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INTRODUCTION

The past two years may go down in history as one of those roller-coaster periods that risk managers commonly call "four-sigma events"—a shorthand for something that is statistically nearly impossible, yet seems to occur every so often. All of the major financial asset markets were affected by the technology boom/bust cycle, shifting central bank policies, and the recent global economic slowdown, exacerbated by the heightened security concerns. This has understandably led to an elevated sensitivity to the risks of investing, in particular regarding the less seasoned and more exotic products. Even the rating agencies have been forced to re-evaluate long-standing ratings methodologies in order to respond to this shifting climate.

Both cash and structured credit markets have been particularly volatile in the wake of multiple economic and political events. Fortunately, The credit derivatives market played an important role in mitigating the response both to September 11 events and to subsequent shocks of the Enron, Argentina, Kmart, and Global Crossing and other defaults. Single-name default swaps continue to be actively used to manage balance sheet risks of banks. We also see interest in these instruments, notably basket default swaps, as a source of return rather than just a hedge. CDO issuance reached $87 billion ($66 billion in arbitrage structures), with investment grade CDOs capturing significant share due to historically wide credit IG and crossover spreads making the arbitrage attractive. Synthetic CDOs backed by default swaps became extremely popular in 2001, sometimes driving the default swap spreads through the cash market.

The proliferation of new credit products marketplace combined with unprecedented underlying credit volatility, emerging prominence of non-traditional credit investors and shifting methodologies at the major rating agencies have accelerated the embrace of quantitative credit tools. As hedge funds, CDO and convertible bond fund managers and non-US investors become more active participants in both the cash and structured credit markets, the reliance on statistical forecasting and analytical pricing models will increasingly drive relative security valuation. And with restive rating agencies now utilizing quantitative models as screening tools, such approaches will surely become more mainstream. Therefore, recognizing which frameworks and inputs have the most predictive power

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becomes an increasingly important issue for investors, structurers, risk managers, and other capital market participants.

In this issue of *QCR Quarterly*, we publish several papers covering a wide range of important topics mentioned above.

The first article shows that economy-wide risk factors that work well for pricing equity portfolios are also significant for pricing investment-grade corporate debt. Identifying the priced sources of macroeconomic risk allows for a decomposition of credit risk premia by means of a familiar beta representation. The three macro factors tell us that corporate bonds reward their holders for tolerating market and default risk and that the resulting premia are then adjusted downward because corporate bonds offer a desirable hedge against recession risk. The model is completed with the addition of sector-specific factors.

The second article introduces a new credit selection model, ESPRI, that uses the information in the historical behavior of equity returns and credit spreads to select corporate bond portfolios. Its authors review the results of an extensive empirical study that forms the basis of this model and show that portfolios selected by the ESPRI procedure significantly outperform a broad-based benchmark on a risk-adjusted basis. They also discuss practical applications of ESPRI—in particular, its ability to predict credit blow-ups.

The third article uses the previously introduced metrics of *excess spread premium* and *spread coverage ratio* to analyze the mezzanine tranches of generic investment grade CDOs. The authors show that by choosing the appropriate subordination for their risk appetite, investors can achieve substantially better performance versus single-name default swaps.

The fourth article takes a look at the fundamental issues regarding the risk measures for complex credit portfolios and structured products. After discussing the well-known approach of Value at Risk (VaR) and explaining why it is not appropriate for default risk, the authors introduce the concept of *coherent risk measures*, which overcome the limits of VaR, and focus on the use of the *expected shortfall*. Furthermore, they show how the recent developments in portfolio credit modeling allow one to apply this analysis to the default risk of a credit portfolio and of the tranches of a CDO.

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DECOMPOSING CREDIT RISK PREMIA[1]

Abstract

In this article, we find that three economy-wide risk factors that work well for pricing equity portfolios are also significant for pricing investment-grade corporate debt. Identifying the priced sources of macroeconomic risk allows for a decomposition of credit risk premia by means of a familiar beta representation.

The three macro factors that we propose tell us that corporate bonds reward their holders for tolerating market and default risk and that the resulting premia are then adjusted downward, since corporate bonds offer a desirable hedge against recession risk. The model is completed with the addition of sector-specific factors.

We estimate time-varying market prices for the identified risk sources and relate the price of recession risk to the Fed fund target and the price of aggregate default risk to KMV's median Expected Default Frequency™ (EDF™) for U.S. investment-grade corporates. In addition, we show that the dynamics of sector-specific risk prices capture well-known market events such as the S&L crisis, the extreme reaction of financials after Russia's default, and the significant widening of utilities during the California energy turmoil.

[1] I would like to thank Arthur Berd, Steve Bergantino, Ganlin Chang, Lev Dynkin, Mark Howard, Aleks Kocic, Guilherme Marone, Ravi Mattu, Prafulla Nabar, Bruce Phelps, Stefano Risa, Gaurav Tejwani, and Minh Trinh for their comments and suggestions.

1. Introduction

Decomposing the reward for bearing credit risk is certainly a challenging task, since it requires establishing a statistically meaningful relation between the performance of credit-sensitive assets and the fundamental sources of systematic risk in the economy. This article is an attempt to establish this link. Our findings allow for a decomposition of credit risk premia and shed some light on the determinants of credit spread behavior over the past several years.

A well-known measure for the performance of a corporate bond is given by its "duration-matched excess return," computed as the total return of the bond minus the total return of a Treasury portfolio with the same duration. This duration-matching procedure subtracts the interest-rate portion of the bond's return process and produces a reasonable measure of the reward received for bearing credit risk.

A naïve way to estimate credit risk premia is to look at the historical performance of a given credit portfolio and compute the sample mean of its realized excess returns. This statistical approach, however, has a major problem. Estimating mean returns with some level of precision requires a very large number of observations. The variance of the sample mean decreases quite slowly as the number of observations increases, so short time-series produce unreliable estimates.

In fixed-income markets, option-adjusted spreads (OASs) have become a standard metric for measuring credit premia. If OASs are a good proxy for conditionally expected excess returns (i.e., the risk premium over the next interval of time), then their time-series mean is going to be a fair estimate of the unconditional (long-run) credit risk premium.

Figure 1 shows time-series means of market-value-weighted excess returns and OASs (both in bp per month) for 16 credit portfolios over the period January 1992-December 2000. The portfolios are constructed from monthly data in the Lehman Brothers Credit Index, intersecting three sectors (financials, industrials and utilities), three quality groups (AA, A, BBB) and two duration buckets (below and above the median duration in the sector/quality cell).[2,3] For example, the excess return series for the FIN_A_SHORT portfolio is constructed as follows. At the beginning of a given month in the sample, we select all A financials with duration below the median duration of all A financials in the index at that time. Then we compute the excess return for the following month as the market-value-weighted excess return of the selected bonds. Figure 1 also reports the average number of bonds in the portfolios, as well as their average (option-adjusted) spread durations.

[2] AA utilities are omitted from our analysis because of the extremely low number of bonds in these cells. This insufficient diversification produces a return behavior that is dominated by idiosyncratic movements.

[3] Step-ups, sinkers, non-U.S., and non-corporate bonds were excluded from the analysis.

It takes only a quick look to realize that the sample means of excess returns must be very imprecise estimates of the true means. Some of the portfolios have negative mean excess returns. Risk-averse investors would not hold a portfolio of corporate bonds if they expected to underperform its duration-matched Treasury counterpart in the long run. Moreover, insofar as lower-quality portfolios have a higher exposure to systematic sources of risk in the economy than higher-quality portfolios, we expect their risk premia to be higher. This does not appear to be the case with the mean excess returns depicted in Figure 1.

OASs look like reasonable proxies for credit risk premia, but the lack of economic content that characterizes a price-to-OAS calculation makes it hard to decompose them. A fundamental problem with both approaches is that they neglect everything we know about asset pricing. In particular, they neglect the fact that in the absence of arbitrage opportunities, the risk premium of a financial asset must be proportional to the covariation between its return and a pricing kernel. This idea is formalized in section 3, in which we also derive a standard beta representation for risk premia. Section 4 discusses the method that we use to identify priced factors in corporate bonds, and section 5 presents a statistical test of model specification. In section 6, we complete the model with sector-specific factors and describe a heuristic procedure to estimate time-varying risk prices. A brief summary and a short list of references are given in sections 7 and 8. Before we enter the more technical aspects of our discussion, section 2 anticipates the main results and offers some necessary background.

Figure 1. **Sample Means: January 1992-December 2000**
Average Excess Returns and Average OASs in bp per month

	Average			
	Excess Return	**OAS**	**Spread Duration**	**Number**
FIN_SHORT_AA	6.17	4.70	2.35	85
FIN_SHORT_A	6.74	5.81	2.72	303
FIN_SHORT_BBB	1.65	9.62	3.10	86
FIN_LONG_AA	0.05	6.07	6.31	86
FIN_LONG_A	1.47	7.60	6.64	304
FIN_LONG_BBB	5.62	11.19	6.75	87
IND_SHORT_AA	4.24	3.96	3.78	80
IND_SHORT_A	3.51	5.85	3.84	285
IND_SHORT_BBB	3.91	9.87	3.97	245
IND_LONG_AA	-4.00	5.03	8.73	80
IND_LONG_A	-7.16	7.18	9.36	285
IND_LONG_BBB	-5.36	11.22	8.75	246
UTI_SHORT_A	4.99	5.60	3.76	72
UTI_SHORT_BBB	6.59	7.84	3.51	87
UTI_LONG_A	-2.43	6.02	7.59	73
UTI_LONG_BBB	1.80	9.11	7.69	88

2. Results and Discussion

2.1. The Framework

We can identify the systematic factors responsible for the observed credit risk premia by imposing some economic structure on the problem. As a first step, we can assume that credit bonds are priced in an economy in which there are no trading strategies that cost zero today, have zero probability of losing money, and pay some positive amount at some future date with positive probability. The rationale behind this no-arbitrage condition is clear: if such strategies existed, investors would identify them and implement them, and prices would adjust enough to make them disappear. This rather innocuous assumption gives us a strong and useful prediction: the risk premium of a financial asset must be proportional to the covariance between its return and a pricing kernel that, for reasons that will become obvious in the next section, is often called “stochastic discount factor” (SDF).

The second modeling assumption we are going to make is that the SDF can be represented as a linear combination of a set of factors that proxy for systematic (economy-wide) sources of risk. These two assumptions alone will provide us with a familiar beta representation, a natural way to decompose the credit premia and attribute their different components to identifiable sources of risk. The ubiquitous CAPM, Merton’s (1973) Intertemporal-CAPM, and Breeden’s (1979) Consumption-CAPM—as well as more recent pricing models developed by Fama and French (1993), Cochrane (1996), Campbell (1996), Jagannathan and Wang (1996), and many others—can all be seen as special cases of this basic framework (see Hodrick and Zhang [2001] for an excellent comparison of several factor models for equity pricing).

Identifying the set of relevant systematic factors is not an easy task. We employ the econometric tool-bag offered by the Generalized Method of Moments (GMM) and its distribution theory to estimate the unknown parameters of a number of factor models and to test whether their pricing errors are statistically significant. The model we eventually select exhibits statistical significance of the factors, successfully passes a model diagnostic test, and, most important, produces a combination of betas and risk prices that are consistent with economic intuition.

2.2. Fama-French Factors

Our model selection establishes a significant connection between credit and equity markets. The 3-factor Fama-French (1993, 1996) model has been shown to perform quite well in pricing equity portfolios. Currently, this is probably the dominant factor model for equity applications, and its popularity has grown to the point that many practitioners now use it instead of CAPM for standard cost-of-capital computations and investment choices. The three Fama-French factors are:

1) **MKT:** the difference between the return on the market-value-weighted portfolio of all stocks traded on NYSE, AMEX, and NASDAQ and the return on the 1-month Treasury bill,

2) **SMB (Small Minus Big):** the difference between the return on a portfolio of small-cap stocks and a portfolio of large-cap stocks,
3) **HML (High Minus Low):** the difference between the return on a portfolio of high book-to-market (B/M) stocks and a portfolio of low B/M stocks.[4]

Researchers have been trying to come up with plausible explanations for the good performance of these factors in order to understand what sources of macroeconomic risks lie behind these three excess returns. Although important steps have recently been taken in this direction (see Hahn and Lee [2001]), the debate is still open. There is, however, a generally accepted interpretation for their effectiveness. The excess return on the market portfolio is simply the traditional CAPM factor and proxies for the fundamental risk of market directionality. The higher the beta on the market portfolio, the higher the average risk premium, since risk-averse investors need to be paid (in expectation) for holding an asset that does well when most assets do well and poorly when most assets do poorly.

The rationale for the remaining two factors requires a slightly more elaborate digression. Using long time-series of equity returns, one can see that small-cap stocks have returned on average more than large-cap stocks, even after controlling for their beta on the market return. The reason small-cap stocks carry this (unconditional) premium is that they tend to perform particularly well in periods when default risk decreases and quite poorly in periods when default risk increases, possibly because of the relatively low amount of collateralizable assets that small-cap companies own. Small-cap stocks are therefore a “bad hedge” against this underlying source of default risk. For investors to hold them in their portfolios, small-cap stocks have to provide a relatively high expected return. An immediate implication is that the time-series of the difference between small-cap returns and large-cap returns (SMB) can be used as a proxy for this source of systematic default risk, and any asset that positively covaries with the SMB series must reward its holder with a risk premium (and, vice versa, any asset that negatively covaries with SMB must have its risk premium decreased accordingly).

Similarly, it is well known that high B/M (value) stocks have experienced higher average returns than low B/M (growth) stocks, even after controlling for market betas. Value companies tend to have high market leverage because of the relatively low market price of their equity. As a consequence, value stocks are more sensitive to the risk of financial distress, and they tend to underperform when economic growth is expected to slow down, exactly at a time when investors would mostly appreciate an extra point of return. Value stocks are therefore a “bad hedge” against business-cycle risk, and investors require a higher expected return in order to hold them. The time-series of the difference between value and growth returns

[4] Interested readers can download these factors and find a detailed description of how they are constructed at http://mba.tuck.dartmouth.edu/pages/faculty/ken.french. We would like to thank Kenneth French for making the factors available.

(HML) can therefore be used as a proxy for this systematic source of risk. Any asset that positively covaries with the HML series must reward its holder with a risk premium, and any asset that negatively covaries with HML must have its risk premium decreased accordingly.

Even if the theoretical rationale behind the Fama-French returns is still the object of extensive research, the fact is that these factors work surprisingly well in empirical tests. Up to now, the asset pricing literature has focused almost exclusively on their ability to price equity portfolios: in this article, we find that MKT, SMB, and HML are significant for the pricing of credit portfolios as well. Quite interestingly, our results will also provide further support for the economic interpretations outlined above.

This connection between credit and equity markets is certainly relevant for investors who hold both credit and equity positions, since it suggests that the hedging and risk management of their portfolios should take these common dependencies into account. Moreover, the interplay between credit and equity markets, together with the predictability of equity returns documented in the literature, suggests that the informational efficiency of stock prices may be used to uncover predictable patterns in credit spread changes. Naik, Trinh, and Rennison (2002, elsewhere in this issue) show that this is much more than a theoretical speculation.

2.3. Decomposing OASs

Our approach allows for an intuitive decomposition of credit risk premia. In the next section, we formally show that every linear factor model has an associated beta representation. This means that the risk premium of any asset can be represented as the sum of K components, where K is the number of factors. The K^{th} component is simply the product between the beta of the asset on the K^{th} factor and the market price of the risk proxied by the K^{th} factor.

In a later section, we propose a heuristic procedure to calibrate time-varying risk prices to OASs. We also allow for sector-specific factors, defined as latent sources of risk that separately affect financials, industrials, and utilities, that are loaded with unit betas by every bond belonging to the sector. This allows us to represent the time-varying risk premium of asset i as

$$OAS_{i,t} \approx \beta_i^{MKT}\lambda_t^{MKT} + \beta_i^{SMB}\lambda_t^{SMB} + \beta_i^{HML}\lambda_t^{HML} + \beta_i^{FIN}\lambda_t^{FIN} + \beta_i^{IND}\lambda_t^{IND} + \beta_i^{UTI}\lambda_t^{UTI}. \quad (1)$$

Figure 2 reports the betas for the 16 credit portfolios described earlier, while Figures 3-5 help visualize the patterns in the betas on the macro factors.

The relative magnitudes of the betas on the Fama-French proxies agree quite well with the economic intuition. Market betas are similar for financials and industrials—though long financials load heavily on the market factor almost independently of their quality—while they are lower for utilities, especially for long portfolios.

