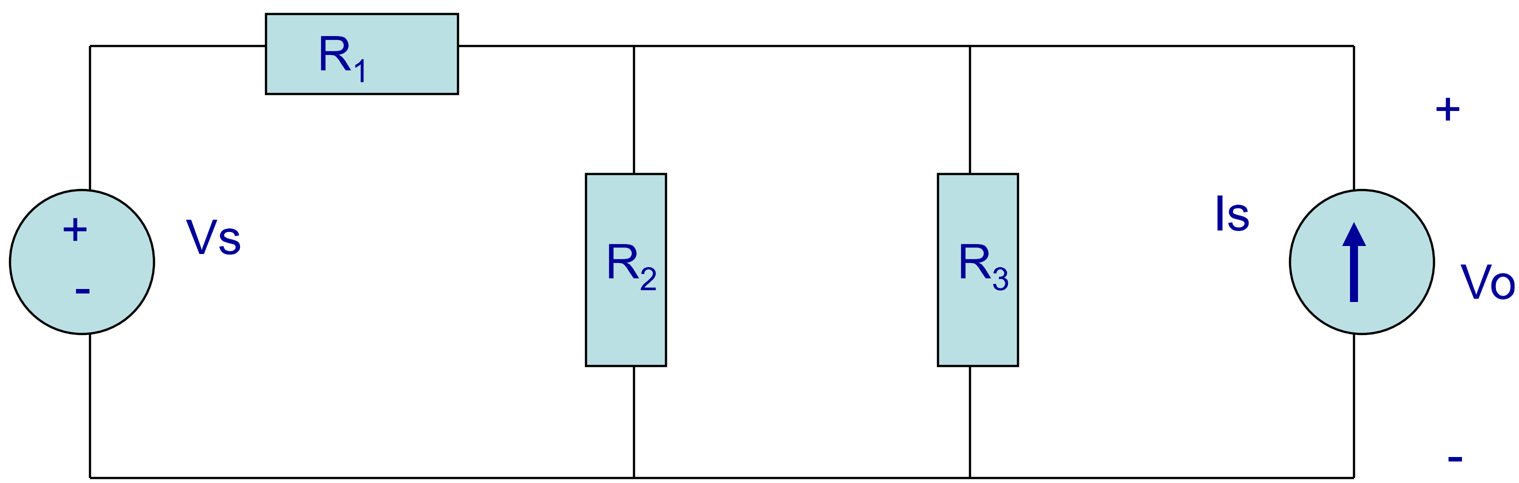
Kirchoff’s Laws

Circuit Definitions

- Node – any point where 2 or more circuit elements are connected together
  - Wires usually have negligible resistance
  - Each node has one voltage (w.r.t. ground)
- Branch – a circuit element between two nodes
- Loop – a collection of branches that form a closed path returning to the same node without going through any other nodes or branches twice

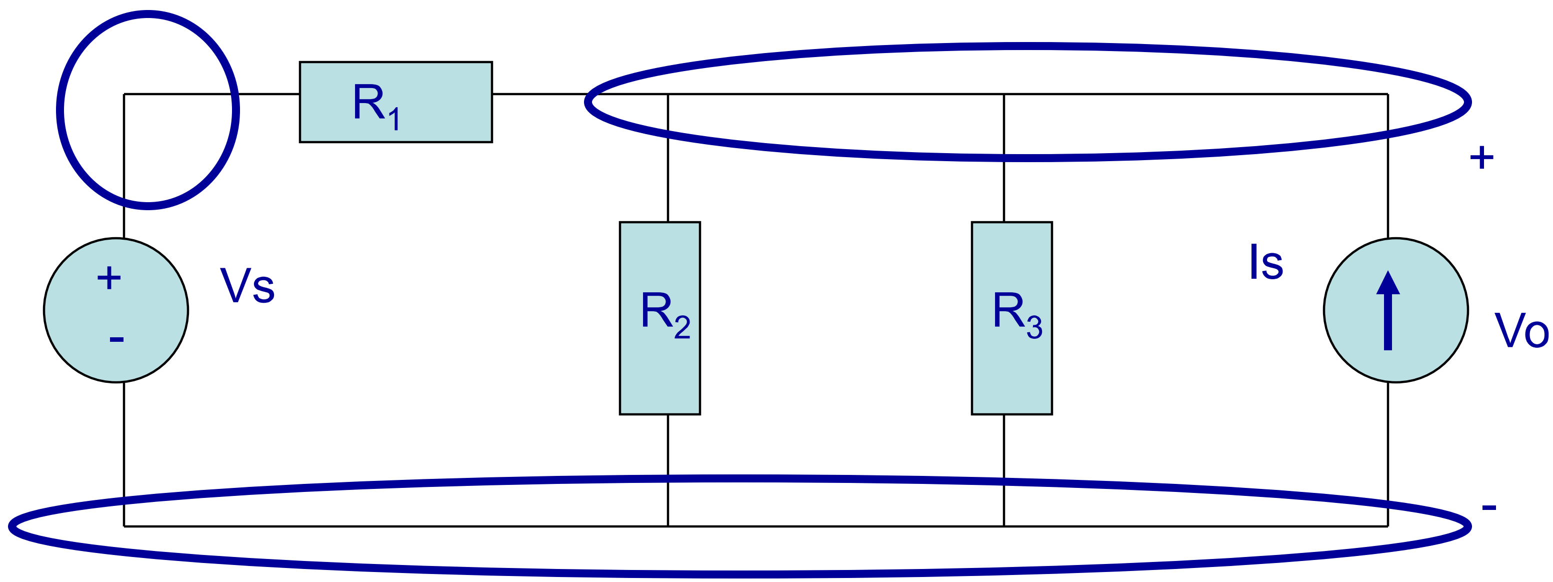
Example

- How many nodes, branches & loops?



