Techniques of
Circuit Analysis
(Chapter 4)

Textbook:

Electric Circuits

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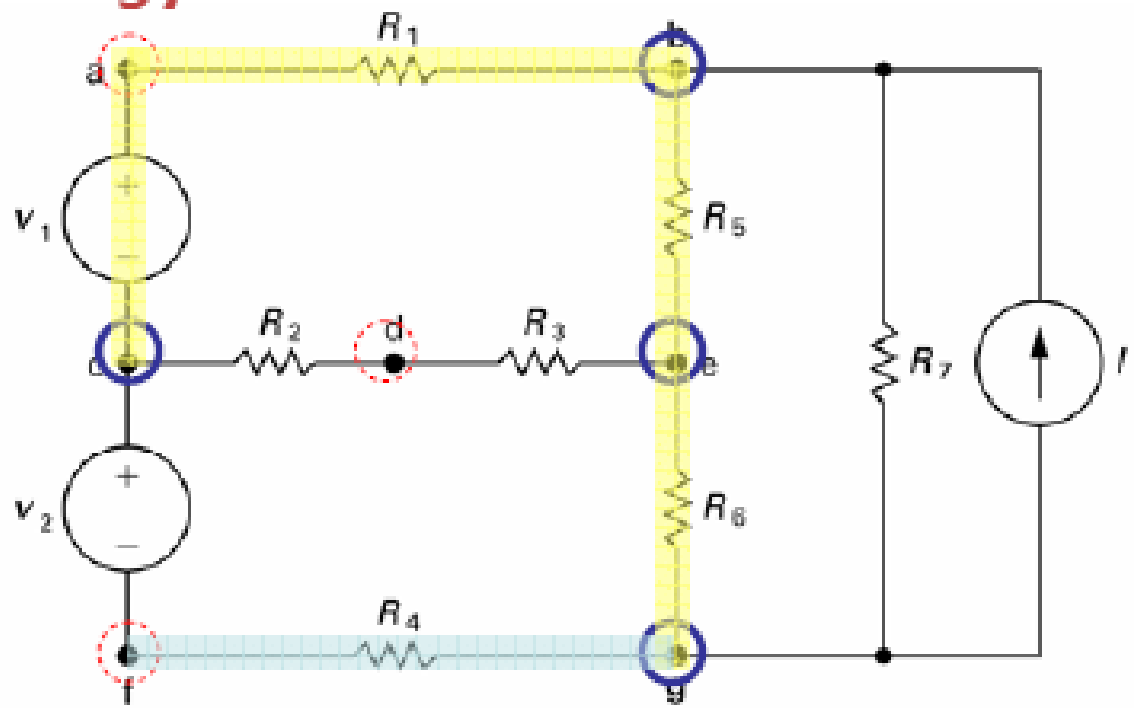
9th Edition.

Outline

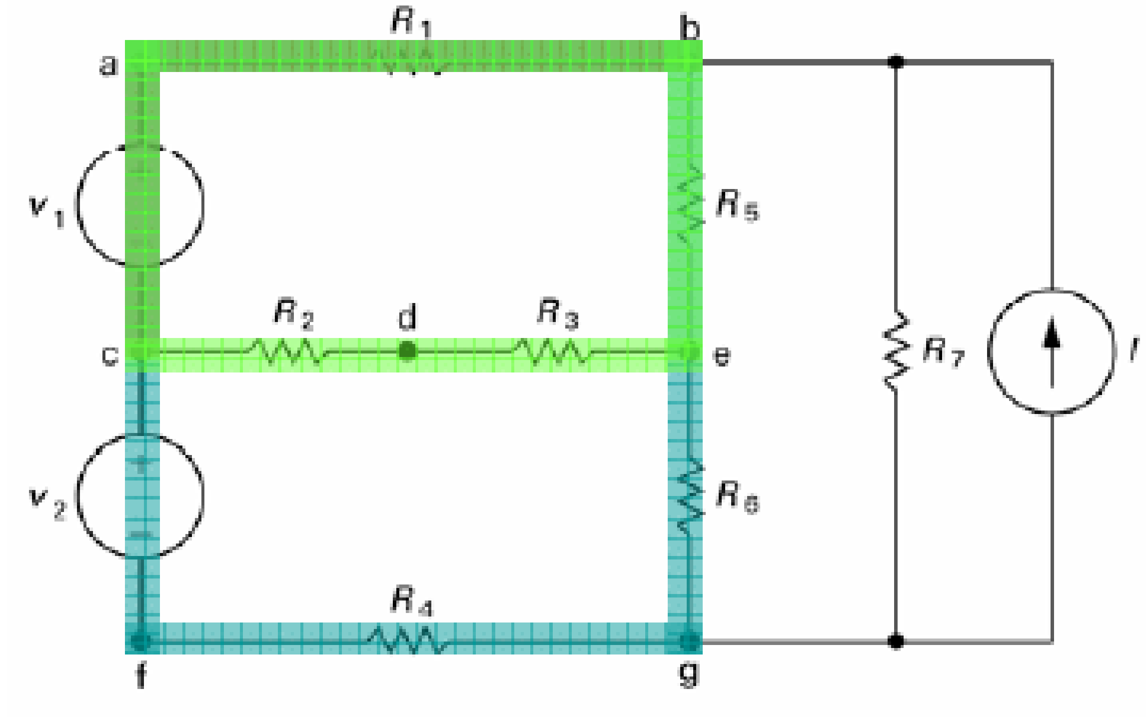
- *The node-voltage method*
- *The mesh-current method*
- *Source transformation*
- *Thevenin & Norton equivalents*
- *Maximum power transfer*
- *Super position*

Terminology



Node	A point where two or more circuit elements join	a
Essential Node	A node where three or more circuit elements join	b
Path	A trace of adjoining basic elements with no elements included more than once	v_1-R_1-R_5-R_6
Branch	A path that connects two nodes	R_4

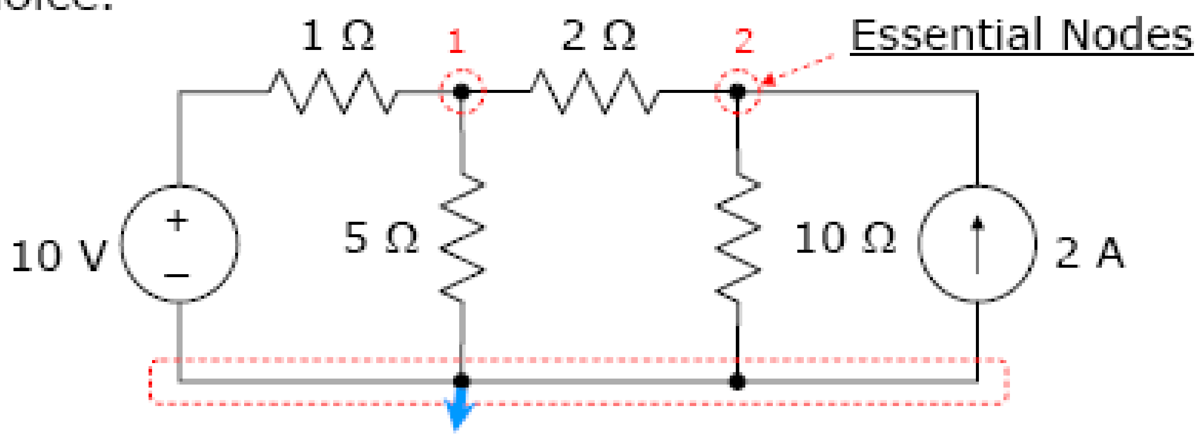
Terminology



Essential branch	A path which connects two essential nodes without passing through an essential node	v_1-R_1
Loop	A path whose last node is the same as the starting node	v_1-R_1-R_5-R_6-R_4-v_2
Mesh	A loop that does not enclose any other loop	v_1-R_1-R_5-R_3-R_2

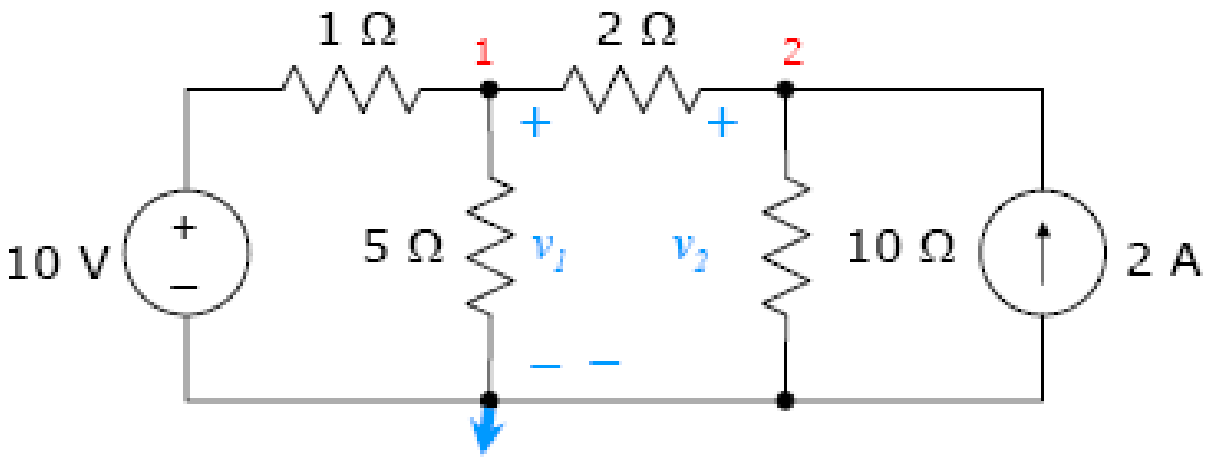
Introduction to node-voltage method

- Can be applied to both planar and non-planar circuits.
- 1st redraw the circuit so that no branches cross over.
- 2nd mark clearly all essential nodes in the circuit.
  - In a circuit with n_e essential nodes, n_e-1 node voltage can be written.
- 3rd select on of the essential nodes to be the reference node.
  - Generally the node with the most branches is a good choice.



Introduction to node-voltage method

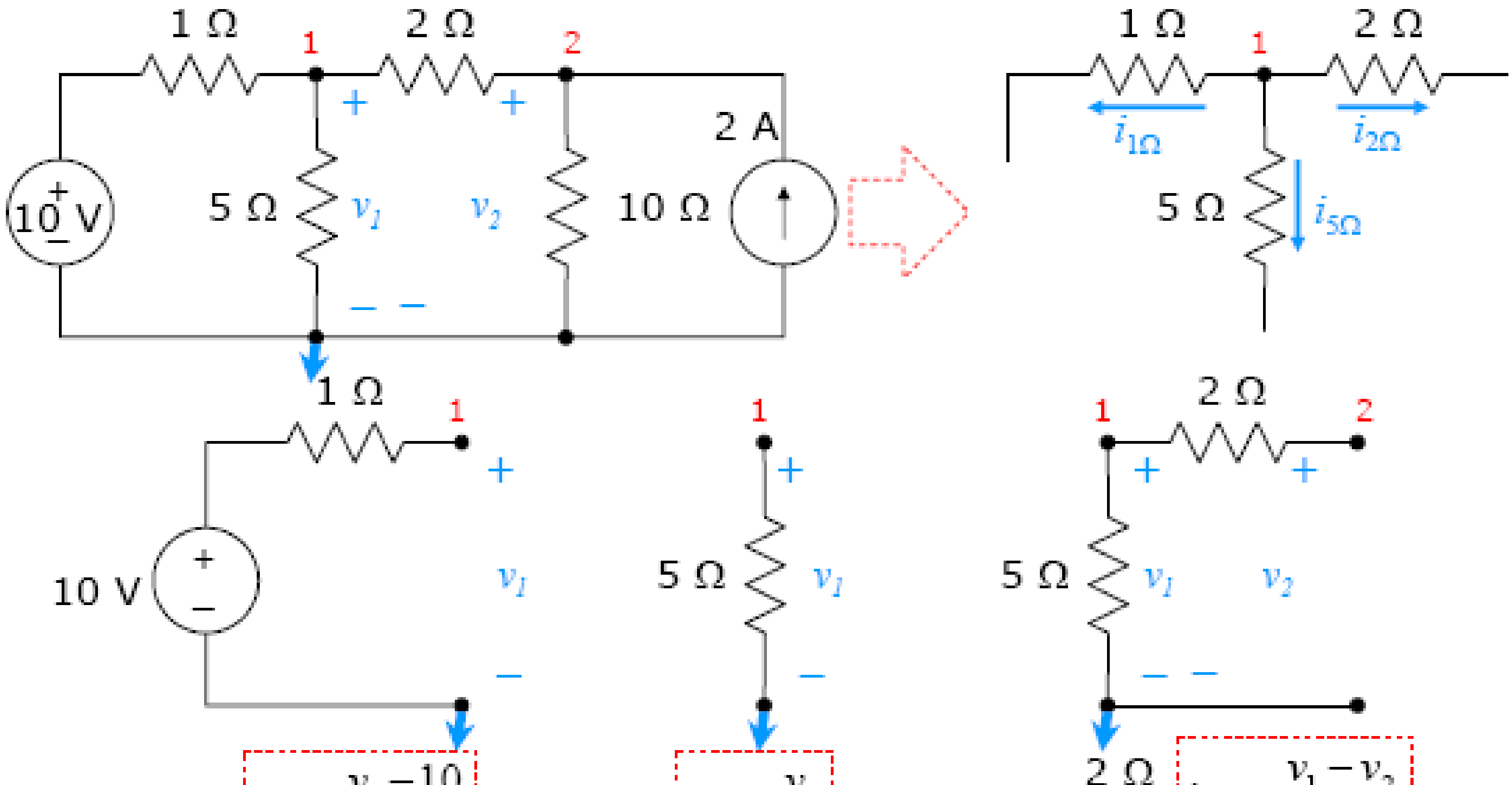
- 4th define the node voltages.
  - Voltage rise from the reference node to a non-reference node.



- 5th generate the node-voltage equations.
  - Write the current leaving each branch connected to a non-reference node as a function of the node voltages.
  - Apply KCL at the nodes by summing the currents.

Introduction to node-voltage method

- At node 1



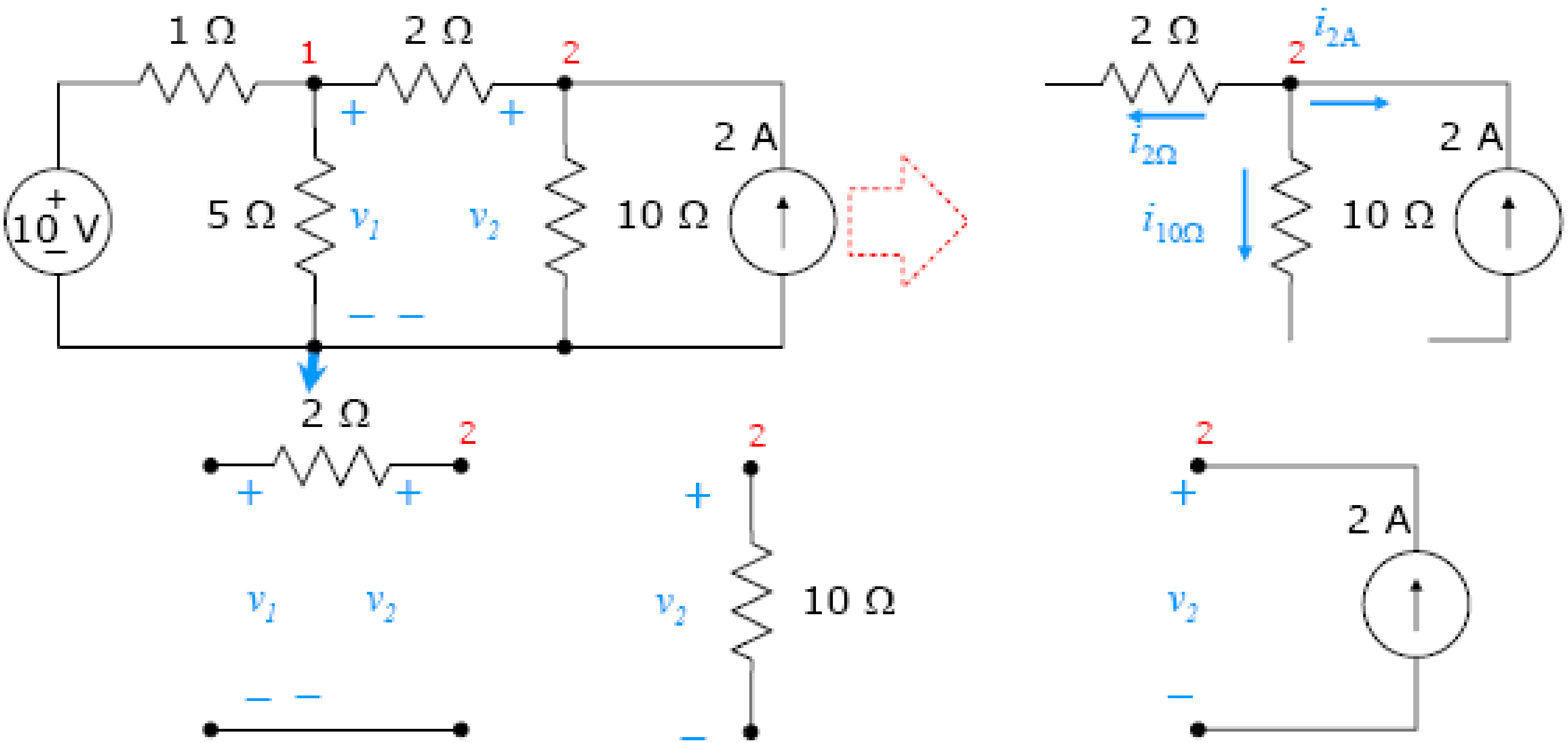
$$i_{1\Omega} = \frac{v_1 - 10}{1}$$

$$i_{5\Omega} = \frac{v_1}{5}$$

$$i_{2\Omega} = \frac{v_1 - v_2}{2}$$

Introduction to node-voltage method

- At node 2



$$i_{2\Omega} = \frac{v_2 - v_1}{2}$$

$$i_{10\Omega} = \frac{v_2}{10}$$

$$i_{2A} = -2$$

Introduction to node-voltage method

$$i_{1\Omega} + i_{5\Omega} + i_{2\Omega} = 0 \quad \Rightarrow \quad \boxed{\frac{v_1 - 10}{1} + \frac{v_1}{5} + \frac{v_1 - v_2}{2} = 0} \quad (1)$$

$$i_{2\Omega} + i_{10\Omega} + i_{2A} = 0 \quad \Rightarrow \quad \boxed{\frac{v_2 - v_1}{2} + \frac{v_2}{10} - 2 = 0} \quad (2)$$

Rearranging the equations

$$\begin{aligned} 17v_1 - 5v_2 &= 100 \\ -5v_1 + 6v_2 &= 20 \end{aligned} \quad \Rightarrow \quad v_1 = 9\frac{1}{11}\,\text{V} \;\; \& \;\; v_2 = 10\frac{10}{11}\,\text{V}$$

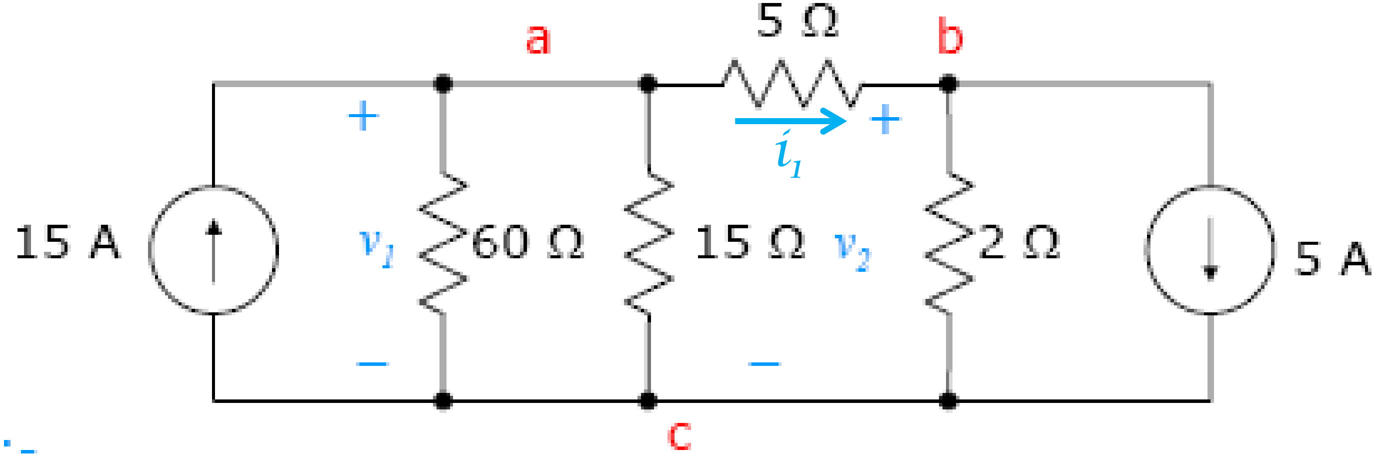
Or Using Matrices

$$\begin{bmatrix} 17 & -5 \\ -5 & 6 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 100 \\ 20 \end{bmatrix} \quad \Rightarrow \quad AV = I \quad \Rightarrow \quad V = A^{-1}I$$

$$\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \frac{1}{(17 \times 6 - 25)} \begin{bmatrix} 6 & 5 \\ 5 & 17 \end{bmatrix} \begin{bmatrix} 100 \\ 20 \end{bmatrix} = \frac{1}{77} \begin{bmatrix} 700 \\ 840 \end{bmatrix}$$

Assessing Objective 1

Use the node voltage method to find v_1, v_2, and i_1.



