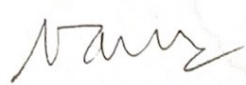
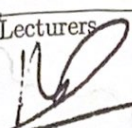
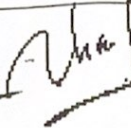
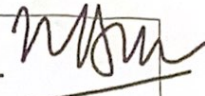


CALCULUS 2 MIDTERM
Semester 2, Academic Year 2022-2023
Duration: 90 minutes

Department of Mathematics  Nguyen Minh Quan	Lecturers    Ta Quoc Bao, Tran Vu Khanh, Nguyen Anh Tu
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Instructions:

- Write the test code MAR086 on your answer sheet.
- You can use two A4 sheets of notes and a calculator. All other documents and electronic devices are forbidden.
- Each question carries 4 points. For the True/False questions, fill in the circles completely.
- Only the answer sheet will be graded.

Part A: True/False Questions

1. The series $\sum_{n=1}^{\infty} (-1)^n (\sqrt{n} - \sqrt{n-1})$ converges. T
2. The series $\sum_{n=1}^{\infty} (-1)^n \frac{(2n)!}{(n!)^2}$ diverges. T
3. The explicit formula of the sequence $\left\{ \frac{2}{25}, \frac{4}{36}, \frac{6}{49}, \frac{8}{64}, \frac{10}{81}, \dots \right\}$ is $a_n = \frac{2n}{(n+4)^2}$. T
4. The sequence $a_n = \frac{(2n)!n^2}{(2n+2)!}$ has limit $\frac{1}{4}$. F
5. If nonzero vectors \mathbf{u} and \mathbf{v} have the same magnitude, then they make equal angle with vector $\mathbf{u} + \mathbf{v}$. F
6. If for some three dimensional vectors $\mathbf{u}, \mathbf{v}, \mathbf{w}$ we have $\mathbf{u} \times \mathbf{v} = \mathbf{u} \times \mathbf{w}$, then $\mathbf{v} = \mathbf{w}$. T
7. The series $\sum_{n=1}^{\infty} \frac{3}{n^2 + 3}$ is divergent. F
8. The series $\sum_{n=2}^{\infty} (-1)^n \frac{n^2 - 1}{n^2 + 3}$ is convergent. F
9. If \mathbf{u} is orthogonal to \mathbf{v} , and \mathbf{v} is orthogonal to \mathbf{w} , then \mathbf{u} is orthogonal to \mathbf{w} . F
10. The series $\sum_{n=1}^{\infty} \frac{n^2}{4^n}$ diverges. F
11. The vector $\mathbf{u} = \langle -1, -5, 7 \rangle$ is perpendicular to both the line $x = 1 + 5t, y = 3 - t, z = \dots$ and the plane $2x + y + z = 12$. F

$$\begin{array}{r} u+v \perp 2u-3v \\ u-v \perp 2u+3v \end{array}$$

12. If $u+v$ is orthogonal to $2u-3v$ and $u-v$ is orthogonal to $2u+3v$, then u is orthogonal to v . \uparrow
13. The series $\sum_{n=1}^{\infty} ne^{-2n^2}$ converges. \uparrow
14. If u is a three dimensional vector, then $(u \cdot i)^2 + (u \cdot j)^2 + (u \cdot k)^2 = |u|^2$.
15. If u, v are three dimensional vectors, then $(u-v) \times (u+v) = 2u \times v$.

Part B: Short Answer Questions

16. Find the limit $\lim_{(x,y) \rightarrow (0,0)} \frac{x^2 + 4xy}{3x^2 + y^2}$ if it exists.
17. Determine the radius and interval of convergence of the power series $\sum_{n=2}^{\infty} \frac{2^n (x-3)^n}{n}$.
18. Let $f(x) = \frac{x^2}{x+2}$. Find the formula for the coefficients c_n in the power series expansion $f(x) = \sum_{n=0}^{\infty} c_n x^n$.
19. Evaluate the series $\sum_{n=1}^{\infty} \frac{2^{2n-1}}{5^n}$.
20. Find all values of a such that $\langle a, a, 2 \rangle \times \langle 1, a, 3 \rangle = \langle 2, -4, 2 \rangle$.
21. Find the area of the triangle T with vertices $P(2, -1, 4)$, $Q(1, 1, -1)$ and $R(-4, 1, 1)$.
22. Find the line through point $(-3, 4, 2)$ that is perpendicular to both $u = \langle 3, 1, 0 \rangle$ and $v = \langle 2, 4, 1 \rangle$.
23. Find the plane passing through the points $P(2, -1, 4)$, $Q(1, 1, -1)$ and $R(-4, 1, 1)$.
24. Find the volume of the parallelepiped determined by $u = \langle 3, 1, 0 \rangle$, $v = \langle 2, 4, 1 \rangle$, $w = \langle 1, 1, 5 \rangle$.
25. Find the length of the two-dimensional curve $r(t) = \langle \cos t + t \sin t, \sin t - t \cos t \rangle$, for $t \in [0, \pi/2]$.

$$uv = u'v + uv'$$