Homework:

(1) Read Sec. 2-10.

(2) 1-6, 16, 20, 29-31, 33, 46, 48, 50

2. Compute your average velocity in the following two cases: (a) You walk 73.2 m at a speed of 1.22 m/s and then run 73.2 m at a speed of 2.85 m/s along a straight track. (b) You walk for 1.0 min at a speed 1.22 m/s and then run for 1.0 min at 3.05 m/s along a straight track. (c) Graph x versus t for both cases and indicate how the average velocity is found on the graph.

(a) the average velocity:
$$v_{avg} = \frac{\text{displacement}}{\text{time interval}}$$

As you walk along a straight line:
$$v_{avg} = \frac{73.2 + 73.2}{\dfrac{73.2}{1.22} + \dfrac{73.2}{2.85}} = 1.71(\text{m/s})$$

(b)
$$v_{avg} = \frac{1.22(m/s) \times 60s + 3.05(m/s) \times 60s}{2 \times 60s} \approx 2.14(m/s)$$

(c)
$$x = v_0 t$$

3. An automobile travels on a straight road for 40 km at 30 km/h. It then continues in the same direction for another 40 km at 60 km/h. (a) What is the average velocity of the car during this 80 km trip? (assume that it moves in the positive x direction) (b) What is the average speed? (c) Graph x versus t and indicate how the average velocity is found on the graph.

Following the definition:

$$v_{avg} = \frac{\text{displacement}}{\text{time interval}}$$

(a) As the car moves in the same direction, so the total displacement is:

$$\Delta x = 40 + 40 = 80(km)$$

The total time:

$$\Delta t = \frac{40}{30} + \frac{40}{60} = 2(h)$$

So:

$$v_{avg} = \frac{\Delta x}{\Delta t} = \frac{80}{2} = 40(km/h)$$

(b) Following the definition: $$s_{avg} = \frac{\text{total distance}}{\text{time interval}}$$

So:

$$d = 40 + 40 = 80(km)$$

$$\Delta t = \frac{40}{30} + \frac{40}{60} = 2(h)$$

$$s_{avg} = \frac{d}{\Delta t} = \frac{80}{2} = 40(km/h)$$

(c) We use the following equation to graph x versus t:

$$x = v_0 t$$

4. A car travels up a hill at a constant speed of 35 km/h and returns down the hill at a constant speed of 60 km/h. Calculate the average speed for the round trip.

Average speed = total distance/time interval

$$D_{up} = D_{down} = D$$

$$s = \frac{D_{up} + D_{down}}{t_{up} + t_{down}} = \frac{D_{up} + D_{down}}{\frac{D_{up}}{s_{up}} + \frac{D_{down}}{s_{down}}} = 2\frac{s_{up} \times s_{down}}{s_{up} + s_{down}}$$

$$s = 44.2(km/h)$$

30. The brakes on your car can slow you at a rate of 5.2 m/s^2.
(a) If you are going 146 km/h and suddenly see a state trooper, what is the minimum time in which you can get your car under the 90 km/h speed limit? (The answer reveals the futility of braking to keep your high speed from being detected with a radar or laser gun).
(b) Graph x versus t and v versus t for such a slowing.

- $a=-5.2\ m/s^2$; $v_0=146$ km/h or $v_0=40.6$ m/s; $v_1=90$ km/h or $v_1=25$ m/s

(a) The minimum time t_{min} must match:

$$v = v_0 + at \le v_1 \xrightarrow{(a<0)} \frac{v_1 - v_0}{a} \le t$$

$$t_{min} = \frac{v_1 - v_0}{a} = \frac{(25 - 40.6)}{-5.2} = 3.0(s)$$

(b) x vs. t and v vs. t:

$$x = v_0 t + \frac{1}{2}at^2 = 40.6t - 2.6t^2; \quad v = v_0 + at = 40.6 - 5.2t$$

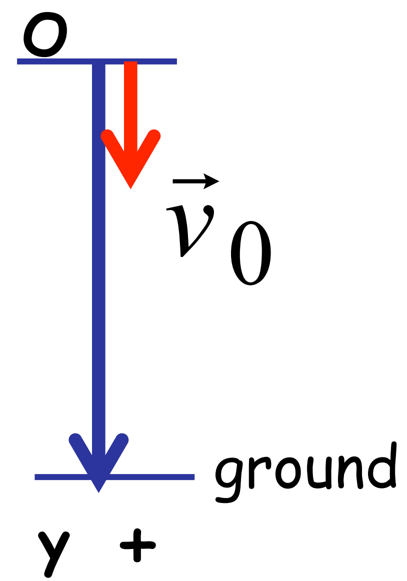
- Visit Online Function Graphers, e.g.: http://www.function-grapher.com/index.php

48. A hoodlum throws a stone vertically downward with an initial speed of 15.0 m/s from the roof of a building, 30.0 m above the ground. (a) How long does it take the stone to reach the ground? (b) What is the speed of the stone at impact?

This is a freely falling object problem:

$$y = y_0 + v_0 t + \frac{1}{2}at^2$$

We choose the positive direction is downward, the origin O at the roof $y_0 = 0$, so:



(a) When the stone hits the ground, we have

$y = 30$ m:
$$30 = 0 + 15t + \frac{1}{2}9.8t^2$$

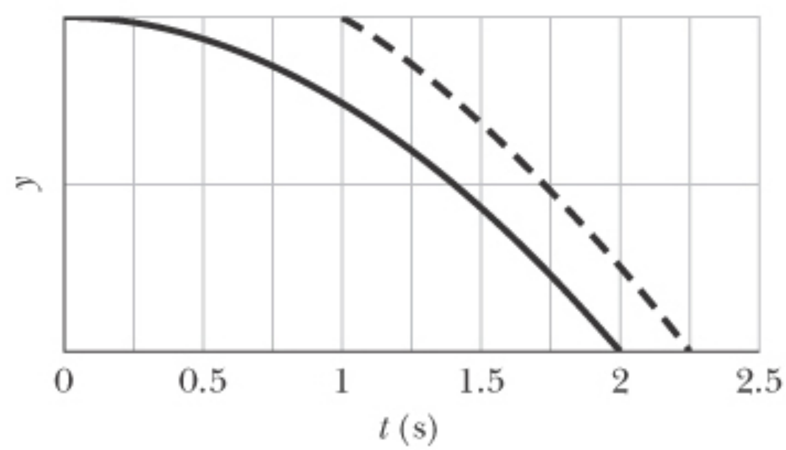
$$\Rightarrow t = 1.38(s)$$

(discard t < 0)

(b)
$$v = v_0 + at = v_0 + gt = 15 + 9.8 \times 1.38 \approx 28.5(m/s)$$

50. At time t=0, apple 1 is dropped from a bridge onto a roadway beneath the bridge; somewhat later, apple 2 is thrown down from the same height. Figure below gives the vertical positions y of the apples versus t during the falling, until both apples have hit the roadway. With approximately what speed is apple2 thrown down?

