Chapter 7

Electromagnetic Waves

7.3. Electromagnetic waves:

7.3.1. Maxwell's Rainbow:

As the figure shows, we now know a wide spectrum (or range) of electromagnetic waves: Maxwell's rainbow. In the wavelength scale in the figure, (and similarly the corresponding frequency scale), each scale marker represents a change in wavelength (and correspondingly in frequency) by a factor of 10.

The scale is open-ended; the wavelengths/frequencies of electromagnetic waves have **no inherent upper or lower bound.**
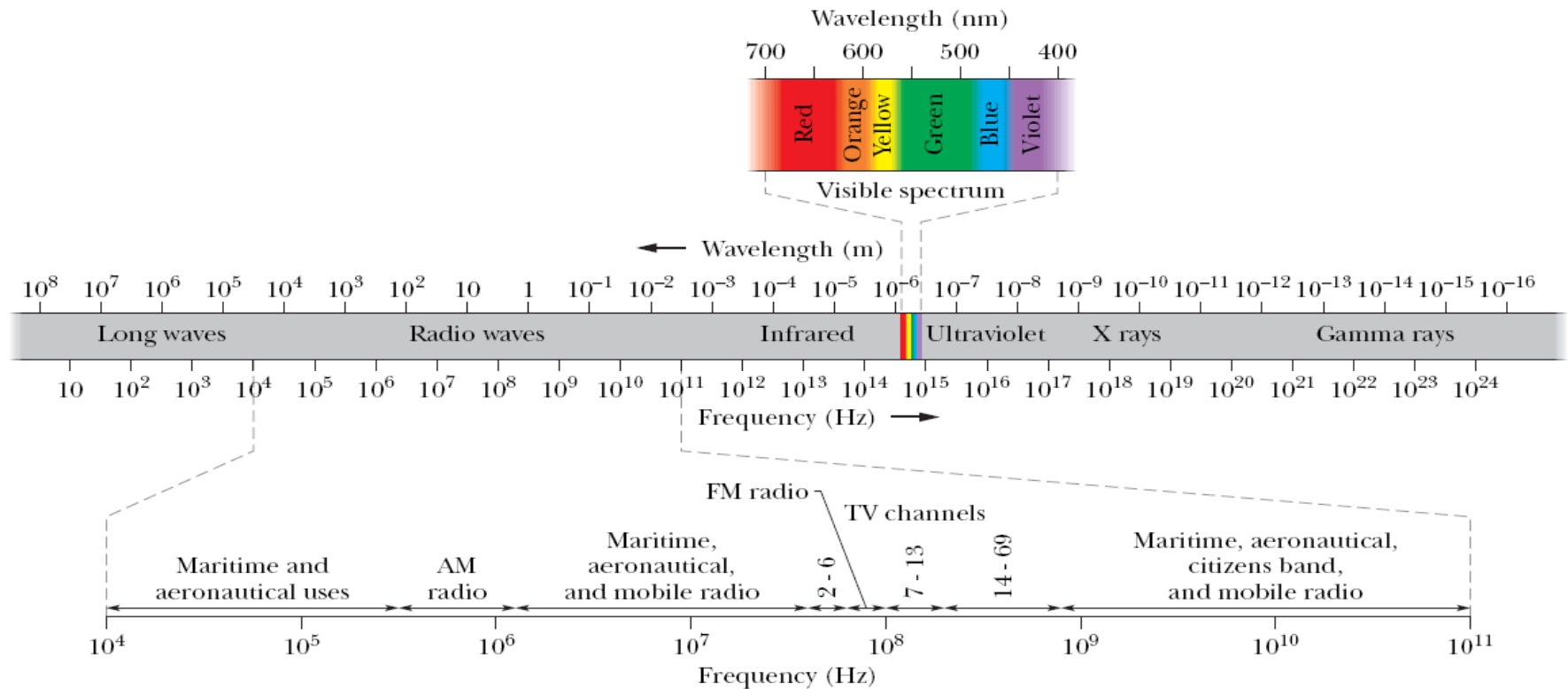

Fig. 33-1 The electromagnetic spectrum.
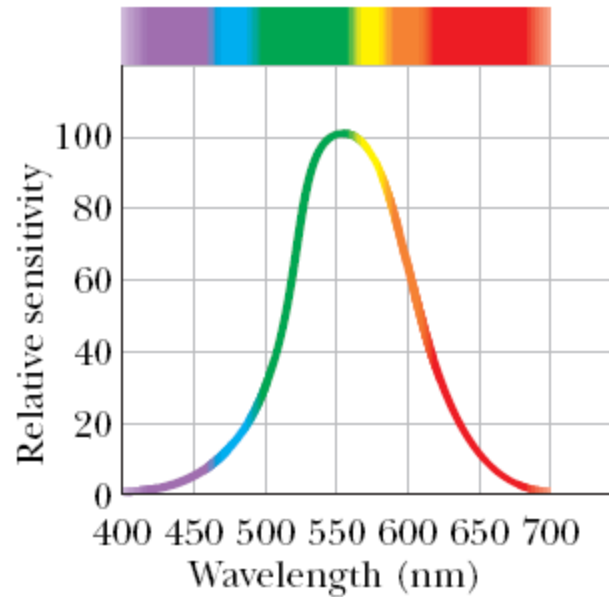
Visible Spectrum:

Fig. 33-2 The relative sensitivity of the average human eye to electromagnetic waves at different wavelengths. This portion of the electromagnetic spectrum to which the eye is sensitive is called *visible light.*
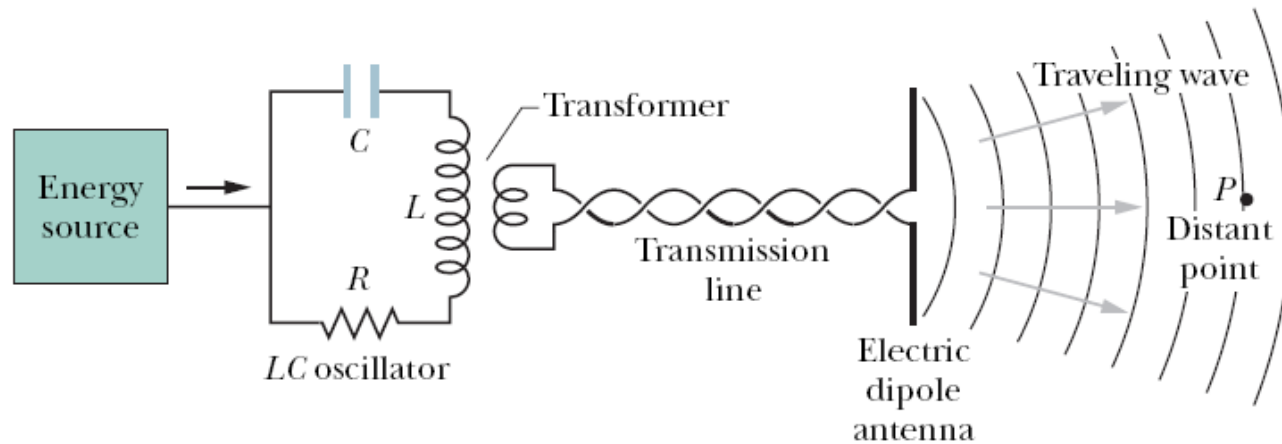
7.3.2. The Traveling Wave, Qualitatively:

Fig. 33-3 An arrangement for generating a traveling electromagnetic wave in the shortwave radio region of the spectrum: an *LC* oscillator produces a sinusoidal current in the antenna, which generates the wave. *P* is a distant point at which a detector can monitor the wave traveling past it.

