Homework

Chapter 1 - Week 2

Có No

Question 1:

1. Determine if the following system is consistent:

$$\begin{cases} x_1 - 4x_3 = 8 \\ 2x_1 - 3x_2 + 2x_3 = 1 \\ 4x_1 - 8x_2 + 12x_3 = 1 \end{cases}$$

$$\begin{cases} x_1 - 4x_3 = 8 \\ 2x_1 - 3x_2 + 2x_3 = 1 \\ 4x_1 - 8x_2 + 12x_3 = 1 \end{cases}$$

$$\left[\begin{array}{ccc|c} 1 & 0 & -4 & 8 \\ 2 & -3 & 2 & 1 \\ 4 & -8 & 12 & 1 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 4 & -8 & 12 & 1 \\ 2 & -3 & 2 & 1 \\ 1 & 0 & -4 & 8 \end{array}\right]$$

(1) $\Rightarrow \begin{cases} x = 35 \\ y = 55/2 \\ z = 27/4 \end{cases}$

$$\left[\begin{array}{ccc|c} 4 & -8 & 12 & 1 \\ 2 & -3 & 2 & 1 \\ 0 & -3 & 10 & -15 \end{array}\right]$$

(2) $\Rightarrow \begin{cases} x = 35 \\ y = 55/2 \\ z = 27/4 \end{cases}$

$R_3' \leftarrow R_3 \times (-2) + R_2$

$$\left[\begin{array}{ccc|c} 4 & -8 & 12 & 1 \\ 0 & -2 & 8 & -1 \\ 0 & -3 & 10 & -15 \end{array}\right]$$

$R_2' \leftarrow R_2 \times (-2) + R_1$

$-3 + (-2) \times x = 0$

$$\begin{array}{cccc} x_1 & x_2 & x_3 & \text{RHS} \\ \left[\begin{array}{ccc|c} 4 & -8 & 12 & 1 \\ 0 & -2 & 8 & -1 \\ 0 & 0 & -2 & \frac{-27}{2} \end{array}\right] \end{array}$$

$$\begin{cases} 4x_1 - 8x_2 + 12x_3 = 1 \\ -2x_2 + 8x_3 = -1 \\ -2x_3 = \frac{-27}{2} \end{cases}$$

$R_3' \leftarrow R_3 + R_2 \times \left(\frac{-3}{2}\right)$

$\Rightarrow$ The system has a unique solution $\Rightarrow$ Consistant

Question 2:

1) Echelon form : d

2) Reduced echelon form : a , b ~~c~~

2. Determine which matrices are in reduced echelon form and which others are only in echelon form.

a. $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}$ b. $\begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

c. $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ d. $\begin{bmatrix} 1 & 1 & 0 & 1 & 1 \\ 0 & 2 & 0 & 2 & 2 \\ 0 & 0 & 0 & 3 & 3 \\ 0 & 0 & 0 & 0 & 4 \end{bmatrix}$

Question 3:

Pivot

a)

$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 4 & 5 & 6 & 7 \\ 6 & 7 & 8 & 9 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 4 & 5 & 6 & 7 \\ 0 & -5 & -10 & -15 \end{bmatrix} \quad R_3 \leftarrow R_1 \times (-6) + R_3$$

$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & -3 & -6 & -9 \\ 0 & -5 & -10 & -15 \end{bmatrix} \quad R_2 \leftarrow R_1 \times (-4) + R_2$$

$-5 \times 2 + -5 = 0$

Pivot

$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & -3 & -6 & -9 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad R_3 \leftarrow R_2 \times \left(\frac{-5}{3}\right) + R_3$$

b)

$$\left[\begin{array}{ccc|c} 1 & 3 & 5 & 7 \\ 3 & 5 & 7 & 9 \\ 5 & 7 & 9 & 1 \end{array}\right] \quad (1)$$

$$\left[\begin{array}{ccc|c} 1 & 3 & 5 & 7 \\ 3 & 5 & 7 & 9 \\ 0 & -8 & -16 & -34 \end{array}\right] \quad (2) \quad R_3 \leftarrow R_1 \times (-5) + R_3$$

$$\left[\begin{array}{ccc|c} 1 & 3 & 5 & 7 \\ 0 & -4 & -8 & -12 \\ 0 & -8 & -16 & -34 \end{array}\right] \quad R_2 \leftarrow R_1 \times (-3) + R_2$$

3. Reduced the matrices to echelon form. Circle the pivot positions in the final matrix and in the original matrix, and list the pivot columns.

