Problems

P 7.1 [a] $R = \dfrac{v}{i} = 25\,\Omega$

[b] $\tau = \dfrac{1}{10} = 100\,\text{ms}$

[c] $\tau = \dfrac{L}{R} = 0.1$

$L = (0.1)(25) = 2.5\,\text{H}$

[d] $w(0) = \dfrac{1}{2}L[i(0)]^2 = \dfrac{1}{2}(2.5)(6.4)^2 = 51.2\,\text{J}$

[e] $w_{\text{diss}} = \displaystyle\int_0^t 1024e^{-20x}\,dx = 1024\frac{e^{-20x}}{-20}\Big|_0^t = 51.2(1-e^{-20t})\,\text{J}$

$\%\,\text{dissipated} = \dfrac{51.2(1-e^{-20t})}{51.2}(100) = 100(1-e^{-20t})$

$\therefore\ \ 100(1-e^{-20t}) = 60 \qquad \text{so} \qquad e^{-20t} = 0.4$

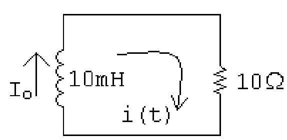
Therefore $\quad t = \dfrac{1}{20}\ln 2.5 = 45.81\,\text{ms}$

P 7.2 [a] Note that there are several different possible solutions to this problem, and the answer to part (c) depends on the value of inductance chosen.

$R = \dfrac{L}{\tau}$

Choose a 10 mH inductor from Appendix H. Then,

$R = \dfrac{0.01}{0.001} = 10\,\Omega$ which is a resistor value from Appendix H.



[b] $i(t) = I_o e^{-t/\tau} = 10e^{-1000t}\,\text{mA}, \qquad t \ge 0$

[c] $w(0) = \dfrac{1}{2}LI_o^2 = \dfrac{1}{2}(0.01)(0.01)^2 = 0.5\,\mu\text{J}$

$w(t) = \dfrac{1}{2}(0.01)(0.01e^{-1000t})^2 = 0.5\times10^{-6}e^{-2000t}$

So $\quad 0.5\times10^{-6}e^{-2000t} = \dfrac{1}{2}w(0) = 0.25\times10^{-6}$

$e^{-2000t} = 0.5 \qquad \text{then} \qquad e^{2000t} = 2$

$\therefore\ \ t = \dfrac{\ln 2}{2000} = 346.57\,\mu\text{s} \qquad \text{(for a 10 mH inductor)}$

P 7.3 [**a**] $i_L(0) = \dfrac{125}{50} = 2.5\,\text{A}$

$$i_o(0^+) = \frac{125}{25} - 2.5 = 5 - 2.5 = 2.5\,\text{A}$$

$$i_o(\infty) = \frac{125}{25} = 5\,\text{A}$$

[**b**] $i_L = 2.5e^{-t/\tau}; \qquad \tau = \dfrac{L}{R} = \dfrac{50 \times 10^{-3}}{25} = 2\,\text{ms}$

$$i_L = 2.5e^{-500t}\,\text{A}$$

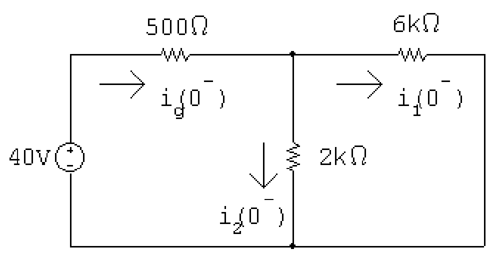
$$i_o = 5 - i_L = 5 - 2.5e^{-500t}\,\text{A}, \qquad t \geq 0^+$$

[**c**] $5 - 2.5e^{-500t} = 3$

$$2 = 2.5e^{-500t}$$

$$e^{500t} = 1.25 \qquad \therefore \quad t = 446.29\,\mu\text{s}$$

P 7.4 [**a**] $t < 0$



$$2\,\text{k}\Omega \| 6\,\text{k}\Omega = 1.5\,\text{k}\Omega$$

Find the current from the voltage source by combining the resistors in series and parallel and using Ohm's law:

$$i_g(0^-) = \frac{40}{(1500 + 500)} = 20\,\text{mA}$$

Find the branch currents using current division:

$$i_1(0^-) = \frac{2000}{8000}(0.02) = 5\,\text{mA}$$

$$i_2(0^-) = \frac{6000}{8000}(0.02) = 15\,\text{mA}$$

[**b**] The current in an inductor is continuous. Therefore,

$$i_1(0^+) = i_1(0^-) = 5\,\text{mA}$$

$$i_2(0^+) = -i_1(0^+) = -5\,\text{mA} \qquad \text{(when switch is open)}$$

[c] $\tau = \frac{L}{R} = \frac{0.4 \times 10^{-3}}{8 \times 10^{3}} = 5 \times 10^{-5}\,\text{s}; \qquad \frac{1}{\tau} = 20{,}000$

$$i_1(t) = i_1(0^+)e^{-t/\tau} = 5e^{-20{,}000t}\,\text{mA}, \qquad t \geq 0$$

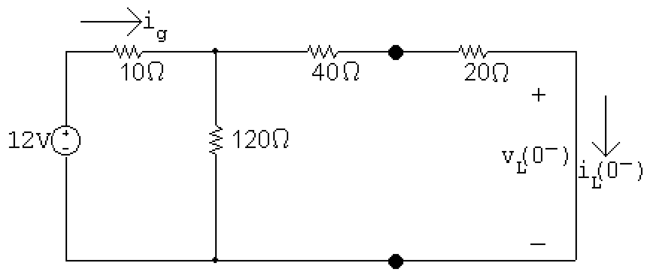
[d] $i_2(t) = -i_1(t) \qquad \text{when} \quad t \geq 0^+$

$$\therefore \quad i_2(t) = -5e^{-20{,}000t}\,\text{mA}, \qquad t \geq 0^+$$

[e] The current in a resistor can change instantaneously. The switching operation forces $i_2(0^-)$ to equal 15 mA and $i_2(0^+) = -5$ mA.

P 7.5 **[a]** $i_o(0^-) = 0 \qquad$ since the switch is open for $t < 0$.

[b] For $t = 0^-$ the circuit is:

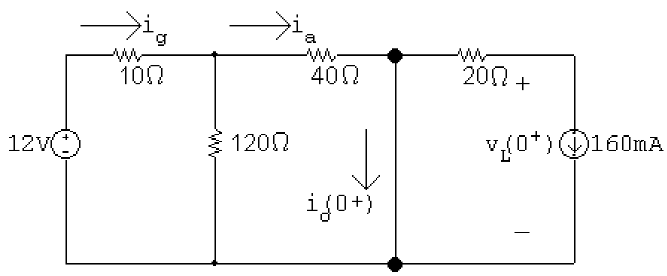


$$120\,\Omega \| 60\,\Omega = 40\,\Omega$$

$$\therefore \quad i_g = \frac{12}{10 + 40} = 0.24\,\text{A} = 240\,\text{mA}$$

$$i_L(0^-) = \left(\frac{120}{180}\right) i_g = 160\,\text{mA}$$

[c] For $t = 0^+$ the circuit is:



$$120\,\Omega \| 40\,\Omega = 30\,\Omega$$

$$\therefore \quad i_g = \frac{12}{10 + 30} = 0.30\,\text{A} = 300\,\text{mA}$$

$$i_a = \left(\frac{120}{160}\right) 300 = 225\,\text{mA}$$

$$\therefore \quad i_o(0^+) = 225 - 160 = 65\,\text{mA}$$

[d] $i_L(0^+) = i_L(0^-) = 160\,\text{mA}$

[e] $i_o(\infty) = i_a = 225\,\text{mA}$

[f] $i_L(\infty) = 0,$ since the switch short circuits the branch containing the 20 Ω resistor and the 100 mH inductor.

[g] $\tau = \dfrac{L}{R} = \dfrac{100 \times 10^{-3}}{20} = 5\,\text{ms}; \qquad \dfrac{1}{\tau} = 200$

$$\therefore \quad i_L = 0 + (160 - 0)e^{-200t} = 160e^{-200t}\,\text{mA}, \qquad t \geq 0$$

[h] $v_L(0^-) = 0$ since for $t < 0$ the current in the inductor is constant

[i] Refer to the circuit at $t = 0^+$ and note:

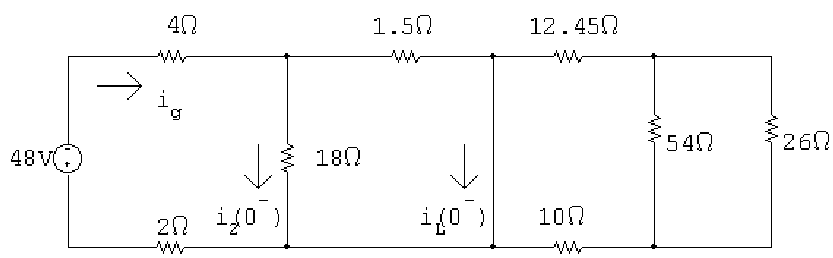
$$20(0.16) + v_L(0^+) = 0; \qquad \therefore \quad v_L(0^+) = -3.2\,\text{V}$$

[j] $v_L(\infty) = 0,$ since the current in the inductor is a constant at $t = \infty$.

[k] $v_L(t) = 0 + (-3.2 - 0)e^{-200t} = -3.2e^{-200t}\,\text{V}, \qquad t \geq 0^+$

[l] $i_o(t) = i_a - i_L = 225 - 160e^{-200t}\,\text{mA}, \qquad t \geq 0^+$

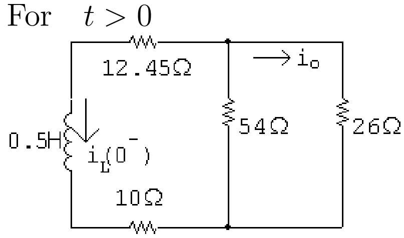
P 7.6 For $t < 0$



$$i_g = \frac{-48}{6 + (18\|1.5)} = -6.5\,\text{A}$$

$$i_L(0^-) = \frac{18}{18 + 1.5}(-6.5) = -6\,\text{A} = i_L(0^+)$$

For $t > 0$



$$i_L(t) = i_L(0^+)e^{-t/\tau}\,\text{A}, \qquad t \geq 0$$

$$\tau = \frac{L}{R} = \frac{0.5}{10 + 12.45 + (54\|26)} = 0.0125\,\text{s}; \qquad \frac{1}{\tau} = 80$$

$$i_L(t) = -6e^{-80t}\,\text{A}, \qquad t \geq 0$$

$$i_o(t) = \frac{54}{80}(-i_L(t)) = \frac{54}{80}(6e^{-80t}) = 4.05e^{-80t}\,\text{V}, \qquad t \geq 0^+$$

P 7.7 **[a]** $i(0) = \dfrac{24}{12} = 2\,\text{A}$

[b] $\tau = \dfrac{L}{R} = \dfrac{1.6}{80} = 20\,\text{ms}$

[c] $i = 2e^{-50t}\,\text{A}, \qquad t \geq 0$

$$v_1 = L\frac{d}{dt}(2e^{-50t}) = -160e^{-50t}\,\text{V} \qquad t \geq 0^+$$

$$v_2 = -72i = -144e^{-50t}\,\text{V} \qquad t \geq 0$$

[d] $w(0) = \dfrac{1}{2}(1.6)(2)^2 = 3.2\,\text{J}$

$$w_{72\Omega} = \int_0^t 72(4e^{-100x})\,dx = 288\frac{e^{-100x}}{-100}\Big|_0^t = 2.88(1 - e^{-100t})\,\text{J}$$

$$w_{72\Omega}(15\,\text{ms}) = 2.88(1 - e^{-1.5}) = 2.24\,\text{J}$$

$$\%\text{ dissipated } = \frac{2.24}{3.2}(100) = 69.92\%$$

P 7.8 $w(0) = \dfrac{1}{2}(10 \times 10^{-3})(5)^2 = 125\,\text{mJ}$

$$0.9w(0) = 112.5\,\text{mJ}$$

$$w(t) = \frac{1}{2}(10 \times 10^{-3})i(t)^2, \qquad i(t) = 5e^{-t/\tau}\,\text{A}$$

$$\therefore \quad w(t) = 0.005(25e^{-2t/\tau}) = 125e^{-2t/\tau})\,\text{mJ}$$

$$w(10\,\mu\text{s}) = 125e^{-20\times10^{-6}/\tau}\,\text{mJ}$$

$$\therefore \quad 125e^{-20\times10^{-6}/\tau} = 112.5 \qquad \text{so} \qquad e^{20\times10^{-6}/\tau} = \frac{10}{9}$$

$$\tau = \frac{20 \times 10^{-6}}{\ln(10/9)} = \frac{L}{R}$$

$$R = \frac{10 \times 10^{-3}\ln(10/9)}{20 \times 10^{-6}} = 52.68\,\Omega$$

P 7.9 **[a]** $w(0) = \frac{1}{2}LI_g^2$

$$w_{\text{diss}} = \int_0^{t_o} I_g^2 R e^{-2t/\tau}\,dt = I_g^2 R \frac{e^{-2t/\tau}}{(-2/\tau)}\Big|_0^{t_o}$$

$$= \frac{1}{2}I_g^2 R\tau(1 - e^{-2t_o/\tau}) = \frac{1}{2}I_g^2 L(1 - e^{-2t_o/\tau})$$

$$w_{\text{diss}} = \sigma w(0)$$

$$\therefore \quad \frac{1}{2}LI_g^2(1 - e^{-2t_o/\tau}) = \sigma\left(\frac{1}{2}LI_g^2\right)$$

$$1 - e^{-2t_o/\tau} = \sigma; \qquad e^{2t_o/\tau} = \frac{1}{(1-\sigma)}$$

$$\frac{2t_o}{\tau} = \ln\left[\frac{1}{(1-\sigma)}\right]; \qquad \frac{R(2t_o)}{L} = \ln[1/(1-\sigma)]$$

$$R = \frac{L\,\ln[1/(1-\sigma)]}{2t_o}$$

[b] $R = \dfrac{(10\times 10^{-3})\ln[1/0.9]}{20\times 10^{-6}}$

$$R = 52.68\,\Omega$$

P 7.10 **[a]** $v_o(t) = v_o(0^+)e^{-t/\tau}$

$$\therefore \quad v_o(0^+)e^{-10^{-3}/\tau} = 0.5v_o(0^+)$$

$$\therefore \quad e^{10^{-3}/\tau} = 2$$

$$\therefore \quad \tau = \frac{L}{R} = \frac{10^{-3}}{\ln 2}$$

$$\therefore \quad L = \frac{10\times 10^{-3}}{\ln 2} = 14.43\,\text{mH}$$

[b] $v_o(0^+) = -10i_L(0^+) = -10(1/10)(30\times 10^{-3}) = -30\,\text{mV}$

$$v_o(t) = -0.03e^{-t/\tau}\,\text{V}$$

$$p_{10\Omega} = \frac{v_o^2}{10} = 9\times 10^{-5}e^{-2t/\tau}$$

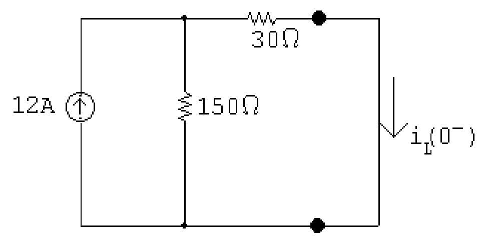
$$w_{10\Omega} = \int_0^{10^{-3}} 9\times 10^{-5}e^{-2t/\tau}\,dt = 4.5\tau\times 10^{-5}(1 - e^{-2\times 10^{-3}/\tau})$$

$$\tau = \frac{1}{1000\ln 2} \qquad \therefore \qquad w_{10\Omega} = 48.69\,\text{nJ}$$

$$w_L(0) = \frac{1}{2}Li_L^2(0) = \frac{1}{2}(14.43 \times 10^{-3})(3 \times 10^{-3})^2 = 64.92\,\text{nJ}$$

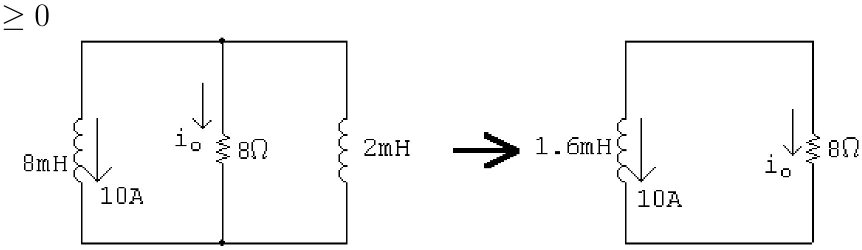
$$\%\text{ diss in 1 ms} = \frac{48.69}{64.92} \times 100 = 75\%$$

P 7.11 **[a]** $t < 0$



$$i_L(0^-) = \frac{150}{180}(12) = 10\,\text{A}$$

$t \geq 0$



$$\tau = \frac{1.6 \times 10^{-3}}{8} = 200 \times 10^{-6}; \qquad 1/\tau = 5000$$

$$i_o = -10e^{-5000t}\,\text{A} \quad t \geq 0$$

[b] $w_{\text{del}} = \frac{1}{2}(1.6 \times 10^{-3})(10)^2 = 80\,\text{mJ}$

[c] $0.95w_{\text{del}} = 76\,\text{mJ}$

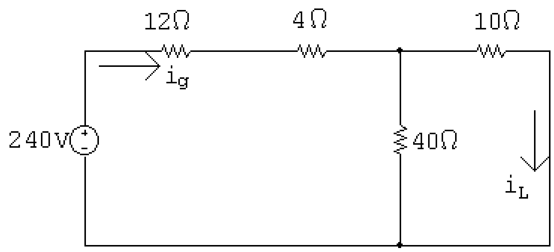
$$\therefore \quad 76 \times 10^{-3} = \int_0^{t_o} 8(100e^{-10{,}000t})\,dt$$

$$\therefore \quad 76 \times 10^{-3} = -80 \times 10^{-3}e^{-10{,}000t}\Big|_0^{t_o} = 80 \times 10^{-3}(1 - e^{-10{,}000t_o})$$

$$\therefore \quad e^{-10{,}000t_o} = 0.05 \quad \text{so} \quad t_o = 299.57\,\mu\text{s}$$

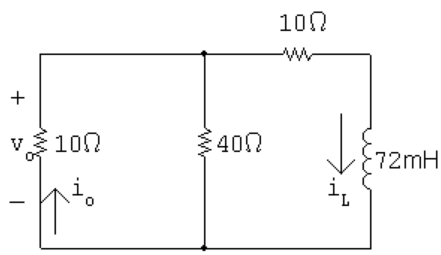
$$\therefore \quad \frac{t_o}{\tau} = \frac{299.57 \times 10^{-6}}{200 \times 10^{-6}} = 1.498 \quad \text{so} \quad t_o \approx 1.498\tau$$

P 7.12 $t < 0$:



$$i_L(0^+) = \frac{240}{16+8} = 10\,\text{A}; \qquad i_L(0^-) = 10\frac{40}{50} = 8\,\text{A}$$

$t > 0$:



$$R_e = \frac{(10)(40)}{50} + 10 = 18\,\Omega$$

$$\tau = \frac{L}{R_e} = \frac{72\times10^{-3}}{18} = 4\,\text{ms}; \qquad \frac{1}{\tau} = 250$$

$$\therefore \quad i_L = 8e^{-250t}\,\text{A}$$

$$v_o = 8i_o = 64e^{-250t}\,\text{V}, \quad t \geq 0^+$$

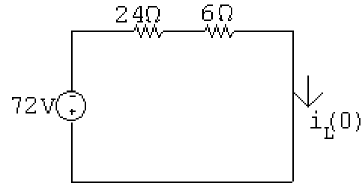
P 7.13 $$p_{40\Omega} = \frac{v_o^2}{40} = \frac{(64)^2}{40}e^{-500t} = 102.4e^{-500t}\,\text{W}$$

$$w_{40\Omega} = \int_0^\infty 102.4e^{-500t}\,dt = 102.4\frac{e^{-500t}}{-500}\Bigg|_0^\infty = 204.8\,\text{mJ}$$

$$w(0) = \frac{1}{2}(72\times10^{-3})(8)^2 = 2304\,\text{mJ}$$

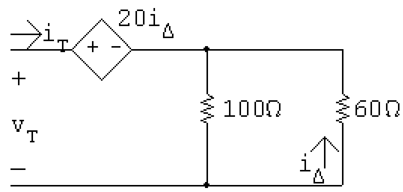
$$\%\text{ diss } = \frac{204.8}{2304}(100) = 8.89\%$$

P 7.14 **[a]** $t < 0$:



$$i_L(0) = -\frac{72}{24+6} = -2.4\,\text{A}$$

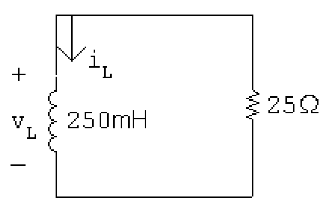
$t > 0$:



$$i_\Delta = -\frac{100}{160}i_T = -\frac{5}{8}i_T$$

$$v_T = 20i_\Delta + i_T\frac{(100)(60)}{160} = -12.5i_T + 37.5i_T$$

$$\frac{v_T}{i_T} = R_{\text{Th}} = -12.5 + 37.5 = 25\,\Omega$$



$$\tau = \frac{L}{R} = \frac{250 \times 10^{-3}}{25} \qquad \frac{1}{\tau} = 100$$

$$i_L = -2.4e^{-100t}\,\text{A}, \qquad t \geq 0$$

[b] $v_L = 250 \times 10^{-3}(240e^{-100t}) = 60e^{-100t}\,\text{V}, \quad t \geq 0^+$

[c] $i_\Delta = 0.625i_L = -1.5e^{-100t}\,\text{A} \qquad t \geq 0^+$

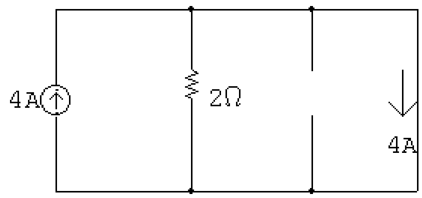
P 7.15 $w(0) = \frac{1}{2}(250 \times 10^{-3})(-2.4)^2 = 720\,\text{mJ}$

$$p_{60\Omega} = 60(-1.5e^{-100t})^2 = 135e^{-200t}\,\text{W}$$

$$w_{60\Omega} = \int_0^\infty 135e^{-200t}\,dt = 135\frac{e^{-200t}}{-200}\Big|_0^\infty = 675\,\text{mJ}$$

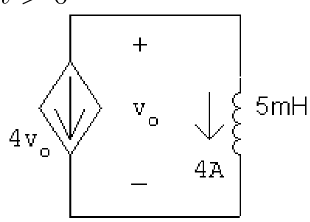
$$\%\text{ dissipated } = \frac{675}{720}(100) = 93.75\%$$

P 7.16 $t < 0$

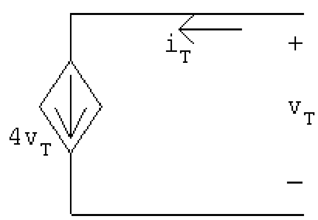


$$i_L(0^-) = i_L(0^+) = 4\,\text{A}$$

$t > 0$

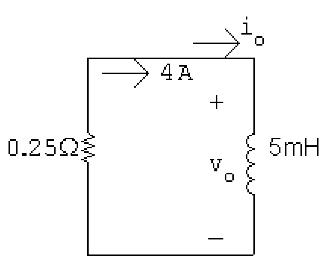


Find Thévenin resistance seen by inductor:



$$i_T = 4v_T; \qquad \frac{v_T}{i_T} = R_{\text{Th}} = \frac{1}{4} = 0.25\,\Omega$$

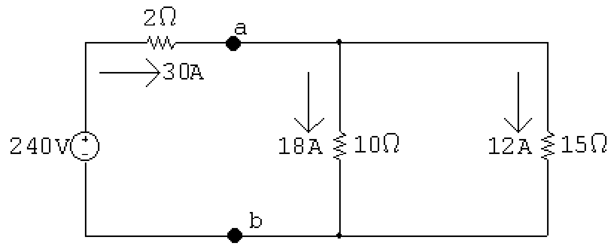
$$\tau = \frac{L}{R} = \frac{5 \times 10^{-3}}{0.25} = 20\,\text{ms}; \qquad 1/\tau = 50$$



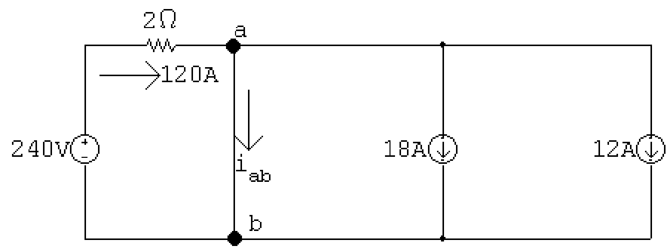
$$i_o = 4e^{-50t}\,\text{A}, \qquad t \geq 0$$

$$v_o = L\frac{di_o}{dt} = (5 \times 10^{-3})(-200e^{-50t}) = -e^{-50t}\,\text{V}, \quad t \geq 0^+$$

P 7.17 **[a]** $t < 0$:

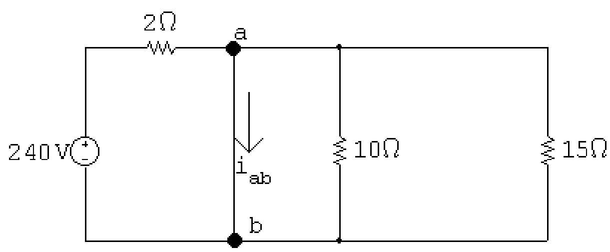


$t = 0^+$:

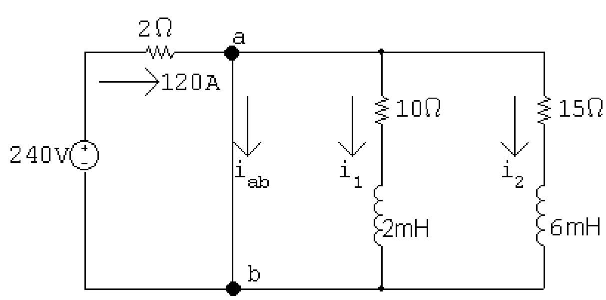


$$120 = i_{\rm ab} + 18 + 12, \qquad i_{\rm ab} = 90\,{\rm A}, \quad t = 0^+$$

[b] At $\quad t = \infty$:



$$i_{\rm ab} = 240/2 = 120\,{\rm A}, \quad t = \infty$$



[c] $i_1(0) = 18, \qquad \tau_1 = \dfrac{2 \times 10^{-3}}{10} = 0.2\,{\rm ms}$

$$i_2(0) = 12, \qquad \tau_2 = \frac{6 \times 10^{-3}}{15} = 0.4\,{\rm ms}$$

$$i_1(t) = 18e^{-5000t}\,{\rm A}, \quad t \geq 0$$

$$i_2(t) = 12e^{-2500t}\,{\rm A}, \quad t \geq 0$$

$$i_{ab} = 120 - 18e^{-5000t} - 12e^{-2500t}\,\text{A}, \quad t \geq 0$$

$$120 - 18e^{-5000t} - 12e^{-2500t} = 114$$

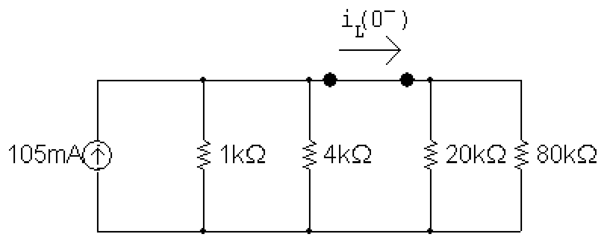
$$6 = 18e^{-5000t} + 12e^{-2500t}$$

Let $\quad x = e^{-2500t} \qquad$ so $\qquad 6 = 18x^2 + 12x$

Solving $\quad x = \dfrac{1}{3} = e^{-2500t}$

$$\therefore \quad e^{2500t} = 3 \qquad \text{and} \qquad t = \frac{\ln 3}{2500} = 439.44\,\mu\text{s}$$

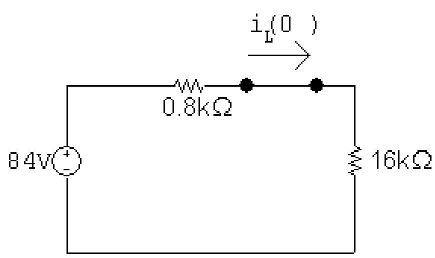
P 7.18 **[a]** $t < 0$



$$1\,\text{k}\Omega \| 4\,\text{k}\Omega = 0.8\,\text{k}\Omega$$

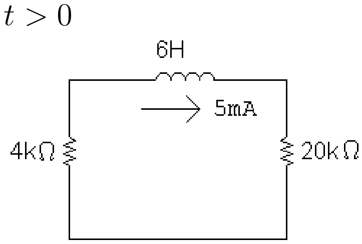
$$20\,\text{k}\Omega \| 80\,\text{k}\Omega = 16\,\text{k}\Omega$$

$$(105 \times 10^{-3})(0.8 \times 10^3) = 84\,\text{V}$$



$$i_L(0^-) = \frac{84}{16{,}800} = 5\,\text{mA}$$

$t > 0$



$$\tau = \frac{L}{R} = \frac{6}{24} \times 10^{-3} = 250\,\mu\text{s}; \qquad \frac{1}{\tau} = 4000$$

$$i_L(t) = 5e^{-4000t}\,\text{mA}, \qquad t \geq 0$$

$$p_{4k} = 25 \times 10^{-6}e^{-8000t}(4000) = 0.10e^{-8000t}\,\text{W}$$

$$w_{\text{diss}} = \int_0^t 0.10e^{-8000x}\,dx = 12.5 \times 10^{-6}[1 - e^{-8000t}]\,\text{J}$$

$$w(0) = \frac{1}{2}(6)(25 \times 10^{-6}) = 75\,\mu\text{J}$$

$$0.10w(0) = 7.5\,\mu\text{J}$$

$$12.5(1 - e^{-8000t}) = 7.5; \qquad \therefore \quad e^{8000t} = 2.5$$

$$t = \frac{\ln 2.5}{8000} = 114.54\,\mu\text{s}$$

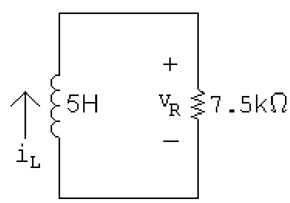
[b] $w_{\text{diss}}(\text{total}) = 75(1 - e^{-8000t})\,\mu\text{J}$

$$w_{\text{diss}}(114.54\,\mu\text{s}) = 45\,\mu\text{J}$$

$$\% = (45/75)(100) = 60\%$$

P 7.19 **[a]** $t > 0$:

$$L_{\text{eq}} = 1.25 + \frac{60}{16} = 5\,\text{H}$$



$$i_L(t) = i_L(0)e^{-t/\tau}\,\text{mA}; \qquad i_L(0) = 2\,\text{A}; \qquad \frac{1}{\tau} = \frac{R}{L} = \frac{7500}{5} = 1500$$

$$i_L(t) = 2e^{-1500t}\,\text{A}, \qquad t \geq 0$$

$$v_R(t) = Ri_L(t) = (7500)(2e^{-1500t}) = 15{,}000e^{-1500t}\,\text{V}, \qquad t \geq 0^+$$

$$v_o = -3.75\frac{di_L}{dt} = 11{,}250e^{-1500t}\,\text{V}, \qquad t \geq 0^+$$

[b] $i_o = \dfrac{-1}{6}\displaystyle\int_0^t 11{,}250e^{-1500x}\,dx + 0 = 1.25e^{-1500t} - 1.25\,\text{A}$

P 7.20 **[a]** From the solution to Problem 7.19,

$$w(0) = \frac{1}{2}L_{\text{eq}}[i_L(0)]^2 = \frac{1}{2}(5)(2)^2 = 10\,\text{J}$$

[b] $w_{\text{trapped}} = \dfrac{1}{2}(10)(1.25)^2 + \dfrac{1}{2}(6)(1.25)^2 = 12.5\,\text{J}$

P 7.21 [a] $R = \dfrac{v}{i} = 8\,\text{k}\Omega$

[b] $\dfrac{1}{\tau} = \dfrac{1}{RC} = 500; \qquad C = \dfrac{1}{(500)(8000)} = 0.25\,\mu\text{F}$

[c] $\tau = \dfrac{1}{500} = 2\,\text{ms}$

[d] $w(0) = \dfrac{1}{2}(0.25 \times 10^{-6})(72)^2 = 648\,\mu\text{J}$

[e] $w_{\text{diss}} = \displaystyle\int_0^{t_o} \frac{(72)^2 e^{-1000t}}{(800)}\,dt$

$$= 0.648\frac{e^{-1000t}}{-1000}\Bigg|_0^{t_o} = 648(1 - e^{-1000t_o})\,\mu\text{J}$$

$\%\text{diss} = 100(1 - e^{-1000t_o}) = 68 \qquad \text{so} \qquad e^{1000t_o} = 3.125$

$\therefore \quad t = \dfrac{\ln 3.125}{1000} = 1139\,\mu\text{s}$

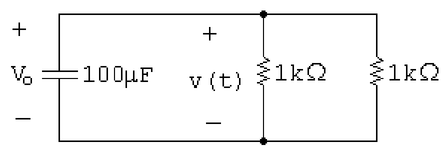
P 7.22 [a] Note that there are many different possible correct solutions to this problem.

$R = \dfrac{\tau}{C}$

Choose a $100\,\mu$F capacitor from Appendix H. Then,

$R = \dfrac{0.05}{100 \times 10^{-6}} = 500\,\Omega$

Construct a $500\,\Omega$ resistor by combining two $1\,\text{k}\Omega$ resistors in parallel:



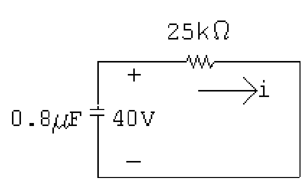
[b] $v(t) = V_o e^{-t/\tau} = 50e^{-20t}\,\text{V}, \qquad t \geq 0$

[c] $50e^{-20t} = 10 \qquad \text{so} \qquad e^{20t} = 5$

$\therefore \quad t = \dfrac{\ln 5}{20} = 80.47\,\text{ms}$

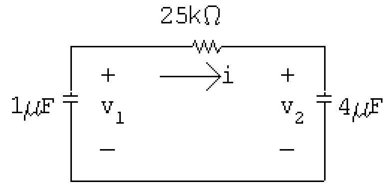
P 7.23 [a] $v_1(0^-) = v_1(0^+) = 40\,\text{V} \qquad\qquad v_2(0^+) = 0$

$C_{\text{eq}} = (1)(4)/5 = 0.8\,\mu\text{F}$



$\tau = (25 \times 10^3)(0.8 \times 10^{-6}) = 20\text{ms}; \qquad \dfrac{1}{\tau} = 50$

$$i = \frac{40}{25{,}000}e^{-50t} = 1.6e^{-50t}\,\text{mA}, \qquad t \geq 0^+$$



$$v_1 = \frac{-1}{10^{-6}}\int_0^t 1.6\times10^{-3}e^{-50x}\,dx + 40 = 32e^{-50t} + 8\,\text{V}, \qquad t \geq 0$$

$$v_2 = \frac{1}{4\times10^{-6}}\int_0^t 1.6\times10^{-3}e^{-50x}\,dx + 0 = -8e^{-50t} + 8\,\text{V}, \qquad t \geq 0$$

[b] $w(0) = \frac{1}{2}(10^{-6})(40)^2 = 800\,\mu\text{J}$

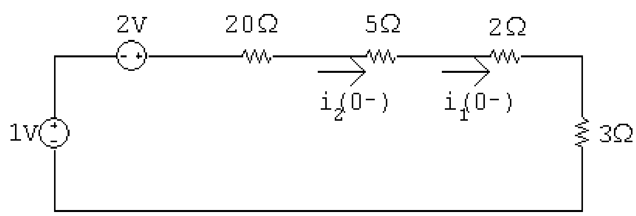
[c] $w_{\text{trapped}} = \frac{1}{2}(10^{-6})(8)^2 + \frac{1}{2}(4\times10^{-6})(8)^2 = 160\,\mu\text{J}.$

The energy dissipated by the 25 kΩ resistor is equal to the energy dissipated by the two capacitors; it is easier to calculate the energy dissipated by the capacitors:

$$w_{\text{diss}} = \frac{1}{2}(0.8\times10^{-6})(40)^2 = 640\,\mu\text{J}.$$

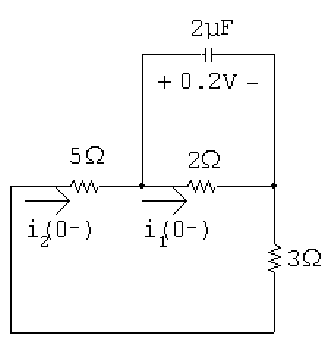
Check: $w_{\text{trapped}} + w_{\text{diss}} = 160 + 640 = 800\,\mu\text{J}; \qquad w(0) = 800\,\mu\text{J}.$

P 7.24 [a] $t < 0$:



$$i_1(0^-) = i_2(0^-) = \frac{3}{30} = 100\,\text{mA}$$

[b] $t > 0$:



$$i_1(0^+) = \frac{0.2}{2} = 100\,\text{mA}$$

$$i_2(0^+) = \frac{-0.2}{8} = -25\,\text{mA}$$

[c] Capacitor voltage cannot change instantaneously, therefore,

$$i_1(0^-) = i_1(0^+) = 100\,\text{mA}$$

[d] Switching can cause an instantaneous change in the current in a resistive branch. In this circuit

$$i_2(0^-) = 100\,\text{mA} \quad \text{and} \quad i_2(0^+) = 25\,\text{mA}$$

[e] $v_c = 0.2e^{-t/\tau}\,\text{V}, \qquad t \geq 0$

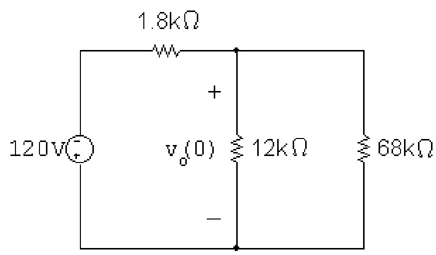
$$\tau = R_eC = 1.6(2 \times 10^{-6}) = 3.2\,\mu\text{s}; \qquad \frac{1}{\tau} = 312{,}500$$

$$v_c = 0.2e^{-312{,}000t}\,\text{V}, \qquad t \geq 0$$

$$i_1 = \frac{v_c}{2} = 0.1e^{-312{,}000t}\,\text{A}, \qquad t \geq 0$$

[f] $i_2 = \dfrac{-v_c}{8} = -25e^{-312{,}000t}\,\text{mA}, \qquad t \geq 0^+$

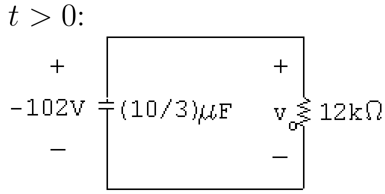
P 7.25 **[a]** $t < 0$:



$$R_{\text{eq}} = 12\,\text{k}\|8\,\text{k} = 10.2\,\text{k}\Omega$$

$$v_o(0) = \frac{10{,}200}{10{,}200 + 1800}(-120) = -102\,\text{V}$$

$t > 0$:



$$\tau = [(10/3) \times 10^{-6})(12{,}000) = 40\,\text{ms}; \qquad \frac{1}{\tau} = 25$$

$$v_o = -102e^{-25t}\,\text{V}, \quad t \geq 0$$

$$p = \frac{v_o^2}{12{,}000} = 867 \times 10^{-3}e^{-50t}\,\text{W}$$

$$w_{\text{diss}} = \int_0^{12\times10^{-3}} 867 \times 10^{-3} e^{-50t}\,dt$$
$$= 17.34 \times 10^{-3}(1 - e^{-50(12\times10^{-3})}) = 7824\,\mu\text{J}$$

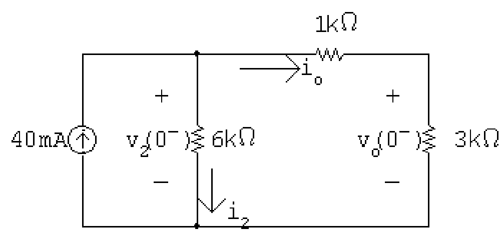
[b] $w(0) = \left(\frac{1}{2}\right)\left(\frac{10}{3}\right)(102)^2 \times 10^{-6} = 17.34\,\text{mJ}$

$0.75w(0) = 13\,\text{mJ}$

$$\int_0^{t_o} 867 \times 10^{-3} e^{-50x}\,dx = 13 \times 10^{-3}$$

$\therefore\quad 1 - e^{-50t_o} = 0.75; \qquad e^{50t_o} = 4; \quad \text{so} \quad t_o = 27.73\,\text{ms}$

P 7.26 **[a]** $t < 0$:



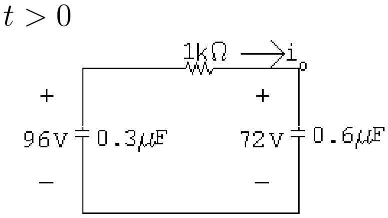
$$i_o(0^-) = \frac{6000}{6000 + 4000}(40\,\text{m}) = 24\,\text{mA}$$

$v_o(0^-) = (3000)(24\,\text{m}) = 72\,\text{V}$

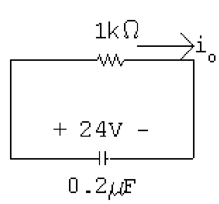
$i_2(0^-) = 40 - 24 = 16\,\text{mA}$

$v_2(0^-) = (6000)(16\,\text{m}) = 96\,\text{V}$

$t > 0$

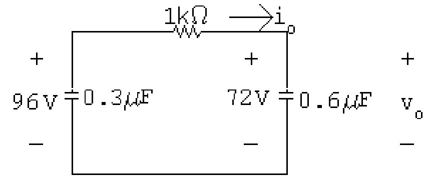


$\tau = RC = (1000)(0.2 \times 10^{-6}) = 200\,\mu\text{s}; \qquad \frac{1}{\tau} = 5000$



$$i_o(t) = \frac{24}{1 \times 10^3} e^{-t/\tau} = 24e^{-5000t}\,\text{mA}, \qquad t \geq 0^+$$

[b]



$$v_o = \frac{1}{0.6 \times 10^{-6}} \int_0^t 24 \times 10^{-3} e^{-5000x}\, dx + 72$$
$$= (40{,}000) \frac{e^{-5000x}}{-5000} \Big|_0^t + 72$$
$$= -8e^{-5000t} + 8 + 72$$
$$v_o = [-8e^{-5000t} + 80]\,\text{V}, \qquad t \geq 0$$

[c] $w_{\text{trapped}} = (1/2)(0.3 \times 10^{-6})(80)^2 + (1/2)(0.6 \times 10^{-6})(80)^2$

$w_{\text{trapped}} = 2880\,\mu\text{J}.$

Check:

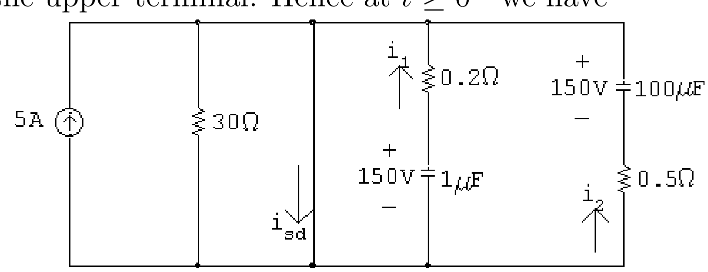
$$w_{\text{diss}} = \frac{1}{2}(0.2 \times 10^{-6})(24)^2 = 57.6\,\mu\text{J}$$

$$w(0) = \frac{1}{2}(0.3 \times 10^{-6})(96)^2 + \frac{1}{2}(0.6 \times 10^{-6})(72)^2 = 2937.6\,\mu\text{J}.$$

$w_{\text{trapped}} + w_{\text{diss}} = w(0)$

$2880 + 57.6 = 2937.6 \qquad$ OK.

P 7.27 [a] At $t = 0^-$ the voltage on each capacitor will be 150 V(5×30), positive at the upper terminal. Hence at $t \geq 0^+$ we have



$$\therefore \quad i_{sd}(0^+) = 5 + \frac{150}{0.2} + \frac{150}{0.5} = 1055\,\text{A}$$

At $t = \infty$, both capacitors will have completely discharged.

$$\therefore \quad i_{sd}(\infty) = 5\,\text{A}$$

[b] $i_{sd}(t) = 5 + i_1(t) + i_2(t)$

$$\tau_1 = 0.2(10^{-6}) = 0.2\,\mu\text{s}$$

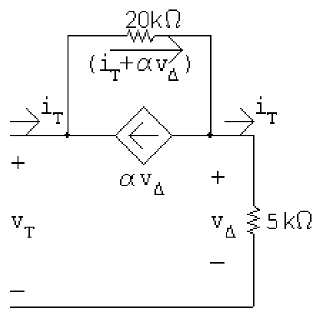
$$\tau_2 = 0.5(100 \times 10^{-6}) = 50\,\mu\text{s}$$

$$\therefore \quad i_1(t) = 750e^{-5\times10^6 t}\,\text{A}, \qquad t \geq 0^+$$

$$i_2(t) = 300e^{-20{,}000t}\,\text{A}, \qquad t \geq 0$$

$$\therefore \quad i_{sd} = 5 + 750e^{-5\times10^6 t} + 300e^{-20{,}000t}\,\text{A}, \qquad t \geq 0^+$$

P 7.28 **[a]**



$$v_T = 20 \times 10^3(i_T + \alpha v_\Delta) + 5 \times 10^3 i_T$$

$$v_\Delta = 5 \times 10^3 i_T$$

$$v_T = 25 \times 10^3 i_T + 20 \times 10^3\alpha(5 \times 10^3 i_T)$$

$$R_{\text{Th}} = 25{,}000 + 100 \times 10^6\alpha$$

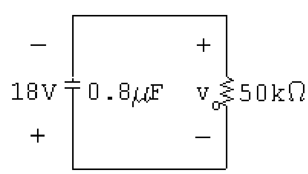
$$\tau = R_{\text{Th}}C = 40 \times 10^{-3} = R_{\text{Th}}(0.8 \times 10^{-6})$$

$$R_{\text{Th}} = 50\,\text{k}\Omega = 25{,}000 + 100 \times 10^6\alpha$$

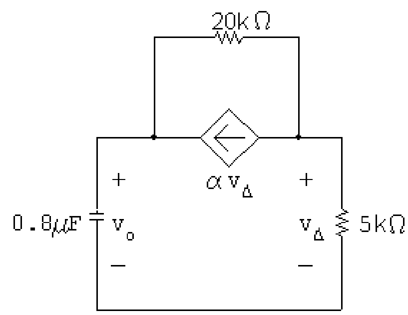
$$\alpha = \frac{25{,}000}{100 \times 10^6} = 2.5 \times 10^{-4}\,\text{A/V}$$

[b] $v_o(0) = (-5 \times 10^{-3})(3600) = -18\,\text{V} \qquad t < 0$

$t > 0$:



$$v_o = -18e^{-25t}\,\text{V}, \quad t \geq 0$$

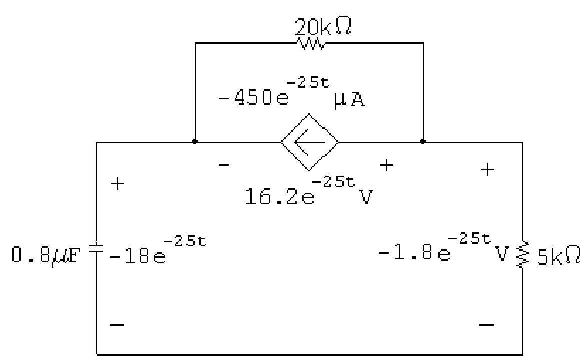


$$\frac{v_\Delta}{5000} + \frac{v_\Delta - v_o}{20{,}000} + 2.5 \times 10^{-4} v_\Delta = 0$$

$$4v_\Delta + v_\Delta - v_o + 5v_\Delta = 0$$

$$\therefore \quad v_\Delta = \frac{v_o}{10} = -1.8e^{-25t}\,\text{V}, \quad t \geq 0^+$$

P 7.29 **[a]**



$$p_{ds} = (16.2e^{-25t})(-450 \times 10^{-6}e^{-25t}) = -7290 \times 10^{-6}e^{-50t}\,\text{W}$$

$$w_{ds} = \int_0^\infty p_{ds}\,dt = -145.8\,\mu\text{J}.$$

$\therefore$ dependent source is delivering $145.8\,\mu$J.

