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Homework Week 4

Question 1

a) $(A - AX)^{-1} = X^{-1} B$

$\Leftrightarrow (A - AX)(A - AX)^{-1} = X^{-1} B (A - AX)$

$\Leftrightarrow \quad I = (A - AX)(X^{-1} B)$

$\Leftrightarrow (AX^{-1} - AXX^{-1}) B = I$

$\Leftrightarrow (Ax^{-1} - A) B = I$

Either $(Ax^{-1} - A) = 1$ or $B = 1$ (1)

$\Rightarrow$ B is invertible matrix

b) From eqt (1), we get

$Ax^{-1} - A = 1$

$\Leftrightarrow \frac{A}{x} - A = 1$

$\Leftrightarrow A - Ax = x$

$\because$ Given that $(A - Ax)$ is invertible

$\therefore$ For X, is invertible

$Ax = B$

$\Rightarrow x = \frac{B}{A} = B \cdot \frac{1}{A} = B \cdot A^{-1}$

Question 2:

2. Find the inverses of the matrices in Exercises, if they exist

a)
$$\begin{pmatrix} 1 & 0 & -2 \\ -3 & 1 & 4 \\ 2 & -3 & 4 \end{pmatrix}$$

b)
$$\begin{pmatrix} 1 & -2 & 1 \\ 4 & -7 & 3 \\ -2 & 6 & -4 \end{pmatrix}$$

c)
$$\begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{pmatrix}$$

a)
$$A = \begin{pmatrix} 1 & 0 & -2 \\ -3 & 1 & 4 \\ 2 & -3 & 4 \end{pmatrix}$$

$$\begin{array}{l} \\ R_1 \times 3 + R_2 \\ R_1 \times (-2) + R_3 \end{array} \left[\begin{array}{ccc|ccc} 1 & 0 & -2 & 1 & 0 & 0 \\ -3 & 1 & 4 & 0 & 1 & 0 \\ 2 & -3 & 4 & 0 & 0 & 1 \end{array}\right]$$

$$\begin{array}{l} \\ \\ R_2 \times 3 + R_3 \end{array} \left[\begin{array}{ccc|ccc} 1 & 0 & -2 & 1 & 0 & 0 \\ 0 & 1 & -2 & 3 & 1 & 0 \\ 0 & -3 & 8 & -2 & 0 & 1 \end{array}\right]$$

$$\frac{1}{2}R_3 \quad \left[\begin{array}{ccc|ccc} 1 & 0 & -2 & 1 & 0 & 0 \\ 0 & 1 & -2 & 3 & 1 & 0 \\ 0 & 0 & 2 & 7 & 3 & 1 \end{array}\right]$$

$$\begin{array}{l} R_3 \times 2 + R_1 \\ R_3 \times 2 + R_2 \end{array} \quad \left[\begin{array}{ccc|ccc} 1 & 0 & -2 & 1 & 0 & 0 \\ 0 & 1 & -2 & 3 & 1 & 0 \\ 0 & 0 & 1 & 7/2 & 3/2 & 1/2 \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 8 & 3 & 1 \\ 0 & 1 & 0 & 10 & 4 & 1 \\ 0 & 0 & 1 & 7/2 & 3/2 & 1/2 \end{array}\right]$$

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$$\Rightarrow A^{-1} = \begin{bmatrix} 8 & 3 & 1 \\ 10 & 4 & 1 \\ 7/2 & 3/2 & 1/2 \end{bmatrix}$$

b)

$$B = \begin{pmatrix} 1 & -2 & 1 \\ 4 & -7 & 3 \\ -2 & 6 & -4 \end{pmatrix}$$

$$\begin{array}{l} R_1 \times 3 + R_3 \\ R_1 \times (-4) + R_2 \\ R_3 \times 2 + R_2 \end{array} \quad \left[\begin{array}{ccc|ccc} 1 & -2 & 1 & 1 & 0 & 0 \\ 4 & -7 & 3 & 0 & 1 & 0 \\ -2 & 6 & -4 & 0 & 0 & 1 \end{array}\right]$$

$$\frac{1}{5}R_3 \quad \left[\begin{array}{ccc|ccc} 1 & 0 & -1 & 3 & 0 & 1 \\ 0 & 1 & -1 & -4 & 1 & 0 \\ 0 & 5 & -5 & 0 & 1 & 2 \end{array}\right]$$

$$R_2 - R_3 \quad \left[\begin{array}{ccc|ccc} 1 & 0 & -1 & 3 & 0 & 1 \\ 0 & 1 & -1 & -4 & 1 & 0 \\ 0 & 1 & -1 & 0 & 1/5 & 2/5 \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & -1 & 3 & 0 & 1 \\ 0 & 1 & -1 & -4 & 1 & 0 \\ 0 & 0 & 0 & -4 & 4/5 & -2/5 \end{array}\right]$$

Since row 3 consists solely of zeros, the determinant $= 0$

Thus, the matrix is not invertable

c)

$$C = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{pmatrix}$$

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$$\begin{array}{c} \\ R_2 - R_1 \\ R_3 - R_2 \end{array} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & -1 & 1 & 0 \\ 0 & 0 & 1 & 0 & -1 & 1 \end{array}\right]$$

$$\Rightarrow \quad C^{-1} = \begin{pmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{pmatrix}$$

Question 3:

$$C^{-1} (A + X) B^{-1} = I_n$$

$$\Leftrightarrow \quad C \left[C^{-1} (A + X) B^{-1} \right] = C \cdot I$$

$$\Leftrightarrow \quad C . C^{-1} . (A + X) B^{-1} = C . I$$

$$\Leftrightarrow \quad I . (A + X) B^{-1} = C . I$$

$$\Rightarrow \quad (A + X) B^{-1} = C$$

$$\Rightarrow \quad (A + X) I = C B$$

$$\Rightarrow \quad A + X = CB$$

$$\Rightarrow \quad X = CB - A$$

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