•Apple 1 hits the roadway at t=2 s:



$$y = y_0 - \frac{1}{2}gt^2$$

$$0 = y_0 - \frac{1}{2}\times 9.8\times 2^2 \;\rightarrow\; y_0 = 19.6\,(\mathrm{m})$$

•Apple 2 is thrown down at t=1 s and hits the roadway at t=2.25 s:

$$y = y_0 + v_0 t - \frac{1}{2}gt^2 \;\rightarrow\; 0 = y_0 + v_0 t_2 - \frac{1}{2}g{t_2}^2$$

$$t_2 = 1.25\,(\mathrm{s}) \;\rightarrow\; \boxed{v_0 \approx -9.6\,(\mathrm{m/s})}$$

Chapter 1 Bases of Kinematics

1.1. Motion in One Dimension
1.2. Motion in Two Dimensions

1.2.1. The Position, Velocity, and Acceleration Vectors

1.2.2. Two-Dimensional Motion with Constant Acceleration. Projectile Motion

1.2.3. Circular Motion. Tangential and Radial Acceleration

1.2.4. Relative Velocity and Relative Acceleration

Vectors (Recall)

R1. Vectors and scalars:

- A vector has magnitude and direction; vectors follow certain rules of combination.
- Some physical quantities that are vector quantities are displacement, velocity, and acceleration.
- Some physical quantities that does not involve direction are temperature, pressure, energy, mass, time. We call them scalars.

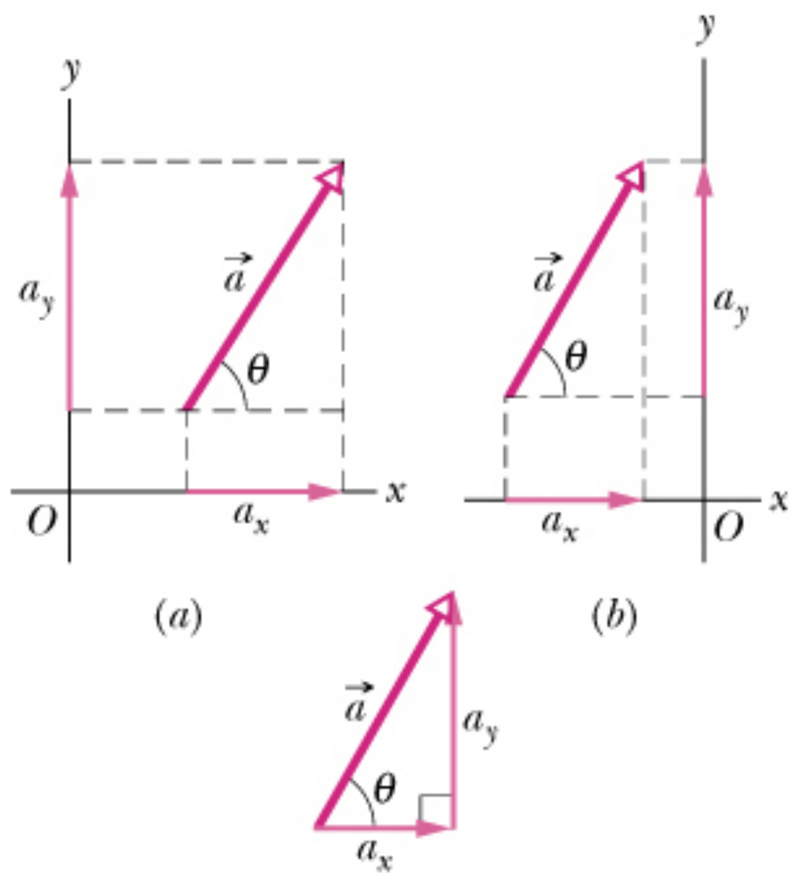
R2. Components of vectors:

- A component of a vector is the projection of the vector on an axis.

$$a_x = a\cos\theta \qquad a_y = a\sin\theta$$

- If we know a vector in component notation (a_x and a_y), we determine it in magnitude-angle notation (a and θ):

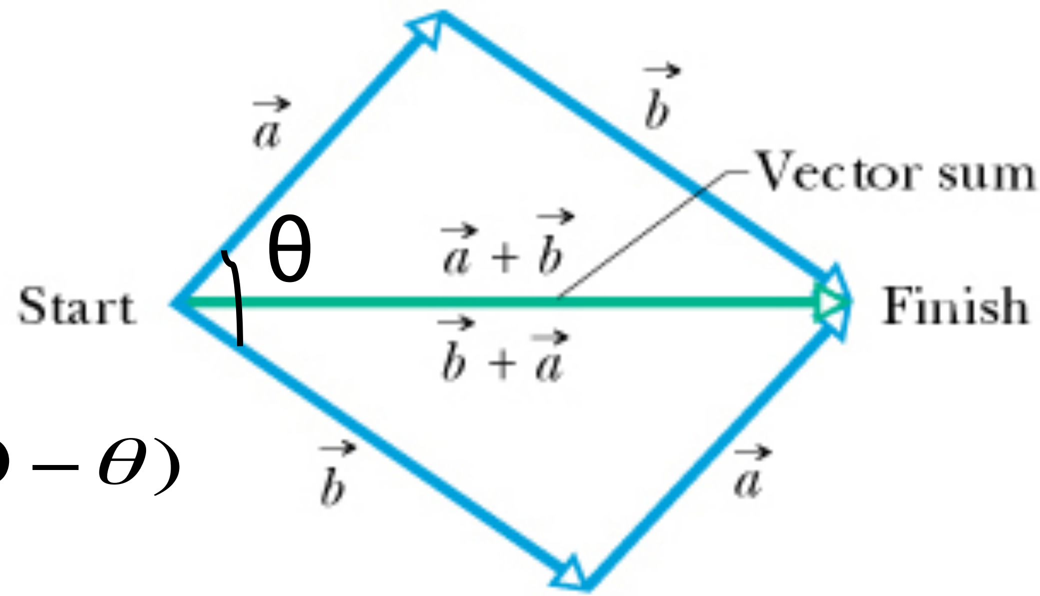
$$a = \sqrt{a_x^2 + a_y^2} \qquad \tan\theta = \frac{a_y}{a_x}$$



R3. Adding vectors:
R3.1. Adding vectors geometrically:

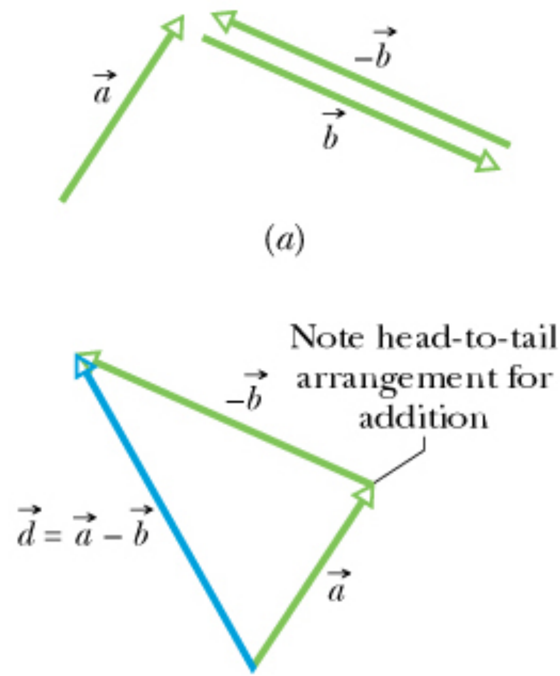
$$\vec{s} = \vec{a} + \vec{b}$$

$$s^2 = a^2 + b^2 - 2ab\cos(180 - \theta)$$



- **Vector subtraction:**

$$\vec{d} = \vec{a} - \vec{b} = \vec{a} + (-\vec{b})$$



R3.2. Adding vectors by components:

$$\vec{a} = a_x\hat{i} + a_y\hat{j}; \quad \vec{b} = b_x\hat{i} + b_y\hat{j}$$

$$\vec{s} = \vec{a} + \vec{b}$$

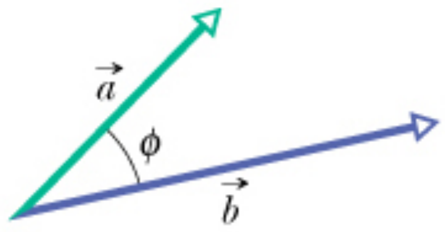
$$s_x = a_x + b_x; \quad s_y = a_y + b_y$$

R4. Multiplying a vector by a vector:

R4.1. The scalar product (the dot product):

$$\vec{a} \bullet \vec{b} = ab\cos\phi$$

$$\vec{a} \bullet \vec{b} = a_x b_x + a_y b_y$$



R4.2. The vector product (the cross product):

$$\vec{c} = \vec{a} \times \vec{b}$$

$$c = ab\sin\phi$$

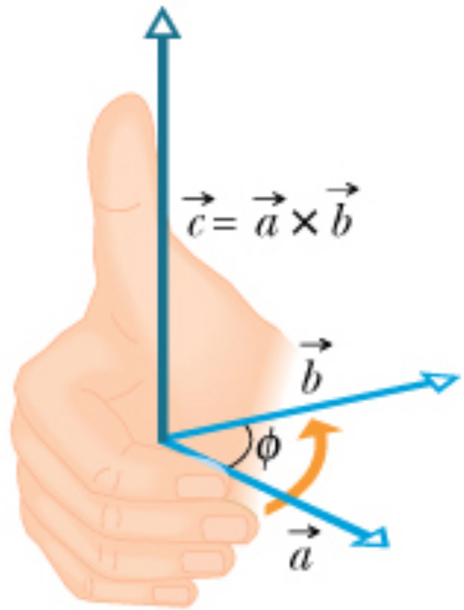
ϕ is the smaller of the two angles between $\vec{a}$ and $\vec{b}$

The direction of $\vec{c}$ is determined by using the right-hand rule:
Your fingers (right-hand) sweep $\vec{a}$ into $\vec{b}$ through the smaller angle between them, your outstretched thumb points in the direction of $\vec{c}$

$$\vec{a} = a_x\hat{i} + a_y\hat{j} + a_z\hat{k}$$

$$\vec{b} = b_x\hat{i} + b_y\hat{j} + b_z\hat{k}$$

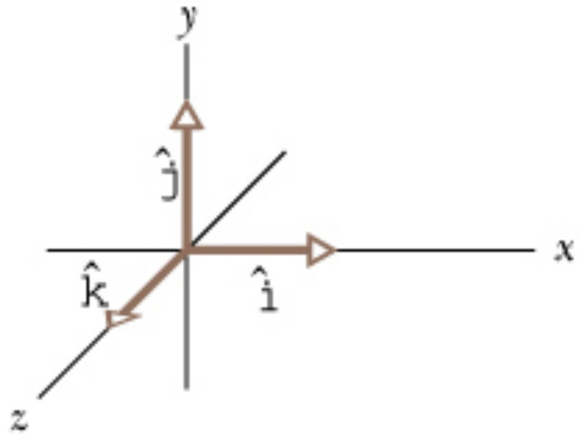
$$\vec{c} = \vec{a} \times \vec{b}$$



$$\vec{c} = (a_y b_z - b_y a_z)\hat{i} + (a_z b_x - b_z a_x)\hat{j} + (a_x b_y - b_x a_y)\hat{k}$$

In the right-handed xyz coordinate system:

$$\hat{k} = \hat{i} \times \hat{j}$$



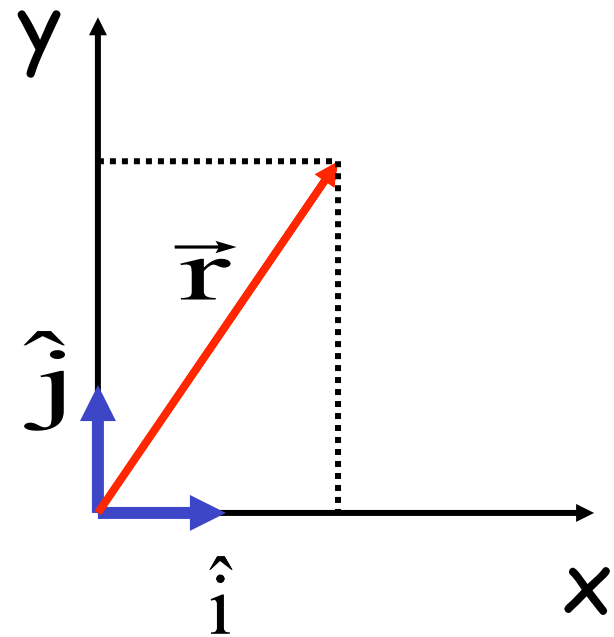
1.2. Motion in Two Dimensions

1.2.1. The Position, Velocity, and Acceleration Vectors

Position and Displacement:

- A particle is located by a position vector:

$$\vec{r} = x\hat{i} + y\hat{j}$$



$x\hat{i}$ and $y\hat{j}$ are vector components of $\vec{r}$

x and y are scalar components of $\vec{r}$

- Displacement: $\Delta\vec{r} = \vec{r}_2 - \vec{r}_1$

$$\Delta\vec{r} = (x_2\hat{i} + y_2\hat{j}) - (x_1\hat{i} + y_1\hat{j})$$

$$\Delta\vec{r} = (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j} = \Delta x\hat{i} + \Delta y\hat{j}$$

- Three dimensions: $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$

$$\Delta\vec{r} = (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j} + (z_2 - z_1)\hat{k} = \Delta x\hat{i} + \Delta y\hat{j} + \Delta z\hat{k}$$

