Ch06 Solution Manual Material Science and Engineering 8th Edition

Material Science & Engineering (HITEC University)

CHAPTER 6

MECHANICAL PROPERTIES OF METALS

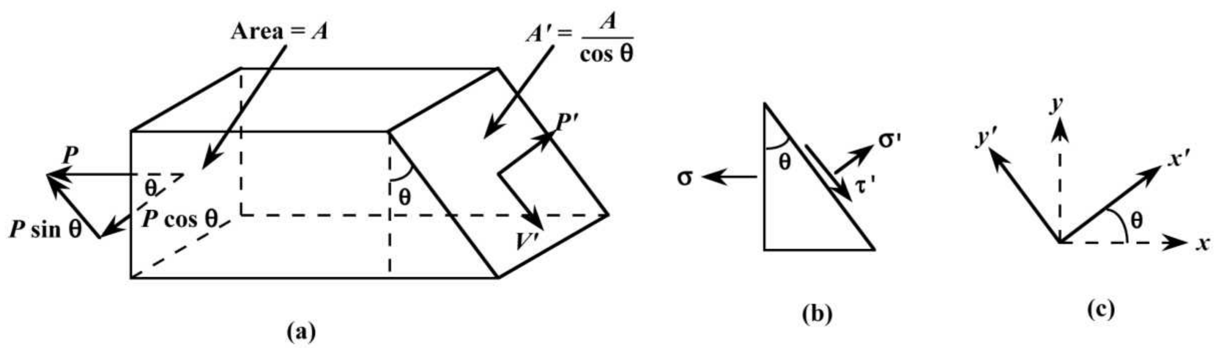
PROBLEM SOLUTIONS

Concepts of Stress and Strain

6.1 Using mechanics of materials principles (i.e., equations of mechanical equilibrium applied to a free-body diagram), derive Equations 6.4a and 6.4b.

Solution

This problem asks that we derive Equations 6.4a and 6.4b, using mechanics of materials principles. In Figure (a) below is shown a block element of material of cross-sectional area A that is subjected to a tensile force P. Also represented is a plane that is oriented at an angle θ referenced to the plane perpendicular to the tensile axis; the area of this plane is $A' = A/\cos\theta$. In addition, and the forces normal and parallel to this plane are labeled as P' and V', respectively. Furthermore, on the left-hand side of this block element are shown force components that are tangential and perpendicular to the inclined plane. In Figure (b) are shown the orientations of the applied stress σ, the normal stress to this plane σ', as well as the shear stress τ' taken parallel to this inclined plane. In addition, two coordinate axis systems in represented in Figure (c): the primed x and y axes are referenced to the inclined plane, whereas the unprimed x axis is taken parallel to the applied stress.



Normal and shear stresses are defined by Equations 6.1 and 6.3, respectively. However, we now chose to express these stresses in terms (i.e., general terms) of normal and shear forces (P and V) as

$$\sigma = \frac{P}{A}$$

$$\tau = \frac{V}{A}$$

For static equilibrium in the x' direction the following condition must be met:

$$\sum F_{x'} = 0$$

which means that

$$P' - P \cos\theta = 0$$

Or that

$$P' = P \cos\theta$$

Now it is possible to write an expression for the stress σ' in terms of P' and A' using the above expression and the relationship between A and A' [Figure (a)]:

$$\sigma' = \frac{P'}{A'}$$

$$= \frac{P\cos\theta}{\frac{A}{\cos\theta}} = \frac{P}{A}\cos^2\theta$$

However, it is the case that $P/A = \sigma$; and, after making this substitution into the above expression, we have Equation 6.4a--that is

$$\sigma' = \sigma \cos^2\theta$$

Now, for static equilibrium in the y' direction, it is necessary that

$$\sum F_{y'} = 0$$

$$= -V' + P \sin\theta$$

Or

$$V' = P \sin\theta$$

We now write an expression for τ' as

$$\tau' = \frac{V'}{A'}$$

And, substitution of the above equation for V' and also the expression for A' gives

$$\tau' = \frac{V'}{A'}$$

$$= \frac{P \sin\theta}{\frac{A}{\cos\theta}}$$

$$= \frac{P}{A} \sin\theta \cos\theta$$

$$= \sigma \sin\theta \cos\theta$$

which is just Equation 6.4b.

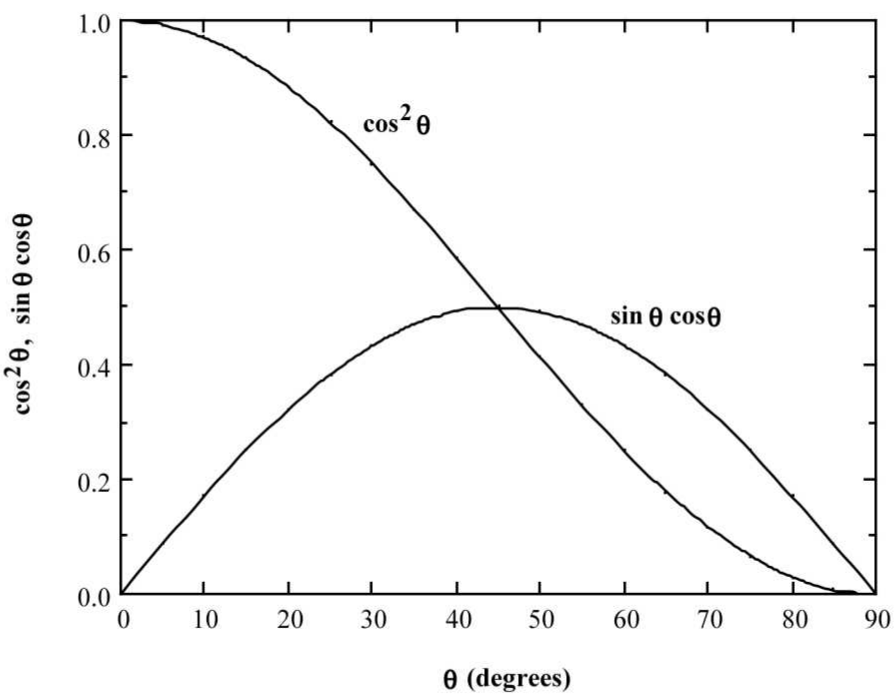
6.2 (a) Equations 6.4a and 6.4b are expressions for normal (σ') and shear (τ') stresses, respectively, as a function of the applied tensile stress (σ) and the inclination angle of the plane on which these stresses are taken (θ of Figure 6.4). Make a plot on which is presented the orientation parameters of these expressions (i.e., $\cos^2\theta$ and $\sin\theta\cos\theta$) versus θ.

(b) From this plot, at what angle of inclination is the normal stress a maximum?

(c) Also, at what inclination angle is the shear stress a maximum?

Solution

(a) Below are plotted curves of $\cos^2\theta$ (for σ') and $\sin\theta\cos\theta$ (for τ') versus θ.



(b) The maximum normal stress occurs at an inclination angle of 0°.

(c) The maximum shear stress occurs at an inclination angle of 45°.

Stress-Strain Behavior

6.3 A specimen of aluminum having a rectangular cross section 10 mm × 12.7 mm (0.4 in. × 0.5 in.) is pulled in tension with 35,500 N (8000 lb_f) force, producing only elastic deformation. Calculate the resulting strain.

<u>Solution</u>

This problem calls for us to calculate the elastic strain that results for an aluminum specimen stressed in tension. The cross-sectional area is just (10 mm) × (12.7 mm) = 127 mm^2 (= 1.27×10^{-4} m^2 = 0.20 $in.^2$); also, the elastic modulus for Al is given in Table 6.1 as 69 GPa (or 69×10^9 N/m^2). Combining Equations 6.1 and 6.5 and solving for the strain yields

$$\varepsilon = \frac{\sigma}{E} = \frac{F}{A_0 E} = \frac{35,500 \text{ N}}{(1.27 \times 10^{-4} \text{ m}^2)(69 \times 10^9 \text{ N/m}^2)} = 4.1 \times 10^{-3}$$

6.4 A cylindrical specimen of a titanium alloy having an elastic modulus of 107 GPa (15.5×10^6 psi) and an original diameter of 3.8 mm (0.15 in.) will experience only elastic deformation when a tensile load of 2000 N (450 lb_f) is applied. Compute the maximum length of the specimen before deformation if the maximum allowable elongation is 0.42 mm (0.0165 in.).

Solution

We are asked to compute the maximum length of a cylindrical titanium alloy specimen (before deformation) that is deformed elastically in tension. For a cylindrical specimen

$$A_0 = \pi \left(\frac{d_0}{2}\right)^2$$

where d_0 is the original diameter. Combining Equations 6.1, 6.2, and 6.5 and solving for l_0 leads to

$$l_0 = \frac{\Delta l}{\varepsilon} = \frac{\Delta l}{\frac{\sigma}{E}} = \frac{\Delta l\, E}{\frac{F}{A_0}} = \frac{\Delta l\, E \pi \left(\frac{d_0}{2}\right)^2}{F} = \frac{\Delta l\, E \pi\, d_0^2}{4F}$$

$$= \frac{(0.42 \times 10^{-3}\ \mathrm{m})(107 \times 10^9\ \mathrm{N/m^2})\,(\pi)(3.8 \times 10^{-3}\ \mathrm{m})^2}{(4)(2000\ \mathrm{N})}$$

$$= 0.255\ \mathrm{m} = 255\ \mathrm{mm}\ (10.0\ \mathrm{in.})$$

6.5 A steel bar 100 mm (4.0 in.) long and having a square cross section 20 mm (0.8 in.) on an edge is pulled in tension with a load of 89,000 N (20,000 lb_f), and experiences an elongation of 0.10 mm (4.0×10^{-3} in.). Assuming that the deformation is entirely elastic, calculate the elastic modulus of the steel.

Solution

This problem asks us to compute the elastic modulus of steel. For a square cross-section, $A_0 = b_0^2$, where b_0 is the edge length. Combining Equations 6.1, 6.2, and 6.5 and solving for E, leads to

$$E = \frac{\sigma}{\varepsilon} = \frac{\dfrac{F}{A_0}}{\dfrac{\Delta l}{l_0}} = \frac{F l_0}{b_0^2 \Delta l}$$

$$= \frac{(89{,}000 \text{ N})(100 \times 10^{-3}\text{ m})}{(20 \times 10^{-3}\text{ m})^2 (0.10 \times 10^{-3}\text{ m})}$$

$$= 223 \times 10^9 \text{ N/m}^2 = 223 \text{ GPa} \ (31.3 \times 10^6 \text{ psi})$$

6.6 Consider a cylindrical titanium wire 3.0 mm (0.12 in.) in diameter and 2.5×10^4 mm (1000 in.) long. Calculate its elongation when a load of 500 N (112 lb_f) is applied. Assume that the deformation is totally elastic.

Solution

In order to compute the elongation of the Ti wire when the 500 N load is applied we must employ Equations 6.1, 6.2, and 6.5. Solving for Δl and realizing that for Ti, E = 107 GPa (15.5×10^6 psi) (Table 6.1),

$$\Delta l = l_0 \varepsilon = l_0 \frac{\sigma}{E} = \frac{l_0 F}{E A_0} = \frac{l_0 F}{E\pi \left(\frac{d_0}{2}\right)^2} = \frac{4 l_0 F}{E \pi d_0^2}$$

$$= \frac{(4)(25\ \text{m})(500\ \text{N})}{\left(107 \times 10^9\ \text{N/m}^2\right)(\pi)(3 \times 10^{-3}\ \text{m})^2} = 0.0165\ \text{m} = 16.5\ \text{mm}\ (0.65\ \text{in.})$$

6.7 For a bronze alloy, the stress at which plastic deformation begins is 275 MPa (40,000 psi), and the modulus of elasticity is 115 GPa (16.7×10^6 psi).

(a) What is the maximum load that may be applied to a specimen with a cross-sectional area of 325 mm^2 (0.5 $in.^2$) without plastic deformation?

(b) If the original specimen length is 115 mm (4.5 in.), what is the maximum length to which it may be stretched without causing plastic deformation?

Solution

(a) This portion of the problem calls for a determination of the maximum load that can be applied without plastic deformation (F_y). Taking the yield strength to be 275 MPa, and employment of Equation 6.1 leads to

$$F_y = \sigma_y A_0 = (275 \times 10^6 \text{ N/m}^2)(325 \times 10^{-6} \text{ m}^2)$$

$$= 89{,}375 \text{ N} \quad (20{,}000 \text{ lb}_f)$$

(b) The maximum length to which the sample may be deformed without plastic deformation is determined from Equations 6.2 and 6.5 as

$$l_i = l_0\left(1 + \frac{\sigma}{E}\right)$$

$$= (115 \text{ mm})\left[1 + \frac{275 \text{ MPa}}{115 \times 10^3 \text{ MPa}}\right] = 115.28 \text{ mm} \quad (4.51 \text{ in.})$$

6.8 A cylindrical rod of copper (E = 110 GPa, 16×10^6 psi) having a yield strength of 240 MPa (35,000 psi) is to be subjected to a load of 6660 N (1500 lb_f). If the length of the rod is 380 mm (15.0 in.), what must be the diameter to allow an elongation of 0.50 mm (0.020 in.)?

Solution

This problem asks us to compute the diameter of a cylindrical specimen of copper in order to allow an elongation of 0.50 mm. Employing Equations 6.1, 6.2, and 6.5, assuming that deformation is entirely elastic

$$\sigma = \frac{F}{A_0} = \frac{F}{\pi\left(\frac{d_0^2}{4}\right)} = E\,\frac{\Delta l}{l_0}$$

Or, solving for d_0

$$d_0 = \sqrt{\frac{4\,l_0 F}{\pi\,E\,\Delta l}}$$

$$= \sqrt{\frac{(4)\left(380 \times 10^{-3}\,\mathrm{m}\right)(6660\ \mathrm{N})}{(\pi)\left(110 \times 10^{9}\ \mathrm{N/m^2}\right)\left(0.50 \times 10^{-3}\,\mathrm{m}\right)}}$$

$$= 7.65 \times 10^{-3}\ \mathrm{m} = 7.65\ \mathrm{mm}\ \ (0.30\ \mathrm{in.})$$

6.9 Compute the elastic moduli for the following metal alloys, whose stress-strain behaviors may be observed in the "Tensile Tests" module of Virtual Materials Science and Engineering (VMSE): (a) titanium, (b) tempered steel, (c) aluminum, and (d) carbon steel. How do these values compare with those presented in Table 6.1 for the same metals?

Solution

The elastic modulus is the slope in the linear elastic region (Equation 6.10) as

$$E = \frac{\Delta\sigma}{\Delta\varepsilon} = \frac{\sigma_2 - \sigma_1}{\varepsilon_2 - \varepsilon_1}$$

Since stress-strain curves for all of the metals/alloys pass through the origin, we make take $\sigma_1 = 0$ and $\varepsilon_1 = 0$. Determinations of σ_2 and ε_2 are possible by moving the cursor to some arbitrary point in the linear region of the curve and then reading corresponding values in the "Stress" and "Strain" windows that are located below the plot.

