# Chapter 2 Electric Energy and Capacitance

One goal of physics is to identify basic forces in our world, such as the electric force as studied in the previous lectures. Experimentally, we discovered that the electric force is conservative and thus has associated electric potential energy. <span style="color:red">Therefore, we can apply the principle of the conservation of mechanical energy for the case of the electric force</span>. This extremely powerful principle allows us to solve problems for which calculations based on the force alone would be very difficult.

# Chapter 2 Electric Energy and Capacitance

## 2.1. Electric Potential and Electric Potential Difference:

### 2.1.1. Electric Potential Energy:

Key issue: Newton's law (for the gravitational force) and Coulomb's law (for the electrostatic force) are mathematically identical. Therefore, the general features for the gravitational force is applicable for the electrostatic force, e.g. the properties of a conservative force

. We can assign an **electric potential energy U**:

$$\Delta U = U_f - U_i = -W$$

W: work done by the electrostatic force on the particles

$$W = -W_{applied}$$

. The reference configuration (U = 0) of a system of charged particles: all particles are infinitely separated from each other

Thus, the potential energy of the system:
$W_\infty$: work done by the electrostatic force during the move in from infinity

$$U = -W_\infty$$

**Checkpoint 1:** A proton moves from point i to f in a uniform electric field directed as shown: (a) does E do the positive or negative work on the proton? (b) does the electric potential energy of the proton increase or decrease?
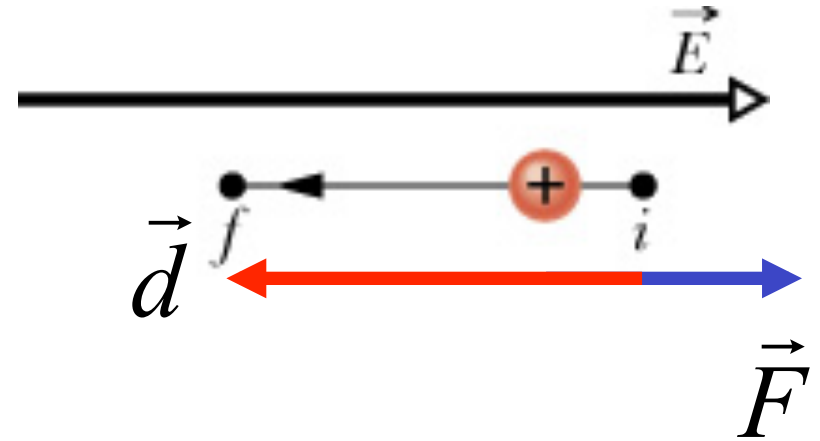


(a) Work done by the electric field E:

$$W = \vec{F}.\vec{d} = q\vec{E}\vec{d} = qEd \cos\theta = -qEd < 0$$

(b)
$$\Delta U = -W > 0$$

➔ U increases

## 2.1.2. Electric Potential and Electric Potential Difference:

- U depends on q: U ~ q

$$V = \frac{U}{q} \qquad (\text{unit}: J/C)$$

- V is the potential energy per unit charge and it is characteristic only of the electric field, called **the electric potential**
- Electric potential difference ΔV between two points:

$$\Delta V = V_f - V_i = \frac{U_f}{q} - \frac{U_i}{q} = \frac{\Delta U}{q}$$

$$\boxed{\Delta V = V_f - V_i = -\frac{W}{q}}$$

- The potential difference between two points is the negative of the work done by the electrostatic force to move <u>a unit charge</u> from one point to other

- We set $U_i = 0$ at infinity:

$$V = -\frac{W_\infty}{q}$$

- We introduce a new unit for V:

  1 volt = 1 joule per coulomb

- We adopt a new conventional unit for E:

$$1\,N/C = \left(1\frac{N}{C}\right)\left(\frac{1V.C}{1J}\right)\left(\frac{1J}{1N.m}\right) = 1V/m$$

- We also define one electron volt that is the energy equal to the work required to move 1 e⁻ through a potential difference of exactly one volt:

$$1eV = 1.60 \times 10^{-19}\ J$$

## Work done by an applied force:

Suppose we move a particle of charge q from point i to f in an electric field by applying a force to it:

The work-kinetic energy theorem gives:

$$\Delta K = K_f - K_i = W_{applied} + W$$

(or you can use: $\Delta K + \Delta U = W_{applied}$ )

If ΔK = 0 (the particle is stationary before and after the move):

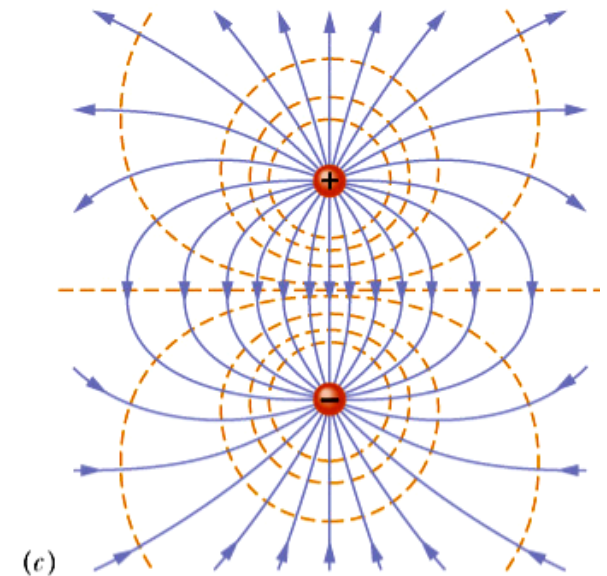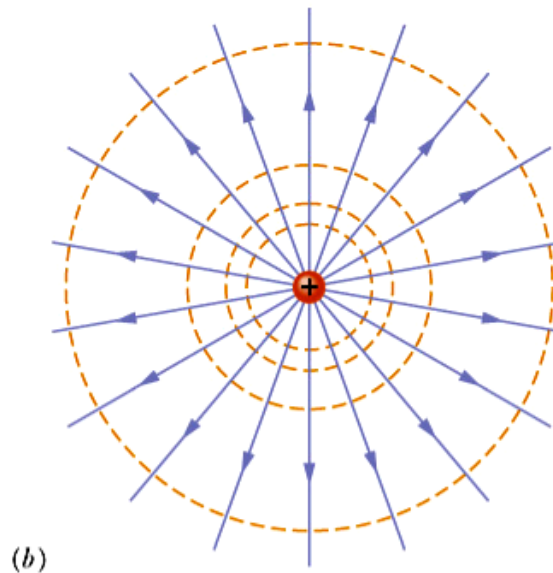$$W_{applied} = -W$$

