

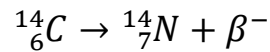
SOLUTION FOR PHYSICS 4

(Final)

January – 2011

2)

a) The nuclear reaction process:



b) The mass effect:

$$\Delta m = M_C - M_N = 14.003241 - 14.003074 = 1.97 \times 10^{-4} (u)$$

The energy released by this reaction:

$$Q = \Delta mc^2 = 1.97 \times 10^{-4} \times 931.5 = 0.183 (MeV)$$

3)

The distance measured on Earth: $L_0 = 4.2 \times 10^6 (m)$

The distance measured by the voyagers aboard the UFO:

$$L = L_0 \sqrt{1 - \frac{u^2}{c^2}} = 4.2 \times 10^6 \sqrt{1 - 0.7^2} \approx 3 \times 10^6 (m)$$

4)

For the Balmer series:

$$\frac{1}{\lambda} = R \left(\frac{1}{n^2} - \frac{1}{m^2} \right) = R \left(\frac{1}{2^2} - \frac{1}{m^2} \right)$$
$$\rightarrow \lambda = \frac{1}{R \left(\frac{1}{2^2} - \frac{1}{m^2} \right)}$$

We have: For visible light, the wavelength is:

$$380 \text{ nm} \leq \lambda \leq 760 \text{ nm}$$

$$\Leftrightarrow 380 \times 10^{-9} \leq \frac{1}{R \left(\frac{1}{2^2} - \frac{1}{m^2} \right)} \leq 760 \times 10^{-9} \Leftrightarrow 0.12 \leq \frac{1}{2^2} - \frac{1}{m^2} \leq 0.23$$

$$\Leftrightarrow 0.02 \leq \frac{1}{m^2} \leq 0.136 \quad 2.77 \leq m \leq 7.07 \quad m = \{3, 4, 5, 6, 7\}$$

Conclusion: All the lines which lie in the visible spectrum belong to Balmer series

5)

a) We have: The boundary condition:

$$\begin{cases} V = 0 \text{ for } -L \leq x \leq L \\ V = \infty \text{ otherwise} \end{cases}$$

$$\text{At } x = -L: \psi(-L) = 0 \Rightarrow \psi(-L) = A \sin(-kL) + B \cos(-kL) = -A \sin(kL) + B \cos(kL) = 0$$

$$\Rightarrow B \cos(kL) = A \sin(kL) \quad (1)$$

$$\text{At } x = L: \psi(L) = 0 \Rightarrow \psi(L) = A \sin(kL) + B \cos(kL) = 0$$

$$\Rightarrow B \cos(kL) = -A \sin(kL) \quad (2)$$

From (1) and (2): $A \sin(kL) = -A \sin(kL)$. This only makes sense if $A = 0$

Therefore: $\psi(x) = B \cos(kx)$

$$\text{At } x = L: \psi(L) = B \cos(kL) = 0 \rightarrow kL = \frac{\pi}{2} + n\pi \rightarrow k = \frac{1}{L} \left(\frac{\pi}{2} + n\pi \right)$$

b) The wave function:

$$\psi(x) = B \cos(kx)$$

The normalization condition:

$$P = \int_{-\infty}^{+\infty} |\psi|^2 dx = \int_{-L}^L |\psi|^2 dx = \int_{-L}^L (B \cos(kx))^2 dx = 1$$

$$\text{Since: } \sin^2 x = \frac{1 - \cos 2x}{2}; \cos^2 x = \frac{1 + \cos 2x}{2}; \sin(2x) = 2 \sin x \cos x$$

$$\Rightarrow P = B^2 \int_{-L}^L \frac{1 + \cos(2kx)}{2} dx = \frac{B^2}{2} \int_{-L}^L (1 + \cos(2kx)) dx = 1$$

$$= \frac{B^2}{2} \left(L + \frac{1}{2k} \sin(2kL) + L - \frac{1}{2k} \sin(2kL) \right) = B^2 L = 1 \rightarrow B = \sqrt{\frac{1}{L}}$$

Conclusion: The wave function of a particle in an infinite deep well one-dimensional well

$$\psi(x) = \sqrt{\frac{1}{L}} \cos(kx) = \sqrt{\frac{1}{L}} \cos\left(\frac{x\pi}{2L} (2n+1)\right)$$

c) For the value of k: $k = \frac{1}{L} \left(\frac{\pi}{2} + n\pi \right) = \frac{\pi}{2L} (2n+1)$

Since k is the wave number:

$$k = \frac{2\pi}{\lambda} \rightarrow \lambda = \frac{2\pi}{k} = \frac{4\pi L}{\pi(2n+1)} = \frac{4L}{2n+1}$$

The momentum of the particle:

$$p = \frac{h}{\lambda} = \frac{h(2n+1)}{4L}$$

The energy of the particle:

$$E = \frac{p^2}{2m} = \frac{h^2(2n+1)^2}{2m(4L)^2} = \frac{h^2(2n+1)^2}{32mL^2} = \frac{(2\pi\hbar)^2(2n+1)^2}{32mL^2} = \frac{\hbar^2\pi^2(2n+1)^2}{8mL^2}$$

