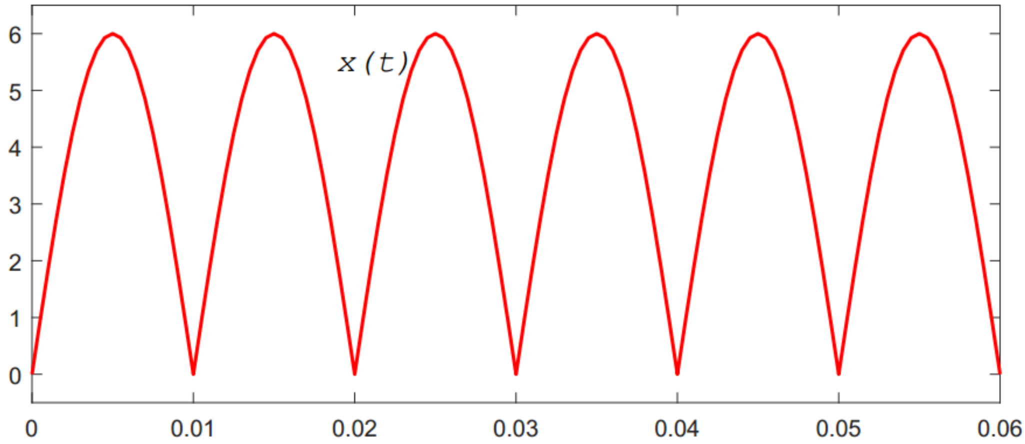
Q1.

a)



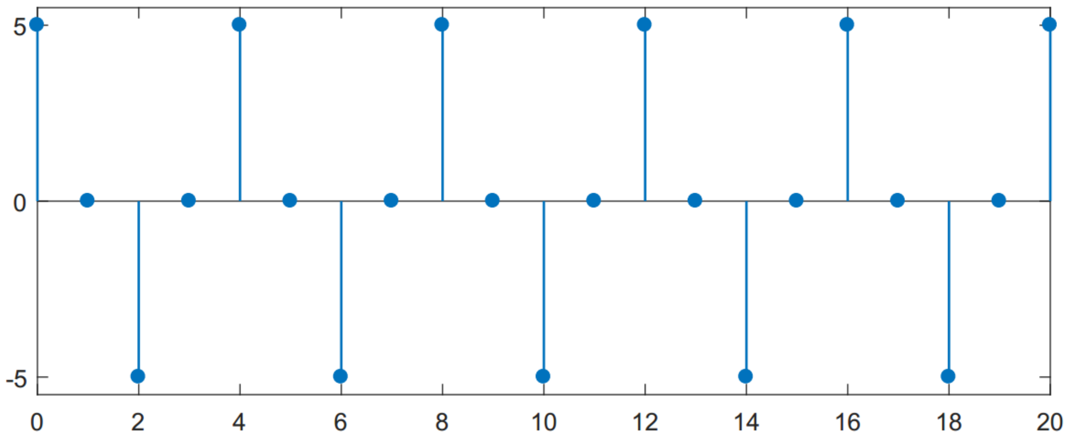
From the figure: $T_0 = 0.01$ (s).

$$P_x = \frac{1}{T_0}\int_{t_0}^{t_0+T_0} |x(t)|^2 dt = \frac{1}{0.01}\int_0^{0.01} |6\sin 100\pi t|^2 dt = 18$$

b)

Sampling time: $T_s = 1/f_s = 1/20 = 0.05$ (s)

Sampled signal: $x[n] = x(nT_s) = 5\cos(10\pi \times 0.05n) = 5\cos(0.5\pi n)$



$$P_x = \frac{1}{N}\sum_{n=0}^{N-1} |x[n]|^2 = \frac{1}{21}\sum_{n=0}^{20} |5\cos(0.5\pi n)|^2 = \frac{275}{21}$$

Q2.

Given that: $y(t) = x(t-2) + x(2-t)$

a)

1. Check for linearity:

$$\text{Let: } \begin{cases} x_1 \xrightarrow{S} y_1 = x_1(t-2) + x_1(2-t) \\ x_2 \xrightarrow{S} y_2 = x_2(t-2) + x_2(2-t) \end{cases}$$

$$\rightarrow a_1y_1 + a_2y_2 = a_1\big(x_1(t-2) + x_1(2-t)\big) + a_2\big(x_2(t-2) + x_2(2-t)\big) \quad (1)$$

Let: $x = a_1x_1 + a_2x_2 \xrightarrow{S} y$

$$\rightarrow y = a_1x_1(t-2) + a_2x_2(t-2) + a_1x_1(2-t) + a_2x_2\,(2-t) \quad (2)$$

From (1) and (2), $a_1y_1 + a_2y_2 = \boldsymbol{S}\{a_1x_1 + a_2x_2\}$, the system is linear.

2. Check for time invariant:

Let: $x(t) \xrightarrow{s} y = x(t-2) + x(2-t)$

$\rightarrow y(t-T) = x(t-T-2) + x(2-t+T)$ (1) (delay the ouput).

