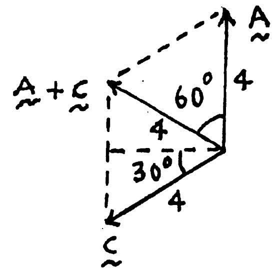
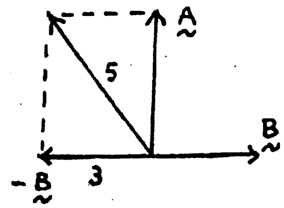
D1.1. (a) $|\mathbf{A}+\mathbf{C}| = 2 \times 4 \times \cos 60^\circ$

$= 4$

Direction is 60° west of north



(b) $|\mathbf{A}-\mathbf{B}| = 5$

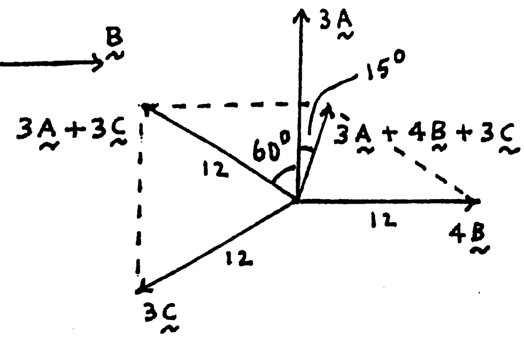


(c) $|3\mathbf{A}+4\mathbf{B}+3\mathbf{C}|$

$= 2 \times 12 \times \cos 75^\circ$

$= 6.212$

Direction is 15° east of north



(d) $\mathbf{B} \bullet (\mathbf{A}-\mathbf{C})$

$= 3 \times (2 \times 4 \times \cos 30^\circ) \times \cos 60^\circ$

$= 10.392$

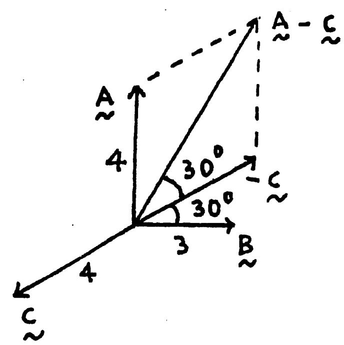
Note also that

$\mathbf{B} \bullet (\mathbf{A}-\mathbf{C}) = \mathbf{B} \bullet \mathbf{A} - \mathbf{B} \bullet \mathbf{C}$

$= 3 \times 4 \times \cos 90^\circ - 3 \times 4 \times \cos 150^\circ$

$= 0 - (-10.392)$

$= 10.392$



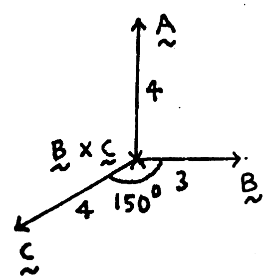
(e) $\mathbf{B} \times \mathbf{C} = 4 \times 3 \times \sin 150^\circ$ into the paper

$= 6$ units into the paper

$\mathbf{A} \times (\mathbf{B} \times \mathbf{C})$

$= 4 \times 6 \times \sin 90^\circ$ toward west

$= 24$ units directed westward



D1.2. (a) $\mathbf{A}+\mathbf{B}-4\mathbf{C} = (3+1-4)\mathbf{a}_1 + (2+1-8)\mathbf{a}_2 + (1-1-12)\mathbf{a}_3$

$= -5\mathbf{a}_2 - 12\mathbf{a}_3$

$|\mathbf{A}+\mathbf{B}-4\mathbf{C}| = \sqrt{25+144} = 13$

(b) $\mathbf{A}+2\mathbf{B}-\mathbf{C} = (3+2-1)\mathbf{a}_1 + (2+2-2)\mathbf{a}_2 + (1-2-3)\mathbf{a}_3$

$= 4\mathbf{a}_1 + 2\mathbf{a}_2 - 4\mathbf{a}_3$

$$\text{Unit vector} = \frac{4\mathbf{a}_1 + 2\mathbf{a}_2 - 4\mathbf{a}_3}{|4\mathbf{a}_1 + 2\mathbf{a}_2 - 4\mathbf{a}_3|} = \frac{1}{3}(2\mathbf{a}_1 + \mathbf{a}_2 - 2\mathbf{a}_3)$$

(c) $\mathbf{A} \bullet \mathbf{C} = 3 \times 1 + 2 \times 2 + 1 \times 3 = 10$

(d) $$\mathbf{B} \times \mathbf{C} = \begin{vmatrix} \mathbf{a}_1 & \mathbf{a}_2 & \mathbf{a}_3 \\ 1 & 1 & -1 \\ 1 & 2 & 3 \end{vmatrix} = 5\mathbf{a}_1 - 4\mathbf{a}_2 + \mathbf{a}_3$$

(e) $$\mathbf{A} \bullet \mathbf{B} \times \mathbf{C} = \begin{vmatrix} 3 & 2 & 1 \\ 1 & 1 & -1 \\ 1 & 2 & 3 \end{vmatrix} = 15 - 8 + 1 = 8$$

D1.3. (a) $$\mathbf{B} \times \mathbf{C} = \begin{vmatrix} \mathbf{a}_1 & \mathbf{a}_2 & \mathbf{a}_3 \\ 2 & 1 & -2 \\ 1 & -1 & 1 \end{vmatrix} = -\mathbf{a}_1 - 4\mathbf{a}_2 - 3\mathbf{a}_3$$

$$\mathbf{A} \times (\mathbf{B} \times \mathbf{C}) = \begin{vmatrix} \mathbf{a}_1 & \mathbf{a}_2 & \mathbf{a}_3 \\ 1 & 2 & 2 \\ -1 & -4 & -3 \end{vmatrix} = 2\mathbf{a}_1 + \mathbf{a}_2 - 2\mathbf{a}_3$$

(b) $$\mathbf{C} \times \mathbf{A} = \begin{vmatrix} \mathbf{a}_1 & \mathbf{a}_2 & \mathbf{a}_3 \\ 1 & -1 & 1 \\ 1 & 2 & 2 \end{vmatrix} = -4\mathbf{a}_1 - \mathbf{a}_2 + 3\mathbf{a}_3$$

$$\mathbf{B} \times (\mathbf{C} \times \mathbf{A}) = \begin{vmatrix} \mathbf{a}_1 & \mathbf{a}_2 & \mathbf{a}_3 \\ 2 & 1 & -2 \\ -4 & -1 & 3 \end{vmatrix} = \mathbf{a}_1 + 2\mathbf{a}_2 + 2\mathbf{a}_3$$

(c) $$\mathbf{A} \times \mathbf{B} = \begin{vmatrix} \mathbf{a}_1 & \mathbf{a}_2 & \mathbf{a}_3 \\ 1 & 2 & 2 \\ 2 & 1 & -2 \end{vmatrix} = -6\mathbf{a}_1 + 6\mathbf{a}_2 - 3\mathbf{a}_3$$

$$\mathbf{C} \times (\mathbf{A} \times \mathbf{B}) = \begin{vmatrix} \mathbf{a}_1 & \mathbf{a}_2 & \mathbf{a}_3 \\ 1 & -1 & 1 \\ -6 & 6 & -3 \end{vmatrix} = -3\mathbf{a}_1 - 3\mathbf{a}_2$$

D1.4. (a) Vector drawn from P_1 to P_2

$= (3-1)\mathbf{a}_x + [1-(-2)]\mathbf{a}_y + (0-2)\mathbf{a}_z$

$= 2\mathbf{a}_x + 3\mathbf{a}_y - 2\mathbf{a}_z$

(b) Vector drawn from P_2 to P_3

$= (5-3)\mathbf{a}_x + (2-1)\mathbf{a}_y + (-2-0)\mathbf{a}_z$

$= 2\mathbf{a}_x + \mathbf{a}_y - 2\mathbf{a}_z$

Straight line distance from P_2 to P_3

$= |2\mathbf{a}_x + \mathbf{a}_y - 2\mathbf{a}_z| = 3$

(c) Vector drawn from P_1 to P_3

$= (5-1)\mathbf{a}_x + [2-(-2)]\mathbf{a}_y + (-2-2)\mathbf{a}_z$

$= 4\mathbf{a}_x + 4\mathbf{a}_y - 4\mathbf{a}_z$

Unit vector along the line from P_1 to P_3

$$= \frac{4\mathbf{a}_x + 4\mathbf{a}_y - 4\mathbf{a}_z}{|4\mathbf{a}_x + 4\mathbf{a}_y - 4\mathbf{a}_z|} = \frac{1}{\sqrt{3}}(\mathbf{a}_x + \mathbf{a}_y - \mathbf{a}_z)$$

D1.5. (a) For $x = 3$, $y = -4$,

$dx = 0$, $dy = 0$

$\therefore d\mathbf{l} = dz\, \mathbf{a}_z$

(b) For $x + y = 0$, $y + z = 1$,

$dx + dy = 0$, $dy + dz = 0$

$\therefore dy = -dz$, $dx = -dy = dz$

$d\mathbf{l} = dz\, \mathbf{a}_x - dz\, \mathbf{a}_y + dz\, \mathbf{a}_z$

$= (\mathbf{a}_x - \mathbf{a}_y + \mathbf{a}_z)\, dz$

(c) $$\frac{dx}{0-2} = \frac{dy}{0-0} = \frac{dz}{1-0}$$

$dx = -2\, dz$, $dy = 0$

$d\mathbf{l} = -2\, dz\, \mathbf{a}_x + dz\, \mathbf{a}_z$

$= (-2\mathbf{a}_x + \mathbf{a}_z)\, dz$

D1.6. (a) $$\frac{dx}{3-1} = \frac{dy}{4-2} = \frac{dz}{0-0}$$

$dy = dx$, $dz = 0$

$y = x + C_1$, $z = C_2$

$2 = 1 + C_1$, $z = 0$

$C_1 = 1$, $C_2 = 0$

Equation is $y = x + 1$, $z = 0$.

(b) $\dfrac{dx}{2-0}=\dfrac{dy}{2-0}=\dfrac{dz}{-1-0}$

$\dfrac{dx}{2}=\dfrac{dy}{2}=-\,dz$

$\dfrac{1}{2}x=\dfrac{1}{2}y+C_1=-z+C_2$

$0=0+C_1=0+C_2$

$C_1=C_2=0$

Equation is $x=y\ =-2z$.

(c) $\dfrac{dx}{3-1}=\dfrac{dy}{-2-1}=\dfrac{dz}{4-1}$

$\dfrac{dx}{2}=\dfrac{dy}{-3}=\dfrac{dz}{3}$

$\dfrac{1}{2}x=-\dfrac{1}{3}y+C_1=\dfrac{1}{3}z+C_2$

$\dfrac{1}{2}=-\dfrac{1}{3}+C_1=\dfrac{1}{3}+C_2$

$C_1=\dfrac{5}{6},\ C_2=\dfrac{1}{6}$

$\dfrac{1}{2}x=-\dfrac{1}{3}y+\dfrac{5}{6}=\dfrac{1}{3}z+\dfrac{1}{6}$

Equation is $3x+2y=5,\ 3x-2z=1$.

D1.7. (a) $(2, 5\pi/6, 3)\rightarrow(2\cos 5\pi/6, 2\sin 5\pi/6, 3)=(-\sqrt{3}, 1, 3)$

(b) $(4, 4\pi/3, -1)\rightarrow(4\cos 4\pi/3, 4\sin 4\pi/3, -1)=(-2, -2\sqrt{3}, -1)$

(c) $(4, 2\pi/3, \pi/6)\rightarrow(4\sin 2\pi/3\cos\pi/6, 4\sin 2\pi/3\sin\pi/6, 4\cos 2\pi/3)=(3, \sqrt{3}, -2)$

(d) $(\sqrt{8}, \pi/4, \pi/3)\rightarrow(\sqrt{8}\sin\pi/4\cos\pi/3, \sqrt{8}\sin\pi/4\sin\pi/3, \sqrt{8}\cos\pi/4)=(1, \sqrt{3}, 2)$

D1.8. (a) $r_c=\sqrt{4+0}=2$

$\tan\phi=\dfrac{0}{-2}=\pi$

$(-2, 0, 1)\rightarrow(2, \pi, 1)$

(b) $r_c=\sqrt{1+3}=2$

$$\tan \phi = \frac{-\sqrt{3}}{1} = \frac{5\pi}{3}$$

$$(1, -\sqrt{3}, -1) \rightarrow (2, 5\pi/3, -1)$$

(c) $r_c = \sqrt{2+2} = 2$

$$\tan \phi = \frac{-\sqrt{2}}{-\sqrt{2}} = \frac{5\pi}{4}$$

$$(-\sqrt{2}, -\sqrt{2}, 3) \rightarrow (2, 5\pi/4, 3)$$

D1.9. (a) $r_s = \sqrt{0+4+0} = 2$

$$\theta = \tan^{-1} \frac{\sqrt{0+4}}{0} = \frac{\pi}{2}$$

$$\phi = \tan^{-1} \frac{-2}{0} = \frac{3\pi}{2}$$

$$(0, -2, 0) \rightarrow (2, \pi/2, 3\pi/2)$$

(b) $r_s = \sqrt{9+3+4} = 4$

$$\theta = \tan^{-1} \frac{\sqrt{9+3}}{2} = \frac{\pi}{3}$$

$$\phi = \tan^{-1} \frac{\sqrt{3}}{-3} = \frac{5\pi}{6}$$

$$(-3, \sqrt{3}, 2) \rightarrow (4, \pi/3, 5\pi/6)$$

(c) $r_s = \sqrt{2+0+2} = 2$

$$\theta = \tan^{-1} \frac{\sqrt{2+0}}{-\sqrt{2}} = \frac{3\pi}{4}$$

$$\phi = \tan^{-1} \frac{0}{-\sqrt{2}} = \pi$$

$$(-\sqrt{2}, 0, -\sqrt{2}) \rightarrow (2, 3\pi/4, \pi)$$

D1.10. (a) $T(x, y, z, 0) = T_0(x^2 + 4z^2)$

Constant temperature surfaces are given by $(x^2 + 4z^2)$ = constant, which are elliptic cylinders.

(b) $T(x, y, z, 0.5) = T_0(4x^2 + 4y^2 + 4z^2)$

Constant temperature surfaces are given by $(x^2 + y^2 + z^2)$ = constant, which are spheres.

(c) $T(x, y, z, 1) = T_0(x^2 + 16y^2 + 4z^2)$

Constant temperature surfaces are given by $(x^2 + 16y^2 + 4z^2)$ = constant, which are ellipsoids.

D1.11. **(a)** $\mathbf{F}(1, 1, 0) = 2\mathbf{a}_x + \mathbf{a}_y + 2\mathbf{a}_z$

Magnitude of $\mathbf{F} = |2\mathbf{a}_x + \mathbf{a}_y + 2\mathbf{a}_z| = 3$

Unit vector along $\mathbf{F} = \frac{1}{3}(2\mathbf{a}_x + \mathbf{a}_y + 2\mathbf{a}_z)$

(b) $\mathbf{F}(x, y, z) = 3 \times \frac{1}{3}(2\mathbf{a}_x + 2\mathbf{a}_y + \mathbf{a}_z) = 2\mathbf{a}_x + 2\mathbf{a}_y + \mathbf{a}_z$

$3x - y = 2, x + z = 2, 2y - z = 1$

Solving, we get $x = 1$, $y = 1$, and $z = 1$

The point is (1, 1, 1).

(c) $\mathbf{F}(x, y, z) = 3\mathbf{a}_z$

$3x - y = 0, x + z = 0, 2y - z = 3$

Solving, we get $x = 0.6$, $y = 1.8$, and $z = -0.6$

The point is (0.6, 1.8, – 0.6).

D1.12. **(a)** $(1, 0, 0) \rightarrow (1, 0, 0)$

$\mathbf{F}(1, 0, 0) = \frac{1}{1}(\cos 0\ \mathbf{a}_r + \sin 0\ \mathbf{a}_\phi) = \mathbf{a}_r = \mathbf{a}_x$

(b) $(1, -1, -3) \rightarrow (\sqrt{2}, 7\pi/4, -3)$

$$\mathbf{F}(\sqrt{2}, 7\pi/4, -3) = \frac{1}{2}\left(\cos\frac{7\pi}{4}\,\mathbf{a}_r + \sin\frac{7\pi}{4}\,\mathbf{a}_\phi\right)$$

$$= \frac{1}{2\sqrt{2}}(\mathbf{a}_r - \mathbf{a}_\phi)$$

$$= \frac{1}{2\sqrt{2}}\left[\left(\frac{1}{\sqrt{2}}\mathbf{a}_x - \frac{1}{\sqrt{2}}\mathbf{a}_y\right) - \left(\frac{1}{\sqrt{2}}\mathbf{a}_x + \frac{1}{\sqrt{2}}\mathbf{a}_y\right)\right]$$

$$= -\frac{1}{2}\mathbf{a}_y$$

(c) $(1, \sqrt{3}, -4) \rightarrow (2, \pi/3, -4)$

$$\mathbf{F}(2, \pi/3, -4) = \frac{1}{4}\left(\cos\frac{\pi}{3}\,\mathbf{a}_r + \sin\frac{\pi}{3}\,\mathbf{a}_\phi\right)$$

$$= \frac{1}{8}(\mathbf{a}_r + \sqrt{3}\,\mathbf{a}_\phi)$$

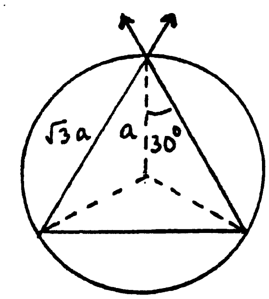
$$= \frac{1}{8}\left[\left(\frac{1}{2}\mathbf{a}_x + \frac{\sqrt{3}}{2}\mathbf{a}_y\right) + \sqrt{3}\left(-\frac{\sqrt{3}}{2}\mathbf{a}_x + \frac{1}{2}\mathbf{a}_y\right)\right]$$

$$= \frac{1}{8}(-\mathbf{a}_x + \sqrt{3}\,\mathbf{a}_y)$$

D1.13. (a) $n = 3$

$$F = 2\frac{4\pi\varepsilon_0}{4\pi\varepsilon_0(\sqrt{3}a)^2}\cos 30^\circ$$

$$= \frac{0.577}{a^2}\text{ N}$$

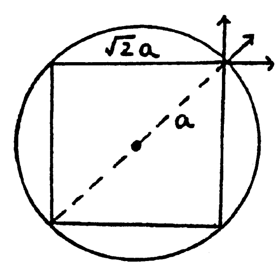


Direction away from the center of the polygon.

(b) $n = 4$

$$F = 2\frac{4\pi\varepsilon_0}{4\pi\varepsilon_0(\sqrt{2}a)^2}\cos 45^\circ + \frac{4\pi\varepsilon_0}{4\pi\varepsilon_0(2a)^2}$$

$$= \frac{0.957}{a^2}\text{ N}$$

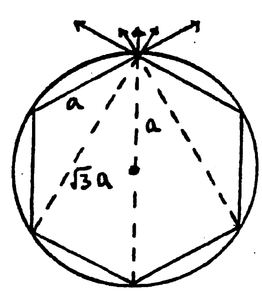


Direction away from the center of the polygon.