Ans.:-

a) $$-15+\frac{v_1}{60}+\frac{v_1}{15}+\frac{v_1-v_2}{5}=0$$

b) $$\frac{v_2-v_1}{5}+\frac{v_2}{2}+5=0$$

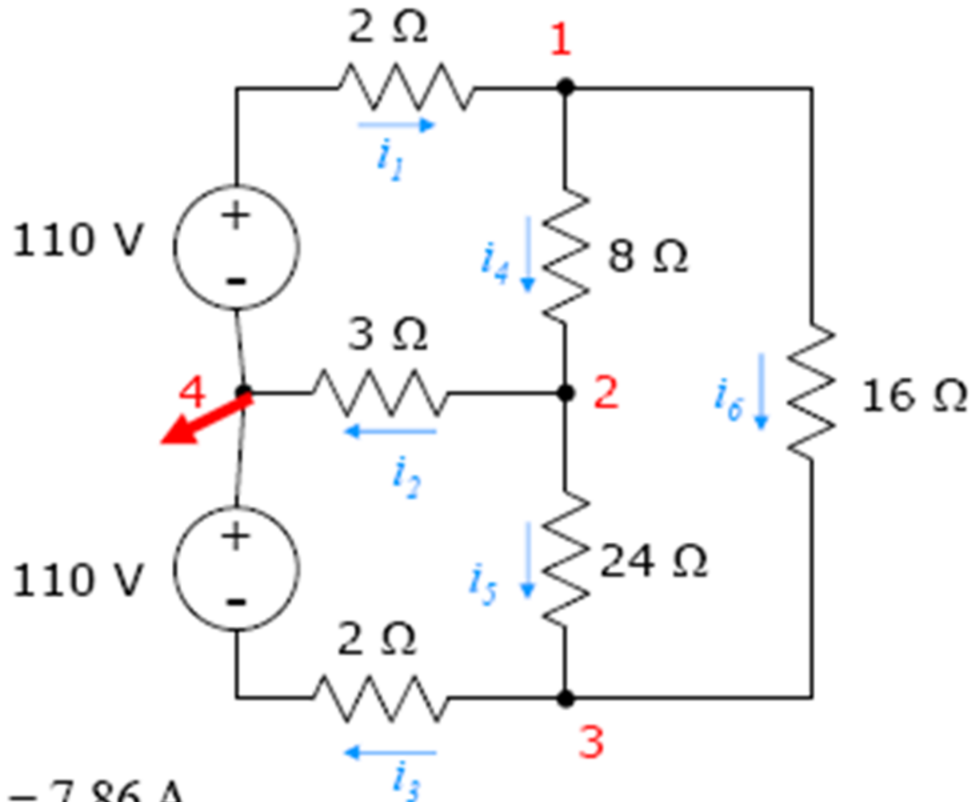
$$v_1 = 60\text{ V}$$

$$v_2 = 10\text{ V}$$

$$i_1 = \frac{60-10}{5} = 10\text{ A}$$

Problem 1

Use the node-voltage method to find the branch currents i_1-i_6.



Ans.:

1) $$\frac{v_1-110}{2}+\frac{v_1-v_2}{8}+\frac{v_1-v_3}{16}=0$$

2) $$\frac{v_2-v_1}{8}+\frac{v_2}{3}+\frac{v_2-v_3}{24}=0$$

3) $$\frac{v_3+110}{2}+\frac{v_3-v_2}{24}+\frac{v_3-v_1}{16}=0$$

$$\begin{bmatrix} 11 & -2 & -1 \\ -3 & 12 & -1 \\ -3 & -2 & 29 \end{bmatrix}\begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}=\begin{bmatrix} 880 \\ 0 \\ -2640 \end{bmatrix}$$

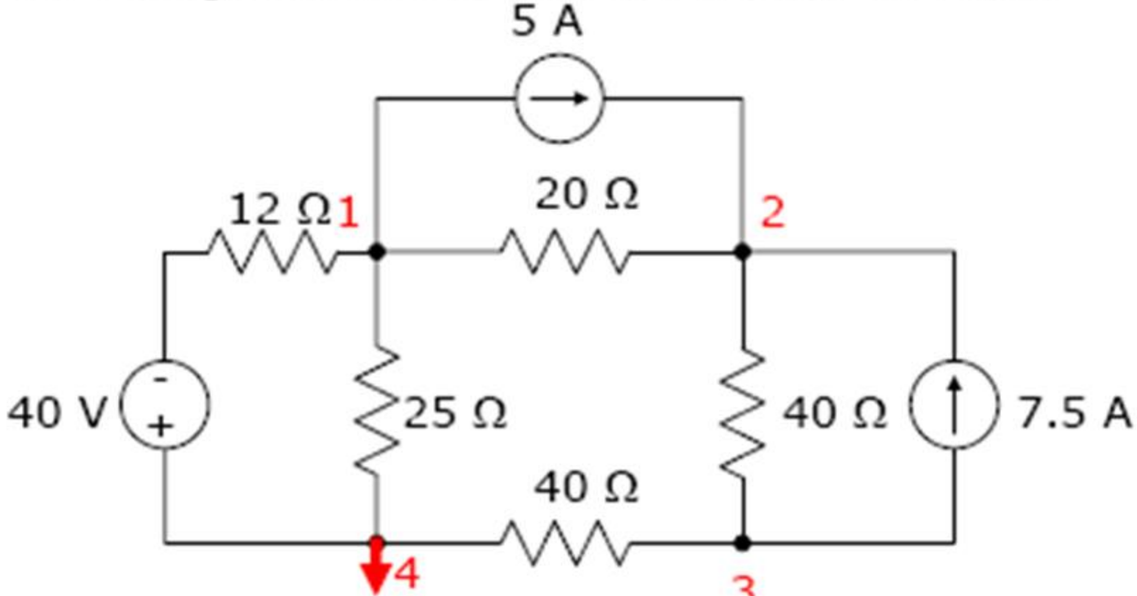
$v_1 = 74.64\text{ V}$ $\quad i_1 = 17.68\text{ A}$ $\quad i_4 = 7.86\text{ A}$

$v_2 = 11.79\text{ V}$ $\quad i_2 = 3.93\text{ A}$ $\quad i_5 = 3.93\text{ A}$

$v_3 = -82.5\text{ V}$ $\quad i_3 = 13.75\text{ A}$ $\quad i_6 = 9.82\text{ A}$

Problem 2

Use the node-voltage method to find the branch currents.



Ans.:

1) $\frac{v_1+40}{12}+\frac{v_1}{25}+5+\frac{v_1-v_2}{20}=0$

2) $\frac{v_2-v_1}{20}+\frac{v_2-v_3}{40}-7.5-5=0$

3) $\frac{v_3-v_2}{40}+\frac{v_3}{40}+7.5=0$

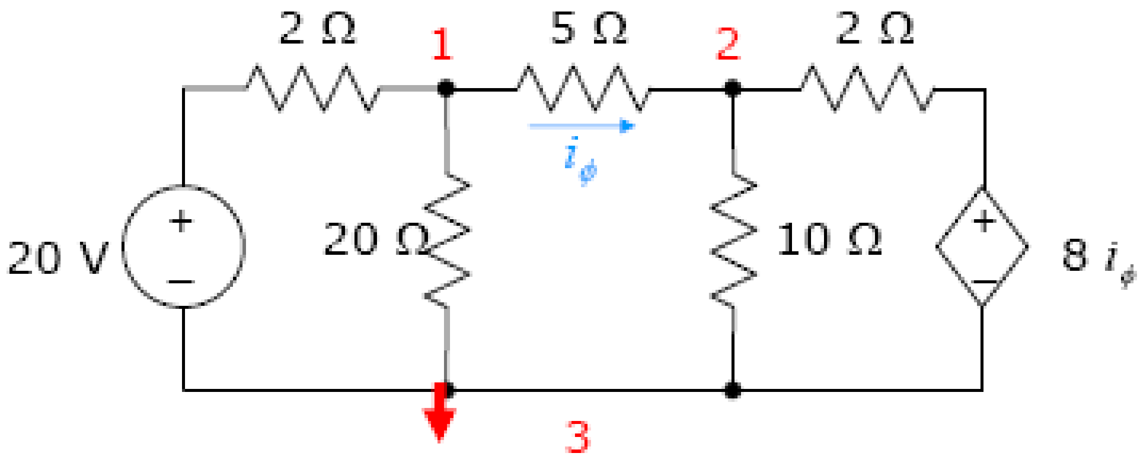
$v_1=-10\text{ V}$

$v_2=132\text{ V}$

$v_3=-84\text{ V}$

Example 1

Use the node-voltage method to find i_ϕ.



Ans.:-

$$\frac{v_1-20}{2}+\frac{v_1}{20}+\frac{v_1-v_2}{5}=0$$

$$\frac{v_2-v_1}{5}+\frac{v_2}{10}+\frac{v_2-8i_\phi}{2}=0$$

$$i_\phi=\frac{v_1-v_2}{5}$$

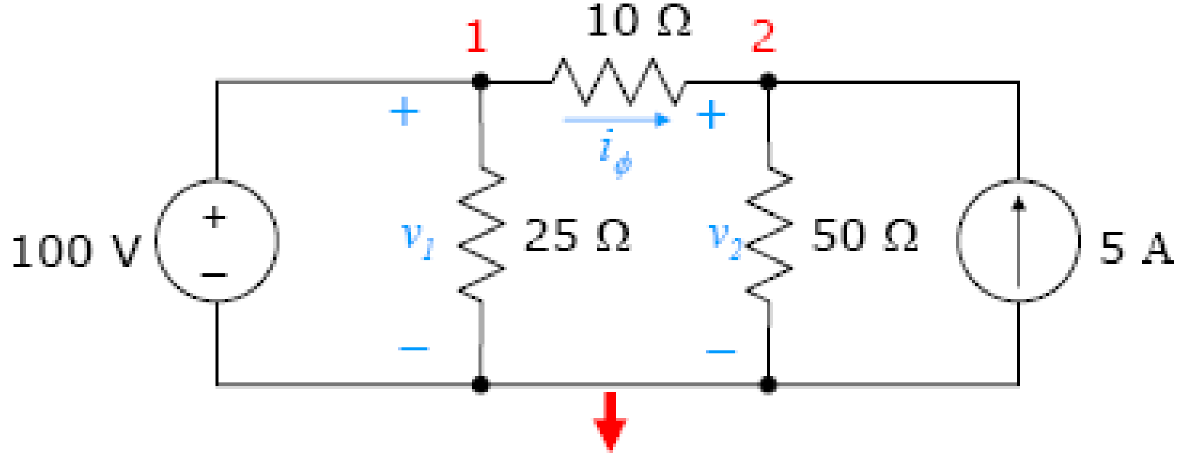
$$v_1=16\text{ V}$$

$$v_2=10\text{ V}$$

$$i_\phi=1.2\text{ A}$$

Special Cases

- When a voltage source is the only element connected between two essential nodes.



$$\frac{v_2 - v_1}{10} + \frac{v_2}{50} - 5 = 0$$

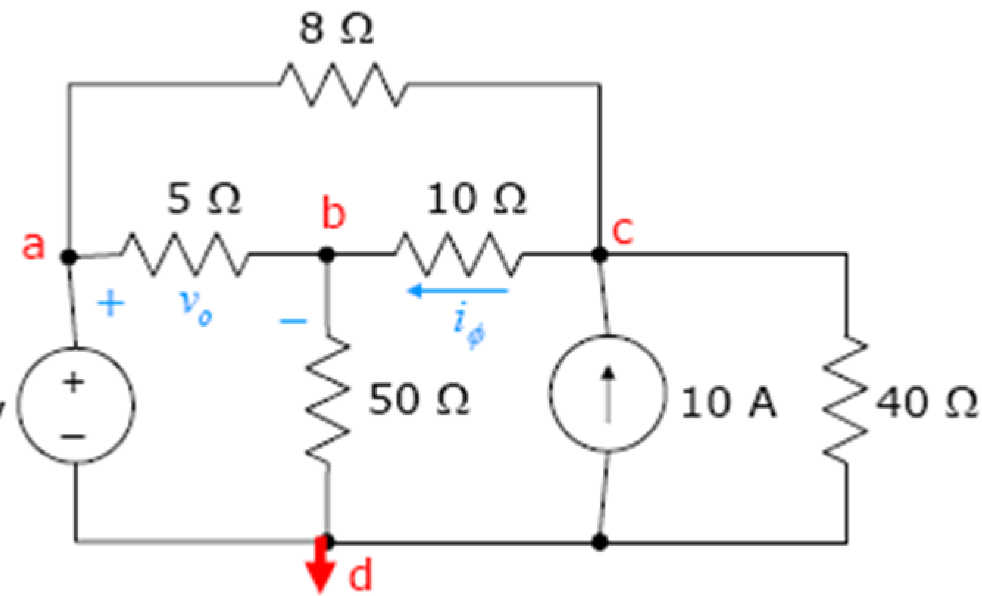
$$v_1 = 100\ \mathrm{V}$$

$$v_2 = 125\ \mathrm{V}$$

Problem 3

Use the node-voltage method to find v_o.

Ans.:-



a) $v_a = 40\text{ V}$

b) $\dfrac{v_b - 40}{5} + \dfrac{v_b}{50} + \dfrac{v_b - v_c}{10} = 0$

c) $\dfrac{v_c - 40}{8} + \dfrac{v_c}{40} + \dfrac{v_c - v_b}{10} - 10 = 0$

$v_a = 40\text{ V}$

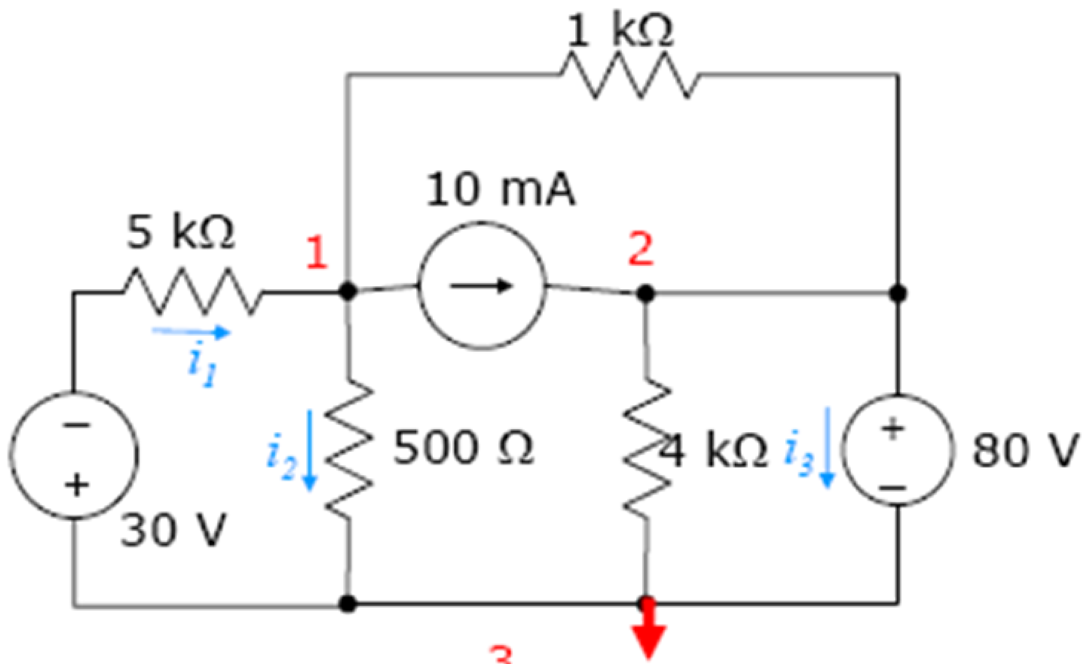
$v_b = 50\text{ V}$

$v_c = 80\text{ V}$

Problem 4

Use the node-voltage method to find i_1, i_2 & i_3.

Ans.:-

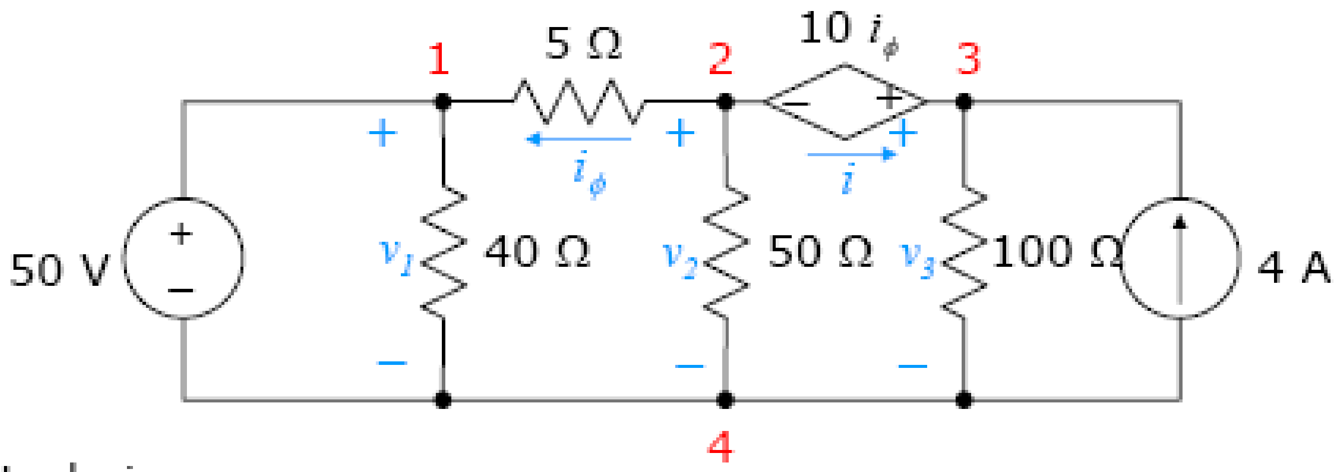


1) $$\frac{v_1+30}{5000}+\frac{v_1}{500}+\frac{v_1-80}{1000}+10\times10^{-3}=0 \qquad v_1=20\ \text{V}$$

Write the equation @ node 2 by yourselves!

Special Cases

- When a dependent voltage source is connected between nodes.



First technique

1) $$v_1 = 50\text{ V}$$

2) $$\frac{v_2 - v_1}{5} + \frac{v_2}{50} + i = 0$$

$+$

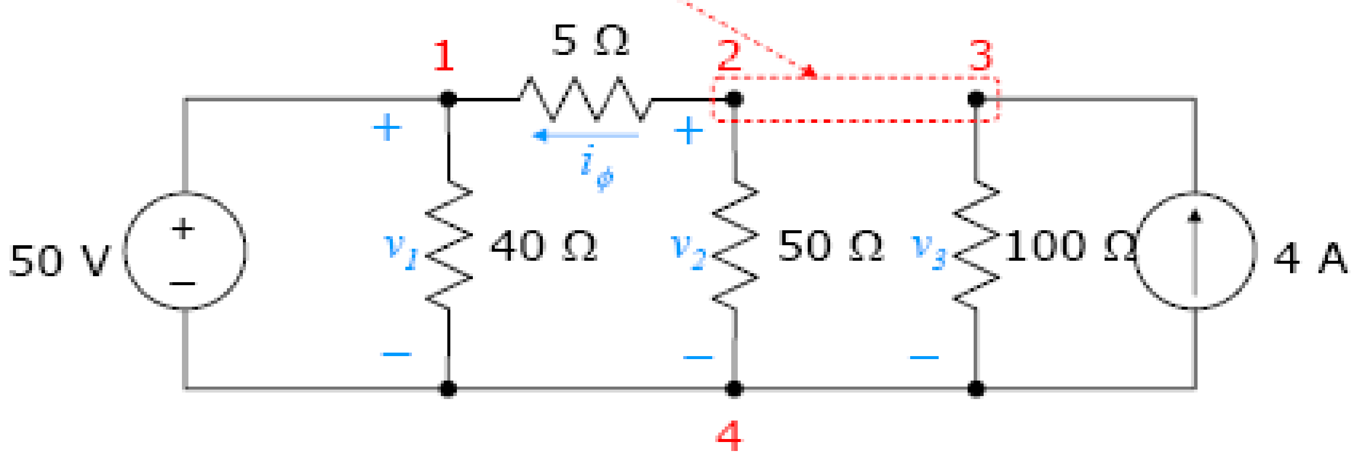
3) $$\frac{v_3}{100} - i - 4 = 0$$

$$\frac{v_2 - v_1}{5} + \frac{v_2}{50} + \frac{v_3}{100} - 4 = 0$$

Special Cases

Second technique (Super Node)

When a voltage source is between two essential nodes, we can combine those nodes to form a **supernode**



@ Super Node $\frac{v_2 - v_1}{5} + \frac{v_2}{50} + \frac{v_3}{100} - 4 = 0$

Need more equations

$v_3 = v_2 + 10i_\phi$ & $i_\phi = \frac{v_2 - v_1}{5} = \frac{v_2 - 50}{5}$

$v_1 = 50\text{ V}$

$v_2 = 60\text{ V}$

$v_3 = 80\text{ V}$

Assessing Objective 2

Use the node-voltage method to find v_o.

