THE INTERNATIONAL UNIVERSITY – VIETNAM NATIONAL UNIVERSITY – HCMC

FINAL EXAMINATION

Semester 2, 2021-2022 • Date: June 14, 2022 • Duration: 120 minutes

SUBJECT: Calculus 2	CODE: MA1109
Vice Head of Department of Mathematics: Nguyễn Minh Quân	Lecturer: Assoc. Prof. Tran Vu Khanh, PhD

INSTRUCTIONS:

Each student is allowed a scientific calculator and a maximum of ONE double-sided sheet of reference material (size A4 or similar) marked with their name and ID. All other documents and electronic devices are forbidden.

Arguments and computations must be detailed so that they are easy to follow.

Please indicate precisely which problem and question you are solving, e.g. Part I: 1a, 2b,.....; Part II: 1, 2, ...

....

Part I. Write answers of the following questions. Do not need to show the work.

1. *[10 points]* The rectangular coordinates of a point are $(1, \sqrt{3}, -2)$. Find the cylindrical and spherical coordinates of the point.

2. *[20 points]* Let $f(x, y) = 2e^{x^2-2y}$.

   a. Find $\nabla f(x, y)$.

   b. Find the tangent plane to the surface $z = f(x, y)$ at the point $(4, 8, 2)$.

   c. Find the linearization $L(x, y)$ of $f(x, y)$ at $(4, 8)$ and use it to approximate $f(4.1, 7.9)$.

   d. Find the directional derivative of f at $(4, 8)$ in the direction toward to the point $(1, 12)$.

Please turn over...

3. *[10 points]* Describe the solid whose volume is given by the integral

$$\int_0^{2\pi} \int_0^{\pi/2} \int_1^2 \rho^2 \sin\phi \, d\rho \, d\phi \, d\theta$$

and evaluate the integral.

4. *[15 points]* Determine whether the statement is true or false.

   a. There exists a function with continuous second-order partial derivatives such that $f_x(x, y) = 2x + y$ and $f_y(x, y) = -x + 2y$.

   b. If f has a local maximum at (a, b) and f is differentiable at (a, b), then $\nabla f(a, b) = \langle 0, 0 \rangle$.

   c. $\int_{-1}^{1} \int_0^1 y^3 e^{x^2+y^2} dx dy = 0$.

   d. $\int_{-C} f(x, y) ds = -\int_C f(x, y) ds$.

   e. $\iint_D \sqrt{x^2 + y^2} dA = \frac{16\pi}{3}$ where D is the disk given by $x^2 + y^2 \leq 4$.

Part II. Show details your work, and indicate answers clearly.

1. *[15 points]* Find the local maximum and minimum values and saddles points of

$$f(x, y) = xy(1 - x - y).$$

2. *[10 points]* Evaluate the integral by reversing the order of integration

$$\int_0^1 \int_{x^2}^1 x^3 \sin(y^3) \, dy \, dx.$$

3. *[10 points]* Evaluate

$$\oint_C e^y \, dx + 2xe^y \, dy$$

where C is the square with sides $x = 0$, $x = 1$, $y = 0$, and $y = 1$; and C is positively orientated. Here, $\oint_C$ denotes the integral along the closed curve C.

4. *[10 points]* Evaluate $\iiint_E x^2 dV$ where E is the solid that lies within the cylinder $x^2 + y^2 = 1$, above the plane $z = 0$, and below the cone $z^2 = 4x^2 + 4y^2$.

—END OF QUESTION PAPER—