Figure 2. **Betas**

	MKT	SMB	HML	FIN	IND	UTI
FIN_SHORT_AA	0.016	0.011	0.013	1	0	0
FIN_SHORT_A	0.027	0.019	0.023	1	0	0
FIN_SHORT_BBB	0.054	0.075	0.022	1	0	0
FIN_LONG_AA	0.084	0.031	0.044	1	0	0
FIN_LONG_A	0.086	0.050	0.059	1	0	0
FIN_LONG_BBB	0.086	0.087	0.080	1	0	0
IND_SHORT_AA	0.017	0.016	0.016	0	1	0
IND_SHORT_A	0.022	0.022	0.015	0	1	0
IND_SHORT_BBB	0.044	0.029	0.011	0	1	0
IND_LONG_AA	0.060	0.031	0.029	0	1	0
IND_LONG_A	0.073	0.042	0.024	0	1	0
IND_LONG_BBB	0.104	0.061	0.030	0	1	0
UTI_SHORT_A	0.014	0.021	0.009	0	0	1
UTI_SHORT_BBB	0.016	0.030	0.019	0	0	1
UTI_LONG_A	0.051	0.036	0.016	0	0	1
UTI_LONG_BBB	0.080	0.054	0.033	0	0	1

Figure 3. **Betas on Macro Factors: Financials**

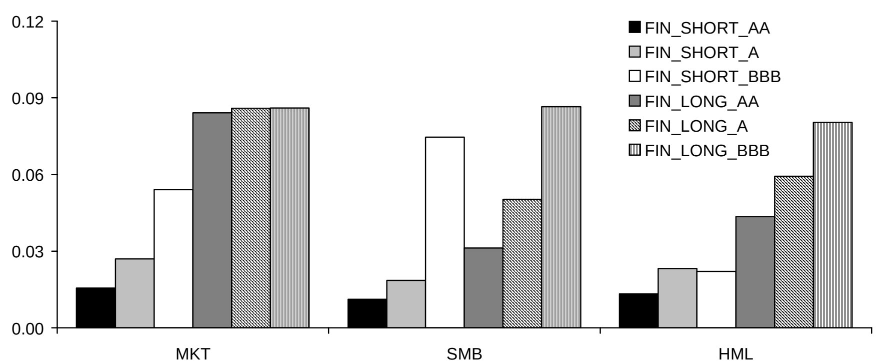


Figure 4. **Betas on Macro Factors: Industrials**

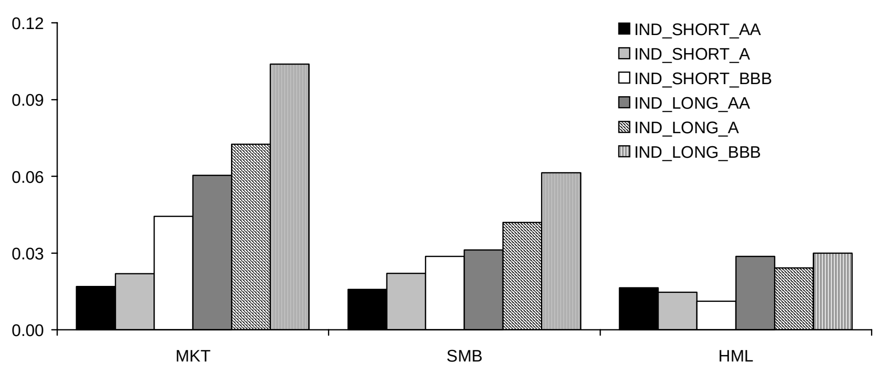
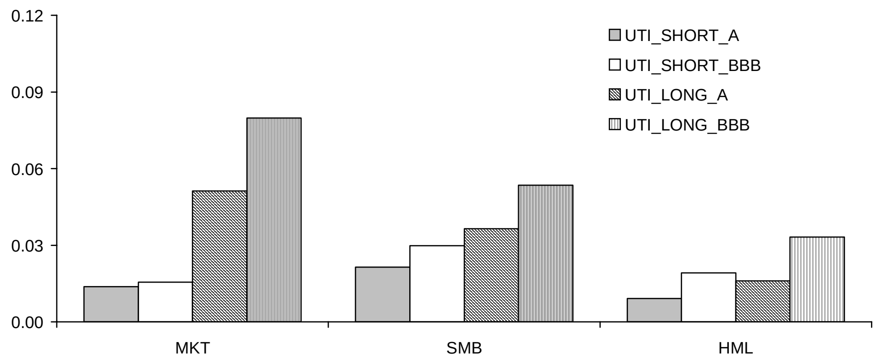


Figure 5. **Betas on Macro Factors: Utilities**



Sensitivity to default risk clearly increases as quality worsens. Betas for AA and A are not very different across industries, while financials load more heavily on the default factor at the low end of the quality spectrum, on both sides of the curve.

Long portfolios have the higher betas on the business-cycle factor. Claims to shorter streams of cash flows are less sensitive to recession risk by definition. In particular, long financials appear to be most sensitive to this factor.

Figure 6 illustrates the time-series of estimated macro risk prices (λ^{MKT}, λ^{SMB}, λ^{HML}). To understand the patterns of these series, we now have to switch from thinking "unconditionally," i.e., on average, to thinking "conditionally," i.e., taking into account what the market knew at each point in time.

λ^{MKT}**:** The price of market risk was fairly stable over the sample period, except for the fact that it fell sharply—and turned negative—right after the major market downturns. This can be seen at the beginning of the sample, as the economy was coming out of the 1991 recession, in the summer of 1998, right after the Russian crisis, after the market correction in the summer of 1999, and after the broad market downturn in September 2000. When the market is expected to recover, positive sensitivity to the market is a characteristic for which investors are willing to pay. The excess demand for assets with a high market beta pushes down the price of this risk.

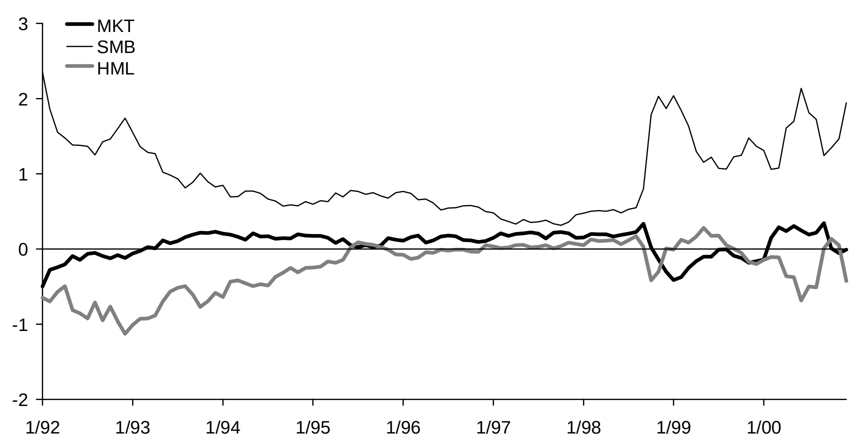
λ^{SMB}**:** The price of default risk was always positive over the sample period and followed a clear downward trend until late 1997. When default risk is low, positive covariation with the excess return of small-cap stocks over large-cap stocks is a characteristic that investors do not dislike as much as they do in a high-default environment. (Remember that small-cap stocks do relatively well in periods when default risk decreases, and vice-versa, which is the reason they carry a premium on average). During the Russian crisis, the price of default risk blew up as investors moved out of low-quality assets, and over the last two years of the decade, it exhibited an upward trend and a pronounced volatility, tracking the large spread fluctuations observed in the credit sector. The price of default risk should be related to the market expectation of aggregate default frequency, to the level of uncertainty about future defaults, and to the risk premium charged by investors for bearing that uncertainty. Figure 7 compares λ^{SMB} with the median Expected Default Frequency (EDF) for U.S. investment-grade corporates (excluding financials), as computed by KMV over the period January 1997-December 2000.[5] It shows that the median EDF measure drives the trend of the market price of default risk and that default uncertainty and/or the associated risk premium increased dramatically as liquidity dried up in August 1998. The graph also suggests that an increased volatility in EDF levels contributed to the large spread movements observed in 2000.

λ^{HML}: The price of distress risk was negative for most of the sample period, reflecting the fact that all of our credit excess returns have negative covariance and positive beta with the HML series (cf. section 3).[6] When value stocks underperform growth stocks (remember, this tends to happen when economic

[5] Source: CreditEdge™, KMV LLC. © 2001-2002 KMV LLC. All rights reserved. CreditEdge, Expected Default Frequency, and EDF are trademarks of KMV LLC.

[6] Betas are slopes in a *multiple* regression; covariance has the same sign as the slope of a *univariate* regression. The two slopes need not have the same sign unless the factors are orthogonal, which is definitely not our case.

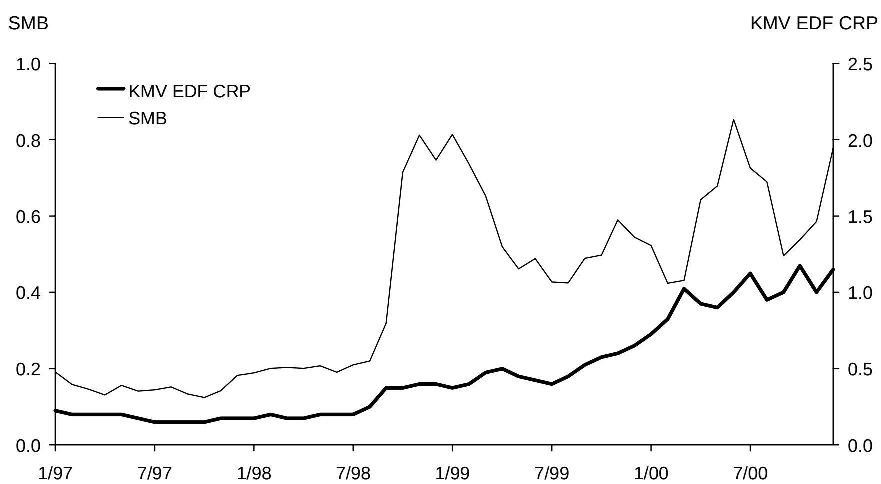
Figure 6. **Time-Varying Prices of Macro Risks: λ^{MKT}, λ^{SMB}, λ^{HML}**



growth is expected to slow and distress risk increases, which is the reason value stocks carry a premium on average), excess returns of credit portfolios tend to be positive (and vice versa). This negative covariance is likely to be the consequence of a flight to quality (from equity to corporates) when a contraction is anticipated and an opposite flight to equity when economic activity is expected to boost. This counter-cyclical behavior makes corporate bonds a good hedge against recession risk, for which investors are willing to pay. Consistent with the interpretation of HML as a business-cycle factor, the price of this risk is closely related to expected economic growth and Fed activity (see Figure 8 for a pictorial comparison). In 1992, monetary policy was loose in an attempt to restart the economy. There was significant uncertainty about future economic growth, and hedging against recession was expensive for investors, i.e., the price of HML risk b1ecame significantly negative. In 1993, the real effects of the easing kicked in, the market anticipated a subsequent Fed tightening, and λ^{HML} started to climb. As predicted, in 1994, the Fed took a drastic U-turn, increasing the Fed fund target from 3% to 6% over the year. The economy was now rapidly expanding, distress risk was decreasing, and negative covariance with the HML series became a cheaper commodity: investors did not need to give up a significant portion of risk premium to hedge recession risk. From 1995 to late 1998, the target was in the 5%-6% bracket, and λ^{HML} also stayed within a range. Since 1998, the price of distress risk incorporated in credit spreads has often led Fed policy and economic activity. In particular, from mid-1999 to mid-2000, as the Fed was still tightening to deflate the tech bubble, credit investors anticipated the subsequent economic slowdown and hedging recession risk became more expensive.[7]

[7] This confirms the recent findings of Saito and Takeda (2000) that corporate bond spreads contain information about future economic activity.

Figure 7. **Median KMV EDF for U.S. Investment-Grade Corporates and λ^{SMB}**



λ^{FIN}, λ^{IND}, λ^{UTI}**:** Figure 9 shows that the price of risk specific to the financial sector followed a downward trend from the S&L crisis up to 1998, when the Russian crisis hit financials harder than either industrials or utilities. At the far right of the plot, the pronounced spike in the price of utility-specific risk at the end of 2000 is largely due to the California energy crisis. Notice that the prices of sector-specific risks are highly correlated both among themselves and with the price of aggregate default risk (λ^{SMB} in Figure 6). In fact, one can think of two components of default risk, one specific to the sector, the other to the broad market.

Figure 8. **Fed Activity and** λ^{HML}

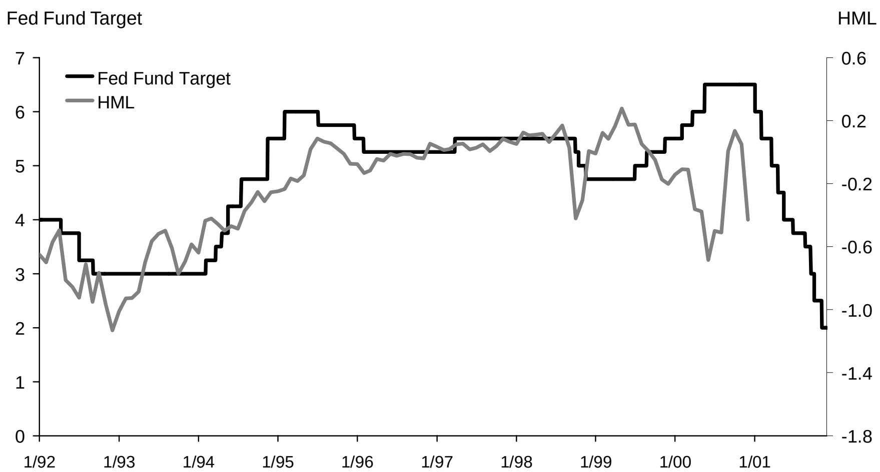
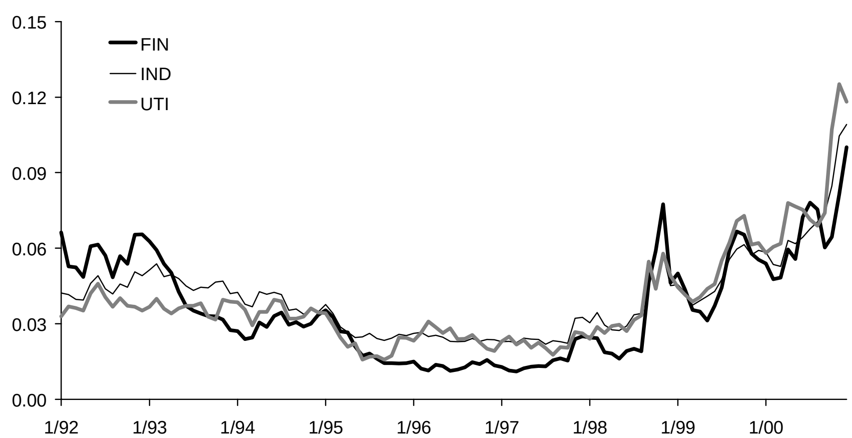


Figure 9. **Time-Varying Prices of Sector-Specific Risks:** λ^{FIN}, λ^{IND}, λ^{UTI}



Not surprisingly, the equilibrium prices for bearing the different components of default risk move in a highly correlated fashion.

Multiplying the betas in Figure 2 by the lambdas in Figures 6 and 9, we can obtain a decomposition of the time-varying risk premia according to equation (1). Figure 10 offers an example of this decomposition, plotting the four components of OAS for the FIN_LONG_AA portfolio. Each component is the product between a beta and the associated risk price, as represented in equation (1). We can now relate the behavior of risk prices outlined above with the performance of this particular portfolio.

Coming out of the S&L crisis and the 1991 economic slowdown, long AA financials were receiving considerable premium because of their exposure to default risk, both at the aggregate and the sector-specific level. Part of this compensation was used to achieve exposure to the broad stock market—expected to rebound—and to hedge uncertainty about future economic growth. When it became clear that the U.S. economy was headed for a stable growth period, the cost of hedging recession risk gradually disappeared. Between mid-1995 and mid-1998, the premium carried by the FIN_LONG_AA portfolio was almost equally due to compensation for market risk, aggregate default risk, and sector-specific risk. Since August 1998, economy-wide and sector-specific default risks have been driving the spread of long AA financials. The high beta on the market (Figure 3), a desirable characteristic in the fall of 1998, has limited the spread widening during that period.

Figure 11 presents the R^2 for the 16 portfolios, i.e., the percentage of OAS variance explained by the model in equation (1).

