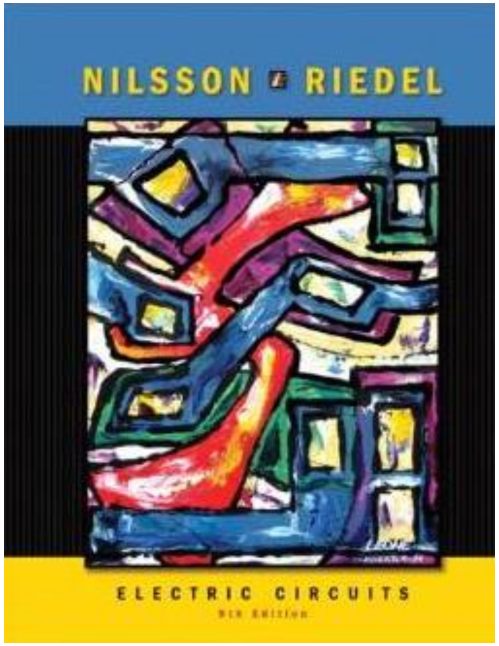
Lecture #13

Balanced Three-Phase Circuits



Text book: Electric Circuits

James W. Nilsson & Susan A. Riedel

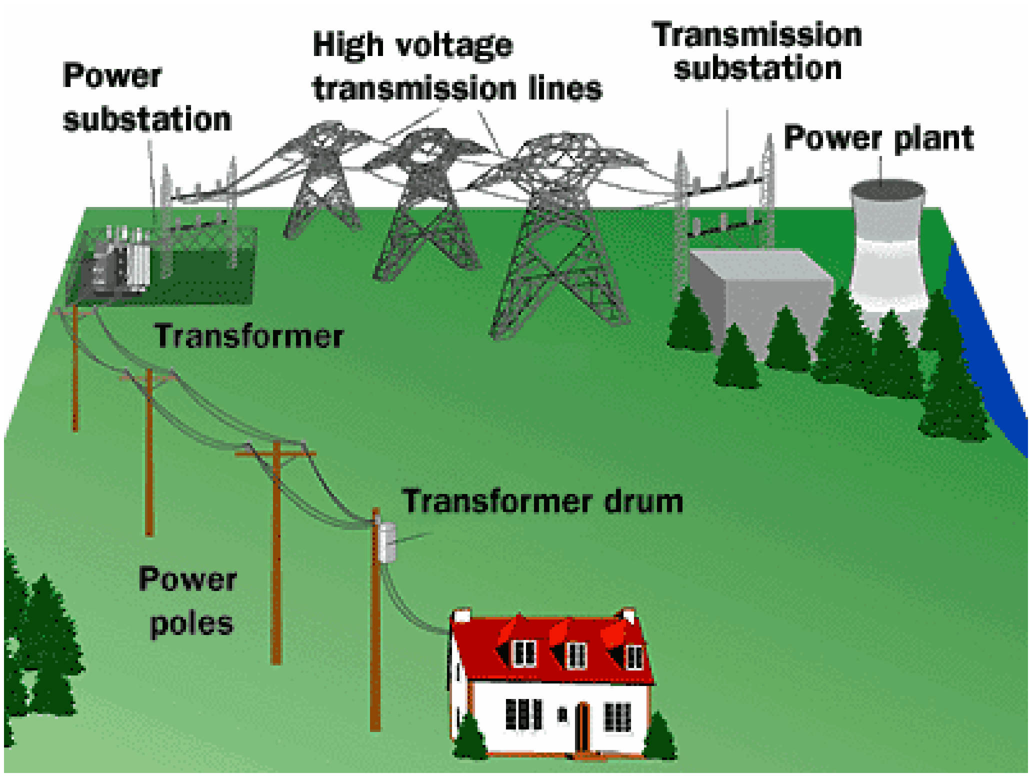
9th Edition.

link: **http://blackboard.hcmiu.edu.vn/**

to download materials

Overview

An electric power distribution system looks like:

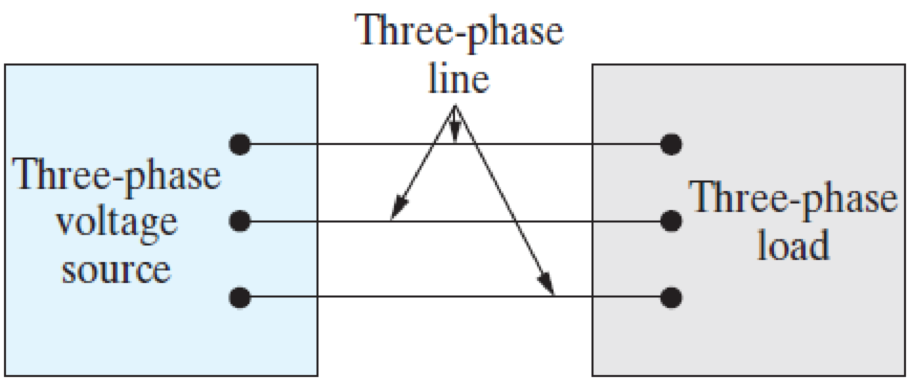


where the power transmission uses “balanced three-phase” configuration.

Why three-phase?

Three-phase circuits are used for generating, transmitting, distributing and using large blocks of electric power.

The basic structure of a three-phase system consist of voltage sources connected to loads by means of transformers and transmission lines. To analyze such a circuit, we can reduce it to a voltage source connected to a load via a line.



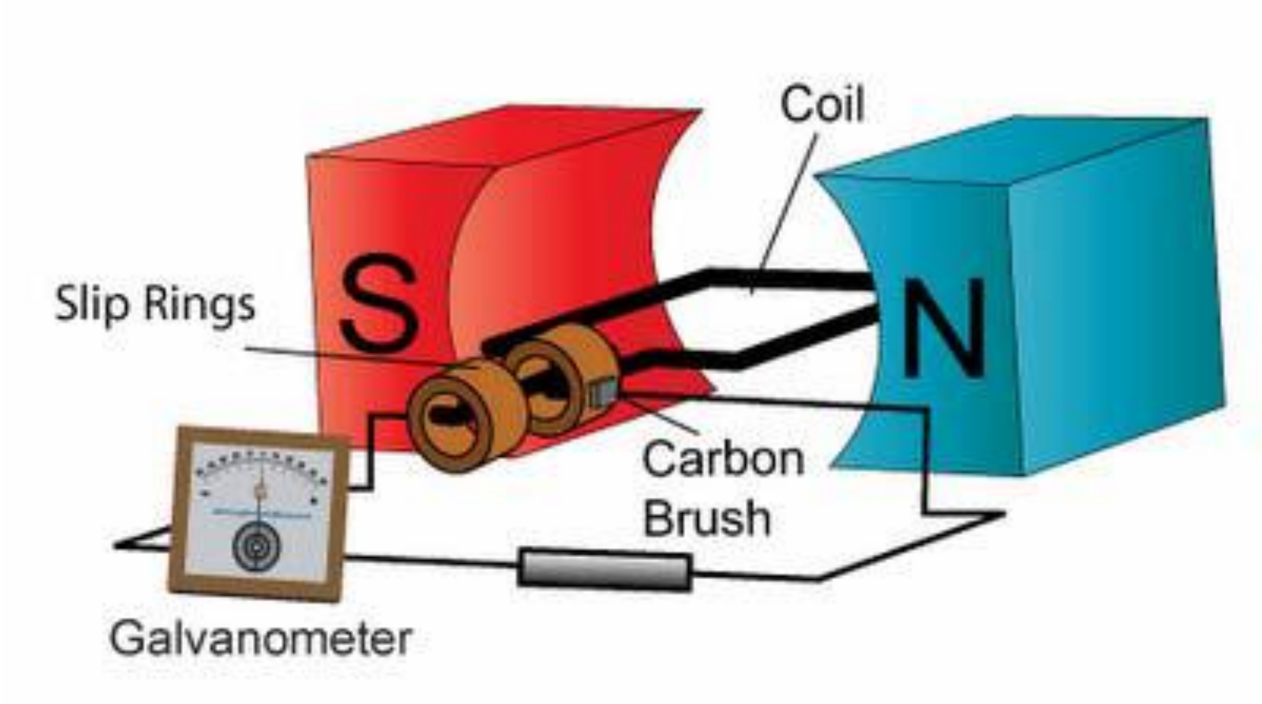
Faraday’s Law

“The EMF induced in a circuit is directly proportional to the time rate of change of magnetic flux through the circuit.”

The EMF can be produced by changing B (induced EMF) or by changing the area, e.g., by moving the wire (motional EMF).

