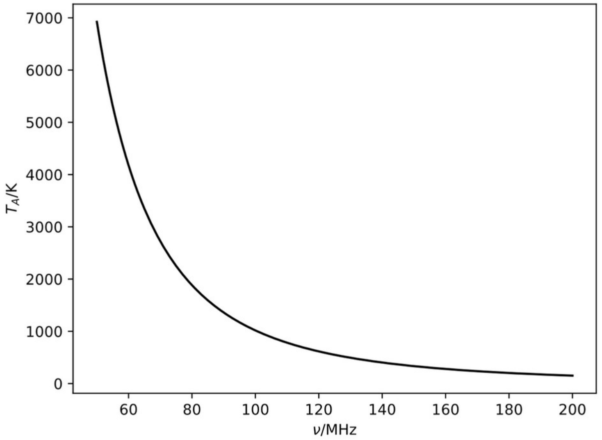
Parameterised Modelling of Antenna Beams for Global 21cm Experiments

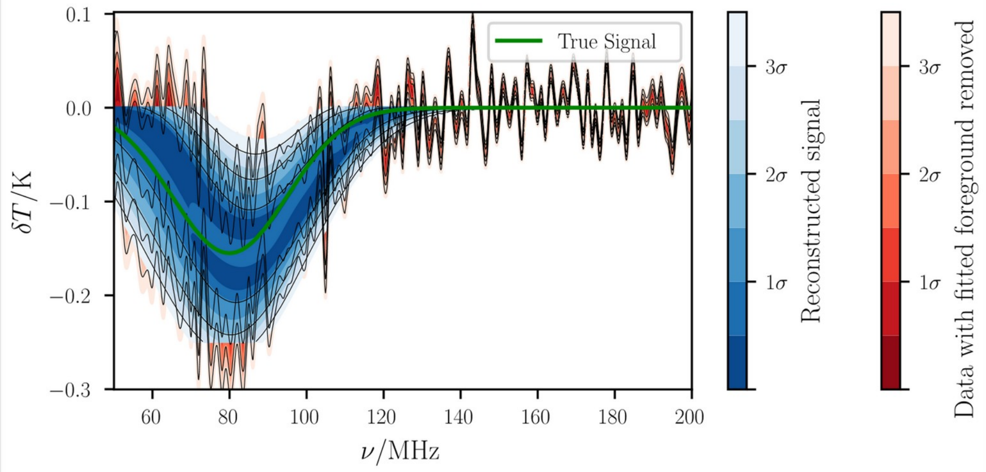
Dominic Anstey | 5th Global 21cm Workshop