(c) $n = 6$

$$F = 2\frac{4\pi\varepsilon_0}{4\pi\varepsilon_0 a^2}\cos 60^\circ + 2\frac{4\pi\varepsilon_0}{4\pi\varepsilon_0(\sqrt{3}a)^2}\cos 30^\circ + \frac{4\pi\varepsilon_0}{4\pi\varepsilon_0(2a)^2}$$

$$= \frac{1.827}{a^2}\text{ N}$$



Direction away from the center of the polygon.

D1.14. From computation similar to that in Ex. 1.6, for $Q_2 = 4\pi\varepsilon_0$ C,

(a) $[\mathbf{E}]_{(0,0,1)} = 1.118(0.316\mathbf{a}_x + 0.949\mathbf{a}_z)$

$$= 0.353\mathbf{a}_x + 1.061\mathbf{a}_z$$

(b) Coordinates of point at the end of the second step are (0.060, 0, 1.191).

(c) Unit vector along **E** at the point in (b) $= 0.264\mathbf{a}_x + 0.965\mathbf{a}_z$

D1.15. **(a)** At the point (0, 0, 0),

$$\mathbf{E} = \left[0 - \frac{-Q(2a)}{4\pi\varepsilon_0(a^2+4a^2)^{3/2}}\right]\mathbf{a}_z$$

$$= \frac{0.0142Q}{\varepsilon_0 a^2}\,\mathbf{a}_z$$

(b) At the point (0, 0, a),

$$\mathbf{E} = \left[\frac{Qa}{4\pi\varepsilon_0(a^2+a^2)^{3/2}} - \frac{-Qa}{4\pi\varepsilon_0(a^2+a^2)^{3/2}}\right]\mathbf{a}_z$$

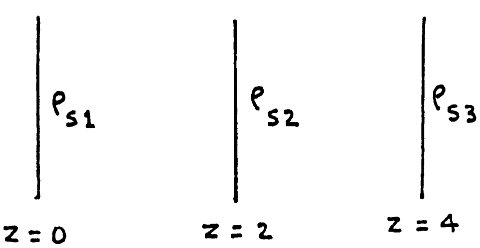
$$= \frac{0.0563Q}{\varepsilon_0 a^2}\,\mathbf{a}_z$$

(c) At the point (0, 0, $3a$),

$$\mathbf{E} = \left[\frac{Q(3a)}{4\pi\varepsilon_0(a^2+9a^2)^{3/2}} + \frac{-Qa}{4\pi\varepsilon_0(a^2+a^2)^{3/2}}\right]\mathbf{a}_z$$

$$= -\frac{0.0206Q}{\varepsilon_0 a^2}\,\mathbf{a}_z$$

D1.16.



From the given values of the electric field intensities at the points (3, 5, 1), (1, –2, 3), and (3, 4, 5), we can write

$$\frac{1}{2\varepsilon_0}(\rho_{S1} - \rho_{S2} - \rho_{S3}) = 0 \qquad \text{or,} \qquad \rho_{S1} - \rho_{S2} - \rho_{S3} = 0$$

$$\frac{1}{2\varepsilon_0}(\rho_{S1} + \rho_{S2} - \rho_{S3}) = 6 \qquad \text{or,} \qquad \rho_{S1} + \rho_{S2} - \rho_{S3} = 12\varepsilon_0$$

$$\frac{1}{2\varepsilon_0}(\rho_{S1} + \rho_{S2} + \rho_{S3}) = 4 \qquad \text{or,} \qquad \rho_{S1} + \rho_{S2} + \rho_{S3} = 8\varepsilon_0$$

Solving, we obtain

(a) $\rho_{S1} = 4\varepsilon_0$ C/m^2
(b) $\rho_{S2} = 6\varepsilon_0$ C/m^2
(c) $\rho_{S3} = -2\varepsilon_0$ C/m^2

Then

(d) $$[\mathbf{E}]_{(-2,\,1,\,-6)} = \frac{1}{2\varepsilon_0}(-\rho_{S1} - \rho_{S2} - \rho_{S3})$$

$$= \frac{1}{2\varepsilon_0}(-4\varepsilon_0 - 6\varepsilon_0 + 2\varepsilon_0)$$

$$= -\,4\mathbf{a}_z \text{ V/m}$$

D1.17. **(a)** $$d\mathbf{F}_1 = I_1\, d\mathbf{l}_1 \times \left(\frac{\mu_0 I_2\, d\mathbf{l}_2 \times \mathbf{R}_{21}}{4\pi R_{21}^3}\right)$$

$$= I_1\, dy\, \mathbf{a}_y \times \left[\frac{\mu_0 I_2\, dx\, \mathbf{a}_x \times (\mathbf{a}_x - \mathbf{a}_y)}{4\pi(\sqrt{2})^3}\right]$$

$$= I_1\, dy\, \mathbf{a}_y \times \left(\frac{-\mu_0 I_2\, dx\, \mathbf{a}_z}{8\sqrt{2}\pi}\right)$$

$$= -\frac{\mu_0 I_1 I_2\, dx\, dy}{8\sqrt{2}\pi}\mathbf{a}_x$$

(b) $$d\mathbf{F}_2 = I_2\, d\mathbf{l}_2 \times \left(\frac{\mu_0 I_1\, d\mathbf{l}_1 \times \mathbf{R}_{12}}{4\pi R_{12}^3}\right)$$

$$= I_2\, dx\, \mathbf{a}_x \times \left[\frac{\mu_0 I_1\, dy\, \mathbf{a}_y \times (-\mathbf{a}_x + \mathbf{a}_y)}{4\pi(\sqrt{2})^3}\right]$$

$$= I_2\, dx\, \mathbf{a}_x \times \left(\frac{\mu_0 I_1\, dy\, \mathbf{a}_z}{8\sqrt{2}\pi}\right)$$

$$= -\frac{\mu_0 I_1 I_2\, dx\, dy}{8\sqrt{2}\pi}\mathbf{a}_y$$

D1.18. For $x = 2y = z^2 + 2$,

$dx = 2\,dy = 2z\,dz$

$d\mathbf{l} = (2z\mathbf{a}_x + z\mathbf{a}_y + \mathbf{a}_z)\,dz$

(a) At the point (2, 1, 0),

$$d\mathbf{l} = dz\,\mathbf{a}_z, \qquad \mathbf{B} = \frac{\mathbf{a}_x - 2\mathbf{a}_y}{5}$$

$$d\mathbf{F} = I\,d\mathbf{l} \times \mathbf{B} = I\,dz\,\mathbf{a}_z \times \frac{(\mathbf{a}_x - 2\mathbf{a}_y)}{5}$$

$$= \frac{I\,dz\,(2\mathbf{a}_x + \mathbf{a}_y)}{5}$$

(b) At the point (3, 1.5, 1),

$$d\mathbf{l} = (2\mathbf{a}_x + \mathbf{a}_y + \mathbf{a}_z)\,dz, \qquad \mathbf{B} = \frac{1.5\mathbf{a}_x - 3\mathbf{a}_y}{11.25}$$

$$d\mathbf{F} = I\,d\mathbf{l} \times \mathbf{B} = I(2\mathbf{a}_x + \mathbf{a}_y + \mathbf{a}_z)\,dz \times \frac{(1.5\mathbf{a}_x - 3\mathbf{a}_y)}{11.25}$$

$$= I\,dz\,\frac{(3\mathbf{a}_x + 1.5\mathbf{a}_y - 7.5\mathbf{a}_z)}{11.25} = \frac{I\,dz\,(2\mathbf{a}_x + \mathbf{a}_y - 5\mathbf{a}_z)}{7.5}$$

(c) At the point (6, 3, 2),

$$d\mathbf{l} = (4\mathbf{a}_x + 2\mathbf{a}_y + \mathbf{a}_z)\,dz, \qquad \mathbf{B} = \frac{3\mathbf{a}_x - 6\mathbf{a}_y}{45}$$

$$d\mathbf{F} = I\,d\mathbf{l} \times \mathbf{B} = I(4\mathbf{a}_x + 2\mathbf{a}_y + \mathbf{a}_z)\,dz \times \frac{(3\mathbf{a}_x - 6\mathbf{a}_y)}{45}$$

$$= I\,dz\,\frac{(6\mathbf{a}_x + 3\mathbf{a}_y - 30\mathbf{a}_z)}{45} = \frac{I\,dz\,(2\mathbf{a}_x + \mathbf{a}_y - 10\mathbf{a}_z)}{15}$$

D1.19. $\mathbf{F} = q\mathbf{v} \times \mathbf{B} = qv_0 \dfrac{d\mathbf{l}}{dl} \times \dfrac{B_0}{3}(2\mathbf{a}_x + 2\mathbf{a}_y - \mathbf{a}_z)$

$$= \frac{qv_0 B_0}{3}\frac{d\mathbf{l}}{dl} \times (2\mathbf{a}_x + 2\mathbf{a}_y - \mathbf{a}_z)$$

(a) For $x = y = -2z$, $dx = dy = -2\,dz$

$$d\mathbf{l} = dz\,(-2\mathbf{a}_x - 2\mathbf{a}_y + \mathbf{a}_z)\,dz$$

$$\frac{d\mathbf{l}}{dl} = \frac{1}{3}(-2\mathbf{a}_x - 2\mathbf{a}_y + \mathbf{a}_z)$$

$$\mathbf{F} = \frac{qv_0B_0}{9}(-2\mathbf{a}_x - 2\mathbf{a}_y + \mathbf{a}_z) \times (2\mathbf{a}_x + 2\mathbf{a}_y - \mathbf{a}_z) = \mathbf{0}$$

$$|\mathbf{F}| = 0$$

(b) For $4x = 4y = z + 9$, $4\,dx = 4\,dy = dz$

$$d\mathbf{l} = \left(\frac{1}{4}\mathbf{a}_x + \frac{1}{4}\mathbf{a}_y + \mathbf{a}_z\right)dz$$

$$\frac{d\mathbf{l}}{dl} = \frac{1}{3\sqrt{2}}(\mathbf{a}_x + \mathbf{a}_y + 4\mathbf{a}_z)$$

$$\mathbf{F} = \frac{qv_0B_0}{9\sqrt{2}}(\mathbf{a}_x + \mathbf{a}_y + 4\mathbf{a}_z) \times (2\mathbf{a}_x + 2\mathbf{a}_y - \mathbf{a}_z)$$

$$= \frac{qv_0B_0}{9\sqrt{2}}(-9\mathbf{a}_x + 9\mathbf{a}_y)$$

$$|\mathbf{F}| = \frac{qv_0B_0}{9\sqrt{2}}\sqrt{81+81} = qv_0B_0$$

(c) For $x = y = 2z^2$, $dx = dy = 4z\,dz$

$$d\mathbf{l} = (4z\mathbf{a}_x + 4z\mathbf{a}_y + \mathbf{a}_z)\,dz = (-4\mathbf{a}_x - 4\mathbf{a}_y + \mathbf{a}_z)\,dz$$

$$\frac{d\mathbf{l}}{dl} = \frac{1}{\sqrt{33}}(-4\mathbf{a}_x - 4\mathbf{a}_y + \mathbf{a}_z)$$

$$\mathbf{F} = \frac{qv_0B_0}{3\sqrt{33}}(-4\mathbf{a}_x - 4\mathbf{a}_y + \mathbf{a}_z) \times (2\mathbf{a}_x + 2\mathbf{a}_y - \mathbf{a}_z)$$

$$= \frac{qv_0B_0}{3\sqrt{33}}(2\mathbf{a}_x - 2\mathbf{a}_y)$$

$$|\mathbf{F}| = \frac{qv_0B_0}{3\sqrt{33}}\sqrt{4+4} = 0.1641\,qv_0B_0$$

D1.20. **(a)** At (1, 2, 2),

$$\mathbf{B} = \frac{\mu_0}{2}\left[J_{S0}\mathbf{a}_z \times \mathbf{a}_x + 2J_{S0}\mathbf{a}_x \times \mathbf{a}_y + (-J_{S0}\mathbf{a}_x) \times \mathbf{a}_z\right]$$

$$= \frac{\mu_0 J_{S0}}{2}(2\mathbf{a}_y + 2\mathbf{a}_z) = \mu_0 J_{S0}(\mathbf{a}_y + \mathbf{a}_z)$$

(b) At (2, –2, –1),

$$\mathbf{B} = \frac{\mu_0}{2}\left[J_{S0}\mathbf{a}_z \times \mathbf{a}_x + 2J_{S0}\mathbf{a}_x \times (-\mathbf{a}_y) + (-J_{S0}\mathbf{a}_x) \times (-\mathbf{a}_z)\right]$$

$$= \frac{\mu_0 J_{S0}}{2}(-2\mathbf{a}_z) = -\mu_0 J_{S0}\mathbf{a}_z$$

(c) At (–2, 1, –2),

$$\mathbf{B} = \frac{\mu_0}{2}\left[J_{S0}\mathbf{a}_z \times (-\mathbf{a}_x) + 2J_{S0}\mathbf{a}_x \times \mathbf{a}_y + (-J_{S0}\mathbf{a}_x) \times (-\mathbf{a}_z)\right]$$

$$= \frac{\mu_0 J_{S0}}{2}(-2\mathbf{a}_y + 2\mathbf{a}_z) = \mu_0 J_{S0}(-\mathbf{a}_y + \mathbf{a}_z)$$

D1.21. $\mathbf{F} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B}) = \mathbf{0}$

$\mathbf{E} = -\mathbf{v} \times \mathbf{B}$

(a) $\mathbf{E} = -v_0(\mathbf{a}_x - \mathbf{a}_y + \mathbf{a}_z) \times \frac{B_0}{3}(\mathbf{a}_x + 2\mathbf{a}_y - 2\mathbf{a}_z)$

$$= -\frac{v_0 B_0}{3}(3\mathbf{a}_y + 3\mathbf{a}_z) = -v_0 B_0(\mathbf{a}_y + \mathbf{a}_z)$$

(b) $\mathbf{E} = -v_0(2\mathbf{a}_x + \mathbf{a}_y + 2\mathbf{a}_z) \times \frac{B_0}{3}(\mathbf{a}_x + 2\mathbf{a}_y - 2\mathbf{a}_z)$

$$= -\frac{v_0 B_0}{3}(-6\mathbf{a}_x + 6\mathbf{a}_y + 3\mathbf{a}_z) = v_0 B_0(2\mathbf{a}_x - 2\mathbf{a}_y - \mathbf{a}_z)$$

(c) For $y = -z = 2x$, $dy = -\,dz = 2\,dx$

$$d\mathbf{l} = \left(-\frac{1}{2}\mathbf{a}_x - \mathbf{a}_y + \mathbf{a}_z\right) dz$$

$$\mathbf{v}_0 = v_0 \frac{d\mathbf{l}}{dl} = \frac{v_0}{3}(-\mathbf{a}_x - 2\mathbf{a}_y + 2\mathbf{a}_z)$$

$$\mathbf{E} = -\frac{v_0}{3}(-\mathbf{a}_x - 2\mathbf{a}_y + 2\mathbf{a}_z) \times \frac{B_0}{3}(\mathbf{a}_x + 2\mathbf{a}_y - 2\mathbf{a}_z)$$

$$= \mathbf{0}$$

D1.22. $v_x = \frac{dx}{dt} = \frac{E_0}{\omega_c B_0}(\omega_c - \omega_c \cos \omega_c t) = \frac{E_0}{B_0}(1 - \cos \omega_c t)$

$$v_y = \frac{dy}{dt} = \frac{E_0}{\omega_c B_0}(\omega_c \sin \omega_c t) = \frac{E_0}{B_0}\sin \omega_c t$$

$$v_z = \frac{dz}{dt} = 0$$

$$\therefore \mathbf{v} = \frac{E_0}{B_0}\left[(1-\cos\omega_c t)\,\mathbf{a}_x + \sin\omega_c t\;\mathbf{a}_y\right]$$

(a) $t = 0;\ \mathbf{v} = \mathbf{0}$

$$\mathbf{F} = qE_0\mathbf{a}_y + q(\mathbf{0} \times B_0\mathbf{a}_z) = qE_0\mathbf{a}_y$$

(b) $t = \dfrac{\pi}{2\omega_c};\ \mathbf{v} = \dfrac{E_0}{B_0}(\mathbf{a}_x + \mathbf{a}_y)$

$$\mathbf{F} = qE_0\mathbf{a}_y + q\frac{E_0}{B_0}(\mathbf{a}_x + \mathbf{a}_y) \times B_0\mathbf{a}_z$$

$$= qE_0\mathbf{a}_x$$

(c) $t = \dfrac{\pi}{\omega_c};\ \mathbf{v} = \dfrac{2E_0}{B_0}\mathbf{a}_x$

$$\mathbf{F} = qE_0\mathbf{a}_y + q\frac{2E_0}{B_0}\mathbf{a}_x \times B_0\mathbf{a}_z$$

$$= -\,qE_0\mathbf{a}_y$$

D2.1. $[\mathbf{E}]_{(1,1,0)} = \mathbf{a}_x + \mathbf{a}_y$

(a) $y = x^2, z = 0$

$$\Delta\mathbf{l} = 0.1\mathbf{a}_x + [(1.1)^2 - 1]\mathbf{a}_y = 0.1\mathbf{a}_x + 0.21\mathbf{a}_y$$

$$q\mathbf{E} \bullet \Delta\mathbf{l} = 10^{-6}(\mathbf{a}_x + \mathbf{a}_y) \bullet (0.1\mathbf{a}_x + 0.21\mathbf{a}_y)$$

$$= 0.31 \times 10^{-6} \text{ J} = 0.31 \ \mu\text{J}$$

(b) $x^2 + y^2 = 2, z = 0$

$$\Delta\mathbf{l} = 0.1\mathbf{a}_x + \left[\sqrt{2-(1.1)^2} - 1\right]\mathbf{a}_y = 0.1\mathbf{a}_x - 0.1112\mathbf{a}_y$$

$$q\mathbf{E} \bullet \Delta\mathbf{l} = 10^{-6}(\mathbf{a}_x + \mathbf{a}_y) \bullet (0.1\mathbf{a}_x - 0.1112\mathbf{a}_y)$$

$$= -\ 0.0112 \times 10^{-6} \text{ J} = -\ 0.0112 \ \mu\text{J}$$

(c) $y = \sin 0.5\pi x, z = 0$

$$\Delta\mathbf{l} = 0.1\mathbf{a}_x + (\sin 0.55\pi - 1)\mathbf{a}_y = 0.1\mathbf{a}_x - 0.0123\mathbf{a}_y$$

$$q\mathbf{E} \bullet \Delta\mathbf{l} = 10^{-6}(\mathbf{a}_x + \mathbf{a}_y) \bullet (0.1\mathbf{a}_x - 0.0123\mathbf{a}_y)$$

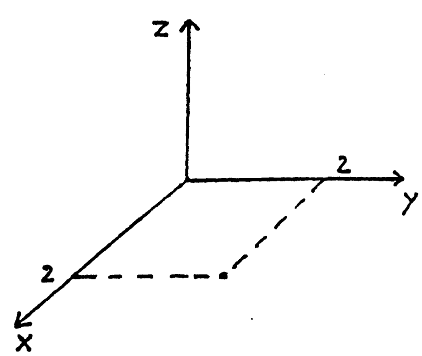
$$= 0.0877 \times 10^{-6} \text{ J} = 0.0877 \ \mu\text{J}$$

