Name:

ID   :

PRINCIPLES OF EE1

Homework #5

IMPORTANT: You should write on **A4 paper** that contains a full and detailed description of all the work done on the homework. Then you must submit the test hand-written by scanning and uploading the file in **pdf** form on Blackboard (Assignment Session). Marks will be deducted if there are sign of violation of regulation and late submission (20% for each day).

Tip: You draw a bounding box or highlight for your final answer. Ex: $Y = ABC + AC = \boxed{ABC}$

Problem 1: (25 marks)

Determine the phasor voltage $\boldsymbol{V}_g$ by using the node-voltage method in the circuit below.

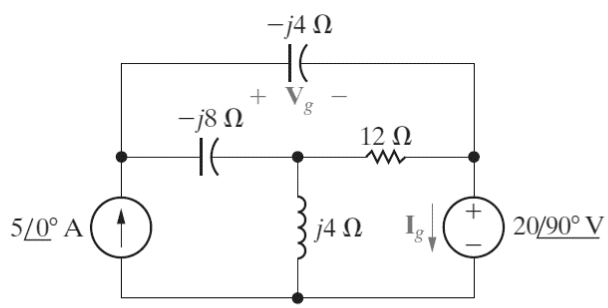
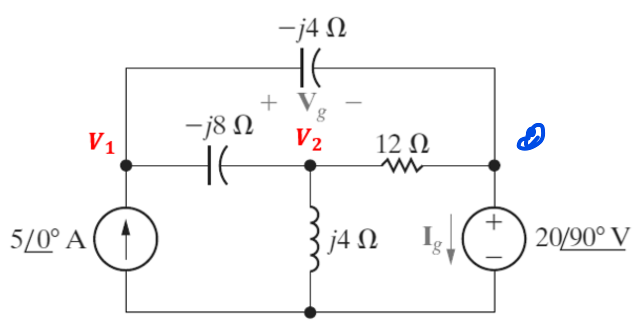


Figure 1

Solution:



Apply node voltage method:

$$\text{At } \mathbf{V}_1: \qquad -5\underline{/0^\circ} + \frac{\mathbf{V}_1 - \mathbf{V}_2}{-j8} + \frac{\mathbf{V}_1 - 20\underline{/90^\circ}}{-j4} = 0$$

$$\text{At } \mathbf{V}_2: \qquad \frac{\mathbf{V}_2 - \mathbf{V}_1}{-j8} + \frac{\mathbf{V}_2}{j4} + \frac{\mathbf{V}_2 - 20\underline{/90^\circ}}{12} = 0$$

$$\mathbf{V}_1\left(\frac{1}{-j8} + \frac{1}{-j4}\right) + \mathbf{V}_2\left(-\frac{1}{-j8}\right) = 5\underline{/0^\circ} + \frac{20\underline{/90^\circ}}{-j4}$$

$$\mathbf{V}_1\left(-\frac{1}{-j8}\right) + \mathbf{V}_2\left(\frac{1}{-j8} + \frac{1}{j4} + \frac{1}{12}\right) = \frac{20\underline{/90^\circ}}{12}$$

We get the result:

$$\boxed{\mathbf{V}_1 = -\frac{8}{3} + j\frac{4}{3} \qquad \mathbf{V}_2 = -8 + j4}$$

So, $\boldsymbol{V_g} = \boldsymbol{V_1} - \mathbf{20\angle 90} = -\frac{\mathbf{8}}{\mathbf{3}} - \frac{\boldsymbol{j}\mathbf{56}}{\mathbf{3}}$ $(\boldsymbol{V})$

Problem 2: (25 marks)

Using superposition to find the current I_L in circuit below.

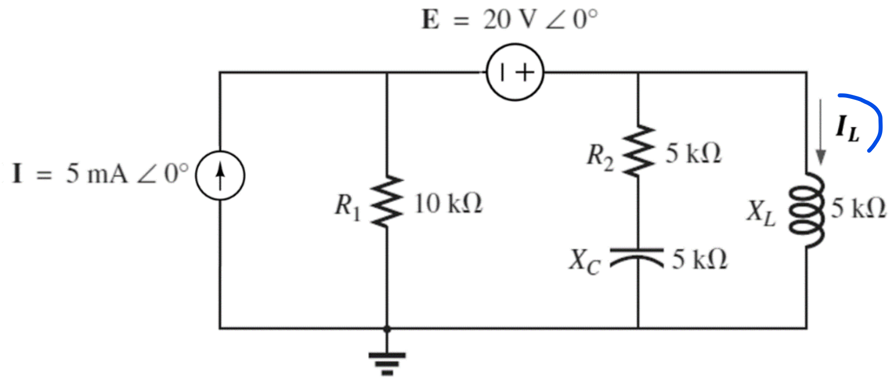
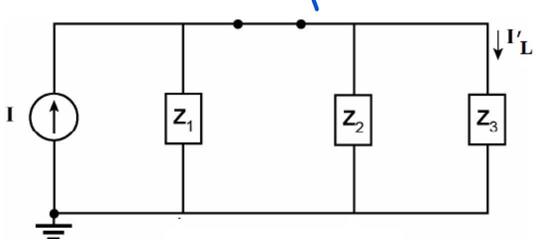


