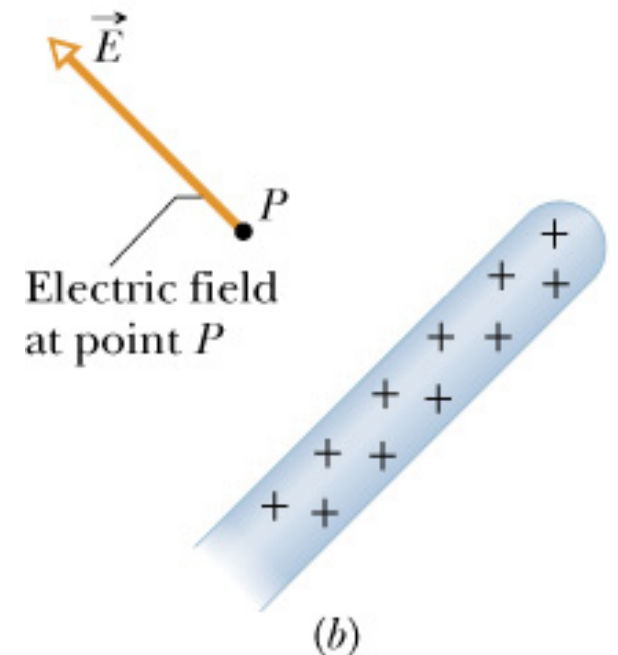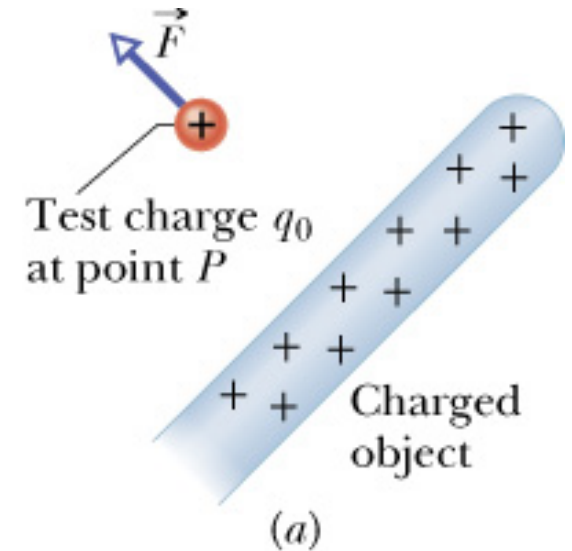# 1.5. The Electric Field. Electric Field Lines:

Experiment: Place q_0 at point P close to a charged object, a force F acting on q_0

Key Question: How can the charged object push q_0 away without touching it?

➢ The charged object sets up an *electric field* in the space surrounding itself that affects q_0

Test charge q_0 at point P

Charged object

(a)

Electric field at point P

(b)

1.5.1. The Electric Field (introduced by Michael Faraday):

The electric field is a vector field and consists of a distribution of vectors. One vector at each point in the region around a charged object
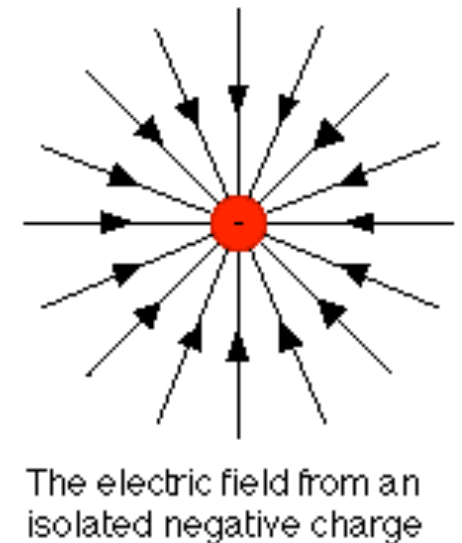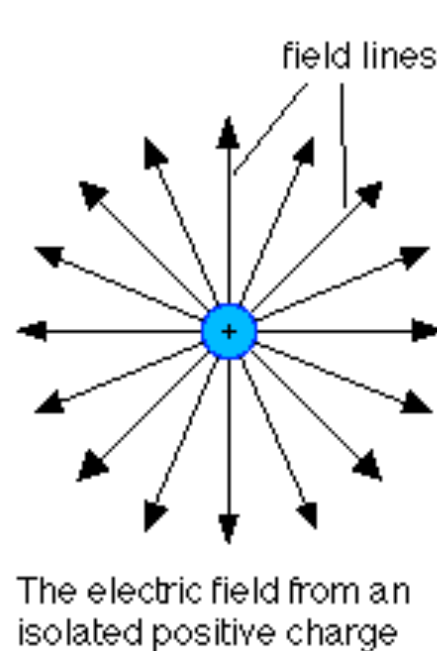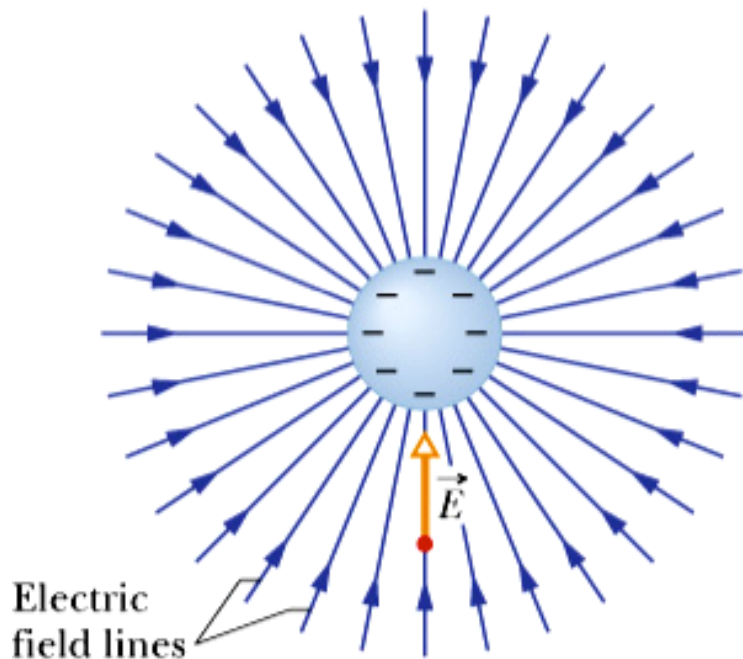
$$\vec{E} = \frac{\vec{F}}{q_0}$$

- the direction of $\vec{E}$ is that of $\vec{F}$ acting on the positive charge
- Unit: N/C

Some Electric Fields

Field Location or Situation	Value (N/C)
At the surface of a uranium nucleus	3×10^{21}
Electric breakdown occurs in air	3×10^{6}
Near a charged comb	10^{3}
Inside the copper wire of household circuits	10^{-2}

1.5.2. Electric Field Lines:

● An electric field can be represented diagrammatically as a set of unbroken lines with arrows on, which are electric field lines

