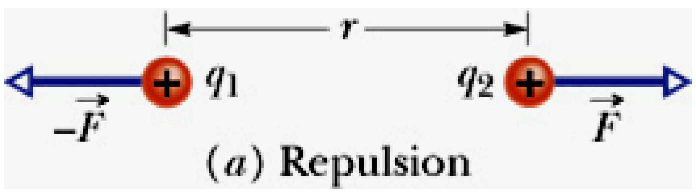
Review

Chapter 1: Electric Fields

Coulomb's Law: $$\boxed{F = k\frac{|q_1||q_2|}{r^2}}$$ (Unit: N)

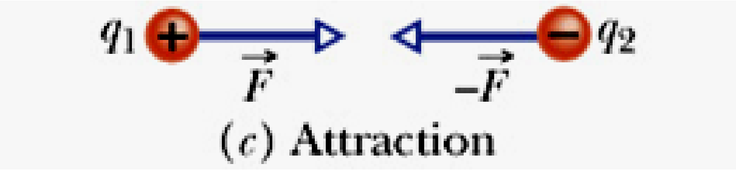
$k = \dfrac{1}{4\pi\varepsilon_0} = 8.99\times10^{9}\ \mathrm{N.m^2/C^2}$: electrostatic constant

$\varepsilon_0 = 8.85\times10^{-12}\ \mathrm{C^2/N/m^2}$: permittivity constant



(a) Repulsion

(b) Repulsion



(c) Attraction

The Principle of Superposition:

$$\boxed{\vec{F}_{1,net} = \vec{F}_{12} + \vec{F}_{13} + \vec{F}_{14} + \ldots + \vec{F}_{1n}}$$

The Electric Field: $$\vec{E} = \frac{\vec{F}}{q_0}$$ $$E = \frac{F}{q_0} = \frac{1}{4\pi\varepsilon_0}\frac{|q|}{r^2}$$

(Unit: N/C)

- The direction of $\vec{E}$:
  - q > 0: directly away from the charge
  - q < 0: toward the charge

The Principle of Superposition: $$\vec{E} = \frac{\vec{F}_0}{q_0} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 + \ldots + \vec{E}_n$$

The electric dipole moment $\vec{p}$ of the dipole:

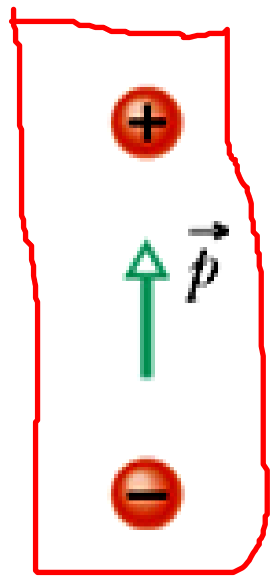
- Magnitude: $$p = q\overset{m}{d}$$

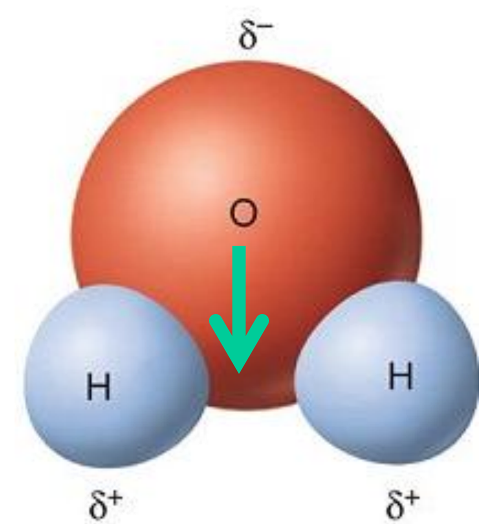
$\rightarrow (x\hat{i} + y\hat{j})$

$\rightarrow 1.6 \times 10^{-19}\,C$

(Unit: C.m)

