1. Vibration:

a. Simple Harmonic Motion: (SHM)

$x = A\cos(\omega t + \varphi)$ $\quad K = m\omega^2$

b. Energy in SHM:

$$E = \frac{1}{2}mv^2 + \frac{1}{2}kx^2 = \frac{1}{2}kA^2$$

c. Physical Pendulum (θ small)

$\theta = \theta_0 \cos(\omega t + \varphi)$

- $\omega = \sqrt{\dfrac{mgd}{I}}$

* Simple pendulum: $\omega = \sqrt{\dfrac{g}{L}}$

d. Damped oscillations

Additional friction force: $F = -bv$

$x = Ae^{(-b/2m)t}\cos(\omega' t + \varphi)$

- $\omega' = \sqrt{\dfrac{k}{m} - \dfrac{b^2}{4m}}$

e. Force oscillation:

Additional force: $F_{ext} = F_0 \cos\omega t$

$A = \cos(\omega t + \varphi)$

$$A = \frac{F_0/m}{\sqrt{(\omega^2 - \omega_0^2)^2 + (b\omega/m)^2}} \;;\; \omega_0 = \sqrt{\frac{k}{m}}$$

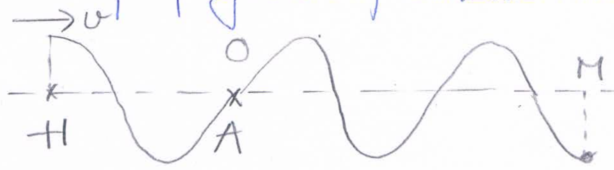
* Resonance:

$\omega = \omega_0 = \sqrt{\dfrac{k}{m}} \rightarrow A_{maximum}$

↳ Resonance

2. Mechanical wave:

Wave: propagation of oscillation in space



$y_A = A\sin\omega t$

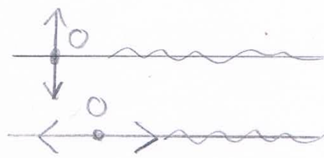
At M: $y = A\sin\left(\omega t - \dfrac{2\pi . OM}{\lambda}\right)$

At H: $y = A\sin\left(\omega t + \dfrac{2\pi . HO}{\lambda}\right)$

- wave length: $\lambda = vT$
- wave number: $k = \dfrac{2\pi}{\lambda}$

a) Type of wave

- Transverse wave
- Longitudinal wave



b) Speed of wave on string

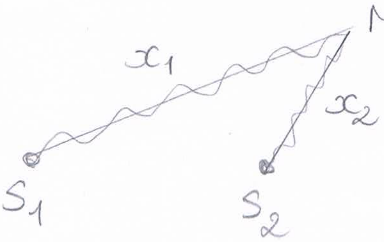
$v = \sqrt{\dfrac{T}{\mu}}$ $\begin{cases} \mu = \text{mass/length unit} \\ T: \text{tension of string} \end{cases}$

c) General form of travelling wave

Wave travelling to the left: $y = f(x + vt)$

wave travelling to the right: $y = f(x - vt)$

Super position and Interference:



At S_1, S_2 : $y = A\sin(\omega t + \overset{0}{\widehat{\varphi}})$

$S_1 \rightsquigarrow M$: $y_1 = A\sin(\omega t - kx_1)$

$S_2 \rightsquigarrow M$ $y_2 = A\sin(\omega t - kx_2)$

- let $x_1 = x_2 + \delta$

$\Rightarrow \phi = k(x_1 - x_2) = k\delta$

$y = y_1 + y_2 = 2A\cos\frac{\phi}{2}\sin(\omega t - kx_1 + \frac{\phi}{2})$

- Constructive interference

$x_1 - x_2 = k\lambda$

- Destructive interference

$x_1 - x_2 = (k + \frac{1}{2})\lambda$

Amplitude:

$2A \cdot \left|\cos\frac{\phi}{2}\right|$

Standing Wave

Given 2 waves in opposite direction:

$y_1 = A\sin(kx - \omega t)$

$y_2 = A\sin(kx + \omega t)$

$y = y_1 + y_2 = 2A\sin(kx)\cos(\omega t)$

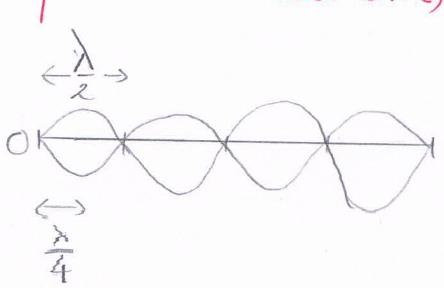
- At node (amplitude is zero)

$x = \frac{n\lambda}{2}$ $(n = 1, 2, 3 \ldots)$

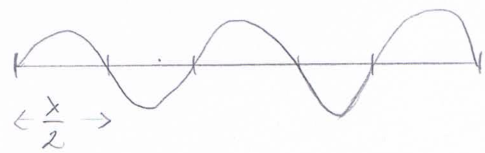
- At anti-node (amplitude is maximum)

$x = (n + \frac{1}{2})\frac{\lambda}{2}$

$(n = 0, 1, 2 \ldots)$



Standing waves in a string fixed at both ends.

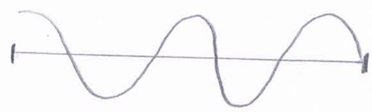


Wavelength of natural pattern: $\lambda_n = \frac{2}{n}$

$L = n\frac{\lambda_n}{2} = \frac{nv}{2f} \Rightarrow f = \frac{nv}{2L}$

$f_1 = \frac{v}{2L}$; $f_n = nf_1$ $(n = 1, 2, 3 \ldots)$

Standing waves in a string opened at ~~both~~ ends



$L = (n + \frac{1}{2})\frac{\lambda}{2} = (n + \frac{1}{2})\frac{v}{2f} \Rightarrow f = (n + \frac{1}{2})\frac{v}{2L}$

$f_1 = \frac{v}{4L}$; $f_n = \cancel{(2n+1)f_1}\ nf_1$

$(n = 1, 3, 5 \ldots)$

Energy Transfer by Waves

$\left.\begin{aligned} U_\lambda &= \frac{1}{4}\mu\lambda\omega^2A^2 \\ K_\lambda &= \frac{1}{4}\mu\lambda\omega^2A^2 \end{aligned}\right\}$ in one λ

$E_\lambda = U_\lambda + K_\lambda = \frac{1}{2}\mu\lambda\omega^2A^2$

Power:

$P = \frac{E_\lambda}{T} = \frac{1}{2}\mu\omega^2A^2v$

Soud wave:

- Audible: 20 - 20.000 Hz
- Intensity: $I = \frac{P}{A}$; $I_0 = 10^{-12}\ W/m$
- Intensity level: $\beta = 10\log\frac{I}{I_0}$

Threshold: hearing: 0dB
pain: 120dB
normal: 50dB

Dopple Effect:

$f' = \frac{v \pm v_0}{v \mp v_s} f$ may

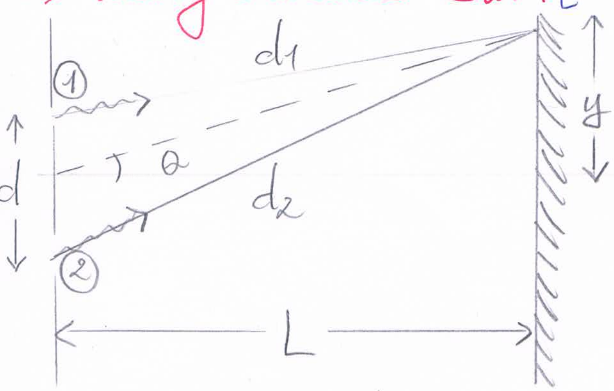
1. Nature of Light

$E = hf$

$v_{light} = 3 \times 10^8 (m/s)$

2. Interference of Light Waves :

a) Young's Double-Slit :



