

Homework: 49, 56, 67, 60, 64, 74 (p. 234-237)

49. A bullet of mass 10g strikes a ballistic pendulum of mass 2kg. The center of mass of the pendulum rises a vertical distance of 12 cm. Assuming that the bullet remains embedded in the pendulum, calculate the bullet's initial speed.

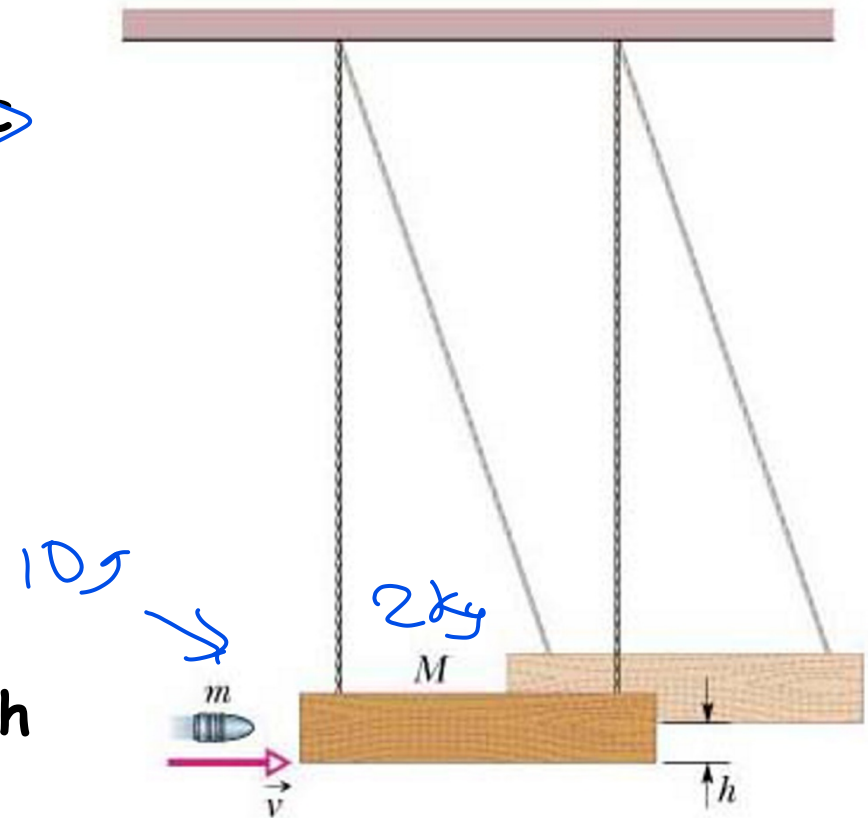
The collision here is a perfectly inelastic collision, the linear momentum of the system bullet + pendulum is conserved because the external impulse J on the system is zero:

$$\Rightarrow mv = (m + M)V \Rightarrow V = \frac{m}{m + M} v$$

After the collision, the mechanical energy of the system bullet-block-Earth is conserved:

$$\frac{1}{2}(m + M)V^2 = (m + M)gh \Rightarrow V = \sqrt{2gh}$$

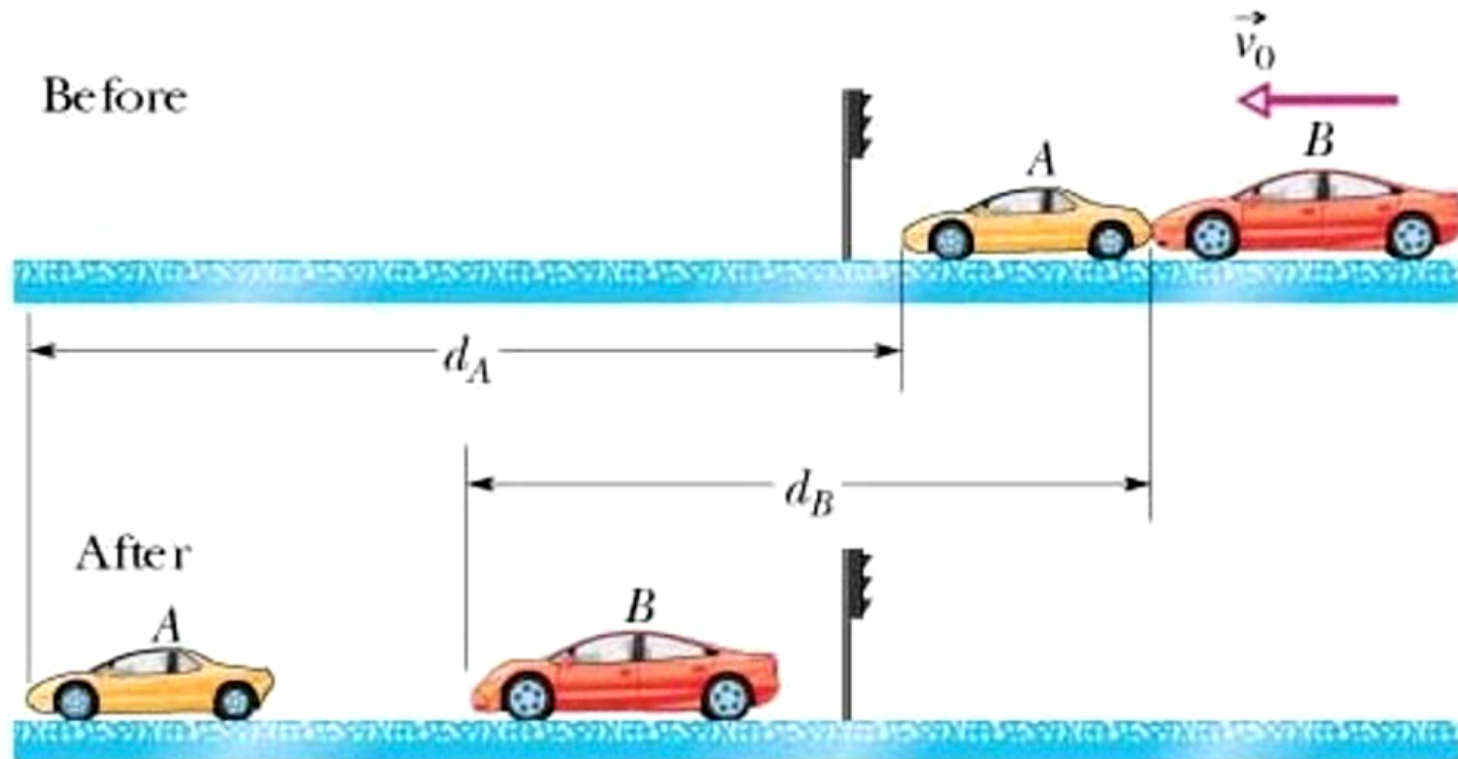
$$\Rightarrow v = \frac{m + M}{m} \sqrt{2gh}$$



Read also Sample Problem 9-8 (page 218)



56. In the "before" part of the figure below, car A (mass 1100 kg) is stopped at a traffic light when it is rear-ended by car B (mass 1400 kg). Both cars then slide with locked wheels until the frictional force from the slick road (with a low μ_k of 0.10) stops them, at distance $d_A = 8.2$ m and $d_B = 6.1$ m. What are the speeds of (a) car A and (b) car B at the start of the sliding, just after the collision? (c) Assuming that linear momentum is conserved during the collision, find the speed of car B just before the collision. (d) Explain why this assumption may be invalid.



$$v^2 - v_0^2 = -2ad \Rightarrow v_0 = \sqrt{2ad} = \sqrt{2\mu_k gd}$$

The magnitude of the acceleration of each car is determined by:

$$a = \frac{F_k}{m} = \frac{\mu_k mg}{m} = \mu_k g$$

Velocities of car A and B after the collision:

$$\Rightarrow v_A = \sqrt{2\mu_k gd_A}; \quad v_B = \sqrt{2\mu_k gd_B}$$

(c) If p conserved: $\Delta p = J = F_{avg} \Delta t = 0$

$$m_B v_{0,B} = m_A v_A + m_B v_B \Rightarrow v_{0,B} = \frac{m_A v_A + m_B v_B}{m_B}$$

(d) p is conserved if the frictional force exerted on the cars from the road is negligible during the collision.

