

# Introduction to Logic, Part I, Chapter I by Patrick Suppes - exercises

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## Exercise 4.

In the following examples determine the truth value of the compound sentences from the given truth values of the component sentences.

- (i) "Galileo was born before Descartes" is true.
- (ii) "Descartes was born in the sixteenth century" is true.
- (iii) "Newton was born before Shakespeare" is false.
- (iv) "Racine was a compatriot of Galileo" is false.
- (a) If Galileo was born before Descartes, then Newton was not born before Shakespeare.  
Answer:  $true \rightarrow \neg false$  is *true*
- (b) If either Racine was a compatriot of Galileo or Newton was born before Shakespeare, then Descartes was born in the sixteenth century. Answer:  
 $(false \vee false) \rightarrow true$  is *true*
- (c) If Racine was not a compatriot of Galileo, then either Descartes was not born in the sixteenth century or Newton was born before Shakespeare.  
Answer:  $\neg false \rightarrow (\neg true \vee false)$  is *false*

## Exercise 5.

Let

$N$  = New York is larger than Chicago

$W$  = New York is north of Washington

$C$  = Chicago is larger than New York

$N$ ,  $W$  are true and  $C$  is false.

Which of the following sentences are true?

- (a)  $N \vee C$  is *true*
- (b)  $N \wedge C$  is *false*
- (c)  $\neg N \wedge \neg C$  is *false*
- (d)  $N \leftrightarrow \neg W \vee C$  is *false*
- (e)  $W \vee \neg C \rightarrow N$  is *true*

- (f)  $(W \vee N) \rightarrow (W \rightarrow -C)$  is *true*
- (g)  $(W \leftrightarrow -N) \leftrightarrow (N \leftrightarrow C)$  is *true*
- (h)  $(W \rightarrow N) \rightarrow [(N \rightarrow -C) \rightarrow (-C \rightarrow W)]$  is *true*

### Exercise 6.

Let

$P$  = Jane Austen was contemporary of Beethoven

$Q$  = Beethoven was a contemporary of Gauss

$R$  = Gauss was a contemporary of Napoleon

$S$  = Napoleon was a contemporary of Julius Caesar

$P$ ,  $Q$ , and  $R$  are true, and  $S$  is false.

Find the truth values of the following sentences:

- (a)  $(P \wedge Q) \wedge R$  is *true*
- (b)  $P \wedge (Q \wedge R)$  is *true*
- (c)  $S \rightarrow P$  is *true*
- (d)  $P \rightarrow S$  is *false*
- (e)  $(P \wedge Q) \wedge (R \wedge S)$  is *false*
- (f)  $P \wedge Q \leftrightarrow R \wedge -S$  is *true*
- (g)  $(P \leftrightarrow Q) \rightarrow (S \leftrightarrow R)$  is *false*
- (h)  $(-P \leftarrow Q) \leftarrow (S \leftarrow R)$  is *true*
- (i)  $(P \rightarrow -Q) \rightarrow (S \leftrightarrow R)$  is *true*
- (j)  $(P \rightarrow Q)[(Q \rightarrow R) \rightarrow (R \rightarrow S)]$  is *false*
- (k)  $P \rightarrow [Q \leftrightarrow (R \rightarrow S)]$  is *false*

### Exercise 7.

Let  $P$  be a sentence such that for any sentence  $Q$  the sentence  $P \vee Q$  is true.

What can be said about the truth value of  $P$ .

Answer:  $P$  is *true*

### Exercise 8.

Let  $P$  be a sentence such that for any sentence  $Q$  the sentence  $P \wedge Q$  is false.

What can be said about the truth value of  $P$ .

Answer:  $P$  is *false*

### Exercise 9.

If  $P \leftrightarrow Q$  is true, what can be said about the truth value of  $P \vee -Q$ ?

Answer:  $P \vee -Q$  is *true*

**Exercise 10.**

- (a)  $P \vee Q$  is **not** a tautology.

$p$	$q$	$p \vee q$
$T$	$T$	$T$
$T$	$F$	$T$
$F$	$T$	$T$
$F$	$F$	$F$

- (b)  $P \vee \neg P$  is a tautology

$p$	$p \vee \neg p$
$T$	$T$
$F$	$T$

- (c)  $P \vee Q \rightarrow Q \vee P$  is a tautology.

$P$	$Q$	$P \vee Q \rightarrow Q \vee P$
$T$	$T$	$T$
$T$	$F$	$T$
$F$	$T$	$T$
$F$	$F$	$T$

- (d)  $P \rightarrow (P \vee Q) \vee R$  is a tautology.

$P$	$Q$	$R$	$P \rightarrow (P \vee Q) \vee R$
$T$	$T$	$T$	$T$
$T$	$T$	$F$	$T$
$T$	$F$	$T$	$T$
$T$	$F$	$F$	$T$
$F$	$F$	$F$	$T$
$F$	$F$	$T$	$T$
$F$	$T$	$F$	$T$
$F$	$T$	$T$	$T$

- (e)  $P \rightarrow (\neg P \rightarrow Q)$  is a tautology.

$P$	$Q$	$P \rightarrow (\neg P \rightarrow Q)$
$T$	$T$	$T$
$T$	$F$	$T$
$F$	$T$	$T$
$F$	$F$	$T$

- (f)  $(P \rightarrow Q) \rightarrow (Q \rightarrow P)$  is **not** a tautology.