One-phase voltage sources

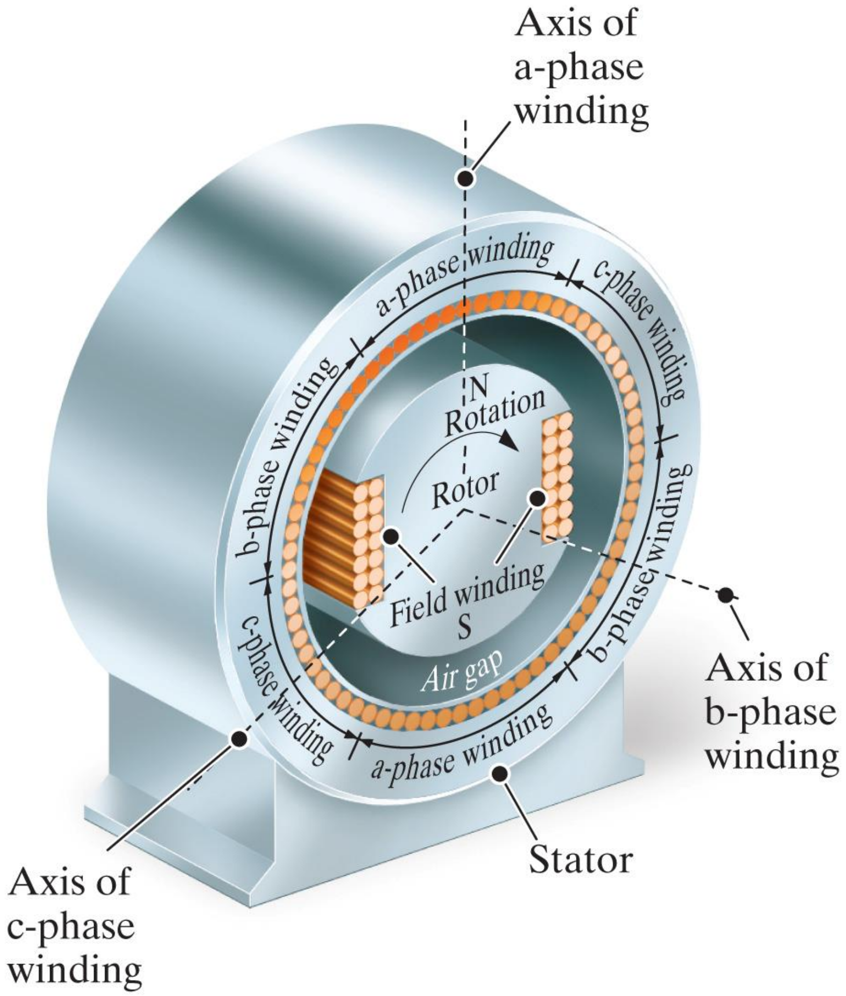
One-phase ac generator: static magnets, one rotating coil, single output voltage $v(t) = V_m \cos\omega t$



(www.ac-motors.us)

Three-phase voltage sources

Three static coils, rotating magnets, three output voltages $v_a(t)$, $v_b(t)$, $v_c(t)$.



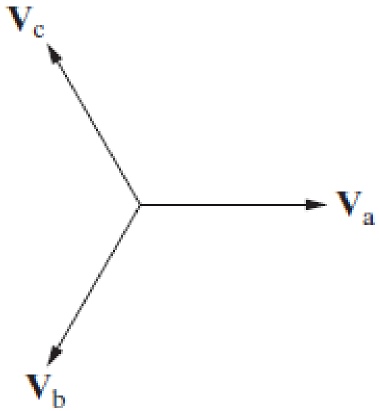
Balanced Three-phase Voltages

Three sinusoidal voltages of the same amplitude, frequency, but differing by 120° phase difference with one another.

There are two possible sequences:

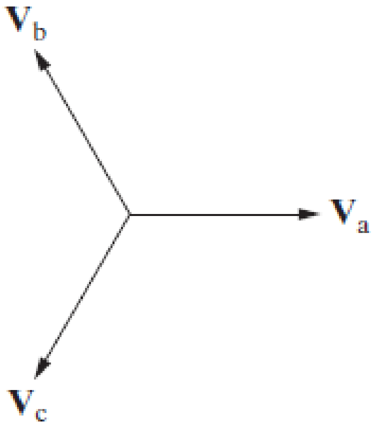
1. abc (positive) sequence: $v_b(t)$ lags $v_a(t)$ by $120°$.
2. acb (negative) sequence: $v_b(t)$ leads $v_a(t)$ by $120°$.

abc (positive) phase sequence



$$V_a = V_m \angle 0^\circ$$
$$V_b = V_m \angle -120^\circ$$
$$V_c = V_m \angle +120^\circ$$

acb (negative) phase sequence



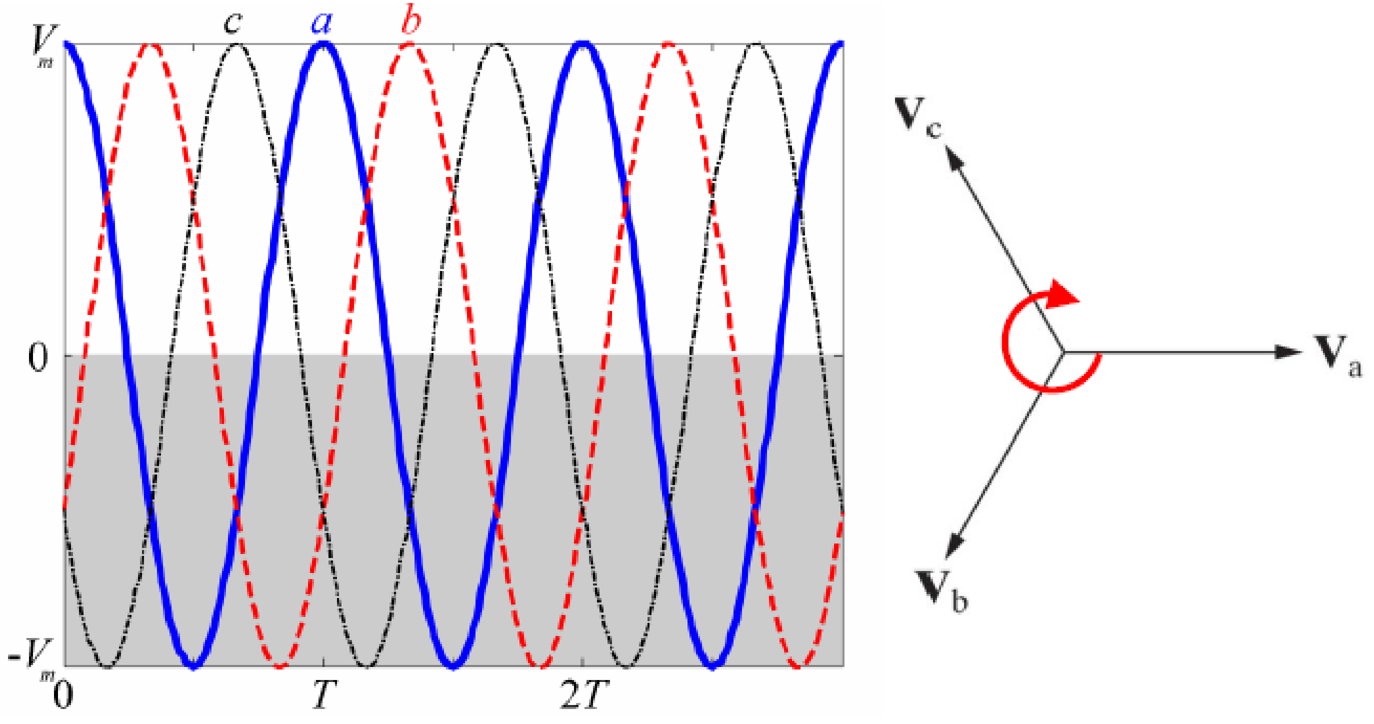
$$V_a = V_m \angle 0^\circ$$
$$V_b = V_m \angle +120^\circ$$
$$V_c = V_m \angle -120^\circ$$

An important characteristic of a set of balanced three-phase voltages is that the sum of the voltages is zero $V_a + V_b + V_c = 0$

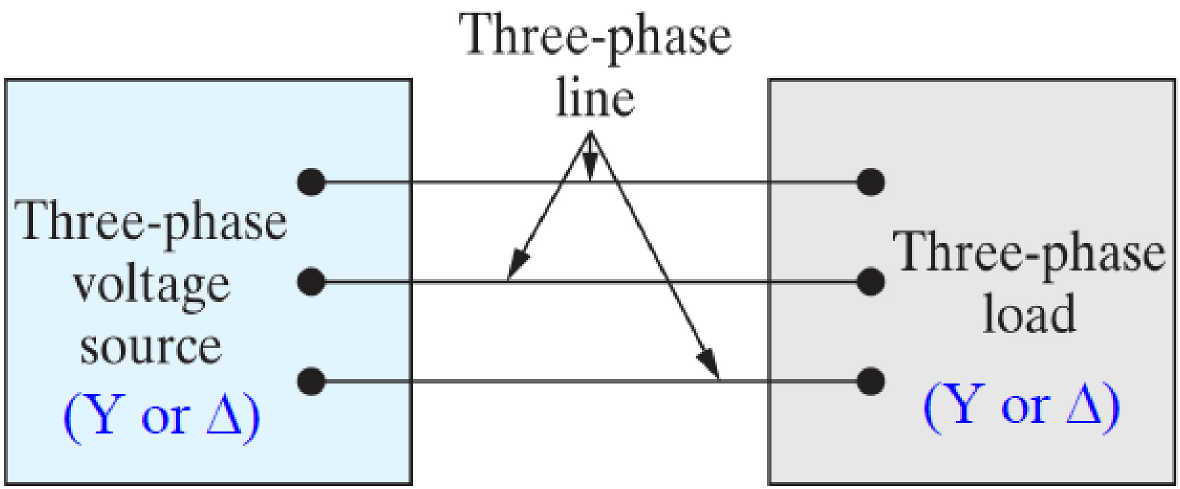
Balanced Three-phase Voltages

abc sequence

- $v_b(t)$ lags $v_a(t)$ by 120° or $T/3$.
- $\mathbf{V}_a = V_m\angle 0^\circ,\ \mathbf{V}_b = V_m\angle -120^\circ,\ \mathbf{V}_c = V_m\angle +120^\circ.$

