# • PROGRAM OF "PHYSICS"

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# PHYSICS I
# (General Mechanics)
## 02 credits (30 periods)

**Chapter 1 Bases of Kinematics** Cơ sở của Động học

- Motion in One Dimension 1 chiều
- Motion in Two Dimensions 2 chiều

**Chapter 2 The Laws of Motion**

**Chapter 3 Work and Mechanical Energy** Năng lượng Cơ học

**Chapter 4 Linear Momentum and Collisions** Động lượng Tuyến tính và Va chạm

**Chapter 5 Rotation of a Rigid Object About a Fixed Axis** Quay của một Vật cứng xung quanh một Trục cố định

**Chapter 6 Static Equilibrium** thăng bằng tĩnh

**Chapter 7 Universal Gravitation** Lực hấp dẫn Toàn cầu

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# PHYSICS I

## Chapter 1 Bases of Kinematics

## A. Motion in One Dimension

### 1. Position, Velocity, and Acceleration

### 2. One-Dimensional Motion with Constant Acceleration

### 3. Freely Falling Objects

## B. Motion in Two Dimensions

### 4. The Position, Velocity, and Acceleration Vectors

### 5. Two-Dimensional Motion with Constant Acceleration. Projectile Motion

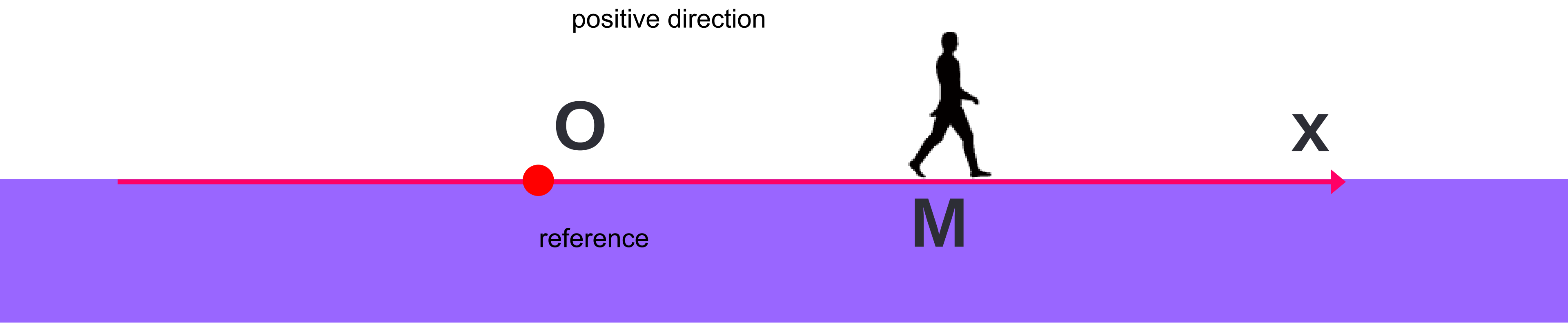
### 6. Circular Motion. Tangential and Radial Acceleration

### 7. Relative Velocity and Relative Acceleration

# Dynamics động lực học

- The branch of physics involving the motion of an object and the relationship between that motion and other physics concepts
- ***Kinematics*** is a part of dynamics
  - In kinematics, you are interested in the *description* of motion
  - *Not* concerned with the cause of the motion

# 1. 1 Position and Displacement

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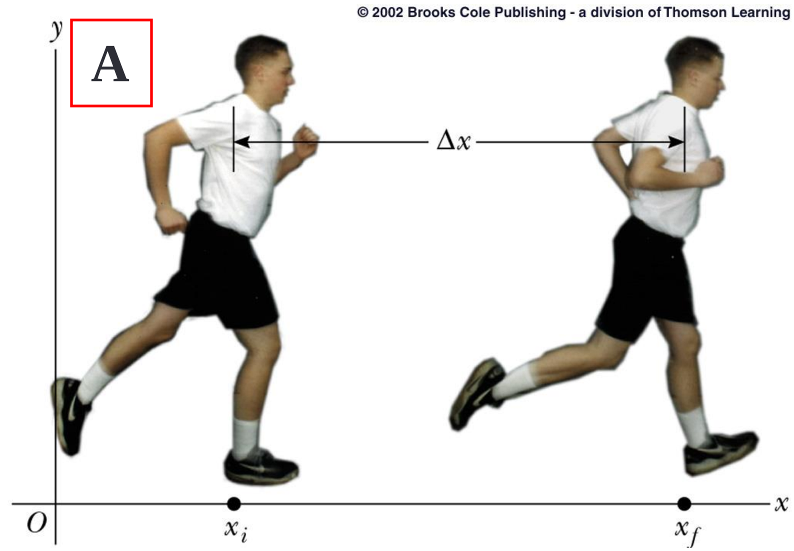
# 1. 1 Position and Displacement

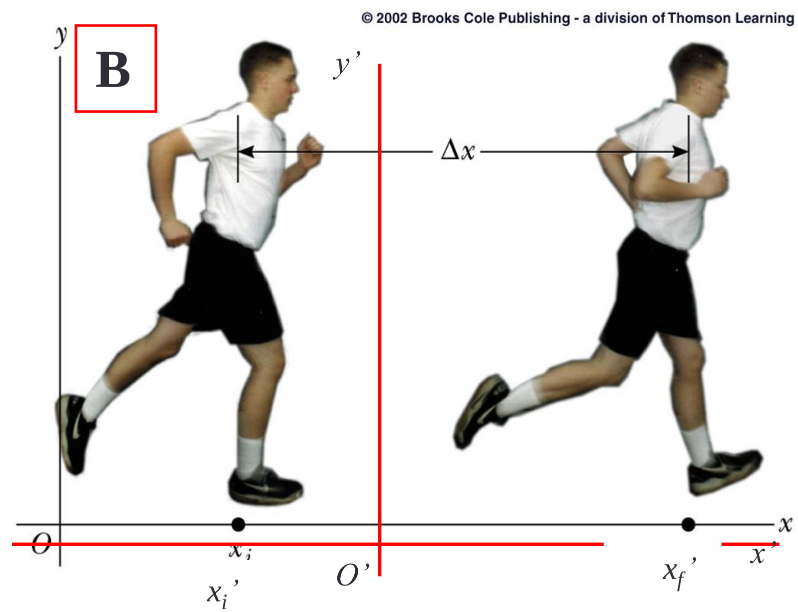
- Position is defined in terms of a **frame of reference**

**Frame A:** $x_i > 0\ ;\ x_f > 0$

**Frame B:** $x'_I < 0\ ;\ x'_f > 0$

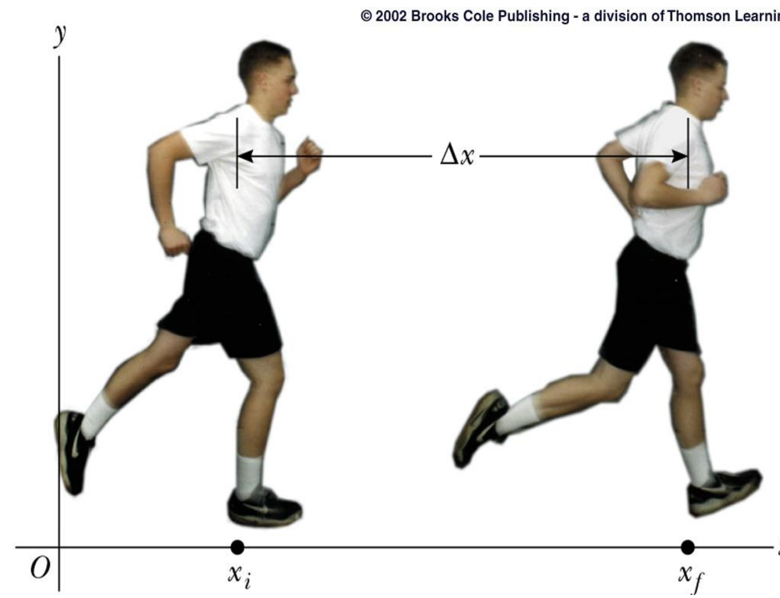
- One dimensional, so generally the x- or y-axis



# Position and Displacement

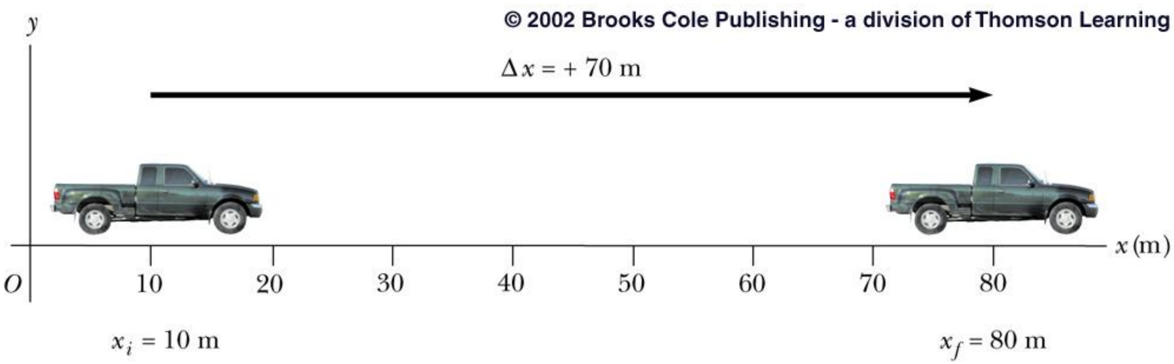
- Position is defined in terms of a **frame of reference**
  - One dimensional, so generally the x- or y-axis
- Displacement measures the change in position
  - Represented as $\Delta x$ (if horizontal) or $\Delta y$ (if vertical)
  - Vector quantity (i.e. needs directional information)
    - + or - is generally sufficient to indicate direction for one-dimensional motion




| system international | Units |
|---|---|
| SI | Meters (m) |
| CGS | Centimeters (cm) |
| US Cust | Feet (ft) |

# Displacement

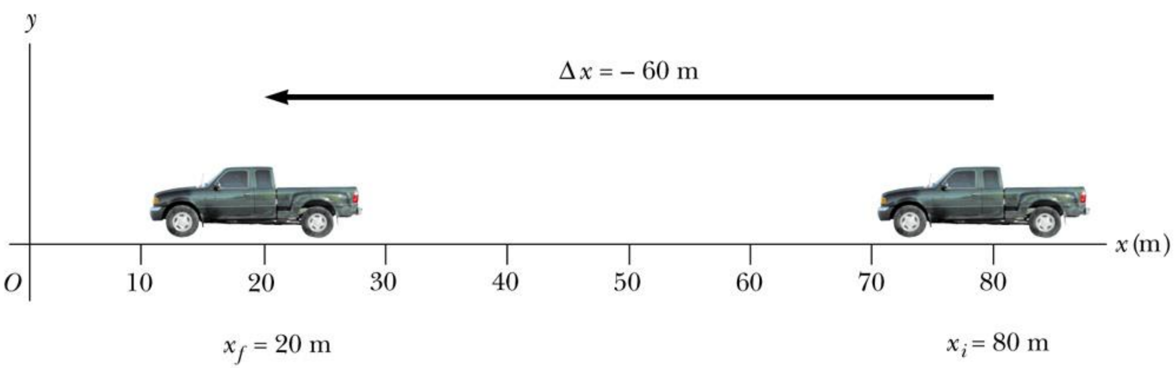
- Displacement measures the change in position
  - represented as $\Delta x$ or $\Delta y$




(a)

$$\Delta x_1 = x_f - x_i$$
$$= 80\,m - 10\,m$$
$$= \underline{+70\,m}$$

✓



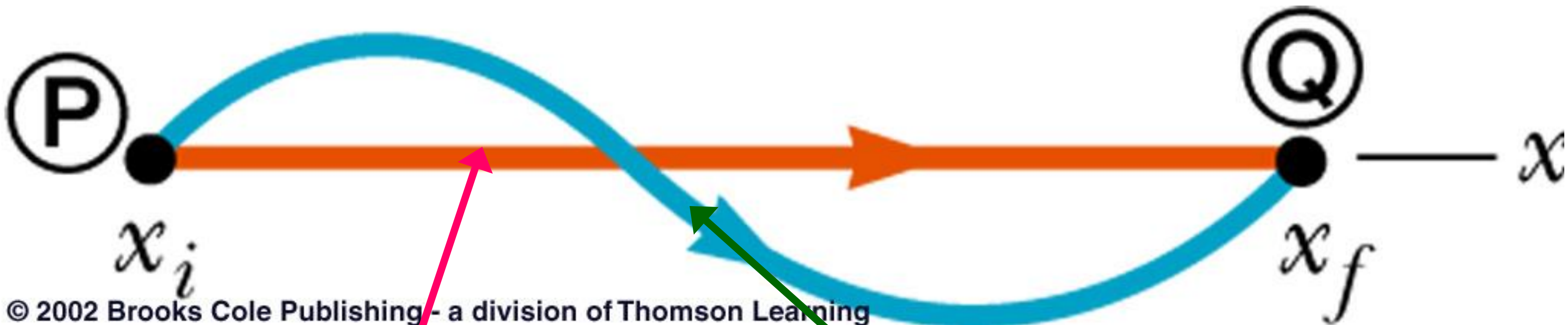

(b)

$$\Delta x_2 = x_f - x_i$$
$$= 20\,m - 80\,m$$
$$= \underline{-60\,m}$$

✓

# Distance or Displacement?

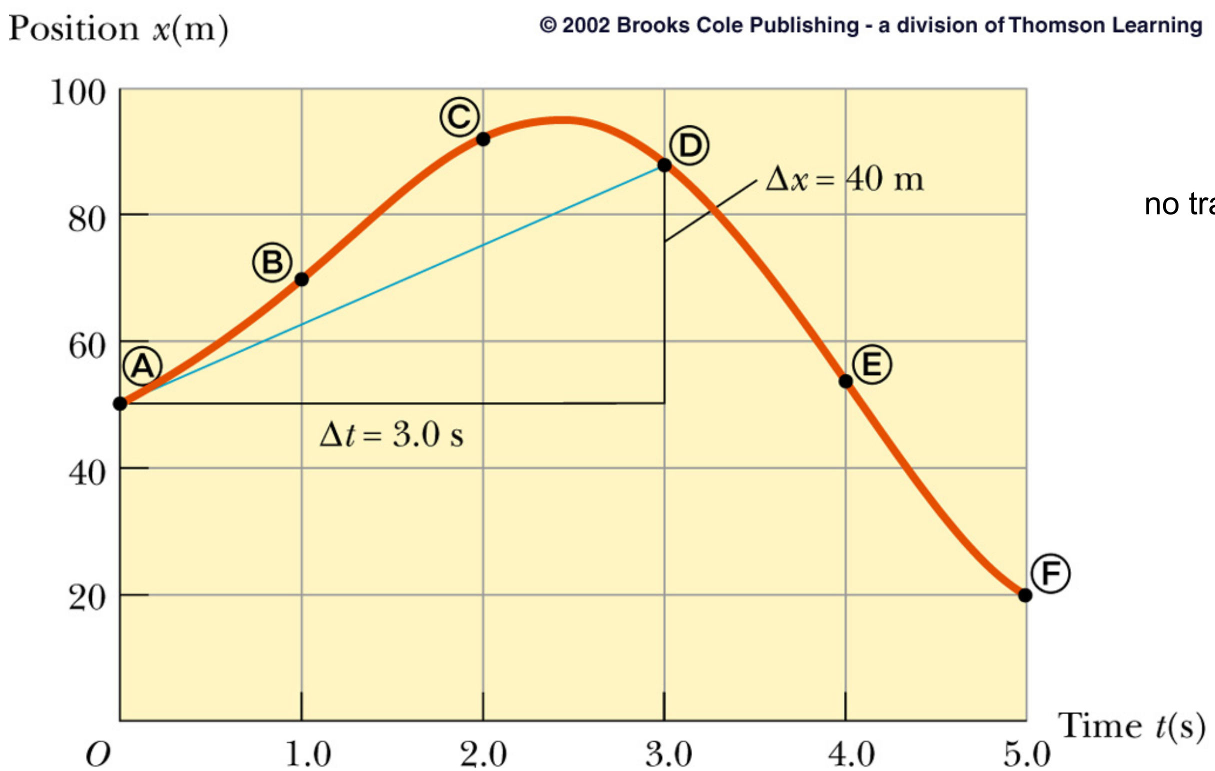
- Distance may be, but is not necessarily, the magnitude of the displacement




Displacement (yellow line)

Distance (blue line)

# Position-time graphs




no trajectory

- Note: position-time graph is <u>not necessarily a straight line</u>, even though the motion is along x-direction

# Test 1

An object (say, car) goes from one point in space to another. After it arrives to its destination, its displacement is

1. either greater than or equal to
2. always greater than
3. always equal to
4. either smaller or equal to
5. either smaller or larger

than the distance it traveled.

# A. Motion in One Dimension

## 1. 2 Velocity vận tốc

### a. Average Velocity

- It takes time for an object to undergo a displacement
- The **average velocity** is rate at which the displacement occurs

$$\boxed{v_{average} = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{\Delta t}}$$

- Direction will be the same as the direction of the displacement ($\Delta t$ is always positive)

# More About Average Velocity

- Units of velocity:

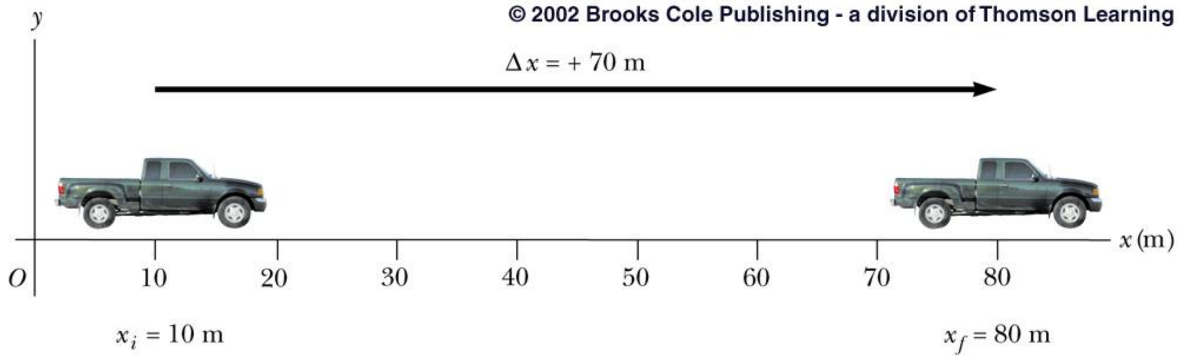
| | Units |
|---|---|
| SI | Meters per second (m/s) |
| CGS | Centimeters per second (cm/s) |
| US Customary | Feet per second (ft/s) |

- Note: other units may be given in a problem, but generally will need to be converted to these

# Example:

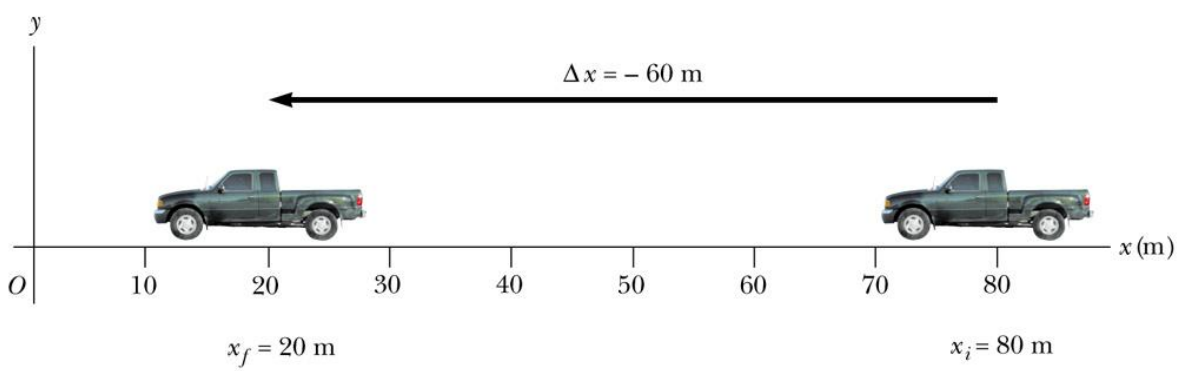
**Suppose that in both cases truck covers the distance in 10 seconds:**






