Simulating, Propagating and Modeling DROs

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Space Exploration, Space Flight

This document has been made in conjunction with Stahl [2020].

Executive Summary

This project will deal with the possibilities of propagating and modeling Distant Retrograde Orbits (DROs). DROs are a family of specific orbits in a two-body system like the Earth-Moon system or the Sun-Jupiter system. Their biggest advantage is their long-term stability that usually withstands the known perturbating forces. And yet, a DRO has never been performed by a man-made object. Due to a lack of already existing research when it comes to modeling DROs, the thesis project shall investigate the possibilities of propagating and modeling DROs.

This is achieved with two major steps: Firstly, a set of elements has to be chosen that is able to propagate DROs efficiently. For this purpose, several possible candidates have to be found which will then be numerically compared with regard to their performance. This will be done with the Tudat (Delft University of Technology [2018]). Secondly, with the newly defined set of elements, a method of modeling DROs shall be found. This can for example be based on a Fourier analysis.

DROs (in the Earth-Moon system) are the best orbit a retrieved asteroid could be put in the future. This is due to the aforementioned stability and the high energy level of DROs with respect to the Earth, which makes it easier (less ΔV-intensive) to put the asteroid there. The thesis project aims to contribute to the tools that can be used for analyzing DROs in order to enable future asteroid retrieval missions.

1. Introduction

Asteroid retrieval and mining has been the dream of many and whoever achieves this is guaranteed to be at a blow the richest person alive. This mission comes with many scientific and engineering problems of which not all can be solved with today's technology. However, the destination orbit of the retrieved asteroid can very well be analyzed with current means. As being the most stable orbit possible (see next section), DROs in the Earth-Moon system are the most suitable destination orbit for such an asteroid. Therefore, analyzing them is of high interest for future missions.

This research proposal is structured as follows: The second section describes the state-of-the-art of DROs that has been found during the literature research. The third section explains the research questions and objectives that the thesis project aims to deal with. The fourth section explains with which methodological means those will be reached. Therefore, the fifth section explains the set-up that the simulations will have. The expected results of the research can be found in the sixth section. The planning of the thesis is in the seventh section with the corresponding Gantt chart being in the appendix. Finally, the findings of this research proposal are concluded in the eighth section.

2. State-of-the-art/Literature Review

When it comes to modeling DROs, an in-depth literature review has been made (Stahl [2020]) in order to determine the state-of-the-art of DROs as well as to reveal remaining scientific questions regarding DROs. The results of the literature review are mentioned briefly in the following paragraphs.

DROs are orbits performed by a third body about the secondary of a two-body system. The direction is opposed to the direction in which primary and secondary are orbiting each other. This orbit is known to be the most stable one among all orbits that third bodies can perform in a two-body system (Henon [1968], Benest [1974], Ming and Shijie [2009], Pires and Winter [2020], Turner [2016], Bezrouk and Parker [2014, 2017], Lam and Whiffen [2005], Lara et al. [2007]).

DROs are usually represented with the Circular Restricted Three-Body Problem (CR3BP). While its mathematics are beyond the scope of this text, its assumptions are briefly mentioned here: The primary and the secondary are orbiting their common barycenter on circular orbits. The third body's mass is negligibly small compared to primary and secondary. The third body's position is given in a coordinate system that has its origin in the barycenter of primary secondary and that rotates such that the primary's and secondary's coordinates do not change. The second usual way of describing DROs is with the coordinates of the Hill problem which alter the CR3BP's assumptions a little bit: The third body is assumed to be *close* to the secondary. Furthermore, the secondary is assumed to have a negligible mass with respect to the primary, which makes the secondary's and third body's masses of the same order. They experience mutual attraction because they are *close* to each other.

Regarding the modeling of DROs the following has been found: There are two modeling techniques that already exist. Firstly, a Fourier series approach that enables calculating position and velocity of a periodic DRO in the CR3BP efficiently (Hirani and Russell [2006]). Secondly, a Lindstedt-series approach that accounts for the librating corrections that non-perfect quasi-periodic DROs are facing. This is done with the assumptions of the Hill problem, which include that the secondary and the third body are *close* to each other, which can not be expected to be true in the general case (Lara [2018, 2019]). None of those existing modeling techniques are flexible enough to take (state- and time-dependent) perturbations of any kind into account. Therefore, a new modeling technique for DROs would be beneficial to have.

