Response of RLC circuit

	Parallel RLC		Series RLC	
	Characteristic Equation: $s^2 + \frac{s}{RC} + \frac{1}{LC} = 0$ $\alpha = \frac{1}{2RC}$ $\quad \omega_0 = \frac{1}{\sqrt{LC}}$		Characteristic Equation: $s^2 + \frac{R}{L}s + \frac{1}{LC} = 0$ $\alpha = \frac{R}{2L}$ $\quad \omega_0 = \frac{1}{\sqrt{LC}}$	
	Natural response	Step response	Natural response	Step response
Over-damped	$v(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t}$ $\begin{cases} v(0^+) = v_0 = A_1 + A_2 \\ \frac{dv(0^+)}{dt} = \frac{i_C(0^+)}{C} = s_1 A_1 + s_2 A_2 \end{cases}$	$i_L(t) = I + A_1 e^{s_1 t} + A_2 e^{s_2 t}$ $\begin{cases} i_L(0^+) = I + A_1 + A_2 \\ \frac{di_L(0^+)}{dt} = \frac{v(0^+)}{L} = s_1 A_1 + s_2 A_2 \end{cases}$	$i(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t}$ $\begin{cases} i(0^+) = A_1 + A_2 \\ \frac{di(0^+)}{dt} = \frac{v_L(0^+)}{L} = s_1 A_1 + s_2 A_2 \end{cases}$	$v_C(t) = V + A_1 e^{s_1 t} + A_2 e^{s_2 t}$ $\begin{cases} v_C(0^+) = V + A_1 + A_2 \\ \frac{dv_C(0^+)}{dt} = \frac{i_C(0^+)}{C} = s_1 A_1 + s_2 A_2 \end{cases}$
Under-damped	$v(t) = B_1 e^{-\alpha t} \cos \omega_d t$ $+ B_2 e^{-\alpha t} \sin \omega_d t$ $\begin{cases} v(0^+) = v_0 = B_1 \\ \frac{dv(0^+)}{dt} = \frac{i_C(0^+)}{C} = -\alpha B_1 + \omega_d B_2 \end{cases}$	$i_L(t) = I + B_1 e^{-\alpha t} \cos \omega_d t$ $+ B_2 e^{-\alpha t} \sin \omega_d t$ $\begin{cases} i_L(0^+) = I + B_1 \\ \frac{di_L(0^+)}{dt} = \frac{v(0^+)}{L} = -\alpha B_1 + \omega_d B_2 \end{cases}$	$i(t) = B_1 e^{-\alpha t} \cos \omega_d t$ $+ B_2 e^{-\alpha t} \sin \omega_d t$ $\begin{cases} i(0^+) = B_1 \\ \frac{di(0^+)}{dt} = \frac{v_L(0^+)}{L} = -\alpha B_1 + \omega_d B_2 \end{cases}$	$v_C(t) = V + B_1 e^{-\alpha t} \cos \omega_d t$ $+ B_2 e^{-\alpha t} \sin \omega_d t$ $\begin{cases} v_C(0^+) = V + B_1 \\ \frac{dv_C(0^+)}{dt} = \frac{i_C(0^+)}{C} = -\alpha B_1 + \omega_d B_2 \end{cases}$
Critically damped	$v(t) = D_1 t e^{-\alpha t} + D_2 e^{-\alpha t}$ $\begin{cases} v(0^+) = v_0 = D_2 \\ \frac{dv(0^+)}{dt} = \frac{i_C(0^+)}{C} = D_1 - \alpha D_2 \end{cases}$	$i_L(t) = I + D_1 t e^{-\alpha t} + D_2 e^{-\alpha t}$ $\begin{cases} i_L(0^+) = I + D_2 \\ \frac{di_L(0^+)}{dt} = \frac{v(0^+)}{L} = D_1 - \alpha D_2 \end{cases}$	$i(t) = D_1 t e^{-\alpha t} + D_2 e^{-\alpha t}$ $\begin{cases} i(0^+) = D_2 \\ \frac{di(0^+)}{dt} = \frac{v_L(0^+)}{L} = D_1 - \alpha D_2 \end{cases}$	$v_C(t) = V + D_1 t e^{-\alpha t} + D_2 e^{-\alpha t}$ $\begin{cases} v_C(0^+) = V + D_2 \\ \frac{dv_C(0^+)}{dt} = \frac{i_C(0^+)}{C} = D_1 - \alpha D_2 \end{cases}$

Note:

+ $\omega_0^2 < \alpha^2$: Over-damped

+ $\omega_0^2 > \alpha^2$: Under-damped

+ $\omega_0^2 = \alpha^2$: Critically damped

For the case of Over-damped and Under-damped, there are two roots: $s_1 = -\alpha + \sqrt{\alpha^2 - \omega_0^2}$ v $s_2 = -\alpha - \sqrt{\alpha^2 - \omega_0^2}$

Especially, in case of Under-damped $\omega_d = \sqrt{\omega_0^2 - \alpha^2}$

For the case of Critically damped, the root is: $s_1 = s_2 = -\alpha$