Homework: 25, 28, 32, 33, 40 (page 532)

25. Determine the average value of the translational kinetic energy of the molecules of an ideal gas at (a) 0.00°C and (b) 100°C. What is the translational kinetic energy per mole of an ideal gas at (c) 0.00°C and (d) 100°C?

(a) The translational kinetic energy per molecule:

1.38×10^{-23}

1.38×10^{-23}

$$\overline{K}=\frac{3}{2}kT$$

average, per molecule, atom: $K=\frac{3}{2}kT$

per mole: $K=\frac{3}{2}RT$

8.31

n moles: $K=\frac{3}{2}nRT$

$T=0+273=273\,\mathrm{K}:$

$$\overline{K}=\frac{3}{2}\times1.38\times10^{-23}\times273=5.65\times10^{-21}\ \mathrm{(J)}$$

(b) see (a): $$\overline{K}=\frac{3}{2}\times1.38\times10^{-23}\times373=7.72\times10^{-21}\ \mathrm{(J)}$$

(c) The translational kinetic energy per mole: $K_{mole}=\overline{K}\times N_A$

$$K_{mole}=5.65\times10^{-21}\times6.02\times10^{23}=3.4\times10^{3}\ \mathrm{(J)}$$

(d) $$K_{mole}=4.7\times10^{3}\ \mathrm{(J)}$$

Note: If a sample of gas has n moles (or N molecules), its total translational kinetic energy is:

$$K_{total} = n \times K_{mole} = n \times N_A \times \overline{K}$$

$$K_{total} = n \times K_{mole} = n \times N_A \times \frac{3}{2}kT = \frac{3}{2}nRT$$

$$\boxed{K_{total} = \frac{3}{2}nRT}$$

$PV = nRT$

28. At what **frequency** would the **wavelength** of **sound** in air be equal to the mean free path of oxygen molecules at 1.0 atm pressure and 0.0°C? take the **diameter** of an oxygen molecule to be 3.0 x 10^{-8} cm.

Mean Free Path:

$$\lambda_{MFP} = \frac{kT}{\sqrt{2}\pi d^2 p}$$

1.28×10^{-23}

$$f = \frac{v_{sound}}{\lambda_{sound}}$$

$$k = 1.38\times10^{-23}\ \text{J/K};\ T = 273\ \text{K};\ p = 1.01\times10^{5}\ \text{Pa};$$

$$d = 3\times10^{-8}\,\text{cm} = 3\times10^{-10}\,\text{m}$$

Frequency of sound in air: $$f_{sound} = \frac{v_{\text{sound in air}}}{\lambda_{sound}} = \frac{v_{\text{sound in air}}}{\lambda_{MFP}}$$

$v_{\text{sound in air}} = 343\ \text{m/s}:$

$$\lambda_{MFP} = 9.33\times10^{-8}\,\text{m}$$

$$f_{sound} = \frac{343\ \text{m/s}}{9.33\times10^{8}\ (\text{m})} \approx 3.68\times10^{9}\,(\text{Hz})\text{ or }3.68\,\text{GHz}$$

32. At 20^{0}C and 750 torr pressure, the mean free paths for argon gas (Ar) and nitrogen (N_2) are λ_{Ar}=9.9x10^{-6} cm and λ_{N2}=27.5x10^{-6} cm. (a) Find the ratio of the diameter of an Ar atom to that of an N_2 molecule. What is the mean free path of Ar at (b) 20^{0}C and 150 torr, and (c) -40^{0}C and 750 torr?

Mean Free Path:

$$\lambda = \frac{kT}{\sqrt{2}\pi d^2 p}$$

$$\lambda \propto \frac{1}{d^2}$$

(a) The ratio d_{Ar} to d_{N2}:

$$\frac{d_{Ar}}{d_{N_2}} = \sqrt{\frac{\lambda_{N_2}}{\lambda_{Ar}}}$$

(b):

$$\lambda_1 = \frac{kT_1}{\sqrt{2}\pi d^2 p_1}; \lambda_2 = \frac{kT_2}{\sqrt{2}\pi d^2 p_2}$$

$$\lambda_2 = \frac{T_2}{T_1} \times \frac{p_1}{p_2} \times \lambda_1$$

$$\frac{\lambda_2}{\lambda_1} = \frac{T_2}{T_1} \times \frac{p_1}{p_2}$$

33. The speeds of 10 molecules are 2.0, 3.0, 4.0,. . . , 11 km/s. What are their (a) average speed and (b) rms speed?

(a) $$\bar{v} = \frac{\sum_{i=1}^{N} v_i}{N} = \frac{2+3+4+...+11}{10} = \frac{65}{10} = 6.5\,(\text{km/s})$$

