Introduction to Logic, Part II, Chapter 9 by Patrick Suppes - notes and exercises

Dominik Lenda

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1 Notes

Principle of extensionality - in axiomatic set theory
$\forall A \forall B(\forall X(X \in A \leftrightarrow X \in B) \rightarrow A = B)$

$\{1, 3, 5\} = \{5, 3, 1\}$

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{Elizabeth II} $\neq$ Elizabeth II

Important difference between $A = A$ and $A \in A$ - the former is always true, while the latter is usually false. Standard systems of axiomatic set theory assert that set cannot be member of itself.
The relation of membership is not symmetric and transitive: $2 \in \{1, 2\}$ $\{1, 2\} \notin 2$

Exercise 1.

Which of the following statements are true (for all sets A, B, and C)?

(a) If $A = B$ and $B = C$, then $A = C$. It is *true* because the relation of identity is transitive.
(b) If $A \in B$ and $B \in C$, then $A \in C$. It is *false* because the relation of membership is **not** transitive.
(c) If $A \subseteq B$ and $B \subseteq C$, then $A \subseteq C$. It is *true* since the relation of inclusion is transitive.