c, $P(F_1|D) = \frac{P(F_1 \cap D)}{P(D)} = \frac{50/300}{11/30}$ | $P(D) = ? \Rightarrow \frac{n(D)}{n(S)} = \frac{50+60}{300} = \frac{11}{30}$

d, ngược lại câu a : $P(F_1 \cap D)$

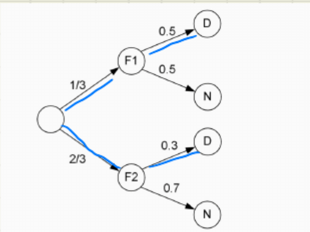
Graph Method

(1 cách để giải Bayesian' theorem)

Example

Factory 1 produced 100 items. Of them, 50 items are defective, 50 are non defective. Factory 2 produced 200 items. Of them, 60 are defective, the rest are non defective. Put all items produced by 1 and 2 together, pick 1 item

a) Find the probability that item is manufactured by 1 and defective.
b) Given that that item is manufactured by 1, find the probability that it is defective.
c) Find the probability that item is defective.
d) Given that this item is defective, find the probability that it is manufactured by Factory1.



a, Probability item manufactored by F_1 and defective

$P(F_1 \cap D) = \frac{1}{3} \times 0.5 = 0.16$

b, $P(D|F_1) = \frac{1/3 \times 0.5}{1/3 \times 0.5 + \frac{1}{3} \times 0.5} = 0.5$

c, $P(D) = \frac{1}{3} \times 0.5 + \frac{2}{3} \times 0.3$

d, $P(F_1|D) = \frac{1/3 \times 0.5}{1/3 \times 0.5 + \frac{2}{3} \times 0.3}$

2021.

Q1. (20pts)

Andy and Bill go target shooting together. Andy shoots first then Bill. Everyone shoots once. Suppose Andy hits the target with probability 0.7, whereas Bill, independently, hits the target with probability **P**. **For index 1: P=0.4**

a) **Given that EXACTLY ONE shot hit the target**, what is the probability that it was **Bill's** shot? **(10 pts)**

b) **Given that the target is hit**, what is the probability that **Andy** hit it? **(10pts)**

a) $P(B) = 0.4$

$P(A) = 0.7$

$$P(B \mid shot) = \frac{P(B \cap 1\ shot)}{P(1\ shot)} \quad (*)$$

$P(1\ shot)$ — A hit, B miss ①; A miss, B hit ② $\cap$ B' shot

$$P(1\ shot) = P(1) + P(2)$$
$$= \underset{hit}{0.7}\,(1 - \underset{miss}{0.4})_{①} + (1 - \overset{miss}{0.7})(\overset{hit}{0.4})_{②}$$
$$= 0.42 + 0.12$$
$$= 0.54$$

$$(*) = \frac{0.12}{0.54} = \frac{2}{9}$$

b)
$$P(A \mid target\ hit) = \frac{P(A \cap target\ hit)}{P(target\ hit)}$$

$P(target\ hit)$ — A hit, B miss $\to 0.42$; B hit, A miss $\to 0.12$; A and B hit $\to 0.4 \times 0.7 = 0.28$

$$= \frac{0.42 + 0.28}{0.42 + 0.12 + 0.42}$$

$$= \frac{0.7}{0.82}$$

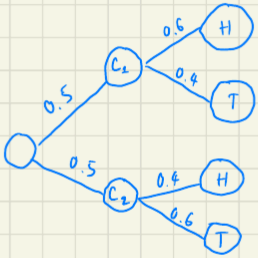
Q2. (20pts)

A man has 2 coins in his pocket, namely coin 1 with P(Head) =**P** and coin 2 with P(Head)=**1-P**

For index 1: P=0.6

a) He picks a coin randomly then toss it once, find the probability that face up is Head

b) He returned the coin in the first pick into the pockets. Then randomly pick the second time and toss. Find the expectation number of head in 2 tosses.



