P6.1. $V(z, t) = 10 \cos (3\pi \times 10^8 t - 2\pi z)$ V

$w = 0.1$ m

$d = 0.01$ m

(a) $E_x = \dfrac{V}{d}$

$= 1000 \cos (3\pi \times 10^8 t - 2\pi z)$ V/m

(b) $H_y = \dfrac{E_x}{\eta}$

To find η, we note that

$$v_p = \frac{3\pi \times 10^8}{2\pi} = 1.5 \times 10^8 \text{ m/s} = \frac{c}{2} = \frac{1}{\sqrt{\mu_0 \cdot 4\varepsilon_0}}$$

$\therefore\ \varepsilon = 4\varepsilon_0$

$$\eta = \sqrt{\frac{\mu}{\varepsilon}} = \sqrt{\frac{\mu_0}{4\varepsilon_0}} = \frac{120\pi}{2} = 60\pi\Omega$$

$$H_y = \frac{50}{3\pi} \cos (3\pi \times 10^8 t - 2\pi z) \text{ A/m}$$

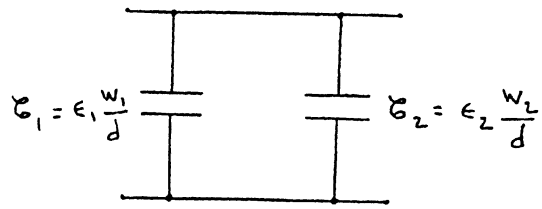
(c) $I = H_y w$

$= \dfrac{5}{3\pi} \cos (3\pi \times 10^8 t - 2\pi z)$ A

(d) $P = VI$

$= \dfrac{50}{3\pi} \cos^2 (3\pi \times 10^8 t - 2\pi z)$ W

P6.2.



$$\mathcal{C}_1 = 4\varepsilon_0 \frac{0.1}{0.01} = 40\varepsilon_0$$

$$\mathcal{C}_2 = 2\varepsilon_0 \frac{0.1}{0.01} = 20\varepsilon_0$$

$$\mathcal{C} = \mathcal{C}_1 + \mathcal{C}_2 = 40\varepsilon_0 + 20\varepsilon_0$$

$$= 60\varepsilon_0 \text{ F/m}$$

From $\mathcal{L}\mathcal{C} = \mu_1\varepsilon_1 = \mu_2\varepsilon_2 = 4\mu_0\varepsilon_0$,

$$\mathcal{L} = \frac{4\mu_0\varepsilon_0}{\mathcal{C}} = \frac{4\mu_0\varepsilon_0}{60\varepsilon_0}$$

$$= \frac{1}{15}\mu_0 \text{ H/m}$$

$$Z_0 = \sqrt{\frac{\mathcal{L}}{\mathcal{C}}} = \sqrt{\frac{\mu_0}{15}\cdot\frac{1}{60\varepsilon_0}}$$

$$= \frac{1}{30}\sqrt{\frac{\mu_0}{\varepsilon_0}} = \frac{120\pi}{30}$$

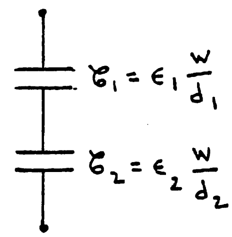
$$= 4\pi\ \Omega$$

P6.3. $C_1 = 9\varepsilon_0 \dfrac{0.2}{0.01} = 180\varepsilon_0$

$$C_2 = 3\varepsilon_0 \frac{0.2}{0.01} = 60\varepsilon_0$$

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{180\varepsilon_0} + \frac{1}{60\varepsilon_0} = \frac{1}{45\varepsilon_0}$$

$$\therefore C = 45\varepsilon_0 \text{ F/m}$$



From $\mathcal{L}C = \mu_1\varepsilon_1 = \mu_2\varepsilon_2 = 9\mu_0\varepsilon_0$,

$$\mathcal{L} = \frac{9\mu_0\varepsilon_0}{C} = \frac{9\mu_0\varepsilon_0}{45\varepsilon_0}$$

$$= 0.2\mu_0 \text{ H/m}$$

$$Z_0 = \sqrt{\frac{\mathcal{L}}{C}} = \sqrt{\frac{0.2\mu_0}{45\varepsilon_0}}$$

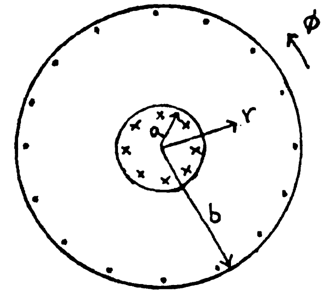
$$= \frac{1}{15}\sqrt{\frac{\mu_0}{\varepsilon_0}} = \frac{120\pi}{15}$$

$$= 8\pi\,\Omega$$

P6.4. Recognizing that

$$\mathbf{E} = E_r(r, z, t)\, \mathbf{a}_r$$

$$\mathbf{H} = H_\phi(r, z, t)\, \mathbf{a}_\phi$$



and substituting in Maxwell's curl equations, we get

$$\begin{vmatrix} \frac{\mathbf{a}_r}{r} & \mathbf{a}_\phi & \frac{\mathbf{a}_z}{r} \\ \frac{\partial}{\partial r} & 0 & \frac{\partial}{\partial z} \\ E_r & 0 & 0 \end{vmatrix} = -\frac{\partial \mathbf{B}}{\partial t} \quad \text{and} \quad \begin{vmatrix} \frac{\mathbf{a}_r}{r} & \mathbf{a}_\phi & \frac{\mathbf{a}_z}{r} \\ \frac{\partial}{\partial r} & 0 & \frac{\partial}{\partial z} \\ 0 & rH_\phi & 0 \end{vmatrix} = \frac{\partial \mathbf{D}}{\partial t}$$

or

$$\frac{\partial E_r}{\partial z} = -\mu \frac{\partial H_\phi}{\partial t} \quad \text{and} \quad \frac{1}{r}\frac{\partial}{\partial z}(rH_\phi) = -\varepsilon \frac{\partial E_r}{\partial t}$$

But from $V(z, t) = \int_{r=a}^{b} E_r(r, z, t)\, dr$ and since $E_r \propto \frac{1}{r}$ from Gauss' law, we have

$E_r(r, z, t) = \dfrac{V(z, t)}{r \ln \frac{b}{a}}$. Also from $I(z, t) = \int_{\phi=0}^{2\pi} H_\phi(r, z, t)\, r\, d\phi$, we have

$H_\phi(r, z, t) = \dfrac{I(z, t)}{2\pi r}$. Substituting these into the differential equations, we get

$$\frac{\partial}{\partial z}\left[\frac{V(z, t)}{r \ln \frac{b}{a}}\right] = -\mu \frac{\partial}{\partial t}\left[\frac{(I(z, t)}{2\pi r}\right]$$

$$\frac{1}{r}\frac{\partial}{\partial z}\left[\frac{(I(z, t)}{2\pi r}\right] = -\varepsilon \frac{\partial}{\partial t}\left[\frac{V(z, t)}{r \ln \frac{b}{a}}\right]$$

or

$$\frac{\partial V}{\partial z} = -\left(\frac{\mu}{2\pi} \ln \frac{b}{a}\right)\frac{\partial I}{\partial t} = -\mathcal{L}\frac{\partial I}{\partial t}$$

$$\frac{\partial I}{\partial z} = -\left(\frac{2\pi\varepsilon}{\ln \frac{b}{a}}\right)\frac{\partial V}{\partial t} = -\mathcal{C}\frac{\partial V}{\partial t}$$

P6.4. (continued)

The power flow along the line is given by

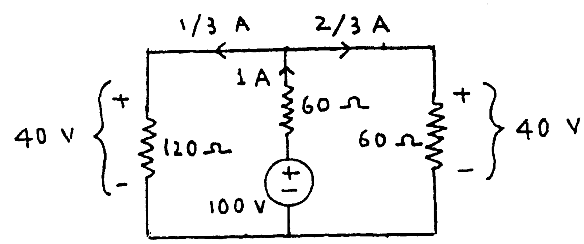
$$P(z, t) = \int_S (\mathbf{E} \times \mathbf{H}) \bullet d\mathbf{S}$$

$$= \int_{r=a}^{b} \int_{\phi=0}^{2\pi} \frac{V(z, t)}{r \ln \frac{b}{a}} \mathbf{a}_r \times \frac{I(z, t)}{2\pi r} \mathbf{a}_\phi \bullet \mathbf{a}_z \, r \, dr \, d\phi$$

$$= \int_{r=a}^{b} \int_{\phi=0}^{2\pi} \frac{V(z, t)\, I(z, t)}{2\pi r \ln \frac{b}{a}} dr \, d\phi$$

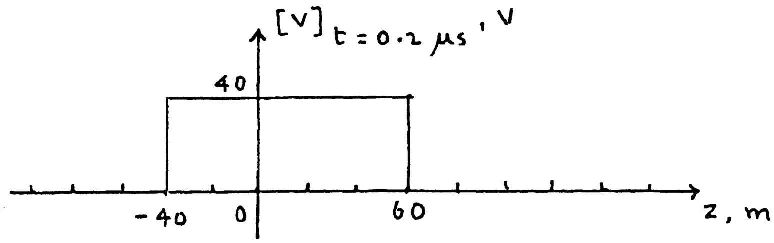
$$= V(z, t)\, I(z, t)$$

P6.5.



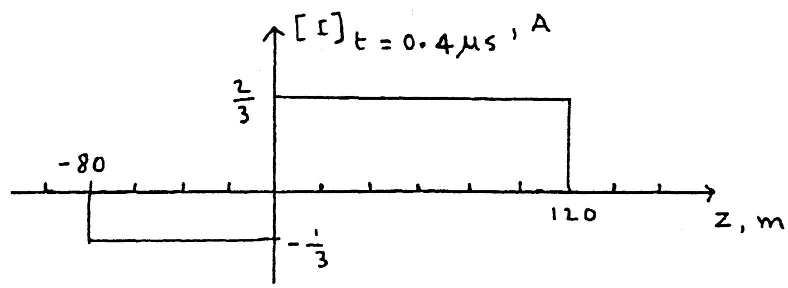
1/3 A
2/3 A
1 A
60 Ω
60 Ω
120 Ω
40 V
40 V
100 V

(a)



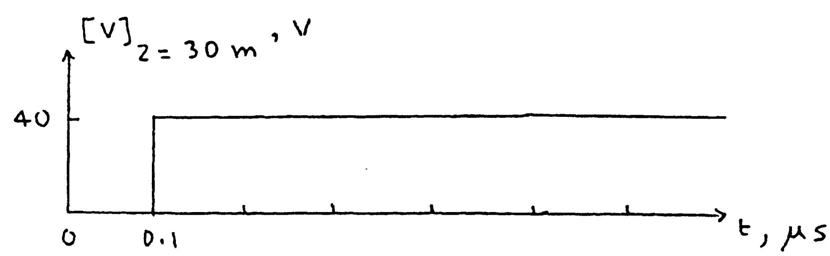
$[V]_{t=0.2\,\mu s}$, V
40
-40
0
60
z, m

(b)



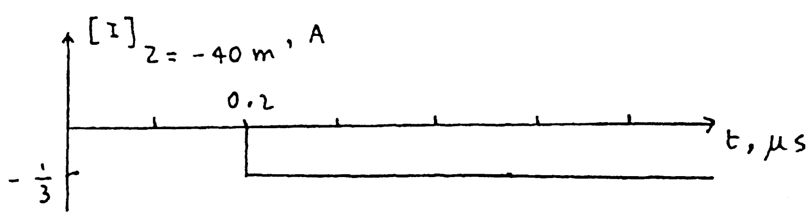
$[I]_{t=0.4\,\mu s}$, A
$\frac{2}{3}$
-80
120
z, m
$-\frac{1}{3}$

(c)



$[V]_{z=30\,m}$, V
40
0
0.1
t, μs

(d)



$[I]_{z=-40\,m}$, A
0.2
t, μs
$-\frac{1}{3}$

P6.6. From the sketches of $[V]_{z=0}$ and $[V]_{z=l}$, we can obtain

$$V^+ = 100 \text{ V}$$

$$V^+ + V^- = 75 \text{ V}$$

$$V^+ + V^- + V^{-+} = 90 \text{ V}$$

$$\therefore \quad V^- = 75 - 100 = -25 \text{ V}$$

$$V^{-+} = 90 - 75 = 15 \text{ V}$$

$$\Gamma_R = \frac{V^-}{V^+} = -\frac{1}{4}, \ \Gamma_S = \frac{V^{-+}}{V^-} = -0.6$$

$$\frac{R_L - Z_0}{R_L + Z_0} = -\frac{1}{4} \qquad \frac{R_g - Z_0}{R_g + Z_0} = -0.6$$

$$5R_L = 3Z_0 \qquad 1.6\, R_g = 0.4\, Z_0$$

$$R_L = \frac{300}{5} = 60\ \Omega \qquad R_g = \frac{100}{4} = 25\ \Omega$$

$$V^+ = V_0 \frac{Z_0}{R_g + Z_0} = \frac{100}{125} V_0 = 100 \text{ V}$$

$$V_0 = 125 \text{ V}$$

Also, $T = 2\ \mu\text{s}$

Thus

$$V_0 = 125 \text{ V}$$

$$R_g = 25\ \Omega$$

$$R_L = 60\ \Omega$$

$$T = 2\ \mu\text{s}$$

P6.7. **(a)** $V_{SS} = \dfrac{100}{40+60} \times 60 = 60\text{ V}$

$$I_{SS} = \frac{100}{40+60} = 1\text{ A}$$

(b) $V^+ + V^- = 60$

$$\frac{1}{75}\left(V^+ - V^-\right) = 1$$

$$V^+ - V^- = 75$$

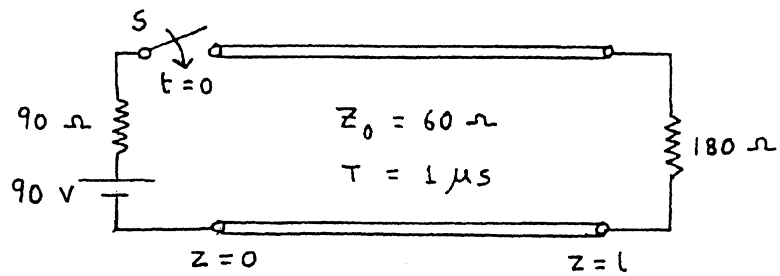
$$\therefore\ V^+ = \frac{135}{2} = 67.5\text{ V}$$

$$I^+ = \frac{V^+}{75} = \frac{67.5}{75} = 0.9\text{ A}$$

(c) $V^- = 60 - 67.5 = -7.5\text{ V}$

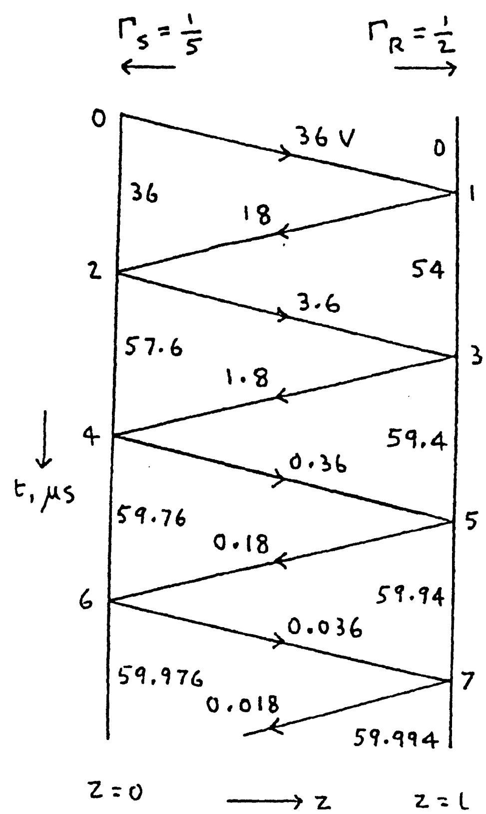
$$I^- = -\frac{V^-}{75} = \frac{7.5}{75} = 0.1\text{ A}$$

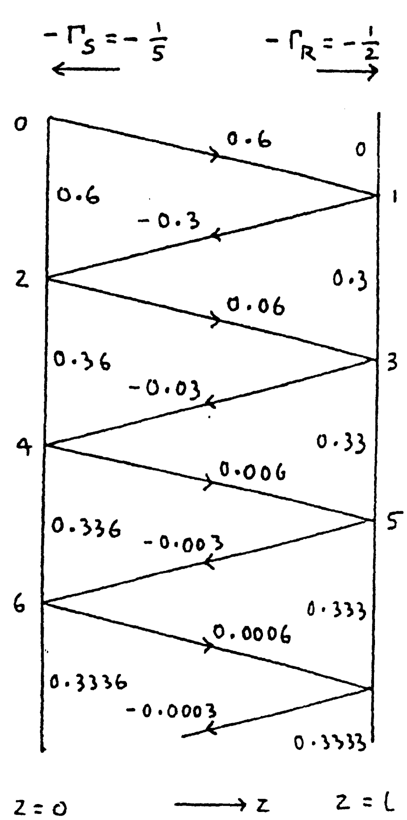
P6.8.