Conclusion: The energy of the particle in state nth:

$$E_n = \frac{\hbar^2\pi^2n^2}{8mL^2} \quad (n = 1, 3, 5, 7, \dots)$$

June – 2012

1)

- a) We have totally 10 different lines can be emitted by an hydrogen atoms:
 $(2 \rightarrow 1; 3 \rightarrow 1; 4 \rightarrow 1; 5 \rightarrow 1; 3 \rightarrow 2; 4 \rightarrow 2; 5 \rightarrow 2; 4 \rightarrow 3; 5 \rightarrow 3; 5 \rightarrow 4)$

Or we can calculate by using the formula:

$$N = \frac{n(n-1)}{2} = \frac{5(5-1)}{2} = 10$$

There are 3 spectral lines are found in the visible spectrum (in Balmer series): $3 \rightarrow 2$; $4 \rightarrow 2$; $5 \rightarrow 2$

b) We have:

The energy state of excitation energy 10.19 eV: $E_f = -13.6 + 10.19 = -3.41 \text{ (eV)}$

$$\Delta E = hf = \frac{hc}{\lambda} = \frac{hc}{4890 \times 10^{-10}} = 2.54 \text{ (eV)}$$

$$\Delta E = E_i - E_f = E_i - (-3.41) = 2.54 \rightarrow E_i = -0.87 \text{ (eV)}$$

The principle quantum number of these state:

$$E_i = \frac{-13.6}{n_i^2} = -0.87 \rightarrow n_i = 4; E_f = \frac{-13.6}{n_f^2} = -3.41 \Rightarrow n_f = 2$$

2)

In Newtonian mechanics: $K_N = \frac{1}{2}m_0u^2$

In special relativity:

$$E = m_0c^2 + K_S \rightarrow K_S = E - m_0c^2 = \gamma m_0c^2 - m_0c^2 = (\gamma - 1)m_0c^2$$

$$\text{With: } \gamma = \frac{1}{\sqrt{1 - \frac{u^2}{c^2}}}$$

Since the relativistic kinetic energy is 50% greater than the Newtonian value:

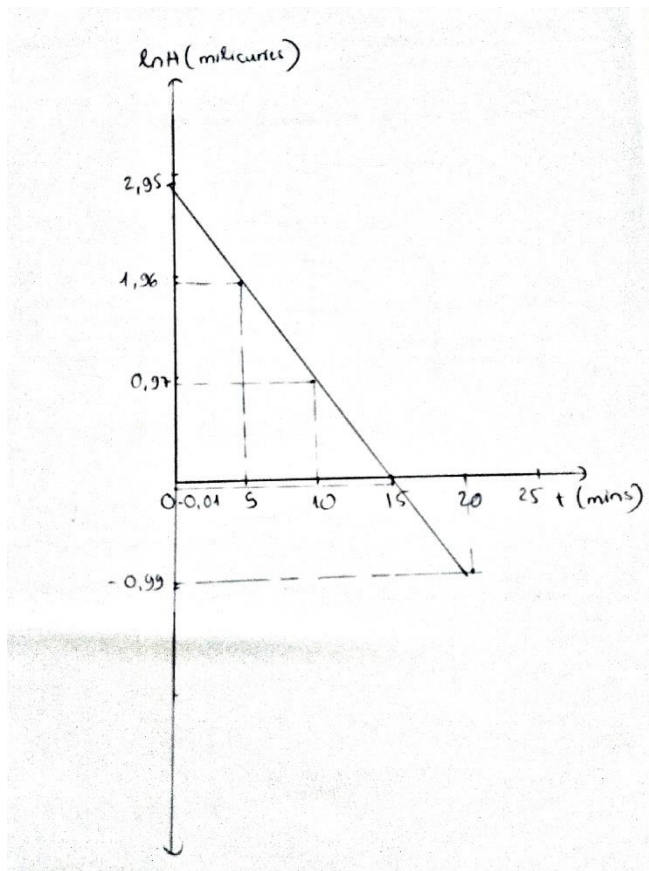
$$\begin{aligned} K_S &= K_N + 50\%K_N = 1.5K_N \Leftrightarrow (\gamma - 1)m_0c^2 \\ &= 1.5 \times \frac{1}{2}m_0u^2 \Leftrightarrow \left(\frac{1}{\sqrt{1 - \frac{u^2}{c^2}}} - 1 \right) c^2 = \frac{3}{4}u^2 \end{aligned}$$

$$\text{Let } \beta = \frac{u}{c}$$

Therefore, the equation becomes:

$$\left(\frac{1}{\sqrt{1 - \beta^2}} - 1 \right) = \frac{3}{4}\beta^2 \Rightarrow \beta = 0.652 \text{ } \delta u = 0.652c$$

3)



We have: The activity of the sample

$$H = \lambda N(t) = \lambda N_0 e^{-\lambda t}$$

$$\rightarrow \ln H = \ln(\lambda N_0 e^{-\lambda t}) = \ln \lambda N_0 + \ln e^{-\lambda t} = \ln \lambda N_0 - \lambda t = a - bt$$

