Final Exam Solution of <u>Option 2</u> – Principles of EE1

Fall semester of 2011

<u>Problem 1 (10 points):</u> For the ideal op amp circuit in Fig. 1, calculate the output voltage v_0.

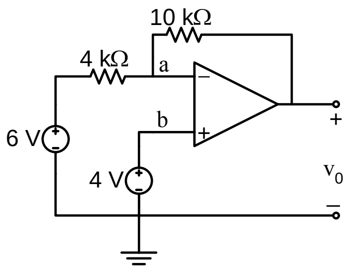


Figure 1 of problem 1

<u>Sol. of prob 1:</u> We may solve this in two ways: using superposition and using nodal analysis.
Method 1: Using superposition, we let $v_o = v_{o1} + v_{o2}$

where v_{o1} is due to the 6-V voltage source, and v_{o2} is due to the 4-V input.
To get v_{o1}, we set the 4-V source equal to zero. Under this condition, the circuit becomes an inverter. Hence:

$$v_{o1} = -\frac{10}{4}(6) = -15 \text{ V}$$

To get v_{o2}, we set the 6-V source equal to zero. The circuit becomes a noninverting amplifier so that

$$v_{o2} = \left(1 + \frac{10}{4}\right)4 = 14 \text{ V}$$

Thus

$$v_o = v_{o1} + v_{o2} = -15 + 14 = -1 \text{ V}$$

Method 2: Applying KCL at node *a*,

$$\frac{6 - v_a}{4} = \frac{v_a - v_o}{10}$$

But $v_a = v_b = 4$, and so

$$\frac{6-4}{4} = \frac{4 - v_o}{10} \qquad \Longrightarrow \qquad 5 = 4 - v_o$$

or $v_o = -1$ V, as before.

Problem 2 (10 points): Determine the current through a 200-μF capacitor whose voltage is shown in Fig. 2a. Draw the current waveform.

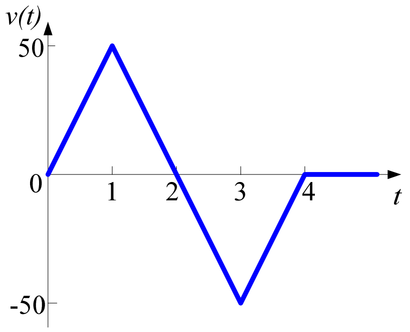


Figure 2a of problem 2

Sol.:

The voltage waveform can be described mathematically as

$$v(t) = \begin{cases} 50t \text{ V} & 0 < t < 1 \\ 100 - 50t \text{ V} & 1 < t < 3 \\ -200 + 50t \text{ V} & 3 < t < 4 \\ 0 & \text{otherwise} \end{cases}$$

Since $i = C\, dv/dt$ and $C = 200\ \mu$F, we take the derivative of v to obtain

$$i(t) = 200 \times 10^{-6} \times \begin{cases} 50 & 0 < t < 1 \\ -50 & 1 < t < 3 \\ 50 & 3 < t < 4 \\ 0 & \text{otherwise} \end{cases}$$

$$= \begin{cases} 10 \text{ mA} & 0 < t < 1 \\ -10 \text{ mA} & 1 < t < 3 \\ 10 \text{ mA} & 3 < t < 4 \\ 0 & \text{otherwise} \end{cases}$$

Thus the current waveform is as shown in Fig 2b

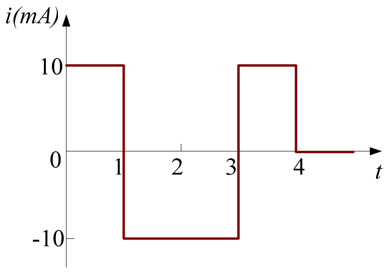


Figure 2b of problem 2

Problem 3 (10 points): write mesh equations in terms of i_1 & i_2 for the circuit shown in Fig. 3