(b) $$v_{rms} = \sqrt{\left(v^2\right)_{avg}} = \sqrt{\frac{\sum_{i=1}^{N} v_i^2}{N}} = 7.1\,(\text{km/s})$$

40. Two containers are at the same temperature. The first contains gas with pressure p_1, molecular mass m_1, and rms speed v_{rms1}. The second contains gas with pressure $1.5p_1$, molecular mass m_2, and average speed $v_{avg2}=2.0v_{rms1}$. Find the mass ratio m_1/m_2.

RMS speed: $v_{rms1} = \sqrt{\dfrac{3RT_1}{m_1}}$

$T_1 = \dfrac{v_1^2 \times m_1}{3R}$

Average speed: $\bar{v}_2 = \sqrt{\dfrac{8RT_2}{\pi m_2}}$

$T_2 = \dfrac{v_2^2 \times \pi m_2}{8R}$

$$T_1 = T_2 \Rightarrow \frac{m_1}{m_2} = \frac{3\pi}{8}\left(\frac{\bar{v}_2}{v_{\text{rms}1}}\right)^2 = 4.71$$

$$\frac{v_1^2 m_1}{3R} = \frac{V_2^2 \times \pi m_2}{8R} \Rightarrow \frac{m_1}{m_2} = \frac{4 \times 3\pi R}{8R} = 4.71$$

$4v_1^2$

Chapter 3 The Kinetic Theory of Gases

3.1. Ideal Gases

3.1.1. Experimental Laws and the Equation of State

3.1.2. Molecular Model of an Ideal Gas

3.2. Mean Free Path

3.3. The Boltzmann Distribution Law and

The Distribution of Molecular Speeds

3.4. The Molar Specific Heats of an Ideal Gas

3.5. The Equipartition-of-Energy Theorem

3.6. The Adiabatic Expansion of an Ideal Gas

3.4. The Molar Specific Heats of an Ideal Gas

Let's consider our ideal gas of n moles that is a monatomic gas, which has individual atoms, e.g. helium, argon, neon.
For a single atom, the average translational KE:

$$\overline{K} = \frac{3}{2}kT$$

The internal energy E_{int} of the gas (no rotational KE for monatomic gases):

$$E_{int} = \sum_{1}^{N} \overline{K} = \frac{3}{2}kT \times nN_A = \frac{3}{2}nRT$$

Recall molar specific heat: $Q = Cn\Delta T$

a. Molar specific heat at constant volume:

- Consider n moles of an ideal gas at state i: p, T, and fixed V → state f: p+Δp, T+ΔT

