Review: theory

1. Mechanical waves

Wave Equation :

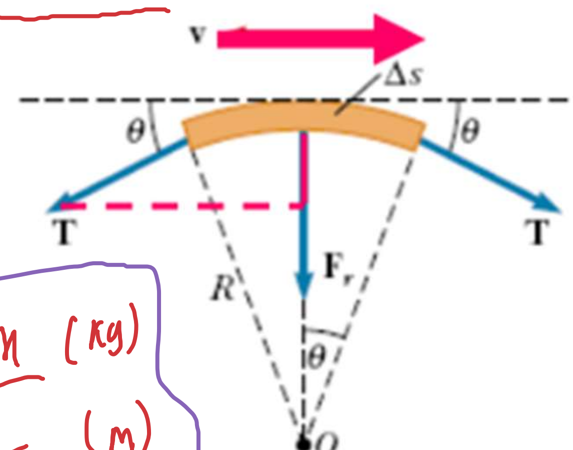
$$y = A\sin\left(\overset{\omega}{\frac{2\pi}{T}}t - \overset{K}{\frac{2\pi}{\lambda}}x\right) = A\sin(\omega t - Kx)$$

The **wavelength :** $\lambda = vT = \frac{v}{f}$ (T: period)

The **wave number** : $K = \frac{2\pi}{\lambda}$; $\omega = \frac{2\pi}{T}$

Speed of wave on a string

$$V = \sqrt{\frac{T}{\mu}}$$



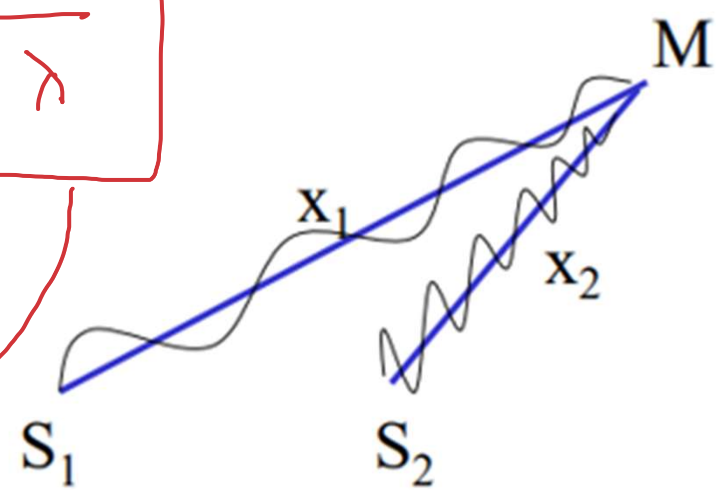
μ: linear mass density

T: string tension force

$$\mu = \frac{m\ (kg)}{L\ (m)}$$

Superposition of waves

$k = \frac{2\pi}{\lambda}$



$$y = y_1 + y_2 = 2A\cos\left(\frac{\phi}{2}\right)\sin(\omega t - Kx_1 + \frac{\phi}{2})$$

$\phi = K(x_1 - x_2) = K\delta$: **Phase difference**

$\delta = x_1 - x_2$: **Path difference**

$\to \Delta x$

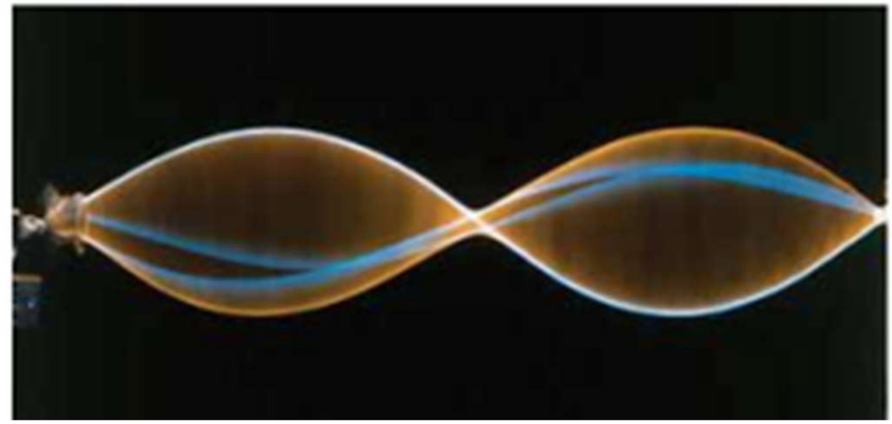
Constructive interference : $\phi = K\delta = k2\pi$; $\frac{2\pi}{\lambda}\delta = k2\pi$; $\boxed{\delta = k\lambda}$

Destructive interference : $\phi = K\delta = (2k+1)\pi$; $\frac{2\pi}{\lambda}\delta = (2k+1)\pi$;

$$\boxed{\delta = \left(k + \frac{1}{2}\right)\lambda}$$

Phase difference $\to \phi = K\Delta x$

2. Standing waves



General consideration

Position of nodes:

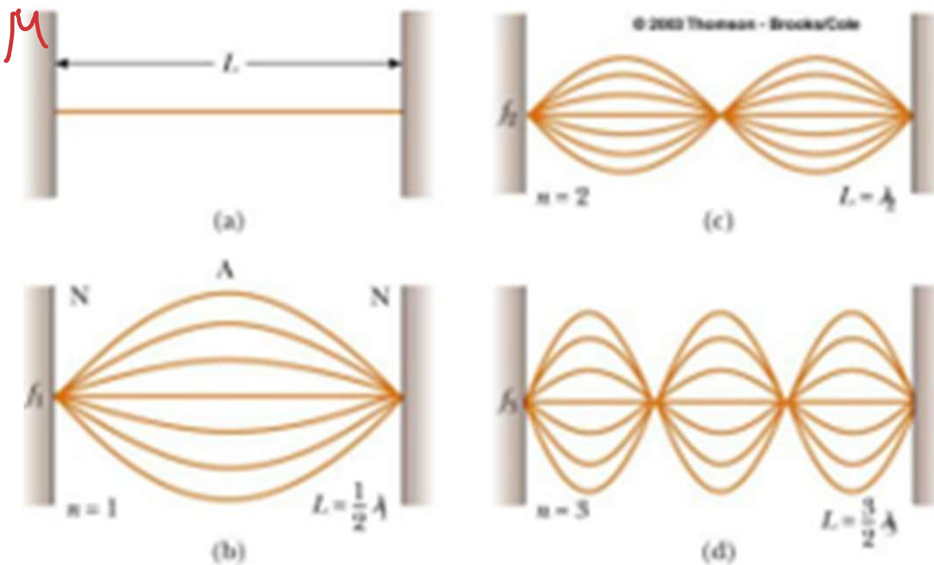
$$\boxed{x = n\lambda/2} \quad (n = 1, 2, 3, \ldots)$$

Position of antinodes:

$$\boxed{x = \left(n + \frac{1}{2}\right)\lambda/2}$$

$v = \sqrt{\frac{T}{\mu}}$

String fixed at both ends



$$\lambda_n = \frac{2L}{n} \qquad n = 1, 2, 3, \ldots$$

$f = \frac{vn}{2L}$

$vT = \frac{v}{f}$

$$f_1 = \frac{1}{2L}\sqrt{\frac{F}{\mu}}; \quad f_n = nf_1 = \frac{n}{2L}\sqrt{\frac{F}{\mu}}$$

(Fundamental frequency)

$\mu = \frac{m}{L}$

Air columns

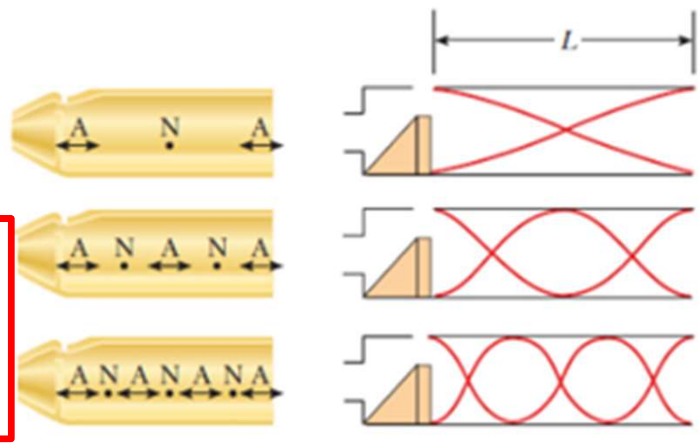
$2\frac{\lambda}{4}$

$\frac{f}{n} = \frac{vn}{2L}$

$\lambda = \frac{v}{f}$

Both open end

$$\boxed{\lambda = \frac{2L}{n}, \quad n = 1, 2, 3, \ldots,}$$



First harmonic: $\lambda_1 = 2L$, $f_1 = \frac{v}{2L}$

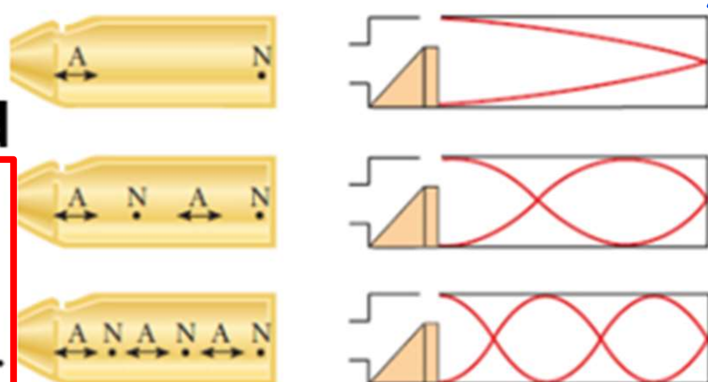
Second harmonic: $\lambda_2 = L$, $f_2 = 2f_1$

