Calibrating Ensemble Forecasting Models with Sparse Data in the Social Sciences

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April 1, 2014

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Abstract

We consider ensemble Bayesian model averaging (EBMA) in the context of small-n prediction tasks in the presence of a large number of component models. With a large number of observations to calibrate ensembles, relatively small numbers of component forecasts, and low rates of missingness, the standard approach to calibrating forecasting ensembles introduced by Raftery et al. (2005) performs well. However, data in the social sciences generally do not fulfill these requirements. In these circumstances, EBMA models may miss-weight components, undermining the advantages of the ensemble approach to prediction. In this article, we explore these issues and introduce a "wisdom of the crowds" parameter to the standard EBMA framework, which improves its performance. Specifically, we show that this solution improves the accuracy of EBMA forecasts in predicting the 2012 US presidential election and the US unemployment rate.

Keywords: Bayesian methods, Election forecasting, Labour Market forecasting, Calibration, Ensembles

1 Introduction

Although accurate prediction of future events is not the primary goal for most social sciences, recent years have witnessed the spreading of systematic forecasting from more traditional topics, such as GDP growth and unemployment, to many new domains, including elections (e.g., Linzer 2013), political instability (e.g., Goldstone et al. 2010), and mass killings (Ulfelder 2012). Several factors have motivated this trend. To begin with, testing predictions about future events against observed outcomes is seen as a stringent validity check of statistical and theoretical models (Ward et al. 2010). In addition, the forecasting of important political, economic, and social events is of great interest to policymakers and the public.

With the proliferation of forecasting efforts, however, comes a need for sensible methods to aggregate and utilize the various scholarly efforts. One attractive solution to this problem is to combine prediction models and create an ensemble forecast. Combining forecasts reduces reliance on any single data source or methodology, and allows for the incorporation of more information than any one model can provide in isolation. Across subject domains, scholars have shown ensemble predictions to be more accurate than any individual component model (Bates and Granger 1969; Armstrong 2001; Raftery et al. 2005).

One approach to combining multiple forecasts is ensemble Bayesian model averaging (EBMA), a method first proposed by Raftery et al. (2005) to combine weather forecasts and introduced for the social sciences in Montgomery et al. (2012a). EBMA combines multiple forecasts using a finite mixture model that generates a weighted predictive probability density function (PDF). EBMA mixture models collate the good parts of existing forecasting models, while avoiding over-fitting to past observations and over-estimating our certainty about forecasts of future events. The hope is that integrating the knowledge and implied uncertainty of a variety of approaches into a combined predictive PDF will result in more accurate and better calibrated forecasts.

In this article, we present several adjustments to the basic EBMA model as specified in Montgomery et al. (2012a) that can aid applied researchers in creating ensemble forecasts in the presence of data-quality challenges common to real-world social science settings. Specifically, we show that EBMA can be adjusted to accommodate small calibration samples, large numbers of candidate components, and missing forecasts. We propose an alteration to the basic model to hedge against the miss-weighting of components resulting from either strong or poor performance in the limited calibration period. After discussing the data-quality challenges commonly experienced in ensemble forecasting, we introduce the basic EBMA model and outline modifications for small samples and missing components in Section 3. In Section 4, we demonstrate how our adjustment to the basic EBMA model improves out-of-sample forecasts in a simulation study and apply the method to predict the 2012 US presidential election and the US unemployment rate.

2 Ensemble prediction with sparse data and multiple forecasts

The concept of ensemble forecasting builds on the basic notion that combining multiple points of view leads to a more accurate picture of reality (c.f., Surowiecki 2004). Among the more famous demonstrations of this phenomenon was a competition to guess the weight of an ox at the West of England Fat Stock and Poultry Exhibition. Galton (1907) famously demonstrated that, while individual entrants were often wildly inaccurate, aggregating the "wisdom of crowds" by using the average guess resulted in a remarkably accurate estimate.

In recent years, the advantages of ensembles have come to play a particularly prominent role in the machine-learning and nonparametric statistics community (Hastie et al. 2009). A wide range of approaches, including neural nets, additive regression trees, and K nearest neighbors, fall under the general umbrella of ensemble approaches. Of particular relevance is the success of boosting (Freund and Schapire 1997; Friedman 2001), bagging (Breiman 1996), random forests (Breiman 2001), and related techniques (e.g., Chipman et al. 2010) to aggregate so-called "weak learners." These approaches to classification and prediction have been advertised as the "best off-the-shelf classifier[s] in the world" (Zhu et al. 2009, 350), and are equally powerful in prediction tasks.

While the advantages of collating information from multiple sources are manifold, it is nevertheless false to assume that more is always better. Not all guesses are equally informative, and naive approaches to collating forecasts risks overvaluing wild guesses and undervaluing unusual forecasts that are nonetheless sometimes correct. The particular ensemble method we are extending is ensemble Bayesian model averaging (EBMA). First proposed by Raftery et al. (2005), EBMA pools forecasts as a weighted combination of predictive PDFs. Rather than selecting some "best model," EBMA collects *all* of the insights from multiple forecasting efforts in a coherent manner via statistical post processing. The weight assigned to each component forecast reflects both its past predictive accuracy and its uniqueness (i.e., the degree to which it makes predictions different from other component models).

In recent years, variants of the EBMA method have been applied to subjects as diverse as inflation (e.g., Wright 2009), stock prices (e.g., Billio et al. 2011), economic growth and policymaking (e.g., Billio et al. 2010), and, climatology (Min et al. 2007). Though EBMA is already in use, research to improve upon the basic model is ongoing. It has, for instance, been adjusted to handle missing data (Fraley et al. 2010; McCandless et al. 2011) and incorporate spatial information (Feldman 2012). Other recent innovations include Möller et al. (2013), who use multiple EBMA models predicting univariate outcomes to create joint predictive distributions of multiple (correlated) dependent variables. All in all, the promise of ensemble forecasting via EBMA has led to multiple efforts to refine the method, although this work has taken place primarily outside of the social sciences.

In this article, we focus on difficulties in calibrating accurate EBMA forecasting models in the context of data-quality challenges especially common in (although not limited to) social science applications. To begin with, the amount and quality of data for calibrating ensembles is far from ideal. EBMA was first developed for use in weather forecasting where the measurement of outcomes is fairly precise and data are abundant. For instance, predicting water surface temperatures in 200 locations across just five days provides 1,000 observations on which model weights can be calibrated. In contrast, forecasting quarterly GDP growth in the United States for five *years* provides only 20 data points.

A second, and related, issue is dimensionality. Prediction tasks often involve many forecasts predicting few outcomes, or even just one. For example, in the field of economics, a wide variety of consulting firms, banks, and international organizations provide forecasts for various economic quantities, such as the unemployment rate, GDP growth, and inflation. Indeed, the Federal Open Market Committee (FOMC) of the US Federal Reserve Board generates over a dozen forecasts for key economic indicators.[1]

A final issue is the inconsistency with which forecasts are issued. Given the lengthy time periods often involved, there are likely to be many missing forecasts in any time window containing a modestly large number of observations. Moreover, we cannot assume that forecasts for any time period from a specific model or team are missing at random. Particularly unsuccessful forecasts may be suppressed, and some forecasting efforts are only active for short time-periods due to poor performance. In addition, forecasts tend to accumulate, with more potential components being available for more proximate time periods.