(a) For the titanium alloy, we selected σ_2 = 404.2 MPa with its corresponding ε_2 = 0.0038. Therefore,

$$E = \frac{\sigma_2 - \sigma_1}{\varepsilon_2 - \varepsilon_1} = \frac{404.2 \text{ MPa} - 0 \text{ MPa}}{0.0038 - 0} = 106{,}400 \text{ MPa} = 106.4 \text{ GPa}$$

The elastic modulus for titanium given in Table 6.1 is 107 GPa, which is in very good agreement with this value.

(b) For the tempered steel, we selected σ_2 = 962.2 MPa with its corresponding ε_2 = 0.0047. Therefore,

$$E = \frac{\sigma_2 - \sigma_1}{\varepsilon_2 - \varepsilon_1} = \frac{962.2 \text{ MPa} - 0 \text{ MPa}}{0.0047 - 0} = 204{,}700 \text{ MPa} = 204.7 \text{ GPa}$$

The elastic modulus for steel given in Table 6.1 is 207 GPa, which is in reasonably good agreement with this value.

(c) For the aluminum, we selected σ_2 = 145.1 MPa with its corresponding ε_2 = 0.0021. Therefore,

$$E = \frac{\sigma_2 - \sigma_1}{\varepsilon_2 - \varepsilon_1} = \frac{145.1 \text{ MPa} - 0 \text{ MPa}}{0.0021 - 0} = 69{,}100 \text{ MPa} = 69.1 \text{ GPa}$$

The elastic modulus for aluminum given in Table 6.1 is 69 GPa, which is in excellent agreement with this value.

(d) For the carbon steel, we selected σ_2 = 129 MPa with its corresponding ε_2 = 0.0006. Therefore,

$$E = \frac{\sigma_2 - \sigma_1}{\varepsilon_2 - \varepsilon_1} = \frac{129 \text{ MPa} - 0 \text{ MPa}}{0.0006 - 0} = 215{,}000 \text{ MPa} = 215 \text{ GPa}$$

The elastic modulus for steel given in Table 6.1 is 207 GPa, which is in reasonable agreement with this value.

6.10 Consider a cylindrical specimen of a steel alloy (Figure 6.21) 10.0 mm (0.39 in.) in diameter and 75 mm (3.0 in.) long that is pulled in tension. Determine its elongation when a load of 20,000 N (4,500 lb_f) is applied.

Solution

This problem asks that we calculate the elongation Δl of a specimen of steel the stress-strain behavior of which is shown in Figure 6.21. First it becomes necessary to compute the stress when a load of 20,000 N is applied using Equation 6.1 as

$$\sigma = \frac{F}{A_0} = \frac{F}{\pi\left(\frac{d_0}{2}\right)^2} = \frac{20,000 \text{ N}}{\pi\left(\frac{10.0 \times 10^{-3} \text{ m}}{2}\right)^2} = 255 \text{ MPa} \quad (37,700 \text{ psi})$$

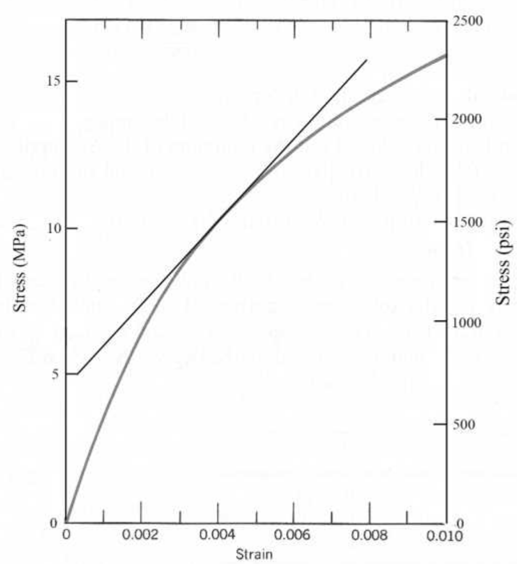
Referring to Figure 6.21, at this stress level we are in the elastic region on the stress-strain curve, which corresponds to a strain of 0.0012. Now, utilization of Equation 6.2 to compute the value of Δl

$$\Delta l = \varepsilon l_0 = (0.0012)(75 \text{mm}) = 0.090 \text{ mm} \ (0.0036 \text{ in.})$$

6.11 Figure 6.22 shows, for a gray cast iron, the tensile engineering stress–strain curve in the elastic region. Determine (a) the tangent modulus at 10.3 MPa (1500 psi), and (b) the secant modulus taken to 6.9 MPa (1000 psi).

Solution

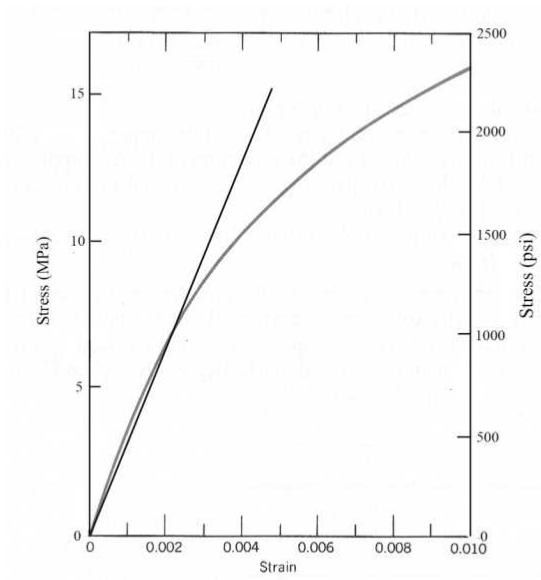
(a) This portion of the problem asks that the tangent modulus be determined for the gray cast iron, the stress-strain behavior of which is shown in Figure 6.22. In the figure below is shown a tangent draw on the curve at a stress of 10.3 MPa (1500 psi).



The slope of this line (i.e., $\Delta\sigma/\Delta\varepsilon$), the tangent modulus, is computed as follows:

$$\frac{\Delta\sigma}{\Delta\varepsilon} = \frac{15\text{ MPa} - 5\text{ MPa}}{0.0074 - 0.0003} = 1410\text{ MPa} = 1.41\text{ GPa}\ (2.04 \times 10^5\text{ psi})$$

(b) The secant modulus taken from the origin is calculated by taking the slope of a secant drawn from the origin through the stress-strain curve at 6.9 MPa (1,000 psi). This secant is drawn on the curve shown below:



The slope of this line (i.e., $\Delta\sigma/\Delta\varepsilon$), the secant modulus, is computed as follows:

$$\frac{\Delta\sigma}{\Delta\varepsilon} = \frac{15\text{ MPa} - 0\text{ MPa}}{0.0047 - 0} = 3190\text{ MPa} = 3.19\text{ GPa} \quad (4.63 \times 10^5\text{ psi})$$

6.12 As noted in Section 3.15, for single crystals of some substances, the physical properties are anisotropic; that is, they are dependent on crystallographic direction. One such property is the modulus of elasticity. For cubic single crystals, the modulus of elasticity in a general [uvw] direction, E_{uvw}, is described by the relationship

$$\frac{1}{E_{uvw}} = \frac{1}{E_{\langle 100\rangle}} - 3\left(\frac{1}{E_{\langle 100\rangle}} - \frac{1}{E_{\langle 111\rangle}}\right)\left(\alpha^2\beta^2 + \beta^2\gamma^2 + \gamma^2\alpha^2\right)$$

where $E_{\langle 100\rangle}$ and $E_{\langle 111\rangle}$ are the moduli of elasticity in [100] and [111] directions, respectively; α, β, and γ are the cosines of the angles between [uvw] and the respective [100], [010], and [001] directions. Verify that the $E_{\langle 110\rangle}$ values for aluminum, copper, and iron in Table 3.3 are correct.

<u>Solution</u>

We are asked, using the equation given in the problem statement, to verify that the modulus of elasticity values along [110] directions given in Table 3.3 for aluminum, copper, and iron are correct. The α, β, and γ parameters in the equation correspond, respectively, to the cosines of the angles between the [110] direction and [100], [010] and [001] directions. Since these angles are 45°, 45°, and 90°, the values of α, β, and γ are 0.707, 0.707, and 0, respectively. Thus, the given equation takes the form

$$\frac{1}{E_{<110>}}$$

$$= \frac{1}{E_{<100>}} - 3\left(\frac{1}{E_{<100>}} - \frac{1}{E_{<111>}}\right)\left[(0.707)^2(0.707)^2 + (0.707)^2(0)^2 + (0)^2(0.707)^2\right]$$

$$= \frac{1}{E_{<100>}} - (0.75)\left(\frac{1}{E_{<100>}} - \frac{1}{E_{<111>}}\right)$$

Utilizing the values of $E_{<100>}$ and $E_{<111>}$ from Table 3.3 for Al

$$\frac{1}{E_{<110>}} = \frac{1}{63.7\ \text{GPa}} - (0.75)\left[\frac{1}{63.7\ \text{GPa}} - \frac{1}{76.1\ \text{GPa}}\right]$$

Which leads to, $E_{<110>}$ = 72.6 GPa, the value cited in the table.

For Cu,

$$\frac{1}{E_{<110>}} = \frac{1}{66.7\ \text{GPa}} - (0.75)\left[\frac{1}{66.7\ \text{GPa}} - \frac{1}{191.1\ \text{GPa}}\right]$$

Thus, $E_{<110>}$ = 130.3 GPa, which is also the value cited in the table.

Similarly, for Fe

$$\frac{1}{E_{<110>}} = \frac{1}{125.0\ \text{GPa}} - (0.75)\left[\frac{1}{125.0\ \text{GPa}} - \frac{1}{272.7\ \text{GPa}}\right]$$

And $E_{<110>}$ = 210.5 GPa, which is also the value given in the table.

6.13 In Section 2.6 it was noted that the net bonding energy E_N between two isolated positive and negative ions is a function of interionic distance r as follows:

$$E_N = -\frac{A}{r} + \frac{B}{r^n} \tag{6.25}$$

where A, B, and n are constants for the particular ion pair. Equation 6.25 is also valid for the bonding energy between adjacent ions in solid materials. The modulus of elasticity E is proportional to the slope of the interionic force–separation curve at the equilibrium interionic separation; that is,

$$E \propto \left(\frac{dF}{dr}\right)_{r_o}$$

Derive an expression for the dependence of the modulus of elasticity on these A, B, and n parameters (for the two-ion system) using the following procedure:

1. Establish a relationship for the force F as a function of r, realizing that

$$F = \frac{dE_N}{dr}$$

2. Now take the derivative dF/dr.

3. Develop an expression for r_0, the equilibrium separation. Since r_0 corresponds to the value of r at the minimum of the E_N-versus-r curve (Figure 2.8b), take the derivative dE_N/dr, set it equal to zero, and solve for r, which corresponds to r_0.

4. Finally, substitute this expression for r_0 into the relationship obtained by taking dF/dr.

Solution

This problem asks that we derive an expression for the dependence of the modulus of elasticity, E, on the parameters A, B, and n in Equation 6.25. It is first necessary to take dE_N/dr in order to obtain an expression for the force F; this is accomplished as follows:

$$F = \frac{dE_N}{dr} = \frac{d\left(-\frac{A}{r}\right)}{dr} + \frac{d\left(\frac{B}{r^n}\right)}{dr}$$

$$= \frac{A}{r^2} - \frac{nB}{r^{(n+1)}}$$

The second step is to set this dE_N/dr expression equal to zero and then solve for r (= r_0). The algebra for this procedure is carried out in Problem 2.14, with the result that

$$r_0 = \left(\frac{A}{nB}\right)^{1/(1 - n)}$$

Next it becomes necessary to take the derivative of the force (dF/dr), which is accomplished as follows:

$$\frac{dF}{dr} = \frac{d\left(\frac{A}{r^2}\right)}{dr} + \frac{d\left(-\frac{nB}{r^{(n+1)}}\right)}{dr}$$

$$= -\frac{2A}{r^3} + \frac{(n)(n + 1)B}{r^{(n+2)}}$$

Now, substitution of the above expression for r_0 into this equation yields

$$\left(\frac{dF}{dr}\right)_{r_0} = -\frac{2A}{\left(\frac{A}{nB}\right)^{3/(1-n)}} + \frac{(n)(n + 1)B}{\left(\frac{A}{nB}\right)^{(n+2)/(1-n)}}$$

which is the expression to which the modulus of elasticity is proportional.

6.14 Using the solution to Problem 6.13, rank the magnitudes of the moduli of elasticity for the following hypothetical X, Y, and Z materials from the greatest to the least. The appropriate A, B, and n parameters (Equation 6.25) for these three materials are tabulated below; they yield E_N in units of electron volts and r in nanometers:

Material	**A**	**B**	**n**
X	2.5	2.0×10^{-5}	8
Y	2.3	8.0×10^{-6}	10.5
Z	3.0	1.5×10^{-5}	9

Solution

This problem asks that we rank the magnitudes of the moduli of elasticity of the three hypothetical metals X, Y, and Z. From Problem 6.13, it was shown for materials in which the bonding energy is dependent on the interatomic distance r according to Equation 6.25, that the modulus of elasticity E is proportional to

$$E \propto -\frac{2A}{\left(\dfrac{A}{nB}\right)^{3/(1-n)}} + \frac{(n)(n+1)B}{\left(\dfrac{A}{nB}\right)^{(n+2)/(1-n)}}$$

For metal X, A = 2.5, B = 2.0 × 10^{-5}, and n = 8. Therefore,

$$E \propto -\frac{(2)(2.5)}{\left[\dfrac{2.5}{(8)\left(2 \times 10^{-5}\right)}\right]^{3/(1-8)}} + \frac{(8)(8+1)\left(2 \times 10^{-5}\right)}{\left[\dfrac{2.5}{(8)\left(2 \times 10^{-5}\right)}\right]^{(8+2)/(1-8)}}$$

$$= 1097$$

For metal Y, A = 2.3, B = 8 × 10^{-6}, and n = 10.5. Hence

$$E \propto -\frac{(2)(2.3)}{\left[\dfrac{2.3}{(10.5)\left(8 \times 10^{-6}\right)}\right]^{3/(1-10.5)}} + \frac{(10.5)(10.5+1)\left(8 \times 10^{-6}\right)}{\left[\dfrac{2.3}{(10.5)\left(8 \times 10^{-6}\right)}\right]^{(10.5+2)/(1-10.5)}}$$

$$= 551$$

And, for metal Z, A = 3.0, B = 1.5 × 10^{-5}, and n = 9. Thus

$$E \propto -\frac{(2)(3.0)}{\left[\dfrac{3.0}{(9)\left(1.5 \times 10^{-5}\right)}\right]^{3/(1-9)}} + \frac{(9)(9+1)\left(1.5 \times 10^{-5}\right)}{\left[\dfrac{3.0}{(9)\left(1.5 \times 10^{-5}\right)}\right]^{(9+2)/(1-9)}}$$

$$= 1024$$

Therefore, metal X has the highest modulus of elasticity.

