Critical Thinking: A Student's Introduction

Chapter 10

A Little Propositional Logic

Propositional Logic

Symbolizing parts of arguments so that one can analyze whole arguments for validity

The method for analyzing arguments for validity involves assigning variables to the different parts of the argument, just like in algebra

Conjunction

When two simple statements are conjoined with an "and," we call it a "conjunction"

We represent each statement as a simple letter, and represent the "and" with an "&"

- Example: "Tina is tall, and Sarah is tall" gets symbolized as "p & q"

Get it?

- p = Tina is tall
- q = Sarah is tall

Truth Tables, 1

Each of the variables has two possible **truth values**

- It could either be true or it could be false

Truth tables allow you to evaluate statements and arguments without knowing truth values by representing all possible truth value combinations

Truth Tables, 2

Recall that we symbolized “Tina is tall, and Sarah is tall” as “p & q”

- We don’t know if they are or not, but we can represent all possibilities this way:

p	q	p & q
T	T	T
T	F	F
F	T	F
F	F	F

Truth Tables, 3

Notice that “p & q” means “both p and q are true”

- Unless both p and q are true, “p & q” will not be true
- That is why, p & q has a T only on the row on which both p and q both have a T as well

p & q is a propositional form that can stand for an infinite number of compound statements

- Example: “The train is late, and the bus is on time”

Conjunction: A Word of Caution

Not every use of the word “and” indicates a compound statement that can be represented by p & q

- Sometimes “and” joins two things within the same simple statement
  - Example: The Knicks and the Bulls are playing each other tonight
    - This can be symbolized as “e”
  - Note: What letter you give it really doesn’t matter, just as long as you are consistent (use the same letter for that statement every time) and don’t use the same letter for two different statements in the same argument

Other Words for “And,” 1

If you see any of the following words, treat them like “and” and symbolize the statement with an “&”

- “But,” “yet,” “while,” “whereas,” “although,” “though,” and “however”

The following compound statements are all correctly symbolized as p & q:

- Tony had steak, *and* Theresa had chicken
- Tony had steak, *but* Theresa had chicken

Other Words for “And,” 2

- Tony had steak, *yet* Theresa had chicken
- Tony had steak, *while* Theresa had chicken
- Tony had steak, *whereas* Theresa had chicken
- Tony had steak, *although* Theresa had chicken
- Tony had steak, *though* Theresa had chicken
- Tony had steak; *however* Theresa had chicken

Using Truth Tables to Examine Validity, 1

Truth tables can be used to evaluate validity because:

- An argument is invalid only when it is possible for its premises to be true and the conclusion false
- Truth tables show us all possible truth values

Using Truth Tables to Examine Validity, 2

We use truth tables to determine all the possible truth values, and then look for a row where all the premises are true but the conclusion is false

- If we find one, the argument is invalid
- If there is no such row, the argument is valid

Example

- Tina is tall
- Sarah is tall
- So, Tina is tall, and Sarah is tall

Using Truth Tables to Examine Validity, 3

Argument can be symbolized as:

- p
- q
- ∴ p & q

First, represent all the statement letters and their truth values

p	q
T	T
T	F
F	T
F	F

Using Truth Tables to Examine Validity, 4

Create the truth table

p	q
T	T
T	F
F	T
F	F

p	q	p & q
T	T	T
T	F	F
F	T	F
F	F	F

Using Truth Tables to Examine Validity, 5

Look for rows where the premises are all true, and see if the conclusion is false in those rows

- If there is such a row, then the argument is invalid
- In this case, the only row with all true premises is the one in which the conclusion is also true
  - Thus, the argument is valid

p	q
T	T
T	F
F	T
F	F

p	q	p & q
T	T	T
T	F	F
F	T	F
F	F	F

Conjunction and Validity: Example 2, 1

Example

- Grass is green
- Therefore, grass is green, and the sky is blue
  - Symbolized as:
    - p
    - ∴ p & q
- Truth table format:

p	q	p & q
T	T	T
T	F	F
F	T	F
F	F	F

Conjunction and Validity: Example 2, 2

Note the following changes in the truth table:

- Premises are marked with an asterisk
- Conclusion is marked with a capital C
- Line across that ultimately allows one to determine validity or invalidity will be circled

p*	q	p & qC
T	T	T
T	F	F
F	T	F
F	F	F

The second line allows us to determine that the argument form is invalid

Negation

Negated statements can easily be represented with a "~"

- If "Sarah is tall" is "p," then "Sarah is not tall" is "~p"
- On a truth table, when p is true, ~p is false and vice versa

Negation: Example A

Argument: Tina is not tall, but Sarah is tall. So, Tina is not tall

- Symbolic form
  - ~p & q
  - ~p
- The truth table for the argument
  - There are no cases in which all the premises are true and the conclusion is false, so we know that the argument form is valid

p	q	~p	~p & q*	~pC
T	T	F	F	F
T	F	F	F	F
F	T	T	T	T
F	F	T	F	T

Negation: Example B, 1

Argument: Frank does not drive a truck. So, Frank does not drive a truck, and Vinny does not drive a minivan.

- Symbolic form
  - ~p
  - ∴ ~p & ~q
- Truth table

p	q	~p	~q	~p*	~p & ~qC
T	T	F	F	F	F
T	F	F	T	F	F
F	T	T	F	T	F
F	F	T	T	T	T

Negation: Example B, 2

- In the highlighted line, the premise is true and the conclusion is false
  - Therefore, the argument form is invalid

p	q	~p	~q	~p*	~p & ~qC
T	T	F	F	F	F
T	F	F	T	F	F
F	T	T	F	T	F
F	F	T	T	T	T

Whole Statement Negations

Compound statements can be negated too

- Example: “It is not the case that Lisa drives a Jeep and Jennifer drives a Jeep” can be symbolized as ~(p & q)

Since “p & q” means “Both p and q are true,” “~(p & q)” means “It is false that both p and q are true”

But don’t distribute (like in math)

- ~(p & q) is not the same as (~p & ~q)
  - ~(p & q) means they are not both true (at least one is false)
  - (~p & ~q) means they are both false

Proof: ~(p & q) ≠ (~p & ~q)

p	q	p & q	~(p & q)
T	T	T	F
T	F	F	T
F	T	F	T
F	F	F	T

p	q	p & q	~p	~q	~(p& q)	~p & ~q
T	T	T	F	F	F	F
T	F	F	F	T	T	F
F	T	F	T	F	T	F
F	F	F	T	T	T	T

Truth Tables with Three Variables: Example A, 1

Argument

- Tina is tall
- Sarah is not tall, but Missy is tall
- So, Tina is tall, and Missy is tall

Symbolic form

- p
- ~q & r
- ∴ p & r

Truth Tables with Three Variables: Example A, 2

Guide columns for the variables

p	q	r
T	T	T
T	T	F
T	F	T
T	F	F
F	T	T
F	T	F
F	F	T
F	F	F

Truth Tables with Three Variables: Example A, 3

Truth table for the argument

p	q	r	~q	p*	~q & r*	p & rC
T	T	T	F	T	F	T
T	T	F	F	T	F	F
T	F	T	T	T	T	T
T	F	F	T	T	F	F
F	T	T	F	F	F	F
F	T	F	F	F	F	F
F	F	T	T	F	T	F
F	F	F	T	F	F	F

Truth Tables with Three Variables: Example A, 4

The argument is valid because:

- The highlighted row contains true premises and a true conclusion
- There are no cases in which both premises are true and the conclusion is false