On behalf of the
REACH collaboration

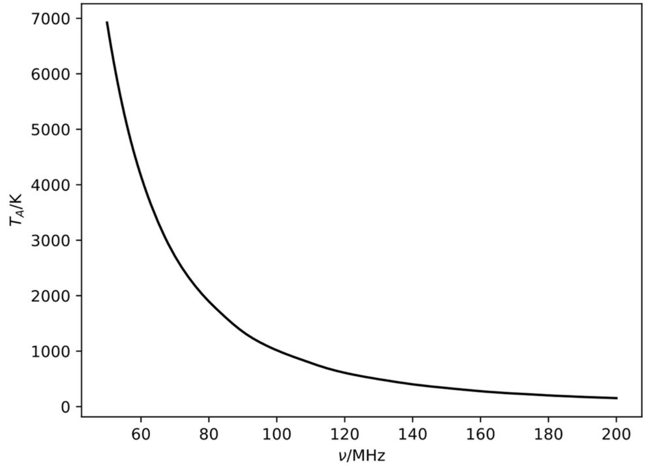
The Issue of Chromaticity

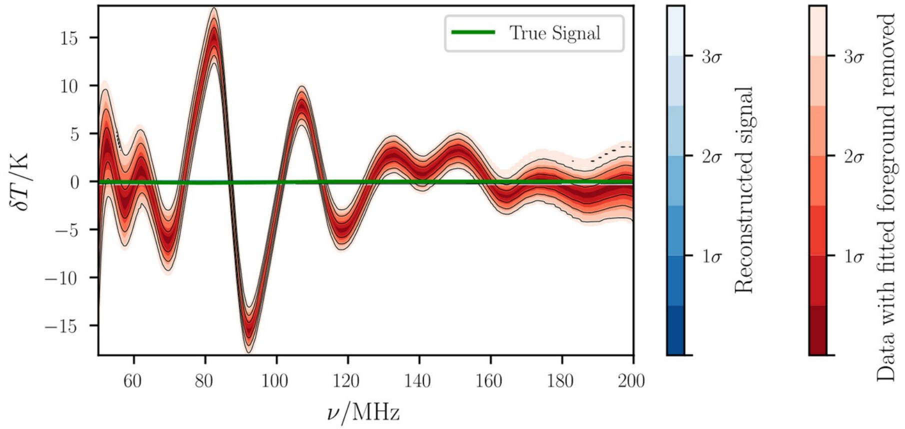
Achromatic antenna





Chromatic antenna

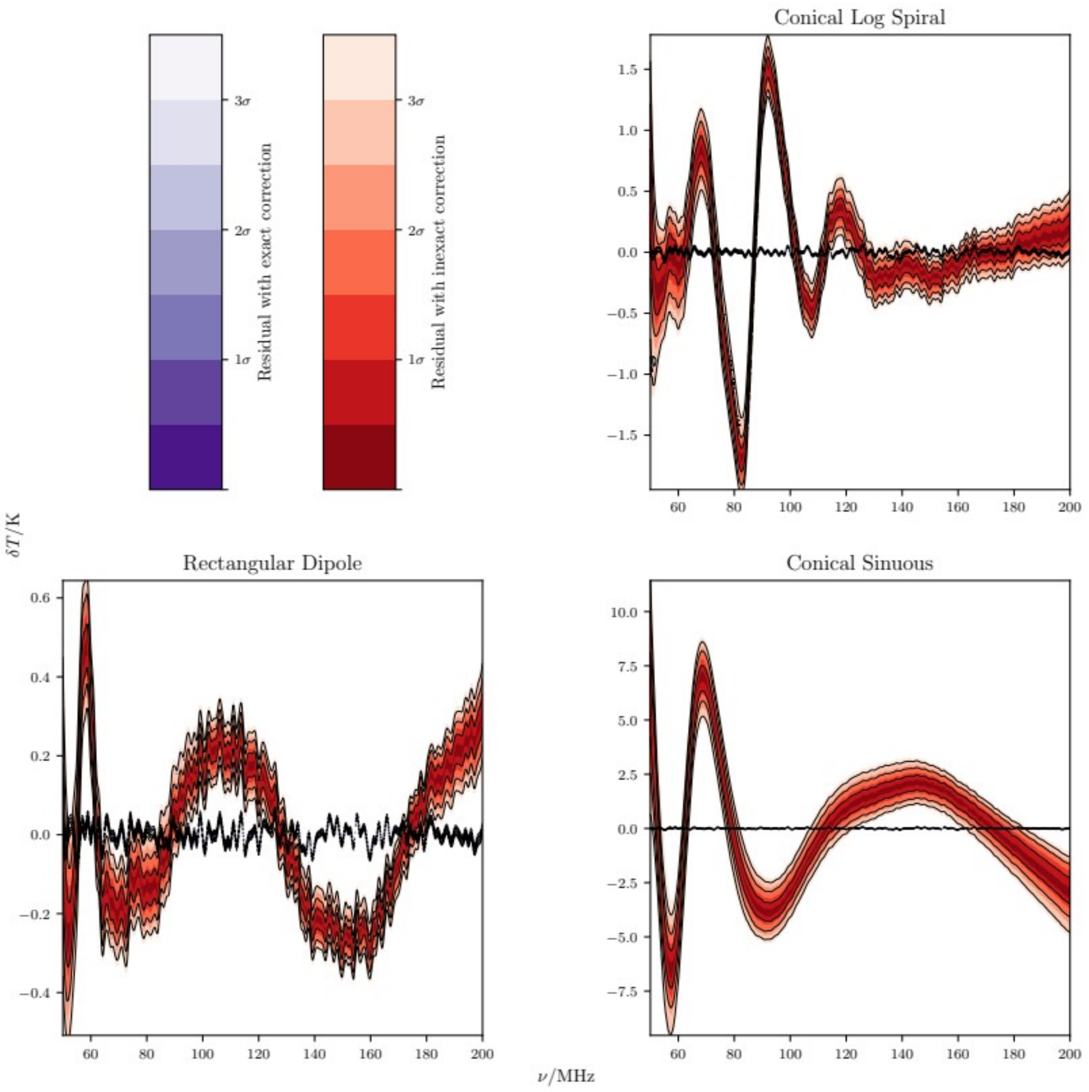




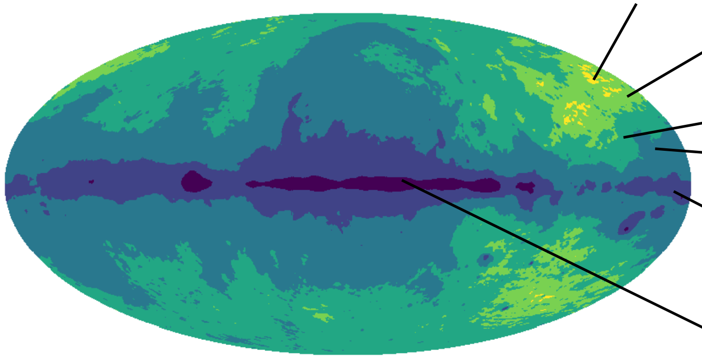
The Issue of Chromaticity

$$B_{\text{factor}}(\nu, t) = \frac{\int D(\Omega, \nu)\, T_{\text{sky}}(\Omega, t)\, d\Omega}{\int D(\Omega, \nu_{\text{ref}})\, T_{\text{sky}}(\Omega, t)\, d\Omega}$$

e.g Mozdzen et al. (2019)
Murray et al. (2022)



Summary of REACH Pipeline



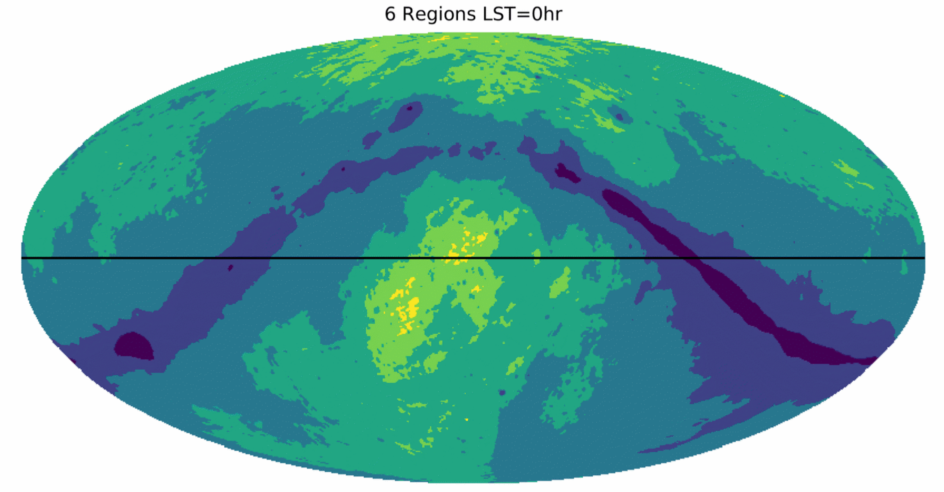
$$T_{\mathrm{F}}\left(\nu,\theta_{\mathrm{F}}\right)=\frac{1}{4\pi}\int_{0}^{4\pi}D\left(\theta,\phi,\nu\right)\times\int_{t_{\mathrm{start}}}^{t_{\mathrm{end}}}\sum_{i=1}^{N}M_{i}\left(\theta,\phi\right)\left(T_{230}\left(\theta,\phi\right)-T_{\mathrm{CMB}}\right)\left(\frac{\nu}{230}\right)^{-\beta_{i}}dt\,d\Omega+T_{\mathrm{CMB}}$$

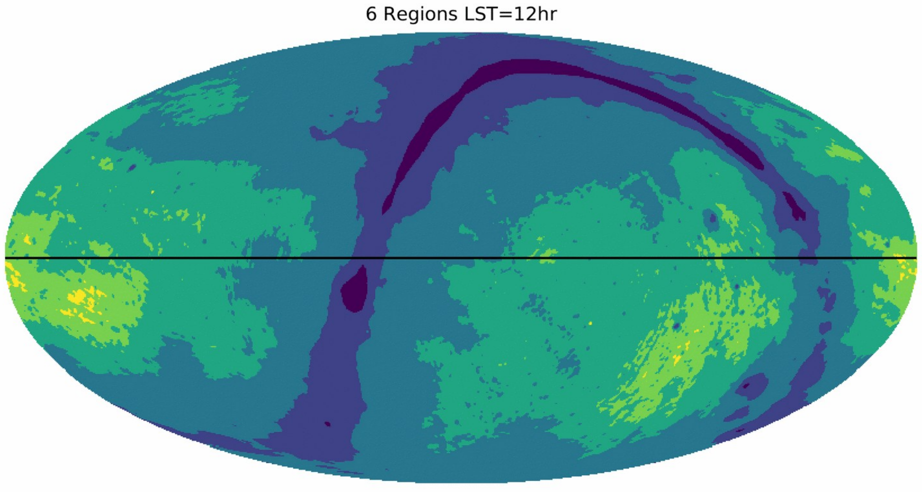
$$\log\mathcal{L}=\sum_{i}-\frac{1}{2}\log\left(2\pi\sigma_{\mathrm{n}}^{2}\right)-\frac{1}{2}\left(\frac{T_{\mathrm{data}}\left(\nu_{i}\right)-\left(T_{\mathrm{F}}\left(\nu_{i},\theta_{\mathrm{F}}\right)+T_{S}\left(\nu_{i},\theta_{\mathrm{S}}\right)\right)}{\sigma_{\mathrm{n}}}\right)^{2}$$

Anstey et al. 2021

Time-Separated Fitting

$$\log \mathcal{L} = \sum_i -\frac{1}{2}\log\left(2\pi\sigma_{\mathrm{n}}^2\right) - \frac{1}{2}\left(\frac{\frac{1}{n_j}\sum_j \left[T_{\mathrm{data}}\left(\nu_i, t_j\right)\right] - \left(\frac{1}{n_j}\sum_j \left[T_{\mathrm{F}}\left(\nu_i, t_j, \theta_{\mathrm{F}}\right)\right] + T_S\left(\nu_i, \theta_{\mathrm{S}}\right)\right)}{\sigma_{\mathrm{n}}}\right)^2$$





Anstey et al. (preprint arXiv:2210.04707)