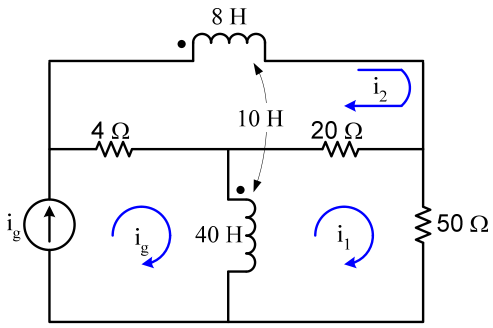


Figure 3 of problem 3

Sol.:

Mesh number #2:

$$8\frac{di_2}{dt} + 10\frac{d}{dt}(i_g - i_1) + 20(i_2 - i_1) + 4(i_2 - i_g) = 0$$

Mesh number #1:

$$40\frac{d}{dt}(i_1 - i_g) - 10\frac{di_2}{dt} + 20(i_1 - i_2) + 50i_1 = 0$$

Problem 4 (10 points): The voltage v = 12cos(60t + 45°) is applied to a 0.1 H inductor. Find the steady-state current through the inductor.

Sol. of problem 4:

For the inductor, $\mathbf{V} = j\omega L\mathbf{I}$, where $\omega = 60$ rad/s and $\mathbf{V} = 12\angle 45^\circ$ V.
Hence

$$\mathbf{I} = \frac{\mathbf{V}}{j\omega L} = \frac{12\angle 45^\circ}{j60 \times 0.1} = \frac{12\angle 45^\circ}{6\angle 90^\circ} = 2\angle -45^\circ \text{ A}$$

Converting this to the time domain,

$$i(t) = 2\cos(60t - 45^\circ) \text{ A}$$

<u>Problem 5 (10 points):</u> Find *v(t)* and *i(t)* in the circuit shown in Fig. 5

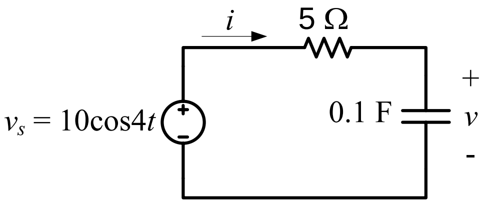


Figure 5 of problem 5

<u>Sol. of problem 5</u>

From the voltage source $10\cos 4t$, $\omega = 4$,

$$\mathbf{V}_s = 10\underline{/0^\circ}\ \text{V}$$

The impedance is

$$\mathbf{Z} = 5 + \frac{1}{j\omega C} = 5 + \frac{1}{j4 \times 0.1} = 5 - j2.5\ \Omega$$

Hence the current

$$\begin{aligned}\mathbf{I} = \frac{\mathbf{V}_s}{\mathbf{Z}} &= \frac{10\underline{/0^\circ}}{5 - j2.5} = \frac{10(5 + j2.5)}{5^2 + 2.5^2} \\ &= 1.6 + j0.8 = 1.789\underline{/26.57^\circ}\ \text{A}\end{aligned} \quad (1)$$

The voltage across the capacitor is

$$\begin{aligned}\mathbf{V} = \mathbf{I}\mathbf{Z}_C = \frac{\mathbf{I}}{j\omega C} &= \frac{1.789\underline{/26.57^\circ}}{j4 \times 0.1} \\ &= \frac{1.789\underline{/26.57^\circ}}{0.4\underline{/90^\circ}} = 4.47\underline{/-63.43^\circ}\ \text{V}\end{aligned} \quad (2)$$

Converting **I** and **V** in Eqs. (1) and (2) to the time domain, we get

$$i(t) = 1.789\cos(4t + 26.57^\circ)\ \text{A}$$
$$v(t) = 4.47\cos(4t - 63.43^\circ)\ \text{V}$$

Notice that $i(t)$ leads $v(t)$ by 90° as expected.

Problem 6 (10 points): Find the input impedance of the circuit in Fig. 6. Assume that the circuit operates at $\omega = 50$ rad/s.