Based on the variation of $\ln H$, we have:

The slope of the plot of $\ln H$: $b = \lambda$

Therefore:

$$\lambda = \frac{(0.97 - 1.96) \times 3.7 \times 10^{10} \times 10^{-3}}{5 \times 60} = -122100 \text{ (s}^{-1}\text{)}$$

The half-life of a sample $^{55}_{24}\text{Cr}$:

$$T_{\text{half}} = \frac{\ln 2}{\lambda} = \frac{\ln 2}{122100} \approx 5.67 (\mu\text{s})$$

4)

Conservation of energy:

$$E_U + E_\gamma = E_{Kr} + E_{Ba} + 3E_n$$

$$\Leftrightarrow K_U + M_U c^2 + K_\gamma = K_{Kr} + M_{Kr} c^2 + K_{Ba} + M_{Ba} c^2 + 3(K_n + M_n c^2)$$

Since $K_U = 0$; $K_\gamma = 6 \text{ MeV}$

$$\Leftrightarrow \sum K_f = M_U c^2 + K_\gamma - M_{Kr} c^2 - M_{Ba} c^2 - 3M_n c^2$$

$$\Leftrightarrow \sum K_f = (M_U - M_{Kr} - M_{Ba} - 3M_n) c^2 + K_\gamma$$

$$\Leftrightarrow \sum K_f = (235.043915 - 89.91972 - 141.91635 - 3 \times 1.008665) \times 931.5 + 6 = 175.44 \text{ (MeV)}$$

5)

The wave function of a particle in a square well: (At the ground level: $n=1$)

$$\psi(x) = \sqrt{\frac{2}{L}} \sin\left(\frac{n\pi x}{L}\right) = \sqrt{\frac{2}{L}} \sin\left(\frac{\pi x}{L}\right)$$

a) The probability of finding particle in the region between 0 and $L/4$

$$\begin{aligned} P &= \int_0^{L/4} |\psi^2| dx \\ &= \frac{2}{L} \int_0^{L/4} \sin^2\left(\frac{\pi x}{L}\right) dx = \frac{2}{L} \int_0^{L/4} \frac{1 - \cos\left(\frac{2\pi x}{L}\right)}{2} dx \\ &= \frac{1}{L} \int_0^{L/4} 1 - \cos\left(\frac{2\pi x}{L}\right) dx \\ &= \frac{1}{L} \left(\frac{L}{4} - \frac{L}{2\pi} \sin\left(\frac{2\pi L}{4L}\right) \right) = \frac{1}{4} - \frac{1}{2\pi} \sin\left(\frac{\pi}{2}\right) \approx 9.1\% \end{aligned}$$

b) The probability of finding particle in the region between $L/4$ and $L/2$

$$\begin{aligned}
P &= \int_{L/4}^{L/2} |\psi|^2 dx \\
&= \frac{2}{L} \int_{L/4}^{L/2} \sin^2\left(\frac{\pi x}{L}\right) dx = \frac{2}{L} \int_{L/4}^{L/2} \frac{1 - \cos\left(\frac{2\pi x}{L}\right)}{2} dx \\
&= \frac{1}{L} \int_{L/4}^{L/2} 1 - \cos\left(\frac{2\pi x}{L}\right) dx \\
&= \frac{1}{L} \left(\frac{L}{2} - \frac{L}{2\pi} \sin(\pi) - \frac{L}{4} + \frac{L}{2\pi} \sin\left(\frac{\pi}{2}\right) \right) = \frac{1}{4} + \frac{1}{2\pi} \left(\sin\left(\frac{\pi}{2}\right) - \sin(\pi) \right) \approx 40.9\%
\end{aligned}$$

c) From the results of part a/ and part b/ we have:

$P_a < P_b$ (9.1% < 40.9%) \Rightarrow Particle is mostly found in the region between $L/4$ and $L/2$

d) The sum of two results calculated in part a/ and b/ (The probability of finding particle in the region between 0 and $L/2$)

$P = P_a + P_b \approx 50\%$ \Rightarrow The probability of finding particle in first-half of the width of the well is 50%

June – 2014

1)

Let the length of cube sides in a frame S is a_0

The volume of a metal cube: $V_0 = a_0^3$

The length measured by an observer in frame S' (move along x-axis)

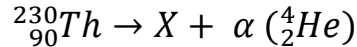
$$a = a_0 \sqrt{1 - \frac{u^2}{c^2}} = \frac{a_0}{\gamma}$$

The volume of a metal cube in frame S'

$$V = a \times a_0^2 = \frac{a_0}{\gamma} \times a_0^2 = \frac{a_0^3}{\gamma} = V_0 \sqrt{1 - \frac{u^2}{c^2}}$$

3)

a) The nuclear reaction process:



Conservation of charge:

$$Z_{\text{Th}} = Z_X + Z_\alpha \Leftrightarrow 90 = Z_X + 2 \Rightarrow Z_X = 88 \text{ (protons)}$$

