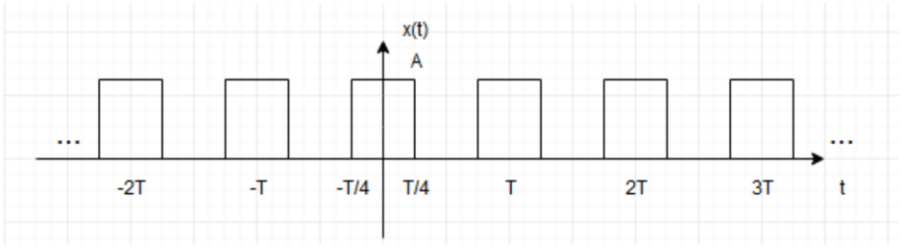
DATE: November 12, 2018 – Duration: 90 minutes

Question 1 (20 marks)

a) Consider the signal $x(t) = e^{-t}[u(t) - u(t-2)]$. Sketch the signal and calculate the energy of the signal $x(t)$.

b) Consider the square wave signal as shown in Figure below



Determine the power of the signal $x(t)$ in terms of A and T.

Question 2 (20 marks)

a) Discuss the linearity and time-invariance properties of the system: $y(t) = x^2(t)$

b) Discuss the linearity and time-invariance properties of the system: $y[n] = x[2n-1] + 3$

Question 3 (20 marks)

The input-output relationship of the discrete-time LTI system is given by

$$y[n] = 4x[n] - 2x[n-1] + 3x[n-2]$$

where $x[n]$ and $y[n]$ are the system input and output, respectively

a) Find the unit impulse response $h[n]$ of the system.

b) Find the output $y[n]$ for the input $x[n] = [2, -1, 1, 3, 1]$.

Question 4 (20 marks)

An LTI analog system has the following unit impulse response

$$h(t) = 2e^{-2t}u(t)$$

Find the output $y(t) = x(t) * h(t) = \int_{-\infty}^{+\infty} x(\tau)h(t-\tau)d\tau$ for the input $x(t) = u(t+2) - u(t-2)$ through the following steps:

a) Plot $x(\tau)$ and $h(t-\tau)$ with respect to τ.

b) Considering the values of t for all possible cases (the two plots do not overlap, they partially overlap, one plot is completely covered by the other), compute y(t) for each case.

Question 5 (20 marks)

Consider an analog system whose input-output relationship is defined by

$$\frac{dy(t)}{dt} + 2y(t) = 6x(t)$$

where $x(t)$ is the input and $y(t)$ is the output.

a) Find the natural response $y^{(h)}(t)$ which is the solution to the homogeneous equation.

b) Assuming that $x(t) = 2u(t)$, find the forced response $y^{(p)}(t)$ for $t \geq 0$. Then, find the total response $y(t)$ for $t \geq 0$ if the initial condition $y(0) = 5$.