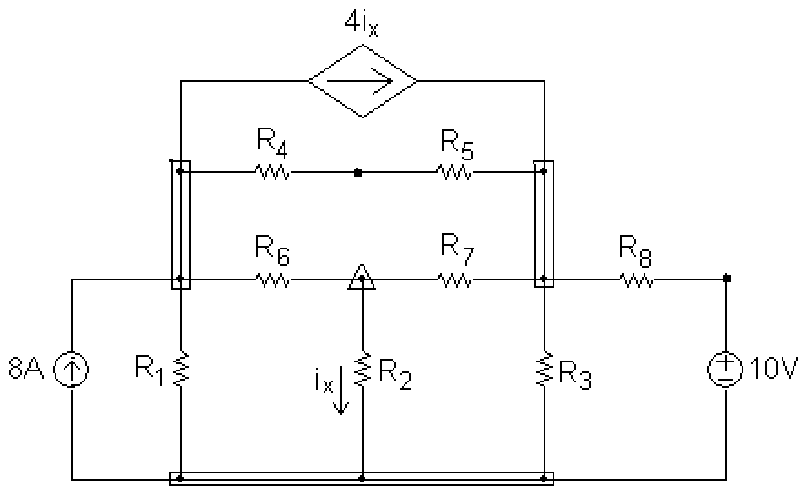
Problems

P 4.1



[**a**] 11 branches, 8 branches with resistors, 2 branches with independent sources, 1 branch with a dependent source

[**b**] The current is unknown in every branch except the one containing the 8 A current source, so the current is unknown in 10 branches.

[**c**] 9 essential branches – $R_4 - R_5$ forms an essential branch as does $R_8 - 10$ V. The remaining seven branches are essential branches that contain a single element.

[**d**] The current is known only in the essential branch containing the current source, and is unknown in the remaining 8 essential branches

[**e**] From the figure there are 6 nodes – three identified by rectangular boxes, two identified with single black dots, and one identified by a triangle.

[**f**] There are 4 essential nodes, three identified with rectangular boxes and one identified with a triangle

[**g**] A mesh is like a window pane, and as can be seen from the figure there are 6 window panes or meshes.

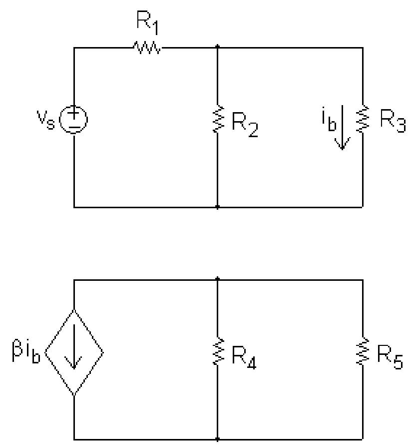
P 4.2 [**a**] From Problem 4.1(d) there are 8 essential branches where the current is unknown, so we need 8 simultaneous equations to describe the circuit.

[**b**] From Problem 4.1(f), there are 4 essential nodes, so we can apply KCL at $(4 - 1) = 3$ of these essential nodes. There would also be a dependent source constraint equation.

[**c**] The remaining 4 equations needed to describe the circuit will be derived from KVL equations.

[d] We must avoid using the topmost mesh and the leftmost mesh. Each of these meshes contains a current source, and we have no way of determining the voltage drop across a current source.

P 4.3



[a] As can be seen from the figure, the circuit has 2 separate parts.

[b] There are 5 nodes – the four black dots and the node betweem the voltage source and the resistor R_1.

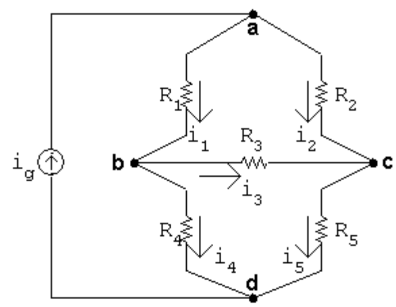
[c] There are 7 branches, each containing one of the seven circuit components.

[d] When a conductor joins the lower nodes of the two separate parts, there is now only a single part in the circuit. There would now be 4 nodes, because the two lower nodes are now joined as a single node. The number of branches remains at 7, where each branch contains one of the seven individual circuit components.

P 4.4 **[a]** There are six circuit components, five resistors and the current source. Since the current is known only in the current source, it is unknown in the five resistors. Therefore there are **five** unknown currents.

[b] There are four essential nodes in this circuit, identified by the dark black dots in Fig. P4.4. At three of these nodes you can write KCL equations that will be independent of one another. A KCL equation at the fourth node would be dependent on the first three. Therefore there are **three** independent KCL equations.

[c]



Sum the currents at any three of the four essential nodes a, b, c, and d. Using nodes a, b, and c we get

$$-i_g + i_1 + i_2 = 0$$

$$-i_1 + i_4 + i_3 = 0$$

$$i_5 - i_2 - i_3 = 0$$

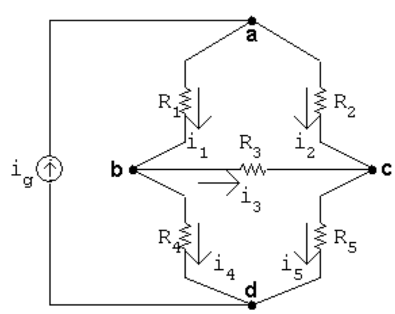
[d] There are three meshes in this circuit: one on the left with the components i_g, R_1, and R_4; one on the top right with components R_1, R_2, and R_3; and one on the bottom right with components R_3, R_4, and R_5. We cannot write a KVL equation for the left mesh because we don't know the voltage drop across the current source. Therefore, we can write KVL equations for the two meshes on the right, giving a total of **two** independent KVL equations.

[e] Sum the voltages around two independent closed paths, avoiding a path that contains the independent current source since the voltage across the current source is not known. Using the upper and lower meshes formed by the five resistors gives

$$R_1 i_1 + R_3 i_3 - R_2 i_2 = 0$$

$$R_3 i_3 + R_5 i_5 - R_4 i_4 = 0$$

P 4.5



[a] At node a: $\quad -i_g + i_1 + i_2 = 0$

At node b: $\quad -i_1 + i_3 + i_4 = 0$

At node c: $\quad -i_2 - i_3 + i_5 = 0$

At node d: $\quad i_g - i_4 - i_5 = 0$

[b] There are many possible solutions. For example, solve the equations at nodes a and d for i_g:

$$i_g = i_4 + i_5 \qquad i_g = i_1 + i_2 \qquad \text{so} \qquad i_1 + i_2 = i_4 + i_5$$

Solve this expression for i_1:

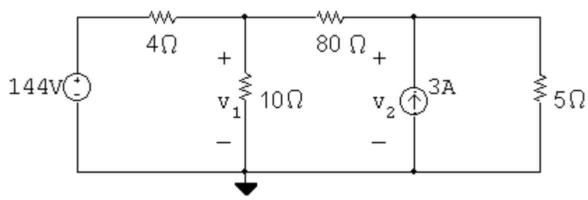
$$i_1 = i_4 + i_5 - i_2$$

Substitute this expression for i_1 into the equation for node b:

$$-(i_4 + i_5 - i_2) + i_3 + i_4 = 0 \qquad \text{so} \qquad -i_2 - i_3 + i_5 = 0$$

The result above is the equation at node c.

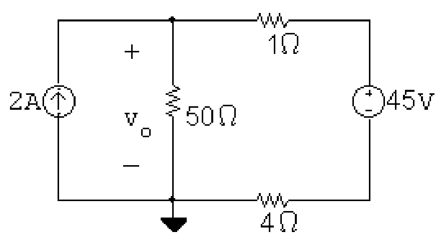
P 4.6



$$\frac{v_1 - 144}{4} + \frac{v_1}{10} + \frac{v_1 - v_2}{80} = 0 \qquad \text{so} \qquad 29v_1 - v_2 = 2880$$
$$-3 + \frac{v_2 - v_1}{80} + \frac{v_2}{5} = 0 \qquad \text{so} \qquad -v_1 + 17v_2 = 240$$

Solving, $v_1 = 100$ V; $\quad v_2 = 20$ V

P 4.7



$$-2 + \frac{v_o}{50} + \frac{v_o - 45}{1 + 4} = 0$$

$$v_o = 50 \text{ V}$$

$$p_{2\text{A}} = -(50)(2) = -100 \text{ W} \quad \text{(delivering)}$$

The 2 A source extracts -100 W from the circuit, because it delivers 100 W to the circuit.

P 4.8 $-6 + \frac{v_1}{40} + \frac{v_1 - v_2}{8} = 0$

$$\frac{v_2 - v_1}{8} + \frac{v_2}{80} + \frac{v_2}{120} + 1 = 0$$

Solving, $v_1 = 120$ V; $v_2 = 96$ V
CHECK:

$$p_{40\Omega} = \frac{(120)^2}{40} = 360 \text{ W}$$

$$p_{8\Omega} = \frac{(120 - 96)^2}{8} = 72 \text{ W}$$

$$p_{80\Omega} = \frac{(96)^2}{80} = 115.2 \text{ W}$$

$$p_{120\Omega} = \frac{(96)^2}{120} = 76.8 \text{ W}$$

$$p_{6\text{A}} = -(6)(120) = -720 \text{ W}$$

$$p_{1\text{A}} = (1)(96) = 96 \text{ W}$$

$$\sum p_{\text{abs}} = 360 + 72 + 115.2 + 76.8 + 96 = 720 \text{ W}$$

$$\sum p_{\text{dev}} = 720 \text{ W} \quad \text{(CHECKS)}$$

P 4.9 Use the lower terminal of the 25 Ω resistor as the reference node.

$$\frac{v_o - 24}{20 + 80} + \frac{v_o}{25} + 0.04 = 0$$

Solving, $v_o = 4$ V

P 4.10 [**a**] From the solution to Problem 4.9 we know $v_o = 4$ V, therefore

$$p_{40\text{mA}} = 0.04 v_o = 0.16 \text{ W}$$

$$\therefore \quad p_{40\text{mA}} \text{ (developed) } = -160 \text{ mW}$$

[**b**] The current into the negative terminal of the 24 V source is

$$i_g = \frac{24 - 4}{20 + 80} = 0.2 \text{ A}$$

$$p_{24\text{V}} = -24(0.2) = -4.8 \text{ W}$$

$$\therefore \quad p_{24\text{V}} \text{ (developed) } = 4800 \text{ mW}$$

[c] $p_{20\Omega} = (0.2)^2(20) = 800$ mW

$p_{80\Omega} = (0.2)^2(80) = 3200$ mW

$p_{25\Omega} = (4)^2/25 = 640$ mW

$\sum p_{\text{dev}} = 4800$ mW

$\sum p_{\text{dis}} = 160 + 800 + 3200 + 640 = 4800$ mW

P 4.11 **[a]** $\dfrac{v_0 - 24}{20 + 80} + \dfrac{v_o}{25} + 0.04 = 0; \quad v_o = 4$ V

[b] Let v_x = voltage drop across 40 mA source

$v_x = v_o - (50)(0.04) = 2$ V

$p_{40\text{mA}} = (2)(0.04) = 80$ mW so $p_{40\text{mA}}$ (developed) $= -80$ mW

[c] Let i_g = be the current into the positive terminal of the 24 V source

$i_g = (4 - 24)/100 = -0.2$ A

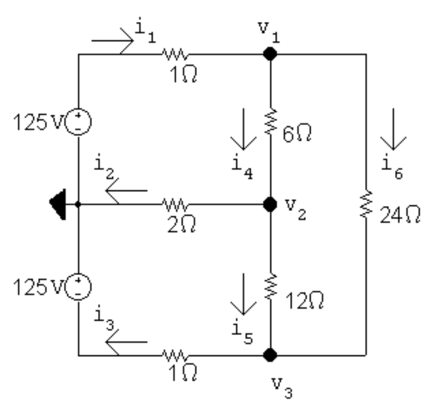
$p_{24\text{V}} = (-0.2)(24) = -4800$ mW so $p_{24\text{V}}$ (developed) $= 4800$ mW

[d] $\sum p_{\text{dis}} = (0.2)^2(20) + (0.2)^2(80) + (4)^2/25 + (0.04)^2(50) + 0.08$

$= 4800$ mW

[e] v_o is independent of any finite resistance connected in series with the 40 mA current source

P 4.12 **[a]**



$$\frac{v_1 - 125}{1} + \frac{v_1 - v_2}{6} + \frac{v_1 - v_3}{24} = 0$$

$$\frac{v_2 - v_1}{6} + \frac{v_2}{2} + \frac{v_2 - v_3}{12} = 0$$

$$\frac{v_3 + 125}{1} + \frac{v_3 - v_2}{12} + \frac{v_3 - v_1}{24} = 0$$

In standard form:

$$\begin{aligned}
v_1\left(\frac{1}{1}+\frac{1}{6}+\frac{1}{24}\right)+v_2\left(-\frac{1}{6}\right)+v_3\left(-\frac{1}{24}\right) &= 125\\
v_1\left(-\frac{1}{6}\right)+v_2\left(\frac{1}{6}+\frac{1}{2}+\frac{1}{12}\right)+v_3\left(-\frac{1}{12}\right) &= 0\\
v_1\left(-\frac{1}{24}\right)+v_2\left(-\frac{1}{12}\right)+v_3\left(\frac{1}{1}+\frac{1}{12}+\frac{1}{24}\right) &= -125
\end{aligned}$$

Solving, $v_1 = 101.24$ V; $v_2 = 10.66$ V; $v_3 = -106.57$ V

Thus, $i_1 = \dfrac{125 - v_1}{1} = 23.76$ A $\quad i_4 = \dfrac{v_1 - v_2}{6} = 15.10$ A

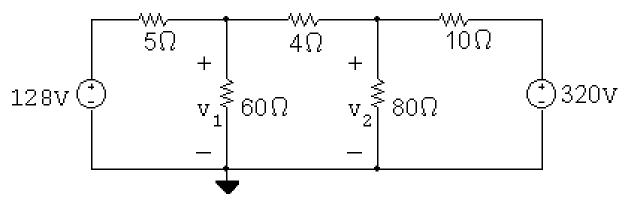
$i_2 = \dfrac{v_2}{2} = 5.33$ A $\quad i_5 = \dfrac{v_2 - v_3}{12} = 9.77$ A

$i_3 = \dfrac{v_3 + 125}{1} = 18.43$ A $\quad i_6 = \dfrac{v_1 - v_3}{24} = 8.66$ A

[b] $\sum P_{\text{dev}} = 125 i_1 + 125 i_3 = 5273.09$ W

$\sum P_{\text{dis}} = i_1^2(1) + i_2^2(2) + i_3^2(1) + i_4^2(6) + i_5^2(12) + i_6^2(24) = 5273.09$ W

P 4.13 **[a]**



$$\begin{aligned}
\frac{v_1 - 128}{5} + \frac{v_1}{60} + \frac{v_1 - v_2}{4} &= 0\\
\frac{v_2 - v_1}{4} + \frac{v_2}{80} + \frac{v_2 - 320}{10} &= 0
\end{aligned}$$

In standard form,

$$\begin{aligned}
v_1\left(\frac{1}{5}+\frac{1}{60}+\frac{1}{4}\right)+v_2\left(-\frac{1}{4}\right) &= \frac{128}{5}\\
v_1\left(-\frac{1}{4}\right)+v_2\left(\frac{1}{4}+\frac{1}{80}+\frac{1}{10}\right) &= \frac{320}{10}
\end{aligned}$$

Solving, $v_1 = 162$ V; $v_2 = 200$ V

$$i_a = \frac{128 - 162}{5} = -6.8 \text{ A}$$

$$i_b = \frac{162}{60} = 2.7 \text{ A}$$

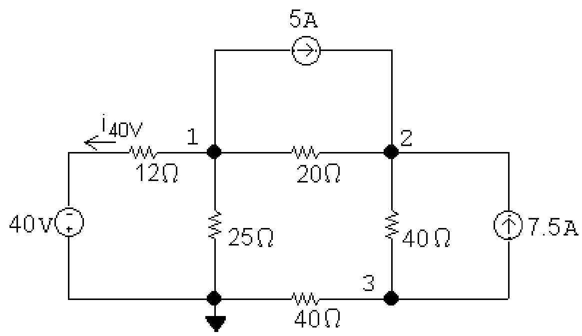
$$i_c = \frac{162 - 200}{4} = -9.5 \text{ A}$$

$$i_{\rm d} = \frac{200}{80} = 2.5 \text{ A}$$

$$i_{\rm e} = \frac{200 - 320}{10} = -12 \text{ A}$$

[b] $p_{128V} = -(128)(-6.8) = 870.4$ W (abs)
$p_{320V} = (320)(-12) = -3840$ W (dev)
Therefore, the total power developed is 3840 W.

P 4.14



$$\frac{v_1 + 40}{12} + \frac{v_1}{25} + \frac{v_1 - v_2}{20} + 5 = 0$$

$$\left[\frac{v_2 - v_1}{20}\right] - 5 + \frac{v_2 - v_1}{40} + -7.5 = 0$$

$$\frac{v_3}{40} + \frac{v_3 - v_2}{40} + 7.5 = 0$$

Solving, $v_1 = -10$ V; $\quad v_2 = 132$ V; $\quad v_3 = -84$ V; $\quad i_{40\rm V} = \dfrac{-10 + 40}{12} = 2.5$ A

$$p_{5\rm A} = 5(v_1 - v_2) = 5(-10 - 132) = -710 \text{ W} \quad \text{(del)}$$

$$p_{7.5\rm A} = (-84 - 132)(7.5) = -1620 \text{ W} \quad \text{(del)}$$

$$p_{40\rm V} = -(40)(2.5) = -100 \text{ W} \quad \text{(del)}$$

$$p_{12\Omega} = (2.5)^2(12) = 75 \text{ W}$$

$$p_{25\Omega} = \frac{v_1^2}{25} = \frac{10^2}{25} = 4 \text{ W}$$

$$p_{20\Omega} = \frac{(v_1 - v_2)^2}{20} = \frac{142^2}{20} = 1008.2 \text{ W}$$

$$p_{40\Omega}(\text{lower}) = \frac{(v_3)^2}{40} = \frac{84^2}{40} = 176.4\ \text{W}$$

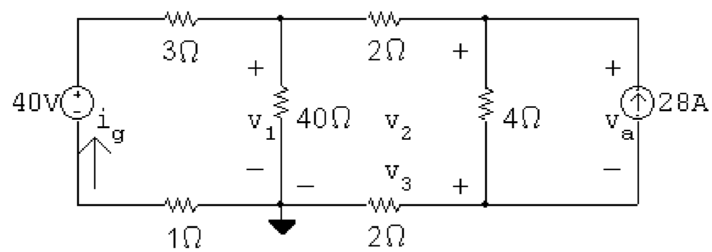
$$p_{40\Omega}(\text{right}) = \frac{(v_2 - v_3)^2}{40} = \frac{216^2}{40} = 1166.4\ \text{W}$$

$$\sum p_{\text{diss}} = 75 + 4 + 1008.2 + 176.4 + 1166.4 = 2430\ \text{W}$$

$$\sum p_{\text{dev}} = 710 + 1620 + 100 = 2430\ \text{W} \qquad \text{(CHECKS)}$$

The total power dissipated in the circuit is 2430 W.

P 4.15 **[a]**



$$\frac{v_1}{40} + \frac{v_1 - 40}{4} + \frac{v_1 - v_2}{2} = 0 \qquad \text{so} \qquad 31v_1 - 20v_2 + 0v_3 = 400$$

$$\frac{v_2 - v_1}{2} + \frac{v_2 - v_3}{4} - 28 = 0 \qquad \text{so} \qquad -2v_1 + 3v_2 - v_3 = 112$$

$$\frac{v_3}{2} + \frac{v_3 - v_2}{4} + 28 = 0 \qquad \text{so} \qquad 0v_1 - v_2 + 3v_3 = -112$$

Solving, $v_1 = 60$ V; $v_2 = 73$ V; $v_3 = -13$ V,

[b] $i_g = \dfrac{40 - 60}{4} = -5$ A

$p_g = (40)(-5) = -200$ W

Thus the 40 V source delivers 200 W of power.

P 4.16 **[a]** $\dfrac{v_o - v_1}{R} + \dfrac{v_o - v_2}{R} + \dfrac{v_o - v_3}{R} + \cdots + \dfrac{v_o - v_n}{R} = 0$

$\therefore\ nv_o = v_1 + v_2 + v_3 + \cdots + v_n$

$\therefore\ v_o = \dfrac{1}{n}[v_1 + v_2 + v_3 + \cdots + v_n] = \dfrac{1}{n}\sum_{k=1}^{n} v_k$

[b] $v_o = \dfrac{1}{3}(100 + 80 - 60) = 40$ V

P 4.17 **[a]** $-25 + \frac{v_1}{40} + \frac{v_1}{160} + \frac{v_1 - v_2}{10} = 0 \quad \text{so} \quad 21v_1 - 16v_2 + 0i_\Delta = 4000$

$$\frac{v_2 - v_1}{10} + \frac{v_2}{20} + \frac{v_2 - 84i_\Delta}{8} = 0 \quad \text{so} \quad -16v_1 + 44v_2 - 1680i_\Delta = 0$$

$$i_\Delta = \frac{v_1}{160} \quad \text{so} \quad v_1 + (0)v_2 - 160i_\Delta = 0$$

Solving, $v_1 = 352$ V; $v_2 = 212$ V; $i_\Delta = 2.2$ A;

$$i_{\text{depsource}} = \frac{212 - 84(2.2)}{8} = 3.4 \text{ A}$$

$$p_{84i_\Delta} = 84(2.2)(3.4) = 628.32 \text{ W(abs)}$$

$$p_{25\text{A}} = -25(352) = -8800 \text{ W(del)}$$

$$\therefore \quad p_{\text{dev}} = 8800 \text{ W}$$

[b] $\sum p_{\text{abs}} = \frac{(352)^2}{40} + \frac{(352)^2}{160} + \frac{(352 - 212)^2}{10} + \frac{(212)^2}{20}$

$$+(3.4)^2(8) + 628.32 = 8800 \text{ W}$$

$$\therefore \quad \sum p_{\text{dev}} = \sum p_{\text{abs}} = 8800 \text{ W}$$

P 4.18 $-3 + \frac{v_o}{200} + \frac{v_o + 5i_\Delta}{10} + \frac{v_o - 80}{20} = 0; \quad i_\Delta = \frac{v_o - 80}{20}$

[a] Solving, $v_o = 50$ V

[b] $i_{\text{ds}} = \frac{v_o + 5i_\Delta}{10}$

$$i_\Delta = (50 - 80)/20 = -1.5 \text{ A}$$

$$\therefore \quad i_{\text{ds}} = 4.25 \text{ A}; \quad 5i_\Delta = -7.5 \text{ V} : \quad p_{\text{ds}} = (-5i_\Delta)(i_{\text{ds}}) = 31.875 \text{ W}$$

[c] $p_{3\text{A}} = -3v_o = -3(50) = -150$ W (del)

$$p_{80\text{V}} = 80i_\Delta = 80(-1.5) = -120 \text{ W} \quad \text{(del)}$$

$$\sum p_{\text{del}} = 150 + 120 = 270 \text{ W}$$

CHECK:

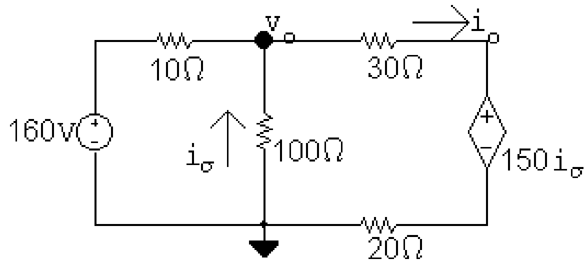
$$p_{200\Omega} = 2500/200 = 12.5 \text{ W}$$

$$p_{20\Omega} = (80 - 50)^2/20 = 900/20 = 45 \text{ W}$$

$$p_{10\Omega} = (4.25)^2(10) = 180.625 \text{ W}$$

$$\sum p_{\text{diss}} = 31.875 + 180.625 + 12.5 + 45 = 270 \text{ W}$$

P 4.19



$$\frac{v_o - 160}{10} + \frac{v_o}{100} + \frac{v_o - 150i_\sigma}{50} = 0; \quad i_\sigma = -\frac{v_o}{100}$$

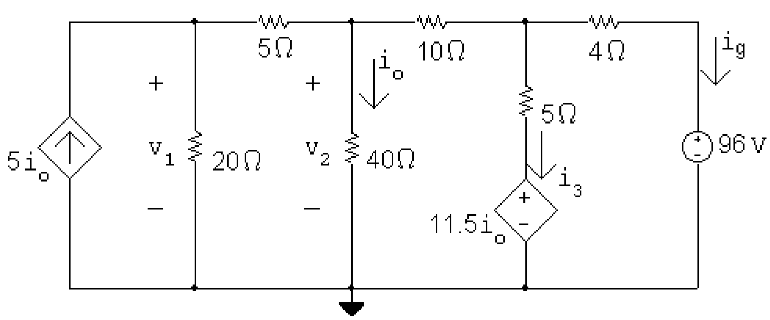
Solving, $v_o = 100$ V; $\quad i_\sigma = -1$ A

$$i_o = \frac{100 - (150)(-1)}{50} = 5 \text{ A}$$

$$p_{150i_\sigma} = 150 i_\sigma i_o = -750 \text{ W}$$

$\therefore$ The dependent voltage source delivers 750 W to the circuit.

P 4.20 [**a**]



$$i_o = \frac{v_2}{40}$$

$$-5i_o + \frac{v_1}{20} + \frac{v_1 - v_2}{5} = 0 \qquad \text{so} \qquad 10v_1 - 13v_2 + 0v_3 = 0$$

$$\frac{v_2 - v_1}{5} + \frac{v_2}{40} + \frac{v_2 - v_3}{10} \qquad \text{so} \qquad -8v_1 + 13v_2 - 4v_3 = 0$$

$$\frac{v_3 - v_2}{10} + \frac{v_3 - 11.5i_o}{5} + \frac{v_3 - 96}{4} = 0 \qquad \text{so} \qquad 0v_1 - 63v_2 + 220v_3 = 9600$$

Solving, $v_1 = 156$ V; $\quad v_2 = 120$ V; $\quad v_3 = 78$ V

[**b**] $i_o = \dfrac{v_2}{40} = \dfrac{120}{40} = 3$ A

$$i_3 = \frac{v_3 - 11.5i_o}{5} = \frac{78 - 11.5(3)}{5} = 8.7 \text{ A}$$

$$i_g = \frac{78 - 96}{4} = -4.5 \text{ A}$$

$$p_{5i_o} = -5i_o v_1 = -5(3)(156) = -2340 \text{ W(dev)}$$

$$p_{11.5i_o} = 11.5 i_o i_3 = 11.5(3)(8.7) = 300.15 \text{ W(abs)}$$

$$p_{96V} = 96(-4.5) = -432 \text{ W(dev)}$$

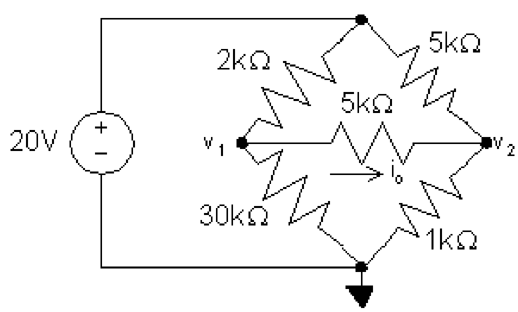
$$\sum p_{\text{dev}} = 2340 + 432 = 2772 \text{ W}$$

CHECK

$$\sum p_{\text{dis}} = \frac{156^2}{20} + \frac{(156-120)^2}{5} + \frac{120^2}{40} + \frac{(120-78)^2}{50} + (8.7)^2(5) + (4.5)^2(4) + 300.15 = 2772 \text{ W}$$

$$\therefore \quad \sum p_{\text{dev}} = \sum p_{\text{dis}} = 2772 \text{ W}$$

P 4.21



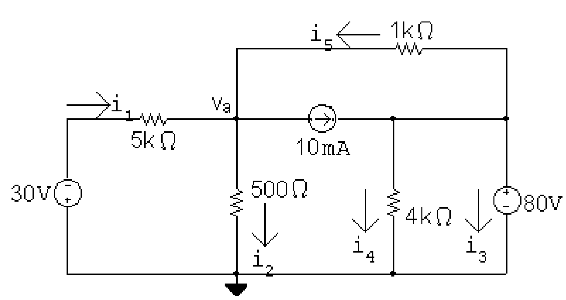
$$\frac{v_1}{30{,}000} + \frac{v_1 - v_2}{5000} + \frac{v_1 - 20}{2000} = 0 \qquad \text{so} \qquad 22v_1 - 6v_2 = 300$$

$$\frac{v_2}{1000} + \frac{v_2 - v_1}{5000} + \frac{v_2 - 20}{5000} = 0 \qquad \text{so} \qquad -v_1 + 7v_2 = 20$$

Solving, $v_1 = 15$ V; $\qquad v_2 = 5$ V

Thus, $i_o = \dfrac{v_1 - v_2}{5000} = 2$ mA

P 4.22 **[a]**



There is only one node voltage equation:

$$\frac{v_a + 30}{5000} + \frac{v_a}{500} + \frac{v_a - 80}{1000} + 0.01 = 0$$

Solving,

$v_a + 30 + 10v_a + 5v_a - 400 + 50 = 0 \qquad \text{so} \qquad 16v_a = 320$

$\therefore \qquad v_a = 20 \text{ V}$

Calculate the currents:

$$\begin{aligned} i_1 &= (-30 - 20)/5000 = -10 \text{ mA} \\ i_2 &= 20/500 = 40 \text{ mA} \\ i_4 &= 80/4000 = 20 \text{ mA} \\ i_5 &= (80 - 20)/1000 = 60 \text{ mA} \end{aligned}$$

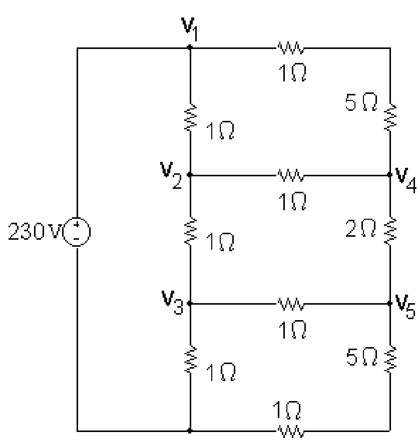
$i_3 + i_4 + i_5 - 10 \text{ mA} = 0 \quad \text{so} \quad i_3 = 0.01 - 0.02 - 0.06 = -0.07 = -70 \text{ mA}$

[**b**]
$$\begin{aligned} p_{30\text{V}} &= (30)(-0.01) = -0.3 \text{ W} \\ p_{10\text{mA}} &= (20 - 80)(0.01) = -0.6 \text{ W} \\ p_{80\text{V}} &= (80)(-0.07) = -5.6 \text{ W} \\ p_{5\text{k}} &= (-0.01)^2(5000) = 0.5 \text{ W} \\ p_{500\Omega} &= (0.04)^2(500) = 0.8 \text{ W} \\ p_{1\text{k}} &= (80 - 20)^2/(1000) = 3.6 \text{ W} \\ p_{4\text{k}} &= (80)^2/(4000) = 1.6 \text{ W} \end{aligned}$$

$\sum p_{\text{abs}} = 0.5 + 0.8 + 3.6 + 1.6 = 6.5 \text{ W}$

$\sum p_{\text{del}} = 0.3 + 0.6 + 5.6 = 6.5 \text{ W (checks!)}$

P 4.23 [**a**]



$$\frac{v_2 - 230}{1} + \frac{v_2 - v_4}{1} + \frac{v_2 - v_3}{1} = 0 \qquad \text{so} \qquad 3v_2 - 1v_3 - 1v_4 + 0v_5 = 230$$

$$\frac{v_3 - v_2}{1} + \frac{v_3}{1} + \frac{v_3 - v_5}{1} = 0 \qquad \text{so} \qquad -1v_2 + 3v_3 + 0v_4 - 1v_5 = 0$$

$$\frac{v_4 - v_2}{1} + \frac{v_4 - 230}{6} + \frac{v_4 - v_5}{2} = 0 \qquad \text{so} \qquad -12v_2 + 0v_3 + 20v_4 - 6v_5 = 460$$

$$\frac{v_5 - v_3}{1} + \frac{v_5}{6} + \frac{v_5 - v_4}{2} = 0 \qquad \text{so} \qquad 0v_2 - 12v_3 - 6v_4 + 20v_5 = 0$$

Solving, $v_2 = 150$ V; $v_3 = 80$ V; $v_4 = 140$ V; $v_5 = 90$ V

$$i_{2\Omega} = \frac{v_4 - v_5}{2} = \frac{140 - 90}{2} = 25 \text{ A}$$

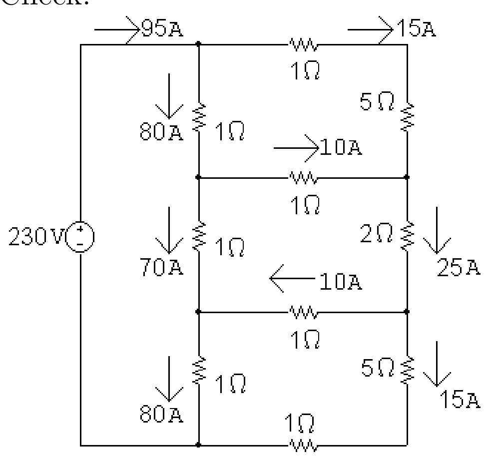
$$p_{2\Omega} = (25)^2(2) = 1250 \text{ W}$$

[b]

$$\begin{aligned} i_{230\text{V}} &= \frac{v_1 - v_2}{1} + \frac{v_1 - v_4}{6} \\ &= \frac{230 - 150}{1} + \frac{230 - 140}{6} = 80 + 15 = 95 \text{ A} \end{aligned}$$

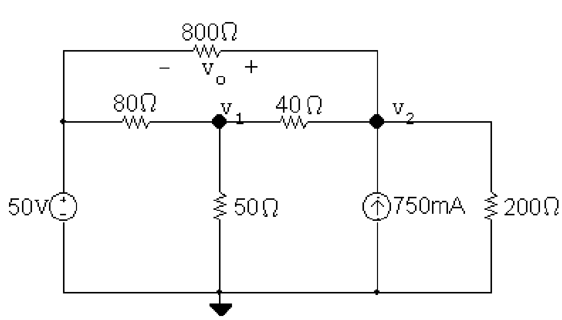
$$p_{230\text{V}} = (230)(95) = 21{,}850 \text{ W}$$

Check:



$$\begin{aligned} \sum P_{\text{dis}} &= (80)^2(1) + (70)^2(1) + (80)^2(1) + (15)^2(6) + (10)^2(1) \\ &\quad + (10)^2(1) + (25)^2(2) + (15)^2(6) = 21{,}850 \text{ W} \end{aligned}$$

P 4.24



The two node voltage equations are:

$$\frac{v_1 - 50}{80} + \frac{v_1}{50} + \frac{v_1 - v_2}{40} = 0$$
$$\frac{v_2 - v_1}{40} - 0.75 + \frac{v_2}{200} + \frac{v_2 - 50}{800} = 0$$

Place these equations in standard form:

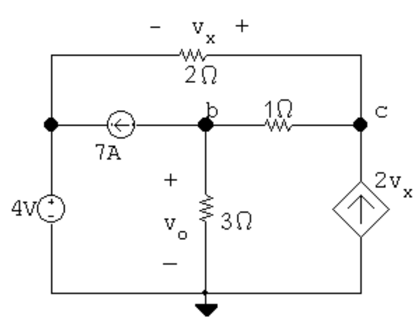
$$v_1\left(\frac{1}{80} + \frac{1}{50} + \frac{1}{40}\right) + v_2\left(-\frac{1}{40}\right) = \frac{50}{80}$$
$$v_1\left(-\frac{1}{40}\right) + v_2\left(\frac{1}{40} + \frac{1}{200} + \frac{1}{800}\right) = 0.75 + \frac{50}{800}$$

Solving, $v_1 = 34$ V; $v_2 = 53.2$ V.
Thus, $v_o = v_2 - 50 = 53.2 - 50 = 3.2$ V.
POWER CHECK:

$$i_g = (50 - 34)/80 + (50 - 53.2)/800 = 196 \text{ m A}$$
$$p_{50\text{V}} = -(50)(0.196) = -9.8 \text{ W}$$
$$p_{80\Omega} = (50 - 34)^2/80 = 3.2 \text{ W}$$
$$p_{800\Omega} = (50 - 53.2)^2/800 = 12.8 \text{ m W}$$
$$p_{40\Omega} = (53.2 - 34)^2/40 = 9.216 \text{ W}$$
$$p_{50\Omega} = 34^2/50 = 23.12 \text{ W}$$
$$p_{200\Omega} = 53.2^2/200 = 14.1512 \text{ W}$$
$$p_{0.75\text{A}} = -(53.2)(0.75) = -39.9 \text{ W}$$

$\sum p_{\text{abs}} = 3.2 + .0128 + 9.216 + 23.12 + 14.1512 = 49.7 \text{ W} = \sum p_{\text{del}} = 9.8 + 39.9 = 49.7$

P 4.25



The two node voltage equations are:

$$7 + \frac{v_\text{b}}{3} + \frac{v_\text{b} - v_\text{c}}{1} = 0$$
$$-2v_x + \frac{v_\text{c} - v_\text{b}}{1} + \frac{v_\text{c} - 4}{2} = 0$$

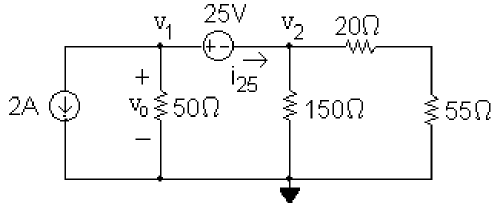
The constraint equation for the dependent source is:
$v_x = v_c - 4$

Place these equations in standard form:

$$\begin{array}{lclclcl} v_b\left(\frac{1}{3}+1\right) & + & v_c(-1) & + & v_x(0) & = & -7 \\ v_b(-1) & + & v_c\left(1+\frac{1}{2}\right) & + & v_x(-2) & = & \frac{4}{2} \\ v_b(0) & + & v_c(1) & + & v_x(-1) & = & 4 \end{array}$$

Solving, $v_c = 9$ V, $v_x = 5$ V, and $v_o = v_b = 1.5$ V

P 4.26 **[a]**



This circuit has a supernode includes the nodes v_1, v_2 and the 25 V source. The supernode equation is

$$2 + \frac{v_1}{50} + \frac{v_2}{150} + \frac{v_2}{75} = 0$$

The supernode constraint equation is

$$v_1 - v_2 = 25$$

Place these two equations in standard form:

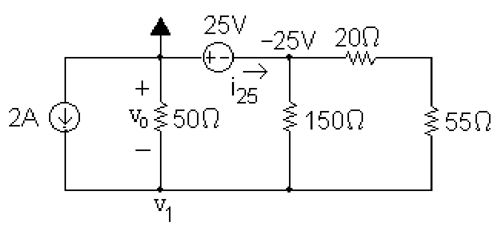
$$\begin{array}{lclcl} v_1\left(\frac{1}{50}\right) & + & v_2\left(\frac{1}{150}+\frac{1}{75}\right) & = & -2 \\ v_1(1) & + & v_2(-1) & = & 25 \end{array}$$

Solving, $v_1 = -37.5$ V and $v_2 = -62.5$ V, so $v_o = v_1 = -37.5$ V.

$$p_{2A} = (2)v_o = (2)(-37.5) = -75 \text{ W}$$

The 2 A source delivers 75 W.

