Homework: 51, 54, 59, 60 (pages 503, 504)

51. A sphere of radius 0.500 m, temperature 27.0°C, and emissivity 0.850 is located in an environment of temperature 77.0°C. At what rate does the sphere (a) emit and (b) absorb thermal radiation? (c) What is the sphere's net rate of energy exchange?

(a)

$$P_{rad} = \sigma \varepsilon A T^4$$

$$A = 4\pi R^2; T = 273 + 27 = 300 \, (K)$$

$$P_{rad} \approx 1.23 \times 10^3 \, (W)$$

(b)

$$P_{abs} = \sigma \varepsilon A T^4_{env}$$

$$T_{env} = 273 + 77 = 350 \, (K)$$

$$P_{abs} \approx 2.27 \times 10^3 \, (W)$$

(c) $\quad P_{net} = P_{abs} - P_{rad} = 2.27 \times 10^3 - 1.23 \times 10^3 = 1.04 \times 10^3 \, (W)$

54. If you were to walk briefly in space without a spacesuit while far from the Sun (as an astronaut does in the movie 2001), you would feel the cold of space – while you radiated energy, you would absorb almost none from your environment. (a) At what rate would you lose energy? (b) How much energy would you lose in 30 s? Assume that your emissivity is 0.90, and estimate other data needed in the calculations.
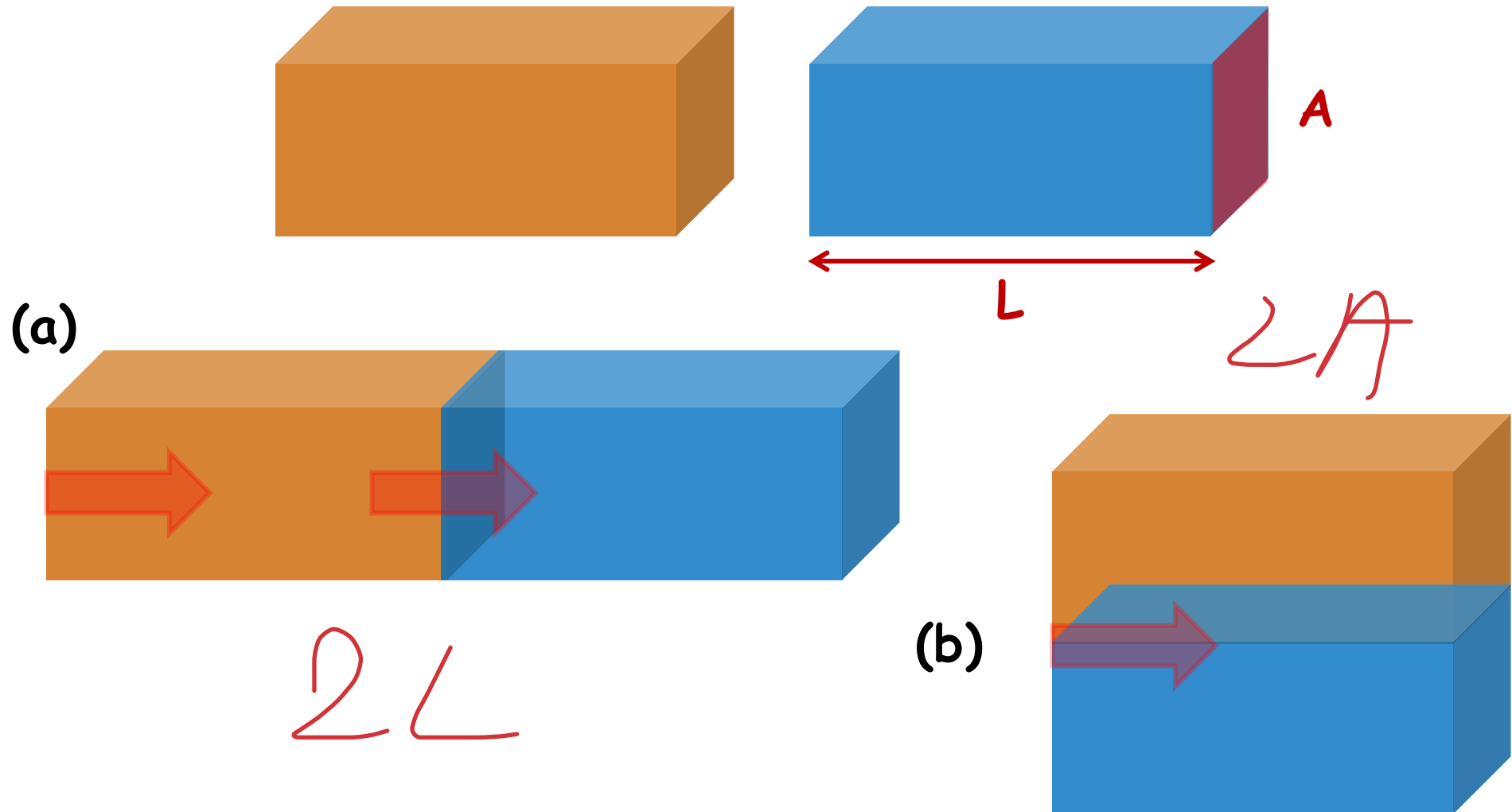
(a) The heat transfer mechanism is radiation:

$$P_{rad} = \sigma \varepsilon A T^4$$

$$P_{rad} = 5.67 \times 10^{-8} \times 0.9 \times 2.0 \times 310^4 = 9.4 \times 10^2 \text{ (W)}$$

(b) The energy lost in 30 s is:

$$E = P_{rad} \times t = 9.4 \times 10^2 \times 30 = 2.8 \times 10^4 \text{ (J)}$$

59. In Figure a, two identical rectangular rods of metal are welded end to end, with a temperature of $T_1 = 0^0C$ on the left side and a temperature of $T_2 = 100^0C$ on the right side. In 2.0 min, 10 J is conducted at a constant rate from the right side to the left side. How much time would be required to conduct 10 J if the rods were welded side to side as in Figure b.
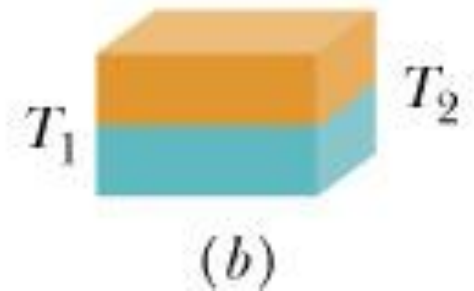

(a)

L

2A

2L

(b)

A

The heat transfer mechanism is conduction:

$$P_{cond} = kA \frac{T_H - T_C}{L}$$

(a)

(b)
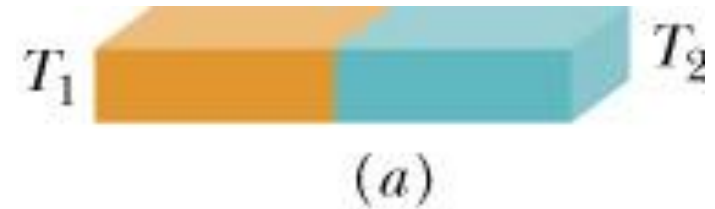
$$P_{cond, a} = kA_a \frac{T_H - T_C}{L_a}$$

$$P_{cond, b} = kA_b \frac{T_H - T_C}{L_b}$$

$$\Rightarrow P_{cond, b} = \frac{A_b}{A_a} \frac{L_a}{L_b} P_{cond, a} = 2 \times 2 \times P_{cond, a} = 4P_{cond, a}$$

so, the requested time is 2.0/4=0.5 min or 30 s.

