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Lecture #1

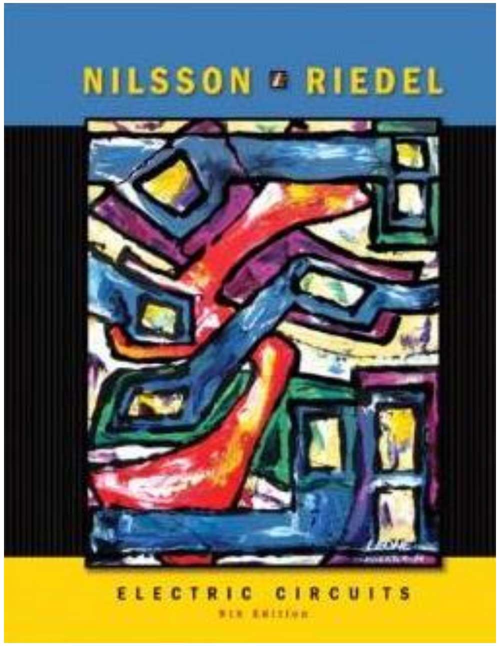
Response of First-Order *RL* and *RC* Circuits

Chapter #7

Text book: Electric Circuits

James W. Nilsson & Susan A. Riedel

9th Edition.

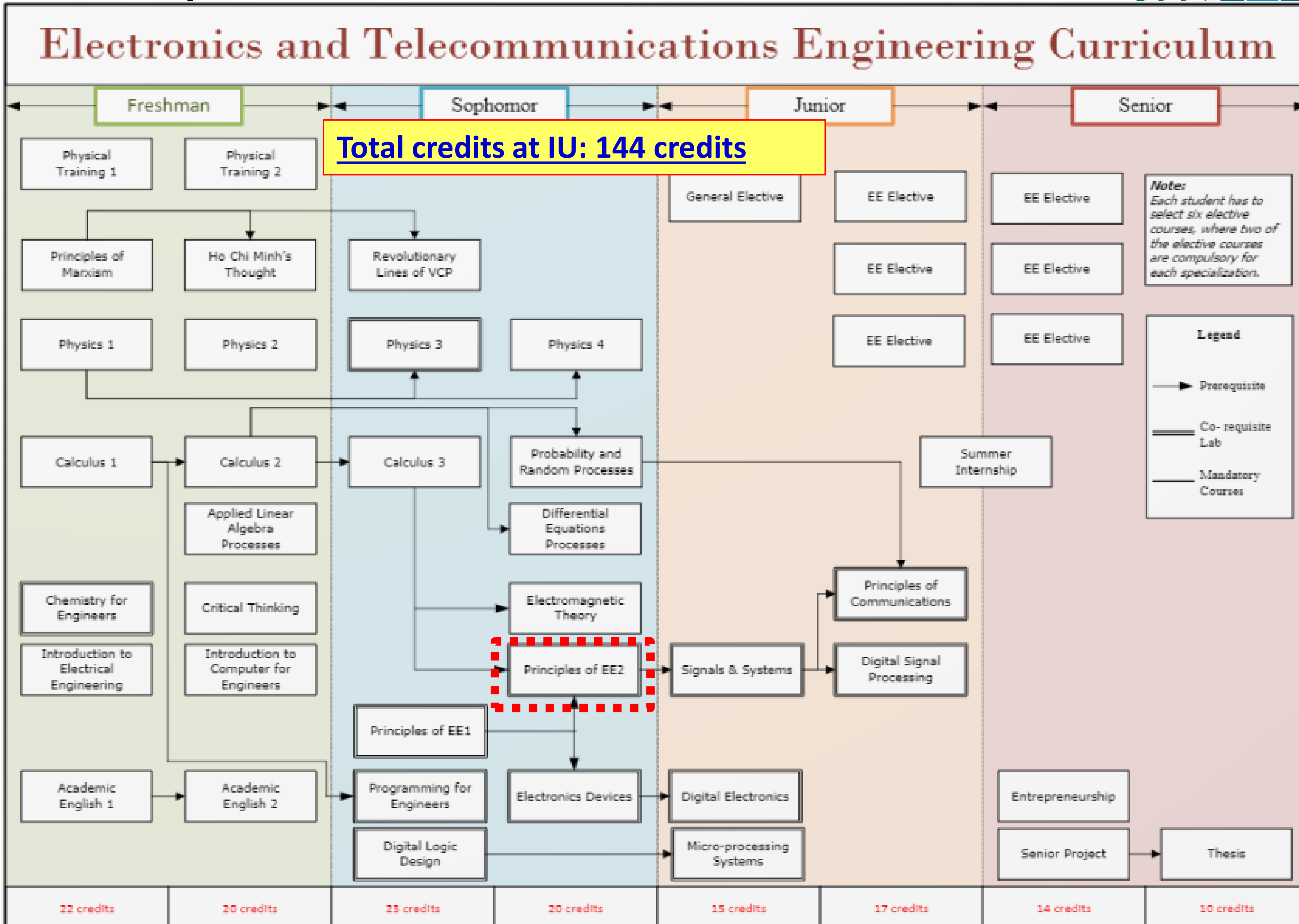


link: **http://blackboard.hcmiu.edu.vn/**

to download materials

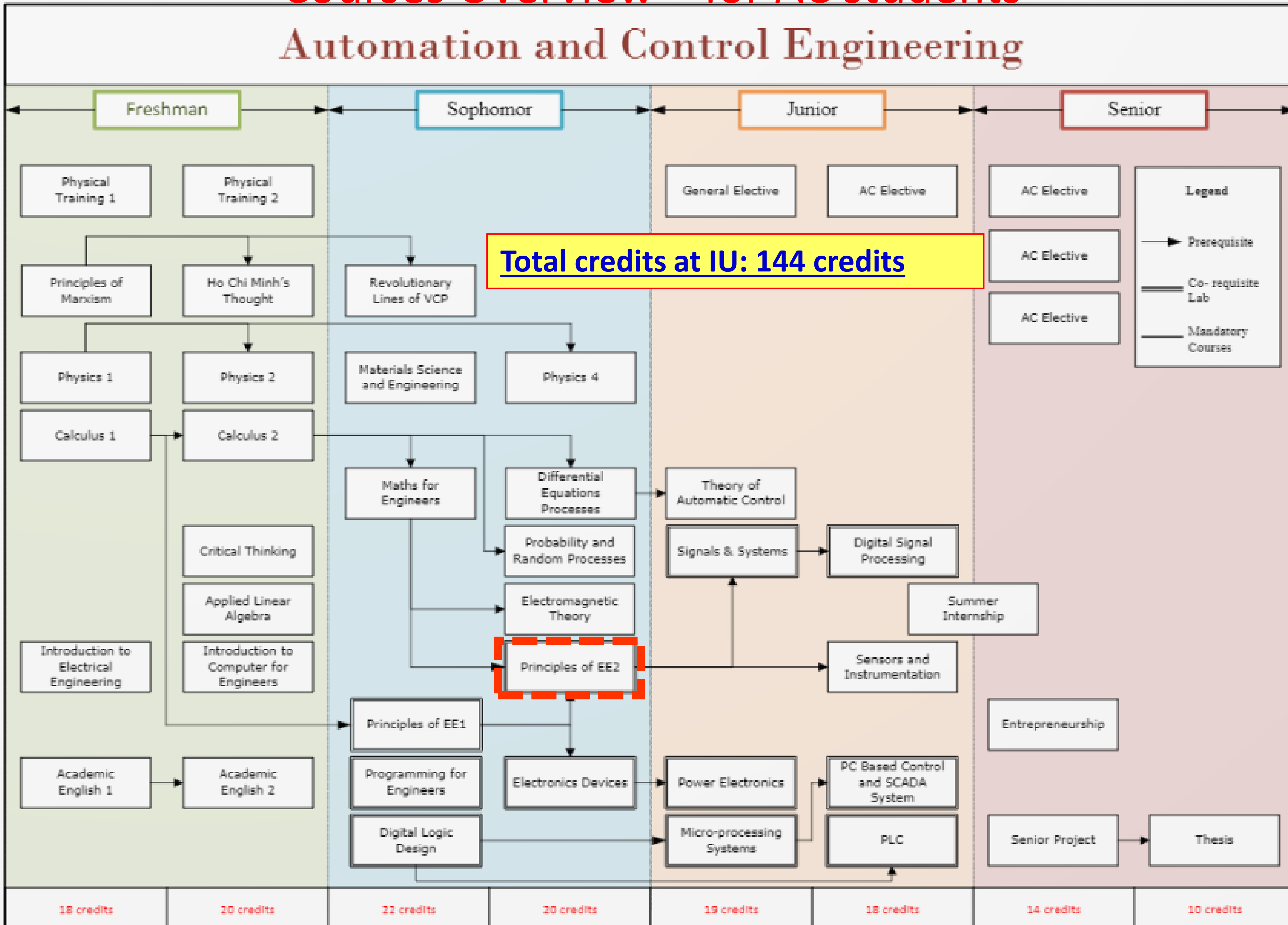
Spring 2019

Courses Overview – for EE students



Spring 2019

Courses Overview – for AC students



Total credits at IU: 144 credits

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Objectives

- Be able to determine the natural response of both RL and RC circuits.
- Be able to determine the step response of both RL and RC circuits.
- Know how to analyze circuits with sequential switching.

Outlines

- The natural response of an RL circuit & an RC circuit
- The step response of RL & RC circuits
- Sequential switching
- Unbounded response

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Overview

- Ch9-10 discuss "steady-state response" of linear circuits to "sinusoidal sources". The math treatment is the same as the "dc response" except for introducing "phasors" and "impedances" in the algebraic equations.

- From now on, we will discuss "transient response" of linear circuits to "step sources" (Ch7-8) and general "time-varying sources" (Ch12-13). The math treatment involves with differential equations and Laplace transform.

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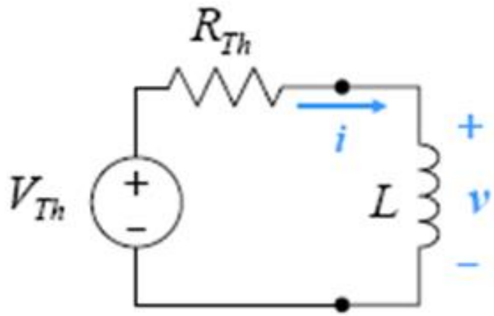
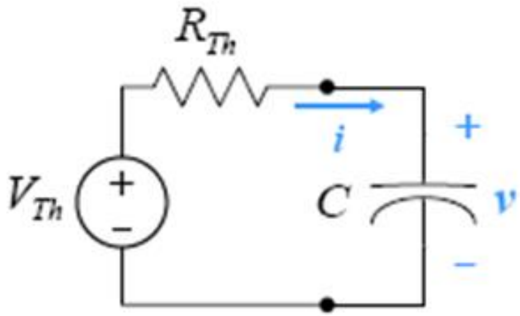
First order circuits

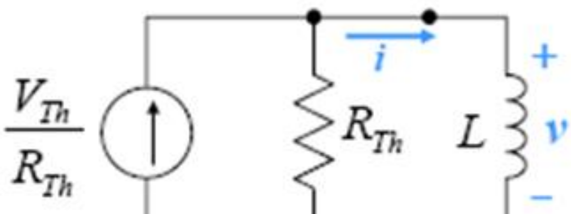
A circuit that can be simplified to a Thévenin (or Norton) equivalent connected to either a single equivalent inductor or capacitor.