a) $P(H) = 0.5 \times 0.6 + 0.5 \times 0.4$
$= 0.5$

b) $E(H) = 0 \times 0.25 + 1 \times 0.5 + 2 \times 0.25$
$= 0.5 + 0.5$
$= 1$

toss twice

$E(H)$	Probability
0	$P(0) = C_2^0 . (0.5)^0 . (0.5)^{2-0} = 0.25$
1	$P(1) = C_2^1 (0.5)^1 . (0.5)^{2-1} = 0.5$
2	$P(2) = C_2^2 (0.5)^2 . (0.5)^0 = 0.25$
	(Apply Binomial Distribution)

$C_n^x \, p^x q^{n-x}$

	Binomial Distribution
Definition	Measure the probability of X successes in n trials where the probability of success in each trial is p
Formula	$P(x) = \binom{n}{x} p^x q^{(n-x)} = \frac{n!}{x!(n-x)!} p^x q^{(n-x)}$ $q = 1 - p$
E(X)	$E(X) = \mu = np$
Var(X)	$Var(X) = \sigma^2 = npq$

The discrete case

1.4 Example

1.4.1 Example 1 (tìm điều kiện của pdf, có ROI)

Given that

$$f(x,y)=\begin{cases} Cx(2x+3y) & (0\le y\le 2 \mid 0\le x\le y) \\ 0 & \text{, otherwise} \end{cases}$$

- Find C so that $f(x,y)$ is probability distribution function
- Change the integration to verify the result
- Find the p.d.f of X and Y

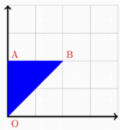


Figure 1: Figure of Example 1.

, tích phân phải = 1

$$\boxed{\iint_{ROI} f(x,y)\,dx\,dy = 1}$$

C1: Tính x trước

$0 \le y \le 2$, $0 \le a \le y \Rightarrow a \le y$

$\Rightarrow a \le y \le 2 \Rightarrow ROI_y\ [a, 2]$

} tích phân theo y (dy)

$$\int_a^2 Ca(2a+3y)\,dy$$

$$= \int (2Ca^2 + 3Cay)\,dy$$

$$= 2Ca^2y + \frac{3}{2}Cay^2 \Big|_a^2$$

$$= 2Ca^2y\Big|_a^2 + \frac{3}{2}Cay^2\Big|_a^2$$

$$= \frac{Ca(7a+6)(2-a)}{2}$$

Tiểu phân theo x :

Thay $a = x$ vào $\dfrac{Ca(7a+6)(2-a)}{2}$ $\left(x = 0 \text{ vì } 0 \leq x \leq y \leq 2 \Rightarrow 0 \leq x \leq 2 \right)$

We have :

$$\int_0^2 \frac{Ca(7a+6)(2-a)}{2}\, dx = \frac{26C}{3} = 1$$

$$\Rightarrow C = \frac{3}{26}$$

C2 : Tính y trước (*)

Assume $y = b$

ROI = $0 \leq b \leq 2 \Rightarrow 0 \leq x \leq b$

$$\int_0^b Cx(2x + 3b)\, dx = \frac{2}{3} Cx^3 \Big|_0^b + \frac{3}{2} Cbx^2 \Big|_0^b$$

$$= \frac{13}{6} Cb^3$$

Thay $b = y$:

$$\int_0^2 \frac{13}{6} y^3\, dy = \frac{26}{3} C = 1$$

$$\Rightarrow C = \frac{3}{26}$$

1.4.2 Example 2

Given that $f(x, y)$ as follows:

$$f = x(2y + A)$$

On the region:

$f(x,y) = x(2y + A)$

Step 1: find ROI:

$0 \le x \le 2$

$0 \le y \le 1$

$y \le x$

$y \le -x+2 \Leftrightarrow x \le 2-y$

$\boxed{0 \le y \le x} \le 2-y$

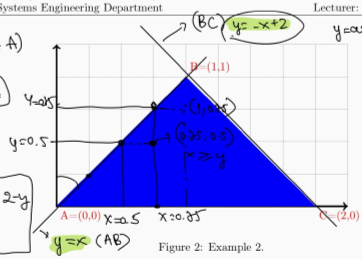


Figure 2: Example 2.

