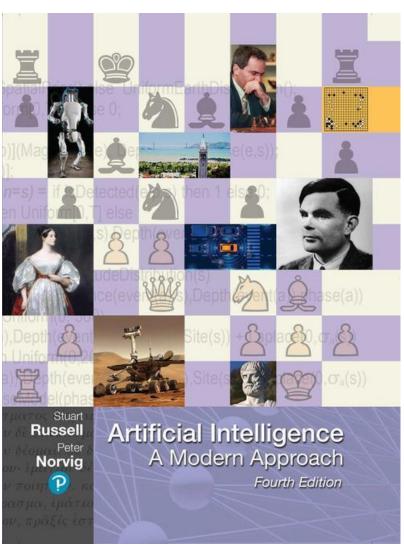
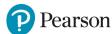
Artificial Intelligence: A Modern Approach

Fourth Edition



Chapter 7

Logical agents



Outline

- Knowledge-based agents
- Wumpus world
- Logic in general—models and entailment
- Propositional (Boolean) logic
- Equivalence, validity, satisfiability
- Inference rules and theorem proving
 - resolution
 - forward chaining
 - backward chaining
- Effective Propositional Model Checking



Knowledge bases

Inference engine domain-independent algorithms

Knowledge base domain-specific content

Knowledge base = set of sentences in a **formal** language

Declarative approach to building an agent (or other system):

Tell it what it needs to know

Then it can Ask itself what to do—answers should follow from the KB

Agents can be viewed at the knowledge level

i.e., what they know, regardless of how implemented

Or at the implementation level

i.e., data structures in KB and algorithms that manipulate them



A simple knowledge-based agent

```
function KB-Agent( percept) returns an action static: KB, a knowledge base t, a counter, initially 0, indicating time Tell(KB, Make-Percept-Sentence( percept, t)) action \leftarrow Ask(KB, Make-Action-Query(t)) Tell(KB, Make-Action-Sentence(action, t)) t \leftarrow t + 1 return action
```

The agent must be able to:

Represent states, actions, etc. Incorporate new percepts
Update internal representations of the world Deduce hidden properties of the world Deduce appropriate actions



Wumpus World PEAS description

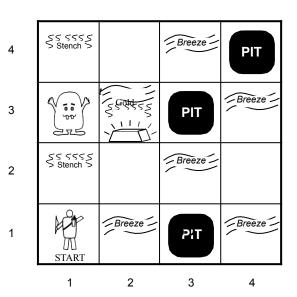
Performance measure gold +1000, death -1000

-1 per step, -10 for using the arrow Environment

Squares adjacent to wumpus are smelly Squares adjacent to pit are breezy Glitter iff gold is in the same square Shooting kills wumpus if you are facing it Shooting uses up the only arrow Grabbing picks up gold if in same square Releasing drops the gold in same square

Actuators Left turn, Right turn, Forward, Grab, Release, Shoot

Sensors Breeze, Glitter, Smell





Observable?



Observable?? No—only local perception

Deterministic??



Observable?? No—only local perception
Deterministic?? Yes—outcomes exactly specified

Episodic??



Observable?? No—only local perception

<u>Deterministic??</u> Yes—outcomes exactly specified

<u>Episodic</u>?? No—sequential at the level of actions

Static??



Observable?? No—only local perception

<u>Deterministic??</u> Yes—outcomes exactly specified

<u>Episodic</u>?? No—sequential at the level of actions

Static?? Yes—Wumpus and Pits do not move

<u>Discrete</u>??



Observable?? No—only local perception

Deterministic?? Yes—outcomes exactly specified

Episodic?? No—sequential at the level of actions

Static?? Yes—Wumpus and Pits do not move

Discrete?? Yes

Single-agent??



Observable?? No—only local perception

<u>Deterministic</u>?? Yes—outcomes exactly specified

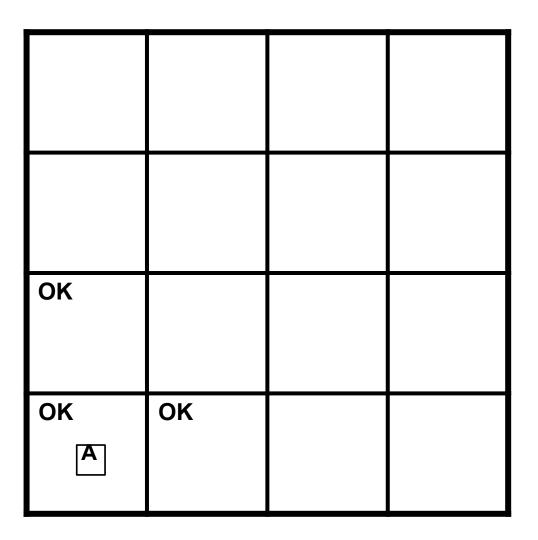
Episodic?? No—sequential at the level of actions

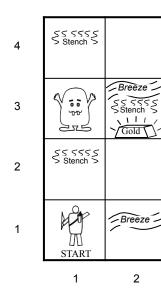
Static?? Yes—Wumpus and Pits do not move

Discrete?? Yes

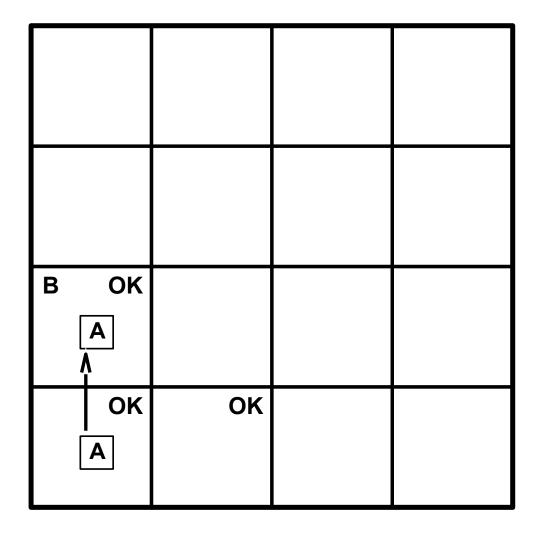
<u>Single-agent??</u> Yes—Wumpus is essentially a natural feature



