Introduction to Computing Section 2 – The Information Layer



Binary Values and Number Systems

Data Representation



PART 1 Binary Values and Number Systems



Number and Computing



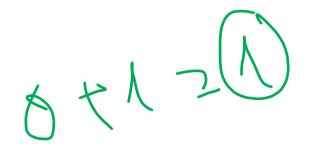
- → All types of information: stored as number
- → Explore the numbers

Natural, negative, rational, irrational numbers and many others (not to the understanding of computing)

Number: A unit of an abstract mathematical system subject to the laws of arithmetic

Numbers





Natural Numbers

Zero and any number obtained by repeatedly adding one to it.

Examples: 100, 0, 45645, 32

Negative Numbers

A value less than 0, with a – sign

Examples: -24, -1, -45645, -32

Numbers



Integers

A natural number, a negative number, zero

Examples: 249, 0, - 45645, - 32

Rational Numbers

An integer or the quotient of two integers

Examples: -249, -1, 0, 3/7, -2/5

Natural Numbers



How many ones are there in 642?

$$600 + 40 + 2$$
?

Or is it

$$384 + 32 + 2$$
?

Or maybe...

Natural Numbers



6 x 10 + 4 . 10 tl.

The base of a number determines the number of digits used in the system and the value of digit positions.

The digits always begin with 0 and continue through one less than the base.

Numbers are written using positional notation.



Numbers are written using positional notation

The rightmost digit its value * the base to the zeroth power

- →The digit to the left of that one: its value*the base to 1st power
- \rightarrow The next digit: its value*the base to 2nd power and so on

Positional notation A system of expressing numbers in which the digits are arranged in succession, the position of each digit has a place value, and the number is equal to the sum of the products of each digit by its place value.



Continuing with our example...
642 in base 10 *positional notation* is:

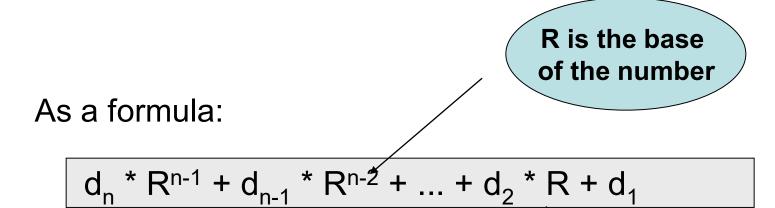
$$6 \times 10^{2} = 6 \times 100 = 600$$

+ $4 \times 10^{1} = 4 \times 10 = 40$
+ $2 \times 10^{\circ} = 2 \times 1 = 2 = 642$ in base 10

This number is in base 10

The power indicates the position of the number





n is the number of digits in the number

d is the digit in the ith position in the number

642 is
$$6_3 * 10^2 + 4_2 * 10 + 2_1$$



What if 642 has the base of 13?

$$+ 6 \times 13^{2} = 6 \times 169 = 1014$$

 $+ 4 \times 13^{1} = 4 \times 13 = 52$
 $+ 2 \times 13^{0} = 2 \times 1 = 2$
 $= 1068 \text{ in base } 10$

642 in base 13 is equivalent to 1068 in base 10

Binary



Decimal is base 10 and has 10 digits:

0,1,2,3,4,5,6,7,8,9

Binary is base 2 and has 2 digits:

0,1

For a number to exist in a given number system, the number system must include those digits. For example, the number 284 only exists in base 9 and higher.

Bases Higher than 10



How are digits in bases higher than 10 represented?

With distinct symbols for 10 and above.

Base 16 has 16 digits: 0,1,2,3,4,5,6,7,8,9,A,B,C,D,E, and F

Converting Octal to Decimal



What is the decimal equivalent of the octal number 642?

$$6 \times 8^{2} = 6 \times 64 = 384$$

 $+ 4 \times 8^{1} = 4 \times 8 = 32$
 $+ 2 \times 8^{\circ} = 2 \times 1 = 2$
 $= 418 \text{ in base } 10$

Converting Hexadecimal to Decimal



What is the decimal equivalent of the hexadecimal number DEF?

D x
$$16^2$$
 = $13 \times 256 = 3328$
+ E x 16^1 = $14 \times 16 = 224$
+ F x 16^0 = $15 \times 1 = 15$
= 3567 in base 10

Remember, the digits in base 16 are 0,1,2,3,4,5,6,7,8,9,A,B,C,D,E,F

Converting Binary to Decimal



What is the decimal equivalent of the binary number 1101110?

$$1 \times 2^{6} = 1 \times 64 = 64$$

$$+ 1 \times 2^{5} = 1 \times 32 = 32$$

$$+ 0 \times 2^{4} = 0 \times 16 = 0$$

$$+ 1 \times 2^{3} = 1 \times 8 = 8$$

$$+ 1 \times 2^{2} = 1 \times 4 = 4$$

$$+ 1 \times 2^{1} = 1 \times 2 = 2$$

$$+ 0 \times 2^{0} = 0 \times 1 = 0$$

$$= 110 \text{ in base } 10$$

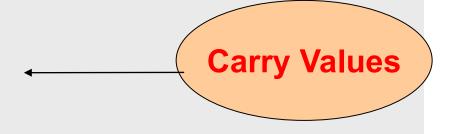
Arithmetic in Binary



Remember that there are only 2 digits in binary, 0 and 1

Position is key, carry values are used:





1100110011+1110111101=?

