REVIEW FINAL EXAM PHYSICS 4

1/ Consider a particle moving in one dimension, which we shall call the *x*-axis.

(a) What does it mean for the wave function of this particle to be *normalized* ?

(b) If the particle described by the wave function $\psi(x) = Ae^{bx}$, where A and b are positive real numbers, is confined to the range $x \geq 0$, determine A (including its units) in function of b so that the wave function is normalized.

IDENTIFY: To describe a real situation, a wave function must be normalizable.

SET UP: $|\psi|^2 dV$ is the probability that the particle is found in volume dV. Since the particle must be *somewhere*, ψ must have the property that $\int |\psi|^2 dV = 1$ when the integral is taken over all space.

EXECUTE: (a) In one dimension, as we have here, the integral discussed above is of the form $\int_{-\infty}^{\infty} |\psi(x)|^2 dx = 1$.

(b) Using the result from part (a), we have $\int_{-\infty}^{\infty} \left(e^{ax}\right)^2 dx = \int_{-\infty}^{\infty} e^{2ax} dx = \left.\frac{e^{2ax}}{2a}\right|_{-\infty}^{\infty} = \infty$. Hence this wave function cannot be normalized and therefore cannot be a valid wave function.

(c) We only need to integrate this wave function of 0 to ∞ because it is zero for $x < 0$. For normalization we have

$$1 = \int_{-\infty}^{\infty} |\psi|^2 dx = \int_0^{\infty} \left(Ae^{-bx}\right)^2 dx = \int_0^{\infty} A^2 e^{-2bx} dx = \left.\frac{A^2 e^{-2bx}}{-2b}\right|_0^{\infty} = \frac{A^2}{2b}, \text{ which gives } \frac{A^2}{2b} = 1, \text{ so } A = \sqrt{2b}.$$

EVALUATE: If b were positive, the given wave function could not be normalized, so it would not be allowable.

2/ (a) The orbital angular momentum of an electron has a magnitude of $L = 4.716 \times 10^{-34}\, kg.m^2/s$. What is the angular-momentum quantum number l for this electron ?

IDENTIFY and SET UP: The magnitude of the orbital angular momentum L is related to the quantum number l by Eq.(41.4): $L = \sqrt{l(l+1)}\hbar$, $1 = 0, 1, 2, \ldots$

EXECUTE: $l(l+1) = \left(\frac{L}{\hbar}\right)^2 = \left(\frac{4.716 \times 10^{-34} \text{ kg}\cdot\text{m}^2/\text{s}}{1.055 \times 10^{-34} \text{ J}\cdot\text{s}}\right) = 20$

And then $l(l+1) = 20$ gives that $l = 4$.

(b) Make a chart showing all the possible sets of quantum numbers l and m_l for the states of the electron in the hydrogen atom when n = 5. How many combinations are there ? What are the energies of these states?

The (l, m_l) combinations are $(0,\ 0)$, $(1,\ 0)$, $(1,\ \pm1)$, $(2,\ 0)$, $(2,\ \pm1)$, $(2,\ \pm2)$, $(3,\ 0)$, $(3,\ \pm1)$, $(3,\ \pm2)$, $(3,\ \pm3)$, $(4,\ 0)$, $(4,\ \pm1)$, $(4,\ \pm2)$, $(4,\ \pm3)$, and $(4,\ \pm4)$, a total of 25.

(b) Each state has the same energy (n is the same), $-\frac{13.60 \text{ eV}}{25} = -0.544$ eV.

3/ (a) The uncertainty in the y-component of a proton's position is 2.0×10^{-12} m. What is the minimum uncertainty in a simultaneous measurement of the y-component of the proton's velocity ?

(b) The uncertainty in the z-component of an e1ectron's velocity is 0.250 m/s. What is the minimum uncertainty in a simultaneous measurement of the z-coordinate of the electron ?