Time-Separated Fitting

$$\frac{1}{n_j}\sum_j \left[T_{\mathrm{data}}\left(\nu_i, t_j\right)\right]$$

$$\frac{1}{n_j}\sum_j \left[T_{\mathrm{F}}\left(\nu_i, t_j, \theta_{\mathrm{F}}\right)\right]$$

$$\theta_{\mathrm{F}}$$

$$T_{\mathrm{data}}\left(\nu_i, t_1\right)$$

$$T_{\mathrm{F}}\left(\nu_i, t_1, \theta_{\mathrm{F}}\right)$$

$$T_{\mathrm{data}}\left(\nu_i, t_2\right)$$

$$T_{\mathrm{F}}\left(\nu_i, t_2, \theta_{\mathrm{F}}\right)$$

$$T_{\mathrm{F}}\left(\nu_i, t_3, \theta_{\mathrm{F}}\right)$$

$$T_{\mathrm{data}}\left(\nu_i, t_3\right)$$

(…)

$$\log \mathcal{L} = \sum_i \sum_j -\frac{1}{2}\log\left(2\pi\sigma_{\mathrm{n}}^2\right) - \frac{1}{2}\left(\frac{T_{\mathrm{data}}\left(\nu_i, t_j\right) - \left(T_{\mathrm{F}}\left(\nu_i, t_j, \theta_{\mathrm{F}}\right) + T_S\left(\nu_i, \theta_{\mathrm{S}}\right)\right)}{\sigma_{\mathrm{n}}}\right)^2$$

Anstey et al. (preprint arXiv:2210.04707)

Antenna Beam Modelling

The antenna beam pattern may not be known exactly in practice:

- Soil permittivity
- Horizon effects
- Imperfections in construction
- Uncertainties in EM solver
- Etc.

Impact of Uncertainties

	Ax	Bx	By	Cx	Cy	Dx	Dy	Ex	gap	h	groundplane_size	R_L	l_x	l_o
Beam A	17.956	78.147	565.57	271.74	454.42	724.74	415.64	894.35	11.208	694.9	14196	29.694	485.65	433.16
Beam B	18	79	566	271	454	725	415	895	11	700	14250	30	486	433
Difference	0.044	0.853	0.43	0.74	0.42	0.26	0.64	0.65	0.208	5.1	54	0.306	0.35	0.16

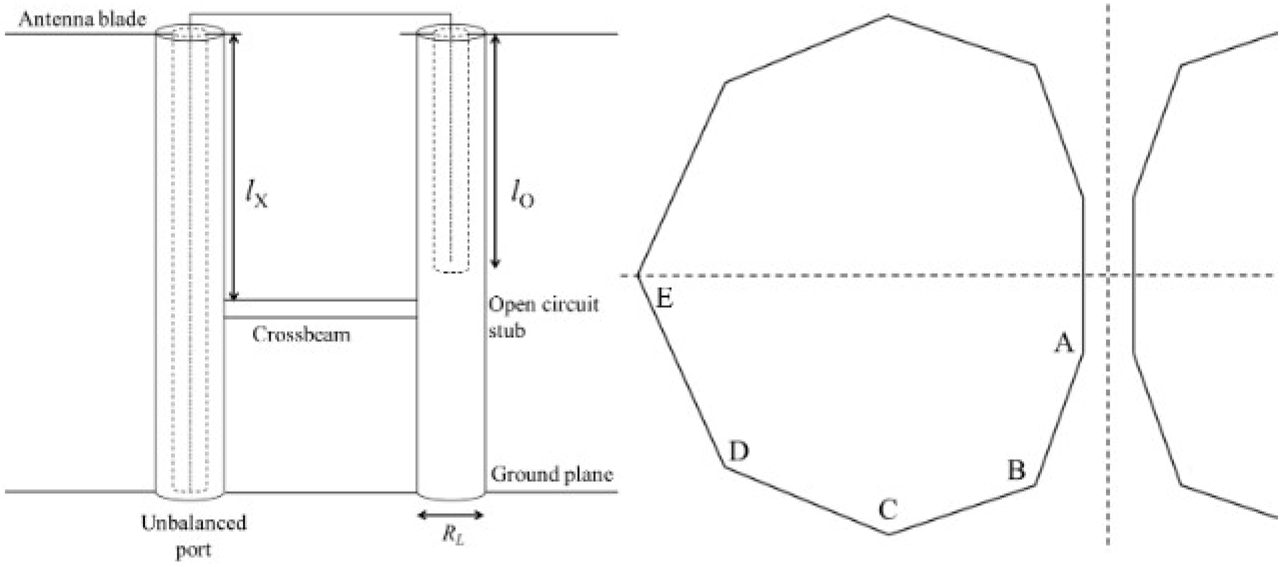
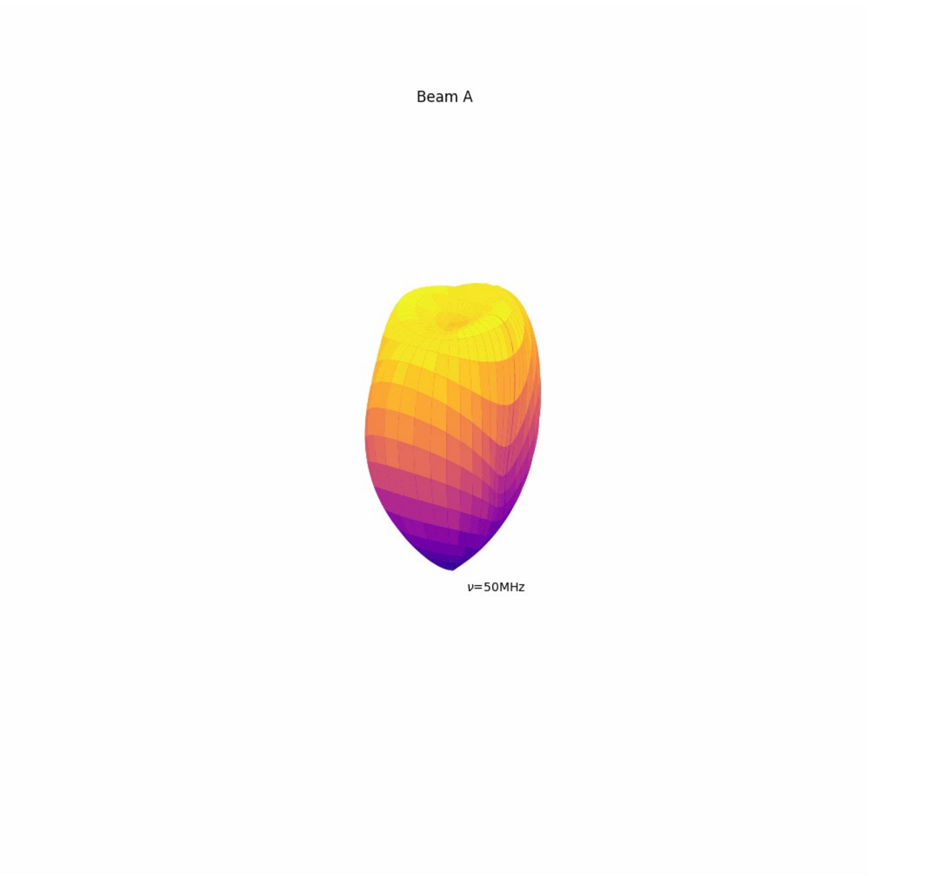
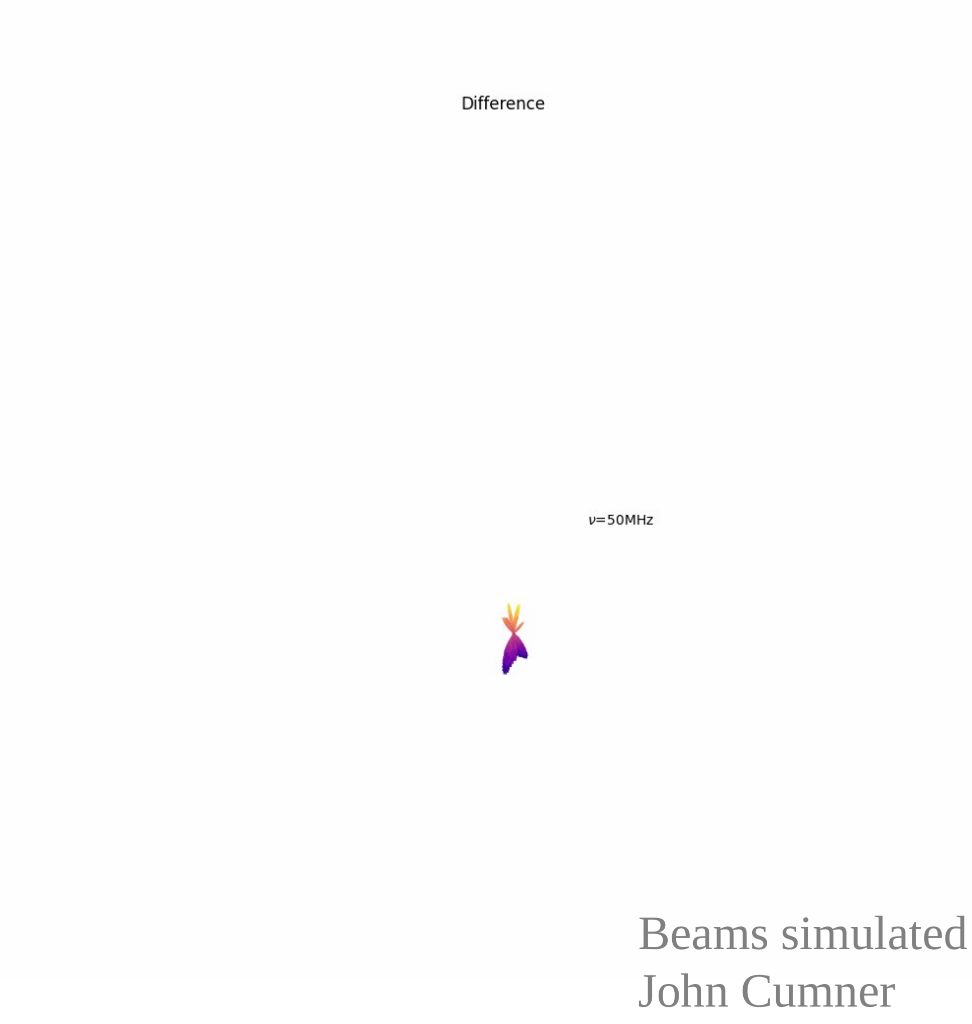