(a)

$$v_{1\,average} = \frac{\Delta x_1}{\Delta t} = \frac{+70m}{10s}$$
$$= \underline{+7\,m/s}$$




(b)

$$v_{2\,average} = \frac{\Delta x_2}{\Delta t} = \frac{-60m}{10s}$$
$$= \underline{-6\,m/s}$$

# Speed

tốc độ

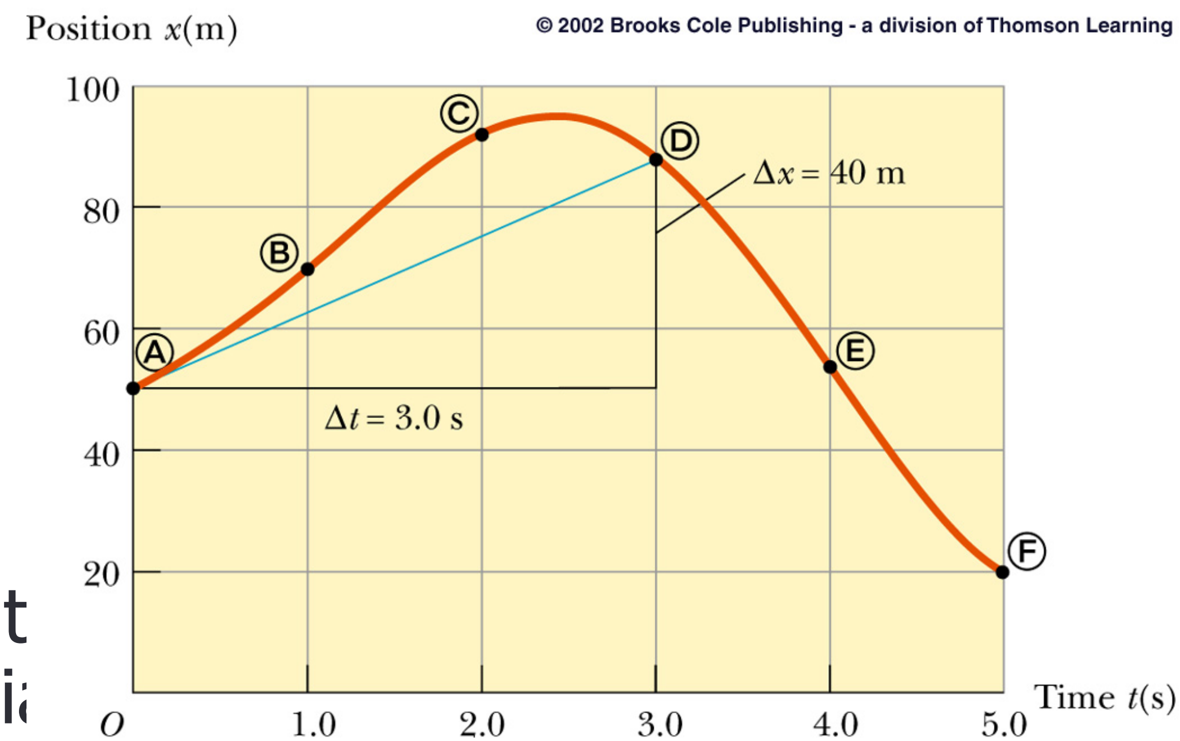
no sign

- Speed is a scalar quantity (no information about sign/direction is need)
  - same units as velocity
  - Average speed = total distance / total time
- Speed is the magnitude of the velocity

# Graphical Interpretation of Average Velocity

- Velocity can be determined from a position-time graph

$$v_{average} = \frac{\Delta x}{\Delta t} = \frac{+40m}{3.0s}$$
$$= \underline{+13\, m/s}$$




- Average velocit
joining the initi

# b. Instantaneous Velocity

right at that time

- Instantaneous velocity is defined as the limit of the average velocity as the time interval becomes infinitesimally short, or as the time interval approaches zero

$$v_{inst} \equiv v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \lim_{\Delta t \to 0} \frac{x - x_i}{\Delta t}$$

$$\boxed{v = x'_t = \frac{dx}{dt}}$$

- The instantaneous velocity indicates what is happening at every point of time

# Instantaneous Velocity

$$v = x'_t = \frac{dx}{dt}$$

The instantaneous velocity equals the first derivative of the position with respect to time

$$\int_{x_0}^{x} dx = \int_{t_0}^{t} v dt \ ; \quad x\Big|_{x_0}^{x} = \int_{t_0}^{t} v dt \ ;$$

$$\boxed{x = x_0 + \int_{t_0}^{t} v dt}$$

# Uniform Velocity

- Uniform velocity is constant velocity :

$$v = \text{const}$$

- The instantaneous velocities are always the same
  - All the instantaneous velocities will also equal the average velocity

# Uniform Velocity

- Uniform velocity is constant velocity :

$$v = \text{const}$$

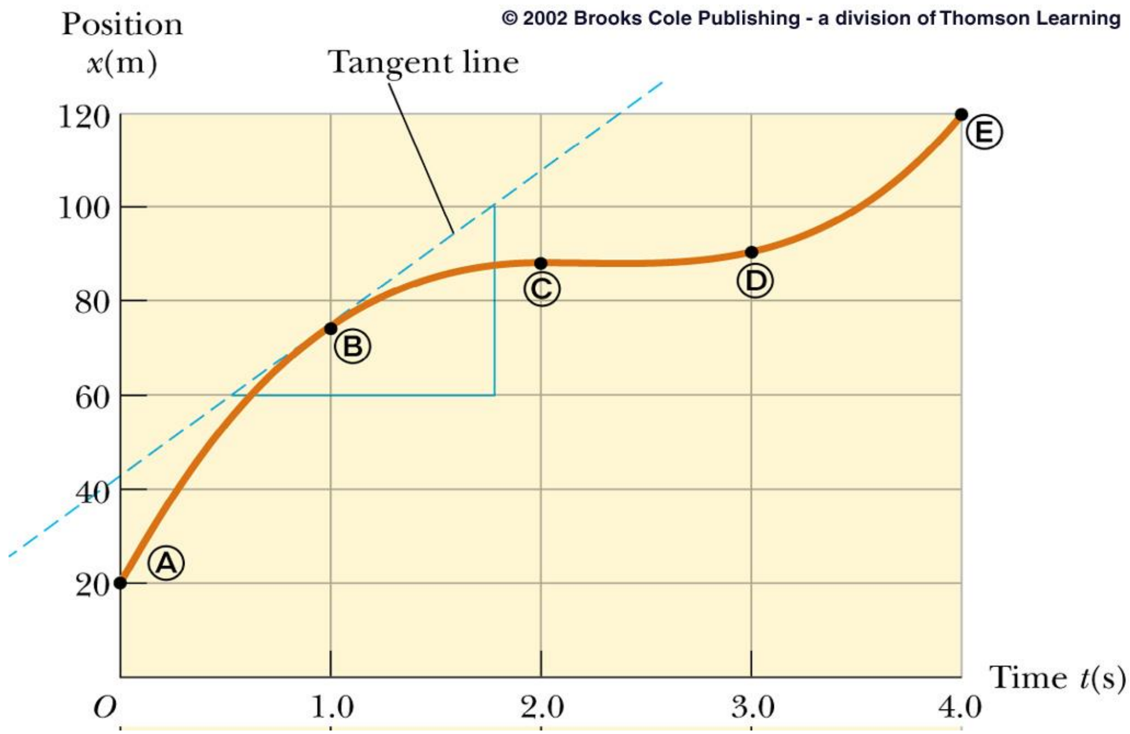
$$x = x_0 + \int_{t_0}^{t} v\,dt \; ; \quad x = x_0 + v\int_{t_0}^{t} dt \; ;$$

$$x = x_0 + v(t - t_0)$$

$$t_0 = 0 \longrightarrow \boxed{x = x_0 + vt}$$

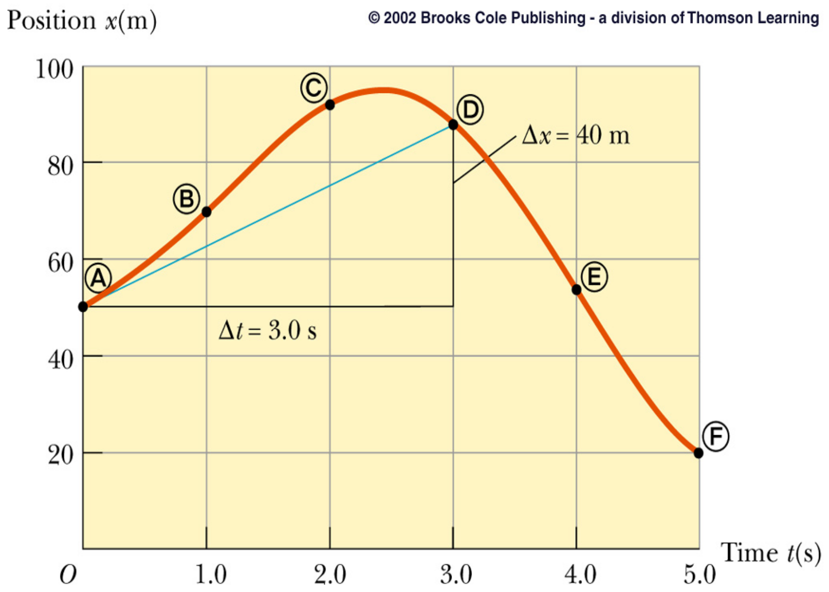
# Graphical Interpretation of Instantaneous Velocity

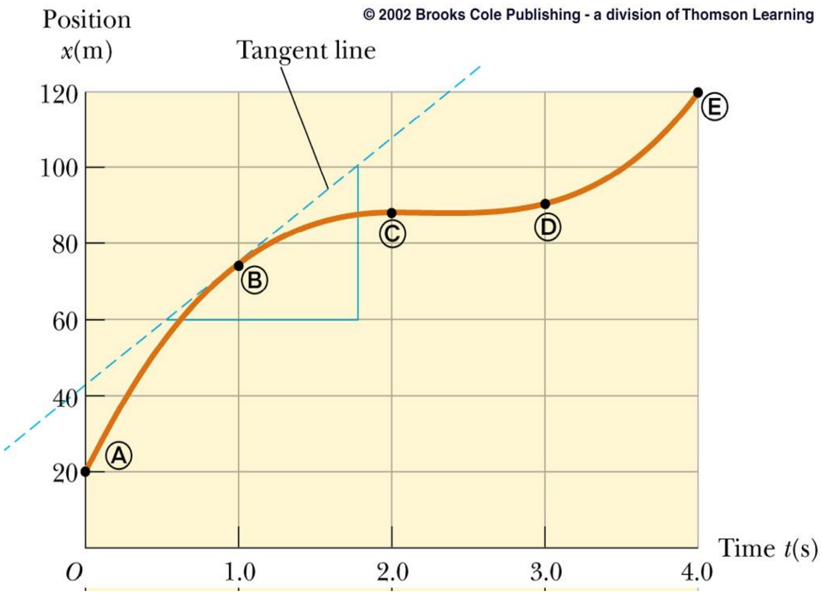
- Instantaneous velocity is the slope of the tangent to the curve at the time of interest




- The instantaneous speed is the magnitude of the instantaneous velocity

# Average vs Instantaneous Velocity



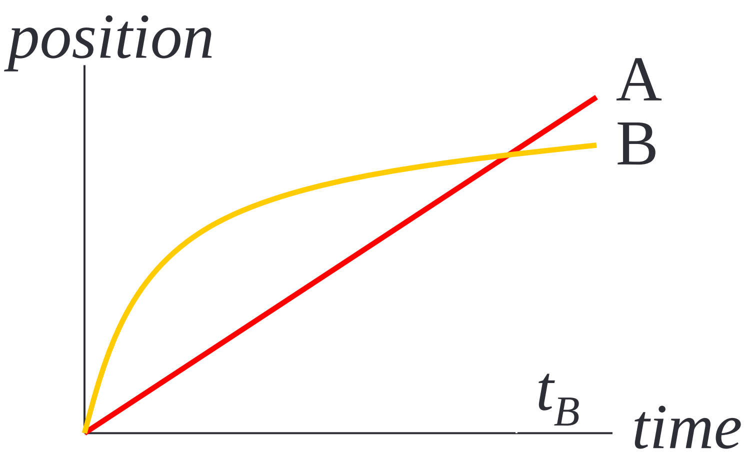

Average velocity

Instantaneous velocity

# Test 2

The graph shows position as a function of time for two trains running on parallel tracks. Which of the following is true:

1. at time $t_B$ both trains have the same velocity
2. both trains speed up all the time
3. both trains have the same velocity at some time before $t_B$
4. train A is longer than train B
5. all of the above statements are true




**Note: the slope of curve B is parallel to line A at some point *t*< *t<sub></sub>*$_B$**

- **A. Motion in One Dimension**
- **1. 3 Acceleration**

a. Average Acceleration

- Changing velocity (non-uniform) means an acceleration is present
- Average acceleration is the rate of change of the velocity

$$\boxed{a_{average} = \frac{\Delta v}{\Delta t} = \frac{v_f - v_i}{\Delta t}}$$

- Average acceleration is a vector quantity (i.e. described by both magnitude and direction)

# Average Acceleration

- When the sign of the velocity and the acceleration are the same (either positive or negative), then the speed is increasing
- When the sign of the velocity and the acceleration are opposite, the speed is decreasing

| | Units |
|---|---|
| SI | Meters per second squared ($m/s^2$) |
| CGS | Centimeters per second squared ($cm/s^2$) |
| US Customary | Feet per second squared ($ft/s^2$) |

# b. Instantaneous Acceleration

- Instantaneous acceleration is the limit of the average acceleration as the time interval goes to zero

$$a_{inst} \equiv a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \lim_{\Delta t \to 0} \frac{v_f - v_i}{\Delta t}$$

$$a = v'_t = x''_t \quad ; \quad a = \frac{dv}{dt} = \frac{d^2x}{dt^2}$$

The instantaneous acceleration equals the first derivative of the velocity and the second derivative of the position with respect to time

# 2. Uniform Acceleration

$$a = v'_t = x''_t \;\; ; \; a = \frac{dv}{dt} = \frac{d^2x}{dt^2}$$

- When the instantaneous accelerations are always the same, the acceleration will be uniform
  - The instantaneous accelerations will all be equal to the average acceleration

# Uniform Acceleration

$a$ = const

$$a = v'_t = x''_t \quad ; \quad a = \frac{dv}{dt} = \frac{d^2x}{dt^2}$$

*WHAT ARE THE FORMULAE FOR V AND X ?*

# Uniform Acceleration

$a = \text{const}$

$$a = v'_t = x''_t \ ; \quad a = \frac{dv}{dt} = \frac{d^2x}{dt^2}$$

$$v = v_0 + \int_{t_0}^{t} a\,dt \ ; \quad v = v_0 + a\int_{t_0}^{t} dt \ ;$$

$$\boxed{v = v_0 + a(t - t_0)}$$

$$x = x_0 + \int_{t_0}^{t} v\,dt = x_0 + \int_{t_0}^{t} \left[v_0 + a(t - t_0)\right] dt$$

# Uniform Acceleration

$a = \text{const}$

$$x = x_0 + \int_{t_0}^{t} \left[ v_0 + a(t - t_0) \right] dt$$

$$= x_0 + \int_{t_0}^{t} v_0 dt + a \int_{t_0}^{t} (t - t_0) dt$$

$$= x_0 + v_0 (t - t_0) + \frac{a}{2} (t - t_0)^2$$

$$t_0 = 0 \longrightarrow \boxed{x = x_0 + v_0 t + \frac{a}{2} t^2}$$

# Uniform Acceleration

$a$ = const

*WHAT IS THE FORMULA BETWEEN V AND X INDEPENDENT OF $t$ ?*

# Uniform Acceleration

$$a = \text{const}$$

$$dv = adt\ ;\quad vdv = \frac{dx}{dt}adt\ ;\quad vdv = adx\ ;$$

$$\int_{v_0}^{v} vdv = \int_{x_0}^{x} adx\ ;\qquad \frac{v^2}{2} - \frac{v_0^2}{2} = \int_{x_0}^{x} adx$$

$$a = \text{const} \longrightarrow \frac{v^2}{2} - \frac{v_0^2}{2} = a\int_{x_0}^{x} dx$$

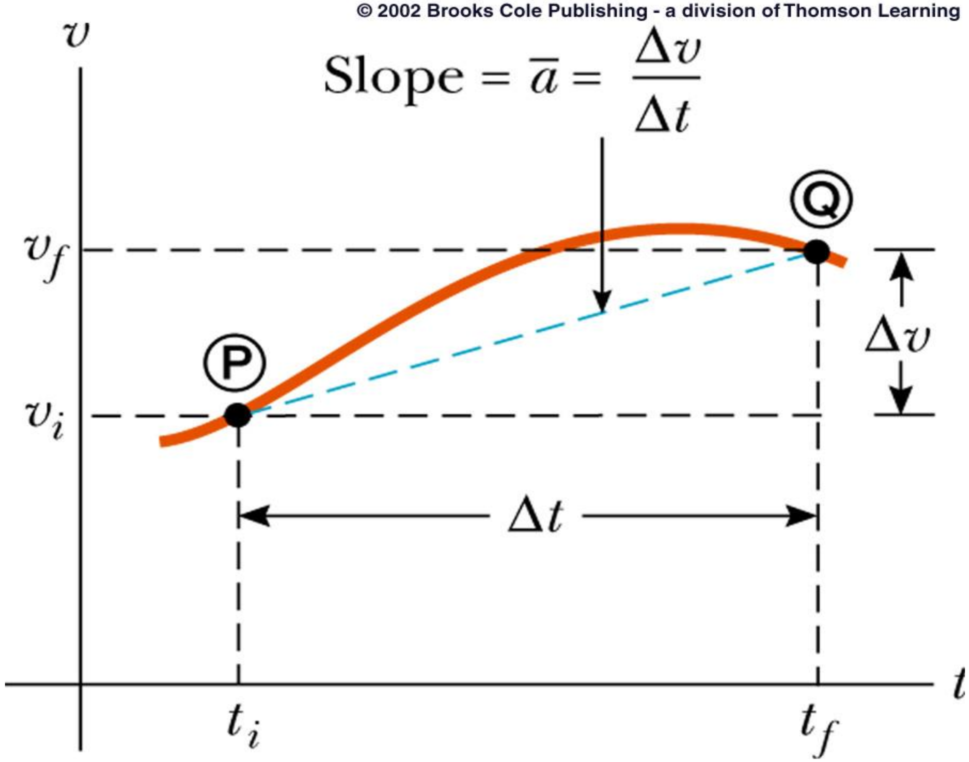
$$\boxed{v^2 - v_0^2 = 2a(x - x_0)}$$

# Graphical Interpretation of Acceleration

- Average acceleration is the slope of the line connecting the initial and final velocities on a velocity-time graph

- Instantaneous acceleration is the slope of the tangent to the curve of the velocity-time graph

