Forces on a Model Rocket

This is Rocket Science
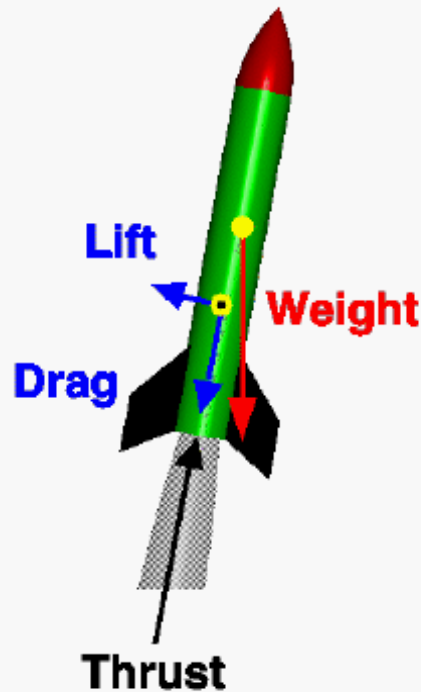
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http://www.grc.nasa.gov/WWW/K-12/aboutltp/EducationalTechnologyApplications.html

Forces on a Model Rocket — Glenn Research Center

Flying model rockets is a relatively inexpensive way for students to learn the basics of aerodynamic forces and the response of vehicles to external forces. Like an aircraft, a model rocket is subjected to the forces of weight, thrust, drag, and lift. There are, however, some important differences in the actions of these forces on a model rocket as opposed to a powered aircraft or a glider:

1. On an airplane, most of the aerodynamic forces are generated by the wings and the tail surfaces. For a model rocket, the aerodynamic forces are generated by both the body and the fins. For both aircraft and model rocket, the aerodynamic forces act through the center of pressure (the yellow dot with the black center on the figure).

2. On an airplane, the **lift force** (the aerodynamic force perpendicular to the flight direction) is used to overcome the weight of the aircraft. On the model rocket, thrust is used in opposition to weight, and lift is used to stabilize and control the direction of flight. As the rocket is buffeted by the wind, it naturally turns back to stable flight if the center of pressure is located below the center of gravity (the plain yellow dot on the figure).

3. While most aircraft have a high lift to drag ratio, the drag of a model rocket is usually much higher than the lift.

4. While the magnitude and direction of the forces remain fairly constant for an aircraft, the magnitude and direction of the forces acting on a model rocket change dramatically during a typical flight.

Separate slides show the forces acting on a model rocket during liftoff, powered flight, the coasting flight going upward and downward, and the final parachute recovery.

A model rocket rotates in flight about the center of gravity (the average location of the weight of the vehicle), just like any other flying object. It is easier to determine the center of gravity for a model rocket than for an aircraft. There are fewer components on a model rocket, and the geometry is much simpler than for an aircraft.

NOTE: *Modern full scale rockets do not usually rely on aerodynamics for stability. Full scale rockets can pivot their exhaust nozzles to provide stability and control. That's why you don't see fins on a Delta, Titan, or Atlas booster.*

Newton's First Law
Applied to Model Rocket Liftoff

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"Every object persists in its state of rest or uniform motion in a straight line unless it is compelled to change that state by forces impressed on it."

Before release:
  Object in state of rest, airspeed zero.
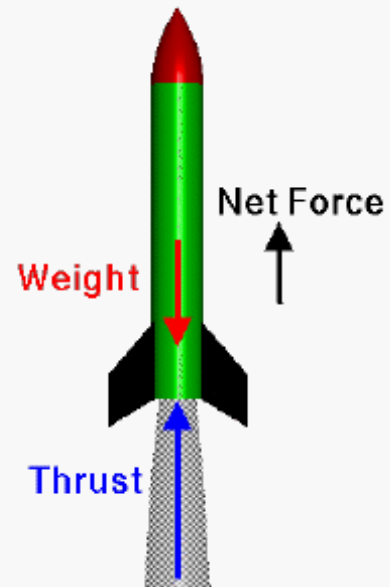
Engine fired:
  Thrust increases from zero.
  Weight decreases slightly as fuel burns.