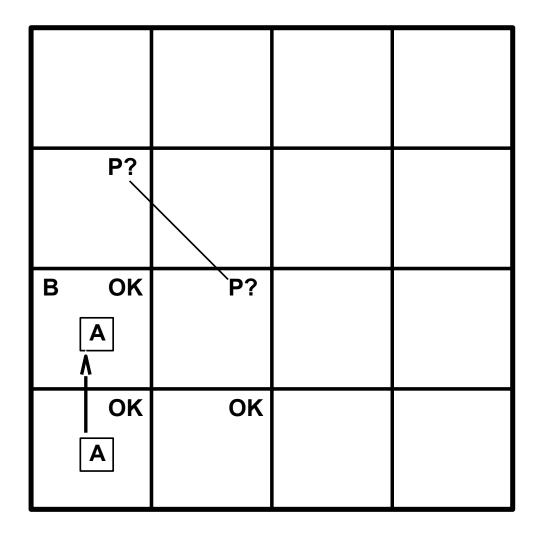




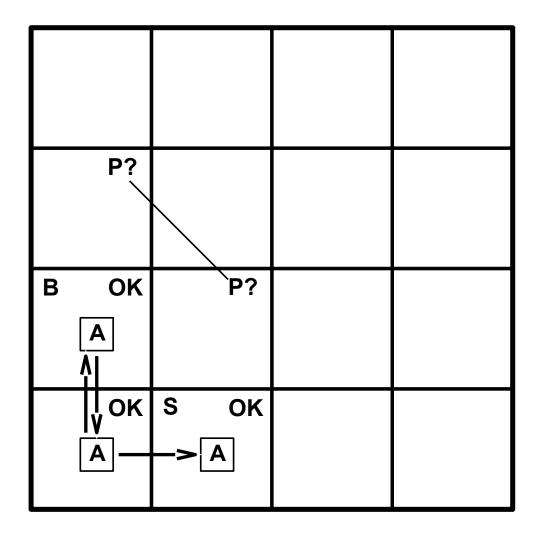




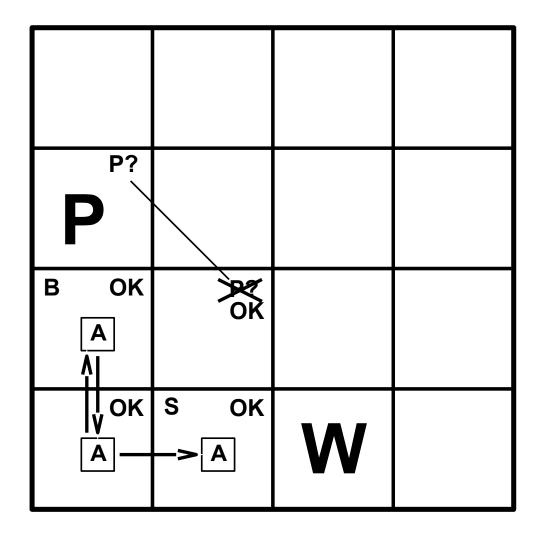




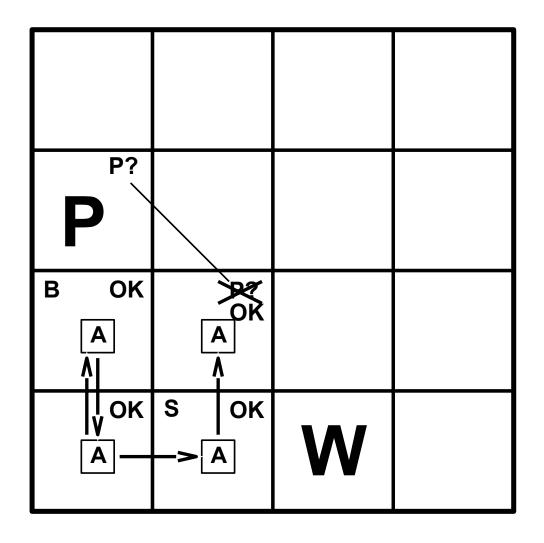




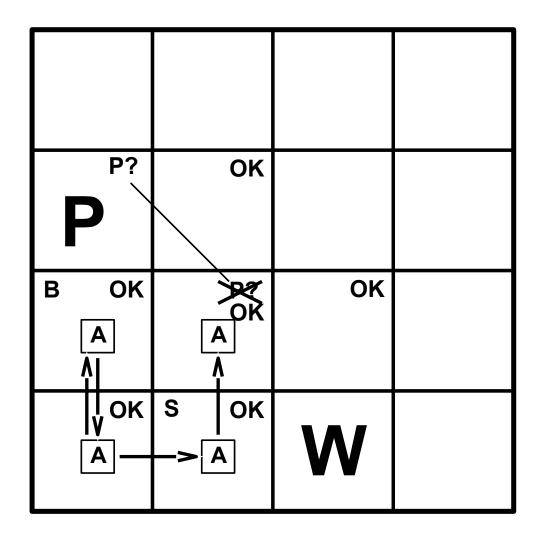




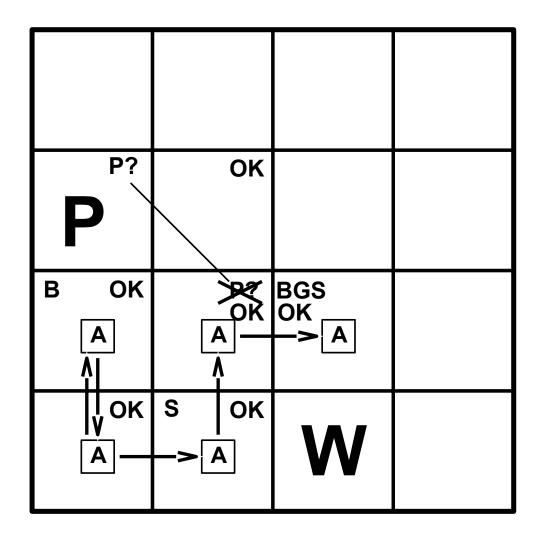






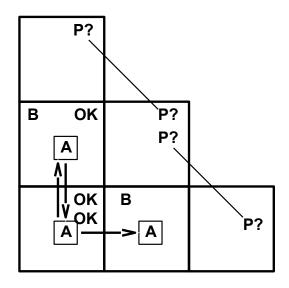






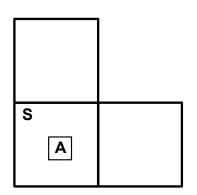


Other tight spots



Breeze in (1,2) and (2,1) \Rightarrow no safe actions

Assuming pits uniformly distributed, (2,2) has pit w/ prob 0.86, vs. 0.31



Smell in (1,1)