Conservation of nucleon number A:

$$A_{\text{Th}} = A_X + A_\alpha \Leftrightarrow 230 = A_X + 4 \rightarrow A_X = 226$$

The number of neutrons for the nuclide X:

$$N_X = A_X - Z_X = 226 - 88 = 138 \text{ (neutrons)}$$

b) Conservation of energy:

$$E_{\text{Th}} = E_X + E_\alpha$$

$$\begin{aligned} \Leftrightarrow K_{\text{Th}} + M_{\text{Th}}c^2 &= K_X + M_Xc^2 + K_\alpha + M_\alpha c^2 \Leftrightarrow K_X + K_\alpha - K_{\text{Th}} \\ &= (M_{\text{Th}} - M_X - M_\alpha)c^2 \end{aligned}$$

The thorium is at rest: $K_{\text{Th}} = 0$

The recoil effect of the daughter nucleus: $K_X = K_\alpha$

\Rightarrow The kinetic energy of the emitted α particle:

$$\begin{aligned} 2K_\alpha &= (M_{\text{Th}} - M_X - M_\alpha)c^2 \\ &= (230.033127 - 226.025403 - 4.002603) \times 931.5 \rightarrow K_\alpha \\ &= 2.385 \text{ (MeV)} \end{aligned}$$

5)

a) We have: The photon energy arising from the transition between two energy levels (from level E_n to level E_m)

$$\begin{aligned} \Delta E = hf = \frac{hc}{\lambda} &= -13.6 \left(\frac{1}{m^2} - \frac{1}{n^2} \right) \text{ (eV)} \Leftrightarrow \frac{1}{\lambda} = \frac{-13.6 \times 1.6 \times 10^{-19}}{hc} \left(\frac{1}{m^2} - \frac{1}{n^2} \right) \\ \Leftrightarrow \frac{1}{\lambda} &= 1.094 \times 10^{-7} \left(\frac{1}{n^2} - \frac{1}{m^2} \right) \Rightarrow R = 1.094 \times 10^7 \text{ (m}^{-1}\text{)} \end{aligned}$$

b) The shortest wavelength in the Lyman series: From $n = \infty$ to $n = 1$

$$\Leftrightarrow \frac{1}{\lambda} = 1.094 \times 10^7 \left(\frac{1}{1^2} - \frac{1}{\infty^2} \right) = \textcolor{red}{+} 1.094 \times 10^7 \Leftrightarrow \lambda = 91.4 \text{ (nm)} \rightarrow \text{Ultraviolet}$$

The longest wavelength in the Paschen series: From $n = 4$ to $n = 3$

$$\Leftrightarrow \frac{1}{\lambda} = 1.094 \times 10^7 \left(\frac{1}{3^2} - \frac{1}{4^2} \right) \Leftrightarrow \lambda = 1880 \text{ (nm)} \rightarrow \text{Infrared}$$

June – 2015

1)

a) The first line of Balmer series: $\lambda = 658 \text{ nm}$

$$\Delta E = hf = \frac{hc}{\lambda} = \frac{hc}{658 \times 10^{-9}} = 1.88 \text{ (eV)}$$

The potential difference for the electron to accelerate:

$$e\Delta V = \Delta E \rightarrow \Delta V = \frac{\Delta E}{e} = 1.88 \text{ (V)}$$

b) We have: H atoms are excited by UV radiation with $\lambda = 100 \text{ nm}$

$$\Delta E = \frac{hc}{\lambda} = \frac{hc}{100 \times 10^{-9}} = 12.42 \text{ (eV)}$$

$$\Delta E = E_i - E_f = -13.6 \left(\frac{1}{n_i^2} - \frac{1}{n_f^2} \right) = -13.6 \left(\frac{1}{n^2} - 1 \right) = 12.42 \Leftrightarrow n = 3$$

Conclusion: That is Lyman spectral line from $n = 3$ to $n = 1$

2)

a) Conservation of charge:

$$Z_{Si} = Z_X + Z_{Mg} \Leftrightarrow 14 = Z_X + 12 \rightarrow Z_X = 2$$

Conservation of nucleon number A:

$$A_{Si} = A_X + A_{Mg} \Leftrightarrow 28 = A_X + 24 \rightarrow A_X = 4$$

The number of neutrons for the nuclide X:

$$N_X = A_X - Z_X = 4 - 2 = 2 \text{ (neutrons)}$$

b) The energy released in this reaction

$$Q = (M_{Si} - M_X - M_{Mg})c^2 = (27.976927 - 4.002603 - 23.985042) \times 931.5 \\ = -9.98 \text{ (MeV)}$$

This energy is transferred into gamma radiation

$$Q = hf = \frac{hc}{\lambda} = 9.98 \text{ (MeV)} \Rightarrow \lambda = 0.124 \text{ (pm)}$$

3)

a) The width of the aperture: $\Delta y = d = 0.5 \text{ mm}$

According to the Heisenberg's uncertainty principle:

$$\Delta p_y \Delta y \geq \hbar \rightarrow \Delta p_y \geq \frac{\hbar}{\Delta y} = \frac{\hbar}{0.5 \times 10^{-3}} = 2.11 \times 10^{-31} \text{ (kg} \cdot \frac{m}{s})$$

