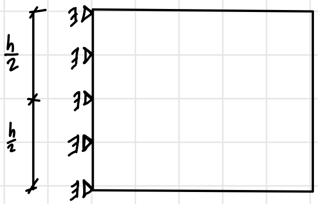
Øving 3
Mek 4

Oppgave 1



a) $w(0) = 0$

Dermed må $w(x=0, y \in [0,h]) = 0$

Vi ser at dette vil stemme uansett fordi vi har at $x=0$. Uavhengig av y så vil u bli til null.

$w(x=0) = 0$ o.k.

b)

$$\varepsilon = \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{bmatrix} = \begin{bmatrix} du/dx \\ dv/dy \\ du/dy + dv/dx \end{bmatrix} = \begin{bmatrix} -2Cvx \\ 2Cx \\ -2C_y + 2C_y \end{bmatrix} = -2Cx \begin{bmatrix} v \\ -1 \\ 0 \end{bmatrix}$$

$$\sigma = C\varepsilon = \begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix} = \frac{E}{1-v^2} \cdot \begin{bmatrix} 1 & v & 0 \\ v & 1 & 0 \\ 0 & 0 & \frac{1}{2}(1-v) \end{bmatrix} \begin{bmatrix} -2Cvx \\ 2Cx \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 2Cx(1-v^2) \\ 0 \end{bmatrix} \frac{E}{(1-v^2)}$$

$$= 2ECx$$

c) $\delta W_{ext} = \delta W_{int}$

$$\delta W_{ext} = \int_V \cancel{\delta u^T b}_{0}\, dV + \int_S \delta u^T t \, dS$$

$$\delta u^T(x=l)\, t = [\, 2ly\,\delta C \quad -\delta C(l^2+vy^2) \,] \begin{bmatrix} \frac{M}{I}x \\ 0 \end{bmatrix} = 2x^2 \cdot \frac{Ml}{I} \cdot \delta C$$

$$\delta W_{ext} = \int_{-\frac{h}{2}}^{\frac{h}{2}} 2x^2 \frac{Ml}{I} \delta C \, t\, dx = 2 \frac{Mtl}{I} \delta C \cdot \left[\frac{1}{3}x^3\right]_{-\frac{h}{2}}^{\frac{h}{2}}$$

$$= \frac{Mth^3 l}{6I} \delta C$$

$$\delta W_{int} = \int_V \delta \varepsilon^T \sigma \, dV$$

$$\delta \varepsilon^T \sigma = -2x\delta C\, [\, v \quad -1 \quad 0 \,] \cdot \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \cdot 2ECx$$

$$= 4x^2 E C \delta C$$

$$\int_0^l \int_{-\frac{h}{2}}^{\frac{h}{2}} 4Ex^2 C\delta C \underbrace{t\, dx\, dy}_{dV} = 4tEC\delta C l \cdot \left[\frac{1}{3}x^3\right]_{-\frac{h}{2}}^{\frac{h}{2}} = \frac{Eth^3 l}{3} C\delta C$$

d) $\delta W_{ext} = \delta W_{int} \Leftrightarrow$ Statisk likevekt

$$\frac{M\cancel{t}\cancel{h^3}\cancel{l}}{\cancel{6}_{2}I} \cancel{\delta C} = \frac{E\cancel{t}\cancel{h^3}\cancel{l}}{\cancel{3}} C\cancel{\delta C}$$

$$C = \frac{M}{2EI}$$

e) $$\sigma = \begin{bmatrix} \frac{M}{I} x \\ 0 \\ 0 \end{bmatrix}$$

$$b_x = b_y = 0$$

$$\left.\begin{aligned} \frac{d \sigma_x}{dx} + \frac{d \tau_{xy}}{dy} + b_x = 0 \\ \frac{d \sigma_y}{dy} + \frac{d \tau_{xy}}{dx} + b_y = 0 \end{aligned}\right\} \text{O.G.}$$

f) For å vurdere kvaliteten til løsningen kan vi bruke disse tre formlene

1) Likevekt ($N^T \sigma b = 0$)
2) Mekaniske randkrav (Cauchy)
3) Kompabilitet ($\varepsilon = \Delta u$)

