AP 8.8 $v_c(t) = v_f + e^{-\alpha t}[B_1' \cos \omega_d t + B_2' \sin \omega_d t], \qquad v_f = 100\,\text{V}$

$$v_c(0^+) = 50\,\text{V}; \qquad \frac{dv_c(0^+)}{dt} = 0; \qquad \text{therefore} \quad 50 = 100 + B_1'$$

$$B_1' = -50\,\text{V}; \qquad 0 = -\alpha B_1' + \omega_d B_2'$$

$$\text{Therefore} \quad B_2' = \frac{\alpha}{\omega_d} B_1' = \left(\frac{8000}{6000}\right)(-50) = -66.67\,\text{V}$$

$$\text{Therefore} \quad v_c(t) = 100 - e^{-8000t}[50 \cos 6000t + 66.67 \sin 6000t]\,\text{V}, \qquad t \geq 0$$

Problems

P 8.1 **[a]** $\alpha = \dfrac{1}{2RC} = \dfrac{10^{12}}{(4000)(10)} = 25{,}000$

$$\omega_o^2 = \frac{1}{LC} = \frac{10^{12}}{(250)(10)} = 4 \times 10^8$$

$$s_{1,2} = -25{,}000 \pm \sqrt{625 \times 10^6 - 400 \times 10^6} = -25{,}000 \pm 15{,}000$$

$$s_1 = -10{,}000 \text{ rad/s}$$

$$s_2 = -40{,}000 \text{ rad/s}$$

[b] overdamped

[c] $\omega_d = \sqrt{\omega_o^2 - \alpha^2}$

$$\therefore \quad \alpha^2 = \omega_o^2 - \omega_d^2 = 4 \times 10^8 - 144 \times 10^6 = 256 \times 10^6$$

$$\alpha = 16 \times 10^3 = 16{,}000$$

$$\frac{1}{2RC} = 16{,}000; \qquad \therefore \quad R = \frac{10^9}{(32{,}000)(10)} = 3125\,\Omega$$

[d] $s_1 = -16{,}000 + j12{,}000$ rad/s; $\qquad s_2 = -16{,}000 - j12{,}000$ rad/s

[e] $\alpha = 4 \times 10^4 = \dfrac{1}{2RC}; \qquad \therefore \quad R = \dfrac{1}{2C(4 \times 10^4)} = 2500\,\Omega$

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P 8.2 **[a]** $i_{\rm R}(0) = \dfrac{15}{200} = 75\,\text{mA}$

$i_{\rm L}(0) = -45\,\text{mA}$

$i_{\rm C}(0) = -i_{\rm L}(0) - i_{\rm R}(0) = 45 - 75 = -30\,\text{mA}$

[b] $\alpha = \dfrac{1}{2RC} = \dfrac{1}{2(200)(0.2 \times 10^{-6})} = 12{,}500$

$$\omega_o^2 = \frac{1}{LC} = \frac{1}{(50 \times 10^{-3})(0.2 \times 10^{-6})} = 10^8$$

$$s_{1,2} = -12{,}500 \pm \sqrt{1.5625 \times 10^8 - 10^8} = -12{,}500 \pm 7500$$

$s_1 = -5000$ rad/s; $\qquad s_2 = -20{,}000$ rad/s

$$v = A_1 e^{-5000t} + A_2 e^{-20{,}000t}$$

$$v(0) = A_1 + A_2 = 15$$

$$\frac{dv}{dt}(0) = -5000A_1 - 20{,}000A_2 = \frac{-30 \times 10^{-3}}{0.2 \times 10^{-6}} = -15 \times 10^4\text{V/s}$$

Solving, $\quad A_1 = 10; \quad A_2 = 5$

$$v = 10e^{-5000t} + 5e^{-20{,}000t}\,\text{V}, \qquad t \geq 0$$

[c]
$$\begin{aligned} i_{\rm C} &= C\frac{dv}{dt} \\ &= 0.2 \times 10^{-6}[-50{,}000e^{-5000t} - 100{,}000e^{-20{,}000t}] \\ &= -10e^{-5000t} - 20e^{-20{,}000t}\,\text{mA} \end{aligned}$$

$$i_{\rm R} = 50e^{-5000t} + 25e^{-20{,}000t}\,\text{mA}$$

$$i_{\rm L} = -i_{\rm C} - i_{\rm R} = -40e^{-5000t} - 5e^{-20{,}000t}\,\text{mA}, \quad t \geq 0$$

P 8.3 $\dfrac{1}{2RC} = \dfrac{1}{2(312.5)(0.2 \times 10^{-6})} = 8000$

$$\frac{1}{LC} = \frac{1}{(50 \times 10^{-3})(0.2 \times 10^{-6})} = 10^8$$

$$s_{1,2} = -8000 \pm \sqrt{8000^2 - 10^8} = -8000 \pm j6000\,\text{rad/s}$$

$\therefore$ response is underdamped

$$v(t) = B_1 e^{-8000t}\cos 6000t + B_2 e^{-8000t}\sin 6000t$$

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$$v(0^+) = 15\,\text{V} = B_1; \qquad i_\text{R}(0^+) = \frac{15}{312.5} = 48\,\text{mA}$$

$$i_\text{C}(0^+) = [-i_\text{L}(0^+) + i_\text{R}(0^+)] = -[-45 + 48] = -3\,\text{mA}$$

$$\frac{dv(0^+)}{dt} = \frac{-3 \times 10^{-3}}{0.2 \times 10^{-6}} = -15{,}000\,\text{V/s}$$

$$\frac{dv(0)}{dt} = -8000B_1 + 6000B_2 = -15{,}000$$

$$6000B_2 = 8000(15) - 15{,}000; \qquad \therefore \quad B_2 = 17.5\,\text{V}$$

$$v(t) = 15e^{-8000t}\cos 6000t + 17.5e^{-8000t}\sin 6000t\,\text{V}, \qquad t \geq 0$$

P 8.4 $$\alpha = \frac{1}{2RC} = \frac{1}{2(250)(0.2 \times 10^{-6})} = 10^4$$

$$\alpha^2 = 10^8; \qquad \therefore \quad \alpha^2 = \omega_o^2$$

Critical damping:

$$v = D_1te^{-\alpha t} + D_2e^{-\alpha t}$$

$$i_R(0^+) = \frac{15}{250} = 60\,\text{mA}$$

$$i_C(0^+) = -[i_L(0^+) + i_R(0^+)] = -[-45 + 60] = -15\,\text{mA}$$

$$v(0) = D_2 = 15$$

$$\frac{dv}{dt} = D_1[t(-\alpha e^{-\alpha t}) + e^{-\alpha t}] - \alpha D_2e^{-\alpha t}$$

$$\frac{dv}{dt}(0) = D_1 - \alpha D_2 = \frac{i_\text{C}(0)}{C} = \frac{-15 \times 10^{-3}}{0.2 \times 10^{-6}} = -75{,}000$$

$$D_1 = \alpha D_2 - 75{,}000 = (10^4)(15) - 75{,}000 = 75{,}000$$

$$v = (75{,}000t + 15)e^{-10{,}000t}\,\text{V}, \qquad t \geq 0$$

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P 8.5 [a] $\dfrac{1}{LC} = 5000^2$

There are many possible solutions. This one begins by choosing $L = 10\,\text{mH}$. Then,

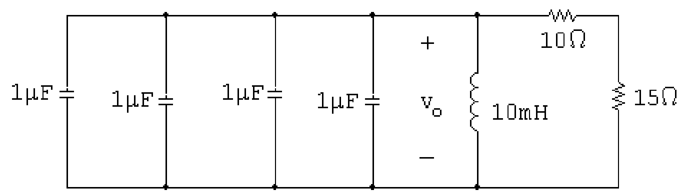
$$C = \frac{1}{(10 \times 10^{-3})(5000)^2} = 4\,\mu\text{F}$$

We can achieve this capacitor value using components from Appendix H by combining four $1\,\mu$F capacitors in parallel.

Critically damped: $\alpha = \omega_0 = 5000$ so $\dfrac{1}{2RC} = 5000$

$$\therefore \quad R = \frac{1}{2(4 \times 10^{-6})(5000)} = 25\,\Omega$$

We can create this resistor value using components from Appendix H by combining a $10\,\Omega$ resistor and a $15\,\Omega$ resistor in series. The final circuit:



[b] $s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2} = -5000 \pm 0$

Therefore there are two repeated real roots at -5000 rad/s.

P 8.6 [a] Underdamped response:

$\alpha < \omega_0$ so $\alpha < 5000$

Therefore we choose a larger resistor value than the one used in Problem 8.5. Choose $R = 100\,\Omega$:

$$\alpha = \frac{1}{2(100)(4 \times 10^{-6})} = 1250$$

$$s_{1,2} = -1250 \pm \sqrt{1250^2 - 5000^2} = -1250 \pm j4841.23 \text{ rad/s}$$

[b] Overdamped response:

$\alpha > \omega_0$ so $\alpha > 5000$

Therefore we choose a smaller resistor value than the one used in Problem 8.5. Choose $R = 20\,\Omega$:

$$\alpha = \frac{1}{2(20)(4 \times 10^{-6})} = 6250$$

$$s_{1,2} = -1250 \pm \sqrt{6250^2 - 5000^2} = -1250 \pm 3750$$

$$= -2500 \text{ rad/s}; \qquad \text{and} \qquad -10{,}000 \text{ rad/s}$$

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P 8.7 **[a]** $\alpha = 8000; \qquad \omega_d = 6000$

$$\omega_d = \sqrt{\omega_o^2 - \alpha^2}$$

$$\therefore \quad \omega_o^2 = \omega_d^2 + \alpha^2 = 36 \times 10^6 + 64 \times 10^6 = 100 \times 10^6$$

$$\frac{1}{LC} = 100 \times 10^6$$

$$C = \frac{1}{(100 \times 10^6)(0.4)} = 25\,\text{nF}$$

[b] $\alpha = \dfrac{1}{2RC}$

$$\therefore \quad R = \frac{1}{2\alpha C} = \frac{1}{(16{,}000)(25 \times 10^{-9})} = 2500\,\Omega$$

[c] $V_o = v(0) = 75\,\text{V}$

[d] $I_o = i_{\text{L}}(0) = -i_{\text{R}}(0) - i_{\text{C}}(0)$

$$i_{\text{R}}(0) = \frac{75}{2500} = 30\,\text{mA}$$

$$i_{\text{C}}(0) = C\frac{dv}{dt}(0) = 25 \times 10^{-9}[6000(-300) - 8000(75)] = -60\,\text{mA}$$

$$\therefore \quad I_o = -30 + 60 = 30\,\text{mA}$$

[e] $i_{\text{C}}(t) = 25 \times 10^{-9}\dfrac{dv(t)}{dt} = e^{-8000t}(48.75\sin 6000t - 60\cos 6000t)\,\text{mA}$

$$i_{\text{R}}(t) = \frac{v(t)}{2500} = e^{-8000t}(30\cos 6000t - 120\sin 6000t)\,\text{mA}$$

$$i_{\text{L}}(t) = -i_{\text{R}}(t) - i_{\text{C}}(t)$$

$$= e^{-8000t}(30\cos 6000t + 71.25\sin 6000t)\,\text{mA}, \quad t \geq 0$$

Check:

$$L\frac{di_{\text{L}}}{dt} = 0.4 \times 10^{-3}e^{-8000t}[187{,}000\cos 6000t - 750{,}000\sin 6000t]$$

$$v(t) = e^{-8000t}[75\cos 6000t - 300\sin 6000t]\,\text{V}$$

P 8.8 **[a]** $-\alpha + \sqrt{\alpha^2 - \omega_o^2} = -250$

$$-\alpha - \sqrt{\alpha^2 - \omega_o^2} = -1000$$

Adding the above equations, $\qquad -2\alpha = -1250$

$$\alpha = 625\,\text{rad/s}$$

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$$\frac{1}{2RC} = \frac{1}{2R(0.1 \times 10^{-6})} = 625$$

$$R = 8\,\text{k}\Omega$$

$$2\sqrt{\alpha^2 - \omega_o^2} = 750$$

$$4(\alpha^2 - \omega_o^2) = 562{,}500$$

$$\therefore \quad \omega_o = 500\,\text{rad/s}$$

$$\omega_o^2 = 25 \times 10^4 = \frac{1}{LC}$$

$$\therefore \quad L = \frac{1}{(25 \times 10^4)(0.1 \times 10^{-6})} = 40\,\text{H}$$

[b] $i_\text{R} = \dfrac{v(t)}{R} = -1e^{-250t} + 4e^{-1000t}\,\text{mA}, \qquad t \geq 0^+$

$$i_\text{C} = C\frac{dv(t)}{dt} = 0.2e^{-250t} - 3.2e^{-1000t}\,\text{mA}, \qquad t \geq 0^+$$

$$i_\text{L} = -(i_\text{R} + i_\text{C}) = 0.8e^{-250t} - 0.8e^{-1000t}\,\text{mA}, \qquad t \geq 0$$

P 8.9 **[a]** $\left(\dfrac{1}{2RC}\right)^2 = \dfrac{1}{LC} = (500)^2$

$$\therefore \quad C = \frac{1}{(500)^2(4)} = 1\,\mu\text{F}$$

$$\frac{1}{2RC} = 500$$

$$\therefore \quad R = \frac{1}{2(500)(10^{-6})} = 1\,\text{k}\Omega$$

$$v(0) = D_2 = 8\,\text{V}$$

$$i_\text{R}(0) = \frac{8}{1000} = 8\text{mA}$$

$$i_\text{C}(0) = -8 + 10 = 2\,\text{mA}$$

$$\frac{dv}{dt}(0) = D_1 - 500D_2 = \frac{2 \times 10^{-3}}{10^{-6}} = 2000\,\text{V/s}$$

$$\therefore \quad D_1 = 2000 + 500(8) = 6000 \ \text{V/s}$$

[b] $v = 6000te^{-500t} + 8e^{-500t}\,\text{V}, \quad t \geq 0$

$$\frac{dv}{dt} = [-3 \times 10^6 t + 2000]e^{-500t}$$

$$i_\text{C} = C\frac{dv}{dt} = (-3000t + 2)e^{-500t}\,\text{mA}, \quad t \geq 0^+$$

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P 8.10 $\alpha = 500/2 = 250$

$$R = \frac{1}{2\alpha C} = \frac{10^6}{(500)(18)} = 1000\,\Omega$$

$$v(0^+) = -11 + 20 = 9\,\text{V}$$

$$i_\text{R}(0^+) = \frac{9}{1000} = 9\,\text{mA}$$

$$\frac{dv}{dt} = 1100e^{-100t} - 8000e^{-400t}$$

$$\frac{dv(0^+)}{dt} = 1100 - 8000 = -6900\,\text{V/s}$$

$$i_\text{C}(0^+) = 2 \times 10^{-6}(-6900) = -13.8\,\text{mA}$$

$$i_\text{L}(0^+) = -[i_\text{R}(0^+) + i_\text{C}(0^+)] = -[9 - 13.8] = 4.8\,\text{mA}$$

P 8.11 **[a]** $2\alpha = 1000; \qquad \alpha = 500\,\text{rad/s}$

$$2\sqrt{\alpha^2 - \omega_o^2} = 600; \qquad \omega_o = 400\,\text{rad/s}$$

$$C = \frac{1}{2\alpha R} = \frac{1}{2(500)(250)} = 4\,\mu F$$

$$L = \frac{1}{\omega_o^2 C} = \frac{1}{(400)^2(4 \times 10^{-6})} = 1.5625\,\text{H}$$

$$i_\text{C}(0^+) = A_1 + A_2 = 45\,\text{mA}$$

$$\frac{di_\text{C}}{dt} + \frac{di_\text{L}}{dt} + \frac{di_\text{R}}{dt} = 0$$

$$\frac{di_\text{C}(0)}{dt} = -\frac{di_\text{L}(0)}{dt} - \frac{di_\text{R}(0)}{dt}$$

$$\frac{di_\text{L}(0)}{dt} = \frac{0}{1.5625} = 0\,\text{A/s}$$

$$\frac{di_\text{R}(0)}{dt} = \frac{1}{R}\frac{dv(0)}{dt} = \frac{1}{R}\frac{i_\text{C}(0)}{C} = \frac{45 \times 10^{-3}}{(250)(4 \times 10^{-6})} = 45\,\text{A/s}$$

$$\therefore \quad \frac{di_\text{C}(0)}{dt} = 0 - 45 = -45\,\text{A/s}$$

$$\therefore \quad 200A_1 + 800A_2 = 45; \qquad A_1 + A_2 = 0.045$$

Solving, $A_1 = -15\,\text{mA}; \qquad A_2 = 60\,\text{mA}$

$$\therefore \quad i_\text{C} = -15e^{-200t} + 60e^{-800t}\,\text{mA}, \qquad t \geq 0^+$$

