Techniques of
Circuit Analysis
(Chapter 4)

Textbook:

Electric Circuits

James W. Nilsson & Susan A. Riedel

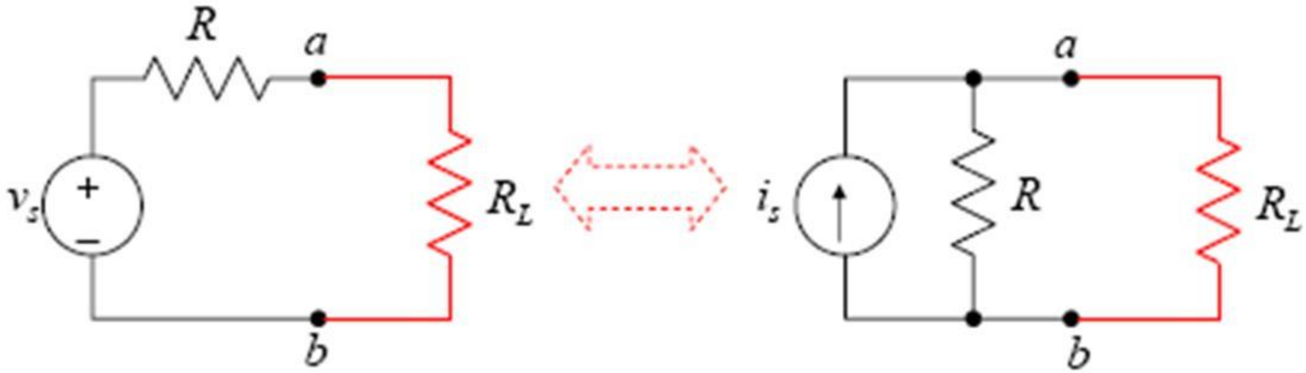
9th Edition.

Outline

- *The node-voltage method*
- *The mesh-current method*
- *Source transformation*
- *Thevenin & Norton equivalents*
- *Maximum power transfer*
- *Super position*

Source Transformation

- A simplification technique that allows a voltage source in series with a resistor to be replaced by a current source in parallel with the same resistor.



$$i_L = \frac{v_s}{R+R_L} \quad \Longleftrightarrow \quad i_L = \frac{R}{R+R_L} i_s$$

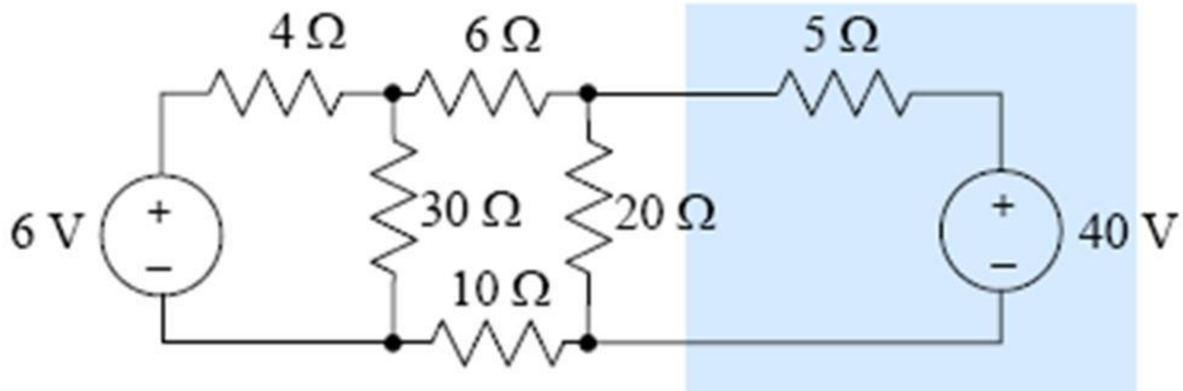
$$\boxed{i_s = \frac{v_s}{R}}$$

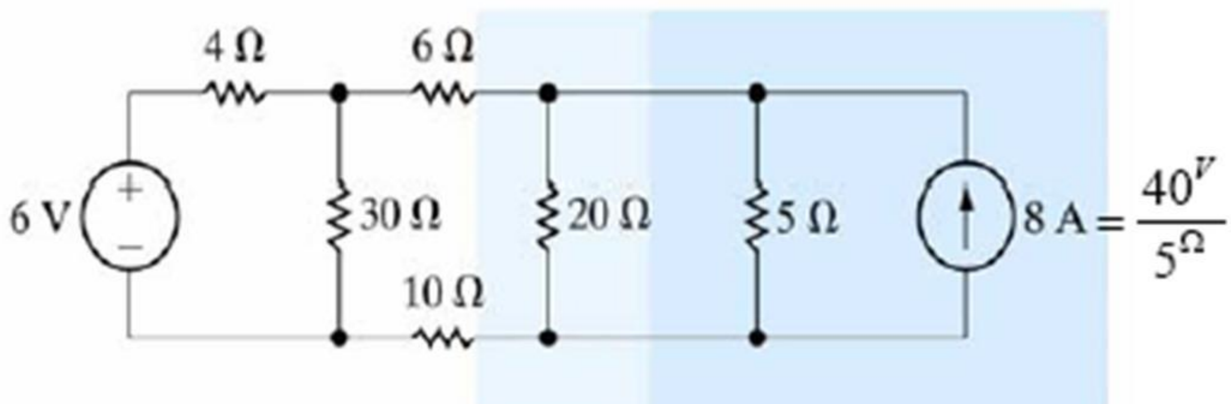
Example 5

Hỏi chỗ nào giữ lại chỗ đó

Determine the power associated with the 6 V source.

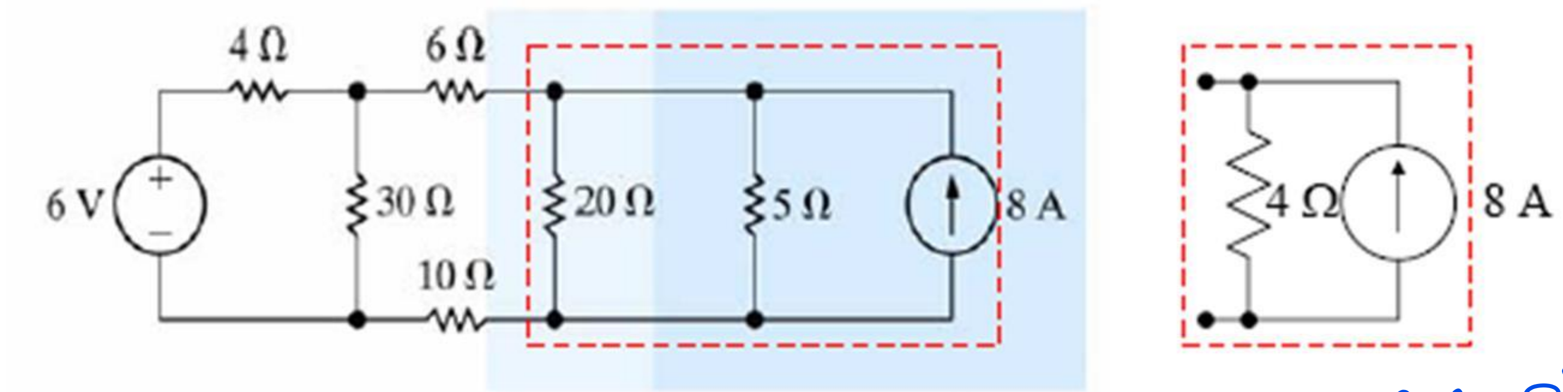
Ans.:-





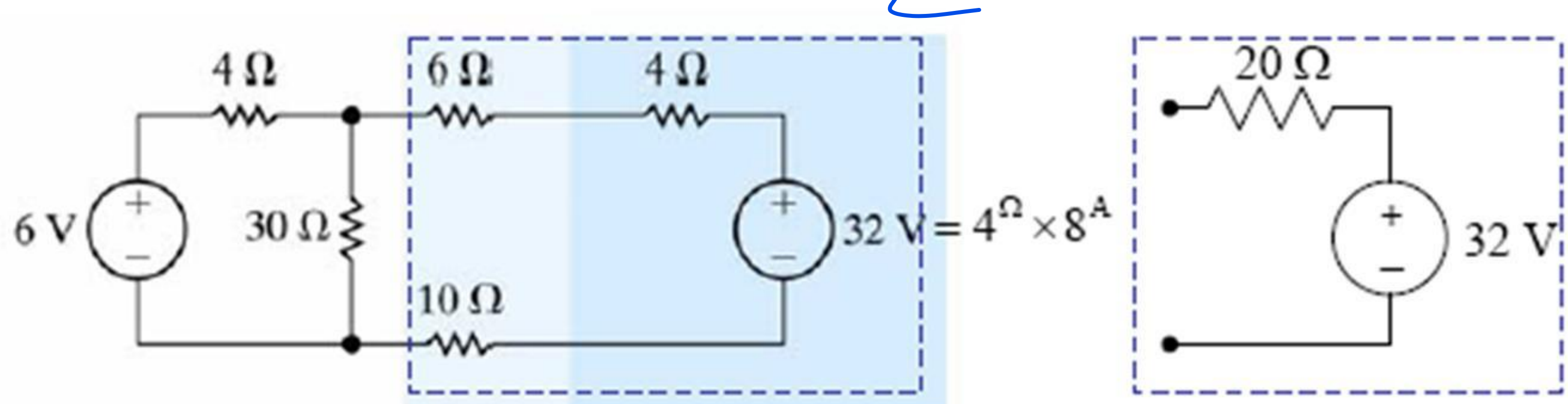
(a) First step

Example (Cont.)