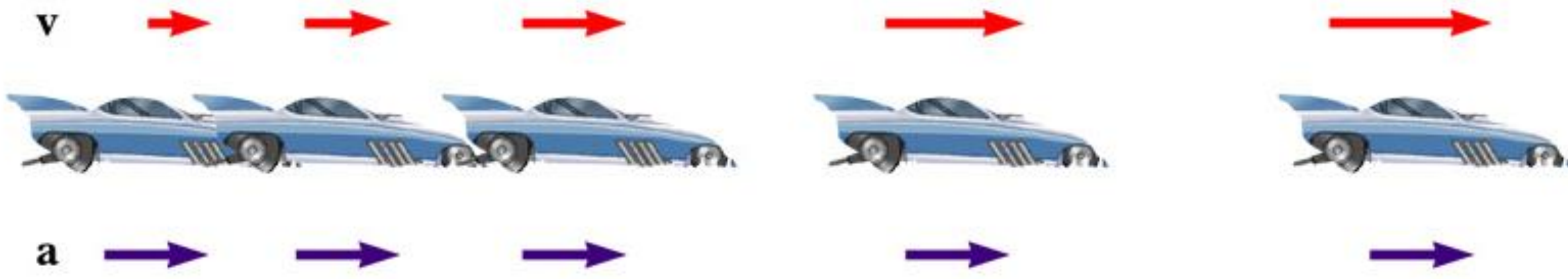




# Example 1: Motion Diagrams

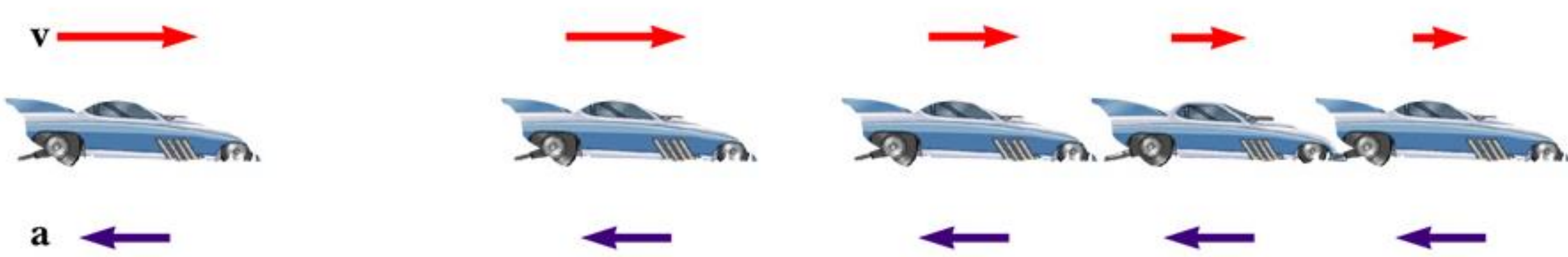

- Uniform velocity (shown by red arrows maintaining the same size)
- Acceleration equals zero

# Example 2:




- Velocity and acceleration are in the same direction
- Acceleration is uniform (blue arrows maintain the same length)
- Velocity is increasing (red arrows are getting longer)

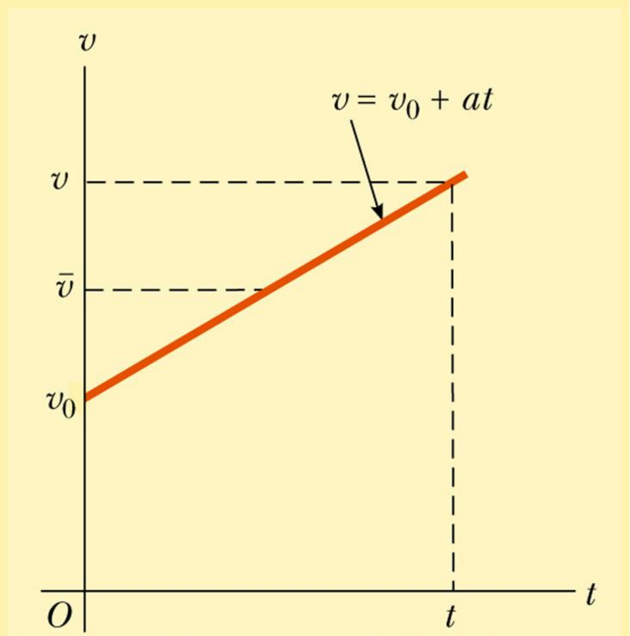
# Example 3:




- Acceleration and velocity are in opposite directions
- Acceleration is uniform (blue arrows maintain the same length)
- Velocity is decreasing (red arrows are getting shorter)

# One-dimensional Motion With Constant Acceleration

- If acceleration is uniform





$a = v'$ thus:

$$v = v_o + at$$

- Shows velocity as a function of acceleration and time

# One-dimensional Motion With Constant Acceleration

- Used in situations with uniform acceleration

$$v = v_o + at$$

$$x = v'$$

$$x = x_o + v_o t + \frac{1}{2} a t^2$$

$$v^2 = v_o^2 + 2a\Delta x$$

# Summary of kinematic equations

**TABLE 2.3** **Equations for Motion in a Straight Line Under Constant Acceleration**

| Equation | Information Given by Equation |
|---|---|
| $v = v_0 + at$ | Velocity as a function of time |
| $\Delta x = \frac{1}{2}(v_0 + v)\,t$ | Displacement as a function of velocity and time |
| $\Delta x = v_0 t + \frac{1}{2}at^2$ | Displacement as a function of time |
| $v^2 = {v_0}^2 + 2a\,\Delta x$ | Velocity as a function of displacement |

*Note:* Motion is along the $x$ axis. At $t = 0$, the velocity of the particle is $v_0$.

# *EXAMPLE 1*

The velocity of a particle moving along the *x* axis varies in time according to the expression $v = (40 - 5t^2)\ m/s$ , where $t$ is in seconds.
(a) Find the average acceleration in the time interval $t$ = 0 to $t$ = 2.0 s.

(a) $v_A = (40 - 5 \times 0^2)\ m/s = 40\ m/s$

$$v_B = (40 - 5 \times 2^2)\ m/s = 20\ m/s$$

$$a_{average} = \frac{v_B - v_A}{t_B - t_A} = \frac{20\ m/s - 40\ m/s}{2{,}0 - 0} = -10\ m/s^2$$

# *EXAMPLE 1*

The velocity of a particle moving along the $x$ axis varies in time according to the expression $v = (40 - 5t^2)\ m/s$, where $t$ is in seconds.

(b) Determine the acceleration at $t = 2.0$ s.

(b) $a = v' = (-10t)\ m/s^2$

$$= (-10 \times 2.0\ s)\ m/s^2 = -20\ m/s^2$$

# 3. Free Fall

- All objects moving under the influence of only gravity are said to be in free fall
- All objects falling near the earth's surface fall with a constant acceleration
- This acceleration is called the acceleration due to gravity, and indicated by $g$

# Acceleration due to Gravity

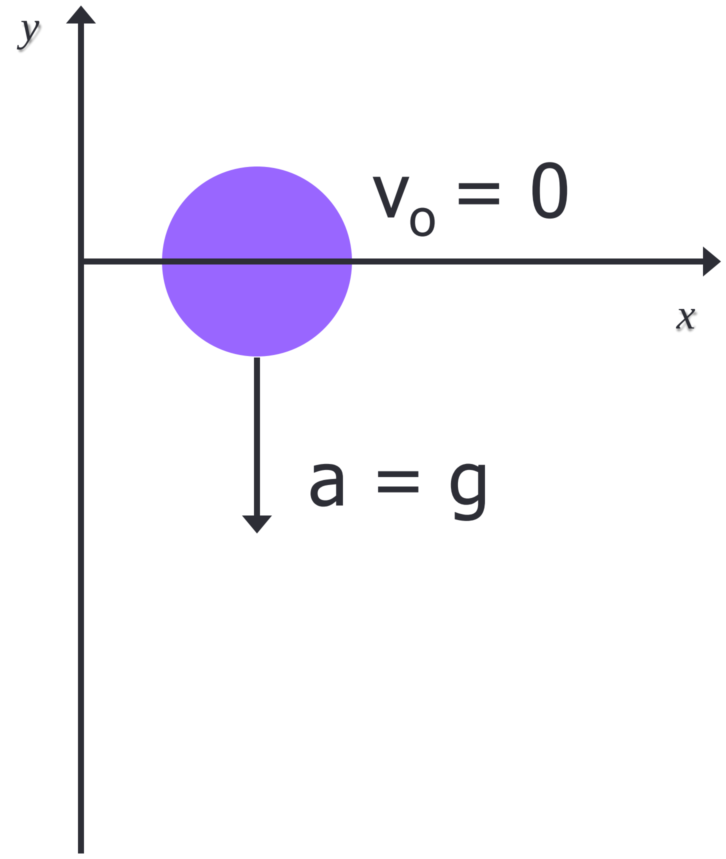
- Symbolized by *g*
- *g* = 9.8 m/s² (can use g = 10 m/s² for estimates)
- *g* is always directed downward
  - toward the center of the earth

# Free Fall -- an Object Dropped

- Initial velocity is zero
- Frame: let up be positive
- Use the kinematic equations
  - Generally use y instead of x since vertical

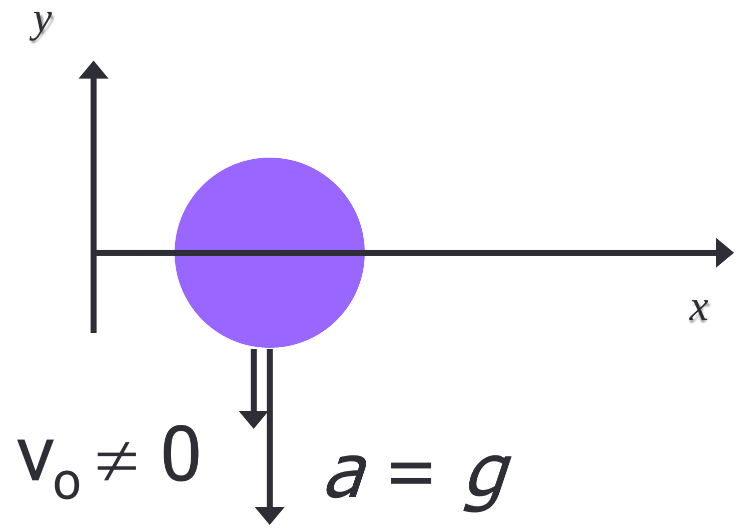
$$y - 0 = y = \frac{1}{2}at^2$$

$$a = -9.8\, m/s^2$$



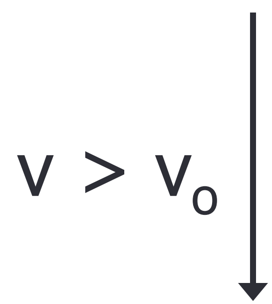
# Free Fall -- an Object Thrown Downward

- $a = g$
  - With upward being positive, acceleration will be negative, g = -9.8 m/s²
- Initial velocity $v_o \neq 0$
  - With upward being positive, initial velocity will be negative



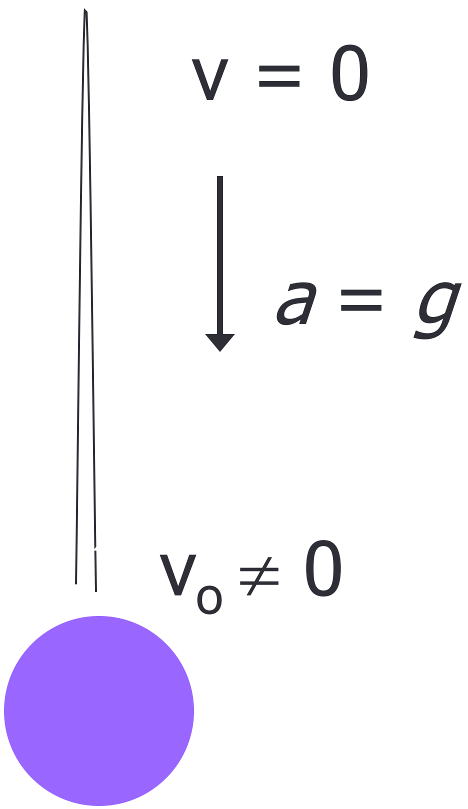

$$\Delta y = y - 0 = y = v_0 + \frac{1}{2}at^2$$

$$a = -9.8\, m/s^2$$



# Free Fall -- object thrown upward

- Initial velocity is upward, so positive
- The instantaneous velocity at the maximum height is zero
- $a = g$ everywhere in the motion
  - g is always downward, negative



# Thrown upward

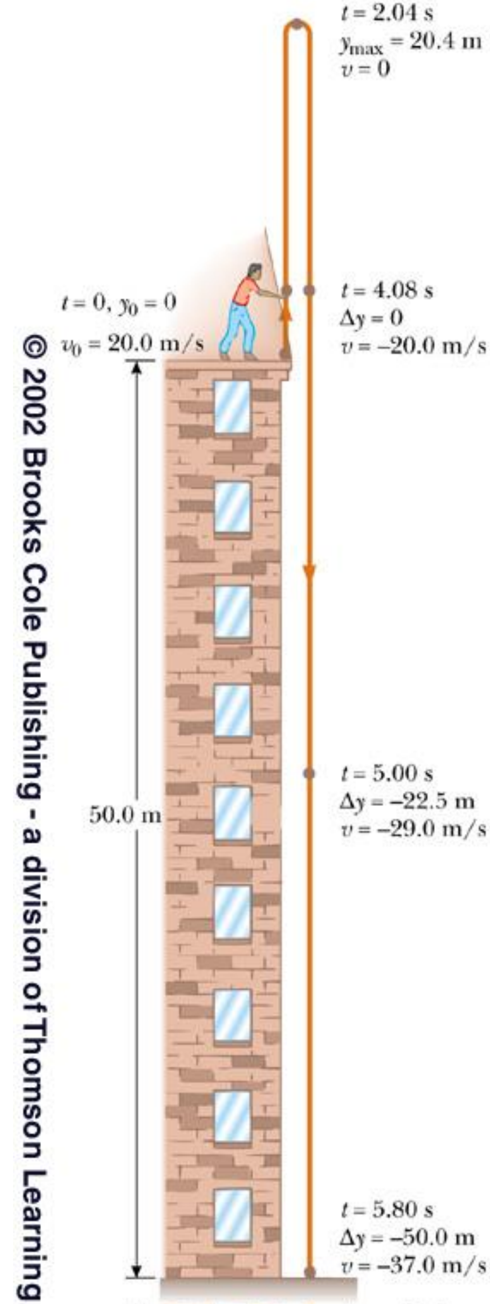
- The motion may be symmetrical
  - then $t_{up} = t_{down}$
  - then $v_f = -v_o$

**why ?**

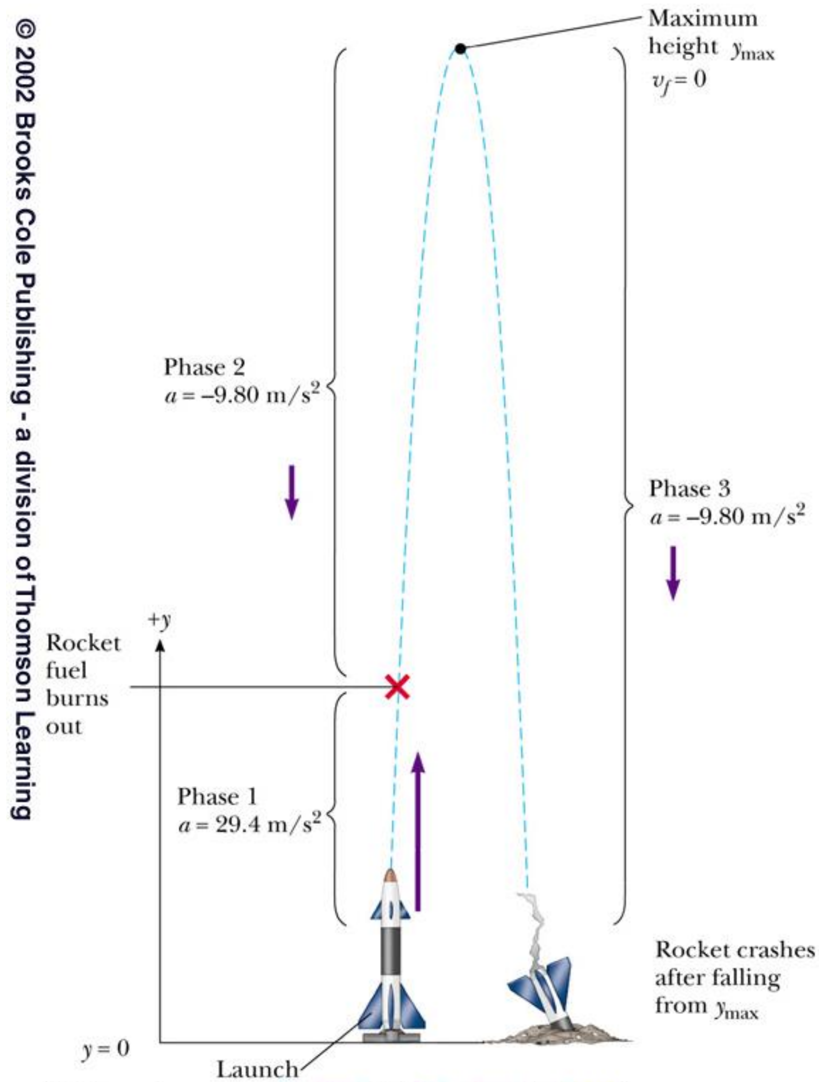
- The motion may not be symmetrical
  - Break the motion into various parts
    - generally up and down

# Non-symmetrical Free Fall

- Need to divide the motion into segments
- Possibilities include
  - Upward and downward portions
  - The symmetrical portion back to the release point and then the non-symmetrical portion



# Combination Motions



# Test 3

A person standing at the edge of a cliff throws one ball straight up and another ball straight down at the same initial speed. Neglecting air resistance, the ball to hit ground below the cliff with greater speed is the one initially thrown

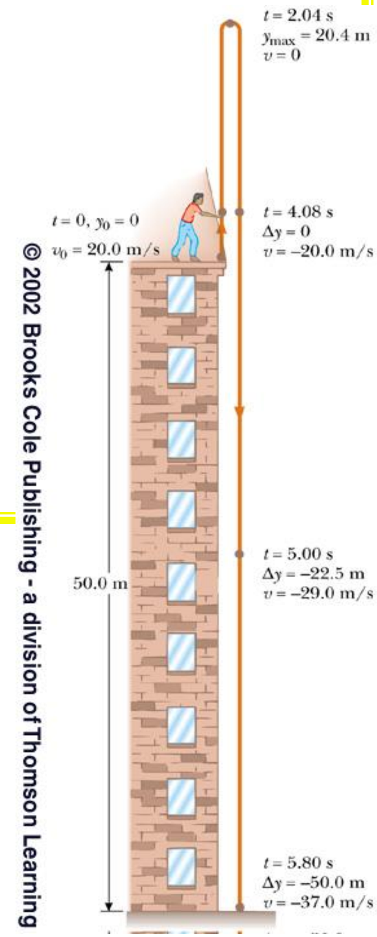
1. upward
2. downward
3. neither – they both hit at the same speed

**Note**: upon the descent, the velocity of an object thrown straight up with an initial velocity $v$ is exactly $-v$ when it passes the point at which it was first released.