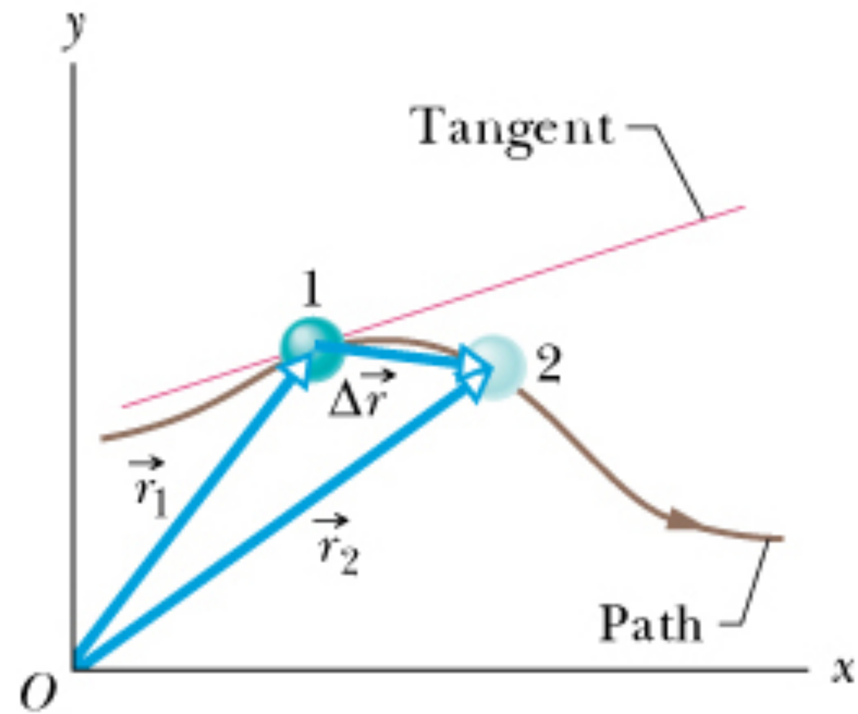
Average Velocity and Instantaneous Velocity:

$$\text{average velocity} = \frac{\text{displacement}}{\text{time interval}}$$

$$\vec{v}_{avg} = \frac{\Delta\vec{r}}{\Delta t}$$

- **Instantaneous Velocity, $\Delta t \to 0$:**

$$\vec{v} = \lim_{\Delta t \to 0} \frac{\Delta\vec{r}}{\Delta t} = \frac{d\vec{r}}{dt}$$



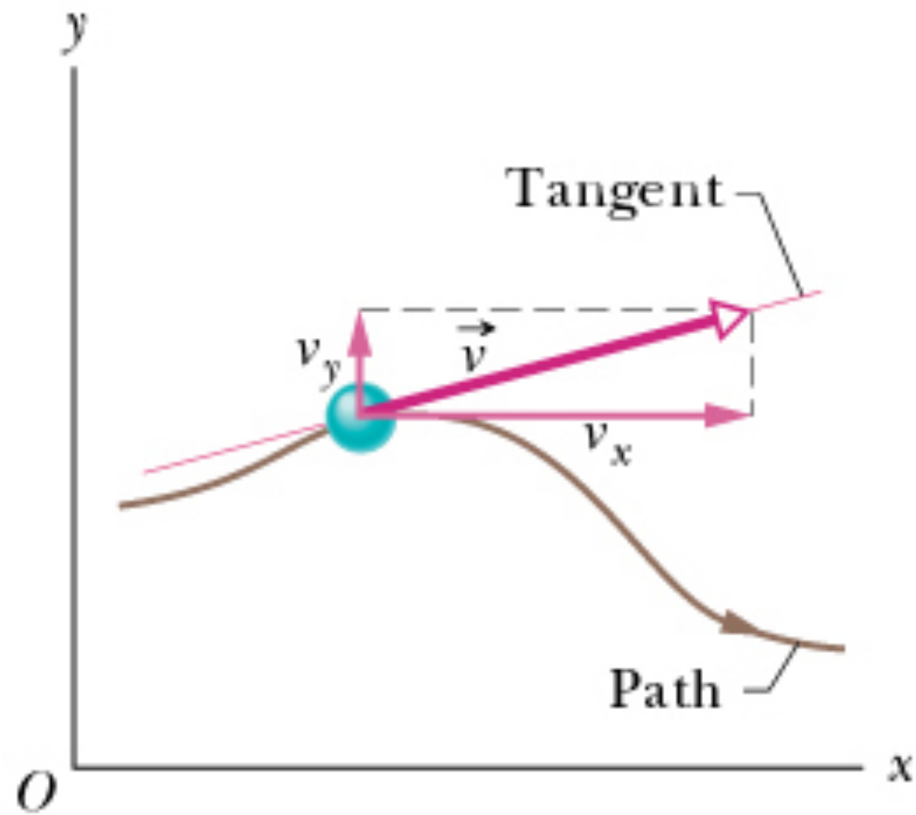
- **The direction of the instantaneous velocity of a particle is always tangent to the particle's path at the particle position.**

$$\vec{v} = \frac{d}{dt}(x\hat{i} + y\hat{j}) = \frac{dx}{dt}\hat{i} + \frac{dy}{dt}\hat{j}$$

$$\vec{v} = v_x\hat{i} + v_y\hat{j}$$

The scalar components of $\vec{v}$

$$v_x = \frac{dx}{dt}, v_y = \frac{dy}{dt}$$



Three dimensions:

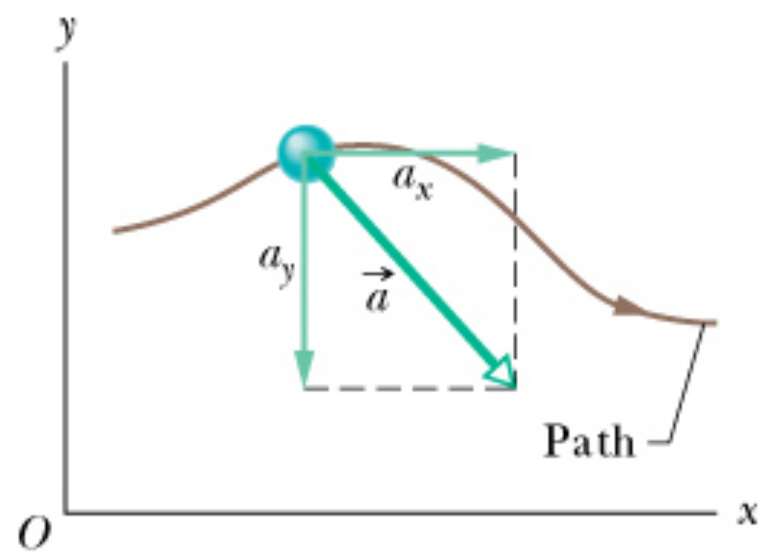
$$\vec{v} = v_x\hat{i} + v_y\hat{j} + v_z\hat{k}$$