Elastic Properties of Materials

6.15 A cylindrical specimen of aluminum having a diameter of 19 mm (0.75 in.) and length of 200 mm (8.0 in.) is deformed elastically in tension with a force of 48,800 N (11,000 lb_f). Using the data contained in Table 6.1, determine the following:

(a) The amount by which this specimen will elongate in the direction of the applied stress.

(b) The change in diameter of the specimen. Will the diameter increase or decrease?

Solution

(a) We are asked, in this portion of the problem, to determine the elongation of a cylindrical specimen of aluminum. Combining Equations 6.1, 6.2, and 6.5, leads to

$$\sigma = E\varepsilon$$

$$\frac{F}{\pi\left(\frac{d_0^2}{4}\right)} = E\,\frac{\Delta l}{l_0}$$

Or, solving for Δl (and realizing that E = 69 GPa, Table 6.1), yields

$$\Delta l = \frac{4F\,l_0}{\pi\,d_0^2 E}$$

$$= \frac{(4)(48{,}800\ \text{N})\left(200 \times 10^{-3}\,\text{m}\right)}{(\pi)\left(19 \times 10^{-3}\,\text{m}\right)^2\left(69 \times 10^9\,\text{N/m}^2\right)} = 5 \times 10^{-4}\ \text{m} = 0.50\ \text{mm}\ \ (0.02\ \text{in.})$$

(b) We are now called upon to determine the change in diameter, Δd. Using Equation 6.8

$$\nu = -\frac{\varepsilon_x}{\varepsilon_z} = -\frac{\Delta d/d_0}{\Delta l/l_0}$$

From Table 6.1, for aluminum, $\nu = 0.33$. Now, solving the above expression for Δd yields

$$\Delta d = -\frac{\nu\,\Delta l\,d_0}{l_0} = -\frac{(0.33)(0.50\ \text{mm})(19\ \text{mm})}{200\ \text{mm}}$$

$$= -1.6 \times 10^{-2}\ \text{mm}\ \ (-6.2 \times 10^{-4}\ \text{in.})$$

The diameter will decrease.

6.16 A cylindrical bar of steel 10 mm (0.4 in.) in diameter is to be deformed elastically by application of a force along the bar axis. Using the data in Table 6.1, determine the force that will produce an elastic reduction of 3×10^{-3} mm (1.2×10^{-4} in.) in the diameter.

Solution

This problem asks that we calculate the force necessary to produce a reduction in diameter of 3×10^{-3} mm for a cylindrical bar of steel. For a cylindrical specimen, the cross-sectional area is equal to

$$A_0 = \frac{\pi d_0^2}{4}$$

Now, combining Equations 6.1 and 6.5 leads to

$$\sigma = \frac{F}{A_0} = \frac{F}{\dfrac{\pi d_0^2}{4}} = E\varepsilon_z$$

And, since from Equation 6.8

$$\varepsilon_z = -\frac{\varepsilon_x}{\nu} = -\frac{\dfrac{\Delta d}{d_0}}{\nu} = -\frac{\Delta d}{\nu d_0}$$

Substitution of this equation into the above expression gives

$$\frac{F}{\dfrac{\pi d_0^2}{4}} = E\left(-\frac{\Delta d}{\nu d_0}\right)$$

And, solving for F leads to

$$F = -\frac{d_0 \Delta d\, \pi E}{4\nu}$$

From Table 6.1, for steel, $\nu = 0.30$ and $E = 207$ GPa. Thus,

$$F = -\frac{\left(10 \times 10^{-3}\,\mathrm{m}\right)\left(-3.0 \times 10^{-6}\,\mathrm{m}\right)(\pi)\left(207 \times 10^{9}\ \mathrm{N/m^2}\right)}{(4)(0.30)}$$

$$= 16{,}250\ \mathrm{N}\ \ (3770\ \mathrm{lb_f})$$

6.17 A cylindrical specimen of some alloy 8 mm (0.31 in.) in diameter is stressed elastically in tension. A force of 15,700 N (3530 lb_f) produces a reduction in specimen diameter of 5×10^{-3} mm (2×10^{-4} in.). Compute Poisson's ratio for this material if its modulus of elasticity is 140 GPa (20.3×10^6 psi).

Solution

This problem asks that we compute Poisson's ratio for the metal alloy. From Equations 6.5 and 6.1

$$\varepsilon_z = \frac{\sigma}{E} = \frac{F}{A_0 E} = \frac{F}{\pi \left(\frac{d_0}{2}\right)^2 E} = \frac{4F}{\pi d_0^2 E}$$

Since the transverse strain ε_x is just

$$\varepsilon_x = \frac{\Delta d}{d_0}$$

and Poisson's ratio is defined by Equation 6.8, then

$$\nu = -\frac{\varepsilon_x}{\varepsilon_z} = -\frac{\Delta d / d_0}{\left(\frac{4F}{\pi d_0^2 E}\right)} = -\frac{d_0 \Delta d \, \pi E}{4F}$$

$$= -\frac{(8 \times 10^{-3}\ \text{m})(-5 \times 10^{-6}\ \text{m})\,(\pi)(140 \times 10^9\ \text{N/m}^2)}{(4)(15,700\ \ \text{N})} = 0.280$$

6.18 A cylindrical specimen of a hypothetical metal alloy is stressed in compression. If its original and final diameters are 20.000 and 20.025 mm, respectively, and its final length is 74.96 mm, compute its original length if the deformation is totally elastic. The elastic and shear moduli for this alloy are 105 GPa and 39.7 GPa, respectively.

Solution

This problem asks that we compute the original length of a cylindrical specimen that is stressed in compression. It is first convenient to compute the lateral strain ε_x as

$$\varepsilon_x = \frac{\Delta d}{d_0} = \frac{20.025 \text{ mm} - 20.000 \text{ mm}}{20.000 \text{ mm}} = 1.25 \times 10^{-3}$$

In order to determine the longitudinal strain ε_z ZH QHHG 3RLVVRQV UDWLR ZKLFK PD EH FRPSXWHG XVLQJ (TXDWLRQ

solving for ν yields

$$\nu = \frac{E}{2G} - 1 = \frac{105 \times 10^3 \text{ MPa}}{(2)\left(39.7 \times 10^3 \text{ MPa}\right)} - 1 = 0.322$$

Now ε_z may be computed from Equation 6.8 as

$$\varepsilon_z = -\frac{\varepsilon_x}{\nu} = -\frac{1.25 \times 10^{-3}}{0.322} = -3.88 \times 10^{-3}$$

Now solving for l_0 using Equation 6.2

$$l_0 = \frac{l_i}{1 + \varepsilon_z}$$

$$= \frac{74.96 \text{ mm}}{1 - 3.88 \times 10^{-3}} = 75.25 \text{ mm}$$

6.19 Consider a cylindrical specimen of some hypothetical metal alloy that has a diameter of 8.0 mm (0.31 in.). A tensile force of 1000 N (225 lb_f) produces an elastic reduction in diameter of 2.8×10^{-4} mm (1.10×10^{-5} in.). Compute the modulus of elasticity for this alloy, given that Poisson's ratio is 0.30.

Solution

This problem asks that we calculate the modulus of elasticity of a metal that is stressed in tension. Combining Equations 6.5 and 6.1 leads to

$$E = \frac{\sigma}{\varepsilon_z} = \frac{F}{A_0 \varepsilon_z} = \frac{F}{\varepsilon_z \pi \left(\frac{d_0}{2}\right)^2} = \frac{4F}{\varepsilon_z \pi d_0^2}$$

From the definition of Poisson's ratio, (Equation 6.8) and realizing that for the transverse strain, $\varepsilon_x = \frac{\Delta d}{d_0}$

$$\varepsilon_z = -\frac{\varepsilon_x}{\nu} = -\frac{\Delta d}{d_0 \nu}$$

Therefore, substitution of this expression for ε_z into the above equation yields

$$E = \frac{4F}{\varepsilon_z \pi d_0^2} = \frac{4F\nu}{\pi d_0 \Delta d}$$

$$= \frac{(4)(1000\text{ N})(0.30)}{\pi \left(8 \times 10^{-3}\text{ m}\right)\left(2.8 \times 10^{-7}\text{ m}\right)} = 1.705 \times 10^{11}\text{ Pa} = 170.5\text{ GPa}\ (24.7 \times 10^6\text{ psi})$$

6.20 A brass alloy is known to have a yield strength of 275 MPa (40,000 psi), a tensile strength of 380 MPa (55,000 psi), and an elastic modulus of 103 GPa (15.0×10^6 psi). A cylindrical specimen of this alloy 12.7 mm (0.50 in.) in diameter and 250 mm (10.0 in.) long is stressed in tension and found to elongate 7.6 mm (0.30 in.). On the basis of the information given, is it possible to compute the magnitude of the load that is necessary to produce this change in length? If so, calculate the load. If not, explain why.

Solution

We are asked to ascertain whether or not it is possible to compute, for brass, the magnitude of the load necessary to produce an elongation of 7.6 mm (0.30 in.). It is first necessary to compute the strain at yielding from the yield strength and the elastic modulus, and then the strain experienced by the test specimen. Then, if

$$\varepsilon(\text{test}) < \varepsilon(\text{yield})$$

deformation is elastic, and the load may be computed using Equations 6.1 and 6.5. However, if

$$\varepsilon(\text{test}) > \varepsilon(\text{yield})$$

computation of the load is not possible inasmuch as deformation is plastic and we have neither a stress-strain plot nor a mathematical expression relating plastic stress and strain. We compute these two strain values as

$$\varepsilon(\text{test}) = \frac{\Delta l}{l_0} = \frac{7.6 \text{ mm}}{250 \text{ mm}} = 0.03$$

and

$$\varepsilon(\text{yield}) = \frac{\sigma_y}{E} = \frac{275 \text{ MPa}}{103 \times 10^3 \text{ MPa}} = 0.0027$$

Therefore, computation of the load is not possible since $\varepsilon(\text{test}) > \varepsilon(\text{yield})$.

6.21 A cylindrical metal specimen 12.7 mm (0.5 in.) in diameter and 250 mm (10 in.) long is to be VXEMHFWHG WR D WHQVLOH VWUHVV RI 0 3D SVL DWKLV VWUHVV OHYHO WKH UHVXOWLQJ GHIRUPDWLRQ ZLOO EH WRWDOO elastic.

(a) If the elongation must be less than 0.080 mm (3.2×10^{-3} in.), which of the metals in Table 6.1 are suitable candidates? Why?

(b) If, in addition, the maximum permissible diameter decrease is 1.2×10^{-3} mm (4.7×10^{-5} in.) when the tensile stress of 28 MPa is applied, which of the metals that satisfy the criterion in part (a) are suitable candidates? Why?

Solution

(a) This part of the problem asks that we ascertain which of the metals in Table 6.1 experience an elongation of less than 0.080 mm when subjected to a tensile stress of 28 MPa. The maximum strain that may be sustained, (using Equation 6.2) is just

$$\varepsilon = \frac{\Delta l}{l_0} = \frac{0.080 \text{ mm}}{250 \text{ mm}} = 3.2 \times 10^{-4}$$

Since the stress level is given (50 MPa), using Equation 6.5 it is possible to compute the minimum modulus of elasticity which is required to yield this minimum strain. Hence

$$E = \frac{\sigma}{\varepsilon} = \frac{28 \text{ MPa}}{3.2 \times 10^{-4}} = 87.5 \text{ GPa}$$

Which means that those metals with moduli of elasticity greater than this value are acceptable candidates—namely, brass, Cu, Ni, steel, Ti and W.

(b) This portion of the problem further stipulates that the maximum permissible diameter decrease is 1.2×10^{-3} mm when the tensile stress of 28 MPa is applied. This translates into a maximum lateral strain $\varepsilon_x(\text{max})$ as

$$\varepsilon_x(\text{max}) = \frac{\Delta d}{d_0} = \frac{-1.2 \times 10^{-3} \text{ mm}}{12.7 \text{ mm}} = -9.45 \times 10^{-5}$$

But, since the specimen contracts in this lateral direction, and we are concerned that this strain be less than 9.45×10^{-5}, then the criterion for this part of the problem may be stipulated as $-\frac{\Delta d}{d_0} < 9.45 \times 10^{-5}$.

Now, Poisson's ratio is defined by Equation 6.8 as

$$\nu = -\frac{\varepsilon_x}{\varepsilon_z}$$

For each of the metal alloys let us consider a possible lateral strain, $\varepsilon_x = \frac{\Delta d}{d_0}$. Furthermore, since the deformation is elastic, then, from Equation 6.5, the longitudinal strain, ε_z is equal to

$$\varepsilon_z = \frac{\sigma}{E}$$

Substituting these expressions for ε_x and ε_z into the definition of Poisson's ratio we have

$$\nu = -\frac{\varepsilon_x}{\varepsilon_z} = -\frac{\frac{\Delta d}{d_0}}{\frac{\sigma}{E}}$$

which leads to the following:

$$-\frac{\Delta d}{d_0} = \frac{\nu\sigma}{E}$$

Using values for ν and E found in Table 6.1 for the six metal alloys that satisfy the criterion for part (a), and for $\sigma = 28$ MPa, we are able to compute a $-\frac{\Delta d}{d_0}$ for each alloy as follows:

$$-\frac{\Delta d}{d_0}(\text{brass}) = \frac{(0.34)(28 \times 10^6 \text{ N/m}^2)}{97 \times 10^9 \text{ N/m}^2} = 9.81 \times 10^{-5}$$

$$-\frac{\Delta d}{d_0}(\text{copper}) = \frac{(0.34)(28 \times 10^6 \text{ N/m}^2)}{110 \times 10^9 \text{ N/m}^2} = 8.65 \times 10^{-5}$$

$$-\frac{\Delta d}{d_0}(\text{titanium}) = \frac{(0.34)(28 \times 10^6 \text{ N/m}^2)}{107 \times 10^9 \text{ N/m}^2} = 8.90 \times 10^{-5}$$

$$-\frac{\Delta d}{d_0}(\text{nickel}) = \frac{(0.31)(28 \times 10^6 \text{ N/m}^2)}{207 \times 10^9 \text{ N/m}^2} = 4.19 \times 10^{-5}$$

$$-\frac{\Delta d}{d_0}(\text{steel}) = \frac{(0.30)(28 \times 10^6 \text{ N/m}^2)}{207 \times 10^9 \text{ N/m}^2} = 4.06 \times 10^{-5}$$

$$-\frac{\Delta d}{d_0}(\text{tungsten}) = \frac{(0.28)(28 \times 10^6 \text{ N/m}^2)}{407 \times 10^9 \text{ N/m}^2} = 1.93 \times 10^{-5}$$

Thus, of the above six alloys, only brass will have a negative transverse strain that is greater than 9.45×10^{-5}. This means that the following alloys satisfy the criteria for both parts (a) and (b) of the problem: copper, titanium, nickel, steel, and tungsten.