Third harmonic: $\lambda_3 = \frac{2}{3}L$, $f_2 = 3f_1$

$\frac{\lambda}{4}$

One closed end

$$\boxed{\lambda = \frac{4L}{n}, \quad n = 1, 3, 5, \ldots}$$



First harmonic: $\lambda_1 = 4L$, $f_1 = \frac{v}{4L}$

Third harmonic: $\lambda_3 = \frac{4}{3}L$, $f_3 = 3f_1$

Fifth harmonic: $\lambda_5 = \frac{4}{5}L$, $f_5 = 5f_1$

n

3. Sound waves

Intensity

$$I = \frac{\Delta E}{A\,\Delta t} = \frac{P}{A}$$

(W/m²)

$4\pi R^2$

Audile range:
20 Hz – 20,000 Hz

Intensity sound level

$$\beta = 10 \log \frac{I}{I_o}$$

Normal conversation's intensity level is about 50 dB.

hướng về

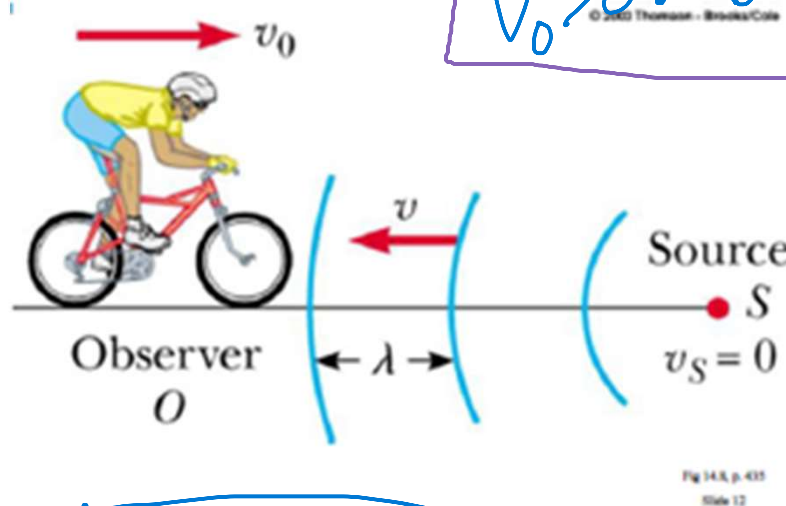
$V_s > 0: V_s \to V_o$

$V_o > 0: V_o \to V_s$

Const : $\mathbf{I_o = 1 \times 10^{-12}\ W/m^2}$

Doppler effect

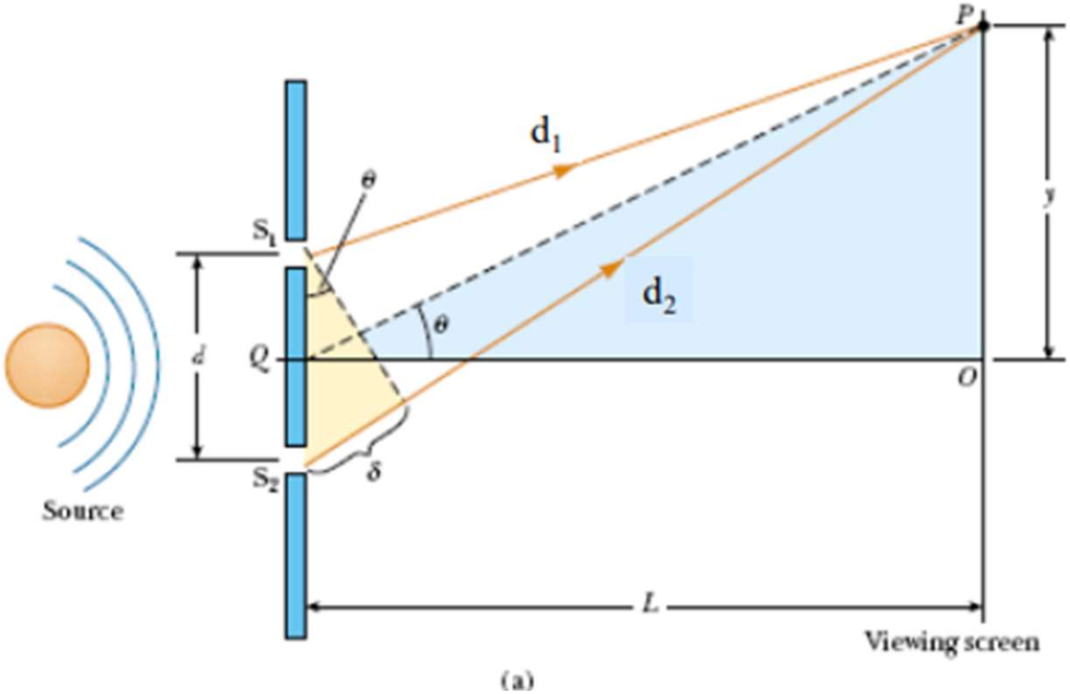
$$\boxed{f' = \frac{V + V_o}{V - V_s} f}$$



v, v_o > 0 observer and source move towards together
v, v_o < 0 observer and source move far away each other