Figure 24 from Cumner et al. 2022

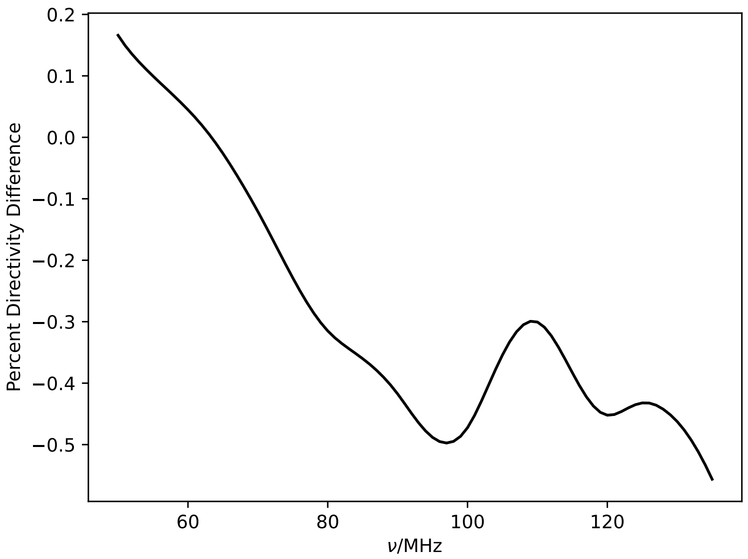
Impact of Uncertainties





Beams simulated by John Cumner

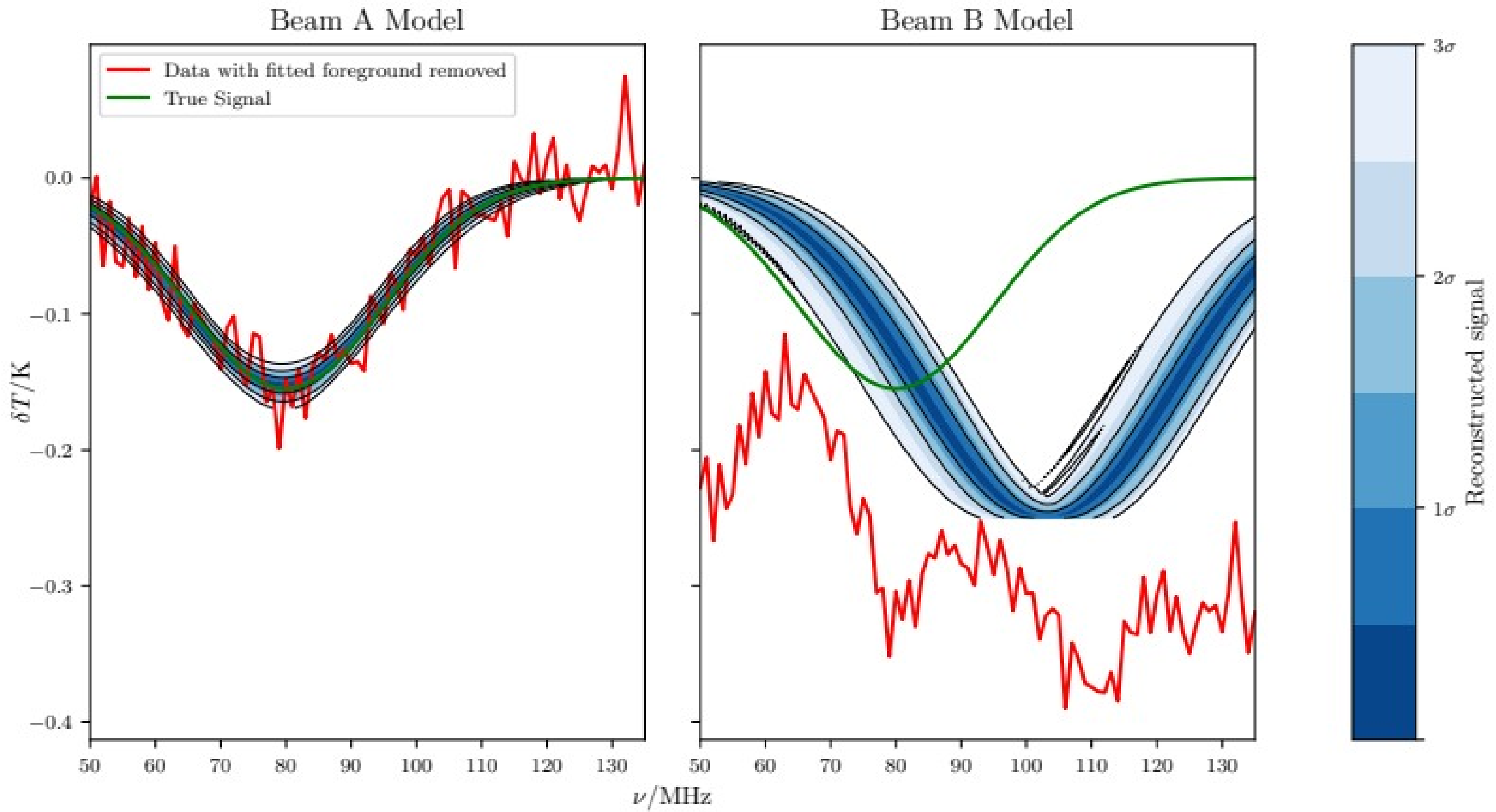
Impact of Uncertainties



- Generate a simulated data set using Beam A
- Fit for the signal correcting for the chromaticity assuming Beam A
- Fit for the signal correcting for the chromaticity assuming Beam B