6.22 Consider the brass alloy for which the stress-strain behavior is shown in Figure 6.12. A cylindrical specimen of this material 6 mm (0.24 in.) in diameter and 50 mm (2 in.) long is pulled in tension with a force of 5000 N (1125 lb_f). If it is known that this alloy has a Poisson's ratio of 0.30, compute: (a) the specimen elongation, and (b) the reduction in specimen diameter.

Solution

(a) This portion of the problem asks that we compute the elongation of the brass specimen. The first calculation necessary is that of the applied stress using Equation 6.1, as

$$\sigma = \frac{F}{A_0} = \frac{F}{\pi\left(\frac{d_0}{2}\right)^2} = \frac{5000 \text{ N}}{\pi\left(\frac{6 \times 10^{-3}\text{ m}}{2}\right)^2} = 177 \times 10^6 \text{ N/m}^2 = 177 \text{ MPa} \quad (25,000 \text{ psi})$$

From the stress-strain plot in Figure 6.12, this stress corresponds to a strain of about 2.0×10^{-3}. From the definition of strain, Equation 6.2

$$\Delta l = \varepsilon l_0 = (2.0 \times 10^{-3})(50 \text{ mm}) = 0.10 \text{ mm} \quad (4 \times 10^{-3} \text{ in.})$$

(b) In order to determine the reduction in diameter Δd, it is necessary to use Equation 6.8 and the definition of lateral strain (i.e., $\varepsilon_x = \Delta d/d_0$) as follows

$$\Delta d = d_0 \varepsilon_x = -d_0 \nu \varepsilon_z = -(6 \text{ mm})(0.30)(2.0 \times 10^{-3})$$

$$= -3.6 \times 10^{-3} \text{ mm} \quad (-1.4 \times 10^{-4} \text{ in.})$$

6.23 A cylindrical rod 100 mm long and having a diameter of 10.0 mm is to be deformed using a tensile load of 27,500 N. It must not experience either plastic deformation or a diameter reduction of more than 7.5×10^{-3} mm. Of the materials listed as follows, which are possible candidates? Justify your choice(s).

Material	**Modulus of Elasticity (GPa)**	**Yield Strength (MPa)**	**Poisson's Ratio**
Aluminum alloy	70	200	0.33
Brass alloy	101	300	0.34
Steel alloy	207	400	0.30
Titanium alloy	107	650	0.34

Solution

This problem asks that we assess the four alloys relative to the two criteria presented. The first criterion is that the material not experience plastic deformation when the tensile load of 27,500 N is applied; this means that the stress corresponding to this load not exceed the yield strength of the material. Upon computing the stress

$$\sigma = \frac{F}{A_0} = \frac{F}{\pi\left(\frac{d_0}{2}\right)^2} = \frac{27{,}500 \text{ N}}{\pi\left(\frac{10 \times 10^{-3}\text{ m}}{2}\right)^2} = 350 \times 10^6 \text{ N/m}^2 = 350 \text{ MPa}$$

Of the alloys listed, the Ti and steel alloys have yield strengths greater than 350 MPa.

Relative to the second criterion (i.e., that Δd be less than 7.5×10^{-3} mm), it is necessary to calculate the change in diameter Δd for these three alloys. From Equation 6.8

$$\nu = -\frac{\varepsilon_x}{\varepsilon_z} = -\frac{\frac{\Delta d}{d_0}}{\frac{\sigma}{E}} = -\frac{E\,\Delta d}{\sigma\, d_0}$$

Now, solving for Δd from this expression,

$$\Delta d = -\frac{\nu\,\sigma\, d_0}{E}$$

For the steel alloy

$$\Delta d = -\frac{(0.30)(350 \text{ MPa})(10 \text{ mm})}{207 \times 10^3 \text{ MPa}} = -5.1 \times 10^{-3} \text{ mm}$$

Therefore, the steel is a candidate.

For the Ti alloy

$$\Delta d = -\frac{(0.34)(350 \text{ MPa})(10 \text{ mm})}{107 \times 10^3 \text{ MPa}} = -11.1 \times 10^{-3} \text{ mm}$$

Hence, the titanium alloy is not a candidate.

6.24 A cylindrical rod 380 mm (15.0 in.) long, having a diameter of 10.0 mm (0.40 in.), is to be subjected to a tensile load. If the rod is to experience neither plastic deformation nor an elongation of more than 0.9 mm (0.035 in.) when the applied load is 24,500 N (5500 lb_f), which of the four metals or alloys listed below are possible candidates? Justify your choice(s).

Material	Modulus of Elasticity (GPa)	Yield Strength (MPa)	Tensile Strength (MPa)
Aluminum alloy	70	255	420
Brass alloy	100	345	420
Copper	110	250	290
Steel alloy	207	450	550

Solution

This problem asks that we ascertain which of four metal alloys will not (1) experience plastic deformation, and (2) elongate more than 0.9 mm when a tensile load of 24,500 N is applied. It is first necessary to compute the VWUHVV XVLQJ (TXDWLRQ ¶ DP DWHULDO WR EH Xsed for this application must necessarily have a yield strength greater than this value. Thus,

$$\sigma = \frac{F}{A_0} = \frac{24,500\ \text{N}}{\pi\left(\dfrac{10.0 \times 10^{-3}\ \text{m}}{2}\right)^2} = 312\ \text{MPa}$$

Of the metal alloys listed, only brass and steel have yield strengths greater than this stress.

Next, we must compute the elongation produced in both brass and steel using Equations 6.2 and 6.5 in order to determine whether or not this elongation is less than 0.9 mm. For brass

$$\Delta l = \frac{\sigma l_0}{E} = \frac{(312\ \text{MPa})(380\ \text{mm})}{100 \times 10^3\ \text{MPa}} = 1.19\ \text{mm}$$

Thus, brass is not a candidate. However, for steel

$$\Delta l = \frac{\sigma l_0}{E} = \frac{(312\ \text{MPa})(380\ \text{mm})}{207 \times 10^3\ \text{MPa}} = 0.57\ \text{mm}$$

Therefore, of these four alloys, only steel satisfies the stipulated criteria.

Tensile Properties

6.25 Figure 6.21 shows the tensile engineering stress–strain behavior for a steel alloy.

(a) What is the modulus of elasticity?

(b) What is the proportional limit?

(c) What is the yield strength at a strain offset of 0.002?

(d) What is the tensile strength?

Solution

Using the stress-strain plot for a steel alloy (Figure 6.21), we are asked to determine several of its mechanical characteristics.

(a) The elastic modulus is just the slope of the initial linear portion of the curve; or, from the inset and using Equation 6.10

$$E = \frac{\sigma_2 - \sigma_1}{\varepsilon_2 - \varepsilon_1} = \frac{(200 - 0)\ \text{MPa}}{(0.0010 - 0)} = 200 \times 10^3\ \text{MPa} = 200\ \text{GPa} \quad (29 \times 10^6\ \text{psi})$$

The value given in Table 6.1 is 207 GPa.

(b) The proportional limit is the stress level at which linearity of the stress-strain curve ends, which is approximately 300 MPa (43,500 psi).

(c) The 0.002 strain offset line intersects the stress-strain curve at approximately 400 MPa (58,000 psi).

(d) The tensile strength (the maximum on the curve) is approximately 515 MPa (74,700 psi).

6.26 A cylindrical specimen of a brass alloy having a length of 60 mm (2.36 in.) must elongate only 10.8 mm (0.425 in.) when a tensile load of 50,000 N (11,240 lb_f) is applied. Under these circumstances, what must be the radius of the specimen? Consider this brass alloy to have the stress-strain behavior shown in Figure 6.12.

Solution

We are asked to calculate the radius of a cylindrical brass specimen in order to produce an elongation of 10.8 mm when a load of 50,000 N is applied. It first becomes necessary to compute the strain corresponding to this elongation using Equation 6.2 as

$$\varepsilon = \frac{\Delta l}{l_0} = \frac{10.8 \text{ mm}}{60 \text{ mm}} = 0.18$$

From Figure 6.12, a stress of 420 MPa (61,000 psi) corresponds to this strain. Since for a cylindrical specimen, stress, force, and initial radius r_0 are related as

$$\sigma = \frac{F}{\pi r_0^2}$$

then

$$r_0 = \sqrt{\frac{F}{\pi \sigma}} = \sqrt{\frac{50,000 \text{ N}}{\pi \left(420 \times 10^6 \text{ N/m}^2\right)}} = 0.0062 \text{ m} = 6.2 \text{ mm} \quad (0.24 \text{ in.})$$

6.27 A load of 85,000 N (19,100 lb_f) is applied to a cylindrical specimen of a steel alloy (displaying the stress–strain behavior shown in Figure 6.21) that has a cross-sectional diameter of 15 mm (0.59 in.).

(a) Will the specimen experience elastic and/or plastic deformation? Why?

(b) If the original specimen length is 250 mm (10 in.), how much will it increase in length when this load is applied?

Solution

This problem asks us to determine the deformation characteristics of a steel specimen, the stress-strain behavior for which is shown in Figure 6.21.

(a) In order to ascertain whether the deformation is elastic or plastic, we must first compute the stress, then locate it on the stress-strain curve, and, finally, note whether this point is on the elastic or plastic region. Thus, from Equation 6.1

$$\sigma = \frac{F}{A_0} = \frac{85,000 \text{ N}}{\pi \left(\dfrac{15 \times 10^{-3} \text{ m}}{2}\right)^2} = 481 \times 10^6 \text{ N/m}^2 = 481 \text{ MPa} \quad (69,900 \text{ psi})$$

The 481 MPa point is beyond the linear portion of the curve, and, therefore, the deformation will be both elastic and plastic.

(b) This portion of the problem asks us to compute the increase in specimen length. From the stress-strain curve, the strain at 481 MPa is approximately 0.0135. Thus, from Equation 6.2

$$\Delta l = \varepsilon l_0 = (0.0135)(250 \text{ mm}) = 3.4 \text{ mm} \quad (0.135 \text{ in.})$$

6.28 A bar of a steel alloy that exhibits the stress-strain behavior shown in Figure 6.21 is subjected to a WHQVLOH ORDG WKH VSHFLPHQ LV ┼‼‼ PP ╚¶┴ LQ◀ ORQJ ╔DQG RI VTXDUH FURVV VHFWLRQ ╢◀ PP ╚‼◀¶→╯ LQ◀ RQ D VLGH◀

(a) Compute the magnitude of the load necessary to produce an elongation of 0.45 mm (0.018 in.).

(b) What will be the deformation after the load has been released?

Solution

(a) We are asked to compute the magnitude of the load necessary to produce an elongation of 0.45 mm for the steel displaying the stress-strain behavior shown in Figure 6.21. First, calculate the strain, and then the corresponding stress from the plot.

$$\varepsilon = \frac{\Delta l}{l_0} = \frac{0.45 \text{ mm}}{300 \text{ mm}} = 1.5 \times 10^{-3}$$

7KLV LV QHDU WKH HQG RI WKH HODVWLF UHJLRQ IURP WKH LQVHW RI)LJXUH ┼.21, this corresponds to a stress of about 300 MPa (43,500 psi). Now, from Equation 6.1

$$F = \sigma A_0 = \sigma b^2$$

in which b is the cross-section side length. Thus,

$$F = (300 \times 10^6 \text{ N/m}^2)(4.5 \times 10^{-3} \text{ m})^2 = 6075 \text{ N} \quad (1366 \text{ lb}_f)$$

(b) After the load is released there will be no deformation since the material was strained only elastically.

6.29 A cylindrical specimen of aluminum having a diameter of 0.505 in. (12.8 mm) and a gauge length of 2.000 in. (50.800 mm) is pulled in tension. Use the load–elongation characteristics tabulated below to complete parts (a) through (f).

Load		**Length**	
N	**lb_f**	**mm**	**in.**
0	0	50.800	2.000
7,330	1,650	50.851	2.002
15,100	3,400	50.902	2.004
23,100	5,200	50.952	2.006
30,400	6,850	51.003	2.008
34,400	7,750	51.054	2.010
38,400	8,650	51.308	2.020
41,300	9,300	51.816	2.040
44,800	10,100	52.832	2.080
46,200	10,400	53.848	2.120
47,300	10,650	54.864	2.160
47,500	10,700	55.880	2.200
46,100	10,400	56.896	2.240
44,800	10,100	57.658	2.270
42,600	9,600	58.420	2.300
36,400	8,200	59.182	2.330
Fracture			

(a) Plot the data as engineering stress versus engineering strain.

(b) Compute the modulus of elasticity.

(c) Determine the yield strength at a strain offset of 0.002.

(d) Determine the tensile strength of this alloy.

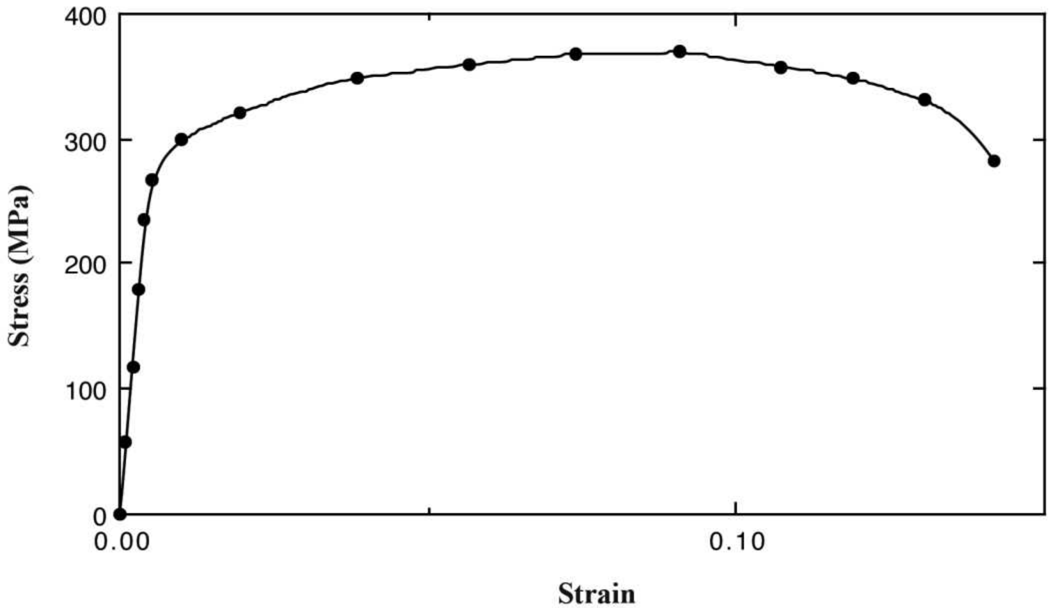
(e) What is the approximate ductility, in percent elongation?

