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The Kantorovich Formulation is an infinited dimensional problem. In order to understand it, we will first consider the finite dimensional problem.(add graphics - DJM)

Suppose we want to find the global minimum of a function, f, with multiple local minima. A common approach is to take the derivative, but that may only give us a local minimum rather than the global minimum we are after. THe Kantorovhich duality proposes a new to find a minimum.

The idea is to find a function, g, that is less than or equal to f everywhere (should this be a.e ???). If there is a point, x, where g touches f, then we can conclude that x is global minimum for f.

Suppose we have 3 suppliers x_1, x_2, x_3 and three buyers y_1, y_2, y_3 and $c(x_i, y_j)$ is the cost of a transaction between supplier j and buyer i. In the context of the Monge problem a buyer can only purchase from one supplier and a supplier can only sell to one buyer. Practically, this may not be ideal though. A buyer may want to purchase from multiple suppliers. The Kantorovich formulation offers a more relaxed option. in which a buyer can purchase from more than one seller and same for the suppliers selling to buyers. There is a condition imposed in both the Monge and Kantorovich formulations: the amount of supplies sold must be equal to the amount of supplies purchased. (Include graphic/example - DJM)

Now we will define a transport map and potential.

Definition (Transport Map)
A transport map $T : \mathbb{R}^d \to \mathbb{R}$ is a borel measure such that … (DJM)

Definition (Outer Probability Measure)
An outer probability measure on $\mathbb{R}^d$ is a real-valued function, μ , defined on the subset of $\mathbb{R}^d$ which satifies the following

- $\mu(\emptyset) = 0$
- $0 \leq \mu[A] \leq \sum_{i=1}^{\infty} \mu[A_i]$ if $A \subset \cup_{i=1}^{\infty} A_i$
- $\mu[\mathbb{R}^d] = 1$

notation
$2^{\mathbb{R}^d}$ is the collection of all subsets of $\mathbb{R}^d$. If x is a set, 2^x is the collection of all subsets of x.