

MIDTERM EXAMINATION

Semester 1, 2023-2024 • Date: November 23, 2023 • Duration: 75 minutes

SUBJECT: Applied Linear Algebra	
Department of Mathematics	Lecturers
Vice Chair:	
Dr. Nguyen Minh Quan	Ta Q Bao, and Tran V Khanh

INSTRUCTIONS:

- Each student is allowed a scientific calculator and a maximum of TWO double-sided sheets of reference material (size A4 or similar) marked with their name and ID. All other documents and electronic devices are forbidden. Show your work in details and indicate answers clearly.

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1. (15 points) Let A and B be 3×3 matrices whose $\det A = 3$ and $\det B = -2$. Compute the following if possible.

- (a) $\det(\frac{1}{3}AA^T)$
- (b) $\det(A^2B^{-1})$
- (c) $\det(3(A^{-1})^TB^{-2})$

2. (40 points) Let $A = \begin{bmatrix} 1 & 1 \\ -2 & -1 \\ 1 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 1 & -1 & -2 \\ 2 & 1 & -2 \end{bmatrix}$, and $C = \begin{bmatrix} 1 & 1 & -3 \\ -1 & 2 & 1 \\ -3 & -1 & 0 \end{bmatrix}$. Find the following if possible.

- (a) $A + 20B$, $B - 5A^T$, BA and $(BA)^{-1}$
- (b) $A + 4C^T$, AC and CA
- (c) $\det(C)$ and $\det(D)$ where $D = \det \begin{bmatrix} 7 & 1 & -3 \\ -7 & 2 & 1 \\ -21 & -1 & 0 \end{bmatrix}$.

3. (10 points) Find the solution to the system of equations, $7x + 14y + 15z = 22$, $2x + 4y + 3z = 5$, and $3x + 6y + 10z = 13$.
4. (20 points) Let

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 2 & 1 \\ 2 & 6 & 0 \end{bmatrix}$$

Determine whether the matrix A has an inverse by finding whether the determinant is non zero. If the determinant is nonzero, find the inverse.

5. (5 points) Show that for any square matrix A satisfying

$$A^2 - 2A + 5I = 0,$$

the inverse of A is $A^{-1} = \frac{1}{5}(2I - A)$.

Verify for the matrix $A = \begin{pmatrix} 1 & 2 \\ -2 & 1 \end{pmatrix}$.