$$\Delta U = U_f - U_i = W_{applied}$$

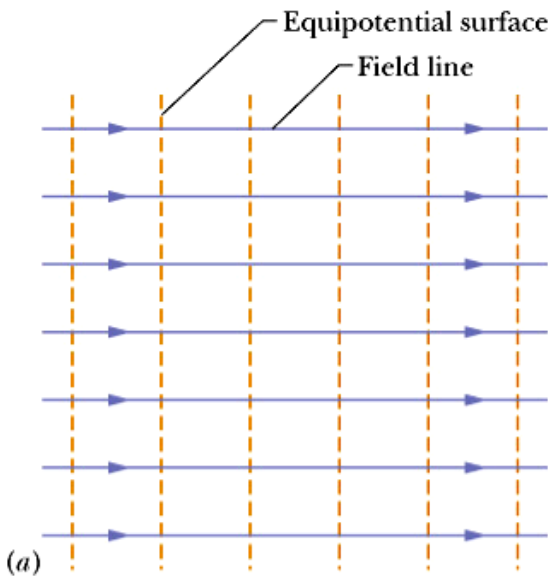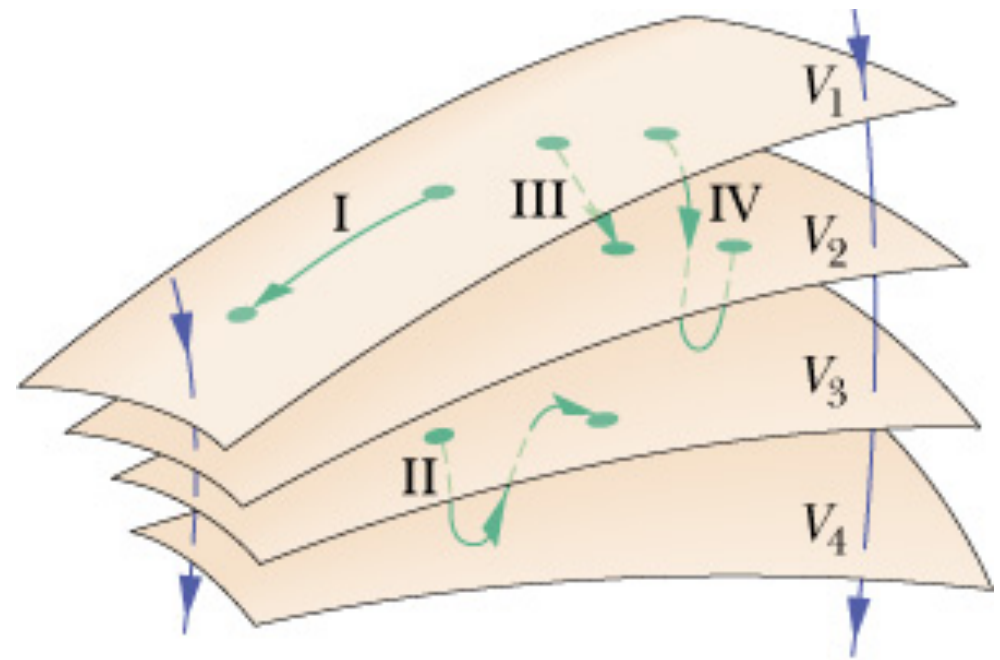$$W_{applied} = q\Delta V$$

## 2.1.3. Equipotential Surfaces:

Concept: Adjacent points with the same electric potential form an **equipotential surface**.

No net work is done on a charged particle by an electric field between two points on the same equipotential surface.

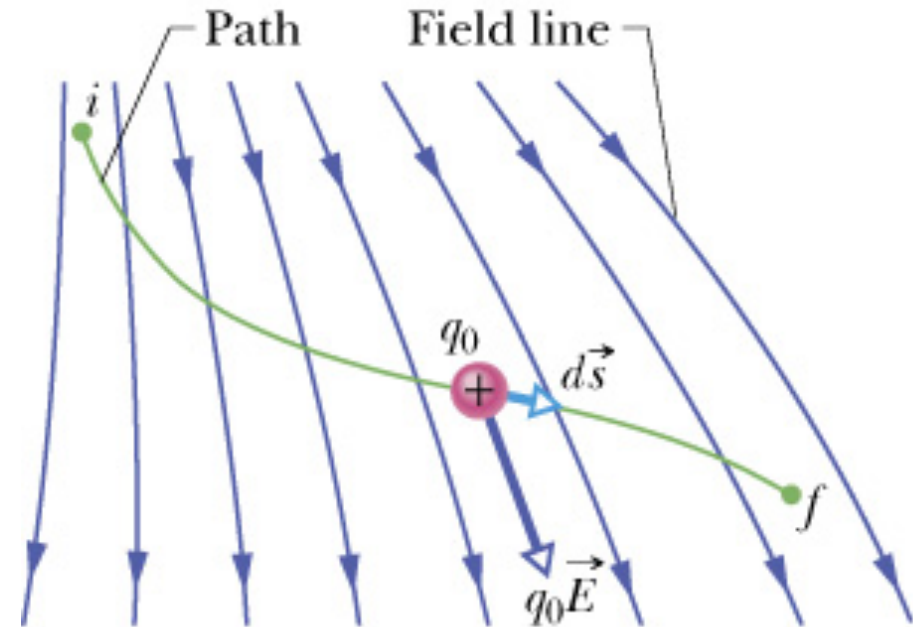Those surfaces are always perpendicular to electric field lines (i.e., to $\vec{E}$ ).