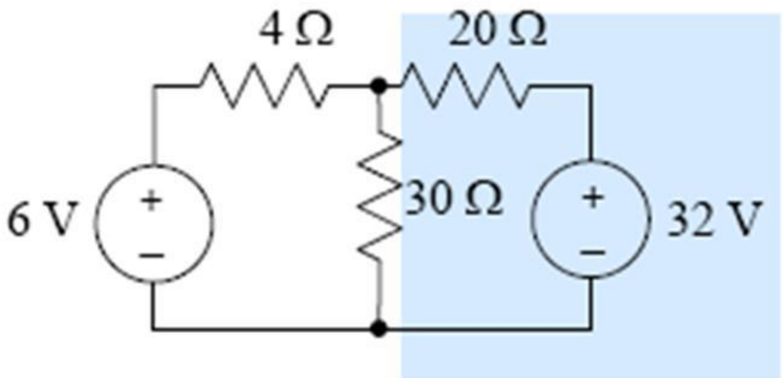
(a) First step

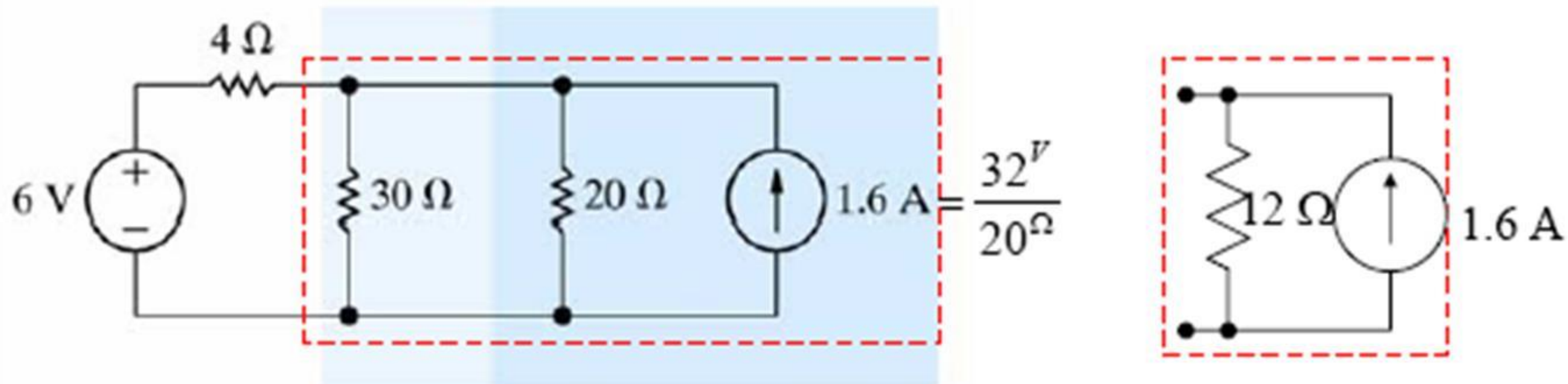
In series



(b) Second step

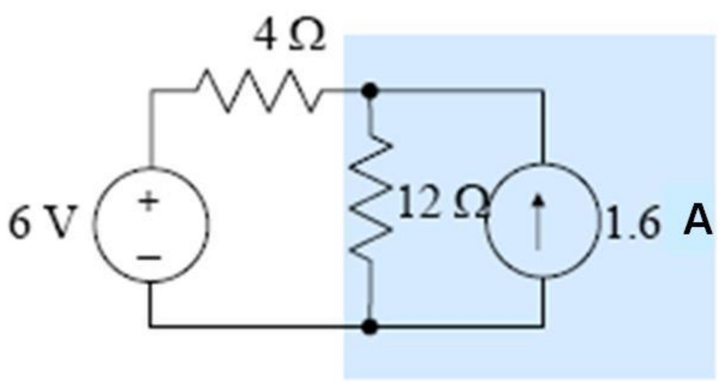
Example(Cont.)

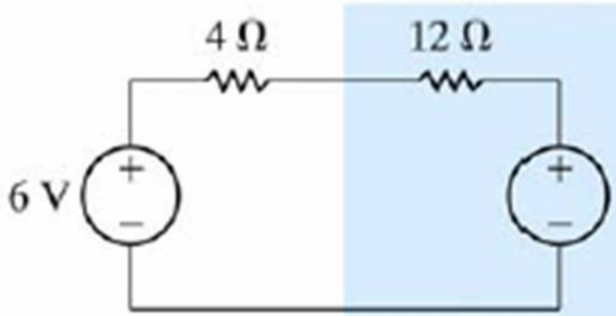




(c) Third step

Example (Cont.)





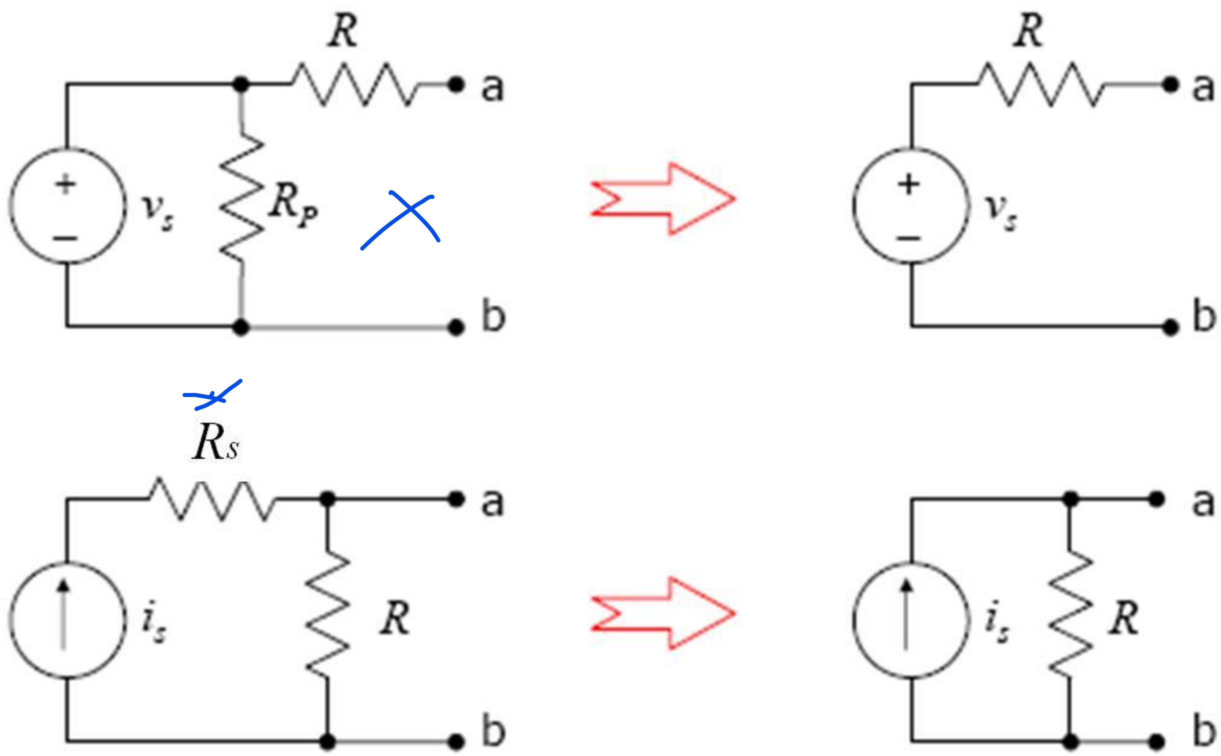
$19.2\ \text{V} = 12^{\Omega} \times 1.6^{A}$

(d) Fourth step

$$\boxed{i_{6V} = \frac{6 - 19.2}{16} = -0.825\ \text{A}}$$

Special Case

- What happens if there is a resistance R_p in parallel with the voltage source or a resistance R_s in series with the current source?