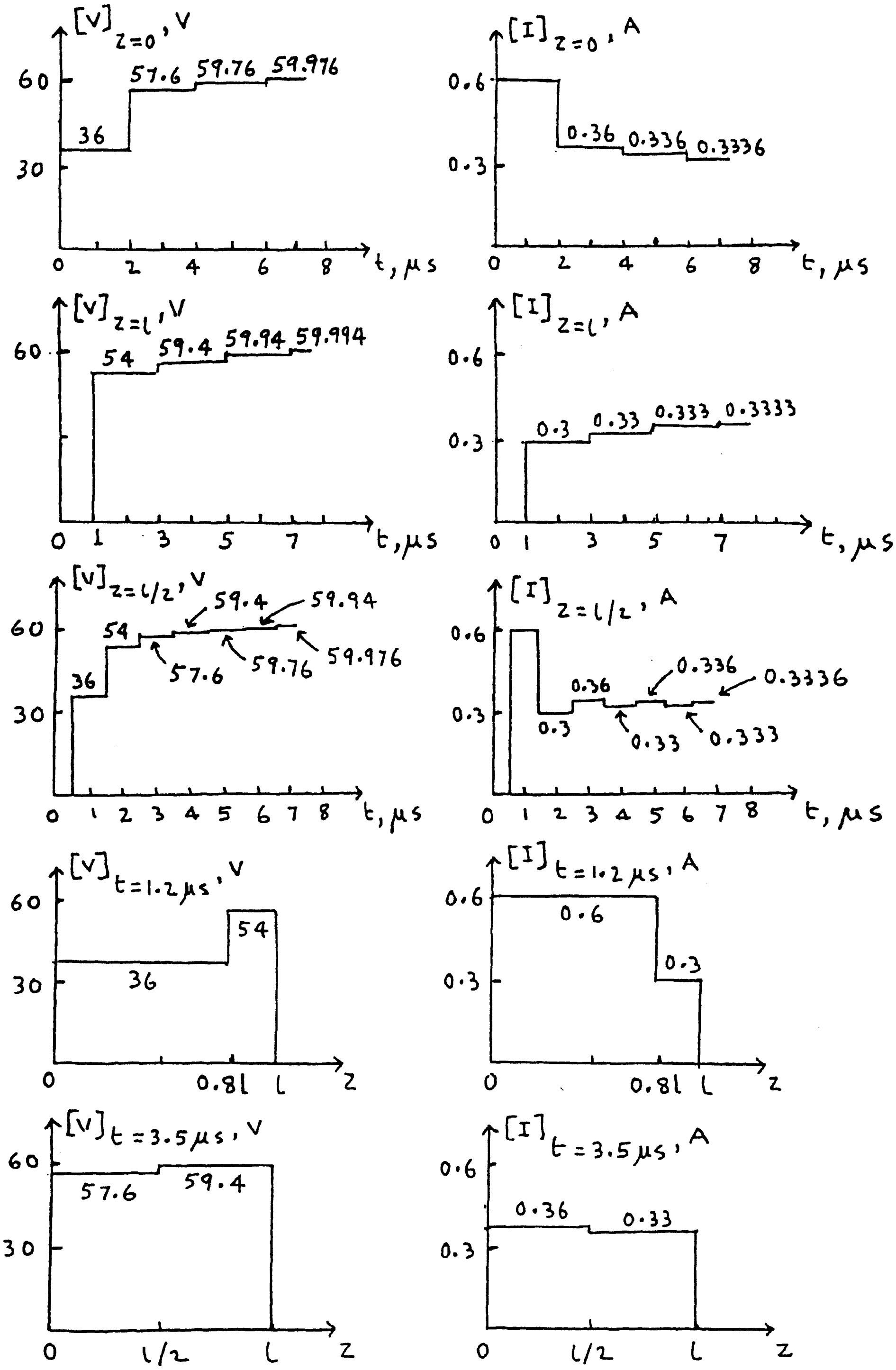
$$V^+ = 90 \times \frac{60}{90+60} = 36 \text{ V} \qquad \Gamma_R = \frac{180-60}{180+60} = \frac{1}{2}$$

$$I^+ = \frac{60}{60} = 1 \text{ A} \qquad \Gamma_S = \frac{90-60}{90+60} = \frac{1}{5}$$

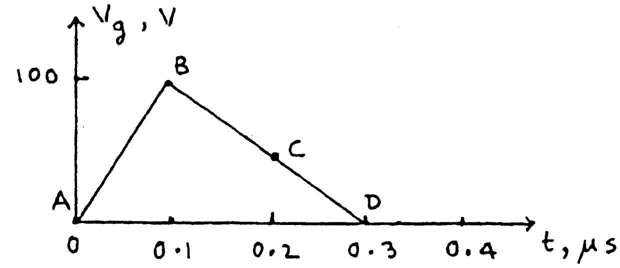




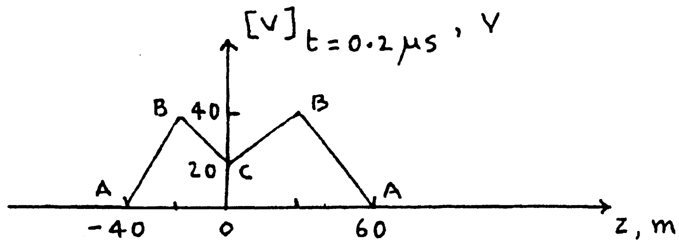
P6.8. (continued)



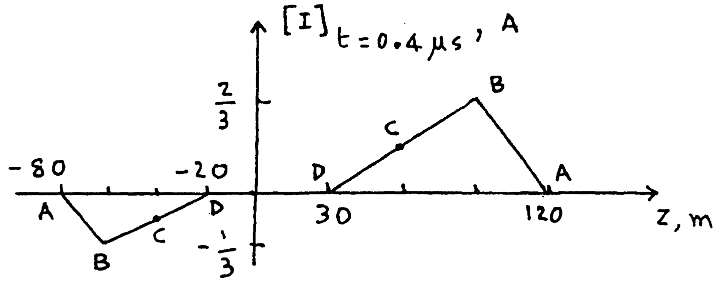
P6.9.


V_g, V
100
A
B
C
D
0
0.1
0.2
0.3
0.4
t, μs

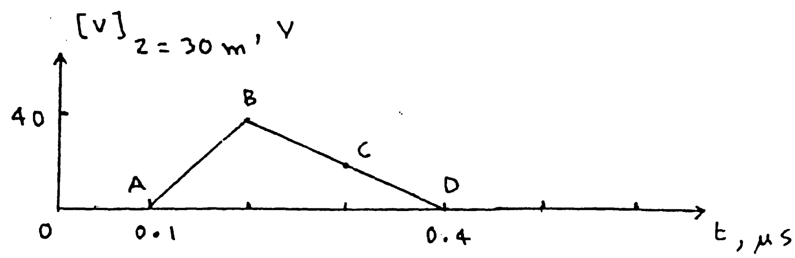
(a)


$[V]_{t=0.2\,\mu s}$, V
B
40
B
20
C
A
A
-40
0
60
z, m

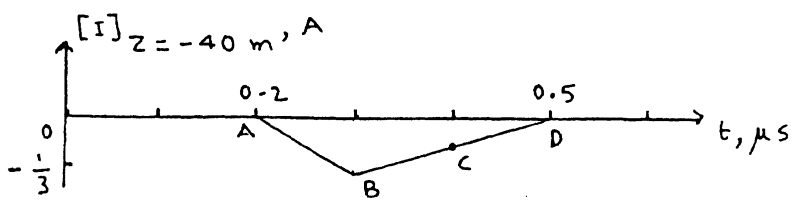
(b)


$[I]_{t=0.4\,\mu s}$, A
$\frac{2}{3}$
B
C
-80
-20
D
A
A
D
30
120
z, m
C
B
$-\frac{1}{3}$

(c)


$[V]_{z=30\,m}$, V
40
B
C
A
D
0
0.1
0.4
t, μs

(d)


$[I]_{z=-40\,m}$, A
0.2
0.5
0
A
D
t, μs
$-\frac{1}{3}$
C
B

P6.10. For voltage source of 0.3 μs duration,

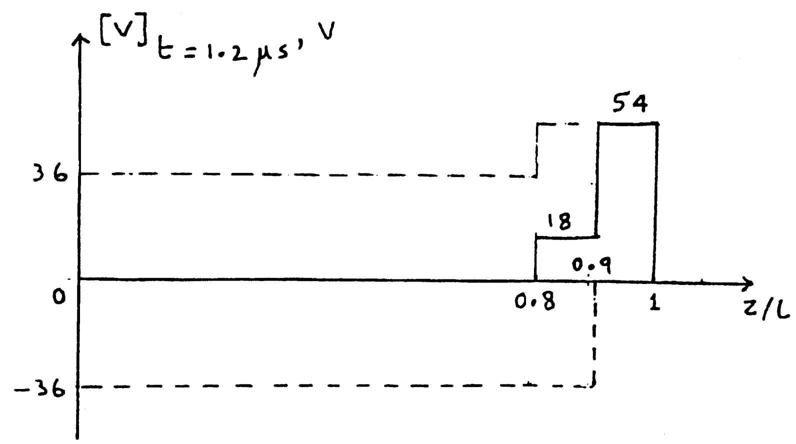
$[V(z)]_{t=1.2\mu s} = [V(z)]_{t=1.2\ \mu s}$ for 90 V source of infinite duration turned on at $t = 0$

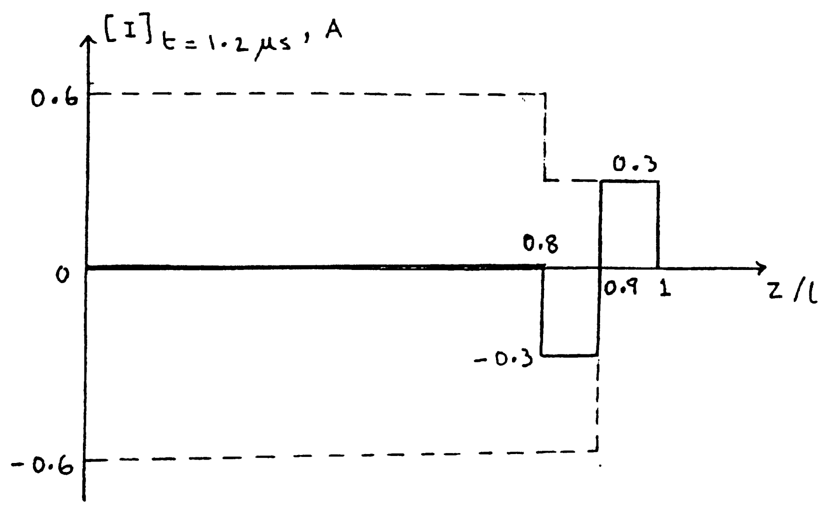
$+ [V(z)]_{t=1.2\ \mu s}$ for –90 V source of infinite duration turned on at $t = 0.3\ \mu$s

$= [V(z)]_{t=1.2\ \mu s}$ for 90 V source of infinite duration turned on at $t = 0$

$+ [V(z)]_{t=0.9\ \mu s}$ for –90 V source of infinite duration turned on at $t = 0$

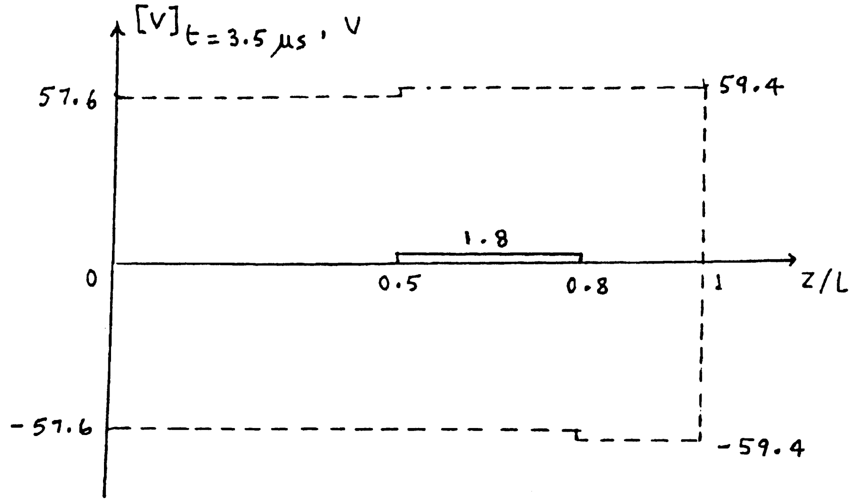
From the bounce diagram of Prob. P6.8, we then obtain the following:

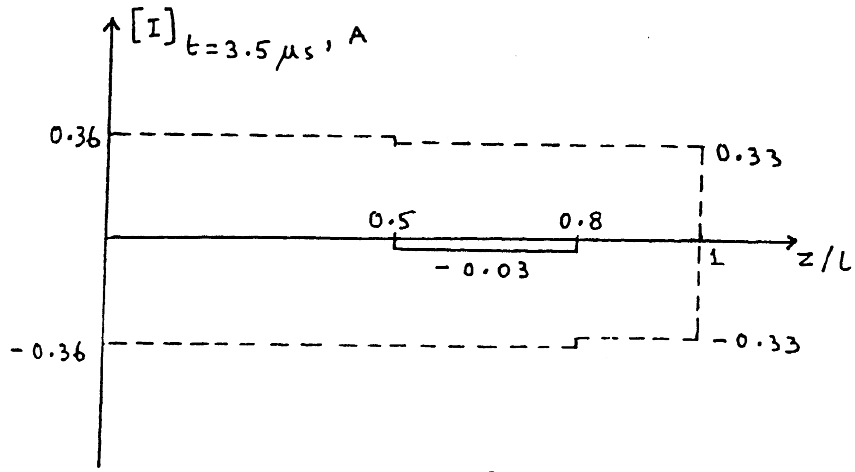




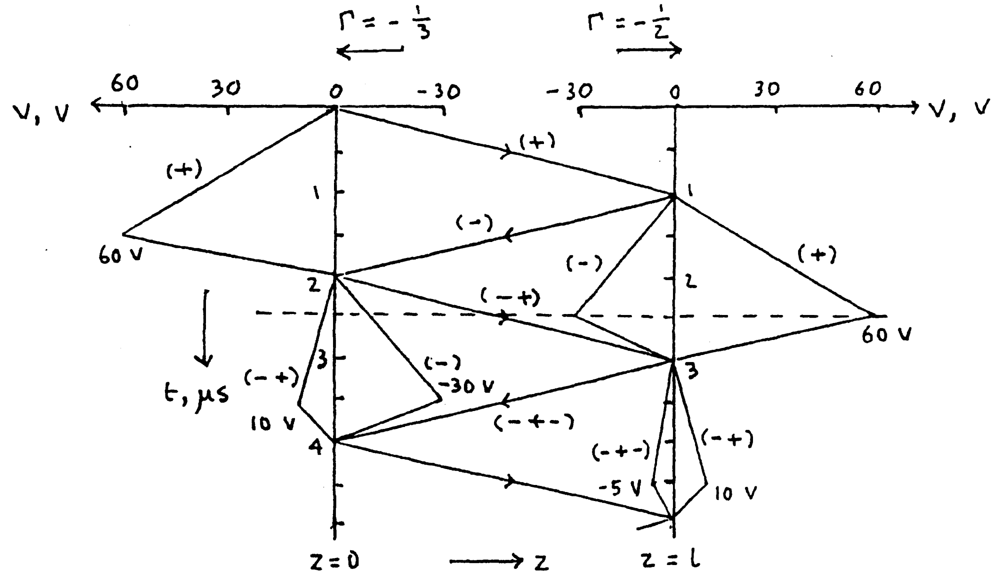
P6.10. (continued)

Similarly for $t = 3.5\ \mu s$, we obtain the following:



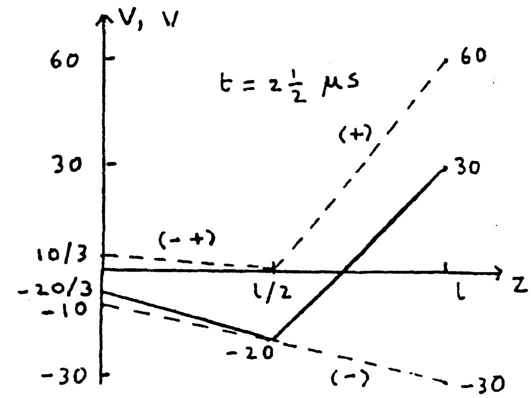


P6.11.



