Introduction to Logic, Part I, Chapter I by Patrick Suppes - exercises

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Exercise 4.

In the following examples determine the truth value of the compound sentences from the given truth values of the component sentences.

(i) "Galileo was born before Descartes" is true.
(ii) "Descartes was born in the sixteenth century" is true.
(iii) "Newton was born before Shakespeare" is false.
(iv) "Racine was a compatriot of Galileo" is false.

(a) If Galileo was born before Descartes, then Newton was not born before Shakespeare.
Answer: $true \to \neg false$ is $true$
(b) If either Racine was a compatriot of Galileo or Newton was born before Shakespeare, then Descartes was born in the sixteenth century. Answer: $(false \vee false) \to true$ is $true$
(c) If Racine was not a compatriot of Galileo, then either Descartes was not born in the sixteenth century or Newton was born before Shakespeare. Answer: $\neg false \to (\neg true \vee false)$ is $false$

Exercise 5.

Let
$N =$ New York is larger than Chicago
$W =$ New York is north of Washington
$C =$ Chicago is larger than New York
N, W are true and C is false.
Which of the following sentences are true?

(a) $N \vee C$ is $true$
(b) $N \wedge C$ is $false$
(c) $-N \wedge -C$ is $false$
(d) $N \leftrightarrow -W \vee C$ is $false$
(e) $W \vee -C \to N$ is $true$

(f) $(W \vee N) \rightarrow (W \rightarrow -C)$ is $true$
(g) $(W \leftrightarrow -N) \leftrightarrow (N \leftrightarrow C)$ is $true$
(h) $(W \rightarrow N) \rightarrow [(N \rightarrow -C) \rightarrow (-C \rightarrow W)]$ is $true$

Exercise 6.

Let
$P =$ Jane Austen was contemporary of Beethoven
$Q =$ Beethoven was a contemporary of Gauss
$R =$ Gauss was a contemporary of Napoleon
$S =$ Napoleon was a contemporary of Julius Caesar
P, Q, and R are true, and S is false.
Find the truth values of the following sentences:

(a) $(P \wedge Q) \wedge R$ is $true$
(b) $P \wedge (Q \wedge R)$ is $true$
(c) $S \rightarrow P$ is $true$
(d) $P \rightarrow S$ is $false$
(e) $(P \wedge Q) \wedge (R \wedge S)$ is $false$
(f) $P \wedge Q \leftrightarrow R \wedge -S$ is $true$
(g) $(P \leftrightarrow Q) \rightarrow (S \leftrightarrow R)$ is $false$
(h) $(-P \leftarrow Q) \leftarrow (S \leftarrow R)$ is $true$
(i) $(P \rightarrow -Q) \rightarrow (S \leftrightarrow R)$ is $true$
(j) $(P \rightarrow Q)[(Q \rightarrow R) \rightarrow (R \rightarrow S)]$ is $false$
(k) $P \rightarrow [Q \leftrightarrow (R \rightarrow S)]$ is $false$

Exercise 7.

Let P be a sentence such that for any sentence Q the sentence $P \vee Q$ is true.
What can be said about the truth value of P.
Answer: P is $true$

Exercise 8.

Let P be a sentence such that for any sentence Q the sentence $P \wedge Q$ is false.
What can be said about the truth value of P.
Answer: P is $false$

Exercise 9.

If $P \leftrightarrow Q$ is true, what can be said about the truth value of $P \vee -Q$?
Answer: $P \vee -Q$ is $true$

Exercise 10.

(a) $P \vee Q$ is **not** a tautology.

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

(b) $P \vee -P$ is a tautology

p	$p \vee -p$
T	T
F	T

(c) $P \vee Q \to Q \vee P$ is a tautology.

P	Q	$P \vee Q \to Q \vee P$
T	T	T
T	F	T
F	T	T
F	F	T

(d) $P \to (P \vee Q) \vee R$ is a tautology.

P	Q	R	$P \to (P \vee Q) \vee R$
T	T	T	T
T	T	F	T
T	F	T	T
T	F	F	T
F	F	F	T
F	F	T	T
F	T	F	T
F	T	T	T

(e) $P \to (-P \to Q)$ is a tautology.

P	Q	$P \to (-P \to Q)$
T	T	T
T	F	T
F	T	T
F	F	T

(f) $(P \to Q) \to (Q \to P)$ is **not** a tautology.

P	Q	$P \to Q) \to (Q \to P$
T	T	T
T	F	T
F	T	F
F	F	T

(g) $[(P \to Q) \leftrightarrow Q] \to P$ is **not** a tautology.

