THE INTERNATIONAL UNIVERSITY(IU) - VNU HCMC

MIDTERM EXAMINATION
PROBABILITY, STATISTICS AND RANDOM PROCESS
Semester 2, 2021-22 • April 2022 • Total duration: 90 minutes

Chair of Mathematics Department	Lecturer
	Dr. Pham Hai Ha

INSTRUCTIONS: Each student is allowed calculators and one double-sided sheet of reference material (size A4 or similar) marked with their name and ID. All other documents and electronic devices are forbidden.

1. *(10 points)* There are currently six TV set and eight DVD players waiting to be repaired at a repair department of a store. On any particular day, the staff of the store can repair a total of six applicances. Compute the probability that 4 TV sets and 2 DVD players will be repaired.

$$\frac{\frac{4}{6} \times \frac{2}{8}}{\frac{6}{14}}$$

2. *(10 points)* Consider a series system of four independent components which fail with probability 0.01, 0.03, 0.05 and 0.05 respectively.

   What is the probability that the system works?

3. *(10 points)* Car production in the United States in 2005 was distributed among car manufacturers as follows.

U.S. car production	Type	Percentage of type by brand	
60%	Domestic	Chrysler	23%
		Ford	31%
		General Motors	46%
40%	Foreign	Honda	20%
		Toyota	32%
		Other	48%

This means that 60% of the cars produced in the United States were manufactured by domestic companies; of them, 23% were Chryslers, 31% were Fords, and 46% were General Motors products.

A 2005 automobile is chosen at random. What is the probability that it is a Toyota car?

1. *(10 points)* $\frac{\binom{6}{4}\binom{8}{2}}{\binom{14}{6}}$
2. *(10 points)* $(0.99)(0.97)(0.95)^2$
3. *(10 points)* $(40\%)(32\%) = 12.8\%$
4. *(10 points)* $(0.5\%)(50\%) + (1\%)(30\%) + (1.5\%)(20\%) = 0.31\%$
5. (a) $P(X = k) = \binom{4}{k} p^k (1-p)^{4-k}$
   (b) $1 - (1-p)^4$
6. (a) $mean = \frac{70}{69}\ln(70)$, $Var = 70 - \left(\frac{70}{69}\ln(70)\right)^2$
   (b) $\frac{140}{69}\ln(70)$
7. $P(9.8 < X < 10.2) = P(-1 < Z < 1) = 2P(Z < 1) - 1 = 0.6826$
8. $\left(\int_4^\infty \frac{1}{3}e^{-\frac{1}{3}x}dx\right)^2$

4. *(10 points)* A factory has three production lines that produce 50%, 30%, and 20%, respectively, of the items made in the factory during a day. It has been found that 0.5% of the items produced in the first line of production are defective, while in the second and third lines, the corresponding proportions are 1% and 1.5%, respectively. Calculate the proportion of defective items produced in the factory during a day.

5. *(20 points)* Suppose that a lottery ticket has probability $p = 0.0001$ of being a winning ticket, independently of other tickets. A gambler buys 4 tickets, hoping this will increase the chance of having at least one winning ticket.

   Let X be the number of wining tickets among 4 tickets that he or she purchases.

   (a) Determine the probability mass function of X, i.e., $P(X = k)$ for $k = 0, \ldots, 4$.

   (b) Compute the probability that there is at least one wining ticket.

6. *(20 points)* The probability density function of weight (in pounds) of packages delivered by a post office is

$$f(x) = \begin{cases} \frac{70}{69x^2} & \text{for } 1 < x < 70 \\ 0 & \text{otherwise} \end{cases}.$$

   (a) Determine the mean and variance of the weight.

   (b) If the shipping cost is \$2 per pound, what is the average shipping cost of a package?

7. *(10 points)* The distance between two points needs to be measured, in meters. The true distance between the points is 10 meters, but due to measurement error we can't measure the distance exactly. Instead, we will observe a value of $10 + \varepsilon$, where the error ε is distributed $\mathcal{N}(0, 0.04)$. So the distance X that we observe is normally distributed $\mathcal{N}(10, 0.04)$. Find the probability that the observed distance is within 0.2 meters of the true distance (10 meters).

8. *(10 points)* The blade and the bearings are important parts of a lathe. The lathe can operate only when both of them work properly. The lifetime of the blade is exponentially distributed with the mean three years; the lifetime of the bearings is also exponentially distributed with the mean three years. Thus the probability density function of both blade and bearing is given by

$$f(x) = \begin{cases} \frac{1}{3}e^{-\frac{1}{3}x} & \text{for } x \geq 0 \\ 0 & \text{otherwise} \end{cases}.$$

   Assume each lifetime is independent. What is the probability that the lathe will operate at least four years?

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   (b) $1 - (1-p)^4$
6. (a) $mean = \frac{70}{69}\ln(70)$, $Var = 70 - \left(\frac{70}{69}\ln(70)\right)^2$

   (b) $\frac{140}{69}\ln(70)$
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Hard

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998

Figure 1: Cumulative standard normal distribution

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