Predicting US presidential elections is, perhaps, the quintessential forecasting task that combines all of these issues. Table 1 represents nearly the entirety of scholarly forecasts that produced more than one true out-of-sample forecast for elections in the 20th century prior to the 2012 election.[2] In this instance, we have only five observations on which to calibrate an ensemble model, while we have nine forecasting models.[3] Moreover, several of the individual forecasts are missing for a significant portion of this period. The forecast of Cuzàn, for instance, is missing for 60% of the elections in this dataset.[4]

While particularly egregious for U.S. presidential elections these data issues are endemic to the social sciences and are far from benign. Important events including economic growth, inflation rates, monetary policy decisions, political elections, and civil conflict are often measured infrequently (annually or quarterly) fundamentally limiting the number of available observations, even when measured for multiple countries. Even more, these data issues are likely to arise in any instance where predictions are made for a single unit observed over a modest time period, which includes making predictions for trade regimes, supply chains, commodity prices, and more.

As we demonstrate below, calibrating large ensemble models on sparse (and even incomplete)

[1] For a recent sample of these forecasts, see: `http://1.usa.gov/zjyisV`.

[2] See, for example: Fair (2009, 2011); Abramowitz (2008); Campbell (2008b); Cuzán and Bundrick (2004, 2008); Hibbs (2012a); Lockerbie (2008); Erikson and Wlezien (2008); Graefe et al. (2010); Holbrook (2008). A recent symposium in *PS: Political Science & Politics* presents and summarizes attempts by a variety of scholars to predict the 2012 US presidential election. In a symposium contribution, we use the in-sample fitted values of the election forecasting models to calibrate the EBMA model (Montgomery et al. 2012b). However, the strength of EBMA is greatest when the model is calibrated on true out-of-sample forecasts as we do here.

[3] The out-of-sample predictions for these models were collected from the individual journal articles, personal websites and symposia introductions. When multiple forecasts were made, we used the authors' preferred forecast, or took the mean if no preference was given (Hibbs 1992; Holbrook 1996; Lewis-Beck and Tien 1996; Wlezien and Erikson 1996; Abramowitz 2000; Hibbs 2000; Campbell and Garand 2000; Campbell 2000, 2001; Hibbs 2004; Campbell 2004, 2005, 2008a; Abramowitz 2012; Campbell 2012; Cuzán 2012; Erikson and Wlezien 2012; Fair 2012; Hibbs 2012b; Holbrook 2012; Lewis-Beck and Tien 2012; Lockerbie 2012).

[4] The prediction by Cuzàn for 2004 stems from the FISCAL model published prior to the 2004 election by Cuzán and Bundrick (2004), while the 2008 prediction comes from the FPRIME short model presented in advance of the election (Cuzán and Bundrick 2008). However, both models are quite similar in their composition.

Table 1: Pre-election forecasts of the percent of the two-party vote going to the incumbent party in US presidential elections

	Year	F	A	C	H	LBRT	L	Hol	EW	Cuz
1	1992	55.70	46.30	47.10	48.90					
2	1996	49.50	56.80	58.10	53.50	54.80		57.20	57.20	
3	2000	50.80	53.20	52.80	53.80	55.40	60.30	60.30	55.20	
4	2004	57.50	53.70	53.80	53.20	49.90	57.60	54.50	52.30	52.80
5	2008	48.10	45.70	52.70	48.20	49.90	41.80	44.30	47.80	48.00

Note: Forecasts were published prior to each election by **F**air, **A**bramowitz, **C**ampbell, **H**ibbs, **L**ewis-**B**eck and **R**ice (1992) and Lewis-Beck and **T**ien (1996-2008)), **L**ockerbie, **Hol**brook, **E**rikson and **W**lezien and **Cuz**àn and Bundrick.

data leads to misspecification of EBMA model weights and decreased out-of-sample predictive performance. In essence, when there are many forecasting models and few observations with which to evaluate them, the probability that some forecasting model will perform unusually well or poorly due to mere *chance* increases markedly. Under these circumstances, ensemble methods in general, and EBMA models in particular, will miss-estimate model weights. Too much weight will be given to poor models that were *randomly* correct, while generally accurate models that were wrong for the small calibration sample will be likewise undervalued.

In light of these difficulties, below we propose several extensions to the baseline EBMA algorithm introduced in Montgomery et al. (2012a), and explore the effect of these modifications on the method's predictive performance. To begin with, we discuss a method proposed by (Fraley et al. 2010) for efficiently handling missing data. Further, we add a simple an intuitive "wisdom of crowds" parameter, that ensures that model weights are more evenly distributed.

3 EBMA for sparse data

As its name suggests, EBMA descends from the Bayesian model averaging (BMA) methodology (c.f., Raftery 1995; Hoeting et al. 1999; Clyde and George 2004), which was first introduced to political science by Bartels (1997) and has been applied in a number of contexts (e.g., Imai and King 2004; Montgomery and Nyhan 2010). A more detailed discussion of the basic EBMA model extended here is provided in Montgomery et al. (2012a).

3.1 Baseline EBMA model

Assume the researcher is interested in predicting event $\mathbf{y}^{t^*}$ for some future time period $t^* \in T^*$, which we term the test period below. In addition, we have a number of different out-of-sample forecasts for similar events $\mathbf{y}^t$ in some past period $t \in T$, which we term the calibration period. The different predictions were generated from K forecasting models or teams, $M_1, M_2, \ldots, M_K$. As we show in our examples below, these predictions might originate from the insights and intuitions of individual subject-experts, traditional statistical models, non-linear classication trees, neural networks, agent based models, or anything in between. Indeed, there is no restriction at all on the kind of forecasting method that can be incorporated into the ensemble, so long as it offers a prediction for a sufficiently large subset of the calibration sample.

For each forecast we have a prior probability distribution $M_k \sim \pi(M_k)$ and the PDF for $\mathbf{y}^t$ is denoted $p(\mathbf{y}^t|M_k)$. Under this model, the predictive PDF for the quantity of interest is $p(\mathbf{y}^{t^*}|M_k)$, the conditional probability for each model is $p(M_k|\mathbf{y}^t) = p(\mathbf{y}^t|M_k)\pi(M_k)/\sum\limits_{k=1}^{K} p(\mathbf{y}^t|M_k)\pi(M_k)$, and the marginal predictive PDF is $p(\mathbf{y}^{t^*}) = \sum\limits_{k=1}^{K} p(\mathbf{y}^{t^*}|M_k)p(M_k|\mathbf{y}^t)$. The prediction via EBMA is thus a weighted average of the component PDFs, and the weight for each model is based on its predictive performance on past observations in period T.

The general EBMA procedure assumes K forecasting models throughout the training (T'), calibration (T), and test (T^*) periods. Each component model is fit on data from the training period T'. The assumption is that, for each component model, the model-specific parameters (if any) will be estimated based on observations in the training period T'. Each component model then generates out-of-sample predicitions for the calibration period T, which represents the time period used to calculate model weights. It is then possible to generate true ensemble out-of-sample forecasts ($\mathbf{f}_k^{t^*}$) for observations in the test period $t^* \in T^*$.

Using these three distinct time periods makes it possible to calibrate the EBMA model on the component models' out-of-sample predictive power, thus implicitly penalizing overly-complex "garbage can" models. Thus, one of the distinct advantages of EBMA is that it does not require researchers to develop metrics to penalize component forecasts for complexity or even to have access to the details of the component forecasting methods themselves. Model weights are calculated purely on their out-of-sample predictive performance in the calibration period. (However, as we show below, this same feature leads to miss-specification of model weights when data is sparse in the calibration period.)