When Thrust is greater than Weight:
  Net force (Thrust – Weight) is positive upward.
  Rocket accelerates upward –– Velocity increases from zero.

Net Force

Weight

Thrust

Sir Isaac Newton first presented his three laws of motion in the "Principia Mathematica Philosophiae Naturalis" in 1686. His first law states that every object will remain at rest or in uniform motion in a straight line unless compelled to change its state by the action of an external force. This is normally taken as the definition of **inertia**. The key point here is that if there is no net force acting on an object (if all the external forces cancel each other out) then the object will maintain a constant velocity. If that velocity is zero, then the object remains at rest. And if an additional external force is applied, the velocity will change because of the force.

A model rocket lifting off from the launch pad is a good example of this principle. Just prior to engine ignition, the velocity of the rocket is zero and the rocket is at rest. If the rocket is sitting on its fins, the weight of the rocket is balanced by the re-action of the earth to the weight as described by Newton's third law of motion. There is no net force on the object, and the rocket would remain at rest indefinitely.
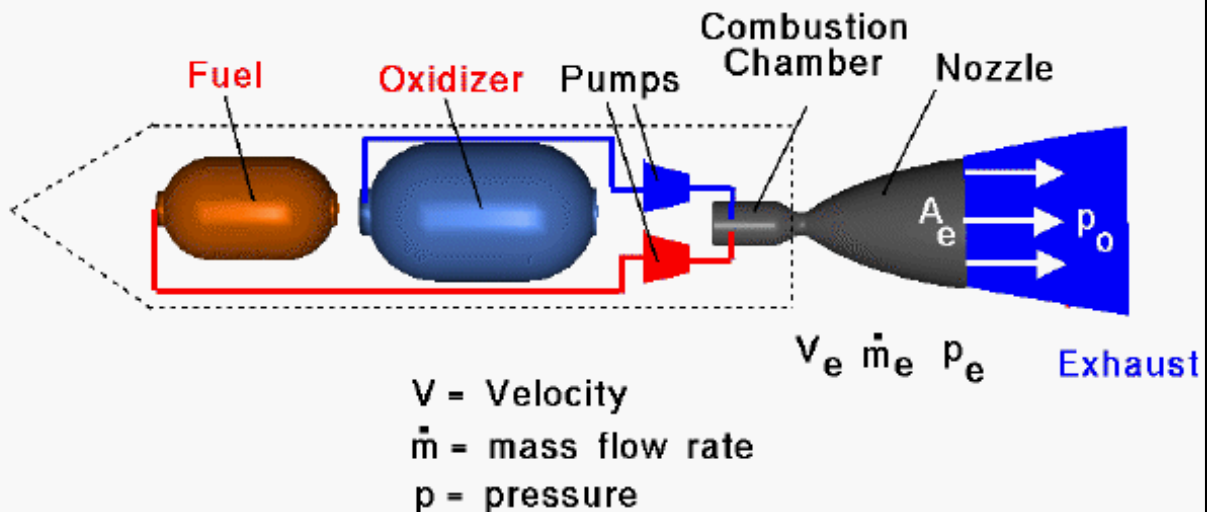
When the engine is ignited, the thrust of the engine creates an additional force opposed to the weight. As long as the thrust is less than the weight, the combination of the thrust and the re-action force through the fins balance the weight and there is no net external force and the rocket stays on the pad. When the thrust is equal to the weight, there is no longer any re-action force through the fins, but the net force on the rocket is still zero. When the thrust is greater than the weight, there is a net external force equal to the thrust minus the weight, and the rocket begins to rise.

The velocity of the rocket increases from zero to some positive value under the acceleration produced by the net external force. But as the rocket velocity increases, it encounters air resistance, or drag, which opposes the motion and increases as the square of the velocity. The thrust of the rocket must be greater than the weight plus the drag for the rocket to continue accelerating. If the thrust becomes equal to the weight plus the drag, the rocket will continue to climb at a fixed velocity, but it will not accelerate.