However, $\Delta p = J = F_{avg} \Delta t \neq 0$

Therefore, the assumption that p is conserved (or $\Delta p = 0$) may be invalid

60. Block A (mass 1.6 kg) slides into block B (mass 2.4 kg), along a frictionless surface. The directions of three velocities before (i) and after (f) the collision are indicated; the corresponding speeds are $v_{Ai} = 5.5 \text{ m/s}$, $v_{Bi} = 2.5 \text{ m/s}$, and $v_{Bf} = 4.9 \text{ m/s}$. What are the (a) speed and (b) direction (left or right) of velocity v_{Af} ? (c) Is the collision elastic?

We choose the positive direction is rightward

(a) The linear momentum of the system (A+B) is conserved (no friction):

$$m_A v_{Ai} + m_B v_{Bi} = m_A v_{Af} + m_B v_{Bf}$$

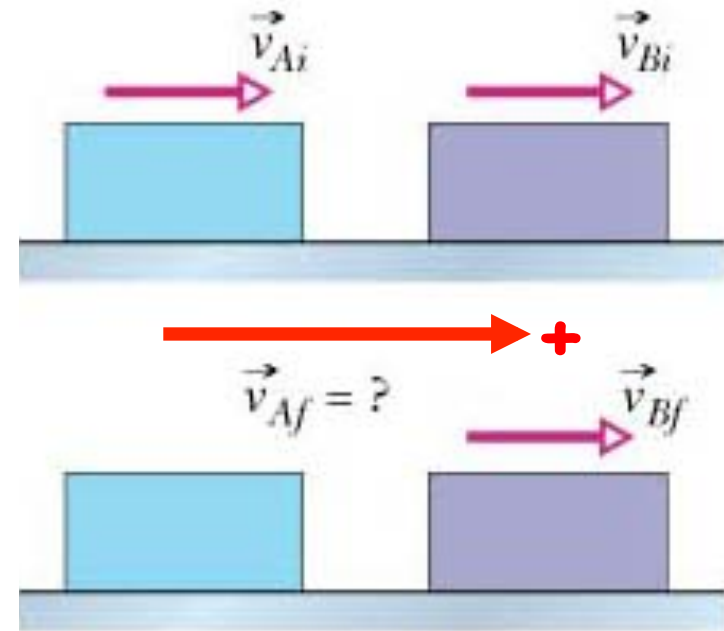
$$v_{Af} = v_{Ai} + \frac{m_B}{m_A} (v_{Bi} - v_{Bf})$$

$$v_{Af} = 5.5 + \frac{2.4}{1.6} (2.5 - 4.9) = 1.9 (\text{m/s})$$

(b) $v_{Af} > 0$, so the v_{Af} direction is to the right

$$(c) \Delta K = K_f - K_i = \frac{1}{2} (m_A v_{Af}^2 + m_B v_{Bf}^2) - \frac{1}{2} (m_A v_{Ai}^2 + m_B v_{Bi}^2) = 0 \text{ (J)}$$

\Rightarrow the collision is elastic



64. A steel ball of mass 2.5 kg is fastened to a cord that is 70cm long and fixed at the far end. The ball is then released when the cord is horizontal. At the bottom of its path, the ball strikes a 2.8 kg steel block initially at rest on a frictionless surface. The collision is elastic. Find (a) the speed of the ball and (b) the speed of the block, both just after the collision.

Conservation of mechanical energy: $U_g = mgh$

$$mgh = mgl = \frac{1}{2}mv^2 \Rightarrow v = \sqrt{2gh}$$

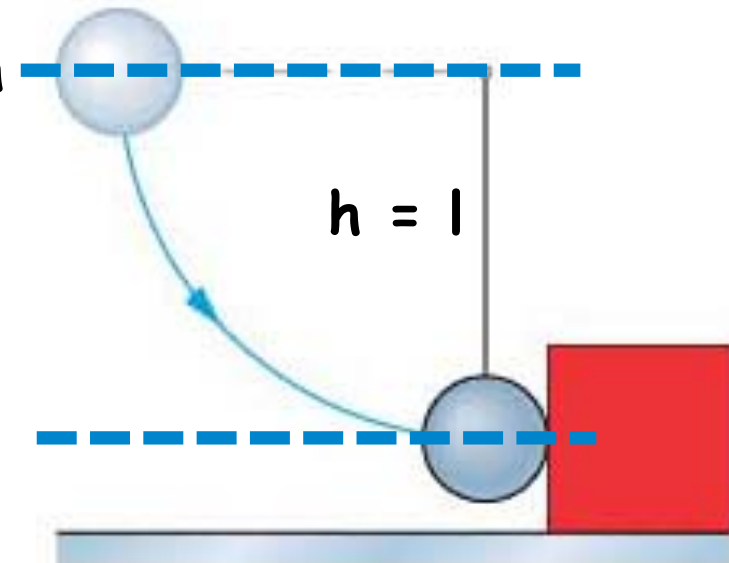
Conservation of linear momentum (no friction):