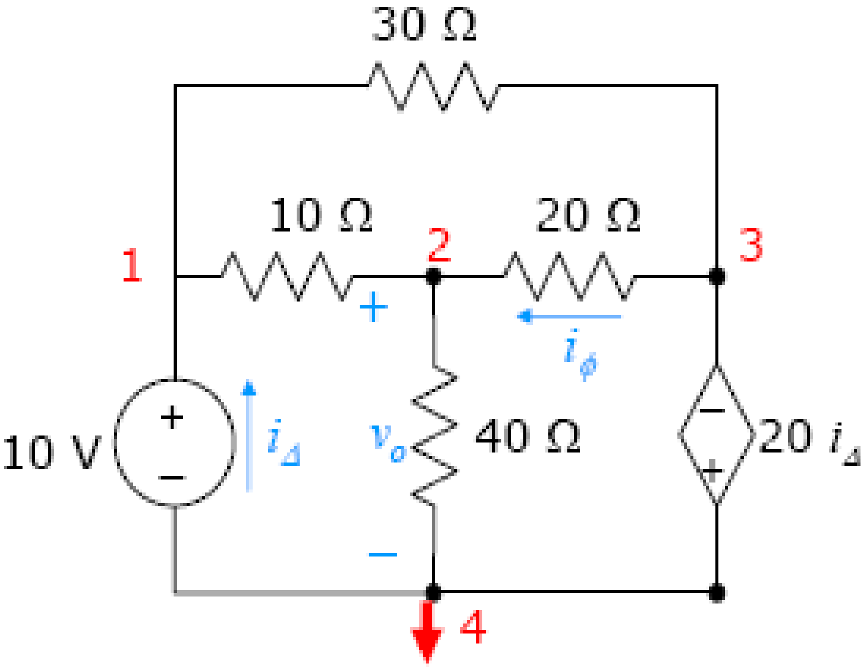
Ans.:-

$$\frac{v_o - 10}{10} + \frac{v_o}{40} + \frac{v_o + 20i_\Delta}{20} = 0$$

$$i_\Delta = i_1 + i_2 = \frac{10 - v_o}{10} + \frac{10 + 20i_\Delta}{30}$$

$v_o = 24\ \text{V}$

$i_\Delta = -3.2\ \text{A}$



Assessing Objective 3

Use the node-voltage method to find i_ϕ.

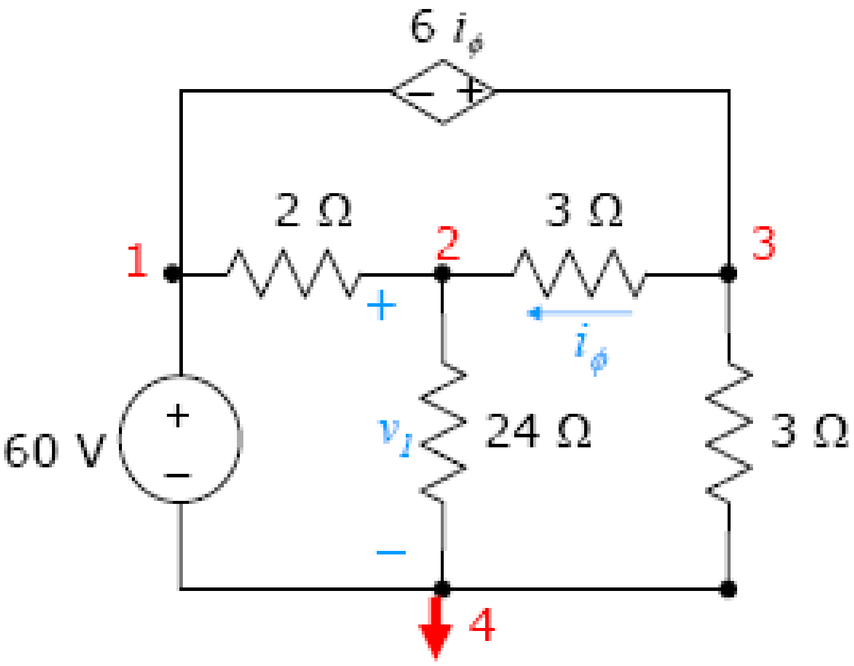
Ans.:-

$$\frac{v_1 - 60}{2} + \frac{v_1}{24} + \frac{v_1 - 6i_\phi - 60}{3} = 0$$

$$i_\phi = \frac{6i_\phi + 60 - v_1}{3}$$

$$v_1 = 48\ \text{V}$$

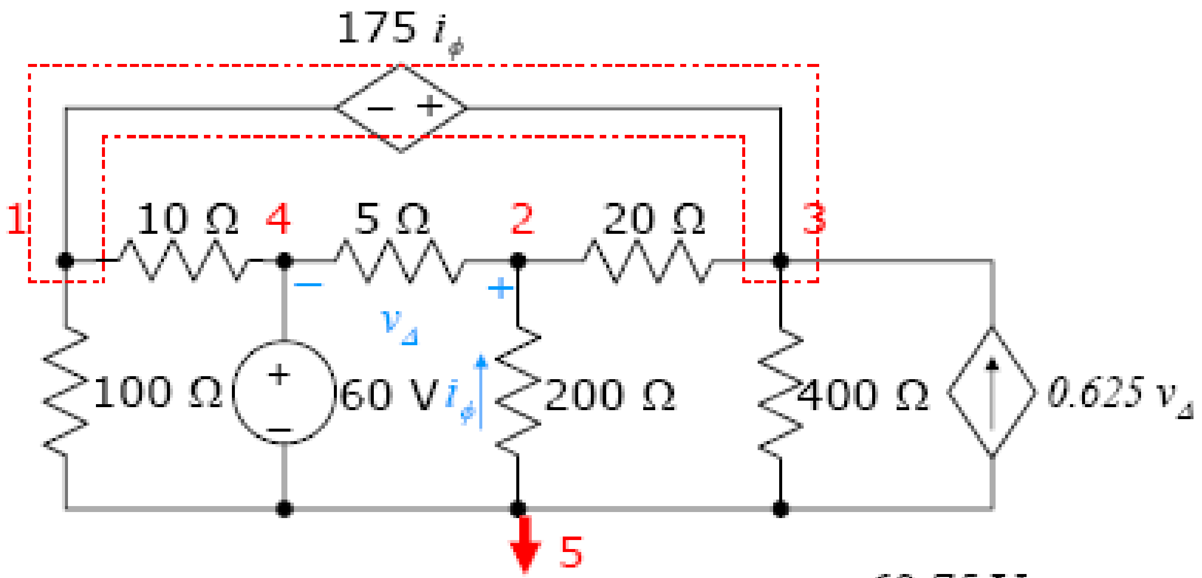
$$i_\phi = -4\ \text{A}$$



Problem 5

Use the node-voltage method to find v_Δ and i_ϕ.

Ans.:-



$$\frac{v_1 - 60}{10} + \frac{v_1}{100} - 0.625 v_\Delta + \frac{v_3}{400} + \frac{v_3 - v_2}{20} = 0$$

$$\frac{v_2 - 60}{5} + \frac{v_2}{200} + \frac{v_2 - v_3}{20} = 0$$

$$i_\phi = \frac{-v_2}{200} \qquad v_\Delta = v_2 - 60$$

$$v_1 = -60.75 \text{ V}$$

$$v_2 = 30 \text{ V}$$

$$v_3 = -87 \text{ V}$$

$$i_\phi = -0.15 \text{ A}$$

$$v_\Delta = -30 \text{ V}$$

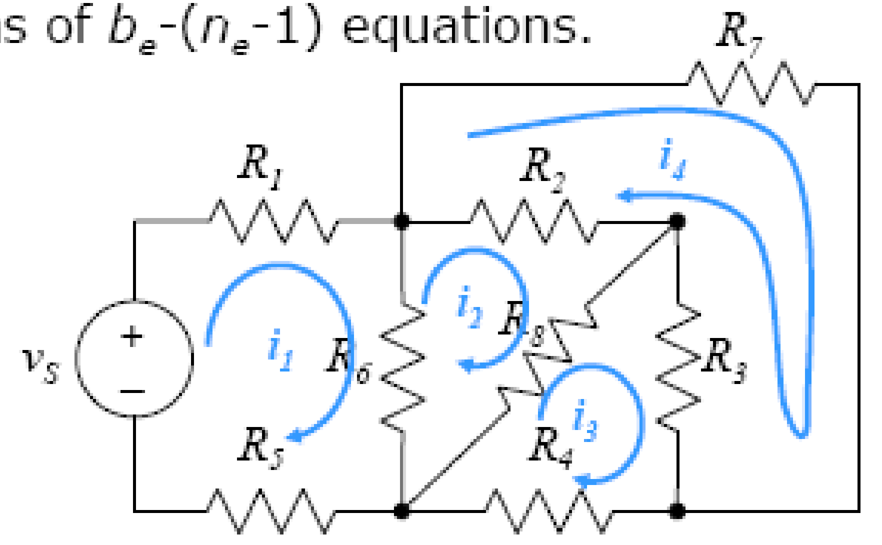
Introduction to Mesh-Current Method

- Applicable only to planar circuits.
- Describe a circuit in terms of b_e-(n_e-1) equations.

b_e= 7

n_e= 4

b_e-(n_e-1)=7-(4-1)=4



- A mesh current is the current that exists only in the perimeter of a mesh.

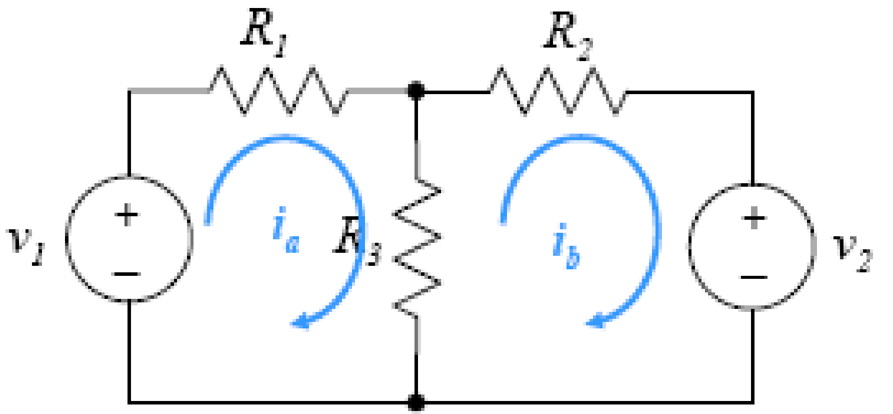
The fact that a mesh current can be a fictitious quantity

<u>Notes:</u> a mesh is a loop with no other loops inside it.

Introduction to Mesh-Current Method

$$-v_1 + i_a R_1 + (i_a - i_b)R_3 = 0$$

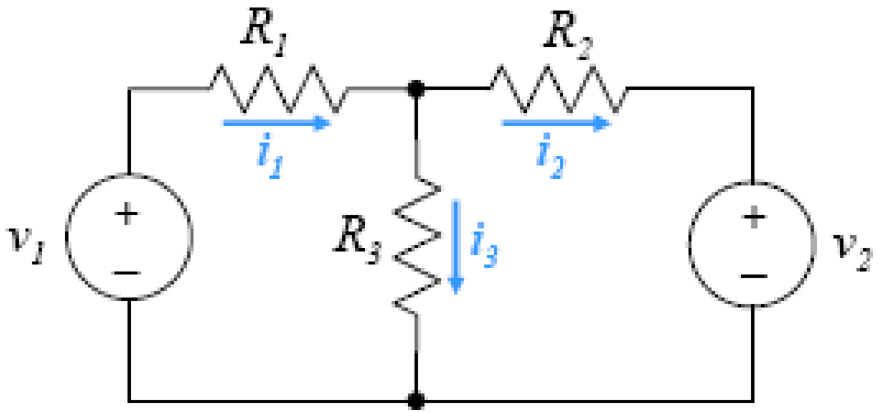
$$(i_b - i_a)R_3 + i_b R_2 + v_2 = 0$$



$$i_1 = i_a$$

$$i_2 = i_b$$

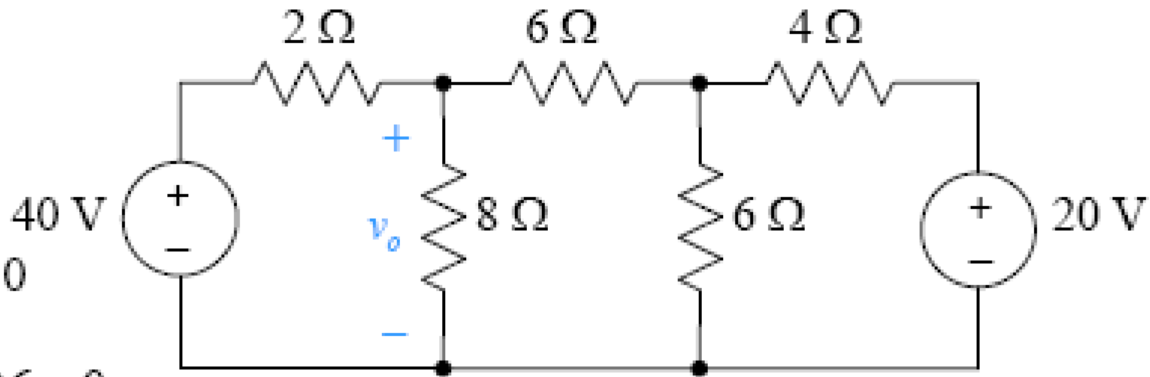
$$i_1 = i_2 + i_3$$



Example 2

Find v_o?

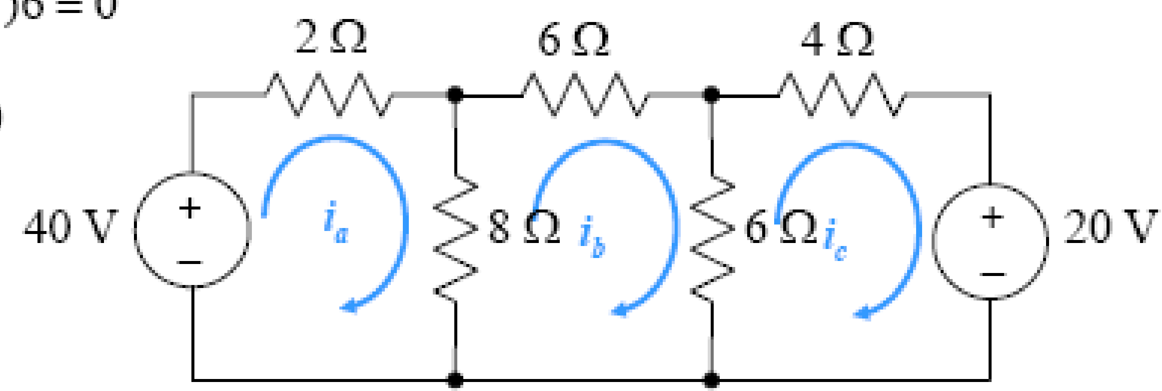
Ans.:-



$$-40 + i_a 2 + (i_a - i_b)8 = 0$$

$$(i_b - i_a)8 + i_b 6 + (i_b - i_c)6 = 0$$

$$(i_c - i_b)6 + i_c 4 + 20 = 0$$



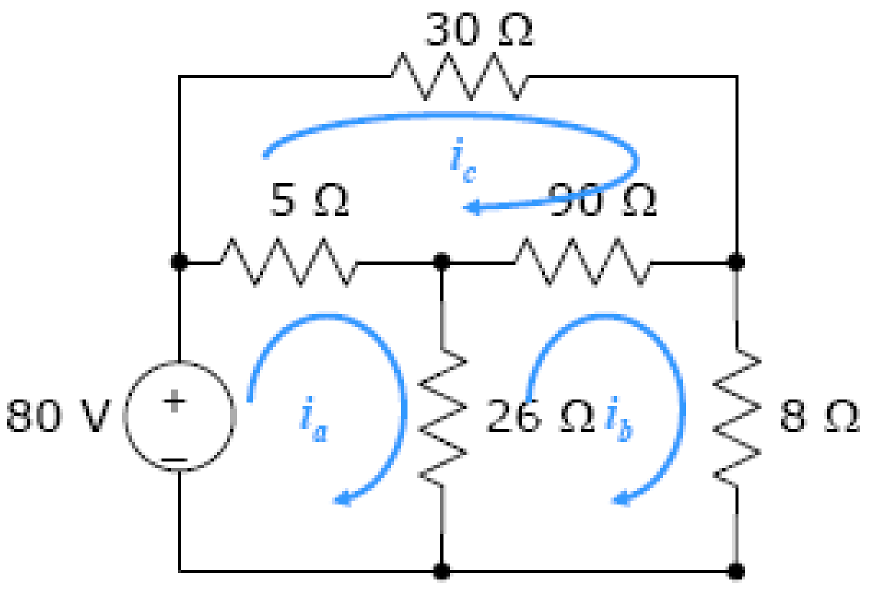
$$\begin{bmatrix} 10 & -8 & 0 \\ -8 & 20 & -6 \\ 0 & -6 & 10 \end{bmatrix} \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} = \begin{bmatrix} 40 \\ 0 \\ -20 \end{bmatrix}$$

$$i_a = 5.6\ \text{A}$$
$$i_b = 2.0\ \text{A}$$
$$i_c = -0.8\ \text{A}$$

Assessing Objective 4

- Power delivered by the 80 V source and power dissipated in the 8 Ω resistor.

Ans.:-



$$-80+(i_a-i_c)5+(i_a-i_b)26=0$$

$$(i_b-i_a)26+(i_b-i_c)90+i_b8=0$$

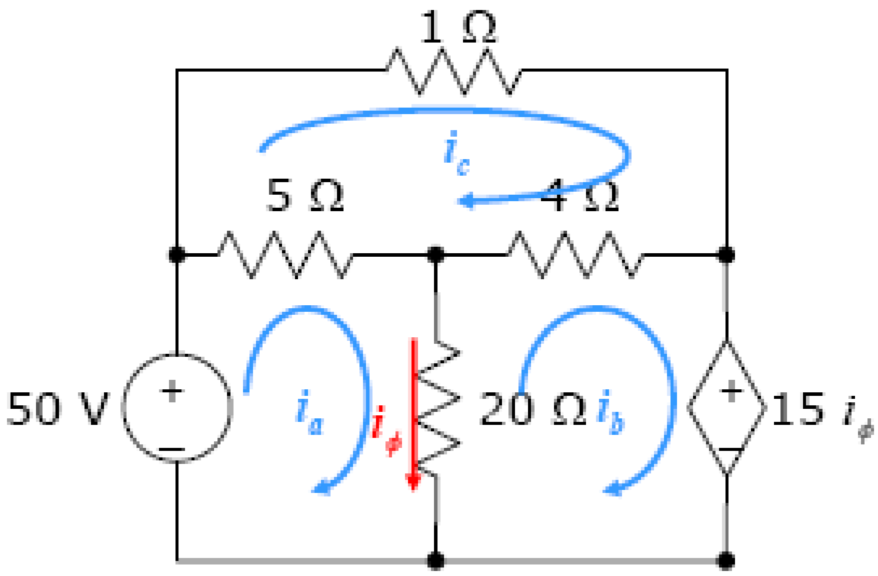
$$(i_c-i_a)5+i_c30+(i_c-i_b)90=0$$

$$\begin{bmatrix} 31 & -26 & -5 \\ -26 & 124 & -90 \\ -5 & -90 & 125 \end{bmatrix}\begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix}=\begin{bmatrix} 80 \\ 0 \\ 0 \end{bmatrix} \Rightarrow \begin{aligned} i_a &= 5.0\text{ A} \\ i_b &= 2.5\text{ A} \\ i_c &= 2.0\text{ A} \end{aligned} \Rightarrow \begin{aligned} P_{80V} &= 400\text{ W} \\ P_{8\Omega} &= 50\text{ W} \end{aligned}$$

Mesh-current method and dependent sources

Find i_ϕ?

