

# Linear Regression

1. A study was done to study the effect of ambient temperature  $x$  on the electric power consumed by a chemical plant  $y$ . Other factors were held constant, and the data were collected from an experimental pilot plant.

$y$ (BTU)	$x$ ( $^{\circ}\text{F}$ )	$y$ (BTU)	$x$ ( $^{\circ}\text{F}$ )
250	27	265	31
285	45	298	60
320	72	267	34
295	58	321	74

- (a) Plot the data.
  - (b) Estimate the slope and intercept in a linear regression model.
  - (c) Predict power consumption for an ambient temperature of  $65^{\circ}\text{F}$ .
2. A study of the amount of rainfall and the quantity of air pollution removed produced the following data:

Daily Rainfall, $x$ (0.01 cm)	Particulate Remove, $y$ ( $\mu\text{g}/\text{m}^3$ )
4.3	126
4.5	121
5.9	116
5.6	118
6.1	114
5.2	118
3.8	132
2.1	141
7.5	108

- (a) Find the equation of the regression line to predict the particulate removed from the amount of daily rainfall.
  - (b) Estimate the amount of particulate removed when the daily rainfall is  $x = 4.8$  units.
3. Heat treating is often used to carburize metal parts such as gears. The thickness of the carburized layer is considered an important feature of the gear, and it contributes to the overall reliability of the part. Because of the critical nature of this feature, a lab test is performed on each furnace load. The test is a destructive one, where an actual part is cross sectioned and soaked in a chemical for a period of time. This test involves running a carbon analysis on the surface of both the gear pitch (top of the gear tooth) and the gear root (between the gear teeth). The data below are the results of the pitch carbon-analysis test for 19 parts.

Soak Time	Pitch	Soak Time	Pitch
0.58	0.013	1.17	0.021
0.66	0.016	1.17	0.019
0.66	0.015	1.17	0.021
0.66	0.016	1.20	0.025
0.66	0.015	2.00	0.025
0.66	0.016	2.00	0.026
1.00	0.014	2.20	0.024
1.17	0.021	2.20	0.025
1.17	0.018	2.20	0.024
1.17	0.019		

- (a) Fit a simple linear regression relating the pitch carbon analysis  $y$  against soak time.
- (b) Construct 95% confidence interval for the slope and intercept

- (c) Test  $H_0 : \beta_1 = 0$  against  $H_1 : \beta_1 \neq 0$  at level of significance  $\alpha = 1\%$
- (d) Test  $H_0 : \beta_1 = 0$
- (e) Find  $R^2$