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[b] By hypothesis

$$v = A_3e^{-200t} + A_4e^{-800t}, \qquad t \geq 0$$

$$v(0) = A_3 + A_4 = 0$$

$$\frac{dv(0)}{dt} = \frac{45 \times 10^{-3}}{4 \times 10^{-6}} = 11{,}250\,\text{V/s}$$

$$-200A_3 - 800A_4 = 11{,}250; \qquad \therefore \quad A_3 = 18.75\,\text{V}; \qquad A_4 = -18.75\,\text{V}$$

$$v = 18.75e^{-200t} - 18.75e^{-800t}\,\text{V}, \qquad t \geq 0$$

[c] $i_{\text{R}}(t) = \dfrac{v}{250} = 75e^{-200t} - 75e^{-800t}\,\text{mA}, \qquad t \geq 0^+$

[d] $i_{\text{L}} = -i_{\text{R}} - i_{\text{C}}$

$$i_{\text{L}} = -60e^{-200t} + 15e^{-800t}\,\text{mA}, \qquad t \geq 0$$

P 8.12 From the form of the solution we have

$$v(0) = A_1 + A_2$$

$$\frac{dv(0^+)}{dt} = -\alpha(A_1 + A_2) + j\omega_d(A_1 - A_2)$$

We know both $v(0)$ and $dv(0^+)/dt$ will be real numbers. To facilitate the algebra we let these numbers be K_1 and K_2, respectively. Then our two simultaneous equations are

$$K_1 = A_1 + A_2$$

$$K_2 = (-\alpha + j\omega_d)A_1 + (-\alpha - j\omega_d)A_2$$

The characteristic determinant is

$$\Delta = \begin{vmatrix} 1 & 1 \\ (-\alpha + j\omega_d) & (-\alpha - j\omega_d) \end{vmatrix} = -j2\omega_d$$

The numerator determinants are

$$N_1 = \begin{vmatrix} K_1 & 1 \\ K_2 & (-\alpha - j\omega_d) \end{vmatrix} = -(\alpha + j\omega_d)K_1 - K_2$$

and $\quad N_2 = \begin{vmatrix} 1 & K_1 \\ (-\alpha + j\omega_d) & K_2 \end{vmatrix} = K_2 + (\alpha - j\omega_d)K_1$

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It follows that $A_1 = \dfrac{N_1}{\Delta} = \dfrac{\omega_d K_1 - j(\alpha K_1 + K_2)}{2\omega_d}$

and $A_2 = \dfrac{N_2}{\Delta} = \dfrac{\omega_d K_1 + j(\alpha K_1 + K_2)}{2\omega_d}$

We see from these expressions that $A_1 = A_2^*$.

P 8.13 By definition, $B_1 = A_1 + A_2$. From the solution to Problem 8.12 we have

$$A_1 + A_2 = \frac{2\omega_d K_1}{2\omega_d} = K_1$$

But K_1 is $v(0)$, therefore, $B_1 = v(0)$, which is identical to Eq. (8.30).
By definition, $B_2 = j(A_1 - A_2)$. From Problem 8.12 we have

$$B_2 = j(A_1 - A_2) = \frac{j[-2j(\alpha K_1 + K_2)]}{2\omega_d} = \frac{\alpha K_1 + K_2}{\omega_d}$$

It follows that

$$K_2 = -\alpha K_1 + \omega_d B_2, \quad \text{but} \quad K_2 = \frac{dv(0^+)}{dt} \quad \text{and} \quad K_1 = B_1.$$

Thus we have

$$\frac{dv}{dt}(0^+) = -\alpha B_1 + \omega_d B_2,$$

which is identical to Eq. (8.31).

P 8.14 **[a]** $\alpha = \dfrac{1}{2RC} = 800$ rad/s

$$\omega_o^2 = \frac{1}{LC} = 10^6$$

$$\omega_d = \sqrt{10^6 - 800^2} = 600 \text{ rad/s}$$

$$\therefore \quad v = B_1 e^{-800t}\cos 600t + B_2 e^{-800t}\sin 600t$$

$$v(0) = B_1 = 30$$

$$i_R(0^+) = \frac{30}{5000} = 6\,\text{mA}; \qquad i_C(0^+) = -12\,\text{mA}$$

$$\therefore \quad \frac{dv}{dt}(0^+) = \frac{-0.012}{125 \times 10^{-9}} = -96{,}000 \text{ V/s}$$

$$-96{,}000 = -\alpha B_1 + \omega_d B_2 = -(800)(30) + 600B_2$$

$$\therefore \quad B_2 = -120$$

$$\therefore \quad v = 30e^{-800t}\cos 600t - 120e^{-800t}\sin 600t\,\text{V}, \qquad t \geq 0$$

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[b] $\dfrac{dv}{dt} = 6000e^{-800t}(13\sin 600t - 16\cos 600t)$

$$\frac{dv}{dt} = 0 \quad \text{when} \quad 16\cos 600t = 13\sin 600t \quad \text{or} \quad \tan 600t = \frac{16}{13}$$

$$\therefore \quad 600t_1 = 0.8885, \qquad t_1 = 1.48\,\text{ms}$$

$$600t_2 = 0.8885 + \pi, \qquad t_2 = 6.72\,\text{ms}$$

$$600t_3 = 0.8885 + 2\pi, \qquad t_3 = 11.95\,\text{ms}$$

[c] $t_3 - t_1 = 10.47\,\text{ms}; \qquad T_d = \dfrac{2\pi}{\omega_d} = \dfrac{2\pi}{600} = 10.47\,\text{ms}$

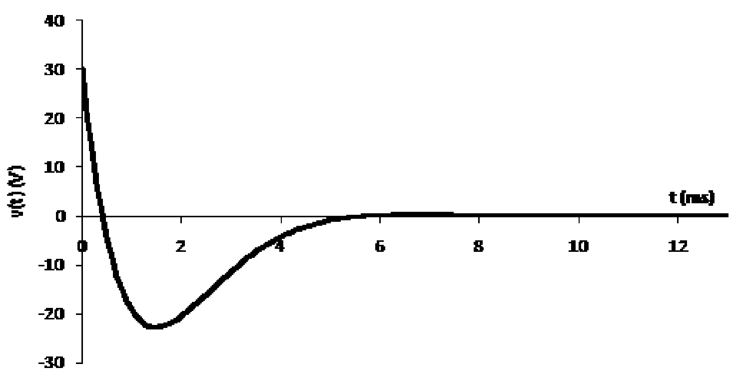
[d] $t_2 - t_1 = 5.24\,\text{ms}; \qquad \dfrac{T_d}{2} = \dfrac{10.48}{2} = 5.24\,\text{ms}$

[e] $v(t_1) = 30e^{-(1.184)}(\cos 0.8885 - 4\sin 0.8885) = -22.7\,\text{V}$

$$v(t_2) = 30e^{-(5.376)}(\cos 4.032 - 4\sin 4.032) = 0.334\,\text{V}$$

$$v(t_3) = 30e^{-(9.56)}(\cos 7.17 - 4\sin 7.17) = -5.22\,\text{mV}$$

[f]



P 8.15 [a] $\alpha = 0; \qquad \omega_d = \omega_o = \sqrt{10^6} = 1000\,\text{rad/s}$

$$v = B_1\cos\omega_o t + B_2\sin\omega_o t; \qquad v(0) = B_1 = 30$$

$$C\frac{dv}{dt}(0) = -i_{\text{L}}(0) = -0.006$$

$$-48{,}000 = -\alpha B_1 + \omega_d B_2 = -0 + 1000B_2$$

$$\therefore \quad B_2 = \frac{-48{,}000}{1000} = -48\,\text{V}$$

$$v = 30\cos 1000t - 48\sin 1000t\,\text{V}, \qquad t \geq 0$$

[b] $2\pi f = 1000; \qquad f = \dfrac{1000}{2\pi} \cong 159.15\,\text{Hz}$

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[c] $\sqrt{30^2 + 48^2} = 56.6\,\text{V}$

P 8.16 [a] $\omega_o^2 = \dfrac{1}{LC} = \dfrac{10^9}{(2.5)(100)} = 4 \times 10^6$

$$\omega_o = 2000 \text{ rad/s}$$

$$\frac{1}{2RC} = 2000; \qquad R = \frac{1}{4000C} = 2500\,\Omega$$

[b] $v(t) = D_1 te^{-5000t} + D_2 e^{-5000t}$

$$v(0) = -15\,\text{V} = D_2$$

$$i_\text{C}(0) = 5 + \frac{15}{2.5} = 11\,\text{mA}$$

$$\frac{dv}{dt}(0) = \frac{i_\text{C}(0)}{C} = \frac{11 \times 10^{-3}}{100 \times 10^{-9}} = 110{,}000$$

$$D_1 - 2000(-15) = 110{,}000 \qquad \text{so} \qquad D_1 = 80{,}000 \text{ V/s}$$

$$\therefore \quad v(t) = (80{,}000t - 15)e^{-2000t}\,\text{V}, \qquad t \geq 0$$

[c] $i_\text{C}(t) = 0$ when $\dfrac{dv}{dt}(t) = 0$

$$\frac{dv}{dt} = (110{,}000 - 160 \times 10^6 t))e^{-2000t}$$

$$\frac{dv}{dt} = 0 \text{ when } 160 \times 10^6 t_1 = 110{,}000; \qquad \therefore \quad t_1 = 687.5\,\mu\text{s}$$

$$v(687.5\mu\text{s}) = (55 - 15)e^{-1.375} = 10.1136\,\text{V}$$

[d] $w(0) = \dfrac{1}{2}(100 \times 10^{-9})(15)^2 + \dfrac{1}{2}(2.5)(0.005)^2 = 42.5\,\mu\text{J}$

$$w(687.5\,\mu\text{s}) = \frac{1}{2}(100 \times 10^{-9})(10.1136)^2 + \frac{1}{2}(2.5)\left(\frac{10.1136}{2500}\right)^2 = 25.571\,\mu\text{J}$$

$$\% \text{ remaining } = \frac{25.571}{42.5}(100) = 60.17\%$$

P 8.17 [a] $\alpha = \dfrac{1}{2RC} = 1250, \qquad \omega_o = 10^3, \qquad$ therefore overdamped

$$s_1 = -500, \qquad s_2 = -2000$$

therefore $\quad v = A_1 e^{-500t} + A_2 e^{-2000t}$

$$v(0^+) = 0 = A_1 + A_2; \qquad \left[\frac{dv(0^+)}{dt}\right] = \frac{i_\text{C}(0^+)}{C} = 98{,}000\,\text{V/s}$$

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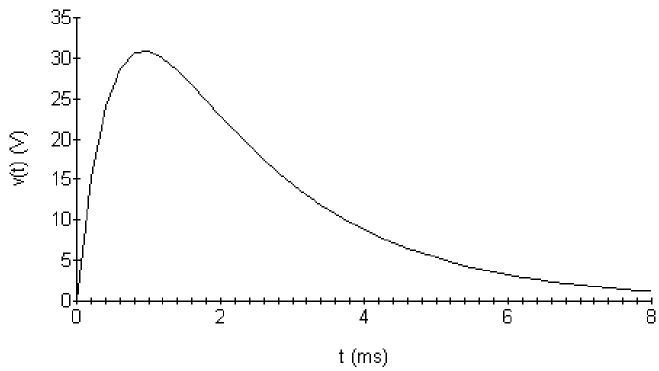
$$\text{Therefore} \quad -500A_1 - 2000A_2 = 98{,}000$$

$$A_1 = \frac{+980}{15}, \quad A_2 = \frac{-980}{15}$$

$$v(t) = \left[\frac{980}{15}\right][e^{-500t} - e^{-2000t}]\,\text{V}, \qquad t \geq 0$$

[**b**]

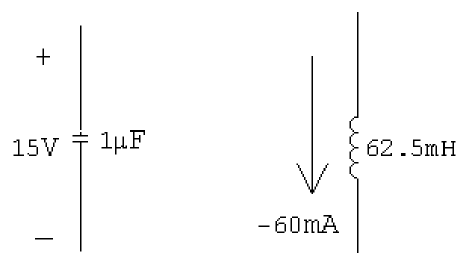


$$\text{Example 8.4:} \quad v_{\text{max}} \cong 74.1\,\text{V} \quad \text{at } 1.4 \text{ ms}$$

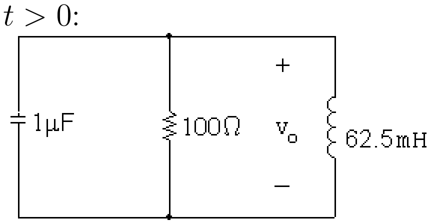
$$\text{Example 8.5:} \quad v_{\text{max}} \cong 36.1\,\text{V} \quad \text{at } 1.0 \text{ ms}$$

$$\text{Problem 8.17:} \quad v_{\text{max}} \cong 30.9 \quad \text{at } 0.92 \text{ ms}$$

P 8.18 $\quad t < 0: \qquad V_o = 15\,\text{V}, \qquad I_o = -60\,\text{mA}$



$t > 0$:



$$i_R(0) = \frac{15}{100} = 150\,\text{mA}; \qquad i_{\text{L}}(0) = -60\,\text{mA}$$

$$i_{\text{C}}(0) = -150 - (-60) = -90\,\text{mA}$$

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$$\alpha = \frac{1}{2RC} = \frac{1}{2(100)(10^{-6})} = 5000 \text{ rad/s}$$

$$\omega_o^2 = \frac{1}{LC} = \frac{1}{(62.5 \times 10^{-3})(10^{-6})} = 16 \times 10^6$$

$$s_{1,2} = -5000 \pm \sqrt{25 \times 10^6 - 16 \times 10^6} = -5000 \pm 3000$$

$$s_1 = -2000 \text{ rad/s}; \qquad s_2 = -8000 \text{ rad/s}$$

$$\therefore \quad v_o = A_1 e^{-2000t} + A_2 e^{-8000t}$$

$$A_1 + A_2 = v_o(0) = 15$$

$$\frac{dv_o}{dt}(0) = -2000A_1 - 8000A_2 = \frac{-90 \times 10^{-3}}{10^{-6}} = -90{,}000$$

Solving, $\qquad A_1 = 5\,\text{V}, \qquad A_2 = 10\,\text{V}$

$$\therefore \quad v_o = 5e^{-2000t} + 10e^{-8000t}\,\text{V}, \qquad t \geq 0$$

P 8.19 $$\omega_o^2 = \frac{1}{LC} = \frac{1}{(62.5 \times 10^{-3})(10^{-6})} = 16 \times 10^6$$

$$\alpha = \frac{1}{2RC} = \frac{1}{2(200)(10^{-6})} = 2500$$

$$s_{1,2} = -2500 \pm \sqrt{2500^2 - 16 \times 10^6} = -2500 \pm j3122.5\text{rad/s}$$

$$v_o(t) = B_1 e^{-2500t} \cos 3122.5t + B_2 e^{-2500t} \sin 3122.5t$$

$$v_o(0) = B_1 = 15\,\text{V}$$

$$i_R(0) = \frac{15}{200} = 75\,\text{mA}$$

$$i_{\text{L}}(0) = -60\,\text{mA}$$

$$i_{\text{C}}(0) = -i_R(0) - i_{\text{L}}(0) = -15\,\text{mA} \quad \therefore \quad \frac{i_{\text{C}}(0)}{C} = -15{,}000$$

$$\frac{dv_o}{dt}(0) = -2500B_1 + 3122.5B_2 = -15{,}000$$

$$\therefore \qquad B_2 = 7.21$$

$$v_o(t) = 15e^{-2500t} \cos 3122.5t + 7.21e^{-2500t} \sin 3122.5t\,\text{V}, \qquad t \geq 0$$

