloy leads to the following

$$C_{Cu} = \frac{C'_{Cu} A_{Cu}}{C'_{Cu} A_{Cu} + C'_{Zn} A_{Zn} + C'_{Pb} A_{Pb}} \times 100$$

$$= \frac{(50.0)(63.55\ g/mol)}{(50.0)(63.55\ g/mol) + (49.7)(65.41\ g/mol) + (0.3)(207.2\ g/mol)} \times 100$$

$$= 49.0\ wt\%$$

$$C_{Zn} = \frac{C'_{Zn} A_{Zn}}{C'_{Cu} A_{Cu} + C'_{Zn} A_{Zn} + C'_{Pb} A_{Pb}} \times 100$$

$$= \frac{(49.7)(65.41\ g/mol)}{(50.0)(63.55\ g/mol) + (49.7)(65.41\ g/mol) + (0.3)(207.2\ g/mol)} \times 100$$

$$= 50.1\ wt\%$$

$$C_{Pb} = \frac{C'_{Pb} A_{Pb}}{C'_{Cu} A_{Cu} + C'_{Zn} A_{Zn} + C'_{Pb} A_{Pb}} \times 100$$

$$= \frac{(0.3)(207.2\ g/mol)}{(50.0)(63.55\ g/mol) + (49.7)(65.41\ g/mol) + (0.3)(207.2\ g/mol)} \times 100$$

$$= 1.0\ wt\%$$

4.14 Calculate the number of atoms per cubic meter in aluminum.

Solution

In order to solve this problem, one must employ Equation 4.2,

$$N = \frac{N_A \rho_{Al}}{A_{Al}}$$

The density of Al (from the table inside of the front cover) is 2.71 g/cm^3, while its atomic weight is 26.98 g/mol. Thus,

$$N = \frac{(6.022 \times 10^{23} \text{ atoms/mol})(2.71 \text{ g/cm}^3)}{26.98 \text{ g/mol}}$$

$$= 6.05 \times 10^{22} \text{ atoms/cm}^3 = 6.05 \times 10^{28} \text{ atoms/m}^3$$

4.15 The concentration of carbon in an iron-carbon alloy is 0.15 wt%. What is the concentration in kilograms of carbon per cubic meter of alloy?

Solution

In order to compute the concentration in kg/m^3 of C in a 0.15 wt% C-99.85 wt% Fe alloy we must employ Equation 4.9 as

$$C_C'' = \frac{C_C}{\frac{C_C}{\rho_C} + \frac{C_{Fe}}{\rho_{Fe}}} \times 10^3$$

From inside the front cover, densities for carbon and iron are 2.25 and 7.87 g/cm^3, respectively; and, therefore

$$C_C'' = \frac{0.15}{\frac{0.15}{2.25\ g/cm^3} + \frac{99.85}{7.87\ g/cm^3}} \times 10^3$$

$$= 11.8\ kg/m^3$$

4.16 Determine the approximate density of a high-leaded brass that has a composition of 64.5 wt% Cu, 33.5 wt% Zn, and 2.0 wt% Pb.

Solution

In order to solve this problem, Equation 4.10a is modified to take the following form:

$$\rho_{ave} = \frac{100}{\frac{C_{Cu}}{\rho_{Cu}} + \frac{C_{Zn}}{\rho_{Zn}} + \frac{C_{Pb}}{\rho_{Pb}}}$$

And, using the density values for Cu, Zn, and Pb—i.e., 8.94 g/cm^3, 7.13 g/cm^3, and 11.35 g/cm^3—(as taken from inside the front cover of the text), the density is computed as follows:

$$\rho_{ave} = \frac{100}{\frac{64.5\ \text{wt\%}}{8.94\ \text{g/cm}^3} + \frac{33.5\ \text{wt\%}}{7.13\ \text{g/cm}^3} + \frac{2.0\ \text{wt\%}}{11.35\ \text{g/cm}^3}}$$

$$= 8.27\ \text{g/cm}^3$$

4.17 Calculate the unit cell edge length for an 85 wt% Fe-15 wt% V alloy. All of the vanadium is in solid solution, and, at room temperature the crystal structure for this alloy is BCC.

