DO Minh Hieu #6

Numerical Methods (ENUME 2018) – Project
Assignment A: Solving linear algebraic equations

1. Determine the smallest value α_N of the parameter α, satisfying the equation $\det\left(\mathbf{A}_N(\alpha)\right)=0$, where:

$$\mathbf{A}(\alpha)=\begin{bmatrix} f^2(\alpha) & -2f(\alpha) & 2f(\alpha) & -2f(\alpha) & \cdots & 2f(\alpha) & -2f(\alpha) \\ -2f(\alpha) & 8 & -8 & 8 & \cdots & -8 & 8 \\ 2f(\alpha) & -8 & 12 & -12 & \cdots & 12 & -12 \\ -2f(\alpha) & 8 & -12 & 16 & \cdots & -16 & 16 \\ 2f(\alpha) & -8 & 12 & -16 & \cdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & & 4(N-1) & -4(N-1) \\ -2f(\alpha) & 8 & -12 & 16 & \cdots & -4(N-1) & 4N \end{bmatrix} \text{ with } f(\alpha)=\frac{2}{\alpha}-2$$

   Repeat calculations for $N=3,10,20$.

2. Make the graphs of the dependence of the determinant of $\mathbf{A}_N(\alpha)$ and of the condition number of $\mathbf{A}_N(\alpha)$ on α for $\alpha\in[\alpha_N-0.01,\alpha_N+0.01]$.

3. Design a MATLAB procedure for solving systems of linear algebraic equations using the Cholesky-Banachiewicz method, denoted here with the acronym CB. Check its correctness via solving, by means of this procedure, several systems of linear algebraic equations whose solutions are known.

4. Apply the CB procedure for solving the following systems of linear algebraic equations:

   $$\mathbf{A}_N(\alpha)\cdot\mathbf{x}=\mathbf{b}_N(\alpha) \text{ with } \mathbf{b}_N(\alpha)=\mathbf{A}_N(\alpha)\cdot\dot{\mathbf{x}}_N \text{ and } \dot{\mathbf{x}}_N=[1,2,\ldots,N]^T$$

   for $N=3,10,20$ and selected values of $\alpha\in[\alpha_N-0.01,\alpha_N+0.01]$.

5. For each solution $\hat{\mathbf{x}}_N$ compute the values of the following accuracy indicators:

   $$\delta_2=\frac{\left\|\hat{\mathbf{x}}_N-\dot{\mathbf{x}}_N\right\|_2}{\left\|\dot{\mathbf{x}}_N\right\|_2} \text{ and } \delta_\infty=\frac{\left\|\hat{\mathbf{x}}_N-\dot{\mathbf{x}}_N\right\|_\infty}{\left\|\dot{\mathbf{x}}_N\right\|_\infty}$$

   Compare the values of δ_2 and δ_∞, computed for the solutions obtained by means of the procedure CB, with the corresponding values of δ_2 and δ_∞, computed for the solutions obtained by means of the operator "\" implemented in MATLAB. Illustrate the results of comparison with the adequate graphs.