

The Operational Amplifier

(Chapter 5)

Textbook:

Electric Circuits

James W. Nilsson & Susan A. Riedel

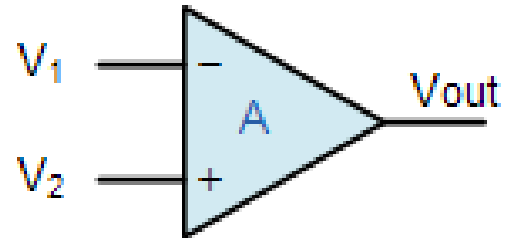
9th Edition.

Objectives

- Be able to name the five **op amp terminals**, and describe and use the **voltage and current constraints** of an **ideal op amp**.
- Be able to **analyze simple circuits** containing ideal op amps, and **recognize** the following op amp circuits: **inverting amplifier, summing amplifier, non inverting amplifier, and difference amplifier**.
- Understand the more **realistic model** for an op amp.

Outline

- Operational amplifier terminals
- Terminal voltages and currents
- The inverting-amplifier circuit
- The summing-amplifier circuit
- The non inverting-amplifier circuit
- The difference-amplifier circuit



Op Amps

- Op Amp \Leftrightarrow operational amplifier.
- An Op Amp is modeled as a voltage controlled voltage source.
- An operational amplifier has a very high input impedance and a very high gain.
- Op amps can be configured in many different ways using resistors and other components.
- Most configurations use feedback.

Applications of Op Amps

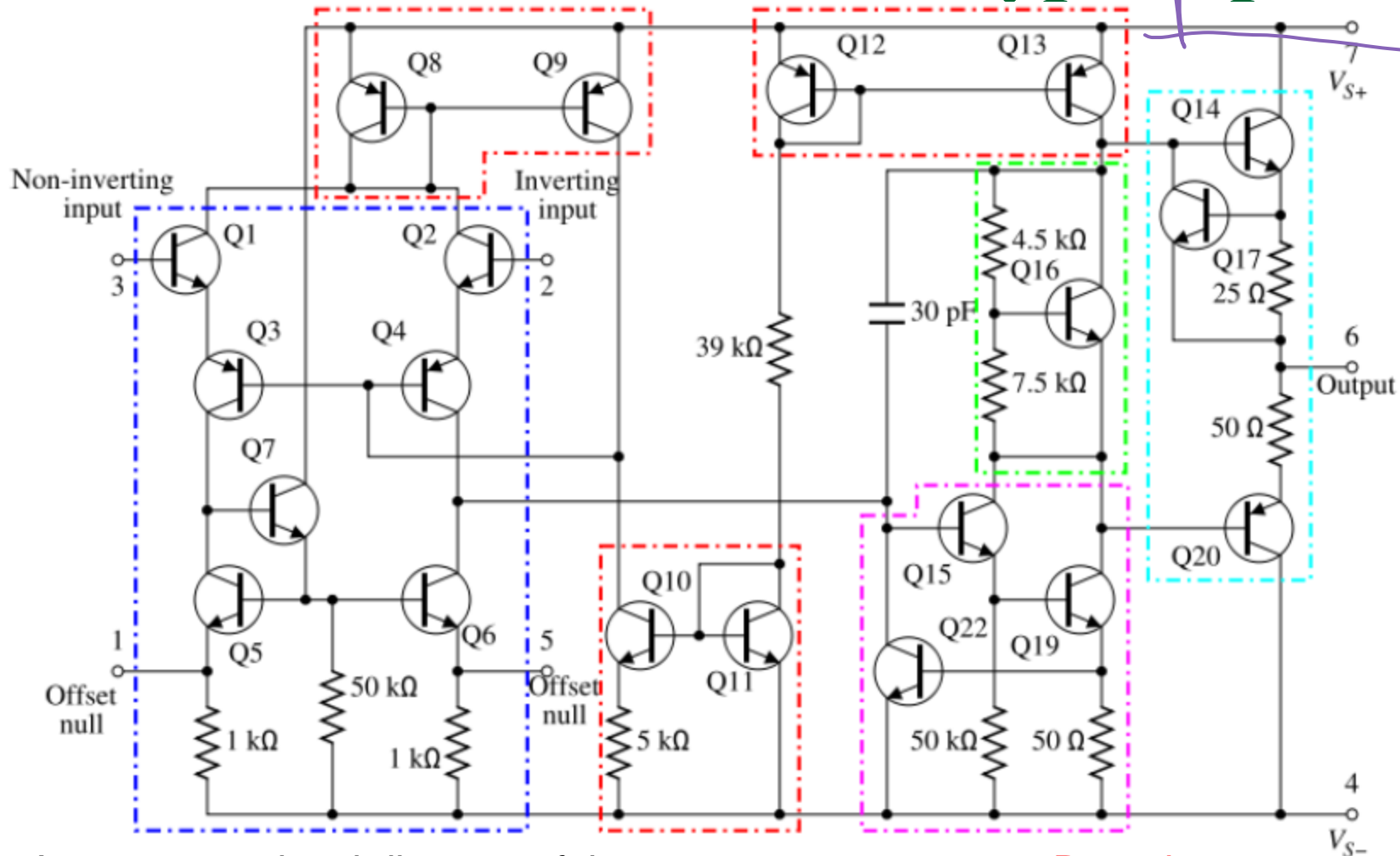
- Amplifiers provide gains in voltage or current.
- Op amps can convert current to voltage.
- Op amps can provide a buffer between two circuits.
- Op amps can be used to implement integrators and differentiators.
- Lowpass and bandpass filters.

Applications of Op Amps

An **op amp** is an active circuit element designed to perform mathematical operations of addition, subtraction, multiplication, division and differentiation.



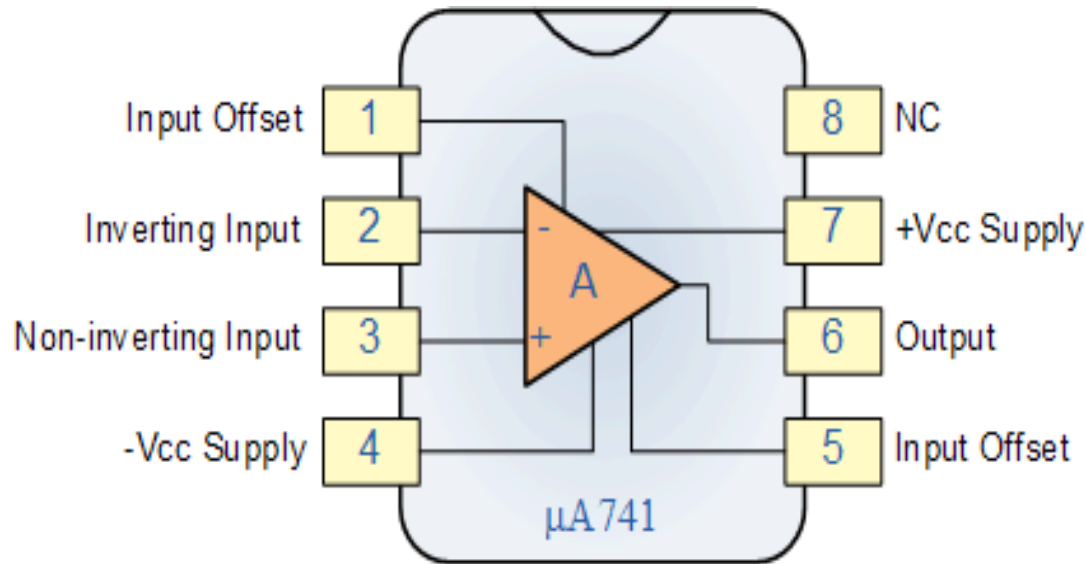
Internal Circuit of 741 Type Op Amp



A component level diagram of the common 741 op-amp. Dotted lines outline: current mirrors (red); differential amplifier (blue); class A gain stage (magenta); voltage level shifter (green); output stage (cyan).

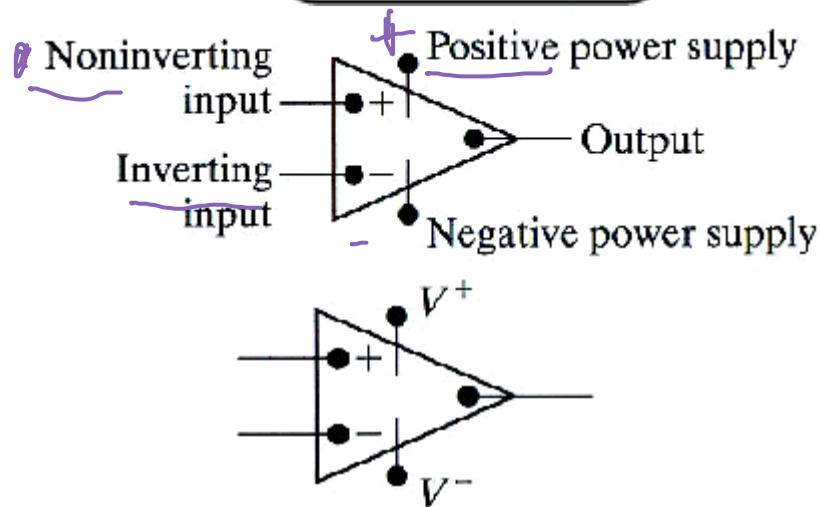
From Daniel Braun

Op Amp Terminals



Terminals of primary interest:

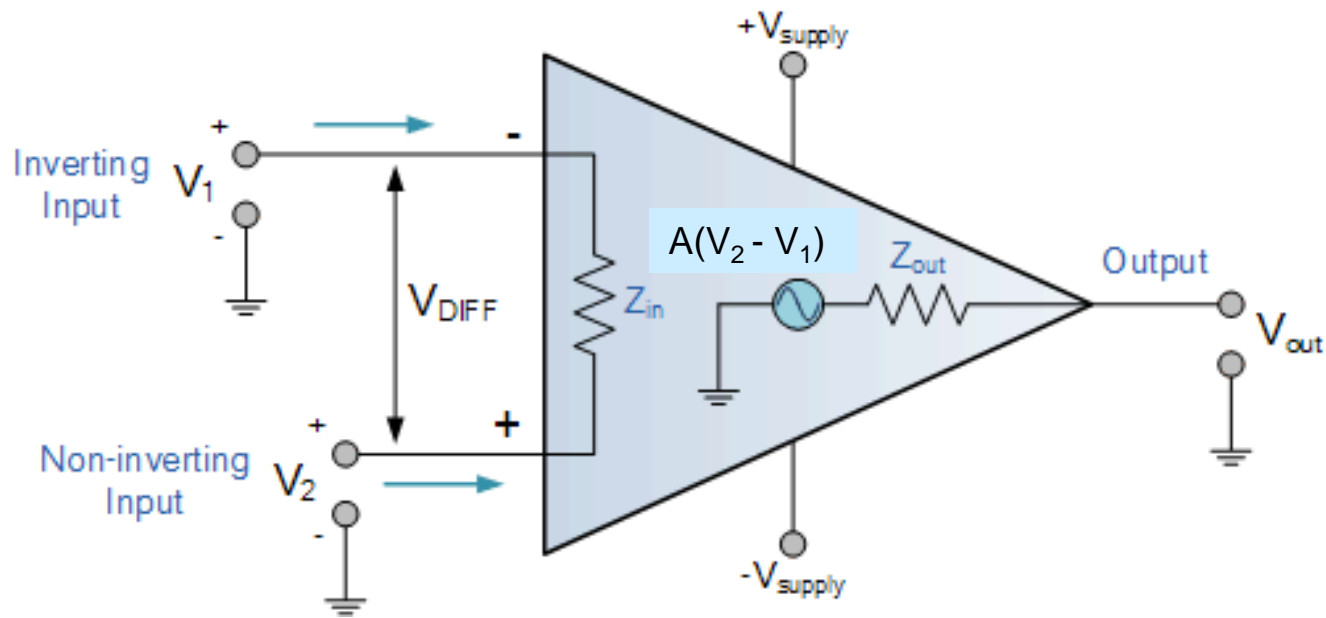
- inverting input
- noninverting input
- output
- positive power supply (+Vcc)
- negative power supply (-Vcc)



Offset null terminals may be used to compensate for a degradation in performance because of aging and imperfections.

The Op Amp Model

An operational amplifier is modeled as a **voltage-controlled voltage source**.



Typical vs. Ideal Op Amps

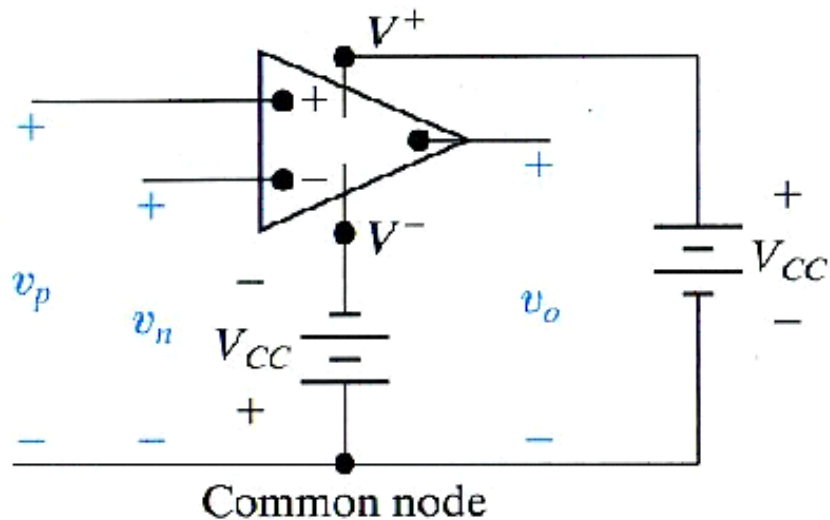
Typical Op Amp:

- The input resistance (impedance) R_{in} is very large (practically infinite).
- The voltage gain A is very large (practically infinite).

Ideal Op Amp:

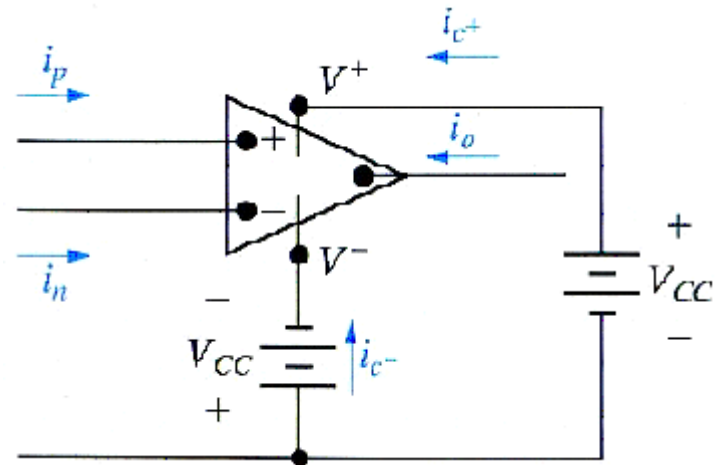
- The input resistance is **infinite**.
- The gain is **infinite**.
- The op amp is in a **negative feedback** configuration.

Terminal Voltages and Currents



Terminal voltage variables

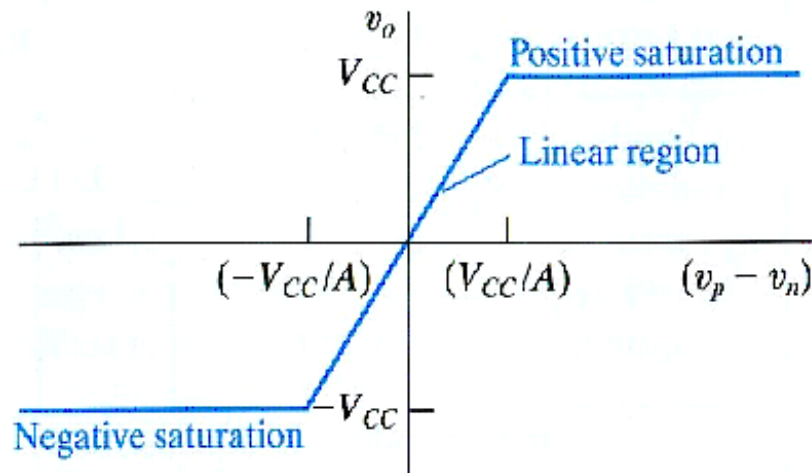
All voltages are considered as voltages rises from the common node.



Terminal current variables

All current reference directions are **into the terminal** of the op-amp.

Terminal Voltages and Currents



The terminal behavior of the op amp as linear circuit element is characterized by constraints on the input voltages and input currents.

Voltage transfer characteristic:

$$v_o = \begin{cases} -V_{CC} & A(v_p - v_n) < -V_{CC} \\ A(v_p - v_n) & -V_{CC} \leq A(v_p - v_n) \leq +V_{CC} \\ +V_{CC} & A(v_p - v_n) > +V_{CC} \end{cases}$$

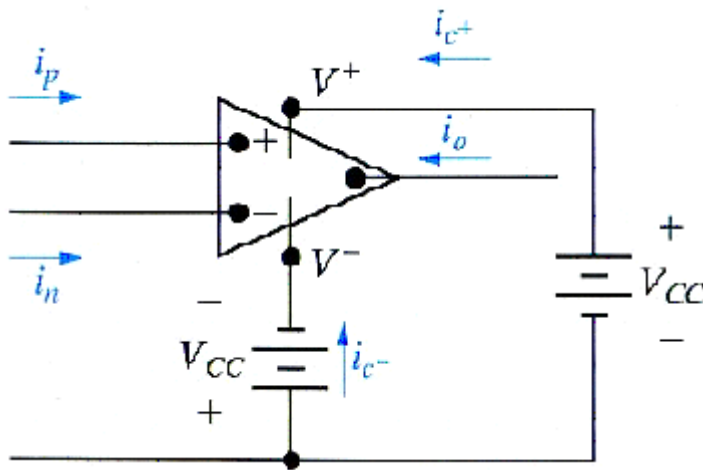
When the magnitude of the input voltage difference ($|v_p - v_n|$) is small, the op amp behaves as a linear device, as the output voltage is a linear function of the input voltages (the output voltage is equal to the difference in its input voltages times the gain, A).