## 2.1.4. Finding the Potential from the Field:

**Problem:** Calculate the potential difference between two points i and f

$$dW = \vec{F}.d\vec{s}$$

$$dW = q_0\vec{E}.d\vec{s}$$

$$W = q_0\int_i^f \vec{E}.d\vec{s}$$



Path    Field line

**We have:**

$$V_f - V_i = -\frac{W}{q_0} = -\int_i^f \vec{E}.d\vec{s}$$

**If we choose $V_i = 0$:**

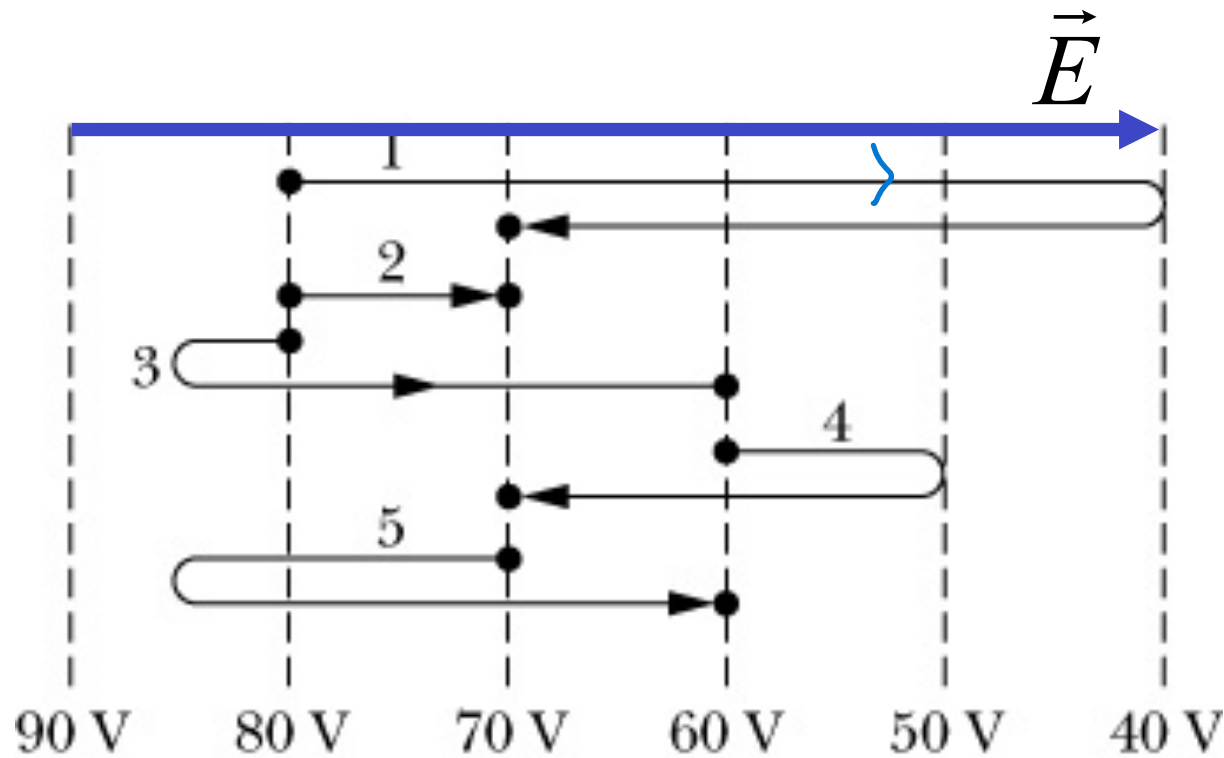$$V = -\int_i^f \vec{E}.d\vec{s}$$

**Checkpoint 3 (page 633):**
The figure shows a set of parallel equipotential surfaces and 5 paths along which we shall move an electron.
(a) the direction of E
(b) for each path, the work we do positive, negative or zero?



90 V   80 V   70 V   60 V   50 V   40 V

(c) Rank the paths according to the work we do, greatest first.

$$V_f - V_i = -\frac{W}{q_0} = -\int_i^f \vec{E}.d\vec{s}$$

a)

$$V_F - V_i = \int_i \vec{E}.d\vec{s}$$

(a) from the left to the right

$$W_{applied} = q\Delta V$$

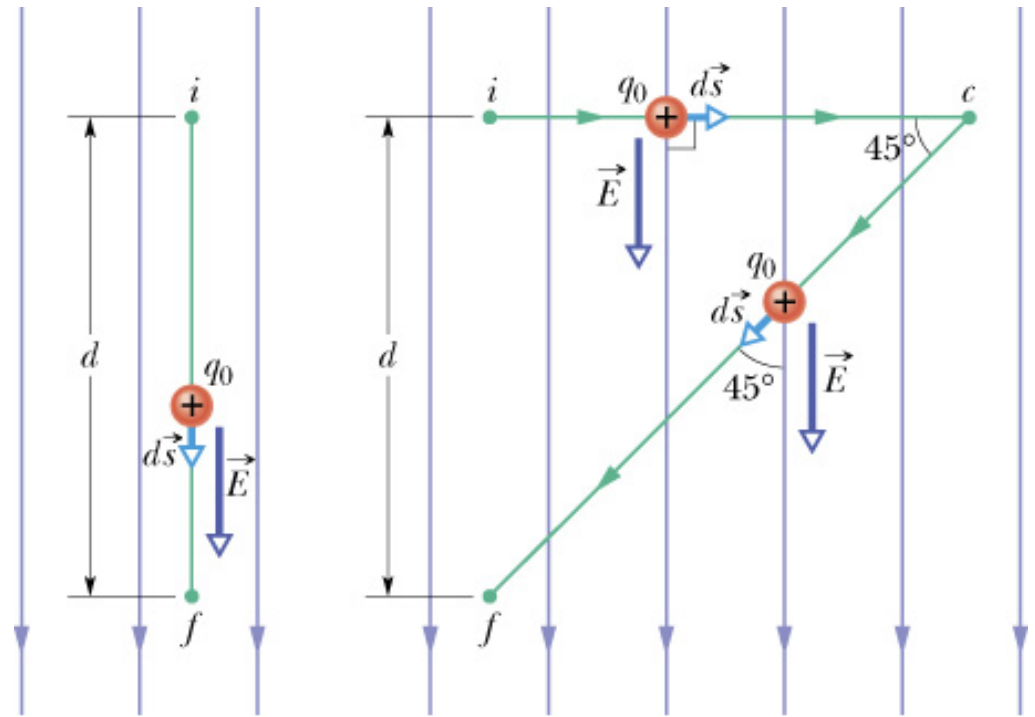(b) negative: 4; positive: 1,2,3,5
(c) 3,1-2-5,4

## 2.2. Potential Difference in a Uniform Electric Field:

**Problem:** Find the potential difference between two points $V_f - V_i$:

$$V_f - V_i = -\frac{W}{q_0} = -\int_i^f \vec{E}.d\vec{s}$$

The test charge $q_0$ moves along the path parallel to the field lines, so:

$$\boxed{V_f - V_i = -Ed}$$



- Electric field lines always point in the direction of decreasing electric potential
- Potential difference between two points does not depend on the path connecting them (electrostatic force is a conservative force)