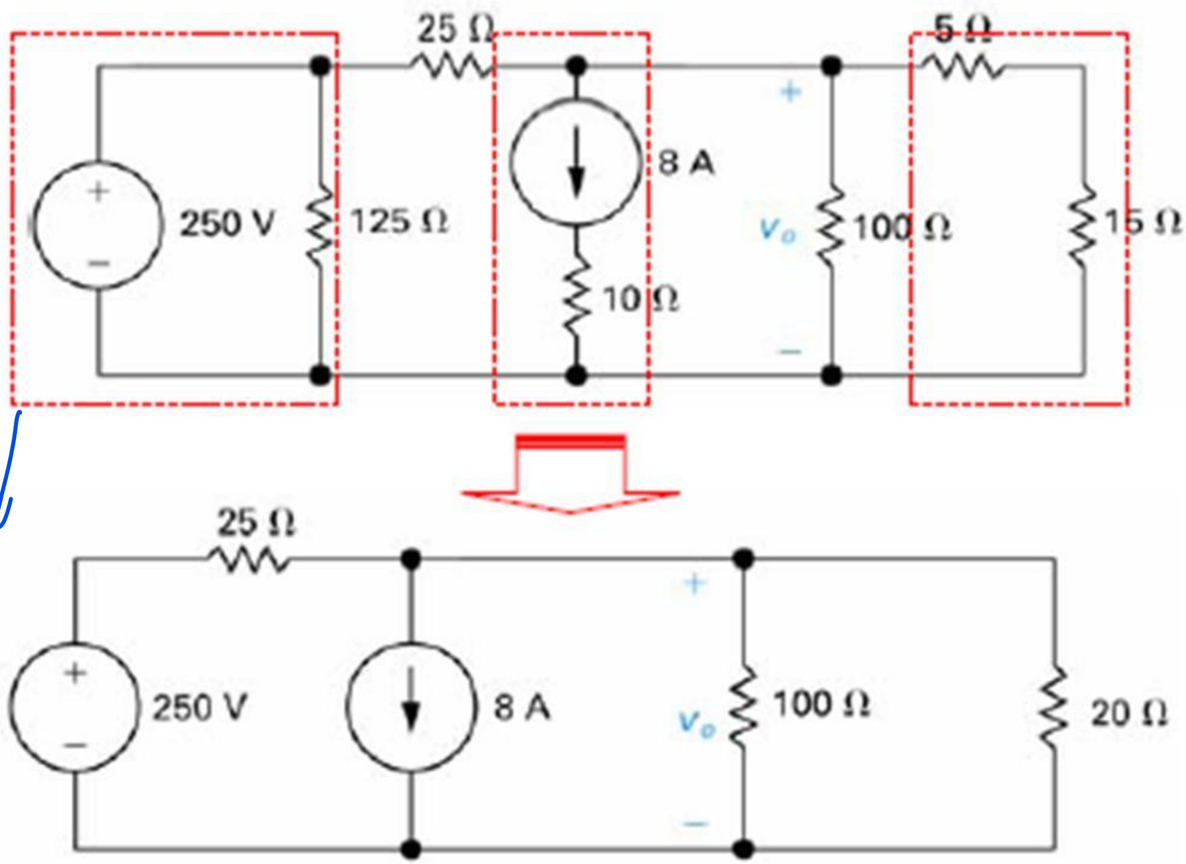


Example 6

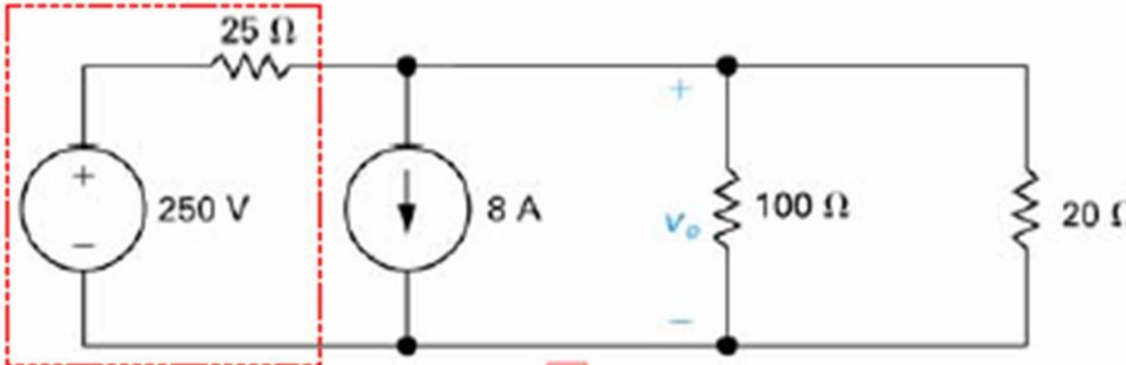
Find v_o?

Ans.:-

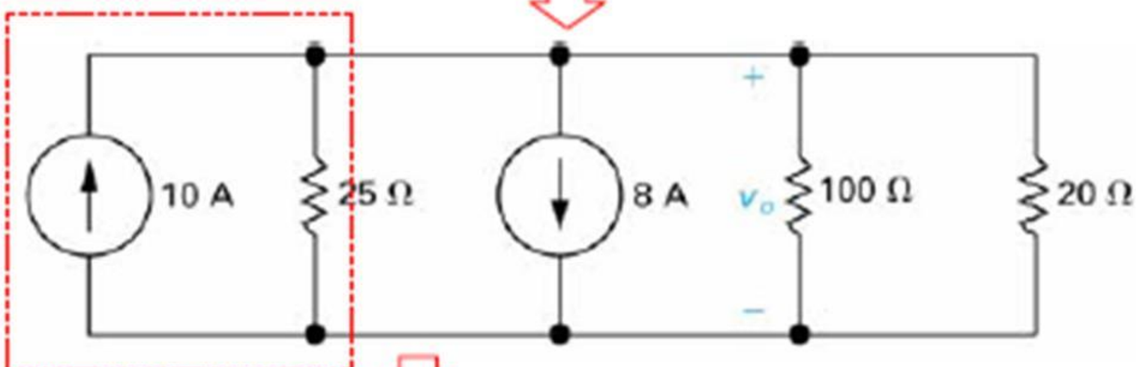
special case



Example (Cont.)



$$10\text{ A} = \frac{250^{\text{V}}}{25^{\Omega}}$$



$$10\text{ A} - 8\text{ A} = 2\text{ A}$$

$$10\,\Omega = \left(\frac{1}{\frac{1}{25}+\frac{1}{100}+\frac{1}{20}}\right)$$

$$\boxed{v_o = 20\text{ V}}$$

Th: thev

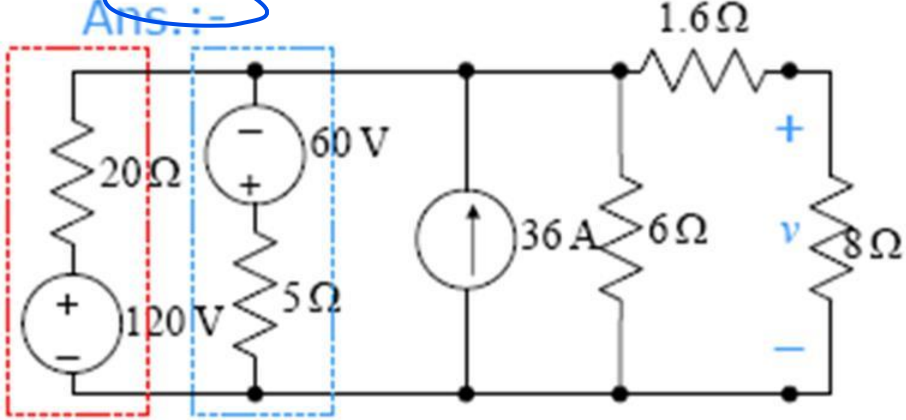
$V = 2 \times 10$

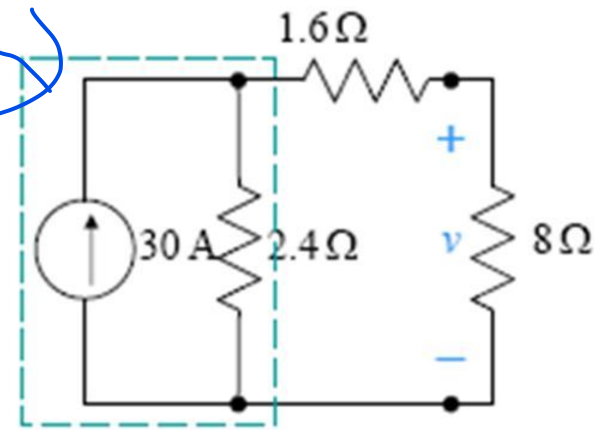
Assessing Objective 9

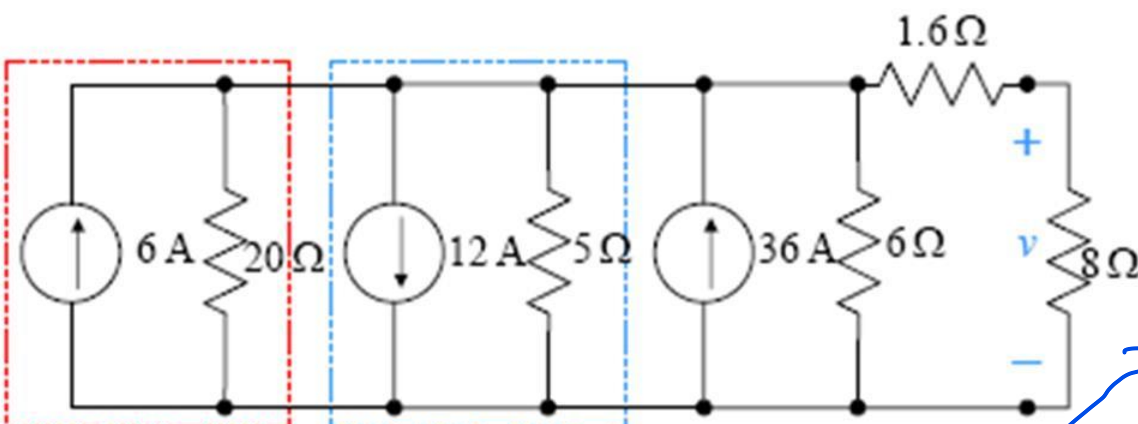
$I = 56 + 6 - 12 = 30$

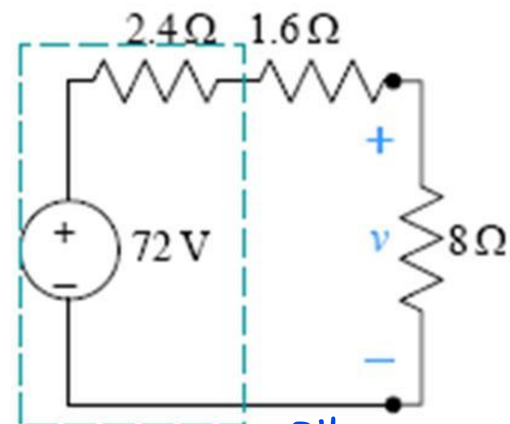
Find v?