Ans.:-



$$-50+(i_a-i_c)5+(i_a-i_b)20=0$$

$$(i_b-i_a)20+(i_b-i_c)4+15i_\phi=0$$

$$(i_c-i_a)5+i_c+(i_c-i_b)4=0$$

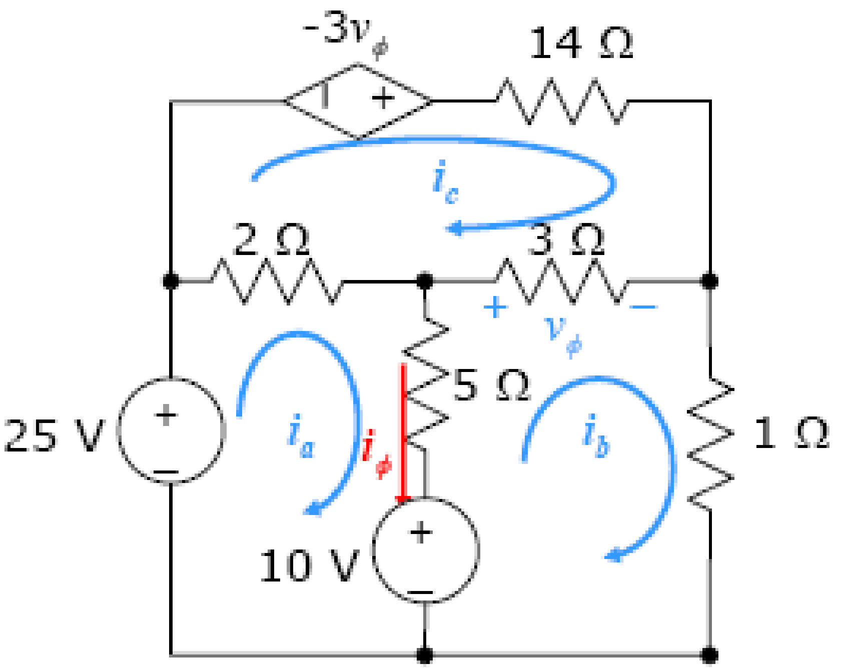
$$i_\phi=i_a-i_b$$

$$\begin{bmatrix}25 & -20 & -5\\ -5 & 9 & -4\\ -5 & -4 & 10\end{bmatrix}\begin{bmatrix}i_a\\ i_b\\ i_c\end{bmatrix}=\begin{bmatrix}50\\ 0\\ 0\end{bmatrix} \Rightarrow \begin{aligned} i_a &= 29.6\text{ A}\\ i_b &= 28.0\text{ A}\\ i_c &= 26.0\text{ A}\end{aligned} \Rightarrow i_\phi = 1.6\text{ A}$$

Assessing Objective 5

Find i_ϕ?

Ans.:-



$$-25+(i_a-i_c)2+(i_a-i_b)5+10=0$$

$$-10+(i_b-i_a)5+(i_b-i_c)3+i_b=0$$

$$(i_c-i_a)2+3v_\phi+i_c14+(i_c-i_b)3=0$$

$$v_\phi=(i_b-i_c)3$$

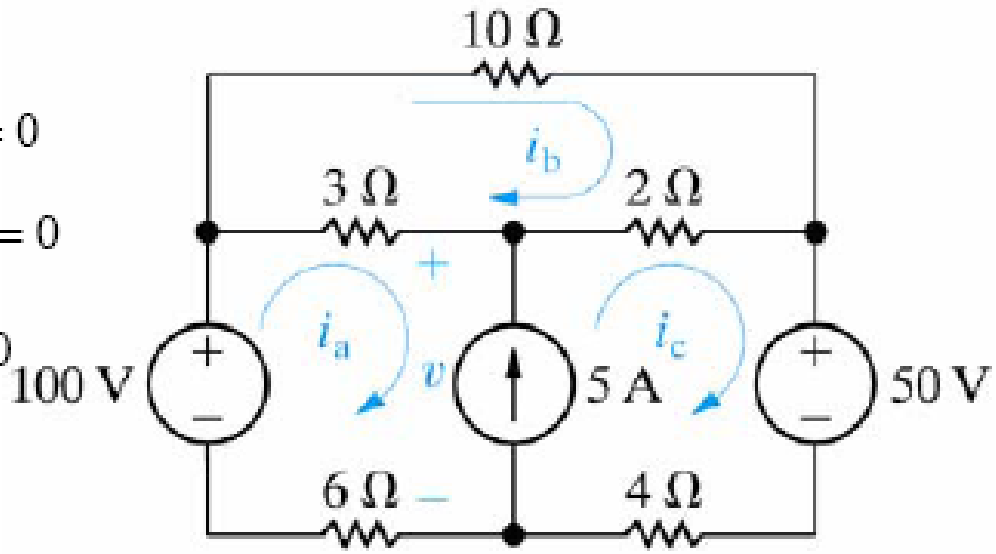
$$\begin{bmatrix}7 & -5 & -2\\ -5 & 9 & -3\\ -2 & 6 & 10\end{bmatrix}\begin{bmatrix}i_a\\ i_b\\ i_c\end{bmatrix}=\begin{bmatrix}15\\ 10\\ 0\end{bmatrix}$$

$$i_a=4\text{ A}$$
$$i_b=3\text{ A}$$
$$i_c=-1\text{ A}$$

$$i_\phi=1\text{ A}$$

Special Cases

- When an independent source is connected between two essential nodes.



a) $-100+(i_a-i_b)3+v+i_a6=0$

b) $(i_b-i_a)3+i_b10+(i_b-i_c)2=0$

c) $-v+(i_c-i_b)2+50+i_c4=0$

$i_c-i_a=5$

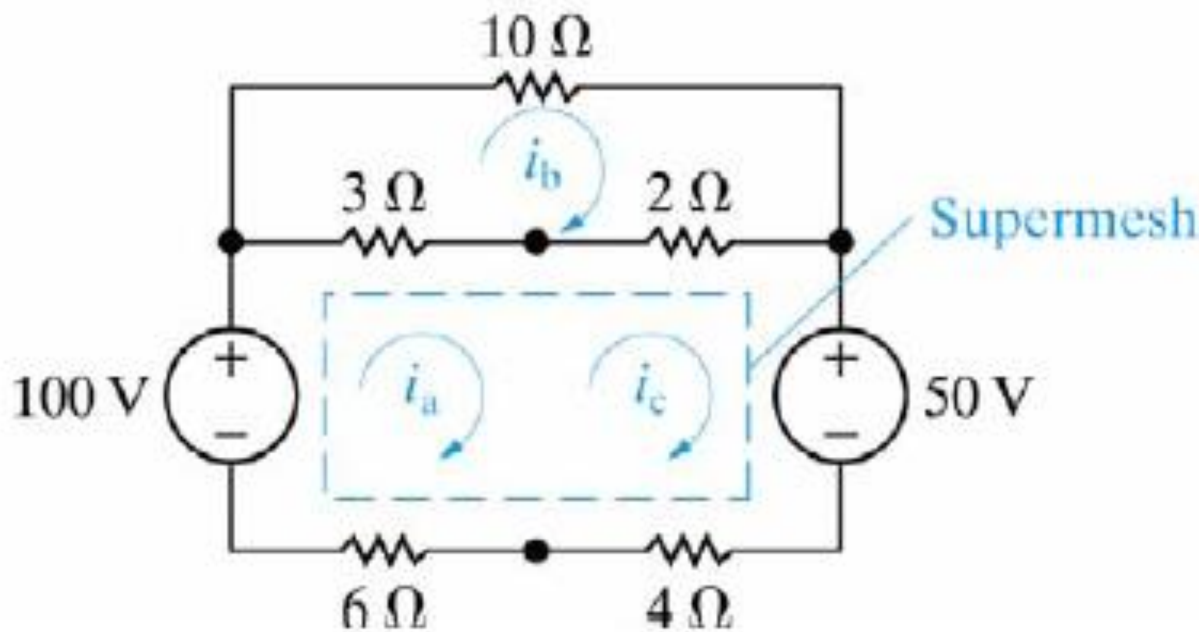
a) + c)

$$-100+(i_a-i_b)3+i_a6+(i_c-i_b)2+50+i_c4=0$$

$$\begin{aligned} 9i_a-5i_b+6i_c&=50\\ -3i_a+15i_b-2i_c&=0\\ -i_a+0i_b+i_c&=5 \end{aligned} \Rightarrow \begin{bmatrix} 9 & -5 & 6\\ -3 & 15 & -2\\ -1 & 0 & 1 \end{bmatrix}\begin{bmatrix} i_a\\ i_b\\ i_c \end{bmatrix}=\begin{bmatrix} 50\\ 0\\ 5 \end{bmatrix} \Rightarrow \begin{aligned} i_a&=1.75\text{ A}\\ i_b&=1.25\text{ A}\\ i_c&=6.75\text{ A} \end{aligned}$$

Special Cases

- Supermesh

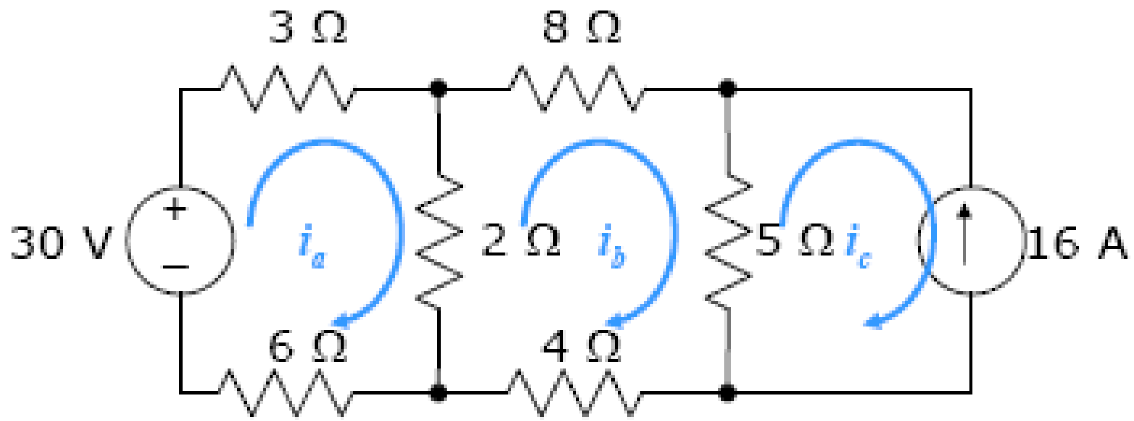


$$-100+(i_a-i_b)3+i_a6+(i_c-i_b)2+50+i_c4=0$$

Assessing Objective 6

- Find the power dissipated in the 2 Ω resistor.

Ans.:-



$$-30 + i_a 3 + (i_a - i_b)2 + i_a 6 = 0$$

$$(i_b - i_a)2 + i_b 8 + (i_b - i_c)5 + i_b 4 = 0$$

$$i_c = -16 \text{ A}$$

$$i_a = 2 \text{ A}$$

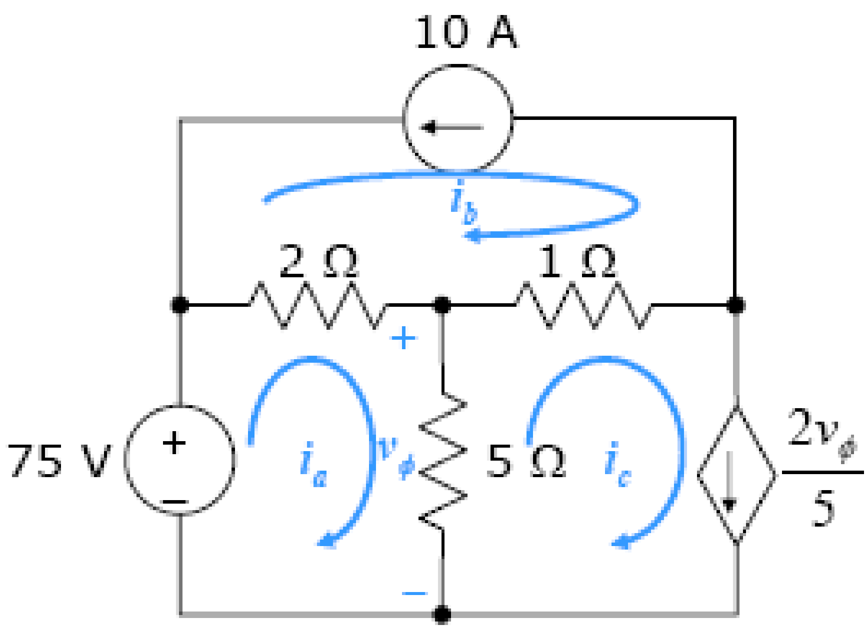
$$i_b = -4 \text{ A}$$

$$P_{2\Omega} = 6^2 2 = 72 \text{ W}$$

Assessing Objective 7

- Find the current i_a.

Ans.:-



$$-75+(i_a-i_b)2+(i_a-i_c)5=0$$

$$i_b=-10\text{ A}$$

$$v_\phi=(i_a-i_c)5$$

$$i_c=\frac{2v_\phi}{5}$$

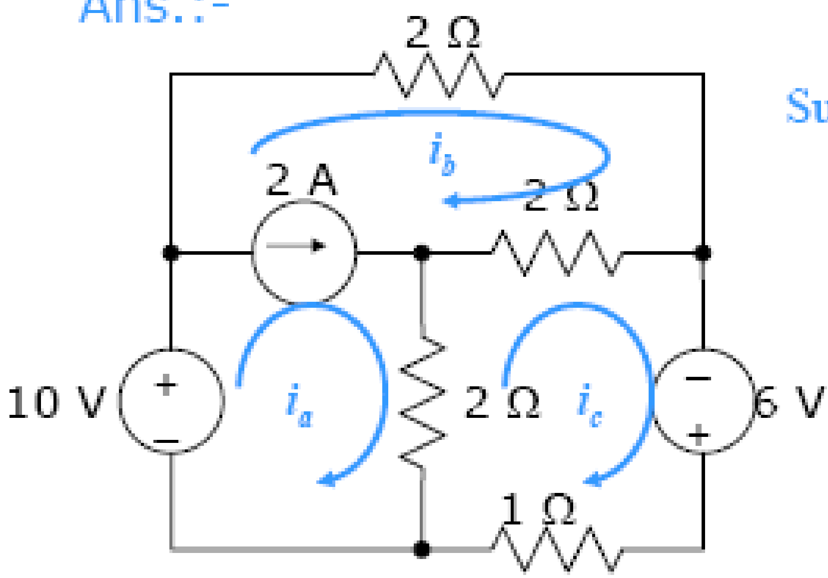
$$i_a=15\text{ A}$$

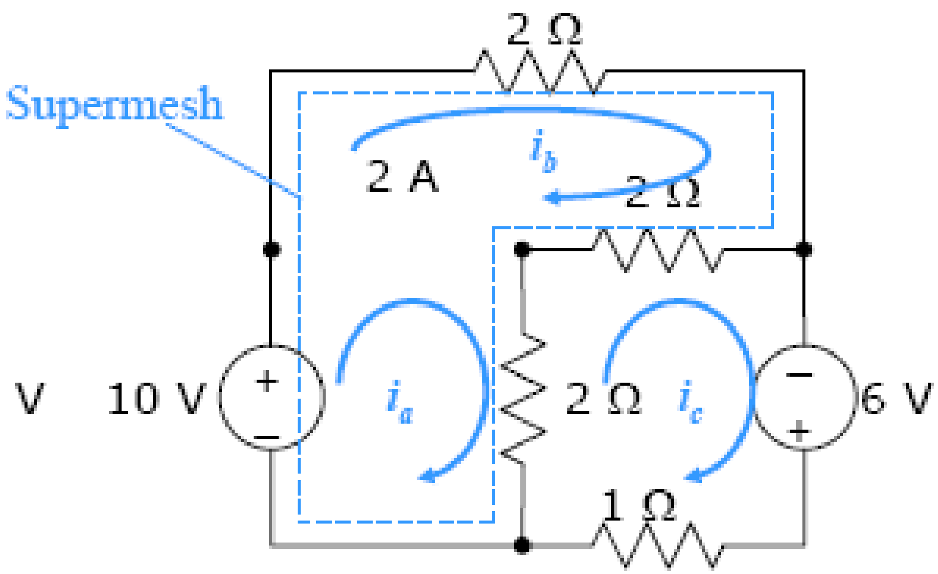
$$i_c=10\text{ A} \qquad v_\phi=25\text{ V}$$

Assessing Objective 8

- Find the power dissipated in the 1 Ω resistor.

Ans.:-





$$-10 + i_b 2 + (i_b - i_c)2 + (i_a - i_c)2 = 0$$

$$(i_c - i_a)2 + (i_c - i_b)2 - 6 + i_c = 0$$

$$i_a - i_b = 2$$

$$i_a = 7 \text{ A}$$

$$i_b = 5 \text{ A}$$

$$i_c = 6 \text{ A}$$

$$P_{2\Omega} = 6^2 1 = 36 \text{ W}$$

Node voltage method vs. Mesh current method

Which method is more efficient?

- Does one of the methods result in fewer simultaneous equations to solve?
- Does the circuit contain <u>supernodes</u>?

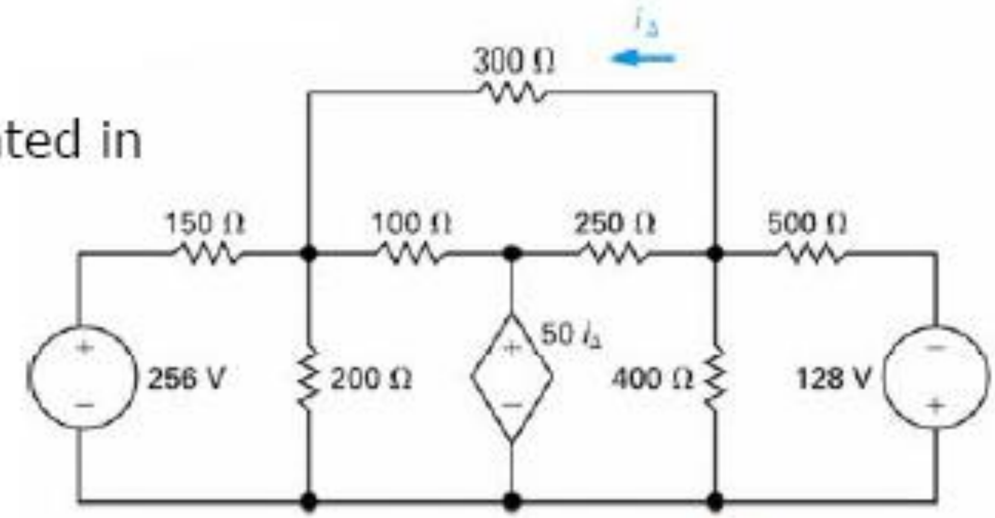
  If so, use the **node-voltage method**.
- Does the circuit contain <u>supermeshes</u>?

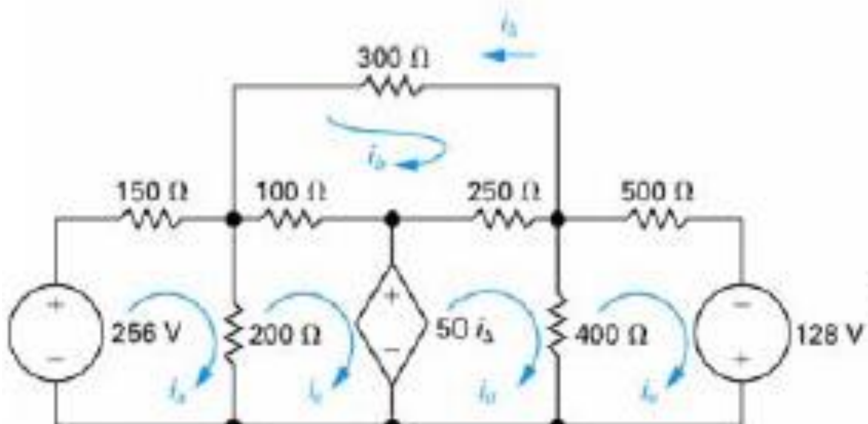
  If so, use the **mesh-current method**.

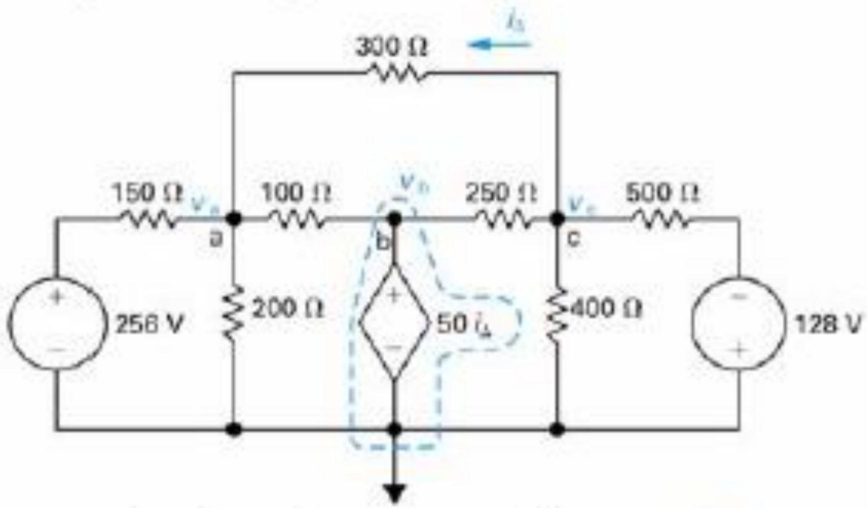
Example 3

Find the power dissipated in the 300 Ω resistor?