 \Rightarrow cannot move \Rightarrow safe

Can use a strategy of coercion: shoot straight ahead

wumpus was there ⇒ dead

wumpus wasn't there ⇒ safe

Logic in general

Logics are formal languages for representing information such that conclusions can be drawn

Syntax defines the sentences in the language

Semantics define the "meaning" of sentences;

i.e., define truth of a sentence in a world

E.g., the language of arithmetic

 $x + 2 \ge y$ is a sentence; $x^2 + y >$ is not a sentence

 $x + 2 \ge y$ is true iff the number x + 2 is no less than the number y

 $x + 2 \ge y$ is true in a world where x = 7, y = 1

 $x + 2 \ge y$ is false in a world where x = 0, y = 6



Entailment

Entailment means that one thing follows from another:

$$KB \mid = a$$

Knowledge base *KB* entails sentence *a*if and only if *a* is true in all worlds where *KB* is true

E.g., the KB containing "the Giants won" and "the Reds won" entails "Either the Giants won or the Reds won"

E.g.,
$$x + y = 4$$
 entails $4 = x + y$

Entailment is a relationship between sentences (i.e., syntax) that is based on semantics

Note: brains process syntax (of some sort)



Models

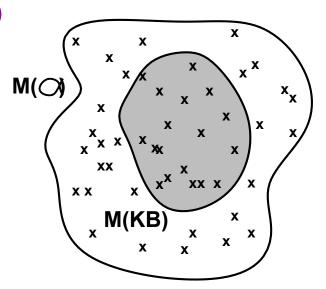
Logicians typically think in terms of models, which are formally structured worlds with respect to which truth can be evaluated

We say m is a model of a sentence a if a is true in m

M(a) is the set of all models of a

Then KB = a if and only if $M(KB) \subseteq M(a)$

E.g. KB = Giants won and Reds won a = Giants won



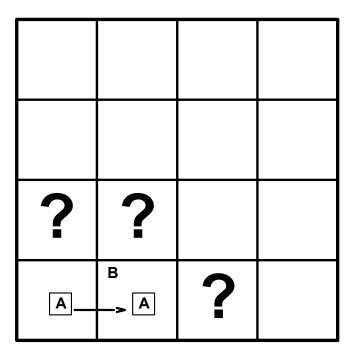


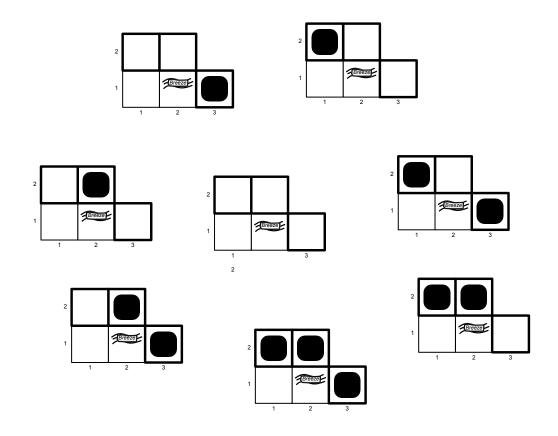
Entailment in the wumpus world

Situation after detecting nothing in [1,1], moving right, breeze in [2,1]

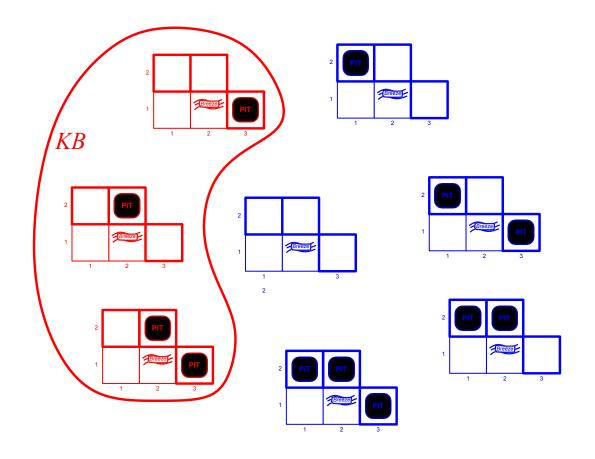
Consider possible models for ?s assuming only pits