**Check:** move $q_0$ following icf: $V_f - V_i = -E(\cos 45^0)\dfrac{d}{\sin 45^0} = -Ed$

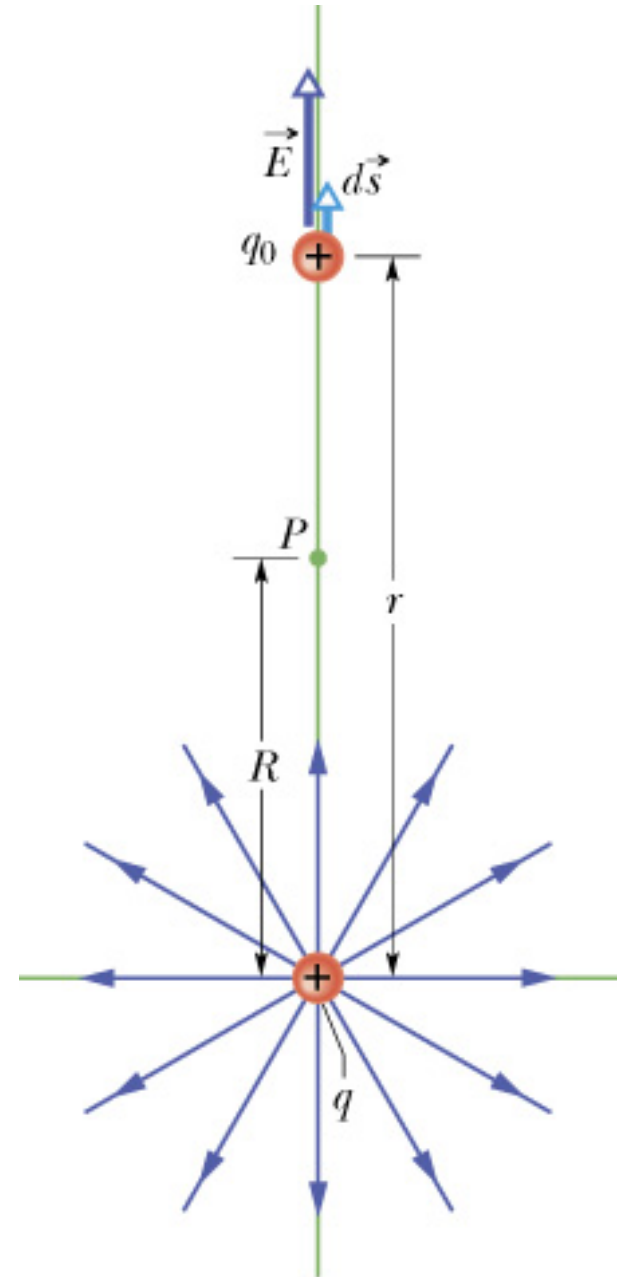# 2.3. Electric Potential and Potential Energy Due to Point Charges:

**Key equation:**

$$V_f - V_i = -\int_i^f \vec{E}.d\vec{s}$$

## 2.3.1. Potential Due to a Point Charge:

➢ **Choose the zero potential at infinity**

➢ **Move $q_0$ along a field line extending radially from point P to infinity, so $\theta = 0^0$**
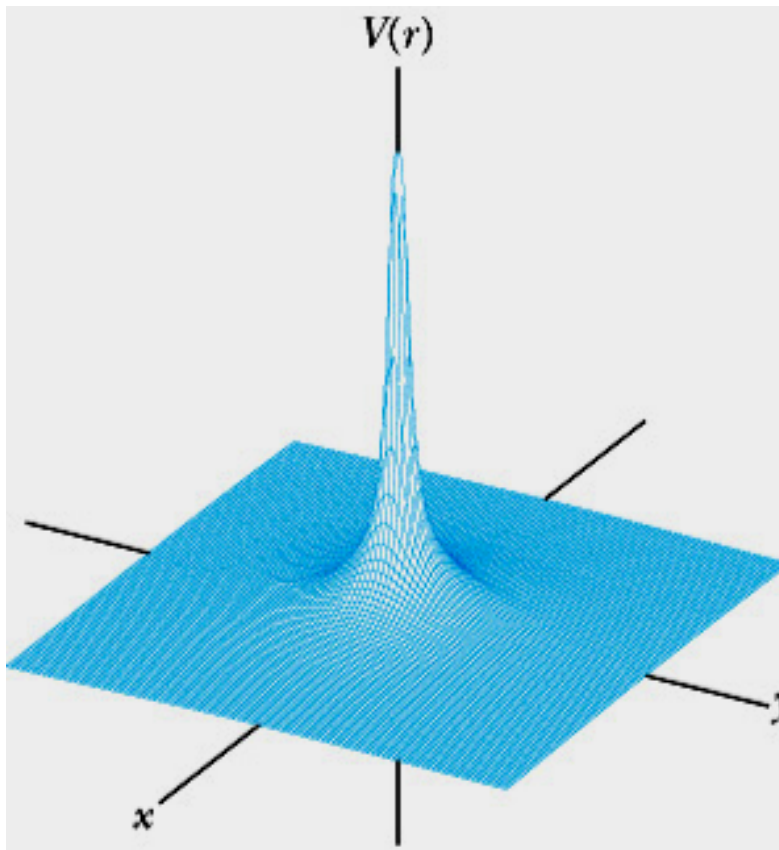
$$V_\infty - V_P = -\int_R^\infty E dr$$

$$V_P = kq \int_R^\infty \frac{1}{r^2} dr = k\frac{q}{R}$$

In a general case:

$$V = k \frac{q}{r}$$

- A positively charged particle produces a positive electric potential, a negatively charged particle produces a negative electric potential



Potentials V(r) at points in the xy plane due to a positive point charge at O

## 2.3.2. Potential Due to A Group of Point Charges:

➢ **Using the superposition principle:**

$$V = \sum_{i=1}^{n} V_i = k \sum_{i=1}^{n} \frac{q_i}{r_i}$$ (an algebraic sum, not a vector sum)

**Checkpoint 4:** The figure shows three arrangements of two protons. Rank the arrangements according to the net electric potential produced at point P by the protons, greatest first.



