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Quantitative Credit Research

# Quantitative Credit Research

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# Basis Trade Screening and Analysis on LehmanLive

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*The CDS basis of a bond is defined as the CDS spread to the bond maturity minus a bond-implied CDS (BCDS) spread. This basis can be used to make relative value comparisons between the bond market and CDS market of an issuer. We previously introduced the LehmanLive Basis Calculator, a single-security analysis tool that calculates BCDS spread and basis for a bond. We introduce two additional LehmanLive tools that provide users with a complete toolkit to identify and analyze basis trades. The first of these, the Basis Screening Tool, can be used to track the BCDS spread and the basis of the bonds in a bond universe defined by the user based on multiple criteria. The second one, the Basis Trade Analysis Tool, can be used for performing scenario analysis for basis trades that include CDS and bond positions.*[1]

## 1. INTRODUCTION

The bond-implied CDS (BCDS) spread is a credit spread that can be compared directly with a CDS spread. It differs from other credit spreads such as the Z-spread and the OAS primarily by explicitly recognizing the recovery rate of the bond. Therefore, the BCDS spread is a more useful concept than either of these spreads in determining whether a basis exists between a bond and CDS of an issuer. Accordingly, we define the CDS basis for a bond as the CDS spread to the bond maturity minus the BCDS spread. Pedersen (2006) explains the BCDS spread, how it is computed, and introduces the LehmanLive Basis Calculator that can be used to compute the BCDS spread and basis for specific bonds. In this article, we introduce two additional LehmanLive tools that build on the BCDS spread concept, and can be used in screening and analyzing basis trades.

The Basis Screening Tool (LehmanLive keyword: BST) is a filtering and monitoring tool for corporate bonds that makes it easier to identify basis trading opportunities. Based on various user input parameters such as sector, rating and maturity, the tool will return a universe of bonds that fit the specified parameters of the search. The user can define many different bond universes this way and save these within the tool to screen the basis of the bonds in different groups. Bonds in a specific universe are displayed in a table format along with many bond-related quantities such as current price, BCDS spread and basis. The tool also calculates statistics based on the bond universe allowing the user to monitor universe rankings for a specific bond's basis and BCDS spread. The user can access the historical time series of BCDS and CDS spreads through the tool demonstrating richness/cheapness of the basis. The tool also displays the CDS curve of an issuer together with the BCDS spread of each of the issuer's bonds to allow for relative value analysis between the bonds of an issuer. Finally, a trade that includes a specific bond in the bond universe can be created by simply clicking on the bond's cusip and launching the Basis Trade Analysis Tool.

The Basis Trade Analysis Tool (LehmanLive keyword: BTA) can be used to perform risk and scenario analysis on basis trades that include CDS, corporate bond and Treasury positions. A trade can be loaded either through the Basis Screening Tool or directly within the tool by adding bond and CDS positions. The tool reports the standard risk measures such as delta and gamma for each position and aggregated across all positions in the trade. A trade can be saved within the tool and easily tracked for future performance. The user can perform

[1] *The tools were implemented by Peili Wang, Joshua Lee and Vineet Enagandula from our Analytics and Technology group. To contact them about any technical problems please email cca_onduty@lehman.com.*

scenario analysis by specifying a time horizon and changing the market variables such as credit curve and bond basis as of that date. The tool will calculate and display profit/loss and carry of each position and of the overall trade for the scenarios defined by the user.

In Section 2, we briefly explain how basis trades can be implemented. In Sections 3 and 4, we describe various features of each tool and illustrate the basic functionality with some simple examples.

## 2. TRADING THE CDS BASIS

From a theoretical point of view, the CDS basis of a bond should be close to zero, and the spread computed for an issuer from a bond price should be approximately the same as the spread in the CDS market. If this is not the case, investors would take advantage of the pricing discrepancy by trading the bond and CDS, creating risk-free profits. This would in turn cause the spreads in the two markets to converge and drive the basis to zero. However, this no-arbitrage relationship between the spreads in CDS and bond markets is based on many simplifying assumptions that do not hold in the real world. In reality, these two spreads are rarely the same and the basis can persistently stay positive or negative. There are some fundamental and market reasons why a basis exists between bond and CDS markets. McAdie and O'Kane (2001a) give a detailed account of these reasons and we refer readers to their article for a better understanding of the factors affecting the basis.

Although it is hard to know what the basis should be for a given issuer, an investor can still form views on the direction of basis moves and trade this view using bonds and CDS of the issuer. A basis value that is not in line with the long-term trends may indicate a genuine relative value trading opportunity.