$$m_1v_{1i} = m_1v_{1f} + m_2v_{2f} \quad (1) \quad U_g = 0$$

$$v_{1i} = \sqrt{2gl}$$

The collision is elastic: $\Delta K = \frac{1}{2}(m_1v_{1f}^2 + m_2v_{2f}^2) - \frac{1}{2}(m_1v_{1i}^2) = 0$

$$m_1v_{1f}^2 + m_2v_{2f}^2 = m_1v_{1i}^2 \quad (2)$$



$$\left\{ \begin{array}{l} m_1 v_{1i} = m_1 v_{1f} + m_2 v_{2f} \quad (1) \\ m_1 v_{1f}^2 + m_2 v_{2f}^2 = m_1 v_{1i}^2 \quad (2) \end{array} \right.$$

$$\left\{ \begin{array}{l} m_1 (v_{1i} - v_{1f}) = m_2 v_{2f} \\ m_1 (v_{1i}^2 - v_{1f}^2) = m_2 v_{2f}^2 \end{array} \right.$$

$$\left\{ \begin{array}{l} m_1 (v_{1i} - v_{1f}) = m_2 v_{2f} \\ v_{1i} + v_{1f} = v_{2f} \end{array} \right.$$

$$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2} v_{1i}; v_{2f} = \frac{2m_1}{m_1 + m_2} v_{1i}$$

74. Two 2.0kg bodies, A and B, collide. The velocities before the collision are $\vec{v}_A = 15\hat{i} + 30\hat{j}$ m/s and $\vec{v}_B = -10\hat{i} + 5\hat{j}$ m/s. After the collision, $\vec{v}'_A = -5\hat{i} + 20\hat{j}$ m/s. What are (a) the final velocity of B and (b) the change in the total kinetic energy (including sign)?

We assume that the total linear momentum of the two bodies is conserved:

$$m_A \vec{v}_A + m_B \vec{v}_B = m_A \vec{v}'_A + m_B \vec{v}'_B$$

$$m_A = m_B :$$

$$\vec{v}'_B = \vec{v}_A + \vec{v}_B - \vec{v}'_A$$

$$\vec{v}'_B = 10\hat{i} + 15\hat{j}$$

(b)