On this slide, we show a schematic of a liquid rocket engine. In a rocket, stored fuel and stored oxidizer are pumped into a combustion chamber where they are mixed and exploded. The hot exhaust is then passed through a nozzle, which accelerates the flow. The exit velocity is determined by the shape of the rocket nozzle and is supersonic. The exit pressure is set by the nozzle shape as well and will only be equal to free stream pressure at some design condition. We must, therefore, use the longer version of the thrust equation to describe the thrust of the system. If the mass flow is denoted by "m dot", the exit velocity by Ve, the pressure at the exit by pe, the free stream pressure by p0, and the nozzle exit area by Ae, the thrust (F) is given by:

$$F = m \ dot * Ve + (pe - p0) * Ae$$

This thrust equation works for both liquid and solid rocket engines. The mass flow rate through the propulsion system is determined by the nozzle design. You can explore the design and operation of a rocket nozzle with our interactive nozzle simulator program which runs on your browser. Notice that there is no free stream mass times free stream velocity term in the thrust equation because no external air is brought on board. Since the oxidizer is carried on board the rocket, rockets can generate thrust in a vacuum where there is no other source of oxygen. That's why a rocket will work in space, where there is no surrounding air, and a gas turbine or propeller will not work. Jets and propellers rely on the atmosphere to provide the oxygen.

In general: $F = G \dfrac{m_1 m_2}{d^2}$

Force equals a gravitational constant times the product of the masses divided by the square of the distance between the masses.

$$g = G \frac{m_{earth}}{d^2_{earth}} = 9.8 \frac{meter}{sec^2} = 32.2 \frac{feet}{sec^2}$$

$$W = m\,g$$

Weight equals mass times gravitational acceleration.

Weight is the force generated by the gravitational attraction of the earth on an airplane or model rocket. Weight is fundamentally different from the aerodynamic forces, lift and drag. Aerodynamic forces are mechanical forces and the vehicle has to be in physical contact with the air which generates the force. The gravitational force is a field force; the source of the force does not have to be in physical contact with the object (the airplane or rocket).

The nature of the gravitational force has been studied by scientists for many years and is still being investigated by theoretical physicists. For an object the size of an airplane flying near the earth, the descriptions given three hundred years ago by Sir Isaac Newton work quite well. Newton published his theory of gravitation with his laws of motion in 1686. The gravitational force between two particles equals a universal constant times the product of the mass of the particles divided by the square of the distance between the particles.

If you have a lot of particles acting on a single particle, you have to add up the contribution of all the individual particles. For objects near the earth, the sum of the mass of all the particles is simply the mass of the earth and the distance is then measured from the center of the earth. On the surface of the earth the distance is about 4000 miles. Scientists have combined the universal gravitational constant, the mass of the earth, and the square of the radius of the earth to form the gravitational acceleration, **g** . On the surface of the earth, it's value is 9.8 meters per square second or 32.2 feet per square second. The weight, or gravitational force, is then just the mass of an object times the gravitational acceleration (W = mg).

The Weight Equation

In general: $F = G \dfrac{m_1 m_2}{d^2}$

Force equals a gravitational constant times the product of the masses divided by the square of the distance between the masses.

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$W = m\,g$

Weight equals mass times gravitational acceleration.

(Continued from page 5)

Since the gravitational constant (g) depends on the square of the distance from the center of the earth, we would expect that the weight of an airplane would decrease as we fly higher. Let's do a test problem to see how much the weight changes. If an airplane is flying at 35000 feet (about 7 miles) the distance to the center of the earth is about 4007 miles.. We can calculate the change in the gravitational constant as the square of (4000/4007) which equals .9965. If the airplane weighs 10000 pounds on the surface of the earth, it weighs 9965 pounds at 35000 feet; it has lost 35 pounds, a very small amount compared to 10000 pounds.

The magnitude of the airplane's weight depends on the mass of all of the parts of the airplane itself, plus the amount of fuel, plus any payload on board (people, baggage, freight, ...). The weight is distributed throughout the airplane, but we can often think of it as collected and acting through a single point called the center of gravity. In flight, the airplane rotates about the center of gravity, but the direction of the weight force always remains toward the center of the earth. During a flight the aircraft burns up its fuel, so the weight of the airplane constantly changes. Also, the distribution of the weight and the center of gravity can change, so the pilot must constantly adjust the controls to keep the airplane balanced.