Figure 2

Solution:

Z'

1/ Replacing the voltage source by short circuit

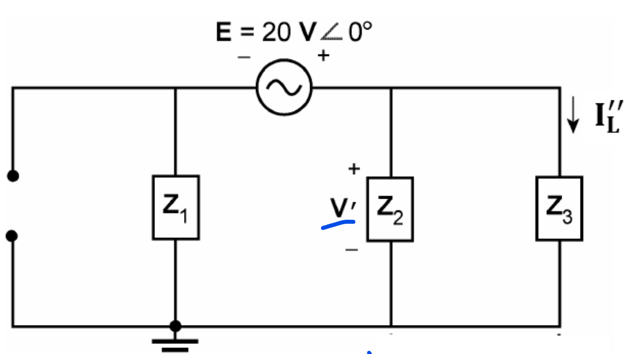


$$\mathbf{Z}' = \mathbf{Z}_1 \parallel \mathbf{Z}_2 = 10\text{ k}\Omega\ \angle 0^\circ \parallel 7.071\text{ k}\Omega\ \angle{-45^\circ} = 4.472\text{ k}\Omega\ \angle{-26.57^\circ}$$

$$\mathbf{I}'_L = \frac{\mathbf{Z}'\mathbf{I}}{\mathbf{Z}' + \mathbf{Z}_3} = \frac{(4.472\text{ k}\Omega\ \angle -26.57^\circ)(5\text{ mA}\ \angle 0^\circ)}{4\text{ k}\Omega - j2\text{ k}\Omega + j5\text{ k}\Omega} = \frac{22.36\text{ mA}\ \angle -26.57^\circ}{5\ \angle 36.87^\circ}$$

$$= 4.472\text{ mA}\ \angle{-63.44^\circ}$$

2/ Replacing the current source by open circuit



Z''

$$\mathbf{Z}'' = \mathbf{Z}_2 \parallel \mathbf{Z}_3$$
$$= 7.071\ \text{k}\Omega\ \angle{-45^\circ} \parallel\ 5\ \text{k}\Omega\ \angle 90^\circ$$
$$= 7.071\ \text{k}\Omega\ \angle 45^\circ$$

$$\mathbf{V}' = \frac{\mathbf{Z}''\mathbf{E}}{\mathbf{Z}''+\mathbf{Z}_1} = \frac{(7.071\ \text{k}\Omega\ \angle 45^\circ)(20\ \text{V}\ \angle 0^\circ)}{(5\ \text{k}\Omega + j5\ \text{k}\Omega) + (10\ \text{k}\Omega)} = \frac{141.42\ \text{V}\ \angle 45^\circ}{15.81\ \angle 18.435^\circ}$$
$$= 8.945\ \text{V}\ \angle 26.565^\circ$$
$$\mathbf{I}'' = \frac{\mathbf{V}'}{\mathbf{Z}_3} = \frac{8.945\ \text{V}\ \angle 26.565^\circ}{5\ \text{k}\Omega\ \angle 90^\circ} = 1.789\ \text{mA}\ \angle{-63.435^\circ} = 0.8\ \text{mA} - j1.6\ \text{mA}$$

So, the current $\boldsymbol{I_L = I'_L + I''_L = 6.26\ mA\angle - 63.43}$

Problem 3: **(25 marks)**

Determine the current $i_x(t)$ I steady state of the following circuit when $v_0(t) = 2\sin(2t)$

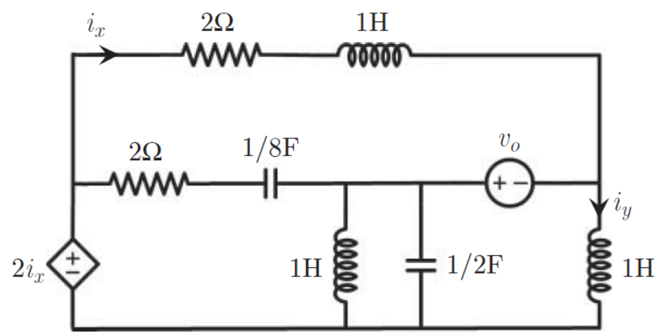
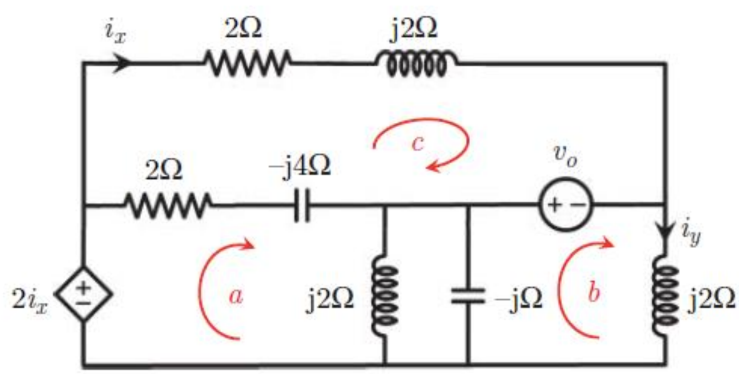