$$v_z = \frac{dz}{dt}$$

Average Acceleration and Instantaneous Acceleration:

$$\text{average acceleration} = \frac{\text{change in velocity}}{\text{time interval}}$$

$$\vec{a}_{avg} = \frac{\Delta\vec{v}}{\Delta t}$$



- **Instantaneous Acceleration, $\Delta t \to 0$:**

$$\vec{a} = \lim_{\Delta t \to 0} \frac{\Delta\vec{v}}{\Delta t} = \frac{d\vec{v}}{dt}$$

$$\vec{a} = a_x\hat{i} + a_y\hat{j}$$

The scalar components of $\vec{a}$

$$a_x = \frac{dv_x}{dt},\ a_y = \frac{dv_y}{dt},\ a_z = \frac{dv_z}{dt}$$

Three dimensions: $$\vec{a} = a_x\hat{i} + a_y\hat{j} + a_z\hat{k};$$

1.2.2. Two-Dimensional Motion with Constant Acceleration. Projectile Motion

$$\vec{a} = a_x\hat{i} + a_y\hat{j} = \text{constant}$$

Key point: To determine velocity and position, we need to determine x and y components of velocity and position

$$\vec{v} = \vec{v}_0 + \vec{a}t = v_x\hat{i} + v_y\hat{j}$$

$$\vec{r} = \vec{r}_0 + \vec{v}_0 t + \frac{1}{2}\vec{a}t^2 = x\hat{i} + y\hat{j}$$

Along the x axis:

$$v_x = v_{0x} + a_x t; \quad x = x_0 + v_{0x}t + \frac{1}{2}a_x t^2$$

Along the y axis:

$$v_y = v_{0y} + a_y t; \quad y = y_0 + v_{0y}t + \frac{1}{2}a_y t^2$$

Sample Problem:

A particle with velocity $\vec{v}_0 = -2.0\hat{i} + 4.0\hat{j}$ (m/s) at t=0 undergoes a constant acceleration $\vec{a}$ of magnitude a = 3.0 m/s^2 at an angle θ= 130^0 from the positive direction of the x axis. What is the particle's velocity $\vec{v}$ at t=5.0 s, in unit-vector notation and in magnitude-angle notation?

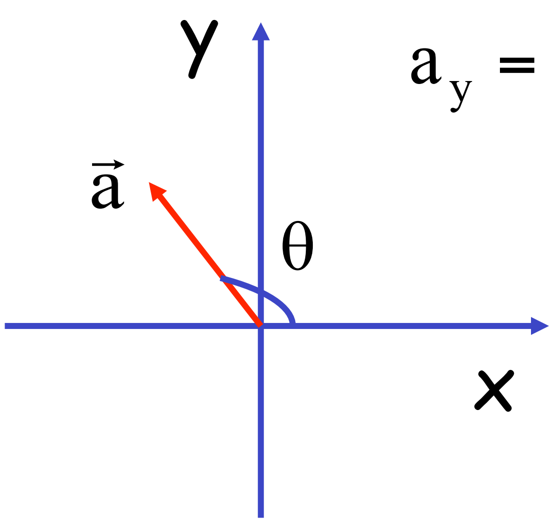
- **Key issues: This is a two-dimensional motion, we must apply equations of straight-line motion separately for motion parallel**

$$v_x = v_{0x} + a_x t; \quad v_y = v_{0y} + a_y t$$

v_{0x}=-2.0 (m/s) and v_{0y}=4.0 (m/s)

$$a_x = a\cos\theta = 3.0\times\cos(130^0) = -1.93\,(m/s^2)$$

$$a_y = a\sin\theta = 3.0\times\sin(130^0) = 2.30\,(m/s^2)$$



- **At t= 5 s:** $v_x = -11.7\,(m/s); \quad v_y = 15.5\,(m/s)$

$$\vec{v} = -11.7\hat{i} + 15.5\hat{j}$$

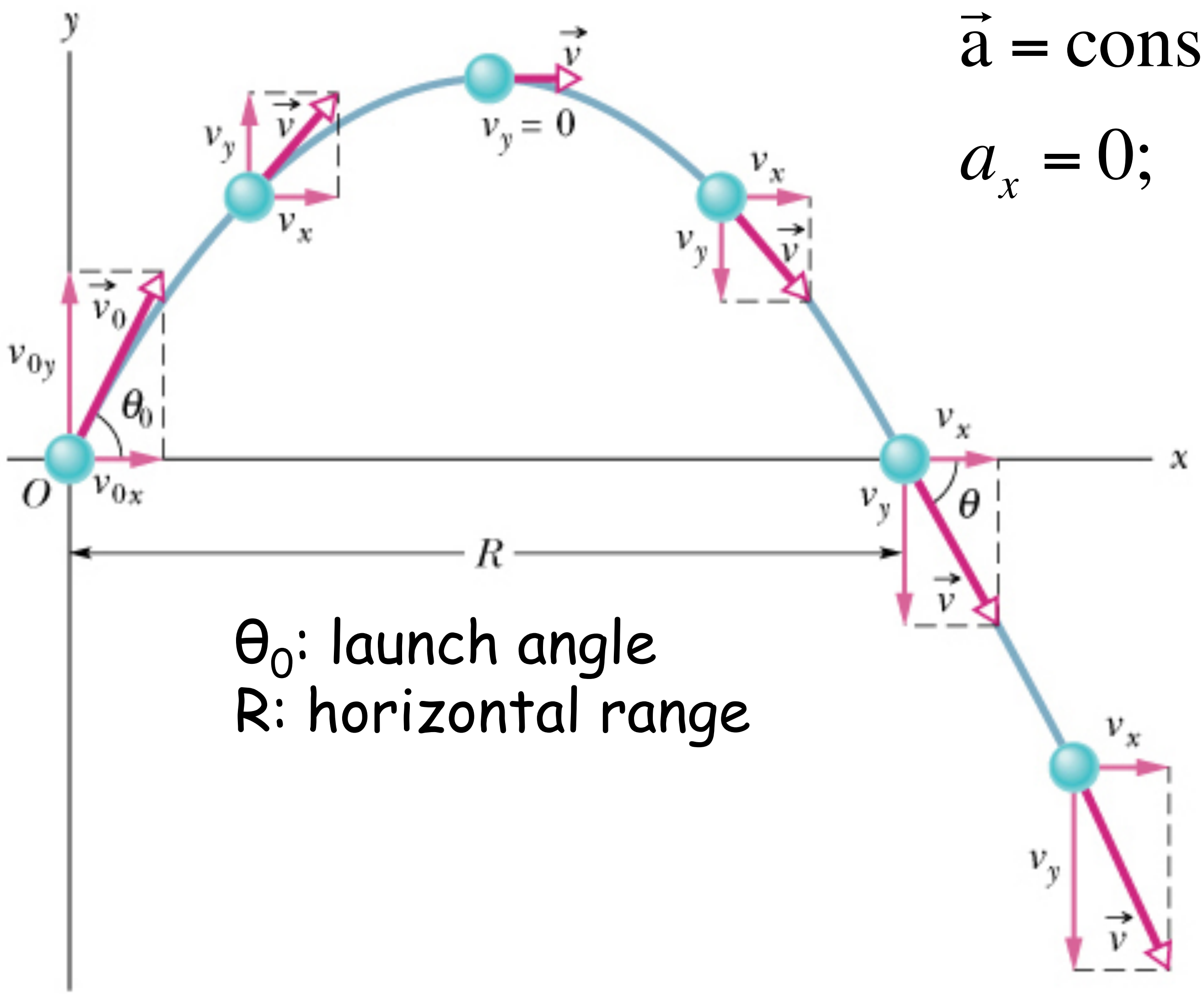
- **The magnitude and angle of $\vec{v}$:** $v = \sqrt{v_x^2 + v_y^2} = 19.4\,(m/s)$