● **Rules to draw electric field lines:**
  – the direction of $\vec{E}$ is tangent to the field lines
  – the magnitude of E is proportional to the number of field-lines per unit area measured in a plane normal to the lines
  – electric field lines extend away from positive charge where they originate and toward negative charge where they terminate
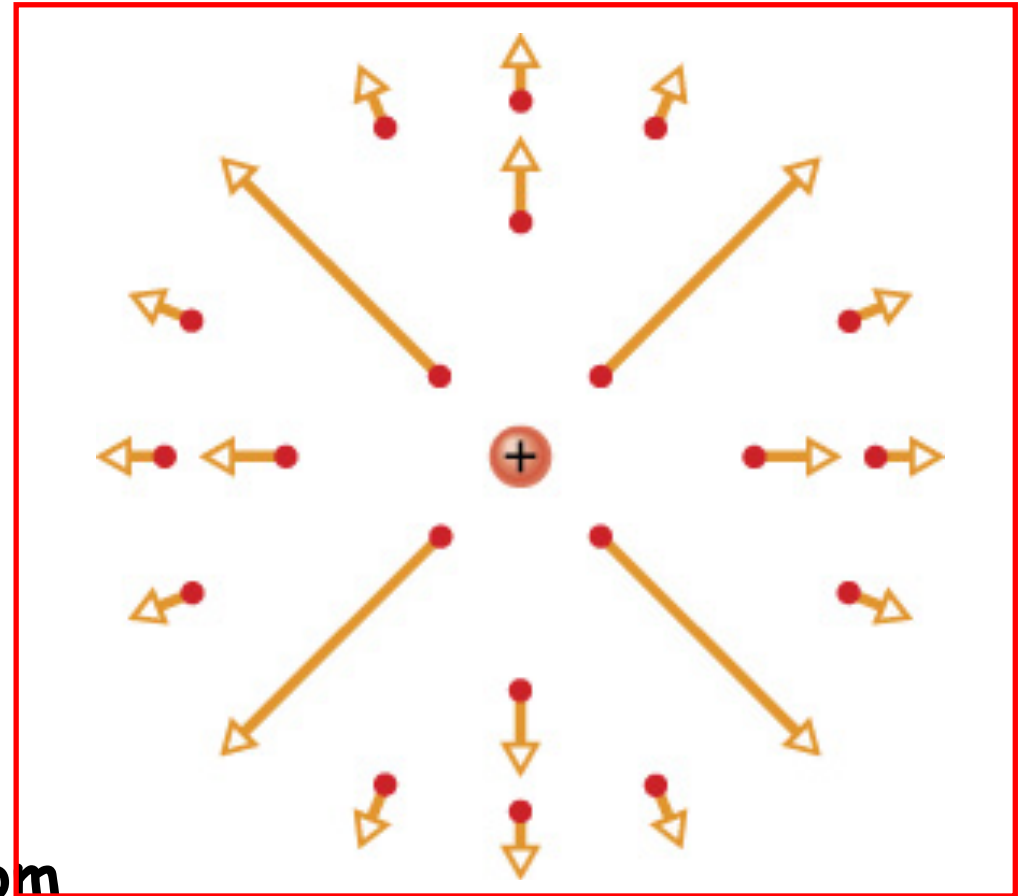

Electric field lines

field lines

The electric field from an isolated positive charge

The electric field from an isolated negative charge

(a) The electric field due to a point charge:

$$F = \frac{1}{4\pi\varepsilon_0}\frac{|q\,|\,q_0\,|}{r^2}$$
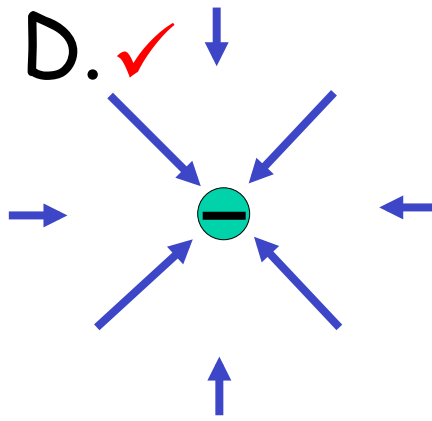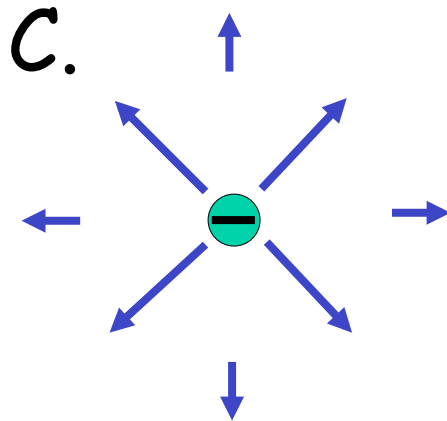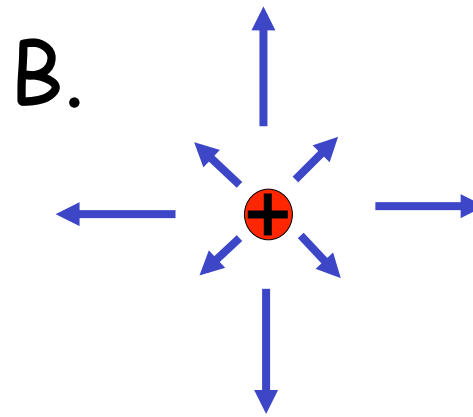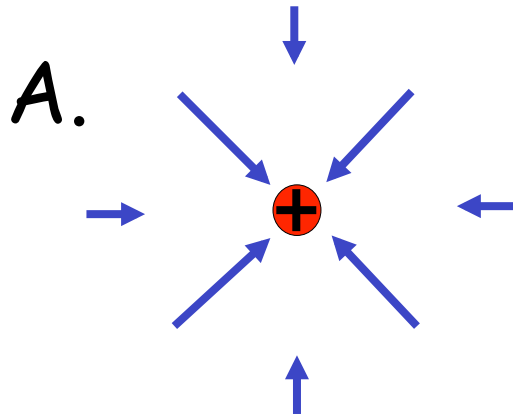
- The **magnitude** of $\vec{E}$:

$$E = \frac{F}{q_0} = \frac{1}{4\pi\varepsilon_0}\frac{|q|}{r^2}$$

- The **direction** of $\vec{E}$:
  - q > 0: directly away from the charge
  - q < 0: toward the charge

Checkpoint: Which diagram can be considered to show the correct electric force on a positive test charge due to a point charge?
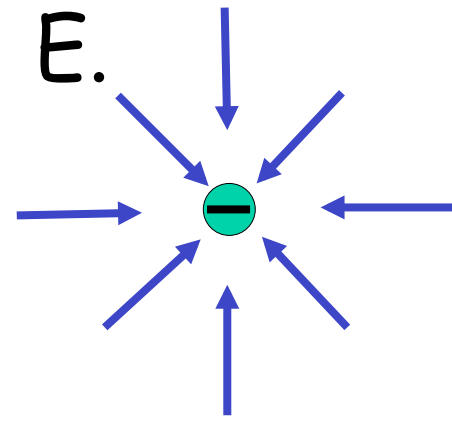
(b) The electric field due to a group of individual charge:

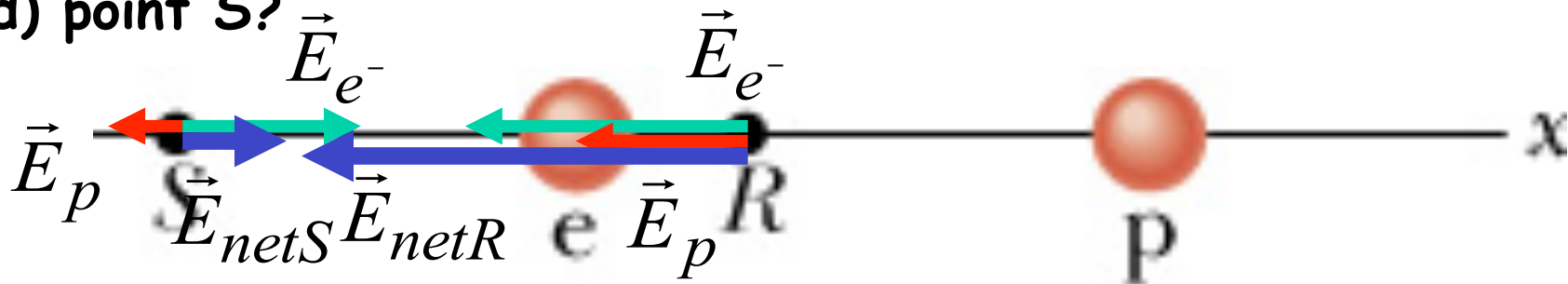- We have n point charges, the net force acting on the test charge q_0:

$$\vec{F}_0 = \vec{F}_{01} + \vec{F}_{02} + \vec{F}_{03} + ... + \vec{F}_{0n}$$

- Therefore, the net $\vec{E}$:

$$\vec{E} = \frac{\vec{F}_0}{q_0} = \frac{\vec{F}_{01}}{q_0} + \frac{\vec{F}_{02}}{q_0} + \frac{\vec{F}_{03}}{q_0} + ... + \frac{\vec{F}_{0n}}{q_0} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 + ... + \vec{E}_n$$

Checkpoint 1 (page 583): See the figure, what is the direction of the electric field due to e⁻ at (a) point S and (b) point R?, what is the direction of the net electric field at (c) point R and (d) point S?

