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EARIN Miniproject 1 – optimization

Project 1. Highway system

Prepare an algorithm that optimizes the network of highways between cities from a given set – assuming that city coordinates are given. Your solution should minimize the following objective function: $f(x) = w_1 t(x) + w_2 d(x)$ where $t(x)$ - total length of highways, $d(x)$ - average length of path between two cities, and w_1 and w_2 are user provided parameters provided.

Input:

- file with coordinates (x_i, y_i) for all n cities,
- w_1 , w_2 – parameter values.

Output: graph describing the highway system.

Optimization task:

Goal function: minimize function $f(x)$

Constrains: x is sequence of path in the highway system

Search space: contains elements which describe where the roads will be located

Cost function: The length of the covered distance and its average value.

The distance between every two cities can be evaluated as Euclidean distance:

$$d_{i,j} = \sqrt{\left(x_i - x_j\right)^2 + \left(y_i - y_j\right)^2}$$

The average length of path between two cities can be evaluated as:

$$d(x) = \frac{1}{n(n-1)} \sum_{i,j} d_{i,j}$$

$d_{i,j}$ is the shortest distance between node *i* and node *j*.

Method:

First, the Prim's algorithm is implemented to find the minimum spanning tree of the network.

After that, each path between every two cities which was not in the system (the list of unused paths is sorted in ascending order based on their weight) is added to the current highway system. Then, the objective function $f_{new}(x)$ is evaluated and compared to $f(x)$ of the current system. If $f_{new}(x) < f(x)$, the system is updated. This step is repeated until the list of unused paths is empty.

Input file:

The input file is text file in the following format

$x_0\ y_0$

$x_1\ y_1$

…

$x_n\ y_n$

$w_1\ w_2$

Example:

1) The simple set of city coordinates:

x	0	0	0.5	1	1.5	2	2.5	3
y	0	2	4	1	0	3	1	5

The graph describing the highway system with

$w_1 = 1, w_2 = 0$

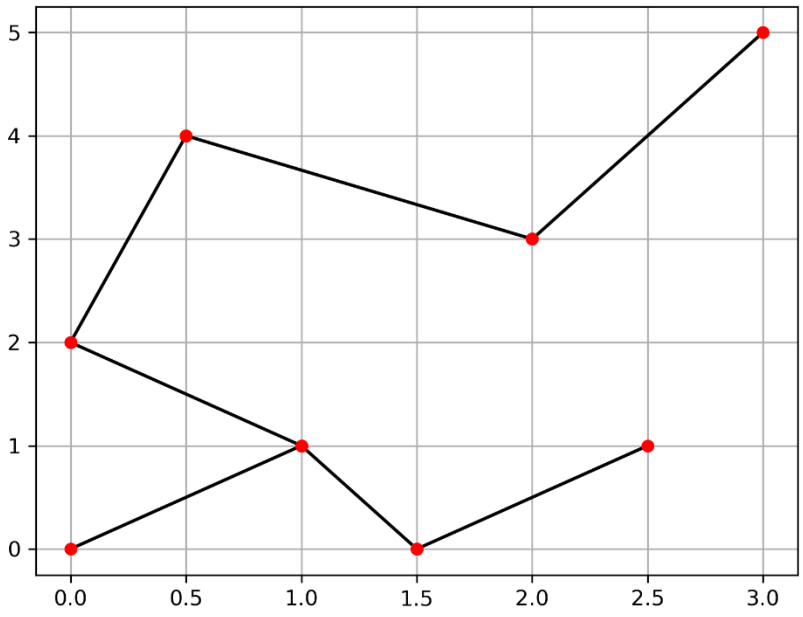


Figure 1.1. $w_1 = 1, w_2 = 0$

$w_1 = 1, w_2 = 1$

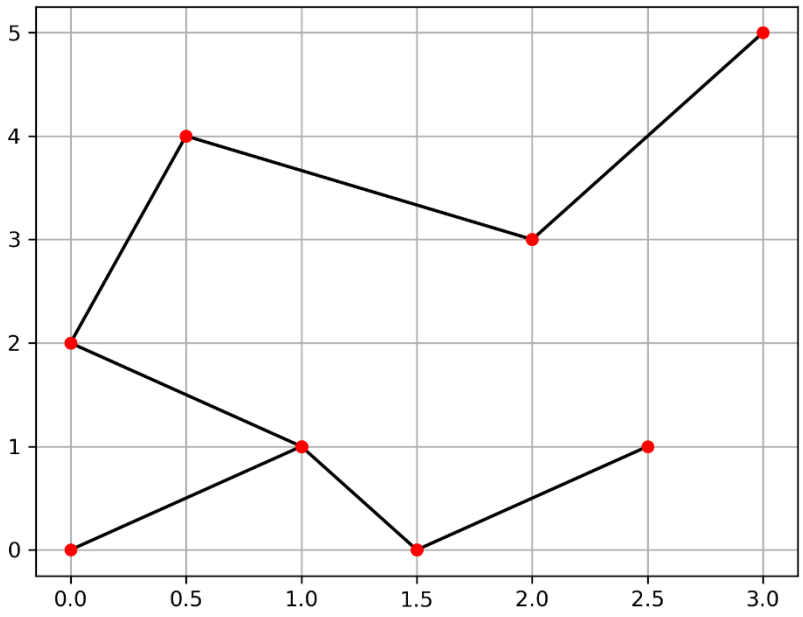


Figure 1.2. $w_1 = 1, w_2 = 1$

$w\_1 = 1, w\_2 = 2$