60. The figure below shows the cross section of a wall made of three layers. The thicknesses of the layers are L_1, $L_2 = 0.750L_1$, and $L_3 = 0.350L_1$. The thermal conductivities are k_1, $k_2 = 0.900k_1$, and $k_3 = 0.800k1$. The temperatures at the left and right sides of the wall Are 30.0^oC and -15.0^oC, respectively. Thermal conduction through the wall has reached the steady state. (a) What is the temperature difference ΔT_2 across layer 2 (between the left and right sides of the layer)? If k_2 were, instead, equal to $1.1k_1$, (b) would the rate at which energy is conducted through the wall be greater than, less than, or the same as previously, and (c) what would be the value of ΔT_2?
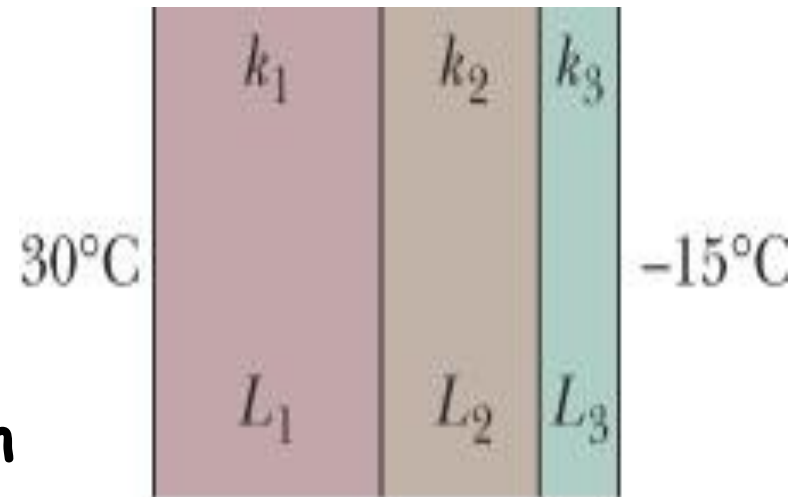
(a)
$$P_{cond} = \frac{A(T_H - T_C)}{\Sigma (L/k)} = \frac{A\Delta T_2}{L_2/k_2}$$

$$\Delta T_2 = \frac{(L_2/k_2)(T_H - T_C)}{\Sigma (L/k)} \approx 16.5^0 C$$

k_1 k_2 k_3

30^oC -15^oC

L_1 L_2 L_3

(b) conductivity k increases → conduction rate increases.

(c) Repeat the calculation in part (a):
$$\Delta T_2 \approx 14.5^0 C$$

Chapter 3 The Kinetic Theory of Gases

Work Done by an Ideal Gas at Constant Temperature

A process at constant temperature is called an isothermal expansion/compression. The equation of state for n moles:
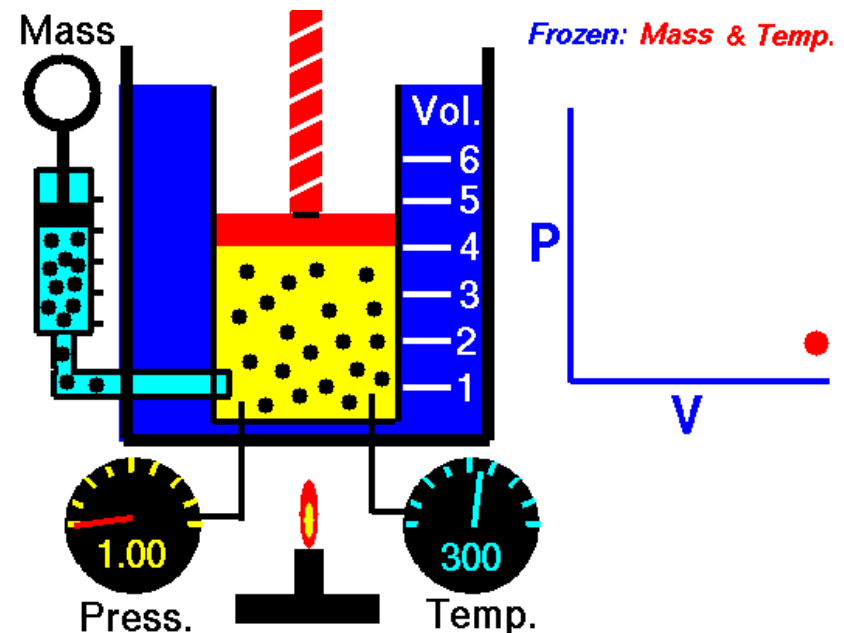
$$p = nRT \, \frac{1}{V} = \underbrace{\text{constant}} \times \frac{1}{V}$$

The work done during an isothermal process:

$$W = \vec{F}\,\vec{d}$$

$$W = \int_{V_i}^{V_f} p\,dV = \int_{V_i}^{V_f} \frac{nRT}{V}\,dV = nRT\,\Big[\ln V\Big]_{V_i}^{V_f}$$

$$\boxed{W = nRT \, \ln \frac{V_f}{V_i}}$$

$T = 320\ \text{K}$

$T = 310\ \text{K}$

$T = 300\ \text{K}$

Mass

Frozen: Mass & Temp.