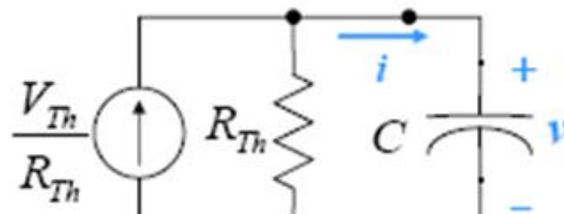
L or C connected to a Thevenin equivalent





L or C connected to a Norton equivalent





In Ch7, the source is either none (natural response) or step source.

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Key points

- Why an RC or RL circuit is charged or discharged as an exponential function of time?
- Why the charging and discharging speed of an RC or RL circuit is determined by RC or L/R?
- What could happen when an energy-storing element (C or L) is connected to a circuit with dependent source?

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The natural response of an *RL* circuit

- Differential equation & solution of a discharging RL circuit.
- Time constant
- Discharging RC circuit

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The natural response of an *RL* circuit

What is natural response?

- ✓ It describes the "discharging" of inductors or capacitors via a circuit of no dependent source.
- ✓ No external source is involved, thus termed as "natural" response.
- ✓ The effect will vanish as t → ∞. The interval within which the natural response matters depends on the element parameters.

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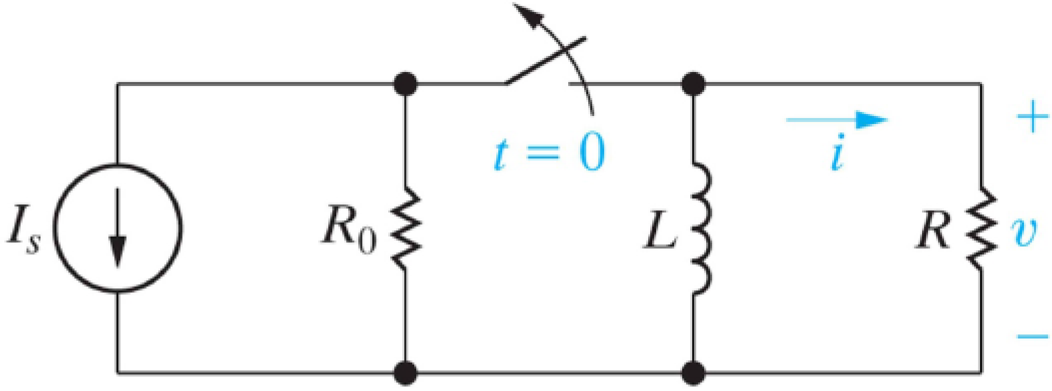
The natural response of an *RL* circuit

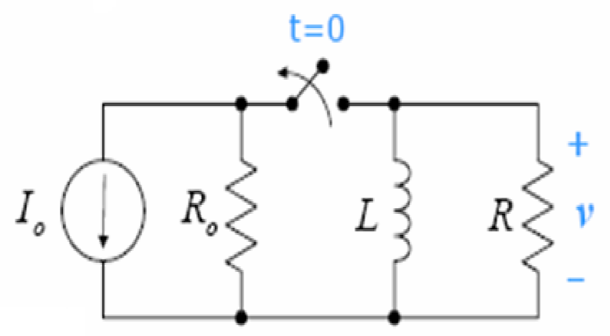
Circuit model of a discharging RL circuit

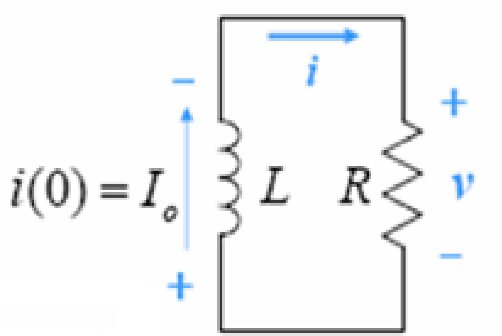
- Consider the following circuit model:



- For $t < 0$, the inductor L is short and carries a current I_s, while R_0 and R carry no current.
- For $t > 0$, the inductor current decreases and the energy is dissipated via R.

The natural response of an *RL* circuit





The switch is closed for a long time and opened at t = 0

t ≤ 0 ⟶ $\dfrac{di}{dt} = 0$ ⟶ v = 0 (short circuit)

All the source current I_0 appears in the inductive branch

t ≥ 0 Apply KVL: $\boxed{L\dfrac{di}{dt} + Ri = 0}$ (1st order differential equation)

Ordinary differential equation (ODE)

Solving the loop current:

$$L\frac{di}{dt} = -Ri \;\longrightarrow\; \frac{di}{i} = -\frac{R}{L}dt$$

$$\int_{i(t_o)}^{i(t)} \frac{di}{i} = -\frac{R}{L}\int_{t_o}^{t} dt \;\longrightarrow\; \ln\left(\frac{i(t)}{i(0)}\right) = -\frac{R}{L}t \;\longrightarrow\; \boxed{i(t) = i(0)e^{-\frac{R}{L}t}}$$

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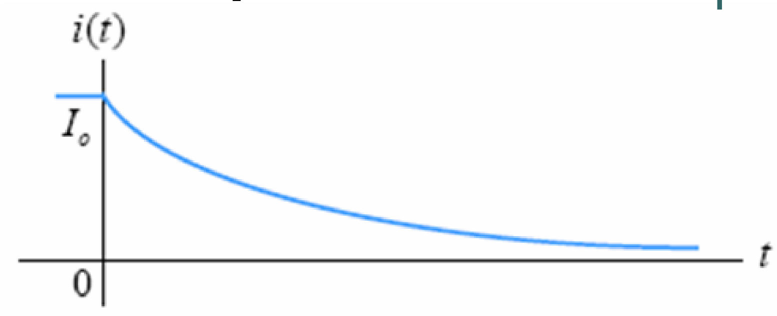
The natural response of an *RL* circuit

Initial condition depends on initial energy of the inductor:



$$i(0^-) = i(0^+) = I_0$$

$$i(t) = I_0 e^{-(R/L)t} \qquad t \geq 0$$

The voltage across the resistor using Ohm's law

$$v = iR = I_0 \mathrm{R} e^{-(R/L)t} \qquad t \geq 0^+$$

$$v(0^-) = 0 \qquad v(0^+) = I_0 R$$

The power dissipated in the resistor

$$p = iv = I_0^2 \mathrm{R} e^{-2(R/L)t} \qquad t \geq 0^+$$

The energy delivered to the resistor during any interval of time after the switch has been opened

$$w = \int_0^t p\,dt = I_o^2 R \int_0^t e^{-2\frac{R}{L}t} dt = I_o^2 R \left. \frac{e^{-2\frac{R}{L}t}}{\frac{-2R}{L}} \right|_0^t$$

$$w = \frac{1}{2} L I_o^2 \left(1 - e^{-2\frac{R}{L}t} \right) \qquad t \geq 0$$

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The time constant (τ)

- The time constant is the coefficient of time (t)

$$\tau = \frac{L}{R} \qquad \text{(seconds)}$$

$$i = I_o e^{-\frac{t}{\tau}} \qquad t \geq 0$$

$$v = iR = I_o R e^{-\frac{t}{\tau}} \qquad t \geq 0^+$$

$$p = iv = I_o^2 R e^{-2\frac{t}{\tau}} \qquad t \geq 0^+$$

$$w = \frac{1}{2} L I_o^2 \left(1 - e^{-2\frac{t}{\tau}}\right) \qquad t \geq 0$$

$$\boxed{w_0 = L I_0^2 / 2}$$

initial energy stored in L

"One time constant after the inductor had begun to release its stored energy to the resistor, the current has been reduced to e^{-1}, or approximately 0.37 of its initial value."

"Long time implies that five or more time constants have elapsed."
≡ Steady State Response

t	$e^{-t/\tau}$	t	$e^{-t/\tau}$
τ	3.6788×10^{-1}	6τ	2.4788×10^{-3}
2τ	1.3534×10^{-1}	7τ	9.1188×10^{-4}
3τ	4.9787×10^{-2}	8τ	3.3546×10^{-4}
4τ	1.8316×10^{-2}	9τ	1.2341×10^{-4}
5τ	6.7379×10^{-3}	10τ	4.5400×10^{-5}

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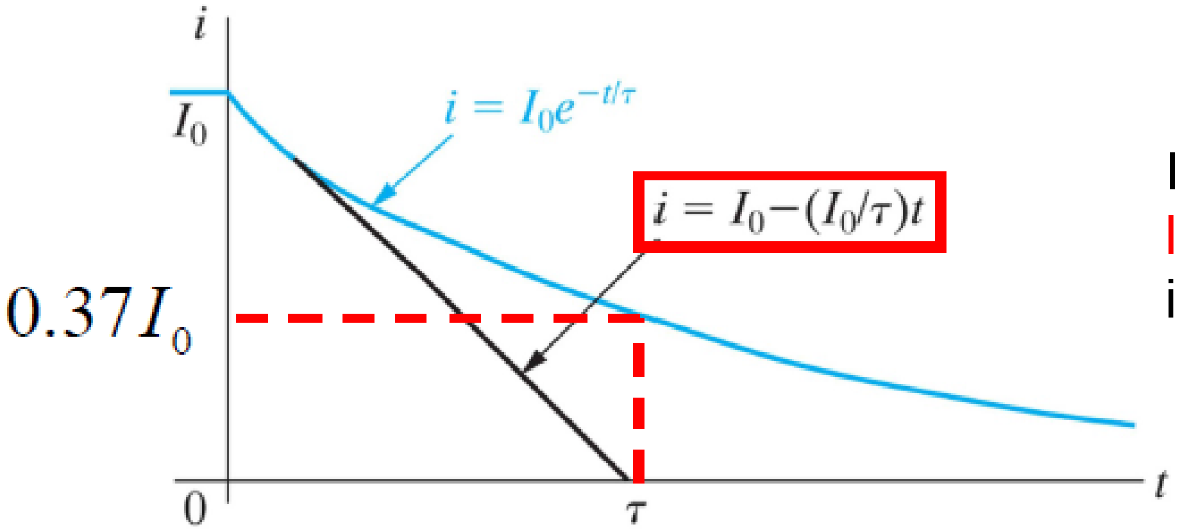
The time constant (τ)



Interpretation of the time constant of the RL circuit

when $\tau = t \rightarrow i = I_0$

If $i(t)$ is approximated by a linear function, it will vanish in one time constant.