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P 8.20 $\omega_o^2 = \frac{1}{LC} = \frac{1}{(62.5 \times 10^{-3})(10^{-6})} = 16 \times 10^6$

$$\alpha = \frac{1}{2RC} = \frac{1}{2(125)(10^{-6})} = 4000$$

$$\therefore \quad \alpha^2 = \omega_o^2 \text{ (critical damping)}$$

$$v_o(t) = D_1 te^{-4000t} + D_2 e^{-4000t}$$

$$v_o(0) = D_2 = 15\,\text{V}$$

$$i_R(0) = \frac{15}{125} = 120\,\text{mA}$$

$$i_{\text{L}}(0) = -60\,\text{mA}$$

$$i_{\text{C}}(0) = -60\,\text{mA}$$

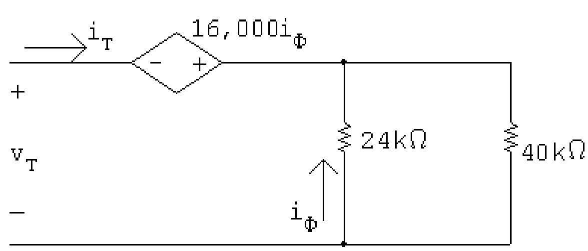
$$\frac{dv_o}{dt}(0) = -4000D_2 + D_1$$

$$\frac{i_{\text{C}}(0)}{C} = \frac{-60 \times 10^{-3}}{10^{-6}} = -60{,}000$$

$$D_1 - 4000D_2 = -60{,}000; \qquad D_1 = 0$$

$$v_o(t) = 15e^{-4000t}\,\text{V}, \qquad t \geq 0$$

P 8.21



$$v_T = -16{,}000 i_\phi + i_T(15{,}000) = -16{,}000 \frac{-i_T(40)}{64} + i_t(15{,}000)$$

$$\frac{v_T}{i_T} = 10{,}000 + 15{,}000 = 25\,\text{k}\Omega$$

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$$V_o = \frac{4000}{5000}(7.5) = 6\,\text{V}; \qquad I_o = 0$$

$$i_{\rm C}(0) = -i_R(0) - i_{\rm L}(0) = -\frac{6}{25{,}000} = -240\,\mu\text{A}$$

$$\frac{i_{\rm C}(0)}{C} = \frac{-240\times 10^{-6}}{4\times 10^{-9}} = -60{,}000$$

$$\omega_o^2 = \frac{1}{LC} = \frac{10^9}{(4)(15.625)} = 16\times 10^6; \qquad \omega_o = 4000\ \text{rad/s}$$

$$\alpha = \frac{1}{2RC} = \frac{10^9}{(2)(4)(25\times 10^3)} = 5000\ \text{rad/s}$$

$\alpha^2 > \omega_0^2$ so the response is overdamped

$$s_{1,2} = -5000 \pm \sqrt{5000^2 - 4000^2} = -5000 \pm 3000\ \text{rad/s}$$

$$v_o = A_1 e^{-2000t} + A_2 e^{-8000t}$$

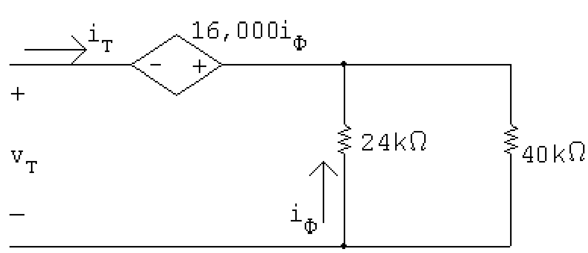
$$v_o(0) = A_1 + A_2 = 6\,\text{V}$$

$$\frac{dv_o}{dt}(0) = -2000A_1 - 8000A_2 = -60{,}000$$

$$\therefore\ A_1 = -2\,\text{V}; \qquad A_2 = 8\,\text{V}$$

$$v_o = 8e^{-8000t} - 2e^{-2000t}\,\text{V}, \qquad t \ge 0$$

P 8.22



$$v_T = -16{,}000 i_\phi + i_T(15{,}000) = -16{,}000\frac{-i_T(40)}{64} + i_t(15{,}000$$

$$\frac{v_T}{i_T} = 10{,}000 + 15{,}000 = 25\,\text{k}\Omega$$

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$$V_o = \frac{4000}{5000}(7.5) = 6\,\text{V}; \qquad I_o = 0$$

$$i_{\text{C}}(0) = -i_R(0) - i_{\text{L}}(0) = -\frac{6}{25{,}000} = -240\,\mu\text{A}$$

$$\frac{i_{\text{C}}(0)}{C} = \frac{-240 \times 10^{-6}}{4 \times 10^{-9}} = -60{,}000$$

$$\omega_o^2 = \frac{1}{LC} = \frac{10^9}{(4)(10)} = 25 \times 10^6; \qquad \omega_o = 5000 \text{ rad/s}$$

$$\alpha = \frac{1}{2RC} = \frac{10^9}{(2)(4)(25 \times 10^3)} = 5000 \text{ rad/s}$$

$\alpha^2 = \omega_0^2$ so the response is critically damped

$$v_o = D_1 t e^{-5000t} + D_2 e^{-5000t}$$

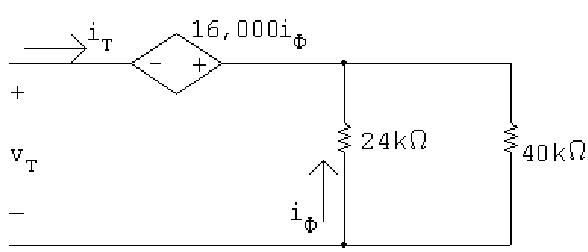
$$v_o(0) = D_2 = 6\,\text{V}$$

$$\frac{dv_o}{dt}(0) = D_1 - \alpha D_2 = -60{,}000$$

$$\therefore \quad D_1 = -60{,}000 + (5000)(6) = -30{,}000 \text{ V/s}$$

$$v_o = -30{,}000 t e^{-5000t} + 6e^{-5000t}\,\text{V}, \qquad t \geq 0$$

P 8.23



$$v_T = -16{,}000 i_\phi + i_T(15{,}000) = -16{,}000\frac{-i_T(40)}{64} + i_t(15{,}000$$

$$\frac{v_T}{i_T} = 10{,}000 + 15{,}000 = 25\,\text{k}\Omega$$

$$V_o = \frac{4000}{5000}(7.5) = 6\,\text{V}; \qquad I_o = 0$$

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$$i_{\rm C}(0) = -i_R(0) - i_{\rm L}(0) = -\frac{6}{25{,}000} = -240\,\mu\text{A}$$

$$\frac{i_{\rm C}(0)}{C} = \frac{-240 \times 10^{-6}}{4 \times 10^{-9}} = -60{,}000$$

$$\omega_o^2 = \frac{1}{LC} = \frac{10^9}{(4)(6.4)} = 6250^2; \qquad \omega_o = 6250 \text{ rad/s}$$

$$\alpha = \frac{1}{2RC} = \frac{10^9}{(2)(4)(25 \times 10^3)} = 5000 \text{ rad/s}$$

$\alpha^2 < \omega_0^2$ so the response is underdamped

$$\omega_d = \sqrt{6250^2 - 5000^2} = 3750 \text{ rad/s}$$

$$v_o = B_1 e^{-5000t} \cos 3750t + B_2 e^{-5000t} \sin 3750t$$

$$v_o(0) = B_1 = 6\,\text{V}$$

$$\frac{dv_o}{dt}(0) = -5000B_1 + 3750B_2 = -60{,}000$$

$$\therefore \quad B_2 = -8\,\text{V}$$

$$v_o = e^{-5000t}(6\cos 3750t - 8\sin 3750t)\,\text{V}, \qquad t \geq 0$$

P 8.24 **[a]** $v = L\left(\frac{di_{\rm L}}{dt}\right) = 16[e^{-20{,}000t} - e^{-80{,}000t}]\,\text{V}, \qquad t \geq 0$

[b] $i_{\rm R} = \frac{v}{R} = 40[e^{-20{,}000t} - e^{-80{,}000t}]\,\text{mA}, \qquad t \geq 0^+$

[c] $i_{\rm C} = I - i_{\rm L} - i_{\rm R} = [-8e^{-20{,}000t} + 32e^{-80{,}000t}]\,\text{mA}, \qquad t \geq 0^+$

P 8.25 **[a]** $v = L\left(\frac{di_{\rm L}}{dt}\right) = 40e^{-32{,}000t}\sin 24{,}000t\,\text{V}, \qquad t \geq 0$

[b] $i_{\rm C}(t) = I - i_{\rm R} - i_{\rm L} = 24 \times 10^{-3} - \frac{v}{625} - i_{\rm L}$

$$= [24e^{-32{,}000t}\cos 24{,}000t - 32e^{-32{,}000t}\sin 24{,}000t]\,\text{mA}, \qquad t \geq 0^+$$

P 8.26 $v = L\left(\frac{di_{\rm L}}{dt}\right) = 960{,}000te^{-40{,}000t}\,\text{V}, \qquad t \geq 0$

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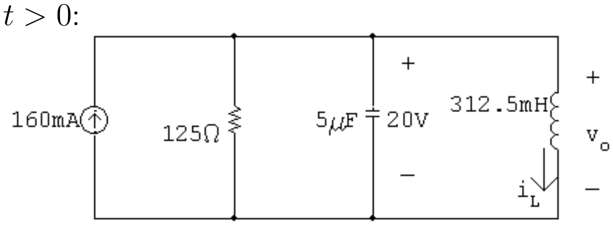
P 8.27 $t < 0$:

$$v_o(0^-) = v_o(0^+) = \frac{625}{781.25}(25) = 20\,\text{V}$$

$$i_\text{L}(0^-) = i_\text{L}(0^+) = 0$$

$t > 0$:



$$-160 \times 10^{-3} + \frac{20}{125} + i_\text{C}(0^+) + 0 = 0; \qquad \therefore \quad i_\text{C}(0^+) = 0$$

$$\frac{1}{2RC} = \frac{1}{2(125)(5 \times 10^{-6})} = 800\,\text{rad/s}$$

$$\omega_o^2 = \frac{1}{LC} = \frac{1}{(312.5 \times 10^{-3})(5 \times 10^{-6})} = 64 \times 10^4$$

$\therefore \quad \alpha^2 = \omega_o^2 \quad$ critically damped

[a] $v_o = V_f + D_1' te^{-800t} + D_2' e^{-800t}$

$$V_f = 0$$

$$\frac{dv_o(0)}{dt} = -800D_2' + D_1' = 0$$

$$v_o(0^+) = 20 = D_2'$$

$$D_1' = 800D_2' = 16{,}000\,\text{V/s}$$

$$\therefore \quad v_o = 16{,}000te^{-800t} + 20e^{-800t}\,\text{V}, \quad t \geq 0^+$$

[b] $i_\text{L} = I_f + D_3' te^{-800t} + D_4' e^{-800t}$

$$i_\text{L}(0^+) = 0; \qquad I_f = 160\,\text{mA}; \qquad \frac{di_\text{L}(0^+)}{dt} = \frac{20}{312.5 \times 10^{-3}} = 64\ \text{A/s}$$

$$\therefore \quad 0 = 160 + D_4'; \qquad D_4' = -160\,\text{mA};$$

$$-800D_4' + D_3' = 64; \qquad D_3' = -64\,\text{A/s}$$

$$\therefore \quad i_\text{L} = 160 - 64{,}000te^{-800t} - 160e^{-800t}\,\text{mA} \qquad t \geq 0$$

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P 8.28 **[a]** $w_L = \int_0^\infty p\,dt = \int_0^\infty v_o i_L\,dt$

$$v_o = 16{,}000te^{-800t} + 20e^{-800t}\,\text{V}$$

$$i_L = 0.16 - 64te^{-800t} - 0.16e^{-800t}\,\text{A}$$

$$p = 3.2e^{-800t} + 2560te^{-800t} - 3840te^{-1600t}$$
$$-1{,}024{,}000t^2e^{-1600t} - 3.2e^{-1600t}\,\text{W}$$

$$w_L = 3.2\int_0^\infty e^{-800t}\,dt + 2560\int_0^\infty te^{-800t}\,dt - 3480\int_0^\infty te^{-1600t}\,dt$$
$$-\ 1{,}024{,}000\int_0^\infty t^2e^{-1600t}\,dt - 3.2\int_0^\infty e^{-1600t}\,dt$$

$$= 3.2\frac{e^{-800t}}{-800}\Big|_0^\infty + \frac{2560}{(800)^2}e^{-800t}(-2560t-1)\Big|_0^\infty$$
$$-\ \frac{3840}{(1600)^2}e^{-1600t}(-1600t-1)\Big|_0^\infty$$
$$-\ \frac{1{,}024{,}000}{(-1600)^3}e^{-1600t}(1600^2t^2+3200t+2)\Big|_0^\infty$$
$$-\ 3.2\frac{e^{-1600t}}{(-1600)}\Big|_0^\infty$$

All the upper limits evaluate to zero hence

$$w_L = \frac{3.2}{800} + \frac{2560}{800^2} - \frac{3840}{1600^2} - \frac{(1{,}024{,}000)(2)}{1600^3} - \frac{3.2}{1600} = 4\,\text{mJ}$$

Note this value corresponds to the final energy stored in the inductor, i.e.

$$w_L(\infty) = \frac{1}{2}(312.5\times10^{-3})(0.16)^2 = 4\,\text{mJ}.$$

[b] $v = 16{,}000te^{-800t} + 20e^{-800t}\,\text{V}$

$$i_R = \frac{v}{125} = 128te^{-800t} + 0.16e^{-800t}\,\text{A}$$

$$p_R = vi_R = 2{,}048{,}000t^2e^{-1600t} + 5120te^{-1600t} + 3.2e^{-1600t}$$

$$w_R = \int_0^\infty p_R\,dt$$

$$= 2{,}048{,}000\int_0^\infty t^2e^{-1600t}\,dt + 5120\int_0^\infty te^{-1600t}\,dt + 3.2\int_0^\infty e^{-1600t}\,dt$$

$$= \frac{2{,}048{,}000e^{-1600t}}{-1600^3}[1600^2t^2+3200t+2]\Big|_0^\infty +$$
$$\frac{5120e^{-1600t}}{1600^2}(-1600t-1)\Big|_0^\infty + \frac{3.2e^{-1600t}}{(-1600)}\Big|_0^\infty$$

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Since all the upper limits evaluate to zero we have

$$w_R = \frac{2{,}048{,}000(2)}{1600^3} + \frac{5120}{1600^2} + \frac{3.2}{1600} = 5\,\text{mJ}$$

[c] $160 = i_R + i_C + i_L \quad \text{(mA)}$

$$i_R + i_L = 160 + 64{,}000te^{-800t}\,\text{mA}$$

$$\therefore \quad i_C = 160 - (i_R + i_L) = -64{,}000te^{-800t}\,\text{mA} = -64te^{-800t}\,\text{A}$$

$$p_C = vi_C = [16{,}000te^{-800t} + 20e^{-800t}][-64te^{-800t}]$$

$$= -1{,}024{,}000t^2e^{-1600t} - 1280e^{-1600t}$$

$$w_C = -1{,}024{,}000\int_0^\infty t^2e^{-1600t}\,dt - 1280\int_0^\infty te^{-1600t}\,dt$$

$$w_C = \frac{-1{,}024{,}000e^{-1600t}}{-1600^3}[1600^2t^2 + 3200t + 2]\Big|_0^\infty - \frac{1280e^{-1600t}}{1600^2}(-1600t - 1)\Big|_0^\infty$$

Since all upper limits evaluate to zero we have

$$w_C = \frac{-1{,}024{,}000(2)}{1600^3} - \frac{1280(1)}{1600^2} = -1\,\text{mJ}$$

Note this 1 mJ corresponds to the initial energy stored in the capacitor, i.e.,

$$w_C(0) = \frac{1}{2}(5\times10^{-6})(20)^2 = 1\,\text{mJ}.$$

Thus $w_C(\infty) = 0\,\text{mJ}$ which agrees with the final value of $v = 0$.

