$$\left[\begin{array}{ccc|c} 2 & 1 & 3 & 1 \\ 1 & 2 & -1 & 0 \\ 1 & -4 & 9 & 2 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & 2 & -1 & 0 \\ 2 & 1 & 3 & 1 \\ 1 & -4 & 9 & 2 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & 2 & -1 & 0 \\ 0 & 3 & -5 & -1 \\ 0 & 6 & -10 & -2 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & 2 & -1 & 0 \\ 0 & 3 & -5 & -1 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & 2 & -1 & 0 \\ 0 & 1 & -5/3 & -1/3 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$A_1 - 2R_2$

$$\left[\begin{array}{ccc|c} 1 & 0 & 7/3 & 2/3 \\ 0 & 1 & -5/3 & -1/3 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$$\begin{cases} x + \frac{7}{3}t = \frac{2}{3} \rightarrow x = \frac{2}{3} - \frac{7}{3}t \\ y - \frac{5}{3}t = \frac{-1}{3} \rightarrow y = \frac{5}{3}t - \frac{1}{3} \\ z = t \end{cases}$$

$$\left[\begin{array}{ccc|c} 2 & 3 & a & b \\ -1 & 2 & c \\ 1 & 3 & -2 & 1 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & 3 & -2 & 1 \\ 0 & +1 & -2 & -c \\ 2 & 3 & a & b \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & 3 & -2 & 1 \\ 0 & 1 & -2 & -c \\ 0 & -3 & a+4 & b-2 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & 3 & -2 & 1 \\ 0 & 1 & -2 & -c \\ 0 & 0 & a-2 & b-3c-2 \end{array}\right]$$

Homogeneous Equation :

$$\left[\begin{array}{cccc|c} 1 & 1 & -1 & 3 & 0 \\ -1 & 4 & 5 & -2 & 0 \\ 1 & 6 & 3 & 4 & 0 \end{array}\right]$$

$$\left[\begin{array}{cccc|c} 1 & 1 & -1 & 3 & 0 \\ 0 & 10 & 8 & 2 & 0 \\ 0 & 5 & 4 & 1 & 0 \end{array}\right]$$

$2R_3 - R_2$

$$\left[\begin{array}{cccc|c} 1 & 1 & -1 & 3 & 0 \\ 0 & 10 & 8 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

$$\left[\begin{array}{cccc|c} 1 & 1 & -1 & 3 & 0 \\ 0 & 1 & \frac{8}{10} & \frac{2}{10} & 0 \\ & & & & 0 \end{array}\right]$$

$$\left[\begin{array}{cccc|c} 1 & 0 & -\frac{9}{5} & \frac{14}{5} & 0 \\ 0 & 1 & \frac{4}{5} & \frac{1}{5} & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

$x_4 = t$, $x_3 = s$

$$x_2 + \frac{4}{5}s + \frac{1}{4}t = 0$$

$$\Rightarrow x_2 = -\frac{4}{5}s - \frac{1}{4}t$$

$$x_1 - \frac{9}{5}s + \frac{14}{5}t = 0$$

$$x_1 = \frac{9}{5}s - \frac{14}{5}t$$

Find the rank of $A = \begin{bmatrix} a & b & 5 \\ 1 & -2 & 1 \end{bmatrix}$

$$= \begin{bmatrix} 1 & -2 & 1 \\ a & b & 5 \end{bmatrix}$$

$R_2 = R_2 - aR_1$

$$= \begin{bmatrix} 1 & -2 & 1 \\ 0 & b+2a & 5-a \end{bmatrix}$$

$$\begin{cases} b+2a=0 \\ 5-a=0 \end{cases} \rightarrow \begin{cases} b=-10 \\ a=5 \end{cases}$$

$$\begin{cases} 2x + y - z + 2w = 1 \\ x + y - 2z - w = -4 \end{cases}$$

$$\left[\begin{array}{cccc|c} 2 & 1 & -1 & 2 & 1 \\ 1 & 1 & -2 & -1 & -4 \end{array}\right]$$

$$\left[\begin{array}{cccc|c} 1 & 1 & -2 & -1 & -4 \\ 2 & 1 & -1 & 2 & 1 \end{array}\right]$$

$R_2 - 2R_1$

$$\left[\begin{array}{cccc|c} 1 & 1 & -2 & -1 & -4 \\ 0 & -1 & 3 & 4 & 9 \end{array}\right]$$

$R_1 + R_2$

$$\left[\begin{array}{cccc|c} 1 & 0 & 1 & 3 & 5 \\ 0 & 1 & -3 & -4 & -9 \end{array}\right]$$

$z = t$
$w = s$
$\Rightarrow y - 3t - 4s = -9$
$\Rightarrow \boxed{y = 3t + 4s - 9}$

$x + t + 3s = 5$
$\rightarrow \boxed{x = 5 - t - 3s}$

$$\begin{cases} 3x - y + 2z - w = 2 \\ y - 2w = -1 \end{cases}$$

$$\left[\begin{array}{cccc|c} 3 & -1 & 2 & -1 & 2 \\ 0 & 1 & 0 & -2 & -1 \end{array}\right]$$

$$\left[\begin{array}{cccc|c} 1 & -\frac{1}{3} & \frac{2}{3} & -\frac{1}{3} & \frac{2}{3} \\ 0 & 1 & 0 & -2 & -1 \end{array}\right]$$

$3R_1 + R_2$

$$\left[\begin{array}{cccc|c} 1 & 0 & 2 & -3 & 1 \\ 0 & 1 & 0 & -2 & -1 \end{array}\right]$$

$z = t$
$w = s$

$y - 2s = -1 \Rightarrow \boxed{y = 2s - 1}$

$x + 2t - 3s = -1$

$\rightarrow x = \boxed{3s - 2t - 1}$

$$
\begin{aligned}
x + 2y + z - w &= 2 \\
x - y + z + w &= 1 \\
2x + y - z &= 1 \\
4x + 2y + z &= 5
\end{aligned}
$$