Regarding transfers to DROs: The transfer to DROs can be categorized into three classes. Transfers from the primary to the DRO (Capdevila et al., Capdevila and Howell [2018], Minghu et al. [2014], Welch et al. [2015], Murakami and Yamanaka [2015]), transfers from the secondary to the DRO (Demeyer and Gurfil [2007], Demeyer et al. [2007], Ocampo and Rosborough [1993], Scott and Spencer [2010a,b], Scott [2010], Zhang et al. [2020]) and transfers from a place or orbit that lies outside of the primary-secondary system (Muirhead and Brophy [2014], Bezrouk and Parker [2018]). The first two classes have been subject of intensive research already, resulting in detailed descriptions of the trajectory properties. The transfer from the outside to a DRO has not been analyzed in detail yet. Therefore, this can be considered an interesting research topic for the thesis project. This is relevant for two different cases: capturing an asteroid into an Earth-Moon DRO and inserting a spacecraft into a planet-moon DRO, for example a Jupiter-Europa DRO. These cases are vastly different since the spacecraft can get a significantly larger ΔV caused by propulsion due to its smaller mass. Therefore, those two transfers are to be investigated.

3. Research Question, Aim/Objectives and Sub-goals

3.1. Research Question(s)

The available literature that has been reviewed in the previous section is pointing towards one major gap when it comes to analyzing DROs. There is no method of efficiently modeling DROs yet. Therefore, the main research question is:

Can a novel set of elements suit the requirements for both, modeling and propagating DROs?

This main research question can be divided into sub-questions. They are:

- Can the elliptical shape of DROs in the Hill problem be exploited to come up with the new set of elements that also suits the more general CR3BP?
- Can the new set of elements help to model and understand DROs better?
- Can the new set of elements help to predict DROs, their long term behavior, and their stability better and more efficiently?
- Does this new set of elements perform better in terms of accuracy and computation time than the existing propagators?
- Does the performance of the new set of elements depend on the mass ratio of the primaries? Since the set of element is developed with the help of the Hill problem which assumes zero mass for the secondary, this is not naturally the case.
- Can complete transfers to DROs be modeled with the new set of elements? The transfer to a DRO usually takes place far outside the range of the DRO. Therefore, their usefulness needs to be evaluated. This can be divided into the different kinds of transfers that are possible for DROs.

3.2. Research Objective

The main research objective of this thesis is:

To develop a new set of elements for DROs using numerical simulations of three-body systems.

This main objective can be divided into several sub-objectives. The first of which is to develop candidates for sets of elements that might perform better in simulating DROs than the known propagators. In order to cover a broad variety of possibilities, different techniques shall be used. For example, the elliptic shape of DROs in the Hill problem case can be exploited or an approach similar to the Encke method can be tried in combination with the Fourier series approach. Before comparing the methods, they have to be implemented in Tudat. They shall be compared to each other and to usual propagation methods first in the CR3BP and then using the full ephemeris. As a result of this comparison, one of the candidates should be selected and used for the further investigation.

To assess, if the new set of elements can help modeling DROs more efficiently than the already know ones, a Fourier analysis similar to the one in Hirani and Russell [2006] shall be performed. Furthermore, the new set of elements shall be used for a stability analysis and a transfer analysis in order to confirm its viability. The work packages that are needed for this can be found in more detail in Section 7.

4. Theoretical Content/Methodology

The goal of the thesis is to come up with a set of elements that is suitable for simulating DROs and coming up with a technique for modeling them. Therefore, the thesis does not confirm or falsify any hypotheses but it is the documented search for an approach to a specific problem. The path to reach this goal itself does contain hypotheses that shall be confirmed, such as the performance of specific sets of elements.

The scientific approach that will be taken for this thesis project is numerical simulation. The candidates for sets of elements will be used to propagate various DROs in different two-body systems. For each scenario, a comparison to a benchmark (a propagation with a step-size as low as possible – meaning that the rounding errors do not exceed the truncation errors) reveals which of the sets of elements performs best and archives the most accurate result with the lowest amount of computational power.

In the next step, a modeling technique for DROs has to be created. This includes, but is not limited to, a Fourier analysis of the elements in the Circular Restricted Three-Body Problem (CR3BP) as performed with the usual CR3BP coordinate set by Hirani and Russell [2006]. Other than that another modeling method has to be come up with in the process of the thesis work.

5. Experimental Set-up

The TU Delft Astrodynamics Toolbox (Tudat) (Delft University of Technology [2018]), whose documentation can be found in Delft University of Technology [2018], is a tool for astrodynamic simulations. It can help answering astrodynamic problems, finding and optimizing trajectories, incorporating perturbations of all kinds (spherical harmonics gravity, radiation pressure, atmospheric drag, ...), etcetera. It consists of C++ libraries that are publicly available, developed mainly at Delft University of Technology. It can be run with C++ and JSON files and recently also with Python.

