Introduction to Logic, Part I, Chapter I by Patrick Suppes - exercises

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Exercise 1.

A classical example of a non-truth-functional connective is that of possibility. For example, the sentence:

1. "It is possible that there is life on Mars."

is true under any liberal interpretation of the notion of possibility; but then so is the sentence:

2. "It is possible that there is not any life on Mars."

On the other hand, the sentence:

3. "It is possible that $2 + 2 = 5$"

is ordinarly regarded as false. Using a exclamation mark "!" for "it is possible that", M for "there is life on Mars" and W for "2 + 2 = 5", we get the following tabular analysis of (1)-(3):

M	$-M$	W	$!M$	$!-M$	$!W$
T	F	F	T	T	F
F	T		T	T	F

The analysis of !M and !-M entails that the only truth-functiona lanalysis of the possibility connective is that for any sentence P, !P is true, but the truth value of !W controverts this; and we see that there is no appriopriate truth-functional analysis.

Give examples and an anlysis to show that the following are not truth-functional connectives:

(a) "Mr. Smith belives that carrots are beasts."

(b) "It is necessary that people eat pudding to stay fit."

This exercise is not finished. In my opinion it has too long introduction and vague goal, so I will not finish it.

Exercise 2.

Which of the truth-functional connectives introduced in this chapter is an approximate synonym of the connective "unless"? (Hint: To say "There will be peace unless there is a major war in the next five years, then there will be peace" is equicalent to saying "If there is not a major war in the next five years, then there will be peace".)

Answer: Negation of implication

Exercise 3.

Translate the following compound sentences into symbolic notation, using letters to stand for atomic sentences.

(a) "Either the fire was produced by arson or it was produced by spontaneous combustion."
$M =$ "the fire was produced by arson",
$L =$ "it was produced by spontaneous combustion"

$$M \vee L$$

(b) "If the water is clear, then either Henry can see the bottom of the pool or he is a nincompoop."
$M =$ "the water is clear",
$L =$ "Henry can see the bottom of the pool",
$S =$ "he is a nincompoop"

$$M \to L \vee S$$

(c) "Either John is not here or Mary is, and Helen certainly is."
$M =$ "John is here",
$L =$ "Mary is here",
$S =$ "Helen certainly is here"

$$-M \vee L \wedge S$$

(d) "If there are more cats than dogs, then there are more horses than dogs and there are fewer snakes than cats."
$M =$ "there are more cats than dogs",
$L =$ "there are more horses than dogs",
$S =$ "there are fewer snakes than cats"

$$M \to (L \wedge S)$$

(e) "The man in the moon is a fake, and if the same is true of Santa Claus, many children are deceived."
$M =$ "The man in the moon is a fake",
$L =$ "the same is true of Santa Claus",
$S =$ "many children are deceived"

$$M \wedge (L \to S)$$

(f) "If either red-heads are lovely or blondes do not have freckles, then logic

is confusing."
$M =$ "red-heads are lovely",
$L =$ "blondes do not have freckles",
$S =$ "logic is confusing"

$$(M \vee L) \rightarrow S$$

(g) "If either housing is scarce or people like to live with their in-laws, and if people do not like to live with their in-laws, then housing is scarce."
$M =$ "housing is scarce",
$L =$ "people like to live with their in-laws",

$$M \vee L \wedge -L \rightarrow M$$

(h) "If John testifies and tells the truth, he will be found guilty; and if he does not testify, he will be found guilty."
$M =$ "John testifies",
$L =$ "John thells the truth",
$S =$ "he will be found guilty"

$$(M \wedge L \rightarrow S) \wedge (-M \rightarrow S)$$

(i) "Either John must testify and tell the truth, or he does not have to testify."
$M =$ "John must testify",
$L =$ "John must tell the truth",

$$(M \wedge L) \vee -M$$

Exercise 4.

In the following examples determine the truth value of the compound sentences from the given truth values of the component sentences.

(i) "Galileo was born before Descartes" is true.
(ii) "Descartes was born in the sixteenth century" is true.
(iii) "Newton was born before Shakespeare" is false.
(iv) "Racine was a compatriot of Galileo" is false.