[b]



This circuit now has only one non-reference essential node where the voltage is not known – note that it is not a supernode. The KCL equation at v_1 is

$$-2 + \frac{v_1}{50} + \frac{v_1 + 25}{150} + \frac{v_1 + 25}{75} = 0$$

Solving, $v_1 = 37.5$ V so $v_0 = -v_1 = -37.5$ V.

$$p_{2\mathrm{A}} = (2)v_o = (2)(-37.5) = -75 \;\; \mathrm{W}$$

The 2 A source delivers 75 W.

[**c**] The choice of a reference node in part (b) resulted in one simple KCL equation, while the choice of a reference node in part (a) resulted in a supernode KCL equation and a second supernode constraint equation. Both methods give the same result but the choice of reference node in part (b) yielded fewer equations to solve, so is the preferred method.

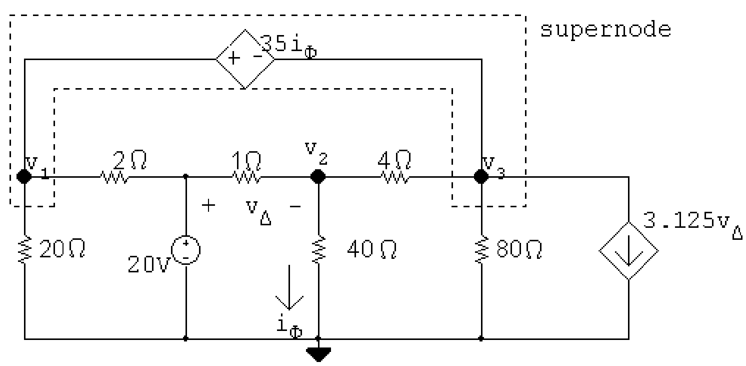
P 4.27 Place $5v_\Delta$ inside a supernode and use the lower node as a reference. Then

$$\frac{v_\Delta - 15}{10} + \frac{v_\Delta}{2} + \frac{v_\Delta - 5v_\Delta}{20} + \frac{v_\Delta - 5v_\Delta}{40} = 0$$

$$12v_\Delta = 60; \qquad v_\Delta = 5 \;\; \mathrm{V}$$

$$v_o = v_\Delta - 5v_\Delta = -4(5) = -20 \;\; \mathrm{V}$$

P 4.28



Node equations:

$$\frac{v_1}{20} + \frac{v_1 - 20}{2} + \frac{v_3 - v_2}{4} + \frac{v_3}{80} + 3.125v_\Delta = 0$$

$$\frac{v_2}{40} + \frac{v_2 - v_3}{4} + \frac{v_2 - 20}{1} = 0$$

Constraint equations:

$$v_\Delta = 20 - v_2$$

$v_1 - 35i_\phi = v_3$

$i_\phi = v_2/40$

Solving, $v_1 = -20.25$ V; $v_2 = 10$ V; $v_3 = -29$ V

Let i_g be the current delivered by the 20 V source, then

$$i_g = \frac{20 - (20.25)}{2} + \frac{20 - 10}{1} = 30.125 \text{ A}$$

p_g (delivered) $= 20(30.125) = 602.5$ W

P 4.29 For the given values of v_3 and v_4:

$$v_\Delta = 120 - v_3 = 120 - 108 = 12 \text{ V}$$

$$i_\phi = \frac{v_4 - v_3}{8} = \frac{81.6 - 108}{8} = -3.3 \text{ A}$$

$$\frac{40}{3} i_\phi = -44 \text{ V}$$

$$v_1 = v_4 + \frac{40}{3} i_\phi = 81.6 - 44 = 37.6 \text{ V}$$

Let i_a be the current from right to left through the dependent voltage source:

$$i_a = \frac{v_1}{20} + \frac{v_1 - v_2}{4} = 1.88 - 20.6 = -18.72 \text{ A}$$

Let i_b be the current supplied by the 120 V source:

$$i_b = \frac{120 - 37.6}{4} + \frac{120 - 108}{2} = 20.6 + 6 = 26.6 \text{ A}$$

Then

$$p_{120\text{V}} = -(120)(26.6) = -3192 \text{ W}$$

$$p_{\text{CCVS}} = [(40/3)(-3.3)](-18.72) = -823.68 \text{ W}$$

$$p_{\text{VCVS}} = (81.6)[1.75(12)] = 1713.6 \text{ W}$$

$$\therefore \sum p_{\text{dev}} = 3192 + 823.68 = 4015.68 \text{ W}$$

The total power dissipated by the resistors is

$$p_R = \frac{(37.6)^2}{2} + \frac{(82.4)^2}{4} + \frac{(12)^2}{2} + \frac{(108)^2}{40}$$

$$= +(3.3)^2(8) + \frac{(81.6)^2}{80} = 2302.08 \text{ W}$$

$$\therefore \quad \sum p_{\text{diss}} = 2302.08 + 1713.6 = 4015.68 \text{ W}$$

Thus, $\sum p_{\text{dev}} = \sum p_{\text{diss}}$; Agree with analyst

P 4.30 From Eq. 4.16, $i_B = v_c/(1+\beta)R_E$

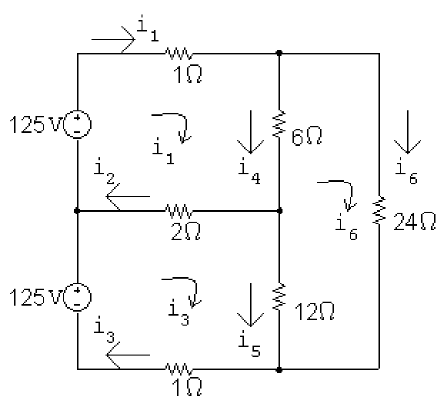
From Eq. 4.17, $i_B = (v_b - V_o)/(1+\beta)R_E$

From Eq. 4.19,

$$i_B = \frac{1}{(1+\beta)R_E}\left[\frac{V_{CC}(1+\beta)R_E R_2 + V_o R_1 R_2}{R_1R_2 + (1+\beta)R_E(R_1+R_2)} - V_o\right]$$

$$= \frac{V_{CC}R_2 - V_o(R_1+R_2)}{R_1R_2 + (1+\beta)R_E(R_1+R_2)} = \frac{[V_{CC}R_2/(R_1+R_2)] - V_o}{[R_1R_2/(R_1+R_2)] + (1+\beta)R_E}$$

P 4.31 [**a**]



The three mesh current equations are:

$$-125 + 1i_1 + 6(i_1 - i_6) + 2(i_1 - i_3) = 0$$

$$24i_6 + 12(i_6 - i_3) + 6(i_6 - i_1) = 0$$

$$-125 + 2(i_3 - i_1) + 12(i_3 - i_6) + 1i_3 = 0$$

Place these equations in standard form:

$$i_1(1+6+2)+i_3(-2)+i_6(-6) = 125$$

$$i_1(-6)+i_3(-12)+i_6(24+12+6) = 0$$

$$i_1(-2)+i_3(2+12+1)+i_6(-12) = 125$$

Solving, $i_1 = 23.76$ A; $i_3 = 18.43$ A; $i_6 = 8.66$ A
Now calculate the remaining branch currents:

$$i_2 = i_1 - i_3 = 5.33 \text{ A}$$

$$i_4 = i_1 - i_6 = 15.10 \text{ A}$$

$$i_5 = i_3 - i_6 = 9.77 \text{ A}$$

[b] $p_{\text{sources}} = p_{\text{top}} + p_{\text{bottom}} = -(125)(23.76) - (125)(18.43)$

$$= -2970 - 2304 = -5274 \text{ W}$$

Thus, the power developed in the circuit is 5274 W.
Now calculate the power absorbed by the resistors:

$p_{1\text{top}} = (23.76)^2(1) = 564.54$ W

$p_2 = (5.33)^2(2) = 56.82$ W

$p_{1\text{bot}} = (18.43)^2(1) = 339.66$ W

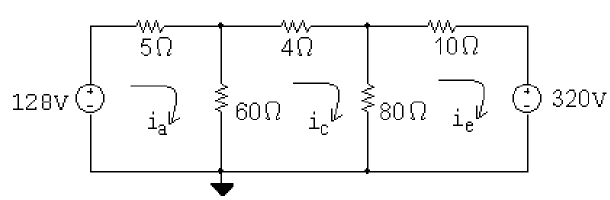
$p_6 = (15.10)^2(6) = 1368.06$ W

$p_{12} = (9.77)^2(12) = 1145.43$ W

$p_{24} = (8.66)^2(24) = 1799.89$ W

The power absorbed by the resistors is
$564.54 + 56.82 + 339.66 + 1368.06 + 1145.43 + 1799.89 = 5274$ W so the power balances.

P 4.32 **[a]**



The three mesh current equations are:

$$-128 + 5i_\mathrm{a} + 60(i_\mathrm{a} - i_\mathrm{c}) = 0$$

$$4i_\mathrm{c} + 80(i_\mathrm{c} - i_\mathrm{e}) + 60(i_\mathrm{c} - i_\mathrm{a}) = 0$$

$$320 + 80(i_\mathrm{e} - i_\mathrm{c}) + 10i_\mathrm{e} = 0$$

Place these equations in standard form:

$$i_\mathrm{a}(5 + 60) + i_\mathrm{c}(-60) + i_\mathrm{e}(0) = 128$$

$$i_\mathrm{a}(-60) + i_\mathrm{c}(4 + 80 + 60) + i_\mathrm{e}(-80) = 0$$

$$i_\mathrm{a}(0) + i_\mathrm{c}(-80) + i_\mathrm{e}(80 + 10) = -320$$

Solving, $i_\mathrm{a} = -6.8$ A; $\quad i_\mathrm{c} = -9.5$ A; $\quad i_\mathrm{e} = -12$ A

Now calculate the remaining branch currents:

$$i_\mathrm{b} = i_\mathrm{a} - i_\mathrm{c} = 2.7 \text{ A}$$

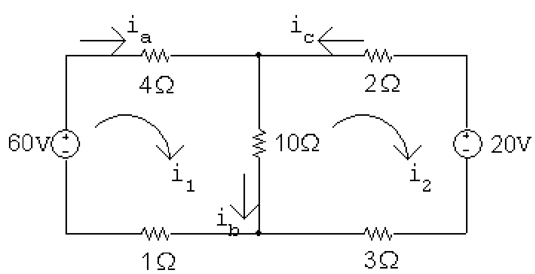
$$i_\mathrm{d} = i_\mathrm{c} - i_\mathrm{e} = 2.5 \text{ A}$$

[b] $p_{128\mathrm{V}} = -(128)i_a = -(128)(-6.8) = 870.4$ W (abs)

$p_{320\mathrm{V}} = (320)i_e = (320)(-12) = -3840$ W (dev)

Thus, the power developed in the circuit is 3840 W. Note that the resistors cannot develop power!

P 4.33 **[a]**



$$60 = 15i_1 - 10i_2$$

$$-20 = -10i_1 + 15i_2$$

Solving, $i_1 = 5.6$ A; $\quad i_2 = 2.4$ A

$i_\mathrm{a} = i_1 = 5.6$ A; $\quad i_\mathrm{b} = i_1 - i_2 = 3.2$ A; $\quad i_\mathrm{c} = -i_2 = -2.4$ A

[b] If the polarity of the 60 V source is reversed, we have

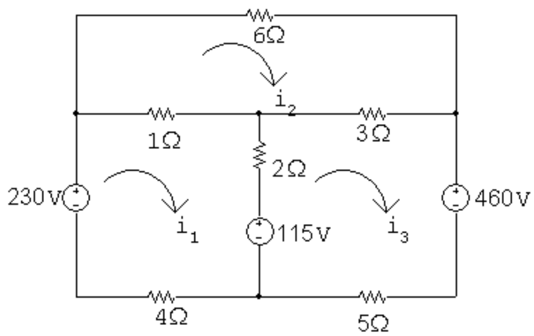
$$-60 = 15i_1 - 10i_2$$

$$-20 = -10i_1 + 15i_2$$

$$i_1 = -8.8 \text{ A} \quad \text{and} \quad i_2 = -7.2 \text{ A}$$

$$i_\text{a} = i_1 = -8.8 \text{ A}; \quad i_\text{b} = i_1 - i_2 = -1.6 \text{ A}; \quad i_\text{c} = -i_2 = 7.2 \text{ A}$$

P 4.34 [**a**]



$$230 - 115 = 7i_1 - 1i_2 - 2i_3$$

$$0 = -1i_1 + 10i_2 - 3i_3$$

$$115 - 460 = -2i_1 - 3i_2 + 10i_3$$

Solving, $i_1 = 4.4$ A; $\quad i_2 = -10.6$ A; $\quad i_3 = -36.8$ A

$$p_{230} = -230i_1 = -1012 \text{ W(del)}$$

$$p_{115} = 115(i_1 - i_3) = 4738 \text{ W(abs)}$$

$$p_{460} = 460i_3 = -16{,}928 \text{ W(del)}$$

$$\therefore \quad \sum p_\text{dev} = 17{,}940 \text{ W}$$

[**b**] $p_{6\Omega} = (10.6)^2(6) = 674.16$ W

$$p_{1\Omega} = (15)^2(1) = 225 \text{ W}$$

$$p_{3\Omega} = (26.2)^2(3) = 2059.32 \text{ W}$$

$$p_{2\Omega} = (41.2)^2(2) = 3394.88 \text{ W}$$

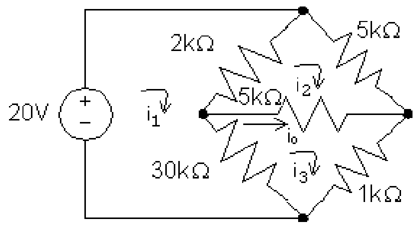
$$p_{4\Omega} = (4.4)^2(4) = 77.44 \text{ W}$$

$$p_{5\Omega} = (36.8)^2(5) = 6771.2 \text{ W}$$

$$\therefore \quad \sum p_\text{abs} = 4738 + 674.16 + 225 + 2059.32 + 3394.88$$

$$+77.44 + 6771.2 = 17{,}940 \text{ W}$$

P 4.35



The three mesh current equations are:

$$-20 + 2000(i_1 - i_2) + 30{,}000(i_1 - i_3) = 0$$

$$5000i_2 + 5000(i_2 - i_3) + 2000(i_2 - i_1) = 0$$

$$1000i_3 + 30{,}000(i_3 - i_1) + 5000(i_3 - i_2) = 0$$

Place these equations in standard form:

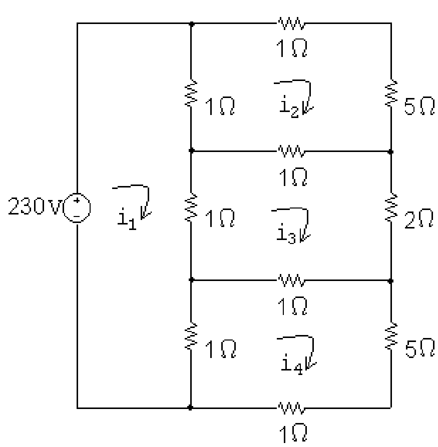
$$i_1(32{,}000) + i_2(-2000) + i_3(-30{,}000) = 20$$

$$i_1(-2000) + i_2(12{,}000) + i_3(-5000) = 0$$

$$i_1(-30{,}000) + i_2(-5000) + i_3(36{,}000) = 0$$

Solving, $i_1 = 5.5$ mA; $i_2 = 3$ mA; $i_3 = 5$ mA
Thus, $i_o = i_3 - i_2 = 2$ mA.

P 4.36 [**a**]



The four mesh current equations are:

$$
\begin{aligned}
-230 + 1(i_1 - i_2) + 1(i_1 - i_3) + 1(i_1 - i_4) &= 0 \\
6i_2 + 1(i_2 - i_3) + 1(i_2 - i_1) &= 0 \\
2i_3 + 1(i_3 - i_4) + 1(i_3 - i_1) + 1(i_3 - i_2) &= 0 \\
6i_4 + 1(i_4 - i_1) + 1(i_4 - i_3) &= 0
\end{aligned}
$$

Place these equations in standard form:

$$
\begin{aligned}
i_1(3) + i_2(-1) + i_3(-1) + i_4(-1) &= 230 \\
i_1(-1) + i_2(8) + i_3(-1) + i_4(0) &= 0 \\
i_1(-1) + i_2(-1) + i_3(5) + i_4(-1) &= 0 \\
i_1(-1) + i_2(0) + i_3(-1) + i_4(8) &= 0
\end{aligned}
$$

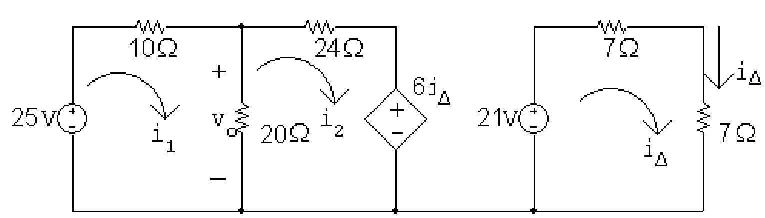
Solving, $i_1 = 95$ A; $i_2 = 15$ A; $i_3 = 25$ A; $i_4 = 15$ A

The power absorbed by the $5\,\Omega$ resistor is

$p_5 = i_3^2(2) = (25)^2(2) = 1250$ W

[b] $p_{230} = -(230)i_1 = -(230)(95) = -21{,}850$ W

P 4.37 **[a]**



$25 = 30i_1 - 20i_2 + 0i_\Delta$

$0 = -20i_1 + 44i_2 + 6i_\Delta$

$21 = 0i_1 + 0i_2 + 14i_\Delta$

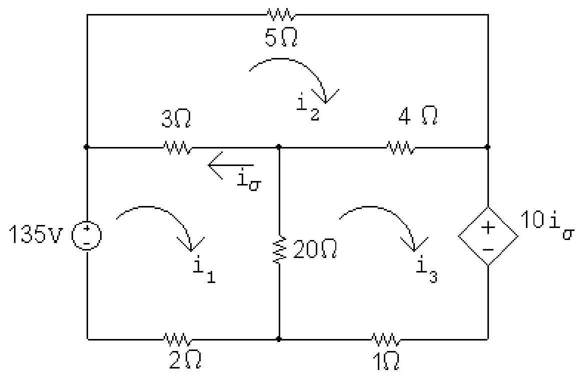
Solving, $i_1 = 1$ A; $i_2 = 0.25$ A; $i_\Delta = 1.5$ A

$v_o = 20(i_1 - i_2) = 20(0.75) = 15$ V

[b] $p_{6i_\Delta} = 6i_\Delta i_2 = (6)(1.5)(0.25) = 2.25$ W (abs)

$\therefore\ p_{6i_\Delta}$ (deliver) $= -2.25$ W

P 4.38



$-135 + 25i_1 - 3i_2 - 20i_3 + 0i_\sigma = 0$

$-3i_1 + 12i_2 - 4i_3 + 0i_\sigma = 0$

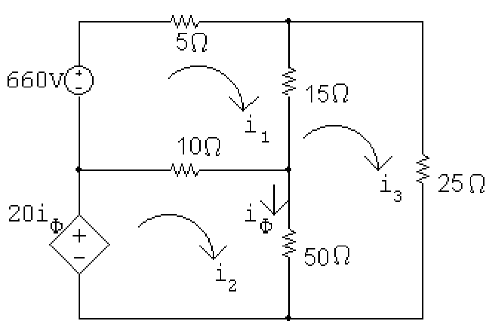
$-20i_1 - 4i_2 + 25i_3 + 10i_\sigma = 0$

$1i_1 - 1i_2 + 0i_3 + 1i_\sigma = 0$

Solving, $i_1 = 64.8$ A $\quad i_2 = 39$ A $\quad i_3 = 68.4$ A $\quad i_\sigma = -25.8$ A

$p_{20\Omega} = (68.4 - 64.8)^2(20) = 259.2$ W

P 4.39



$660 = 30i_1 - 10i_2 - 15i_3$

$20i_\phi = -10i_1 + 60i_2 - 50i_3$

$0 = -15i_1 - 50i_2 + 90i_3$

$i_\phi = i_2 - i_3$

Solving, $i_1 = 42$ A; $\quad i_2 = 27$ A; $\quad i_3 = 22$ A; $\quad i_\phi = 5$ A

$20i_\phi = 100$ V

$p_{20i_\phi} = -100i_2 = -100(27) = -2700$ W

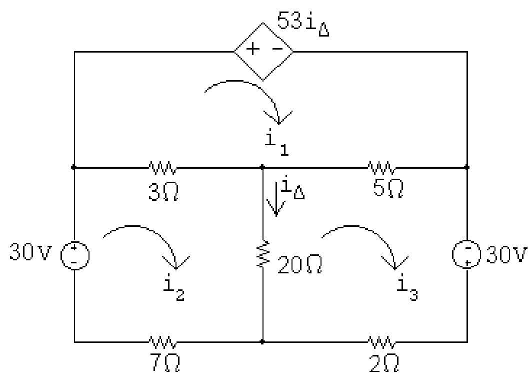
$\therefore \quad p_{20i_\phi}$ (developed) $= 2700$ W

CHECK:

$p_{660\text{V}} = -660(42) = -27{,}720$ W (dev)

$$\begin{aligned} \therefore \quad \sum P_{\text{dev}} &= 27{,}720 + 2700 = 30{,}420 \text{ W} \\ \sum P_{\text{dis}} &= (42)^2(5) + (22)^2(25) + (20)^2(15) + (5)^2(50) + \\ &\quad (15)^2(10) \\ &= 30{,}420 \text{ W} \end{aligned}$$

P 4.40



Mesh equations:

$53i_\Delta + 8i_1 - 3i_2 - 5i_3 = 0$

$0i_\Delta - 3i_1 + 30i_2 - 20i_3 = 30$

$0i_\Delta - 5i_1 - 20i_2 + 27i_3 = 30$

Constraint equations:

$i_\Delta = i_2 - i_3$

Solving, $i_1 = 110$ A; $\quad i_2 = 52$ A; $\quad i_3 = 60$ A; $\quad i_\Delta = -8$ A

$p_{\text{depsource}} = 53i_\Delta i_1 = (53)(-8)(110) = -46{,}640$ W

Therefore, the dependent source is developing 46,640 W.

CHECK:

$p_{30\text{V}} = -30i_2 = -1560$ W (left source)

$p_{30\text{V}} = -30i_3 = -1800$ W (right source)

$\sum p_{\text{dev}} = 46{,}640 + 1560 + 1800 = 50$ k W

$p_{3\Omega} = (110 - 52)^2(3) = 10{,}092$ W

$p_{5\Omega} = (110 - 60)^2(5) = 12{,}500$ W

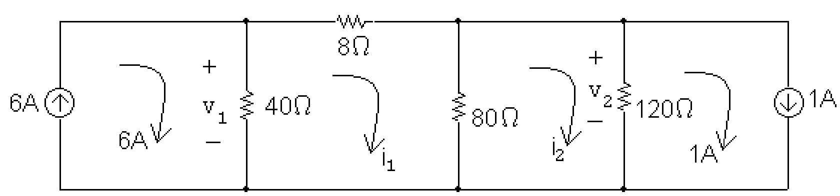
$p_{20\Omega} = (-8)^2(20) = 1280$ W

$p_{7\Omega} = (52)^2(7) = 18{,}928$ W

$p_{2\Omega} = (60)^2(2) = 7200$ W

$\sum p_{\text{diss}} = 10{,}092 + 12{,}500 + 1280 + 18{,}928 + 7200 = 50$ kW

P 4.41



Mesh equations:

$128i_1 - 80i_2 = 240$

$-80i_1 + 200i_2 = 120$

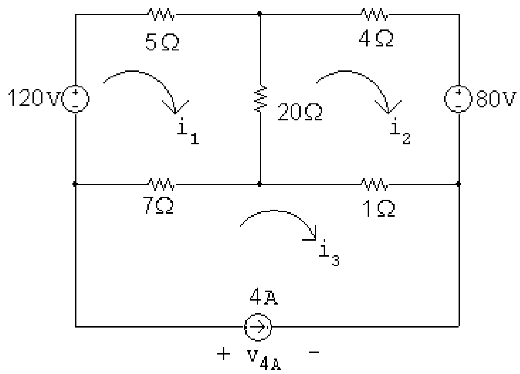
Solving,

$i_1 = 3$ A; $\qquad i_2 = 1.8$ A

Therefore,

$v_1 = 40(6 - 3) = 120$ V; $\qquad v_2 = 120(1.8 - 1) = 96$ V

P 4.42



$$120 = 32i_1 - 20i_2 - 7i_3$$

$$-80 = -20i_1 + 25i_2 - 1i_3$$

$$-4 = 0i_1 + 0i_2 + 1i_3$$

Solving, $i_1 = 1.55$ A; $i_2 = -2.12$ A; $i_3 = -4$ A

[a] $v_{4\text{A}} = 7(-4 - 1.55) + 1(-4 + 2.12)$

$$= -40.73 \text{ V}$$

$p_{4\text{A}} = 4v_{4\text{A}} = 4(-40.73) = -162.92$ W

Therefore, the 4 A source delivers 162.92 W.

[b] $p_{120\text{V}} = -120(1.55) = -186$ W

$p_{80\text{V}} = -80(-2.12) = 169.6$ W

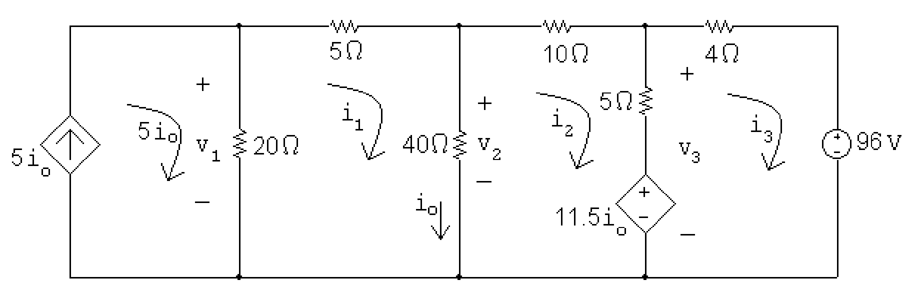
Therefore, the total power delivered is $162.92 + 186 + 169.6 = 518.52$ W

[c] $\sum p_{\text{resistors}} = (1.55)^2(5) + (2.12)^2(4) + (3.67)^2(20) + (5.55)^2(7) + (1.88)^2(1)$

$$= 518.52 \text{ W}$$

$\sum p_{\text{abs}} = 518.52 \text{ W} = \sum p_{\text{del}}$ (CHECKS)

P 4.43 **[a]**



Mesh equations:

$$65i_1 - 40i_2 + 0i_3 - 100i_o = 0$$

$$-40i_1 + 55i_2 - 5i_3 + 11.5i_o = 0$$

$$0i_1 - 5i_2 + 9i_3 - 11.5i_o = 0$$

$$-1i_1 + 1i_2 + 0i_3 + 1i_o = 0$$

Solving,

$i_1 = 7.2$ A; $\quad i_2 = 4.2$ A; $\quad i_3 = -4.5$ A; $\quad i_o = 3$ A

Therefore,

$v_1 = 20[5(3) - 7.2] = 156$ V; $\quad v_2 = 40(7.2 - 4.2) = 120$ V

$v_3 = 5(4.2 + 4.5) + 11.5(3) = 78$ V

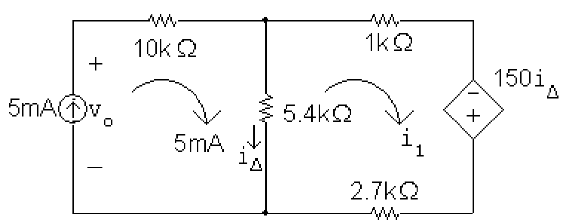
[b] $p_{5i_o} = -5i_ov_1 = -5(3)(156) = -2340$ W

$p_{11.5i_o} = 11.5i_o(i_2 - i_3) = 11.5(3)(4.2 + 4.5) = 300.15$ W

$p_{96\text{V}} = 96i_3 = 96(-4.5) = -432$ W

Thus, the total power dissipated in the circuit, which equals the total power developed in the circuit is $2340 + 432 = 2772$ W.

P 4.44 **[a]**

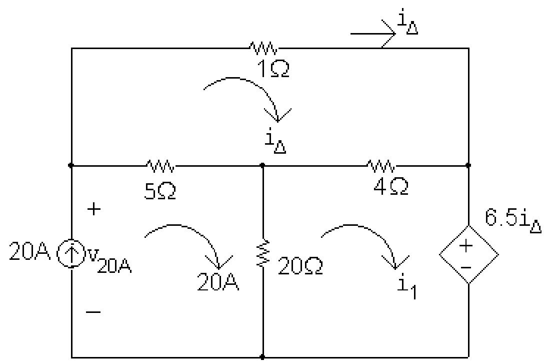


The mesh current equation for the right mesh is:
$5400(i_1 - 0.005) + 3700i_1 - 150(0.005 - i_1) = 0$
Solving, $\quad 9250i_1 = 27.75 \quad \therefore \quad i_1 = 3$ mA
Then, $\quad i_\Delta = 5 - 3 = 2$ mA

[b] $v_o = (0.005)(10{,}000) + (5400)(0.002) = 60.8$ V
$p_{5\text{mA}} = -(60.8)(0.005) = -304$ mW
Thus, the 5 mA source delivers 304 mW

[c] $p_{\text{dep source}} = -150i_\Delta i_1 = (-150)(0.002)(0.003) = -0.9$ mW
The dependent source delivers 0.9 mW.