Figure 10**. OAS Decomposition: FIN_LONG_AA**

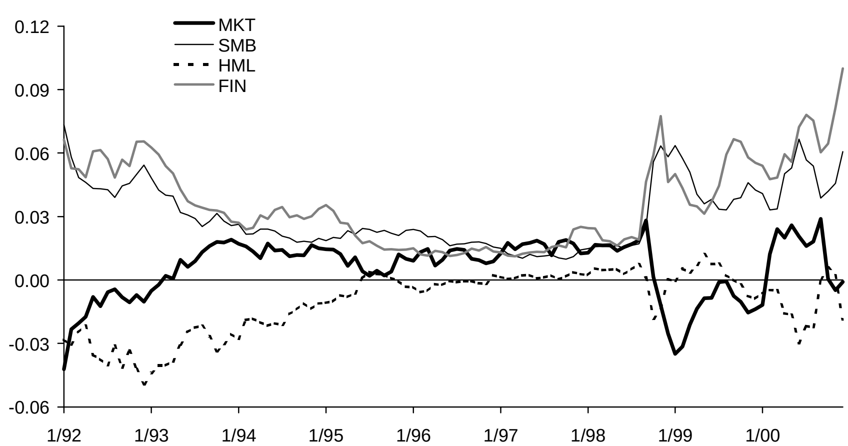


Figure 11. **R^2** (%)

	R^2
FIN_SHORT_AA	93
FIN_SHORT_A	96
FIN_SHORT_BBB	99
FIN_LONG_AA	98
FIN_LONG_A	96
FIN_LONG_BBB	99
IND_SHORT_AA	82
IND_SHORT_A	98
IND_SHORT_BBB	84
IND_LONG_AA	91
IND_LONG_A	94
IND_LONG_BBB	91
UTI_SHORT_A	91
UTI_SHORT_BBB	88
UTI_LONG_A	89
UTI_LONG_BBB	93

3. Linear Factor Models and Beta Representations

One of the most influential propositions of modern asset pricing theory (sometimes referred to as the Fundamental Theorem of Pricing) is that, in the absence of arbitrage opportunities, the price vector p_{t-1} and the payoff vector x_t of a set of financial assets must obey the relation

$$p_{t-1} = E_{t-1}[M_t x_t] \ , \tag{2}$$

where M_t is a stochastic discount factor (SDF) and E_{t-1} denotes conditional expectation at time *(t-1)* (see Harrison and Kreps [1979]). Equation (2) states something that is powerful and intuitive at the same time. If there were no uncertainty, the payoffs x_t would be known at *(t-1)*, and, in order to avoid arbitrage opportunities, M_t would have to equal the price of a one-period risk-free bond, i.e., the one-period discount factor. When the pay-offs x_t are uncertain, this notion of discounting still applies in expectation, and the discount factor itself becomes random (stochastic).

We can divide both sides of equation (2) by p_{t-1} (element by element) to get

$$1 = E_{t-1}[M_t R_t],$$

where $R_t = x_t / p_{t-1}$ represents the vector of gross rates of return. One can directly write down this equation simply by observing that the price of a gross rate of return is one by definition.

More interesting for our study, the price at *(t-1)* of any vector of excess returns r_t is zero—an excess return can always be seen as generated by a self-financing strategy—so that equation (2) becomes

$$0 = E_{t\text{-}1}\left[M_t r_t\right]. \tag{3}$$

Equation (3) will be the basis for all the analysis in this article. In view of the estimation that we are going to perform in a later section, it is convenient to take expectations of both sides of equation (3) to obtain the unconditional pricing relation

$$0 = E\left[M_t r_t\right]. \tag{4}$$

Using the definition of covariance $Cov_t[X,Y]=E_t[XY]-E_t[X]E_t[Y]$, we can solve equation (3) for $E_{t-1}[r_t]$ and conclude that expected excess returns must be proportional to their covariance with the SDF:

$$E_{t-1}\left[r_t\right] = \frac{-\ Cov_{t-1}\left[M_t, r_t\right]}{E_{t-1}\left[M_t\right]}. \tag{5}$$

Up to now, we have only assumed the absence of arbitrage opportunities. Most readers who are familiar with CAPM-type restrictions will probably find a beta representation of equation (5) more familiar. To get there, we need to introduce our second assumption, namely that the SDF can be represented as a linear combination of systematic factors.[8] Let us write

$$M_t = a_{t-1} - b_{t-1}' F_t, \tag{6}$$

where a_{t-1} is an intercept, b_{t-1} is a vector of K parameters (both a_{t-1} and b_{t-1} are known at *(t-1)*), and F_t is a vector of K systematic factors. Substituting (6) into (5), some simple algebra gives

$$E_{t-1}\left[r_t\right] = \beta_{t-1}\Lambda_{t-1},$$

where $\beta_{t-1} = Cov_{t-1}[r_t,F_t]*Cov_{t-1}[F_t,F_t]^{-1}$ is the vector of K conditional betas of r_t on the factors F_t, and $\Lambda_{t-1}=Cov_{t-1}[F_t,F_t]*b_{t-1} / E_{t-1}[M_t]$ is the vector of K conditional prices of risks associated with the factors. This linear representation summarizes the core of risk-return analysis: risk premia are related to the covariation between asset returns and the priced sources of risk in the economy.

[8] This assumption does not really cost us much in terms of generality. For example, if the SDF were a quadratic function of Y, i.e., $M = aY + bY^2$, we could just redefine $Z=Y^2$ and write $M = aY + bZ$, a linear function of Y and Z.

Starting from equation (4) and restricting the parameters of the SDF to be constant,

$$M_t = a - b' F_t, \tag{7}$$

the same steps lead to the unconditional beta representation

$$E[r_t] = \beta\Lambda$$

where $\beta = Cov[r_t,F_t]*Cov[F_t,F_t]^{-1}$ is the vector of K unconditional betas (i.e., it is the vector of slopes in a multiple regression of r on F) and $\Lambda = Cov[F_t,F_t]*b / E[M_t]$ is the vector of unconditional risk prices.

At this point, we already have the theory we need. In summary, we have assumed that:

1) there are no long-lived arbitrage opportunities in financial markets, and
2) the pricing kernel can be represented as a linear combination of a set of risk factors.

We are now ready to make use of the restrictions derived in this section and let the data help us identify the relevant risk factors.

4. Selecting the Macro Factors

How can we estimate the unknown parameters of a factor model? What metric should we use to compare the performance of alternative sets of factors? How can we control for the time-varying liquidity and volatility states of the market? We now tackle these important steps of our analysis.

4.1. GMM and the Hansen-Jagannathan Distance

In the previous section, we derived the unconditional restriction (see equations (4) and (7))

$$0 = E[(a - b' F_t)\, r_t], \tag{8}$$

where r_t is a vector of N excess returns that we are asking the model to price correctly. The Generalized Method of Moments (GMM) suggests an intuitive estimation strategy: pick those values of a and b that minimize the model's pricing errors. In other words, let us pick a and b to make the vector

$$g(a,b) \equiv \frac{1}{T}\sum_{t=1}^{T} [(a - b'F_t)\, r_t]$$

as close as possible to zero. Since we are not using the risk-free rate as a test asset, it turns out that the mean SDF *(E[M])* is not identified, and we can arbitrarily normalize one of the parameters. A convenient normalizaton is $a=1$, so that we really want to minimize

$$g(b) \equiv \frac{1}{T}\sum_{t=1}^{T} [(1 - b'F_t)\, r_t].$$

One way to perform this minimization is to give equal importance to the pricing of each one of the test assets and simply solve the unweighted least squares problem

$$\min_b \; g(b)'g(b). \tag{9}$$

The seminal work of Hansen (1982) shows that the optimal GMM estimator solves the weighted problem[9]

$$\min_b \; g(b)'S^{-1}g(b), \tag{10}$$

where S, often called the spectral density matrix, is a consistent estimate of $Cov(g(b))$. Intuitively, this result says that when we try to minimize the pricing errors of a model, we should really put more weight on those portfolios that display a lower variance, since their return processes are going to be more informative about the parameters of interest.

It is straightforward to show that the solution to (10) is given by

$$b_{GMM} = \left(d'S^{-1}d\right)^{-1} d'S^{-1}\frac{1}{T}\sum_{t=1}^{T} r(t),$$

where

$$d = \frac{1}{T}\sum_{t=i}^{T} r_t\, F_t'.$$

Since a consistent estimate of S requires residuals from the model and this, in turn, requires an estimate of b, optimal GMM is usually implemented in two stages. In the first stage, an estimate of b is obtained through an arbitrary weighting scheme (often using equal weights as in problem (9)); then a consistent estimate of S is obtained, and b_{GMM} is computed in the second stage.

Unfortunately, optimal GMM is not a viable estimation technique when one needs to select a model from alternative specifications. Different models produce different residuals from the first stage and, therefore, different

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[9] "Optimal" here means that it is consistent and asymptotically efficient.

estimates of the spectral density matrix S. It follows that the minimized values of problem (10) cannot be used to run a horse race among alternative specifications.

Hansen and Jagannathan (1997) provide a solution to this problem. They show that if we use the second moments of the test assets as the weighting matrix in the GMM problem, the solution b_{HJ} minimizes the distance between the SDF implied by the model and the set of all SDFs that correctly price the test assets. Formally, if we denote this distance with δ, Hansen and Jagannathan show that

$$\delta = g(b_{HJ})' W^{-1} g(b_{HJ}),$$

where b_{HJ} solves

$$\min_b \; g(b)' W^{-1} g(b), \tag{11}$$

and

$$W = \frac{1}{T} \sum_{t=1}^{T} r_t \, r_t'.$$

A comparison with (10) confirms that (11) is just a GMM problem with a weighting scheme provided by the matrix of second moments of the test assets. Therefore, following our earlier results, we have

$$b_{HJ} = \left(d' W^{-1} d\right)^{-1} d' W^{-1} \frac{1}{T} \sum_{t=1}^{T} r(t). \tag{12}$$

The important point here is that W is independent of the chosen specification. Thus, comparing the values of δ produced by alternative models is a valid selection methodology.

In order to test for the statistical significance of the estimates, we need to know their standard errors. It can be shown that the covariance matrix of b_{HJ} is given by[10]

$$Cov[b_{HJ}] = \frac{1}{T} \left(d' W^{-1} d\right)^{-1} d' W^{-1} S \, W^{-1} d \left(d' W^{-1} d\right)^{-1}. \tag{13}$$

4.2. Controlling for Time-Varying Liquidity and Volatility

Risk premia can vary through time, responding to changes in the state of the economy. In particular, fixed-income markets are known to go through periods of high and low liquidity. It is likely that the representative investor will demand a higher level of expected compensation for bearing credit risk in periods in which the expected cost of a roundtrip in and out of a position is higher. Similarly, changing forecasts of future volatility can affect the attitude toward risk-taking.

Even when we estimate an unconditional model such as the one described in equation (8), we can bring in conditional information to improve upon the estimate of the unknown parameters. Suppose z_t is a state variable summarizing the liquidity of the market at time t. Multiplying both sides of equation (3) by z_{t-1} and then taking expectation gives

$$0 = E[M_t r_t z_{t-1}]. \tag{14}$$

Analogously, if w_t is a state variable describing the time-t forecast of future volatility levels, we have:

$$0 = E[M_t r_t w_{t-1}]. \tag{15}$$

If r_t is the N-vector of excess returns that we are trying to price, equations (14) and (15) give two more sets of restrictions that we can stack together with (8) to obtain a system of $3N$ conditions:

$$\begin{aligned} 0 &= E[M_t \ r_t] \\ 0 &= E[M_t \ r_t z_{t-1}] \\ 0 &= E[M_t \ r_t w_{t-1}]. \end{aligned} \tag{16}$$

Notice that the first N conditions are asking the model to price the original excess returns correctly, while the remaining conditions are asking that the return surprises be unforecastable by the liquidity and volatility state variables.[11]

Another way to look at the system in (16) is that we are asking the model to price not only the original excess returns r_t, but also a set of actively managed portfolios with returns equal to $r_t \, z_{t-1}$ and $r_t \, w_{t-1}$. This interpretation should convince the reader that we can still apply the GMM formulas developed in the previous section to estimate and test the significance of the unknown parameters b. We have simply tripled the number of portfolios that we are asking our factor model to price.

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[10] This is just an application of the "delta method." See Cochrane (2001) for details.

[11] $M_t \, r_t$ is the vector of innovations for period t. Therefore, $E[M_t \, r_t \, z_{t-1}]$ is the numerator of the slope coefficient in a forecasting regression of the innovations at t on z_{t-1}. Asking this coefficient to be zero is equivalent to asking that the pricing innovations cannot be forecast by the liquidity state of the market. The same reasoning holds for w_t.

We carry out the race among alternative specifications by requiring the candidate models to price 48 portfolios, i.e., the original 16 excess returns plus the same series scaled by:

1) the difference between the on- and off-the-run 5-year Treasury rate (liquidity state) and
2) the level of the VIX index, an index of implied volatility produced by the Chicago Board Options Exchange (volatility state).

4.3. Model Selection and Parameter Estimates

We estimated a large number of models, combining market factors (equity and fixed-income), default factors (changes in spreads between high- and low-quality bonds at various points along the curve), interest-rate factors (changes in level and slope of the off-the-run Treasury yield curve), volatility factors (constructed from both equity and fixed-income data), liquidity factors, and equity factors.

The three Fama-French factors give the lowest H-J distance. Figure 12 provides the GMM estimates of b_{MKT}, b_{SMB}, and b_{HML} (see equation (12)), their standard errors (see equation (13)), and their ratios, showing that these three factors are statistically significant for pricing the test assets.

5. Model Diagnostic

Empirical modeling is about describing data behavior through a set of relations that are often (though not necessarily) derived from an underlying theory. These relations generally involve unknown parameters that must be estimated. A sensible model is always over-identified, i.e., it contains a number of unknown parameters that is smaller than the number of restrictions. If we were free to pick parameters to satisfy every relation without error, our model would be meaningless.

A direct implication of this line of thought is that the number of extra restrictions that a model imposes, i.e., the difference between the number of specified relations and the number of free parameters, can (and should) be used to test the statistical relevance of the model itself. Intuitively, we want to ask the question: are the model errors too large to have been generated by sample variation? If the answer is yes, the model should be rejected.

Figure 12. **Parameter Estimates**

	MKT	**SMB**	**HML**
b	0.095	0.066	0.152
s.e	0.040	0.032	0.030
b/s.e.	2.390	2.050	4.990

Recall that the minimized weighted sum of squares in the Hansen-Jagannathan problem is equal to δ, the distance between the SDF implied by the model and the set of all SDFs that correctly price the assets.

Now the question of whether the chosen model is correctly pricing the test assets can be statistically reformulated in the context of the H-J distance. We need to test the hypothesis $\delta=0$. Jagannathan and Wang (1996) show that under the null hypothesis $\delta=0$,

$$T\delta^2 \xrightarrow{d} \sum_{i=1}^{N-K} \theta_i \nu_i \quad \text{as} \quad T \to \infty,$$

where the θ_i 's can be consistently estimated by the first N-K eigenvalues of the matrix

$$S^{1/2} W^{1/2\,\prime} \left(I_N - W^{1/2} d \left(d' W d \right)^{-1} d' W^{1/2\,\prime} \right) W^{1/2} S^{1/2\,\prime},$$

and the ν_is are independent chi-square random variables with one degree of freedom. We simulate 100,000 times the test statistic to obtain its distribution and compute the p-value for testing the null hypothesis $\delta=0$.

The p-value for our 3-factor model is 9.35%, so that we fail to reject the model at a standard test size.

6. Sector-Specific Factors and Time-Varying Risk Prices

Once we have identified the macro factors that appear to be most significant for pricing our credit portfolios, we complete the set of underlying risks by adding sector-specific latent factors loaded by unit betas. We then use the available OAS series to estimate time-varying risk prices.

Our underlying assumption is that credit risk premia are generated by constant betas and time-varying risk prices, as described in equation (1). Referring to equations (5) and (6), the reader can check that sufficient conditions for this specification are that the parameters of the SDF, b_t, are time varying, while the conditional covariances between factors and returns are constant and equal to their unconditional counterparts.

To estimate the risk prices plotted in Figures 6 and 9, we simply run period-by-period cross-sectional regressions of the portfolio OASs on the constant betas depicted in Figure 2. Although this procedure is clearly heuristic, the resulting risk prices are supported by the economic interpretations offered in section 2.

7. Summary

After running a horse race among alternative factor models, we have concluded that the three Fama-French factors—MKT, SMB, and HML—significantly affect the pricing of U.S. investment-grade corporate bonds. The interpretation of these economy-wide factors as proxies for market, default, and business-cycle risk, already established in equity literature, is reinforced by the informational content of bond prices in the Lehman Brothers Credit Index.

After completing the model with sector-specific factors, we were able to decompose the credit risk premia (proxied by option-adjusted spreads) for several credit portfolios and estimate the time-varying compensation that investors received for the different sources of risk over the period 1992-2000. We have shown that some of the major events in the credit markets during this period can be related to the dynamics of prices for aggregate and sector-specific default risks. Moreover, we have established a relation between the cost of hedging business-cycle risk and the dynamics of the Fed fund target.