4. Light

Young's double-slit interference



Path difference:

$$\boxed{\delta \approx d \sin\theta}$$

$$\boxed{\delta \approx d \frac{y}{L}}$$

Bright region:

$$\boxed{\delta = d \sin\theta = k\lambda}$$

$$\boxed{y_{BRIGHT} = k \frac{L}{d} \lambda = ki}$$

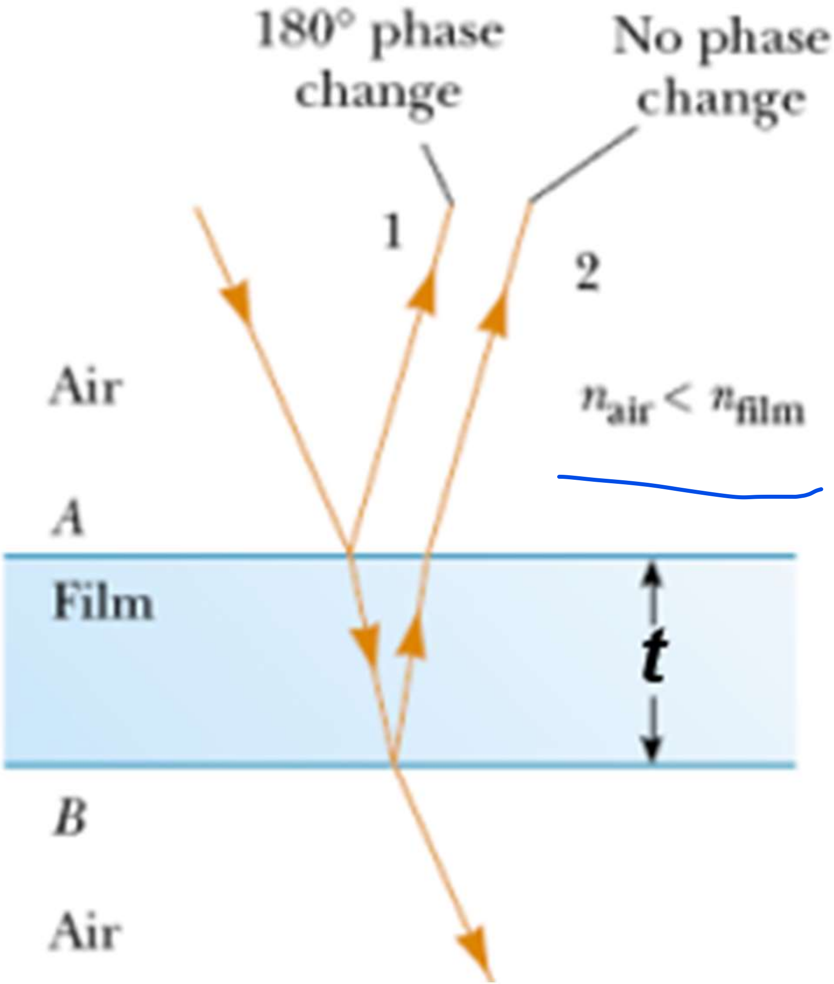
$$\boxed{i = \frac{L}{d} \lambda}$$

Dark region:

$$\boxed{\delta = d \sin\theta = (k + \frac{1}{2})\lambda}$$

$$\boxed{y_{DARK} = \left(k + \frac{1}{2}\right) i}$$

Thin-film interference



Refraction index: $n = \dfrac{c}{v}$

$$\lambda_n = \frac{v}{f} = \frac{c}{nf}; \quad \boxed{\lambda_n = \frac{\lambda}{n}}$$