Ans.:-



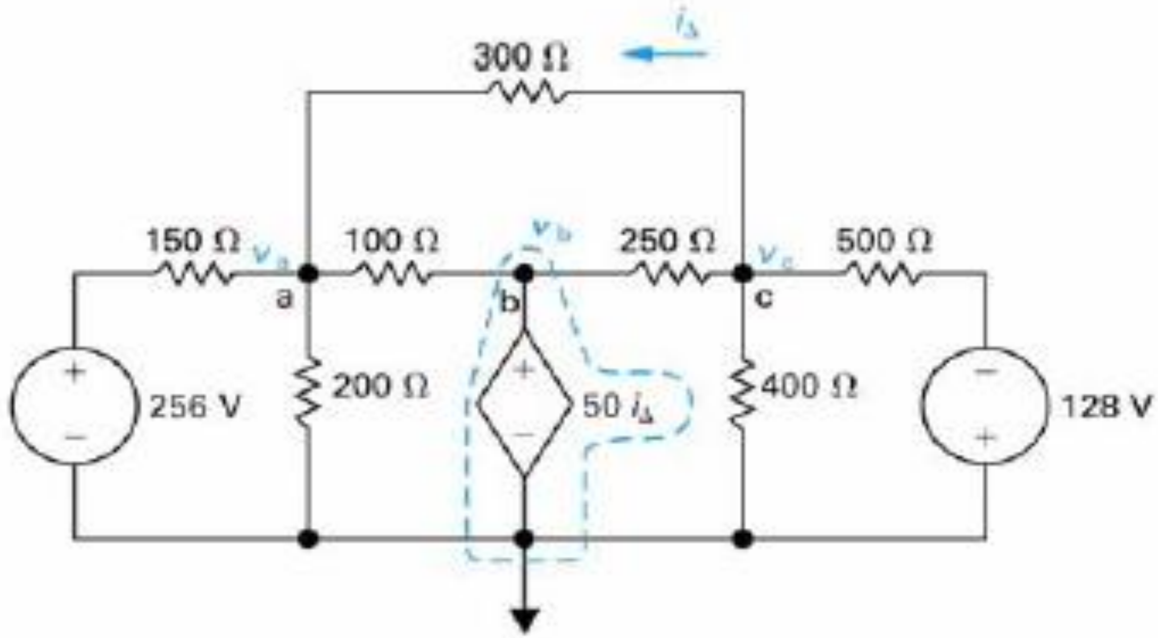




Mesh current equations = 5

Node voltage equations = 3

Example (Cont.)



$$\frac{v_a - 256}{150} + \frac{v_a}{200} + \frac{v_a - v_c}{300} + \frac{v_a - v_b}{100} = 0$$

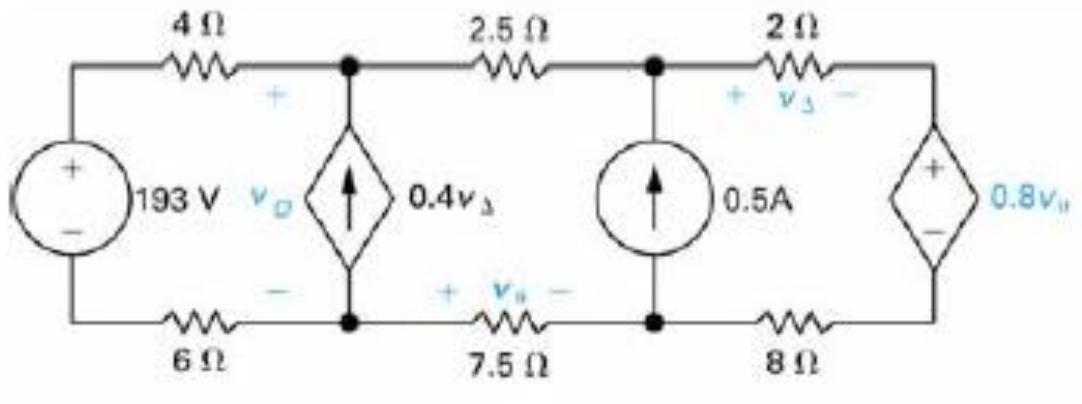
$$\frac{v_c - 128}{500} + \frac{v_c}{400} + \frac{v_c - v_b}{250} + \frac{v_c - v_a}{300} = 0$$

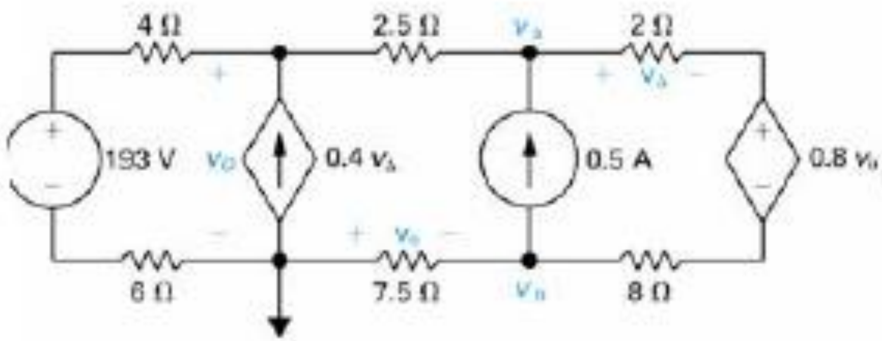
$$v_b = 50 i_\Delta$$

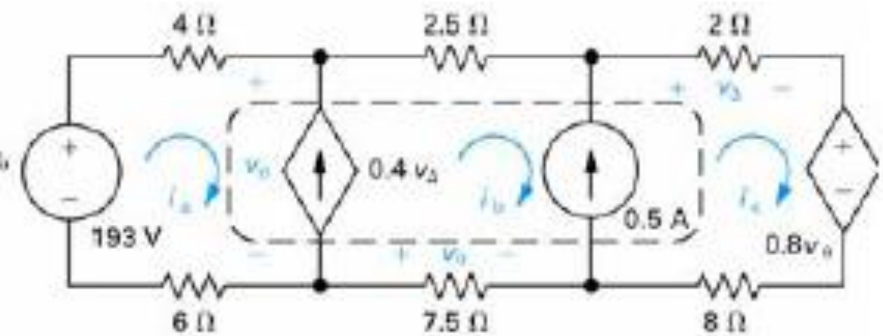
Example 4

Find v_o?

Ans.:-



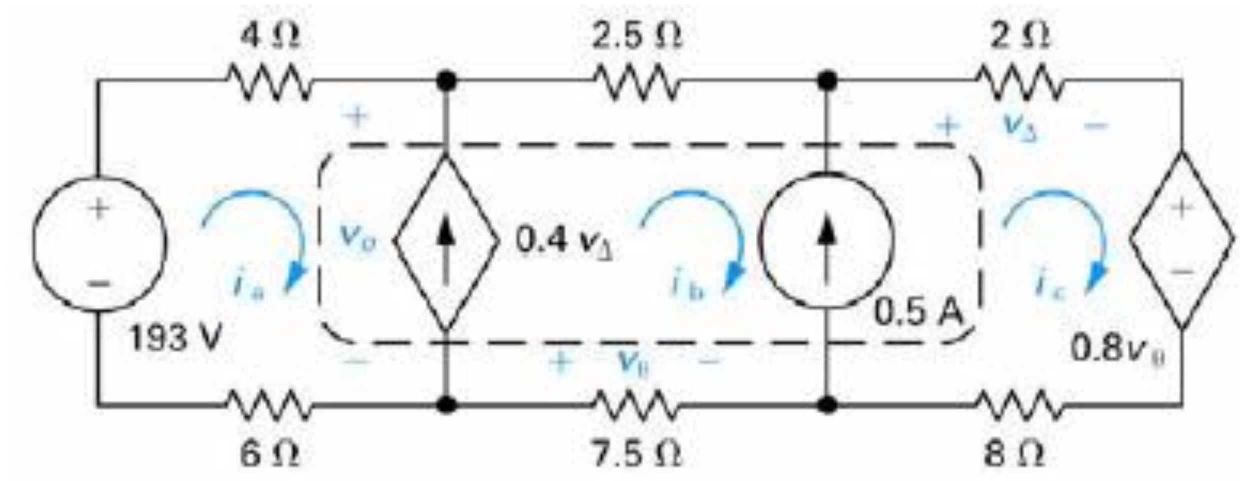




Node voltage equations = 3

Mesh current equations = 1

Example (Cont.)



$$-193 + i_a 4 + i_b 2.5 + i_c 2 + 0.8 v_\theta + i_c 8 + i_b 7.5 + i_a 6 = 0$$

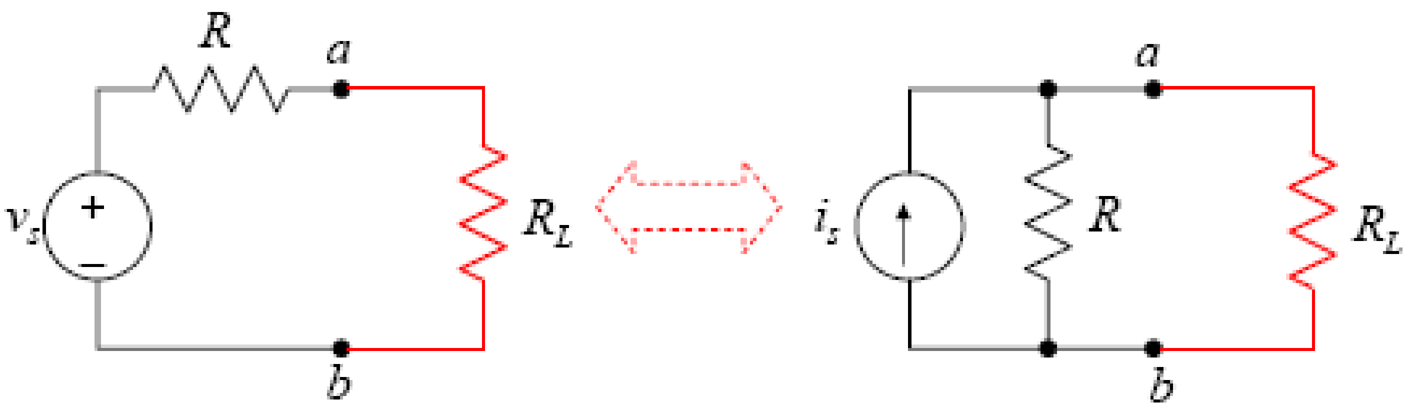
$$i_b - i_a = 0.4 v_\Delta \qquad v_\Delta = i_c 2$$

$$i_c - i_b = 0.5 \qquad v_\theta = -i_b 7.5$$

$$i_a = 2\ \mathrm{A}$$

Source Transformation

- A simplification technique that allows a voltage source in series with a resistor to be replaced by a current source in parallel with the same resistor.



$$i_L = \frac{v_s}{R+R_L}$$

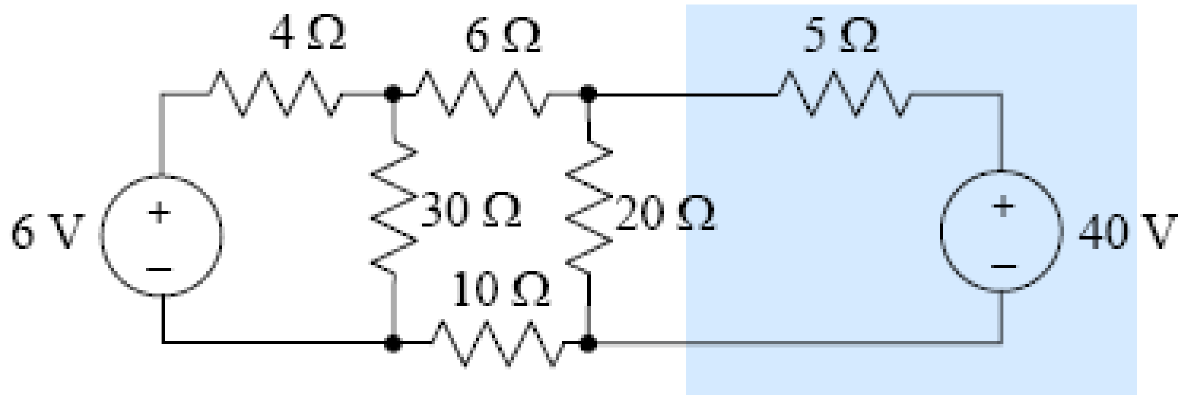
$$i_L = \frac{R}{R+R_L} i_s$$

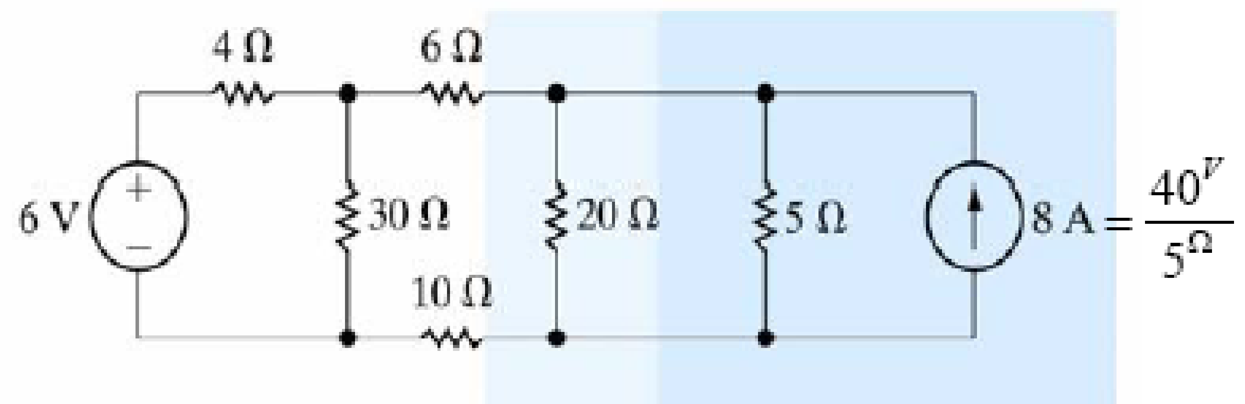
$$\boxed{i_s = \frac{v_s}{R}}$$

Example 5

Determine the power associated with the 6 V source.

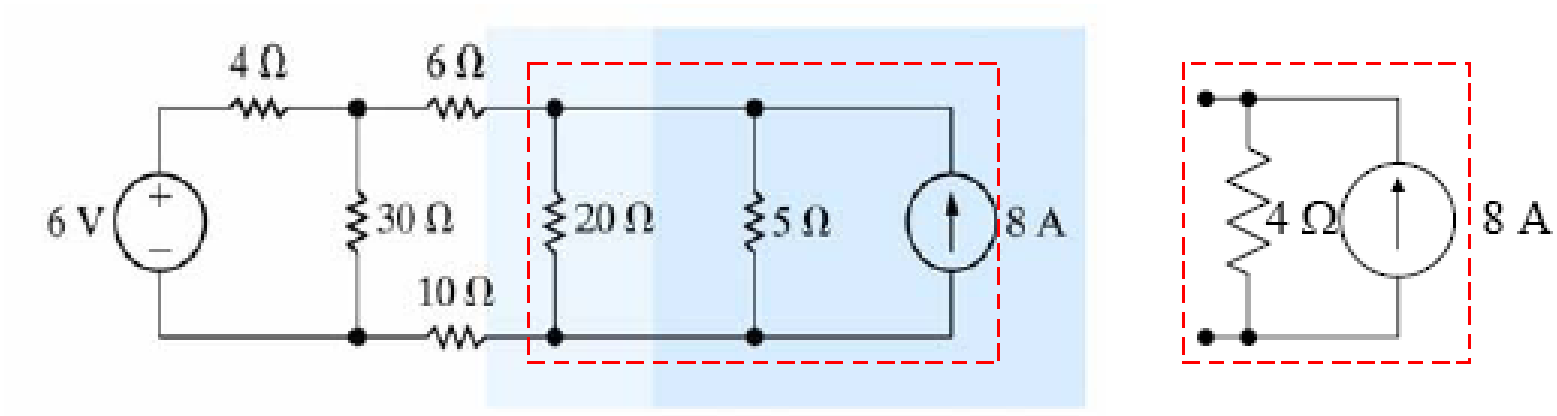
Ans.:-



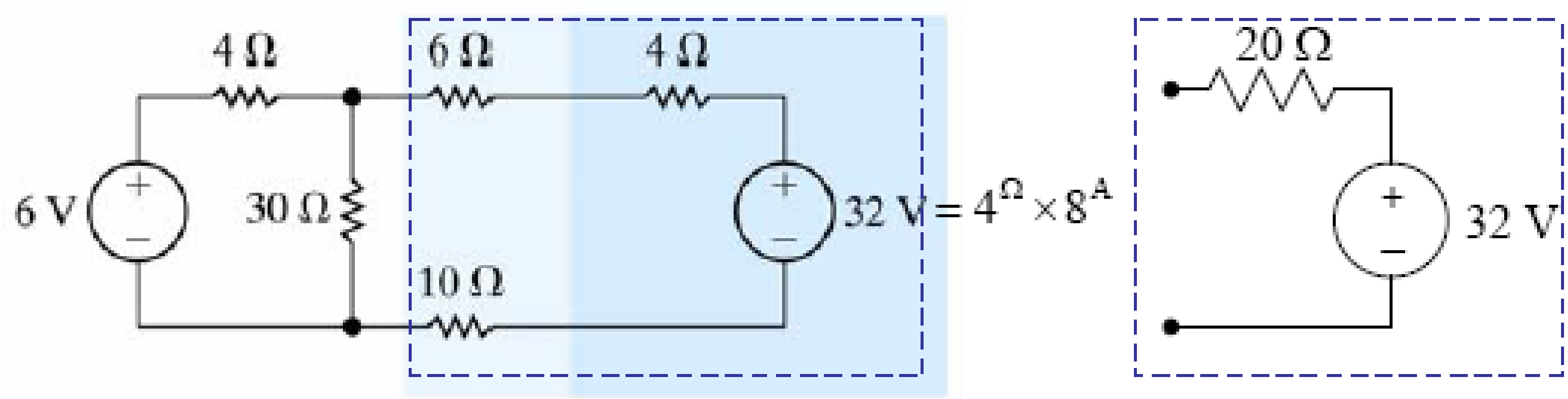


(a) First step

Example (Cont.)

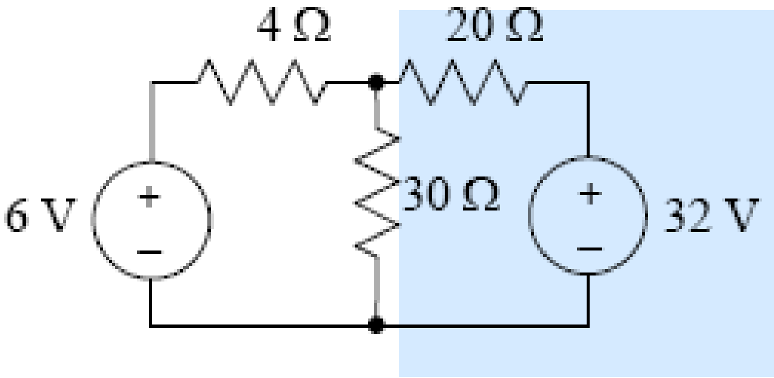


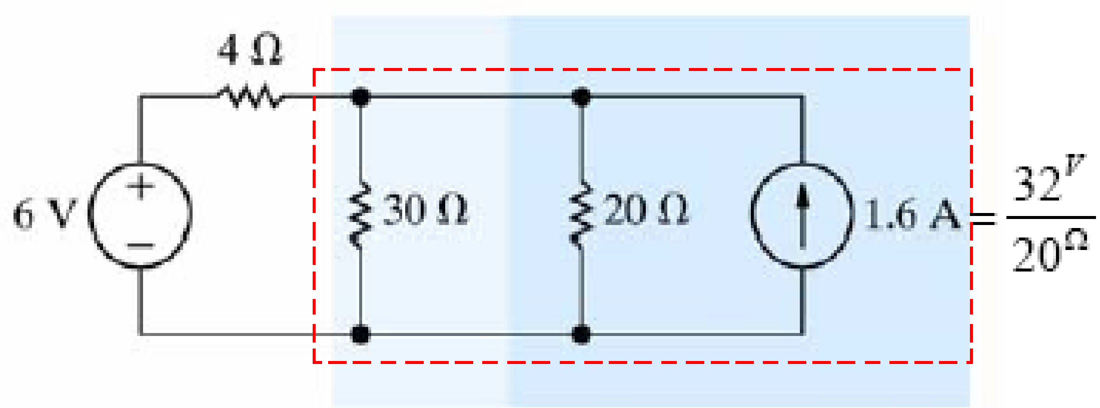
(a) First step



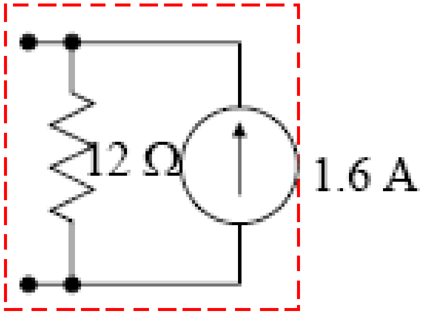
(b) Second step

Example(Cont.)

