THE INTERNATIONAL UNIVERSITY – VIETNAM NATIONAL UNIVERSITY – HCMC

CALCULUS 2 MIDTERM

Semester 2, Academic Year 2022-2023

Duration: 90 minutes

Department of Mathematics	Lecturers
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Instructions:

- **Write the test code MAR086 on your answer sheet.**
- You can use two A4 sheets of notes and a calculator. All other documents and electronic devices are forbidden.
- Each question carries 4 points. For the True/False questions, fill in the circles completely.
- Only the answer sheet will be graded.

Part A: True/False Questions

1. The series $\sum_{n=1}^{\infty}(-1)^n(\sqrt{n}-\sqrt{n-1})$ converges.
2. The series $\sum_{n=1}^{\infty}(-1)^n\dfrac{(2n)!}{(n!)^2}$ diverges.
3. The explicit formula of the sequence $\{\frac{2}{25},\frac{4}{36},\frac{6}{49},\frac{8}{64},\frac{10}{81},\dots\}$ is $a_n=\dfrac{2n}{(n+4)^2}$.
4. The sequence $a_n=\dfrac{(2n)!n^2}{(2n+2)!}$ has limit $\frac{1}{4}$.
5. If nonzero vectors $\mathbf{u}$ and $\mathbf{v}$ have the same magnitude, then they make equal angle with vector $\mathbf{u}+\mathbf{v}$.
6. If for some three dimensional vectors $\mathbf{u}$, $\mathbf{v}$, $\mathbf{w}$ we have $\mathbf{u}\times\mathbf{v}=\mathbf{u}\times\mathbf{w}$, then $\mathbf{v}=\mathbf{w}$.
7. The series $\sum_{n=1}^{\infty}\dfrac{3}{n^2+3}$ is divergent.
8. The series $\sum_{n=2}^{\infty}(-1)^n\dfrac{n^2-1}{n^2+3}$ is convergent.
9. If $\mathbf{u}$ is orthogonal to $\mathbf{v}$, and $\mathbf{v}$ is orthogonal to $\mathbf{w}$, then $\mathbf{u}$ is orthogonal to $\mathbf{w}$.
10. The series $\sum_{n=1}^{\infty}\dfrac{n^2}{4^n}$ diverges.
11. The vector $\mathbf{u}=\langle -1,-5,7\rangle$ is perpendicular to both the line $x=1+5t, y=3-t, z=\dots$ and the plane $2x+y+z=12$.

$u+v \perp 2u-3v$

$u-v \perp 2u+3v$

12. If $\mathbf{u}+\mathbf{v}$ is orthogonal to $2\mathbf{u}-3\mathbf{v}$ and $\mathbf{u}-\mathbf{v}$ is orthogonal to $2\mathbf{u}+3\mathbf{v}$, then $\mathbf{u}$ is orthogonal to $\mathbf{v}$.

13. The series $\sum_{n=1}^{\infty} ne^{-2n^2}$ converges.

14. If $\mathbf{u}$ is a three dimensional vector, then $(\mathbf{u}\cdot\mathbf{i})^2+(\mathbf{u}\cdot\mathbf{j})^2+(\mathbf{u}\cdot\mathbf{k})^2 = |\mathbf{u}|^2$.

15. If $\mathbf{u}$, $\mathbf{v}$ are three dimensional vectors, then $(\mathbf{u}-\mathbf{v})\times(\mathbf{u}+\mathbf{v}) = 2\mathbf{u}\times\mathbf{v}$.

Part B: Short Answer Questions

16. Find the limit $\lim_{(x,y)\to(0,0)} \dfrac{x^2+4xy}{3x^2+y^2}$ if it exists.

17. Determine the radius and interval of convergence of the power series $\sum_{n=2}^{\infty} \dfrac{2^n(x-3)^n}{n}$.

18. Let $f(x) = \dfrac{x^2}{x+2}$. Find the formula for the coefficients c_n in the power series expansion $f(x) = \sum_{n=0}^{\infty} c_n x^n$.

19. Evaluate the series $\sum_{n=1}^{\infty} \dfrac{2^{2n-1}}{5^n}$.

20. Find all values of a such that $\langle a, a, 2\rangle \times \langle 1, a, 3\rangle = \langle 2, -4, 2\rangle$.

21. Find the area of the triangle T with vertices $P(2,-1,4), Q(1,1,-1)$ and $R(-4,1,1)$.

22. Find the line through point $(-3,4,2)$ that is perpendicular to both $\mathbf{u} = \langle 3,1,0\rangle$ and $\mathbf{v} = \langle 2,4,1\rangle$.

23. Find the plane passing through the points $P(2,-1,4), Q(1,1,-1)$ and $R(-4,1,1)$.

24. Find the volume of the parallepiped determined by $\mathbf{u} = \langle 3,1,0\rangle, \mathbf{v} = \langle 2,4,1\rangle, \mathbf{w} = \langle 1,1,5\rangle$.

25. Find the length of the two-dimensional curve $\mathbf{r}(t) = \langle \cos t + t\sin t, \sin t - t\cos t\rangle$, for $t \in [0, \pi/2]$.

$uv = u'v + uv'$.