[d] $i_s = 160\,\text{mA}$

$$p_s(\text{del}) = 160v\,\text{mW}$$

$$= 0.16[16{,}000te^{-800t} + 20e^{-800t}]$$

$$= 3.2e^{-800t} + 2560te^{-800t}\,\text{W}$$

$$w_s = 3.2\int_0^\infty e^{-800t}\,dt + \int_0^\infty 2560te^{-800t}\,dt$$

$$= \frac{3.2e^{-800t}}{-800}\Big|_0^\infty + \frac{2560e^{-800t}}{800^2}(-800t - 1)\Big|_0^\infty$$

$$= \frac{3.2}{800} + \frac{2560}{800} = 8\,\text{mJ}$$

[e] $w_L = 4\,\text{mJ} \quad \text{(absorbed)}$

$$w_R = 5\,\text{mJ} \quad \text{(absorbed)}$$

$$w_C = 1\,\text{mJ} \quad \text{(delivered)}$$

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$$w_S = 8\,\text{mJ} \quad \text{(delivered)}$$

$$\sum w_{\text{del}} = w_{\text{abs}} = 9\,\text{mJ}.$$

P 8.29 $$\omega_o^2 = \frac{1}{LC} = \frac{1}{(50\times10^{-3})(0.2\times10^{-6})} = 10^8; \qquad \omega_o = 10^4\ \text{rad/s}$$

$$\alpha = \frac{1}{2RC} = \frac{1}{2(200)(0.2\times10^{-6})} = 12{,}500\ \text{rad/s} \qquad \therefore\ \text{overdamped}$$

$$s_{1,2} = -12{,}500 \pm \sqrt{(12{,}500)^2 - 10^8} = -12{,}500 \pm 7500\ \text{rad/s}$$

$$s_1 = -5000\ \text{rad/s}; \qquad s_2 = -20{,}000\ \text{rad/s}$$

$$I_f = 60\,\text{mA}$$

$$i_{\text{L}} = 60\times10^{-3} + A_1'e^{-5000t} + A_2'e^{-20{,}000t}$$

$$\therefore\ -45\times10^{-3} = 60\times10^{-3} + A_1' + A_2'; \qquad A_1' + A_2' = -105\times10^{-3}$$

$$\frac{di_{\text{L}}}{dt} = -5000A_1' - 20{,}000A_2' = \frac{15}{0.05} = 300$$

Solving, $A_1' = -120\,\text{mA}$; $A_2' = 15\,\text{mA}$

$$i_{\text{L}} = 60 - 120e^{-5000t} + 15e^{-20{,}000t}\,\text{mA}, \quad t \geq 0$$

P 8.30 $$\alpha = \frac{1}{2RC} = \frac{1}{2(312.5)(0.2\times10^{-6})} = 8000; \qquad \alpha^2 = 64\times10^6$$

$\omega_o = 10^4$ underdamped

$$s_{1,2} = -8000 \pm j\sqrt{8000^2 - 10^8} = -8000 \pm j6000\ \text{rad/s}$$

$$i_{\text{L}} = 60\times10^{-3} + B_1'e^{-8000t}\cos 6000t + B_2'e^{-8000t}\sin 6000t$$

$$-45\times10^{-3} = 60\times10^{-3} + B_1' \qquad \therefore\ B_1' = -105\,\text{mA}$$

$$\frac{di_{\text{L}}}{dt}(0) = -8000B_1' + 6000B_2' = 300$$

$$\therefore\ B_2' = -90\,\text{mA}$$

$$i_{\text{L}} = 60 - 105e^{-8000t}\cos 6000t - 90e^{-8000t}\sin 6000t\,\text{mA}, \quad t \geq 0$$

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P 8.31 $\alpha = \dfrac{1}{2RC} = \dfrac{1}{2(250)(0.2\times10^{-6})} = 10^4$

$\alpha^2 = 10^4 = \omega_o^2 \qquad$ critical damping

$i_{\rm L} = I_f + D_1'te^{-10^4t} + D_2'e^{-10^4t} = 60\times10^{-3} + D_1'te^{-10^4t} + D_2'e^{-10^4t}$

$i_{\rm L}(0) = -45\times10^{-3} = 60\times10^{-3} + D_2'; \qquad \therefore\ D_2' = -105\,\text{mA}$

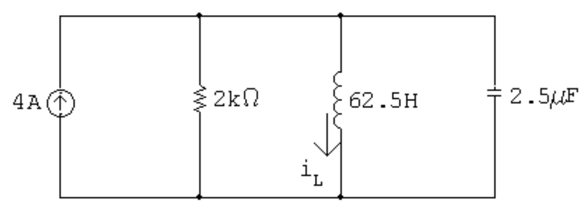
$\dfrac{di_{\rm L}}{dt}(0) = -10^4D_2' + D_1' = 300\,\text{A/s}$

$\therefore\ D_1' = 300 + 10^4(-105\times10^{-3}) = -750\,\text{A/s}$

$i_{\rm L} = 60 - 750{,}000te^{-10^4t} - 105e^{-10^4t}\,\text{mA}, \quad t\geq0$

P 8.32 $t<0: \qquad i_{\rm L}(0^-) = \dfrac{-15}{3000} = -5\,\text{mA}; \qquad v_{\rm C}(0^-) = 0\,\text{V}$

The circuit reduces to:



$i_{\rm L}(\infty) = 4\,\text{mA}$

$\omega_o^2 = \dfrac{1}{LC} = \dfrac{10^6}{(62.5)(2.5)} = 6400; \qquad \omega_o = 80\ \text{rad/s}$

$\alpha = \dfrac{1}{2RC} = \dfrac{10^6}{(4000)(2.5)} = 100$

$s_{1,2} = -100 \pm \sqrt{100^2 - 80^2} = -100 \pm 60$

$s_1 = -40\ \text{rad/s}; \qquad s_2 = -160\ \text{rad/s}$

$i_{\rm L} = I_f + A_1'e^{-40t} + A_2'e^{-160t}$

$i_{\rm L}(\infty) = I_f = 4\text{mA}$

$i_{\rm L}(0) = A_1' + A_2' + I_f = -5\,\text{mA}$

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$\therefore \quad A_1' + A_2' + 4 = -5 \quad \text{so} \quad A_1' + A_2' = -9\,\text{mA}$

$$\frac{di_L}{dt}(0) = 0 = -40A_1 - 160A_2'$$

Solving, $\quad A_1' = -12\,\text{mA}, \qquad A_2' = 3\,\text{mA}$

$$i_L = 4 - 12e^{-40t} + 3e^{-160t}\,\text{mA}, \qquad t \geq 0$$

P 8.33 $\quad v_C(0^+) = \frac{1}{2}(240) = 120\,\text{V}$

$$i_L(0^+) = 60\,\text{mA}; \qquad i_L(\infty) = \frac{240}{5} \times 10^{-3} = 48\,\text{mA}$$

$$\alpha = \frac{1}{2RC} = \frac{10^6}{2(2500)(5)} = 40$$

$$\omega_o^2 = \frac{1}{LC} = \frac{10^6}{400} = 2500$$

$\alpha^2 = 1600; \qquad \alpha^2 < \omega_o^2; \qquad \therefore \quad$ underdamped

$$s_{1,2} = -40 \pm j\sqrt{2500 - 1600} = -40 \pm j30 \text{ rad/s}$$

$$\begin{aligned} i_L &= I_f + B_1'e^{-\alpha t}\cos\omega_d t + B_2'e^{-\alpha t}\sin\omega_d t \\ &= 48 + B_1'e^{-40t}\cos 30t + B_2'e^{-40t}\sin 30t \end{aligned}$$

$$i_L(0) = 48 + B_1'; \qquad B_1' = 60 - 48 = 12\,\text{mA}$$

$$\frac{di_L}{dt}(0) = 30B_2' - 40B_1' = \frac{120}{80} = 1.5 = 1500 \times 10^{-3}$$

$$\therefore \quad 30B_2' = 40(12) \times 10^{-3} + 1500 \times 10^{-3}; \qquad B_2' = 66\,\text{mA}$$

$$\therefore \quad i_L = 48 + 12e^{-40t}\cos 30t + 66e^{-40t}\sin 30t\,\text{mA}, \qquad t \geq 0$$

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P 8.34 $\alpha = \dfrac{1}{2RC} = \dfrac{1}{2(400)(1.25\times 10^{-6})} = 1000$

$$\omega_o^2 = \frac{1}{LC} = \frac{1}{(1.25\times 10^{-6})(1.25)} = 64\times 10^4$$

$$s_{1,2} = -1000 \pm \sqrt{1000^2 - 64\times 10^4} = -1000 \pm 600\ \text{rad/s}$$

$$s_1 = -400\ \text{rad/s}; \qquad s_2 = -1600\ \text{rad/s}$$

$$v_o(\infty) = 0 = V_f$$

$$\therefore\quad v_o = A_1' e^{-400t} + A_2' e^{-1600t}$$

$$v_o(0) = 12 = A_1' + A_2'$$

Note: $\qquad i_C(0^+) = 0$

$$\therefore\quad \frac{dv_o}{dt}(0) = 0 = -400A_1' - 1600A_2'$$

Solving, $\qquad A_1' = 16\,\text{V}, \qquad A_2' = -4\,\text{V}$

$$v_o(t) = 16e^{-400t} - 4e^{-1600t}\,\text{V}, \qquad t \geq 0$$

P 8.35 **[a]** $i_o = I_f + A_1' e^{-400t} + A_2' e^{-1600t}$

$$I_f = \frac{12}{400} = 30\text{mA}; \qquad i_o(0) = 0$$

$$0 = 30\times 10^{-3} + A_1' + A_2', \qquad \therefore\quad A_1' + A_2' = -30\times 10^{-3}$$

$$\frac{di_o}{dt}(0) = \frac{12}{1.25} = -400A_1' - 1600A_2'$$

Solving, $\qquad A_1' = -32\,\text{mA}; \qquad A_2' = 2\,\text{mA}$

$$i_o = 30 - 32e^{-400t} + 2e^{-1600t}\,\text{mA}, \quad t \geq 0$$

[b] $\dfrac{di_o}{dt} = [12.8e^{-400t} - 3.2e^{-1600t}]$

$$v_o = L\frac{di_o}{dt} = 16e^{-400t} - 4e^{-1600t}\,\text{V}, \quad t \geq 0$$

This agrees with the solution to Problem 8.34.

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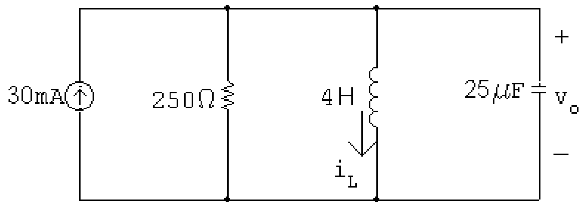
P 8.36 $i_{\rm L}(0^-) = i_{\rm L}(0^+) = \dfrac{7.5}{250} = 30\,{\rm mA}$

For $t > 0$



$$i_{\rm L}(0^-) = i_{\rm L}(0^+) = 30\,{\rm mA}$$

$$\alpha = \frac{1}{2RC} = 80\,{\rm rad/s}; \qquad \omega_o^2 = \frac{1}{LC} = 10^4 \quad {\rm so} \quad \omega_o = 100\ {\rm rad/s}$$

$$\omega_d = \sqrt{100^2 - 80^2} = 60\,{\rm rad/s}$$

$$v_o(\infty) = 0 = V_f; \qquad B_1' = v(0) = 0$$

$$v_o = e^{-80t} B_2' \sin 60t$$

$$i_{\rm C}(0^+) = -30 + 30 + 0 = 0$$

$$\therefore \quad \frac{dv_o}{dt} = 0$$

$$\frac{dv_o}{dt}(0) = -\alpha B_1' + \omega_d B_2' = 0 + 60B_2' = 0$$

$$\therefore \quad B_1' = 0; \qquad B_2' = 0$$

$$\therefore \quad v_o = 0 \text{ for } t \geq 0$$

Note: $\quad v_o(0) = 0; \qquad v_o(\infty) = 0; \qquad \dfrac{dv_o(0)}{dt} = 0$

Hence, the 30 mA current circulates between the current source and the ideal inductor in the equivalent circuit. In the original circuit, the 7.5 V source sustains a current of 30 mA in the inductor. This is an example of a circuit going directly into steady state when the switch is closed. There is no transient period, or interval.

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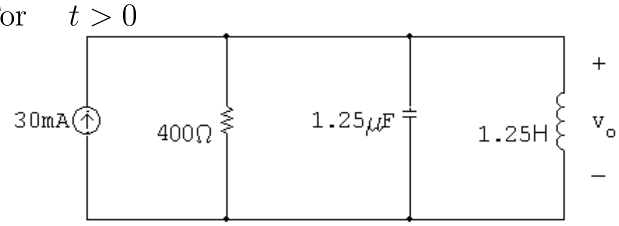
P 8.37 For $t > 0$



$$\alpha = \frac{1}{2RC} = 1000; \qquad \frac{1}{LC} = 64 \times 10^4$$

$$s_{1,2} = -1000 \pm 600 \text{ rad/s}$$

$$s_1 = -400\,\text{rad/s}; \qquad s_2 = -1600\,\text{rad/s}$$

$$v_o = V_f + A_1'e^{-400t} + A_2'e^{-1600t}$$

$$V_f = 0; \qquad v_o(0^+) = 0; \qquad i_C(0^+) = 30\,\text{mA}$$

$$\therefore \quad A_1' + A_2' = 0$$

$$\frac{dv_o(0^+)}{dt} = \frac{i_C(0^+)}{1.25 \times 10^{-6}} = 24{,}000\,\text{V/s}$$

$$\frac{dv_o(0^+)}{dt} = -400A_1' - 1600A_2' = 24{,}000$$

Solving,

$$A_1' = 20\,\text{V}; \qquad A_2' = -20\,\text{V}$$

$$v_o = 20e^{-400t} - 20e^{-1600t}\,\text{V}, \qquad t \geq 0$$

P 8.38 [**a**] From the solution to Prob. 8.37 $s_1 = -400\,\text{rad/s}$ and $s_2 = -1600\,\text{rad/s}$, therefore

$$i_o = I_f + A_1'e^{-400t} + A_2'e^{-1600t}$$

$$I_f = 30\,\text{mA}; \qquad i_o(0^+) = 0; \qquad \frac{di_o(0^+)}{dt} = 0$$

$$\therefore \quad 0 = 30 \times 10^{-3} + A_1' + A_2'; \qquad -400A_1' - 1600A_2' = 0$$

Solving

$$A_1' = -40\,\text{mA}; \qquad A_2' = 10\,\text{mA}$$

$$\therefore \quad i_o = 30 - 40e^{-400t} + 10e^{-1600t}\,\text{mA}, \qquad t \geq 0$$

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[**b**] $\dfrac{di_o}{dt} = 16e^{-400t} - 16e^{-1600t}$

$$v_o = L\frac{di_o}{dt} = 20e^{-400t} - 20e^{-1600t}\,\text{V}, \qquad t \geq 0$$

This agrees with the solution to Problem 8.27.

P 8.39 [**a**] $-\alpha + \sqrt{\alpha^2 - \omega_0^2} = -4000; \qquad -\alpha - \sqrt{\alpha^2 - \omega_0^2} = -16{,}000$

$$\therefore \quad \alpha = 10{,}000 \text{ rad/s}, \qquad \omega_0^2 = 64 \times 10^6$$

$$\alpha = \frac{R}{2L} = 10{,}000; \qquad R = 20{,}000L$$

$$\omega_o^2 = \frac{1}{LC} = 64 \times 10^6; \qquad L = \frac{10^9}{64 \times 10^6(31.25)} = 0.5\,\text{H}$$

$$R = 10{,}000\ \Omega$$

[**b**] $i(0) = 0$

$$L\frac{di(0)}{dt} = v_c(0); \qquad \frac{1}{2}(31.25) \times 10^{-9} v_c^2(0) = 9 \times 10^{-6}$$

$$\therefore \quad v_c^2(0) = 576; \qquad v_c(0) = 24\,\text{V}$$

$$\frac{di(0)}{dt} = \frac{24}{0.5} = 48\,\text{A/s}$$

[**c**] $i(t) = A_1 e^{-4000t} + A_2 e^{-16{,}000t}$

$$i(0) = A_1 + A_2 = 0$$

$$\frac{di(0)}{dt} = -4000A_1 - 16{,}000A_2 = 48$$

Solving,

$$\therefore \quad A_1 = 4\,\text{mA}; \qquad A_2 = -4\,\text{mA}$$

$$i(t) = 4e^{-4000t} - 4e^{-16{,}000t}\,\text{mA}, \qquad t \geq 0$$

[**d**] $\dfrac{di(t)}{dt} = -16e^{-4000t} + 64e^{-16{,}000t}$

$$\frac{di}{dt} = 0 \text{ when } 64e^{-16{,}000t} = 16e^{-4000t}$$

$$\text{or } e^{12{,}000t} = 4$$

$$\therefore \quad t = \frac{\ln 4}{12{,}000} = 115.52\,\mu\text{s}$$

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[e] $i_{\max} = 4e^{-0.4621} - 4e^{-1.8484} = 1.89\,\text{mA}$

[f] $v_L(t) = 0.5\dfrac{di}{dt} = [-8e^{-1000t} + 32e^{-4000t}]\,\text{V}, \quad t \geq 0^+$

P 8.40 [a] $\dfrac{1}{LC} = 20{,}000^2$

There are many possible solutions. This one begins by choosing $L = 1\,\text{mH}$. Then,

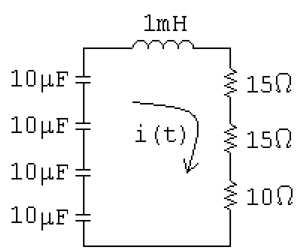
$$C = \frac{1}{(1 \times 10^{-3})(20{,}000)^2} = 2.5\,\mu\text{F}$$

We can achieve this capacitor value using components from Appendix H by combining four $10\,\mu$F capacitors in series.

