CALCULUS 1 – FINAL EXAMINATION

Semester 2, 2021-22 − Thursday 23 June 2022 − Total duration: 85 minutes

Student's name:	Student ID:

Chair of Mathematics Department	Lecturers	Proctor
Prof. Pham Huu Anh Ngoc		

INSTRUCTIONS: Each student is allowed one double-sided sheet of reference material (size A4 or similar) marked with their name and ID. Calculators are allowed in Part 2 only, *not* in Part 1. All other documents and electronic devices are forbidden. Please choose the correct answers on the answer sheet at the end of Part 1.

PART 1: MULTIPLE CHOICE QUESTIONS – 45 Minutes – 60 points

1. The absolute maximum value of the function $f(x) = -12x - 3x^2 + 2x^3 + 3$ on the interval $[-2, 3]$ is

   (A) -1 (B) 9 (C) 10 (D) 5

2. Let $f(x) = (\sin x)^x$. Find $f'(x)$.

   (A) $(x\cot x + \ln(\sin x))(\sin x)^x$
   (B) $x(\sin x)^{x-1}$
   (C) $(\sin x)^x$
   (D) $(\cos x)^x$

3. Suppose that $f(0) = 0$ and $f'(x) \geq 3$ for all values of x. How small can $f(3)$ possibly be?

   (A) -12 (B) 9 (C) 7 (D) 5

4. Let $f(x) = \lfloor x \rfloor$ be the greatest integer that is not greater than x. Find $f'\left(\frac{5}{2}\right)$.

   (A) Does not exist (B) $\frac{5}{2}$ (C) 1 (D) 0

5. Evaluate $\lim\limits_{x\to\infty} \dfrac{\ln(\sqrt{x})}{e^x}$

   (A) Does not exist (B) e^2 (C) -1 (D) 0

6. A particle moves in a straight line and its velocity is given by $v(t) = 2t^2 + t - 3$ and its initial position is $s(0) = 2$. Find its position function $s(t)$.

   (A) $s(t) = \dfrac{2t^3}{3} + \dfrac{t^2}{2} - 3t + 2$
   (B) $s(t) = \dfrac{2t^2}{3} + 3t + 2$
   (C) $s(t) = t^3 + 2t + 2$
   (D) $s(t) = 2t + 3$

7. Find the length of the arc $y = \frac{2}{3}x^{\frac{3}{2}}$ between $x = 0$ and $x = 1$.

(A) $\frac{2}{3}$ (B) $\frac{2}{3}(2\sqrt{2}-1)$ (C) 1 (D) None of them

8. Evaluate $\lim_{x\to 0}\left(1+x^2\right)^{\frac{1}{x}}$

(A) $\sqrt{e}$ (B) ∞ (C) e (D) 1

9. The area of the region enclosed by the curve $y = 5x - 2x^2$ and the line $y = x$ is

(A) None of them (B) $\frac{8}{3}$ (C) $-\frac{1}{6}$ (D) $\frac{32}{3}$

10. The value of $\int_0^1 \frac{1}{x^2+4x+3}\,dx$ is

(A) None of them (B) $\ln\frac{3}{2}$ (C) $\frac{1}{2}\ln\frac{3}{2}$ (D) $\ln\frac{\sqrt{3}}{2}$

11. The value of limit $\lim_{x\to 0}\frac{\int_0^x \sqrt[3]{t^2}\,dt}{x^{5/3}}$ is

(A) $\frac{1}{3}$ (B) 0 (C) $-\frac{2}{3}$ (D) $\frac{3}{5}$

12. Given $F(x) = \int_0^{x^2} \sqrt{t+\cos(\pi t)}\,dt$, the value of $F'(1)$ is

(A) $\sqrt{1+\pi}$ (B) $2\sqrt{\pi}$ (C) 0 (D) $2\sqrt{\pi-1}$

13. The value of $\int_0^\infty \frac{x^2}{\sqrt{1+x^3}}\,dx$ is

(A) $\frac{1}{\sqrt{3}}$ (B) $\frac{2}{3}$ (C) 0 (D) divergent

14. If $f(1) = 5$ and $\int_0^1 xf'(x)dx = 1$, then the value of $\int_0^1 f(x)dx$ is

(A) 3 (B) 4 (C) 5 (D) None of them

15. The region R enclosed by the curves $y = 2+x^2$, $y = x$, $x = 0$ and $x = 1$ is rotated about the x-axis. Find the volume of the resulting solid

