SS 2019/11

Q1.

a)

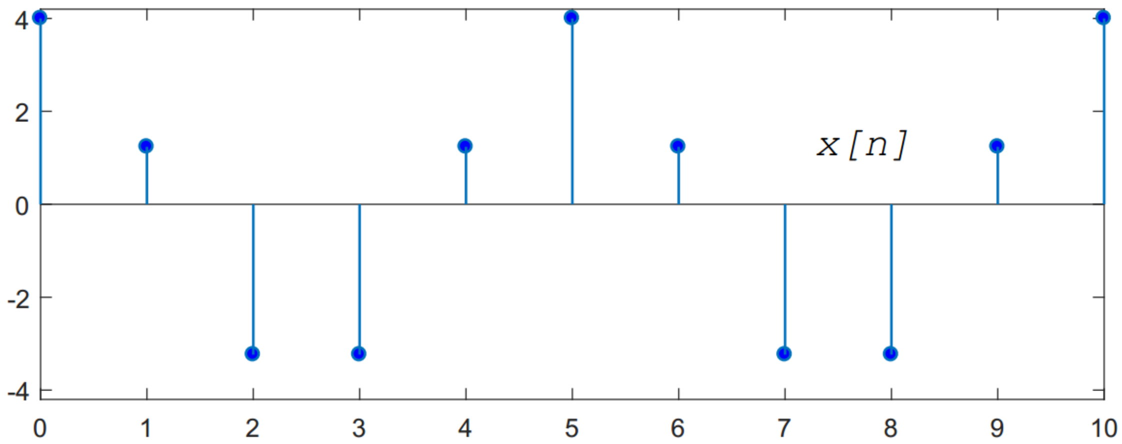
$$E_x = \int_{-\infty}^{+\infty} |x(t)|^2 dt = \int_{-1}^{1} |1|^2 dt + \int_{1}^{2} |-1|^2 dt = 3$$

(Reader sketches the signal by yourself)

b)

Sampling time: $T_s = 1/f_s = 1/20 = 0.05$ (s)

Sampled signal: $x[n] = x(nT_s) = 4\cos(8\pi \times 0.05n) = 4\cos(0.4\pi n)$



$$P_x = \frac{1}{N}\sum_{n=0}^{N-1} |x[n]|^2 = \frac{1}{11}\sum_{n=0}^{10} |4\cos(0.4\pi n)|^2 = \frac{96}{11}$$

Q2.

Given that: $y(t) = 2x(t) + x(2t)$

a) Check for linearity:

Let: $\begin{cases} x_1 \xrightarrow{s} y_1 = 2x_1(t) + x_1(2t) \\ x_2 \xrightarrow{s} y_2 = 2x_2(t) + x_2(2t) \end{cases}$

$\rightarrow a_1y_1 + a_2y_2 = a_1\big(2x_1(t) + x_1(2t)\big) + a_2\big(2x_2(t) + x_2(2t)\big)$ (1)

Let: $x = a_1x_1 + a_2x_2 \xrightarrow{s} y$

$\rightarrow y = 2\big(a_1x_t(t) + a_2x_2(t)\big) + a_1x_1(2t) + a_2x_2(t)$ (2)

From (1) and (2), $a_1y_1 + a_2y_2 = \boldsymbol{S}\{a_1x_1 + a_2x_2\}$, the system is linear.

a) Check for time invariant:

Let: $x(t) \xrightarrow{s} y = 2x(t) + x(2t)$

$\rightarrow y(t - T) = 2x(t - T) + x(2t - 2T)$ (1) (delay the ouput).

Let: $x_T(t) = x(t - T) \xrightarrow{s} y_T$

$\rightarrow y_T = 2x_T(t) + x_T(2t) = 2x(t - T) + x(2t - T)$(2)

Since, (1) $\neq$ (2), therefore, the system is time variant.

b)

Assume that $|x(t)| \leq M, M$ is finite for all t.

We have: $|y(t)| = |2x(t) + x(2t)| \leq |2x(t)| + |x(2t)| \leq 2M + M = 3M$

$\rightarrow |y(t)| \leq 3M$

Therefore, with bounded input, the output will be bounded, which leads to the system is BIBO system.

Q3.

a) Given that: $y[n] = 2x[n] - x[n-1] + x[n-2]$

$$y[n] = x[n] * h[n] = \sum_{k=-\infty}^{+\infty} x[n-k]h[k] = \sum_{k=0}^{2} x[n-k]h[k]$$
$$= x[n]h[0] + x[n-1]h[1] + x[n-2]h[2] \ (1)$$
$$= 2x[n] - x[n-1] + x[n-2] \ (2)$$

Compare (1) and (2), we obtain: $h[n] = [2, -1,\ 1]$

Given that: $x[n] = [1, 0, 2, -1, 3]$

$+\ y[0] = 2x[0] - x[-1] + x[-2] = 2 \times 1 - 0 + 0 = 2$

$+\ y[1] = 2x[1] - x[0] + x[-1] = 2 \times 0 - 1 + 0 = -1$

$+\ y[2] = 2x[2] - x[1] + x[0] = 2 \times 2 - 0 + 1 = 5$

$+\ y[3] = 2x[3] - x[2] + x[1] = 2 \times (-1) - 2 + 0 = -4$

$+\ y[4] = 2x[4] - x[3] + x[2] = 2 \times 3 - (-1) + 2 = 9$

$+\ y[5] = 2x[5] - x[4] + x[3] = 2 \times 0 - 3 + (-1) = -4$

$+\ y[6] = 2x[6] - x[5] + x[4] = 2 \times 0 - 0 + 3 = 3$

Therefore, $y[n] = [2, -1, 5, -4, 9, -4, 3]$

b)