Critically damped: $\quad \alpha = \omega_0 = 20{,}000 \quad$ so $\quad \dfrac{R}{2L} = 20{,}000$

$$\therefore \quad R = 2(10^{-3})(20{,}000) = 40\,\Omega$$

We can create this resistor value using components from Appendix H by combining a $10\,\Omega$ resistor and two $15\,\Omega$ resistors in series. The final circuit:



[b] $s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2} = -20{,}000 \pm 0$

Therefore there are two repeated real roots at $-20{,}000$ rad/s.

P 8.41 [a] Underdamped response:

$\alpha < \omega_0 \quad$ so $\quad \alpha < 20{,}000$

Therefore we choose a larger resistor value than the one used in Problem 8.40 to give a smaller value of α. For convenience, pick $\alpha = 16{,}000$ rad/s:

$\alpha = \dfrac{R}{2L} = 16{,}000 \quad$ so $\quad R = 2(16{,}000)(10^{-3}) = 32\,\Omega$

We can create a $32\,\Omega$ resistance by combining a $10\,\Omega$ resistor and a $22\,\Omega$ resistor in series.

$$s_{1,2} = -16{,}000 \pm \sqrt{16{,}000^2 - 20{,}000^2} = -16{,}000 \pm j12{,}000 \text{ rad/s}$$

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[**b**] Overdamped response:

$\alpha > \omega_0$ so $\alpha > 20{,}000$

Therefore we choose a smaller resistor value than the one used in Problem 8.40. Choose $R = 50\,\Omega$, which can be created by combining two $100\,\Omega$ resistors in parallel:

$$\alpha = \frac{R}{2L} = 25{,}000$$

$$s_{1,2} = -25{,}000 \pm \sqrt{25{,}000^2 - 20{,}000^2} = -25{,}000 \pm 15{,}000$$

$$= -10{,}000 \text{ rad/s}; \qquad \text{and} \qquad -40{,}000 \text{ rad/s}$$

P 8.42 $\alpha = 2000\,\text{rad/s}; \qquad \omega_d = 1500\,\text{rad/s}$

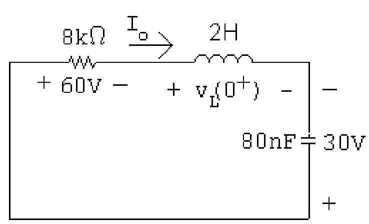
$$\omega_o^2 - \alpha^2 = 225 \times 10^4; \qquad \omega_o^2 = 625 \times 10^4; \quad w_o = 25{,}000\,\text{rad/s}$$

$$\alpha = \frac{R}{2L} = 2000; \qquad R = 4000L$$

$$\frac{1}{LC} = 625 \times 10^4; \qquad L = \frac{1}{(625 \times 10^4)(80 \times 10^{-9})} = 2\,\text{H}$$

$$\therefore \quad R = 8\,\text{k}\Omega$$

$$i(0^+) = B_1 = 7.5\,\text{mA}; \qquad \text{at } t = 0^+$$



$$60 + v_L(0^+) - 30 = 0; \qquad \therefore \qquad v_L(0^+) = -30\,\text{V}$$

$$\frac{di(0^+)}{dt} = \frac{-30}{2} = -15\,\text{A/s}$$

$$\therefore \quad \frac{di(0^+)}{dt} = 1500B_2 - 2000B_1 = -15$$

$$\therefore \quad 1500B_2 = 2000(7.5 \times 10^{-3}) - 15; \qquad \therefore \quad B_2 = 0\,\text{A}$$

$$\therefore \quad i = 7.5e^{-2000t} \sin 1500t\,\text{mA}, \quad t \geq 0$$

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P 8.43 From Prob. 8.42 we know v_c will be of the form

$$v_c = B_3 e^{-2000t}\cos 1500t + B_4 e^{-2000t}\sin 1500t$$

From Prob. 8.42 we have

$$v_c(0) = -30\,\text{V} = B_3$$

and

$$\frac{dv_c(0)}{dt} = \frac{i_\text{C}(0)}{C} = \frac{7.5\times 10^{-3}}{80\times 10^{-9}} = 93.75\times 10^3$$

$$\frac{dv_c(0)}{dt} = 1500B_4 - 2000B_3 = 93{,}750$$

$$\therefore\quad 1500B_4 = 2000(-30) + 93{,}750; \qquad B_4 = 22.5\,\text{V}$$

$$v_c(t) = -30e^{-2000t}\cos 1500t + 22.5e^{-2000t}\sin 1500t\,\text{V} \qquad t \geq 0$$

P 8.44 **[a]** $\omega_o^2 = \dfrac{1}{LC} = \dfrac{10^9}{(125)(0.32)} = 25\times 10^6$

$$\alpha = \frac{R}{2L} = \omega_o = 5000\,\text{rad/s}$$

$$\therefore\quad R = (5000)(2)L = 1250\,\Omega$$

[b] $i(0) = i_\text{L}(0) = 6\,\text{mA}$

$$v_\text{L}(0) = 15 - (0.006)(1250) = 7.5\,\text{V}$$

$$\frac{di}{dt}(0) = \frac{7.5}{0.125} = 60\ \text{A/s}$$

[c] $v_\text{C} = D_1 te^{-5000t} + D_2 e^{-5000t}$

$$v_\text{C}(0) = D_2 = 15\,\text{V}$$

$$\frac{dv_\text{C}}{dt}(0) = D_1 - 5000D_2 = \frac{i_\text{C}(0)}{C} = \frac{-i_\text{L}(0)}{C} = -18{,}750$$

$$\therefore\quad D_1 = 56{,}250\ \text{V/s}$$

$$v_\text{C} = 56{,}250te^{-5000t} + 15e^{-5000t}\,\text{V}, \qquad t \geq 0$$

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P 8.45 $\omega_o^2 = \dfrac{1}{LC} = \dfrac{1}{(10)(4 \times 10^{-3})} = 25$

$$\alpha = \frac{R}{2L} = \frac{80}{2(10)} = 4; \qquad \alpha^2 = 16$$

$$\alpha^2 < \omega_o^2 \quad \therefore \qquad \text{underdamped}$$

$$s_{1,2} = -4 \pm j\sqrt{9} = -4 \pm j3 \text{ rad/s}$$

$$i = B_1 e^{-4t} \cos 3t + B_2 e^{-4t} \sin 3t$$

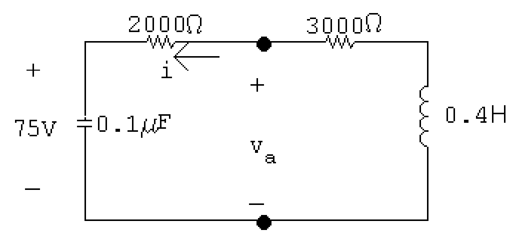
$$i(0) = B_1 = -240/100 = -2.4\,\text{A}$$

$$\frac{di}{dt}(0) = 3B_2 - 4B_1 = 0$$

$$\therefore \quad B_2 = -3.2\,\text{A}$$

$$i = -2.4e^{-4t} \cos 3t - 3.2 \sin 3t\,\text{A}, \qquad t \geq 0$$

P 8.46 **[a]** For $t > 0$:



Since $i(0^-) = i(0^+) = 0$

$v_a(0^+) = 75\,\text{V}$

[b] $v_a = 2000i + 10^7 \displaystyle\int_0^t i\,dx + 75$

$$\frac{dv_a}{dt} = 2000\frac{di}{dt} + 10^7 i$$

$$\frac{dv_a(0^+)}{dt} = 2000\frac{di(0^+)}{dt} + 10^7 i(0^+) = 2000\frac{di(0^+)}{dt}$$

$$-L\frac{di(0^+)}{dt} = 75$$

$$\frac{di(0^+)}{dt} = -2.5(75) = -187.5\,\text{A/s}$$

$$\therefore \quad \frac{dv_a(0^+)}{dt} = -375{,}000\,\text{V/s}$$

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[**c**] $\alpha = \dfrac{R}{2L} = \dfrac{5000}{0.8} = 6250\,\text{rad/s}$

$$\omega_o^2 = \frac{1}{LC} = \frac{10^6}{(0.4)(0.1)} = 25 \times 10^6$$

$$s_{1,2} = -6250 \pm \sqrt{6250^2 - 25 \times 10^6} = -6250 \pm 3750\,\text{rad/s}$$

$$\therefore \quad s_1 = -2500\,\text{rad/s}; \qquad s_2 = -10{,}000\,\text{rad/s}$$

Overdamped:

$$v_a = A_1 e^{-2500t} + A_2 e^{-10{,}000t}$$

$$v_a(0) = A_1 + A_2 = 75\,\text{V}$$

$$\frac{dv_a(0)}{dt} = -2500A_1 - 10{,}000A_2 = -375{,}000; \qquad \therefore \quad A_1 = 50\,\text{V}, \qquad A_2 = 25\,\text{V}$$

$$v_a = 50e^{-2500t} + 25e^{-10{,}000t}\,\text{V}, \quad t \geq 0^+$$

P 8.47 [**a**] $t < 0$:

$$i_o = \frac{80}{800} = 100\,\text{mA}; \qquad v_o = 500i_o = (500)(0.01) = 50\,\text{V}$$

$t > 0$:

$$\alpha = \frac{R}{2L} = \frac{500}{2(2.5 \times 10^{-3})} = 10^5 \text{ rad/s}$$

$$\omega_o^2 = \frac{1}{LC} = \frac{1}{(2.5 \times 10^{-3})(40 \times 10^{-9})} = 100 \times 10^8$$

$$\alpha^2 = \omega_o^2 \quad \therefore \quad \text{critically damped}$$

$$\therefore \quad i_o(t) = D_1 t e^{-10^5 t} + D_2 e^{-10^5 t}$$

$$i_o(0) = D_2 = 100\,\text{mA}$$

$$\frac{di_o}{dt}(0) = -\alpha D_2 + D_1 = 0$$

$$\therefore \quad D_1 = 10^5(100 \times 10^{-3}) = 10{,}000$$

$$i_o(t) = 10{,}000te^{-10^5 t} + 0.1e^{-10^5 t}\,\text{A}, \qquad t \geq 0^+$$

[**b**] $v_o(t) = D_3 t e^{-10^5 t} + D_4 e^{-10^5 t}$

$$v_o(0) = D_4 = 50$$

$$C\frac{dv_o}{dt}(0) = -0.1$$

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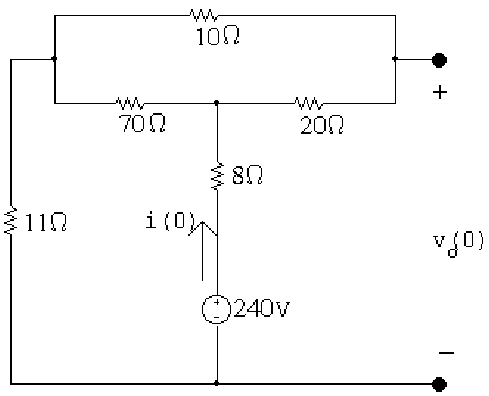
$$\frac{dv_o}{dt}(0) = \frac{-0.1}{40 \times 10^{-9}} = -25 \times 10^5 \text{ V/s} = -\alpha D_4 + D_3$$

$$\therefore \qquad D_3 = 10^5(50) - 25 \times 10^5 = 25 \times 10^5$$

$$v_o(t) = 25 \times 10^5 te^{-10^5 t} + 50e^{-10^5 t} \text{ V}, \quad t \geq 0^+$$

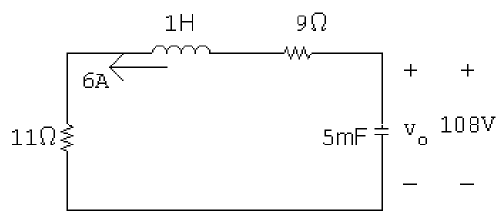
P 8.48 $t < 0$:



$$i(0) = \frac{240}{8 + 30\|70 + 11} = \frac{240}{40} = 6 \text{ A}$$

$$v_o(0) = 240 - 8(6) - \frac{70}{100}(6)(20) = 108 \text{ V}$$

$t > 0$:



$$\alpha = \frac{R}{2L} = \frac{20}{2(1)} = 10, \qquad \alpha^2 = 100$$

$$\omega_o^2 = \frac{1}{LC} = \frac{1}{(1)(5 \times 10^{-3})} = 200$$

$$\omega_o^2 > \alpha^2 \quad \text{underdamped}$$

$$s_{1,2} = -100 \pm \sqrt{100 - 200} = -10 \pm j10 \text{ rad/s}$$

$$v_o = B_1 e^{-10t} \cos 10t + B_2 e^{-10t} \sin 10t$$

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$$v_o(0) = B_1 = 108\,\text{V}$$

$$C\frac{dv_o}{dt}(0) = -6, \qquad \frac{dv_o}{dt} = \frac{-6}{5\times 10^{-3}} = -1200\,\text{V/s}$$

$$\frac{dv_o}{dt}(0) = -10B_1 + 10B_2 = -1200$$

$$10B_2 = -1200 + 10B_1 = -1200 + 1080; \qquad B_2 = -120/10 = -12\,\text{V}$$

$$\therefore \quad v_o = 108e^{-10t}\cos 10t - 12e^{-10t}\sin 10t\,\text{V}, \qquad t \geq 0$$

P 8.49 $i_C(0) = 0; \qquad v_o(0) = 50\,\text{V}$

$$\alpha = \frac{R}{2L} = \frac{8000}{2(160\times 10^{-3})} = 25{,}000\,\text{rad/s}$$

$$\omega_o^2 = \frac{1}{LC} = \frac{1}{(160\times 10^{-3})(10\times 10^{-9})} = 625\times 10^6$$

$$\therefore \quad \alpha^2 = \omega_o^2; \qquad \text{critical damping}$$

$$v_o(t) = V_f + D_1' te^{-25{,}000t} + D_2' e^{-25{,}000t}$$

$$V_f = 250\,\text{V}$$

$$v_o(0) = 250 + D_2' = 50; \qquad D_2' = -200\,V$$

$$\frac{dv_o}{dt}(0) = -25{,}000D_2' + D_1' = 0$$

$$D_1' = 25{,}000D_2' = -5\times 10^6 \text{ V/s}$$

$$v_o = 250 - 5\times 10^6 te^{-25{,}000t} - 200e^{-25{,}000t}\,\text{V}, \quad t \geq 0$$

P 8.50 $\alpha = \dfrac{R}{2L} = 2000\,\text{rad/s}$

$$\omega_o^2 = \frac{1}{LC} = \frac{1}{(62.5\times 10^{-3})(6.25\times 10^{-6})} = 256\times 10^4$$

$$s_{1,2} = -2000 \pm \sqrt{4\times 10^6 - 256\times 10^4} = -2000 \pm j1200\,\text{rad/s}$$