Determining Rocket Weight
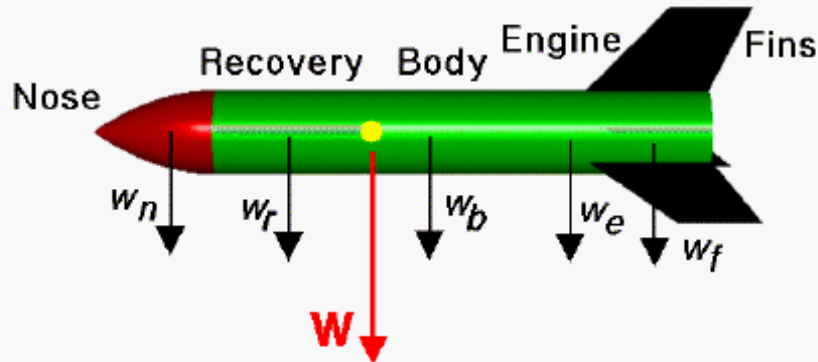
Glenn Research Center

Each component has some weight $w_i = m_i g$ where m_i is the component mass and where g is the gravitational acceleration.

Total rocket weight W is the sum of the component weights.

$$W = w_n + w_r + w_b + w_e + w_f$$

Discrete: $W = \sum_i^n w_i$ Differential: $W = \int w(x)\, dx$

Weight is the force generated by the gravitational attraction of the earth on the model rocket. The mass (and weight) is actually distributed throughout the rocket and for some problems it is important to know the distribution. But for rocket trajectory and stability, we only need to be concerned with the total weight and the location of the center of gravity. The center of gravity is the average location of the mass of the rocket.

How do engineers determine the rocket weight?

In general, determining the weight is a complicated procedure requiring the use of calculus. We'll discuss the general answer at the bottom of this page. This figure shows a simplified version of this calculation which can be used by secondary students. For this figure, we assume that we already know the weight of each of the major parts of the rocket (the nose, recovery system, body tube, engine, and fins). The individual component weight is simply the mass of the component times the gravitational constant, **g** , which is 32.2 ft/square sec in English units and 9.8 meters/square sec in metric units. The total weight of the rocket is simply the sum of all the individual weights of the components.

We can generalize the addition of component weights and the result is the "discrete" version shown at the lower left of the figure. The zig-zag symbol is the greek letter sigma - which indicates summation. The "i" below the sigma is an index, and the index goes from 1 to some number "n", which is the total number of parts. So this equation says that the weight of the rocket is equal to the sum of the weight of "n" parts. This equation would work for "n" discrete parts, but what if the parts are not discrete? What if we had a continuous change of mass from front to
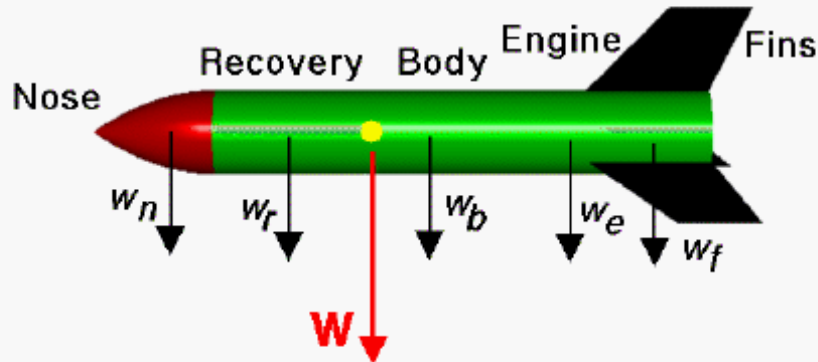
Determining Rocket Weight — Glenn Research Center

Each component has some weight $w_i = m_i g$ where m_i is the component mass and where g is the gravitational acceleration.

Total rocket weight $\mathbf{W}$ is the sum of the component weights.

$$\mathbf{W} = w_n + w_r + w_b + w_e + w_f$$

Discrete: $\mathbf{W} = \sum_i^n w_i$ Differential: $\mathbf{W} = \int w(x)\,dx$

back (as occurs for a balsa wood nose)? The continuous change can be computed using integral calculus, as shown in the "differential" equation at the lower right. The w(x) symbol represents the weight which is some function of distance x. If we are given the form of the function, there are methods to integrate the equation. If we don't know the actual functional form, we can still numerically integrate the equation using a spread sheet by dividing the distance up into a number of small distance segments and determining the average value of the mass (or weight) over that small segment.

