Calculus 2
Fall 2023

Homework

Chapter 1

Week 1

1. Consider the following sequence $\{a_n\}$ with the few first terms given as

$$\{-\frac{1}{2}, \frac{16}{3}, -\frac{81}{4}, \frac{256}{5}, -\frac{625}{6}, ...\}$$

Find a formula for the general term a_n.

2. If \$600 is invested at 4% interest, compounded annually. Find the size of investment after 7 years.

3. Determine the limits of the following sequences

a) $a_n = \frac{3n^3}{n^3+1}$

b) $b_n = \left(\frac{5+n}{n}\right)^n$

c) $c_n = n^{1/n}$

d) $d_n = \ln(n^3+1) - \ln(3n^3+10n)$

Solution

a) Consider function $f(x) = \frac{3x^3}{x^3+1}$, we see that $f(n) = a_n$ and $\lim_{x\to\infty} f(x) = 3$. Hence $\lim_{n\to\infty} a_n = 3$

b) Consider $\lim_{n\to\infty} \ln(b_n) = \lim_{n\to\infty} n \ln(\frac{5+n}{n}) = \lim_{n\to\infty} \frac{\ln(\frac{5+n}{n})}{1/n}$. Using L'Hopital we get $\lim_{n\to\infty} \ln(b_n) = \lim_{n\to\infty} \frac{5}{1+5/n} = 5$. So $\lim_{n\to\infty} b_n = e^5$

c) Similarly to b) we have $\lim_{n\to\infty} c_n = 1$

d) Note that $d_n = \ln(n^3+1) - \ln(3n^3+10n) = \ln(\frac{n^3+1}{3n^3+10n})$. So $\lim_{n\to\infty} d_n = -\ln(3)$

4. Using squeeze Theorem find the limit of the sequence

$$a_n = \frac{\sin(2n)}{2^n}$$

Solution Since $-1 \leq \sin(2n) \leq 1$ for all n, hence, $-1/2^n \leq \frac{\sin(2n)}{2^n} \leq 1/2^n$. Using squeeze theorem we have $\lim_{n\to\infty} a_n = 0$

Week 2

1. Check the following series if the series is convergent or divergent

a) $\sum_{n=0}^{\infty} 2^{1-3n}3^{n+2}$

b) $\sum_{n=1}^{\infty} \frac{3}{n^2+7n+12}$

Solution

a) We have

$$\sum_{n=0}^{\infty} 2^{1-3n}3^{n+2} = 18\sum_{n=0}^{\infty}(\frac{3}{8})^n$$

Since $r = 3/8 < 1$ then $\sum_{n=0}^{\infty}(\frac{3}{8})^n$ is convergent, so $\sum_{n=0}^{\infty} 2^{1-3n}3^{n+2}$ is convergent.

b) Note that

$$\frac{3}{n^2+7n+12} = 3(\frac{1}{n+3} - \frac{1}{n+4})$$

So

$$S_n = \sum_{k=1}^{n} \frac{3}{k^2+7k+12} = 3[\sum_{k=1}^{n}(\frac{1}{k+3} - \frac{1}{k+4})] = \frac{3}{4} - \frac{3}{n+4}$$

We get,

$$\lim_{n\to\infty} S_n = \frac{3}{4}$$

Hence, the series is convergent and $\sum_{n=1}^{\infty} \frac{3}{n^2+7n+12} = \frac{3}{4}$

2 Using integral test to check the following series if the series converges or diverges.

a) $\sum_{n=1}^{\infty} \frac{1}{n^{\pi}}$

b) $\sum_{n=0}^{\infty} \frac{2}{5n+3}$

c) $\sum_{n=0}^{\infty} \frac{n^2}{n^3+1}$

d) $\sum_{n=0}^{\infty} \frac{1}{n^2+4}$

Solution

a) Since $p = \pi > 1$, hence the series converges

b) we have $a_n > 0$ and $a_n > a_{n+1}$. Now compute the integral for the test.

$$\int_1^\infty \frac{2}{5x+3}dx = \infty$$

So the series diverges.

c) Consider the function $f(x) = \frac{x^2}{x^3+1}$, hence, $f'(x) = \frac{x(2-x^3)}{(x^3+1)^2}$. So $f(x)$ is decreasing if $x > \sqrt[3]{2}$. Now compute the integral for the test

$$\int_1^\infty \frac{x^2}{x^3+1}dx = \infty$$

So the series diverges.