3 Boolean choices ⇒ 8 possible models



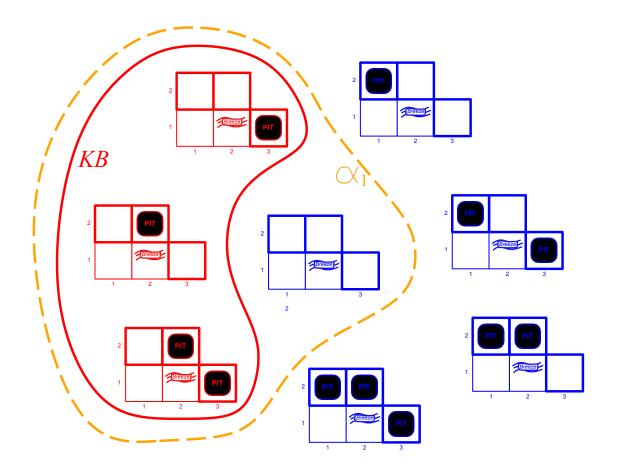






KB = wumpus-world rules + observations

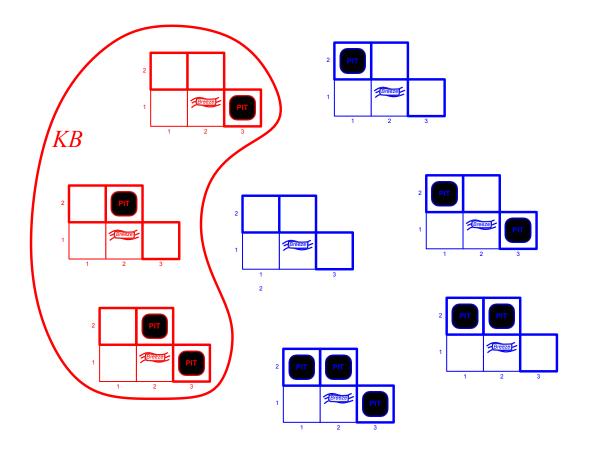




KB = wumpus-world rules + observations

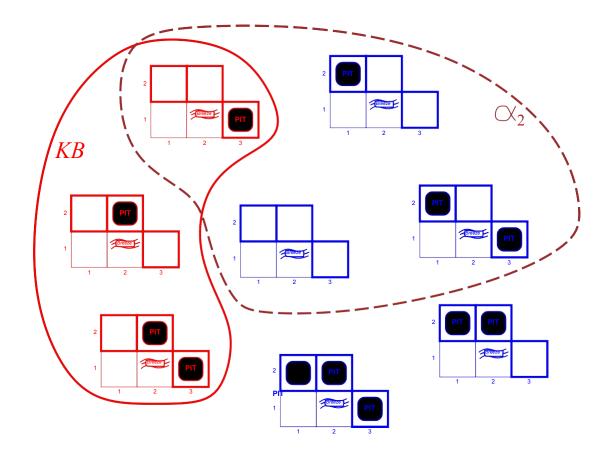
 a_1 = "[1,2] is safe", KB |= a_1 , proved by model checking





KB = wumpus-world rules + observations





KB = wumpus-world rules + observations

$$a_2$$
 = "[2,2] is safe", KB |= a_2



Inference

 $KB f_{-i}$ a = sentence a can be derived from <math>KB by procedure i

Consequences of KB are a haystack; a is a needle. Entailment = needle in haystack; inference = finding it

Soundness: i is sound if whenever KB f-- $_i$ a, it is also true that KB = a

Completeness: i is complete if whenever KB = a, it is also true that $KB f_{-i}$ a

Preview: we will define a logic (first-order logic) which is expressive enough to say almost anything of interest, and for which there exists a sound and complete inference procedure.

That is, the procedure will answer any question whose answer follows from what is known by the KB.



Propositional logic: Syntax

Propositional logic is the simplest logic—illustrates basic ideas. The proposition symbols P_1 , P_2 etc are sentences

If S is a sentence, $\neg S$ is a sentence (negation)

If S_1 and S_2 are sentences, $S_1 \wedge S_2$ is a sentence (conjunction)

If S_1 and S_2 are sentences, $S_1 \lor S_2$ is a sentence (disjunction)

If S_1 and S_2 are sentences, $S_1 \Rightarrow S_2$ is a sentence (implication)

If S_1 and S_2 are sentences, $S_1 \Leftrightarrow S_2$ is a sentence (biconditional)



Propositional logic: Semantics

Each model specifies true/false for each proposition symbol

E.g.
$$P_{1,2}$$
 p $P_{2,2}$

(With these symbols, 8 follows) le models, can be enumerated automatically.)

Rules for evaluating truth with respect to a model m:

Simple recursive process evaluates an arbitrary sentence, e.g.,

$$\neg P_{1,2} \land (P_{2,2} \lor P_{3,1}) = true \land (false \lor true) = true \land true = true$$



Truth tables for connectives

P	Q	$\neg P$	$P \wedge Q$	$P \lor Q$	$P \Rightarrow Q$	$P \Leftrightarrow Q$
false	false	true	false	false	true	true
false	true	true	false	true	true	false
true	false	false	false	true	false	false
true	true	false	true	true	true	true



Wumpus world sentences

Let $P_{i,j}$ be true if there is a pit in [i, j]. Let $B_{i,j}$ be true if there is a breeze in [i, j].

$$eg P \\
eg B_{1,1}^{1,1} \\
eg B_{2,1}^{2,1}$$

"Pits cause breezes in adjacent squares"



Wumpus world sentences

Let $P_{i,j}$ be true if there is a pit in [i, j]. Let $B_{i,j}$ be true if there is a breeze in [i, j].

$$\neg P$$
 $\neg B^{1,1}$
 $B^{1,1}$
 $B^{2,1}$

"Pits cause breezes in adjacent squares"

$$\begin{array}{cccc} B_{1,1} & \Leftrightarrow & (P_{1,2} \vee P_{2,1}) \\ & \Leftrightarrow & \\ B_{2,1} & & (P_{1,1} \vee P_{2,2} \vee P_{2,2}) \end{array}$$

"A square is breezy if and only if there is an adjacent pit"

Truth tables for inference

$B_{1,1}$	$B_{2,1}$	$P_{1,1}$	$P_{1,2}$	$P_{2,1}$	$P_{2,2}$	$P_{3,1}$	R_1	R_2	R_3	R_4	R_5	KB
false	true	true	true	true	false	false						
false	false	false	false	false	false	true	true	true	false	true	false	false
false	true	false	false	false	false	false	true	true	false	true	true	false
false	true	false	false	false	false	true	true	true	true	true	true	<u>true</u>
false	true	false	false	false	true	false	true	true	true	true	true	<u>true</u>
false	true	false	false	false	true	true	true	true	true	true	true	<u>true</u>
false	true	false	false	true	false	false	true	false	false	true	true	false
											•	
true	false	true	true	false	true	false						

Enumerate rows (different assignments to symbols), if KB is true in row, check that a is too



Inference by enumeration

Depth-first enumeration of all models is sound and complete

```
function TT-Entails?(KB, a) returns true or false
   inputs: KB, the knowledge base, a sentence in propositional logic
            a, the guery, a sentence in propositional logic
   symbols \leftarrow a list of the proposition symbols in KB and a
   return TT-Check-All(KB, a, symbols, [])
function TT-Check-All(KB, a, symbols, model) returns true or false
   if Empty?(symbols) then
        if PL-True?(KB, model) then return PL-True?(a, model)
        else return true
   else do
       P \leftarrow \text{First(symbols)}; rest \leftarrow \text{Rest(symbols)}
       return TT-Check-All(KB, a, rest, Extend(P, true, model))
       and
                 TT-Check-All(KB, a, rest, Extend(P, false, model))
```

