P4.1. Denoting the equal and opposite velocities to be $\pm v_0\mathbf{a}_x$ before the application of the field $E_0\mathbf{a}_x$, we obtain the velocities after the application of the field to be

$$\mathbf{v}_1 = -\left(v_0 + \frac{|e|E_0}{m}t\right)\mathbf{a}_x \text{ for the accelerating electron, and}$$

$$\mathbf{v}_2 = \left(v_0 - \frac{|e|E_0}{m}t\right)\mathbf{a}_x \text{ for the decelerating electron}$$

where e and m are the charge and mass of the electron, respectively. The kinetic energies are

$$\frac{1}{2}mv_1^2 = \frac{1}{2}mv_0^2 + \frac{|e|E_0 t}{2}\left(2v_0 + \frac{|e|E_0 t}{m}\right)$$

$$\frac{1}{2}mv_2^2 = \frac{1}{2}mv_0^2 - \frac{|e|E_0 t}{2}\left(2v_0 - \frac{|e|E_0 t}{m}\right)$$

Thus the gain in the kinetic energy by the accelerating electron is greater than the loss in the kinetic energy of the decelerating electron.

P4.2. **(a)** $m \dfrac{d\mathbf{v}_d}{dt} + \dfrac{m}{\tau}\,\mathbf{v}_d = e\mathbf{E}_0 \cos \omega t$

$$j\omega m \bar{\mathbf{v}}_d + \frac{m}{\tau}\bar{\mathbf{v}}_d = e\bar{\mathbf{E}}_0$$

$$\bar{\mathbf{v}}_d\left(\frac{m}{\tau} + j\omega m\right) = e\bar{\mathbf{E}}_0$$

$$\bar{\mathbf{v}}_d = \frac{\tau e\bar{\mathbf{E}}_0}{m(1+j\omega\tau)} = \frac{\tau e\mathbf{E}_0}{m\sqrt{1+\omega^2\tau^2}}\, e^{-j\tan^{-1}\omega\tau}$$

$$\therefore\ \mathbf{v}_d = \frac{\tau e}{m\sqrt{1+\omega^2\tau^2}}\,\mathbf{E}_0 \cos(\omega t - \tan^{-1}\omega\tau)$$

(b) $\tan^{-1}\omega\tau = \dfrac{\pi}{4},\ \omega\tau = 1$

From Eqs. (4.6) and (4.12),

$$\tau = \frac{\mu_e m}{|e|} = \frac{\sigma m}{N_e|e|^2}$$

$$= \frac{6.1\times10^7\times9.107\times10^{-31}}{5.86\times10^{28}\times1.602^2\times10^{-38}}$$

$$= 3.694\times10^{-14}\ \text{s}$$

$$\therefore\ \omega = \frac{1}{\tau} = \frac{1}{3.694\times10^{-14}} = 2.707\times10^{13}\ \text{rad/s}$$

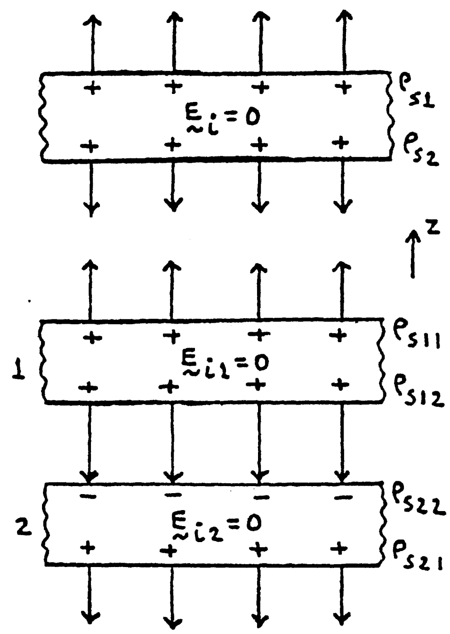
$$f = \frac{\omega}{2\pi} = \frac{2.707\times10^{13}}{2\pi} = 0.431\times10^{13}\ \text{Hz}$$

The drift velocity magnitude at this frequency is $\frac{\tau|e||\mathbf{E}_0|}{m\sqrt{2}}$. Hence, the mobility at this frequency is $\frac{\tau|e|}{m\sqrt{2}}$. Since the mobility at zero frequency is $\frac{\tau|e|}{m}$, the required ratio is $\frac{1}{\sqrt{2}}$.

P4.3. (a) $\mathbf{E}_i = -\dfrac{\rho_{S1}}{2\varepsilon_0}\mathbf{a}_z + \dfrac{\rho_{S2}}{2\varepsilon_0}\mathbf{a}_z$

$= 0$

$\rho_{S1} = \rho_{S2} = \dfrac{\rho_{S0}}{2}$



(b) $\mathbf{E}_{i1} = \left(-\dfrac{\rho_{S11}}{2\varepsilon_0} + \dfrac{\rho_{S12}}{2\varepsilon_0} + \dfrac{\rho_{S22}}{2\varepsilon_0} + \dfrac{\rho_{S21}}{2\varepsilon_0}\right)\mathbf{a}_z$

$= 0$

$\mathbf{E}_{i2} = \left(-\dfrac{\rho_{S11}}{2\varepsilon_0} - \dfrac{\rho_{S12}}{2\varepsilon_0} - \dfrac{\rho_{S22}}{2\varepsilon_0} + \dfrac{\rho_{S21}}{2\varepsilon_0}\right)\mathbf{a}_z$

$= 0$

$-\rho_{S11} + \rho_{S12} + \rho_{S22} + \rho_{S21} = 0$ (1)

$-\rho_{S11} - \rho_{S12} - \rho_{S22} + \rho_{S21} = 0$ (2)

Also

$\rho_{S11} + \rho_{S12} = \rho_{S1}$ (3)

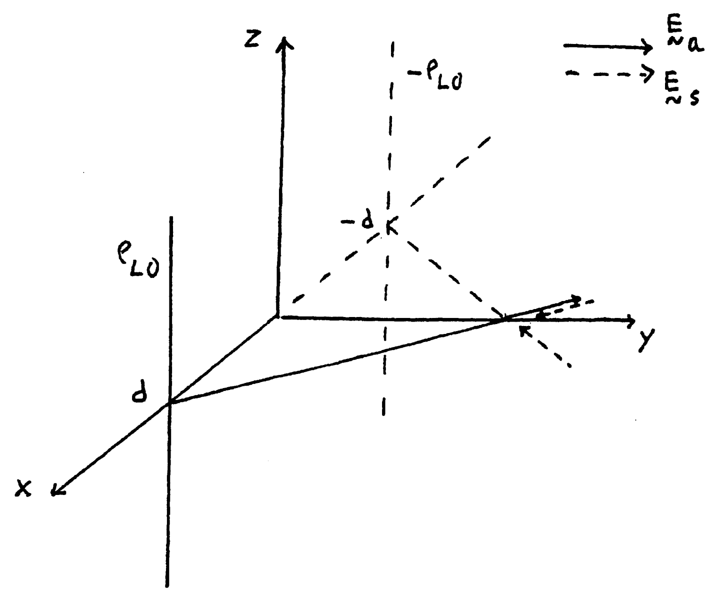
$\rho_{S21} + \rho_{S22} = \rho_{S2}$ (4)

Solving these four equations for the four unknowns, we get

$\rho_{S11} = \rho_{S21} = \dfrac{1}{2}(\rho_{S1} + \rho_{S2})$

$\rho_{S12} = -\rho_{S22} = \dfrac{1}{2}(\rho_{S1} - \rho_{S2})$

P4.4.



Denoting the applied field to be $\mathbf{E}_a$, and applying the expression for the electric field due to an infinitely long line charge to that passing through $(d, 0, 0)$, we have

$$\mathbf{E}_a = \frac{\rho_{L0}\left[(x-d)\mathbf{a}_x + y\mathbf{a}_y\right]}{2\pi\varepsilon_0\left[(x-d)^2 + y^2\right]}$$

For the total field inside the conductor to be zero,

$$\begin{aligned}[\mathbf{E}_s]_{x<0} &= -[\mathbf{E}_a]_{x>0} \\ &= -\frac{\rho_{L0}\left[(x-d)\mathbf{a}_x + y\mathbf{a}_y\right]}{2\pi\varepsilon_0\left[(x-d)^2 + y^2\right]}\end{aligned}$$

This field is produced by a surface charge on the conductor surface $x = 0$. From symmetry considerations, we can write $\mathbf{E}_s$ for $x > 0$ by substituting $-x$ for x and $-\mathbf{a}_x$ for $\mathbf{a}_x$ in the expression for $\mathbf{E}_s$ for $x < 0$. Thus

$$\begin{aligned}[\mathbf{E}_s]_{x>0} &= -\frac{\rho_{L0}\left[(-x-d)(-\mathbf{a}_x) + y\mathbf{a}_y\right]}{2\pi\varepsilon_0\left[(-x-d)^2 + y^2\right]} \\ &= -\frac{\rho_{L0}\left[(x+d)\mathbf{a}_x + y\mathbf{a}_y\right]}{2\pi\varepsilon_0\left[(x+d)^2 + y^2\right]}\end{aligned}$$

P4.4. (continued)

This is the same as the field due to an infinitely long line charge of uniform density $-\rho_{L0}$ situated parallel to the z-axis and passing through $(-d, 0, 0)$. Thus the total field outside the conductor is the superposition of the fields due to the line charge through $(d, 0, 0)$ and an "image" line charge through $(-d, 0, 0)$. It is given by

$$[\mathbf{E}]_{x>0} = \frac{\rho_{L0}}{2\pi\varepsilon_0}\left\{\frac{(x-d)\mathbf{a}_x + y\mathbf{a}_y}{(x-d)^2+y^2} - \frac{(x+d)\mathbf{a}_x + y\mathbf{a}_y}{(x+d)^2+y^2}\right\}$$

Note that on the conductor surface $x = 0$,

$$[\mathbf{E}]_{x=0+} = -\frac{\rho_{L0}}{\pi\varepsilon_0}\frac{d}{y^2+d^2}\mathbf{a}_x$$

is normal to the surface.

P4.5. From the figure, magnitude of torque

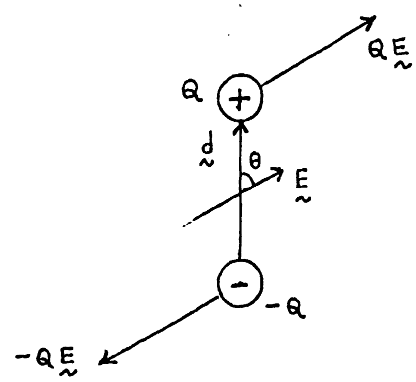
$= QEd \sin\theta$

Direction of torque is into the paper.

$\therefore \boldsymbol{\tau} = QEd \sin\theta$ into the paper

$= Q\mathbf{d} \times \mathbf{E}$

$= \mathbf{p} \times \mathbf{E}$



For a dipole consisting of 1 μC at $(0, 0, 10^{-3})$ and -1 μC at $(0, 0, -10^{-3})$,

$$\mathbf{p} = 10^{-6}(2 \times 10^{-3}\,\mathbf{a}_z)$$

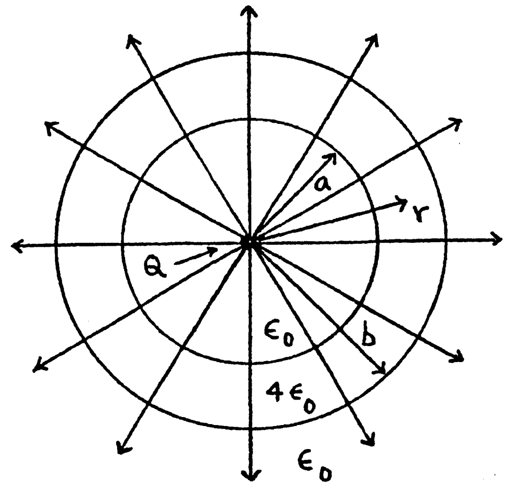
$$= 2 \times 10^{-9}\,\mathbf{a}_z$$

For $\mathbf{E} = 10^3(2\mathbf{a}_x - \mathbf{a}_y - 2\mathbf{a}_z)$,

$$\boldsymbol{\tau} = 2 \times 10^{-9}\,\mathbf{a}_z \times 10^3(2\mathbf{a}_x - \mathbf{a}_y - 2\mathbf{a}_z)$$

$$= 2 \times 10^{-6}(\mathbf{a}_x + 2\mathbf{a}_y) \text{ N-m}$$

P4.6. Because of spherical symmetry, the field lines remain radially directed away from the point charge, as in the absence of the dielectric shell. Thus



(a) $\mathbf{D} = \dfrac{Q}{4\pi r^2}\,\mathbf{a}_r$ everywhere

$$\mathbf{E} = \begin{cases} \dfrac{Q}{4\pi\varepsilon_0 r^2}\,\mathbf{a}_r & \text{for } r < a \\ \dfrac{Q}{16\pi\varepsilon_0 r^2}\,\mathbf{a}_r & \text{for } a < r < b \\ \dfrac{Q}{4\pi\varepsilon_0 r^2}\,\mathbf{a}_r & \text{for } r > b \end{cases}$$

(b) Inside the dielectric shell,

$$\mathbf{P} = \mathbf{D} - \varepsilon_0\mathbf{E}$$

$$= \frac{Q}{4\pi r^2}\,\mathbf{a}_r - \frac{Q}{16\pi r^2}\,\mathbf{a}_r$$

$$= \frac{3Q}{16\pi r^2}\,\mathbf{a}_r$$

P4.7.

