3.4.2. Conservative Forces (gravitational and spring force) and Potential Energy

The work done by a force F=F(x):

$$W = \int_{x_i}^{x_f} F(x)dx$$

$$\Delta U = -W = -\int_{x_i}^{x_f} F(x)dx$$

a. Gravitational Potential Energy

$$\Delta U = -W = -\int_{y_i}^{y_f} (-mg)dy = mg(y_f - y_i)$$

At a certain height y: $\Delta U = U - U_i = mg(y - y_i)$

If we take U_i to be the gravitational PE of the system when it is in a reference configuration in which the object is at a reference point y_i and U_i=0 and y_i=0, we have:

$$\boxed{U(y) = mgy \quad \text{(gravitational potential energy)}}$$

→ U(y) depends only on the vertical position y of the object relative to the reference point y=0.

b. Elastic Potential Energy

$$\Delta U = -\int_{x_i}^{x_f} (-kx)dx = \frac{1}{2}kx_f^2 - \frac{1}{2}kx_i^2$$

If we choose the reference configuration to be when the spring at its relaxed length, x_i=0 and U_i=0:

$$\boxed{U = \frac{1}{2}kx^2 \quad \text{(elastic potential energy)}}$$

3.5. Conservation of Mechanical Energy

The mechanical energy of a system = its PE U + the kinetic energy K of all objects of the system:

$$E_{mec} = K + U$$

If we only consider conservative forces that cause energy transfers within the system and assume that the system is isolated (i.e., no external forces):

$$\Delta K = W$$

$$\Delta U = -W$$

$$\Rightarrow \Delta K = -\Delta U$$

$$K_2 - K_1 = -(U_2 - U_1)$$

$$\boxed{K_2 + U_2 = K_1 + U_1}$$

In an isolated system, the kinetic energy and potential energy can change but the mechanical energy of the system is a constant.

$$\Delta E_{mec} = \Delta K + \Delta U = 0$$

3.6. Work Done on a System by an External Force. Conservation of Energy

3.6.1. Work Done on a System by an External Force

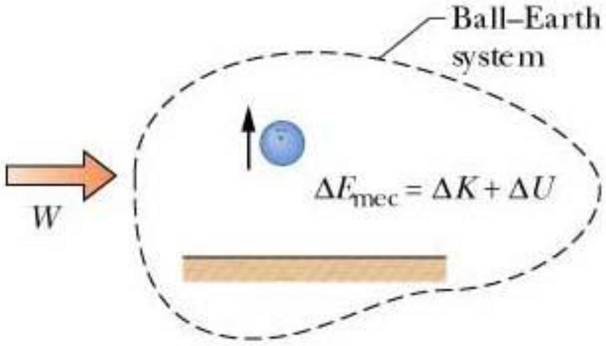
Recall: Work is energy is transferred to or from an object by means of a force acting on the object.

For a system: Work is energy is transferred to or from a system by means of an external force acting on that system.

a. No friction involved:

$$W = \Delta E_{mechanical} = \Delta K + \Delta U$$

→the work done by an external force on a system is equal to the change in the mechanical energy of the system.



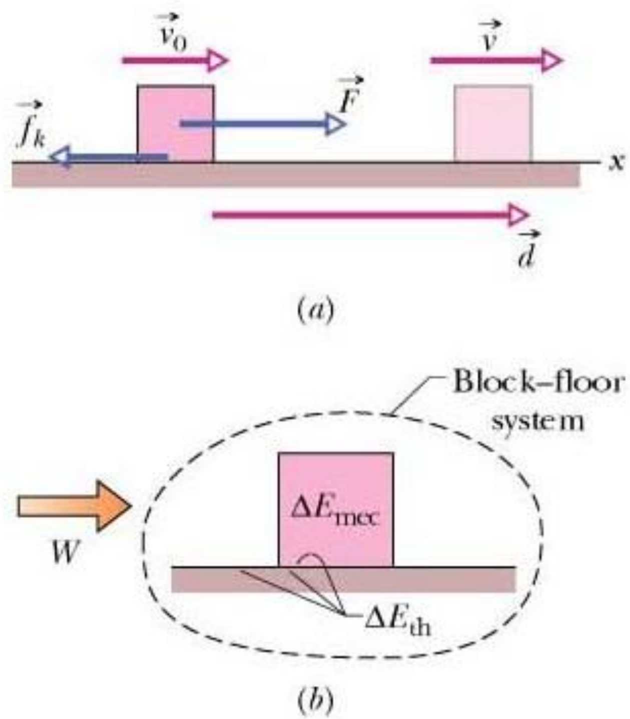
b. Friction involved:

$$W = \Delta E_{\text{mechanical}} + \Delta E_{\text{thermal}}$$

where

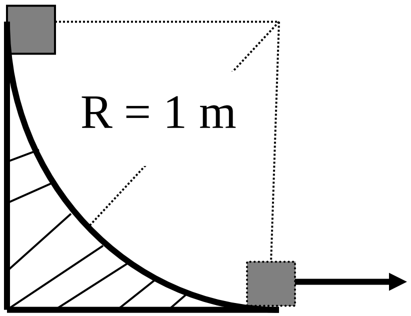
$$\Delta E_{\text{thermal}} = f_k d \text{ (increase in thermal energy by friction)}$$

f_k: the frictional force



Positive work done on the block-floor system = A change in the block's mechanical energy + A change in the thermal energy of the block and floor.

Example: A block of mass 1.0 kg is released from rest and slides down a rough track of radius R = 1.0 m (Figure 1). If the speed of the block at the bottom of the track is 3.0 m/s, what is the work done by the frictional force acting on the block? (Final exam, June 2014)



$$W = \Delta E_{\text{mechanical}} + \Delta E_{\text{thermal}}$$

W: work done by external (applied) forces
In this case: W = 0

$$|W_{\text{friction}}| = \Delta E_{\text{thermal}} = -\Delta E_{\text{mechanical}}$$

$$\Delta E_{\text{mechanical}} = \Delta K + \Delta U = E_{\text{mechanical, bottom}} - E_{\text{mechanical, top}}$$

$$\Delta E_{\text{mechanical}} = \left(0 + \frac{1}{2}mv^2\right) - (mgh - 0)$$ (We choose U_{bottom} = 0)

$$|W_{\text{friction}}| = mgh - \frac{1}{2}mv^2 = 1.0(9.8 \times 1.0 - \frac{1}{2} \times 3^2) = 5.3\ (\text{J})$$

3.6.2. Conservation of Energy

The law of conservation of energy:

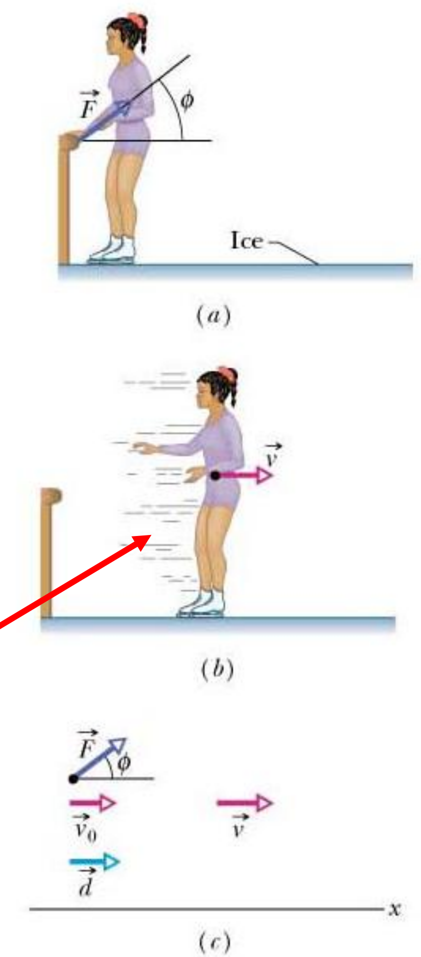
The total energy E of a system can change only by amounts of energy that are transferred to or from the system.

$$W = \Delta E_{\text{mechanical}} + \Delta E_{\text{thermal}} + \Delta E_{\text{internal}}$$

Isolated systems:

No energy transfer to or from the systems.

$$\Delta E_{\text{mechanical}} + \Delta E_{\text{thermal}} + \Delta E_{\text{internal}} = 0$$



If an external force acts on the system but does no work, i.e. no energy transfer to the system, the force can change the KE or PE of the system.