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INTRODUCING LEHMAN BROTHERS ESPRI: A CREDIT SELECTION MODEL USING *E*QUITY RETURNS AS *SPR*EAD *I*NDICATORS

1. Introduction

In this article, we present the results of an extensive empirical study that investigates the performance of investment strategies based on bond selection using the information contained in equity market movements and credit spreads. A close correlation between contemporaneously measured equity returns and movements in credit spreads is naturally expected, especially for issuers with low credit ratings. Indeed, as the structural models of corporate bond pricing (such as the one proposed by Merton [1974][1]) postulate, one can value corporate debt and equity as two claims contingent on the same variable—namely, the assets of a firm. This does not imply, however, that the *historical* performance of equity and credit spreads is informative about the future behavior of spreads. The aim of this article is to document the empirical evidence on the performance of credit portfolios selected using such historical information.

During the past decade, the evidence of predictability in equity markets has been widely documented in the empirical finance literature. This evidence suggests that stock prices tend to follow short-run reversal, to continue past trend (momentum) until 6 months and a year, and to go through reversal beyond 2-5 years.[2] Some theoretical models based on behavioral assumptions have been developed to explain these features.[3] For example, it has been argued that the evidence of predictability in equity markets can be explained by various behavioral biases. These include the representativeness bias (investors tend to extrapolate and generalize from singular observations), conservatism (investors do not change their mind easily), overconfidence (investors overestimate the precision of their information signals), and self-attribution (investors attribute good returns to their skill but bad returns to luck). A number of models have been developed to explain equity market predictability in the context of rational asset pricing as well. In these models, equity market predictability is a reflection of time-varying risk premia. The debate about whether rational investor behavior can lead to the patterns in asset returns that we observe is ongoing.

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In this article, we present the results of a study that aims to explore whether the predictability seen in equity markets carries over to corporate bond markets. Given the evidence on equity market predictability and the fact that, contemporaneously, corporate bond and equity returns are likely to be correlated, especially for firms with low credit ratings, it is quite natural to look for bond returns predictability and for any cross-market effects between equity and bonds. Below, we present the results of our investigation of trend and cross-market effects using the extensive Lehman Brothers bond index data (in particular, option-adjusted spreads data) and equity data to develop some understanding of these effects.

We document the evidence for such cross-market spillover in the U.S. corporate bond market for the period 1994-2001. When we look at the effect of equity returns and control for rating and duration, a portfolio of bonds with past high equity returns tends to outperform a portfolio of bonds with past low equity returns. Furthermore, this effect is especially strong for bonds that are trading at above-average spread levels compared with their peer groups. For example, in the case of A-rated medium-duration bonds, the high spread, high equity return portfolio outperforms the benchmark (an equally weighted portfolio of all A-rated medium-duration bonds) by an average of 6 basis points and outperforms the high spread, low equity returns portfolio by 7 basis points per month. Results for BBB-rated bonds are even more striking. For these bonds, the high spread, high equity return portfolio outperforms the benchmark and the high spread, low equity return portfolio by an average of 20 basis points and 43 basis points per month, respectively. This outperformance is also significant on a risk-adjusted basis, with typical information ratios from long-short strategies in excess of 0.5. This evidence forms the basis of the current implementation of our model of credit selection (ESPRI) that uses equity returns and spread levels as bond selection instruments.

The article is organized as follows. In section 2, we introduce basic hypotheses that we examine in our empirical study and the methodology for the study. Section 3 describes the data we use. After presenting the main results of the study in section 4, we describe how this evidence forms the basis for our credit selection model ESPRI in section 5. In section 6, we present some evidence of the success of ESPRI in capturing large spread widening events. We conclude our study in section 7.

2. The Basic Hypotheses and Methodology

We have structured the study of predictability in credit markets as an examination of the following hypotheses regarding the excess return (over duration-matched Treasuries) performance of appropriately chosen corporate bond portfolios.

- Hypothesis H1: Bonds with low spreads tend to outperform bonds with high spread in the near future. A confirmation of this would be interpreted as evidence of a momentum effect for spreads in the bond market.
- Hypothesis H2: Bonds with high spreads tend to outperform bonds with low spreads. A confirmation of this would be consistent with mean-reversion of spreads in the bond market.

- Hypothesis H3: There is a cross-momentum effect between the equity and the bond market. Bonds of issuers with improving fundamentals (captured by the equity information) should outperform; bonds of issuers with worsening fundamentals (captured by the equity information) should underperform.
- Hypothesis H4: There is a cross-reversal effect between the equity and the bond market. Bonds of issuers with improving fundamentals (captured by the equity information) should underperform; bonds of issuers with worsening fundamentals (captured by the equity information) should outperform.
- Hypothesis H5: There is a cross-market effect between the equity and the bond market. Bonds of issuers with improving fundamentals (captured by the equity information) should outperform, especially if they are trading at high spreads. Conversely, bonds of issuers with worsening fundamentals (captured by the equity information) should underperform if they are trading at low spreads. Furthermore, bonds of issuers with worsening fundamentals should underperform if they are trading at high spreads (negative momentum), and bonds of issuers with improving fundamentals should outperform if they are trading at high spreads (quality effect).

All the above hypotheses raise the question of whether particular variables are useful in predicting spread movements. For example, if H2 is true, then bonds with high spreads would outperform both the broader markets and bonds with low spreads in the future. Our empirical test of various hypotheses, therefore, considers the performance of hypothetical investment strategies that seek to benefit from the predictability that would exist if the hypotheses were true. For example, if H2 were true, then going long the bonds with high spreads and short the bonds with low spreads within a homogeneous class of bonds should, on average, generate positive excess returns without too much risk.

To examine whether a variable has predictive value for spread movements, we first define a universe of bonds with similar risk characteristics. This universe is taken to be bonds of a given rating in a given duration bucket. We consider A and BBB rated bonds and, in both these rating categories, create three equal-sized duration buckets. Then, all bonds in a given universe are sorted according to the variable in question (e.g., option-adjusted spreads). We construct three equally weighted portfolios: the top X% of these bonds (portfolio H), the middle 1-2X% of these bonds (portfolio M) and the bottom X% of these bonds (portfolio L). For spreads, we chose X = 33 and for equity X = 20. We compare the excess returns over duration-matched Treasuries of these sorted portfolios with the excess returns over Treasuries on a benchmark that is taken to be the equally weighted portfolio of all bonds in the universe. If the sorting variables (e.g., spread level or past equity returns) are uninformative, the sort is equivalent to a random sort. In this case, the excess returns on the sorted portfolios over Treasuries should not differ from those of the benchmark, and the information ratio should be close to zero. The annualized information ratio is defined as the average outperformance divided by the annualized standard deviation.

Hypothesis H5 raises the question of whether spread levels and equity returns can *jointly* be used to predict spread movements. To examine this hypothesis, The information ratio is a useful statistic to measure active management performance. It is defined as the ratio of the average of active return (the excess return over a benchmark used by the passive investors) over the annualized active risk (the standard deviation).

$$IR = \frac{Mean(R_i)}{Stdev(R_i)}\sqrt{T}$$

where R is the monthly active return and T is the number of return horizon (1 month) in a year; thus, T=12.

we consider the performance of portfolios that result from a double sorting. In this procedure, we first sort bonds in the universe according to their spread levels, and then within each spread category, we sort the bonds according to the return on their equities over the previous months. Thus, each spread portfolio is split into three portfolios. For instance, portfolio H is divided into the top 20% (portfolio HH), the middle 60% of (portfolio HM), and the bottom 20% (portfolio HL). We define the average outperformance of a given portfolio (e.g., HH) as the time-series average of the difference in (equally weighted) excess returns between the portfolio in that category (HH) and the equally weighted portfolio of all bonds in the appropriate rating-duration bucket. In this way, we are always concerned with the performance of the excess returns over Treasuries (as opposed to total returns) of the portfolio in question versus the excess returns of the benchmark.

We present results for the case in which the holding period of various portfolios is three months. To compute the performance of various portfolios held for longer than one month, we use the methodology of overlapping portfolios of Jegadeesh and Titman (1993). According to this methodology, we compute the average excess returns of several overlapping portfolios currently in existence during the return period. For a 3-month holding period, at any point in time (except at the beginning of the sample), there are three portfolios, the portfolio being constructed in the current month and in the two previous months. The monthly performance of the strategy is then taken to be the average excess return on these portfolios in a given month.

Finally, we assume that there are no transaction costs, as the objective here is to examine the data in the simplest possible way.

3. Data

Our study of the U.S. corporate bond market covers the sample period May 1994-December 2001. We have constructed an extensive database that uses our bond index data from the U.S. Aggregate Index and equity data from Lehman Brothers' global equity databases. We have matched the majority of the bonds to the stock of the issuer and tracked the relevant corporate events (take-overs, mergers, spin-offs, etc.). This was necessary to ensure that each corporate bond is associated with the relevant equity in each month in the sample.

We have restricted our study to corporate bonds satisfying the following criteria: belonging to the Lehman Brothers Corporate Bond Index (investment grade); fixed coupon rate; bullet bonds, callable bonds, putable bonds; senior unsecured debt; trader price quote (only trader quoted prices are used in estimation, any estimated or matrix prices are excluded); maturity between 3 and 30 years; day count 30/360 and semiannual paying bonds.

The overall universe for our study is, therefore, the set of all such bonds for which we can find a matching publicly traded equity stock. These bonds are then categorized by rating and duration to create a homogeneous universe within which the above-mentioned sorted portfolios are formed.

4. Results

4.1. Mean-Reversion in Spreads

We first explore hypotheses H1 and H2. Recall that these hypotheses concern the effect of the level of spreads on the future excess return of bonds trading at different spread levels. If H1 is true, bonds with high spreads are likely to underperform in the future, while if H2 is true, then these bonds are “cheap” bonds that might be attractive to value investors. These bonds with high spreads would be expected to outperform if spreads are mean reverting, or at least if wide spreads in themselves do not imply further widening. Conversely, bonds with low spreads would be expected to underperform if H2 is true.

We explore that hypothesis by sorting the bonds according to their option-adjusted spreads. Thus, for each month, we construct the three portfolios H, M, and L and report the average outperformance (difference of excess returns over duration-matched Treasuries on the portfolio less the excess return over Treasuries on the benchmark) over a three-month holding period.

The results are presented in Figure 1. This table suggests that the evidence favors the hypothesis of mean-reversion in spreads (i.e., H2) rather than that of spread momentum (i.e., H1). For example, over our sample period, for the subuniverse of A-rated medium-duration bonds (Figure 1), the H portfolio (bonds with widest spreads) would have outperformed the benchmark by 5 basis points per month. Moreover, the average outperformance of this portfolio over the L portfolio (bond with tightest spreads) would have been 9 basis points per month. The BBB-rated H portfolio outperforms the benchmark and the L portfolio by an average of 3 basis points and 6 basis points per month, respectively. The results are similar for long-duration bonds and for short-duration bonds, although they are weaker for the latter.

The past level of spreads alone brings significant information ratios for A-rated bonds but not for BBB-rated bonds. This would suggest that the evidence for mean-reversion is stronger for A-rated bonds than for BBB bonds. Therefore, using only spreads, we accept H2 for A-rated bonds and reject H1. We are unable to accept either H1 or H2 for BBB-rated bonds.

Figure 1. **Results for Single Sort on Past OAS Level**

		BBB Bonds		A Bonds	
Duration	**OAS Level**	**Avg. Rtn**	**Ann. IR**	**Avg. Rtn**	**Ann. IR**
Long	*High*	3.1	0.2	5.2	0.5
	Medium	1.0	0.1	-0.9	-0.3
	Low	-4.1	-0.3	-4.4	-0.5
Medium	*High*	2.6	0.2	5.2	0.9
	Medium	1.0	0.2	-0.9	-0.4
	Low	-3.6	-0.4	-4.3	-0.9
Short	*High*	0.0	0.0	2.6	0.5
	Medium	1.6	0.2	0.1	0.1
	Low	-2.0	-0.2	-2.7	-0.6

Instead of looking at the past level of spreads, one could look at the past change of spreads or, rather, the past excess return of the bond. Our experiments (not reported) suggest that this would not have generated statistically or economically significant results.

4.2. Using Equity Returns in Bond Selection

We now turn to the strategy of investing based on equity signals. Several questions are of interest. Is the equity performance informative for spreads? Should we buy or sell bonds for which the equity has outperformed? Should we buy or sell bonds for which the equity has underperformed? These questions can be addressed by examining hypotheses H3, H4, and H5.

Recall that hypothesis H3 states that bonds should outperform the broader market if the equities of the issuer have been outperforming the broader market in the previous periods. Hypothesis H4 states exactly the opposite. The assumption underlying H3 is that good equity performance is unconditionally good news for the bond or past bad equity performance is unconditionally bad for the bond. For example, this would result if a momentum effect is present in equity markets and there is a significant contemporaneous co-movement between equity prices and spreads. To examine which of these hypotheses is supported by the data, bonds in a given rating-duration bucket are sorted based on past 3-month total equity return.[4] For instance, for a bond that we want to select on January 1, 2002, we compute the equity return during the months of October, November, and December 2001. Then, every month, we construct the top, middle, and bottom portfolios (H, M, and L) and compute the average monthly excess returns of these portfolios over the benchmark of an equally weighted portfolio of all bonds in the rating-duration bucket. We also assess the risk-adjusted performance of various portfolios by computing the annualized information ratio for a 3-month holding period.

[4] We have also investigated 1-month, 6-month, 9-month, and 1-year equity returns. As the holding period increases, the excess returns on various portfolios tend to get smaller, but the evidence for the effects mentioned above is still present at these horizons.

As Figure 2 shows, there is evidence of a significant equity-momentum effect for both A-rated and BBB-rated bonds. For medium-duration bonds, the H portfolio of A-rated bonds (bonds with strongest equity returns in the previous period) outperforms the L portfolio (bonds with weakest equity returns in the previous period) by 4 basis points per month. For BBB-rated bonds, the H portfolio outperforms the bottom portfolio by 22 basis points per month.

The results are better for long-duration bonds. The H portfolio of A-rated bonds is outperforming the L portfolio by 19 basis points per month. For BBB-rated bonds, the H portfolio outperforms the bottom portfolio by 30 basis points per month.

The results are also significant for BBB-rated short-duration bonds. The H portfolio outperforms the bottom portfolio by 32 basis points per month. It is, however, weaker for A-rated bonds short duration (3 basis points per month between the H and L portfolios). Overall, this suggests that there is robust cross-market spill-over between equity and bond markets and that the data are consistent with H3 (equity momentum spill-over into credit spreads). We reject H4.

4.3. Combining Equity Returns and Spreads

Past equity returns give good results, but these could be improved if we use some value indicators. The use of two selection variables instead of one should add robustness to our results, as we are not relying on information coming from only one market. We therefore also investigate the performance of portfolios that are formed using the level of spread at the beginning of the investment period in addition to the historical equity returns. The intuition for considering both the spread level and past equity returns in portfolio selection is that high equity returns in the past may be an especially effective signal for bonds that are already trading at wide spread levels and, conversely, for low equity returns. The effect of outperformance should be stronger with the high spread category (more room to tighten), whereas the effect of underperformance could be stronger for low spread (more room to widen).

Figure 2. **Results for Single Sort on 3-Month Equity Returns**

		BBB Bonds		A Bonds	
Duration	**Equity Rtn**	**Avg. Rtn**	**Ann. IR**	**Avg. Rtn**	**Ann. IR**
Long	*High*	15.5	1.4	7.5	1.5
	Medium	0.1	0.0	1.6	0.7
	Low	-14.8	-0.9	-11.9	-1.4
Medium	*High*	10.3	1.7	1.5	0.5
	Medium	0.8	0.2	0.5	0.3
	Low	-12.4	-1.1	-3.0	-0.6
Short	*High*	11.5	1.7	1.3	0.4
	Medium	3.0	0.5	0.2	0.1
	Low	-20.9	-1.0	-1.8	-0.3

As mentioned in Section 2, we use double sorting to look at the combined effect of spread levels and equity returns. First, the universe is sorted on spread levels, and then within each spread category, we sort the bonds on the basis of the historical equity returns of their issuers, forming nine portfolios. Figure 3 illustrates that for every spread category, the portfolios with high equity returns outperforms the portfolio with low equity returns. For example, in the case of A-rated medium-duration bonds (Figure 3), the high spread, high equity return portfolio (the HH portfolio) outperforms the benchmark and the high spread, low equity returns portfolio (the HL portfolio) by an average of 6 basis points and 7 basis points per month, respectively. The BBB-rated HH portfolio outperforms the benchmark and the HL portfolio by an average of 20 basis points and 43 basis points per month, respectively.