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Procedures to get natural response of RL, RC circuits

1. Find the equivalent circuit.
2. Find the initial conditions: initial current I_0 through the equivalent inductor, or initial voltage V_0 across the equivalent capacitor.
3. Find the time constant of the circuit by the values of the equivalent R, L, C:

$$\tau = L/R, \text{ or } RC;$$

4. Directly write down the solutions:

$$i(t) = I_0 e^{-(t/\tau)}, \; v(t) = V_0 e^{-(t/\tau)}.$$

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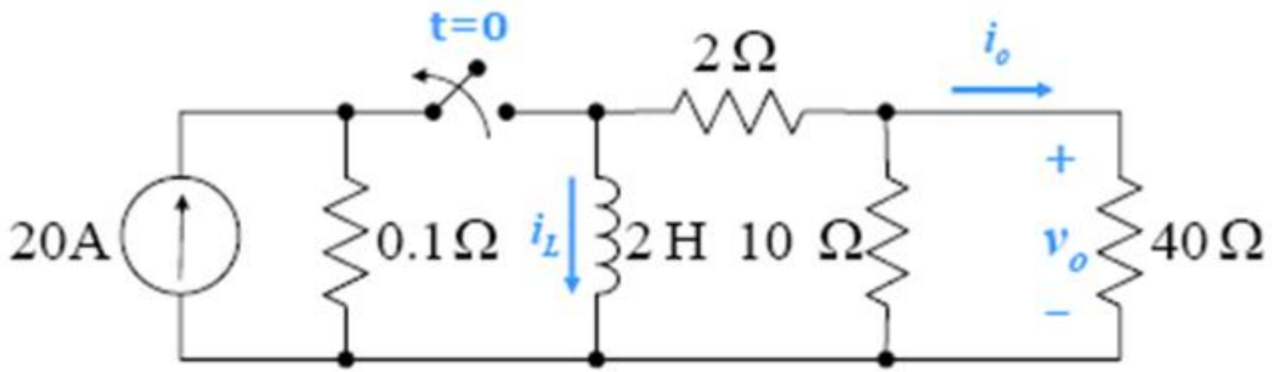
Example 1

- The switch in the circuit has been closed for a long time before it is opened at t=0. Find:
  a) $i_L(t)$ for $t \geq 0$.
  b) $i_o(t)$ for $t \geq 0^+$.
  c) $v_o(t)$ for $t \geq 0^+$.
  d) The percentage of the total energy stored in the 2 H inductor that is dissipated in the 10 Ω resistor.



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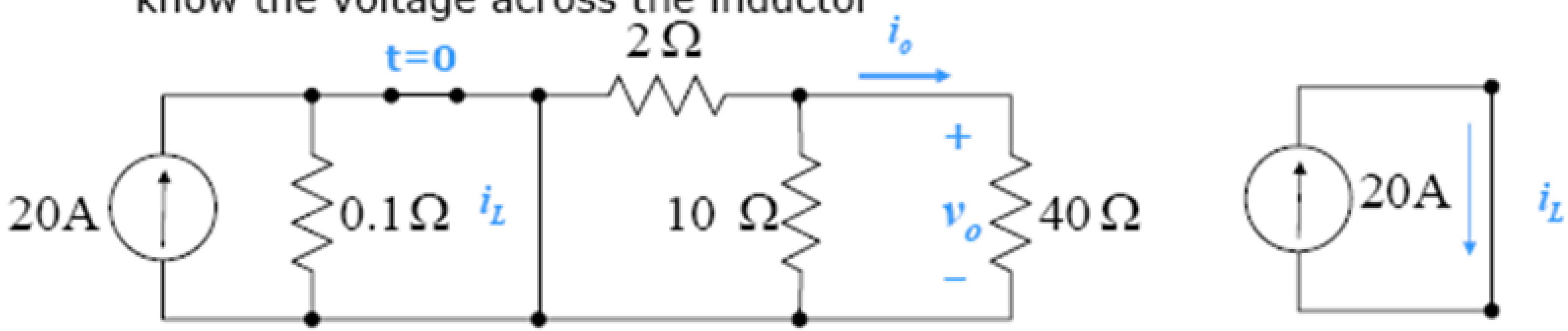
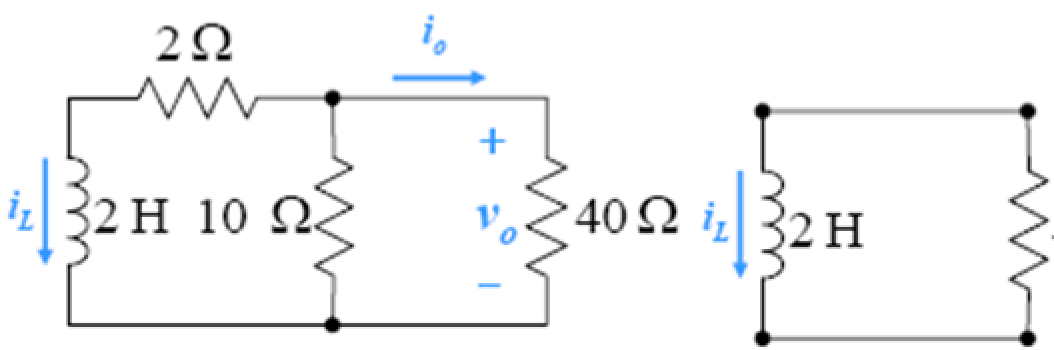
Example 1 - Solution

a) The switch has been closed for a long time prior to t=0, so we know the voltage across the inductor



$t = 0^-$ $\qquad i_L(0^-) = 20\text{ A} \rightarrow i_L(0^+) = 20\text{ A}$



$R_{eq} = 10 // 40 + 2 = 10\,\Omega$

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Example 1 – Solution (cont)

$$i = I_o e^{-\frac{t}{\tau}}$$

$$I_o = 20\ \text{A}$$

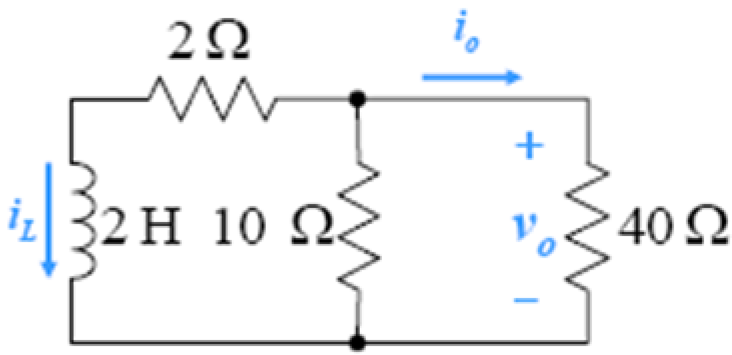
$$\tau = \frac{L}{R} = \frac{2}{10} = 0.2\ s$$

$$i = 20e^{-5t} \quad t \ge 0$$

b) i_o(t)

$$i_o = -i_L \frac{10}{10+40}$$

$$i_o(t) = -4e^{-5t} \quad t \ge 0^+$$



c) v_o(t)

$$v_o(t) = -40i_o = -160e^{-5t} \quad t \ge 0^+$$

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Example 1 – Solution (cont)

d) $p_{10\Omega}$?

$$p_{10\Omega}(t) = \frac{v_o^2}{10} = 2560e^{-10t} \quad \text{W}$$

$$w_{10\Omega}(t) = \int_o^\infty 2560e^{-10t}\,dt = 256\ \text{J}$$

Initial energy stored in the 2 H inductor is

$$w_{10\Omega}(0) = \frac{1}{2}Li^2(0) = \frac{1}{2}(2)(400) = 400\ J$$

$$\frac{256}{400}(100) = 64\%$$

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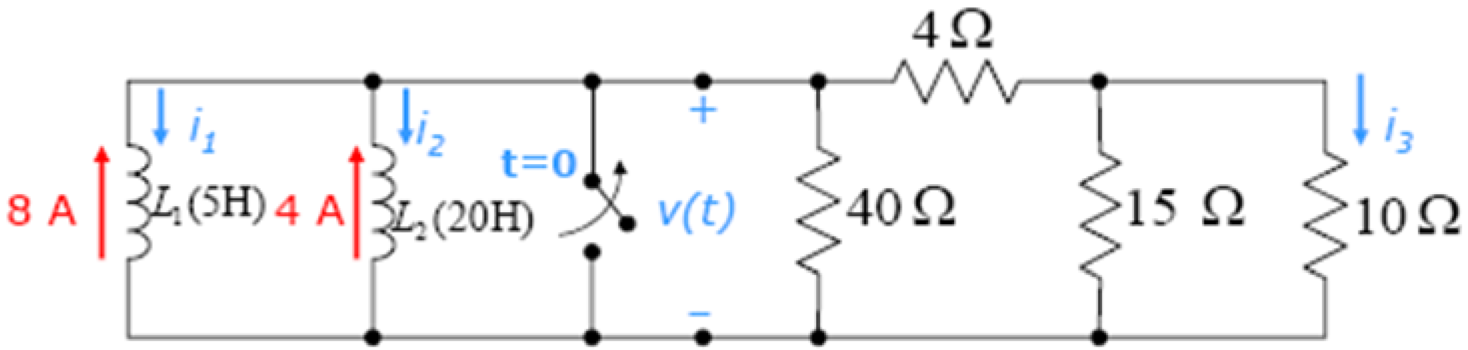
Example 2



a) Find i_1 , i_2 and i_3 .
b) Calculate the initial energy stored in the parallel inductors.
c) Calculate the energy stored in the inductor as t → ∞
d) Show that the total energy delivered to the resistive network equals to the difference between the result obtained in (b) and (c).

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Example 2 – Solution

a) The switch has been closed for a long time prior to t=0, so we know the voltage across the inductor

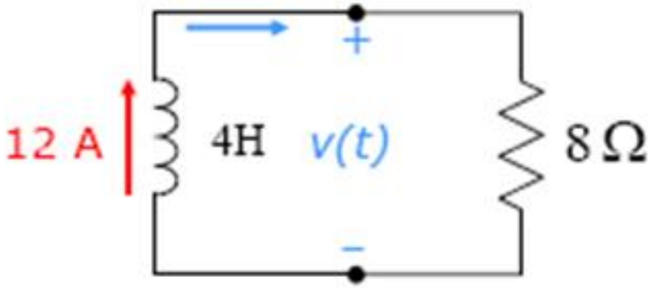
$$R_{eq} = (10 // 15 + 4) // 40 = 8\,\Omega$$

$$L_{eq} = 5 // 20 = 4\,\text{H}$$

$$i(t) = I_o e^{-\frac{R}{L}t}$$

$$i(t) = 12e^{-2t}\ \text{A} \quad t \geq 0$$

$$v(t) = 8i(t) = 96e^{-2t}\ \text{V} \quad t \geq 0^+$$



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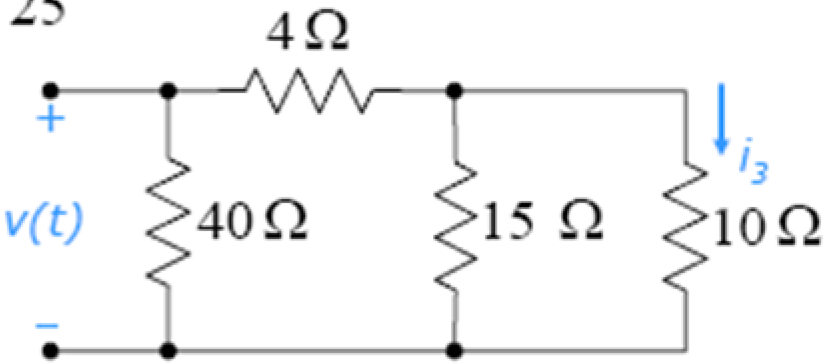
Example 2 – Solution (cont)

$$i(t) = \frac{1}{L}\int_0^t v(t)dt + i(t_o)$$

$$i_1 = \frac{1}{5}\int_0^t 96e^{-2x}dx - 8 = 1.6 - 9.6e^{-2t}\text{A.} \quad t \geq 0$$

$$i_2 = \frac{1}{20}\int_0^t 96e^{-2x}dx - 4 = -1.6 - 2.4e^{-2t}\text{A.} \quad t \geq 0$$

$$i_3 = \frac{v(t)}{4 + \frac{10\times 15}{10+15}} \times \frac{15}{10+15} = \frac{v(t)}{10}\times\frac{15}{25} = 5.76e^{-2t}\text{A.} \quad \text{t} \geq 0^+$$