- Direction: from the negative to the positive





Electric Field of a Continuous Charge Distribution:

The principle to calculate E:

✓ Find an expression for dq:

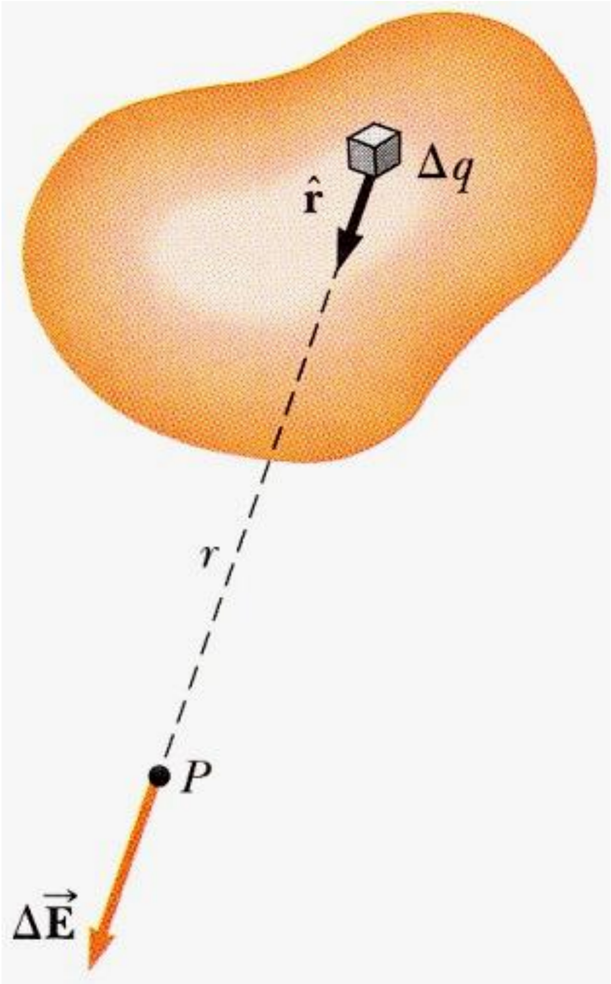
- dq = λdl for a line distribution
- dq = σdA for a surface distribution
- dq = ρdV for a volume distribution

✓ Calculate dE:

$$d\vec{E} = \frac{dq}{4\pi\varepsilon_0 r^2}\hat{r}$$

✓ Add up (integrate the contributions) over the whole distribution, varying the displacement as needed:

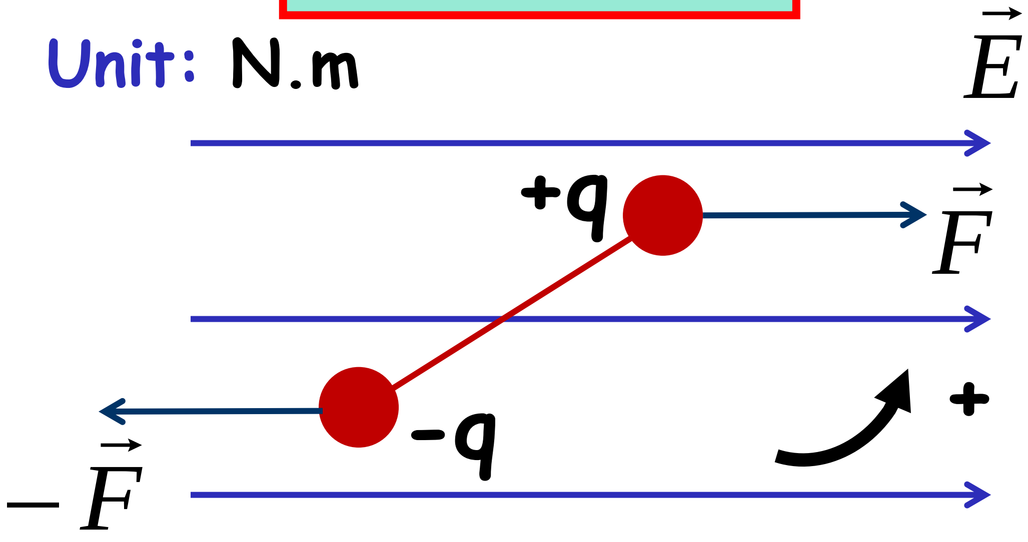
$$\vec{E} = \int d\vec{E}$$



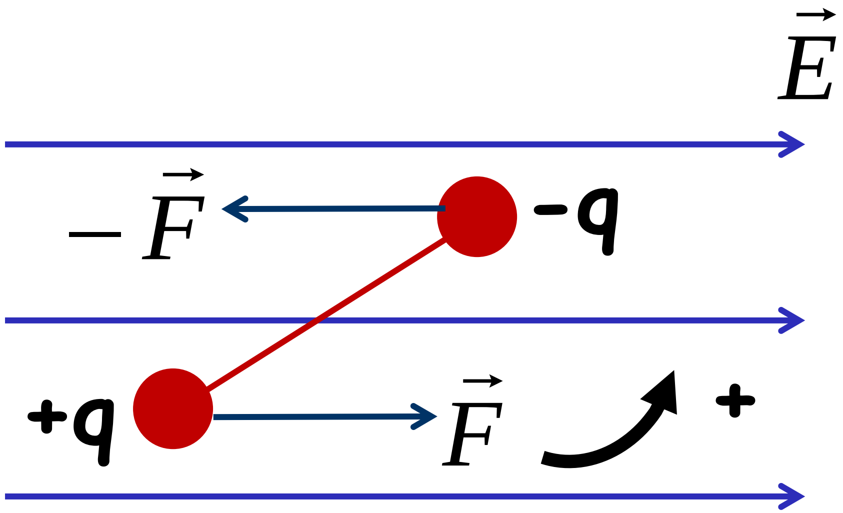
A Dipole in an Electric Field:

$$\boxed{\begin{aligned}\vec{\tau} &= \vec{p}\times\vec{E}\\ \tau &= pE\sin\theta\end{aligned}}$$

Unit: N.m



$$\tau = -pE\sin\theta$$



$$\tau = +pE\sin\theta$$

Potential Energy of an Electric Dipole:

$$\Delta U = -W$$ (W : work done by the electric field)

$$U = -\vec{p}\vec{E}$$

$$U = -pE\cos\theta$$

- Choose U = 0 at $\theta = 90^0$, then calculate U at $\theta \neq 90^0$

- Work done by the field from θ_i to θ_f:

$$W = -(U_{\theta_f} - U_{\theta_i})$$

- Work done by the applied torque (of the applied force):

$$W_a = -W = U_{\theta_f} - U_{\theta_i}$$

Electric flux:

$$\Phi = \oint \vec{E}\, d\vec{A}$$

Unit: $N.m^2/C$

Gauss' Law:

$$\varepsilon_0 \Phi = q_{enc}$$

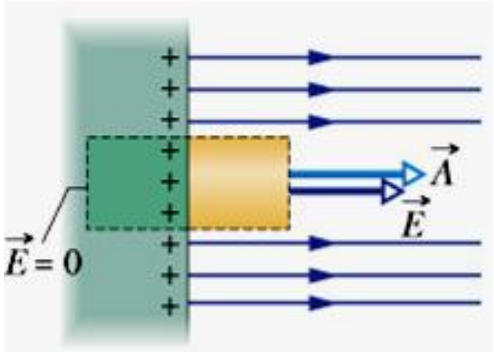
q_{enc} : the net charge enclosed in the surface