(a) If Galileo was born before Descartes, then Newton was not born before Shakespeare.
Answer: $true \rightarrow \neg false$ is $true$
(b) If either Racine was a compatriot of Galileo or Newton was born before Shakespeare, then Descartes was born in the sixteenth century. Answer: $(false \vee false) \rightarrow true$ is $true$
(c) If Racine was not a compatriot of Galileo, then either Descartes was not born in the sixteenth century or Newton was born before Shakespeare.

Answer: $\neg false \rightarrow (\neg true \vee false)$ is $false$

Exercise 5.

Let
$N =$ New York is larger than Chicago
$W =$ New York is north of Washington
$C =$ Chicago is larger than New York
N, W are true and C is false.
Which of the following sentences are true?

(a) $N \vee C$ is $true$
(b) $N \wedge C$ is $false$
(c) $-N \wedge -C$ is $false$
(d) $N \leftrightarrow -W \vee C$ is $false$
(e) $W \vee -C \rightarrow N$ is $true$
(f) $(W \vee N) \rightarrow (W \rightarrow -C)$ is $true$
(g) $(W \leftrightarrow -N) \leftrightarrow (N \leftrightarrow C)$ is $true$
(h) $(W \rightarrow N) \rightarrow [(N \rightarrow -C) \rightarrow (-C \rightarrow W)]$ is $true$

Exercise 6.

Let
$P =$ Jane Austen was contemporary of Beethoven
$Q =$ Beethoven was a contemporary of Gauss
$R =$ Gauss was a contemporary of Napoleon
$S =$ Napoleon was a contemporary of Julius Caesar
P, Q, and R are true, and S is false.
Find the truth values of the following sentences:

(a) $(P \wedge Q) \wedge R$ is $true$
(b) $P \wedge (Q \wedge R)$ is $true$
(c) $S \rightarrow P$ is $true$
(d) $P \rightarrow S$ is $false$
(e) $(P \wedge Q) \wedge (R \wedge S)$ is $false$
(f) $P \wedge Q \leftrightarrow R \wedge -S$ is $true$
(g) $(P \leftrightarrow Q) \rightarrow (S \leftrightarrow R)$ is $false$
(h) $(-P \leftarrow Q) \leftarrow (S \leftarrow R)$ is $true$
(i) $(P \rightarrow -Q) \rightarrow (S \leftrightarrow R)$ is $true$
(j) $(P \rightarrow Q)[(Q \rightarrow R) \rightarrow (R \rightarrow S)]$ is $false$
(k) $P \rightarrow [Q \leftrightarrow (R \rightarrow S)]$ is $false$

Exercise 7.

Let P be a sentence such that for any sentence Q the sentence $P \vee Q$ is true. What can be said about the truth value of P.

Answer: P is $true$

Exercise 8.

Let P be a sentence such that for any sentence Q the sentence $P \wedge Q$ is false.
What can be said about the truth value of P.
Answer: P is $false$

Exercise 9.

If $P \leftrightarrow Q$ is true, what can be said about the truth value of $P \vee -Q$?
Answer: $P \vee -Q$ is $true$

Exercise 10.

(a) $P \vee Q$ is **not** a tautology.

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

(b) $P \vee -P$ is a tautology

p	$p \vee -p$
T	T
F	T

(c) $P \vee Q \to Q \vee P$ is a tautology.

P	Q	$P \vee Q \to Q \vee P$
T	T	T
T	F	T
F	T	T
F	F	T

(d) $P \to (P \vee Q) \vee R$ is a tautology.

P	Q	R	$P \to (P \vee Q) \vee R$
T	T	T	T
T	T	F	T
T	F	T	T
T	F	F	T
F	F	F	T
F	F	T	T
F	T	F	T
F	T	T	T

(e) $P \to (-P \to Q)$ is a tautology.

P	Q	$P \to (-P \to Q)$
T	T	T
T	F	T
F	T	T
F	F	T

(f) $(P \to Q) \to (Q \to P)$ is **not** a tautology.

P	Q	$P \to Q) \to (Q \to P$
T	T	T
T	F	T
F	T	F
F	F	T

(g) $[(P \to Q) \leftrightarrow Q] \to P$ is **not** a tautology.

P	Q	$[(P \to Q) \leftrightarrow Q] \to P$
T	T	T
T	F	T
F	T	F
F	F	T

(h) $P \to [Q \to (Q \to P)]$ is a tautology.