(a)

$$\begin{bmatrix} D_x \\ D_y \\ D_z \end{bmatrix} = \varepsilon_0 \begin{bmatrix} 3 & 1 & 1 \\ 1 & 3 & 1 \\ 1 & 1 & 3 \end{bmatrix} \begin{bmatrix} E_0 \\ E_0 \\ 0 \end{bmatrix} = \begin{bmatrix} 4\varepsilon_0 E_0 \\ 4\varepsilon_0 E_0 \\ 2\varepsilon_0 E_0 \end{bmatrix}$$

$\mathbf{D} = 2\varepsilon_0 E_0 (2\mathbf{a}_x + 2\mathbf{a}_y + \mathbf{a}_z)$

D is not parallel to **E**.

(b)

$$\begin{bmatrix} D_x \\ D_y \\ D_z \end{bmatrix} = \varepsilon_0 \begin{bmatrix} 3 & 1 & 1 \\ 1 & 3 & 1 \\ 1 & 1 & 3 \end{bmatrix} \begin{bmatrix} E_0 \\ -E_0 \\ 0 \end{bmatrix} = \begin{bmatrix} 2\varepsilon_0 E_0 \\ -2\varepsilon_0 E_0 \\ 0 \end{bmatrix}$$

$\mathbf{D} = 2\varepsilon_0 E_0 (\mathbf{a}_x - \mathbf{a}_y) = 2\varepsilon_0 \mathbf{E}$

D is parallel to **E**.

(c)

$$\begin{bmatrix} E_x \\ E_y \\ E_z \end{bmatrix} = \frac{1}{\varepsilon_0} \begin{bmatrix} 3 & 1 & 1 \\ 1 & 3 & 1 \\ 1 & 1 & 3 \end{bmatrix}^{-1} \begin{bmatrix} D_0 \\ D_0 \\ -2D_0 \end{bmatrix}$$

$$= \frac{1}{20\varepsilon_0} \begin{bmatrix} 8 & -2 & -2 \\ -2 & 8 & -2 \\ -2 & -2 & 8 \end{bmatrix} \begin{bmatrix} D_0 \\ D_0 \\ -2D_0 \end{bmatrix} = \frac{1}{2\varepsilon_0} \begin{bmatrix} D_0 \\ D_0 \\ -2D_0 \end{bmatrix}$$

$$\mathbf{E} = \frac{D_0}{2\varepsilon_0} (\mathbf{a}_x + \mathbf{a}_y - 2\mathbf{a}_z) = \frac{\mathbf{D}}{2\varepsilon_0}$$

D is parallel to **E**.

P4.8. For $\mathbf{E} = E_x\mathbf{a}_x + E_y\mathbf{a}_y$,

$$\begin{bmatrix} D_x \\ D_y \\ D_z \end{bmatrix} = \begin{bmatrix} \varepsilon_{xx} & \varepsilon_{xy} & 0 \\ \varepsilon_{yx} & \varepsilon_{yy} & 0 \\ 0 & 0 & \varepsilon_{zz} \end{bmatrix} \begin{bmatrix} E_x \\ E_y \\ 0 \end{bmatrix} = \begin{bmatrix} \varepsilon_{xx}E_x + \varepsilon_{xy}E_y \\ \varepsilon_{yx}E_x + \varepsilon_{yy}E_y \\ 0 \end{bmatrix}$$

For $\mathbf{D}$ to be parallel to $\mathbf{E}$, $\dfrac{D_x}{E_x}$ must be equal to $\dfrac{D_y}{E_y}$.

$$\therefore \quad \frac{\varepsilon_{xx}E_x + \varepsilon_{xy}E_y}{E_x} = \frac{\varepsilon_{yx}E_x + \varepsilon_{yy}E_y}{E_y}$$

$$\varepsilon_{xy}\left(\frac{E_y}{E_x}\right)^2 + (\varepsilon_{xx} - \varepsilon_{yy})\left(\frac{E_y}{E_x}\right) - \varepsilon_{yx} = 0$$

$$\frac{E_y}{E_x} = \frac{(\varepsilon_{yy} - \varepsilon_{xx}) \pm \sqrt{(\varepsilon_{xx} - \varepsilon_{yy})^2 + 4\varepsilon_{xy}\varepsilon_{yx}}}{2\varepsilon_{xy}}$$

The corresponding effective permittivities are given by

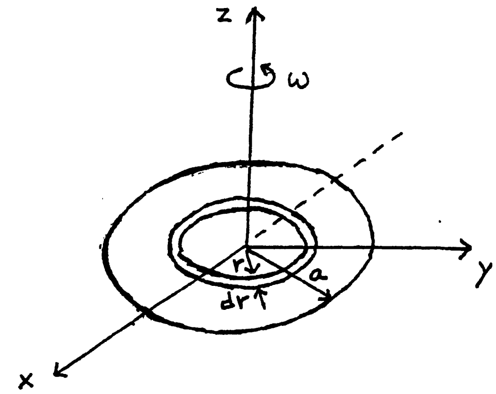
$$\varepsilon_{\text{eff}} = \frac{D_x}{E_x} = \frac{\varepsilon_{xx}E_x + \varepsilon_{xy}E_y}{E_x}$$

$$= \varepsilon_{xx} + \varepsilon_{xy}\frac{E_y}{E_x}$$

$$= \varepsilon_{xx} + \frac{1}{2}\left[(\varepsilon_{yy} - \varepsilon_{xx}) \pm \sqrt{(\varepsilon_{xx} - \varepsilon_{yy})^2 + 4\varepsilon_{xy}\varepsilon_{yx}}\right]$$

$$= \frac{1}{2}\left[(\varepsilon_{xx} + \varepsilon_{yy}) \pm \sqrt{(\varepsilon_{xx} - \varepsilon_{yy})^2 + 4\varepsilon_{xy}\varepsilon_{yx}}\right]$$

P4.9. Consider a ring of radius $r < a$ and width dr, as shown in the figure. Then the amount of charge in the ring is $\frac{Q}{\pi a^2}(2\pi r\, dr) = \frac{2Qr}{a^2}dr$. This amount of charge passes through the width dr of the ring in one revolution of the disk. Since the number of revolutions per second is $\omega/2\pi$, the amount of charge passing through the width in one second, that is, the current in the ring, is $\left(\frac{2Qr}{a^2}dr\right)\left(\frac{\omega}{2\pi}\right) = \frac{\omega Qr}{\pi a^2}dr$. Thus, the dipole moment of the ring is given by



$$d\mathbf{m} = \left(\frac{\omega Qr}{\pi a^2}dr\right)(\pi r^2)\mathbf{a}_z$$

$$= \frac{\omega Qr^3}{a^2}dr\,\mathbf{a}_z$$

The dipole moment of the entire disk is then given by

$$\mathbf{m} = \int_{r=0}^{a} d\mathbf{m} = \int_{r=0}^{a} \frac{\omega Qr^3}{a^2}dr\,\mathbf{a}_z$$

$$= \frac{\omega Q}{a^2}\left[\frac{r^4}{4}\right]_0^a \mathbf{a}_z$$

$$= \frac{\omega Qa^2}{4}\mathbf{a}_z$$

P4.10. Let $\mathbf{B} = B_x\mathbf{a}_x + B_y\mathbf{a}_y + B_z\mathbf{a}_z$

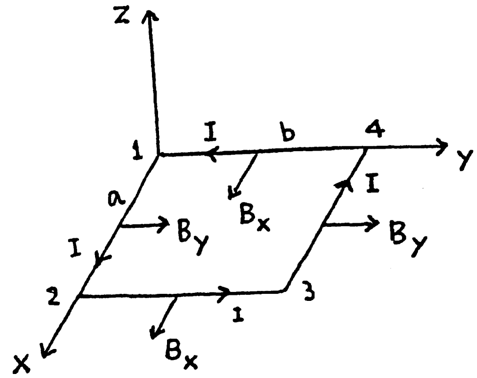
Then the only forces contributing to the torque are

$IaB_y\mathbf{a}_z$ on side 12,

$-IaB_y\mathbf{a}_z$ on side 34,

$-IbB_x\mathbf{a}_z$ on side 23, and

$IbB_x\mathbf{a}_z$ on side 41.



$$\therefore \boldsymbol{\tau} = -IaB_yb\mathbf{a}_x + IbB_xa\mathbf{a}_y$$

$$= Iab(B_x\mathbf{a}_y - B_y\mathbf{a}_x)$$

$$= Iab\mathbf{a}_z \times (B_x\mathbf{a}_x + B_y\mathbf{a}_y + B_z\mathbf{a}_z)$$

$$= \mathbf{m} \times \mathbf{B}$$

For a circular loop of radius 1 mm in the xy plane and with current 0.1 A flowing in the sense of increasing ϕ,

$$\mathbf{m} = 0.1 \times 10^{-6}\pi\,\mathbf{a}_z = 10^{-7}\pi\,\mathbf{a}_z$$

For $\mathbf{B} = 10^{-5}(2\mathbf{a}_x - 2\mathbf{a}_y + \mathbf{a}_z)$ Wb/m^2,

$$\boldsymbol{\tau} = 10^{-7}\pi\,\mathbf{a}_z \times 10^{-5}(2\mathbf{a}_x - 2\mathbf{a}_y + \mathbf{a}_z)$$

$$= 2 \times 10^{-12}\pi\,(\mathbf{a}_x + \mathbf{a}_y) \text{ N-m}$$

P4.11. $\mu = \frac{B}{H} = \frac{\mu_0 k H^2}{H} = \mu_0 k H$

$$\mu_r = \frac{\mu}{\mu_0} = kH$$

$$\chi_m = \mu_r - 1 = kH - 1$$

$$\mathbf{M} = \chi_m \mathbf{H} = (kH - 1)\mathbf{H}$$

P4.12.

$$\begin{bmatrix} B_x \\ B_y \\ B_z \end{bmatrix} = k\mu_0 \begin{bmatrix} 7 & 6 & 0 \\ 6 & 12 & 0 \\ 0 & 0 & 3 \end{bmatrix} \begin{bmatrix} 3H_0 \\ -2H_0 \\ 0 \end{bmatrix}$$

$$= k\mu_0 \begin{bmatrix} 9H_0 \\ -6H_0 \\ 0 \end{bmatrix}$$

$$\begin{aligned} \mathbf{B} &= k\mu_0 H_0(9\mathbf{a}_x - 6\mathbf{a}_y) \\ &= 3k\mu_0 H_0(3\mathbf{a}_x - 2\mathbf{a}_y) \\ &= 3k\mu_0 \mathbf{H} \end{aligned}$$

$$\mu_{\text{eff}} = \frac{\mathbf{B}}{\mathbf{H}} = 3k\mu_0$$

P4.13. Computing the propagation parameters for $\sigma = 10^{-3}$ S/m, $\varepsilon = 6\varepsilon_0$, $\mu = \mu_0$, and $f = 1$ MHz, as in Ex. 4.5 of the text, we have

$$\alpha = 0.05334 \text{ Np/m}$$

$$\beta = 0.07401 \text{ rad/m}$$

$$|\bar{\eta}| = 86.5477\ \Omega$$

$$\underline{/\bar{\eta}} = 35.7825^\circ = 0.1988\pi$$

$$|\bar{\eta}|\frac{J_{S0}}{2} = 86.5477 \times 0.1 = 8.655$$

Thus

$$\mathbf{E} = 8.655e^{\mp 0.0533z} \cos(2\pi \times 10^6 t \mp 0.074z + 0.1988\pi)\ \mathbf{a}_x \text{ V/m} \qquad \text{for } z \gtrless 0$$

$$\mathbf{H} = \pm 0.1e^{\mp 0.0533z} \cos(2\pi \times 10^6 t \mp 0.074z)\ \mathbf{a}_y \text{ A/m} \qquad \text{for } z \gtrless 0$$

P4.14. $\mathbf{J}_{S1} = -J_{S0} \cos 2\pi \times 10^6 t \, \mathbf{a}_x, \ z = 0$

$\mathbf{J}_{S2} = -kJ_{S0} \sin 2\pi \times 10^6 t \, \mathbf{a}_x, \ z = d$

(a) Minimum value of d is such that $\beta d = \frac{\pi}{2}$ or $d = \frac{\lambda}{4}$

From Prob. P4.13,

$$\lambda = \frac{2\pi}{0.07401} = 84.896 \text{ m}$$

$$\therefore d_{\min} = \frac{84.896}{4} = 21.224 \text{ m}$$

Then $k = e^{\alpha d_{\min}}$

$$= e^{0.05334 \times 21.224} = 3.10$$

(b) $\mathbf{E}$ for $z > d_{\min}$ is

$$[8.655 e^{-0.0533z} \cos(2\pi \times 10^6 t - 0.074z + 0.1988\pi)$$

$$+ 8.655 \times 3.10 e^{-0.0533(z - 21.224)}$$

$$\cdot \sin(2\pi \times 10^6 t - 0.074z + \pi/2 + 0.1988\pi)] \, \mathbf{a}_x$$

$$= 91.82 e^{-0.0533z} \cos(2\pi \times 10^6 t - 0.074z + 0.1988\pi) \, \mathbf{a}_x \text{ V/m}$$

