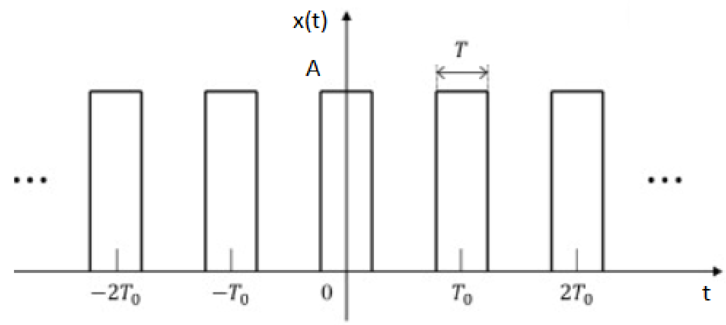
Problem 1

Consider the signal



a) Find the exponential Fourier series , i.e., determine $X[k]$ such that:

$$x(t) = \sum_{k=-\infty}^{+\infty} X[k] e^{jk\omega_0 t}$$

b) Find the trigonometric Fourier series.

Problem 2

The periodic signal $x(t)$ is given in **Figure 1**:

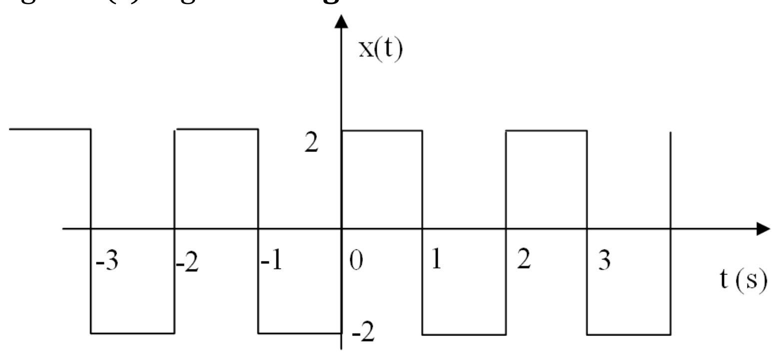


Figure 1

a) Determine the power of the signal $x(t)$.
b) Find the exponential Fourier coefficients of the signal $x(t)$.

Problem 3

Consider the signal

$$x(t) = |6\cos(100\pi t)|$$

a) Sketch the signal $x(t)$.
b) Find the exponential Fourier series , i.e., determine $X[k]$ such that:

$$x(t) = \sum_{k=-\infty}^{+\infty} X[k] e^{jk\omega_0 t}$$

Problem 4

a) Consider a high-pass filter with the frequency response $H(\omega) = \frac{j\omega}{1+j\omega}$. Determine the steady state output $y(t)$ of the system for the input $x(t) = 2\cos(2t) + \cos(100t)$.
b) Let:

$$x(t) = \begin{cases} 1, & -1 \le t \le 1 \\ 0, & \text{elsewhere} \end{cases}$$

Using the definition of the Fourier transform find the Fourier transform $X(\omega)$ of the signal $x(t)$.

Problem 5

a) Consider filter with the frequency response $H(\omega) = \frac{1+j0.01\omega}{1+j0.1\omega}$. Determine (steady-state) output $y(t)$ of the system for the input $x(t) = 5 + 2\cos(5t) - \cos(200t)$.
b) Let:

$$x(t) = \begin{cases} 1, & -2 \le t \le 2 \\ 0, & \text{elsewhere} \end{cases}$$

Using the definition of the Fourier transform, find the Fourier transform $X(\omega)$ of the signal $x(t)$.

Problem 6

Consider the LTI system having the unit impulse response:

$$h(t) = e^{-5t}u(t)$$

a) Find the frequency response $H(\omega)$.
b) Find the response $y(t)$ for the input signal

$$x(t) = 5 + 2\cos(2t - 30°) - 4\sin(10t + 20°)$$

Problem 7

a) Consider the following signal:

$$x(t) = 6\cos(20t) + 4\cos(100t)$$

Find and sketch the spectrum $X(\omega)$, and determine the power of signal $x(t)$.
b) The modulated signal $y(t)$ is defined by

$$y(t) = [12 + x(t)]\cos(500t)$$

Find and sketch the spectrum $Y(\omega)$, and calculate the power of $y(t)$.

Problem 8

Consider a causal LTI system with frequency response

$$H(j\omega) = \frac{1}{j\omega + 3}$$

For a particular input $x(t)$ this system produces the output

$$y(t) = e^{-3t}u(t) - e^{-4t}u(t)$$

Determine $x(t)$.

Problem 9

Consider a casual LTI system whose response (output) is $y(t) = e^{-5t}u(t)$ for the input $x(t) = e^{-2t}u(t)$.

a) Find the transfer function $H(s) = \dfrac{Y(s)}{X(s)}$ and the unit impulse response $h(t)$.

b) Find the zero-state response $y(t)$ of the system if the input signal is $x(t) = 2e^{-4t}$ $u(t)$.

Problem 10

Consider the causal filter with the transfer function

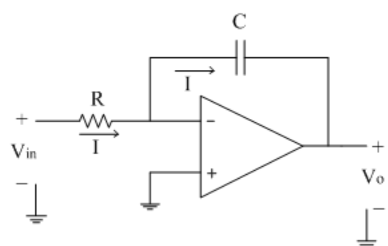
$$H(s) = \frac{Y(s)}{X(s)} = \frac{100s + 10000}{(s + 10)(s + 1000)}$$

a) Determine the expression of the magnitude response in dB and use the Bode plot to sketch the magnitude response.

b) Find the zero-state response $y(t)$ of the system if the input signal is $x(t) = e^{-50t}u(t)$.

Problem 11

Consider the following circuit with $R = 10\ \text{k}\Omega$, $C = 2\ \mu\text{F}$.



a) Find the transfer function $H(s) = \frac{V_o(s)}{V_i(s)}$.

b) Find the frequency response $H(\omega)$.

c) Sketch the magnitude response $|H(\omega)|$ and phase response $\angle H(\omega)$.

d) Find $y(t)$ for $x(t) = 5\cos(10t) + 6\cos(20t)$.

Problem 12

In **Figure 2**, the op-amp is ideal and $R_1 = 10\ \text{k}\Omega$, $R_2 = 100\ \text{k}\Omega$, $C = 0.1\ \mu\text{F}$.

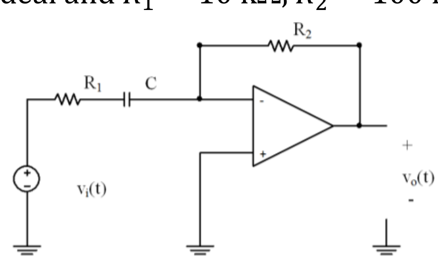


Figure 2

a) Find the transfer function $H(s) = \frac{V_o(s)}{V_i(s)}$ where $V_i(s)$, $V_o(s)$ are the Laplace transforms of $v_i(t), v_o(t)$, respectively.

b) Find the frequency response $H(\omega)$ and calculate the magnitude response at the given frequencies in Table below. Sketch the magnitude $A(\omega)$ in dB.

ω (rad/s)	1	10	100	1000	10000
$\lvert H(\omega)\rvert$					
$A(\omega) = 20\log_{10}\lvert H(\omega)\rvert$					