Đỗ Minh Duy - ITITSB22029

Hypothesis Testing

1. $Z = \dfrac{\bar{X} - \mu}{\frac{\sigma}{\sqrt{n}}} = \dfrac{57.5 - 50}{\frac{20}{\sqrt{64}}} = 3$

- If $|z| > 2.576$ → reject the null hypotesis
- If $|z| \leq 2.576$ → fail to reject the null hypothesis.

In this case, $|3| > 2.576$ ⇒ reject the null hypothesis.

⇒ the population mean is not equal to 50 at the 0.01 level of significance

2. $n = 16$, sample mean $\hat{\mu} = 7.2$ | $\sigma = 1.2$

$H_0 : \mu \geq 7.6$

$H_1 : \mu < 7.6$

$$p\text{-value} = P_{H_0}\left(\sqrt{n}\cdot\frac{\bar{X} - \mu}{\sigma} < \sqrt{n}\,\frac{\hat{\mu} - \mu}{\sigma}\right)$$

$$= P_{H_0}\left(\sqrt{n}\cdot\frac{\bar{X} - \mu}{\sigma} < \sqrt{16}\cdot\frac{7.2 - 7.6}{1.2}\right)$$

$$= P_{H_0}\left(\sqrt{n}\,\frac{\bar{X} - \mu}{\sigma} < -1.333\right) = 1 - \phi(1.333) = 0.0913$$

(3) Population mean, $\mu = 5.5$ ounces

Sample mean, $\bar{x} = 5.23$ ounces

Sample size, $n = 64$

Alpha, $\alpha = 0.05$.

standard deviation, $\sigma = 0.24$ ounce

$H_0 : \mu = 5.5$ ounces

$H_A : \mu < 5.5$ ounces

$$Z_{stat} = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}}$$

$$Z_{stat} = \frac{5.23 - 5.5}{\frac{0.24}{\sqrt{64}}} = -9$$

Now, $Z_{critical}$ at 0.05 level of significance $= -1.645$

Since $Z_{stat} < Z_{critical}$, we reject the null hypothesis and accept the alternate hypothesis

Thus, we conclude that cheddar popcorn weighed less than 5.5 ounces.

(4) Sample size, $n = 2500$

Sample mean $= 2.95$

$H_0 : \bar{x} = 3$

$H_a : \bar{x} < 3$

mean difference $= 2.95 - 3 = -0.05$

$$\text{Std err of mean} = \frac{\sigma}{\sqrt{n}} = 0.02$$

$$Z = \frac{\text{mean difference}}{\text{std error}} = -2.5$$

Level of significance $= 5\%$

For one tailed test at 5% significance level $Z_{critical} = -1.645$

Since $-2.5 < -1.645$ we reject null hypothesis.

these data are strong enough, at 5% level of significance, to establish the claim of the toothpaste advertisement.