or

$$\varepsilon_0 \oint \vec{E}\, d\vec{A} = q_{enc}$$

$q_{enc} > 0$: the net flux is outward

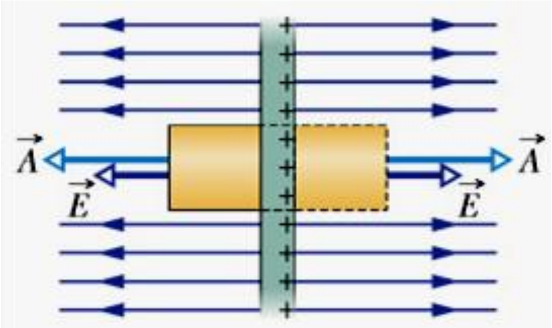
$q_{enc} < 0$: the net flux is inward



- **Electric field due to a charged conductor:**

$$\boxed{E = \frac{\sigma}{\varepsilon_0}}$$

- **Electric field due to non-conducting sheet:**



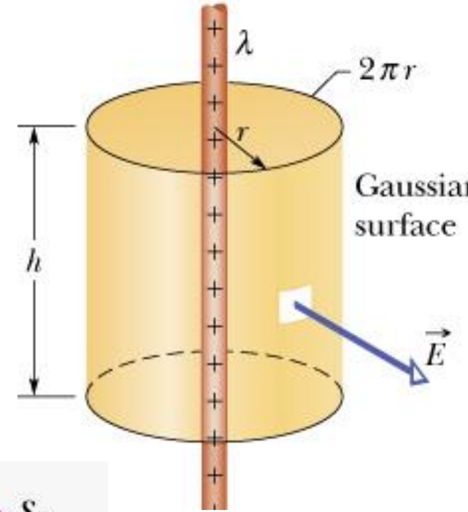
$$\boxed{E = \frac{\sigma}{2\varepsilon_0}}$$

σ : surface charge density (C/m^2)

- Electric field due to a very long, uniformly charged, cylindrical plastic rod :

$$\boxed{E = \frac{\lambda}{2\pi\varepsilon_0 r}}$$

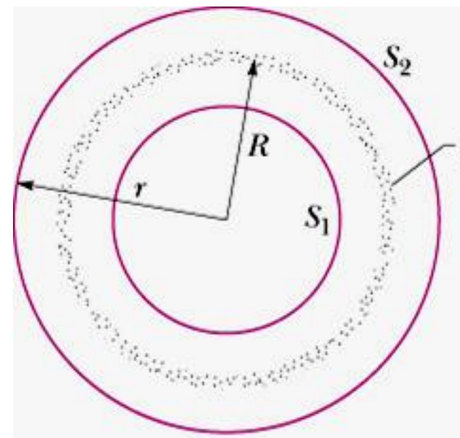
λ : linear charge density (C/m)



- A thin, uniformly charged spherical shell:

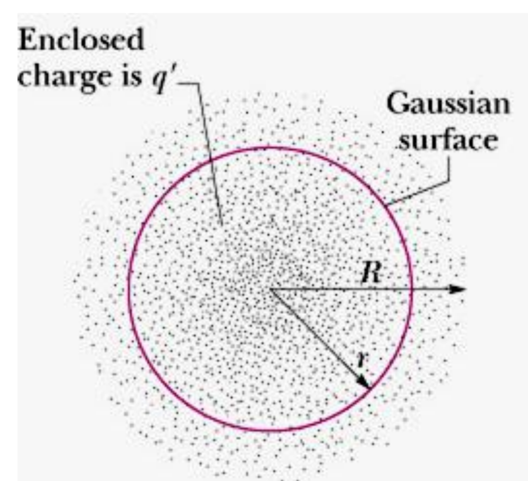
$$\boxed{E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2} \quad (r \geq R)}$$

$$E = 0 \quad (r < R)$$



- A uniformly charged sphere:

$$\boxed{E = \left(\frac{q}{4\pi\varepsilon_0 R^3}\right) r \quad (r \leq R)}$$



The electric field at a distance r > R: the charge sphere acts like a point charge at the center