P4.15. (a) $e^{-28.65\alpha} = e^{-1}$, $\alpha = \dfrac{1}{28.65} = 0.0349$

$$111.2\beta = 2\pi,\ \beta = \frac{2\pi}{111.2} = 0.0565$$

$$\bar{\gamma} = 0.0349 + j0.0565$$

(b) $|\bar{\eta}| = 59.4$

$$\underline{/\bar{\gamma}} = \tan^{-1}\frac{0.0565}{0.0349} = 58.3^\circ$$

Since $\overline{\gamma\eta} = j\omega\mu$,

$$\underline{/\bar{\eta}} = 90^\circ - \underline{/\bar{\gamma}} = 31.7^\circ$$

$$\therefore\ \bar{\eta} = 59.4\underline{/31.7^\circ}$$

(c) $\sigma = \text{Re}\left[\dfrac{\bar{\gamma}}{\bar{\eta}}\right] = \text{Re}\left[\dfrac{0.0349 + j0.0565}{59.4\underline{/31.7^\circ}}\right]$

$$= \text{Re}\frac{0.0664\underline{/58.3^\circ}}{59.4\underline{/31.7^\circ}} = \text{Re}\ (0.001118\underline{/26.6^\circ})$$

$$= 0.001118 \cos 26.6^\circ$$

$$= 0.001 = 10^{-3}\ \text{S/m}$$

$$\varepsilon = \frac{1}{\omega}\text{Im}\frac{\bar{\gamma}}{\bar{\eta}} = \frac{1}{2\pi \times 5 \times 10^5}\ \text{Im}\ (0.001118\underline{/26.6^\circ})$$

$$= \frac{1}{10^6\pi}(0.001118 \sin 26.6^\circ) = \frac{5 \times 10^{-4}}{10^6\pi}$$

$$= \frac{10^{-9}}{2\pi} = 18 \times \frac{10^{-9}}{36\pi} = 18\varepsilon_0$$

$$\mu = \frac{\overline{\gamma\eta}}{j\omega} = \frac{|\overline{\gamma\eta}|}{\omega} = \frac{0.0664 \times 59.4}{2\pi \times 5 \times 10^5}$$

$$= 1.26 \times 10^{-6} = 4\pi \times 10^{-7} = \mu_0$$

P4.16. $\mathbf{J}_S = -0.2 \cos 2\pi \times 10^6 t \cos 4\pi \times 10^6 t \, \mathbf{a}_x$

$$= -0.1 (\cos 2\pi \times 10^6 t + \cos 6\pi \times 10^6 t) \, \mathbf{a}_x$$

Thus, $\mathbf{J}_S$ consists of two components, one at $f = f_1 = 10^6$ Hz and the second at $f = f_2 = 3 \times 10^6$ Hz. Using superposition, we find **E** and **H** for each component and add them to get the complete solution.

For $f = f_1 = 10^6$ Hz, from Prob. P5.19,

$$\mathbf{E}_1 = 4.3275 e^{\mp 0.0533 z} \cos (2\pi \times 10^6 t \mp 0.074 z + 0.1988\pi) \, \mathbf{a}_x \ \text{V/m} \qquad \text{for } z \gtrless 0$$

$$\mathbf{H}_1 = \pm 0.05 e^{\mp 0.0533 z} \cos (2\pi \times 10^6 t \mp 0.074 z) \, \mathbf{a}_y \ \text{A/m} \qquad \text{for } z \gtrless 0$$

Following in the same manner as in Prob. P5.19 for $f = f_2 = 3 \times 10^6$ Hz, we have

$$\alpha = 0.07 \ \text{Np/m}$$

$$\beta = 0.1691$$

$$|\bar{\eta}| = 129.42 \ \Omega$$

$$\angle \bar{\eta} = 0.125\pi$$

$$|\bar{\eta}| \frac{0.1}{2} = 6.471$$

Thus

$$\mathbf{E}_2 = 6.471 e^{\mp 0.07 z} \cos (6\pi \times 10^6 t \mp 0.1691 z + 0.125\pi) \, \mathbf{a}_x \ \text{V/m} \qquad \text{for } z \gtrless 0$$

$$\mathbf{H}_2 = \pm 0.05 e^{\mp 0.07 z} \cos (6\pi \times 10^6 t \mp 0.1691 z) \, \mathbf{a}_y \ \text{A/m} \qquad \text{for } z \gtrless 0$$

Adding the two sets of fields, we have

$$\begin{aligned}\mathbf{E} &= \mathbf{E}_1 + \mathbf{E}_2 \\ &= \left[4.3275 e^{\mp 0.0533 z} \cos (2\pi \times 10^6 t \mp 0.074 z + 0.1988\pi)\right. \\ &\quad \left. + 6.471 e^{\mp 0.07 z} \cos (6\pi \times 10^6 t \mp 0.1691 z + 0.125\pi)\right] \mathbf{a}_x \ \text{V/m} \qquad \text{for } z \gtrless 0\end{aligned}$$

$$\begin{aligned}\mathbf{H} &= \mathbf{H}_1 + \mathbf{H}_2 \\ &= \pm 0.05 \left[e^{\mp 0.0533 z} \cos (2\pi \times 10^6 t \mp 0.074 z)\right. \\ &\quad \left. + e^{\mp 0.07 z} \cos (6\pi \times 10^6 t \mp 0.1691 z)\right] \mathbf{a}_y \ \text{A/m} \qquad \text{for } z \gtrless 0\end{aligned}$$

P4.17. For $\mathbf{H} = H_0 e^{-z} \cos(2\pi \times 10^6 t - 2z)\, \mathbf{a}_x$,

$$\bar{\gamma} = \alpha + j\beta = 1 + j2$$

$$\bar{\eta} = \frac{j\omega\mu}{\bar{\gamma}} = \frac{j2\pi \times 10^6 \times 4\pi \times 10^{-7}}{1 + j2}$$

$$= \frac{0.8\pi^2 \angle 90^\circ}{\sqrt{5} \angle 63.43^\circ} = 3.531 \angle 26.57^\circ$$

$\therefore$ $\mathbf{H} = 3.531 H_0 e^{-z} \cos(2\pi \times 10^6 t - 2z + 0.1476\pi)\, \mathbf{a}_y$

$\mathbf{P} = \mathbf{E} \times \mathbf{H}$

$$= 1.7655 H_0^2 e^{-2z} [\cos 0.1476\pi + \cos(4\pi \times 10^6 t - 4z - 0.1476\pi)]\, \mathbf{a}_z$$

(a) $\langle P_z \rangle = 1.7655 H_0^2 e^{-2z} \cos 0.1476\pi$

$$= 1.5791 H_0^2 e^{-2z} \text{ W/m}^2$$

(b) Time-average power dissipated in the given volume

$$= 1.5791 H_0^2 \left(1 - e^{-2}\right)$$

$$= 1.3654 H_0^2 \text{ W}$$

P4.18. $\mathbf{E} = 10 \cos (3\pi \times 10^7 t - 0.2\pi x)\, \mathbf{a}_z$ V/m

(a) $f = \dfrac{\omega}{2\pi} = \dfrac{3\pi \times 10^7}{2\pi} = 1.5 \times 10^7$ Hz

$= 15$ MHz

(b) $\lambda = \dfrac{2\pi}{\beta} = \dfrac{2\pi}{0.2\pi} = 10$ m

(c) $v_p = \dfrac{\omega}{\beta} = \dfrac{3\pi \times 10^7}{0.2\pi} = 15 \times 10^7$

$= 1.5 \times 10^8$ m/s

(d) $v_p = 1.5 \times 10^8 = \dfrac{1}{\sqrt{\mu_0 \varepsilon}} = \dfrac{1}{\sqrt{\mu_0 \varepsilon_0 \varepsilon_r}}$

$$1.5 \times 10^8 = \frac{3 \times 10^8}{\sqrt{\varepsilon_r}}$$

$$\sqrt{\varepsilon_r} = 2$$

$$\varepsilon_r = 4$$

(e) $\eta = \sqrt{\dfrac{\mu_0}{\varepsilon_0 \varepsilon_r}} = \dfrac{120\pi}{2} = 60\pi$

$$\therefore\ \mathbf{H} = -\frac{10}{60\pi} \cos (3\pi \times 10^7 t - 0.2\pi x)\, \mathbf{a}_y$$

Note $\mathbf{E} \times \mathbf{H}$ is in the $+x$ direction, since $\mathbf{a}_z \times (-\mathbf{a}_y) = \mathbf{a}_x$. Thus

$$\mathbf{H} = -\frac{1}{6\pi} \cos (3\pi \times 10^7 t - 0.2\pi x)\, \mathbf{a}_y$$

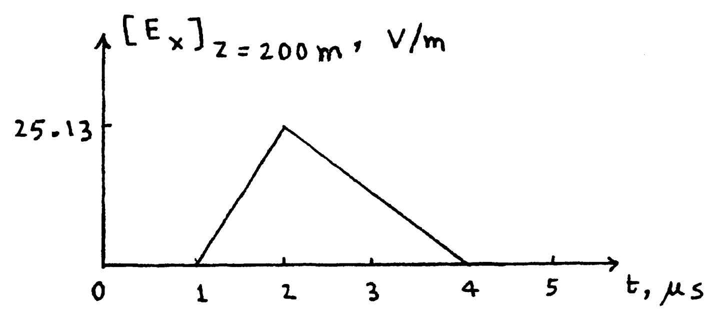
P4.19. For $\varepsilon = 2.25\varepsilon_0$ and $\mu = \mu_0$,

$$v_p = \frac{1}{\sqrt{\mu_0 \times 2.25\varepsilon_0}} = 2 \times 10^8 \text{ m/s}$$

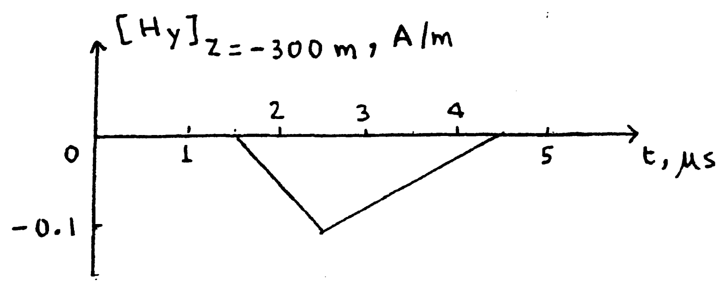
$\therefore$ 1 μs $\leftrightarrow$ 200 m

$$\eta = \sqrt{\frac{\mu_0}{2.25\varepsilon_0}} = 80\pi\,\Omega$$

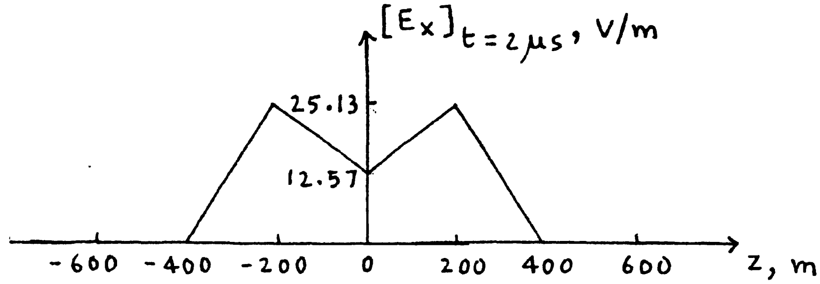
(a)



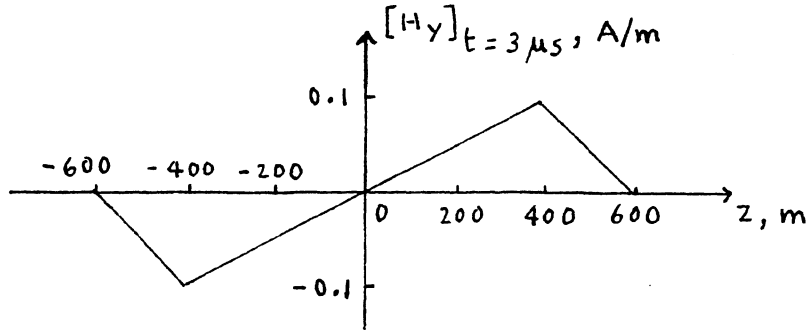
(b)



(c)



(d)



P4.20. From the given plots,

$$v_p = \frac{50 \times 10^{-2}}{10 \times 10^{-9}} = 5 \times 10^7 \text{ m/s}$$

$$\eta = \frac{9\pi}{0.2} = 45\pi \ \Omega$$

Thus

$$\frac{1}{\sqrt{\mu\varepsilon}} = 5 \times 10^7$$

$$\sqrt{\frac{\mu}{\varepsilon}} = 45\pi$$

$$\mu = \frac{45\pi}{5 \times 10^7} = 9\pi \times 10^{-7} = 2.25\mu_0$$

$$\varepsilon = \frac{1}{5 \times 10^7 \times 45\pi} = \frac{10^{-9}}{2.25\pi} = 16\varepsilon_0$$