(a)                                    (b)                                    (c)
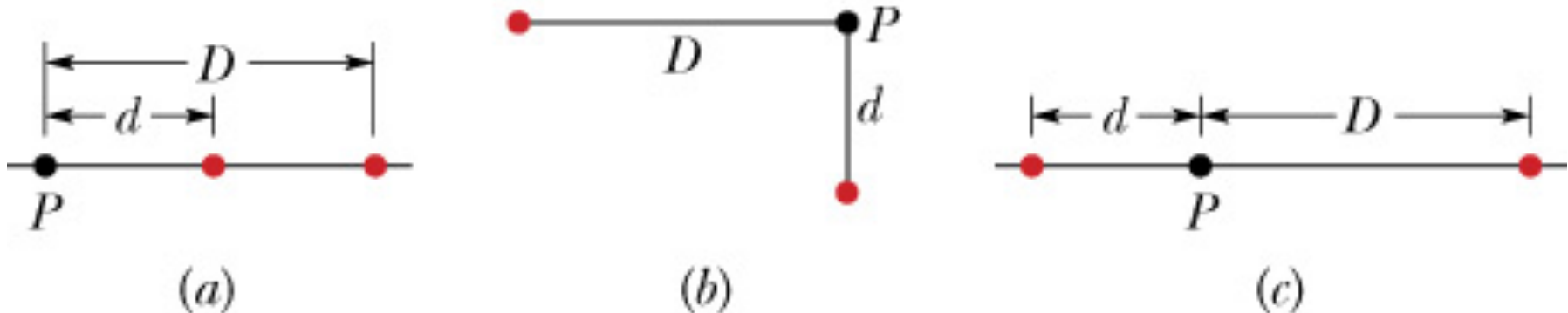
Use of the formula above gives the same rank

$$V = k\frac{q}{d} + k\frac{q}{D}$$

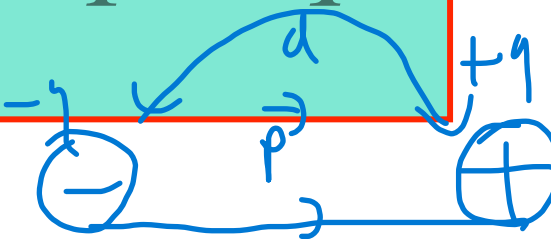## 2.3.3. Potential Due to an Electric Dipole:
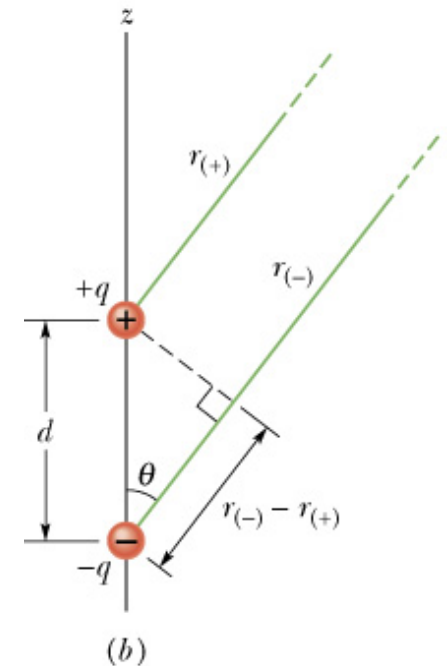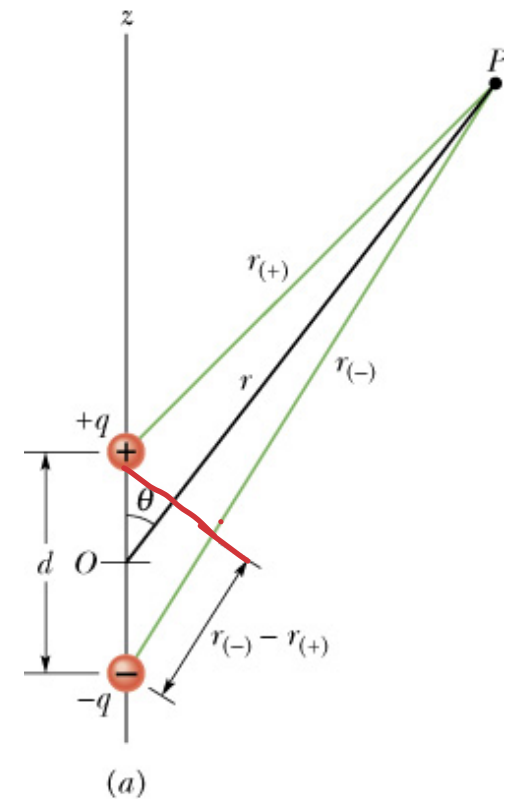
**Problem:** Calculate V at point P

$$V = \sum_{i=1}^{2} V_i = V_{(+)} + V_{(-)} = k \left( \frac{q}{r_{(+)}} + \frac{-q}{r_{(-)}} \right)$$

$$= kq \frac{r_{(-)} - r_{(+)}}{r_{(-)} r_{(+)}}$$

**If r >> d:**

$$r_{(-)} - r_{(+)} \approx d \cos\theta \text{ and } r_{(-)} r_{(+)} \approx r^2$$

$$V = k \frac{p \cos\theta}{r^2}; \quad p = qd$$

(a)

(b)

## Induced Dipole Moment:

**+ No external electric field**: in some molecules, the centers of the positive and negative charges coincide, thus no dipole moment is set up

Charged electrons

positive nucleus
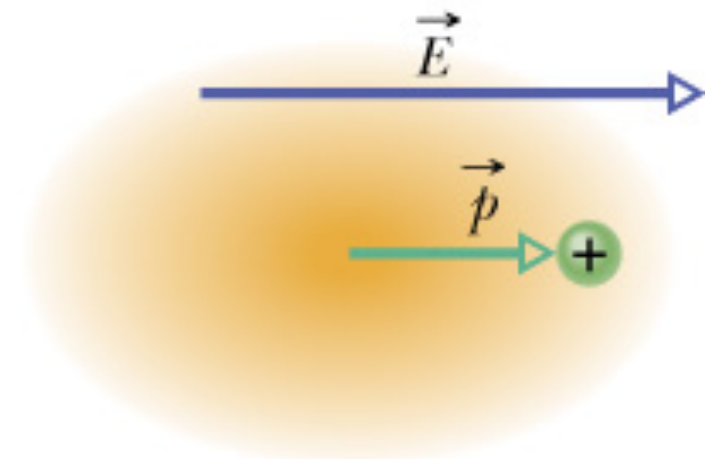
(a)

**+ Presence of an external E**: the field distorts the electron orbits and hence separates the centers of positive and negative charge, the electrons tend to be shifted in a direction opposite the field. This shift sets up a dipole moment, the so-called **induced dipole moment**. The atom or molecule is called to be **polarized** by the field.

