

**Homework: 53, 56 (p. 270-271); 26, 38 (p. 299-300)**

**Final exam: Chapter 7, 8, 9, 10, 11 (textbook)**

**Read 11-12 Precession of a Gyroscope**

53. The figure below shows a uniform disk that can rotate around its center like a merry-go-round. The disk has a radius of 2.0 cm and a mass of 20.0 grams and is initially at rest. Starting at time  $t=0$ , two forces are to be applied tangentially to the rim as indicated, so that at time  $t=1.25$  s the disk has an angular velocity of 250 rad/s counterclockwise. Force  $F_1$  has a magnitude of 0.1 N. What is magnitude  $F_2$ ?

$$\tau_{net} = I\alpha$$

$$\sum F_i d_i = \tau_{net} = F_2 R - F_1 R = I\alpha$$

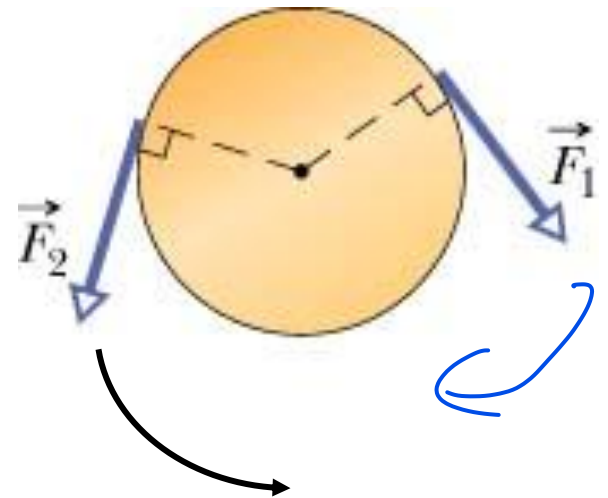
$$(1) \quad F_2 = \frac{I\alpha}{R} + F_1$$

For a uniform disk:

$$I = \frac{1}{2} MR^2 \quad (2)$$

For rotation:  $\omega = \omega_0 + \alpha t = \alpha t \Rightarrow \alpha = \frac{\omega}{t}$

$$F_2 = \frac{MR\omega}{2t} + F_1 = \frac{(20.0 \times 10^{-3})(2.0 \times 10^{-2})(250)}{2 \times 1.25} + 0.1 = 0.14 \text{ (N)}$$



$$\omega_0 = 0$$

(3)

56. The figure shows particles 1 and 2, each of mass  $m$ , attached to the ends of a rigid massless rod of length  $L_1 + L_2$ , with  $L_1 = 20$  cm and  $L_2 = 80$  cm. The rod is held horizontally on the fulcrum and then released. What are the magnitudes of the initial accelerations of (a) particle 1 and (b) particle 2?

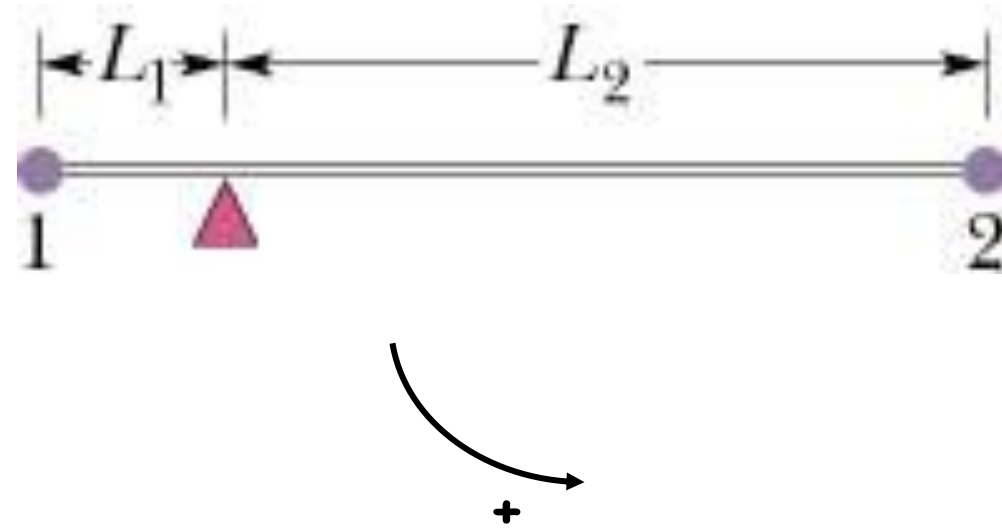
$$\tau_{net} = mgL_1 - mgL_2 = \underline{I\alpha}$$

$$I = mL_1^2 + mL_2^2$$

$$\Rightarrow \alpha = \frac{g(L_1 - L_2)}{L_1^2 + L_2^2}$$

$$\alpha = \frac{9.8(0.2 - 0.8)}{0.2^2 + 0.8^2} = -8.65 \text{ (rad/s}^2\text{)}$$