Her KE increases due to internal transfers from the biochemical energy in her muscles

Chapter 4 Linear Momentum and Collisions

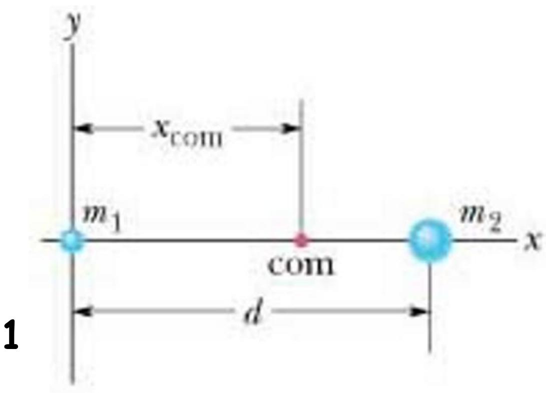
4.1. The Center of Mass. Newton's Second Law for a System of Particles
4.2. Linear Momentum and Its Conservation
4.3. Collision and Impulse
4.4. Momentum and Kinetic Energy in Collisions

4.1. The Center of Mass. Newton's Second Law for a System of Particles

4.1.1. The center of mass

a. Systems of Particles

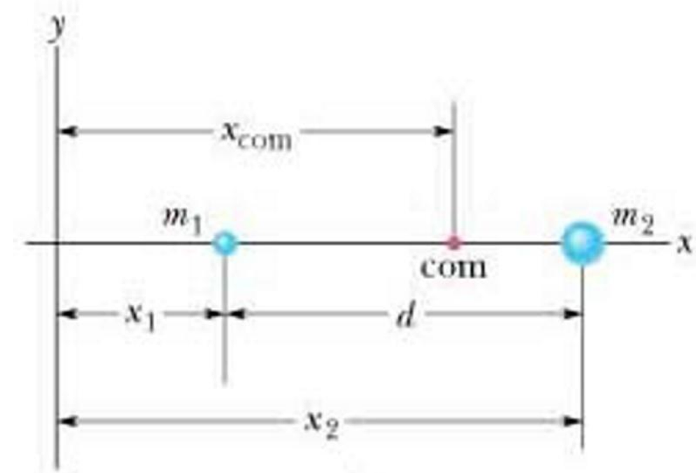
•Consider a system of 2 particles of masses m_1 and m_2 separated by distance d:



(a)

$$x_{com} = \frac{m_2}{m_1 + m_2} d$$

•If m_1 at x_1 and m_2 at x_2:



(b)

$$\boxed{x_{com} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2} = \frac{m_1 x_1 + m_2 x_2}{M}}$$

where M is the total mass of the system

•If the system has n particles that are strung out along the x axis:

$$\boxed{x_{com} = \frac{m_1 x_1 + m_2 x_2 + \ldots + m_n x_n}{M} = \frac{1}{M}\sum_{i=1}^{n} m_i x_i}$$

•If the n particles are distributed in three dimensions:

$$\boxed{x_{com} = \frac{1}{M}\sum_{i=1}^{n} m_i x_i, \quad y_{com} = \frac{1}{M}\sum_{i=1}^{n} m_i y_i, \quad z_{com} = \frac{1}{M}\sum_{i=1}^{n} m_i z_i}$$

•If the position of particle i is given by a vector:

$$\vec{r}_i = x_i\hat{i} + y_i\hat{j} + z_i\hat{k}$$

$$\vec{r}_{com} = x_{com}\hat{i} + y_{xom}\hat{j} + z_{com}\hat{k}$$

•The center of mass of the system is determined by:

$$\boxed{\vec{r}_{com} = \frac{1}{M}\sum_{i=1}^{n} m_i \vec{r}_i}$$

b. Solid Bodies

$$\boxed{x_{com} = \frac{1}{M}\int x\,dm, \quad y_{com} = \frac{1}{M}\int y\,dm, \quad z_{com} = \frac{1}{M}\int z\,dm}$$

where M is the mass of the object

•For uniform objects, their density are:

$$\rho = \frac{dm}{dV} = \frac{M}{V}$$

$$\Rightarrow dm = \left(\frac{M}{V}\right)dV$$

$$\boxed{x_{com} = \frac{1}{V}\int x\,dV, \quad y_{com} = \frac{1}{V}\int y\,dV, \quad z_{com} = \frac{1}{V}\int z\,dV}$$

Sample Problem (p. 204)

