Numerical Solutions to the Lorentz Force Equation for a charged particle in a uniform Electromagnetic field

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Abstract

This study details the process for a numerical solution to the lorentz force law for a charged particle in uniform electric and magnetic fields. The differential equation was broken down into 6 first order ordinary differential equations before they were implimented in the Scilab language. The vector components of electric and magnetic fields, and the initial conditions were tweaked to generate exotic plots for the trajectories of the charged particle. These trajectories were visualized in three dimensional scatter plots in Scilab software.

1 Introduction

The Lorentz Force law gives the total force on a charged particle present in an electromagnetic field by algebraically summing over the components of the electric force and the magnetic force. Analytical solutions are possible but quite exhausting and unsuited for plots of these kinds. For the Numerical solutions, the Scilab language and the Scilab interpreter was used. The reason for the choice of this software package was the ease with which the differential equations could be implimented to be solved, and the fact that the software is open-source and cross-platform.

2 Results and Discussions

Let us begin our discussion with the vector form of the Lorentz force law:

$$\vec{F} = q(\vec{E} + \vec{v} \times \vec{B}) \tag{1}$$

Substituting in the newton's second law would give us the following differential equation:

$$\frac{d^2\vec{r}}{dt^2} = \frac{q}{m}(\vec{E} + \vec{v} \times \vec{B}) \tag{2}$$

The above is a vector differential equation. For us to be able to encode it into a programming language, we need to resolve it into it's components. Doing that, and substituting in the definition of the cross product would break that differential equation into 3 second order ordinary differential equations.

$$\frac{d^2x}{dt^2} = \frac{q}{m}(E_x + \frac{dy}{dt}B_z - \frac{dz}{dt}B_y) \tag{3}$$
$$\frac{d^2y}{dt^2} = \frac{q}{m}(E_y + \frac{dz}{dt}B_x - \frac{dx}{dt}B_z) \tag{4}$$
$$\frac{d^2z}{dt^2} = \frac{q}{m}(E_z + \frac{dx}{dt}B_y - \frac{dy}{dt}B_x) \tag{5}$$

These differential equations as they are right now cannot be solved numerically. They need to be broken down further into first order differential equations. We can do that through a simple velocity substitution. At the end, we would have a total of 6 first order ordinary differential equations that we would need to solve simultaneously to get the trajectory of the particle.

$$\frac{dx}{dt} = V_x \tag{6}$$
$$\frac{dy}{dt} = V_y \tag{7}$$
$$\frac{dz}{dt} = V_z \tag{8}$$
$$\tag{9}$$

These substitutions themselves act as differential equations that need to be solved to obtain the particle co-ordinates at different times.

$$\frac{d}{dt}V_x = \frac{q}{m}(E_x + V_yB_z - V_zB_y) \tag{10}$$
$$\frac{d}{dt}V_y = \frac{q}{m}(E_y + V_zB_x - V_xB_z) \tag{11}$$
$$\frac{d}{dt}V_z = \frac{q}{m}(E_z + V_xB_y - V_yB_x) \tag{12}$$

One of the benefits of using the Scilab language for numerical computations is that it treats all objects as matrices by default. A scalar is just a zero dimensional matrix, while a tensor is just a higher dimensional one. This lets us encode all these differential equations as different rows of a matrix, thus making the simultaneous solutions convenient. Our solution would also be a matrix, with each row giving solution to the corresponding differential equation. The initial conditions and the magnitude of the electric and magnetic fields can easily be tweaked to produce exotic results. Some of the plots by varying these conditions have been attached with this document.