P 4.45



Mesh equations:

$$10i_\Delta - 4i_1 = 0$$

$$-4i_\Delta + 24i_1 + 6.5i_\Delta = 400$$

Solving, $i_1 = 15$ A; $\qquad i_\Delta = 16$ A

$$v_{20\text{A}} = 1i_\Delta + 6.5i_\Delta = 7.5(16) = 120 \text{ V}$$

$$p_{20\text{A}} = -20v_{20\text{A}} = -(20)(120) = -2400 \text{ W (del)}$$

$$p_{6.5i_\Delta} = 6.5i_\Delta i_1 = (6.5)(16)(15) = 1560 \text{ W (abs)}$$

Therefore, the independent source is developing 2400 W, all other elements are absorbing power, and the total power developed is thus 2400 W.
CHECK:

$$p_{1\Omega} = (16)^2(1) = 256 \text{ W}$$

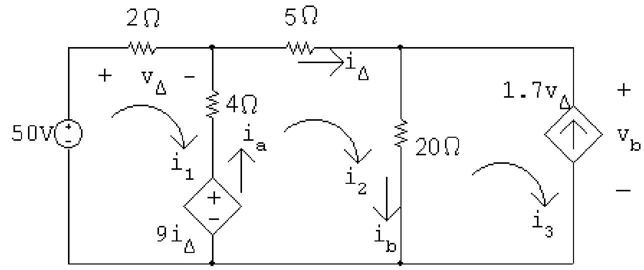
$$p_{5\Omega} = (20 - 16)^2(5) = 80 \text{ W}$$

$$p_{4\Omega} = (1)^2(4) = 4 \text{ W}$$

$$p_{20\Omega} = (20 - 15)^2(20) = 500 \text{ W}$$

$$\sum p_{\text{abs}} = 1560 + 256 + 80 + 4 + 500 = 2400 \text{ W (CHECKS)}$$

P 4.46 [**a**]



Mesh equations:

$$-50 + 6i_1 - 4i_2 + 9i_\Delta = 0$$

$$-9i_\Delta - 4i_1 + 29i_2 - 20i_3 = 0$$

Constraint equations:

$i_\Delta = i_2; \qquad i_3 = -1.7v_\Delta; \qquad v_\Delta = 2i_1$

Solving, $i_1 = -5$ A; $\quad i_2 = 16$ A; $\quad i_3 = 17$ A; $\quad v_\Delta = -10$ V

$$9i_\Delta = 9(16) = 144 \text{ V}$$

$$i_\text{a} = i_2 - i_1 = 21 \text{ A}$$

$$i_\text{b} = i_2 - i_3 = -1 \text{ A}$$

$$v_\text{b} = 20i_\text{b} = -20 \text{ V}$$

$$p_{50\text{V}} = -50i_1 = 250 \text{ W (absorbing)}$$

$$p_{9i_\Delta} = -i_\text{a}(9i_\Delta) = -(21)(144) = -3024 \text{ W (delivering)}$$

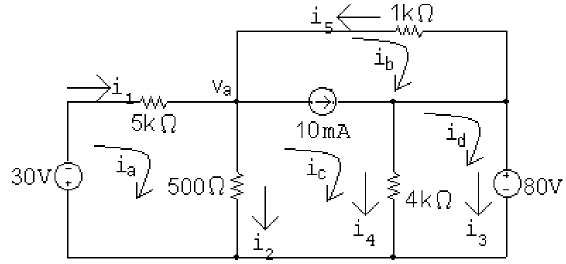
$$p_{1.7\text{V}} = -1.7v_\Delta v_\text{b} = i_3 v_\text{b} = (17)(-20) = -340 \text{ W (delivering)}$$

[**b**] $\sum P_\text{dev} = 3024 + 340 = 3364$ W

$$\sum P_\text{dis} = 250 + (-5)^2(2) + (21)^2(4) + (16)^2(5) + (-1)^2(20)$$

$$= 3364 \text{ W}$$

P 4.47 [**a**]



Supermesh equations:

$$1000i_b + 4000(i_c - i_d) + 500(i_c - i_a) = 0$$

$$i_c - i_b = 0.01$$

Two remaining mesh equations:

$$5500i_a - 500i_c = -30$$

$$4000i_d - 4000i_c = -80$$

In standard form,

$$-500i_a + 1000i_b + 4500i_c - 4000i_d = 0$$

$$0i_a - 1i_b + 1i_c + 0i_d = 0.01$$

$$5500i_a + 0i_b - 500i_c + 0i_d = -30$$

$$0i_a + 0i_b - 4000i_c + 4000i_d = -80$$

Solving:

$$i_a = -10 \text{ mA}; \qquad i_b = -60 \text{ mA}; \qquad i_c = -50 \text{ mA}; \qquad i_d = -70 \text{ mA}$$

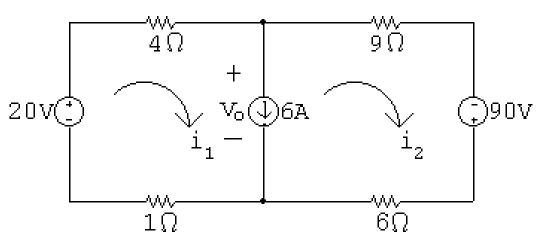
Then,

$$i_1 = i_a = -10 \text{ mA}; \qquad i_2 = i_a - i_c = 40 \text{ mA}; \qquad i_3 = i_d = -70 \text{ mA}$$

[b] $p_{\text{sources}} = 30(-0.01) + [1000(-0.06)](0.01) + 80(-0.07) = -6.5 \text{ W}$

$$p_{\text{resistors}} = 1000(0.06)^2 + 5000(0.01)^2 + 500(0.04)^2$$
$$+4000(-0.05 + 0.07)^2 = 6.5 \text{ W}$$

P 4.48



$$-20 + 4i_1 + 9i_2 - 90 + 6i_2 + 1i_1 = 0; \quad i_1 - i_2 = 6$$

Solving, $i_1 = 10 \text{ A}; \quad i_2 = 4 \text{ A}$

$$p_{20V} = -20i_1 = -200 \text{ W (diss)}$$

$$p_{4\,\Omega} = (10)^2(4) = 400 \text{ W}$$

$p_{1\,\Omega} = (10)^2(1) = 100\ \text{W}$

$p_{9\,\Omega} = (4)^2(9) = 144\ \text{W}$

$p_{6\,\Omega} = (4)^2(6) = 96\ \text{W}$

$v_o = 9(4) - 90 + 6(4) = -30\ \text{V}$

$p_{6A} = 6v_o = -180\ \text{W}$

$p_{90V} = -90i_2 = -360\ \text{W}$

$\sum p_{\text{dev}} = 200 + 180 + 360 = 740\ \text{W}$

$\sum p_{\text{diss}} = 400 + 100 + 144 + 96 = 740\ \text{W}$

Thus the total power dissipated is 740 W.

P 4.49 [**a**] Summing around the supermesh used in the solution to Problem 4.48 gives

$-60 + 4i_1 + 9i_2 - 90 + 6i_2 + 1i_1 = 0; \qquad i_1 - i_2 = 6$

$\therefore\ \ i_1 = 12\ \text{A}; \qquad i_2 = 6\ \text{A}$

$p_{60\text{V}} = -60(12) = -720\ \text{W (del)}$

$v_o = 9(6) - 90 + 6(6) = 0\ \text{V}$

$p_{6\text{A}} = 6v_o = 0\ \text{W}$

$p_{90\text{V}} = -90i_2 = -540\ \text{W (del)}$

$\sum p_{\text{diss}} = (12)^2(4+1) + (6)^2(9+6) = 1260\ \text{W}$

$\sum p_{\text{dev}} = 720 + 0 + 540 = 1260\ \text{W} = \sum p_{\text{diss}}$

[**b**] With 6 A current source replaced with a short circuit

$5i_1 = 60; \qquad 15i_2 = 90$

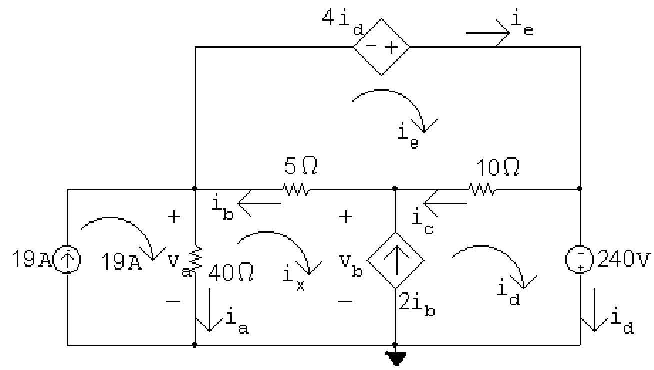
Solving,

$i_1 = 12\ \text{A}, \qquad i_2 = 6\ \text{A}$

$\therefore\ \ \sum P_{\text{sources}} = -(60)(12) - (90)(6) = -1260\ \text{W}$

[**c**] A 6 A source with zero terminal voltage is equivalent to a short circuit carrying 6 A.

P 4.50 [**a**]



$$-4i_d + 10(i_e - i_d) + 5(i_e - i_x) = 0$$

$$5(i_x - i_e) + 10(i_d - i_e) - 240 + 40(i_x - 19) = 0$$

$$i_d - i_x = 2i_b = 2(i_e - i_x)$$

Solving, $i_d = 10$ A; $\quad i_e = 18$ A; $\quad i_x = 26$ A

$i_a = 19 - i_x = -7$ A; $\quad i_b = i_e - i_x = -8$ A; $\quad i_c = i_e - i_d = 8$ A;

[**b**] $v_\mathrm{a} = 40i_\mathrm{a} = -280$ V; $\qquad v_\mathrm{b} = 5i_b + 40i_a = -320$ V

$$p_{19\mathrm{A}} = -19v_a = 5320 \text{ W}$$

$$p_{4i_d} = -4i_d i_e = -720 \text{ W}$$

$$p_{2i_a} = -2i_b v_b = -5120 \text{ W}$$

$$p_{240\mathrm{V}} = -240i_d = -2400 \text{ W}$$

$$p_{40\Omega} = (7)^2(40) = 1960 \text{ W} =$$

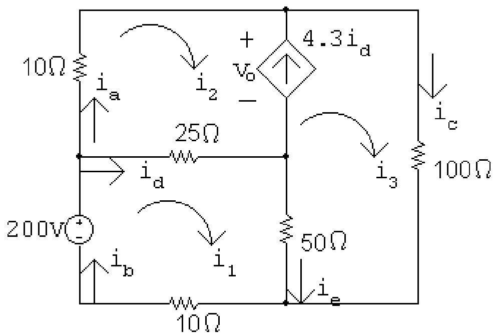
$$p_{5\Omega} = (8)^2(5) = 320 \text{ W}$$

$$p_{10\Omega} = (8)^2(10) = 640 \text{ W}$$

$$\sum P_\mathrm{gen} = 720 + 5120 + 2400 = 8240 \text{ W}$$

$$\sum P_\mathrm{diss} = 5320 + 1960 + 320 + 640 = 8240 \text{ W}$$

P 4.51 [**a**]



$$200 = 85i_1 - 25i_2 - 50i_3$$

$$0 = -75i_1 + 35i_2 + 150i_3 \qquad \text{(supermesh)}$$

$$i_3 - i_2 = 4.3(i_1 - i_2)$$

Solving, $i_1 = 4.6$ A; $\quad i_2 = 5.7$ A; $\quad i_3 = 0.97$ A

$i_{\rm a} = i_2 = 5.7$ A; $\quad i_{\rm b} = i_1 = 4.6$ A

$i_{\rm c} = i_3 = 0.97$ A; $\quad i_{\rm d} = i_1 - i_2 = -1.1$ A

$i_{\rm e} = i_1 - i_3 = 3.63$ A

[**b**] $10i_2 + v_o + 25(i_2 - i_1) = 0$

$$\therefore \quad v_o = -57 - 27.5 = -84.5 \text{ V}$$

$$p_{4.3i_{\rm d}} = -v_o(4.3i_{\rm d}) = -(-84.5)(4.3)(-1.1) = -399.685 \text{ W(dev)}$$

$$p_{200\rm V} = -200(4.6) = -920 \text{ W(dev)}$$

$$\begin{aligned} \sum P_{\rm dev} &= 1319.685 \text{ W} \\ \sum P_{\rm dis} &= (5.7)^2 10 + (1.1)^2(25) + (0.97)^2 100 + (4.6)^2(10) + \\ &\quad (3.63)^2(50) \\ &= 1319.685 \text{ W} \end{aligned}$$

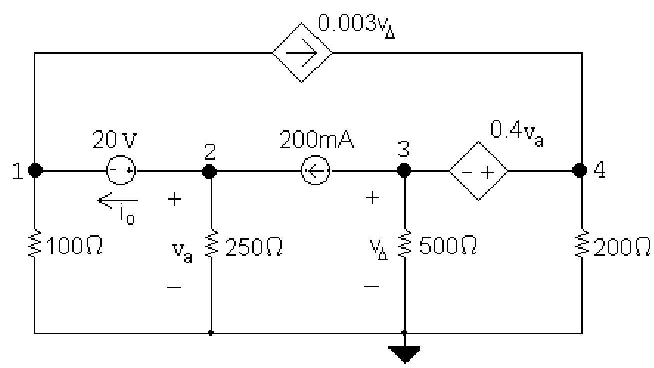
$$\therefore \quad \sum P_{\rm dev} = \sum P_{\rm dis} = 1319.685 \text{ W}$$

P 4.52 [**a**] The node voltage method requires summing the currents at two supernodes in terms of four node voltages and using two constraint equations to reduce the system of equations to two unknowns. If the connection at the bottom of the circuit is used as the reference node, then the voltages controlling the dependent sources are node voltages. This makes it easy to formulate the constraint equations. The current in the 20 V source is obtained by summing the currents at either terminal of the source.

The mesh current method requires summing the voltages around the two meshes not containing current sources in terms of four mesh currents. In addition the voltages controlling the dependent sources must be expressed in terms of the mesh currents. Thus the constraint equations are more complicated, and the reduction to two equations and two unknowns involves more algebraic manipulation. The current in the 20 V source is found by subtracting two mesh currents.

Because the constraint equations are easier to formulate in the node voltage method, it is the preferred approach.

[b]



Node voltage equations:

$$\frac{v_1}{100} + 0.003v_\Delta + \frac{v_2}{250} - 0.2 = 0$$

$$0.2 + \frac{v_3}{100} + \frac{v_4}{200} - 0.003v_\Delta = 0$$

Constraints:

$v_2 = v_a$; $\quad v_3 = v_\Delta$; $\quad v_4 - v_3 = 0.4v_a$; $\quad v_2 - v_1 = 20$

Solving, $v_1 = 24$ V; $\quad v_2 = 44$ V; $\quad v_3 = -72$ V; $\quad v_4 = -54$ V.

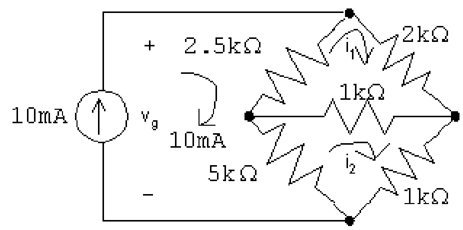
$$i_o = 0.2 - \frac{v_2}{250} = 24 \text{ m A}$$

$$p_{20\text{V}} = 20(0.024) = 480 \text{ m W}$$

Thus, the 20 V source absorbs 480 mW.

P 4.53 **[a]** There are three unknown node voltages and only two unknown mesh currents. Use the mesh current method to minimize the number of simultaneous equations.

[**b**]



The mesh current equations:

$$2500(i_1 - 0.01) + 2000i_1 + 1000(i_1 - i_2) = 0$$

$$5000(i_2 - 0.01) + 1000(i_2 - i_1) + 1000i_2 = 0$$

Place the equations in standard form:

$$i_1(2500 + 2000 + 1000) + i_2(-1000) = 25$$

$$i_1(-1000) + i_2(5000 + 1000 + 1000) = 50$$

Solving, $i_1 = 6$ mA; $\qquad i_2 = 8$ mA

Find the power in the 1 kΩ resistor:

$i_{1k} = i_1 - i_2 = -2$ m A

$p_{1k} = (-0.002)^2(1000) = 4$ mW

[**c**] No, the voltage across the 10 A current source is readily available from the mesh currents, and solving two simultaneous mesh-current equations is less work than solving three node voltage equations.

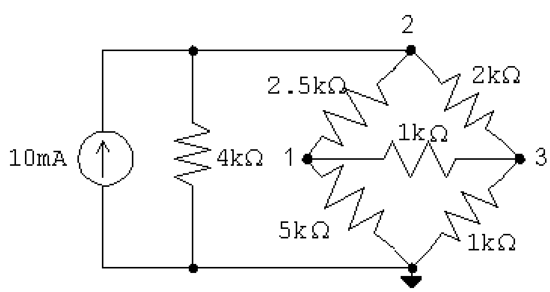
[**d**] $v_g = 2000i_1 + 1000i_2 = 12 + 8 = 20$ V

$p_{10\text{mA}} = -(20)(0.01) = -200$ m W

Thus the 10 mA source develops 200 mW.

P 4.54 [**a**] There are three unknown node voltages and three unknown mesh currents, so the number of simultaneous equations required is the same for both methods. The node voltage method has the advantage of having to solve the three simultaneous equations for one unknown voltage provided the connection at either the top or bottom of the circuit is used as the reference node. Therefore recommend the node voltage method.

[**b**]



The node voltage equations are:

$$\begin{aligned}
\frac{v_1}{5000}+\frac{v_1-v_2}{2500}+\frac{v_1-v_3}{1000} &= 0\\
-0.01+\frac{v_2}{4000}+\frac{v_2-v_1}{2500}+\frac{v_2-v_3}{2000} &= 0\\
\frac{v_3-v_1}{1000}+\frac{v_3-v_2}{2000}+\frac{v_3}{1000} &= 0
\end{aligned}$$

Put the equations in standard form:

$$\begin{aligned}
v_1\left(\frac{1}{5000}+\frac{1}{2500}+\frac{1}{1000}\right)+v_2\left(-\frac{1}{2500}\right)+v_3\left(-\frac{1}{1000}\right) &= 0\\
v_1\left(-\frac{1}{2500}\right)+v_2\left(\frac{1}{4000}+\frac{1}{2500}+\frac{1}{2000}\right)+v_3\left(-\frac{1}{2000}\right) &= 0.01\\
v_1\left(-\frac{1}{1000}\right)+v_2\left(-\frac{1}{2000}\right)+v_3\left(\frac{1}{2000}+\frac{1}{1000}+\frac{1}{1000}\right) &= 0
\end{aligned}$$

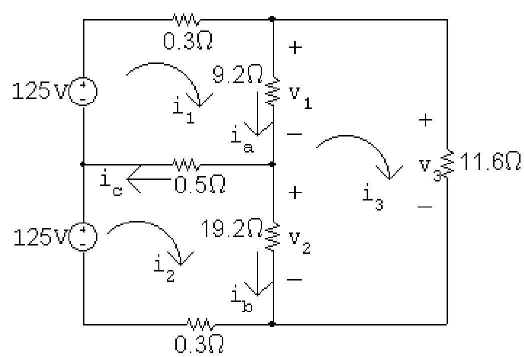
Solving, $v_1 = 6.67$ V; $v_2 = 13.33$ V; $v_3 = 5.33$ V

$p_{10\mathrm{m}} = -(13.33)(0.01) = -133.33$ m W

Therefore, the 10 mA source is developing 133.33 mW

P 4.55 **[a]** Both the mesh-current method and the node-voltage method require three equations. The mesh-current method is a bit more intuitive due to the presence of the voltage sources. We choose the mesh-current method, although technically it is a toss-up.

[b]



$$\begin{aligned}
125 &= 10i_1 - 0.5i_2 - 9.2i_3\\
125 &= -0.5i_1 + 20i_2 - 19.2i_3\\
0 &= -9.2i_1 - 19.2i_2 + 40i_3
\end{aligned}$$

Solving, $i_1 = 32.25$ A; $i_2 = 26.29$ A; $i_3 = 20.04$ A

$$\begin{aligned}
v_1 &= 9.2(i_1 - i_3) = 112.35 \text{ V}\\
v_2 &= 19.2(i_2 - i_3) = 120.09 \text{ V}\\
v_3 &= 11.6i_3 = 232.44 \text{ V}
\end{aligned}$$

[**c**] $p_{R1} = (i_1 - i_3)^2(9.2) = 1371.93$ W

$p_{R2} = (i_2 - i_3)^2(19.2) = 751.13$ W

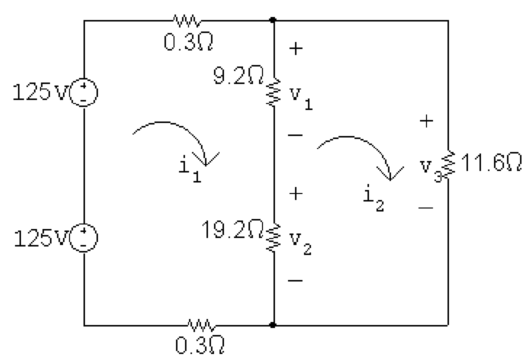
$p_{R3} = i_3^2(11.6) = 4657.52$ W

[**d**] $\sum p_{\text{dev}} = 125(i_1 + i_2) = 7317.72$ W

$\sum p_{\text{load}} = 6780.58$ W

$$\%\text{ delivered } = \frac{6780.58}{7317.72} \times 100 = 92.66\%$$

[**e**]



$250 = 29i_1 - 28.4i_2$

$0 = -28.4i_1 + 40i_2$

Solving, $i_1 = 28.29$ A; $\quad i_2 = 20.09$ A

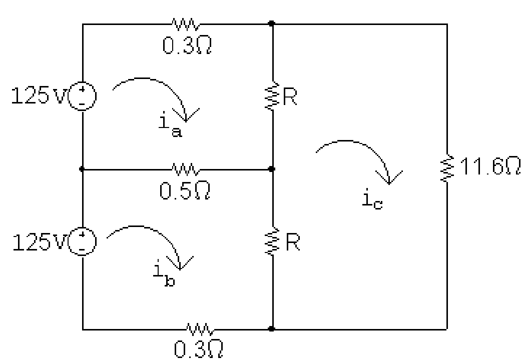
$i_1 - i_2 = 8.2$ A

$v_1 = (8.2)(9.2) = 75.44$ V

$v_2 = (8.2)(19.2) = 157.44$ V

Note v_1 is low and v_2 is high. Therefore, loads designed for 125 V would not function properly, and could be damaged.

P 4.56



The mesh current equations:

$$125 = (R + 0.8)i_{\rm a} - 0.5i_{\rm b} - Ri_{\rm c}$$

$$125 = -0.5i_{\rm a} + (R + 0.8)i_{\rm b} - Ri_{\rm c}$$

$$\therefore \quad (R + 0.8)i_{\rm a} - 0.5i_{\rm b} - Ri_{\rm c} = -0.5i_{\rm a} + (R + 0.8)i_{\rm b} - Ri_{\rm c}$$

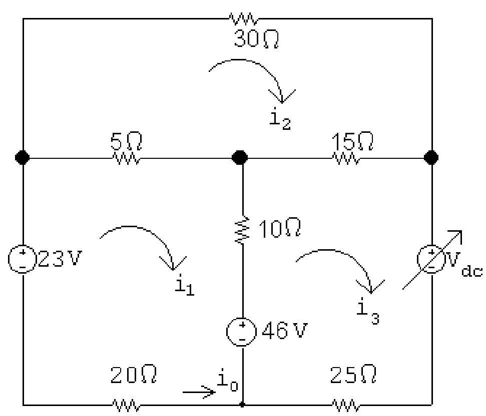
$$\therefore \quad (R + 0.8)i_{\rm a} - 0.5i_{\rm b} = -0.5i_{\rm a} + (R + 0.8)i_{\rm b}$$

$$\therefore \quad (R + 1.3)i_{\rm a} = (R + 1.3)i_{\rm b}$$

Thus

$$i_{\rm a} = i_{\rm b} \qquad \text{so} \qquad i_o = i_{\rm b} - i_{\rm a} = 0$$

P 4.57 **[a]**



Write the mesh current equations. Note that if $i_o = 0$, then $i_1 = 0$:

$$-23 + 5(-i_2) + 10(-i_3) + 46 = 0$$

$$30i_2 + 15(i_2 - i_3) + 5i_2 = 0$$

$$V_{\rm dc} + 25i_3 - 46 + 10i_3 + 15(i_3 - i_2) = 0$$

Place the equations in standard form:

$$i_2(-5) + i_3(-10) + V_{\rm dc}(0) = -23$$

$$i_2(30 + 15 + 5) + i_3(-15) + V_{\rm dc}(0) = 0$$

$$i_2(-15) + i_3(25 + 10 + 15) + V_{\rm dc}(1) = 46$$

Solving, $\quad i_2 = 0.6$ A; $\quad i_3 = 2$ A; $\quad V_{\rm dc} = -45$ V

Thus, the value of $V_{\rm dc}$ required to make $i_o = 0$ is -45 V.

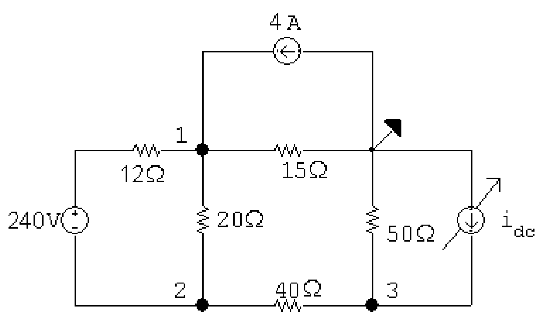
[b] Calculate the power:

$$\begin{aligned}
p_{23\text{V}} &= -(23)(0) = 0 \text{ W} \\
p_{46\text{V}} &= -(46)(2) = -92 \text{ W} \\
p_{\text{Vdc}} &= (-45)(2) = -90 \text{ W} \\
p_{30\Omega} &= (30)(0.6)^2 = 10.8 \text{ W} \\
p_{5\Omega} &= (5)(0.6)^2 = 1.8 \text{ W} \\
p_{15\Omega} &= (15)(2-0.6)^2 = 29.4 \text{ W} \\
p_{10\Omega} &= (10)(2)^2 = 40 \text{ W} \\
p_{20\Omega} &= (20)(0)^2 = 0 \text{ W} \\
p_{25\Omega} &= (25)(2)^2 = 100 \text{ W}
\end{aligned}$$

$$\sum p_{\text{dev}} = 92 + 90 = 182 \text{ W}$$

$$\sum p_{\text{dis}} = 10.8 + 1.8 + 29.4 + 40 + 0 + 100 = 182 \text{ W(checks)}$$

P 4.58 Choose the reference node so that a node voltage is identical to the voltage across the 4 A source; thus:



Since the 4 A source is developing 0 W, v_1 must be 0 V.

Since v_1 is known, we can sum the currents away from node 1 to find v_2; thus:

$$\frac{0-(240+v_2)}{12} + \frac{0-v_2}{20} + \frac{0}{15} - 4 = 0$$

$$\therefore \quad v_2 = -180 \text{ V}$$

Now that we know v_2 we sum the currents away from node 2 to find v_3; thus:

$$\frac{v_2+240-0}{12} + \frac{v_2-0}{20} + \frac{v_2-v_3}{40} = 0$$

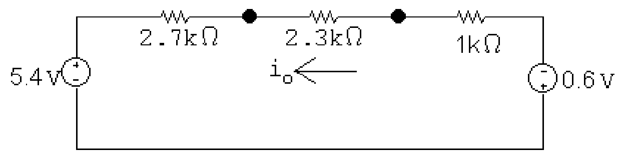
$$\therefore \quad v_3 = -340 \text{ V}$$

Now that we know v_3 we sum the currents away from node 3 to find $i_{\rm dc}$; thus:

$$\frac{v_3}{50} + \frac{v_3 - v_2}{40} = i_{\rm dc}$$

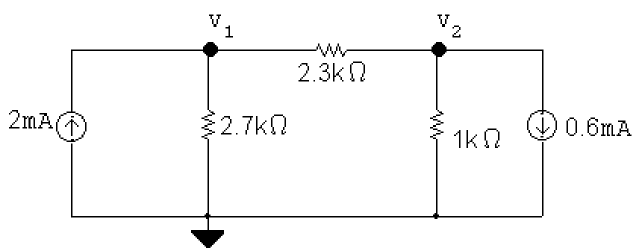
$$\therefore \quad i_{\rm dc} = -10.8 \text{ A}$$

P 4.59 **[a]** Apply source transformations to both current sources to get



$$i_o = \frac{-(5.4 + 0.6)}{2700 + 2300 + 1000} = -1 \text{ mA}$$

[b]



The node voltage equations:

$$-2 \times 10^{-3} + \frac{v_1}{2700} + \frac{v_1 - v_2}{2300} = 0$$

$$\frac{v_2}{1000} + \frac{v_2 - v_1}{2300} + 0.6 \times 10^{-3} = 0$$

Place these equations in standard form:

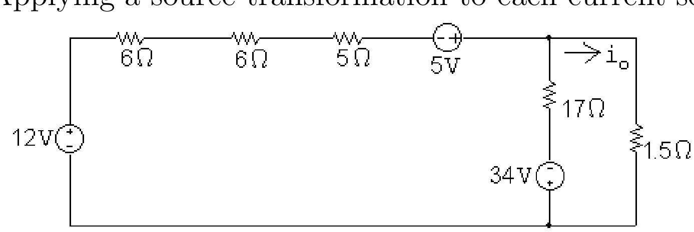
$$v_1\left(\frac{1}{2700} + \frac{1}{2300}\right) + v_2\left(-\frac{1}{2300}\right) = 2 \times 10^{-3}$$

$$v_1\left(-\frac{1}{2300}\right) + v_2\left(\frac{1}{1000} + \frac{1}{2300}\right) = -0.6 \times 10^{-3}$$

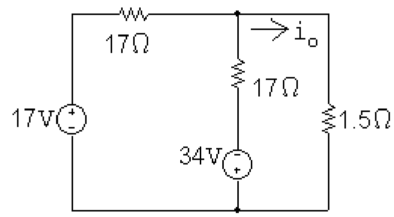
Solving, $v_1 = 2.7$ V; $\qquad v_2 = 0.4$ V

$$\therefore \quad i_o = \frac{v_2 - v_1}{2300} = -1 \text{ mA}$$

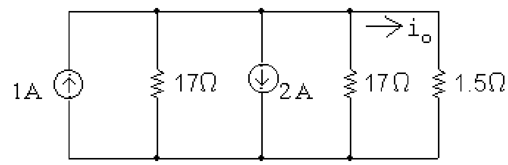
P 4.60 **[a]** Applying a source transformation to each current source yields



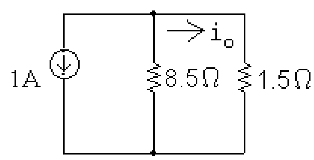
Now combine the 12 V and 5 V sources into a single voltage source and the 6 Ω, 6 Ω and 5 Ω resistors into a single resistor to get



Now use a source transformation on each voltage source, thus

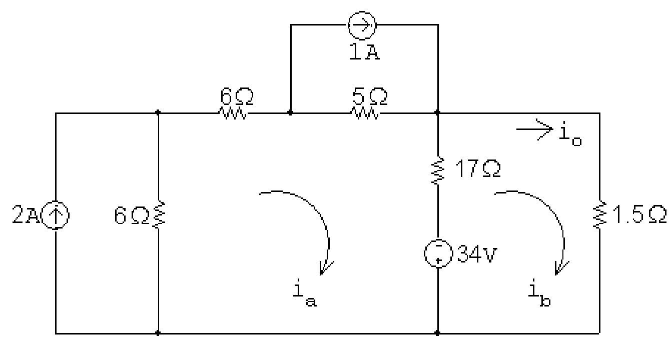


which can be reduced to



$$\therefore \quad i_o = -\frac{8.5}{10}(1) = -0.85 \text{ A}$$

[**b**]

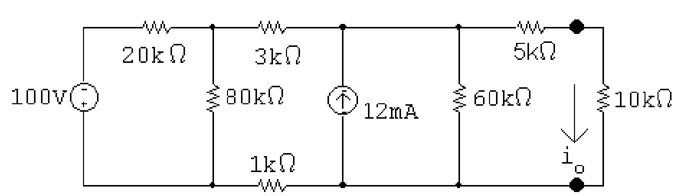


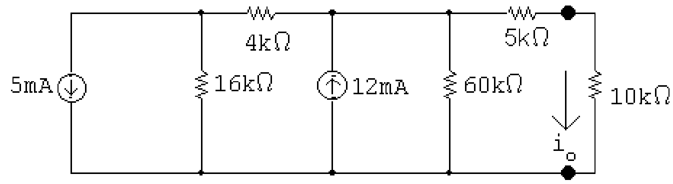
$$34i_{\rm a} - 17i_{\rm b} = 12 + 5 + 34 = 51$$

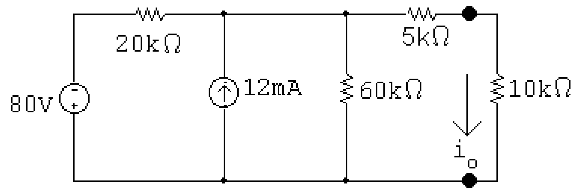
$$-17i_{\rm a} + 18.5i_{\rm b} = -34$$

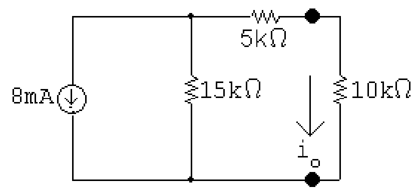
Solving, $i_{\rm b} = -0.85$ A $= i_o$

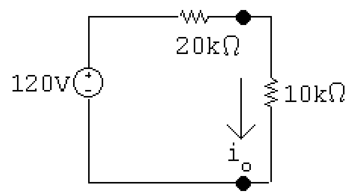
P 4.61 [**a**]





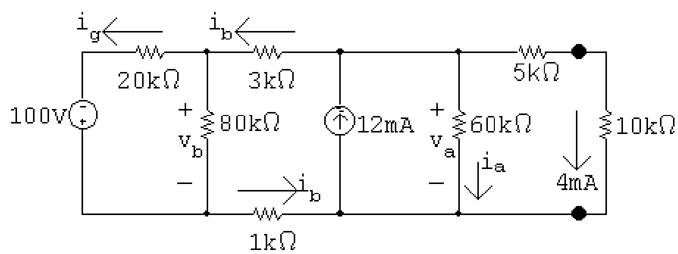






$$i_o = \frac{120}{30{,}000} = 4\ \text{mA}$$

[**b**]



$$
\begin{aligned}
v_\text{a} &= (15{,}000)(0.004) = 60\ \text{V} \\
i_\text{a} &= \frac{v_\text{a}}{60{,}000} = 1\ \text{mA} \\
i_\text{b} &= 12 - 1 - 4 = 7\ \text{mA} \\
v_\text{b} &= 60 - (0.007)(4000) = 32\ \text{V} \\
i_g &= 0.007 - \frac{32}{80{,}000} = 6.6\ \text{mA} \\
p_{100\text{V}} &= -(100)(6.6 \times 10^{-3}) = -660\ \text{mW}
\end{aligned}
$$

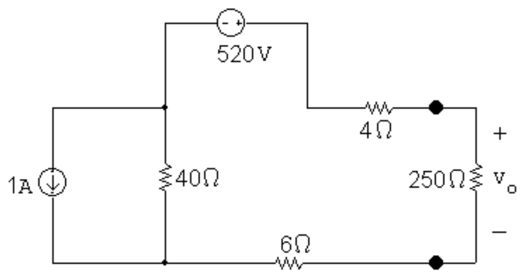
Check:

$$p_{12\text{mA}} = -(60)(12 \times 10^{-3}) = -720 \text{ mW}$$

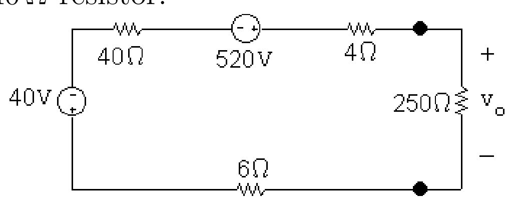
$$\sum P_{\text{dev}} = 660 + 720 = 1380 \text{ mW}$$

$$\sum P_{\text{dis}} = (20{,}000)(6.6 \times 10^{-3})^2 + (80{,}000)(0.4 \times 10^{-3})^2 + (4000)(7 \times 10^{-3})^2 + (60{,}000)(1 \times 10^{-3})^2 + (15{,}000)(4 \times 10^{-3})^2 = 1380 \text{ mW}$$

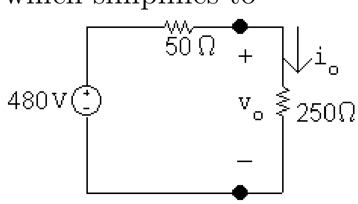
P 4.62 **[a]** First remove the $16\,\Omega$ and $260\,\Omega$ resistors:



Next use a source transformation to convert the 1 A current source and $40\,\Omega$ resistor:

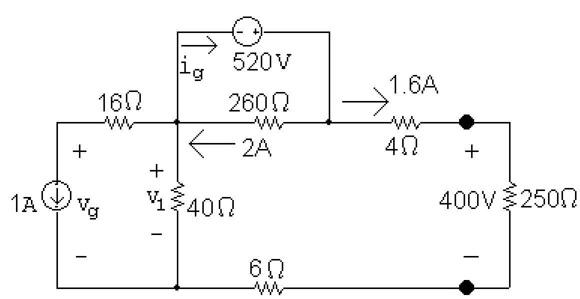


which simplifies to



$$\therefore \quad v_o = \frac{250}{300}(480) = 400 \text{ V}$$

[b] Return to the original circuit with $v_o = 400$ V:



$$i_g = \frac{520}{260} + 1.6 = 3.6 \text{ A}$$

$p_{520\text{V}} = -(520)(3.6) = -1872$ W

Therefore, the 520 V source is developing 1872 W.