Some electromagnetic waves, including x rays, gamma rays, and visible light, are *radiated (emitted)* from sources that are of atomic or nuclear size. Figure 33-3 shows the generation of such waves. At its heart is an *LC oscillator,* which establishes an angular frequency w(=1/√(LC)). Charges and currents in this circuit vary sinusoidally at this frequency.

$R \ll \lambda/2\pi$

$l < \lambda/8$

E

$E \sim 1/R^3$

$R < \lambda/2\pi$

H

$H \sim 1/R^2$

Einduction

$E \sim 1/R^2$

Fig. 33-4

Greatest magnitudes
(a)

(h)

(b)

Zero magnitudes

(g)

(c)

(f)

(d)

Greatest magnitudes
(e)
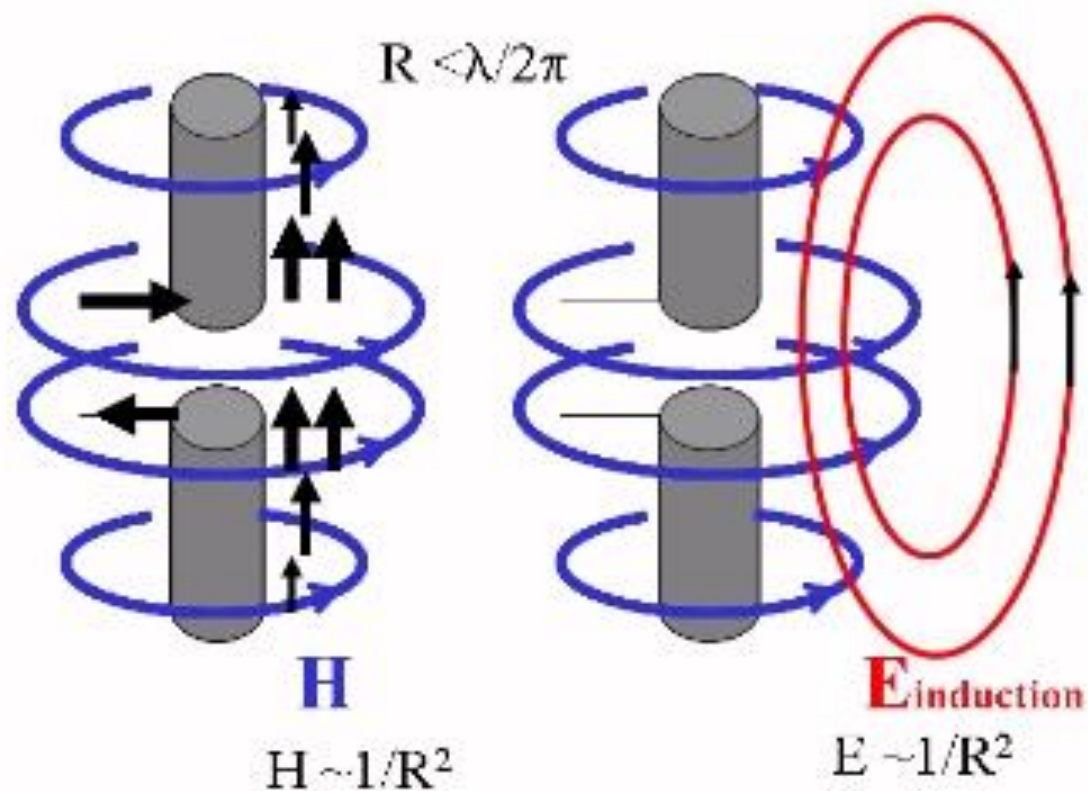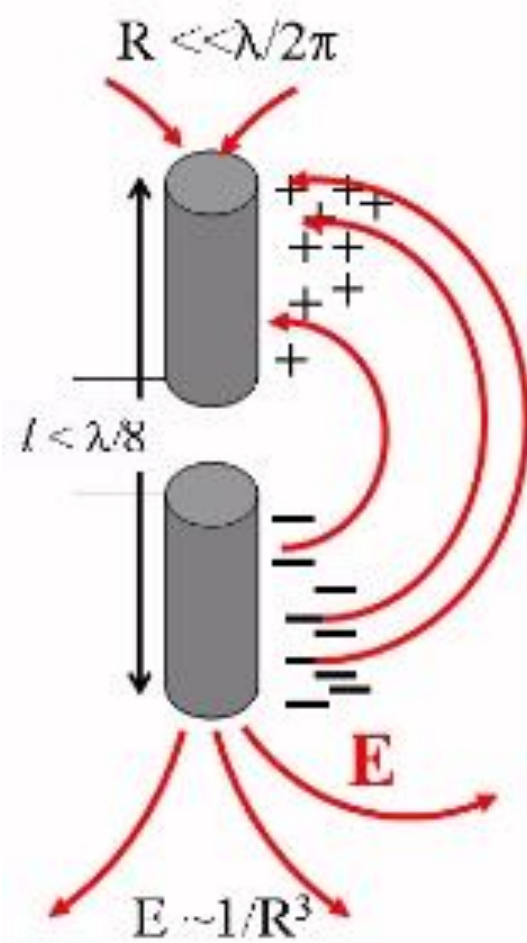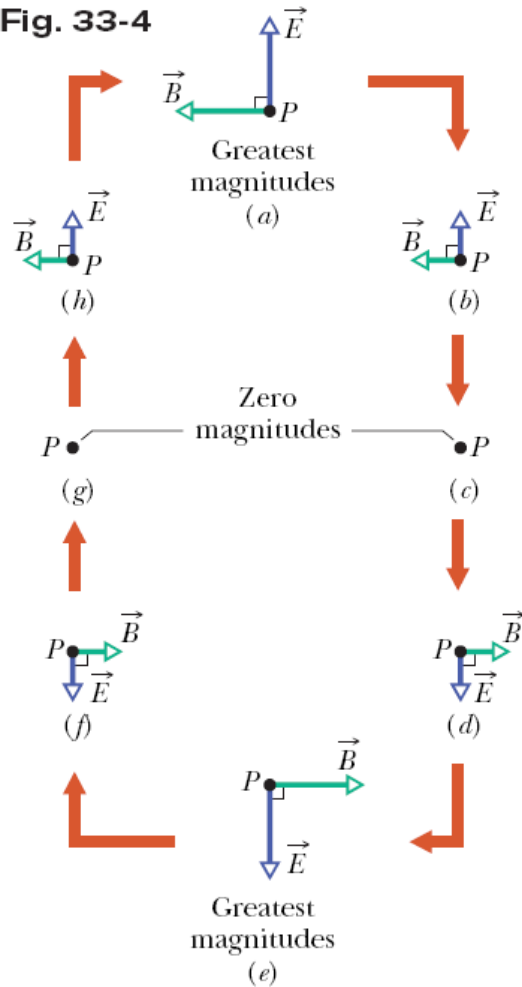
Figure 33-4 shows how the electric field and the magnetic field change with time as one wavelength of the wave sweeps past the distant point P in the last figure; in each part of Fig. 33-4, the wave is traveling directly out of the page.

