

Introduction to Logic, P. Suppes - exercises

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Exercise 4.

In the following examples determine the truth value of the compound sentences from the given truth values of the component sentences.

- (i) "Galileo was born before Descartes" is true.
- (ii) "Descartes was born in the sixteenth century" is true.
- (iii) "Newton was born before Shakespeare" is false.
- (iv) "Racine was a compatriot of Galileo" is false.
- (a) If Galileo was born before Descartes, then Newton was not born before Shakespeare.
Answer: $true \rightarrow \neg false$ is *true*
- (b) If either Racine was a compatriot of Galileo or Newton was born before Shakespeare, then Descartes was born in the sixteenth century. Answer:
 $(false \vee false) \rightarrow true$ is *true*
- (c) If Racine was not a compatriot of Galileo, then either Descartes was not born in the sixteenth century or Newton was born before Shakespeare.
Answer: $\neg false \rightarrow (\neg true \vee false)$ is *false*

Exercise 5.

Let

N = New York is larger than Chicago

W = New York is north of Washington

C = Chicago is larger than New York

N , W are true and C is false.

Which of the following sentences are true?

- (a) $N \vee C$ is *true*
- (b) $N \wedge C$ is *false*
- (c) $\neg N \wedge \neg C$ is *false*
- (d) $N \leftrightarrow \neg W \vee C$ is *false*
- (e) $W \vee \neg C \rightarrow N$ is *true*
- (f) $(W \vee N) \rightarrow (W \rightarrow \neg C)$ is *true*
- (g) $(W \leftrightarrow \neg N) \leftrightarrow (N \leftrightarrow C)$ is *true*
- (h) $(W \rightarrow N) \rightarrow [(N \rightarrow \neg C) \rightarrow (\neg C \rightarrow W)]$ is *true*

Exercise 6.

Let

P = Jane Austen was contemporary of Beethoven

Q = Beethoven was a contemporary of Gauss

R = Gauss was a contemporary of Napoleon

S = Napoleon was a contemporary of Julius Caesar

P , Q , and R are true, and S is false.

Find the truth values of the following sentences:

- (a) $(P \wedge Q) \wedge R$ is *true*
- (b) $P \wedge (Q \wedge R)$ is *true*
- (c) $S \rightarrow P$ is *true*
- (d) $P \rightarrow S$ is *false*
- (e) $(P \wedge Q) \wedge (R \wedge S)$ is *false*
- (f) $P \wedge Q \leftrightarrow R \wedge \neg S$ is *true*
- (g) $(P \leftrightarrow Q) \rightarrow (S \leftrightarrow R)$ is *false*
- (h) $(\neg P \leftrightarrow Q) \leftrightarrow (S \leftrightarrow R)$ is *true*
- (i) $(P \rightarrow \neg Q) \rightarrow (S \leftrightarrow R)$ is *true*
- (j) $(P \rightarrow Q)[(Q \rightarrow R) \rightarrow (R \rightarrow S)]$ is *false*
- (k) $P \rightarrow [Q \leftrightarrow (R \rightarrow S)]$ is *false*

Exercise 7.

Let P be a sentence such that for any sentence Q the sentence $P \vee Q$ is true.

What can be said about the truth value of P .

Answer: P is *true*

Exercise 8.

Let P be a sentence such that for any sentence Q the sentence $P \wedge Q$ is false.

What can be said about the truth value of P .

Answer: P is *false*

Exercise 9.

If $P \leftrightarrow Q$ is true, what can be said about the truth value of $P \vee \neg Q$?

Answer: $P \vee \neg Q$ is *true*