The electric field due to an electric dipole:

$|q_1| = |q_2|$

$|q_1| < |q_2|$

$\vec{E}$

q_2

$\vec{E}$

q_1
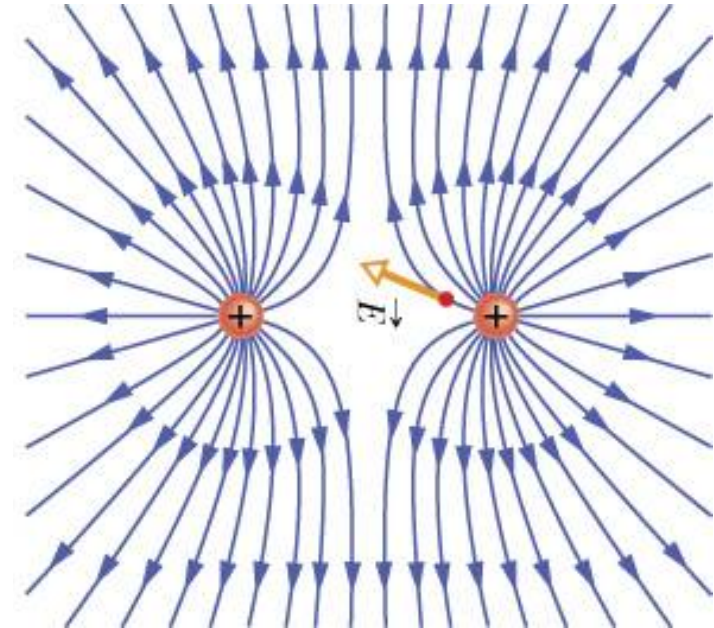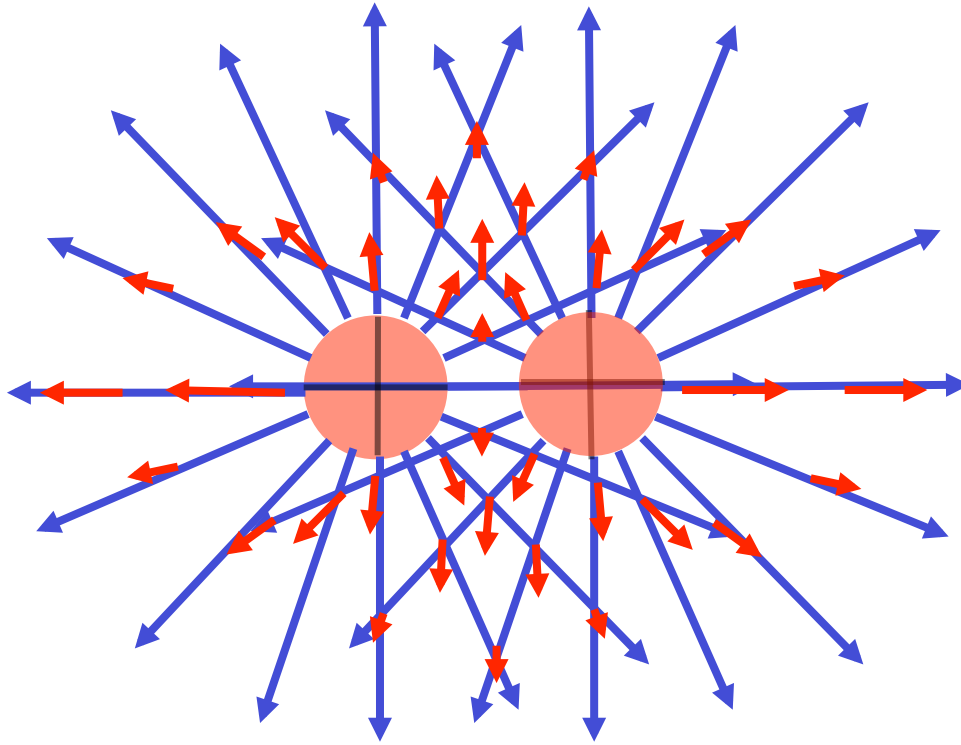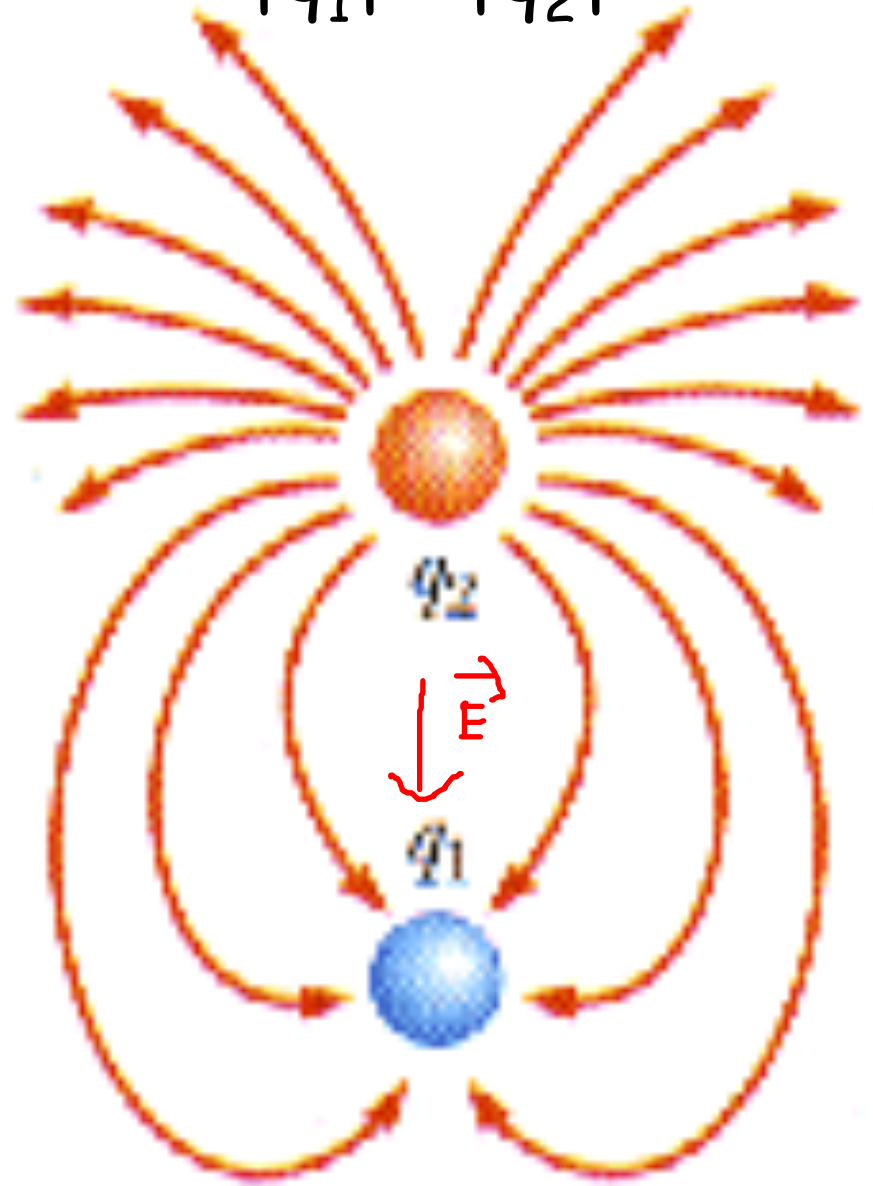
simulation

Problem:

• *An electric dipole*: two charged particles of q but of opposite sign, separated by d
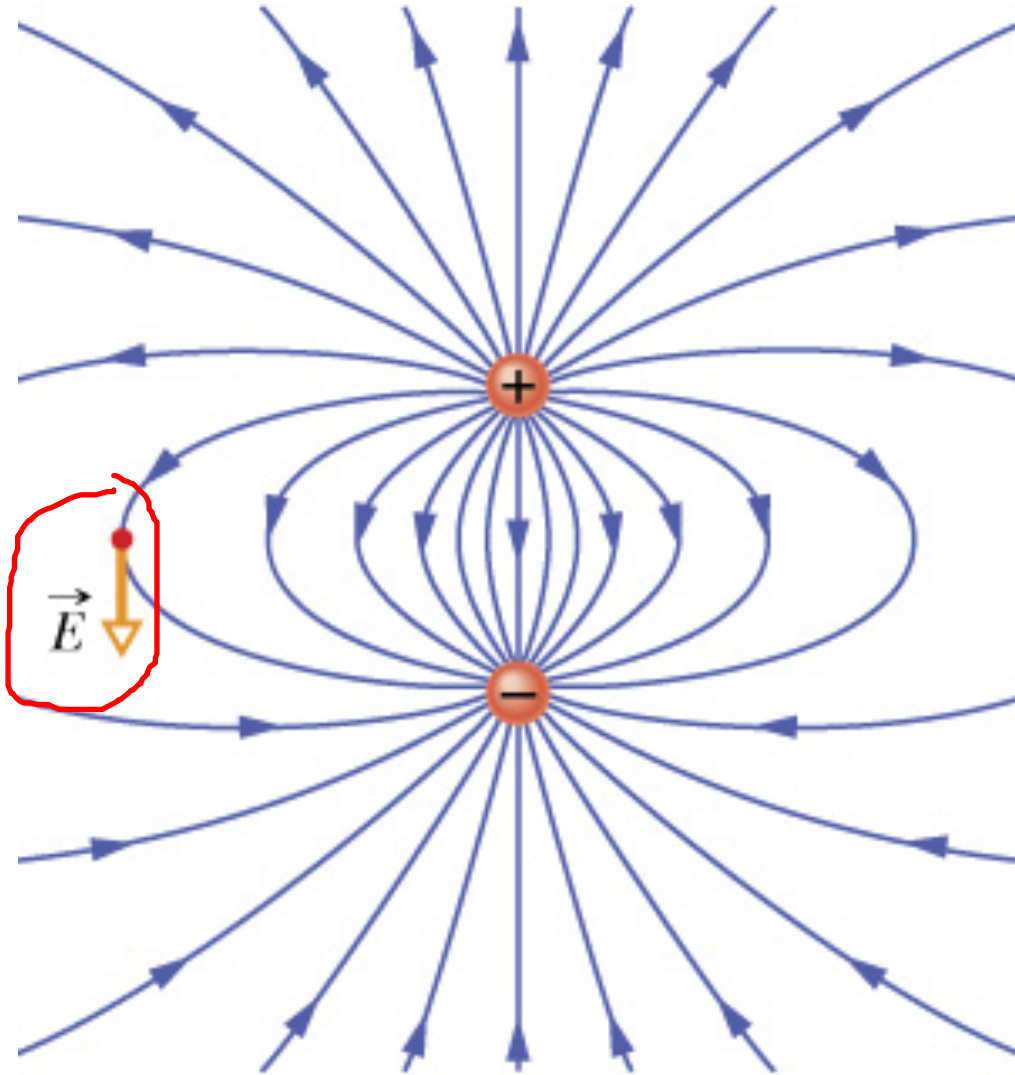• Calculate E due to the electric dipole at point P

$$E = E_{(+)} - E_{(-)} = \frac{1}{4\pi\varepsilon_0} \frac{q}{r_{(+)}^2} - \frac{1}{4\pi\varepsilon_0} \frac{q}{r_{(-)}^2}$$

$$E = \frac{q}{4\pi\varepsilon_0 z^2}\left[\left(1 - \frac{d}{2z}\right)^{-2} - \left(1 + \frac{d}{2z}\right)^{-2}\right]$$

• Usually, we consider z >> d, so d/2z << and apply the binomial theorem (see Appendix E),

(a)

(b)
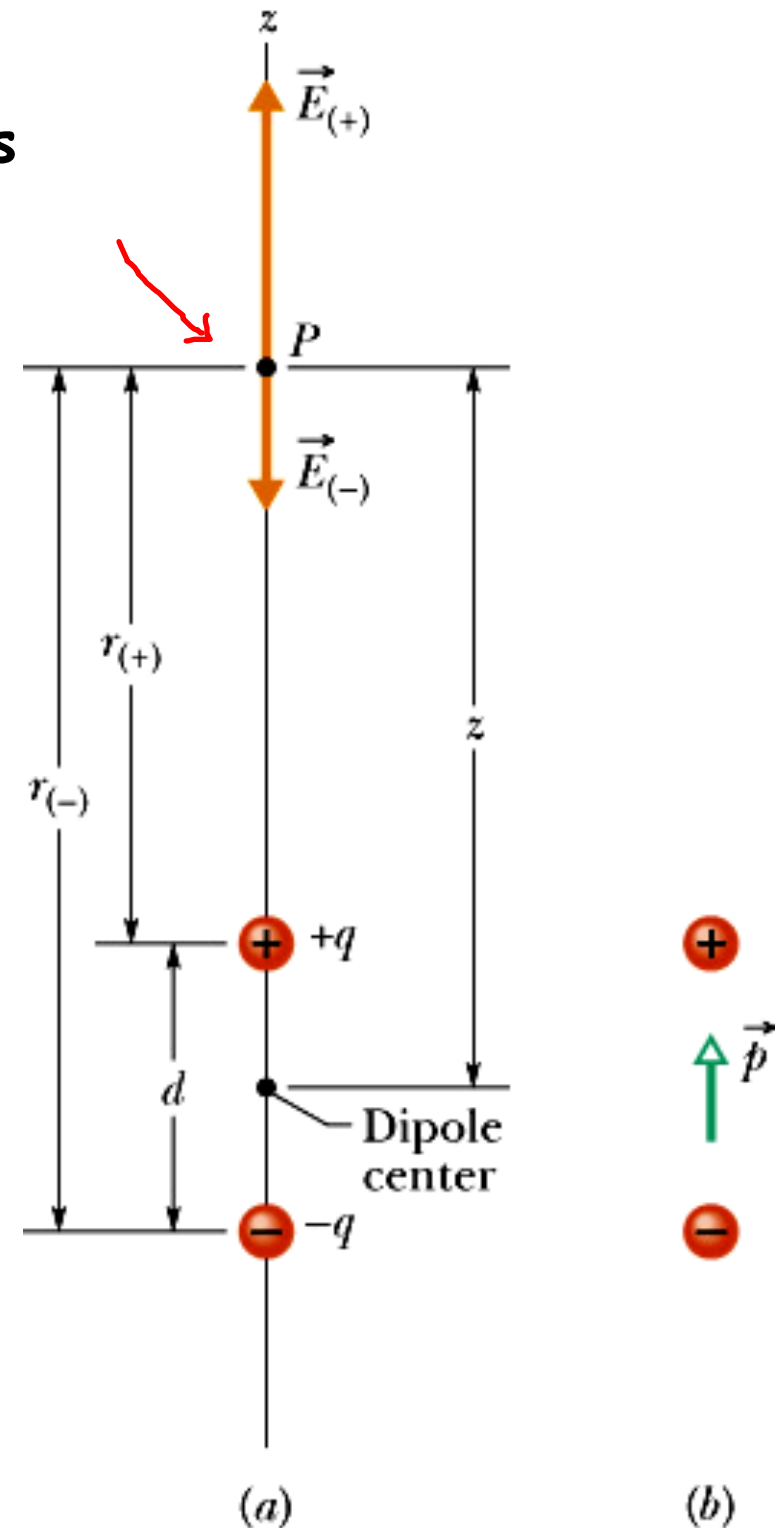
$$E = \frac{1}{2\pi\varepsilon_0} \frac{qd}{z^3}$$

The electric dipole moment $\vec{p}$ **of the dipole:**

  – Magnitude:

$$\boxed{p = qd : \text{an intrinsic property of a dipole}}$$

  • Unit: **C.m**

  – Direction: from the **negative to the positive**, the direction can be used to specify the orientation of the dipole

$$E = \frac{1}{2\pi\varepsilon_0} \frac{p}{z^3} \qquad (1)$$

Note: $E \sim 1/z^3$ is applicable for all <u>distant</u> points regardless of whether they lie on the dipole axis (d << z)