Determine the center of mass of the plate

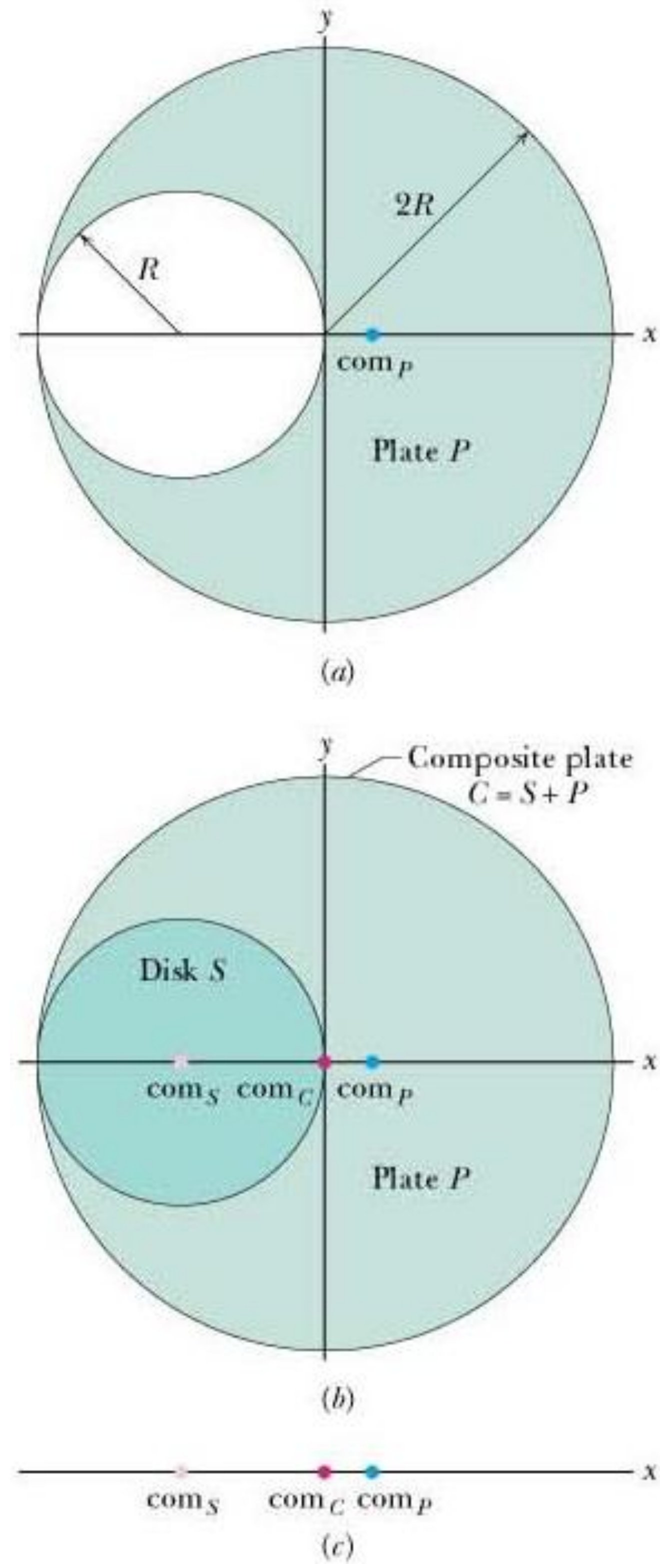
$$x_{S+P} = \frac{m_S x_S + m_P x_P}{m_S + m_P} = 0$$

$$x_P = -x_S \frac{m_S}{m_P}$$

$$\frac{m_S}{m_P} = \frac{\rho_S}{\rho_P} \times \frac{thickness_S}{thickness_P} \times \frac{area_S}{area_P} = \frac{\pi R^2}{\pi (2R)^2 - \pi R^2} = \frac{1}{3}$$

$$x_S = -R$$

$$\boxed{\Rightarrow x_P = \frac{1}{3} R}$$



4.1.2. Newton's Second Law for a System of Particles

$$\boxed{\vec{F}_{net} = M\vec{a}_{com} \qquad (1)}$$

$\vec{F}_{net}$: the net force of all external forces

$\vec{a}_{com}$: the acceleration of the center of mass of the system

M : the total mass of the system

$$\boxed{F_{net,x} = Ma_{com,x} \qquad F_{net,y} = Ma_{com,y} \qquad F_{net,z} = Ma_{com,z}}$$

Proof of Equation (1):

$$M\vec{r}_{com} = \sum_{i=1}^{n} m_i \vec{r}_i$$

$$\Rightarrow M\vec{v}_{com} = \sum_{i=1}^{n} m_i \vec{v}_i ; \quad M\vec{a}_{com} = \sum_{i=1}^{n} m_i \vec{a}_i = \sum_{i=1}^{n} \vec{F}_i = \vec{F}_{net}$$

4.2. Linear Momentum and Its Conservation

a. Linear Momentum

The linear momentum of a particle is a vector quantity $\vec{p}$ defined as:

$$\boxed{\vec{p} = m\vec{v}}$$ **(Unit: kg m/s)**

where m and $\vec{v}$ are the mass and the velocity of the particle, respectively.

