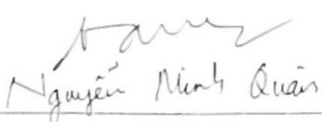



**THE INTERNATIONAL UNIVERSITY(IU)  
VIETNAM NATIONAL UNIVERSITY - HCMC**

**FINAL EXAMINATION  
PROBABILITY, STATISTICS AND RANDOM PROCESS**  
Semester 1, 2022-23 • January 2023 • Total duration: 90 minutes

Chair of Mathematics Department	Lecturer
 Nguyễn Minh Quân	 Dr. Pham Hai Ha

**INSTRUCTIONS:** Each student is allowed calculators, statistical tables and one double-sided sheet of reference material (size A4 or similar) marked with their name and ID. All other documents and electronic devices are forbidden.

1. (20 points) Consider a Markov chain  $(X_n)_{n \geq 0}$  with state space  $S = \{1, 2\}$  and transition matrix

$$P = \begin{array}{c} \text{To} \\ \begin{array}{cc} & \begin{array}{c} 1 \\ 2 \end{array} \\ \begin{array}{c} \text{From} \\ 1 \\ 2 \end{array} & \begin{bmatrix} 0.5 & 0.5 \\ 0.3 & 0.7 \end{bmatrix} \end{array}$$

- (a) Compute  $P(X_3 = 2 | X_0 = 1)$  and  $P(X_5 = 2, X_3 = 2 | X_0 = 1)$ .  
 (b) Given initial distribution  $P(X_0 = 1) = 0.4$ ,  $P(X_0 = 2) = 0.6$ . Evaluate  $E(X_2)$ .  
 (c) Determine the stationary distribution of this Markov chain.
2. (10 points) Consider a Markov chain  $(X_n)_{n \geq 0}$  with state space  $S = \{1, 2, 3, 4\}$  and transition matrix

$$P = \begin{array}{c} \text{To} \\ \begin{array}{cccc} & 1 & 2 & 3 & 4 \\ \begin{array}{c} \text{From} \\ 1 \\ 2 \\ 3 \\ 4 \end{array} & \begin{bmatrix} 0.25 & 0.5 & 0.25 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

Which state(s) is (are) recurrent?

3. (10 points) The data show the number of licensed nuclear reactors in the United States for a recent 15-year period. Compute the sample mean, sample standard deviation, median and mode.

104	104	104	104	104
107	109	109	109	110
109	111	112	111	109

PLEASE TURN OVER

4. (10 points) In a random sample of 100 shoppers, the customer spend an average of 600 thousand vietnam dong per visit at a Vincom Center. The standard deviation of the population is 50 thousand vietnam dong. Find the 99% confidence interval of the true mean.
5. (10 points) In a study of 500 accidents that required treatment in an emergency room, 10 occurred at work. Find the 90% confidence interval of the true proportion of accidents that occurred at work.
6. (10 points) A random sample of the final score in Calculus I is below.

72 79 80 74 82 79 82 78 60 75

Is there sufficient evidence to conclude that the variance in score differs from 40 at level of significance  $\alpha = 0.05$ ? Assume that the score is normally distributed.

7. (10 points) The deflection temperature (in Celcius degree) under load for two different types of plastic pipe is being investigated. The deflection temperature for type 1 and type 2 are suppose to be normally distributed with standard deviation  $\sigma_1 = 0.02$  and  $\sigma_2 = 0.025$  respectively. For type 1, a random sample of 10 pipe specimens are tested and provides sample mean  $\bar{x}_1 = 16$  while a sample of 20 pipe specimens tested for type 2 has sample mean  $\bar{x}_2 = 17$ .

At level of significant  $\alpha = 1\%$ , do the data support the claim that the deflection temperature under load for type 2 pipe is strictly greater than that of type 1?

8. (20 points) The data to study the deflection (mm) of particleboard from stress levels of relative humidity are shown below:

$x = \text{Stress level (\%)}$	54	58	61	62	68	70	72	75	78
$y = \text{Deflection (mm)}$	16	19	14	13	14	15	11	12	11

- (a) Fit the simple linear regression model using the method of least squares.
- (b) Find the estimate of the mean deflection if the stress level is 65%.

THE END