Source light wave : $y = A.\sin\omega t$

① ⇝ E : $y_1 = A\sin(\omega t - kd_1)$

② ⇝ E : $y_2 = A\sin(\omega t - kd_2)$

At E : $y = y_1 + y_2 = 2A\cos\left(k\dfrac{d_2 - d_1}{2}\right)\sin\left(\omega t + k\dfrac{d_1 + d_2}{2}\right)$

$\delta = d_2 - d_1 \approx d.\tan\theta \approx d\dfrac{y}{L}$

• At bright fringes : $\to A_{max}$, $\to I_{max}$

$\Delta d = k\lambda$; $y_{bright} = ki$

• At dark fringes : $\to A_{min} = 0$, $\to I_{min} = 0$

$\Delta d = (k + \frac{1}{2})\lambda$; $y_{dark} = (k + \frac{1}{2})\,i$

$i = \dfrac{L\lambda}{d}$

Electric field :

① ⇝ E : $E_1 = E_0\sin(\omega t - kd_1)$

② ⇝ E : $E_2 = E_0\sin(\omega t - kd_2)$

At E : $E = 2E_0\cos\left(k\dfrac{d_2 - d_1}{2}\right).\sin\left(\omega t + k\dfrac{d_1 + d_2}{2}\right)$

Intensity of wave :

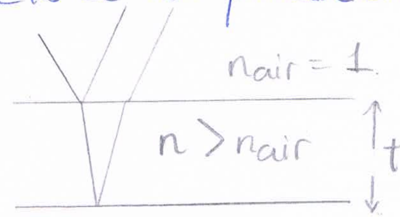
$I = I_0\cos^2\left(k\dfrac{\delta}{2}\right)$; $\delta \approx d.\sin\theta$

$I = I_0\cos^2\left(\dfrac{\pi d\sin\theta}{\lambda}\right)$; $\sin\theta \approx \tan\theta \approx \dfrac{y}{L}$

$I = I_0\cos^2\left(\dfrac{\pi d y}{\lambda L}\right)$

b) Thin film :

If light goes from $n_1 \to n_2$ $(n_1 < n_2)$ there is a 180° phase change



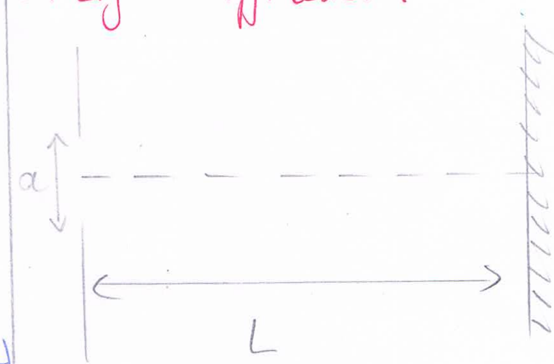
• Constructive interference :

$2nt = (m + \frac{1}{2})\lambda$

• Destructive interference

$2nt = m\lambda_n$ $(m = 0, 1, 2 \ldots)$

3. Light Diffraction :



General condition for destructive interference

$\sin\theta = m\dfrac{\lambda}{a}$ $(m = \pm 1, \pm 2 \ldots)$

$\lambda \ll a$: $\sin \approx \tan \approx \dfrac{y_m}{L}$

Position of dark fringes :

$y_m = m\dfrac{L\lambda}{a}$

$(m = \pm 1, \pm 2 \ldots)$

* Intensity of Single-slit

$I = I_0 \cdot \left[\dfrac{\sin(\pi a\sin\theta/\lambda)}{\pi a\sin\theta/\lambda}\right]^2$