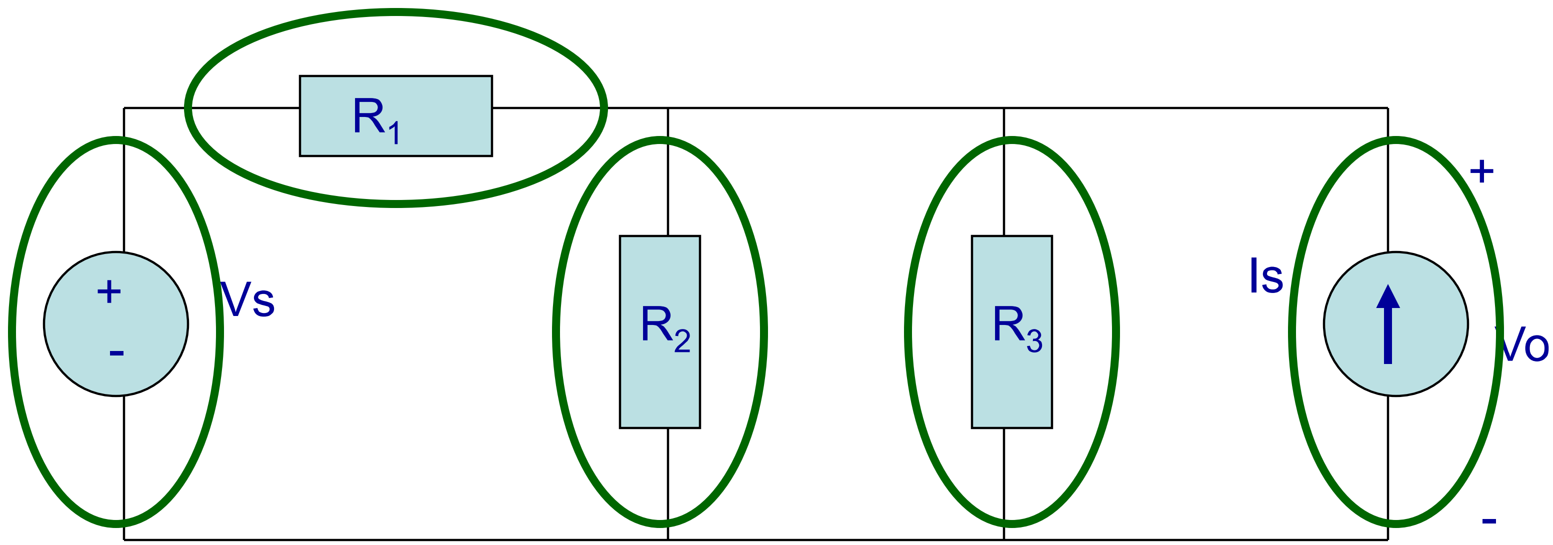
Example

- Three nodes



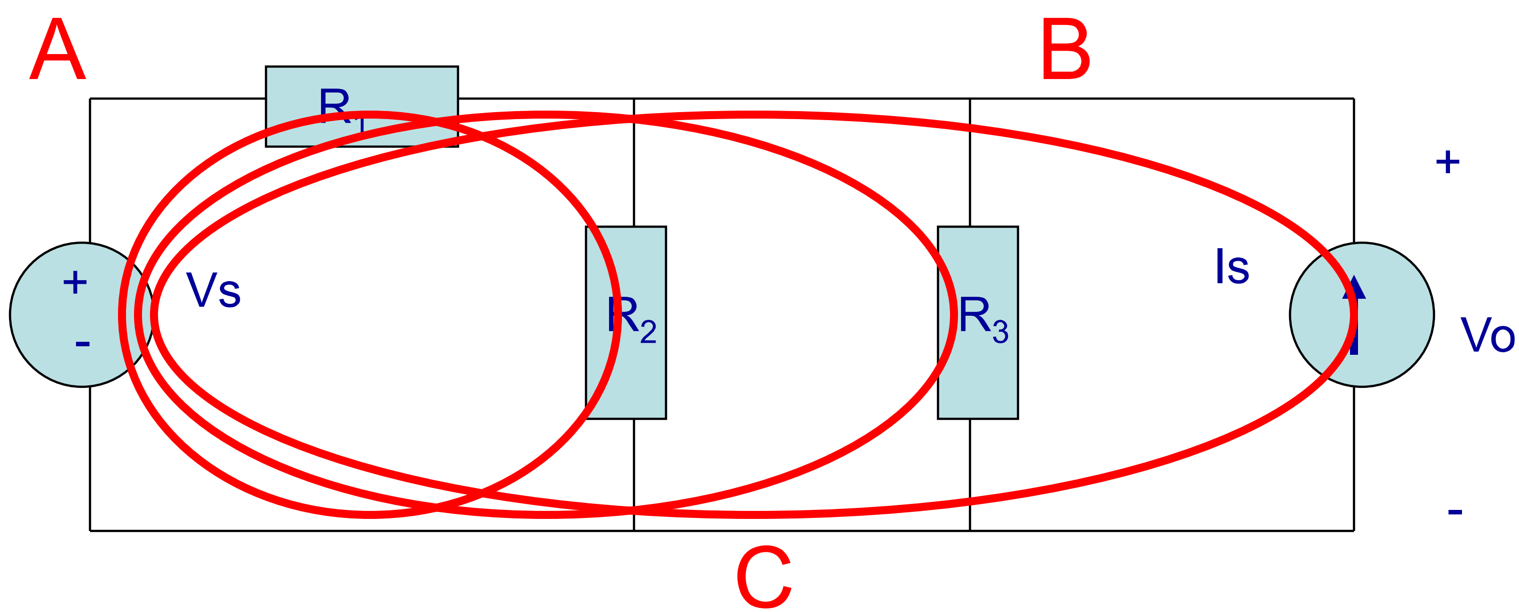
Example

- 5 Branches



Example

- Three Loops, if starting at node A

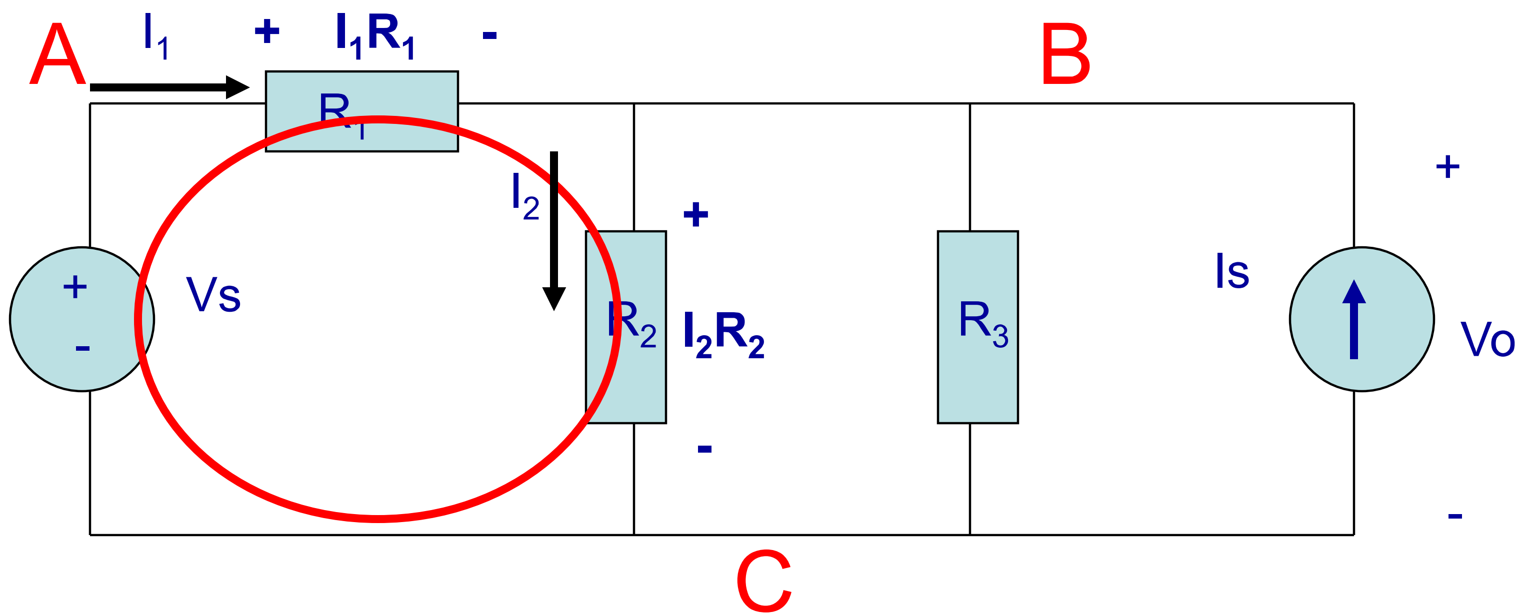


Kirchoff's Voltage Law (KVL)

- The algebraic sum of voltages around each loop is zero
  - Beginning with one node, add voltages across each branch in the loop (if you encounter a + sign first) and subtract voltages (if you encounter a – sign first)
- Σ voltage drops - Σ voltage rises = 0
- Or Σ voltage drops = Σ voltage rises

Example

- Kirchoff's Voltage Law around 1st Loop

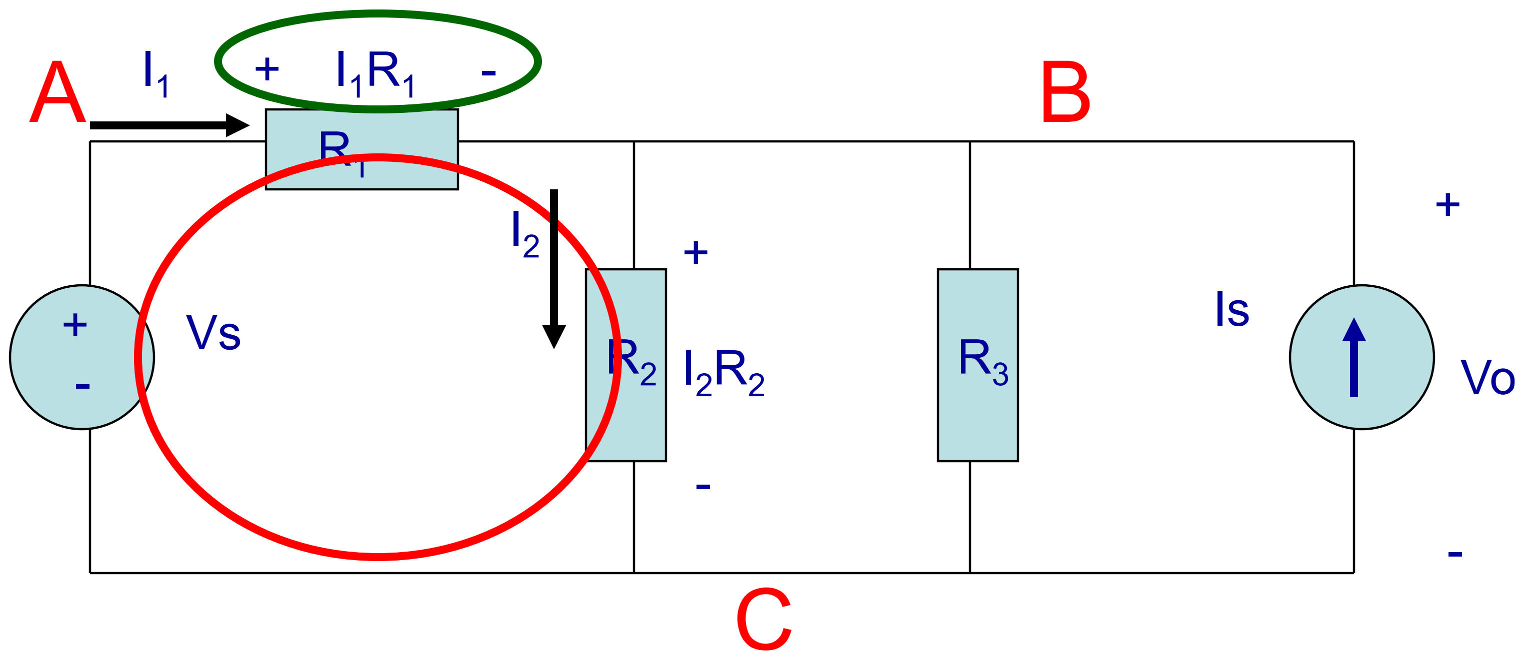


Assign current variables and directions

Use Ohm's law to assign voltages and polarities consistent with passive devices (current enters at the + side)

Example

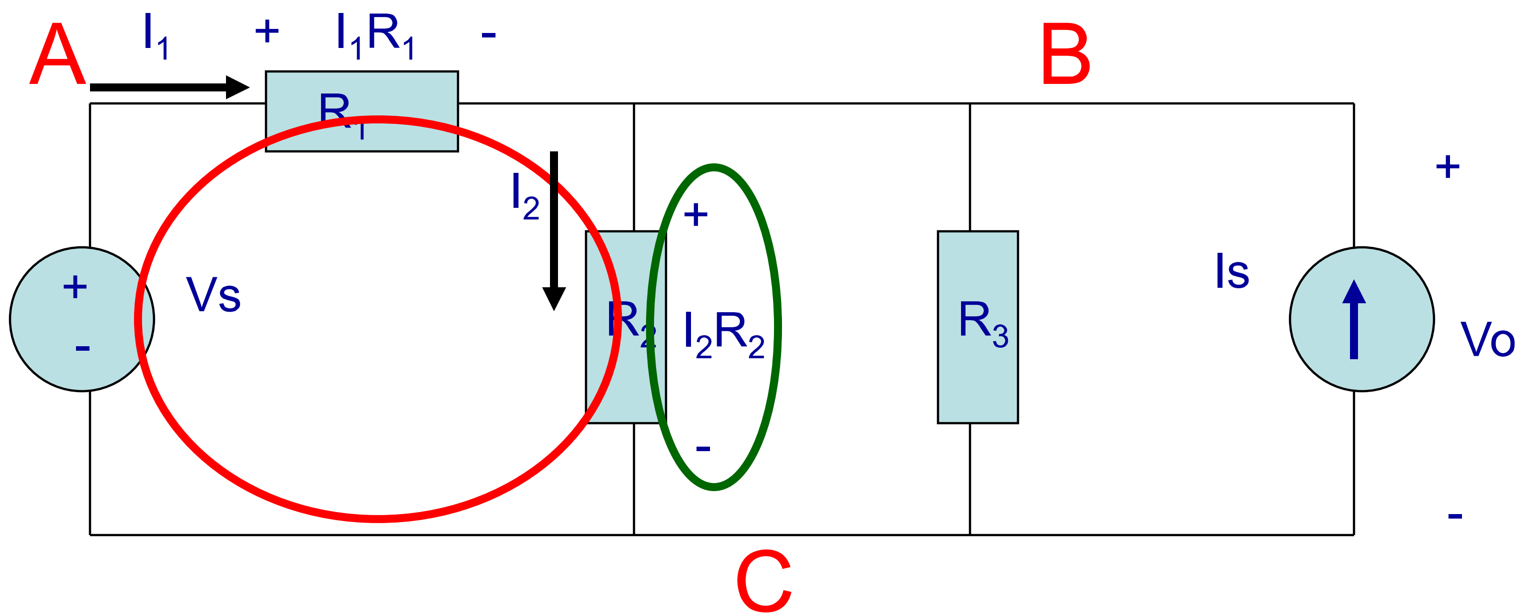
- Kirchoff's Voltage Law around 1st Loop