Terminal Voltages and Currents

For ideal op amp:

Input voltage constraint: $V_p = V_n$

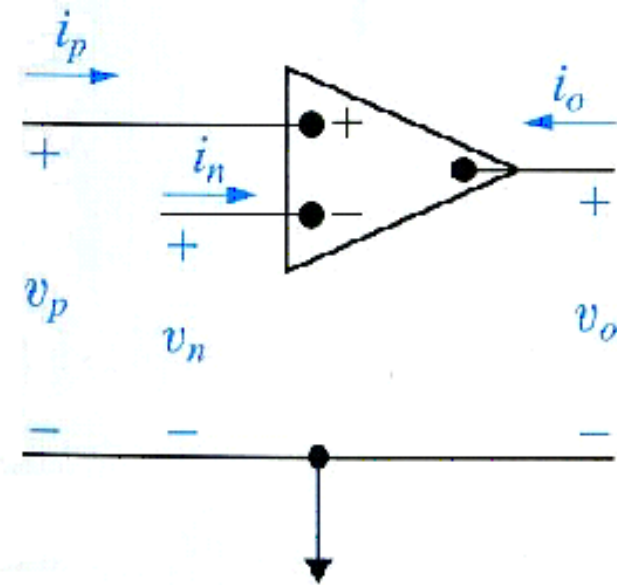
Input current constraint: $i_p = i_n = 0$



Apply Kirchhoff's current law

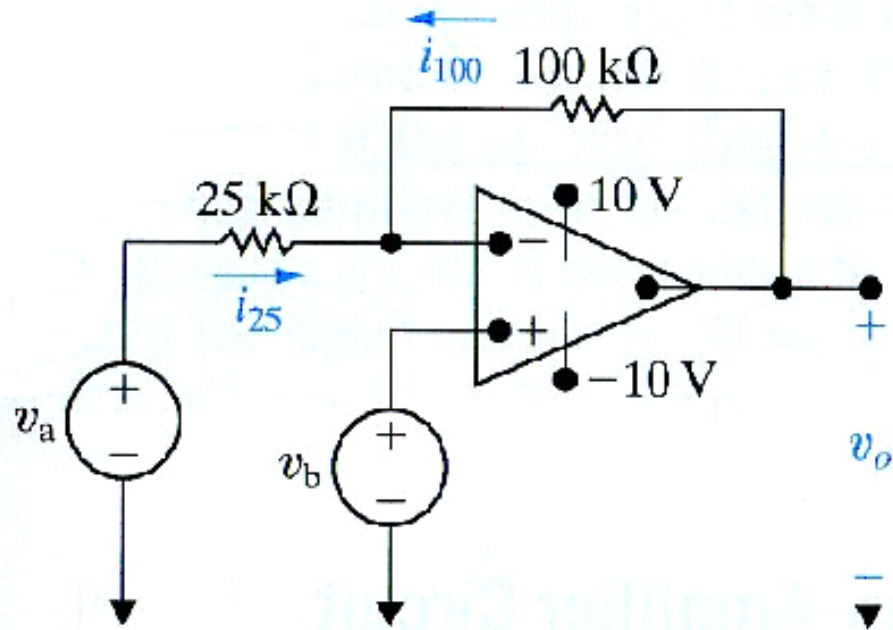
$$i_p + i_n + i_o + i_{c^+} + i_{c^-} = 0$$

$$\rightarrow i_o = -(i_{c^+} + i_{c^-})$$



Even though the current at the input terminal is negligible, there are still appreciable current at the output terminal.

Example 1



- (a) Calculate v_o if $v_a = 1\text{ V}$ and $v_b = 0\text{ V}$.
- (b) Calculate v_o if $v_a = 1\text{ V}$ and $v_b = 2\text{ V}$.
- (c) If $v_a = 1.5\text{ V}$, specify the range of v_b that avoids amplifier saturation.

Sol. of Example 1

a) A **negative feedback** path exists from the **op amp's output to its inverting input** through the $100\text{ k}\Omega$ resistor, \rightarrow assume the op amp is working in linear operating region. \rightarrow write a **node-voltage equation** at the inverting input terminal. The voltage at the inverting input terminal is 0, as $v_p = v_b = 0$ from the connected voltage source, and $v_n = v_p$ from the voltage constraint. The node-voltage equation at v_n is: $i_{25} + i_{100} = i_n$

$$i_{25} = (v_a - v_n)/25 = 1/25\text{ mA}$$

$$i_{100} = (v_o - v_n)/100 = v_o/100\text{ mA}$$

The current constraint requires $i_n = 0$. Substituting the values for the three currents into the node-voltage equation, we obtain


$$\frac{1}{25} + \frac{v_o}{100} = 0. \quad \Rightarrow \quad v_o = -4\text{ V}$$

$$\begin{aligned} \text{b) } V_p = v_b = v_n = 2\text{ V} \quad &\Rightarrow \quad i_{25} = \frac{v_a - v_n}{25} = \frac{1 - 2}{25} = -\frac{1}{25}\text{ mA}, \\ & i_{100} = \frac{v_o - v_n}{100} = \frac{v_o - 2}{100}\text{ mA}, \end{aligned} \quad \Rightarrow \quad \begin{aligned} i_{25} &= -i_{100} \\ v_o &= 6\text{ V} \end{aligned}$$

Sol. of Example 1

c) $v_n = v_p = v_b$, and $i_{25} = -i_{100}$

$$v_a = 1.5 \text{ V}$$


$$\frac{1.5 - v_b}{25} = -\frac{v_o - v_b}{100}.$$

Solving for v_b as a function of v_o gives

$$v_b = \frac{1}{5}(6 + v_o).$$

Now, if the amplifier is to be within the linear region of operation,
 $-10 \text{ V} \leq v_o \leq 10 \text{ V}.$

Substituting these limits on v_o into the expression for v_b , we see that v_b is limited to

$$-0.8 \text{ V} \leq v_b \leq 3.2 \text{ V}.$$

The Basic Inverting Amplifier

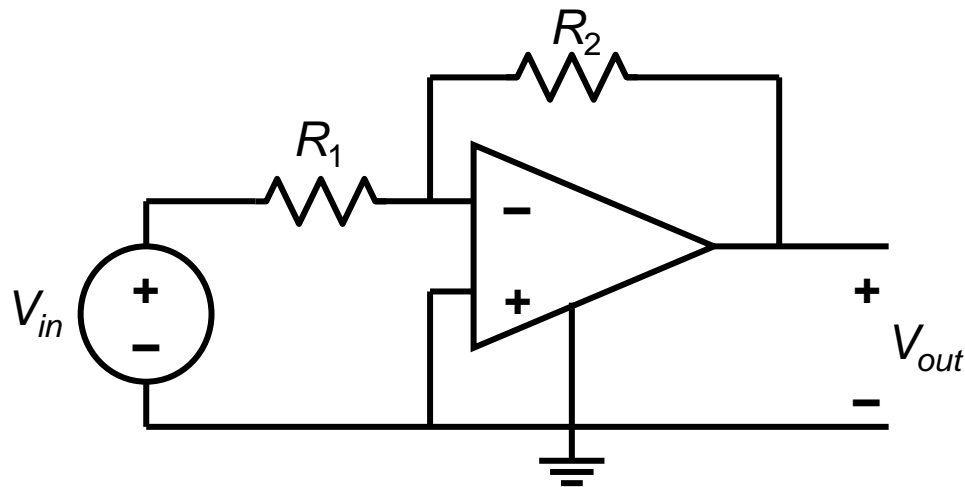
Consequences of the Ideal

- Infinite input resistance means the current into the inverting (–) input is zero:

$$i_- = 0$$

- Infinite gain means the difference between v_+ and v_- is zero:

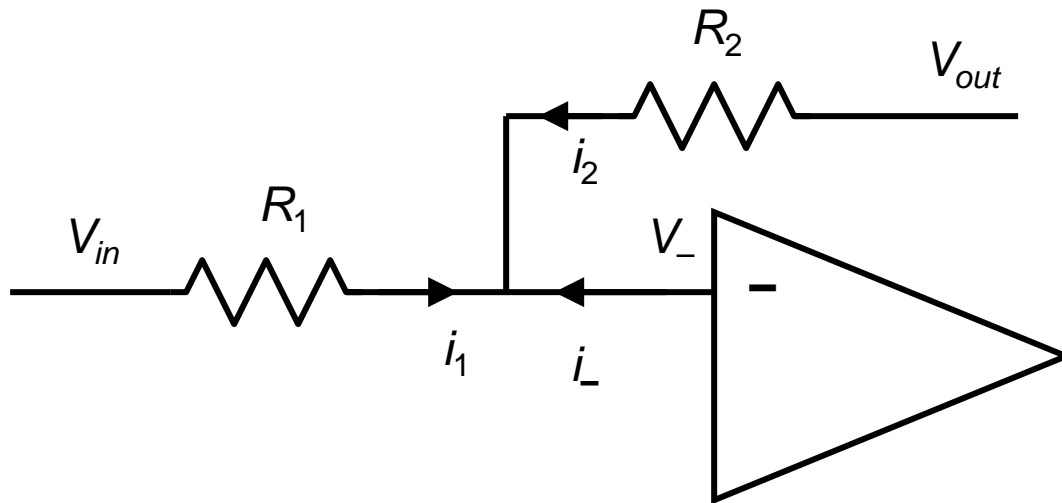
$$v_+ - v_- = 0$$



The Basic Inverting Amplifier

Solving the Amplifier Circuit

Apply KCL at the inverting (–) input:



$$i_1 + i_2 + i_- = 0$$

$$i_- = 0$$

$$i_1 = \frac{V_{in} - V_-}{R_1} = \frac{V_{in}}{R_1}$$

$$i_2 = \frac{V_{out} - V_-}{R_2} = \frac{V_{out}}{R_2}$$

The Basic Inverting Amplifier

Solve for V_{out}

- From KCL

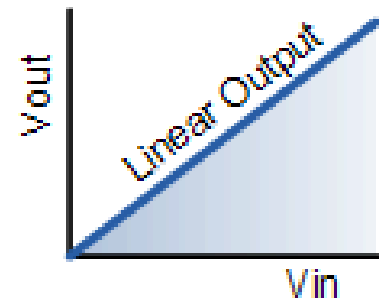
$$i_1 + i_2 + i_- = 0$$

$$\frac{V_{in}}{R_1} + \frac{V_{out}}{R_2} + 0 = 0$$

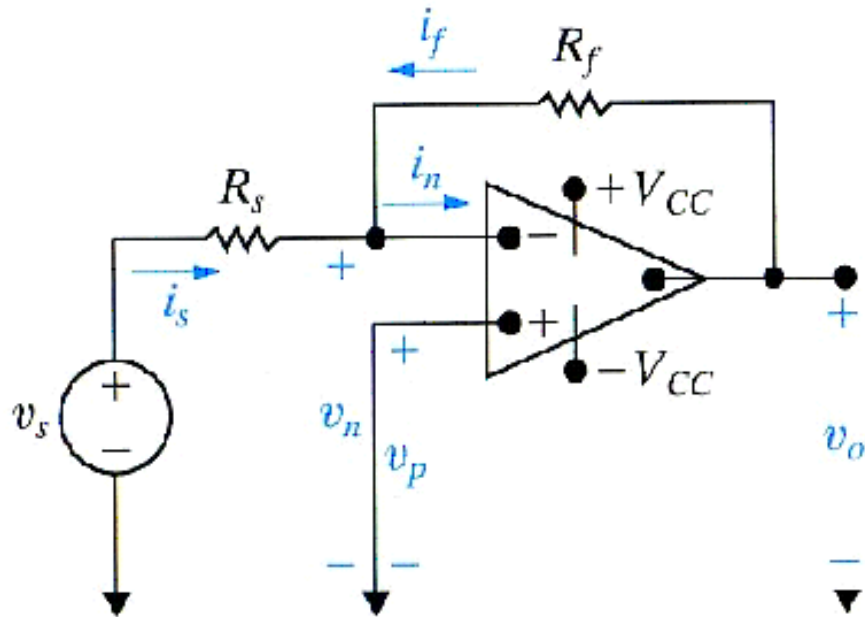
$$\frac{V_{in}}{R_1} = -\frac{V_{out}}{R_2}$$

- Thus, the amplifier gain is

$$\frac{V_{out}}{V_{in}} = -\frac{R_2}{R_1}$$



The Inverting Amplifier Circuit



$$v_o = -\frac{R_f}{R_s} v_s$$

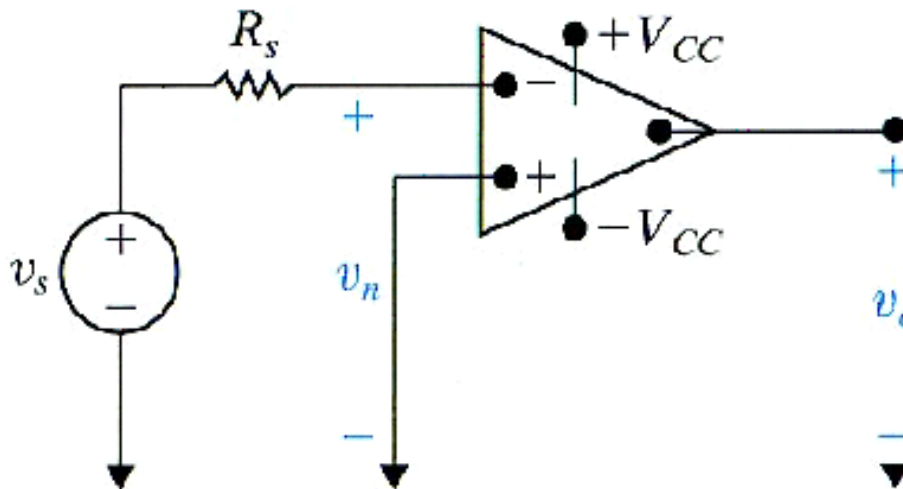
The output voltage is an inverted, scaled replica of the input.

R_f is negative feedback of the circuit.

Upper limit on the gain: $\frac{R_f}{R_s} < \left| \frac{V_{CC}}{v_s} \right|$

The Inverting Amplifier Circuit

When R_f is removed, the feedback path is opened and the amplifier is called *open loop*.



$$v_o = -Av_s$$

A is called the open-loop gain of the op amp

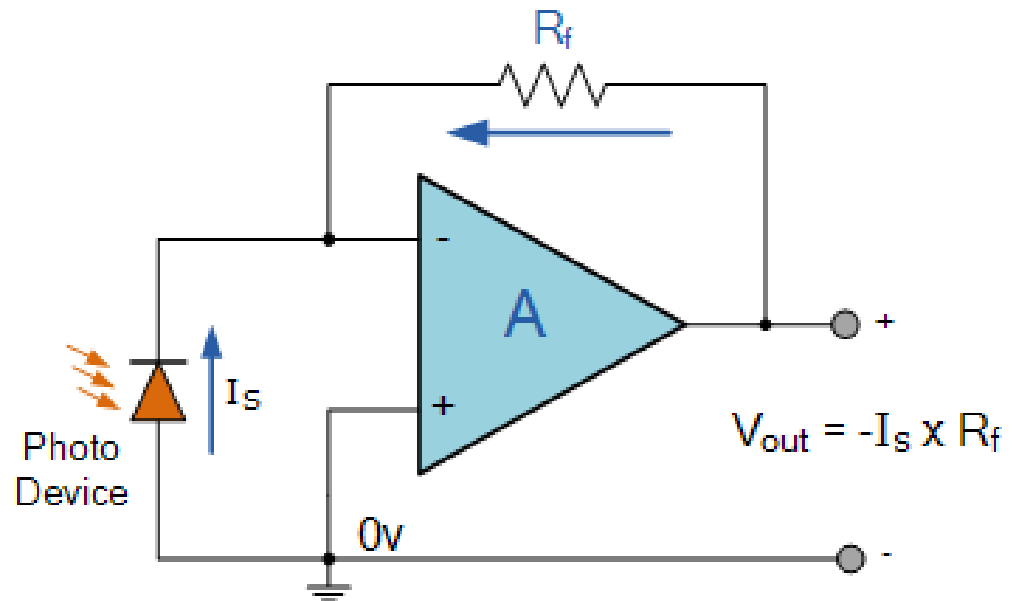
The negative sign in the equation indicates an inversion of the output signal with respect to the input as it is 180° out of phase.

The Inverting Amplifier Circuit

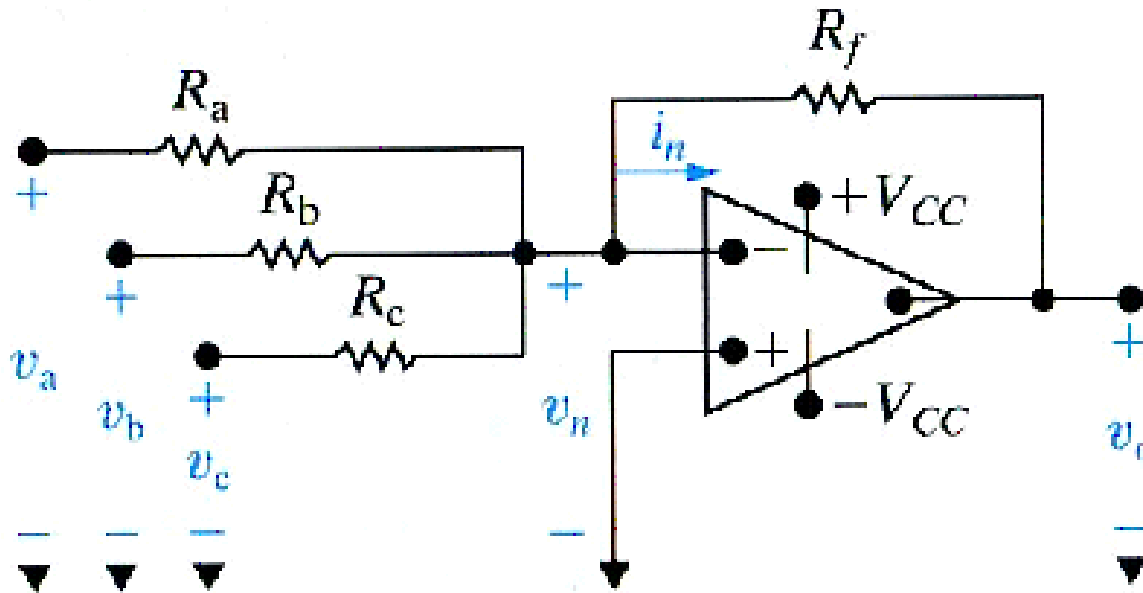
Transresistance Amplifier Circuit

Application of an inverting amplifier is that of a "transresistance amplifier" circuit. A "transimpedance amplifier", is basically a **current-to-voltage converter** (Current "in" and Voltage "out"). They can be used in low-power applications to convert a very small current generated by a photo-diode or photo-detecting device etc, into a usable output voltage which is proportional to the input current.

The output voltage is proportional to the amount of input current generated by the photo-diode.



