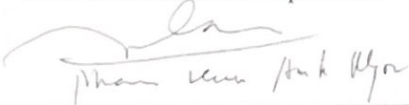## THE INTERNATIONAL UNIVERSITY(IU) - VNU HCMC

MIDTERM EXAMINATION
PROBABILITY, STATISTICS AND RANDOM PROCESS

Semester 1, 2022-23 • November 2022 • Total duration: 90 minutes

Chair of Mathematics Department	Lecturer
Pham um Anh Ngoc	Dr. Pham Hai Ha

INSTRUCTIONS: Each student is allowed calculators one double-sided sheet of reference material (size A4 or similar) marked with their name and ID and statistical tables. All other documents and electronic devices are forbidden.

1. *(10 points)* Flip a fair coin 4 times. Find the probability that the number of heads is greater than or equal to the number of tails.

2. *(10 points)* The percentages of people with each of the four blood types (O, A, B, and AB) in a region are as follows:

Blood type	A	B	AB	O
Percentage	30	12	3	55

   Select randomly a person in this region. Given that his/her blood type is either B or AB, find the (conditional) probability that his/her blood type is B.

3. *(10 points)* In a box, there is 2 blue balls and 18 green balls. Select randomly without replacement two balls from the box. What is the probability that the second ball selected is blue.

4. *(10 points)* A company has two stores of TV, one is located in Hanoi and another is in Hochiminh city. At the store in Hanoi, 20% of TV are defective. The percentage of TV which are defective in Hochiminh city is 15%. Choose randomly a store and from this store select randomly a TV. The selected TV is tested and found to be defective, what is the probability that it comes from the store in Hanoi.

5. *(20 points)* The probability function of a discrete random variable X has the form

$$p(x) = P(X = x) = c(x^2 + 3|x| + 1), \quad \text{for } x = -2, -1, 0, 1, 2.$$

   (a) Find c.

   (b) Compute $P(-2 \leq X < 1)$.

   (c) Evaluate $E(X)$ and $Var(X)$.

—— PLEASE TURN OVER ——

6. *(20 points)* The borrowing period, in days, for a particular book at a University library can be regarded as random variable X which has normal distribution with mean $\mu = 8$ and standard deviation $\sigma = 2$. A book need to be return within 10 days.

   (a) Compute $P(X > 10)$ - the probability that a new borrower returns the book after 10 days.

   (b) For a late return, the borrower has to pay a penalty of \$5. Otherwise, the borrower pays \$0. Evaluate the average payment of a borrower.

7. *(10 points)* Jack has invested \$1000 in product A and \$2000 in product B. He expects that if project A is success, he get a profit of \$800 and lose his money that he invested in A if A is unsuccessful. For project B, a successfull investement yields a profit of \$1000 and a uncessesfull of B makes him lose his money invested in B. He estimates the probability of success as following

		Project B	
		successful	unsuccessful
Project A	successful	0.6	0.05
	unsuccessful	0.25	0.1

Let X and Y be the his profit from project A and B respectively. Remark that X and Y can take negative value when he loses his money.

   (a) Determine the probability mass functions of X and Y.

   (b) Calculate $E(X)$ and $E(Y)$ - the profits each project.

   (c) Compute $E(X+Y)$ - the average of the overall profit from two projects.

8. *(10 points)* Consider a system of 4 components with structure as following

Suppose that all four components operate independently. Let T_1, T_2, T_3 and T_4 are lifetime or time to failure (in years) of the component 1, 2, 3, 4 respectively. Their probability density functions are given by

$$f_{T_1}(x) = \begin{cases} 0.1e^{-0.1x} & \text{for } x > 0 \\ 0 & \text{otherwise} \end{cases}, \qquad f_{T_2}(x) = \begin{cases} 0.2e^{-0.2x} & \text{for } x > 0 \\ 0 & \text{otherwise} \end{cases}$$

$$f_{T_3}(x) = \begin{cases} 0.2e^{-0.2x} & \text{for } x > 0 \\ 0 & \text{otherwise} \end{cases}, \qquad f_{T_4}(x) = \begin{cases} 0.1e^{-0.1x} & \text{for } x > 0 \\ 0 & \text{otherwise} \end{cases}$$

   (a) Compute $p_1 = P(T_1 > 1)$ - the probability that the component 1 lasts more than 1 year.

   (b) Evaluate the probability that the system lasts more than 1 year.

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