P	Q	$P \to [Q \to (Q \to P)]$
T	T	T
T	F	T
F	T	T
F	F	T

(i) $P \wedge Q \to P \vee R$ is a tautology.

P	Q	R	$P \wedge Q \to P \vee R$
T	T	T	T
T	T	F	T
T	F	T	T
T	F	F	T
F	F	F	T
F	F	T	T
F	T	F	T
F	T	T	T

(j) $[P \vee (-P \wedge Q)] \vee (-P \wedge -Q)$ is a tautology.

P	Q	$[P \vee (-P \wedge Q)] \vee (-P \wedge -Q)$
T	T	T
T	F	T
F	T	T
F	F	T

(k) $P \wedge Q \to (P \leftrightarrow Q \vee R)$ is a tautology.

P	Q	R	$P \wedge Q \to (P \leftrightarrow Q \vee R)$
T	T	T	T
T	T	F	T
T	F	T	T
T	F	F	T
F	F	F	T
F	F	T	T
F	T	F	T
F	T	T	T

(l) $[P \wedge Q \to (P \wedge -P \to Q \vee -Q)] \wedge (Q \to Q)$ is a tautology.

P	Q	$[P \wedge Q \to (P \wedge -P \to Q \vee -Q)] \wedge (Q \to Q)$
T	T	T
T	F	T
F	T	T
F	F	T

Exercise 11.

If P and Q are distinct atomic sentences, which of the following are tautologies?

(a) $P \leftrightarrow Q$ is **not** a tautology.

P	Q	$P \leftrightarrow Q$
T	T	T
T	F	F
F	T	F
F	F	T

(b) $P \leftrightarrow P \vee P$ is a tautology.

P	$P \leftrightarrow P \vee P$
T	T
F	T

(c) $P \vee Q \leftrightarrow Q \vee P$ is a tautology.

P	Q	$P \vee Q \leftrightarrow Q \vee P$
T	T	T
T	F	T
F	T	T
F	F	T

(d) $(P \to Q) \leftrightarrow (Q \to P)$ is **not** a tautology.

P	Q	$(P \to Q) \leftrightarrow (Q \to P)$
T	T	T
T	F	F
F	T	F
F	F	T

(e) $(P \leftrightarrow P) \leftrightarrow P$ is a tautology.

P	$(P \leftrightarrow P) \leftrightarrow P$
T	T
F	T

Exercise 12.

On the basis of ordinary usage construct truth tables for the sentential connectives used in the following examples:

(a) Not both P and Q.

P	Q	$-(P \wedge Q)$
T	T	F
T	F	T
F	T	T
F	F	T

(b) Neither P nor Q.

P	Q	$-(P \vee Q)$
T	T	F
T	F	F
F	T	F
F	F	T

Exercise 13.

Give examples of sentences P and Q (not necessarily atomic) such that the following compound sentences are tautologies.

(a) Sentence $W = P \wedge Q$ is **not** a tautology. Assumption $P = P \vee -P$ and $Q = Q \vee -Q$ changes W into a tautology.

P	Q	$P \wedge Q$	$(P \vee -P) \wedge (Q \vee -Q)$
T	T	T	T
T	F	F	T
F	T	F	T
F	F	F	T

(b) Sentence $W = P \vee (P \wedge -Q)$ is **not** a tautology. Assumption $P = P \vee -P$ changes W into a tautology.

P	Q	$P \vee (P \wedge -Q)$	$(P \vee -P) \vee (Q \wedge -Q)$
T	T	T	T
T	F	T	T
F	T	F	T
F	F	F	T

(c) Sentence $W = P \to P \wedge -Q$ is **not** a tautology. Assumption $Q = -P$ changes W into a tautology.

P	Q	$P \to P \wedge -Q$	$P \to P \wedge -(-P)$
T	T	F	T
T	F	T	T
F	T	T	T
F	F	T	T

(d) Sentence $W = P \to -P$ is **not** a tautology. Assumption $P = -(P \vee -P)$ changes W into a tautology.

P	$P \to -P$	$-(P \vee -P) \to -(-(P \vee -P))$
T	F	T
F	T	T

Exercise 14.

Is there any sentence P such that $P \wedge -P$ is a tautology?
Answer: No such sentence exists.

Exercise 15.

If P and Q are distinct atomic sentences, the sentence $P \wedge Q$ tautologically implies which of the following?