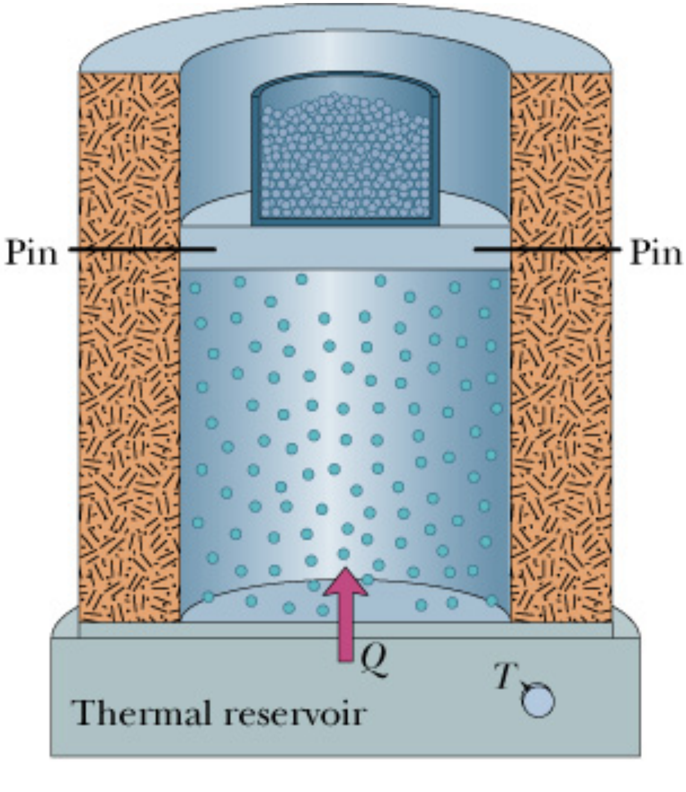
$$\boxed{Q = nC_V\Delta T}$$

C_V is a constant and called the molar specific heat at constant volume.

$$\boxed{\Delta E_{\text{int}} = Q - W = nC_V\Delta T - W = \frac{3}{2}nR\Delta T}$$

$$\text{Since } W = 0 \Rightarrow \boxed{C_V = \frac{3}{2}R = 12.5\ \text{J mol}^{-1}\ \text{K}^{-1}}$$

Note: For diatomic and polyatomic gases, their C_V is greater than that of monatomic gases.



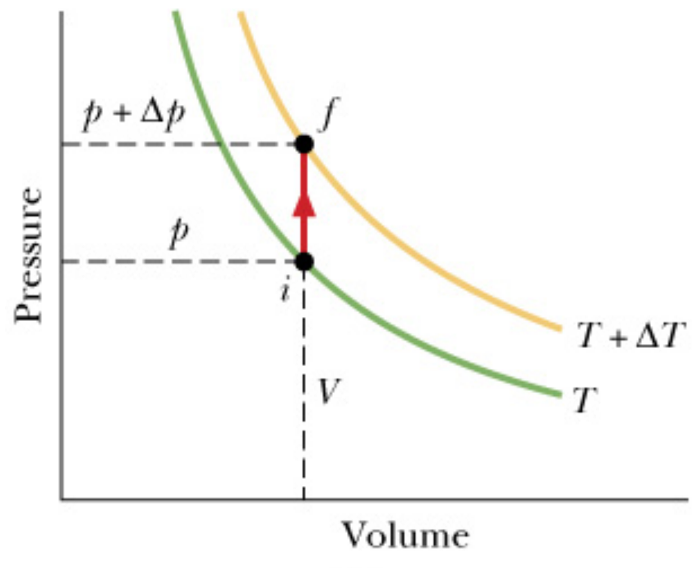
(a)

So, the change in internal energy can be calculated by:

$$\Delta E_{\text{int}} = \frac{3}{2}nR\Delta T$$

or

$$\Delta E_{\text{int}} = nC_V\Delta T$$



(b)

b. Molar specific heat at constant pressure:

$$\boxed{Q = nC_p \Delta T}$$

C_p is the molar specific heat at constant pressure.

