3. Additive rule

① $P(A) = 0.3$; $P(B) = 0.2$ and $P(AB) = 0.1$

a) $P(A') = 1 - P(A) = 1 - 0.3 = 0.7$

b) $P(A \cap B') = P(A) - P(A \cap B) = 0.3 - 0.1 = 0.2$

c) $P(A' \cap B) = P(B) - P(A \cap B) = 0.2 - 0.1 = 0.1$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$
$= 0.3 + 0.2 - 0.1 = 0.4$

d) $P[(A \cup B)'] = 1 - P(A \cup B)$
$= 1 - 0.4 = 0.6$

e) $P(A \cup B') = P(A) + P(B') - P(A \cap B$
$= 0.3 + 1 - P(B) - 0.2$
$= 0.3 + 1 - 0.2 - 0.2$
$= 0.9$

② $P(A') = \frac{14}{100} = 0.14$;

$P(AB) = \frac{70}{100} = 0.7$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$
$= 0.86 + 0.79 - 0.7 = 0.95$

$P(A) = \frac{86}{100} = 0.86$

$P(B) = \frac{79}{100} = 0.79$

③

26	25	24	9	8	7	6

$n(S) = 26 \times 25 \times 24 \times 9 \times 8 \times 7 \times 6 = 47174400$

5	25	24	8	7	6	4

$n(A) = 5 \times 25 \times 24 \times 8 \times 7 \times 6 \times 4 = 4032000$

$P(A) = \frac{n(A)}{n(S)} = \frac{4032000}{47174400} = 0.085$

④ a) $P(A) + P(B) + P(C) = 1$

$\Rightarrow 0.99 + 0.001 + P(C) = 1$

$\Rightarrow P(C) = 0.009$

b) $P(B) + P(B') = 1$

$\Rightarrow 0.001 + P(B') = 1$

$\Rightarrow P(B') = 0.999$

c) $P(B \cup C) = P(B) + P(C)$
$= 0.001 + 0.009$
$= 0.01$

4. Conditional probability - Multiplication rule.

① a) $P(A \cap B) = \frac{70}{100} = 0.7$

b) $P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{0.7}{0.79} \approx 0.88$

$P(B) = \frac{79}{100} = 0.79$

(2) A: the event that the voter who voted for Republican is a Democrat

B: the event that the voter who voted for Republican

$$P(A/B) = \frac{P(AB)}{P(B)} = \frac{50/1000}{\frac{680-60+50}{1000}} = 0.085$$

(3) a) $\{(1,1),(2,2),(3,3),(4,4),(5,5),(6,6)\} = A$

$$P(A) = \frac{6}{36} = \frac{1}{6}$$

b) - sum of 4 or less $\{(1,1),(1,2),(1,3),(2,1),(2,2)(3,1)\} = F$

$$P(F) = \frac{6}{36} = \frac{1}{6}$$

- double rolled $\{(1,1),(2,2)\} = E$

~~$P(E) = \frac{2}{36}$~~ ; $E \cap F = \{(1,1),(2,2)\}$

$$P(E \cap F) = \frac{2}{36}$$

$$P(E/F) = \frac{P(E \cap F)}{P(F)} = \frac{2/36}{1/6} = \frac{1}{3}$$

c) outcomes $= (1,6),(2,6),(3,6),(4,6),(5,6),(6,6),(6,1),(6,2),(6,3),(6,4),(6,5)$

$$\text{Probability} = \frac{11}{36}$$

d) n(faces are different) $= 30$

$$P(\text{faces are different}) = \frac{30}{36}$$

n(faces at least one 6 and faces are different) $= 10$

$$P(\text{faces at least one 6 and faces are different}) = \frac{10}{36}$$

$$P(\text{at least on dice roll is a 6} / \text{faces are different}) = \frac{P(\text{at least one dice is a 6 and faces different})}{P(\text{faces are different})} = \frac{1}{3}$$

(4) $A = \{(5,1), (5,2), (5,3), (5,4), (5,5), (5,6), (1,5), (2,5), (3,5), (4,5), (6,5)\}$

$B = \{(3,1), (3,2), (3,3), (3,4), (3,5), (3,6), (1,3), (2,3), (4,3), (5,3), (6,3)\}$

$A \cap B = \{(3,5), (5,3)\}$

$P(A \cap B) = \frac{2}{36}$

$P(B) = \frac{7}{36}$

$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{2/36}{7/36} = \frac{2}{7}$