Determining Center of Gravity – cg   Glenn Research Center
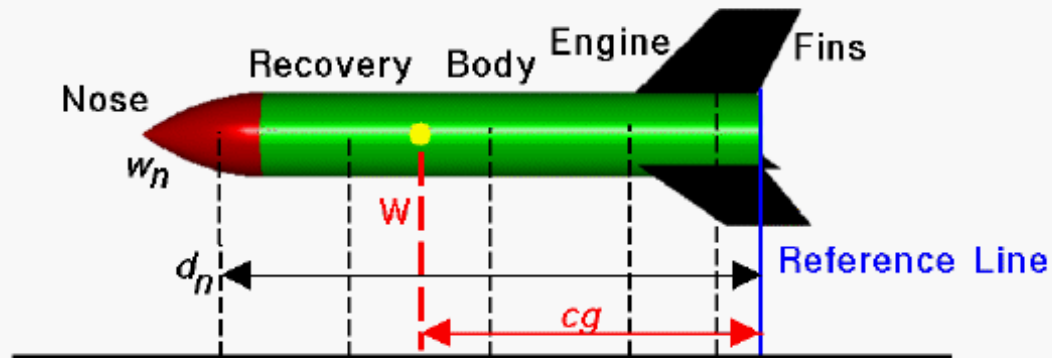
Each component has some weight w_i located some distance d_i from reference line.

Distance cg times the weight W equals the sum of the component distance times component weight.

$$cg\, W = d_n w_n + d_r w_r + d_b w_b + d_e w_e + d_f w_f$$

As a model rocket flies through the air, it rotates about a point called the **center of gravity**. The center of gravity is the average location of the weight of the rocket. The mass (and weight) is actually distributed throughout the rocket. And for some problems, it is important to know the distribution. But for rocket trajectory and maneuvering, we need to be concerned with only the total weight and the location of the center of gravity. How would you determine the location of the center of gravity?

Calculating cg

You can calculate the center of gravity. But in general, this is a complicated procedure requiring the use of calculus. This figure shows a simplified version of the center of gravity calculation that you can use. We assume that we already know the weight and location, relative to some reference location, of each of the major parts of the rocket: the nose, recovery system, body tube, fins, and engine. The **total weight** (W) of the rocket is simply the sum of all the individual weights of the components. Since the **center of gravity** (cg) is an average location of the weight, we can say that the weight of the whole rocket times the location of the center of gravity is equal to the sum of the weight of each component times the distance of that component from the reference location. (This is equivalent to saying the center of gravity is the mass-weighted average of the component locations.)
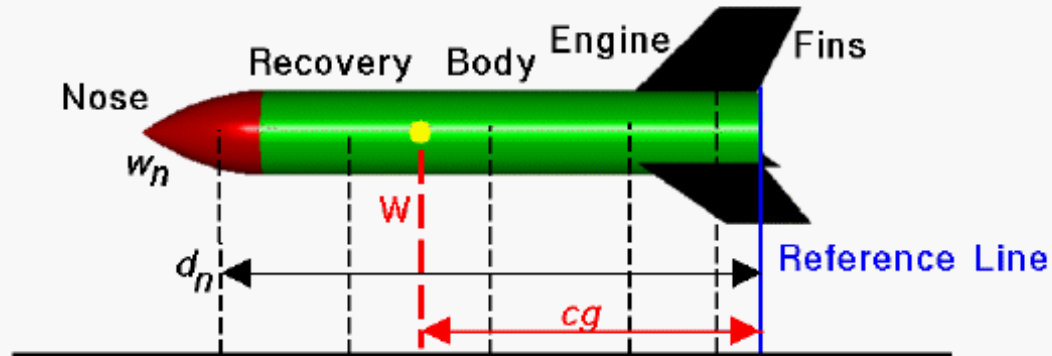
Components' Location

On the slide, we show the weight and distance of the nose relative to the reference line. The location of the reference line is arbitrary (it could be measured from the tip of the nose instead

Determining Center of Gravity – cg

Nose · Recovery · Body · Engine · Fins

Reference Line

w_n

W

d_n

cg

Each component has some weight w_i located some distance d_i from reference line.