MATLAB (MathWorks Inc. [2020]), originally *MATrix LABoratories*, is a software created by the company *The MathWorks, Inc.* that aims to offer an easy solution for scientific programming. It can handle matrices, vectors, imaginary numbers, calculations of any kind, etcetera. It can be used in an object-oriented way as well as without

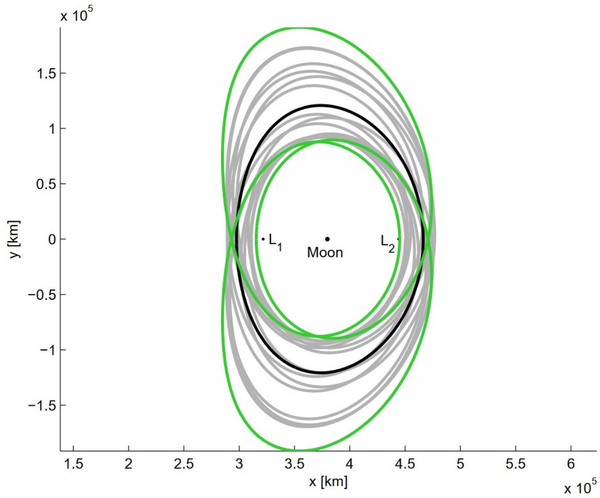

Figure 1: Periodic DRO (black), aperiodic DRO (grey), and period-3 DRO (green) in the Earth-Moon system. Capdevila et al.

the use of objects at all. For the thesis, the main benefit of MATLAB will be the many functionalities regarding plotting that help to present the achieved knowledge in an understandable way.

Furthermore, MATLAB is perfectly able to run simple simulations as finding the initial conditions for DROs or calculating the monodromy matrix of one orbit. More complex tasks as trajectory planning or simulations that include perturbations are left to Tudat which can not only handle perturbations efficiently and with less coding effort, but works more efficiently in general because it is based on C++.

In order to represent a larger number of different problems, the simulations will be done with different mass ratios of the primaries as for example 1:330000 (Sun-Earth), 1:1000 (Sun-Jupiter) or 1:100 (Earth-Moon). The performed orbit will be varying from a perfectly periodic DRO to a highly aperiodic DRO, which can be seen in Figure 1. The more regular the DRO is, the more it is expected to be represented well by a set of elements that is specialized on DROs.

6. Results, Outcome and Relevance

The result of the thesis work will be a set of elements with which DROs can be propagated efficiently on the one hand and a modeling technique for DROs on the other hand. Both of them are of inherent importance for future space missions that involve DROs as for example asteroid retrieval missions.

Validation and verification of the newly defined set of elements is done by applying it to a real world problem with the full ephemeris and examining if the performance is indeed better than the one of already known sets of elements.

7. Project Planning and Gantt Chart

The first phase of the thesis project is coming up with several candidates for sets of elements and selecting the best performing one. Therefore, the first step is to develop those candidates and calculate their state derivatives so that they can be implemented in Matlab and Tudat – which is the second step. The third and the fourth step will be to simulate those elements in various scenarios in order to evaluate the best of them.

The second phase of the thesis project is to analyze DROs with this new set of elements. This includes as a first step the modeling of DROs and secondly a stability analysis of DROs. Finally, the possibilities of simulating transfers to DROs are to be explored.

The last work package includes finalizing the thesis report and the thesis defense preparation. A small overall buffer is included, although the work packages themselves are meant to have their own buffer time as well as time for report writing.

The Gantt chart is to be found in Appendix A. Its legend is:

- **WP1:** Defining sets of elements
  - **WP1.1:** By exploiting the elliptic shape of DROs
  - **WP1.2:** Computing state derivatives of determined elements
  - **WP1.3:** Validate state derivatives with Matlab
- **WP2:** Implementation in Tudat
  - **WP2.1:** Install Tudat
  - **WP2.2:** Repeat basics of Tudat
  - **WP2.3:** Implementation in Tudat
- **WP3:** Verification and performance evaluation in CR3BP
  - **WP3.1:** Setting up example test cases in CR3BP
  - **WP3.2:** Evaluate performance of different propagators
- **WP4:** Verification and performance evaluation with full ephemeris
  - **WP4.1:** Setting up three-dimensional example test cases
  - **WP4.2:** Evaluate performance of different propagators
- **WP5:** Modeling of DROs with new elements
  - **WP5.1:** Fourier analysis of periodic DROs
  - **WP5.2:** Other modeling techniques
- **WP6:** Stability analysis with new elements
- **WP7:** Transfer with new elements
  - **WP7.1:** Transfers from the primary
  - **WP7.2:** Transfers from the secondary
  - **WP7.3:** Transfers form the outside of the two-body system
- **WP8:** Final reporting, buffer, and thesis defense
  - **WP8.1:** Final changes to report
  - **WP8.2:** Buffer
  - **WP8.3:** Defense preparation

8. Conclusions

To conclude, the aim of the approaching thesis work is to come up with a coherent method of describing, simulating, and modeling DROs. This shall be achieved by developing a set of elements that can be used to efficiently propagate DROs. With this set of elements, DROs can expectedly be analyzed in a better way than before. This is what happens in the second phase of the thesis project.