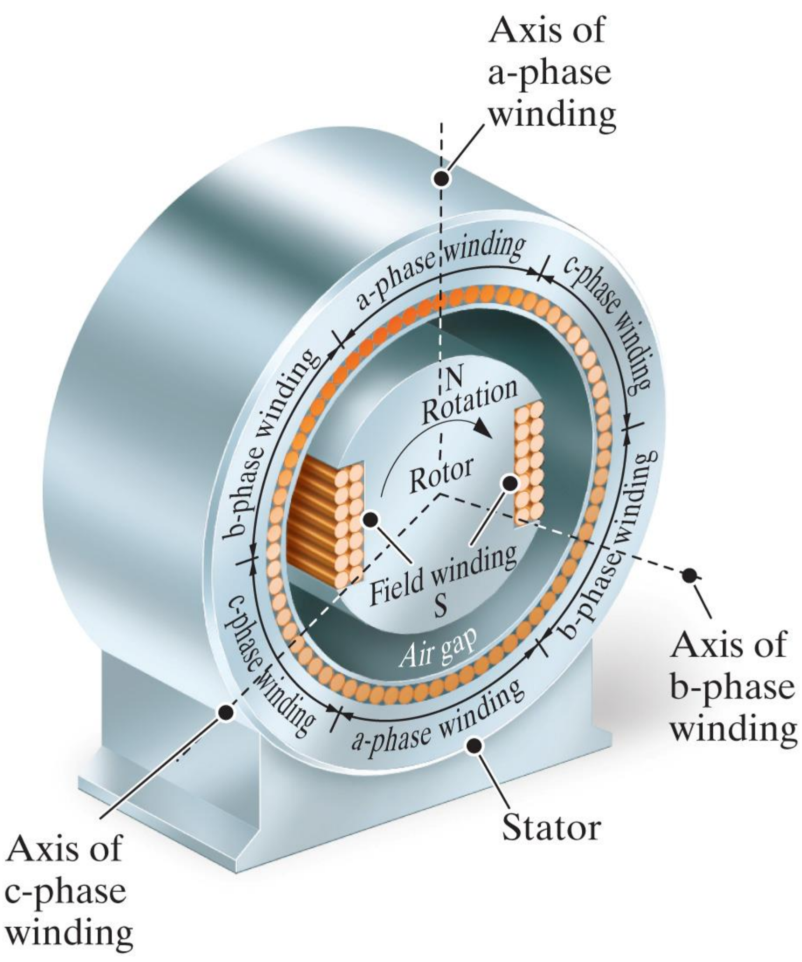


Three-phase systems



- Source-load can be connected in four configurations: Y-Y, Y-Δ, Δ-Y, Δ-Δ.
- It's sufficient to analyze Y-Y, while the others can be treated by Δ-Y and Y-Δ transformations.

Balanced Three-phase Voltages



A three-phase voltage source is a generator with three separate windings distributed around the periphery of the stator.

Each winding comprises one phase of the generator.

The rotor of the generator is an electromagnet driven at synchronous speed by a prime mover, such as a steam or gas turbine.

Rotation of the electromagnet induces a sinusoidal voltage in each winding.

The phase windings are designed so that the sinusoidal voltages induced in them are equal in amplitude and out of phase with each other by 120º.

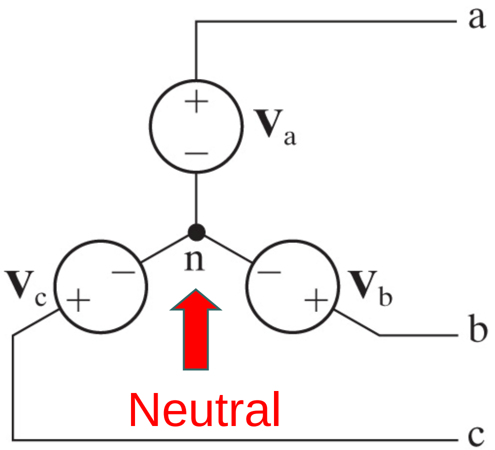
The phase windings are stationary with respect to the rotating electromagnet, so the frequency of the voltage induced in each winding is the same.

Balanced Three-phase Voltages

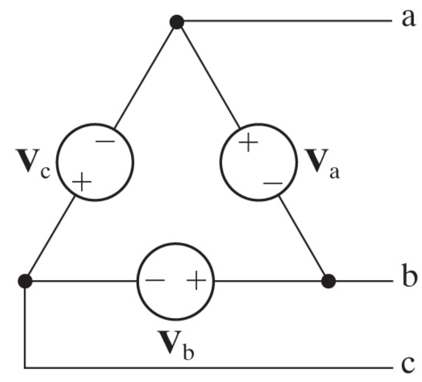
Ideal Y- and Δ-connected voltage sources

- There are two ways of interconnecting the separate phase windings to form a three-phase source:

Y configuration



Δ configuration



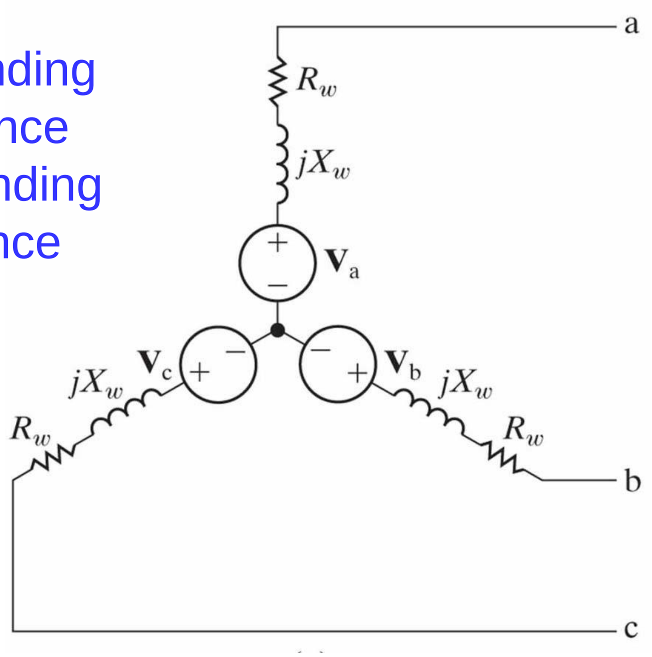
n is the neutral terminal of the source. The neutral terminal may or may not be available for external connections

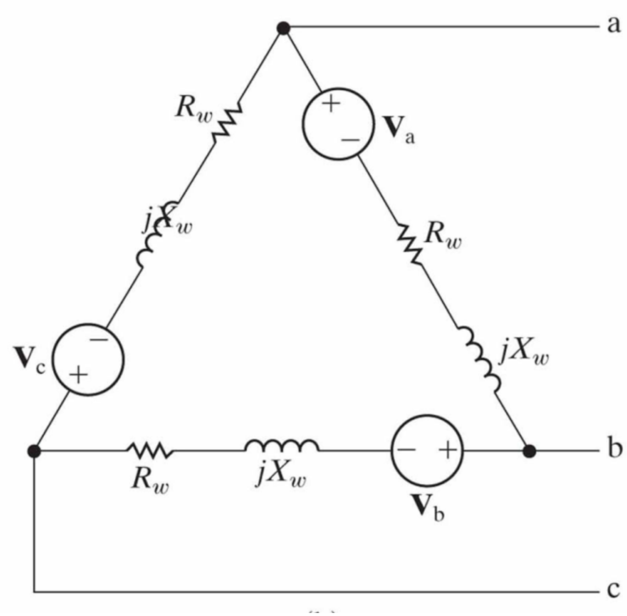
Balanced Three-phase Voltages

Real Y- and Δ-connected voltage sources

- Three-phase sources and loads can be either Y or Δ-connected, therefore, there are 4 different configuration between source and load: Y – Y, Y – Δ, Δ – Y, Δ – Δ.

R_w: winding resistance
X_w : winding reactance



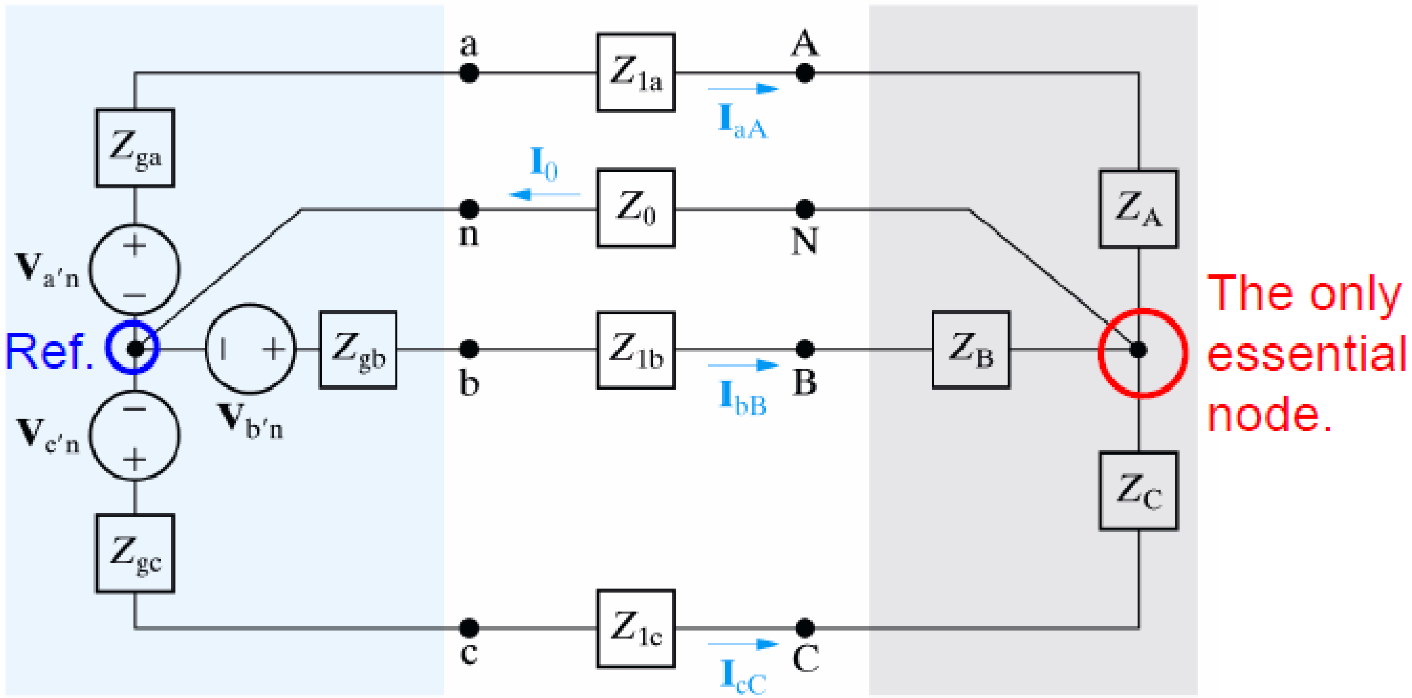


A model of a three-phase source with winding impedance

Internal impedance of a generator is usually inductive (due to the use of coils).