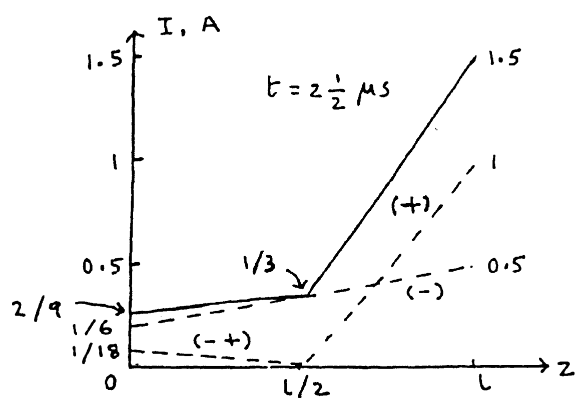
Γ = -1/3
Γ = -1/2
V, V
60
30
0
-30
-30
0
30
60
V, V
(+)
(+)
1
(-)
60 V
(-)
(+)
2
2
(- +)
60 V
t, μs
3
(-)
-30 V
3
(- +)
10 V
(- + -)
4
(- + -)
(- +)
-5 V
10 V
z = 0
z
z = l

(a)



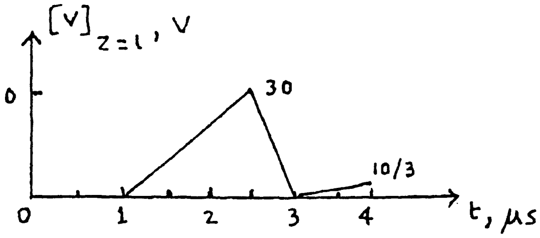
V, V
60
t = 2½ μs
60
(+)
30
30
10/3
(- +)
-20/3
-10
l/2
l
z
-20
-30
(-)
-30

(b)



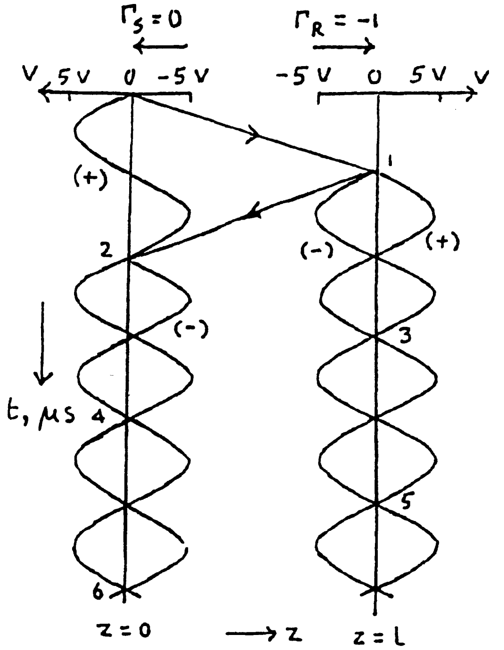
I, A
1.5
1.5
t = 2½ μs
1
1
(+)
0.5
1/3
0.5
(-)
2/9
1/6
1/18
(- +)
0
l/2
l
z

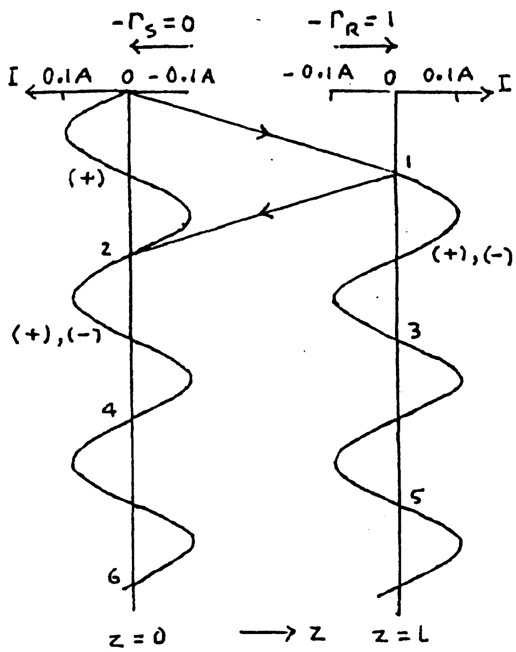
(c)

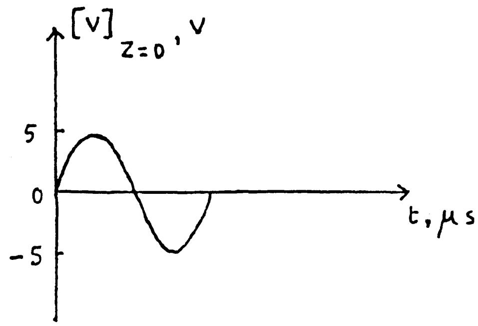


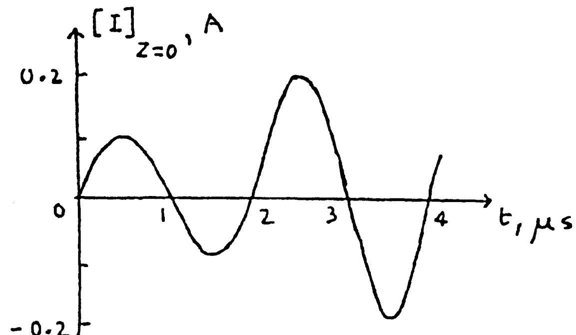
[V]z=l, V
30
30
10/3
0
1
2
3
4
t, μs

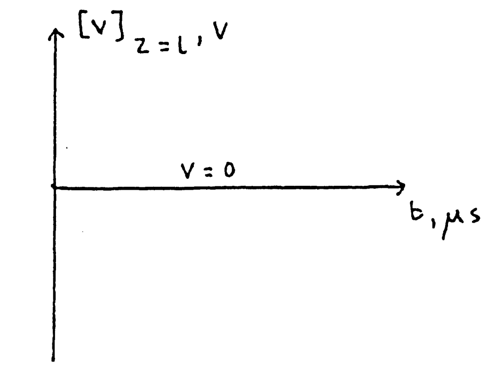
P6.12.

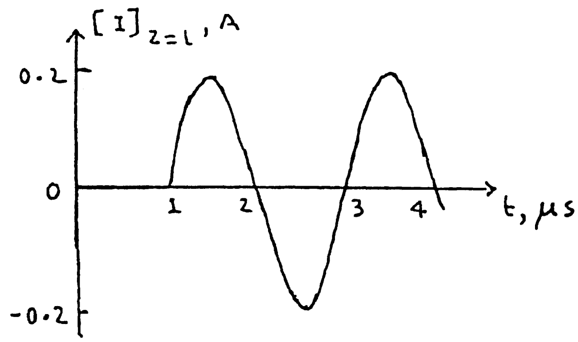

$\Gamma_S = 0$
$\Gamma_R = -1$
V 5 V 0 −5 V
−5 V 0 5 V V
1
(+)
2
(−)
(+)
(−)
3
t, μs 4
5
6
z = 0
→ z
z = l


$-\Gamma_S = 0$
$-\Gamma_R = 1$
I 0.1A 0 −0.1A
−0.1A 0 0.1A I
1
(+)
2
(+), (−)
(+), (−)
3
4
5
6
z = 0
→ z
z = l

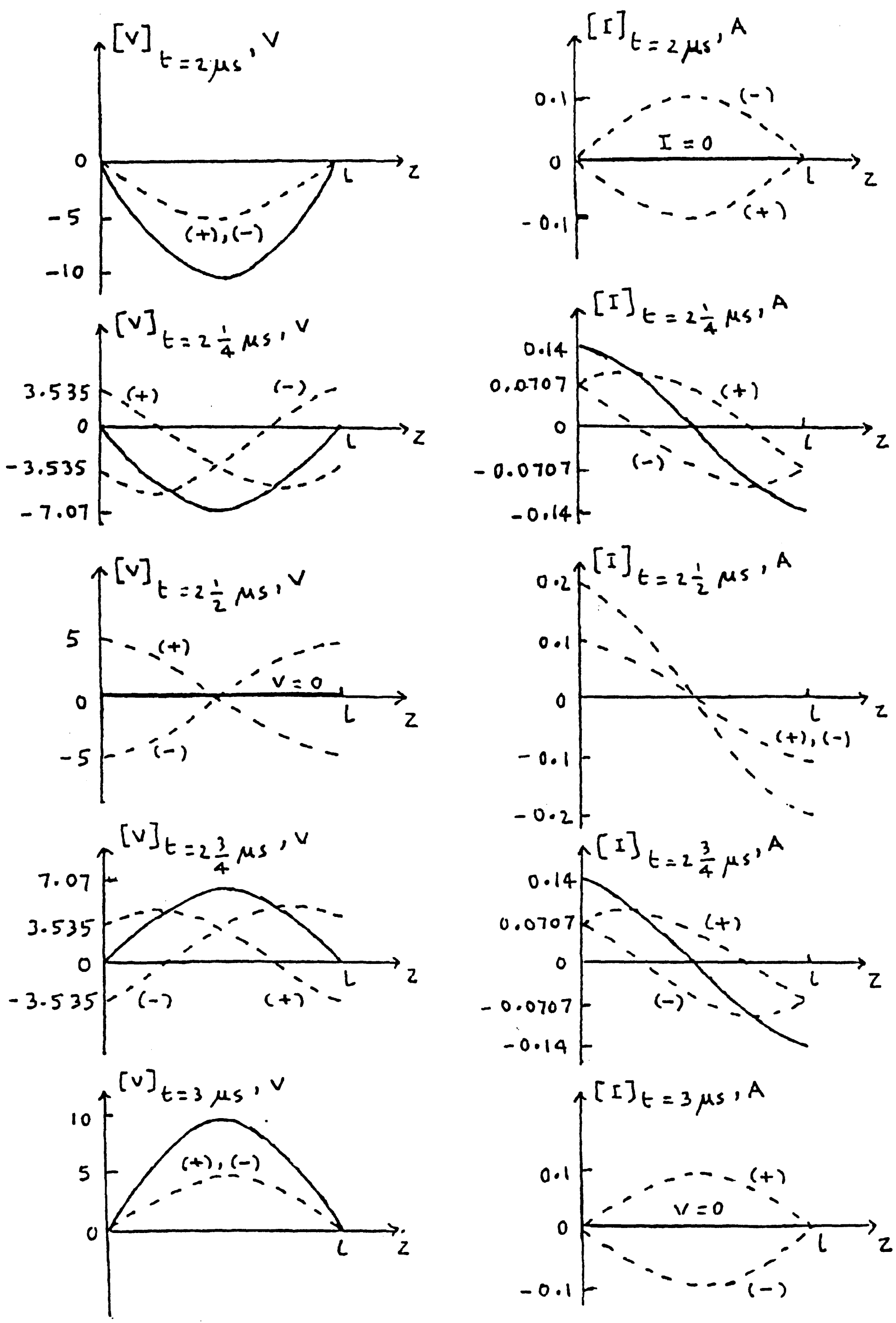

$[V]_{z=0}$, V
5
0
−5
t, μs


$[I]_{z=0}$, A
0.2
0
−0.2
1
2
3
4
t, μs

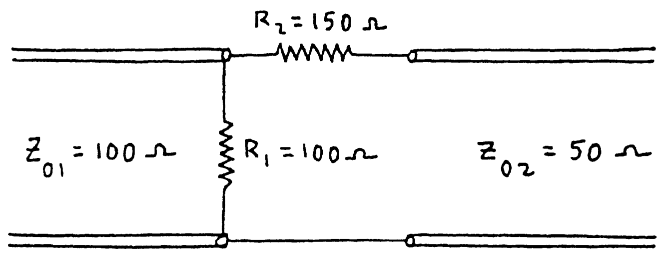

$[V]_{z=l}$, V
V = 0
t, μs

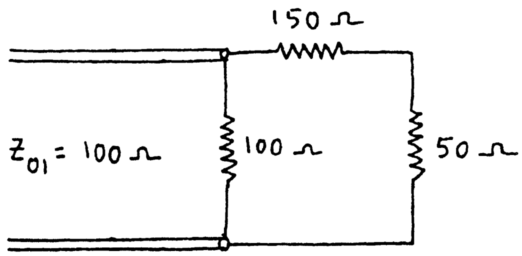

$[I]_{z=l}$, A
0.2
0
−0.2
1
2
3
4
t, μs

P6.12. (continued)



P6.13.





$$\Gamma = \frac{\frac{200}{3} - 100}{\frac{200}{3} + 100} = -\frac{1}{5}$$

$$\tau_V = 1 + \Gamma = 1 - \frac{1}{5} = \frac{4}{5}$$

$$\tau_C = 1 + \Gamma = 1 + \frac{1}{5} = \frac{6}{5}$$

$$\tau_{V_{\text{eff}}} = \frac{50}{50 + 150}\tau_V = \frac{1}{4} \times \frac{4}{5} = \frac{1}{5}$$

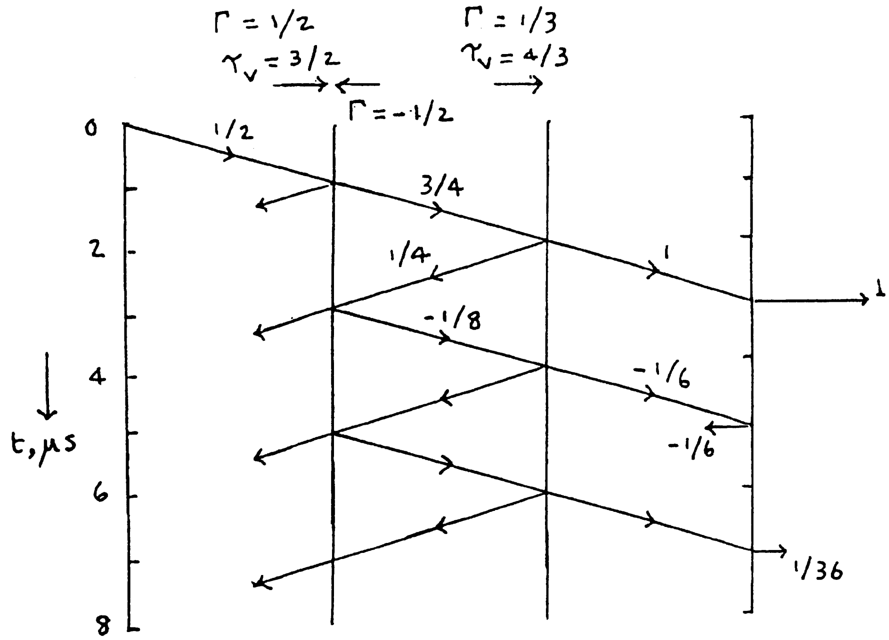
$$\tau_{C_{\text{eff}}} = \frac{100}{100 + 200}\tau_C = \frac{1}{3} \times \frac{6}{5} = \frac{2}{5}$$

Thus

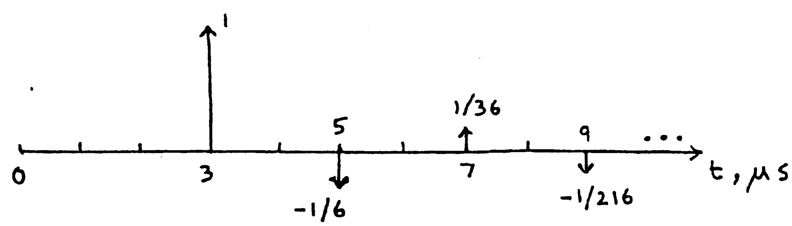
$V^- = -0.2\ V^+$, $\qquad I^- = -\dfrac{V^-}{Z_{01}} = 0.002\ V^+$

$V^{++} = 0.1\ V^+$, $\qquad I^{++} = 0.004\ V^+$

P6.14.



(a)



$$V_o(t) = \sum_{n=0,1,2,\ldots}^{\infty} \left(-\frac{1}{6}\right)^n \delta(t - 2\times10^{-6}n - 3\times10^{-6})$$

(b) For $V_g(t) = \cos \omega t$,

$$V_o(t) = \sum_{n=0}^{\infty}\left(-\frac{1}{6}\right)^n \cos \omega(t - 2\times10^{-6}n - 3\times10^{-6})$$

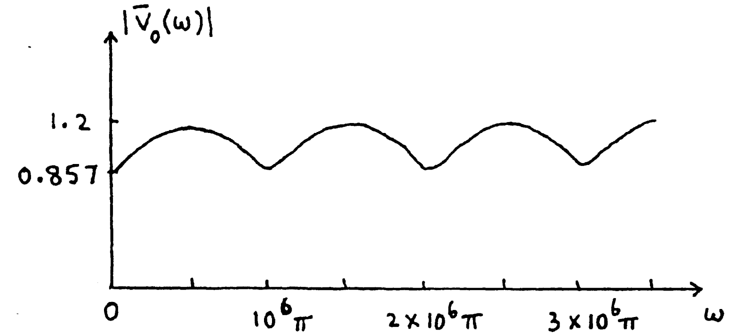
$$\bar{V}_o(\omega) = \sum_{n=0}^{\infty}\left(-\frac{1}{6}\right)^n e^{-j\omega(2\times10^{-6}n + 3\times10^{-6})}$$

$$= e^{-j\omega 3\times10^{6}} \sum_{n=0}^{\infty}\left(-\frac{1}{6}e^{-j\omega 2\times10^{-6}}\right)^n$$

P6.14. (continued)

$$= \frac{e^{-j\omega 3\times10^{-6}}}{1+\frac{1}{6}e^{-j\omega 2\times10^{-6}}}$$

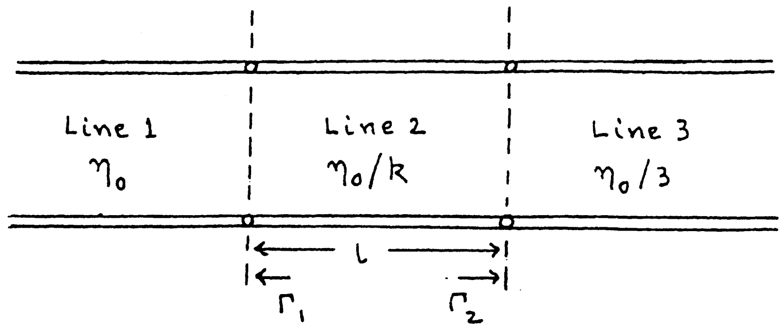
$$\left|\bar{V}_o(\omega)\right| = \frac{1}{\left|1+\frac{1}{6}e^{-j\omega 2\times10^{-6}}\right|}$$



$$\left|\bar{V}_o(\omega)\right|_{\max} = \frac{1}{1-\frac{1}{6}} = \frac{6}{5} = 1.2$$

$$\left|\bar{V}_o(\omega)\right|_{\min} = \frac{1}{1+\frac{1}{6}} = \frac{6}{7} = 0.857$$