Starting at node A, add the 1st voltage drop: + I_1R_1

Example

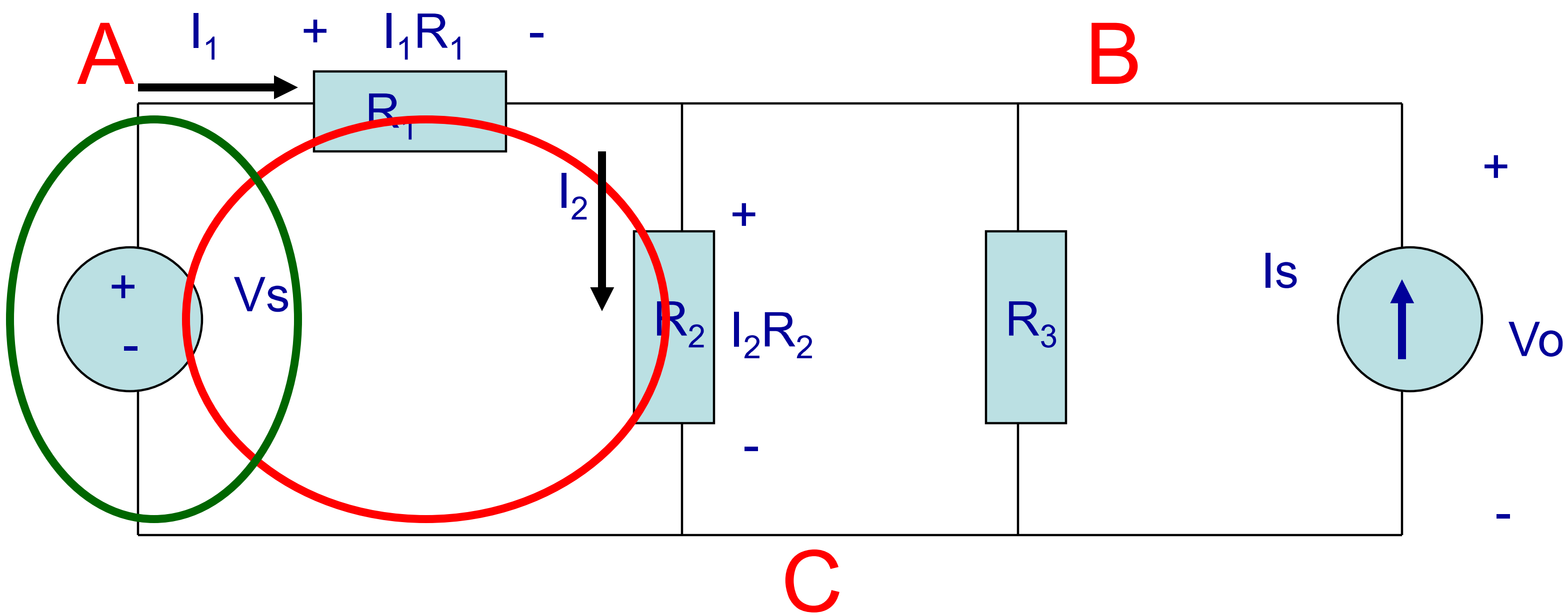
- Kirchoff's Voltage Law around 1st Loop



Add the voltage drop from B to C through R_2: $+ I_1R_1 + I_2R_2$

Example

- Kirchoff's Voltage Law around 1st Loop

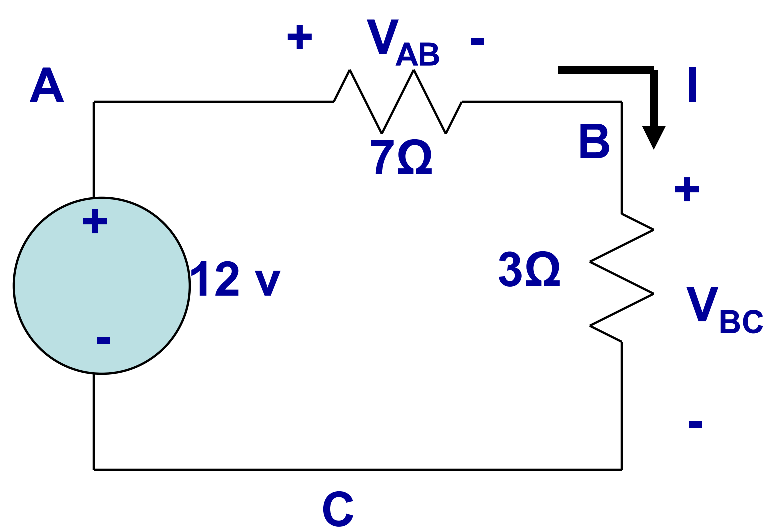


Subtract the voltage rise from C to A through Vs: $+ I_1R_1 + I_2R_2 - Vs = 0$

Notice that the sign of each term matches the polarity encountered 1st

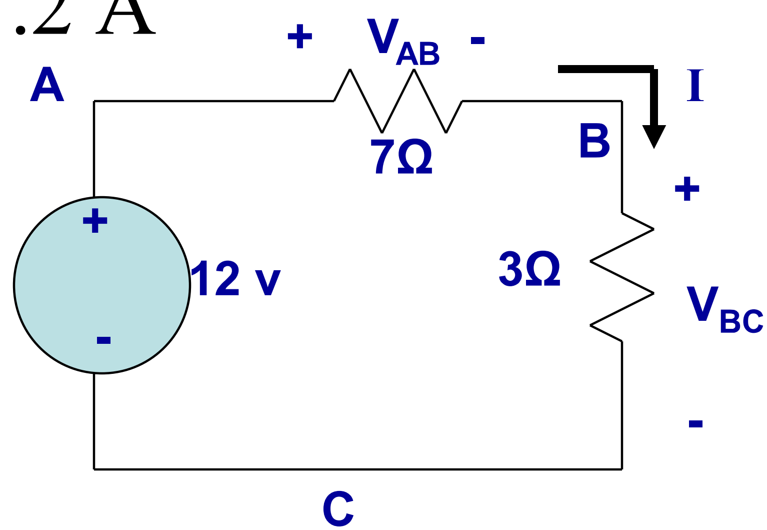
Circuit Analysis

- When given a circuit with sources and resistors having fixed values, you can use Kirchoff's two laws and Ohm's law to determine all branch voltages and currents



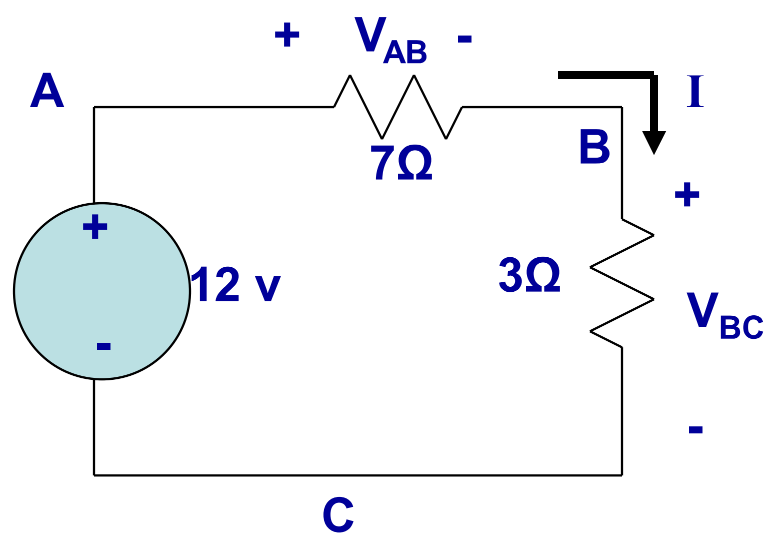
Circuit Analysis

- By Ohm's law: $V_{AB} = I \cdot 7\Omega$ and $V_{BC} = I \cdot 3\Omega$
- By KVL: $V_{AB} + V_{BC} - 12\text{ v} = 0$
- Substituting: $I \cdot 7\Omega + I \cdot 3\Omega - 12\text{ v} = 0$
- Solving: $I = 1.2\text{ A}$



