EE2 – Fall 2020
Midterm Exam SOLUTION

A. Select the correct answer (30 marks)

"Step Response of an R-C Circuit with a DC voltage V_S".

1. The current in the R-C circuit at a time $t = 0^+$ is?

a) V_S/R
b) R/V_S
c) V_S
d) R

Ans.: a

Explanation: The capacitor never allows sudden changes in voltage, it will act as a short circuit at $t = 0^+$. So the current in the circuit at $t = 0^+$ is V_S/R.

2. In an R-C circuit, when the switch is closed, the response _____________

a) do not vary with time
b) decays with time
c) rises with time
d) first increases and then decreases

Ans.: b

Explanation: In a R-C circuit, when the switch is closed, the response decays with time that is the response V_S/R decreases with increase in time.

3. A series R-C circuit consists of resistor of 10 Ω and capacitor of 0.1 F as shown in the Fig. 1. A constant voltage $V_S = 20$ V is applied to the circuit at $t = 0$. What is the current in the circuit at $t = 0$? Assume at $t = 0$, switch S is closed.

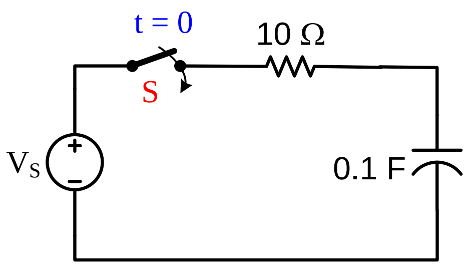


Fig. 1

a) 1A
b) 2A
c) 3A
d) 4A

Ans.: b.

Explanation: At $t = 0$, switch S is closed. Since the capacitor does not allow sudden changes in voltage, the current in the circuit is $i = V_S/R = 20/10 = 2$A. At $t = 0$, $i = 2$A.

4. The current equation in the circuit shown in the question 3 is?

a) $i = 2(e^{-2t})$ A

b) $i = 2(e^{2t})$ A
c) $i = 2(-e^{-2t})$ A
d) $i = 2(-e^{2t})$ A

Ans.: a
Explanation: At $t = 0$, switch S is closed. Since the capacitor does not allow sudden changes in voltage, the current in the circuit is $i = V_S/R = 20/10 = 2A$. At $t = 0$, $i = 2A$. The current equation is $i = 2(e^{-2t})$ A.

5. Determine the voltage across the capacitor in the circuit shown in the question 3 is?
a) $V_C = 20(1 - e^{-t})$ V
b) $V_C = 20(1 + e^{t})$ V
c) $V_C = 20(1 - e^{t})$ V
d) $V_C = 20(1 + e^{-t})$ V

Ans.: a
Explanation:
The expression of voltage across capacitor in the circuit $V_C = V_S(1\text{-}e^{-t/RC}) = 20(1 - e^{-t})$ V.

"Step Response of an R-L Circuit with a DC voltage V_S"

6. The value of the time constant in the R-L circuit is?
a) L/R
b) R/L
c) R
d) L

Ans.: a
Explanation: The time constant of a function $(V_S/R)e^{-(R/L)t}$ is the time at which the exponent of e is unity where e is the base of the natural logarithms. The term L/R is called the time constant and is denoted by 'τ'.

7. After how many time constants, the transient part reaches more than 99 percent of its final value?
a) 2
b) 3
c) 4
d) 5

Ans.: d
Explanation: After five time constants, the transient part of the response reaches more than 99 percent of its final value.

8. A series R-L circuit with R = 30 Ω and L = 15 H has a constant voltage V_S = 60V applied at t = 0 as shown in the Fig. 2. Determine the current (A) in the circuit at $t = 0^+$.

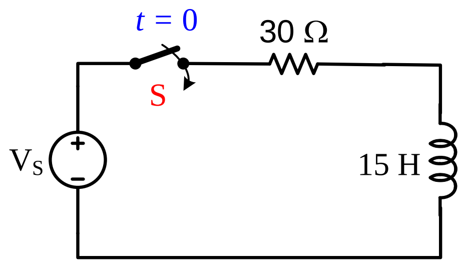


Fig. 2

a) 1A
b) 2A
c) 3A
d) 0A

Ans.: d
Explanation: Since the inductor never allows sudden changes in currents. At $t = 0^+$ that just after the initial state the current in the circuit is zero.

