Đỗ Minh Duy - ITITSB22029

1. $A(1,2)$; $B(4,3)$

i) $\overrightarrow{AB} = (3;1)$

ii) parametric equations:

$x = x_0 + at = 1 + 3t$

$y = y_0 + at = 2 + t$

Symmetric equations:

$$\frac{x - x_0}{a} = \frac{y - y_0}{b}$$

$$(=) \frac{x-1}{3} = \frac{y-2}{1}$$

$$=) \frac{x-1}{3} = y - 2$$

2. $f(x) = x^2 + 2x$, $x \geqslant -1$

$y = x^2 + 2x$

$(=) y = x^2 + 2x + 1 - 1$

$(=) y = (x+1)^2 - 1$

$\rightarrow x = (y+1)^2 - 1$

$(=) (x+1) = (y+1)^2$

$(=) \pm\sqrt{x+1} = y + 1$

$=) y = -1 \pm \sqrt{x+1}$

The inverse of the function:

$f^{-1}(x) = -1 \pm \sqrt{x+1}$

3. $f(x) = \sqrt{2017 - \sin(x+1)}$

i) $\cdot D = R$

we have:

$-1 \leqslant \sin(x+1) \leqslant 1$

$(=) 2016 \leqslant 2017 - \sin(x+1) \leqslant 2018$

$(=) 12\sqrt{14} \leqslant \sqrt{2017 - \sin(x+1)} \leqslant \sqrt{2018}$

$\cdot R = [12\sqrt{14} ; \sqrt{2018}]$

ii) $f(x) = g(h(x))$

$h(x) = \sin(x+1)$

$g(x) = \sqrt{2017 - x}$

4. Radius $= 2$

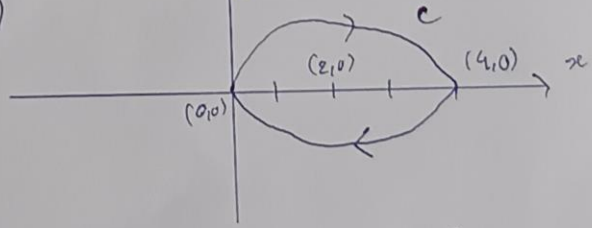
$C(2;0)$

i) $(x-2)^2 + y^2 = 4$

ii) Yes, we can write:

$y = \sqrt{4 - (x-2)^2}$

iii)



Let $x = r.\cos\theta$, $y = r.\sin\theta$

Now $(r.\cos\theta - 2)^2 + (r.\sin\theta)^2 = 4$

$=) r^2\cos^2\theta + 4 - 4r\cos\theta + r^2\sin^2\theta = 4$

$=) r^2 - 4r\cos\theta = 0$

$=) r = 4\cos(\theta)$

⑤ $\lim\limits_{x\to 2} \dfrac{\sqrt{6-x}-2}{\sqrt{3-x}-1}$

$= \lim\limits_{x\to 2} \dfrac{(6-x-4)(\sqrt{3-x}+1)}{(3-x-1)(\sqrt{6-x}+2)} = \lim\limits_{x\to 2} \dfrac{(2-x)(\sqrt{3-x}+1)}{(2-x)(\sqrt{6-x}+2)}$

$= \lim\limits_{x\to 2} \dfrac{\sqrt{3-x}+1}{\sqrt{6-x}+2} = \dfrac{\sqrt{3-2}+1}{\sqrt{6-2}+2} = \dfrac{2}{4} = \dfrac{1}{2}$

⑥ $\lim\limits_{x\to 0} x\cos(\ln|x|)$

We have $0 \le \cos(\ln|x|) \le 1$

$(=)\ 0.x \le x.\cos(\ln|x|) \le 1.x$

$(=)\ 0 \le x.\cos(\ln|x|) \le x$

By the Squeeze Theorem:

$\lim\limits_{x\to 0} 0 = 0$; $\lim\limits_{x\to 0} x = 0$

~~Hence~~ thus, $\lim\limits_{x\to 0} x\cos(\ln|x|) = 0$

⑦ $f(x) = \begin{cases} 2x-1 & \text{if } x < -1 \\ x^2+1 & \text{if } -1 \le x \le 1 \\ x+1 & \text{if } x > 1 \end{cases}$

- When $x < -1$, then $f(x) = 2x-1$ is continuous at every $x<-1$
- when $-1 < x < 1$, then $f(x) = x^2+1$ is continuous at every $-1<x<1$
- when $x > 1$, then $f(x) = x+1$ is continuous at every $x > 1$

Then we have:

- $f(-1) = (-1)^2 + 1 = 2$
- $\lim\limits_{x\to -1^-} f(x) = \lim\limits_{x\to -1^-} (2x-1) = 2.(-1) - 1 = -3$
- $\lim\limits_{x\to -1^+} f(x) = \lim\limits_{x\to -1^+} (x^2+1) = (-1)^2+1 = 2$

$\Rightarrow f(-1) = \lim\limits_{x\to -1^+} f(x) \ne \lim\limits_{x\to -1^-} f(x)$

$\Rightarrow f(x)$ is not continuous at $x = -1$

- $f(1) = 1^2 + 1 = 2$
- $\lim\limits_{x\to 1^-} f(x) = \lim\limits_{x\to 1^-} (x^2+1) = 2$
- $\lim\limits_{x\to 1^+} f(x) = \lim\limits_{x\to 1^+} (x+1) = 2$

$\Rightarrow f(1) = \lim\limits_{x\to 1^-} f(x) = \lim\limits_{x\to 1^+} f(x)$

$\Rightarrow f(x)$ is continuous at every $x = 1$

* Conclusion:
  - iv is true.

⑨ $x^3 - x\sin x - 1 = x\sqrt{x+2}$ (1)

(1) $\Leftrightarrow x^3 - x\sin x - 1 - x\sqrt{x+2} = 0$ (2)

Denote $f(x) = x^3 - x\sin x - 1 - x\sqrt{x+2}$

Then $f(x)$ is continuous on $[0;2]$

$f(0) = 0 - 0 - 1 - 0 = -1 < 0$

$f(2) = 8 - 2\sin(2) - 1 - 2\sqrt{4}$

$= 8 - 2\sin(2) - 5$

$= 3 - 2\sin(2) = 2.93 > 0$

$f(x)$ is continuous on $[0;2]$ and $f(0) < 0 < f(2)$

By the Intermediate Value Theorem, there is a number $c \in (0;2)$ such that $f(c) = 0$

$\Rightarrow$ (2) has a real root in $c \in (0;2)$

⑧ $g(x) = \begin{cases} \cos x & \text{if } x < 0 \\ 0 & \text{if } x = 0 \\ 1 - x^2 & \text{if } x > 0 \end{cases}$

i) • $\lim\limits_{x\to 0^-} f(x) = \lim\limits_{x\to 0^-} (\cos x) = 1$

• $\lim\limits_{x\to 0^+} f(x) = \lim\limits_{x\to 0^+} (1-x^2) = 1$

• $f(0) = 0$

$\Rightarrow \lim\limits_{x\to 0^-} f(x) = \lim\limits_{x\to 0^+} f(x) \ne f(0)$.

~~Hence~~ Thus, $g(x)$ is discontinuous at $x = 0$

ii)