Chapter 2: Electric Energy and Capacitance

- **Electric Potential and Electric Potential Difference:**

$$\boxed{V = \frac{U}{q} \quad (\text{unit: J/C, V})}$$

$$\boxed{\Delta V = V_f - V_i = \frac{\Delta U}{q} = -\frac{W}{q}}$$

W: work done by the electric force

- **Calculating the Electric Potential Difference between 2 Points i and f from the Electric Field:**

$$\boxed{V_f - V_i = -\int_i^f \vec{E}.d\vec{s}}$$

- **Potential difference in a uniform electric field:**

$$\boxed{V_f - V_i = -Ed}$$

- **Potential due to a point charge:**

$$\boxed{V = k\frac{q}{r}}$$

- Potential due to a group of point charges:

$$\boxed{V = \sum_{i=1}^{n} V_i = k \sum_{i=1}^{n} \frac{q_i}{r_i}}$$

(an algebraic sum, not a vector sum)

- Calculating the Electric Field from the Potential:

$$\boxed{\vec{E} = -\nabla V}$$

$$\nabla = \frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}$$

- Electric Potential Energy of a System of Point Charges:

  - Two charges:

$$\boxed{W_{\text{applied}} = U_{\text{system}} = q_2 V = k\frac{q_1 q_2}{r}}$$

  - Three charges:

$$\boxed{U = U_{12} + U_{13} + U_{23} = k\left(\frac{q_1 q_2}{r_{12}} + \frac{q_1 q_3}{r_{13}} + \frac{q_2 q_3}{r_{23}}\right)}$$

- Electric Potential due to Continuous Charge Distributions:

See the principle to calculate the electric field due to a continuous charge distribution

$$V = \int dV = k\int \frac{dq}{r}$$

• Capacitance. Capacitors in Parallel and in Series:

$$\boxed{q = CV}$$

C: Capacitance of the capacitor
unit: F

• A Parallel-Plate Capacitor: $\boxed{C = \dfrac{\varepsilon_0 A}{d}}$

• Capacitors in Parallel: $\boxed{C_{eq} = \sum_{i=1}^{n} C_i}$

• Capacitors in Series: $\boxed{\dfrac{1}{C_{eq}} = \sum_{i=1}^{n} \dfrac{1}{C_i}}$

potential E

• Energy Stored in a Charged Capacitor: $\boxed{U = \dfrac{q^2}{2C} = \dfrac{1}{2} CV^2}$

• Energy density: $\boxed{u = \dfrac{1}{2} \varepsilon_0 E^2}$

- Capacitor with a Dielectric: $\boxed{C' = \kappa C_{air}}$

- Dielectrics and Gauss' Law:

$$\boxed{\oint \vec{D} d\vec{A} = q; \vec{D} = \varepsilon_0 \kappa \vec{E} : \text{electric displacement}}$$

Chapter 3: Current and Resistance. Direct Current Circuits

- Electric Current: $\boxed{i = \dfrac{dq}{dt}}$ (Unit: A)

- Current Density: $\boxed{J = \dfrac{i}{A}}$

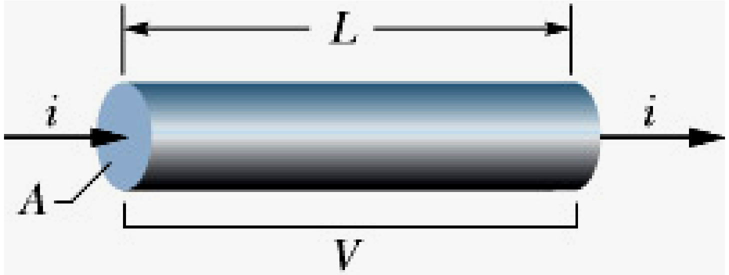
- Drift Speed: $\boxed{\vec{J} = (ne)\vec{v}_d}$ ne: charge density (C/m^3)