[c] $v_1 = -520 + 1.6(4 + 250 + 6) = -104$ V

$v_g = v_1 - 1(16) = -104 - 16 = -120$ V

$p_{1\text{A}} = (1)(-120) = -120$ W

Therefore the 1 A source is developing 120 W.

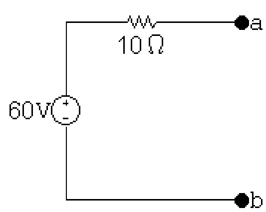
[d] $\sum p_{\text{dev}} = 1872 + 120 = 1992$ W

$$\sum p_{\text{diss}} = (1)^2(16) + \frac{(104)^2}{40} + \frac{(520)^2}{260} + (1.6)^2(260) = 1992 \text{ W}$$

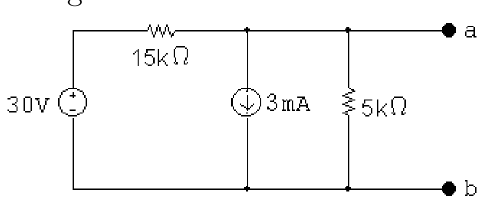
$\therefore \quad \sum p_{\text{diss}} = \sum p_{\text{dev}}$

P 4.63 $v_{\text{Th}} = \dfrac{30}{40}(80) = 60$ V

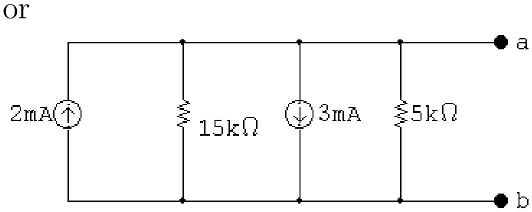
$$R_{\text{Th}} = 2.5 + \frac{(30)(10)}{40} = 10\,\Omega$$



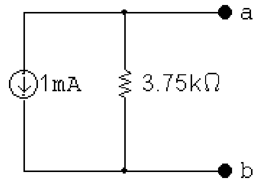
P 4.64 First we make the observation that the 10 mA current source and the 10 kΩ resistor will have no influence on the behavior of the circuit with respect to the terminals a,b. This follows because they are in parallel with an ideal voltage source. Hence our circuit can be simplified to



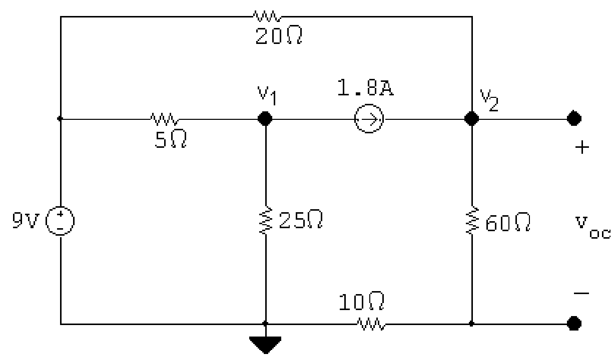
or



Therefore the Norton equivalent is



P 4.65 [**a**] Open circuit:

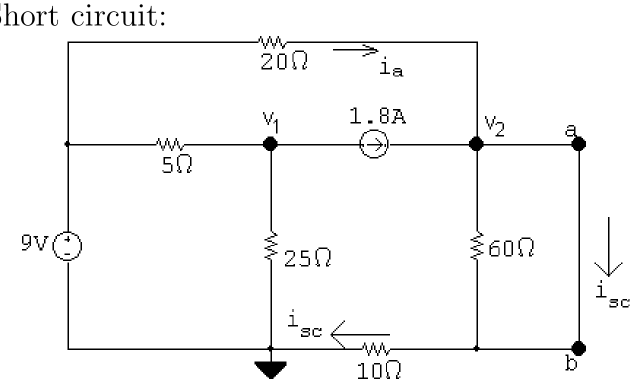


$$\frac{v_2 - 9}{20} + \frac{v_2}{70} - 1.8 = 0$$

$$v_2 = 35 \text{ V}$$

$$v_{\text{Th}} = \frac{60}{70} v_2 = 30 \text{ V}$$

Short circuit:



$$\frac{v_2 - 9}{20} + \frac{v_2}{10} - 1.8 = 0$$

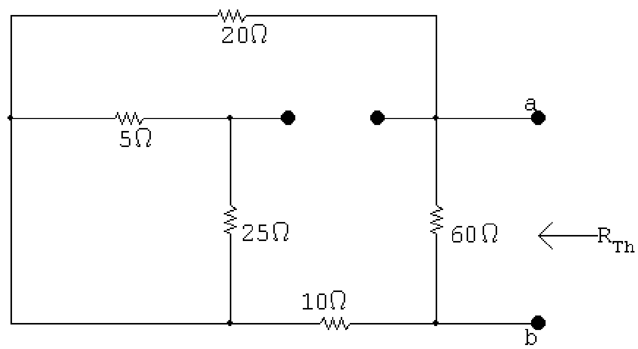
$$\therefore \quad v_2 = 15 \text{ V}$$

$$i_a = \frac{9 - 15}{20} = -0.3 \text{ A}$$

$$i_{sc} = 1.8 - 0.3 = 1.5 \text{ A}$$

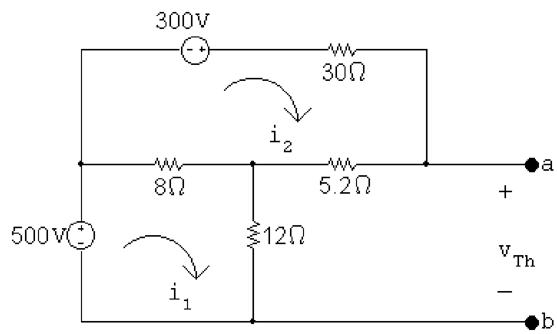
$$R_{\text{Th}} = \frac{30}{1.5} = 20\,\Omega$$

[b]



$R_{\mathrm{Th}} = (20 + 10\|60 = 20\,\Omega$ (CHECKS)

P 4.66 After making a source transformation the circuit becomes



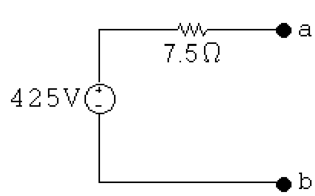
$500 = 20i_1 - 8i_2$

$300 = -8i_1 + 43.2i_2$

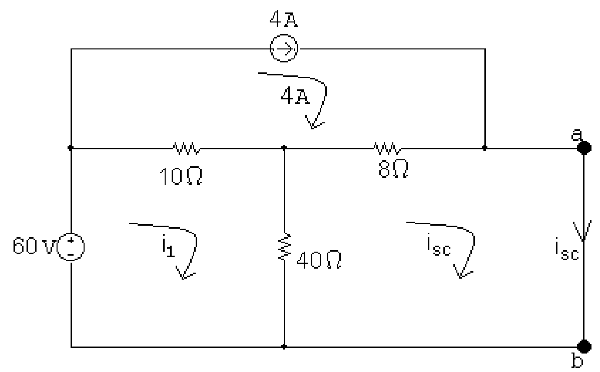
$\therefore \quad i_1 = 30$ A and $i_2 = 12.5$ A

$v_{\mathrm{Th}} = 12i_1 + 5.2i_2 = 425$ V

$R_{\mathrm{Th}} = (8\|12 + 5.2)\|30 = 7.5\,\Omega$



P 4.67

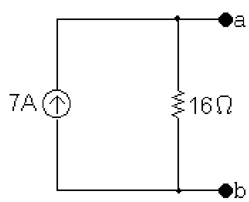


$$50i_1 - 40i_{\rm sc} = 60 + 40$$

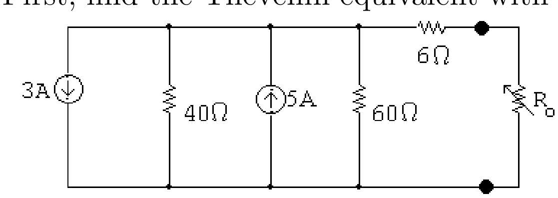
$$-40i_1 + 48i_{\rm scs} = 32$$

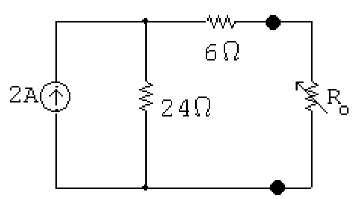
Solving, $\quad i_{\rm sc} = 7\ \text{A}$

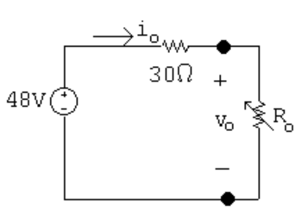
$$R_{\rm Th} = 8 + \frac{(10)(40)}{50} = 16\,\Omega$$



P 4.68 First, find the Thévenin equivalent with respect to R_o.

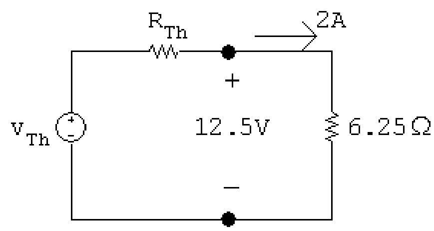




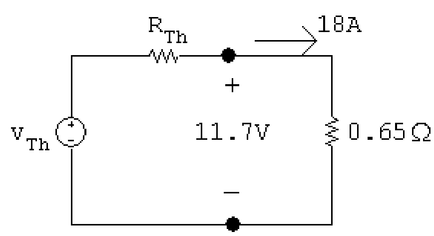


$R_o(\Omega)$	i_o(A)	v_o(V)
10	1.2	12
15	1.067	16
22	0.923	20.31
33	0.762	25.14
47	0.623	29.30
68	0.490	33.31

P 4.69



$12.5 = v_{\rm Th} - 2R_{\rm Th}$



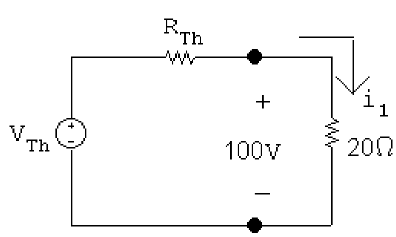
$11.7 = v_{\rm Th} - 18R_{\rm Th}$

Solving the above equations for $V_{\rm Th}$ and $R_{\rm Th}$ yields

$v_{\rm Th} = 12.6$ V, $\qquad R_{\rm Th} = 50$ mΩ

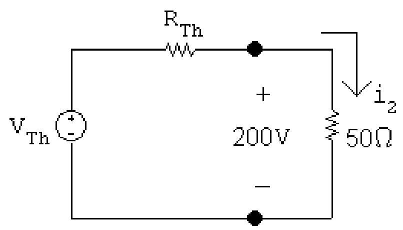
$\therefore \quad I_N = 252$ A, $\qquad R_N = 50$ mΩ

P 4.70



$i_1 = 100/20 = 5$ A

$100 = v_{\text{Th}} - 5R_{\text{Th}}, \qquad v_{\text{Th}} = 100 + 5R_{\text{Th}}$

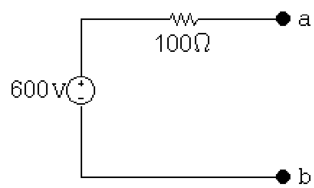


$i_2 = 200/50 = 4\ \text{A}$

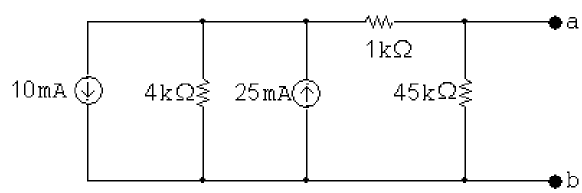
$200 = v_{\text{Th}} - 4R_{\text{Th}}, \qquad v_{\text{Th}} = 200 + 4R_{\text{Th}}$

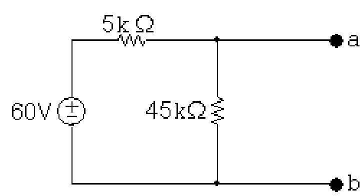
$\therefore\ \ 100 + 5R_{\text{Th}} = 200 + 4R_{\text{Th}} \qquad \text{so} \qquad R_{\text{Th}} = 100\Omega$

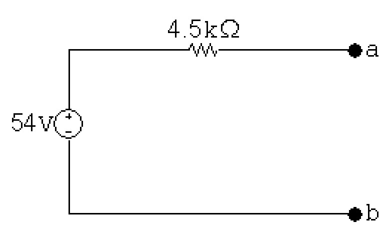
$v_{\text{Th}} = 100 + 500 = 600\ \text{V}$



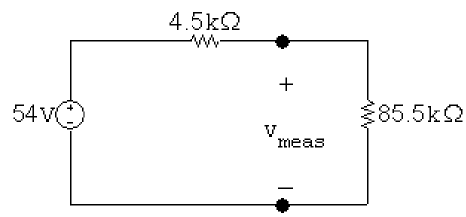
P 4.71 **[a]** First, find the Thévenin equivalent with respect to a,b using a succession of source transformations.







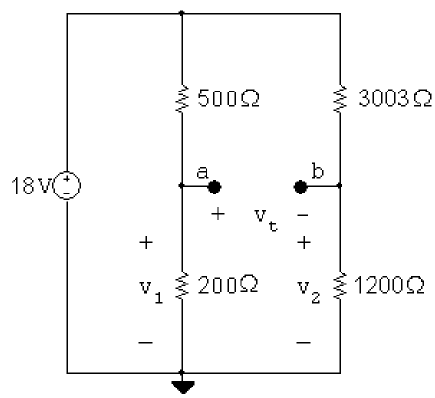
$\therefore\ \ v_{\text{Th}} = 54\ \text{V} \qquad R_{\text{Th}} = 4.5\ \text{k}\Omega$



$$v_{\text{meas}} = \frac{54}{90}(85.5) = 51.3 \text{ V}$$

[b] $\%\text{error} = \left(\frac{51.3 - 54}{54}\right) \times 100 = -5\%$

P 4.72

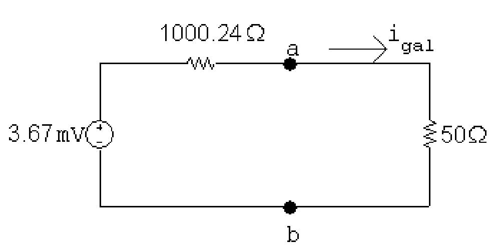


$$v_1 = \frac{200}{700}(18) = 5.143 \text{ V}$$

$$v_2 = \frac{1200}{4203}(18) = 5.139 \text{ V}$$

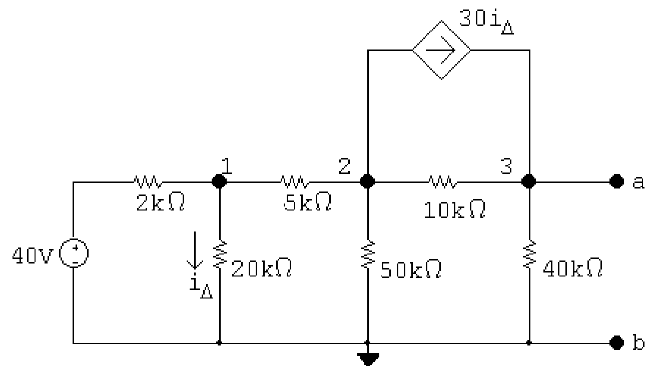
$$v_{\text{Th}} = v_1 - v_2 = 5.143 - 5.139 = 3.67 \text{ mV}$$

$$R_{\text{Th}} = \frac{(500)(200)}{700} + \frac{(3003)(1200)}{4203} = 1000.24\,\Omega$$



$$i_{\text{gal}} = \frac{3.67 \times 10^{-3}}{1050.24} = 3.5\,\mu\text{A}$$

P 4.73



The node voltage equations are:

$$\frac{v_1 - 40}{2000} + \frac{v_1}{20{,}000} + \frac{v_1 - v_2}{5000} = 0$$

$$\frac{v_2 - v_1}{5000} + \frac{v_2}{50{,}000} + \frac{v_2 - v_3}{10{,}000} + 30\frac{v_1}{20{,}000} = 0$$

$$\frac{v_3 - v_2}{10{,}000} + \frac{v_3}{40{,}000} - 30\frac{v_1}{20{,}000} = 0$$

In standard form:

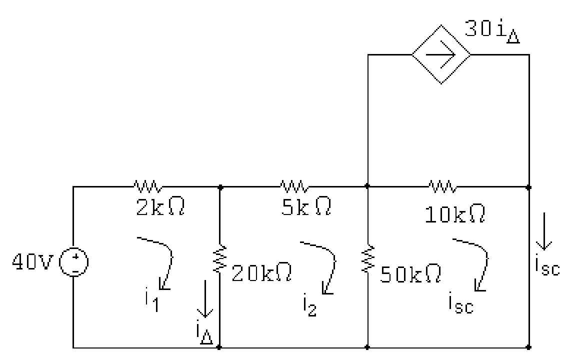
$$v_1\left(\frac{1}{2000} + \frac{1}{20{,}000} + \frac{1}{5000}\right) + v_2\left(-\frac{1}{5000}\right) + v_3(0) = \frac{40}{2000}$$

$$v_1\left(-\frac{1}{5000} + \frac{30}{20{,}000}\right) + v_2\left(\frac{1}{5000} + \frac{1}{50{,}000} + \frac{1}{10{,}000}\right) + v_3\left(-\frac{1}{10{,}000}\right) = 0$$

$$v_1\left(-\frac{30}{20{,}000}\right) + v_2\left(-\frac{1}{10{,}000}\right) + v_3\left(\frac{1}{10{,}000} + \frac{1}{40{,}000}\right) = 0$$

Solving, $v_1 = 24$ V; $v_2 = -10$ V; $v_3 = 280$ V

$V_{\mathrm{Th}} = v_3 = 280$ V



The mesh current equations are:

$$-40 + 2000i_1 + 20{,}000(i_1 - i_2) = 0$$
$$5000i_2 + 50{,}000(i_2 - i_{\rm sc}) + 20{,}000(i_2 - i_1) = 0$$
$$50{,}000(i_{\rm sc} - i_2) + 10{,}000(i_{\rm sc} - 30i_\Delta) = 0$$

The constraint equation is:

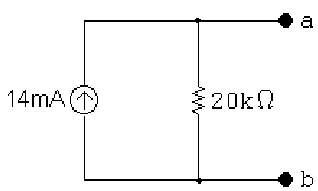
$$i_\Delta = i_1 - i_2$$

Put these equations in standard form:

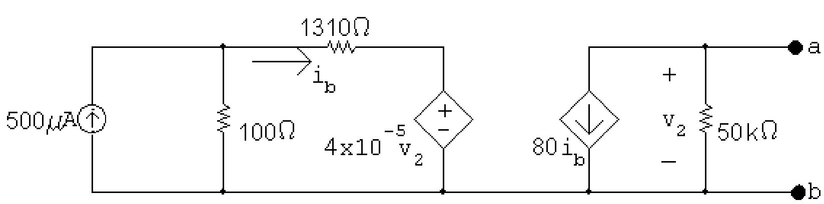
$$i_1(22{,}000) + i_2(-20{,}000) + i_{\rm sc}(0) + i_\Delta(0) = 40$$
$$i_1(-20{,}000) + i_2(75{,}000) + i_{\rm sc}(-50{,}000) + i_\Delta(0) = 0$$
$$i_1(0) + i_2(-50{,}000) + i_{\rm sc}(60{,}000) + i_\Delta(-300{,}000) = 0$$
$$i_1(-1) + i_2(1) + i_{\rm sc}(0) + i_\Delta(1) = 0$$

Solving, $i_1 = 13.6$ mA; $i_2 = 12.96$ mA; $i_{\rm sc} = 14$ mA; $i_\Delta = 640\,\mu$A

$$R_{\rm Th} = \frac{280}{0.014} = 20\ {\rm k}\Omega$$



P 4.74



OPEN CIRCUIT

$$v_2 = -80i_b(50 \times 10^3) = -40 \times 10^5 i_b$$

$$4 \times 10^{-5} v_2 = -160i_b$$

$$1310i_b + 4 \times 10^{-5} v_2 = 1310i_b - 160i_b = 1150i_b$$

So $1150i_b$ is the voltage across the $100\,\Omega$ resistor.

From KCL at the top left node, $500\,\mu{\rm A} = \dfrac{1150i_b}{100} + i_b = 12.5i_b$

$$\therefore \quad i_b = \frac{500 \times 10^{-6}}{12.5} = 40\,\mu\text{A}$$

$$v_{\text{Th}} = -40 \times 10^5(40 \times 10^{-6}) = -160 \text{ V}$$

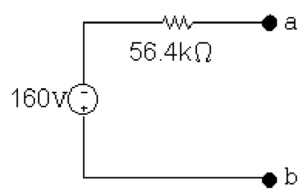
SHORT CIRCUIT

$$v_2 = 0; \qquad i_{\text{sc}} = -80 i_b$$

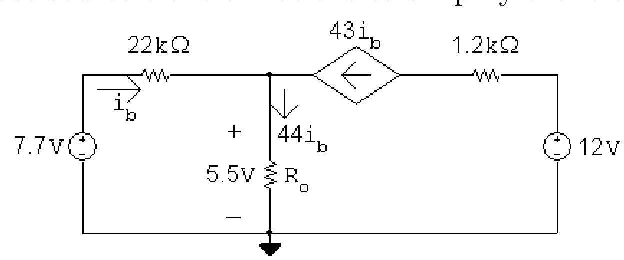
$$i_b = \frac{100}{100 + 1310}(500 \times 10^{-6}) = 35.46\,\mu\text{A}$$

$$i_{\text{sc}} = -80(35.46) = -2837\,\mu\text{A}$$

$$R_{\text{Th}} = \frac{-160}{-2837 \times 10^{-6}} = 56.4 \text{ k}\Omega$$



P 4.75 [**a**] Use source transformations to simplify the left side of the circuit.



$$i_b = \frac{7.7 - 5.5}{22{,}000} = 0.1 \text{ mA}$$

Let $\quad R_o = R_{\text{meter}} \| 1.3 \text{ k}\Omega = 5.5/4.4 = 1.25 \text{ k}\Omega$

$$\therefore \quad \frac{(R_{\text{meter}})(1.3)}{R_{\text{metere}} + 1.3} = 1.25; \qquad R_{\text{meter}} = \frac{(1.25)(1.3)}{0.05} = 32.5 \text{ k}\Omega$$

[**b**] Actual value of v_e:

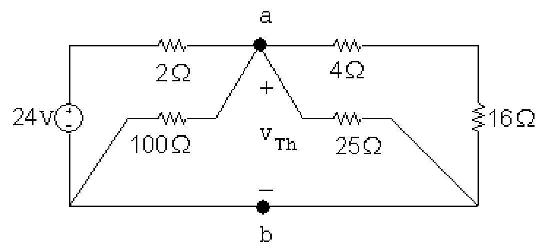
$$i_b = \frac{7.7}{22 + (44)(1.3)} = 0.0972 \text{ mA}$$

$$v_e = 44 i_b (1.3) = 5.56 \text{ V}$$

$$\% \text{ error } = \left(\frac{5.5 - 5.56}{5.56}\right) \times 100 = -1.1\%$$

P 4.76 [a] Find the Thévenin equivalent with respect to the terminals of the ammeter. This is most easily done by first finding the Thévenin with respect to the terminals of the 4.8 Ω resistor.

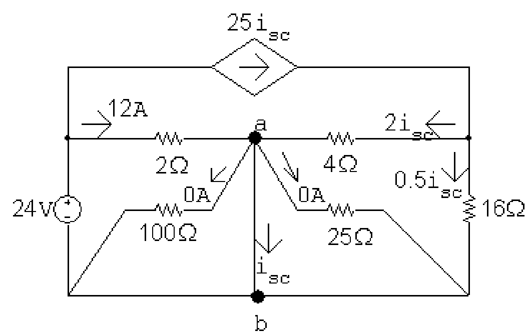
Thévenin voltage: note i_ϕ is zero.



$$\frac{v_{\rm Th}}{100} + \frac{v_{\rm Th}}{25} + \frac{v_{\rm Th}}{20} + \frac{v_{\rm Th} - 16}{2} = 0$$

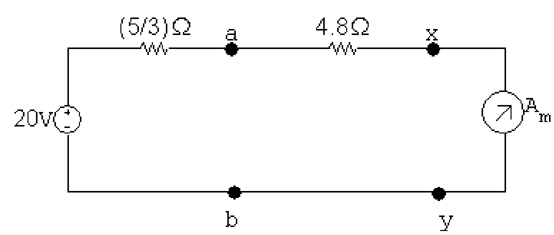
Solving, $v_{\rm Th} = 20$ V.

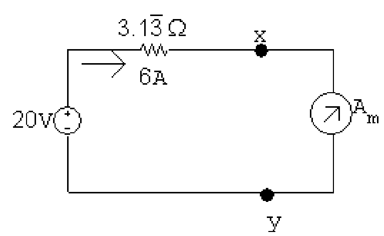
Short-circuit current:



$$i_{\rm sc} = 12 + 2i_{\rm sc}, \qquad \therefore \quad i_{\rm sc} = -12 \text{ A}$$

$$R_{\rm Th} = \frac{20}{-12} = -(5/3)\,\Omega$$

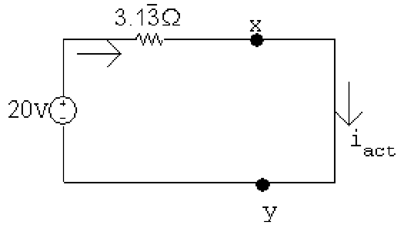




$$R_{\rm total} = \frac{20}{6} = 3.33\,\Omega$$

$$R_{\rm meter} = 3.33 - 3.13 = 0.2\,\Omega$$

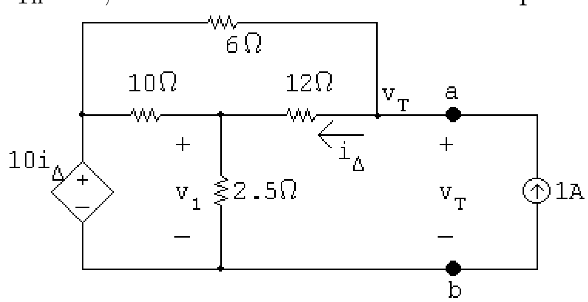
[**b**] Actual current:



$$i_{\text{actual}} = \frac{20}{3.13} = 6.38 \text{ A}$$

$$\% \text{ error } = \frac{6 - 6.38}{6.38} \times 100 = -6\%$$

P 4.77 $V_{\text{Th}} = 0$, since circuit contains no independent sources.



$$\frac{v_1 - 10i_\Delta}{10} + \frac{v_1}{2.5} + \frac{v_1 - v_{\text{T}}}{12} = 0$$

$$\frac{v_{\text{T}} - v_1}{12} + \frac{v_{\text{T}} - 10i_\Delta}{6} - 1 = 0$$

$$i_\Delta = \frac{v_{\text{T}} - v_1}{12}$$

In standard form:

$$v_1\left(\frac{1}{10} + \frac{1}{2.5} + \frac{1}{12}\right) + v_{\text{T}}\left(-\frac{1}{12}\right) + i_\Delta(-1) = 0$$

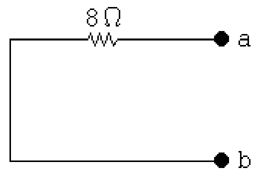
$$v_1\left(-\frac{1}{12}\right) + v_{\text{T}}\left(\frac{1}{12} + \frac{1}{6}\right) + i_\Delta\left(-\frac{10}{6}\right) = 1$$

$$v_1(1) + v_{\text{T}}(-1) + i_\Delta(12) = 0$$

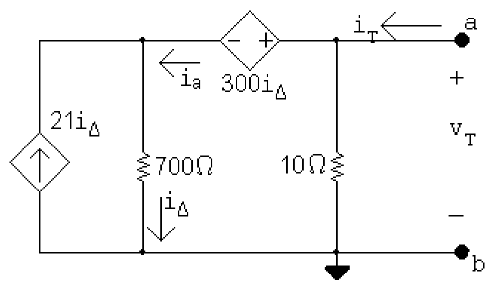
Solving,

$$v_1 = 2 \text{ V}; \qquad v_{\text{T}} = 8 \text{ V}; \qquad i_\Delta = 0.5 \text{ A}$$

$$\therefore \quad R_{\text{Th}} = \frac{v_{\text{T}}}{1\ \text{A}} = 8\,\Omega$$



P 4.78 $V_{\text{Th}} = 0$ since there are no independent sources in the circuit. Thus we need only find R_{Th}.



$$i_{\text{T}} = \frac{v_{\text{T}}}{10} + i_{\text{a}}$$

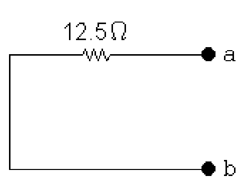
$$i_{\text{a}} = i_{\Delta} - 21i_{\Delta} = -20i_{\Delta}$$

$$i_{\Delta} = \frac{v_{\text{T}} - 300i_{\Delta}}{700}, \qquad 1000i_{\Delta} = v_{\text{T}}$$

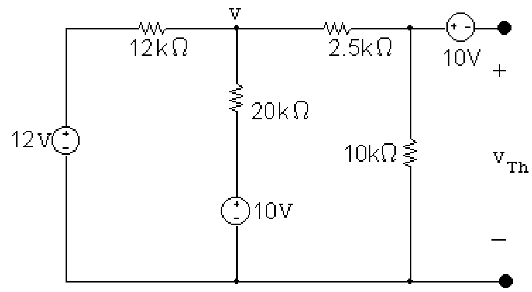
$$\therefore \quad i_{\text{T}} = \frac{v_{\text{T}}}{10} - 20\frac{v_{\text{T}}}{1000} = 0.08v_{\text{T}}$$

$$\frac{v_{\text{T}}}{i_{\text{T}}} = 1/0.08 = 12.5\,\Omega$$

$$\therefore \quad R_{\text{Th}} = 12.5\,\Omega$$



P 4.79 [**a**]

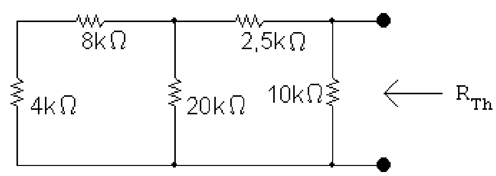


$$\frac{v-12}{12{,}000}+\frac{v-10}{20{,}000}+\frac{v}{12{,}500}=0$$

Solving, $v = 7.03125$ V

$$v_{10k}=\frac{10{,}000}{12{,}500}(7.03125)=5.625 \text{ V}$$

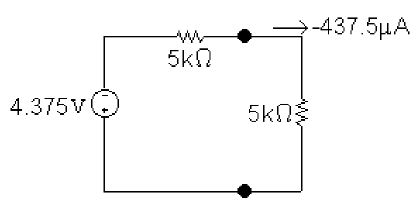
$$\therefore \quad V_{\rm Th}=v-10=-4.375 \text{ V}$$



$$R_{\rm Th}=[(12{,}000\|20{,}000)+2500]=5 \text{ k}\Omega$$

$$R_o=R_{\rm Th}=5 \text{ k}\Omega$$

[**b**]



$$p_{\max}=(-437.5\times10^{-6})^2(5000)=957\,\mu\text{W}$$

[**c**] The resistor closest to 5 kΩ from Appendix H has a value of 4.7 kΩ. Use voltage division to find the voltage drop across this load resistor, and use the voltage to find the power delivered to it:

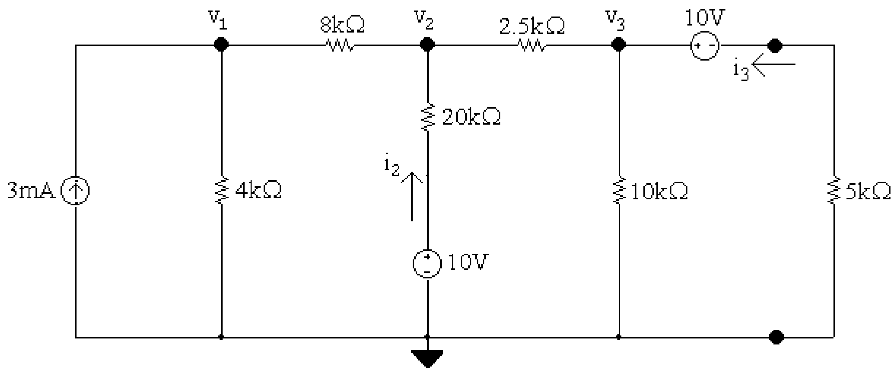
$$v_{4.7k}=\frac{4700}{4700+5000}(-4.375)=-2.12 \text{ V}$$

$$p_{4.7k}=\frac{(-2.12)^2}{4700}=956.12\,\mu\text{W}$$

The percent error between the maximum power and the power delivered to the best resistor from Appendix H is

$$\%\text{ error}=\left(\frac{956}{957}-1\right)(100)=-0.1\%$$

P 4.80 Write KCL equations at each of the labeled nodes, place them in standard form, and solve:



At v_1: $$-3\times 10^{-3} + \frac{v_1}{4000} + \frac{v_1 - v_2}{8000} = 0$$

At v_2: $$\frac{v_2 - v_1}{8000} + \frac{v_2 - 10}{20{,}000} + \frac{v_2 - v_3}{2500} = 0$$

At v_3: $$\frac{v_3 - v_2}{2500} + \frac{v_3}{10{,}000} + \frac{v_3 - 10}{5000} = 0$$

Standard form:

$$v_1\left(\frac{1}{4000} + \frac{1}{8000}\right) + v_2\left(-\frac{1}{8000}\right) + v_3(0) = 0.003$$

$$v_1\left(-\frac{1}{8000}\right) + v_2\left(\frac{1}{8000} + \frac{1}{20{,}000} + \frac{1}{2500}\right) + v_3\left(-\frac{1}{2500}\right) = \frac{10}{20{,}000}$$

$$v_1(0) + v_2\left(-\frac{1}{2500}\right) + v_3\left(\frac{1}{2500} + \frac{1}{10{,}000} + \frac{1}{5000}\right) = \frac{10}{5000}$$

Calculator solution:

$v_1 = 10.890625$ V $\qquad v_2 = 8.671875$ V $\qquad v_3 = 7.8125$ V

Calculate currents:

$$i_2 = \frac{10 - v_2}{20{,}000} = 66.40625\,\mu\text{ A} \qquad i_3 = \frac{10 - v_3}{5000} = 437.5\,\mu\text{ A}$$

Calculate power delivered by the sources:

$$p_{3\text{mA}} = (3\times 10^{-3})v_1 = (3\times 10^{-3})(10.890625) = 32.671875\text{ mW}$$

$$p_{10\text{Vmiddle}} = i_2(10) = (66.40625 \times 10^{-6})(10) = 0.6640625 \text{ mW}$$

$$p_{10\text{Vtop}} = i_3(10) = (437.5 \times 10^{-6})(10) = 4.375 \text{ mW}$$

$$p_{\text{deliveredtotal}} = 32.671875 + 0.6640625 + 4.375 = 37.7109375 \text{ mW}$$

Calculate power absorbed by the 5 kΩ resistor and the percentage power:

$$p_{5\text{k}} = i_3^2(5000) = (437.5 \times 10^{-6})^2(5000) = 0.95703125 \text{ mW}$$

% delivered to R_o: $\quad \dfrac{0.95793125}{37.7109375}(100) = 2.54\%$

P 4.81 [**a**] Since $0 \leq R_o \leq \infty$ maximum power will be delivered to the 6 Ω resistor when $R_o = 0$.

