**DO Minh Hieu #6**

# Numerical Methods (ENUME 2018) – Project
## Assignment B: Approximation of function

**1**. Make the graphs of the function $f(x) = -\sin(\pi x) e^{-x}$; on each graph mark the following points:

$$\{y_n = f(x_n) | n = 1, 2, ..., N\}, \text{ where } x_n = -1 + 2\frac{n-1}{N-1} \text{ for } N = 5, 10, 15$$

**2**. Design a MATLAB procedure for approximation of the function $f(x)$ on the basis of the data $\{(x_n, y_n) | n = 1, ..., N\}$, using the method of least squares (LS) and the base functions:

$$P_k(x) \text{ for } k = 1, 2, ..., K: \; P_0(x) = 1, \; P_1(x) = x, \; P_k(x) = \frac{2k-1}{k} x P_{k-1}(x) - \frac{k-1}{k} P_{k-2}(x) \text{ for } k > 1$$

(Legendre polynomials).

For solving the system of normal equations, use the method of Cholesky-Banachiewicz and the operator "\" implemented in MATLAB (for comparison). Check the correctness of the procedure for several pairs of the values of the parameters $N$ and $K$.

**3**. Apply the developed procedure for a systematic study of the dependence of approximation accuracy on the parameters $N$ and $K$; use the following indicators for this purpose:

$$\delta_2(K, N) = \frac{\left\| \hat{f}(x; K, N) - f(x) \right\|_2}{\left\| f(x) \right\|_2} \quad \text{and} \quad \delta_\infty(K, N) = \frac{\left\| \hat{f}(x; K, N) - f(x) \right\|_\infty}{\left\| f(x) \right\|_\infty}$$

where $\hat{f}(x; K, N)$ is the approximating function obtained for a given pair of the values of $N$ and $K$. Make the graphs of the dependence of $\delta_2(K, N)$ and $\delta_\infty(K, N)$ on $N$ for several values of $K$ and the graphs of the dependence of $\delta_2(K, N)$ and $\delta_\infty(K, N)$ on $K$ for several values of $N$.

**4**. Repeat the above-described study for the data corrupted with pseudorandom additive errors $\{\Delta\tilde{y}_n | n = 1, ..., N\}$ following the normal distribution with the zero mean and variance $\sigma_y^2$:

$$\{(x_n, \tilde{y}_n) | n = 1, ..., N\}, \text{ where } \tilde{y}_n = y_n + \Delta\tilde{y}_n$$

Use the MATLAB function ***randn*** for generation of the errors. Assess the influence of errors on the accuracy of approximation by analysing the graphs of the dependence of $\delta_2(K, N)$ and $\delta_\infty(K, N)$ on $\sigma_y^2$.