9. The expression of current obtained from the circuit in terms of differentiation from the circuit shown in the question 8?

a) $\frac{di}{dt} + i = 4$

b) $\frac{di}{dt} + 2i = 0$

c) $\frac{di}{dt} + 2i = 4$

d) $\frac{di}{dt} - 2i = 4$

Ans.: c
Explanation: Let the i be the current flowing through the circuit. By applying Kirchhoff's voltage law, we get $15di/dt + 30i = 60 \Rightarrow di/dt + 2i = 4$.

10. Determine the voltage across the inductor in the circuit shown in the question 8 is?
a) $V_L = 60(-e^{-2t})$ V
b) $V_L = 60(e^{2t})$ V
c) $V_L = 60(e^{-2t})$ V
d) $V_L = 60(-e^{2t})$ V

Ans.: c
Explanation: Time constant τ = L/R = 15/30 = 0.5 (s).
Voltage across the inductor $V_L = V_S \times e^{-t/\tau} = 60(e^{-2t})$V.

"Step Response of an R-L-C Circuit"

11. For an R-L-C circuit, if the roots of an equation are real and unequal, then the response will be?
a) critically damped
b) under damped
c) over damped
d) damped

Ans.: c
Explanation: If the roots of an equation are real and unequal, then the response will be over damped response. Over damped response of a system is defined as the system returns (exponentially decays) to equilibrium without oscillating.

12. If the roots of an equation are complex conjugate, then the response will be?
a) over damped
b) critically damped
c) damped
d) under damped

Ans.: d
Explanation: If the roots of an equation are complex conjugate, then the response will be under damped response. Damping is an influence within or upon an oscillatory system that has the effect of reducing, restricting or preventing its oscillations.

13. If the roots of an equation are real and equal, then the response will be?
a) over damped
b) damped
c) critically damped
d) under damped

Ans.: c
Explanation: If the roots of an equation are real and equal, then the response will be critically damped response. For a critically damped system, the system returns to equilibrium as quickly as possible without oscillating.

14. The circuit shown in the Fig. 3 consists of resistance, capacitance and inductance in series with a source V_S = 100 V when the switch S is closed at t = 0. Find the equation obtained from the circuit in terms of current i?

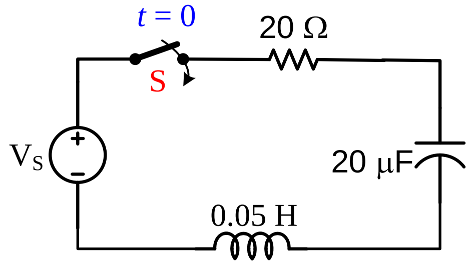


Fig. 3

a) $100 = 20i + 0.05\frac{di}{dt} + \frac{1}{20\times10^{-6}}\int idt$

b) $100 = 20i - 0.05\frac{di}{dt} + \frac{1}{20\times10^{-6}}\int idt$

c) $100 = 20i + 0.05\frac{di}{dt} - \frac{1}{20\times10^{-6}}\int idt$

d) $100 = 20i - 0.05\frac{di}{dt} - \frac{1}{20\times10^{-6}}\int idt$

Ans.: a
Explanation: At t = 0, switch S is closed when the 100V source is applied to the circuit and results in the following differential equation. $100 = 20i + 0.05\frac{di}{dt} + \frac{1}{20\times10^{-6}}\int idt$

15. At time t = 0, the value of current in the circuit shown in the question 14 is?
a) 1A
b) 2A
c) 3A
d) 0A

Ans.: d
Explanation: At t = 0 that is initially the current flowing through the circuit is zero that is i = 0. So, i = 0.

B. Problems:

Problem 3 (20 marks): For the following Laplace function $F(s)$, find the inverse Laplace transform $f(t)$

$$F(s) = \frac{10s^3 + 40s^2 + 40s + 6}{s^4 + 6s^3 + 11s^2 + 6s}$$

$$F(s) = \frac{10s^3 + 40s^2 + 40s + 6}{s^4 + 6s^3 + 11s^2 + 6s} = \frac{10s^3 + 40s^2 + 40s + 6}{s(s+1)(s+2)(s+3)} = \frac{A}{s} + \frac{B}{s+1} + \frac{C}{s+2} + \frac{D}{s+3}$$

$$A = \left.\frac{10s^3 + 40s^2 + 40s + 6}{(s+1)(s+2)(s+3)}\right|_{s=0} = 1$$

$$B = \left.\frac{10s^3 + 40s^2 + 40s + 6}{s(s+2)(s+3)}\right|_{s=-1} = 2$$

$$C = \left.\frac{10s^3 + 40s^2 + 40s + 6}{s(s+1)(s+3)}\right|_{s=-2} = 3$$

$$D = \left.\frac{10s^3 + 40s^2 + 40s + 6}{s(s+1)(s+2)}\right|_{s=-3} = 4$$

$$f(t) = [1 + 2e^{-t} + 3e^{-2t} + 4e^{-3t}]u(t)$$

Problem 1 (25 marks): Given the circuit in below Fig. 4.

a) Construct the s – domain circuit from the given circuit.
b) Find the expression of voltage $v_0(t)$ from the circuit in below figure by means of Laplace transform.