[b] $$w_{5\text{k}} = \int_0^\infty (5000)(0.36 \times 10^{-3}e^{-25t})^2\,dt = 648 \times 10^{-6}\int_0^\infty e^{-50t}\,dt = 12.96\,\mu\text{J}$$

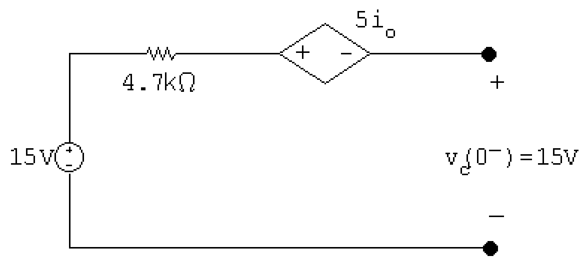
$$w_{20\text{k}} = \int_0^\infty \frac{(16.2e^{-25t})^2}{20{,}000}\,dt = 13{,}122 \times 10^{-6}\int_0^\infty e^{-50t}\,dt = 262.44\,\mu\text{J}$$

$$w_c(0) = \frac{1}{2}(0.8 \times 10^{-6})(18)^2 = 129.6\,\mu\text{J}$$

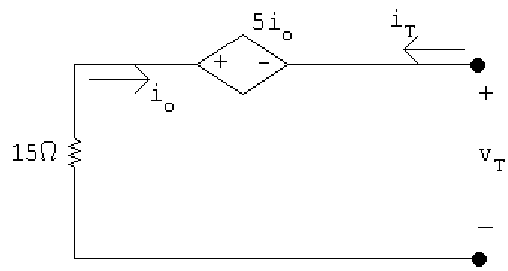
$$\sum w_{\text{diss}} = 12.96 + 262.44 = 275.4\,\mu\text{J}$$

$$\sum w_{\text{dev}} = 145.8 + 129.6 = 275.4\,\mu\text{J}.$$

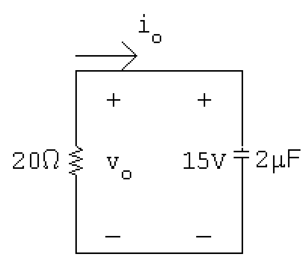
P 7.30 $t < 0$



$t > 0$



$$v_T = -5i_o - 15i_o = -20i_o = 20i_T \quad \therefore \quad R_{\text{Th}} = \frac{v_T}{i_T} = 20\,\Omega$$

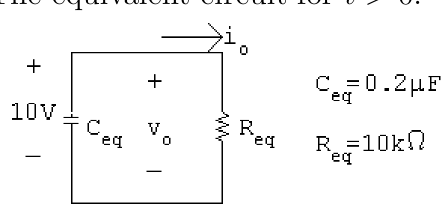


$$\tau = RC = 40\,\mu\text{s}; \qquad \frac{1}{\tau} = 25{,}000$$

$$v_o = 15e^{-25{,}000t}\,\text{V}, \qquad t \geq 0$$

$$i_o = -\frac{v_o}{20} = -0.75e^{-25{,}000t}\,\text{A}, \qquad t \geq 0^+$$

P 7.31 **[a]** The equivalent circuit for $t > 0$:



$$\tau = 2\,\text{ms}; \qquad 1/\tau = 500$$

$v_o = 10e^{-500t}\,\text{V}, \qquad t \geq 0$

$i_o = e^{-500t}\,\text{mA}, \qquad t \geq 0^+$

$i_{24\text{k}\Omega} = e^{-500t}\left(\dfrac{16}{40}\right) = 0.4e^{-500t}\,\text{mA}, \qquad t \geq 0^+$

$p_{24\text{k}\Omega} = (0.16 \times 10^{-6}e^{-1000t})(24{,}000) = 3.84e^{-1000t}\,\text{mW}$

$w_{24\text{k}\Omega} = \displaystyle\int_0^\infty 3.84 \times 10^{-3}e^{-1000t}\,dt = -3.84 \times 10^{-6}(0-1) = 3.84\,\mu\text{J}$

$w(0) = \dfrac{1}{2}(0.25 \times 10^{-6})(40)^2 + \dfrac{1}{2}(1 \times 10^{-6})(50)^2 = 1.45\,\text{mJ}$

$\%\text{ diss }(24\,\text{k}\Omega) = \dfrac{3.84 \times 10^{-6}}{1.45 \times 10^{-3}} \times 100 = 0.26\%$

[b] $p_{400\Omega} = 400(1 \times 10^{-3}e^{-500t})^2 = 0.4 \times 10^{-3}e^{-1000t}$

$w_{400\Omega} = \displaystyle\int_0^\infty p_{400}\,dt = 0.40\,\mu\text{J}$

$\%\text{ diss }(400\,\Omega) = \dfrac{0.4 \times 10^{-6}}{1.45 \times 10^{-3}} \times 100 = 0.03\%$

$i_{16k\Omega} = e^{-500t}\left(\dfrac{24}{40}\right) = 0.6e^{-500t}\,\text{mA}, \quad t \geq 0^+$

$p_{16k\Omega} = (0.6 \times 10^{-3}e^{-500t})^2(16{,}000) = 5.76 \times 10^{-3}e^{-1000t}\,\text{W}$

$w_{16k\Omega} = \displaystyle\int_0^\infty 5.76 \times 10^{-3}e^{-1000t}\,dt = 5.76\,\mu\text{J}$

$\%\text{ diss }(16\,\text{k}\Omega) = 0.4\%$

[c] $\sum w_{\text{diss}} = 3.84 + 5.76 + 0.4 = 10\,\mu\text{J}$

$w_{\text{trapped}} = w(0) - \sum w_{\text{diss}} = 1.45 \times 10^{-3} - 10 \times 10^{-6} = 1.44\,\text{mJ}$

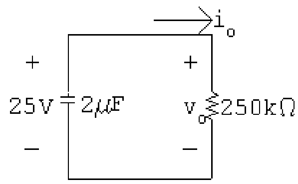
$\%\text{ trapped} = \dfrac{1.44}{1.45} \times 100 = 99.31\%$

Check: $\quad 0.26 + 0.03 + 0.4 + 99.31 = 100\%$

P 7.32 **[a]** $C_e = \dfrac{(2+1)6}{2+1+6} = 2\,\mu\text{F}$

$v_o(0) = -5 + 30 = 25\,\text{V}$

$\tau = (2 \times 10^{-6})(250 \times 10^3) = 0.5\,\text{s}; \qquad \dfrac{1}{\tau} = 2$



$$v_o = 25e^{-2t}\,\text{V}, \qquad t \geq 0^+$$

[b] $w_o = \frac{1}{2}(3 \times 10^{-6})(30)^2 + \frac{1}{2}(6 \times 10^{-6})(5)^2 = 1425\,\mu\text{J}$

$$w_{\text{diss}} = \frac{1}{2}(2 \times 10^{-6})(25)^2 = 625\,\mu\text{J}$$

$$\%\text{ diss } = \frac{625}{1425} \times 100 = 43.86\%$$

[c] $i_o = \dfrac{v_o}{250 \times 10^{-3}} = 100e^{-2t}\,\mu\text{A}$

$$v_1 = -\frac{1}{6 \times 10^{-6}}\int_0^t 100 \times 10^{-6} e^{-2x}\,dx - 5 = -16.67\int_0^t e^{-2x}\,dx - 5$$

$$= -16.67\frac{e^{-2x}}{-2}\Big|_0^t - 5 = 8.33e^{-2t} - 13.33\,\text{V} \qquad t \geq 0$$

[d] $v_1 + v_2 = v_o$

$$v_2 = v_o - v_1 = 25e^{-2t} - 8.33e^{-2t} + 13.33 = 16.67e^{-2t} + 13.33\,\text{V} \quad t \geq 0$$

[e] $w_{\text{trapped}} = \frac{1}{2}(6 \times 10^{-6})(13.33)^2 + \frac{1}{2}(3 \times 10^{-6})(13.33)^2 = 800\,\mu\text{J}$

$$w_{\text{diss}} + w_{\text{trapped}} = 625 + 800 = 1425\,\mu\text{J} \qquad \text{(check)}$$

P 7.33 **[a]** From Eqs. (7.35) and (7.42)

$$i = \frac{V_s}{R} + \left(I_o - \frac{V_s}{R}\right)e^{-(R/L)t}$$

$$v = (V_s - I_oR)e^{-(R/L)t}$$

$$\therefore \quad \frac{V_s}{R} = 4; \qquad I_o - \frac{V_s}{R} = 4$$

$$V_s - I_oR = -80; \qquad \frac{R}{L} = 40$$

$$\therefore \quad I_o = 4 + \frac{V_s}{R} = 8\,\text{A}$$

Now since $V_s = 4R$ we have

$$4R - 8R = -80; \qquad R = 20\,\Omega$$

$$V_s = 80\,\text{V}; \qquad L = \frac{R}{40} = 0.5\,\text{H}$$

[b] $i = 4 + 4e^{-40t}; \qquad i^2 = 16 + 32e^{-40t} + 16e^{-80t}$

$$w = \frac{1}{2}Li^2 = \frac{1}{2}(0.5)[16 + 32e^{-40t} + 16e^{-80t}] = 4 + 8e^{-40t} + 4e^{-80t}$$

$$\therefore \quad 4 + 8e^{-40t} + 4e^{-80t} = 9 \quad \text{or} \quad e^{-80t} + 2e^{-40t} - 1.25 = 0$$

Let $x = e^{-40t}$:

$x^2 + 2x - 1.25 = 0; \quad$ Solving, $\quad x = 0.5; \quad x = -2.5$

But $x \geq 0$ for all t. Thus,

$e^{-40t} = 0.5; \qquad e^{40t} = 2; \qquad t = 25\ln 2 = 17.33\,\text{ms}$

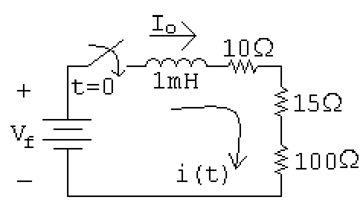
P 7.34 **[a]** Note that there are many different possible solutions to this problem.

$$R = \frac{L}{\tau}$$

Choose a 1 mH inductor from Appendix H. Then,

$$R = \frac{0.001}{8 \times 10^{-6}} = 125\,\Omega$$

Construct the resistance needed by combining $100\,\Omega$, $10\,\Omega$, and $15\,\Omega$ resistors in series:



[b] $i(t) = I_f + (I_o - I_f)e^{-t/\tau}$

$$I_o = 0\,\text{A}; \qquad I_f = \frac{V_f}{R} = \frac{25}{125} = 200\,\text{mA}$$

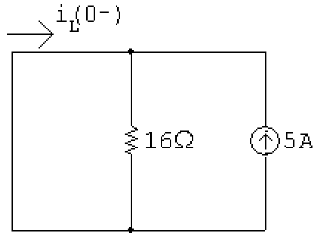
$$\therefore \quad i(t) = 200 + (0 - 200)e^{-125{,}000t}\,\text{mA} = 200 - 200e^{-125{,}000t}\,\text{mA}, \qquad t \geq 0$$

[c] $i(t) = 0.2 - 0.2e^{-125{,}000t} = (0.75)(0.2) = 0.15$

$$e^{-125{,}000t} = 0.25 \qquad \text{so} \qquad e^{125{,}000t} = 4$$

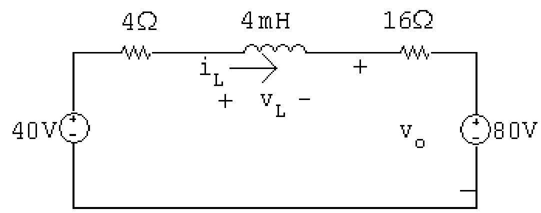
$$\therefore \quad t = \frac{\ln 4}{125{,}000} = 11.09\,\mu\text{s}$$

P 7.35 **[a]** $t < 0$



$i_L(0^-) = -5\,\text{A}$

$t > 0$



$$i_L(\infty) = \frac{40 - 80}{4 + 16} = -2\,\text{A}$$

$$\tau = \frac{L}{R} = \frac{4 \times 10^{-3}}{4 + 16} = 200\,\mu\text{s}; \qquad \frac{1}{\tau} = 5000$$

$$i_L = i_L(\infty) + [i_L(0^+) - i_L(\infty)]e^{-t/\tau}$$

$$= -2 + (-5 + 2)e^{-5000t} = -2 - 3e^{-5000t}\,\text{A}, \qquad t \geq 0$$

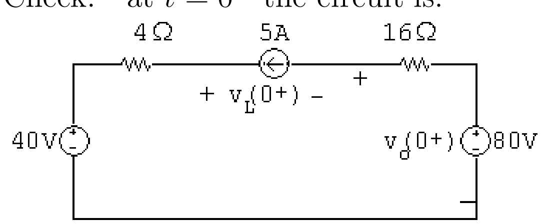
$$v_o = 16i_L + 80 = 16(-2 - 3e^{-5000t}) + 80 = 48 - 48e^{-5000t}\,\text{V}, \qquad t \geq 0$$

[b] $v_L = L\dfrac{di_L}{dt} = 4 \times 10^{-3}(-5000)[-3e^{-5000t}] = 60e^{-5000t}\,\text{V}, \qquad t \geq 0^+$

$v_L(0^+) = 60\,\text{V}$

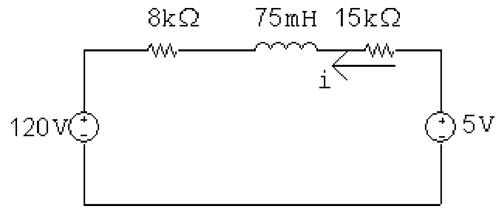
From part (a) $\qquad v_o(0^+) = 0\,\text{V}$

Check: at $t = 0^+$ the circuit is:



$$v_L(0^+) = 40 + (5\,\text{A})(4\,\Omega) = 60\,\text{V}, \qquad v_o(0^+) = 80 - (16\,\Omega)(5\,\text{A}) = 0\,\text{V}$$

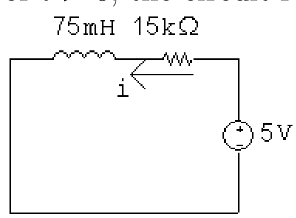
P 7.36 [a] For $t < 0$, calculate the Thévenin equivalent for the circuit to the left and right of the 75 mH inductor. We get



$$i(0^-) = \frac{5-120}{15\,\mathrm{k} + 8\,\mathrm{k}} = -5\,\mathrm{mA}$$

$$i(0^-) = i(0^+) = -5\,\mathrm{mA}$$

[b] For $t > 0$, the circuit reduces to



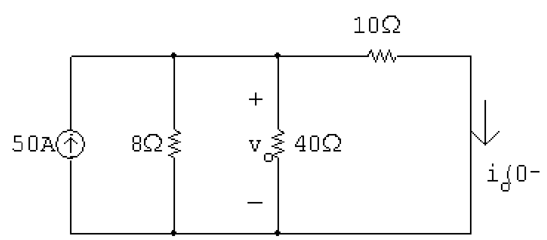
Therefore $i(\infty) = 5/15{,}000 = 0.333\,\mathrm{mA}$

[c] $\tau = \dfrac{L}{R} = \dfrac{75 \times 10^{-3}}{15{,}000} = 5\,\mu\mathrm{s}$

[d] $i(t) = i(\infty) + [i(0^+) - i(\infty)]e^{-t/\tau}$

$$= 0.333 + [-5 - 0.333]e^{-200{,}000t} = 0.333 - 5.333e^{-200{,}000t}\,\mathrm{mA}, \qquad t \geq 0$$

P 7.37 [a] $t < 0$



KVL equation at the top node:

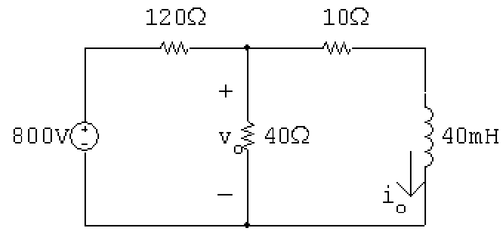
$$50 = \frac{v_o}{8} + \frac{v_o}{40} + \frac{v_o}{10}$$

Multiply by 40 and solve:

$$2000 = (5 + 1 + 4)v_o; \qquad v_o = 200\,\mathrm{V}$$

$$\therefore \quad i_o(0^-) = \frac{v_o}{10} = 200/10 = 20\,\mathrm{A}$$

$t > 0$



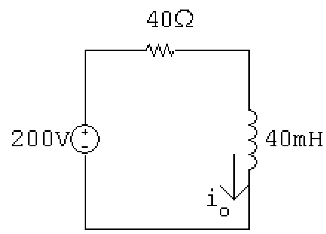
Use voltage division to find the Thévenin voltage:

$$V_{\text{Th}} = v_o = \frac{40}{40+120}(800) = 200\,\text{V}$$

Remove the voltage source and make series and parallel combinations of resistors to find the equivalent resistance:

$$R_{\text{Th}} = 10 + 120\|40 = 10 + 30 = 40\,\Omega$$

The simplified circuit is:



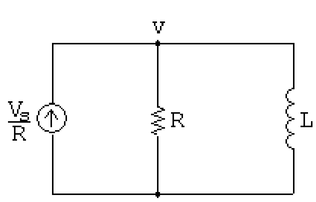
$$\tau = \frac{L}{R} = \frac{40\times 10^{-3}}{40} = 1\,\text{ms}; \qquad \frac{1}{\tau} = 1000$$

$$i_o(\infty) = \frac{200}{40} = 5\,\text{A}$$

$$\therefore\ \ i_o = i_o(\infty) + [i_o(0^+) - i_o(\infty)]e^{-t/\tau}$$
$$= 5 + (20-5)e^{-1000t} = 5 + 15e^{-1000t}\,\text{A}, \qquad t \geq 0$$

[b]
$$\begin{aligned} v_o &= 10i_o + L\frac{di_o}{dt} \\ &= 10(5+15e^{-1000t}) + 0.04(-1000)(15e^{-1000t}) \\ &= 50 + 150e^{-1000t} - 600e^{-1000t} \\ v_o &= 50 - 450e^{-1000t}\,\text{V}, \qquad t \geq 0^+ \end{aligned}$$

P 7.38 [a]



$$-\frac{V_s}{R} + \frac{v}{R} + \frac{1}{L}\int_0^t v\,dt + I_o = 0$$

Differentiating both sides,

$$\frac{1}{R}\frac{dv}{dt} + \frac{1}{L}v = 0$$

$$\therefore \qquad \frac{dv}{dt} + \frac{R}{L}v = 0$$

[b] $\dfrac{dv}{dt} = -\dfrac{R}{L}v$

$$\frac{dv}{dt}dt = -\frac{R}{L}v\,dt \qquad \text{so} \qquad dv = -\frac{R}{L}v\,dt$$

$$\frac{dv}{v} = -\frac{R}{L}dt$$

$$\int_{V_o}^{v(t)} \frac{dx}{x} = -\frac{R}{L}\int_0^t dy$$

$$\ln\frac{v(t)}{V_o} = -\frac{R}{L}t$$

$$\therefore \qquad v(t) = V_o e^{-(R/L)t} = (V_s - RI_o)e^{-(R/L)t}$$

P 7.39 **[a]** $v_o(0^+) = -I_gR_2; \qquad \tau = \dfrac{L}{R_1 + R_2}$

$$v_o(\infty) = 0$$

$$v_o(t) = -I_gR_2e^{-[(R_1+R_2)/L]t}\,\text{V}, \qquad t \geq 0^+$$

[b] $v_o(0^+) \to \infty$, and the duration of $v_o(t) \to$ zero

[c] $v_{sw} = R_2i_o; \qquad \tau = \dfrac{L}{R_1 + R_2}$

$$i_o(0^+) = I_g; \qquad i_o(\infty) = I_g\frac{R_1}{R_1 + R_2}$$

Therefore $\qquad i_o(t) = \frac{I_gR_1}{R_1+R_2} + \left[I_g - \frac{I_gR_1}{R_1+R_2}\right]e^{-[(R_1+R_2)/L]t}$

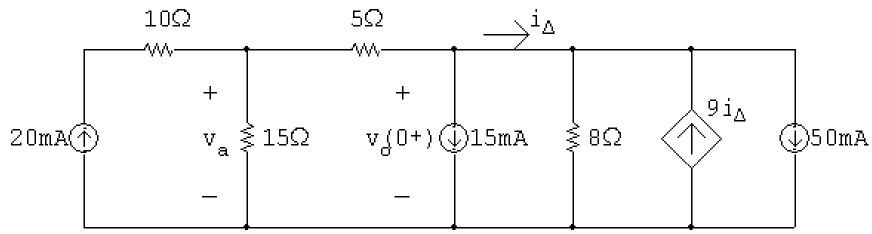
$$i_o(t) = \frac{R_1I_g}{(R_1+R_2)} + \frac{R_2I_g}{(R_1+R_2)}e^{-[(R_1+R_2)/L]t}$$

Therefore $\qquad v_{sw} = \frac{R_1I_g}{(1+R_1/R_2)} + \frac{R_2I_g}{(1+R_1/R_2)}e^{-[(R_1+R_2)/L]t}, \qquad t \geq 0^+$

[d] $|v_{sw}(0^+)| \to \infty; \qquad$ duration $\to 0$

P 7.40 Opening the inductive circuit causes a very large voltage to be induced across the inductor L. This voltage also appears across the switch (part [d] of Problem 7.39), causing the switch to arc over. At the same time, the large voltage across L damages the meter movement.

P 7.41 $t > 0$; calculate $v_o(0^+)$



$$\frac{v_a}{15} + \frac{v_a - v_o(0^+)}{5} = 20 \times 10^{-3}$$

$$\therefore \quad v_a = 0.75v_o(0^+) + 75 \times 10^{-3}$$

$$15 \times 10^{-3} + \frac{v_o(0^+) - v_a}{5} + \frac{v_o(0^+)}{8} - 9i_\Delta + 50 \times 10^{-3} = 0$$

$$13v_o(0^+) - 8v_a - 360i_\Delta = -2600 \times 10^{-3}$$

$$i_\Delta = \frac{v_o(0^+)}{8} - 9i_\Delta + 50 \times 10^{-3}$$

$$\therefore \quad i_\Delta = \frac{v_o(0^+)}{80} + 5 \times 10^{-3}$$

$$\therefore \quad 360i_\Delta = 4.5v_o(0^+) + 1800 \times 10^{-3}$$

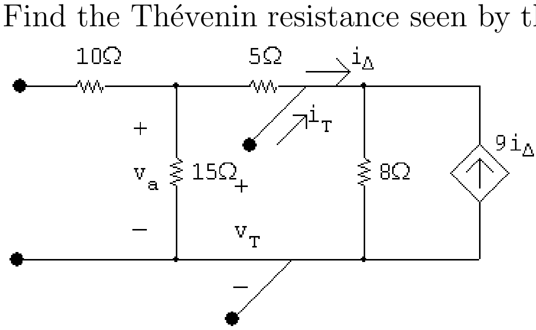
$$8v_a = 6v_o(0^+) + 600 \times 10^{-3}$$

$$\therefore \quad 13v_o(0^+) - 6v_o(0^+) - 600 \times 10^{-3} - 4.5v_o(0^+) -$$
$$1800 \times 10^{-3} = -2600 \times 10^{-3}$$

$$2.5v_o(0^+) = -200 \times 10^{-3}; \qquad v_o(0^+) = -80\,\text{mV}$$

$$v_o(\infty) = 0$$

Find the Thévenin resistance seen by the 4 mH inductor:



$$i_T = \frac{v_T}{20} + \frac{v_T}{8} - 9i_\Delta$$

$$i_\Delta = \frac{v_T}{8} - 9i_\Delta \qquad \therefore \quad 10i_\Delta = \frac{v_T}{8}; \qquad i_\Delta = \frac{v_T}{80}$$

$$i_T = \frac{v_T}{20} + \frac{10v_T}{80} - \frac{9v_T}{80}$$

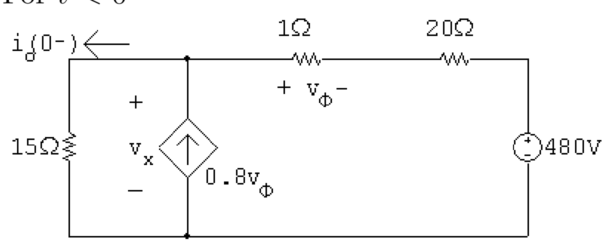
$$\frac{i_T}{v_T} = \frac{1}{20} + \frac{1}{80} = \frac{5}{80} = \frac{1}{16}\,\mathrm{S}$$

$$\therefore \quad R_{\mathrm{Th}} = 16\Omega$$

$$\tau = \frac{4\times 10^{-3}}{16} = 0.25\,\mathrm{ms}; \qquad 1/\tau = 4000$$

$$\therefore \quad v_o = 0 + (-80 - 0)e^{-4000t} = -80e^{-4000t}\,\mathrm{mV}, \qquad t \geq 0^+$$

P 7.42 For $t < 0$



$$\frac{v_x}{15} - 0.8v_\phi + \frac{v_x - 480}{21} = 0$$

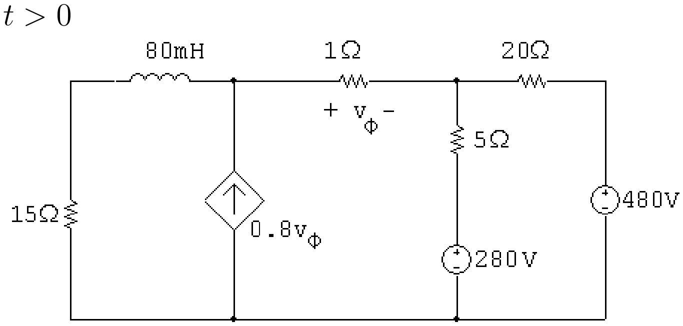
$$v_\phi = \frac{v_x - 480}{21}$$

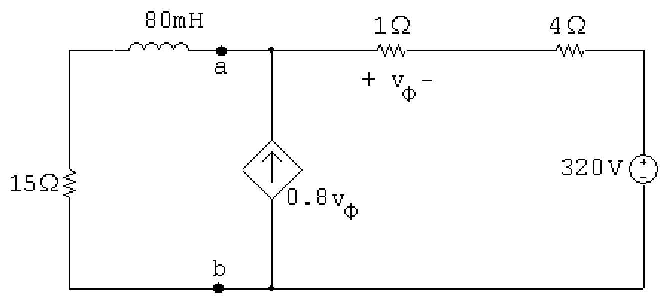
$$\frac{v_x}{15} - 0.8\left(\frac{v_x - 480}{21}\right) + \left(\frac{v_x - 480}{21}\right)$$

$$= \frac{v_x}{15} + 0.2\left(\frac{v_s - 480}{21}\right) = 21v_x + 3(v_x - 480) = 0$$

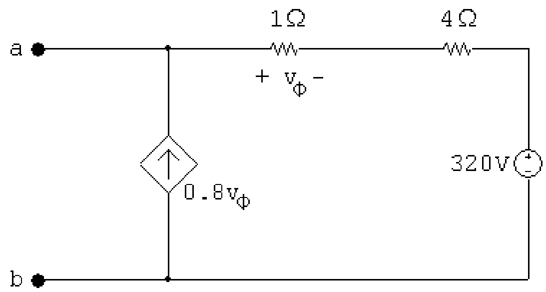
$$\therefore \qquad 24v_x = 1440 \quad \text{so} \quad v_x = 60\,\mathrm{V} \qquad i_o(0^-) = \frac{v_x}{15} = 4\,\mathrm{A}$$

$t > 0$

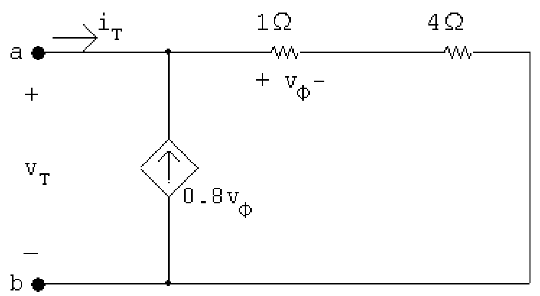




Find Thévenin equivalent with respect to a, b



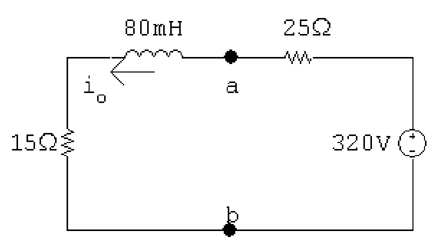
$$\frac{V_{\text{Th}} - 320}{5} - 0.8\left(\frac{V_{\text{Th}} - 320}{5}\right) = 0 \qquad V_{\text{Th}} = 320\,\text{V}$$



$$v_T = (i_T + 0.8v_\phi)(5) = \left(i_T + 0.8\frac{v_T}{5}\right)(5)$$

$$v_T = 5i_T + 0.8v_T \qquad \therefore \quad 0.2v_T = 5i_T$$

$$\frac{v_T}{i_T} = R_{\text{Th}} = 25\,\Omega$$

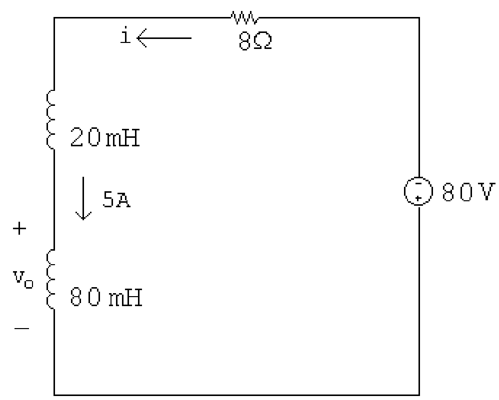


$$i_o(\infty) = 320/40 = 8\,\text{A}$$

$$\tau = \frac{80 \times 10^{-3}}{40} = 2\,\text{ms}; \qquad 1/\tau = 500$$

$$i_o = 8 + (4 - 8)e^{-500t} = 8 - 4e^{-500t}\,\text{A}, \qquad t \geq 0$$

P 7.43 For $t < 0$, $\quad i_{80\text{mH}}(0) = 50\,\text{V}/10\,\Omega = 5\,\text{A}$
For $t > 0$, after making a Thévenin equivalent we have



$$i = \frac{V_s}{R} + \left(I_o - \frac{V_s}{R}\right)e^{-t/\tau}$$

$$\frac{1}{\tau} = \frac{R}{L} = \frac{8}{100 \times 10^{-3}} = 80$$

$$I_o = 5\,\text{A}; \qquad I_f = \frac{V_s}{R} = \frac{-80}{8} = -10\,\text{A}$$

$$i = -10 + (5 + 10)e^{-80t} = -10 + 15e^{-80t}\,\text{A}, \qquad t \geq 0$$

$$v_o = 0.08\frac{di}{dt} = 0.08(-1200e^{-80t}) = -96e^{-80t}\,\text{V}, \qquad t \geq 0^+$$

