FINAL REVIEW
PHYSIC 1

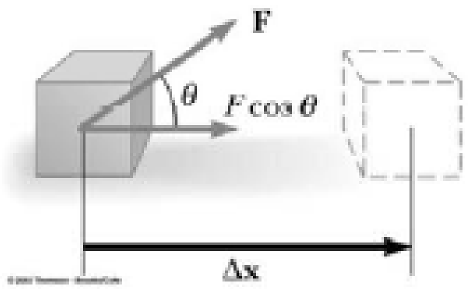
Ta Minh Tri

CHAPTER 3

1 Work Done by Force. Power

- Provides a link between force and energy
- The work, *W*, done by a constant force on an object is defined as the product of the *component of the force along the direction of displacement* and *the magnitude of the displacement*

$$\boxed{W = (F\cos\theta)\Delta x}$$



- (F cos θ) is the component of the force in the direction of the displacement

Δx is the displacement

(F cos θ) is the component of the force in the direction of the displacement

- **WORK DONE BY THE GRAVITATIONAL FORCE**

Divide the path into small segments $\overrightarrow{\Delta s}$

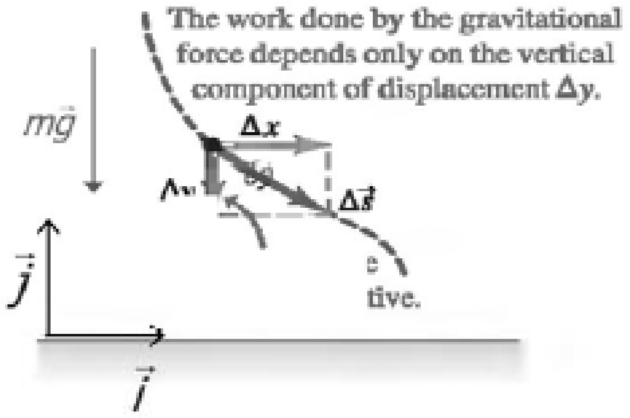
$$m\vec{g} = -mg\vec{j}$$

$$\overrightarrow{\Delta s} = \Delta x\vec{i} + \Delta y\vec{j}$$

$$\Delta W = \vec{F}\overrightarrow{\Delta s} = m\vec{g}(\Delta x\vec{i} + \Delta y\vec{j})$$

$$= m\vec{g}(\Delta x\vec{i}) + m\vec{g}(\Delta y\vec{j})$$

$$= 0 - mg\Delta y$$



The total work done by the gravitational force :

$$W = -mg(y_2 - y_1)$$

The work done by the gravitational force does not depend on the path; it depends only on the vertical distance

Work done by spring force

$$\boxed{W = \frac{1}{2}kx_i^2 - \frac{1}{2}kx_f^2}$$

Conservation of Energy :

$$E_i = E_f$$

$$K_i + U_i = K_f + U_f$$

CHAPTER 3

- **Kinetic Energy Theorem**

(or The work–kinetic energy theorem)

" The net work done on a particle by external forces equals the change in kinetic energy of the particle"

$$\boxed{W_{net} = \Delta KE = KE_f - KE_i}$$

Speed will increase if work is positive

Speed will decrease if work is negative

Kinetic energy

$$\boxed{KE = \frac{1}{2}mv^2}$$

CHAPTER 3

Q1 (20 pts) A worker pushes a 12-kg block, starting from rest, on a horizontal frictionless plane. His force is 45 N and parallel to the plane. Use the work – kinetic energy theorem to compute the block's speed when its displacement is 10 m. 8.66 m/s

e2

$v_i = 0$; $d = 10$; $m = 12\,kg$ | $F = 45N$

$K_f - K_i = W$

Work-kinetic energy theorem:

$$\Delta K = \sum W$$

$K = \frac{1}{2} m v^2$

$$(1) <=> K_f - K_i = F \cdot \Delta x \cdot cos(\vec{F}, \Delta \vec{x}) \qquad (1)$$

Starting from rest: $v_i = 0m/s$

$$m = 12kg$$

$$F = 45N$$

$$(1) <=> \frac{1}{2} \times 12 \times v_f^2 - 0 = 45 \cdot 10 \cdot cos(0) = 450J$$

$$=> v_f = 5\sqrt{3} = 8.66m/s$$

CHAPTER 3

Q5. (20 marks) Two balls A and B of mass M and $3M$, respectively, hang from a ceiling on strings with negligible mass. Drag the ball A to the left so that it is raised to a height $h_0 = 1$ m and then released (**Figure 3**). At its lowest position, the ball A collides with the ball B and then they stick together.

a) What are the velocity of A before collision and the velocity of the system (A and B) after collision?

b) What is a maximum height that the two balls swing to the right?