$\vec{E}$

$\vec{p}$
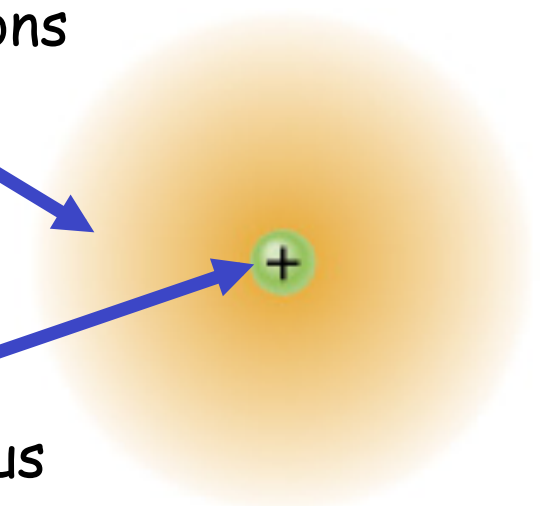
(b)

## 2.3.4. Calculating the Field from the Potential:

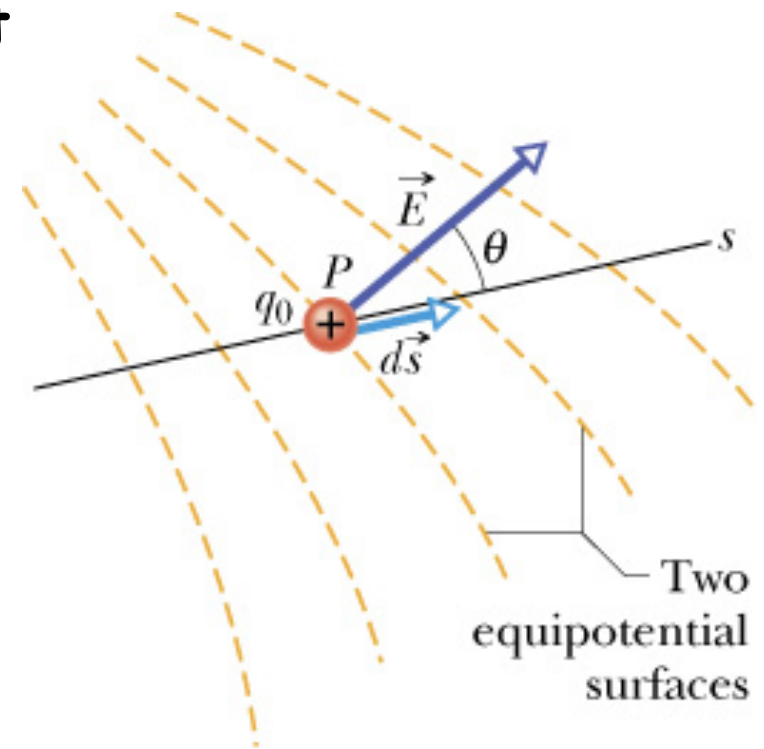**Problem:** $q_0$ moves through a displacement $\vec{ds}$ from one equipotential surface to the adjacent surface:

$$W = -q_0 dV = q_0 E(\cos\theta)ds$$

$$E\cos\theta = -\frac{dV}{ds}$$

$\dfrac{dV}{ds}$ : the **gradient** of the electric potential in the $\vec{ds}$ direction

$E_s$ = Ecosθ is the component of $\vec{E}$ in the direction of $\vec{ds}$ :

$$E_s = -\frac{\partial V}{\partial s}\text{(partial derivative)}$$

**In Cartesian coordinates:**

$$E_x = -\frac{\partial V}{\partial x}; E_y = -\frac{\partial V}{\partial y}; E_z = -\frac{\partial V}{\partial z}$$

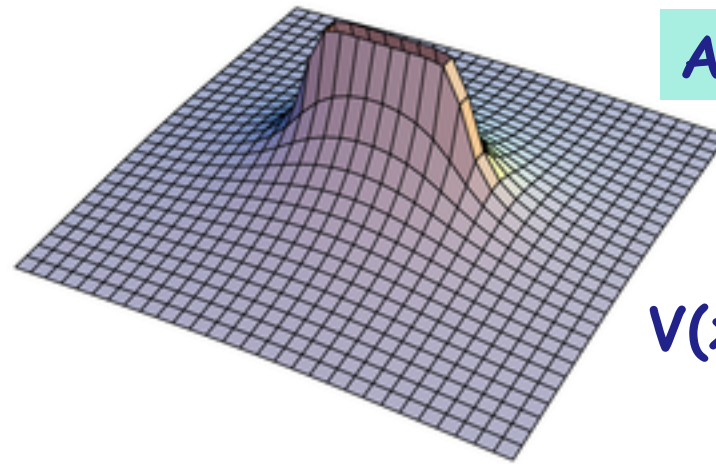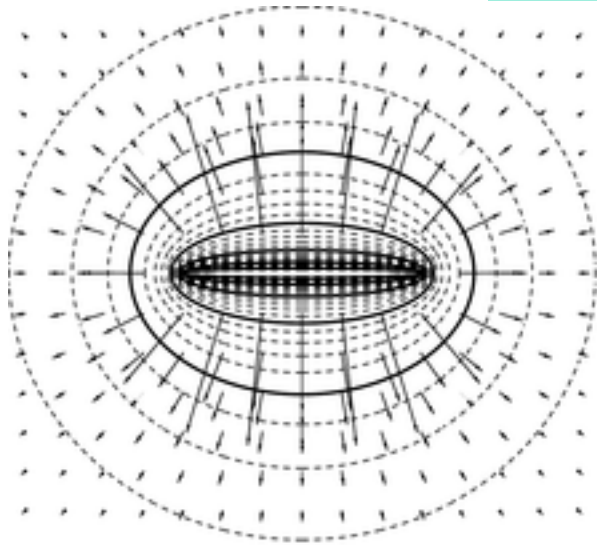$$\vec{E} = -\nabla V$$