P 7.44 **[a]** Let v be the voltage drop across the parallel branches, positive at the top node, then

$$-I_g + \frac{v}{R_g} + \frac{1}{L_1}\int_0^t v\,dx + \frac{1}{L_2}\int_0^t v\,dx = 0$$

$$\frac{v}{R_g} + \left(\frac{1}{L_1} + \frac{1}{L_2}\right)\int_0^t v\,dx = I_g$$

$$\frac{v}{R_g} + \frac{1}{L_e}\int_0^t v\,dx = I_g$$

$$\frac{1}{R_g}\frac{dv}{dt} + \frac{v}{L_e} = 0$$

$$\frac{dv}{dt} + \frac{R_g}{L_e}v = 0$$

Therefore $\quad v = I_g R_g e^{-t/\tau}; \qquad \tau = L_e/R_g$

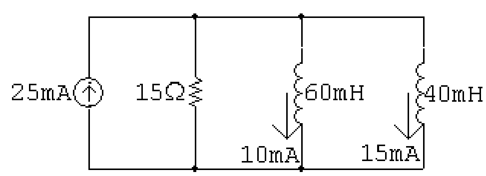
Thus

$$i_1 = \frac{1}{L_1}\int_0^t I_g R_g e^{-x/\tau}\,dx = \frac{I_g R_g}{L_1}\frac{e^{-x/\tau}}{(-1/\tau)}\bigg|_0^t = \frac{I_g L_e}{L_1}(1 - e^{-t/\tau})$$

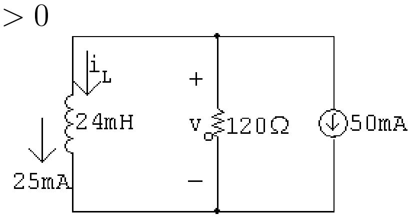
$$i_1 = \frac{I_g L_2}{L_1 + L_2}(1 - e^{-t/\tau}) \quad \text{and} \quad i_2 = \frac{I_g L_1}{L_1 + L_2}(1 - e^{-t/\tau})$$

[b] $i_1(\infty) = \dfrac{L_2}{L_1 + L_2}I_g; \qquad i_2(\infty) = \dfrac{L_1}{L_1 + L_2}I_g$

P 7.45 [a] $t < 0$



$t > 0$



$$i_L(0^-) = i_L(0^+) = 25\,\text{mA}; \qquad \tau = \frac{24 \times 10^{-3}}{120} = 0.2\,\text{ms}; \qquad \frac{1}{\tau} = 5000$$

$$i_L(\infty) = -50\,\text{mA}$$

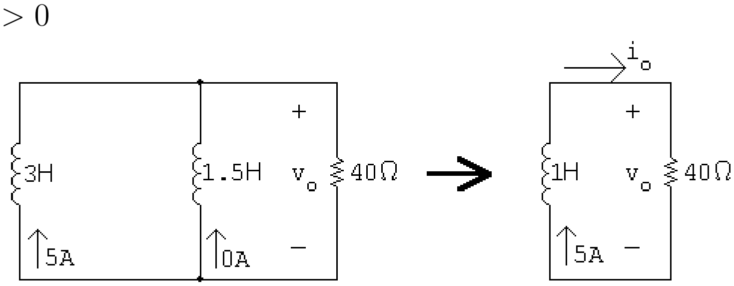
$$i_L = -50 + (25 + 50)e^{-5000t} = -50 + 75e^{-5000t}\,\text{mA}, \qquad t \geq 0$$

$$v_o = -120[75 \times 10^{-3}e^{-5000t}] = -9e^{-5000t}\,\text{V}, \qquad t \geq 0^+$$

[b] $i_1 = \dfrac{1}{60 \times 10^{-3}}\displaystyle\int_0^t -9e^{-5000x}\,dx + 10 \times 10^{-3} = (30e^{-5000t} - 20)\,\text{mA}, \qquad t \geq 0$

[c] $i_2 = \dfrac{1}{40 \times 10^{-3}}\displaystyle\int_0^t -9e^{-5000x}\,dx + 15 \times 10^{-3} = (45e^{-5000t} - 30)\,\text{mA}, \qquad t \geq 0$

P 7.46 $t > 0$



$$\tau = \frac{1}{40}$$

$$i_o = 5e^{-40t}\,\text{A}, \qquad t \geq 0$$

$$v_o = 40i_o = 200e^{-40t}\,\text{V}, \qquad t > 0^+$$

$$200e^{-40t} = 100; \qquad e^{40t} = 2$$

$$\therefore \quad t = \frac{1}{40}\ln 2 = 17.33\,\text{ms}$$

P 7.47 **[a]** $w_{\text{diss}} = \dfrac{1}{2}L_e i^2(0) = \dfrac{1}{2}(1)(5)^2 = 12.5\,\text{J}$

[b] $i_{3H} = \dfrac{1}{3}\displaystyle\int_0^t (200)e^{-40x}\,dx - 5$

$$= 1.67(1 - e^{-40t}) - 5 = -1.67e^{-40t} - 3.33\,\text{A}$$

$$i_{1.5H} = \frac{1}{1.5}\int_0^t (200)e^{-40x}\,dx + 0$$

$$= -3.33e^{-40t} + 3.33\,\text{A}$$

$$w_{\text{trapped}} = \frac{1}{2}(4.5)(3.33)^2 = 25\,\text{J}$$

[c] $w(0) = \dfrac{1}{2}(3)(5)^2 = 37.5\,\text{J}$

P 7.48 **[a]** $v = I_sR + (V_o - I_sR)e^{-t/RC} \qquad i = \left(I_s - \dfrac{V_o}{R}\right)e^{-t/RC}$

$$\therefore \quad I_sR = 40, \qquad V_o - I_sR = -24$$

$$\therefore \quad V_o = 16\,\text{V}$$

$$I_s - \frac{V_o}{R} = 3 \times 10^{-3}; \qquad I_s - \frac{16}{R} = 3 \times 10^{-3}; \qquad R = \frac{40}{I_s}$$

$$\therefore \quad I_s - 0.4I_s = 3 \times 10^{-3}; \qquad I_s = 5\,\text{mA}$$

$$R = \frac{40}{5} \times 10^3 = 8\,\text{k}\Omega$$

$$\frac{1}{RC} = 2500; \quad C = \frac{1}{2500R} = \frac{10^{-3}}{20 \times 10^3} = 50\,\text{nF}; \quad \tau = RC = \frac{1}{2500} = 400\,\mu\text{s}$$

[b] $v(\infty) = 40\,\text{V}$

$$w(\infty) = \frac{1}{2}(50 \times 10^{-9})(1600) = 40\,\mu\text{J}$$

$$0.81w(\infty) = 32.4\,\mu\text{J}$$

$$v^2(t_o) = \frac{32.4 \times 10^{-6}}{25 \times 10^{-9}} = 1296; \qquad v(t_o) = 36\,\text{V}$$

$$40 - 24e^{-2500t_o} = 36; \qquad e^{2500t_o} = 6; \qquad \therefore \quad t_o = 716.70\,\mu\text{s}$$

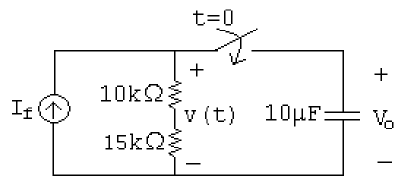
P 7.49 **[a]** Note that there are many different possible solutions to this problem.

$$R = \frac{\tau}{C}$$

Choose a $10\,\mu$H capacitor from Appendix H. Then,

$$R = \frac{0.25}{10 \times 10^{-6}} = 25\,\text{k}\Omega$$

Construct the resistance needed by combining $10\,\text{k}\Omega$ and $15\,\text{k}\Omega$ resistors in series:



[b] $v(t) = V_f + (V_o - V_f)e^{-t/\tau}$

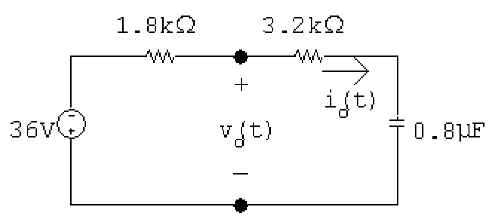
$V_o = 100\,\text{V}$; $\qquad V_f = (I_f)(R) = (1 \times 10^{-3})(25 \times 10^3) = 25\,\text{V}$

$\therefore \quad v(t) = 25 + (100 - 25)e^{-4t}\,\text{V} = 25 + 75e^{-4t}\,\text{V}, \qquad t \geq 0$

[c] $v(t) = 25 + 75e^{-4t} = 50 \qquad \text{so} \qquad e^{-4t} = \dfrac{1}{3}$

$$\therefore \quad t = \frac{\ln 3}{4} = 274.65\,\text{ms}$$

P 7.50 **[a]**



$$i_o(0^+) = \frac{-36}{5000} = -7.2\,\text{mA}$$

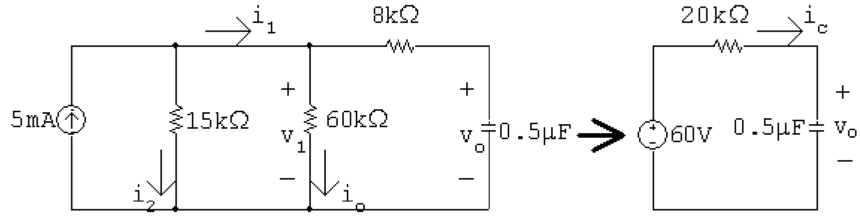
[b] $i_o(\infty) = 0$

[c] $\tau = RC = (5000)(0.8 \times 10^{-6}) = 4\,\text{ms}$

[d] $i_o = 0 + (-7.2)e^{-250t} = -7.2e^{-250t}\,\text{mA}, \qquad t \geq 0^+$

[e] $v_o = -[36 + 1800(-7.2 \times 10^{-3}e^{-250t})] = -36 + 12.96e^{-250t}\,\text{V}, \qquad t \geq 0^+$

P 7.51 [a] Simplify the circuit for $t > 0$ using source transformation:



Since there is no source connected to the capacitor for $t < 0$

$v_o(0^-) = v_o(0^+) = 0\,\text{V}$

From the simplified circuit,

$v_o(\infty) = 60\,\text{V}$

$\tau = RC = (20 \times 10^3)(0.5 \times 10^{-6}) = 10\,\text{ms} \qquad 1/\tau = 100$

$v_o = v_o(\infty) + [v_o(0^+) - v_o(\infty)]e^{-t/\tau} = (60 - 60e^{-100t})\,\text{V}, \qquad t \geq 0$

[b] $i_c = C\dfrac{dv_o}{dt}$

$i_c = 0.5 \times 10^{-6}(-100)(-60e^{-100t}) = 3e^{-100t}\,\text{mA}$

$v_1 = 8000i_c + v_o = (8000)(3 \times 10^{-3})e^{-100t} + (60 - 60e^{-100t}) = 60 - 36e^{-100t}\,\text{V}$

$i_o = \dfrac{v_1}{60 \times 10^3} = 1 - 0.6e^{-100t}\,\text{mA}, \quad t \geq 0^+$

[c] $i_1(t) = i_o + i_c = 1 + 2.4e^{-100t}\,\text{mA}, \quad t \geq 0^+$

[d] $i_2(t) = \dfrac{v_1}{15 \times 10^3} = 4 - 2.4e^{-100t}\,\text{mA}, \quad t \geq 0^+$

[e] $i_1(0^+) = 1 + 2.4 = 3.4\,\text{mA}$

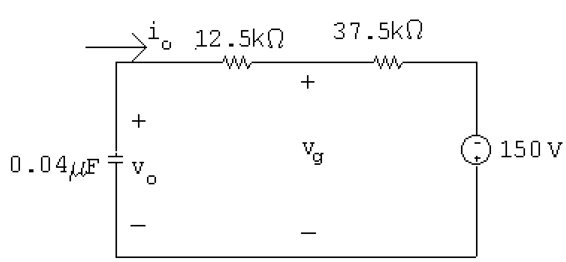
At $t = 0^+$:

$R_e = 15\,\text{k}\|60\,\text{k}\|8\,\text{k} = 4800\,\Omega$

$v_1(0^+) = (5 \times 10^{-3})(4800) = 24\,\text{V}$

$i_1(0^+) = \dfrac{v_1(0^+)}{60{,}000} + \dfrac{v_1(0^+)}{8000} = 0.4\,\text{m} + 3\,\text{m} = 3.4\,\text{mA} \quad \text{(checks)}$

P 7.52 [a] $v_o(0^-) = v_o(0^+) = 120\,\text{V}$



$v_o(\infty) = -150\,\text{V}; \qquad \tau = 2\,\text{ms}; \qquad \dfrac{1}{\tau} = 500$

$$v_o = -150 + (120 - (-150))e^{-500t}$$

$$v_o = -150 + 270e^{-500t}\,\text{V}, \qquad t \geq 0$$

[b] $i_o = -0.04 \times 10^{-6}(-500)[270e^{-500t}] = 5.4e^{-500t}\,\text{mA}, \qquad t \geq 0^+$

[c] $v_g = v_o - 12.5 \times 10^3 i_o = -150 + 202.5e^{-500t}\,\text{V}$

[d] $v_g(0^+) = -150 + 202.5 = 52.5\,\text{V}$

Checks:

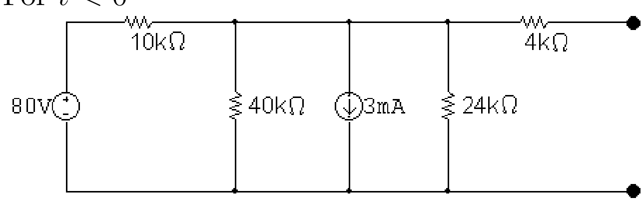
$$v_g(0^+) = i_o(0^+)[37.5 \times 10^3] - 150 = 202.5 - 150 = 52.5\,\text{V}$$

$$i_{50k} = \frac{v_g}{50\text{k}} = -3 + 4.05e^{-500t}\,\text{mA}$$

$$i_{150k} = \frac{v_g}{150\text{k}} = -1 + 1.35e^{-500t}\,\text{mA}$$

$$-i_o + i_{50k} + i_{150k} + 4 = 0 \qquad \text{(ok)}$$

P 7.53 For $t < 0$



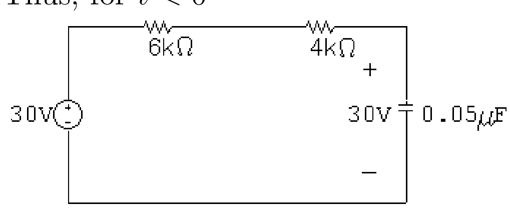
Simplify the circuit:

$80/10{,}000 = 8\,\text{mA}, \qquad 10\,\text{k}\Omega \| 40\,\text{k}\Omega \| 24\,\text{k}\Omega = 6\,\text{k}\Omega$

$8\,\text{mA} - 3\,\text{mA} = 5\,\text{mA}$

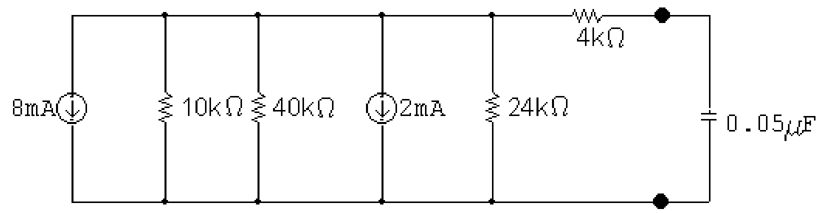
$5\,\text{mA} \times 6\,\text{k}\Omega = 30\,\text{V}$

Thus, for $t < 0$



$\therefore \quad v_o(0^-) = v_o(0^+) = 30\,\text{V}$

$t > 0$



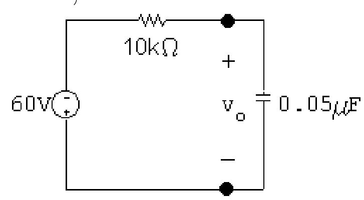
Simplify the circuit:

$8\,\text{mA} + 2\,\text{mA} = 10\,\text{mA}$

$10\,\text{k}\|40\,\text{k}\|24\,\text{k} = 6\,\text{k}\Omega$

$(10\,\text{mA})(6\,\text{k}\Omega) = 60\,\text{V}$

Thus, for $t > 0$



$$v_o(\infty) = -10 \times 10^{-3}(6 \times 10^3) = -60\,\text{V}$$

$$\tau = RC = (10\,\text{k})(0.05\,\mu) = 0.5\,\text{ms}; \qquad \frac{1}{\tau} = 2000$$

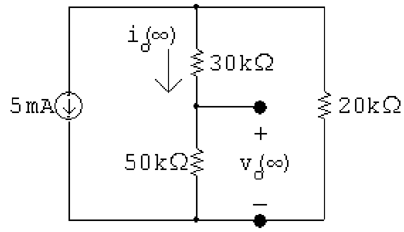
$$v_o = v_o(\infty) + [v_o(0^+) - v_o(\infty)]e^{-t/\tau} = -60 + [30 - (-60)]e^{-2000t}$$

$$= -60 + 90e^{-2000t}\,\text{V} \qquad t \geq 0$$

P 7.54 $t < 0$:

$$i_o(0^-) = \frac{20}{100}(10 \times 10^{-3}) = 2\,\text{mA}; \qquad v_o(0^-) = (2 \times 10^{-3})(50{,}000) = 100\,\text{V}$$

$t = \infty$:

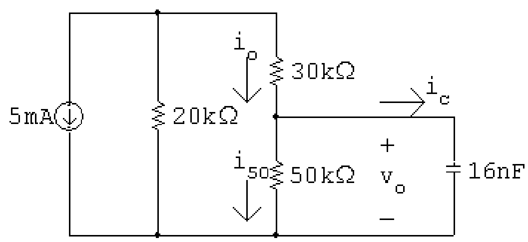


$$i_o(\infty) = -5 \times 10^{-3}\left(\frac{20}{100}\right) = -1\,\text{mA}; \qquad v_o(\infty) = i_o(\infty)(50{,}000) = -50\,\text{V}$$

$$R_{\text{Th}} = 50\,\text{k}\Omega \| 50\,\text{k}\Omega = 25\,\text{k}\Omega; \qquad C = 16\,\text{nF}$$

$$\tau = (25{,}000)(16 \times 10^{-9}) = 0.4\,\text{ms}; \qquad \frac{1}{\tau} = 2500$$

$$\therefore \quad v_o(t) = -50 + 150e^{-2500t}\,\text{V}, \qquad t \geq 0$$



$$i_c = C\frac{dv_o}{dt} = -6e^{-2500t}\,\text{mA}, \qquad t \geq 0^+$$

$$i_{50k} = \frac{v_o}{50{,}000} = -1 + 3e^{-2500t}\,\text{mA}, \qquad t \geq 0^+$$

$$i_o = i_c + i_{50k} = -(1 + 3e^{-2500t})\,\text{mA}, \qquad t \geq 0^+$$

P 7.55 **[a]** $v_c(0^+) = 50\,\text{V}$

[b] Use voltage division to find the final value of voltage:

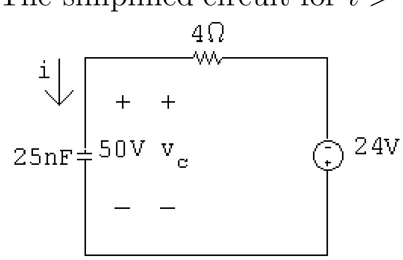
$$v_c(\infty) = \frac{20}{20+5}(-30) = -24\,\text{V}$$

[c] Find the Thévenin equivalent with respect to the terminals of the capacitor:

$$V_{\text{Th}} = -24\,\text{V}, \qquad R_{\text{Th}} = 20\|5 = 4\,\Omega,$$

Therefore $\quad \tau = R_{\text{eq}}C = 4(25 \times 10^{-9}) = 0.1\,\mu\text{s}$

The simplified circuit for $t > 0$ is:



[d] $$i(0^+) = \frac{-24 - 50}{4} = -18.5\,\text{A}$$

[e] $v_c = v_c(\infty) + [v_c(0^+) - v_c(\infty)]e^{-t/\tau}$

$$= -24 + [50 - (-24)]e^{-t/\tau} = -24 + 74e^{-10^7 t}\,\text{V}, \qquad t \geq 0$$

[f] $i = C\dfrac{dv_c}{dt} = (25 \times 10^{-9})(-10^7)(74e^{-10^7 t}) = -18.5e^{-10^7 t}\,\text{A}, \qquad t \geq 0^+$

P 7.56 **[a]** Use voltage division to find the initial value of the voltage:

$$v_c(0^+) = v_{9k} = \frac{9\,\text{k}}{9\,\text{k} + 3\,\text{k}}(120) = 90\,\text{V}$$

[b] Use Ohm's law to find the final value of voltage:

$$v_c(\infty) = v_{40k} = -(1.5 \times 10^{-3})(40 \times 10^3) = -60\,\text{V}$$

[c] Find the Thévenin equivalent with respect to the terminals of the capacitor:

$$V_{\text{Th}} = -60\,\text{V}, \qquad R_{\text{Th}} = 10\,\text{k} + 40\,\text{k} = 50\,\text{k}\Omega$$

$$\tau = R_{\text{Th}}C = 1\,\text{ms} \;= 1000\,\mu\text{s}$$

[d] $v_c = v_c(\infty) + [v_c(0^+) - v_c(\infty)]e^{-t/\tau}$

$$= -60 + (90 + 60)e^{-1000t} = -60 + 150e^{-1000t}\,\text{V}, \quad t \geq 0$$

We want $v_c = -60 + 150e^{-1000t} = 0$:

Therefore $\quad t = \dfrac{\ln(150/60)}{1000} = 916.3\,\mu\text{s}$

P 7.57 Use voltage division to find the initial voltage:

$$v_o(0) = \frac{60}{40 + 60}(50) = 30\,\text{V}$$

Use Ohm's law to find the final value of voltage:

$$v_o(\infty) = (-5\,\text{mA})(20\,\text{k}\Omega) = -100\,\text{V}$$

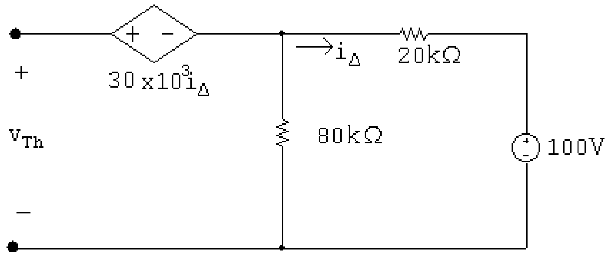
$$\tau = RC = (20 \times 10^3)(250 \times 10^{-9}) = 5\,\text{ms}; \qquad \frac{1}{\tau} = 200$$

$$v_o = v_o(\infty) + [v_o(0^+) - v_o(\infty)]e^{-t/\tau}$$

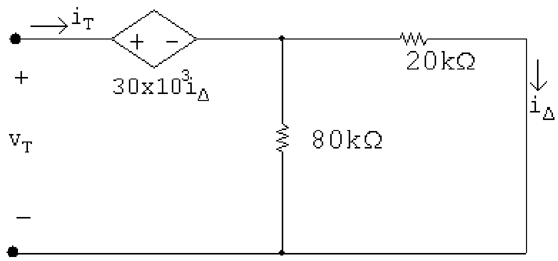
$$= -100 + (30 + 100)e^{-200t} = -100 + 130e^{-200t}\,\text{V}, \qquad t \geq 0$$

P 7.58 For $t < 0$, $v_o(0) = 80\,\text{V}$

$t > 0$:



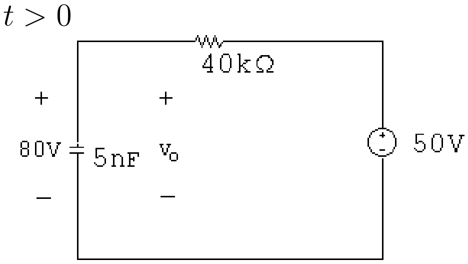
$$v_{\text{Th}} = 30 \times 10^3 i_\Delta + 0.8(100) = 30 \times 10^3 \left(\frac{-100}{100 \times 10^3}\right) + 80 = 50\,\text{V}$$



$$v_T = 30 \times 10^3 i_\Delta + 16 \times 10^3 i_T = 30 \times 10^3 (0.8) i_T + 16 \times 10^3 i_T = 40 \times 10^3 i_T$$

$$R_{\text{Th}} = \frac{v_T}{i_T} = 40\,\text{k}\Omega$$

$t > 0$



$$v_o = 50 + (80 - 50)e^{-t/\tau}$$

$$\tau = RC = (40 \times 10^3)(5 \times 10^{-9}) = 200 \times 10^{-6}; \qquad \frac{1}{\tau} = 5000$$

$$v_o = 50 + 30e^{-5000t}\,\text{V}, \quad t \geq 0$$

P 7.59 $v_o(0) = 50\,\text{V}$; $\quad v_o(\infty) = 80\,\text{V}$

$$R_{\text{Th}} = 16\,\text{k}\Omega$$

$$\tau = (16)(5 \times 10^{-6}) = 80 \times 10^{-6}; \qquad \frac{1}{\tau} = 12{,}500$$

$$v = 80 + (50 - 80)e^{-12{,}500t} = 80 - 30e^{-12{,}500t}\,\text{V}, \quad t \geq 0$$