Analysis of the Y-Y circuit

General Y-Y circuit model



Z_{ga}, Z_{gb}, Z_{gc} : the internal impedance associated with each phase winding of the voltage generator.

Z_{la}, Z_{lb}, Z_{lc} : the impedance of the lines connecting a phase of the source to a phase of the load

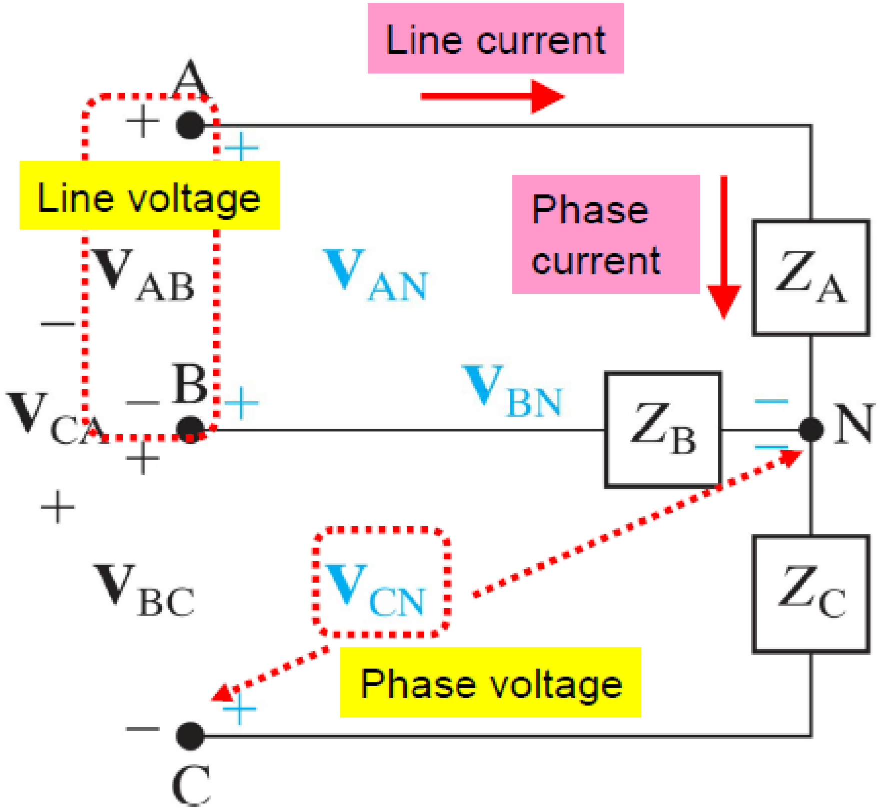
Z_0 : the impedance of the neutral conductor connecting the source neutral to the load neutral

Z_A, Z_B, Z_C : the impedance of each phase of the load.

Analysis of the Y-Y circuit

Unknowns to be solved

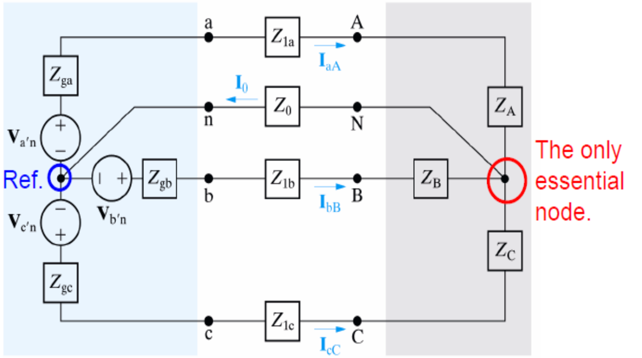
- Line (line-to-line) voltage: voltage across any pair of lines.
- Phase (line-to-neutral) voltage: voltage across a single phase.



- For Y-connected load, line current equals phase current.

Analysis of the Y-Y circuit

Solution to general three-phase circuit



No matter it's balanced or imbalanced three- phase circuit, KCL leads to one equation:

$$\mathbf{I}_0 = \mathbf{I}_{aA} + \mathbf{I}_{bB} + \mathbf{I}_{cC}, \Rightarrow$$

$$\frac{\mathbf{V}_N}{Z_0} = \frac{\mathbf{V}_{a'n} - \mathbf{V}_N}{Z_{ga} + Z_{1a} + Z_A} + \frac{\mathbf{V}_{b'n} - \mathbf{V}_N}{Z_{gb} + Z_{1b} + Z_B} + \frac{\mathbf{V}_{c'n} - \mathbf{V}_N}{Z_{gc} + Z_{1c} + Z_C} \ldots (1),$$

Impedance of neutral line.

Total impedance along line aA.

Total impedance along line bB.

Total impedance along line cC.

which is sufficient to solve V_N (thus the entire circuit).

Analysis of the Y-Y circuit

Solution to “balanced” three-phase circuit

- For balanced three-phase circuits,

1. $\{\mathbf{V}_{a'n}, \mathbf{V}_{b'n}, \mathbf{V}_{c'n}\}$ have equal magnitude and $120°$ relative phases;
2. $\{Z_{ga} = Z_{gb} = Z_{gc}\}$, $\{Z_{1\mathrm{a}} = Z_{1\mathrm{b}} = Z_{1\mathrm{c}}\}$, $\{Z_A = Z_B = Z_C\}$;
   $\Rightarrow$ total impedance along any line is the same
   $Z_{ga} + Z_{1\mathrm{a}} + Z_A = \ldots = Z_\phi$.

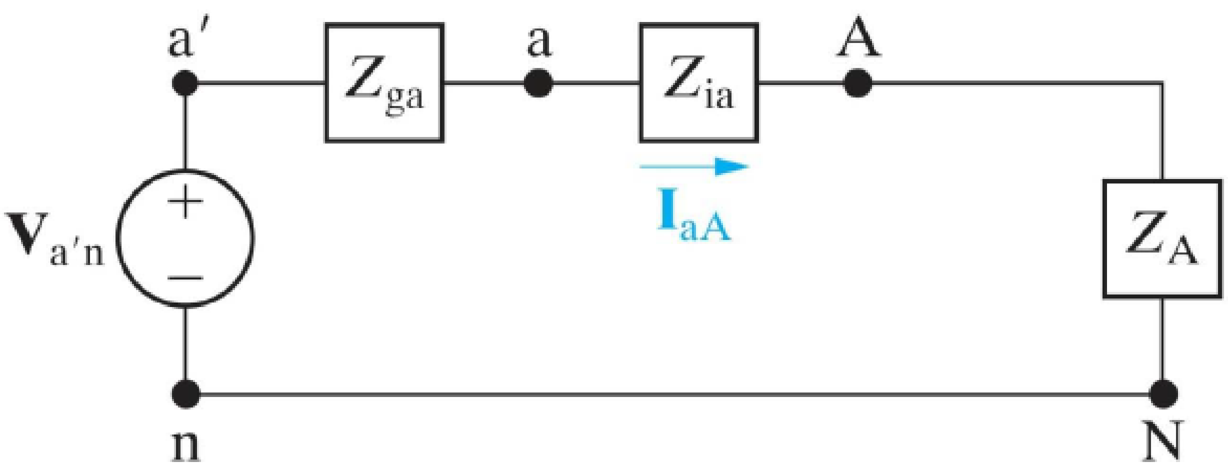
- Eq. (1) becomes: $\dfrac{\mathbf{V}_N}{Z_0} = \dfrac{\mathbf{V}_{a'n} - \mathbf{V}_N}{Z_\phi} + \dfrac{\mathbf{V}_{b'n} - \mathbf{V}_N}{Z_\phi} + \dfrac{\mathbf{V}_{c'n} - \mathbf{V}_N}{Z_\phi}$,

$$\Rightarrow \mathbf{V}_N\left(\frac{1}{Z_0} + \frac{3}{Z_\phi}\right) = \frac{\mathbf{V}_{a'n} + \mathbf{V}_{b'n} + \mathbf{V}_{c'n}}{Z_\phi} = 0, \quad \mathbf{V}_N = 0.$$

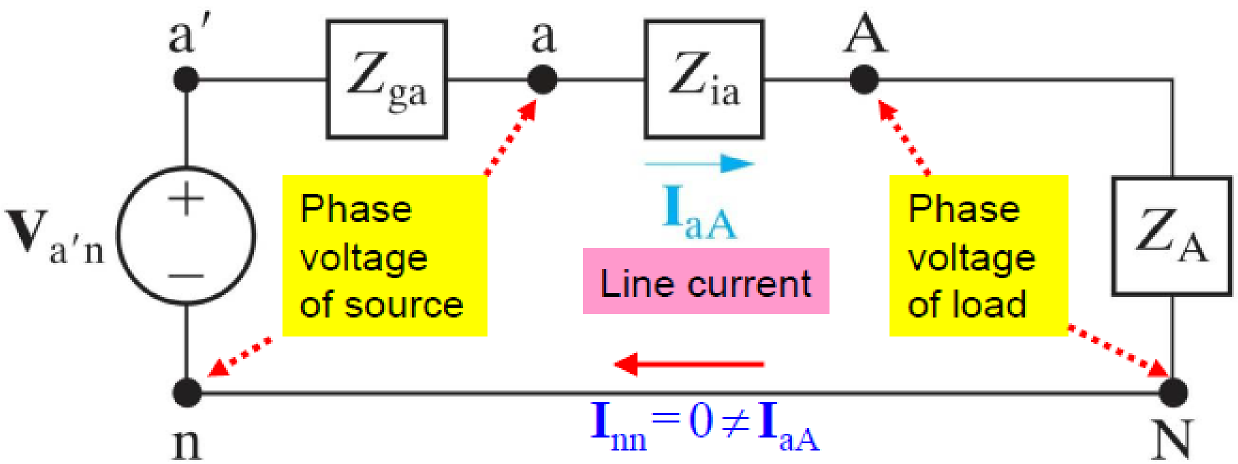
Analysis of the Y-Y circuit

Meaning of the solution

- $\mathbf{V}_N = 0$ means no voltage difference between nodes $\boldsymbol{n}$ and $\boldsymbol{N}$ in the presence of Z_0. $\Rightarrow$ Neutral line is both short $(v = 0)$ and open $(i = 0)$.
- The three-phase circuit can be separated into 3 one-phase circuits (open), while each of them has a short between nodes $\boldsymbol{n}$ and $\boldsymbol{N}$.