The uncertainty in the component of the electron's velocity:

$$\Delta p_y = m \Delta v_y \rightarrow \Delta v_y = \frac{\Delta p_y}{m} = \frac{2.11 \times 10^{-31}}{9.11 \times 10^{-31}} = 0.231 \text{ (} \frac{m}{s} \text{)}$$

b) There is the uncertainty in momentum of the electron \Rightarrow Affecting on its direction when it strikes the screen \Rightarrow Electron diffraction

$$\text{We have: } K = \frac{1}{2}mv^2 = \frac{p^2}{2m} = eV$$

$$\rightarrow p = \sqrt{2meV} = \sqrt{2 \times 9.11 \times 10^{-31} \times 1.6 \times 10^{-19} \times 15 \times 10^3} \\ = 6.612 \times 10^{-23} \text{ (kg} \cdot \frac{m}{s})$$

The De Broglie's wavelength of the electron:

$$\lambda = \frac{h}{p} = \frac{6.62 \times 10^{-23}}{6.612 \times 10^{-23}} = 10.02 \text{ (pm)}$$

The first minima (dark fringe) in electron diffraction:

$$d \sin \theta = \lambda \rightarrow \sin \theta = \frac{\lambda}{d} = \frac{10.2 \times 10^{-12}}{0.5 \times 10^{-3}} = 2.04 \times 10^{-8}$$

We have:

$$\sin \theta \sim \tan \theta = \frac{\Delta p_y}{\Delta p_x} = \frac{\Delta y_{screen}}{L} = 2.04 \times 10^{-8} \rightarrow \Delta y_{screen} = 6.12 \text{ (nm)}$$

Conclusion: The position of the electron on the screen will spread over all direction.

4)

a) We have: The normalization condition:

$$\begin{aligned}
 P &= \int_{-\infty}^{+\infty} \psi^2(x) dx = \int_{-\infty}^0 \psi^2(x) dx + \int_0^{+\infty} \psi^2(x) dx = 1 \\
 \Leftrightarrow \int_{-\infty}^0 A^2 e^{2bx} dx + \int_0^{+\infty} A^2 e^{-2bx} dx &= \frac{A^2}{2b} (e^0 - e^{-2b\infty}) - \frac{A^2}{2b} (e^{-2b\infty} - e^0) = 1 \\
 \Leftrightarrow \frac{A^2}{2b} (1 - 0) - \frac{A^2}{2b} (0 - 1) &= \frac{2A^2}{2b} = 1 \Leftrightarrow A = \sqrt{2}
 \end{aligned}$$

b) The probability of finding this particle within 50cm of the origin:

$$\begin{aligned}
 P &= \int_{-0.25}^{0.25} \psi^2(x) dx \\
 &= \int_{-0.25}^0 \psi^2(x) dx + \int_0^{0.25} \psi^2(x) dx = \int_{-0.25}^0 A^2 e^{2bx} dx + \int_0^{0.25} A^2 e^{-2bx} dx \\
 &= \int_{-0.25}^0 2e^{4x} dx + \int_0^{0.25} 2e^{-4x} dx = \frac{1}{2} (e^0 - e^{-1}) - \frac{1}{2} (e^{-1} - e^0) \approx 63.21\%
 \end{aligned}$$

5)

a) The length of a spacecraft runway measured by an observer on the Earth:

$$L = 3600 \text{ (m)}$$

\Rightarrow The length of the runway measured by a pilot on a spacecraft:

$$L = L_0 \sqrt{1 - \frac{u^2}{c^2}} \rightarrow L_0 = \frac{L}{\sqrt{1 - \frac{u^2}{c^2}}} = \frac{3600}{\sqrt{1 - \left(\frac{4 \times 10^7}{3 \times 10^8}\right)^2}} = 3632.433 \text{ (m)}$$

b) The time that takes him to travel from one end of the runway to the other end:

$$\Delta t_0 = \frac{L_0}{u} = \frac{3632.433}{4 \times 10^7} = 90.81 \text{ } (\mu m)$$

June – 2016

1)

When a hydrogen atom absorbs a photon, it jumps from the ground level to the higher level

$$E_n = E_1 + \varepsilon = -13.6 + 12.75 = -0.85(eV)$$

$$E_n = \frac{-13.6}{n^2} = -0.85 \rightarrow n = 4$$

a) The shortest wavelength of the photons: From $n = 4$ to $n = 1$

$$\begin{aligned} \Delta E = hf = \frac{hc}{\lambda} &= E_i - E_f = -13.6 \left(\frac{1}{n_i^2} - \frac{1}{n_f^2} \right) = -13.6 \left(\frac{1}{4^2} - \frac{1}{1^2} \right) \\ &= 12.75 \text{ (eV)} \Leftrightarrow \lambda_{min} = 97.4 \text{ (nm)} \end{aligned}$$