# *EXAMPLE 2*

A stone thrown from the top of a building is given an initial velocity of 20.0 m/s straight upward. The building is 50.0 m high, and the stone just misses the edge of the roof on its way down, as shown in figure . Using as the time the stone leaves the thrower’s hand at position , determine

(a) the time at which the stone reaches its maximum height



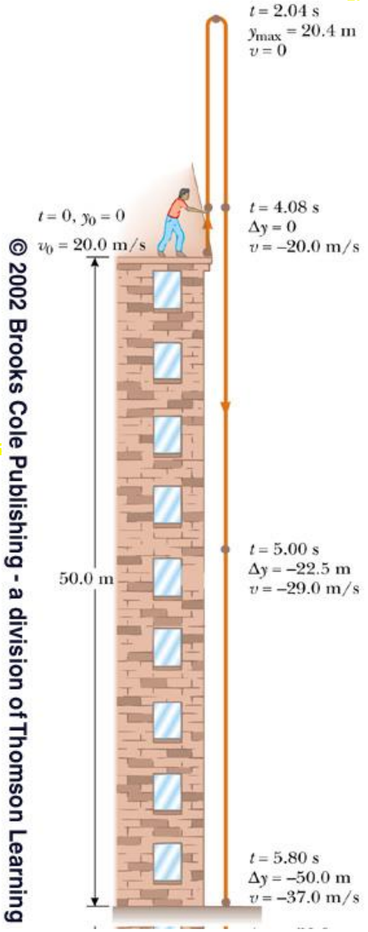

(a)

$$v = v_0 + at = 20.0\ \text{m/s} + (-\ 9.80\ \text{m/s}^2)t = 0$$

$$t = \frac{20.0\ m/s}{9.80\ m/s^2} = 2.04\ s$$

# *EXAMPLE 2*

A stone thrown from the top of a building is given an initial velocity of 20.0 m/s straight upward. The building is 50.0 m high, and the stone just misses the edge of the roof on its way down, as shown in figure .
Using as the time the stone leaves the thrower's hand at position , determine
(b) the maximum height





(b)

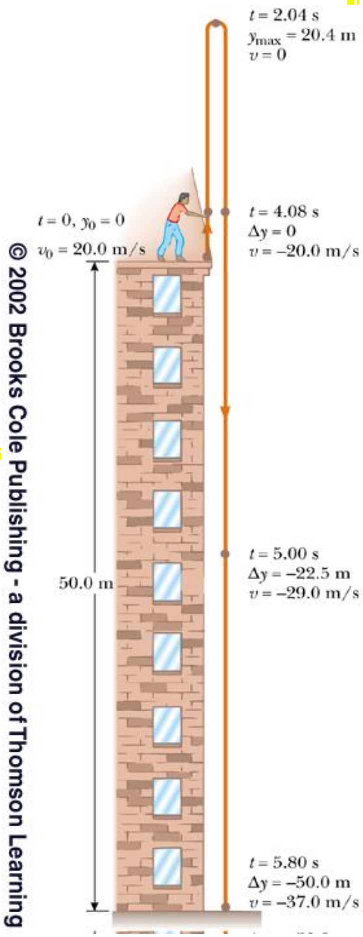
$$y_B = v_0 t + \frac{1}{2} a t^2$$

$$= (20.0 \text{ m/s}) \times (2.04 \text{ s}) + \frac{1}{2}(-9.80 \text{ m/s}^2) \times (2.04 \text{ s})^2$$

$$y_B = 20.4\ m$$

# *EXAMPLE 2*

A stone thrown from the top of a building is given an initial velocity of 20.0 m/s straight upward. The building is 50.0 m high, and the stone just misses the edge of the roof on its way down, as shown in figure .
Using as the time the stone leaves the thrower's hand at position , determine
(b) the maximum height





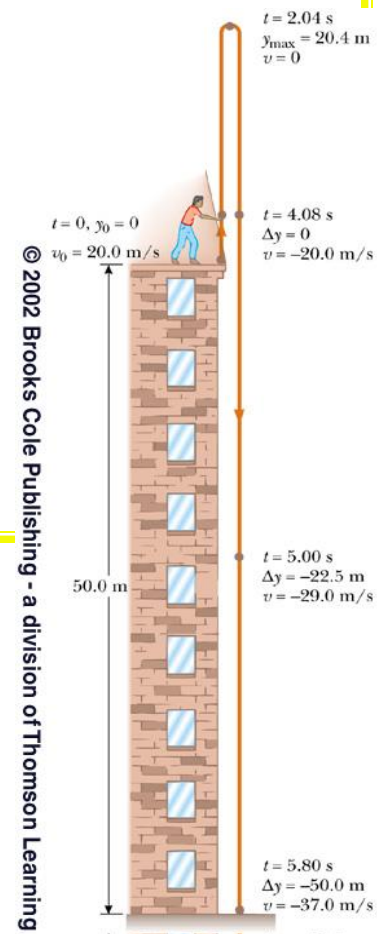
Other method : $v^2 - v_0^2 = 2a(y - y_0)$

$$0^2 - (20.0)^2 = 2(-9.80)(y - y_0)$$

$$y - y_0 = 20.4\ m$$

# EXAMPLE 2

A stone thrown from the top of a building is given an initial velocity of 20.0 m/s straight upward. The building is 50.0 m high, and the stone just misses the edge of the roof on its way down, as shown in figure . Using as the time the stone leaves the thrower's hand at position , determine (c) the time at which the stone returns to the height from which it was thrown



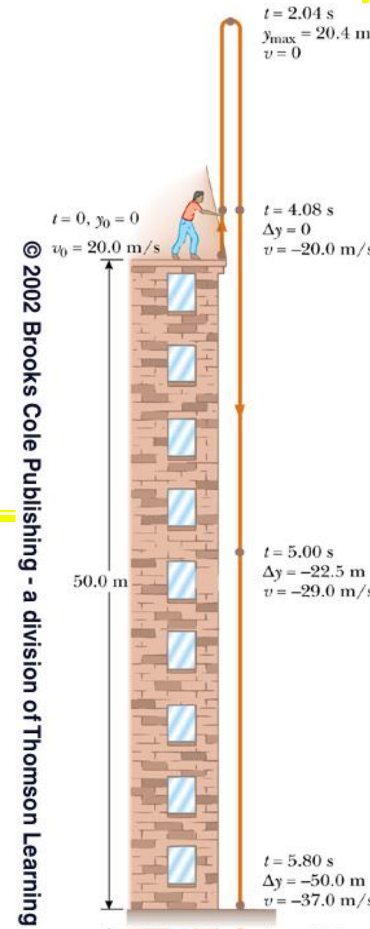

(c) $$y_C - y_0 = 0 = v_0 t + \frac{1}{2} a t^2$$

$$t = 0$$

$$t = \frac{-2v_0}{a} = \frac{-2 \times 20.0\ m/s}{-9.80\ m/s^2} = 4.08\ s$$

# EXAMPLE 2

A stone thrown from the top of a building is given an initial velocity of 20.0 m/s straight upward. The building is 50.0 m high, and the stone just misses the edge of the roof on its way down, as shown in figure . Using as the time the stone leaves the thrower's hand at position , determine
(d) the velocity of the stone at the stone returns to the height from which it was thrown




(d) $v_C = v_0 + at$

$$= 20.0\text{ m/s} + (-9.80\text{ m/s}^2) \times (4.08\text{ s})$$

$$= -20.0\ m/s$$

# EXAMPLE 2

A stone thrown from the top of a building is given an initial velocity of 20.0 m/s straight upward. The building is 50.0 m high, and the stone just misses the edge of the roof on its way down, as shown in figure . Using as the time the stone leaves the thrower's hand at position , determine
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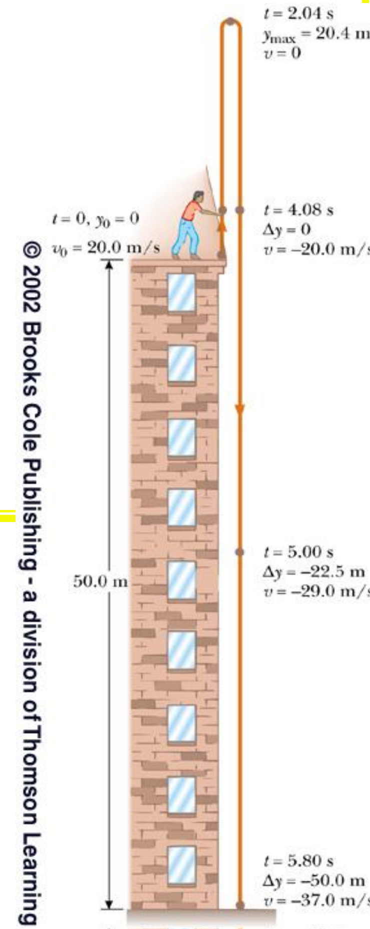



Other method : $v^2 - v_0^2 = 2a(y - y_0)$

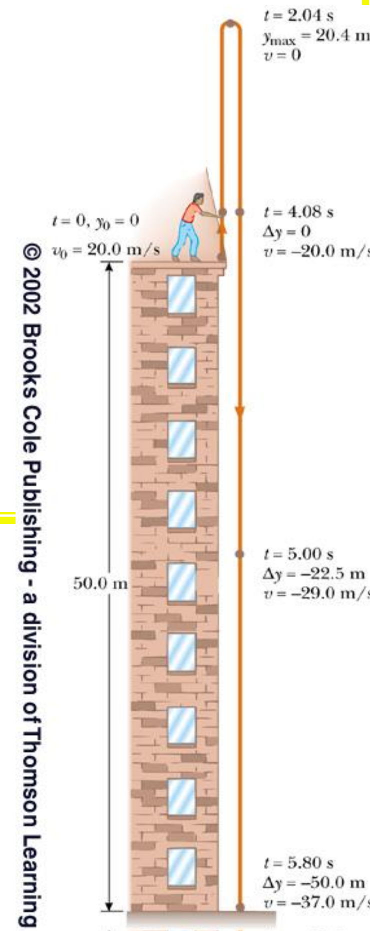
$$v^2 - v_0^2 = 2a(y_0 - y_0) = 0$$

$$v = \pm v_0 = \pm 20.0\ m/s$$

# *EXAMPLE 2*

A stone thrown from the top of a building is given an initial velocity of 20.0 m/s straight upward. The building is 50.0 m high, and the stone just misses the edge of the roof on its way down, as shown in figure . Using as the time the stone leaves the thrower's hand at position , determine

(e) the velocity and position of the stone at $t = 5.00$ s




(e) $v = v_0 + at$

$$= 20.0 \text{ m/s} + (-9.80 \text{ m/s}^2) \times (5.00 \text{ s})$$

$$= -29.0\ m/s$$

# EXAMPLE 2

A stone thrown from the top of a building is given an initial velocity of 20.0 m/s straight upward. The building is 50.0 m high, and the stone just misses the edge of the roof on its way down, as shown in figure .
Using as the time the stone leaves the thrower's hand at position , determine
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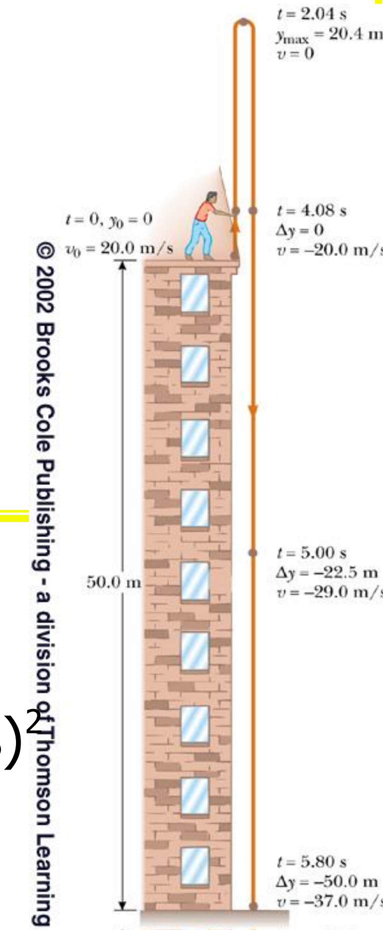



(e) $y_B = v_0 t + \frac{1}{2} a t^2$

$= (20.0 \text{ m/s}) \times (5.00 \text{ s}) + \frac{1}{2}(-9.80 \text{ m/s}^2) \times (5.00 \text{ s})^2$

$= -22.5\ m$

## PROBLEM 1

A person walks first at a constant speed of 5.00 m/s along a straight line from point *A* to point *B* and then back along the line from *B* to *A* at a constant speed of 3.00 m/s. What are (a) her average speed over the entire trip and (b) her average velocity over the entire trip?

## SOLUTION (a)

**From A to B :** $\bar{v}_1 = \dfrac{AB}{t_1}$ **From B to A :** $\bar{v}_2 = \dfrac{AB}{t_2}$

**Average speed over the entire trip :**

$$\bar{v} = \frac{2AB}{t} = \frac{2AB}{t_1 + t_2} = \frac{2AB}{AB / v_1 + AB / v_2} = \frac{2}{1 / v_1 + 1 / v_2}$$

$$= \frac{2}{1 / 5.00\ m / s + 1 / 3.00\ m / s} = 1.88\ m / s$$

## PROBLEM 1

A person walks first at a constant speed of 5.00 m/s along a straight line from point *A* to point *B* and then back along the line from *B* to *A* at a constant speed of 3.00 m/s. What are (a) her average speed over the entire trip and (b) her average velocity over the entire trip?

## SOLUTION (b)

**Average velocity over the entire trip :**

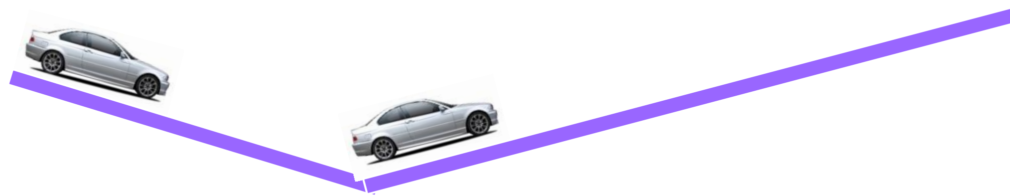
$$v = \frac{\Delta x}{t} = \frac{0}{t} = 0$$

**PROBLEM 2**

**A car starts from rest and accelerates at 0.500 m/s² while moving down an inclined plane 9.00 m long. When it reaches the bottom, the car rolls up another plane, where, after moving 15.0 m, it comes to rest. (a) What is the speed of the car at the bottom of the first plane ?**

**SOLUTION**

(a)



$$v^2 - v_0^2 = 2a(x - x_0)$$

$$v^2 - 0^2 = 2 \times 0.5\ m/s^2 \times 9.00\ m$$

$$v = \pm 3.00\ m/s$$

## PROBLEM 2

**A car starts from rest and accelerates at 0.500 m/s2 while moving down an inclined plane 9.00 m long. When it reaches the bottom, the car rolls up another plane, where, after moving 15.0 m, it comes to rest. (b) How long does it take to roll down the first plane?**

## SOLUTION (b)

$$v = v_0 + at$$

$$t = \frac{v - v_0}{a} = \frac{3.00\ m/s - 0}{0.05\ m/s^2} = 60.0\ s$$

## PROBLEM 2

**A car starts from rest and accelerates at 0.500 m/s2 while moving down an inclined plane 9.00 m long. When it reaches the bottom, the car rolls up another plane, where, after moving 15.0 m, it comes to rest. (c) What is the acceleration along the second plane?**

**SOLUTION** (c) $v'^2 - v^2 = 2a'(x' - x'_0)$

$$a' = \frac{v'^2 - v^2}{2(x' - x'_0)} = \frac{0^2 - (3.00\ m/s)^2}{2 \times 15.0\ m}$$

$$= -0.300\ m/s^2$$

Velocity is uniformly decreasing

## PROBLEM 2

**A car starts from rest and accelerates at 0.500 m/s2 while moving down an inclined plane 9.00 m long. When it reaches the bottom, the car rolls up another plane, where, after moving 15.0 m, it comes to rest. (d) What is the car's speed 8.00 m along the second plane?**

**SOLUTION** (d)

$$v_1^2 - v^2 = 2a'(x_1 - x'_0)$$

$$v_1^2 - (3.00\ m/s)^2 = 2 \times (-0.300\ m/s^2) \times 8.00\ m$$

$$v_1 = \pm 2.05\ m/s$$

# Chapter 1 Bases of Kinematics

## B. Motion in Two Dimensions

4. The Position, Velocity, and Acceleration Vectors
5. Two-Dimensional Motion with Constant Acceleration. Projectile Motion
6. Circular Motion. Tangential and Radial Acceleration
7. Relative Velocity and Relative Acceleration

# Chapter 1 Bases of Kinematics

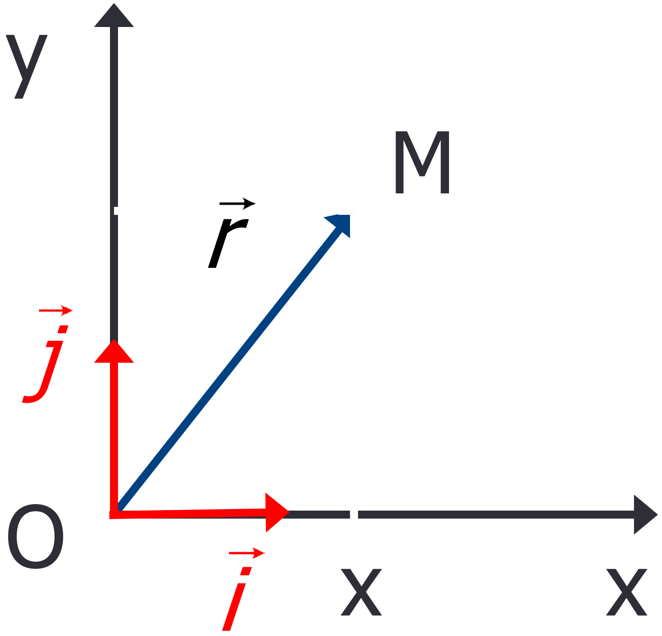
## B. Motion in Two Dimensions

4. The Position, Velocity, and Acceleration Vectors

# 4.1 Displacement

- The position of an object is described by its position vector:

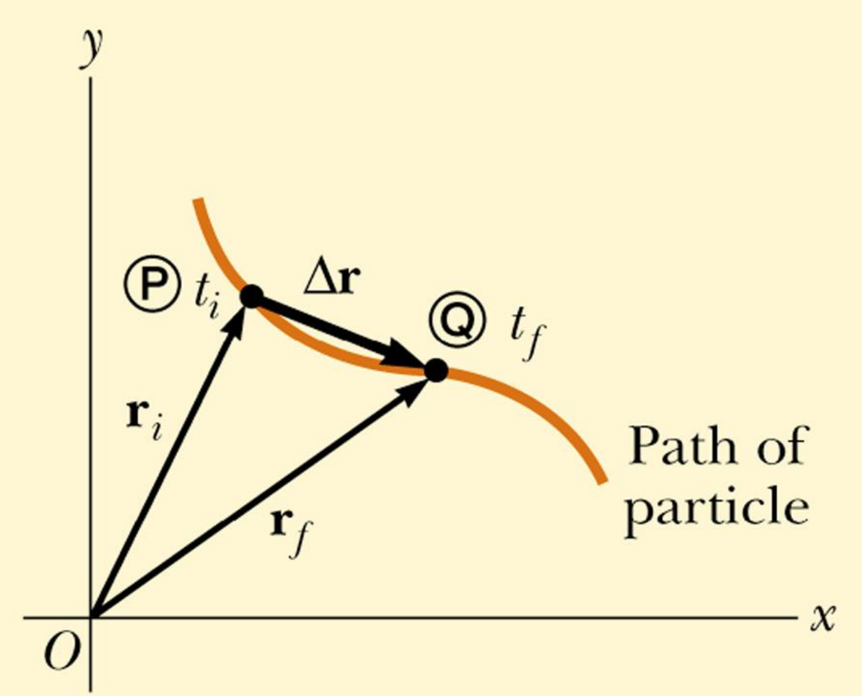
$$\vec{r} = x\vec{i} + y\vec{j}$$



# 4.1 Displacement

- The position of an object is described by its position vector $\vec{r}$
- The **displacement** of the object is defined as the ***change in its position***

$$\Delta\vec{r} = \vec{r}_f - \vec{r}_i$$



# 4.2 Velocity

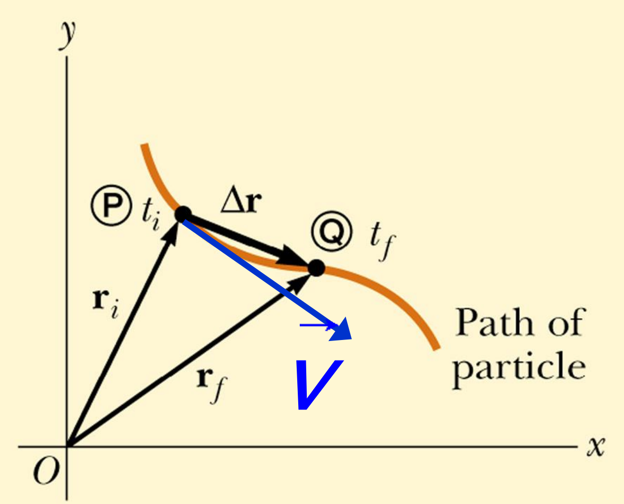
The instantaneous velocity is the limit of the average velocity as Δt approaches zero

