

Introduction to Logic, Part I, Chapter I by Patrick Suppes - exercises

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Exercise 2.

Which of the truth-functional connectives introduced in this chapter is an approximate synonym of the connective "unless"? (Hint: To say "There will be peace unless there is a major war in the next five years, then there will be peace" is equivalent to saying "If there is not a major war in the next five years, then there will be peace".)

Answer: Negation of implication

Exercise 3.

Translate the following compound sentences into symbolic notation, using letters to stand for atomic sentences.

- (a) "Either the fire was produced by arson or it was produced by spontaneous combustion."

M = "the fire was produced by arson",

L = "it was produced by spontaneous combustion"

$$M \vee L$$

- (b) "If the water is clear, then either Henry can see the bottom of the pool or he is a nincompoop."

M = "the water is clear",

L = "Henry can see the bottom of the pool",

S = "he is a nincompoop"

$$M \rightarrow L \vee S$$

- (c) "Either John is not here or Mary is, and Helen certainly is."

M = "John is here",

L = "Mary is here",

S = "Helen certainly is here"

$$\neg M \vee L \wedge S$$

- (d) "If there are more cats than dogs, then there are more horses than dogs and there are fewer snakes than cats."
 M = "there are more cats than dogs",
 L = "there are more horses than dogs",
 S = "there are fewer snakes than cats"

$$M \rightarrow (L \wedge S)$$

- (e) "The man in the moon is a fake, and if the same is true of Santa Claus, many children are deceived."
 M = "The man in the moon is a fake",
 L = "the same is true of Santa Claus",
 S = "many children are deceived"

$$M \wedge (L \rightarrow S)$$

- (f) "If either red-heads are lovely or blondes do not have freckles, then logic is confusing."
 M = "red-heads are lovely",
 L = "blondes do not have freckles",
 S = "logic is confusing"

$$(M \vee L) \rightarrow S$$

- (g) "If either housing is scarce or people like to live with their in-laws, and if people do not like to live with their in-laws, then housing is scarce."
 M = "housing is scarce",
 L = "people like to live with their in-laws",

$$M \vee L \wedge \neg L \rightarrow M$$

- (h) "If John testifies and tells the truth, he will be found guilty; and if he does not testify, he will be found guilty."
 M = "John testifies",
 L = "John tells the truth",
 S = "he will be found guilty"

$$(M \wedge L \rightarrow S) \wedge (\neg M \rightarrow S)$$

- (i) "Either John must testify and tell the truth, or he does not have to testify."
 M = "John must testify",
 L = "John must tell the truth",

$$(M \wedge L) \vee \neg M$$

Exercise 4.

In the following examples determine the truth value of the compound sentences from the given truth values of the component sentences.

- (i) "Galileo was born before Descartes" is true.
 - (ii) "Descartes was born in the sixteenth century" is true.
 - (iii) "Newton was born before Shakespeare" is false.
 - (iv) "Racine was a compatriot of Galileo" is false.
- (a) If Galileo was born before Descartes, then Newton was not born before Shakespeare.
Answer: $true \rightarrow \neg false$ is *true*
 - (b) If either Racine was a compatriot of Galileo or Newton was born before Shakespeare, then Descartes was born in the sixteenth century. Answer:
 $(false \vee false) \rightarrow true$ is *true*
 - (c) If Racine was not a compatriot of Galileo, then either Descartes was not born in the sixteenth century or Newton was born before Shakespeare.
Answer: $\neg false \rightarrow (\neg true \vee false)$ is *false*

Exercise 5.

Let

N = New York is larger than Chicago

W = New York is north of Washington

C = Chicago is larger than New York

N , W are true and C is false.

Which of the following sentences are true?

- (a) $N \vee C$ is *true*
- (b) $N \wedge C$ is *false*
- (c) $\neg N \wedge \neg C$ is *false*
- (d) $N \leftrightarrow \neg W \vee C$ is *false*
- (e) $W \vee \neg C \rightarrow N$ is *true*
- (f) $(W \vee N) \rightarrow (W \rightarrow \neg C)$ is *true*
- (g) $(W \leftrightarrow \neg N) \leftrightarrow (N \leftrightarrow C)$ is *true*
- (h) $(W \rightarrow N) \rightarrow [(N \rightarrow \neg C) \rightarrow (\neg C \rightarrow W)]$ is *true*

Exercise 6.