For long-duration bonds (Figure 4), we find the same patterns: the A-rated HH portfolio outperforms the benchmark and the HL portfolio by an average of

Figure 3. **Results for Single and Double Sorting on Past Level of Spreads and Past 3-Month Equity Returns—Medium Duration**, May 1994-November 2001, 3-Month Investment Horizon
Average Excess Return over Universe, bp, *Annualized Information Ratio*

	BBB-Rated Bonds				A-Rated Bonds			
	3-Mo. Equity Return			Single	3-Mo. Equity Return			Single
Double Sort OAS Level	*Top 20%*	*Mid 60%*	*Btm 20%*	**Sort**	*Top 20%*	*Mid 60%*	*Btm 20%*	**Sort**
Top 33%	**20.4**	**6.9**	**-22.3**	**2.6**	**5.7**	**7.0**	**-1.5**	**5.2**
	1.6	*0.5*	*-0.6*	*0.2*	*0.7*	*1.3*	*-0.1*	*0.9*
Mid 33%	**7.5**	**2.7**	**-9.9**	**1.0**	**-0.7**	**-0.1**	**-3.0**	**-0.9**
	1.0	*0.4*	*-1.1*	*0.2*	*-0.2*	*0.0*	*-0.6*	*-0.4*
Btm 33%	**1.4**	**-4.1**	**-11.2**	**-3.6**	**-2.4**	**-3.1**	**-8.3**	**-4.3**
	0.1	*-0.5*	*-1.0*	*-0.4*	*-0.4*	*-0.6*	*-1.5*	*-0.9*
Single Sort	**10.3**	**0.8**	**-12.4**		**1.5**	**0.5**	**-3.0**	
	1.7	*0.2*	*-1.1*		*0.5*	*0.3*	*-0.6*	

Figure 4. **Results for Single and Double Sorting on Past Level of Spreads and Past 3-Month Equity Returns—Long Duration,** May 1994-November 2001, 3-Month Investment Horizon
Average Excess Return over Universe, bp, *Annualized Information Ratio*

	BBB-Rated Bonds				A-Rated Bonds			
	3-Mo. Equity Return			Single	3-Mo. Equity Return			Single
Double Sort OAS Level	*Top 20%*	*Mid 60%*	*Btm 20%*	**Sort**	*Top 20%*	*Mid 60%*	*Btm 20%*	**Sort**
Top 33%	**31.3**	**3.3**	**-24.7**	**3.1**	**19.2**	**8.4**	**-17.5**	**5.2**
	1.3	*0.2*	*-0.6*	*0.2*	*1.8*	*1.0*	*-0.8*	*0.5*
Mid 33%	**12.4**	**2.5**	**-13.4**	**1.0**	**4.9**	**-0.5**	**-6.6**	**-0.9**
	1.0	*0.3*	*-0.9*	*0.1*	*0.6*	*-0.1*	*-0.7*	*-0.3*
Btm 33%	**4.5**	**-2.6**	**-12.4**	**-4.1**	**-2.4**	**-3.2**	**-9.4**	**-4.4**
	0.2	*-0.2*	*-0.8*	*-0.3*	*-0.2*	*-0.4*	*-1.2*	*-0.5*
Single Sort	**15.5**	**0.1**	**-14.8**		**7.5**	**1.6**	**-11.9**	
	1.4	*0.0*	*-0.9*		*1.5*	*0.7*	*-1.4*	

19 basis points and 37 basis points per month, respectively. The BBB-rated HH portfolio outperforms the benchmark and the HL portfolio by an average of 31 basis points and 56 basis points per month, respectively. Qualitatively, the same effect is present for other spread buckets, although, intuitively, the effect is strongest in BBB-rated bonds in the high spread category. Figure 5 gives broadly consistent results for short-duration bonds.

It is also interesting to note that the HL portfolio is often an underperforming portfolio, probably because of the inclusion of potential future downgraded bonds. Financially weak firms with high spreads and worsening equity returns seem not to recover.

Figure 6 summarizes some descriptive statistics for the portfolios of interest. In particular, we provide the average duration, number of bonds, OAS, and amount outstanding. We see that these portfolios are reasonable along these dimensions.

Figure 5. **Results for Single and Double Sorting on Past Level of Spreads and Past 3-Month Equity Returns—Short Duration,** May 1994-November 2001, 3-Month Investment Horizon
Average Excess Return over Universe, bp, *Annualized Information Ratio*

	BBB-Rated Bonds				A-Rated Bonds			
	3-Mo. Equity Return			Single	3-Mo. Equity Return			Single
Double Sort OAS Level	*Top 20%*	*Mid 60%*	*Btm 20%*	**Sort**	*Top 20%*	*Mid 60%*	*Btm 20%*	**Sort**
Top 33%	**24.0**	**2.2**	**-34.6**	**0.0**	**4.6**	**4.4**	**-2.6**	**2.6**
	1.7	*0.1*	*-0.6*	*0.0*	*1.1*	*1.1*	*-0.1*	*0.5*
Mid 33%	**7.4**	**4.1**	**-8.7**	**1.6**	**-0.4**	**0.2**	**-0.5**	**0.1**
	0.9	*0.5*	*-0.6*	*0.2*	*-0.1*	*0.1*	*-0.2*	*0.1*
Btm 33%	**3.2**	**0.5**	**-10.0**	**-2.0**	**-0.7**	**-2.8**	**-4.7**	**-2.7**
	0.3	*0.1*	*-0.7*	*-0.2*	*-0.1*	*-0.6*	*-0.9*	*-0.6*
Single Sort	**11.5**	**3.0**	**-20.9**		**1.3**	**0.2**	**-1.8**	
	1.7	*0.5*	*-1.0*		*0.4*	*0.1*	*-0.3*	

Figure 6. **Portfolio Statistics**

		BBB-Rated Bonds				A-Rated Bonds			
Duration	**ESPRI Portfolio**	**Avg. Count**	**Avg. Size** ($ mn)	**Avg. Duration**	**Avg. OAS**	**Avg. Count**	**Avg. Size** ($ mn)	**Avg. Duration**	**Avg. OAS**
Long	*HH*	16	247.2	9.2	150.1	14	304.8	8.9	215.3
	LH	15	266.3	8.6	89.1	14	288.4	8.6	116.4
	HL	17	263.0	9.3	152.8	16	267.6	9.0	229.2
	LL	17	247.3	8.5	90.5	16	278.2	8.7	117.9
Medium	*HH*	16	251.1	5.4	123.1	15	224.6	5.6	186.1
	LH	16	293.6	5.4	74.1	14	270.8	5.6	97.8
	HL	17	280.2	5.5	128.8	15	235.5	5.6	212.8
	LL	17	259.4	5.5	73.5	16	283.1	5.6	98.4
Short	*HH*	15	252.1	3.4	108.8	14	222.8	3.7	187.7
	LH	15	275.5	3.0	56.4	14	261.9	3.2	80.9
	HL	18	286.1	3.5	120.8	15	234.4	3.7	357.7
	LL	17	276.9	2.9	54.2	16	259.8	3.2	76.9

Figure 7 explores the robustness of the patterns exhibited in Figures 3-5. We show the performance of the portfolios of interest over the two half-period sub-samples and for the case in which the universe of bonds is restricted to those we define as liquid (specifically, those bonds less than three years old and falling into the upper half of the distribution of amount outstanding). The patterns mentioned before are broadly present in these sub-samples, signifying the robustness of our results.

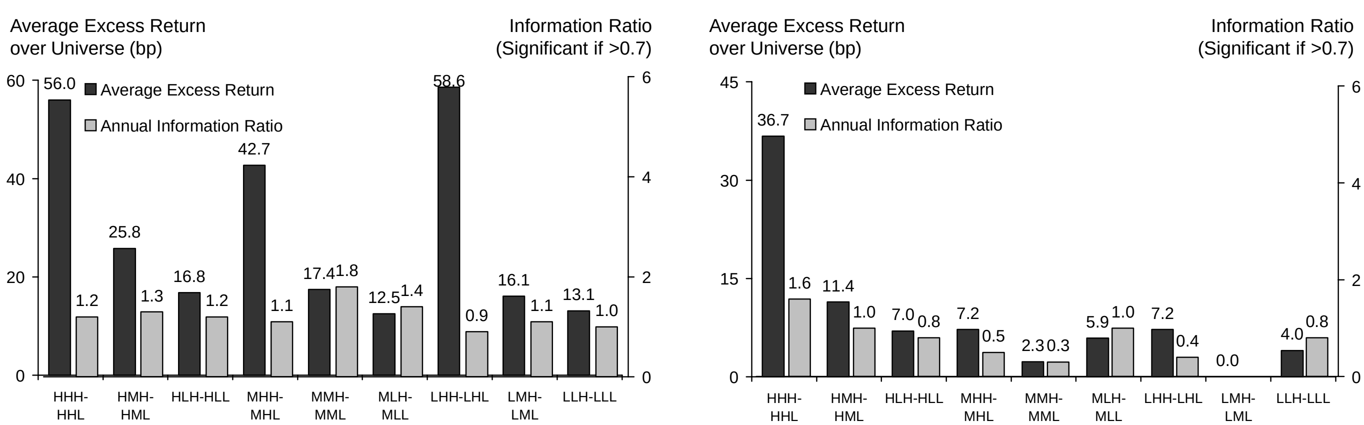
In Figure 8, we report the average excess returns over Treasuries on a long-short strategy in which one goes long the high return portfolio and goes short the low

Figure 7. **Robustness Summary**

		BBB-Rated Bonds						A-Rated Bonds					
		Liquid Bonds Top Half in Size, Under 3 Years Old		Sub-Periods				Liquid Bonds Top Half in Size, Under 3 Years Old		Sub-Periods			
	ESPRI			May 94-Dec 97		Jan 98-Dec 01				May 94-Dec 97		Jan 98-Dec 01	
Duration	**Portfolio**	**Avg Rtn**	**Ann IR**	**Avg Rtn**	**Ann IR**	**Avg Rtn**	**Ann IR**	**Avg Rtn**	**Ann IR**	**Avg Rtn**	**Ann IR**	**Avg Rtn**	**Ann IR**
Long	*HH*	32.0	1.3	23.4	1.6	38.6	1.3	16.7	1.5	13.0	2.5	18.2	1.6
	LH	1.6	0.1	-3.0	-0.3	12.5	0.5	-1.8	-0.1	-4.1	-0.8	0.3	0.0
	HL	-23.9	-0.6	2.6	0.1	-65.3	-1.2	-22.4	-0.8	-4.4	-0.4	-32.1	-1.1
	LL	-12.3	-0.7	-6.1	-0.7	-14.2	-0.8	-7.9	-0.8	-6.6	-1.4	-9.8	-1.0
Medium	*HH*	20.4	2.1	9.3	1.5	25.0	2.2	5.7	0.6	3.1	1.2	5.0	0.5
	LH	2.7	0.2	-2.9	-0.7	6.2	0.5	-0.7	-0.1	-2.7	-1.0	-1.4	-0.2
	HL	-39.5	-1.2	-3.8	-0.3	-45.5	-1.0	-3.5	-0.2	-0.9	-0.2	-4.7	-0.3
	LL	-6.3	-0.5	-8.6	-1.3	-11.7	-0.9	-7.4	-1.1	-5.0	-1.6	-10.5	-1.5
Short	*HH*	15.9	1.1	8.7	2.4	27.9	2.1	5.6	1.0	0.2	0.1	7.7	1.7
	LH	3.2	0.3	-3.6	-0.9	10.4	0.9	-0.9	-0.1	-2.0	-1.0	-0.7	-0.1
	HL	-54.1	-1.1	-0.2	-0.1	-85.6	-1.2	-7.2	-0.4	3.0	1.8	-12.5	-0.5
	LL	1.9	0.2	-4.6	-1.1	-12.6	-0.8	-1.6	-0.3	-3.9	-1.6	-4.7	-0.9

Figure 8. **Historical Performance of Trading High Minus Low Equity Return for USD Bonds**

a. **U.S. BBB-Rated Bonds: High Equity—Low Equity Performance** b. **US A-Rated Bonds: High Equity—Low Equity Performance**



return portfolio in each spread category for each rating-duration bucket. These average returns are all positive and have economically significant information ratios across all the portfolios for BBB-rated bonds and most portfolios for A-rated bonds.

5. The ESPRI Model for Credit Selection

The ESPRI model (**E**quity returns as **SPR**ead **I**ndicators) employs this double-sorting approach to select bond portfolios. In the current implementation, we have selected the widest and the tightest spread categories, where the spread information is the most significant. Thus, the portfolios that are expected to outperform given historical evidence are the high spread, high equity return and low spread, high return portfolios, while high spread, low return and low spread, low return portfolios are expected to underperform broader markets.

The reason for including both the high- and low-spread portfolios in the ESPRI output is that these two categories of bonds have different macro-risk profiles. When the risk appetite in the marketplace is normal, the stock market rises, or the yield curve is steepening, the HH portfolio does particularly well relative to the benchmark and the LL portfolio does especially poorly. However, under bad market conditions, a flight-to-quality effect takes hold. We find that when the stock market falls or when the yield curve is flattening, the strategy of going long LH and short HL performs well. The LH portfolio is selected as a long portfolio despite the fact that, on its own, it does not significantly outperform the benchmark. This is because of its defensive characteristic. Bonds with tight spreads and good equity returns are more resistant to a market drop (quality effect), whereas bonds with high spreads and worsening fundamentals (bad equity performance) are going to underperform (negative equity momentum). All in all, if we do not want to take a view on market conditions, then the strategy of going long both HH and LH and going short both HL and LL seems appropriate.

The excess returns generated by going long HH and LH and short the HL and LL portfolios [½ (*HH* + *LH* – *HL* – *LL*)] are reported on Figure 9 for the past nine months. Figure 10 presents the performance of this combined strategy for the last nine months and for the whole sample period. Both the average outperformance and the risk-adjusted outperformance are economically significant.

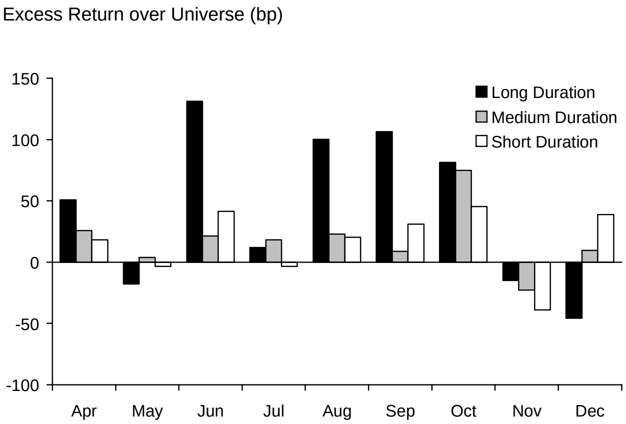
6. Capturing Large Spread Widenings

As a further way of measuring the performance of the *ESPRI* model, we examine its effectiveness at predicting large increases in OAS—or so-called credit blow-ups.

We define large spread widenings as total widenings of more than a certain threshold value over the three months following a bond's *ESPRI* classification. Counting such events through the usual sample period of May 1994 to November 2001, Figure 10 shows the percentage "captured" in *ESPRI* underperform portfolios (the upper, solid line) and the percentage misallocated to outperform portfolios (the lower, dashed line) for different sizes of jumps (the x-axis). These should be interpreted as follows. Each month, *ESPRI* sorts the universe into 27 portfolios of

Figure 9. **Performance of Combined ESPRI Recommendations,** Last 9 Months

a. **U.S. A Rated Bonds**



b. **U.S. BBB Rated Bonds**

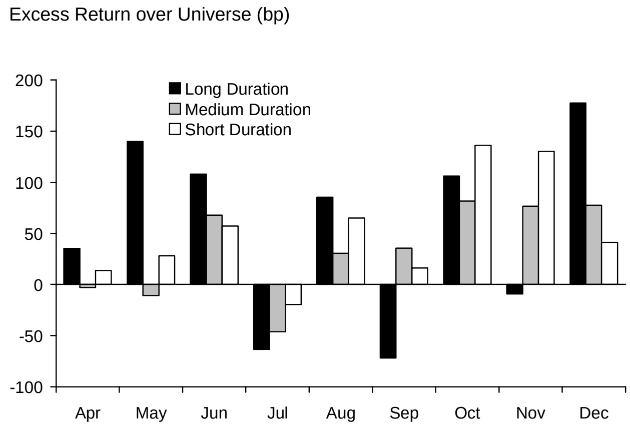


Figure 10. **Performance of Combined ESPRI Recommendations**

			Full Period		**Last Nine Months**	
Denomination	**Rating**	**Duration**	**Average Monthly Excess Return** (bp)	**Annualized Information Ratio**	**Average Monthly Excess Return** (bp)	**Annualized Information Ratio**
U.S.	BBB	*Long*	36.4	1.4	60.6	2.6
		Medium	27.6	1.4	45.3	3.3
		Short	35.9	1.0	65.9	4.2
	A	*Long*	21.9	1.6	43.2	2.3
		Medium	6.5	0.8	12.9	1.7
		Short	5.6	0.5	12.6	1.7

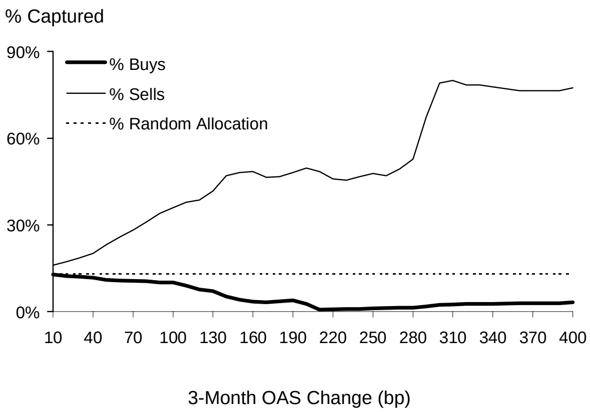
which two are classified underperform (HL & LL) and two outperform. The total number of bonds receiving an underperform label is, therefore, $2 \times 33\% \times 20\% = 13.3\%$, and similarly for outperforms. So a random allocation of bonds would result, on average, in 13.3% of credit blow-ups being designated underperform, 13.3% outperform, and, hence, 73.3% as neutral. The values below should, therefore, be compared to 13.3%—the dotted horizontal line marks this level (Figure 11).