Ans.:-









voltage divider

$$\boxed{v = 72\frac{8}{12} = \underline{48\text{ V}}}$$

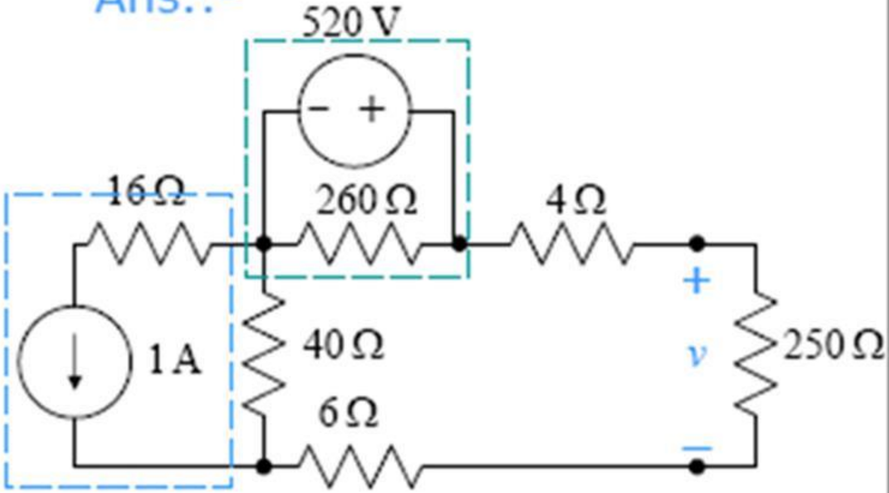
$1 : \left(\frac{1}{20} + \frac{1}{5} + \frac{1}{6}\right) R$

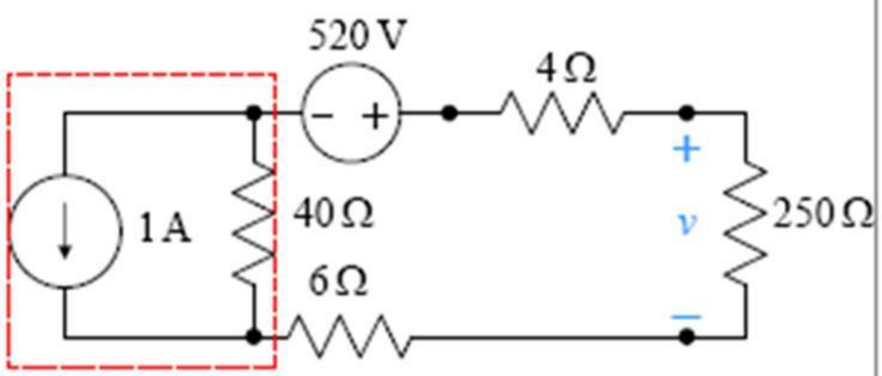
$u = IR = 12$

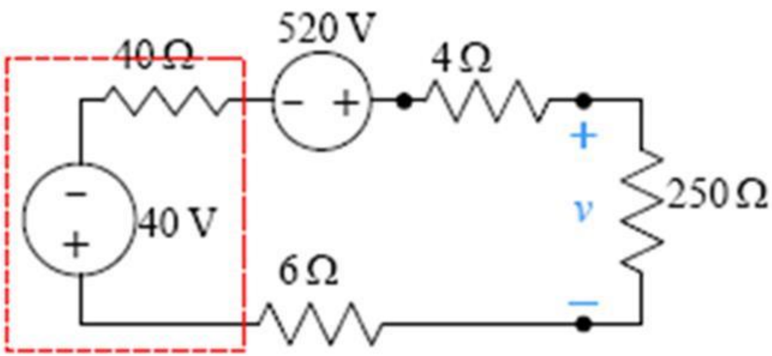
Problem 6

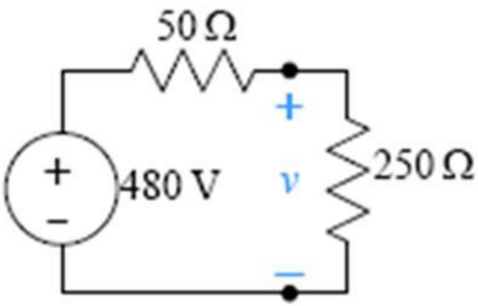
Find v?

Ans.:-









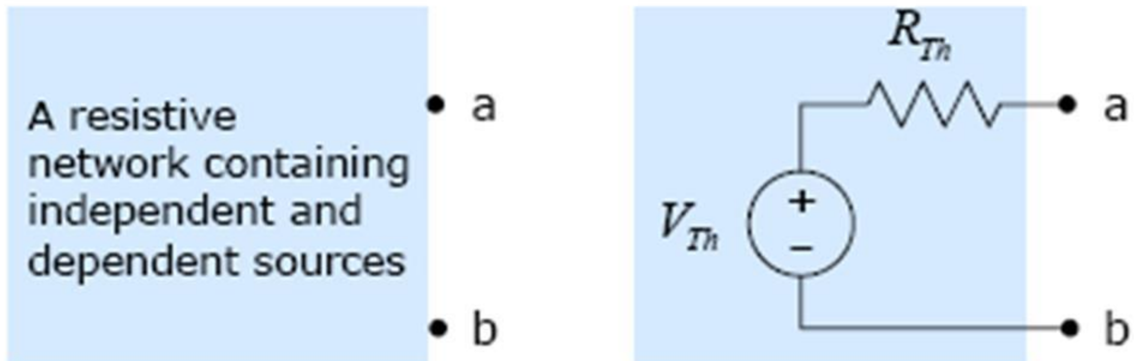
$$\boxed{v = 480\frac{250}{300} = 400\text{ V}}$$

Thévenin and Norton Equivalents

- Used when you want to concentrate on what happens at a specific pair of terminals.

- They are circuit simplification techniques that focus on terminal behavior.

Thévenin equivalent circuit



V_{Th} is the open-circuit voltage in the original circuit.
R_{Th} is the ratio of the open-circuit voltage to the short-circuit current.

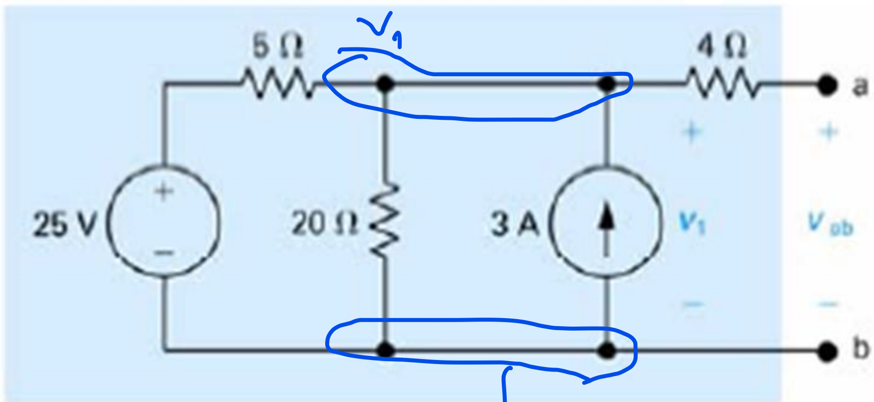
$$\boxed{R_{Th} = \frac{V_{Th}}{i_{sc}}}$$

Check open → $V_{th} = V_{ab}$

Thévenin equivalent circuit

node V_1

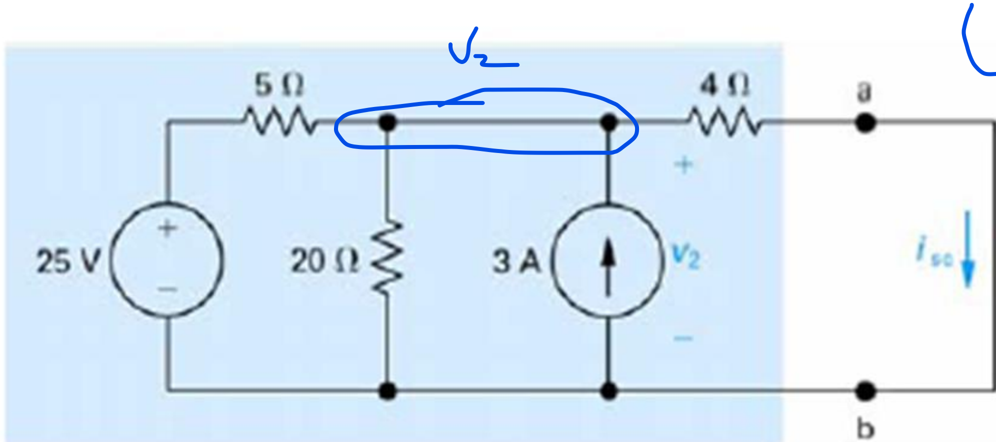
V_{Th} is the open-circuit voltage in the original circuit.



$$\frac{v_1-25}{5}+\frac{v_1}{20}-3=0$$

$$v_1 = 32\text{ V} = V_{ab}$$

$$\boxed{V_{Th}=32\text{ V}}$$



$$\frac{v_2-25}{5}+\frac{v_2}{20}-3+\frac{v_2}{4}=0$$

$$v_2 = 16\text{ V}$$

$$\boxed{i_{sc}=\frac{16}{4}=4\text{ A}}$$

$$\boxed{R_{Th}=\frac{V_{Th}}{i_{sc}}=\frac{32}{4}=8\,\Omega}$$

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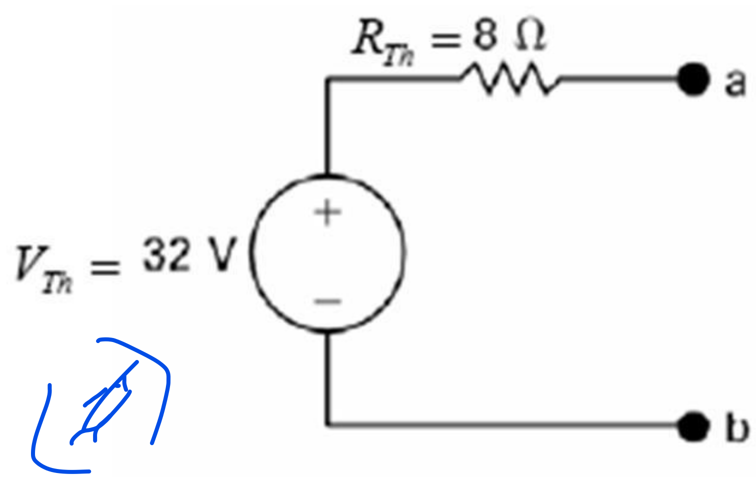
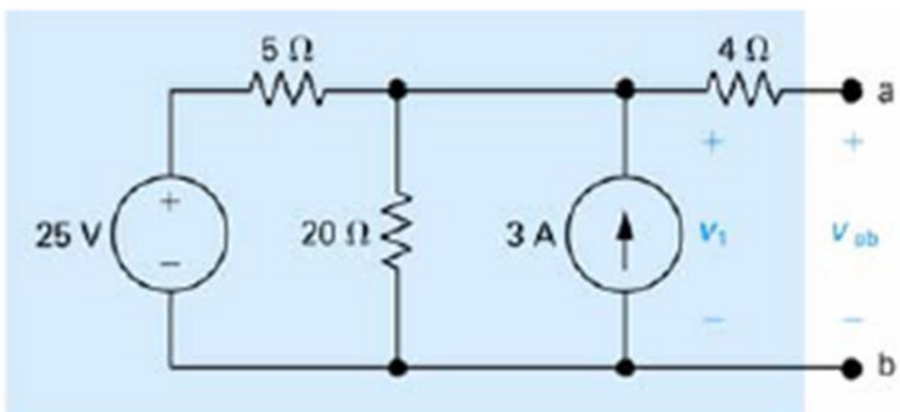
Thévenin equivalent circuit