$$K_f = \frac{1}{2} m_A v'^2_A + \frac{1}{2} m_B v'^2_B$$

$$K_i = \frac{1}{2} m_A v^2_A + \frac{1}{2} m_B v^2_B$$

$$\Delta K = K_f - K_i = -500 \text{ (J)}$$

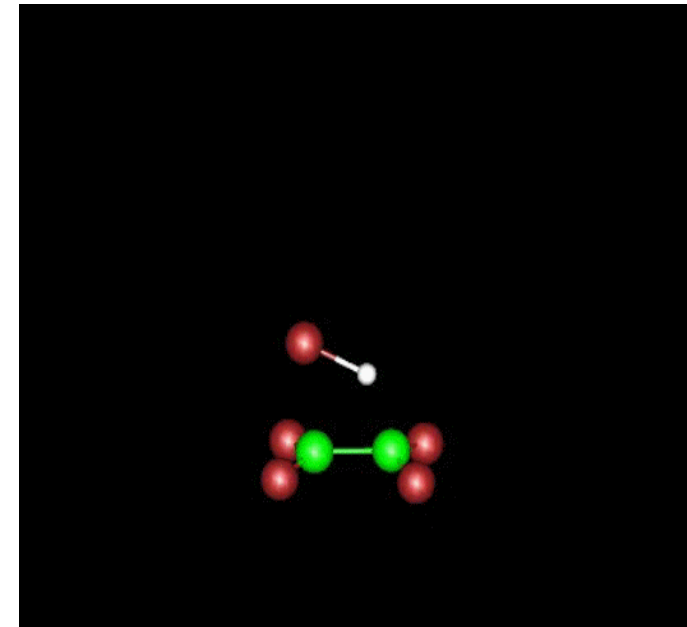
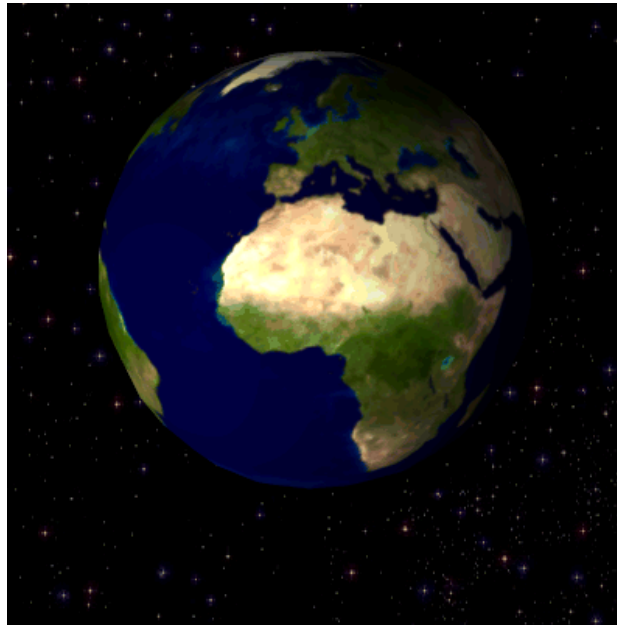
→ The collision here is an inelastic collision since KE is not a constant.

Part C Dynamics and Statics of Rigid Body
Chapter 5 Rotation of a Rigid Body
About a Fixed Axis

- 5.1. Rotational Variables**
- 5.2. Rotation with Constant Angular Acceleration**
- 5.3. Kinetic Energy of Rotation, Rotational Inertia**
- 5.4. Torque, and Newton's Second Law for Rotation**
- 5.5. Work and Rotational Kinetic Energy**
- 5.6. Rolling Motion of a Rigid Body**
- 5.7. Angular Momentum of a Rotating Rigid Body**
- 5.8. Conservation of Angular Momentum**

Overview

- We have studied the motion of translation, in which objects move along a straight or curved line.
- In this chapter, we will examine the motion of rotation, in which objects turn about an axis.



5.1. Rotational variables:

We study the rotation of a rigid body about a fixed axis.

Rigid bodies: Bodies can rotate with all its parts locked together and without any change in its shape.

Fixed axis: A fixed axis means the rotational axis does not move.

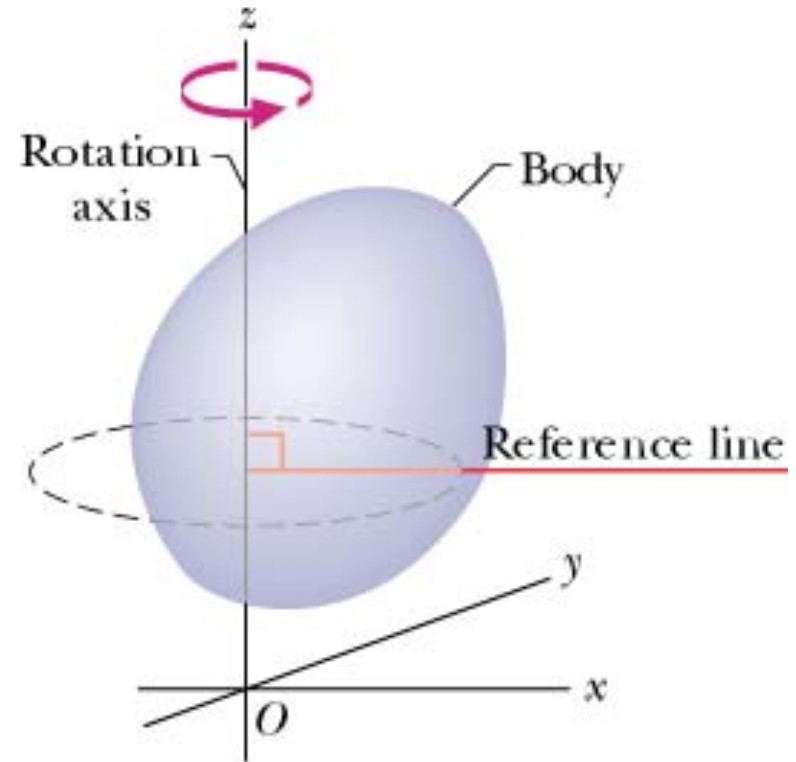
Angular Position:

Reference line: To determine the angular position, we must define a reference line, which is fixed in the body, perpendicular to the rotation axis and rotating with the body.

The angular position of this line is the angle of the line relative to a fixed direction.

$$\theta = \frac{s}{r}$$

θ : radians (rad)



$$1 \text{ rev} = 360^\circ = 2\pi \text{ rad}$$

Angular Displacement

$$\Delta\theta = \theta_2 - \theta_1$$

Convention:

- $\Delta\theta > 0$ in the counterclockwise direction.
- $\Delta\theta < 0$ in the clockwise direction.

Angular Velocity

Average angular velocity: $\omega_{\text{avg}} = \frac{\theta_2 - \theta_1}{t_2 - t_1} = \frac{\Delta\theta}{\Delta t}$

Instantaneous angular velocity: $\omega = \lim_{\Delta t \rightarrow 0} \frac{\Delta\theta}{\Delta t} = \frac{d\theta}{dt}$

Unit: rad/s or rev/s or rpm; (rev: revolution)

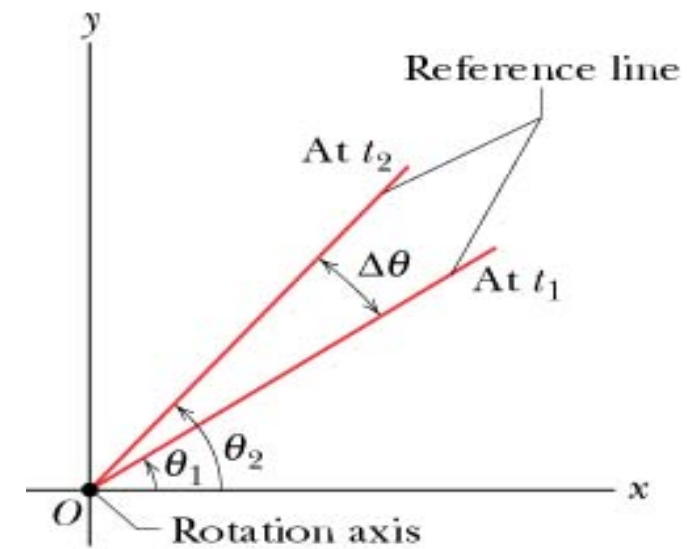
Angular Acceleration

Average angular acceleration: $\alpha_{\text{avg}} = \frac{\omega_2 - \omega_1}{t_2 - t_1} = \frac{\Delta\omega}{\Delta t}$

Instantaneous angular acceleration: $\alpha = \lim_{\Delta t \rightarrow 0} \frac{\Delta\omega}{\Delta t} = \frac{d\omega}{dt}$

Unit: rad/s² or rev/s²

Note: Angular displacement, velocity, and acceleration can be treated as vectors (see page 246).



5.2. Rotation with Constant Angular Acceleration

For one dimensional motion: $v = \frac{dx}{dt}$; $a = \frac{dv}{dt}$

Let's change variable names: $x \rightarrow \theta$, $v \rightarrow \omega$, $a \rightarrow \alpha$

$$\omega = \omega_0 + \alpha t$$

$$\theta = \theta_0 + \omega_0 t + \frac{1}{2} \alpha t^2$$

$$\omega^2 - \omega_0^2 = 2\alpha(\theta - \theta_0)$$

Turns
↓
radians

Checkpoint 2 (p. 248): In four situations, a rotating body has angular position $\theta(t)$ given by (a) $\theta = 3t - 4$, (b) $\theta = -5t^3 + 4t^2 + 6$, (c) $\theta = 2/t^2 - 4/t$, and (d) $\theta = 5t^2 - 3$. To which situations do the angular equations above apply? (d)