We define a **negative basis trade** as a trade in which the investor buys the bond and buys protection in the CDS market. This trade is usually appropriate for situations when the CDS basis is negative and the investor expects it to increase and become closer to zero. A negative basis may indicate that the spread in the bond market is higher than the spread in the CDS market, and therefore the bond is relatively cheap. An investor executing a negative basis trade is positioning for either the bond price to increase and the bond spread to decrease relative to CDS spreads, or the CDS spreads to increase relative to the bond spread. In either case, the value of the basis will increase. The profit/loss of the trade is more sensitive to relative changes in CDS and bond prices rather than the absolute change.

A **positive basis trade** is defined as a trade in which the investor shorts the bond and sells protection in the CDS market. This strategy can be used when the CDS basis is positive and the investor expects it to decrease. A positive basis indicates that the bond is relatively expensive and the bond spread is lower than the spread in the CDS market for the issuer. A positive basis trade would be profitable if the bond price decreases causing the bond spread to increase relative to CDS spreads, or if the CDS spreads decrease relative to bond spreads.

One issue when implementing basis trades is the choice of CDS notional in the trade. A CDS is a par instrument but bonds usually trade at a discount or premium to par. An investor may trade a CDS notional equal to the full face value of the bond. In such a trade, if the bond traded is a discount bond, the investor will be over-hedged in the case of an immediate default. Similarly, a premium bond position will be under-hedged. Bond investments in a basis trade are usually funded and the investor borrows money in the repo market to fund the position. Therefore, an investor would usually like to protect the initial investment in the case of default. In that case, CDS notional can be chosen to hedge the initial full price of the bond. The hedge ratio of CDS also depends on the expected recovery rate and is given by:

$$\text{CDS Hedge Ratio} = (\text{Full Price} - \text{Recovery}) / (100\% - \text{Recovery})$$

If CDS notional is chosen using the CDS hedge ratio above and actual recovery is the same as the expected recovery, the payoff from the CDS position will exactly offset the loss in the initial bond investment. That will not be the case if the actual recovery is different from the expected recovery, and the position may end up with a loss or gain. A third alternative is to choose the CDS notional of protection equal to the market price of the bond. The specific strategy to choose CDS notional depends on the investor preferences and the profit/loss profile of the trade will depend on this choice.

A basis trade is usually implemented to take a view on the credit spreads. Hence, an investor would want to hedge against the interest rate risk and be exposed only to credit changes. This can be done by entering into an interest rate swap that offsets the interest rate risk in the combined bond and CDS position. Interest rate risk can also be hedged with Treasury bonds. Since these bonds are assumed to have no credit risk, they will not have any credit exposure and can be used to hedge away the interest rate risk. This is the approach we use in implementing and analyzing trades with the Basis Trade Analysis Tool.

A basis trade has many risks, and there may be different basis trades that can be implemented in the same situation depending on the risk preferences of the investor. These considerations are beyond the scope of this paper, and we give only some simple illustrations here. See McAdie and O'Kane (2001b) for an explanation of the risks in basis trades and different types of basis trades that can be implemented.

In the following sections, we will present an example basis trade in the context of introducing the Basis Screening and Basis Trade Analysis tools.

## 3. MONITORING AND IDENTIFYING BASIS TRADES WITH THE BASIS SCREENING TOOL

The easiest way to access the Basis Screening Tool is to use the keyword field in the upper right-hand corner of the LehmanLive screen and enter BST. The tool can also be accessed by following the link Fixed Income->Credit->Analytic Toolkit on LehmanLive. A link to the tool is available under the "CDS and Bond Tools" section.

When the tool is launched, a front page is displayed where the user can choose to load a previously saved screen or create a new screen. Figure 1 shows a sample front page. We use the term "screen" to denote a collection of bonds that the user filters based on some criteria, and saves within the tool and monitors. The screens that were previously created and saved by the user are shown under the **My Screens** heading. The listings under the **Shared Screens** heading show the screens created and shared by a Lehman Brothers employee. A new screen can be created by clicking on the **Screener** tab at the top of the tool.

