Homework Week 3

Question 1:

$(m \times n)(n \times p) = (mp)$

1. Answer the following questions

a) If a matrix A is 5×3 and the product AB is 5×7, what is the size of B?

b) How many rows does B have if BC is a 3×4 matrix?

a) A is an $m \times n$ matrix and matrix product AB is an $m \times p$ matrix

$\Rightarrow$ B is an $~~m~~\,n \times p$ matrix

$\Rightarrow$ B is a 3×7 matrix

b) If BC is a 3×4 matrix, then B has 3 rows and C has 4 columns

Question 2:

2. Let $A = \begin{pmatrix} 2 & 5 \\ -3 & 1 \end{pmatrix}$ and $B = \begin{pmatrix} 4 & -5 \\ 3 & c \end{pmatrix}$. What is value of c such that $AB = BA$?

$B = \begin{pmatrix} 4 & -5 \\ 3 & c \end{pmatrix}$ and $A = \begin{pmatrix} 2 & 5 \\ -3 & 1 \end{pmatrix}$

$AB = \begin{pmatrix} 23 & -10+5c \\ -9 & 15+c \end{pmatrix}$

$BA = \begin{pmatrix} 23 & 15 \\ 6-3c & 15+c \end{pmatrix}$

Because $AB = BA$

$\Rightarrow \begin{cases} -10+5c = 15 \\ 6-3c = -9 \end{cases}$

$\Rightarrow c = 5$

Question 3:

3. Let $A = \begin{pmatrix} 3 & -6 \\ -1 & 2 \end{pmatrix}$. Find matrix B such that $AB = 0$

$A = \begin{pmatrix} 3 & -6 \\ -1 & 2 \end{pmatrix}$ let $B = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$

$$AB = \begin{pmatrix} 3 & -6 \\ -1 & 2 \end{pmatrix}\begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

$$= \begin{pmatrix} 3a-6c & 3b-6d \\ -a+2c & -b+2d \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

$$\Rightarrow \begin{cases} 3a-6c = 0 \\ 3b-6d = 0 \\ -a+2c = 0 \\ -b+2d = 0 \end{cases} \Leftrightarrow \begin{cases} a = 2c \\ b = 2d \end{cases}$$

$\therefore$ $B = \begin{pmatrix} 2c & 2d \\ c & d \end{pmatrix}$ where c and d are free variables

Question 4:

4. Consider the following system of equation

$$\begin{cases} 3x_1 + x_2 + x_3 = 3 \\ x_1 - x_2 - x_3 = 1 \\ x_1 + 2x_2 + 2x_3 = 1 \end{cases}$$

Denote $x = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$ the vector solution of the equation. Express your solution in the form $x = v + tu$, where v and u are column vector in three dimensions.

$$\begin{cases} 3x_1 + x_2 + x_3 = 3 \\ x_1 - x_2 - x_3 = 1 \\ x_1 + 2x_2 + 2x_3 = 1 \end{cases}$$

$$\left[\begin{array}{ccc|c} 3 & 1 & 1 & 3 \\ 1 & -1 & -1 & 1 \\ 1 & 2 & 2 & 1 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 3 & 1 & 1 & 3 \\ 1 & -1 & -1 & 1 \\ 0 & -5 & -5 & 0 \end{array}\right] \quad R_3 \leftarrow (-3)R_3 + R_1$$

$$\left[\begin{array}{ccc|c} 3 & 1 & 1 & 3 \\ 0 & 4 & 4 & 0 \\ 0 & -5 & -5 & 0 \end{array}\right] \quad R_2 \leftarrow (-3)R_2 + R_1$$

$$\left[\begin{array}{ccc|c} 3 & 1 & 1 & 3 \\ 0 & 4 & 4 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \quad R_3 \leftarrow R_2 \times \frac{5}{4} + R_3$$

$$\begin{cases} 3x_1 + x_2 + x_3 = 3 \\ 4x_2 + 4x_3 = 0 \\ x_3 = x_3 \quad (\text{free variable}) \end{cases}$$

$4(x_2 + x_3) = 0$

$\Rightarrow x_2 + x_3 = 0$

$\Rightarrow x_2 = -x_3$

Introduce parameter s for free variable x_3

$$\begin{cases} x_1 = 1 \\ x_2 = -s \\ x_3 = s \end{cases}$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + s \begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix}$$