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$$v_o = V_f + A_1' e^{-800t} + A_2' e^{-3200t}$$

$$v_o(0) = 0 = V_f + A_1' + A_2'$$

$$v_o(\infty) = 60\,\text{V}; \qquad \therefore \quad A_1' + A_2' = -60$$

$$\frac{dv_o(0)}{dt} = 0 = -800A_1' - 3200A_2'$$

$$\therefore \quad A_1' = -80\,\text{V}; \qquad\qquad A_2' = 20\,\text{V}$$

$$v_o = 60 - 80e^{-800t} + 20e^{-3200t}\,\text{V}, \quad t \geq 0$$

P 8.51 $\alpha = \dfrac{R}{2L} = 2000\,\text{rad/s}$

$$\omega_o^2 = \frac{1}{LC} = \frac{1}{(62.5 \times 10^{-3})(4 \times 10^{-6})} = 4 \times 10^6 \qquad \therefore \quad \omega_o = 2000 \text{ rad/s}$$

The response is therefore critically damped

$$v_o = V_f + D_1' te^{-2000t} + D_2' e^{-2000t}$$

$$v_o(0) = 0 = V_f + D_2'$$

$$v_o(\infty) = 60\,\text{V}; \qquad \therefore \quad D_2' = -60\,\text{V}$$

$$\frac{dv_o(0)}{dt} = 0 = D_1' - \alpha D_2'$$

$$\therefore \quad D_1' = (2000)(-60) = -120{,}000 \text{ V/s}$$

$$v_o = 60 - 120{,}000te^{-2000t} - 60e^{-2000t}\,\text{V}, \quad t \geq 0$$

P 8.52 $\alpha = \dfrac{R}{2L} = 2000\,\text{rad/s}$

$$\omega_o^2 = \frac{1}{LC} = \frac{1}{(62.5 \times 10^{-3})(2.56 \times 10^{-6})} = 625 \times 10^4 \qquad \therefore \quad \omega_o = 2500 \text{ rad/s}$$

The response is therefore underdamped.

$$\omega_d = \sqrt{2500^2 - 2000^2} = 1500 \text{ rad/s}$$

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$$v_o = V_f + B_1' e^{-2000t} \cos 1500t + B_2' e^{-2000t} \sin 1500t$$

$$v_o(0) = 0 = V_f + B_1'$$

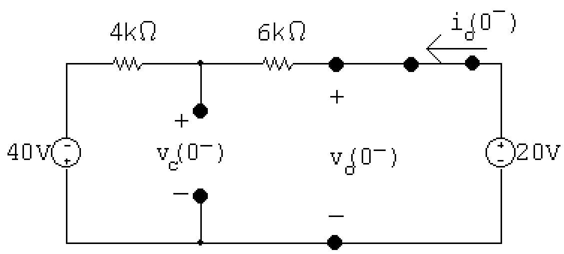
$$v_o(\infty) = 60\,\mathrm{V}; \qquad \therefore \quad B_1' = -60\,\mathrm{V}$$

$$\frac{dv_o(0)}{dt} = 0 = -2000B_1' + 1500B_2'$$

$$\therefore \quad B_2' = -80\,\mathrm{V}$$

$$v_o = \ \mathrm{V}, \quad t \geq 0$$

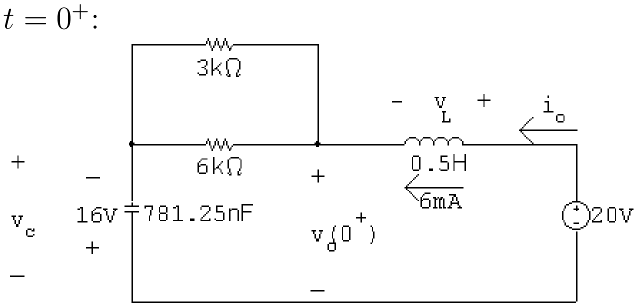
P 8.53 **[a]** $t < 0$:



$$i_o(0^-) = \frac{60}{10{,}000} = 6\,\mathrm{mA}$$

$$v_C(0^-) = 20 - (6000)(0.006) = -16\,\mathrm{V}$$

$t = 0^+$:



$$3\,\mathrm{k}\Omega \| 6\,\mathrm{k}\Omega = 2\,\mathrm{k}\Omega$$

$$\therefore \quad v_o(0^+) = (0.006)(2000) - 16 = 12 - 16 = -4\,\mathrm{V}$$

and $v_L(0^+) = 20 - (-4) = 24\,\mathrm{V}$

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[b] $v_o(t) = 2000i_o + v_{\rm C}$

$$\frac{dv_o}{dt}(t) = 2000\frac{di_o}{dt} + \frac{dv_{\rm C}}{dt}$$

$$\frac{dv_o}{dt}(0^+) = 2000\frac{di_o}{dt}(0^+) + \frac{dv_{\rm C}}{dt}(0^+)$$

$$v_L(0^+) = L\frac{di_o}{dt}(0^+)$$

$$\frac{di_o}{dt}(0^+) = \frac{v_L(0^+)}{L} = \frac{24}{0.5} = 48 \text{ A/s}$$

$$C\frac{dv_c}{dt}(0^+) = i_o(0^+)$$

$$\therefore \quad \frac{dv_c}{dt}(0^+) = \frac{6 \times 10^{-3}}{781.25 \times 10^{-9}} = 7680$$

$$\therefore \quad \frac{dv_o}{dt}(0^+) = 2000(48) + 7680 = 103{,}680 \text{ V/s}$$

[c] $\omega_o^2 = \dfrac{1}{LC} = 2.56 \times 10^6; \qquad \omega_o = 1600 \text{ rad/s}$

$$\alpha = \frac{R}{2L} = 2000 \text{ rad/s}$$

$\alpha^2 > \omega_o^2$ overdamped

$$s_{1,2} = -2000 \pm j1200 \text{ rad/s}$$

$$v_o(t) = V_f + A_1'e^{-800t} + A_2'e^{-3200t}$$

$$V_f = v_o(\infty) = 20\,\text{V}$$

$$20 + A_1' + A_2' = -4; \qquad -800A_1' - 3200A_2' = 103{,}680$$

Solving $A_1' = 11.2; \qquad A_2' = -35.2$

$$\therefore \quad v_o(t) = 20 + 11.2e^{-800t} - 35.2e^{-3200t}\,\text{V}, \qquad t \geq 0^+$$

P 8.54 **[a]** Let i be the current in the direction of the voltage drop $v_o(t)$. Then by hypothesis

$$i = i_f + B_1'e^{-\alpha t}\cos\omega_d t + B_2'e^{-\alpha t}\sin\omega_d t$$

$$i_f = i(\infty) = 0, \qquad i(0) = \frac{V_g}{R} = B_1'$$

Therefore $i = B_1'e^{-\alpha t}\cos\omega_d t + B_2'e^{-\alpha t}\sin\omega_d t$

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$$L\frac{di(0)}{dt} = 0, \qquad \text{therefore} \quad \frac{di(0)}{dt} = 0$$

$$\frac{di}{dt} = [(\omega_d B_2' - \alpha B_1')\cos\omega_d t - (\alpha B_2' + \omega_d B_1')\sin\omega_d t]\, e^{-\alpha t}$$

Therefore $\quad \omega_d B_2' - \alpha B_1' = 0; \qquad B_2' = \frac{\alpha}{\omega_d} B_1' = \frac{\alpha}{\omega_d}\frac{V_g}{R}$

Therefore

$$v_o = L\frac{di}{dt} = -\left\{L\left(\frac{\alpha^2 V_g}{\omega_d R} + \frac{\omega_d V_g}{R}\right)\sin\omega_d t\right\} e^{-\alpha t}$$

$$= -\left\{\frac{LV_g}{R}\left(\frac{\alpha^2}{\omega_d} + \omega_d\right)\sin\omega_d t\right\} e^{-\alpha t}$$

$$= -\frac{V_g L}{R}\left(\frac{\alpha^2 + \omega_d^2}{\omega_d}\right) e^{-\alpha t}\sin\omega_d t$$

$$= -\frac{V_g L}{R}\left(\frac{\omega_o^2}{\omega_d}\right) e^{-\alpha t}\sin\omega_d t$$

$$= -\frac{V_g L}{R\omega_d}\left(\frac{1}{LC}\right) e^{-\alpha t}\sin\omega_d t$$

$$v_o = -\frac{V_g}{RC\omega_d} e^{-\alpha t}\sin\omega_d t\,\text{V}, \quad t \geq 0$$

[b] $\displaystyle \frac{dv_o}{dt} = -\frac{V_g}{\omega_d RC}\{\omega_d\cos\omega_d t - \alpha\sin\omega_d t\}e^{-\alpha t}$

$$\frac{dv_o}{dt} = 0 \quad \text{when} \quad \tan\omega_d t = \frac{\omega_d}{\alpha}$$

Therefore $\quad \omega_d t = \tan^{-1}(\omega_d/\alpha) \quad$ (smallest t)

$$t = \frac{1}{\omega_d}\tan^{-1}\left(\frac{\omega_d}{\alpha}\right)$$

P 8.55 **[a]** From Problem 8.54 we have

$$v_o = \frac{-V_g}{RC\omega_d} e^{-\alpha t}\sin\omega_d t$$

$$\alpha = \frac{R}{2L} = \frac{4800}{2(64 \times 10^{-3})} = 37{,}500\,\text{rad/s}$$

$$\omega_o^2 = \frac{1}{LC} = \frac{1}{(64 \times 10^{-3})(4 \times 10^{-9})} = 3906.25 \times 10^6$$

$$\omega_d = \sqrt{\omega_o^2 - \alpha^2} = 50\,\text{krad/s}$$

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$$\frac{-V_g}{RC\omega_d} = \frac{-(-72)}{(4800)(4 \times 10^{-9})(50 \times 10^3)} = 75$$

$$\therefore \quad v_o = 75e^{-37{,}500t} \sin 50{,}000t \, \text{V}$$

[b] From Problem 8.54

$$t_d = \frac{1}{\omega_d} \tan^{-1}\left(\frac{\omega_d}{\alpha}\right) = \frac{1}{50{,}000} \tan^{-1}\left(\frac{50{,}000}{37{,}500}\right)$$

$$t_d = 18.55 \, \mu\text{s}$$

[c] $v_{\max} = 75e^{-0.0375(18.55)} \sin[(0.05)(18.55)] = 29.93 \, \text{V}$

[d] $R = 480 \, \Omega; \qquad \alpha = 3750 \, \text{rad/s}$

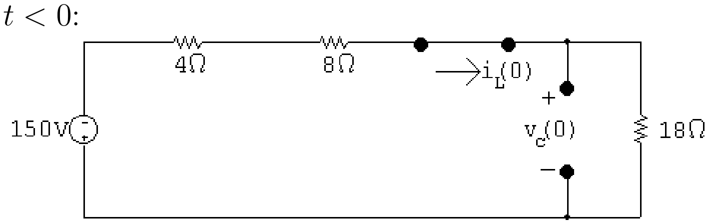
$$\omega_d = 62{,}387.4 \, \text{rad/s}$$

$$v_o = 601.08e^{-3750t} \sin 62{,}387.4t \, \text{V}, \quad t \geq 0$$

$$t_d = 24.22 \, \mu\text{s}$$

$$v_{\max} = 547.92 \, \text{V}$$

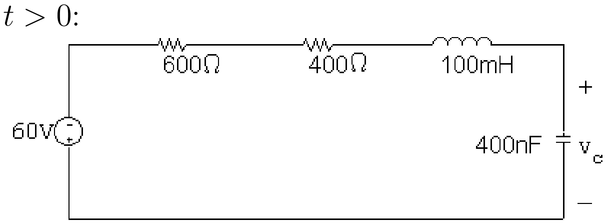
P 8.56 $t < 0$:



$$i_{\text{L}}(0) = \frac{-150}{30} = -5 \, \text{A}$$

$$v_{\text{C}}(0) = 18 i_{\text{L}}(0) = -90 \, \text{V}$$

$t > 0$:



$$\alpha = \frac{R}{2L} = \frac{10}{2(0.1)} = 50 \text{ rad/s}$$

$$\omega_o^2 = \frac{1}{LC} = \frac{1}{(0.1)(2 \times 10^{-3})} = 5000$$

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$\omega_o > \alpha^2 \quad \therefore \qquad \text{underdamped}$

$$s_{1,2} = -50 \pm \sqrt{50^2 - 5000} = -50 \pm j50$$

$$v_c = 60 + B_1' e^{-50t} \cos 50t + B_2' e^{-50t} \sin 50t$$

$$v_c(0) = -90 = 60 + B_1' \quad \therefore \qquad B_1' = -150$$

$$C\frac{dv_c}{dt}(0) = -5; \qquad \frac{dv_c}{dt}(0) = \frac{-5}{2 \times 10^{-3}} = -2500$$

$$\frac{dv_c}{dt}(0) = -50B_1' + 50B_2 = -2500 \quad \therefore \qquad B_2' = -200$$

$$v_c = 60 - 150e^{-50t} \cos 50t - 200e^{-50t} \sin 50t\,\text{V}, \quad t \geq 0$$

P 8.57 **[a]** $v_c = V_f + [B_1' \cos \omega_d t + B_2' \sin \omega_d t]\, e^{-\alpha t}$

$$\frac{dv_c}{dt} = [(\omega_d B_2' - \alpha B_1') \cos \omega_d t - (\alpha B_2' + \omega_d B_1') \sin \omega_d t]e^{-\alpha t}$$

Since the initial stored energy is zero,

$$v_c(0^+) = 0 \quad \text{and} \quad \frac{dv_c(0^+)}{dt} = 0$$

It follows that $\quad B_1' = -V_f \quad$ and $\quad B_2' = \dfrac{\alpha B_1'}{\omega_d}$

When these values are substituted into the expression for $[dv_c/dt]$, we get

$$\frac{dv_c}{dt} = \left(\frac{\alpha^2}{\omega_d} + \omega_d\right) V_f e^{-\alpha t} \sin \omega_d t$$

But $\quad V_f = V \quad$ and $\quad \dfrac{\alpha^2}{\omega_d} + \omega_d = \dfrac{\alpha^2 + \omega_d^2}{\omega_d} = \dfrac{\omega_o^2}{\omega_d}$

Therefore $\quad \dfrac{dv_c}{dt} = \left(\dfrac{\omega_o^2}{\omega_d}\right) V e^{-\alpha t} \sin \omega_d t$

[b] $\dfrac{dv_c}{dt} = 0 \quad$ when $\quad \sin \omega_d t = 0, \quad$ or $\quad \omega_d t = n\pi$

where $\quad n = 0, 1, 2, 3, \ldots$

Therefore $\quad t = \dfrac{n\pi}{\omega_d}$

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[c] When $t_n = \dfrac{n\pi}{\omega_d}, \quad \cos\omega_d t_n = \cos n\pi = (-1)^n$

and $\sin\omega_d t_n = \sin n\pi = 0$

Therefore $v_c(t_n) = V[1 - (-1)^n e^{-\alpha n\pi/\omega_d}]$

[d] It follows from [c] that

$$v(t_1) = V + Ve^{-(\alpha\pi/\omega_d)} \quad \text{and} \quad v_c(t_3) = V + Ve^{-(3\alpha\pi/\omega_d)}$$

Therefore $\dfrac{v_c(t_1) - V}{v_c(t_3) - V} = \dfrac{e^{-(\alpha\pi/\omega_d)}}{e^{-(3\alpha\pi/\omega_d)}} = e^{(2\alpha\pi/\omega_d)}$

But $\dfrac{2\pi}{\omega_d} = t_3 - t_1 = T_d, \quad$ thus $\alpha = \dfrac{1}{T_d}\ln\dfrac{[v_c(t_1) - V]}{[v_c(t_3) - V]}$

P 8.58 $\dfrac{1}{T_d}\ln\left\{\dfrac{v_c(t_1) - V}{v_c(t_3) - V}\right\}; \qquad T_d = t_3 - t_1 = \dfrac{3\pi}{7} - \dfrac{\pi}{7} = \dfrac{2\pi}{7}\,\text{ms}$

$$\alpha = \frac{7000}{2\pi}\ln\left[\frac{63.84}{26.02}\right] = 1000; \qquad \omega_d = \frac{2\pi}{T_d} = 7000\,\text{rad/s}$$

$$\omega_o^2 = \omega_d^2 + \alpha^2 = 49 \times 10^6 + 10^6 = 50 \times 10^6$$

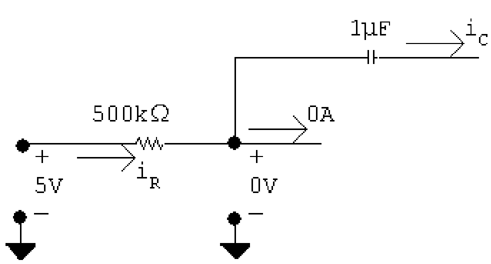
$$L = \frac{1}{(50 \times 10^6)(0.1 \times 10^{-6})} = 200\,\text{mH}; \qquad R = 2\alpha L = 400\,\Omega$$

P 8.59 At $t = 0$ the voltage across each capacitor is zero. It follows that since the operational amplifiers are ideal, the current in the $500\,\text{k}\Omega$ is zero. Therefore there cannot be an instantaneous change in the current in the $1\,\mu\text{F}$ capacitor. Since the capacitor current equals $C(dv_o/dt)$, the derivative must be zero.