Solution

,Q RUGHU WR VROYH WKLV SUREOHP LW LV QHFHVVDU\ WR HPSOR\ (TXDWLRQ ◀ LQ WKLV H[SUHVVLRQ GHQVLW\ DQG DWomic weight will be averages for the alloy—that is

$$\rho_{ave} = \frac{nA_{ave}}{V_C N_A}$$

Inasmuch as the unit cell is cubic, then $V_C = a^3$, then

$$\rho_{ave} = \frac{nA_{ave}}{a^3 N_A}$$

And solving this equation for the unit cell edge length, leads to

$$a = \left(\frac{nA_{ave}}{\rho_{ave} N_A} \right)^{1/3}$$

Expressions for A_{ave} and ρ_{ave} are found in Equations 4.11a and 4.10a, respectively, which, when incorporated into the above expression yields

$$a = \left[\frac{n \left(\frac{100}{\frac{C_{Fe}}{A_{Fe}} + \frac{C_V}{A_V}} \right)}{\left(\frac{100}{\frac{C_{Fe}}{\rho_{Fe}} + \frac{C_V}{\rho_V}} \right) N_A} \right]^{1/3}$$

Since the crystal structure is BCC, the value of n in the above expression is 2 atoms per unit cell. The atomic weights for Fe and V are 55.85 and 50.94 g/mol, respectively (Figure 2.6), whereas the densities for the Fe and V are 7.87 g/cm^3 and 6.10 g/cm^3 (from inside the front cover). Substitution of these, as well as the concentration values stipulated in the problem statement, into the above equation gives

$$a = \left[\frac{(2 \text{ atoms/unit cell}) \left(\dfrac{100}{\dfrac{85 \text{ wt\%}}{55.85 \text{ g/mol}} + \dfrac{15 \text{ wt\%}}{50.94 \text{ g/mol}}} \right)}{\left(\dfrac{100}{\dfrac{85 \text{ wt\%}}{7.87 \text{ g/cm}^3} + \dfrac{15 \text{ wt\%}}{6.10 \text{ g/cm}^3}} \right) \left(6.022 \times 10^{23} \text{ atoms/mol}\right)} \right]^{1/3}$$

$$= 2.89 \times 10^{-8} \text{ cm} = 0.289 \text{ nm}$$

4.18 Some hypothetical alloy is composed of 12.5 wt% of metal A and 87.5 wt% of metal B. If the densities of metals A and B are 4.27 and 6.35 g/cm^3, respectively, whereas their respective atomic weights are 61.4 and 125.7 g/mol, determine whether the crystal structure for this alloy is simple cubic, face-centered cubic, or body-centered cubic. Assume a unit cell edge length of 0.395 nm.

Solution

,Q RUGHU WR VROYH WKLV SUREOHP LW LV QHFHVVDU\ WR HP SOR\ (TXDWLRQ ◀ LQ WKLV H[SUHVVLRQ GHQVLW\ DQG DWRP LF weight will be averages for the alloy—that is

$$\rho_{ave} = \frac{nA_{ave}}{V_C N_A}$$

Inasmuch as for each of the possible crystal structures, the unit cell is cubic, then $V_C = a^3$, or

$$\rho_{ave} = \frac{nA_{ave}}{a^3 N_A}$$

And, in order to determine the crystal structure it is necessary to solve for n, the number of atoms per unit cell. For n =1, the crystal structure is simple cubic, whereas for n values of 2 and 4, the crystal structure will be either BCC or FCC, respectively. When we solve the above expression for n the result is as follows:

$$n = \frac{\rho_{ave} a^3 N_A}{A_{ave}}$$

Expressions for A_{ave} and ρ_{ave} are found in Equations 4.11a and 4.10a, respectively, which, when incorporated into the above expression yields

$$n = \frac{\left(\dfrac{100}{\dfrac{C_A}{\rho_A} + \dfrac{C_B}{\rho_B}} \right) a^3 N_A}{\left(\dfrac{100}{\dfrac{C_A}{A_A} + \dfrac{C_B}{A_B}} \right)}$$

Substitution of the concentration values (i.e., C_A = 12.5 wt% and C_B = 87.5 wt%) as well as values for the other parameters given in the problem statement, into the above equation gives

$$n = \frac{\left(\frac{100}{\frac{12.5\ \text{wt\%}}{4.27\ \text{g/cm}^3} + \frac{87.5\ \text{wt\%}}{6.35\ \text{g/cm}^3}}\right)(3.95 \times 10^{-8}\ \text{nm})^3(6.022 \times 10^{23}\ \text{atoms/mol})}{\left(\frac{100}{\frac{12.5\ \text{wt\%}}{61.4\ \text{g/mol}} + \frac{87.5\ \text{wt\%}}{125.7\ \text{g/mol}}}\right)}$$

$$= 2.00\ \text{atoms/unit cell}$$

Therefore, on the basis of this value, the crystal structure is body-centered cubic.

4.19 For a solid solution consisting of two elements (designated as 1 and 2), sometimes it is desirable to determine the number of atoms per cubic centimeter of one element in a solid solution, N_1, given the concentration of that element specified in weight percent, C_1. This computation is possible using the following expression:

$$N_1 = \frac{N_A C_1}{\frac{C_1 A_1}{\rho_1} + \frac{A_1}{\rho_2}(100 - C_1)} \tag{4.18}$$

where

N_A = Avogadro's number

ρ_1 and ρ_2 = densities of the two elements

A_1 = the atomic weight of element 1

Derive Equation 4.18 using Equation 4.2 and expressions contained in Section 4.4.

