SS 2018/04

Q1.

a)

Given that:

$$x(t) = \begin{cases} 4, & 0 \le t \le 6 \\ -2, & -4 \le t < 0 \\ 0, & otheWise \end{cases}$$

$$E_x = \int_{-\infty}^{+\infty} |x(t)|^2 dt = \int_0^6 |4|^2 dt + \int_{-4}^{0} |-2|^2 dt = 112$$

We have: $x(t) = 4\big(u(t) - u(t-6)\big) + (-2)\big(u(t+4) - u(t)\big)$

$= -2u(t+4) + 6u(t) - 4u(t-6)$

(Reader sketches the signal by yourself)

b)

Given that: $x[n] = 0.5^n u[n]$

$$E_x = \sum_{n=-\infty}^{+\infty} |x[n]|^2 = \sum_{n=0}^{+\infty} |0.5^n|^2 = \sum_{n=0}^{+\infty} 0.25^n = \frac{4}{3}$$

Q2.

a) Given that: $y(t) = x(2t-1) + x(t)$

1. Check for linearity:

Let: $\begin{cases} x_1 \xrightarrow{s} y_1 = x_1(2t-1) + x_1(t) \\ x_2 \xrightarrow{s} y_2 = x_2(2t-1) + x_2(t) \end{cases}$

$\rightarrow a_1y_1 + a_2y_2 = a_1\big(x_1(2t-1) + x_1(t)\big) + a_2\big(x_2(2t-1) + x_2(t)\big)$ (1)

Let: $x = a_1x_1 + a_2x_2 \xrightarrow{s} y = a_1x_1(2t-1) + a_2x_2(2t-1) + a_1x_1(t) + a_2x_2(t)$ (2)

From (1) and (2), $a_1y_1 + a_2y_2 = \boldsymbol{S}\{a_1x_1 + a_2x_2\}$, the system is linear.

2. Check for time invariant:

Let: $x(t) \xrightarrow{s} y = x(2t-1) + x(t)$

$\rightarrow y(t-T) = x(2t-2T-1) + x(t-T)$ (1) (delay the ouput).

Let: $x_T(t) = x(t-T) \xrightarrow{s} y_T = x_T(2t-1) + x_T(t) = x(2t-T-1) + x(t-T)$ (2)

Since, $(1) \ne (2)$, therefore, the system is time variant.

b) Given that: $y(t) = 3x(t-1) + 2$

1. Check for linearity:

Let: $\begin{cases} x_1 \xrightarrow{s} y_1 = 3x_1(t-1) + 2 \\ x_2 \xrightarrow{s} y_2 = 3x_2(t-1) + 2 \end{cases}$

$\rightarrow a_1y_1 + a_2y_2 = a_1(3x_1(t-1) + 2) + a_2(3x_2(t-1) + 2)$ (1)

Let: $x = a_1x_1 + a_2x_2 \xrightarrow{s} y = 3\big(a_1x_1(t-1) + a_2x_2(t-1)\big) + 2$ (2)

From (1) and (2), $a_1y_1 + a_2y_2 \ne \boldsymbol{S}\{a_1x_1 + a_2x_2\}$, the system is nonlinear.

2. Check for time invariant:

Let: $x(t) \xrightarrow{s} y = 3x(t-1) + 2$

$\rightarrow y(t-T) = 3x(t-T-1) + 2$ (1) (delay the ouput).

Let: $x_T(t) = x(t-T) \xrightarrow{s} y_T = 3x_T(t-1) + 2 = 3x(t-T-1) + 2$ (2)

Since, $(1) = (2)$, therefore, the system is time invariant.

Q3.

a) Given that: $y[n] = x[2n]$

We have:

$$+\ y[0] = x[0] = 4$$
$$+\ y[1] = x[2] = 3$$
$$+\ y[2] = x[4] = 3$$
$$+\ y[3] = x[6] = 5$$

Therefore, $y[n] = [4, 3, 3, 5]$

Let: $x[n] \xrightarrow{S} y[n] = x[2n]$

$\rightarrow y[n-N] = x[2n-2N]$ (1) (delay the ouput).