D2.2. (a) From (0, 0, 0) to (2, 0, 0),

$y = 0, d\mathbf{l} = dx\,\mathbf{a}_x$

$\mathbf{F} = \mathbf{0}, \mathbf{F} \bullet d\mathbf{l} = 0$

$\int \mathbf{F} \bullet d\mathbf{l} = 0$



(b) From (0, 2, 0) to (2, 2, 0)

$y = 2, d\mathbf{l} = dx\,\mathbf{a}_x$

$\mathbf{F} = 2(\mathbf{a}_x + \mathbf{a}_y)$

$\mathbf{F} \bullet d\mathbf{l} = 2(\mathbf{a}_x + \mathbf{a}_y) \bullet dx\,\mathbf{a}_x = 2\,dx$

$$\int \mathbf{F} \bullet d\mathbf{l} = \int_0^2 2\,dx = 4$$

(c) From (2, 0, 0) to (2, 2, 0)

$x = 2, d\mathbf{l} = dy\,\mathbf{a}_y$

$\mathbf{F} = y(\mathbf{a}_x + \mathbf{a}_y)$

$\mathbf{F} \bullet d\mathbf{l} = y(\mathbf{a}_x + \mathbf{a}_y) \bullet dy\,\mathbf{a}_y = y\,dy$

$$\int \mathbf{F} \cdot d\mathbf{l} = \int_0^2 y\,dy = 2$$

D2.3. $[\mathbf{B}]_{(1,2,1)} = 2\mathbf{a}_x - \mathbf{a}_y$

(a) For the $x = 1$ plane, $\mathbf{a}_n = \pm\,\mathbf{a}_x$, $\Delta\mathbf{S} = \pm\,0.001\mathbf{a}_x$

$$|\mathbf{B} \cdot \Delta\mathbf{S}| = |(2\mathbf{a}_x - \mathbf{a}_y) \cdot (\pm\,0.001\mathbf{a}_x)| = 2 \times 10^{-3} \text{ Wb}$$

(b) From Example 1.3, for the surface $2x^2 + y^2 = 6$,

$$\mathbf{a}_n = \pm \frac{dz\,\mathbf{a}_z \times dx\,(\mathbf{a}_x - \mathbf{a}_y)}{|dz\,\mathbf{a}_z \times dx\,(\mathbf{a}_x - \mathbf{a}_y)|} = \pm\frac{1}{\sqrt{2}}(\mathbf{a}_x + \mathbf{a}_y)$$

$$\Delta\mathbf{S} = \pm\,\frac{0.001}{\sqrt{2}}(\mathbf{a}_x + \mathbf{a}_y)$$

$$|\mathbf{B} \cdot \Delta\mathbf{S}| = (2\mathbf{a}_x - \mathbf{a}_y) \cdot \left[\pm\frac{0.001}{\sqrt{2}}(\mathbf{a}_x + \mathbf{a}_y)\right]$$

$$= \frac{1}{\sqrt{2}} \times 10^{-3} \text{ Wb}$$

(c) $\Delta\mathbf{S} = \dfrac{0.001}{3}(2\mathbf{a}_x + \mathbf{a}_y + 2\mathbf{a}_z)$

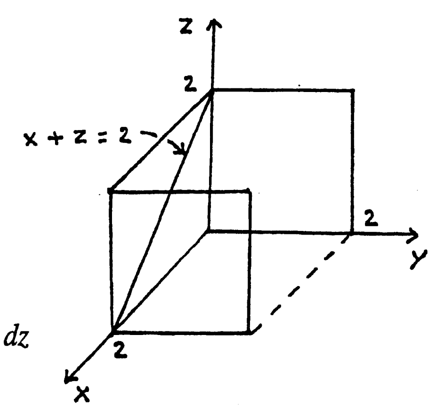
$$|\mathbf{B} \cdot \Delta\mathbf{S}| = (2\mathbf{a}_x - \mathbf{a}_y) \cdot \frac{0.001}{3}(2\mathbf{a}_x + \mathbf{a}_y + 2\mathbf{a}_z)$$

$$= 10^{-3} \text{ Wb}$$

D2.4. **(a)** $x = 0$, $\mathbf{a}_n = \pm\,\mathbf{a}_x$

$\mathbf{A} = 0$, $\mathbf{A} \cdot d\mathbf{S} = 0$

$$\int \mathbf{A} \cdot d\mathbf{S} = 0$$



(b) $x = 2$, $\mathbf{a}_n = \pm\,\mathbf{a}_x$

$\mathbf{A} = 2(\mathbf{a}_x + \mathbf{a}_y)$, $d\mathbf{S} = \pm\,dy\,dz\,\mathbf{a}_x$

$\mathbf{A} \cdot d\mathbf{S} = 2(\mathbf{a}_x + \mathbf{a}_y) \cdot (\pm\,dy\,dz\,\mathbf{a}_x) = \pm\,2\,dy\,dz$

$$\left|\int \mathbf{A} \cdot d\mathbf{S}\right| = \int_{y=0}^{2} \int_{z=0}^{2} 2\,dy\,dz = 8$$

(c) $y = 0$, $\mathbf{a}_n = \pm\,\mathbf{a}_y$

$\mathbf{A} = x(\mathbf{a}_x + \mathbf{a}_y)$, $d\mathbf{S} = \pm\,dz\,dx\,\mathbf{a}_y$

$$\mathbf{A} \bullet d\mathbf{S} = x(\mathbf{a}_x + \mathbf{a}_y) \bullet (\pm\, dz\, dx\, \mathbf{a}_y) = \pm\, x\, dx\, dz$$

$$\left|\int \mathbf{A} \bullet d\mathbf{S}\right| = \int_{x=0}^{2} \int_{z=0}^{2} x\, dx\, dz = 4$$

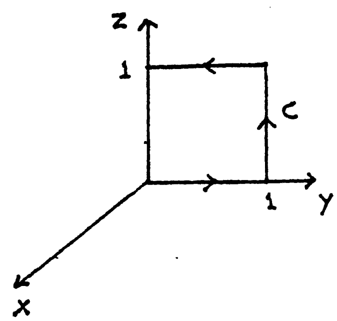
(d) From (c), $\mathbf{A} \bullet d\mathbf{S} = \pm\, x\, dx\, dz$

$$\left|\int \mathbf{A} \bullet d\mathbf{S}\right| = \int_{x=0}^{2} \int_{z=0}^{2-x} x\, dx\, dz$$

$$= \int_0^2 x(2 - x)dx$$

$$= \frac{4}{3}$$

D2.5. $\mathbf{B} = B_0(\sin \omega t\, \mathbf{a}_x - \cos \omega t\, \mathbf{a}_y)$

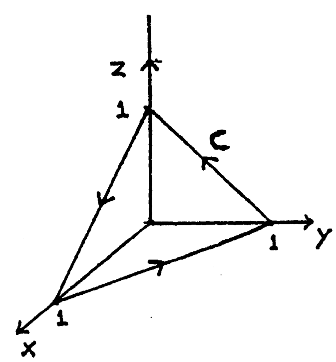


(a) $\int_S \mathbf{B} \bullet d\mathbf{S} = B_0 \sin \omega t$

$$\oint_C \mathbf{E} \bullet d\mathbf{l} = -\frac{d}{dt}(B_0 \sin \omega t)$$

$$= -\omega B_0 \cos \omega t \text{ V}$$

(b) $\int_S \mathbf{B} \bullet d\mathbf{S}$



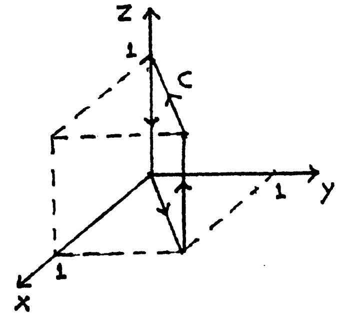
$$= \frac{1}{2}B_0 \sin \omega t - \frac{1}{2}B_0 \cos \omega t$$

$$= \frac{1}{\sqrt{2}}B_0 \sin\left(\omega t - \frac{\pi}{4}\right)$$

$$\oint_C \mathbf{E} \bullet d\mathbf{l} = -\frac{d}{dt}\left[\frac{1}{\sqrt{2}} B_0 \sin\left(\omega t - \frac{\pi}{4}\right)\right]$$

$$= -\frac{\omega B_0}{\sqrt{2}} \cos\left(\omega t - \frac{\pi}{4}\right) \text{ V}$$

(c) $\int_S \mathbf{B} \bullet d\mathbf{S}$



$$= B_0 \sin \omega t + B_0 \cos \omega t$$

$$= \sqrt{2} B_0 \sin\left(\omega t + \frac{\pi}{4}\right)$$

$$\oint_C \mathbf{E} \cdot d\mathbf{l} = -\frac{d}{dt}\left[\sqrt{2}\, B_0 \sin\left(\omega t + \frac{\pi}{4}\right)\right]$$

$$= -\sqrt{2}\, \omega B_0 \cos\left(\omega t + \frac{\pi}{4}\right) \text{ V}$$

D2.6. (a) $\mathbf{B} = B_0 t\, \mathbf{a}_z$

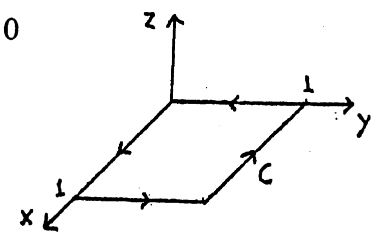
$$\frac{d\psi}{dt} = B_0 \text{ is positive}$$

$\therefore$ Induced emf is negative.

(b) $\mathbf{B} = B_0 \cos\,(2\pi t + 60°)\, \mathbf{a}_z$

$$\frac{d\psi}{dt} = -2\pi B_0 \sin\,(2\pi t + 60°) \text{ is negative at } t = 0$$

$\therefore$ Induced emf is positive.

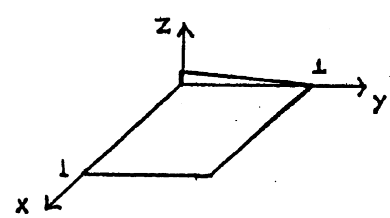


(c) $\mathbf{B} = B_0 e^{-t^2}\, \mathbf{a}_z$

$$\frac{d\psi}{dt} = -2tB_0 e^{-t^2} \text{ is zero at } t = 0$$

$\therefore$ Induced emf is zero.

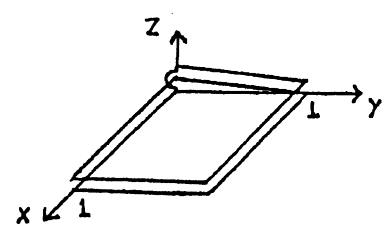
D2.7. (a) $\psi = B_0 \cos \omega t$



$$\text{emf} = -\frac{d}{dt}(B_0 \cos \omega t)$$

$$= \omega B_0 \sin \omega t \text{ V}$$

(b) $\psi = 2B_0 \cos \omega t$



$$\text{emf} = -\frac{d}{dt}(2B_0 \cos \omega t)$$

$$= 2\omega B_0 \sin \omega t \text{ V}$$

(c) For $z = 0.01$, $\phi = 1000\pi(0.01) = 10\pi$

Thus the helical path has 5 turns.

$$\psi = 5\left(\pi \times \frac{1}{\pi}\right) B_0 \cos \omega t = 5B_0 \cos \omega t$$

$$\text{emf} = -\frac{d}{dt}(5B_0 \cos \omega t)$$

$$= 5\omega B_0 \sin \omega t \text{ V}$$

D2.8. For all cases, $\int \mathbf{D} \cdot d\mathbf{S} = D_z(0.1) = 0.1\varepsilon_0 E_z$

$$I_d = \frac{d}{dt}\int \mathbf{D} \cdot d\mathbf{S} = \frac{d}{dt}(0.1\varepsilon_0 E_z) = 0.1\varepsilon_0 \frac{dE_z}{dt}$$

$$= 0.1\varepsilon_0 \frac{d}{dt}(E_0 t e^{-t^2})$$

$$= 0.1\varepsilon_0 E_0 (e^{-t^2} - 2t^2 e^{-t^2})$$

$$= 0.1\varepsilon_0 E_0 (1 - 2t^2) e^{-t^2}$$

(a) $t = 0$

$$I_d = 0.1\varepsilon_0 E_0 \text{ A}$$

(b) $t = \frac{1}{\sqrt{2}}$ s

$$I_d = 0.1\varepsilon_0 E_0 (1-1) e^{-1/2} = 0$$

(c) $t = 1$ s

$$I_d = 0.1\varepsilon_0 E_0 (1-2) e^{-1}$$

$$= -\,0.1 e^{-1} \varepsilon_0 E_0 \text{ A}$$

D2.9. **(a)** $-I + I_{23} + \frac{d}{dt}\oint_{S_2} \mathbf{D} \cdot d\mathbf{S} = 0$

$$-I + I_{23} - 2I = 0$$

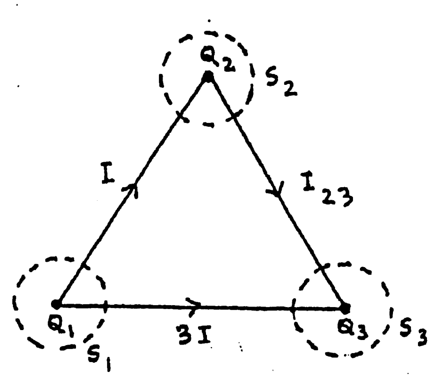
$$I_{23} = 3I \text{ A}$$

(b) $I + 3I + \frac{d}{dt}\oint_{S_1} \mathbf{D} \cdot d\mathbf{S} = 0$

$$\frac{d}{dt}\oint_{S_1} \mathbf{D} \cdot d\mathbf{S} = -4I \text{ A}$$

(c) $-3I - I_{23} + \frac{d}{dt}\oint_{S_3} \mathbf{D} \cdot d\mathbf{S} = 0$

$$\frac{d}{dt}\oint_{S_3} \mathbf{D} \cdot d\mathbf{S} = 3I + I_{23} = 3I + 3I = 6I \text{ A}$$

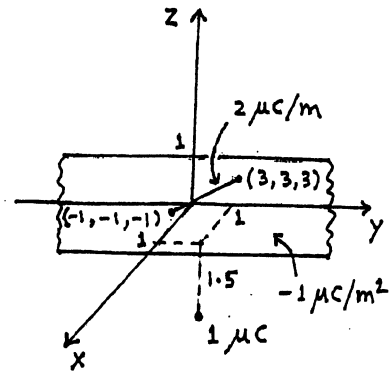


D2.10. (a) $\oint_S \mathbf{D} \cdot d\mathbf{S}$

$= 10^{-6} + 3\sqrt{3}(2 \times 10^{-6}) + 8(-10^{-6})$

$= 3.3923 \times 10^{-6}$ C

$= 3.3923\ \mu$C



(b) $\oint_S \mathbf{D} \cdot d\mathbf{S}$

$= 10^{-6} + (\sqrt{2}+1)\sqrt{3}(2 \times 10^{-6}) + 8(-10^{-6})$

$= 1.3631 \times 10^{-6}$ C

$= 1.3631\ \mu$C

(c) $\oint_C \mathbf{D} \cdot d\mathbf{S}$

$= 2\sqrt{3}(2 \times 10^{-6}) + 10(-10^{-6})$

$= -3.0718 \times 10^{-6}$ C

$= -3.0718\ \mu$C

D2.11. $\psi_1 + \psi_2 + \psi_3 = \psi_0$. Let ψ_1 be the smallest in all cases.

(a) $\psi_1 + (\psi_1 + a) + (\psi_1 + 2a) = \psi_0 \rightarrow 3\psi_1 + 3a = \psi_0$

$\psi_1 + (\psi_1 + a) - (\psi_1 + 2a) = 0 \rightarrow \psi_1 - a = 0$

$\psi_1 = a = \dfrac{\psi_0}{6}$

$\therefore$ Smallest value is $\dfrac{1}{6}\psi_0$.

(b) Let $\dfrac{1}{\psi_2} = \dfrac{1}{\psi_1} - a$ and $\dfrac{1}{\psi_3} = \dfrac{1}{\psi_1} - 2a$, where $a > 0$. Then

$$\psi_1 + \frac{1}{\frac{1}{\psi_1} - a} + \frac{1}{\frac{1}{\psi_1} - 2a} = \psi_0 \qquad (1)$$

$$\psi_1 + \frac{1}{\frac{1}{\psi_1} - a} - \frac{1}{\frac{1}{\psi_1} - 2a} = 0 \qquad (2)$$

From (2),

$$\psi_1 + \frac{\psi_1}{1 - a\psi_1} - \frac{\psi_1}{1 - 2a\psi_1} = 0$$

$$(1 - 3a\psi_1 + 2a^2\psi_1^2) + (1 - 2a\psi_1) - (1 - a\psi_1) = 0$$

$$2a^2\psi_1^2 - 4a\psi_1 + 1 = 0$$

$$a\psi_1 = \frac{4 \pm \sqrt{16-8}}{4} = 1 \pm \sqrt{1/2}$$

Then from (1),

$$\psi_1 + \frac{\psi_1}{1-(1\pm\sqrt{1/2})} + \frac{\psi_1}{1-2(1\pm\sqrt{1/2})} = \psi_0$$

$$\psi_1\left(1 \mp \sqrt{2} - \frac{1}{1\pm\sqrt{2}}\right) = \psi_0$$

$$\psi_1 \frac{-2}{1\pm\sqrt{2}} = \psi_0$$

Ruling out the negative value, we get

$$\psi_1 = \frac{\sqrt{2}-1}{2}\psi_0 = \frac{\psi_0}{2+2\sqrt{2}}$$

Thus the required value is $\frac{1}{2+2\sqrt{2}}\psi_0$.

(c) Let $\ln \psi_2 = \ln \psi_1 + \ln a$. Then $\ln \psi_3 = \ln \psi_1 + 2 \ln a$.

