Exercise 1:

$x_0, x_1, x_2 \in \{0, 1\}$

$$E(x_0, x_1, x_2) = \phi_0(x_0) + \phi_1(x_1) + \phi_2(x_2) + \phi_p(x_0, x_1) + \phi_p(x_1, x_2) + \phi_p(x_0, x_2)$$

a) Evaluate $E(x_0, x_1, x_2)$ by hand for all possible configurations of x_0, x_1, x_2

i) all down-state $(0,0,0)$

$E(0,0,0) = 0{,}1 + 0{,}8 + 0{,}9 + 0 = 1{,}8$

ii) all up-state $(1,1,1)$

$E(1,1,1) = 0{,}9 + 0{,}1 + 0{,}1 + 0 = 1{,}1$

iii) x_1 up-state, x_2, x_3 ~~different~~ arbitrary

$E(1,0,1) = 0{,}9 + 0{,}8 + 0{,}1 + 2 = 3{,}8$

$E(1,1,0) = 0{,}9 + 0{,}1 + 0{,}9 + 2 = 3{,}9$

$E(1,0,0) = 0{,}9 + 0{,}8 + 0{,}9 + 2 = 4{,}6$

iv) x_1 down state

$E(0,0,1) = 0{,}1 + 0{,}8 + 0{,}9 + 2 = 3{,}8$

$E(0,1,0) = 0{,}1 + 0{,}1 + 0{,}9 + 2 = 3{,}1$

$E(0,1,1) = 0{,}1 + 0{,}1 + 0{,}1 + 2 = 2{,}3$

b) Which configuration of x_0, x_1, x_2 minimizes $E(x_0, x_1, x_2)$?

$E(1,1,1)$ and $E(0,0,0)$ are the 2 lowest energies. They minimize $E_{pott} = \sum \phi_p$. Because the upper state is more likely on 2 particles $E(1,1,1)$ minimizes E.

Exercise 2:

$$\psi_{ij}(z_i, z_j) = \begin{cases} \alpha & \text{if } z_i = z_j \\ \beta & \text{if } z_i \neq z_j \end{cases}$$

$$E(z) = \sum_{i,j} \psi_{ij}(z_i, z_j)$$

Find the configuration which minimizes E for:

a) $\alpha = 0$, $\beta = 1$:

for minimal E, ψ needs to be 0.

ψ is 0 for $z_i = z_j$

↳ neighbors need to be in the same state

$E = 0$ for
$$\begin{array}{ccccc}\downarrow & -0- & \downarrow & -0- & \downarrow\\ \downarrow & & \downarrow & & \downarrow\\ \downarrow & & \downarrow & & \downarrow\end{array}$$

all particles in down state (0)

$E = 0$ for
$$\begin{array}{ccc}\uparrow & \uparrow & \uparrow\\ \uparrow & \uparrow & \uparrow\\ \uparrow & \uparrow & \uparrow\end{array}$$

all particles in up state (1)

b) $\alpha = 1$, $\beta = 0$:

$\psi = 0$ for $z_i \neq z_j$

↳ neighbors need to be in different states

configuration 1

$E = 0$ for
$$\begin{array}{cccccc}\uparrow & -0- & \downarrow & -0- & \uparrow & \downarrow\\ \downarrow & & \uparrow & & \downarrow & \uparrow\\ \uparrow & & \downarrow & & \uparrow & \downarrow\\ \downarrow & & \uparrow & & \downarrow & \uparrow\end{array}$$

configuration 2

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Exercise 3: $\psi_{ij}(z_i, z_j) = \begin{cases} \alpha & \text{if } z_i = z_j \\ \beta & \text{if } z_i \neq z_j \end{cases}$

Is it possible to find Z such that $E(Z) = 0$ for ~~regar~~

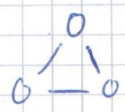
a) $\alpha = 0$, $\beta = 1$ regardless of the structure.
It is possible, because if all particles are in the same state ψ for any neighbor is 0.

Z all up-state

or

Z all down-state ~~minimizes~~ fullfills $E(Z) = 0$

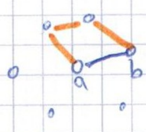
b) $\beta = 0$, $\alpha = 1$ regardless of structure.
This is not possible ~~e.g.~~ i.e. for the configuration of exercise 1



we can not achieve that all neighbors are in different states.

The „chain" which connects particle a to the neighbor b needs to be odd:

"chain": 3



Maybe the consequence is, that a particle needs to have an even number of neighbors

b