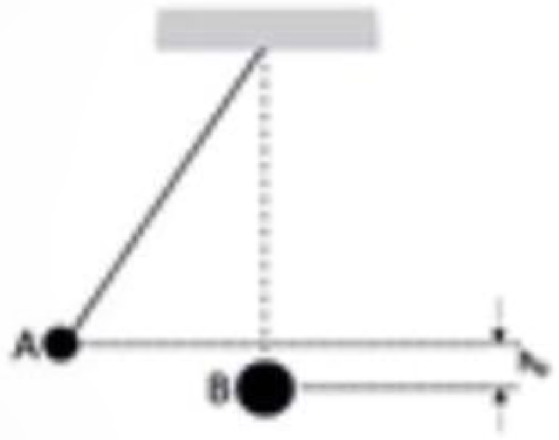


Figure 3

CHAPTER 4

Linear momentum: (động lượng)

$$\boxed{\vec{p} = m\vec{v}}$$

$\Delta\vec{p} = m\,\Delta\vec{v}$

Impulse:

$$I = \Delta\vec{p} = \int d\,\vec{p} = \int_{t_i}^{t_f} \vec{F}\,\Delta t$$

Conservation of linear momentum:

$$\boxed{\overrightarrow{p_i} = \overrightarrow{p_f}}$$

CHAPTER 4

ONE DIMENSION

Perfectly Inelastic Collisions:

- What amount of KE lost during collision?

$$KE_{before} = \frac{1}{2}m_1 v_{1i}^2 + \frac{1}{2}m_2 v_{2i}^2$$

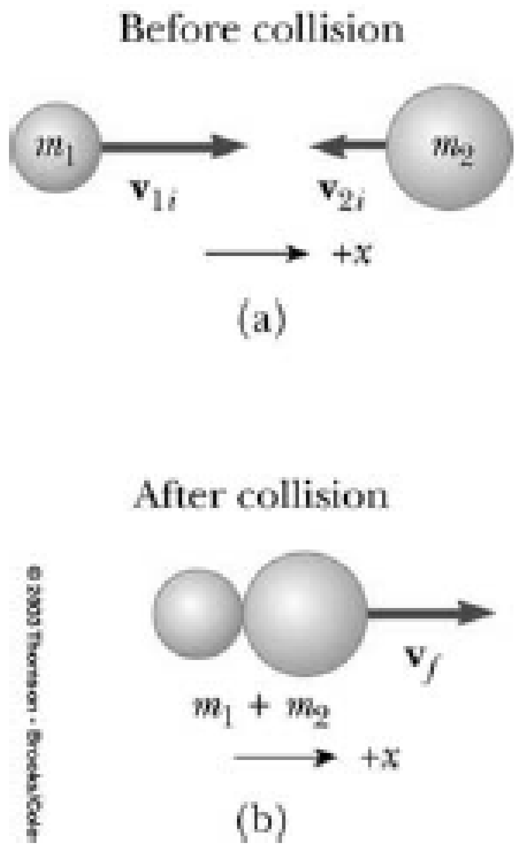
$$= \frac{1}{2}(1000\ kg)(50\ m/s)^2 = 1.25\times 10^6\ J$$

$$KE_{after} = \frac{1}{2}(m_1 + m_2)v_f^2$$

$$= \frac{1}{2}(2500\ kg)(20\ m/s)^2 = 0.50\times 10^6\ J$$

$$\Delta KE_{lost} = \underline{0.75\times 10^6\ J}$$

lost in heat/"gluing"/sound/…



Elastic collisions

both momentum and kinetic energy are conserved

Typically have two unknowns

$$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}$$

$$\frac{1}{2}m_1 v_{1i}^2 + \frac{1}{2}m_2 v_{2i}^2 = \frac{1}{2}m_1 v_{1f}^2 + \frac{1}{2}m_2 v_{2f}^2$$

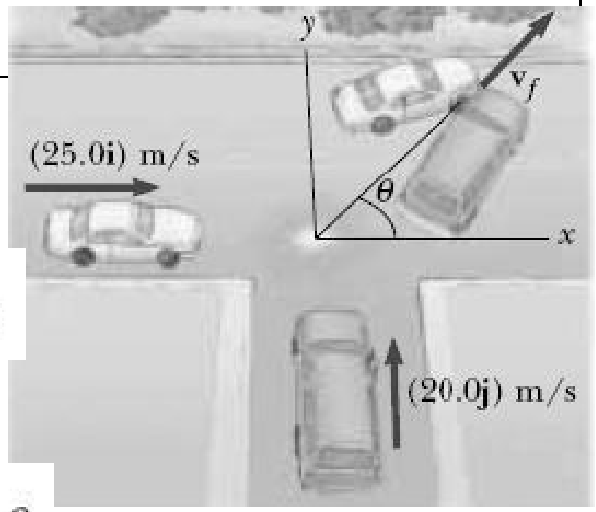
Solve the equations simultaneously

CHAPTER 4

TWO DIMENSION

PROBLEM 2

A 1 500-kg car traveling east with a speed of 25.0 m/s collides at an intersection with a 2 500-kg van traveling north at a speed of 20.0 m/s.
Find the direction and magnitude of the velocity of the wreckage after the collision, assuming that the vehicles stick together after the collision.