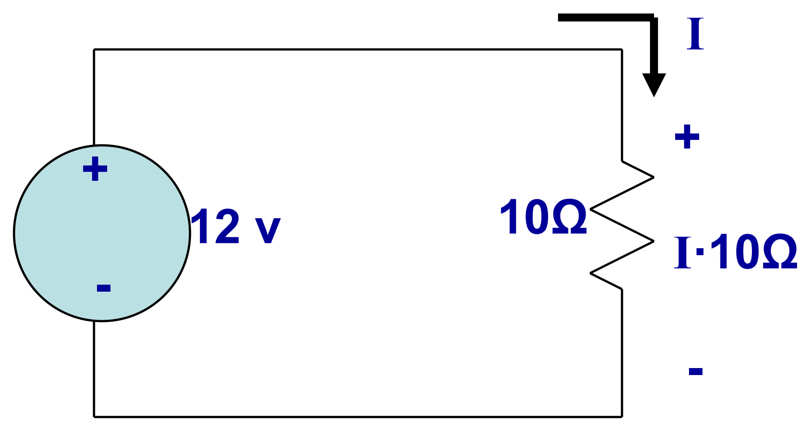
Circuit Analysis

- Since $V_{AB} = I \cdot 7\Omega$ and $V_{BC} = I \cdot 3\Omega$
- And $I = 1.2\ \mathrm{A}$
- So $V_{AB} = 8.4\ \mathrm{v}$ and $V_{BC} = 3.6\ \mathrm{v}$



Series Resistors

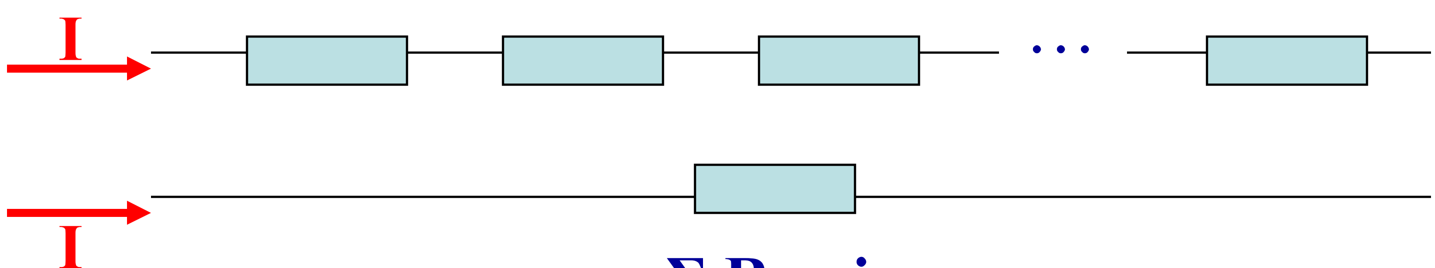
- KVL: $+I \cdot 10\Omega - 12\text{ v} = 0$, So $I = 1.2$ A
- From the viewpoint of the source, the 7 and 3 ohm resistors in series are equivalent to the 10 ohms



Series Resistors

- To the rest of the circuit, series resistors can be replaced by an equivalent resistance equal to the sum of all resistors

Series resistors (same current through all)



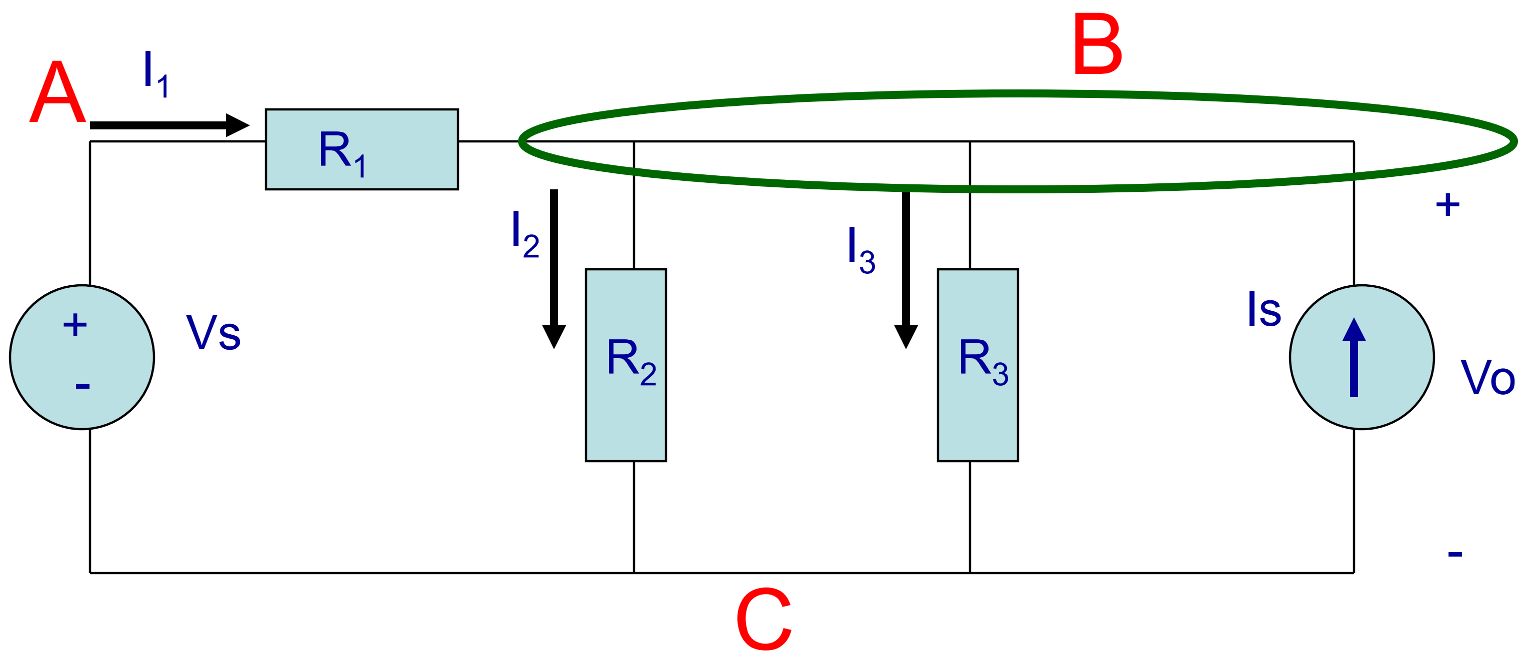
Σ Rseries

Kirchoff’s Current Law (KCL)

- The algebraic sum of currents entering a node is zero
  - Add each branch current entering the node and subtract each branch current leaving the node
- Σ currents in - Σ currents out = 0
- Or Σ currents in = Σ currents out

Example

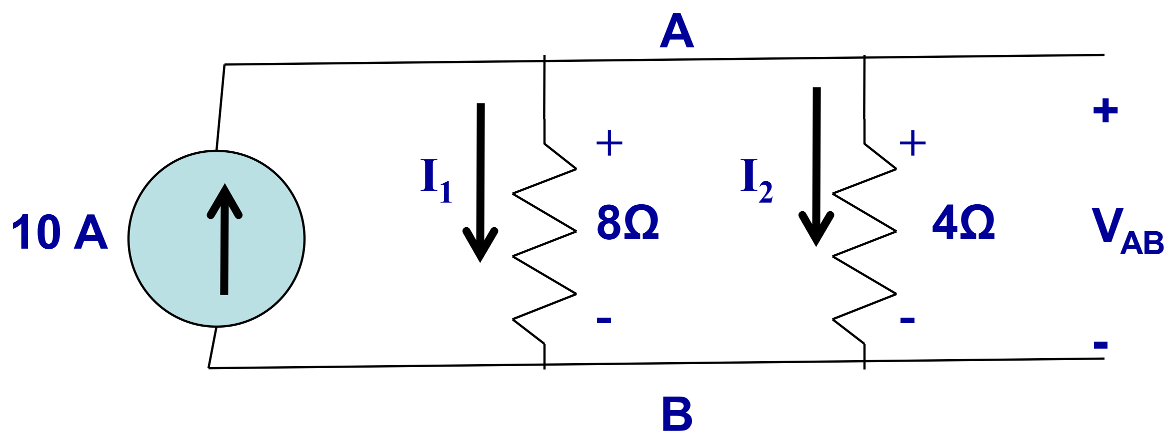
- Kirchoff's Current Law at B



Assign current variables and directions

Add currents in, subtract currents out: $I_1 - I_2 - I_3 + Is = 0$

Circuit Analysis



By KVL: $-I_1 \cdot 8\Omega + I_2 \cdot 4\Omega = 0$

Solving: $I_2 = 2 \cdot I_1$

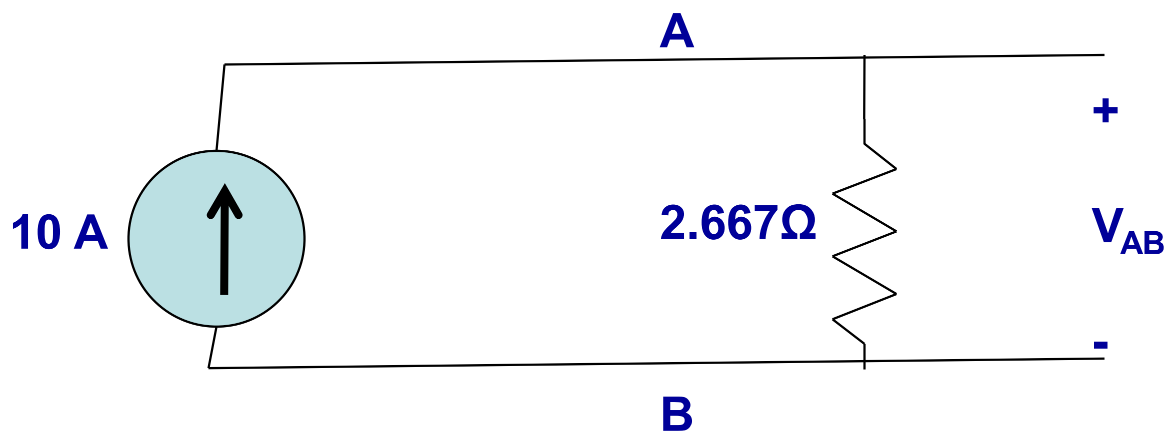
By KCL: $10A = I_1 + I_2$

Substituting: $10A = I_1 + 2 \cdot I_1 = 3 \cdot I_1$

So $I_1 = 3.33$ A and $I_2 = 6.67$ A

And $V_{AB} = 26.33$ volts

Circuit Analysis



By Ohm's Law: $V_{AB} = 10\ A \cdot 2.667\ \Omega$

So $V_{AB} = 26.67$ volts

Replacing two parallel resistors (8 and 4 Ω) by one equivalent one produces the same result from the viewpoint of the rest of the circuit.