Applied Linear Algebra homework

a) $\begin{bmatrix} 1 & 2 & 3 & 4 \\ 4 & 5 & 6 & 7 \\ 6 & 7 & 8 & 9 \end{bmatrix}$

b) $\begin{bmatrix} 1 & 3 & 5 & 7 \\ 3 & 5 & 7 & 9 \\ 5 & 7 & 9 & 1 \end{bmatrix}$

$$\left[\begin{array}{ccc|c} 1 & 3 & 5 & 7 \\ 0 & -4 & -8 & -12 \\ 0 & 0 & 0 & -10 \end{array}\right] \quad \text{RHS} \qquad R_3 \leftarrow -2R_2 + R_3$$

li - column $\left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} ; \begin{bmatrix} 3 \\ -4 \\ 0 \end{bmatrix} \right\}$

Question 4:

4. Find the general solutions of the systems whose augmented matrices

a) $\left[\begin{array}{cccc} 1 & 3 & 4 & 7 \\ 3 & 9 & 7 & 6 \end{array}\right]$

b) $\left[\begin{array}{cccc} 3 & -4 & 2 & 0 \\ -9 & 12 & -6 & 0 \\ -6 & 8 & -4 & 0 \end{array}\right]$

a) $\left[\begin{array}{ccc|c} 1 & 3 & 4 & 7 \\ 3 & 9 & 7 & 6 \end{array}\right]$

$$\left[\begin{array}{ccc|c} 1 & 3 & 4 & 7 \\ 0 & 0 & -5 & -15 \end{array}\right] \qquad R_2 \leftarrow -3R_1 + R_2$$

$$\Leftrightarrow \begin{cases} x_1 + 3x_2 + 4x_3 = 7 \\ -5x_3 = -15 \end{cases}$$

$$\Leftrightarrow \begin{cases} x_1 + 3x_2 = -5 \\ x_3 = 3 \end{cases}$$

Introduce parameter s for free variable x_2

$$\begin{cases} x_1 = -5 - 3s \\ x_2 = s \\ x_3 = 3 \end{cases}$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -5 \\ 0 \\ 3 \end{bmatrix} + s \begin{bmatrix} -3 \\ 1 \\ 0 \end{bmatrix}$$

b) $\begin{bmatrix} 3 & -4 & 2 & 0 \\ -9 & 12 & -6 & 0 \\ -6 & 8 & -4 & 0 \end{bmatrix}$

$\begin{bmatrix} 3 & -4 & 2 & 0 \\ -9 & 12 & -6 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ $R_3 \leftarrow R_1 \times 2 + R_3$

$\begin{bmatrix} 3 & -4 & 2 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ $R_2 \leftarrow R_1 \times 3 + R_2$

$$\begin{cases} 3x_1 - 4x_2 + 2x_3 = 0 \\ x_2 = x_2 \\ x_3 = x_3 \end{cases}$$

$$\Leftrightarrow \begin{cases} x_1 = \frac{4}{3}x_2 - \frac{2}{3}x_3 \\ x_2 = x_2 \\ x_3 = x_3 \end{cases} \text{ (free variable)}$$

Introduce parameter s and t for free variable x_2 and x_3 respectively

$$\begin{cases} x_1 = 4/3\, s - 2/3\, t \\ x_2 = s \\ x_3 = t \end{cases}$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 4/3 \\ 1 \\ 0 \end{bmatrix} s + \begin{bmatrix} -2/3 \\ 0 \\ 1 \end{bmatrix} t$$