$$\vec{a} = \vec{a}_r + \vec{a}_t; a_r = \frac{v^2}{r}; a_t = \alpha r$$



(a)  $a_{1t} = \alpha L_1 = -8.65 \times 0.2 = -1.73 \text{ (m/s}^2\text{)}$

(b)  $a_{2t} = \alpha L_2 = -8.65 \times 0.8 = -6.92 \text{ (m/s}^2\text{)}$

26. At the instant of the figure below, a 2.0 kg particle P has a position vector  $\vec{r}$  of magnitude 5.0 m and angle  $\theta_1 = 45^\circ$  and a velocity vector  $\vec{v}$  of magnitude 4.0 m/s and angle  $\theta_2 = 30^\circ$ . Force  $\vec{F}$ , of magnitude 2.0 N and angle  $\theta_3 = 30^\circ$ , acts on P. All three vectors lie in the xy plane. About the origin, what are the (a) magnitude and (b) direction of the angular momentum of P and the (c) magnitude and (d) direction of the torque acting on P?

$$\vec{l} = \vec{r} \times \vec{p}$$

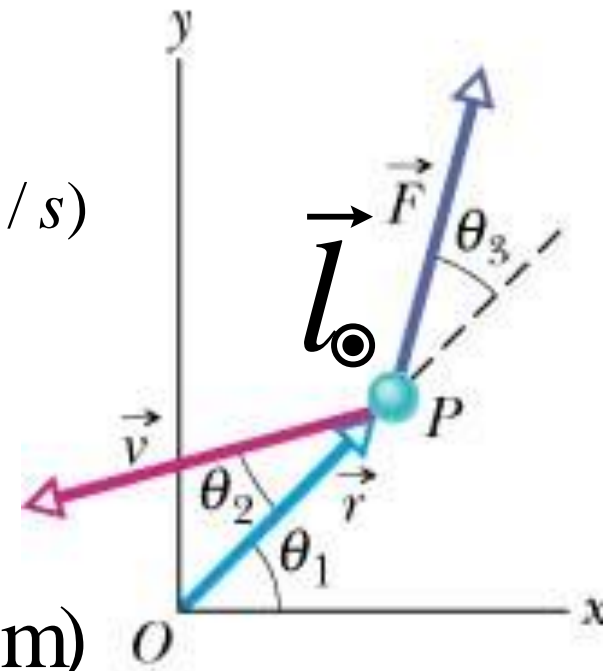
(a)  $l = rmv \sin \theta_2 = 5.0 \times 2.0 \times 4.0 \times \sin(30) = 20 \text{ (kg m}^2 \text{ / s)}$

(b) Using the right-hand rule,  $\vec{l}$  points out of the page and it is perpendicular to the figure plane.

$$\vec{\tau} = \vec{r} \times \vec{F}$$

(c)  $\tau = rF \sin \theta_3 = 5.0 \times 2.0 \times \sin(30) = 5 \text{ (N m)}$

(d)  $\vec{\tau}$  points out of the page and it is perpendicular to the figure plane.



38. A sanding disk with rotational inertia  $8.6 \times 10^{-3} \text{ kg}\cdot\text{m}^2$  is attached to an electric drill whose motor delivers a torque of magnitude  $16 \text{ N}\cdot\text{m}$  about the central axis of the disk. About that axis and with the torque applied for  $33 \text{ ms}$  (milliseconds), what is the magnitude of the (a) angular momentum and (b) angular velocity of the disk?

(a) Using Newton's second law for rotation:

$$\tau_{avg} = \frac{\Delta L}{\Delta t}$$

$$\Rightarrow \Delta L = L - L_0 = L = \tau_{avg} \Delta t$$

$$L = 16 \times 33 \times 10^{-3} = 0.528 \text{ (Nms)} \text{ or } 0.528 \text{ (kg m}^2/\text{s)}$$

(b)

$$L = I\omega \Rightarrow \omega = \frac{L}{I}$$

$$\omega = \frac{0.528}{8.6 \times 10^{-3}} = 61.4 \text{ (rad/s)}$$



# Chapter 6 Equilibrium and Elasticity

6.1. Equilibrium

6.2. The Center of Gravity and Conditions for Stable Equilibrium

6.3. Elasticity

## 6.1. Equilibrium

Examples: (1) a book resting on a table, (2) a hockey puck sliding with constant velocity across a frictionless surface, (3) the rotating blades of a ceiling fan, and (4) the wheel of a bicycle traveling along a straight path at constant speed.

a. The requirements of Equilibrium of the objects above:

1. The linear momentum  $\vec{P}$  of its center of mass is constant.

$$\vec{F}_{net} = \frac{d\vec{P}}{dt} = 0$$

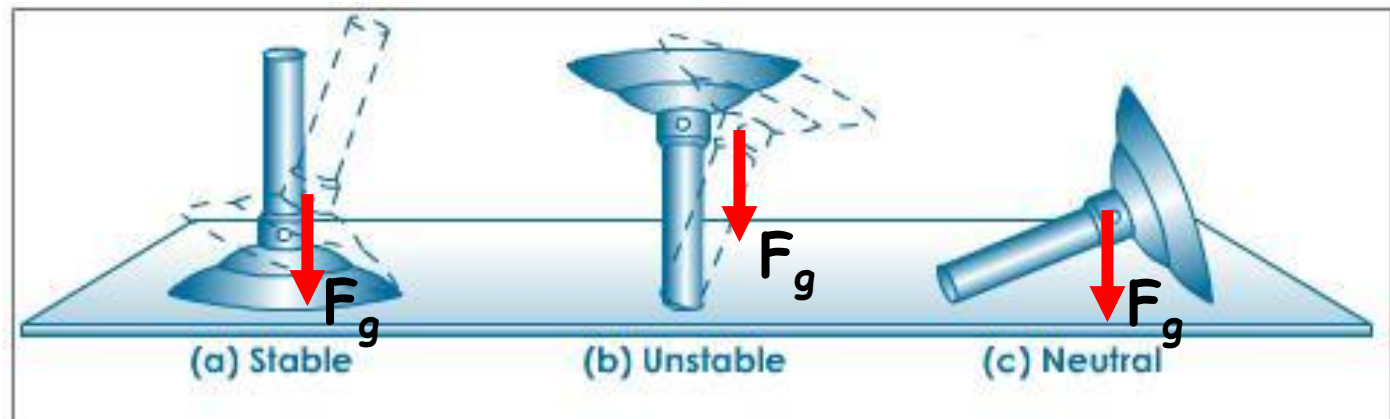
2. Its angular momentum  $\vec{L}$  about its COM, or about any other point, is also a constant.

$$\vec{\tau}_{net} = \frac{d\vec{L}}{dt} = 0$$

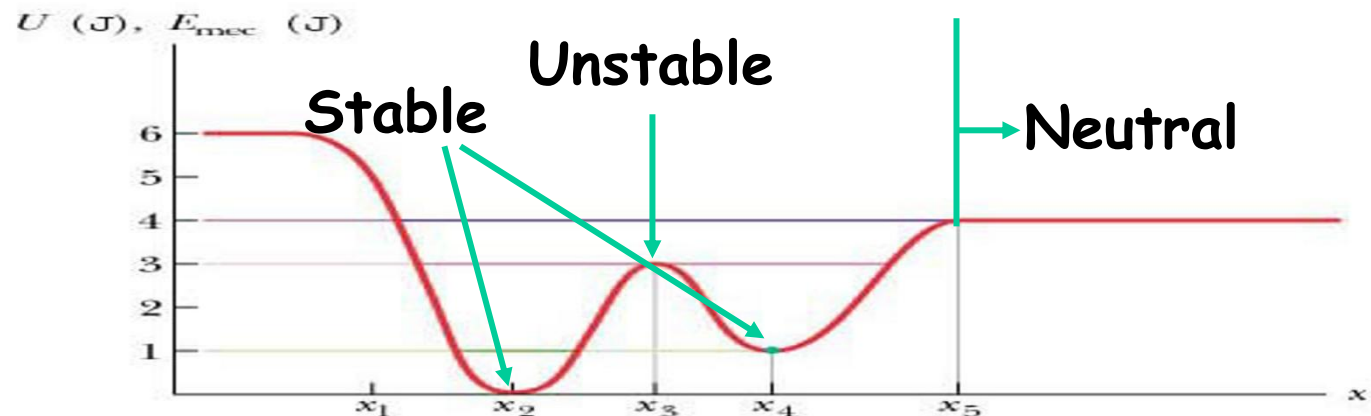
b. **Static equilibrium:** Objects are not moving either in translation or in rotation. In these examples above, the book resting on the table is in static equilibrium.

A body may be in one of three states of static equilibrium: neutral, stable, and unstable:

- **Stable Equilibrium**: A body is in stable equilibrium if it returns to its equilibrium position after it has been displaced *slightly*.
- **Unstable Equilibrium**: A body is in unstable equilibrium if it does not return to its equilibrium position and does not remain in the displaced position after it has been displaced slightly.
- **Neutral Equilibrium**: A body is in neutral equilibrium if it stays in the displaced position after it has been displaced slightly.



Equilibrium of a Bunsen burner



Potential Energy Curve

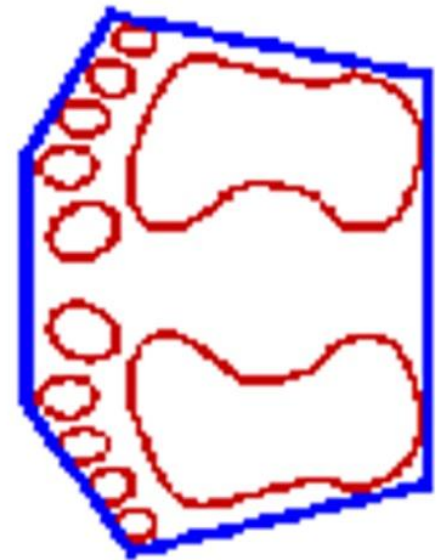


## 6.2. The Center of Gravity and Conditions for Stable Equilibrium

- **Center of Gravity (COG):** The point of a body at which the gravitational force can be considered to act and which undergoes no internal motion.
- If the gravitational acceleration  $g$  is the same for all elements of a body, then the body's COG is coincident with its COM.
- Recall Center of Mass (COM): 
$$\vec{r}_{com} = \frac{1}{M} \sum_{i=1}^n m_i \vec{r}_i$$
- **Base of Support:** It is the area formed by a perimeter around the supporting parts of an object in balance on a surface.