Thus, $\psi_2 = \psi_1 a$ and $\psi_3 = \psi_1 a^2$.

$$\psi_1 + a\psi_1 + a^2\psi_1 = \psi_0 \qquad (1)$$

$$\psi_1 + a\psi_1 - a^2\psi_1 = 0 \qquad (2)$$

From (2), $1 + a - a^2 = 0$ or $a^2 - a - 1 = 0$

$$a = \frac{1+\sqrt{1+4}}{2} = \frac{1+\sqrt{5}}{2}$$

Then from (1),

$$\psi_1 = \frac{\psi_0}{1+a+a^2} = \frac{\psi_0}{1+\frac{1+\sqrt{5}}{2}+\frac{6+2\sqrt{5}}{4}}$$

$$= \frac{1}{3+\sqrt{5}}\psi_0$$

D2.12. **(a)** At Q_3,

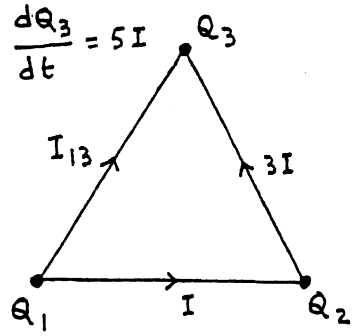
$$-3I - I_{13} + \frac{dQ_3}{dt} = 0$$

$$I_{13} = 5I - 3I = 2I$$

Then at Q_1,

$$I + I_{13} + \frac{dQ_1}{dt} = 0$$

$$\frac{dQ_1}{dt} = -I - 2I = -3I \text{ C/s}$$



(b) At Q_2,

$$-I + 3I + \frac{dQ_2}{dt} = 0$$

$$\frac{dQ_2}{dt} = -2I \text{ C/s}$$

(c) From (a),

current flowing from Q_1 to $Q_3 = 2I$ A

D2.13. From symmetry considerations and Gauss' law for the electric field in integral form, the displacement flux emanating from one side of the regular solid

$$= \frac{\text{Total flux emanating from the solid}}{\text{Number of sides}}$$

(a) Tetrahedron:

Number of sides = 4

Volume = $0.11785a^3$

Flux from one side = $\rho_0 \times \dfrac{0.11785a^3}{4} = 0.0295a^3\rho_0$

(b) Cube:

Number of sides = 6

Volume = a^3

Flux from one side = $\rho_0 \times \dfrac{a^3}{6} = 0.1667a^3\rho_0$

(c) Octahedron:

Number of sides = 8

Volume = $0.4714a^3$

Flux from one side = $\rho_0 \times \dfrac{0.4714a^3}{8} = 0.0589a^3\rho_0$

D2.14. $\oint_C \mathbf{H} \cdot d\mathbf{l} = \int_S \mathbf{J} \cdot d\mathbf{S}$

$= J_0 \times$ cross-sectional area of the wire

Then from symmetry,

$$\oint_{\text{one side}} \mathbf{H} \cdot d\mathbf{l} = \frac{\oint_C \mathbf{H} \cdot d\mathbf{l}}{\text{Number of sides}}$$

$$= J_0 \frac{\text{Cross-sectional area of the wire}}{\text{Number of sides}}$$

(a) Equilateral triangle:

$$\text{Area} = \frac{1}{2} a^2 \sin 60^\circ = \frac{\sqrt{3}\, a^2}{4} \doteq 0.433 a^2$$

Number of sides = 3

$$\int_{\text{one side}} \mathbf{H} \cdot d\mathbf{l} = J_0 \frac{0.433 a^2}{3} = 0.1443 J_0 a^2$$

(b) Square:

Area = a^2

Number of sides = 4

$$\int_{\text{one side}} \mathbf{H} \cdot d\mathbf{l} = J_0 \frac{a^2}{4} = 0.25 J_0 a^2$$

(c) Octagon:

$$\text{Area} = 8 \times \frac{a^2}{4} \tan 67.5^\circ = 4.8284 a^2$$

Number of sides = 8

$$\int_{\text{one side}} \mathbf{H} \cdot d\mathbf{l} = J_0 \frac{4.8284 a^2}{8} = 0.6036 J_0 a^2$$

D3.1. $\mathbf{E} = E_0 \cos(6\pi \times 10^8 t - 2\pi z)\,\mathbf{a}_x$ V/m

$$\frac{\partial E_x}{\partial z} = -\frac{\partial B_y}{\partial t}$$

$$\frac{\partial B_y}{\partial t} = -\frac{\partial E_x}{\partial z} = -2\pi E_0 \sin(6\pi \times 10^8 t - 2\pi z)$$

At $t = 10^{-8}$ s, $\dfrac{\partial B_y}{\partial t} = -2\pi E_0 \sin(6\pi - 2\pi z)$

$$= 2\pi E_0 \sin 2\pi z$$

(a) $z = 0,\ \dfrac{\partial B_y}{\partial t} = 2\pi E_0 \sin 0 = 0$

(b) $z = \dfrac{1}{4},\ \dfrac{\partial B_y}{\partial t} = 2\pi E_0 \sin \dfrac{\pi}{2} = 2\pi E_0$

(c) $z = \dfrac{2}{3},\ \dfrac{\partial B_y}{\partial t} = 2\pi E_0 \sin \dfrac{4\pi}{3} = -\sqrt{3}\,\pi E_0$

D3.2. $\mathbf{A} = xy^2\mathbf{a}_x + zx\mathbf{a}_y + x^2yz\mathbf{a}_z$

(a) $\dfrac{\partial A_x}{\partial(-y)} + \dfrac{\partial A_y}{\partial x} = \dfrac{\partial A_y}{\partial x} - \dfrac{\partial A_x}{\partial y}$

$$= z - 2xy = 1 - (2 \times 1 \times 1) = -1$$

(b) $\dfrac{\partial A_y}{\partial(-z)} + \dfrac{\partial A_z}{\partial y} = \dfrac{\partial A_z}{\partial y} - \dfrac{\partial A_y}{\partial z}$

$$= x^2z - x = (1^2 \times 1) - 1 = 0$$

(c) $\dfrac{\partial A_z}{\partial(-x)} + \dfrac{\partial A_x}{\partial z} = \dfrac{\partial A_x}{\partial z} - \dfrac{\partial A_z}{\partial x}$

$$= 0 - 2xyz = -2 \times 1 \times 1 \times (-1) = 2$$

D3.3. $\mathbf{J} = \mathbf{0},\ \mathbf{H} = H_0 e^{-(3 \times 10^8 t - z)^2}\mathbf{a}_y$

$$\frac{\partial H_y}{\partial z} = -\frac{\partial D_x}{\partial t}$$

$$\frac{\partial D_x}{\partial t} = -\frac{\partial H_y}{\partial z} = -2(3 \times 10^8 t - z)H_0 e^{-(3 \times 10^8 t - z)^2}$$

(a) $z = 2,\ t = 10^{-8},\ \dfrac{\partial D_x}{\partial t} = -2H_0 e^{-1} = -0.7358H_0$

(b) $z = 3, t = \frac{1}{3} \times 10^{-8}, \frac{\partial D_x}{\partial t} = 4H_0 e^{-4} = 0.0733H_0$

(c) $z = 3, t = 10^{-8}, \frac{\partial D_x}{\partial t} = 0$

D3.4. $\mathbf{A} = yz\mathbf{a}_x + xy\mathbf{a}_y + xyz^2\mathbf{a}_z$

$$\frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z} = 0 + x + 2xyz = x(1 + 2yz)$$

(a) At $(1, 1, -1)$, $\frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z} = 1(1 - 2) = -1$

(b) At $\left(1, 1, -\frac{1}{2}\right)$, $\frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z} = 1(1 - 1) = 0$

(c) At $(1, 1, 1)$, $\frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z} = 1(1 + 2) = 3$

D3.5. $\frac{\partial D_x}{\partial x} + \frac{\partial D_y}{\partial y} + \frac{\partial D_z}{\partial z} = \rho = 0$

$$\frac{\partial D_x}{\partial x} + \frac{\partial D_y}{\partial y} = D_0$$

$$\frac{\partial D_y}{\partial y} = 3\frac{\partial D_z}{\partial z}$$

Solving, we get

(a) $\frac{\partial D_x}{\partial x} = 4D_0$

(b) $\frac{\partial D_y}{\partial y} = -3D_0$

(c) $\frac{\partial D_z}{\partial z} = -D_0$

D3.6. $\nabla \bullet \mathbf{J} = J_0(2x + 2y + 2z)$

$$\frac{\partial \rho}{\partial t} = -\nabla \bullet \mathbf{J} = -2J_0(x + y + z)$$

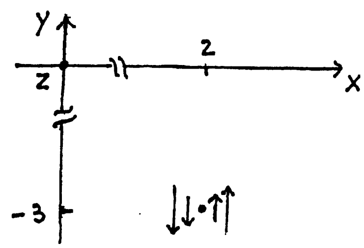
(a) At (0.02, 0.01, 0.01), $\frac{\partial \rho}{\partial t} = -0.08 J_0$

(b) At (0.02, −0.01, −0.01), $\frac{\partial \rho}{\partial t} = 0$

(c) At (−0.02, −0.01, 0.01), $\frac{\partial \rho}{\partial t} = 0.04 J_0$

D3.7. $\mathbf{A} = (x^2 - 4)\mathbf{a}_y$

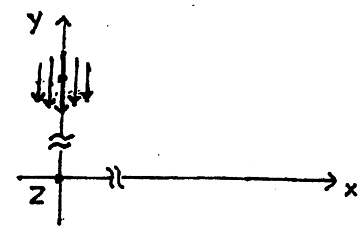
(a) At the point (2, −3, 1):



Curl meter rotates in the cw sense when placed with its axis along the z-axis.

∴ z-component of curl is positive.

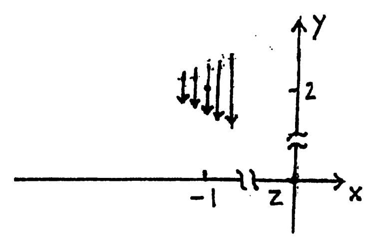
(b) At the point (0, 2, 4):



Curl meter does not rotate when placed with its axis along the z-axis.

∴ z-component of curl is zero.

(c) At the point (−1, 2, −1):

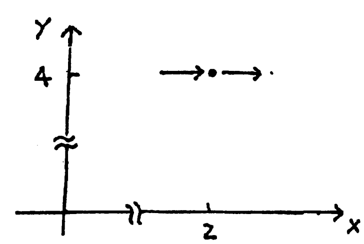


Curl meter rotates in the ccw sense when placed with its axis along the z-axis.

∴ z-component of curl is negative.

D3.8. $\mathbf{A} = (x - 2)^2\, \mathbf{a}_x$

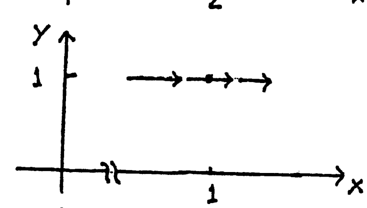
(a) At the point (2, 4, 3):



Balloon does not expand or contract.

∴ Divergence is zero.

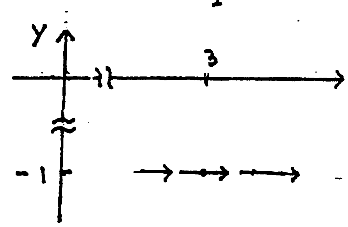
(b) At the point (1, 1, −1):



Balloon contracts.

∴ Divergence is negative.

(c) At the point (3, −1, 4):



Balloon expands.

∴ Divergence is positive.

D3.9. $\nabla \times \mathbf{A} = \begin{vmatrix} \mathbf{a}_x & \mathbf{a}_y & \mathbf{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 0 & x & \sqrt{3}y \end{vmatrix} = \sqrt{3}\mathbf{a}_x + \mathbf{a}_z$

$$\oint_C \mathbf{A} \cdot d\mathbf{l} = \int_S (\nabla \times \mathbf{A}) \cdot d\mathbf{S}$$

(a) $\int_S \nabla \times \mathbf{A} \cdot d\mathbf{S} = \int_S \nabla \times \mathbf{A} \cdot dS\, \mathbf{a}_z$

$$= \int_S dS = 2^2 = 4$$

(b) $\int_S \nabla \times \mathbf{A} \cdot d\mathbf{S} = \int_S \nabla \times \mathbf{A} \cdot dS\, \mathbf{a}_z$

$$= \int_S dS = \pi\left(\frac{1}{\sqrt{\pi}}\right)^2 = 1$$

(c) $\int_S \nabla \times \mathbf{A} \cdot d\mathbf{S} = \int_S \nabla \times \mathbf{A} \cdot dS\, \mathbf{a}_x$

$$= \sqrt{3}\int_S dS = \sqrt{3} \times \left(\frac{2 \times 2 \times \sin 60^\circ}{2}\right) = 3$$

D3.10. $\mathbf{A} = x\mathbf{a}_x + y\mathbf{a}_y + z\mathbf{a}_z$

$$\oint_S \mathbf{A} \cdot d\mathbf{S} = \int_V (\nabla \cdot \mathbf{A})\, dv = \int_V 3\, dv = 3\int_V dv$$
$$= 3 \times \text{volume bounded by } S$$

(a) volume $= 1^3 = 1$

$$\oint_S \mathbf{A} \cdot d\mathbf{S} = 3$$

(b) volume $= \pi\left(\frac{1}{\sqrt{\pi}}\right)^2 \times 2 = 2$

$$\oint_S \mathbf{A} \cdot d\mathbf{S} = 6$$

(c) volume $= \frac{4}{3}\pi\left[\frac{1}{(\pi)^{1/3}}\right]^3 = \frac{4}{3}$

$$\oint_V \mathbf{A} \cdot d\mathbf{S} = 4$$

D3.11. **(a)** $(0.05y - t)^2 = \left(t - \frac{y}{20}\right)^2$

$\mathbf{v}_p = 20\mathbf{a}_y$ m/s

(b) $u(t + 0.02x) = u\left(t + \frac{x}{50}\right)$

$\mathbf{v}_p = -50\mathbf{a}_x$ m/s

(c) $\cos\,(2\pi \times 10^8 t - 2\pi z) = \cos\left[2\pi \times 10^8\left(t - \frac{z}{10^8}\right)\right]$

$\mathbf{v}_p = 10^8\mathbf{a}_z$ m/s

D3.12. $f(z, t) = f\left(t - \frac{z}{200}\right)$

$= f\left[\left(t - \frac{z}{200}\right) - \frac{0}{200}\right]$

$= f\left(0, t - \frac{z}{200}\right)$

(a) $f(300, 2) = f\left(0, 2 - \frac{300}{200}\right)$

$= f(0, 0.5) = 0.25A$

(b) $f(-200, 0.4) = f\left(0, 0.4 + \frac{200}{200}\right)$

$= f(0, 1.4) = 0.6A$

(c) $f(100, 0.5) = f\left(0, 0.5 - \frac{100}{200}\right)$

$= f(0, 0) = 0$

D3.13. $g(z, t) = g\left(t + \frac{z}{100}\right)$

$= g\left[\left(t + \frac{z}{100}\right) + \frac{0}{100}\right]$

$= g\left(0, t + \frac{z}{100}\right)$

(a) $g(200, 0.2) = g\left(0, 0.2 + \frac{200}{100}\right)$

$$= g(0, 2.2) = 0.9A$$

(b) $g(-300, 3.4) = g\left(0, 3.4 - \dfrac{300}{100}\right)$

$$= g(0, 0.4) = 0.4A$$

(c) $g(100, 0.6) = g\left(0, 0.6 + \dfrac{100}{100}\right)$

$$= g(0, 1.6) = A$$

D3.14. **(a)** $\omega = \dfrac{\partial\phi}{\partial t} = \dfrac{3\pi}{0.1\times10^{-6}} = 3\pi \times 10^7$

$$f = \frac{\omega}{2\pi} = \frac{3\pi\times10^7}{2\pi} = 1.5 \times 10^7 \text{ Hz} = 15 \text{ MHz}$$

(b) $\beta = \dfrac{\partial\phi}{\partial z} = \dfrac{0.04\pi}{1} = 0.04\pi$

$$\lambda = \frac{2\pi}{\beta} = \frac{2\pi}{0.04\pi} = 50 \text{ m}$$

(c) $f = \dfrac{v_p}{\lambda} = \dfrac{3\times10^8}{25} = 12 \times 10^6 \text{ Hz} = 12 \text{ MHz}$

(d) $\lambda = \dfrac{v_p}{f} = \dfrac{3\times10^8}{5\times10^6} = 60 \text{ m}$

D3.15. $\mathbf{H} = H_0 \cos(6\pi\times10^8 t + 2\pi y)\, \mathbf{a}_x$ A/m

(a) In view of the argument $(6\pi\times10^8 t + 2\pi y)$ for the cosine function, the direction of propagation of the wave is the $-y$ direction. Hence the required unit vector is $-\mathbf{a}_y$.

(b) $\mathbf{H}(t = 0, y = 0) = H_0\, \mathbf{a}_x$

$\therefore$ The required unit vector is $\mathbf{a}_x$.

(c) Since $\mathbf{E} \times \mathbf{H}$ must be directed along $-\mathbf{a}_y$, and $-\mathbf{a}_z \times \mathbf{a}_x = -\mathbf{a}_y$, the required unit vector is $-\mathbf{a}_z$.

D3.16. For $\mathbf{J}_{S2} = -kJ_{S0} \sin \omega t\, \mathbf{a}_x$, $z = \lambda/4$

$$\mathbf{E}_2 = \begin{cases} k\dfrac{\eta_0 J_{S0}}{2} \cos(\omega t - \beta z)\, \mathbf{a}_x & \textit{for } z > \dfrac{\lambda}{4} \\ -k\dfrac{\eta_0 J_{S0}}{2} \cos(\omega t + \beta z)\, \mathbf{a}_x & \textit{for } z < \dfrac{\lambda}{4} \end{cases}$$

$\mathbf{E} = \mathbf{E}_1 + \mathbf{E}_2$

$$= \begin{cases} (1+k)\dfrac{\eta_0 J_{S0}}{2} \cos(\omega t - \beta z)\, \mathbf{a}_x & \textit{for } z > \dfrac{\lambda}{4} \\ (1-k)\dfrac{\eta_0 J_{S0}}{2} \cos(\omega t + \beta z)\, \mathbf{a}_x & \textit{for } z < 0 \end{cases}$$

$$\frac{\text{Amplitude of } \mathbf{E} \text{ for } z > \lambda/4}{\text{Amplitude of } \mathbf{E} \text{ for } z < \lambda/4} = \frac{|1+k|}{|1-k|}$$

(a) $\dfrac{|1+k|}{|1-k|} = \dfrac{1}{3}$

$9(1 + 2k + k^2) = 1 - 2k + k^2$

$8k^2 + 20k + 8 = 0$

$(2k + 1)(k + 2) = 0$

$k = -\dfrac{1}{2}$ or -2

$\therefore k = -\dfrac{1}{2}$

(b) $\dfrac{|1+k|}{|1-k|} = 3$

$1 + 2k + k^2 = 9(1 - 2k + k^2)$

$8k^2 - 20k + 8 = 0$

$(2k - 1)(k - 2) = 0$

$k = \dfrac{1}{2}$ or 2

$\therefore k = \dfrac{1}{2}$

(c) $\dfrac{|1+k|}{|1-k|} = 7$

$1 + 2k + k^2 = 49(1 - 2k + k^2)$

$48k^2 - 100k + 48 = 0$

$(4k - 3)(3k - 4) = 0$

$k = \dfrac{3}{4}$ or $\dfrac{4}{3}$

$\therefore k = \dfrac{3}{4}$

D3.17. The two fields are equal in amplitude and differ in direction by 90°. The phase difference is $-2\pi z + 3\pi z$, or πz.

(a) At (3, 4, 0), the phase difference is zero.

$\mathbf{F}_1 + \mathbf{F}_2$ is linearly polarized.

(b) At (3, –2, 0.5), the phase difference is 0.5π.

$\mathbf{F}_1 + \mathbf{F}_2$ is circularly polarized.