$$\tan(\theta) = \frac{v_y}{v_x} \approx -1.33 \Rightarrow \theta = 127^0$$

Projectile motion

$\vec{a} = \text{constant}$

$a_x = 0; \quad a_y = -g$



θ_0: launch angle
R: horizontal range

Projectile Motion: A particle moves in a vertical plane with some initial velocity but its acceleration is always the free-fall acceleration, the motion of this particle is called projectile motion.

- Ox, horizontal motion (no acceleration, $a_x = 0$):

$$v_x = v_{0x} = v_0 \cos\theta_0 = \text{constant}$$

$$x = x_0 + v_{0x} t = x_0 + v_0 \cos\theta_0 t$$

- Oy, vertical motion (free fall, $a_y = -g$ if the positive y direction is upward):

$$v_y = v_{0y} + a_y t = v_0 \sin\theta_0 - gt$$

$$y = y_0 + v_0 \sin\theta_0 t - \frac{1}{2} g t^2$$

- The equation of the path:

$$y = (\tan\theta_0)x - \frac{gx^2}{2(v_0\cos\theta_0)^2}$$

- Horizontal range: $$R = \frac{v_0^2}{g}\sin 2\theta_0$$

Example: A projectile is shot from the edge of a cliff 115m above ground level with an initial speed of 65.0 m/s at an angle of 35^0 with the horizontal (see the figure below). Determine:
(a) the maximum height of the projectile above the cliff;
(b) the projectile velocity when it strikes the ground (point P);
(c) point P from the base of the cliff (distance X).

(a) At its maximum height:

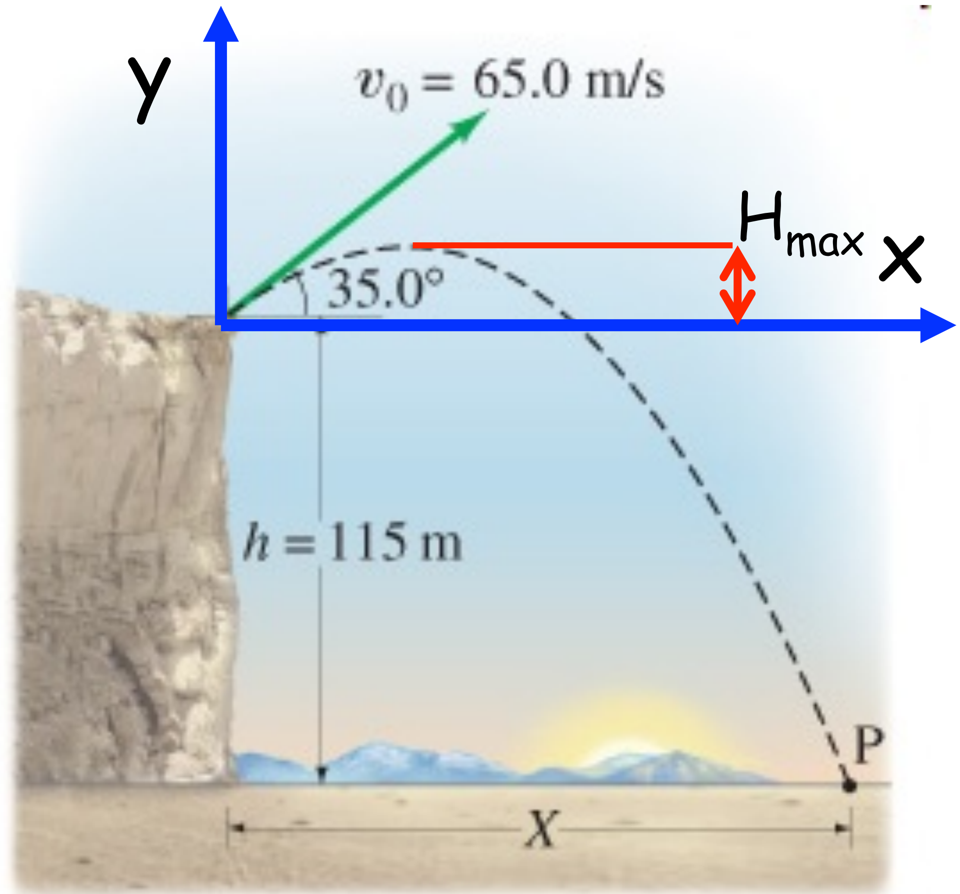
$$v_y = v_0 \sin\theta_0 - gt = 0$$

$$t = \frac{v_0 \sin\theta_0}{g}$$

$$y = y_0 + v_0 \sin\theta_0 t - \frac{1}{2} g t^2$$

$$\boxed{y = \frac{(v_0 \sin\theta_0)^2}{2g} = H_{max}}$$

$$H_{max} = 70.9\,(m)$$



(b) its velocity:

