Expectation - Variance

1. A coin is biased such that a head is three times as likely to occur as a tail. Find the expected number of tails when this coin is tossed twice.

2. By investing in a particular stock, a person can make a profit in one year of $4000 with probability 0.3 or take a loss of $1000 with probability 0.7. What is this person's expected gain?

3. A large industrial firm purchases several new word processors at the end of each year, the exact number depending on the frequency of repairs in the previous year. Suppose that the number of word processors, X, purchased each year has the following probability distribution:

x	0	1	2	3
$p(x)$	1/10	3/10	2/5	1/5

   If the cost of the desired model is $1200 per unit and at the end of the year a refund of $50X^2$ dollars will be issued, how much can this firm expect to spend on new word processors during this year?

4. Nick has invested $1000 in each of two products. He expects that each successful investment will yield a profit of $800, while if an investment is unsuccessful he will lose his money. A successful outcome in each investment is independent of the other investment being successful. If he estimates that the probability of success in each of the two investments is 0.6, find the probability mass function, the expected value of Nick's profit.

5. If a dealer's profit, in units of $5000, on a new automobile can be looked upon as a random variable X having the density function

$$f(x) = \begin{cases} 2(1-x), & 0 < x < 1 \\ 0 & elsewhere \end{cases}$$

   find the average profit per automobile and $Var(X)$.

6. The repair time, in hours, for a certain type of laptop is a continuous variable with density function

$$f(x) = \begin{cases} 0.5, & 0 < x < 2 \\ 0 & elsewhere \end{cases}$$

   (a) What is the expected time to repair a laptop of this type when it breaks down?

   (b) If the repair cost depends on the time that the repair takes and, when this time is x hours, the associated cost in dollars is estimated to be $30 + 5x$, find the expected cost for each repair.

7. Suppose that a lottery ticket has probability $p = 0.0001$ of being a winning ticket, independently of other tickets. A gambler buys 4 tickets, hoping this will increase the chance of having at least one winning ticket. Let X be the number of wining tickets among 4 tickets that he or she purchases. Evaluate $E(X)$ and $Var(X)$.

8. The probability density function of the time to failure an electronic device (in hours) is

$$f(x) = \begin{cases} \frac{1}{2000}e^{-\frac{x}{2000}} & \text{if } x > 0 \\ 0 & \text{otherwise} \end{cases}.$$

   (a) Compute mean and variance for the lifetime of the device.

   (b) There is a test after 3000 hours of using. The fee of test is $20. If the device has failed then we need to replace it by a new one with a cost of $100. Otherwise, we continue to use the device without replacement. Evaluate the average of total cost for the test.