Foreground known exactly:
GSM base map scaled by -2.55 spectral index

Impact of Uncertainties



Antenna Beam Modelling

$$D\left(\Omega, \nu, \bar{\theta}\right) = \sum_{j=1}^{M} \theta_j X_j \left(\Omega, \nu\right)$$

$$D\left(\Omega, \nu, \bar{\theta}\right) = \sum_{j=1}^{M} \Gamma\left(\nu, \bar{\theta}_j\right) Y_j\left(\Omega\right)$$

$$T_{\mathrm{F}}\left(\nu, \theta_{\mathrm{F}}\right) = \frac{1}{4\pi} \int_0^{4\pi} D\left(\theta, \phi, \nu\right) \times \int_{t_{\mathrm{start}}}^{t_{\mathrm{end}}} \sum_{i=1}^{N} M_i\left(\theta, \phi\right) \left(T_{230}\left(\theta, \phi\right) - T_{\mathrm{CMB}}\right) \left(\frac{\nu}{230}\right)^{-\beta_i} dt d\Omega + T_{\mathrm{CMB}}$$

$$\Downarrow$$

$$T_{\mathrm{F}}\left(\nu, \theta_{\mathrm{F}}, \theta_{\mathrm{A}}\right) = \frac{1}{4\pi} \int_0^{4\pi} \sum_{j=1}^{M} \Gamma\left(\nu, \theta_{\mathrm{A}}\right) Y_j\left(\theta, \phi\right) \times \int_{t_{\mathrm{start}}}^{t_{\mathrm{end}}} \sum_{i=1}^{N} M_i\left(\theta, \phi\right) \left(T_{230}\left(\theta, \phi\right) - T_{\mathrm{CMB}}\right) \left(\frac{\nu}{230}\right)^{-\beta_i} dt d\Omega + T_{\mathrm{CMB}}$$

Beam Normalisation

$$T_{\mathrm{F}}(\nu) = \frac{1}{4\pi}\int_0^{4\pi} D(\Omega,\nu)\int_{t_{\mathrm{start}}}^{t_{\mathrm{end}}} \left(T_{\mathrm{base}}(\Omega) - T_{\mathrm{CMB}}\right)\left(\frac{\nu}{\nu_{\mathrm{base}}}\right)^{-\beta(\Omega)} dt\,d\Omega + T_{\mathrm{CMB}}$$

Provided basis functions are all normalised, can use polynomials

0^{th} order coefficients of all basis functions sum to 1

$$\sum_{j=1}^{M}\theta_{j,0} = 1$$

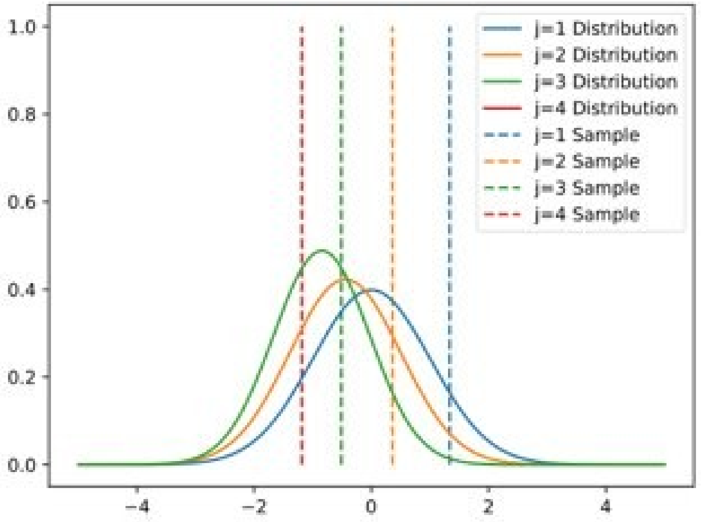
For every other order, coefficients of each basis function must sum to zero

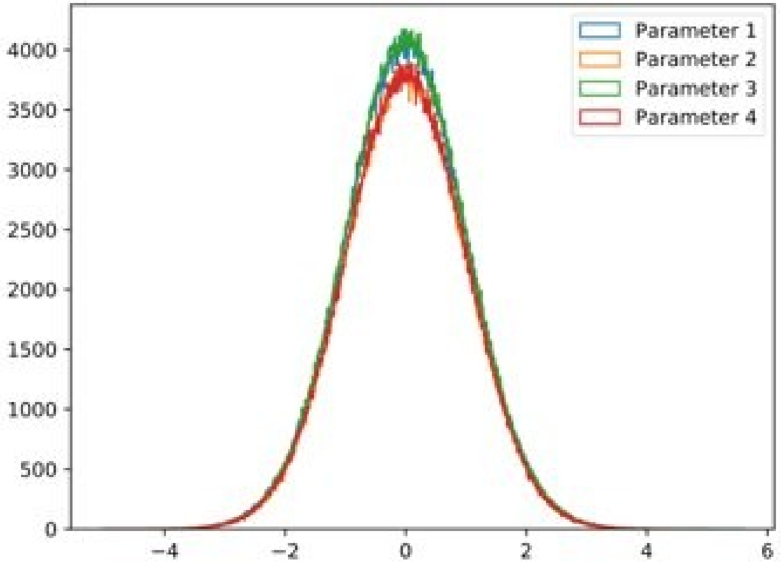
$$\sum_{j=1}^{M}\theta_{j,k\neq 0} = 0$$

Beam Normalisation

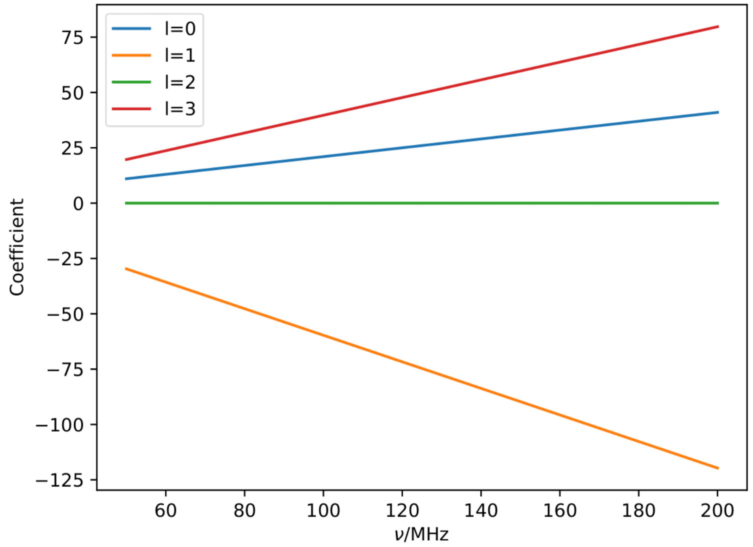
$$\mu_j = \frac{-\Sigma_{l=1}^{j}\theta_l}{N_{\text{basis}} - j + 1}$$