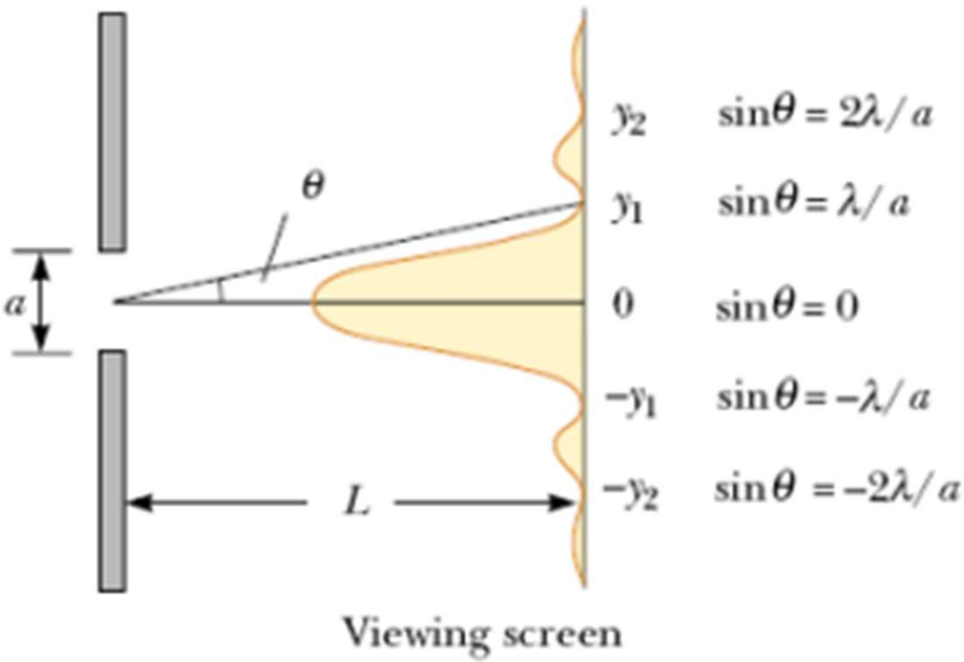
Constructive interference:

$$\boxed{2nt = (m + \frac{1}{2})\lambda}$$

Destructive interference:

$$\boxed{2nt = m\lambda_n} \qquad m = 0; 1; 2; 3; \ldots$$

Single-slit diffraction



Condition for destructive interference:

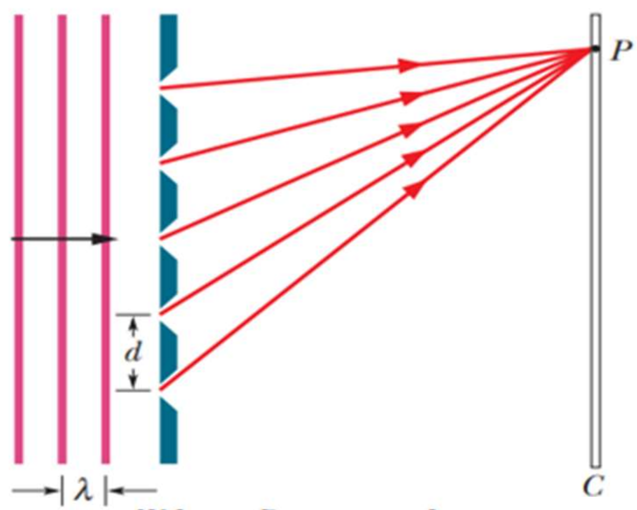
$$\boxed{\sin\theta = m\frac{\lambda}{a}; \quad m = \pm 1; \pm 2; \pm 3; \ldots}$$

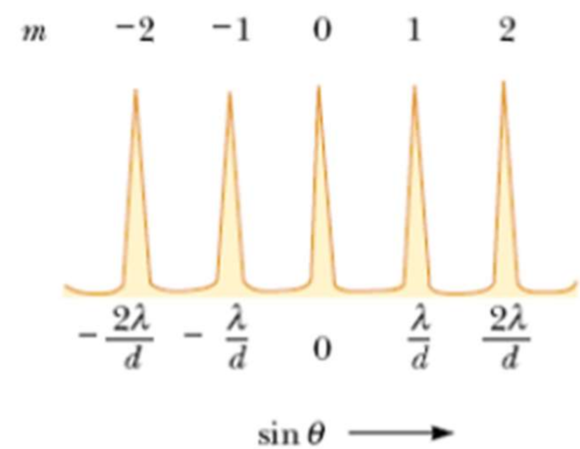
Position of dark fringes:

$$\boxed{y_m = m\frac{L\lambda}{a}; \quad m = \pm 1; \pm 2; \pm 3; \ldots}$$

$\boxed{\text{Grating}}$

$d\sin\theta$
$= K\lambda$





Condition for maxima: $\boxed{d\sin\theta = m\lambda \quad m = 0; 1; 2; \ldots}$

Review: problems

1 2

Problem 1: What are (a) the lowest frequency, (b) the second lowest frequency, and (c) the third lowest frequency for standing waves on a wire that is 10.0 m long, has a mass of 100 g, and is stretched under a tension of 250 N?