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Example 2 – Solution (cont)

b) Calculate the initial energy stored in the parallel inductors

$$w = \frac{1}{2}Li^2 = \frac{1}{2}(5)(64) + \frac{1}{2}(20)(16) = 320\ \text{J}$$

c) Determine how much energy is stored in the inductor as $t \to \infty$

$$t \to \infty \quad i_1 = 1.6 \qquad\qquad t \to \infty \quad i_2 = -1.6$$

$$w = \frac{1}{2}Li^2 = \frac{1}{2}(5)(1.6)^2 + \frac{1}{2}(20)(-16)^2 = 32\ \text{J}$$

d) Show that total energy delivered to the resistive network equals the difference between the results obtained in (b) and (c)

$$w = \int_0^\infty p\,dt = \int_0^\infty iv\,dt = \int_0^\infty 1152e^{-4t}\,dt = 288\ \text{J}$$

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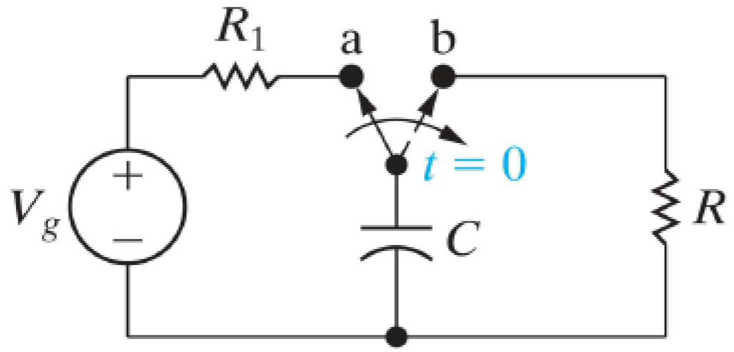
The natural response of an *RC* circuit

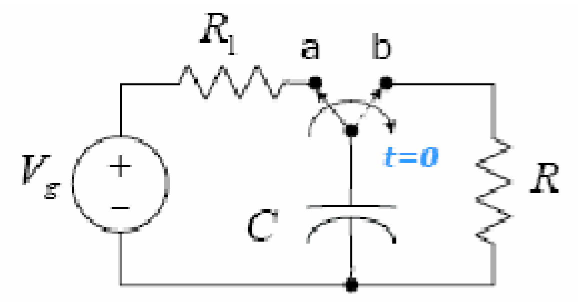
Circuit model of a discharging RC circuit

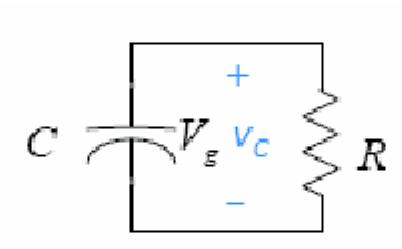
- Consider the following circuit model:



- For $t<0$, C is open and biased by a voltage V_g, while R_1 and R carry no current.
- For $t>0$, the capacitor voltage decreases and the energy is dissipated via R.

The natural response of an *RC* circuit





Assume the switch has been in position a for a long time:

t ≤ 0 ⟶ $\dfrac{dv}{dt} = 0$ ⟶ i = 0 (open circuit)

$\boxed{v_C = V_g}$

t ≥ 0 Apply node voltage technique:

$$C\frac{dv}{dt} + \frac{v}{R} = 0$$

first-order ODE for $v(t)$:

$$v(t) = v(0)e^{-\frac{t}{RC}} \quad t \geq 0$$

$$v(0^-) = v(0) = v(0^+) = V_g = V_o \qquad \tau = RC$$

$$\boxed{v(t) = V_0 e^{-t/\tau} \qquad t \geq 0}$$

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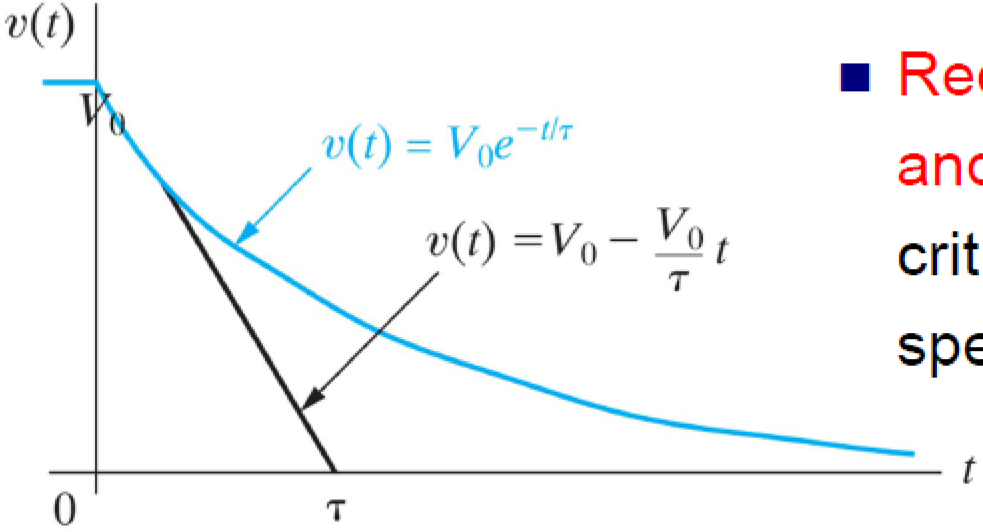
The natural response of an *RC* circuit

$\Rightarrow v(t) = V_0 e^{-(t/\tau)}$, where $\tau = RC$ …time constant



- Reducing R (loss) and parasitic C is critical for high-speed circuits.

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The natural response of an *RC* circuit

The current goes through the resistor

$$i(t)=\frac{v(t)}{R}=\frac{V_0}{R}e^{-(t/\tau)} \qquad t\geq 0^+$$

The power dissipated in the resistor

$$p=vi=\frac{V_0^2}{R}e^{-2(t/\tau)} \qquad t\geq 0^+$$

The energy delivered to the resistor

$$w=\int_0^t p\,dt=\int_0^t \frac{V_0^2}{R}e^{-2(t/\tau)}dt=\frac{1}{2}CV_0^2\left(1-e^{-2(t/\tau)}\right) \qquad t\geq 0$$

$\boxed{w_0=\frac{CV_0^2}{2}}$ initial energy stored in C

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Procedures to get natural response of RL, RC circuits

1. Find the equivalent circuit.
2. Find the initial conditions: initial current I_0 through the equivalent inductor, or initial voltage V_0 across the equivalent capacitor.
3. Find the time constant of the circuit by the values of the equivalent R, L, C:

$$\tau = L/R, \text{ or } RC;$$

4. Directly write down the solutions:

$$i(t) = I_0 e^{-(t/\tau)}, \; v(t) = V_0 e^{-(t/\tau)}.$$

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Example 3

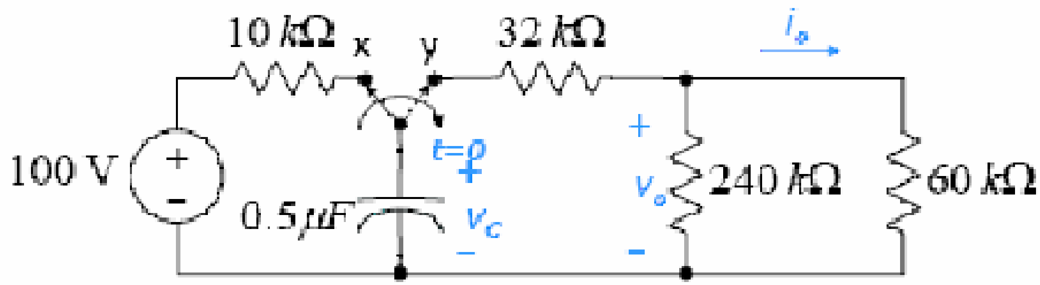
Find:

a) $v_C(t)$ for $t \geq 0$,

b) $v_o(t)$ for $t \geq 0^+$,

c) $i_o(t)$ for $t \geq 0^+$, and

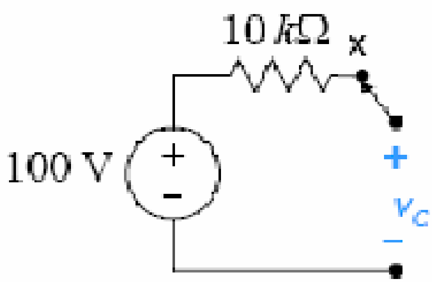
d) the total energy dissipated in the 60 kΩ resistor.



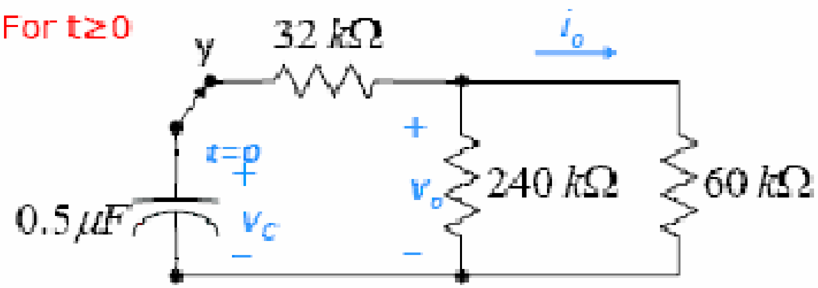
a) $v_C(t)$ for $t \geq 0$

$v_C = V_s = V_o = 100\ V$

When the switch is closed for a long time, the capacitor is an open circuit



For t≥0



$R = 32k + 240k\,//\,60k = 80k\Omega$

$\tau = RC = 80k \times 0.5\mu = 0.04$ $\quad V_o = 100V$

$v_C(t) = 100e^{-25t}$ $t \geq 0$

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Example 3 (cont)

b) $v_o(t)$ for $t \geq 0^+$

$$v_o(t) = \frac{100}{80 \times 10^3} e^{-25t} \times 48 \times 10^3 = 60e^{-25t}\ \text{V} \quad t \geq 0^+$$

c) $i_o(t)$ for $t \geq 0^+$

$$i_o(t) = \frac{60}{60 \times 10^3} e^{-25t} = 0.001e^{-25t}\ \text{A}$$

d) The total energy dissipated in the 60 kΩ resistor

$$p = iv = 0.06e^{-50t}\ \text{W}$$

$$w = \int_o^\infty p\,dt = \int_o^\infty 0.06e^{-50t}\,dt = 1.2\ \text{mJ}$$

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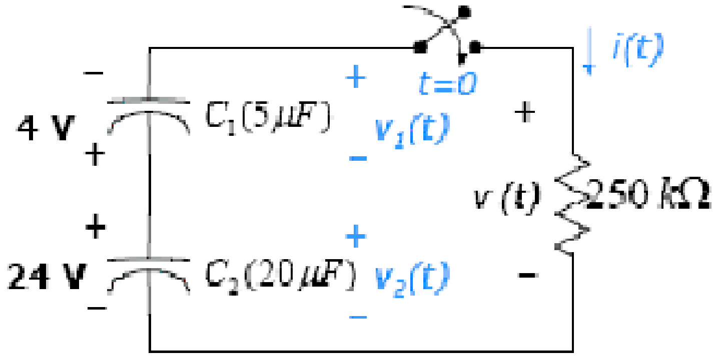
Example 4



a) Find $v_1(t)$, $v_2(t)$, and $v(t)$ for $t \geq 0$

$V_{s1} = -4\text{ V} \qquad V_{s2} = 24\text{ V}$

For $t \geq 0$

$$C_{eq} = \frac{5\mu \times 20\mu}{5\mu + 20\mu} = 4\mu F$$

$$\tau = RC_{eq} = 250\times 10^3 \times 4\times 10^{-6} = 1s$$

$$v(t) = (24-4)e^{-t} = 20e^{-t}\text{ V} \quad t \geq 0$$

$$i(t) = \frac{v(t)}{R} = \frac{20e^{-t}}{250\times 10^3} = 80e^{-t}\ \mu A \quad t \geq 0^+$$

$$v_1(t) = -\frac{1}{C_1}\int_0^t i(t)dt + v_1(0) = -\frac{10^6}{5}\int_0^t 80\times 10^{-6}e^{-t}dt - 4 = \left(16e^{-t} - 20\right)V \quad t \geq 0$$

$$v_2(t) = -\frac{1}{C_2}\int_0^t i(t)dt + v_2(0) = -\frac{10^6}{20}\int_0^t 80\times 10^{-6}e^{-t}dt + 24 = \left(4e^{-t} + 20\right)V \quad t \geq 0$$