P	Q	$[(P \to Q) \leftrightarrow Q] \to P$
T	T	T
T	F	T
F	T	F
F	F	T

(h) $P \to [Q \to (Q \to P)]$ is a tautology.

P	Q	$P \to [Q \to (Q \to P)]$
T	T	T
T	F	T
F	T	T
F	F	T

(i) $P \wedge Q \to P \vee R$ is a tautology.

P	Q	R	$P \wedge Q \to P \vee R$
T	T	T	T
T	T	F	T
T	F	T	T
T	F	F	T
F	F	F	T
F	F	T	T
F	T	F	T
F	T	T	T

(j) $[P \vee (-P \wedge Q)] \vee (-P \wedge -Q)$ is a tautology.

P	Q	$[P \vee (-P \wedge Q)] \vee (-P \wedge -Q)$
T	T	T
T	F	T
F	T	T
F	F	T

(k) $P \wedge Q \to (P \leftrightarrow Q \vee R)$ is a tautology.

P	Q	R	$P \wedge Q \to (P \leftrightarrow Q \vee R)$
T	T	T	T
T	T	F	T
T	F	T	T
T	F	F	T
F	F	F	T
F	F	T	T
F	T	F	T
F	T	T	T

(l) $[P \wedge Q \to (P \wedge -P \to Q \vee -Q)] \wedge (Q \to Q)$ is a tautology.

P	Q	$[P \wedge Q \to (P \wedge -P \to Q \vee -Q)] \wedge (Q \to Q)$
T	T	T
T	F	T
F	T	T
F	F	T

Exercise 11.

If P and Q are distinct atomic sentences, which of the following are tautologies?

(a) $P \leftrightarrow Q$ is **not** a tautology.

P	Q	$P \leftrightarrow Q$
T	T	T
T	F	F
F	T	F
F	F	T

(b) $P \leftrightarrow P \vee P$ is a tautology.

P	$P \leftrightarrow P \vee P$
T	T
F	T

(c) $P \vee Q \leftrightarrow Q \vee P$ is a tautology.

P	Q	$P \vee Q \leftrightarrow Q \vee P$
T	T	T
T	F	T
F	T	T
F	F	T

(d) $(P \to Q) \leftrightarrow (Q \to P)$ is **not** a tautology.

P	Q	$(P \to Q) \leftrightarrow (Q \to P)$
T	T	T
T	F	F
F	T	F
F	F	T

(e) $(P \leftrightarrow P) \leftrightarrow P$ is a tautology.

P	$(P \leftrightarrow P) \leftrightarrow P$
T	T
F	T

Exercise 12.

On the basis of ordinary usage construct truth tables for the sentential connectives used in the following examples:

(a) Not both P and Q.

P	Q	$-(P \wedge Q)$
T	T	F
T	F	T
F	T	T
F	F	T

(b) Neither P nor Q.

P	Q	$-(P \vee Q)$
T	T	F
T	F	F
F	T	F
F	F	T

Exercise 13.

Give examples of sentences P and Q (not necessarily atomic) such that the following compound sentences are tautologies.

(a) Sentence $W = P \wedge Q$ is **not** a tautology. Assumption $P = P \vee -P$ and $Q = Q \vee -Q$ changes W into a tautology.

P	Q	$P \wedge Q$	$(P \vee -P) \wedge (Q \vee -Q)$
T	T	T	T
T	F	F	T
F	T	F	T
F	F	F	T

(b) Sentence $W = P \vee (P \wedge -Q)$ is **not** a tautology. Assumption $P = P \vee -P$ changes W into a tautology.

P	Q	$P \vee (P \wedge -Q)$	$(P \vee -P) \vee (Q \wedge -Q)$
T	T	T	T
T	F	T	T
F	T	F	T
F	F	F	T

(c) Sentence $W = P \to P \wedge -Q$ is **not** a tautology. Assumption $Q = -P$ changes W into a tautology.

P	Q	$P \to P \wedge -Q$	$P \to P \wedge -(-P)$
T	T	F	T
T	F	T	T
F	T	T	T
F	F	T	T

(d) Sentence $W = P \to -P$ is **not** a tautology. Assumption $P = -(P \vee -P)$ changes W into a tautology.

P	$P \to -P$	$-(P \vee -P) \to -(-(P \vee -P))$
T	F	T
F	T	T

Exercise 14.

Is there any sentence P such that $P \wedge -P$ is a tautology?
Answer: No such sentence exists.

Exercise 15.