Oxygen

8+
8n

+ Hydrogen

+ Hydrogen

$\vec{p}$

Electrons from hydrogen
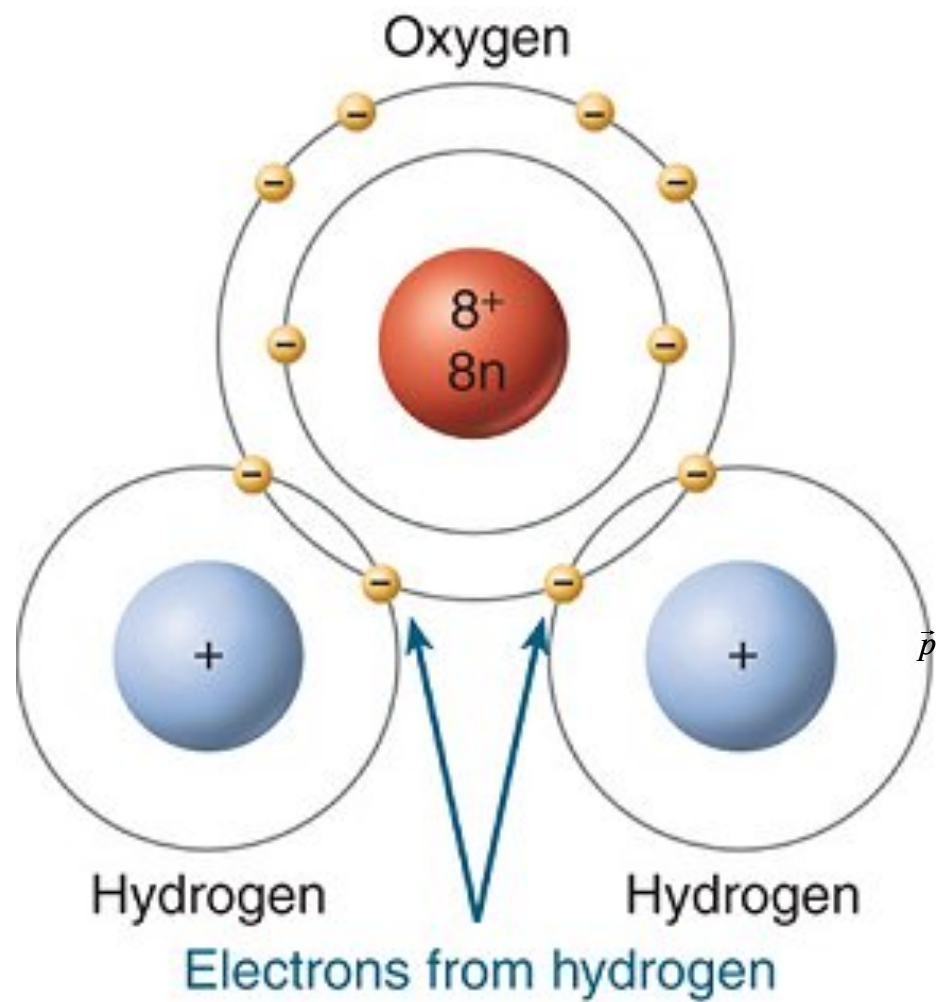
(a) Electron shells in a
water molecule

δ^-

O

$\vec{p}$

H
δ^+

H
δ^+

(b) Distribution of partial
charges in a water
molecule

1.6. Electric Field of a Continuous Charge Distribution:

Concept: Charge distributions consist of an enormous number (e.g., billions) of closely spaced point charges spreading along a line, over a surface (this section) or within a volume (next lecture).
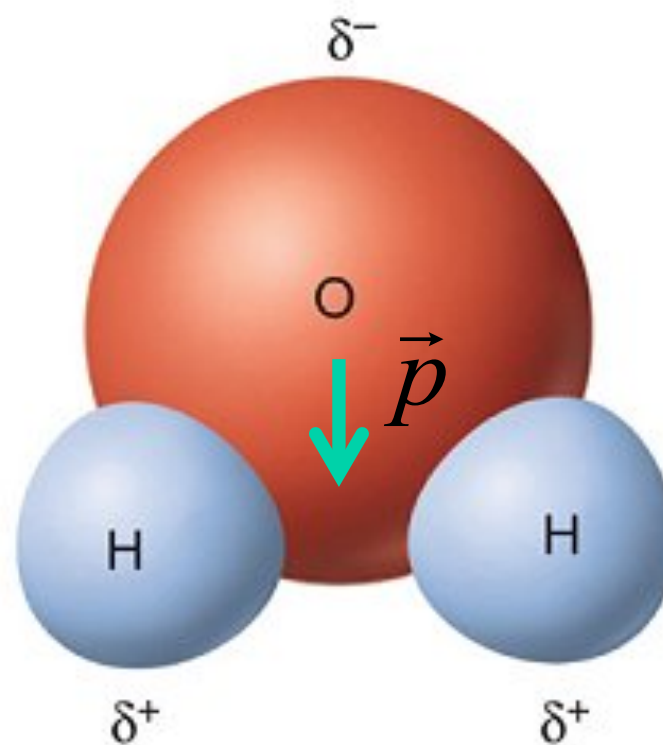
The principle to calculate E:

✓ Find an expression for dq:
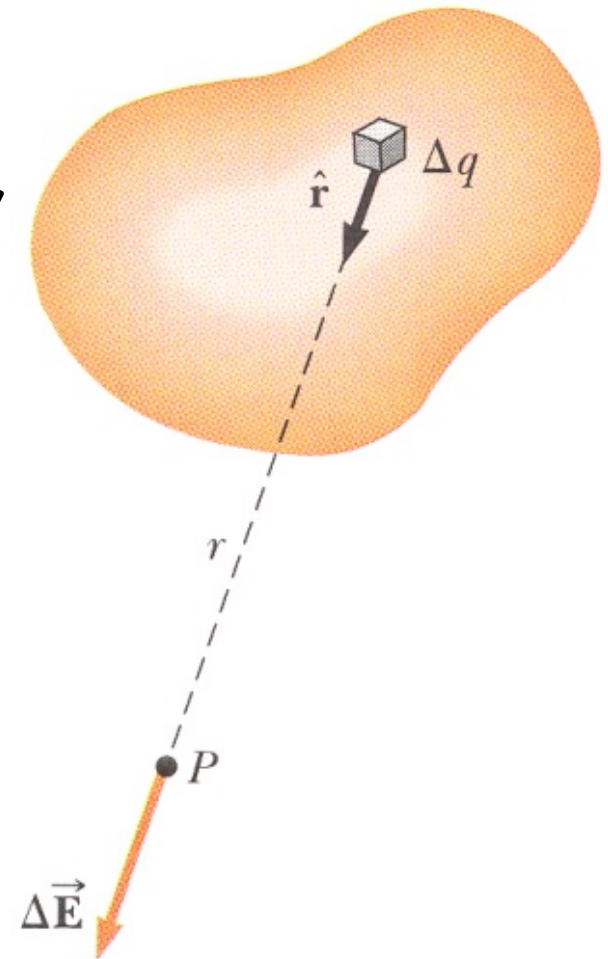   - dq = Λdl for a **line** distribution
   - dq = σdA for a **surface** distribution
   - dq = ρdV for a **volume** distribution

✓ Calculate dE:

$$d\vec{E} = \frac{dq}{4\pi\varepsilon_0 r^2}\hat{r}$$

✓ Add up (integrate the contributions) over the whole distribution, varying the displacement as needed:
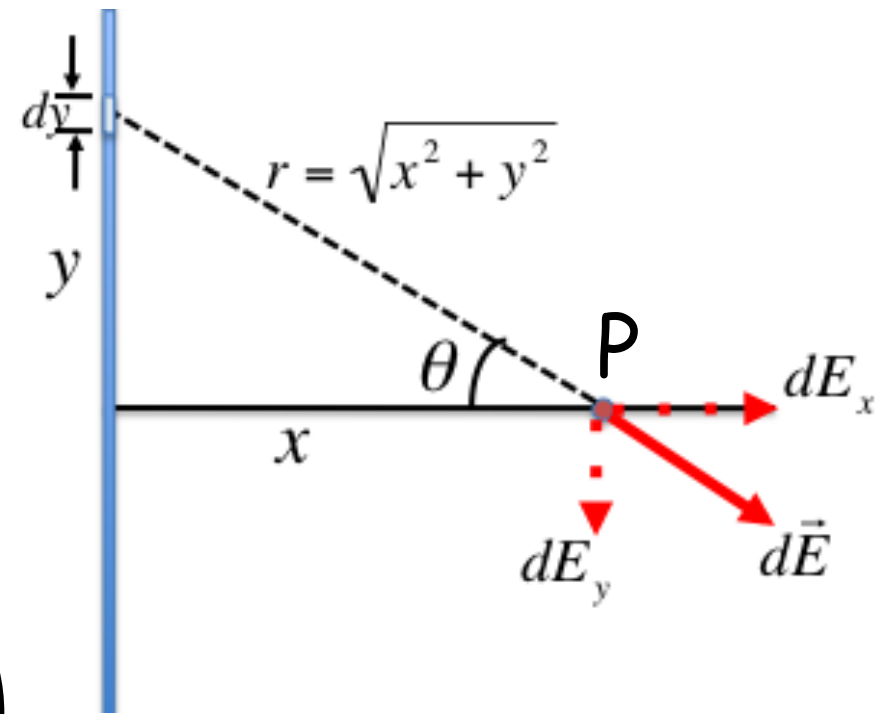
$$\vec{E} = \int d\vec{E}$$

