HO CHI MINH CITY INTERNATIONAL UNIVERSITY

MIDTERM EXAMINATION

Academic year 2022-2023, Semester 1
Duration: 90 minutes

SUBJECT: **Differential Equations (MA024IU)**	
Head of Department of Mathematics	Lecturer:
Signature:	Signature:
Prof. Pham Huu Anh Ngoc	Full name: Pham Huu Anh Ngoc

Instructions:

- *Each student is allowed a scientific calculator and a maximum of two double-sided sheets of reference material (size A4 or similar), stapled together and marked with their name and ID. All other documents and electronic devices are forbidden.*

Question 1. *(15 marks)* (**Logistic Equation**) The number $N(t)$ of supermarkets throughout the country that are using a computerized checkout system is described by the initial value problem:

$$\frac{dN}{dt} = N(1 - 0.0005N), \quad N(0) = 1.$$

How many supermarkets are expected to adopt the new technology when $t = 10$?

Question 2. *(15 marks)* (i) Show that the differential equation

$$\big(e^y + 5x^4y + 2022x\big)dx + \big(x^5 + xe^y + y^2\big)dy = 0,$$

is exact. Solve the differential equation.

(ii) *(10 marks)* Solve the following differential equation

$$x^2\frac{dy}{dx} = y^2 - xy^2.$$

Question 3. *(20 marks)* Find the solution to the initial value problem

$$(x+1)y' + (x+2)y = 2xe^{-x}, \qquad y(0) = 2022.$$

Question 4. *(20 marks)* Find the general solution of the following differential equation

$$y'' - 5y' + 6y = e^{3x}x.$$

Question 5. a) *(10 marks)* Find $\alpha \in \mathbb{R}$ such that $y_1(x) := (x-4)^\alpha$ is a solution of the following differential equation

$$(x-4)^2y'' - 5(x-4)y' + 9y = 0, \qquad x \in (4, \infty).$$

b) *(10 marks)* Find the general solution of the above differential equation.

END.

SOLUTIONS:

Question 1. The limiting value of the population is $1,000,000$. The population will reach $500,000$ in 5.29 months.
Question 2. The given differential equation is rewritten as

$$(e^{2y}dx + xde^{2y}) - \cos(xy)(xdy + ydx) + dy^2 = 0.$$

Then, we get

$$d(e^{2y}x) - \cos(xy)d(xy) + dy^2 = d(e^{2y}x) + d(-\sin(xy)) + dy^2 = 0.$$

Therefore,

$$d(e^{2y}x - \sin(xy) + y^2) = 0.$$

Thus the general solution is given by

$$e^{2y}x - \sin(xy) + y^2 = C.$$

Question 3. Consider the differential equation

$$y' - (\sin x)y = 2\sin x.$$

The integrating factor is given by $I(x) = e^{\cos x}$. Thus, we get

$$e^{\cos x}y' - e^{\cos x}(\sin x)y = 2e^{\cos x}\sin x.$$

This gives

$$\frac{d}{dx}(e^{\cos x}y) = 2\int e^{\cos x}\sin x dx = -2e^{\cos x} + C.$$

Therefore, the general solution is

$$y(x) = -2 + \frac{C}{e^{\cos x}}.$$

Since $y(\frac{\pi}{2}) = 1$, the particular solution is $y(x) = -2 + \frac{3}{e^{\cos x}}$.
Question 4. a) The form of a particular solution of the differential equation

$$y'' - 4y' + 3y = e^{2x}(x^3 + 1) + e^x(x + 1)$$

is given by

$$y_p(x) = e^{2x}(Ax^3 + Bx^2 + Cx + D) + e^x(Ex^2 + Fx).$$

The general solution of the differential equation

$$y'' - 4y' + 3y = e^x(x + 1)$$

is given by

$$y(x) = c_1e^x + c_2e^{3x} - e^x(\frac{1}{4}x^2 + \frac{3}{4}x).$$

Question 5. a) $a = b = q, q \in \mathbb{R}$.
b) Note that $y_1(x) = x + 1$ is a particular solution of the differential equation

$$(1 - 2x - x^2)y'' + 2(x + 1)y' - 2y = 0.$$

By the Liouville formula, $y_2(x) = x^2 + x + 2$ is a solution of this equation such that y_1, y_2 are linearly independent. So, the general solution is given by

$$y(x) = c_1(x + 1) + c_2(x^2 + x + 2).$$