 $O(2^n)$ for n symbols; problem is co-NP-complete



Logical equivalence

Two sentences are logically equivalent iff true in same models:

$$a \equiv \beta$$
 if and only if $a \models \beta$ and $\beta \models a$

$$(a \land \beta) \equiv (\beta \land a) \quad \text{commutativity of } \land \\ (a \lor \beta) \equiv (\beta \lor a) \quad \text{commutativity of } \lor \quad ((a \land \beta) \land \gamma) \equiv (a \land (\beta \lor \gamma)) \quad \text{associativity of } \lor \\ \land \quad ((a \lor \beta) \lor \gamma) \equiv (a \lor (\beta \lor \gamma)) \quad \text{associativity of } \lor \\ \neg (\neg a) \equiv a \quad \text{double-negation elimination} \\ (a \Rightarrow \beta) \equiv (\neg \beta \Rightarrow \neg a) \quad \text{contraposition} \\ (a \Rightarrow \beta) \equiv (\neg a \lor \beta) \quad \text{implication elimination} \\ (a \Rightarrow \beta) \equiv ((a \Rightarrow \beta) \land (\beta \Rightarrow a)) \quad \text{biconditional elimination} \\ \neg (a \land \beta) \equiv (\neg a \lor \neg \beta) \quad \text{De Morgan} \\ \neg (a \lor \beta) \equiv (\neg a \land \neg \beta) \quad \text{De Morgan} \\ (a \land (\beta \lor \gamma)) \equiv ((a \land \beta) \lor (a \land \gamma)) \quad \text{distributivity of } \land \quad \text{over} \\ (a \lor (\beta \land \gamma)) \equiv ((a \lor \beta) \land (a \lor \gamma)) \quad \text{distributivity of } \lor \quad \text{over} \land$$

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Validity and satisfiability

A sentence is valid if it is true in all models,

e.g., True,
$$A \lor \neg A$$
, $A \Rightarrow A$, $(A \land (A \Rightarrow B))$
 $\Rightarrow B$

Validity is connected to inference via the Deduction Theorem:

$$KB = a$$
 if and only if $(KB \Rightarrow a)$ is valid

A sentence is satisfiable if it is true in some model

e.g.,
$$A \lor B$$
, C

A sentence is unsatisfiable if it is true in no

models e.g.,
$$A \land \neg A$$

Satisfiability is connected to inference via the following:

 $KB \mid = a$ if and only if $(KB \land \neg a)$ is unsatisfiable

i.e., prove *a* by *reductio* ad absurdum



Proof methods

Proof methods divide into (roughly) two kinds:

Application of inference rules

- Legitimate (sound) generation of new sentences from old
- Proof = a sequence of inference rule applications
 Can use inference rules as operators in a standard search alg.
- Typically require translation of sentences into a normal form

Model checking

truth table enumeration (always exponential in n) improved backtracking, e.g.,

Davis—Putnam—Logemann—Loveland heuristic search in model space (sound but incomplete)

e.g., min-conflicts-like hill-climbing algorithms



Resolution

Conjunctive Normal Form (CNF—universal) conjunction of disjunctions of literals

cla uses

E.g.,
$$(A \lor \neg B) \land (B \lor \neg C \lor \neg D)$$

Resolution inference rule (for CNF): complete for propositional logic

Resolution is sound and complete for propositional logic



Conversion to CNF

$$B_{1,1} \Leftrightarrow (P_{1,2} \vee P_{2,1})$$

1. Eliminate \Leftrightarrow , replacing $a \Leftrightarrow \beta$ with $(a \Rightarrow \beta) \land (\beta \Rightarrow \beta)$

a).
$$(B_{1,1} \Rightarrow (P_{1,2} \lor P_{2,1})) \land ((P_{1,2} \lor P_{2,1})) \Rightarrow$$

2. Eliminate \Rightarrow , replacing $a \Rightarrow \beta$ with $\neg a \lor \beta$.

$$(\neg B_{1,1} \lor P_{1,2} \lor P_{2,1}) \land (\neg (P_{1,2} \lor P_{2,1}) \lor B_{1,1})$$

3. Move ¬ inwards using de Morgan's rules and double-negation:

$$(\neg B_{1,1} \lor P_{1,2} \lor P_{2,1}) \land ((\neg P_{1,2} \land \neg P_{2,1}) \lor B_{1,1})$$

4. Apply distributivity law (\vee over \wedge) and flatten:

$$(\neg B_{1,1} \lor P_{1,2} \lor P_{2,1}) \land (\neg P_{1,2} \lor B_{1,1}) \land (\neg P_{2,1} \lor B_{1,1})$$



Resolution algorithm

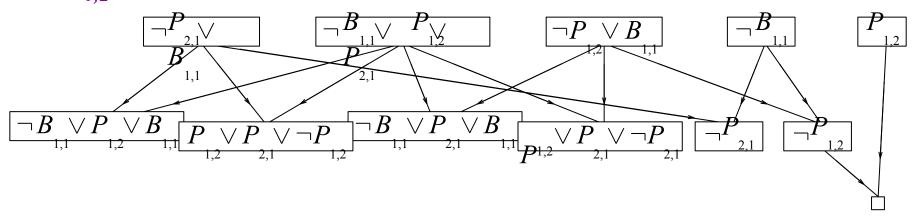
Proof by contradiction, i.e., show $KB \land \neg a$ unsatisfiable

```
function PL-Resolution(KB, a) returns true or false
   inputs: KB, the knowledge base, a sentence in propositional
   logic
             a, the query, a sentence in propositional logic
   clauses \leftarrow the set of clauses in the CNF representation of KB
   \wedge \neg a
   new \leftarrow \{\}
   loop do
        for each C_i, C_i in clauses do
             resolvents \leftarrow PL-Resolve(C_i, C_j)
if resolvents contains the empty clause then return
             true new \leftarrow new \cup resolvents
        if new \subseteq clauses then return
        false clauses ← clauses ∪ new
```



