Sample Problem

A 5 kg block of ice at $0°$C is added to an insulated container partially filled with 10.0 kg of water at $15°$C. Given that the specific heat capacity to make the temperature of 1 kg of water to increase 1°C is $c_{\text{water}} = 4190$ J/kg.°C, while the hidden heat for 1 kg of ice to melt completely is $L_{\text{ice}} = 3.33 \times 10^5$ J/kg.

(a) Find the final temperature, neglecting the heat capacity of the container.

(b) Find the mass of the ice that was melted.

Sample Solution

The Physics: Part (a) is tricky because the ice does not entirely melt in this example. When there is any doubt concerning whether there will be a complete phase change, some preliminary calculations are necessary. First, find the total energy required to melt the ice, Q_{melt}, and then find Q_{water}, the maximum energy that can be delivered by the water above $0°$C. If the energy delivered by the water is high enough, all the ice melts. If not, there will usually be a final mixture of ice and water at $0°$C, unless the ice starts at a temperature far below $0°$C, in which case all the liquid water freezes.

The Solution:

(a) Find the equilibrium temperature. First, compute the amount of energy necessary to completely melt the ice: $Q_{\text{melt}} = m_{\text{ice}} L_{\text{ice}} = (5\text{ kg})(3.33 \times 10^5 \text{J/kg}) = 1.67 \times 10^6$ J.

Next, calculate the maximum energy that can be lost by the initial mass of liquid water without freezing it: $Q_{\text{water}} = m_{\text{water}} c_{\text{water}} (T_2 - T_1) = m_{\text{water}} c_{\text{water}} \Delta T = (10\text{ kg})(4190\text{ J/kg.°C})(0° - 15°) \approx -6.3 \times 10^5$ J.

This result is less than half the energy necessary to melt all the ice, so the final state of the system is a mixture of water and ice at the freezing point: $T = 0°$C.

(b) Compute the mass of ice melted. Set the total available energy equal to the heat of fusion of m kg of ice, mL_f , and solve for m: $6.3 \times 10^5\text{ J} = mL_f = m(3.33 \times 10^5 \text{J/kg})$; Thus, $m = 1.89$ kg.

The Remarks: If this problem is solved assuming (wrongly) that all the ice melts, a final temperature of $T = -16.5°$C is obtained. The only way that could happen is if the system were not isolated, contrary to the statement of the problem.

An extra questions could be asked sometimes to check for a deep understanding; for example: What effect would doubling the initial amount of liquid water have on the amount of ice melted? Calculate and Explain.

The correct answer will get awarded points!

Notes for the Midterm Exam

1. The total time for doing the exam is 120 minutes, excluding the preparation time.
2. The total points for the exam is 100 points. However, some questions will give extra (awarded) points if answered correctly, and these awarded points will be added to your total/final grade.
3. The ideal solution should include the following items:
   - A summary of the problem,
   - A draw a simple scratch diagram to represent the problem,
   - Symbolically solving the problem first, then the final numbers and conclusions coming later after substituting all given numbers to the final symbolic formulae.
   - All Equations in use must come along with explanations; don't just write down the mathematics only without any given reasons.