$$\sigma_j = \sigma\sqrt{1 - \frac{j-1}{(M-1)\,(M-j-1)}},$$

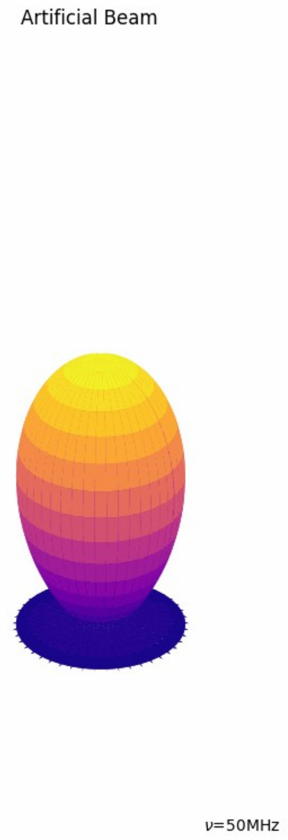




Example Beam

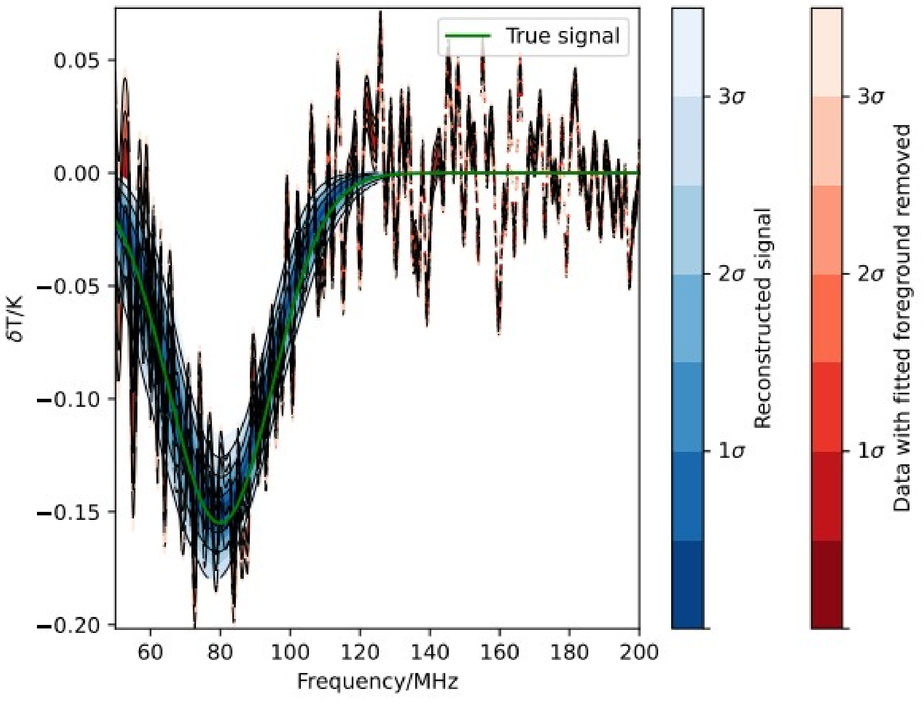


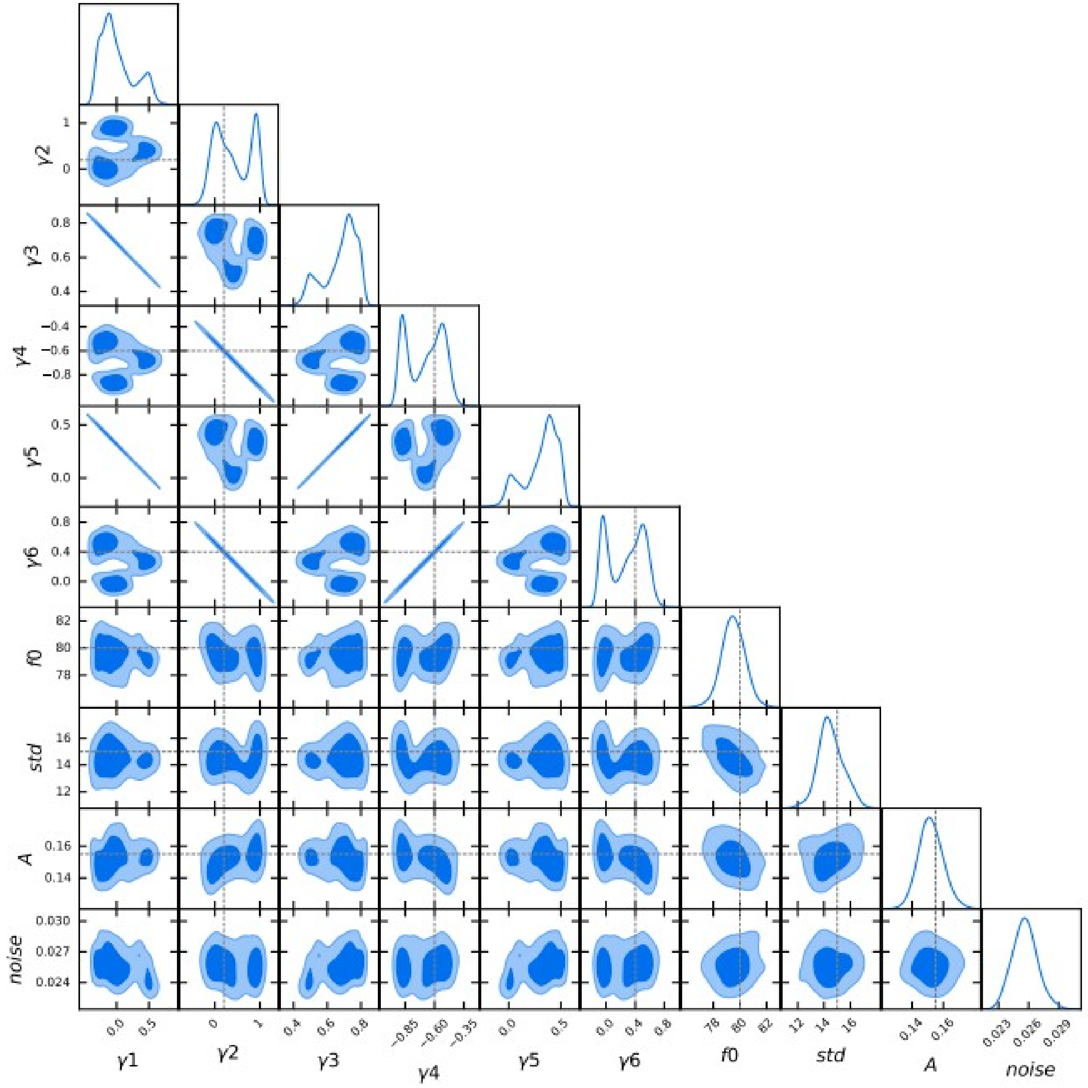
	l=0	l=1	l=2	l=3
a_0	1.0	0.3	0.0	-0.3
a_1	0.2	-0.6	0.0	0.4