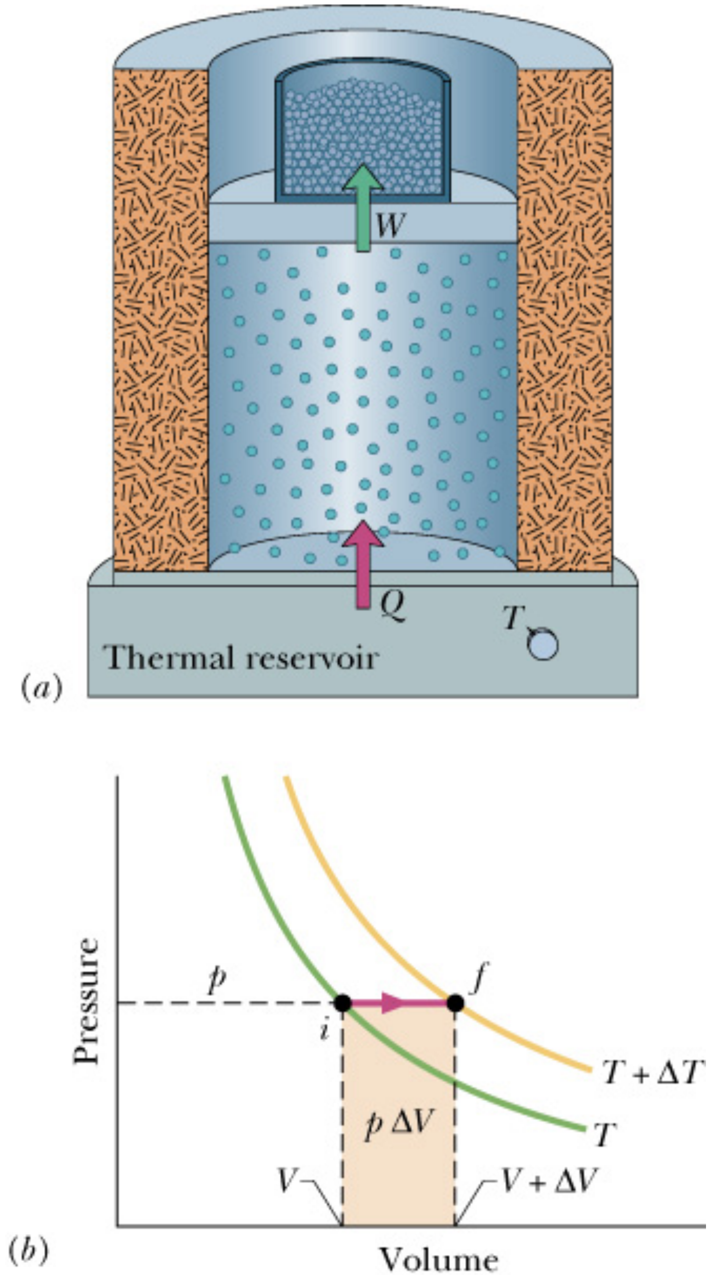
$$\Delta E_{\text{int}} = Q - W$$

$$W = p\Delta V = nR\Delta T$$

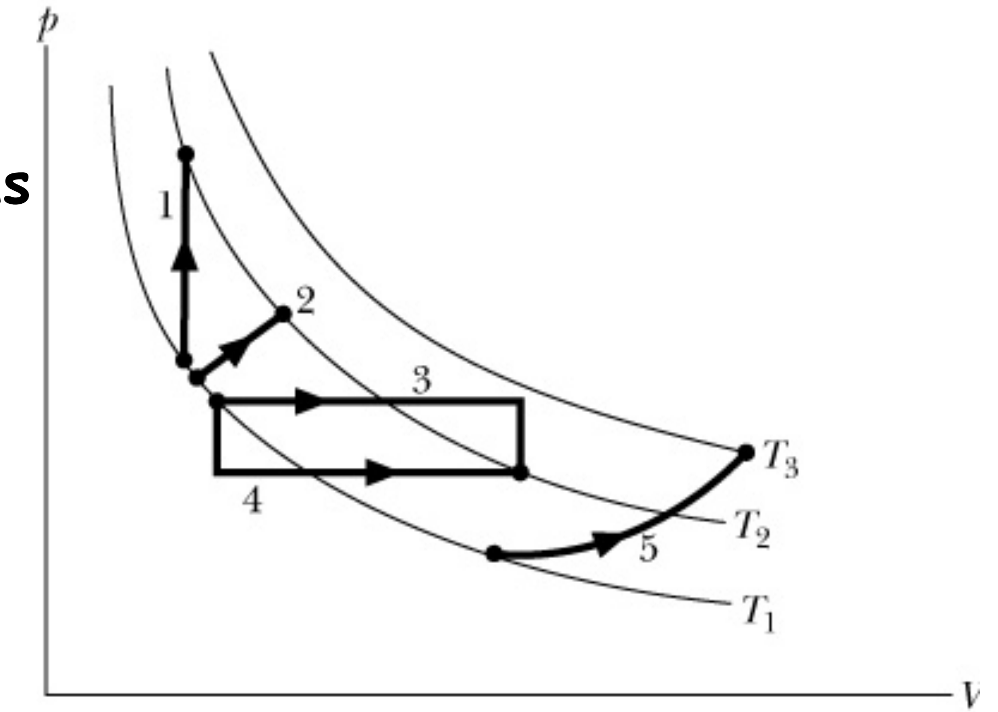
$$\Rightarrow \frac{3}{2} nR\Delta T = nC_p \Delta T - nR\Delta T$$

