

KCL at ~~node~~ node $-v_i$:

$$\frac{v^- - v_s}{R_1} + \frac{v^- - v_o}{R_2} = 0$$

$$\Leftrightarrow \left. \begin{aligned} \frac{-v_i - v_s}{R_1} + \frac{-v_i - v_o}{R_2} &= 0 \\ \text{• With } A \ v_o &= A v_i \end{aligned} \right\}$$

$$\Rightarrow \frac{-v_i - v_s}{R_1} + \frac{-v_i - A v_i}{R_2} = 0$$

$$\Leftrightarrow \frac{v_s}{R_1} = \frac{v_i}{R_1} + \frac{v_i}{R_2} + \frac{A v_i}{R_2} = -v_i \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{A}{R_2} \right)$$

• Feedback network:

$$v_f = v_s - v_i = -v_i \left(1 + \frac{R_1}{R_2} + \frac{A R_1}{R_2} \right) - v_i$$

$$= -v_i \left(2 + \frac{R_1}{R_2} + \frac{A R_1}{R_2} \right)$$

$$= -\frac{v_o}{A} \left(2 + \frac{R_1}{R_2} + \frac{A R_1}{R_2} \right)$$

$$\frac{v_o}{v_f} = -A / \left(2 + \frac{R_1}{R_2} + \frac{A R_1}{R_2} \right)$$

$$= \frac{-1}{\frac{2 + R_1/R_2}{A} + \frac{R_1}{R_2}} = \frac{1}{\beta}$$

$$\Rightarrow \beta = -\frac{R_1}{R_2}, \text{ because } \frac{2 + R_1/R_2}{A} \rightarrow 0$$