The Summing Amplifier Circuit



$$v_o = -\left(\frac{R_f}{R_a} v_a + \frac{R_f}{R_b} v_b + \frac{R_f}{R_c} v_c \right)$$

The output voltage of a summing amplifier is an inverted, scaled sum of the voltages applied to the input of the amplifier.

Example 3.1:

Find the output voltage of the following Summing Amplifier circuit

Solution:

Using the previously found formula for the gain of the circuit

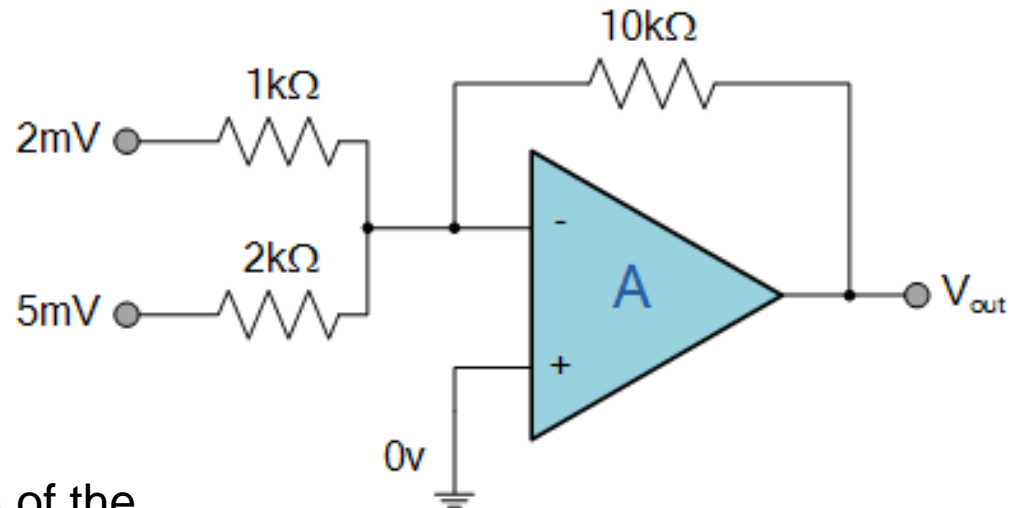
$$\text{Gain (A}_v\text{)} = \frac{V_{\text{out}}}{V_{\text{in}}} = -\frac{R_f}{R_{\text{in}}}$$

we can now substitute the values of the resistors in the circuit as follows,

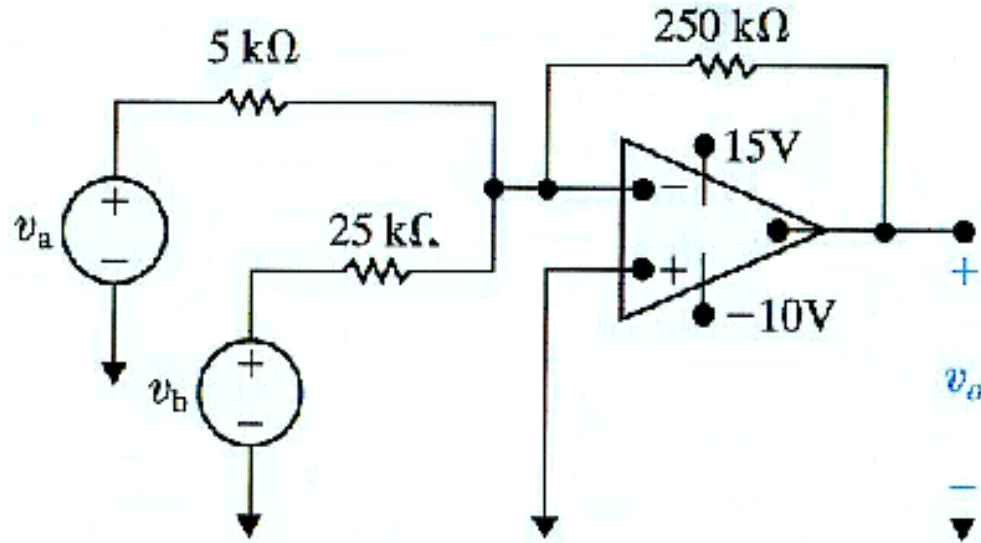
$$\left\{ \begin{array}{l} A_1 = \frac{10\text{k}\Omega}{1\text{k}\Omega} = -10 \\ A_2 = \frac{10\text{k}\Omega}{2\text{k}\Omega} = -5 \end{array} \right.$$



$$\begin{aligned} V_{\text{out}} &= (A_1 \times V_1) + (A_2 \times V_2) \\ V_{\text{out}} &= (-10(2\text{mV})) + (-5(5\text{mV})) = -45\text{mV} \end{aligned}$$



Example 3.2



- (a) Find v_o with $v_a = 0.1\text{ V}$ and $v_b = 0.25\text{ V}$.
- (b) If $v_b = 0.25\text{ V}$, how large can v_a be before the op amp saturates.
- (c) If $v_a = 0.1\text{ V}$, how large can v_b be before the op amp saturates.
- (d) Repeat (a), (b), and (c) with the polarity of v_b reversed.

Sol. of example 3

[a] This is an inverting summing amplifier so

$$v_o = (-R_f/R_a)v_a + (-R_f/R_b)v_b = -(250/5)v_a - (250/25)v_b = -50v_a - 10v_b$$

Substituting the values for v_a and v_b :

$$v_o = -50(0.1) - 10(0.25) = -5 - 2.5 = -7.5 \text{ V}$$

[b] Substitute the value for v_b into the equation for v_o from part (a) and use the negative power supply value:

$$v_o = -50v_a - 10(0.25) = -50v_a - 2.5 = -10 \text{ V}$$

$$\text{Therefore } 50v_a = 7.5, \quad \text{so } v_a = 0.15 \text{ V}$$

[c] Substitute the value for v_a into the equation for v_o from part (a) and use the negative power supply value:

$$v_o = -50(0.10) - 10v_b = -5 - 10v_b = -10 \text{ V};$$

$$\text{Therefore } 10v_b = 5, \quad \text{so } v_b = 0.5 \text{ V}$$

Sol. of example 3

[d] The effect of reversing polarity is to change the sign on the v_b term in each equation from negative to positive.

Repeat part (a):

$$v_o = -50v_a + 10v_b = -5 + 2.5 = -2.5 \text{ V}$$

Repeat part (b):

$$v_o = -50v_a + 2.5 = -10 \text{ V}; \quad 50v_a = 12.5, \quad v_a = 0.25 \text{ V}$$

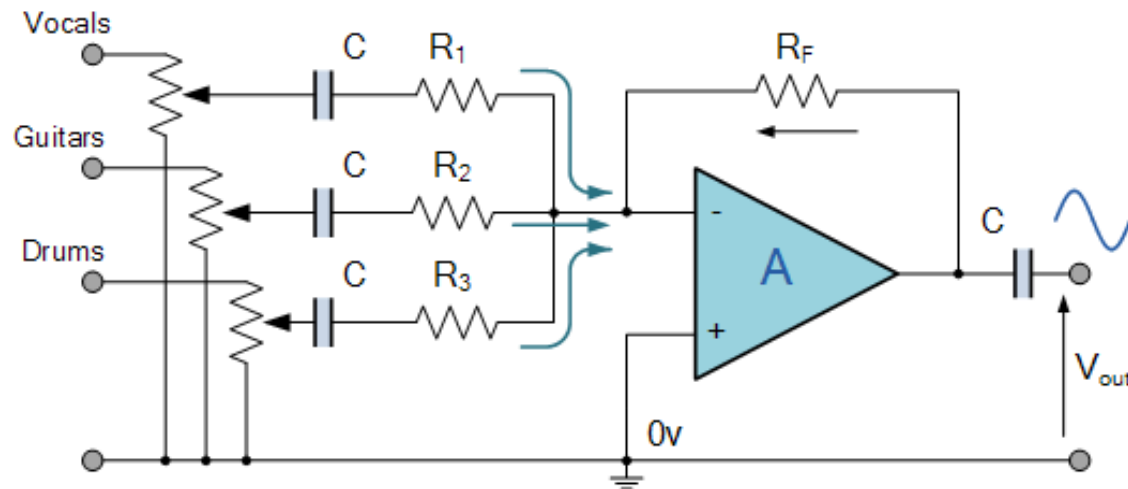
Repeat part (c), using the value of the positive power supply:

$$v_o = -5 + 10v_b = 15 \text{ V}; \quad 10v_b = 20; \quad v_b = 2.0 \text{ V}$$

Summing Amplifier Applications

Summing Amplifier Audio Mixer

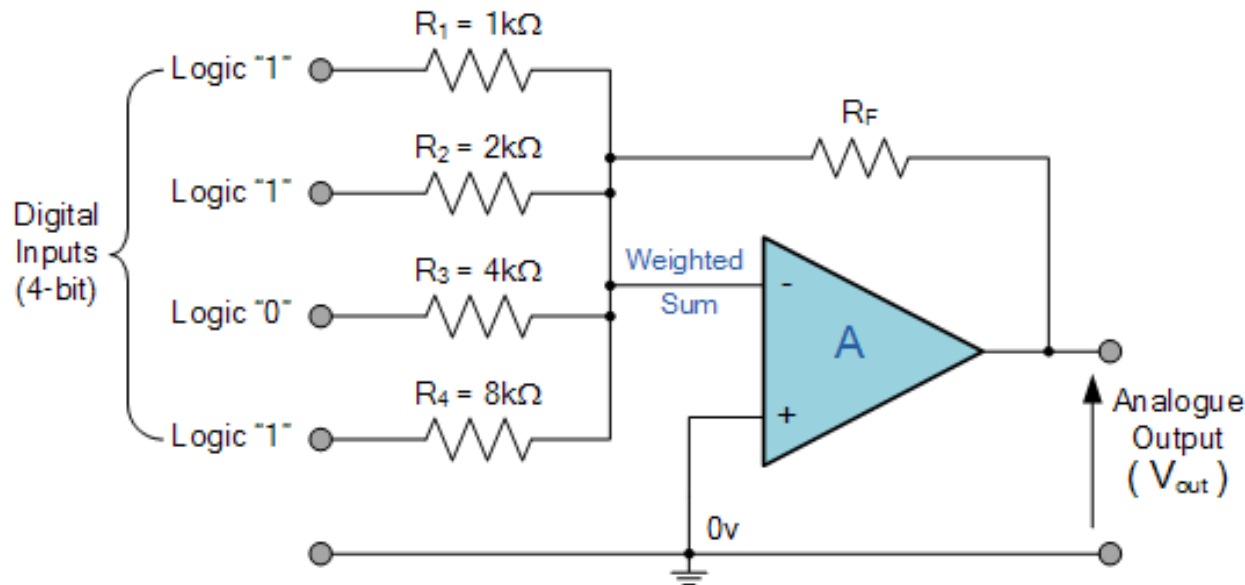
If the input resistances of a summing amplifier are connected to potentiometers the individual input signals can be mixed together by varying amounts.



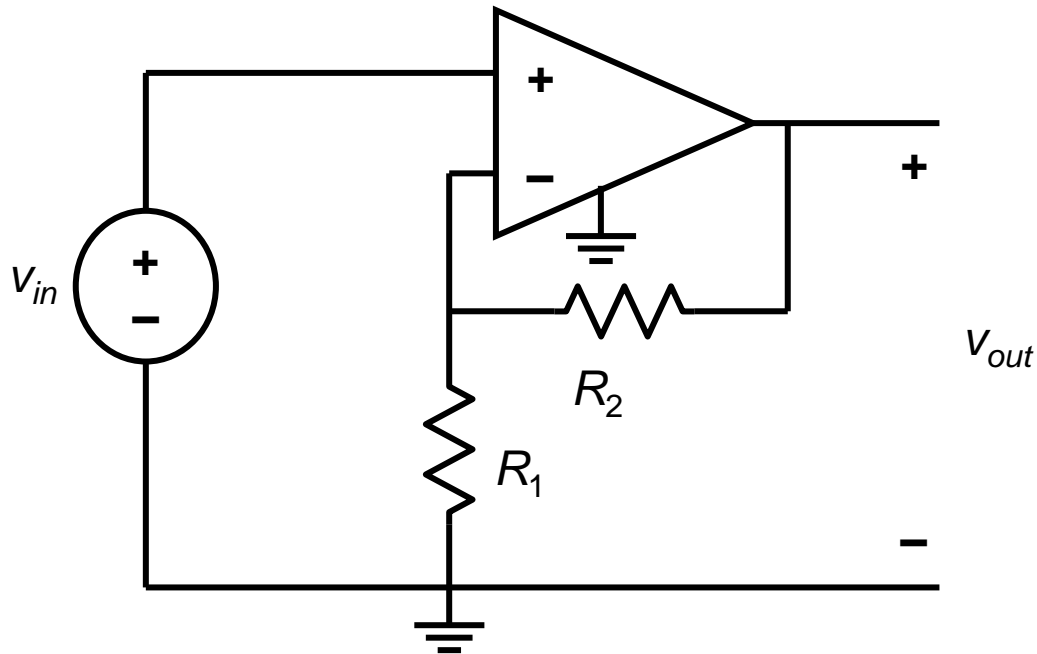
Summing Amplifier Applications

Digital to Analogue Converter (DAC summing amplifier circuit)

Another useful application of a **Summing Amplifier** is as a weighted sum digital-to-analogue converter. If the input resistors, R_{in} of the summing amplifier double in value for each input, for example, $1\text{k}\Omega$, $2\text{k}\Omega$, $4\text{k}\Omega$, $8\text{k}\Omega$, $16\text{k}\Omega$, etc, then a digital logical voltage, either a logic level "0" or a logic level "1" on these inputs will produce an output which is the weighted sum of the digital inputs. Consider the circuit below.

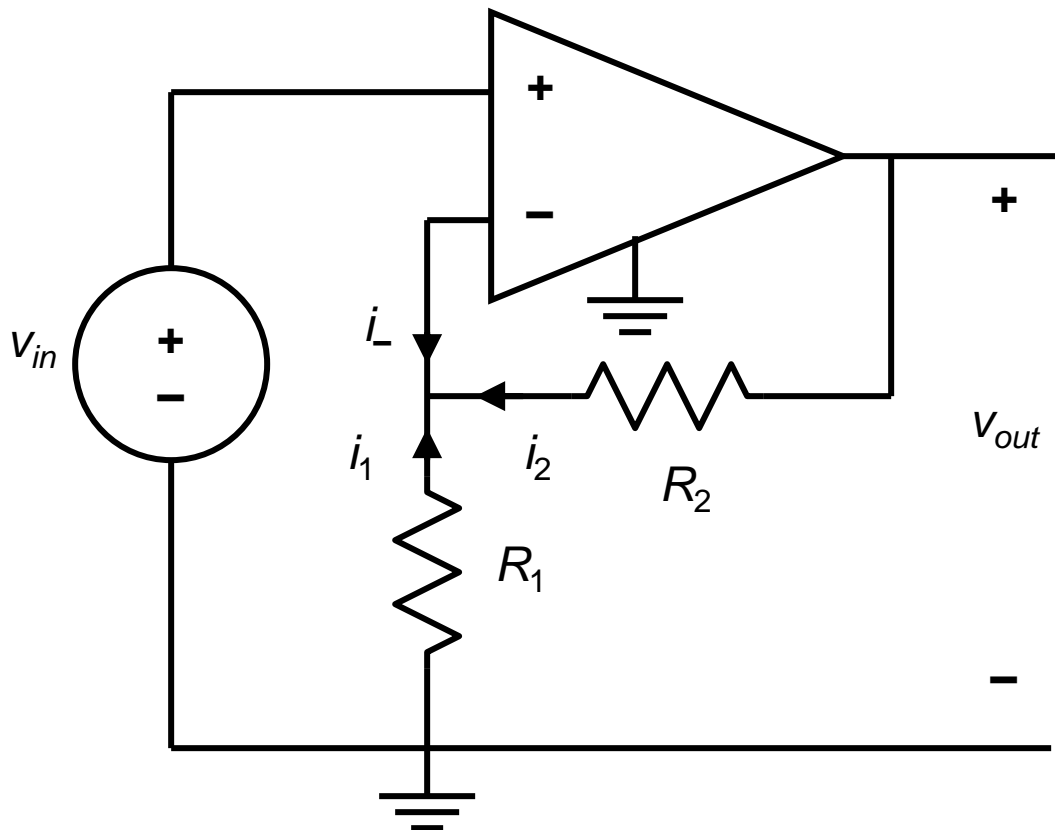


The Non-Inverting Amplifier



The Non-Inverting Amplifier

KCL at the Inverting Input



$$i_- = 0$$

$$i_1 = \frac{-v_-}{R_1} = \frac{-v_{in}}{R_1}$$

$$i_2 = \frac{v_{out} - v_-}{R_2}$$
$$= \frac{v_{out} - v_{in}}{R_2}$$

The Non-Inverting Amplifier

Solve for v_{out}

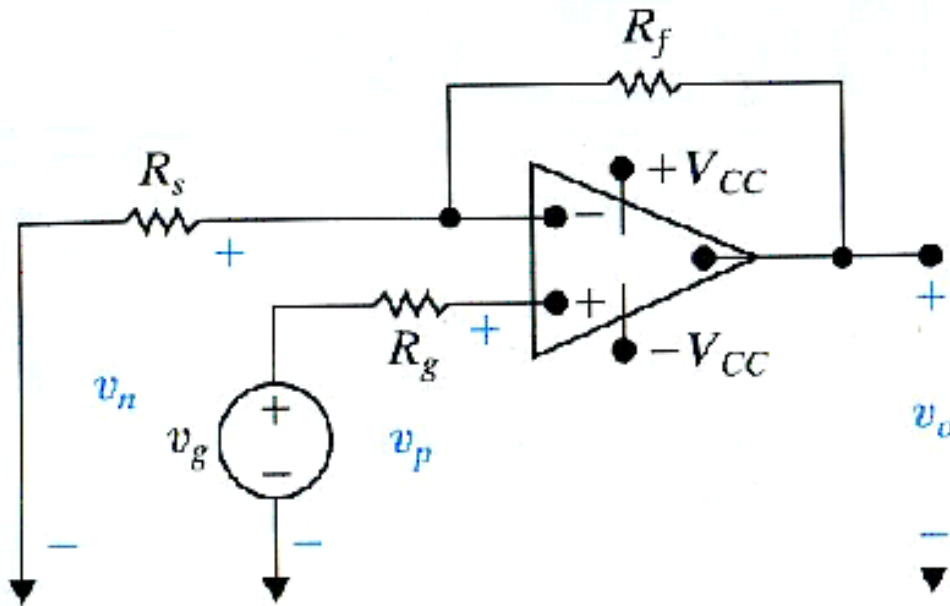
$$i_1 + i_2 + i_- = 0$$

$$\frac{-v_{in}}{R_1} + \frac{v_{out} - v_{in}}{R_2} = 0$$

$$v_{out} = v_{in} \left(1 + \frac{R_2}{R_1} \right)$$

- Hence, the non-inverting amplifier has a gained output ($>$ unity) relative to the resistance ratio

