HO CHI MINH CITY INTERNATIONAL UNIVERSITY

MIDTERM TEST (Group 1)

Semester 3, Academic year 2018-2019

Duration: 90 minutes

SUBJECT: Calculus 2	
Chair of Department of Mathematics	Lecturer:
Signature:	Signature:
Full name:	Full name: Assoc.Prof. Mai Duc Thanh

<u>**Instructions:**</u>

- **Each student is allowed a maximum of two double-sided sheets of reference material (of size A4 or similar). All other documents and electronic devices, except scientific calculators, are not allowed.**
- **Each question carries 20 marks.**

Question 1. Find the following limits:

$$a)\quad \lim_{n\to\infty}\big(\ln(6n^2+n+1)-\ln(n^2+2n+5)\big) \qquad\qquad b)\quad \lim_{n\to\infty} n(\sqrt[n]{e}-1)$$

Question 2. Determine whether the given series is convergent or divergent:

$$a)\quad \sum_{n=1}^{\infty}\frac{\ln n}{n^2} \qquad\qquad b)\quad \sum_{n=1}^{\infty}\sin(\frac{1}{n})$$

Question 3. Find a power series representation for the function $f(x)=\dfrac{x}{(1+2x)^2}$ and determine the radius of convergence of the power series.

Question 4. Determine whether the following two lines are parallel, intersecting, or skew. If they are skew, find the distance between them

$$L_1: \qquad x=1+t, \quad y=1+6t, \quad z=2t$$

and

$$L_2: \qquad 1+2s, \quad y=5+15s, \quad z=-2+6s$$

Question 5. (a) Find the limit of the given vector function

$$\lim_{t\to 0}\left\langle \frac{\sqrt{1+t}-\sqrt{1-t}}{t}, t^2+2, \frac{1}{t}-\frac{1}{t^2+t}\right\rangle.$$

(b) Find parametric equations for the tangent line to the curve $\mathbf{r}(t) =< t\cos t, t\sin t, t >$, $\quad 0\le t\le 2\pi$ at the point $(0,\pi/2,\pi/2)$.

———END OF QUESTIONS———

CALCULUS 2
Solutions for Mid-term Test

Question 1. a)

$$\lim_{n\to\infty}(\ln(6n^2+n+1)-\ln(n^2+2n+5)) = \lim_{n\to\infty}\ln\frac{6n^2+n+1}{n^2+2n+5} = \ln\left(\lim_{n\to\infty}\frac{6+1/n+1/n^2}{1+2/n+5/n^2}\right) = \ln 6$$

b) We have

$$\begin{aligned}\lim_{n\to\infty} n(\sqrt[n]{e}-1) &= \lim_{x\to\infty} x(e^{1/x}-1)\\ &= \lim_{x\to\infty}\frac{e^{1/x}-1}{1/x} = \lim_{x\to\infty}\frac{e^{1/x}(1/x)'}{(1/x)'}\\ &= e^0 = 1.\end{aligned}$$

Question 2. a) Set

$$f(x) = \frac{\ln x}{x^2}, \quad x \geq 3.$$

Then $f(x)$ is continuous, decreasing, and $f(n) = a_n$. Since

$$\begin{aligned}\int_3^\infty \frac{\ln x}{x^2}dx &= \int_3^\infty \ln x d(-1/x) = -\ln x/x\Big|_3^\infty + \int_3^\infty 1/x^2 dx\\ &= \ln 3/3 - 1/x\Big|_3^\infty = (\ln 3+1)/3.\end{aligned}$$

This implies the series $\sum_{n=3}^\infty (\ln n)/n^2$ is convergent so is the series $\sum_{n=1}^\infty (\ln n)/n^2$.

b) We have

$$\lim_{n\to\infty}\frac{\sin 1/n}{1/n} = 1$$

The series $\sum 1/n$ diverges, by the limit comparison test, the given series diverges.

Question 3. We have

$$\frac{1}{1-x} = \sum_{n=0}^\infty x^n, \quad |x| < 1.$$

So

$$\frac{1}{1+2x} = \frac{1}{1-(-2x)} = \sum_{n=0}^\infty (-2x)^n = \sum_{n=0}^\infty (-2)^n x^n, \quad |x| < 1/2.$$

Using differentiation

$$\frac{d}{dx}\frac{1}{1+2x} = \frac{-2}{(1+2x)^2} = \sum_{n=1}^\infty (-2)^n n x^{n-1}, \quad |x| < 1/2$$

so

$$\frac{x}{(1+2x)^2} = \sum_{n=1}^{\infty}(-2)^{n-1}nx^n, \quad |x| < 1/2.$$

$R = 1/2$.

Question 4. Let (α) be the plane containing (L_1) and parallel to (L_2). Then the distance between the skew lines (L_1) and (L_2) is equal to the distance from $M(1,5,-2)$ on (L_2) to (α).

The normal vector n of (α) can be chosen as

$$n =< 1,6,2 > \times < 2,15,6 >=< 6,-2,3 > .$$

Hence, the plane has the equation

$$6(x-1) - 2(y-1) + 3z = 0$$

or

$$6x - 2y + 3z - 4 = 0.$$

Therefore, the distance is

$$d = \frac{|6 \times 1 - 2 \times 5 + 3 \times (-2) - 4|}{\sqrt{6^2 + 2^2 + 3^2}} = 2.$$

Question 5. (a) It holds that

$$\lim_{t\to 0} \frac{\sqrt{1+t} - \sqrt{1-t}}{t} = \lim_{t\to 0} \frac{(1+t)-(1-t)}{t(\sqrt{1+t}+\sqrt{1-t})} = \lim_{t\to 0} \frac{2}{(\sqrt{1+t}+\sqrt{1-t})} = 1$$

$$\lim_{t\to 0}\left(\frac{1}{t} - \frac{1}{t^2+t}\right) = \lim_{t\to 0}\frac{t^2+t-t}{t(t^2+t)} = \lim_{t\to 0}\frac{1}{t+1} = 1.$$

So

$$\lim_{t\to 0}\left\langle \frac{\sqrt{1+t}-\sqrt{1-t}}{t}, t^2+2, \frac{1}{t} - \frac{1}{t^2+t} \right\rangle =< 1,2,1 >$$

(b) $\mathbf{r}(t) =< t\cos t, t\sin t, t >, \quad 0 \le t \le 2\pi.$

It holds that

$$\mathbf{r}'(t) =< \cos t - t\sin t, \sin t + t\cos t, 1 >, \quad 0 \le t \le 2\pi.$$

The point $A(0, \pi/2, \pi/2)$ on the curve corresponds to $t = \pi/2$. So $\mathbf{r}'(\pi/2) =< -\pi/2, 1, 1 >$. Thus the tangent line has equations

$$x = -(\pi/2)t, \quad y = \pi/2 + t, \quad z = \pi/2 + t.$$