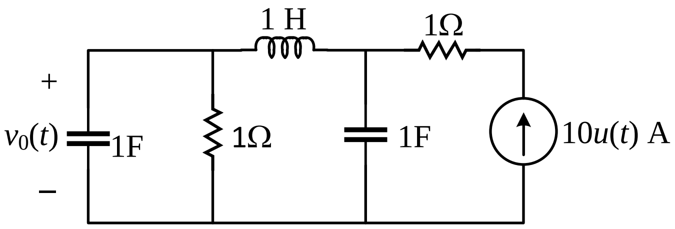
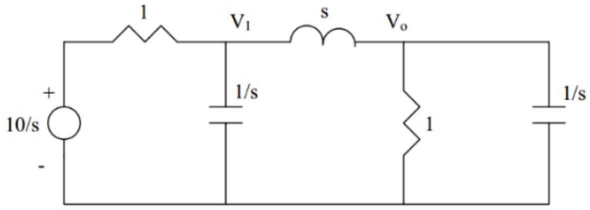


Fig. 4: Figure for Problem 2

a) The equivalent s – domain circuit is as follow



b)

At node 1: $\frac{10}{s} = sV_1 + \frac{V_1 - V_0}{s} \rightarrow 10 = (s^2+1)V_1 - V_o$ (1)

At node o: $\frac{V_1 - V_o}{s} = V_o + sV_o \rightarrow V_1 = (s^2+s+1)V_o$ (2)

From (1) and (2), we have:

$$10 = (s^2+1)(s^2+s+1)V_o - V_o = s(s^3+s^2+2s+1)V_o \rightarrow V_o = \frac{10}{s(s^3+s^2+2s+1)}$$

$$V_o = \frac{10}{s(s^3+s^2+2s+1)} = \frac{A}{s} + \frac{B}{s+0.57} + \frac{C}{s+0.215-1.31j} + \frac{C^*}{s+0.215+1.31j}$$

$$A = \left.\frac{10}{(s^3+s^2+2s+1)}\right|_{s=0} = 10$$

$$B = \left.\frac{10}{s(s+0.215-1.31j)(s+0.215+1.31j)}\right|_{s=-0.57} = -9.524$$

$$C = \left.\frac{10}{s(s+0.57)(s+0.215+1.31j)}\right|_{s=-0.215+1.31j} = -0.216 + 2.11j = 2.12\angle 95.85°$$

$$v_0(t) = [10 - 9.524e^{-0.57t} + 4.24e^{-0.215t}\cos(1.31t + 95.85°)]u(t) \quad (V)$$

Problem 2 (25 marks): The switch in the circuit seen in following figure (Fig. 5) has been in position **a** for a long time. At $t = 0$ it moves instantaneously to position **b**. Knowing that $V = 24V,\ R_1 = 3\Omega, R_2 = 5\Omega, R_3 = 20\Omega, C = 0.1F, L = 5.625H$.