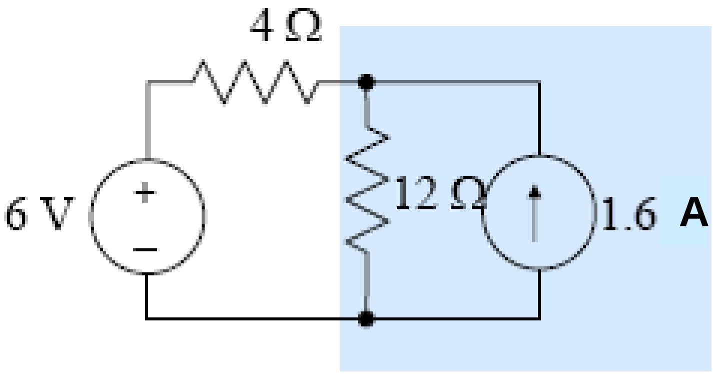


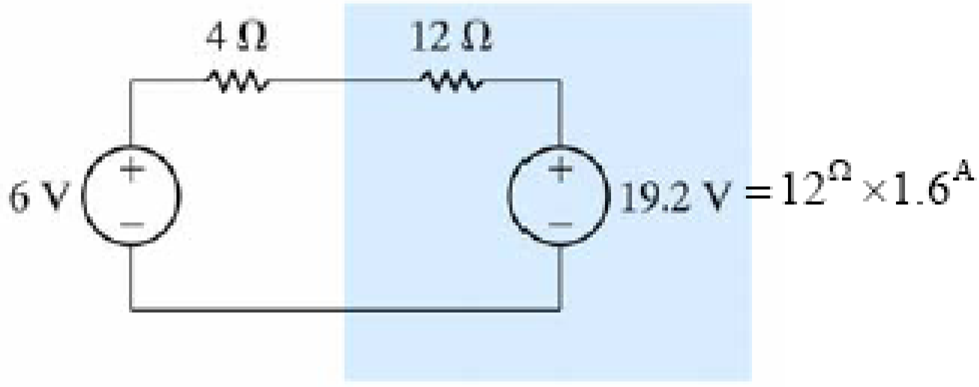


(c) Third step



Example(Cont.)



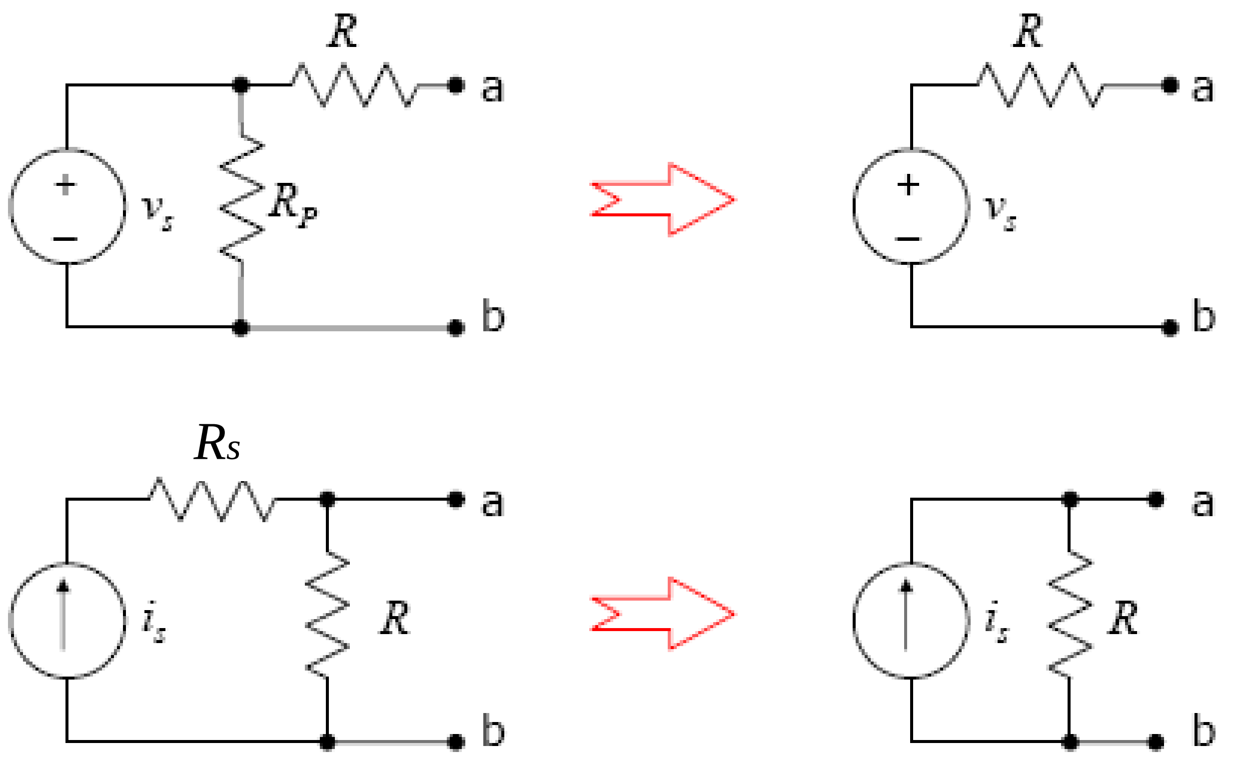


(d) Fourth step

$$i_{6V} = \frac{6-19.2}{16} = -0.825\ \mathrm{A}$$

Special Case

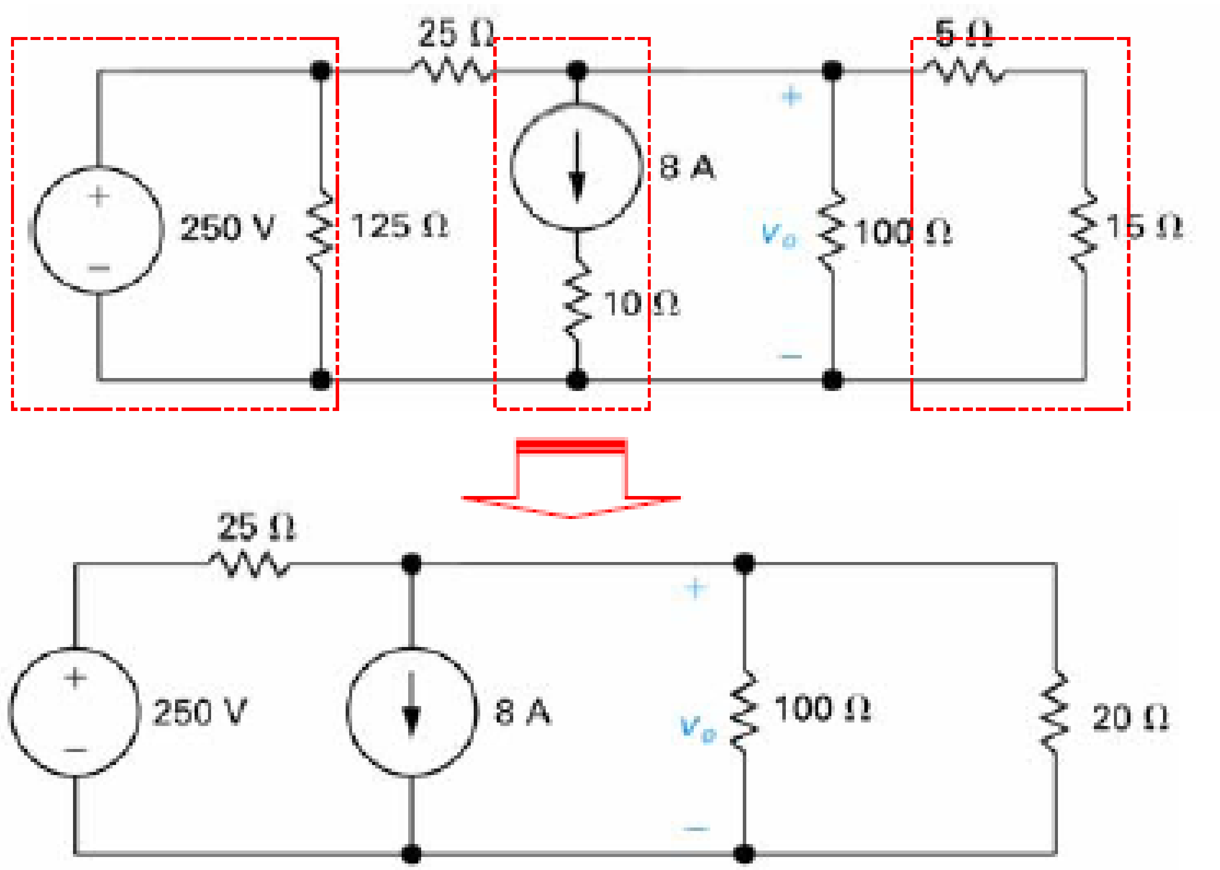
- What happens if there is a resistance R_P in parallel with the voltage source or a resistance R_s in series with the current source?



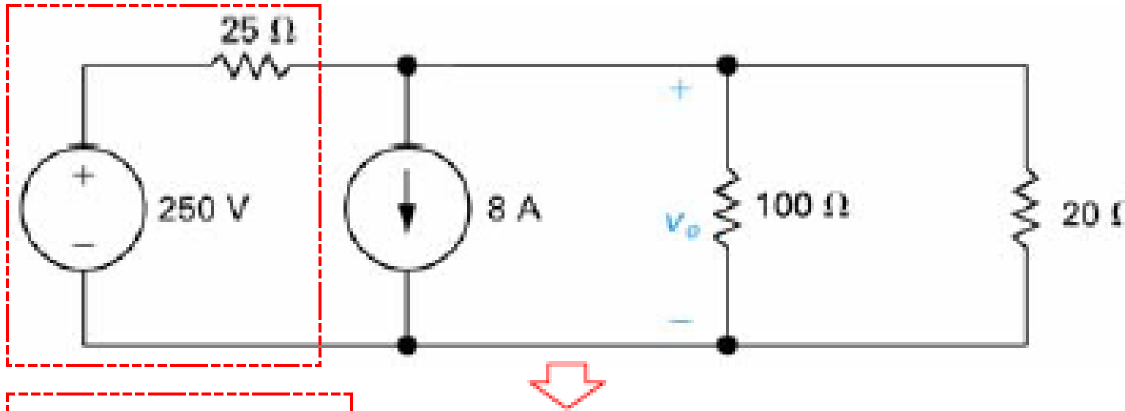
Example 6

Find v_o?

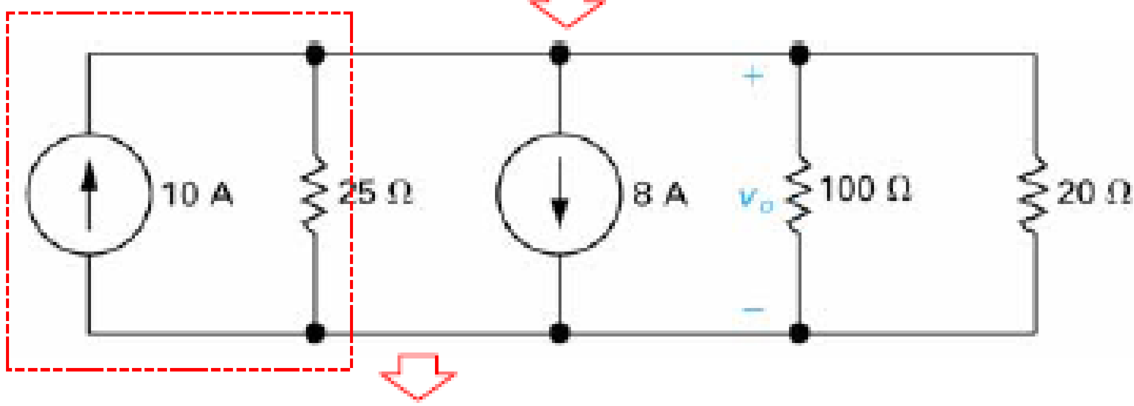
Ans.:-



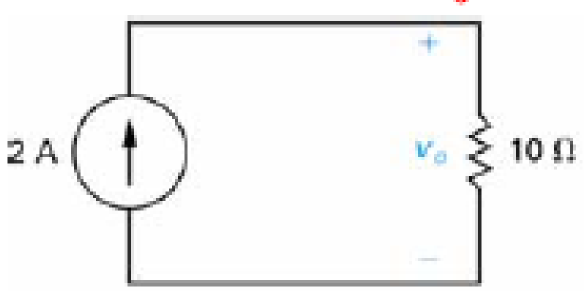
Example (Cont.)



$$10\,\mathrm{A} = \frac{250^{\mathrm{V}}}{25^{\Omega}}$$



$$10\,\mathrm{A} - 8\,\mathrm{A} = 2\,\mathrm{A}$$



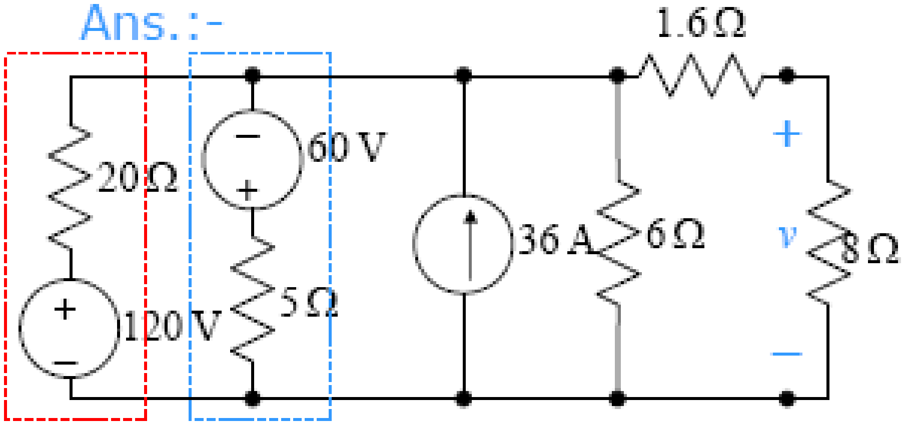
$$= \left(\frac{1}{\frac{1}{25} + \frac{1}{100} + \frac{1}{20}} \right)$$

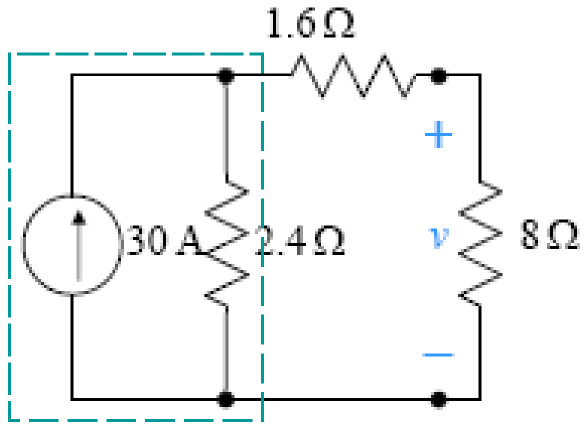
$$\boxed{v_o = 20\,\mathrm{V}}$$

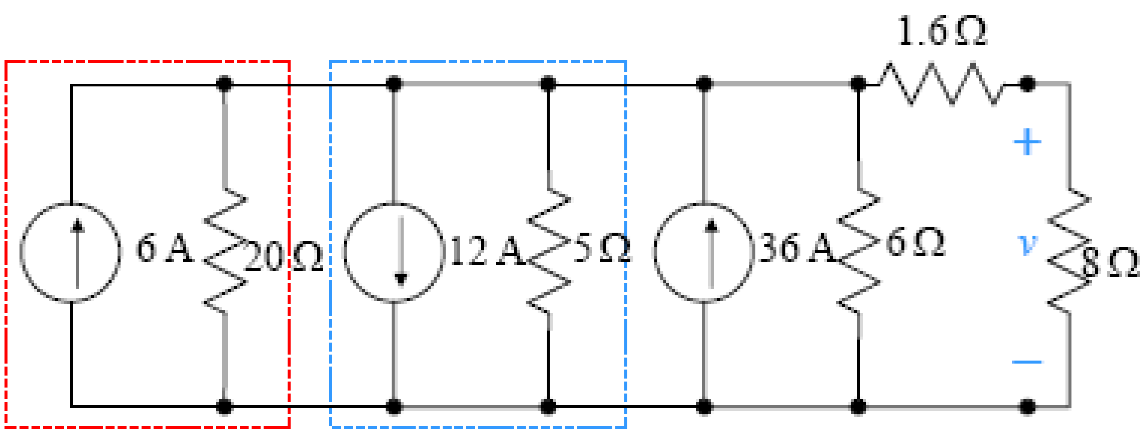
Assessing Objective 9

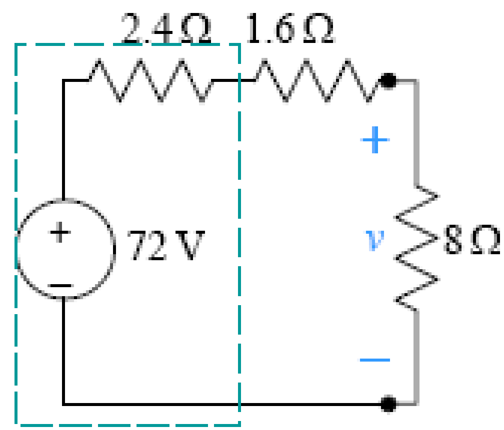
Find v?

Ans.:-







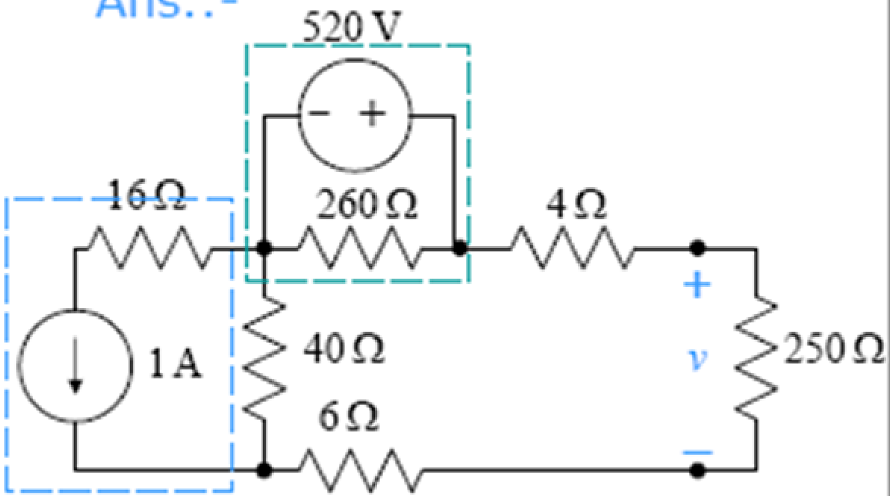


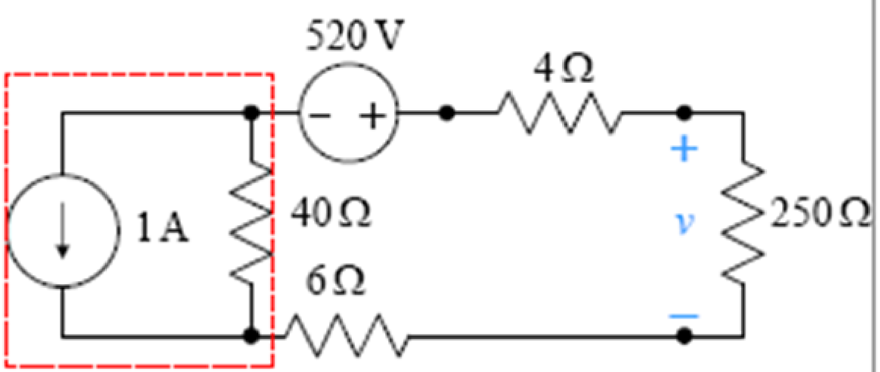
$$\boxed{v = 72\frac{8}{12} = 48\ \text{V}}$$

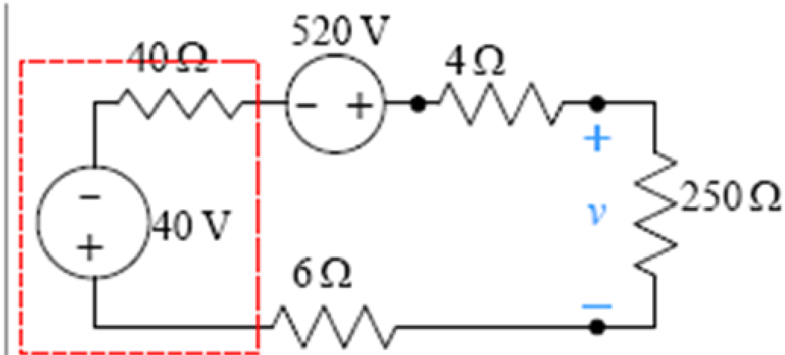
Problem 6

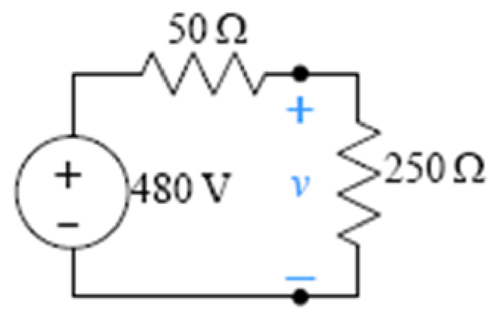
Find v?