The longest wavelength of the photon: From $n = 4$ to $n = 3$

$$\begin{aligned} \Delta E = hf = \frac{hc}{\lambda} &= E_i - E_f = -13.6 \left(\frac{1}{n_i^2} - \frac{1}{n_f^2} \right) = -13.6 \left(\frac{1}{4^2} - \frac{1}{3^2} \right) = \\ 0.661 \text{ (eV)} &\Leftrightarrow \lambda_{max} = 1879.2 \text{ (nm)} \end{aligned}$$

b) For the shortest wavelength: $\lambda_{min} = 97.4 \text{ (nm)} \Rightarrow$ Lyman series

For the longest wavelength: $\lambda_{max} = 1879.2 \text{ (nm)} \Rightarrow$ Paschen series

2)

a) The number of nuclei in the sample at $t = 0$:

$$N_0 = N_A \frac{m}{M} = N_A \frac{0.0035}{11} = 1.92 \times 10^{20} \text{ (nuclei)}$$

b) After 8 hours, the number of nuclei:

$$\begin{aligned} N(t) = N_0 2^{-\frac{t}{T_{half}}} &\Leftrightarrow 53.76 \times 10^6 = 1.92 \times 10^{20} 2^{-\frac{8 \times 3600}{T_{half}}} \\ &\Leftrightarrow T_{half} \approx 690.65 \text{ (s)} \end{aligned}$$

3)

a) We have: The normalization condition:

$$\begin{aligned}
 P &= \int_{-\infty}^{+\infty} \psi^2(x) dx = \int_{-\infty}^0 \psi^2(x) dx + \int_0^{+\infty} \psi^2(x) dx = 1 \\
 \Leftrightarrow \int_{-\infty}^0 A^2 e^{2bx} dx + \int_0^{+\infty} A^2 e^{-2bx} dx &= \frac{A^2}{2b} (e^0 - e^{-2b\infty}) - \frac{A^2}{2b} (e^{-2b\infty} - e^0) = 1 \\
 \Leftrightarrow \frac{A^2}{2b} (1 - 0) - \frac{A^2}{2b} (0 - 1) &= \frac{2A^2}{2b} = 1 \Rightarrow A = \sqrt{2}
 \end{aligned}$$

b) The probability of finding this particle within 50cm of the origin:

$$\begin{aligned}
 P &= \int_{-0.25}^{0.25} \psi^2(x) dx \\
 &= \int_{-0.25}^0 \psi^2(x) dx + \int_0^{0.25} \psi^2(x) dx = \int_{-0.25}^0 A^2 e^{2bx} dx + \int_0^{0.25} A^2 e^{-2bx} dx \\
 &= \int_{-0.25}^0 2e^{4x} dx + \int_0^{0.25} 2e^{-4x} dx = \frac{1}{2} (e^0 - e^{-1}) - \frac{1}{2} (e^{-1} - e^0) \approx 63.21\%
 \end{aligned}$$

4)

The length of an object when it moves: $L = 74 \text{ (m)}$

The length of an object when it is stationary:

$$L = L_0 \sqrt{1 - \frac{u^2}{c^2}} \rightarrow L_0 = \frac{L}{\sqrt{1 - \frac{u^2}{c^2}}} = \frac{74}{\sqrt{1 - 0.6^2}} = 92.5 \text{ (m)}$$

5)

a) The wavelength of the emitted photon:

$$E = \frac{hc}{\lambda} \Rightarrow \lambda = \frac{hc}{E} = \frac{6.62 \times 10^{-34} \times 3 \times 10^8}{2.58 \times 1.6 \times 10^{-19}} = 4.81 \times 10^{-7} \text{ (m)} = 0.418 \text{ (}\mu\text{m)}$$

b) According to the uncertainty principle of energy:

$$\Delta E_{\min} \Delta t = \hbar \Rightarrow \Delta E_{\min} = \frac{\hbar}{\Delta t} = \frac{\hbar}{1.64 \times 10^{-7}} = 6.43 \times 10^{-28} (J)$$

2018

1)

The longest wavelength of the Lyman series: From $n = 2$ to $n = 1$

$$\Delta E = hf = \frac{hc}{\lambda} = E_i - E_f = -13.6 \left(\frac{1}{n_i^2} - \frac{1}{n_f^2} \right) = -13.6 \left(\frac{1}{2^2} - \frac{1}{1^2} \right) = 10.2 (eV)$$

$$\Rightarrow \lambda_{\max} = 121.8 (nm)$$

The shortest wavelength of the Lyman series: From $n = \infty$ to $n = 1$

$$\begin{aligned} \Delta E = hf = \frac{hc}{\lambda} = E_i - E_f &= -13.6 \left(\frac{1}{n_i^2} - \frac{1}{n_f^2} \right) = -13.6 \left(\frac{1}{\infty^2} - \frac{1}{1^2} \right) \\ &= 3.4 (eV) \end{aligned}$$

$$\Rightarrow \lambda_{\min} = 91.3 (nm)$$

Since $91.3 \leq \lambda \leq 121.8 \rightarrow$ Lyman series falls in ultraviolet region of the electromagnetic spectrum