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Example 4 (cont)

b) Calculate the initial energy stored in the capacitor C_1 and C_2

$$w_1 = \frac{1}{2}CV^2 = \frac{1}{2}(5\times10^{-6})(4)^2 = 40\ \mu J$$
$$w_2 = \frac{1}{2}CV^2 = \frac{1}{2}(20\times10^{-6})(24)^2 = 5760\ \mu J$$

$$w_{total} = w_1 + w_2 = 40\ \mu J + 5760\ \mu J = 5800\ \mu J$$

c) Calculate how much energy is stored in the Capacitors as t → ∞

$$w_1 = \frac{1}{2}CV^2 = \frac{1}{2}(5\times10^{-6})(-20)^2 = 1000\ \mu J$$
$$w_2 = \frac{1}{2}CV^2 = \frac{1}{2}(20\times10^{-6})(20)^2 = 4000\ \mu J$$
$$w_{total} = w_1 + w_2 = 1000\ \mu J + 4000\ \mu J = 5000\ \mu J$$

d) Show that the total energy delivered to the 250 kΩ resistor is the difference between the results obtained in (b) and (c)

$$p = iv = 1.6e^{-2t}\ \text{mW}$$

$$w = \int_0^\infty p\,dt = \int_0^\infty 1.6\times10^{-3}e^{-2t}\,dt = 800\ \mu J$$

$$800\ \mu J = (5800\ \mu J - 5000\ \mu J)$$

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The Step Response of RL and RC Circuits

1. Charging an RC circuit
2. Charging an RL circuit

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What is step response?

- The response of a circuit to the sudden application of a constant voltage or current source, describing the charging behavior of the circuit.
- Step (charging) response and natural (discharging) response show how the signal in a digital circuit switches between Low and High with time.

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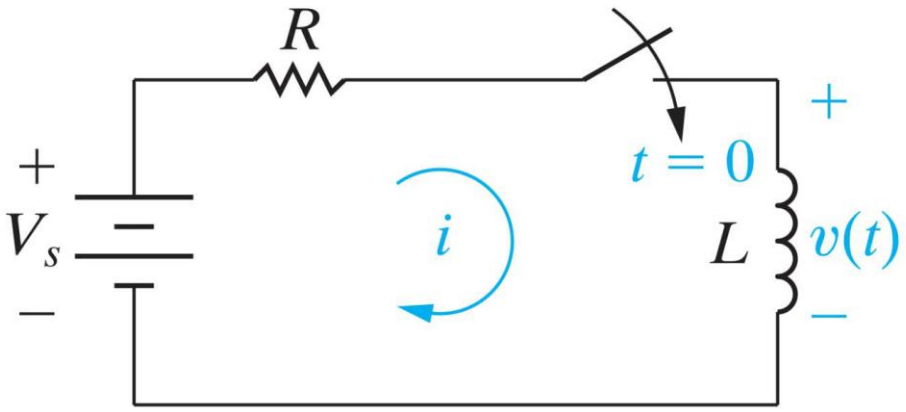
The step response of an *RL* circuit

Energy stored in the inductor at the time the switch is closed is given in terms of a nonzero initial current *i(0)*.



For $\boxed{t \geq 0}$

KVL $V_s = iR + L\dfrac{di}{dt}$

$$\frac{di}{dt} = \frac{-Ri + V_s}{L} = -\frac{R}{L}\left(i - \frac{V_s}{R}\right) \rightarrow di = -\frac{R}{L}\left(i - \frac{V_s}{R}\right)dt \rightarrow \frac{di}{i - \frac{V_s}{R}} = -\frac{R}{L}dt$$

$$\int_{i(0)=I_o}^{i(t)} \frac{di}{i - \frac{V_s}{R}} = -\frac{R}{L}\int_0^t dt \rightarrow \ln\left(\frac{i(t) - \left(\frac{V_s}{R}\right)}{I_o - \left(\frac{V_s}{R}\right)}\right) = -\frac{R}{L}t \rightarrow \frac{i(t) - \left(\frac{V_s}{R}\right)}{I_o - \left(\frac{V_s}{R}\right)} = e^{-\frac{R}{L}t}$$

$$\boxed{i(t) = \frac{V_s}{R} + \left(I_o - \frac{V_s}{R}\right)e^{-\frac{R}{L}t}}$$

SEE

The step response of an *RL* circuit

If at $t=0 \rightarrow I_o = 0$

$$i(t)=\frac{V_s}{R}+\left(I_o-\frac{V_s}{R}\right)e^{-\frac{R}{L}t} \rightarrow i(t)=\frac{V_s}{R}\left(1-e^{-\frac{R}{L}t}\right)$$

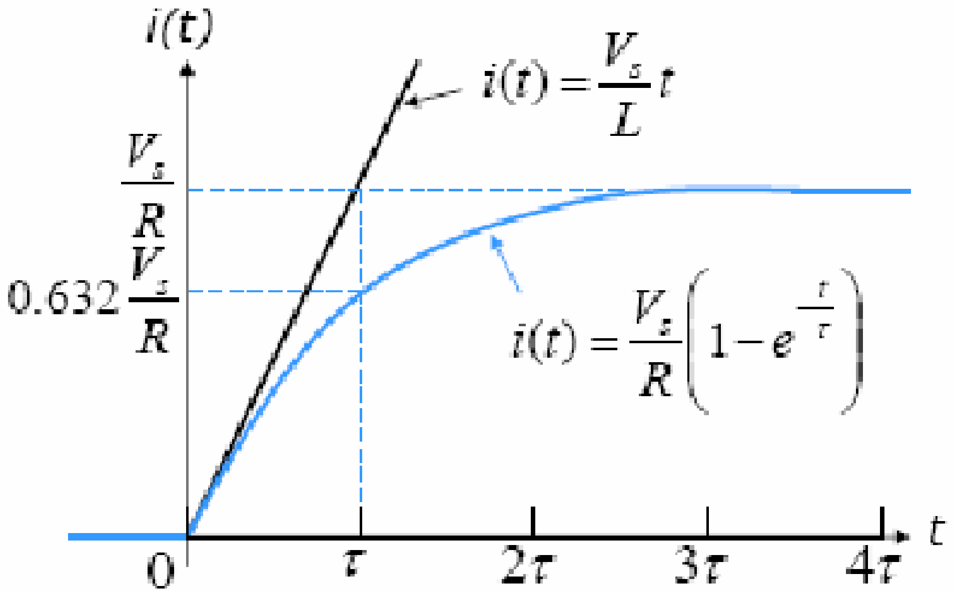
At $t=\infty \rightarrow i(\infty)=\frac{V_s}{R}$

At $t=\tau=\frac{L}{R}$

$$i(t)=\frac{V_s}{R}\left(1-e^{-1}\right)\approx 0.6321\frac{V_s}{R}$$

$$\frac{di}{dt}=\frac{-V_s}{R}\left(\frac{-1}{\tau}\right)e^{-t/\tau}=\frac{V_s}{L}e^{-t/\tau}$$

$$\frac{di}{dt}(0)=\frac{V_s}{L}$$



SEE

The step response of an *RL* circuit

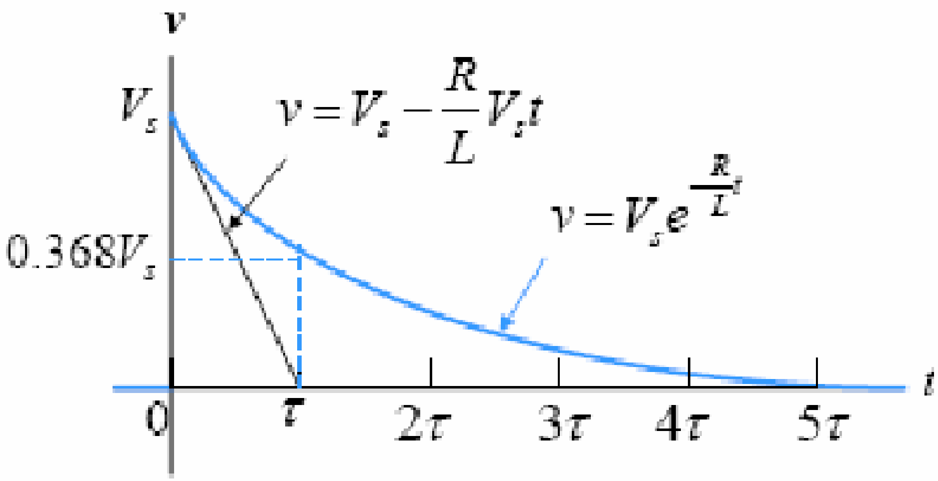
$$v = L\left(\frac{-R}{L}\right)\left(I_o - \frac{V_s}{R}\right)e^{-\frac{R}{L}t} = (V_s - I_oR)e^{-\frac{R}{L}t}$$

$$v(0^-) = 0 \qquad\qquad v(0^+) = V_s - I_oR$$

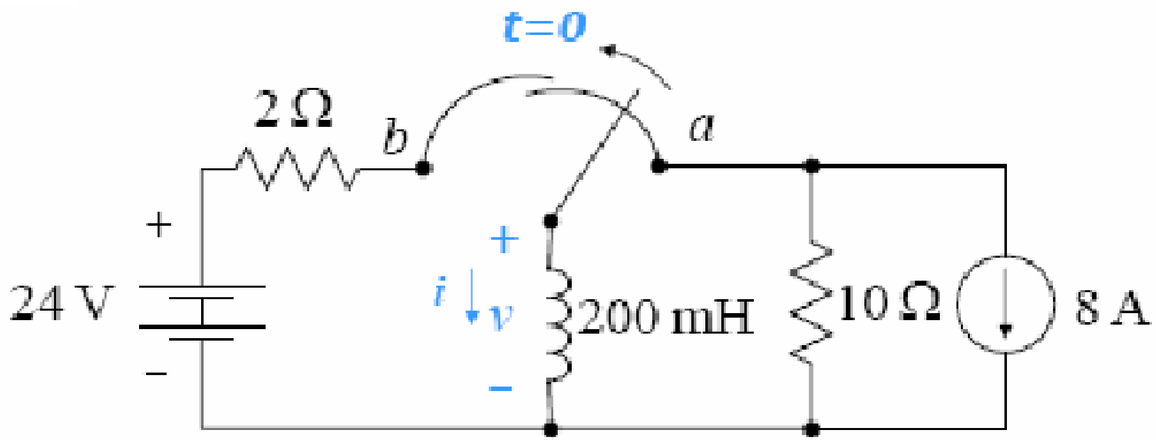
If at $t = 0 \rightarrow I_o = 0$

$$v = V_se^{-\frac{R}{L}t}$$

At $t = \infty \rightarrow v(\infty) = 0$



Example 5



Make-before-break Switch= The connection at position b is established before the connection at position a is broken, so there is no interruption of current through the inductor.

a) Find the expression for *i(t)* for $t \geq 0$.