$$\boxed{R_{Th} = \frac{V_{Th}}{i_{sc}}} = \frac{32}{4} = 8\,\Omega$$

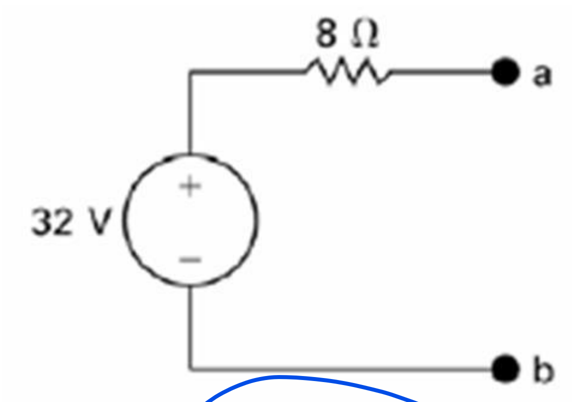




The Norton equivalent circuit

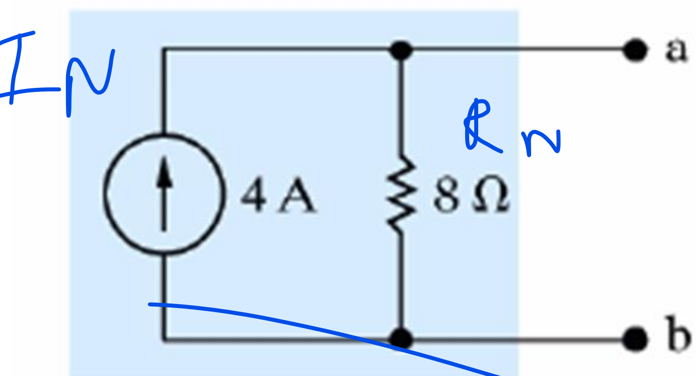
- Consists of an independent current source in parallel with the Norton equivalent resistance.
- Can be derived from Thévenin equivalent circuit simply by making a source transformation.

Source Trans



Thévenin Equivalent Circuit

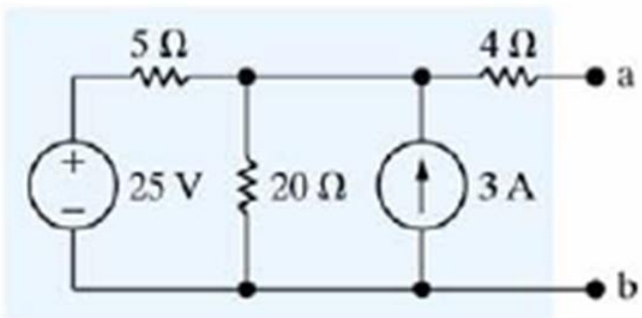
I_N



R_n

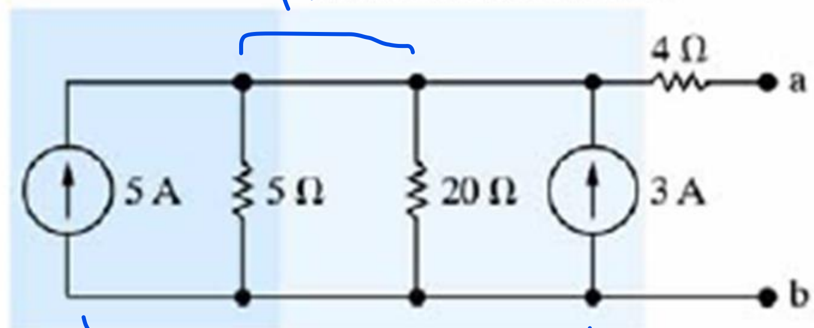
Norton Equivalent Circuit

The Norton equivalent circuit



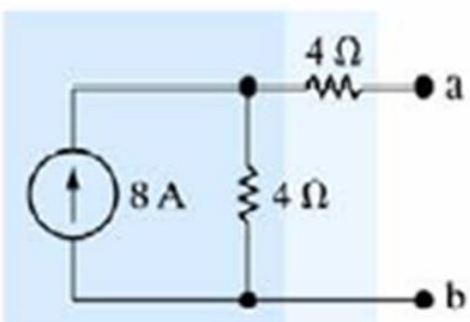
4Ω

Step 1:
Source transformation

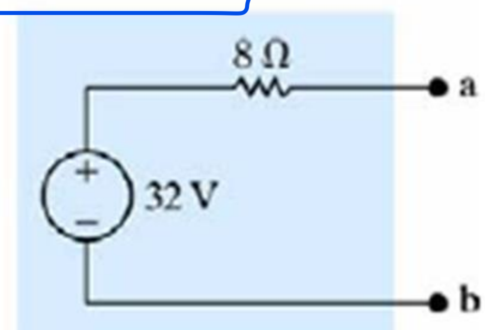


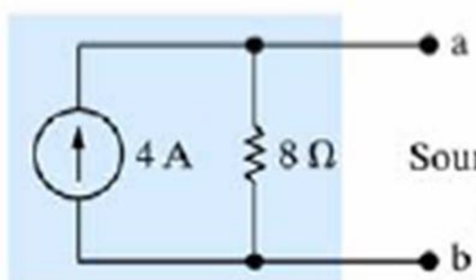
8A

Step 2:
Parallel sources and
parallel resistors combined



Step 3:
Source transformation; series
resistors combined, producing
the Thévenin equivalent circuit





Step 4:
Source transformation, producing
the Norton equivalent circuit

Example 7

$i = \frac{5 - 3V}{2000}$

Find V_{Th} & R_{Th}?

ans.:-

1st open circuit to evaluate V_{Th}

$\boxed{i_x = 0}$

$$v = -(20i)(25) = -500i$$

$$-5 + i2000 + 3v = 0$$

$$\boxed{i = \frac{5 - 3v}{2000}}$$

$$v = -5\text{ V}$$

2nd short circuit to evaluate R_{Th}

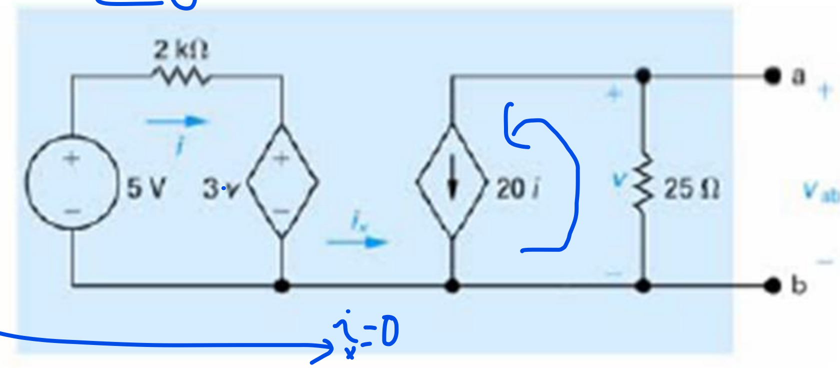
$$v = 0\text{ V}$$

$$i_{sc} = -20i$$

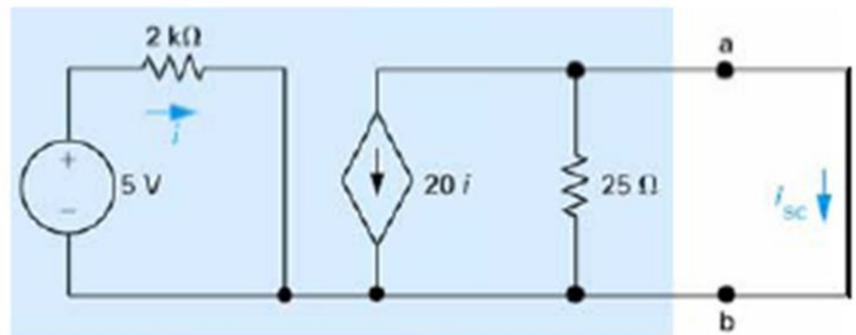
$$i = \frac{5}{2000} = 2.5\text{ mA}$$

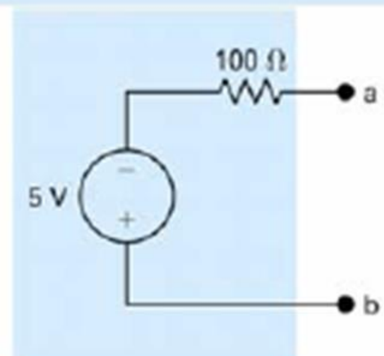
$$i_{sc} = -50\text{ mA}$$

$$\boxed{R_{Th} = \frac{V_{Th}}{i_{sc}} = 100\,\Omega}$$



i=0





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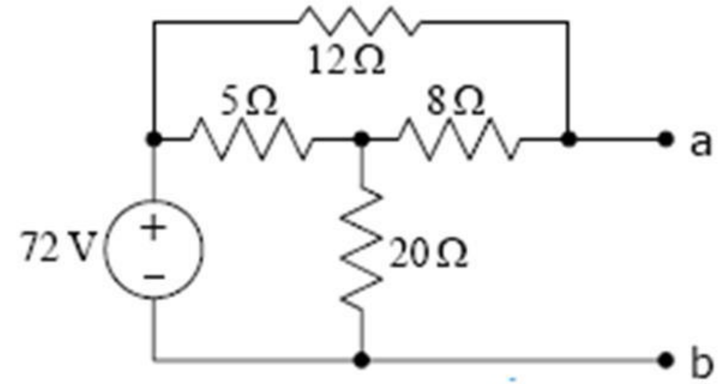
Assessing Objective 10