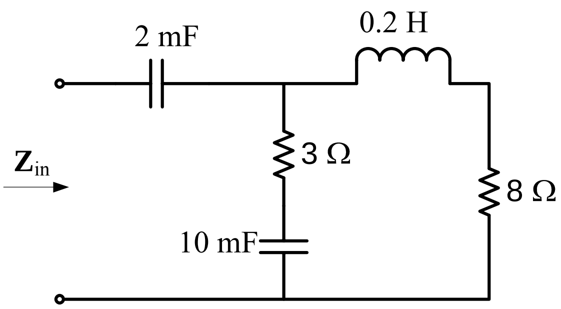


Figure 6 of problem 6

Sol. of problem 6:

Let

Z_1 = Impedance of the 2 mF capacitor
Z_2 = Impedance of the 3 Ω resistor in series with the 10 mF capacitor
Z_3 = Impedance of the 0.2 H inductor in series with the 8 Ω resistor
Then

$$\mathbf{Z}_1 = \frac{1}{j\omega C} = \frac{1}{j50 \times 2 \times 10^{-3}} = -j10\ \Omega$$

$$\mathbf{Z}_2 = 3 + \frac{1}{j\omega C} = 3 + \frac{1}{j50 \times 10 \times 10^{-3}} = (3 - j2)\ \Omega$$

$$\mathbf{Z}_3 = 8 + j\omega L = 8 + j50 \times 0.2 = (8 + j10)\ \Omega$$

The input impedance is

$$\mathbf{Z}_{\text{in}} = \mathbf{Z}_1 + \mathbf{Z}_2 \parallel \mathbf{Z}_3 = -j10 + \frac{(3 - j2)(8 + j10)}{11 + j8}$$

$$= -j10 + \frac{(44 + j14)(11 - j8)}{11^2 + 8^2} = -j10 + 3.22 - j1.07\ \Omega$$

Thus,

$$\mathbf{Z}_{\text{in}} = 3.22 - j11.07\ \Omega$$

Problem 7 (10 points): Find the steady-state expression for $v_o(t)$ in the circuit shown by using the technique of source transformations. The sinusoidal voltage sources are

$v_1 = 240\cos(4000t + 53.13°)$ V,

$v_2 = 96\sin 4000t$ V.

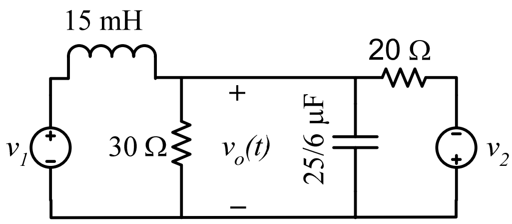


Figure 7 of problem 7

Sol. of problem 7:

$$\mathbf{V}_1 = 240\underline{/53.13^\circ} = 144 + j192\,\mathrm{V}$$

$$\mathbf{V}_2 = 96\underline{/-90^\circ} = -j96\,\mathrm{V}$$

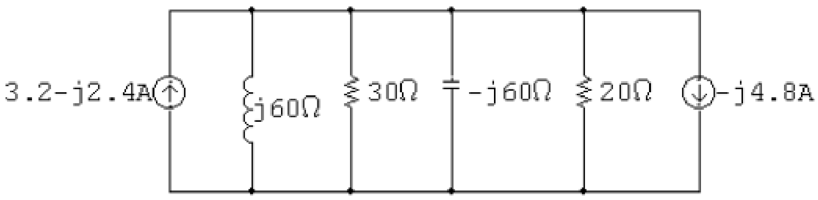
$$j\omega L = j(4000)(15 \times 10^{-3}) = j60\,\Omega$$

$$\frac{1}{j\omega C} = -j\frac{6 \times 10^6}{(4000)(25)} = -j60\,\Omega$$

Perform a source transformation:

$$\frac{\mathbf{V}_1}{j60} = \frac{144 + j192}{j60} = 3.2 - j2.4\,\mathrm{A}$$

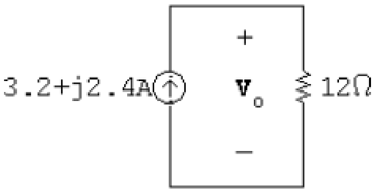
$$\frac{\mathbf{V}_2}{20} = -j\frac{96}{20} = -j4.8\,\mathrm{A}$$