Let $g_k(\mathbf{y}|\mathbf{f}_k^{s|t,t^*})$ represent the predictive PDF of component k, which may be the original prediction from the forecast model or some bias-corrected forecast. The EBMA PDF is a finite mixture of the K component PDFs, denoted $p(\mathbf{y}|\mathbf{f}_1^{s|t}, \ldots, \mathbf{f}_K^{s|t}) = \sum\limits_{k=1}^{K} w_k g_k(\mathbf{y}|\mathbf{f}_k^{s|t})$, where $w_k \in [0,1]$ are model probabilities, $p(M_k|\mathbf{y}^t)$, and $\sum_{k=1}^{K} w_k = 1$. The ensemble predictive PDF with this notation is then $p(y|f_1^{t^*}, \ldots, f_K^{t^*}) = \sum\limits_{k=1}^{K} w_k g_k(y|f_k^{t^*})$.[5] For the applications below, we assume $g_k(\mathbf{y}|\mathbf{f}_k^{s|t}) = N(\mathbf{f}_k^t, \sigma^2)$, where σ is a common variance across components.[6] Thus, the ultimate predictive distribution for some observation y^{t^*} is

$$p(y|f_1^{s|t^*}, \ldots, f_K^{s|t^*}) = \sum_{k=1}^{K} w_k N(f_k^{t^*}, \sigma^2). \tag{1}$$

This is a weighted mixture of K normal distributions each with means determined by $\mathbf{f}^{t^*}$ and scaled by the model weights $\mathbf{w} = (w_1, \ldots, w_k)$.

3.2 Model estimation

Since the component model forecasts, $f_1^t, \ldots, f_k^t$, are pre-determined, the EBMA model is fully specified by estimating model weights, $\mathbf{w}$ and the common variance parameter σ^2. We estimate these using maximum likelihood methods (Raftery et al. 2005). The log-likelihood function,

$$\mathcal{L}(\mathbf{w}, \sigma^2) = \sum_t \log\left(\sum_{k=1}^{K} w_k N(f_k^t, \sigma^2)\right), \tag{2}$$

cannot be maximized analytically. Instead, we follow Raftery et al. (2005) and use an EM algorithm to calibrate the weights, an approach made possible by recognizing the EBMA here as a finite mixture model (McLachlan and Peel 2000). Specifically, the unobserved quantities z_k^t are introduced, which represent the probability that observation y^t is "best" predicted by model k. These

[5] Past applications have statistically post-processed the predictions for out-of-sample bias reduction and treated these adjusted predictions as a component model. Raftery et al. (2005) propose approximating the conditional PDF as a normal distribution centered at a linear transformation of the individual forecast, $g_k(\mathbf{y}|\mathbf{f}_k^{s|t}) = N(a_{k0} + a_{k1}\mathbf{f}_k^t, \sigma^2)$. However, in the presence of sparse data, including the additional $\mathbf{a}$ parameters risks over-fitting and reduced predictive performance. We therefore use a simpler formulation.

[6] The model presented here assumes that errors are distributed normally and a common variance term. However, neither of these assumptions are strictly necessary. EBMA has been extended to handle multiple distributional forms, including zero-inflated gamma, discrete quanitative outcomes, and binary outcomes (Sloughter et al. 2007, 2010; Montgomery et al. 2012a). Likewise, the method allows for estimating distinct variance parameters, although in practice this is not usually advantageous for limited samples.

unobserved quantities are estimated (E-step) in the algorithm using the formula

$$\hat{z}_k^{(j+1)t} = \frac{\hat{w}_k^{(j)} p^{(j)}(y|f_k^t)}{\sum\limits_{k=1}^{K} \hat{w}_k^{(j)} p^{(j)}(y|f_k^t)}, \tag{3}$$

where the superscript j refers to the jth iteration of the EM algorithm. Note that $w_k^{(j)}$ is the estimate of w_k in the jth iteration and $p^{(j)}(.)$ is shown in Equation (1).

Using the estimates of $z_k^{s|t}$, one can then calculate the maximizing value (the M step) for the component weights as

$$\hat{w}_k^{(j+1)} = \frac{1}{n} \sum_t \hat{z}_k^{(j+1)t}, \tag{4}$$

where n represents the number of observations in the calibration dataset. Finally, the common variance term is estimated as

$$\hat{\sigma}^{2(j+1)} = \frac{1}{n} \sum_t \sum_{k=1}^{K} \hat{z}_k^{(j+1)t} (y - f_k^t)^2. \tag{5}$$

The E and M steps are iterated until the improvement in the log-likelihood is no larger than some pre-defined tolerance. The algorithm is started with the assumption that all component models are equally likely to be the best forecast, i.e. $w_k = \frac{1}{K} \; \forall \; k \in [1, \ldots, K]$ and $\sigma^2 = 1$.

3.3 Adjustments for sparse data

When ensembles are calibrated on very few observations, there is an increased chance that EBMA may incorrectly weight component models in a way that reduces out-of-sample performance due to unusually poor or strong predictive performance in the limited calibration sample. With small samples, random chance alone can lead some specific models to perform quite well, even if their long-term predictive power is quite weak. This is especially true when the short calibration period is combined with missing observations in component model predictions. (Adjustments to the baseline model to accomodate missing components are provided in Appendix A.) To ameliorate these problems, we therefore introduce a "wisdom of crowds" parameters, that serves to distribute model weights more evenly than they would be otherwise. As we show below, in practice this can significantly improve the out-of-sample performance of ensemble models in the context of sparse data.

To improve the performance of EBMA in the context of sparse data, we propose a "wisdom of crowds" parameter, $c \in [0, 1]$, that reflects our prior belief that all models should receive some, but not necessarily equal, weight. We rescale z_k^t to have a minimum value $\frac{c}{K}$. This states that there is, at a minimum, a $\frac{c}{K}$ probability that observation t is correctly represented by each model k. Since $\sum\limits_{k=1}^{K} z_k^t = 1$, this implies that $z_k^t \in [\frac{c}{K}, (1-c)]$. To achieve this, we replace Equation (4) above with

$$\hat{z}_k^{(j+1)t} = \frac{c}{K} + (1-c)\frac{\hat{w}_k^{(j)} p^{(j)}(y|f_k^t)}{\sum\limits_{k=1}^{K} \hat{w}_k^{(j)} p^{(j)}(y|f_k^t)}. \tag{6}$$

Conceptually, c resembles a shrinkage or regularization prior in a fully Bayesian setup which serves to prevent over-fitting the available training data. Indeed, in a proper Bayesian framework, several classes of priors could serve the same purposes.[7]

Note that when $c = 1$, all models are considered equally informative about the outcome and $w_k = \frac{1}{K} \forall K$. Thus, we see that the arithmetic mean or median of component forecasts for time period t represents a special case where $c = 1$.[8] Likewise, the EBMA discussed in Montgomery et al. (2012a) represents a special case of this more general model where $c = 0$.

4 Simulations and Applications

The introduction of the "wisdom of crowds" parameter to the base EBMA model is designed to improve out-of-sample predictive performance in the context of data-quality challenges common to social science applications. In particular, it is designed to address poor weight calibrations that are likely when the size of the calibration sample is small, when component models with missing predictions in the calibration sample are included, and when the number of component forecasts for which weights must be estimated is large. We argue that these issues increases the miss-estimation of component model weights and decrease the predictive performance of EBMA.

To justify these claims, we present the results of a simulation study of our modified EBMA algorithm and two empirical applications of the adjusted method. We begin with a simulation that illustrates the reduced predictive performance of the baseline EBMA model in the circumstances described above and illustrates the improvements that result from our proposed modification. We

[7] A helpful reviewer notes that a symmetric Dirichlet prior on the model weights might serve the same purpose, but may prove to be more general. While we believe that this approach appears plausible, we leave this proposed variant to future research.