Let: $x_N[n] = x[n-N] \xrightarrow{S} y_N = x_N[2n] = x[2n-N]$ (2)

Since, (1) $\neq$ (2), therefore, the system is time variant.

b) Given that: $h[n] = [3, 0, \ -2]$

$$y[n] = x[n] * h[n] = \sum_{k=-\infty}^{+\infty} x[n-k]h[k] = \sum_{k=0}^{2} x[n-k]h[k]$$
$$= x[n]h[0] + x[n-1]h[1] + x[n-2]h[2]$$
$$= 3x[n] - 2x[n-2]$$

c) Given that: $x[n] = [1, 2, 0, -1, -2]$

$$+\ y[0] = 3x[0] - 2x[-2] = 3 \times 1 - 2 \times 0 = 3$$
$$+\ y[1] = 3x[1] - 2x[-1] = 3 \times 2 - 2 \times 0 = 6$$
$$+\ y[2] = 3x[2] - 2x[0] = 3 \times 0 - 2 \times 1 = -2$$
$$+\ y[3] = 3x[3] - 2x[1] = 3 \times (-1) - 2 \times 2 = -7$$
$$+\ y[4] = 3x[4] - 2x[2] = 3 \times (-2) - 2 \times 0 = -6$$
$$+\ y[5] = 3x[5] - 2x[3] = 3 \times 0 - 2 \times (-1) = 2$$
$$+\ y[6] = 3x[6] - 2x[4] = 3 \times 0 - 2 \times (-2) = 4$$

Therefore, $y[n] = [3, 6, -2, -7, -6, 2, 4]$

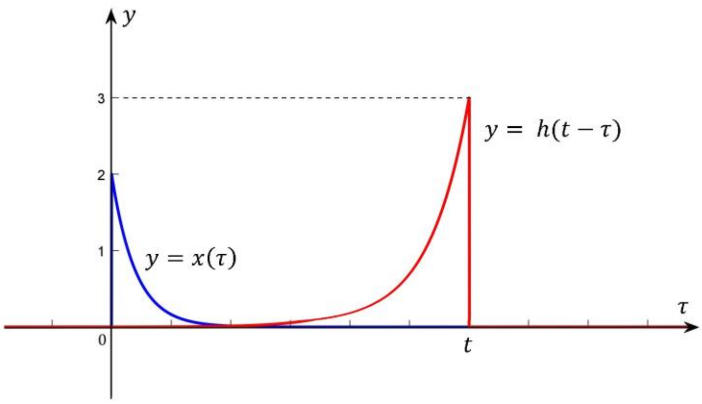
Q4.

Given that: $h(t) = 3e^{-2t}u(t)$

a)

For input $x(t) = 2e^{-4t}u(t)$

We first sketch the graph of $x(\tau)$ and $h(t-\tau)$, for any value of $t > 0$.



For $t < 0$, $x(\tau)$ and $h(t-\tau)$ does not overlap, $x(\tau)h(t-\tau) = 0$ that leads to $y(t) = 0$.

For $t \geq 0$, the output becomes:

$$y(t) = \int_{-\infty}^{+\infty} x(\tau)h(t-\tau)d\tau = \int_{0}^{t} 2e^{-4\tau}.3e^{-2(t-\tau)}d\tau$$

$$= 6e^{-2t}\int_{0}^{t} e^{-2\tau}d\tau = 10e^{-3t}\left(\frac{e^{-2\tau}}{-2}\right)\Bigg|_{0}^{t}$$

$$= 6e^{-2t}\left(\frac{e^{-2t}-1}{-2}\right) = 3e^{-2t}(1-e^{-2t})$$

Thus,

$$y(t) = \begin{cases} 0 & , \quad t < 0 \\ 3e^{-2t}(1-e^{-2t}), & t \geq 0 \end{cases}$$

b)

For $x_1(t) = 2e^{-4t}u(t) \rightarrow y_1(t) = \begin{cases} 0 & , \quad t < 0 \\ 3e^{-2t}(1-e^{-2t}), & t \geq 0 \end{cases}$

We have:

$$x(t) = e^{-4t}[u(t) - u(t-2)] = e^{-4t}u(t) - e^{-4(t-2)-8}u(t-2)$$

$$= \frac{1}{2}x_1(t) - e^{-8}x_1(t-2)$$

Due to the properties of LTI system, the output is given by:

$$y(t) = \frac{1}{2}y_1(t) - e^{-8}y_1(t-2)$$

$$= \begin{cases} 0 & , \quad t < 0 \\ \frac{3}{2}e^{-2t}(1-e^{-2t}) & , 0 \leq t < 2 \\ \frac{3}{2}e^{-2t}(1-e^{-2t}) - 3e^{-8}e^{-2(t-2)}\left(1-e^{-2(t-2)}\right), & 2 \leq t \end{cases}$$

$$= \begin{cases} 0 & , \quad t < 0 \\ \frac{3}{2}e^{-2t}(1-e^{-2t}) & , 0 \leq t < 2 \\ \frac{3}{2}e^{-2t}(1+e^{-2t}-2e^{-4}), & 2 \leq t \end{cases}$$