Find V_{Th} & R_{Th}? $V_{th} = V_8 + V_{20}$



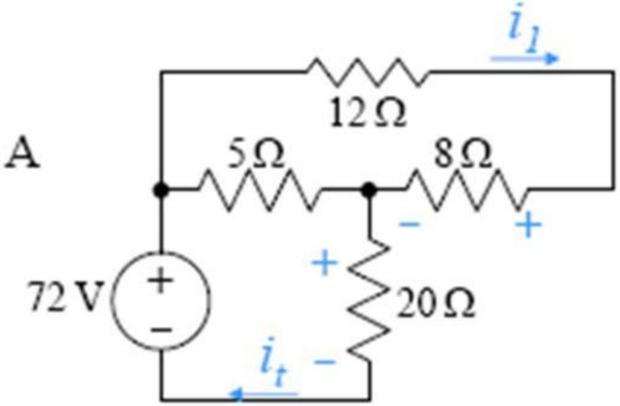
Ans.:-

1st open circuit to evaluate V_{Th}

$$R_{eq} = (12\,\Omega + 8\,\Omega) // (5\,\Omega) + 20\,\Omega$$

$$R_{eq} = 4\,\Omega + 20\,\Omega = 24\,\Omega$$

$$i_t = 72/24 = 3\text{ A} \qquad i_1 = 3\frac{5}{12+8+5} = 0.6\text{ A}$$



$$V_{Th} = 0.6\times 8 + 3\times 20 = 64.8\text{ V}$$

2nd short circuit to evaluate R_{Th}

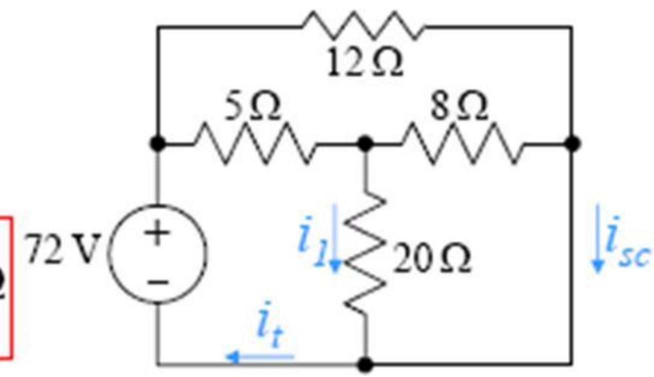
$$R_{eq} = \left[(8\Omega // 20\Omega) + 5\Omega\right] // 12\Omega$$

$$R_{eq} = 5.66\Omega \qquad i_t = 72/5.66 = 12.72\text{ A}$$

i_5

$$i_1 = 12.72\left(\frac{12}{10\frac{5}{7}+12}\right)\times\frac{8}{20+8} = 1.92\text{A}$$

$$i_{sc} = 12.72 - 1.92 = 10.8\text{A} \qquad \boxed{R_{Th} = \frac{V_{Th}}{i_{sc}} = 6\,\Omega}$$



5+8//20

short

open

$R_{Th} = R_{ab}$

Evaluating R_{Th} using source deactivating

- Useful if the network contains only independent sources.

- 1st deactivate all independent sources and then calculate the resistance seen looking into the network at the designated terminal pair.
  - A voltage source is deactivated by replacing it with a short circuit.
  - A current source is deactivated by replacing it with an open circuit.

Only for Independent source

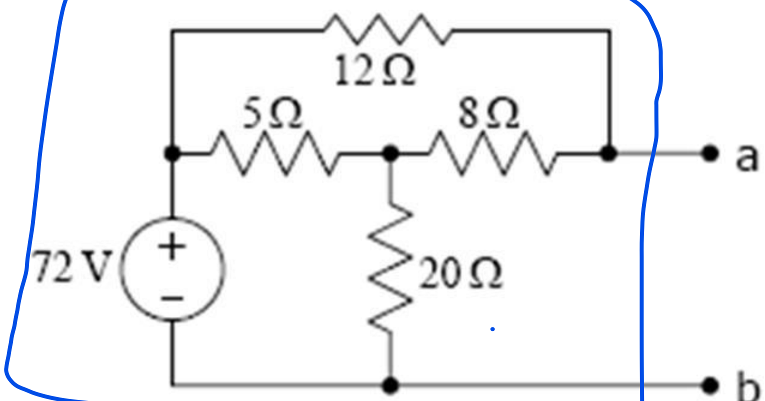
Example 8

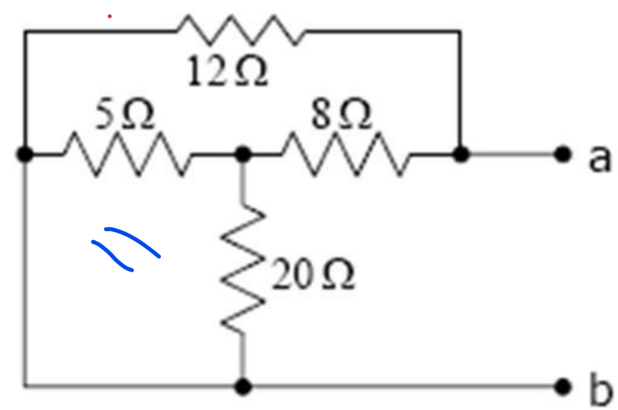
Find V_{Th} & R_{Th}?

Ans.:-

$V_{Th} = 64.8\,\text{V}$ (previous result)

Rth





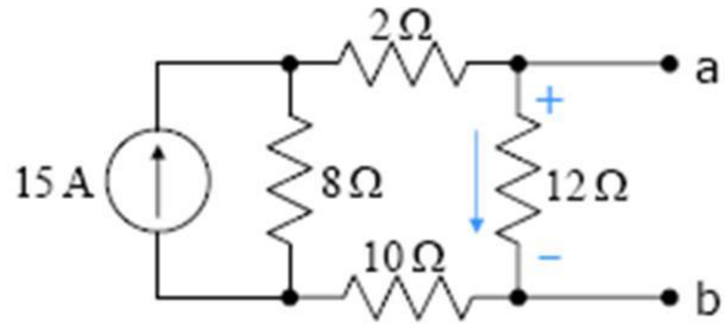
$$R_{Th} = \left[(5\Omega // 20\Omega) + 8\Omega\right] // 12\Omega$$

$$R_{Th} = 6\Omega$$

Assessing Objective 11

Find I_N & R_N?

Ans.:-



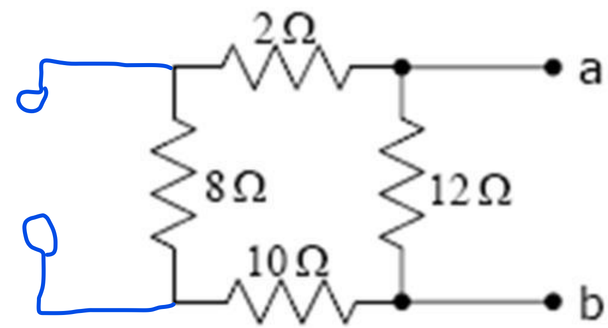
$$R_N = (2\Omega + 8\Omega + 10\Omega) // 12\Omega = 7.5\Omega$$

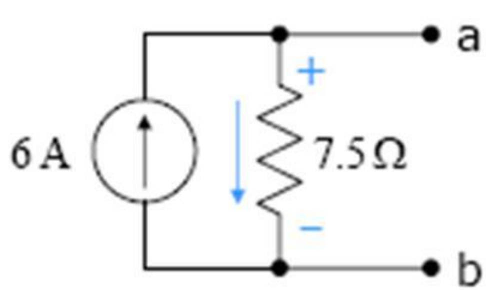
$$V_{Th} = 15\frac{8}{2+10+12+8}\times 12 = 45\text{V}$$

I_{12}

$$I_N = \frac{45}{7.5} = 6A$$

open

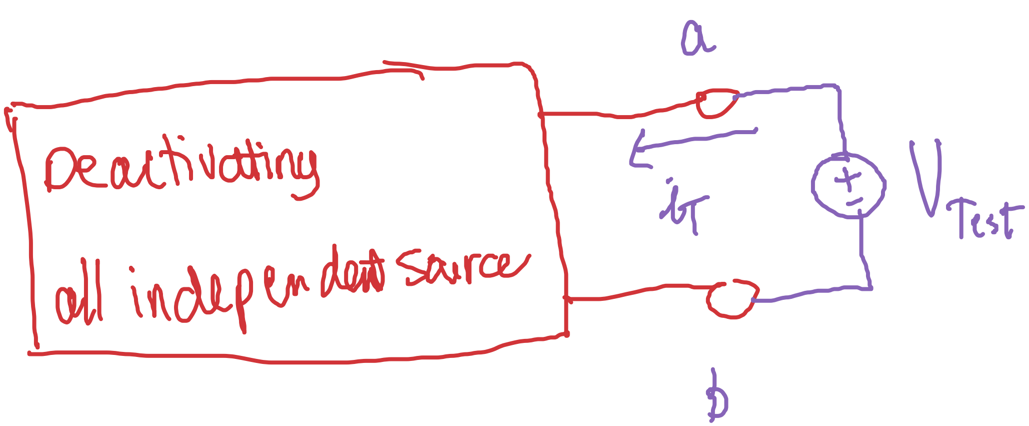




Evaluating R_{Th} using test source → Đưa nguồn thử

- First deactivate all independent sources, and we then apply either a test voltage source or a test current source to the Thévenin terminals a,b.

- The Thévenin resistance equals the ratio of the voltage across the test source to the current delivered by the test source.



apply any circuit

Example 9

Previous example $V_{th} = -5V$

Find V_{Th} & R_{Th}?