$P$	$Q$	$(P \rightarrow Q) \rightarrow (Q \rightarrow P)$
$T$	$T$	$T$
$T$	$F$	$T$
$F$	$T$	$F$
$F$	$F$	$T$

- (g)  $[(P \rightarrow Q) \leftrightarrow Q] \rightarrow P$  is **not** a tautology.

$P$	$Q$	$[(P \rightarrow Q) \leftrightarrow Q] \rightarrow P$
$T$	$T$	$T$
$T$	$F$	$T$
$F$	$T$	$F$
$F$	$F$	$T$

(h)  $P \rightarrow [Q \rightarrow (Q \rightarrow P)]$  is a tautology.

$P$	$Q$	$P \rightarrow [Q \rightarrow (Q \rightarrow P)]$
$T$	$T$	$T$
$T$	$F$	$T$
$F$	$T$	$T$
$F$	$F$	$T$

(i)  $P \wedge Q \rightarrow P \vee R$  is a tautology.

$P$	$Q$	$R$	$P \wedge Q \rightarrow P \vee R$
$T$	$T$	$T$	$T$
$T$	$T$	$F$	$T$
$T$	$F$	$T$	$T$
$T$	$F$	$F$	$T$
$F$	$F$	$F$	$T$
$F$	$F$	$T$	$T$
$F$	$T$	$F$	$T$
$F$	$T$	$T$	$T$

(j)  $[P \vee (-P \wedge Q)] \vee (-P \wedge -Q)$  is a tautology.

$P$	$Q$	$[P \vee (-P \wedge Q)] \vee (-P \wedge -Q)$
$T$	$T$	$T$
$T$	$F$	$T$
$F$	$T$	$T$
$F$	$F$	$T$

(k)  $P \wedge Q \rightarrow (P \leftrightarrow Q \vee R)$  is a tautology.

$P$	$Q$	$R$	$P \wedge Q \rightarrow (P \leftrightarrow Q \vee R)$
$T$	$T$	$T$	$T$
$T$	$T$	$F$	$T$
$T$	$F$	$T$	$T$
$T$	$F$	$F$	$T$
$F$	$F$	$F$	$T$
$F$	$F$	$T$	$T$
$F$	$T$	$F$	$T$
$F$	$T$	$T$	$T$

(l)  $[P \wedge Q \rightarrow (P \wedge -P \rightarrow Q \vee -Q)] \wedge (Q \rightarrow Q)$  is a tautology.

$P$	$Q$	$[P \wedge Q \rightarrow (P \wedge -P \rightarrow Q \vee -Q)] \wedge (Q \rightarrow Q)$
$T$	$T$	$T$
$T$	$F$	$T$
$F$	$T$	$T$
$F$	$F$	$T$

### Exercise 11.

If P and Q are distinct atomic sentences, which of the following are tautologies?

- (a)  $P \leftrightarrow Q$  is **not** a tautology.

$P$	$Q$	$P \leftrightarrow Q$
$T$	$T$	$T$
$T$	$F$	$F$
$F$	$T$	$F$
$F$	$F$	$T$

- (b)  $P \leftrightarrow P \vee P$  is a tautology.

$P$	$P \leftrightarrow P \vee P$
$T$	$T$
$F$	$T$

- (c)  $P \vee Q \leftrightarrow Q \vee P$  is a tautology.

$P$	$Q$	$P \vee Q \leftrightarrow Q \vee P$
$T$	$T$	$T$
$T$	$F$	$T$
$F$	$T$	$T$
$F$	$F$	$T$

- (d)  $(P \rightarrow Q) \leftrightarrow (Q \rightarrow P)$  is **not** a tautology.

$P$	$Q$	$(P \rightarrow Q) \leftrightarrow (Q \rightarrow P)$
$T$	$T$	$T$
$T$	$F$	$F$
$F$	$T$	$F$
$F$	$F$	$T$

- (e)  $(P \leftrightarrow P) \leftrightarrow P$  is a tautology.

$P$	$(P \leftrightarrow P) \leftrightarrow P$
$T$	$T$
$F$	$T$

### Exercise 12.

On the basis of ordinary usage construct truth tables for the sentential connectives used in the following examples:

- (a) Not both P and Q.

$P$	$Q$	$\neg(P \wedge Q)$
$T$	$T$	$F$
$T$	$F$	$T$
$F$	$T$	$T$
$F$	$F$	$T$

(b) Neither P nor Q.

$P$	$Q$	$-(P \vee Q)$
$T$	$T$	$F$
$T$	$F$	$F$
$F$	$T$	$F$
$F$	$F$	$T$

### Exercise 13.

Give examples of sentences P and Q (not necessarily atomic) such that the following compound sentences are tautologies.

(a)  $P \wedge Q$  is **not** a tautology.

$P$	$Q$	$P \wedge Q$	$(P_1 \vee -P_2) \wedge (Q_1 \vee -Q_2)$
$T$	$T$	$T$	$T$
$T$	$F$	$F$	$T$
$F$	$T$	$F$	$T$
$F$	$F$	$F$	$T$

Changing atomic sentences  $P$  and  $Q$  into compound sentences  $P_1 \vee -P_2$  and  $Q_1 \vee -Q_2$  makes the whole sentence tautology.