If P and Q are distinct atomic sentences, the sentence $P \wedge Q$ tautologically implies which of the following?

(a) P

P	Q	$P \wedge Q \to P$
T	T	T
T	F	T
F	T	T
F	F	T

Answer: $P \wedge Q$ tautologically implies P.

(b) Q

P	Q	$P \wedge Q \to Q$
T	T	T
T	F	T
F	T	T
F	F	T

Answer: $P \wedge Q$ tautologically implies Q.

(c) $P \vee Q$

P	Q	$P \wedge Q \to P \vee Q$
T	T	T
T	F	T
F	T	T
F	F	T

Answer: $P \wedge Q$ tautologically implies $P \vee Q$.

(d) $P \wedge -Q$

P	Q	$P \wedge Q \to P \wedge -Q$
T	T	F
T	F	T
F	T	T
F	F	T

Answer: $P \wedge Q$ **does not** tautologically imply $P \wedge -Q$.

(e) $-P \vee Q$

P	Q	$P \wedge Q \to -P \vee Q$
T	T	T
T	F	T
F	T	T
F	F	T

Answer: $P \wedge Q$ tautologically implies $-P \vee Q$.

(f) $-Q \to P$

P	Q	$P \wedge Q \to (-Q \to P)$
T	T	T
T	F	T
F	T	T
F	F	T

Answer: $P \wedge Q$ tautologically implies $-Q \to P$.

(g) $P \leftrightarrow Q$

P	Q	$P \wedge Q \to (P \leftrightarrow Q)$
T	T	T
T	F	T
F	T	T
F	F	T

Answer: $P \wedge Q$ tautologically implies $P \leftrightarrow Q$.

Exercise 16.

If P and Q are distinct atomic sentences, the sentence $-P \vee Q$ tautologically implies which of the following?

(a) P

P	Q	$-P \vee Q \to P$
T	T	T
T	F	T
F	T	F
F	F	F

Answer: $-P \vee Q$ **does not** tautologically imply P.

(b) $Q \to P$

P	Q	$-P \vee Q \to (Q \to P)$
T	T	T
T	F	T
F	T	F
F	F	T

Answer: $-P \vee Q$ **does not** tautologically imply $Q \to P$.

(c) $P \to Q$

P	Q	$-P \vee Q \rightarrow (P \rightarrow Q)$
T	T	T
T	F	T
F	T	T
F	F	T

Answer: $-P \vee Q$ tautologically implies $P \rightarrow Q$.

(d) $-Q \rightarrow -P$

P	Q	$-P \vee Q \rightarrow (-Q \rightarrow -P)$
T	T	T
T	F	T
F	T	T
F	F	T

Answer: $-P \vee Q$ tautologically implies $-Q \rightarrow -P$.

(e) $-P \wedge Q$

P	Q	$-P \vee Q \rightarrow -P \wedge Q$
T	T	F
T	F	T
F	T	T
F	F	F

Answer: $-P \vee Q$ **does not** tautologically imply $-P \wedge Q$.

Exercise 17.

If P and Q are distinct atomic sentences, the sentence P is tautologically equivalent to which of the following?

(a) $P \vee Q$

P	Q	$P \leftrightarrow P \vee Q$
T	T	T
T	F	T
F	T	F
F	F	T

Answer: P is **not** tautologically equivalent to $P \vee Q$.

(b) $P \vee -P$

P	$P \leftrightarrow P \vee -P$
T	T
F	F

Answer: P is **not** tautologically equivalent to $P \vee -P$.

(c) $P \wedge P$

P	$P \leftrightarrow P \wedge P$
T	T
F	T

Answer: P is tautologically equivalent to $P \wedge P$.

(d) $P \rightarrow P$

P	$P \leftrightarrow P \rightarrow P$
T	T
F	F

Answer: P is **not** tautologically equivalent to $P \rightarrow P$.

(e) $-P \rightarrow P$

P	$P \leftrightarrow -P \rightarrow P$
T	T
F	T

Answer: P is tautologically equivalent to $-P \rightarrow P$.

(f) $P \rightarrow -P$

P	$P \leftrightarrow P \rightarrow -P$
T	F
F	F

Answer: P is **not** tautologically equivalent to $P \rightarrow -P$.

(g) $Q \vee -Q \rightarrow P$

P	Q	$P \leftrightarrow (Q \vee -Q \rightarrow P)$
T	T	T
T	F	T
F	T	T
F	F	T

Answer: P is tautologically equivalent to $Q \vee -Q \rightarrow P$.