P6.15.



$$\Gamma_1\Gamma_2 = -\frac{1}{15},\ T_2 = 1\ \mu\text{s}$$

$$\left(\frac{\eta_0 - \dfrac{\eta_0}{k}}{\eta_0 + \dfrac{\eta_0}{k}}\right)\left(\frac{\dfrac{\eta_0}{3} - \dfrac{\eta_0}{k}}{\dfrac{\eta_0}{3} + \dfrac{\eta_0}{k}}\right) = -\frac{1}{15}$$

$$\left(\frac{k-1}{k+1}\right)\left(\frac{k-3}{k+3}\right) = -\frac{1}{15}$$

$$15(k^2 - 4k + 3) = -(k^2 + 4k + 3)$$

$$16k^2 - 56k + 48 = 0$$

$$2k^2 - 7k + 6 = 0$$

$$(2k - 3)\ (k - 2) = 0$$

$$k = \frac{3}{2} \text{ or } 2$$

$$\varepsilon_2 = 2.25\varepsilon_0 \text{ or } 4\varepsilon_0$$

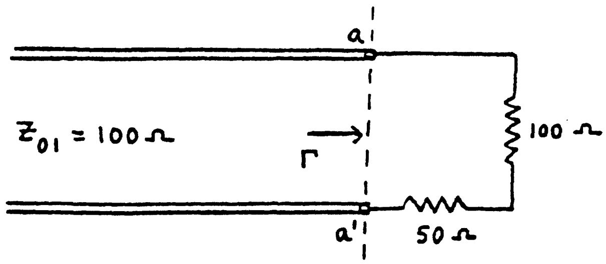
Then $v_{p2} = \dfrac{1}{\sqrt{\mu_0\varepsilon_2}} = \dfrac{c}{1.5}$ or $\dfrac{c}{2} = 2 \times 10^8$ m/s or 1.5×10^8 m/s

$$l = v_{p2}T_2 = 2 \times 10^8 \times 10^{-6} \text{ or } 1.5 \times 10^8 \times 10^{-6} = 200 \text{ m or } 150 \text{ m}$$

$\therefore$ Minimum value of $l = 150$ m corresponding to permittivity $\varepsilon_2 = 4\varepsilon_0$.

$$A = 1\left(1 + \frac{\frac{1}{2} - 1}{\frac{1}{2} + 1}\right)\left(1 + \frac{\frac{1}{3} - \frac{1}{2}}{\frac{1}{3} + \frac{1}{2}}\right) = \left(\frac{2}{3}\right)\left(\frac{4}{5}\right) = \frac{8}{15}$$

P6.16.



$$\Gamma = \frac{150-100}{150+100} = \frac{50}{250} = \frac{1}{5}$$

$$\tau_V = 1 + \Gamma = \frac{6}{5}$$

$$\tau_{V_{\text{eff2}}} = \frac{6}{5} \times \frac{100}{150} = \frac{4}{5}$$

$$\tau_{V_{\text{eff3}}} = \frac{6}{5} \times \frac{50}{150} = \frac{2}{5}$$

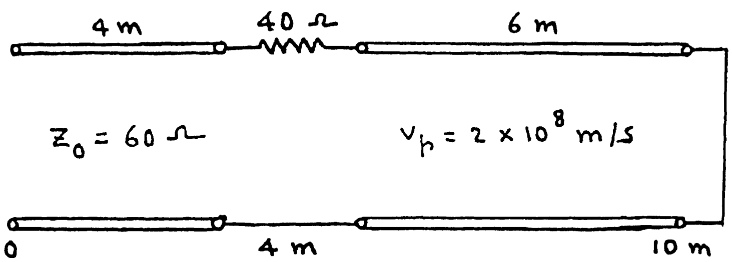
$$\tau_I = 1 - \Gamma = 1 - \frac{1}{5} = \frac{4}{5}$$

(a) Power reflected into line 1 $= \Gamma^2 P = \frac{1}{25} P$

(b) Power transmitted into line 2 $= \tau_{V_{\text{eff2}}} \tau_I \, P = \frac{16}{25} P$

(c) Power transmitted into line 3 $= \tau_{V_{\text{eff3}}} \tau_I \, P = \frac{8}{25} P$

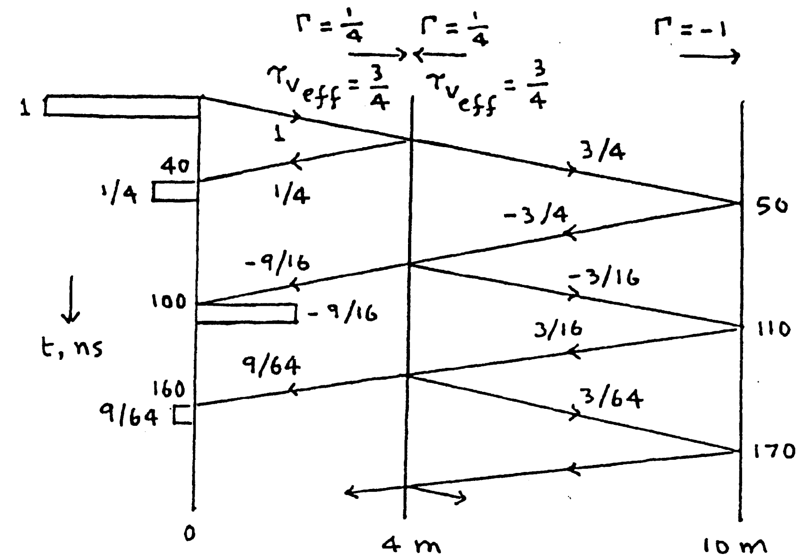
P6.17.

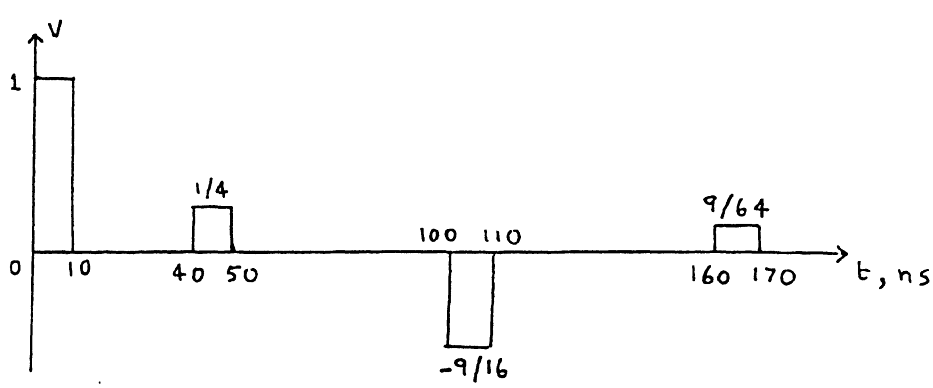


$$\Gamma = \frac{100-60}{100+60} = \frac{40}{160} = \frac{1}{4}$$

$$\tau_V = 1 + \Gamma = \frac{5}{4}$$

$$\tau_{V_{\text{eff}}} = \tau_V \frac{60}{60+40} = \frac{5}{4} \times \frac{3}{5} = \frac{3}{4}$$





P6.18. (a) $I_L = I^+ + I^-$

$$= \frac{V_0}{2Z_0} + \frac{V_0}{2Z_0} - \frac{V_0}{Z_0} e^{-(Z_0/L)(t-T)} \text{ for } t > T$$

$$1.73 = \frac{100}{100} + \frac{100}{100} - \frac{100}{50} e^{-(50/0.1)(t_1 - 10^{-3})}$$

$$1.73 = 2 - 2e^{-500(t_1 - 10^{-3})}$$

$$2e^{-500(t_1 - 10^{-3})} = 2 - 1.73 = 0.27$$

$$e^{-500(t_1 - 10^{-3})} = 0.135$$

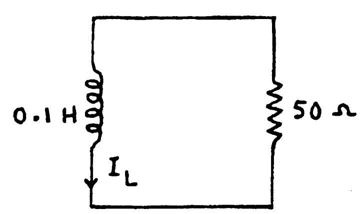
$$500(t_1 - 10^{-3}) = 2.00$$

$$t_1 - 10^{-3} = \frac{2}{500} = 4 \times 10^{-3}$$

$$t_1 = 5 \times 10^{-3} \text{ s} = 5 \text{ ms}$$

(b) $$0.1 \frac{dI_L}{dt} + 50 I_L = 0$$

$$\frac{dI_L}{dt} + 500 I_L = 500$$



$$I_L = Ce^{-500t}$$

$$1.73 = Ce^{-500(5 \times 10^{-3})}$$

$$C = 1.73 e^{500(5 \times 10^{-3})}$$

$$I_L = 1.73 e^{-500(t - 5 \times 10^{-3})}$$

$$0.636 = 1.73 e^{-500(t_2 - 5 \times 10^{-3})}$$

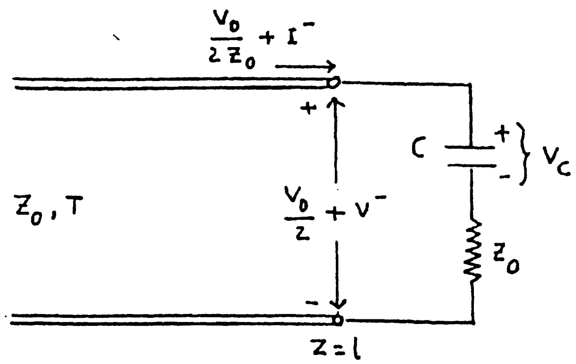
$$e^{-500(t_2 - 5 \times 10^{-3})} = \frac{0.636}{1.73} = 0.368$$

$$500(t_2 - 5 \times 10^{-3}) = 1.00$$

$$t_2 - 5 \times 10^{-3} = \frac{1}{500} = 2 \times 10^{-3}$$

$$t_2 = 7 \times 10^{-3} \text{ s} = 7 \text{ ms}$$

P6.19. (a)



$$\frac{V_0}{2} + V^- = \frac{1}{C}\int\left(\frac{V_0}{2Z_0} + I^-\right)dt + Z_0\left(\frac{V_0}{2Z_0} + I^-\right)$$

$$\frac{V_0}{2} + V^- = \frac{1}{C}\int\left(\frac{V_0}{2Z_0} - \frac{V^-}{Z_0}\right)dt + \left(\frac{V_0}{2} - V^-\right)$$

$$\frac{dV^-}{dt} = \frac{1}{C}\left(\frac{V_0}{2Z_0} - \frac{V^-}{Z_0}\right) - \frac{dV^-}{dt}$$

$$\frac{dV^-}{dt} + \frac{1}{2CZ_0}V^- = \frac{V_0}{4CZ_0} \qquad \text{for } t > T$$

(b) $V^- = \frac{V_0}{2} + Ae^{-(1/2CZ_0)t}$

Since $[V_C]_{t=T} = 0$ (initial condition),

$$\frac{V_0}{2} + V^- = Z_0\left(\frac{V_0}{2Z_0} + I^-\right) \text{ at } t = T$$

$$\frac{V_0}{2} + V^- = \frac{V_0}{2} - V^-$$

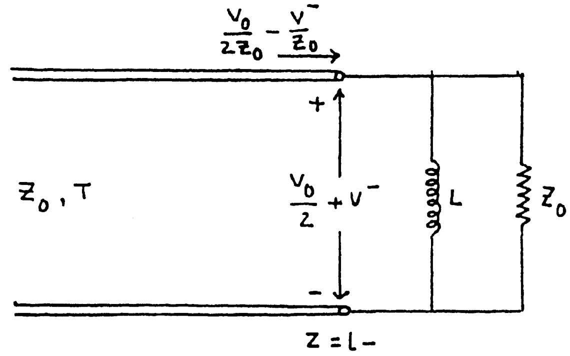
$V^- = 0$ at $t = T$

$$0 = \frac{V_0}{2} + Ae^{-(1/2CZ_0)T}$$

$$A = -\frac{V_0}{2}e^{(1/2CZ_0)T}$$

$$V^-(l, t) = \frac{V_0}{2} - \frac{V_0}{2}e^{-(1/2CZ_0)(t-T)} \qquad \text{for } t > T$$

P6.20.



$$\frac{V_0}{2Z_0} - \frac{V^-}{Z_0} = \frac{1}{L}\int\left(\frac{V_0}{2} + V^-\right)dt + \frac{1}{Z_0}\left(\frac{V_0}{2} + V^-\right)$$

$$-\frac{1}{Z_0}\frac{dV^-}{dt} = \frac{1}{L}\left(\frac{V_0}{2} + V^-\right) + \frac{1}{Z_0}\frac{dV^-}{dt}$$

$$\frac{2L}{Z_0}\frac{dV^-}{dt} + V^- = -\frac{V_0}{2}$$

$$V^-(t) = -\frac{V_0}{2} + Ae^{-(Z_0/2L)t}$$

At $t = T+$, current through L is zero.

$$\therefore \left[\frac{V_0}{2Z_0} - \frac{V^-}{Z_0}\right]_{t=T+} = \frac{1}{Z_0}\left[\frac{V_0}{2} + V^-\right]_{t=T+}$$

$$[V^-]_{t=T+} = 0$$

$$0 = -\frac{V_0}{2} + Ae^{-(Z_0/2L)T}$$

$$A = \frac{V_0}{2}e^{(Z_0/2L)T}$$

$$V^-(t) = -\frac{V_0}{2} + \frac{V_0}{2}e^{-(Z_0/2L)(t-T)}$$

$$[V]_{z=l+} = [V]_{z=l^-} = \frac{V_0}{2} + V^-$$
$$= \frac{V_0}{2}e^{-(Z_0/2L)(t-T)} \qquad \text{for } t \geq T$$

P6.21. (a) Since for $t > 2T$, the voltage varies exponentially, the circuit element is either L or C.

At $t = 2T_1$, the voltage is double, indicating that the reflected wave voltage is equal to the incident wave voltage initially. Thus the reflection coefficient at the junction is +1, corresponding to an open circuit.

$\therefore$ The circuit element is L (zero initial current in L is equivalent to open circuit).

(b) In the steady state, L behaves like a short circuit. The line voltage at $z = 0$ is $V_0 \dfrac{Z_{02}}{Z_{01} + Z_{02}}$. Hence

$$\frac{Z_{02}}{Z_{01} + Z_{02}} = 0.25$$

$$0.75 Z_{02} = 0.25 Z_{01}$$

$$\frac{Z_{02}}{Z_{01}} = \frac{1}{3}$$

P6.22. At $t = 0$, $V^+ + V^- = 10$

$$\frac{1}{50}(V^+ - V^-) = 0$$

$$V^+ = V^- = 5$$

Thus, voltage incident on the nonlinear element at $t = 0$ is 5 V. Since the element is passive, it cannot produce a reflected wave having $|V^-| > 5$. Therefore, V is positive such that the characteristic is $V = 50\,I^2$. Thus the (–) wave voltage resulting from the incidence of the (+) wave on the nonlinear element is given by the solution of

$$5 + V^- = 50\left(\frac{5}{50} - \frac{V^-}{50}\right)^2$$

$$50(5 + V^-) = (5 - V^-)^2$$

$$(V^-)^2 - 60\,V^- - 225 = 0$$

$$V^- = \frac{60 \pm \sqrt{3600 + 900}}{2} = \frac{60 \pm 67.08}{2}$$
$$= 68.54 \text{ or } -3.54 \text{ V}$$

Ruling out 68.54 V, we obtain $V^- = -3.54$ V.

Voltage across the nonlinear element at $t = 0+$ is $5 - 3.54 = 1.46$ V.

