Markov Chain

November 28, 2023

1. Consider a Markov chain with transition matrix

$$\mathrm{P} = \begin{array}{c} \\ \text{From} \\ \begin{matrix} 1 \\ 2 \\ 3 \end{matrix} \end{array} \begin{array}{c} \text{To} \\ \begin{matrix} 1 & 2 & 3 \end{matrix} \\ \begin{bmatrix} 0 & 1 & 0 \\ 0.1 & 0.2 & 0.7 \\ 0.4 & 0 & 0.6 \end{bmatrix} \end{array}$$

   Compute

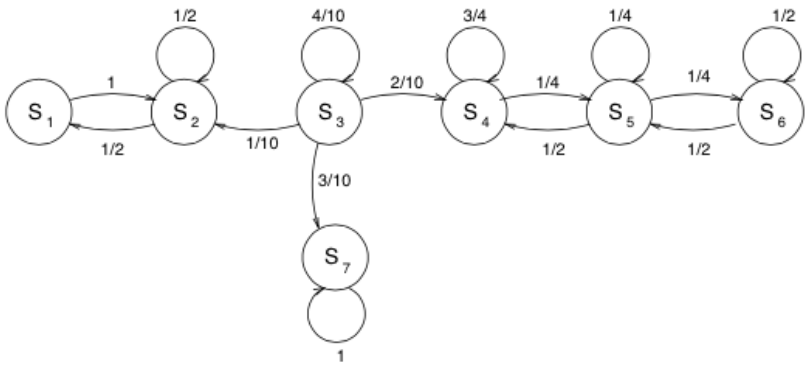
   (a) $P(X_2 = 3, X_1 = 3|X_0) = 1$
   (b) $P(X_4 = 2, X_3 = 1|X_0 = 2)$
   (c) $P(X_3 = 1|X_0 = 3)$
   (d) $P(X_5 = 1, X_2 = 0|X_1 = 3)$

2. A Markov chain $X_n, n \geq 0$ with states 0, 1, 2, has the transition probability matrix

$$P = \begin{bmatrix} 1/2 & 1/3 & 1/6 \\ 0 & 1/3 & 2/3 \\ 1/2 & 0 & 1/2 \end{bmatrix}$$

   (a) Compute $P(X_3 = 2|X_1 = 0)$.
   (b) Determine $P(X_5 = 1, X_2 = 0|X_0 = 2)$.
   (c) Suppose that the initial distribution of X_0 is $\pi^{(0)} = \begin{pmatrix} 0.25 & 0.35 & 0.4 \end{pmatrix}$. Find the unconditional distribution of X_2 and then compute $E(X_2)$.
   (d) Find the stationary distribution of this Markov chain

3. Consider the Markov chain below.



   For all parts of this problem, the process is in state 3 immediately before the first transition.

   (a) Classify all states of the Markov chain.
   (b) Determine all recurrence class and indicate which one is periodic.