$y = ax + b$

$1 = a1 + b$
$2 = a0 + b$
$\Rightarrow b = 2$

$1 = a + 2 \Rightarrow a = -1$

$0.25 \le -1 + 2$
$y \le -x + 2$

- Find C so that $f(x, y)$ is probability distribution function
- Change the integration to verify the result
- Find the probability regarding to the area $ABCEFD$

C1: Assume $y = b$

$f(x, y) = x(2y + A)$

Step 1: find ROI:

$0 \leqslant x \leqslant 2$ ①
$0 \leqslant y \leqslant 1$ ②
$y \leqslant x$
$\Rightarrow y \leqslant -x + 2 \Leftrightarrow x \leqslant 2 - y$

①②③④ $\Rightarrow 0 \leqslant y \leqslant x \leqslant 2 - y$

$y = ax + b$

Đường AB: $x = y$ ③

Đường BC: $\begin{cases} a1 + b = 1 \quad (*) \\ a0 + b = 2 \end{cases}$

$\Rightarrow b = 2$

thế vào $(*) \Rightarrow a = -1$

$\Rightarrow BC = -x + 2$ ④

ROI of x : $y \le x \le 2 - y$

$\Leftrightarrow b \le x \le 2 - b$

We have :

$$\int_b^{2-b} x\,(2b + A)\,dx = bx^2 \Big|_2^{2-b} + \frac{1}{2} A x^2 \Big|_b^{2-b}$$

$$= -2(A + 2b)(b - 1)$$

Replace $b = y$:

$$\int_0^1 -2\,(A + 2y)(y - 1)\,dy = A + \frac{2}{3} = 1$$

$\Rightarrow A = \frac{1}{3} \to$ Điều kiện để $f(x, y)$ trở thành pdf

C2 : Assume $x = a$

- $0 \le x \le 1 \Rightarrow 0 \le y \le x$
- $1 \le x \le 2 \Rightarrow 0 \le y \le 2 - x$

When $0 \le a \le 1 \Rightarrow y \in [0, a]$

$$\int_0^a a\,(2y + A)\,dy = ay^2 \Big|_0^a + aAy \Big|_0^a = a^2(A + a) \quad (1)$$

When $1 \le a \le 2 \Rightarrow y \in [0, 2 - a]$

$$\int_0^{2-a} a\,(2y + A)\,dy = ay^2 \Big|_0^{2-a} + aAy \Big|_0^{2-a}$$

$$= -a(a - 2)(A - a + 2) \quad (2)$$

Replace $a = x$:

①② $\Rightarrow \int_0^1 x^2 (A + x)\,dx + \int_1^2 -x(x-2)(A - x + 2)\,dx = \frac{A}{3} + \frac{1}{4} + \frac{2}{3}A + \frac{5}{12} = 1$

$\Rightarrow A = \frac{1}{3}$

·) Find probability regarding to Area ABCEFD

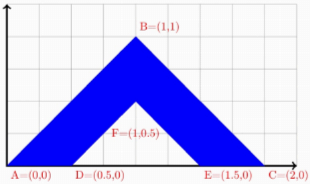


Figure 3: Example 2.

2021.