P4.21. $\dfrac{\sigma}{\omega\varepsilon} = \dfrac{10^{-6} \times 36\pi}{2\pi \times 10^{6} \times 3 \times 10^{-9}} = 6 \times 10^{-3} << 1$

$\therefore$ The material behaves like an imperfect dielectric.

$$\alpha \approx \frac{\sigma}{2}\sqrt{\frac{\mu}{\varepsilon}}\left(1 - \frac{\sigma^2}{8\omega^2\varepsilon^2}\right)$$

$$\approx \frac{\sigma}{2}\sqrt{\frac{\mu}{\varepsilon}} = \frac{10^{-6}}{2} \times \frac{120\pi}{\sqrt{3}} = 1.0883 \times 10^{-4} \text{ m}^{-1}$$

$$\beta \approx \omega\sqrt{\mu\varepsilon}\left(1 + \frac{\sigma^2}{8\omega^2\varepsilon^2}\right)$$

$$\approx \omega\sqrt{\mu\varepsilon} = \frac{2\pi \times 10^{6} \times \sqrt{3}}{3 \times 10^{8}} = 0.036276 \text{ rad/m}$$

$$v_p \approx \frac{1}{\sqrt{\mu\varepsilon}}\left(1 + \frac{\sigma^2}{8\omega^2\varepsilon^2}\right)$$

$$\approx \frac{1}{\sqrt{\mu\varepsilon}} = \frac{3 \times 10^{8}}{\sqrt{3}} = 1.732 \times 10^{8} \text{ m/s}$$

$$\lambda \approx \frac{1}{f\sqrt{\mu\varepsilon}}\left(1 - \frac{\sigma^2}{8\omega^2\varepsilon^2}\right)$$

$$\approx \frac{1}{f\sqrt{\mu\varepsilon}} = \frac{3 \times 10^{8}}{10^{6} \times \sqrt{3}} = 173.21 \text{ m}$$

$$\bar{\eta} \approx \sqrt{\frac{\mu}{\varepsilon}}\left[\left(1 - \frac{3}{8}\frac{\sigma^2}{\omega^2\varepsilon^2}\right) + j\frac{\sigma}{2\omega\varepsilon}\right]$$

$$\approx \frac{120\pi}{\sqrt{3}}(1 + j0.003)$$

$$= (217.66 + j0.653)\ \Omega$$

Distance in which fields are attenuated by the factor e^{-1}

$$= \frac{1}{\alpha} = \frac{10^{4}}{1.0883} \text{ m} = 9.189 \times 10^{3} \text{ m} = 9.189 \text{ km}$$

P4.22. $\sigma = 4$ S/m, $\varepsilon = 80\varepsilon_0$, $\mu = \mu_0$

(a) $f = 10$ GHz, $\dfrac{\sigma}{\omega\varepsilon} = \dfrac{4 \times 36\pi}{2\pi \times 10^{10} \times 80 \times 10^{-9}} = \dfrac{9}{100} << 1$

The medium behaves like an imperfect dielectric.

$$\alpha \approx \frac{\sigma}{2}\sqrt{\frac{\mu}{\varepsilon}} = \frac{4}{2}\sqrt{\frac{\mu_0}{80\varepsilon_0}} = \frac{2 \times 120\pi}{\sqrt{80}} = 84.3 \text{ Np/m}$$

$$\delta = \frac{1}{\alpha} = \frac{1}{84.3} = 0.0119 \text{ m} = 11.9 \text{ mm}$$

$$\beta \approx \omega\sqrt{\mu\varepsilon} = 2\pi \times 10^{10} \times \frac{\sqrt{80}}{3 \times 10^8} = 1873 \text{ rad/m}$$

$$\lambda = \frac{2\pi}{\beta} = 3.355 \times 10^{-3} \text{ m} = 3.355 \text{ mm}$$

$$v_p = \frac{\omega}{\beta} = \frac{2\pi \times 10^{10}}{1873} = 0.3355 \times 10^8 \text{ m/s}$$

$$\bar{\eta} \approx \sqrt{\frac{\mu}{\varepsilon}} = \frac{120\pi}{\sqrt{80}} = 42.15\ \Omega$$

(b) $f = 100$ kHz, $\dfrac{\sigma}{\omega\varepsilon} = \dfrac{4 \times 36\pi}{2\pi \times 10^5 \times 80 \times 10^{-9}} = 9 \times 10^3 >> 1$

The medium behaves like a good conductor.

$$\alpha \approx \sqrt{\pi f \mu \sigma} = \sqrt{\pi \times 10^5 \times 4\pi \times 10^{-7} \times 4} = 0.4\pi \text{ m}^{-1}$$

$$\delta = \frac{1}{\alpha} = \frac{1}{0.4\pi} = 0.796 \text{ m}$$

$$\beta \approx \sqrt{\pi f \mu \sigma} = 0.4\pi \text{ rad/m}$$

$$\lambda = \frac{2\pi}{\beta} = 5 \text{ m}$$

$$v_p = \frac{\omega}{\beta} = \frac{2\pi \times 10^5}{0.4\pi} = 5 \times 10^5 \text{ m/s}$$

$$\bar{\eta} \approx \sqrt{\frac{\pi f \mu}{\sigma}}(1+j) = \sqrt{\frac{\pi \times 10^5 \times 4\pi \times 10^{-7}}{4}}(1+j)$$

$$= 0.1\pi(1+j)\ \Omega$$

P4.23.. $[\mathbf{H}]_{z=0} = 0.1 \cos^3 2\pi \times 10^8 t\, \mathbf{a}_y$ A/m

(a) $\sigma = 0$, $\varepsilon = 9\varepsilon_0$, $\mu = \mu_0$. The medium is a perfect dielectric.

$$v_p = \frac{1}{\sqrt{\mu_0 \cdot 9\varepsilon_0}} = 10^8 \text{ m/s},\ \eta = \sqrt{\frac{\mu_0}{9\varepsilon_0}} = 40\pi\,\Omega$$

$$\therefore \mathbf{E}(z, t) = 40\pi \times 0.1 \cos^3\left(2\pi \times 10^8 t - \frac{2\pi \times 10^8}{10^8} z\right) \mathbf{a}_x$$

$$= 4\pi \cos^3 (2\pi \times 10^8 t - 2\pi z)\, \mathbf{a}_x \text{ V/m}$$

(b) $\sigma = 10^{-3}$ S/m, $\varepsilon = 9\varepsilon_0$, $\mu = \mu_0$

$$\cos^3 2\pi \times 10^8 t = \frac{3}{4} \cos 2\pi \times 10^8 t + \frac{1}{4} \cos 6\pi \times 10^8 t$$

$\dfrac{10^{-3} \times 36\pi}{2\pi \times 10^8 \times 9 \times 10^{-9}} = \dfrac{1}{50} << 1$. The medium is an imperfect dielectric at both frequencies.

$$\alpha \approx \frac{\sigma}{2}\sqrt{\frac{\mu}{\varepsilon}} = \frac{10^{-3}}{2}\sqrt{\frac{\mu_0}{9\varepsilon_0}} = 0.02\pi;\ \bar{\eta} \approx \sqrt{\frac{\mu}{\varepsilon}} = 40\pi\,\Omega$$

$$\beta \approx \omega\sqrt{\mu\varepsilon} = 2\pi \text{ for } \omega = 2\pi \times 10^8$$

$$\therefore \mathbf{E}(z, t) \approx 4\pi e^{-0.02\pi z} \cos^3 (2\pi \times 10^8 t - 2\pi z)\, \mathbf{a}_x \text{ V/m}$$

(c) $\sigma = 10$ S/m, $\varepsilon = 9\varepsilon_0$, $\mu = \mu_0$

$\dfrac{10 \times 36\pi}{6\pi \times 10^8 \times 9 \times 10^{-9}} = \dfrac{200}{3} >> 1$. The medium is a good conductor at both frequencies.

$$\alpha = \beta \approx \sqrt{\pi f \mu \sigma},\ \bar{\eta} \approx \sqrt{\frac{2\pi f \mu}{\sigma}}\,\underline{/45^\circ}$$

For $f = 10^8$, $\alpha = \beta \approx \sqrt{\pi \times 10^8 \times 4\pi \times 10^{-7} \times 10} = 20\pi$

$$\bar{\eta} \approx \sqrt{\frac{2\pi \times 10^8 \times 4\pi \times 10^{-7}}{10}}\,\underline{/45^\circ} = 2\sqrt{2}\pi\,\underline{/45^\circ}\ \Omega$$

P4.23. (continued)

For $f = 3 \times 10^8$, $\alpha = \beta \approx 20\sqrt{3}\pi$, $\bar{\eta} \approx 2\sqrt{6}\pi\underline{/45^\circ}$

$$\mathbf{E}(z, t) \approx [0.67e^{-20\pi z} \cos (2\pi \times 10^8 t - 20\pi z + \pi/4)$$

$$+ 0.38e^{-34.64\pi z} \cos (6\pi \times 10^8 t - 34.64\pi z + \pi/4)]\, \mathbf{a}_x$$

where $0.67 = \frac{3}{4} \times 0.1 \times 2\sqrt{2}\pi$

and $0.38 = \frac{1}{4} \times 0.1 \times 2\sqrt{6}\pi$

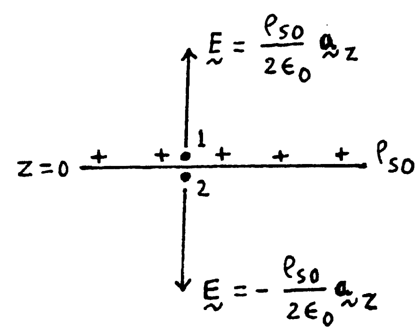
P4.24. Example 1.9

$\mathbf{a}_n \cdot (\mathbf{D}_1 - \mathbf{D}_2)$

$$= \mathbf{a}_z \cdot \left[\frac{\rho_{S0}}{2}\,\mathbf{a}_z - \left(-\frac{\rho_{S0}}{2}\,\mathbf{a}_z\right)\right]$$

$$= \mathbf{a}_z \cdot \rho_{S0}\mathbf{a}_z$$

$$= \rho_{S0}$$



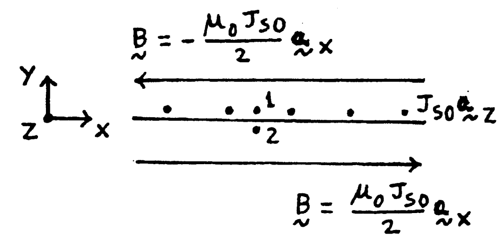
Example 1.12.

$\mathbf{a}_n \times (\mathbf{H}_1 - \mathbf{H}_2)$

$$= \mathbf{a}_y \times \left[-\frac{J_{S0}}{2}\,\mathbf{a}_x - \frac{J_{S0}}{2}\,\mathbf{a}_x\right]$$

$$= -J_{S0}\mathbf{a}_y \times \mathbf{a}_x$$

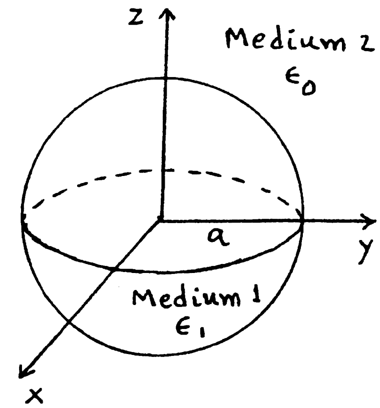
$$= J_{S0}\mathbf{a}_z$$



P4.25. At $r = a$,

$$E_{\theta 1} = E_{\theta 2}$$

$$D_{r1} = D_{r2}$$



Thus

$$-E_{01} \sin\theta = -E_{02}\left(1 - \frac{a^3}{4a^3}\right)\sin\theta \qquad (1)$$

$$\varepsilon_1 E_{01} \cos\theta = \varepsilon_0 E_{02}\left(1 + \frac{a^3}{2a^3}\right)\cos\theta \qquad (2)$$

