Chapter 4: Fourier Series

1. Full Range Series

$$x(t) = \frac{1}{2}a_0 + \sum_{k=1}^{+\infty} a_k \cos(k\omega_0 t) + \sum_{k=1}^{+\infty} b_k \sin(k\omega_0 t) \qquad \text{(Eq 4.1)}$$

Where:

$$a_0 = \frac{2}{T_0}\int_{t_0}^{t_0+T_0} x(t)dt\,;\ a_k = \frac{2}{T_0}\int_{t_0}^{t_0+T_0} x(t)\cos(k\omega_0 t)\,dt\,;\ b_k = \frac{2}{T_0}\int_{t_0}^{t_0+T_0} x(t)\sin(k\omega_0 t)\,dt$$

Odd function: $a_0 = a_k = 0$, and

$$b_k = \frac{4}{T_0}\int_0^{T_0/2} x(t)\sin(k\omega_0 t)\,dt$$

Even function: $b_k = 0$, and

$$a_0 = \frac{4}{T_0}\int_0^{T_0/2} x(t)dt\,;\ a_k = \frac{4}{T_0}\int_0^{T_0/2} x(t)\cos(k\omega_0 t)\,dt$$

Parseval's identity:

$$\frac{1}{T_0}\int_{t_0}^{t_0+T_0} |x(t)|^2 dt = \frac{1}{4}a_0^2 + \frac{1}{2}\sum_{k=1}^{+\infty}(a_k^2 + b_k^2) \qquad \text{(Eq 4.2)}$$

2. Half Range Series

2. 1. Half Range Sine Series:

$$x(t) = \sum_{k=1}^{+\infty} b_k \sin\left(\frac{k\pi t}{L}\right);\ b_k = \frac{2}{L}\int_0^L x(t)\sin\left(\frac{k\pi t}{L}\right)dt$$

2. 2. Half Range Cosine Series:

$$x(t) = \frac{1}{2}a_0 + \sum_{k=1}^{+\infty} a_k \cos\left(\frac{k\pi t}{L}\right);\ \ a_0 = \frac{2}{L}\int_0^L x(t)dt\,;\ \ a_k = \frac{2}{L}\int_0^L x(t)\cos\left(\frac{k\pi t}{L}\right)dt$$

3. Frequently Used Formulas

$$I_1 = \int (at+b)\sin ct\,dt = -\frac{at+b}{c}\cos ct + \frac{a}{c^2}\sin ct$$

$$I_2 = \int (at+b)\cos ct\,dt = \frac{at+b}{c}\sin ct + \frac{a}{c^2}\cos ct$$

$$I_3 = \int \sin(at+b)\sin(ct+d)\,dt = \frac{1}{2}\left(\frac{\sin(t(a-c)+b-d)}{a-c} - \frac{\sin(t(a+c)+b-d)}{a+c}\right)$$

$$I_4 = \int \cos(at+b)\cos(ct+d)\,dt = \frac{1}{2}\left(\frac{\sin(t(a-c)+b-d)}{a-c} + \frac{\sin(t(a+c)+b-d)}{a+c}\right)$$

$$I_5 = \int \sin(at+b)\cos(ct+d)\,dt = -\frac{1}{2}\left(\frac{\cos(t(a-c)+b-d)}{a-c} + \frac{\cos(t(a+c)+b-d)}{a+c}\right)$$

$\cos k\pi = (-1)^k$

$\sin k\pi = 0$