Let

P = Jane Austen was contemporary of Beethoven

Q = Beethoven was a contemporary of Gauss

R = Gauss was a contemporary of Napoleon

S = Napoleon was a contemporary of Julius Caesar

P , Q , and R are true, and S is false.

Find the truth values of the following sentences:

- (a) $(P \wedge Q) \wedge R$ is *true*
- (b) $P \wedge (Q \wedge R)$ is *true*
- (c) $S \rightarrow P$ is *true*
- (d) $P \rightarrow S$ is *false*
- (e) $(P \wedge Q) \wedge (R \wedge S)$ is *false*
- (f) $P \wedge Q \leftrightarrow R \wedge \neg S$ is *true*
- (g) $(P \leftrightarrow Q) \rightarrow (S \leftrightarrow R)$ is *false*
- (h) $(\neg P \leftarrow Q) \leftarrow (S \leftarrow R)$ is *true*
- (i) $(P \rightarrow \neg Q) \rightarrow (S \leftrightarrow R)$ is *true*
- (j) $(P \rightarrow Q)[(Q \rightarrow R) \rightarrow (R \rightarrow S)]$ is *false*
- (k) $P \rightarrow [Q \leftrightarrow (R \rightarrow S)]$ is *false*

Exercise 7.

Let P be a sentence such that for any sentence Q the sentence $P \vee Q$ is true.
What can be said about the truth value of P .

Answer: P is *true*

Exercise 8.

Let P be a sentence such that for any sentence Q the sentence $P \wedge Q$ is false.
What can be said about the truth value of P .

Answer: P is *false*

Exercise 9.

If $P \leftrightarrow Q$ is true, what can be said about the truth value of $P \vee \neg Q$?

Answer: $P \vee \neg Q$ is *true*

Exercise 10.

- (a) $P \vee Q$ is **not** a tautology.

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

- (b) $P \vee \neg P$ is a tautology

p	$p \vee \neg p$
T	T
F	T

(c) $P \vee Q \rightarrow Q \vee P$ is a tautology.

P	Q	$P \vee Q \rightarrow Q \vee P$
T	T	T
T	F	T
F	T	T
F	F	T

(d) $P \rightarrow (P \vee Q) \vee R$ is a tautology.

P	Q	R	$P \rightarrow (P \vee Q) \vee R$
T	T	T	T
T	T	F	T
T	F	T	T
T	F	F	T
F	F	F	T
F	F	T	T
F	T	F	T
F	T	T	T

(e) $P \rightarrow (-P \rightarrow Q)$ is a tautology.

P	Q	$P \rightarrow (-P \rightarrow Q)$
T	T	T
T	F	T
F	T	T
F	F	T

(f) $(P \rightarrow Q) \rightarrow (Q \rightarrow P)$ is **not** a tautology.

P	Q	$(P \rightarrow Q) \rightarrow (Q \rightarrow P)$
T	T	T
T	F	T
F	T	F
F	F	T

(g) $[(P \rightarrow Q) \leftrightarrow Q] \rightarrow P$ is **not** a tautology.

P	Q	$[(P \rightarrow Q) \leftrightarrow Q] \rightarrow P$
T	T	T
T	F	T
F	T	F
F	F	T

(h) $P \rightarrow [Q \rightarrow (Q \rightarrow P)]$ is a tautology.

P	Q	$P \rightarrow [Q \rightarrow (Q \rightarrow P)]$
T	T	T
T	F	T
F	T	T
F	F	T

(i) $P \wedge Q \rightarrow P \vee R$ is a tautology.

P	Q	R	$P \wedge Q \rightarrow P \vee R$
T	T	T	T
T	T	F	T
T	F	T	T
T	F	F	T
F	F	F	T
F	F	T	T
F	T	F	T
F	T	T	T

- (j) $[P \vee (-P \wedge Q)] \vee (-P \wedge -Q)$ is a tautology.

P	Q	$[P \vee (-P \wedge Q)] \vee (-P \wedge -Q)$
T	T	T
T	F	T
F	T	T
F	F	T

- (k) $P \wedge Q \rightarrow (P \leftrightarrow Q \vee R)$ is a tautology.

