HO CHI MINH CITY INTERNATIONAL UNIVERSITY

MIDTERM EXAMINATION

Academic year 2022-2023, Semester 1
Duration: 120 minutes

SUBJECT: **Differential Equations (MA024IU)**	
Head of Department of Mathematics Signature: Professor Pham Huu Anh Ngoc	Lecturer: Signature: Full name: Pham Huu Anh Ngoc

Instructions:

- *Each student is allowed a scientific calculator and a maximum of two double-sided sheets of reference material (size A4 or similar), stapled together and marked with their name and ID. All other documents and electronic devices are forbidden.*

Question 1. *(20 marks)* Two chemicals A and B are combined to form a chemical C. The rate, or velocity, of the reaction is proportional to the product of the instantaneous amounts of A and B not converted to chemical C. Initially, there are 40 grams of A and 50 grams of B and for each gram of B, 2 grams of A is used. It is observed that 10 grams of C is formed in 5 minutes. How much is formed in 20 minutes? What is the limiting amount of C after a long time? How much of chemicals A and B remain after a long time?

Question 2. *(20 marks)* Solve the following differential equation

$$\big(\sin x - 3x^2y^2 + y^2\big)dx + \big(3y^2 - 2x^3y + 2xy\big)dy = 0.$$

Question 3. *(20 marks)* Find the solution to the initial value problem

$$(x+1)y' + (2x+3)y = x^2e^{-2x}, \qquad y(0) = 1.$$

Question 4. *(20 marks)* Find a particular solution of the following differential equation

$$y'' - 5y' + 6y = x^2 + xe^{3x}.$$

Question 5. *(20 marks)* Find the general solution of the following differential equation

$$x^2y'' - 7xy' + 12y = x^2 + 1, \qquad x \in (0, \infty).$$

END.

SOLUTIONS:

Question 1. The limiting value of the population is $1,000,000$. The population will reach $500,000$ in 5.29 months.

Question 2. The given differential equation is rewritten as

$$(e^{2y}dx + xde^{2y}) - \cos(xy)(xdy + ydx) + dy^2 = 0.$$

Then, we get

$$d(e^{2y}x) - \cos(xy)d(xy) + dy^2 = d(e^{2y}x) + d(-\sin(xy)) + dy^2 = 0.$$

Therefore,

$$d(e^{2y}x - \sin(xy) + y^2) = 0.$$

Thus the general solution is given by

$$e^{2y}x - \sin(xy) + y^2 = C.$$

Question 3. Consider the differential equation

$$y' - (\sin x)y = 2\sin x.$$

The integrating factor is given by $I(x) = e^{\cos x}$. Thus, we get

$$e^{\cos x}y' - e^{\cos x}(\sin x)y = 2e^{\cos x}\sin x.$$

This gives

$$\frac{d}{dx}(e^{\cos x}y) = 2\int e^{\cos x}\sin x dx = -2e^{\cos x} + C.$$

Therefore, the general solution is

$$y(x) = -2 + \frac{C}{e^{\cos x}}.$$

Since $y(\frac{\pi}{2}) = 1$, the particular solution is $y(x) = -2 + \frac{3}{e^{\cos x}}$.

Question 4. a) The form of a particular solution of the differential equation

$$y'' - 4y' + 3y = e^{2x}(x^3 + 1) + e^x(x + 1)$$

is given by

$$y_p(x) = e^{2x}(Ax^3 + Bx^2 + Cx + D) + e^x(Ex^2 + Fx).$$

The general solution of the differential equation

$$y'' - 4y' + 3y = e^x(x + 1)$$

is given by

$$y(x) = c_1e^x + c_2e^{3x} - e^x(\frac{1}{4}x^2 + \frac{3}{4}x).$$

Question 5. a) $a = b = q, q \in \mathbb{R}$.
b) Note that $y_1(x) = x + 1$ is a particular solution of the differential equation

$$(1 - 2x - x^2)y'' + 2(x + 1)y' - 2y = 0.$$

By the Liouville formula, $y_2(x) = x^2 + x + 2$ is a solution of this equation such that y_1, y_2 are linearly independent. So, the general solution is given by

$$y(x) = c_1(x + 1) + c_2(x^2 + x + 2).$$