For $t < 0$ $\quad I_o = -8\,\text{A}$

For $t \geq 0$ $\quad \tau = \frac{L}{R} = \frac{200 \times 10^{-3}}{2} = 0.1s$

$$i(t) = \frac{V_s}{R} + \left(I_o - \frac{V_s}{R}\right)e^{-\frac{R}{L}t} = 12 + (-8 - 12)e^{-10t} = 12 - 20e^{-10t}$$

SEE

Example 5 (cont)

b) What is the initial voltage across the inductor just after the switch has been moved to position b?

$$v = L\frac{di}{dt} = 0.2\left(200e^{-10t}\right) = 40e^{-10t}\text{V} \qquad t \geq 0^+$$

$$v(0^+) = 40\text{V}$$

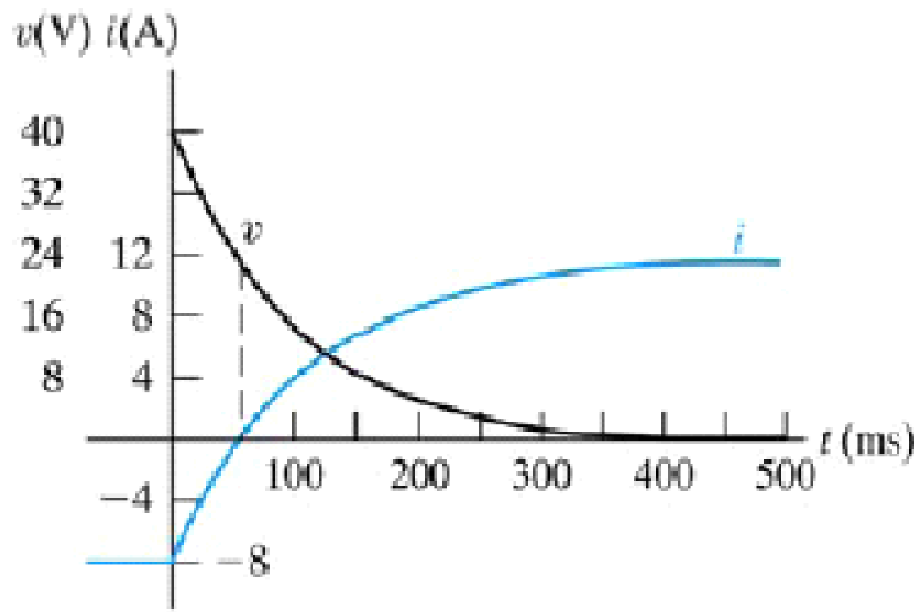
c) How many milliseconds after the switch has been moved does the inductor voltage equal to 24 V?

$$24 = 40e^{-10t}$$

$$t = \frac{1}{10}\ln\frac{40}{24} = 51.08\times10^{-3}$$

$$t = 51.08\text{ ms}$$

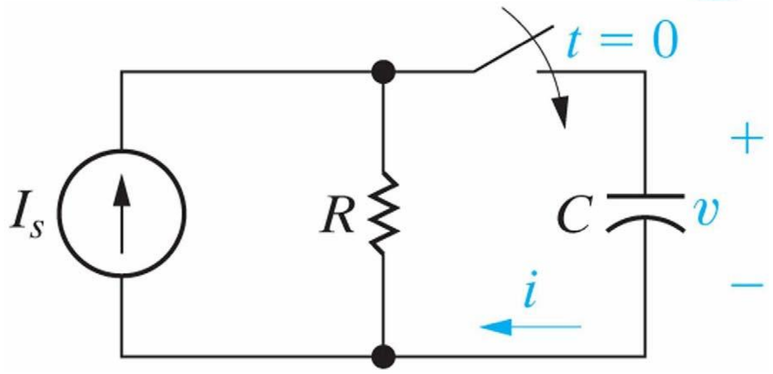
d) Plot $i(t)$ & $v(t)$?



The step response of an *RC* circuit

SEE

Initial condition depends on initial energy of the capacitor: $V(0^+) = v(0^-) \equiv V_0$



Apply KCL: $$C\frac{dv}{dt} + \frac{v_C}{R} = I_s$$

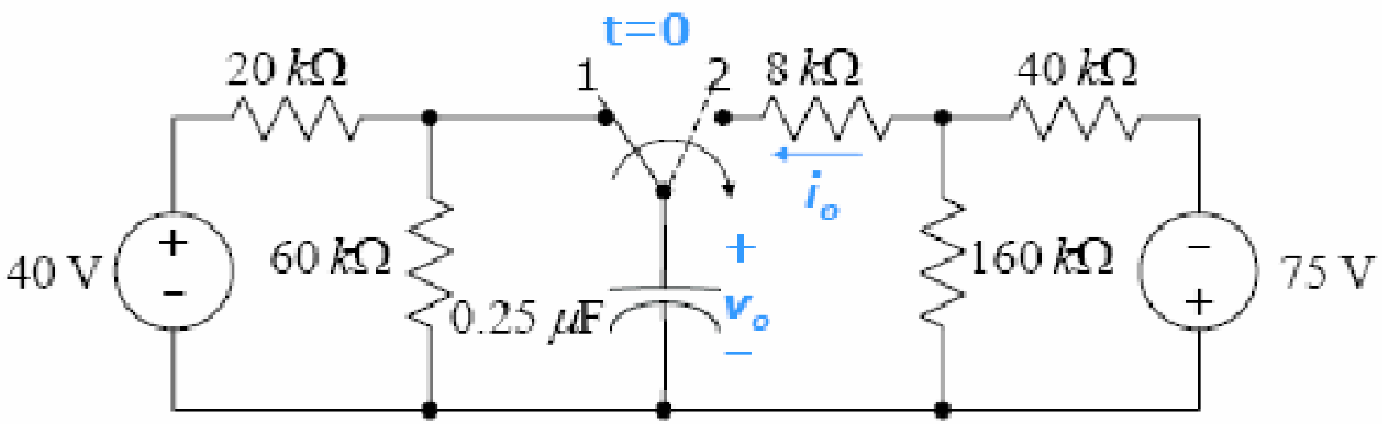
$$\boxed{v_C(t) = I_sR + (V_0 - I_sR)e^{-t/RC}}, \quad t \geq 0$$

$$i(t) = C\frac{dv_C}{dt} = C(V_0 - I_sR)\left(\frac{-1}{RC}\right)e^{-t/RC}$$

$$\boxed{i(t) = \left(I_s - \frac{V_0}{R}\right)e^{-t/RC}}, \quad t \geq 0^+$$

SEE

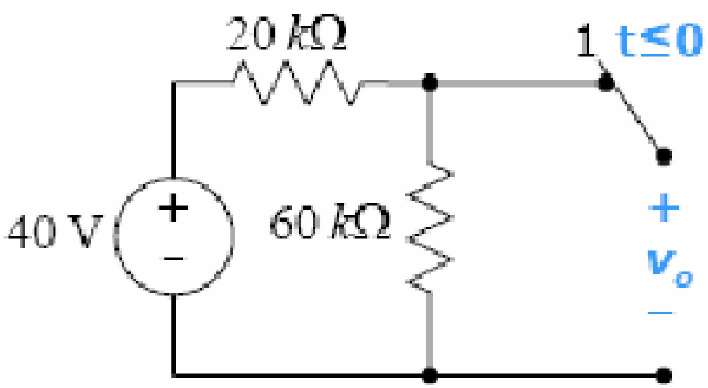
Example 6



a) Find $v_o(t)$ for $t \geq 0$

$$\frac{dv_C}{dt} = 0 \dashrightarrow i_C = 0 \dashrightarrow \text{(open circuit)}$$

$$v_o = 40\frac{60k}{20k + 60k} = 30\ \text{V}$$



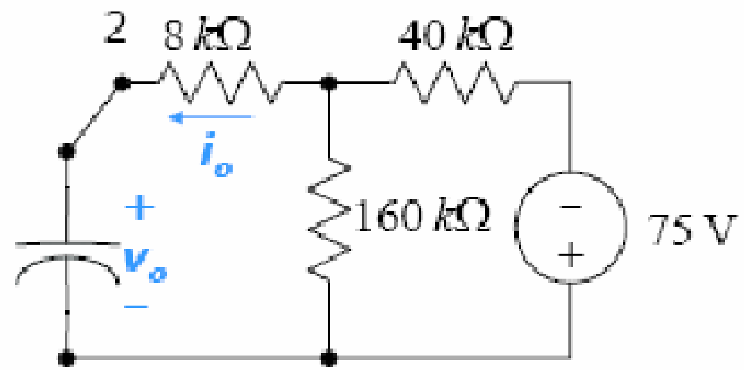
SEE

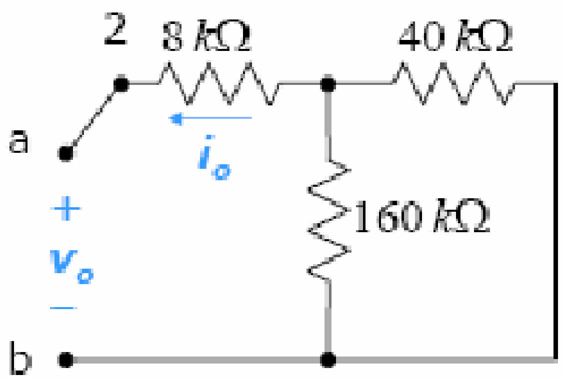
Example 6 (cont)