[8] The mean or median would be equivalent depending on if the posterior mean or median is used to make a point prediction.

then apply our method, first, to the prediction of the 2012 US presidential election, and, second, to the prediction of the US unemployment rate.[9]

4.1 Simulation study

In this section, we conduct a simulation study of the adjusted EBMA model proposed above. This study serve two purposes. First, it demonstrates the challenges inherent in ensemble forecasting when calibration samples are small and the number of forecasting models is large.[10] Second, it explores the extent to which our modified EBMA algorithm ameliorates these difficulties. In addition, we provide some guidance regarding the selection of c.

The simulations are designed to reflect the "best possible" world for the baseline EBMA model. The distribution of outcomes is drawn precisely from the mixture distribution shown in Equation (1), where $\sigma^2 = 1$ and the individual component forecasts are drawn from the multivariate normal distribution $N(\mathbf{0}_K, \mathbf{I}_K)$. Moreover, we assume that the true data-generating process, both in-sample and out-of sample, involves *only* the K forecasting models which are themselves estimated with perfect precision. The "true" model weights for each simulation are drawn from a Dirichlet distribution with K categories and concentration parameter $\alpha = (10, 5, 3, \frac{1}{K-3})$ when $K > 3$ and $\alpha = (10, 5, 3)$ when $K = 3$.[11] This ensures that the model weights always sum to 1, but that there is still some heterogeneity. We varied the size of the calibration sample (n_T), the number of component forecasts (K), and the wisdom of crowds parameter (c). The c parameter is used only for model estimation and plays no role in the creation of the simulated data itself.

For each simulation, we generate component forecasts for both the calibration and test period. We fit an EBMA model as specified above to the calibration sample data only. We then generate out-of-sample predictions for the 250 observations in the test period using the fitted EBMA model and compare the forecasts to the true values from the simulated data.

We begin by examining the accuracy of the baseline EBMA ($c = 0$) predictive PDF shown in Equation (1) for different values of K (the number of components) and n_T (the calibration sample

[9] All files needed for these analyses will be made available online at the time of publication. Our modified EBMA algorithm will be released as part of the R package 'EBMAforecast', which is hosted online with the Comprehensive R Archiving Network (CRAN) (Montgomery et al. 2013).

[10] To reduce the parameter space for these simulations, we limit ourselves here to exploring the role of calibration sample sizes and number of component forecasts. We do not consider issues of missingness.

[11] Implicitly, this simulation framework states that the first model will generally be the most "correct" as it has the highest concentration parameter of 10. In models where $K > 3$, on the other hand, the additional models will become increasingly uninformative about the outcome as their weight is decreased. However, as a helpful reviewer notes, adding hundreds of models that essentially *never* provide correct answers would lead to a different simulation outcome. In this hypothetical case, where many models are by definition inappropriate and uninformative, giving more models more weight will provide an inferior solution.

Table 2: Parameters for simulation

Parameter	Meaning	Values
n_T	Sample size in calibration period T	3-15,20,25,35,45,55,65,85,100
n_{T^*}	Sample size in test period T^*	250
K	# of component forecasts	3,5,7,9,11,13,15
σ^2	Common variance component	1
α	Weight concentration parameter	$(10, 5, 3, \frac{1}{K-3})$
c	Wisdom of crowds parameter	0,0.01,0.02,0.03,0.04,0.05,0.075,0.1 0.15,0.2,0.3,0.5
M	Simulations at each setting	100

size). To evaluate the forecasts, we focus here on the continuous rank probability score (CRPS) for several reasons.[12] The CRPS has been widely used to evaluate forecasts of continuous outcomes, and its many advantages as a proper scoring rule have been discussed elsewhere (Hersbach 2000; Gneiting and Raftery 2007; Brandt et al. 2011). One of the main advantages of the CRPS over other scoring rules is that it can be interpreted as the integral over all possible Brier scores (Brier 1950) and takes into account the uncertainty of forecasts (i.e., the predictive distributions rather than the point prediction in isolation). The CRPS ranges from 0 to n_{T^*}, with smaller numbers indicating a better forecast performance.[13] (The mathematical details behind the CRPS can be found in Appendix B.)

Figure 1 shows the out-of-sample performance of the EBMA method, as measured by CRPS, against the ratio of the number of component models included to the size of the calibration period (i.e., $\frac{K}{n_T}$). As one can see, the performance of the EBMA model depends significantly on this ratio. As the number of component models included increases or the calibration sample size decreases, CRPS rises; that is, the predictive performance of the ensembles decreases as a function of $\frac{K}{n_T}$. Note that in this figure, $c = 0$ for all models.

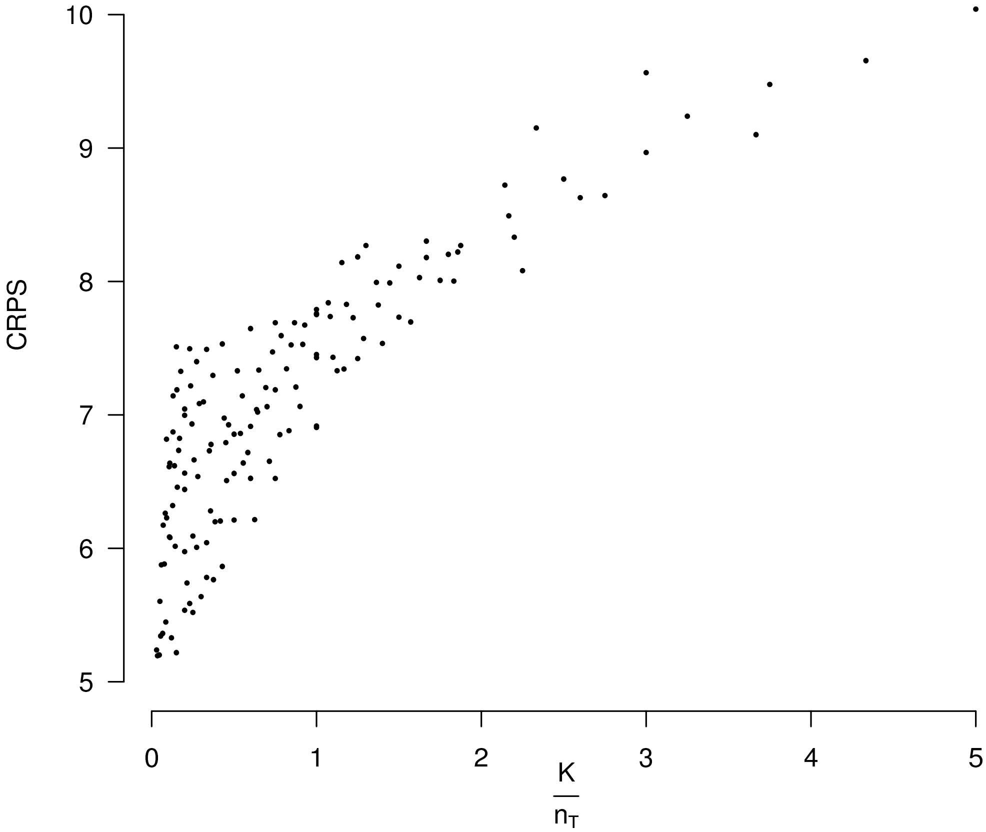
Thinking of each parameter in isolation, Figure 1 shows that the predictive power of the EBMA model decreases as the number of components in the true data-generating process increases. That is, as the number of model parameters that *must* be correctly estimated to make accurate predictions increases, the quality of the forecast goes down. Furthermore, CRPS is a decreasing function of n_T; i.e. the performance of the baseline EBMA model improves as the calibration sample grows.[14]

[12] Note that it is only possible to use the CRPS because we are comparing multiple EBMA models, for which we have the entire predictive PDF. For the applied examples below, we have only point estimates from the component models and cannot evaluate their individual performance with CRPS.