Vol.
—6
—5
—4
—3
—2
—1

P

V

1.00

Press.

300

Temp.
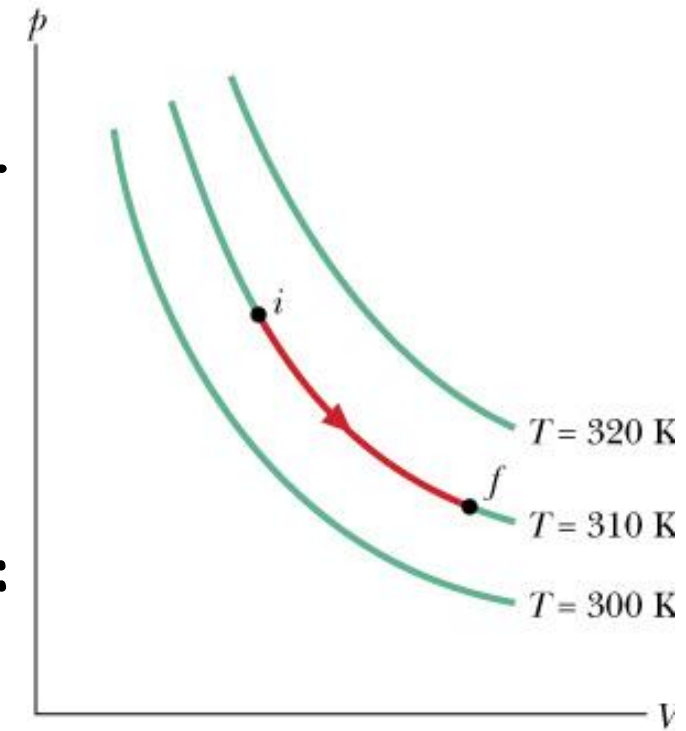
Summary

The equations below allows us to calculate work done by the gas for three special cases:

$$W = \int_{V_i}^{V_f} p \, dV$$

1) If $V = $ constant (isochoric): $\qquad W = 0$

2) If $p = $ constant (isobaric): $\quad W = p(V_f - V_i) = p\Delta V$

3) If $T = $ constant (isothermal): $W = nRT \, \ln \dfrac{V_f}{V_i}$

Checkpoint 1: An ideal gas has an initial pressure of 3 pressure units and an initial volume of 4 volume units. The table gives the final pressure and volume of the gas (in those same units) in 5 processes. Which processes start and end on the same isotherm?

T const

	a	b	c	d	e
p	12	6	5	4	1
V	1	2	7	3	12
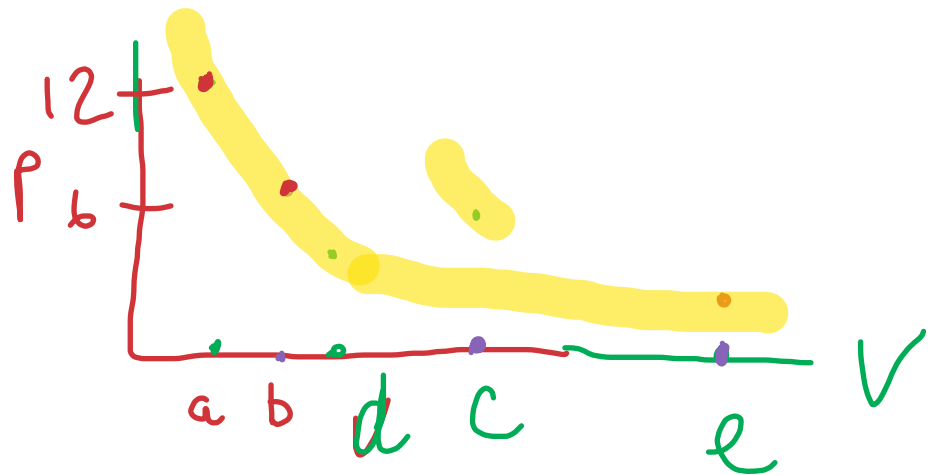
$\Rightarrow a, b, d, e$

3.1.2. Molecular Model for an Ideal Gas

In this model:

1. The molecules obey Newton's laws of motion.

2. The molecules move in all direction with equal probability.

3. There is no interactions between molecules (no collisions between molecules).

4. The molecules undergo elastic collisions with the walls.

Simulations:

http://www.phy.ntnu.edu.tw/ntnujava/index.php?topic=25

http://www2.biglobe.ne.jp/~norimari/science/JavaApp/Mole/e-gas.html

a. Pressure, Temperature, and RMS Speed

Problem: Let n moles of an ideal gas be confined in a cubical box of volume V, (see the figure below). The walls of the box are held at temperature T.