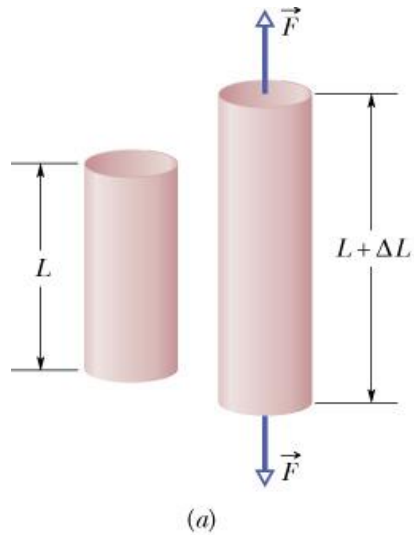
### • Conditions for Stable Equilibrium:

- The body should have a broad base of support.
- Center of gravity of the body should be as low as possible.
- Vertical line drawn from the center of gravity should fall within the base of support.

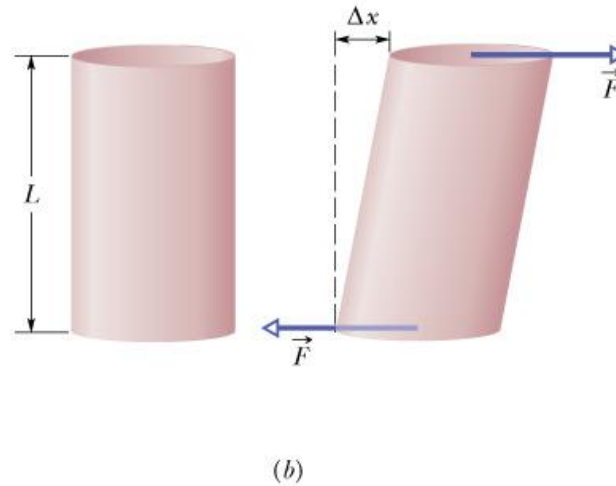


## 6.3. Elasticity

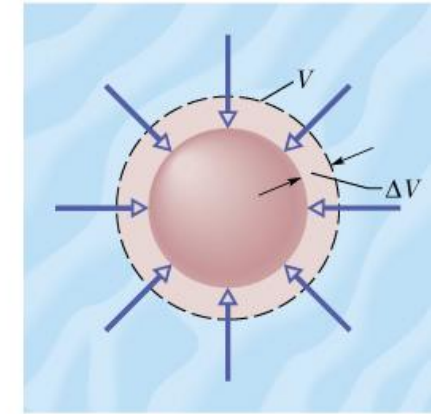
It is the physical property of a material that returns to its original shape once deforming forces (stress) are removed.



**Tensile stress**



**Shearing stress**



**Hydraulic stress**

**Read text (p. 315-318)**

# Chapter 7 Gravitation

7.1. Newton's Law of Gravitation

7.2. Kepler's Laws

7.3. Gravitational Lensing Effect

Formation of the solar system

Asteroid Hit in Russia on Feb 15 2013

## 7.1. Newton's Law of Gravitation

$$F = G \frac{m_1 m_2}{r^2}$$

$G = 6.67 \times 10^{-11} \text{ N m}^2/\text{kg}^2$ : the gravitational constant

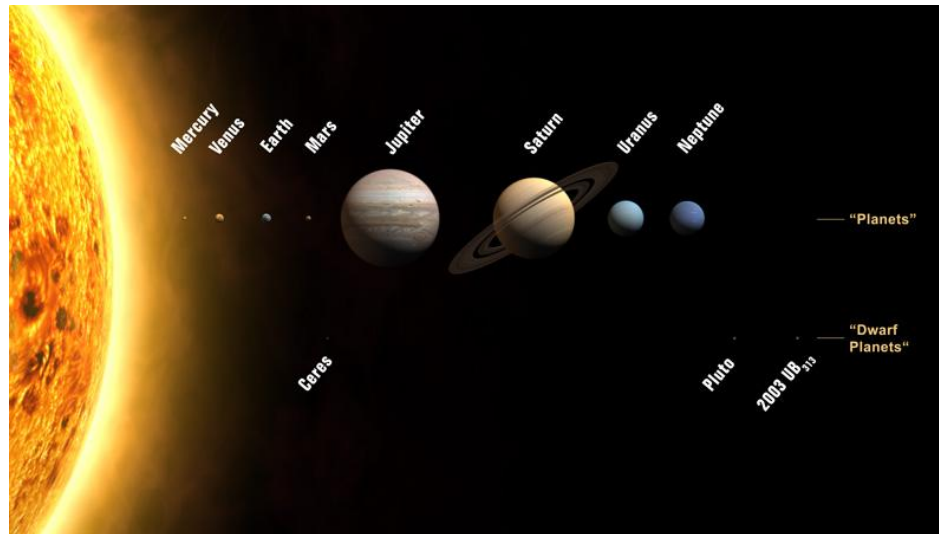
• Gravitational Potential Energy:

$$U = -\frac{GMm}{r}$$

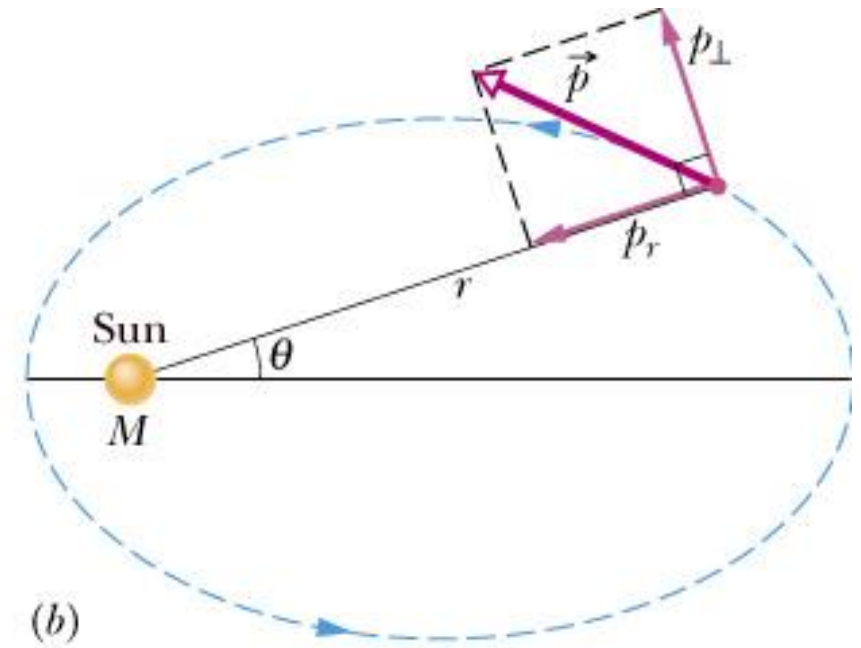
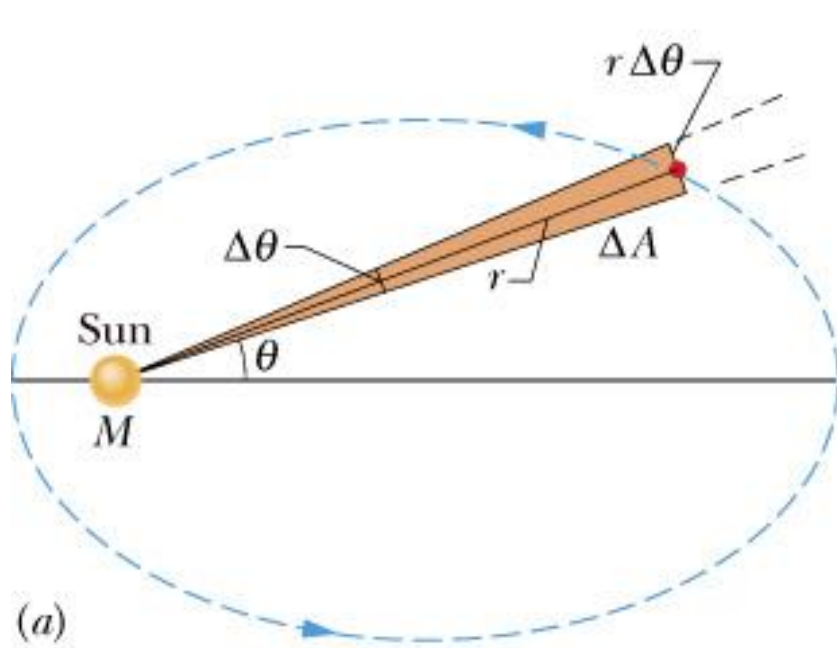
$U = 0$  for  $r = \infty$

## 7.2. Kepler's Laws

1. The law of orbits: All planets move in elliptical orbits, with the Sun at one focus.



**2. The law of areas:** A line that connects a planet to the Sun sweeps out equal areas in the plane of the planet's orbit in equal time intervals; that is, the rate  $dA/dt$  at which it sweeps out area  $A$  is constant.