- The direction of the instantaneous velocity is along a line that is tangent to the path of the particle and in the direction of motion

$$\vec{v} = \lim_{\Delta t \to 0} \frac{\Delta \vec{r}}{\Delta t} = \frac{d\vec{r}}{dt}$$

$$= \frac{d}{dt}\left(x\vec{i} + y\vec{j}\right)$$

$$= \frac{dx}{dt}\vec{i} + \frac{dy}{dt}\vec{j}$$





$$\boxed{\vec{V} = V_X\vec{i} + V_Y\vec{j}}$$

# 4.3 Acceleration

The instantaneous acceleration is the limit of the average acceleration as Δt approaches zero

$$\vec{a} = \lim_{\Delta t \to 0} \frac{\Delta \vec{v}}{\Delta t} = \frac{d\vec{v}}{dt} = \frac{d}{dt}\left( \frac{dx}{dt}\vec{i} + \frac{dy}{dt}\vec{j} \right)$$

$$= \frac{dv_X}{dt}\vec{i} + \frac{dv_Y}{dt}\vec{j}$$

$$\boxed{\vec{a} = a_X \vec{i} + a_Y \vec{j}}$$

## Chapter 1 Bases of Kinematics

## B. Motion in Two Dimensions

## 5. Two-Dimensional Motion with Constant Acceleration. Projectile Motion

### 5.1 Two-Dimensional Motion with Constant Acceleration

$$\vec{a} = const$$

# Vector acceleration is a const :

$$\vec{a} = a_X \vec{i} + a_Y \vec{j} = const \ ; \ a_X = const \ ; a_Y = const$$

$$a_X = \frac{dv_X}{dt} ; a_Y = \frac{dv_Y}{dt}$$

$$v_X = a_X t + v_{X0} ; v_Y = a_Y t + v_{Y0}$$

$$\vec{v} = (a_X t + v_{X0}) \vec{i} + (a_Y t + v_{Y0}) \vec{j}$$

$$\vec{v} = (a_X \vec{i} + a_Y \vec{j}) t + (v_{X0} \vec{i} + v_{Y0} \vec{j})$$

$$\boxed{\vec{v} = \vec{a} t + \vec{v}_0}$$

We can demonstrate :

$$\vec{r} = \vec{r}_0 + \vec{v}_0 t + \frac{\vec{a}}{2} t^2$$

( To compare with motion in one dimension :

$$x = x_o + v_o t + \frac{1}{2} a t^2 \quad )$$

# Chapter 1 Bases of Kinematics

# B. Motion in Two Dimensions

## 5. Two-Dimensional Motion with Constant Acceleration. Projectile Motion

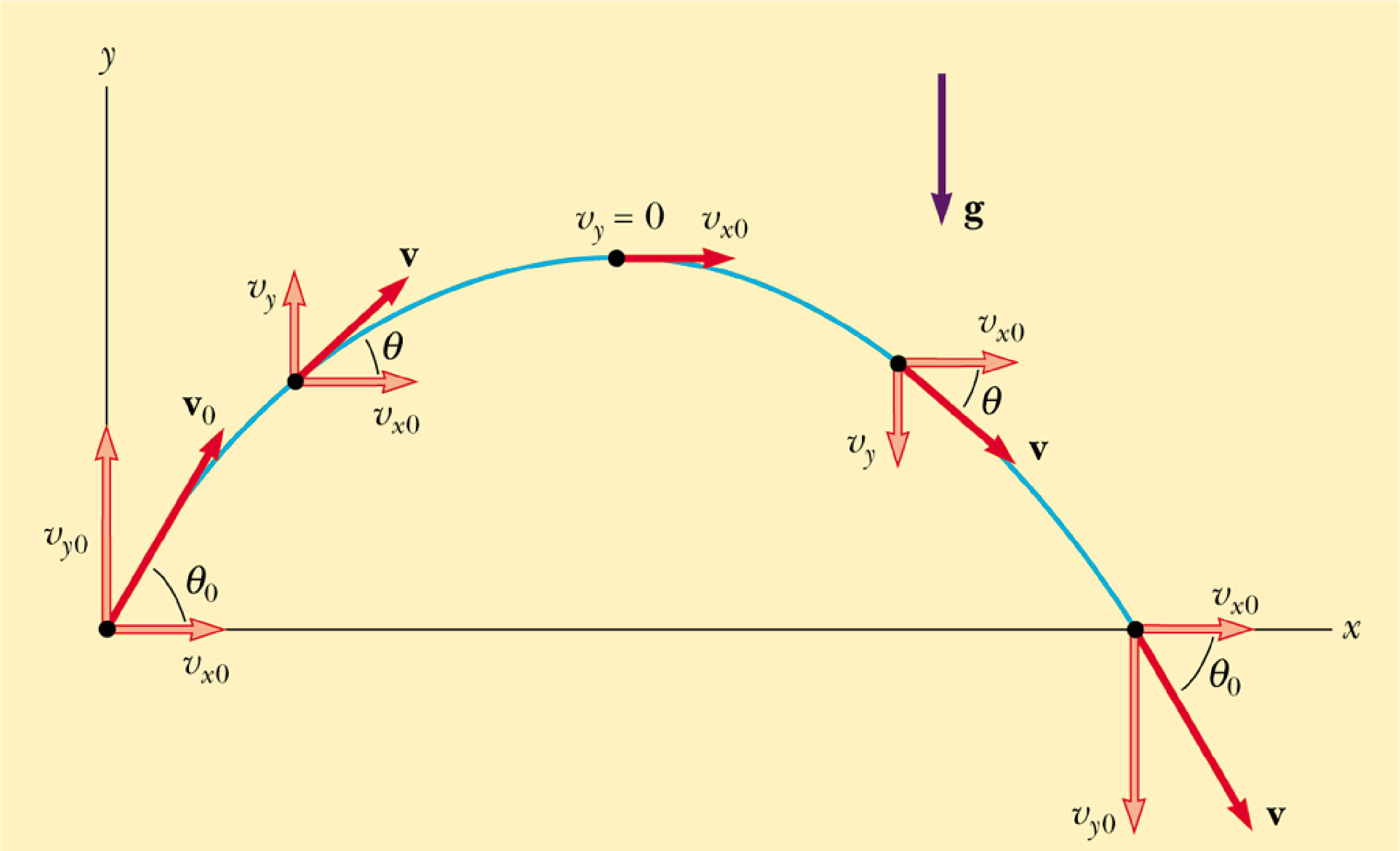
### 5.1 Two-Dimensional Motion with Constant Acceleration

### 5.2 Projectile Motion

$$\vec{a} = \vec{g} = const$$

# 5.2 Projectile Motion

$$\vec{a} = \vec{g} = const$$

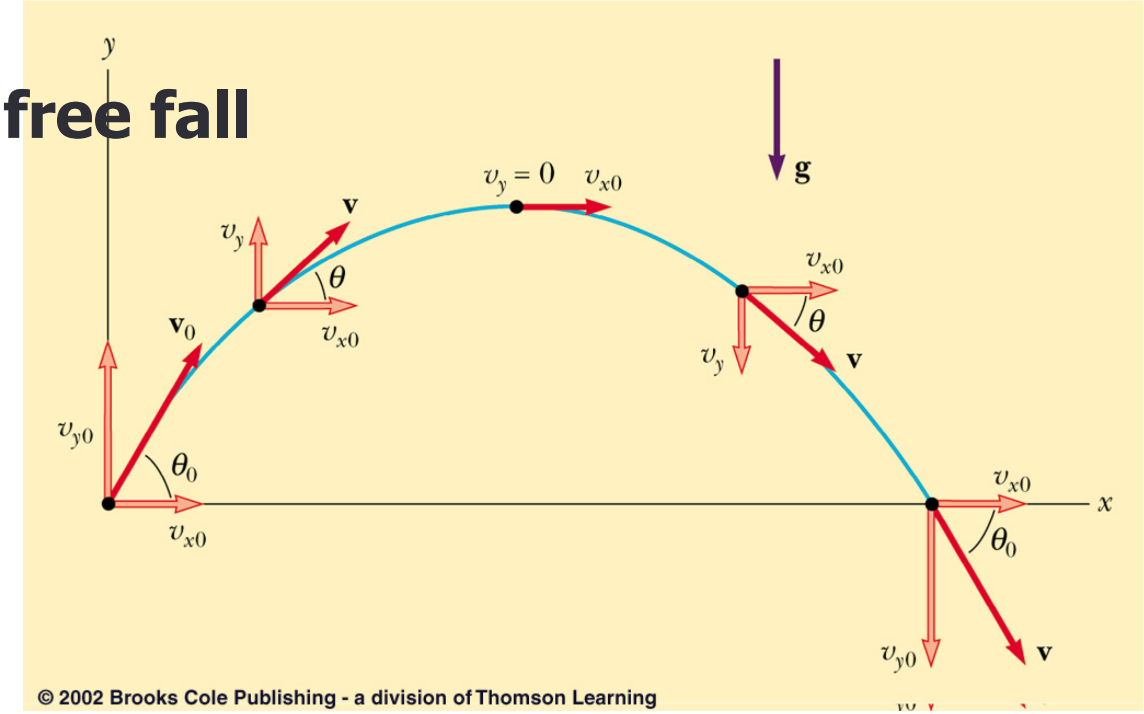


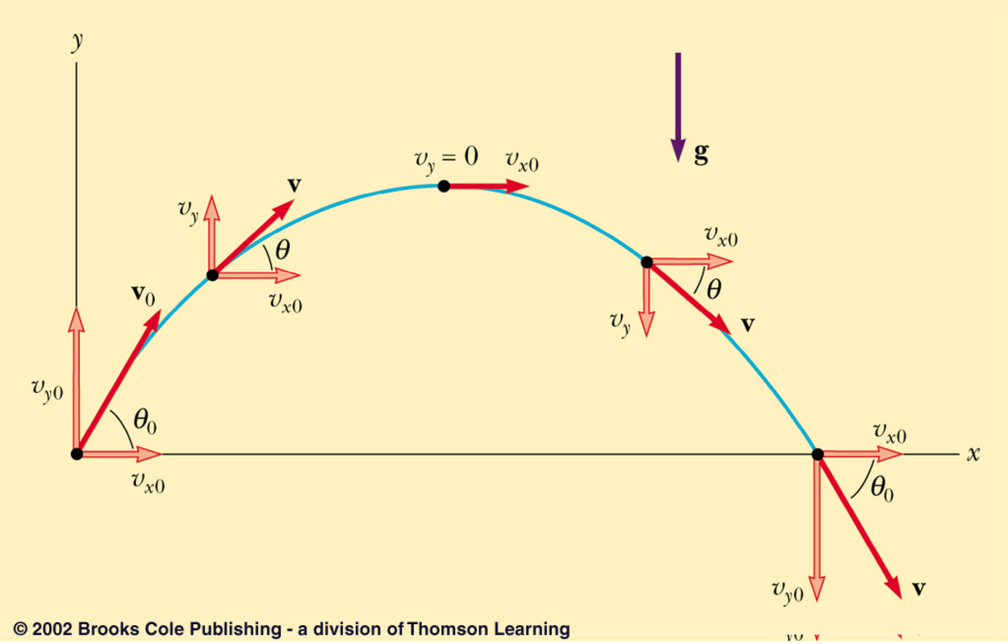
# Rules of Projectile Motion

- Introduce coordinate frame: y is up
- The x- and y-components of motion can be treated independently
- Velocities (incl. initial velocity) can be broken down into its x- and y-components
- The x-direction is uniform motion

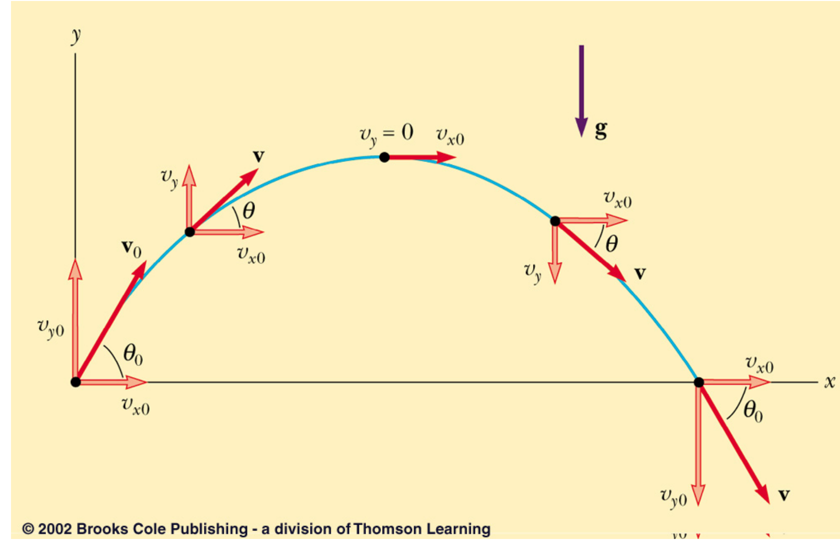
  $a_x = 0$
- The y-direction is free fall

  $|a_y| = g$






- x-direction
  - $a_x = 0$
  - $v_{xo} = v_o \cos\theta_o = v_x = \text{constant}$
  - $x = v_{xo}t$
    - This is the only operative equation in the x-direction since there is uniform velocity in that direction





- **y-direction**
  - $v_{yo} = v_o \sin\theta_o$
  - **take the positive direction as upward**
  - **then: free fall problem**
    - **only then: $a_y$ = -g (in general, $|a_y|$ = g)**
  - **uniformly accelerated motion, so the motion equations all hold**

$$v_{yo} = v_o \sin\theta_o$$

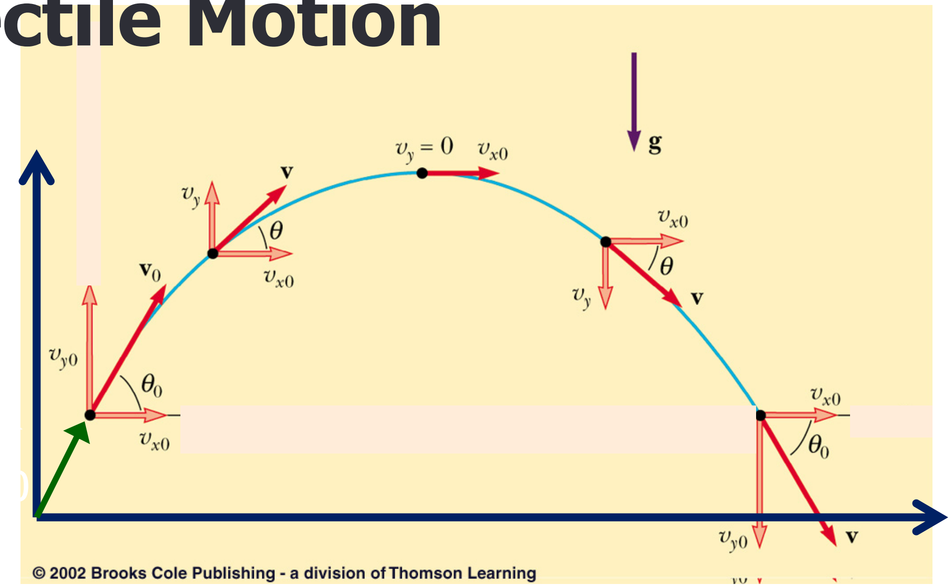
# Rules of Projectile Motion

$$\vec{v} = \vec{a}t + \vec{v}_0 = \vec{g}t + \vec{v}_0$$

On Ox and Oy :

$$v_X = 0 \times t + v_{X0} = v_{X0}$$

$$v_Y = -gt + v_{Y0}$$




$$\vec{r} = \vec{r}_0 + \vec{v}_0 t + \frac{\vec{a}}{2}t^2 = \vec{r}_0 + \vec{v}_0 t + \frac{\vec{g}}{2}t^2$$

On Ox and Oy :

$$x = x_0 + v_{X0}t$$

$$y = y_0 + v_{Y0}t - \frac{1}{2}gt^2$$

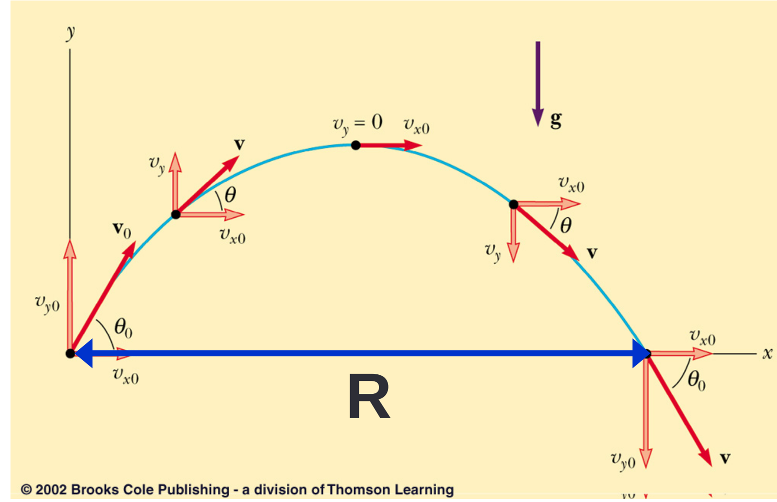
# Velocity of the Projectile

- The velocity of the projectile at any point of its motion is the vector sum of its x and y components at that point

$$v = \sqrt{v_x^2 + v_y^2} \quad \text{and} \quad \theta = \tan^{-1}\frac{v_y}{v_x}$$

# 5.2 Projectile Motion

$$\vec{a} = \vec{g} = const$$




On Ox and Oy :

$$x = x_0 + v_{x0}t = v_{x0}t$$

$$y = y_0 + v_{Y0}t - \frac{1}{2}gt^2 = v_{Y0}t - \frac{1}{2}gt^2$$

$$= \frac{v_{Y0}}{v_{X0}}x - \frac{1}{2}\frac{g}{(v_{X0})^2}x^2 = (\tan\theta_0)x - \frac{g}{2(v_0\cos\theta_0)^2}x^2$$

$$y = (\tan\theta_0)x - \frac{g}{2(v_0\cos\theta_0)^2}x^2$$ $\rightarrow$ trajectory: **parabola**

The horizontal range R:

$$y = (\tan\theta_0)x - \frac{g}{2(v_0\cos\theta_0)^2}x^2 = 0$$

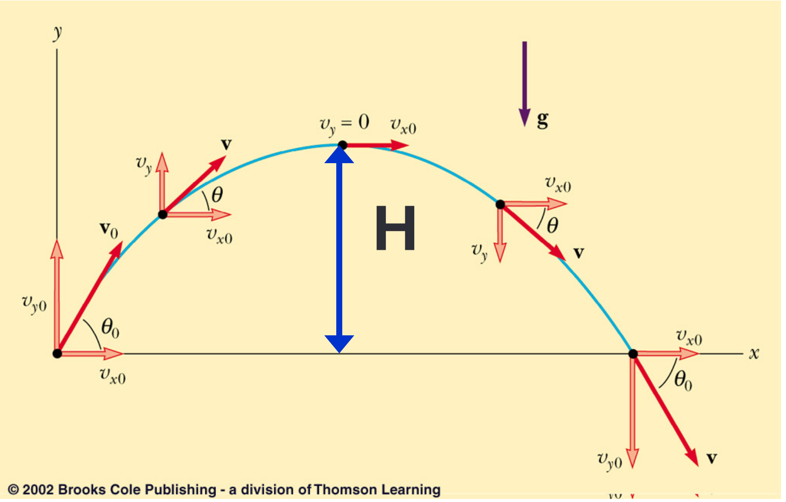
$$x = 0$$

$$x \equiv R = \frac{v_0^2}{g}\sin 2\theta_0$$

R max: $\theta_0 = 45^o$

# 5.2 Projectile Motion

$$\vec{a} = \vec{g} = const$$




$$y = (\tan\theta_0)x - \frac{g}{2(v_0\cos\theta_0)^2}x^2$$

Maximum height H:

$$y' = \left[(\tan\theta_0)x - \frac{g}{2(v_0\cos\theta_0)^2}x^2\right]'$$

$$= \tan\theta_0 - \frac{g}{(v_0\cos\theta_0)^2}x = 0\,; \quad x = \frac{R}{2} = \frac{v_0^2}{2g}\sin 2\theta_0$$

$$H = (\tan\theta_0)\frac{v_0^2}{2g}\sin 2\theta_0 - \frac{g}{2(v_0\cos\theta_0)^2}\left[\frac{v_0^2}{2g}\sin 2\theta_0\right]^2$$