Let: $x_T(t) = x(t-T) \xrightarrow{s} y_T$

$$\rightarrow y_T = x_T(t-2) + x_T(2-t) = x(t-T-2) + \ x(-t-T-2)(2)$$

Since, $(1) \neq (2)$, therefore, the system is time variant.

b)

Assume that $|x(t)| \leq M, M$ is finite for all t.

We have: $|y(t)| = |x(t-2) + x(2-t)| \leq |x(t-2)| + |x(2-t)| \leq M + M = 2M$

$$\rightarrow |y(t)| \leq 2M$$

Therefore, with bounded input, the output will be bounded, which leads to the system is BIBO system.

Q3.

Given that: $h[n] = [2, -1, 0, 2]$

$$y[n] = x[n] * h[n] = \sum_{k=-\infty}^{+\infty} x[n-k]h[k] = \sum_{k=0}^{3} x[n-k]h[k]$$

$$= x[n]h[0] + x[n-1]h[1] + x[n-2]h[2] + x[n-3]h[3]$$

$$= 2x[n] - x[n-1] + 2x[n-3]$$

c) Given that: $x[n] = [2, 1, 0, 1, 3]$

$$+ \ y[0] = 2x[0] - x[-1] + 2x[-3] = 2 \times 2 - 0 + 2 \times 0 = 4$$

$$+ \ y[1] = 2x[1] - x[0] + 2x[-2] = 2 \times 1 - 2 + 2 \times 0 = 0$$

$$+ \ y[2] = 2x[2] - x[1] + 2x[-1] = 2 \times 0 - 1 + 2 \times 0 = -1$$

$$+ \ y[3] = 2x[3] - x[2] + 2x[0] = 2 \times 1 - 0 + 2 \times 2 = 6$$

$$+ \ y[4] = 2x[4] - x[3] + 2x[1] = 2 \times 3 - 1 + 2 \times 1 = 7$$

$$+ \ y[5] = 2x[5] - x[4] + 2x[2] = 2 \times 0 - 3 + 2 \times 0 = -3$$

$$+ \ y[6] = 2x[6] - x[5] + 2x[3] = 2 \times 0 - 0 + 2 \times 1 = 2$$

$$+ \ y[7] = 2x[7] - x[6] + 2x[4] = 2 \times 0 - 0 + 2 \times 3 = 6$$

Therefore, $y[n] = [4, 0, -1, 6, 7, -3, 2, 6]$

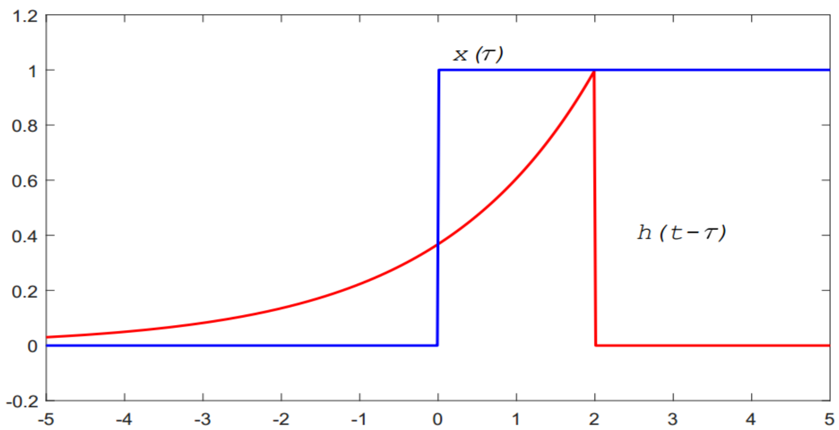
Q4.

Given that: $h(t) = e^{-t/2}u(t)$

a)

For $t < 0$, $h(t)$ and $x(t)$ does not overlap.

For $t \geq 0$:



$$y(t) = \int_{-\infty}^{+\infty} x(\tau)h(t-\tau)d\tau = \int_{0}^{t} 1. e^{-(t-\tau)/2} d\tau = e^{-t/2} \int_{0}^{t} e^{-\tau/2} d\tau = 2e^{-t/2}\left(1 - e^{-t/2}\right)$$

Thus,

$$y_1(t) = y(t) = \begin{cases} 0 & , \quad t < 0 \\ 2e^{-t/2}\left(1 - e^{-t/2}\right), & \quad t \geq 0 \end{cases}$$

b)

We have: $x_2(t) = 2u(t) - 2u(t-3) = 2x_1(t) - 2x_1(t-3)$

By the properties of LTI system, the output $y_2(t)$ is given by:

$$\begin{aligned} y_2(t) &= 2y_1(t) - 2y_1(t-3) \\ &= \begin{cases} 0 & , \quad t < 0 \\ 4e^{-t/2}\left(1 - e^{-t/2}\right) & , 0 \leq t < 3 \\ 4e^{-t/2}\left(1 - e^{-t/2}\right) - 4e^{-(t-3)/2}\left(1 - e^{-(t-3)/2}\right), & \quad t \geq 3 \end{cases} \end{aligned}$$