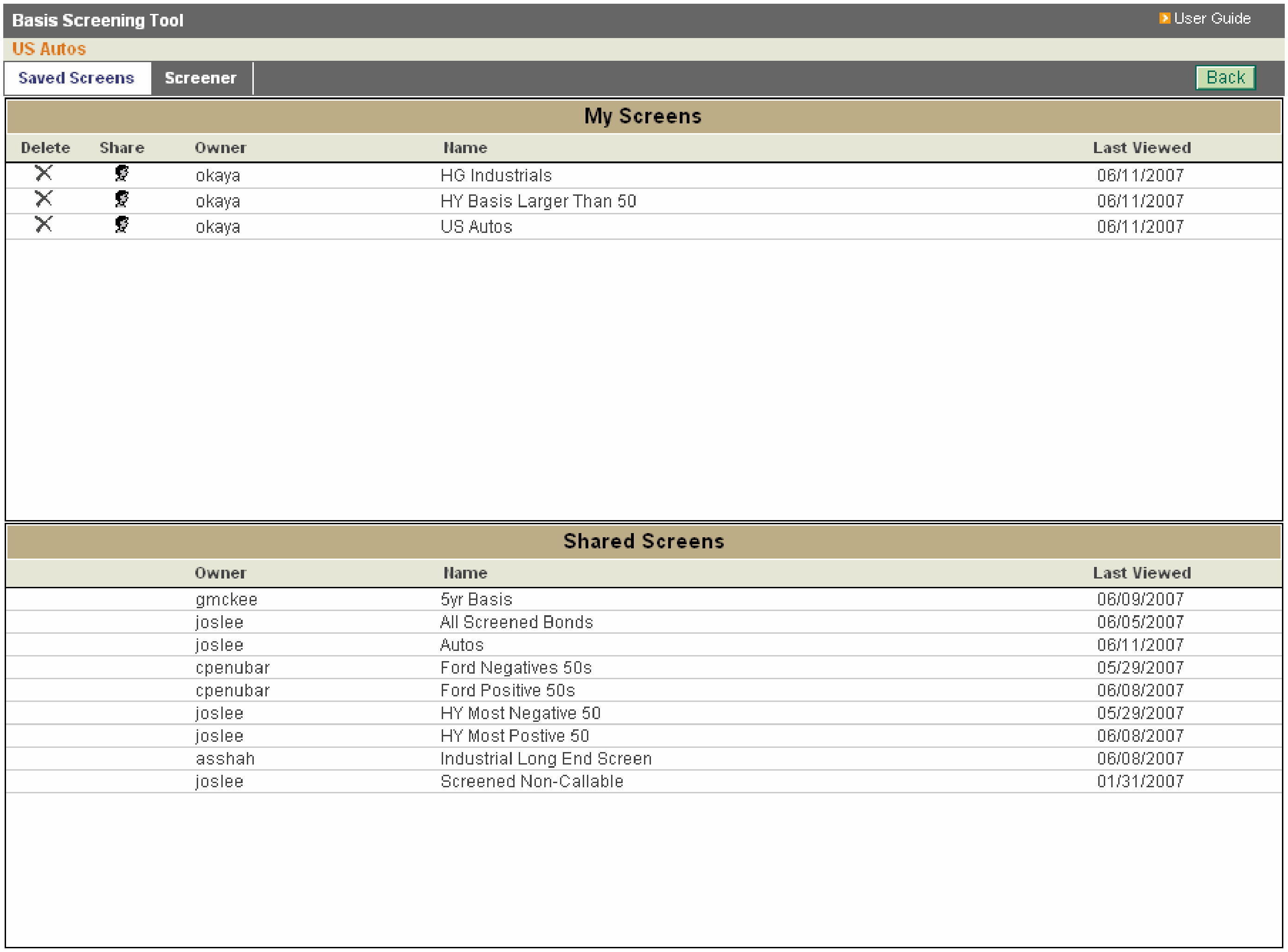
### Creating and Saving a New Screen

In the Screener window, the Screen Inputs tab on the left-hand side shows the screening parameters that are used to search for bonds satisfying user-specified criteria (Figure 2).

The initial bond universe to perform the search can be chosen as the names in Lehman Brothers' High Yield or Investment Grade Index. In order to narrow the screen or search for crossover securities, the user can set ratings parameters. In addition, the user can specify a particular Lehman Brothers sector of which the issuer must be a member.

Bond-specific parameters can be set based on a specific maturity range and deal size. These two limit the returns on the screen so that the less liquid securities in an issuer's smaller deals do not come through on the search. Furthermore, the trading volume can be specified which will further increase the probability of the screen coming back with the most liquid bonds. Lehman Brothers' CDS levels have been integrated into the Basis Screening Tool permitting a user to reverse engineer basis trades by searching based on CDS spread. Finally, since specific issuers may present better opportunities for basis trades, the screening tool allows a user to identify issuer tickers on which they would like to focus.