SOLUTION

Stick together : perfectly inelastic collision

$$\sum p_{xi} = (1\,500 \text{ kg})(25.0 \text{ m/s}) = 3.75 \times 10^4 \text{ kg}\cdot\text{m/s}$$

$$\sum p_{xf} = (4\,000 \text{ kg})v_f \cos\theta$$

$$3.75 \times 10^4 \text{ kg}\cdot\text{m/s} = (4\,000 \text{ kg})v_f \cos\theta$$

$$\sum p_{yi} = \sum p_{yf}$$

$$(2\,500 \text{ kg})(20.0 \text{ m/s}) = (4\,000 \text{ kg})v_f \sin\theta$$

$$5.00 \times 10^4 \text{ kg}\cdot\text{m/s} = (4\,000 \text{ kg})v_f \sin\theta$$

PROBLEM 2

A 1 500-kg car traveling east with a speed of 25.0 m/s collides at an intersection with a 2 500-kg van traveling north at a speed of 20.0 m/s.
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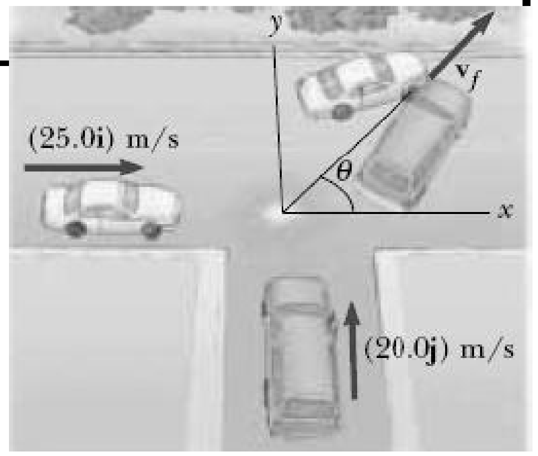

SOLUTION

$$3.75 \times 10^4 \text{ kg}\cdot\text{m/s} = (4\,000 \text{ kg})v_f \cos\theta$$

$$5.00 \times 10^4 \text{ kg}\cdot\text{m/s} = (4\,000 \text{ kg})v_f \sin\theta$$

$$\frac{\sin\theta}{\cos\theta} = \tan\theta = \frac{5.00 \times 10^4}{3.75 \times 10^4} = 1.33$$

$$\theta = \boxed{53.1^\circ}$$

$$v_f = \frac{5.00 \times 10^4 \text{ kg}\cdot\text{m/s}}{(4\,000 \text{ kg})\sin 53.1^\circ} = \boxed{15.6 \text{ m/s}}$$

CHAPTER 5

Notation	Linear Translational	Angular Rotational
Basic quantities	x (m) v (m/s) a (m/s²)	θ (rad) ω (rad/s) α (rad/s²)
Basic formula	a const $v = v_0 + at$ $x = x_0 + v_0 t + \frac{1}{2}at^2$ $v^2 - v_0^2 = 2a\Delta x$	α const $\omega = \omega_0 + \alpha t$ $\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$ $\omega^2 - \omega_0^2 = 2\alpha\Delta\theta$
Inertia	mass: m (kg)	Moment of inertia Rotational inertia ❖ $I = \sum mR^2$ (kg×m²) ❖ I = ...

Notation	Linear Translational	Angular Rotational
Speeding up **Slowing down**	**a·v** **$\vec{a}\cdot\vec{v}$**	**α·ω** **$\vec{\alpha}\cdot\vec{\omega}$**
Force **vs** **Torque**	Newton's 2nd law: $\vec{F} = m\vec{a}$ (N)	$\vec{\tau} = \vec{r}\times\vec{F}$ or τ = Fd (d: moment/lever arm) Newton's 2nd law: $\vec{\tau} = I\times\vec{\alpha}$ (N·m)
Convention of (+) direction	y up x to the right	Counterclockwise