Resolution example

$$KB = (B_{1,1} \Leftrightarrow (P_{1,2} \lor P_{2,1})) \land \neg B_{1,1} \ a = \neg P_{1,2}$$





Forward and backward chaining

Horn Form (restricted)

Horn clause =

- proposition symbol; or
- (conjunction of symbols) \Rightarrow symbol E.g., $C \land (B \Rightarrow A) \land (C \land D \Rightarrow B)$

Modus Ponens (for Horn Form): complete for Horn KBs

$$a_1, \ldots, a_n, \qquad a_1 \wedge \cdots \wedge a_n$$

 $\Rightarrow \beta \beta$

Can be used with forward chaining or backward chaining. These algorithms are very natural and run in linear time



Forward chaining

Idea: fire any rule whose premises are satisfied in the *KB*, add its conclusion to the *KB*, until query is found

$$P \Rightarrow Q$$

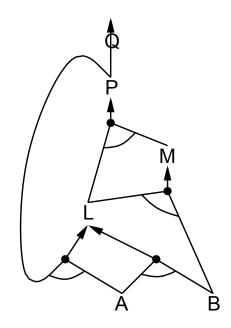
$$L \land M \Rightarrow$$

$$P \quad B \land L$$

$$\Rightarrow M \quad A \land$$

$$R \Rightarrow B \rightarrow$$

$$B$$

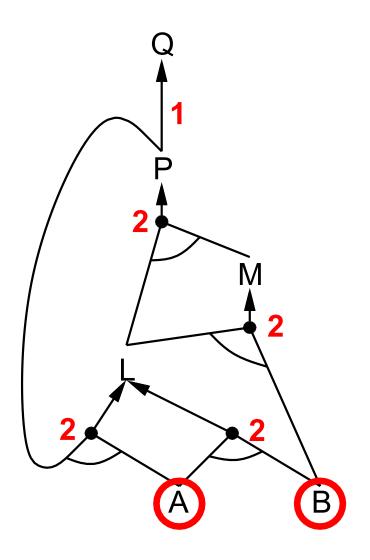




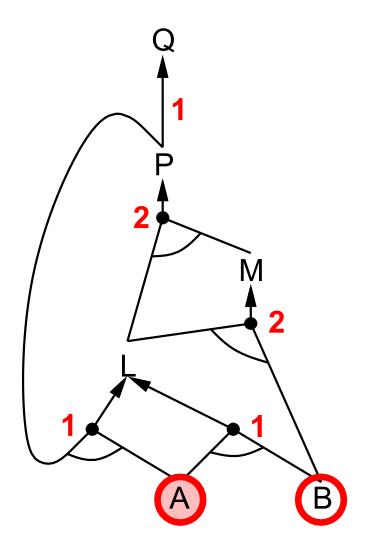
Forward chaining algorithm

```
function PL-FC-Entails?(KB, q) returns true or false
  inputs: KB, the knowledge base, a set of propositional Horn clauses
           q, the query, a proposition symbol
  local variables: count, a table, indexed by clause, initially the number of
                    premises inferred, a table, indexed by symbol, each
                    entry initially false agenda, a list of symbols, initially
                    the symbols known in KB
  while agenda is not empty
       do p \leftarrow Pop(agenda)
      unless inferred[p] do
           inferred[p] \leftarrow true
           for each Horn clause c in whose premise p appears do
               decrement count[c]
               if count[c] = 0 then do
                   if Head[c] = q then return true
                   Push(Head[c], agenda)
  return false
```