1.6.1. Electric Field Due to a Line of Charge:

Problem: Calculate E of a line of uniform positive charge at point P . Consider a differential element of charge dq:

$$dq = \lambda dy$$

λ : linear charge density (C/m)

$$dE = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r^2} = \frac{1}{4\pi\varepsilon_0}\frac{\lambda dy}{\left(x^2 + y^2\right)}$$

$r = \sqrt{x^2 + y^2}$

- Each element has 2 components: **dEcosθ** parallel to x and **dEsinθ** perpendicular to x.
- All perpendicular components of the elements will cancel out each other, so:

$$E = \int dE\cos\theta = \int_{-l/2}^{+l/2}\frac{1}{4\pi\varepsilon_0}\frac{x\lambda}{\left(x^2 + y^2\right)^{3/2}}dy$$

(see App. E, A-11):

$$\int \frac{dx}{(x^2 + a^2)^{3/2}} = \frac{x}{a^2(x^2 + a^2)^{1/2}}$$

$$E = \frac{\lambda}{2\pi\varepsilon_0 x} \frac{\frac{l}{2}}{\sqrt{x^2 + \left(\frac{l}{2}\right)^2}}$$

1.6.2. Electric Field Due to a Ring of Charge:
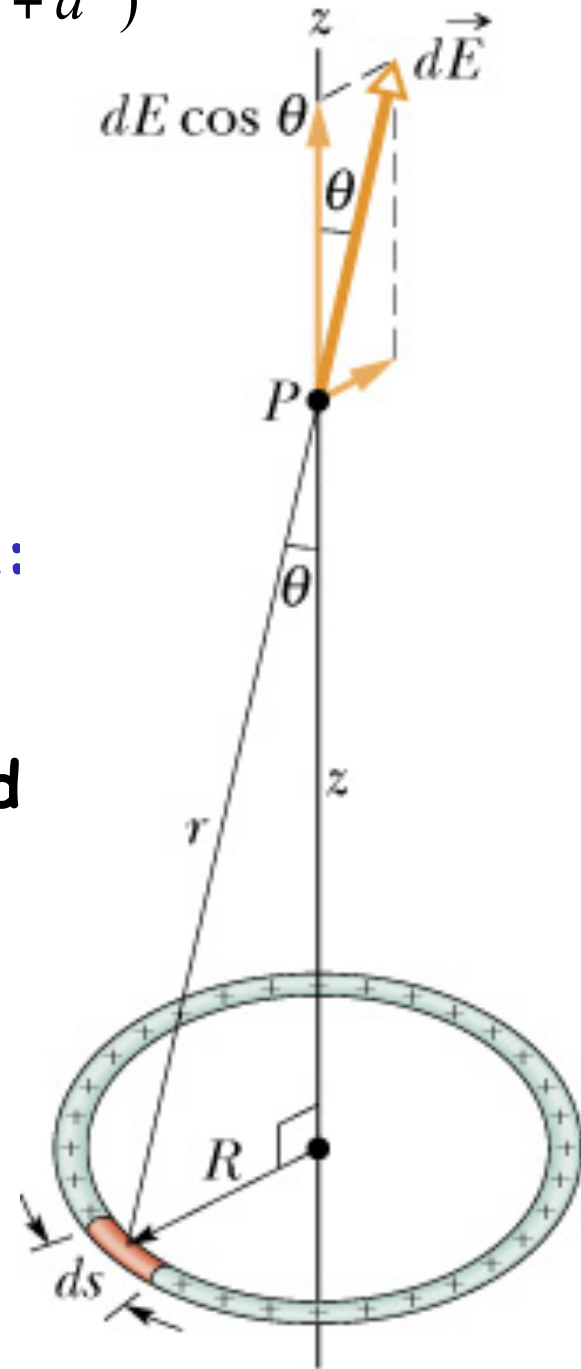
Problem: Calculate E of a ring of uniform positive charge at point P
. Consider a differential element of charge d

$$dq = \lambda ds$$

$$\lambda : \text{linear charge density (C/m)}$$

$$dE = \frac{1}{4\pi\varepsilon_0} \frac{dq}{r^2} = \frac{1}{4\pi\varepsilon_0} \left(\frac{\lambda ds}{z^2 + R^2}\right)$$

$$E = \int dE \cos\theta = \int_0^{2\pi R} \frac{1}{4\pi\varepsilon_0} \frac{z\lambda}{(z^2 + R^2)^{3/2}} ds$$

We have $q = 2\pi R \times \lambda$

$$E = \frac{qz}{4\pi\varepsilon_0 (z^2 + R^2)^{3/2}}$$

• If z >> R (distant points):

$$E = \frac{q}{4\pi\varepsilon_0 z^2} \quad \text{(like a point charge)}$$

1.6.3. Electric Field Due to a Charged Disk:

Problem: Calculate E at point P of a circular plastic disk with a uniform surface density of positive charge.
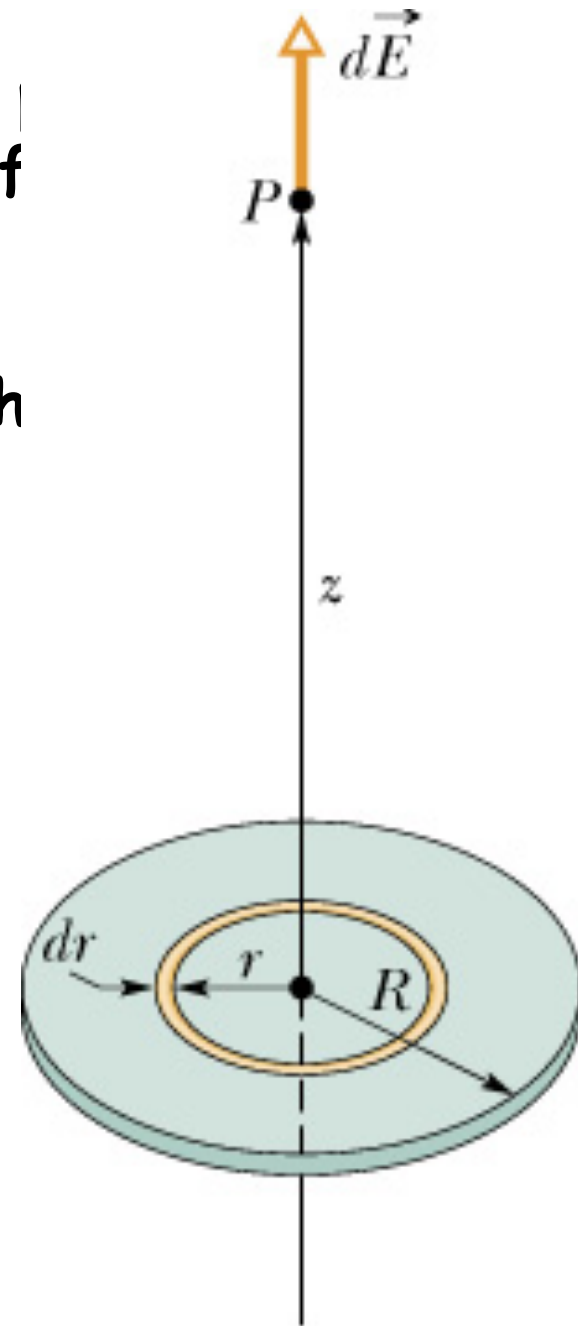
Consider a ring with radius r and radial width dr:

$$dE = \frac{z\,dq}{4\pi\varepsilon_0 \left(z^2 + r^2\right)^{3/2}}$$

$$dq = \sigma dA = \sigma 2\pi r\,dr$$

σ : surface charge density (C/m^2)

$$E = \int_0^R dE = \frac{\sigma}{2\varepsilon_0}\left(1 - \frac{z}{\sqrt{z^2 + R^2}}\right)$$

1.7. Charge in an Electric Field:

## 1.7.1. A Point Charge in an Electric Field:

A charge particle in an external field $\vec{E}$ experiences a force:

$$\vec{F} = q\vec{E} \text{ (q includes its sign)}$$

Checkpoint 3 (page 592): see the figure below (a) what is the direction of the electrostatic force on e^-? (b) in which direction will e^- accelerate if it is moving parallel to y before it encounters E? (c) If, instead, e^- is initially move rightward, will its speed increase, decrease or remain constant?

