1 Introduction

The goal of the current code is to implement a simple temperature profile through the combustion chamber ablative + wall and the nozzle graphite. The temperature profile is based on the simple thermal resistance network concept at steady-state conditions (combustion chamber temperature, pressure, and O/F ratio at t= 8.4 seconds), where the energy through each layer is preserved.

The current code accurately calculates the gas-side convective heat transfer coefficient, which allows for an implicit equation for the inner wall temperature to be solved for each axial position (using fsolve). Once this is calculated, the temperatures for each successive material layer can be found easily at each axial position, and the heat flux for each position is likewise known.

2 The combustion chamber section

This analysis is only done for one axial position of y = 6.675 cm (the CC-radius can easily be modified to accommodate design changes). Using the relevant rocketCEA transport properties + other parameters needed in the Bartz equation, the heat transfer coefficient can be defined in terms of the unknown inner wall temperature. Then, the first part of the thermal resistance network can be implemented [1, p4]:

$$Q^* = h_1 A(T_{\infty,1} - T_1) = \frac{T_{\infty,1} - T_{\infty,2}}{R_{total}} \tag{1}$$

where h_1 = gas-side convective heat transfer coefficient, A = area (in cylindrical coordinates = $2\pi r_{in} L$, $T_{\infty,1}$ = steady state combustion chamber temperature, $T_{\infty,2}$ = exterior atmospheric temperature, and R_{total} is defined as

$$R_{total} = R_{CC-convection} + ... + R_{material-n} + ... + R_{ATM-convection} \tag{2}$$

Equivalently, (2) is expressed in cylindrical coordinates as

$$R_{total} = \frac{1}{h_1 2\pi r_{in} L} + ln(\frac{r_{mid}/r_{in}}{2\pi k_a}) + ln(\frac{r_{out}/r_{mid}}{2\pi k_b}) + \frac{1}{h_2 2\pi r_{out} L} \tag{3}$$

where r_{in} = radius of inner wall, r_{mid} = radius of first material boundary, r_{out} = outer radius, k_a = thermal conductivity of inner material, k_b = thermal conductivity of outer material, h_2 = atmosphere-side convective heat transfer coefficient.

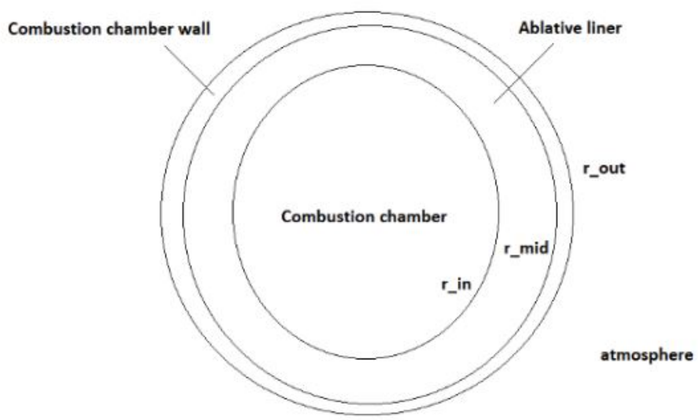

Figure 1: Cross section view of thermal resistance network in combustion chamber

Knowing all parameters in (1) except for T_1 (inner wall temperature), which cannot be solved explicitly since it appears inside h_1 and on the left side, equation (1) is solved implicitly for T_1 (using fsolve in scipy.optimize). In the equation, the area parameter L can be canceled out. The values for the thermal conjunctivitis, inner radius, ablative thickness, atmospheric heat transfer coefficient can all be easily modified. Once T_1 is known, the heat flux is known, so we can use

$$Q^* = h_1 A(T_{\infty,1} - T_1) = k_1 A \frac{T_1 - T_2}{L_1} = k_2 A \frac{T_2 - T_3}{L_2} = h_2 A(T_2 - T_{\infty,2}) \tag{4}$$

to easily find T_2 and T_3, the temperatures at the ablative-wall boundary and the temperature on the exterior wall. This completes the thermal network, and thus the temperature profile for the combustion chamber, given an appropriate $r_i n$, ablative thickness, k_a, k_b, etc. This model assumes a linear temperature change between material layers.