Solution

This problem asks that we derive Equation 4.18, using other equations given in the chapter. The concentration of component 1 in atom percent (C_1') is just $100\,c_1'$ where c_1' is the atom fraction of component 1. Furthermore, c_1' is defined as $c_1' = N_1/N$ where N_1 and N are, respectively, the number of atoms of component 1 and total number of atoms per cubic centimeter. Thus, from the above discussion the following holds:

$$N_1 = \frac{C_1' N}{100}$$

Substitution into this expression of the appropriate form of N from Equation 4.2 yields

$$N_1 = \frac{C_1' N_A \rho_{ave}}{100 A_{ave}}$$

And, finally, substitution into this equation expressions for C_1' (Equation 4.6a), ρ_{ave} (Equation 4.10a), A_{ave} (Equation 4.11a), and realizing that $C_2 = (C_1 - 100)$, and after some algebraic manipulation we obtain the desired expression:

$$N_1 = \frac{N_A C_1}{\frac{C_1 A_1}{\rho_1} + \frac{A_1}{\rho_2}(100 - C_1)}$$

4.20 Gold forms a substitutional solid solution with silver. Compute the number of gold atoms per cubic centimeter for a silver-gold alloy that contains 10 wt% Au and 90 wt% Ag. The densities of pure gold and silver are 19.32 and 10.49 g/cm^3, respectively.

Solution

To solve this problem, employment of Equation 4.18 is necessary, using the following values:

$C_1 = C_{Au} = 10$ wt%

$\rho_1 = \rho_{Au} = 19.32$ g/cm^3

$\rho_2 = \rho_{Ag} = 10.49$ g/cm^3

$A_1 = A_{Au} = 196.97$ g/mol

Thus

$$N_{Au} = \frac{N_A C_{Au}}{\frac{C_{Au} A_{Au}}{\rho_{Au}} + \frac{A_{Au}}{\rho_{Ag}}(100 - C_{Au})}$$

$$= \frac{(6.022 \times 10^{23} \text{ atoms/mol})(10 \text{ wt\%})}{\frac{(10 \text{ wt\%})(196.97 \text{ g/mol})}{19.32 \text{ g/cm}^3} + \frac{196.97 \text{ g/mol}}{10.49 \text{ g/cm}^3}(100 - 10 \text{ wt\%})}$$

$$= 3.36 \times 10^{21} \text{ atoms/cm}^3$$

4.21 Germanium forms a substitutional solid solution with silicon. Compute the number of germanium atoms per cubic centimeter for a germanium-silicon alloy that contains 15 wt% Ge and 85 wt% Si. The densities of pure germanium and silicon are 5.32 and 2.33 g/cm^3, respectively.

Solution

To solve this problem, employment of Equation 4.18 is necessary, using the following values:

$C_1 = C_{Ge} = 15$ wt%

$\rho_1 = \rho_{Ge} = 5.32$ g/cm^3

$\rho_2 = \rho_{Si} = 2.33$ g/cm^3

$A_1 = A_{Ge} = 72.64$ g/mol

Thus

$$N_{Ge} = \frac{N_A C_{Ge}}{\frac{C_{Ge} A_{Ge}}{\rho_{Ge}} + \frac{A_{Ge}}{\rho_{Si}}(100 - C_{Ge})}$$

$$= \frac{(6.022 \times 10^{23} \text{ atoms/mol})(15 \text{ wt\%})}{\frac{(15 \text{ wt\%})(72.64 \text{ g/mol})}{5.32 \text{ g/cm}^3} + \frac{72.64 \text{ g/mol}}{2.33 \text{ g/cm}^3}(100 - 15 \text{ wt\%})}$$

$$= 3.16 \times 10^{21} \text{ atoms/cm}^3$$

4.22 Sometimes it is desirable to be able to determine the weight percent of one element, C_1, that will produce a specified concentration in terms of the number of atoms per cubic centimeter, N_1, for an alloy composed of two types of atoms. This computation is possible using the following expression:

$$C_1 = \frac{100}{1 + \frac{N_A \rho_2}{N_1 A_1} - \frac{\rho_2}{\rho_1}} \tag{4.19}$$

where

N_A = Avogadro's number

ρ_1 and ρ_2 = densities of the two elements

A_1 and A_2 = the atomic weights of the two elements

Derive Equation 4.19 using Equation 4.2 and expressions contained in Section 4.4.

