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$$a_n = \frac{1}{\pi}\int_0^\pi |\sin(x)|\cos(nx)\,dx \qquad \begin{matrix}\alpha = x & \alpha-\beta = x(1-n)\\ \beta = nx & \alpha+\beta = x(1+n)\end{matrix}$$

$$= \frac{1}{\pi}\int_0^\pi \frac{1}{2}\left(\sin(x-nx)+\sin(x+nx)\right)$$

$$= \frac{1}{2\pi}\int_0^\pi \left(-\sin((n-1)x)+\sin((n+1)x)\right)$$

$$= \frac{1}{2\pi}\left[\frac{\cos((n-1)x)}{(n-1)} - \frac{\cos((n+1)x)}{(n+1)}\right]_0^\pi$$

$$= \frac{1}{2\pi}\left(\frac{\cos((n-1)\pi)}{(n-1)} - \frac{\cos((n+1)\pi)}{(n+1)} - \frac{1}{(n-1)} + \frac{1}{(n+1)}\right)$$

$$= \frac{1}{2\pi}\left(\frac{(-1)^n}{(n-1)} - \frac{(-1)^n}{(n+1)} - \frac{1}{(n-1)} + \frac{1}{(n+1)}\right)$$

$$= \frac{1}{2\pi(n-1)(n+1)}\left((-1)^n(n+1) - (-1)^n(n-1) - (n+1) + (n-1)\right)$$

$$= \frac{1}{2\pi}\left(\cancel{(-1)^n\cdot n} + (-1)^n - \cancel{(-1)^n\cdot n} + (-1)^n - \cancel{n} - 1 + \cancel{n} - 1\right)$$

$$= \frac{1}{2\pi}\left(2(-1)^n - 2\right)$$

$$= \frac{1}{\pi}\left((-1)^n - 1\right)$$

Oppgave 3

$$u_t(x,t) = c^2 u_{xx}(x,t) \qquad t \geq 0 \qquad x \in [0,\pi]$$

$$u(0,t) = u(\pi,t) = 0 \qquad t > 0$$

$$u(x,0) = \frac{\pi}{2} - \left|x - \frac{\pi}{2}\right| \qquad x \in [0,\pi]$$

c) Her har du en funksjon på s og må spalte den opp og få til en form på t

$$F(s) = \frac{1}{s^2+2s+17}$$

$$= \frac{1}{(s+1)^2+4^2}$$

$$\Rightarrow f(t) = e^{-t}\,\frac{\sin(4t)}{4}$$

Oppgave 2

a) Vi ser at de spør om Fourier series, altså f~, så vi må se hvis den er "even" eller "odd". Her så spør de ikke å bruke den spesifikke formelen så vi finner c_n, og husk å forkorte og bruke sin og cos reglene. Den er også ikke odde

$$f(x) = 1+x$$

$$= \frac{1}{2\pi}\int_{-\pi}^{\pi}(1+x)e^{-inx} = \frac{1}{2\pi}\left[\int_{-\pi}^{\pi} e^{-inx} + \int_{-\pi}^{\pi} x e^{-inx}\right] = \frac{1}{2\pi}\left[\frac{e^{-inx}}{-kx}\Big|_{-\pi}^{\pi} + \frac{xe^{-ikx}}{-in}\Big|_{-\pi}^{\pi} - \int_{-\pi}^{\pi}\frac{e^{-inx}}{-in}\right]$$

$$= \frac{1}{2\pi}\left[\frac{e^{-inx}}{-in}\Big|_{-\pi}^{\pi} + \frac{xe^{-inx}}{-in}\Big|_{-\pi}^{\pi} - \frac{e^{-inx}}{(in)^2}\Big|_{-\pi}^{\pi}\right]$$

$$= \frac{1}{2\pi}\left[-\frac{e^{-in\pi}}{in} + \frac{e^{in\pi}}{in} - \frac{\pi e^{-in\pi}}{in} - \frac{\pi e^{inx}}{in} - \frac{e^{-in\pi}}{(in)^2} + \frac{e^{in\pi}}{(in)^2}\right]$$

$$= \left[\frac{1}{\pi in}\left(\frac{e^{in\pi}-e^{-in\pi}}{2}\right) - \frac{1}{in}\left(\frac{e^{-in\pi}+e^{in\pi}}{2}\right) + \frac{1}{(in)^2\pi}\left(\frac{e^{in\pi}-e^{in\pi}}{2}\right)\right.$$

$$= \frac{1}{\pi in}\left(\cancel{\sin(n\pi)}\right) - \frac{1}{in}\left(\cos(n\pi)\right) + \cancel{\frac{1}{(in)^2\pi}\left(\sin(n\pi)\right)}$$

$$0 \qquad\qquad 0$$

$$= \frac{\cos(n\pi)}{in} = \frac{(-1)^n}{-in}$$

$$f \sim \sum \frac{(-1)^n}{-in}\cdot e^{inx} \qquad c_0 = \frac{(-1)^0}{-i0} = \text{ugyldig}$$

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$$= \frac{1}{2\pi}\left[\frac{e^{-ikx}}{-ik}\Big|_{-\pi}^{\pi} + \frac{xe^{-ikx}}{-ik}\Big|_{-\pi}^{\pi} - \frac{e^{-ikx}}{(ik)^2}\Big|_{-\pi}^{\pi}\right]$$

$$= \frac{1}{2\pi}\left[-\frac{e^{-ik\pi}}{ik} + \frac{e^{ik\pi}}{ik} - \frac{\pi e^{-ik\pi}}{ik} - \frac{\pi e^{ikx}}{ik} - \frac{e^{-ik\pi}}{(ik)^2} + \frac{e^{ik\pi}}{(ik)^2}\right]$$

$$= \left[\frac{1}{\pi ik}\left(\frac{e^{ik\pi} - e^{-ik\pi}}{2}\right) - \frac{1}{ik}\left(\frac{e^{-ik\pi} + e^{ik\pi}}{2}\right) + \frac{1}{(ik)^2\pi}\left(\frac{e^{ik\pi} - e^{ik\pi}}{2}\right)\right.$$

$$= \frac{1}{\pi ik}\big(\underbrace{\cancel{\sin(k\pi)}}_{0}\big) - \frac{1}{ik}\big(\cos(k\pi)\big) + \cancel{\frac{1}{(ik)^2\pi}\big(\sin(k\pi)\big)}_{0}$$

$$= \frac{\cos(k\pi)}{ik} = \frac{(-1)^n}{-ik}$$

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