We see that *ESPRI* is able to capture a significant proportion of these events, getting better as the size of the widening increases. For example, for A-rated bonds, *ESPRI* captures 75% of all 300 bp widenings, misclassifying just 5%. For BBBs, *ESPRI* captures approximately 55% of 300 bp widenings and misallocates about 5%,

Figure 11. **Capturing Large Spread Widenings**

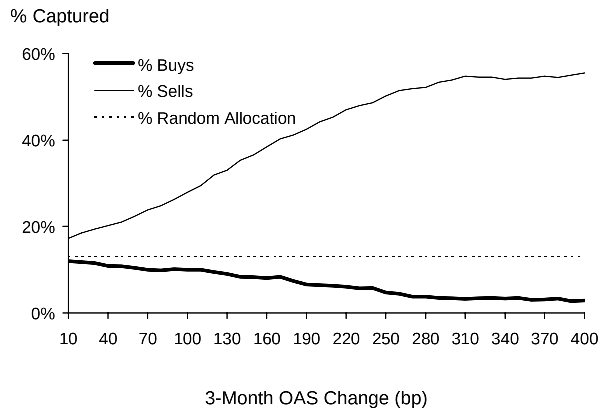
a. **A-Rated Bonds**

OAS Increase During Subsequent 3 Months	Total No. of Occurrences In Sample Period
100	600
300	70
500	30



b. **BBB-Rated Bonds**

OAS Increase During Subsequent 3 Months	Total No. of Occurrences In Sample Period
100	2200
300	500
500	220



leaving 40% as neutral. It should be borne in mind that by the nature of the model, 73.3% of all bonds are "forced" into a neutral category, so these figures should be compared with 13.3% rather than viewed as absolute figures on their own.

7. Summary

Our objective was to investigate predictability in the credit market and, in particular, the effect of the equity market on corporate bond valuation. We have documented, using a simple strategy based on screening and sorting corporate bonds, that it was possible to outperform our benchmark and generate good information ratios. Using past levels of option-adjusted spreads, past equity returns, and the combination of both, we are able to select outperforming and underperforming portfolios.

The applications of ESPRI to the credit world are several: the model can be used in portfolio management for bond and sector selection/weighting. The ESPRI approach is complementary to fundamental research, as it could work as a filtering procedure for credit analysts. Furthermore, ESPRI can be used in some instances to anticipate credit downgrades or upgrades, rendering it appropriate in certain risk management applications. While fundamental analysis can provide depth, a filtering tool such as ESPRI can offer breadth of analysis, which has become increasingly important with the boom in corporate debt issuance globally in the past few years. Combining both could be helpful in generating positive alpha.

ESPRI should also be valuable combined with a risk model, to control for risk exposure and tracking errors of bond portfolios, and with derivatives such as total return swaps, to mix asset allocation and security selection. A detailed understanding of which instruments have significant predictive content for spread movements is obviously of great importance to credit portfolio managers. Through the current and future versions of ESPRI, we expect to assist portfolio managers in developing such an understanding.

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SPREAD PREMIA FOR PORTFOLIO TRANCHES[1]

Tranches of synthetic CDOs allow investors to obtain tailored risk exposure to large and diversified credit portfolios. Using our previously introduced metrics of *spread premium* and *spread coverage ratio*, we compare the mezzanine tranches of a generic investment grade portfolio with single-name default swaps. By choosing the appropriate subordination for their risk appetite, investors can substantially increase returns versus default swaps.

Introduction

By investing in structured credit products, investors can leverage the spread premium they receive in return for assuming part of the credit risk on a pool of issuers. Buy-and-hold investors benefit from the tension between the market-implied issuer default probabilities that drive the pricing and the estimated default rate of the underlying issuer pool, which will typically be based on average historical issuer default probabilities.

The market for portfolio credit derivatives has its base in the market for individual default swaps, since these are the building blocks for many synthetic structures. When making an investment decision on portfolio credit products, it is important to compare them on a risk-return basis with single-name default swaps.

In this article, we extend our earlier study[2] from default baskets to synthetic CDO portfolio tranches. In particular, we use the same method as in the earlier study, basing our comparison between tranches and single-name default swaps on the spread premium and the spread coverage ratio. Recall that the spread premium is defined as the difference between the spread that an investor receives and the actuarially fair spread, which is the spread at which the expected premium payments equal the expected loss. The coverage ratio is the ratio of the two spreads. For a more detailed discussion of these concepts, see the article mentioned above.

Structure of the Underlying Portfolio

The risk of capital losses to a portfolio tranche is determined by the probability distribution of issuer defaults in the portfolio. One of the factors that affect this is the default probability of the individual issuers. We assume a trade horizon of five years and use the idealized default probabilities provided by Standard and Poor's for our risk assessment; they are shown in Figure 1.

The portfolio we analyze is a generic proxy for a pool of investment grade credits that might be used in a synthetic CDO. It consists of 100 issuers with equal notional amounts, distributed across ratings as shown in Figure 2.

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[1] We thank Sunita Ganapati for suggestions and comments.

[2] See "Leveraging Spread Premia with Default Baskets" by O'Kane and Schloegl in the October 2001 issue of *QCR Quarterly.*

To compare tranches with single-name default swaps, we need some estimate of average default swap spreads by rating category. Names in the same rating category can trade at dramatically different levels, so a generic estimate, by its very nature, must be quite rough. Nevertheless, from a universe of about 700 quoted default swap spreads, we come to the estimates for five-year spreads given in Figure 3.

Another factor that affects the portfolio loss distribution is the choice of asset correlation. For simplicity, we use a flat correlation for all the assets in the pool. In their CDO evaluation methodology, Standard and Poor's use an implied correlation of around 30% for issuers in the same industry sector and zero for issuers in different industry sectors. In other words, a flat correlation of 30% for the whole portfolio would be consistent with the assumption that all credits belong to the same industry sector, i.e., that there is no diversification across sectors. Based on our experience with average issuer correlations, we regard a flat correlation of 30% for the whole portfolio as being on the high side, particularly given that CDO portfolios are generally required to be diversified across industries. Therefore, for our base case, we use a correlation of 25% and also consider the cases of 20% and 30% correlation.

The risk profile of a portfolio tranche depends on its subordination, as given by its lower boundary, and its leverage, which is determined by the width of the tranche. We consider a portfolio with the capital structure shown in Figure 4. Our analysis will focus on the two mezzanine tranches.

Figure 1. **Five-Year Idealized Historical Default Probabilities by Rating**

	AA+	AA	AA-	A+	A	A-	BBB+	BBB	BBB-	BB+	BB
Probability	0.36%	0.76%	0.88%	0.98%	1.11%	1.33%	1.84%	2.50%	4.39%	7.87%	11.25%

Source: Standard And Poor's.

Figure 2. **Rating Distribution of Proxy Portfolio**

	AA+	AA	AA-	A+	A	A-	BBB+	BBB	BBB-
Number of Assets	4	6	3	6	18	20	23	15	5

Figure 3. **Estimated Average 5-Year Default Swap Spreads by Rating**

	AA+	AA	AA-	A+	A	A-	BBB+	BBB	BBB-	BB+	BB
Avg. 5-Year Spread	19	28	37	47	61	75	116	164	265	349	463

Figure 4. **Capital Structure of Synthetic CDO,** %

Tranche Name	Lower Threshold	Upper Threshold
Senior	9.25	100.0
Upper Mezzanine	7.75	9.25
Lower Mezzanine	4.25	7.75
Equity	0.00	4.25

The equity tranche absorbs losses of up to 4.25% of the total portfolio notional, after which the lower mezzanine incurs capital losses. The lower mezzanine tranche suffers a total loss once 7.75% of the portfolio notional has been lost, after which the upper mezzanine is affected; that, in turn, absorbs another 1.5% of the portfolio notional in losses.

Another important parameter is the recovery rate that is assumed for the issuer defaults. The recovery rate determines how many defaults a tranche can withstand before incurring losses, cf. Figure 5. At the same time, it connects the single-name default swap spreads to market-implied default probabilities.

For the purpose of this study, we assume that all the default swaps in the underlying portfolio reference senior unsecured debt and, therefore, use a recovery rate of 40%. This leads to the expected losses for single-name default swaps shown in Figure 6.

Loss Distributions and Statistics

Based on the individual default probabilities, we can compute the portfolio loss distribution using a copula-based model of default. Figure 7 shows the loss distribution of the portfolio for an issuer correlation of 25%. The loss distributions for the other two correlation cases are qualitatively similar; the tail of the distribution lengthens as the correlation increases. The expected percentage loss of the portfolio over the 5-year horizon is 0.98%; this is independent of the correlation.

From the portfolio loss distribution, it is straightforward to derive the loss distribution for each of the tranches.[3] In Figures 8 and 9, we show histograms for the loss distributions of the two mezzanine tranches. The tranche loss distributions are much more binary than that of the portfolio. Based on the idealized historical

[3] Cf. "Modelling Credit: Theory and Practice," by O'Kane and Schloegl, Lehman Brothers Analytical Research Series, February 2001.

Figure 5. **Number of Defaults Withstood by Tranche by Recovery Rate**

Recovery Rate	**0%**	**20%**	**40%**	**60%**	**80%**
Lower Mezzanine	4	5	7	10	21
Upper Mezzanine	7	9	12	19	38

Figure 6. **Expected Loss for Single-Name Default Swap with 40% Recovery Rate by Rating**

	AA+	**AA**	**AA-**	**A+**	**A**	**A-**	**BBB+**	**BBB**	**BBB-**	**BB+**	**BB**
Expected Loss	0.21%	0.45%	0.53%	0.59%	0.67%	0.80%	1.10%	1.50%	2.63%	4.72%	6.75%

default probabilities, there is a high probability of not suffering any loss at all. At the same time, there is a non-negligible probability that the complete tranche will be wiped out. Note that the probability masses shown in the middle three columns are spread out evenly between losses of 1% and 99%.

We show summary statistics for the two tranches in Figure 10. Note that the expected loss is correlation dependent. This is because the tranche loss is a non-linear function of the portfolio loss. The lowest row shows the probability of incurring a capital loss on the tranche.

In order to use our comparison metrics, we must assess the riskiness of each tranche relative to the single-name default swaps. Note that we are comparing the two loss distributions shown in Figures 8 and 9 with the binary default distribution of a single issuer for each rating class; there is no unique way of doing this.

Figure 7. **Portfolio Loss Distribution for an Issuer Correlation of 25%, Based on Idealized Historical Default Probabilities**

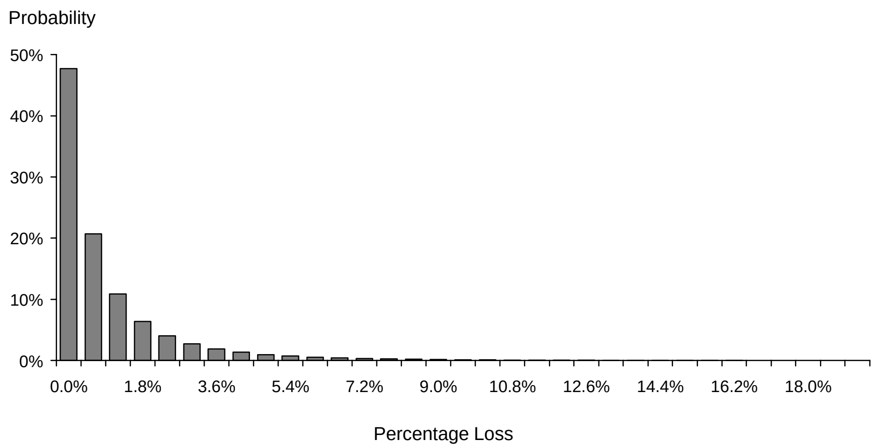

Figure 8. **Lower Mezzanine Loss Distribution**

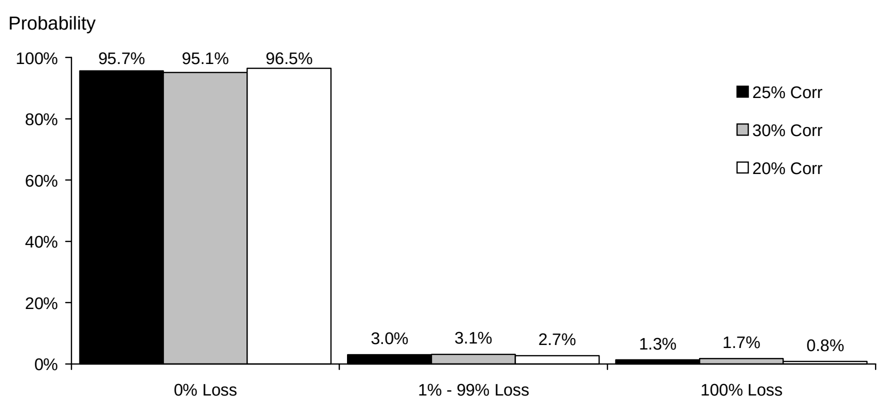

Figure 9. **Upper Mezzanine Loss Distribution**

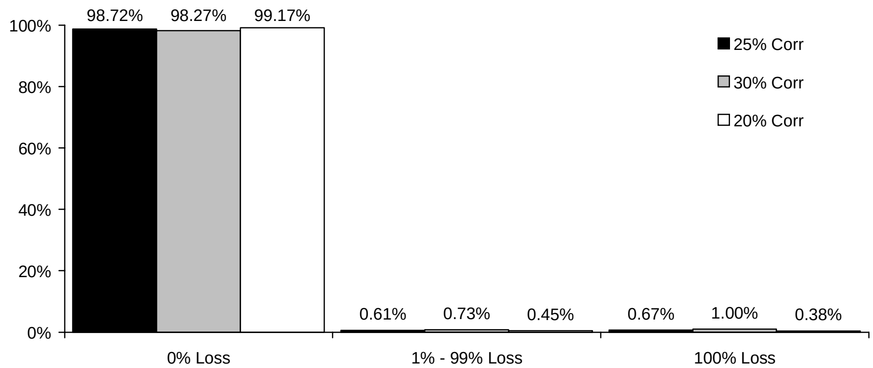

Figure 10. **Summary Statistics for Tranche Loss Distributions,** %

Tranche	**Lower Mezzanine**			**Upper Mezzanine**		
Correlation	**20%**	**25%**	**30%**	**20%**	**25%**	**30%**
Expected Loss	1.89	2.50	3.04	0.54	0.90	1.28
Probability of Loss	3.53	4.28	4.86	0.83	1.28	1.73

One method is to compare the probabilities of incurring a loss (this is advocated by Standard and Poor's for CDO tranche evaluation). Figures 1 and 10 show that for the base case of 25% correlation, the loss probability of the lower mezzanine tranche (4.28%) lies between those for issuers with ratings of BBB (2.5%) and BBB- (4.39%) and is closest to that of the BBB- rating. The loss probability of the upper mezzanine (1.28%) lies between those for issuers with ratings of A (1.11%) and A- (1.33%) and is closest to the A- rating.

Note, however, that comparing loss probabilities does not take into account the magnitude of the loss. For the individual issuer, a default event leads to a loss of 60%, as we are assuming a 40% recovery rate. The size of the loss on the tranche can vary anywhere from 0% to 100%, and this depends crucially on its width. An alternative approach that takes this into account is to compare based on the expected loss (this is, in fact, advocated by Moody's). Once more considering the base case, the expected loss of the lower mezzanine tranche (2.5%) is again between that of the BBB (1.5%) and the BBB- (2.63%) rating category. The upper mezzanine tranche (0.9%) lies between the A- (0.8%) and the BBB+ (1.1%) category.

Spread Coverage Ratios and Spread Premium

Following this analysis, the upper mezzanine tranche is comparable with an issuer with rating A-, and the lower mezzanine is slightly better than an issuer with rating BBB-. Indicative bid spreads for protection on the two tranches of our generic portfolio in a newly issued synthetic CDO would be around 330 bp for the lower mezzanine tranche and 210 bp for the upper mezzanine. Note that the weighted average spread of the portfolio is 97 bp.