From (1), $E_{02} = \frac{4}{3} E_{01}$

Then from (2), $\varepsilon_1 E_{01} = \frac{4}{3}\varepsilon_0 E_{01} \times \frac{3}{2}$

$$\varepsilon_1 = 2\varepsilon_0$$

P4.26. Let

$$\mathbf{E}_1 = \mathbf{E}_{t1} + \mathbf{E}_{n1} \quad (1)$$

$$\mathbf{E}_2 = \mathbf{E}_{t2} + \mathbf{E}_{n2} \quad (2)$$

where t and n denote 'tangential' and 'normal,' respectively. Then from the boundary conditions at the interface between the two media

$$\mathbf{E}_{t1} - \mathbf{E}_{t2} = 0 \quad (3)$$

$$D_{n1} - D_{n2} = 0 \quad (4)$$

Then subtracting (2) from (1), we have

$$\begin{aligned}\mathbf{E}_1 - \mathbf{E}_2 &= \mathbf{E}_{n1} - \mathbf{E}_{n2} \\ &= (E_{n1} - E_{n2})\,\mathbf{a}_n\end{aligned}$$

so that $(\mathbf{E}_1 - \mathbf{E}_2)$ is in the direction normal to the boundary. Thus, unit vector normal to the boundary is given by

$$\begin{aligned}\mathbf{i}_n &= \frac{\mathbf{E}_1 - \mathbf{E}_2}{|\mathbf{E}_1 - \mathbf{E}_2|} \\ &= \frac{E_0(4\mathbf{a}_x + 2\mathbf{a}_y + 5\mathbf{a}_z) - 3E_0(\mathbf{a}_x + \mathbf{a}_z)}{|E_0(4\mathbf{a}_x + 2\mathbf{a}_y + 5\mathbf{a}_z) - 3E_0(\mathbf{a}_x + \mathbf{a}_z)|} \\ &= \frac{\mathbf{a}_x + 2\mathbf{a}_y + 2\mathbf{a}_z}{3}\end{aligned}$$

$$E_{n1} = \mathbf{E}_1 \bullet \mathbf{a}_n = \frac{E_0}{3}(4 + 4 + 10) = 6E_0$$

$$E_{n2} = \mathbf{E}_2 \bullet \mathbf{a}_n = E_0\,(1 + 2) = 3E_0$$

Since $\dfrac{E_{n1}}{E_{n2}} = \dfrac{D_{n1}/\varepsilon_1}{D_{n2}/\varepsilon_2} = \dfrac{\varepsilon_2}{\varepsilon_1}$ in view of (4), we get

$$\frac{\varepsilon_2}{\varepsilon_1} = \frac{6E_0}{3E_0} = 2$$

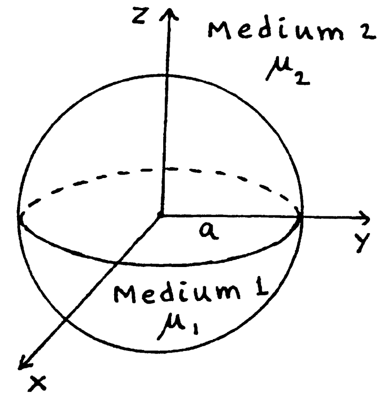
$$\varepsilon_2 = 2\varepsilon_0$$

$\therefore$ The required permittivity is $2\varepsilon_0$.

P4.27. At $r = a$,

$$B_{r1} = B_{r2}$$

$$H_{\theta 1} = H_{\theta 2}$$



Thus

$$B_{01} \cos\theta = B_{02}\left(1 + 1.94\frac{a^3}{a^3}\right)\cos\theta \qquad (1)$$

$$-\frac{B_{01}}{\mu_1}\sin\theta = -\frac{B_{02}}{\mu_0}\left(1 - 0.97\frac{a^3}{a^3}\right)\cos\theta \qquad (2)$$

From (1), $B_{01} = 2.94 B_{02}$

Then from (2),

$$\frac{2.94 B_{02}}{\mu_1} = \frac{B_{02}}{\mu_0} \times 0.03$$

$$\mu_1 = \frac{2.94}{0.03}\mu_0$$

$$= 98\mu_0$$

P4.28. From the solution to Prob. P4.4,

$$[\mathbf{E}]_{x>0} = \frac{\rho_{L0}}{2\pi\varepsilon_0}\left[\frac{(x-d)\mathbf{a}_x + y\mathbf{a}_y}{(x-d)^2+y^2} - \frac{(x+d)\mathbf{a}_x + y\mathbf{a}_y}{(x+d)^2+y^2}\right]$$

$$[\mathbf{E}]_{x=0+} = \frac{\rho_{L0}}{2\pi\varepsilon_0}\left[\frac{-d\mathbf{a}_x + y\mathbf{a}_y}{d^2+y^2} - \frac{d\mathbf{a}_x + y\mathbf{a}_y}{d^2+y^2}\right]$$

$$= -\frac{\rho_{L0}d}{\pi\varepsilon_0(d^2+y^2)}\mathbf{a}_x$$

which is completely normal to the conductor. Thus the boundary condition of zero tangential component of electric field on the conductor surface is satisfied.

From the boundary condition for the normal component of **D**,

$$\rho_S = \mathbf{a}_n \bullet \mathbf{D} = \mathbf{a}_x \bullet [\mathbf{D}]_{x=0+}$$

$$= -\frac{\rho_{L0}d}{\pi(d^2+y^2)}$$

The total induced surface charge per unit width in the z-direction

$$= \int_{y=-\infty}^{\infty}\int_{z=0}^{1}\left[-\frac{\rho_{L0}d}{\pi(d^2+y^2)}\right]dy\,dz$$

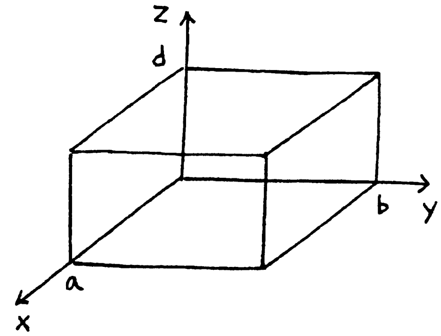
$$= -\frac{\rho_{L0}d}{\pi}\left[\frac{1}{d}\tan^{-1}\frac{y}{d}\right]_{-\infty}^{\infty}$$

$$= -\rho_{L0}$$

P4.29. $\mathbf{E} = E_0 \sin \frac{\pi x}{a} \sin \frac{\pi z}{d} \cos \omega t \, \mathbf{a}_y$

$$\mathbf{H} = H_{01} \sin \frac{\pi x}{a} \cos \frac{\pi z}{d} \sin \omega t \, \mathbf{a}_x$$

$$- H_{02} \cos \frac{\pi x}{a} \sin \frac{\pi z}{d} \sin \omega t \, \mathbf{a}_z$$



Using $\rho_S = \mathbf{a}_n \bullet \mathbf{D} = \mathbf{a}_n \bullet 4\varepsilon_0 \mathbf{E}_0$, we obtain

$$[\rho_S]_{x=0} = \mathbf{a}_x \bullet 4\varepsilon_0 [\mathbf{E}]_{x=0} = 0$$

$$[\rho_S]_{x=a} = -\mathbf{a}_x \bullet 4\varepsilon_0 [\mathbf{E}]_{x=a} = 0$$

$$[\rho_S]_{y=0} = \mathbf{a}_y \bullet 4\varepsilon_0 [\mathbf{E}]_{y=0} = 4\varepsilon_0 E_0 \sin \frac{\pi x}{a} \sin \frac{\pi z}{d} \cos \omega t$$

$$[\rho_S]_{y=b} = -\mathbf{a}_y \bullet 4\varepsilon_0 [\mathbf{E}]_{y=b} = - 4\varepsilon_0 E_0 \sin \frac{\pi x}{a} \sin \frac{\pi z}{d} \cos \omega t$$

$$[\rho_S]_{z=0} = \mathbf{a}_z \bullet 4\varepsilon_0 [\mathbf{E}]_{z=0} = 0$$

$$[\rho_S]_{z=d} = -\mathbf{a}_z \bullet 4\varepsilon_0 [\mathbf{E}]_{z=d} = 0$$

Using $\mathbf{J}_S = \mathbf{a}_n \times \mathbf{H}$, we obtain

$$[\mathbf{J}_S]_{x=0} = \mathbf{a}_x \times [\mathbf{H}]_{x=0} = H_{02} \sin \frac{\pi z}{d} \sin \omega t \, \mathbf{a}_y$$

$$[\mathbf{J}_S]_{x=a} = -\mathbf{a}_x \times [\mathbf{H}]_{x=a} = H_{02} \sin \frac{\pi z}{d} \sin \omega t \, \mathbf{a}_y$$

$$[\mathbf{J}_S]_{y=0} = \mathbf{a}_y \times [\mathbf{H}]_{y=0} = -H_{02} \cos \frac{\pi x}{a} \sin \frac{\pi z}{d} \sin \omega t \, \mathbf{a}_x$$

$$- H_{01} \sin \frac{\pi x}{a} \cos \frac{\pi z}{d} \sin \omega t \, \mathbf{a}_z$$

$$[\mathbf{J}_S]_{y=b} = -\mathbf{a}_y \times [\mathbf{H}]_{y=b} = H_{02} \cos \frac{\pi x}{a} \sin \frac{\pi z}{d} \sin \omega t \, \mathbf{a}_x$$

$$+ H_{01} \sin \frac{\pi x}{a} \cos \frac{\pi z}{d} \sin \omega t \, \mathbf{a}_z$$

$$[\mathbf{J}_S]_{z=0} = \mathbf{a}_z \times [\mathbf{H}]_{z=0} = H_{01} \sin \frac{\pi x}{a} \sin \omega t \, \mathbf{a}_y$$

$$[\mathbf{J}_S]_{z=d} = -\mathbf{a}_z \times [\mathbf{H}]_{z=d} = H_{01} \sin \frac{\pi x}{a} \sin \omega t \, \mathbf{a}_y$$

P4.30. For $z > 0$: $\bar{\gamma} = 0 + j\dfrac{2\pi \times 10^6}{3 \times 10^8} = j\dfrac{\pi}{150}$

$$\bar{\eta} = 120\pi\underline{/0^\circ}$$

For $z < 0$: $\bar{\gamma} = 0.05334 + j0.07401$

$$\bar{\eta} = 86.5477\underline{/35.783^\circ}$$

Let the complex electric field at $z = 0+$ be $\bar{E}_x(0+) = \bar{E}_0$. Then from continuity of E_x at $z = 0$, $\bar{E}_x(0-) = \bar{E}_x(0+) = \bar{E}_0$. Proceeding further, we have

$$\bar{H}_y(0+) = \frac{\bar{E}_x(0+)}{120\pi} = \frac{\bar{E}_0}{120\pi}$$

$$H_y(0-) = -\frac{\bar{E}_x(0-)}{86.5477\underline{/35.783^\circ}} = -\frac{\bar{E}_0}{86.5477\underline{/35.783^\circ}}$$

From the boundary condition $\mathbf{a}_z \times (\mathbf{H}_1 - \mathbf{H}_2) = \mathbf{J}_S$ at $z = 0$, we can write

$$\mathbf{a}_z \times [\bar{H}_y(0+) - \bar{H}_y(0-)]\mathbf{a}_y = -0.2\mathbf{a}_x$$

$$\bar{E}_0\left(\frac{1}{120\pi} + \frac{1}{86.5477\underline{/35.783^\circ}}\right) = 0.2$$

$$\bar{E}_0[0.002653 + (0.009373 - j0.006756)] = 0.2$$

$$\bar{E}_0 = \frac{0.2}{0.01203 - j0.006756} = \frac{0.2}{0.0138\underline{/-29.318^\circ}} = 14.493\underline{/29.318^\circ}$$

Thus

$$\bar{E}_x(0+) = \bar{E}_x(0-) = 14.493\underline{/29.318^\circ} = 14.493\underline{/0.163\pi}$$

$$\bar{H}_y(0+) = \frac{14.493\underline{/0.163\pi}}{120\pi} = 0.0384\underline{/0.163\pi}$$

$$\bar{H}_y(0-) = -\frac{14.493\underline{/29.318^\circ}}{86.5477\underline{/35.783^\circ}} = 0.1675\underline{/173.535^\circ} = 0.1675\underline{/0.964\pi}$$

$$\mathbf{E} = \begin{cases} 14.493 \cos\left(2\pi \times 10^6 t - \dfrac{\pi}{150}z + 0.163\pi\right)\mathbf{a}_x & \text{for } z > 0 \\ 14.493e^{0.05334z} \cos(2\pi \times 10^6 t + 0.07401z + 0.163\pi)\,\mathbf{a}_x & \text{for } z < 0 \end{cases}$$

$$\mathbf{H} = \begin{cases} 0.0384 \cos\left(2\pi \times 10^6 t - \dfrac{\pi}{150}z + 0.163\pi\right)\mathbf{a}_y & \text{for } z > 0 \\ 0.1675e^{0.05334z} \cos(2\pi \times 10^6 t + 0.07401z + 0.964\pi)\,\mathbf{a}_y & \text{for } z < 0 \end{cases}$$

P4.31. For $z > 0$, $\varepsilon = 2.25\varepsilon_0$, $\mu = \mu_0$

$$\beta = \frac{6\pi \times 10^8}{2 \times 10^8} = 3\pi,\ \eta = \frac{\eta_0}{1.5} = 80\pi$$

For $z < 0$, $\varepsilon = 4\varepsilon_0$, $\mu = \mu_0$

$$\beta = \frac{6\pi \times 10^8}{1.5 \times 10^8} = 4\pi,\ \eta = \frac{\eta_0}{2} = 60\pi$$

Thus, let

$\mathbf{E}_1 = E_0 \cos(6\pi \times 10^8 t - 3\pi z)\,\mathbf{a}_x$ for $z > 0$

Then from continuity of E_x at $z = 0$,

$\mathbf{E}_2 = E_0 \cos(6\pi \times 10^8 t + 4\pi z)\,\mathbf{a}_x$ for $z < 0$

Proceeding further, we have

$\mathbf{H}_1 = \dfrac{E_0}{80\pi} \cos(6\pi \times 10^8 t - 3\pi z)\,\mathbf{a}_y$ for $z > 0$