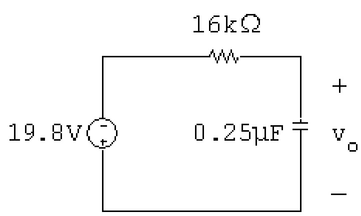
P 7.60 For $t > 0$

$$V_{\text{Th}} = (-25)(16{,}000)i_{\text{b}} = -400 \times 10^3 i_{\text{b}}$$

$$i_{\text{b}} = \frac{33{,}000}{80{,}000}(120 \times 10^{-6}) = 49.5\,\mu\text{A}$$

$$V_{\text{Th}} = -400 \times 10^3(49.5 \times 10^{-6}) = -19.8\,\text{V}$$

$$R_{\text{Th}} = 16\,\text{k}\Omega$$



$$v_o(\infty) = -19.8\,\text{V}; \qquad v_o(0^+) = 0$$

$$\tau = (16,000)(0.25 \times 10^{-6}) = 4\,\text{ms}; \qquad 1/\tau = 250$$

$$v_o = -19.8 + 19.8e^{-250t}\,\text{V}, \qquad t \geq 0$$

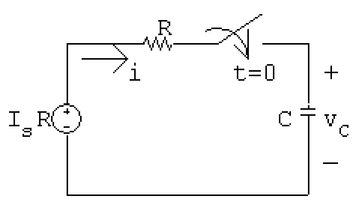
$$w(t) = \frac{1}{2}(0.25 \times 10^{-6})v_o^2 = w(\infty)(1 - e^{-250t})^2\,\text{J}$$

$$(1 - e^{-250t})^2 = \frac{0.36w(\infty)}{w(\infty)} = 0.36$$

$$1 - e^{-250t} = 0.6$$

$$e^{-250t} = 0.4 \quad \therefore \qquad t = 3.67\,\text{ms}$$

P 7.61 [**a**]



$$I_s R = Ri + \frac{1}{C}\int_{0^+}^{t} i\,dx + V_o$$

$$0 = R\frac{di}{dt} + \frac{i}{C} + 0$$

$$\therefore \quad \frac{di}{dt} + \frac{i}{RC} = 0$$

[b] $\dfrac{di}{dt} = -\dfrac{i}{RC}; \qquad \dfrac{di}{i} = -\dfrac{dt}{RC}$

$$\int_{i(0^+)}^{i(t)} \frac{dy}{y} = -\frac{1}{RC}\int_{0^+}^{t} dx$$

$$\ln\frac{i(t)}{i(0^+)} = \frac{-t}{RC}$$

$$i(t) = i(0^+)e^{-t/RC}; \qquad i(0^+) = \frac{I_sR - V_o}{R} = \left(I_s - \frac{V_o}{R}\right)$$

$$\therefore \quad i(t) = \left(I_s - \frac{V_o}{R}\right)e^{-t/RC}$$

P 7.62 **[a]** Let i be the current in the clockwise direction around the circuit. Then

$$V_g = iR_g + \frac{1}{C_1}\int_0^t i\,dx + \frac{1}{C_2}\int_0^t i\,dx$$

$$= iR_g + \left(\frac{1}{C_1} + \frac{1}{C_2}\right)\int_0^t i\,dx = iR_g + \frac{1}{C_e}\int_0^t i\,dx$$

Now differentiate the equation

$$0 = R_g\frac{di}{dt} + \frac{i}{C_e} \quad \text{or} \quad \frac{di}{dt} + \frac{1}{R_gC_e}i = 0$$

Therefore $\quad i = \dfrac{V_g}{R_g}e^{-t/R_gC_e} = \dfrac{V_g}{R_g}e^{-t/\tau}; \qquad \tau = R_gC_e$

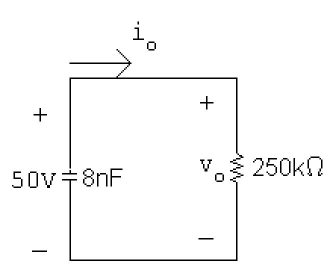
$$v_1(t) = \frac{1}{C_1}\int_0^t \frac{V_g}{R_g}e^{-x/\tau}\,dx = \frac{V_g}{R_gC_1}\,\frac{e^{-x/\tau}}{-1/\tau}\Bigg|_0^t = -\frac{V_gC_e}{C_1}(e^{-t/\tau} - 1)$$

$$v_1(t) = \frac{V_gC_2}{C_1 + C_2}(1 - e^{-t/\tau}); \qquad \tau = R_gC_e$$

$$v_2(t) = \frac{V_gC_1}{C_1 + C_2}(1 - e^{-t/\tau}); \qquad \tau = R_gC_e$$

[b] $v_1(\infty) = \dfrac{C_2}{C_1 + C_2}V_g; \qquad v_2(\infty) = \dfrac{C_1}{C_1 + C_2}V_g$

P 7.63 **[a]** For $t > 0$:



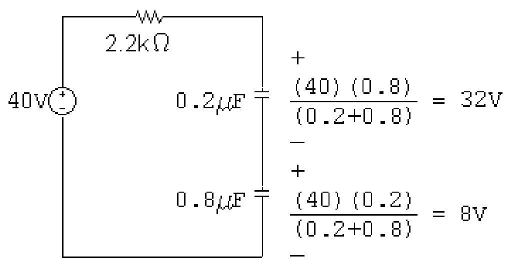
$$\tau = RC = 250 \times 10^3 \times 8 \times 10^{-9} = 2\,\text{ms}; \qquad \frac{1}{\tau} = 500$$

$$v_o = 50e^{-500t}\,\text{V}, \qquad t \geq 0^+$$

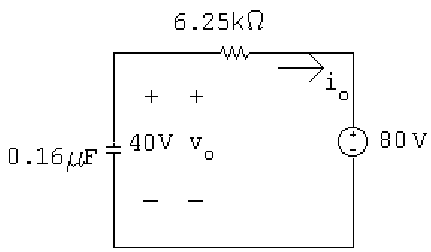
[b] $$i_o = \frac{v_o}{250{,}000} = \frac{50e^{-500t}}{250{,}000} = 200e^{-500t}\,\mu\text{A}$$

$$v_1 = \frac{-1}{40 \times 10^{-9}} \times 200 \times 10^{-6} \int_0^t e^{-500x}\,dx + 50 = 10e^{-500t} + 40\,\text{V}, \quad t \geq 0$$

P 7.64 **[a]** $t < 0$



$t > 0$



$$v_o(0^-) = v_o(0^+) = 40\,\text{V}$$

$$v_o(\infty) = 80\,\text{V}$$

$$\tau = (0.16 \times 10^{-6})(6.25 \times 10^3) = 1\,\text{ms}; \qquad 1/\tau = 1000$$

$$v_o = 80 - 40e^{-1000t}\,\text{V}, \qquad t \geq 0$$

[b] $$i_o = -C\frac{dv_o}{dt} = -0.16 \times 10^{-6}[40{,}000e^{-1000t}]$$

$$= -6.4e^{-1000t}\,\text{mA}; \qquad t \geq 0^+$$

[c] $$v_1 = \frac{-1}{0.2 \times 10^{-6}} \int_0^t -6.4 \times 10^{-3} e^{-1000x}\,dx + 32$$

$$= 64 - 32e^{-1000t}\,\text{V}, \qquad t \geq 0$$

[d] $$v_2 = \frac{-1}{0.8 \times 10^{-6}} \int_0^t -6.4 \times 10^{-3} e^{-1000x}\,dx + 8$$

$$= 16 - 8e^{-1000t}\,\text{V}, \qquad t \geq 0$$

[e] $$w_{\text{trapped}} = \frac{1}{2}(0.2 \times 10^{-6})(64)^2 + \frac{1}{2}(0.8 \times 10^{-6})(16)^2 = 512\,\mu\text{J}.$$

P 7.65 **[a]** $L_{\text{eq}} = \dfrac{(3)(15)}{3+15} = 2.5\,\text{H}$

$$\tau = \frac{L_{\text{eq}}}{R} = \frac{2.5}{7.5} = \frac{1}{3}\,\text{s}$$

$$i_o(0) = 0; \qquad i_o(\infty) = \frac{120}{7.5} = 16\,\text{A}$$

$$\therefore\ \ i_o = 16 - 16e^{-3t}\,\text{A}, \qquad t \geq 0$$

$$v_o = 120 - 7.5i_o = 120e^{-3t}\,\text{V}, \qquad t \geq 0^+$$

$$i_1 = \frac{1}{3}\int_0^t 120e^{-3x}\,dx = \frac{40}{3} - \frac{40}{3}e^{-3t}\,\text{A}, \qquad t \geq 0$$

$$i_2 = i_o - i_1 = \frac{8}{3} - \frac{8}{3}e^{-3t}\,\text{A}, \qquad t \geq 0$$

[b] $i_o(0) = i_1(0) = i_2(0) = 0$, consistent with initial conditions.
$v_o(0^+) = 120$ V, consistent with $i_o(0) = 0$.

$$v_o = 3\frac{di_1}{dt} = 120e^{-3t}\,\text{V}, \qquad t \geq 0^+$$

or

$$v_o = 15\frac{di_2}{dt} = 120e^{-3t}\,\text{V}, \qquad t \geq 0^+$$

The voltage solution is consistent with the current solutions.

$$\lambda_1 = 3i_1 = 40 - 40e^{-3t}\ \text{Wb-turns}$$

$$\lambda_2 = 15i_2 = 40 - 40e^{-3t}\ \text{Wb-turns}$$

$\therefore\ \ \lambda_1 = \lambda_2$ as it must, since

$$v_o = \frac{d\lambda_1}{dt} = \frac{d\lambda_2}{dt}$$

$$\lambda_1(\infty) = \lambda_2(\infty) = 40\ \text{Wb-turns}$$

$$\lambda_1(\infty) = 3i_1(\infty) = 3(40/3) = 40\ \text{Wb-turns}$$

$$\lambda_2(\infty) = 15i_2(\infty) = 15(8/3) = 40\ \text{Wb-turns}$$

$\therefore\ \ i_1(\infty)$ and $i_2(\infty)$ are consistent with $\lambda_1(\infty)$ and $\lambda_2(\infty)$.

P 7.66 **[a]** $L_{\text{eq}} = 5 + 10 - 2.5(2) = 10\,\text{H}$

$$\tau = \frac{L}{R} = \frac{10}{40} = \frac{1}{4}; \qquad \frac{1}{\tau} = 4$$

$$i = 2 - 2e^{-4t}\,\text{A}, \quad t \geq 0$$

[b] $v_1(t) = 5\dfrac{di_1}{dt} - 2.5\dfrac{di}{dt} = 2.5\dfrac{di}{dt} = 2.5(8e^{-4t}) = 20e^{-4t}\,\text{V}, \quad t \geq 0^+$

[c] $v_2(t) = 10\dfrac{di_1}{dt} - 2.5\dfrac{di}{dt} = 7.5\dfrac{di}{dt} = 7.5(8e^{-4t}) = 60e^{-4t}\,\text{V}, \quad t \geq 0^+$

[d] $i(0) = 2 - 2 = 0,$ which agrees with initial conditions.

$$80 = 40i_1 + v_1 + v_2 = 40(2 - 2e^{-4t}) + 20e^{-4t} + 60e^{-4t} = 80\,\text{V}$$

Therefore, Kirchhoff's voltage law is satisfied for all values of $t \geq 0$. Thus, the answers make sense in terms of known circuit behavior.

P 7.67 [a] $L_{\text{eq}} = 5 + 10 + 2.5(2) = 20\,\text{H}$

$$\tau = \frac{L}{R} = \frac{20}{40} = \frac{1}{2}; \qquad \frac{1}{\tau} = 2$$

$$i = 2 - 2e^{-2t}\,\text{A}, \quad t \geq 0$$

[b] $v_1(t) = 5\dfrac{di_1}{dt} + 2.5\dfrac{di}{dt} = 7.5\dfrac{di}{dt} = 7.5(4e^{-2t}) = 30e^{-2t}\,\text{V}, \quad t \geq 0^+$

[c] $v_2(t) = 10\dfrac{di_1}{dt} + 2.5\dfrac{di}{dt} = 12.5\dfrac{di}{dt} = 12.5(4e^{-2t}) = 50e^{-2t}\,\text{V}, \quad t \geq 0^+$

[d] $i(0) = 0,$ which agrees with initial conditions.

$$80 = 40i_1 + v_1 + v_2 = 40(2 - 2e^{-2t}) + 30e^{-2t} + 50e^{-2t} = 80\,\text{V}$$

Therefore, Kirchhoff's voltage law is satisfied for all values of $t \geq 0$. Thus, the answers make sense in terms of known circuit behavior.

P 7.68 [a] From Example 7.10,

$$L_{\text{eq}} = \frac{L_1L_2 - M^2}{L_1 + L_2 + 2M} = \frac{50 - 25}{15 + 10} = 1\,\text{H}$$

$$\tau = \frac{L}{R} = \frac{1}{20}; \qquad \frac{1}{\tau} = 20$$

$$\therefore \quad i_o(t) = 4 - 4e^{-20t}\,\text{A}, \qquad t \geq 0$$

[b] $v_o = 80 - 20i_o = 80 - 80 + 80e^{-20t} = 80e^{-20t}\,\text{V}, \qquad t \geq 0^+$

[c] $v_o = 5\dfrac{di_1}{dt} - 5\dfrac{di_2}{dt} = 80e^{-20t}\,\text{V}$

$$i_o = i_1 + i_2$$

$$\frac{di_o}{dt} = \frac{di_1}{dt} + \frac{di_2}{dt} = 80e^{-20t}\ \text{A/s}$$

$$\therefore \quad \frac{di_2}{dt} = 80e^{-20t} - \frac{di_1}{dt}$$

$$\therefore \quad 80e^{-20t} = 5\frac{di_1}{dt} - 400e^{-20t} + 5\frac{di_1}{dt}$$

$$\therefore \quad 10\frac{di_1}{dt} = 480e^{-20t}; \qquad di_1 = 48e^{-20t}\,dt$$

$$\int_0^{t_1} dx = \int_0^t 48e^{-20y}\,dy$$

$$i_1 = \frac{48}{-20}e^{-20y}\Big|_0^t = 2.4 - 2.4e^{-20t}\,\text{A}, \qquad t \geq 0$$

[d] $i_2 = i_o - i_1 = 4 - 4e^{-20t} - 2.4 + 2.4e^{-20t}$

$$= 1.6 - 1.6e^{-20t}\,\text{A}, \qquad t \geq 0$$

[e] $i_o(0) = i_1(0) = i_2(0) = 0$, consistent with zero initial stored energy.

$$v_o = L_{\text{eq}}\frac{di_o}{dt} = 1(80)e^{-20t} = 80e^{-20t}\,\text{V}, \qquad t \geq 0^+ \text{ (checks)}$$

Also,

$$v_o = 5\frac{di_1}{dt} - 5\frac{di_2}{dt} = 80e^{-20t}\,\text{V}, \qquad t \geq 0^+ \text{ (checks)}$$

$$v_o = 10\frac{di_2}{dt} - 5\frac{di_1}{dt} = 80e^{-20t}\,\text{V}, \qquad t \geq 0^+ \text{ (checks)}$$

$v_o(0^+) = 80\,\text{V}$, which agrees with $i_o(0^+) = 0\,\text{A}$

$i_o(\infty) = 4\,\text{A}; \qquad i_o(\infty)L_{\text{eq}} = (4)(1) = 4$ Wb-turns

$i_1(\infty)L_1 + i_2(\infty)M = (2.4)(5) + (1.6)(-5) = 4$ Wb-turns (ok)

$i_2(\infty)L_2 + i_1(\infty)M = (1.6)(10) + (2.4)(-5) = 4$ Wb-turns (ok)

Therefore, the final values of i_o, i_1, and i_2 are consistent with conservation of flux linkage. Hence, the answers make sense in terms of known circuit behavior.

P 7.69 **[a]** From Example 7.10,

$$L_{\text{eq}} = \frac{L_1L_2 - M^2}{L_1 + L_2 + 2M} = \frac{0.125 - 0.0625}{0.75 + 0.5} = 50\,\text{mH}$$

$$\tau = \frac{L}{R} = \frac{1}{5000}; \qquad \frac{1}{\tau} = 5000$$

$$\therefore \quad i_o(t) = 40 - 40e^{-5000t}\,\text{mA}, \qquad t \geq 0$$

[b] $v_o = 10 - 250i_o = 10 - 250(0.04 + 0.04e^{-5000t} = 10e^{-5000t}\,\text{V}, \qquad t \geq 0^+$

[**c**] $v_o = 0.5\frac{di_1}{dt} - 0.25\frac{di_2}{dt} = 10e^{-5000t}\,\text{V}$

$i_o = i_1 + i_2$

$$\frac{di_o}{dt} = \frac{di_1}{dt} + \frac{di_2}{dt} = 200e^{-5000t}\ \text{A/s}$$

$$\therefore\quad \frac{di_2}{dt} = 200e^{-5000t} - \frac{di_1}{dt}$$

$$\therefore\quad 10e^{-5000t} = 0.5\frac{di_1}{dt} - 50e^{-5000t} + 0.25\frac{di_1}{dt}$$

$$\therefore\quad 0.75\frac{di_1}{dt} = 60e^{-5000t}; \qquad di_1 = 80e^{-5000t}\,dt$$

$$\int_0^{t_1} dx = \int_0^t 80e^{-5000y}\,dy$$

$$i_1 = \frac{80}{-5000}e^{-5000y}\Big|_0^t = 16 - 16e^{-5000t}\,\text{mA}, \qquad t \geq 0$$

[**d**] $i_2 = i_o - i_1 = 40 - 40e^{-5000t} - 16 + 16e^{-5000t}$

$$= 24 - 24e^{-5000t}\,\text{mA}, \qquad t \geq 0$$

[**e**] $i_o(0) = i_1(0) = i_2(0) = 0$, consistent with zero initial stored energy.

$$v_o = L_{\text{eq}}\frac{di_o}{dt} = (0.05)(200)e^{-5000t} = 10e^{-5000t}\,\text{V}, \qquad t \geq 0^+ \text{ (checks)}$$

Also,

$$v_o = 0.5\frac{di_1}{dt} - 0.25\frac{di_2}{dt} = 10e^{-5000t}\,\text{V}, \qquad t \geq 0^+ \text{ (checks)}$$

$$v_o = 0.25\frac{di_2}{dt} - 0.25\frac{di_1}{dt} = 10e^{-5000t}\,\text{V}, \qquad t \geq 0^+ \text{ (checks)}$$

$v_o(0^+) = 10\,\text{V}$, which agrees with $i_o(0^+) = 0\,\text{A}$

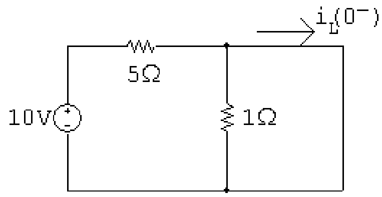
$$i_o(\infty) = 40\,\text{mA}; \qquad i_o(\infty)L_{\text{eq}} = (0.04)(0.05) = 2\ \text{mWb-turns}$$

$$i_1(\infty)L_1 + i_2(\infty)M = (16\,\text{m})(500) + (24\,\text{m})(-250) = 2\ \text{mWb-turns (ok)}$$

$$i_2(\infty)L_2 + i_1(\infty)M = (24\,\text{m})(250) + (16\,\text{m})(-250) = 2\ \text{mWb-turns (ok)}$$

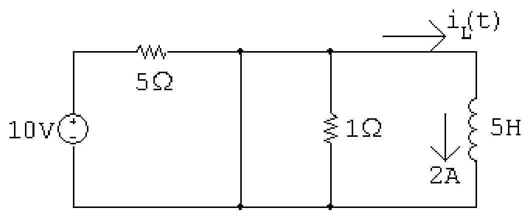
Therefore, the final values of i_o, i_1, and i_2 are consistent with conservation of flux linkage. Hence, the answers make sense in terms of known circuit behavior.

P 7.70 $t < 0$:



$$i_L(0^-) = 10\,\text{V}/5\,\Omega = 2\,\text{A} = i_L(0^+)$$

$0 \leq t \leq 5$:

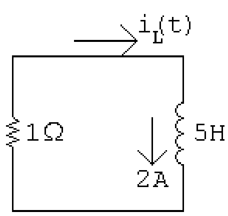


$$\tau = 5/0 = \infty$$

$$i_L(t) = 2e^{-t/\infty} = 2e^{-0} = 2$$

$$i_L(t) = 2\,\text{A} \quad 0 \leq t \leq 5\,\text{s}$$

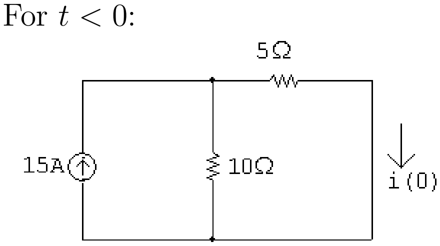
$5 \leq t < \infty$:



$$\tau = \frac{5}{1} = 5\,\text{s}; \qquad 1/\tau = 0.2$$

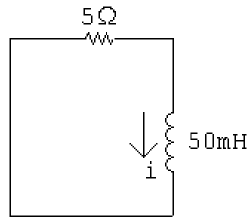
$$i_L(t) = 2e^{-0.2(t-5)}\,\text{A}, \quad t \geq 5\,\text{s}$$

P 7.71 For $t < 0$:



$$i(0) = \frac{10}{15}(15) = 10\,\text{A}$$

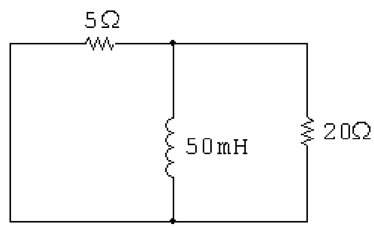
$0 \le t \le 10\,\text{ms}$:



$$i = 10e^{-100t}\,\text{A}$$

$$i(10\,\text{ms}) = 10e^{-1} = 3.68\,\text{A}$$

$10\,\text{ms} \le t \le 20\,\text{ms}$:



$$R_{\text{eq}} = \frac{(5)(20)}{25} = 4\,\Omega$$

$$\frac{1}{\tau} = \frac{R}{L} = \frac{4}{50 \times 10^{-3}} = 80$$

$$i = 3.68e^{-80(t-0.01)}\,\text{A}$$

$20\,\text{ms} \le t < \infty$:

$$i(20\,\text{ms}) = 3.68e^{-80(0.02-0.01)} = 1.65\,\text{A}$$

$$i = 1.65e^{-100(t-0.02)}\,\text{A}$$

$$v_o = L\frac{di}{dt}; \qquad L = 50\,\text{mH}$$

$$\frac{di}{dt} = 1.65(-100)e^{-100(t-0.02)} = -165e^{-100(t-0.02)}$$

$$v_o = (50 \times 10^{-3})(-165)e^{-100(t-0.02)}$$

$$= -8.26e^{-100(t-0.02)}\,\text{V}, \qquad t > 20^{+}\,\text{ms}$$

$$v_o(25\,\text{ms}) = -8.26e^{-100(0.025-0.02)} = -5.013\,\text{V}$$

P 7.72 From the solution to Problem 7.71, the initial energy is

$$w(0) = \frac{1}{2}(50\,\text{mH})(10\,\text{A})^2 = 2.5\,\text{J}$$

$$0.04w(0) = 0.1\,\text{J}$$

$$\therefore \quad \frac{1}{2}(50 \times 10^{-3})i_L^2 = 0.1 \quad \text{so} \quad i_L = 2\,\text{A}$$

Again, from the solution to Problem 7.73, t must be between 10 ms and 20 ms since

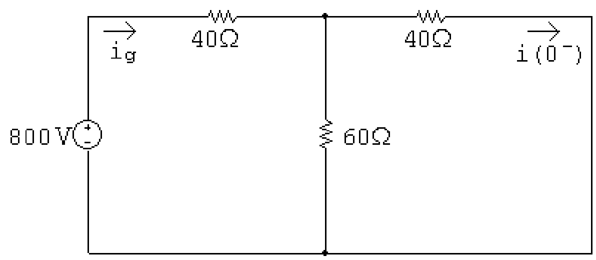
$$i(10\,\text{ms}) = 3.68\,\text{A} \quad \text{and} \quad i(20\,\text{ms}) = 1.65\,\text{A}$$

For $10\,\text{ms} \le t \le 20\,\text{ms}$:

$$i = 3.68e^{-80(t-0.01)} = 2$$

$$e^{80(t-0.01)} = \frac{3.68}{2} \quad \text{so} \quad t - 0.01 = 0.0076 \quad \therefore \quad t = 17.6\,\text{ms}$$

P 7.73 [a] $t < 0$:



Using Ohm's law,

$$i_g = \frac{800}{40 + 60\|40} = 12.5\,\text{A}$$

Using current division,

$$i(0^-) = \frac{60}{60+40}(12.5) = 7.5\,\text{A} = i(0^+)$$

[b] $0 \le t \le 1\,\text{ms}$:

$$i = i(0^+)e^{-t/\tau} = 7.5e^{-t/\tau}$$

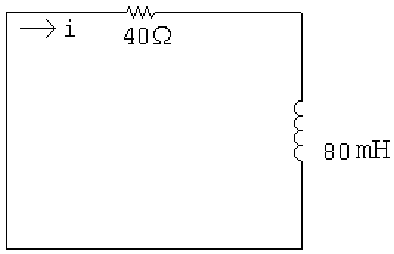
$$\frac{1}{\tau} = \frac{R}{L} = \frac{40 + 120\|60}{80 \times 10^{-3}} = 1000$$

$$i = 7.5e^{-1000t}$$

$$i(200\mu\text{s}) = 7.5e^{-10^3(200\times10^{-6})} = 7.5e^{-0.2} = 6.14\,\text{A}$$