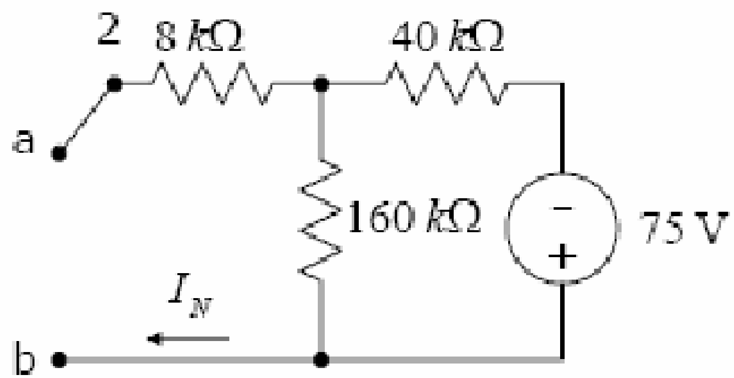
For t ≥ 0

$$v_C(t) = I_s R + (V_o - I_s R)e^{-\frac{t}{RC}}$$

Apply Source Transformation Or Norton Equivalent





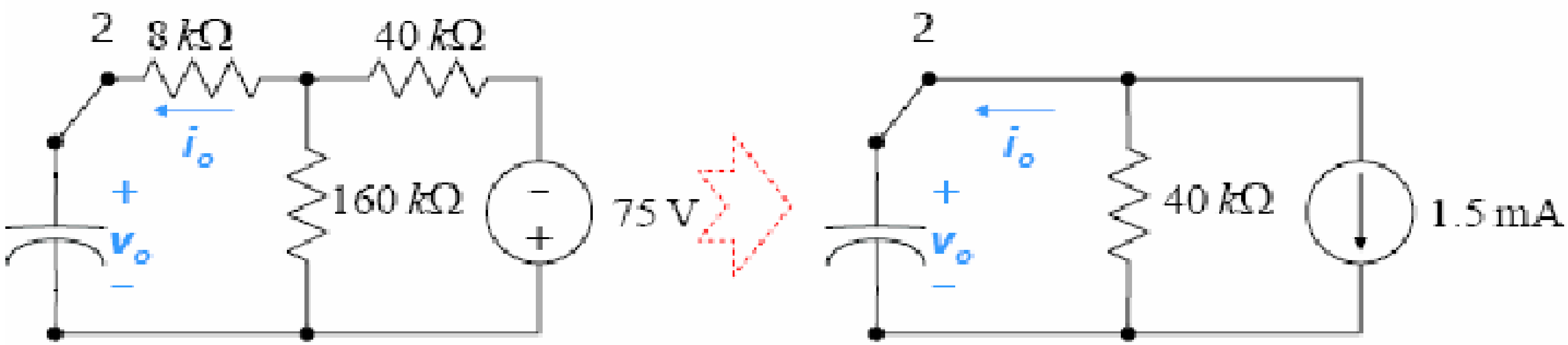


$R_N = R_{Th} = 40\ k\Omega$

$V_{Th} = 60\ \text{V} \dashrightarrow I_N = \dfrac{60^V}{40^{k\Omega}} = 1.5\ \text{mA}$

SEE

Example 6 (cont)



$$v_C(t) = I_s R + (V_o - I_s R)e^{-\frac{t}{RC}}$$

$V_o = v_o = 30\,V$ $\quad R = R_N = 40\,k\Omega$ $\quad I = I_S = -1.5mA$ $\quad RC = 0.01$

$$v_o(t) = v_C(t) = -60 + (30 + 60)e^{-100t} = -60 + 90e^{-100t} \qquad t \geq 0$$

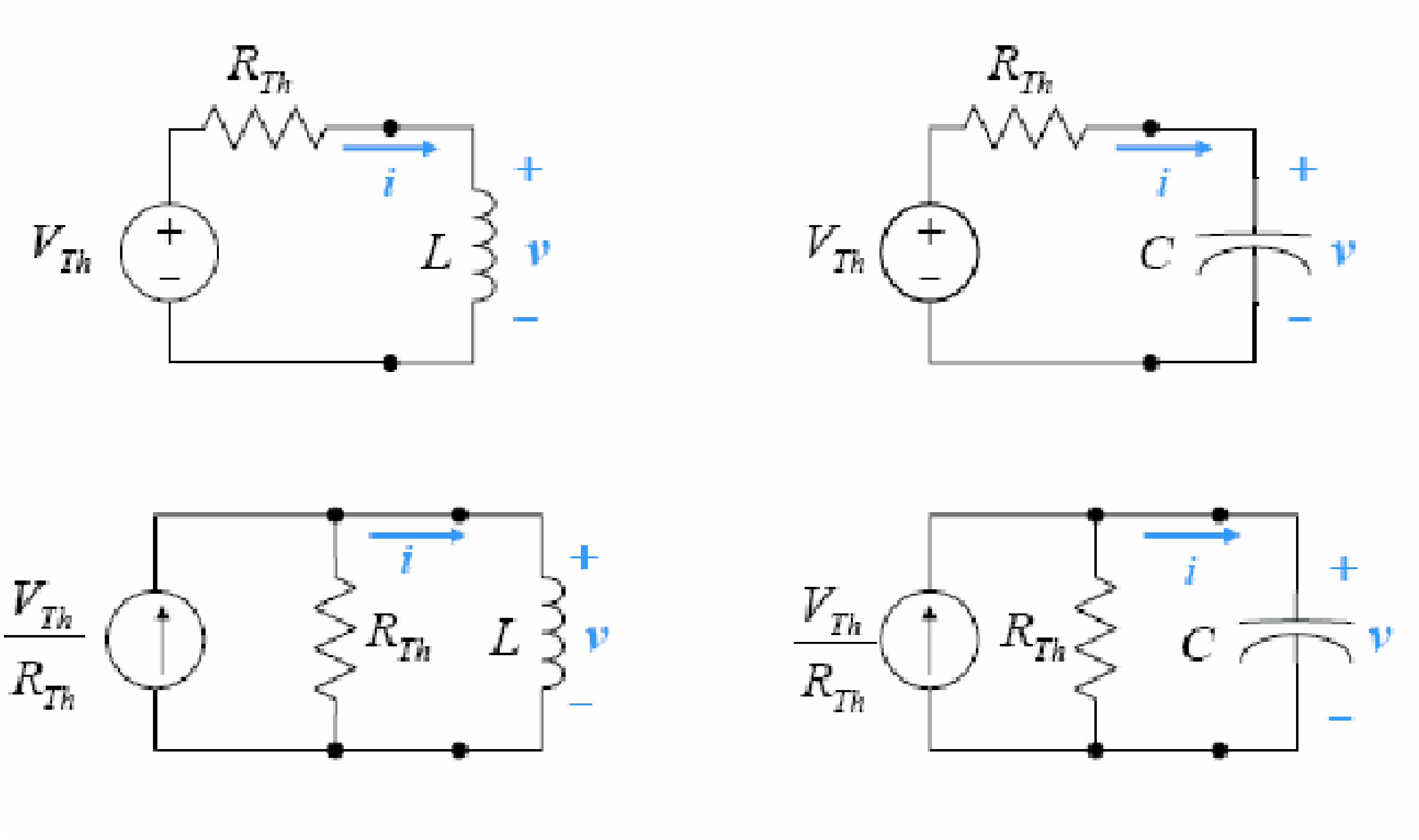
b) Find $i_o(t)$ for t ≥ 0^+

$$i_o(t) = C\frac{dv_o}{dt} = (0.25\times10^{-6})(-9000\times10^{-100t})$$

$$i_o(t) = -2.25e^{-100t}\ \text{mA}$$

SEE

A general solution for natural & step responses



SEE

A general solution for natural & step responses

$$x(t) = x_f + [x(t_0) - x_f]e^{-\left(\frac{t-t_0}{\tau}\right)}$$

x(t) the unknown variable as a function of time

x_f the final value of the variable

$x(t_0)$ the initial value of the variable

t_0 time of switching

T time constant

Procedure:

1) Identify the variable of interest of the circuit. For RC circuits, it is best to choose v_C ; for RL circuit, it is best to choose i_L.

2) Determine the initial value of the variable.($v_c(t_0)$ in case of RC circuit and $i_L(t_0)$ in case of RL circuits)

3) Calculate the final value of the variable (value at t = ∞)

4) Calculate the time constant for the circuit.

Sequential Switching

SEE

- Switching occurs more than once in a circuit.
- The time reference for switching cannot be t = 0.

Procedure for sequential switching problem

(1) Obtain the initial value $x(t_0)$

(2) Apply the techniques described previously to find current and voltage value.

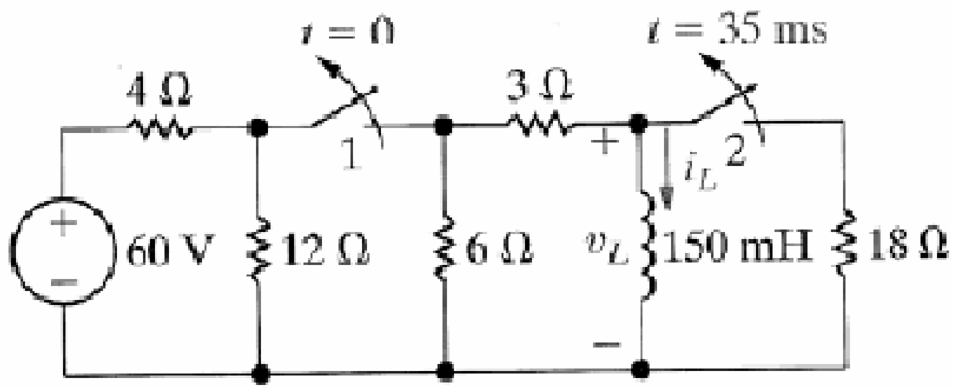
(3) Redraw the circuit that pertains to each time interval and repeat step (1).

Note: Since inductive current I_L and capacitive voltage V_C can change instantaneously at the time of switching, these value should be solved first for sequential switching problem.

Example 8

At t=0 switch 1 is opened; Then, 35 ms later, switch2 is opened

Find



a) $i_L(t)$ for $0 \le t \le 35$ ms.

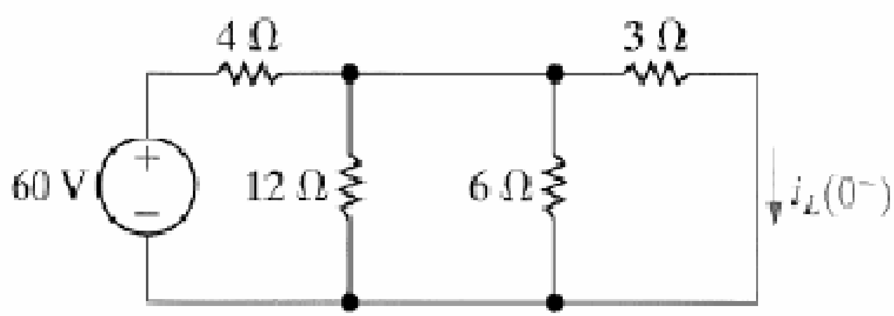
Both switches are initially closed for a long time

$$R_{eq} = \frac{1}{\frac{1}{12}+\frac{1}{6}+\frac{1}{3}} = \frac{12}{7}\,\Omega$$

$$6 // 12 = 4\,\Omega$$

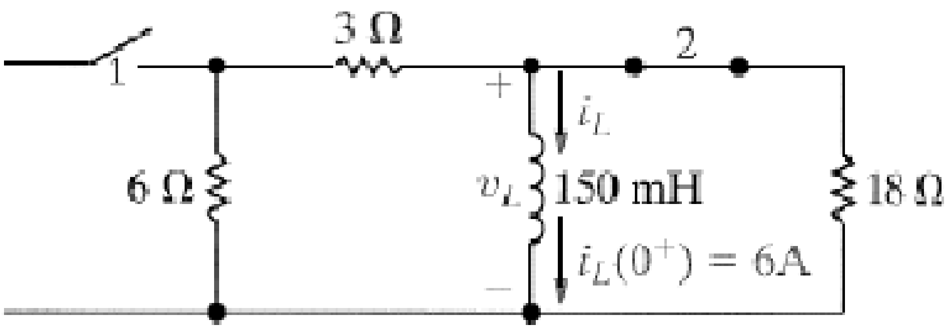
$$i_L(0^-) = \frac{60}{\frac{12}{7}+4} \times \frac{4}{4+3} = 6\text{ A}$$

Switch 1 is open



Example 8 (cont)

