Principles of EE1 Quiz #2 – solution

Wednesday class - Fall 2018

Problem 1. **(30 points)** The currents i_a & i_b in the circuit in Fig. 1 are 4 A & 2 A, respectively

a) Find i_g. **(10 points)**
b) Find the power dissipated in each resistor **(10 points)**
c) Find v_g. **(10 points)**

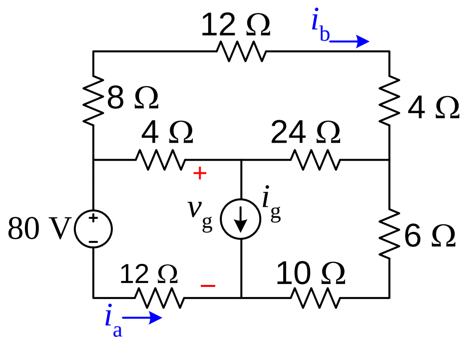
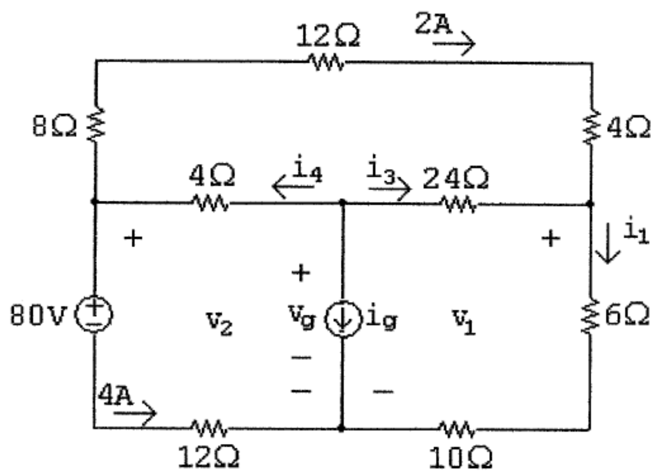


Fig. 1

Sol:

a)



$v_2 = 80 + 4(12) = 128\,\text{V}; \qquad v_1 = 128 - (8 + 12 + 4)(2) = 80\,\text{V}$

$$i_1 = \frac{v_1}{6+10} = \frac{80}{16} = 5\,\text{A}; \qquad i_3 = i_1 - 2 = 5 - 2 = 3\,\text{A}$$

$v_g = v_1 + 24 i_3 = 80 + 24(3) = 152\,\text{V}$

$i_4 = 2 + 4 = 6\,\text{A}$

$i_g = -i_4 - i_3 = -6 - 3 = -9\,\text{A}$

b) Calculate power using the formula $p = Ri^2$:

$$p_{8\,\Omega} = (8)(2)^2 = 32\text{ W}; \qquad p_{12\,\Omega} = (12)(2)^2 = 48\text{ W}$$

$$p_{4\,\Omega} = (4)(2)^2 = 16\text{ W}; \qquad p_{4\,\Omega} = (4)(6)^2 = 144\text{ W}$$

$$p_{24\,\Omega} = (24)(3)^2 = 216\text{ W}; \qquad p_{6\,\Omega} = (6)(5)^2 = 150\text{ W}$$

$$p_{10\,\Omega} = (10)(5)^2 = 250\text{ W}; \qquad p_{12\,\Omega} = (12)(4)^2 = 192\text{ W}$$

c) v_g = 152 V

Problem 2. **(40 points)** Use mesh analysis to find the current I_o in the circuit of Fig. 2

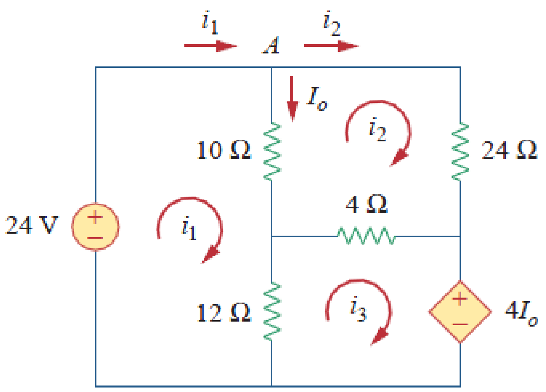


Fig. 2

Sol.:

We apply KVL to the three meshes in turn. For mesh 1,

$$-24 + 10(i_1 - i_2) + 12(i_1 - i_3) = 0 \text{ or } \quad 11i_1 - 5i_2 - 6i_3 = 12 \qquad (1)$$

For mesh 2, $\quad 24i_2 + 4(i_2 - i_3) + 10(i_2 - i_1) = 0$ or $\quad -5i_1 + 19i_2 - 2i_3 = 0 \qquad (2)$

For mesh 3, $4I_o + 12(i_3 - i_1) + 4(i_3 - i_2) = 0$, But at node A, $I_o = i_1 - i_2$, so that

$$4(i_1 - i_2) + 12(i_3 - i_1) + 4(i_3 - i_2) = 0\text{ , or } \quad -i_1 - i_2 + 2i_3 = 0 \qquad (3)$$

In matrix form, Eqs. (1) to (3) become

$$\begin{bmatrix} 11 & -5 & -6 \\ -5 & 19 & -2 \\ -1 & -1 & 2 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 12 \\ 0 \\ 0 \end{bmatrix}$$