(a) P

P	Q	$P \wedge Q \to P$
T	T	T
T	F	T
F	T	T
F	F	T

Answer: $P \wedge Q$ tautologically implies P.

(b) Q

P	Q	$P \wedge Q \to Q$
T	T	T
T	F	T
F	T	T
F	F	T

Answer: $P \wedge Q$ tautologically implies Q.

(c) $P \vee Q$

P	Q	$P \wedge Q \to P \vee Q$
T	T	T
T	F	T
F	T	T
F	F	T

Answer: $P \wedge Q$ tautologically implies $P \vee Q$.

(d) $P \wedge -Q$

P	Q	$P \wedge Q \to P \wedge -Q$
T	T	F
T	F	T
F	T	T
F	F	T

Answer: $P \wedge Q$ **does not** tautologically imply $P \wedge -Q$.

(e) $-P \vee Q$

P	Q	$P \wedge Q \to -P \vee Q$
T	T	T
T	F	T
F	T	T
F	F	T

Answer: $P \wedge Q$ tautologically implies $-P \vee Q$.

(f) $-Q \to P$

P	Q	$P \wedge Q \to (-Q \to P)$
T	T	T
T	F	T
F	T	T
F	F	T

Answer: $P \wedge Q$ tautologically implies $-Q \to P$.

(g) $P \leftrightarrow Q$

P	Q	$P \wedge Q \to (P \leftrightarrow Q)$
T	T	T
T	F	T
F	T	T
F	F	T

Answer: $P \wedge Q$ tautologically implies $P \leftrightarrow Q$.

Exercise 16.

If P and Q are distinct atomic sentences, the sentence $-P \vee Q$ tautologically implies which of the following?

(a) P

P	Q	$-P \vee Q \to P$
T	T	T
T	F	T
F	T	F
F	F	F

Answer: $-P \vee Q$ **does not** tautologically imply P.

(b) $Q \to P$

P	Q	$-P \vee Q \to (Q \to P)$
T	T	T
T	F	T
F	T	F
F	F	T

Answer: $-P \vee Q$ **does not** tautologically imply $Q \to P$.

(c) $P \to Q$

P	Q	$-P \vee Q \to (P \to Q)$
T	T	T
T	F	T
F	T	T
F	F	T

Answer: $-P \vee Q$ tautologically implies $P \to Q$.

(d) $-Q \to -P$

P	Q	$-P \vee Q \to (-Q \to -P)$
T	T	T
T	F	T
F	T	T
F	F	T

Answer: $-P \vee Q$ tautologically implies $-Q \to -P$.

(e) $-P \wedge Q$

P	Q	$-P \vee Q \to -P \wedge Q$
T	T	F
T	F	T
F	T	T
F	F	F

Answer: $-P \vee Q$ **does not** tautologically imply $-P \wedge Q$.

Exercise 17.

If P and Q are distinct atomic sentences, the sentence P is tautologically equivalent to which of the following?

(a) $P \vee Q$

P	Q	$P \leftrightarrow P \vee Q$
T	T	T
T	F	T
F	T	F
F	F	T

Answer: P is **not** tautologically equivalent to $P \vee Q$.

(b) $P \vee -P$

P	$P \leftrightarrow P \vee -P$
T	T
F	F

Answer: P is **not** tautologically equivalent to $P \vee -P$.

(c) $P \wedge P$

P	$P \leftrightarrow P \wedge P$
T	T
F	T

Answer: P is tautologically equivalent to $P \wedge P$.

(d) $P \rightarrow P$

P	$P \leftrightarrow P \rightarrow P$
T	T
F	F

Answer: P is **not** tautologically equivalent to $P \rightarrow P$.

(e) $-P \rightarrow P$

P	$P \leftrightarrow -P \rightarrow P$
T	T
F	T

Answer: P is tautologically equivalent to $-P \rightarrow P$.

(f) $P \rightarrow -P$

P	$P \leftrightarrow P \rightarrow -P$
T	F
F	F

Answer: P is **not** tautologically equivalent to $P \rightarrow -P$.

(g) $Q \vee -Q \to P$

P	Q	$P \leftrightarrow (Q \vee -Q \to P)$
T	T	T
T	F	T
F	T	T
F	F	T

Answer: P is tautologically equivalent to $Q \vee -Q \to P$.