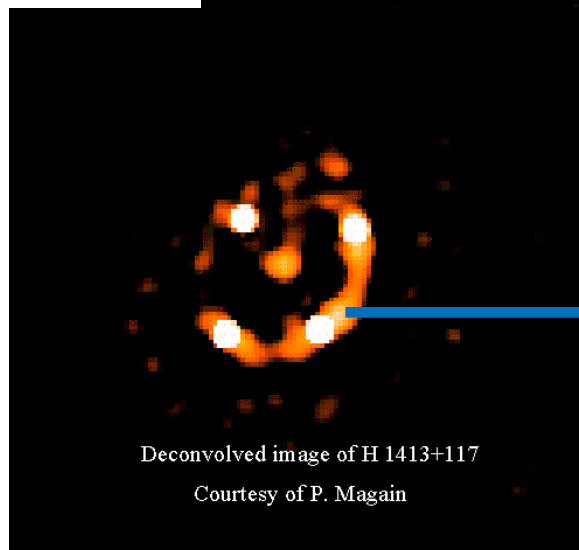
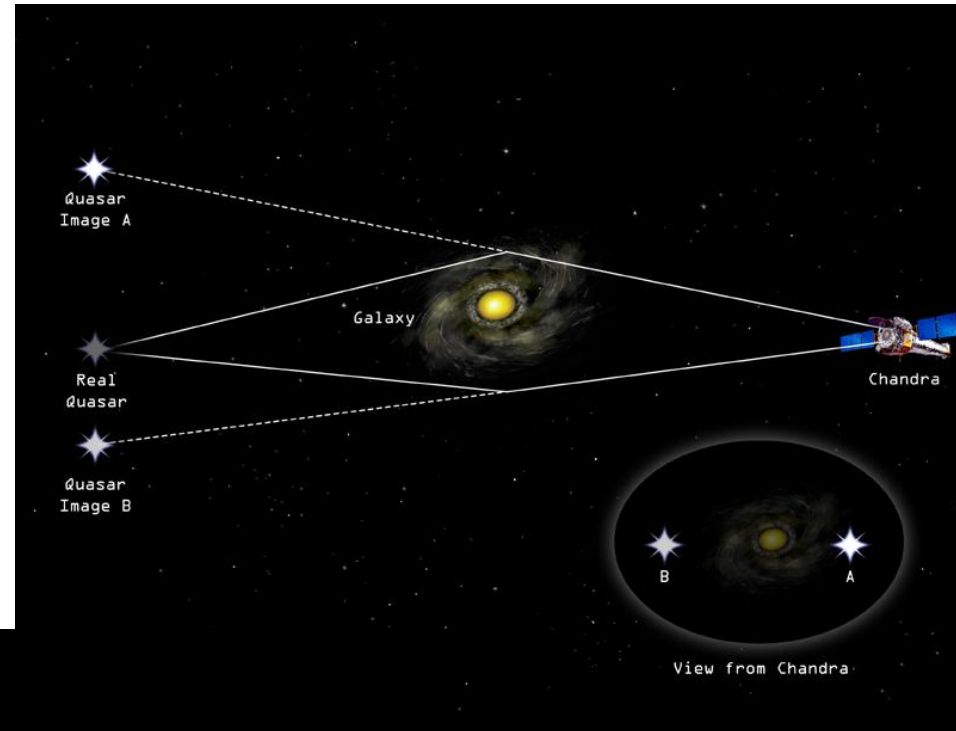
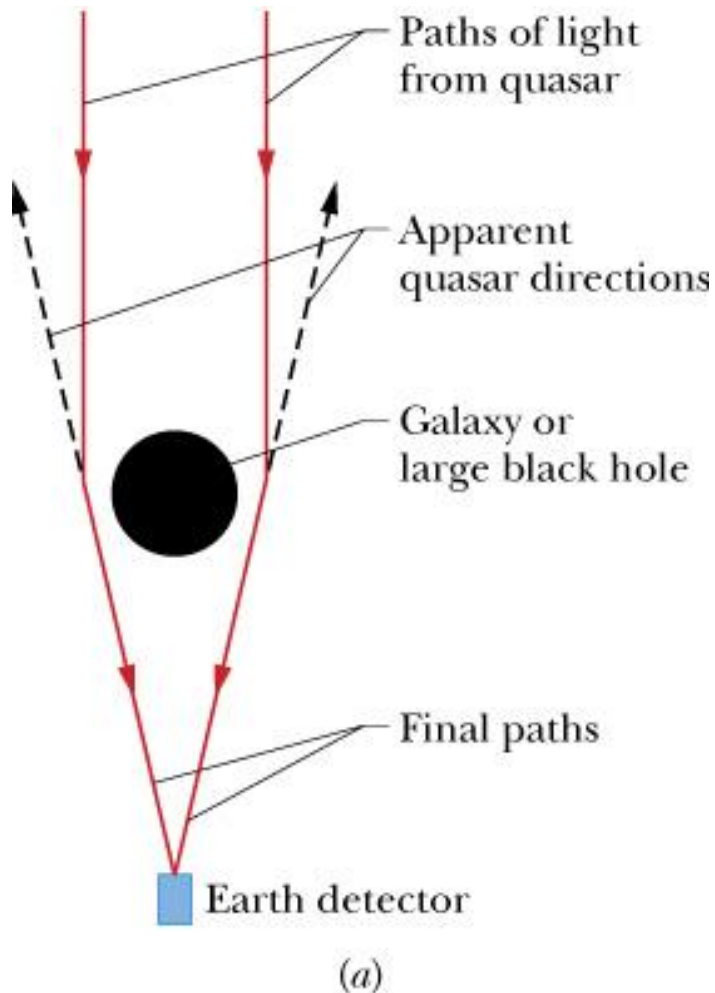


**3. The law of periods:** The square of the period of any planet is proportional to the cube of the semimajor axis of its orbit.

$$T^2 = \left( \frac{4\pi^2}{GM} \right) r^3$$

## 7.3. Gravitational Lensing Effect

According to Einstein's general theory of relativity, when light passes near a massive object (e.g., Earth), the path of the light bends slightly because of the curvature of space there. This effect is called Gravitational Lensing.