Example Beam Fitting Tests

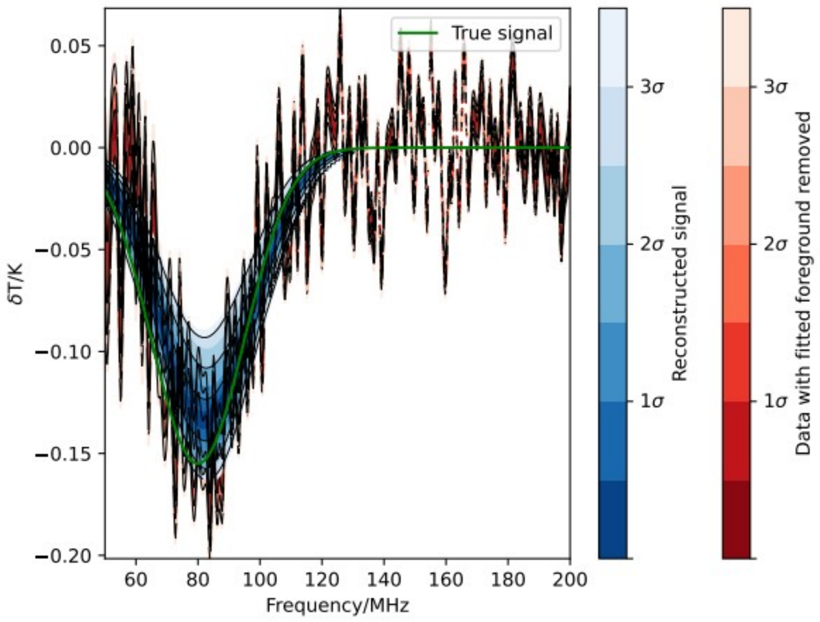
Foreground model fixed

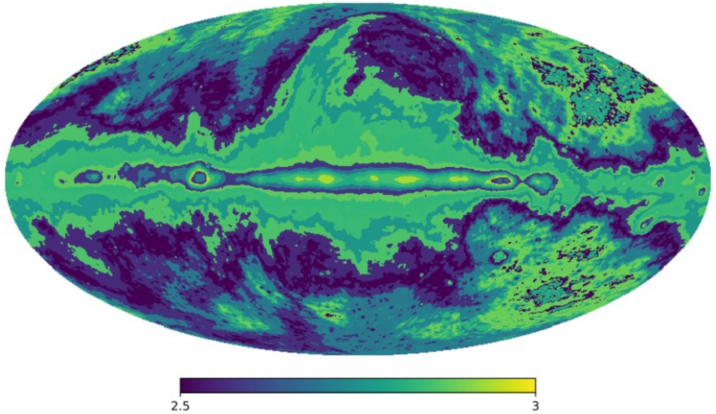


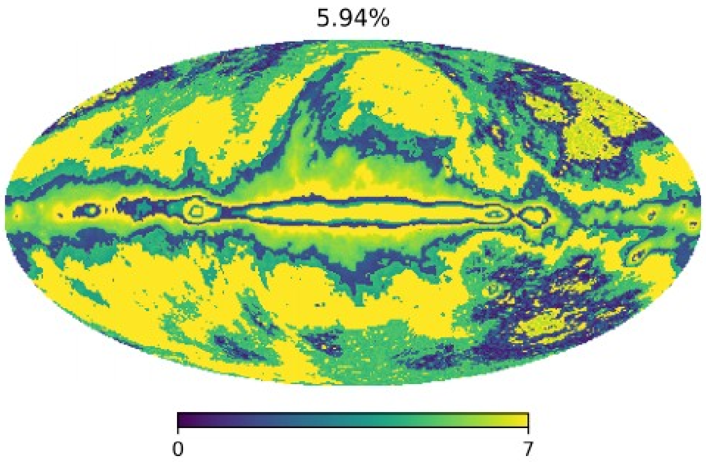


Example Beam Fitting Tests

Fitting foreground together with antenna

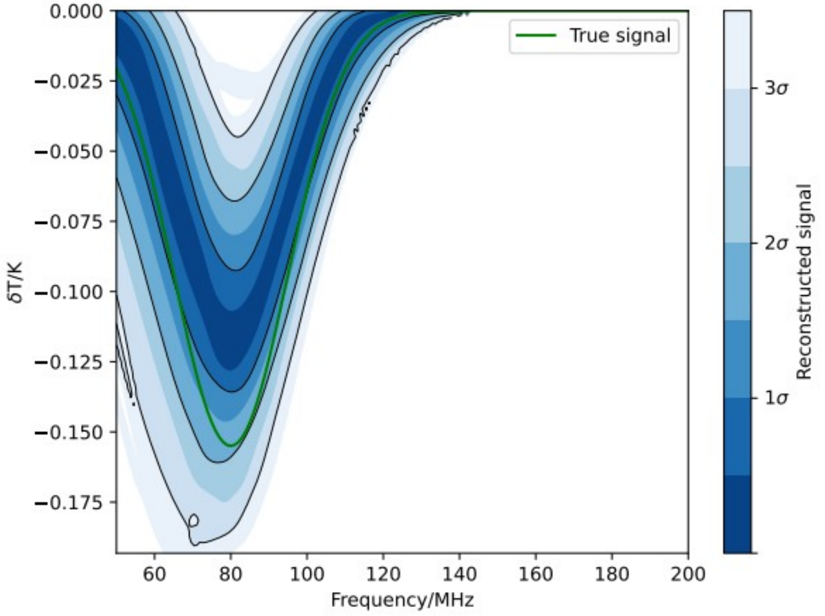


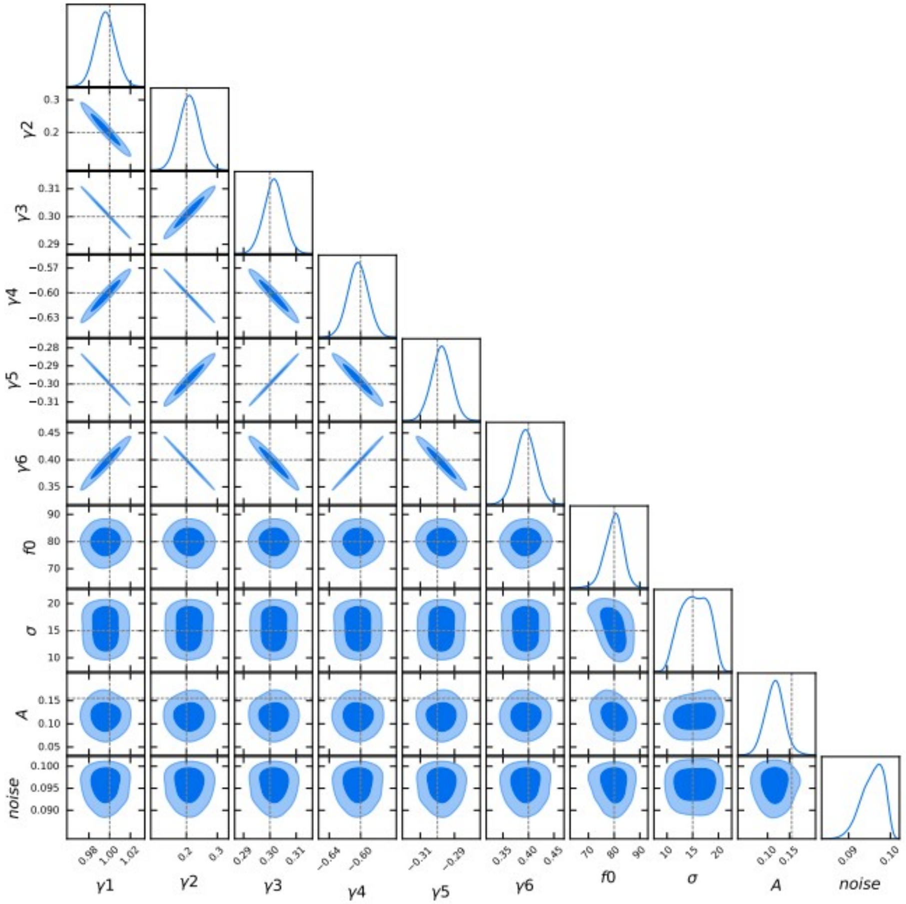




Example Beam Fitting Tests

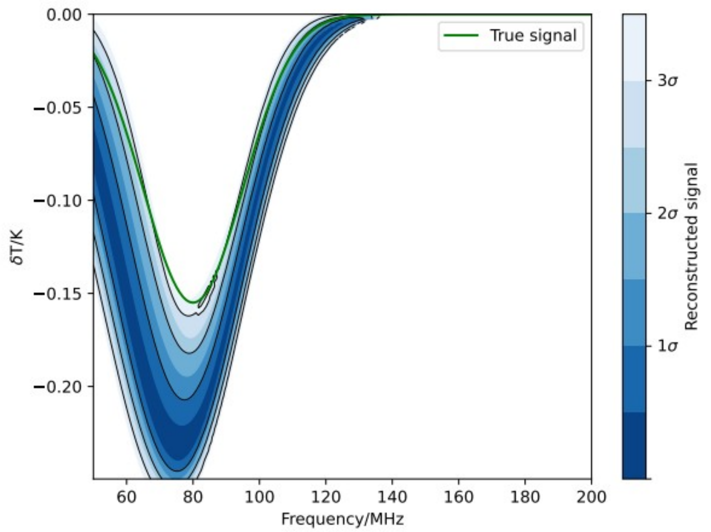
Foreground model fixed, 2 time bins

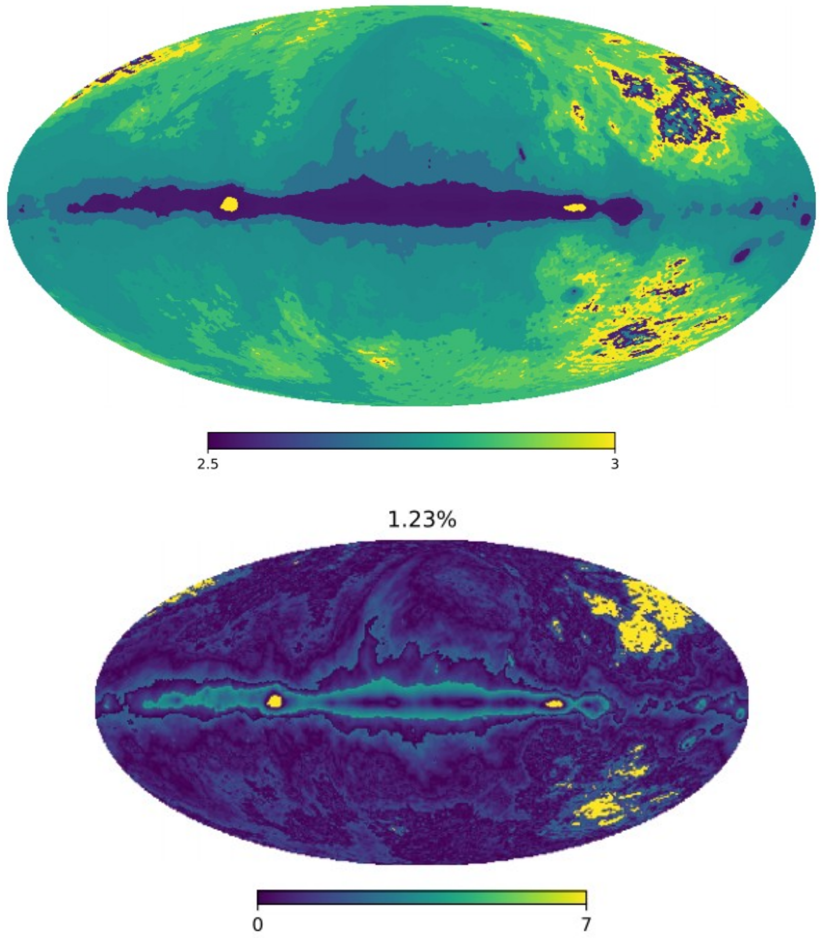




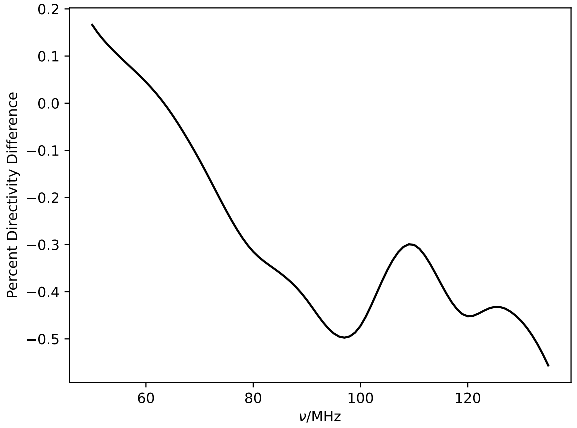
Example Beam Fitting Tests

Fitting foreground together with antenna, 12 time bins

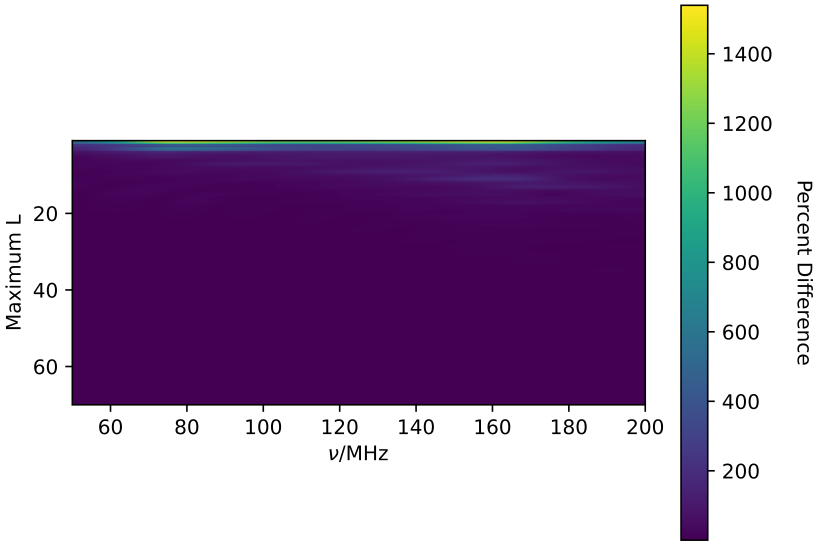




Parameter Numbers