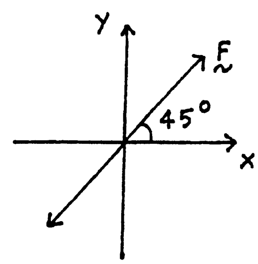
(c) At (–2, 1, 1), the phase difference is π.

$\mathbf{F}_1 + \mathbf{F}_2$ is linearly polarized.

(d) At (–1, –3, 0.2), the phase difference is 0.2π.

$\mathbf{F}_1 + \mathbf{F}_2$ is elliptically polarized.

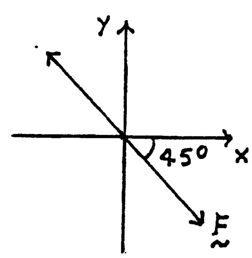
D3.18. (a) $\mathbf{F}$ is linearly polarized if its components are in phase, or out of phase by 180°, that is, for values of α equal to 60° and 240°.



For $\alpha = 60°$,

$$\mathbf{F} = 1 \cos(\omega t + 60°)\, \mathbf{a}_x + 1 \cos(\omega t + 60°)\, \mathbf{a}_y$$

The polarization is along a line lying in the first and third quadrants.



For $\alpha = 240°$,

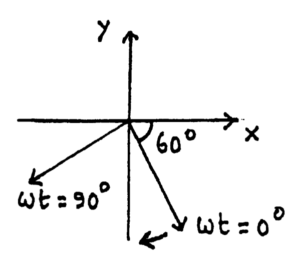
$$\mathbf{F} = 1 \cos(\omega t + 60°)\, \mathbf{a}_x + 1 \cos(\omega t + 240°)\, \mathbf{a}_y$$

$$= 1 \cos(\omega t + 60°)\, \mathbf{a}_x - 1 \cos(\omega t + 60°)\, \mathbf{a}_y$$

The polarization is along a line lying in the second and fourth quadrants.

Thus the required value of α is 240°.

(b) $\mathbf{F}$ is circularly polarized if its components are out of phase by 90°. Note that their amplitudes are equal and they are perpendicular in direction. Thus the possible values of α between 0° and 360° are 150° and 330°.

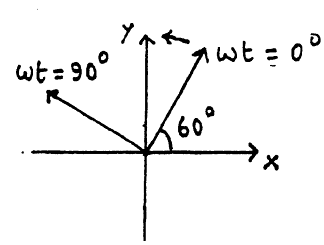


For $\alpha = 150°$,

$$\mathbf{F} = 1 \cos(\omega t + 60°)\, \mathbf{a}_x + 1 \cos(\omega t + 150°)\, \mathbf{a}_y$$

$$= 1 \cos(\omega t + 60°)\, \mathbf{a}_x - 1 \sin(\omega t + 60°)\, \mathbf{a}_y$$

The vector rotates from the $+y$-direction toward the $+x$-direction with time.



For $\alpha = 330°$,

$$\mathbf{F} = 1 \cos(\omega t + 60°)\, \mathbf{a}_x + 1 \cos(\omega t + 330°)\, \mathbf{a}_y$$

$= 1 \cos(\omega t + 60^\circ)\, \mathbf{a}_x + 1 \sin(\omega t + 60^\circ)\, \mathbf{a}_y$

The vector rotates from the $+x$-direction toward the $+y$-direction with time.

Thus the required value of α is 330°.

(c) From part (b), the required value of α is 150°.

D3.19. $\mathbf{H} = H_0 \cos(6\pi \times 10^7 t - 0.2\pi z)\, \mathbf{a}_y$

$\mathbf{E} = -\eta_0 H_0 \cos(6\pi \times 10^7 t - 0.2\pi z)\, \mathbf{a}_x$

$\mathbf{P} = \mathbf{E} \times \mathbf{H} = \eta_0 H_0^2 \cos^2(6\pi \times 10^7 t - 0.2\pi z)\, \mathbf{a}_z$

(a) Instantaneous power flow across a surface of area 1 m^2 in the $z = 0$ plane at $t = 0$ is

$\eta_0 H_0^2 = 120\pi\, H_0^2$

(b) Instantaneous power flow across a surface of area 1 m^2 in the $z = 0$ plane at $t = \frac{1}{8}\ \mu$s is

$\eta_0^2 H_0^2 \cos^2(7.5\pi)\, \mathbf{a}_z = 0$

(c) Time-average power flow across a surface of area 1 m^2 in the $z = 0$ plane is

$< \eta_0 H_0^2 \cos^2 6\pi \times 10^7 t > = \frac{1}{2}\, \eta_0 H_0^2 = 60\pi\, H_0^2$

D3.20. **(a)** $< A \sin \omega t \sin 3\omega t >$

$= < \frac{A}{2}(\cos 2\omega t - \cos 4\omega t) >$

$= \frac{A}{2}[< \cos 2\omega t > - < \cos 4\omega t >]$

$= 0$

(b) $< A (\cos^2 \omega t - 0.5 \sin^2 2\omega t) >$

$= A < \cos^2 \omega t > - 0.5\, A < \sin^2 2\omega t >$

$= 0.5A < 1 + \cos 2\omega t > - 0.25\, A < 1 - \cos 4\omega t >$

$= 0.5A - 0.25A$

$= 0.25A$

(c) $\sin^3 \omega t = \sin \omega t \sin^2 \omega t$

$= \sin \omega t \left(\frac{1 - \cos 2\omega t}{2}\right) = \frac{1}{2} \sin \omega t - \frac{1}{2} \sin \omega t \cos 2\omega t$

$$= \frac{1}{2} \sin \omega t - \frac{1}{4} (\sin 3\omega t - \sin \omega t)$$

$$= \frac{3}{4} \sin \omega t - \frac{1}{4} \sin 3\omega t$$

$$\sin^6 \omega t = \left(\frac{3}{4} \sin \omega t - \frac{1}{4} \sin 3\omega t\right)^2$$

$$= \frac{9}{16} \sin^2 \omega t + \frac{1}{16} \sin^2 3\omega t - \frac{3}{8} \sin \omega t \sin 3\omega t$$

$$= \frac{9}{16} \sin^2 \omega t + \frac{1}{16} \sin^2 3\omega t + \frac{3}{16} (\cos 4\omega t - \cos 2\omega t)$$

$$< \sin^6 \omega t > = \frac{9}{32} + \frac{1}{32} = \frac{5}{16}$$

$$< A \sin^6 \omega t > = \frac{5A}{16} = 0.3125A$$

D4.1. $J_c = \frac{I}{A} = \frac{0.1}{10^{-4}} = 10^3$ A/m^2

(a) For copper, $\sigma = 5.8 \times 10^7$ S/m

$$E = \frac{J_c}{\sigma} = \frac{10^3}{5.8 \times 10^7} = 17.24 \times 10^{-6} \text{ V/m}$$

$$= 17.24\ \mu\text{V/m}$$

(b) $\sigma = (\mu_h + \mu_e) N_e |e|$

$$= (1700 + 3600) \times 10^{-4} \times 2.5 \times 10^{19} \times 1.6022 \times 10^{-19}$$

$$= 2.1229 \text{ S/m}$$

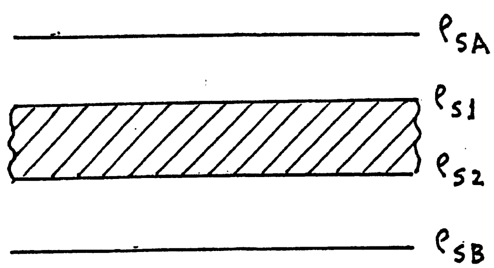
$$E = \frac{J_c}{\sigma} = \frac{10^3}{2.1229} = 471.1 \text{ V/m}$$

(c) $R = \frac{l}{\sigma A} = 1$

$$\sigma = \frac{l}{A} = \frac{1}{\pi \times 10^{-6}} = \frac{10^6}{\pi}$$

$$E = \frac{J_c}{\sigma} = \frac{10^3}{10^6/\pi} = \pi \times 10^{-3} \text{ V/m} = 3.14 \text{ mV/m}$$

D4.2.



From charge neutrality in the slab,

$$\rho_{S1} + \rho_{S2} = 0 \qquad (1)$$

For the electric field intensity inside the slab to be zero,

$$\frac{\rho_{SA}}{2\varepsilon_0} + \frac{\rho_{S1}}{2\varepsilon_0} - \frac{\rho_{S2}}{2\varepsilon_0} - \frac{\rho_{SB}}{2\varepsilon_0} = 0$$

or, $\rho_{S1} - \rho_{S2} = \rho_{SB} - \rho_{SA} \qquad (2)$

From (1) and (2), we obtain

(a) $\rho_{S1} = \frac{1}{2}(\rho_{SB} - \rho_{SA})$

(b) $\rho_{S2} = \frac{1}{2}(\rho_{SA} - \rho_{SB})$

D4.3. (a) $\mathbf{D} = \rho_{S0}\mathbf{a}_z = 10^{-6}\,\mathbf{a}_z$ C/m^2

(b) $\mathbf{E} = \frac{\mathbf{D}}{4\varepsilon_0} = \frac{10^{-6} \times 36\pi}{4 \times 10^{-9}}\,\mathbf{a}_z = 9000\pi\,\mathbf{a}_z$ V/m

(c) $\mathbf{P} = \mathbf{D} - \varepsilon_0\mathbf{E} = \mathbf{D} - \frac{\mathbf{D}}{4} = \frac{3}{4}\,\mathbf{D} = 0.75 \times 10^{-6}\,\mathbf{a}_z$ C/m^2

D4.4. (a) $\mathbf{E} = E_0\mathbf{a}_z$

$$\begin{bmatrix} D_x \\ D_y \\ D_z \end{bmatrix} = \varepsilon_0 \begin{bmatrix} 8 & 2 & 0 \\ 2 & 5 & 0 \\ 0 & 0 & 9 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ E_0 \end{bmatrix} = \varepsilon_0 \begin{bmatrix} 0 \\ 0 \\ 9E_0 \end{bmatrix}$$

$\mathbf{D} = 9\varepsilon_0 E_0\mathbf{a}_z = 9\varepsilon_0\mathbf{E}$

$\varepsilon_{\text{eff}} = 9\varepsilon_0,\ \varepsilon_{r\text{eff}} = 9$

(b) $\mathbf{E} = E_0(\mathbf{a}_x - 2\mathbf{a}_y)$

$$\begin{bmatrix} D_x \\ D_y \\ D_z \end{bmatrix} = \varepsilon_0 \begin{bmatrix} 8 & 2 & 0 \\ 2 & 5 & 0 \\ 0 & 0 & 4 \end{bmatrix} \begin{bmatrix} E_0 \\ -2E_0 \\ 0 \end{bmatrix} = \varepsilon_0 \begin{bmatrix} 4E_0 \\ -8E_0 \\ 0 \end{bmatrix}$$

$\mathbf{D} = 4\varepsilon_0 E_0(\mathbf{a}_x - 2\mathbf{a}_y) = 4\varepsilon_0\mathbf{E}$

$\varepsilon_{\text{eff}} = 4\varepsilon_0,\ \varepsilon_{r\text{eff}} = 4$

(c) $\mathbf{E} = E_0(2\mathbf{a}_x + \mathbf{a}_y)$

$$\begin{bmatrix} D_x \\ D_y \\ D_z \end{bmatrix} = \varepsilon_0 \begin{bmatrix} 8 & 2 & 0 \\ 2 & 5 & 0 \\ 0 & 0 & 4 \end{bmatrix} \begin{bmatrix} 2E_0 \\ E_0 \\ 0 \end{bmatrix} = \varepsilon_0 \begin{bmatrix} 18E_0 \\ 9E_0 \\ 0 \end{bmatrix}$$

$\mathbf{D} = 9\varepsilon_0 E_0(2\mathbf{a}_x + \mathbf{a}_y) = 9\varepsilon_0\mathbf{E}$

$\varepsilon_{\text{eff}} = 9\varepsilon_0,\ \varepsilon_{r\text{eff}} = 9$

D4.5. (a) Number of revolutions per second $= \frac{1}{10^{-3}} = 1000$

Amount of charge passing per second $= 1000 \times 10^{-6} = 10^{-3}$ C

$\therefore I = 10^{-3}$ A

$$\text{Area of the loop} = \pi\left(\frac{1}{\sqrt{\pi}}\times 10^{-3}\right)^2 = 10^{-6}\ \text{m}^2$$

$$\mathbf{m} = 10^{-3}\times 10^{-6}\,\mathbf{a}_z = 10^{-9}\,\mathbf{a}_z\ \text{A-m}^2$$

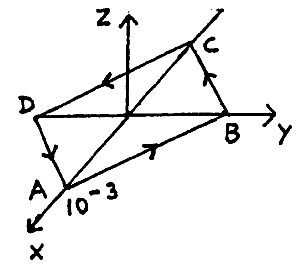
(b) Area of the loop $= (\sqrt{2}\times 10^{-3})^2$

$$= 2\times 10^{-6}\ \text{m}^2$$

$$I = 0.1\ \text{A}$$

$$\mathbf{m} = 0.1\times 2\times 10^{-6}\,\mathbf{a}_z$$

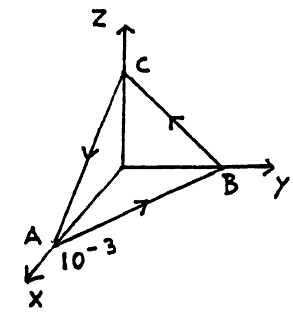
$$= 2\times 10^{-7}\,\mathbf{a}_z\ \text{A-m}^2$$



(c) Area of the loop

$$= \frac{1}{2}\times\sqrt{2}\times 10^{-3}\times\frac{\sqrt{3}}{2}\times\sqrt{2}\times 10^{-3}$$

$$= \frac{\sqrt{3}}{2}\times 10^{-6}\ \text{m}^2$$



Unit vector normal to the loop

$$= \frac{1}{\sqrt{3}}(\mathbf{a}_x + \mathbf{a}_y + \mathbf{a}_z)$$

$$I = 0.1\ \text{A}$$

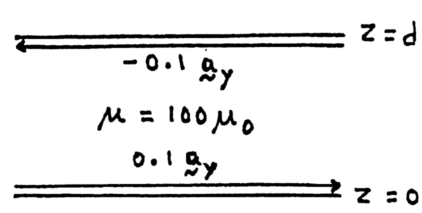
$$\mathbf{m} = 0.1\times\frac{\sqrt{3}}{2}\times 10^{-6}\times\frac{1}{\sqrt{3}}(\mathbf{a}_x + \mathbf{a}_y + \mathbf{a}_z)$$

$$= 5\times 10^{-8}\,(\mathbf{a}_x + \mathbf{a}_y + \mathbf{a}_z)\ \text{A-m}^2$$

D4.6. (a) $\mathbf{H} = 0.1\mathbf{a}_y \times \mathbf{a}_z = 0.1\mathbf{a}_x$ A/m

(b) $\mathbf{B} = \mu\mathbf{H} = 100\times 4\pi\times 10^{-7}\times 0.1\mathbf{a}_x$

$$= 4\pi\times 10^{-6}\,\mathbf{a}_x\ \text{Wb/m}^2$$



(c) $\mathbf{M} = \dfrac{\mathbf{B}}{\mu_0} - \mathbf{H} = \dfrac{4\pi\times 10^{-6}\mathbf{a}_x}{4\pi\times 10^{-7}} - 0.1\mathbf{a}_x$

$$= 10\mathbf{a}_x - 0.1\mathbf{a}_x = 9.9\,\mathbf{a}_x\ \text{A/m}$$

D4.7. From computation as in Ex. 4.5,

(a) $\bar{\gamma} = (0.00083 + j\,0.00476)\ \text{m}^{-1}$

$\bar{\eta} = 163.54\underline{/9.9^\circ}\ \Omega$

(b) $\bar{\gamma} = (77.84 + j\,202.86)\ \text{m}^{-1}$

$\bar{\eta} = 36.34\underline{/20.99^\circ}\ \Omega$

D4.8. $\bar{\gamma} = (0.05 + j\,0.1)\ \text{m}^{-1},\ f = 10^6\ \text{Hz},\ \mu = \mu_0$

(a) $\alpha d = 1$

$$d = \frac{1}{\alpha} = \frac{1}{0.05} = 20\ \text{m}$$

(b) $\beta d = 1$

$$d = \frac{1}{\beta} = \frac{1}{0.1} = 10\ \text{m}$$

(c) $d = v_p t = \dfrac{\omega}{\beta} t$

$$= \frac{2\pi \times 10^6}{0.1} \times 10^{-6} = 62.83\ \text{m}$$

(d) From $\bar{\gamma}\,\bar{\eta} = j\omega\mu$

$$\bar{\eta} = \frac{j\omega\mu}{\bar{\gamma}} = \frac{j2\pi \times 10^6 \times 4\pi \times 10^{-7}}{0.05 + j0.1}$$

$$= \frac{0.8\pi^2\underline{/90^\circ}}{0.1118\underline{/63.435^\circ}} = 70.62\ \underline{/26.565^\circ}$$

$$\frac{\text{Amplitude of } \mathbf{E}}{\text{Amplitude of } \mathbf{H}} = 70.62\ \Omega$$

(e) Phase difference between **E** and **H**

$= 26.565^\circ = 0.1476\pi$

D4.9. $\mathbf{H} = H_0 e^{-z} \cos\,(6\pi \times 10^7 t - \sqrt{3}\,z)\ \mathbf{a}_y$ V/m

$\bar{\gamma} = 1 + j\sqrt{3}$

From $\bar{\gamma}\,\bar{\eta} = j\omega\mu = j\,6\pi \times 10^7 \times 4\pi \times 10^{-7} = j24\pi^2$,

$$\bar{\eta} = \frac{j24\pi^2}{1 + j\sqrt{3}} = \frac{24\pi^2 e^{j\pi/2}}{2e^{j\pi/3}} = 12\pi^2 e^{\,j\pi/6}$$

$$\therefore\ \mathbf{E} = 12\pi^2 H_0 e^{-z} \cos\left(6\pi \times 10^7 t - \sqrt{3}z + \frac{\pi}{6}\right) \mathbf{a}_x\ \text{A/m}$$

$$\mathbf{P} = \mathbf{E} \times \mathbf{H}$$

$$= 12\pi^2 H_0^2 e^{-2z} \cos\left(6\pi \times 10^7 t - \sqrt{3}z + \frac{\pi}{6}\right)$$

$$\cdot \cos\,(6\pi \times 10^7 t - \sqrt{3}\,z)\ \mathbf{a}_z \text{ W/m}^2$$

$$= 6\pi^2 H_0^2 e^{-2z}\left[\cos\frac{\pi}{6} + \cos\left(12\pi \times 10^7 t - 2\sqrt{3}z + \frac{\pi}{6}\right)\right] \mathbf{a}_z \text{ W/m}^2$$

(a) Instantaneous power flow across a surface of area 1 m^2 in the $z = 0$ plane at $t = 0$ is $6\pi^2 H_0^2 \times 2\cos\frac{\pi}{6} = 102.57\, H_0^2$ W