5.3. Kinetic Energy of Rotation

a. Linear and Angular Variable Relationship

• The position: $s = \theta r$

where angle θ measured in rad; s : distance along a circular arc; r : radius of the circle

• The speed: $\frac{ds}{dt} = \frac{d\theta}{dt} r$

$$v = \omega r$$

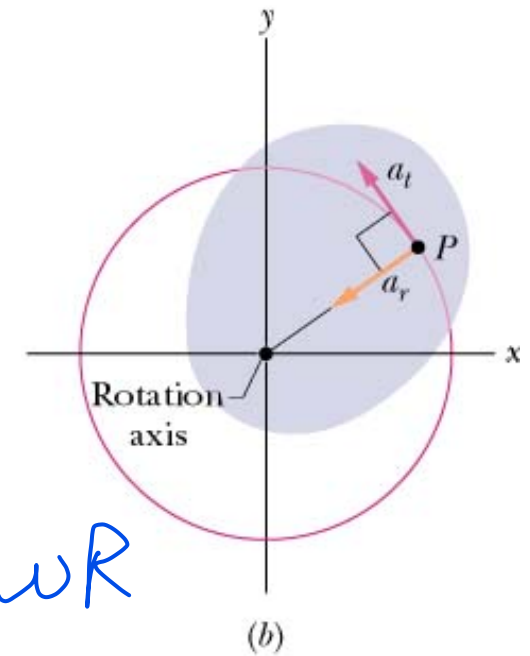
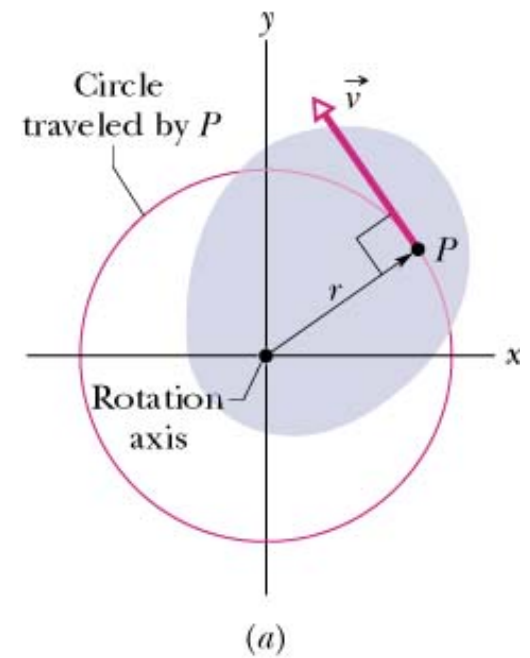
where ω in radian measure

The period of revolution: $T = \frac{2\pi r}{v} = \frac{2\pi}{\omega}$

• The Acceleration: $\frac{dv}{dt} = \frac{d\omega}{dt} r$

Tangential acceleration: $a_t = \alpha r$

Radial acceleration: $a_r = \frac{v^2}{r} = \omega^2 r$



$$v = \omega R$$

b. Kinetic Energy of Rotation:

The KE of a rotating rigid body is calculated by adding up the kinetic energies of all the particles:

$$K = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 + \frac{1}{2}m_3v_3^2 + \dots = \sum \frac{1}{2}m_iv_i^2$$

$$K = \sum \frac{1}{2}m_i(\omega r)^2 = \frac{1}{2}\omega^2 \sum m_ir_i^2$$

$I = \sum m_ir_i^2$: Rotational Inertia (or moment of inertia),

Unit for I: kg m²

$$K = \frac{1}{2}I\omega^2$$

c. Calculating the Rotational Inertia:

- If the rigid body consists of a few particles:

$$I = \sum m_i r_i^2$$

$$I = I$$

- For continuous bodies:

$$I = \int r^2 dm$$

- Parallel-Axis Theorem: The theorem allows us to calculate I of a body of mass M about a given axis if we already know I_{com} :

$$I = I_{com} + Mh^2$$

h : the perpendicular distance between the given axis and the axis through the center of mass of the body.

