Midterm (2023-2024): LAB-Differential equations

1.

a) Write the sets

$$S_1 = \{-4, -2, 0, 3, 5, 7, 12\}$$

and

$$S_2 = \{-1, -2, 0, 5, 9, 11, 12\}$$

Find (i) $S_1 \cup S_2$, (ii) $S_1 \cap S_2$, (iii) remove element number -4 and 3 from S_1

b) Write the list

$$L = [\sin(\pi/6), e^3, 1, \ln(5), \cos(\pi/4), \sqrt{8}]$$

How many members in L. Add element π^3 into L. Remove the third element from L

2. Define the function in Maple

$$f(x) = \frac{1 - \cos(x^2 - 1)}{x^4 - 1}$$

a) Find $f(1/2)$ and $\lim_{x \to 1} f(x)$

b) Find $\lim_{x \to -1^-} f(x)$.

c) Calculate the derivative of $f(x)$

3. Plot the graphs of the functions

a) $r = 5 - \sin(3\theta)$ with $\theta = 0..2\pi$ on the polar coordinate

b) plot the graphs of the function $z = \frac{12\cos((x^2+y^2)/4)}{3+x^2+y^2}$ on three dimensions

4.

a) Let $F(x, y) = x^3y^2 - y^3x^2 + \sin(xy)$. Find $\frac{\partial F}{\partial y}$, $\frac{\partial^2 F}{\partial x^2}$ and $\frac{\partial^3 F}{\partial x^2 \partial y^2}$, and using D differential operator find $\frac{\partial F}{\partial x \partial y}(1/2, -1)$

b) Compute $\int_0^{\pi/2} \frac{\cos(x)\sin(x)}{\sqrt{\cos^2(x)+16}} dx$

5. Solving the equation

a) $x^3 \sin(x) + x^2 - 5x + 1 = 0$, find the root of this equation

b) Solving the system equations

$$\begin{aligned} 3x - y + 4z - 2 &= 0 \\ 17x + 2y + z - 14 &= 0 \\ x + 12y - 77z - 54 &= 0 \end{aligned}$$