→ Einstein's ring

# Review

• Work and Kinetic Energy:

$$\Delta K = K_f - K_i = W$$

$$W = \Delta K$$

• Conservative and Non-conservative Forces

• Mechanical Energy:

$$E_{mec} = K + U$$

$$U(y) = mgy \quad (\text{gravitational potential energy})$$

$$U = \frac{1}{2} kx^2 \quad (\text{elastic potential energy})$$

For an isolated system:

$$K_1 + U_1 = K_2 + U_2$$

$$\Delta E_{mec} = 0$$

• Center of mass:

$$\vec{r}_{com} = \frac{1}{M} \sum_{i=1}^n m_i \vec{r}_i$$

• Collision and Impulse:

$$\Delta \vec{p} = \vec{J} \quad (\text{linear momentum - impulse theorem})$$

$$J = \Delta p = m \Delta v$$

$$J = F_{avg} \Delta t$$



• **Conservation of Linear Momentum:**

For an isolated system:  $\vec{P} = \text{constant}$

$$p = p'$$

• **Inelastic Collisions:**

$$\vec{p}_{1i} + \vec{p}_{2i} = \vec{p}_{1f} + \vec{p}_{2f}$$

$$KE \neq \text{constant}$$

• **Elastic Collisions:**

$$\vec{p}_{1i} + \vec{p}_{2i} = \vec{p}_{1f} + \vec{p}_{2f}$$

$$K_{1i} + K_{2i} = K_{1f} + K_{2f}$$

• **Rotation:**

$$\omega = \omega_0 + \alpha t$$

$$\theta = \theta_0 + \omega_0 t + \frac{1}{2} \alpha t^2$$

$$\omega^2 - \omega_0^2 = 2\alpha(\theta - \theta_0)$$

$$v = \omega r$$

$$a_t = \alpha r$$

$$a_r = \frac{v^2}{r} = \omega^2 r$$

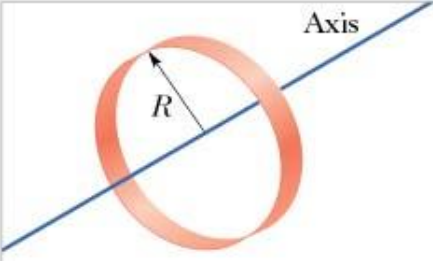
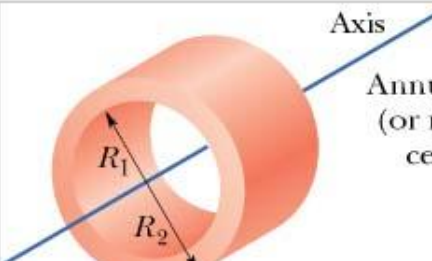
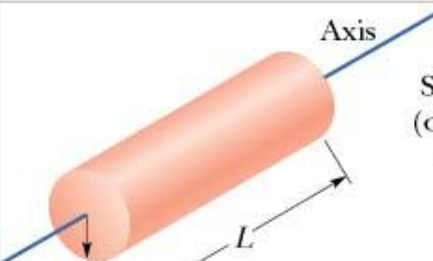
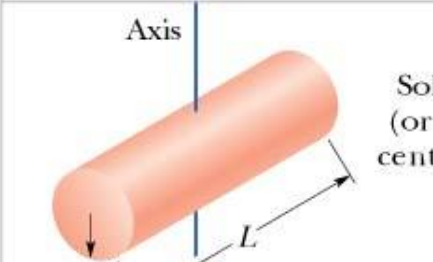
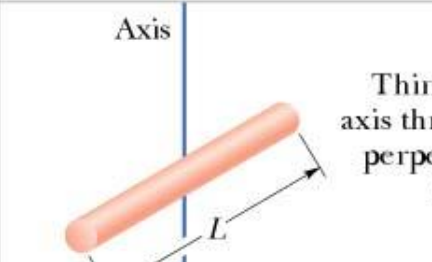
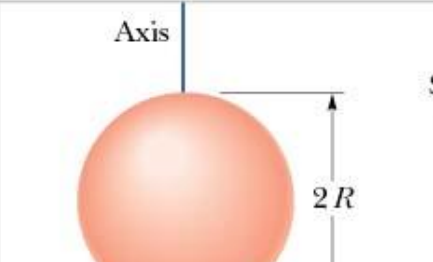
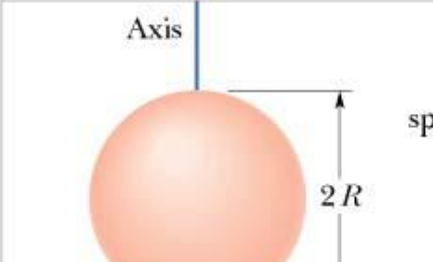
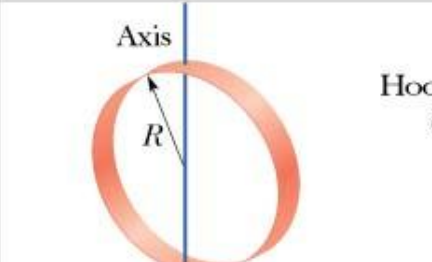
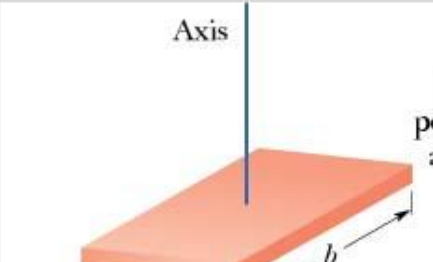
$$v \sim \omega$$

