HO CHI MINH CITY INTERNATIONAL UNIVERSITY

MIDTERM EXAMINATION

Academic year 2011-2012, Semester 1
Duration: 90 minutes

SUBJECT: **Differential Equations**	
Chair of Department of Mathematics	Lecturer:
Signature:	Signature:
Full name: Prof. Phan Quoc Khanh	Full name: Dr. Pham Huu Anh Ngoc

Instructions:

- *Open-book examination. Laptops NOT allowed.*

Question 1. *(25 marks) Solve the following differential equation*

$$(e^x \ln y + x + 1)dx + (\frac{e^x}{y} + 2y)dy = 0.$$

Question 2. *(25 marks) Find the solution to the initial value problem*

$$xy' + 2y = 4e^{x^2}, \quad x \geq 1; \qquad y(1) = 1 + 2e.$$

Question 3. *(25 marks)* Find the general solution of the differential equation

$$y'' + 3y' - 28y = -11e^{-7x}.$$

Question 4. *(25 marks)* Solve the following differential equation in two different ways

$$(x^2 - 1)y'' + 2xy' = 0.$$

END.

SOLUTIONS:

Question 1. Note that

$$0 = (e^x \ln y + x + 1)dx + (\frac{e^x}{y} + 2y)dy = (\ln y de^x + e^x d\ln y) + d(x^2/2 + x) + dy^2 = d(e^x \ln y + x^2/2 + x + y^2).$$

Thus the general solution is given by

$$e^x \ln y + x^2/2 + x + y^2 = C.$$

Question 2. The given equation is written as

$$y' + \frac{2}{x}y = \frac{4e^{x^2}}{x}, \quad x \geq 1; \qquad y(1) = 1 + 2e.$$

The integrating factor is given by $I(x) = x^2$. Thus, we get

$$x^2y' + 2xy = 4xe^{x^2}, \quad x \geq 1.$$

This gives

$$\frac{d}{dx}(x^2y) = 4xe^{x^2}, \quad x \geq 1.$$

Therefore, the general solution is

$$y(x) = \frac{2e^{x^2} + C}{x^2}.$$

Since $y(1) = 1 + 2e$, the particular solution is $y(x) = \frac{2e^{x^2}+1}{x^2}$.

Question 3.

The general solution of the corresponding homogeneous equation $y'' + 3y' - 28y = 0$ is

$$y(x) = c_1e^{4x} + c_2e^{-7x}.$$

A particular solution of $y'' + 3y' - 28y = -11e^{-7x}$ is $y_p(x) = xe^{-7x}$. Thus the general solution of the equation $y'' + 3y' - 28y = -11e^{-7x}$, is given by

$$y(x) = xe^{-7x} + c_1e^{4x} + c_2e^{-7x}.$$

Question 4.

Solution 1: Note that $y_1(x) = 1$ is a particular solution of the given differential equation. By the Liouville formula, $y_2(x) = \ln|\frac{x-1}{x+1}|$ is another solution such that y_1, y_2 are linearly independent. So, the general solution is given by

$$y(x) = c_1 + c_2 \ln\left|\frac{x-1}{x+1}\right|.$$

Solution 2: Setting $z = y'$, we get a linear first order differential equation

$$(x^2 - 1)z' + 2xz = 0.$$

This gives

$$y' = z = \frac{c}{x^2 - 1}.$$

Thus,

$$y(x) = c_1 + c_2 \ln\left|\frac{x-1}{x+1}\right|.$$