From the historical default probabilities, we can compute the actuarially fair spreads for single name default swaps. These are the spreads for which the expected premium payments exactly offset the expected loss of the protection seller. In effect, the expected loss is being annuitized using the risky PV01 derived from the idealized historical default probabilities. The actuarially fair spreads are shown in Figure 11.

We obtain the spread premium and the spread coverage ratio for the single-name default swaps by combining Figures 3 and 11.

For the tranches, we use our model for generating the portfolio loss distribution to derive the actuarially fair spread. This is correlation dependent. In conjunction with the already mentioned bid spreads of 330 bp and 210 bp, this lets us compute the spread coverage ratio and spread premium. In Figures 13 and 14, we show the

Figure 11. **Actuarially Fair Five-Year Default Swap Spreads by Rating Category**

	AA+	AA	AA-	A+	A	A-	BBB+	BBB	BBB-	BB+	BB
Spread	4	9	10	12	13	16	22	30	53	98	145

Figure 12. **Spread Coverage Ratio and Spread Premium for Single-Name Default Swaps by Rating Category**

	AA+	AA	AA-	A+	A	A-	BBB+	BBB	BBB-	BB+	BB
Ratio	4.47	3.12	3.52	4.05	4.67	4.77	5.33	5.54	5.02	3.56	3.19
Premium	14	19	26	36	48	59	94	134	212	251	318

Figure 13. **Spread Coverage Ratio and Spread Premium for Lower Mezzanine Tranche Compared with a Default Swap on an Issuer with Rating BBB-**

	Lower Mezzanine Tranche			Default Swap on BBB- Name
Correlation	**20%**	**25%**	**30%**	**NA**
Loss Probability	3.53%	4.28%	4.86%	4.39%
Expected Loss	1.89%	2.50%	3.04%	2.63%
Bid Spread	330	330	330	265
Actuarial Spread	36	48	58	53
Spread Ratio	9.23	6.91	5.65	5.02
Spread Premium	294	282	272	212

Figure 14. **Spread Coverage Ratio and Spread Premium for Upper Mezzanine Tranche Compared with a Default Swap on an Issuer with Rating A-**

	Upper Mezzanine Tranche			Default Swap on A- Name
Correlation	**20%**	**25%**	**30%**	**NA**
Loss Probability	0.83%	1.28%	1.73%	1.33%
Expected Loss	0.54%	0.90%	1.28%	0.80%
Bid Spread	210	210	210	75
Actuarial Spread	10	17	24	16
Spread Ratio	20.93	12.42	8.65	4.77
Spread Premium	200	193	186	59

results for each of the tranches, together with those of the comparable single-name default swap (note that apparent discrepancies are due to rounding).

Comparison and Conclusions

We see that the spread premium and the spread coverage ratio show a fair amount of variation as a function of correlation. The tranches become riskier as correlation increases. Correspondingly, the excess spread and the coverage ratio decrease. The investor has implicitly taken a short position in the asset correlation.

The synthetic CDO tranches outperform single-name default swaps both in terms of spread premium and spread coverage ratio. This is true even in the pessimistic correlation scenario, i.e. even if we assume no diversification across industry sectors (30% correlation). The lower mezzanine tranche offers a sizeable nominal spread pick-up (330 bp on the tranche versus 265 bp on a similar quality default swap) while improving on the coverage ratio (6.91 versus 5.02 in the base case).

For the upper mezzanine tranche, the results are even more striking: the investor earns a spread of 210 bp, versus the 75 bp paid on A- rated issuers, while increasing the coverage ratio substantially, from 4.77 to 12.42 in the base case. Even in the pessimistic scenario, the coverage ratio is almost doubled.

Clearly, an investor taking exposure to the portfolio tranches is being compensated generously for the default risk vis-à-vis single name products. This must be weighed against the drawback of reduced liquidity in what are essentially buy-and-hold investments. Nonetheless, we believe that synthetic tranches offer an attractive alternative to single-name default swaps, while mitigating the exposure to idiosyncratic issuer risk.

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COHERENT RISK MEASURES APPLIED TO THE DEFAULT RISK OF CREDIT PORTFOLIOS AND CDO TRANCHES

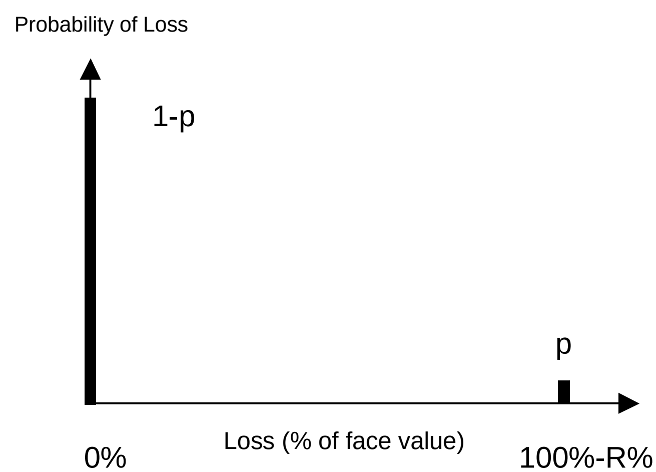
With the high number of recent defaults in the credit markets, there has been an increased desire to analyze default risk at the level of single assets, portfolios, and structured credit products such as CDOs. We discuss the well-known approach of Value at Risk (VaR) and explain why it is not appropriate for default risk. We therefore introduce the concept of *coherent risk measures*, which overcome the limits of VaR, and focus on the use of the *expected shortfall*. Recent developments in portfolio credit modeling mean that we can now apply this to the analysis of the default risk of a credit portfolio and of the tranches of a CDO.

Introduction

Recent events in the credit markets, in particular the high number of investment grade defaults, have focused investors on how to analyze credit default risk at both a single asset and a portfolio level. At the same time, the considerable growth in the use of credit derivatives has increased the need for methods to analyze default risk at the level of structured credit transactions such as synthetic CDOs.[1] These structured credit products have added new risk profiles to investors' portfolios that have posed a challenge to existing credit risk management approaches.

[1] For a description of basket default swaps and synthetic CDOs, refer to the Lehman Brothers publication *Credit Derivatives Explained*, Dominic O'Kane, March 2001. A comprehensive description of the CDO market is provided in *Collateralized Debt Obligations: Market, Structure, and Valuation*, Sunita Ganapati, June 1998.

Figure 1. **Loss Distribution of a Single Asset with a Default Probability *p* and an Assumed Recovery Rate *R***



There are two approaches to credit risk management. The first and most common approach, generally used by asset managers who are required to mark-to-market their portfolios, is to estimate the price distribution of the portfolio at some future date, which is then used to generate some risk measure such as the Value at Risk (VaR). This approach generally applies to portfolios of investment grade credits in which the risk of default is very low—since assets will be sold as soon as they are downgraded into sub-investment grade, it is hoped that default can be avoided. As a result, the investor's risk is to changes in credit spreads that will cause the value of the portfolio to fall. In this approach, the risk of default, leading to actual capital losses on either coupons or principal, is often ignored.

The second approach is to examine default risk—the likelihood of losing interest or principal due to default. This risk is the focus of buy-and-hold investors who are not required to mark-to-market, those investors who buy high-yield assets, and banks who hold large portfolios of loans on balance sheet. Other investors who take exposure to structured credit products such as cash flow and synthetic CDOs are also concerned about the default characteristics of the underlying collateral in the deal and how these affect the default risk profile of the CDO. Finding a good way to measure portfolio default risk is the main focus of this article.

The starting point for analyzing the default risk of a portfolio is the single asset. The risk of any single asset or portfolio can be captured by the shape of its loss distribution. For single assets, computing the loss distribution is trivial (Figure 1). The asset defaults with probability p resulting in a loss of par minus the recovery rate, otherwise the asset survives with no loss incurred.

For portfolios of assets, the shape of the loss distribution is also a function of the default correlation between the assets. Default correlation measures the tendency of different assets to default together. The mean of the loss distribution of a portfolio is simply the weighted average mean of the loss distributions for the single assets—it is independent of the default correlation, depending only on the default probability and assumed recovery rate of the assets. The loss distributions of structured credit assets such as CDOs are also driven by the default probability and recovery rates of the assets, as well as the level of default correlation.

Calculating the shape of the loss distribution for a portfolio of assets is non-trivial. Through their role in rating structured credit products, rating agencies have developed their own approaches to assessing the default credit risk of portfolios. Their aim is to compute the probability and magnitude of potential losses to the investor. This is based on historical default rates for issuers with the same credit quality as those in the underlying portfolio. For example, Moody's uses a binomial model to capture the loss distribution and, hence, the risk of a CDO tranche. The correlation between issuers is taken into account by mapping the number of issuers to a corresponding number of independent ones using the diversity score.

Recently, Standard and Poor's has introduced a new CDO evaluation model in which the loss distribution of a portfolio of correlated issuers is simulated. This model uses the copula function approach with a Gaussian copula, cf. Naldi and Mashal (2001). It is worthwhile to note that Moody's bases its rating assessment on the expected loss of a tranche, while Standard and Poor's considers the probability of a loss.

In this article, we briefly discuss a model that we have developed in order to analyze portfolio default risk. We discuss the different risk measures that may be used to capture default risk at an asset and portfolio level and apply these methods to analyzing the risk of tranches of synthetic CDOs.

Methodology

To generate loss distributions, we require a way of modeling correlated defaults. One choice gaining favor in the credit derivatives market is the use of a Merton-style model of correlated default in which default occurs when the asset value of a firm falls below a certain threshold. In such a model, the default correlation arises through the correlation of the asset values of the various issuers. The correlation structure is defined using a specific choice of copula function. A copula is a function that specifies how the joint default dependence structure depends on the marginal default distributions of the individual issuers. We can therefore write the joint default probability as a function of the individual default probabilities and the asset-value correlation of issuers A and B, i.e.,

$$P_{AB} = \Phi(P_A, P_B, \rho) \tag{1}$$

where $\Phi\ (x,\ y)$ is the cumulative density function that depends on the type of copula function used. Choosing a normal copula implies that $\Phi\ (x, y, \rho)$ is the cdf of the bivariate normal distribution with correlation coefficient ρ.

VaR as a Measure of Default Risk

A tool widely used by risk managers is the Value at Risk (VaR) concept. By definition, the VaR of the portfolio to, say, a 95% confidence level gives a lower bound on the portfolio loss that will not be breached in any but the 5% worst outcomes—it is the smallest loss that can occur in the worst 5% of all outcomes.

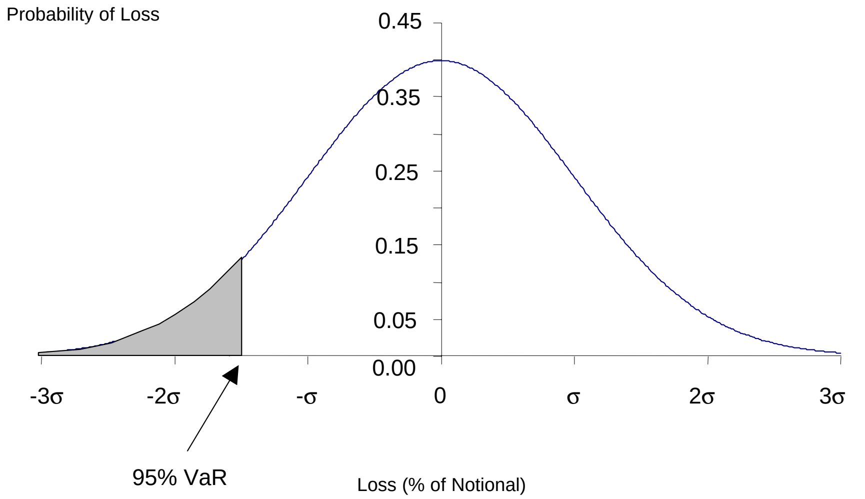
Technically, if X is a random variable with a distribution function F describing a credit exposure (losses are negative), then the VaR is related to the quantiles of the distribution of X. For $0 < \alpha < 1$, the corresponding quantile q_α is given by

$$q_\alpha = \min\ \{x \in \mathbb{R}\ |\ F\ (x) \geq \alpha\ \} \tag{2}$$

Note that we have given the definition of the lower α-quantile. If the distribution of X is continuous, the quantile is characterized by the fact that

$$P\left[X \leq q_\alpha\right] = \alpha \tag{3}$$

Figure 2. **Value at Risk for a Normal Loss Distribution with Mean Zero and Standard Deviation σ**
We have used a 95% confidence limit, i.e., a = 5.0%



If the distribution of X is not continuous, the quantile is not unique. If we have an asset with a 5% probability of default and an assumed recovery rate of 40%, it is not clear whether the 95% VaR quantile should start at a loss of 60% or 0%. Equally, for a given quantile, the confidence limit may not unique. For example, the loss distribution of a single asset is not continuous—if the asset's probability of default is 7% and the recovery rate is 60%, then the 95% confidence limit VaR equals 40%. However, this is also the VaR for confidence limits from 93% to 100%.

Since the loss amount is negative, the VaR to the confidence level $1 - \alpha$ is defined as

$$\mathrm{VaR}_{1-\alpha} = -q_{\alpha} \tag{4}$$

In effect, the VaR tells us where the tail of the loss distribution begins. At the same time, the VaR concept suffers from the drawback that it ignores the behavior of the distribution **within the tail**. In the world of credit, this problem is particularly pressing, because we deal with large losses that have a very low probability of occurring. For example, an asset with a recovery rate of 40% and a default probability of 4% will have an expected loss of 2.40%, a maximum potential loss of 60%, and using a confidence limit of 95%, a VaR of 0%. This is because the smallest loss that can occur in 5% of the worst outcomes is zero—in 4% of outcomes, we lose 60%, but in the other 1% of outcomes, we don't default and so lose nothing.

Another well-known drawback of VaR is the fact that it is not amenable to the aggregation of risks and, in fact, can penalize perfectly reasonable diversification strategies. Technically, if X and Y denote two random variables that describe risky exposures and ρ denotes a mapping that associates with each random variable a measure of its risk, then we would expect the risk of the exposure $X + Y$ to be less than the sum of the individual exposures:

$$\rho(X+Y) \le \rho(X) + \rho(Y) \tag{5}$$

This sub-additivity is the mathematical expression of the fact that diversification reduces risk. Value at Risk is not sub-additive and can, therefore, penalize diversification.

A classical example of this phenomenon is the following. Suppose we invest in a bond that defaults with a probability of 1% and has a recovery rate of 40%. Because the probability of a default is less than 5%, the VaR of this position at a 95% confidence level is zero. On the other hand, if we invest the same money in an equally weighted portfolio of 100 independent issuers with the same credit risk profile, a simple calculation using the binomial distribution shows that the VaR of this portfolio is 1.8% of the portfolio notional (corresponding to three defaults). However, if we invest in only one bond, there is a 1% probability of losing 60% of our investment, whereas for the diversified portfolio, the probability of incurring such a large loss is effectively zero. VaR would imply that the single asset is a safer investment than the portfolio.

Beyond Value-at-Risk: The Expected Shortfall

In response to this drawback, extensions of VaR have been developed. In an influential paper, Artzner et al. (1999) developed the framework of so-called coherent risk measures, whose properties include sub-additivity in accordance with equation (5). One coherent risk measure that we find useful is the expected shortfall of a portfolio. Loosely speaking, this is the expected loss of the portfolio, conditional on experiencing a loss beyond the VaR bound. In Figure 2, it corresponds to the average loss in the shaded tail of the loss distribution.

If the distribution of X is continuous, the expected shortfall (ES) can be written as (note again that losses have a negative sign)

$$\mathrm{ES}_\alpha = -E\left[X \mid X \le q_\alpha\right] \tag{6}$$

Unfortunately, if the distribution of X is not continuous, as is indeed the case in all our applications, there are certain technical difficulties to be overcome because the definition given in equation (6) will not lead to a sub-additive risk measure. This is related to the fact that the quantiles of the distribution of X are not unique and the mass of the set $\{X \le q_\alpha\}$ may be different from α. The technical difficulties can be overcome by modifying equation (6) to

$$\mathrm{ES}_\alpha = -\frac{1}{\alpha}\left\{E\left[X 1_{\{X \le q_\alpha\}}\right] + q_\alpha\left(\alpha - F\left(q_\alpha\right)\right)\right\} \tag{7}$$

Note that the first summand on the right-hand side of equation (7) looks somewhat like a conditional expectation, whereas the second summand corrects for the discontinuity of X. We won't go into more detail here, instead referring the interested reader to Acerbi and Tasche (2001).

The advantage of expected shortfall over VaR is that it takes into account the behavior of the distribution within the tail. As VaR is effectively a boundary on the tail, expected shortfall is a more conservative measure of risk. Finally, because expected shortfall is a coherent risk measure, it is guaranteed to behave suitably under aggregation.