The Non-inverting Amplifier Circuit

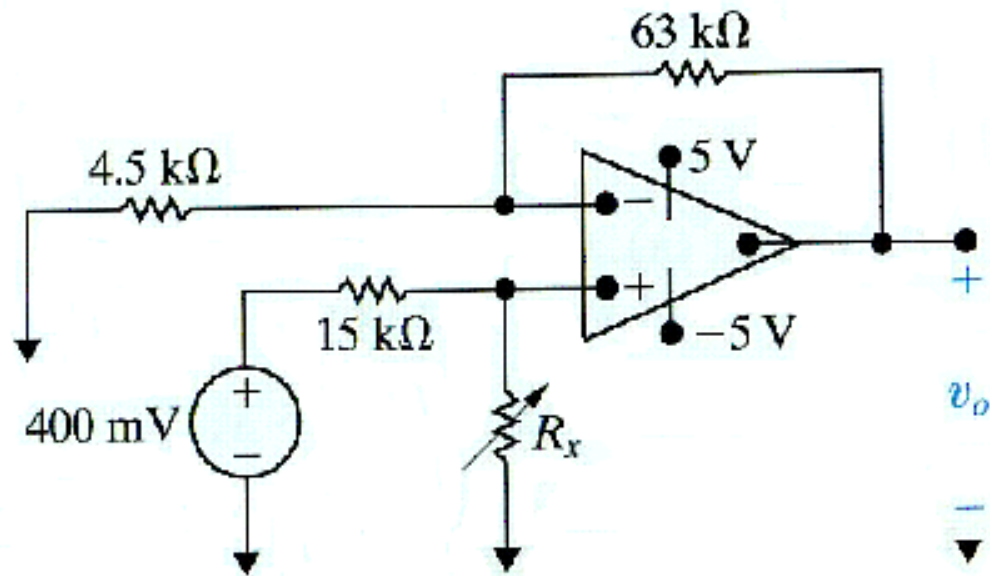


$$v_o = \frac{R_s + R_f}{R_s} v_g$$

Requirement for operation in the linear region:

$$\frac{R_s + R_f}{R_s} < \left| \frac{V_{CC}}{V_g} \right|$$

Example 4



- (a) Find the output voltage when $R_x = 60 \text{ k}\Omega$
- (b) How large can R_x be before the amplifier saturates.

Sol. of example 4

- [a] Write a node voltage equation at v_n ; remember that for an ideal op amp, the current into the op amp at the inputs is zero:

$$\frac{v_n}{4500} + \frac{v_n - v_o}{63,000} = 0$$

Solve for v_o in terms of v_n by multiplying both sides by 63,000 and collecting terms:

$$14v_n + v_n - v_o = 0 \quad \text{so} \quad v_o = 15v_n$$

Now use voltage division to calculate v_p . We can use voltage division because the op amp is ideal, so no current flows into the non-inverting input terminal and the 400 mV divides between the 15 k Ω resistor and the R_x resistor:

$$v_p = \frac{R_x}{15,000 + R_x}(0.400)$$

Now substitute the value $R_x = 60 \text{ k}\Omega$:

Sol. of example 4

$$v_p = \frac{60,000}{15,000 + 60,000}(0.400) = 0.32 \text{ V}$$

Finally, remember that for an ideal op amp, $v_n = v_p$, so substitute the value of v_p into the equation for v_o

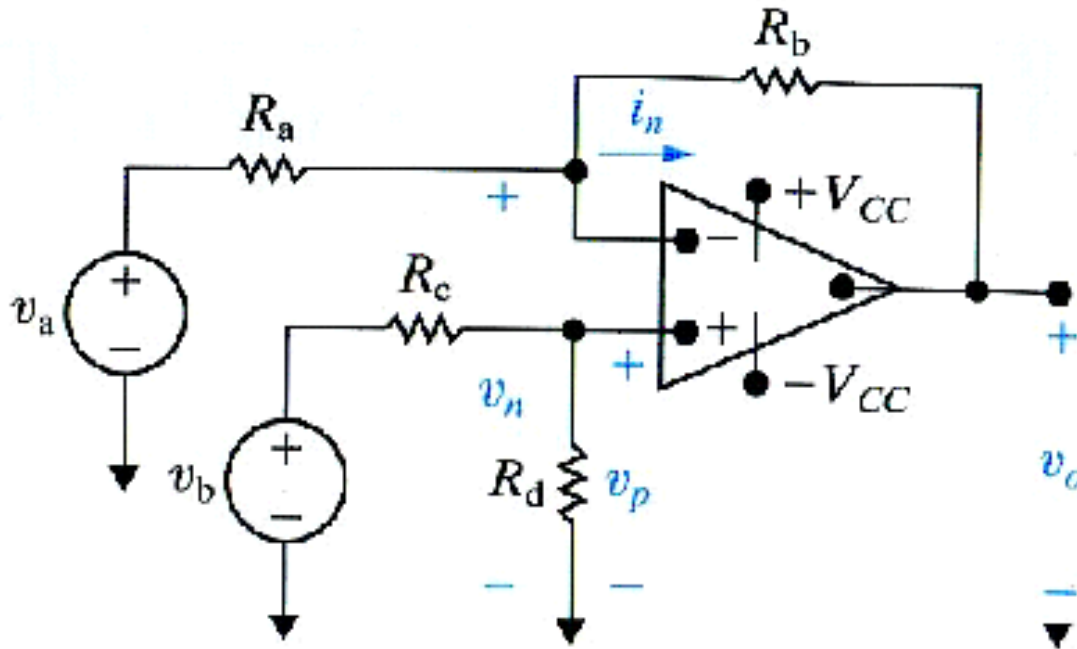
$$v_o = 15v_n = 15v_p = 15(0.32) = 4.8 \text{ V}$$

- [b] Substitute the expression for v_p into the equation for v_o and set the resulting equation equal to the positive power supply value:

$$v_o = 15 \left(\frac{0.4R_x}{15,000 + R_x} \right) = 5$$

$$15(0.4R_x) = 5(15,000 + R_x) \quad \text{so} \quad R_x = 75 \text{ k}\Omega$$

The Difference Amplifier Circuit



$$v_o = \frac{R_d(R_a + R_b)}{R_a(R_c + R_d)}v_b - \frac{R_b}{R_a}v_a$$

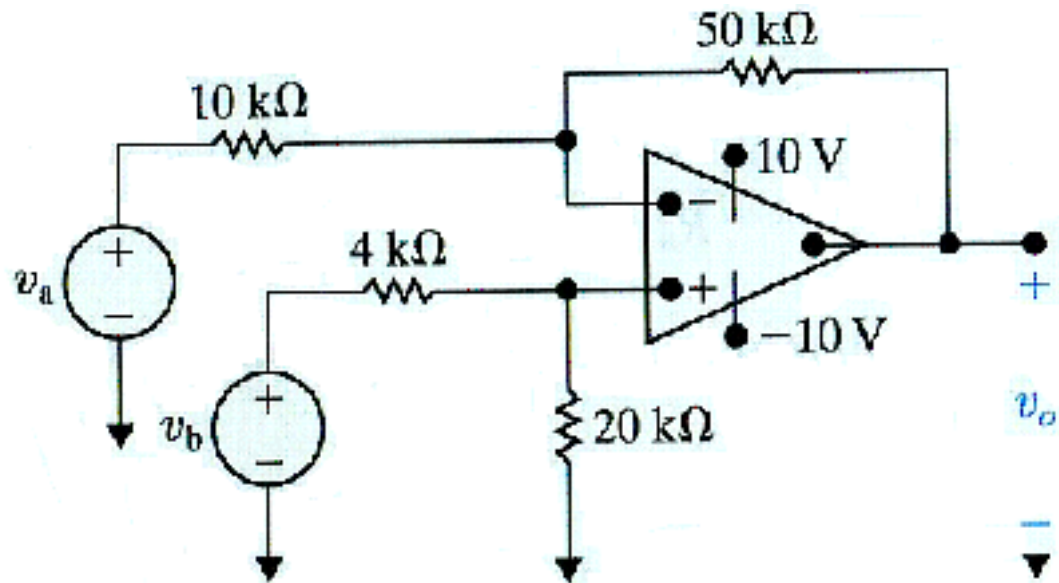
(*)

If set $\frac{R_a}{R_b} = \frac{R_c}{R_d}$ then

$$v_o = \frac{R_b}{R_a}(v_b - v_a)$$

The output voltage of a difference amplifier is a scaled replica of the difference between the two input voltages. The scaling is controlled by the external resistors.