Equivalent one-phase circuit



- Directly giving the line current & phase voltages:

$$\mathbf{I}_{aA} = \frac{\mathbf{V}_{a'n} - \cancel{\mathbf{V}_N}}{\left(Z_{ga} + Z_{1a} + Z_A\right) = Z_\phi}, \ \mathbf{V}_{AN} = \mathbf{I}_{aA} Z_A, \ \mathbf{V}_{an} = \mathbf{I}_{aA}\left(Z_{1a} + Z_A\right).$$

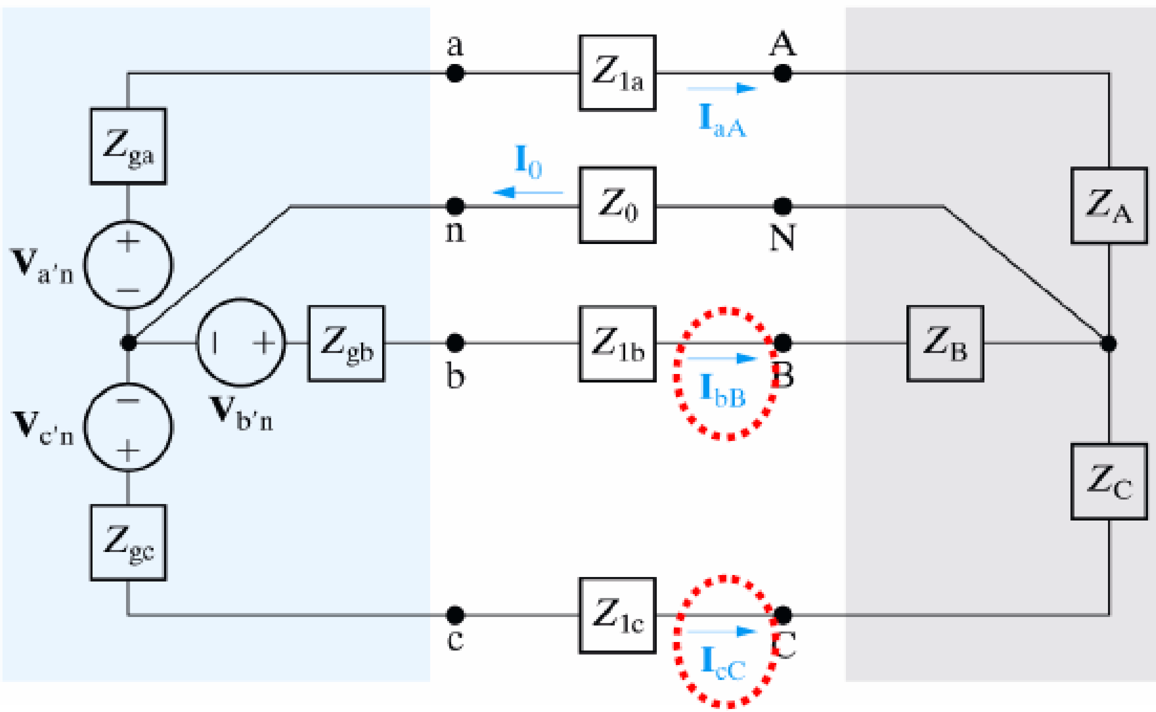
- Unknowns of phases b, c can be determined by the fixed (abc or acb) sequence relation.

Analysis of the Y-Y circuit

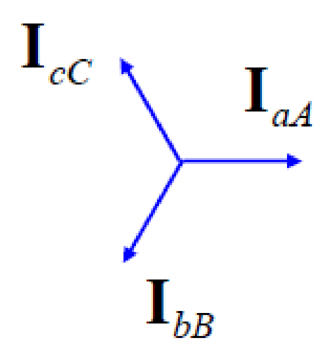
The 3 line and phase currents in abc sequence

Given $\mathbf{I}_{aA} = \mathbf{V}_{a'n} / Z_{\phi}$, the other 2 line currents are:

$$\mathbf{I}_{bB} = \frac{\mathbf{V}_{b'n}}{Z_{\phi}} = \mathbf{I}_{aA} \angle -120^{\circ}, \quad \mathbf{I}_{cC} = \frac{\mathbf{V}_{c'n}}{Z_{\phi}} = \mathbf{I}_{aA} \angle 120^{\circ},$$



which still follow the abc sequence relation.

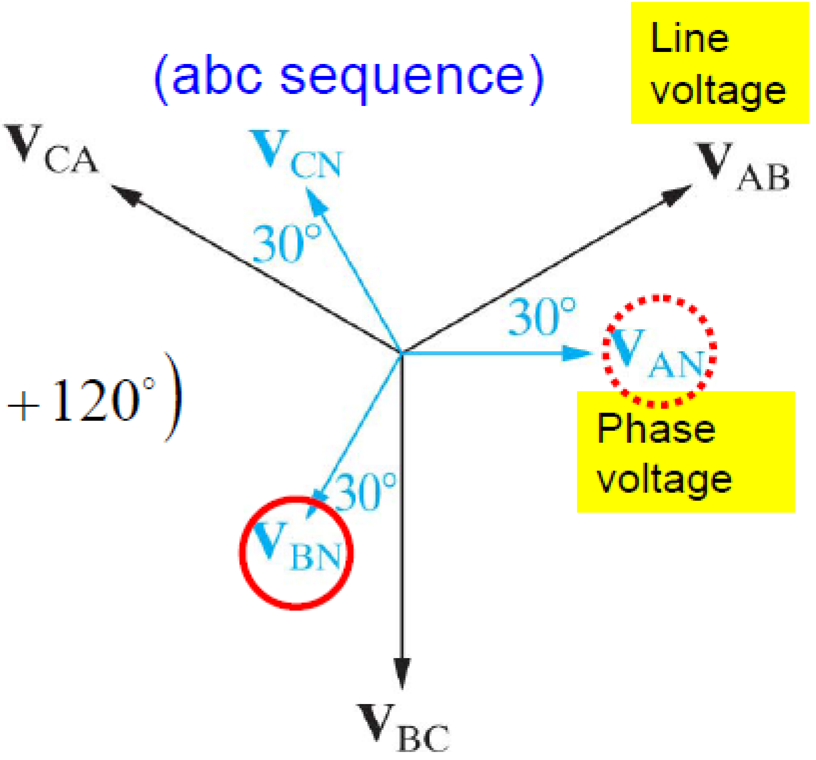


Analysis of the Y-Y circuit

The phase & line voltages of the load in **abc seq**.

$$\mathbf{V}_{AN} = \mathbf{V}_{a'n}\frac{Z_A}{Z_\phi}, \mathbf{V}_{BN} = \mathbf{V}_{b'n}\frac{Z_B}{Z_\phi} = \mathbf{V}_{AN}\angle -120^\circ, \mathbf{V}_{CN} = \mathbf{V}_{AN}\angle 120^\circ.$$

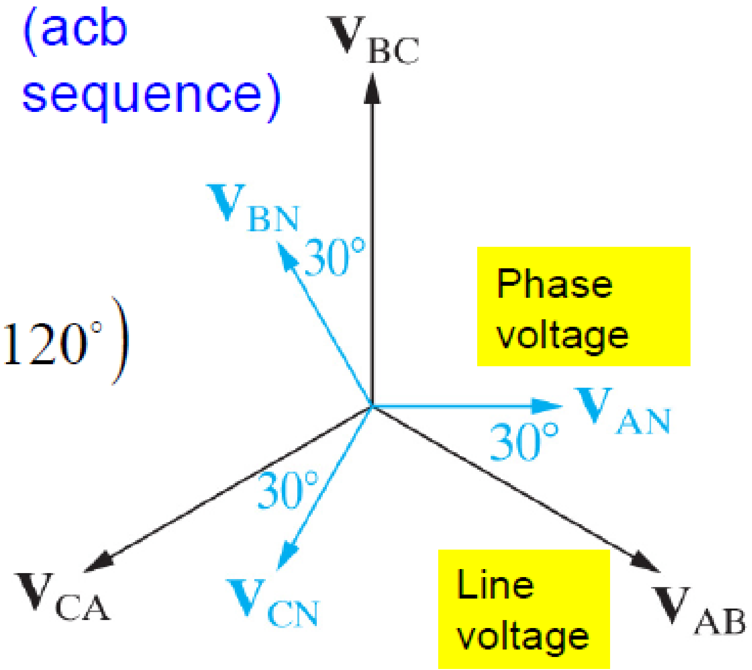
$$\begin{aligned}
\mathbf{V}_{AB} &= \mathbf{V}_{AN} - \mathbf{V}_{BN} \\
&= \mathbf{V}_{AN} - \left(\mathbf{V}_{AN}\angle -120^\circ\right) \\
&= \sqrt{3}\mathbf{V}_{AN}\angle +30^\circ, \\
\mathbf{V}_{BC} &= \left(\mathbf{V}_{AN}\angle -120^\circ\right) - \left(\mathbf{V}_{AN}\angle +120^\circ\right) \\
&= \sqrt{3}\mathbf{V}_{AN}\angle -90^\circ, \\
\mathbf{V}_{CA} &= \left(\mathbf{V}_{AN}\angle +120^\circ\right) - \mathbf{V}_{AN} \\
&= \sqrt{3}\mathbf{V}_{AN}\angle +150^\circ.
\end{aligned}$$



Analysis of the Y-Y circuit

The phase & line voltages of the load in **acb seq**.

$$
\begin{aligned}
\mathbf{V}_{AB} &= \mathbf{V}_{AN} - \mathbf{V}_{BN} \\
&= \mathbf{V}_{AN} - \left(\mathbf{V}_{AN}\angle +120^{\circ}\right) \\
&= \sqrt{3}\mathbf{V}_{AN}\angle -30^{\circ}, \\
\mathbf{V}_{BC} &= \left(\mathbf{V}_{AN}\angle +120^{\circ}\right) - \left(\mathbf{V}_{AN}\angle -120^{\circ}\right) \\
&= \sqrt{3}\mathbf{V}_{AN}\angle +90^{\circ}, \\
\mathbf{V}_{CA} &= \left(\mathbf{V}_{AN}\angle -120^{\circ}\right) - \mathbf{V}_{AN} \\
&= \sqrt{3}\mathbf{V}_{AN}\angle -150^{\circ}.
\end{aligned}
$$



- Line voltages are $\sqrt{3}$ times bigger, leading (abc) or lagging (acb) the phase voltages by $30°$.