The longest wavelength of the Paschen series: From $n = 4$ to $n = 3$

$$\Delta E = hf = \frac{hc}{\lambda} = E_i - E_f = -13.6 \left(\frac{1}{n_i^2} - \frac{1}{n_f^2} \right) = -13.6 \left(\frac{1}{4^2} - \frac{1}{3^2} \right) = 0.66 (eV)$$

$$\Rightarrow \lambda_{\max} = 1879.2 (nm)$$

The shortest wavelength of the Lyman series: From $n = \infty$ to $n = 3$

$$\begin{aligned} \Delta E = hf = \frac{hc}{\lambda} = E_i - E_f &= -13.6 \left(\frac{1}{n_i^2} - \frac{1}{n_f^2} \right) = -13.6 \left(\frac{1}{\infty^2} - \frac{1}{3^2} \right) \\ &= 1.51 (eV) \end{aligned}$$

$$\Rightarrow \lambda_{\min} = 822.1 (nm)$$

Since $822.1 \leq \lambda \leq 1879.2 \rightarrow$ Paschen series falls in infrared region of the electromagnetic spectrum

2)

a) We have: The normalization condition:

$$\begin{aligned}
 P &= \int_{-\infty}^{+\infty} \psi^2(x) dx = \int_{-\infty}^0 \psi^2(x) dx + \int_0^{+\infty} \psi^2(x) dx = 1 \\
 \Leftrightarrow \int_{-\infty}^0 A^2 e^{2bx} dx + \int_0^{+\infty} A^2 e^{-2bx} dx &= \frac{A^2}{2b} (e^0 - e^{-2b\infty}) - \frac{A^2}{2b} (e^{-2b\infty} - e^0) = 1 \\
 \Leftrightarrow \frac{A^2}{2b} (1 - 0) - \frac{A^2}{2b} (0 - 1) &= \frac{2A^2}{2b} = 1 \Rightarrow A = \sqrt{2}
 \end{aligned}$$

b) The probability of finding this particle within 50cm of the origin:

$$\begin{aligned}
 P &= \int_{-0.25}^{0.25} \psi^2(x) dx \\
 &= \int_{-0.25}^0 \psi^2(x) dx + \int_0^{0.25} \psi^2(x) dx = \int_{-0.25}^0 A^2 e^{2bx} dx + \int_0^{0.25} A^2 e^{-2bx} dx \\
 &= \int_{-0.25}^0 2e^{4x} dx + \int_0^{0.25} 2e^{-4x} dx = \frac{1}{2} (e^0 - e^{-1}) - \frac{1}{2} (e^{-1} - e^0) \approx 63.21\%
 \end{aligned}$$

3)

a) Since n is an integer (n = 1,2,3,...) \Rightarrow Only certain value of energy is evaluated \Rightarrow The energy of this electron is quantized.

b) We have

$$\begin{aligned}
 \Delta E = E_4 - E_1 &= \frac{16\pi^2 \hbar^2}{2mL^2} - \frac{\pi^2 \hbar^2}{2mL^2} = \frac{15\pi^2 \hbar^2}{2mL^2} \\
 &= \frac{15\pi^2 \hbar^2}{2 \times 9.1 \times 10^{-31} \times (0.125 \times 10^{-9})^2} = 361.4(eV)
 \end{aligned}$$

c) We have:

$$\Delta E = hf = \frac{hc}{\lambda} = 361.4(eV) \Rightarrow \lambda = 3.43(nm) \Rightarrow \text{X-rays}$$

4)

The Lorentz factor: $\gamma = \frac{1}{\sqrt{1-\frac{u^2}{c^2}}} = \frac{1}{\sqrt{1-0.99^2}} = 7.08$

a) The kinetic energy of the electron:

$$\begin{aligned} E &= m_0 c^2 + K_S \rightarrow K_S = E - m_0 c^2 = \gamma m_0 c^2 - m_0 c^2 = (\gamma - 1) m_0 c^2 \\ &\rightarrow K_S = (7.08 - 1) \times 9.11 \times 10^{-31} \times (3 \times 10^8)^2 = 4.992 \times 10^{-13} (J) \\ &= 3.12 (MeV) \end{aligned}$$

b) The kinetic energy of the electron (in classical mechanics)

$$\begin{aligned} K_N &= \frac{1}{2} m_0 u^2 = \frac{1}{2} \times 9.11 \times 10^{-31} \times (0.99 \times 3 \times 10^8)^2 = 4.018 \times 10^{-14} (J) \\ &= 0.25 (MeV) \end{aligned}$$

The difference between the classical value and relativistic value:

$$\%K = \frac{K_S - K_N}{K_S} = 91.98 \%$$

Conclusion: There is an error when we use classical mechanics to calculate the kinetic energy of the electron when its velocity reaches to the light speed.