(a) the opposite direction of $\vec{E}$

(b) same as (a)

(c) decrease
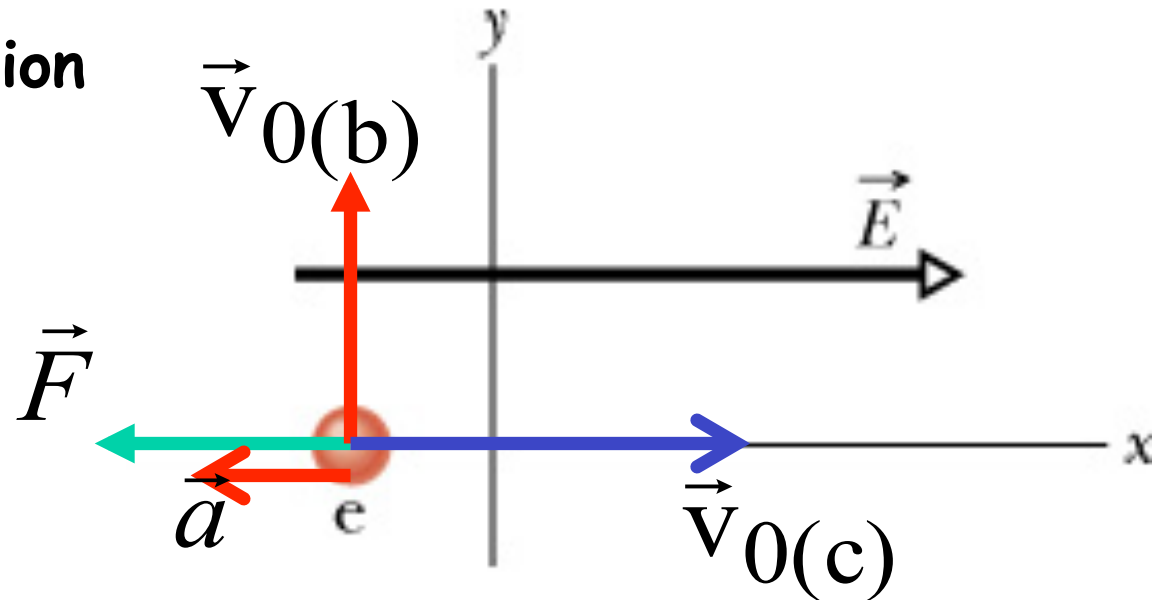
1.7.2. A Dipole in an Electric Field:

Problem: Consider a dipole (e.g., a molecule of water) in a uniform external electric field $\vec{E}$, determine the net torque acting on the dipole.

$$\vec{\tau} = \vec{r} \times \vec{F}$$
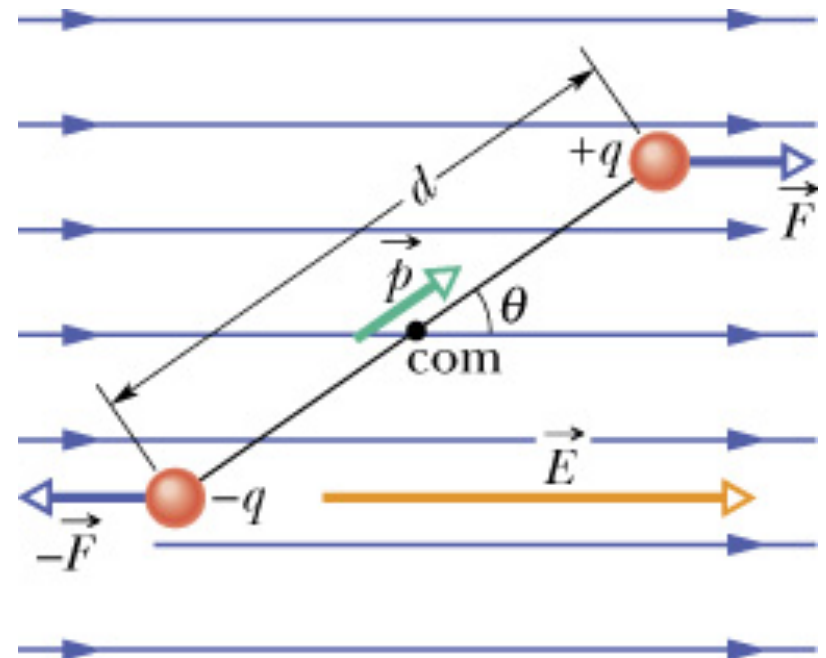
The magnitude of the torque:

$$\tau = Fx\sin\theta + F(d-x)\sin\theta$$

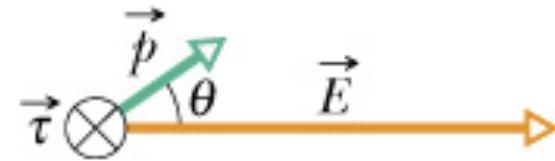$$= Fd\sin\theta = qdE\sin\theta$$

We have the dipole moment:

$$p = qd$$

$$\tau = pE\sin\theta$$

(Unit: N.m)

(a)

(b)

$$\vec{\tau} = \vec{p} \times \vec{E}$$

Note: in the figure above, the torque tends to rotate the dipole in the clockwise direction, so:

$$\tau = -pE \sin \theta$$

⬓ Potential Energy of an Electric Dipole:

Problem: Calculate the potential energy U of a dipole in an external electric field

● Choose U = 0 at θ = 90⁰, then calculate U at θ ≠ 90⁰

$$\Delta U = U_\theta - U_{90} = -W \text{ (work done by the field)}$$

$$U = -\int_{90^0}^{\theta} \tau \, d\theta = \int_{90^0}^{\theta} pE \sin \theta \, d\theta = -pE \cos \theta$$

$$U = -\vec{p}\vec{E}$$

(Unit: J)

- $\theta = 0^0$: U is minimum → the dipole is in stable equilibrium
- $\theta = 180^0$: U is maximum → the dipole is in unstable equilibrium
- Work done by the field from θ_i to θ_f:

$$W = -(U_{\theta_f} - U_{\theta_i})$$

- Work done by the applied torque (of the applied force):
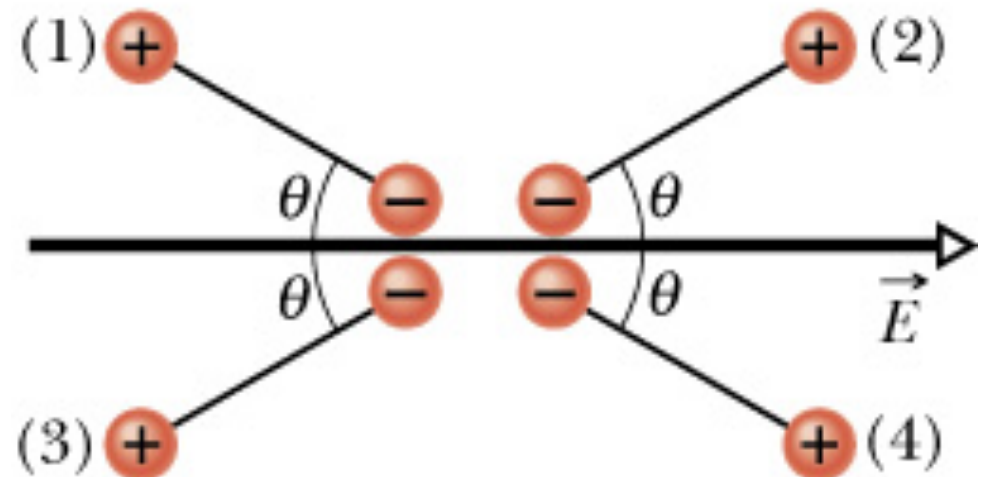
$$W_a = -W = U_{\theta_f} - U_{\theta_i}$$

Checkpoint 4 (page 595): Rank the orientations according to (a) the magnitude of the torque on the dipole and (b) the potential energy of the dipole, greatest first.