P 8.60 **[a]** From Example 8.13 $\dfrac{d^2v_o}{dt^2} = 2$

therefore $\dfrac{dg(t)}{dt} = 2, \quad g(t) = \dfrac{dv_o}{dt}$

$$g(t) - g(0) = 2t; \qquad g(t) = 2t + g(0); \qquad g(0) = \frac{dv_o(0)}{dt}$$



$$i_R = \frac{5}{500} \times 10^{-3} = 10\,\mu\text{A} = i_C = -C\frac{dv_o(0)}{dt}$$

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$$\frac{dv_o(0)}{dt} = \frac{-10 \times 10^{-6}}{1 \times 10^{-6}} = -10 = g(0)$$

$$\frac{dv_o}{dt} = 2t - 10$$

$$dv_o = 2t\,dt - 10\,dt$$

$$v_o - v_o(0) = t^2 - 10t; \qquad v_o(0) = 8\,\text{V}$$

$$v_o = t^2 - 10t + 8, \qquad 0 \le t \le t_{\text{sat}}$$

[b] $t^2 - 10t + 8 = -9$

$$t^2 - 10t + 17 = 0$$

$$t \cong 2.17\,\text{s}$$

P 8.61 Part (1) — Example 8.14, with R_1 and R_2 removed:

[a] $R_{\text{a}} = 100\,\text{k}\Omega; \qquad C_1 = 0.1\,\mu\text{F}; \qquad R_{\text{b}} = 25\,\text{k}\Omega; \qquad C_2 = 1\,\mu\text{F}$

$$\frac{d^2v_o}{dt^2} = \left(\frac{1}{R_{\text{a}}C_1}\right)\left(\frac{1}{R_{\text{b}}C_2}\right)v_g; \qquad \frac{1}{R_{\text{a}}C_1} = 100 \quad \frac{1}{R_{\text{b}}C_2} = 40$$

$$v_g = 250 \times 10^{-3}; \qquad \text{therefore} \quad \frac{d^2v_o}{dt^2} = 1000$$

[b] Since $v_o(0) = 0 = \dfrac{dv_o(0)}{dt}$, our solution is $v_o = 500t^2$

The second op-amp will saturate when

$$v_o = 6\,\text{V}, \quad \text{or} \quad t_{\text{sat}} = \sqrt{6/500} \cong 0.1095\,\text{s}$$

[c] $\dfrac{dv_{o1}}{dt} = -\dfrac{1}{R_{\text{a}}C_1}v_g = -25$

[d] Since $v_{o1}(0) = 0, \quad v_{o1} = -25t\,\text{V}$

At $t = 0.1095\,\text{s}, \quad v_{o1} \cong -2.74\,\text{V}$

Therefore the second amplifier saturates before the first amplifier saturates. Our expressions are valid for $0 \le t \le 0.1095\,\text{s}$. Once the second op-amp saturates, our linear model is no longer valid.

Part (2) — Example 8.14 with $v_{o1}(0) = -2\,\text{V}$ and $v_o(0) = 4\,\text{V}$:

[a] Initial conditions will not change the differential equation; hence the equation is the same as Example 8.14.

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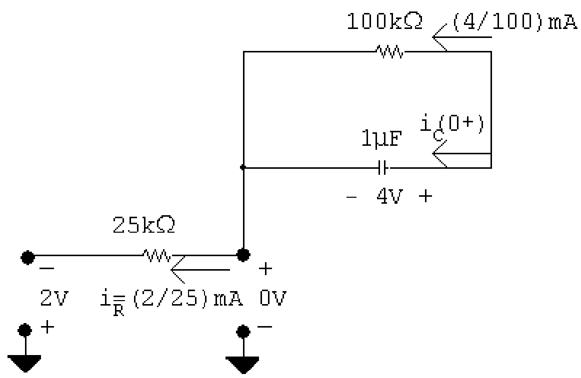
[b] $v_o = 5 + A_1' e^{-10t} + A_2' e^{-20t}$ (from Example 8.14)

$$v_o(0) = 4 = 5 + A_1' + A_2'$$



$$\frac{4}{100} + i_C(0^+) - \frac{2}{25} = 0$$

$$i_C(0^+) = \frac{4}{100}\,\text{mA} = C\frac{dv_o(0^+)}{dt}$$

$$\frac{dv_o(0^+)}{dt} = \frac{0.04 \times 10^{-3}}{10^{-6}} = 40\,\text{V/s}$$

$$\frac{dv_o}{dt} = -10A_1' e^{-10t} - 20A_2' e^{-20t}$$

$$\frac{dv_o}{dt}(0^+) = -10A_1' - 20A_2' = 40$$

Therefore $-A_1' - 2A_2' = 4$ and $A_1' + A_2' = -1$
Thus, $A_1' = 2$ and $A_2' = -3$

$$v_o = 5 + 2e^{-10t} - 3e^{-20t}\,\text{V}$$

[c] Same as Example 8.14:

$$\frac{dv_{o1}}{dt} + 20v_{o1} = -25$$

[d] From Example 8.14:

$v_{o1}(\infty) = -1.25\,\text{V}$; $v_1(0) = -2\,\text{V}$ (given)

Therefore

$$v_{o1} = -1.25 + (-2 + 1.25)e^{-20t} = -1.25 - 0.75e^{-20t}\,\text{V}$$

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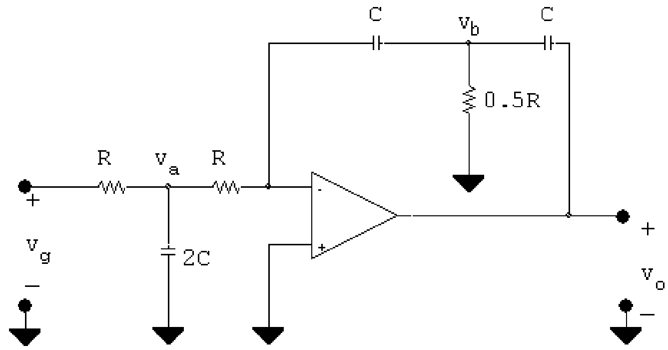
P 8.62 [**a**]



$$2C\frac{dv_a}{dt} + \frac{v_a - v_g}{R} + \frac{v_a}{R} = 0$$

(1) Therefore $\displaystyle \frac{dv_a}{dt} + \frac{v_a}{RC} = \frac{v_g}{2RC}$

$$\frac{0 - v_a}{R} + C\frac{d(0 - v_b)}{dt} = 0$$

(2) Therefore $\displaystyle \frac{dv_b}{dt} + \frac{v_a}{RC} = 0, \quad v_a = -RC\frac{dv_b}{dt}$

$$\frac{2v_b}{R} + C\frac{dv_b}{dt} + C\frac{d(v_b - v_o)}{dt} = 0$$

(3) Therefore $\displaystyle \frac{dv_b}{dt} + \frac{v_b}{RC} = \frac{1}{2}\frac{dv_o}{dt}$

From (2) we have $\displaystyle \frac{dv_a}{dt} = -RC\frac{d^2v_b}{dt^2}$ and $\displaystyle v_a = -RC\frac{dv_b}{dt}$

When these are substituted into (1) we get

(4) $\displaystyle -RC\frac{d^2v_b}{dt^2} - \frac{dv_b}{dt} = \frac{v_g}{2RC}$

Now differentiate (3) to get

(5) $\displaystyle \frac{d^2v_b}{dt^2} + \frac{1}{RC}\frac{dv_b}{dt} = \frac{1}{2}\frac{d^2v_o}{dt^2}$

But from (4) we have

(6) $\displaystyle \frac{d^2v_b}{dt^2} + \frac{1}{RC}\frac{dv_b}{dt} = -\frac{v_g}{2R^2C^2}$

Now substitute (6) into (5)

$$\frac{d^2v_o}{dt^2} = -\frac{v_g}{R^2C^2}$$

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[**b**] When $R_1C_1 = R_2C_2 = RC$: $\dfrac{d^2v_o}{dt^2} = \dfrac{v_g}{R^2C^2}$

The two equations are the same except for a reversal in algebraic sign.

[**c**] Two integrations of the input signal with one operational amplifier.

P 8.63 [**a**] $\dfrac{d^2v_o}{dt^2} = \dfrac{1}{R_1C_1R_2C_2}v_g$

$$\frac{1}{R_1C_1R_2C_2} = \frac{10^{-6}}{(100)(400)(0.5)(0.2)\times 10^{-6}\times 10^{-6}} = 250$$

$$\therefore \quad \frac{d^2v_o}{dt^2} = 250v_g$$

$0 \le t \le 0.5^-$:

$v_g = 80\,\text{mV}$

$$\frac{d^2v_o}{dt^2} = 20$$

Let $\quad g(t) = \dfrac{dv_o}{dt}, \qquad$ then $\quad \dfrac{dg}{dt} = 20 \quad$ or $\quad dg = 20\,dt$

$$\int_{g(0)}^{g(t)} dx = 20\int_0^t dy$$

$$g(t) - g(0) = 20t, \quad g(0) = \frac{dv_o}{dt}(0) = 0$$

$$g(t) = \frac{dv_o}{dt} = 20t$$

$$dv_o = 20t\,dt$$

$$\int_{v_o(0)}^{v_o(t)} dx = 20\int_0^t x\,dx; \qquad v_o(t) - v_o(0) = 10t^2, \quad v_o(0) = 0$$

$$v_o(t) = 10t^2\,\text{V}, \quad 0 \le t \le 0.5^-$$

$$\frac{dv_{o1}}{dt} = -\frac{1}{R_1C_1}v_g = -20v_g = -1.6$$

$$dv_{o1} = -1.6\,dt$$

$$\int_{v_{o1}(0)}^{v_{o1}(t)} dx = -1.6\int_0^t dy$$

$$v_{o1}(t) - v_{o1}(0) = -1.6t, \qquad v_{o1}(0) = 0$$

$$v_{o1}(t) = -1.6t\,\text{V}, \qquad 0 \le t \le 0.5^-$$

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$0.5^+ \le t \le t_{\text{sat}}$:

$$\frac{d^2v_o}{dt^2} = -10, \qquad \text{let} \quad g(t) = \frac{dv_o}{dt}$$

$$\frac{dg(t)}{dt} = -10; \qquad dg(t) = -10\,dt$$

$$\int_{g(0.5^+)}^{g(t)} dx = -10\int_{0.5}^{t} dy$$

$$g(t) - g(0.5^+) = -10(t - 0.5) = -10t + 5$$

$$g(0.5^+) = \frac{dv_o(0.5^+)}{dt}$$

$$C\frac{dv_o}{dt}(0.5^+) = \frac{0 - v_{o1}(0.5^+)}{400 \times 10^3}$$

$$v_{o1}(0.5^+) = v_o(0.5^-) = -1.6(0.5) = -0.80\,\text{V}$$

$$\therefore \quad C\frac{dv_{o1}(0.5^+)}{dt} = \frac{0.80}{0.4 \times 10^3} = 2\,\mu\text{A}$$

$$\frac{dv_{o1}}{dt}(0.5^+) = \frac{2 \times 10^{-6}}{0.2 \times 10^{-6}} = 10\,\text{V/s}$$

$$\therefore \quad g(t) = -10t + 5 + 10 = -10t + 15 = \frac{dv_o}{dt}$$

$$\therefore \quad dv_o = -10t\,dt + 15\,dt$$

$$\int_{v_o(0.5^+)}^{v_o(t)} dx = \int_{0.5^+}^{t} -10y\,dy + \int_{0.5^+}^{t} 15\,dy$$

$$v_o(t) - v_o(0.5^+) = -5y^2 \Big|_{0.5}^{t} + 15y \Big|_{0.5}^{t}$$

$$v_o(t) = v_o(0.5^+) - 5t^2 + 1.25 + 15t - 7.5$$

$$v_o(0.5^+) = v_o(0.5^-) = 2.5\,\text{V}$$

$$\therefore \quad v_o(t) = -5t^2 + 15t - 3.75\,\text{V}, \qquad 0.5^+ \le t \le t_{\text{sat}}$$

$$\frac{dv_{o1}}{dt} = -20(-0.04) = 0.8, \qquad 0.5^+ \le t \le t_{\text{sat}}$$

$$dv_{o1} = 0.8\,dt; \qquad \int_{v_{o1}(0.5^+)}^{v_{o1}(t)} dx = 0.8\int_{0.5^+}^{t} dy$$

$$v_{o1}(t) - v_{o1}(0.5^+) = 0.8t - 0.4; \qquad v_{o1}(0.5^+) = v_{o1}(0.5^-) = -0.8\,\text{V}$$

$$\therefore \quad v_{o1}(t) = 0.8t - 1.2\,\text{V}, \qquad 0.5^+ \le t \le t_{\text{sat}}$$

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Summary:

$0 \le t \le 0.5^{-}\text{s}: \qquad v_{o1} = -1.6t\,\text{V}, \quad v_o = 10t^2\,\text{V}$

$0.5^{+}\text{s} \le t \le t_{\text{sat}}: \qquad v_{o1} = 0.8t - 1.2\,\text{V}, \quad v_o = -5t^2 + 15t - 3.75\,\text{V}$

[b] $-12.5 = -5t_{\text{sat}}^2 + 15t_{\text{sat}} - 3.75$

$\therefore \quad 5t_{\text{sat}}^2 - 15t_{\text{sat}} - 8.75 = 0$

Solving, $\qquad t_{\text{sat}} = 3.5\,\text{sec}$

$v_{o1}(t_{\text{sat}}) = 0.8(3.5) - 1.2 = 1.6\,\text{V}$

P 8.64 $\quad \tau_1 = (10^6)(0.5 \times 10^{-6}) = 0.50\,\text{s}$

$$\frac{1}{\tau_1} = 2; \qquad \tau_2 = (5 \times 10^6)(0.2 \times 10^{-6}) = 1\,\text{s}; \qquad \therefore \quad \frac{1}{\tau_2} = 1$$

$$\therefore \quad \frac{d^2v_o}{dt^2} + 3\frac{dv_o}{dt} + 2v_o = 20$$

$$s^2 + 3s + 2 = 0$$

$$(s+1)(s+2) = 0; \qquad s_1 = -1, \quad s_2 = -2$$

$$v_o = V_f + A_1'e^{-t} + A_2'e^{-2t}; \qquad V_f = \frac{20}{2} = 10\,\text{V}$$

$$v_o = 10 + A_1'e^{-t} + A_2'e^{-2t}$$

$$v_o(0) = 0 = 10 + A_1' + A_2'; \qquad \frac{dv_o}{dt}(0) = 0 = -A_1' - 2A_2'$$

$$\therefore \quad A_1' = -20, \qquad A_2' = 10\,\text{V}$$

$$v_o(t) = 10 - 20e^{-t} + 10e^{-2t}\,\text{V}, \qquad 0 \le t \le 0.5\,\text{s}$$

$$\frac{dv_{o1}}{dt} + 2v_{o1} = -1.6; \qquad \therefore \quad v_{o1} = -0.8 + 0.8e^{-2t}\,\text{V}, \quad 0 \le t \le 0.5\,\text{s}$$

$$v_o(0.5) = 10 - 20e^{-0.5} + 10e^{-1} = 1.55\,\text{V}$$

$$v_{o1}(0.5) = -0.8 + 0.8e^{-1} = -0.51\,\text{V}$$

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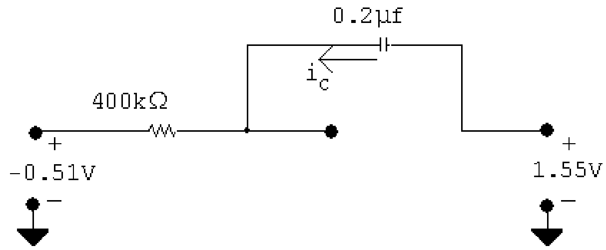
At $t = 0.5\,\text{s}$



$$i_C = \frac{0 + 0.51}{400 \times 10^3} = 1.26\,\mu\text{A}$$

$$C\frac{dv_o}{dt} = 1.26\,\mu\text{A}; \qquad \frac{dv_o}{dt} = \frac{1.26}{0.2} = 6.32\,\text{V/s}$$