Figure 3

<u>Solution:</u>

Redraw the circuit in phasor domain



$-j2$

$V_o = -j2$

We have:

$\boldsymbol{V}_0 = 2\angle -90 = -j2\ (V)$

$(j2) \parallel (-j) = -j2\ (\Omega)$

Apply mesh analysis for Mesh a, b, c:

Mesh (b): $-j2(\boldsymbol{I}_b - \boldsymbol{I}_a) - j2 + j2\boldsymbol{I}_b = 0 \quad \Rightarrow \boldsymbol{I}_a = 1\ (A)$

Mesh (c): $(2 + j2)\boldsymbol{I}_c + j2 + (2 - j4)(\boldsymbol{I}_c - \boldsymbol{I}_a) = 0 \Rightarrow \boldsymbol{I}_c = 1 - j\ (A)$

⇨ $\boldsymbol{I}_x = \boldsymbol{I}_c = 1 - j = \sqrt{2}\angle(-45)$

So, $\boldsymbol{i_x(t) = \sqrt{2}\cos(2t - 45)}$ **(A)**

Problem 4: **(25 marks)**

Find the Norton equivalent circuit at terminal a-b.

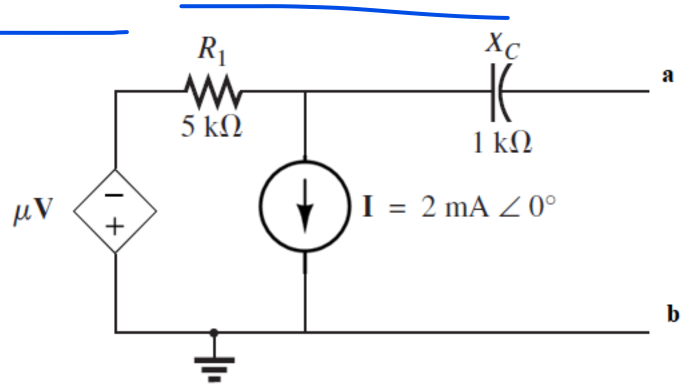
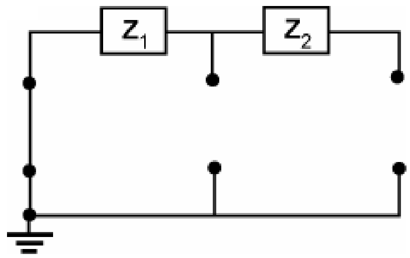


Figure 4

<u>Solution:</u>

Replacing voltage source by short circuit and current source by open circuit to find Z_N

Zn

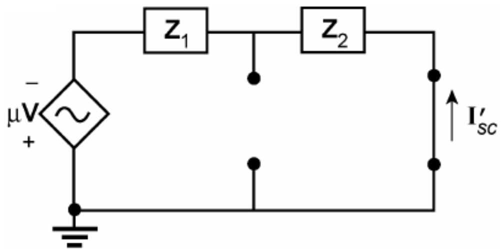


$\mathbf{Z}_1 = 5\ \text{k}\Omega\ \angle 0°$, $\mathbf{Z}_2 = 1\ \text{k}\Omega\ \angle{-90°}$

$\leftarrow \mathbf{Z}_N = \mathbf{Z}_1 + \mathbf{Z}_2 = 5\ \text{k}\Omega - j1\ \text{k}\Omega$

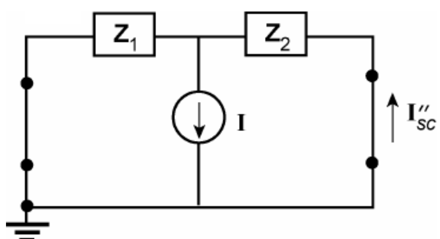
$= \mathbf{5.1\ k\Omega\ \angle{-11.31°}}$

Then, apply superposition to find $\boldsymbol{I_N} = \boldsymbol{I'_{sc}} + \boldsymbol{I''_{sc}}$



$$\mathbf{I'}_{sc} = \frac{\mu\mathbf{V}}{\mathbf{Z}_1 + \mathbf{Z}_2} = \frac{(20)(2\ \text{V}\ \angle 0°)}{5.1\,\text{k}\Omega\ \angle -11.31°}$$

$$= 7.843\ \text{mA}\ \angle 11.31°$$



$$\mathbf{I''}_{sc} = \frac{\mathbf{Z}_1(\mathbf{I})}{\mathbf{Z}_1 + \mathbf{Z}_2}$$

$$= \frac{(5\,\text{k}\Omega\ \angle 0°)(2\ \text{mA}\ \angle 0°)}{5.1\,\text{k}\Omega\ \angle -11.31°}$$

$$= 1.96\ \text{mA}\ \angle 11.31°$$

So, $\boldsymbol{I_N} = \boldsymbol{I'_{sc}} + \boldsymbol{I''_{sc}} = \mathbf{9.81\ \boldsymbol{mA}\ \angle 11.31}$