Comparison of Value at Risk and Expected Shortfall for Credit Portfolios

In this section, we take an example portfolio of defaultable assets and, using a model of correlated default, we compute the VaR and expected shortfall. The aim is to highlight the differences between the two approaches.

Arguably, the most important inputs to a model for the portfolio loss distribution are the individual issuer default probabilities. For our risk assessment, we use the idealized historical default probabilities that Standard and Poor's provides for CDO evaluation, which are tabulated in Figure 3. We consider these to be relatively conservative estimates of default probabilities based on historical data.

To use these idealized historical default probabilities, we must differentiate between assets according to rating. Our base case is an equally weighted portfolio of 200 issuers with ratings evenly divided across all the rating categories from AA to BBB-, i.e., with 25 issuers in each of the eight rating categories. The reason we go from AA to BBB- is that these are the investment grade ratings at which an issuer can be expected to trade at a positive spread to LIBOR on a floating basis.

The loss distributions for the whole portfolio (base case) plus two sub-portfolios (all BB and all AA) are shown in Figures 5 and 7. The expected percentage loss of each portfolio is independent of the correlation; we obtain 2.5% for the base case,

Figure 3. **Idealized Historical Default Probabilities By Rating, %**

	1Y	2Y	3Y	4Y	5Y	6Y	7Y	8Y	9Y	10Y
AA	0.111	0.242	0.394	0.565	0.757	0.968	1.198	1.445	1.710	1.990
AA-	0.136	0.290	0.464	0.659	0.875	1.113	1.372	1.650	1.946	2.259
A+	0.136	0.303	0.501	0.728	0.984	1.265	1.570	1.896	2.242	2.604
A	0.136	0.317	0.542	0.808	1.111	1.448	1.814	2.204	2.614	3.041
A-	0.145	0.358	0.632	0.959	1.330	1.737	2.173	2.632	3.108	3.597
BBB+	0.225	0.532	0.911	1.352	1.841	2.368	2.921	3.492	4.074	4.661
BBB	0.225	0.638	1.182	1.814	2.500	3.215	3.941	4.667	5.383	6.084
BBB-	0.544	1.357	2.317	3.344	4.387	5.415	6.410	7.360	8.261	9.112

Source: Standard and Poor's.

Figure 4. **Portfolio Loss Distributions Over a 10 Year Time Horizon with Zero Correlation**

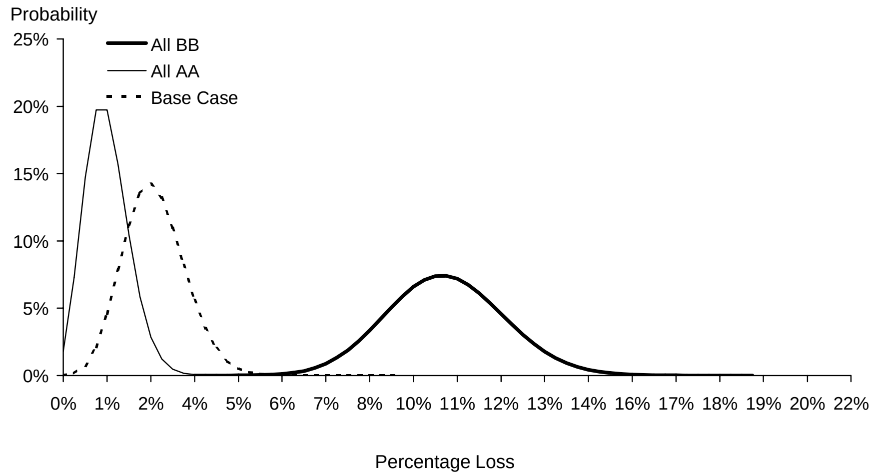
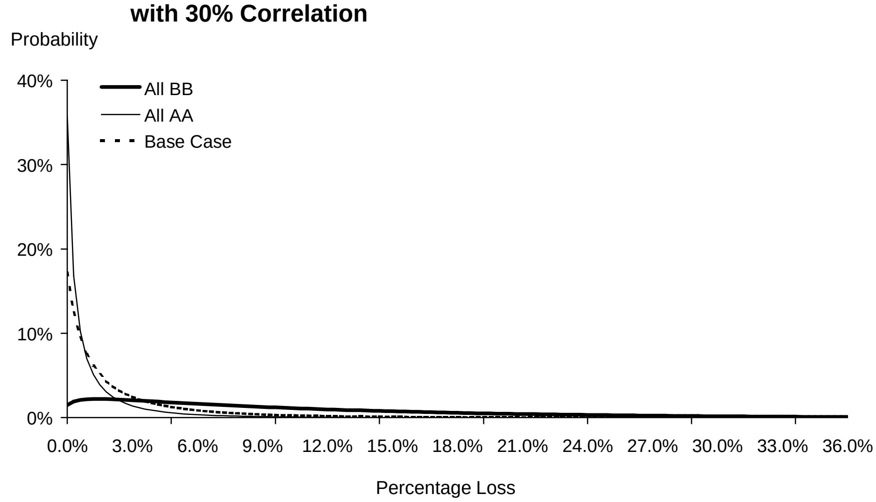


Figure 5. **Portfolio Loss Distributions over a 10-Year Time Horizon with 30% Correlation**



1.19% for the AA, and 10.48% for the BB portfolio. To calculate these loss distributions, we have assumed two flat correlation structures—Figure 4 shows 0% asset-value correlation and Figure 5 shows 30% asset-value correlation.

The effect of the increase in correlation is striking. For zero correlation, the loss distributions are essentially binomial. However, as the correlation is increased, the probability of assets defaulting jointly increases, thereby pushing out the tails of the distributions. At the same time, a higher default correlation also implies that assets are more likely to survive together, resulting in an increase in the likelihood of no loss. As a result, the distributions become more monotonic.

In Figure 6, we have tabulated the VaR and the expected shortfall for each of the three portfolios. By its definition, the expected shortfall is always greater than the VaR. Both measures of risk increase at higher correlation; note that the difference between the two is more pronounced for 30% correlation, reflecting the long tail behavior of the distribution at higher correlation.

Comparison of Value at Risk and Expected Shortfall for CDO Tranches

The loss distribution of a tranche is a function of the portfolio loss distribution. The tranche is described by two boundaries, k_1 and k_2, given as percentages of the total portfolio notional. For example, a mezzanine tranche might range from 3% to 7%. An investor selling protection on this tranche must indemnify the protection buyer for any loss due to default after 3% of the portfolio notional has been lost. At the same time, the liability of the investor can never exceed 4% = 7% – 3% of the total notional.

For a given portfolio, the risk profile of a tranche is a function of its seniority, as determined by the lower boundary k_1, and its leverage, as given by the tranche width $k_2 - k_1$. The payoff diagram is shown in Figure 7.

Before embarking on the analysis of the loss distribution, let us briefly describe how a synthetic tranche deal would be structured in swap format. The investor bears the credit risk on part of the capital structure of a credit portfolio, say 7% to 10%. This implies that as soon as the cumulative loss on the portfolio exceeds 7% of the total notional, the investor pays the difference between par and the recovery rate for each defaulted asset to the protection buyer. The investor must make these payments until the cumulative loss reaches 10% of the portfolio notional. In return, the investor is paid a periodic spread on the outstanding notional of the underlying tranche, which is reduced each time the tranche is affected by a default event.

For a fixed portfolio, the risk profile of a tranche depends on its seniority and its leverage, as determined by the lower tranche boundary and the distance between the upper and the lower boundary, respectively. Clearly, lower seniority means

Figure 6. **Comparison of Var and Expected Shortfall for Three Portfolios of Credit Names Assuming a Correlation of 0% and 30% and a Confidence Limit of 95%,** %

	0% Correlation		30% Correlation	
Portfolio	**VaR**	**Expected Shortfall**	**VaR**	**Expected Shortfall**
BASE (all assets)	3.90	4.37	9.60	14.20
AA only	2.10	2.57	5.10	8.69
BB only	13.20	13.90	29.10	35.58

Figure 7. **Tranche Loss Payoff as a Function of the Portfolio Loss**

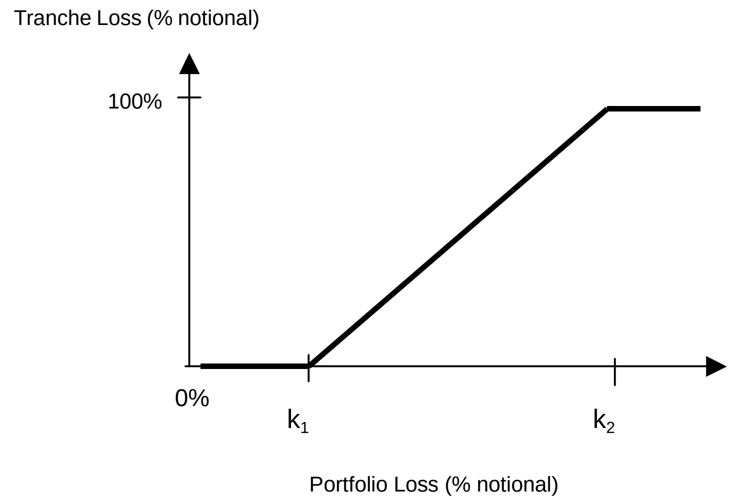


Figure 8. **CDO Tranche Structure, %**

Tranche	Lower Loss Bound	Upper Loss Bound
Super Senior	8.0	100.0
Senior	5.0	8.0
Mezzanine	2.5	5.0
Equity	0.0	2.5

that there is a smaller protective cushion for a tranche, thereby increasing its riskiness. At the same time, an increase in the leverage of the tranche, as measured by a decrease in its width, means that it takes fewer defaults to eliminate it once the lower boundary has been breached. This makes the loss distribution more bimodal, also increasing its risk.

Because the leverage of a tranche increases its risk, the underlying portfolio is typically selected to consist of better-quality issuers than when investing in an unleveraged portfolio. We consider a mezzanine and a senior tranche on an underlying portfolio of 200 equally weighted AA rated issuers. The capital structure of the CDO tranches is shown in Figure 8. Note that there is a super senior tranche (a tranche above a senior tranche) which, due to its very high credit quality, we will exclude from our analysis.

In Figures 9 and 11, we show the loss histograms for the senior and mezzanine tranches for different values of the asset correlation. Note that these distributions are much more bimodal than the portfolio distribution. While the probability of incurring losses is low, these losses can be quite significant, reaching 100% of the tranche notional.

Figure 9. **Loss Histogram for Senior Tranche, 5% - 8% on Portfolio of AA Rated Issuers**

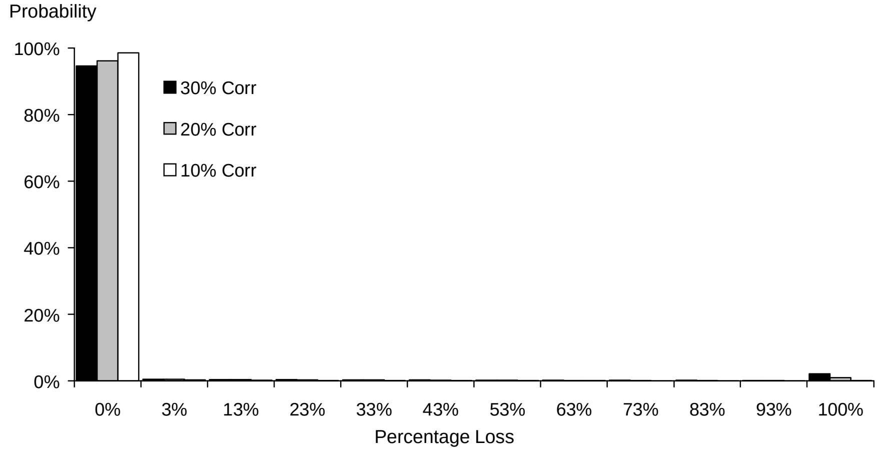
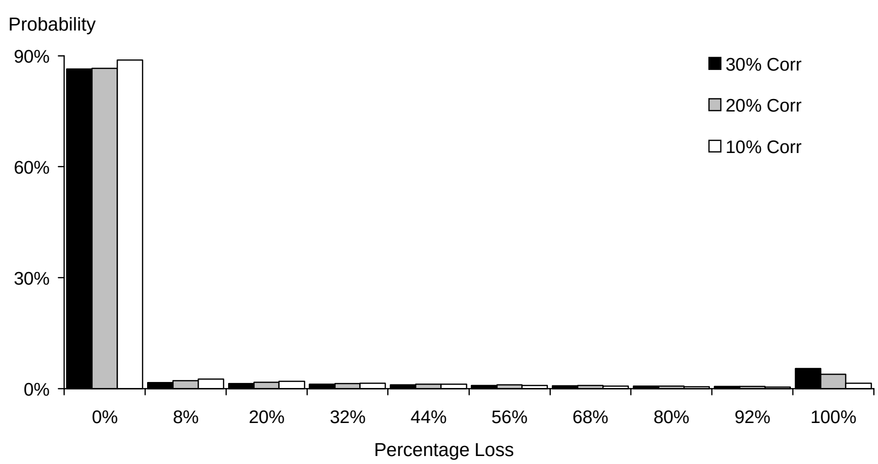


Figure 10. **Summary of Statistics for Senior Tranche**

Correlation	**10%**	**20%**	**30%**
Expected Loss	0.53%	2.07%	3.44%
95% Value at Risk	0.00%	0.00%	3.33%
95% Expected Shortfall	10.55%	41.38%	68.43%
Probability of Zero Loss	98.56%	96.15%	94.59%

Figure 11. **Loss Histogram for Mezzanine Tranche 2.5% - 5% on Portfolio of AA Rated Issuers**



In Figure 10, we see the summary statistics of the senior tranche. The risk to the tranche clearly increases with correlation, as the increased likelihood of joint defaults makes it more likely that the lower loss bound of 5% will be exceeded. Note, however, that it takes a relatively large number (16.66=200 × 5%/(100% – 40%)) of defaults before the tranche incurs losses. In particular, the expected loss is no longer independent of the correlation. Recall that the expected loss for the unleveraged portfolio is 2.5% for the base case and 1.19% for the portfolio of AA rated issuers. We see that there is a large probability of not incurring any loss, and, correspondingly, the expected loss of the tranche is small.

Note also that the VaR is actually zero in two of the correlation cases, while the expected shortfall is quite substantial. This illustrates the need for a risk measure that succeeds in capturing the behavior of the tail of the distribution.

The situation for the mezzanine tranche is qualitatively similar. As the mezzanine tranche is riskier than the senior one, the investor would be compensated by a significantly higher spread. For the tranche to take any loss, nine or more out of the portfolio of 200 AA rated issuers have to default. The fact that the expected shortfall equals 100% for a correlation of 30% means that in the 5% worst cases, the tranche would be completely eliminated by defaults. This must be set against the fact that the probability of incurring no loss on the tranche is between 86% and 89% for the different correlation values.

These numbers reflect the bimodal nature of this tranche loss distribution and make clear that while expected shortfall is a sensible and coherent risk measure, it does not capture the full risk profile of the tranche. This is no surprise—it's difficult to capture the richness of the loss distribution in one number.

Conclusions

Better tools for risk management of credit default risk are an important requirement for credit risk managers. We have described a model that could be used to capture portfolio credit default risk and discussed what risk measures are appropriate. Value at Risk fails the test as a risk-measure since it is inappropriate for the analysis of the risks of rare events (such as default) for small numbers of assets. It also fails the test of sub-additivity. The coherent risk measure expected shortfall is a better candidate, and we have used the example of a large credit portfolio to compare both measures.

Figure 12. **Summary of Statistics for Mezzanine Tranche**

Correlation	**10%**	**20%**	**30%**
Expected Loss	4.78%	7.57%	8.75%
95% Value at Risk	44.0%	80.00%	100.0%
95% Expected Shortfall	73.16%	96.84%	100.0%
Probability of Zero Loss	88.83%	86.59%	86.38%

We have also shown how the nature of the risk profile to which a credit investor is exposed changes when investing in a tranche of a portfolio instead of the portfolio as a whole. By leveraging the credit exposure, the earned spread can be substantially increased, with the buy-and-hold investor profiting from the tension between the market-implied loss distribution and one based on idealized historical probabilities, such as those used for rating tranches. At the same time, the investor is trading the smooth loss profile of the portfolio as a whole for a more bimodal one. In the situation we have considered, the loss distribution for the mezzanine tranche is qualitatively similar to that of the senior one, albeit with more risk in return for the higher spread that is earned.

The risk management implication of the shape of the loss distributions is that it is important to analyze their tail behavior, which requires a risk measure such as the expected shortfall. However, a basic observation is that it is impossible to characterize the full shape of a loss distribution with just one number. So while it is right for risk managers to focus on the tail of the distribution, which the expected shortfall does, perhaps by adding a second dimension such as the expected loss, a more complete, yet parsimonious, description of the credit default risk profile can be obtained.

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