We obtain the determinants as

$$\Delta = \begin{array}{c|ccc|c} & 11 & -5 & -6 & \\ & -5 & 19 & -2 & \\ & -1 & -1 & 2 & \\ - & 11 & -5 & -6 & + \\ - & -5 & 19 & -2 & + \\ - & & & & + \end{array}$$

$= 418 - 30 - 10 - 114 - 22 - 50 = 192$

$$\Delta_1 = \begin{vmatrix} 12 & -5 & -6 \\ 0 & 19 & -2 \\ 0 & -1 & 2 \\ 12 & -5 & -6 \\ 0 & 19 & -2 \end{vmatrix} = 456 - 24 = 432$$

$$\Delta_2 = \begin{vmatrix} 11 & 12 & -6 \\ -5 & 0 & -2 \\ -1 & 0 & 2 \\ 11 & 12 & -6 \\ -5 & 0 & -2 \end{vmatrix} = 24 + 120 = 144$$

$$\Delta_3 = \begin{vmatrix} 11 & -5 & 12 \\ -5 & 19 & 0 \\ -1 & -1 & 0 \\ 11 & -5 & 12 \\ -5 & 19 & 0 \end{vmatrix} = 60 + 228 = 288$$

We calculate the mesh currents using Cramer's rule as

$$i_1 = \frac{\Delta_1}{\Delta} = \frac{432}{192} = 2.25 \text{ A}, \qquad i_2 = \frac{\Delta_2}{\Delta} = \frac{144}{192} = 0.75 \text{ A}, \; i_3 = \frac{\Delta_3}{\Delta} = \frac{288}{192} = 1.5 \text{ A}$$

Thus, $I_o = i_1 - i_2 = 1.5$ A.

Problem 3. **(30 points)** Find the Norton equivalent circuit of the circuit in Fig. 3 at terminals a-b.

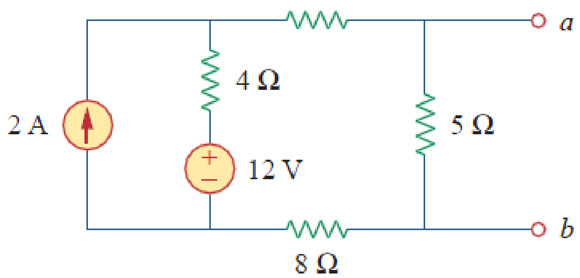


Fig. 3

__Sol.:__ We find in the same way we find R_{Th} in the Thevenin equivalent circuit.
Deactivate all the independent sources → the circuit in Fig. 3(a), from which we find $R_{Th} = R_N$

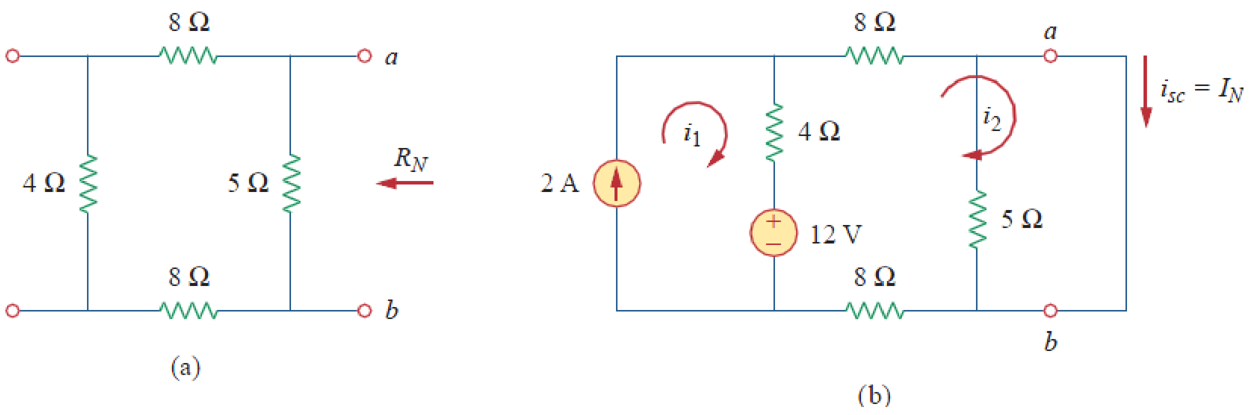


Fig. 3(a) R_N , (b) $I_N = i_{sc}$

$$R_N = 5 \parallel (8 + 4 + 8) = 5 \parallel 20 = \frac{20 \times 5}{25} = 4\ \Omega$$

To find I_N we short-circuit terminals a & b, as shown in Fig. 3(b). We ignore the 5-Ω resistor because it has been short-circuited. Applying mesh analysis, we obtain

$$i_1 = 2\text{ A}, \qquad 20i_2 - 4i_1 - 12 = 0 \;\rightarrow\; i_2 = 1\text{ A} = i_{sc} = I_N$$

Thus, the Norton equivalent circuit is as shown in Fig. 4.

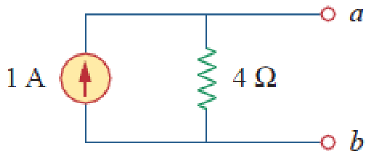


Fig. 4