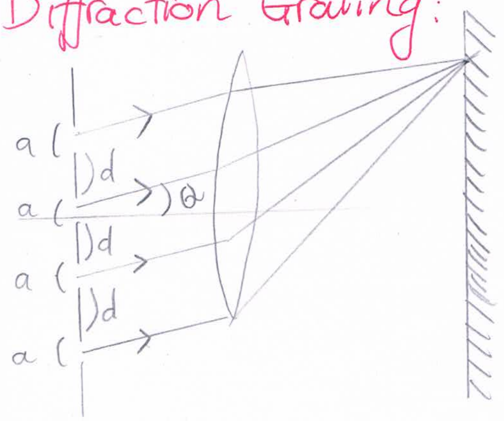
• $\theta = 0 \to I_{max}$

• $\sin\theta = m\dfrac{\lambda}{a}$ $(m \neq 0) \to I_{min}$

• $a \gg \lambda$: $\sin\theta \ll 1 \to$ geometrical focus

• $a < \lambda$: $\theta \approx \pi/2 \to$ No see fringe

Diffraction Grating:



Condition for maxima in the interference at angle θ:

$$d \cdot \sin\theta = m\lambda \quad (m = 0; 1; 2 \ldots)$$

d: grating space

a: width of slits

Circular Apertures:

- Dark ring: $\sin\theta_1 = 1.22 \frac{\lambda}{D}$

  $\sin\theta_2 = 2.23 \frac{\lambda}{D}$
- Bright ring: $\sin\theta = 1.63 \frac{\lambda}{D}; \ 2.68 \frac{\lambda}{D}$

Diffraction of X-ray:

Difference in travelling route of $\left\{ \begin{array}{l} \text{lower} \\ \text{upper} \end{array} \right.$ plane

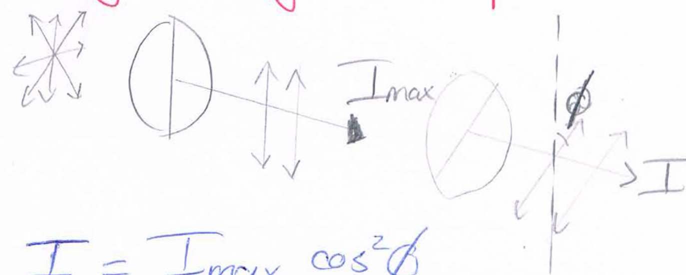
$$\delta = 2d\sin\theta$$

- d: distance of planes

POLARIZATION OF LIGHT WAVES

- Speed of light: $c = \frac{E}{B}$
- Wave length (vacuum): $\lambda = \frac{c}{f}$

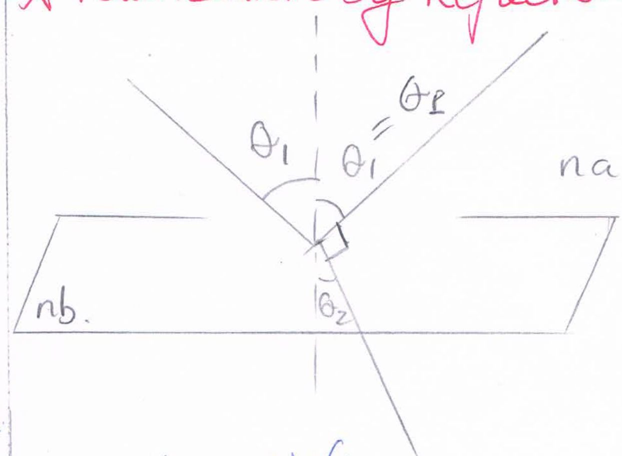
* Intensity of light transmitted through analyzer and polarizer



$$I = I_{max} \cos^2\phi$$

- $\phi = 0° \rightarrow I = I_{max}$
- $\phi = 45° \rightarrow I = \frac{1}{2} I_{max}$
- $\phi = 90° \rightarrow I = 0$

* Polarization by Reflection:



$$n = \frac{n_b}{n_a} = \frac{\sin\theta_p}{\sin\theta_2} = \tan\theta_p$$

$(\theta_p + \theta_2 = 90°)$

Reflection and Refraction:

- index of refraction: $n = \frac{c}{v}$
- Law of reflection: $\theta_a = \theta_r$
- Law of refraction: $n_a \sin\theta_a = n_b \sin$