Newton's second law is expressed in terms of momentum:

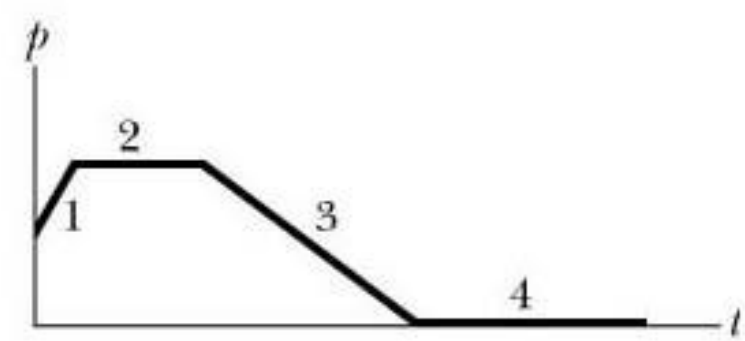
$$\boxed{\vec{F}_{net} = \frac{d\vec{p}}{dt}}$$

where $\vec{F}_{net}$ is the net external force on the particle.

Checkpoint 3 (p. 210)

(a) rank the magnitude of forces

(b) in which region is the particle slowing?



1, 3, 2, 4; 3

•For a system of particles:

$$\vec{P} = \vec{p}_1 + \vec{p}_2 + \ldots + \vec{p}_n = m_1\vec{v}_1 + m_2\vec{v}_2 + \ldots + m_n\vec{v}_n$$

$$\boxed{\vec{P} = M\vec{v}_{com}}$$

→The linear momentum of a system of particles is equal to the product of the total mass M of the system and the velocity of the center of mass.

$$\Rightarrow \frac{d\vec{P}}{dt} = M\frac{d\vec{v}_{com}}{dt} = M\vec{a}_{com}$$

$$\boxed{\vec{F}_{net} = \frac{d\vec{P}}{dt}}$$

b. Conservation of Linear Momentum:

If the net external force acting on a system of particles is zero,

$\vec{F}_{net} = 0$

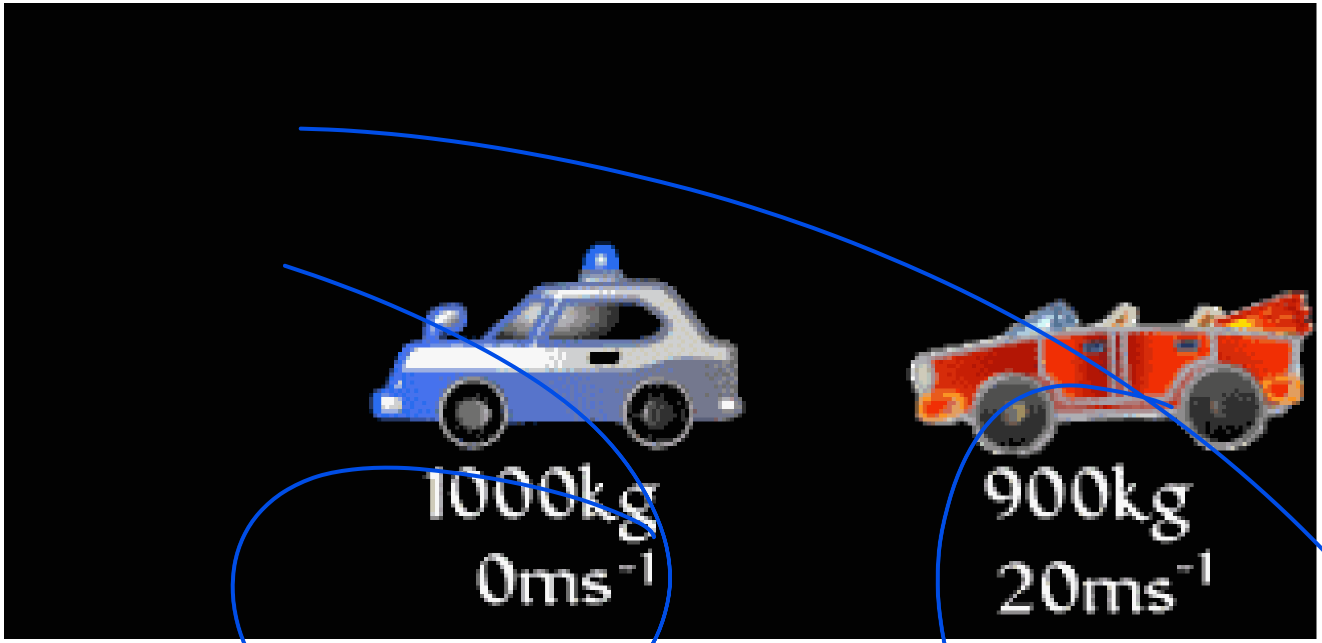
$$\vec{P} = \text{constant}$$

If $F_{net,X} = 0 \; (\mathrm{X} = \mathrm{x, y, or z})$:

$$P_X = \text{constant}$$

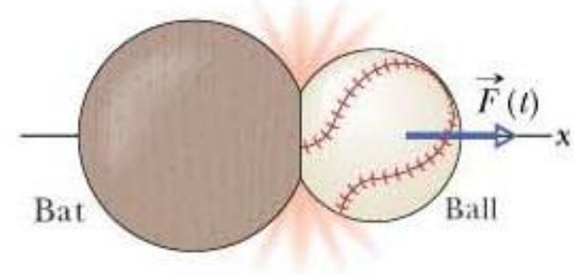
<u>Question:</u> Why do we need momentum?

Because <u>momentum</u> provides us a tool for studying collision of 2 or more objects.



4.3. Collision and Impulse

- Consider a collision between a bat and a ball:

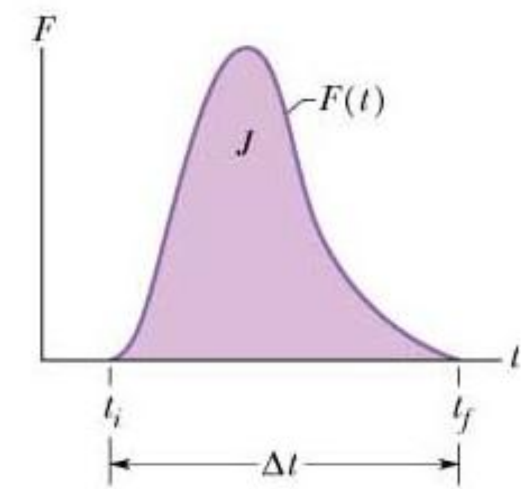


The change in the ball's momentum is:

$$d\vec{p} = \vec{F}(t)dt$$

from a time t_i to a time t_f:

$$\int_{t_i}^{t_f} d\vec{p} = \int_{t_i}^{t_f} \vec{F}(t)dt$$



(a)

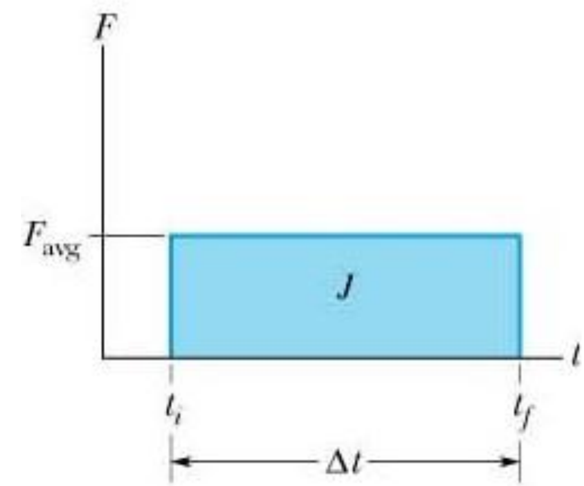
- The impulse of the collision is defined by:

$$\boxed{\vec{J} = \int_{t_i}^{t_f} \vec{F}(t)dt}$$ (Unit: kg m/s)

$$\Delta\vec{p} = \vec{J}$$

the impulse of the object

the change in the object's momentum



(b)