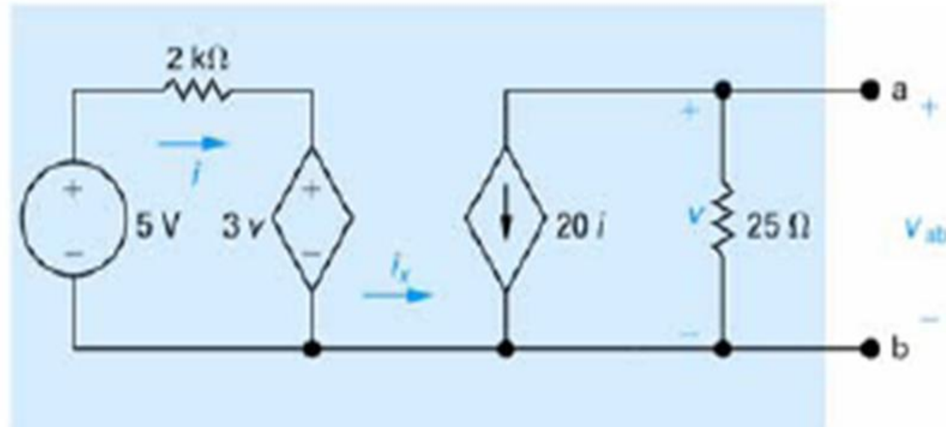
Ans.:-

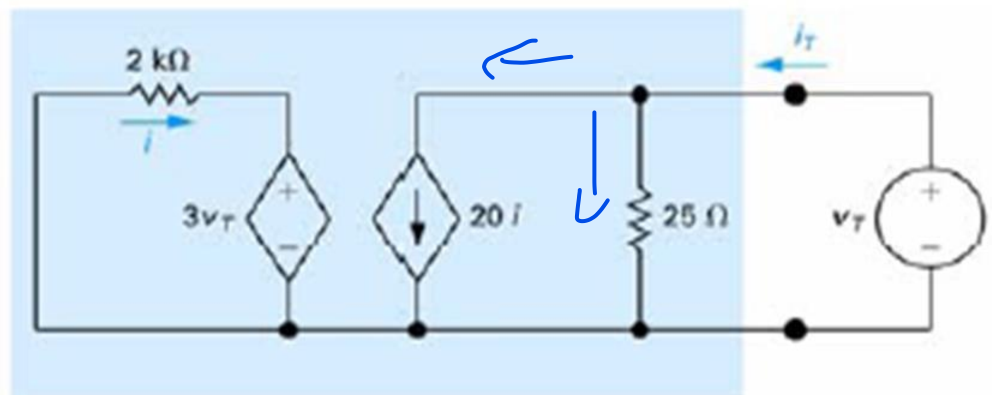
$$3v_T = -i2000$$

$$i_T = 20i + \frac{v_T}{25}$$

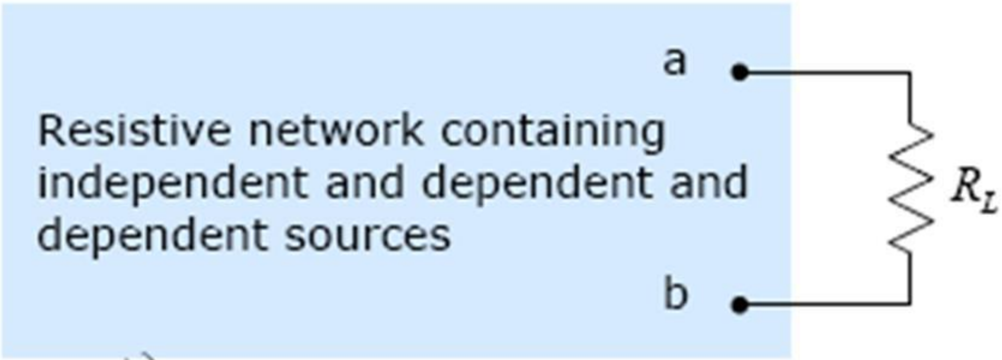
$$i_T = -20\frac{3v_T}{2000} + \frac{v_T}{25} = 0.01v_T$$

$$R_{Th} = \frac{v_T}{i_T} = 100\Omega$$





Maximum Power Transfer



$$p = i^2 R_L = \left(\frac{V_{Th}}{R_{Th} + R_L} \right)^2 R_L$$

$$\frac{dp}{dR_L} = V_{Th}^2 \left(\frac{(R_{Th} + R_L)^2 - R_L \cdot 2(R_{Th} + R_L)}{(R_{Th} + R_L)^4} \right)$$

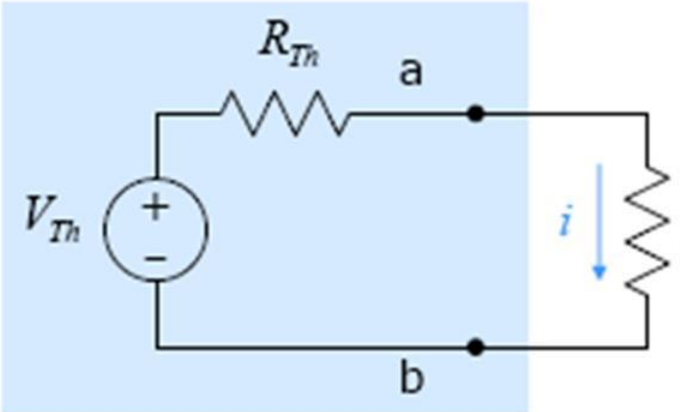
To maximize the function,
the derivative should be equal to zero

$$(R_{Th} + R_L)^2 = R_L \cdot 2(R_{Th} + R_L)$$

$$R_{Th} + R_L = 2R_L$$

$$\boxed{R_{Th} = R_L}$$

$$p_{max} = \frac{V_{Th}^2 R_L}{(2R_L)^2} = \frac{V_{Th}^2}{4R_L}$$



Example 10

, Rth

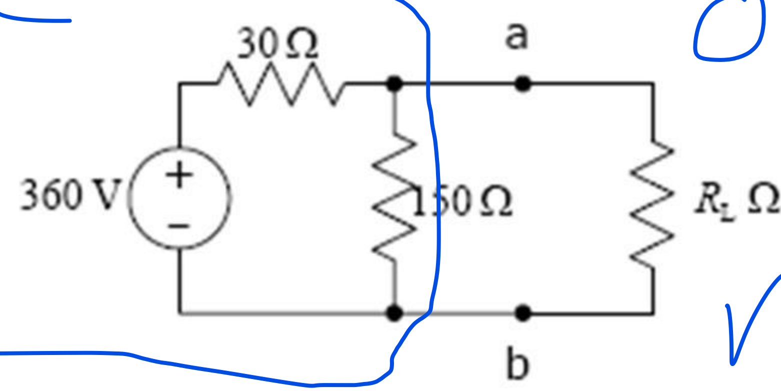
a) Find R_L to achieve maximum power at R_L.
b) Calculate maximum power at R_L.
c) Find the % of power from the source is delivered to R_L.

Ans.:-

a) $V_{Th} = 360 \dfrac{150}{150+30} = 300\ \text{V}$

$R_{Th} = 150 // 30 = 25\ \Omega$

$R_L = R_{Th} = 25\Omega$



Vab
open
remove
Load

b) $p = \dfrac{V_{Th}^2}{4R_L} = 900\ \text{W}$

NOT RL consider

c) $p_s = \dfrac{V_s^2}{R_{eq}} = \dfrac{360^2}{51.43} = 2520\ \text{W}$

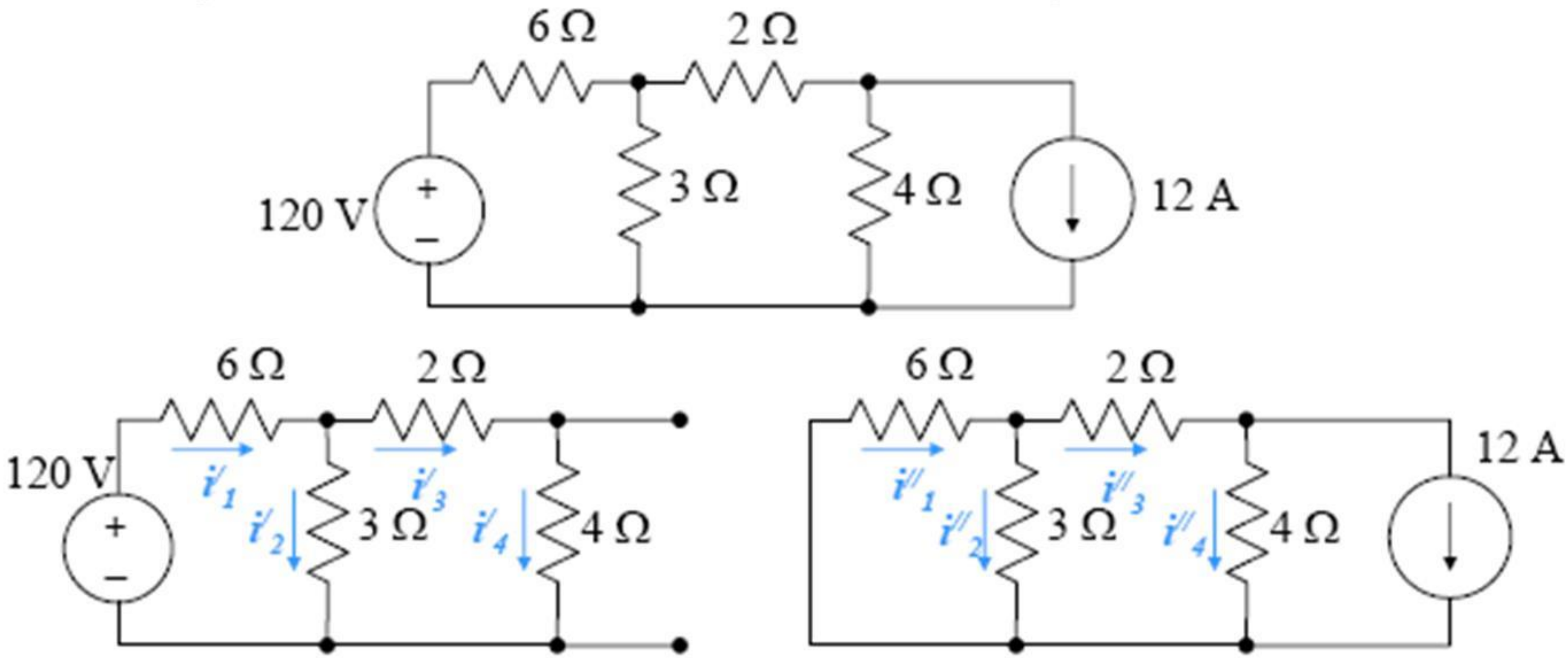
$\% p = \dfrac{900}{2520} \times 100 = 35.71\%$

$R_{eq} = 25//150 + 30$

Superposition

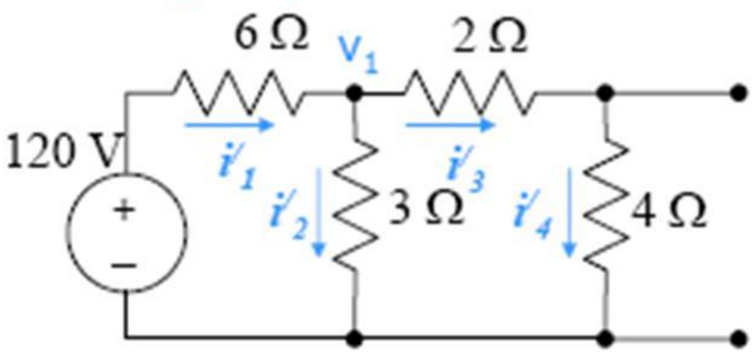
2 Source

- A linear system obeys the principle of superposition, which states that whenever a linear system is excited, or driven, by more than one independent source of energy, the total response is the sum of the individual responses.



giữ cho 1 nguồn.