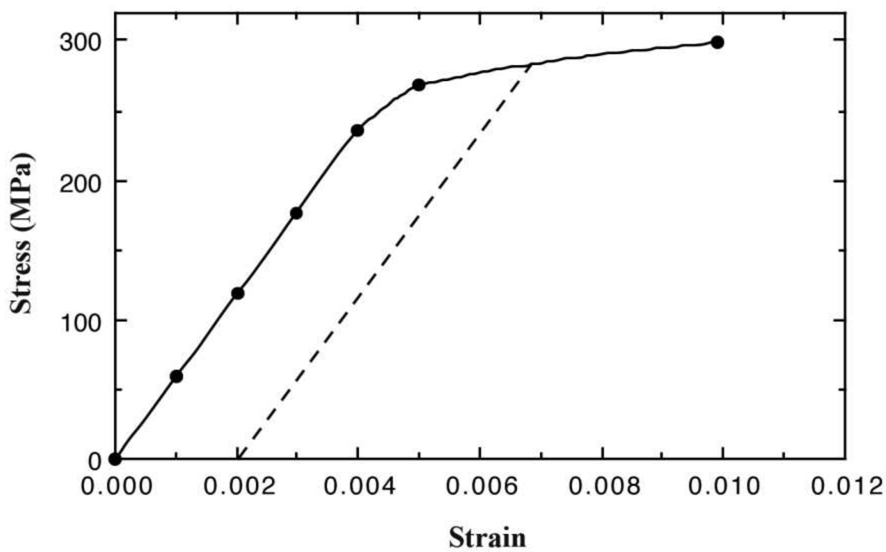
(f) Compute the modulus of resilience.

Solution

This problem calls for us to make a stress-strain plot for aluminum, given its tensile load-length data, and then to determine some of its mechanical characteristics.

(a) The data are plotted below on two plots: the first corresponds to the entire stress-strain curve, while for the second, the curve extends to just beyond the elastic region of deformation.





(b) The elastic modulus is the slope in the linear elastic region (Equation 6.10) as

$$E = \frac{\Delta\sigma}{\Delta\varepsilon} = \frac{200\text{ MPa} - 0\text{ MPa}}{0.0032 - 0} = 62.5 \times 10^3\text{ MPa} = 62.5\text{ GPa} \quad (9.1 \times 10^6\text{ psi})$$

(c) For the yield strength, the 0.002 strain offset line is drawn dashed. It intersects the stress-strain curve at approximately 285 MPa (41,000 psi).

(d) The tensile strength is approximately 370 MPa (54,000 psi), corresponding to the maximum stress on the complete stress-strain plot.

(e) The ductility, in percent elongation, is just the plastic strain at fracture, multiplied by one-hundred. The WRWDOIUDFWXUH VWUDLQ DWIUDFWXUH LV ‼◀¶├↑ VXEWUDFWLQJ RXW the elastic strain (which is about 0.005) leaves a plastic strain of 0.160. Thus, the ductility is about 16%EL.

(f) From Equation 6.14, the modulus of resilience is just

$$U_r = \frac{\sigma_y^2}{2E}$$

which, using data computed above gives a value of

$$U_r = \frac{(285\ \text{MPa})^2}{(2)\left(62.5 \times 10^3\ \text{MPa}\right)} = 0.65\ \text{MN/m}^2 = 0.65 \times 10^6\ \text{N/m}^2 = 6.5 \times 10^5\ \text{J/m}^3 \quad \left(93.8\ \text{in.-lb}_f/\text{in.}^3\right)$$

6.30 A specimen of ductile cast iron having a rectangular cross section of dimensions 4.8 mm × 15.9 mm (3/16 in. × 5/8 in.) is deformed in tension. Using the load-elongation data tabulated below, complete problems (a) through (f).

Load		**Length**	
N	**lb_f**	**mm**	**in.**
0	0	75.000	2.953
4,740	1,065	75.025	2.954
9,140	2,055	75.050	2.955
12,920	2,900	75.075	2.956
16,540	3,720	75.113	2.957
18,300	4,110	75.150	2.959
20,170	4,530	75.225	2.962
22,900	5,145	75.375	2.968
25,070	5,635	75.525	2.973
26,800	6,025	75.750	2.982
28,640	6,440	76.500	3.012
30,240	6,800	78.000	3.071
31,100	7,000	79.500	3.130
31,280	7,030	81.000	3.189
30,820	6,930	82.500	3.248
29,180	6,560	84.000	3.307
27,190	6,110	85.500	3.366
24,140	5,430	87.000	3.425
18,970	4,265	88.725	3.493
	Fracture		

(a) Plot the data as engineering stress versus engineering strain.

(b) Compute the modulus of elasticity.

(c) Determine the yield strength at a strain offset of 0.002.

(d) Determine the tensile strength of this alloy.

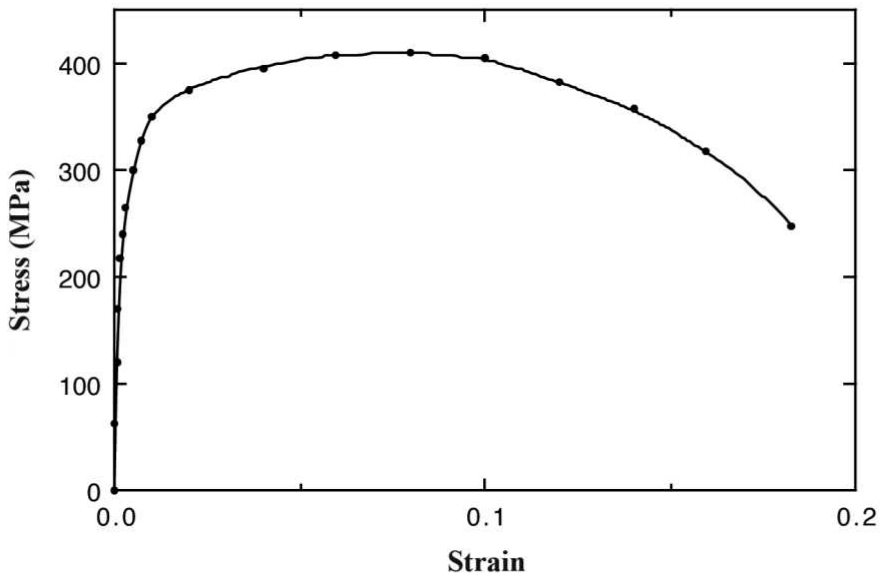
(e) Compute the modulus of resilience.

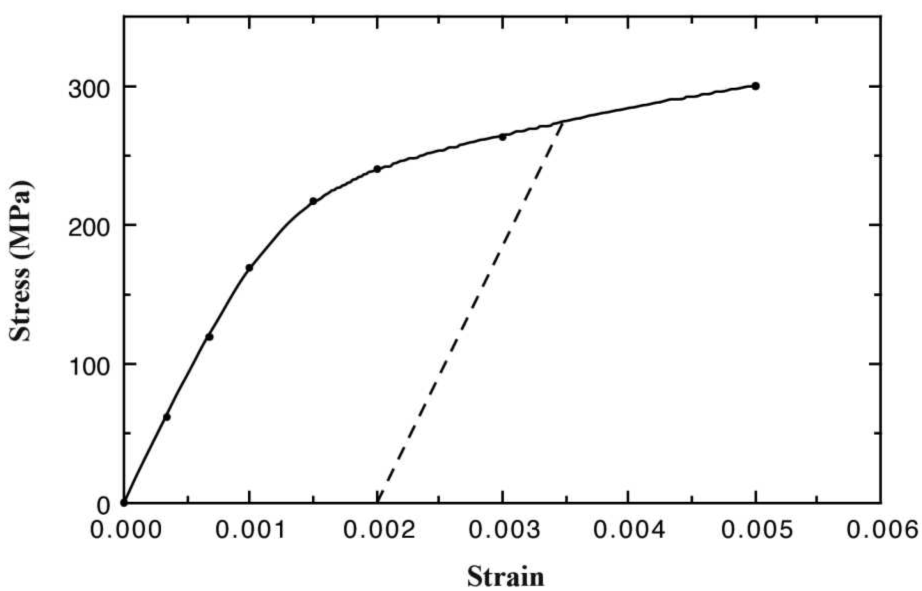
(f) What is the ductility, in percent elongation?

Solution

This problem calls for us to make a stress-strain plot for a ductile cast iron, given its tensile load-length data, and then to determine some of its mechanical characteristics.

(a) The data are plotted below on two plots: the first corresponds to the entire stress-strain curve, while for the second, the curve extends just beyond the elastic region of deformation.





(b) The elastic modulus is the slope in the linear elastic region (Equation 6.10) as

$$E = \frac{\Delta\sigma}{\Delta\varepsilon} = \frac{100\ \text{MPa} - 0\ \text{MPa}}{0.0005 - 0} = 200 \times 10^{3}\ \text{MPa} = 200\ \text{GPa} \qquad (29 \times 10^{6}\ \text{psi})$$

(c) For the yield strength, the 0.002 strain offset line is drawn dashed. It intersects the stress-strain curve at approximately 280 MPa (40,500 psi).

(d) The tensile strength is approximately 410 MPa (59,500 psi), corresponding to the maximum stress on the complete stress-strain plot.

(e) From Equation 6.14, the modulus of resilience is just

$$U_r = \frac{\sigma_y^2}{2E}$$

which, using data computed above, yields a value of

$$U_r = \frac{(280 \times 10^{6}\ \text{N/m}^2)^2}{(2)\left(200 \times 10^{9}\ \text{N/m}^2\right)} = 1.96 \times 10^{5}\ \text{J/m}^3 \quad \left(28.3\ \text{in.-lb}_f/\text{in.}^3\right)$$

(f) The ductility, in percent elongation, is just the plastic strain at fracture, multiplied by one-hundred. The WRWDOIUDFWXUH VWUDLQ DWIUDFWXUH LV !!◄¶←↑ VXEWUDFWLQJ RXWWKH HODVWLF VWUDLQ ZKLFK LV DERXW!◄!!¶♀ OHDYHV D SODVWLF Vtrain of 0.184. Thus, the ductility is about 18.4%EL.

6.31 For the titanium alloy, whose stress strain behavior may be observed in the "Tensile Tests" module of Virtual Materials Science and Engineering (VMSE), determine the following:

(a) the approximate yield strength (0.002 strain offset),

(b) the tensile strength, and

(c) the approximate ductility, in percent elongation.

How do these values compare with those for the two Ti-6Al-4V alloys presented in Table B.4 of Appendix B?

<u>Solution</u>

(a) It is possible to do a screen capture and then print out the entire stress-strain curve for the Ti alloy. The intersection of a straight line parallel to the initial linear region of the curve and offset at a strain of 0.002 with this curve is at approximately 720 MPa.

(b) The maximum reading in the stress window located below the plot as the curser point is dragged along the stress-strain curve is 1000 MPa, the value of the tensile strength.

(c) The approximate percent elongation corresponds to the strain at fracture multiplied by 100 (i.e., 12%) minus the maximum elastic strain (i.e., value of strain at which the linearity of the curve ends multiplied by 100—in WKLV FDVH DERXW!◀◘ ♀ WKLV JLYHV D YDOXH RI DERXW¶¶◀◘ (/◀

From Table B.4 in Appendix B, yield strength, tensile strength, and percent elongation values for the anneal Ti-6Al-4V are 830 MPa, 900 MPa, and 14%EL, while for the solution heat treated and aged alloy, the corresponding values are 1103 MPa, 1172 MPa, and 10%EL. Thus, tensile strength and percent elongation values for the VMSE alloy are slightly lower than for the annealed material in Table B.4 (720 vs 830 MPa, and 11.5 vs. 14 %EL), whereas the tensile strength is slightly higher (1000 vs. 900 MPa).

6.32 For the tempered steel alloy, whose stress strain behavior may be observed in the "Tensile Tests" module of Virtual Materials Science and Engineering (VMSE), determine the following:

(a) the approximate yield strength (0.002 strain offset),

(b) the tensile strength, and

(c) the approximate ductility, in percent elongation.

How do these values compare with those for the oil-quenched and tempered 4140 and 4340 steel alloys presented in Table B.4 of Appendix B?

Solution

(a) It is possible to do a screen capture and then print out the entire stress-strain curve for the tempered steel alloy. The intersection of a straight line parallel to the initial linear region of the curve and offset at a strain of 0.002 with this curve is at approximately 1430 MPa.

(b) The maximum reading in the stress window located below the plot as the curser point is dragged along the stress-strain curve is 1656 MPa, the value of the tensile strength.

(c) The approximate percent elongation corresponds to the strain at fracture multiplied by 100 (i.e., 14.8%) minus the maximum elastic strain (i.e., value of strain at which the linearity of the curve ends multiplied by 100—in WKLV FDVH DERXW‼◄◘ ♀ WKLV JLYHV D YDOXH RI DERXW¶├◄!◘ (/ ◄

For the oil-quenched and tempered 4140 and 4340 steel alloys, yield strength values presented in Table B.4 RI $ SSHQGL[%DUH ¶↑→‼ 0 3D DQG ¶├┴‼ 0 3D UHVSHFWLYHO WKHVH YDOXHV DUH VRP HZ KDWOJ HU WKDQ WKH ¶├┬‼ 0 3D IRU the tempered steel alloy of VMSE. Tensile strength values for these 4140 and 4340 alloys are, respectively 1720 MPa and 1760 MPa (compared to 1656 MPa for the VMSE steel). And, finally, the respective ductilities for the 4140 and 4340 alloys are 11.5%EL and 12%EL, which are slightly lower than the 14%EL value for the VMSE steel.

6.33 For the aluminum alloy, whose stress strain behavior may be observed in the "Tensile Tests" module of Virtual Materials Science and Engineering (VMSE), determine the following:

(a) the approximate yield strength (0.002 strain offset),

(b) the tensile strength, and

(c) the approximate ductility, in percent elongation.

How do these values compare with those for the 2024 aluminum alloy (T351 temper) presented in Table B.4 of Appendix B?

Solution

(a) It is possible to do a screen capture and then print out the entire stress-strain curve for the aluminum alloy. The intersection of a straight line parallel to the initial linear region of the curve and offset at a strain of 0.002 with this curve is at approximately 300 MPa.

(b) The maximum reading in the stress window located below the plot as the curser point is dragged along the stress-strain curve is 484 MPa, the value of the tensile strength.

(c) The approximate percent elongation corresponds to the strain at fracture multiplied by 100 (i.e., 22.4%) minus the maximum elastic strain (i.e., value of strain at which the linearity of the curve ends multiplied by 100—in WKLV FDVH DERXW ! ◀ ◘ ♀ WKLV J LYHV D YDOXH RI DERXW ¶ ◀ ◘ (/ ◀

For the 2024 aluminum alloy (T351 temper), the yield strength value presented in Table B.4 of Appendix B is 325, which is slightly larger than the 300 MPa for the aluminum alloy of VMSE. The tensile strength value for the 2024-T351 is 470 MPa (compared to 484 MPa for the VMSE alloy). And, finally, the ductility for 2024-T351 is 20%EL, which is about the same as for the VMSE aluminum (21.9%EL).

6.34 For the (plain) carbon steel alloy, whose stress strain behavior may be observed in the "Tensile Tests" module of Virtual Materials Science and Engineering (VMSE), determine the following:

(a) the approximate yield strength,

(b) the tensile strength, and

(c) the approximate ductility, in percent elongation.