From the given $x(n)$ and $h(n)$, we have the following convolution table:

	$h_0 = 1$	$h_1 = 1$	$h_2 = 1$	$h_3 = 1$
$x_0 = 8$	8	8	8	8
$x_1 = 4$	4	4	4	4
$x_2 = 2$	2	2	2	2
$x_3 = 1$	1	1	1	1

Using this convolution table, we obtain the result:

$+\ y[0] = h_0x_0 = 8$

$+\ y[1] = h_0x_1 + h_1x_0 = 8 + 4 = 12$

$+\ y[2] = h_0x_2 + h_1x_1 + h_2x_0 = 8 + 4 + 2 = 14$

$+\ y[3] = h_0x_3 + h_1x_2 + h_2x_1 + h_3x_0 = 8 + 4 + 2 + 1 = 15$

$+\ y[4] = h_1x_3 + h_2x_2 + h_3x_1 = 4 + 2 + 1 = 7$

$+\ y[5] = h_2x_3 + h_3x_2 = 2 + 1 = 3$

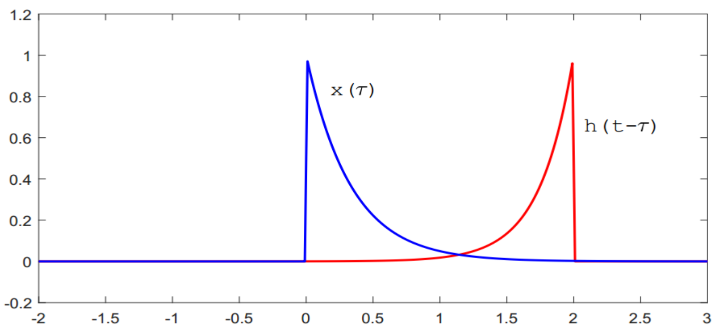
$+\ y[6] = h_3x_3 = 1$

Therefore, $y[n] = [8, 12, 14, 15, 7, 3, 1]$

Q4.

Given that: $h(t) = e^{-4t}u(t)$

a)



For $t < 0$, $x(\tau)$ and $h(t-\tau)$does not overlap $\rightarrow y(t) = 0$.

For $t \geq 0$:

$$y(t) = \int_{-\infty}^{+\infty} x(\tau)h(t-\tau)d\tau = \int_0^t e^{-3\tau}.e^{-4(t-\tau)}d\tau = e^{-4t}\int_0^t e^{\tau}d\tau = e^{-4t}(e^t - 1)$$

Thus,

$$y(t) = \begin{cases} 0 \qquad\qquad\quad , & t < 0 \\ e^{-4t}(e^t - 1), & t \geq 0 \end{cases}$$

b)

With $x_1(t) = e^{-3t}u(t) \rightarrow y_1(t) = e^{-4t}(e^t - 1)u(t)$

For $x(t) = e^{-3t}u(t-1) = e^{-3}e^{-3(t-1)}u(t-1) = e^{-3}x_1(t-1)$

By properties of LTI system, the output $y(t)$ is given by:

$$y(t) = e^{-3}y_1(t-1) = e^{-3}e^{-4(t-1)}(e^{t-1} - 1)u(t-1)$$

Q5.

Given that: $$\frac{d^2y}{dt^2} + 10\frac{dy}{dt} + 24y = 6x \ (*)$$

a)

From $(*)$, we obtain homogeneous equation:

$$\frac{d^2y}{dt^2} + 10\frac{dy}{dt} + 24y = 0$$

$$\rightarrow D^2y + 10Dy + 24y = 0$$

$$\leftrightarrow y(D^2 + 10D + 24) = 0$$

Therefore, the characteristic polynomial: $P(D) = D^2 + 10D + 24$

Let: $P(D) = 0 \leftrightarrow D^2 + 10D + 24 = 0 \leftrightarrow D = -4 \vee D = -6$

Thus, the natural response is:

$$y_h = C_1e^{-4t} + C_2e^{-6t}, \qquad C_1, C_2 \text{ are arbitrary constants}$$

b)

For $t > 0$, $(*)$ equivalent with:

$$\frac{d^2y}{dt^2} + 10\frac{dy}{dt} + 24y = 6 \times 8$$

Assume that the forced response has the form: $y_p = K \rightarrow y_p' = 0 \rightarrow y_p'' = 0$.

Substitute into the above equation:

$$0 + 10 \times 0 + 24K = 48 \leftrightarrow K = 2$$

Therefore, the total response is:$y(t) = y_h + y_p = C_1e^{-4t} + C_2e^{-6t} + 2$

With the initial conditions:

$$\begin{cases} y(0) = 5 \\ y'(0) = 0 \end{cases} \rightarrow \begin{cases} C_1 + C_2 + 2 = 5 \\ -4C_1 - 6C_2 = 0 \end{cases} \leftrightarrow \begin{cases} C_1 = 9 \\ C_2 = -6 \end{cases}$$

Thus, the total response the system is:

$$y(t) = 9e^{-4t} - 6e^{-6t} + 2$$