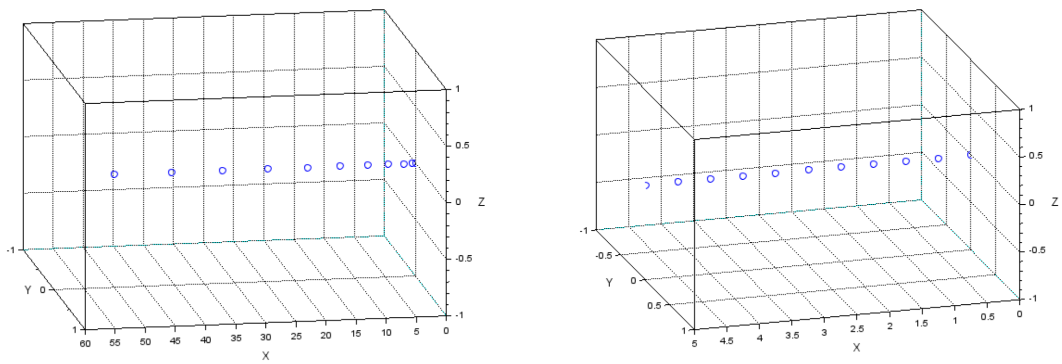

Figure 1: The image on the left gives the motion of the particle under a uniform electric field while the image on the right gives a particle travelling at a constant velocity.

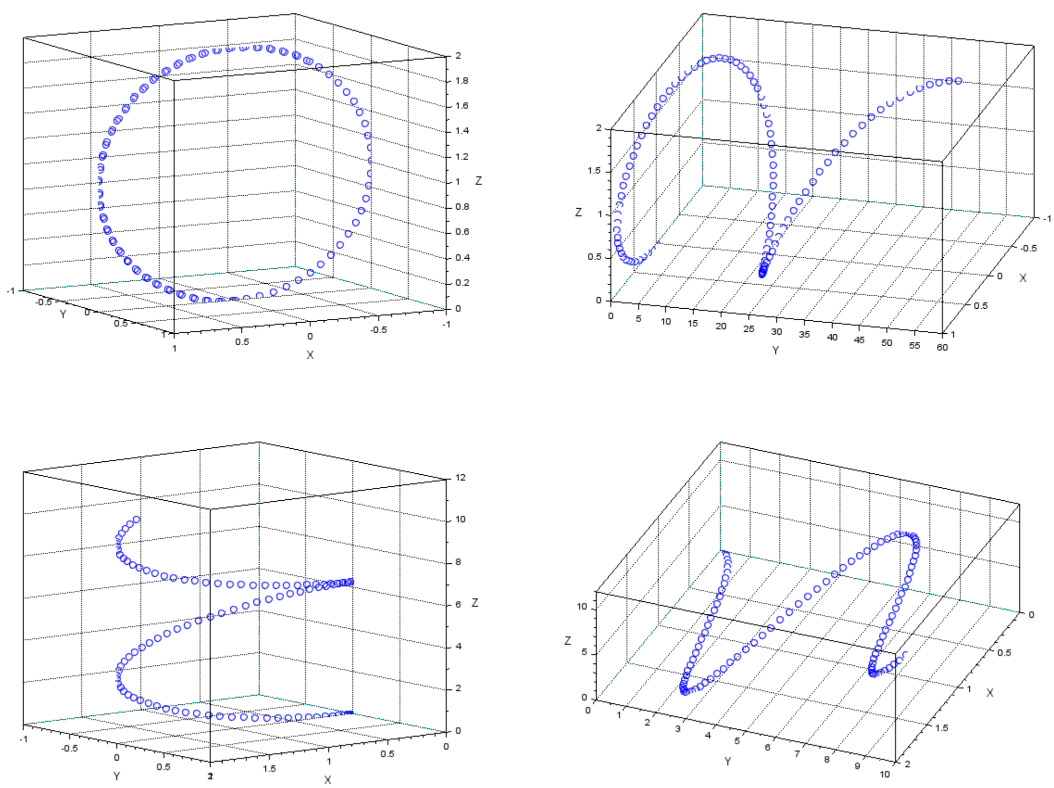

Figure 2: These exotic trajectories of the charged particle were obtained by varying the electric and magnetic field components, and the initial velocity components.