Ans.:-









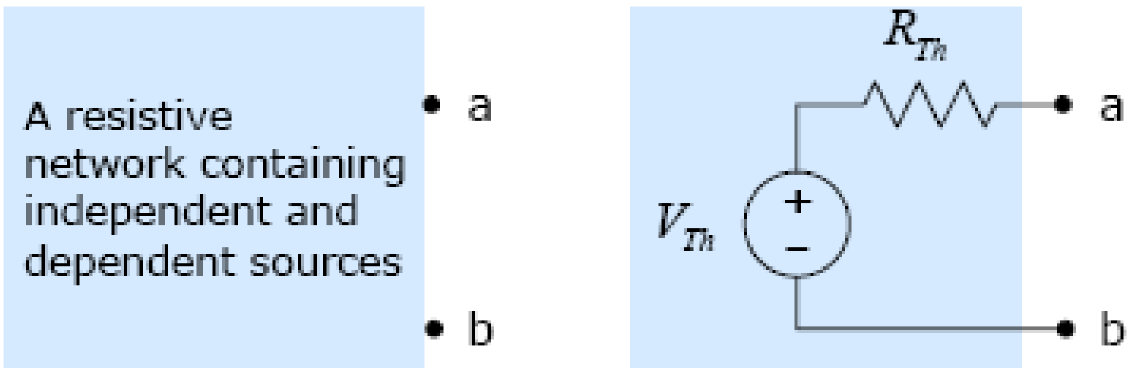
$$\boxed{v = 480\frac{250}{300} = 400\ \text{V}}$$

Thévenin and Norton Equivalents

- Used when you want to concentrate on what happens at a specific pair of terminals.

- They are circuit simplification techniques that focus on terminal behavior.

Thévenin equivalent circuit

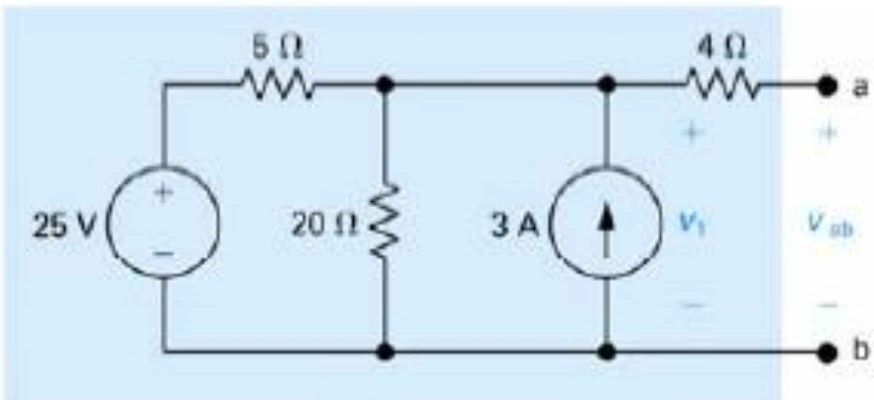


V_{Th} is the open-circuit voltage in the original circuit.
R_{Th} is the ratio of the open-circuit voltage to the short-circuit current.

$$R_{Th} = \frac{V_{Th}}{i_{sc}}$$

Thévenin equivalent circuit

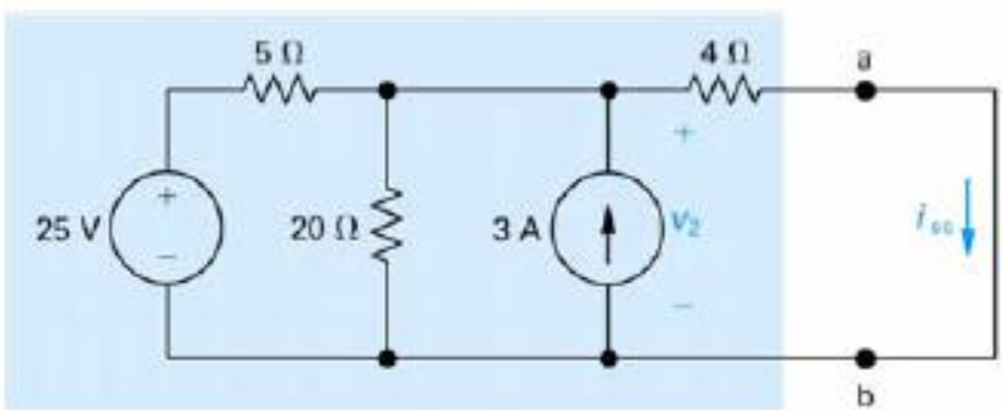
V_{Th} is the open-circuit voltage in the original circuit.



$$\frac{v_1 - 25}{5} + \frac{v_1}{20} - 3 = 0$$

$$v_1 = 32\ \text{V}$$

$$V_{Th} = 32\ \text{V}$$



$$\frac{v_2 - 25}{5} + \frac{v_2}{20} - 3 + \frac{v_2}{4} = 0$$

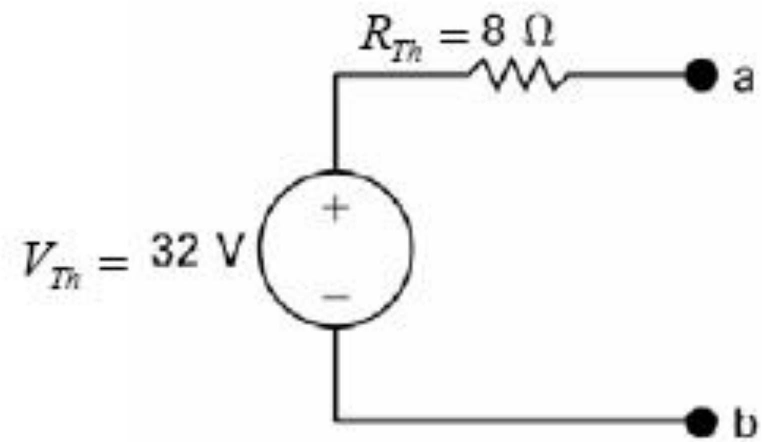
$$v_2 = 16\ \text{V}$$

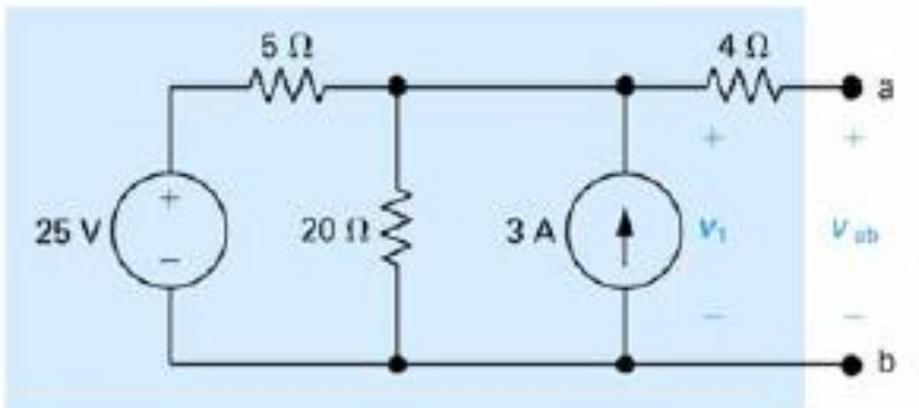
$$i_{sc} = \frac{16}{4} = 4\ \text{A}$$

$$R_{Th} = \frac{V_{Th}}{i_{sc}} = \frac{32}{4} = 8\ \Omega$$

Thévenin equivalent circuit

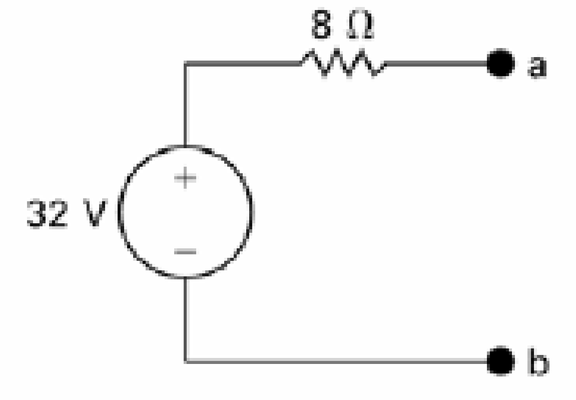
$$R_{Th} = \frac{V_{Th}}{i_{sc}} = \frac{32}{4} = 8\,\Omega$$



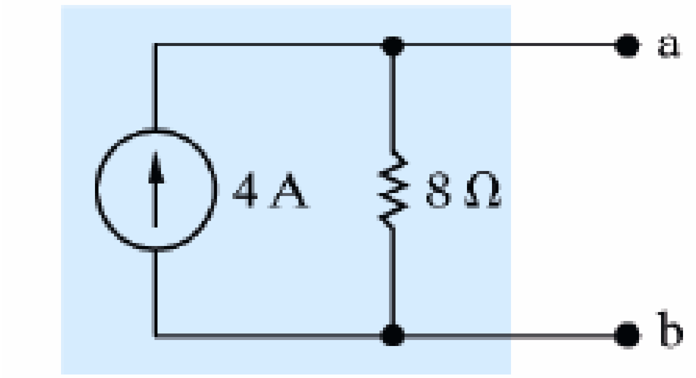


The Norton equivalent circuit

- Consists of an independent current source in parallel with the Norton equivalent resistance.
- Can be derived from Thévenin equivalent circuit simply by making a source transformation.

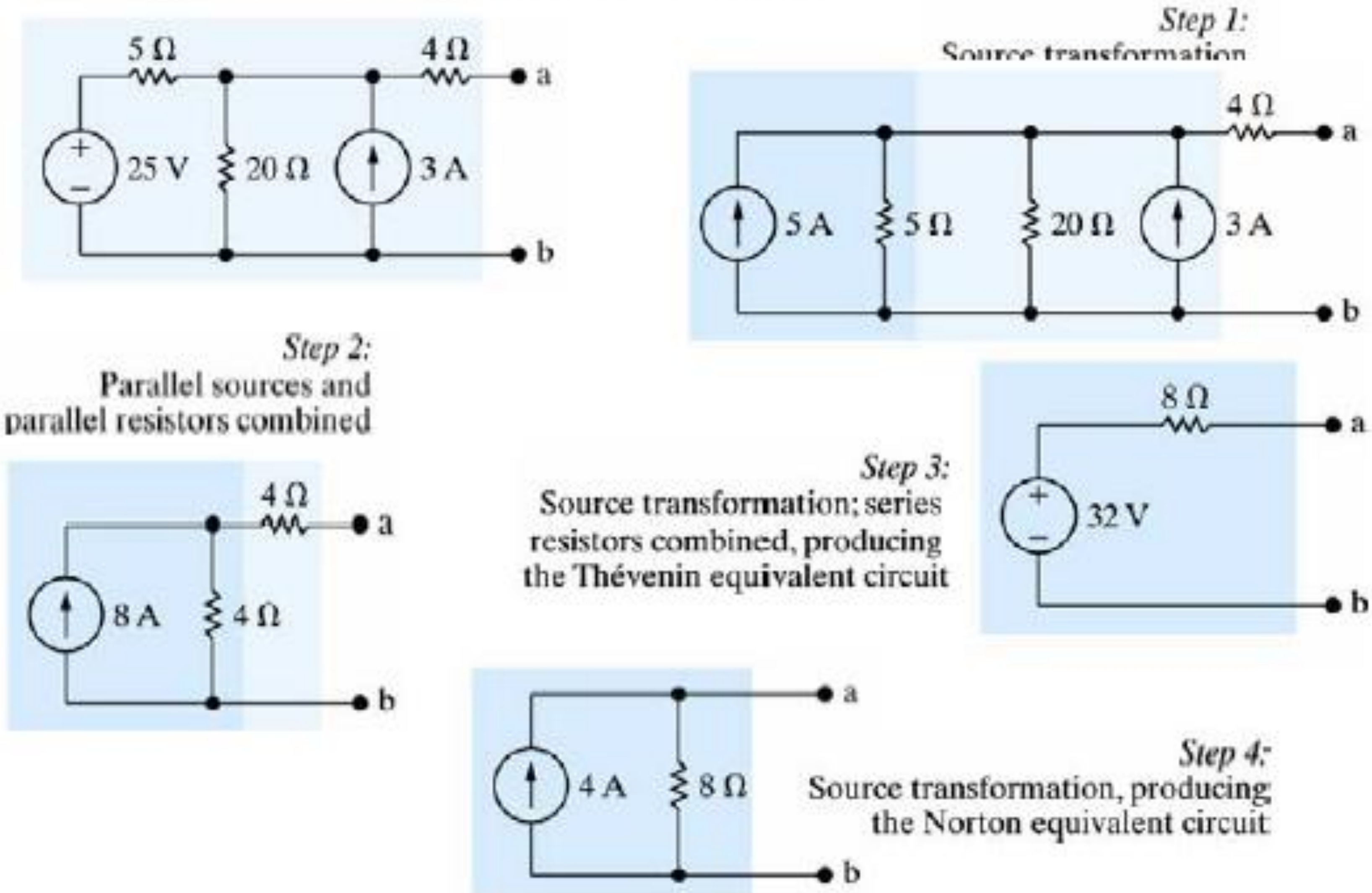


Thévenin Equivalent Circuit



Norton Equivalent Circuit

The Norton equivalent circuit



Example 7

Find V_{Th} & R_{Th}?

ans.:-

1^{st} open circuit to evaluate V_{Th}

$$i_x = 0$$

$$v = -(20i)(25) = -500i$$

$$-5 + i2000 + 3v = 0$$

$$i = \frac{5-3v}{2000}$$

$$v = -5\ \mathrm{V}$$

2^{nd} short circuit to evaluate R_{Th}

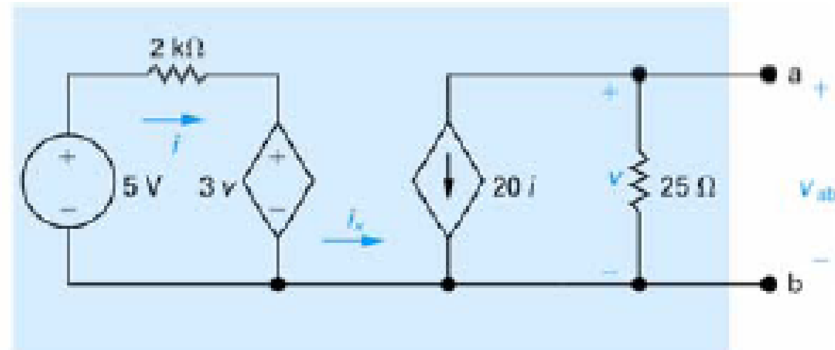
$$v = 0\ \mathrm{V}$$

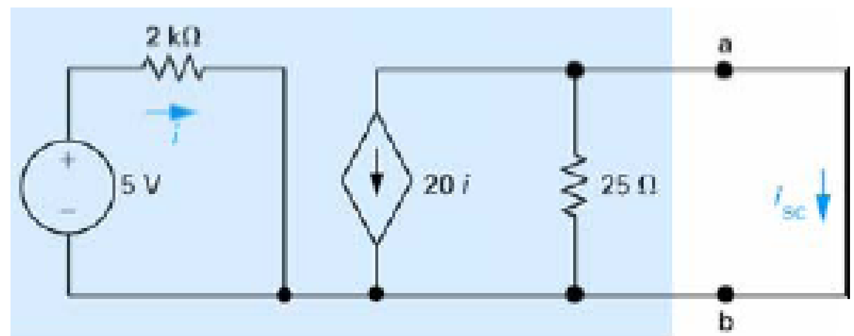
$$i_{sc} = -20i$$

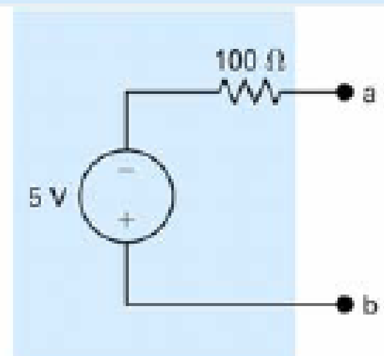
$$i = \frac{5}{2000} = 2.5\ \mathrm{mA}$$

$$i_{sc} = -50\ \mathrm{mA}$$

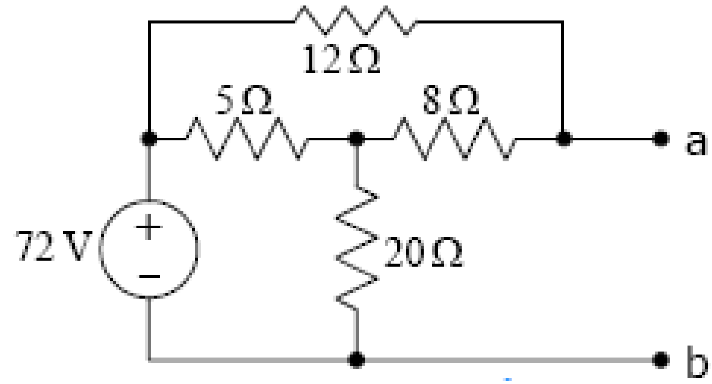
$$\boxed{R_{Th} = \frac{V_{Th}}{i_{sc}} = 100\ \Omega}$$







Assessing Objective 10



Find V_{Th} & R_{Th}?

Ans.:-

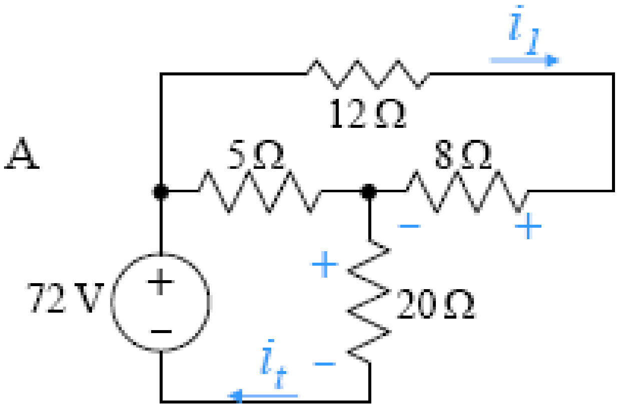
1st open circuit to evaluate V_{Th}

$$R_{eq} = (12\,\Omega + 8\,\Omega) // (5\,\Omega) + 20\,\Omega$$

$$R_{eq} = 4\,\Omega + 20\,\Omega = 24\,\Omega$$

$$i_t = 72/24 = 3\text{ A} \qquad i_1 = 3\frac{5}{12+8+5} = 0.6\text{ A}$$

$$V_{Th} = 0.6\times 8 + 3\times 20 = 64.8\text{ V}$$



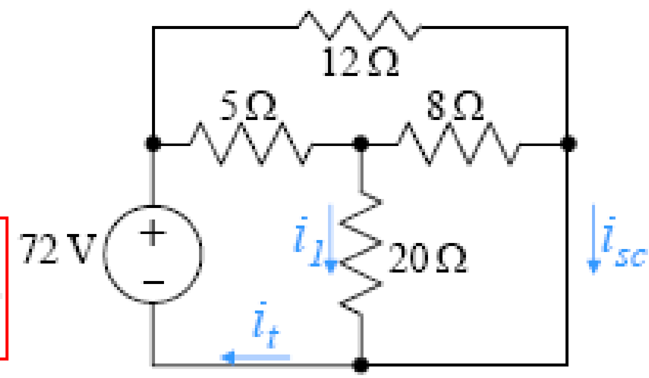
2nd short circuit to evaluate R_{Th}

$$R_{eq} = \left[(8\Omega // 20\Omega) + 5\Omega\right] // 12\Omega$$

$$R_{eq} = 5.66\Omega \qquad i_t = 72/5.66 = 12.72\text{ A}$$

$$i_1 = 12.72\left(\frac{12}{10\frac{5}{7}+12}\right)\times\frac{8}{20+8} = 1.92\text{A}$$

$$i_{sc} = 12.72 - 1.92 = 10.8\text{A} \qquad \boxed{R_{Th} = \frac{V_{Th}}{i_{sc}} = 6\,\Omega}$$



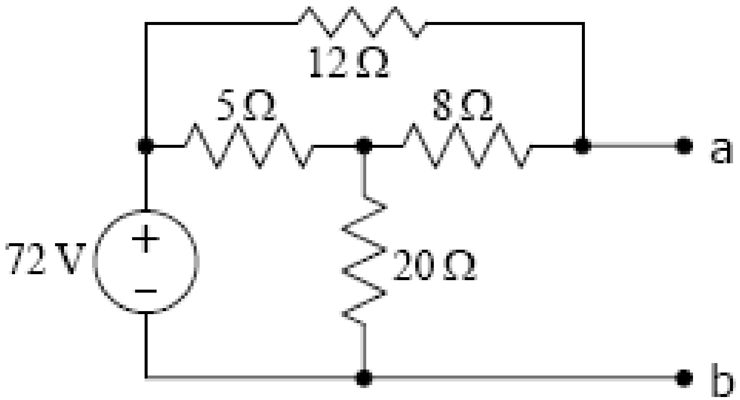
Evaluating R_{Th} using source deactivating

- Useful if the network contains only independent sources.

- 1^{st} deactivate all independent sources and then calculate the resistance seen looking into the network at the designated terminal pair.
  - A voltage source is deactivated by replacing it with a short circuit.
  - A current source is deactivated by replacing it with an open circuit.

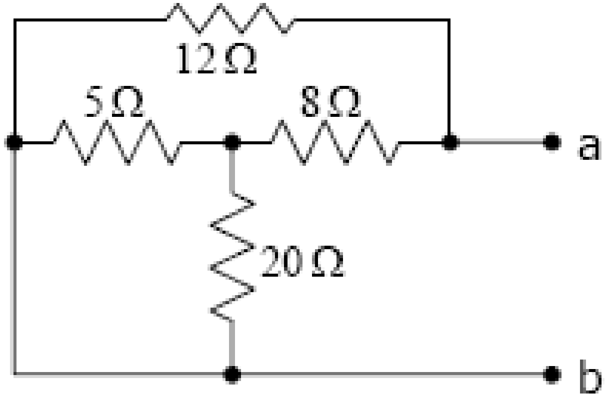
Example 8

Find V_{Th} & R_{Th}?