Solution

The number of atoms of component 1 per cubic centimeter is just equal to the atom fraction of component 1 (c_1') times the total number of atoms per cubic centimeter in the alloy (N). Thus, using the equivalent of Equation 4.2, we may write

$$N_1 = c_1' N = \frac{c_1' N_A \rho_{ave}}{A_{ave}}$$

Realizing that

$$c_1' = \frac{C_1'}{100}$$

and

$$C_2' = 100 - C_1'$$

and substitution of the expressions for ρ_{ave} and A_{ave}, Equations 4.10b and 4.11b, respectively, leads to

$$N_1 = \frac{c_1' N_A \rho_{ave}}{A_{ave}}$$

$$= \frac{N_A C_1' \rho_1 \rho_2}{C_1' \rho_2 A_1 + (100 - C_1')\rho_1 A_2}$$

And, solving for C_1'

$$C_1' = \frac{100\ N_1 \rho_1 A_2}{N_A \rho_1 \rho_2 - N_1 \rho_2 A_1 + N_1 \rho_1 A_2}$$

Substitution of this expression for C_1' into Equation 4.7a, which may be written in the following form

$$C_1 = \frac{C_1' A_1}{C_1' A_1 + C_2' A_2} \times 100$$

$$= \frac{C_1' A_1}{C_1' A_1 + (100 - C_1') A_2} \times 100$$

yields

$$C_1 = \frac{100}{1 + \frac{N_A \rho_2}{N_1 A_1} - \frac{\rho_2}{\rho_1}}$$

the desired expression.

4.23 Molybdenum forms a substitutional solid solution with tungsten. Compute the weight percent of molybdenum that must be added to tungsten to yield an alloy that contains 1.0×10^{22} Mo atoms per cubic centimeter. The densities of pure Mo and W are 10.22 and 19.30 g/cm^3, respectively.

Solution

To solve this problem, employment of Equation 4.19 is necessary, using the following values:

$N_1 = N_{Mo} = 10^{22}$ atoms/cm^3
$\rho_1 = \rho_{Mo} = 10.22$ g/cm^3
$\rho_2 = \rho_W = 19.30$ g/cm^3
$A_1 = A_{Mo} = 95.94$ g/mol
$A_2 = A_W = 183.84$ g/mol

Thus

$$C_{Mo} = \frac{100}{1 + \frac{N_A \rho_W}{N_{Mo} A_{Mo}} - \frac{\rho_W}{\rho_{Mo}}}$$

$$= \frac{100}{1 + \frac{(6.022 \times 10^{23} \text{ atoms/mol})(19.30 \text{ g/cm}^3)}{(10^{22} \text{ atoms/cm}^3)(95.94 \text{ g/mol})} - \left(\frac{19.30 \text{ g/cm}^3}{10.22 \text{ g/cm}^3}\right)}$$

$$= 8.91 \text{ wt\%}$$

4.24 Niobium forms a substitutional solid solution with vanadium. Compute the weight percent of niobium that must be added to vanadium to yield an alloy that contains 1.55×10^{22} Nb atoms per cubic centimeter. The densities of pure Nb and V are 8.57 and 6.10 g/cm^3, respectively.

Solution

To solve this problem, employment of Equation 4.19 is necessary, using the following values:

$N_1 = N_{Nb} = 1.55 \times 10^{22}$ atoms/cm^3
$\rho_1 = \rho_{Nb} = 8.57$ g/cm^3
$\rho_2 = \rho_V = 6.10$ g/cm^3
$A_1 = A_{Nb} = 92.91$ g/mol
$A_2 = A_V = 50.94$ g/mol

Thus

$$C_{Nb} = \frac{100}{1 + \frac{N_A \rho_V}{N_{Nb} A_{Nb}} - \frac{\rho_V}{\rho_{Nb}}}$$

$$= \frac{100}{1 + \frac{(6.022 \times 10^{23} \text{ atoms/mol})(6.10 \text{ g/cm}^3)}{(1.55 \times 10^{22} \text{ atoms/cm}^3)(92.91 \text{ g/mol})} - \left(\frac{6.10 \text{ g/cm}^3}{8.57 \text{ g/cm}^3}\right)}$$

$$= 35.2 \text{ wt\%}$$

4.25 Silver and palladium both have the FCC crystal structure, and Pd forms a substitutional solid solution for all concentrations at room temperature. Compute the unit cell edge length for a 75 wt% Ag–25 wt% Pd alloy. The room-temperature density of Pd is 12.02 g/cm^3, and its atomic weight and atomic radius are 106.4 g/mol and 0.138 nm, respectively.