(b) Time-average power flow across a surface of area 1 m^2 in the $z = 0$ plane is $6\pi^2 H_0^2 \times \cos\frac{\pi}{6} = 51.28\, H_0^2$ W

(c) Time-average power flow across a surface of area 1 m^2 in the $z = 1$ plane is $6\pi^2 H_0^2 e^{-2} \times \cos\frac{\pi}{6} = 6.94\, H_0^2$ W

D4.10. **(a)** $$\frac{1/\sqrt{\mu_0 \varepsilon}}{1/\sqrt{\mu_0 \varepsilon_0}} = \frac{1}{3}$$

$$\sqrt{\frac{\varepsilon_0}{\varepsilon}} = \frac{1}{3},\ \frac{\varepsilon}{\varepsilon_0} = 9$$

$$\varepsilon_r = 9$$

(b) $$2\pi f_0 \sqrt{\mu_0 \varepsilon} = 2\pi(2f_0)\sqrt{\mu_0 \varepsilon_0}$$

$$\varepsilon = 4\varepsilon_0$$

$$\varepsilon_r = 4$$

(c) $$\frac{1}{f\sqrt{\mu_0 \varepsilon}} = \frac{2}{3}\,\frac{1}{f\sqrt{\mu_0 \varepsilon_0}}$$

$$\sqrt{\frac{\varepsilon}{\varepsilon_0}} = \frac{3}{2},\ \frac{\varepsilon}{\varepsilon_0} = 2.25$$

$$\varepsilon_r = 2.25$$

(d) $$\frac{E_0}{\sqrt{\mu/\varepsilon}} = 4\,\frac{E_0}{\sqrt{\mu_0/\varepsilon_0}}$$

$$\sqrt{\frac{\varepsilon}{\varepsilon_0}} = 4,\ \frac{\varepsilon}{\varepsilon_0} = 16$$

$\varepsilon_r = 16$

D4.11. $f = 10^5$ Hz, $\alpha(2.5) = \pi$ or $\alpha = \dfrac{\pi}{2.5}$

(a) $\beta = \alpha = \dfrac{\pi}{2.5}$

$$d = \frac{2\pi}{\beta} = \frac{2\pi}{\pi/2.5} = 5 \text{ m}$$

(b) $d = v_p \times 10^{-6} = \dfrac{\omega}{\beta} \times 10^{-6}$

$$= \frac{2\pi \times 10^5}{\pi/2.5} \times 10^{-6} = 0.5 \text{ m}$$

(c) Since $v_p = \sqrt{\dfrac{4\pi f}{\mu\sigma}}$, $v_p \propto \sqrt{f}$

$$\therefore d = 0.5 \times \sqrt{\frac{10^4}{10^5}} = \frac{0.5}{\sqrt{10}}$$

$$= 0.1581 \text{ m}$$

D4.12. (a) $\mathbf{E}_1 = E_0 e^{-0.4\pi z} \cos (2\pi \times 10^5 t - 0.4\pi z)\, \mathbf{a}_x$

$\alpha = \beta = 0.4\pi$

Material is good conductor.

(b) $\mathbf{E}_2 = E_0 e^{-2\pi \times 10^{-5} z} \cos (2\pi \times 10^5 t - 2\pi \times 10^{-3} z)\, \mathbf{a}_x$

$\alpha = 2\pi \times 10^{-5}$, $\beta = 2\pi \times 10^{-3}$

$$\frac{\alpha}{\beta} = \frac{2\pi \times 10^{-5}}{2\pi \times 10^{-3}} = 10^{-2} << 1$$

Material is imperfect dielectric, since from (4.94) and (4.95) or from (4.110a) and (4.110b), $\alpha/\beta << 1$ if $\sigma/\omega\varepsilon << 1$.

(c) $\mathbf{E}_3 = E_0 e^{-0.004z} \cos (2\pi \times 10^5 t - 0.01z)\, \mathbf{a}_x$

$\alpha = 0.004$, $\beta = 0.01$

$$\frac{\alpha}{\beta} = \frac{0.004}{0.01} = 0.4 \text{ (not equal to 1 or << 1)}$$

Material is neither a good conductor nor an imperfect dielectric.

D4.13. $\rho_S = \mathbf{a}_n \cdot \mathbf{D}$

$$= \begin{cases} |\mathbf{D}| \text{ if } \mathbf{D} \text{ pointing away from the surface} \\ -|\mathbf{D}| \text{ if } \mathbf{D} \text{ pointing toward the surface} \end{cases}$$

(a) $\mathbf{D} = D_0(\mathbf{a}_x - 2\mathbf{a}_y + 2\mathbf{a}_z)$ pointing away from the surface;

$$\rho_S = |\mathbf{D}| = |D_0(\mathbf{a}_x - 2\mathbf{a}_y + 2\mathbf{a}_z)| = 3D_0$$

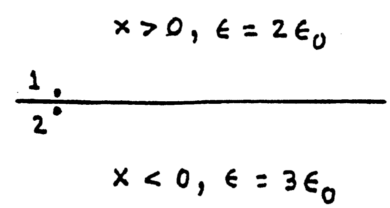
(b) $\mathbf{D} = D_0(\mathbf{a}_x + \sqrt{3}\,\mathbf{a}_z)$ pointing toward the surface;

$$\rho_S = -|\mathbf{D}| = -|D_0(\mathbf{a}_x + \sqrt{3}\,\mathbf{a}_z)| = -2D_0$$

(c) $\mathbf{D} = D_0(0.8\mathbf{a}_x + 0.6\mathbf{a}_y)$ pointing away from the surface;

$$\rho_S = |\mathbf{D}| = |D_0(0.8\mathbf{a}_x + 0.6\mathbf{a}_y)| = D_0$$

D4.14. $\mathbf{E}_1 = E_0(2\mathbf{a}_x + \mathbf{a}_y)$



(a) $$\frac{E_{x1}}{E_{x2}} = \frac{D_{x1}/2\varepsilon_0}{D_{x2}/3\varepsilon_0}$$

$$= \frac{3}{2}\frac{D_{x1}}{D_{x2}}$$

$$= 1.5$$

(b) $$E_{x2} = \frac{E_{x1}}{1.5} = \frac{2E_0}{1.5} = \frac{4}{3}E_0$$

$$E_{y2} = E_{y1} = E_0$$

$$\mathbf{E}_2 = E_0\left(\frac{4}{3}\mathbf{a}_x + \mathbf{a}_y\right)$$

$$\frac{E_1}{E_2} = \frac{E_0\sqrt{4+1}}{E_0\sqrt{\frac{16}{9}+1}} = \frac{\sqrt{5}}{\sqrt{25/9}}$$

$$= \frac{3}{\sqrt{5}}$$

(c) $$\frac{D_1}{D_2} = \frac{2\varepsilon_0 E_1}{3\varepsilon_0 E_2} = \frac{2}{3}\frac{E_1}{E_2}$$

$$= \frac{2}{3} \times \frac{3}{\sqrt{5}} = \frac{2}{\sqrt{5}}$$

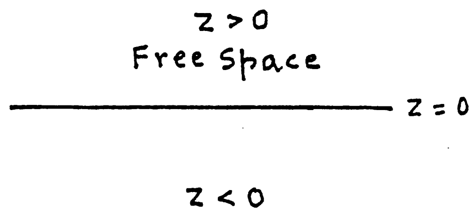
D4.15. (a) $\mathbf{J}_S = \mathbf{a}_n \times \mathbf{H}$

At $t = 0$,

$$\mathbf{H}(0,0,0+) = H_0(3\mathbf{a}_x - 4\mathbf{a}_y)$$

$$\mathbf{J}_S(0,0,0) = \mathbf{a}_z \times H_0(3\mathbf{a}_x - 4\mathbf{a}_y)$$

$$= H_0(4\mathbf{a}_x + 3\mathbf{a}_y)$$



(b) $H_x(0,0,0+) = H_x(0,0,0-) = 10H_0$

$$H_y(0,0,0+) = H_y(0,0,0-) = 0$$

$$B_z(0,0,0+) = B_z(0,0,0-) = 20\mu_0 H_0$$

$$H_z(0,0,0+) = \frac{1}{\mu_0} B_z(0,0,0+) = 20H_0$$

$$\therefore \mathbf{H}(0,0,0+) = 10H_0(\mathbf{a}_x + 2\mathbf{a}_z)$$

(c) $$\frac{B(0,0,0-)}{B(0,0,0+)} = \frac{20\mu_0 H(0,0,0-)}{\mu_0 H(0,0,0+)}$$

$$= 20\frac{H_0\sqrt{100+1}}{10H_0\sqrt{1+4}}$$

$$= 8.989$$

D4.16. **(a)** For $\sigma = 10^{-3}$ S/m, $\varepsilon = 6\varepsilon_0$, $\mu = \mu_0$, and $f = 10^6$ Hz,

$$\bar{\eta} = 86.5477\angle 35.7825^\circ\ \Omega$$

$$\bar{\Gamma} = \frac{\bar{\eta}_2 - \bar{\eta}_1}{\bar{\eta}_2 + \bar{\eta}_1} = \frac{86.5477\angle 35.7825^\circ - 377}{86.5477\angle 35.7825^\circ + 377}$$

$$= \frac{-306.7888 + j50.6053}{447.2112 + j50.6053} = \frac{310.9345\angle 170.633^\circ}{450.0653\angle 6.456^\circ}$$

$$= 0.6909\angle 164.177^\circ$$

$$\bar{\tau} = 1 + \bar{\Gamma} = 1 + 0.6909\angle 164.177^\circ$$

$$= 0.3353 + j\,0.1884$$

$$= 0.3846\angle 29.331^\circ$$

(b) For $\sigma = 4$ S/m, $\varepsilon = 80\varepsilon_0$, $\mu = \mu_0$, and $f = 10^6$ Hz,

$$\bar{\eta} = 1.405\angle 44.968^\circ\ \Omega$$

For $\sigma = 10^{-3}$ S/m, $\varepsilon = 80\varepsilon_0$, $\mu = \mu_0$, and $f = 10^6$ Hz,

$$\bar{\eta} = 41.632\angle 6.34^\circ\ \Omega$$

$$\bar{\Gamma} = \frac{\bar{\eta}_2 - \bar{\eta}_1}{\bar{\eta}_2 + \bar{\eta}_1} = \frac{41.632\underline{/6.34^\circ} - 1.405\underline{/44.968^\circ}}{41.632\underline{/6.34^\circ} + 1.405\underline{/44.968^\circ}}$$

$$= \frac{40.3833 + j3.6044}{42.3714 + j5.5903} = \frac{40.5438\underline{/5.1004^\circ}}{42.7386\underline{/7.5159^\circ}}$$

$$= 0.9486\underline{/-2.4155^\circ}$$

$$\bar{\tau} = 1 + \bar{\Gamma} = 1 + 0.9486\underline{/-2.4155^\circ}$$

$$= 1.9478 - j\,0.04$$

$$= 1.948\underline{/-1.177^\circ}$$

D4.17. $\Gamma = \dfrac{1 - \sqrt{\varepsilon_2/\varepsilon_1}}{1 + \sqrt{\varepsilon_2/\varepsilon_1}}$, $\tau = \dfrac{2}{1 + \sqrt{\varepsilon_2/\varepsilon_1}}$

(a) $\Gamma = -\frac{1}{3}$

$$\frac{1 - \sqrt{\varepsilon_2/\varepsilon_1}}{1 + \sqrt{\varepsilon_2/\varepsilon_1}} = -\frac{1}{3}$$

$$4 = 2\sqrt{\varepsilon_2/\varepsilon_1}$$

$$\frac{\varepsilon_2}{\varepsilon_1} = 4$$

(b) $\tau = 0.4$

$$\frac{2}{1 + \sqrt{\varepsilon_2/\varepsilon_1}} = 0.4$$

$$\sqrt{\frac{\varepsilon_2}{\varepsilon_1}} = \frac{1.6}{0.4} = 4$$

$$\frac{\varepsilon_2}{\varepsilon_1} = 16$$

(c) $\tau = 6\Gamma$

$$\frac{2}{1 + \sqrt{\varepsilon_2/\varepsilon_1}} = 6\,\frac{1 - \sqrt{\varepsilon_2/\varepsilon_1}}{1 + \sqrt{\varepsilon_2/\varepsilon_1}}$$

$$\sqrt{\frac{\varepsilon_2}{\varepsilon_1}} = \frac{2}{3}$$

$$\frac{\varepsilon_2}{\varepsilon_1} = \frac{4}{9}$$

D5.1. $\mathbf{a}_n = \dfrac{\nabla(2x^2+2y^2+z^2}{|\nabla(2x^2+2y^2+z^2|}$

$$= \frac{4x\mathbf{a}_x + 4y\mathbf{a}_y + 2z\mathbf{a}_z}{|4x\mathbf{a}_x + 4y\mathbf{a}_y + 2z\mathbf{a}_z|} = \frac{2x\mathbf{a}_x + 2y\mathbf{a}_y + z\mathbf{a}_z}{|2x\mathbf{a}_x + 2y\mathbf{a}_y + z\mathbf{a}_z|}$$

(a) At $(\sqrt{2}, \sqrt{2}, 0)$,

$$\mathbf{a}_n = \frac{2\sqrt{2}\mathbf{a}_x + 2\sqrt{2}\mathbf{a}_y}{|2\sqrt{2}\mathbf{a}_x + 2\sqrt{2}\mathbf{a}_y|} = \frac{\mathbf{a}_x + \mathbf{a}_y}{\sqrt{2}}$$

(b) At $(1, 1, 2)$,

$$\mathbf{a}_n = \frac{2\mathbf{a}_x + 2\mathbf{a}_y + 2\mathbf{a}_z}{|2\mathbf{a}_x + 2\mathbf{a}_y + 2\mathbf{a}_z|} = \frac{\mathbf{a}_x + \mathbf{a}_y + \mathbf{a}_z}{\sqrt{3}}$$

(c) At $(1, \sqrt{2}, \sqrt{2})$,

$$\mathbf{a}_n = \frac{2\mathbf{a}_x + 2\sqrt{2}\mathbf{a}_y + \sqrt{2}\mathbf{a}_z}{|2\mathbf{a}_x + 2\sqrt{2}\mathbf{a}_y + \sqrt{2}\mathbf{a}_z|} = \frac{\sqrt{2}\mathbf{a}_x + 2\mathbf{a}_y + \mathbf{a}_z}{\sqrt{7}}$$

D5.2. **(a)** Maximum rate of increase of Φ_1

$$= |\nabla\Phi_1|_{(3,\,4,\,12)}$$

$$= |2x\mathbf{a}_x + 2y\mathbf{a}_y + 2z\mathbf{a}_z|_{(3,\,4,\,12)}$$

$$= |6\mathbf{a}_x + 8\mathbf{a}_y + 24\mathbf{a}_z|$$

$$= 26$$

(b) Maximum rate of increase of Φ_2

$$= |\nabla\Phi_2|_{(3,\,4,\,12)}$$

$$= |\mathbf{a}_x + 2\mathbf{a}_y + 2\mathbf{a}_z|_{(3,\,4,\,12)}$$

$$= 3$$

(c) Rate of increase of Φ_1 along the direction of the maximum rate of increase of Φ_2

$$= \left[\nabla\Phi_1 \bullet \frac{\nabla\Phi_2}{|\nabla\Phi_2|}\right]_{(3,4,12)}$$

$$= (6\mathbf{a}_x + 8\mathbf{a}_y + 24\mathbf{a}_z) \bullet \frac{(\mathbf{a}_x + 2\mathbf{a}_y + 2\mathbf{a}_z)}{3}$$

$$= \frac{1}{3}(6 + 16 + 48) = 23\frac{1}{3}$$

D5.3. (a) $\nabla^2(x^2yz^3)$

$$= \frac{\partial^2}{\partial x^2}(x^2yz^3) + \frac{\partial^2}{\partial y^2}(x^2yz^3) + \frac{\partial^2}{\partial z^2}(x^2yz^3)$$

$$= 2yz^3 + 6x^2yz$$

(b) $\nabla^2\left(\frac{\sin\phi}{r}\right)$

$$= \frac{1}{r}\frac{\partial}{\partial r}\left[r\frac{\partial}{\partial r}\left(\frac{\sin\phi}{r}\right)\right] + \frac{1}{r^2}\frac{\partial^2}{\partial \phi^2}\left(\frac{\sin\phi}{r}\right) + \frac{\partial^2}{\partial z^2}\left(\frac{\sin\phi}{r}\right)$$

$$= \frac{1}{r^3}\sin\phi - \frac{1}{r^3}\sin\phi = 0$$

(c) $\nabla^2(r^2\cos\theta)$

$$= \frac{1}{r^2}\frac{\partial}{\partial r}\left[r^2\frac{\partial}{\partial r}(r^2\cos\theta)\right] + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left[\sin\theta\frac{\partial}{\partial\theta}(r^2\cos\theta)\right]$$

$$+ \frac{1}{r^2\sin^2\theta}\frac{\partial^2}{\partial\phi^2}(r^2\cos\theta)$$

$$= \frac{1}{r^2}\frac{\partial}{\partial r}(2r^3\cos\theta) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}(-r^2\sin^2\theta)$$

$$= 6\cos\theta - 2\cos\theta$$

$$= 4\cos\theta$$

D5.4. $$V_A - V_B = \int_A^B \mathbf{E}\cdot d\,\mathbf{l}$$

$$= \int_A^B (yz\mathbf{a}_x + zx\mathbf{a}_y + xy\mathbf{a}_z)\cdot(dx\,\mathbf{a}_x + dy\,\mathbf{a}_y + dz\,\mathbf{a}_z)$$

$$= \int_A^B yz\,dx + zx\,dy + xy\,dz$$

$$= \int_A^B d(xyz)$$

$$= [xyz]_A^B$$

(a) For $A(2, 1, 1)$ and $B(1, 4, 0.5)$,

$V_A - V_B = 2 - 2 = 0$

(b) For $A(2, 2, 2)$ and $B(1, 1, 1)$,

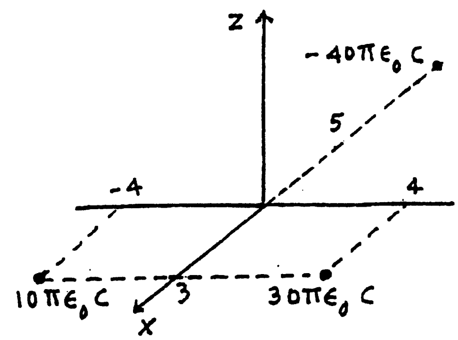
$$V_A - V_B = 1 - 8 = -7 \text{ V}$$

(c) For $A(5, 1, 0.2)$ and $B(1, 2, 3)$,

$$V_A - V_B = 6 - 1 = 5 \text{ V}$$

D5.5. **(a)** All point charges are equidistant from any point on the z-axis. Therefore, the potential is equal to the sum of the three point charges divided by $4\pi\varepsilon_0 R$ where R is the distance from one of the point charges to $(0, 0, 3.2)$.

Thus the potential at $(0, 0, 3.2)$ is zero.