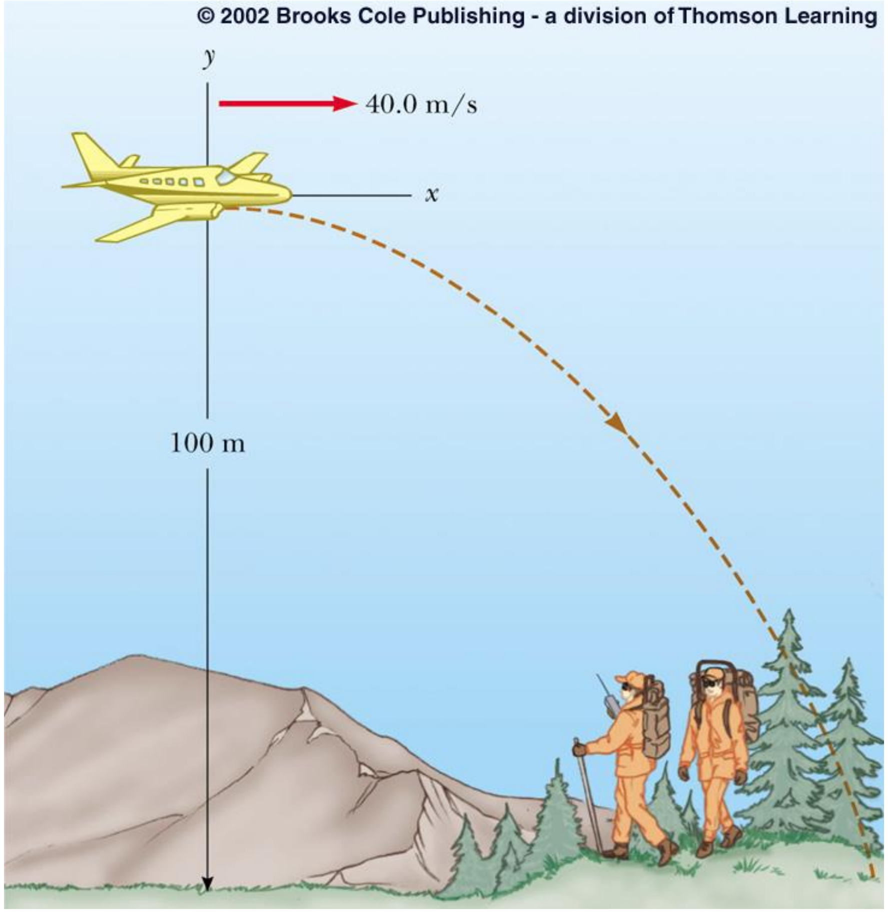
$$H = \frac{(v_0\sin\theta_0)^2}{2g}$$

Free fall:

$$v^2 - (v_0\sin\theta_0)^2 = 0^2 - (v_0\sin\theta_0)^2 = -2gH$$

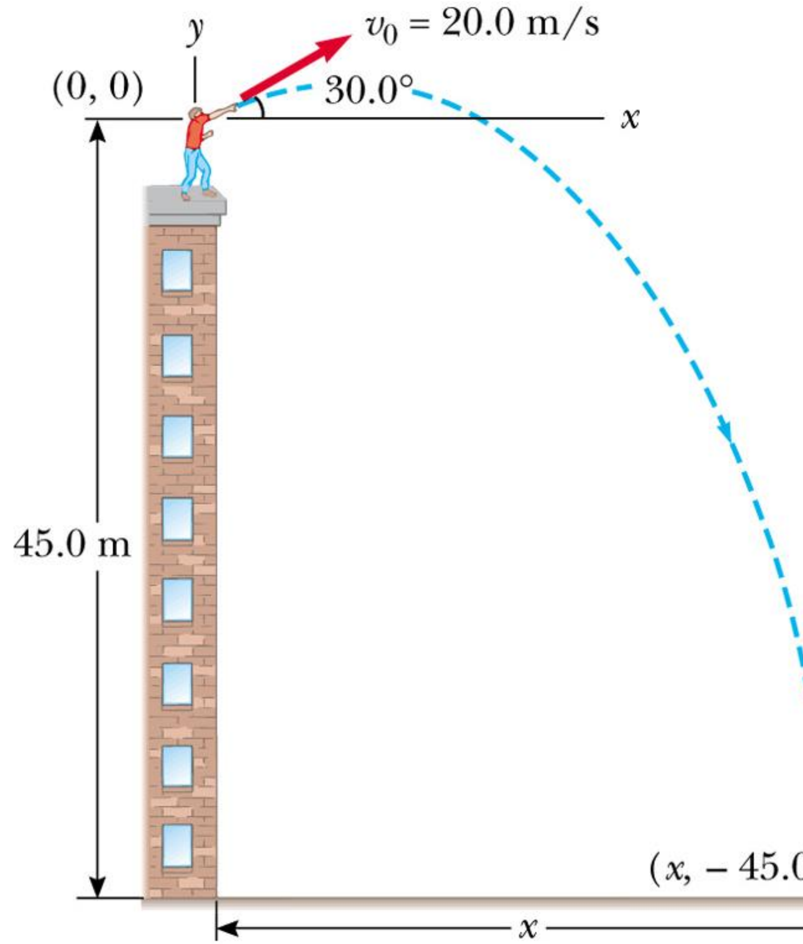
# Examples of Projectile Motion:

- An object may be fired horizontally
- The initial velocity is all in the x-direction
  - $v_o = v_x$ and $v_y = 0$
- All the general rules of projectile motion apply



# Non-Symmetrical Projectile Motion

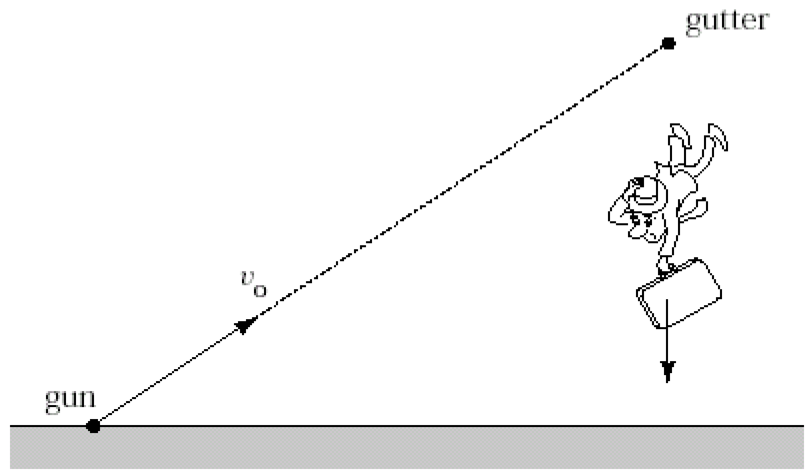
- Follow the general rules for projectile motion
- Break the y-direction into parts
  - up and down
  - symmetrical back to initial height and then the rest of the height



# Test 4

Consider the situation depicted here. A gun is accurately aimed at a dangerous criminal hanging from the gutter of a building. The target is well within the gun's range, but the instant the gun is fired and the bullet moves with a speed $v_o$, the criminal lets go and drops to the ground. What happens? The bullet

1. hits the criminal regardless of the value of $v_o$.
2. hits the criminal only if $v_o$ is large enough.
3. misses the criminal.



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1. hits the criminal regardless of the value of $v_o$. ✓
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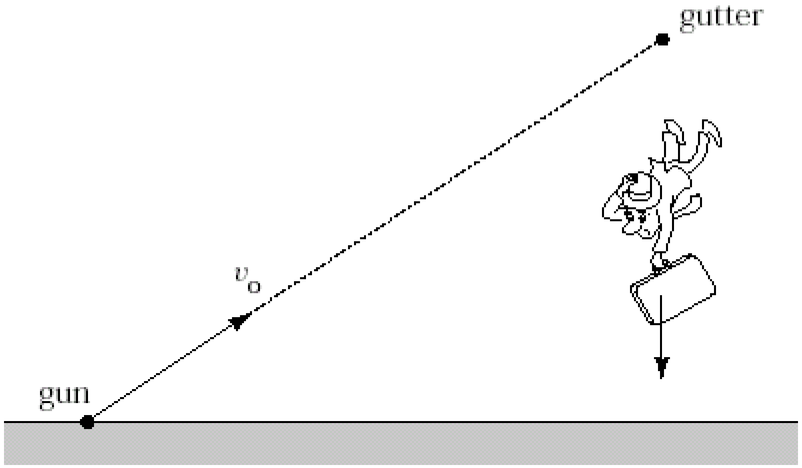



**Note:** The downward acceleration of the bullet and the criminal are identical, so the bullet will hit the target – they both "fall" the same distance!

# EXAMPLE 3

A long-jumper leaves the ground at an angle of 20.0° above the horizontal and at a speed of 11.0 m/s.
(a) How far does he jump in the horizontal direction ?
(Assume his motion is equivalent to that of a particle.)

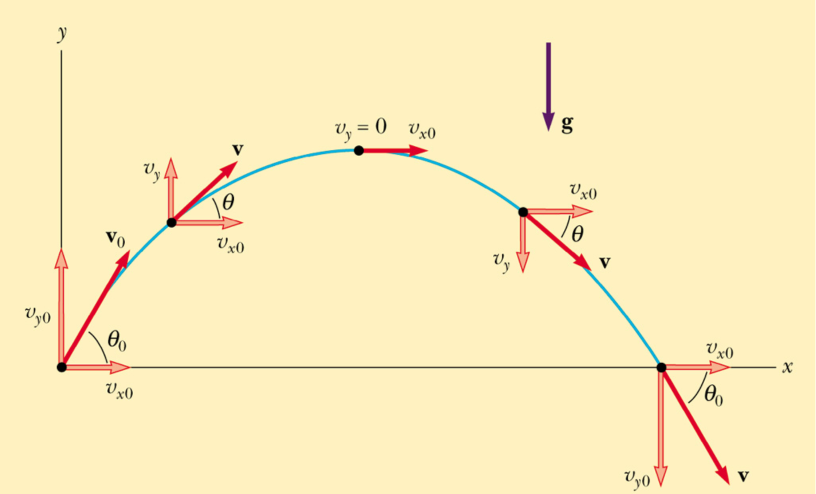
(a) $$v_{xo} = v_o \cos\theta_o$$

$$x = v_{x0}t = (v_0 \cos\theta_0)t$$

$$v_y = v_o \sin\theta_o - gt$$

At the top:

$$v_y = v_o \sin\theta_o - gt = 0$$

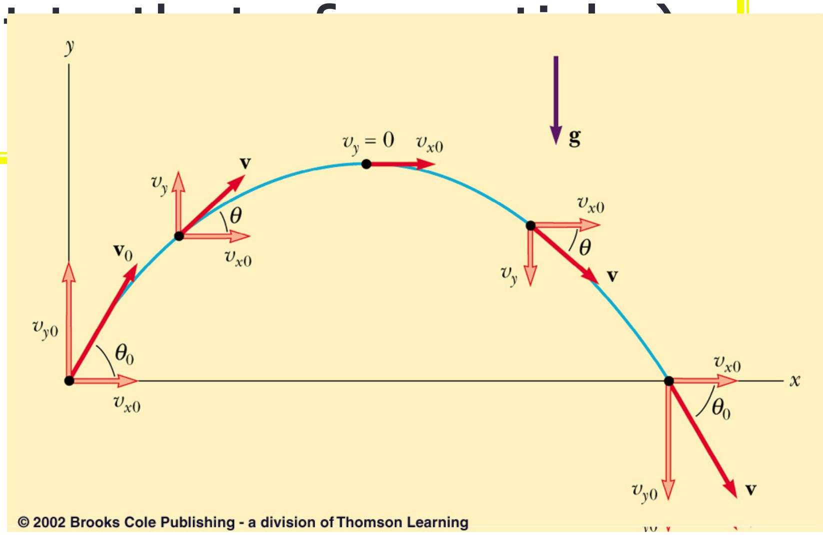


# EXAMPLE 3

A long-jumper leaves the ground at an angle of 20.0° above the horizontal and at a speed of 11.0 m/s.

(a) How far does he jump in the horizontal direction ?

(Assume his motion is equivalent




At the top:

$$v_y = v_o \sin\theta_o - gt' = 0$$

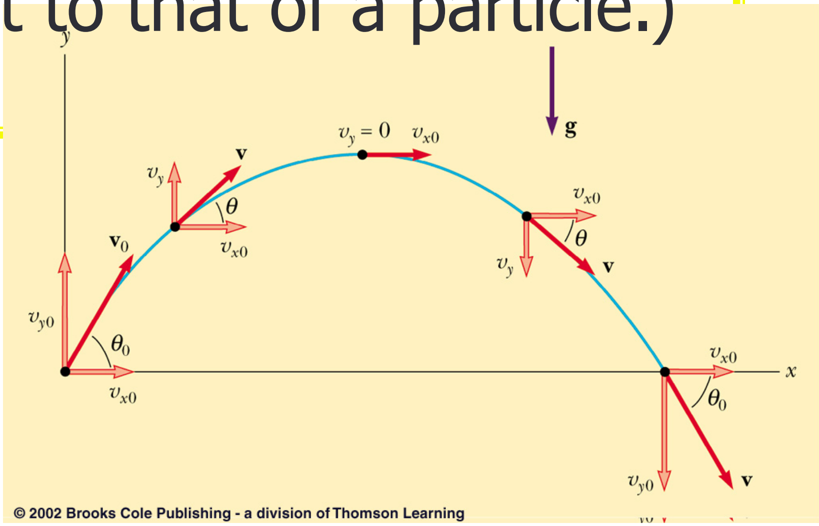
$$t' = \frac{v_o \sin\theta_o}{g}$$

$$= \frac{(11.0\ m/s)\sin 20^0 0}{9.80\ m/s^2} = 0.384\ s$$

# EXAMPLE 3

A long-jumper leaves the ground at an angle of 20.0° above the horizontal and at a speed of 11.0 m/s.

(a) How far does he jump in the horizontal direction ? (Assume his motion is equivalent to that of a particle.)




At the ground:

$$t = 2t' = 2 \times 0.384\ s = 0.768\ s$$

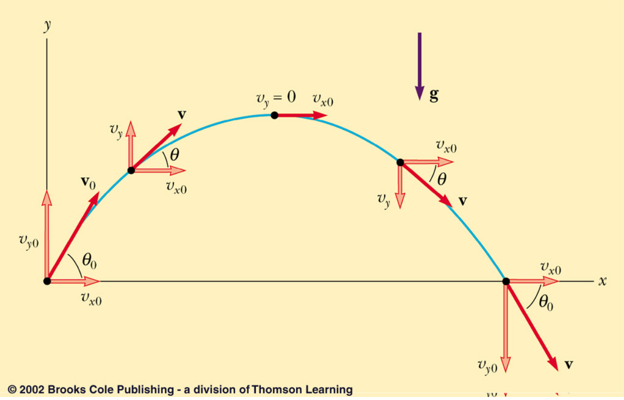
$$x = v_{x0}t \ = (v_0 \cos\theta_0)t$$

$$= 11.0\ m/s \times \cos 22^0 0 \times 0.768\ s = 7.94\ m$$

# EXAMPLE 3

A long-jumper leaves the ground at an angle of 20.0° above the horizontal and at a speed of 11.0 m/s.

(b) What is the maximum height reached ?




(b) At the top:

$$y_{MAX} = (v_o \sin\theta_o)t - \frac{1}{2}gt^2$$

$$= (11.0\ m/s)\sin 20.0^0 \times 0.384\ s$$

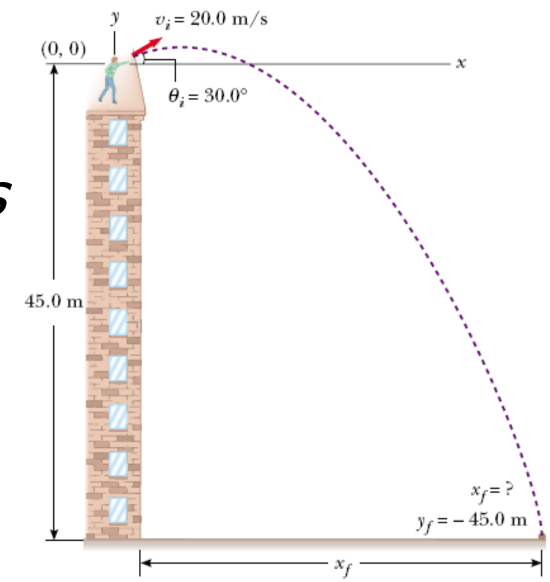
$$-\frac{1}{2} \times 9.80\ m/s^2 \times (0.384\ s)^2 = 0.722\ m$$

## PROBLEM 3

A stone is thrown from the top of a building upward at an angle of 30.0° to the horizontal and with an initial speed of 20.0 m/s, as shown in the figure. If the height of the building is 45.0 m,

**(a)** how long is it before the stone hits the ground?

## SOLUTION (a)




$$v_{X0} = v_0 \cos\theta = 20m/s \times \cos 30^0 = 17.3m/s$$

$$v_{Y0} = v_0 \sin\theta = 20m/s \times \sin 30^0 = 10.0m/s$$

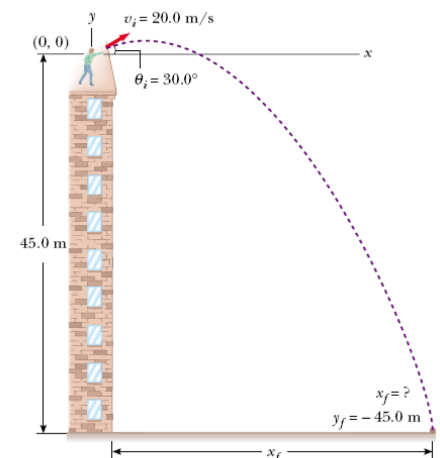
$$y = y_0 + v_{Y0}t - \frac{1}{2}gt^2$$

$$-45.0\ m = 0 + (10.0m/s)t - \frac{1}{2}(9.80m/s^2)t^2\ ;\ t = 4.22\ s$$

## PROBLEM 3

A stone is thrown from the top of a building upward at an angle of 30.0° to the horizontal and with an initial speed of 20.0 m/s, as shown in the figure. If the height of the building is 45.0 m,

**(b)** What is the speed of the stone just before it strikes the ground?




## SOLUTION (b)

$$v_{X0} = 17.3m/s \qquad v_{Y0} = 10.0m/s$$

$$v_X = v_{X0} = 17.3m/s$$

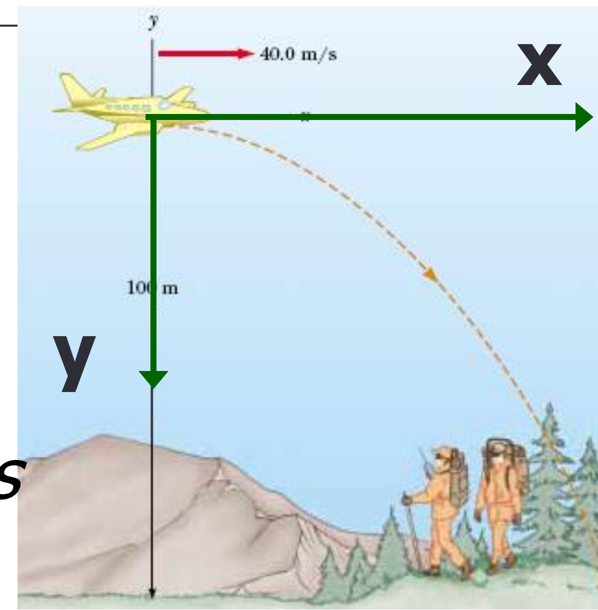
$$v_Y = -gt + v_{Y0} = -9.80m/s^2 \times 4.22s + 10.0m/s = -31.4m/s$$

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{(17.3m/s)^2 + (-31.4m/s)^2} = 35.9m/s$$

## PROBLEM 4

An Alaskan rescue plane drops a package of emergency rations to a stranded party of explorers, as shown in the figure.

