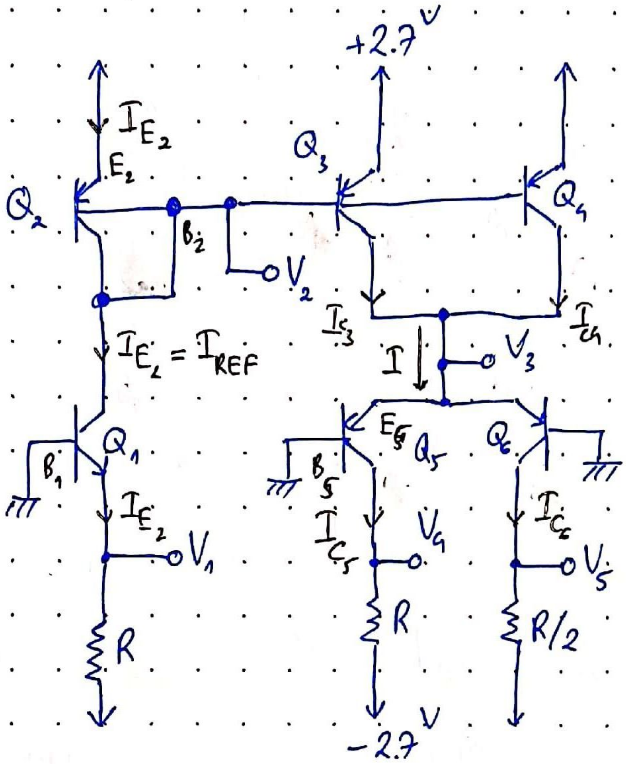
Date: 12/3/20

Nguyen Bang Dang Huy

Analog Electronics

Homework #1

Problem 1



- $V_{E_2B_2} = 0.7^V \Rightarrow V_2 = +2.7^V - V_{E_2B_2} = 2^V$
- $V_1 = 0^V - V_{B_1E_1} = -0.7^V$
- $I_{REF} = I_{E_2} = \dfrac{V_1 - (-2.7^V)}{50^{k\Omega}} = 0.04^{mA}$

Due to the characteristic of the current mirror circuit, $I_{C3} = I_{C4} = I_{C5} = I_{C6} = I_{REF}$

- $I = I_{C3} + I_{C4} = 0.08^{mA}$
- $V_3 = 0^V + V_{E_5B_5} = 0.7^V$
- $V_4 = -2.7^V + I_{C5} R = -2.7^V + 0.04^{mA} \times 50^{k\Omega} = -0.7^V$
- $V_5 = -2.7^V + I_{C6} R/2 = -2.7^V + 0.04^{mA} \times 25^{k\Omega} = -1.7^V$

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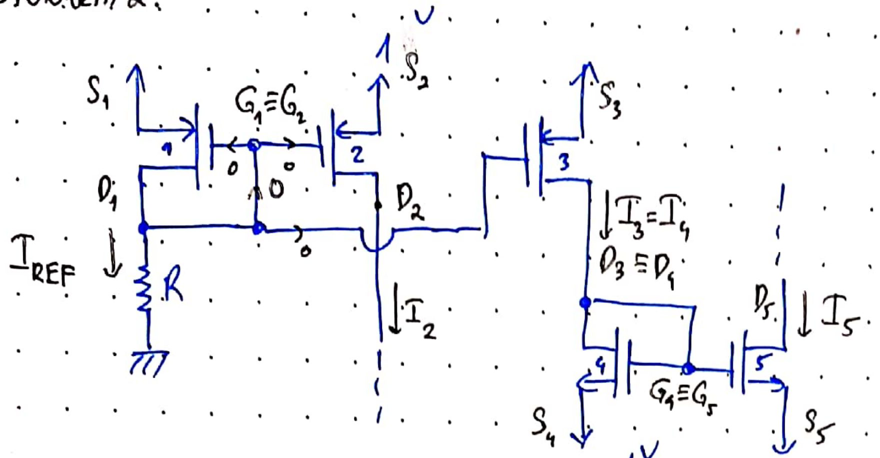
Problem 2.



• Assume that all transistors are in saturation

a) $V'_{An} = |V'_{Ap}| = 5^{V/\mu m} \Rightarrow V_{An} = V_{Ap} = V'_{An} \times L = 5 \times 0.5 = 2.5^V$

$V_{D2,max} = 0.8^V \Rightarrow V_{S2} - V_{D2,max} = V_{SD_2,min} = 1 - 0.8^V = 0.2^V$

• Q_2 is PMOS $\Rightarrow V_{SD2,min} = |V_{OV_2}| = 0.2^V$

$\Leftrightarrow V_{SG2} - |V_{tp}| = 0.2^V \Leftrightarrow V_{SG2} = 0.2 + 0.5 = 0.7^V = V_{SG1}$

