INTERNATIONAL UNIVERSITY (IU) - VNU HCMC

ANSWERKEY MIDTERM EXAMINATION
PROBABILITY, STATISTICS AND RANDOM PROCESS
Semester 2, 2023-24 • April 2024 • Total duration: 90 minutes

Approval by the Mathematics Department	Lecturer
	Dr. Pham Hai Ha

INSTRUCTIONS: Each student is allowed calculators one double-sided sheet of reference material (size A4 or similar) marked with their name and ID and statistical tables. All other documents and electronic devices are forbidden.

1. *(10 points)* Among the customers of an insurance company, the percentages of those who have home insurance, liability insurance and car insurance are respectively, 35%, 15%, and 40%. Furthermore, 8% have both home and liability insurances; 25% have both home and car insurances; 10% have both liability and car insurances; 3% of the customers have all three types of insurance. Calculte the percentage of customers who have exactly 2 types of insurance with the company. 34%

2. *(10 points)* Roll a fair dice three times. Considers the events:

   - A: the outcome of the second roll is a two
   - B: two rolls out of the three resulted in a two

   (a) Calculate $P(B) = \frac{15}{6^3}$ and $P(B|A) = \frac{5}{18}$.

   (b) Are A and B independent? no

3. *(20 points)* There are two boxes. The first box contains 10 while and 6 red balls while the second box contains 4 white and 7 red balls. Select randomly a ball from the first box and place it in the second box. Then choose randomly a ball from the second box.

   (a) What is the probability that the ball selected from the second box is red? $\frac{10*7+6*8}{16*12} \approx 61\%$

   (b) Given that the ball selected from the second box is red, what is the conditional probability that the ball selected from the first box is also red? $= \frac{48}{118} \approx 40\%$

4. *(20 points)* Let X be a discrete random variable with probability mass function

$$P(X = x) = \frac{x}{c}, \quad \text{for } x = 2, 4, 7, 10$$

   Find the value of $c = 23$ and compute $P(X \leq 7 | X \geq 4) = \frac{\frac{4}{c} + \frac{7}{c}}{\frac{4}{c} + \frac{7}{c} + \frac{10}{c}} = \frac{11}{21} \approx 0.52$.

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5. *(30 points)* The waiting time X (in hour) for a bus at a given station is a continuous random variable with probability density function

$$f(x) = \begin{cases} Ce^{-3x} & \text{if } x \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

   (a) Find the value of $C = 3$.

   (b) Compute the mean $\frac{1}{3}$ and variance $\frac{1}{9}$ of X.

   (c) Alice has been waiting for a bus at this station to go to her company for 10 minuites. If there is no bus in the next 20 minuites then she will be late. In the case that Alice is late, she has to pay 10$ due to the company rule. Determine her expected payment. $10P(X > 1/2|X > 1/6) = 10P(X > 1/3)$ \$

6. *(10 points)* The daily demand for a product in a shop (in kilograms) has normal distribution with mean 80 and variance 36. Compute the probability that the demand tomorrow is greater than 90 kilograms. $P(X > 90) = P(Z > \frac{90-80}{\sqrt{36}}) \approx 1 - \phi(1.67) \approx 0.049$

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