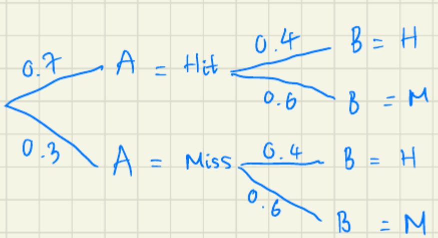
Q1. (20pts)

Andy and Bill go target shooting together. Andy shoots first then Bill. Everyone shoots once. Suppose Andy hits the target with probability 0.7, whereas Bill, independently, hits the target with probability **P**. **For index 1: P=0.4**

a) **Given that EXACTLY ONE shot hit the target**, what is the probability that it was **Bill's** shot? **(10 pts)**

b) **Given that the target is hit**, what is the probability that **Andy hit it? (10pts)**

a,



$P(B \cap 1\text{ hit}) = 0.3 \times 0.4 = 0.12$

$P(1\text{ hit}) = 0.7 \times 0.6 + 0.3 \times 0.4 = 0.54$

$P(B = H \mid 1\text{ shot}) = \dfrac{0.12}{0.54} = \dfrac{2}{9}$

b, $P(A = H) = 0.7 \times 0.4 + 0.7 \times 0.6 = 0.7$

$P(H) = 0.7 \times 0.4 + 0.7 \times 0.6 + 0.3 \times 0.4 = 0.82$

$P(H \cap A) = \dfrac{0.7}{0.82} = \dfrac{35}{41} = 0.85$

Q2. (20pts)
A man has 2 coins in his pocket, namely coin 1 with P(Head) =**P** and coin 2 with P(Head)=**1-P**

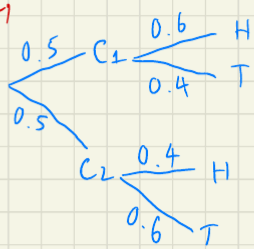
For index 1: P=0.6

For index 2: P=0.7

a) He picks a coin randomly then toss it once, find the probability that face up is Head

b) He returned the coin in the first pick into the pockets. Then randomly pick the second time and toss. Find the expectation number of head in 2 tosses.

a)



$$P(H) = 0.5 \times 0.6 + 0.5 \times 0.4 = 0.5$$

b)

Integration	Probability
0	$C_2^0 \times 0.5^0 \times 0.5^{2-0} = 0.25$
1	$C_2^1 \times 0.5^1 \times 0.5^{2-1} = 0.5$
2	$C_2^2 \times 0.5^2 \times 0.5^{2-2} = 0.25$

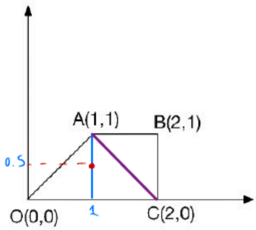
Expectation of head in 2 tosses:

$$E = 0.25 \times 0 + 0.5 \times 1 + 0.25 \times 2 = 1$$

Q3. (20pts)

Given that

$$f(x,y) = \{K(x + y) \text{ on } OABC \ 0, \text{ otherwise}$$



a) Find C so that f(x,y) is a distribution function (10pts)

b) ~~Find probability under OAC (10pts)~~

b, Change the verifi

a)

OA : $y \leq x$

AB : $y = 1$

BC : $x = 2$

OC : $y = 0$

$0 \leq y \leq 1$

$0 \leq x \leq 2$

$y \leq x$ (1)

$\Rightarrow 0 \leq y \leq x \leq 2$

$\Rightarrow 0 \leq y \leq 2 - x$ (2)

Assume $y = b$

(1) (2)

$\Rightarrow b \leq x \leq 2 - b$

$$\int_b^{2-b} K(x+b)\,dx = -\frac{3}{2}Kb^2 + 2Kb + 2K$$

Replace $b = y$:

$$\int_0^1 k(x+y)\,dy = \int_0^1 -\frac{3}{2}ky^2 + 2ky + 2k$$

$$= \frac{5}{2}k$$

$f(x, y)$ is pdf $\Rightarrow \frac{5}{2}k = 1$

$$k = 1 : \frac{5}{2}$$

$$= \frac{2}{5}$$

Assume $x = a$:

$$\int_0^{2-a} k(a+y)\,dy = \frac{-a^2 - 2a + 4}{5} . k$$

Replace $a = x$:

$$\int_0^2 k . \frac{-x^2 - 2x + 4}{5} = \frac{5}{2}k$$