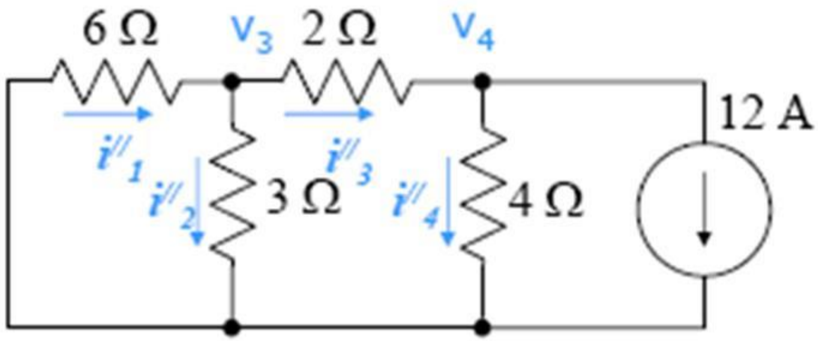
Superposition



$$\frac{v_1-120}{6}+\frac{v_1}{3}+\frac{v_1}{2+4}=0 \rightarrow v_1=30$$

$$\boxed{i_1'=\frac{120-30}{6}=15\text{A}} \qquad \boxed{i_2'=\frac{30}{3}=10\text{A}}$$

$$\boxed{i_3'=i_4'=\frac{30}{6}=5\text{A}}$$



$$\frac{v_3}{6}+\frac{v_3}{3}+\frac{v_3-v_4}{2}=0 \qquad v_3=-12$$

$$\frac{v_4-v_3}{2}+\frac{v_4}{4}+12=0 \qquad v_4=-24$$

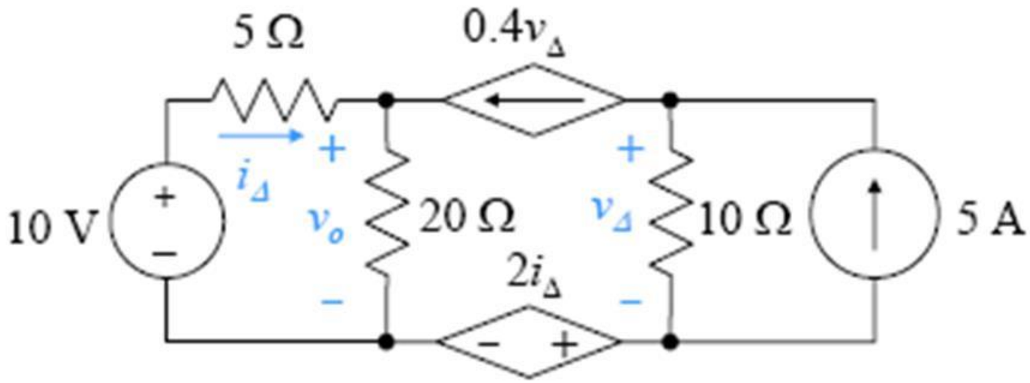
$$\boxed{i_1''=\frac{12}{6}=2\text{A}} \quad \boxed{i_2''=\frac{-12}{3}=-4\text{A}} \quad \boxed{i_3''=\frac{-12+24}{2}=6\text{A}} \quad \boxed{i_4''=\frac{-24}{4}=-6\text{A}}$$

$$\boxed{i_1=i_1'+i_1''=15+2=17\text{A}} \qquad \boxed{i_2=i_2'+i_2''=10-4=6\text{A}}$$

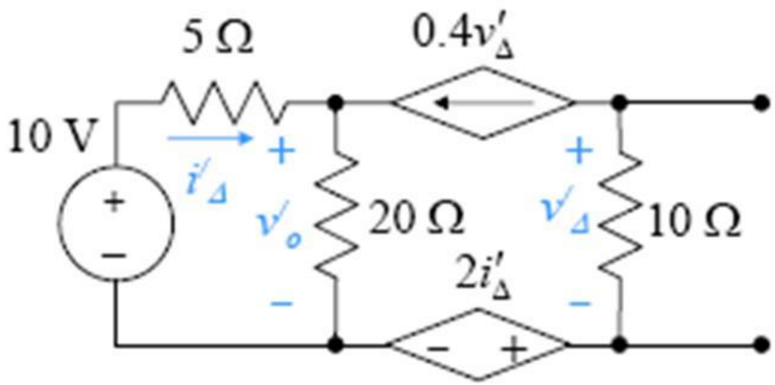
$$\boxed{i_3=i_3'+i_3''=5+6=11\text{A}} \qquad \boxed{i_4=i_4'+i_4''=5-6=-1\text{A}}$$

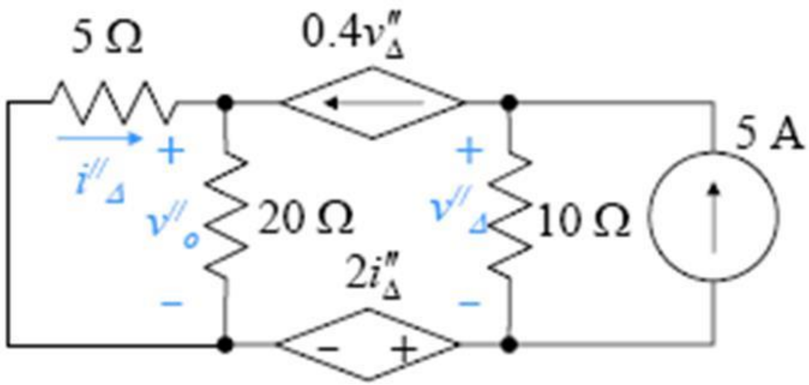
Example 11

Apply superposition to find v_o



Ans.:-





Nguồn độc lập => de activate 2

Example (Cont.)

$$v'_\Delta = -(0.4v'_\Delta)10 \dashrightarrow v'_\Delta = 0$$

$$v'_o = \frac{10}{5+20}20 = 8\text{ V}$$

Node

$$\frac{v''_o}{5} + \frac{v''_o}{20} - 0.4v''_\Delta = 0$$

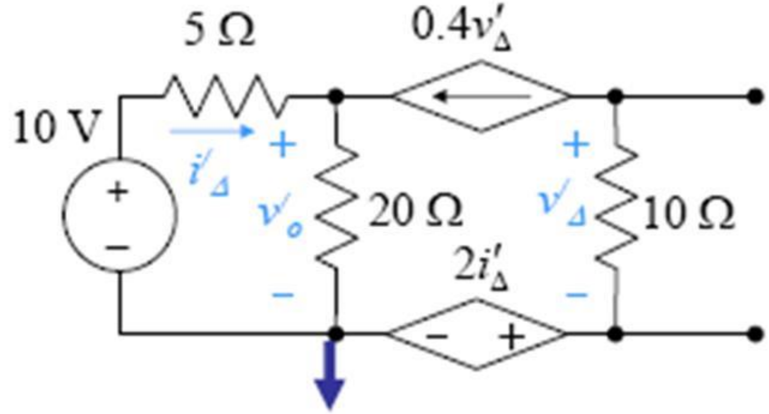
$$0.4v''_\Delta + \frac{v_b - 2i''_\Delta}{10} - 5 = 0$$

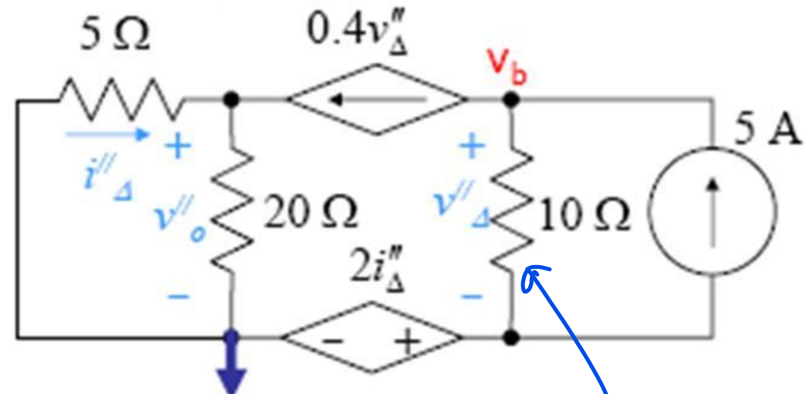
$$v_b = v''_\Delta + 2i''_\Delta$$, $v''_o = -i''_\Delta \times 5$

$$v''_\Delta = 10\text{ V}$$

$$v''_o = 16\text{ V}$$

$$v_o = v'_o + v''_o = 24\text{ V}$$





$V_b \sim 2i'_\Delta$