$\mathbf{H}_2 = -\dfrac{E_0}{60\pi} \cos(6\pi \times 10^8 t + 4\pi z)\,\mathbf{a}_y$ for $z < 0$

From the boundary condition $\mathbf{a}_z \times (\mathbf{H}_1 - \mathbf{H}_2) = \mathbf{J}_S$ at $z = 0$, we can write

$$\mathbf{a}_z \times \left(\frac{E_0}{80\pi} + \frac{E_0}{60\pi}\right) \cos 6\pi \times 10^8 t\ \mathbf{a}_y = -0.2 \cos 6\pi \times 10^8 t\,\mathbf{a}_x$$

or, $\dfrac{7E_0}{240\pi} = 0.2$, $E_0 = \dfrac{48\pi}{7}$

Thus the required solutions are

$$\mathbf{E} = \begin{cases} \dfrac{48\pi}{7} \cos(6\pi \times 10^8 t - 3\pi z)\,\mathbf{a}_x \text{ V/m} & \text{for } z > 0 \\ \dfrac{48\pi}{7} \cos(6\pi \times 10^8 t + 4\pi z)\,\mathbf{a}_x \text{ V/m} & \text{for } z < 0 \end{cases}$$

$$\mathbf{H} = \begin{cases} \dfrac{3}{35} \cos(6\pi \times 10^8 t - 3\pi z)\,\mathbf{a}_y \text{ A/m} & \text{for } z > 0 \\ -\dfrac{4}{35} \cos(6\pi \times 10^8 t + 4\pi z)\,\mathbf{a}_y \text{ A/m} & \text{for } z < 0 \end{cases}$$

P4.32. For $\sigma = 10^{-4}$ S/m, $\varepsilon = 5\varepsilon_0$, $\mu = \mu_0$, $f = 1.5 \times 10^5$ Hz,

$$\bar{\gamma} = (6.283 + j9.425) \times 10^{-3}$$

$$\bar{\eta} = 104.559\underline{/33.69^\circ}$$

$$\therefore\ \bar{\Gamma} = \frac{\bar{\eta} - \eta_0}{\bar{\eta} + \eta_0} = \frac{104.559\underline{/33.69^\circ} - 377}{104.559\underline{/33.69^\circ} + 377}$$

$$= \frac{-290.002 + j57.999}{463.998 + j57.999} = \frac{295.745\underline{/168.69^\circ}}{467.609\underline{/7.125^\circ}}$$

$$= 0.6325\underline{/161.565^\circ}$$

$$\tau = 1 + \bar{\Gamma} = 1 + 0.6325\underline{/161.565^\circ}$$

$$= 0.4 + j0.2$$

$$= 0.4472\underline{/26.565^\circ}$$

$$\mathbf{E}_r = 0.6325E_0 \cos\,(3\pi \times 10^5 t + 10^{-3}\pi z + 0.8976\pi)\ \mathbf{a}_x \text{ V/m}$$

$$\mathbf{E}_t = 0.4472E_0 e^{-6.283\times10^{-3}z}$$

$$\cdot \cos\,(3\pi \times 10^5 t - 9.425 \times 10^{-3}\pi z + 0.1476\pi)\ \mathbf{a}_x \text{ V/m}$$

P4.33. $E_0 \cos^3 (3\pi \times 10^5 t - 10^{-3}\pi t)\, \mathbf{a}_x$

$$= \frac{3E_0}{4} \cos (3\pi \times 10^5 t - 10^{-3}\pi t)\, \mathbf{a}_x$$

$$+ \frac{E_0}{4} \cos (9\pi \times 10^5 t - 3 \times 10^{-3}\pi t)\, \mathbf{a}_x$$

For $f = 1.5 \times 10^5$ Hz,

$$\bar{\gamma} = (6.283 + j9.425) \times 10^{-3}, \ \bar{\eta} = 104.559\underline{/33.69^\circ}$$

$$\bar{\Gamma} = 0.6325\underline{/161.565^\circ}, \ \tau = 0.4472\underline{/26.72^\circ}$$

For $f = 4.5 \times 10^5$ Hz,

$$\bar{\gamma} = (7.894 + j22.504) \times 10^{-3}$$

$$\bar{\eta} = 148.983\underline{/19.33^\circ}$$

$$\bar{\Gamma} = \frac{\bar{\eta} - \eta_0}{\bar{\eta} + \eta_0} = \frac{148.983\underline{/19.33^\circ} - 377}{148.983\underline{/19.33^\circ} + 377}$$

$$= \frac{-236.416 + j49.315}{517.584 + j49.315} = \frac{241.505\underline{/168.217^\circ}}{519.928\underline{/5.443^\circ}}$$

$$= 0.4645\underline{/162.774^\circ}$$

$$\tau = 1 + \bar{\Gamma} = 1 + 0.4645\underline{/162.774^\circ}$$

$$= 0.5563 + j0.1376$$

$$= 0.5731\underline{/13.893^\circ}$$

$$\mathbf{E}_r = [0.4744E_0 \cos (3\pi \times 10^5 t + 10^{-3}\pi z + 0.8976\pi)\, \mathbf{a}_x$$

$$+ 0.1161E_0 \cos (9\pi \times 10^5 t + 3 \times 10^{-3}\pi z + 0.9043\pi)\, \mathbf{a}_x] \text{ V/m}$$

$$\mathbf{E}_t = [0.3354E_0 e^{-6.283\times 10^{-3} z}$$

$$\cdot \cos (3\pi \times 10^5 t - 9.425 \times 10^{-3}\pi z + 0.1476\pi)\, \mathbf{a}_x$$

$$+ 0.1433E_0 e^{-7.894\times 10^{-3} z}$$

$$\cdot \cos (9\pi \times 10^5 t - 22.504 \times 10^{-3}\pi z + 0.0772\pi)\, \mathbf{a}_x] \text{ V/m}$$

P4.34. Medium 1:

$$\bar{E}_x = E_0 e^{-j\pi z} + \bar{E}_1^- e^{j\pi z}$$

$$\bar{H}_y = \frac{1}{\eta_0}\left(E_0 e^{-j\pi z} - \bar{E}_1^- e^{j\pi z}\right)$$

Medium 2:

$$\bar{E}_x = \bar{E}_2^+ e^{-j3\pi z} + \bar{E}_2^- e^{j3\pi z}$$

$$\bar{H}_y = \frac{3}{\eta_0}\left(\bar{E}_2^+ e^{-j3\pi z} - \bar{E}_2^- e^{j3\pi z}\right)$$

Medium 3:

$$\bar{E}_x = \bar{E}_3^+ e^{-j2\pi z}$$

$$\bar{H}_y = \frac{2}{\eta_0}\bar{E}_3^+ e^{-j2\pi z}$$

Using boundary conditions, we have

$$E_0 + \bar{E}_1^- = \bar{E}_2^+ + \bar{E}_2^- \qquad \bar{E}_2^+ + \bar{E}_2^- = -\bar{E}_3^+ e^{-j2\pi/3}$$

$$E_0 - \bar{E}_1^- = 3(\bar{E}_2^+ - \bar{E}_2^-) \qquad 3(\bar{E}_2^+ - \bar{E}_2^-) = -2\bar{E}_3^+ e^{-j2\pi/3}$$

Solving, we get

$$\bar{E}_1^- = -\tfrac{1}{3}E_0, \quad \bar{E}_2^+ = \tfrac{5}{9}E_0, \quad \bar{E}_2^- = \tfrac{1}{9}E_0, \quad \bar{E}_3^+ = -\tfrac{2}{3}E_0 e^{j2\pi/3}$$

Thus

$$\bar{E}_{x1} = E_0\left(e^{-j\pi z} - \tfrac{1}{3}e^{j\pi z}\right)$$

$$\bar{H}_{y1} = \frac{E_0}{\eta_0}\left(e^{-j\pi z} + \tfrac{1}{3}e^{j\pi z}\right)$$

$$\bar{E}_{x2} = \frac{E_0}{9}\left(5e^{-j3\pi z} + e^{j3\pi z}\right)$$

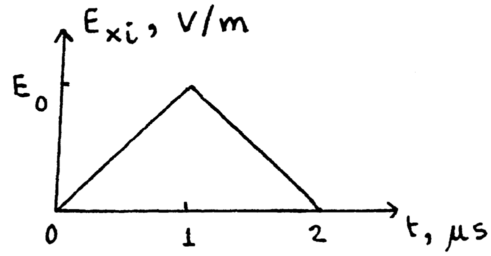
$$\bar{H}_{y2} = \frac{E_0}{3\eta_0}\left(5e^{-j3\pi z} - e^{j3\pi z}\right)$$

$$\bar{E}_{x3} = -\tfrac{2}{3}E_0 e^{j2\pi/3} e^{-j2\pi z}$$

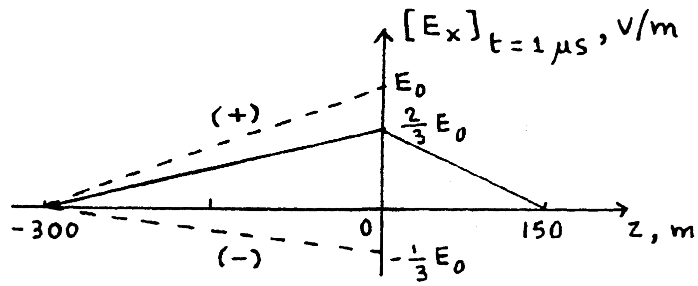
$$\bar{H}_{y3} = -\frac{4}{3\eta_0}E_0 e^{j2\pi/3} e^{-j2\pi z}$$

P4.35. $\bar{\Gamma} = \dfrac{1-\sqrt{\varepsilon_2/\varepsilon_1}}{1+\sqrt{\varepsilon_2/\varepsilon_1}} = \dfrac{1-2}{1+2} = -\dfrac{1}{3}$

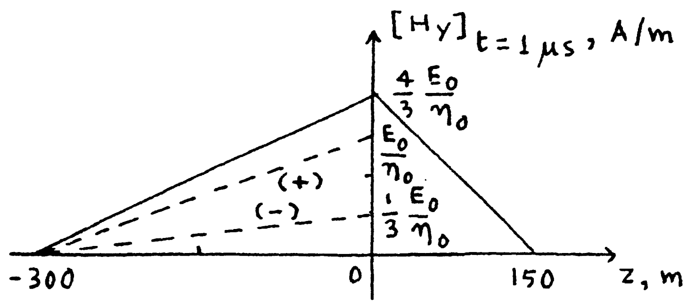
$$\bar{\tau} = \frac{2}{1+\sqrt{\varepsilon_2/\varepsilon_1}} = \frac{2}{1+2} = \frac{2}{3}$$



(a)



(b)



P4.36. Let the reflected wave electric field be

$$\mathbf{E}_r = E_1 \cos(\omega t - \beta z + \phi)\, \mathbf{a}_x$$

Then using the boundary condition that the tangential component of the total electric field be zero on the perfect conductor surface, we have

$$E_0 \cos \omega t + E_1 \cos(\omega t + \phi) = 0$$

$$E_1 = -E_0,\ \phi = 0$$

$$\therefore\ \mathbf{E}_r = -E_0 \cos(\omega t + \beta z)\, \mathbf{a}_x$$

$$\mathbf{H}_r = \frac{E_0}{\eta} \cos(\omega t + \beta z)\, \mathbf{a}_y$$

The total fields are

$$\mathbf{E} = \mathbf{E}_i + \mathbf{E}_r = [E_0 \cos(\omega t - \beta z) - E_0 \cos(\omega t + \beta z)]\, \mathbf{a}_x$$

$$= 2E_0 \sin \omega t \sin \beta z\, \mathbf{a}_x$$

$$\mathbf{H} = \mathbf{H}_i + \mathbf{H}_r = \left[\frac{E_0}{\eta} \cos(\omega t - \beta z) + \frac{E_0}{\eta} \cos(\omega t + \beta z)\right] \mathbf{a}_y$$

$$= 2\, \frac{E_0}{\eta} \cos \omega t \cos \beta z\, \mathbf{a}_y$$

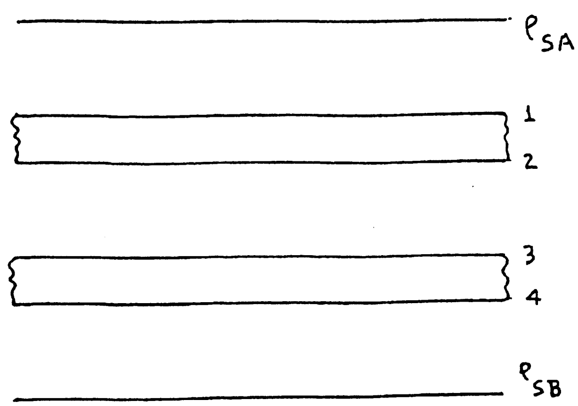
Finally,

$$[\mathbf{J}_S]_{z=0} = -\mathbf{a}_z \times [\mathbf{H}]_{z=0}$$

$$= -\mathbf{a}_z \times \frac{2E_0}{\eta} \cos \omega t\, \mathbf{a}_y$$

$$= \frac{2E_0}{\eta} \cos \omega t\, \mathbf{a}_x$$

R4.1.