At a distant point, such as P, the curvature of the waves is small enough to neglect it. At such points, the wave is said to be a *plane wave*.

Here are some key features regardless of how the waves are generated:

1. The electric and magnetic fields and are always perpendicular to the direction in which the wave is traveling. The wave is a *transverse wave*.

2. The electric field is always perpendicular to the magnetic field.

3. The cross product always gives the direction in which the wave travels.

4. The fields always vary sinusoidally. The fields vary with the same frequency and are *in phase* with each other.

We can write the electric and magnetic fields as sinusoidal functions of position x (along the path of the wave) and time t :

$$E = E_m \sin(kx - \omega t),$$

$$B = B_m \sin(kx - \omega t),$$

Here E_m and B_m are the amplitudes of the fields and, ω and k are the angular frequency and angular wave number of the wave, respectively.

All electromagnetic waves, including visible light, have the same speed c in vacuum.

The speed of the wave (in vacuum) is given by c.

$$c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} \qquad \text{(wave speed)},$$

Its value is about 3.0×10^8 m/s.

Quantitatively:

The dashed rectangle of dimensions dx and h in Fig. 33-6 is fixed at point P on the x axis and in the xy plane.

As the electromagnetic wave moves rightward past the rectangle, the magnetic flux B through the rectangle changes and—according to Faraday's law of induction—induced electric fields appear throughout the region of the rectangle. We take $\mathbf{E}$ and $\mathbf{E} + \mathbf{dE}$ to be the induced fields along the two long sides of the rectangle. These induced electric fields are, in fact, the electrical component of the electromagnetic wave.

(a) Wavefronts — Ray

(b) Electric component — Magnetic component
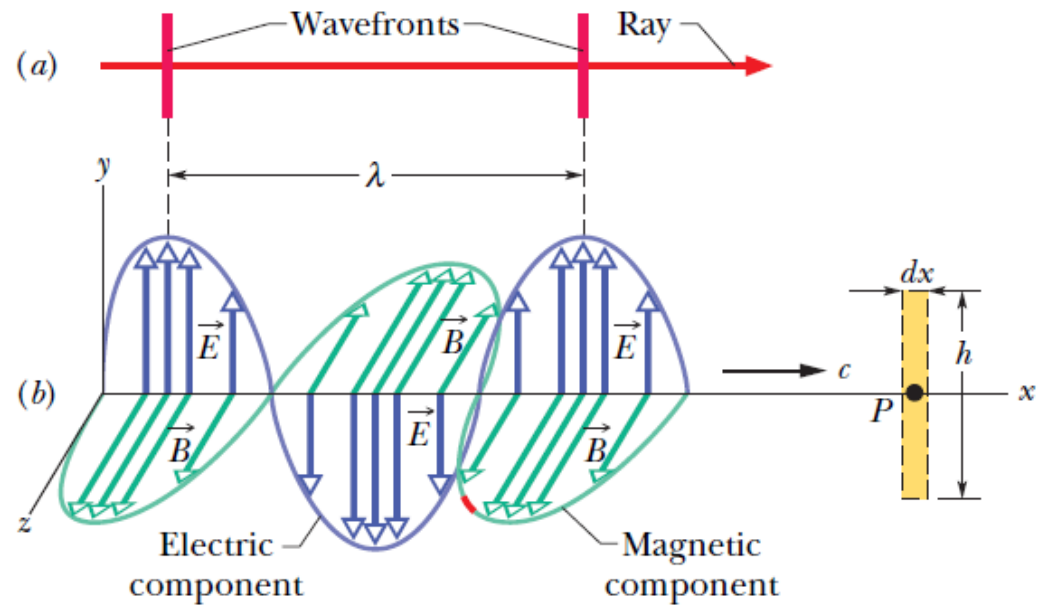
$$\oint \vec{E} \cdot d\vec{s} = -\frac{d\Phi_B}{dt} = (E + dE)h - Eh = h\,dE.$$

$$\frac{d\Phi_B}{dt} = h\,dx\,\frac{dB}{dt} \implies h\,dE = -h\,dx\,\frac{dB}{dt} \implies \frac{dE}{dx} = -\frac{dB}{dt}.$$

$$\frac{\partial E}{\partial x} = kE_m\cos(kx - \omega t)$$

$$\frac{\partial B}{\partial t} = -\omega B_m\cos(kx - \omega t).$$

$$kE_m\cos(kx - \omega t) = \omega B_m\cos(kx - \omega t).$$

$$\boxed{\frac{E_m}{B_m} = c \qquad \text{(amplitude ratio)},}$$

The oscillating electric field induces an oscillating and perpendicular magnetic field.
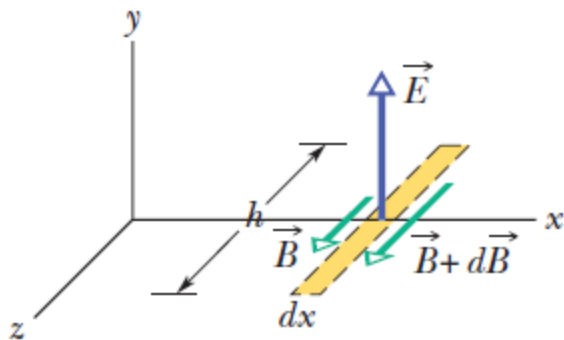

Fig. 33-7 The sinusoidal variation of the electric field through this rectangle, located (but not shown) at point *P* in *Fig. 33-5b*, **E** induces magnetic fields along the rectangle.The instant shown is that of Fig. 33-6: is decreasing in magnitude, and the magnitude of the induced magnetic field is greater on the right side of the rectangle than on the left.