**(a)** If the plane is traveling horizontally at 40.0 m/s and is 100 m above the ground, where does the package strike the ground relative to the point at which it was released?




**SOLUTION** (a)

$$x = vt = (40.0 m/s)t$$

$$y = -\frac{1}{2}gt^2 = (-4.90 m/s^2)t^2$$

$$-100\ m = (-4.90 m/s^2)t^2\ ;\ t = 4.52\ s$$

$$x = (40.0 m/s) \times 4.52 s = 181\ m$$

## PROBLEM 4

An Alaskan rescue plane drops a package of emergency rations to a stranded party of explorers, as shown in the figure.
**(b)** What are the horizontal and vertical components of the velocity of the package just before it hits the ground?

## SOLUTION (b)

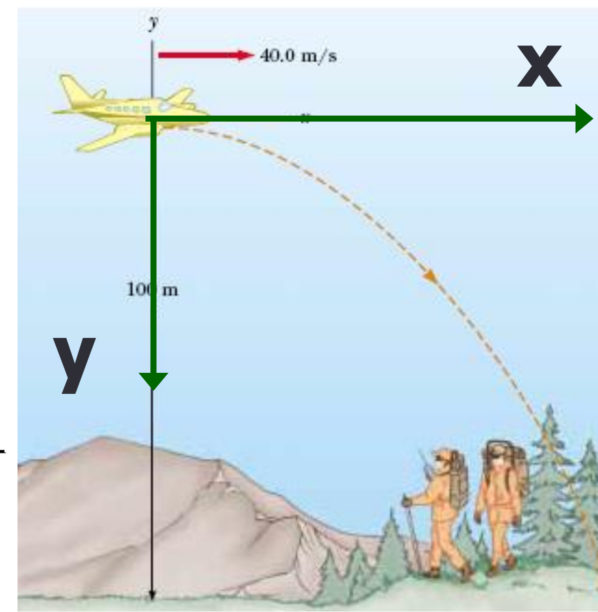
$$v_X = 40.0 m/s$$

$$v_Y = -gt + v_{Y0} = -9.80 m/s^2 \times 4.52 s + 0$$

$$= -44.3 m/s$$

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{(40.0 m/s)^2 + (-44.3 m/s)^2}$$

$$= 59.9 m/s$$



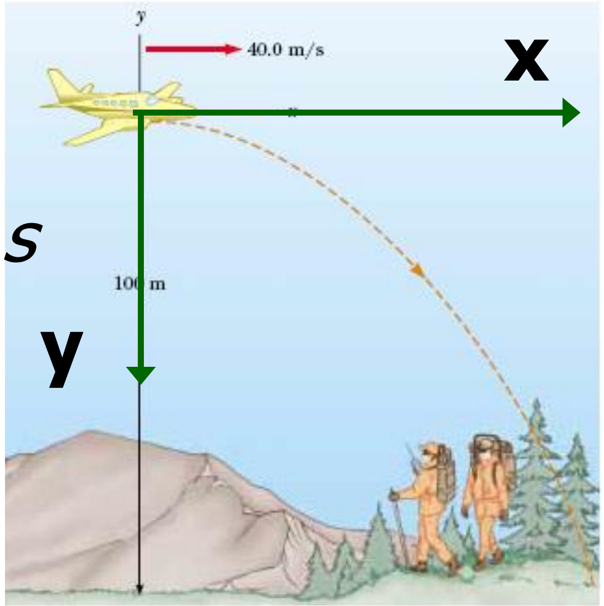
## PROBLEM 4

An Alaskan rescue plane drops a package of emergency rations to a stranded party of explorers, as shown in the figure.
**(c)** Where is the plane when the package hits the ground?

## SOLUTION (c)

$$x = vt = (40.0 m/s)t \; ; \; t = 4.52 \, s$$

$$x = vt = (40.0 m/s) \times 4.52 s = 59.9 m/s$$




**The plane is directly over the package.**

# Chapter 1 Bases of Kinematics

## A. Motion in One Dimension

## B. Motion in Two Dimensions

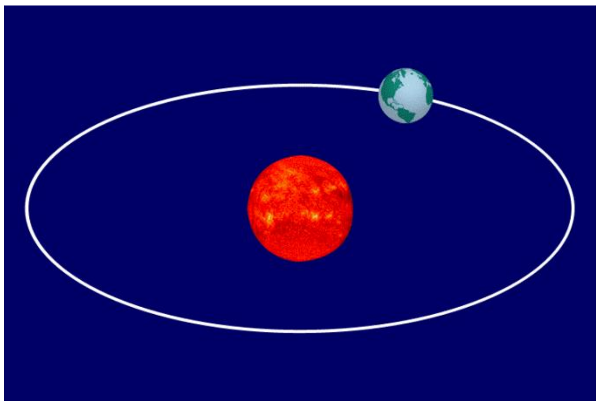
### 4. The Position, Velocity, and Acceleration Vectors

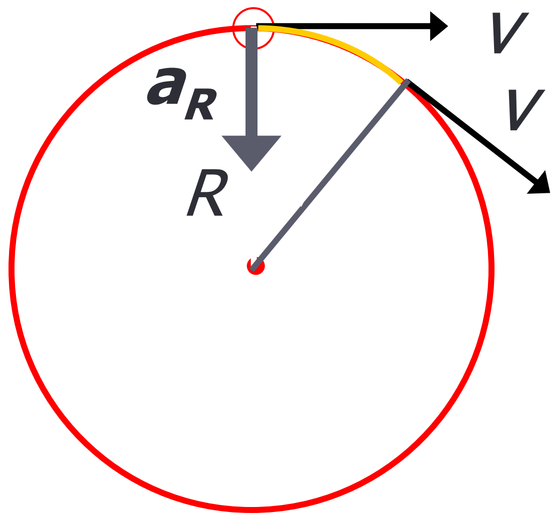
### 5. Two-Dimensional Motion with Constant Acceleration. Projectile Motion

### 6. Circular Motion. Tangential and Radial Acceleration

# 6. Circular Motion.
# Tangential and Radial Acceleration

## 6.1 Review of uniform circular motion






Magnitude of velocity does not change

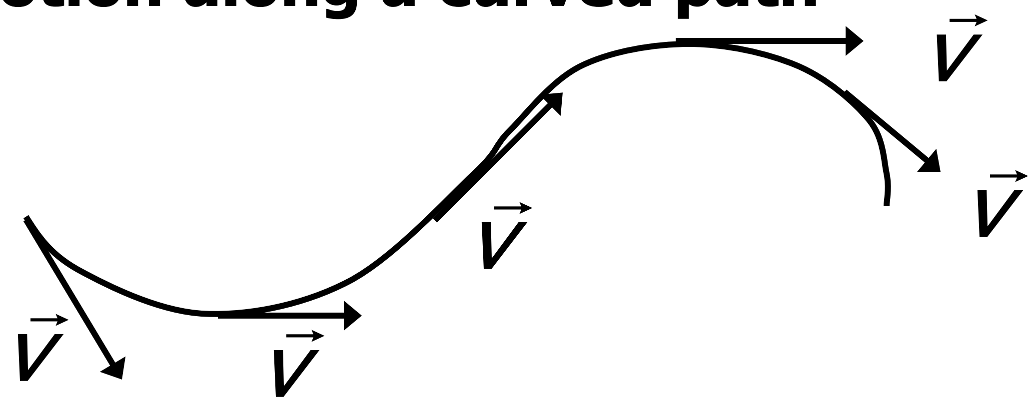
Acceleration vector : perpendicular to the path and always points toward the center of the circle

**centripetal (center-seeking) acceleration**

Magnitude : $\boxed{a_R = \dfrac{V^2}{R} = R\omega^2 = const}$

# 6. Circular Motion.
# Tangential and Radial Acceleration

## 6.2 Motion along a curved path



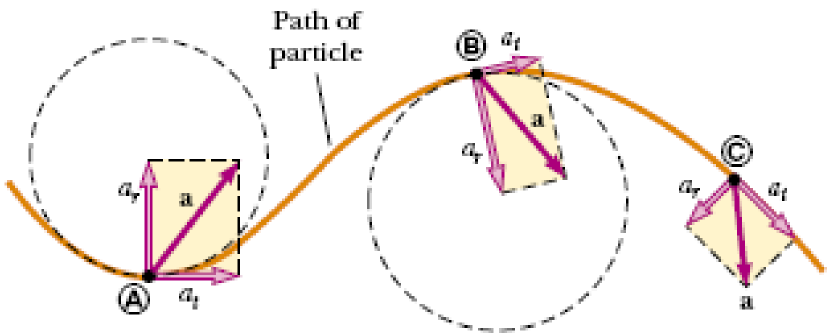

**Velocity vector** : changes both **in magnitude** and **in direction** at every point

**Acceleration vector $\rightarrow$ two component vectors:**

**a radial component vector a$_R$**

**and a tangential component vector a$_T$**

$$\boxed{\vec{a} = \vec{a}_R + \vec{a}_T}$$



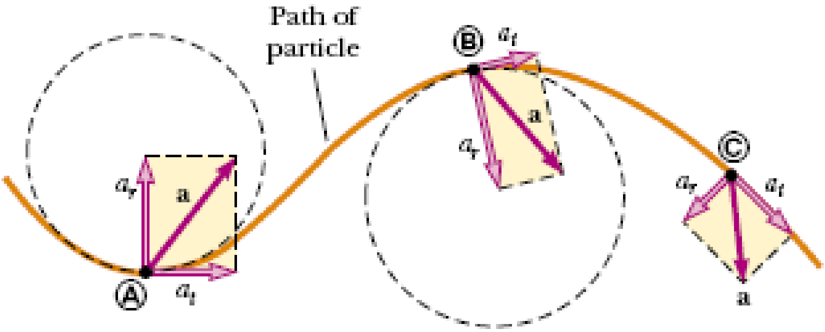

• The tangential acceleration causes the change in the **speed** of the particle: parallel to the instantaneous velocity, magnitude :

$$a_T = \frac{d\left|\vec{v}\right|}{dt}$$

• The radial acceleration arises from the change in **direction** of the velocity vector : perpendicular to the path, magnitude :

$$a_R = \frac{v^2}{R}$$

(***R*** : radius of curvature of the path at the point)




$$a_T = \frac{d\left|\vec{v}\right|}{dt}$$

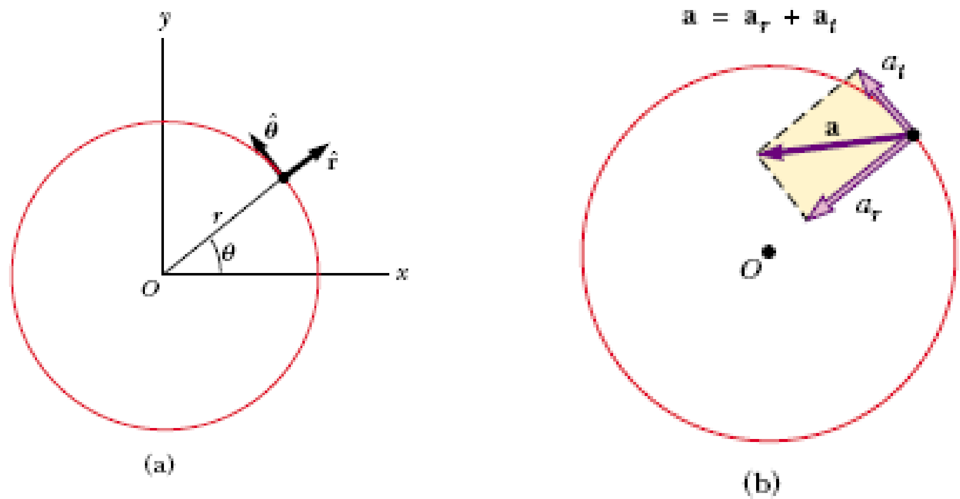
$$a_R = \frac{v^2}{R}$$

- The magnitude of the acceleration vector :

$$a = \sqrt{a_T^2 + a_R^2}$$

**Uniform circular motion** ( $v$ is constant ) : $\boldsymbol{a_T = 0}$
$\rightarrow$ the acceleration is always **completely radial**

- **Unit vectors**




$\hat{r}$ is a unit vector lying along the radius vector and directed radially outward from the center of the circle

$\hat{\theta}$ is a unit vector tangent to the circle in the direction of increasing θ

Both $\hat{r}$ and $\hat{\theta}$ move along with the particle

$$\boxed{\vec{a} = \vec{a}_R + \vec{a}_T = \frac{d\left|\vec{v}\right|}{dt}\hat{\theta} - \frac{v^2}{R}\hat{r}}$$

# *EXAMPLE 4*

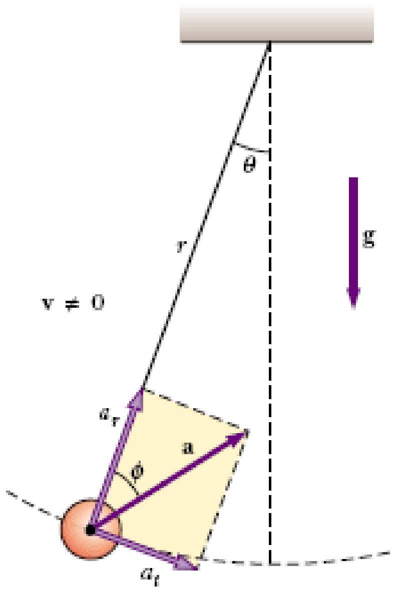
A ball tied to the end of a string 0.50 m in length swings in A vertical circle under the influence of gravity, as shown in figure. When the string makes an angle 20° with the vertical, the ball has a speed of 1.5 m/s.

**(a)** Find the magnitude of the radial component of acceleration at this instant.

(a)

$$a_R = \frac{v^2}{R}$$

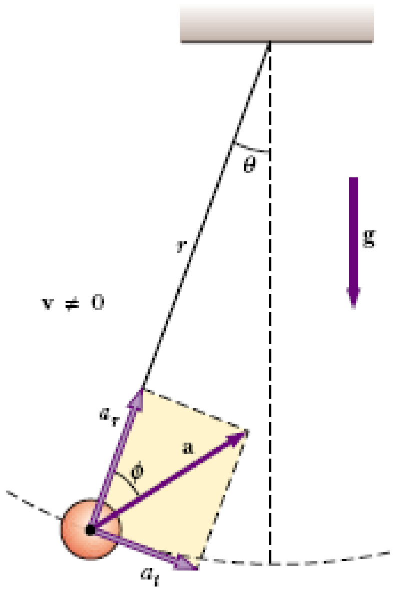
$$= \frac{(1.8\ m/s)^2}{0.50\ m} = 4.5 m/s^2$$



# *EXAMPLE 4*

A ball tied to the end of a string 0.50 m in length swings in a vertical circle under the influence of gravity, as shown in figure. When the string makes an angle 20° with the vertical, the ball has a speed of 1.5 m/s.

**(b)** What is the magnitude of the tangential acceleration when $\theta$ = 20°?



(b)

$$m\vec{a} = m\vec{g} + \vec{T}$$

$$ma_T = mg\sin\theta + 0$$

$$a_T = g\sin\theta$$

$$= (9.8m/s^2)\sin 20^0 = 3.4m/s^2$$

# *EXAMPLE 4*

A ball tied to the end of a string 0.50 m in length swings in a vertical circle under the influence of gravity, as shown in figure. When the string makes an angle 20° with the vertical, the ball has a speed of 1.5 m/s.

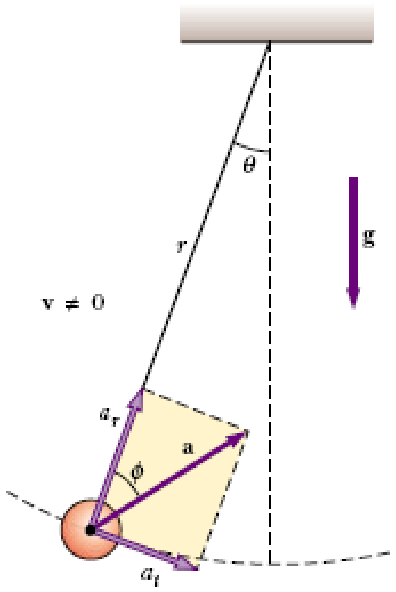
**(c)** Find the magnitude and direction of the total acceleration at $\theta = 20°$.

(c)

$$a = \sqrt{a_T^2 + a_R^2}$$

$$= \sqrt{4.5^2 + 3.4^2}\ m/s^2 = 5.6\ m/s^2$$

$$\varphi = \tan^{-1}\frac{a_T}{a_R} = \tan^{-1}\frac{3.4}{4.5} = 37^0$$

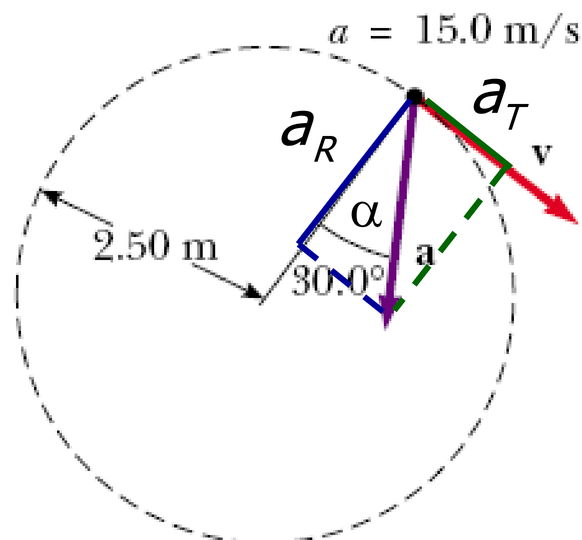


## PROBLEM 5

The figure shows the total acceleration and velocity of a particle moving clockwise in a circle of radius 2.50 at a given instant of time. At this instant, find

**(a)** the radial acceleration **(b)** the speed of the particle

**(c)** its tangential acceleration




## SOLUTION (a)

$$a_R = a\cos\alpha$$

$$= 15.0\ m/s^2 \times \cos 30^0 = 2.3\ m/s^2$$

(b) $$a_R = \frac{v^2}{R}\ ;\ |v| = \sqrt{Ra_R}$$

$$= \sqrt{2.5 \times 2.3} = 2.4\ m/s$$

(c) $$a_T = a\sin\alpha$$

$$= 15.0\ m/s^2 \times \sin 30^0 = 7.50\ m/s^2$$

# PHYSICS I

- **Chapter 1 Bases of Kinematics**
- **A. Motion in One Dimension**
- **1. Position, Velocity, and Acceleration**
- **2. One-Dimensional Motion with Constant Acceleration**
- **3. Freely Falling Objects**
- **B. Motion in Two Dimensions**
- **4. The Position, Velocity, and Acceleration Vectors**
- **5. Two-Dimensional Motion with Constant Acceleration. Projectile Motion**
- **6. Circular Motion. Tangential and Radial Acceleration**
- **7. Relative Velocity and Relative Acceleration**

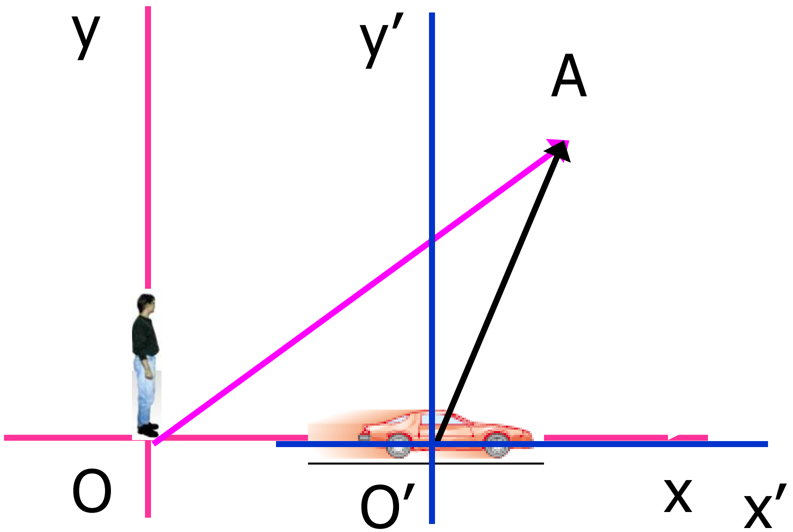
# 7. Relative Velocity and Relative Acceleration

## 7.1 Relative velocity

Consider a particle at point A

An observer in reference frame $S$

An observer in reference frame $S'$ moving to the right relative to $S$ with a constant velocity $v_0$ .