Switch 1 is open

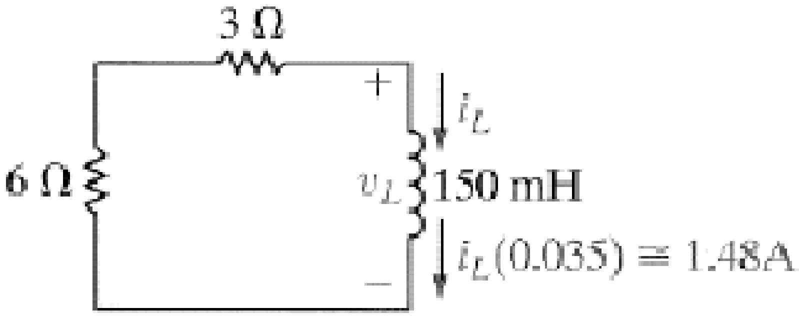


$$R_{eq} = 9//18 = 6\Omega$$

$$\tau = \frac{L}{R} = \frac{150\times10^{-3}}{6} = 0.025\,s$$

$$i_L(t) = 6e^{-40t}\ \text{A} \qquad 0 \le t \le 35\ \text{ms}$$

b) $i_L(t)$ for $t \ge 35$ ms.



$$i_L(35\ ms) = 6e^{-40\left(35\times10^{-3}\right)} = 1.48\ \text{A}$$

$$\tau = \frac{L}{R} = \frac{150\times10^{-3}}{9} = 0.016666\,s$$

$$i_L(t) = 1.48e^{-60(t-0.035)}\ \text{A} \qquad t \ge 35\ \text{ms}$$

SEE

Example 8 (cont)

c) What % of the initial energy stored in the 150 mH inductor is dissipated in the 18 Ω resistor.

$$w_{in.} = \frac{1}{2}Li^2 = \frac{1}{2}\times 150\times 10^{-3}\times(6)^2 = 2.7\,J$$

$$v_L = L\frac{di}{dt} = -36e^{-40t}$$

$$P = \frac{v_L^2}{18} = 72e^{-80t}\ \text{W}$$

$$w = \int_0^{0.035} 72e^{-80t}\,dt = 845.27\ \text{mJ}$$

$$\%w = \frac{845.27\ \text{mJ}}{2700\ \text{mJ}}\times 100 = 31.31\%$$

SEE

Example 8 (cont)

d) Repeat (c) for the 3 Ω resistor

$0 \le t \le 35\,\text{ms}$

$$v_{3\Omega} = \left(\frac{v_L}{9}\right)(3) = \frac{1}{3}v_L = -12e^{-40t}\ \text{V}$$

$$w_{3\Omega} = \int_0^{0.035} \frac{144}{3}e^{-80t}dt = 563.51\,\text{mJ}$$

$t > 35\,\text{ms}$

$$i_{3\Omega} = i_L = 1.48e^{-60(t-0.035)}\ \text{A}$$

$$w_{3\Omega} = \int_{0.035}^{\infty} i_{3\Omega}^2 \times 3dt = 54.73\,\text{mJ}$$

$$w_{3\Omega}(\text{total}) = 563.51\,\text{mJ} + 54.73\,\text{mJ} = 618.24\,\text{mJ}$$

$$\%w = \frac{618.24\,\text{mJ}}{2700\,\text{mJ}} \times 100 = 22.90\%$$

Unbounded Response

SEE

Definition and reason of unbounded response

- An unbounded response means the voltages or currents increase with time without limit.

- It occurs when the Thévenin resistance is negative ($R_{Th} < 0$), which is possible when the first-order circuit contains dependent sources.

SEE

Unbounded Response

A circuit response may grow, rather than decay, exponentially with time.

This type of response is called an **unbounded response**.

It may happen when the circuit contains dependent source.

In this case, the Thevenin equivalent with respect to the terminals of either an inductor or a capacitor may be negative, which resulting in a negative time constant.

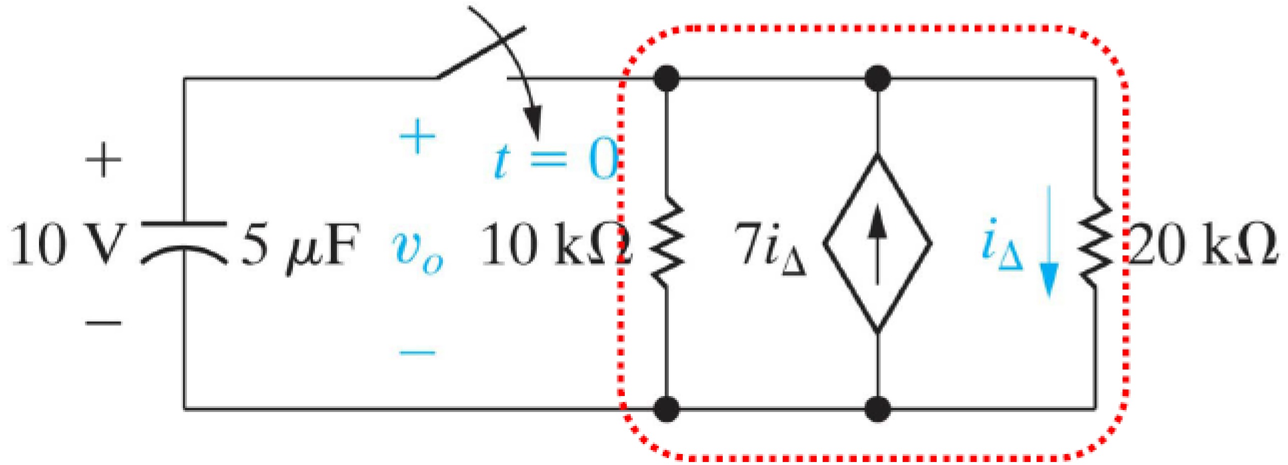
To solve the circuit which have unbounded response, we need to derive the differential equation that describes the circuit containing the negative R_{th}.

SEE

Unbounded Response

Example

- Q: $v_o(t) = ?$ for $t \geq 0$.



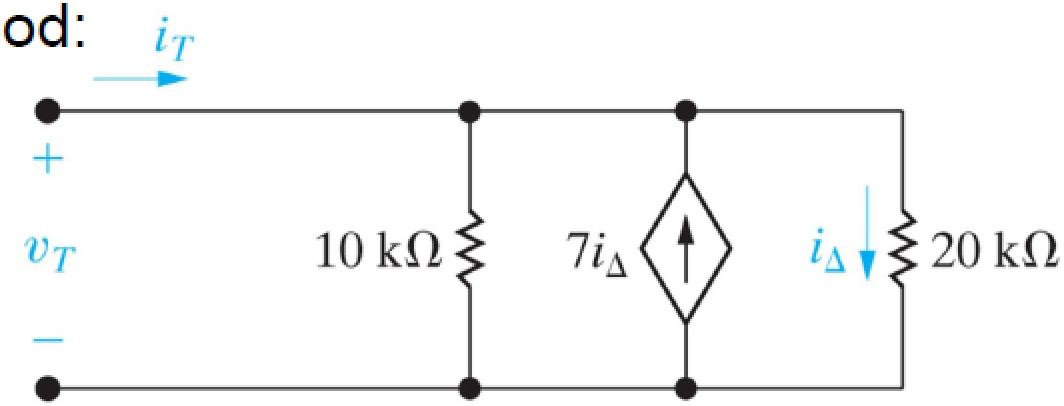
- For $t > 0$, the capacitor seems to "discharge" (not really, to be discussed) via a circuit with a current-controlled current source, which can be represented by a Thévenin equivalent.

Unbounded Response

SEE

Example

- Since there is no independent source, $V_{Th}=0$, while R_{Th} can be determined by the test source method:



$$i_T = \frac{v_T}{10\text{ k}\Omega} + \frac{v_T}{20\text{ k}\Omega} - 7\frac{v_T}{20\text{ k}\Omega},$$

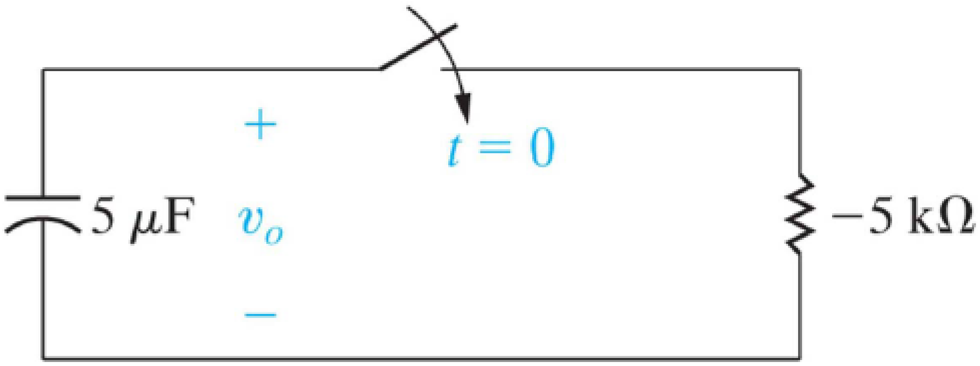
$$\Rightarrow R_{Th} = v_T / i_T = -5\text{ k}\Omega < 0.$$

SEE

Unbounded Response

Example

- For $t \geq 0$, the equivalent circuit and governing differential equation become:



$$V_0 = v_o(0^+) = 10\text{ V},\ \tau = RC = -25\text{ ms} < 0,$$

$$\Rightarrow v_o(t) = V_0 e^{-t/\tau} = 10\boxed{e^{+40t}}\text{ V}.$$ ..grow without limit.

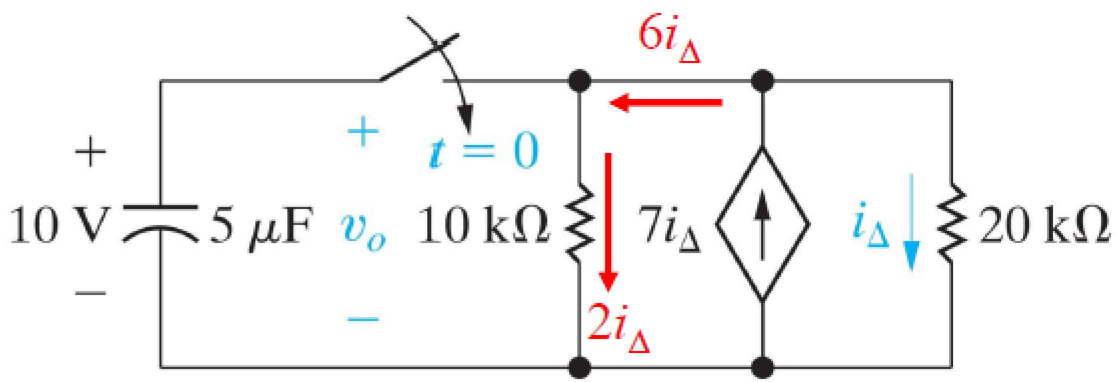
Unbounded Response

SEE

Example

Why the voltage is unbounded?

- Since 10-kΩ, 20-kΩ resistors are in parallel, $\Rightarrow$ $i_{10k\Omega} = 2i_\Delta$, the capacitor is actually charged (not discharged) by a current of $4i_\Delta$!
- Charging effect will increase v_o, which will in turn increase the charging current $(i_\Delta = v_o/20\ k\Omega)$ and v_o itself. The positive feedback makes v_o soaring.



Lesson for circuit designers & device fabrication engineers

Undesired interconnection between a capacitor and a sub-circuit with dependent source (e.g. transistor) could be catastrophic!