[13] Technically, it ranges from 0 to 1 for each of the n_{T^*} observations in the test sample.

[14] Examining each parameter in isolation supports this claim, although the relationship is highly interactive (results not shown).

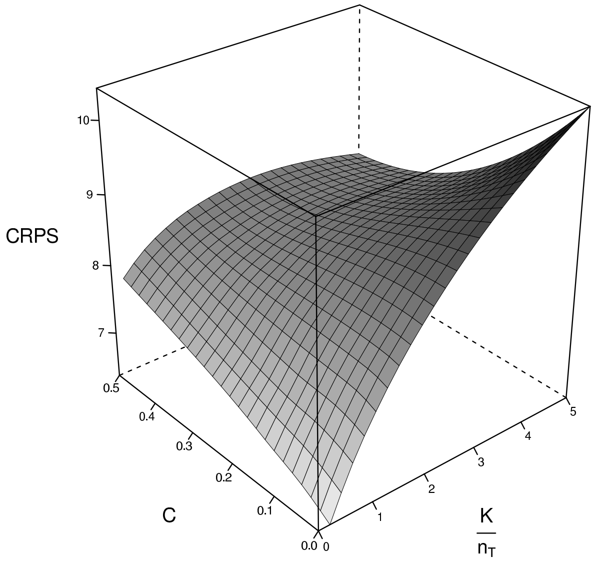
Figure 1: CRPS with varying number of models and observations in calibration period



See Table 2 for a full listing of the parameter values considered in our simulations. These results represent all simulations where $c = 0$, which is the baseline EBMA model. Note that the EBMA model performs signifcantly less well when the ratio of component forecasts to the sample size in the calibration period $\frac{K}{n_T}$ is high.

The remaining question, then, is to what degree adding the "wisdom of crowds" parameter to the baseline model improves performance. To answer, we examined the out-of-sample predictive performance of EBMA for differing values of c. Figure 2 shows a smoothed plot of the median CRPS for differing values of the ratio $\frac{K}{n_T}$ and c in our simulation. Darker gray tones depict higher CRPS levels. The plot shows that while CRPS generally still increases with higher values of $\frac{K}{n_T}$, including a "wisdom of crowds" parameter within the EBMA algorithm can help when $\frac{K}{n_T}$ is large.

Figure 2: CRPS with "wisdom of crowds" parameter



The figure shows a smoothed relationship between the median CRPS for differing values of the ratio $\frac{K}{n_T}$ and c. Darker gray tones depict higher CRPS levels. Not that while CRPS generally still increases with higher values of $\frac{K}{n_T}$, including a "wisdom of crowds" parameter within the EBMA algorithm can help when $\frac{K}{n_T}$ is large.

There are two aspects of Figure 2 that are particularly salient. First, note that the addition of c to the base EBMA model does not uniformly aid out-of-sample performance. When the calibration sample is modestly large and there are few models to calibrate, the addition of c uniformly decreases model performance. However, with small calibration samples and modestly large numbers of component models, the addition of c aids predictive performance. Second, the relationship between c and CRPS is non-monotonic. CRPS decreases for small to modest values of c, but eventually begins to rise. While this is far from a complete analysis of the simulated data, it does serve the limited purposes of demonstrating that, in some circumstances, the "wisdom of crowds" parameter aids prediction.

Generally speaking, in choosing c the researcher faces a trade-off between possible overweighting certain models based on the performance in short calibration periods and moving too far into the direction of a simple average. A relatively small c seems to generally aid performance. Based on our examination of the broader set of simulations, we generally recommend the selection of values of $c \in [0, 0.1]$.[15] Higher settings of c will be preferred as the ratio $\frac{K}{n_T}$ goes up. The simulations favor smaller values of c as the ratio of K to n_T decreases. In our experience based on the simulations, as well as cross-validation studies during the applications presented below, we feel a c value at 0.05 is a good default choice, although the "best" value of c may vary from application to application. It may be preferable to many researchers to choose the value of c based on a k-fold cross-validation study of the calibration sample.

Bearing these results in mind, we now turn to examining how these methods work in two areas that exemplify forecasting in the social sciences. The first is predicting the vote for the incumbent-party candidate in United States presidential elections , and the second is the prediction of unemployment in the United States. Both areas have well developed forecasting traditions in the scholarly and policy communities.

4.2 United States presidential elections

We now turn to the task of combining expert predictions of presidential elections.[16] This example provides a clear illustration of the difficulties of creating ensemble forecasts in the social sciences and allows us to further illustrate the advantages of generating predictive PDFs when focusing on a limited number of important events.

Using the forecasts shown in Table 1, we fit an EBMA model with $c = 0.05$. The model weights and in-sample fit statistics for the ensemble and its components are shown in Table 3. As can be seen, the EBMA model assigns the majority of weight to the Abramowitz model with the model by Hibbs receiving the second largest weight. These weights are based on the performance of each model in forecasting the incumbent vote share in the presidential elections between 1992 and 2008. The Cuzàn and Bundrick model is weighted to such a small degree because only out-

[15] As we show in our empirical examples below, even very small settings of c can improve out-of-sample predictive performance when there are many models.

[16] See also Montgomery et al. (2012b).

of-sample predictions for 2004 and 2008 were available.

Table 3: Model weights and in-sample fit statistics for EBMA model of US presidential elections (1992–2008)

	EBMA Weight	RMSE	MAE
EBMA		1.92	1.49
Fair	0.02	5.53	4.58
Abramowitz	0.80	1.98	1.68
Campbell	0.02	3.63	3.08
Hibbs	0.06	2.31	2.18
Lewis-Beck, Rice, and Tien	0.06	2.87	2.16
Lockerbie	0.00	7.33	6.97
Holbrook	0.01	5.50	4.45
Erikson and Wlezien	0.02	2.90	2.50
Cuzàn	0.00	1.65	1.65

Figure 3 shows the posterior predictive distribution for the 2008 election (top) and, based on the forecasts from each of the component models, the 2012 election. Component models' predictive distributions are shown in color (scaled by their respective weight), while the EBMA predictive distribution is shown in black. The bold dash displays the point prediction for the EBMA model. The vertical dotted line depicts the actual election results in 2008 and 2012.[17]