3

m

For standing waves of string fixed at both ends:

- $\lambda_n = 2L/n,$
- $f_n = v/\lambda_n = nv/2L$

$\lambda = \frac{2L}{n}$

$f = \frac{v}{\lambda} = \frac{vn}{2L}$

The speed of wave is given by:

$$v = \sqrt{\tau/\mu} = \sqrt{\tau L/M},$$

$v = \sqrt{\frac{T}{\mu}}$

$\mu = \frac{m}{L}$

$$\Rightarrow f_n = \frac{n}{2L}\sqrt{\frac{\tau L}{M}} = \frac{n}{2}\sqrt{\frac{\tau}{LM}}$$

$m = 0.1$

$L = 10m$

$T = 250N$

⇨ (a) f_1 = 7.91 Hz; (b) f_2 = 15.8 Hz; f_3 = 23.7 Hz

Problem 2: Organ pipe A, with both ends open, has a fundamental frequency of 300 Hz. The third harmonic of organ pipe B, with one end open, has the same frequency as the second harmonic of pipe A. Sound speed v = 344 m/s. How long are (a) pipe A and (b) pipe B?

$\lambda = \frac{v}{f}$ $L =$ $\lambda = \frac{2L}{n}$

(a) For pipe A with both ends open, the fundamental frequency:

$$f_1^A = v\,/\,(2L_A)$$ $\rightarrow f = \frac{vn}{2L}$

$$\boldsymbol{\rightarrow L_A = v\,/\,(2\,f_1^{\,A}) = 344/2\text{x}300 = 0.572\ (\text{m})}$$

The frequency of second harmonic for pipe A: $f = \frac{v}{2L} = 300.7\ (\text{Hz})$

$$f_2^A = 2f_1^A = 2v\,/\,(2L_A)$$ $\simeq 600$ Hz

(b) For pipe B with one end open, the frequency of third harmonic:

$$f_3^B = 3f_1^B = 3v\,/\,(4L_B)$$

Since $f_2^A = f_3^B$: $600 = \frac{3v}{4L_B}$ $\frac{v}{f} = \lambda = \frac{4L}{n}$

$$2v/(2L_A) = 3v\,/\,(4L_B)$$

$$\boldsymbol{\rightarrow L_B = 3L_A/4 = 0.429\ (\text{m})}$$

↓

600 Hz

Problem 3: A 1.0 W point source emits sound waves isotropically. Assuming that the energy of the waves is conserved, find the intensity (a) 1.0 m from the source and (b) 2.5 m from the source.

P

m

m

Using the formula: $\boxed{I = \frac{P}{A}}$ (W/m²)

With $\boxed{P = 1.0\ \text{W}}$ and $\boxed{A = 4\pi R^2}$ → S mặt cầu

a) $R = 1.0$ m → $I = 0.08$ W/m²

b) $R = 2.5$ m → I = 0.013 W/m²

Problem 4: A person is standing a certain distance from four noisy juicers and hear a sound level of 77dB in a juice shop. What sound level would this person experience if three of them are turned off?

First of all, find the sound intensity that four equally noisy juicers produce using the sound level formula

$I_0 = 10^{-12}$

$$77\text{dB} = \beta = 10 \log\left(\frac{I}{I_0}\right)$$

$$\rightarrow \quad \boxed{I = 5.01 \times 10^{-5}\,\mathrm{W/m^2}} \rightarrow \text{const}$$