$\nabla$ or $\vec{\nabla}$ : gradient

$\nabla$ : Nabla symbol

$$\nabla = \frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}$$

$$V = 2x^2 + 3yz + 4z^3$$

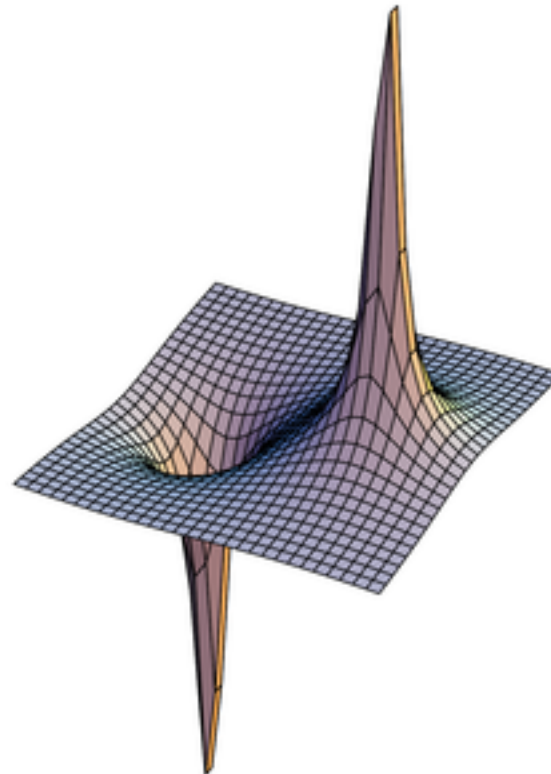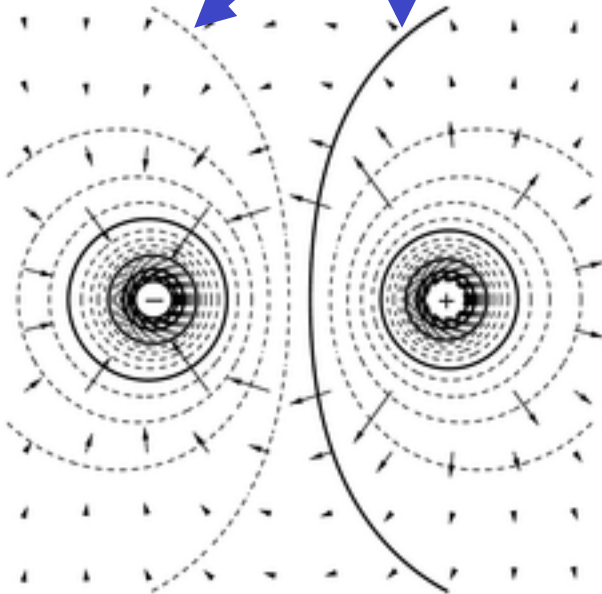$$E_x = -4x \; ; \; E_y = -3z \; ; \; E_z = -12z^2 - 3y$$

# Two Dimensional Field and Potential



A uniformly charged rod

$V(x,y)$

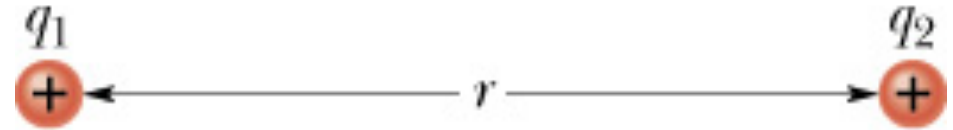Equipotential curves

A dipole

$V(x,y)$

## 2.3.5. Electric Potential Energy of a System of Point Charges:

$$\Delta U = U_f - U_i = -W = W_{applied}$$

**Problem:** Calculate the electric potential energy of a system of charges due to the electric field produced by those same charges

**Consider a simple case:** two point charges held a fixed distance r



**We define:** *The electric potential energy of a system of fixed point charges is equal to the work that must be done by an external agent to assemble the system, bringing each charge in from an infinite distance*

The work by an applied force to bring q2 in from infinity:

$$W_{applied} = q_2 \Delta V = q_2 (V_f - V_\infty)$$

$$W_{applied} = U_{system} = q_2 V = k\frac{q_1 q_2}{r}$$

If the system consists of three charges, we calculate **U for each pair of charges** and **sum** the terms algebraically:

$$U = U_{12} + U_{13} + U_{23} = k(\frac{q_1 q_2}{r_{12}} + \frac{q_1 q_3}{r_{13}} + \frac{q_2 q_3}{r_{23}})$$

## 2.4. Electric Potential Due to Continuous Charge Distributions

<u>Method:</u> **We calculate the potential dV at point P due to a differential element dq, then integrate over the entire charge distribution**

$$dV = k\frac{dq}{r}$$

$$V = \int dV = k\int \frac{dq}{r}$$
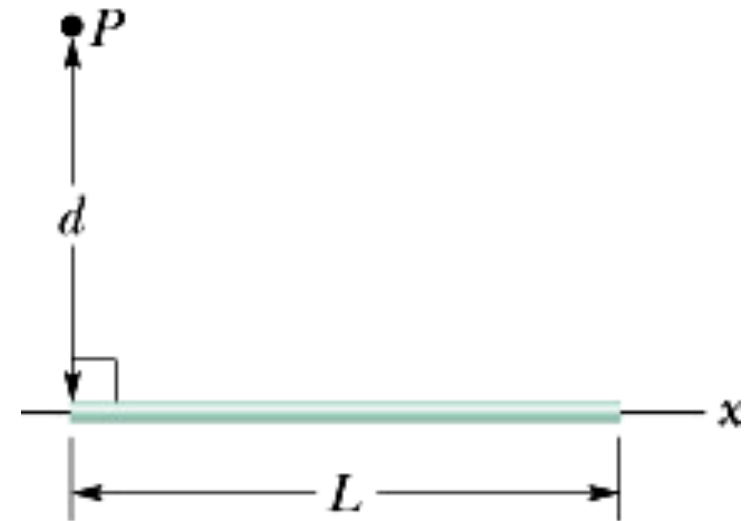
## 2.4.1. Line of Charge:

$$dq = \lambda dx$$

$$dV = k\frac{dq}{r} = k\frac{\lambda dx}{\left(x^2 + d^2\right)^{1/2}}$$

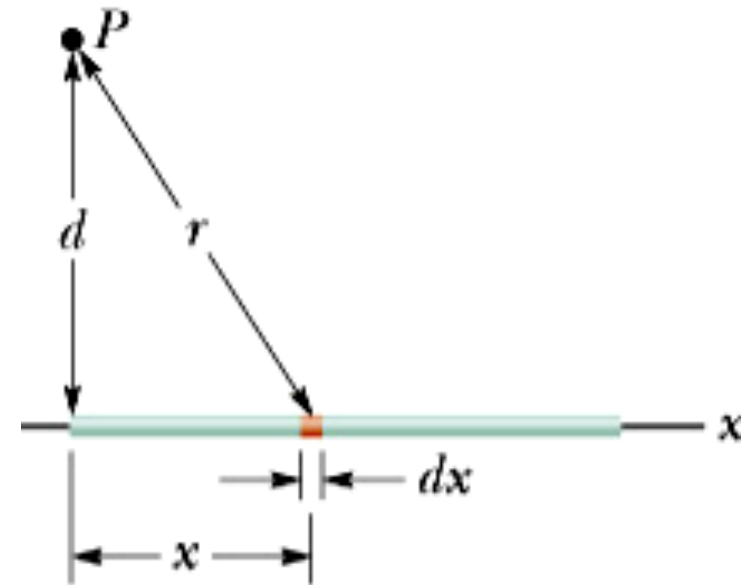$$V = \int_0^L k\frac{\lambda dx}{\left(x^2 + d^2\right)^{1/2}}$$

