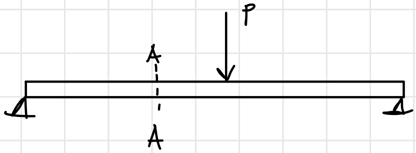
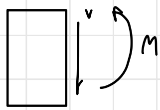
Øving 2
TKT 4134

Oppgave 1

b)





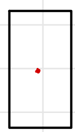
$$\sigma_M = \frac{M}{I} \cdot y$$

$$I = \frac{1}{12} b h^3 = \frac{1}{12} \cdot 1 \cdot 1^3 = \frac{1}{12}$$

$$M = -10^8 \ Nm$$

$$\sigma_M = \frac{10^8}{\frac{1}{12}} = 1{,}2 \cdot 10^9 \ y \ N/m^3$$

$$\tau_{xy} = \frac{V \cdot S}{I \cdot b}$$



$$S = A_1 \cdot y_{c1}$$
$$= \left(\frac{h}{2} - y\right) b \cdot \left(y + \frac{1}{2}\left(\frac{h}{2} - y\right)\right)$$
$$= b\left(\frac{h}{2} - y\right)\left(y + \frac{h}{4} - \frac{y}{2}\right)$$
$$= b\left(\cancel{\frac{yh}{2}} + \frac{h^2}{8} - \cancel{\frac{yh}{4}} - y^2 - \cancel{\frac{yh}{4}} + \frac{y^2}{4}\right)$$
$$= b\left(\frac{h^2}{8} - \frac{3y^2}{4}\right)$$
$$= \frac{bh^2}{8}\left(1 - \left(\frac{2y}{h}\right)^2\right)$$

$$\tau_{xy} = \frac{10^8 \cdot \frac{bh^2}{8}\left(1 - \left(\frac{2y}{h}\right)^2\right)}{\frac{1}{12} \cdot 1}$$

$$= \frac{10^8 \cdot \frac{1}{8} \cdot \left(1 - \left(\frac{2y}{1}\right)^2\right)}{\frac{1}{12} \cdot 1}$$

$$= 1{,}5 \cdot 10^8 \left(1 - \left(\frac{2y}{1}\right)^2\right)$$

c) $\sigma_{1,2} = \frac{\sigma_x}{2} \pm \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + \tau_{xy}}$

$\alpha = \arctan\left(\frac{\sigma_1 - \sigma_2}{\tau_{xy}}\right)$

Point A: $\sigma_x = -1.2 \cdot 10^9 \cdot \frac{1}{2} = -600\,MPa$ $\tau_{xy} = 0$ $\sigma_2 = -600$ $\sigma_1 = 0$ $\alpha = 90°$

Point E: $\sigma_x = 1.2 \cdot 10^9 \cdot \frac{1}{2} = 600\,MPa$ $\tau_{xy} = 0$ $\sigma_2 = 600$ $\sigma_1 = 0$ $\alpha = 0°$

Point B: $\sigma_x = -1.2 \cdot 10^9 \cdot \frac{1}{4} = -300\,MPa$ $\tau_{xy} = 1.5 \cdot 10^8 \cdot \frac{3}{4} = 112{,}5$

$\sigma_{1,2} = \frac{-300}{2} \pm \sqrt{\left(\frac{300}{2}\right)^2 + 112{,}5^2} = 37.5,\ -337.5$

$\alpha = \arctan\left(\frac{37.5 + 337.5}{112{,}5}\right) = 71.6°$

Point D: $\sigma_x = 300\,MPa$ $\tau_{xy} = 112{,}5$

$\sigma_{1,2} = -37.5,\ 337.5$

$\alpha = 90 - 71.6 = 18.4°$

Point C: $\sigma_x = 0$ $\tau_{xy} = 150$

$\sigma_{1,2} = 150,\ -150$

$\alpha = 45°$

Oppgave 2

a) $\sigma_x = -\frac{P_0}{h} y$

$\sigma_y = \left(\frac{\rho g h}{w} - \frac{2 P_0 h^2}{w^3} \right) x + \left(\frac{P_0 h}{w^2} - \rho g \right) y$

$\tau_{xy} = -\frac{P_0 h}{w^2} x$

$$\frac{d\sigma_x}{dx} + \frac{d\tau_{xy}}{dy} + b_x = 0$$

$$0 + 0 + b_x = 0 \quad , \; b_x = 0 \qquad \text{O.K.}$$

$$\frac{d\sigma_y}{dy} + \frac{d\tau_{xy}}{dx} + b_y = 0$$

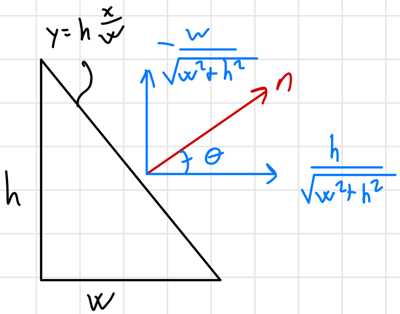
$$\frac{P_0 h}{w^2} - \rho g - \frac{P_0 h}{w^2} + \rho g = 0 \qquad \text{O.K.}$$

b) Boundary condition

$y = h \cdot \frac{x}{w}$

Cauchy law

$$n = \begin{bmatrix} \cos\theta \\ -\sin\theta \end{bmatrix} = \frac{1}{\sqrt{w^2+h^2}} \begin{bmatrix} h \\ -w \end{bmatrix}$$



$$\sigma = \begin{bmatrix} \sigma_x & \tau_{xy} \\ \tau_{xy} & \sigma_y \end{bmatrix} = -\frac{P_0}{w} \times \begin{bmatrix} 1 & \frac{h}{w} \\ \frac{h}{w} & \frac{h^2}{w^2} \end{bmatrix}$$