Solution

First of all, the atomic radii for Ag (using the table inside the front cover) and Pd are 0.144 and 0.138 nm, respectively. Also, using Equation 3.5 it is possible to compute the unit cell volume, and inasmuch as the unit cell is cubic, the unit cell edge length is just the cube root of the volume. However, it is first necessary to calculate the density and average atomic weight of this alloy using Equations 4.10a and 4.11a. Inasmuch as the densities of silver and palladium are 10.49 g/cm^3 (as taken from inside the front cover) and 12.02 g/cm^3, respectively, the average density is just

$$\rho_{ave} = \frac{100}{\frac{C_{Ag}}{\rho_{Ag}} + \frac{C_{Pd}}{\rho_{Pd}}}$$

$$= \frac{100}{\frac{75 \text{ wt\%}}{10.49 \text{ g/cm}^3} + \frac{25 \text{ wt\%}}{12.02 \text{ g/cm}^3}}$$

$$= 10.83 \text{ g/cm}^3$$

And for the average atomic weight

$$A_{ave} = \frac{100}{\frac{C_{Ag}}{A_{Ag}} + \frac{C_{Pd}}{A_{Pd}}}$$

$$= \frac{100}{\frac{75 \text{ wt\%}}{107.9 \text{ g/mol}} + \frac{25 \text{ wt\%}}{106.4 \text{ g/mol}}}$$

$$= 107.5 \text{ g/mol}$$

Now, V_C is determined from Equation 3.5 as

$$V_C = \frac{nA_{ave}}{\rho_{ave} N_A}$$

$$= \frac{(4 \text{ atoms/unit cell})(107.5 \text{ g/mol})}{(10.83 \text{ g/cm}^3)(6.022 \times 10^{23} \text{ atoms/mol})}$$

$$= 6.59 \times 10^{-23} \text{ cm}^3\text{/unit cell}$$

And, finally

$$a = (V_C)^{1/3}$$

$$= (6.59 \times 10^{-23} \text{cm}^3\text{/unit cell})^{1/3}$$

$$= 4.04 \times 10^{-8} \text{ cm} = 0.404 \text{ nm}$$

Dislocations—Linear Defects

4.26 Cite the relative Burgers vector–dislocation line orientations for edge, screw, and mixed dislocations.

Solution

The Burgers vector and dislocation line are perpendicular for edge dislocations, parallel for screw dislocations, and neither perpendicular nor parallel for mixed dislocations.

Interfacial Defects

4.27 For an FCC single crystal, would you expect the surface energy for a (100) plane to be greater or less than that for a (111) plane? Why? (Note: You may want to consult the solution to Problem 3.54 at the end of Chapter 3.)

Solution

The surface energy for a crystallographic plane will depend on its packing density [i.e., the planar density (Section 3.11)]—that is, the higher the packing density, the greater the number of nearest-neighbor atoms, and the more atomic bonds in that plane that are satisfied, and, consequently, the lower the surface energy. From the solution to Problem 3.54, planar densities for FCC (100) and (111) planes are $\frac{1}{4R^2}$ and $\frac{1}{2R^2\sqrt{3}}$, respectively—that is $\frac{0.25}{R^2}$ and $\frac{0.29}{R^2}$ (where R is the atomic radius). Thus, since the planar density for (111) is greater, it will have the lower surface energy.

4.28 For a BCC single crystal, would you expect the surface energy for a (100) plane to be greater or less than that for a (110) plane? Why? (Note: You may want to consult the solution to Problem 3.55 at the end of Chapter 3.)

Solution

The surface energy for a crystallographic plane will depend on its packing density [i.e., the planar density (Section 3.11)]—that is, the higher the packing density, the greater the number of nearest-neighbor atoms, and the more atomic bonds in that plane that are satisfied, and, consequently, the lower the surface energy. From the solution to Problem 3.55, the planar densities for BCC (100) and (110) are $\frac{3}{16R^2}$ and $\frac{3}{8R^2\sqrt{2}}$, respectively—that is $\frac{0.19}{R^2}$ and $\frac{0.27}{R^2}$. Thus, since the planar density for (110) is greater, it will have the lower surface energy.

4.29 (a) For a given material, would you expect the surface energy to be greater than, the same as, or less than the grain boundary energy? Why?

(b) The grain boundary energy of a small-angle grain boundary is less than for a high-angle one. Why is this so?

Solution

(a) The surface energy will be greater than the grain boundary energy. For grain boundaries, some atoms on one side of a boundary will bond to atoms on the other side; such is not the case for surface atoms. Therefore, there will be fewer unsatisfied bonds along a grain boundary.

(b) The small-angle grain boundary energy is lower than for a high-angle one because more atoms bond across the boundary for the small-angle, and, thus, there are fewer unsatisfied bonds.

4.30 (a) Briefly describe a twin and a twin boundary.

(b) Cite the difference between mechanical and annealing twins.

Solution

(a) A twin boundary is an interface such that atoms on one side are located at mirror image positions of those atoms situated on the other boundary side. The region on one side of this boundary is called a twin.

(b) Mechanical twins are produced as a result of mechanical deformation and generally occur in BCC and HCP metals. Annealing twins form during annealing heat treatments, most often in FCC metals.

4.31 For each of the following stacking sequences found in FCC metals, cite the type of planar defect that exists:

(a) . . . A B C A B C B A C B A . . .

(b) . . . A B C A B C B C A B C . . .