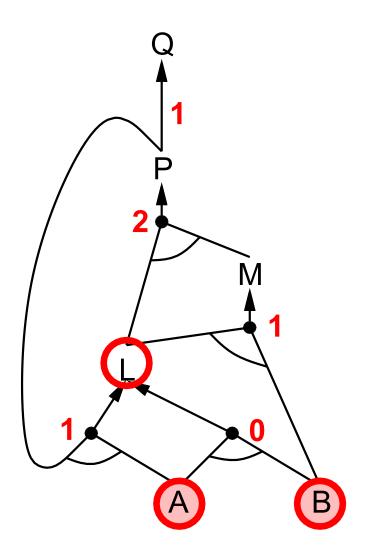




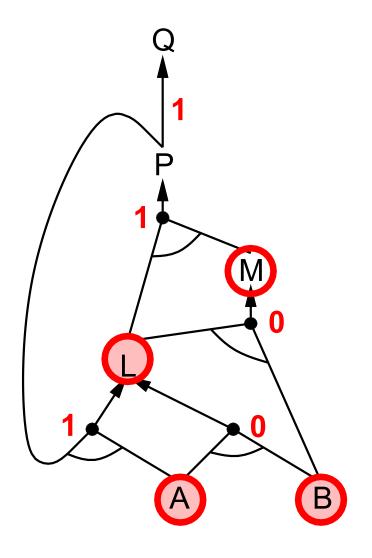




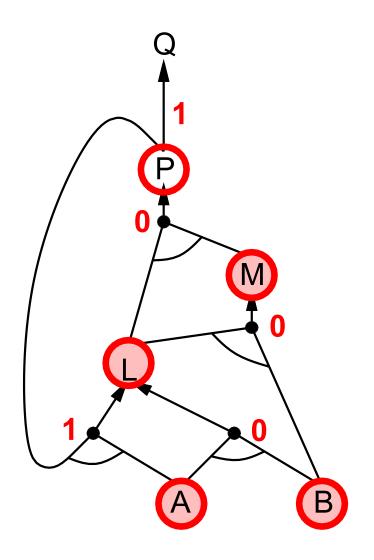




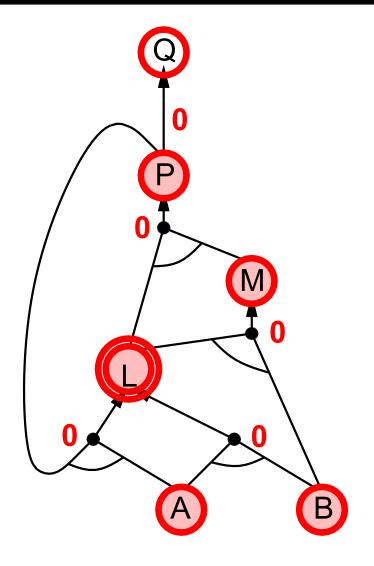




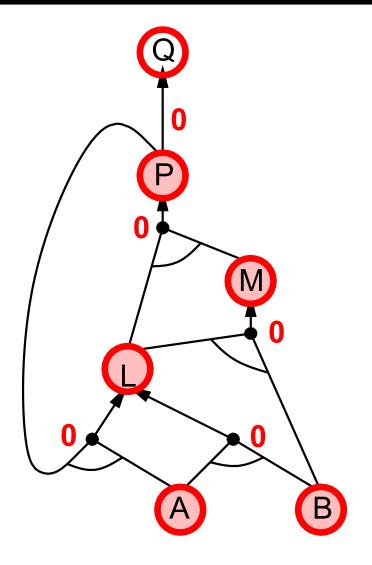




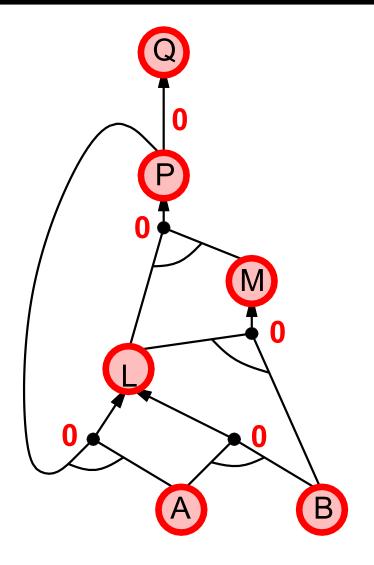














Proof of completeness

FC derives every atomic sentence that is entailed by KB

- 1. FC reaches a fixed point where no new atomic sentences are derived
- 2. Consider the final state as a model m, assigning true/false to symbols
- 3. Every clause in the original KB is true in mProof: Suppose a clause $a_1 \land \ldots \land a_k \Rightarrow b$ is false in m Then $a_1 \land \ldots \land a_k$ is true in m and b is false in m Therefore the algorithm has not reached a fixed point!
- 4. Hence m is a model of KB
- 5. If KB = q, q is true in every model of KB, including m

General idea: construct any model of KB by sound inference, check a



Backward chaining

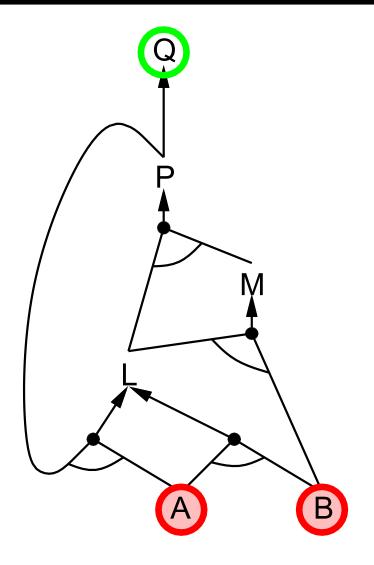
Idea: work backwards from the query

q: to prove q by BC, check if q is known already, or prove by BC all premises of some rule concluding q Avoid loops: check if new subgoal is already on the goal

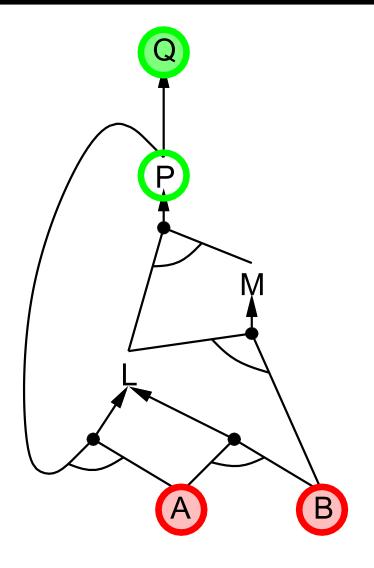
stack Avoid repeated work: check if new subgoal