Solution

(a) It is possible to do a screen capture and then print out the entire stress-strain curve for the plain carbon steel alloy. Inasmuch as the stress-strain curve displays the yield point phenomenon, we take the yield strength as the lower yield point, which, for this steel, is about 225 MPa.

(b) The maximum reading in the stress window located below the plot as the curser point is dragged along the stress-strain curve is 274 MPa, the value of the tensile strength.

(c) The approximate percent elongation corresponds to the strain at fracture multiplied by 100 (i.e., 43.0%) minus the maximum elastic strain (i.e., value of strain at which the linearity of the curve ends multiplied by 100—in WKLV FDVH DERXW!◄◘ ♀ WKLV J LYHV D YDOXH RI DERXW ◄◘ (/ ◄

6.35 A cylindrical metal specimen having an original diameter of 12.8 mm (0.505 in.) and gauge length of 50.80 mm (2.000 in.) is pulled in tension until fracture occurs. The diameter at the point of fracture is 6.60 mm (0.260 in.), and the fractured gauge length is 72.14 mm (2.840 in.). Calculate the ductility in terms of percent reduction in area and percent elongation.

Solution

This problem calls for the computation of ductility in both percent reduction in area and percent elongation. Percent reduction in area is computed using Equation 6.12 as

$$\%RA = \frac{\pi\left(\frac{d_0}{2}\right)^2 - \pi\left(\frac{d_f}{2}\right)^2}{\pi\left(\frac{d_0}{2}\right)^2} \times 100$$

in which d_0 and d_f are, respectively, the original and fracture cross-sectional areas. Thus,

$$\%RA = \frac{\pi\left(\frac{12.8\text{ mm}}{2}\right)^2 - \pi\left(\frac{6.60\text{ mm}}{2}\right)^2}{\pi\left(\frac{12.8\text{ mm}}{2}\right)^2} \times 100 = 73.4\%$$

While, for percent elongation, we use Equation 6.11 as

$$\%EL = \left(\frac{l_f - l_0}{l_0}\right) \times 100$$

$$= \frac{72.14\text{ mm} - 50.80\text{ mm}}{50.80\text{ mm}} \times 100 = 42\%$$

6.36 Calculate the moduli of resilience for the materials having the stress–strain behaviors shown in Figures 6.12 and 6.21.

Solution

This problem asks us to calculate the moduli of resilience for the materials having the stress-strain behaviors shown in Figures 6.12 and 6.21. According to Equation 6.14, the modulus of resilience U_r is a function of the yield strength and the modulus of elasticity as

$$U_r = \frac{\sigma_y^2}{2E}$$

The values for σ_y and E for the brass in Figure 6.12 are determined in Example Problem 6.3 as 250 MPa (36,000 psi) and 93.8 GPa (13.6×10^6 psi), respectively. Thus

$$U_r = \frac{(250\ \text{MPa})^2}{(2)\left(93.8 \times 10^3\ \text{MPa}\right)} = 3.32 \times 10^5\ \text{J/m}^3 \quad \left(48.2\ \text{in.-lb}_f/\text{in.}^3\right)$$

Values of the corresponding parameters for the steel alloy (Figure 6.21) are determined in Problem 6.25 as 400 MPa (58,000 psi) and 200 GPa (29×10^6 psi), respectively, and therefore

$$U_r = \frac{(400\ \text{MPa})^2}{(2)\left(200 \times 10^3\ \text{MPa}\right)} = 4.0 \times 10^5\ \text{J/m}^3 \quad \left(58\ \text{in.-lb}_f/\text{in.}^3\right)$$

6.37 Determine the modulus of resilience for each of the following alloys:

	Yield Strength	
Material	**MPa**	**psi**
Steel alloy	550	80,000
Brass alloy	350	50,750
Aluminum alloy	250	36,250
Titanium alloy	800	116,000

Use modulus of elasticity values in Table 6.1.

Solution

The moduli of resilience of the alloys listed in the table may be determined using Equation 6.14. Yield strength values are provided in this table, whereas the elastic moduli are tabulated in Table 6.1.

For steel

$$U_r = \frac{\sigma_y^2}{2E}$$

$$= \frac{(550 \times 10^6 \text{ N/m}^2)^2}{(2)\left(207 \times 10^9 \text{ N/m}^2\right)} = 7.31 \times 10^5 \text{ J/m}^3 \quad \left(107 \text{ in.-lb}_f/\text{in.}^3\right)$$

For the brass

$$U_r = \frac{(350 \times 10^6 \text{ N/m}^2)^2}{(2)\left(97 \times 10^9 \text{ N/m}^2\right)} = 6.31 \times 10^5 \text{ J/m}^3 \quad \left(92.0 \text{ in.-lb}_f/\text{in.}^3\right)$$

For the aluminum alloy

$$U_r = \frac{(250 \times 10^6 \text{ N/m}^2)^2}{(2)\left(69 \times 10^9 \text{ N/m}^2\right)} = 4.53 \times 10^5 \text{ J/m}^3 \quad \left(65.7 \text{ in.-lb}_f/\text{in.}^3\right)$$

And, for the titanium alloy

$$U_r = \frac{(800 \times 10^6 \text{ N/m}^2)^2}{(2)\left(107 \times 10^9 \text{ N/m}^2\right)} = 30.0 \times 10^5 \text{ J/m}^3 \quad \left(434 \text{ in.-lb}_f/\text{in.}^3\right)$$

6.38 A brass alloy to be used for a spring application must have a modulus of resilience of at least 0.75 MPa (110 psi). What must be its minimum yield strength?

Solution

The modulus of resilience, yield strength, and elastic modulus of elasticity are related to one another WKURXJK (TXDWLRQ ... WKH YDOXH RI E for brass given in Table 6.1 is 97 GPa. Solving for σ_y from this expression yields

$$\sigma_y = \sqrt{2U_r E} = \sqrt{(2)(0.75\ \text{MPa})\left(97 \times 10^3\ \text{MPa}\right)}$$

$$= 381\ \text{MPa}\ \ (55{,}500\ \text{psi})$$

True Stress and Strain

6.39 Show that Equations 6.18a and 6.18b are valid when there is no volume change during deformation.

Solution

To show that Equation 6.18a is valid, we must first rearrange Equation 6.17 as

$$A_i = \frac{A_0 l_0}{l_i}$$

Substituting this expression into Equation 6.15 yields

$$\sigma_T = \frac{F}{A_i} = \frac{F}{A_0}\left(\frac{l_i}{l_0}\right) = \sigma\left(\frac{l_i}{l_0}\right)$$

But, from Equation 6.2

$$\varepsilon = \frac{l_i}{l_0} - 1$$

Or

$$\frac{l_i}{l_0} = \varepsilon + 1$$

Thus,

$$\sigma_T = \sigma\left(\frac{l_i}{l_0}\right) = \sigma(\varepsilon + 1)$$

For Equation 6.18b

$$\varepsilon_T = \ln(1 + \varepsilon)$$

is valid since, from Equation 6.16

$$\varepsilon_T = \ln\left(\frac{l_i}{l_0}\right)$$

and

$$\frac{l_i}{l_0} = \varepsilon + 1$$

from above.

6.40 Demonstrate that Equation 6.16, the expression defining true strain, may also be represented by

$$\epsilon_T = \ln\left(\frac{A_0}{A_i}\right)$$

when specimen volume remains constant during deformation. Which of these two expressions is more valid during necking? Why?

Solution

This problem asks us to demonstrate that true strain may also be represented by

$$\epsilon_T = \ln\left(\frac{A_0}{A_i}\right)$$

Rearrangement of Equation 6.17 leads to

$$\frac{l_i}{l_0} = \frac{A_0}{A_i}$$

Thus, Equation 6.16 takes the form

$$\epsilon_T = \ln\left(\frac{l_i}{l_0}\right) = \ln\left(\frac{A_0}{A_i}\right)$$

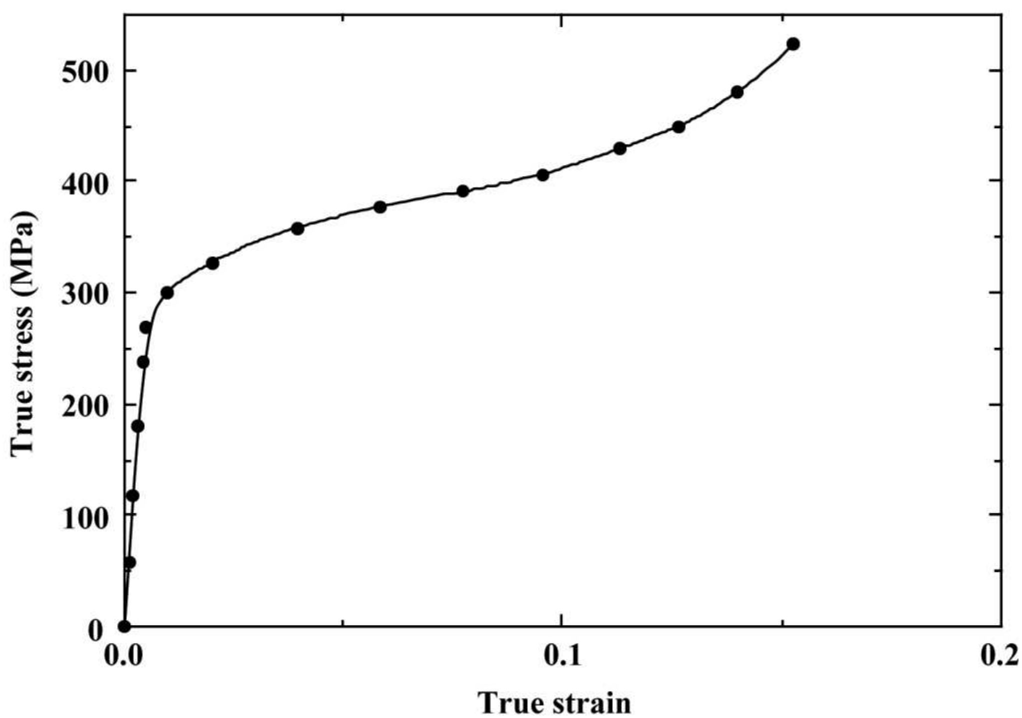
The expression $\epsilon_T = \ln\left(\frac{A_0}{A_i}\right)$ is more valid during necking because A_i is taken as the area of the neck.

6.41 Using the data in Problem 6.28 and Equations 6.15, 6.16, and 6.18a, generate a true stress–true WWDLQ SORWIRU DOXPLQXP (TXDWLRQ 7D EHFRPHV LQYDOLG SDVW WKH SRLQW DW ZKLFK QHFNLQJ EHJLQV WKHUHIRUH measured diameters are given below for the last four data points, which should be used in true stress computations.

Load		Length		Diameter	
N	**lb$_f$**	**mm**	**in.**	**mm**	**in.**
46,100	10,400	56.896	2.240	11.71	0.461
42,400	10,100	57.658	2.270	10.95	0.431
42,600	9,600	58.420	2.300	10.62	0.418
36,400	8,200	59.182	2.330	9.40	0.370

Solution

These true stress-strain data are plotted below.



6.42 A tensile test is performed on a metal specimen, and it is found that a true plastic strain of 0.20 is produced when a true stress of 575 MPa ([illegible] psi) is applied; for the same metal, the value of K in Equation 6.19 is 860 MPa (125,000 psi). Calculate the true strain that results from the application of a true stress of 600 MPa (87,000 psi).

Solution

It first becomes necessary to solve for n in Equation 6.19. Taking logarithms of this expression and after rearrangement we have

$$n = \frac{\log \sigma_T - \log K}{\log \varepsilon_T}$$

And, incorporating values of the parameters provided in the problem statement leads to

$$n = \frac{\log (575 \text{ MPa}) - \log (860 \text{ MPa})}{\log (0.20)} = 0.250$$

Expressing ε_T as the dependent variable (Equation 6.19), and then solving for its value from the data stipulated in the problem statement, leads to

$$\varepsilon_T = \left(\frac{\sigma_T}{K}\right)^{1/n} = \left(\frac{600 \text{ MPa}}{860 \text{ MPa}}\right)^{1/0.250} = 0.237$$

6.43 For some metal alloy, a true stress of 415 MPa (60,175 psi) produces a plastic true strain of 0.475. How much will a specimen of this material elongate when a true stress of 325 MPa (46,125 psi) is applied if the original length is 300 mm (11.8 in.)? Assume a value of 0.25 for the strain-hardening exponent n.

Solution

Solution of this problem requires that we utilize Equation 6.19. It is first necessary to solve for K from the given true stress and strain. Rearrangement of this equation yields

$$K = \frac{\sigma_T}{(\varepsilon_T)^n} = \frac{415 \text{ MPa}}{(0.475)^{0.25}} = 500 \text{ MPa} \quad (72,500 \text{ psi})$$

Next we must solve for the true strain produced when a true stress of 325 MPa is applied, also using Equation 6.19. Thus

$$\varepsilon_T = \left(\frac{\sigma_T}{K}\right)^{1/n} = \left(\frac{325 \text{ MPa}}{500 \text{ MPa}}\right)^{1/0.25} = 0.179 = \ln\left(\frac{l_i}{l_0}\right)$$

Now, solving for l_i gives

$$l_i = l_0 e^{0.179} = (300 \text{ mm})\, e^{0.179} = 358.8 \text{ mm} \quad (14.11 \text{ in.})$$

And finally, the elongation Δl is just

$$\Delta l = l_i - l_0 = 358.8 \text{ mm} - 300 \text{ mm} = 58.8 \text{ mm} \quad (2.31 \text{ in.})$$

6.44 The following true stresses produce the corresponding true plastic strains for a brass alloy:

True Stress (psi)	**True Strain**
50,000	0.10
60,000	0.20

What true stress is necessary to produce a true plastic strain of 0.25?

Solution

For this problem, we are given two values of ϵ_T and σ_T, from which we are asked to calculate the true stress which produces a true plastic strain of 0.25. Employing Equation 6.19, we may set up two simultaneous equations with two unknowns (the unknowns being K and n), as

$$\log (50{,}000 \text{ psi}) = \log K + n \log (0.10)$$

$$\log (60{,}000 \text{ psi}) = \log K + n \log (0.20)$$

Solving for n from these two expressions yields

$$n = \frac{\log (50{,}000) - \log (60{,}000)}{\log (0.10) - \log (0.20)} = 0.263$$

and for K

$$\log K = 4.96 \text{ or } K = 10^{4.96} = 91{,}623 \text{ psi}$$

Thus, for $\epsilon_T = 0.25$

$$\sigma_T = K (\epsilon_T)^n = (91{,}623 \text{ psi})(0.25)^{0.263} = 63{,}700 \text{ psi} \quad (440 \text{ MPa})$$

6.45 For a brass alloy, the following engineering stresses produce the corresponding plastic engineering strains, prior to necking:

Engineering Stress (MPa)	Engineering Strain
235	0.194
250	0.296

On the basis of this information, compute the engineering stress necessary to produce an engineering strain of 0.25.