P	Q	R	$P \wedge Q \rightarrow (P \leftrightarrow Q \vee R)$
T	T	T	T
T	T	F	T
T	F	T	T
T	F	F	T
F	F	F	T
F	F	T	T
F	T	F	T
F	T	T	T

- (l) $[P \wedge Q \rightarrow (P \wedge -P \rightarrow Q \vee -Q)] \wedge (Q \rightarrow Q)$ is a tautology.

P	Q	$[P \wedge Q \rightarrow (P \wedge -P \rightarrow Q \vee -Q)] \wedge (Q \rightarrow Q)$
T	T	T
T	F	T
F	T	T
F	F	T

Exercise 11.

If P and Q are distinct atomic sentences, which of the following are tautologies?

- (a) $P \leftrightarrow Q$ is **not** a tautology.

P	Q	$P \leftrightarrow Q$
T	T	T
T	F	F
F	T	F
F	F	T

- (b) $P \leftrightarrow P \vee P$ is a tautology.

P	$P \leftrightarrow P \vee P$
T	T
F	T

- (c) $P \vee Q \leftrightarrow Q \vee P$ is a tautology.

P	Q	$P \vee Q \leftrightarrow Q \vee P$
T	T	T
T	F	T
F	T	T
F	F	T

- (d) $(P \rightarrow Q) \leftrightarrow (Q \rightarrow P)$ is **not** a tautology.

P	Q	$(P \rightarrow Q) \leftrightarrow (Q \rightarrow P)$
T	T	T
T	F	F
F	T	F
F	F	T

- (e) $(P \leftrightarrow P) \leftrightarrow P$ is a tautology.

P	$(P \leftrightarrow P) \leftrightarrow P$
T	T
F	T

Exercise 12.

On the basis of ordinary usage construct truth tables for the sentential connectives used in the following examples:

- (a) Not both P and Q.

P	Q	$\neg(P \wedge Q)$
T	T	F
T	F	T
F	T	T
F	F	T

- (b) Neither P nor Q.

P	Q	$\neg(P \vee Q)$
T	T	F
T	F	F
F	T	F
F	F	T

Exercise 13.

Give examples of sentences P and Q (not necessarily atomic) such that the following compound sentences are tautologies.

- (a) Sentence $W = P \wedge Q$ is **not** a tautology. Assumption $P = P \vee -P$ and $Q = Q \vee -Q$ changes W into a tautology.

P	Q	$P \wedge Q$	$(P \vee -P) \wedge (Q \vee -Q)$
T	T	T	T
T	F	F	T
F	T	F	T
F	F	F	T

- (b) Sentence $W = P \vee (P \wedge -Q)$ is **not** a tautology. Assumption $P = P \vee -P$ changes W into a tautology.

P	Q	$P \vee (P \wedge -Q)$	$(P \vee -P) \vee (Q \wedge -Q)$
T	T	T	T
T	F	T	T
F	T	F	T
F	F	F	T

- (c) Sentence $W = P \rightarrow P \wedge -Q$ is **not** a tautology. Assumption $Q = -P$ changes W into a tautology.

P	Q	$P \rightarrow P \wedge -Q$	$P \rightarrow P \wedge -(-P)$
T	T	F	T
T	F	T	T
F	T	T	T
F	F	T	T

- (d) Sentence $W = P \rightarrow -P$ is **not** a tautology. Assumption $P = -(P \vee -P)$ changes W into a tautology.

P	$P \rightarrow -P$	$-(P \vee -P) \rightarrow -(-(P \vee -P))$
T	F	T
F	T	T

Exercise 14.

Is there any sentence P such that $P \wedge -P$ is a tautology?

Answer: No such sentence exists.

Exercise 15.

If P and Q are distinct atomic sentences, the sentence $P \wedge Q$ tautologically implies which of the following?

- (a) P

P	Q	$P \wedge Q \rightarrow P$
T	T	T
T	F	T
F	T	T
F	F	T

Answer: $P \wedge Q$ tautologically implies P .

(b) Q

P	Q	$P \wedge Q \rightarrow Q$
T	T	T
T	F	T
F	T	T
F	F	T

Answer: $P \wedge Q$ tautologically implies Q .

(c) $P \vee Q$

P	Q	$P \wedge Q \rightarrow P \vee Q$
T	T	T
T	F	T
F	T	T
F	F	T

Answer: $P \wedge Q$ tautologically implies $P \vee Q$.