Distance cg times the weight W equals the sum of the component distance times component weight.

$$cg\ W = d_n w_n + d_r w_r + d_b w_b + d_e w_e + d_f w_f$$

of from the base of the rocket as shown in the figure) but we have to remember where it is. The cg is a distance measured from the reference line as well. The "distance" of the nose (dn) is the distance of the center of gravity of the nose relative to the reference line. So we have to be able to calculate or determine the center of gravity of the nose and each of the other rocket components. For some simple shapes, finding the cg, or average location of the weight, is quite simple. For example, when viewed perpendicular to the axis, the body tube is rectangular. The center of gravity is on the axis, halfway between the end planes. There is a technique for determining the center of gravity of any general shape, and the details of this technique is given on another page.

Determining cg Mechanically

For a model rocket, there is a simple mechanical way to determine the center of gravity for each component or for the entire rocket:

If we just balance the component (or the entire rocket) using a string or an edge, the point at which the component (or rocket) is balanced is the center of gravity. (Just like balancing a pencil on your finger!) Obviously, we could not use this procedure for a large rocket like the Space Shuttle, but it works quite well for a model.
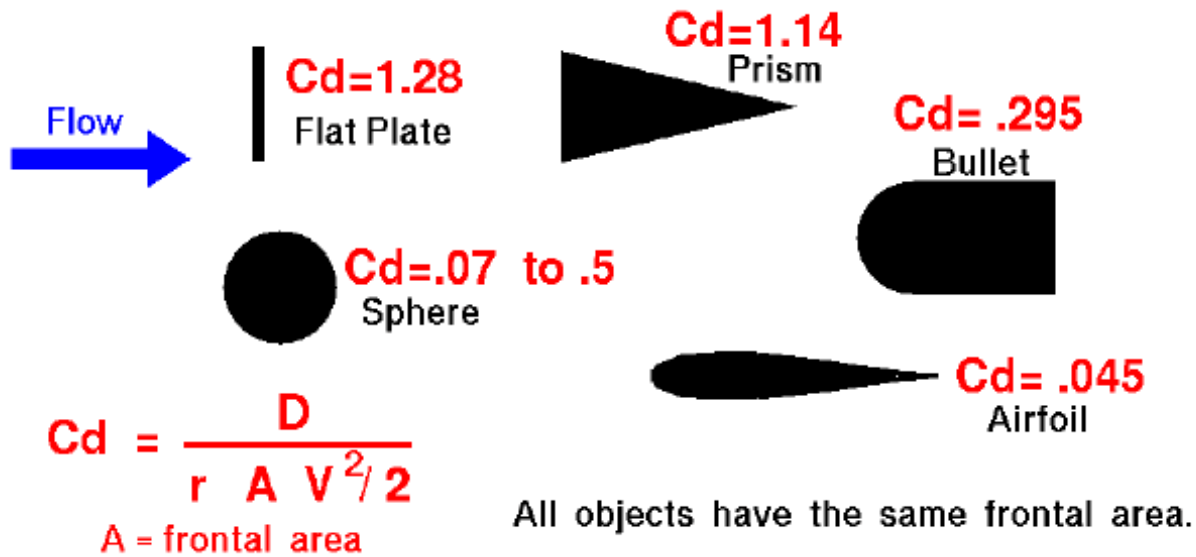
Another, more complicated way, is to hang the model from some point (the corner of a fin, for example) and drop a weighted string from the same point. Draw a line on the rocket along the string. Repeat the procedure from another point on the rocket (the nose, for example). You now have two lines drawn on the rocket. The center of gravity is the point where the lines intersect. This procedure works well for irregularly shaped objects that are hard to balance.

