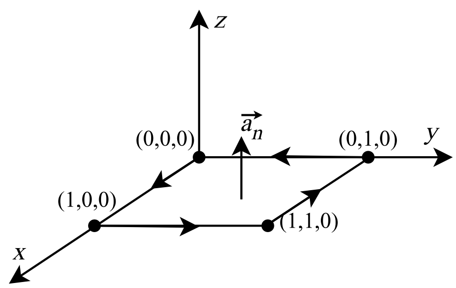
Midterm: March, 2013

Solved by **Le Diep Phi**

November 12, 2020

Question 1



From the figure and applying the Right Hand Rule give us $d\mathbf{S} = \mathbf{a_n} dS = \hat{\mathbf{z}} dS$

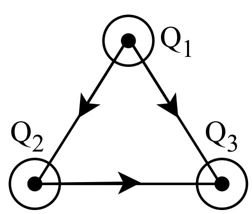
The magnetic flux Ψ due to the magnetic field $\mathbf{B}$ crossing the area $S = 1$ is

$$\begin{aligned}\Psi &= \int_S \mathbf{B} \cdot d\mathbf{S} = \int_S B_0 \cos(2\pi t + \pi/3)\hat{\mathbf{z}} \cdot \hat{\mathbf{z}} dS \\ &= B_0 \cos(2\pi t + \pi/3) \int_S 1\, dS = B_0 \cos(2\pi t + \pi/3) S \\ &= B_0 \cos(2\pi t + \pi/3)\end{aligned}$$

Therefore, the induced electromotive force is

$$\text{emf} = -\frac{d\Psi}{dt} = 2\pi B_0 \sin(2\pi t + \pi/3)$$

Question 2



a) Applying the KCL at charge Q_1, immediately gives us

$$I_{12} + I_{13} + \frac{d}{dt}\int_{S1} \mathbf{D_1} \cdot d\mathbf{S_1} = 0 \Rightarrow I_{13} = -I_{12} - \frac{d}{dt}\int_{S1} \mathbf{D_1} \cdot d\mathbf{S_1} = -1 - (-2) = 1\text{A}$$

b) Applying the KCL at charge Q_3, immediately gives us

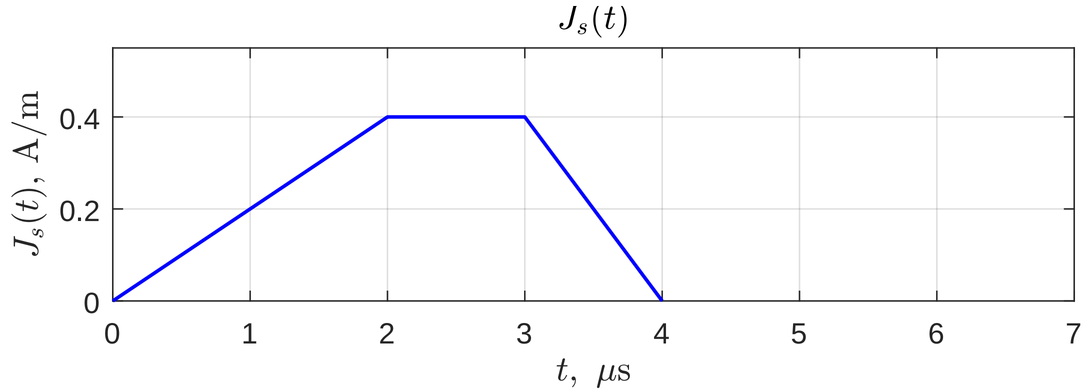
$$-I_{13} - I_{23} + \frac{d}{dt}\int_{S3} \mathbf{D_3} \cdot d\mathbf{S_3} = 0 \Rightarrow I_{23} = I_{13} + \frac{d}{dt}\int_{S3} \mathbf{D_3} \cdot d\mathbf{S_3} = -1 + 2 = 1\text{A}$$

c) Applying the KCL at charge Q_2, immediately gives us

$$-I_{12} + I_{23} + \frac{d}{dt}\int_{S2} \mathbf{D_2} \cdot d\mathbf{S_2} = 0 \Rightarrow \frac{d}{dt}\int_{S2} \mathbf{D_2} \cdot d\mathbf{S_2} = I_{12} - I_{23} = 1 - 1 = 0\text{A}$$