3. Using the Divergence Test to determine if the following series diverges or conclude that the Divergence Test is inconclusive

a) $\sum_{n=1}^{\infty} \frac{3^n+1}{2^n}$

b) $\sum_{n=1}^{\infty} \frac{1}{n^2}$

c) $\sum_{n=2}^{\infty} \frac{n}{\ln(n)}$

Solution

c) Since $\lim_{n\to\infty} a_n = \infty$ hence the series diverges

4. Using comparison test or limit comparison test to determine if the following series diverges or converges

a) $\sum_{n=1}^{\infty} (\frac{1}{n^2} + 1)^2$

b) $\sum_{n=1}^{\infty} \frac{4}{n^2-2n-3}$

c) $\sum_{n=1}^{\infty} \frac{n^3}{2n^4-1}$

d) $\sum_{n=2}^{\infty} \frac{\ln(n)}{n^3}$

Solution

a) We see that for all $n \geq 1$, it holds

$$(\frac{1}{n^2}+1)^2 = \frac{1}{n^2} + \frac{2}{n} + 1 < \frac{3}{n} + 1$$

However, the series

$$\sum_{i=1}^{n} \frac{3}{n}$$

is divergent and also

$$\sum_{i=1}^{n} 1 = \infty$$

So the series $\sum_{n=1}^{\infty} (\frac{1}{n^2}+1)^2$ also diverges.
Alternative. We check

$$\lim_{n\to\infty} (\frac{1}{n^2}+1)^2 = 1 \neq 0$$

So the series is divergent.

b) We see that for $n \geq 7$, it holds $n^2 - 2n - 3 > 0$. Therefore, the series terms are positive, decreasing. We have $\sum_{n=1}^{\infty} \frac{4}{n^2}$ converges and

$$\lim_{n\to\infty} \frac{a_n}{b_n} = \lim_{n\to\infty} \frac{4n^2}{n^2 - 2n - 3} = 1 < \infty$$

So $\sum_{n=1}^{\infty} \frac{4}{n^2-2n-3}$ also converges.

c) we have $\frac{n^3}{2n^4-1} > \frac{1}{2n}$. But $\sum_{n=1}^{\infty} \frac{1}{2n}$ diverges. So the series diverges.

d) Note that for all $n \geq 2$, we have $\ln(n) < n$. So

$$\frac{\ln(n)}{n^3} < \frac{1}{n^2}$$

Since $\sum_{n=2}^{\infty} \frac{1}{n^2}$ converges, hence the series converges.

5. Determine whether the following series converges or diverges

a) $\sum_{n=1}^{\infty} \sqrt{\frac{n+1}{n}}$

b) $\sum_{n=0}^{\infty} \frac{10}{n^2+9}$

c) $\sum_{n=2}^{\infty} \frac{4}{n \ln^2(n)}$

Solution

a) Since $\lim_{n\to\infty} a_n = 1 \neq 0$ hence the series diverges

b) Using integration test the series is convergent.

c) Using integration test $\int_2^{\infty} \frac{4}{x\ln^2(x)}dx = \frac{4}{\ln(2)}$ hence the series is convergent.

6 Consider the series

$$\sum_{n=2}^{\infty} \frac{1}{n\ln^p(n)}$$

where p is a real number.

i) Using the integral test to determine the value of p for which the series converges

ii) Does the series converge faster for $p = 2$ or $p = 3$? Explain.