Let the surface charge densities be ρ_{S1}, ρ_{S2}, ρ_{S3}, and ρ_{S4} on the surfaces 1, 2, 3, and 4, respectively. Then from charge neutrality in each conductor,

$$\rho_{S1} + \rho_{S2} = 0 \qquad (1)$$

$$\rho_{S3} + \rho_{S4} = 0 \qquad (2)$$

For the electric fields inside the two slabs to be zero,

$$\frac{1}{2\varepsilon_0}(\rho_{SA} + \rho_{S1} - \rho_{S2} - \rho_{S3} - \rho_{S4} - \rho_{SB}) = 0$$

$$\frac{1}{2\varepsilon_0}(\rho_{SA} + \rho_{S1} + \rho_{S2} + \rho_{S3} - \rho_{S4} - \rho_{SB}) = 0$$

or $\quad \rho_{S1} - \rho_{S2} - \rho_{S3} - \rho_{S4} = \rho_{SB} - \rho_{SA} \qquad (3)$

$$\rho_{S1} + \rho_{S2} + \rho_{S3} - \rho_{S4} = \rho_{SB} - \rho_{SA} \qquad (4)$$

From (1) – (4), we obtain

$$\rho_{S1} = \frac{1}{2}(\rho_{SB} - \rho_{SA})$$

$$\rho_{S2} = \frac{1}{2}(\rho_{SA} - \rho_{SB})$$

$$\rho_{S3} = \frac{1}{2}(\rho_{SB} - \rho_{SA})$$

$$\rho_{S4} = \frac{1}{2}(\rho_{SA} - \rho_{SB})$$

R4.2.

$$\begin{bmatrix} D_x \\ D_y \\ D_z \end{bmatrix} = \varepsilon_0 \begin{bmatrix} 6.5 & 1.5 & 0 \\ 1.5 & 2.5 & 0 \\ 0 & 0 & 2 \end{bmatrix} \begin{bmatrix} E_x \\ E_y \\ E_z \end{bmatrix}$$

$$D_x = \varepsilon_0(6.5E_x + 1.5E_y)$$

$$D_y = \varepsilon_0(1.5E_x + 2.5E_y)$$

For characteristic polarizations involving x- and y-components only,

$$\frac{D_x}{D_y} = \frac{6.5E_x + 1.5E_y}{1.5E_x + 2.5E_y} = \frac{E_x}{E_y}$$

$$\frac{1.3\frac{E_x}{E_y} + 0.3}{0.3\frac{E_x}{E_y} + 0.5} = \frac{E_x}{E_y}$$

$$3\left(\frac{E_x}{E_y}\right)^2 - 8\frac{E_x}{E_y} - 3 = 0$$

$$\frac{E_x}{E_y} = \frac{8 \pm \sqrt{64+36}}{6}$$

$$= \frac{8 \pm 10}{6}$$

$$= 3 \text{ or } -\frac{1}{3}$$

The characteristic polarizations are given by

$$\mathbf{E}_1 = E_{01}(3\mathbf{a}_x + \mathbf{a}_y)$$

$$\mathbf{E}_2 = E_{02}(\mathbf{a}_x - 3\mathbf{a}_y)$$

In terms of the two characteristic polarizations, let

$$\mathbf{E} = E_0(\mathbf{a}_x - \mathbf{a}_y)$$

$$= E_{01}(3\mathbf{a}_x + \mathbf{a}_y) + E_{02}(\mathbf{a}_x - 3\mathbf{a}_y)$$

Then

$$3E_{01} + E_{02} = E_0$$

$$E_{01} - 3E_{02} = -E_0$$

Solving, we obtain

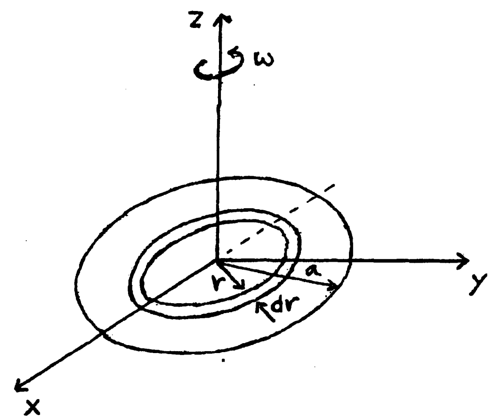
R4.2. (continued)

$$E_{01} = \frac{E_0}{5}, E_{02} = \frac{2E_0}{5}$$

Thus

$$\begin{aligned}\mathbf{E} &= E_0(\mathbf{a}_x - \mathbf{a}_y)\\ &= \frac{E_0}{5}(3\mathbf{a}_x + \mathbf{a}_y) + \frac{2E_0}{5}(\mathbf{a}_x - 3\mathbf{a}_y)\end{aligned}$$

R4.3. Let the charge density be



$$\rho_S = kr$$

Then

$$\int_{r=0}^{a} \int_{\phi=0}^{2\pi} kr \cdot r\, d\phi\, dr = Q$$

or, $$\frac{2\pi k\, a^3}{3} = Q$$

$$k = \frac{3Q}{2\pi a^3}$$

$$\rho_S = \frac{3Qr}{2\pi a^3}$$

Amount of charge in a ring of radius r and width dr is

$$\frac{3Qr}{2\pi a^3}(2\pi r\, dr) = \frac{3Qr^2}{a^3} dr$$

$$\text{Current in the ring} = \left(\frac{3Qr^2}{a^3} dr\right)\left(\frac{\omega}{2\pi}\right)$$

$$= \frac{3\omega Qr^2}{2\pi a^3} dr$$

Dipole moment of the ring is given by

$$d\mathbf{m} = \left(\frac{3\omega Qr^2}{2\pi a^3} dr\right)(\pi r^2)\,\mathbf{a}_z$$

$$= \frac{3\omega Qr^4}{2a^3} dr\, \mathbf{a}_z$$

Dipole moment of the entire disk is

$$\mathbf{m} = \int_{r=0}^{a} d\mathbf{m} = \int_{r=0}^{a} \frac{3\omega Qr^4}{2a^3} dr\, \mathbf{a}_z$$

$$= \frac{3\omega Q}{2a^3}\left[\frac{r^5}{5}\right]_0^a \mathbf{a}_z$$

$$= \frac{3\omega Qa^2}{10}\mathbf{a}_z$$

R4.4. From the given electric field

$$\mathbf{E} = 8.4\, e^{-0.0432z} \cos\,(4\pi \times 10^6 t - 0.1829z)\, \mathbf{a}_x$$

we note that

$$\omega = 4\pi \times 10^6$$

$$\bar{\gamma} = \alpha + j\beta = 0.0432 + j0.1829$$

Now, since

$$\bar{\gamma}\bar{\eta} = \sqrt{j\omega\mu(\sigma + j\omega\varepsilon)} \cdot \sqrt{\frac{j\omega\mu}{\sigma + j\omega\varepsilon}}$$

$$= j\omega\mu = j\omega\mu_0$$

we have

$$\bar{\eta} = \frac{j\omega\mu_0}{\bar{\gamma}} = \frac{j4\pi \times 10^6 \times 4\pi \times 10^{-7}}{0.0432 + j0.1829}$$

$$= \frac{1.6\pi^2 e^{j\pi/2}}{0.1879 e^{j0.4262\pi}}$$

$$= 84 e^{j0.0738\pi}$$

Thus

$$\mathbf{H} = \frac{8.4}{|\bar{\eta}|} e^{-0.0432z} \cos\,(4\pi \times 10^6 t - 0.1829z - \angle\bar{\eta})\, \mathbf{a}_y$$

$$= 0.1 e^{-0.0432z} \cos\,(4\pi \times 10^6 t - 0.1829z - 0.0738\pi)\, \mathbf{a}_y \text{ A/m}$$

Proceeding further, we observe that

$$\frac{\bar{\gamma}}{\bar{\eta}} = \sqrt{j\omega\mu(\sigma + j\omega\varepsilon)} \cdot \sqrt{\frac{\sigma + j\omega\varepsilon}{j\omega\mu}}$$

$$= \sigma + j\omega\varepsilon$$

R4.4. (continued)

Therefore

$$\sigma + j\omega\varepsilon = \frac{0.1879\, e^{j0.4262\pi}}{84\, e^{j0.0738\pi}}$$

$$= 2.2369 \times 10^{-3}\, e^{j0.3524\pi}$$

$$= 2.2369 \times 10^{-3}\,(0.4473 + j0.8944)$$

$$= 10^{-3} + j2 \times 10^{-3}$$

so that

$$\sigma = 10^{-3} \text{ S/m}$$

$$\omega\varepsilon = 2 \times 10^{-3}$$

$$\varepsilon = \frac{2 \times 10^{-3}}{4\pi \times 10^{6}} = \frac{10^{-9}}{2\pi}$$

$$= 18\varepsilon_0$$

R4.5. From the given sketches of the fields for the triangular pulse source, we can obtain for medium 2,

$$v_{p2} = \frac{225 \text{ m}}{3\ \mu\text{s}} = 0.75 \times 10^8 \text{ m/s} = \frac{c}{4}$$

$$\eta_2 = \frac{E_0}{E_0 / 60\pi} = 60\pi = \frac{\eta_0}{2}$$

Now, we know that the pulse duration in medium 1 is 3 μs and because of continuity of tangential electric field at $z = 0$, the peak value of E_x for $z < 0$ is the same as that for $z > 0$. Thus, for medium 1,

$$v_{p1} = \frac{150 \text{ m}}{3\ \mu\text{s}} = 0.5 \times 10^8 \text{ m/s} = \frac{c}{6}$$

$$\eta_1 = \frac{E_0}{E_0 / 80\pi} = 80\pi = \frac{2}{3}\eta_0$$

Now, for $\mathbf{J}_S = -J_{S0} \cos 6\pi \times 10^8 t\, \mathbf{a}_x$ A/m, let

$$\mathbf{E}_1 = E_0 \cos\left(6\pi \times 10^8 t + \frac{6\pi \times 10^8}{0.5 \times 10^8} z\right) \mathbf{a}_x$$

$$= E_0 \cos (6\pi \times 10^8 t + 12\pi z)\, \mathbf{a}_x \text{ V/m for } z < 0$$

Then

$$\mathbf{E}_2 = E_0 \cos\left(6\pi \times 10^8 t - \frac{6\pi \times 10^8}{0.75 \times 10^8} z\right) \mathbf{a}_x$$

$$= E_0 \cos (6\pi \times 10^8 t - 8\pi z)\, \mathbf{a}_x \text{ V/m for } z > 0$$

$$\mathbf{H}_1 = -\frac{E_0}{80\pi} \cos (6\pi \times 10^8 t + 12\pi z)\, \mathbf{a}_y \text{ A/m for } z < 0$$

$$\mathbf{H}_2 = \frac{E_0}{60\pi} \cos (6\pi \times 10^8 t - 8\pi z)\, \mathbf{a}_y \text{ A/m for } z > 0$$

R4.5. (continued)

From the boundary conditions for **H** at $z = 0$

$$\mathbf{a}_z \times \left\{ [\mathbf{H}_2]_{z=0+} - [\mathbf{H}_1]_{z=0-} \right\} = \mathbf{J}_S$$

or,

$$\mathbf{a}_z \times \left(\frac{E_0}{60\pi} \cos 6\pi \times 10^8 t \, \mathbf{a}_y + \frac{E_0}{80\pi} \cos 6\pi \times 10^8 t \, \mathbf{a}_y \right)$$

$$= -J_{S0} \cos 6\pi \times 10^8 t \, \mathbf{a}_x$$

$$E_0 \left(\frac{1}{60\pi} + \frac{1}{80\pi} \right) = J_{S0}$$

$$E_0 = \frac{240\pi}{7} J_{S0}$$

Thus, for $z < 0$,

$$\mathbf{E}_1 = \frac{240\pi}{7} J_{S0} \cos (6\pi \times 10^8 t + 12\pi z) \, \mathbf{a}_x \text{ V/m}$$

$$\mathbf{H}_1 = -\frac{3}{7} J_{S0} \cos (6\pi \times 10^8 t + 12\pi z) \, \mathbf{a}_y \text{ A/m}$$

and for $z > 0$,

$$\mathbf{E}_2 = \frac{240\pi}{7} J_{S0} \cos (6\pi \times 10^8 t - 8\pi z) \, \mathbf{a}_x \text{ V/m}$$

$$\mathbf{H}_2 = \frac{4}{7} J_{S0} \cos (6\pi \times 10^8 t - 8\pi z) \, \mathbf{a}_y \text{ A/m}$$

Proceeding further, we obtain the instantaneous Poynting vectors on either side of the sheet:

$$\mathbf{P}_1 = \mathbf{E}_1 \times \mathbf{H}_1$$

$$= -\frac{720\pi}{49} J_{S0}^2 \cos^2 (6\pi \times 10^8 t + 12\pi z) \, \mathbf{a}_z \text{ W/m}^2 \text{ for } z < 0$$

R4.5. (continued)

$$\mathbf{P}_2 = \mathbf{E}_2 \times \mathbf{H}_2$$

$$= \frac{960\pi}{49} J_{S0}^2 \cos^2 (6\pi \times 10^8 t - 8\pi z)\, \mathbf{a}_z \text{ W/m}^2 \text{ for } z > 0$$

