CHAPTER 3 THE KINETIC THEORY OF GASES

Homework: 32, 40, 42, 44, 46, 54, 56, 78 in Chapter 19

32. At 20^0C and 750 torr pressure, the mean free paths for argon gas (Ar) and nitrogen (N_2) are λ_{Ar}=9.9x10^{-6} cm and λ_{N2}=27.5x10^{-6} cm. (a) Find the ratio of the diameter of an Ar atom to that of an N_2 molecule. What is the mean free path of Ar at (b) 20^0C and 150 torr, and (c) -40^0C and 750 torr?

Mean Free Path: $$\lambda = \frac{kT}{\sqrt{2}\pi d^2 p}$$

(a) The ratio d_{Ar} to d_{N2}: $$\frac{d_{Ar}}{d_{N_2}} = \sqrt{\frac{\lambda_{N_2}}{\lambda_{Ar}}}$$

(b): $$\lambda_1 = \frac{kT_1}{\sqrt{2}\pi d^2 p_1}; \lambda_2 = \frac{kT_2}{\sqrt{2}\pi d^2 p_2}$$

$$\lambda_2 = \frac{T_2}{T_1} \times \frac{p_1}{p_2} \times \lambda_1$$

40. Two containers are at the same temperature. The first contains gas with pressure p_1, molecular mass m_1, and rms speed v_{rms1}. The second contains gas with pressure $1.5p_1$, molecular mass m_2, and average speed $v_{avg2}=2.0v_{rms1}$. Find the mass ratio m_1/m_2.

RMS speed: $$v_{rms1} = \sqrt{\frac{3RT_1}{m_1}}$$

Average speed: $$\bar{v}_2 = \sqrt{\frac{8RT_2}{\pi m_2}}$$

$$T_1 = T_2 \Rightarrow \frac{m_1}{m_2} = \frac{3\pi}{8}\left(\frac{\bar{v}_2}{v_{\mathrm{rms1}}}\right)^2 = 4.71$$

42. What is the internal energy of 2.0 mol of an ideal monatomic gas at 273 K?

$$E = nC_V T$$

$$C_V = \frac{3}{2}R = 12.5 \text{ J mol}^{-1} \text{ K}^{-1}$$

$$E = 2.0 \times 12.5 \times 273 = 6825 \text{ (J)}$$

$$E \approx 6.8 \text{ (kJ)}$$

46. Under constant pressure, the temperature of 3.0 mol of an ideal monatomic gas is raised 15.0 K. What are (a) the work W done by the gas, (b) the energy transferred as heat Q, (c) the change ΔE_{int} of the gas, and (d) the change ΔK in the average KE per atom?

(a) At constant pressure:

$$W = p\Delta V = nR\Delta T = 3.0\times 8.31\times 15.0 \approx 374\ \text{(J)}$$

(b) $$Q = nC_p\Delta T = n\times\frac{5}{2}R\times\Delta T = \frac{5}{2}W \approx 935\ \text{(J)}$$

(c) We use the first law of thermodynamics:

$$\Delta E_{\text{int}} = Q - W \qquad (\text{or } \Delta E_{\text{int}} = nC_V\Delta T = \frac{3}{2}nR\Delta T)$$

$$\Delta E_{\text{int}} = 935 - 374 = 561\ \text{(J)}$$

(d) For a monatomic gas: $K_{\text{avg}} = \frac{3}{2}kT \Rightarrow \Delta K_{\text{avg}} = \frac{3}{2}k\Delta T$

$$\Delta K_{\text{avg}} = \frac{3}{2}\times 1.38\times 10^{-23}\times 15.0 \approx 3.1\times 10^{-22}\ \text{(J)}$$

54. We know that for an adiabatic process $pV^{\gamma} = \text{constant}$
Evaluate "constant" for an adiabatic process involving exactly 2.0 mol of an ideal gas passing through the state having exactly p=1.5 atm and T=300 K. Assume a diatomic gas whose molecules rotate but do not oscillate.

$$1\,\text{atm} = 1.01\times 10^{5}\ (\text{Pa})$$

Equation of state: $pV = nRT$

$$V = \frac{nRT}{p} = \frac{2.0\times 8.31\times 300}{1.5\times 1.01\times 10^{5}} \approx 0.033\ (\text{m}^3)$$

$$\gamma = \frac{C_p}{C_V} = \frac{C_V + R}{C_V} = \frac{\frac{f}{2}R + R}{\frac{f}{2}R}$$

For a diatomic gas, f=5: $\gamma = \dfrac{7}{5}$

$$\text{constant} = pV^{\gamma} = 1.5\times 1.01\times 10^{5}\times 0.033^{\frac{7}{5}} = 1.28\times 10^{3}(\text{N/m}^2\times(\text{m}^3)^{1.4})$$

$$\text{constant} = 1.28\times 10^{3}(\text{N m}^{2.2})$$

56. Suppose 1.0L of a gas with γ=1.30, initially at 285 K and 1.0 atm, is suddenly compressed adiabatically to half its initial volume. Find its final (a) pressure and (b) temperature. (c) If the gas is then cooled to 273 K at constant pressure, what is its final volume?

$$V_f = \frac{1}{2} V_i$$

$$p_i V_i^{\gamma} = p_f V_f^{\gamma};$$

$$p_f = p_i \left(\frac{V_i}{V_f} \right)^{\gamma}$$

$$T_f = T_i \left(\frac{V_i}{V_f} \right)^{\gamma - 1}$$

$$pV = nRT, p = \text{constant} \Rightarrow \frac{V'_f}{V_f} = \frac{T'_f}{T_f}$$

78. (a) An ideal gas initially at pressure p_0 undergoes a free expansion until its volume is 3.0 times its initial volume. What then is the ratio of its pressure to p_0? (b) The gas is next slowly and adiabatically compressed back to its original volume. The pressure after compression is $(3.0)^{1/3}p_0$. Is the gas monatomic, diatomic, or polyatomic? (c) What is the ratio of the average kinetic energy per molecule in this final state to that in the initial state?

(a) $$p_0V_0 = p_1V_1; V_1 = 3V_0 \Rightarrow p_1 = \frac{1}{3}p_0$$

(b) $$p_1V_1^\gamma = p'_1V_0^\gamma$$

$$p'_1 = p_1\left(\frac{V_1}{V_0}\right)^\gamma = \frac{1}{3}p_0 3^\gamma = 3^{\gamma-1}p_0$$

$$\Rightarrow \gamma - 1 = \frac{1}{3} \Rightarrow \gamma = \frac{4}{3} = \frac{C_p}{C_V} = \frac{C_V + R}{C_V} = \frac{\frac{f}{2}+1}{\frac{f}{2}} = \frac{f+2}{f}$$

$$f = 6 : \text{polyatomic}$$

(c) $$K_{avg} = \frac{3}{2}kT$$

$$r = \frac{K'_{avg}}{K_{avg}} = \frac{T'_1}{T_0}$$

$$r = \frac{T'_1}{T_0} = \frac{p'_1 V_1'}{p_0 V_0} = \frac{p'_1}{p_0} = 3^{1/3} = 1.44 \text{ (since } V_1' = V_0)$$