(d) $P \wedge \neg Q$

P	Q	$P \wedge Q \rightarrow P \wedge \neg Q$
T	T	F
T	F	T
F	T	T
F	F	T

Answer: $P \wedge Q$ **does not** tautologically imply $P \wedge \neg Q$.

(e) $\neg P \vee Q$

P	Q	$P \wedge Q \rightarrow \neg P \vee Q$
T	T	T
T	F	T
F	T	T
F	F	T

Answer: $P \wedge Q$ tautologically implies $\neg P \vee Q$.

(f) $\neg Q \rightarrow P$

P	Q	$P \wedge Q \rightarrow (\neg Q \rightarrow P)$
T	T	T
T	F	T
F	T	T
F	F	T

Answer: $P \wedge Q$ tautologically implies $\neg Q \rightarrow P$.

(g) $P \leftrightarrow Q$

P	Q	$P \wedge Q \rightarrow (P \leftrightarrow Q)$
T	T	T
T	F	T
F	T	T
F	F	T

Answer: $P \wedge Q$ tautologically implies $P \leftrightarrow Q$.

Exercise 16.

If P and Q are distinct atomic sentences, the sentence $\neg P \vee Q$ tautologically implies which of the following?

(a) P

P	Q	$\neg P \vee Q \rightarrow P$
T	T	T
T	F	T
F	T	F
F	F	F

Answer: $\neg P \vee Q$ **does not** tautologically imply P .

(b) $Q \rightarrow P$

P	Q	$\neg P \vee Q \rightarrow (Q \rightarrow P)$
T	T	T
T	F	T
F	T	F
F	F	T

Answer: $\neg P \vee Q$ **does not** tautologically imply $Q \rightarrow P$.

(c) $P \rightarrow Q$

P	Q	$\neg P \vee Q \rightarrow (P \rightarrow Q)$
T	T	T
T	F	T
F	T	T
F	F	T

Answer: $\neg P \vee Q$ tautologically implies $P \rightarrow Q$.

(d) $\neg Q \rightarrow \neg P$

P	Q	$\neg P \vee Q \rightarrow (\neg Q \rightarrow \neg P)$
T	T	T
T	F	T
F	T	T
F	F	T

Answer: $\neg P \vee Q$ tautologically implies $\neg Q \rightarrow \neg P$.

(e) $\neg P \wedge Q$

P	Q	$\neg P \vee Q \rightarrow \neg P \wedge Q$
T	T	F
T	F	T
F	T	T
F	F	F

Answer: $\neg P \vee Q$ **does not** tautologically imply $\neg P \wedge Q$.

Exercise 17.

If P and Q are distinct atomic sentences, the sentence P is tautologically equivalent to which of the following?

(a) $P \vee Q$

P	Q	$P \leftrightarrow P \vee Q$
T	T	T
T	F	T
F	T	F
F	F	T

Answer: P is **not** tautologically equivalent to $P \vee Q$.

(b) $P \vee \neg P$

P	$P \leftrightarrow P \vee \neg P$
T	T
F	T

Answer: P is **not** tautologically equivalent to $P \vee \neg P$.

(c) $P \wedge P$

P	$P \leftrightarrow P \wedge P$
T	T
F	F

Answer: P is tautologically equivalent to $P \wedge P$.

(d) $P \rightarrow P$

P	$P \leftrightarrow P \rightarrow P$
T	T
F	T

Answer: P is **not** tautologically equivalent to $P \rightarrow P$.

(e) $\neg P \rightarrow P$

P	$P \leftrightarrow \neg P \rightarrow P$
T	T
F	T

Answer: P is tautologically equivalent to $\neg P \rightarrow P$.

(f) $P \rightarrow \neg P$

P	$P \leftrightarrow P \rightarrow \neg P$
T	F
F	F

Answer: P is **not** tautologically equivalent to $P \rightarrow \neg P$.

(g) $Q \vee \neg Q \rightarrow P$

P	Q	$P \leftrightarrow (Q \vee \neg Q \rightarrow P)$
T	T	T
T	F	T
F	T	T
F	F	T

Answer: P is tautologically equivalent to $Q \vee \neg Q \rightarrow P$.