Thus, the time-average power radiated by the sheet per unit area of the sheet

$$= \langle\mathbf{P}_1\rangle \cdot (-\mathbf{a}_z) + \langle\mathbf{P}_2\rangle \cdot \mathbf{a}_z$$

$$= \frac{1}{2}\left(\frac{720\pi}{49} + \frac{960\pi}{49}\right) J_{S0}^2$$

$$= \frac{120\pi}{7} J_{S0}^2 \text{ W}$$

R4.6. For $3x + 4y = 12, z = 0$

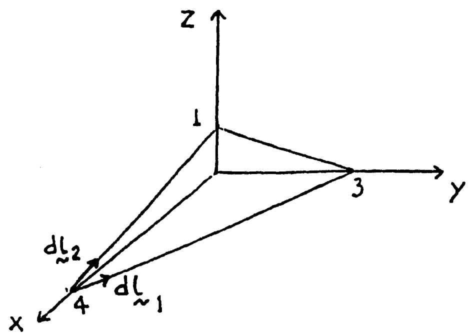
$$3\,dx + 4\,dy = 0,\ dz = 0$$

$$\therefore\ d\mathbf{l}_1 = dx\left(-\mathbf{a}_x + \frac{3}{4}\,\mathbf{a}_y\right)$$

For $3x + 12z = 12,\ y = 0$

$$3\,dx + 12\,dz = 0,\ dy = 0$$

$$\therefore\ d\mathbf{l}_2 = dx\left(-\mathbf{a}_x + \frac{1}{4}\,\mathbf{a}_z\right)$$



Unit vector normal to the surface is given by

$$\mathbf{a}_n = \frac{d\mathbf{l}_1 \times d\mathbf{l}_2}{|d\mathbf{l}_1 \times d\mathbf{l}_2|} = \frac{\left(-\mathbf{a}_x + \frac{3}{4}\,\mathbf{a}_y\right) \times \left(-\mathbf{a}_x + \frac{1}{4}\,\mathbf{a}_z\right)}{\left|\left(-\mathbf{a}_x + \frac{3}{4}\,\mathbf{a}_y\right) \times \left(-\mathbf{a}_x + \frac{1}{4}\,\mathbf{a}_z\right)\right|}$$

$$= \frac{3\mathbf{a}_x + 4\mathbf{a}_y + 12\mathbf{a}_z}{13}$$

Let $\mathbf{D} = D_0\mathbf{a}_n$. Then, since $\rho_{S0} = \mathbf{a}_n \cdot \mathbf{D}$, we have

$$\rho_{S0} = \mathbf{a}_n \cdot D_0\mathbf{a}_n = D_0$$

Thus $\mathbf{D} = \rho_{S0}\,\dfrac{3\mathbf{a}_x + 4\mathbf{a}_y + 12\mathbf{a}_z}{13}$

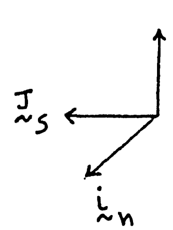
From $\mathbf{a}_n \times \mathbf{H} = \mathbf{J}_S$, $\mathbf{J}_S$ is perpendicular to $\mathbf{H}$ (and $\mathbf{a}_n$).

$\mathbf{a}_n$, $\mathbf{H}$, and $\mathbf{J}_S$ form a right-handed set of orthogonal vectors. Hence, we can write

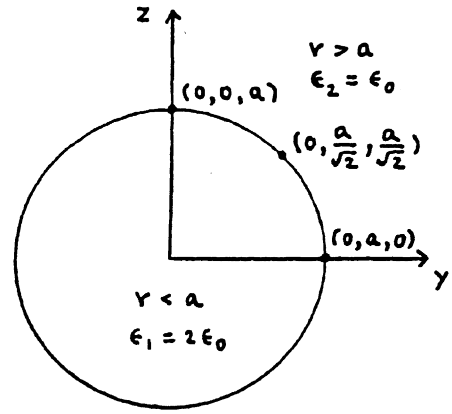
$$\mathbf{H} = \mathbf{J}_S \times \mathbf{a}_n$$

$$= J_{S0}(4\mathbf{a}_x - 3\mathbf{a}_y) \times \frac{1}{13}\,(3\mathbf{a}_x + 4\mathbf{a}_y + 12\mathbf{a}_z)$$

$$= -\frac{J_{S0}}{13}\,(36\mathbf{a}_x + 48\mathbf{a}_y - 25\mathbf{a}_z)$$



R4.7. $\mathbf{E}_1 = E_0\mathbf{a}_z$



(a) At $(0, 0, a)$

$\mathbf{a}_n = \mathbf{a}_z$

$\therefore$ $\mathbf{E}_1$ is entirely normal.

$\mathbf{D}_2 = \mathbf{D}_1 = 2\varepsilon_0 E_1 = 2\varepsilon_0 E_0 \mathbf{a}_z$

$\mathbf{E}_2 = \mathbf{D}_2/\varepsilon_2 = 2E_0\mathbf{a}_z$

(b) At $(0, a, 0)$, $\mathbf{a}_n = \mathbf{a}_y$, $\mathbf{E}_1$ is entirely tangential.

$\therefore$ $\mathbf{E}_2 = \mathbf{E}_1 = E_0\mathbf{a}_z$

(c) At $\left(0, a/\sqrt{2}, a/\sqrt{2}\right)$, $\mathbf{a}_n = \dfrac{1}{\sqrt{2}}(\mathbf{a}_y + \mathbf{a}_z)$

Let $\mathbf{E}_2 = E_{2x}\mathbf{a}_x + E_{2y}\mathbf{a}_y + E_{2z}\mathbf{a}_z$

Then from $\mathbf{a}_n \times (\mathbf{E}_2 - \mathbf{E}_1) = \mathbf{0}$, we have

$$\begin{vmatrix} \mathbf{a}_x & \mathbf{a}_y & \mathbf{a}_z \\ 0 & 1 & 1 \\ E_{2x} & E_{2y} & E_{2z} - E_0 \end{vmatrix} = \mathbf{0}$$

or,

$$E_{2z} - E_0 - E_{2y} = 0 \qquad (1)$$

$$E_{2x} = 0 \qquad (2)$$

From $\mathbf{a}_n \bullet (\mathbf{D}_2 - \mathbf{D}_1) = 0$, we have

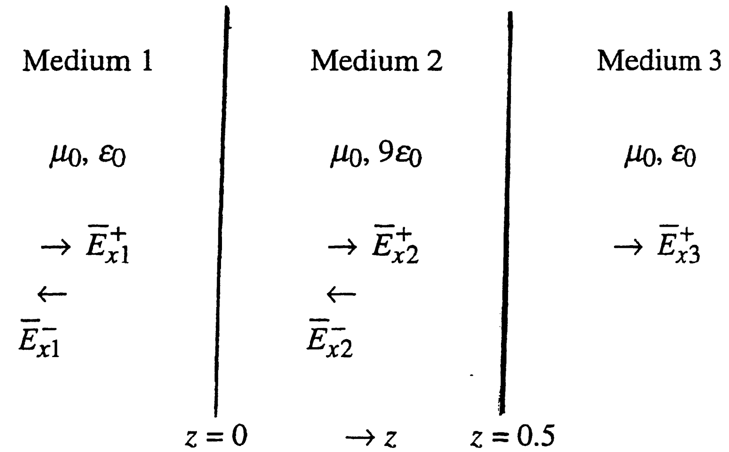
$(\mathbf{a}_y + \mathbf{a}_z) \bullet [\varepsilon_0 E_{2x}\mathbf{a}_x + \varepsilon_0 E_{2y}\mathbf{a}_y + (\varepsilon_0 E_{2z} - 2\varepsilon_0 E_0)\mathbf{a}_z] = 0$

or, $E_{2y} + E_{2z} - 2E_0 = 0$ $\qquad (3)$

Solving (1) and (3), we obtain $E_{2y} = \frac{1}{2}E_0$, $E_{2z} = \frac{3}{2}E_0$

Thus, $\mathbf{E}_2 = \dfrac{E_0}{2}(\mathbf{a}_y + 3\mathbf{a}_z)$

R4.8.



Let $\beta_1 = \omega\sqrt{\mu_0\varepsilon_0} = \frac{\omega}{c}$ be the phase constant in medium 1. Then $\beta_2 = \omega\sqrt{\mu_0 \cdot 9\varepsilon_0} = 3\beta_1$ and $\beta_3 = \beta_1$. Let $\eta_1 = \sqrt{\frac{\mu_0}{\varepsilon_0}} = \eta_0$ be the intrinsic impedance of medium 1. Then $\eta_2 = \sqrt{\frac{\mu_0}{9\varepsilon_0}} = \frac{\eta_1}{3}$ and $\eta_3 = \eta_1$. We can thus write the following equations for the total fields in the three media:

$$\bar{E}_{x1} = \bar{E}_{x1}^{+} e^{-j\beta_1 z} + \bar{E}_{x1}^{-} e^{j\beta_1 z}$$

$$\bar{E}_{x2} = \bar{E}_{x2}^{+} e^{-j3\beta_1 z} + \bar{E}_{x2}^{-} e^{j3\beta_1 z}$$

$$\bar{E}_{x3} = \bar{E}_{x3}^{+} e^{-j\beta_1 z}$$

$$\bar{H}_{y1} = \frac{1}{\eta_1}\left(\bar{E}_{x1}^{+} e^{-j\beta_1 z} - \bar{E}_{x1}^{-} e^{j\beta_1 z}\right)$$

$$\bar{H}_{y2} = \frac{3}{\eta_1}\left(\bar{E}_{x2}^{+} e^{-j3\beta_1 z} - \bar{E}_{x2}^{-} e^{j3\beta_1 z}\right)$$

$$\bar{H}_{y3} = \frac{1}{\eta_1}\bar{E}_{x3}^{+} e^{-j\beta_1 z}$$

Matching the boundary conditions at $z = 0$ and $z = 0.5$, we have

$$\bar{E}_{x1}^{+} + \bar{E}_{x1}^{-} = \bar{E}_{x2}^{+} + \bar{E}_{x2}^{-} \qquad (1)$$

$$\frac{1}{\eta_1}\left(\bar{E}_{x1}^{+} - \bar{E}_{x1}^{-}\right) = \frac{3}{\eta_1}\left(\bar{E}_{x2}^{+} - \bar{E}_{x2}^{-}\right) \qquad (2)$$

R4.8. (continued)

$$\bar{E}_{x2}^{+}e^{-j1.5\beta_1} + \bar{E}_{x2}^{-}e^{j1.5\beta_1} = \bar{E}_{x3}^{+}e^{-j0.5\beta_1} \tag{3}$$

$$\frac{3}{\eta_1}\left(\bar{E}_{x2}^{+}e^{-j1.5\beta_1} - \bar{E}_{x2}^{-}e^{j1.5\beta_1}\right) = \frac{1}{\eta_1}\bar{E}_{x3}^{+}e^{-j0.5\beta_1} \tag{4}$$

From (3) and (4), we get

$$\bar{E}_{x2}^{+}e^{-j1.5\beta_1} + \bar{E}_{x2}^{-}e^{j1.5\beta_1} = 3\left(\bar{E}_{x2}^{+}e^{-j1.5\beta_1} - \bar{E}_{x2}^{-}e^{j1.5\beta_1}\right)$$

$$4\bar{E}_{x2}^{-}e^{j1.5\beta_1} = 2\bar{E}_{x2}^{+}e^{-j1.5\beta_1}$$

$$\bar{E}_{x2}^{-} = 0.5\bar{E}_{x2}^{+}e^{-j3\beta_1}$$

Substituting in (1) and (2), we have

$$\bar{E}_{x1}^{+} + \bar{E}_{x1}^{-} = \bar{E}_{x2}^{+}\left(1 + 0.5e^{-j3\beta_1}\right)$$

$$\bar{E}_{x1}^{+} - \bar{E}_{x1}^{-} = 3\bar{E}_{x2}^{+}\left(1 - 0.5e^{-j3\beta_1}\right)$$

For no reflection at the interface $z = 0$, $\bar{E}_{x1}^{-}$ must be zero. Then

$$1 + 0.5e^{-j3\beta_1} = 3(1 - 0.5e^{-j3\beta_1})$$

$$2e^{-j3\beta_1} = 2$$

$$e^{-j3\beta_1} = 1$$

Ignoring $\beta_1 = 0$, which corresponds to zero frequency, we obtain

$$3\beta_1 = 2\pi, 4\pi, 6\pi, \ldots$$

or, $\beta_2 = 2\pi, 4\pi, 6\pi, \ldots$

$$\frac{6\pi f}{c} = 2\pi, 4\pi, 6\pi, \ldots$$

R4.8. (continued)

$$f = \frac{c}{3}, \frac{2c}{3}, c, \ldots$$

$$= 10^8, 2 \times 10^8, 3 \times 10^8, \ldots$$

Thus the required minimum frequency is 10^8 Hz. Proceeding further, for a nonsinusoidal periodic wave to undergo no reflection at $z = 0$, its Fourier components must be $10^8, 2 \times 10^8, 3 \times 10^8, \ldots$ Hz. Therefore the required maximum value of its period must be 1/fundamental component, or $1/10^8 = 10^{-8}$ s.