DO Minh Hieu #6

Numerical Methods (ENUME 2018) – Project
Assignment C: Solving ordinary differential equations

1. Solve the equation:

$$y' = -y + 2te^{-t+2} \text{ for } t \in [0,10] \text{ and } y(0) = 0$$

using the implicit method Lobatto IIID of order 2 defined by the following table of coefficients:

$$\begin{array}{c|cc} 0 & \frac{1}{2} & \frac{1}{2} \\ 1 & -\frac{1}{2} & \frac{1}{2} \\ \hline & \frac{1}{2} & \frac{1}{2} \end{array}$$

Compare the numerical solution, obtained for the constant integration step $h = 0.01$, with the solution obtained by means of the MATLAB function ***ode45***. Choose the values of its parameters *RelTol* and *AbsTol* in such a way as to maximise the accuracy of the solution; denote the most accurate solution with $\dot{\mathbf{y}}(t)$ and use it as the reference hereinafter.

2. Carry out a systematic study of the dependence of the accuracy of numerical solution on the integration step h; use the following accuracy indicators for this purpose:

$$\delta_2(h) = \frac{\left\|\hat{\mathbf{y}}(t;h) - \dot{\mathbf{y}}(t,h)\right\|_2}{\left\|\dot{\mathbf{y}}(t,h)\right\|_2} \quad \text{and} \quad \delta_\infty(h) = \frac{\left\|\hat{\mathbf{y}}(t;h) - \dot{\mathbf{y}}(t,h)\right\|_\infty}{\left\|\dot{\mathbf{y}}(t,h)\right\|_\infty}$$

where $\hat{\mathbf{y}}(t;h)$ is the numerical solution obtained for the integration step h, and $\dot{\mathbf{y}}(t,h)$ is the corresponding reference solution. Make the graphs of $\delta_2(h)$ and $\delta_\infty(h)$.

3. Carry out the analogous study for the explicit Euler method. Add the graphs of $\delta_2(h)$ and $\delta_\infty(h)$, made for this method, to those made at point 2.