Solution

i) In order to the series converges, the integral

$$\int_2^{\infty} \frac{1}{x\ln^p(x)}dx$$

must exist. We have

$$\int \frac{1}{x\ln^p(x)}dx = \frac{1}{1-p}\ln^{1-p}(x)$$

So we have $1 - p < 0$ hence $p > 1$

ii) The series converges faster for $p = 3$ since the term of the series get smaller faster.

Chapter 2

Week 4

1. Determine the dot product of vectors a and b given as follows

i) $a = (9, 5, -4, 2)$; $b = (-3, -2, 7, -1)$

(ii) $a = (0, 4, -2), b = 2i - j + 7k$

(iii) $||a|| = 5, ||b|| = 3/7$ and the angle between the two vectors is $\theta = \pi/12$

2. Determine the angle between the following two vectors

i) $u = (1, 0, 3); v = (1, -4, 2)$

ii) $a = i + 3j - 2k; b = (-9, 1, -5)$

3. Cross product

i) Find the cross product of vectors $a = (3, -1, 5); b = (0, 4, -2)$

ii) Find a vector such that it is orthogonal to the plane containing the points

$$P = (3, 0, 1); Q = (4, -2, 1); Q = (5, 3, -1)$$

iii) Check if the vectors $u = (1, 2, -4); v = (-5, 3, -7); w = (-1, 4, 2)$ are in the same plane?

4. For the given vectors u and v, calculate $proj_v u$, and $comp_v u$

a) $u =< 13, 0, 26 >$, and $v =< 4, -1, -3 >$

b) $u =< -8, 0, 2 >$, and $v =< 1, 3, -3 >$

c) $u = 5i + j - 5k$, and $v = -i + j - 2k$

5. Find the area of the parallelogram that has two adjacent sides u and v.

a) $u = 2i - j - 2k, v = 3i + 2j - k$

b) $u = 8i + 2j - 3k, v = 2i + 4j - 4k$

6. For the given points A, B, and C, find the area of the triangle with vertices A, B, and C

$$A = (1, 2, 3), B = (5, 1, 5), C = (2, 3, 3)$$

7. Find equations of the following lines.

a) the line through $(1, -3, 4)$ that is parallel to the line $r(t) =< 3 + 4t, 5 - t, 7 >$

b) The line through $(-3, 4, 2)$ that is perpendicular to both $u =< 1, 1, -5 >$, and $v =< 0, 4, 0 >$

8. Find the equation of the line segment between $P_0(3,-1,4)$, and $P_1=(0,5,2)$

9. Find an equation of the plane.

a) The plane through the point $(5,3,5)$ and with normal vector $2i+j-k$

b) The plane through the point $(2,0,1)$ and perpendicular to the

$$x=3t, y=2-t, z=3+4t$$

10. Is the line through $(-2,4,0)$ and $(1,1,1)$ perpendicular to the line through $(2,3,4)$ and $(3,-1,-8)$?

11. The plane through the point $(1,-1,-1)$ and parallel to the plane $5x-y-z=5$

12. The plane that contains the line $x=1+t, y=2-t, z=4-3t$ and is parallel to the plane $5x+2y+z=1$.

13. The plane through the points $(2,1,2),(3,-8,6)$, and $(-2,-3,1)$

14. Find the domain of the vector function

$$r(t)=\cos(t)i+\ln(t)j+\frac{1}{t-2}k$$

15. Let

$$r(t)=<te^{-t},\frac{t^3+t}{2t^3-1},t\sin(1/t)>.$$

Find $\lim_{t\to\infty} r(t)$.