11011110000

Subtracting Binary Numbers



When you borrow one from the next left digit of the number you are subtracting, you borrow one power of the base.

Every time you borrow in a binary subtraction, you borrow 2.

EX:

1010111-111011=?

0011100

Subtracting Octal Numbers



Every time you borrow in a Octal subtraction, you borrow 8.

EX:

Power of 2 Number System



Binary	Octal	Decimal
000	0	0
001	1	1
010	2	2
011	3	3
100	4	4
101	5	5
110	6	6
111	7	7
1000	10	8
1001	11	9
1010	12	10

345

3:011

4:100

5:101

345: 011100101

Converting Binary to Octal



- Groups of Three (from right)
- Convert each group

10101011 is 253 in base 8

Converting Binary to Hexadecimal



- Groups of Four (from right)
- Convert each group

10101011 is AB in base 16

Converting Decimal to Other Bases



Algorithm for converting base 10 to other bases

While the quotient is **not** zero:

- * Divide the decimal number by the new base
- * Make the remainder the next digit to the left in the answer
- * Replace the original dividend with the quotient

Converting Decimal to Hexadecimal

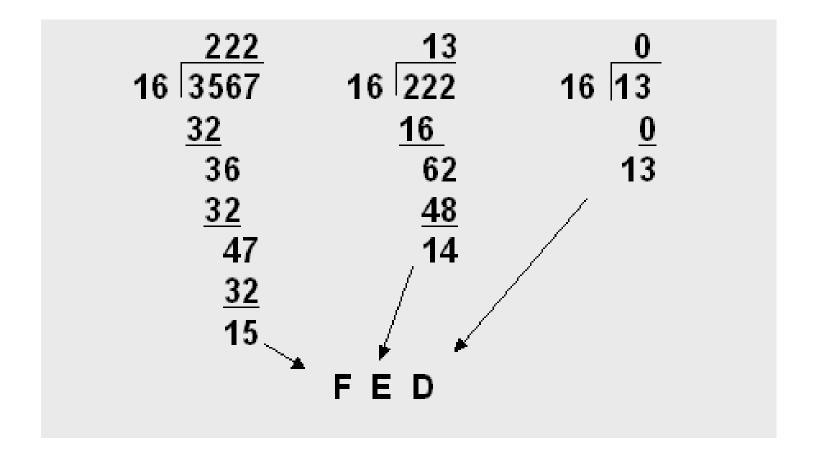


Try a Conversion

The base 10 number 3567 is what number in base 16?

Converting Decimal to Hexadecimal





Binary and Computers



Binary computers have storage units called binary digits or bits

Low Voltage = 0

High Voltage = 1

all bits have 0 or 1

Binary and Computers



Byte 8 bits

The number of bits in a word determines the word length of the computer, but it is usually a multiple of 8

- 32-bit machines
- 64-bit machines etc.

Converting Binary to Decimal



What is the decimal equivalent of the binary number 1101110?

```
1 \times 2^{6} = 1 \times 64 = 64
+ 1 \times 2^{5} = 1 \times 32 = 32
+ 0 \times 2^{4} = 0 \times 16 = 0
+ 1 \times 2^{3} = 1 \times 8 = 8
+ 1 \times 2^{2} = 1 \times 4 = 4
+ 1 \times 2^{1} = 1 \times 2 = 2
+ 0 \times 2^{0} = 0 \times 1 = 0
= 110 \text{ in base } 10
```

Practice



1. Match the solution with the problem.

Problem		Solution	
1/	1110011+11001	A. 10001100	
2/	1010101+10101	B. 10011110	
3/	1111111+11111	C. 1101010	
4/	1110011-111	D. 1011110	
5/	1100101-111	E. 1010001	
6/	1010110-101	F. 1101100	

Practice



2. Convert the following numbers from the base shown to base 10.

- a. 111 (base 2)
- b. 777 (base 8)
- c. FEC (base 16)
- d. 777 (base 16)
- e. 111 (base 8)

3. Convert the following binary numbers to octal and hexadecimal

- a. 111110110
- b. 1000001
- c. 10000010
- d. 1100010

Practice



4. Perform the following Octal subtractions

- a. 23656-763
- b. 2346-527
- c. 3736-777

5. Perform the following Hexadecimal subtractions

- a. ABC-DF
- a. 9988-AB
- b. C9F8-1F92

DONEEEEEEEEEEEEEE