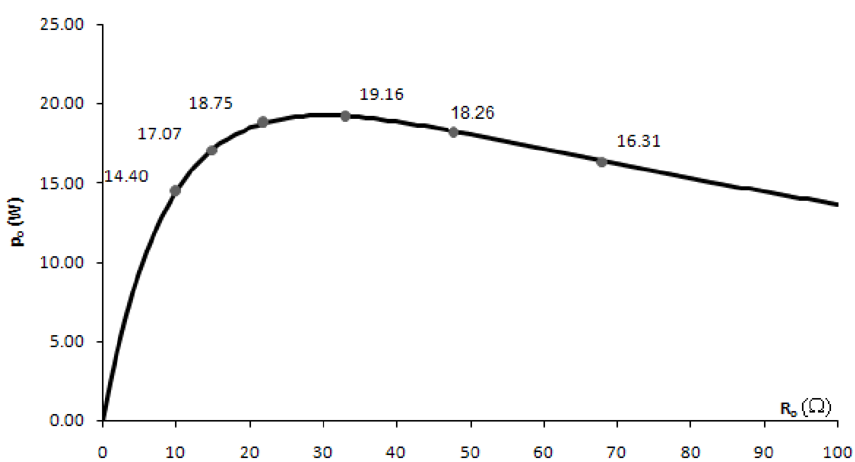
[**b**] $P = \dfrac{30^2}{6} = 150$ W

P 4.82 [**a**] From the solution to Problem 4.68 we have

$R_o(\Omega)$	$p_o(\text{W})$
10	14.4
15	17.07
22	18.75
33	19.16
47	18.26
68	16.31

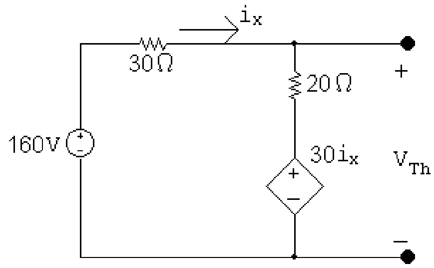
The 33 Ω resistor dissipates the most power, because its value is closest to the Thévenin equivalent resistance of the circuit.

[**b**]



[c] $R_o = 33\,\Omega$, $p_o = 19.16$ W. Compare this to $R_o = R_{\text{Th}} = 30\,\Omega$, which then gives the maximum power delivered to the load, p_o (max) = 19.2 W.

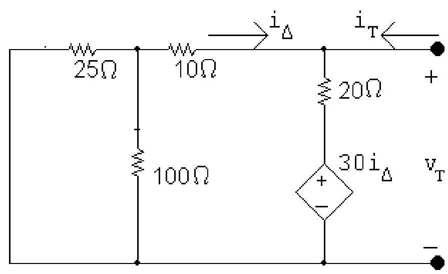
P 4.83 We begin by finding the Thévenin equivalent with respect to R_o. After making a couple of source transformations the circuit simplifies to



$$i_\Delta = \frac{160 - 30i_\Delta}{50}; \qquad i_\Delta = 2\text{ A}$$

$$V_{\text{Th}} = 20i_\Delta + 30i_\Delta = 50i_\Delta = 100\text{ V}$$

Using the test-source method to find the Thévenin resistance gives

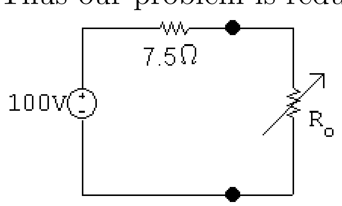


$$i_{\text{T}} = \frac{v_{\text{T}}}{30} + \frac{v_{\text{T}} - 30(-v_{\text{T}}/30)}{20}$$

$$\frac{i_{\text{T}}}{v_{\text{T}}} = \frac{1}{30} + \frac{1}{10} = \frac{4}{30} = \frac{2}{15}$$

$$R_{\text{Th}} = \frac{v_{\text{T}}}{i_{\text{T}}} = \frac{15}{2} = 7.5\,\Omega$$

Thus our problem is reduced to analyzing the circuit shown below.



$$p = \left(\frac{100}{7.5 + R_o}\right)^2 R_o = 250$$

$$\frac{10^4}{R_o^2 + 15R_o + 56.25}R_o = 250$$

$$\frac{10^4 R_o}{250} = R_o^2 + 15R_o + 56.25$$

$$40R_o = R_o^2 + 15R_o + 56.25$$

$$R_o^2 - 25R_o + 56.25 = 0$$

$$R_o = 12.5 \pm \sqrt{156.25 - 56.25} = 12.5 \pm 10$$

$$R_o = 22.5\,\Omega$$

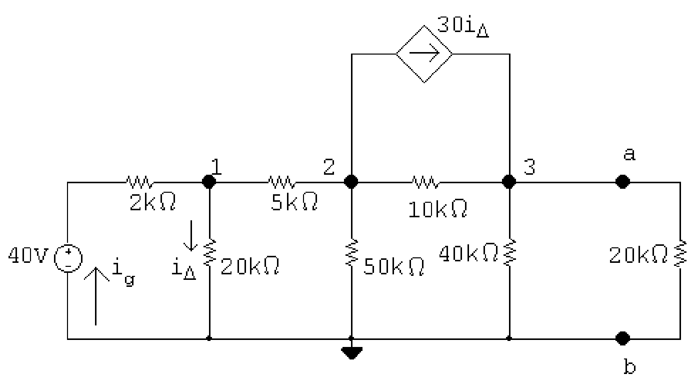
$$R_o = 2.5\,\Omega$$

P 4.84 [**a**] From the solution of Problem 4.73 we have $R_{\text{Th}} = 20$ kΩ and $V_{\text{Th}} = 280$ V. Therefore

$$R_o = R_{\text{Th}} = 20 \text{ k}\Omega$$

[**b**] $p = \dfrac{(140)^2}{20{,}000} = 980$ mW

[**c**]



The node voltage equations are:

$$\begin{aligned}
\frac{v_1 - 40}{2000} + \frac{v_1}{20{,}000} + \frac{v_1 - v_2}{5000} &= 0\\
\frac{v_2 - v_1}{5000} + \frac{v_2}{50{,}000} + \frac{v_2 - v_3}{10{,}000} + 30i_\Delta &= 0\\
\frac{v_3 - v_2}{10{,}000} + \frac{v_3}{40{,}000} - 30i_\Delta + \frac{v_3}{20{,}000} &= 0
\end{aligned}$$

The dependent source constraint equation is:

$$i_\Delta = \frac{v_1}{20{,}000}$$

Place these equations in standard form:

$$v_1\left(\frac{1}{2000}+\frac{1}{20{,}000}+\frac{1}{5000}\right)+v_2\left(-\frac{1}{5000}\right)+v_3(0)+i_\Delta(0)=\frac{40}{2000}$$

$$v_1\left(-\frac{1}{4000}\right)+v_2\left(\frac{1}{4000}+\frac{1}{50{,}000}+\frac{1}{10{,}000}\right)+v_3\left(-\frac{1}{10{,}000}\right)+i_\Delta(30)=0$$

$$v_1(0)+v_2\left(-\frac{1}{10{,}000}\right)+v_3\left(\frac{1}{10{,}000}+\frac{1}{40{,}000}+\frac{1}{20{,}000}\right)+i_\Delta(-30)=0$$

$$v_1\left(\frac{-1}{20{,}000}\right)+v_2(0)+v_3(0)+i_\Delta(1)=0$$

Solving, $v_1 = 18.4$ V; $v_2 = -31$ V; $v_3 = 140$ V; $i_\Delta = 920\,\mu$A

Calculate the power:

$$i_g=\frac{40-18.4}{2000}=10.8 \text{ mA}$$

$p_{40\text{V}} = -(40)(10.8\times10^{-3}) = -432$ mW

$p_{\text{dep source}} = (v_2 - v_3)(30i_\Delta) = -4719.6$ mW

$\sum p_{\text{dev}} = 432 + 4719.6 = 5151.6$ mW

$$\%\text{ delivered } = \frac{980\times10^{-3}}{5151.6\times10^{-3}}\times100 = 19.02\%$$

[d] There are two resistor values in Appendix H that fit the criterion – 18 kΩ and 22 kΩ. Let's use the Thévenin equivalent circuit to calculate the power delivered to each in turn, first by calculating the current through the load resistor and then using the current to calculate to power delivered to the load:

$$i_{18\text{k}}=\frac{280}{20{,}000+18{,}000}=7.368 \text{ m A}$$

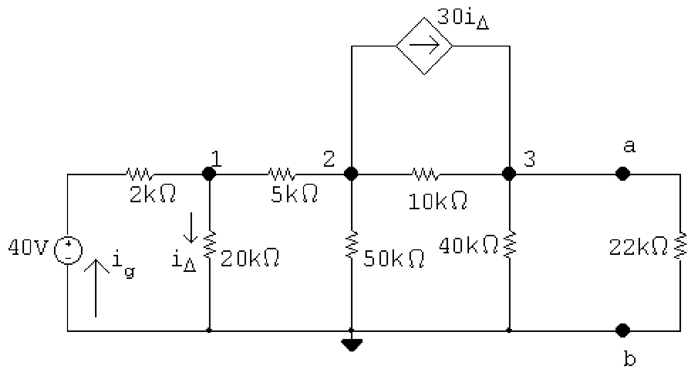
$$p_{18\text{k}}=(7.368)^2(18{,}000)=977.17 \text{ m W}$$

$$i_{22\text{k}}=\frac{280}{20{,}000+22{,}000}=6.667 \text{ m A}$$

$$p_{22\text{k}}=(6.667)^2(22{,}000)=977.88 \text{ m W}$$

We select the 22 kΩ resistor, as the power delivered to it is closer to the maximum power of 980 mW.

[e] Now substitute the 22 kΩ resistor into the original circuit and calculate the power developed by the sources in this circuit:



The node voltage equations are:

$$\frac{v_1 - 40}{2000} + \frac{v_1}{20{,}000} + \frac{v_1 - v_2}{5000} = 0$$

$$\frac{v_2 - v_1}{5000} + \frac{v_2}{50{,}000} + \frac{v_2 - v_3}{10{,}000} + 30i_\Delta = 0$$

$$\frac{v_3 - v_2}{10{,}000} + \frac{v_3}{40{,}000} - 30i_\Delta + \frac{v_3}{22{,}000} = 0$$

The dependent source constraint equation is:

$$i_\Delta = \frac{v_1}{20{,}000}$$

Place these equations in standard form:

$$v_1\left(\frac{1}{2000} + \frac{1}{20{,}000} + \frac{1}{5000}\right) + v_2\left(-\frac{1}{5000}\right) + v_3(0) + i_\Delta(0) = \frac{40}{2000}$$

$$v_1\left(-\frac{1}{5000}\right) + v_2\left(\frac{1}{5000} + \frac{1}{50{,}000} + \frac{1}{10{,}000}\right) + v_3\left(-\frac{1}{10{,}000}\right) + i_\Delta(30) = 0$$

$$v_1(0) + v_2\left(-\frac{1}{10{,}000}\right) + v_3\left(\frac{1}{10{,}000} + \frac{1}{40{,}000} + \frac{1}{22{,}000}\right) + i_\Delta(-30) = 0$$

$$v_1\left(\frac{-1}{20{,}000}\right) + v_2(0) + v_3(0) + i_\Delta(1) = 0$$

Solving, $v_1 = 18.67$ V; $v_2 = -30$ V; $v_3 = 146.67$ V; $i_\Delta = 933.3\,\mu$A

Calculate the power:

$$i_g = \frac{40 - 18.67}{2000} = 10.67 \text{ mA}$$

$$p_{40\text{V}} = -(40)(10.67 \times 10^{-3}) = -426.67 \text{ mW}$$

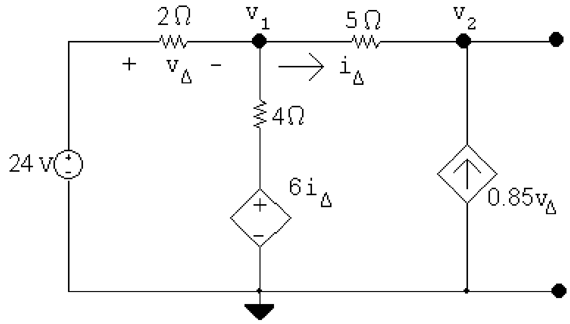
$$p_{\text{dep source}} = (v_2 - v_3)(30i_\Delta) = -4946.67 \text{ mW}$$

$$\sum p_{\text{dev}} = 426.67 + 4946.67 = 5373.33 \text{ mW}$$

$$p_L = (146.67)^2/22{,}000 = 977.78 \text{ mW}$$

$$\%\text{ delivered} = \frac{977.78 \times 10^{-3}}{5373.33 \times 10^{-3}} \times 100 = 18.20\%$$

P 4.85 [a] Open circuit voltage



Node voltage equations:

$$\frac{v_1 - 24}{2} + \frac{v_1 - 6i_\Delta}{4} + \frac{v_1 - v_2}{5} = 0$$

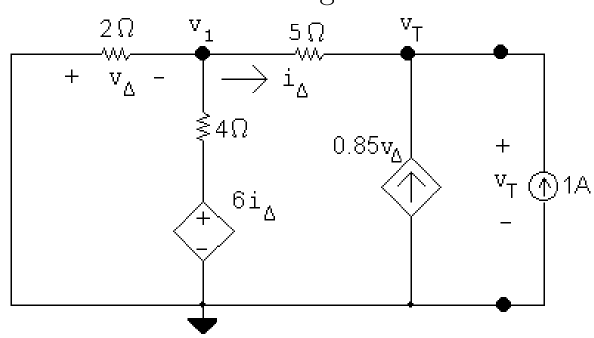
$$\frac{v_2 - v_1}{5} - 0.85v_\Delta = 0$$

Constraint equations:

$$i_\Delta = \frac{v_1 - v_2}{5}; \qquad v_\Delta = 24 - v_1$$

Solving, $v_2 = 84\text{ V} = v_{\text{Th}}$

Thévenin resistance using a test source:



$$\frac{v_1}{2} + \frac{v_1 - 6i_\Delta}{4} + \frac{v_1 - v_T}{5} = 0$$

$$\frac{v_T - v_1}{5} - 0.85v_\Delta - 1 = 0$$

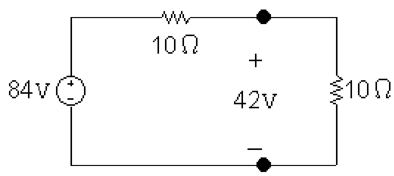
$$i_\Delta = \frac{v_1 - v_T}{5}; \qquad v_\Delta = -v_1$$

Solving, $v_T = 10$

$$R_{\text{Th}} = \frac{v_T}{1} = 10\,\Omega$$

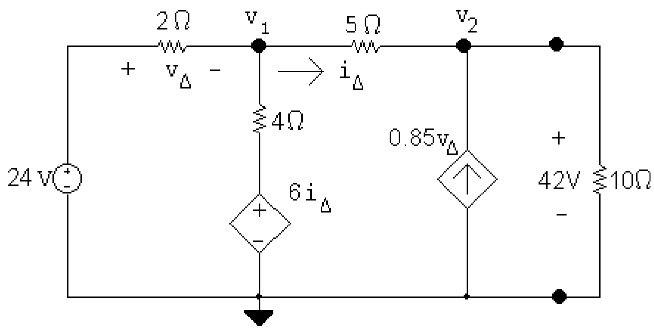
$$\therefore \quad R_o = R_{\text{Th}} = 10\,\Omega$$

[**b**]



$$p_{\max} = \frac{(42)^2}{10} = 176.4 \text{ W}$$

[**c**]



$$\frac{v_1 - 24}{2} + \frac{v_1 - 6i_\Delta}{4} + \frac{v_1 - 42}{5} = 0$$

$$i_\Delta = \frac{v_1 - 42}{5}$$

Solving, $v_1 = 12$ V; $\quad i_\Delta = -6$ A; $\quad v_\Delta = 24 - v_1 = 24 - 12 = 12$ V

$$i_{24\text{V}} = \frac{24 - v_1}{2} = 6 \text{ A}$$

$$p_{24\text{V}} = -24 i_{24\text{V}} = -24(6) = -144 \text{ W}$$

$$i_{\text{CCVS}} = \frac{v_1 - 6i_\Delta}{4} = 12 \text{ A}$$

$$p_{\text{CCVS}} = [6(-6)](12) = -432 \text{ W}$$

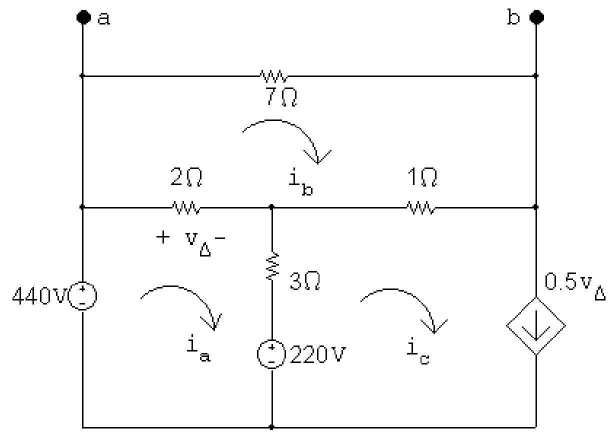
$$p_{\text{VCCS}} = -[0.85(12)](42) = -428.4 \text{ W}$$

$$\sum p_{\text{dev}} = 144 + 432 + 428.4 = 1004.4 \text{ W}$$

$$\% \text{ delivered } = \frac{176.4}{1004.4} \times 100 = 17.56\%$$

P 4.86 Find the Thévenin equivalent with respect to the terminals of R_o.

Open circuit voltage:



$$(440 - 220) = 5i_a - 2i_b - 3i_c$$

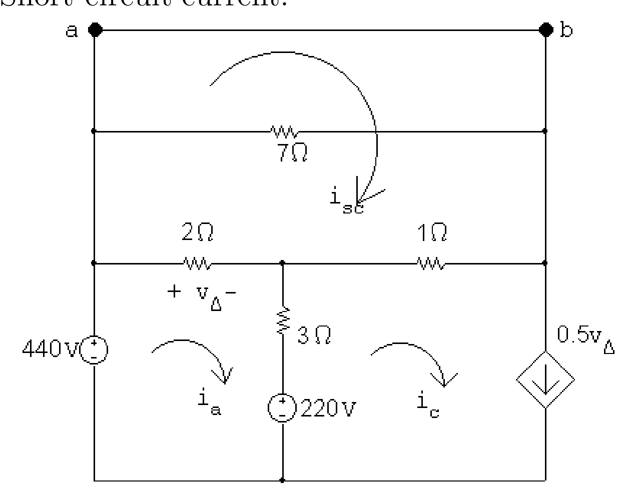
$$0 = -2i_a + 10i_b - 1i_c$$

$$i_c = 0.5v_\Delta; \qquad v_\Delta = 2(i_a - i_b)$$

Solving, $i_b = 26.4$ A

$$\therefore \quad v_{\text{Th}} = 7i_b = 184.8 \text{ V}$$

Short circuit current:



$$440 - 220 = 5i_a - 2i_{\text{sc}} - 3i_c$$

$$0 = -2i_a + 3i_{\text{sc}} - 1i_c$$

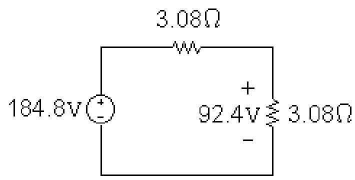
$$i_c = 0.5v_\Delta; \qquad v_\Delta = 2(i_a - i_{\text{sc}})$$

Solving, $i_{\text{sc}} = 60$ A; $i_a = 80$ A; $i_c = 20$ A

$R_{\text{Th}} = v_{\text{Th}}/i_{\text{sc}} = 184.8/60 = 3.08\,\Omega$

$R_o = 3.08\,\Omega$

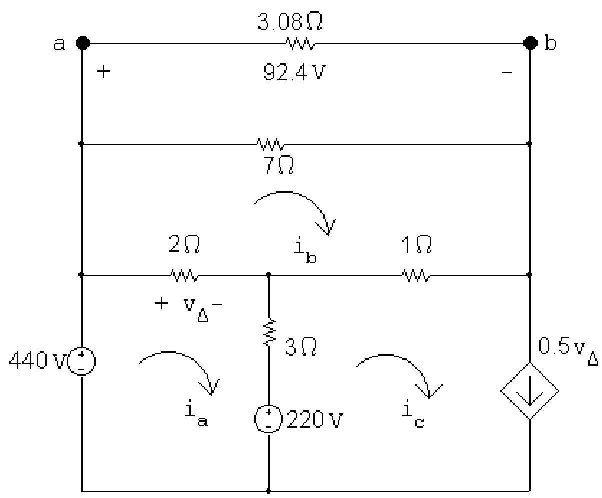
Therefore, the Thévenin equivalent circuit configured for maximum power to the load is



From this circuit,

$$p_{\max} = \frac{(92.4)^2}{3.08} = 2772 \text{ W}$$

With R_o equal to $3.08\,\Omega$ the original circuit becomes



$440 - 220 = 5i_a - 2i_b - 3i_c$

$i_c = 0.5v_\Delta$; $\quad v_\Delta = 2(i_a - i_b)$

$-92.4 = -2i_a + 3i_b - 1i_c$

Solving, $i_a = 88.4$ A; $\quad i_b = 43.2$ A; $\quad i_c = 45.2$ A

$v_\Delta = 2(88.4 - 43.2) = 90.4$ V

$p_{440\text{V}} = -(440)(88.4) = -38{,}896$ W

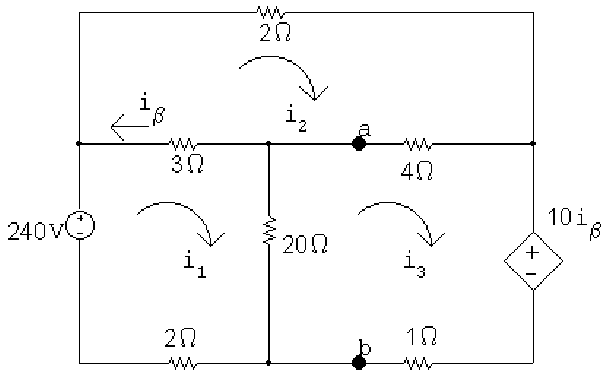
$$p_{220\text{V}} = (220)(88.4 - 45.2) = 9504 \text{ W}$$

$$p_{\text{dep.source}} = (440 - 92.4)[0.5(90.4)] = 15{,}711.52 \text{ W}$$

Therefore, only the 440 V source supplies power to the circuit, and the power supplied is 38,896 W.

$$\% \text{ delivered } = \frac{2772}{38{,}896} = 7.13\%$$

P 4.87 **[a]** Find the Thévenin equivalent with respect to the terminals of R_L.
Open circuit voltage:



The mesh current equations are:

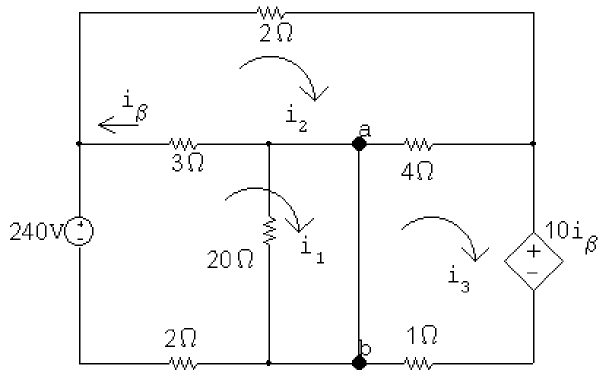
$$\begin{aligned}
-240 + 3(i_1 - i_2) + 20(i_1 - i_3) + 2i_1 &= 0 \\
2i_2 + 4(i_2 - i_3) + 3(i_2 - i_1) &= 0 \\
10i_\beta + 1i_3 + 20(i_3 - i_1) + 4(i_3 - i_2) &= 0
\end{aligned}$$

The dependent source constraint equation is:
$i_\beta = i_2 - i_1$
Place these equations in standard form:

$$\begin{aligned}
i_1(3 + 20 + 2) + i_2(-3) + i_3(-20) + i_\beta(0) &= 240 \\
i_1(-3) + i_2(2 + 4 + 3) + i_3(-4) + i_\beta(0) &= 0 \\
i_1(-20) + i_2(-4) + i_3(1 + 20 + 4) + i_\beta(10) &= 0 \\
i_1(-1) + i_2(1) + i_3(0) + i_\beta(-1) &= 0
\end{aligned}$$

Solving, $i_1 = 99.6$ A; $i_2 = 78$ A; $i_3 = 100.8$ A; $i_\beta = 21.6$ A
$V_{\text{Th}} = 20(i_1 - i_3) = -24$ V

Short-circuit current:



The mesh current equations are:

$$\begin{aligned}
-240 + 3(i_1 - i_2) + 2i_1 &= 0\\
2i_2 + 4(i_2 - i_3) + 3(i_2 - i_1) &= 0\\
10i_\beta + 1i_3 + 4(i_3 - i_2) &= 0
\end{aligned}$$

The dependent source constraint equation is:

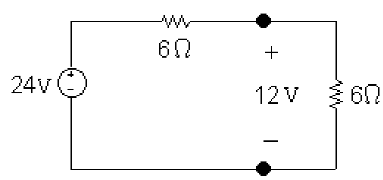
$i_\beta = i_2 - i_1$

Place these equations in standard form:

$$\begin{aligned}
i_1(3+2) + i_2(-3) + i_3(0) + i_\beta(0) &= 240\\
i_1(-3) + i_2(2+4+3) + i_3(-4) + i_\beta(0) &= 0\\
i_1(0) + i_2(-4) + i_3(4+1) + i_\beta(10) &= 0\\
i_1(-1) + i_2(1) + i_3(0) + i_\beta(-1) &= 0
\end{aligned}$$

Solving, $\quad i_1 = 92$ A; $\quad i_2 = 73.33$ A; $\quad i_3 = 96$ A; $\quad i_\beta = 18.67$ A

$i_{\rm sc} = i_1 - i_3 = -4$ A; $\qquad R_{\rm Th} = \dfrac{V_{\rm Th}}{i_{\rm sc}} = \dfrac{-24}{-4} = 6\,\Omega$

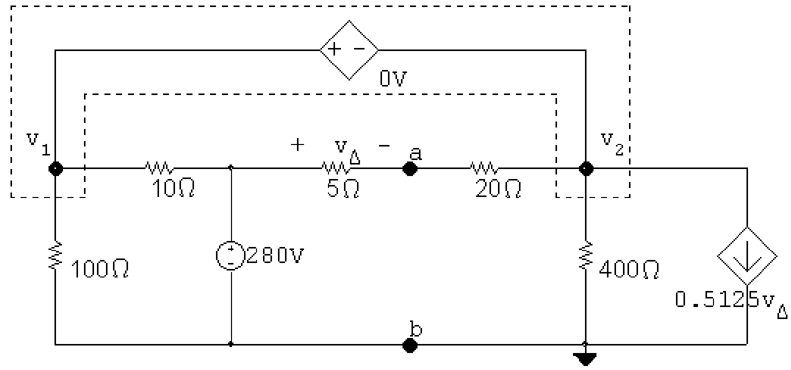


$R_{\rm L} = R_{\rm Th} = 6\,\Omega$

[b] $p_{\max} = \dfrac{12^2}{6} = 24$ W

P 4.88 [a] First find the Thévenin equivalent with respect to R_o.

Open circuit voltage: $i_\phi = 0$; $50i_\phi = 0$



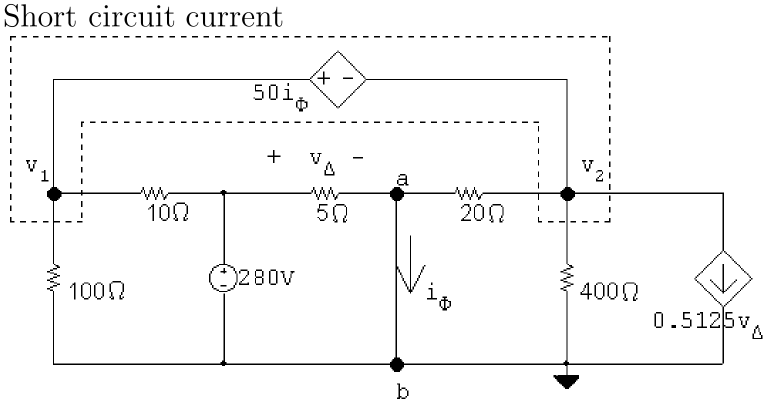
$$\frac{v_1}{100} + \frac{v_1 - 280}{10} + \frac{v_1 - 280}{25} + \frac{v_1}{400} + 0.5125v_\Delta = 0$$

$$v_\Delta = \frac{(280 - v_1)}{25}5 = 56 - 0.2v_1$$

$$v_1 = 210 \text{ V}; \qquad v_\Delta = 14 \text{ V}$$

$$V_{\text{Th}} = 280 - v_\Delta = 280 - 14 = 266 \text{ V}$$

Short circuit current



$$\frac{v_1}{100} + \frac{v_1 - 280}{10} + \frac{v_2}{20} + \frac{v_2}{400} + 0.5125(280) = 0$$

$$v_\Delta = 280 \text{ V}$$

$$v_2 + 50i_\phi = v_1$$

$$i_\phi = \frac{280}{5} + \frac{v_2}{20} = 56 + 0.05v_2$$

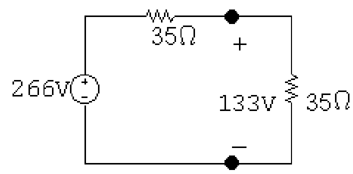
$$v_2 = -968 \text{ V}; \qquad v_1 = -588 \text{ V}$$

$$i_\phi = i_{\text{sc}} = 56 + 0.05(-968) = 7.6 \text{ A}$$

$$R_{\text{Th}} = V_{\text{Th}}/i_{\text{sc}} = 266/7.6 = 35\,\Omega$$

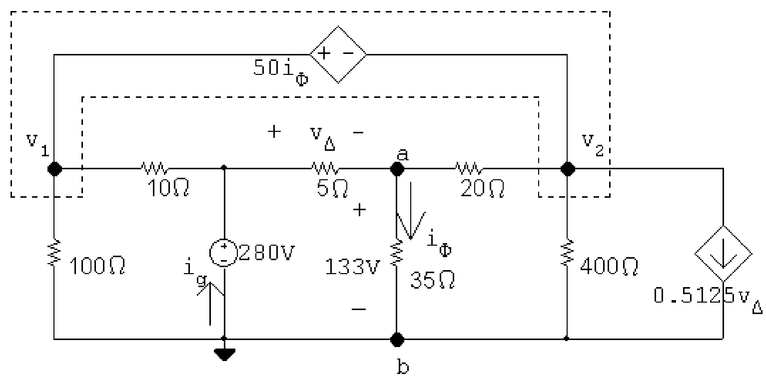
$$\therefore \quad R_o = 35\,\Omega$$

[**b**]



$$p_{\max} = (133)^2/35 = 505.4 \text{ W}$$

[**c**]



$$\frac{v_1}{100} + \frac{v_1 - 280}{10} + \frac{v_2 - 133}{20} + \frac{v_2}{400} + 0.5125(280 - 133) = 0$$

$$v_2 + 50i_\phi = v_1; \qquad i_\phi = 133/35 = 3.8 \text{ A}$$

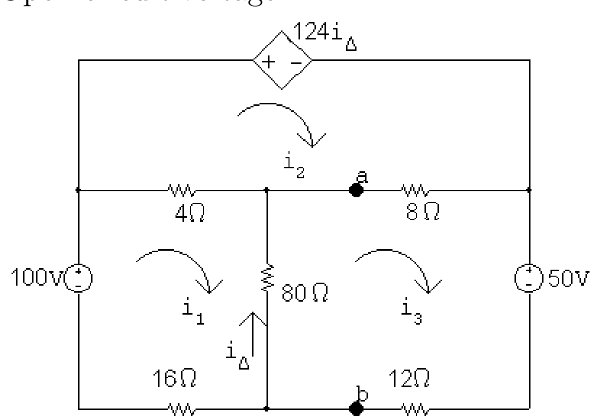
Therefore, $v_1 = -189$ V and $v_2 = -379$ V; thus,

$$i_g = \frac{280 - 133}{5} + \frac{280 + 189}{10} = 76.30 \text{ A}$$

$$p_{280\text{V}} \text{ (dev)} = (280)(76.3) = 21{,}364 \text{ W}$$

P 4.89 [**a**] We begin by finding the Thévenin equivalent with respect to the terminals of R_o.

