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<u>Dirac Masses</u>

Suppose $a \ \epsilon \ \mathbb{R}^3(\mathbb{R}^d, X)$ where X is some arbitrary set.

Say We want to know if a particle is in the set E. (include graphic - DJM)

- $\delta_a(E) = 1 \quad a \ \in \ E$
- $\delta_a(E) = 0 \quad a \ \notin \ E$

δ_a: a dirac mass concentrated at a

- $\delta_a(\not\emptyset) = 0$
- $\delta_a(\cup_{i=1}^{\infty} E_i) = \sum_{i=1}^{\infty} E_i \quad E_i = E_j = \not\emptyset \ \ when \ \ i \neq j$
  – i.e a can only be present in one.

- $\delta_a[\mathbb{R}^d] = 1$

We want to check if the dirac mass is a probability measure. Often times in probability (??? DJM) we are looking at the average of dirac masses.

<u>Examples</u>

- $\mu = \frac{1}{n} \sum_{i=1}^{n} \delta_{a_i} = \frac{\delta_{a_1} + \delta_{a_2} + \ldots + \delta_{a_n}}{n}$ Here we are counting how many particles are in A and dividing by n to average.

- $\mu[A] = \frac{k}{n}$ where k = (number of particles)

Notation

- $\Omega :$ Probability space (a set)
- $\mathbb{P} :$ Probability measure on $\Omega \implies \mathbb{P} : \Sigma \to [0, 1]$
- $\Sigma :$ a σ-algebra on Ω
- $\mathbb{R}^d :$ a vector space of d dimensions

- B: borel σ-algebra on $\mathbb{R}^d$

(add graphic - DJM)

Defintions

- Borel σ-algebra : the smallest σ-algebra containing the open subsets in $\mathbb{R}^d$
- Boret Set : any element of a Borel σ-algebra

Aside

$\Theta \subset \mathbb{R}$ is open IFF Θ is a union of open intervals

OT broad overview

1. start with a map $T : \Omega \to \mathbb{R}^d$
2. construct a set $B \subset \mathbb{R}^d$
3. Pull the set back to Ω to get $T^{-1}(B) \subset \Omega$
4. Find a probability measure $\mu(B) = \mathbb{P}[T^{-1}(B)]$ where μ is called the *law* of T