② Continuous random variable

① $12 \text{ minutes} = \frac{12}{60} = 0.2 \text{ hours}$

$P(X < 0.2) = \lim_{x \to 0.2} P(X \leq x) = 1 - e^{-8 \times 0.2} = 0.7981$

or $\int_{-\infty}^{0.2} f(x)dx = \int_{-\infty}^{0.2} 8e^{-8x} = 0.7981$

②

a) $P(X > 3000) = \int_{3000}^{+\infty} \frac{e^{-x/1000}}{1000} dx = -e^{-x/1000} \Big|_{3000}^{+\infty} = 0 + e^{-3} = 0.0498$

b) $P(1000 < X < 2000) = \int_{1000}^{2000} \frac{e^{-x/1000}}{1000} dx = -e^{-x/1000} \Big|_{1000}^{2000} = 0.2325$

c) $P(0 < X < 1000) = \int_{0}^{1000} \frac{e^{-x/1000}}{1000} dx = -e^{-x/1000} \Big|_{0}^{1000} = 0.6321$

d) $P(X < x) = 0.10 \Rightarrow \int_{0}^{x} \frac{e^{-x/1000}}{1000} dx = 0.10 \Rightarrow -e^{-x/1000} \Big|_{0}^{x} = 0.10$

$\Rightarrow 1 - e^{-x/1000} = 0.10 \Rightarrow e^{-x/1000} = 0.9$

$\Rightarrow -x/1000 = \ln(0.9) \Rightarrow x = 105.36$

③ $P(X > 50) = \int_{50}^{50.25} 2 \, dx = 0.5$

③ Expectation - Variance

①
- $P(H) = 3P(T) = \frac{3}{4}$
- $P(H) + P(T) = 1$

$\Rightarrow 4P(T) = 1$

$\Rightarrow P(T) = \frac{1}{4}$

$P(T=0) = P(HH) = \frac{3}{4} \times \frac{3}{4} = \frac{9}{16}$

$P(T=1) = P(HT, TH) = \frac{3}{4} \times \frac{1}{4} + \frac{3}{4} \times \frac{1}{4} = \frac{6}{16} = \frac{3}{8}$

$P(T=2) = P(TT) = \frac{1}{4} \times \frac{1}{4} = \frac{1}{16}$

② $P(X = 4000) = 0.3$

$P(X = -1000) = 0.7$

$E(X) = \sum x f(x) = 4000 \times 0.3 + (-1000) \times 0.7 = 500\$$