$0.5\,\text{s} \leq t \leq \infty$:

$$\frac{d^2v_o}{dt^2} + 3\frac{dv_o}{dt} + 2 = -10$$

$$v_o(\infty) = -5$$

$$\therefore \quad v_o = -5 + A_1' e^{-(t-0.5)} + A_2' e^{-2(t-0.5)}$$

$$1.55 = -5 + A_1' + A_2'$$

$$\frac{dv_o}{dt}(0.5) = 6.32 = -A_1' - 2A_2'$$

$$\therefore \quad A_1' + A_2' = 6.55; \qquad -A_1' - 2A_2' = 6.32$$

Solving,

$$A_1' = 19.42; \qquad A_2' = -12.87$$

$$\therefore \quad v_o = -5 + 19.42e^{-(t-0.5)} - 12.87e^{-2(t-0.5)}\,\text{V}, \qquad 0.5 \leq t \leq \infty$$

$$\frac{dv_{o1}}{dt} + 2v_{o1} = 0.8$$

$$\therefore \quad v_{o1} = 0.4 + (-0.51 - 0.4)e^{-2(t-0.5)} = 0.4 - 0.91e^{-2(t-0.5)}\,\text{V}, \qquad 0.5 \leq t \leq \infty$$

P 8.65 **[a]**
$$\begin{aligned} f(t) &= \text{inertial force + frictional force + spring force} \\ &= M[d^2x/dt^2] + D[dx/dt] + Kx \end{aligned}$$

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[b] $$\frac{d^2x}{dt^2} = \frac{f}{M} - \left(\frac{D}{M}\right)\left(\frac{dx}{dt}\right) - \left(\frac{K}{M}\right)x$$

Given $\quad v_A = \dfrac{d^2x}{dt^2}, \quad$ then

$$v_B = -\frac{1}{R_1C_1}\int_0^t \left(\frac{d^2x}{dy^2}\right)dy = -\frac{1}{R_1C_1}\frac{dx}{dt}$$

$$v_C = -\frac{1}{R_2C_2}\int_0^t v_B\,dy = \frac{1}{R_1R_2C_1C_2}x$$

$$v_D = -\frac{R_3}{R_4}\cdot v_B = \frac{R_3}{R_4R_1C_1}\frac{dx}{dt}$$

$$v_E = \left[\frac{R_5+R_6}{R_6}\right]v_C = \left[\frac{R_5+R_6}{R_6}\right]\cdot\frac{1}{R_1R_2C_1C_2}\cdot x$$

$$v_F = \left[\frac{-R_8}{R_7}\right]f(t), \qquad v_A = -(v_D + v_E + v_F)$$

Therefore $$\frac{d^2x}{dt^2} = \left[\frac{R_8}{R_7}\right]f(t) - \left[\frac{R_3}{R_4R_1C_1}\right]\frac{dx}{dt} - \left[\frac{R_5+R_6}{R_6R_1R_2C_1C_2}\right]x$$

Therefore $$M = \frac{R_7}{R_8}, \qquad D = \frac{R_3R_7}{R_8R_4R_1C_1} \quad \text{and} \quad K = \frac{R_7(R_5+R_6)}{R_8R_6R_1R_2C_1C_2}$$

Box Number	Function
1	inverting and scaling
2	summing and inverting
3	integrating and scaling
4	integrating and scaling
5	inverting and scaling
6	noninverting and scaling

P 8.66 [a] Given that the current response is underdamped, we know i will be of the form

$$i = I_f + [B_1'\cos\omega_d t + B_2'\sin\omega_d t]e^{-\alpha t}$$

where $$\alpha = \frac{R}{2L}$$

and $$\omega_d = \sqrt{\omega_o^2 - \alpha^2} = \sqrt{\frac{1}{LC} - \alpha^2}$$

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The capacitor will force the final value of i to be zero, therefore $I_f = 0$.
By hypothesis $i(0^+) = V_{dc}/R$; therefore $B_1' = V_{dc}/R$.
At $t = 0^+$ the voltage across the primary winding is approximately zero; hence $di(0^+)/dt = 0$.
From our equation for i we have

$$\frac{di}{dt} = [(\omega_d B_2' - \alpha B_1')\cos\omega_d t - (\omega_d B_1' + \alpha B_2')\sin\omega_d t]e^{-\alpha t}$$

Hence

$$\frac{di(0^+)}{dt} = \omega_d B_2' - \alpha B_1' = 0$$

Thus

$$B_2' = \frac{\alpha}{\omega_d}B_1' = \frac{\alpha V_{dc}}{\omega_d R}$$

It follows directly that

$$i = \frac{V_{dc}}{R}\left[\cos\omega_d t + \frac{\alpha}{\omega_d}\sin\omega_d t\right]e^{-\alpha t}$$

[b] Since $\omega_d B_2' - \alpha B_1' = 0$, it follows that

$$\frac{di}{dt} = -(\omega_d B_1' + \alpha B_2')e^{-\alpha t}\sin\omega_d t$$

But $\quad \alpha B_2' = \dfrac{\alpha^2 V_{dc}}{\omega_d R} \quad$ and $\quad \omega_d B_1' = \dfrac{\omega_d V_{dc}}{R}$

Therefore

$$\omega_d B_1' + \alpha B_2' = \frac{\omega_d V_{dc}}{R} + \frac{\alpha^2 V_{dc}}{\omega_d R} = \frac{V_{dc}}{R}\left[\frac{\omega_d^2 + \alpha^2}{\omega_d}\right]$$

But $\quad \omega_d^2 + \alpha^2 = \omega_o^2 = \dfrac{1}{LC}$

Hence

$$\omega_d B_1' + \alpha B_2' = \frac{V_{dc}}{\omega_d RLC}$$

Now since $\quad v_1 = L\dfrac{di}{dt} \quad$ we get

$$v_1 = -L\frac{V_{dc}}{\omega_d RLC}e^{-\alpha t}\sin\omega_d t = -\frac{V_{dc}}{\omega_d RC}e^{-\alpha t}\sin\omega_d t$$

[c] $v_c = V_{dc} - iR - L\dfrac{di}{dt}$

$$iR = V_{dc}\left(\cos\omega_d t + \frac{\alpha}{\omega_d}\sin\omega_d t\right)e^{-\alpha t}$$

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$$v_c = V_{dc} - V_{dc}\left(\cos\omega_d t + \frac{\alpha}{\omega_d}\,\sin\omega_d t\right)e^{-\alpha t} + \frac{V_{dc}}{\omega_d RC}e^{-\alpha t}\sin\omega_d t$$

$$= V_{dc} - V_{dc}e^{-\alpha t}\cos\omega_d t + \left(\frac{V_{dc}}{\omega_d RC} - \frac{\alpha V_{dc}}{\omega_d}\right)e^{-\alpha t}\sin\omega_d t$$

$$= V_{dc}\left[1 - e^{-\alpha t}\cos\omega_d t + \frac{1}{\omega_d}\left(\frac{1}{RC} - \alpha\right)e^{-\alpha t}\sin\omega_d t\right]$$

$$= V_{dc}\left[1 - e^{-\alpha t}\cos\omega_d t + Ke^{-\alpha t}\sin\omega_d t\right]$$

P 8.67 $v_{sp} = V_{dc}\left[1 - \frac{a}{\omega_d RC}e^{-\alpha t}\,\sin\omega_d t\right]$

$$\frac{dv_{sp}}{dt} = \frac{-aV_{dc}}{\omega_d RC}\,\frac{d}{dt}[e^{-\alpha t}\sin\omega_d t]$$

$$= \frac{-aV_{dc}}{\omega_d RC}[-\alpha e^{-\alpha t}\sin\omega_d t + \omega_d e^{-\alpha t}\cos\omega_d t]$$

$$= \frac{aV_{dc}e^{-\alpha t}}{\omega_d RC}[\alpha\,\sin\omega_d t - \omega_d\,\cos\omega_d t]$$

$$\frac{dv_{sp}}{dt} = 0 \quad \text{when} \quad \alpha\,\sin\omega_d t = \omega_d\,\cos\omega_d t$$

$$\text{or} \quad \tan\omega_d t = \frac{\omega_d}{\alpha}; \qquad \omega_d t = \tan^{-1}\left(\frac{\omega_d}{\alpha}\right)$$

$$\therefore \quad t_{\max} = \frac{1}{\omega_d}\,\tan^{-1}\left(\frac{\omega_d}{\alpha}\right)$$

Note that because $\tan\theta$ is periodic, i.e., $\tan\theta = \tan(\theta \pm n\pi)$, where n is an integer, there are an infinite number of solutions for t where $dv_{sp}/dt = 0$, that is

$$t = \frac{\tan^{-1}(\omega_d/\alpha) \pm n\pi}{\omega_d}$$

Because of $e^{-\alpha t}$ in the expression for v_{sp} and knowing $t \geq 0$ we know v_{sp} will be maximum when t has its smallest positive value. Hence

$$t_{\max} = \frac{\tan^{-1}(\omega_d/\alpha)}{\omega_d}.$$

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P 8.68 **[a]** $v_c = V_{dc}[1 - e^{-\alpha t}\cos\omega_d t + Ke^{-\alpha t}\sin\omega_d t]$

$$\frac{dv_c}{dt} = V_{dc}\frac{d}{dt}[1 + e^{-\alpha t}(K\ \sin\omega_d t - \cos\omega_d t)]$$

$$= V_{dc}\{(-\alpha e^{-\alpha t})(K\ \sin\omega_d t - \cos\omega_d t) +$$

$$e^{-\alpha t}[\omega_d K\ \cos\omega_d t + \omega_d\ \sin\omega_d t]\}$$

$$= V_{dc}e^{-\alpha t}[(\omega_d - \alpha K)\ \sin\omega_d t + (\alpha + \omega_d K)\ \cos\omega_d t]$$

$$\frac{dv_c}{dt} = 0 \quad \text{when} \quad (\omega_d - \alpha K)\ \sin\omega_d t = -(\alpha + \omega_d K)\ \cos\omega_d t$$

$$\text{or} \quad \tan\omega_d t = \left[\frac{\alpha + \omega_d K}{\alpha K - \omega_d}\right]$$

$$\therefore \quad \omega_d t \pm n\pi = \tan^{-1}\left[\frac{\alpha + \omega_d K}{\alpha K - \omega_d}\right]$$

$$t_c = \frac{1}{\omega_d}\left\{\tan^{-1}\left(\frac{\alpha + \omega_d K}{\alpha K - \omega_d}\right) \pm n\pi\right\}$$

$$\alpha = \frac{R}{2L} = \frac{4\times 10^3}{6} = 666.67\,\text{rad/s}$$

$$\omega_d = \sqrt{\frac{10^9}{1.2} - (666.67)^2} = 28{,}859.81\,\text{rad/s}$$

$$K = \frac{1}{\omega_d}\left(\frac{1}{RC} - \alpha\right) = 21.63$$

$$t_c = \frac{1}{\omega_d}\left\{\tan^{-1}(-43.29) + n\pi\right\} = \frac{1}{\omega_d}\{-1.55 + n\pi\}$$

The smallest positive value of t occurs when $n = 1$, therefore

$t_{c\,\max} = 55.23\,\mu\text{s}$

[b] $v_c(t_{c\max}) = 12[1 - e^{-\alpha t_{c\max}}\cos\omega_d t_{c\max} + Ke^{-\alpha t_{c\max}}\sin\omega_d t_{c\max}]$

$$= 262.42\,\text{V}$$

[c] From the text example the voltage across the spark plug reaches its maximum value in $53.63\,\mu$s. If the spark plug does not fire the capacitor voltage peaks in $55.23\,\mu$s. When v_{sp} is maximum the voltage across the capacitor is 262.15 V. If the spark plug does not fire the capacitor voltage reaches 262.42 V.

P 8.69 **[a]** $w = \frac{1}{2}L[i(0^+)]^2 = \frac{1}{2}(5)(16)\times 10^{-3} = 40\,\text{mJ}$

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[b] $\alpha = \frac{R}{2L} = \frac{3 \times 10^3}{10} = 300\,\text{rad/s}$

$$\omega_d = \sqrt{\frac{10^9}{1.25} - (300)^2} = 28{,}282.68\,\text{rad/s}$$

$$\frac{1}{RC} = \frac{10^6}{0.75} = \frac{4 \times 10^6}{3}$$

$$t_{\text{max}} = \frac{1}{\omega_d}\tan^{-1}\left(\frac{\omega_d}{\alpha}\right) = 55.16\,\mu\text{s}$$

$$v_{sp}\,(t_{\text{max}}) = 12 - \frac{12(50)(4 \times 10^6)}{3(28{,}282.68)}e^{-\alpha t_{\text{max}}}\sin\omega_d t_{\text{max}} = -27{,}808.04\,\text{V}$$

[c] $v_c\,(t_{\text{max}}) = 12[1 - e^{-\alpha t_{\text{max}}}\cos\omega_d t_{\text{max}} + Ke^{-\alpha t_{\text{max}}}\sin\omega_d t_{\text{max}}]$

$$K = \frac{1}{\omega_d}\left[\frac{1}{RC} - \alpha\right] = 47.13$$

$$v_c\,(t_{\text{max}}) = 568.15\,\text{V}$$

P 8.70 **[a]** $v_c = V_{dc}[1 - e^{-\alpha t}\cos\omega_d t + Ke^{-\alpha t}\sin\omega_d t]$

$$\frac{dv_c}{dt} = V_{dc}\frac{d}{dt}[1 + e^{-\alpha t}(K\sin\omega_d t - \cos\omega_d t)]$$

$$= V_{dc}\{(-\alpha e^{-\alpha t})(K\sin\omega_d t - \cos\omega_d t) +$$
$$e^{-\alpha t}[\omega_d K\cos\omega_d t + \omega_d\sin\omega_d t]\}$$
$$= V_{dc}e^{-\alpha t}[(\omega_d - \alpha K)\sin\omega_d t + (\alpha + \omega_d K)\cos\omega_d t]$$

$$\frac{dv_c}{dt} = 0 \quad \text{when} \quad (\omega_d - \alpha K)\sin\omega_d t = -(\alpha + \omega_d K)\cos\omega_d t$$

$$\text{or} \quad \tan\omega_d t = \left[\frac{\alpha + \omega_d K}{\alpha K - \omega_d}\right]$$

$$\therefore\ \omega_d t \pm n\pi = \tan^{-1}\left[\frac{\alpha + \omega_d K}{\alpha K - \omega_d}\right]$$

$$t_c = \frac{1}{\omega_d}\left\{\tan^{-1}\left(\frac{\alpha + \omega_d K}{\alpha K - \omega_d}\right) \pm n\pi\right\}$$

$$\alpha = \frac{R}{2L} = \frac{3}{2(5 \times 10^{-3})} = 300\,\text{rad/s}$$

$$\omega_d = \sqrt{\frac{10^9}{1.25} - (300)^2} = 28{,}282.68\,\text{rad/s}$$

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$$K = \frac{1}{\omega_d}\left(\frac{1}{RC} - \alpha\right) = 47.13$$

$$t_c = \frac{1}{\omega_d}\{-1.56 + n\pi\}$$

The smallest positive value of t occurs when $n = 1$, therefore

$$t_{c\,\max} = 55.91\,\mu\text{s}$$

[b] $v_c(t_{c\,\max}) = 12[1 - e^{-\alpha t_{c\,\max}} \cos\omega_d t_{c\,\max} + Ke^{-\alpha t_{c\,\max}} \sin\omega_d t_{c\,\max}] = 568.28\,\text{V}$

[c] From Problem 8.69, the voltage across the spark plug reaches its maximum value in $55.16\,\mu$s. If the spark plug does not fire the capacitor voltage peaks in $55.91\,\mu$s. When v_{sp} is maximum the voltage across the capacitor is 568.15 V. If the spark plug does not fire the capacitor voltage reaches 568.28 V.

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