Parallel Resistors

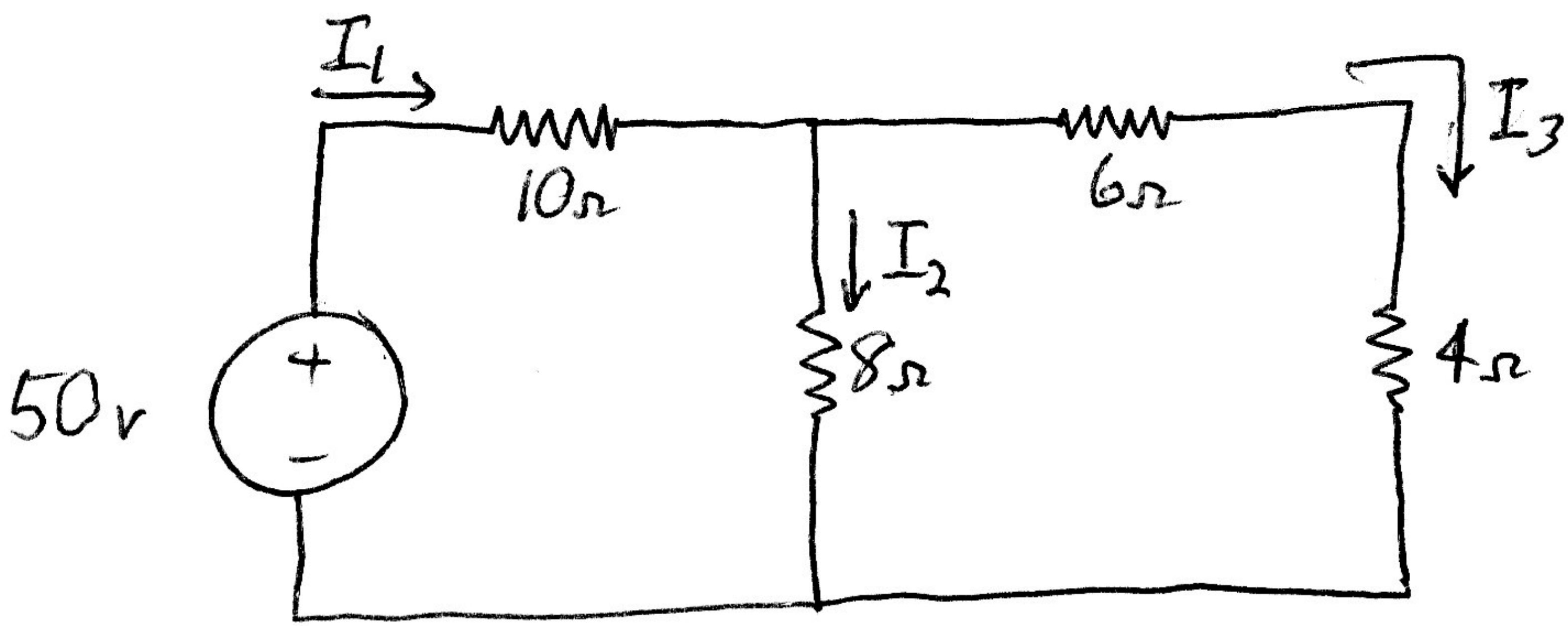
- The equivalent resistance for any number of resistors in parallel (i.e. they have the same voltage across each resistor):

$$Req = \frac{1}{1/R_1 + 1/R_2 + \cdots + 1/R_N}$$

- For two parallel resistors:

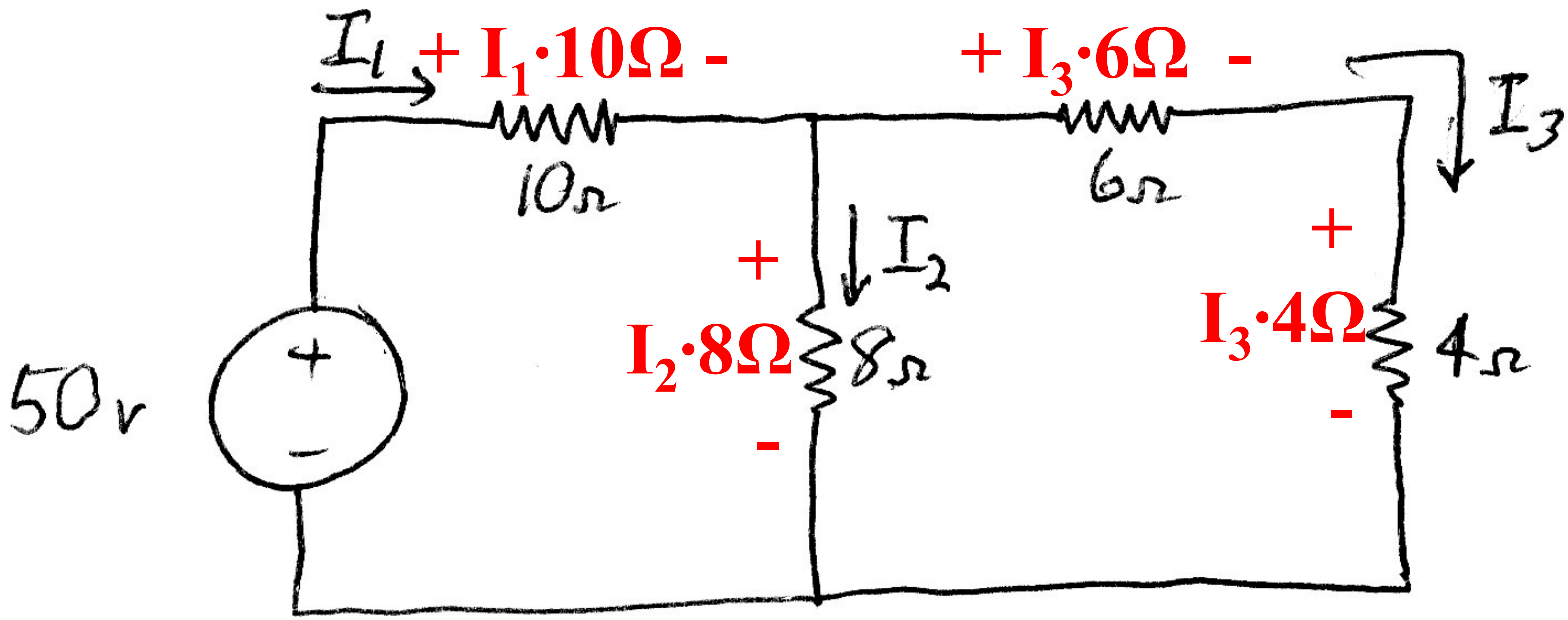
$$Req = R_1 \cdot R_2/(R_1+R_2)$$

Example Circuit



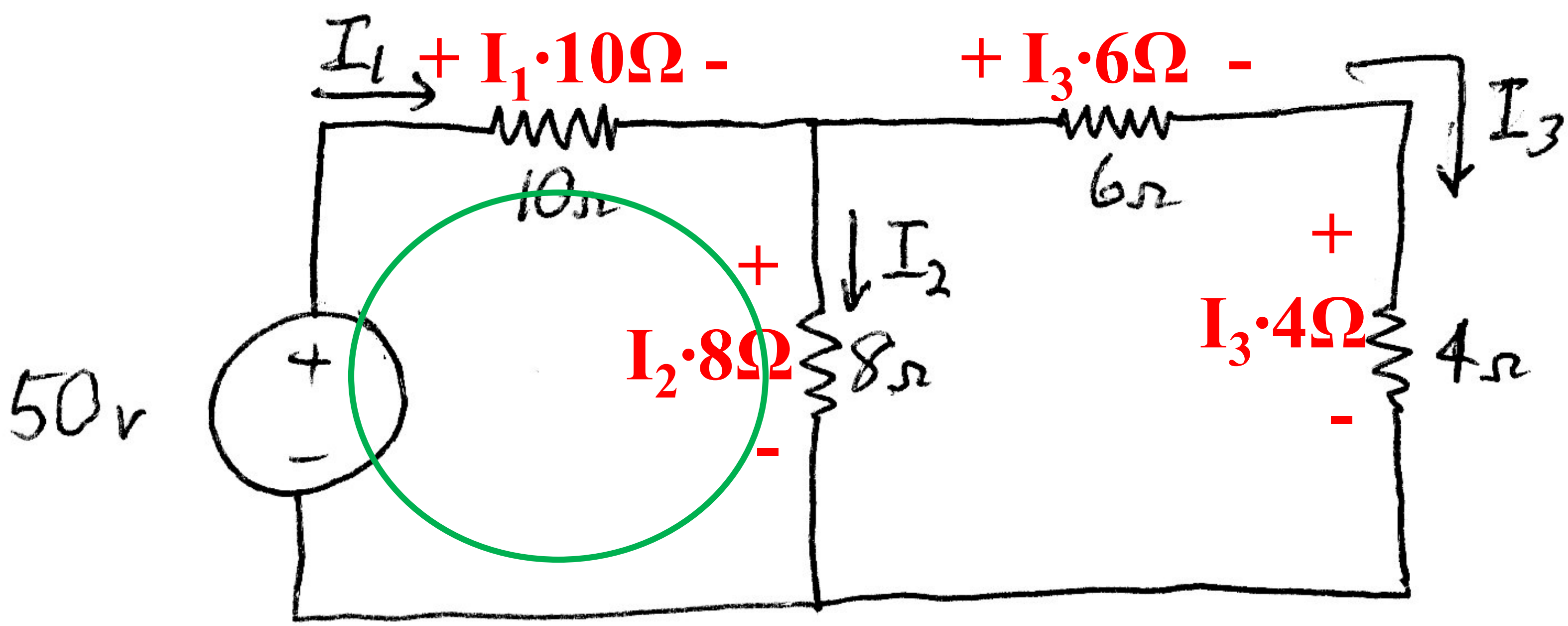
Solve for the currents through each resistor
And the voltages across each resistor

