

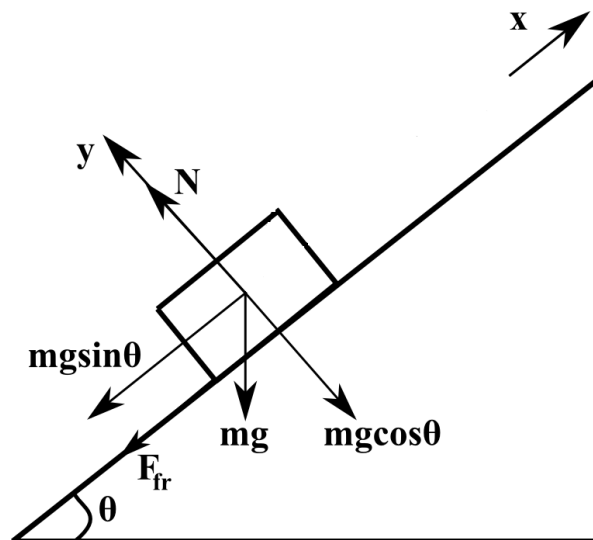
### Answer on Question 52652, Physics, Other

#### Question:

A  $2\text{ kg}$  box is projected with an initial speed of  $3\text{ m/s}$  up a rough plane inclined at  $60^\circ$  above horizontal. The coefficient of kinetic friction is  $0.3$ .

- a) What is the energy dissipated by friction as the box slides up the plane?
- b) What is the speed of the box when it again reaches its initial position?

#### Solution:



Let's write all forces that acts on a box:

$$m\vec{g} + \vec{N} + \vec{F}_{fr} = m\vec{a}$$

Then projected the forces on axis  $x$  and  $y$ :

$$-mg\sin\theta - F_{fr} = ma,$$

$$N - mg\cos\theta = 0.$$

By the definition, the friction force is  $F_{fr} = \mu_k N = \mu_k mg\cos\theta$ , and we can find the acceleration of the box from the first equation:

$$-mg\sin\theta - \mu_k mg\cos\theta = ma,$$

$$a = -g(\sin\theta + \mu_k \cos\theta).$$

a) The energy dissipated by friction as the box slides up the plane is equal to the work done on the box by the friction force as the box slides up the plane:

$$W_{fr} = F_{fr}s = \mu_k mg \cos \theta,$$

where,  $s$  is the distance, that the box slides up the plane before it stops momentarily (when  $v = 0$ ).

We can find  $s$  from the kinematic equation:

$$v^2 = v_0^2 + 2as.$$

Because  $v = 0$  we get:

$$s = -\frac{v_0^2}{2a} = \frac{v_0^2}{2g(\sin \theta + \mu_k \cos \theta)} = \frac{\left(3 \frac{m}{s}\right)^2}{2 \cdot 9.8 \frac{m}{s^2} \cdot (\sin 60^\circ + 0.3 \cdot \cos 60^\circ)} = 0.452m.$$

As we know  $s$  we can find the energy dissipated by friction as the box slides up the plane:

$$W_{fr} = F_{fr}s = \mu_k mg \cos \theta = 0.3 \cdot 2kg \cdot 9.8 \frac{m}{s^2} \cdot 0.452m \cdot \cos 60^\circ = 1.33J.$$

b) In order to find the speed of the box when it again reaches its initial position we use the law of conservation of energy:

$$KE + PE = W_{grav.force} - W_{fr},$$

where,  $KE$  and  $PE$  is the kinetic energy and the potential energy of the box at initial position,  $W_{grav.force}$  is the work done on the box by the gravitational force,  $W_{fr}$  is the work done on the box by the friction force as the box slides down to its initial position, respectively.

Let's obtain the work done on the box by the gravitational force:

$$W_{grav.force} = \int_{start}^{end} \vec{F}_g d\vec{s},$$

where,  $d\vec{s}$  is a vector along inclined rough plane in the direction of motion, and  $\vec{F}_g$  is the gravitational force pointing down vertically.

So, we obtain:

$$\vec{F}_g d\vec{s} = mg ds \cos\left(\frac{\pi}{2} + \theta\right) = -mg ds \sin\theta,$$

$$W_{grav.force} = \int_s^0 \vec{F}_g d\vec{s} = \int_s^0 -mg \sin\theta ds = mg s \sin\theta.$$

Then, we can substitute  $W_{grav.force}$  to the equation for the law of conservation of energy and obtain  $v$ :

$$\frac{1}{2}mv^2 = mg s \sin\theta - \mu_k mg s \cos\theta,$$

$$v = \sqrt{2gs(\sin\theta - \mu_k \cos\theta)} = \sqrt{2 \cdot 9.8 \frac{m}{s^2} \cdot 0.452m \cdot (\sin 60^\circ - 0.3 \cdot \cos 60^\circ)} = \\ = 2.52 \frac{m}{s}.$$

**Answer:**

a)  $W_{fr} = 1.33J$ .

b)  $v = 2.52 \frac{m}{s}$ .