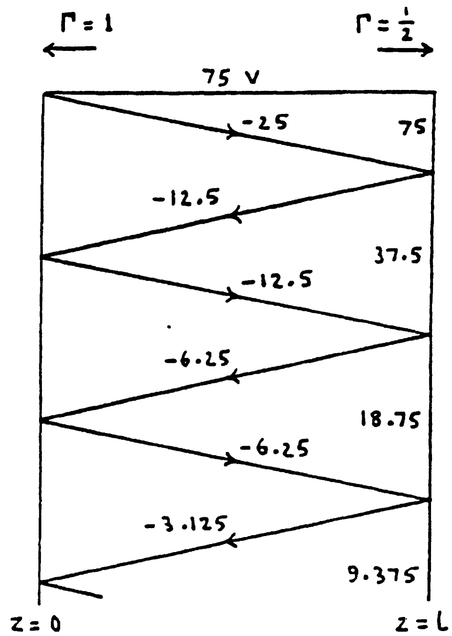
P6.23. (a) $V_{SS} = \dfrac{100}{50+150} \times 150 = 75$ V

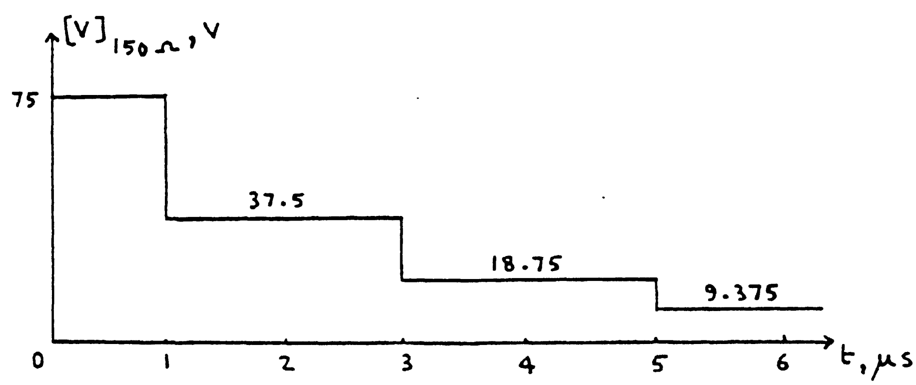
$$I_{SS} = \frac{100}{50+150} = 0.5 \text{ A}$$

$0.5 + I^{+} = 0$ (boundary condition)

$$0.5 + \frac{V^{+}}{50} = 0$$

$$V^{+} = -25 \text{ V}$$





P6.23. (continued)

(b) $W_e = \frac{1}{2} \times \frac{75^2}{50} \times 10^{-6} = 56.25 \times 10^{-6}$ J

$W_m = \frac{1}{2} \times 0.5^2 \times 50 \times 10^{-6} = 6.25 \times 10^{-6}$ J

$W = W_e + W_m = 62.5 \times 10^{-6}$ J

Energy dissipated in the 150 Ω resistor

$$= \frac{75^2}{150} \times 10^{-6} + \frac{37.5^2}{150} \times 2 \times 10^{-6} + \frac{18.75^2}{150} \times 2 \times 10^{-6}$$

$$+ \frac{9.375^2}{150} \times 2 \times 10^{-6} + \ldots$$

$$= 37.5 \times 10^{-6} + \frac{37.5^2 \times 2 \times 10^{-6}}{150}\left(1 + \frac{1}{4} + \frac{1}{16} + \ldots\right)$$

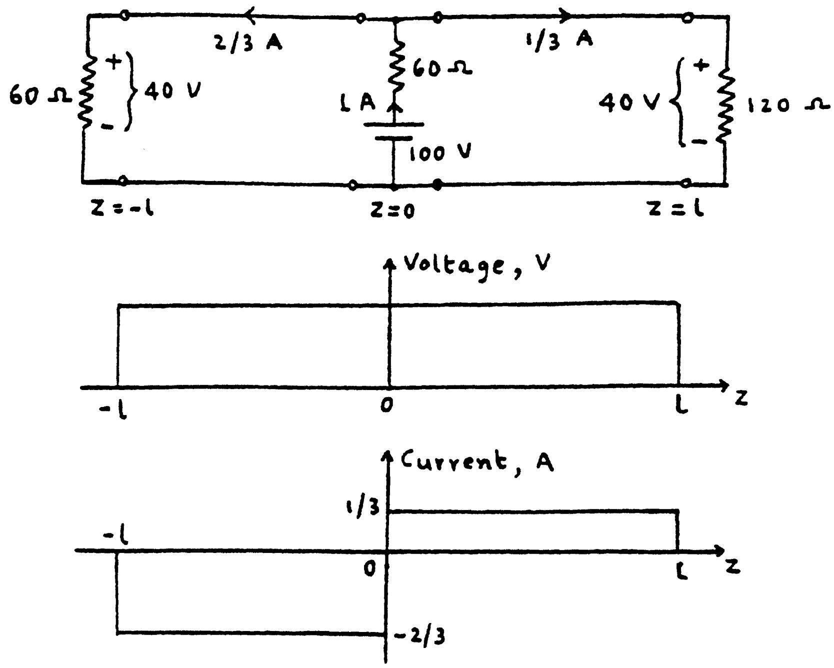
$$= 37.5 \times 10^{-6} + \frac{37.5^2 \times 2 \times 10^{-6}}{150} \times \frac{4}{3}$$

$$= 37.5 \times 10^{-6} + 25 \times 10^{-6}$$

$$= 62.5 \times 10^{-6} \text{ J}$$

= initial stored energy in the line

P6.24. (a)

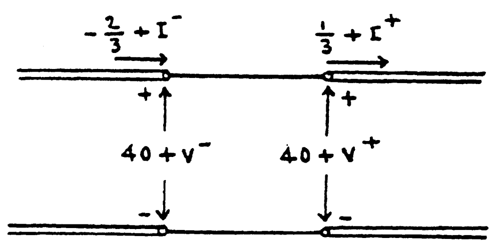


(b) $W_e = \frac{1}{2}\times\frac{40^2}{60}\times 10^{-6} + \frac{1}{2}\times\frac{40^2}{60}\times 10^{-6} = \frac{80}{3}\times 10^{-6}$ J

$W_m = \frac{1}{2}\times\frac{4}{9}\times 60\times 10^{-6} + \frac{1}{2}\times\frac{1}{9}\times 60\times 10^{-6} = \frac{50}{3}\times 10^{-6}$ J

$W = W_e + W_m = \frac{130}{3}\times 10^{-6}$ J

(c)



$40 + V^- = 40 + V^+ \rightarrow V^- = V^+$

$-\frac{2}{3} + I^- = \frac{1}{3} + I^+$

P6.24.. (continued)

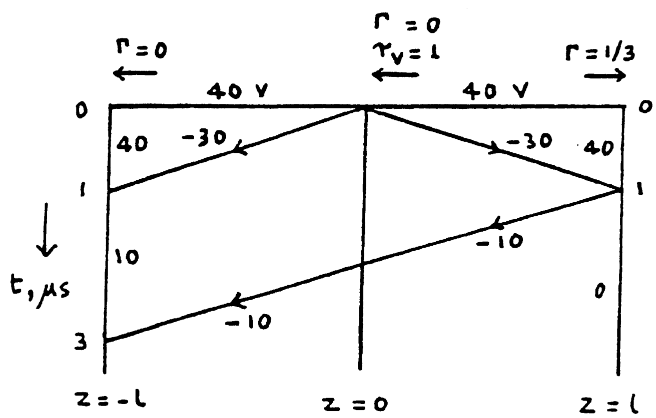
$$-\frac{2}{3}-\frac{V^-}{60}=\frac{1}{3}+\frac{V^+}{60}$$

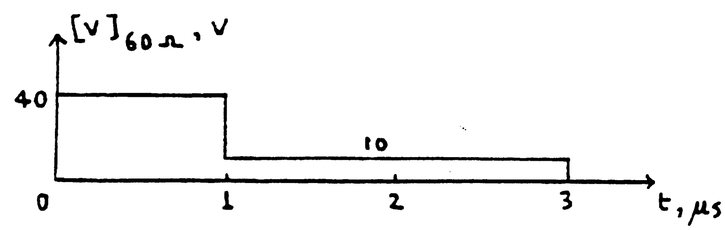
$$-\frac{2}{3}-\frac{V^+}{60}=\frac{1}{3}+\frac{V^+}{60}$$

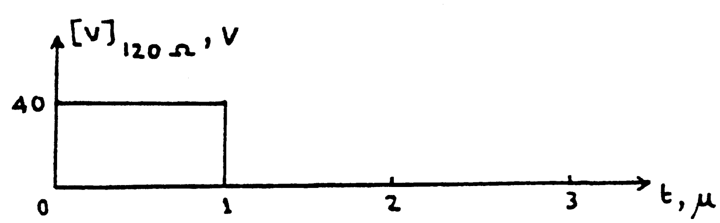
$$\frac{V^+}{30}=-1$$

$$V^+=-30, \qquad V^-=-30$$

$$I^+=-\frac{1}{2}, \qquad I^-=\frac{1}{2}$$







P6.24. (continued)

(d) Energy dissipated

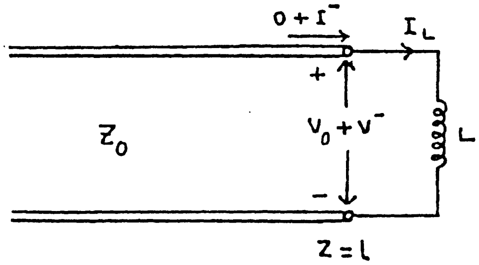
$$= \frac{40^2}{60} \times 10^{-6} + \frac{10^2}{60} \times 2 \times 10^{-6} + \frac{40^2}{120} \times 10^{-6}$$

$$= \left(\frac{80}{3} + \frac{10}{3} + \frac{40}{3} \right) \times 10^{-6}$$

$$= \frac{130}{3} \times 10^{-6} \text{ J}$$

= Initial energy stored in the lines.

P6.25. (a)



$$V_0 + V^- = L\frac{d}{dt}(0 + I^-)$$

$$V_0 + V^- = L\frac{d}{dt}\left(-\frac{V^-}{Z_0}\right)$$

$$\frac{L}{Z_0}\frac{dV^-}{dt} + V^- = -V_0$$

$$V^- = -V_0 + Ae^{-(Z_0/L)t}$$

$$I^- = -\frac{V^-}{Z_0} = \frac{V_0}{Z_0} - \frac{A}{Z_0}e^{-(Z_0/L)t}$$

$$[I_L]_{t=0-} = 0 \rightarrow [0 + I^-]_{t=0+} = 0 \rightarrow [I^-]_{t=0+} = 0$$

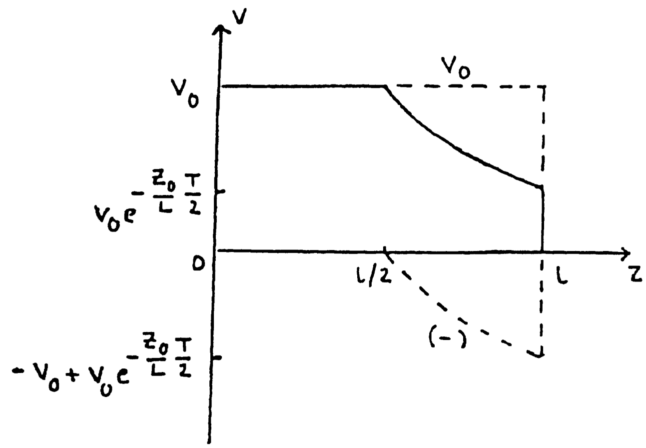
$$\therefore 0 = \frac{V_0}{Z_0} - \frac{A}{Z_0} \rightarrow A = V_0$$

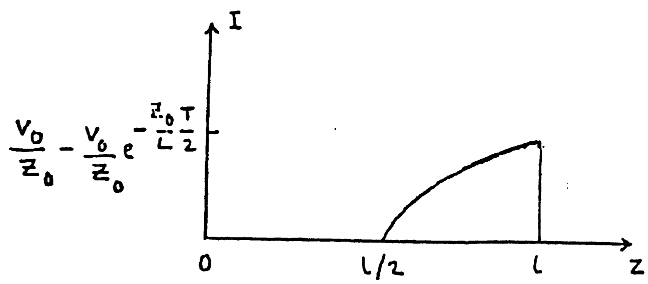
$$V^- = -V_0 + V_0 e^{-(Z_0/L)t}$$

$$V(l, t) = V_0 + V^- = V_0 e^{-(Z_0/L)t}$$

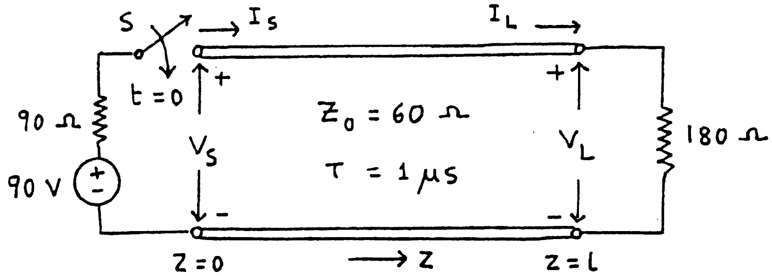
$$I(l, t) = 0 + I^- = \frac{V_0}{Z_0} - \frac{V_0}{Z_0}e^{-(Z_0/L)t}$$

P6.25. (continued)



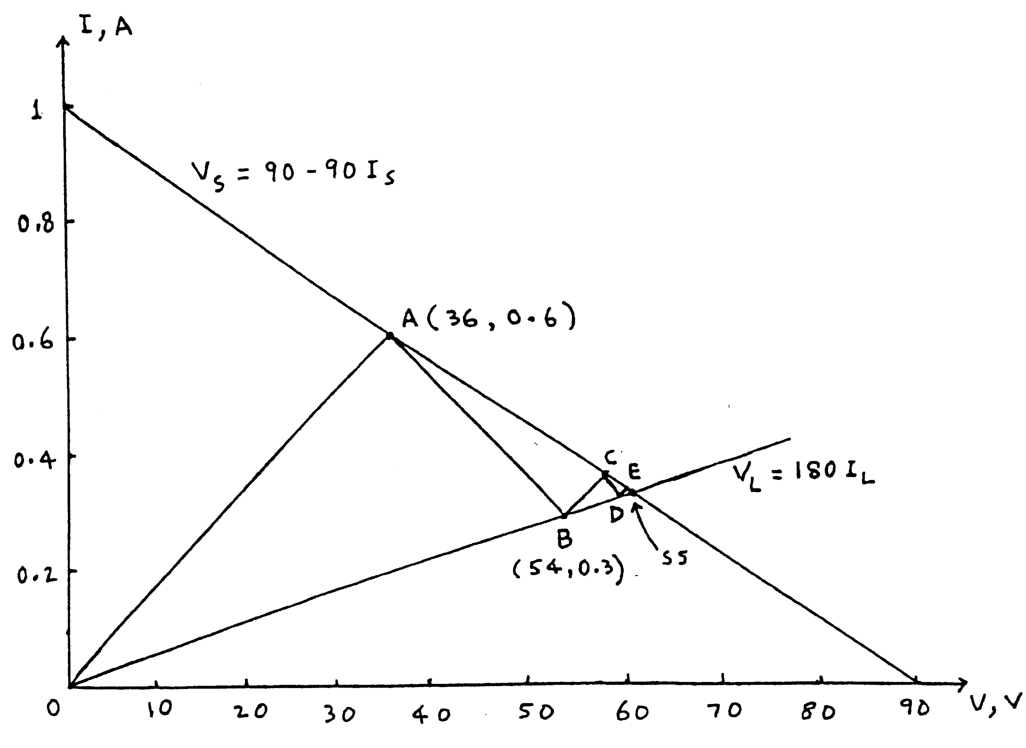


P6.26.