Open circuit voltage



The mesh current equations are:

$$-100 + 4(i_1 - i_2) + 80(i_1 - i_3) + 16i_1 = 0$$

$$124i_\Delta + 8(i_2 - i_3) + 4(i_2 - i_1) = 0$$

$$50 + 12i_3 + 80(i_3 - i_1) + 8(i_3 - i_2) = 0$$

The constraint equation is:
$i_\Delta = i_3 - i_1$
Place these equations in standard form:

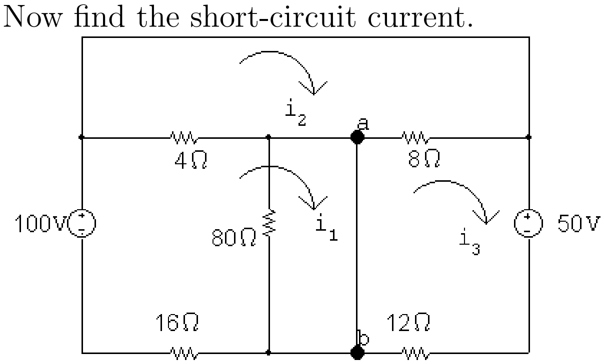
$$i_1(4 + 80 + 16) + i_2(-4) + i_3(-80) + i_\Delta(0) = 100$$

$$i_1(-4) + i_2(8 + 4) + i_3(-8) + i_\Delta(124) = 0$$

$$i_1(-80) + i_2(-8) + i_3(12 + 80 + 8) + i_\Delta(0) = -50$$

$$i_1(1) + i_2(0) + i_3(-1) + i_\Delta(1) = 0$$

Solving, $i_1 = 4.7$ A; $i_2 = 10.5$ A; $i_3 = 4.1$ A; $i_\Delta = -0.6$ A
Also, $V_{\text{Th}} = v_{\text{ab}} = -80i_\Delta = 48$ V
Now find the short-circuit current.



Note with the short circuit from a to b that i_Δ is zero, hence $124i_\Delta$ is also zero.

The mesh currents are:

$$-100 + 4(i_1 - i_2) + 16i_1 = 0$$

$$8(i_2 - i_3) + 4(i_2 - i_1) = 0$$

$$50 + 12i_3 + 8(i_3 - i_2) = 0$$

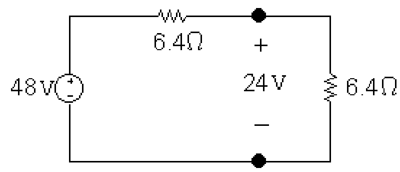
Place these equations in standard form:

$$i_1(4 + 16) + i_2(-4) + i_3(0) = 100$$

$$i_1(-4) + i_2(8 + 4) + i_3(-8) = 0$$

$$i_1(0) + i_2(-8) + i_3(12 + 8) = -50$$

Solving, $i_1 = 5$ A; $i_2 = 0$ A; $i_3 = -2.5$ A
Then, $i_{\text{sc}} = i_1 - i_3 = 7.5$ A
$R_{\text{Th}} = 48/7.5 = 6.4\,\Omega$



For maximum power transfer $R_o = R_{\text{Th}} = 6.4\,\Omega$

[**b**] $p_{\max} = \dfrac{24^2}{6.4} = 90$ W

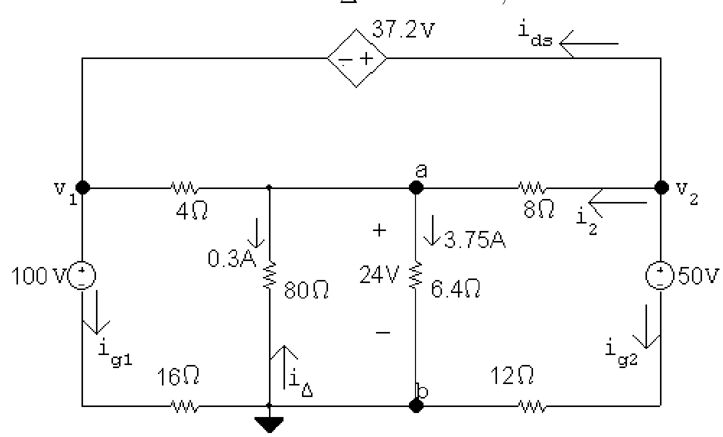
[**c**] The resistor from Appendix H that is closest to the Thévenin resistance is 10 Ω. To calculate the power delivered to a 10 Ω load resistor, calculate the current using the Thévenin circuit and use it to find the power delivered to the load resistor:

$$i_{10} = \frac{48}{6.4 + 10} = 2.927 \text{ A}$$

$$p_{10} = 10(2.927)^2 = 85.7 \text{ W}$$

Thus, using a 10 Ω resistor selected from Appendix H will cause 85.7 W of power to be delivered to the load, compared to the maximum power of 90 W that will be delivered if a 6.4 Ω resistor is used.

P 4.90 From the solution of Problem 4.89 we know that when R_o is 6.4 Ω, the voltage across R_o is 24 V, positive at the upper terminal. Therefore our problem reduces to the analysis of the following circuit. In constructing the circuit we have used the fact that i_Δ is −0.3 A, and hence $124i_\Delta$ is −37.2 V.



Using the node voltage method to find v_1 and v_2 yields

$$4.05 + \frac{24 - v_1}{4} + \frac{24 - v_2}{8} = 0$$

$$2v_1 + v_2 = 104.4; \qquad v_1 + 37.2 = v_2$$

Solving, $v_1 = 22.4$ V; $\qquad v_2 = 59.6$ V.

It follows that

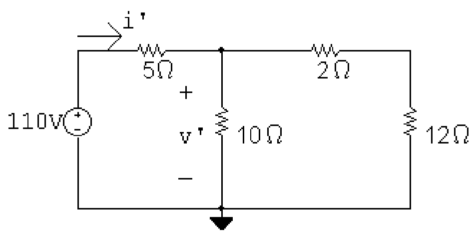
$$
\begin{aligned}
i_{g_1} &= \frac{22.4-100}{16} = -4.85\ \text{A} \\
i_{g_2} &= \frac{59.6-50}{12} = 0.8\ \text{A} \\
i_2 &= \frac{59.6-24}{8} = 4.45\ \text{A} \\
i_{\text{ds}} &= -4.45-0.8 = -5.25\ \text{A} \\
p_{100\text{V}} &= 100 i_{g_1} = -485\ \text{W} \\
p_{50\text{V}} &= 50 i_{g_2} = 40\ \text{W} \\
p_{\text{ds}} &= 37.2 i_{\text{ds}} = -195.3\ \text{W}
\end{aligned}
$$

$$\therefore\ \sum p_{\text{dev}} = 485 + 195.3 = 680.3\ \text{W}$$

$$\therefore\ \%\ \text{delivered} = \frac{90}{680.3}(100) = 13.23\%$$

$\therefore$ 13.23% of developed power is delivered to load

P 4.91 **[a]** 110 V source acting alone:

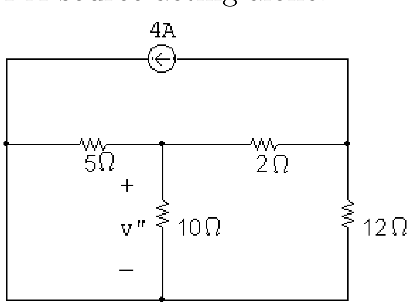


$$R_e = \frac{10(14)}{24} = \frac{35}{6}\ \Omega$$

$$i' = \frac{110}{5+35/6} = \frac{132}{13}\ \text{A}$$

$$v' = \left(\frac{35}{6}\right)\left(\frac{132}{13}\right) = \frac{770}{13}\ \text{V} = 59.231\ \text{V}$$

4 A source acting alone:

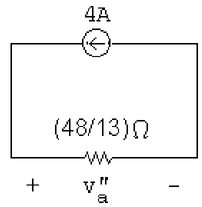


$$5\,\Omega \| 10\,\Omega = 50/15 = 10/3\,\Omega$$

$10/3 + 2 = 16/3\,\Omega$

$16/3\|12 = 48/13\,\Omega$

Hence our circuit reduces to:



It follows that

$v_a'' = 4(48/13) = (192/13)$ V

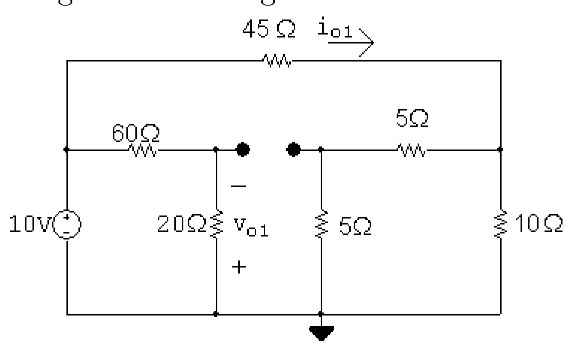
and

$$v'' = \frac{-v_a''}{(16/3)}(10/3) = -\frac{5}{8}v_a'' = -(120/13) \text{ V} = -9.231 \text{ V}$$

$$\therefore \qquad v = v' + v'' = \frac{770}{13} - \frac{120}{13} = 50 \text{ V}$$

[b] $p = \dfrac{v^2}{10} = 250$ W

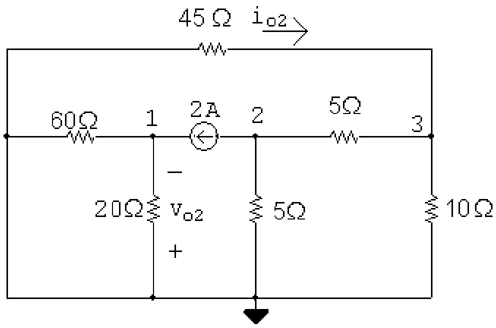
P 4.92 Voltage source acting alone:



$$i_{o1} = \frac{10}{45 + (5+5)\|10} = \frac{10}{45+5} = 0.2 \text{ A}$$

$$v_{o1} = \frac{20}{20+60}(-10) = -2.5 \text{ V}$$

Current source acting alone:



$$\frac{v_2}{5} + 2 + \frac{v_2 - v_3}{5} = 0$$

$$\frac{v_3}{10} + \frac{v_3 - v_2}{5} + \frac{v_3}{45} = 0$$

Solving, $v_2 = -7.25$ V $= v_{o2}$; $\qquad v_3 = -4.5$ V

$$i_{o2} = -\frac{v_3}{45} = -0.1 \text{ A}$$

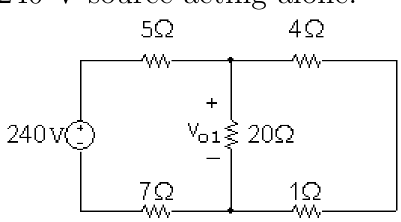
$$i_{20} = \frac{60\|20}{20}(2) = 1.5 \text{ A}$$

$$v_{o2} = -20i_{20} = -20(1.5) = -30 \text{ V}$$

$$\therefore \quad v_o = v_{o1} + v_{o2} = -2.5 - 30 = -32.5 \text{ V}$$

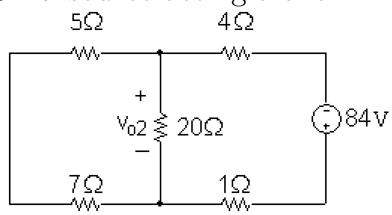
$$i_o = i_{o1} + i_{o2} = 0.2 + 0.1 = 0.3 \text{ A}$$

P 4.93 240 V source acting alone:



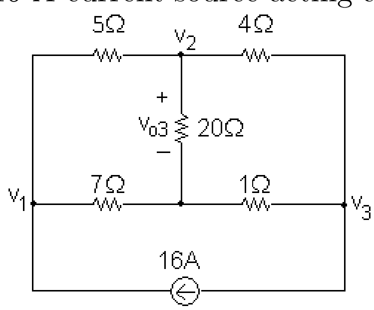
$$v_{o1} = \frac{20\|5}{5 + 7 + 20\|5}(240) = 60 \text{ V}$$

84 V source acting alone:



$$v_{o2} = \frac{20\|12}{1+4+20\|12}(-84) = -50.4 \text{ V}$$

16 A current source acting alone:



$$\frac{v_1 - v_2}{5} + \frac{v_1}{7} - 16 = 0$$

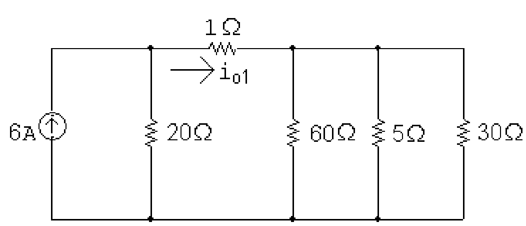
$$\frac{v_2 - v_1}{5} + \frac{v_2}{20} + \frac{v_2 - v_3}{4} = 0$$

$$\frac{v_3 - v_2}{4} + \frac{v_3}{1} + 16 = 0$$

Solving, $v_2 = 18.4$ V $= v_{o3}$. Therefore,

$$v_o = v_{o1} + v_{o2} + v_{o3} = 60 - 50.4 + 18.4 = 28 \text{ V}$$

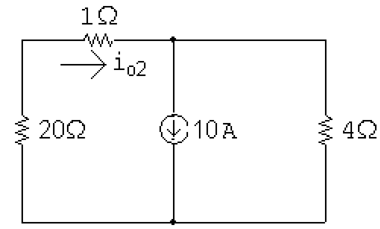
P 4.94 6 A source:



$$30\,\Omega\|5\,\Omega\|60\,\Omega = 4\,\Omega$$

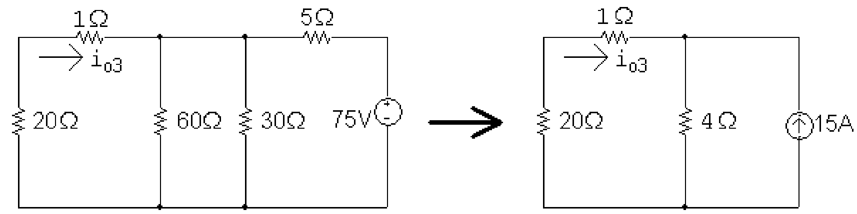
$$\therefore \quad i_{o1} = \frac{20}{20+5}(6) = 4.8 \text{ A}$$

10 A source:



$$i_{o2} = \frac{4}{25}(10) = 1.6 \text{ A}$$

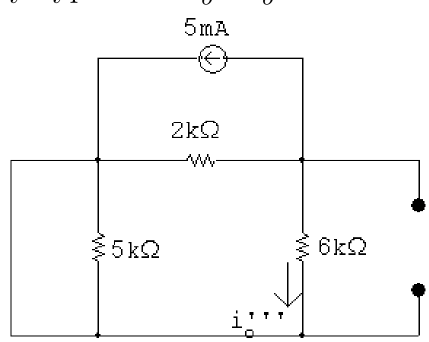
75 V source:



$$i_{o3} = -\frac{4}{25}(15) = -2.4 \text{ A}$$

$$i_o = i_{o1} + i_{o2} + i_{o3} = 4.8 + 1.6 - 2.4 = 4 \text{ A}$$

P 4.95 [**a**] By hypothesis $i_o' + i_o'' = 3$ mA.



$$i_o''' = -5\frac{(2)}{(8)} = -1.25 \text{ mA}; \qquad \therefore \quad i_o = 3.5 - 1.25 = 2.25 \text{ mA}$$

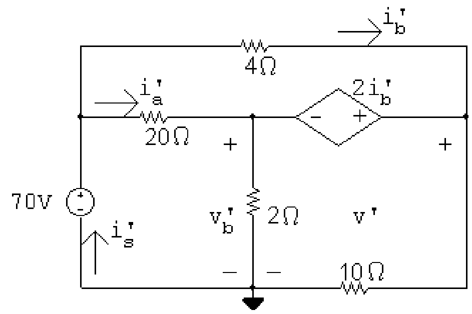
[**b**] With all three sources in the circuit write a single node voltage equation.

$$\frac{v_b}{6} + \frac{v_b - 8}{2} + 5 - 10 = 0$$

$$\therefore \quad v_b = 13.5 \text{ V}$$

$$i_o = \frac{v_b}{6} = 2.25 \text{ mA}$$

P 4.96 70-V source acting alone:



$$v' = 70 - 4i_b'$$

$$i_s' = \frac{v_b'}{2} + \frac{v'}{10} = i_a' + i_b'$$

$$70 = 20i_a' + v_b'$$

$$i_a' = \frac{70 - v_b'}{20}$$

$$\therefore \quad i_b' = \frac{v_b'}{2} + \frac{v'}{10} - \frac{70 - v_b'}{20} = \frac{11}{20}v_b' + \frac{v'}{10} - 3.5$$

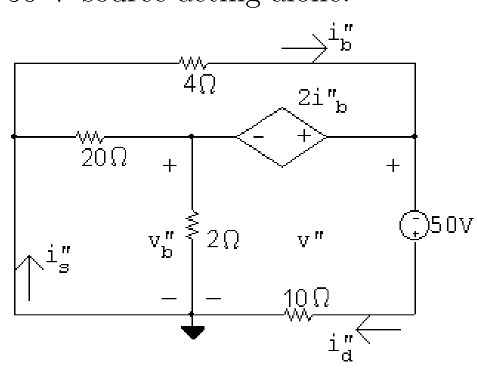
$$v' = v_b' + 2i_b'$$

$$\therefore \quad v_b' = v' - 2i_b'$$

$$\therefore \quad i_b' = \frac{11}{20}(v' - 2i_b') + \frac{v'}{10} - 3.5 \quad \text{or} \quad i_b' = \frac{13}{42}v' - \frac{70}{42}$$

$$\therefore \quad v' = 70 - 4\left(\frac{13}{42}v' - \frac{70}{42}\right) \quad \text{or} \quad v' = \frac{3220}{94} = \frac{1610}{47}\ \text{V} = 34.255\ \text{V}$$

50-V source acting alone:



$$v'' = -4i_b''$$

$$v'' = v_b'' + 2i_b''$$

$$v'' = -50 + 10i_d''$$

$$\therefore \quad i_d'' = \frac{v'' + 50}{10}$$

$$i_s'' = \frac{v_b''}{2} + \frac{v'' + 50}{10}$$

$$i_b'' = \frac{v_b''}{20} + i_s'' = \frac{v_b''}{20} + \frac{v_b''}{2} + \frac{v'' + 50}{10} = \frac{11}{20}v_b'' + \frac{v'' + 50}{10}$$

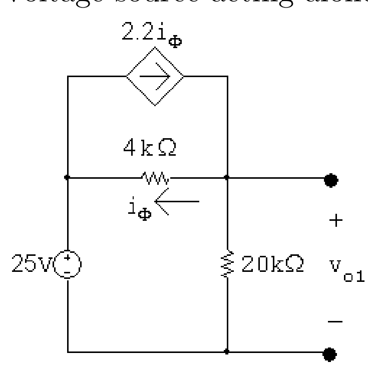
$$v_b'' = v'' - 2i_b''$$

$$\therefore \quad i_b'' = \frac{11}{20}(v'' - 2i_b'') + \frac{v'' + 50}{10} \qquad \text{or} \qquad i_b'' = \frac{13}{42}v'' + \frac{100}{42}$$

Thus, $v'' = -4\left(\frac{13}{42}v'' + \frac{100}{42}\right)$ or $v'' = -\frac{200}{47}\text{ V} = -4.255\text{ V}$

Hence, $v = v' + v'' = \frac{1610}{47} - \frac{200}{47} = \frac{1410}{47} = 30\text{ V}$

P 4.97 Voltage source acting alone:

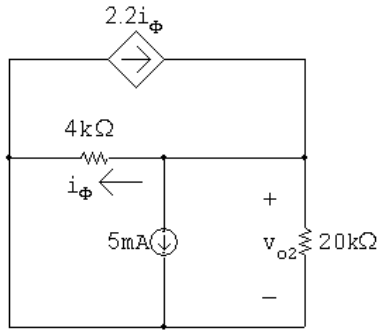


$$\frac{v_{o1} - 25}{4000} + \frac{v_{o1}}{20{,}000} - 2.2\left(\frac{v_{o1} - 25}{4000}\right) = 0$$

Simplifying $5v_{o1} - 125 + v_{o1} - 11v_{o1} + 275 = 0$

$$\therefore \quad v_{o1} = 30\text{ V}$$

Current source acting alone:



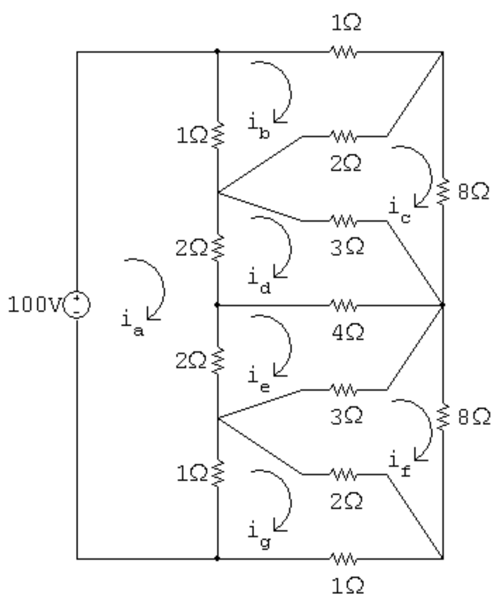
$$\frac{v_{o2}}{4000} + \frac{v_{o2}}{20{,}000} + 0.005 - 2.2\left(\frac{v_{o2}}{4000}\right) = 0$$

Simplifying $\qquad 5v_{o2} + v_{o2} + 100 - 11v_{o2} = 0$

$\therefore \quad v_{o2} = 20$ V

$v_o = v_{o1} + v_{o2} = 30 + 20 = 50$ V

P 4.98



$$\begin{aligned}
100 &= 6i_a - 1i_b + 0i_c - 2i_d - 2i_e + 0i_f - 1i_g \\
0 &= -1i_a + 4i_b - 2i_c + 0i_d + 0i_e + 0i_f + 0i_g \\
0 &= 0i_a - 2i_b + 13i_c - 3i_d + 0i_e + 0i_f + 0i_g \\
0 &= -2i_a + 0i_b - 3i_c + 9i_d - 4i_e + 0i_f + 0i_g \\
0 &= -2i_a + 0i_b + 0i_c - 4i_d + 9i_e - 3i_f + 0i_g \\
0 &= 0i_a + 0i_b + 0i_c + 0i_d - 3i_e + 13i_f - 2i_g \\
0 &= -1i_a + 0i_b + 0i_c + 0i_d + 0i_e - 2i_f + 4i_g
\end{aligned}$$

A computer solution yields

$i_a = 30$ A; $i_e = 15$ A;

$i_b = 10$ A; $i_f = 5$ A;

$i_c = 5$ A; $i_g = 10$ A;

$i_d = 15$ A

$\therefore \quad i = i_d - i_e = 0$ A

CHECK: $p_{1\text{T}} = p_{1\text{B}} = (i_b)^2 = (i_g)^2 = 100$ W

$p_{1\text{L}} = (i_a - i_b)^2 = (i_a - i_g)^2 = 400$ W

$p_{2\text{C}} = 2(i_b - i_c)^2 = (i_g - i_f)^2 = 50$ W

$p3 = 3(i_c - i_d)^2 = 3(i_e - i_f)^2 = 300$ W

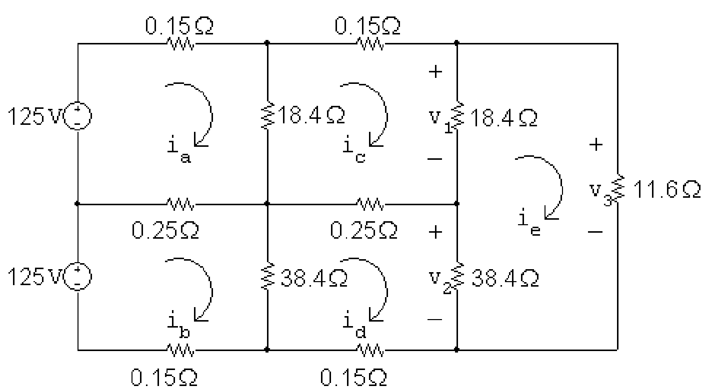
$p4 = 4(i_d - i_e)^2 = 0$ W

$p8 = 8(i_c)^2 = 8(i_f)^2 = 200$ W

$p_{2\text{L}} = 2(i_a - i_d)^2 = 2(i_a - i_e)^2 = 450$ W

$$\begin{aligned}\sum p_{\text{abs}} &= 100 + 400 + 50 + 200 + 300 + 450 + 0 + 450 + 300 + \\ &\quad 200 + 50 + 400 + 100 = 3000 \text{ W} \\ \sum p_{\text{gen}} &= 100 i_a = 100(30) = 3000 \text{ W (CHECKS)}\end{aligned}$$

P 4.99



The mesh equations are:

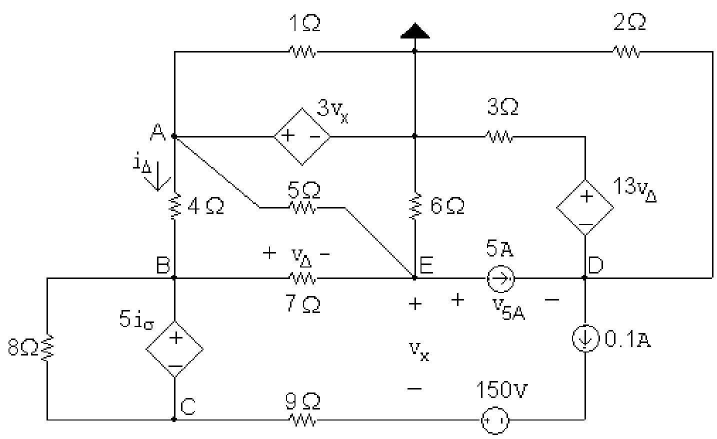
$$\begin{aligned}-125 + 0.15 i_\text{a} + 18.4(i_\text{a} - i_\text{c}) + 0.25(i_\text{a} - i_\text{b}) &= 0 \\ -125 + 0.25(i_\text{b} - i_\text{a}) + 38.4(i_\text{b} - i_\text{d}) + 0.15 i_\text{b} &= 0 \\ 0.15 i_\text{c} + 18.4(i_\text{c} - i_\text{e}) + 0.25(i_\text{c} - i_\text{d}) + 18.4(i_\text{c} - i_\text{a}) &= 0 \\ 0.15 i_\text{d} + 38.4(i_\text{d} - i_\text{b}) + 0.25(i_\text{d} - i_\text{c}) + 38.4(i_\text{d} - i_\text{e}) &= 0 \\ 11.6 i_\text{e} + 38.4(i_\text{e} - i_\text{d}) + 18.4(i_\text{e} - i_\text{c}) &= 0\end{aligned}$$

Place these equations in standard form:

$$
\begin{aligned}
i_a(18.8) + i_b(-0.25) + i_c(-18.4) + i_d(0) + i_e(0) &= 125 \\
i_a(-0.25) + i_b(38.8) + i_c(0) + i_d(-38.4) + i_e(0) &= 125 \\
i_a(-18.4) + i_b(0) + i_c(37.2) + i_d(-0.25) + i_e(-18.4) &= 0 \\
i_a(0) + i_b(-38.4) + i_c(-0.25) + i_d(77.2) + i_e(-38.4) &= 0 \\
i_a(0) + i_b(0) + i_c(-18.4) + i_d(-38.4) + i_e(68.4) &= 0
\end{aligned}
$$

Solving,
$i_a = 32.77$ A; $i_b = 26.46$ A; $i_c = 26.33$ A; $i_d = 23.27$ A; $i_e = 20.14$ A
Find the requested voltages:
$v_1 = 18.4(i_c - i_e) = 113.90$ V
$v_2 = 38.4(i_d - i_e) = 120.19$ V
$v_3 = 11.6 i_e = 233.62$ V

P 4.100



KCL equations at nodes B, D, and E:

$$\frac{v_B - v_A}{4} + \frac{v_B - v_E}{7} - 0.1 = 0$$

$$0.1 + \frac{v_D}{2} + \frac{v_D + 13v_\Delta}{3} - 5 = 0$$

$$\frac{v_E - v_B}{7} + \frac{v_E - v_A}{5} + \frac{v_E}{6} + 5 = 0$$

Multiply the first equation by 28, the second by 6, and the third by 42 to get

$$-7v_A + 11v_B - 4v_E = 2.8$$

$$5v_D + 26v_\Delta = 29.4$$

$$-8.4v_{\rm A} - 6v_{\rm B} + 21.4v_{\rm E} = -210$$

Constraint equations:

$$v_{\rm A} = 3v_x; \qquad v_x = v_{\rm E} - v_{\rm C} - 0.9; \qquad v_\Delta = v_{\rm B} - v_{\rm E}$$

$$v_\sigma = \frac{v_{\rm A} - v_{\rm B}}{4} = 0.25v_{\rm A} - 0.25v_{\rm B}; \qquad 5i_\sigma = v_{\rm B} = v_{\rm C}$$

Use the constraint equations to solve for $v_{\rm A}, v_{\rm B}$ and v_Δ in terms of $v_{\rm C}$ and $v_{\rm E}$:

$$v_{\rm A} = 3v_{\rm E} - 3v_{\rm C} - 2.7$$

$$v_{\rm B} = \frac{15}{9}v_{\rm E} - \frac{11}{9}v_{\rm C} - 1.5$$

$$v_\Delta = \frac{6}{9}v_{\rm E} - \frac{11}{9}v_{\rm C} - 1.5$$

Substitute these three expressions into the previous three equations to yield:

$$68v_{\rm C} + 0v_{\rm D} - 60v_{\rm E} = 3.6$$

$$-286v_{\rm C} + 45v_{\rm D} + 156v_{\rm E} = 615.6$$

$$292.8v_{\rm C} + 0v_{\rm D} - 124.2v_{\rm E} = -2175.12$$

Solving,

$$v_{\rm C} = -14.3552 \text{ V}; \qquad v_{\rm D} = -20.9474 \text{ V}; \qquad v_{\rm C} = 16.3293 \text{ V}$$

From the circuit diagram,

$$p_{5\rm A} = 5v_{5\rm A} = 5(v_{\rm E} - v_{\rm D}) = 23.09 \text{ W}$$

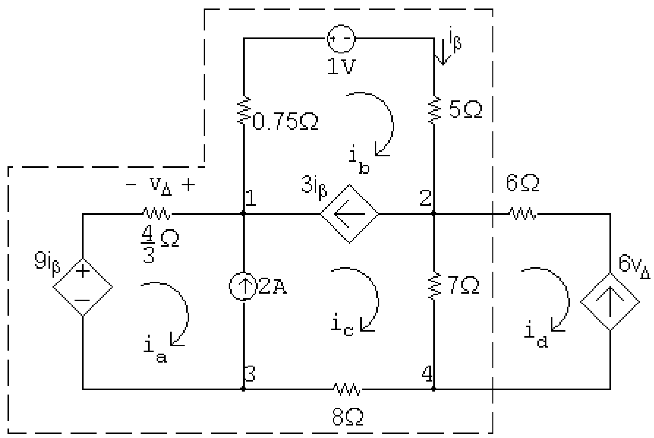
Therefore the 5 A source is absorbing 23.09 W of power.

P 4.101 **[a]** In studying the circuit in Fig. P4.101 we note it contains six meshes and six essential nodes. Further study shows that by replacing the parallel resistors with their equivalent values the circuit reduces to four meshes and four essential nodes as shown in the following diagram.

The node Voltage approach will require solving three node Voltage equations along with equations involving v_Δ and i_β.

The mesh-current approach will require writing one supermesh equation plus three constraint equations involving the three current sources. Thus at the outset we know the supermesh equation can be reduced to a single unknown current. Since we are interested in the power developed by the 1 V source, we will retain the mesh current $i_{\rm b}$ and eliminate the mesh currents $i_{\rm a}$, $i_{\rm c}$ And $i_{\rm d}$.

The supermesh is denoted by the dashed line in the following figure.



[b] Summing the voltages around the supermesh yields

$$-9i_\beta + \frac{4}{3}i_{\rm a} + 0.75i_{\rm b} + 1 + 5i_{\rm b} + 7(i_{\rm c} - i_{\rm d}) + 8i_{\rm c} = 0$$

Note that $i_\beta = i_{\rm b}$; make that substitution and multiply the equation by 12:

$$-108i_{\rm b} + 16i_{\rm a} + 9i_{\rm b} + 12 + 60i_{\rm b} + 84(i_{\rm c} - i_{\rm d}) + 96i_{\rm c} = 0$$

or

$$16i_{\rm a} - 39i_{\rm b} + 180i_{\rm c} - 84i_{\rm d} = -12$$

Use the following constraints:

$$i_{\rm a} - i_{\rm c} = -2; \qquad\qquad i_{\rm b} - i_{\rm c} = 3i_{\rm b}$$

$$\therefore \qquad i_{\rm a} = -2 + i_{\rm c} = -2 - 2i_{\rm b}$$

Therefore,

$$16(-2 - 2i_{\rm b}) - 39i_{\rm b} + 180(-2i_{\rm b}) - 84i_{\rm d} = -12$$

so

$$-431i_{\rm b} - 84i_{\rm d} = 20$$

Finally use the following constraint:

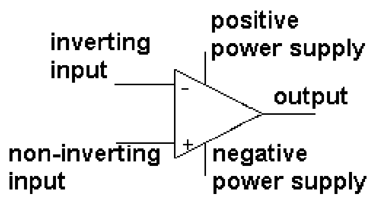
$$i_{\rm d} = -6v_\Delta = -6\left(-\frac{4}{3}i_{\rm a}\right) = 8i_{\rm a} = -16 - 16i_{\rm b}$$

Thus,

$$-431i_{\rm b} - 84(-16 - 16i_{\rm b}) = 20$$

Problems

P 5.1 **[a]** The five terminals of the op amp are identified as follows:



[b] The input resistance of an ideal op amp is infinite, which constrains the value of the input currents to 0. Thus, $i_n = 0$ A.

[c] The open-loop voltage gain of an ideal op amp is infinite, which constrains the difference between the voltage at the two input terminals to 0. Thus, $(v_p - v_n) = 0$.

[d] Write a node voltage equation at v_n:

$$\frac{v_n + 3}{5000} + \frac{v_n - v_o}{15{,}000} = 0$$

But $v_p = 0$ and $v_n = v_p = 0$. Thus,

$$\frac{3}{5000} - \frac{v_o}{15{,}000} = 0 \quad \text{so} \quad v_o = 9 \text{ V}$$

P 5.2 $v_o = -(0.5 \times 10^{-3})(10{,}000) = -5 \text{ V}$

$$\therefore \quad i_o = \frac{v_o}{5000} = \frac{-5}{5000} = -1\,\text{mA}$$

P 5.3 $\dfrac{v_\text{b} - v_\text{a}}{20{,}000} + \dfrac{v_\text{b} - v_o}{100{,}000} = 0,$ therefore $v_o = 6v_\text{b} - 5v_\text{a}$

[a] $v_\text{a} = 4$ V, $v_\text{b} = 0$ V, $v_o = -15$ V (sat)

[b] $v_\text{a} = 2$ V, $v_\text{b} = 0$ V, $v_o = -10$ V

[c] $v_\text{a} = 2$ V, $v_\text{b} = 1$ V, $v_o = -4$ V

[d] $v_\text{a} = 1$ V, $v_\text{b} = 2$ V, $v_o = 7$ V

[e] $v_\text{a} = 1.5$ V, $v_\text{b} = 4$ V, $v_o = 15$ V (sat)

[f] If $v_\text{b} = 1.6$ V, $v_o = 9.6 - 5v_\text{a} = \pm 15$

$$\therefore \quad -1.08 \text{ V} \le v_\text{a} \le 4.92 \text{ V}$$

P 5.4 $v_p = \dfrac{3000}{3000+6000}(3) = 1\ \text{V} = v_n$

$$\frac{v_n - 5}{10{,}000} + \frac{v_n - v_o}{5000} = 0$$

$$(1-5) + 2(1 - v_o) = 0$$

$$v_o = -1.0\ \text{V}$$

$$i_{\text{L}} = \frac{v_o}{4000} = -\frac{1}{4000} = -250 \times 10^{-6}$$

$$i_{\text{L}} = -250\,\mu\text{A}$$

P 5.5 Since the current into the inverting input terminal of an ideal op-amp is zero, the voltage across the $2.2\,\text{M}\Omega$ resistor is $(2.2 \times 10^6)(3.5 \times 10^{-6})$ or 7.7 V. Therefore the voltmeter reads 7.7 V.