Shape Effects on Drag — Glenn Research Center

The drag coefficient is a number which aerodynamicists use to model all of the complex dependencies of drag on shape inclination, and some flow conditions. The drag coefficient (Cd) is equal to the drag (D) divided by the quantity: density (r) times reference area (A) times one half of the velocity (V) squared. This slide shows some typical values of the drag coefficient for a variety of shapes. The values shown here were determined experimentally by placing models in a wind tunnel and measuring the amount of drag and the tunnel conditions of velocity and density. The drag equation was then used to produce the coefficient. The projected frontal area of each object was used as the reference area. A flat plate has Cd = 1.28, a wedge shaped prism with the wedge facing downstream has Cd = 1.14, a sphere has a Cd that varies from .07 to .5, a bullet Cd = .295, and a typical airfoil Cd = .045.

We can study the effect of shape on drag by comparing the values of drag coefficient for any two objects as long as the same reference area is used and the Mach number and Reynolds numbers are matched. All of the drag coefficients on this slide were produced in low speed (subsonic) wind tunnels and at similar Reynolds number, except as noted. A quick comparison shows that a flat plate gives the highest drag and a streamlined symmetric airfoil gives the lowest drag, by a factor of almost 30! **Shape has a very large effect on the amount of drag produced.** The drag coefficient for a sphere is given with a range of values because the drag on a sphere is highly dependent on Reynolds number. (Flow past a sphere, or cylinder, goes through a number of transitions with velocity. At very low velocity, a stable pair of vortices are formed on the downwind side. As velocity increases, the vortices become unstable and are alternately shed downstream. As velocity is increased even more, the boundary layer transitions to chaotic turbulent flow with vortices of many different scales being shed in a turbulent wake from the body. Each of these flow regimes produce a different amount of drag on the sphere.) Comparing the flat plate and the prism, and the sphere and the bullet, we see that the downstream shape can be modified to reduce drag.

$$Cd = \frac{D}{r\ A\ V^2/2}$$

Coefficient **Cd** contains all the complex dependencies
and is usually determined experimentally.
Choice of reference area **A** affects the value of **Cd**.

For an aircraft: $Cd = Cd_0 + \dfrac{Cl^2}{\pi\ Ar\ e}$

(aircraft) = (skin friction + form) **+** (induced)

The drag coefficient is a number that aerodynamicists use to model all of the complex dependencies of drag on shape, inclination, and some flow conditions. This equation is simply a rearrangement of the drag equation where we solve for the drag coefficient in terms of the other variables. The drag coefficient (Cd) is equal to the drag (D) divided by the quantity: density (r) times half the velocity (V) squared times the reference area (A).

This equation gives us a way to determine a value for the drag coefficient. In a controlled environment (wind tunnel) we can set the velocity, density, and area and measure the drag produced. Through division we arrive at a value for the drag coefficient. As pointed out on the drag equation slide, the choice of reference area (wing area, frontal area, surface area, ...) will affect the actual numerical value of the drag coefficient that is calculated. When reporting drag coefficient values, it is important to specify the reference area that is used to determine the coefficient. We can predict the drag that will be produced under a different set of velocity, density (altitude), and area conditions using the drag equation.

The drag coefficient contains not only the complex dependencies of object shape and inclination, but also the effects of air viscosity and compressibility. To correctly use the drag coefficient, we must be sure that the viscosity and compressibility effects are the same between our measured case and the predicted case. Otherwise, the prediction will be inaccurate. For very low speeds (< 200 mph) the compressibility effects are negligible. At higher speeds, it becomes important to match Mach numbers between the two cases. Mach number is the ratio of the velocity to the speed of sound. At supersonic speeds, shock waves will be present in the flow field and we must be sure to account for the wave drag in the drag coefficient. So it is completely incorrect to measure a drag coefficient at some low speed (say 200 mph) and

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(aircraft) = (skin friction + form) + (induced)

apply that drag coefficient at twice the speed of sound (approximately 1,400 mph, Mach = 2.0). It is even more important to match air viscosity effects. The important matching parameter for viscosity is the **Reynolds number** that expresses the ratio of inertial forces to viscous forces. In our discussions on the sources of drag, recall that skin friction drag depends directly on the viscous interaction of the object and the flow. If the Reynolds number of the experiment and flight are close, then we properly model the effects of the viscous forces relative to the inertial forces. If they are very different, we do not correctly model the physics of the real problem and will predict an incorrect drag.