Opgave 2

a) $u(x=0)=0$ Bottom Left
$u(x=a)=0$ Bottom Right
$u(y=a)=0$ Top Left
$u(y=a, x=a)=0$ Top Right

$\left.\right\}$ O.C.

b) $$\delta W_{ext} = \int_V \delta u^T b \, dV + \int_S \delta u^T t \, dS$$

$$\delta W_{int} = \int \delta \varepsilon^T \sigma \, dV$$

$$\varepsilon = \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{bmatrix} = \begin{bmatrix} du/dx \\ dv/dy \\ du/dy + dv/dx \end{bmatrix} = \begin{bmatrix} 0 \\ \delta C \frac{\pi}{a} \sin(\frac{\pi}{a}x)\cos(\frac{\pi}{a}y) \\ \delta C \frac{\pi}{a} \cos(\frac{\pi}{a}x)\sin(\frac{\pi}{a}y) \end{bmatrix} = \delta C \frac{\pi}{a} \begin{bmatrix} 0 \\ \sin(\frac{\pi}{a}x)\cos(\frac{\pi}{a}y) \\ \cos(\frac{\pi}{a}x)\sin(\frac{\pi}{a}y) \end{bmatrix}$$

$$\sigma = E \cdot \varepsilon \div \begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix} = \frac{E}{(1-v^2)} \cdot \begin{bmatrix} 1 & v & 0 \\ v & 1 & 0 \\ 0 & 0 & \frac{1}{2}(1-v) \end{bmatrix} \begin{bmatrix} 0 \\ \sin(\frac{\pi}{a}x)\cos(\frac{\pi}{a}y) \\ \cos(\frac{\pi}{a}x)\sin(\frac{\pi}{a}y) \end{bmatrix} \delta C \frac{\pi}{a}$$

$$= \frac{\delta C E \pi}{a(1-v^2)} \begin{bmatrix} v \sin(\frac{\pi}{a}x)\cos(\frac{\pi}{a}y) \\ \sin(\frac{\pi}{a}x)\cos(\frac{\pi}{a}y) \\ \frac{1}{2}(1-v)\cos(\frac{\pi}{a}x)\sin(\frac{\pi}{a}y) \end{bmatrix}$$

$$\delta W_{ext} = \int_V \delta u^T b \, dV + \underbrace{\cancel{\int_S \delta u^T t \, dS}}_{0} = \int_V \begin{bmatrix} 0 & C \sin(\frac{\pi}{a}x)\sin(\frac{\pi}{a}y) \end{bmatrix} \begin{bmatrix} 0 \\ b_1 \sin(\frac{\pi}{a}x)\sin(\frac{\pi}{a}y) \end{bmatrix} dV$$

$$= \int_V b_1 C \sin^2(\tfrac{\pi}{a}x)\sin^2(\tfrac{\pi}{a}y) \, dV = \int_0^a \int_0^a b_1 \delta C \sin^2(\tfrac{\pi}{a}x)\sin^2(\tfrac{\pi}{a}y) \, t \, dx \, dy$$

$$= b_1 \delta C t \int_0^a \sin^2(\tfrac{\pi}{a}x) \, dx \int_0^a \sin^2(\tfrac{\pi}{a}y) \, dy$$