• $I_{D1} = I_{REF} = \frac{1}{2} \mu_p C_{ox} \frac{W_1}{L_1} (V_{SG1} - |V_{tp}|)^2 (1 + |\lambda| V_{SD_1})$

$\Leftrightarrow 20^{\mu A} = \frac{1}{2} 100 \frac{W_1}{L_1} V_{OV1}^2 (1 + \frac{1}{|V_A|} V_{SG1})$

$20^{\mu A} = \frac{1}{2} 100 \frac{W_1}{0.5} 0.2^2 (1 + \frac{1}{2.5} 0.7)$

$\Leftrightarrow W_1 \approx 3.9^{\mu m}$

• $I_{D2} = \frac{1}{2} k'_p \frac{W_2}{L_2} (V_{SG2} - |V_{tp}|)^2 (1 + |\lambda| V_{SD2})$

$\Rightarrow \frac{I_{D2}}{I_{REF}} = \frac{W_2}{W_1} \cdot \frac{1 + |\lambda| V_{SD2}}{1 + |\lambda| V_{SG1}} \Leftrightarrow 5 = \frac{W_2}{3.9} \cdot \frac{1 + 0.4 \times 0.2}{1 + 0.4 \times 0.7} \Leftrightarrow W_2 \approx 23.1^{\mu m}$

• $V_{O5min} = -0.8^V \Leftrightarrow V_{DS_5,min} = -0.8^V - (-1)^V = 0.2^V = V_{OV5}$ (NMOS)

$\Leftrightarrow V_{GS5} - V_{tn} = V_{OV5} = 0.2^V$

$\Rightarrow V_{GS_5} = 0.2^V + 0.5^V = 0.7^V = V_{GS_4} = V_{DS_4}$

$V_{D_4} = -1 + V_{DS_4} = -0.3^V = V_{D_3} \Rightarrow V_{SD_3} = 1 - (-0.3) = 1.3^V$

- Q_3 is PMOS $\Rightarrow \dfrac{I_{D_3}}{I_{REF}} = \dfrac{W_3}{W_1} \dfrac{1 + |\lambda| V_{SD3}}{1 + |\lambda| V_{SG1}} \Leftrightarrow \dfrac{40}{20} = \dfrac{W_3}{W_1} \dfrac{1 + 0.4 \times 1.3}{1 + 0.4 \times 0.7}$

  $\Leftrightarrow W_3 \approx 6.6^{\mu m}$
- Q_4 is NMOS

  $\Rightarrow I_4 = \dfrac{1}{2} k_n' V_{ov_4}^2 (1 + \lambda V_{DS4}) \dfrac{W_4}{L_4}$

  $\Leftrightarrow 40 = \dfrac{1}{2} 400 \dfrac{W_4}{0.5} 0.2^2 (1 + 0.4 \times 0.7)$

  $\Leftrightarrow W_4 \approx 2^{\mu m}$
- Q_5 is NMOS

  $\Rightarrow \dfrac{I_5}{I_4} = \dfrac{W_5}{W_4} \dfrac{1 + |\lambda| V_{DS5}}{1 + |\lambda| V_{GS4}} \Leftrightarrow 2 = \dfrac{W_5}{2} \dfrac{1 + 0.4 \times 0.2}{1 + 0.4 \times 0.7} \Leftrightarrow W_5 \approx 4.7^{\mu m}$
- $V_{SG_1} = 0.7^V \Rightarrow V_{D_1} = V_{G_1} = 1^V - 0.7^V = 0.3^V$

  $R = \dfrac{V_{D_1} - 0}{I_{REF}} = \dfrac{0.3^V}{20^{\mu A}} = 15^{k\Omega}$

b) $r_{o2} = \dfrac{V_A}{I_{D2}} = \dfrac{2.5^V}{100^{\mu A}} = 25^{k\Omega}$

$r_{o5} = \dfrac{V_A}{I_{D5}} = \dfrac{2.5^V}{80^{\mu A}} = 31.25^{k\Omega}$

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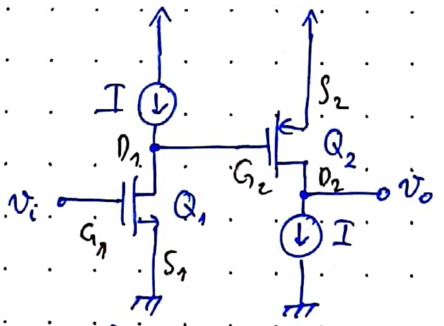
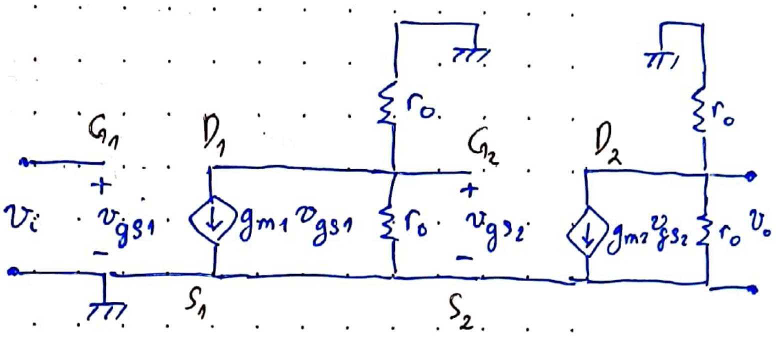
Problem 3.