$$v_x = v_0\cos\theta_0 = 53.25\,(\mathrm{m/s})$$

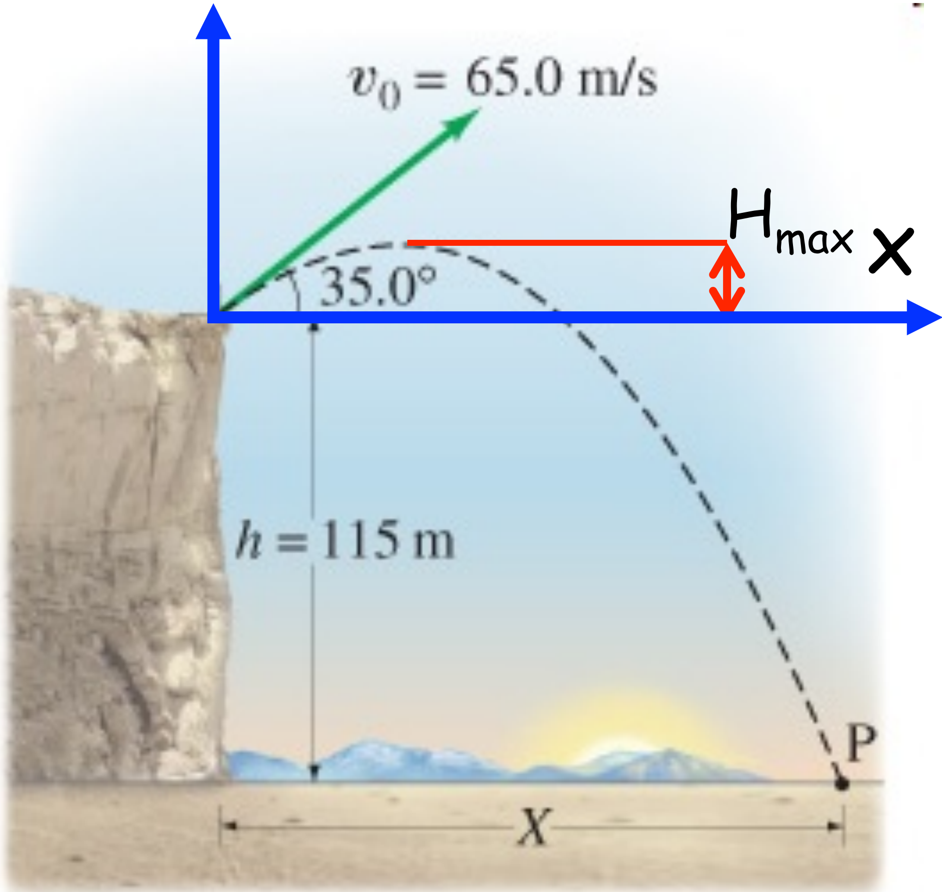
$$v_y^2 - v_{0y}^2 = 2a\Delta y$$

$$v_{0y} = v_0\sin\theta_0 = 37.28\,(\mathrm{m/s})$$

$$a = -9.8\,(\mathrm{m/s^2})$$

$$\Delta y = y_P - y_0 = -115\,(\mathrm{m})$$

$$v_y = -60.36\,(\mathrm{m/s})$$



$$\boxed{\vec{v} = 53.25(\mathrm{m/s})\,\hat{i} - 60.36\,(\mathrm{m/s})\,\hat{j}}$$

(c) Calculate X: $v_y = v_0 \sin\theta + at \Rightarrow t = \dfrac{v_0 \sin\theta \text{ - } v_y}{g}$

$$t = 9.96\,(\mathrm{s})$$

$$\boxed{X = v_x t = v_0\cos\theta_0 t = 530.37\,(\mathrm{m})}$$

Note: for (b), we can solve as follows:

$$y = y_0 + v_0 \sin\theta_0 t - \frac{1}{2} g t^2$$

$$\frac{1}{2} g t^2 - v_0 \sin\theta_0 t + y_P = 0$$

quadratic equation:

$$4.9t^2 - 37.28t - 115 = 0$$

$$\Rightarrow t = 9.96\,(\mathrm{s})$$

$$v_y = v_0 \sin\theta + at = v_0 \sin\theta - gt$$

$$v_y = -60.33\,(\mathrm{m/s})$$

then for (c), use t = 9.96 (s)

Sample Problem (page 70):

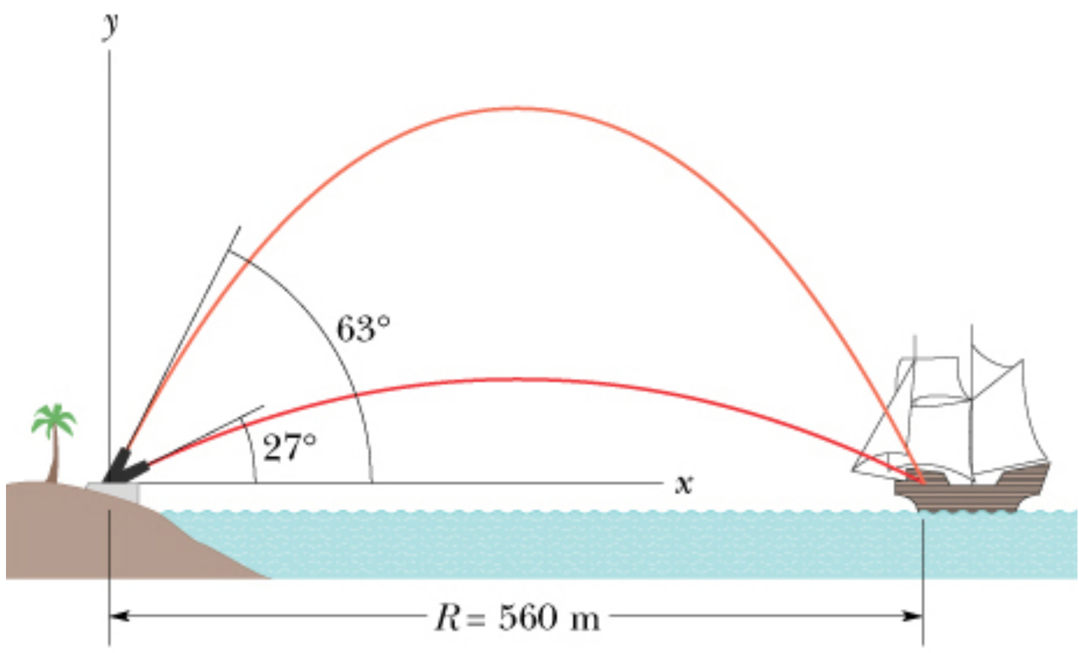
Figure below shows a pirate ship 560 m from a fort defending the harbor entrance of an island. A defense cannon, located at sea level, fires balls at initial speed v_0= 82 m/s. (a) At what angle θ_0 from the horizontal must a ball be fired to hit the ship? (b) How far should the pirate ship be from the cannon if it is to be beyond the maximum range of the cannon balls?

(a)

