Question 4: (25 Marks)

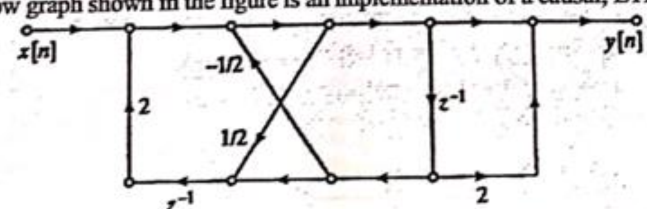
The 2-sided z-transform of transfer function x(n) of a system is given by

$$X(z)=\frac{z^{-1}}{(1-3z^{-1})(1-5z^{-1})}$$

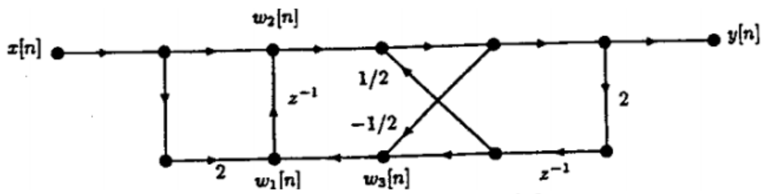
a. Determine all possible ROCs for X(z)
b. For each ROC in (a), find x(n)
c. Discuss the stability and causality of each case.
d. Sketch the pole and zero pattern then draft the frequency response of the system.
e. Realize the canonical form of the system.

Question 5: (25 Marks)

The flow graph shown in the figure is an implementation of a causal, LTI system



a. Determine the difference equation relating to the input signal x(n) to the y(n)
b. Find and sketch the pole/zero pattern of the system. Is the system stable?
c. Determine y(2) if x(n)=(1/2)ⁿ u(n).



$$(1)\ w_1[n] = 2x[n] + w_3[n]$$

$$(2)\ w_2[n] = x[n] + w_1[n-1]$$

$$(3)\ w_3[n] = -\frac{1}{2}y[n] + 2y[n-1]$$

$$(4)\ y[n] = w_2[n] + y[n-1]$$

Z-transform of the above equations, substituting and rearranging terms, we get:

$$(1-\frac{1}{2}z^{-1}-2z^{-2})Y(z) = (2z^{-1}+1)X(z).$$

inverse Z-transforming, we get the following difference equation:

$$y[n]-\frac{1}{2}y[n-1]-2y[n-2] = x[n]+2x[n-1].$$

the system function is given by:

$$H(z) = \frac{1+2z^{-1}}{1-\frac{1}{2}z^{-1}-2z^{-2}}.$$

It has poles at

$$z = -\frac{8}{1-\sqrt{33}} \text{ and } z = -\frac{8}{1+\sqrt{33}}$$

which are outside the unit circle, therefore the system is NOT stable.

(c)

$y[2] = x[2] + 2x[1] + \frac{1}{2}y[1] + 2y[0]$

$y[0] = x[0] = 1$

$y[1] = x[1] + 2x[0] + \frac{1}{2}y[0] = \frac{1}{2} + 2 + \frac{1}{2} = 3$

$$y[2] = \frac{1}{4} + 1 + \frac{3}{2} + 2 = \frac{19}{4}.$$