Ans.:-

$$V_{Th} = 64.8\ \text{V}$$



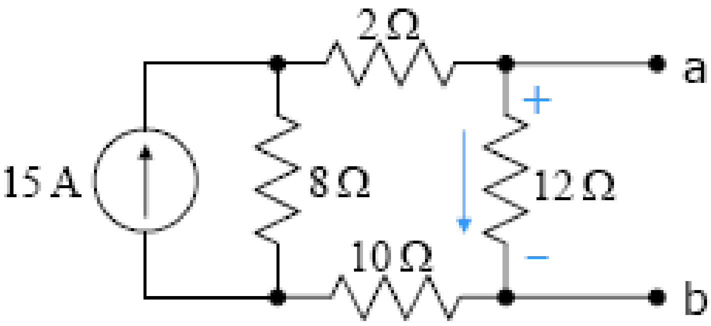
$$R_{Th} = \left[(5\Omega // 20\Omega) + 8\Omega\right] // 12\Omega$$

$$R_{Th} = 6\Omega$$

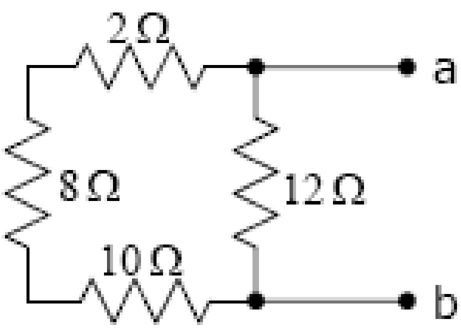
Assessing Objective 11

Find I_N & R_N?

Ans.:-

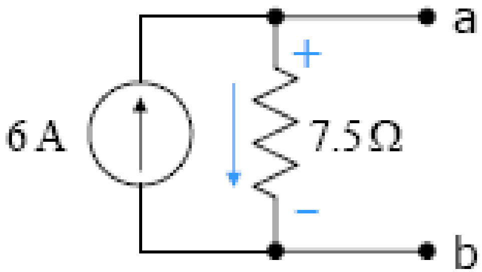


$$R_N = (2\Omega + 8\Omega + 10\Omega) // 12\Omega = 7.5\Omega$$



$$V_{Th} = 15 \frac{8}{2+10+12+8} \times 12 = 45V$$

$$I_N = \frac{45}{7.5} = 6A$$



Evaluating R_{Th} using test source

- First deactivate all independent sources, and we then apply either a test voltage source or a test current source to the Thévenin terminals a,b.

- The Thévenin resistance equals the ratio of the voltage across the test source to the current delivered by the test source.

Example 9

Find V_{Th} & R_{Th}?

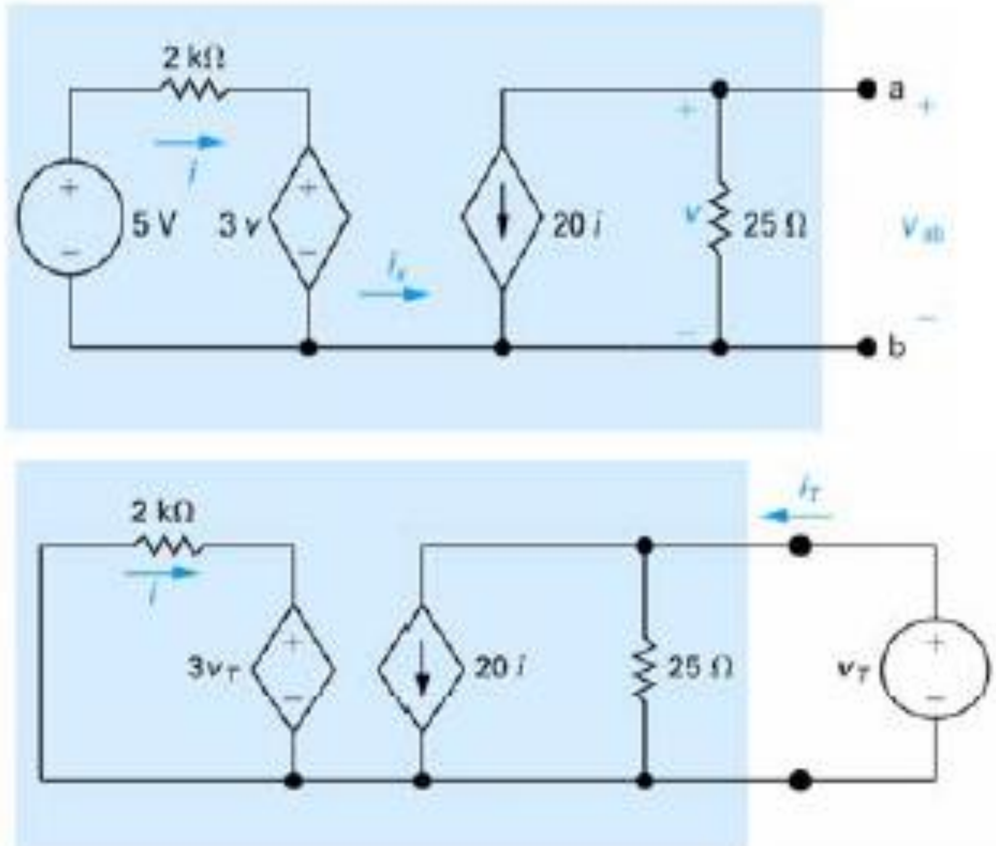
Ans.:-

$$3v_T = -i2000$$

$$i_T = 20i + \frac{v_T}{25}$$

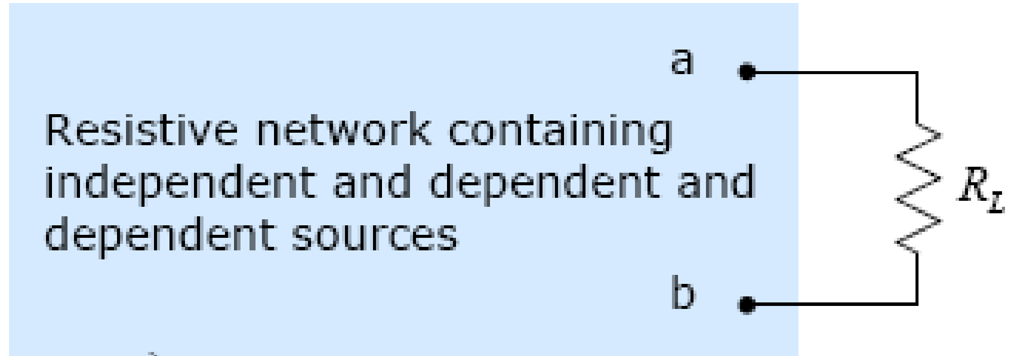
$$i_T = -20\frac{3v_T}{2000} + \frac{v_T}{25} = 0.01v_T$$

$$R_{Th} = \frac{v_T}{i_T} = 100\Omega$$



Maximum Power Transfer

$$p = i^2 R_L = \left(\frac{V_{Th}}{R_{Th} + R_L} \right)^2 R_L$$



$$\frac{dp}{dR_L} = V_{Th}^2 \left(\frac{(R_{Th} + R_L)^2 - R_L \cdot 2(R_{Th} + R_L)}{(R_{Th} + R_L)^4} \right)$$

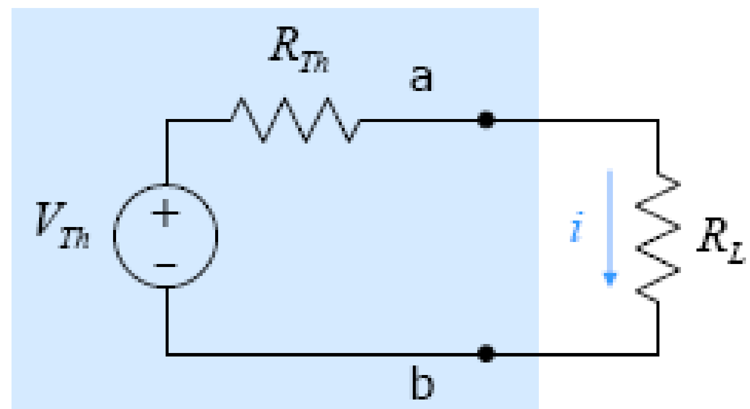
To maximize the function,
the derivative should be equal to zero

$$(R_{Th} + R_L)^2 = R_L \cdot 2(R_{Th} + R_L)$$

$$R_{Th} + R_L = 2R_L$$

$$\boxed{R_{Th} = R_L}$$

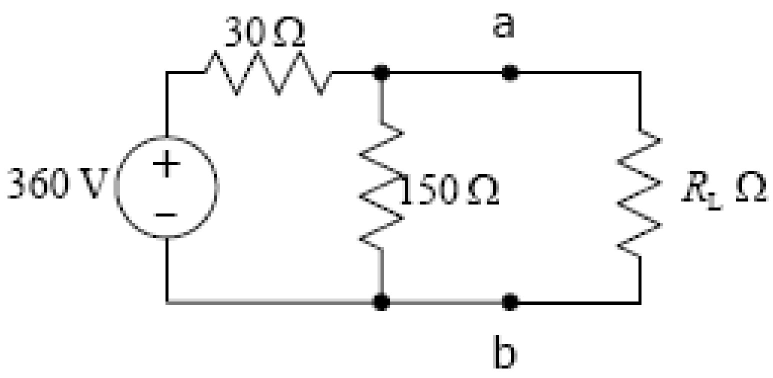
$$p_{max} = \frac{V_{Th}^2 R_L}{(2R_L)^2} = \frac{V_{Th}^2}{4R_L}$$



Example 10

a) Find R_L to achieve maximum power at R_L.
b) Calculate maximum power at R_L.
c) Find the % of power from the source is delivered to R_L.

Ans.:-



a) $V_{Th} = 360 \dfrac{150}{150+30} = 300 \text{ V}$

$R_{Th} = 150 // 30 = 25\ \Omega$

$R_L = R_{Th} = 25\Omega$

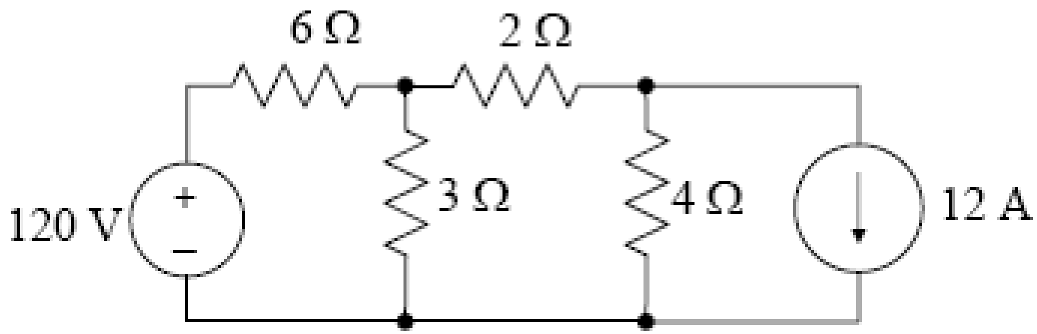
b) $p = \dfrac{V_{Th}^2}{4R_L} = 900 \text{ W}$

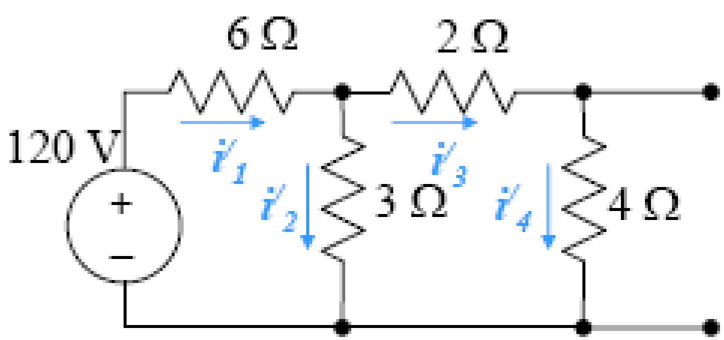
c) $p_s = \dfrac{V_s^2}{R_{eq}} = \dfrac{360^2}{51.43} = 2520 \text{ W}$

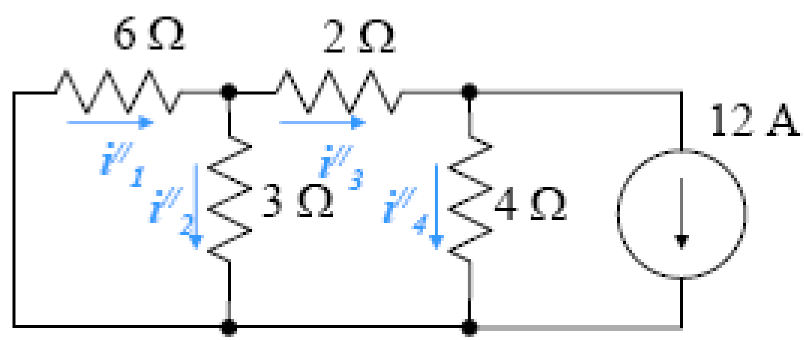
$\% p = \dfrac{900}{2520} \times 100 = 35.71\%$

Superposition

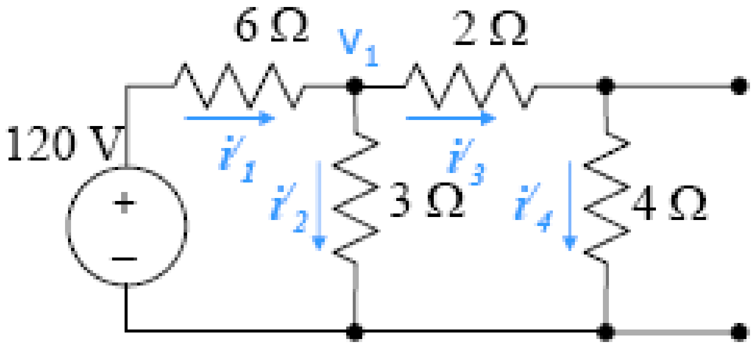
- A linear system obeys the principle of superposition, which states that whenever a linear system is excited, or driven, by more than one independent source of energy, the total response is the sum of the individual responses.







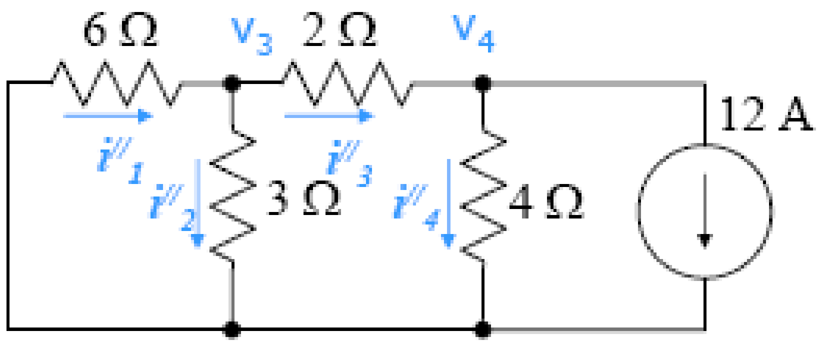
Superposition



$$\frac{v_1-120}{6}+\frac{v_1}{3}+\frac{v_1}{2+4}=0 \quad v_1=30$$

$$\boxed{i_1'=\frac{120-30}{6}=15\text{A}} \quad \boxed{i_2'=\frac{30}{3}=10\text{A}}$$

$$\boxed{i_3'=i_4'=\frac{30}{6}=5\text{A}}$$



$$\frac{v_3}{6}+\frac{v_3}{3}+\frac{v_3-v_4}{2}=0$$

$$\frac{v_4-v_3}{2}+\frac{v_4}{4}+12=0$$

$$v_3=-12$$

$$v_4=-24$$

$$\boxed{i_1''=\frac{12}{6}=2\text{A}} \quad \boxed{i_2''=\frac{-12}{3}=-4\text{A}} \quad \boxed{i_3''=\frac{-12+24}{2}=6\text{A}} \quad \boxed{i_4''=\frac{-24}{4}=-6\text{A}}$$

$$\boxed{i_1=i_1'+i_1''=15+2=17\text{A}}$$

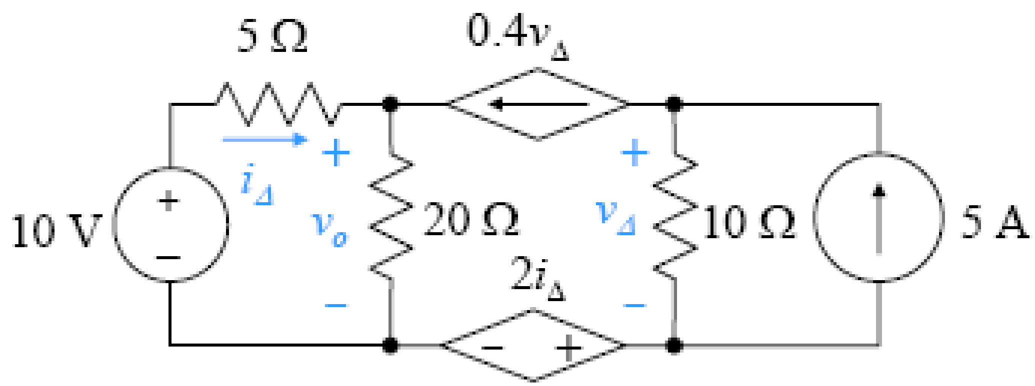
$$\boxed{i_3=i_3'+i_3''=5+6=11\text{A}}$$

$$\boxed{i_2=i_2'+i_2''=10-4=6\text{A}}$$

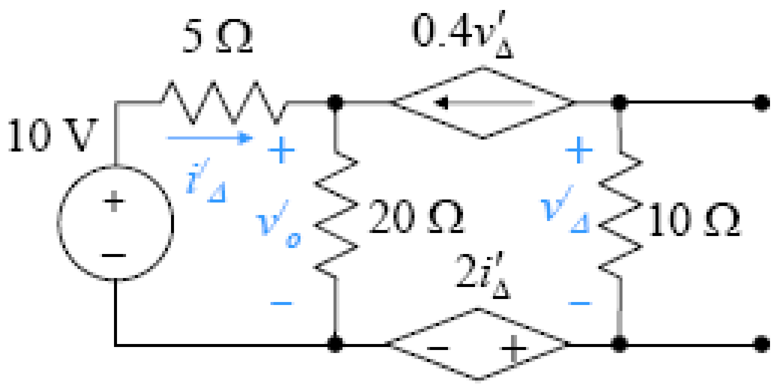
$$\boxed{i_4=i_4'+i_4''=5-6=-1\text{A}}$$

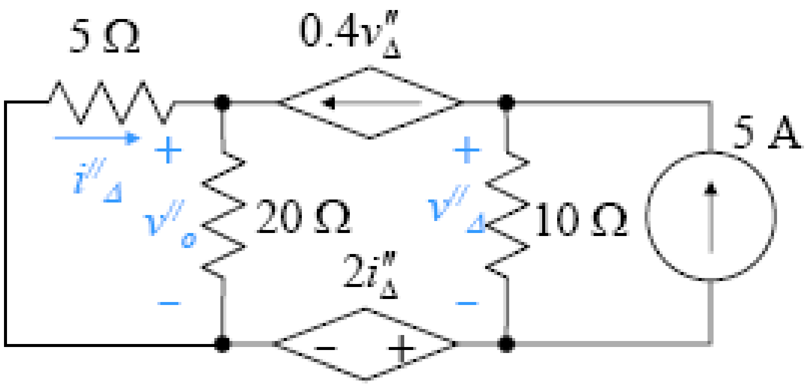
Example 11

Apply superposition to find v_o



Ans.:-

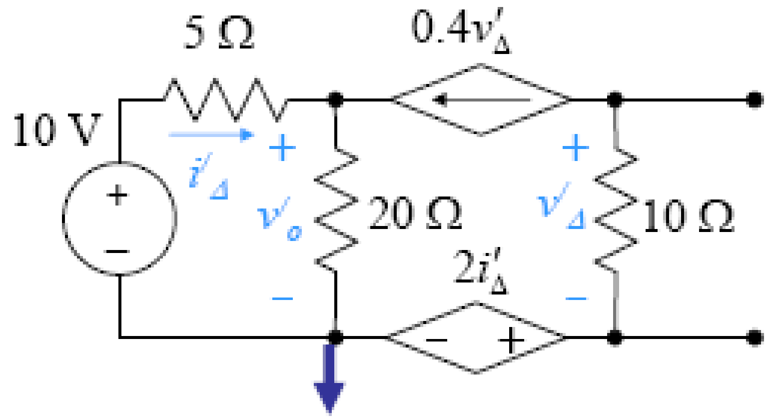




Example (Cont.)

$$v'_\Delta = -(0.4v'_\Delta)10 \longrightarrow v'_\Delta = 0$$

$$v'_o = \frac{10}{5+20}20 = 8\text{ V}$$



$$\frac{v''_o}{5} + \frac{v''_o}{20} - 0.4v''_\Delta = 0$$

$$0.4v''_\Delta + \frac{v_b - 2i''_\Delta}{10} - 5 = 0$$

$$v_b = v''_\Delta + 2i''$$

$$v''_\Delta = 10\text{ V}$$

$$v''_o = 16\text{ V}$$

$$v_o = v'_o + v''_o = 24\text{ V}$$