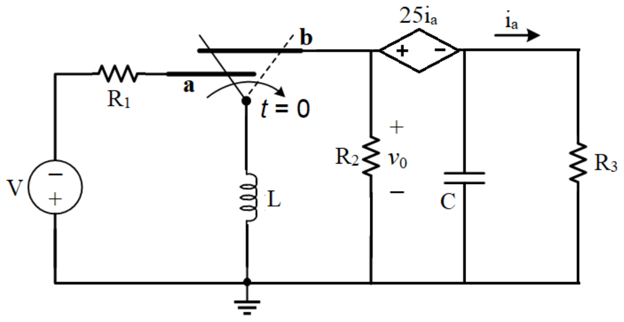


Fig. 5: Figure for Problem 2

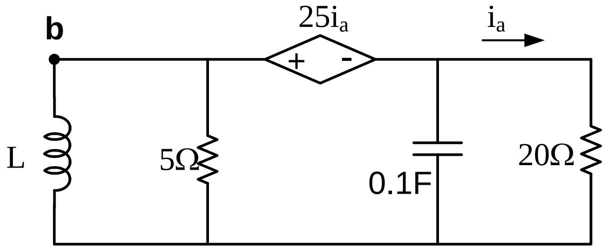
a) Find $V_o(s)$

b) Find $v_0(t)$

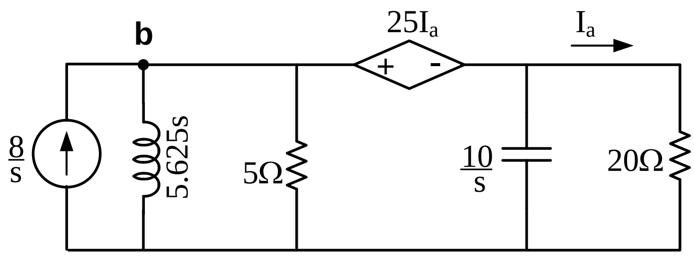
Sol: Consider the initial condition of the circuit, at t = 0. Then, the current running thru inductor L is

$i_L(0^-) = i_L(0^+) = \frac{24}{3} = 8\ (A)$ (directed upward).

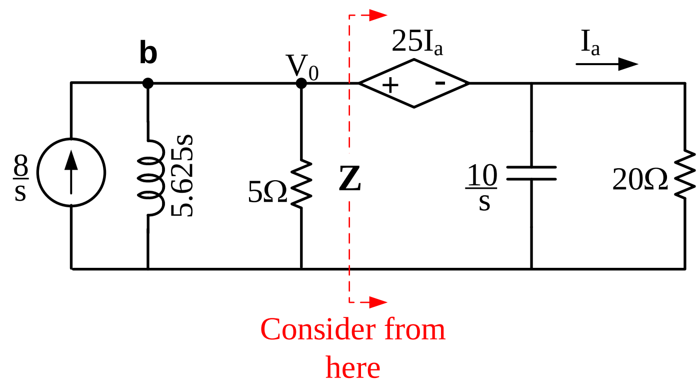
When the switch moved to the b position (since at $t^+ = 0$), the circuit will be (in time domain)

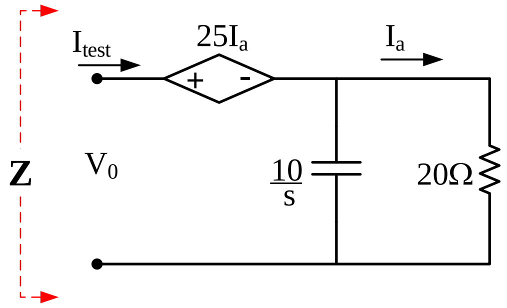


So in the s-domain, the circuit will be:



Consider the circuit as following

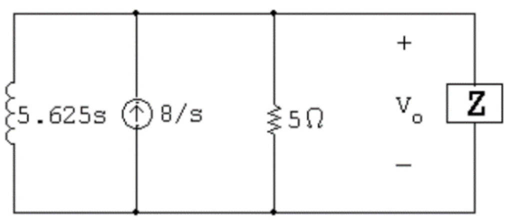




$$V_0 = 25 \times I_a + \left[\frac{20(10/s)}{20 + 10/s}\right] I_{test} = \frac{25 \times I_{test}(10/s)}{20 + 10/s} + I_{test}\left(\frac{200}{10 + 20s}\right)$$

$$\rightarrow \frac{V_0}{I_{test}} = Z = \frac{250 + 200}{10 + 20s} = \frac{45}{2s + 1}$$

Then we have:



$$\frac{V_o}{5} + \frac{V_o(2s + 1)}{45} + \frac{V_o}{5.625s} = \frac{8}{s} \rightarrow \frac{[9s + (2s + 1)s + 8]V_o}{45s} = \frac{8}{s}$$

$$V_o[2s^2 + 10s + 8] = 360 \rightarrow V_o = \frac{360}{2s^2 + 10s + 8} = \frac{180}{s^2 + 5s + 4}$$

b) Find $v_o(t)$

$$V_o = \frac{180}{s^2 + 5s + 4} = \frac{180}{(s + 1)(s + 4)} = \frac{A}{s + 1} + \frac{B}{s + 4}$$

$$A = \left.\frac{180}{(s + 4)}\right|_{s=-1} = 60$$

$$B = \left.\frac{180}{(s + 1)}\right|_{s=-4} = -60$$

$$V_o = \frac{60}{s + 1} - \frac{60}{s + 4} \rightarrow v_o(t) = 60(e^{-t} - e^{-4t})u(t)\ (V)$$