Now, copy the stacking sequences and indicate the position(s) of planar defect(s) with a vertical dashed line.

Solution

(a) The interfacial defect that exists for this stacking sequence is a twin boundary, which occurs at the indicated position.

The stacking sequence on one side of this position is mirrored on the other side.

(b) The interfacial defect that exists within this FCC stacking sequence is a stacking fault, which occurs between the two lines.

Within this region, the stacking sequence is HCP.

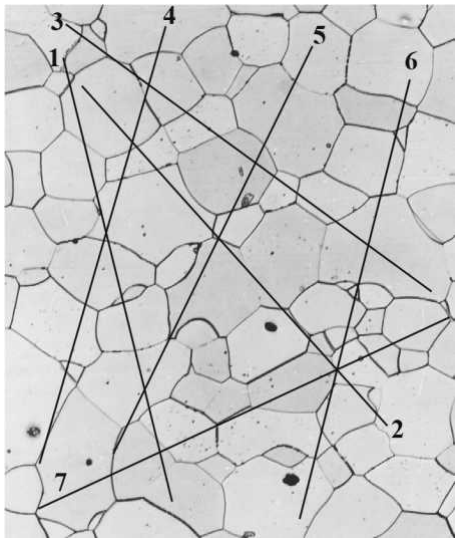
Grain Size Determination

4.32 (a) Using the intercept method, determine the average grain size, in millimeters, of the specimen ZKRVHPLFURVWUXFWXUHVKRZQLQ)LJXUH/◀/♂E♀XVHDWHVWOLQHVHJPHQWVDWUDQGRPOLQH segments.

(b) Estimate the ASTM grain size number for this material.

Solution

(a) Below is shown the photomicrograph of Figure 4.14(b), on which seven straight line segments, each of ZKLFKLVDSSUR[LPDWHO\FRQVWUXFWHGDFURVVWKHPLFURVWUXFWXUH¶VJUDLQV“7”.



In order to determine the average grain diameter, it is necessary to count the number of grains intersected by each of these line segments. These data are tabulated below.

Line Number	No. Grains Intersected
1	11
2	10
3	9
4	8.5
5	7
6	10
7	8

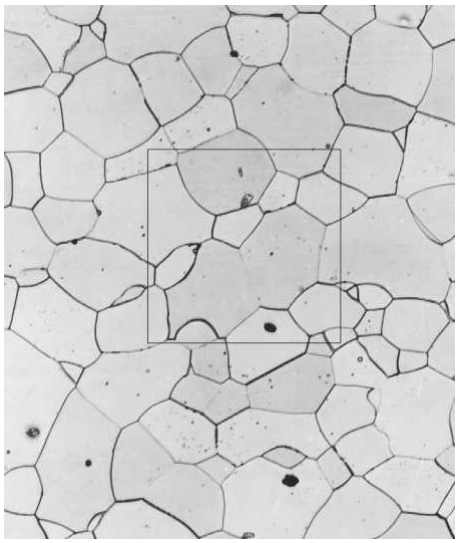
The average number of grain boundary intersections for these lines was 9.1. Therefore, the average line length intersected is just

$$\frac{60 \text{ mm}}{9.1} = 6.59 \text{ mm}$$

Hence, the average grain diameter, d, is

$$d = \frac{\text{ave. line length intersected}}{\text{magnification}} = \frac{6.59 \text{ mm}}{100} = 6.59 \times 10^{-2} \text{ mm}$$

(b) This portion of the problem calls for us to estimate the ASTM grain size number for this same material. The average grain size number, n, is related to the number of grains per square inch, N, at a magnification of 100× according to Equation 4.16. Inasmuch as the magnification is 100×, the value of N is measured directly from the micrograph. The photomicrograph on which has been constructed a square 1 in. on a side is shown below.



The total number of complete grains within this square is approximately 10 (taking into account grain fractions). Now, in order to solve for n in Equation 4.16, it is first necessary to take logarithms as

$$\log N = (n - 1) \log 2$$

From which n equals

$$n = \frac{\log N}{\log 2} + 1$$

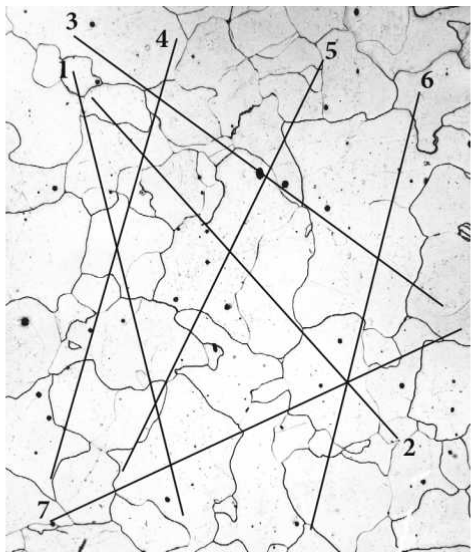
$$= \frac{\log 10}{\log 2} + 1 = 4.3$$

4.33 (a) Employing the intercept technique, determine the average grain size for the steel specimen whose microstructure is shown in Figure 9.25(a); use at least seven straight-line segments.