Analysis of the Y-Y circuit

Load balance $V_N=0$

$$I_{aA} = \frac{V_{a'n} - V_N}{Z_A + Z_{1a} + Z_{ga}} = \frac{V_{a'n}}{Z_\phi}$$

$$I_{bB} = \frac{V_{b'n} - V_N}{Z_B + Z_{1b} + Z_{gb}} = \frac{V_{b'n}}{Z_\phi}$$

$$I_{cC} = \frac{V_{c'n} - V_N}{Z_C + Z_{1c} + Z_{gc}} = \frac{V_{c'n}}{Z_\phi}$$

Line-to-line and line-to-neutral voltages

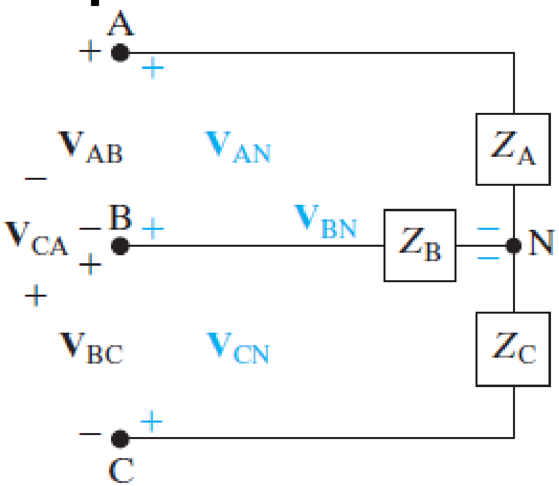
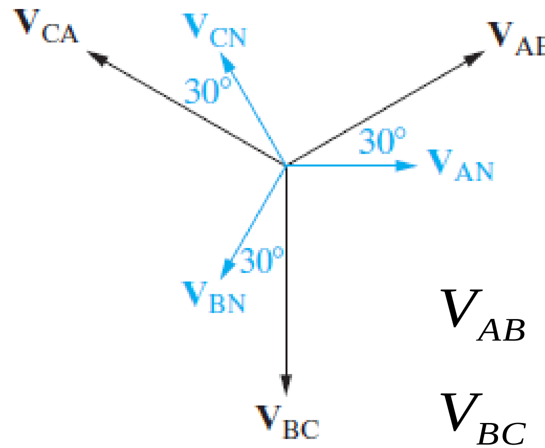


Figure 11.8 ▲ Line-to-line and line-to-neutral voltages.

$$V_{AB} = V_{AN} - V_{BN}$$

$$V_{BC} = V_{BN} - V_{CN}$$

$$V_{CA} = V_{CN} - V_{AN}$$



$$V_{AB} = \sqrt{3} V_{\phi} \angle 30^{o}$$

$$V_{BC} = \sqrt{3} V_{\phi} \angle -90^{o}$$

$$V_{CA} = \sqrt{3} V_{\phi} \angle 150^{o}$$

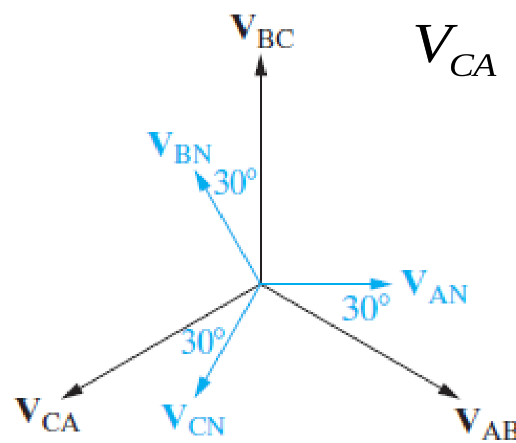
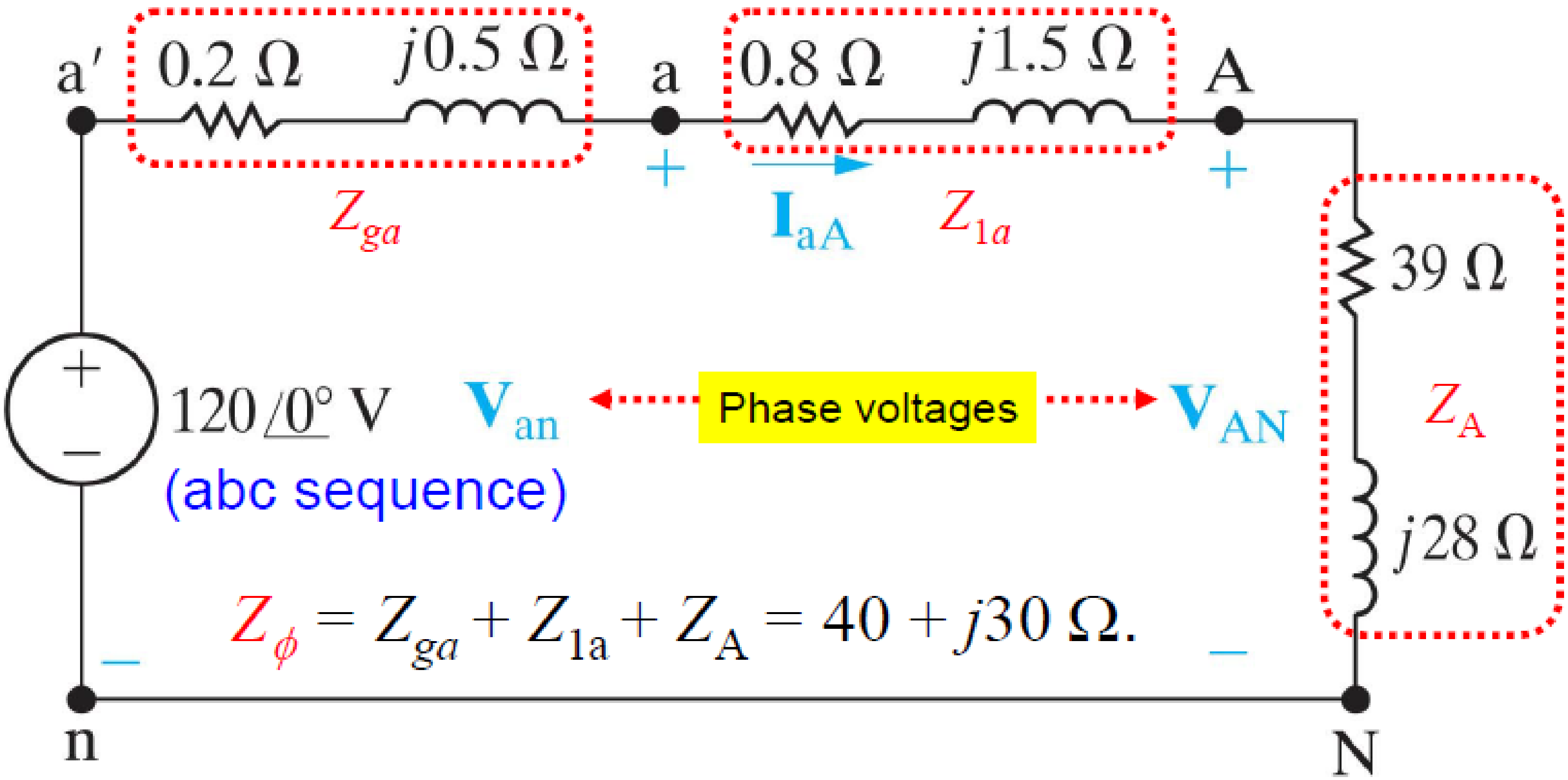


Figure 11.9 ▲ Phasor diagrams showing the relationship between line-to-line and line-to-neutral voltages in a balanced system. (a) The abc sequence. (b) The acb sequence.

Example

Q: What are the line currents, phase and line voltages of the load and source, respectively?



Example

- The 3 line currents (of both load & source) are:

$$\mathbf{I}_{aA} = \frac{\mathbf{V}_{a'n}}{Z_{ga} + Z_{1a} + Z_A} = \frac{120\angle 0^\circ}{40 + j30} = \left(2.4\angle -36.87^\circ\right)\text{A},$$

$$\mathbf{I}_{bB} = \mathbf{I}_{aA}\angle -120^\circ = \left(2.4\angle -156.87^\circ\right)\text{A},$$

$$\mathbf{I}_{cC} = \mathbf{I}_{aA}\angle +120^\circ = \left(2.4\angle +83.13^\circ\right)\text{A}.$$

- The 3 phase voltages of the load are:

$$\mathbf{V}_{AN} = \mathbf{I}_{aA} Z_A = \left(2.4\angle -36.87^\circ\right)\left(39 + j28\right) = \left(115.22\angle -1.19^\circ\right)\ \text{V}.$$

$$\mathbf{V}_{BN} = \mathbf{V}_{AN}\angle -120^\circ = \left(115.22\angle -121.19^\circ\right)\text{V},$$