a) Small-signal equivalent circuit,

($r_{o1} = r_{o2} = r_o$ because $I_{D_1} = I_{D_2} = I$)



We have

$$\begin{cases} v_{gs2} = -g_{m1} v_{gs1} \dfrac{r_o}{2} \\ v_o = -g_{m2} v_{gs2} \dfrac{r_o}{2} = g_{m1} g_{m2} v_i \dfrac{r_o^2}{4} \end{cases}$$

$$\Rightarrow v_o / v_i = g_{m1} \cdot g_{m2} \frac{r_o^2}{4} = A_{vo} \qquad (*)$$

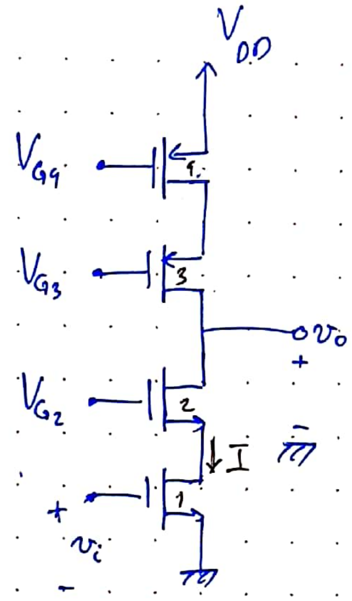
b) $V_{OV1} = |V_{OV2}| = |V_{OV}| \Rightarrow g_{m1} = g_{m2} = g_m = \dfrac{2I_D}{|V_{OV}|} = \dfrac{2I}{|V_{OV}|} \quad (1)$

$$r_o = \frac{V_A}{I_D} = \frac{V_A}{I} \qquad (2)$$

$$\left.\begin{array}{l} (1)(2) \text{ to } (*) \Rightarrow A_{vo} = \dfrac{4I^2}{V_{OV}^2} \dfrac{V_A^2}{4I^2} = \dfrac{V_A^2}{V_{OV}^2} \\ |V_A| = 4^V \text{ and } A_{vo} = 500 \text{ V/V} \end{array}\right\}$$

$$\Rightarrow |V_{OV}| \approx 0.18^V$$

Problem 4.



- $A_v = -g_m R_o \Leftrightarrow -300 = -1.5 R_o \Leftrightarrow R_o = 200^{k\Omega}$
- We have: $R_o = [g_{m2} r_{o2} r_{o1}] \| [g_{m3} r_{o3} r_{o4}]$
- Since $r_{o2} = r_{o1} = r_{o3} = r_{o4} = r_o = \dfrac{|V_A|}{I}$

$$g_{m2} = g_{m3} = g_{m1} = g_m = \frac{2I}{|V_{ov}|}$$

$$\Rightarrow R_o = \frac{g_m r_o^2}{2} \Leftrightarrow 200^{k\Omega} = \frac{1}{2} \times 1.5^{mA/V} \times r_o^2 \Leftrightarrow r_o \approx 16.3^{k\Omega}$$

- And: $I = \dfrac{g_m |V_{ov}|}{2} = \dfrac{1.5 \times 0.5}{2} = 0.375^{mA}$

$$r_o = \frac{|V_A|}{I} \Leftrightarrow |V_A'| L = r_o I \Leftrightarrow 5L = 16.3 \times 0.375$$

$$\Leftrightarrow L \approx 1.22^{\mu m}$$

- Q_1 and Q_2 are NMOS

$$I = \frac{1}{2} \mu_n C_{ox} \left(\frac{W}{L}\right)_1 V_{ov}^2 \Leftrightarrow 0.375 = \frac{1}{2} \times 500 \left(\frac{W}{L}\right)_1 0.5^2$$

$$\Leftrightarrow \left(\frac{W}{L}\right)_1 = 6 = \left(\frac{W}{L}\right)_2$$

- Q_3 and Q_4 are PMOS

$\Rightarrow I = \frac{1}{2} \mu_p C_{ox} \left(\frac{W}{L}\right)_3 V_{ov}^2 \Leftrightarrow 0.375 = \frac{1}{2} \times 200 \times 0.5^2 \left(\frac{W}{L}\right)_3$

$\Leftrightarrow \left(\frac{W}{L}\right)_3 = 15 = \left(\frac{W}{L}\right)_4$