$$a \sim \alpha$$

$$s \sim \theta$$

# • Rotational KE:

$$K = \frac{1}{2} I \omega^2$$

 <p>Hoop about central axis</p> <p><math>I = MR^2</math></p> <p>(a)</p>	 <p>Annular cylinder (or ring) about central axis</p> <p><math>I = \frac{1}{2} M(R_1^2 + R_2^2)</math></p> <p>(b)</p>	 <p>Solid cylinder (or disk) about central axis</p> <p><math>I = \frac{1}{2} MR^2</math></p> <p>(c)</p>
 <p>Solid cylinder (or disk) about central diameter</p> <p><math>I = \frac{1}{4} MR^2 + \frac{1}{12} ML^2</math></p> <p>(d)</p>	 <p>Thin rod about axis through center perpendicular to length</p> <p><math>I = \frac{1}{12} ML^2</math></p> <p>(e)</p>	 <p>Solid sphere about any diameter</p> <p><math>I = \frac{2}{5} MR^2</math></p> <p>(f)</p>
 <p>Thin spherical shell about any diameter</p> <p><math>I = \frac{2}{3} MR^2</math></p> <p>(g)</p>	 <p>Hoop about any diameter</p> <p><math>I = \frac{1}{2} MR^2</math></p> <p>(h)</p>	 <p>Slab about perpendicular axis through center</p> <p><math>I = \frac{1}{12} M(a^2 + b^2)</math></p> <p>(i)</p>

• Torque:

$$\vec{\tau} = \vec{r} \times \vec{F}$$

• Newton's second law for rotation:

$$\tau_{\text{net}} = I\alpha \quad (I: \text{kg.m}^2)$$

• For a point mass:

$$I = mr^2$$

• For a rigid body: I depends on the object shape

• Work - (rotational) KE Theorem:

$$\Delta K = K_f - K_i = \frac{1}{2} I \omega_f^2 - \frac{1}{2} I \omega_i^2 = W$$

$$= \Delta K$$

Counterclockwise  
 $\Rightarrow +$  and sin

• Rolling motion:

$$v_{\text{com}} = \omega R$$

$$a_{\text{com}} = \alpha R$$

$$K = \frac{1}{2} I_{\text{com}} \omega^2 + \frac{1}{2} M v_{\text{com}}^2$$

Rotational KE

Translational KE

• Angular Momentum:

$$\vec{l} = \vec{r} \times \vec{p}$$

$$l = rmv \sin \phi$$

• Newton's Second Law in Angular Form:

$$\vec{\tau}_{net} = \frac{d\vec{L}}{dt}$$

• Angular Momentum of a Rigid Body:

$$L = I\omega$$

• Conservation of Angular Momentum:

If no net external torque acts on the system:

$$\vec{L} = \text{constant}$$

$$\vec{L}_i = \vec{L}_f$$

$$I_i\omega_i = I_f\omega_f$$

$$L = L'$$

## More Corresponding Variables and Relations for Translational and Rotational

Translational		Rotational	
Force	$\vec{F}$	Torque	$\vec{\tau} = \vec{r} \times \vec{F}$
Translational momentum	$\vec{p}_{\text{sys}}$	Rotational momentum	$\vec{\ell} = \vec{r} \times \vec{p}$
Translational momentum <sup>a</sup>	$\vec{p}_{\text{sys}} = \sum \vec{p}_i$	Rotational momentum <sup>a</sup>	$\vec{L} = \sum \vec{\ell}_i$
Translational momentum <sup>a</sup>	$\vec{p}_{\text{sys}} = M\vec{v}_{\text{com}}$	Rotational momentum <sup>b</sup>	$\vec{L} = I\vec{\omega}$
Newton's Second Law <sup>a</sup>	$\sum \vec{F}^{\text{ext}} = \frac{d\vec{p}_{\text{sys}}}{dt}$	Newton's Second Law <sup>a</sup>	$\sum \vec{\tau}^{\text{ext}} = \frac{d\vec{L}}{dt}$
Conservation law <sup>c</sup>	$\vec{p}_{\text{sys}} = \text{a constant}$	Conservation law <sup>c</sup>	$\vec{L} = \text{a constant}$

<sup>a</sup>For systems of particles, including rigid bodies.

<sup>b</sup>For a rigid body about a fixed axis, with  $L$  being the component along that axis.

<sup>c</sup>For a closed, isolated system ( $\vec{F}^{\text{net}} = 0$ ,  $\vec{\tau}^{\text{net}} = 0$ ).