P 5.6 [a] $i_2 = \dfrac{150 \times 10^{-3}}{2000} = 75\,\mu\text{A}$

$$v_1 = -40 \times 10^3 i_2 = -3\,\text{V}$$

[b] $\dfrac{v_1}{20{,}000} + \dfrac{v_1}{40{,}000} + \dfrac{v_1 - v_o}{50{,}000} = 0$

$$\therefore\quad v_o = 4.75 v_1 = -14.25\ \text{V}$$

[c] $i_2 = 75\,\mu\text{A}$, (from part [a])

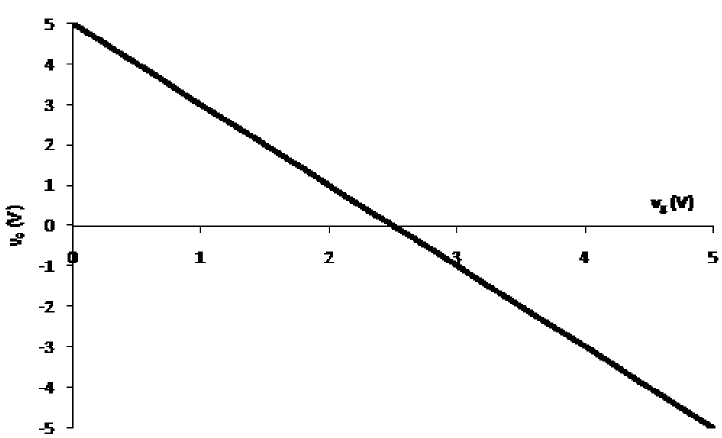
[d] $i_o = \dfrac{-v_o}{25{,}000} + \dfrac{v_1 - v_o}{50{,}000} = 795\,\mu\ \text{A}$

P 5.7 [a] First, note that $v_n = v_p = 2.5\ \text{V}$

Let v_{o1} equal the voltage output of the op-amp. Then

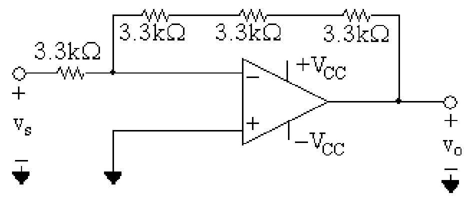
$$\frac{2.5 - v_g}{5000} + \frac{2.5 - v_{o1}}{10{,}000} = 0, \qquad \therefore\quad v_{o1} = 7.5 - 2v_g$$

Also note that $v_{o1} - 2.5 = v_o, \qquad \therefore\quad v_o = 5 - 2v_g$

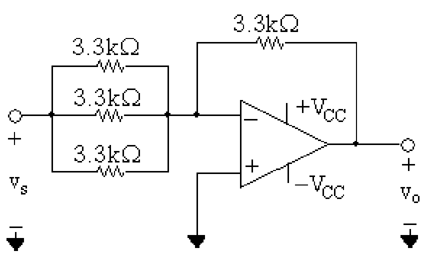


[b] Yes, the circuit designer is correct!

P 5.8 [a] The gain of an inverting amplifier is the ratio of the feedback resistor to the input resistor. If the gain of the inverting amplifier is to be 3, the feedback resistor must be 3 times as large as the input resistor. There are many possible designs that use a resistor value chosen from Appendix H. We present two here that use 3.3 kΩ resistors. Use a single 3.3 kΩ resistor as the input resistor, and use three 3.3 kΩ resistors in series as the feedback resistor to give a total of 9.9 kΩ.

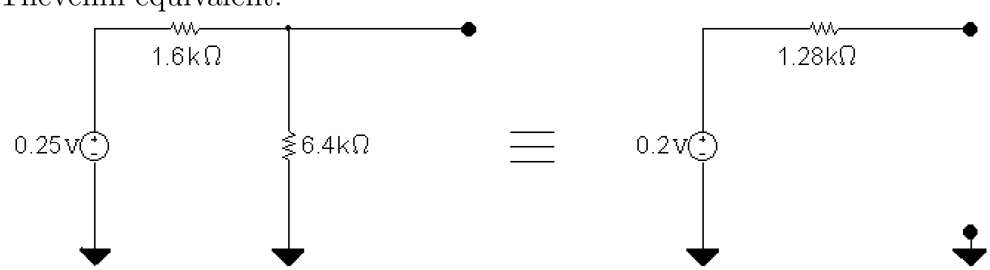


Alternately, use a single 3.3 kΩ resistor as the feedback resistor and use three 3.3 kΩ resistors in parallel as the input resistor to give a total of 1.1 kΩ.



[b] To amplify a 5 V signal without saturating the op amp, the power supply voltages must be greater than or equal to the product of the input voltage and the amplifier gain. Thus, the power supplies should have a magnitude of $(5)(3) = 15$ V.

P 5.9 [a] Replace the combination of v_g, 1.6 kΩ, and the 6.4 kΩ resistors with its Thévenin equivalent.



Then $v_o = \dfrac{-[12 + \sigma 50]}{1.28}(0.20)$

At saturation $v_o = -5$ V; therefore

$$-\left(\frac{12 + \sigma 50}{1.28}\right)(0.2) = -5, \quad \text{or} \quad \sigma = 0.4$$

Thus for $0 \le \sigma \le 0.40$ the operational amplifier will not saturate.

[b] When $\sigma = 0.272, \qquad v_o = \dfrac{-(12 + 13.6)}{1.28}(0.20) = -4\ \text{V}$

Also $\dfrac{v_o}{10} + \dfrac{v_o}{25.6} + i_o = 0$

$$\therefore\ i_o = -\frac{v_o}{10} - \frac{v_o}{25.6} = \frac{4}{10} + \frac{4}{25.6}\ \text{mA} = 556.25\,\mu\text{A}$$

P 5.10 **[a]** Let v_Δ be the voltage from the potentiometer contact to ground. Then

$$\frac{0 - v_g}{2000} + \frac{0 - v_\Delta}{50{,}000} = 0$$

$$-25v_g - v_\Delta = 0, \qquad \therefore\ v_\Delta = -25(40 \times 10^{-3}) = -1\ \text{V}$$

$$\frac{v_\Delta}{\alpha R_\Delta} + \frac{v_\Delta - 0}{50{,}000} + \frac{v_\Delta - v_o}{(1-\alpha)R_\Delta} = 0$$

$$\frac{v_\Delta}{\alpha} + 2v_\Delta + \frac{v_\Delta - v_o}{1-\alpha} = 0$$

$$v_\Delta\left(\frac{1}{\alpha} + 2 + \frac{1}{1-\alpha}\right) = \frac{v_o}{1-\alpha}$$

$$\therefore\ v_o = -1\left[1 + 2(1-\alpha) + \frac{(1-\alpha)}{\alpha}\right]$$

When $\alpha = 0.2, \qquad v_o = -1(1 + 1.6 + 4) = -6.6\ \text{V}$

When $\alpha = 1, \qquad v_o = -1(1 + 0 + 0) = -1\ \text{V}$

$$\therefore\ -6.6\ \text{V} \le v_o \le -1\ \text{V}$$

[b] $-1\left[1 + 2(1-\alpha) + \dfrac{(1-\alpha)}{\alpha}\right] = -7$

$$\alpha + 2\alpha(1-\alpha) + (1-\alpha) = 7\alpha$$

$$\alpha + 2\alpha - 2\alpha^2 + 1 - \alpha = 7\alpha$$

$$\therefore\ 2\alpha^2 + 5\alpha - 1 = 0 \quad \text{so} \quad \alpha \cong 0.186$$

P 5.11 $v_o = -\left[\dfrac{R_\text{f}}{4000}(0.2) + \dfrac{R_\text{f}}{5000}(0.15) + \dfrac{R_\text{f}}{20{,}000}(0.4)\right]$

$$-6 = -0.1 \times 10^{-3} R_\text{f}; \qquad R_\text{f} = 60\,\text{k}\Omega; \qquad \therefore\ 0 \le R_\text{f} \le 60\,\text{k}\Omega$$

P 5.12 **[a]** This circuit is an example of an inverting summing amplifier.

[b] $v_o = -\frac{220}{44}v_\mathrm{a} - \frac{220}{27.5}v_\mathrm{b} - \frac{220}{80}v_\mathrm{c} = -5 - 12 + 11 = -6 \text{ V}$

[c] $v_o = -6 - 8v_\mathrm{b} = \pm 10$

$$\therefore \quad v_\mathrm{b} = -0.5 \text{ V} \quad \text{when} \quad v_o = 10 \text{ V};$$

$$v_\mathrm{b} = 2 \text{ V} \quad \text{when} \quad v_o = -10 \text{ V}$$

$$\therefore \quad -0.5 \text{ V} \le v_\mathrm{b} \le 2 \text{ V}$$

P 5.13 We want the following expression for the output voltage:

$$v_o = -(3v_a + 5v_b + 4v_c + 2v_d)$$

This is an inverting summing amplifier, so each input voltage is amplified by a gain that is the ratio of the feedback resistance to the resistance in the forward path for the input voltage. Pick a feedback resistor with divisors of 3, 5, 4, and 2 – say $60\,\mathrm{k}\Omega$:

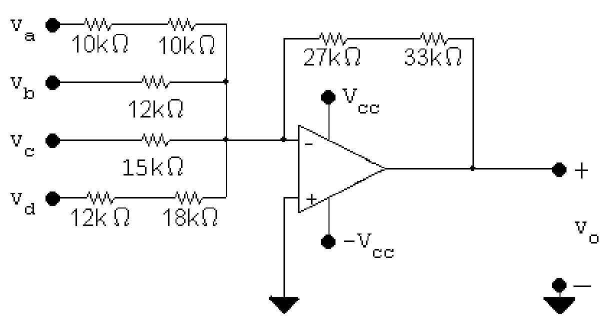
$$v_o = -\left[\frac{60\mathrm{k}}{R_\mathrm{a}}v_\mathrm{a} + \frac{60\mathrm{k}}{R_\mathrm{b}}v_\mathrm{b} + \frac{60\mathrm{k}}{R_\mathrm{c}}v_\mathrm{c} + \frac{60\mathrm{k}}{R_\mathrm{d}}v_\mathrm{d}\right]$$

Solve for each input resistance value to yield the desired gain:

$$\therefore \quad R_\mathrm{a} = 60{,}000/3 = 20\,\mathrm{k}\Omega \qquad R_\mathrm{c} = 60{,}000/4 = 15\,\mathrm{k}\Omega$$

$$R_\mathrm{b} = 60{,}000/5 = 12\,\mathrm{k}\Omega \qquad R_\mathrm{d} = 60{,}000/2 = 30\,\mathrm{k}\Omega$$

Now create the 5 resistor values needed from the realistic resistor values in Appendix H. Note that $R_\mathrm{b} = 12\,\mathrm{k}\Omega$ and $R_\mathrm{c} = 15\,\mathrm{k}\Omega$ are already values from Appendix H. Create $R_\mathrm{f} = 60\,\mathrm{k}\Omega$ by combining $27\,\mathrm{k}\Omega$ and $33\,\mathrm{k}\Omega$ in series. Create $R_\mathrm{a} = 20\,\mathrm{k}\Omega$ by combining two $10\,\mathrm{k}\Omega$ resistors in series. Create $R_\mathrm{d} = 30\,\mathrm{k}\Omega$ by combining $18\,\mathrm{k}\Omega$ and $12\,\mathrm{k}\Omega$ in series. Of course there are many other acceptable possibilities. The final circuit is shown here:



P 5.14 [a] Write a KCL equation at the inverting input to the op amp:

$$\frac{v_\mathrm{d} - v_\mathrm{a}}{40{,}000} + \frac{v_\mathrm{d} - v_\mathrm{b}}{22{,}000} + \frac{v_\mathrm{d} - v_\mathrm{c}}{100{,}000} + \frac{v_\mathrm{d}}{352{,}000} + \frac{v_\mathrm{d} - v_o}{220{,}000} = 0$$

Multiply through by 220,000, plug in the values of input voltages, and rearrange to solve for v_o:

$$v_o = 220{,}000\left(\frac{4}{40{,}000} + \frac{-1}{22{,}000} + \frac{-5}{100{,}000}\right.$$

$$\left. + \frac{8}{352{,}000} + \frac{8}{220{,}000}\right) = 14 \text{ V}$$

[**b**] Write a KCL equation at the inverting input to the op amp. Use the given values of input voltages in the equation:

$$\frac{8 - v_\mathrm{a}}{40{,}000} + \frac{8-9}{22{,}000} + \frac{8-13}{100{,}000} + \frac{8}{352{,}000} + \frac{8 - v_o}{220{,}000} = 0$$

Simplify and solve for v_o:

$$44 - 5.5v_\mathrm{a} - 10 - 11 + 5 + 8 - v_o = 0 \quad \text{so} \quad v_o = 36 - 5.5v_\mathrm{a}$$

Set v_o to the positive power supply voltage and solve for v_a:

$$36 - 5.5v_\mathrm{a} = 15 \quad \therefore \quad v_\mathrm{a} = 3.818 \text{ V}$$

Set v_o to the negative power supply voltage and solve for v_a:

$$36 - 5.5v_\mathrm{a} = -15 \quad \therefore \quad v_\mathrm{a} = 9.273 \text{ V}$$

Therefore,

$$3.818 \text{ V} \le v_\mathrm{a} \le 9.273 \text{ V}$$

P 5.15 [**a**] $$\frac{8-4}{40{,}000} + \frac{8-9}{22{,}000} + \frac{8-13}{100{,}000} + \frac{8}{352{,}000} + \frac{8 - v_0}{R_f} = 0$$

$$\frac{8 - v_o}{R_f} = -2.7272 \times 10^{-5} \quad \text{so} \quad R_f = \frac{8 - v_o}{-2.727 \times 10^{-5}}$$

For $v_o = 15$ V, $R_f = 256.7\,\text{k}\Omega$

For $v_o = -15$ V, $R_f < 0$ so this solution is not possible.

[**b**] $$i_o = -(i_f + i_{10\text{k}}) = -\left[\frac{15-8}{256.7 \times 10^3} + \frac{15}{10{,}000}\right] = -1527\,\mu\text{A}$$

P 5.16 [**a**] The circuit shown is a non-inverting amplifier.

[**b**] We assume the op amp to be ideal, so $v_n = v_p = 3$ V. Write a KCL equation at v_n:

$$\frac{3}{40{,}000} + \frac{3 - v_o}{80{,}000} = 0$$

Solving,

$$v_o = 9 \text{ V}.$$

P 5.17 **[a]** This circuit is an example of the non-inverting amplifier.

[b] Use voltage division to calculate v_p:

$$v_p = \frac{10{,}000}{10{,}000 + 30{,}000} v_s = \frac{v_s}{4}$$

Write a KCL equation at $v_n = v_p = v_s/4$:

$$\frac{v_s/4}{4000} + \frac{v_s/4 - v_o}{28{,}000} = 0$$

Solving,

$$v_o = 7v_s/4 + v_s/4 = 2v_s$$

[c] $2v_s = 8 \quad \text{so} \quad v_s = 4 \text{ V}$

$$2v_s = -12 \quad \text{so} \quad v_s = -6 \text{ V}$$

Thus, $-6 \text{ V} \le v_s \le 4 \text{ V}$.

P 5.18 **[a]** $v_p = v_n = \frac{68}{80} v_g = 0.85 v_g$

$$\therefore \quad \frac{0.85v_g}{30{,}000} + \frac{0.85v_g - v_o}{63{,}000} = 0;$$

$$\therefore \quad v_o = 2.635v_g = 2.635(4), \quad v_o = 10.54 \text{ V}$$

[b] $v_o = 2.635v_g = \pm 12$

$$v_g = \pm 4.55 \text{ V}, \qquad -4.55 \le v_g \le 4.55 \text{ V}$$

[c] $\frac{0.85v_g}{30{,}000} + \frac{0.85v_g - v_o}{R_\mathrm{f}} = 0$

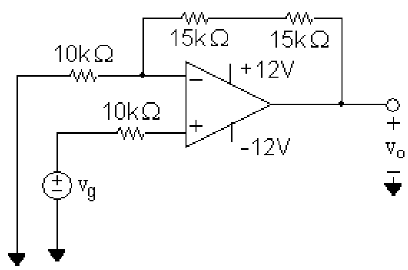
$$\left(\frac{0.85R_\mathrm{f}}{30{,}000} + 0.85\right) v_g = v_o = \pm 12$$

$$\therefore \quad 1.7R_\mathrm{f} + 51 = \pm 360; \qquad 1.7R_\mathrm{f} = 360 - 51; \qquad R_\mathrm{f} = 181.76\,\mathrm{k}\Omega$$

P 5.19 **[a]** From the equation for the non-inverting amplifier,

$$\frac{R_s + R_f}{R_s} = 4 \quad \text{so} \quad R_s + R_f = 4R_s \quad \text{and therefore} \quad R_f = 3R_s$$

Choose $R_f = 30\,\mathrm{k}\Omega$ and implement this choice from components in Appendix H by combining two $15\,\mathrm{k}\Omega$ resistors in series. Choose $R_s = R_g = 10\,\mathrm{k}\Omega$, which is a component in Appendix H. The resulting non-inverting amplifier circuit is shown here:



[**b**] $v_o = 4v_g = 12 \qquad \text{so} \qquad v_g = 3\ \text{V}$

$v_o = 4v_g = -12 \qquad \text{so} \qquad v_g = -3\ \text{V}$

Therefore,

$-3\ \text{V} \le v_g \le 3\ \text{V}$

P 5.20 [**a**] This circuit is an example of a non-inverting summing amplifier.

[**b**] Write a KCL equation at v_p and solve for v_p in terms of v_s:

$$\frac{v_p - v_s}{15{,}000} + \frac{v_p - 6}{30{,}000} = 0$$

$$2v_p - 2v_s + v_p - 6 = 0 \qquad \text{so} \qquad v_p = 2v_s/3 + 2$$

Now write a KCL equation at v_n and solve for v_o:

$$\frac{v_n}{20{,}000} + \frac{v_n - v_o}{60{,}000} = 0 \qquad \text{so} \qquad v_o = 4v_n$$

Since we assume the op amp is ideal, $v_n = v_p$. Thus,

$$v_o = 4(2v_s/3 + 2) = 8v_s/3 + 8$$

[**c**] $8v_s/3 + 8 = 16 \qquad \text{so} \qquad v_s = 3\ \text{V}$

$8v_s/3 + 8 = -12 \qquad \text{so} \qquad v_s = -7.5\ \text{V}$

Thus, $-7.5\ \text{V} \le v_s \le 3\ \text{V}$.

P 5.21 [**a**] This is a non-inverting summing amplifier.

[**b**] $$\frac{v_p - v_\text{a}}{13 \times 10^3} + \frac{v_p - v_\text{b}}{27 \times 10^3} = 0$$

$$\therefore \quad 40v_p = 27v_\text{a} + 13v_\text{b} \qquad \text{so} \qquad v_p = 0.675v_a + 0.325v_b$$

$$\frac{v_n}{11{,}000} + \frac{v_n - v_o}{110{,}000} = 0$$

$$\therefore \quad v_o = 11v_n = 11v_p = 11(0.675v_a + 0.325v_b)$$

$$= 11[0.675(0.8) + 0.325(0.4)] = 7.37\ \text{V}$$

[c] $v_p = v_n = \dfrac{v_o}{11} = 0.667\ \text{V}$

$$i_{\text{a}} = \frac{v_{\text{a}} - v_p}{13 \times 10^3} = 10\,\mu\text{A}$$

$$i_{\text{b}} = \frac{v_{\text{b}} - v_p}{27 \times 10^3} = -10\,\mu\text{A}$$

[d] 7.425 for v_{a};   3.575 for v_{b}

P 5.22 **[a]** $\dfrac{v_p - v_{\text{a}}}{R_{\text{a}}} + \dfrac{v_p - v_{\text{b}}}{R_{\text{b}}} + \dfrac{v_p - v_{\text{c}}}{R_{\text{c}}} = 0$

$$\therefore\quad v_p = \frac{R_{\text{b}}R_{\text{c}}}{D}v_{\text{a}} + \frac{R_{\text{a}}R_{\text{c}}}{D}v_{\text{b}} + \frac{R_{\text{a}}R_{\text{b}}}{D}v_{\text{c}}$$

where $D = R_{\text{b}}R_{\text{c}} + R_{\text{a}}R_{\text{c}} + R_{\text{a}}R_{\text{b}}$

$$\frac{v_n}{20{,}000} + \frac{v_n - v_o}{100{,}000} = 0$$

$$\left(\frac{100{,}000}{20{,}000} + 1\right) v_n = 6v_n = v_o$$

$$\therefore\quad v_o = \frac{6R_{\text{b}}R_{\text{c}}}{D}v_{\text{a}} + \frac{6R_{\text{a}}R_{\text{c}}}{D}v_{\text{b}} + \frac{6R_{\text{a}}R_{\text{b}}}{D}v_{\text{c}}$$

By hypothesis,

$$\frac{6R_{\text{b}}R_{\text{c}}}{D} = 1; \qquad \frac{6R_{\text{a}}R_{\text{c}}}{D} = 2; \qquad \frac{6R_{\text{a}}R_{\text{b}}}{D} = 3$$

Then

$$\frac{6R_{\text{a}}R_{\text{b}}/D}{6R_{\text{a}}R_{\text{c}}/D} = \frac{3}{2} \qquad \text{so} \qquad R_{\text{b}} = 1.5R_{\text{c}}$$

But from the circuit

$$R_{\text{b}} = 15\,\text{k}\Omega \qquad \text{so} \qquad R_{\text{c}} = 10\,\text{k}\Omega$$

Similarly,

$$\frac{6R_{\text{b}}R_{\text{c}}/D}{6R_{\text{a}}R_{\text{b}}/D} = \frac{1}{3} \qquad \text{so} \qquad 3R_{\text{c}} = R_{\text{a}}$$

Thus,

$$R_{\text{a}} = 30\,\text{k}\Omega$$

[b] $v_o = 1(0.7) + 2(0.4) + 3(1.1) = 4.8\ \text{V}$

$$v_n = v_o/6 = 0.8\ \text{V} = v_p$$

$$i_{\text{a}} = \frac{v_{\text{a}} - v_p}{30{,}000} = \frac{0.7 - 0.8}{30{,}000} = -3.33\,\mu\text{A}$$

$$i_b = \frac{v_b - v_p}{15{,}000} = \frac{0.4 - 0.8}{15{,}000} = -26.67\,\mu\text{A}$$

$$i_c = \frac{v_c - v_p}{10{,}000} = \frac{1.1 - 0.8}{10{,}000} = 30\,\mu\text{A}$$

Check:

$i_a + i_b + i_c = 0?$ $\qquad -3.33 - 26.67 + 30 = 0$ (checks)

P 5.23 [**a**] $\dfrac{v_p - v_a}{R_a} + \dfrac{v_p - v_b}{R_b} + \dfrac{v_p - v_c}{R_c} + \dfrac{v_p}{R_g} = 0$

$$\therefore\quad v_p = \frac{R_bR_cR_g}{D}v_a + \frac{R_aR_cR_g}{D}v_b + \frac{R_aR_bR_g}{D}v_c$$

where $\quad D = R_bR_cR_g + R_aR_cR_g + R_aR_bR_g + R_aR_bR_c$

$$\frac{v_n}{R_s} + \frac{v_n - v_o}{R_f} = 0$$

$$v_n\left(\frac{1}{R_s} + \frac{1}{R_f}\right) = \frac{v_o}{R_f}$$

$$\therefore\quad v_o = \left(1 + \frac{R_f}{R_s}\right)v_n = kv_n$$

where $\quad k = \left(1 + \dfrac{R_f}{R_s}\right)$

$$v_p = v_n$$

$$\therefore\quad v_o = kv_p$$

or

$$v_o = \frac{kR_gR_bR_c}{D}v_a + \frac{kR_gR_aR_c}{D}v_b + \frac{kR_gR_aR_b}{D}v_c$$

$$\frac{kR_gR_bR_c}{D} = 6 \qquad \frac{kR_gR_aR_c}{D} = 3 \qquad \frac{kR_gR_aR_b}{D} = 4$$

$$\therefore\quad \frac{R_b}{R_a} = \frac{6}{3} = 2 \qquad \frac{R_c}{R_b} = \frac{3}{4} = 0.75 \qquad \frac{R_c}{R_a} = \frac{6}{4} = 1.5$$

Since $\quad R_a = 1\,\text{k}\Omega \qquad R_b = 2\,\text{k}\Omega \qquad R_c = 1.5\,\text{k}\Omega$

$$\therefore\quad D = [(2)(1.5)(3) + (1)(1.5)(3) + (1)(2)(3) + (1)(2)(1.5)] \times 10^9 = 22.5 \times 10^9$$

$$\frac{k(3)(2)(1.5) \times 10^9}{22.5 \times 10^9} = 6$$

$$k = \frac{135 \times 10^9}{9 \times 10^9} = 15$$

$$\therefore \quad 15 = 1 + \frac{R_\text{f}}{R_s}$$

$$\frac{R_\text{f}}{R_s} = 14$$

$$R_\text{f} = (14)(15{,}000) = 210\,\text{k}\Omega$$

[b] $v_o = 6(0.5) + 3(2.5) + 4(1) = 14\ \text{V}$

$$v_n = v_p = \frac{14.5}{15} = 0.967\ \text{V}$$

$$i_\text{a} = \frac{0.5 - 0.967}{1000} = -466.67\,\mu\text{A}$$

$$i_\text{b} = \frac{2.5 - 0.967}{2000} = 766.67\,\mu\text{A}$$

$$i_\text{c} = \frac{1 - 0.967}{1500} = 22.22\,\mu\text{A}$$

$$i_g = \frac{0.967}{3000} = 322.22\,\mu\text{A}$$

$$i_s = \frac{v_n}{15{,}000} = \frac{0.967}{15{,}000} = 64.44\,\mu\text{A}$$

P 5.24 **[a]** Assume v_a is acting alone. Replacing v_b with a short circuit yields $v_p = 0$, therefore $v_n = 0$ and we have

$$\frac{0 - v_\text{a}}{R_\text{a}} + \frac{0 - v_o'}{R_\text{b}} + i_n = 0, \qquad i_n = 0$$

Therefore

$$\frac{v_o'}{R_\text{b}} = -\frac{v_\text{a}}{R_\text{a}}, \qquad v_o' = -\frac{R_\text{b}}{R_\text{a}} v_\text{a}$$

Assume v_b is acting alone. Replace v_a with a short circuit. Now

$$v_p = v_n = \frac{v_\text{b} R_\text{d}}{R_\text{c} + R_\text{d}}$$

$$\frac{v_n}{R_\text{a}} + \frac{v_n - v_o''}{R_\text{b}} + i_n = 0, \qquad i_n = 0$$

$$\left(\frac{1}{R_\text{a}} + \frac{1}{R_\text{b}}\right)\left(\frac{R_\text{d}}{R_\text{c} + R_\text{d}}\right) v_\text{b} - \frac{v_o''}{R_\text{b}} = 0$$

$$v_o'' = \left(\frac{R_\text{b}}{R_\text{a}} + 1\right)\left(\frac{R_\text{d}}{R_\text{c} + R_\text{d}}\right) v_\text{b} = \frac{R_\text{d}}{R_\text{a}}\left(\frac{R_\text{a} + R_\text{b}}{R_\text{c} + R_\text{d}}\right) v_\text{b}$$

$$v_o = v_o' + v_o'' = \frac{R_\text{d}}{R_\text{a}}\left(\frac{R_\text{a} + R_\text{b}}{R_\text{c} + R_\text{d}}\right) v_\text{b} - \frac{R_\text{b}}{R_\text{a}} v_\text{a}$$

[b] $\dfrac{R_\text{d}}{R_\text{a}}\left(\dfrac{R_\text{a}+R_\text{b}}{R_\text{c}+R_\text{d}}\right)=\dfrac{R_\text{b}}{R_\text{a}},\qquad \text{therefore}\quad R_\text{d}(R_\text{a}+R_\text{b})=R_\text{b}(R_\text{c}+R_\text{d})$

$R_\text{d}R_\text{a}=R_\text{b}R_\text{c},\qquad \text{therefore}\quad \dfrac{R_\text{a}}{R_\text{b}}=\dfrac{R_\text{c}}{R_\text{d}}$

When $\dfrac{R_\text{d}}{R_\text{a}}\left(\dfrac{R_\text{a}+R_\text{b}}{R_\text{c}+R_\text{d}}\right)=\dfrac{R_\text{b}}{R_\text{a}}$

Eq. (5.22) reduces to $v_o=\dfrac{R_\text{b}}{R_\text{a}}v_\text{b}-\dfrac{R_\text{b}}{R_\text{a}}v_\text{a}=\dfrac{R_\text{b}}{R_\text{a}}(v_\text{b}-v_\text{a}).$

P 5.25 **[a]** $v_o=\dfrac{R_\text{d}(R_\text{a}+R_\text{b})}{R_\text{a}(R_\text{c}+R_\text{d})}v_\text{b}-\dfrac{R_\text{b}}{R_\text{a}}v_\text{a}=\dfrac{120(24+75)}{24(130+120)}(5)-\dfrac{75}{24}(8)$

$v_o=9.9-25=-15.1\ \text{V}$

[b] $\dfrac{v_1-8}{24{,}000}+\dfrac{v_1-15.1}{75{,}000}=0\qquad \text{so}\qquad v_1=2.4\ \text{V}$

$i_\text{a}=\dfrac{8-2.4}{24{,}000}=233\,\mu\ \text{A}$

$R_\text{ina}=\dfrac{v_\text{a}}{i_\text{a}}=\dfrac{8}{233\times10^{-6}}=34.3\,\text{k}\Omega$

[c] $R_\text{in b}=R_\text{c}+R_\text{d}=250\,\text{k}\Omega$

P 5.26 Use voltage division to find v_p:

$$v_p=\frac{2000}{2000+8000}(5)=1\ \text{V}$$

Write a KCL equation at v_n and solve it for v_o:

$$\frac{v_n-v_a}{5000}+\frac{v_n-v_o}{R_f}=0\qquad\text{so}\qquad\left(\frac{R_f}{5000}+1\right)v_n-\frac{R_f}{5000}v_a=v_o$$

Since the op amp is ideal, $v_n=v_p=1\text{V}$, so

$$v_o=\left(\frac{R_f}{5000}+1\right)-\frac{R_f}{5000}v_a$$

To satisfy the equation,

$$\left(\frac{R_f}{5000}+1\right)=5\qquad\text{and}\qquad\frac{R_f}{5000}=4$$

Thus, $R_f=20\ \text{k}\Omega$.

P 5.27 $v_p = \dfrac{v_\text{b} R_\text{b}}{R_\text{a} + R_\text{b}} = v_n$

$$\frac{v_n - v_\text{a}}{4700} + \frac{v_n - v_o}{R_\text{f}} = 0$$

$$v_n\left(\frac{R_\text{f}}{4700} + 1\right) - \frac{v_\text{a} R_\text{f}}{4700} = v_o$$

$$\therefore\quad \left(\frac{R_\text{f}}{4700} + 1\right)\frac{R_\text{b}}{R_\text{a} + R_\text{b}} v_\text{b} - \frac{R_\text{f}}{4700} v_\text{a} = v_o$$

$$\therefore\quad \frac{R_\text{f}}{4700} = 10; \qquad R_\text{f} = 47\,\text{k}\Omega \qquad \text{(a value from Appendix H)}$$

$$R_\text{a} + R_\text{b} = 220\,\text{k}\Omega$$

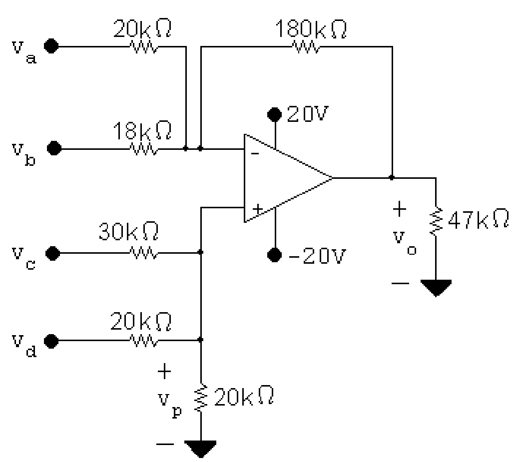
Thus,

$$\left(1 + \frac{47}{4700}\right)\left(\frac{R_\text{b}}{220{,}000}\right) = 10$$

$$\therefore\qquad R_\text{b} = 200\,\text{k}\Omega \qquad \text{and} \qquad R_\text{a} = 220 - 200 = 20\,\text{k}\Omega$$

Use two $100\,\text{k}\Omega$ resistors in series for R_b and use two $10\,\text{k}\Omega$ resistors in series for R_a.

P 5.28 **[a]**



$$\frac{v_p}{20{,}000} + \frac{v_p - v_\text{c}}{30{,}000} + \frac{v_p - v_\text{d}}{20{,}000} = 0$$

$$\therefore\quad 8v_p = 2v_\text{c} + 3v_\text{d} = 8v_n$$

$$\frac{v_n - v_a}{20{,}000} + \frac{v_n - v_b}{18{,}000} + \frac{v_n - v_o}{180{,}000} = 0$$

$$\begin{aligned} \therefore \quad v_o &= 20v_n - 9v_a - 10v_b \\ &= 20[(1/4)v_c + (3/8)v_d] - 9v_a - 10v_b \\ &= 20(0.75 + 1.5) - 9(1) - 10(2) = 16 \text{ V} \end{aligned}$$

[b] $v_o = 5v_c + 30 - 9 - 20 = 5v_c + 1$

$$\pm 20 = 5v_c + 1$$

$$\therefore \quad v_b = -4.2 \text{ V} \qquad \text{and} \qquad v_b = 3.8 \text{ V}$$

$$\therefore \quad -4.2 \text{ V} \le v_b \le 3.8 \text{ V}$$

P 5.29 $v_p = 1000i_b$

$$\frac{1000i_b}{R_a} + \frac{1000i_b - v_o}{R_f} - i_a = 0$$

$$\therefore \qquad 1000i_b\left(\frac{1}{R_a} + \frac{1}{R_f}\right) - i_a = \frac{v_o}{R_f}$$

$$\therefore \qquad 1000i_b\left(1 + \frac{R_f}{R_a}\right) - R_f i_a = v_o$$

By hypopthesis, $v_o = 5000(i_b - i_a)$. Therefore,

$R_f = 5\,\text{k}\Omega$ (use two $10\,\text{k}\Omega$ resistors in parallel)

$$1000\left(1 + \frac{R_f}{R_a}\right) = 5000 \qquad \text{so} \qquad R_a = 1250\,\Omega$$

To construct the $1250\,\Omega$ resistor, combine a $1.2\,\text{k}\Omega$ resistor in series with a parallel combination of two $100\,\Omega$ resistors.