Solution

For this problem we first need to convert engineering stresses and strains to true stresses and strains so that the constants K and n in Equation 6.19 may be determined. Since $\sigma_T = \sigma(1 + \varepsilon)$ then

$$\sigma_{T1} = (235\ \text{MPa})(1 + 0.194) = 280\ \text{MPa}$$

$$\sigma_{T2} = (250\ \text{MPa})(1 + 0.296) = 324\ \text{MPa}$$

Similarly for strains, since $\varepsilon_T = \ln(1 + \varepsilon)$ then

$$\varepsilon_{T1} = \ln\ (1\ + 0.194) = 0.177$$

$$\varepsilon_{T2} = \ln\ (1\ +\ 0.296) = 0.259$$

Taking logarithms of Equation 6.19, we get

$$\log \sigma_T\ = \log K\ +\ n \log \varepsilon_T$$

which allows us to set up two simultaneous equations for the above pairs of true stresses and true strains, with K and n as unknowns. Thus

$$\log\ (280)\ = \log K\ + n \log\ (0.177)$$

$$\log\ (324)\ = \log K\ + n \log\ (0.259)$$

Solving for these two expressions yields K = 543 MPa and n = 0.383.

Now, converting $\varepsilon = 0.25$ to true strain

$$\varepsilon_T = \ln (1 + 0.25) = 0.223$$

The corresponding σ_T to give this value of ε_T (using Equation 6.19) is just

$$\sigma_T = K\varepsilon_T^n = (543 \text{ MPa})(0.223)^{0.383} = 306 \text{ MPa}$$

Now converting this value of σ_T to an engineering stress using Equation 6.18a gives

$$\sigma = \frac{\sigma_T}{1 + \varepsilon} = \frac{306 \text{ MPa}}{1 + 0.25} = 245 \text{ MPa}$$

6.46 Find the toughness (or energy to cause fracture) for a metal that experiences both elastic and plastic deformation. Assume Equation 6.5 for elastic deformation, that the modulus of elasticity is 172 GPa (25×10^6 psi), and that elastic deformation terminates at a strain of 0.01. For plastic deformation, assume that the relationship between stress and strain is described by Equation 6.19, in which the values for K and n are 6900 MPa (1×10^6 psi) and 0.30, respectively. Furthermore, plastic deformation occurs between strain values of 0.01 and 0.75, at which point fracture occurs.

Solution

This problem calls for us to compute the toughness (or energy to cause fracture). The easiest way to do this is to integrate both elastic and plastic regions, and then add them together.

$$\text{Toughness} = \int \sigma\, d\varepsilon$$

$$= \int_0^{0.01} E\varepsilon\, d\varepsilon + \int_{0.01}^{0.75} K\varepsilon^n\, d\varepsilon$$

$$= \left.\frac{E\varepsilon^2}{2}\right|_0^{0.01} + \left.\frac{K}{(n+1)}\varepsilon^{(n+1)}\right|_{0.01}^{0.75}$$

$$= \frac{172 \times 10^9 \text{ N/m}^2}{2}(0.01)^2 + \frac{6900 \times 10^6 \text{ N/m}^2}{(1.0 + 0.3)}\left[(0.75)^{1.3} - (0.01)^{1.3}\right]$$

$$= 3.65 \times 10^9 \text{ J/m}^3 \ \left(5.29 \times 10^5 \text{ in.-lb}_f/\text{in.}^3\right)$$

6.47 For a tensile test, it can be demonstrated that necking begins when

$$\frac{d\sigma_T}{d\varepsilon_T} = \sigma_T \qquad (6.26)$$

Using Equation 6.19, determine the value of the true strain at this onset of necking.

Solution

Let us take the derivative of Equation 6.19, set it equal to σ_T, and then solve for ε_T from the resulting expression. Thus

$$\frac{d\left[K(\varepsilon_T)^n\right]}{d\varepsilon_T} = Kn(\varepsilon_T)^{(n-1)} = \sigma_T$$

However, from Equation 6.19, $\sigma_T = K(\varepsilon_T)^n$, which, when substituted into the above expression, yields

$$Kn(\varepsilon_T)^{(n - 1)} = K(\varepsilon_T)^n$$

Now solving for ε_T from this equation leads to

$$\varepsilon_T = n$$

as the value of the true strain at the onset of necking.

6.48 Taking the logarithm of both sides of Equation 6.19 yields

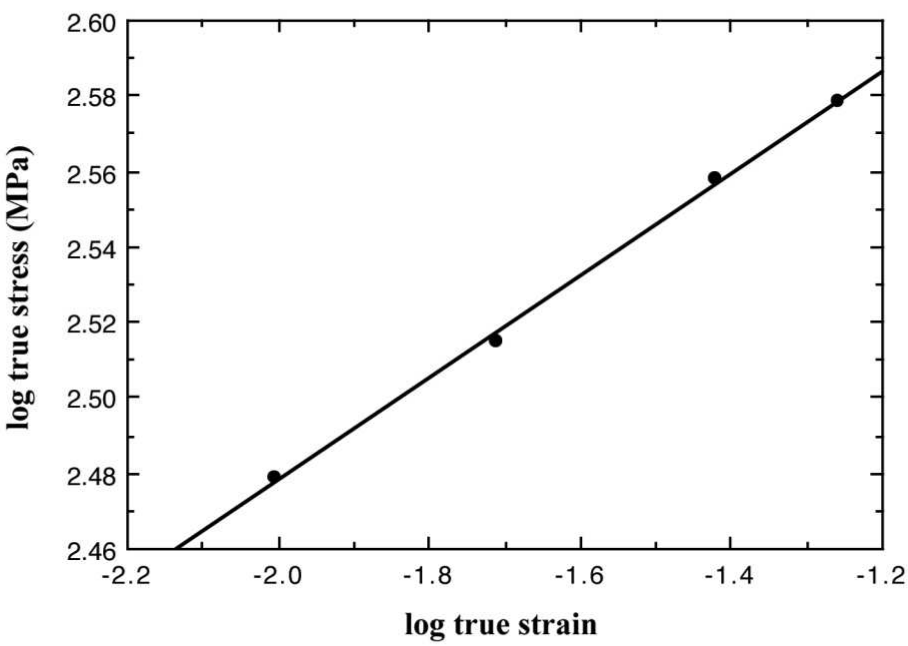
$$\log \sigma_T = \log K + n \log \epsilon_T \qquad (6.27)$$

Thus, a plot of log σ_T versus log ϵ_T in the plastic region to the point of necking should yield a straight line having a slope of n and an intercept (at log σ_T = 0) of log K.

Using the appropriate data tabulated in Problem 6.29, make a plot of log σ_T versus log ϵ_T and determine the values of n and K. It will be necessary to convert engineering stresses and strains to true stresses and strains using Equations 6.18a and 6.18b.

Solution

This problem calls for us to utilize the appropriate data from Problem 6.29 in order to determine the values of n and K for this material. From Equation 6.27 the slope and intercept of a log σ_T versus log ε_T plot will yield n and log K, respectively. However, Equation 6.19 is only valid in the region of plastic deformation to the point of necking; thus, only the 7th, 8th, 9th, and 10th data points may be utilized. The log-log plot with these data points is given below.



The slope yields a value of 0.136 for n, whereas the intercept gives a value of 2.7497 for log K, and thus K = $10^{2.7497}$ = 562 MPa.

Elastic Recovery After Plastic Deformation

6.49 A cylindrical specimen of a brass alloy 7.5 mm (0.30 in.) in diameter and 90.0 mm (3.54 in.) long is pulled in tension with a force of 6000 N (1350 lb_f); the force is subsequently released.

(a) Compute the final length of the specimen at this time. The tensile stress–strain behavior for this alloy is shown in Figure 6.12.

(b) Compute the final specimen length when the load is increased to 16,500 N (3700 lb_f) and then released.

Solution

(a) In order to determine the final length of the brass specimen when the load is released, it first becomes necessary to coPSXWH WKH DSSOLHG VWUHVV XVLQJ (TXDWLRQ ¶ WKXV

$$\sigma = \frac{F}{A_0} = \frac{F}{\pi\left(\frac{d_0}{2}\right)^2} = \frac{6000\ \text{N}}{\pi\left(\frac{7.5 \times 10^{-3}\ \text{m}}{2}\right)^2} = 136\ \text{MPa}\ \ (19,000\ \text{psi})$$

Upon locating this point on the stress-VWUDLQ FXUYH) LJXUH ¶ ZH QRWH WKDW LW LV LQ WKH OLQHDU HODVWLF UHJLRQ therefore, when the load is released the specimen will return to its original length of 90 mm (3.54 in.).

(b) In this portion of the problem we are asked to calculate the final length, after load release, when the load is increased to 16,500 N (3700 lb_f). Again, computing the stress

$$\sigma = \frac{16,500\ \text{N}}{\pi\left(\frac{7.5 \times 10^{-3}\ \text{m}}{2}\right)^2} = 373\ \text{MPa}\ \ (52,300\ \text{psi})$$

The point on the stress-strain curve corresponding to this stress is in the plastic region. We are able to estimate the amount of permanent strain by drawing a straight line parallel to the linear elastic region; this line intersects the strain axis at a strain of about 0.08 which is the amount of plastic strain. The final specimen length l_i may be determined from a rearranged form of Equation 6.2 as

$$l_i = l_0(1 + \varepsilon) = (90\ \text{mm})(1 + 0.08) = 97.20\ \text{mm}\ (3.82\ \text{in.})$$

6.50 A steel alloy specimen having a rectangular cross section of dimensions 12.7 mm × 6.4 mm (0.5 in. × 0.25 in.) has the stress–strain behavior shown in Figure 6.21. If this specimen is subjected to a tensile force of 38,000 N (8540 lb_f) then

(a) Determine the elastic and plastic strain values.

(b) If its original length is 460 mm (18.0 in.), what will be its final length after the load in part (a) is applied and then released?

Solution

(a) We are asked to determine both the elastic and plastic strain values when a tensile force of 38,000 N (8540 lb_f) is applied to the steel specimen and then released. First it becomes necessary to determine the applied

WWHVVXVLQJ (TXDWLRQ ¶ WKXV

$$\sigma = \frac{F}{A_0} = \frac{F}{b_0 d_0}$$

where b_0 and d_0 are cross-sectional width and depth (12.7 mm and 6.4 mm, respectively). Thus

$$\sigma = \frac{38,000 \text{ N}}{(12.7 \times 10^{-3} \text{ m})(6.4 \times 10^{-3} \text{ m})} = 468 \times 10^6 \text{ N/m}^2 = 468 \text{ MPa} \ (68,300 \text{ psi})$$

From Figure 6.21, this point is in the plastic region so the specimen will be both elastic and plastic strains. The total strain at this point, ε_t, is about 0.010. We are able to estimate the amount of permanent strain recovery ε_e from Hooke's law, Equation 6.5 as

$$\varepsilon_e = \frac{\sigma}{E}$$

And, since E = 207 GPa for steel (Table 6.1)

$$\varepsilon_e = \frac{468 \text{ MPa}}{207 \times 10^3 \text{ MPa}} = 0.00226$$

The value of the plastic strain, ε_p is just the difference between the total and elastic strains; that is

$$\varepsilon_p = \varepsilon_t - \varepsilon_e = 0.010 - 0.00226 = 0.00774$$

(b) If the initial length is 460 mm (18.0 in.) then the final specimen length l_i may be determined from a rearranged form of Equation 6.2 using the plastic strain value as

$$l_i = l_0(1 + \varepsilon_p) = (460 \text{ mm})(1 + 0.00774) = 463.6 \text{ mm } (18.14 \text{ in.})$$

Hardness

6.51 (a) A 10-mm-diameter Brinell hardness indenter produced an indentation 1.62 mm in diameter in a steel alloy when a load of 500 kg was used. Compute the HB of this material.

(b) What will be the diameter of an indentation to yield a hardness of 450 HB when a 500 kg load is used?

Solution

(a) We are asked to compute the Brinell hardness for the given indentation. It is necessary to use the equation in Table 6.5 for HB, where P = 500 kg, d = 1.62 mm, and D = 10 mm. Thus, the Brinell hardness is computed as

$$HB = \frac{2P}{\pi D\left[D - \sqrt{D^2 - d^2}\right]}$$

$$= \frac{(2)(500\ \text{kg})}{(\pi)(10\ \text{mm})\left[10\ \text{mm} - \sqrt{(10\ \text{mm})^2 - (1.62\ \text{mm})^2}\right]} = 241$$

(b) This part of the problem calls for us to determine the indentation diameter d which will yield a 450 HB when P = 500 kg. Solving for d from the equation in Table 6.5 gives

$$d = \sqrt{D^2 - \left[D - \frac{2P}{(HB)\pi D}\right]^2}$$

$$= \sqrt{(10\text{mm})^2 - \left[10\ \text{mm} - \frac{(2)(500\ \text{kg})}{(450)(\pi)(10\ \text{mm})}\right]^2} = 1.19\ \text{mm}$$

6.52 Estimate the Brinell and Rockwell hardnesses for the following:

(a) The naval brass for which the stress–strain behavior is shown in Figure 6.12.

(b) The steel alloy for which the stress–strain behavior is shown in Figure 6.21.

Solution

This problem calls for estimations of Brinell and Rockwell hardnesses.

(a) For the brass specimen, the stress-strain behavior for which is shown in Figure 6.12, the tensile strength is 450 MPa (65,000 psi). From Figure 6.19, the hardness for brass corresponding to this tensile strength is about 125 HB or 70 HRB.

(b) The steel alloy (Figure 6.21) has a tensile strength of about 515 MPa (74,700 psi) [Problem 6.25(d)]. This corresponds to a hardness of about 160 HB or ~90 HRB from the line for steels in Figure 6.19. Alternately, using Equation 6.20a

$$\mathrm{HB} = \frac{\mathrm{TS(MPa)}}{3.45} = \frac{515 \text{ MPa}}{3.45} = 149$$

6.53 Using the data represented in Figure 6.19, specify equations relating tensile strength and Brinell hardness for brass and nodular cast iron, similar to Equations 6.20a and 6.20b for steels.

Solution

These equations, for a straight line, are of the form

$$TS = C + (E)(HB)$$

where TS is the tensile strength, HB is the Brinell hardness, and C and E are constants, which need to be determined.

