FINAL 23 JAN

(20 points) **Consider a Markov chain $(x_n)n \geq 0$with state space § = { 1 ,2 } and transition matrix**

$$P = \begin{bmatrix} 0.5 & 0.5 \\ 0.3 & 0.7 \end{bmatrix}$$

(a) Compute $P(X_3 = 2 \mid X_0 = 1)$ and $P(X_5 = 2, X_3 = 2 \mid X_0 = 1)$

$P(X_3 = 2 \mid X_0 = 1) = p_{12}^{(3)} = 0.62$

$P(X_5 = 2, X_3 = 2 \mid X_0 = 1) = p_{12}^{(3)} \times p_{22}^{(2)} = 0.3968$

(b) Given initial distribution $P(X_0 = 1) = 0.4, P(X_0 = 2) = 0.6$. Evaluate $E(X_2)$

$E(X_2) = \pi^{(0)} \times p^{(2)} = [0.376 \quad 0.624]$

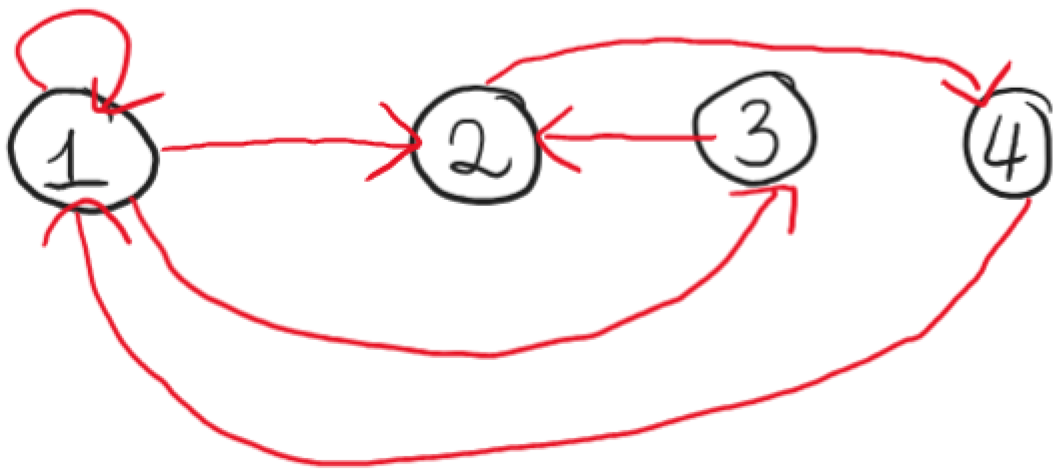
(c) Determine the stationary distribution of this Markov chain.

$$\begin{cases} \pi1 + \pi2 = 1 \\ p_{11}\pi1 + p_{21}\pi2 = \pi1 \end{cases} \Rightarrow \begin{cases} \pi1 = 3/8 \\ \pi2 = 5/8 \end{cases}$$

(10 points) **Consider a Markov chain (X,)„>0 with state space § = {1,2,3,4} and transition matrix**

$$P = \begin{bmatrix} 0.25 & 0.5 & 0.25 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

Which stale(s) is (are) recurrent?



Recurrent states are 1, 2, 3, 4

(10 points) **The data show the number of licensed nuclear reactors in the United States for a recent 15-year period. Compute the sample mean, sample standard deviation, median and mode.**

104 104 104 104 104

107 109 109 109 110

109 111 112 111 109

$$\bar{x} = 107.7(3)\ s_x = 2.9633\ Med = 109\ Mode = 104{,}109$$

(10 points) **In a random sample of 100 shoppers, the customer spends an average of 600 thousand Vietnam dong per visit at a Vincom Center. The standard deviation of the population is 50 thousand Vietnam dong. Find the 99% confidence interval of the true mean.**

$$ME = z_{\frac{\alpha}{2}} \times \frac{\sigma}{\sqrt{n}} = 12.9\ (Vietnam\ dong)$$

IC = (587.1 , 612.9) (Vietnam dong)

(10points) **In a study of 500 accidents that required treatment in an emergency room, 10 occurred at work. Find the 90% confidence interval of the true proportion of accidents that occurred at work.**

$$ME = z_{\frac{\alpha}{2}} \times \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = 0.0103$$

IC = (0.0097 , 0.0303)

(10points) **A random sample of the final score in Calculus I is below.**

72 79 80 74 82 79 82 78 60 75

Is there sufficient evidence to conclude that the variance in score differs from 40 at level of significance α = 0.05? Assume that the score is normally distributed.

$$H_0: \sigma^2 = 40, H_a: \sigma^2 \neq 40$$

$$\chi_0^2 = \frac{(n-1)s^2}{\sigma^2} = 9.6705 < \chi^2_{\frac{\alpha}{2}, n-1} = 19.023$$

Does not have enough evidence to reject H_0 => The variance does not different from 40

(10points) **The deflection temperature (in Celsius degree) under load for two different types of plastic pipe is being investigated. The deflection temperature for type I and type 2 are supposed to be normally distributed with standard deviation = 0.02 and 0.025 respectively. For type 1, a random sample of 10 pipe specimens are tested and provides sample mean = 16 while a sample of 20 pipe specimens tested for type 2 has sample mean = 17.**
At level of significant α = 1%, does the data support the claim that the deflection temperature under load for type 2 pipe is strictly greater than that of type 1 ?

$$H_0: \mu_1 = \mu_2 , H_a: \mu_1 \neq \mu_2$$

$$|z_0| = \frac{\bar{x} - \bar{y}}{\sqrt{\frac{\sigma_x^2}{n_x} + \frac{\sigma_y^2}{n_y}}} = 118.469 > z_{\frac{\alpha}{2}} = 2.58$$

Have enough evidence to reject H_0 => The deflection temperature under load does different from 2 types

(20 points) **The data to study the deflection (mm) of particleboard from stress levels of relative humidity are shown below:**

x = Stress level (%) 54 58 61 62 68 70 72 75 78

y = Deflection (mm) 16 19 14 13 14 15 11 12 1 1

(a) Fit the simple linear regression model using the method of least squares.

$$b_1 = \frac{n\Sigma xy - \Sigma x \times \Sigma y}{n\Sigma x^2 - (\sum x)^2} = -0.241$$

$$b_0 = \bar{y} - b_1\bar{x} \; = \; 29.933$$

y =29.933 - 0.241x (mm,%)

(b) Find the estimate of the mean deflection if the stress level is 65%.

y = 29.933 - 0.241 x 65 =14.268 (mm)