$2 = a/2$

$$= \frac{b_1 t a^2}{4} \delta C$$

$$\delta W_{int} = \int_V \delta \varepsilon^T \sigma \, dV$$

$$= \int_V \left[\delta\varepsilon_x \quad \delta\varepsilon_y \quad \delta\gamma_{xy} \right] \cdot \begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix} dV = \int_V \underbrace{\cancel{\delta\varepsilon_x \sigma_x}}_{0} + \delta\varepsilon_y \sigma_y + \delta\gamma_{xy} \tau_{xy} \, dV$$

$$= \int_V C \delta C \frac{E}{1-\nu^2} \left(\frac{\pi}{a}\right)^2 \cdot \left(\sin^2\left(\frac{\pi}{a}x\right) \cos^2\left(\frac{\pi}{a}y\right) + \frac{1}{2}(1-\nu) \cdot \cos^2\left(\frac{\pi}{a}x\right) \sin^2\left(\frac{\pi}{a}y\right) \right) dV$$

$$= C\delta C \frac{Et}{1-\nu^2} \left(\frac{\pi}{a}\right)^2 \int_0^a \int_0^a \underbrace{\sin^2\left(\frac{\pi}{a}x\right)}_{a/2} \underbrace{\cos^2\left(\frac{\pi}{a}y\right)}_{a/2} + \frac{1}{2}(1-\nu) \underbrace{\cos^2\left(\frac{\pi}{a}x\right)}_{a/2} \underbrace{\sin^2\left(\frac{\pi}{a}y\right)}_{a/2} dx\, dy$$

$$= C\delta C \frac{Et}{1-\nu^2} \left(\frac{\pi}{a}\right)^2 \cdot \left(\frac{a}{2} \frac{a}{2} + \frac{1}{2}(1-\nu) \frac{a}{2} \frac{a}{2} \right)$$

$$= C\delta C \frac{Et}{1-\nu^2} \frac{\pi^2}{a^2} \left(\frac{a^2}{4} + \frac{1}{2}(1-\nu) \frac{a^2}{2} \right)$$

$$= C\delta C \frac{Et}{1-\nu^2} \frac{\pi^2}{4} \left(1 + \frac{1}{2} - \frac{\nu}{2} \right)$$

$$= C\delta C \frac{Et}{1-\nu^2} \frac{\pi^2}{4} \left(\frac{3}{2} - \frac{\nu}{2} \right)$$

$$= C\delta C \frac{3-\nu}{1-\nu^2} \frac{\pi^2}{8} Et$$

$$\delta W_{int} = \delta W_{ext}$$

$$C \cancel{\delta C} \frac{3-\nu}{1-\nu^2} \frac{\pi^2}{\cancel{8}\, 2} Et = \frac{b_1 t a^2}{\cancel{4}} \cancel{\delta C}$$

$$C \frac{3-\nu}{1-\nu^2} \pi^2 Et = b_1 t a$$

$$C = (b_1 t a) \cdot (1-\nu^2) \cdot \frac{2}{(3-\nu)\pi^2 Et}$$

c)

$$\sigma = \frac{\sigma C E \pi}{a(1-\nu^2)} \begin{bmatrix} \nu \sin(\frac{\pi}{a}x)\cos(\frac{\pi}{a}y) \\ \sin(\frac{\pi}{a}x)\cos(\frac{\pi}{a}y) \\ \frac{1}{2}(1-\nu)\cos(\frac{\pi}{a}x)\sin(\frac{\pi}{a}y) \end{bmatrix}$$

$$C = (b_1 a)\cdot(1-\nu^2)\cdot \frac{2}{(3-\nu)\pi^2 E}$$

$$\sigma_x = \frac{2b\, \cancel{a}_1}{3-\nu\, \pi^{\cancel{2}} \cancel{E} t} \cdot \cancel{(1-\nu^2)} \cdot \frac{\cancel{E}\cancel{\pi}}{\cancel{a}\cancel{(1-\nu^2)}} \cdot \nu \sin(\frac{\pi}{a}x)\cos(\frac{\pi}{a}y)$$

$$= \frac{2b_1 \quad \nu}{\pi\cdot(3-\nu)} \sin(\frac{\pi}{a}x)\cos(\frac{\pi}{a}y)$$