(b) Estimate the ASTM grain size number for this material.

Solution

(a) Below is shown the photomicrograph of Figure 9.25(a), on which seven straight line segments, each of which is 60 mm long has been constructed; the lines are labeled "1" through "7".



In order to determine the average grain diameter, it is necessary to count the number of grains intersected by each of these line segments. These data are tabulated below.

Line Number	No. Grains Intersected
1	7
2	7
3	7
4	8
5	10
6	7
7	8

The average number of grain boundary intersections for these lines was 8.7. Therefore, the average line length intersected is just

$$\frac{60\ \text{mm}}{8.7} = 6.9\ \text{mm}$$

Hence, the average grain diameter, d, is

$$d = \frac{\text{ave. line length intersected}}{\text{magnification}} = \frac{6.9\ \text{mm}}{90} = 0.077\ \text{mm}$$

(b) This portion of the problem calls for us to estimate the ASTM grain size number for this same material. The average grain size number, n, is related to the number of grains per square inch, N, at a magnification of 100× according to Equation 4.16. However, the magnification of this micrograph is not 100×, but rather 90×. Consequently, it is necessary to use Equation 4.17

$$N_M\left(\frac{M}{100}\right)^2 = 2^{n-1}$$

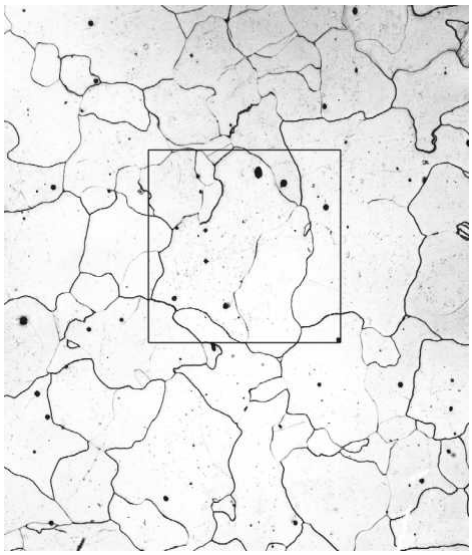
where N_M = the number of grains per square inch at magnification M, and n is the ASTM grain size number. Taking logarithms of both sides of this equation leads to the following:

$$\log N_M + 2\log\left(\frac{M}{100}\right) = (n-1)\log 2$$

Solving this expression for n gives

$$n = \frac{\log N_M + 2\log\left(\frac{M}{100}\right)}{\log 2} + 1$$

The photomicrograph on which has been constructed a square 1 in. on a side is shown below.



From Figure 9.25(a), N_M is measured to be approximately 7, which leads to

$$n = \frac{\log 7 + 2 \log\left(\frac{90}{100}\right)}{\log 2} + 1$$

$$= 3.5$$

4.34 For an ASTM grain size of 8, approximately how many grains would there be per square inch at

(a) a magnification of 100, and

(b) without any magnification?

Solution

(a) This part of problem asks that we compute the number of grains per square inch for an ASTM grain size of 8 at a magnification of 100×. All we need do is solve for the parameter N in Equation 4.16, inasmuch as n = 8. Thus

$$N = 2^{n-1}$$

$$= 2^{8-1} = 128 \text{ grains/in.}^2$$

(b) Now it is necessary to compute the value of N for no magnification. In order to solve this problem it is necessary to use Equation 4.17:

$$N_M \left(\frac{M}{100} \right)^2 = 2^{n-1}$$

where N_M = the number of grains per square inch at magnification M, and n is the ASTM grain size number. Without any magnification, M in the above equation is 1, and therefore,

$$N_1 \left(\frac{1}{100} \right)^2 = 2^{8-1} = 128$$

And, solving for N_1, N_1 = 1,280,000 grains/in.^{2}.

4.35 Determine the ASTM grain size number if 25 grains per square inch are measured at a magnification of 600.

Solution

This problem asks that we determine the ASTM grain size number if 8 grains per square inch are measured at a magnification of 600. In order to solve this problem we make use of Equation 4.17:

$$N_M \left(\frac{M}{100} \right)^2 = 2^{n-1}$$

where N_M = the number of grains per square inch at magnification M, and n is the ASTM grain size number. Solving the above equation for n, and realizing that $N_M = 8$, while $M = 600$, we have

$$n = \frac{\log N_M + 2 \log \left(\frac{M}{100} \right)}{\log 2} + 1$$

$$= \frac{\log 8 + 2 \log \left(\frac{600}{100} \right)}{\log 2} + 1 = 9.2$$

4.36 Determine the ASTM grain size number if 20 grains per square inch are measured at a magnification of 50.

Solution

This problem asks that we determine the ASTM grain size number if 20 grains per square inch are measured at a magnification of 50. In order to solve this problem we make use of Equation 4.17—viz.