Combine the parallel impedances:

$$Y = \frac{1}{j60} + \frac{1}{30} + \frac{1}{-j60} + \frac{1}{20} = \frac{j5}{j60} = \frac{1}{12}$$

$$Z = \frac{1}{Y} = 12\,\Omega$$



$$\mathbf{V}_o = 12(3.2 + j2.4) = 38.4 + j28.8\,\text{V} = 48\underline{/36.87^\circ}\,\text{V}$$

$$v_o = 48\cos(4000t + 36.87^\circ)\,\text{V}$$

Problem 8 (15 points): Use the node-voltage method to find the steady state expression for $v(t)$ in the circuit shown (Fig. 8). The sinusoidal sources are $i_s = 10\cos\omega t$ A and $v_s = 100\sin\omega t$ V, where $\omega = 50$ krad/s.

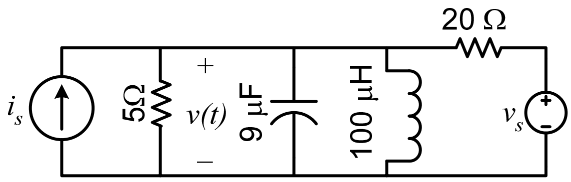
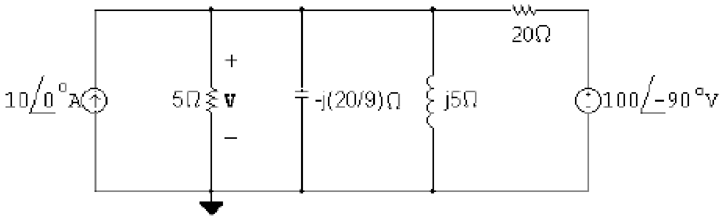


Figure 8 of problem 8

Sol. of problem 8

The phasor domain circuit is as shown in the following diagram:



The node voltage equation is

$$-10 + \frac{\mathbf{V}}{5} + \frac{\mathbf{V}}{-j(20/9)} + \frac{\mathbf{V}}{j5} + \frac{\mathbf{V} - 100\underline{/-90^\circ}}{20} = 0$$

$$\text{Therefore}\quad \mathbf{V} = 10 - j30 = 31.62\underline{/-71.57^\circ}$$

$$\text{Therefore}\quad v = 31.62\cos(50{,}000t - 71.57^\circ)\,\text{V}$$

Problem 9 (15 points): Use the mesh-current method to find the phasor current **I** in the circuit shown (Fig. 9).

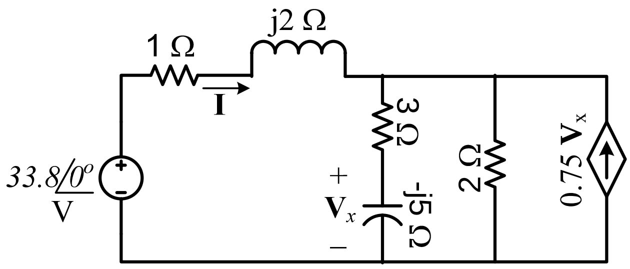


Figure 9 of problem 9

Sol. of problem 9:

Let $\mathbf{I}_a$, $\mathbf{I}_b$, and $\mathbf{I}_c$ be the three clockwise mesh currents going from left to right. Summing the voltages around meshes a and b gives

$33.8 = (1 + j2)\mathbf{I}_a + (3 - j5)(\mathbf{I}_a - \mathbf{I}_b)$

and

$0 = (3 - j5)(\mathbf{I}_b - \mathbf{I}_a) + 2(\mathbf{I}_b - \mathbf{I}_c)$.

But

$\mathbf{V}_x = -j5(\mathbf{I}_a - \mathbf{I}_b)$,

therefore

$\mathbf{I}_c = -0.75[-j5(\mathbf{I}_a - \mathbf{I}_b)]$.

Solving for $\mathbf{I} = \mathbf{I}_a = 29 + j2 = 29.07\underline{/3.95^\circ}$ A.