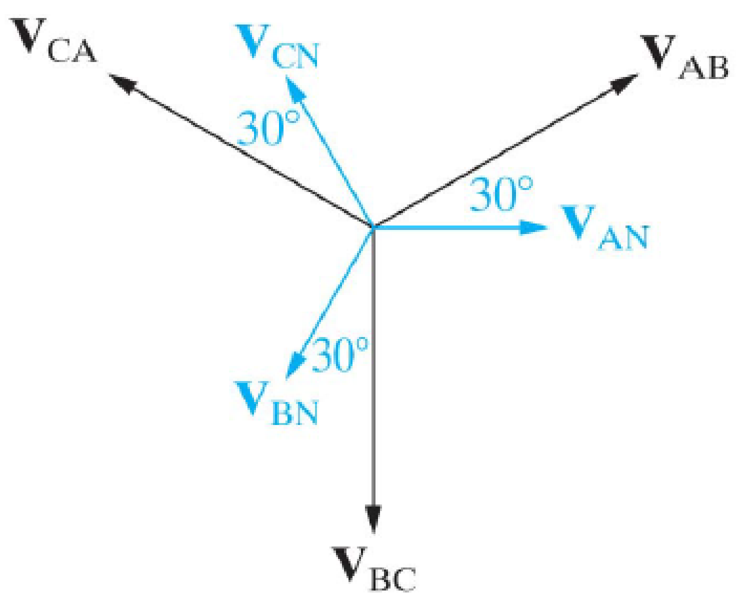
$$\mathbf{V}_{CN} = \mathbf{V}_{AN}\angle +120^\circ = \left(115.22\angle +118.81^\circ\right)\text{V}.$$

- The 3 line voltages of the load are:

$$\mathbf{V}_{AB} = \boxed{\left(\sqrt{3}\angle 30^\circ\right)}\mathbf{V}_{AN}$$
$$= \left(\sqrt{3}\angle 30^\circ\right)\left(115.22\angle -1.19^\circ\right)$$
$$= \left(199.58\angle +28.81^\circ\right)\mathrm{V},$$

$$\mathbf{V}_{BC} = \mathbf{V}_{AB}\angle -120^\circ$$
$$= \left(199.58\angle -91.19^\circ\right)\mathrm{V},$$

$$\mathbf{V}_{CA} = \mathbf{V}_{AB}\angle +120^\circ$$
$$= \left(199.58\angle +148.81^\circ\right)\mathrm{V}.$$



- The 3 phase voltages of the source are:

$$\mathbf{V}_{an} = \mathbf{V}_{a'n} - \mathbf{I}_{aA} Z_{ga} = 120 - \left(2.4\angle -36.87^\circ\right)\left(0.2 + j0.5\right)$$
$$= \left(118.9\angle -0.32^\circ\right) \text{V},$$
$$\mathbf{V}_{bn} = \mathbf{V}_{an}\angle -120^\circ = \left(118.9\angle -120.32^\circ\right) \text{V},$$
$$\mathbf{V}_{cn} = \mathbf{V}_{an}\angle +120^\circ = \left(118.9\angle +119.68^\circ\right) \text{V}.$$

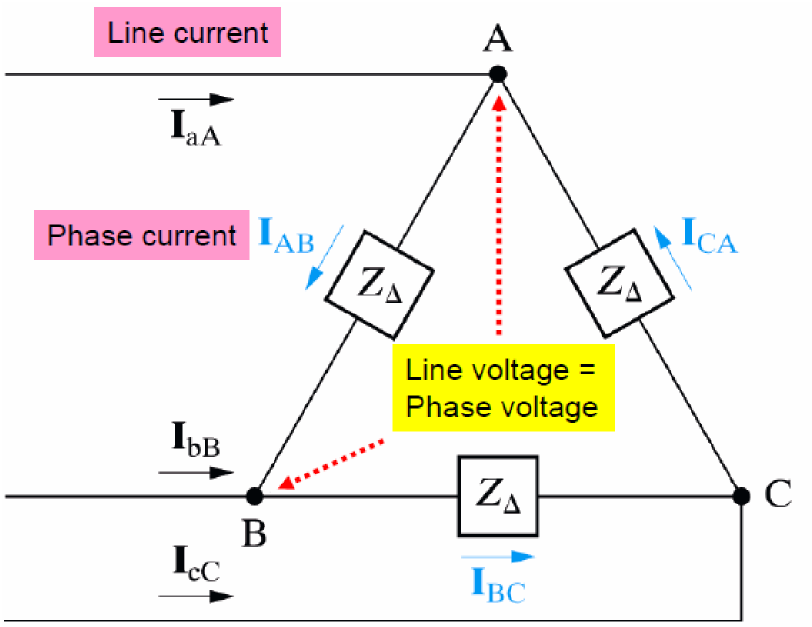
- The three line voltages of the source are:

$$\mathbf{V}_{ab} = \boxed{\left(\sqrt{3}\angle 30^\circ\right)} \mathbf{V}_{an} = \left(\sqrt{3}\angle 30^\circ\right)\left(118.9\angle -0.32^\circ\right)$$
$$= \left(205.94\angle +29.68^\circ\right) \text{V},$$
$$\mathbf{V}_{bc} = \mathbf{V}_{ab}\angle -120^\circ = \left(205.94\angle -90.32^\circ\right) \text{V},$$
$$\mathbf{V}_{ca} = \mathbf{V}_{ab}\angle +120^\circ = \left(205.94\angle +149.68^\circ\right) \text{V}.$$

Analysis of the Y-Δ Circuit

Δ-Y transformation for balanced 3-phase load

Load in Δ configuration



The impedance of each leg in Y-configuration (Z_Y) is one-third of that in Δ-configuration (Z_Δ):

$$Z_1 = \frac{Z_b Z_c}{Z_a + Z_b + Z_c},$$

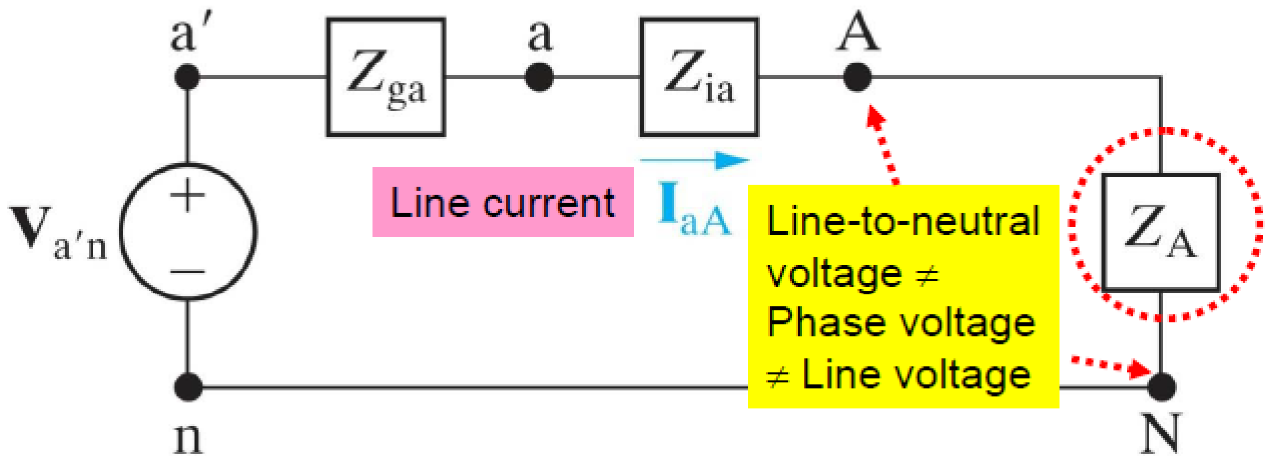
$$Z_2 = \frac{Z_c Z_a}{Z_a + Z_b + Z_c},$$

$$Z_3 = \frac{Z_a Z_b}{Z_a + Z_b + Z_c}.$$

$$\Rightarrow Z_Y = \frac{Z_\Delta Z_\Delta}{3 Z_\Delta} = \boxed{\frac{Z_\Delta}{3}}.$$

Equivalent one-phase circuit

The 1-phase equivalent circuit in Y-Y config. continues to work if Z_A is replaced by $Z_\Delta/3$:



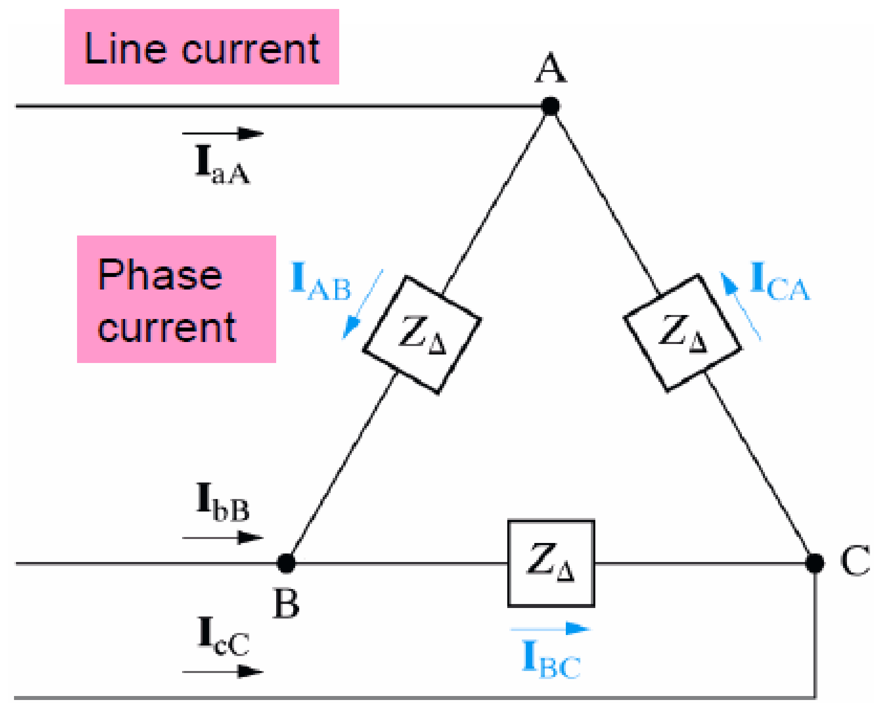
directly giving the line current: $\mathbf{I}_{aA} = \dfrac{\mathbf{V}_{a'n}}{Z_{ga} + Z_{1a} + Z_A}$,