[c] $i(1\,\text{ms}) = 7.5e^{-1} = 2.7591\,\text{A}$

$1\,\text{ms} \le t < \infty$:



$$\frac{1}{\tau} = \frac{R}{L} = \frac{40}{80 \times 10^{-3}} = 500$$

$$i = i(1\,\text{ms})e^{-(t-1\,ms)/\tau} = 2.7591e^{-500(t-0.001)}\,\text{A}$$

$$i(6\text{ms}) = 2.7591e^{-500(0.005)} = 2.7591e^{-2.5} = 226.48\,\text{mA}$$

[d] $0 \le t \le 1\,\text{ms}$:

$$i = 7.5e^{-1000t}$$

$$v = L\frac{di}{dt} = (80 \times 10^{-3})(-1000)(7.5e^{-1000t}) = -600e^{-1000t}\,\text{V}$$

$$v(1^{-}\text{ms}) = -600e^{-1} = -220.73\,\text{V}$$

[e] $1\,\text{ms} \le t \le \infty$:

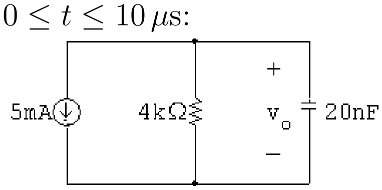
$$i = 2.759e^{-500(t-0.001)}$$

$$v = L\frac{di}{dt} = (80 \times 10^{-3})(-500)(2.759e^{-500(t-0.001)})$$

$$= -110.4e^{-500(t-0.001)}\,\text{V}$$

$$v(1^{+}\text{ms}) = -110.4\,\text{V}$$

P 7.74 $0 \le t \le 10\,\mu\text{s}$:

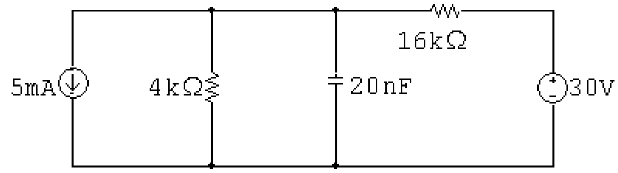


$$\tau = RC = (4 \times 10^{3})(20 \times 10^{-9}) = 80\,\mu\text{s}; \qquad 1/\tau = 12{,}500$$

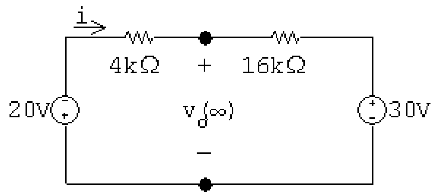
$$v_o(0) = 0\,\text{V}; \qquad v_o(\infty) = -20\,\text{V}$$

$$v_o = -20 + 20e^{-12{,}500t}\,\text{V} \qquad 0 \le t \le 10\,\mu\text{s}$$

$10\,\mu\text{s} \leq t < \infty$:



$t \to \infty$:



$$i = \frac{-50\,\text{V}}{20\,\text{k}\Omega} = -2.5\,\text{mA}$$

$$v_o(\infty) = (-2.5 \times 10^{-3})(16{,}000) + 30 = -10\,\text{V}$$

$$v_o(10\,\mu\text{s}) = -20 + 20^{-0.125} = -2.35\,\text{V}$$

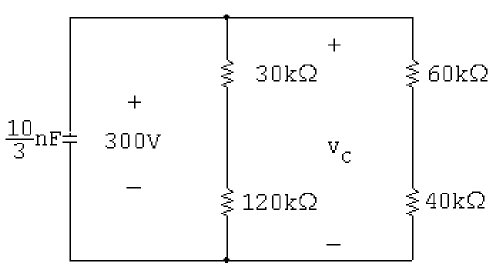
$$v_o = -10 + (-2.35 + 10)e^{-(t\,-\,10\times 10^{-6})/\tau}$$

$$R_{\text{Th}} = 4\,\text{k}\Omega \| 16\,\text{k}\Omega = 3.2\,\text{k}\Omega$$

$$\tau = (3200)(20 \times 10^{-9}) = 64\,\mu\text{s}; \qquad 1/\tau = 15{,}625$$

$$v_o = -10 + 7.65e^{-15{,}625(t\,-\,10\times 10^{-6})} \qquad 10\,\mu\text{s} \leq t < \infty$$

P 7.75 $0 \leq t \leq 200\,\mu\text{s}$;

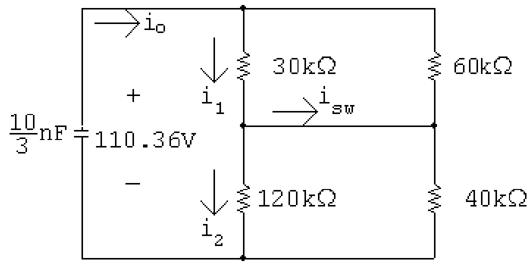


$$R_e = 150 \| 100 = 60\,\text{k}\Omega; \qquad \tau = \left(\frac{10}{3} \times 10^{-9}\right)(60{,}000) = 200\,\mu\text{s}$$

$$v_c = 300e^{-5000t}\,\text{V}$$

$$v_c(200\,\mu\text{s}) = 300e^{-1} = 110.36\,\text{V}$$

$200\,\mu\text{s} \le t < \infty$:



$$R_e = 30\|60 + 120\|40 = 20 + 30 = 50\,\text{k}\Omega$$

$$\tau = \left(\frac{10}{3} \times 10^{-9}\right)(50{,}000) = 166.67\,\mu\text{s}; \qquad \frac{1}{\tau} = 6000$$

$$v_c = 110.36e^{-6000(t-200\,\mu s)}\ \text{V}$$

$$v_c(300\,\mu\text{s}) = 110.36e^{-6000(100\,\mu\text{S})} = 60.57\,\text{V}$$

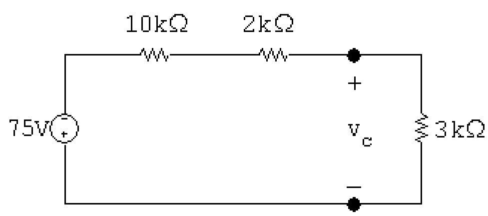
$$i_o(300\,\mu\text{s}) = \frac{60.57}{50{,}000} = 1.21\,\text{mA}$$

$$i_1 = \frac{60}{90}i_o = \frac{2}{3}i_o; \qquad i_2 = \frac{40}{160}i_o = \frac{1}{4}i_o$$

$$i_{\text{sw}} = i_1 - i_2 = \frac{2}{3}i_o - \frac{1}{4}i_o = \frac{5}{12}i_o = \frac{5}{12}(1.21 \times 10^{-3}) = 0.50\,\text{mA}$$

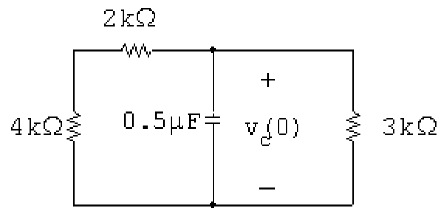
P 7.76 Note that for $t > 0$, $v_o = (4/6)v_c$, where v_c is the voltage across the $0.5\,\mu$F capacitor. Thus we will find v_c first.

$t < 0$



$$v_c(0) = \frac{3}{15}(-75) = -15\,\text{V}$$

$0 \le t \le 800\,\mu\text{s}$:



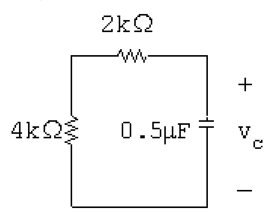
$$\tau = R_eC, \qquad R_e = \frac{(6000)(3000)}{9000} = 2\,\text{k}\Omega$$

$$\tau = (2 \times 10^3)(0.5 \times 10^{-6}) = 1\,\text{ms}, \qquad \frac{1}{\tau} = 1000$$

$$v_c = -15e^{-1000t}\,\text{V}, \qquad t \ge 0$$

$$v_c(800\,\mu\text{s}) = -15e^{-0.8} = -6.74\,\text{V}$$

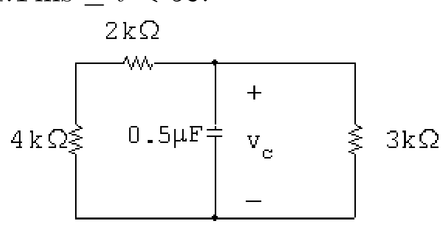
$800\,\mu\text{s} \le t \le 1.1\,\text{ms}$:



$$\tau = (6 \times 10^3)(0.5 \times 10^{-6}) = 3\,\text{ms}, \qquad \frac{1}{\tau} = 333.33$$

$$v_c = -6.74e^{-333.33(t-800\times10^{-6})}\,\text{V}$$

$1.1\,\text{ms} \le t < \infty$:



$$\tau = 1\,\text{ms}, \qquad \frac{1}{\tau} = 1000$$

$$v_c(1.1\text{ms}) = -6.74e^{-333.33(1100-800)10^{-6}} = -6.74e^{-0.1} = -6.1\,\text{V}$$

$$v_c = -6.1e^{-1000(t-1.1\times10^{-3})}\,\text{V}$$

$$v_c(1.5\text{ms}) = -6.1e^{-1000(1.5-1.1)10^{-3}} = -6.1e^{-0.4} = -4.09\,\text{V}$$

$$v_o = (4/6)(-4.09) = -2.73\,\text{V}$$

P 7.77 $w(0) = \frac{1}{2}(0.5 \times 10^{-6})(-15)^2 = 56.25\,\mu\text{J}$

$0 \leq t \leq 800\,\mu$s:

$v_c = -15e^{-1000t}; \qquad v_c^2 = 225e^{-2000t}$

$p_{3k} = 75e^{-2000t}$ mW

$$w_{3k} = \int_0^{800\times10^{-6}} 75 \times 10^{-3} e^{-2000t}\,dt$$
$$= 75 \times 10^{-3} \frac{e^{-2000t}}{-2000}\Big|_0^{800\times10^{-6}}$$
$$= -37.5 \times 10^{-6}(e^{-1.6} - 1) = 29.93\,\mu\text{J}$$

$1.1\,\text{ms} \leq t \leq \infty$:

$v_c = -6.1e^{-1000(t-1.1\times10^{-3})}\,\text{V}; \qquad v_c^2 = 37.19e^{-2000(t-1.1\times10^{-3})}$

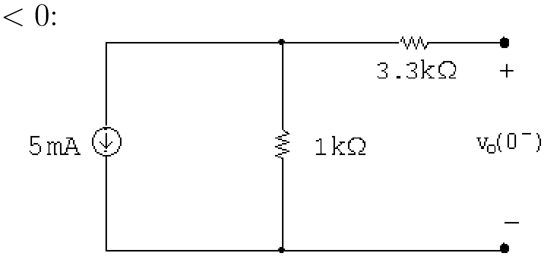
$p_{3k} = 12.4e^{-2000(t-1.1\times10^{-3})}$ mW

$$w_{3k} = \int_{1.1\times10^{-3}}^{\infty} 12.4 \times 10^{-3} e^{-2000(t-1.1\times10^{-3})}\,dt$$
$$= 12.4 \times 10^{-3} \frac{e^{-2000(t-1.1\times10^{-3})}}{-2000}\Big|_{1.1\times10^{-3}}^{\infty}$$
$$= -6.2 \times 10^{-6}(0 - 1) = 6.2\,\mu\text{J}$$

$w_{3k} = 29.93 + 6.2 = 36.13\,\mu\text{J}$

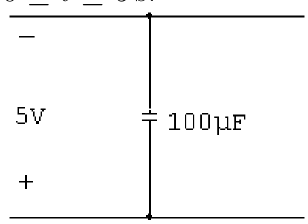
$$\% = \frac{36.13}{56.25}(100) = 64.23\%$$

P 7.78 $t < 0$:



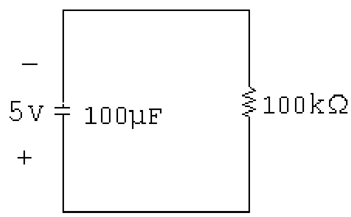
$v_c(0^-) = -(5)(1000) \times 10^{-3} = -5\,\text{V} = v_c(0^+)$

$0 \le t \le 5\,\mathrm{s}$:



$$\tau = \infty; \qquad 1/\tau = 0; \qquad v_o = -5e^{-0} = -5\,\mathrm{V}$$

$5\,\mathrm{s} \le t < \infty$:



$$\tau = (100)(0.1) = 10\,\mathrm{s}; \qquad 1/\tau = 0.1; \qquad v_o = -5e^{-0.1(t-5)}\,\mathrm{V}$$

Summary:

$$v_o = -5\,\mathrm{V}, \qquad 0 \le t \le 5\,\mathrm{s}$$

$$v_o = -5e^{-0.1(t-5)}\,\mathrm{V}, \qquad 5\,\mathrm{s} \le t < \infty$$

P 7.79 **[a]** $0 \le t \le 2.5\,\mathrm{ms}$

$$v_o(0^+) = 80\,\mathrm{V}; \qquad v_o(\infty) = 0$$

$$\tau = \frac{L}{R} = 2\,\mathrm{ms}; \qquad 1/\tau = 500$$

$$v_o(t) = 80e^{-500t}\,\mathrm{V}, \qquad 0^+ \le t \le 2.5^-\,\mathrm{ms}$$

$$v_o(2.5^-\,\mathrm{ms}) = 80e^{-1.25} = 22.92\,\mathrm{V}$$

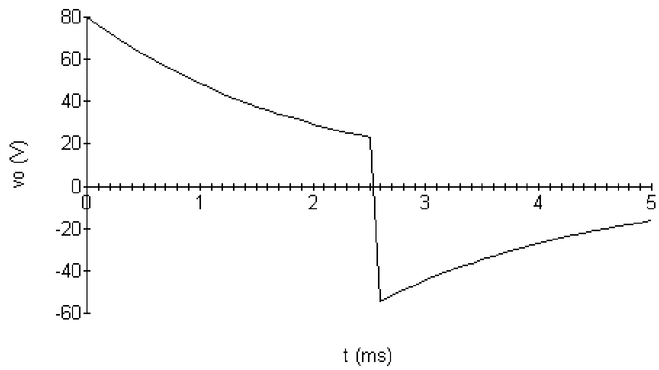
$$i_o(2.5^-\,\mathrm{ms}) = \frac{(80 - 22.92)}{20} = 2.85\,\mathrm{A}$$

$$v_o(2.5^+\,\mathrm{ms}) = -20(2.85) = -57.08\,\mathrm{V}$$

$$v_o(\infty) = 0; \qquad \tau = 2\,\mathrm{ms}; \qquad 1/\tau = 500$$

$$v_o = -57.08e^{-500(t-0.0025)}\,\mathrm{V} \qquad t \ge 2.5^+\,\mathrm{ms}$$

[**b**]



[**c**] $v_o(5\,\text{ms}) = -16.35\,\text{V}$

$$i_o = \frac{+16.35}{20} = 817.68\,\text{mA}$$

P 7.80 [**a**] $i_o(0) = 0; \qquad i_o(\infty) = 25\,\text{mA}$

$$\frac{1}{\tau} = \frac{R}{L} = \frac{2000}{250} \times 10^3 = 8000$$

$$i_o = (25 - 25e^{-8000t})\,\text{mA}, \qquad 0 \leq t \leq 75\,\mu\text{s}$$

$$v_o = 0.25\frac{di_o}{dt} = 50e^{-8000t}\,\text{V}, \qquad 0 \leq t \leq 75\,\mu s$$

$75\,\mu\text{s} \leq t < \infty$:

$$i_o(75\mu\text{s}) = 25 - 25e^{-0.6} = 11.28\,\text{mA}; \qquad i_o(\infty) = 0$$

$$i_o = 11.28e^{-8000(t-75\times10^{-6})}\,\text{mA}$$

$$v_o = (0.25)\frac{di_o}{dt} = -22.56e^{-8000(t-75\mu s)}$$

$$\begin{aligned} \therefore\quad t < 0: \qquad v_o &= 0 \\ 0 \leq t \leq 75\,\mu\text{s}: \qquad v_o &= 50e^{-8000t}\,\text{V} \\ 75\,\mu\text{s} \leq t < \infty: \qquad v_o &= -22.56e^{-8000(t-75\mu s)} \end{aligned}$$

[**b**] $v_o(75^-\,\mu\text{s}) = 50e^{-0.6} = 27.44\,\text{V}$

$$v_o(75^+\mu\text{s}) = -22.56\,\text{V}$$

[**c**] $i_o(75^-\,\mu\text{s}) = i_o(75^+\,\mu\text{s}) = 11.28\,\text{mA}$

P 7.81 **[a]** $0 \leq t \leq 1\,\text{ms}$:

$$v_c(0^+) = 0; \qquad v_c(\infty) = 50\,\text{V};$$

$$RC = 400 \times 10^3(0.01 \times 10^{-6}) = 4\,\text{ms}; \qquad 1/RC = 250$$

$$v_c = 50 - 50e^{-250t}$$

$$v_o = 50 - 50 + 50e^{-250t} = 50e^{-250t}\,\text{V}, \qquad 0 \leq t \leq 1\,\text{ms}$$

$1\,\text{ms} \leq t < \infty$:

$$v_c(1\,\text{ms}) = 50 - 50e^{-0.25} = 11.06\,\text{V}$$

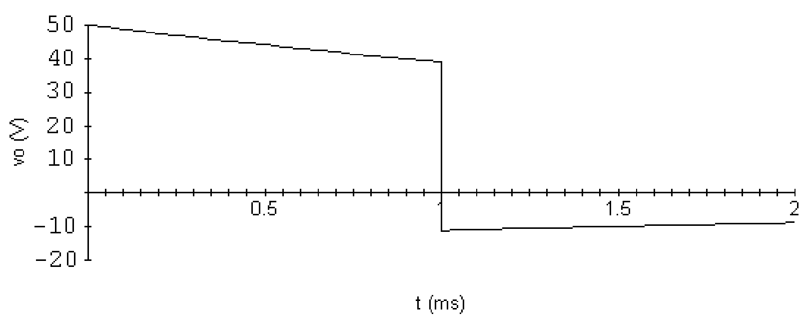
$$v_c(\infty) = 0\,\text{V}$$

$$\tau = 4\,\text{ms}; \qquad 1/\tau = 250$$

$$v_c = 11.06e^{-250(t-0.001)}\,\text{V}$$

$$v_o = -v_c = -11.06e^{-250(t-0.001)}\,\text{V}, \qquad t \geq 1\,\text{ms}$$

[b]



P 7.82 **[a]** $t < 0; \qquad v_o = 0$

$0 \leq t \leq 4\,\text{ms}$:

$$\tau = (200 \times 10^3)(0.025 \times 10^{-6}) = 5\,\text{ms}; \qquad 1/\tau = 200$$

$$v_o = 100 - 100e^{-200t}\,\text{V}, \qquad 0 \leq t \leq 4\,\text{ms}$$

$$v_o(4\,\text{ms}) = 100(1 - e^{-0.8}) = 55.07\,\text{V}$$

$4\,\text{ms} \leq t \leq 8\,\text{ms}$:

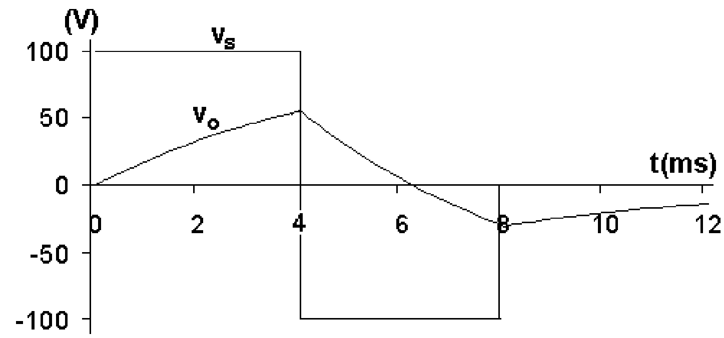
$$v_o = -100 + 155.07e^{-200(t-0.004)}\,\text{V}, \quad 4\,\text{ms} \leq t \leq 8\,\text{ms}$$

$$v_o(8\,\text{ms}) = -100 + 155.07e^{-0.8} = -30.32\,\text{V}$$

$t \geq 8\,\text{ms}$:

$$v_o = -30.32e^{-200(t-0.008)}\,\text{V}, \quad t \geq 8\,\text{ms}$$

[b]



[c] $t \leq 0: \qquad v_o = 0$

$0 \leq t \leq 4\,\text{ms}$:

$$\tau = (50 \times 10^3)(0.025 \times 10^{-6}) = 1.25\,\text{ms} \quad 1/\tau = 800$$

$$v_o = 100 - 100e^{-800t}\,\text{V}, \qquad 0 \leq t \leq 4\,\text{ms}$$

$$v_o(4\,\text{ms}) = 100 - 100e^{-3.2} = 95.92\,\text{V}$$

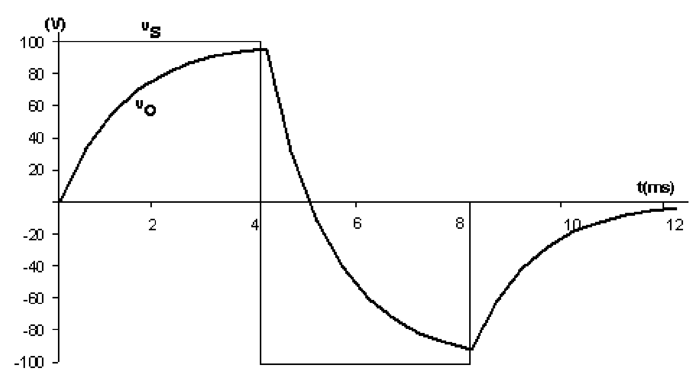
$4\,\text{ms} \leq t \leq 8\,\text{ms}$:

$$v_o = -100 + 195.92e^{-800(t-0.004)}\,\text{V}, \qquad 4\,\text{ms} \leq t \leq 8\,\text{ms}$$

$$v_o(8\,\text{ms}) = -100 + 195.92e^{-3.2} = -92.01\,\text{V}$$

$t \geq 8\,\text{ms}$:

$$v_o = -92.01e^{-800(t-0.008)}\,\text{V}, \quad t \geq 8\,\text{ms}$$



P 7.83 [a] $\tau = RC = (20{,}000)(0.2 \times 10^{-6}) = 4\,\text{ms}; \qquad 1/\tau = 250$

$$i_o = v_o = 0 \qquad t < 0$$

$$i_o(0^+) = 20\left(\frac{16}{20}\right) = 16\,\text{mA}, \qquad i_o(\infty) = 0$$

$$\therefore \quad i_o = 16e^{-250t}\,\text{mA} \qquad 0^+ \leq t \leq 2^-\,\text{ms}$$

$$i_{16k\Omega} = 20 - 16e^{-250t}\,\text{mA}$$

$$\therefore \quad v_o = 320 - 256e^{-250t}\,\text{V} \qquad 0^+ \le t \le 2^-\,\text{ms}$$

$$v_c = v_o - 4 \times 10^3 i_o = 320 - 320e^{-250t}\,\text{V} \qquad 0 \le t \le 2\,\text{ms}$$

$$v_c(2\,\text{ms}) = 320 - 320e^{-0.5} = 125.91\,\text{V}$$

$$\therefore \quad i_o(2^+\,\text{ms}) = 16e^{-0.5} = 9.7\,\text{mA}$$

$$i_o(\infty) = 0$$

$$v_c = 125.91e^{-250(t-0.002)}, \quad t \ge 2\,\text{ms}$$

$$i_o = C\frac{dv_c}{dt} = (0.2 \times 10^{-6})(-250)(125.91)e^{-250(t-0.002)}$$

$$= -6.3e^{-250(t-0.002)}\,\text{mA}, \quad t \ge 2^+\,\text{ms}$$

$$v_o = 4000i_o + v_c = 100.73e^{-250(t-0.002)}\,\text{V} \qquad t \ge 2^+\,\text{ms}$$

