THE INTERNATIONAL UNIVERSITY – VIETNAM NATIONAL UNIVERSITY – HCMC

FINAL EXAMINATION

Semester I, 2023-2024 • Date: Jan, 2024 • Duration: **75 minutes**

SUBJECT: Applied Linear Algebra	
Department of Mathematics Vice Chair: Assoc.Prof. Nguyen Minh Quan	Lecturers Dr. Ta Q Bao, Assoc.Prof. Tran V Khanh

INSTRUCTIONS:

- *Each student is allowed a scientific calculator and a maximum of TWO double-sided sheets of reference material (size A4 or similar) marked with their name and ID. All other documents and electronic devices are forbidden.*

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Question 1. *(20 pts)* Determine, with explanation, whether the set is a subspace of the given vector space $\mathbb{R}^3$

a) $H = \{(x_1, x_2, x_3) : x_1 = x_2 + 2x_3\}$.

b) $W = \{(x_1, x_2, x_3) : x_1 x_2 = x_3\}$.

Question 2. *(30 pts)* In $\mathbb{R}^3$, let a set of vector $S = \{u_1, u_2, u_3\}$, where $u_1 = (1, 1, 2)$ $u_2 = (1, -1, -1)$, and $u_3 = (2, 1, 1)$.

a) Determine whether S is a basis of vector space $\mathbb{R}^3$?

b) Write vector $v = (1, 0, 2)$ as a linear combination of vectors u_1, u_2, and u_3.

Question 3. *(20 pts)* Find rank and nullspace of the following matrix

$$A = \begin{pmatrix} 2 & 4 & -2 & 1 \\ -2 & -5 & 7 & 3 \\ 3 & 7 & -8 & 6 \end{pmatrix}.$$

Question 4. *(20 pts)* Find eigenvalues and eigenvectors of the following matrix

$$A = \begin{pmatrix} 2 & 2 \\ 1 & 3 \end{pmatrix}.$$

Question 5. *(10 pts)* Find the transition matrix from basic T to basic S in $\mathbb{R}^2$ below

$$T = \{(-3, 2), (4, -2)\} \quad \text{and} \quad S = \{(-1, 2), (2, -2)\}.$$

—END OF QUESTION PAPER—

Q1. a) H is a subspace
b) W is not a subspace

Q2.

a) Consider $A = \begin{pmatrix} 1 & 1 & 2 \\ 1 & -1 & 1 \\ 2 & -1 & 1 \end{pmatrix}$, since det(A)=3 then S is a basic

b) $A^{-1} = \begin{pmatrix} 0 & -1 & 1 \\ 1/3 & -1 & 1/3 \\ 1/3 & 1 & -2/3 \end{pmatrix}$, and $v = 2u_1 + u_2 - u_3$

Q3. $A \sim \begin{pmatrix} 2 & 4 & -2 & 1 \\ 0 & -1 & 5 & 4 \\ \underline{0} & \underline{0} & 0 & \underline{17/2} \end{pmatrix}$. $Rank(A) = 3, Null(A) = \{(-9, 5, 1, 0)\}$

Q4. The eigenvalues are $\lambda_1 = 4, \lambda_2 = 1$. The eigenvectors are $v_1 = (1, 1)$ and $v_2 = (1, 2)$

Q5. $P = \begin{pmatrix} -1 & 2 \\ -2 & 3 \end{pmatrix}$