CHAPTER 5

Notation	Linear Translational	Angular Rotational
Energy $E = K + U$	$K = \frac{1}{2}mv^2$ (J)(eV) $U_g = mgh$ (y up, 0 at …) $U_{el} = \frac{1}{2}kx^2$ (J)(eV)	$K = \frac{1}{2}I\omega^2$ (J)(eV)
Work	$W = \vec{F}\cdot\Delta\vec{x}$ or $\int_{x_i}^{x_f} \vec{F}(x)\cdot d\vec{x}$ (J)(eV)	$W = \vec{\tau}\cdot\Delta\vec{\theta}$ or $\int_{\theta_i}^{\theta_f} \vec{\tau}(\theta)\cdot d\vec{\theta}$ (J)(eV)
Power	$P=\frac{W}{\Delta t}=\vec{F}\cdot\vec{v}$ (J/s)(W)	$P=\frac{W}{\Delta t}=\vec{\tau}\cdot\vec{\omega}$ (J/s)(W)
Momentum	$\vec{p}=m\vec{v}$ (kg·m/s)	$\vec{L} = \vec{r}\times\vec{p}$ $\vec{L}=I\vec{\omega}$ (kg·m²/s)

Notation	Linear Translational	Angular Rotational
Impulse	$\vec{I}=\Delta\vec{p}=\vec{F}\Delta t = \int_{t_i}^{t_f} \vec{F}(t)dt$ ❖ $\vec{F}_{net}=\frac{\Delta p}{\Delta t}$ or $\frac{dp}{dt}$	$\vec{I}=\Delta\vec{L}=\vec{\tau}\Delta t = \int_{t_i}^{t_f} \vec{\tau}(t)dt$ ❖ $\vec{\tau}_{net}=\frac{\Delta \vec{L}}{\Delta t}$ or $\frac{d\vec{L}}{dt}$
Momentum conservation	$\vec{F}_{net}=\vec{0} \Rightarrow \Delta\vec{p}=\vec{0}$ ⇨ $\sum\vec{p}_i=\sum\vec{p}_f$ ⇨ $\sum m_i v_i=\sum m_f v_f$	$\vec{\tau}_{net}=\vec{0} \Rightarrow \Delta\vec{L}=\vec{0}$ ⇨ $\sum\vec{L}_i=\sum\vec{L}_f$ ⇨ $\sum I_i\omega_i=\sum I_f\omega_f$

CHAPTER 5

Pure rotation relationship:

$$s = R\theta$$
$$v = R\omega$$
$$a_T = R\alpha$$
$$a_R = \frac{v^2}{R}$$
$$a = \sqrt{a_T^2 + a_R^2}$$

Rolling motion relationship:

$$s_{cm} = R\theta$$
$$v_{cm} = R\omega$$
$$a_{cm} = R\alpha$$

Total kinetic energy (translation + rotation)

$$K = \frac{1}{2} I_{cm}\omega^2 + \frac{1}{2} M v_{cm}^2$$

$$\omega = 240\,\frac{\text{rev}}{\text{min}} \times \frac{1\,\text{min}}{60\,\text{s}} \times \frac{2\pi\ \text{rads}}{1\,\text{rev}} = 8\pi\,\text{rad/s} \approx 25.1\,\text{rad/s}$$

Radial component : $a_R = \frac{v^2}{R} = R\omega^2$

$\rightarrow$ **Change of direction of** $\vec{V}$

Tangential component $a_T = R\alpha$

$\rightarrow$ **Change of magnitude of** $\vec{V}$

$$\boxed{\tau = Fd}$$

N.m **N** **m**

I **:** the **moment of inertia** of the rotating particle about the Δ axis

$\rightarrow$ $\boxed{\tau = I\alpha}$ $\Leftrightarrow$ $\vec{F} = m\vec{a}$

(rotational analog of Newton's second law for a rigid body)

MOMENT OF INERTIA IS GIVEN

CHAPTER 5

$$\tau = Fd$$

$\downarrow$ N.m, $\downarrow$ N, $\downarrow$ m

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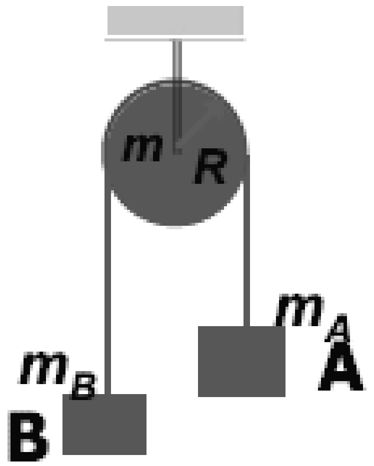
(rotational analog of Newton's second law for a rigid body)