5)

a) The decay constant:

$$T_{half} = \frac{\ln 2}{\lambda} \rightarrow \lambda = \frac{\ln 2}{T_{half}} = 3.75 \times 10^{-4}$$

The initial activity:

$$\begin{aligned} \frac{dN(0)}{dt} &= -\lambda N(0) \Leftrightarrow -7.56 \times 10^{11} = -3.75 \times 10^{-4} N(0) \Leftrightarrow N(0) \\ &= 2.016 \times 10^{15} (nuclei) \end{aligned}$$

b) After 30.8 minutes, the number of nuclei:

$$\begin{aligned} N(t) &= N_0 2^{-\frac{t}{T_{half}}} = 2.016 \times 10^{15} 2^{-\frac{t}{T_{half}}} = 2.016 \times 10^{15} \times 2^{-1} \\ &= 1.008 \times 10^{15} (nuclei) \end{aligned}$$

The activity at this time:

$$\frac{dN(t)}{dt} = -\lambda N(t) = -3.75 \times 10^{-4} \times 1.008 \times 10^{15} = -3.78 \times 10^{11} (Bq)$$

January - 2019

1)

The longest wavelength of the Balmer series: From $n = 3$ to $n = 2$

$$\Delta E = hf = \frac{hc}{\lambda} = E_i - E_f = -13.6 \left(\frac{1}{n_i^2} - \frac{1}{n_f^2} \right) = -13.6 \left(\frac{1}{3^2} - \frac{1}{2^2} \right) = 1.88 (eV)$$

$$\Rightarrow \lambda_{max} = 660.8 (nm)$$

The shortest wavelength of the Balmer series: From $n = \infty$ to $n = 2$

$$\begin{aligned} \Delta E = hf = \frac{hc}{\lambda} = E_i - E_f &= -13.6 \left(\frac{1}{n_i^2} - \frac{1}{n_f^2} \right) = -13.6 \left(\frac{1}{\infty^2} - \frac{1}{2^2} \right) \\ &= 3.4 (eV) \end{aligned}$$

$$\Rightarrow \lambda_{min} = 365.4 (nm)$$

Since $365.4 \leq \lambda \leq 660.8 \rightarrow$ Balmer series falls in visible and ultraviolet region of the electromagnetic spectrum

2)

a) We have: The probability of finding particle from $0 \leq x \leq \frac{L}{a}$

$$\begin{aligned} P &= \int_0^{L/a} |\psi^2| dx = \int_0^{L/a} \frac{2}{L} \sin^2 \left(\frac{n\pi x}{L} \right) dx = \frac{1}{L} \int_0^{L/a} \left(1 - \cos \left(\frac{2n\pi x}{L} \right) \right) dx \\ &= \frac{1}{L} \left(\frac{L}{a} - \frac{L}{2n\pi} \left(\sin \left(\frac{2n\pi}{a} \right) + \sin 0 \right) \right) = \frac{1}{a} - \frac{1}{2n\pi} \sin \left(\frac{2n\pi}{a} \right) \end{aligned}$$

b) When $n \rightarrow \infty$:

$$P = \lim_{n \rightarrow \infty} \frac{1}{a} - \frac{1}{2n\pi} \sin \left(\frac{2n\pi}{a} \right) = \lim_{n \rightarrow \infty} \frac{1}{a} - \lim_{n \rightarrow \infty} \frac{1}{2n\pi} \sin \left(\frac{2n\pi}{a} \right) = \frac{1}{a}$$

The probability is the same as the interval of finding particle ($1/a$) \Rightarrow Fitting with the classical result

3)

a) We have:

$$\Delta E = E_2 - E_1 = \frac{4h^2}{8mL^2} - \frac{h^2}{8mL^2} = \frac{3h^2}{8mL^2} = \frac{3 \times (6.62 \times 10^{-34})^2}{8 \times 1.67 \times 10^{-27} \times (10 \times 10^{-15})^2} = 9.84 \times 10^{-13} (J)$$

Also: The wavelength of the emitted photon

$$\Delta E = hf = \frac{hc}{\lambda} = 9.84 \times 10^{-13} (J) \Rightarrow \lambda = 20.2 (pm)$$

b) Since this wavelength is 20.2 pm \Rightarrow This photon is in the gamma region of the electromagnetic spectrum

4)

In classical physics:

$$s = u\Delta t_0 = 0.99 \times 3 \times 10^8 \times 0.1237 \times 10^{-6} = 36.74 (m)$$

In special relativity:

$$\Delta t = \frac{\Delta t_0}{\sqrt{1 - \frac{u^2}{c^2}}} = \frac{0.1237}{\sqrt{1 - 0.99^2}} = 0.876 (\mu m)$$

$$s = u\Delta t = 0.99 \times 3 \times 10^8 \times 0.876 \times 10^{-6} = 260.43 (m)$$

5)

a) The number of ^{40}K nuclei (and thus atom) in this banana:

$$N = 0.0117\% N_A \frac{m}{M} = 0.0117\% N_A \frac{0.6}{40} = 1.057 \times 10^{18} (nuclei)$$

b) The decay constant:

$$T_{half} = \frac{\ln 2}{\lambda} \rightarrow \lambda = \frac{\ln 2}{T_{half}} = 1.76 \times 10^{-17}$$

The activity R of this banana

$$R = \lambda N = 1.76 \times 10^{-17} \times 1.057 \times 10^{18} = 18.58 (Bq)$$

