INTERNATIONAL UNIVERSITY-VNUHCM

FINAL EXAMINATION

Semester 1, Academic Year 2021-2022

Duration: 90 minutes (online)

SUBJECT: Calculus 2	
Department of Mathematics	Lecturer
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Instructions:

- **You have to write by hands your solutions, and then scan it into a single PDF file and submit it to Blackboard.**
- **Your full name, student ID and your signature must be given on top of the first page of your solution draft.**
- **You are given an additional time of 10 minutes for submission.**
- **Each question carries 20 marks.**

Question 1. Let $f(x,y) = \ln(9 - x^2 - y) + \sqrt{y}$.

(a) Find and sketch the domain of $f(x,y)$.

(b) Find the directional derivative of the function $f(x,y)$ at the point $(1,1)$ in the direction of the vector $\mathbf{v} =< 3,4 >$.

Question 2. Find the local maximum and minimum values and saddles point(s) of the function

$$f(x,y) = x^3 - 24xy + 8y^3.$$

Question 3. (a) Evaluate the double integral

$$I = \iint_D \frac{y}{x}\, dA, \quad D = \{(x,y) \mid \quad 1 \le x \le 2,\ 0 \le y \le 2x^2\}.$$

(b) Estimate the volume of the solid that lies below the surface $z = \ln(x^2 + y^2 + 1)$ and above the rectangle $R = [0,4] \times [0,2]$ by using a Riemann sum with $m = 4, n = 2$ and the sample point to be the upper right corner of each square.

Question 4. Given a force field $\mathbf{F}(x,y) = \dfrac{xy}{10}\mathbf{i} + \dfrac{y}{5}\mathbf{j}$.

(a) Sketch the vector field $\mathbf{F}(x,y)$.

(b) Find the work done by the force field $\mathbf{F}(x,y)$ in moving a particle along the parabola $y = 2x^3$ from the point $(0,0)$ to the point $(1,2)$.

Question 5. Given the vector field $\mathbf{F}(x,y,z) = -z\mathbf{j} + (y - z^2)\mathbf{k}$.

(a) Find curl $\mathbf{F}$ and div $\mathbf{F}$

(b) Evaluate the surface integral $\iint_S \mathbf{F} \cdot dS$, where S is the part of the sphere $x^2 + y^2 + z^2 = 1$ in the first octant with orientation toward the origin.

———END OF QUESTIONS———