If we do not know the F(t) function, we can use:

$$\boxed{J = F_{avg}\Delta t}$$

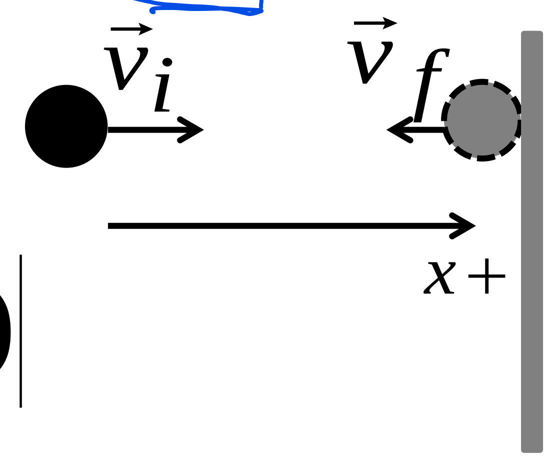
$F = \dfrac{J}{\Delta t}$

Examples:

1. A 0.70 kg ball is moving horizontally with a speed of 5.0 m/s when it strikes a vertical wall. The ball rebounds with a speed of 2.0 m/s. What is the magnitude of the change in linear momentum of the ball?

$$\vec{p} = m\vec{v}; \quad \Delta\vec{p} = m\Delta\vec{v}$$

Since the ball is moving horizontally, therefore, this is one dimensional motion:



$$\Delta p_x = m\Delta v_x$$

$$|\Delta p_x| = m|\Delta v_x| = m|(v_f - v_i)|$$

$$v_f = -2\text{ m/s};\ v_i = 5\text{ m/s} : |\Delta p_x| = |0.7\times(\text{-}2\text{ - }5)| = 4.9\text{ (kg m/s)}$$

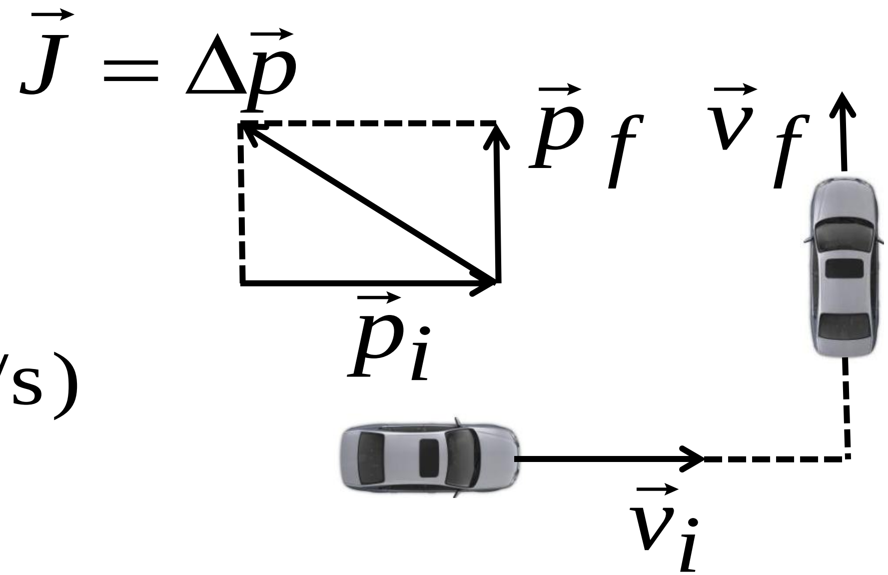
rebounds

$$\Delta\vec{p} = m\Delta\vec{v} = (-4.9\text{ kg.m/s})\,\hat{i}$$

2. A 1500-kg car travelling at a speed of 5.0 m/s makes a 90^0 turn in a time of 3.0 s and emerges from this turn with a speed of 3.0 m/s:

(a) What is the magnitude of the impulse that acts on the car during this turn? Draw the impulse vector.

(b) What is the magnitude of the average force on the car during this turn? (Final exam, June 2014)



(a)

$$\vec{J} = \Delta\vec{p} = \vec{p}_f - \vec{p}_i$$

$$J = \sqrt{p_f^2 + p_i^2} \approx 8746 \text{ (kg m/s)}$$

(b)

$$F_{\text{avg}} = \frac{J}{\Delta t} = \frac{8746}{3} \approx 2915 \text{ (N)}$$

Homework: 24, 56 (pages 191-194);
2, 5, 13, 14, 22, 25, 38 (pages 230-233)