Truth Tables with Three Variables: Example B, 1

Argument
- ˜(p & q)
- ˜q & r
- ∴ ˜p

p	q	r	~q	p & q	~(p & q)*	~q & r*	~pC
T	T	T	F	T	F	F	F
T	T	F	F	T	F	F	F
T	F	T	T	F	T	T	F
T	F	F	T	F	T	F	F
F	T	T	F	F	T	F	T
F	T	F	F	F	T	F	T
F	F	T	T	F	T	T	T
F	F	F	T	F	T	F	T

Truth Tables with Three Variables: Example B, 2

This argument is invalid because both of the premises are true and the conclusion is false in the highlighted row

- It does not matter that in row 8 both of the premises are true and the conclusion is true

Disjunctions, 1

Two or more statements set apart, usually by the word "or"

- Example: "Frank is angry or Hank is tired"
  - Symbolized as p v q
    - To make things easier, don't ever use the letter "v" to symbolize a simple statement

Disjunctions, 2

"Or" can have two possible senses

- **Exclusive sense**: "A or b" means "a or b, but not both"
- **Nonexclusive sense**: "A or b" means "at least a or b, but maybe both"
  - The convention is to use "or" in its nonexclusive sense

Disjunction: Truth Table

p	q	p v q
T	T	T
T	F	T
F	T	T
F	F	F

Disjunction: Example

Argument

- Frank is angry or Hank is tired
- So, Frank is angry

Symbolic form

- p v q
- ∴ p

p	q	p v q*	pC
T	T	T	T
T	F	T	T
F	T	T	F
F	F	F	F

The highlighted instance of all true premises and a false conclusion establishes that the argument is invalid

Be Careful with Negations of Disjunctions, 1

Frank is not angry or Hank is tired
- ~a v t

Frank is not angry or Hank is not tired
- ~a v ~t

It's not the case that Frank is angry or Hank is tired
- ~(a v t)

Be Careful with Negations of Disjunctions, 2

Neither is Frank angry nor is Hank tired
- ~(a v t)

Frank is not angry and Hank is not tired
- (~a & ~t)

Argument That Involves Disjunction, Negation, and Conjunction

Argument

- It's not the case that Frank is angry or Hank is tired
- So, Frank is not angry and Hank is not tired

Symbolic form

- ~(p v q)
- ∴ ~p & ~q

p	q	~p	~q	p v q	~ (p v q)*	~p & ~qC
T	T	F	F	T	F	F
T	F	F	T	T	F	F
F	T	T	F	T	F	F
F	F	T	T	F	T	T

The argument is valid

Conditional (if, then) statements

Example: "If it rained, then the ground is wet"

- Where:
  - r = it rained (**antecedent**)
  - w = the ground is wet (**consequent**)
- Symbolic form: r → w

Conditional Statements: Truth Table

"p→q" means "every time p is true, q is true" or "when p is true, q is true"

So, only when the antecedent is true and the consequent is false, is it the case that the conditional is false?

p	q	p→q
T	T	T
T	F	F
F	T	T
F	F	T

Be Careful with Negation and Conditionals, 1

The following arguments have different meanings:

- If it did not rain, then the game was played
  - ~p→q
- If it did not rain, then the game was not played
  - ~p→~q
- It is not the case that if it rained then the game was played
  - ~(p→q)

Be Careful with Negations and Conditionals, 2

p	q	~p	~q	p→q	~p→q	~p→~q	~(p→q)
T	T	F	F	T	T	T	F
T	F	F	T	F	T	T	T
F	T	T	F	T	T	F	F
F	F	T	T	T	F	T	F

Three-Variable Argument Involving a Conditional

Argument

- ~(p→q)
- q v r
- ∴ q→p

p	q	r	p→q	~(p→q)*	q v r*	q→pC
T	T	T	T	F	T	T
T	T	F	T	F	T	T
T	F	T	F	T	T	T
T	F	F	F	T	F	T
F	T	T	T	F	T	F
F	T	F	T	F	T	F
F	F	T	T	F	T	T
F	F	F	T	F	F	T

This is a valid argument as the highlighted row is the only row that has true premises and a true conclusion