Summary part (a)

$$i_o = 0 \qquad t < 0$$

$$i_o = 16e^{-250t}\,\text{mA} \qquad (0^+ \le t \le 2^-\,\text{ms})$$

$$i_o = -6.3e^{-250(t-0.002)}\,\text{mA} \qquad t \ge 2^+\,\text{ms}$$

$$v_o = 0 \qquad t < 0$$

$$v_o = 320 - 256e^{-250t}\,\text{V}, \qquad 0^+ \le t \le 2^-\,\text{ms}$$

$$v_o = 100.73e^{-250(t-0.002)}\,\text{V}, \qquad t \ge 2^+\,\text{ms}$$

[b] $i_o(0^-) = 0$

$$i_o(0^+) = 16\,\text{mA}$$

$$i_o(2^-\,\text{ms}) = 16e^{-0.5} = 9.7\,\text{mA}$$

$$i_o(2^+\,\text{ms}) = -6.3\,\text{mA}$$

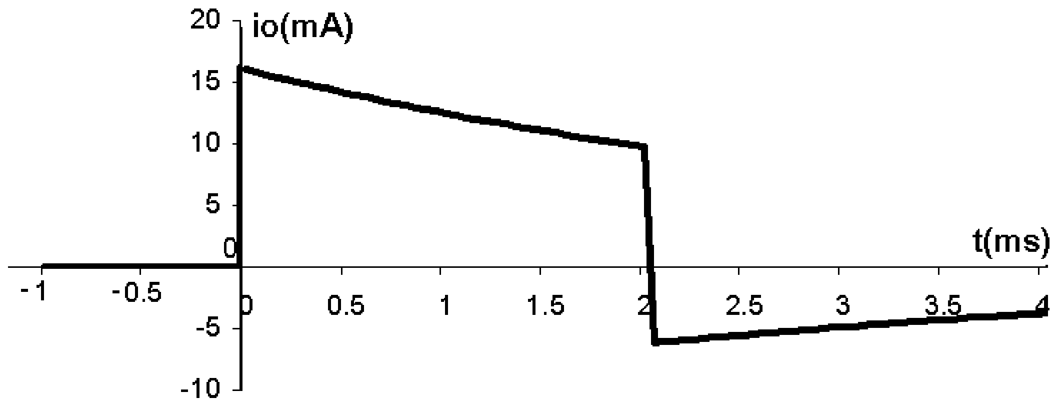
[c] $v_o(0^-) = 0$

$$v_o(0^+) = 64\,\text{V}$$

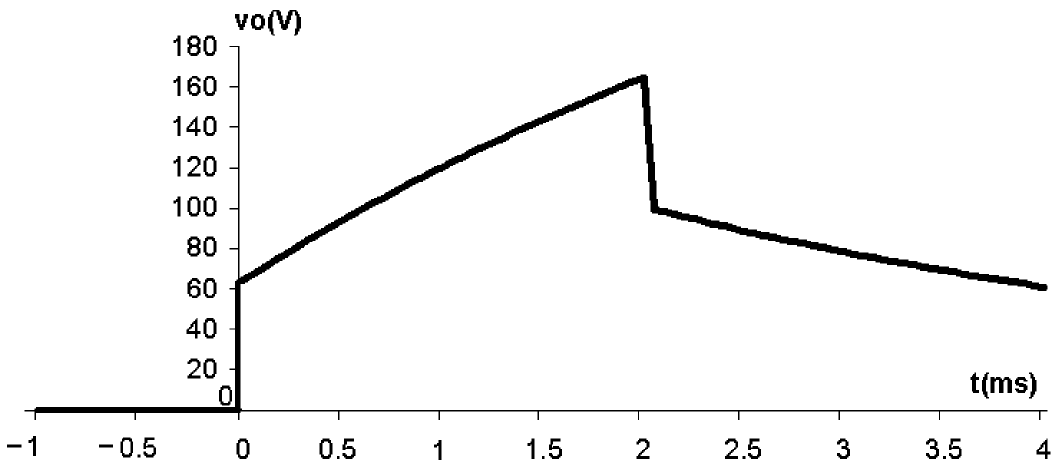
$$v_o(2^-\,\text{ms}) = 320 - 256e^{-0.5} = 164.73\,\text{V}$$

$$v_o(2^+\,\text{ms}) = 100.73$$

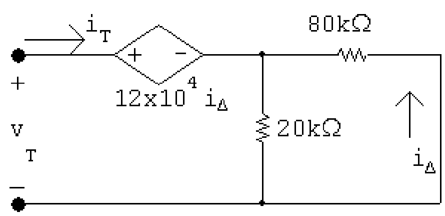
[d]



[e]



P 7.84 $t > 0$:



$$v_T = 12 \times 10^4 i_\Delta + 16 \times 10^3 i_T$$

$$i_\Delta = -\frac{20}{100} i_T = -0.2 i_T$$

$$\therefore \quad v_T = -24 \times 10^3 i_T + 16 \times 10^3 i_T$$

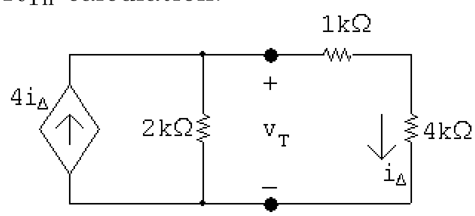
$$R_{\text{Th}} = \frac{v_T}{i_T} = -8\,\text{k}\Omega$$

$$\tau = RC = (-8 \times 10^3)(2.5 \times 10^{-6}) = -0.02 \quad 1/\tau = -50$$

$$v_c = 20e^{50t}\,\text{V}; \qquad 20e^{50t} = 20{,}000$$

$$50t = \ln 1000 \quad \therefore \quad t = 138.16\,\text{ms}$$

P 7.85 Find the Thévenin equivalent with respect to the terminals of the capacitor.
R_{Th} calculation:

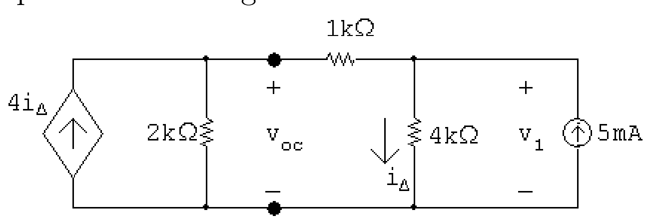


$$i_T = \frac{v_T}{2000} + \frac{v_T}{5000} - 4\frac{v_T}{5000}$$

$$\therefore \quad \frac{i_T}{v_T} = \frac{5+2-8}{10{,}000} = -\frac{1}{10{,}000}$$

$$\frac{v_T}{i_T} = -\frac{10{,}000}{1} = -10\,\text{k}\Omega$$

Open circuit voltage calculation:



The node voltage equations:

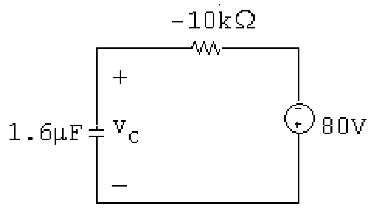
$$\frac{v_{\text{oc}}}{2000} + \frac{v_{\text{oc}} - v_1}{1000} - 4i_\Delta = 0$$

$$\frac{v_1 - v_{\text{oc}}}{1000} + \frac{v_1}{4000} - 5\times 10^{-3} = 0$$

The constraint equation:

$$i_\Delta = \frac{v_1}{4000}$$

Solving, $v_{\text{oc}} = -80\,\text{V}, \quad v_1 = -60\,\text{V}$



$v_c(0) = 0; \qquad v_c(\infty) = -80\,\text{V}$

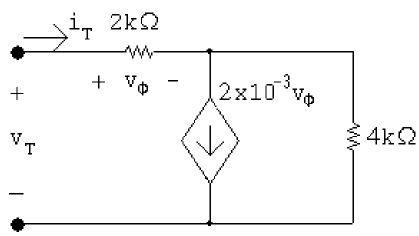
$\tau = RC = (-10{,}000)(1.6 \times 10^{-6}) = -16\,\text{ms}; \qquad \frac{1}{\tau} = -62.5$

$v_c = v_c(\infty) + [v_c(0^+) - v_c(\infty)]e^{-t/\tau} = -80 + 80e^{62.5t} = 14{,}400$

Solve for the time of the maximum voltage rating:

$e^{62.5t} = 181; \qquad 62.5t = \ln 181; \qquad t = 83.09\,\text{ms}$

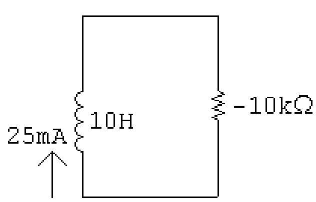
P 7.86



$v_T = 2000i_T + 4000(i_T - 2 \times 10^{-3}v_\phi) = 6000i_T - 8v_\phi$

$= 6000i_T - 8(2000i_T)$

$\frac{v_T}{i_T} = -10{,}000$

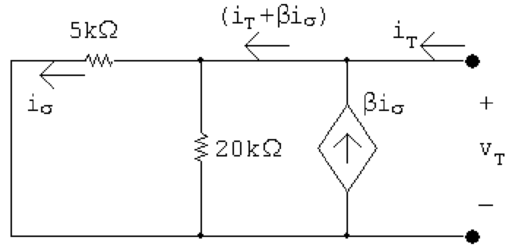


$\tau = \frac{10}{-10{,}000} = -1\,\text{ms}; \qquad 1/\tau = -1000$

$i = 25e^{1000t}\,\text{mA}$

$\therefore \quad 25e^{1000t} \times 10^{-3} = 5; \qquad t = \frac{\ln 200}{1000} = 5.3\,\text{ms}$

P 7.87 [**a**]



Using Ohm's law,

$v_T = 5000 i_\sigma$

Using current division,

$$i_\sigma = \frac{20{,}000}{20{,}000 + 5000}(i_T + \beta i_\sigma) = 0.8 i_T + 0.8\beta i_\sigma$$

Solve for i_σ:

$i_\sigma(1 - 0.8\beta) = 0.8 i_T$

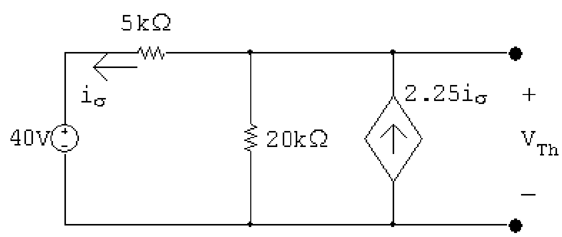
$$i_\sigma = \frac{0.8 i_T}{1 - 0.8\beta}; \qquad v_T = 5000 i_\sigma = \frac{4000 i_T}{(1 - 0.8\beta)}$$

Find β such that $R_{\text{Th}} = -5\,\text{k}\Omega$:

$$R_{\text{Th}} = \frac{v_T}{i_T} = \frac{4000}{1 - 0.8\beta} = -5000$$

$1 - 0.8\beta = -0.8 \qquad \therefore \quad \beta = 2.25$

[**b**] Find V_{Th};



Write a KCL equation at the top node:

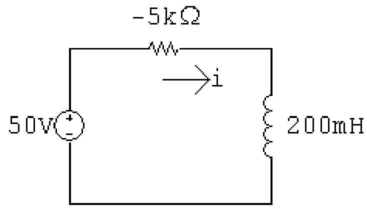
$$\frac{V_{\text{Th}} - 40}{5000} + \frac{V_{\text{Th}}}{20{,}000} - 2.25 i_\sigma = 0$$

The constraint equation is:

$$i_\sigma = \frac{(V_{\text{Th}} - 40)}{5000} = 0$$

Solving,

$V_{\text{Th}} = 50\,\text{V}$



Write a KVL equation around the loop:

$$50 = -5000i + 0.2\frac{di}{dt}$$

Rearranging:

$$\frac{di}{dt} = 250 + 25{,}000i = 25{,}000(i + 0.01)$$

Separate the variables and integrate to find i;

$$\frac{di}{i + 0.01} = 25{,}000\,dt$$

$$\int_0^i \frac{dx}{x + 0.01} = \int_0^t 25{,}000\,dx$$

$$\therefore \quad i = -10 + 10e^{25{,}000t}\,\text{mA}$$

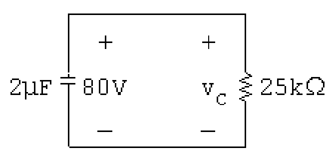
$$\frac{di}{dt} = (10 \times 10^{-3})(25{,}000)e^{25{,}000t} = 250e^{25{,}000t}$$

Solve for the arc time:

$$v = 0.2\frac{di}{dt} = 50e^{25{,}000t} = 45{,}000; \qquad e^{25{,}000t} = 900$$

$$\therefore \quad t = \frac{\ln 900}{25{,}000} = 272.1\,\mu\text{s}$$

P 7.88 **[a]**



$$\tau = (25)(2) \times 10^{-3} = 50\,\text{ms}; \qquad 1/\tau = 20$$

$$v_c(0^+) = 80\,\text{V}; \qquad v_c(\infty) = 0$$

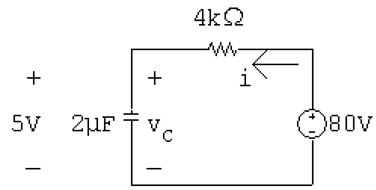
$$v_c = 80e^{-20t}\,\text{V}$$

$$\therefore \quad 80e^{-20t} = 5; \qquad e^{20t} = 16; \qquad t = \frac{\ln 16}{20} = 138.63\,\text{ms}$$

[**b**] $0^+ \le t \le 138.63^-$ ms:

$$i = (2 \times 10^{-6})(-1600e^{-20t}) = -3.2e^{-20t}\,\text{mA}$$

$t \ge 138.63^+$ ms:



$$\tau = (2)(4) \times 10^{-3} = 8\,\text{ms}; \qquad 1/\tau = 125$$

$$v_c(138.63^+\,\text{ms}) = 5\,\text{V}; \qquad v_c(\infty) = 80\,\text{V}$$

$$v_c = 80 - 75e^{-125(t-0.13863)}\,\text{V}, \qquad t \ge 138.63\,\text{ms}$$

$$i = 2 \times 10^{-6}(9375)e^{-125(t-0.13863)}$$
$$= 18.75e^{-125(t-0.13863)}\,\text{mA}, \qquad t \ge 138.63^+\,\text{ms}$$

[**c**] $80 - 75e^{-125\Delta t} = 0.85(80) = 68$

$$80 - 68 = 75e^{-125\Delta t} = 12$$

$$e^{125\Delta t} = 6.25; \qquad \Delta t = \frac{\ln 6.25}{125} \cong 14.66\,\text{ms}$$

P 7.89 [**a**] $RC = (25 \times 10^3)(0.4 \times 10^{-6}) = 10\,\text{ms}; \qquad \frac{1}{RC} = 100$

$$v_o = 0, \quad t < 0$$

[**b**] $0 \le t \le 250\,\text{ms}$:

$$v_o = -100 \int_0^t -0.20\,dx = 20t\,\text{V}$$

[**c**] $250\,\text{ms} \le t \le 500\,\text{ms}$;

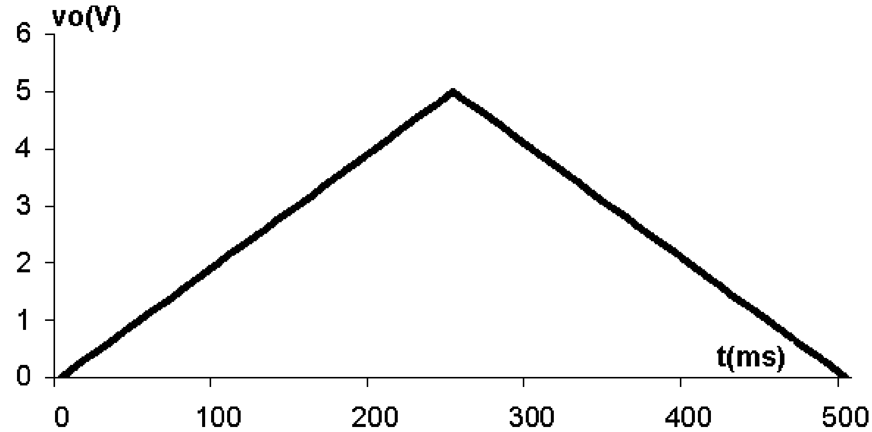
$$v_o(0.25) = 20(0.25) = 5\,\text{V}$$

$$v_o(t) = -100 \int_{0.25}^t 0.20\,dx + 5 = -20(t - 0.25) + 5 = -20t + 10\,\text{V}$$

[**d**] $t \ge 500\,\text{ms}$:

$$v_o(0.5) = -10 + 10 = 0\,\text{V}$$

$$v_o(t) = 0\,\text{V}$$



P 7.90 [a] $v_o = 0, \qquad t < 0$

$$RC = (25 \times 10^3)(0.4 \times 10^{-6}) = 10\,\text{ms} \qquad \frac{1}{RC} = 100$$

[b] $R_fC_f = (5 \times 10^6)(0.4 \times 10^{-6}) = 2; \qquad \dfrac{1}{R_fC_f} = 0.5$

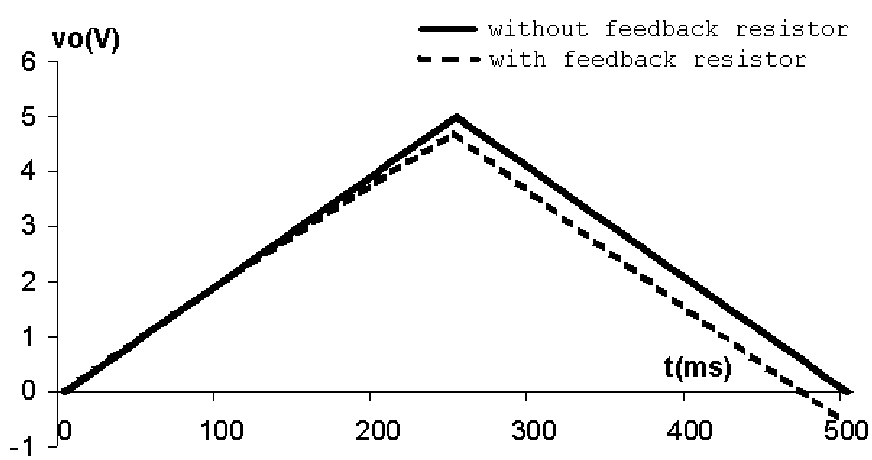
$$v_o = \frac{-5 \times 10^6}{25 \times 10^3}(-0.2)[1 - e^{-0.5t}] = 40(1 - e^{-0.5t})\,\text{V}, \qquad 0 \leq t \leq 250\,\text{ms}$$

[c] $v_o(0.25) = 40(1 - e^{-0.125}) \cong 4.70\,\text{V}$

$$\begin{aligned} v_o &= \frac{-V_mR_f}{R_s} + \frac{V_mR_f}{R_s}(2 - e^{-0.125})e^{-0.5(t-0.25)} \\ &= -40 + 40(2 - e^{-0.125})e^{-0.5(t-0.25)} \\ &= -40 + 44.70e^{-0.5(t-0.25)}\,\text{V}, \qquad 250\,\text{ms} \leq t \leq 500\,\text{ms} \end{aligned}$$

[d] $v_o(0.5) = -40 + 44.70e^{-0.125} \cong -0.55\,\text{V}$

$$v_o = -0.55e^{-0.5(t-0.5)}\,\text{V}, \qquad t \geq 500\,\text{ms}$$



P 7.91 $$v_o = -\frac{1}{R(0.5\times 10^{-6})}\int_0^t 4\,dx + 0 = \frac{-4t}{R(0.5\times 10^{-6})}$$

$$\frac{-4(15\times 10^{-3})}{R(0.5\times 10^{-6})} = -10$$

$$\therefore \qquad R = \frac{-4(15\times 10^{-3})}{-10(0.5\times 10^{-6})} = 12\,\mathrm{k}\Omega$$

P 7.92 $$v_o = \frac{-4t}{R(0.5\times 10^{-6})} + 6 = \frac{-4(40\times 10^{-3})}{R(0.5\times 10^{-6})} + 6 = -10$$

$$\therefore \qquad R = \frac{-4(40\times 10^{-3})}{-16(0.5\times 10^{-6})} = 20\,\mathrm{k}\Omega$$

P 7.93 [a] $RC = (1000)(800\times 10^{-12}) = 800\times 10^{-9}; \qquad \frac{1}{RC} = 1{,}250{,}000$

$0 \le t \le 1\,\mu$s:

$$v_g = 2\times 10^6 t$$

$$v_o = -1.25\times 10^6 \int_0^t 2\times 10^6 x\,dx + 0$$

$$= -2.5\times 10^{12}\frac{x^2}{2}\Big|_0^t = -125\times 10^{10}t^2\,\mathrm{V}, \quad 0\le t\le 1\,\mu\mathrm{s}$$

$$v_o(1\,\mu\mathrm{s}) = -125\times 10^{10}(1\times 10^{-6})^2 = -1.25\,\mathrm{V}$$

$1\,\mu\mathrm{s} \le t \le 3\,\mu$s:

$$v_g = 4 - 2\times 10^6 t$$

$$v_o = -125\times 10^4\int_{1\times 10^{-6}}^t (4 - 2\times 10^6 x)\,dx - 1.25$$

$$= -125\times 10^4\left[4x\Big|_{1\times 10^{-6}}^t - 2\times 10^6\frac{x^2}{2}\Big|_{1\times 10^{-6}}^t\right] - 1.25$$

$$= -5\times 10^6 t + 5 + 125\times 10^{10}t^2 - 1.25 - 1.25$$

$$= 125\times 10^{10}t^2 - 5\times 10^6 t + 2.5\,\mathrm{V}, \quad 1\,\mu\mathrm{s}\le t\le 3\,\mu\mathrm{s}$$

$$v_o(3\,\mu\mathrm{s}) = 125\times 10^{10}(3\times 10^{-6})^2 - 5\times 10^6(3\times 10^{-6}) + 2.5$$

$$= -1.25$$

$3\,\mu\mathrm{s} \le t \le 4\,\mu$s:

$$v_g = -8 + 2\times 10^6 t$$

$$v_o = -125\times 10^4\int_{3\times 10^{-6}}^t (-8 + 2\times 10^6 x)\,dx - 1.25$$

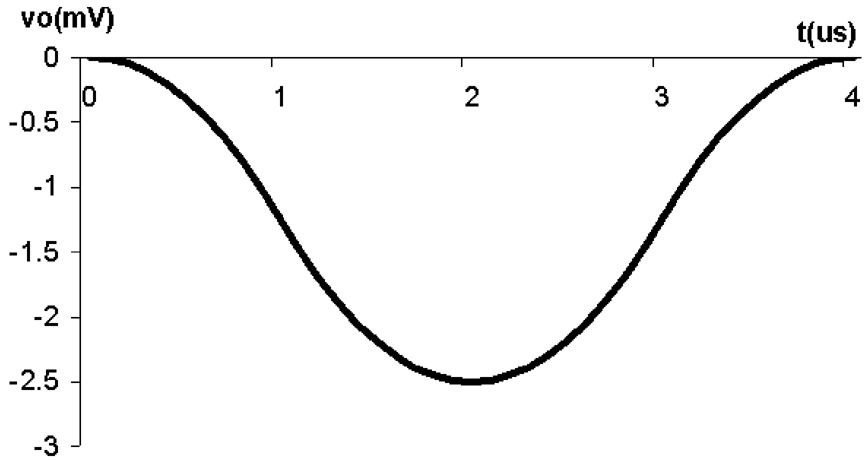
$$= -125\times 10^4\left[-8x\Big|_{3\times 10^{-6}}^t + 2\times 10^6\frac{x^2}{2}\Big|_{3\times 10^{-6}}^t\right] - 1.25$$

$$= 10^7 t - 30 - 125\times 10^{10}t^2 + 11.25 - 1.25$$

$$= -125\times 10^{10}t^2 + 10^7 t - 20\,\mathrm{V}, \quad 3\,\mu\mathrm{s}\le t\le 4\,\mu\mathrm{s}$$

$$v_o(4\,\mu\text{s}) = -125 \times 10^{10}(4 \times 10^{-6})^2 + 10^7(4 \times 10^{-6}) - 20 = 0$$

[b]



[c] The output voltage will also repeat. This follows from the observation that at $t = 4\,\mu$s the output voltage is zero, hence there is no energy stored in the capacitor. This means the circuit is in the same state at $t = 4\,\mu$s as it was at $t = 0$, thus as v_g repeats itself, so will v_o.

P 7.94 **[a]** $\dfrac{C dv_p}{dt} + \dfrac{v_p - v_\text{b}}{R} = 0;$ therefore $\dfrac{dv_p}{dt} + \dfrac{1}{RC}v_p = \dfrac{v_\text{b}}{RC}$

$$\frac{v_n - v_\text{a}}{R} + C\frac{d(v_n - v_o)}{dt} = 0;$$

$$\text{therefore} \quad \frac{dv_o}{dt} = \frac{dv_n}{dt} + \frac{v_n}{RC} - \frac{v_\text{a}}{RC}$$

But $v_n = v_p$

$$\text{Therefore} \quad \frac{dv_n}{dt} + \frac{v_n}{RC} = \frac{dv_p}{dt} + \frac{v_p}{RC} = \frac{v_\text{b}}{RC}$$

$$\text{Therefore} \quad \frac{dv_o}{dt} = \frac{1}{RC}(v_\text{b} - v_\text{a}); \qquad v_o = \frac{1}{RC}\int_0^t (v_\text{b} - v_\text{a})\,dy$$

[b] The output is the integral of the difference between v_b and v_a and then scaled by a factor of $1/RC$.

[c] $v_o = \dfrac{1}{RC}\displaystyle\int_0^t (v_\text{b} - v_\text{a})\,dx$

$$RC = (50 \times 10^3)(10 \times 10^{-9}) = 0.5\,\text{ms}$$

$$v_\text{b} - v_\text{a} = -25\,\text{mV}$$

$$v_o = \frac{1}{0.0005}\int_0^t -25 \times 10^{-3} dx = -50t$$

$$-50t_\text{sat} = -6; \qquad t_\text{sat} = 120\,\text{ms}$$

P 7.95 The equation for an integrating amplifier:

$$v_o = \frac{1}{RC}\int_0^t (v_\mathrm{b} - v_\mathrm{a})\,dy + v_o(0)$$

Find the values and substitute them into the equation:

$$RC = (100 \times 10^3)(0.05 \times 10^{-6}) = 5\,\mathrm{ms}$$

$$\frac{1}{RC} = 200; \qquad v_\mathrm{b} - v_\mathrm{a} = -15 - (-7) = -8\,\mathrm{V}$$

$$v_o(0) = -4 + 12 = 8\,\mathrm{V}$$

$$v_o = 200\int_0^t -8\,dx + 8 = (-1600t + 8)\,\mathrm{V}, \quad 0 \le t \le t_\mathrm{sat}$$

RC circuit analysis for v_2:

$$v_2(0^+) = -4\,\mathrm{V}; \qquad v_2(\infty) = -15\,\mathrm{V}; \qquad \tau = RC = (100\,\mathrm{k})(0.05\,\mu) = 5\,\mathrm{ms}$$

$$v_2 = v_2(\infty) + [v_2(0^+) - v_2(\infty)]e^{-t/\tau}$$

$$= -15 + (-4 + 15)e^{-200t} = -15 + 11e^{-200t}\,\mathrm{V}, \quad 0 \le t \le t_\mathrm{sat}$$

$$v_f + v_2 = v_o \quad \therefore \quad v_f = v_o - v_2 = 23 - 1600t - 11e^{-200t}\,\mathrm{V}, \quad 0 \le t \le t_\mathrm{sat}$$

Note that

$$-1600t_\mathrm{sat} + 8 = -20 \quad \therefore \quad t_\mathrm{sat} = \frac{-28}{-1600} = 17.5\,\mathrm{ms}$$

so the op amp operates in its linear region until it saturates at 17.5 ms.