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt},$$

$$\oint \vec{B} \cdot d\vec{s} = -(B + dB)h + Bh = -h\, dB.$$

$$\Phi_E = (E)(h\, dx), \qquad \frac{d\Phi_E}{dt} = h\, dx\, \frac{dE}{dt}$$

$$-h\, dB = \mu_0 \varepsilon_0 \left(h\, dx\, \frac{dE}{dt} \right)$$

$$-\frac{\partial B}{\partial x} = \mu_0 \varepsilon_0 \frac{\partial E}{\partial t}.$$

$$-kB_m \cos(kx - \omega t) = -\mu_0 \varepsilon_0 \omega E_m \cos(kx - \omega t),$$

$$\frac{E_m}{B_m} = \frac{1}{\mu_0 \varepsilon_0 (\omega/k)} = \frac{1}{\mu_0 \varepsilon_0 c}.$$

$$c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} \qquad \text{(wave speed)},$$

7.4. Energy Transport and the Poynting Vector:

The direction of the Poynting vector $\vec{S}$ of an electromagnetic wave at any point gives the wave's direction of travel and the direction of energy transport at that point.

$$\vec{S} = \frac{1}{\mu_0} \vec{E} \times \vec{B} \qquad \text{(Poynting vector).}$$

$$S = \left(\frac{\text{energy/time}}{\text{area}}\right)_{\text{inst}} = \left(\frac{\text{power}}{\text{area}}\right)_{\text{inst}}.$$

$$S = \frac{1}{\mu_0} EB, \qquad \Rightarrow \qquad S = \frac{1}{c\mu_0} E^2$$

$$I = S_{\text{avg}} = \left(\frac{\text{energy/time}}{\text{area}}\right)_{\text{avg}} = \left(\frac{\text{power}}{\text{area}}\right)_{\text{avg}} = \frac{1}{c\mu_0}[E^2]_{\text{avg}} = \frac{1}{c\mu_0}[E_m^2 \sin^2(kx - \omega t)]_{\text{avg}}.$$

$$E_{\text{rms}} = \frac{E_m}{\sqrt{2}}. \qquad \Rightarrow \qquad I = \frac{1}{c\mu_0} E_{\text{rms}}^2.$$

The energy density u $(= \tfrac{1}{2}\varepsilon_0 E^2)$ within an electric field, can be written as:

$$u_E = \tfrac{1}{2}\varepsilon_0 E^2 = \tfrac{1}{2}\varepsilon_0(cB)^2. = \tfrac{1}{2}\varepsilon_0 \frac{1}{\mu_0 \varepsilon_0} B^2 = \frac{B^2}{2\mu_0}$$

The energy emitted by light source S must pass through the sphere of radius r.

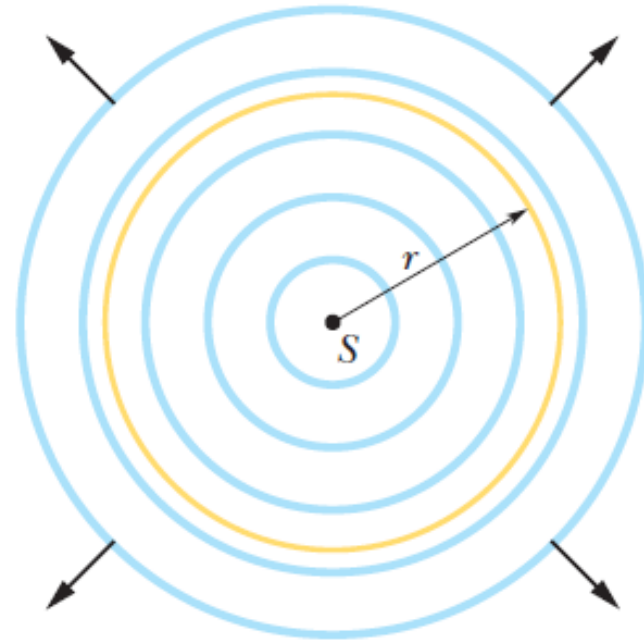$$I = \frac{\text{power}}{\text{area}} = \frac{P_s}{4\pi r^2}$$

Fig. 33-8 A point source S emits electromagnetic waves uniformly in all directions. The spherical wavefronts pass through an imaginary sphere of radius r that is centered on S.

7.5. Radiation Pressure:

Electromagnetic waves have linear momentum and thus can exert a pressure on an object when shining on it.

During the interval *Δt, the object gains* an energy *ΔU* from the radiation. If the object is free to move and that the radiation is entirely **absorbed** (taken up) by the object, then the momentum change *Δp* is given by:

$$\Delta p = \frac{\Delta U}{c} \quad \text{(total absorption)},$$

If the radiation is entirely reflected back along its original path, the magnitude of the momentum change of the object is twice that given above, or

$$\Delta p = \frac{2\,\Delta U}{c} \quad \text{(total reflection back along path).}$$

Since $F = \dfrac{\Delta p}{\Delta t}$. and $I = \dfrac{\text{power}}{\text{area}} = \dfrac{\text{energy/time}}{\text{area}}$ it follows that:

$$F = \frac{IA}{c} \quad \text{(total absorption).}$$

AND

$$F = \frac{2IA}{c} \quad \text{(total reflection back along path).}$$

Finally, the radiation pressure in the two cases are:

$$p_r = \frac{I}{c} \quad \text{(total absorption)}$$

$$p_r = \frac{2I}{c} \quad \text{(total reflection back along path).}$$

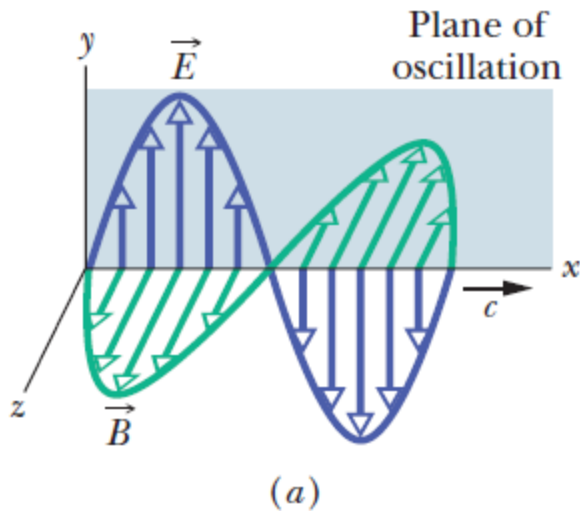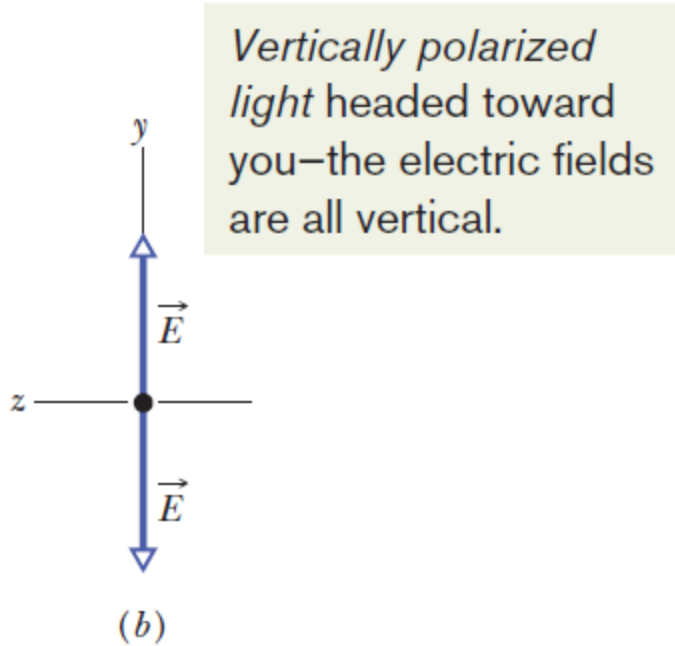# 7.6. Polarization, Reflection and Refraction:

(a)

(b)

Fig. 33-9 (*a*) The plane of oscillation of a polarized electromagnetic wave. (*b*) To represent the polarization, we view the plane of oscillation head-on and indicate the directions of the oscillating electric field with a double arrow.