$$R = \frac{v_0^2}{g}\sin 2\theta_0$$

$$2\theta = \sin^{-1}\frac{gR}{v_0^2}$$

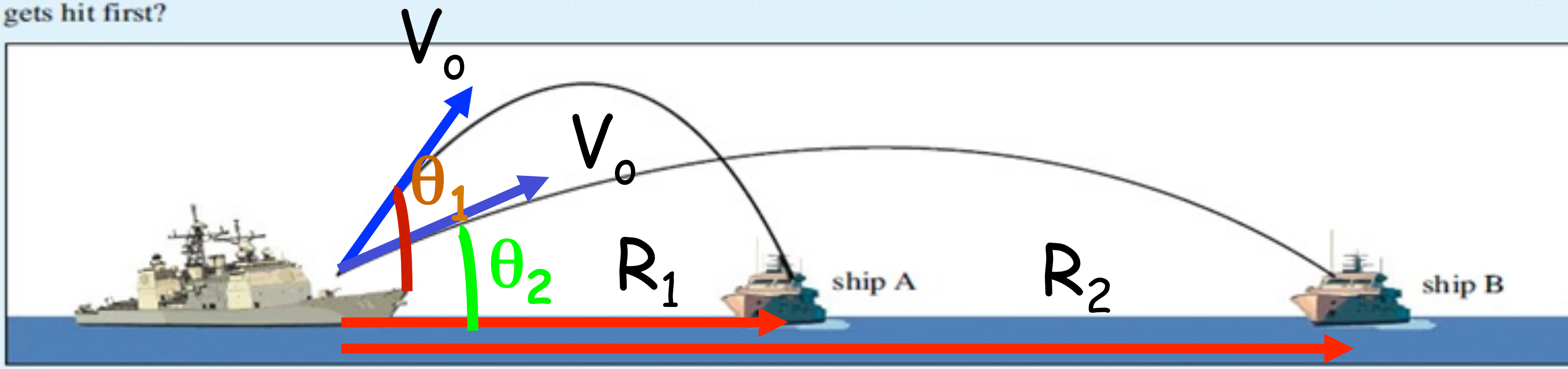
$$\theta \approx 27^0 \text{ or } \theta \approx 63^0$$



(b)

$$R_{max} = \frac{v_0^2}{g}\sin 2\theta_0 = \frac{82^2}{9.8}\sin(2\times 45) = 686\,(\text{m})$$

A battleship simultaneously fires two shells at enemy ships. If the shells follow the parabolic trajectories shown, which ship gets hit first?



1. Ship A
2. Both at the same time
3. Ship B*
4. Need more information

$$x = x_0 + v_0\cos\theta_0 t$$

$$\Rightarrow t_1 = \frac{R_1}{v_0\cos\theta_1}; t_2 = \frac{R_2}{v_0\cos\theta_2}$$ ➔ ~~It is likely answer 4~~

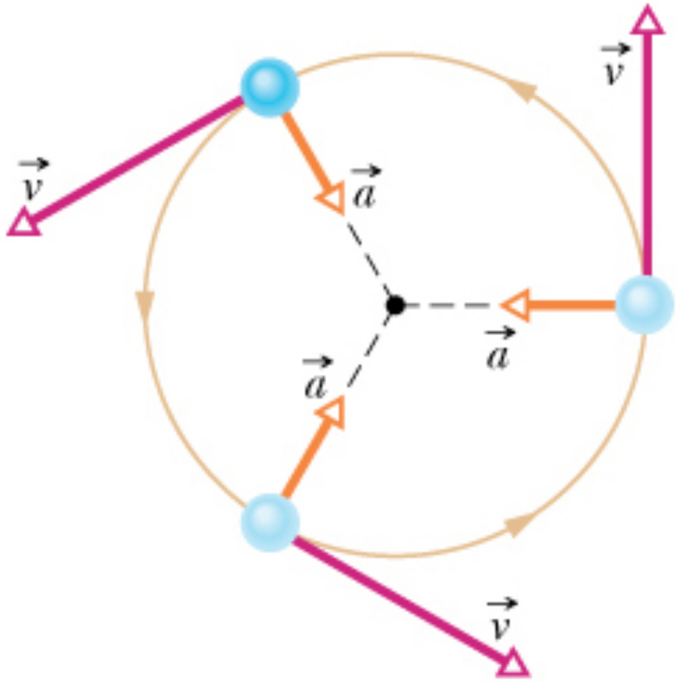
However

$v_y = v_0 \sin\theta_0 - gt$ when the shells hit the ships, $v_y = -v_0 \sin\theta_0$

$$\Rightarrow t_1 = \frac{2v_0 \sin\theta_1}{g}; t_2 = \frac{2v_0 \sin\theta_2}{g}$$ $\rightarrow \theta_1 > \theta_2$ ➔ $t_2 < t_1$: the answer is B

the farther ship gets hit first

1.2.3. Circular Motion. Tangential and Radial Acceleration



Uniform Circular Motion:

A particle moves around a circle or a circular arc at constant speed. The particle is accelerating with a centripetal acceleration:

$$a = \frac{v^2}{r}$$

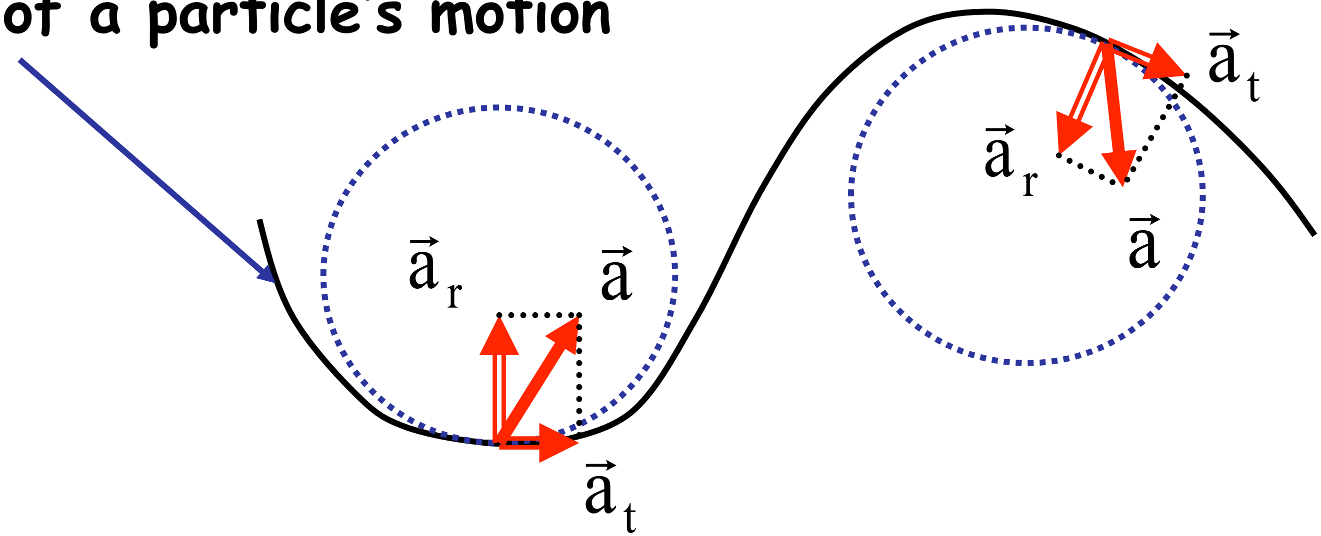
Where r is the radius of the circle
v the speed of the particle

$$T = \frac{2\pi r}{v}$$ (T: period)

1.2.3. Circular Motion. Tangential and Radial Acceleration

Tangential and radial acceleration: If the speed is not constant, then there is also a tangential acceleration.

The path of a particle's motion



$$\boxed{\vec{a} = \vec{a}_r + \vec{a}_t}$$

Radial (centripetal) acceleration

Tangential acceleration

Homework: 6, 7, 11, 20, 27, 29, 54, 58, 66