$$t = -\frac{1}{\sqrt{w^2+h^2}} \cdot \frac{P_0}{w} \times \cdot \begin{bmatrix} 1 & \frac{h}{w} \\ \frac{h}{w} & \frac{h^2}{w^2} \end{bmatrix} \begin{bmatrix} h \\ -w \end{bmatrix} = \frac{1}{\sqrt{w^2+h^2}} \frac{P_0}{w} \times \begin{bmatrix} h-h \\ \frac{h^2}{w} - \frac{h^2}{w} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$t = \underset{\substack{\uparrow \\ \text{Cauchy}}}{\sigma \cdot n} \underset{\substack{\uparrow \\ \text{Svant}}}{=} 0$$

c) $$\sigma_y = \left(\frac{\rho g h}{w} - \frac{2P_0 h^2}{w^3}\right)x + \left(\frac{P_0 h}{w^2} - \rho g\right)y$$

Highest compression at h

$x=0$

$$\sigma_y = 0 + \left(\frac{P_0 h}{w^2} - \rho g\right)y$$

$$= \left(\frac{P_0 h}{w^2} - \rho g\right)h$$

$$= \frac{P_0 h^2}{w^2} - \rho g h$$

$\sigma(x=0) < 0$ $\qquad$ $\rho = \rho_w \cdot 2.4$

$$w^2 = \frac{1}{2.4} h^2$$

$$w = \sqrt{\frac{1}{2.4} h} = 0.645 h$$

d)

$$\Sigma F_x = \frac{1}{2} P_0 h - \frac{1}{2} \frac{P_0 h}{w} \cdot w = 0 \qquad \text{o.k.}$$

$$\Sigma F_y = F_g + \int_0^w \sigma_y \, dx$$

$$= \frac{wh}{2} \rho g + \int_0^w \left(\frac{\rho g h}{w} - \frac{2 P_0 h^2}{w^3} \right) x + \frac{P_0 h^2}{w^2} - \rho g h \, dx$$

$$= \frac{wh}{2} \rho g + \frac{\rho g h}{2} w - \frac{P_0 h^2}{w} + \frac{P_0 h^2}{w} - \rho g h w$$

$$= 0 \qquad \text{o.k.}$$

$$\Sigma M_c = \frac{1}{2} \frac{P_0 h}{w} \cdot w \cdot \frac{h}{3} - \frac{P_0 h^2}{w^2} \cdot w \cdot \left(\frac{w}{2} - \frac{w}{3} \right)$$

$$= \frac{1}{6} P_0 h - \frac{1}{6} P_0 h = 0 \qquad \text{o.k.}$$

Oppgave 3

a) $\frac{d\sigma_x}{\cancel{dx}} + \frac{d\tau_{xy}}{dy} + b_x = 0 \quad \Rightarrow \quad b_x = -\frac{d\tau_{xy}}{dy}$

$\frac{d\sigma_y}{\cancel{dy}} + \frac{d\tau_{xy}}{dx} + b_y = 0 \quad \Rightarrow \quad b_y = -\frac{d\tau_{xy}}{dx}$

Kan se at τ_{xy} er en konstant for at det skal stemme.

$$\tau_{xy} t = -P_0$$

$$\tau_{xy} = -\frac{P_0}{t} = \text{konstant}$$

b) $\varepsilon_x = \frac{du}{dx} = \frac{1}{E}(\sigma_x - \nu\sigma_y) = 0 \qquad \sigma_x = \sigma_y = 0$

$\varepsilon_y = \frac{dv}{dy} = \frac{1}{E}(\sigma_y - \nu\sigma_x) = 0$

$\gamma_{xy} = \left(\frac{du}{dx} + \frac{dv}{dy}\right) = \frac{2(1+\nu)}{E}\tau_{xy} = -\frac{P_0}{Gt}$

$\frac{d\tau_{xy}}{dy} = C_1 \Rightarrow u = C_1 y + C_2$

$\frac{d\tau_{xy}}{dx} = C_3 \Rightarrow v = C_3 x + C_4$

$u(x \neq 0, y=0) = C_2 = 0$

$v(x=0, y=0) = C_4 = 0$

$v(x \neq 0, y=0) = C_3 = 0$

$\gamma_{xy} = -\frac{P_0}{Gt} = C_1$

Forskyvningen er dermed $u(y) = -\frac{P_0}{Gt} \cdot y \;, \; v = 0$

c) $e_x = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \quad e_y = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

$$e_x^* = \begin{bmatrix} \cos\theta \\ \sin\theta \end{bmatrix} \quad e_y^* = \begin{bmatrix} -\sin\theta \\ \cos\theta \end{bmatrix}$$

$$a = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$$

$$\sigma^* = a\sigma a^T = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} \sigma_x & \tau_{xy} \\ \tau_{xy} & \sigma_y \end{bmatrix} \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

Legger den inn i kalkulator for å regne den ut
Med $\theta = 30^\circ$ $\quad \sigma_x = \sigma_y = 0$ for vi

$$\sigma_x^* = -\frac{\sqrt{3}}{2}\frac{P_0}{t} \qquad \sigma_y^* = \frac{\sqrt{3}}{2}\frac{P_0}{t} \qquad \tau_{xy} = -\frac{P_0}{2t}$$

d) $$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \pm \tau_{xy}$$

$$= \pm \left(-\frac{P_0}{t}\right)$$

$$\alpha = \arctan\left(\frac{\tau_{xy}}{\sigma_1 - \sigma_y}\right) = \arctan(1) = 45^\circ$$