$$\boxed{C_p = \frac{3}{2}R + R = \frac{5}{2}R}$$

$$\boxed{\boxed{C_p = C_V + R}}$$

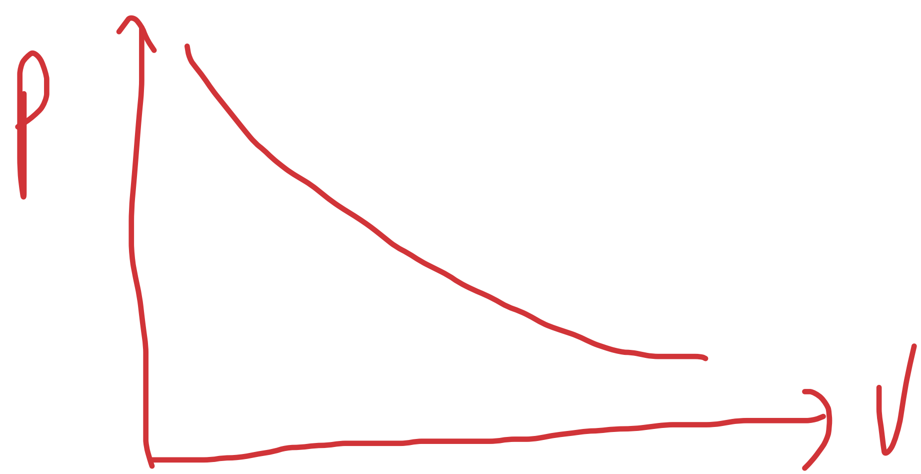


Checkpoint 4 (p. 522): The figure here shows 5 paths traversed by a gas on a p-V diagram. Rank the paths according to the change in internal energy of the gas, greatest first.



$$\Delta E_{\mathrm{int}} = \frac{3}{2} nR\Delta T$$

$$T_3 > T_2 > T_1$$



Example: (Problem 8, page 530) Suppose 1.8 mol of an ideal gas is taken from a volume of 3.0 m^3 to a volume of 1.5 m^3 via an isothermal compression at 30°C. (a) How much energy is transferred as heat during the compression, and (b) is the transfer to or from the gas?

(a) We have: $$\Delta E_{int} = Q - W = nC_V\Delta T = 0$$

$Q>0$ $\quad Q<0$

$\Delta E = 0$

An isothermal process: T=constant

$$\Delta E_{int} = 0 \Rightarrow Q = W = nRT \ln\left(\frac{V_f}{V_i}\right)$$

Work done by the gas for isotherm:

8.31

$$W = nRT \ln \frac{V_f}{V_i}$$

$$Q = W = 1.8 \times 8.31 \times (30+273) \times \ln \frac{1.5}{3.0} \approx -3142\ (J)$$

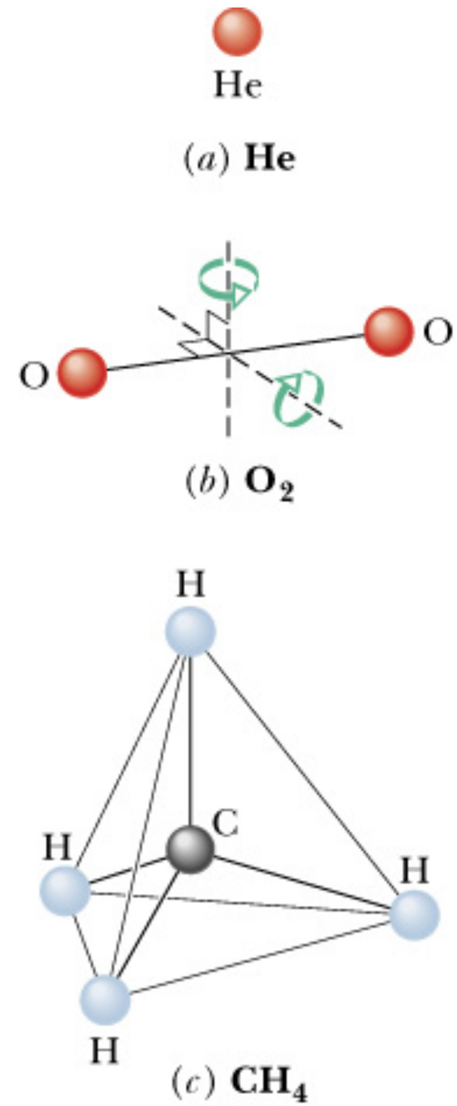
(b) Q<0: heat transferred from the gas to env