3 The nozzle section

The analysis described in the previous section is done for all axial positions of the nozzle portion, with the primary difference being the need to account for a varying radius along the length of the nozzle. We used the nozzle design from the PDR slides:

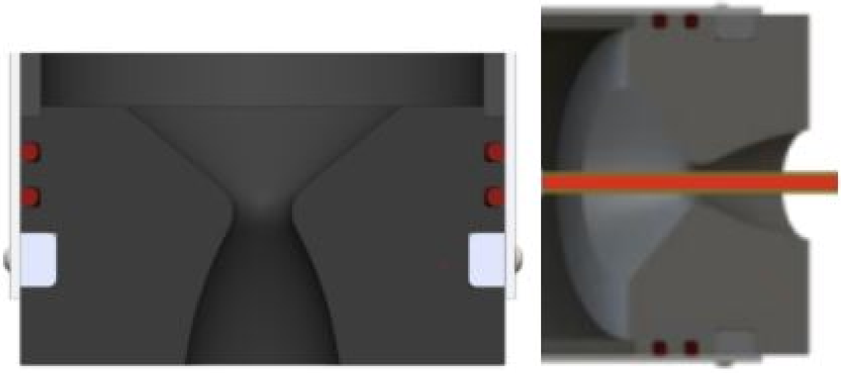

Figure 2: Cross sectional views of most recent Houbolt Jr. Nozzle Configuration

Thus, the first layer is the graphite, with a thickness = cylindrical radius (assumed to be equal to the CC radius) - y-position from the nozzle contour csv file, and the second layer is a thin shell of CC wall. The equations for the thermal resistance model are the same as for the combustion chamber analysis, but the function loops through a list of varying r_{in}, and the thermal conductivity for graphite is taken into account.

4 Sample Code Output

```
--------------------------------
COMBUATION CHAMBER THERMAL PROFILE
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COMBUSTION CHAMBER HTC =  1235.0744095557668

COMBUSTION CHAMBER TEMP=  2777.857329

COMBUSTION CHAMBER INNER RADIUS=  0.06675

COMBUSTION CHAMBER INNER WALL TEMP =  2728.4180387901197

CC ABLATIVE-WALL BOUNDARY TEMP =  1516.886249805516

CC OUTER WALL TEMP =  1515.5980387893803


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NOZZLE TEMPERATURE PROFILES BY AXIAL POSITIONS, GOING DOWNSTREAM
--------------------------------
HTC= 1237.5762477418748 loc radius=  0.066675 T_1=  2710.684593559076 T_2=  2709.437622830913 T_3=  270
7.683796209024


HTC= 1237.5762477418748 loc radius=  0.066675 T_1=  2710.684593559076 T_2=  2709.437622830913 T_3=  270
7.683796209024


HTC= 5332.198864761493 loc radius=  0.029617953 T_1=  2750.05542332143 T_2=  1649.125208921093 T_3=  164
5.9976711595427
```

Figure 3: Sample output for Python code, continued for all axial nozzle positions

5 Next Steps

Using NASA SP-8093 [2], the q-star method for finding the local ablation rate can be used once q-star is determined experimentally with known heat fluxes during in-person ablative testing. From there, using

$$Q^* = \frac{q_{tot}}{\dot{x_a}\rho} \tag{5}$$

where q_{tot} is the heat flux determined for each axial position previously, ρ is the density of the ablative/graphite and Q^* is the experimentally determined heat of ablation parameter, the ablation rate $\dot{x_a}$ can be determined for each axial position.

6 Conclusion

Varying the ablative thickness, thermal conductivity and nozzle contour file will allow us to find a configuration which produces a satisfactory combustion chamber wall temperature and ablation rate.

7 References

1 https://www.sfu.ca/~mbahrami/ENSC%20388/Notes/Staedy%20Conduction%20Heat%20Transfer.pdf

2 https://ntrs.nasa.gov/api/citations/19770023227/downloads/19770023227.pdf