(b) $$V(x, 0, 0) = \frac{1}{4\pi\varepsilon_0}\left(\frac{40\pi\varepsilon_0}{\sqrt{(x-3)^2+16}} - \frac{40\pi\varepsilon_0}{x+5}\right)$$

$$= 10\left(\frac{1}{\sqrt{x^2 - 6x + 25}} - \frac{1}{x+5}\right)$$

Setting $\dfrac{dV}{dx} = 10\left[-\dfrac{x-3}{(x^2-6x+25)^{3/2}} + \dfrac{1}{(x+5)^2}\right] = 0,$

we have

$$\frac{x-3}{(x^2-6x+25)^{3/2}} = \frac{1}{(x+5)^2}$$

Noting that the potential is positive for $x > 0$ and negative for $x < 0$, we look for a solution greater than zero, which satisfies this equation. Using a calculator, or a personal computer, we obtain

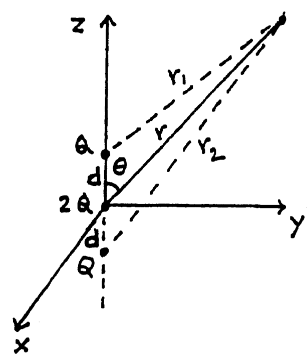
$$x = 3.872 \text{ m}$$

(c) $$V(3.872, 0, 0) = 10\left[\frac{1}{\sqrt{x^2 - 6x + 25}} - \frac{1}{x+5}\right]_{x=3.872}$$

$$= 1.3155 \text{ V}$$

D5.6. (a) $V = \dfrac{2Q}{4\pi\varepsilon_0 r} + \dfrac{Q}{4\pi\varepsilon_0 r_1} + \dfrac{Q_1}{4\pi\varepsilon_0 r_2}$

$$= \frac{Q}{4\pi\varepsilon_0}\left[\frac{2}{r} + \frac{1}{\sqrt{r^2 + d^2 - 2rd\cos\theta}} + \frac{1}{\sqrt{r^2 + d^2 + 2rd\cos\theta}}\right]$$



$$= \frac{Q}{4\pi\varepsilon_0}\left[\frac{2}{r} + \frac{1}{r}\left(1 + \frac{d^2}{r^2} - \frac{2d}{r}\cos\theta\right)^{-1/2} + \frac{1}{r}\left(1 + \frac{d^2}{r^2} + \frac{2d}{r}\cos\theta\right)^{-1/2}\right]$$

$$= \frac{Q}{4\pi\varepsilon_0 r}\left\{2 + \left[1 - \frac{1}{2}\left(\frac{d^2}{r^2} - \frac{2d}{r}\cos\theta\right) + \frac{3}{8}\left(\frac{d^2}{r^2} - \frac{2d}{r}\cos\theta\right)^2 - \cdots\right] + \left[1 - \frac{1}{2}\left(\frac{d^2}{r^2} + \frac{2d}{r}\cos\theta\right) + \frac{3}{8}\left(\frac{d^2}{r^2} + \frac{2d}{r}\cos\theta\right)^2 - \cdots\right]\right\}$$

$$= \frac{Q}{4\pi\varepsilon_0 r}\left\{2 + 2 + \text{terms involving powers of } \frac{d}{r}\right\}$$

$$\approx \frac{Q}{\pi\varepsilon_0 r}$$

(b) $V = \dfrac{Q}{4\pi\varepsilon_0 r_1} + \dfrac{Q}{4\pi\varepsilon_0 r_2} - \dfrac{2Q}{4\pi\varepsilon_0 r}$

$$= \frac{Q}{4\pi\varepsilon_0 r}\left\{-2 + \left[1 - \frac{1}{2}\left(\frac{d^2}{r^2} - \frac{2d}{r}\cos\theta\right) + \frac{3}{8}\left(\frac{d^2}{r^2} - \frac{2d}{r}\cos\theta\right)^2 - \cdots\right] + \left[1 - \frac{1}{2}\left(\frac{d^2}{r^2} + \frac{2d}{r}\cos\theta\right) + \frac{3}{8}\left(\frac{d^2}{r^2} + \frac{2d}{r}\cos\theta\right)^2 - \cdots\right]\right\}$$

$$= \frac{Q}{4\pi\varepsilon_0 r}\left\{-2 + 2 - \frac{d^2}{r^2} + \frac{3d^2}{r^2}\cos^2\theta + \text{terms involving powers of } \frac{d}{r}\right\}$$

$$\approx \frac{Qd^2}{4\pi\varepsilon_0 r^3}\,(3\cos^2\theta - 1)$$

D5.7. $V = V_0\left(\frac{x}{d}\right)^{4/3}$ for $0 < x < d$

(a) $[V]_{x=d/8} = V_0\left(\frac{1}{8}\right)^{4/3} = V_0\left(\frac{1}{2}\right)^{4} = \frac{V_0}{16}$

(b) $\mathbf{E} = -\nabla V = -\frac{\partial V}{\partial x}\mathbf{a}_x = -\frac{4V_0}{3}\left(\frac{x}{d}\right)^{1/3}\left(\frac{1}{d}\right)\mathbf{a}_x = -\frac{4V_0}{3d}\left(\frac{x}{d}\right)^{1/3}\mathbf{a}_x$

$[\mathbf{E}]_{x=d/8} = -\frac{4V_0}{3d}\left(\frac{1}{8}\right)^{1/3} = -\frac{2V_0}{3d}\mathbf{a}_x$

(c) $\rho = -\varepsilon_0\nabla^2 V = -\varepsilon_0\frac{\partial^2 V}{\partial x^2} = -\varepsilon_0\frac{\partial}{\partial x}\left[\frac{4V_0}{3d}\left(\frac{x}{d}\right)^{1/3}\right] = -\varepsilon_0\frac{4V_0}{9d^2}\left(\frac{x}{d}\right)^{-2/3}$

$[\rho]_{x=d/8} = -\frac{4\varepsilon_0 V_0}{9d^2}\left(\frac{1}{8}\right)^{-2/3} = -\frac{4\varepsilon_0 V_0}{9d^2}(64)^{1/3} = -\frac{16\varepsilon_0 V_0}{9d^2}$

(d) $[\rho_S]_{x=d} = -\mathbf{a}_x \bullet [\varepsilon_0\mathbf{E}]_{x=d} = -\mathbf{a}_x \bullet \left(-\frac{4\varepsilon_0 V_0}{3d}\mathbf{a}_x\right) = \frac{4\varepsilon_0 V_0}{3d}$

D5.8. (a) Capacitance per unit area $= \frac{\varepsilon}{d}$

$\therefore\ \frac{2.25\varepsilon_0}{d} = 10^{-9}$

$d = \frac{2.25\varepsilon_0}{10^{-9}} = \frac{2.25\times 10^{-9}}{36\pi\times 10^{-9}} = 0.0199\ \text{m} = 1.99\ \text{cm}$

(b) Capacitance per unit length $= \frac{2\pi\varepsilon}{\ln\frac{b}{a}}$

$\therefore\ \frac{2\pi\times 2.25\varepsilon_0}{\ln\frac{b}{a}} = 10^{-10}$

$\ln\frac{b}{a} = \frac{2\pi\times 2.25\times 10^{-9}}{36\pi\times 10^{-10}} = 1.25$

$\frac{b}{a} = 3.4903$

(c) $C = \lim_{b\to\infty}\frac{4\pi\varepsilon_0}{\left(\frac{1}{a}-\frac{1}{b}\right)} = 4\pi\varepsilon_0 a = 10^{-11}$

$a = \frac{10^{-11}}{4\pi\varepsilon_0} = \frac{36\pi\times 10^{-11}}{4\pi\times 10^{-9}} = 0.09\ \text{m} = 9\ \text{cm}$

D5.9. (a) $\mathcal{C} = \dfrac{2\pi\varepsilon}{\ln \frac{b}{a}} = \dfrac{2\pi\varepsilon}{\ln 3}$

$$\therefore \quad \frac{2\pi\varepsilon}{\cosh^{-1}\left(\dfrac{a^2 + b^2 - d^2}{2ab}\right)} = \frac{2.5\pi\varepsilon}{\ln 3}$$

$$\cosh^{-1}\left(\frac{a^2 + b^2 - d^2}{2ab}\right) = 0.8 \ln 3 = 0.8789$$

$$\frac{a^2 + b^2 - d^2}{2ab} = \frac{1}{2}(e^{0.8789} + e^{-0.8789}) = 1.4117$$

$$\frac{1 + 9 - d^2}{6} = 1.4117$$

$$10 - d^2 = 8.4704$$

$$d^2 = 10 - 8.4704 = 1.5296$$

$$d = 1.2368 \text{ cm}$$

(b) Since $\mathcal{LC} = \mu\varepsilon$, if $\mathcal{L}$ becomes 1.25 times its original value, $\mathcal{C}$ becomes 0.8 times its original value.

$\therefore$ Percentage change in $\mathcal{C} = -20$.

D5.10. (a) $$N = \frac{\displaystyle\int_{r=0}^{0.8a}\int_{\phi=0}^{2\pi} \frac{I_0 e}{\pi a^2}(1 - e^{-r^2/a^2})\mathbf{a}_z \bullet r\, dr\, d\phi\, \mathbf{a}_z}{I_0}$$

$$= \frac{2\pi e}{\pi a^2}\left[\frac{r^2}{2} + \frac{a^2 e^{-r^2/a^2}}{2}\right]_0^{0.8a}$$

$$= \frac{e}{a^2}(0.64a^2 + a^2 e^{-0.64} - a^2)$$

$$= 0.4547$$

(b) $N = 1$, since the entire current I_0 flowing in the inner conductor is enclosed.

(c) $$N = \frac{I_0 - \dfrac{I_0}{9\pi a^2}\pi(20.25a^2 - 16a^2)}{I_0}$$

$$= \frac{4.75a^2}{9a^2}$$

$$= 0.5278$$

D5.11. **(a)** Maximum frequency is

$$f = \frac{1}{20\pi\sqrt{\mu\varepsilon}\, l}$$

$$= \frac{1}{20\pi\sqrt{4\pi\times10^{-7}\times(10^{-9}/36\pi)}\times0.1}$$

$$= 47.746 \times 10^{6}\text{ Hz}$$

$$= 47.746\text{ MHz}$$

(b) $L = \mu_0 \dfrac{dl}{w}$

$$= \frac{4\pi\times10^{-7}\times10^{-2}\times0.1}{0.1}$$

$$= 4\pi\times10^{-9}\text{ H}$$

(c) From (5.104), the required ratio is

$$\frac{\omega\sqrt{\mu_0\varepsilon_0}\, l}{\tan \omega\sqrt{\mu_0\varepsilon_0}\, l}$$

$$= \frac{0.1}{\tan 0.1}$$

$$= 0.9967$$

D5.12. $B = 1.5 + 5\times10^{-5}H$ for $1500 \le H \le 3000$

(a) $A = 4\text{ cm}^2$, $l = 30$ cm, $H = 1800$ A/m

$$B = 1.5 + 5\times10^{-5}\times1800 = 1.59$$

$$\mathcal{R} = \frac{NI_0}{\psi} = \frac{Hl}{BA} = \frac{1800\times0.3}{1.59\times4\times10^{-4}}$$

$$= 849{,}057\text{ A-t/Wb}$$

(b) $A = 2\text{ cm}^2$, $l = 20$ cm, $NI = 500$ A-t

$$H = \frac{NI}{l} = \frac{500}{0.2} = 2500$$

$$B = 1.5 + 5\times10^{-5}\times2500 = 1.625$$

$$\mathcal{R} = \frac{NI_0}{\psi} = \frac{NI}{BA} = \frac{500}{1.625\times2\times10^{-4}}$$

$$= 1{,}538{,}462\text{ A-t/Wb}$$

(c) $A = 5\text{ cm}^2$, $l = 25$ cm, $\psi = 8\times10^{-4}$ Wb

$$B = \frac{\psi}{A} = \frac{8 \times 10^{-4}}{5 \times 10^{-4}} = 1.6$$

$$H = \frac{1.6 - 1.5}{5 \times 10^{-5}} = \frac{10^4}{5} = 2000 \text{ A/m}$$

$$\mathcal{R} = \frac{NI_0}{\psi} = \frac{Hl}{\psi} = \frac{2000 \times 0.25}{8 \times 10^{-4}}$$

$$= 625{,}000 \text{ A-t/Wb}$$

D5.13. Reluctance of leg 1, $\mathcal{R}_1 = \dfrac{0.2}{4000 \times 4\pi \times 10^{-7} \times 3 \times 10^{-4}}$

$$= 132{,}629 \text{ A-t/Wb}$$

Reluctance of leg 2, $\mathcal{R}_2 = \dfrac{0.1}{4000 \times 4\pi \times 10^{-7} \times 6 \times 10^{-4}}$

$$= 33{,}157 \text{ A-t/Wb}$$

Reluctance of leg 3, $\mathcal{R}_3$ = Reluctance of core + Reluctance of gap

$$= 132{,}629 + \frac{0.2 \times 10^{-3}}{4\pi \times 10^{-7} \times 3 \times 10^{-4}}$$

$$= 132{,}629 + 530{,}516$$

$$= 663{,}145 \text{ A-t/Wb}$$

(a) Reluctance $= \mathcal{R}_1 + \dfrac{\mathcal{R}_2\mathcal{R}_3}{\mathcal{R}_2 + \mathcal{R}_3}$

$$= 132{,}629 + \frac{33{,}157 \times 663{,}145}{33{,}157 + 663{,}145}$$

$$= 164{,}207 \text{ A-t/Wb}$$

(b) Reluctance $= \mathcal{R}_2 + \dfrac{\mathcal{R}_1\mathcal{R}_3}{\mathcal{R}_1 + \mathcal{R}_3}$

$$= 33{,}157 + \frac{132{,}629 \times 663{,}145}{132{,}629 + 663{,}145}$$

$$= 143{,}681 \text{ A-t/Wb}$$

(c) Reluctance $= \mathcal{R}_3 + \dfrac{\mathcal{R}_1\mathcal{R}_2}{\mathcal{R}_1 + \mathcal{R}_2}$

$$= 663{,}145 + \frac{132{,}629 \times 33{,}157}{132{,}629 + 33{,}157}$$

$$= 689{,}671 \text{ A-t/Wb}$$

D5.14. (a) $\mathbf{F}_e = -\frac{1}{2}\frac{\varepsilon_0 A V^2}{x^2}\,\mathbf{a}_x$

$= -\frac{1}{2}\varepsilon_0 \frac{0.01\times 100}{(0.01)^2}\,\mathbf{a}_x$

$= -5000\varepsilon_0 \mathbf{a}_x$ N

(b) $\mathbf{F}_e = -\frac{1}{2}\frac{4\varepsilon_0 A V^2}{x^2}\,\mathbf{a}_x$

$= -20{,}000\varepsilon_0 \mathbf{a}_x$ N

(c) Assuming thickness of dielectric material to be t (= 0.005, a constant here), we can write

$$W_e = \frac{1}{2}\,\varepsilon\left(\frac{Q}{A\varepsilon}\right)^2 At + \frac{1}{2}\,\varepsilon_0\left(\frac{Q}{A\varepsilon_0}\right)^2 A(x-t)$$

$$\frac{dW_e}{dx} = \frac{1}{2}\frac{Q^2}{A\varepsilon_0}$$

$$\mathbf{F}_e = -\frac{1}{2}\frac{Q^2}{A\varepsilon_0}\,\mathbf{a}_x$$

To compute Q, we find C first:

$$\frac{1}{C} = \frac{0.005}{0.01\varepsilon_0} + \frac{0.005}{0.04\varepsilon_0} = \frac{2.5}{4\varepsilon_0}$$

$$C = 1.6\varepsilon_0$$

$$Q = CV = 16\varepsilon_0$$

$$\mathbf{F}_e = -\frac{1}{2}\times\frac{256\varepsilon_0^2}{0.01\varepsilon_0}\,\mathbf{a}_x$$

$= -12{,}800\varepsilon_0 \mathbf{a}_x$ N

D6.1. $w = 0.2$ m, $d = 0.01$ m

$$\varepsilon = 2.25\varepsilon_0,\ \mu = \mu_0,\ \eta = \sqrt{\frac{\mu_0}{2.25\varepsilon_0}} = \frac{\eta_0}{1.5} = 80\pi\ \Omega$$

(a) $E_x = 300\pi$ V/m

$V = E_x d = 3\pi$ V

$$H_y = \frac{E_x}{\eta} = 3.75 \text{ A/m}$$

$I = H_y w = 0.75$ A

$P = VI = 2.25\pi$ W

(b) $H_y = 7.5$ A/m

$I = H_y w = 1.5$ A

$E_x = \eta H_y = 600\pi$ V/m

$V = E_x d = 6\pi$ V

$P = VI = 9\pi$ W

(c) $V = 4\pi$ V

$$E_x = \frac{V}{d} = 400\pi \text{ V/m}$$

$$H_y = \frac{E_x}{\eta} = 5 \text{ A/m}$$

$I = H_y w = 1$ A

$P = VI = 4\pi$ W

(d) $I = 0.5$ A

$$H_y = \frac{I}{w} = 2.5 \text{ A/m}$$

$E_x = \eta H_y = 200\pi$ V/m

$V = E_x d = 2\pi$ V

$P = VI = \pi$ W

D6.2. **(a)** $$Z_0 = \frac{\eta}{2\pi}\ln\frac{b}{a}$$

$$50 = \frac{120\pi}{\sqrt{2.56} \times 2\pi}\ \ln\frac{b}{a}$$

$$\ln\frac{b}{a}=\frac{4}{3}$$

$$\frac{b}{a}=e^{4/3}=3.794$$

(b) $$Z_0=\frac{\eta}{2\pi}\ln\frac{b}{a}$$

$$75=\frac{120\pi}{\sqrt{2.25}\times 2\pi}\ \ln\frac{b}{a}$$

$$\ln\frac{b}{a}=1.875$$

$$\frac{b}{a}=e^{1.875}=6.521$$

(c) $$Z_0=\frac{\eta}{\pi}\cosh^{-1}\frac{d}{a}$$

$$300=\frac{120\pi}{\pi}\cosh^{-1}\frac{d}{a}$$

$$\cosh^{-1}\frac{d}{a}=2.5$$

$$\frac{d}{a}=\cosh 2.5=6.132$$

D6.3. $V^+=\dfrac{100}{40+60}\times 60=60\text{ V},\ \Gamma_S=\dfrac{40-60}{40+60}=-0.2$