3.5. The Equipartition-of-Energy Theorem

Every kind of molecule has a certain number *f* of *degrees of freedom*. For each degree of freedom in which a molecule can store energy, the average internal energy is $\frac{1}{2}kT$ per molecule.

Molecule	Example	Degrees of freedom: Translational	Rotational	Total (f)
Monatomic	He	3	0	3
Diatomic	O_2	3	2	5
Polyatomic	CH_4	3	3	6

$$\boxed{C_V = \left(\frac{f}{2}\right)R \qquad C_p = C_V + R}$$



(a) He
(b) O_2
(c) CH_4

3.6. The Adiabatic Expansion of an Ideal Gas

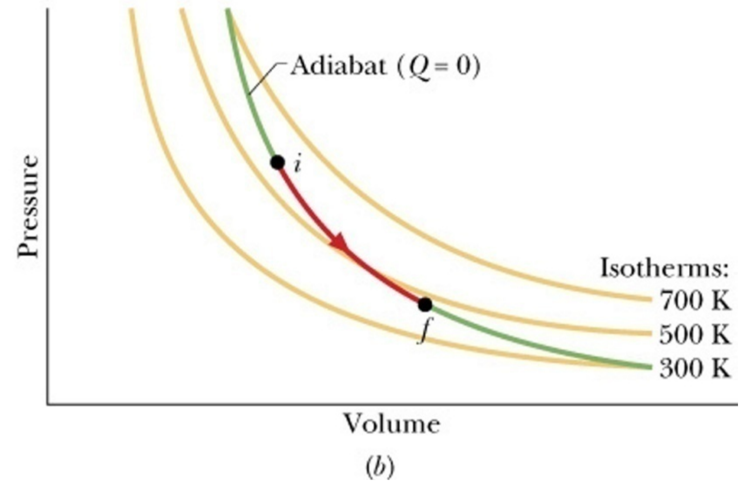
What is an adiabatic process?: a process for which Q = 0

$$\boxed{pV^{\gamma} = \text{constant}}$$

$$\boxed{\text{where } \gamma = C_p/C_V}$$

Equation of state: $pV = nRT$

$$\boxed{TV^{\gamma-1} = \text{constant}}$$



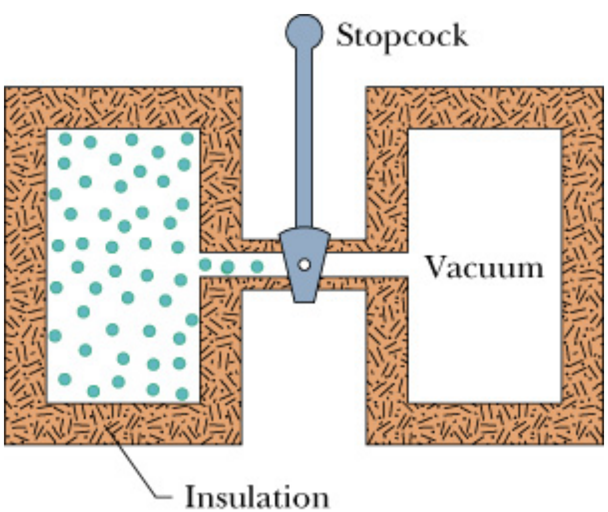
Proof of the equations above, see p. 526-527 (homework)

Free expansions:

$$\text{Recall: } Q = W = 0$$

$$\Delta E_{\text{int}} = 0 \Rightarrow T_i = T_f$$

$$\boxed{p_i V_i = p_f V_f}$$



Homework: 42, 44, 46, 54, 56, 78 (p. 533-535)