EXECUTE: (a) $m\Delta x \Delta v_x = \frac{h}{2\pi}$. $\Delta v_x = \frac{h}{2\pi m \Delta x} = \frac{6.63 \times 10^{-34} \text{ J}\cdot\text{s}}{2\pi(1.67 \times 10^{-27} \text{ kg})(2.0 \times 10^{-12} \text{ m})} = 3.2 \times 10^4$ m/s

(b) $\Delta x = \frac{h}{2\pi m \Delta v_x} = \frac{6.63 \times 10^{-34} \text{ J}\cdot\text{s}}{2\pi(9.11 \times 10^{-31} \text{ kg})(0.250 \text{ m/s})} = 4.6 \times 10^{-4}$ m

4/ Consider a wave function given by $\psi(x) = A\sin kx$, where $k = \frac{2\pi}{\lambda}$ and A is a real constant.

(a) For what values of x is there the highest probability of finding the particle described by this wave function ? Explain.

(b) For which values of x is the probability zero ? Explain.

EXECUTE: (a) The probability is highest where $\sin kx = 1$ so $kx = 2\pi x/\lambda = n\pi/2$, $n = 1, 3, 5, \ldots$
$x = n\lambda/4$, $n = 1, 3, 5, \ldots$ so $x = \lambda/4, 3\lambda/4, 5\lambda/4, \ldots$

(b) The probability of finding the particle is zero where $|\psi|^2 = 0$, which occurs where $\sin kx = 0$ and
$kx = 2\pi x/\lambda = n\pi$, $n = 0, 1, 2, \ldots$
$x = n\lambda/2$, $n = 0, 1, 2, \ldots$ so $x = 0, \lambda/2, \lambda, 3\lambda/2, \ldots$

5/ Knowing that the energy of a particle of mass m in a one-dimension box is given by $E = \frac{h^2}{8mL}n^2$, where L is the width of the box and n is an integer.

(a) Find the excitation energy from the ground level to the third excited level for an electron confined to a box that has a width of 0.125 nm.

(b) The electron makes a transition from the n = 1 to n = 4 level by absorbing a photon. Calculate the wavelength of this photon.

(a) The third excited state is $n = 4$, so

$$\Delta E = (4^2 - 1)\frac{h^2}{8mL^2} = \frac{15(6.626\times10^{-34}\ \text{J}\cdot\text{s})^2}{8(9.11\times10^{-31}\ \text{kg})(0.125\times10^{-9}\ \text{m})^2} = 5.78\times10^{-17}\ \text{J} = 361\ \text{eV}.$$

(b) $$\lambda = \frac{hc}{\Delta E} = \frac{(6.63\times10^{-34}\ \text{J}\cdot\text{s})(3.0\times10^{8}\ \text{m/s})}{5.78\times10^{-17}\ \text{J}} = 3.44\ \text{nm}$$

6/ Consider the nuclear reaction ${}^4_2\text{He} + {}^7_3\text{Li} \rightarrow {}^A_Z\text{X} + {}^1_0\text{n}$ where X is a nuclide.

(a) How many protons and neutrons are there in the nuclide X ?

(b) Calculate the energy (in MeV) of this reaction ?

The mass of ${}^4_2\text{He}$, ${}^7_3\text{Li}$, ${}^A_Z\text{X}$, and ${}^1_0\text{n}$ is, respectively, 4.002603u, 7.016003u, 10.811100u, and 1.008665u, with $1\text{u} = 931.5\text{MeV}/c^2$.

(a) $Z = 3 + 2 - 0 = 5$ and $A = 4 + 7 - 1 = 10$.

(b) The nuclide is a boron nucleus, and $m_{\text{He}} + m_{\text{Li}} - m_{\text{n}} - m_{\text{B}} = -3.00\times10^{-3}$ u, and so 2.79 MeV of energy is absorbed.

7/ Measurements on a certain isotope tell you that the decay rate decreases from 8318 decays/min to 3091 decays/min in 4.00 days. What is the half-life of this isotope ?

$$A = A_0 e^{-\lambda t} = A_0 e^{-t(\ln 2)/T_{1/2}}. \quad -\frac{(\ln 2)t}{T_{½}} = \ln(A/A_0).$$

$$T_{½} = -\frac{(\ln 2)t}{\ln(A/A_0)} = -\frac{(\ln 2)(4.00\ \text{days})}{\ln(3091/8318)} = 2.80\ \text{days}$$

8/ Consider two observers, O and O', where O' travels with a constant velocity *v* with respect to O along their common *x*-*x'* axis.