PROBLEM 4

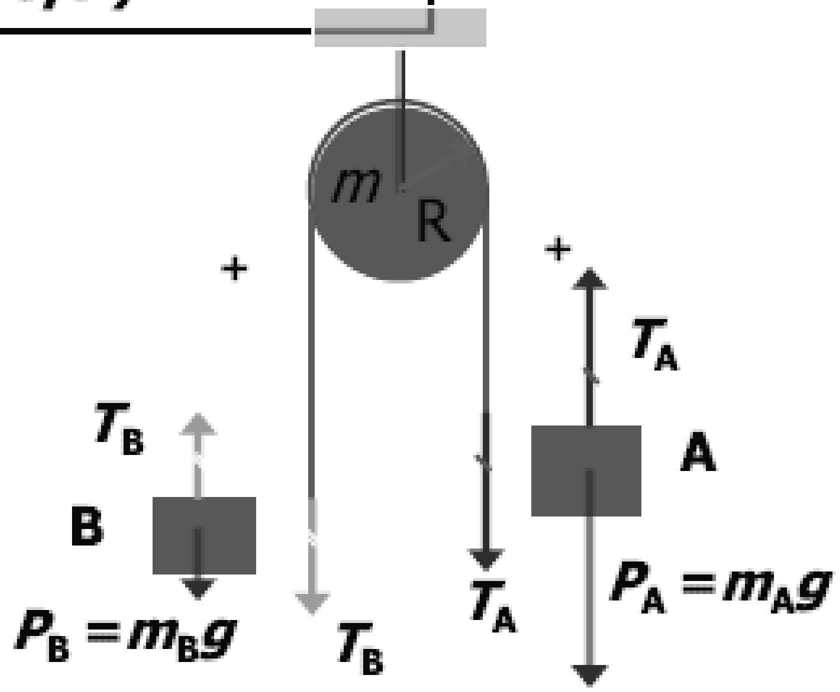
Two blocks having masses m_1 and m_2 are connected to each other by a light cord that passes over one frictionless pulley, having a moment of inertia I and radius R.
Find the acceleration of each block and the tensions T_1 , T_2 in the cord. (Assume no slipping between cord and pulleys.)



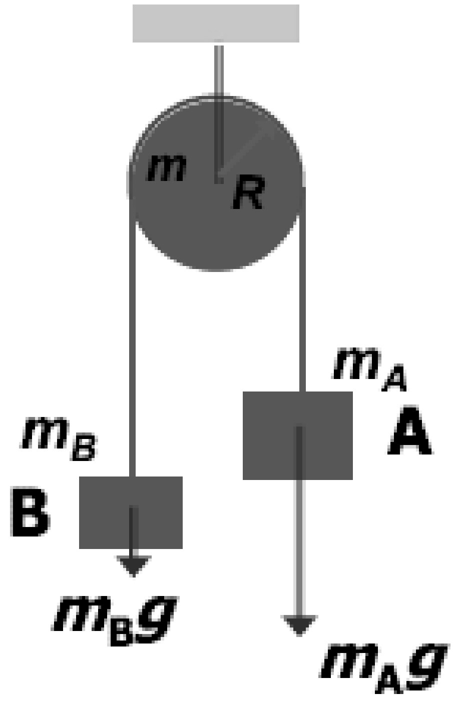
SOLUTION

- **For A :** $m_A g - T_A = m_A a$ **(1)**
- **For B :** $T_B - m_B g = m_B a$ **(2)**
- **For the pulley :**

$\tau = T_A R - T_B R = I\alpha$ **(3)**



$$a = \frac{m_A g - m_B g}{m_A + m_B + \frac{I}{R^2}}$$



- **Notes :**

$$\text{Acceleration} = \frac{\text{Acting force}}{\text{System's Inertia}}$$

- **With :** $I = \frac{1}{2}mR^2 \rightarrow a = \dfrac{m_A g - m_B g}{m_A + m_B + \frac{1}{2}m}$

CHAPTER 5

PROBLEM 5

Two blocks having masses m_A and m_B = 5.5 kg are connected to each other by a light cord that passes over one frictionless pulley, which is a thin-walled hollow cylinder and has a mass of 1.0 kg. The system begins to move from rest. After 2.0 s, the speed of A and B is 10 m/s Find m_A and the tensions T_A , T_B in the cord.

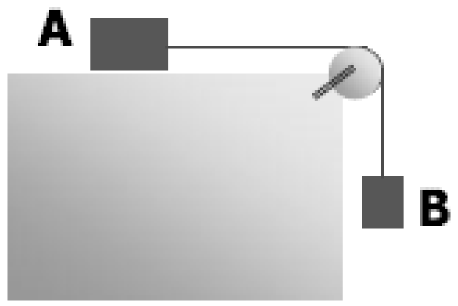
$$\boxed{\tau = Fd}$$

τ → **N.m**, F → **N**, d → **m**

I **:** the **moment of inertia** of the rotating particle about the Δ axis

$$\rightarrow \boxed{\tau = I\alpha} \Leftrightarrow \vec{F} = m\vec{a}$$

(rotational analog of Newton's second law for a rigid body)

