THE INTERNATIONAL UNIVERSITY(IU) - VIETNAM NATIONAL UNIVERSITY HCMC

MIDTERM EXAMINATION

November, 2011

Duration: 90 minutes

SUBJECT: CALCULUS 3	
Head of Dept. Maths.	Lecturer:
Signature:	Signature:
Prof. DrSc. Phan Q. Khanh	Dr. N.N. Hai

INSTRUCTIONS: • Answer ALL questions in Section A and TWO questions in Section B.

- Open-book examination. Computers and laptops prohibited.
- Exchanging documents strictly prohibited.

PART A

Question A1. *[15 marks]* (a) Express $z = 1 + \sqrt{3}j$ in the polar form.

(b) Find $(1 + \sqrt{3}j)^4$.

Question A2. *[15 marks]* Determine

(a) $\mathcal{L}\{e^{-2t}\sin 3t\}$ (b) $\mathcal{L}^{-1}\{\frac{4s-5}{s^2-s-2}\}$.

Question A3. *[15 marks]* (a) Find the real and imaginary parts of the function

$$f(z) = \bar{z}\,\mathrm{Re}(z) + z^2 + \mathrm{Im}(z).$$

(b) Let a, b, and c be real constants. Determine a relation among the coefficients that will guarantee that the function $\phi(x, y) = ax^2 + bxy + cy^2$ is harmonic.

Question A4. *[15 marks]* Use Laplace transforms to solve the differential equation:

$$\frac{d^2y}{dt^2} + 6\frac{dy}{dt} + 13y = 0$$

given that when $t = 0$, $y = 3$ and $dy/dt = 7$.

PART B

Question B1 *[20 marks]* (a) Represent the complex number

$$\frac{1+2j}{3-4j} + \frac{2-j}{5j}$$

in the form $a + bj$.

(b) Show that the function $g(z) = e^y \cos x + je^y \sin x$ is nowhere differentiable.

Question B2 *[20 marks]* Solve the simultaneous equations

$$\frac{dy}{dt} + \frac{dz}{dt} + y(t) + z(t) = 1$$
$$\frac{dy}{dt} + z(t) = e^t$$

given that $y(0) = -1,\ z(0) = 2$.

Question B3 *[20 marks]* Find the cubic roots of $z = -1 + j$.

(b) Find the Laurent expansion for

$$\frac{z}{(z-1)(z-3)}$$

about the point $z = 1$.

END OF QUESTION PAPER