Key question: What is the connection between the pressure p exerted by the gas and the speed of the molecules?

First, we consider a cubical box of edge length L, containing n moles of an ideal gas. A molecule of mass m and velocity v is about to collide with the shaded wall.

For an elastic collision, the particle's momentum (=m.v) along the x axis is reserved and change with an amount:

$$\Delta p_x = (-mv_x) - (mv_x) = -2mv_x$$
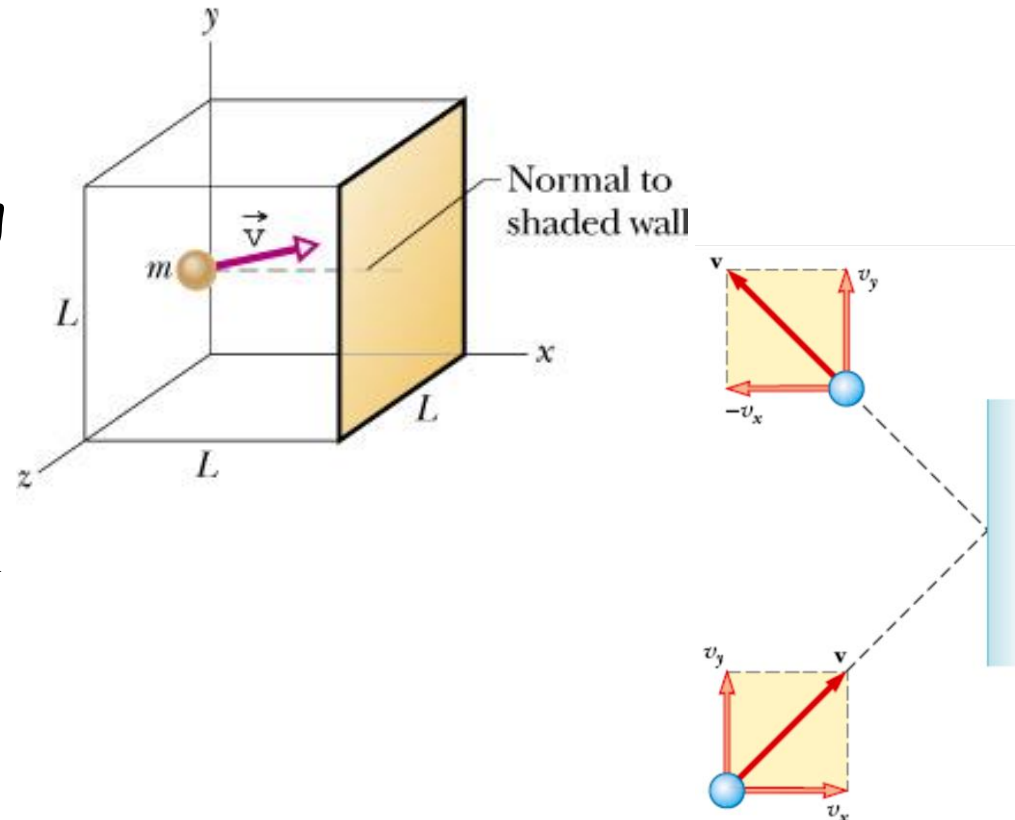
The average rate at which momentum is delivered to the shaded wall by this molecule:

$$\frac{\Delta p_x}{\Delta t} = \frac{2mv_x}{2L/v_x} = \frac{mv_x^2}{L}$$

Recall: $\qquad \vec{F} = m\vec{a} = m\frac{d\vec{v}}{dt} = \frac{d(m\vec{v})}{dt} = \frac{d\vec{p}}{dt}$

$$F_{x,1} = \frac{mv_x^2}{L}$$

The pressure exerted on the wall by this single molecule:

$$p_1 = \frac{F_{x,1}}{L^2}$$

For N molecules, the total pressure p:

$$p = \frac{F_x}{L^2} = \frac{mv_{x,1}^2/L + mv_{x,2}^2/L + ... + mv_{x,N}^2/L}{L^2}$$