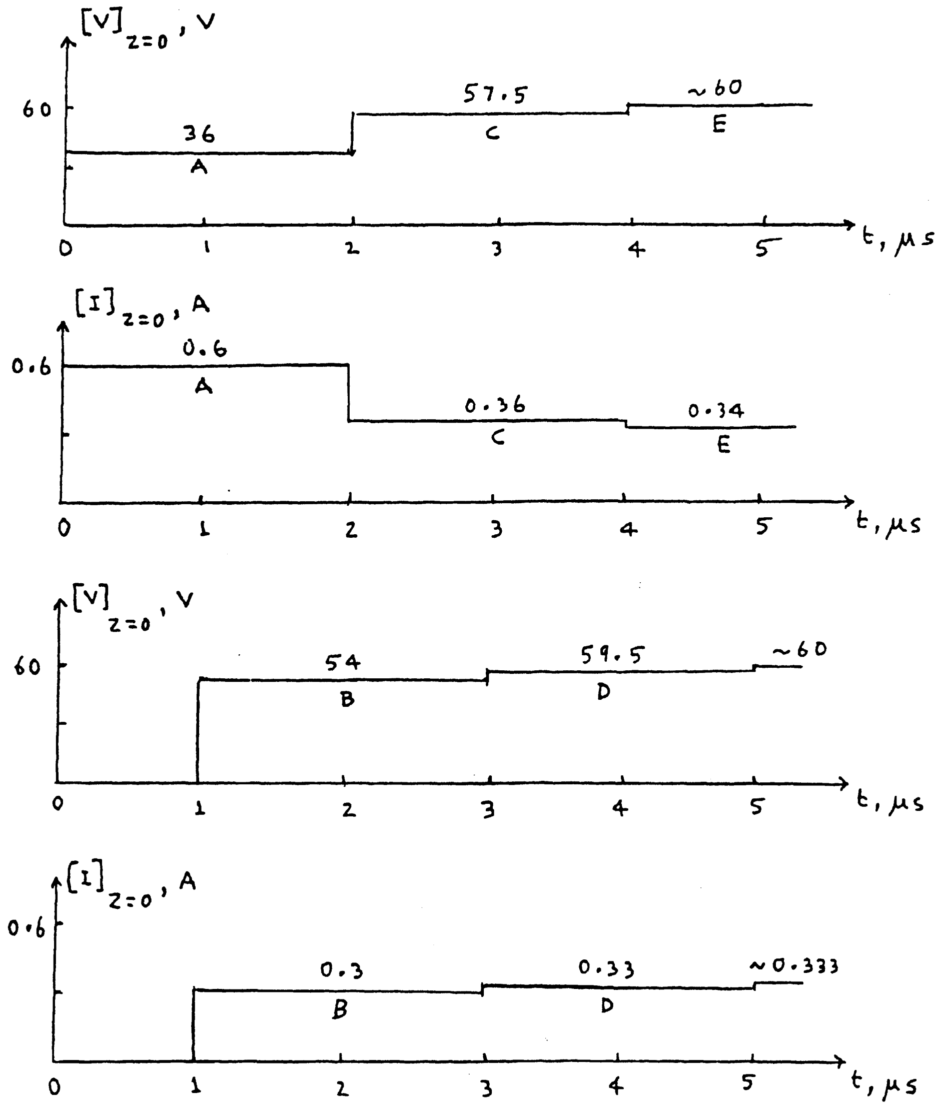


Characteristic at $z = 0$ is $V_S = 90 - 90I_S$

Characteristic at $z = l$ is $V_L = 180I_L$



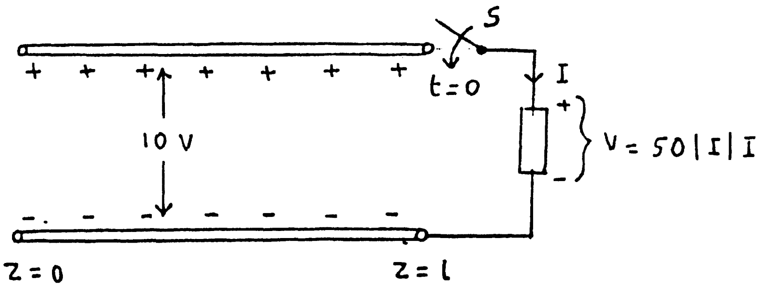
From graph of load line construction,



$V_{SS} = 60$ V

$I_{SS} = 0.333$ A

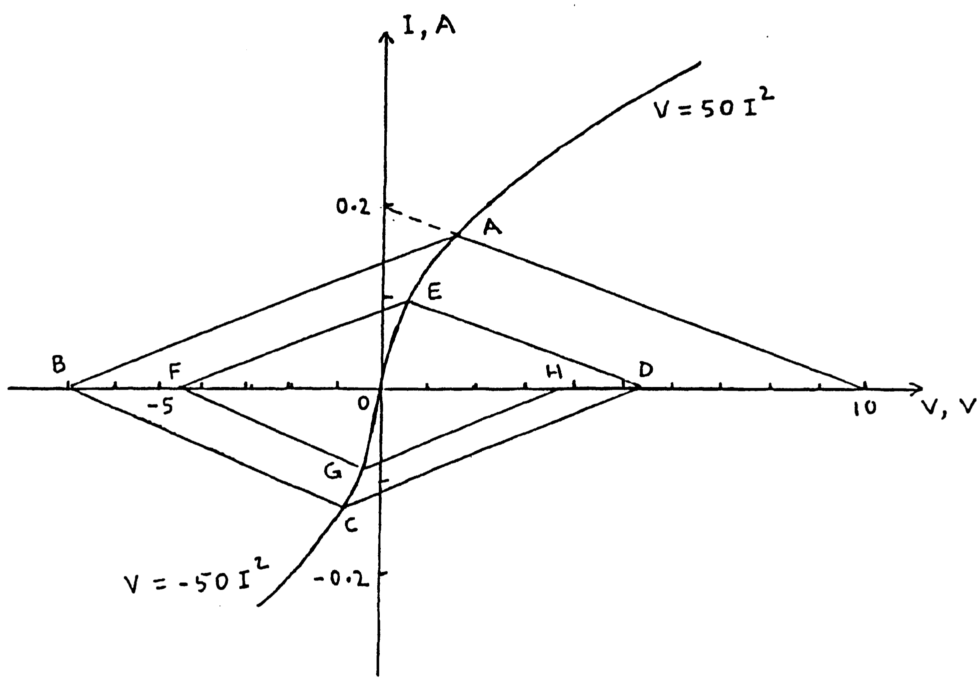
P6.27.



Load characteristic is $V = 50|I|I$

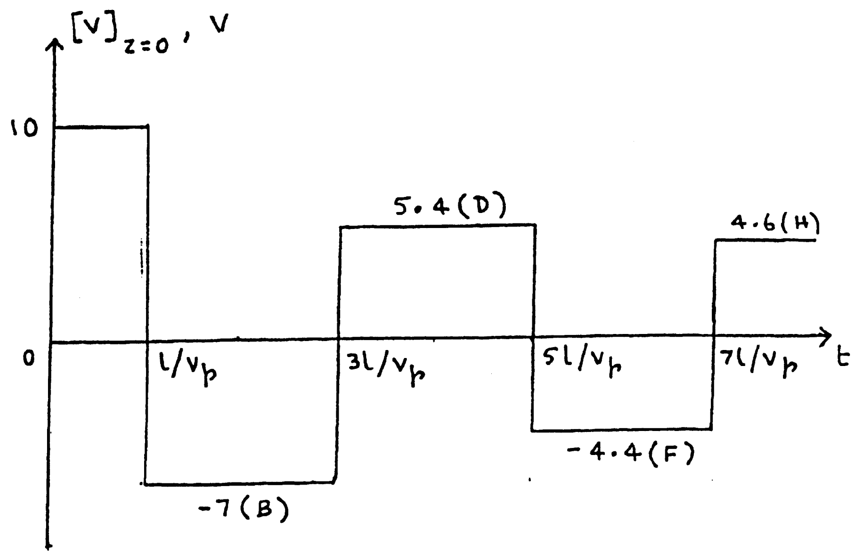
$$\text{or, } V = \begin{cases} 50I^2 & \text{for } I > 0 \\ -50I^2 & \text{for } I < 0 \end{cases}$$

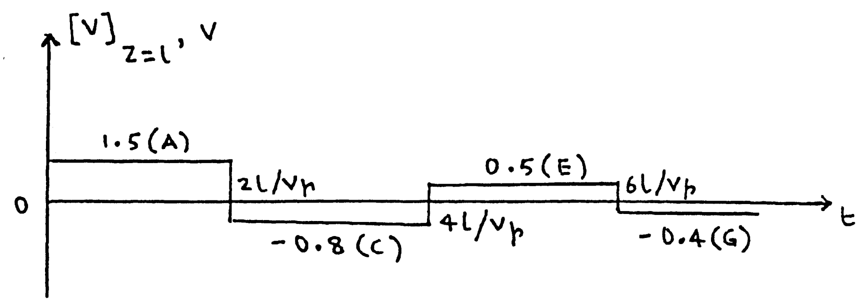
Characteristic at $z = 0$ is open circuit, or, $I = 0$.



P6.27. (continued)

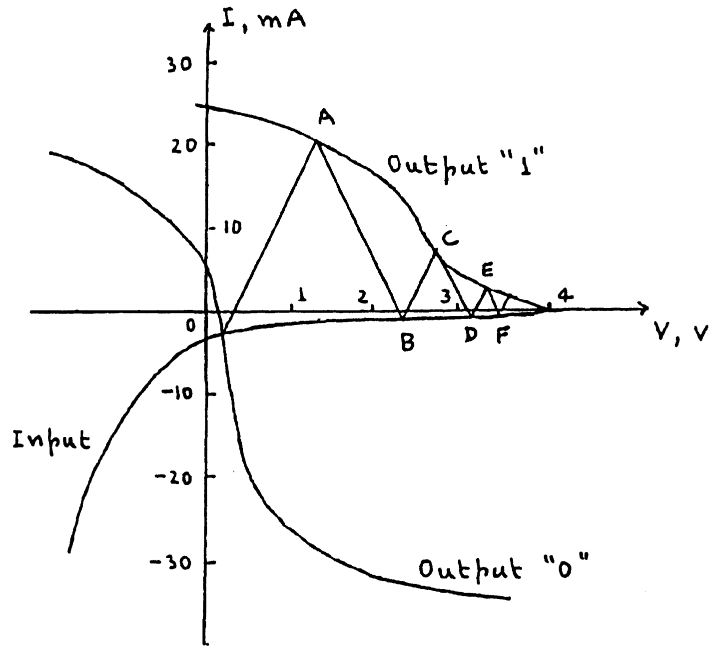
From graph of load line construction,



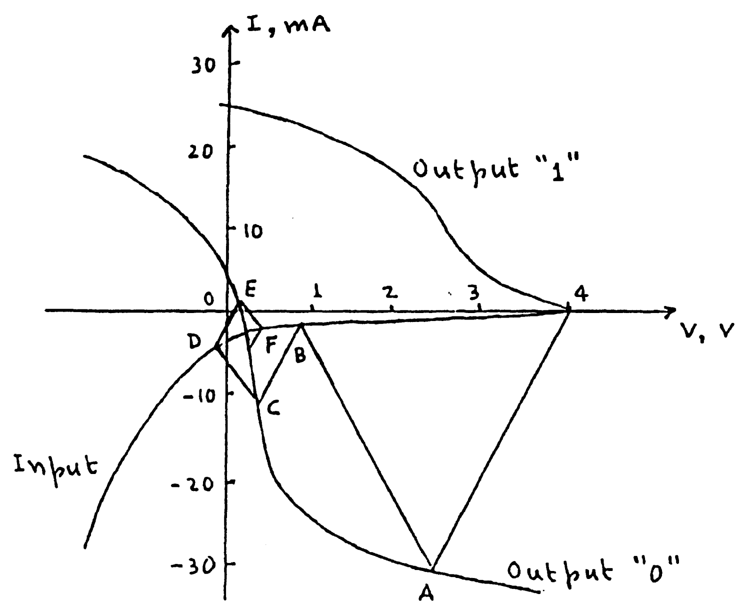


P6.28. $Z_0 = 50\ \Omega$

For "0" to "1" transition,

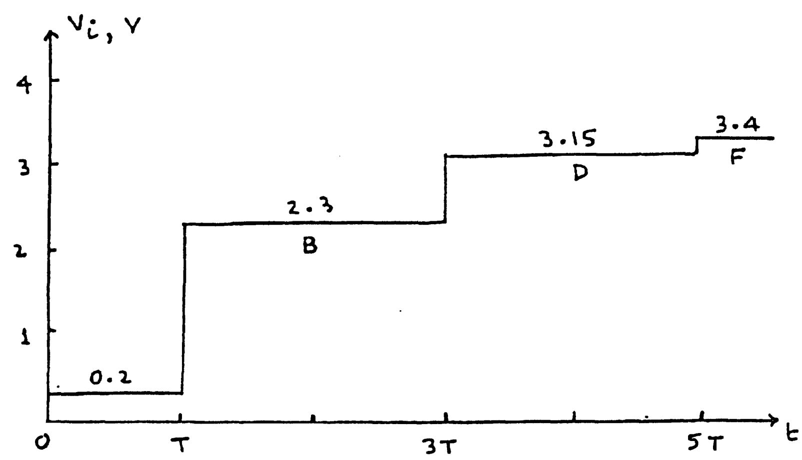


For "1" to "0" transition,

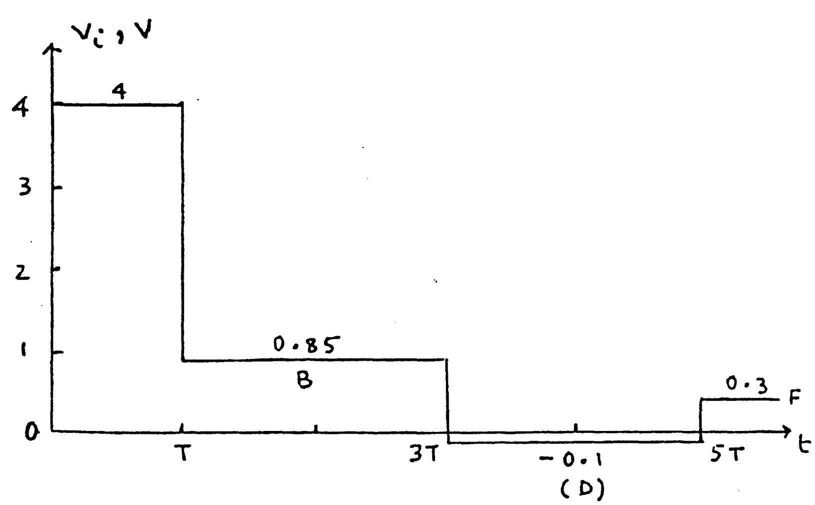


P6.28. (continued)

From the load line construction for "0" to "1" transition,



From the load line construction for "1" to "0" transition,



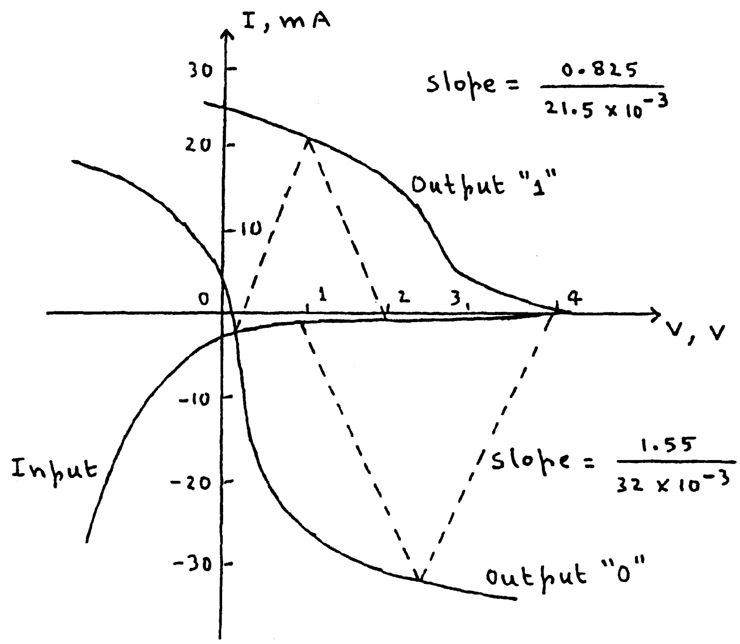
P6.29. From load line constructions,

(a) Minimum value of Z_0 for transition from "0" to "1" for $V_i = 2$ V at $t = T+$ is

$$\frac{0.825}{21.5 \times 10^{-3}} = 38.4\ \Omega$$

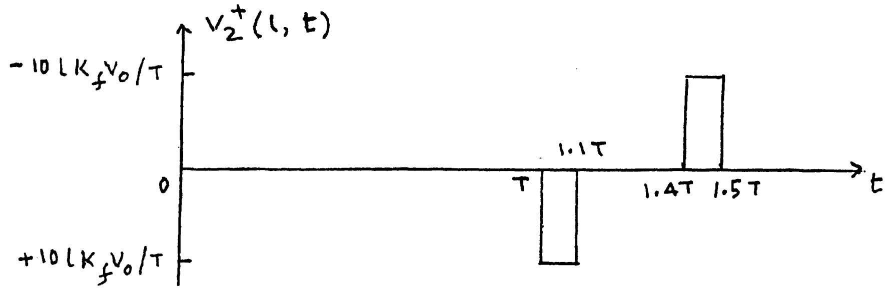
(b) Minimum value of Z_0 for transition from "1" to "0" for $V_i = 1$ V at $t = T+$ is

$$\frac{1.55}{32 \times 10^{-3}} = 48.4\ \Omega$$



P6.30. **(a)** $V_2^+(z,t) = zK_f V_1'(t - z/v_p) = zK_f V_1'\left(t - \frac{z}{l}T\right)$

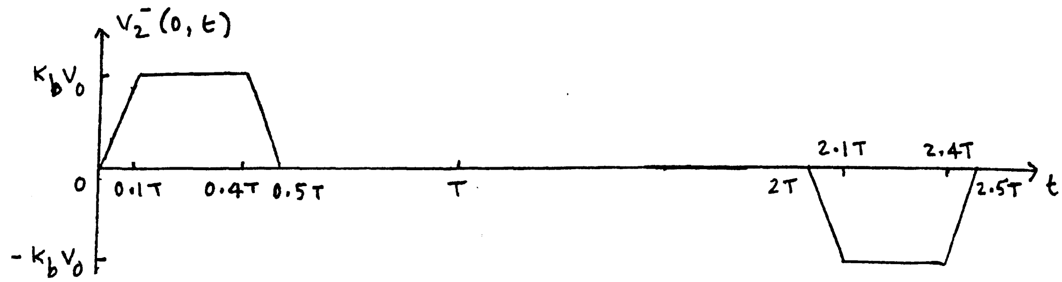
$V_2^+(l,t) = lK_f V_1'(t - T)$



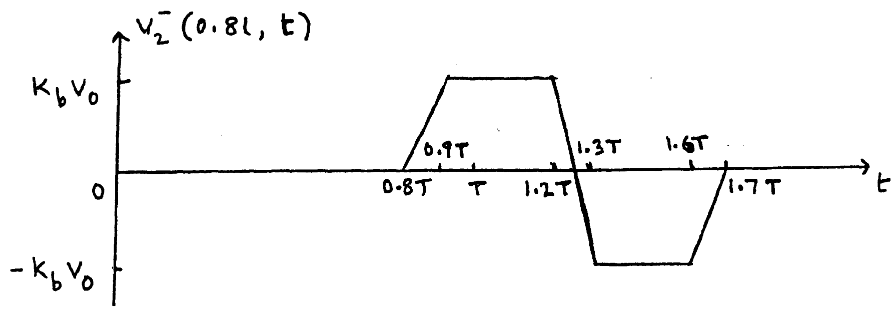
(b) $V_2^-(z,t) = K_b\left[V_1(t - z/v_p) - V_1(t - 2l/v_p + z/v_p)\right]$

$$= K_b\left[V_1\left(t - \frac{z}{l}T\right) - V_1\left(t - 2T + \frac{z}{l}T\right)\right]$$