P 5.30 $v_o = \dfrac{R_d(R_a + R_b)}{R_a(R_c + R_d)}v_b - \dfrac{R_b}{R_a}v_a$

By hypothesis: $R_b/R_a = 4$; $R_c + R_d = 470\,\text{k}\Omega$; $\dfrac{R_d(R_a + R_b)}{R_a(R_c + R_d)} = 3$

$$\therefore \quad \frac{R_d}{R_a}\frac{(R_a + 4R_a)}{470{,}000} = 3 \quad \text{so} \quad R_d = 282\,\text{k}\Omega; \quad R_c = 188\,\text{k}\Omega$$

Create $R_{\rm d} = 282\,\mathrm{k\Omega}$ by combining a $270\,\mathrm{k\Omega}$ resistor and a $12\,\mathrm{k\Omega}$ resistor in series. Create $R_{\rm c} = 188\,\mathrm{k\Omega}$ by combining a $120\,\mathrm{k\Omega}$ resistor and a $68\,\mathrm{k\Omega}$ resistor in series. Also, when $v_o = 0$ we have

$$\frac{v_n - v_{\rm a}}{R_{\rm a}} + \frac{v_n}{R_{\rm b}} = 0$$

$$\therefore \quad v_n\left(1 + \frac{R_{\rm a}}{R_{\rm b}}\right) = v_{\rm a}; \qquad v_n = 0.8v_{\rm a}$$

$$i_{\rm a} = \frac{v_{\rm a} - 0.8v_{\rm a}}{R_{\rm a}} = 0.2\frac{v_{\rm a}}{R_{\rm a}}; \qquad R_{\rm in} = \frac{v_{\rm a}}{i_{\rm a}} = 5R_{\rm a} = 22\,\mathrm{k\Omega}$$

$$\therefore \quad R_{\rm a} = 4.4\,\mathrm{k\Omega}; \qquad R_{\rm b} = 17.6\,\mathrm{k\Omega}$$

Create $R_{\rm a} = 4.4\,\mathrm{k\Omega}$ by combining two $2.2\,\mathrm{k\Omega}$ resistors in series. Create $R_{\rm b} = 17.6\,\mathrm{k\Omega}$ by combining a $12\,\mathrm{k\Omega}$ resistor and a $5.6\,\mathrm{k\Omega}$ resistor in series.

P 5.31 $$v_p = \frac{1500}{9000}(-18) = -3\ \mathrm{V} = v_n$$

$$\frac{-3+18}{1600} + \frac{-3 - v_o}{R_{\rm f}} = 0$$

$$\therefore \quad v_o = 0.009375R_{\rm f} - 3$$

$$v_o = 9\ \mathrm{V}; \qquad R_{\rm f} = 1280\,\Omega$$

$$v_o = -9\ \mathrm{V}; \qquad R_{\rm f} = -640\,\Omega$$

But $\quad R_{\rm f} \geq 0, \qquad \therefore \quad R_{\rm f} = 1.28\,\mathrm{k\Omega}$

P 5.32 **[a]** $$v_p = \frac{\alpha R_{\rm g}}{\alpha R_{\rm g} + (R_{\rm g} - \alpha R_{\rm g})}v_{\rm g}$$

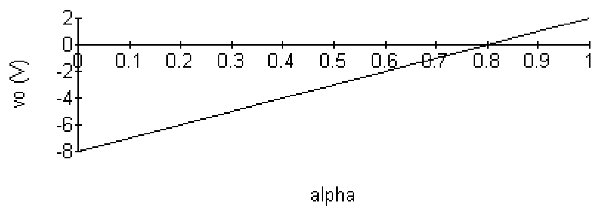
$$v_n = v_p = \alpha v_g$$

$$\frac{v_n - v_g}{R_1} + \frac{v_n - v_o}{R_{\rm f}} = 0$$

$$(v_n - v_g)\frac{R_{\rm f}}{R_1} + v_n - v_o = 0$$

$$\begin{aligned} v_o &= \left(1 + \frac{R_{\rm f}}{R_{\rm g}}\right)\alpha v_{\rm g} - \frac{R_{\rm f}}{R_1}v_{\rm g} \\ &= (\alpha v_g - v_g)4 + \alpha v_g \\ &= [(\alpha - 1)4 + \alpha]v_g \\ &= (5\alpha - 4)v_g \\ &= (5\alpha - 4)(2) = 10\alpha - 8 \end{aligned}$$

α	v_o	α	v_o	α	v_o
0.0	-8 V	0.4	-4 V	0.8	0 V
0.1	-7 V	0.5	-3 V	0.9	1 V
0.2	-6 V	0.6	-2 V	1.0	2 V
0.3	-5 V	0.7	-1 V		



[**b**] Rearranging the equation for v_o from (a) gives

$$v_o = \left(\frac{R_f}{R_1} + 1\right) v_g \alpha + -\left(\frac{R_f}{R_1}\right) v_g$$

Therefore,

$$\text{slope } = \left(\frac{R_f}{R_1} + 1\right) v_g; \qquad \text{intercept } = -\left(\frac{R_f}{R_1}\right) v_g$$

[**c**] Using the equations from (b),

$$-6 = \left(\frac{R_f}{R_1} + 1\right) v_g; \qquad 4 = -\left(\frac{R_f}{R_1}\right) v_g$$

Solving,

$$v_g = -2\text{ V}; \qquad \frac{R_f}{R_1} = 2$$

P 5.33 $$A_{\text{cm}} = \frac{(20)(50) - (50)R_x}{20(50 + R_x)}$$

$$A_{\text{dm}} = \frac{50(20 + 50) + 50(50 + R_x)}{2(20)(50 + R_x)}$$

$$\frac{A_{\text{dm}}}{A_{\text{cm}}} = \frac{R_x + 120}{2(20 - R_x)}$$

$$\therefore \quad \frac{R_x + 120}{2(20 - R_x)} = \pm 1000 \quad \text{for the limits on the value of } R_x$$

If we use $+1000 \quad R_x = 19.93\,\text{k}\Omega$

If we use $-1000 \quad R_x = 20.07\,\mathrm{k}\Omega$

$19.93\,\mathrm{k}\Omega \le R_x \le 20.07\,\mathrm{k}\Omega$

P 5.34 **[a]** $A_{\mathrm{dm}} = \dfrac{(24)(26) + (25)(25)}{(2)(1)(25)} = 24.98$

[b] $A_{\mathrm{cm}} = \dfrac{(1)(24) - 25(1)}{1(25)} = -0.04$

[c] $\mathrm{CMRR} = \left|\dfrac{24.98}{0.04}\right| = 624.50$

P 5.35 **[a]** $v_p = v_s, \qquad v_n = \dfrac{R_1 v_o}{R_1 + R_2}, \qquad v_n = v_p$

Therefore $\quad v_o = \left(\dfrac{R_1 + R_2}{R_1}\right) v_s = \left(1 + \dfrac{R_2}{R_1}\right) v_s$

[b] $v_o = v_s$

[c] Because $v_o = v_s$, thus the output voltage follows the signal voltage.

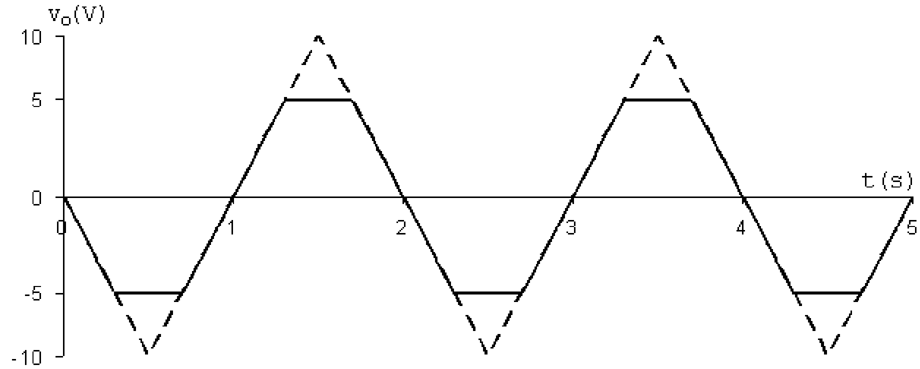
P 5.36 It follows directly from the circuit that $v_o = -(120/7.5)v_g = -16v_g$
From the plot of v_g we have $\quad v_g = 0, \quad t < 0$

$$
\begin{aligned}
v_g &= t & 0 \le t \le 0.5 \\
v_g &= 1 - t & 0.5 \le t \le 1.5 \\
v_g &= t - 2 & 1.5 \le t \le 2.5 \\
v_g &= 3 - t & 2.5 \le t \le 3.5 \\
v_g &= t - 4 & 3.5 \le t \le 4.5, \quad \text{etc.}
\end{aligned}
$$

Therefore

$$
\begin{aligned}
v_o &= -16t & 0 \le t \le 0.5 \\
v_o &= 16t - 16 & 0.5 \le t \le 1.5 \\
v_o &= 32 - 16t & 1.5 \le t \le 2.5 \\
v_o &= 16t - 48 & 2.5 \le t \le 3.5 \\
v_o &= 64 - 16t & 3.5 \le t \le 4.5, \quad \text{etc.}
\end{aligned}
$$

These expressions for v_o are valid as long as the op amp is not saturated. Since the peak values of v_o are ± 5, the output is clipped at ± 5. The plot is shown below.



P 5.37 $v_p = \frac{5.6}{8.0}v_g = 0.7v_g = 7\sin(\pi/3)t$ V

$$\frac{v_n}{15{,}000} + \frac{v_n - v_o}{75{,}000} = 0$$

$6v_n = v_o; \qquad v_n = v_p$

$\therefore \quad v_o = 42\sin(\pi/3)t$ V $\qquad 0 \leq t \leq \infty$

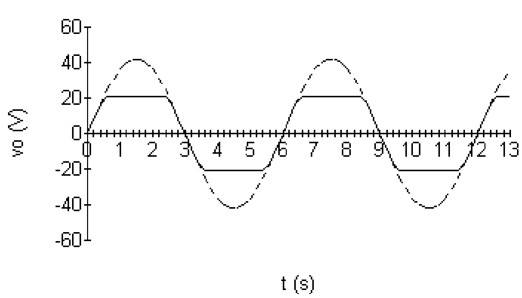
$v_o = 0 \qquad t \leq 0$

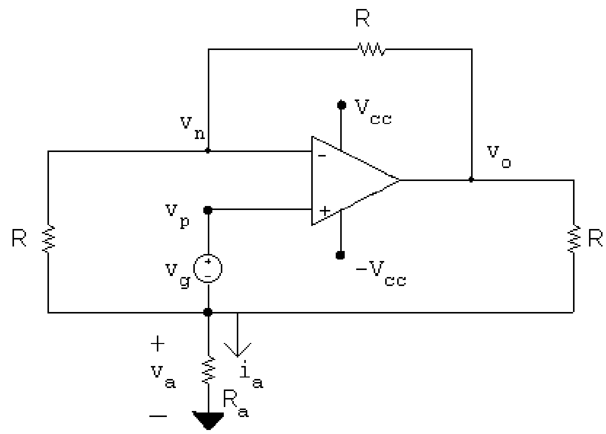
At saturation

$42\sin\left(\frac{\pi}{3}\right)t = \pm 21; \qquad \sin\frac{\pi}{3}t = \pm 0.5$

$\therefore \quad \frac{\pi}{3}t = \frac{\pi}{6}, \quad \frac{5\pi}{6}, \quad \frac{7\pi}{6}, \quad \frac{11\pi}{6}, \quad$ etc.

$t = 0.50\,\text{s}, \quad 2.50\,\text{s}, \quad 3.50\,\text{s}, \quad 5.50\,\text{s}, \quad$ etc.





P 5.38 **[a]**

$$\frac{v_n - v_a}{R} + \frac{v_n - v_o}{R} = 0$$

$$2v_n - v_a = v_o$$

$$\frac{v_a}{R_a} + \frac{v_a - v_n}{R} + \frac{v_a - v_o}{R} = 0$$

$$v_a\left[\frac{1}{R_a} + \frac{2}{R}\right] - \frac{v_n}{R} = \frac{v_o}{R}$$

$$v_a\left(2 + \frac{R}{R_a}\right) - v_n = v_o$$

$$v_n = v_p = v_a + v_g$$

$$\therefore \quad 2v_n - v_a = 2v_a + 2v_g - v_a = v_a + 2v_g$$

$$\therefore \quad v_a - v_o = -2v_g \qquad (1)$$

$$2v_a + v_a\left(\frac{R}{R_a}\right) - v_a - v_g = v_o$$

$$\therefore \quad v_a\left(1 + \frac{R}{R_a}\right) - v_o = v_g \qquad (2)$$

Now combining equations (1) and (2) yields

$$-v_a\frac{R}{R_a} = -3v_g$$

$$\text{or} \quad v_a = 3v_g\frac{R_a}{R}$$

$$\text{Hence} \quad i_a = \frac{v_a}{R_a} = \frac{3v_g}{R} \qquad \text{Q.E.D.}$$

[b] At saturation $v_o = \pm\, \mathrm{V_{cc}}$

$$\therefore \quad v_{\mathrm{a}} = \pm\, \mathrm{V_{cc}} - 2v_g \qquad (3)$$

and

$$\therefore \quad v_{\mathrm{a}}\left(1 + \frac{R}{R_{\mathrm{a}}}\right) = \pm\, \mathrm{V_{cc}} + v_g \qquad (4)$$

Dividing Eq (4) by Eq (3) gives

$$1 + \frac{R}{R_{\mathrm{a}}} = \frac{\pm\, \mathrm{V_{cc}} + v_g}{\pm\, \mathrm{V_{cc}} - 2v_g}$$

$$\therefore \quad \frac{R}{R_{\mathrm{a}}} = \frac{\pm\, \mathrm{V_{cc}} + v_g}{\pm\, \mathrm{V_{cc}} - 2v_g} - 1 = \frac{3v_g}{\pm\, \mathrm{V_{cc}} - 2v_g}$$

$$\text{or} \quad R_{\mathrm{a}} = \frac{(\pm\, \mathrm{V_{cc}} - 2v_g)}{3v_g}R \qquad \text{Q.E.D.}$$

P 5.39 **[a]** $p_{16\,\mathrm{k\Omega}} = \dfrac{(320 \times 10^{-3})^2}{(16 \times 10^3)} = 6.4\,\mu\mathrm{W}$

[b] $v_{16\,\mathrm{k\Omega}} = \left(\dfrac{16}{64}\right)(320) = 80\,\mathrm{mV}$

$$p_{16\,\mathrm{k\Omega}} = \frac{(80 \times 10^{-3})^2}{(16 \times 10^3)} = 0.4\,\mu\mathrm{W}$$

[c] $\dfrac{p_{\mathrm{a}}}{p_{\mathrm{b}}} = \dfrac{6.4}{0.4} = 16$

[d] Yes, the operational amplifier serves several useful purposes:

- First, it enables the source to control 16 times as much power delivered to the load resistor. When a small amount of power controls a larger amount of power, we refer to it as *power amplification*.
- Second, it allows the full source voltage to appear across the load resistor, no matter what the source resistance. This is the *voltage follower* function of the operational amplifier.
- Third, it allows the load resistor voltage (and thus its current) to be set without drawing any current from the input voltage source. This is the *current amplification* function of the circuit.

P 5.40 **[a]** Assume the op-amp is operating within its linear range, then

$$i_L = \frac{8}{4000} = 2\,\mathrm{mA}$$

For $R_L = 4\,\mathrm{k\Omega}$ $\qquad v_o = (4 + 4)(2) = 16\ \mathrm{V}$

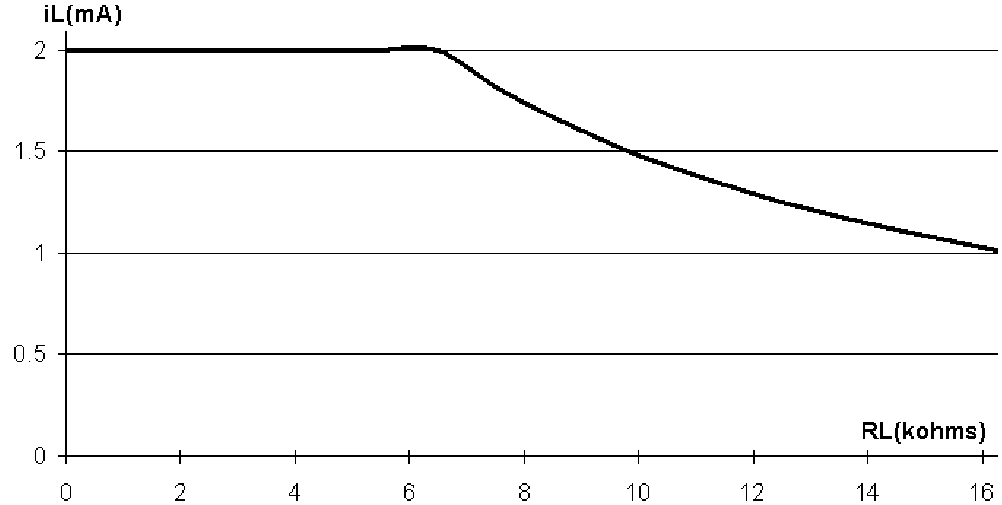
Now since $v_o < 20\ \mathrm{V}$ our assumption of linear operation is correct, therefore

$$i_L = 2\,\mathrm{mA}$$

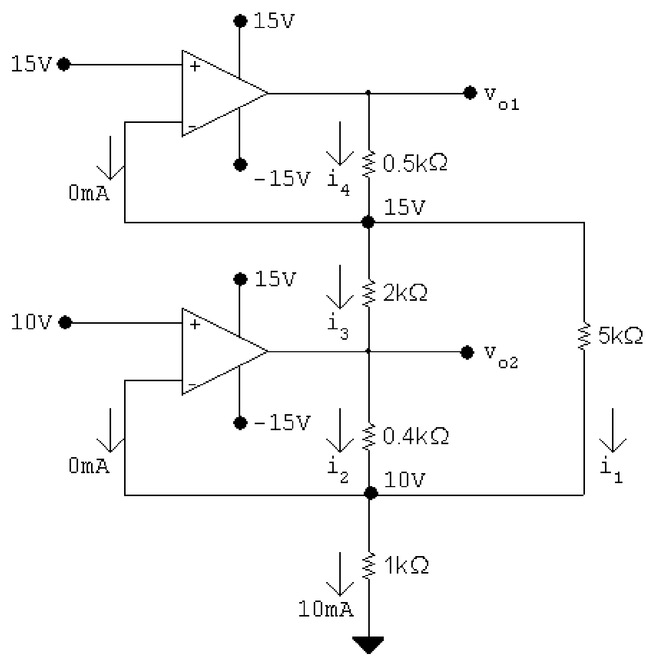
[b] $20 = 2(4 + R_L)$; $\qquad R_L = 6\,\text{k}\Omega$

[c] As long as the op-amp is operating in its linear region i_L is independent of R_L. From (b) we found the op-amp is operating in its linear region as long as $R_L \leq 6\,\text{k}\Omega$. Therefore when $R_L = 6\,\text{k}\Omega$ the op-amp is saturated. We can estimate the value of i_L by assuming $i_p = i_n \ll i_L$. Then $i_L = 20/(4000 + 16{,}000) = 1\,\text{mA}$. To justify neglecting the current into the op-amp assume the drop across the 50 kΩ resistor is negligible, since the input resistance to the op-amp is at least $500\,\text{k}\Omega$. Then $i_p = i_n = (8 - 4)/(500 \times 10^3) = 8\,\mu\text{A}$. But $8\,\mu\text{A} \ll 1\,\text{mA}$, hence our assumption is reasonable.

[d]



P 5.41



$$i_1 = \frac{15 - 10}{5000} = 1\,\text{mA}$$

$$i_2 + i_1 + 0 = 10\,\text{mA}; \qquad i_2 = 9\,\text{mA}$$

$$v_{o2} = 10 + (400)(9) \times 10^{-3} = 13.6\ \text{V}$$

$$i_3 = \frac{15 - 13.6}{2000} = 0.7\,\text{mA}$$

$$i_4 = i_3 + i_1 = 1.7\,\text{mA}$$

$v_{o1} = 15 + 1.7(0.5) = 15.85\ \text{V}$

P 5.42 **[a]** Let v_{o1} = output voltage of the amplifier on the left. Let v_{o2} = output voltage of the amplifier on the right. Then

$$v_{o1} = \frac{-47}{10}(1) = -4.7\ \text{V}; \qquad v_{o2} = \frac{-220}{33}(-0.15) = 1.0\ \text{V}$$

$$i_\text{a} = \frac{v_{o2} - v_{o1}}{1000} = 5.7\,\text{mA}$$

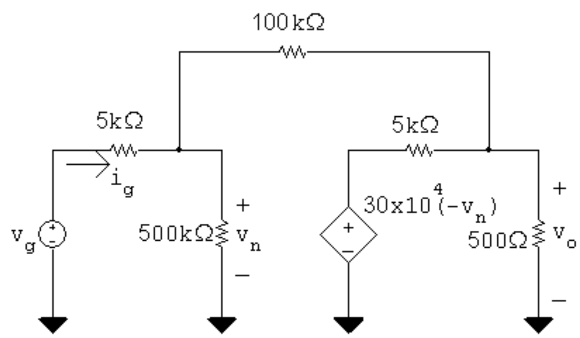
[b] $i_\text{a} = 0$ when $v_{o1} = v_{o2}$ so from (a) $v_{o2} = 1$ V

Thus

$$\frac{-47}{10}(v_\text{L}) = 1$$

$$v_\text{L} = -\frac{10}{47} = -212.77\ \text{mV}$$

P 5.43 **[a]** Replace the op amp with the model from Fig. 5.15:



Write two node voltage equations, one at the left node, the other at the right node:

$$\frac{v_n - v_g}{5000} + \frac{v_n - v_o}{100{,}000} + \frac{v_n}{500{,}000} = 0$$

$$\frac{v_o + 3 \times 10^5 v_n}{5000} + \frac{v_o - v_n}{100{,}000} + \frac{v_o}{500} = 0$$

Simplify and place in standard form:

$$106v_n - 5v_o = 100v_g$$

$$(6 \times 10^6 - 1)v_n + 221v_o = 0$$

Let $v_g = 1$ V and solve the two simultaneous equations:

$$v_o = -19.9844\ \text{V}; \qquad v_n = 736.1\,\mu\text{V}$$

Thus the voltage gain is $v_o/v_g = -19.9844$.

[b] From the solution in part (a), $v_n = 736.1\,\mu$V.

[**c**] $i_g = \dfrac{v_g - v_n}{5000} = \dfrac{v_g - 736.1 \times 10^{-6} v_g}{5000}$

$$R_g = \frac{v_g}{i_g} = \frac{5000}{1 - 736.1 \times 10^{-6}} = 5003.68\,\Omega$$

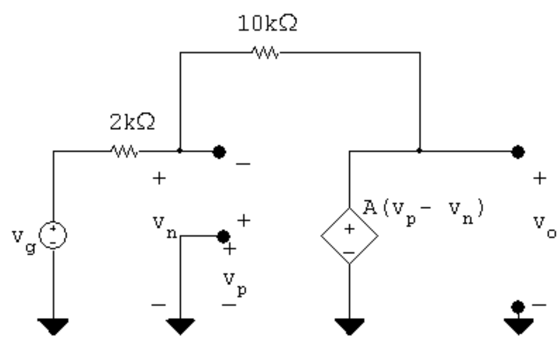
[**d**] For an ideal op amp, the voltage gain is the ratio between the feedback resistor and the input resistor:

$$\frac{v_o}{v_g} = -\frac{100{,}000}{5000} = -20$$

For an ideal op amp, the difference between the voltages at the input terminals is zero, and the input resistance of the op amp is infinite. Therefore,

$$v_n = v_p = 0 \text{ V}; \qquad R_g = 5000\,\Omega$$

P 5.44 [**a**]



$$\frac{v_n - v_g}{2000} + \frac{v_n - v_o}{10{,}000} = 0$$

$$\therefore \quad v_o = 6v_n - 5v_g$$

Also $\quad v_o = A(v_p - v_n) = -Av_n$

$$\therefore \quad v_n = \frac{-v_o}{A}$$

$$\therefore \quad v_o\left(1 + \frac{6}{A}\right) = -5v_g$$

$$v_o = \frac{-5A}{(6 + A)} v_g$$

[**b**] $v_o = \dfrac{-5(194)(1)}{200} = -4.85 \text{ V}$

[**c**] $v_o = \dfrac{-5}{1 + (6/A)}(1) = -5 \text{ V}$

[d] $\dfrac{-5A}{A+6}(1) = -0.99(5) \qquad \text{so} \qquad -5A = -4.95(A+6)$

$\therefore \quad -0.05A = -29.7 \qquad \text{so} \qquad A = 594$

P 5.45 **[a]** $\dfrac{v_n}{16{,}000} + \dfrac{v_n - v_g}{800{,}000} + \dfrac{v_n - v_o}{200{,}000} = 0 \quad \text{or} \quad 55v_n - 4v_o = v_g \qquad \text{Eq (1)}$

$$\frac{v_o}{20{,}000} + \frac{v_o - v_n}{200{,}000} + \frac{v_o - 50{,}000(v_p - v_n)}{8000} = 0$$

$$36v_o - v_n - 125 \times 10^4(v_p - v_n) = 0$$

$$v_p = v_g + \frac{(v_n - v_g)(240)}{800} = (0.7)v_g + (0.3)v_n$$

$$36v_o - v_n - 125 \times 10^4[(0.7)v_g - (0.7)v_n] = 0$$

$$36v_o + 874{,}999v_n = 875{,}000v_g \qquad \text{Eq (2)}$$

Let $v_g = 1$ V and solve Eqs. (1) and (2) simultaneously:

$v_n = 999.446\,\text{mV} \qquad \text{and} \qquad v_o = 13.49\ \text{V}$

$$\therefore \qquad \frac{v_o}{v_g} = 13.49$$

[b] From part (a), $v_n = 999.446$ mV.

$$v_p = (0.7)(1000) + (0.3)(999.446) = 999.834\,\text{mV}$$

[c] $v_p - v_n = 387.78\,\mu\text{V}$

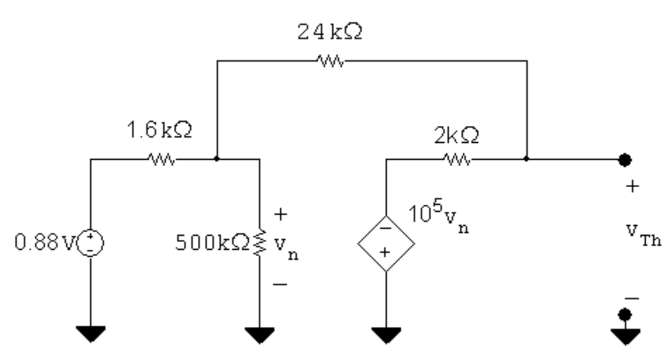
[d] $i_g = \dfrac{(1000 - 999.83)10^{-3}}{24 \times 10^3} = 692.47\,\text{pA}$

[e] $\dfrac{v_g}{16{,}000} + \dfrac{v_g - v_o}{200{,}000} = 0, \qquad \text{since } v_n = v_p = v_g$

$$\therefore \quad v_o = 13.5v_g, \qquad \frac{v_o}{v_g} = 13.5$$

$$v_n = v_p = 1\ \text{V}; \qquad v_p - v_n = 0\ \text{V}; \qquad i_g = 0\ \text{A}$$

P 5.46 **[a]**

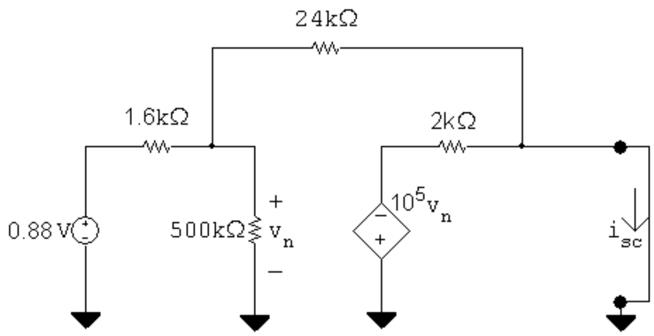


$$\frac{v_n - 0.88}{1600} + \frac{v_n}{500{,}000} + \frac{v_n - v_{Th}}{24{,}000} = 0$$

$$\frac{v_{\text{Th}} + 10^5 v_n}{2000} + \frac{v_{\text{Th}} - v_n}{24{,}000} = 0$$

Solving, $v_{\text{Th}} = -13.198$ V

Short-circuit current calculation:

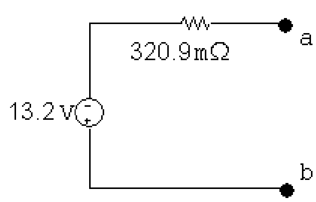


$$\frac{v_n}{500{,}000} + \frac{v_n - 0.88}{1600} + \frac{v_n - 0}{24{,}000} = 0$$

$\therefore \quad v_n = 0.8225$ V

$$i_{\text{sc}} = \frac{v_n}{24{,}000} - \frac{10^5}{2000} v_n = -41.13 \text{ A}$$

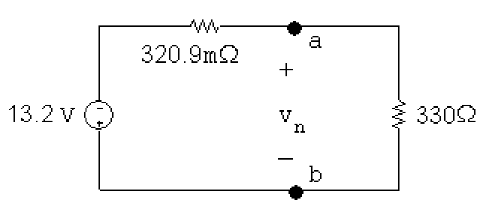
$$R_{\text{Th}} = \frac{v_{\text{Th}}}{i_{\text{sc}}} = 320.9\,\text{m}\Omega$$



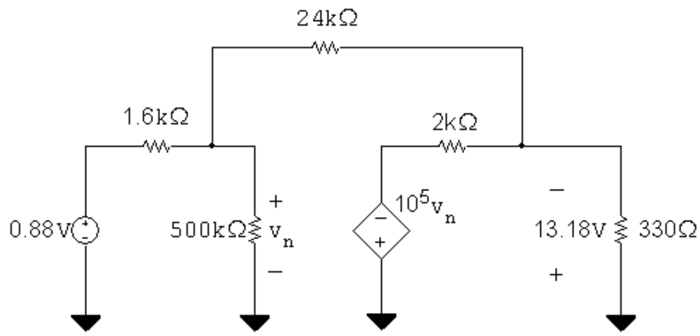
[b] The output resistance of the inverting amplifier is the same as the Thévenin resistance, i.e.,

$$R_o = R_{\text{Th}} = 320.9\,\text{m}\Omega$$

[c]



$$v_o = \left(\frac{330}{330.3209}\right)(-13.2) = -13.18 \text{ V}$$



$$\frac{v_n - 0.88}{1600} + \frac{v_n}{500{,}000} + \frac{v_n + 13.18}{24{,}000} = 0$$

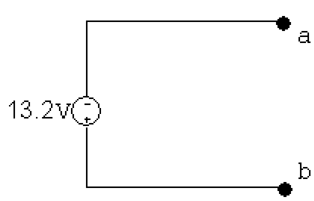
$$\therefore \quad v_n = 942\,\mu\text{V}$$

$$i_g = \frac{0.88 - 942 \times 10^{-6}}{1600} = 549.41\,\mu\text{A}$$

$$R_g = \frac{0.88}{i_g} = 1601.71\,\Omega$$

P 5.47 **[a]** $v_{\text{Th}} = -\dfrac{24{,}000}{1600}(0.88) = -13.2\ \text{V}$

$R_{\text{Th}} = 0$, since op-amp is ideal



[b] $R_o = R_{\text{Th}} = 0\,\Omega$

[c] $R_g = 1.6\,\text{k}\Omega$ since $v_n = 0$

P 5.48 From Eq. 5.57,

$$\frac{v_{\text{ref}}}{R + \Delta R} = v_n \left(\frac{1}{R + \Delta R} + \frac{1}{R - \Delta R} + \frac{1}{R_f} \right) - \frac{v_o}{R_f}$$

Substituting Eq. 5.59 for $v_p = v_n$:

$$\frac{v_{\text{ref}}}{R + \Delta R} = \frac{v_{\text{ref}} \left(\frac{1}{R+\Delta R} + \frac{1}{R-\Delta R} + \frac{1}{R_f} \right)}{(R - \Delta R)\left(\frac{1}{R+\Delta R} + \frac{1}{R-\Delta R} + \frac{1}{R_f} \right)} - \frac{v_o}{R_f}$$

Rearranging,

$$\frac{v_o}{R_f} = v_{\text{ref}} \left(\frac{1}{R - \Delta R} - \frac{1}{R + \Delta R} \right)$$

Thus,

$$v_o = v_{\text{ref}} \left(\frac{2\Delta R}{R^2 - \Delta R^2} \right) R_f$$

P 5.49 [**a**] Use Eq. 5.61 to solve for R_f; note that since we are using 1% strain gages, $\Delta = 0.01$:

$$R_f = \frac{v_o R}{2\Delta v_{\text{ref}}} = \frac{(5)(120)}{(2)(0.01)(15)} = 2\,\text{k}\Omega$$

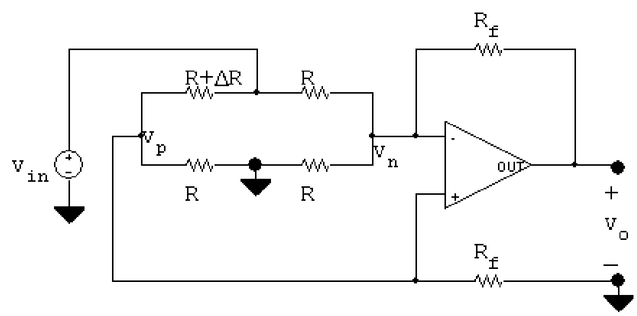
[**b**] Now solve for Δ given $v_o = 50$ mV:

$$\Delta = \frac{v_o R}{2R_f v_{\text{ref}}} = \frac{(0.05)(120)}{2(2000)(15)} = 100 \times 10^{-6}$$

The change in strain gage resistance that corresponds to a 50 mV change in output voltage is thus

$$\Delta R = \Delta R = (100 \times 10^{-6})(120) = 12\ \text{m}\Omega$$

P 5.50 [**a**]



Let $R_1 = R + \Delta R$

$$\frac{v_p}{R_f} + \frac{v_p}{R} + \frac{v_p - v_{\text{in}}}{R_1} = 0$$

$$\therefore \quad v_p \left[\frac{1}{R_f} + \frac{1}{R} + \frac{1}{R_1} \right] = \frac{v_{\text{in}}}{R_1}$$

$$\therefore \quad v_p = \frac{RR_f v_{\text{in}}}{RR_1 + R_f R_1 + R_f R} = v_n$$

$$\frac{v_n}{R} + \frac{v_n - v_{\text{in}}}{R} + \frac{v_n - v_o}{R_f} = 0$$

$$v_n \left[\frac{1}{R} + \frac{1}{R} + \frac{1}{R_f} \right] - \frac{v_o}{R_f} = \frac{v_{\text{in}}}{R}$$

$$\therefore \quad v_n \left[\frac{R + 2R_f}{RR_f} \right] - \frac{v_{\text{in}}}{R} = \frac{v_o}{R_f}$$