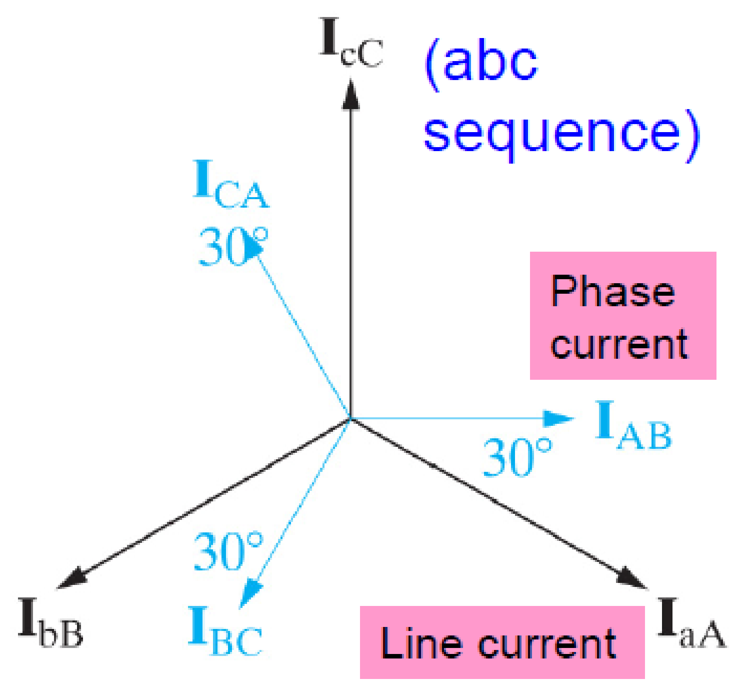
and line-to-neutral voltage: $\mathbf{V}_{AN} = \mathbf{I}_{aA} Z_A$.

The 3 phase currents of the load in abc seq.

- Can be solved by 3 node equations once the 3 line currents $\mathbf{I}_{aA}$, $\mathbf{I}_{bB}$, $\mathbf{I}_{cC}$ are known:

$$\mathbf{I}_{aA} = \mathbf{I}_{AB} - \mathbf{I}_{CA},\ \mathbf{I}_{bB} = \mathbf{I}_{BC} - \mathbf{I}_{AB},\ \mathbf{I}_{cC} = \mathbf{I}_{CA} - \mathbf{I}_{BC}.$$





$$I_{AB} = I_{\phi}\angle 0^{o}$$

$$I_{BC} = I_{\phi}\angle -120^{o}$$

$$I_{CA} = I_{\phi}\angle +120^{o}$$

$$I_{aA} = I_{AB} - I_{CA} = \sqrt{3}I_{\phi}\angle -30^{o}$$

$$I_{bB} = I_{BC} - I_{AB} = \sqrt{3}I_{\phi}\angle -150^{o}$$

$$I_{cC} = I_{CA} - I_{BC} = \sqrt{3}I_{\phi}\angle 90^{o}$$

Power Calculations in Balanced Three-Phase Circuits

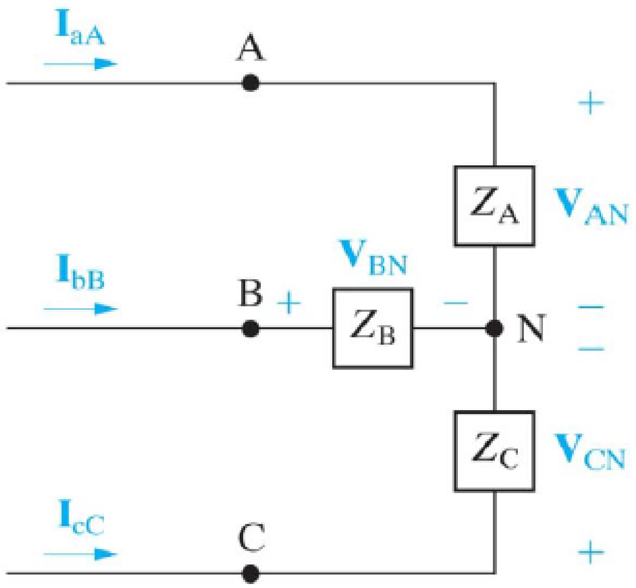
Average power of balanced Y-Load

- The average power delivered to Z_A is:

$$P_A = V_\phi I_\phi \cos\theta_\phi,$$

$$\begin{cases} V_\phi \equiv |\mathbf{V}_{AN}| = V_L / \sqrt{3}, \\ I_\phi \equiv |\mathbf{I}_{aA}| = I_L, \\ \theta_\phi \equiv \angle V_\phi - \angle I_\phi = \angle Z_A. \end{cases}$$

(rms value)



- The total power delivered to the Y-Load is:

$$P_{tot} = 3P_A = 3V_\phi I_\phi \cos\theta_\phi = \sqrt{3} V_L I_L \cos\theta_\phi.$$

Power Calculations in Balanced Three-Phase Circuits

Complex power of a balanced Y-Load

- The reactive powers of one phase and the entire Y-Load are:

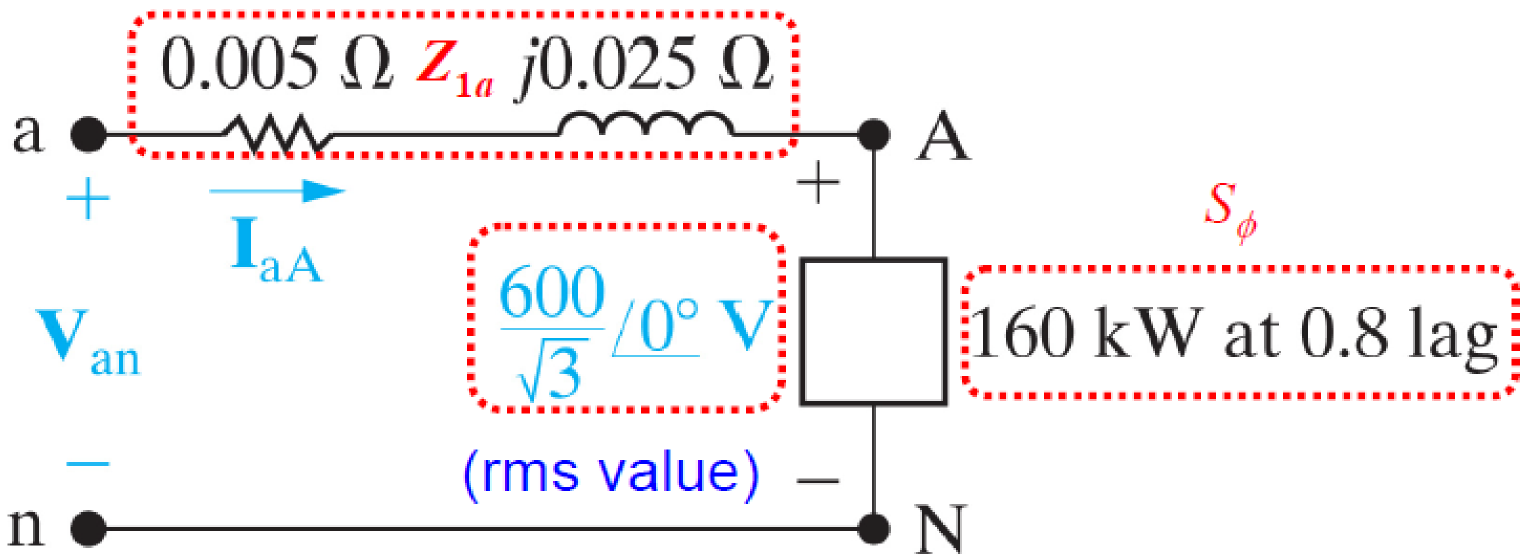
$$\begin{cases} Q_\phi = V_\phi I_\phi \sin\theta_\phi, \\ Q_{tot} = 3V_\phi I_\phi \sin\theta_\phi = \sqrt{3} V_L I_L \sin\theta_\phi. \end{cases}$$

- The complex powers of one phase and the entire Y-Load are:

$$\begin{cases} S_\phi = P_\phi + jQ_\phi = V_\phi I_\phi e^{j\theta_\phi} = \mathbf{V}_\phi \mathbf{I}_\phi^*; \\ S_{tot} = 3S_\phi = 3V_\phi I_\phi e^{j\theta_\phi} = \sqrt{3} V_L I_L e^{j\theta_\phi}. \end{cases}$$

Example 2

- Q: What are the complex powers provided by the source and dissipated by the line of a-phase?
- The equivalent one-phase circuit in Y-Y configuration is:



Example 2

- The line current of a-phase can be calculated by the complex power is:

$$S_\phi = \mathbf{V}_\phi \mathbf{I}_\phi^*, \quad (160 + j120)10^3 = \frac{600}{\sqrt{3}} \mathbf{I}_{aA}^*,$$

(P/|S| = 0.8; Q = SQRT(|S|^2-P^2)

$$\Rightarrow \mathbf{I}_{aA} = \left(577.35\angle -36.87^\circ\right)\text{A}.$$

- The a-phase voltage of the source is:

$$\mathbf{V}_{an} = \mathbf{V}_{AN} + \mathbf{I}_{aA} Z_{1a}$$

$$= 600/\sqrt{3} + \left(577.35\angle -36.87^\circ\right)(0.005 + j0.025)$$

$$= \left(357.51\angle 1.57^\circ\right)\text{V}.$$

Example 2

- The complex power provided by the source of a-phase is:

$$S_{an} = \mathbf{V}_{an}\mathbf{I}_{aA}^{*} = \left(357.51\angle 1.57^{\circ}\right)\left(577.35\angle 36.87^{\circ}\right)$$
$$= \left(206.41\angle 38.44^{\circ}\right)\text{kVA.}$$

- The complex power dissipated by the line of a-phase is:

$$S_{aA} = \left|\mathbf{I}_{aA}\right|^{2} Z_{1a} = \left(577.35\right)^{2}\left(0.005 + j0.025\right)$$
$$= \left(8.50\angle 78.66^{\circ}\right)\text{kVA.}$$