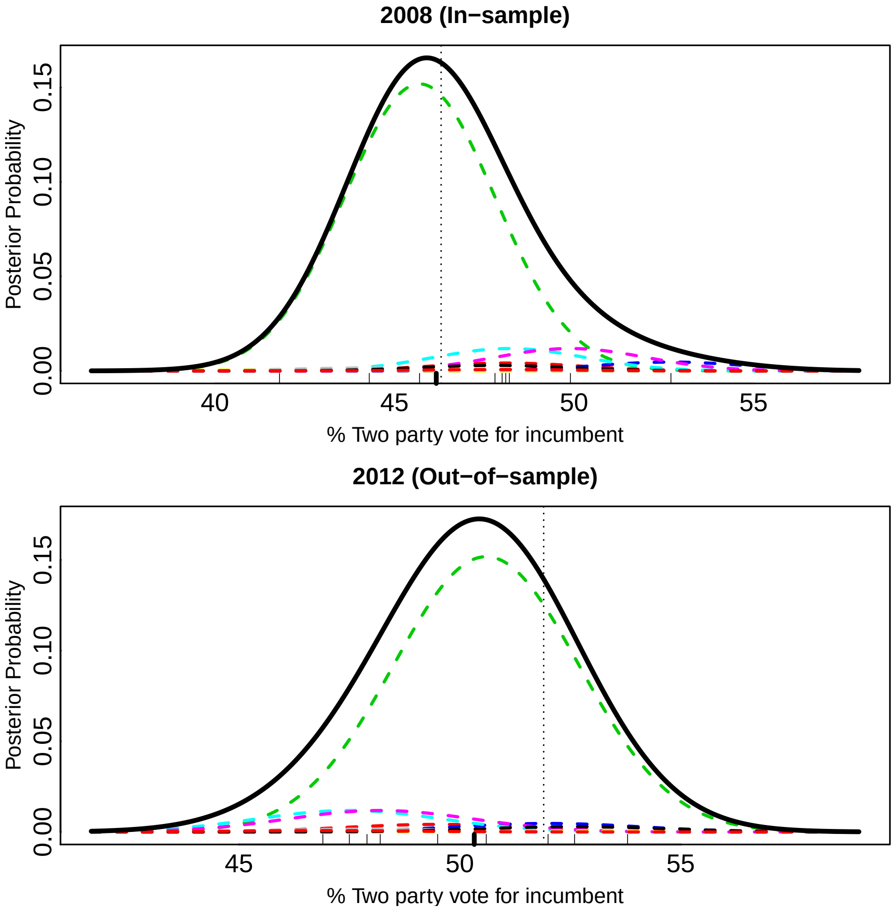
The EBMA model ($c = 0.05$) predicted 50.3% of the two-party vote share for Obama in 2012. This resulted in a reasonably large absolute error of 1.7%.[18] Figure 3 shows that the EBMA prediction performs better than the majority of component forecasts for the 2012 election, with only three providing a more accurate estimate. However, we believe it is important to note that it is difficult to evaluate EBMA against its components using just one out-of-sample observation.

A more important comparison is to examine how EBMA performs versus other methods of aggregating forecasts for the 2012 election. Table 4 shows the point predictions and absolute errors associated with the simple arithmetic mean, the median, and EBMA models fit with $c = 0$, $c = 0.05$, and $c = 0.1$. While the differences in model weights are relatively small, the EBMA prediction for 2012 was 49% and 50.1% for EBMA model with $c = 0$ and $c = 0.1$ respectively –

[17] We emphasize again that these are the predictive PDFs generated not by the component models, but estimated as $g_k(y|f_k^{t^*}) = N(f_k^{t^*}, \sigma^2)$ in Equation 1.

[18] The EBMA model assigned considerable weight to models predicting a Romney *victory*, especially Hibbs (2012b) and Lewis-Beck and Tien (2012).

Figure 3: Predictive ensemble PDFs of incumbent-part vote share in US Presidential Elections



The figure shows the density functions for each of the component models in different colors and scaled by their respective weight. The black curve is the density of the EBMA prediction, with the bold dash indicating the EBMA point prediction. The vertical dashed lines show the actual result of the 2008 and 2012 elections.

Table 4: Comparing Model Results for US Presidential Elections 2012

	Mean	Median	EBMA ($c = 0$)	EBMA ($c = 0.05$)	EBMA ($c = 0.1$)
2012 prediction	49.9	49.5	49	50.3	50.1
Absolute Error	2	2.4	2.9	1.6	1.8

both considerably worse than the prediction with $c = 0.05$.

EBMA with a "wisdom of crowds" parameter of 0.05 also did considerably better relative to naive approaches to aggregation. The mean of the component models' predictions was 49.9% and the median prediction was 49.5$\%$. In essence, while we believe prediction methods should be evaluated on more than one observation, this example again signifies the utility of EBMA in forecasting tasks and, in particular, the improvements the "wisdom of crowds" parameter offers to out-of-sample predictions in the context of sparse data.

4.3 Quarterly unemployment in the United States

Forecasting macroeconomic variables is a quite common exercise in the fields of economics and statistics. Policymakers and businesses both have enormous interest in the calculation of accurate forecasts of economic variables. These forecasts are often created using a wide variety of statistical models; however, professional forecasts are often based only on expert knowledge.[19] The majority of scholars employ time-series models, most commonly applying autoregressive integrated moving average (ARIMA) and vector autoregressive (VAR) models. The sophistication and complexity of forecasting models has increased considerably over time. In particular, non-linear dynamic models have gained prominence including threshold autoregressive models, Markov switching autoregressive models and smooth transition autoregression (e.g., Elliott and Timmermann 2008). More recently, forecasters have added Bayesian VAR models and state-space models to their arsenal (De Gooijer and Hyndman 2006).

Unsurprisingly, given the large number of ongoing forecasts, scholars have attempted to improve predictive accuracy by combining forecasts (e.g., Bates and Granger 1969; Elliott and Timmermann 2008). Recently, EBMA and related Bayesian model averaging methods have been successfully employed to create ensemble forecasts of various macroeconomic indicators such as

[19] For a more comprehensive overview of forecasting economic variables, see Elliott and Timmermann (2008) and De Gooijer and Hyndman (2006). See also Brandt and Williams (2007) and Brandt (2012) for work on forecasting multiple time series.

inflation and exchange rates (e.g., Wright 2008, 2009).

Policymakers too have come to rely on ensemble forecasts of a sort. The desire to aggregate the collective wisdom of multiple forecasting teams is apparent in the *Survey of Professional Forecasters (SPF)* published by the Federal Reserve Bank of Philadelphia. The *SPF* includes forecasts for a large number of macroeconomic variables in the United States, including the unemployment rate, inflation, and GDP growth.[20] In the first month of every quarter, a survey is sent to selected forecasters who return it by the middle of the second month of the quarter. Forecasts are made for the current quarter as well as several quarters into the future. A significant amount of attention is given to the average (usually the median) reported forecast.

This plethora of predictions seems ideal for applying EBMA. Nonetheless, it is plagued by the issues discussed in Section 2. Even with quarterly measures, there are relatively few observations, many forecasting teams, and a significant number of missing observations. This setting, therefore, provides a test bed for the adjusted EBMA model presented above.

We focus on forecasting the civilian unemployment rate (UNEMP) as published by the *SPF*. For this application, we selected the forecast horizon to be four quarters into the future; i.e. predictions made in the first quarter of 2002 are for the first quarter of 2003 and so on. In total, the *SPF* data on unemployment contains forecasts by 569 different teams. However, for any quarter, the average number of teams making a prediction for four quarters into the future is quite small, and the majority of observations for any given quarter are missing.[21]

To provide a meaningful benchmark for our adjusted EBMA model, we also include in the ensemble the “Green Book” forecasts produced by the Federal Reserve. These forecasts are made by the research staff of the Board of Governors and are handed out prior to meetings of the Federal Reserve Open Market Committee (FOMC).[22]

Taking the *SPF* and Green Book unemployment forecasts, we calibrate an ensemble model for

