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① $CI = \bar{x} \pm z\frac{s}{\sqrt{n}}$

Sample mean $= 3.15$

$n = 5$

- For a 95% CI:

$CI = 3.15 \pm 1.96\left(\frac{0.1}{\sqrt{5}}\right) = 3.15 \pm 0.088$

- For a 99% CI:

$CI = 3.15 \pm 2.58\left(\frac{0.1}{\sqrt{5}}\right) = 3.15 \pm 0.115$

③ $E = 0.0065$

$z = 1.96$

$\sigma = 0.0015$

$n = \left(\frac{z \cdot \sigma}{E}\right)^2 = 34.5744 = 35$

⑤ $n = 9$

$\bar{X} = 1.0056$

$s^2 = \frac{\Sigma(x_i - \bar{x})^2}{n-1} = 0.00060278$

$s = \sqrt{0.00060278} = 0.0246$

$\bar{x} - t_{\alpha/2}\frac{s}{\sqrt{n}} < \mu < \bar{x} + t_{\alpha/2}\frac{s}{\sqrt{n}}$

$1.0056 - 2.896\frac{0.0246}{\sqrt{9}} < \mu < 1.0056 + 2.896\frac{0.0246}{\sqrt{9}}$

$0.9819 < \mu < 1.0293$

② $\bar{x} \pm z^* \frac{\sigma}{\sqrt{n}}$

σ : population standard deviation $= 11.3$

n : Sample size $= 81$

$\bar{x}$: sample mean $= 74.6$

z^* = Critical z-value (two-tailed) $= \pm 1.645$

$74.6 \pm (1.645)\frac{11.3}{\sqrt{81}} \approx 74.6 \pm 2.065$

$= (74.6 - 2.065, 74.6 + 2.065)$

$= (72.535, 76.665)$

④ $\bar{X} = 3103$ g

$s = 696$ g

$n = 186$

confidence level $= 95\%$

$n \geq 30$, so the distribution of sample mean is approximate normal

$E = t_{\alpha/2}\frac{s}{\sqrt{n}} = 1.984 \cdot \frac{696}{\sqrt{186}} = 100.025$

$\Rightarrow \bar{x} \pm E = 3103 \pm 100.025$

$= (3002.975; 3203.025)$

⑥ Sample proportion (p) $= \frac{228}{1000} = 0.228$

standard error (SE) $= \sqrt{\frac{p(1-p)}{n}} = 0.014$

a) $CI = p \pm z \cdot SE$

$= 0.228 \pm 2.58 \times 0.014 = (0.198, 0.258)$

$E = 0.05$

$1 - p = 1 - 0.228 = 0.772$

$n = \left(\frac{2.58}{0.05}\right)^2 \times 0.228 \times 0.772$

$= 467$

b)

(7) $n = 20$ (sample size)

$\bar{X} = 72$ (sample mean)

$S^2 = 16$ (Sample variance)

Assuming the scored to be normally distributed 98% confidence interval for σ^2 (Formula)

$$\frac{(n-1)s^2}{\chi^2_{\alpha/2}} < \sigma^2 < \frac{(n-1)s^2}{\chi^2_{1-\alpha/2}}$$

$$\frac{(19 \times 16)}{36.1908} < \sigma^2 < \frac{19 \times 16}{7.6327}$$

$$\Rightarrow 8.3999 < \sigma^2 < 39.8286$$

(8)

observation	X	X^2
1	140	19600
2	136	18496
3	150	22500
4	144	20736
5	148	21904
6	152	23104
7	138	19044
8	141	19881
9	143	20449
10	151	22801

sum = 1443 , 208515

a) $S^2 = \frac{1}{n-1}\left(\Sigma X^2 - \frac{1}{n}\left(\sum^n X\right)^2\right)$

$= \frac{1}{9}\left(208515 - \frac{1443^2}{10}\right)$

$= 32.2333$

b) $\alpha = 0.01$ and $df = 9$

$X_L^2 = X^2_{1-\alpha/2,\,n-1} = 1.7349$

$X_U^2 = X^2_{\alpha/2,\,n-1} = 23.5894$

$CI = \left(\frac{(n-1)s^2}{X^2_{\alpha/2,n-1}} ; \frac{(n-1)s^2}{X^2_{1-\alpha/2,n-1}}\right)$

$= (12.2979, 167.2111)$