16. Find the length of the curve

a) $r(t)=<t,3\cos(t),3\sin(t)>, \quad -5\le t\le 5$

b) $r(t)=\sqrt{2}ti+e^tj+e^{-t}k, \quad 0\le t\le 1$

Chapter 3

Week 5

1. Find the domain of the given functions.

a) $f(x, y) = \sqrt{x^2 - 2y}$

b) $f(x, y) = \ln(2x - 3y + 1)$

2. Find limits of the following functions

(a) $\lim_{(x,y)\to(2,1)} \frac{x^2-2xy}{x^2-4y^2}$

(b) $\lim_{(x,y)\to(0,0)} \frac{x-4y}{6y+7x}$

(c) $\lim_{(x,y)\to(0,0)} \frac{x^2-y^6}{xy^3}$

3. Find the first order partial derivatives of the following functions

a) $f(x, y, z) = 4x^3y^2 - e^zy^4 + \frac{z^3}{x^2} + 4y - x^{16}$

b) $f(x, y) = \frac{x^2}{y^2+1} - \frac{y^2}{x^2+y}$

Week 6

1. Find an equation of the plane tangent to the following at the given point

a) $x^2 + y + z = 1$; $P(1, 1, 1)$

b) $x^2 + y^3 + z^4 = 2$; $Q(1, 0, 1)$

c) $xy + xz + yz - 12 = 0$; $R(2, 0, 6)$

2. Find the linear approximation to the function f at the given point and estimate the given function value.

a) $f(x, y) = xy + x - y$; $P(2, 3)$; estimate $f(2.1, 2.99)$

b) $f(x, y) = \sqrt{x^2 + y^2}$; $Q(3, -4)$; estimate $f(3.06, -3.92)$

c) $f(x, y, z) = \ln(1 + x + y + 2z)$; $R(0, 0, 0)$; estimate $f(0.1, -0.2, 0.2)$

3. Find the following derivatives.

a. $z = x^2y, -xy^2, x = t^2, y = t^{-2}$. Find dz/dt

b. $z = e^{x+y}, x = st, y = s + t$. Find $\frac{\partial z}{\partial t}$ and $\frac{\partial z}{\partial s}$

c. $y\ln(x^2 + y^2 + 4) = 0$, find dy/dx

Week 7

1. Consider the function $f(x, y) = 8 = \frac{x^2}{2} - y^2$. Find the directional derivative at $(2, 0)$ in the corresponding directions by the unit vectors $u =< \sqrt{2}/2, \sqrt{2}/2 >$, $v =< -\sqrt{2}/2, \sqrt{2}/2 >$, and $w =< -\sqrt{2}/2, -\sqrt{2}/2 >$.

Solution We hav $f_x = -x, f_y = -2y$, and hence, $\nabla f(2, 0) =< -2, 0 >$. So we get

- $D_u f(2, 0) = \nabla f(2, 0) \cdot u =< -2, 0 >< \frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2} >= -\sqrt{2}$
- $D_v f(2, 0) = \nabla f(2, 0) \cdot v =< -2, 0 >< -\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2} >= \sqrt{2}$
- $D_w f(2, 0) = \nabla f(2, 0) \cdot u =< -2, 0 >< -\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2} >= \sqrt{2}$

2. Use the second derivatives test to show that function f has a local minimum or maximum

a) $f(x, y) = x^2 + 2y^2 - 4x + 4y + 6$

b) $f(x, y) = xy(x - 2)(y + 3)$

c) $f(x, y) = 2xye^{-x^2-y^2}$

Solution

a) We have $f_x(x, y) = 2x-4$, $f_y(x, y) = 4y+4$, and $f_{xx}(x, y) = 2$, $f_{xy}(x, y) = 0$, $f_{yy}(x, y) = 4$. Then

$$D(2, -1) = f_{xx}(2, -1)f_{yy}(2, -1) - [f_{xy}(2, -1)]^2 = 8 > 0$$

and $f_{xx}(x, y) = 2 > 0$. So f has local minimum at $(2, -1)$.

c) We have

$$f_x(x,y) = 2(1-2x^2)ye^{-x^2-y^2} \qquad f_y(x,y) = 2(1-2y^2)xe^{-x^2-y^2}$$
$$f_{xx}(x,y) = 4(2x^2-3)xye^{-x^2-y^2} \qquad f_{xy}(x,y) = 2(1-2x^2)(1-2y^2)e^{-x^2-y^2}$$
$$f_{yy}(x,y) = 4(2y^2-3)xye^{-x^2-y^2}$$