One way to solve for C and E is analytically--establishing two equations using TS and HB data points on the plot, as

$$(TS)_1 = C + (E)(BH)_1$$
$$(TS)_2 = C + (E)(BH)_2$$

Solving for E from these two expressions yields

$$E = \frac{(TS)_1 - (TS)_2}{(HB)_2 - (HB)_1}$$

For nodular cast iron, if we make the arbitrary choice of $(HB)_1$ and $(HB)_2$ as 200 and 300, respectively, then, from Figure 6.19, $(TS)_1$ and $(TS)_2$ take on values of 600 MPa (87,000 psi) and 1100 MPa (160,000 psi), respectively. Substituting these values into the above expression and solving for E gives

$$E = \frac{600 \text{ MPa} - 1100 \text{ MPa}}{200 \text{ HB} - 300 \text{ HB}} = 5.0 \text{ MPa/HB} \quad (730 \text{ psi/HB})$$

Now, solving for C yields

$$C = (TS)_1 - (E)(BH)_1$$

$$= 600 \text{ MPa} - (5.0 \text{ MPa/HB})(200 \text{ HB}) = -400 \text{ MPa} \; (-59{,}000 \text{ psi})$$

Thus, for nodular cast iron, these two equations take the form

$$TS(MPa) = -400 + 5.0 \times HB$$

$$TS(psi) = -59{,}000 + 730 \times HB$$

Now for brass, we take $(HB)_1$ and $(HB)_2$ as 100 and 200, respectively, then, from Figure 7.31, $(TS)_1$ and $(TS)_2$ take on values of 370 MPa (54,000 psi) and 660 MPa (95,000 psi), respectively. Substituting these values into the above expression and solving for E gives

$$E = \frac{370 \text{ MPa} - 660 \text{ MPa}}{100 \text{ HB} - 200 \text{ HB}} = 2.9 \text{ MPa/HB} \ (410 \text{ psi/HB})$$

Now, solving for C yields

$$C = (TS)_1 - (E)(BH)_1$$

$$= 370 \text{ MPa} - (2.9 \text{ MPa/HB})(100 \text{ HB}) = 80 \text{ MPa} \ (13{,}000 \text{ psi})$$

Thus, for brass these two equations take the form

$$TS(MPa) = 80 + 2.9 \times HB$$

$$TS(psi) = 13{,}000 + 410 \times HB$$

Variability of Material Properties

6.54 Cite five factors that lead to scatter in measured material properties.

Solution

The five factors that lead to scatter in measured material properties are the following: (1) test method; (2) variation in specimen fabrication procedure; (3) operator bias; (4) apparatus calibration; and (5) material inhomogeneities and/or compositional differences.

6.55 Below are tabulated a number of Rockwell B hardness values that were measured on a single steel specimen. Compute average and standard deviation hardness values.

83.3	80.7	86.4
88.3	84.7	85.2
82.8	87.8	86.9
86.2	83.5	84.4
87.2	85.5	86.3

Solution

The average of the given hardness values is calculated using Equation 6.21 as

$$\overline{\text{HRB}} = \frac{\sum_{i=1}^{15} \text{HRB}_i}{15}$$

$$= \frac{83.3 + 88.3 + 82.8 \; . \; . \; . \; . + 86.3}{15} = 85.3$$

And we compute the standard deviation using Equation 6.22 as follows:

$$s = \sqrt{\frac{\sum_{i=1}^{15} \left(\text{HRB}_i - \overline{\text{HRB}}\right)^2}{15 - 1}}$$

$$= \left[\frac{(83.3 - 85.3)^2 + (88.3 - 85.3)^2 + . \; . \; . \; . + (86.3 - 85.3)^2}{14}\right]^{1/2}$$

$$= \sqrt{\frac{60.31}{14}} = 2.08$$

Design/Safety Factors

6.56 Upon what three criteria are factors of safety based?

Solution

The criteria upon which factors of safety are based are (1) consequences of failure, (2) previous experience, (3) accuracy of measurement of mechanical forces and/or material properties, and (4) economics.

6.57 Determine working stresses for the two alloys that have the stress–strain behaviors shown in Figures 6.12 and 6.21.

Solution

The working stresses for the two alloys the stress-strain behaviors of which are shown in Figures 6.12 and 6.21 are calculated by dividing the yield strength by a factor of safety, which we will take to be 2. For the brass alloy (Figure 6.12), since σ_y = 250 MPa (36,000 psi), the working stress is 125 MPa (18,000 psi), whereas for the steel alloy (Figure 6.21), σ_y = 400 MPa (58,000 psi), and, therefore, σ_w = 200 MPa (29,000 psi).

DESIGN PROBLEMS

6.D1 A large tower is to be supported by a series of steel wires. It is estimated that the load on each wire will be 11,100 N (2500 lb_f). Determine the minimum required wire diameter assuming a factor of safety of 2 and a yield strength of 1030 MPa (150,000 psi).

Solution

For this problem the working stress is computed using Equation 6.24 with N = 2, as

$$\sigma_w = \frac{\sigma_y}{2} = \frac{1030 \text{ MPa}}{2} = 515 \text{ MPa} \quad (75{,}000 \text{ psi})$$

Since the force is given, the area may be determined from Equation 6.1, and subsequently the original diameter d_0 may be calculated as

$$A_0 = \frac{F}{\sigma_w} = \pi\left(\frac{d_0}{2}\right)^2$$

And

$$d_0 = \sqrt{\frac{4F}{\pi \sigma_w}} = \sqrt{\frac{(4)(11{,}100 \text{ N})}{\pi\left(515 \times 10^6 \text{ N/m}^2\right)}}$$

$$= 5.23 \times 10^{-3} \text{ m} = 5.23 \text{ mm } (0.206 \text{ in.})$$

6.D2 (a) Gaseous hydrogen at a constant pressure of 1.013 MPa (10 atm) is to flow within the inside of a thin-walled cylindrical tube of nickel that has a radius of 0.1 m. The temperature of the tube is to be 300°C and the pressure of hydrogen outside of the tube will be maintained at 0.01013 MPa (0.1 atm). Calculate the minimum wall thickness if the diffusion flux is to be no greater than 1×10^{-7} mol/m^2-s. The concentration of hydrogen in the nickel, C_H (in moles hydrogen per m^3 of Ni) is a function of hydrogen pressure, P_{H_2} (in MPa) and absolute temperature (T) according to

$$C_H = 30.8\sqrt{p_{H_2}}\,\exp\left(-\frac{12.3 \text{ kJ/mol}}{RT}\right) \tag{6.28}$$

Furthermore, the diffusion coefficient for the diffusion of H in Ni depends on temperature as

$$D_H = 4.76 \times 10^{-7} \exp\left(-\frac{39.56 \text{ kJ/mol}}{RT}\right) \tag{6.29}$$

(b) For thin-walled cylindrical tubes that are pressurized, the circumferential stress is a function of the pressure difference across the wall (Δp), cylinder radius (r), and tube thickness (Δx) as

$$\sigma = \frac{r\,\Delta p}{4\,\Delta x} \tag{6.30}$$

Compute the circumferential stress to which the walls of this pressurized cylinder are exposed.

(c) The room-temperature yield strength of Ni is 100 MPa (15,000 psi) and, furthermore, σ_y diminishes about 5 MPa for every 50°C rise in temperature. Would you expect the wall thickness computed in part (b) to be suitable for this Ni cylinder at 300°C? Why or why not?

(d) If this thickness is found to be suitable, compute the minimum thickness that could be used without any deformation of the tube walls. How much would the diffusion flux increase with this reduction in thickness? On the other hand, if the thickness determined in part (c) is found to be unsuitable, then specify a minimum thickness that you would use. In this case, how much of a diminishment in diffusion flux would result?

Solution

(a) This portion of the problem asks for us to compute the wall thickness of a thin-walled cylindrical Ni tube at 300°C through which hydrogen gas diffuses. The inside and outside pressures are, respectively, 1.1013 and 0.01013 MPa, and the diffusion flux is to be no greater than 1×10^{-7} mol/m^2-s. This is a steady-state diffusion problem, which necessitates that we employ Equation 5.3. The concentrations at the inside and outside wall faces may be determined using Equation 6.28, and, furthermore, the diffusion coefficient is computed using Equation 6.29. Solving for Δx (using Equation 5.3)

$$\Delta x = -\frac{D\,\Delta C}{J}$$

$$= -\frac{1}{1 \times 10^{-7}\ \text{mol/m}^2-\text{s}} \times$$

$$\left(4.76 \times 10^{-7}\right) \exp\left(-\frac{39{,}560\ \text{J/mol}}{(8.31\ \text{J/mol-K})(300 + 273\ \text{K})}\right) \times$$

$$(30.8)\ \exp\left(-\frac{12{,}300\ \text{J/mol}}{(8.31\ \text{J/mol-K})(300 + 273\ \text{K})}\right)\left(\sqrt{0.01013\ \text{MPa}} - \sqrt{1.1013\ \text{MPa}}\right)$$

$$= 0.0025\ \text{m} = 2.5\ \text{mm}$$

(b) Now we are asked to determine the circumferential stress:

$$\sigma = \frac{r\,\Delta p}{4\,\Delta x}$$

$$= \frac{(0.10\ \text{m})(1.013\ \text{MPa} - 0.01013\ \text{MPa})}{(4)(0.0025\ \text{m})}$$

$$= 10.0\ \text{MPa}$$

(c) Now we are to compare this value of stress to the yield strength of Ni at 300°C, from which it is possible to determine whether or not the 2.5 mm wall thickness is suitable. From the information given in the problem, we may write an equation for the dependence of yield strength (σ_y) on temperature (T) as follows:

$$\sigma_y = 100\ \text{MPa} - \frac{5\ \text{MPa}}{50°\text{C}}\left(T - T_r\right)$$

where T_r is room temperature and for temperature in degrees Celsius. Thus, at 300°C

$$\sigma_y = 100\ \text{MPa} - (0.1\ \text{MPa/°C})\ (300°\text{C} - 20°\text{C}) = 72\ \text{MPa}$$

Inasmuch as the circumferential stress (10 MPa) is much less than the yield strength (72 MPa), this thickness is entirely suitable.

(d) And, finally, this part of the problem asks that we specify how much this thickness may be reduced and still retain a safe design. Let us use a working stress by dividing the yield stress by a factor of safety, according to Equation 6.24. On the basis of our experience, let us use a value of 2.0 for N. Thus

$$\sigma_w = \frac{\sigma_y}{N} = \frac{72 \text{ MPa}}{2} = 36 \text{ MPa}$$

Using this value for σ_w and Equation 6.30, we now compute the tube thickness as

$$\Delta x = \frac{r\,\Delta p}{4\sigma_w}$$

$$= \frac{(0.10 \text{ m})(1.013 \text{ MPa} - 0.01013 \text{ MPa})}{4(36 \text{ MPa})}$$

$$= 0.00070 \text{ m} = 0.70 \text{ mm}$$

Substitution of this value into Fick's first law we calculate the diffusion flux as follows:

$$J = -D\frac{\Delta C}{\Delta x}$$

$$= -\left(4.76 \times 10^{-7}\right) \exp\left[-\frac{39{,}560 \text{ J/mol}}{(8.31 \text{ J/mol-K})(300 + 273 \text{ K})}\right] \times$$

$$\frac{(30.8)\exp\left[-\frac{12{,}300 \text{ J/mol}}{(8.31 \text{ J/mol-K})(300 + 273 \text{ K})}\right]\left(\sqrt{0.01013 \text{ MPa}} - \sqrt{1.013 \text{ MPa}}\right)}{0.0007 \text{ m}}$$

$$= 3.53 \times 10^{-7} \text{ mol/m}^2\text{-s}$$

Thus, the flux increases by approximately a factor of 3.5, from 1×10^{-7} to 3.53×10^{-7} mol/m^2-s with this reduction in thickness.

6.D3 Consider the steady-state diffusion of hydrogen through the walls of a cylindrical nickel tube as described in Problem 6.D2. One design calls for a diffusion flux of 5×10^{-8} mol/m^2-s, a tube radius of 0.125 m, and inside and outside pressures of 2.026 MPa (20 atm) and 0.0203 MPa (0.2 atm), respectively; the maximum allowable temperature is 450°C. Specify a suitable temperature and wall thickness to give this diffusion flux and yet ensure that the tube walls will not experience any permanent deformation.

Solution

This problem calls for the specification of a temperature and cylindrical tube wall thickness that will give a diffusion flux of 5×10^{-8} mol/m^2-V IRU WKH GLIIXVLRQ RI K\GURJHQ LQ QLFNHO WKH WXEH UDGLXV LV ◄ ↑ P DQG WKH LQVLGH and outside pressures are 2.026 and 0.0203 MPa, respectively. There are probably several different approaches that may be used; and, of course, there is not one unique solution. Let us employ the following procedure to solve this problem: (1) assume some wall thickness, and, then, using Fick's first law for diffusion (which also employs (TXDWLRQV ↑◄ DQG ◄ ☼ FRPSXWH WKH WHPSHUDWXUH DW ZKLFK WKH GLIIXVLRQ IOX[LV WKDW UHTXLUHG ♂♀ FRPSXWH WKH \LHOG strength of the nickel at this temperature using the dependence of yield strength on temperature as stated in Problem ◄ ⊥ ♂♀ FDOFXODWH WKH FLUFXPIHUHQWLDO VWUHVV RQ WKH WXEH ZDOOV XVLQJ (TXDWLRQ ◄!! DQG ♂♀ FRPSDUH WKH \LHOG strength and circumferential stress values--the yield strength should probably be at least twice the stress in order to make certain that no permanent deformation occurs. If this condition is not met then another iteration of the procedure should be conducted with a more educated choice of wall thickness.

As a starting point, let us arbitrarily choose a wall thickness of 2 mm (2×10^{-3} m). The steady-state diffusion equation, Equation 5.3, takes the form

$$J = -D \frac{\Delta C}{\Delta x}$$

$$= 5 \times 10^{-8} \text{ mol/m}^2\text{-s}$$

$$= -(4.76 \times 10^{-7}) \exp\left[-\frac{39{,}560 \text{ J/mol}}{(8.31 \text{ J/mol-K})(T)}\right] \times$$

$$\frac{(30.8) \exp\left[-\frac{12{,}300 \text{ J/mol}}{(8.31 \text{ J/mol-K})(T)}\right]\left(\sqrt{0.0203 \text{ MPa}} - \sqrt{2.026 \text{ MPa}}\right)}{0.002 \text{ m}}$$

Solving this expression for the temperature T gives T = 514 K = 241°C; this value is satisfactory inasmuch as it is less than the maximum allowable value (450°C).

7KH QH[W VWHS LV WR FRPSXWH WKH VWUHVV RQ WKH ZDOO XVLQJ (TXDWLRQ ◄!! WKXV

$$\sigma = \frac{r\,\Delta p}{4\,\Delta x}$$

$$= \frac{(0.125\ \text{m})(2.026\ \text{MPa} - 0.0203\ \text{MPa})}{(4)\left(2\ \times 10^{-3}\ \text{m}\right)}$$

$$= 31.3\ \text{MPa}$$

Now, the yield strength (σ_y) of Ni at this temperature may be computed using the expression

$$\sigma_y = 100\ \text{MPa} - \frac{5\ \text{MPa}}{50°\text{C}}\left(T - T_r\right)$$

where T_r is room temperature. Thus,

$$\sigma_y = 100\ \text{MPa} - (0.1\ \text{MPa/°C})(241°\text{C} - 20°\text{C}) = 77.9\ \text{MPa}$$

Inasmuch as this yield strength is greater than twice the circumferential stress, wall thickness and temperature values of 2 mm and 241°C are satisfactory design parameters.