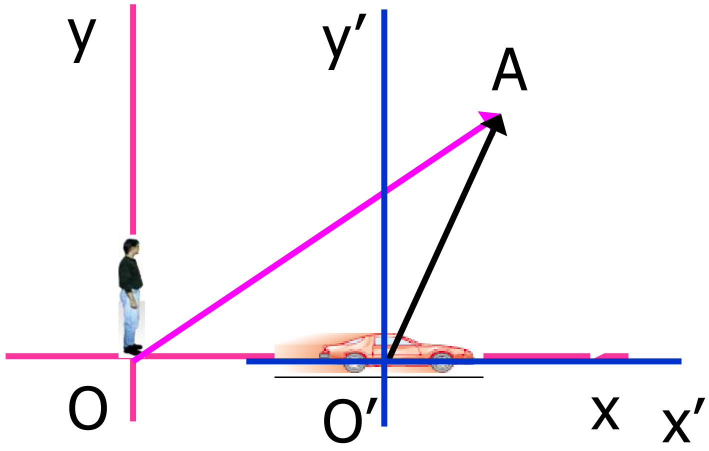



$$\overrightarrow{OA} = \overrightarrow{OO'} + \overrightarrow{O'A}\,; \quad \overrightarrow{r'} = \vec{r} - \overrightarrow{OO'}\,;$$

$$\overrightarrow{OO'} = \vec{v}_0 t \longrightarrow \boxed{\overrightarrow{r'} = \vec{r} - \overrightarrow{v_0} t}$$

Differentiate with respect to time : $\dfrac{d\,\overrightarrow{r'}}{dt} = \dfrac{d\,\vec{r}}{dt} - \overrightarrow{v_0}$

$$\longrightarrow \boxed{\overrightarrow{v'} = \vec{v} - \overrightarrow{v_0}}$$



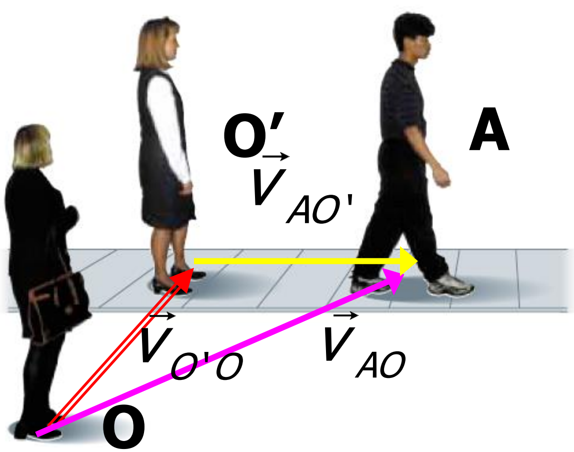

$$\vec{r}\,' = \vec{r} - \vec{v}_0 t$$

$$\vec{v}\,' = \vec{v} - \vec{v}_0$$

Galilean transformation equations

- **PRACITCE :** $\vec{v} = \vec{v}\,' + \vec{v}_0$

$$\overrightarrow{V_{AO}} = \overrightarrow{V_{AO'}} + \overrightarrow{V_{O'O}}$$




The woman standing on the beltway sees the walking man pass by at a slower speed than the woman standing on the stationary floor does

Transformation for acceleration

$$\boxed{\overrightarrow{v'} = \vec{v} - \overrightarrow{v_0}} \longrightarrow \frac{d\,\overrightarrow{v'}}{dt} = \frac{d\,\vec{v}}{dt} - \frac{d\,\overrightarrow{v_0}}{dt}$$

$$\boxed{\overrightarrow{a'} = \vec{a}}$$

The acceleration of the particle measured by an observer in the frame of reference S is the same as that measured by any other observer moving with constant velocity relative to the frame S.

# Test 5

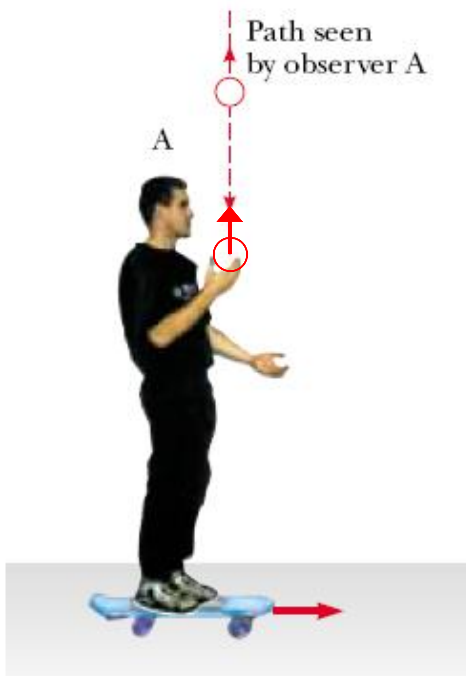
A passenger in a car traveling at 60 km/h pours a cup of coffee for the driver. The path of the coffee as it moves from a Thermos bottle into a cup as seen by the passenger is

1. nearly vertical into the cup, just as if the passenger were standing on the ground pouring it
2. a parabolic path
3. an oblique line

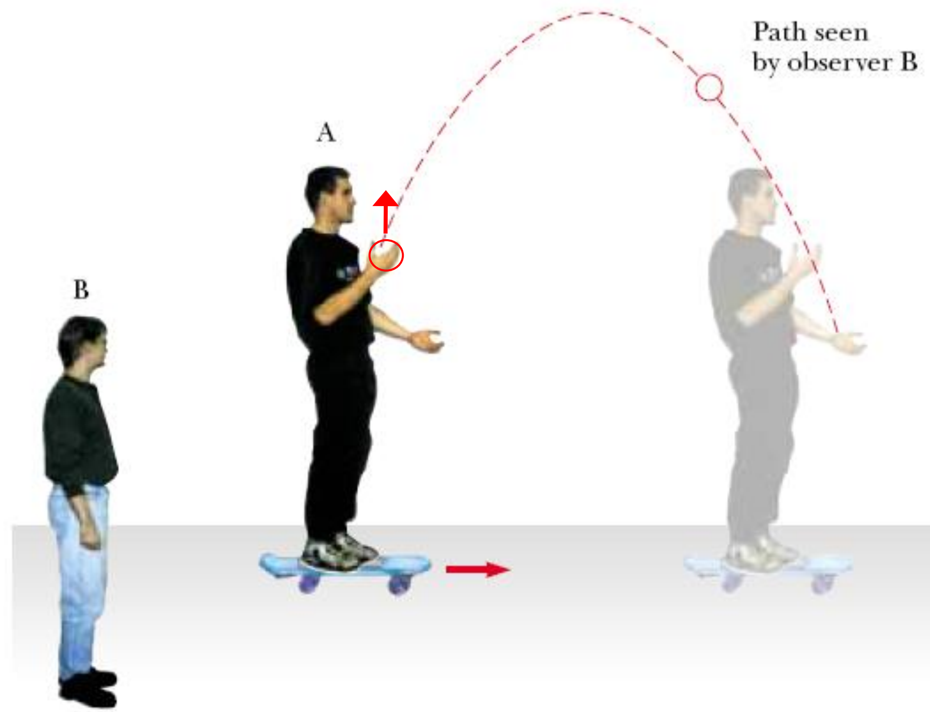
# Test 6

A passenger in a car traveling at 60 km/h pours a cup of coffee for the driver. The path of the coffee as it moves from a Thermos bottle into a cup as seen by someone standing beside the road and looking in the window of the car as it drives past is

1. nearly vertical into the cup, just as if the passenger were standing on the ground pouring it
2. a parabolic path
3. an oblique line




Observer A on a moving vehicle throws a ball upward and sees it rise and fall in **a straight-line path**.

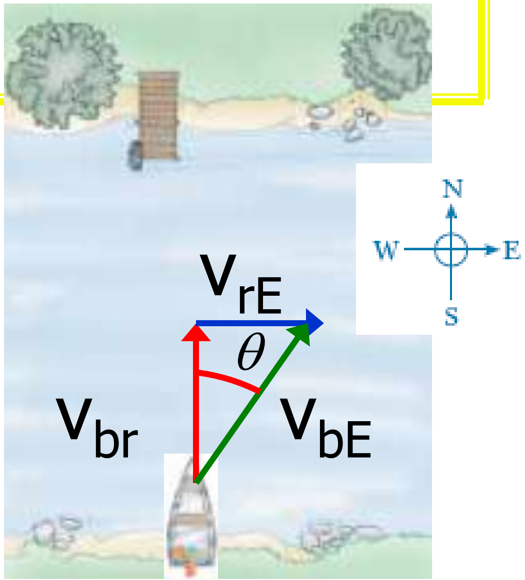



Stationary observer B sees a **parabolic path** for the same ball.

# *EXAMPLE 5*

A boat heading due north crosses a wide river with a speed of 10.0 km/h relative to the water. The water in the river has a uniform speed of 5.00 km/h due east relative to the Earth.

**(a)** Determine the velocity of the boat relative to an observer standing on either bank.




(a) $\overrightarrow{V_{bE}} = \overrightarrow{V_{br}} + \overrightarrow{V_{rE}}$ ; $V_{bE} = \sqrt{V_{br}^2 + V_{rE}^2}$

$$v_{bE} = \sqrt{(10.0\ km/h)^2 + (5.00\ km/h)^2} = 11.2\ km/h$$

Direction of $\overrightarrow{V_{bE}}$

$$\tan\theta = \frac{V_{rE}}{V_{br}} = \frac{5.00\ km/h}{10.0\ km/h}\ ;\ \theta = 26.6^0$$

# *EXAMPLE 5*

A boat heading due north crosses a wide river with a speed of 10.0 km/h relative to the water. The water in the river has a uniform speed of 5.00 km/h due east relative to the Earth.

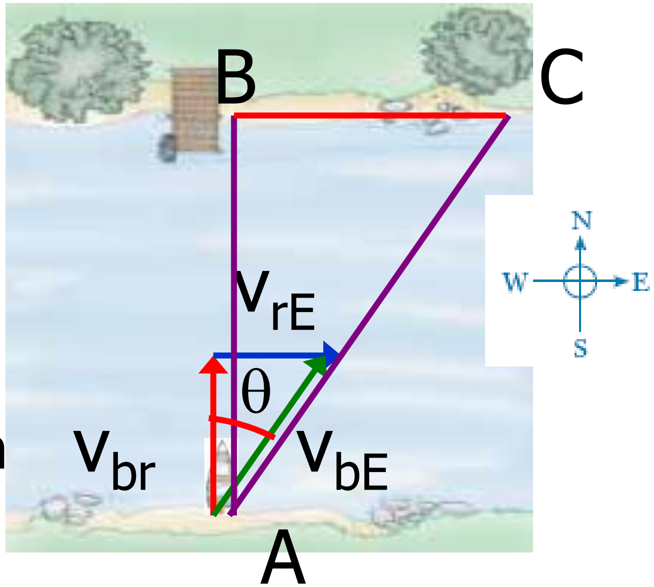
**(b)** If the width of the river is 3.0 km, find the time it takes the boat to cross it.

(b) $\cos\theta = \dfrac{v_{br}}{v_{bE}} = \dfrac{10.0\ km/h}{11.2\ km/h}$;

$$AC = \frac{AB}{\cos\theta} = \frac{3.00\ km}{(10.0\ km/h)/(11.2\ km/h)}$$

$$= 3.36\ km$$

$$t = \frac{AC}{v_{bE}} = \frac{3.36\ km}{11.2\ km/h} = 0.30\ h = 18.0\ \text{min}$$



# *EXAMPLE 5*

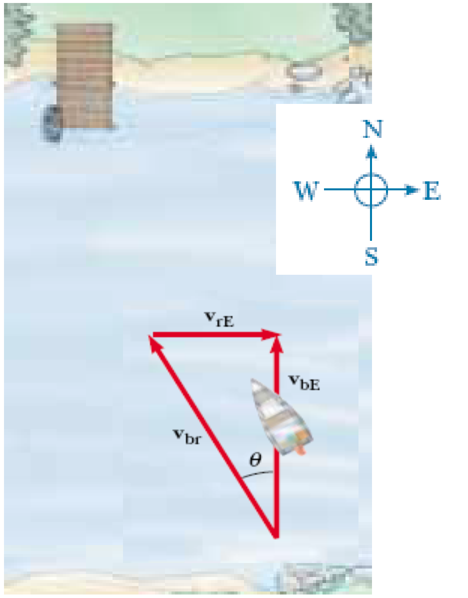
A boat heading due north crosses a wide river with a speed of 10.0 km/h relative to the water. The water in the river has a uniform speed of 5.00 km/h due east relative to the Earth.

**(c)** If this boat is to travel due north, as shown in the figure, what should its heading be? Find the time it takes the boat to cross the river. The width of the river is 3.0 km

(c) The boat must head upstream in order to travel directly northward across the river

$$\sin\theta = \frac{v_{rE}}{v_{br}} = \frac{5.00\ km/h}{10.0\ km/h}\ ;\ \theta = 30.0^0$$

The boat must steer a course 30.0° west of north



# *EXAMPLE 5*

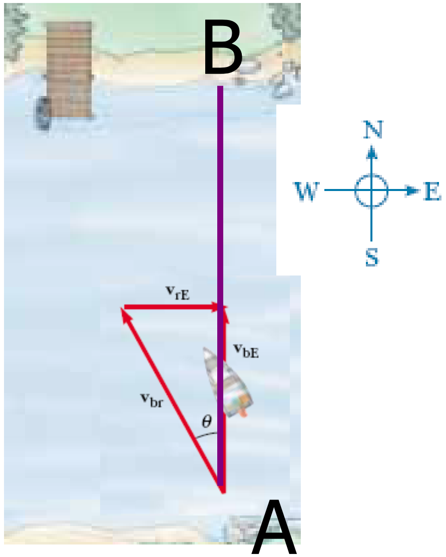
A boat heading due north crosses a wide river with a speed of 10.0 km/h relative to the water. The water in the river has a uniform speed of 5.00 km/h due east relative to the Earth.

**(c)** If this boat is to travel due north, as shown in the figure, what should its heading be? Find the time it takes the boat to cross the river. The width of the river is 3.0 km

(c) $$v_{bE} = v_{br}\cos\theta$$

$$= (10.0\ km/h)\cos 30.0^0 = 8.66\ km/h$$

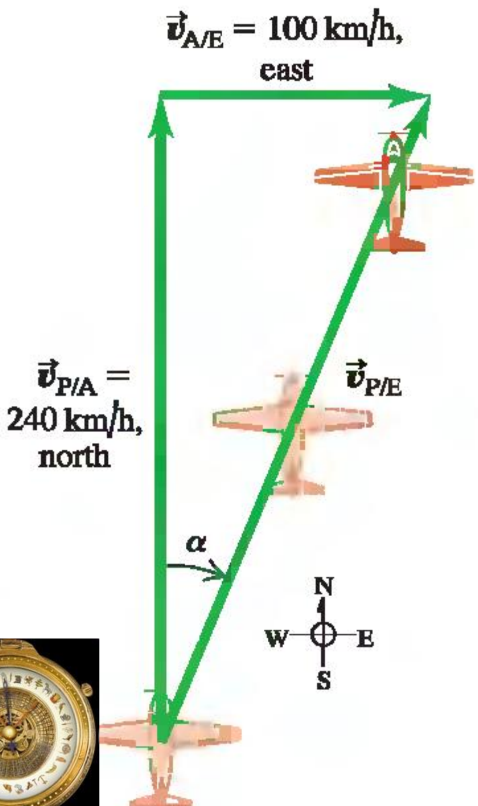
$$t = \frac{AB}{v_{bE}} = \frac{3.00\ km}{8.66\ km/h} = 21.0\ \text{min}$$



## PROBLEM 6

The compass of an airplane indicates that it is headed due north, and its airspeed indicator shows that it is moving through the air at 240 km/h. There is a wind of 100 km/h from west to east.
**(a)** What is the velocity of the airplane relative to the earth?

## SOLUTION




**(a)** $\vec{v}_{P/A} = 240\ \text{km/h}$ due north

$\vec{v}_{A/E} = 100\ \text{km/h}$ due east

$$\vec{v}_{P/E} = \vec{v}_{P/A} + \vec{v}_{A/E}$$

$$v_{P/E} = \sqrt{(240\ \text{km/h})^2 + (100\ \text{km/h})^2} = 260\ \text{km/h}$$

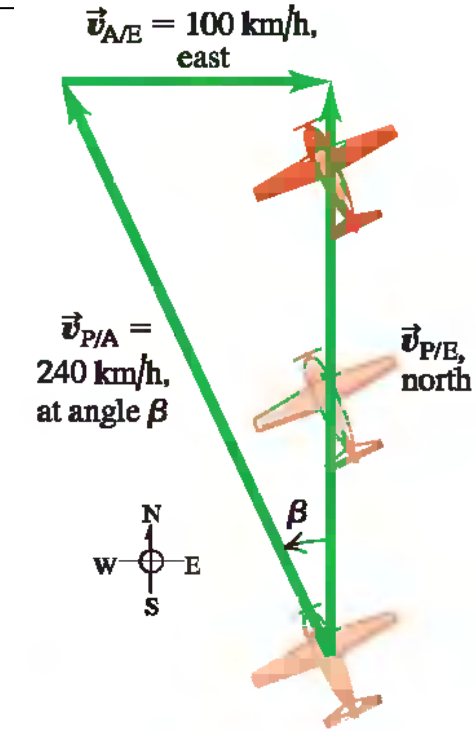
$$\alpha = \arctan\left(\frac{100\ \text{km/h}}{240\ \text{km/h}}\right) = 23^\circ\ \text{E of N}$$

## PROBLEM 6

The compass of an airplane indicates that it is headed due north, and its airspeed indicator shows that it is moving through the air at 240 km/h. There is a wind of 100 km/h from west to east.

**(b)** In what direction should the pilot head to travel due north? What will be her velocity relative to the earth?

## SOLUTION




**(b)** $\vec{v}_{P/E} = \vec{v}_{P/A} + \vec{v}_{A/E}$

| | | |
|---|---|---|
| $\vec{v}_{P/E}$ | = magnitude unknown | due north |
| $\vec{v}_{P/A}$ | = 240 km/h | direction unknown |
| $\vec{v}_{A/E}$ | = 100 km/h | due east |

$$v_{P/E} = \sqrt{(240\ \mathrm{km/h})^2 - (100\ \mathrm{km/h})^2} = 218\ \mathrm{km/h}^2$$

$$\beta = \arcsin\left(\frac{100\ \mathrm{km/h}}{240\ \mathrm{km/h}}\right) = 25°$$