So critical points are:

$$A_1 = (0,0), A_2 = (\frac{1}{\sqrt{2}}, = \frac{1}{\sqrt{2}}), A_3 = (-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}), A_4 = (\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}), A_5 = (-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}})$$

We get

- $D(A_1) = -4 < 0 \Rightarrow$ sadle point
- $D(A_2) > 0, f_{xx}(A_2) < 0 \Rightarrow$ local maximum
- $D(A_3) > 0, f_{xx}(A_3) < 0 \Rightarrow$ local maximum
- $D(A_4) > 0, f_{xx}(A_4) > 0 \Rightarrow$ local minimum
- $D(A_5) > 0, f_{xx}(A_5) > 0 \Rightarrow$ local minimum

3. Find the absolute maximum and minimum values of the function over the given region R

a) $f(x,y) = 4 + 2x^2 + y^2, \quad R = \{(x,y) : -1 \le x \le 1, -1 \le y \le 1\}$

b) $f(x,y) = xy - 8x - y^2 + 12y + 160, \quad R = \{(x,y) : 0 \le x \le 15, 0 \le y \le 15 - x\}$

c) $f(x,y) = x^2 + y^2 - 2y + 1, \quad R = \{(x,y) : x^2 + y^2 \le 4\}$

Solution

a) We have $f_x(x,y) = 4x = 0; f_y(x,y) = 2y = 0$, hence $(0,0)$ is a critical point and $f(0,0) = 4$.

- On the sides $y = -1, -1 \le x \le 1$, and $y = 1, -1 \le x \le 1$, we have

$$f(x,1) = f(x,-1) = 2x^2 + 5 = \begin{cases} 5, & x = 0 \\ 7, & x = -1 \quad \text{or} \quad x = -1 \end{cases}$$

- On the sides $x = -1, -1 \le y \le 1$, and $x = 1, -1 \le y \le 1$, we have

$$f(-1,y) = f(1,y) = x^2 + 6 = \begin{cases} 6, & y = 0 \\ 7, & y = -1 \quad \text{or} \quad y = -1 \end{cases}$$

So minimum value is 4 at $(0,0)$ and maximum value is 7 at $(\pm 1, \pm 1)$

b) We have $f_x = y - 8; f_y = x - 2y + 12$. Hence critical point is $(4,8)$. This point is in the interior of R. So it is a candidate for local of an extreme value of f

$$f(4,8) = 192$$

Consider boundary on R, we consider each edge of R separately.

- Let C_1 be a line segment

$$\{(x,y) : y = 0, 0 \leq x \leq 15\}$$

set $g_1(x) = f(x,0) = 160 - 8x, 0 \leq x \leq 15$. This function has no critical point. We have

$$g_1(0) = f(0,0) = 160; g_1(15) = f(15,0) = 40$$

- Let C_2 be a line segment

$$\{(x,y) : x = 0, 0 \leq y \leq 15\}$$

set $g_2(y) = f(0,y) = -y^2 + 12y + 160, 0 \leq x \leq 15$. Hence, $g_2^{'}(y) = -2y + 12 = 0 \Rightarrow y = 6$. We have

$$g_2(6) = f(0,6) = 196$$

$$g_2(0) = f(0,0) = 160$$

$$g_2(15) = f(0,15) = 115$$

- Let C_3 be a line segment

$$\{(x,y) : y = 15 - x, 0 \leq x \leq 15\}$$

set $g_3(x) = f(x, 15-x) = -2x^2 + 25x + 115, 0 \leq x \leq 15$. Similarly we also obtain

$$\begin{aligned} f(6.25, 8.75) &= 193.125 \\ f(15,0) &= 40 \\ f(0,15) &= 115 \end{aligned}$$

Compare all these values we get the absolute minimum is 40, the absolute maximum is 196.

c) Similarly to questions a) and b) we have: $maxf = f(0,-2) = 9$; $minf = f(0,1) = 1$