$V_2^-(0,t) = K_b\left[V_1(t) - V_1(t - 2T)\right]$



(c) $V_2^-(0.8l, t) = K_b\left[V_1(t - 0.8T) - V_1(t - 1.2T)\right]$



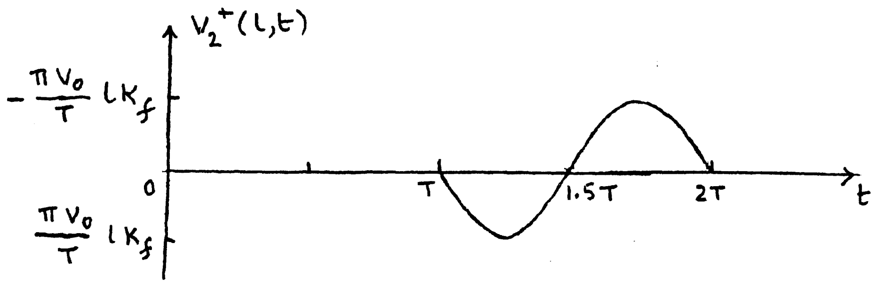
P6.31. **(a)** $V_2^+(l, t) = \frac{1}{2} z K_f V_g'(t - z/v_p)$

$$= \frac{1}{2} z K_f V_g'\left(t - \frac{z}{l}T\right)$$

$$V_g'(t) = \begin{cases} \frac{4\pi}{T} V_0 \sin\frac{\pi t}{T} \cos\frac{\pi t}{T} & \text{for } 0 < t < T \\ 0 & \text{otherwise} \end{cases}$$

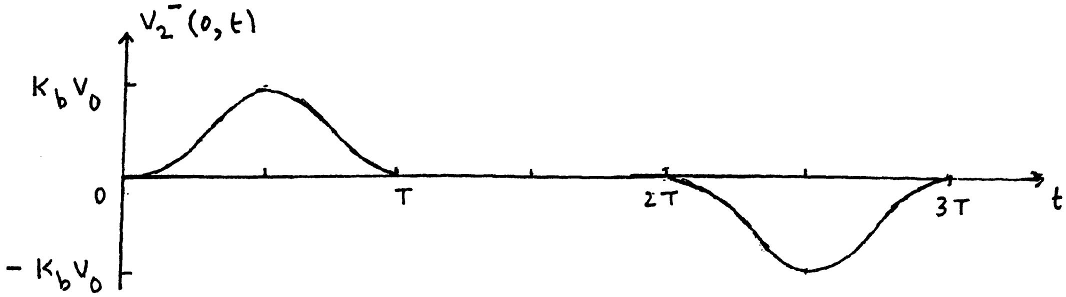
$$= \begin{cases} \frac{2\pi}{T} V_0 \sin\frac{2\pi t}{T} & \text{for } 0 < t < T \\ 0 & \text{otherwise} \end{cases}$$

$$V_2^+(l, t) = \begin{cases} \frac{\pi}{T} V_0 l K_f \sin\frac{2\pi}{T}(t - T) & \text{for } T < t < 2T \\ 0 & \text{otherwise} \end{cases}$$



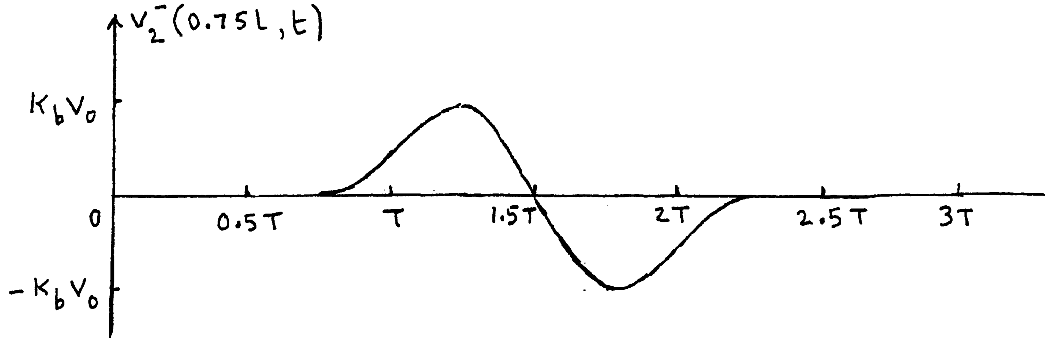
(b) $V_2^-(z, t) = K_b\left[V_1(t - z/v_p) - V_1(t - 2l/v_p + z/v_p)\right]$

$$= K_b\left[V_1\left(t - \frac{z}{l}T\right) - V_1\left(t - 2T + \frac{z}{l}T\right)\right]$$



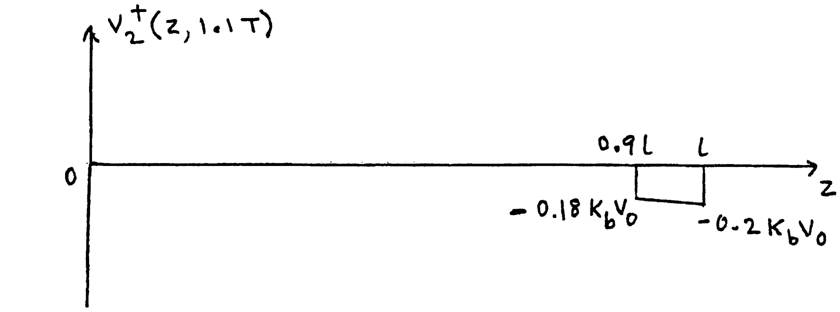
P6.31. (continued)

(c)

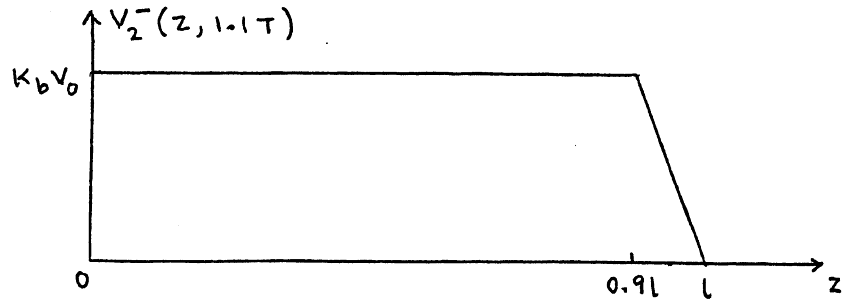


P6.32. (a) $$\frac{lK_f V_0}{T_0} = \frac{l}{v_p}\frac{v_p K_f V_0}{(0.2T)} = -\frac{K_b V_0}{25\times 0.2} = -\frac{1}{5}K_b V_0$$

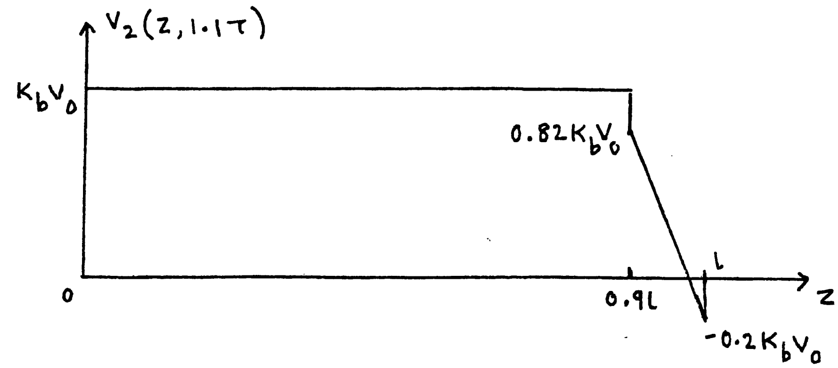
From Fig. 6.59,



(b) From Fig. 6.61,



(c)



R6.1. Equations (6.12a) and (6.12b) can be written as

$$\lim_{\Delta z \to 0} \frac{V\left(z+\frac{\Delta z}{2}, t\right) - V\left(z-\frac{\Delta z}{2}, t\right)}{\Delta z} = -\mathcal{L}\frac{\partial I(z,t)}{\partial t} \qquad (1)$$

$$\lim_{\Delta z \to 0} \frac{I\left(z+\frac{\Delta z}{2}, t\right) - I\left(z-\frac{\Delta z}{2}, t\right)}{\Delta z} = -\mathcal{C}\frac{\partial V(z,t)}{\partial t} \qquad (2)$$

Equation (1) can be thought of as the sum of the two equations,

$$\lim_{\Delta z \to 0} \frac{V\left(z+\frac{\Delta z}{2}, t\right) - V(z,t)}{\Delta z} = \lim_{\Delta z \to 0} -\frac{1}{2}\mathcal{L}\frac{\partial I\left(z+\frac{\Delta z}{2}, t\right)}{\partial t}$$

$$\lim_{\Delta z \to 0} \frac{V(z,t) - V\left(z-\frac{\Delta z}{2}, t\right)}{\Delta z} = \lim_{\Delta z \to 0} -\frac{1}{2}\mathcal{L}\frac{\partial I\left(z-\frac{\Delta z}{2}, t\right)}{\partial t}$$

which together with (2) give the equivalent circuit of Fig. 6.84(a).

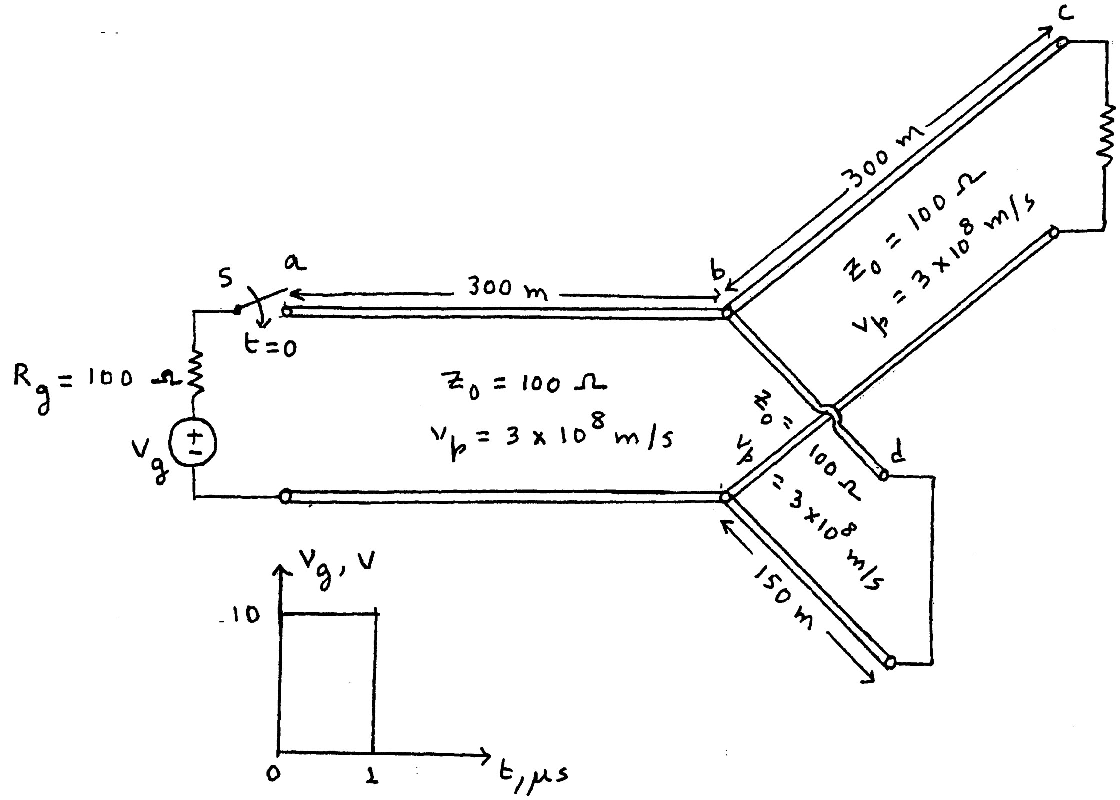
Alternatively, Eq. (2) can be thought of as the sum of the two equations,

$$\lim_{\Delta z \to 0} \frac{I\left(z+\frac{\Delta z}{2}, t\right) - I(z,t)}{\Delta z} = \lim_{\Delta z \to 0} -\frac{1}{2}\mathcal{C}\frac{\partial V\left(z+\frac{\Delta z}{2}, t\right)}{\partial t}$$

$$\lim_{\Delta z \to 0} \frac{I(z,t) - I\left(z-\frac{\Delta z}{2}, t\right)}{\Delta z} = \lim_{\Delta z \to 0} -\frac{1}{2}\mathcal{C}\frac{\partial V\left(z-\frac{\Delta z}{2}, t\right)}{\partial t}$$

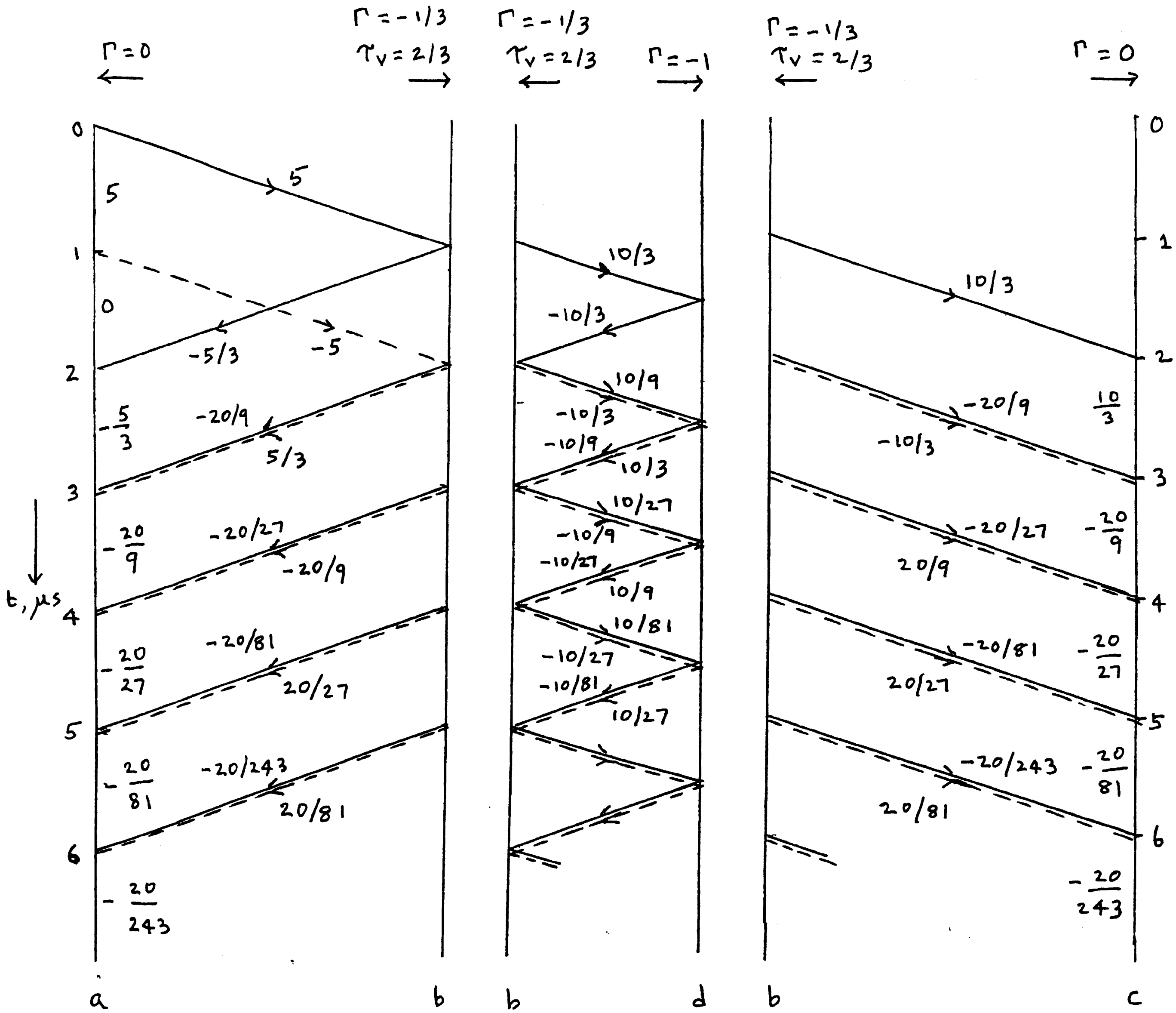
which together with (1) give the equivalent circuit of Fig. 6.84(b).

R6.2.



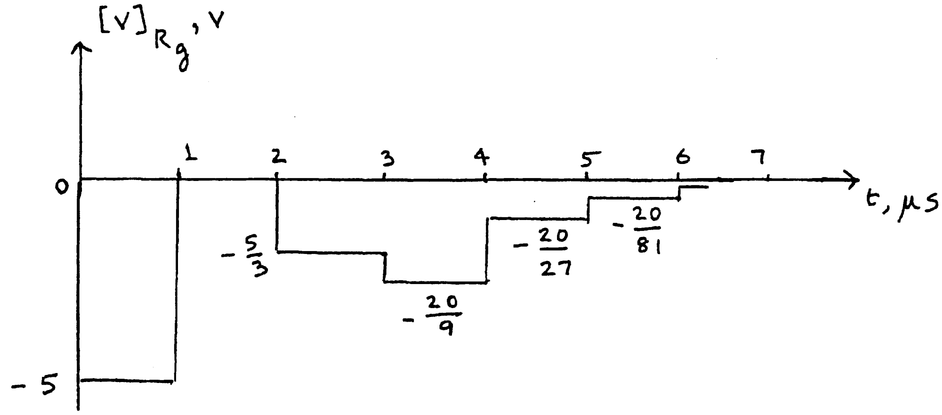
At the junction of the three lines, the effective load for each line looking toward the junction is 100 Ω in parallel with 100 Ω = 50 Ω. Hence, $\Gamma = -\frac{1}{3}$ and $\tau_V = \frac{2}{3}$. The voltage bounce diagrams for the three lines can be drawn as follows.