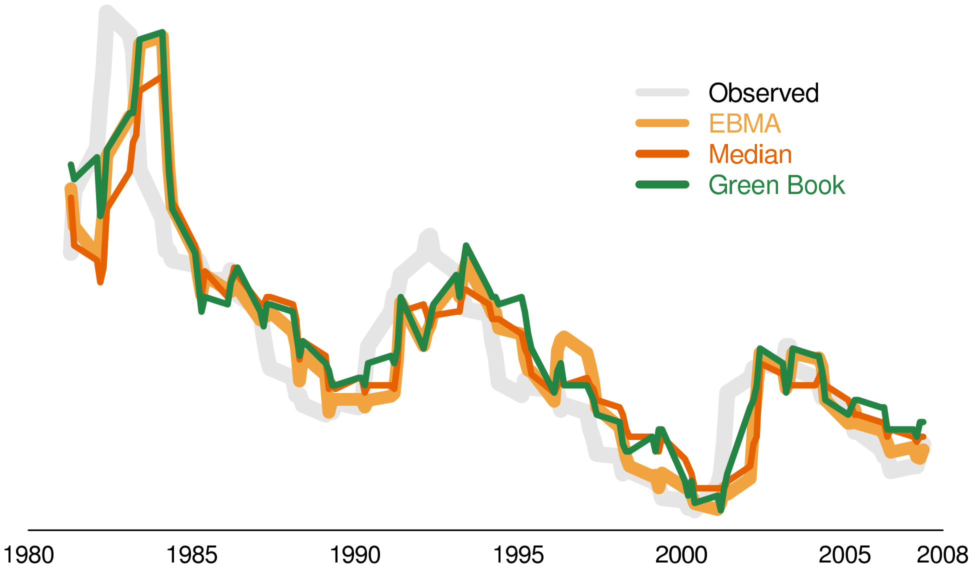
[20] The *SPF* was first administered in 1968 by the American Statistical Association and the National Bureau of Economic Research (NBER). Since 1990, however, it is run by the Federal Reserve Bank of Philadelphia. `http://www.phil.frb.org/research-and-data/real-time-center/survey-of-professional-forecasters/`

[21] On average only 8.4% of all teams make a forecast for any one quarter.

[22] One issue with forecast evaluation in many domains in economics is that the macroeconomic data (i.e. our “true observations”) are revised regularly. The unemployment rate for a given quarter at that time is generally an estimate that is subject to revision when better data becomes available. When evaluating forecasts, it is thus important whether predictions are compared to the outcome data for each quarter available at the time or whether the revised and most recent data is used. As Croushore and Stark (2001) describe, depending on the forecast exercise, it can make a difference whether the forecast models are evaluated using “real-time” (original estimate) or the “latest available” (revised) data. We have decided here to use the “latest available” data and do not believe that this choice affects our results, as all predictions are evaluated against the same data and EBMA is a mixture of the component forecast models. Thus the component models and our benchmark model are estimated and evaluated on the same data.

each period t, using forecaster performance over the past ten quarters. Only forecasts that had made predictions for five of these quarters were included in the ensemble. Thus, the EBMA model uses only 144 models out of a possible 293 forecasting models that made predictions during the period studied. Due to missing data early in the time series, and the fact that Green Book forecasts are sequestered for five years, we generate forecasts beginning in the third quarter of 1983 and running through the fourth quarter of 2007.

Figure 4: Observed and forecasted US unemployment (1981-2007)



One approach to evaluating the performance of EBMA is to compare its predictive accuracy to that made by other systematic forecasting efforts and methods of generating ensemble predictions. Specifically, we compare EBMA's ($c = 0.05$) predictive accuracy to (1) the Green Book, (2) the median forecaster prediction and (3) the mean forecaster prediction.[23]

Figure 4 shows a visual representation of the Green Book, median SPF and the EBMA (with $c = 0.05$) forecasts over time, as well as the true unemployment rate. As was noted above and is clearly visible, the SPF and Green Book forecasts are quite similar. Baghestani (2008) states that the Green Book forecast is slightly biased to over-predict the unemployment rate. In some periods EBMA is able to correct this bias; however, given the similarity of component models,

[23] Note that the EBMA model is calculated on only the subset of forecasts that have made a sufficiently large number of recent predictions to calibrate model weights. Thus, the median forecast and the ensemble forecast will not be the same even when $c = 1$.

the improvement in that direction is rather small. In general, however, it is clearly visible that the EBMA forecast is closer to the actual rate than the median SPF or the Green Book forecast.

Table 5: Comparing adjusted EBMA models with Green Book, median, and mean forecasts of US unemployment (1981–2007)

	MAE	RMSE	MAD	RMSLE	MAPE	MEAPE	MRAE	PW
EBMA (c=0)	0.54	0.74	0.37	0.093	8.37	6.49	**0.73**	**27.36**
EBMA (c=0.05)	**0.54**	0.74	**0.37**	**0.093**	**8.33**	**6.30**	0.75	**27.36**
EBMA (c=0.1)	0.54	0.74	0.35	0.093	8.40	6.44	0.76	28.30
EBMA (c=1)	0.61	0.80	0.46	0.102	9.72	8.92	0.95	46.23
Green Book	0.57	**0.73**	0.43	0.093	9.37	8.81	1.00	45.28
Forecast Median	0.62	0.81	0.47	0.103	9.83	8.87	0.98	47.17
Forecast Mean	0.61	0.80	0.46	0.102	9.71	9.06	0.93	46.23

Note: Definitions of model fit statistics are provided in the Appendix. The model(s) with the lowest score for each metric are shown in bold. Differences between model performance may not be obvious due to rounding.

Table 5 formally compares these models to EBMA models with c =0, 0.05, 0.1, and 1 respectively. To do this, we focus on eight model fit indices available in the literature (Brandt et al. 2011).[24] The eight metrics we use are: mean absolute error (MAE), root mean squared error (RMSE), median absolute deviation (MAD), root mean squared logarithmic error (RMSLE), mean absolute percentage error (MAPE), median absolute percentage error (MEAPE), median relative absolute error (MRAE) and percent worse (PW). The latter two metrics are measured relative to a naive model, simply predicting the future rate of unemployment as being the same as the current rate of unemployment. Further details for these metrics are shown in Appendix B.

The bolded cells in each column of Table 5 indicate the model that performed "best" as measured by each metric. With one exception (the Green Book outperforms the ensemble by 0.01 on RMSE), the EBMA model outperforms both the Green Book forecasts and the unweighted mean and median forecast on every metric. Moreover, these results confirm that the c parameter is best set to a small number. In general, the model with $c = 0.05$ performs best (or is tied for best) on six out of eight of these metrics.

We now turn to evaluating the performance of the ensemble relative to its 144 component forecasts. It is important to note that many of these forecasters make predictions in a relatively

[24] We are unable to use CRPS here because we do not have the predictive PDFs for the Green Book, SPF Mean, or SPF Median models.

Table 6: Comparing predictive accuracy of EBMA and component models with eight metrics

	Number of Predictions Made			
Number of metrics on which EBMA performed better	5 –10	11-30	31–60	> 60
7 – 8	0.59	0.74	0.68	0.75
5 – 6	0.12	0.09	0.18	0.25
2 – 4	0.06	0.08	0.05	0.00
0 – 1	0.22	0.09	0.09	0.00
Number of components	49	65	22	8

The rows of the table show the number of metrics by which EBMA outperforms components, while columns show the number of forecasts made by these models. The values in each cell of the table are the proportion of component models falling into that category (columns will sum to unity). Note that EBMA performs very well against its components, especially those that make many predictions.

small subset of cases. That is, each model k offers forecasts for only a subset of cases $n_k \subset n$. To create a fair comparison, therefore, we calculate these fit indices only for $n_k \forall k \in [1, K]$. By this measure, the EBMA model performs very well. Table 6 provides a summary of these results. The rows of the table show the number of metrics by which EBMA outperforms components, while columns show the number of forecasts made by these models. The values in each cell of the table are the proportion of component models in each column that fall into each row (columns will sum to unity). For instance, the top-left cell represents cases where EBMA is better on at least seven out of eight metrics for component models making between one and ten predictions. Approximately 60% of models that make 5-10 predictions fall into this category.

