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(5) Sample size: $n = 20$

the null hypothesis $H_0: \sigma^2 = 36$

the alternative hypothesis $H_1: \sigma^2 \lesssim 36$

the level of significance is $\alpha = 0.05$

- The test statistics is: $X^2 = \frac{(n-1)s^2}{\sigma^2}$
- $s^2 = 4.51 \times 4.51 = 20.34$

$$X^2 = \frac{(20-1).\ 20.34}{36} = 10.735$$

- The critical region is: $X^2 < X^2_{1-\alpha}$
- Using Chi-Square table, for $\alpha = 0.05$ and $v = 19$ degrees of freedom we get:

$$X^2_{0.95} = 10.117$$

- The p-value is: $p\text{-value} = P(X^2 > 10.735)$
$= 0.067$
- The p-value is greater than the level of significance, so we fail to reject the null hypothesis and conclude that $\sigma = \sqrt{36} = 6$

(6) Sample size $n = 12$

$\sigma_0 = 1.40$; $\sigma^2 = (1.40)^2 = 1.96$

Sample variance: $S = 1.75$

The null hypothesis: $H_0: \sigma^2 = 1.96$

The alternative hypothesis: $H_1: \sigma^2 > 1.96$

the level of significance: $\alpha = 0.01$

the statistic: $X^2 = \frac{(n-1)s^2}{\sigma_0^2} = \frac{(12-1)\ 1.75^2}{1.40^2}$
$= 17.19$

- the critical region is $X^2(\text{observed}) > X^2_{0.01;11}$

where $X^2_{0.01;11}$ is the upper 0.01 point of X^2-distribution with 11 degrees of freedom.

From the X^2 we see that $X^2_{0.01;11} = 24.725$.

So, $X^2(\text{Observed}) < 24.725$

- Therefore, we fail to reject H_0 and conclude that the standard deviation of the contributions of all standard workers is not greater than that of all workers living in the city

(7) $n_A = 10$. Sample Mean:

$$\hat{\mu}_A = \frac{1}{n_A}\sum_{i=1}^{n_A} x_i = 11.17$$

$\sigma_A^2 = 0.09$.

$n_B = 8$. Sample mean:

$$\hat{\mu}_B = \frac{1}{n_B}\sum_{i=1}^{n_B} y_i = 11.9875$$

$\sigma_B^2 = 0.16$

- $H_0: \mu_A = \mu_B$
- $H_1: \mu_A \neq \mu_B$

Test: $\left.\frac{\bar{X}-\bar{Y}}{\sqrt{\frac{\sigma_A^2}{n_A}+\frac{\sigma_B^2}{n_B}}}\right|_{H_0} \sim AN(0,1)$

$$p\text{-value} = P_{H_0}\left(\left|\frac{\bar{X}-\bar{Y}}{\sqrt{\frac{\sigma_A^2}{n_A}+\frac{\sigma_B^2}{n_B}}}\right| > \left|\frac{\hat{\mu}_A-\hat{\mu}_B}{\sqrt{\frac{\sigma_A^2}{n_A}+\frac{\sigma_B^2}{n_B}}}\right|\right)$$

$$= P_{H_0}\left(\left|\frac{\bar{X}-\bar{Y}}{\sqrt{\frac{\sigma_A^2}{n_A}+\frac{\sigma_B^2}{n_B}}}\right| > \left|\frac{11.17-11.9875}{\sqrt{\frac{0.09}{10}+\frac{0.16}{8}}}\right|\right)$$

$$= P_{H_0}\left(\left|\frac{\bar{X}-\bar{Y}}{\sqrt{\frac{\sigma_A^2}{n_A}+\frac{\sigma_B^2}{n_B}}}\right| > 4.8\right) = 2P(Z > 4.8)$$

≈ 0

$\Rightarrow$ the p-value is very close to zero so we reject the null hypothesis.

(8) $\hat{\mu}_A = 47700$, $\sigma_A^2 = 2400^2$

$n_A = 16$

$n_B = 16$

$\hat{\mu}_B = 46400$, $\sigma_B^2 = 2200^2$

- estimate pooled variance : $\sigma_p = \dfrac{(n_A - 1)\sigma_A^2 + (n_B - 1)\sigma_B^2}{n_A + n_B - 2} = 5\,300\,000$

we want to prove that the professor is right. therefore, define hypotesis:

$H_0 : \mu_A \leq \mu_B$

$H_1 : \mu_A > \mu_B$

$$p\text{-value} = P_{H_0}\left(\frac{\bar{X} - \bar{Y}}{\sqrt{\sigma_p^2\left(\frac{1}{n_A} + \frac{1}{n_B}\right)}} > \frac{\hat{\mu}_A - \hat{\mu}_B}{\sqrt{\sigma_p^2\left(\frac{1}{n_A} + \frac{1}{n_B}\right)}} \right)$$

$$= P_{H_0}\left(\frac{\bar{X} - \bar{Y}}{\sqrt{\sigma_p^2\left(\frac{1}{n_A} + \frac{1}{n_B}\right)}} > \frac{47800 - 46400}{\sqrt{5300000\left(\frac{1}{16} + \frac{1}{16}\right)}} \right)$$

$$= P_{H_0}\left(\frac{\bar{X} - \bar{Y}}{\sqrt{\sigma_p^2\left(\frac{1}{n_A} + \frac{1}{n_B}\right)}} > 1.5972 \right) = P(T > 1.5972)$$

≈ 0.06

Depending on significance level that we take, we can not reject or reject the null hypothesis. therefore, we could approve or reject proffessor claim.