THE INTERNATIONAL UNIVERSITY(IU)
VIETNAM NATIONAL UNIVERSITY - HCMC

MIDTERM EXAMINATION
PROBABILITY, STATISTICS AND RANDOM PROCESS
Semester 1, 2021-22 • November 2021 • Total duration: 90 minutes

Chair of Mathematics Department	Lecturer
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INSTRUCTIONS: Each student is allowed calculators, statistical table for standard normal distribution and one double-sided sheet of reference material (size A4 or similar) marked with their name and ID. All other documents and electronic devices are forbidden.

1. *(10 points)* An insurance company classifies the arriving claims into four categories, depending on the size of these claims. From past data, it has been estimated that the first two categories, associated with the largest claims, arrive with the same probability, a claim from the third category is twice as much probable, and a claim from the fourth category is three times as much probable compared to a claim from the first or the second group. What is the probability that the next claim to arrive at the company will not be of the first category?

2. *(10 points)* On the way driving back home from school, An has to pass through two sets of traffic lights. The probability that she has to stop at the first is 0.3, while the probability that she has to stop at the second one is 0.40. and the probability that she does not stop at a traffic light is 0.40. Calculate the probability that she has to stop at least at one set of traffic lights.

3. *(10 points)* A telecommunications system transmits binary signals (0 or 1). The system includes a transmitter that emits the signals and a receiver which receives those signals. The probability that the receiver registers a signal 1 when the transmitter has sent a signal 1 is 99%, while the probability that the receiver registers a signal 0 when the transmitter has sent a signal 0 is 98%. Signals are transmitted every second. The probability that the signal 0 is transmitted is twice as the signal 1.

   The last signal has been registered as 1. Find the probability that the original signal transmitted was also 1.

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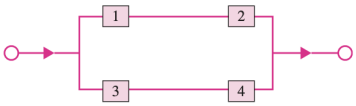
4. *(10 points)* Consider an electrical system comprises four components as following



Each component functions independently of one another. The probabilities that the component 1, 2, 3, 4 work are, respectively, 0.9, 0.92, 0.85, and 0.95.

What is the probability that the system works?

5. *(20 points)* A small internet company sells ebooks. The company has paid \$100 to obtain a new book electronically and sells each copy of the book at a price of \$50. The number of sales X for this book is a random variable with probability mass function

x	0	1	2	3	4	5
$P(X=x)$	$\frac{1}{15}$	$\frac{3}{15}$	$\frac{4}{15}$	$\frac{4}{15}$	$\frac{2}{15}$	$\frac{1}{15}$

(a) Find the probability mass function of the company's profit (or loss).

(b) What is the probability that the company will make a profit but that will be less than \$80?

(c) Compute the expected company's profit.

6. *(20 points)* The borrowing period (in days) for a particular book at a University library can be regarded as a continuous random variable X with probability density function

$$f(x) = \begin{cases} cx, & 0 \le x \le 5 \\ 0 & otherwise \end{cases}.$$

(a) Determine c.

(b) What is the probability that a book is returned within 2 days?

(c) Four students borrow a book. Find the probability that at least two of them return the book within 2 days.

7. *(20 points)* The daily demand for rice in a shop (in kilograms) has the normal distribution with mean $\mu = 800$ and standard deviation $\sigma = 45$. The current stock of the shop is 950 kg

(a) Find the probability that all this stock will be sold within a day.

(b) What is the probability that after the end of the next working day, the shop will have at least 100 kg of rice still in stock?

****** END ******

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998

Figure 1: Cumulative standard normal distribution