$$p = \left(\frac{m}{L^3}\right)(v_{x,1}^2 + v_{x,2}^2 + \ldots + v_{x,N}^2)$$

The average value of the square of the x components of all the molecular speeds:

$$\overline{v_x^2} = \frac{v_{x,1}^2 + v_{x,2}^2 + \ldots + v_{x,N}^2}{N}$$

$$\Rightarrow p = \frac{nmN_A}{L^3}\overline{v_x^2}$$

Since $M = mN_A$: the molar mass of the gas

$V = L^3$:

$$p = \frac{nM}{V}\overline{v_x^2}$$

For any molecule: $v^2 = v_x^2 + v_y^2 + v_z^2$

As all molecules move in random directions: $\overline{v_x^2} = \frac{1}{3}\overline{v^2}$

$$p = \frac{nM}{3V} \overline{v^2}$$

The square root of $\overline{v^2}$ is called the root-mean-square speed:

$$\sqrt{\overline{v^2}} = v_{rms}$$

$$\boxed{p = \frac{nMv_{rms}^2}{3V}}$$

This relationship shows us how the pressure of the gas (a macroscopic quantity) depends on the speed of the molecules (a microscopic quantity)

Combining with the equation of state: $\quad pV = nRT$

$$\boxed{v_{rms} = \sqrt{\frac{3RT}{M}}}$$

b. Translational Kinetic Energy

• Consider a single molecule of an ideal gas moving around in the box (see Section a) .

$$\overline{K} = \frac{1}{2}m\overline{v^2} = \frac{1}{2}m\overline{\overline{v}^2} = \frac{1}{2}mv_{rms}^2$$

$$\overline{K} = \left(\frac{1}{2}m\right)\frac{3RT}{M} = \frac{1}{2}\frac{3RT}{M/m}$$

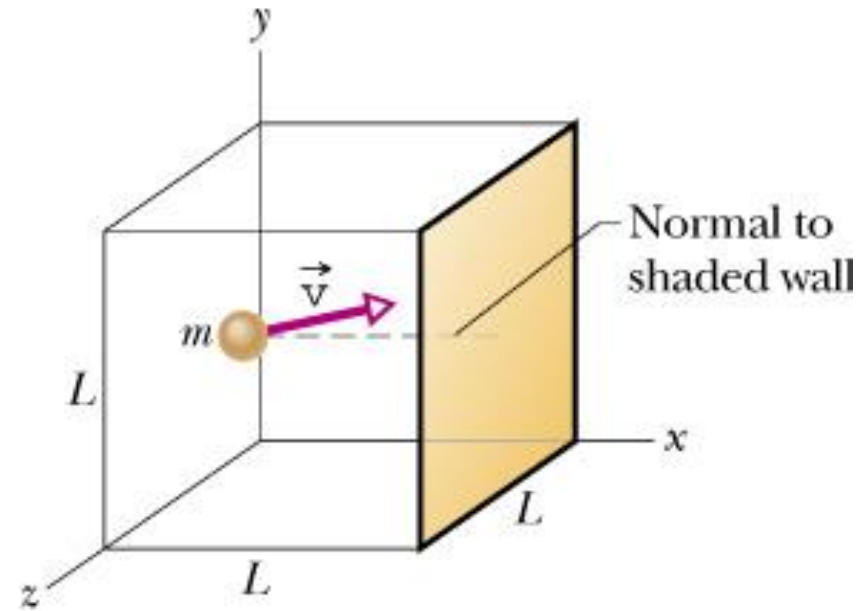$$\overline{K} = \frac{3RT}{2N_A}$$

Recall:

The Boltzmann constant k: $k = \dfrac{R}{N_A}$

3 dimensions

$$\boxed{\Rightarrow \overline{K} = \frac{3}{2}kT}$$ $\Rightarrow \overline{K}$ does not depend on the mass of the molecule

$$\overline{v_x^2} = \overline{v_y^2} = \overline{v_z^2} = \frac{1}{3}\overline{v^2} \Rightarrow \frac{1}{2}m\overline{v_x^2} = \frac{1}{2}m\overline{v_y^2} = \frac{1}{2}m\overline{v_z^2} = \frac{1}{2}kT$$

Homework: 13, 14, 18, 20, 24 (p. 531-532)