$$\sigma_y = \frac{2b_1 \cancel{t}\cancel{a}}{(3-\nu)\pi^2 \cancel{E}\cancel{t}} \cancel{(1-\nu^2)} \cdot \frac{\cancel{E}\cancel{\pi}}{\cancel{a(1-\nu^2)}} \cdot \sin(\frac{\pi}{a}x)\cos(\frac{\pi}{a}y)$$

$$= \frac{2b_1}{\pi(3-\nu)} \sin(\frac{\pi}{a}x)\cos(\frac{\pi}{a}y)$$

$$\tau_{xy} = \frac{2b_1\cancel{a}}{(3-\nu)\pi^2\cancel{E}} \cancel{(1-\nu^2)} \frac{\cancel{E\pi}}{\cancel{a(1-\nu^2)}} \cdot \left(\frac{1}{2}(1-\nu)\cos(\frac{\pi}{a}x)\sin(\frac{\pi}{a}y)\right)$$

$$= \frac{\cancel{2}b_1}{\pi(3-\nu)} \left(\frac{1}{\cancel{2}}(1-\nu)\cos(\frac{\pi}{a}x)\sin(\frac{\pi}{a}y)\right)$$

$$= \frac{b_1(1-\nu)}{\pi(3-\nu)} \cos(\frac{\pi}{a}x)\sin(\frac{\pi}{a}y)$$

d) $$\frac{d\sigma_x}{dx} = 2\frac{b_1 v}{3-v}\cos\left(\frac{\pi}{a}x\right)\cos\left(\frac{\pi}{a}y\right)$$

$$\frac{d\sigma_y}{dy} = -2\frac{b_1}{3-v}\sin\left(\frac{\pi}{a}x\right)\sin\left(\frac{\pi}{a}y\right)$$

$$\frac{d\tau_{xy}}{dx} = \frac{1-v}{3-v}b_1\cos\left(\frac{\pi}{a}x\right)\cos\left(\frac{\pi}{a}y\right)$$

$$\frac{d\tau_{xy}}{dy} = \frac{1-v}{3-v}b_1\sin\left(\frac{\pi}{a}x\right)\sin\left(\frac{\pi}{a}y\right)$$

Eq. x-direction

$$\frac{d\sigma_x}{dx} + \frac{d\tau_{xy}}{dy} + b_x = 0$$

$$2\frac{b_1 v}{3-v}\cos\left(\frac{\pi}{a}x\right)\cos\left(\frac{\pi}{a}y\right) + \frac{1-v}{3-v}b_1\cos\left(\frac{\pi}{a}x\right)\cos\left(\frac{\pi}{a}y\right) \neq 0 \quad \text{Not o.k.}$$

Eq. y-direction

$$\frac{d\sigma_y}{dy} + \frac{d\tau_{xy}}{dy} + b_y = 0$$

$$-2\frac{b_1}{3-v}\sin\left(\frac{\pi}{a}x\right)\sin\left(\frac{\pi}{a}y\right) - \frac{1-v}{3-v}b_1\sin\left(\frac{\pi}{a}x\right)\sin\left(\frac{\pi}{a}y\right) + b_1\sin\left(\frac{\pi}{a}x\right)\sin\left(\frac{\pi}{a}y\right) = 0 \qquad \text{o.k.}$$

No local eq. in x-direction.

Global eq
Check for x-direction because of local eq in x-direction is not o.k.

$$\Sigma F_x = \int_0^a\int_0^a\left(\frac{d\sigma_x}{dx} + \frac{d\tau_{xy}}{dy}\right) + dx\,dy = \frac{1+v}{3-v}b_1 + \int_0^a\int_0^a \underbrace{\cos\frac{\pi}{a}x}_{0}\ \underbrace{\cos\frac{\pi}{a}y}_{0}\,dx\,dy$$

$$= 0 \qquad \text{o.k.}$$