(see Appendix E, integral 17, page A-11)

$$V = k\lambda \ln\left[\frac{L + \left(L^2 + d^2\right)^{1/2}}{d}\right]$$
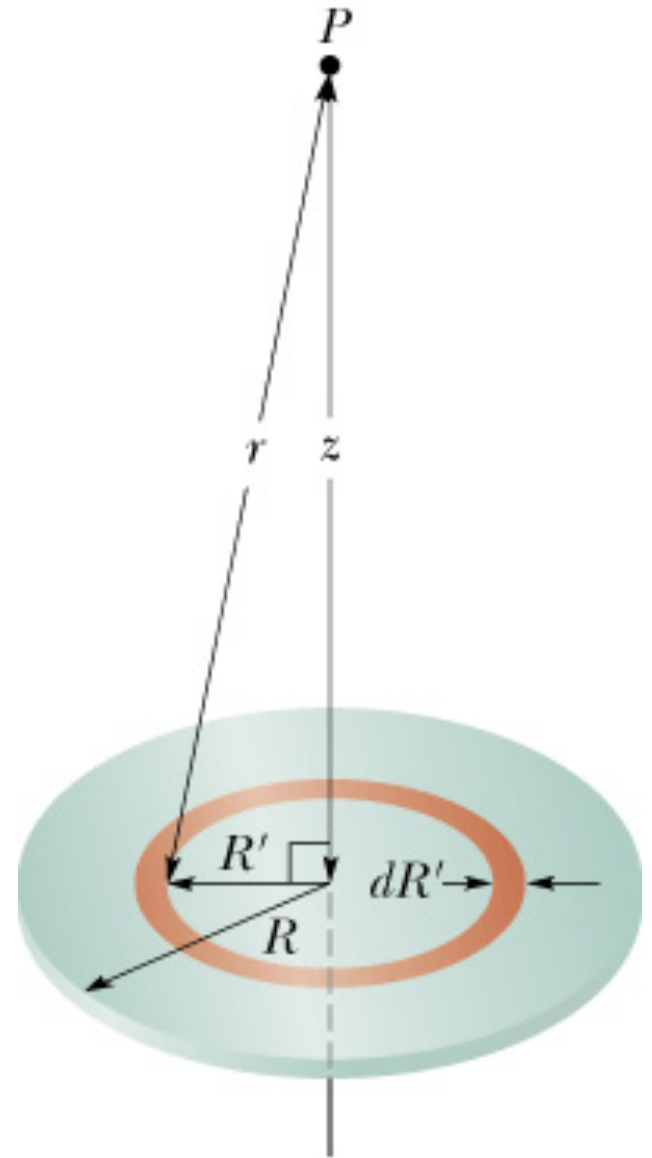
No direction care



(a)



(b)

## 2.4.2. Charged Disk:

$$dq = \sigma(2\pi R')(dR')$$

$$dV = k\frac{dq}{r} = k\frac{\sigma(2\pi R')(dR')}{\sqrt{z^2 + R'^2}}$$

$$V = \frac{1}{4\pi\varepsilon_0}\int_0^R \frac{\sigma(2\pi R')(dR')}{\sqrt{z^2 + R'^2}}$$

$$V = \frac{\sigma}{2\varepsilon_0}\left(\sqrt{z^2 + R^2} - z\right)$$

## 2.5. Electric Potential of a Charged Isolated Conductor:

Using Gauss' law, we prove the following conclusion:

no communicate *(a)*

+ **An excess charge placed on an isolated conductor will distribute itself on the surface of that conductor so that all points of the conductor (on the surface or inside) have the same potential.**

$$V_f - V_i = -\int_i^f \vec{E} d\vec{s}$$

Electric field at the surface is perpendicular to the surface and it is zero inside the conductor, so:

$$V_f = V_i$$


(a)

(b)

(c)
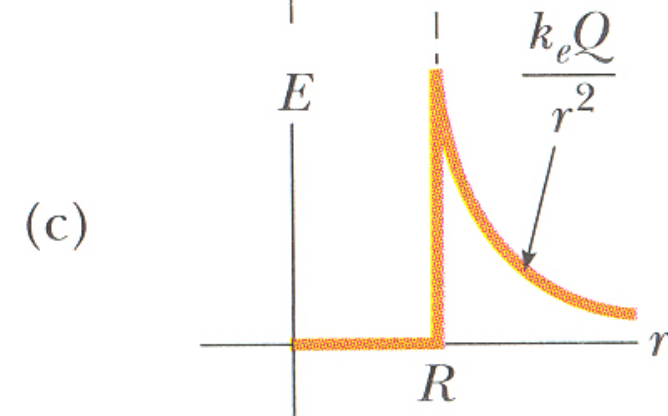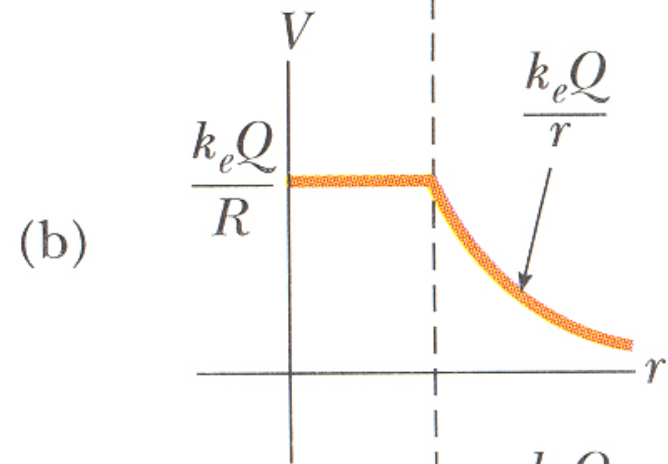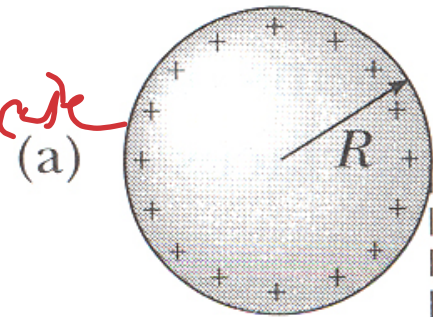
# A large spark jupms to a car's body and then exists by moving across the insulating left front tire, leaving the person inside unharmed



metal cage

Homework: 1, 6, 8, 11, 14, 18, 24, 28, 29, 35, 43, 59, 60, 64 (page 648-653)