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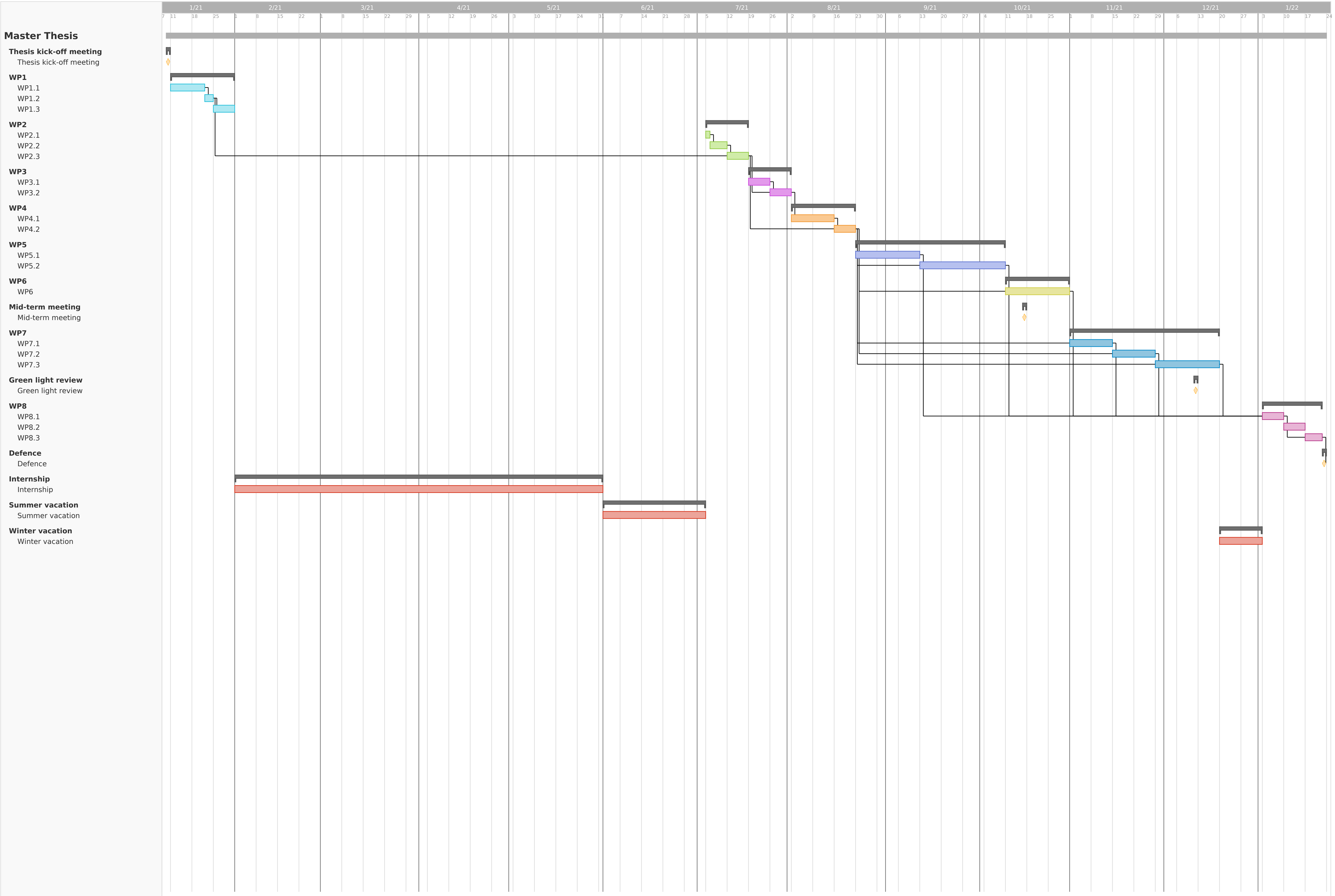
A. Gantt Chart

(The Gantt Chart is inserted on the next page.)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Master Thesis
Thesis kick-off meeting
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Defence
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Internship
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Summer vacation
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