- Resistance: $\boxed{R = \dfrac{V}{i}}$ (Unit: Ω)

- Resistivity: $\boxed{\rho = \dfrac{E}{J}}$ (Unit: Ωm)

$J = \dfrac{E}{\rho}$

- Conductivity: $$\boxed{\sigma = \frac{1}{\rho}}$$ (Unit: $(\Omega m)^{-1}$)



- Calculating Resistance from Resistivity: $$\boxed{R = \rho \frac{L}{A}}$$

- Ohm's Law: $$\boxed{i = \frac{V}{R}}$$ or $$\boxed{\vec{E} = \rho \vec{J}}$$

- Power in Electric Circuits: $$\boxed{P = iV = i^2 R = \frac{V^2}{R}}$$ (Unit: W)

- Emf: $$\boxed{\mathcal{E} = \frac{dW}{dq}}$$ (Unit: V)

- Power of an emf device: $$\boxed{P = i\mathcal{E}}$$ (Unit: W)

• Kirchhoff's Rules:

• Loop Rule (Voltage Law):

$$\sum_{i=1}^{n} \varepsilon_i + \sum_{j=1}^{m} i_j R_j = 0$$

Important Notes:

• For a move through a resistance in the direction of the current, the change in potential is –iR; in the opposite direction it is +iR (resistance rule)

• For a move through an ideal emf device in the direction of the emf arrow, the change in potential is +ε; in the opposite direction it is -ε (emf rule)

• Junction Rule (Current Law):

$$\sum i_{\text{entering}} = \sum i_{\text{leaving}}$$

- Resistors:
  - Resistors in Series:

$$R_{eq} = \sum_{j=1}^{n} R_j$$

  - Resistors in Parallel:

$$\frac{1}{R_{eq}} = \sum_{j=1}^{n} \frac{1}{R_j}$$

- The relationship between Power and Potential:
  - The net rate P of energy transfer from the emf device to the charge carriers:

$$P = iV$$

  - The dissipation rate of energy due to the internal resistance r of the emf device:

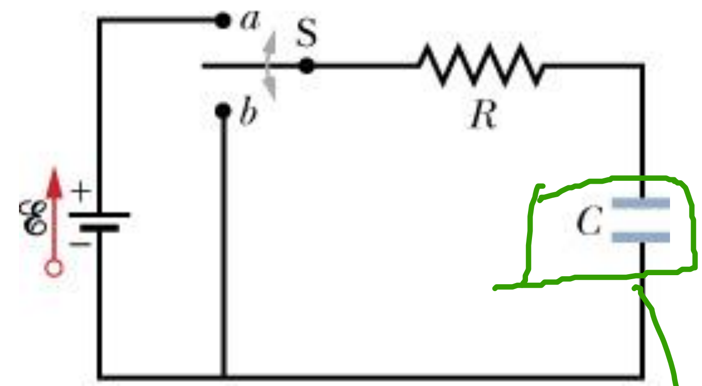
$$P_r = i^2 r$$

  - The power of the emf device:

$$P_{emf} = i\varepsilon$$

- **RC Circuits:**

  - **Charging a Capacitor:**



$q = CV$

$$q = C\varepsilon(1 - e^{-t/RC})$$

$q = it$

$$i = \frac{dq}{dt} = \left(\frac{\varepsilon}{R}\right)e^{-t/RC}$$

i_0

$$V_C = \frac{q}{C} = \varepsilon(1 - e^{-t/RC})$$

V_0

The time constant: $\tau = RC$ (Unit: s)

  - **Discharging a Capacitor:**

$$q = \boxed{q_0}e^{-t/RC}$$

$$i = \frac{dq}{dt} = -\left(\frac{q_0}{RC}\right)e^{-t/RC}$$

$$V_C = \boxed{V_0}e^{-t/RC}$$

$q = CV\ (c)$

$= it\ (c)$