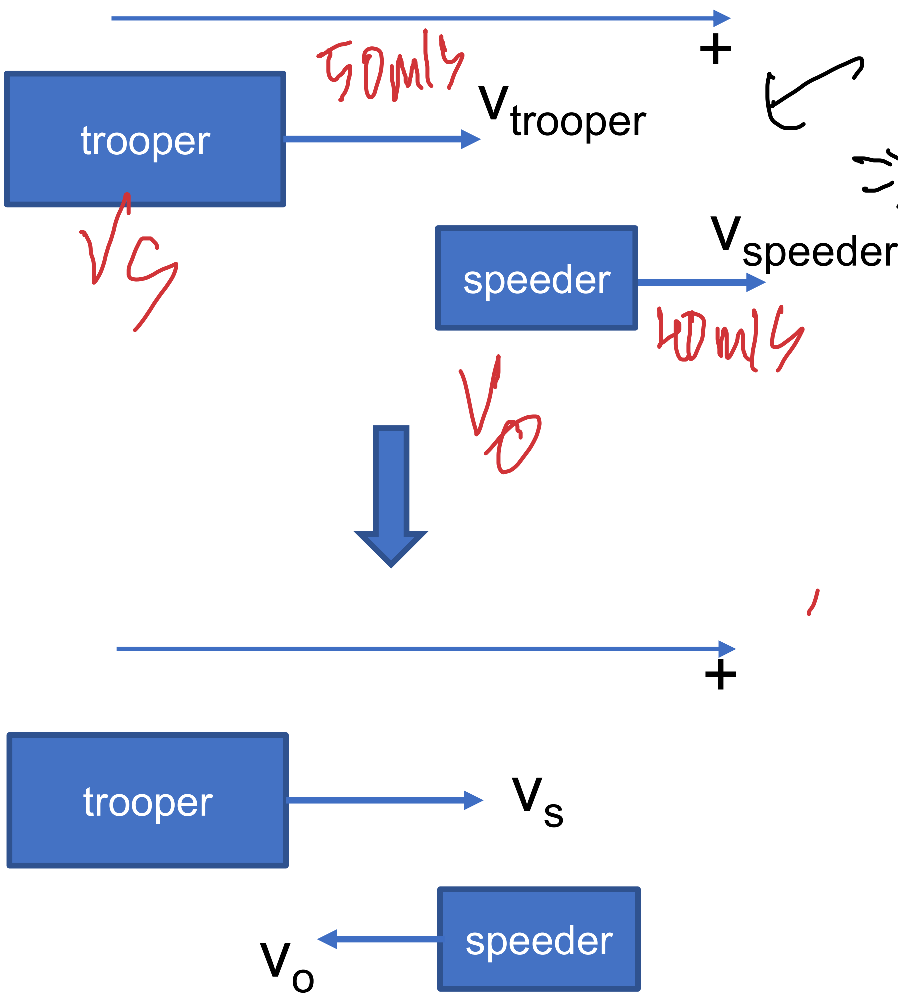
The sound intensity of one juicer as

$$I_{one} = \frac{I}{4} = 1.25 \times 10^{-5}\,(\mathrm{W/m^2})$$

Again, use the sound level formula and find the level for a single machine

$$\rightarrow \quad \beta_1 = \boxed{70.9\,\mathrm{dB}}$$

Problem 5: A state trooper with speed 50 m/s chases a speeder with speed 40 m/s along a straight road. The siren on the trooper's vehicle produces sound at a frequency of 500 Hz. What is the Doppler shift in the frequency heard by the speeder?



$V_S \rightarrow V_O$

$\Rightarrow V_S > 0$

$$\boxed{f' = \frac{V + V_O}{V - V_S} f}$$

Trooper (source) and speeder (observer) move towards each other:

$v_o > 0$ and $v_s > 0$

But: $v_o = - v_{speeder}$ and $v_s = v_{trooper}$

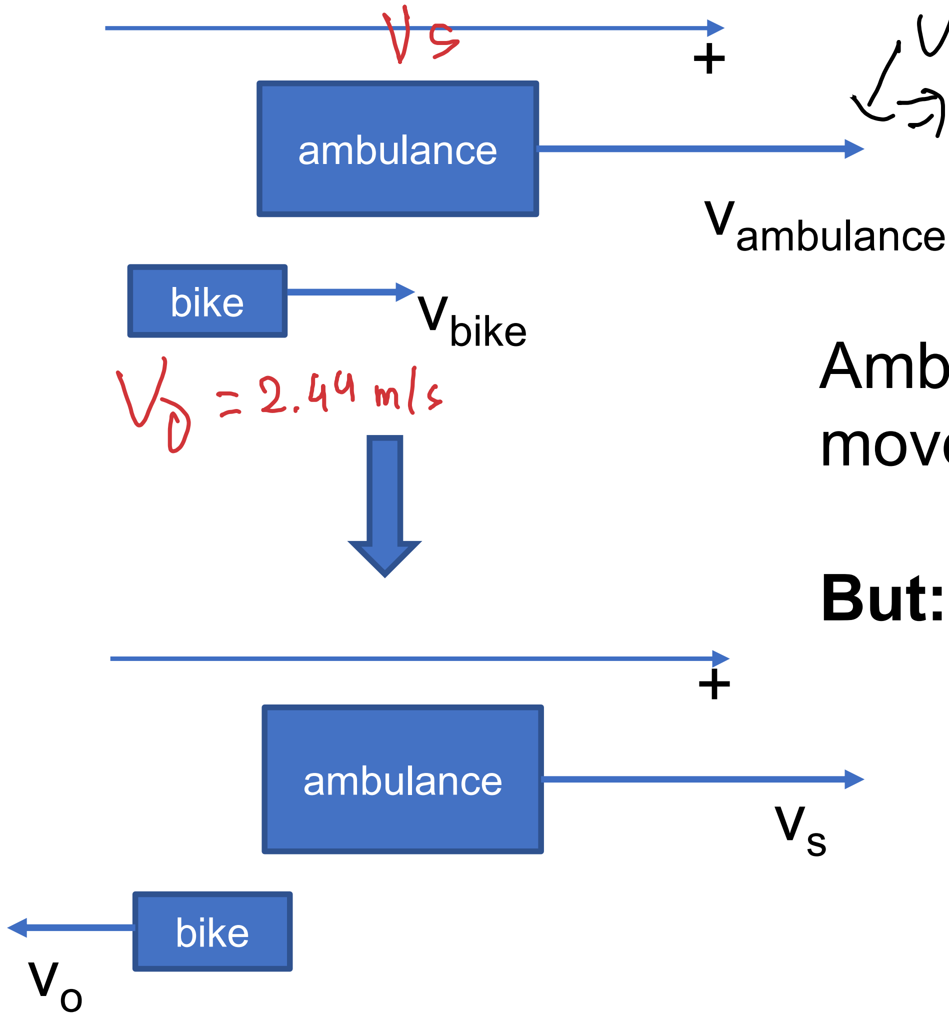
$$f' = \frac{v - v_{speeder}}{v - v_{trooper}} f$$