(A) $\frac{\pi}{16}$ (B) $\frac{26\pi}{5}$ (C) $\frac{\pi}{3}$ (D) 2π

16. The value of $\int_0^{\pi/2} \cos^3 x\,dx$ is

(A) -2 (B) 0 (C) $\frac{1}{2}$ (D) $\frac{2}{3}$

17. On which interval the function $f(x) = \frac{x^2}{x^2+3}$ is strictly increasing?

(A) $(0,+\infty)$ (B) $(-\infty,0)$ (C) $[-1,1]$ (D) None of them

18. Which of the following integrals has the Riemann sum by dividing the interval $[1,2]$ into n equal subintervals with the right hand endpoints

$$\frac{1}{n}\sum_{i=1}^{n} e^{1+\frac{i}{n}}.$$

(A) $\int_0^1 e^x\,dx$ (B) $\int_0^1 e^{1+x}\,dx$ (C) $\int_1^2 e^{1+x}\,dx$ (D) $\int_1^2 e^x\,dx$

19. Consider the equation $x^4+4x+c=0$ with $c<3$. Then the equation

(A) has 2 real roots (B) has only one real root (C) has no real root (D) None of them

20. In the partial fraction decomposition

$$\frac{1}{x^2+2x} = \frac{A}{x} + \frac{Bx+C}{x^2+2},$$

the value of B is

(A) $\sqrt{2}$ (B) $-\frac{1}{2}$ (C) $\sqrt{3}$ (D) 2

ANSWER SHEET OF PART 1

Student Name: .....
Student ID: .....

1	(A)	(B)	(C)	(D)
2	(A)	(B)	(C)	(D)
3	(A)	(B)	(C)	(D)
4	(A)	(B)	(C)	(D)
5	(A)	(B)	(C)	(D)
6	(A)	(B)	(C)	(D)
7	(A)	(B)	(C)	(D)
8	(A)	(B)	(C)	(D)
9	(A)	(B)	(C)	(D)
10	(A)	(B)	(C)	(D)
11	(A)	(B)	(C)	(D)
12	(A)	(B)	(C)	(D)
13	(A)	(B)	(C)	(D)
14	(A)	(B)	(C)	(D)
15	(A)	(B)	(C)	(D)
16	(A)	(B)	(C)	(D)
17	(A)	(B)	(C)	(D)
18	(A)	(B)	(C)	(D)
19	(A)	(B)	(C)	(D)
20	(A)	(B)	(C)	(D)

– END OF PART 1 –

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PART 2: WRITTEN ANSWERS • 40 Minutes • 40 points

Write your answers on this paper. Ask for extra paper if you need more space. Each question carries 10 points. You must explain your answers in detail; no points will be given for the answer alone. You can use a calculator when working on these questions.

1. (10 points) A university campus suffers an outbreak of an infectious disease. The percentage of students infected by the disease after t days can be modelled by the function $p(t) = 5te^{-0.1t}$ for $0 \leq t \leq 30$. After how many days is the percentage of students infected a maximum?

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2. (10 points) Let $I = \int_0^1 e^{-x^2} dx$. Divide the interval $[0,1]$ into 4 equal subintervals and use the trapezoidal rule to approximate the value of I.

3. (10 points) Use the Newton's method to find an approximate value of $\sqrt{2}$ (i.e., solution of the equation $x^2 = 2$) correct to six decimal places, starting with $x_1 = 1$.

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4. (10 points) Let R be the bounded region enclosed by the curves $y = 12 - x^2$, $y = x$, and $x = 0$.

   (a) Find the area of the region R,

   (b) Find the volume of the solid generated by revolving the region R about the x-axis.