Vertically polarized light headed toward you–the electric fields are all vertical.
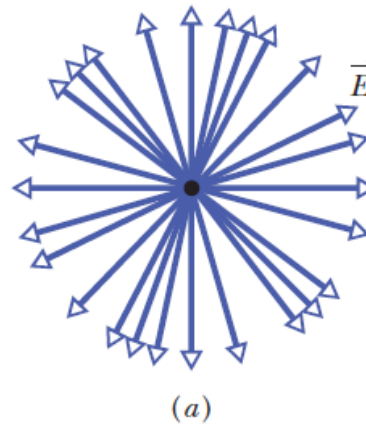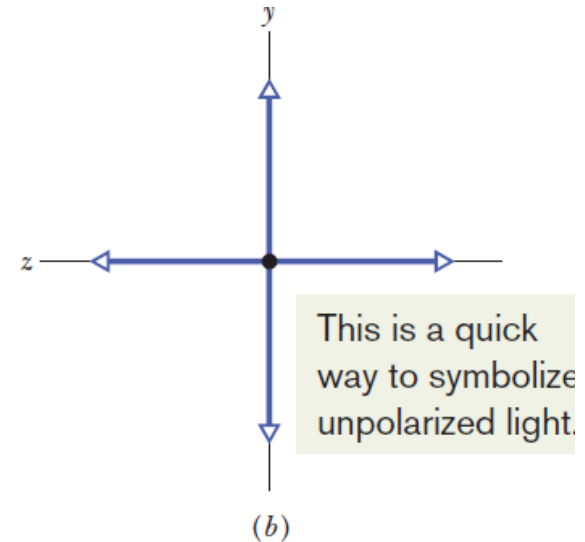
7.6.1. Polarization:

An electric field component parallel to the polarizing direction is passed (*transmitted*) by a polarizing sheet; a component perpendicular to it is absorbed.

Unpolarized light headed toward you—the electric fields are in all directions in the plane.

This is a quick way to symbolize unpolarized light.

(a)

(b)

Fig. 33-10 (*a*) Unpolarized light consists of waves with randomly directed electric fields. Here the waves are all traveling along the same axis, directly out of the page, and all have the same amplitude E. (*b*) A second way of representing unpolarized light—the light is the superposition of two polarized waves whose planes of oscillation are perpendicular to each other.

If the intensity of original unpolarized light is I_o, then the intensity of the emerging light through the polarizer, I, is half of that.

$$I = \tfrac{1}{2}I_0.$$

Intensity of Polarized Light:

Suppose now that the light reaching a polarizing sheet is already polarized.

Figure 33-12 shows a polarizing sheet in the plane of the page and the electric field of such a polarized light wave traveling toward the sheet (and thus prior to any absorption).

We can resolve E into two components relative to the polarizing direction of the sheet: parallel component E_y is transmitted by the sheet, and perpendicular component E_z is absorbed. Since θ is the angle between and the polarizing direction of the sheet, the transmitted parallel component is

$$E_y = E \cos \theta.$$

Since

$$I = E_{rms}^2/c\mu_0$$

$$I = I_0 \cos^2 \theta.$$

The sheet's polarizing axis is vertical, so only vertical components of the electric fields pass.

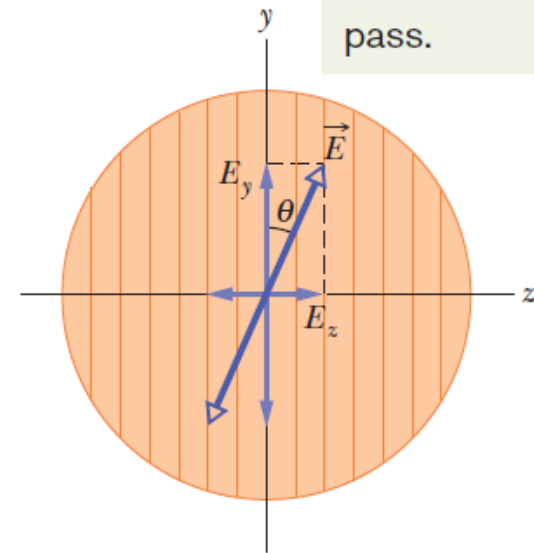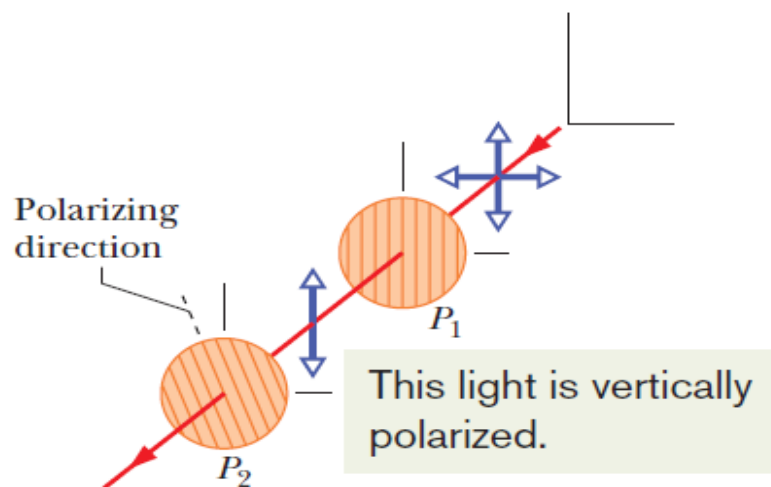**Fig. 33-12** Polarized light approaching a polarizing sheet. The electric field $\vec{E}$ of the light can be resolved into components E_y (parallel to the polarizing direction of the sheet) and E_z (perpendicular to that direction). Component E_y will be transmitted by the sheet; component E_z will be absorbed.

Fig. 33-13 The light transmitted by polarizing sheet P_1 is vertically polarized, as represented by the vertical double arrow. The amount of that light that is then transmitted by polarizing sheet P_2 depends on the angle between the polarization direction of that light and the polarizing direction of P_2 (indicated by the lines drawn in the sheet and by the dashed line).

Polarizing direction

This light is vertically polarized.

The sheet's polarizing axis is tilted, so only a fraction of the intensity passes.