P 7.96 Use voltage division to find the voltage at the non-inverting terminal:

$$v_p = \frac{80}{100}(-45) = -36\,\mathrm{V} = v_n$$

Write a KCL equation at the inverting terminal:

$$\frac{-36 - 14}{80{,}000} + 2.5 \times 10^{-6}\frac{d}{dt}(-36 - v_o) = 0$$

$$\therefore \quad 2.5 \times 10^{-6}\frac{dv_o}{dt} = \frac{-50}{80{,}000}$$

Separate the variables and integrate:

$$\frac{dv_o}{dt} = -250 \quad \therefore \quad dv_o = -250dt$$

$$\int_{v_o(0)}^{v_o(t)} dx = -250 \int_0^t dy \quad \therefore \quad v_o(t) - v_o(0) = -250t$$

$$v_o(0) = -36 + 56 = 20\,\text{V}$$

$$v_o(t) = -250t + 20$$

Find the time when the voltage reaches 0:

$$0 = -250t + 20 \quad \therefore \quad t = \frac{20}{250} = 80\,\text{ms}$$

P 7.97 [a] T_2 is normally ON since its base current i_{b2} is greater than zero, i.e., $i_{b2} = V_{CC}/R$ when T_2 is ON. When T_2 is ON, $v_{ce2} = 0$, therefore $i_{b1} = 0$. When $i_{b1} = 0$, T_1 is OFF. When T_1 is OFF and T_2 is ON, the capacitor C is charged to V_{CC}, positive at the left terminal. This is a stable state; there is nothing to disturb this condition if the circuit is left to itself.

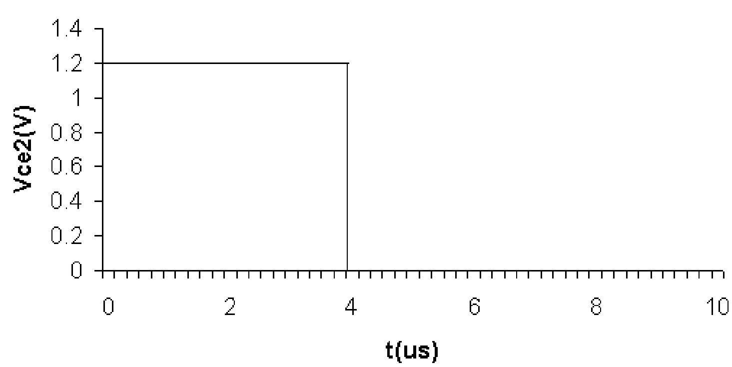
[b] When S is closed momentarily, v_{be2} is changed to $-V_{CC}$ and T_2 snaps OFF. The instant T_2 turns OFF, v_{ce2} jumps to $V_{CC}R_1/(R_1 + R_L)$ and i_{b1} jumps to $V_{CC}/(R_1 + R_L)$, which turns T_1 ON.

[c] As soon as T_1 turns ON, the charge on C starts to reverse polarity. Since v_{be2} is the same as the voltage across C, it starts to increase from $-V_{CC}$ toward $+V_{CC}$. However, T_2 turns ON as soon as $v_{be2} = 0$. The equation for v_{be2} is $v_{be2} = V_{CC} - 2V_{CC}e^{-t/RC}$. $v_{be2} = 0$ when $t = RC \ln 2$, therefore T_2 stays OFF for $RC \ln 2$ seconds.

P 7.98 [a] For $t < 0$, $v_{ce2} = 0$. When the switch is momentarily closed, v_{ce2} jumps to

$$v_{ce2} = \left(\frac{V_{CC}}{R_1 + R_L}\right) R_1 = \frac{6(5)}{25} = 1.2\,\text{V}$$

T_2 remains open for $(23{,}083)(250) \times 10^{-12} \ln 2 \cong 4\,\mu\text{s}$.

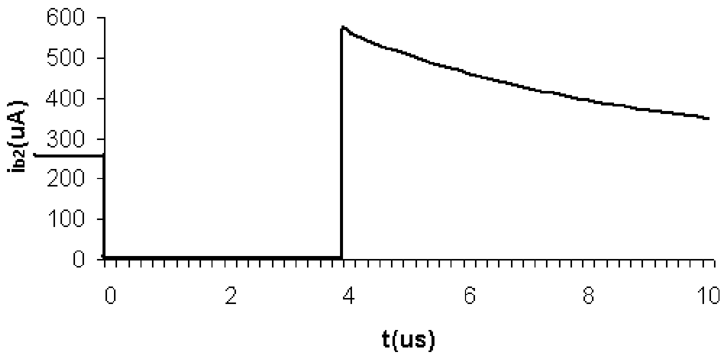


[b] $i_{b2} = \dfrac{V_{CC}}{R} = 259.93\,\mu\text{A}, \qquad -5 \leq t \leq 0\,\mu\text{s}$

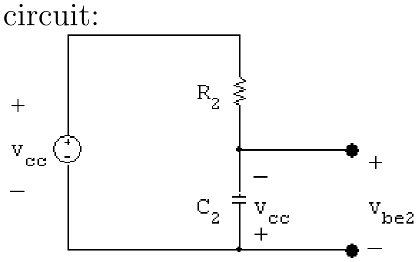
$i_{b2} = 0, \qquad 0 < t < RC\ln 2$

$$i_{b2} = \frac{V_{CC}}{R} + \frac{V_{CC}}{R_L}e^{-(t-RC\ln 2)/R_LC}$$

$$= 259.93 + 300e^{-0.2\times10^6(t-4\times10^{-6})}\,\mu\text{A}, \qquad RC\ln 2 < t$$

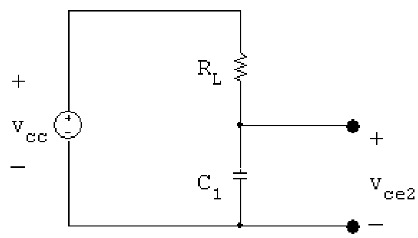


P 7.99 [a] While T_2 has been ON, C_2 is charged to V_{CC}, positive on the left terminal. At the instant T_1 turns ON the capacitor C_2 is connected across $b_2 - e_2$, thus $v_{be2} = -V_{CC}$. This negative voltage snaps T_2 OFF. Now the polarity of the voltage on C_2 starts to reverse, that is, the right-hand terminal of C_2 starts to charge toward $+V_{CC}$. At the same time, C_1 is charging toward V_{CC}, positive on the right. At the instant the charge on C_2 reaches zero, v_{be2} is zero, T_2 turns ON. This makes $v_{be1} = -V_{CC}$ and T_1 snaps OFF. Now the capacitors C_1 and C_2 start to charge with the polarities to turn T_1 ON and T_2 OFF. This switching action repeats itself over and over as long as the circuit is energized. At the instant T_1 turns ON, the voltage controlling the state of T_2 is governed by the following circuit:



It follows that $v_{be2} = V_{CC} - 2V_{CC}e^{-t/R_2C_2}$.

[b] While T_2 is OFF and T_1 is ON, the output voltage v_{ce2} is the same as the voltage across C_1, thus



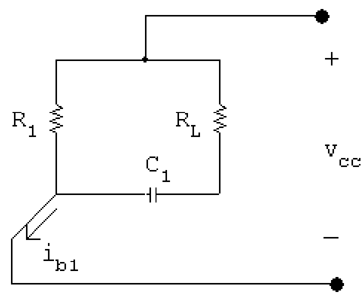
It follows that $v_{\rm ce2} = V_{CC} - V_{CC}e^{-t/R_{\rm L}C_1}$.

[c] T_2 will be OFF until $v_{\rm be2}$ reaches zero. As soon as $v_{\rm be2}$ is zero, $i_{\rm b2}$ will become positive and turn T_2 ON. $v_{\rm be2} = 0$ when $V_{CC} - 2V_{CC}e^{-t/R_2C_2} = 0$, or when $t = R_2C_2 \ln 2$.

[d] When $t = R_2C_2 \ln 2$, we have

$$v_{\rm ce2} = V_{CC} - V_{CC}e^{-[(R_2C_2\ \ln 2)/(R_{\rm L}C_1)]} = V_{CC} - V_{CC}e^{-10\ \ln 2} \cong V_{CC}$$

[e] Before T_1 turns ON, $i_{\rm b1}$ is zero. At the instant T_1 turns ON, we have

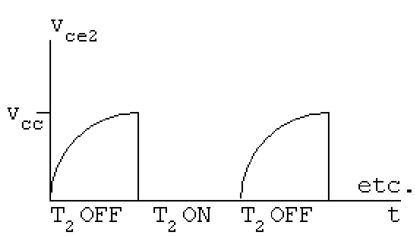


$$i_{b1} = \frac{V_{CC}}{R_1} + \frac{V_{CC}}{R_{\rm L}}e^{-t/R_{\rm L}C_1}$$

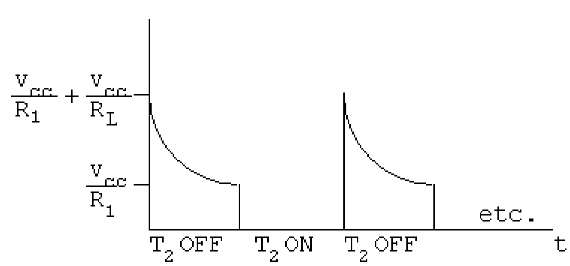
[f] At the instant T_2 turns back ON, $t = R_2C_2 \ln 2$; therefore

$$i_{\rm b1} = \frac{V_{CC}}{R_1} + \frac{V_{CC}}{R_{\rm L}}e^{-10\ \ln 2} \cong \frac{V_{CC}}{R_1}$$

[g]



[h]



P 7.100 **[a]** $t_{\text{OFF2}} = R_2C_2 \ln 2 = 18 \times 10^3(2 \times 10^{-9}) \ln 2 \cong 25\,\mu\text{s}$

[b] $t_{\text{ON2}} = R_1C_1 \ln 2 \cong 25\,\mu\text{s}$

[c] $t_{\text{OFF1}} = R_1C_1 \ln 2 \cong 25\,\mu\text{s}$

[d] $t_{\text{ON1}} = R_2C_2 \ln 2 \cong 25\,\mu\text{s}$

[e] $i_{\text{b1}} = \dfrac{9}{3} + \dfrac{9}{18} = 3.5\,\text{mA}$

[f] $i_{\text{b1}} = \dfrac{9}{18} + \dfrac{9}{3}e^{-6\ln 2} \cong 0.5469\,\text{mA}$

[g] $v_{\text{ce2}} = 9 - 9e^{-6\ln 2} \cong 8.86\,\text{V}$

P 7.101 **[a]** $t_{\text{OFF2}} = R_2C_2 \ln 2 = (18 \times 10^3)(2.8 \times 10^{-9}) \ln 2 \cong 35\,\mu\text{s}$

[b] $t_{\text{ON2}} = R_1C_1 \ln 2 \cong 37.4\,\mu\text{s}$

[c] $t_{\text{OFF1}} = R_1C_1 \ln 2 \cong 37.4\,\mu\text{s}$

[d] $t_{\text{ON1}} = R_2C_2 \ln 2 = 35\,\mu\text{s}$

[e] $i_{\text{b1}} = 3.5\,\text{mA}$

[f] $i_{\text{b1}} = \dfrac{9}{18} + 3e^{-5.6\ln 2} \cong 0.562\,\text{mA}$

[g] $v_{\text{ce2}} = 9 - 9e^{-5.6\ln 2} \cong 8.81\,\text{V}$

Note in this circuit T_2 is OFF $35\,\mu$s and ON $37.4\,\mu$s of every cycle, whereas T_1 is ON $35\,\mu$s and OFF $37.4\,\mu$s every cycle.

P 7.102 If $R_1 = R_2 = 50R_{\text{L}} = 100\,\text{k}\Omega$, then

$$C_1 = \frac{48 \times 10^{-6}}{100 \times 10^3 \ln 2} = 692.49\,\text{pF}; \qquad C_2 = \frac{36 \times 10^{-6}}{100 \times 10^3 \ln 2} = 519.37\,\text{pF}$$

If $R_1 = R_2 = 6R_{\text{L}} = 12\,\text{k}\Omega$, then

$$C_1 = \frac{48 \times 10^{-6}}{12 \times 10^3 \ln 2} = 5.77\,\text{nF}; \qquad C_2 = \frac{36 \times 10^{-6}}{12 \times 10^3 \ln 2} = 4.33\,\text{nF}$$

Therefore $692.49\,\text{pF} \le C_1 \le 5.77\,\text{nF}$ and $519.37\,\text{pF} \le C_2 \le 4.33\,\text{nF}$

P 7.103 **[a]** We want the lamp to be in its nonconducting state for no more than 10 s, the value of t_o:

$$10 = R(10 \times 10^{-6}) \ln \frac{1-6}{4-6} \quad \text{and} \quad R = 1.091\,\text{M}\Omega$$

[b] When the lamp is conducting

$$V_{\text{Th}} = \frac{20 \times 10^3}{20 \times 10^3 + 1.091 \times 10^6}(6) = 0.108\,\text{V}$$

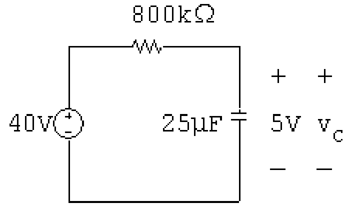
$$R_{\text{Th}} = 20\,\text{k}\|1.091\,\text{M} = 19{,}640\,\Omega$$

So,

$$(t_c - t_o) = (19{,}640)(10 \times 10^{-6}) \ln \frac{4 - 0.108}{1 - 0.108} = 0.289\,\text{s}$$

The flash lasts for 0.289 s.

P 7.104 **[a]** At $t = 0$ we have



$$\tau = (800)(25) \times 10^{-3} = 20\ \text{sec}; \qquad 1/\tau = 0.05$$

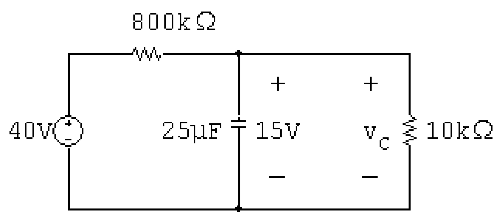
$$v_c(\infty) = 40\,\text{V}; \qquad v_c(0) = 5\,\text{V}$$

$$v_c = 40 - 35e^{-0.05t}\,\text{V}, \qquad 0 \le t \le t_o$$

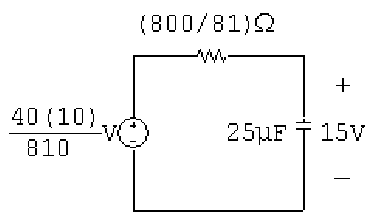
$$40 - 35e^{-0.05t_o} = 15; \qquad \therefore\ e^{0.05t_o} = 1.4$$

$$t_o = 20\ \ln 1.4\,\text{s} = 6.73\,\text{s}$$

At $t = t_o$ we have



The Thévenin equivalent with respect to the capacitor is



$$\tau = \left(\frac{800}{81}\right)(25) \times 10^{-3} = \frac{20}{81}\,\text{s}; \qquad \frac{1}{\tau} = \frac{81}{20} = 4.05$$

$$v_c(t_o) = 15\,\text{V}; \qquad v_c(\infty) = \frac{40}{81}\,\text{V}$$

$$v_c(t) = \frac{40}{81} + \left(15 - \frac{40}{81}\right)e^{-4.05(t-t_o)}\,\text{V} = \frac{40}{81} + \frac{1175}{81}e^{-4.05(t-t_o)}$$

$$\therefore\ \frac{40}{81} + \frac{1175}{81}e^{-4.05(t-t_o)} = 5$$

$$\frac{1175}{81}e^{-4.05(t-t_o)} = \frac{365}{81}$$

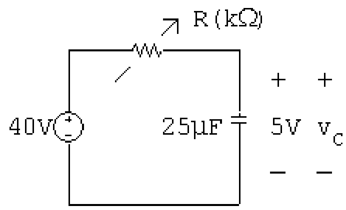
$$e^{4.05(t-t_o)} = \frac{1175}{365} = 3.22$$

$$t - t_o = \frac{1}{4.05}\ \ln 3.22 \cong 0.29\,\text{s}$$

One cycle = 7.02 seconds.

$N = 60/7.02 = 8.55$ flashes per minute

[b] At $t = 0$ we have



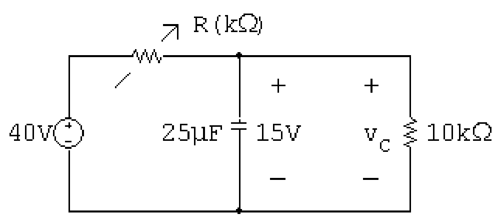
$$\tau = 25R \times 10^{-3}; \qquad 1/\tau = 40/R$$

$$v_c = 40 - 35e^{-(40/R)t}$$

$$40 - 35e^{-(40/R)t_o} = 15$$

$$\therefore\ \ t_o = \frac{R}{40}\ln 1.4, \qquad R \quad \text{in} \quad \text{k}\Omega$$

At $t = t_o$:



$$v_{\text{Th}} = \frac{10}{R+10}(40) = \frac{400}{R+10}; \qquad R_{\text{Th}} = \frac{10R}{R+10}\,\text{k}\Omega$$

$$\tau = \frac{(25)(10R)\times 10^{-3}}{R+10} = \frac{0.25R}{R+10}; \qquad \frac{1}{\tau} = \frac{4(R+10)}{R}$$

$$v_c = \frac{400}{R+10} + \left(15 - \frac{400}{R+10}\right)e^{-\frac{4(R+10)}{R}(t-t_o)}$$

$$\therefore\ \ \frac{400}{R+10} + \left[\frac{15R-250}{R+10}\right]e^{-\frac{4(R+10)}{R}(t-t_o)} = 5$$

$$\text{or} \quad \left(\frac{15R-250}{R+10}\right)e^{-\frac{4(R+10)}{R}(t-t_o)} = \frac{5R-350}{(R+10)}$$

$$\therefore \quad e^{\frac{4(R+10)}{R}(t-t_o)} = \frac{3R-50}{R-70}$$

$$\therefore \quad t - t_o = \frac{R}{4(R+10)} \ln\left(\frac{3R-50}{R-70}\right)$$

At 12 flashes per minute $\quad t_o + (t - t_o) = 5\,\mathrm{s}$

$$\therefore \quad \underbrace{\frac{R}{40}\ln 1.4}_{\text{dominant term}} + \frac{R}{4(R+10)} \ln\left(\frac{3R-50}{R-70}\right) = 5$$

Start the trial-and-error procedure by setting $(R/40)\ \ln 1.4 = 5$, then $R = 200/(\ln 1.4)$ or $594.40\,\mathrm{k\Omega}$. If $R = 594.40\,\mathrm{k\Omega}$ then $t - t_o \cong 0.29\,\mathrm{s}$. Second trial set $(R/40)\ \ln 1.4 = 4.7\,\mathrm{s}$ or $R = 558.74\,\mathrm{k\Omega}$.

With $\quad R = 558.74\,\mathrm{k\Omega}, \quad t - t_o \cong 0.30\,\mathrm{s}$

This procedure converges to $R = 559.3\,\mathrm{k\Omega}$.

P 7.105 **[a]** $t_o = RC \ln\left(\frac{V_{\min} - V_s}{V_{\max} - V_s}\right) = (3700)(250 \times 10^{-6}) \ln\left(\frac{-700}{-100}\right)$

$$= 1.80\,\mathrm{s}$$

$$t_c - t_o = \frac{RCR_{\mathrm{L}}}{R + R_{\mathrm{L}}} \ln\left(\frac{V_{\max} - V_{\mathrm{Th}}}{V_{\min} - V_{\mathrm{Th}}}\right)$$

$$\frac{R_{\mathrm{L}}}{R + R_{\mathrm{L}}} = \frac{1.3}{1.3 + 3.7} = 0.26; \qquad RC = (3700)(250\,10^{-6}) = 0.925\,\mathrm{s}$$

$$V_{\mathrm{Th}} = \frac{1000(1.3)}{1.3 + 3.7} = 260\,\mathrm{V}; \qquad R_{\mathrm{Th}} = 3.7\,\mathrm{k} \| 1.3\,\mathrm{k} = 962\,\Omega$$

$$\therefore \quad t_c - t_o = (0.925)(0.26)\ln(640/40) = 0.67\,\mathrm{s}$$

$$\therefore \quad t_c = 1.8 + 0.67 = 2.47\,\mathrm{s}$$

$$\text{flashes/min} \ = \frac{60}{2.47} = 24.32$$

[b] $0 \leq t \leq t_o$:

$$v_L = 1000 - 700e^{-t/\tau_1}$$

$$\tau_1 = RC = 0.925\,\mathrm{s}$$

$t_o \leq t \leq t_c$:

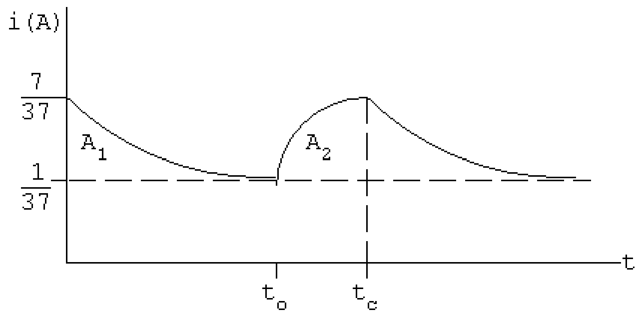
$$v_L = 260 + 640e^{-(t-t_o)/\tau_2}$$

$$\tau_2 = R_{\mathrm{Th}}C = 962(250) \times 10^{-6} = 0.2405\,\mathrm{s}$$

$$0 \le t \le t_o: \qquad i = \frac{1000 - v_L}{3700} = \frac{7}{37}e^{-t/0.925}\,\mathrm{A}$$

$$t_o \le t \le t_c: \qquad i = \frac{1000 - v_L}{3700} = \frac{74}{370} - \frac{64}{370}e^{-(t-t_o)/0.2405}$$

Graphically, i versus t is



The average value of i will equal the areas $(A_1 + A_2)$ divided by t_c.

$$\therefore \quad i_{\text{avg}} = \frac{A_1 + A_2}{t_c}$$

$$\begin{aligned} A_1 &= \frac{7}{37}\int_0^{t_o} e^{-t/0.925}\,dt \\ &= \frac{6.475}{37}(1 - e^{-\ln 7}) = 0.15 \text{ A–s} \\ A_2 &= \int_{t_o}^{t_c} \frac{74 - 64e^{-(t-t_o)/0.2405}}{370}\,dt \\ &= \frac{74}{370}(t_c - t_o) + \frac{15.392}{370}(e^{-\ln 16} - 1) \\ &= \frac{17.797}{370}\ln 16 - \frac{15.392}{370}(1 - e^{-\ln 16}) \\ &= 0.09436 \text{ A–s} \end{aligned}$$

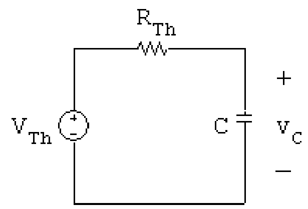
$$i_{\text{avg}} = \frac{(0.15 + 0.09436)}{0.925\ln 7 + 0.2405\ln 16}(1000) = 99.06\,\mathrm{mA}$$

[c] $P_{\text{avg}} = (1000)(99.06 \times 10^{-3}) = 99.06\,\mathrm{W}$

$$\text{No. of kw hrs/yr} = \frac{(99.06)(24)(365)}{1000} = 867.77$$

$$\text{Cost/year} = (867.77)(0.05) = 43.39 \text{ dollars/year}$$

P 7.106 **[a]** Replace the circuit attached to the capacitor with its Thévenin equivalent, where the equivalent resistance is the parallel combination of the two resistors, and the open-circuit voltage is obtained by voltage division across the lamp resistance. The resulting circuit is



$$R_{\rm Th} = R\|R_{\rm L} = \frac{RR_{\rm L}}{R + R_{\rm L}}; \qquad V_{\rm Th} = \frac{R_{\rm L}}{R + R_{\rm L}} V_s$$

From this circuit,

$$v_{\rm C}(\infty) = V_{\rm Th}; \qquad v_{\rm C}(0) = V_{\max}; \qquad \tau = R_{\rm Th}C$$

Thus,

$$v_{\rm C}(t) = V_{\rm Th} + (V_{\max} - V_{\rm Th})e^{-(t-t_o)/\tau}$$

where

$$\tau = \frac{RR_{\rm L}C}{R + R_{\rm L}}$$

[b] Now, set $v_{\rm C}(t_c) = V_{\min}$ and solve for $(t_c - t_o)$:

$$V_{\rm Th} + (V_{\max} - V_{\rm Th})e^{-(t_c-t_o)/\tau} = V_{\min}$$

$$e^{-(t_c-t_o)/\tau} = \frac{V_{\min} - V_{\rm Th}}{V_{\max} - V_{\rm Th}}$$

$$\frac{-(t_c - t_o)}{\tau} = \ln \frac{V_{\min} - V_{\rm Th}}{V_{\max} - V_{\rm Th}}$$

$$(t_c - t_o) = -\frac{RR_{\rm L}C}{R + R_{\rm L}} \ln \frac{V_{\min} - V_{\rm Th}}{V_{\max} - V_{\rm Th}} = \frac{RR_{\rm L}C}{R + R_{\rm L}} \ln \frac{V_{\max} - V_{\rm Th}}{V_{\min} - V_{\rm Th}}$$

P 7.107 **[a]** $0 \le t \le 0.5$:

$$i = \frac{21}{60} + \left(\frac{30}{60} - \frac{21}{60}\right) e^{-t/\tau} \qquad \text{where } \tau = L/R.$$

$$i = 0.35 + 0.15e^{-60t/L}$$

$$i(0.5) = 0.35 + 0.15e^{-30/L} = 0.40$$

$$\therefore \quad e^{30/L} = 3; \qquad L = \frac{30}{\ln 3} = 27.31\,\text{H}$$

[b] $0 \le t \le t_r$, where t_r is the time the relay releases:

$$i = 0 + \left(\frac{30}{60} - 0\right) e^{-60t/L} = 0.5e^{-60t/L}$$

$$\therefore \quad 0.4 = 0.5e^{-60t_r/L}; \qquad e^{60t_r/L} = 1.25$$

$$t_r = \frac{27.31 \ln 1.25}{60} \cong 0.10\,\text{s}$$