Đỗ Minh Duy - ITITSB22029

Week 6

① a) $A = \begin{pmatrix} 1 & -4 & 9 & -7 \\ -1 & 2 & -4 & 1 \\ 5 & -6 & 10 & 7 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & -4 & 9 & -7 \\ 0 & -2 & 5 & 6 \\ 0 & 14 & -35 & 42 \end{pmatrix}$

$\xrightarrow[-2R_3-14R_2]{} \begin{pmatrix} 1 & -4 & 9 & -7 \\ 0 & -2 & 5 & 6 \\ 0 & 0 & 0 & 0 \end{pmatrix}$

Rank(A) = 2

Nullity (A) = 4 - 2 = 2

b) $B = \begin{pmatrix} 1 & 3 & -2 & 4 \\ 0 & 1 & -1 & 2 \\ -2 & -6 & 4 & -8 \end{pmatrix} \xrightarrow[R_3+2R_1]{} \begin{pmatrix} 1 & 3 & -2 & 4 \\ 0 & 1 & -1 & 2 \\ 0 & 0 & 0 & 0 \end{pmatrix}$

Rank (B) = 2

Nullity (B) = 4 - 2 = 2

② a) $A = \begin{pmatrix} 2 & -3 & 1 \\ 5 & 10 & 6 \\ 8 & -7 & 5 \end{pmatrix} \xrightarrow[R_3-4R_1]{2R_2-5R_1} \begin{pmatrix} 2 & -3 & 1 \\ 0 & 35 & 7 \\ 0 & 5 & 1 \end{pmatrix} \xrightarrow[7R_3-R_2]{} \begin{pmatrix} 2 & -3 & 1 \\ 0 & 5 & 1 \\ 0 & 0 & 0 \end{pmatrix}$

Basis of a row Space : $\left\{ \begin{bmatrix} 2 \\ -3 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 5 \\ 1 \end{bmatrix} \right\}$; Rank(A) = 2

b) $B = \begin{pmatrix} -2 & -4 & 4 & -5 \\ 3 & 6 & -6 & -4 \\ -2 & -4 & 4 & 9 \end{pmatrix} \xrightarrow[R_3-R_1]{2R_2+3R_1} \begin{pmatrix} -2 & -4 & 4 & -5 \\ 0 & 0 & 0 & -23 \\ 0 & 0 & 0 & 14 \end{pmatrix} \xrightarrow[-23R_3-14R_2]{}$

$\begin{pmatrix} -2 & -4 & 4 & -5 \\ 0 & 0 & 0 & -23 \\ 0 & 0 & 0 & 0 \end{pmatrix}$ Basis of a row Space $\left\{ \begin{bmatrix} -2 \\ -4 \\ 4 \\ 5 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \\ -23 \end{bmatrix} \right\}$

; Rank(B) = 2

③ a) $S = \{(4,3,2), (0,3,2), (0,0,2)\}$

Let $u = (8,3,8)$

Let $u = c_1(4,3,2) + c_2(0,3,2) + c_3(0,0,2)$

$$\begin{cases} 4c_1 = 8 \\ 3c_1 + 3c_2 = 3 \\ 2c_1 + 2c_2 + 2c_3 = 8 \end{cases} \Rightarrow \begin{cases} c_1 = 2 \\ c_2 = -1 \\ c_3 = 3 \end{cases}$$

$W = \{(2s-t, s, t, s) \mid s, t \in R\}$

$= \{s(2,1,0,1) + t(-1,0,1,0) \mid s,t \in R\}$

Basis $= \left\{ \begin{bmatrix} 2 \\ 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \\ 1 \\ 0 \end{bmatrix} \right\}$, Dim $= 2$.

b) $c_1(1,0,0) + c_2(1,1,0) + c_3(1,1,1) = (0,0,0)$

$$\Rightarrow \begin{cases} c_1 + c_2 + c_3 = 0 \\ c_2 + c_3 = 0 \\ c_3 = 0 \end{cases} \Rightarrow c_1 = c_2 = c_3 = 0$$

$\Rightarrow$ S is linearly ~~in-dependently~~ $\Rightarrow$ S is a basis of R^3

$(8,3,8) = (c_1 + c_2 + c_3, c_2 + c_3, c_3)$

$$\Rightarrow \begin{cases} c_1 + c_2 + c_3 = 8 \\ c_2 + c_3 = 3 \\ c_3 = 8 \end{cases} \Rightarrow \begin{cases} c_1 = 5 \\ c_2 = -5 \\ c_3 = 8 \end{cases}$$

$\Rightarrow (8,3,8) = 5(1,0,0) - 5(1,1,0) + 8(1,1,1)$

c) zero vector means that the vector are not linearly independent => the zero vector can be represented as a scalar multiple of any other vectors, which violates the definition of linear independance therefore, It is not possible to express any vector

(4)

$$H = \left\{ \begin{pmatrix} a - 4b - 2c \\ 2a + 5b - 4c \\ -a + 2c \\ -3a + 7b + 6c \end{pmatrix}, a, b, c \in K \right)$$

$$\Rightarrow H = \left\{ a \begin{bmatrix} 1 \\ 2 \\ -1 \\ -3 \end{bmatrix} + b \begin{bmatrix} -4 \\ 5 \\ 0 \\ 7 \end{bmatrix} + c \begin{bmatrix} -2 \\ -4 \\ 2 \\ 6 \end{bmatrix} : a, b, c \in R \right\}$$

$$H = \text{Span} \left\{ \begin{bmatrix} 1 \\ 2 \\ -1 \\ -3 \end{bmatrix} \begin{bmatrix} -4 \\ 5 \\ 0 \\ 7 \end{bmatrix} \begin{bmatrix} -2 \\ -4 \\ 2 \\ 6 \end{bmatrix} \right\}$$

$$\begin{bmatrix} 1 & -4 & -2 \\ 2 & 5 & -4 \\ -1 & 0 & 2 \\ -3 & 7 & 6 \end{bmatrix} \xrightarrow[\substack{R_3 + R_1 \\ R_4 + 3R_1}]{R_2 - 2R_1} \begin{bmatrix} 1 & -4 & -2 \\ 0 & 13 & 0 \\ 0 & -4 & 0 \\ 0 & -5 & 0 \end{bmatrix} \xrightarrow[4R_4 - 5R_3]{} \begin{bmatrix} 1 & -4 & -2 \\ 0 & 13 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\text{Basis for } H = \left\{ \begin{bmatrix} 1 \\ 2 \\ -1 \\ -3 \end{bmatrix}, \begin{bmatrix} -4 \\ 5 \\ 0 \\ 7 \end{bmatrix} \right\} \Rightarrow \text{Dim}(H) = 2$$