(a) $V(0.5l,\ 1.7\ \mu\text{s})=48\text{ V}$

$$V^+ + V^- = 48$$

$$V^- = -12$$

$$\Gamma_R=\frac{V^-}{V^+}=\frac{-12}{60}=-0.20$$

$$\frac{R_L-60}{R_L+60}=-0.20$$

$$R_L-60=-0.2\,R_L-12$$

$$1.2R_L=48$$

$$R_L=40\ \Omega$$

(b) $V(0.6l,\ 2.8\ \mu\text{s})=76\text{ V}$

$$V^+ + V^- + V^{-+} = 76$$

$$60 + V^- + \Gamma_S V^- = 76$$

$$0.8V^- = 16$$

$$V^- = 20$$

$$\Gamma_R = \frac{V^-}{V^+} = \frac{20}{60} = \frac{1}{3}$$

$$\frac{R_L - 60}{R_L + 60} = \frac{1}{3}$$

$$3R_L - 180 = R_L + 60$$

$$2R_L = 240$$

$$R_L = 120\ \Omega$$

(c) $I(0.3l, 4.4\ \mu s) = 1\text{ A}$

$$\frac{1}{Z_0}(V^+ - V^- + V^{-+} - V^{-+-} + V^{-+-+}) = 1$$

$$60(1 - \Gamma_R + \Gamma_R\Gamma_S - \Gamma_R^2\Gamma_S + \Gamma_R^2\Gamma_S^2) = 60$$

$$-\Gamma_R + \Gamma_R\Gamma_S - \Gamma_R^2\Gamma_S + \Gamma_R^2\Gamma_S^2 = 0$$

$$\Gamma_R(1 - \Gamma_S)(1 + \Gamma_R\Gamma_S) = 0$$

Since $(1 - \Gamma_S) \neq 0$ and $(1 + \Gamma_R\Gamma_S)$ cannot be equal to zero, $\Gamma_R = 0$.

$$\therefore\ R_L = Z_0 = 60\ \Omega$$

(d) $I(0.4l, \infty) = 2.5\text{ A}$

$$\frac{100}{40 + R_L} = 2.5$$

$$100 = 100 + 2.5R_L$$

$$R_L = 0$$

D6.4. $V_{SS}^+ + V_{SS}^- = 30 \qquad (1)$

$$\frac{V_{SS}^+}{75} - \frac{V_{SS}^-}{75} = 1.2$$

$$V_{SS}^+ - V_{SS}^- = 90 \qquad (2)$$

Solving (1) and (2), we have

(a) $V_{SS}^{+} = 60$ V

(b) $V_{SS}^{-} = -30$ V

(c) $I_{SS}^{+} = \frac{60}{75} = 0.8$ A

(d) $I_{SS}^{-} = -\frac{-30}{75} = 0.4$ A

D6.5. $V_L = 50I_L^2$

$$V^+ + V^- = 50\left(\frac{V^+ - V^-}{50}\right)^2 = \frac{1}{50}(V^+ - V^-)^2$$

$$50(V^+ + V^-) = (V^+ - V^-)^2$$

$$(V^-)^2 - (50 + 2V^+)V^- + (V^+)^2 - 50V^+ = 0$$

$$V^- = \frac{(50 + 2V^+) \pm \sqrt{(50 + 2V^+)^2 - 4[(V^+)^2 - 50V^+]}}{2}$$

$$= \frac{(50 + 2V^+) \pm \sqrt{2500 + 400V^+}}{2}$$

$$= (25 + V^+) \pm \sqrt{625 + 100V^+}$$

(a) $V^+ = 36$

$V^- = 61 \pm \sqrt{4225}$

$= 61 \pm 65 = -4$ or 126

$V^- = -4$ V

(b) $V^+ = 50$

$V^- = 75 \pm \sqrt{5625}$

$= 75 \pm 75 = 0$ or 150

$V^- = 0$

(c) $V^+ = 66$

$V^- = 91 \pm \sqrt{7225}$

$= 91 \pm 85 = 6$ or 176

$V^{-} = 6\text{ V}$

D6.6. **(a)** $\Gamma = \frac{1}{5}$

$$\frac{Z_{02} - Z_{01}}{Z_{02} + Z_{01}} = \frac{1}{5}$$

$$4Z_{02} = 6Z_{01}$$

$$\frac{Z_{02}}{Z_{01}} = 1.5$$

(b) $\tau_V = \frac{1}{5}$

$$\frac{2Z_{02}}{Z_{02} + Z_{01}} = \frac{1}{5}$$

$$9Z_{02} = Z_{01}$$

$$\frac{Z_{02}}{Z_{01}} = \frac{1}{9}$$

(c) $V^{-} = \frac{1}{5}V^{++}$

$$\Gamma = \frac{1}{5}\,\tau_V = \frac{1}{5}\,(1 + \Gamma)$$

$$\Gamma = \frac{1}{4}$$

$$\frac{Z_{02} - Z_{01}}{Z_{02} + Z_{01}} = \frac{1}{4}$$

$$3Z_{02} = 5Z_{01}$$

$$\frac{Z_{02}}{Z_{01}} = \frac{5}{3}$$

(d) $I^{-} = \frac{1}{5}\,I^{++}$

$$-\Gamma = \frac{1}{5}\,\tau_C = \frac{1}{5}\,(1 - \Gamma)$$

$$\Gamma = -\frac{1}{4}$$

$$\frac{Z_{02} - Z_{01}}{Z_{02} + Z_{01}} = -\frac{1}{4}$$

$$5Z_{02} = 3Z_{01}$$

$$\frac{Z_{02}}{Z_{01}} = \frac{3}{5}$$

D6.7. For $V_i(t) = \cos \omega t$,

$$V_o(t) = \frac{1}{4}\sum_{n=0}^{\infty}\left(\frac{1}{3}\right)^n \cos \omega(t - 2\times 10^{-6}n - 3\times 10^{-6})$$

$$\bar{V}_o(\omega) = \frac{1}{4}\sum_{n=0}^{\infty}\left(\frac{1}{3}\right)^n e^{-j\omega(2\times 10^{-6}n + 3\times 10^{-6})}$$

$$= \frac{1}{4}e^{-j\omega 3\times 10^{-6}} \sum_{n=0}^{\infty}\left(\frac{1}{3}e^{-j2\times 10^{-6}\omega}\right)^n$$

$$= \frac{\frac{1}{4}e^{-j\omega 3\times 10^{-6}}}{1 - \frac{1}{3}e^{-j2\times 10^{-6}\omega}}$$

$$|\bar{V}_o(\omega)| = \frac{1/4}{\left|1 - \frac{1}{3}e^{-j2\times 10^{-6}\omega}\right|}$$

(a) $\omega = 10^6\pi$

$$|\bar{V}_o| = \frac{1/4}{\left|1 - \frac{1}{3}e^{-j2\pi}\right|} = \frac{1/4}{2/3}$$

$$= \frac{3}{8} = 0.375$$

(b) $\omega = 1.25\times 10^6\pi$

$$|\bar{V}_o| = \frac{1/4}{\left|1 - \frac{1}{3}e^{-j2.5\pi}\right|}$$

$$= \frac{1/4}{\left|1 + j\frac{1}{3}\right|}$$

$$= \frac{0.25}{\sqrt{10/9}} = 0.2372$$

(c) $\omega = 1.5\times 10^6\pi$

$$|\bar{V}_o| = \frac{1/4}{\left|1-\frac{1}{3}e^{-j3\pi}\right|} = \frac{1/4}{4/3}$$

$$= \frac{3}{16} = 0.1875$$

D6.8. Let line 1 be the line from which the (+) wave is incident. Then, effective load for line 1 is $\frac{Z_0}{n}$.

$$\Gamma = \frac{\frac{Z_0}{n} - Z_0}{\frac{Z_0}{n} + Z_0} = \frac{1-n}{1+n}$$

$$\tau_V = 1 + \Gamma = 1 + \frac{1-n}{1+n} = \frac{2}{1+n}$$

$\tau_{V_{\text{eff}}}$ into each of the n lines $= \frac{1}{n}\tau_V = \frac{2}{n(1+n)}$

$$\tau_I = 1 - \Gamma = 1 - \frac{1-n}{1+n} = \frac{2n}{1+n}$$

$\tau_{I_{\text{eff}}}$ into each of the n lines $= \tau_I = \frac{2}{1+n}$

Power reflected into line 1 $= \Gamma^2 P = \left(\frac{1-n}{1+n}\right)^2 P$

Power transmitted into each of the n lines $= \tau_V \tau_{I_{\text{eff}}} P = \frac{4}{(1+n)^2} P$

(a) $n = 2$

Reflected power $= \left(\frac{1-2}{1+2}\right)^2 P = \frac{1}{9}P$

Transmitted power into each of the 2 lines $= \frac{4}{3^2}P = \frac{4}{9}P$

(b) $n = 3$

Reflected power $= \left(\frac{1-3}{1+3}\right)^2 P = \frac{1}{4}P$

Transmitted power into each of the 3 lines $= \frac{4}{4^2}P = \frac{1}{4}P$

(c) $n = 9$

Reflected power $= \left(\frac{1-9}{1+9}\right)^2 P = 0.64\, P$

Transmitted power into each of the 9 lines $= \frac{4}{10^2}P = 0.04P$

D6.9. $V^- = -\frac{V_0}{2} + Ae^{-\frac{Z_0}{L}t} = -10 + Ae^{-\frac{50}{L}t}$

$$\left[\frac{V_0}{2Z_0} - \frac{V^-}{Z_0}\right]_{t=T} = I_L(0-)$$

$$[V^-]_{t\,=\,1\,\mu s} = 50[0.2 - I_L(0-)]$$

$$= 10 - 50I_L(0-)$$

$$10 - 50I_L(0-) = -10 + Ae^{-\frac{50}{L} \times 10^{-6}}$$

$$A = [20 - 50I_L(0-)]\ e^{\frac{50}{L} \times 10^{-6}}$$

$$\therefore\ V^-(l, t) = -10 + [20 - 50I_L(0-)]\,e^{\frac{-50}{L}(t - 10^{-6})}$$

Voltage across the inductor at $t = 2\ \mu$s is

$$V(l, 2\ \mu s) = 10 - 10 + [20 - 50I_L(0-)]\,e^{-\frac{5 \times 10^{-5}}{L}}$$

$$= [20 - 50I_L(0-)]\,e^{-\frac{5 \times 10^{-5}}{L}}$$

(a) $L = 0.1$ mH, $I_L(0-) = 0$

$$V(l, 2\ \mu s) = 20e^{-0.5} = 12.13 \text{ V}$$

(b) $L = 0.1$ mH, $I_L(0-) = 0.05$ A

$$V(l, 2\ \mu s) = (20 - 50 \times 0.05)e^{-0.5}$$

$$= 17.5e^{-0.5} = 10.61 \text{ V}$$

(c) $L = 0.05$ mH, $I_L(0-) = 0.1$ A

$$V(l, 2\ \mu s) = (20 - 50 \times 0.1)e^{-1}$$

$$= 15e^{-1} = 5.52 \text{ V}$$

D6.10. (a) The capacitor behaves initially like a short circuit.

$\therefore\ \Gamma$ at junction is -1.

$V(0, 2\text{ ns}+) = 0$

(b) At $t = \infty$, the capacitor behaves like an open circuit.

$$V(0, \infty) = \frac{20}{50+150} \times 150 = 15 \text{ V}$$

(c) From (a) and (b) and from the time constant at the junction,

$$V(0, t) = 15 - 15e^{-(t - 2 \times 10^{-9}) / (37.5 \times 40 \times 10^{-12})}$$

$$= 15 - 15e^{-(t - 2 \times 10^{-9}) / (1.5 \times 10^{-9})} \text{ for } t > 2 \times 10^{-9}$$

$$\therefore V(0, 3 \text{ ns}) = 15 - 15e^{-2/3}$$

$$= 15 - 7.7013$$

$$= 7.2987 \text{ V}$$

Note that the time constant at the junction is due to $C(= 40 \text{ pF})$ in parallel with $50\ \Omega \parallel 150\ \Omega$ (or $37.5\ \Omega$).

D6.11. **(a)** $V^+(l/2, 0.25\ \mu\text{s}) = V^+(l/4, 0) = 37.5$ V

$V^-(l/2, 0.25\ \mu\text{s}) = V^-(3l/4, 0) = 0$

$V(l/2, 0.25\ \mu\text{s}) = 37.5$ V

(b) $I(l/2, 0.25\ \mu\text{s}) = \dfrac{37.5}{50} - \dfrac{0}{50} = 0.75$ A

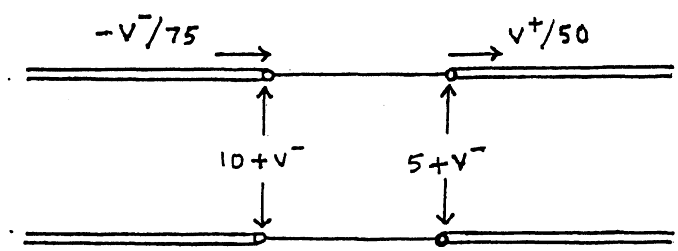
(c) $V^+(l/4, 1\ \mu\text{s}) = V^-(3l/4, 0) = 0$

$V^-(l/4, 1\ \mu\text{s}) = V^+(3l/4, 0) = 25$ V

$V(l/4, 1\ \mu\text{s}) = 25$ V

(d) $I(l/4, 1\ \mu\text{s}) = \dfrac{0}{50} - \dfrac{25}{50} = -\,0.5$ A

D6.12. **(a)** At $t = 0+$



$10 + V^- = 5 + V^+$

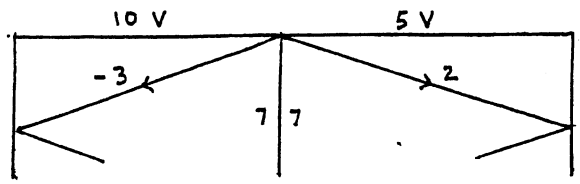
$$-\frac{V^-}{75} = \frac{V^+}{50} \rightarrow V^- = -1.5V^+$$

$$10 - 1.5V^+ = 5 + V^+$$

$$2.5V^+ = 5$$

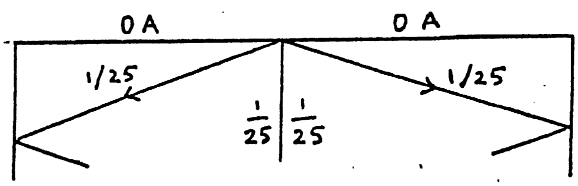
$$V^+ = 2, V^- = -3$$

$$I^+ = \frac{V+}{50} = \frac{1}{25}, I^- = -\frac{V-}{75} = \frac{1}{25}$$



$\therefore$ The required line voltage = 7 V

(b)



The required line current = $\frac{1}{25}$ A = 0.04 A

(c) Energy stored in the system

$$= \frac{1}{2} \times \frac{10^2}{75} \times 10^{-6} + \frac{1}{2} \times \frac{5^2}{50} \times 10^{-6}$$

$$= \left(\frac{2}{3} + \frac{1}{4}\right) \times 10^{-6} \text{ J}$$

$$= \frac{11}{12} \, \mu\text{J}$$

D6.13. The input and output characteristics are

$$12 = 10I_S + V_S$$

$$V_L = 50I_L|I_L|$$

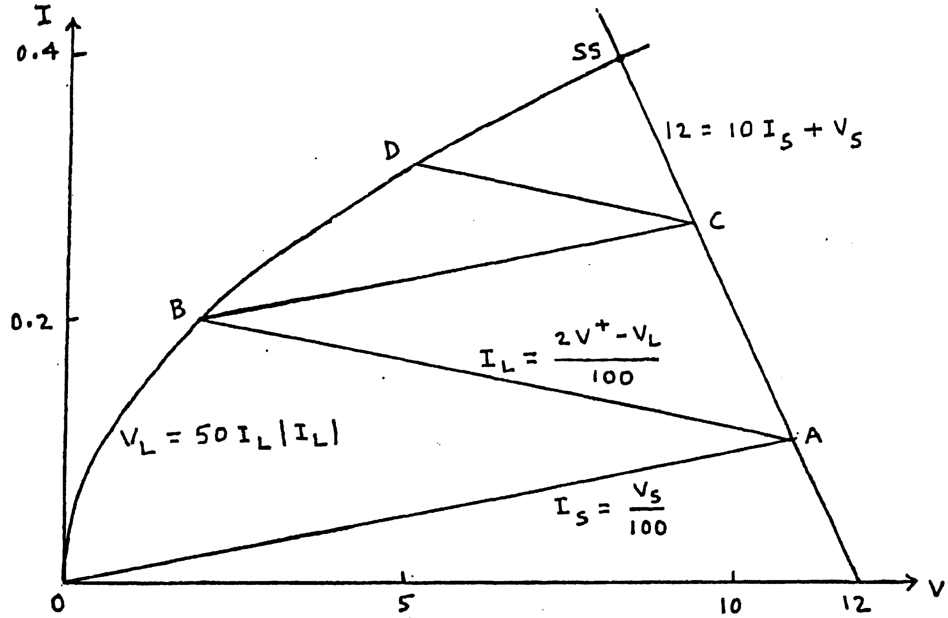
Noting that $Z_0 = 100$ and carrying out the graphical construction as in Fig. 6.48, we obtain

(a) $V_L(t = 2 \, \mu\text{s}) = 2$ V (point B)

(b) $V_S(t = 3 \, \mu\text{s}) = 9.3$ V (point C)

(c) $V_L(t = 4\ \mu s) = 5$ V (point D)

(d) $V_L(t = \infty) = 8$ V (point SS)



D6.14. $\mathcal{L} = 0.9\ \mu$H/m

$C = 40$ pF/m

$\mathcal{L}_m = 0.093\ \mu$H/m

$C_m = 4$ pF/m

$T_0 = 0.2$T

$$Z_0 = \sqrt{\frac{\mathcal{L}}{C}} = \sqrt{\frac{0.9\times10^{-6}}{40\times10^{-12}}} = 150\ \Omega$$

$$v_p = \frac{1}{\sqrt{\mathcal{L}C}} = \frac{1}{\sqrt{0.9\times10^{-6}\times40\times10^{-12}}} = \frac{1}{6}\times10^9 \text{ m/s}$$

(a) $$K_f = \frac{1}{2}\left(C_m Z_0 - \frac{\mathcal{L}_m}{Z_0}\right)$$

$$= \frac{1}{2}\left(4\times10^{-12}\times150 - \frac{0.093\times10^{-6}}{150}\right)$$

$$= -10^{-11} = -0.01 \text{ ns/m}$$

(b) $$K_b = \frac{1}{4}v_p\left(C_m Z_0 + \frac{\mathcal{L}_m}{Z_0}\right)$$

$$= \frac{1}{4}\times\frac{10^9}{6}\left(4\times10^{-12}\times150 + \frac{0.093\times10^{-6}}{150}\right)$$

$= 0.0508$

(c) $V_2^+(0, 0.01T) = 0$

$V_2^-(0, 0.01T) = \frac{0.01}{0.2} K_b V_0 = 0.0025 V_0$

$V_2(0, 0.01T) = 0 + 0.0025 V_0 = 0.0025 V_0$

(d) $V_2^+(l, 1.1T) = l K_f \frac{V_0}{T_0} = v_p T K_f \frac{V_0}{0.2T}$

$= -\frac{10^9}{6} \times 10^{-11} \times \frac{V_0}{0.2}$

$= -0.0083 V_0$

$V_2^-(l, 1.1T) = 0$

$V_2(l, 1.1T) = -0.0083 V_0 + 0 = -0.0083 V_0$

(e) $V_2^+(0.5l, 0.6T) = \frac{0.5 l K_f V_0}{T_0} = -0.0042 V_0$

$V_2^-(0.5l, 0.6T) = \frac{1}{2} K_b V_0 = 0.0254 V_0$

$V_2(0.5l, 0.6T) = -0.0042 V_0 + 0.0254 V_0$

$= 0.0212 V_0$