Example 5



- (a) In the difference amplifier shown, $v_b = 4\text{ V}$. What range of values for v_a will result in linear operation.
- (b) Repeat (a) with $20\text{ k}\Omega$ resistor decreased to $8\text{ k}\Omega$

Sol. of example 5

- [a] Since this is a difference amplifier, we can use the expression for the output voltage in terms of the input voltages and the resistor values given in Eq. (*)

$$v_o = \frac{20(60)}{10(24)}v_b - \frac{50}{10}v_a$$

Simplify this expression and substitute in the value for v_b :

$$v_o = 5(v_b - v_a) = 20 - 5v_a$$

Set this expression for v_o to the positive power supply value:

$$20 - 5v_a = 10 \text{ V} \quad \text{so} \quad v_a = 2 \text{ V}$$

Now set the expression for v_o to the negative power supply value:

$$20 - 5v_a = -10 \text{ V} \quad \text{so} \quad v_a = 6 \text{ V}$$

Therefore $2 \leq v_a \leq 6 \text{ V}$

Sol. of example 5

[b] Begin as before by substituting the appropriate values into Eq. (*)

$$v_o = \frac{8(60)}{10(12)}v_b - 5v_a = 4v_b - 5v_a$$

Now substitute the value for v_b :

$$v_o = 4(4) - 5v_a = 16 - 5v_a$$

Set this expression for v_o to the positive power supply value:

$$16 - 5v_a = 10 \text{ V} \quad \text{so} \quad v_a = 1.2 \text{ V}$$

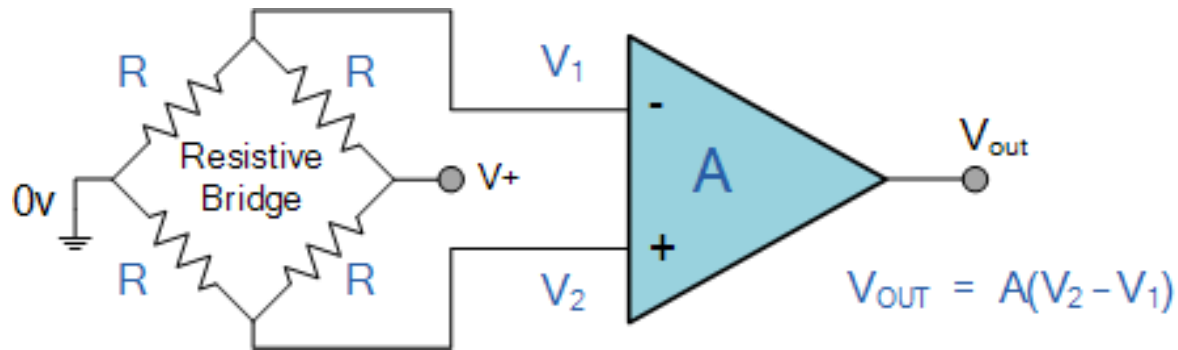
Now set the expression for v_o to the negative power supply value:

$$16 - 5v_a = -10 \text{ V} \quad \text{so} \quad v_a = 5.2 \text{ V}$$

Therefore $1.2 \leq v_a \leq 5.2 \text{ V}$

The Difference Amplifier Circuit: Applications

Bridge Amplifier

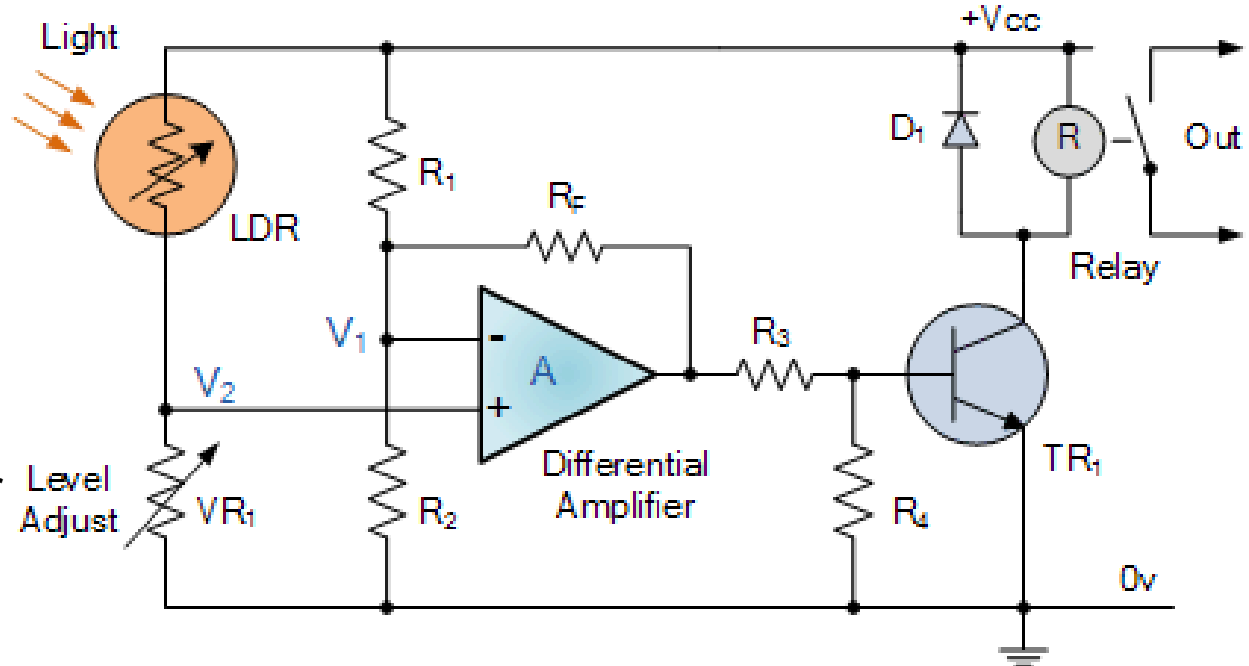


The standard Differential Amplifier circuit now becomes a differential voltage comparator by "Comparing" one input voltage to the other.

By connecting one input to a fixed voltage reference set up on one leg of the resistive bridge network and the other to either a "Thermistor" or a "Light Dependant Resistor" the amplifier circuit can be used to detect either low or high levels of temperature or light as the output voltage becomes a linear function of the changes in the active leg of the resistive bridge.

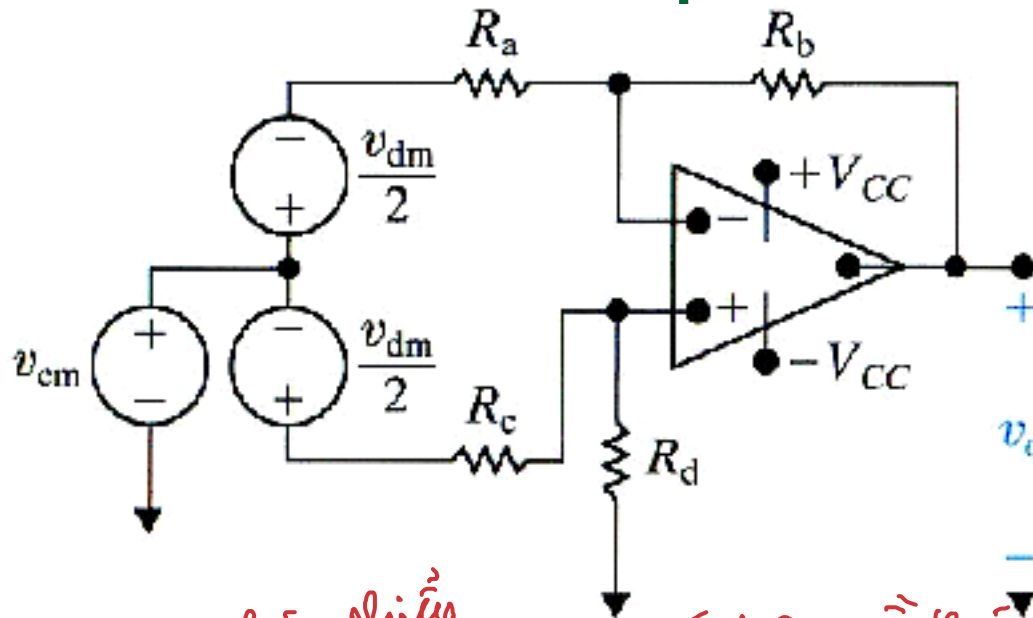
The Difference Amplifier Circuit: Applications

The circuit acts as a light-activated switch → turns the output relay either "ON" or "OFF" as the light level detected by the LDR resistor exceeds or falls below a pre-set value at V_2 determined by the position of VR_1 . A fixed voltage reference is applied to the inverting input terminal V_1 via the R_1 - R_2 voltage divider network and the variable voltage (proportional to the light level) applied to the non-inverting input terminal V_2 . It is also possible to detect temperature using this type of circuit by simply replacing the Light Dependant Resistor (LDR) with a thermistor. By interchanging the positions of VR_1 and the LDR, the circuit can be used to detect either light or dark, or heat or cold using a thermistor.



Light Activated Switch:

The difference amplifier – Another perspective



$$v_{dm} = v_b - v_a$$

$$v_{cm} = (v_a + v_b) / 2$$

$$v_a = v_{cm} - \frac{1}{2}v_{dm},$$

$$v_b = v_{cm} + \frac{1}{2}v_{dm}.$$

tín liên nhiên

tín hiệu cần khuếch đại.

$$v_o = \left[\frac{R_a R_d - R_b R_c}{R_a (R_c + R_d)} \right] v_{cm} + \left[\frac{R_d (R_a + R_b) + R_b (R_c + R_d)}{2 R_a (R_c + R_d)} \right] v_{dm} \Rightarrow v_o = A_{cm} v_{cm} + A_{dm} v_{dm}$$

An ideal differential amplifier has zero common mode gain and non-zero (usually large) differential mode gain.

In practical applications, the differential mode signal contains information of interest, whereas the common mode signal is the noise found in all electric signals.