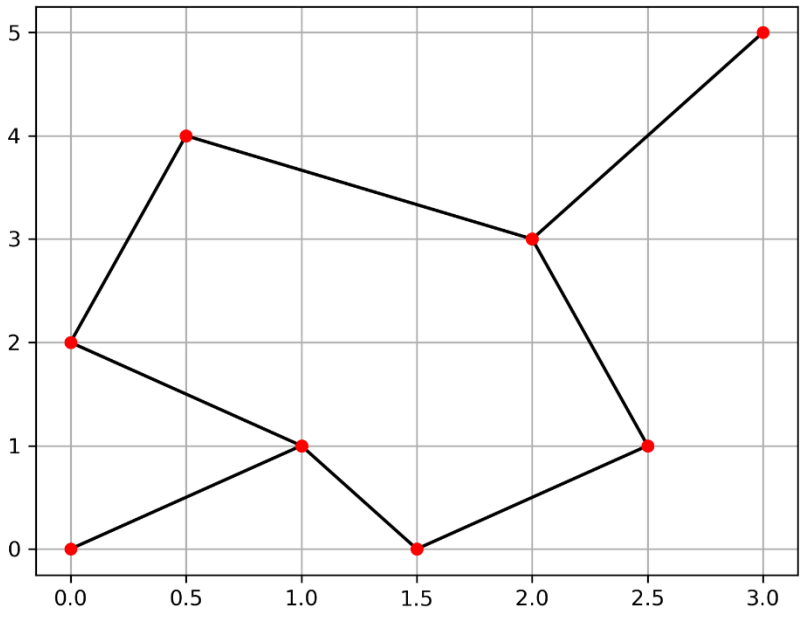


Figure 1.3. $w_1 = 1, w_2 = 2$

$w_1 = 0, w_2 = 1$

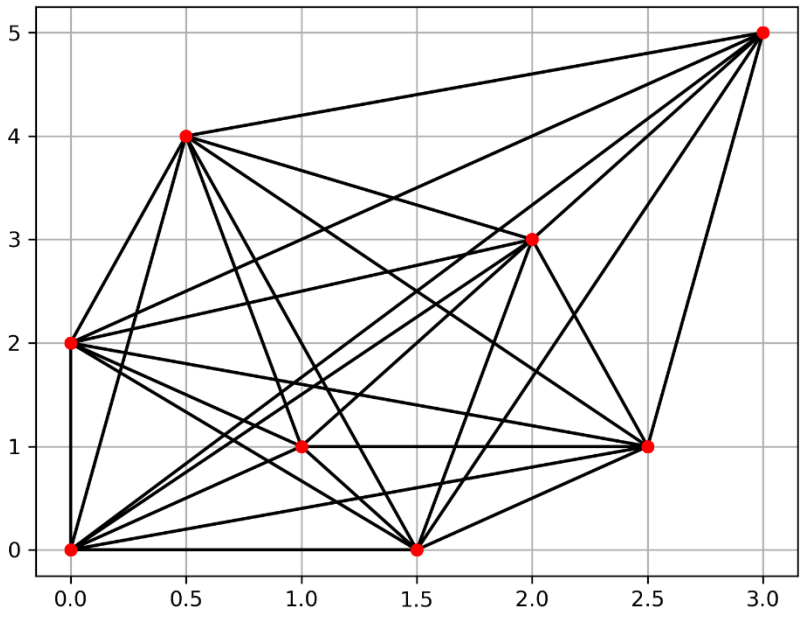


Figure 1.4. $w_1 = 0, w_2 = 1$

2) The example which has bigger problem size
The set of city coordinates can be found in the file “data.txt”