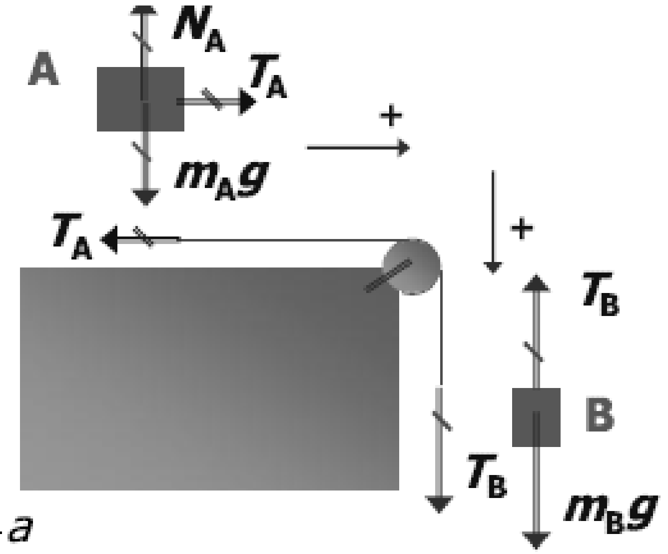


SOLUTION

- **For A :** $T_A = m_A a$
- **For B :** $m_B g - T_B = m_B a$
- **For the pulley :**

$$\tau = T_B R - T_A R = I\alpha$$

$$\alpha = a_T / R \equiv a / R$$



$$(T_B - T_A)R = I\frac{a}{R};\quad T_B - T_A = \frac{I}{R^2}a$$

$$(m_B g - m_B a) - m_A a = \frac{I}{R^2}a;\quad a = \frac{m_B g}{m_A + m_B + \frac{I}{R^2}}$$

$$a = \frac{m_B g}{m_A + m_B + \frac{mR^2}{2R^2}} = \frac{m_B g}{m_A + m_B + \frac{m}{2}}$$

$$a = \frac{m_B g}{m_A + m_B + \frac{mR^2}{2R^2}} = \frac{m_B g}{m_A + m_B + \frac{m}{2}};\quad m_A = m_B\left(\frac{g}{a} - 1\right) - \frac{m}{2}$$

$$v = at + v_0 = at;\quad a = \frac{v}{t} = \frac{10\ m/s}{2{,}0\ s} = 5{,}0\ m/s^2$$

$$m_A = 5{,}5\ kg \times \left(\frac{10}{5{,}0} - 1\right) - \frac{1{,}0}{2};\quad \boxed{m_A = 5{,}0\ kg}$$

$$T_A = m_A a = 5{,}0\ kg \times 5{,}0\ m/s^2;\quad \boxed{T_A = 25\ N}$$

$$m_B g - T_B = m_B a;\ T_B = m_B(g - a)$$

$$= 5{,}5\ kg \times (10 - 5{,}0)\ m/s^2$$

$$\boxed{T_B = 27{,}5\ N}$$

CHAPTER 5

$$\boxed{\tau = Fd}$$

N.m   N   m

I **:** the **moment of inertia** of the rotating particle about the Δ axis

$$\rightarrow \boxed{\tau = I\alpha} \Leftrightarrow \vec{F} = m\vec{a}$$

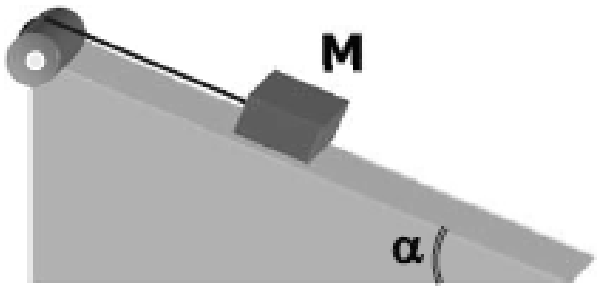
(rotational analog of Newton's second law for a rigid body)

The **rotational kinetic energy** of a object : $\boxed{K = \frac{1}{2} I\omega^2}$

To compare with the linear motion : $K = \frac{1}{2} mv^2$

PROBLEM 8 A block with mass m = 2.00 kg slides down a surface inclined 30^0 to the horizontal. A string attached to the block is wrapped around a flywheel on a fixed axis at O. The flywheel is a hollow cylinder and has mass m = 2.00 kg. The string pulls without slipping.
(a) What is the acceleration of the block down the plane?
(b) What is the tension in the string?