4. Lagrange multipliers: Find the absolute maximum and minimum values of the functions

a) $f(x,y) = xy^2$ subject to $g(x,y) = x^2 + xy + y^2 - 4$

b) $f(x,y) = x + 2y$ subject to $g(x,y) = x^2 + y^2 = 4$

Solution

a) We have $\nabla f(x,y) =< 2x, 2y >; \nabla g(x,y) =< 2x+y, x+2y >$. So

$$\begin{cases} 2x = \lambda(2x+y) \\ 2y = \lambda(x+2y) \\ x^2 + xy + y^2 - 4 = 0 \end{cases}$$

Hence, $(x-y)(2-\lambda) = 0$. So we get two candidates $(\frac{2}{\sqrt{3}}, \frac{2}{\sqrt{3}})$ and $(-\frac{2}{\sqrt{3}}), -\frac{2}{\sqrt{3}}$. Substituting $\lambda = 2$ into the first equation we get $y = -x$ then from constrain equation we obtain

$$x^2 - 4 = 0 \Rightarrow x = \pm 2$$

these values give two addition points $(2,-2); (-2,2)$. We now have

$$f(\frac{2}{\sqrt{3}}, \frac{2}{\sqrt{3}}) = f(-\frac{2}{\sqrt{3}}, -\frac{2}{\sqrt{3}}) = 14/3$$

and

$$f(2,-2) = f(-2,2) = 10$$

So the maximum is 10, and minimum is 14/3

b) Do similarly to question a) we get

$$minf = f(-\frac{2}{\sqrt{5}}, -\frac{4}{\sqrt{5}}) = -2\sqrt{5}$$

$$maxf = f(\frac{2}{\sqrt{5}}, \frac{4}{\sqrt{5}}) = 2\sqrt{5}$$

Chapter 4

Week 8

1 Evaluate the following integrals

a) $\int\int_R xydA$, R is bounded by $x = 0, y = 2x + 1, y = -2x + 5$

b) $\int\int_R (x + y)dA$, R is the region in the first quadrant bounded by $x = 0, y = x^2, y = 8 - x^2$

c) $\int\int_R x^2ydA$, R is the region in quadrants 1 and 4 bounded by the semicircle of radius 4 centered at $(0, 0)$.

short answers

a)

$$= \int_0^1 \int_{2x+1}^{-2x+5} xydydx = 2$$

b)

$$= \int_0^2 \int_{x^2}^{8-x^2} (x + y)dydx = 152/3$$

c)

$$= \int_{-4}^4 \int_0^{\sqrt{16-x^2}} x^2ydxdy = 0$$

2. Find the volume of the following solids.

a) The solid bounded by the cylinder $z = 2 - y^2$, the xy-plane, the xz-plane, and the planes $y = x$ and $x = 1$

b) The solid bounded between the cylinder $z = 2\sin^2(x)$ and the xy-plane over the region $R = \{(x, y) : 0 \leq x \leq y \leq \pi\}$

short answers

a)

$$V = \int_0^1 \int_0^x (2 - y^2)dydx = 11/12$$

b)

$$V = \int_0^\pi \int_0^y (2\sin^2(x)dxdy = \pi^2/2$$

3. Use double integrals to compute the area of the following regions.

a) The region bounded by the parabola $y = x^2$ and the line $y = 4$

b) The region bounded by the parabola $y = x^2$ and the line $y = x + 2$

c) The region in the first quadrant bounded by $y = e^x$ and $x = \ln 2$

Short answers

a)
$$A = \int_{-2}^{2} \int_{x^2}^{4} 1 dydx = 32/3$$

b)
$$A = \int_{-1}^{2} \int_{x^2}^{x+2} 1 dydx = 9/2$$

c)
$$A = \int_{0}^{\ln(2)} \int_{0}^{e^x} 1 dydx = 1$$

4. Evaluate the following integrals using polar coordinates

a) $\int\int_R (x^2 + y^2) dA$, $\quad R = \{(r, \theta) : 0 \leq r \leq 4, 0 \leq \theta \leq 2\pi\}$