- 1) has already been proved true, or
- 2) has already failed

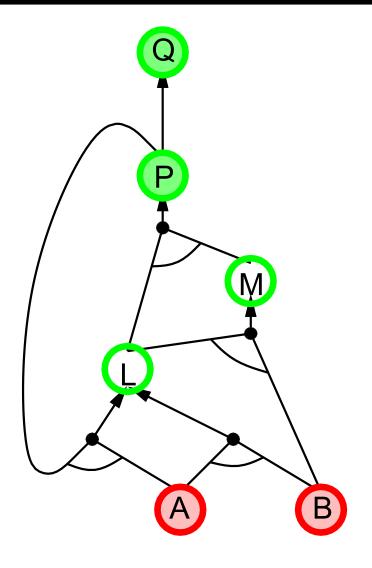




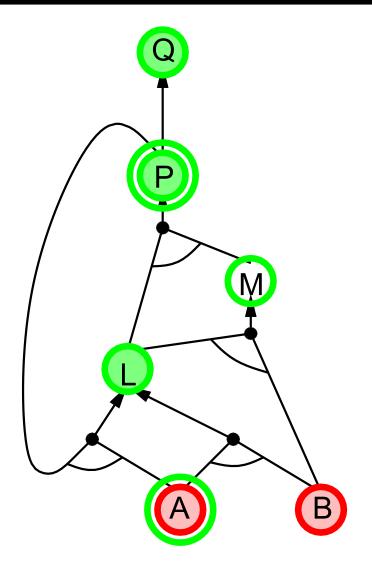




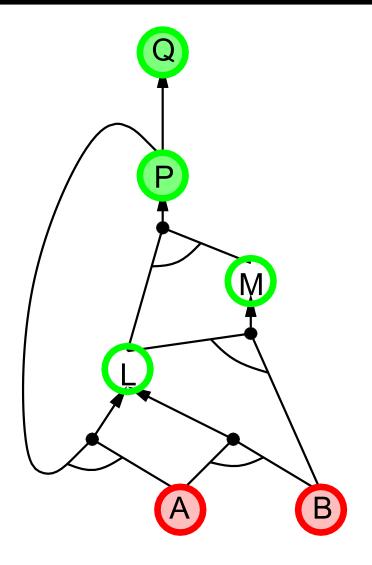




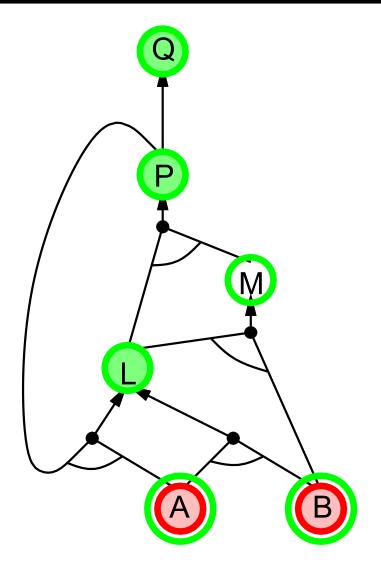




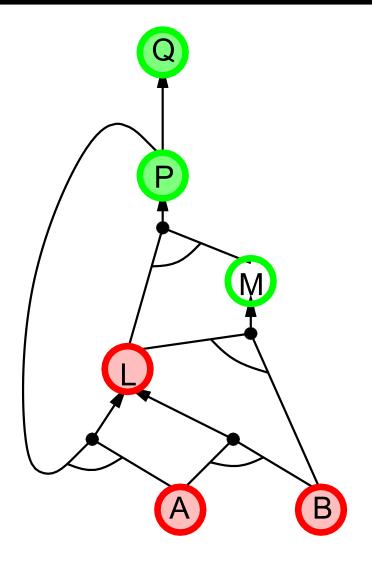




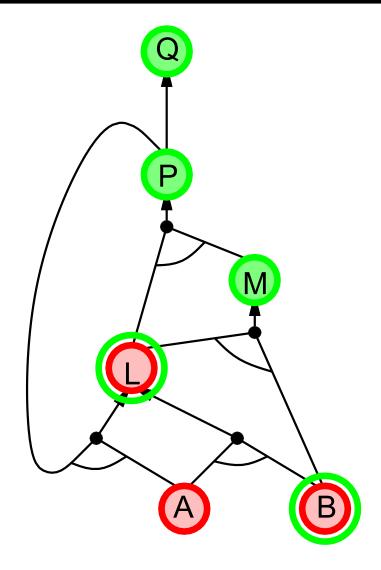




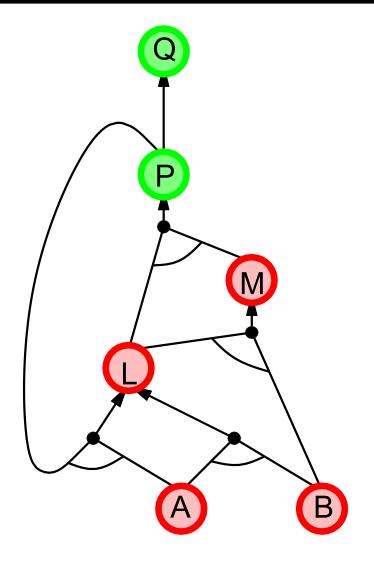




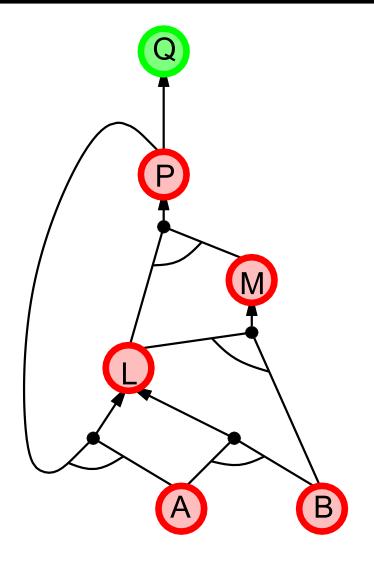




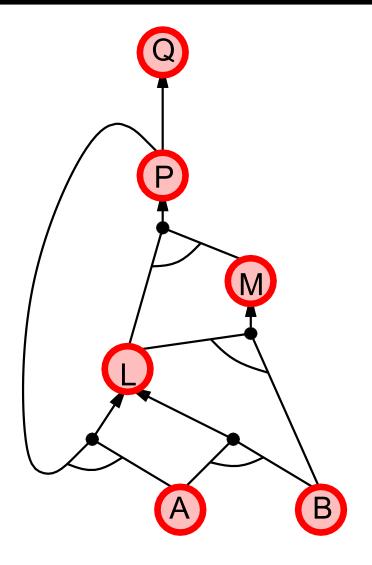














Forward vs. backward chaining

FC is data-driven, cf. automatic, unconscious processing, e.g., object recognition, routine decisions

May do lots of work that is irrelevant to the goal

BC is goal-driven, appropriate for

problem-solving,

e.g., Where are my keys? How do I get into a PhD program?

Complexity of BC can be much less than linear in size of KB



Effective Propositional Model Checking

Davis—Putnam algorithm with three improvements over TT-ENTAILS

- Early termination: detect T/F
- Pure symbol heuristic: same sign in all clauses
- Unit Clause heuristic: clause with one literal

Local search algorithms such as hill-climbing & simulated annealing.

In recent years, there has been a great deal of experimentation to find a good balance between greediness and randomness.

WALKSAT: On every iteration, the algorithm picks an unsatisfied clause and picks a symbol in the clause to flip.



Summary

Logical agents apply inference to a knowledge base to derive new information and make decisions

Basic concepts of logic:

- syntax: formal structure of sentences
- semantics: truth of sentences wrt models
- entailment: necessary truth of one sentence given another
- inference: deriving sentences from other sentences
- soundess: derivations produce only entailed sentences
- completeness: derivations can produce all entailed sentences

Wumpus world requires the ability to represent partial and negated information, reason by cases, etc.

Forward, backward chaining are linear-time, complete for Horn clauses. Resolution is complete for propositional logic. Propositional logic lacks expressive power.

Local search methods (WALKSAT) find solutions (sound but not complete).