SOLUTION



(a)

$$\frac{1}{2} I\omega^2 + \frac{1}{2} mv^2 - 0 = mgx \sin\alpha$$

$$\frac{1}{2}(mR^2)\left(\frac{v}{R}\right)^2 + \frac{1}{2} mv^2 - 0 = mgx \sin\alpha$$

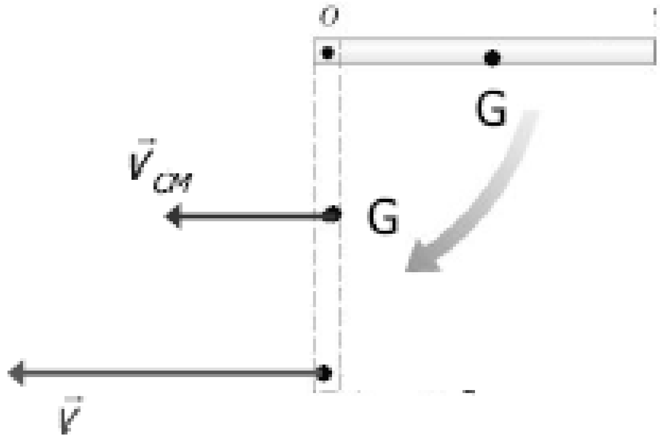
$$v^2 = gx \sin\alpha \; ; \quad 2v'v = gv \sin\alpha \; ;$$

$$\boxed{a = \frac{1}{2} g \sin\alpha} \quad a = \frac{1}{2} \times (9.81 \; m/s^2).\sin 30^0 = 2.45 \; m/s^2$$

PROBLEM 6

A uniform rod of length L and mass M is free to rotate on a frictionless pin passing through one end. The rod is released from rest in the horizontal position.
(b) Determine the linear speed of the center of mass and the linear speed of the lowest point on the rod when it is in the vertical position.

SOLUTION



(a) $\omega_f = \sqrt{\frac{3g}{L}}$

(b) $v_{CM} = R\omega = \frac{L}{2}\sqrt{\frac{3g}{L}}$; $\quad v_{CM} = \frac{1}{2}\sqrt{3gL}$

$$v = R'\omega = L\sqrt{\frac{3g}{L}} \; ; \quad \boxed{v = \sqrt{3gL}}$$

CHAPTER 5

ROLLING MOTION OF THE RIGID OBJECT

$$\boxed{\tau = Fd}$$

N.m N m

I : the **moment of inertia** of the rotating particle about the Δ axis

$$\rightarrow \boxed{\tau = I\alpha} \Leftrightarrow \vec{F} = m\vec{a}$$

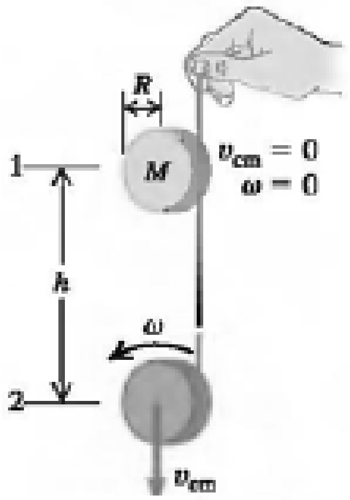
(rotational analog of Newton's second law for a rigid body)

$$\boxed{K = \frac{1}{2}I_{CM}\omega^2 + \frac{1}{2}Mv_{CM}^2}$$

PROBLEM 9 A primitive yo-yo is made by wrapping a string several times around a solid cylinder with mass M and radius R. You hold the end of the string stationary while releasing the cylinder with no initial motion. The string unwinds but does not slip or stretch as the cylinder drops and rotates. Use energy considerations to find the speed of the center of mass of the solid cylinder after it has dropped a distance h.

SOLUTION

The kinetic energy at point 2 :



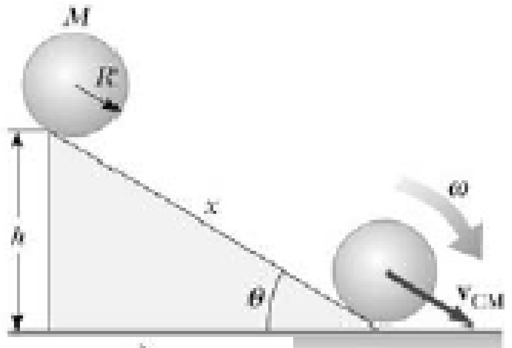
$$K_2 = \frac{1}{2}Mv_{cm}^2 + \frac{1}{2}\left(\frac{1}{2}MR^2\right)\left(\frac{v_{cm}}{R}\right)^2$$

$$= \frac{3}{4}Mv_{cm}^2$$

Conservation of energy : $K_1 + U_1 = K_2 + U_2$

$$0 + Mgh = \frac{3}{4}Mv_{cm}^2 + 0 \longrightarrow v_{cm} = \sqrt{\frac{4}{3}gh}$$

PROBLEM 10 For the solid sphere shown in the figure, calculate the linear speed of the center of mass at the bottom of the incline and the magnitude of the linear acceleration of the center of mass.