b) $\int\int_R xy dA$, $\quad R = \{(x, y) : x^2 + y^2 \leq 9, y \geq 0\}$

c) $\int_0^3 \int_0^{\sqrt{9-x^2}} \sqrt{x^2 + y^2} dydx$

Short answers

a)
$$= \int_0^{2\pi} \int_0^4 r^2 r dr d\theta = 128\pi$$

b)
$$= \int_0^{\pi} \int_0^3 (r \cos\theta)(r \sin\theta) r dr d\theta = 0$$

c)
$$= \int_0^{\pi/2} \int_0^3 r^2 r dr d\theta = 9\pi/2$$

5. Evaluate each double integral over the region R by converting it to an iterated integral.

a) $\int\int(x^2+xy)dA, \quad R=\{(x,y):0\le x\le 3, 1\le y\le 4\}$

b) $\int\int \frac{x}{1+xy}dA, \quad R=\{(x,y):0\le x\le 1, 0\le y\le 1\}$

Chapter 5: Vector Calculus

Week 9

1. Find the gradient field $F=\nabla f$ for the following potential functions f

a) $f(x,y)=x^2y-y^2x$

b) $f(x,y,z)=\ln(1+x^2+y^2+z^2)$

2. The temperature of the circular plate $R=\{(x,y):x^2+y^2\le 1\}$ is

$$f(x,y)=100(x^2+2y^2)$$

Find the average temperature along the edge of the plate.

3. Evaluate

$$\int_C (xy+2z)ds$$

on the following line segments.

a) The line segment from $P(1,0,0)$ to $Q(0,1,1)$

b) The line segment from $Q(0,1,1)$ to $P(1,0,0)$

4. Evaluate line Integrals of Vector Fields

$$\int_C \mathbf{F}\cdot\mathbf{T}ds \quad \text{or} \quad \int_C \mathbf{F}\cdot d\mathbf{r}$$

with $\mathbf{F}=<y-x,x>$ on the following paths in R^2

a) The quarter-circle C_1 from $P(0,1)$ to $Q(1,0)$

b) The quarter-circle $-C_1$ from $Q(1,0)$ to $P(0,1)$

c) the path C_2 from $P(0,1)$ to $Q(1,0)$ via two line segments through $O(0,0)$

5. Evaluate the following line integrals along the curve C.

a) $\int_C \frac{x}{x^2+y^2} ds$, $\quad C$ is the line segment from $(1,1)$ to $(10,10)$

b) $\int_C (xy)^{1/3} ds$, $\quad C$ is the curve $y = x^2$, for $0 \leq x \leq 1$

6. Given the following vector fields and oriented curves C, evaluate $\int_C \mathbf{F} \cdot \mathbf{T} ds$

a) $\mathbf{F} =< -y, x >$, on the parabola $y = x^2$ from $(0,0)$ to $(1,1)$

b)
$$\mathbf{F} = \frac{< x, y >}{(x^2+y^2)^{3/2}}$$
on the curve $\mathbf{r}(t) =< t^2, 3t^2 >$, for $1 \leq t \leq 2$

Week 10

1. Determine whether the following vector fields are conservative

a) $\mathbf{F} =< e^x \cos y, -e^x \sin y >$

b) $\mathbf{F} =< 2xy - z^2, x^2 + 2z, 2y - 2xz >$

2. Using Green's theorem to evaluate Line integral. Assume all curves are oriented counterclockwise.

a) $\int_C (4x^3 + \sin y^2) dy - (4y^3 + \cos x^2) dx$
where C is the boundary of the disk $R = \{(x,y) : x^2 + y^2 \leq 4\}$

b) $\int_C < 3y + 1, 4x^2 + 3 > \cdot d\mathbf{r}$
where C is the boundary of the rectangle with vertices $(0,0), (4,0), (4,2), (0,2)$

c) $\int_C xe^y dx + x dy$,
where C is the boundary of the region bounded by the curves $y = x^2, x = 2$, and the x-axis.