(a) A meterstick makes an angle of 30° with respect to the *x'*-axis of O'. What must be the value of *v* if the meterstick makes an angle of 45° with respect to the *x*-axis of O.

Ans. We have:

$$L'_y = L' \sin\theta' = (1\text{ m})\sin 30^\circ = 0.5\text{ m} \qquad L'_x = L'\cos\theta' = (1\text{ m})\cos 30^\circ = 0.866\text{ m}$$

Since there will be a length contraction only in the x-x' direction,

$$L_y = L'_y = 0.5\text{ m} \qquad L_x = L'_x\sqrt{1-(v^2/c^2)} = (0.866\text{ m})\sqrt{1-(v^2/c^2)}$$

Since $\tan\theta = L_y/L_x$,

$$\tan 45^\circ = 1 = \frac{0.5\text{ m}}{(0.866\text{ m})\sqrt{1-(v^2/c^2)}}$$

Solving, $v = 0.816c$.

(b) What is the length of the meterstick as measured by O'?

Use the Pythagorean theorem or, more simply,

$$L = \frac{L_y}{\sin 45^\circ} = \frac{0.5\text{ m}}{\sin 45^\circ} = 0.707\text{ m}$$

9/ A proton (rest mass $1.67\times10^{-27}\text{ kg}$) has total energy that is 4.00 times its rest energy. What are

(a) the kinetic energy of the proton;

(b) the speed of the proton?

EXECUTE: **(a)** $E = mc^2 + K$, so $E = 4.00mc^2$ means $K = 3.00mc^2 = 4.50\times10^{-10}$ J

(b) $E^2 = (mc^2)^2 + (pc)^2$; $E = 4.00mc^2$, so $15.0(mc^2)^2 = (pc)^2$

$p = \sqrt{15}mc = 1.94\times10^{-18}$ kg · m/s

(c) $E = mc^2/\sqrt{1-v^2/c^2}$

$E = 4.00mc^2$ gives $1 - v^2/c^2 = 1/16$ and $v = \sqrt{15/16}c = 0.968c$

10/ An electron has a velocity v = 0.990c.

(a) Calculate the kinetic energy in MeV of the electron.

(b) Compare this with the classical value for kinetic energy at this velocity. What is your observation?

The mass of an electron is $9.11\times10^{-31}\text{ kg}$.

$$\begin{aligned}
\mathrm{KE_{rel}} &= (\gamma - 1)\,\mathrm{mc^2} \\
&= (7.0888 - 1)\,(9.11\times10^{--31}\ \mathrm{kg})\,(3.00\times10^{8}\ \mathrm{m/s})^2 \\
&= 4.99\times10^{-13}\ \mathrm{J}
\end{aligned}$$

.

$$\begin{aligned}
\mathrm{KE_{rel}} &= (4.99\times10^{-13}\ \mathrm{J})\left(\tfrac{1\ \mathrm{MeV}}{1.60\times10^{--13}\ \mathrm{J}}\right) \\
&= 3.12\ \mathrm{MeV}
\end{aligned}$$

$$
\begin{aligned}
\mathrm{KE_{class}} &= \tfrac{1}{2}\mathrm{mv}^2 \\
&= \tfrac{1}{2}\left(9.00\times10^{--31}\ \mathrm{kg}\right)(0.990)^2\left(3.00\times10^{8}\ \mathrm{m/s}\right)^2 \\
&= 4.02\times10^{--14}\ \mathrm{J}
\end{aligned}
$$

$$
\begin{aligned}
\mathrm{KE_{class}} &= 4.02\times10^{--14}\ \mathrm{J}\left(\tfrac{1\ \mathrm{MeV}}{1.60\times10^{--13}\ \mathrm{J}}\right) \\
&= 0.251\ \mathrm{MeV}
\end{aligned}
$$

As might be expected, since the velocity is 99.0% of the speed of light, the classical kinetic energy is significantly off from the correct relativistic value. Note also that the classical value is much smaller than the relativistic value. In fact, $\mathrm{KE_{rel}}/\mathrm{KE_{class}} = 12.4$ here. This is some indication