$w_1 = 1, w_2 = 0$

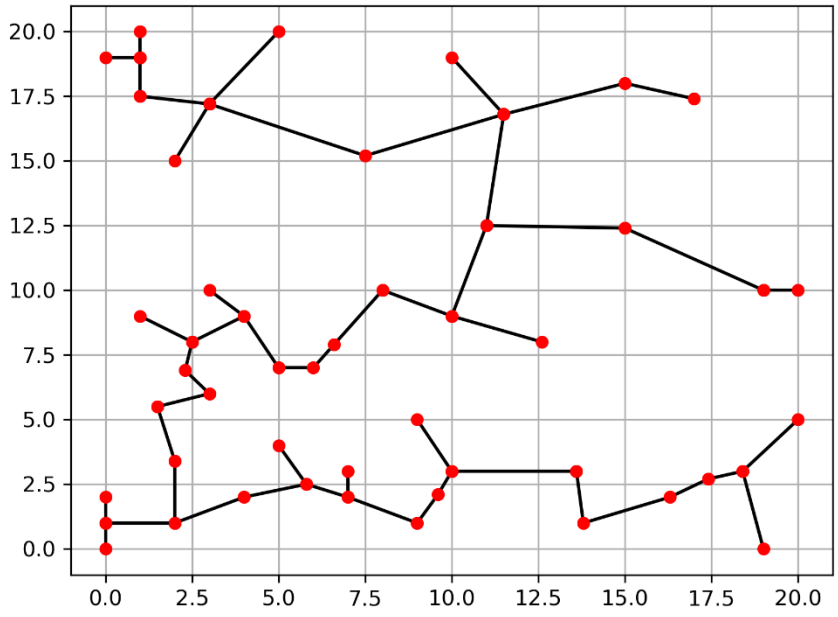


Figure 2.1. $w_1 = 1, w_2 = 0$

$w_1 = 1, w_2 = 1$

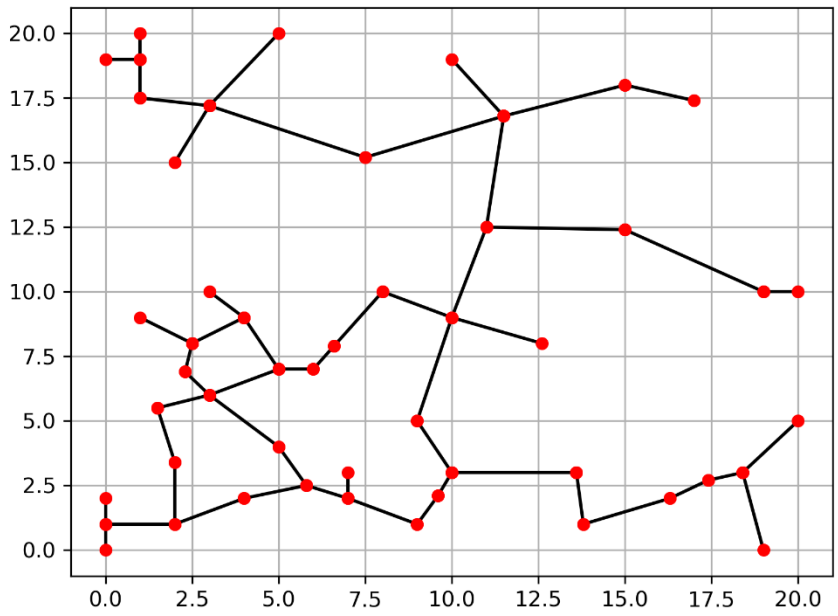


Figure 2.2. $w_1 = 1, w_2 = 1$

$w_1 = 1, w_2 = 2$

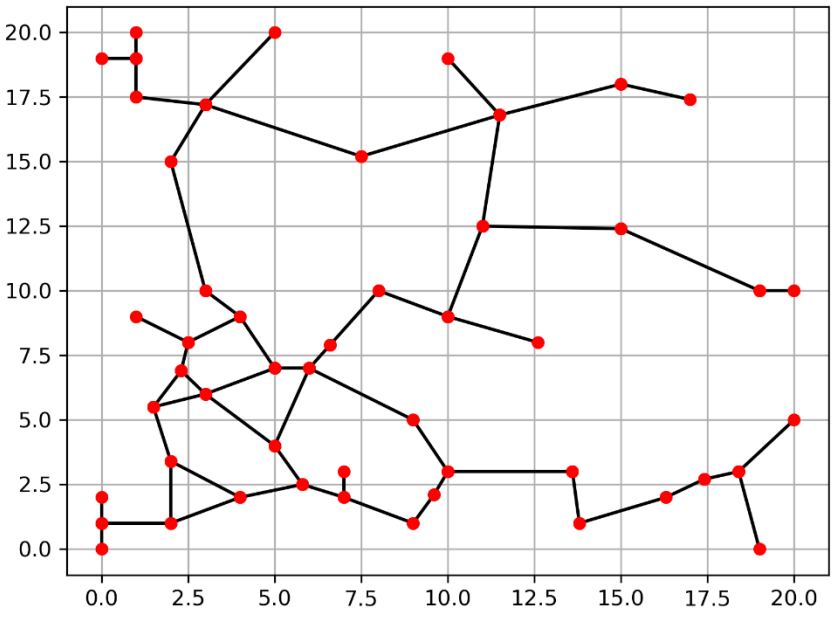
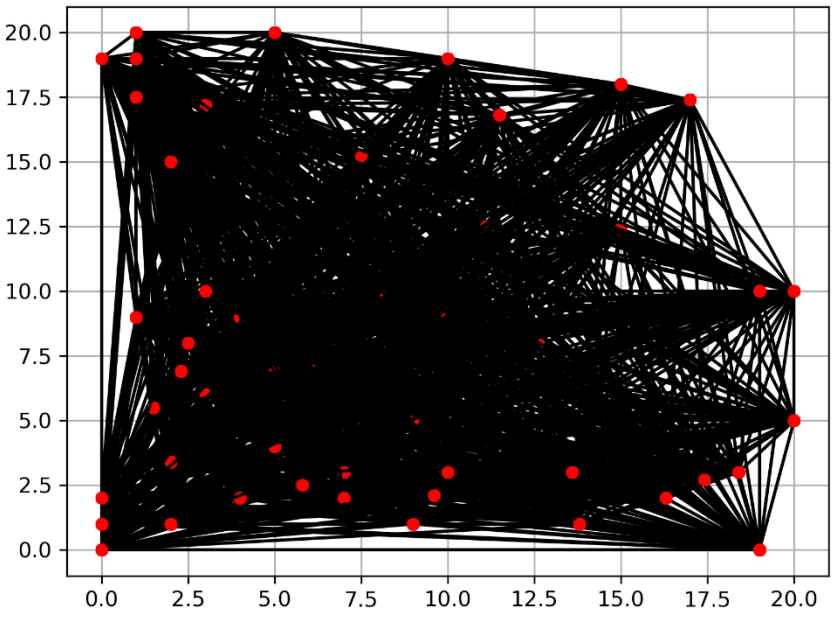


Figure 2.3. $w_1 = 1, w_2 = 2$

$w_1 = 0, w_2 = 1$



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