Fig. 33-14 (*a*) Overlapping polarizing sheets transmit light fairly well when their polarizing directions have the same orientation, but (*b*) they block most of the light when they are crossed. (*Richard Megna/Fundamental Photographs.*)

(*a*)
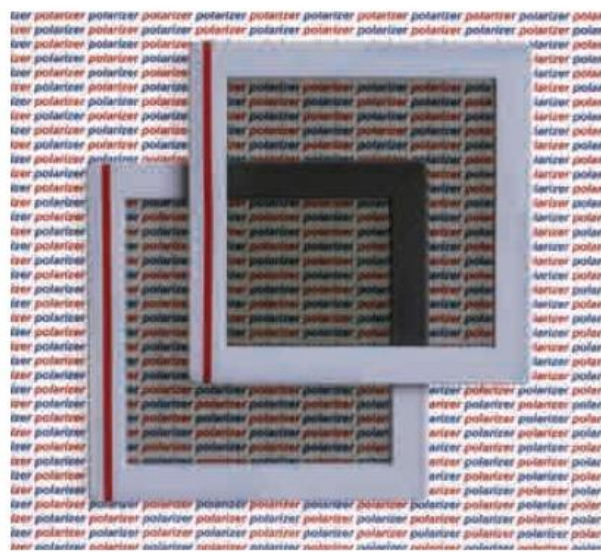
(*b*)
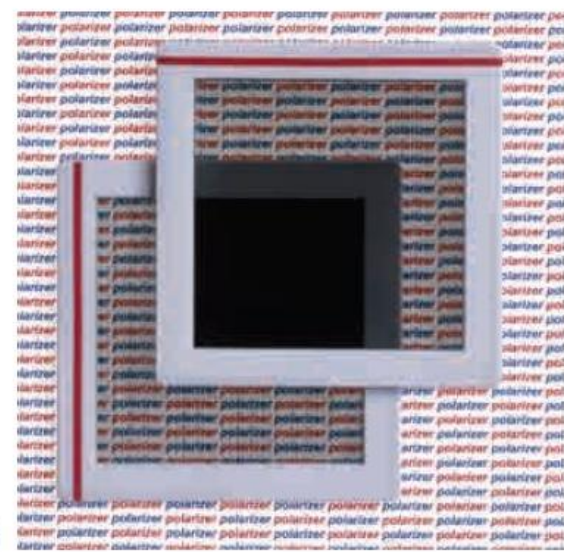
7.6.2. Reflection and Refraction:

$$\theta_1' = \theta_1 \qquad \text{(reflection)}.$$

$$n_2 \sin \theta_2 = n_1 \sin \theta_1 \qquad \text{(refraction)}.$$

The index of refraction, n, of a medium is equal to c/v, where v is the speed of light in that medium and c is its speed in vacuum. In the refraction law, each of the symbols n_1 and n_2 is a dimensionless constant, called the **index of refraction,** that is associated with a medium involved in the refraction. The refraction law is also called **Snell's law**.
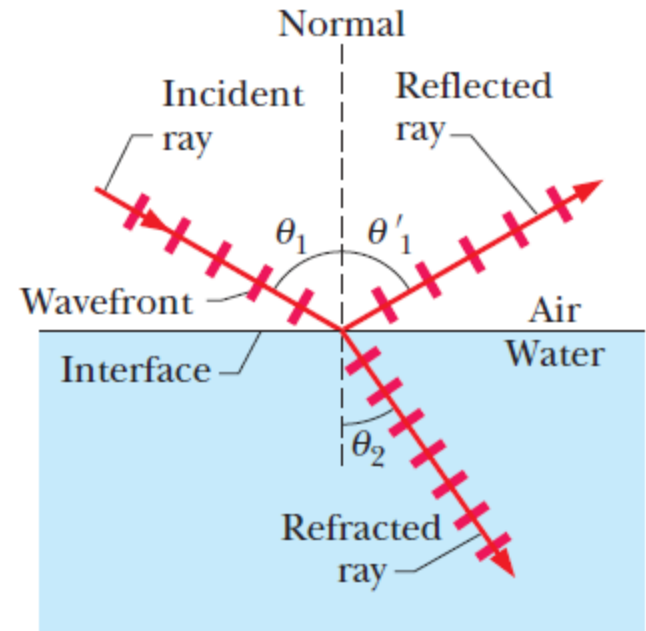

Fig. 33-16 (Continued) (b) A ray representation of (a). The angles of incidence (θ_1), reflection (θ_1'), and refraction (θ_2) are marked.

Table 33-1

Some Indexes of Refraction[a]

| Medium	Index	Medium	Index
Vacuum	Exactly 1	Typical crown glass	1.52
Air (STP)[b]	1.00029	Sodium chloride	1.54
Water (20°C)	1.33	Polystyrene	1.55
Acetone	1.36	Carbon disulfide	1.63
Ethyl alcohol	1.36	Heavy flint glass	1.65
Sugar solution (30%)	1.38	Sapphire	1.77
Fused quartz	1.46	Heaviest flint glass	1.89
Sugar solution (80%)	1.49	Diamond	2.42

[a]For a wavelength of 589 nm (yellow sodium light).

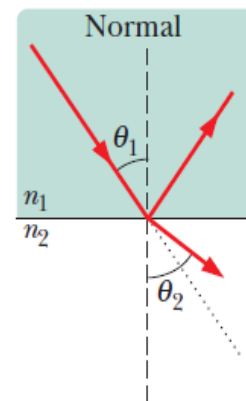[b]STP means "standard temperature (0°C) and pressure (1 atm)."

Normal

n_1
n_2

θ_1

θ_2
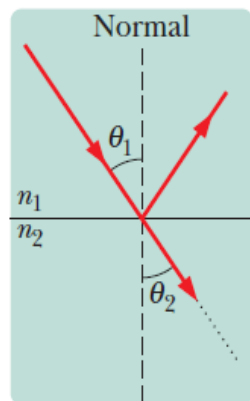
$n_2 = n_1$

(a)

If the indexes match, there is no direction change.

Normal

n_1
n_2

θ_1

θ_2
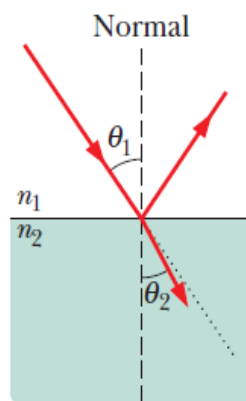
$n_2 > n_1$

(b)

If the next index is greater, the ray is bent *toward* the normal.

Normal

n_1
n_2

θ_1

θ_2

$n_2 < n_1$

(c)

If the next index is less, the ray is bent *away from* the normal.