Example Circuit



Using Ohm’s law, add polarities and expressions for each resistor voltage

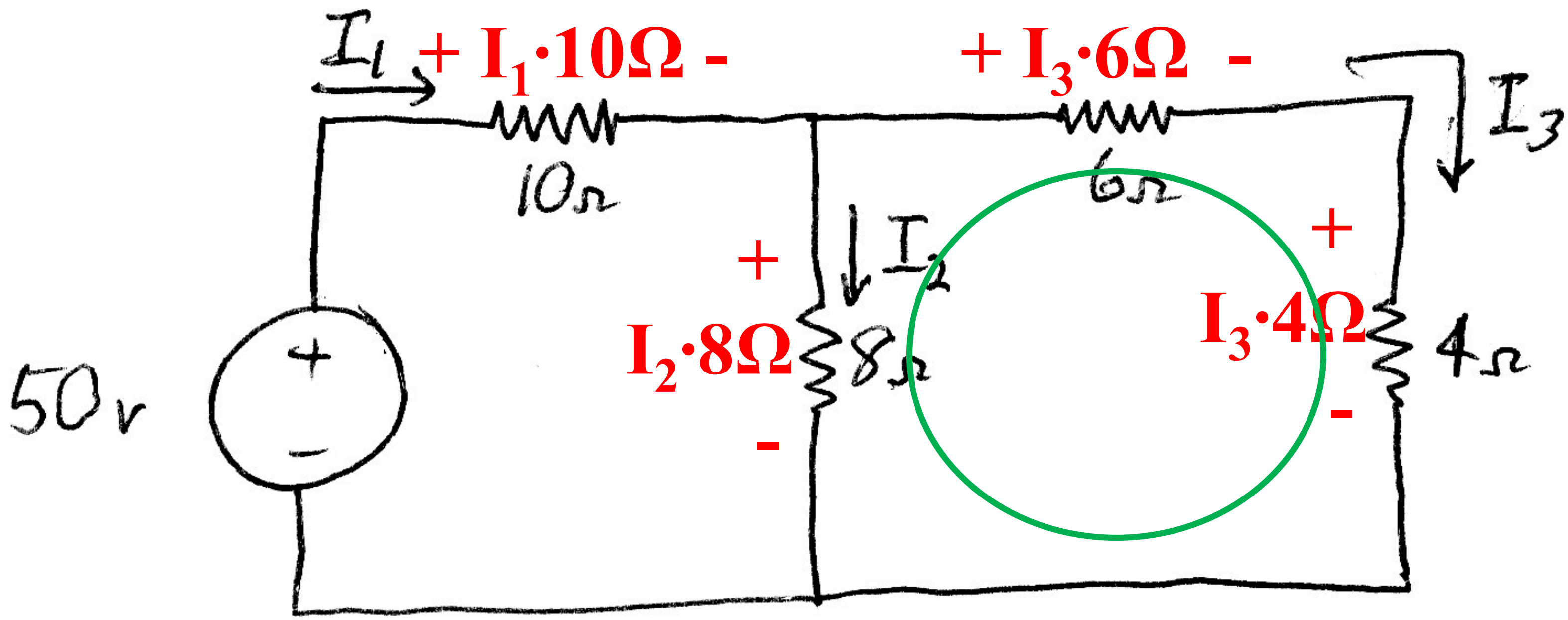
Example Circuit



Write 1st Kirchoff's voltage law equation

$$-50\ v + I_1 \cdot 10\Omega + I_2 \cdot 8\Omega = 0$$

Example Circuit

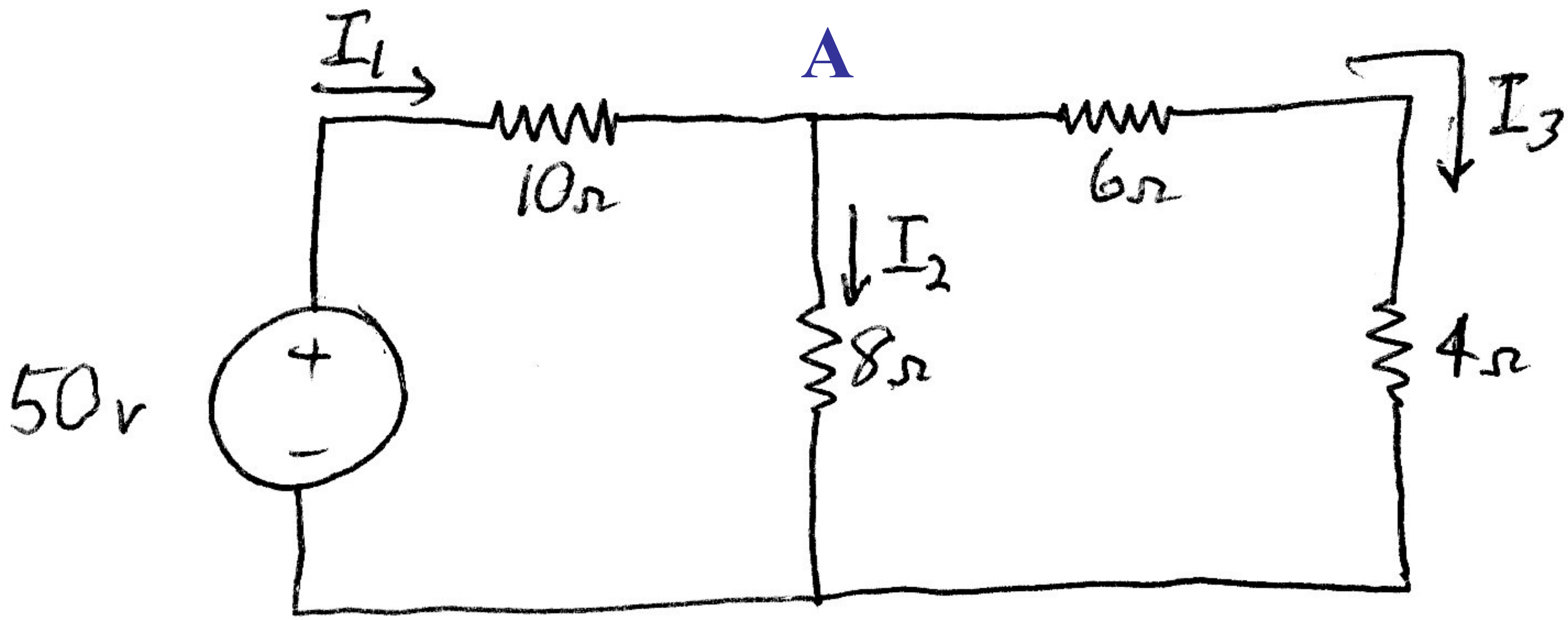


Write 2nd Kirchoff's voltage law equation

$$-I_2 \cdot 8\Omega + I_3 \cdot 6\Omega + I_3 \cdot 4\Omega = 0$$

$$\text{or } I_2 = I_3 \cdot (6+4)/8 = 1.25 \cdot I_3$$

Example Circuit



Write Kirchoff's current law equation at A

$$+I_1 - I_2 - I_3 = 0$$

Example Circuit

- We now have 3 equations in 3 unknowns, so we can solve for the currents through each resistor, that are used to find the voltage across each resistor
- Since $I_1 - I_2 - I_3 = 0$, $I_1 = I_2 + I_3$
- Substituting into the 1st KVL equation

$$-50\text{ v} + (I_2 + I_3)\cdot 10\Omega + I_2\cdot 8\Omega = 0$$

$$\text{or } I_2\cdot 18\ \Omega + I_3\cdot 10\ \Omega = 50\text{ volts}$$

Example Circuit

- But from the 2nd KVL equation, $I_2 = 1.25 \cdot I_3$
- Substituting into 1st KVL equation:

$(1.25 \cdot I_3) \cdot 18\ \Omega + I_3 \cdot 10\ \Omega = 50$ volts

Or: $I_3 \cdot 22.5\ \Omega + I_3 \cdot 10\ \Omega = 50$ volts

Or: $I_3 \cdot 32.5\ \Omega = 50$ volts

Or: $I_3 = 50$ volts$/32.5\ \Omega$

Or: $I_3 = 1.538$ amps

Example Circuit

- Since $I_3 = 1.538$ amps

  $I_2 = 1.25 \cdot I_3 = 1.923$ amps
- Since $I_1 = I_2 + I_3$, $I_1 = 3.461$ amps
- The voltages across the resistors:

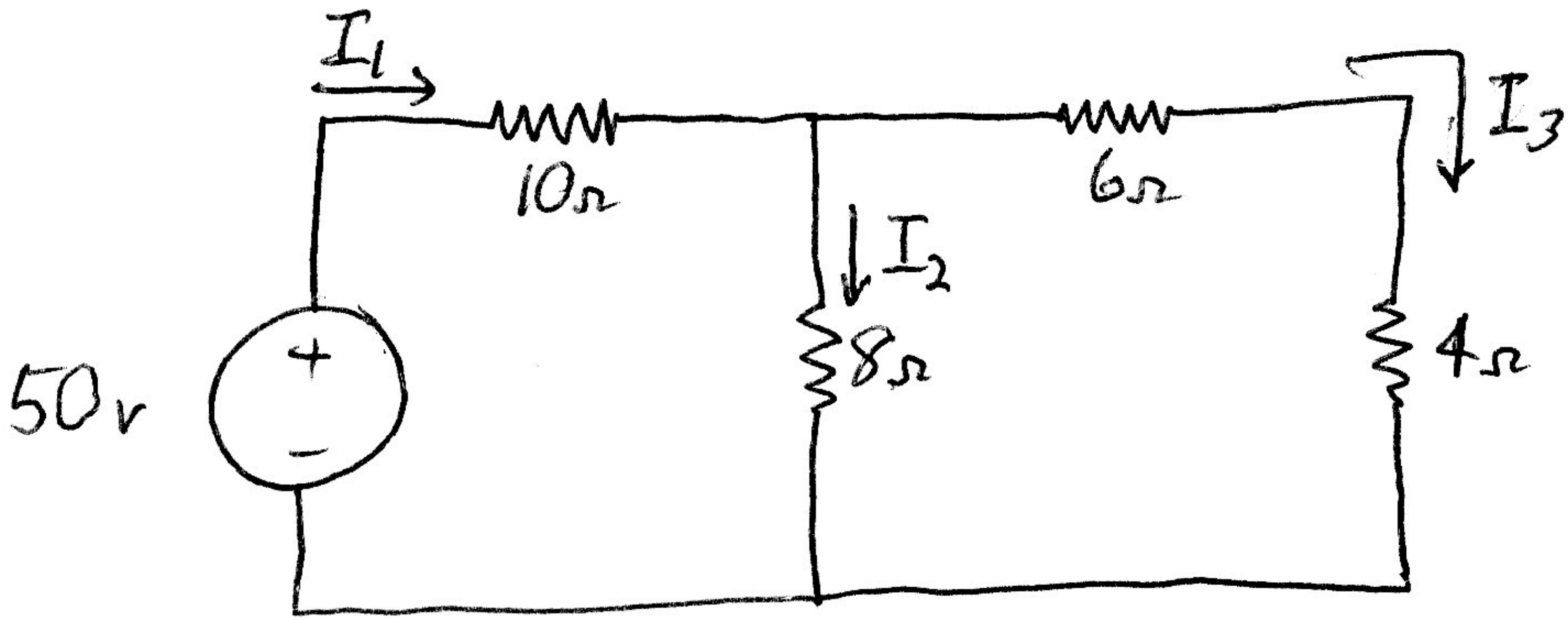
  $I_1 \cdot 10\Omega = 34.61$ volts

  $I_2 \cdot 8\Omega = 15.38$ volts

  $I_3 \cdot 6\Omega = 9.23$ volts

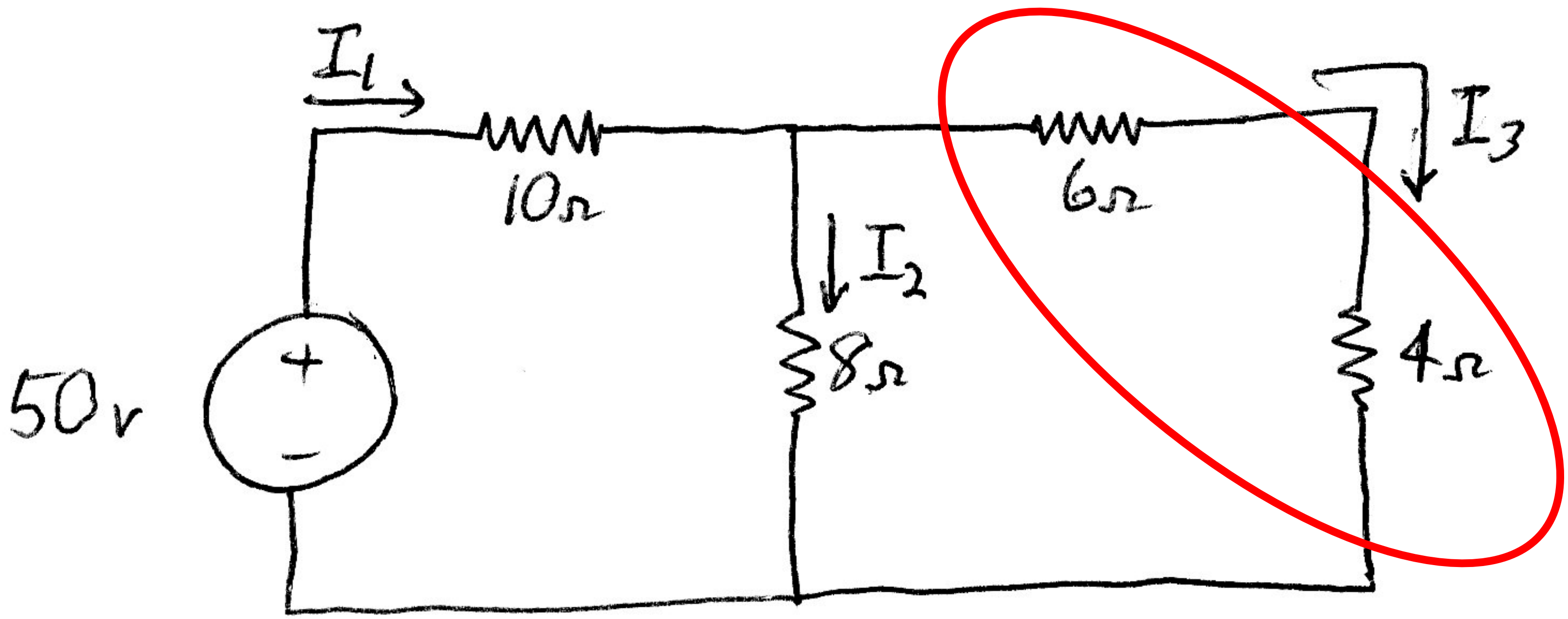
  $I_3 \cdot 4\Omega = 6.15$ volts

Example Circuit



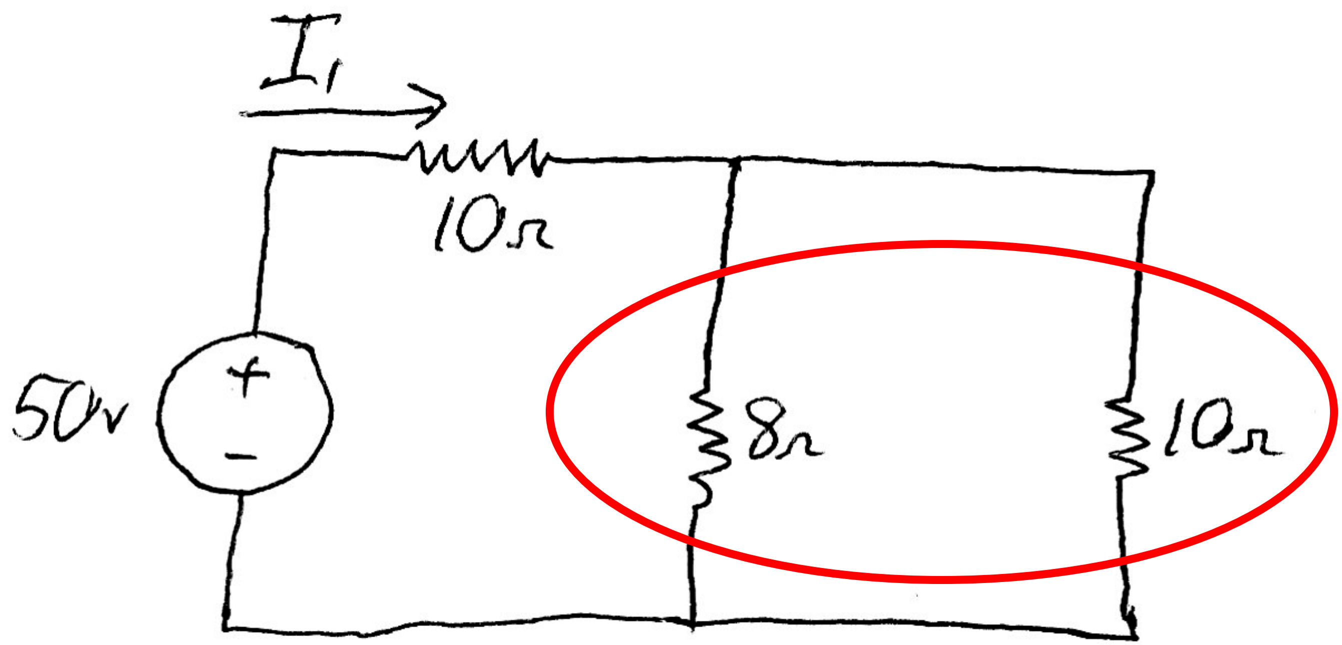
Solve for the currents through each resistor And the voltages across each resistor using Series and parallel simplification.

Example Circuit



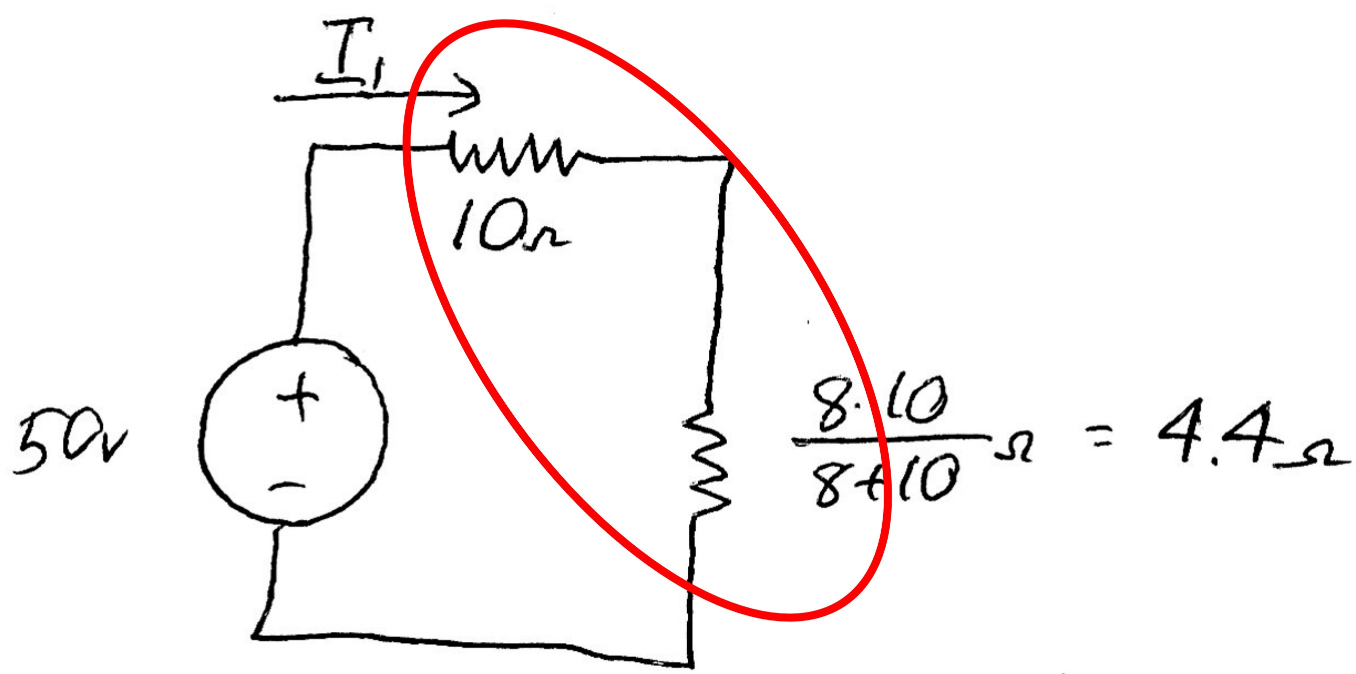
The 6 and 4 ohm resistors are in series, so are combined into 6+4 = 10Ω