R6.2. (continued)

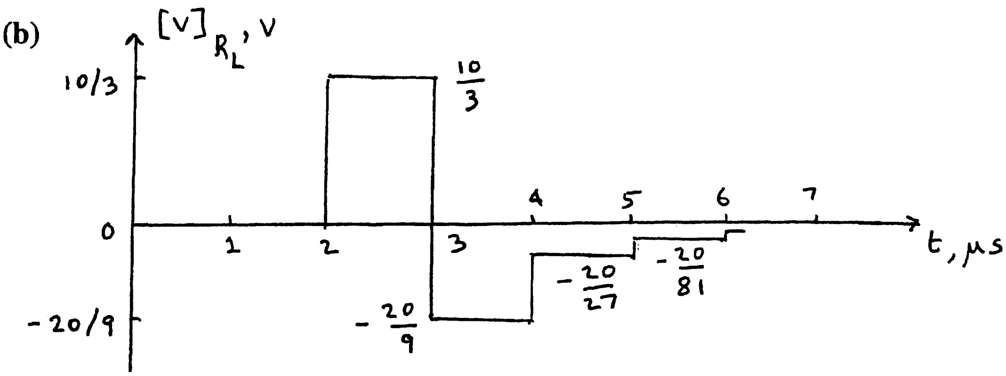


R6.2. (continued)

(a)



(b)



(c) Energy supplied by the voltage source

$$= 10 \times \frac{5}{100} \times 10^{-6} = \frac{1}{2}\ \mu\text{J}$$

Energy dissipated in R_L

$$= \frac{10^{-6}}{100}\left[\left(\frac{10}{3}\right)^2 + \left(-\frac{20}{9}\right)^2 + \left(-\frac{20}{27}\right)^2 + \left(-\frac{20}{81}\right)^2 + \ldots\right]$$

$$= \frac{10^{-6}}{100}\left[\frac{100}{9} + \frac{400}{81}\left(1 + \frac{1}{9} + \frac{1}{81} + \ldots\right)\right]$$

$$= \frac{10^{-6}}{100}\left[\frac{100}{9} + \frac{400}{81} \times \frac{1}{1-1/9}\right]$$

$$= 10^{-6}\left(\frac{1}{9} + \frac{4}{81} \times \frac{9}{8}\right)$$

$$= \frac{1}{6}\ \mu\text{J}$$

R6.2. (continued)

Energy dissipated in R_g

$$= \frac{10^{-6}}{100}\left[5^2 + \left(-\frac{5}{3}\right)^2 + \left(-\frac{20}{9}\right)^2 + \left(-\frac{20}{27}\right)^2 + \left(-\frac{20}{81}\right)^2 + \ldots\right]$$

$$= \frac{10^{-6}}{100}\left[25 + \frac{25}{9} + \frac{400}{81}\left(1 + \frac{1}{9} + \frac{1}{81} + \ldots\right)\right]$$

$$= \frac{10^{-6}}{100}\left[25 + \frac{25}{9} + \frac{400}{81} \times \frac{9}{8}\right]$$

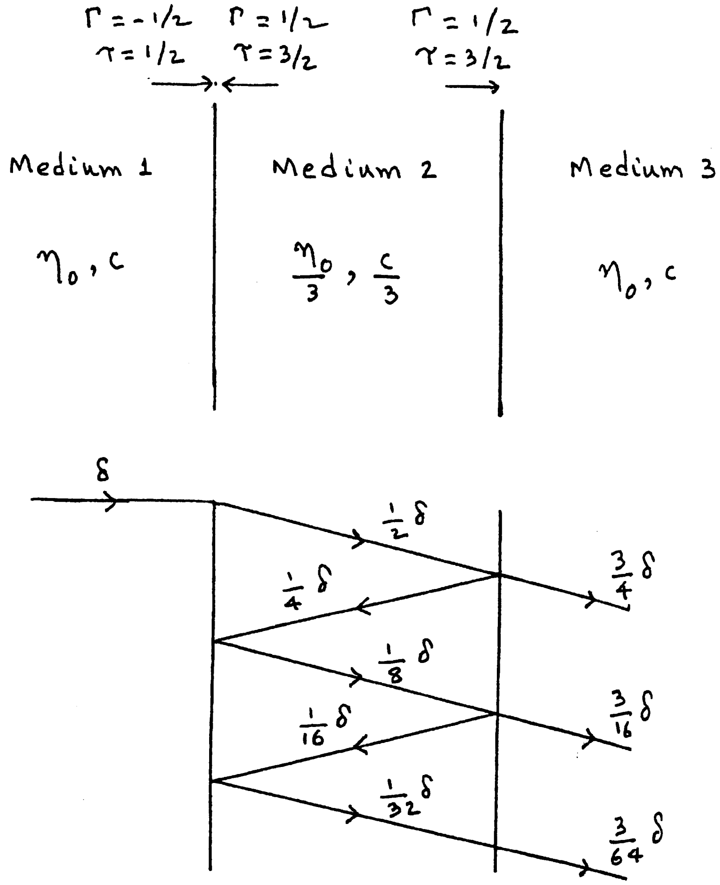
$$= \frac{10^{-6}}{100}\left(\frac{250}{9} + \frac{50}{9}\right)$$

$$= \frac{1}{3}\ \mu\text{J}$$

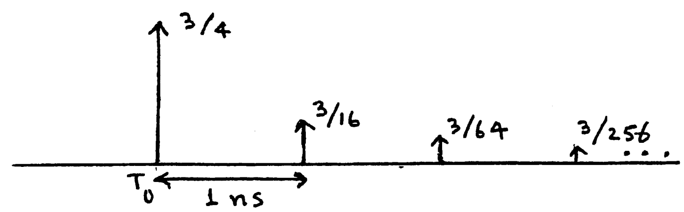
Energy supplied by the voltage source

= Energy dissipated in R_g + Energy dissipated in R_L

R6.3.



(a)



$$\text{Travel time in medium 2} = \frac{5\times 10^{-2}}{10^{8}} = 5\times 10^{-10}\ \text{s}$$

$$= 0.5\ \text{ns}$$

$$E_{xo}(t) = \frac{3}{4}\sum_{n=1,2,3,\ldots}^{\infty}\left(\frac{1}{4}\right)^{n}\delta(t-10^{-9}n-T_0)$$

R6.3. (continued)

(b) Minimum value of f is one for which the period is the time interval between successive impulses, because then for a periodic sequence of unit impulses for $E_{xi}(t)$, each impulse strength in the periodic sequence for $E_{xo}(t)$ will be

$$\frac{3}{4}+\frac{3}{16}+\frac{3}{64}+\frac{3}{256}+\cdots$$

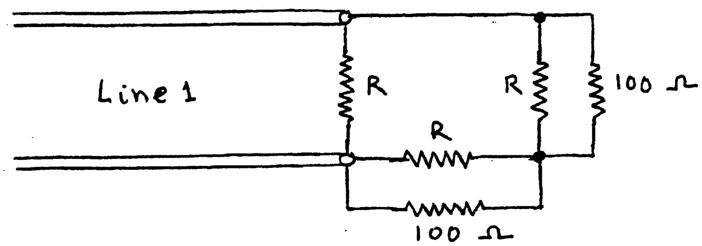
$$= \frac{3}{4}\left(1+\frac{1}{4}+\frac{1}{4^2}+\frac{1}{4^3}+\cdots\right)$$

$$= \frac{3}{4}\times\frac{1}{1-\frac{1}{4}}=\frac{3}{4}\times\frac{4}{3}=1$$

Thus $E_{xo}(t)$ will consist of a periodic sequence of unit impulses of the same frequency as that of $E_{xi}(t)$.

$$\therefore \; f=\frac{1}{10^{-9}}=10^9 \text{ Hz} = 1 \text{ GHz}$$

R6.4. (a)



Effective load for Line 1

$$= R \text{ in parallel with } \left(\frac{100R}{100+R} + \frac{100R}{100+R}\right)$$

$$= \frac{1}{\frac{1}{R} + \frac{100+R}{200R}} = \frac{200R}{300+R}$$

For no reflected wave into line 1

$$\frac{200R}{300+R} = 100$$

$$R = 300\ \Omega$$

(b) $\Gamma = 0,\ \tau_v = \tau_c = 1$

$\tau_{v_{\text{eff}}}$ for each 100 Ω resistor

$$= \frac{1}{2}\tau_v = \frac{1}{2}$$

$\tau_{c_{\text{eff}}}$ for each 100 Ω resistor

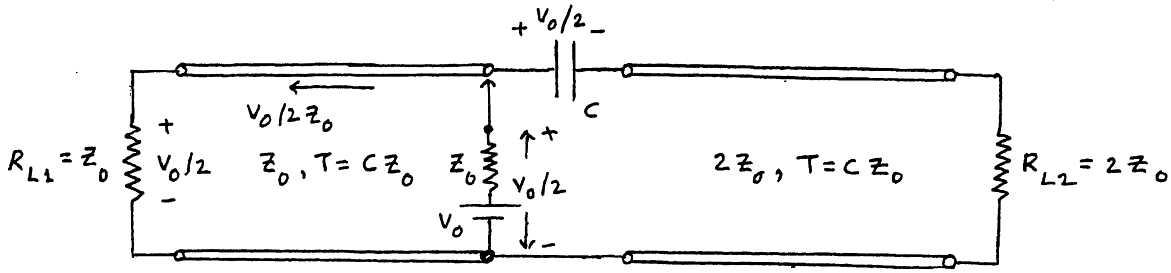
$$= \tau_c \times \frac{300}{300+150} \times \frac{300}{300+100}$$

$$= 1 \times \frac{300}{450} \times \frac{300}{400} = \frac{1}{2}$$

∴ Power transmitted into each of line 2 and 3

$$= \frac{1}{2} \times \frac{1}{2} \times P = \frac{1}{4}P$$

R6.5. (a)



At $t = 0-$,

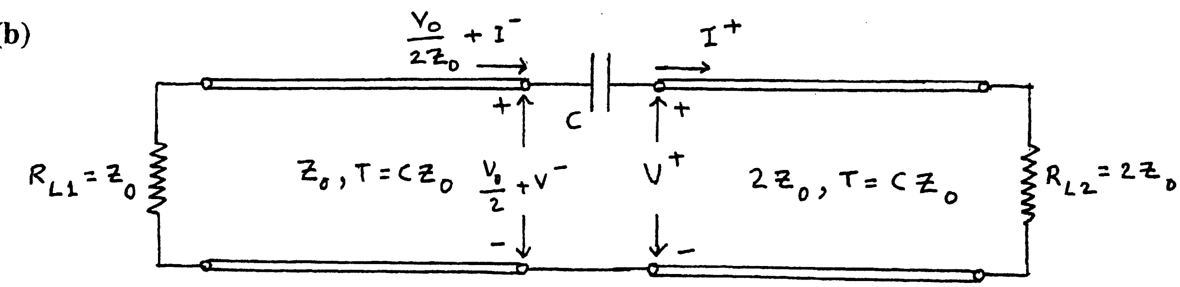
Energy stored in the system

$$= \frac{1}{2}C\left(\frac{V_0}{2}\right)^2 + \frac{1}{2Z_0}\left(\frac{V_0}{2}\right)^2 CZ_0 + \frac{1}{2}\left(\frac{V_0}{2Z_0}\right)^2 Z_0(CZ_0)$$

$$= \frac{1}{8}CV_0^2 + \frac{1}{8}CV_0^2 + \frac{1}{8}CV_0^2$$

$$= \frac{3}{8}CV_0^2$$

(b)



$$I^+ = \frac{V_0}{2Z_0} + I^-$$

$$\frac{V^+}{2Z_0} = \frac{V_0}{2Z_0} - \frac{V^-}{Z_0}$$

$$V^- = -\frac{V^+}{2} - \frac{V_0}{2}$$

$$C\frac{d}{dt}\left(\frac{V_0}{2} + V^- - V^+\right) = I^+$$

$$C\frac{d}{dt}\left(\frac{V_0}{2} - \frac{V^+}{2} - \frac{V_0}{2} - V^+\right) = \frac{V^+}{2Z_0}$$

R6.5. (continued)

$$3CZ_0 \frac{dV^+}{dt} + V^+ = 0$$

$$V^+ = Ae^{-\frac{1}{3CZ_0}t}$$

I.C. $$\left[\frac{V_0}{2} + V^- - V^+\right]_{t=0} = \frac{V_0}{2}$$

$$\left[-\frac{V^+}{2} - V^+\right]_{t=0} = \frac{V_0}{2}$$

$$\left[V^+\right]_{t=0} = -\frac{1}{3}V_0$$

$$\therefore \quad V^+ = -\frac{1}{3}V_0 e^{-\frac{1}{3CZ_0}t}$$

$$V^- = -\frac{V_0}{2} + \frac{1}{6}V_0 e^{-\frac{1}{3CZ_0}t}$$

$$\text{Voltage across } R_{L1} = \begin{cases} \dfrac{V_0}{2} & \text{for } t < T \\ \dfrac{1}{6}V_0 e^{-\frac{1}{3CZ_0}(t-T)} & \text{for } t > T \end{cases}$$

$$\text{Voltage across } R_{L2} = \begin{cases} 0 & \text{for } t < T \\ -\dfrac{1}{3}V_0 e^{-\frac{1}{3CZ_0}(t-T)} & \text{for } t > T \end{cases}$$

(c) Energy dissipated in R_{L1} for $t > 0$

$$= \left(\frac{V_0}{2}\right)^2 \frac{1}{Z_0}(CZ_0) + \frac{1}{Z_0}\int_T^\infty \frac{V_0^2}{36} e^{-\frac{2}{3CZ_0}(t-T)}\, dt$$

$$= \frac{1}{4}CV_0^2 + \frac{V_0^2}{36Z_0}\left[\frac{e^{-\frac{2}{3CZ_0}(t-T)}}{-\frac{2}{3CZ_0}}\right]_{t=T}^{\infty}$$

R6.5. (continued)

$$= \frac{1}{4}CV_0^2 + \frac{V_0^2}{36Z_0} \times \frac{3CZ_0}{2}$$

$$= \frac{7}{24}CV_0^2$$

Energy dissipated in R_{L2} for $t > 0$

$$= \frac{1}{2Z_0}\int_T^\infty \frac{V_0^2}{9} e^{-\frac{2}{3CZ_0}(t-T)}\, dt$$

$$= \frac{V_0^2}{18Z_0}\left[\frac{e^{-\frac{2}{3CZ_0}(t-T)}}{-\frac{2}{3CZ_0}}\right]_{t=T}^{\infty}$$

$$= \frac{V_0^2}{18Z_0} \times \frac{3CZ_0}{2}$$

$$= \frac{1}{12}CV_0^2$$

Sum of the two energies dissipated

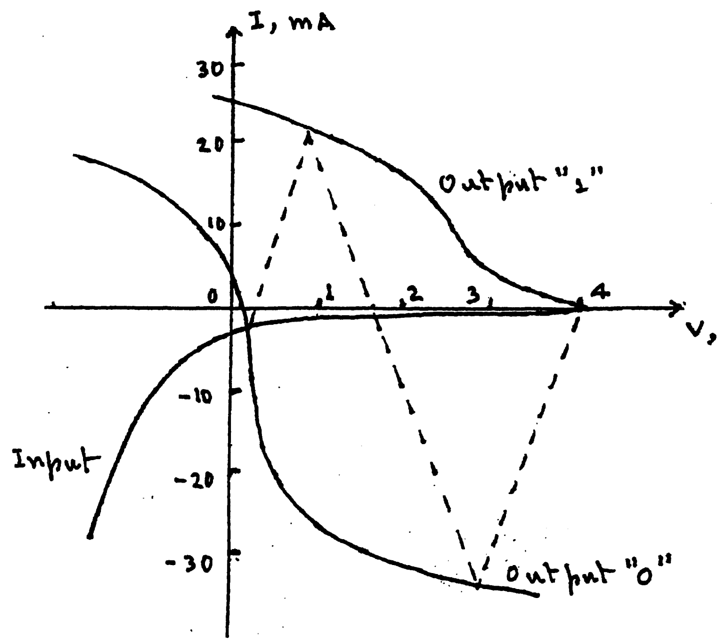
$$= \left(\frac{7}{24} + \frac{1}{12}\right)CV_0^2 = \frac{3}{8}CV_0^2$$

= Energy stored in the system at $t = 0-$

R6.6. From the load line construction, the required value of Z_0 is

$$\frac{0.725}{22\times10^{-3}} \text{ or } \frac{1.15}{35\times10^{-3}} \approx 33\ \Omega$$

Value of the voltage ≈ 1.65 V



R6.7. For $V_g(t) = V_0 \cos 2\pi ft$,

$$V_2(0, t) = V_2^-(0, t)$$

$$= \frac{1}{2} K_b [V_0 \cos 2\pi ft - V_0 \cos 2\pi f(t - 2T)]$$

For $f = 1/4T$, $4\pi fT = \pi$,

$$V_2(0, t) = \frac{1}{2} K_b [V_0 \cos 2\pi ft - V_0 \cos (2\pi ft - \pi)]$$

$$= K_b V_0 \cos 2\pi ft$$

Amplitude of $V_2(0, t) = K_b$

Q.E.D.

$$V_2(l, t) = V_2^+(l, t)$$

$$= \frac{1}{2} l K_f V_g'(t - T)$$

$$= \frac{1}{2} l K_f [-2\pi f V_0 \sin 2\pi f(t - T)]$$

$$= l\, |K_f|\, \pi f V_0 \cos 2\pi ft$$

Amplitude of $V_2(l, t) = l\pi f\, |K_f|\, V_0$

Q.E.D.