Fig. 33-17 Refraction of light traveling from a medium with an index of refraction n_1 into a medium with an index of refraction n_2. (a) The beam does not bend when $n_2 = n_1$; the refracted light then travels in the *undeflected direction* (the dotted line), which is the same as the direction of the incident beam. The beam bends (b) toward the normal when $n_2 > n_1$ and (c) away from the normal when $n_2 < n_1$.
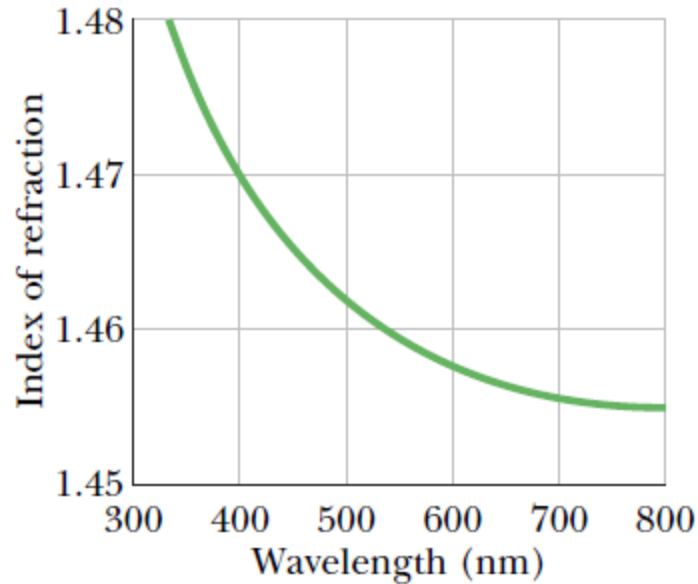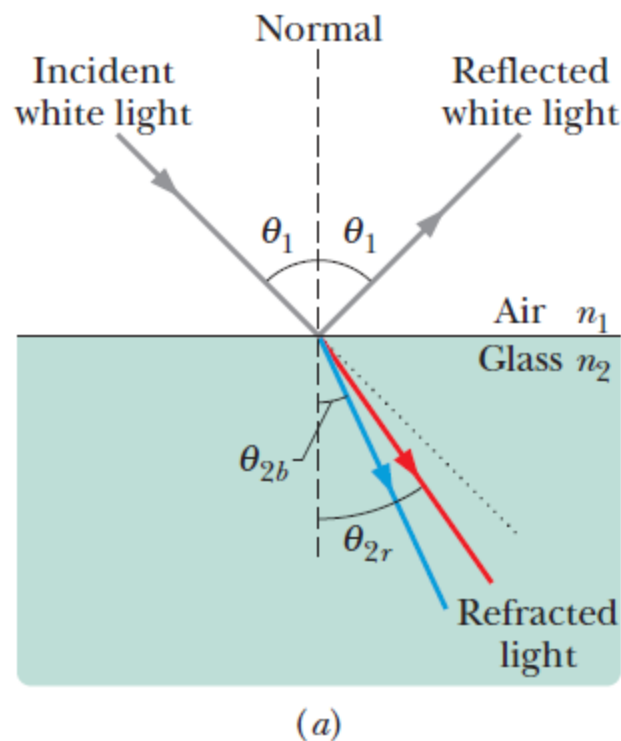
Chromatic Dispersion:

Fig. 33-18 The index of refraction as a function of wavelength for fused quartz. The graph indicates that a beam of short-wavelength light, for which the index of refraction is higher, is bent more upon entering or leaving quartz than a beam of long-wavelength light.

The index of refraction n encountered by light in any medium except vacuum depends on the wavelength of the light.

The dependence of n on wavelength implies that when a light beam consists of rays of different wavelengths, the rays will be refracted at different angles by a surface; that is, the light will be spread out by the refraction.

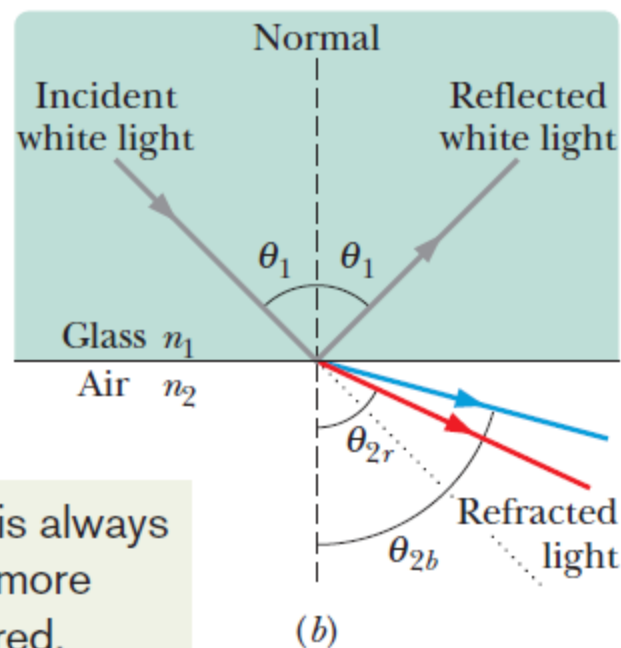This spreading of light is called **chromatic dispersion**.

