INTERNATIONAL UNIVERSITY-VNUHCM

FINAL EXAMINATION

Semester 2, Academic Year 2015-2016
Duration: 120 minutes

SUBJECT: **Calculus 2**	
Department of Mathematics	Lecturers:
Vice-Chair:	Signature:
Assoc. Prof. Pham H. A. Ngoc	Asoc. Prof. M.D. Thanh, T.T. Duong, Ph.D.

Each student is allowed a maximum of two double-sided sheets of reference material (of size A4 or similar). All other documents and electronic devices, except scientific calculators, are forbidden.

Question 1. (*20 marks*)
a) Find the first partial derivatives of the function $f(x, y) = 2x^3y^2 + x + y^2 - 3y$.

b) Find an equation of the tangent plane to the surface

$$z = 2x^2 + 3y^2 - x - 2y$$

at the point $(1, 1, 2)$.

Question 2. Let $f(x, y) = (3x + y)(1 + xy)$.

a) (*10 marks*) Find the local maximum, minimum values, and saddle point(s) of $f(x, y)$.

b) (*15 marks*) Find the absolute maximum and minimum points and values of $f(x, y)$ on the closed square $\mathcal{D} = [0, 1] \times [0, 1]$.

Question 3. (*20 marks*)
a) Evaluate the iterated integral

$$\int_0^2 \int_0^1 (2x + 3y^2)\, dx\, dy.$$

b) Evaluate the double integral

$$I = \iint_D (6x^2 - 2y)\, dA, \quad D = \{(x, y) \mid \quad 0 \le x \le 1, 0 \le y \le 2x^2\}.$$

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Question 4. (*20 marks*)
a) Evaluate the line integral

$$\int_C (2x + y)\ ds$$

where C is the line segment from $(1, 0)$ to $(3, 2)$.

b) Evaluate the line integral $\int_C \mathbf{F} \cdot d\mathbf{r}$, where

$$\begin{aligned}
&\mathbf{F}(x, y, z) = xy\mathbf{i} + y^2\mathbf{j} + z\mathbf{k}, \\
&\mathbf{r}(t) = \sin t\mathbf{i} + \cos t\mathbf{j} + 2t\mathbf{k}, \quad 0 \leq t \leq 2\pi.
\end{aligned}$$

Question 5. Let

$$\mathbf{F}(x, y, z) = (e^{x^2} + 3x)\mathbf{i} + (2x - \sin(y^3))\mathbf{j} + (x - z^2)\mathbf{k}.$$

a) (*10 marks*) Find curl(F) and div(F).

b) (*5 marks*) Apply Stokes's theorem, if needed, to evaluate the line integral

$$\int_{\mathcal{C}} \mathbf{F} \cdot d\mathbf{r},$$

where $\mathcal{C}$ is the triangular loop PQR in the direction $P \to Q, Q \to R, R \to P$ where $P = (2, 0, 0), Q = (0, 3, 0)$, and $R = (0, 0, 4)$.

——— END OF QUESTIONS ———

SOLUTIONS OF FINAL EXAM: CALCULUS 2, SEMESTER 2, 2015-16

Question 1. (a) $f_x = 6x^2y^2 + 1,\ f_y = 4x^3y + 2y - 3$.

(b) $z_x(1,1) = 4(1) - 1 = 3, z_y(1,1) = 6(1) - 2 = 4$. Hence, an equation for the tangent plane is $z = 2 + 3(x-1) + 4(y-1)$ or $3x + 4y - z = 5$.

Question 2. $f(x,y) = 3x + y + 3x^2y + xy^2, f_x = 3 + 6xy + y^2, f_y = 2xy + 3x^2 + 1$.
(a) From $f_x = 0 = f_y$, $y^2 = 9x^2$, and $1 + 2xy = -3x^2 \leq 0$. Therefore $y = -3x$. Replace this in $f_y = -3x^2 + 1 = 0$, we have $x = \pm\frac{1}{\sqrt{3}}$, and the critical points are $(\frac{1}{\sqrt{3}}, -\sqrt{3})$ and $(-\frac{1}{\sqrt{3}}, \sqrt{3})$.

$$D = (6y)(2x) - (6x + 2y)^2 = -36x^2 - 4y^2 - 12xy < 0$$

at the critical points. Hence these are saddle points.

(b) The absolute minimum and maximum are on the boundary.
$f(x,0) = 3x, f(x,1) = 3x^2 + 4x + 1, f(0,y) = y, f(1,y) = y^2 + 4y + 3$. Since $0 \leq x, y \leq 1$, absolute minimum value is 0 at $(0,0)$, and absolute maximum value is 8 at $(1,1)$.

Question 3. a) Evaluate the iterated integral

$$\int_0^2\int_0^1 (2x + 3y^2)\ dxdy = \int_0^2 (x^2 + 3y^2x)\Big|_0^1 dy = \int_0^2 (1 + 3y^2)dy = (y + y^3)\Big|_0^2 = 2 + 2^3 = 10.$$

b) Evaluate the double integral

$$I = \iint_D (6x^2 - 2y)\ dA, \quad D = \{(x,y) \mid \quad 0 \leq x \leq 1, 0 \leq y \leq 2x^2\}$$

$$I = \int_0^1\int_0^{2x^2} (6x^2 - 2y)\ dydx = \int_0^1 (6x^2y - y^2)\Big|_0^{2x^2}\ dx$$

$$= \int_0^1 (12x^4 - 4x^4)\ dx = \int_0^1 8x^4\ dx = (8/5)x^5\Big|_0^1 = 8/5$$

Question 4. (a) $c(t) = (1-t)(1,0) + t(3,2) = (1 + 2t, 2t), \|c'(t)\| = \|(2,2)\| = 2\sqrt{2}$.

$$\int_C (2x + y)\ ds = 2\sqrt{2}\int_0^1 [2(1+2t) + 2t]\ dt = 2\sqrt{2}[2+3] = 10\sqrt{2}.$$

(b) $F(r(t)) = (\sin t\cos t, \cos^2 t, 2t), r'(t) = (\cos t, -\sin t, 2)$. Therefore

$$\int_C \mathbf{F}\cdot d\mathbf{r} = \int_0^{2\pi} F(r(t))\cdot r'(t)\ dt = \int_0^{2\pi} 4t\ dt = 8\pi^2.$$

Question 5. (a) $\mathrm{div}(F) = 2xe^{x^2} + 3 - 3y^2\cos(y^3) - 2z$, $\mathrm{curl}(F) = \mathrm{curl}(0, 2x, x) = (0, -1, 2)$.

(b) Parametrize the triangle ΔPQR as the graph of the function $z = 4 - 2x - \frac{4}{3}y$, where $x \in [0,2], y \in [0, 3 - \frac{3}{2}x]$

$$\int_{\mathcal{C}} \mathbf{F}\cdot d\mathbf{r} = \int_0^2\int_0^{3-\frac{3}{2}x} \mathbf{curl}(F)\cdot(-z_x, -z_y, 1)\ dy\ dx = \int_0^2\int_0^{3-\frac{3}{2}x} \frac{2}{3}\ dy\ dx = \int_0^2 (2-x)dx = 2.$$