Random variables

1 Discrete random variables

1. Classify the following random variables as discrete or continuous:
   (a) X: the number of automobile accidents per year in Virginia.
   (b) Y : the length of time to play 18 holes of golf.
   (c) M: the amount of milk produced yearly by a particular cow.
   (d) N: the number of eggs laid each month by a hen.
   (e) P: the number of building permits issued each month in a certain city.
   (f) Q: the weight of grain produced per acre.

2. The sample space of a random experiment is $\{a, b, c, d, e, f\}$, and each outcome is equally likely. A random variable is defined as follows:

outcome	*a*	*b*	*c*	*d*	*e*	*f*
x	0	0	1.5	1.5	2	3

   Determine the probability mass function of X. Use the probability mass function to determine the following probabilities:

   $a.\, P(X = 1.5)$ $\quad$ $b.\, P(0.5 < X < 2.7)$ $\quad$ $c.\, P(0 \leq X < 2)$ $\quad$ $d.\, P(X > 3)$

3. A shipment of 20 similar laptop computers to a retail outlet contains 3 that are defective. If a school makes a random purchase of 2 of these computers, find the probability distribution for the number of defectives.

4. An assembly consists of two mechanical components. Suppose that the probabilities that the first and second components meet specifications are 0.95 and 0.98. Assume that the components are independent. Determine the probability mass function of the number of components in the assembly that meet specifications.

5. The distributor of a machine for cytogenics has developed a new model. The company estimates that when it is introduced into the market, it will be very successful with a probability 0.6, moderately successful with a probability 0.3, and not successful with probability 0.1. The estimated yearly profit associated with the model being very successful is \$15 million and with it being moderately successful is \$5 million; not successful would result in a loss of \$500,000. Let X be the yearly profit of the new model. Determine the probability mass function of X.

6. Errors in an experimental transmission channel are found when the transmission is checked by a certifier that detects missing pulses. The number of errors found in an eight - bit byte is a random variable with the following cumulative distribution function:

$$F(x) = \begin{cases} 0 & \text{if } x < 1 \\ 0.7 & \text{if } 1 \leq x < 4 \\ 0.9 & \text{if } 4 \leq x < 7 \\ 1 & \text{if } x \geq 7 \end{cases}$$

   Determine the probability mass function.

2 Continuous random variable

1. The waiting time, in hours, between successive speeders spotted by a radar unit is a continuous random variable with cumulative distribution function

$$F(x) = \begin{cases} 0, & x < 0 \\ 1 - e^{-8x}, & x \geq 0 \end{cases}.$$

   Find the probability of waiting less than 12 minutes between successive speeders

2. The probability density function of the time to failure of an electronic component in a copier (in hours) is

$$f(x) = \begin{cases} 0, & x < 0 \\ \frac{1}{1000} e^{-x/1000}, & x \geq 0 \end{cases}.$$

   Determine the probability that

   (a) A component lasts more than 3000 hours before failure.

   (b) A component fails in the interval from 1000 to 2000 hours.

   (c) A component fails before 1000 hours.

   (d) Determine the number of hours at which 10% of all components have failed.

3. The probability density function of the net weight in pounds of a packaged chemical herbicide is

$$f(x) = \begin{cases} 2, & 49.75 \leq x \leq 50.25 \\ 0 & \text{elsewhere} \end{cases}.$$

   Determine the probability that a package weighs more than 50 pounds.