Fig. 33-19 Chromatic dispersion of white light. The blue component is bent more than the red component. (*a*) Passing from air to glass, the blue component ends up with the smaller angle of refraction. (*b*) Passing from glass to air, the blue component ends up with the greater angle of refraction. Each dotted line represents the direction in which the light would continue to travel if it were not bent by the refraction.
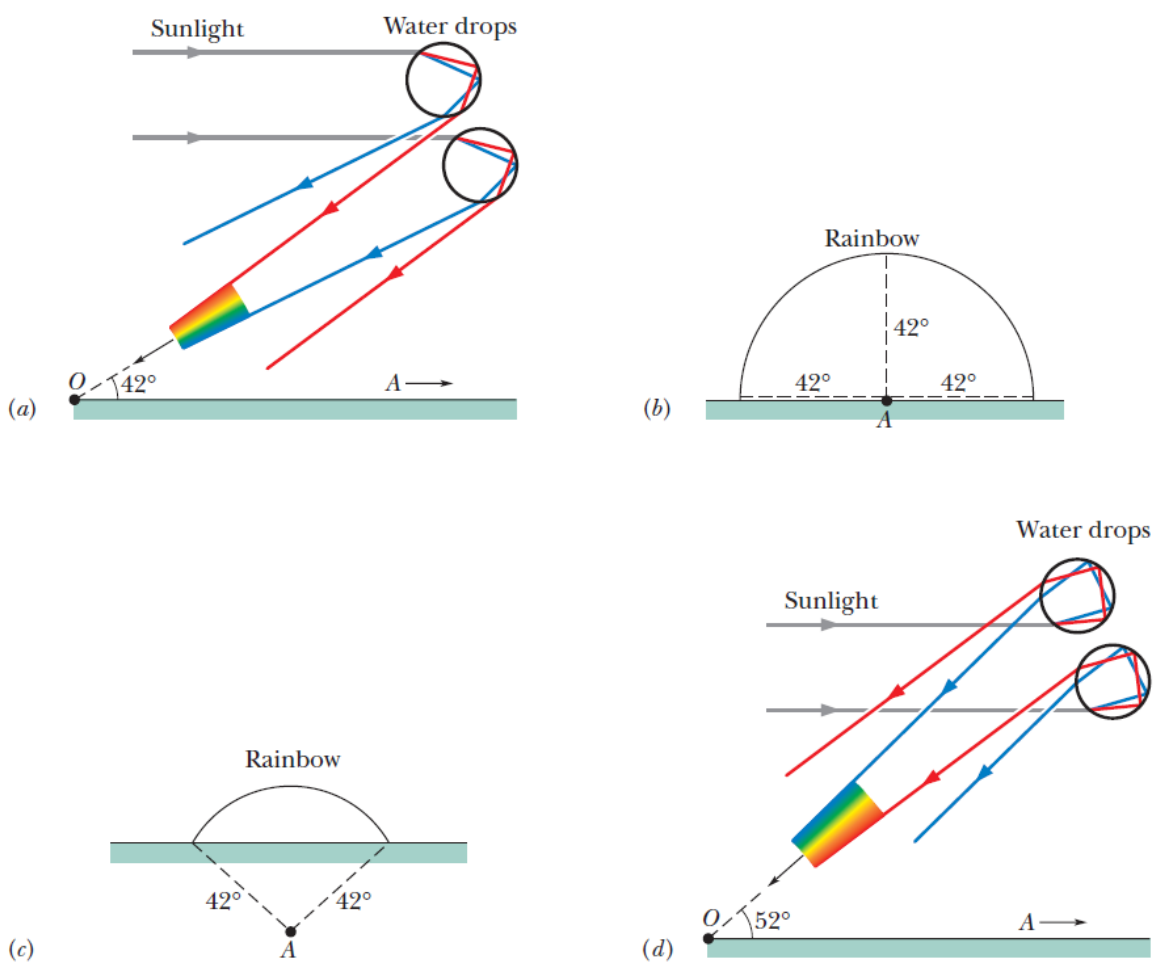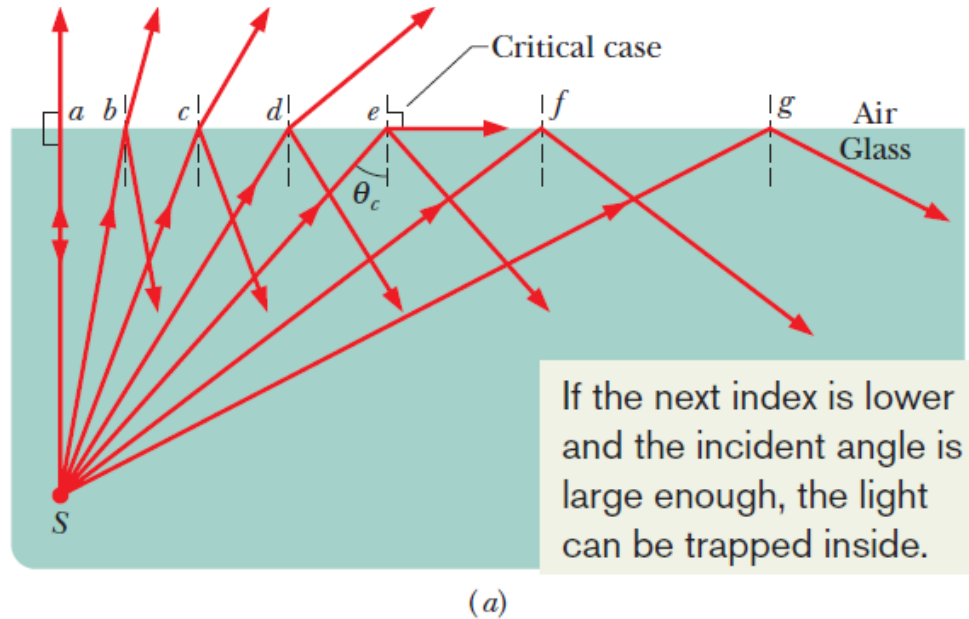
Chromatic Dispersion and Rainbow:

Fig. 33-21 (*a*) The separation of colors when sunlight refracts into and out of falling raindrops leads to a primary rainbow. The antisolar point *A* is on the horizon at the right. The rainbow colors appear at an angle of 42° from the direction of *A*. (*b*) Drops at 42° from *A* in any direction can contribute to the rainbow. (*c*) The rainbow arc when the Sun is higher (and thus *A* is lower). (*d*) The separation of colors leading to a secondary rainbow.

Total Internal Reflection:

Fig. 33-23 (a) Total internal reflection of light from a point source S in glass occurs for all angles of incidence greater than the critical angle θ_c. At the critical angle, the refracted ray points along the air–glass interface. (b) A source in a tank of water. (Ken Kay/Fundamental Photographs)

Critical case

a b   c   d   e   f   g   Air
Glass

If the next index is lower and the incident angle is large enough, the light can be trapped inside.

S

θ_c

(a)

(b)

For angles of incidence larger than θ_c, such as for rays f and g, there is no refracted ray and all the light is reflected; this effect is called **total internal reflection.**
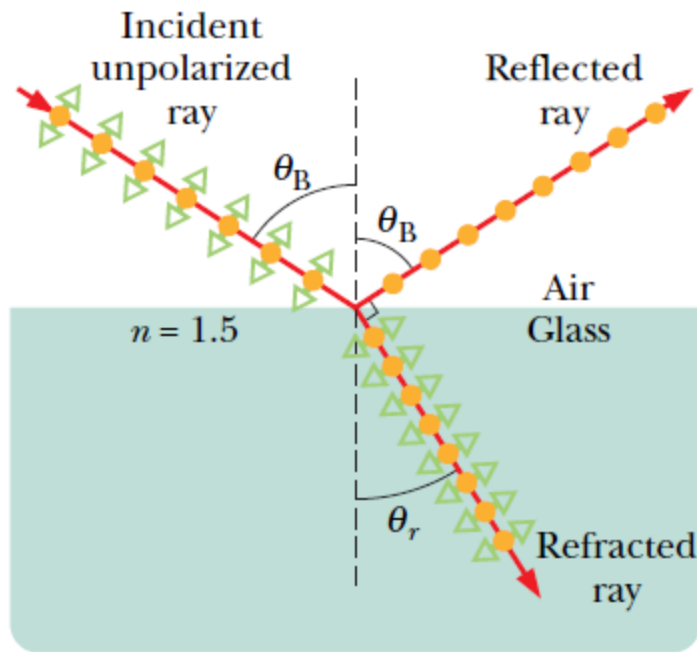
For the critical angle, $\qquad n_1 \sin \theta_c = n_2 \sin 90°,$

Which means that $\qquad\qquad \theta_c = \sin^{-1} \dfrac{n_2}{n_1} \qquad$ (critical angle).

Polarization by Reflection:

Incident unpolarized ray

Reflected ray

θ_B

θ_B

Air

Glass

$n = 1.5$

θ_r

Refracted ray

● Component perpendicular to page
⟷ Component parallel to page

Fig. 33-25 A ray of unpolarized light in air is incident on a glass surface at the Brewster angle θ_B. The electric fields along that ray have been resolved into components perpendicular to the page (the plane of incidence, reflection, and refraction) and components parallel to the page. The reflected light consists only of components perpendicular to the page and is thus polarized in that direction. The refracted light consists of the original components parallel to the page and weaker components perpendicular to the page; this light is partially polarized.

$$\theta_B + \theta_r = 90°.$$

$$n_1 \sin \theta_B = n_2 \sin \theta_r.$$

$$n_1 \sin \theta_B = n_2 \sin(90° - \theta_B) = n_2 \cos \theta_B,$$

$$\theta_B = \tan^{-1} \frac{n_2}{n_1} \qquad \text{(Brewster angle).}$$