$$N_M \left(\frac{M}{100} \right)^2 = 2^{n-1}$$

where N_M = the number of grains per square inch at magnification M, and n is the ASTM grain size number. Solving the above equation for n, and realizing that $N_M = 20$, while $M = 50$, we have

$$n = \frac{\log N_M + 2 \log \left(\frac{M}{100} \right)}{\log 2} + 1$$

$$= \frac{\log 20 + 2 \log \left(\frac{50}{100} \right)}{\log 2} + 1 = 3.3$$

DESIGN PROBLEMS

Specification of Composition

4.D1 Aluminum–lithium alloys have been developed by the aircraft industry to reduce the weight and improve the performance of its aircraft. A commercial aircraft skin material having a density of 2.55 g/cm^3 is desired. Compute the concentration of Li (in wt%) that is required.

Solution

Solution of this problem requires the use of Equation 4.10a, which takes the form

$$\rho_{ave} = \frac{100}{\frac{C_{Li}}{\rho_{Li}} + \frac{100 - C_{Li}}{\rho_{Al}}}$$

inasmuch as $C_{Li} + C_{Al} = 100$. According to the table inside the front cover, the respective densities of Li and Al are 0.534 and 2.71 g/cm^3. Upon solving for C_{Li} from the above equation, we get

$$C_{Li} = \frac{100\ \rho_{Li}(\rho_{Al} - \rho_{ave})}{\rho_{ave}(\rho_{Al} - \rho_{Li})}$$

And incorporating specified values into the above equation leads to

$$C_{Li} = \frac{(100)(0.534\ \text{g/cm}^3)(2.71\ \text{g/cm}^3 - 2.55\ \text{g/cm}^3)}{(2.55\ \text{g/cm}^3)(2.71\ \text{g/cm}^3 - 0.534\ \text{g/cm}^3)}$$

$$= 1.540\ \text{wt\%}$$

4.D2 Iron and vanadium both have the BCC crystal structure and V forms a substitutional solid solution in Fe for concentrations up to approximately 20 wt% V at room temperature. Determine the concentration in weight percent of V that must be added to iron to yield a unit cell edge length of 0.289 nm.

Solution

To begin, it is necessary to employ Equation 3.5, and solve for the unit cell volume, V_C, as

$$V_C = \frac{nA_{ave}}{\rho_{ave} N_A}$$

where A_{ave} and ρ_{ave} are the atomic weight and density, respectively, of the Fe-V alloy. Inasmuch as both of these materials have the BCC crystal structure, which has cubic symmetry, V_C is just the cube of the unit cell length, a. That is

$$V_C = a^3 = (0.289\,\text{nm})^3$$

$$= (2.89 \times 10^{-8}\,\text{cm})^3 = 2.414 \times 10^{-23}\,\text{cm}^3$$

It is now necessary to construct expressions for A_{ave} and ρ_{ave} in terms of the concentration of vanadium, C_V, using Equations 4.11a and 4.10a. For A_{ave} we have

$$A_{ave} = \frac{100}{\frac{C_V}{A_V} + \frac{(100 - C_V)}{A_{Fe}}}$$

$$= \frac{100}{\frac{C_V}{50.94\,\text{g/mol}} + \frac{(100 - C_V)}{55.85\,\text{g/mol}}}$$

whereas for ρ_{ave}

$$\rho_{ave} = \frac{100}{\frac{C_V}{\rho_V} + \frac{(100 - C_V)}{\rho_{Fe}}}$$

$$= \frac{100}{\frac{C_V}{6.10\,\text{g/cm}^3} + \frac{(100 - C_V)}{7.87\,\text{g/cm}^3}}$$

Within the BCC unit cell there are 2 equivalent atoms, and thus, the value of n in the above expression may be written in terms of the concentration of V in weight percent as follows:

$$V_C = 2.414 \times 10^{-23}\ \text{cm}^3$$

$$= \frac{nA_{ave}}{\rho_{ave} N_A}$$

$$= \frac{(2\ \text{atoms/unit cell})\left[\dfrac{100}{\dfrac{C_V}{50.94\ \text{g/mol}} + \dfrac{(100 - C_V)}{55.85\ \text{g/mol}}}\right]}{\left[\dfrac{100}{\dfrac{C_V}{6.10\ \text{g/cm}^3} + \dfrac{(100 - C_V)}{7.87\ \text{g/cm}^3}}\right]\left(6.022 \times 10^{23}\ \text{atoms/mol}\right)}$$

And solving this expression for C_V leads to $C_V = 12.9$ wt%.