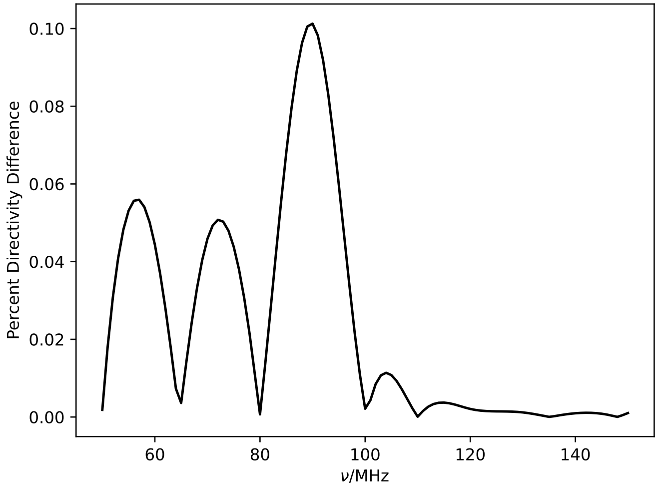


Spherical Harmonic basis functions



>2556 basis functions

Gram-Schmidt basis functions



21 basis functions

Next Steps

- **Analytical Marginalisation of Linear Parameters**

  e.g Tauscher et al. (2021), Murray et al. (2022)

- **Machine Learning**

Conclusions

- Summarised current state of REACH pipeline
- Discussed the impact of beam uncertainties
- Showed how antenna beam modelling can be implemented in the REACH pipeline
- Discussed challenges with antenna beam modelling and how they can be addressed, in particular:
    - Beam normalisation
    - Parameter degeneracies

Antenna EM simulations provided by John Cumner and Quentin Gueuning