Question 3

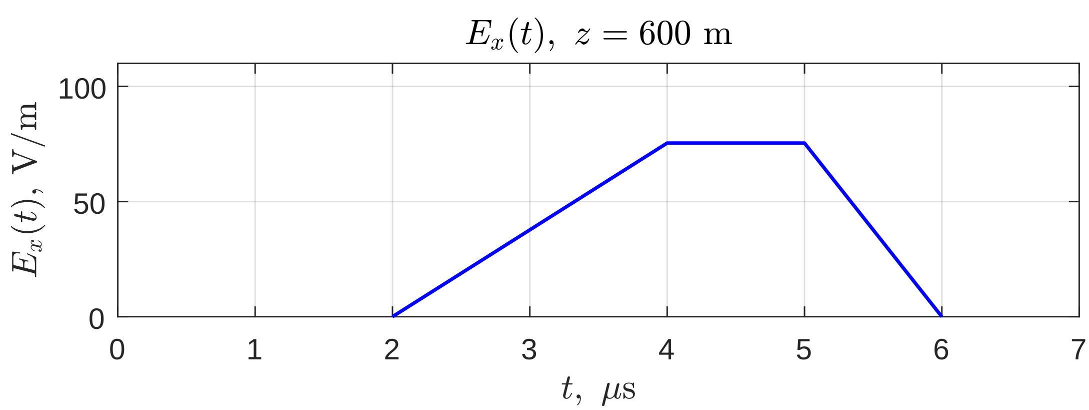
The problem give us



a) For $z = 600$ m the wave equation for electric field is given by

$$E_x(t) = \frac{1}{2}\eta_0 J_s\left(t - \frac{z}{v_p}\right) = 60\pi J_s(t - 2\times 10^{-6})$$

Therefore, the graph of $E_x(t)$ for $z = 600$ m as shown in the below



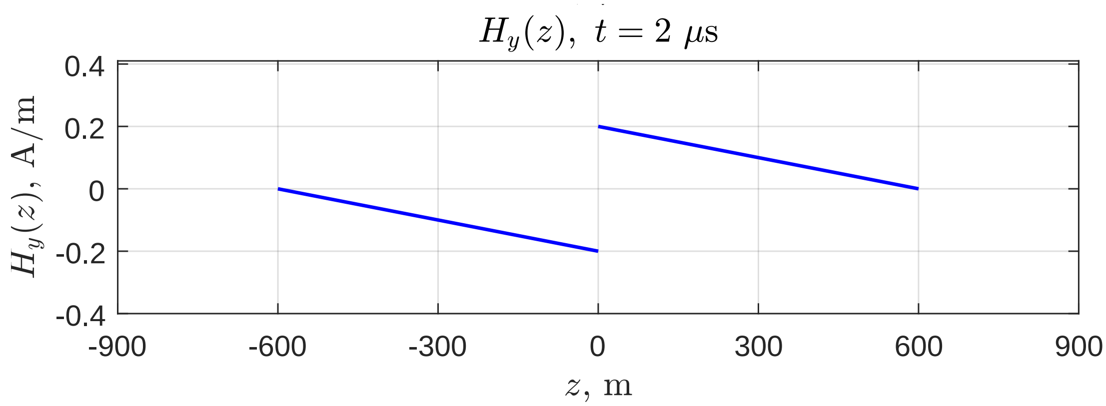
b) The wave equation for magnetic field is given by

$$H_y(z) = \pm\frac{1}{2}J_s\left(t \mp \frac{z}{v_p}\right)$$

For $t = 2\ \mu$s the equation of magnetic field becomes

$$H_y(z) = \begin{cases} \frac{1}{2}J_s\left(2\times 10^{-6} - \frac{z}{3\times 10^{8}}\right) & \text{if } z > 0 \\ -\frac{1}{2}J_s\left(2\times 10^{-6} + \frac{z}{3\times 10^{8}}\right) & \text{if } z < 0 \end{cases}$$

Therefore, the graph of $H_y(z)$ for $t = 2\ \mu$s as shown in the below



Question 4

The problem give us

$$\mathbf{H} = H_0\cos(6\pi\times 10^8 t - 2\pi y)\hat{\mathbf{z}}\ (\text{A/m})$$

a) The unit vectors along the direction of propagation of the wave is $\mathbf{a_P} = \hat{\mathbf{y}}$

b) The unit vectors along the direction of magnetic field is $\mathbf{a_H} = \hat{\mathbf{z}}$

c) Since we have $\mathbf{a_P} = \mathbf{a_E}\times\mathbf{a_H} \Leftrightarrow \hat{\mathbf{y}} = \mathbf{a_E}\times\hat{\mathbf{z}}$. Therefore, the unit vectors along the direction of electric field must be $\mathbf{a_E} = -\hat{\mathbf{x}}$

d) The wave length of the given wave equation is

$$\lambda = \frac{2\pi}{\beta} = \frac{2\pi}{2\pi} = 1\ \text{m}$$

e) The equation for electric field is

$$\mathbf{E} = \eta_0 H_0\cos(6\pi\times 10^8 t - 2\pi y)(-\hat{\mathbf{x}})\ (\text{V/m})$$

At $t = 0, z = 0$, we have:

$$\begin{cases} \mathbf{H}(0,0) = H_0\hat{\mathbf{z}} \\ \mathbf{E}\,(0,0) = \eta_0 H_0(-\hat{\mathbf{x}}) \end{cases}$$

Therefore, the poynting vector is

$$\mathbf{P} = \mathbf{E} \times \mathbf{H} = \eta_0 H_0(-\hat{\mathbf{x}}) \times H_0\hat{\mathbf{z}} = \eta_0 H_0^2\hat{\mathbf{y}}$$

which indicates that the magnitude is $P = |\mathbf{P}| = \eta_0 H_0^2$ and the direction along the positive of y-axis or $\mathbf{a_P} = \hat{\mathbf{y}}$

Question 5

The Maxwell's equations in integral form give us the relationship between field and source over a region in space but not directly at a particular point. However, Maxwell's equations in differential form can apply directly to field vectors and source densities at a given point. Therefore, we need to study both of the forms to fully understand the basic principles of electromagnetic wave propagation.