Notably, the relative superiority of EBMA to its components is somewhat less for components that provide few forecasts. This reflects the fact that, with so many forecasts, some are likely to be more accurate than the ensemble by chance alone. Additionally, when the number of forecasts is low, it is likely that a given model received less weight than it "deserves" given the model's performance.[25] However, Table 6 shows that, across a large number of forecasts, EBMA significantly outperforms its components. That is, when moving to the right in the table, the values in the lower cells decrease. It is also worth noting that only six out of the total 144 components outperform EBMA on every metric.

[25] See Appendix A for a discussion of how EBMA handles missing component forecasts.

5 Discussion

Ensemble Bayesian model averaging is a principled way of combining forecasts to improve prediction accuracy. However, the calibration of such models in the social sciences is often hindered by the quality as well as availability of data. First, in many forecasting exercises the number of forecasting models is large, yet the number of observations on which the EBMA model can be calibrated is small. This creates problems for the estimation of model weights, as it is likely that overly-high weights are assigned to models that perform well over this particular period. Second, many predictive models do not provide forecasts for all observations in the sample, as some forecasts may be missing or the time-periods for which forecasts were made are different for different models. In the standard EBMA model introduced in Montgomery et al. (2012a), missing observations in component model predictions are not allowed.

In this article, we address both of these issues to make EBMA more applicable for researchers and predictioneers in the social sciences. After reviewing the standard EBMA framework, we proceeded to introduce a "wisdom of the crowds" parameter into the model, which forces EBMA to put some minimal weight on all component models. Adding this constant aids the calibration of EBMA when the number of observations in the calibration period is small.

After explaining our adjustments, we illustrated its advantages via simulation. We then apply the modified EBMA model in two prediction exercises. We use the out-of-sample forecasts of nine prediction models of presidential elections from 1992 to 2008 to calibrate an ensemble model. We use the model to make an informed prediction for the 2012 elections based on a weighted combination of the component predictions. This example neatly illustrates the common difficulties facing forecasters in the social sciences, and provides an illustrative example for applied researchers going forward. In a second example, we use EBMA to combine predictions of the unemployment rate in the US from the Survey of Professional Forecasters as well as the Green Book. As we show, even when a large number of forecasts are missing for any given quarter, EBMA generally outperforms the Green Book, SPF component models, as well as the median and mean SPF forecast.

A comprehensive approach to the data problems raised in Section 2 would be to estimate the "wisdom of crowds" parameter within the EBMA algorithm specifically for each forecasting application. So far we have refrained from doing this as we are concerned with the number of parameters being estimated on relatively small numbers of observations (i.e. limited degrees of freedom). In addition, simple solutions have so far failed because of issues of identifiability. Future extensions of this model should aim to adjust the EBMA algorithm further to make estimation of c possible.

In addition, future research should investigate alternative imputation techniques within the

EBMA algorithm to handle missing data. The approach presented in Appendix A follows Fraley et al. (2010). This is an improvement in that it allows for the inclusion of models with missing predictions, but components with missingness are severely down-weighted. Moreover, the algorithm shown in Appendix A was developed in the context of meteorological sciences, where weather stations may fail to report observations randomly. In the social sciences, however, different approaches may be more appropriate. One possible direction would be to implement imputation of missing observations via copula methods within the EBMA framework.

Ensembles are a useful approach to aggregate predictive information. Even a decade ago there was not a lot of predictive information in need of aggregation in the social sciences. With the advent of newer approaches, more widely available data, and predictive heuristics, the arena of predictive forecasts has expanded considerably. The EBMA approach is now not only feasible in the social sciences, but also increasingly advantageous.

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Appendix A: EM-Algorithm for missing data

To accommodate missing values within the EBMA procedure, we follow Fraley et al. (2010) and modify the EM algorithm as follows. Define

$$\mathcal{A}^t = \{i|\text{ensemble member i available at time t}\},$$

which is simply the indicators of the list of components that provide forecasts for observation y^t. For convenience, define $\tilde{z}_k^{(j+1)t} \equiv \sum\limits_{k \in A^t} \hat{w}_k^{(j)} p^{(j)}(y|f_k^t) / \sum\limits_{k \in A^t} w_k^{(j)}$. Equation (3) above is then replaced with

$$\hat{z}_k^{(j+1)t} = \begin{cases} \hat{w}_k^{(j)} p^{(j)}(y|f_k^t)/\tilde{z}_k^{(j+1)t} \text{ if } k \in \mathcal{A}^t \\ 0 \text{ if } k \notin \mathcal{A}^t \end{cases} \tag{7}$$

The M steps in Equations (4) and (5) are likewise replaced with

$$\hat{w}_k^{(j+1)} = \frac{\sum\limits_t \hat{z}_k^{(j+1)t}}{\sum\limits_t \sum\limits_{k=1}^K \hat{z}_k^{(j+1)t}} \tag{8}$$

and

$$\hat{\sigma}^{2(j+1)} = \frac{\sum\limits_t \sum\limits_{k=1}^K \hat{z}_k^{(j+1)t} (y - f_k^t)^2}{\sum\limits_t \sum\limits_{k=1}^K \hat{z}_k^{(j+1)t}}. \tag{9}$$

In essence, the likelihood is renormalized given the missing ensemble observations prior to maximization. Using the adjustments above, the EBMA algorithm now allows for missing observations in the component predictions.

Appendix B: Predictive Metrics

Let x be some prediction of an event, for example a prediction model for the US presidential election. Now let $p(x)$ denote the PDF associated with forecast x and x_a be the actual observed values. The continuous rank probability score CRPS for forecast x and outcome y is then:

$$CRPS = CRPS(P, y) = \int_{-\infty}^{\infty} [P(x) - P_a(x)]^2 dx \tag{10}$$

where P and P_a are cumulative distribution functions, such that:

$$P(x) = \int_{-\infty}^{x} p(y) dy \tag{11}$$

and

$$P_a(x) = H(x - x_a) \tag{12}$$

H(x) denotes the Heaviside function where $H(x) = 0$ for $x < 0$ and $H(x) = 1$ for $x \geq 0$ (Hersbach 2000). The CRPS ranges from zero to one, with the best forecast models scoring closer to zero.[26]

Denote the forecast of observation i as f_i and the observed outcome as y_i. We define the *absolute error* as $e_i \equiv |f_i - y_i|$ and the *absolute percentage error* as $a_i \equiv e_i/|y_i| \times 100$. Finally, for each observation we have predictions from naive forecast, r_i, that serves as a baseline for comparison. In the example in the main text, this naive model is simply the lagged observation. We can therefore define $b_i \equiv |r_i - y_i|$.[27]

Denoting the median of some vector $\mathbf{x}$ as $med(\mathbf{x})$, and the standard indicator function as $I(.)$,

[26] The notation here is borrowed from Hersbach (2000), and Gneiting et al. (2007).

[27] See Brandt et al. (2011) for additional discussion of comparative fit metrics.

we define the following heuristic statistics:

$$
\begin{aligned}
\text{MAE} &= \frac{\sum_1^n e_i}{n} \\
\text{RMSE} &= \sqrt{\frac{\sum_1^n e_i^2}{n}} \\
\text{MAD} &= \text{med}(\mathbf{e}) \\
\text{RMSLE} &= \sqrt{\frac{\sum_1^n \left(\ln(f_i+1) - \ln(y_i+1)\right)^2}{n}} \\
\text{MAPE} &= \frac{\sum_1^n a_i}{n} \\
\text{MEAPE} &= \text{med}(\mathbf{a}) \\
\text{MRAE} &= \text{med}\left(\frac{e_1}{b_1}, \ldots, \frac{e_n}{b_n}\right) \\
\text{PW} &= \frac{\sum_1^n I(e_i > b_i)}{n} \times 100
\end{aligned}
$$