$$f' = \frac{344 - 40}{344 - 50} 500 = 517\ (Hz)$$

$\Delta f = 17$ Hz

Problem 6. An ambulance with a siren emitting a whine at 1600 Hz overtakes and passes a cyclist pedaling a bike at 2.44 m/s. After being passed, the cyclist hears a frequency of 1590 Hz. How fast is the ambulance moving?

f_s f'



$V_o \to V_s$

$V_o > 0$

$$\boxed{f' = \frac{V + V_o}{V - V_s} f}$$

Ambulance (source) and bike (observer) move far away each other:

$$\mathbf{v_o < 0 \text{ and } v_s < 0}$$

But: $\mathbf{v_o = -v_{bike}}$ **and** $\mathbf{v_s = v_{ambulance}}$

$$f' = \frac{v + v_{bike}}{v + v_{ambulance}} f$$

$$v_{ambulance} = \frac{f}{f'}(v + v_{bike}) - v$$

$$= 4.61\ m/s$$

Problem 7: A double-slit arrangement produces bright interference fringes for sodium light at a wavelength of 589 nm. The fringes are angularly separated by 0.30° near the center of the pattern. What is the angular fringe separation if the entire arrangement is immersed in water, which has an index of refraction of 1.33?

n

$n = 1{,}33$

air $\Rightarrow n = 1$

Condition for bright fringes: $d \sin\theta = m\lambda$

$\sin\theta \approx \theta$

If the angle is small: $\theta = m\lambda/d$

The angular separation of two adjacent maxima $\Delta\theta = \lambda/d.$

$n = 1$

In air/vacuum: λ = 589 nm and Δθ = 0.3°

$n = 1{,}33$

In water: λ' = λ/n and Δθ' = λ'/d

→ Δθ'/ Δθ = λ'/λ = 1/n

→ Δθ' = 0.23°

$$\Delta\theta = \frac{\lambda'}{d}$$

$$\boxed{\lambda'_w = \frac{\lambda_{air}}{n_w}} = \frac{31}{7\times10^7}$$

L

Problem 8: A thin film suspended in air is 0.410 μm thick and is illuminated with white light incident perpendicularly on its surface. The index of refraction of the film is 1.50. A visible light that is reflected from the two surfaces of the film undergoes fully constructive interference.

Replacing the film by another film with the index of refraction is 1.30. Find the minimum thickness of the new film so that the same visible light undergoes fully constructive interference.

With n_2 = 1.5, for constructive interference:

$$2n_2L = (m + 1/2)\lambda \Rightarrow \lambda = \frac{2n_2L}{m+1/2} = \frac{1230\text{ nm}}{m+1/2} \quad (m = 0, 1, 2, \ldots)$$

Visible light: 380 nm < λ < 700 nm, so **λ = 492 nm** (with m =1)

m = 1

With n_2 = 1.3:

$$2n_2L = (m + 1/2)\lambda \Rightarrow L = \lambda/(4n_2) = 94.6\text{ nm} = 0.0946\ \mu\text{m}$$

(minimum thickness: m = 0)

? 0 492nm

Bắt đầu từ 0.5 λ

Problem 9: visible light is incident perpendicularly on a grating with 315 rulings/mm. What is the longest wavelength that can be seen in the fifth-order diffraction?

For the grating diffraction maximum Vân sáng

$$m\lambda = d \sin\theta.$$

$-1 \to 1$

$$\sin\theta = \frac{m\lambda}{d}$$

$m = 5$

To be seen in the fifth-order diffraction: $\frac{5\lambda}{d} \leq 1$

$d = 315$

Longest wavelength: $\lambda = d/5$ with d = $10^{-3}/315$

→ **λ = 6.35x10^{-7} m = 635 nm**

$d \sin\theta = k\lambda$ $\qquad \sin\theta \leq 1$

$\Rightarrow \frac{5\lambda}{d} = \frac{k\lambda}{d}$

$\Rightarrow 5\lambda = d \to \lambda = \frac{10^{-3}/315}{5}$