Example Circuit



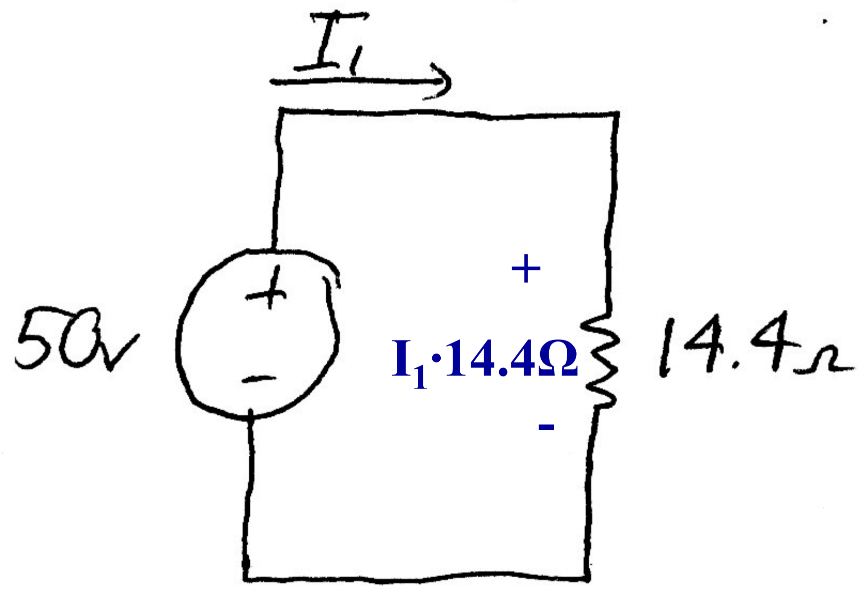
The 8 and 10 ohm resistors are in parallel, so are combined into 8·10/(8+10) =14.4 Ω

Example Circuit



The 10 and 4.4 ohm resistors are in series, so are combined into 10+4 = 14.4Ω

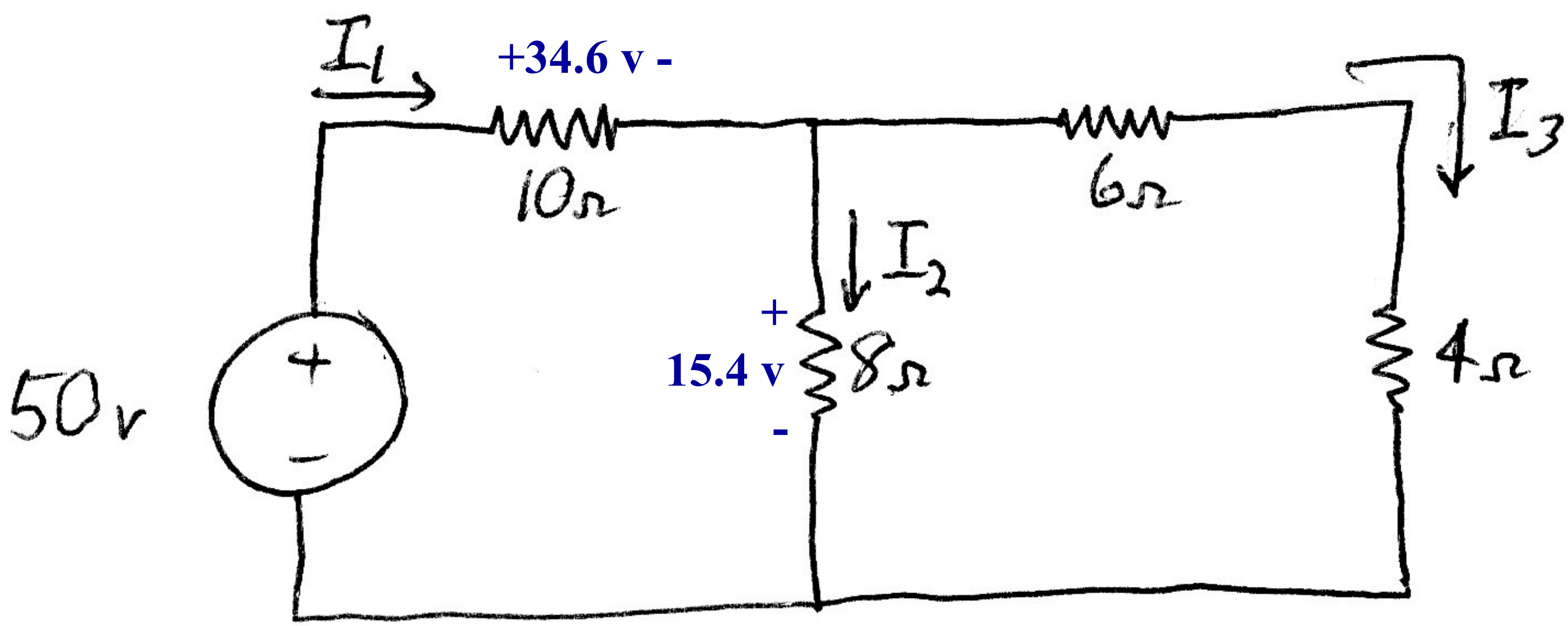
Example Circuit



Writing KVL, $I_1 \cdot 14.4\Omega - 50\text{ v} = 0$

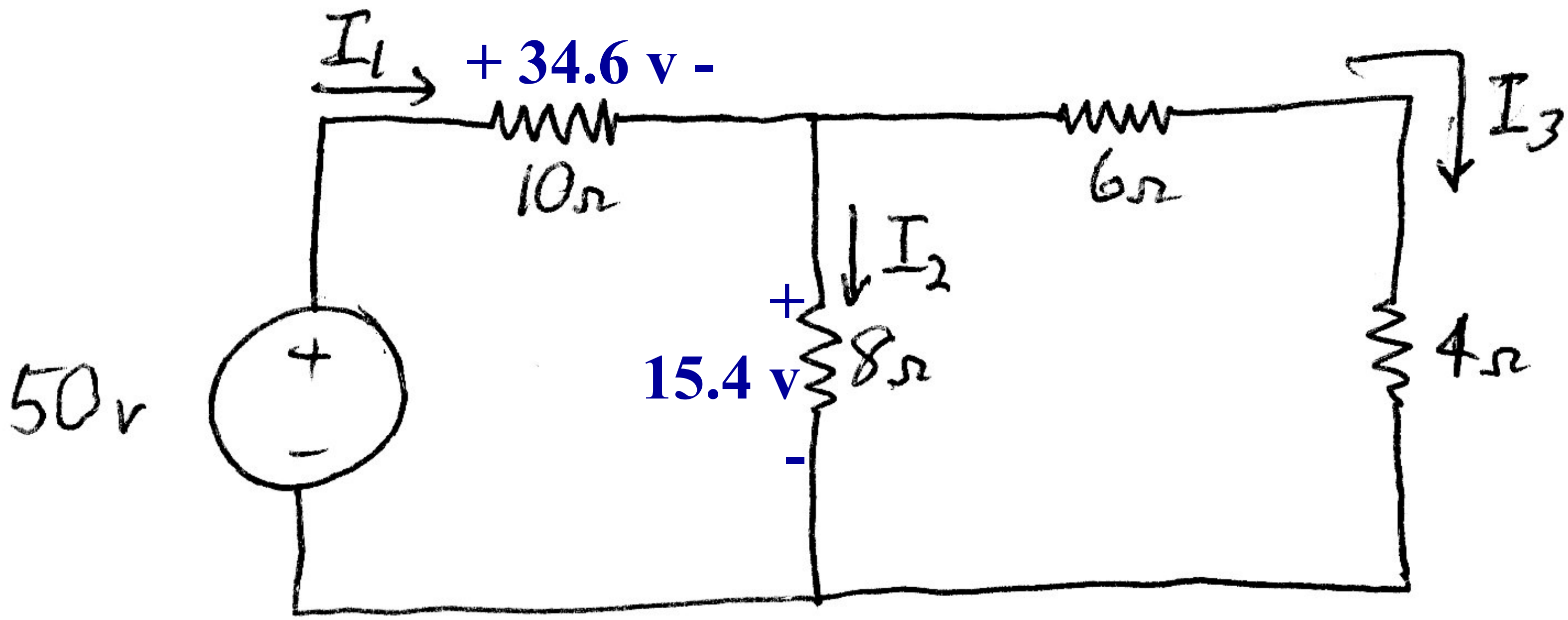
Or $I_1 = 50\text{ v} / 14.4\Omega = 3.46\text{ A}$

Example Circuit



If $I_1 = 3.46$ A, then $I_1 \cdot 10\ \Omega = 34.6$ v
So the voltage across the $8\ \Omega = 15.4$ v

Example Circuit



If $I_2 \cdot 8\ \Omega = 15.4$ v, then $I_2 = 15.4/8 = 1.93$ A

By KCL, $I_1 - I_2 - I_3 = 0$, so $I_3 = I_1 - I_2 = 1.53$ A