SOLUTION

$$\boxed{K = \frac{1}{2}I_{CM}\omega^2 + \frac{1}{2}Mv_{CM}^2} \qquad \boxed{v_{CM} = R\omega}$$

$$K = \frac{1}{2}I_{CM}\left(\frac{v_{CM}}{R}\right)^2 + \frac{1}{2}Mv_{CM}^2 \qquad K = \frac{1}{2}\left(\frac{I_{CM}}{R^2} + M\right)v_{CM}^2$$

Work–kinetic energy theorem : $\frac{1}{2}\left(\frac{I_{CM}}{R^2} + M\right)v_{CM}^2 = Mgh$

$$\longrightarrow v_{CM} = \left(\frac{2gh}{1 + I_{CM}/MR^2}\right)^{1/2} \qquad v_{CM} = \left(\frac{2gh}{1 + \frac{2/5MR^2}{MR^2}}\right)^{1/2} = \left(\frac{10}{7}gh\right)^{1/2}$$

$$v_{CM}^2 = 2a_{CM}x \longrightarrow a_{CM} = \frac{5}{7}g\sin\theta$$

CHAPTER 5

The angular momentum of the whole object:

$$L_z = \sum_I m_i r_i^2 \omega = \left(\sum_I m_i r_i^2 \right) \omega$$

$$\boxed{L_z = I\omega}$$

5.4 Conservation of Angular Momentum

From : $\boxed{\vec{\tau} = \frac{d\vec{L}}{dt}}$

If: $\vec{\tau} = \vec{0} \longrightarrow \frac{d\vec{L}}{dt} = \vec{0}\ ;\ \vec{L} = \overrightarrow{const}$

The total angular momentum of a system is constant in both magnitude and direction if the resultant external torque acting on the system is zero.

$$\boxed{\vec{L}_i = \vec{L}_f}$$

$$\boxed{I_i \omega_i = I_f \omega_f}$$

Three Conservation Laws for an Isolated System

- **Conservation of energy :** $\boxed{K_i + U_i = K_f + U_f}$
- **Conservation of linear momentum :** $\boxed{\vec{p}_i = \vec{p}_f}$
- **Conservation of angular momentum :** $\boxed{\vec{L}_i = \vec{L}_f}$

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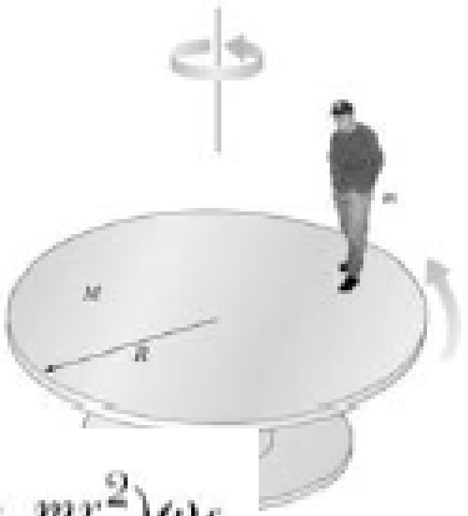
- **Conservation of energy :** $\boxed{K_i + U_i = K_f + U_f}$
- **Conservation of linear momentum :** $\boxed{\vec{p}_i = \vec{p}_f}$
- **Conservation of angular momentum :** $\boxed{\vec{L}_i = \vec{L}_f}$

CHAPTER 5

PROBLEM 14 A horizontal platform in the shape of a circular disk rotates in a horizontal plane about a frictionless vertical axle. The platform has a mass $M = 100$ kg and a radius $R = 2.0$ m. A student whose mass is $m = 60$ kg walks slowly from the rim of the disk toward its center. If the angular speed of the system is 2.0 rad/s when the student is at the rim, what is the angular speed when he has reached a point $r = 0.50$ m from the center?

SOLUTION



$$I_i = I_{pi} + I_{si} = \tfrac{1}{2}MR^2 + mR^2$$

$$I_f = I_{pf} + I_{sf} = \tfrac{1}{2}MR^2 + mr^2$$

$$I_i\omega_i = I_f\omega_f \longrightarrow \left(\tfrac{1}{2}MR^2 + mR^2\right)\omega_i = \left(\tfrac{1}{2}MR^2 + mr^2\right)\omega_f$$

$$\omega_f = \left(\frac{\tfrac{1}{2}MR^2 + mR^2}{\tfrac{1}{2}MR^2 + mr^2}\right)\omega_i = \boxed{4.1 \text{ rad/s}}$$