$$\tau = -pE\sin\theta$$
$$U = -pE\cos\theta$$

(a) All tie
(b) 1=3 > 2=4

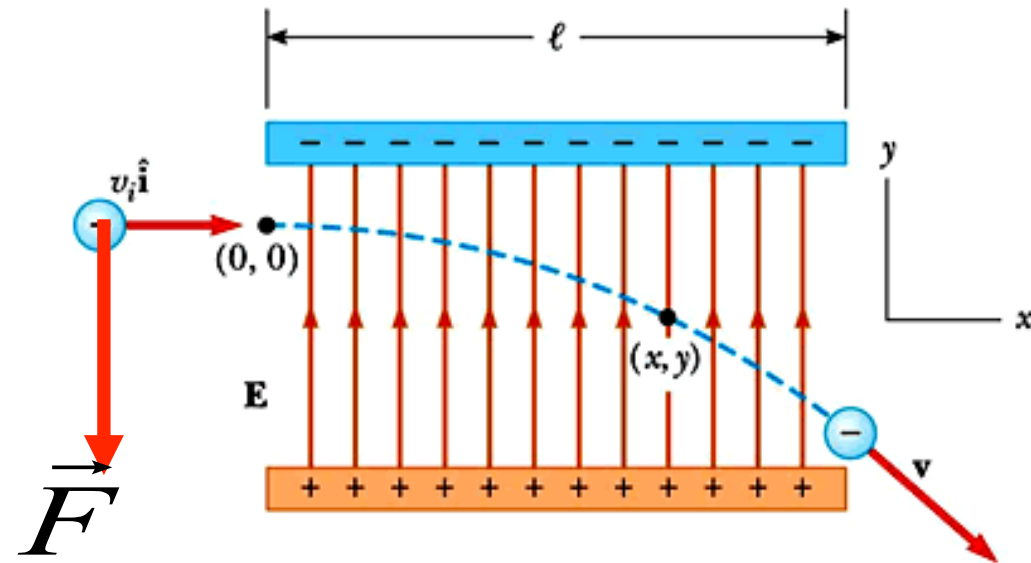# 1.8. Motion of Charged Particles in a Uniform Electric Field:

(using the equations for a projectile motion)

+ x axis: v_x = constant

$$x = l = v_x t$$

+ y axis: a_y = F/m = qE/m

$$y = \frac{1}{2} a_y t^2 \Rightarrow y = \frac{qEl^2}{2mv_x^2}$$
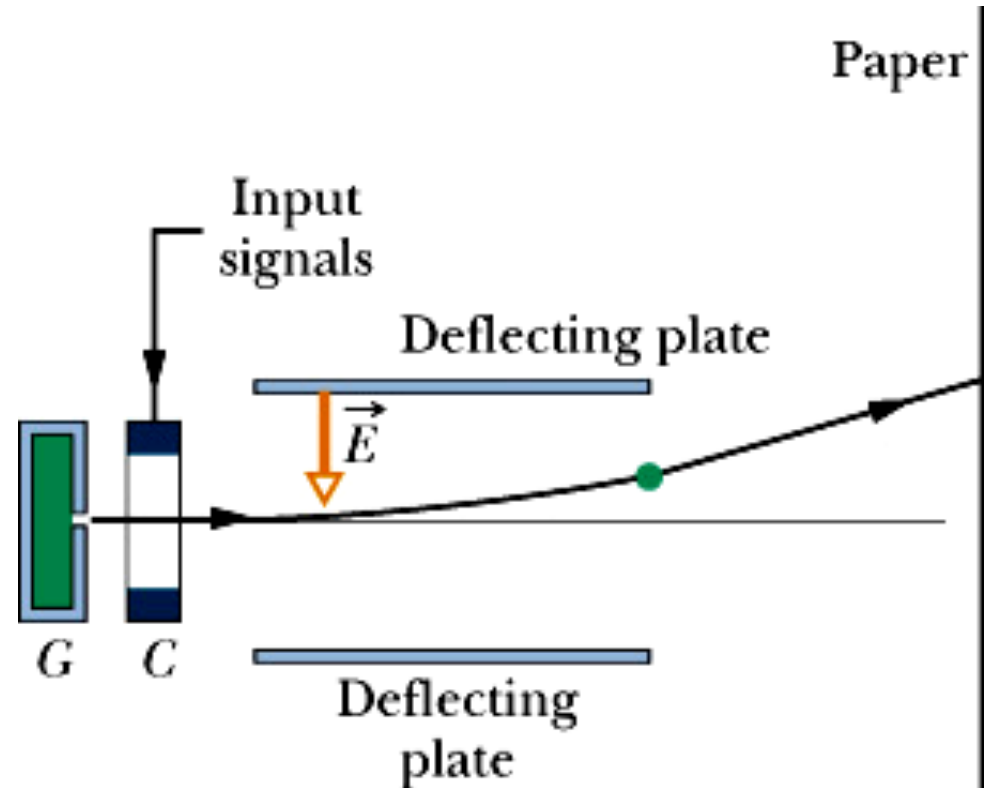
Application in Ink-Jet Printing:

- Drops are shot out from generator G

- Receiving a charge q in Charge Unit C

- An input signal from a computer controls q given to each drop

- The electric field between two conducting deflecting plates is held constant, and the position of the drop on the paper is determined by q
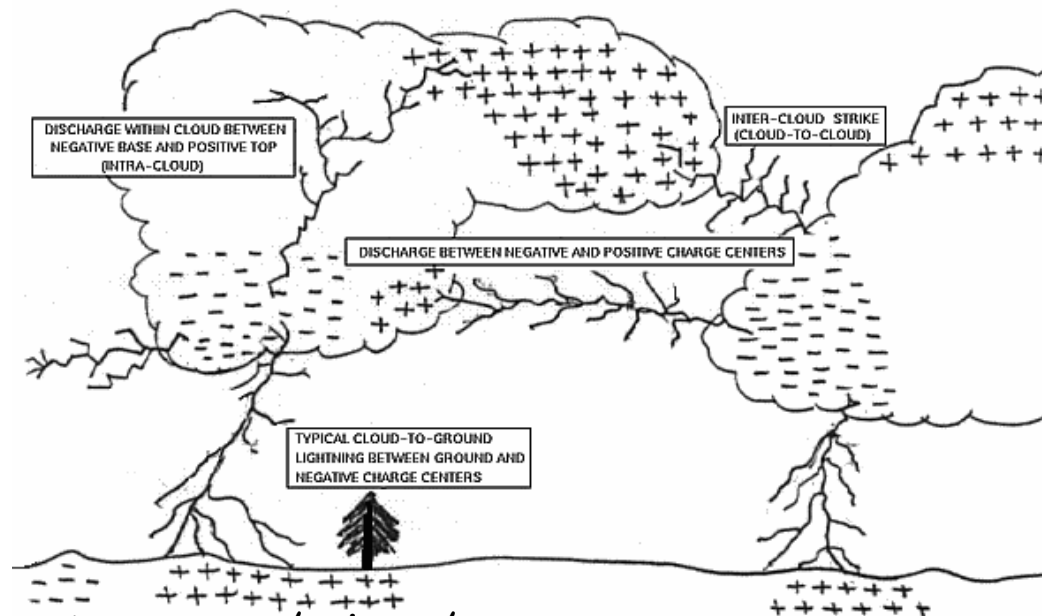
Input signals

Deflecting plate

$\vec{E}$

Paper

G   C

Deflecting plate

© Jeremy Holmes Photography.com

Electrical breakdown and lightning:

Electrical breakdown: If E in air exceeds a critical value E_c, the field removes electrons from the atoms in air. The air then conducts electric current produced by the freed electrons. The electrons collide with any atoms in their path, causing these atoms to emit light. So, we can see the paths which are called "sparks".

Lightning is the result of electric breakdown in air:

DISCHARGE WITHIN CLOUD BETWEEN NEGATIVE BASE AND POSITIVE TOP (INTRA-CLOUD)

INTER-CLOUD STRIKE (CLOUD-TO-CLOUD)

DISCHARGE BETWEEN NEGATIVE AND POSITIVE CHARGE CENTERS

TYPICAL CLOUD-TO-GROUND LIGHTNING BETWEEN GROUND AND NEGATIVE CHARGE CENTERS

movie

http://lightning.nsstc.nasa.gov/primer/

Homework: 1, 5, 14, 15, 19, 23, 27, 31, 35, 44, 54, 56, 59 (page 598-603)