**Figure 1. Front Page of the LehmanLive Basis Screening Tool**




| Delete | Share | Owner | Name | Last Viewed |
|---|---|---|---|---|
| | | okaya | HG Industrials | 06/11/2007 |
| | | okaya | HY Basis Larger Than 50 | 06/11/2007 |
| | | okaya | US Autos | 06/11/2007 |

| Owner | Name | Last Viewed |
|---|---|---|
| gmckee | 5yr Basis | 06/09/2007 |
| joslee | All Screened Bonds | 06/05/2007 |
| joslee | Autos | 06/11/2007 |
| cpenubar | Ford Negatives 50s | 05/29/2007 |
| cpenubar | Ford Positive 50s | 06/08/2007 |
| joslee | HY Most Negative 50 | 05/29/2007 |
| joslee | HY Most Postive 50 | 06/08/2007 |
| asshah | Industrial Long End Screen | 06/08/2007 |
| joslee | Screened Non-Callable | 01/31/2007 |

*Source: LehmanLive Basis Screening Tool.*

The basis screening option allows the user to screen basis trades via current and historical average and z-score levels for Basis (Maturity Matched CDS – BCDS) and Price Basis (CDS Implied Bond Price – Bond Price). This screening mechanism also allows a user to set rules to view Basis as a percentage or percentile of the entire return universe, which allows a user to quickly and easily isolate the most realistic positive and negative basis trades.

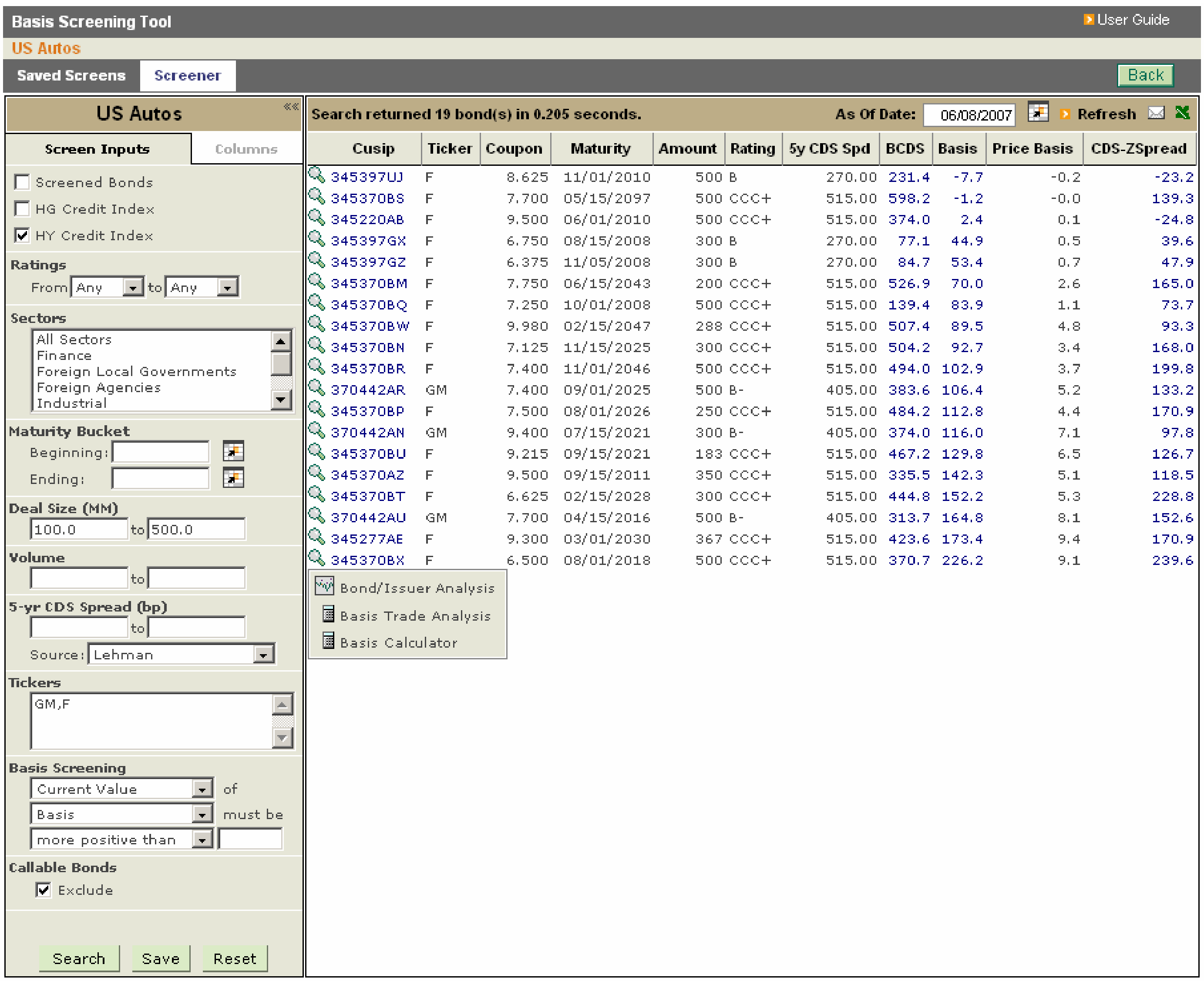
After setting the search parameters, a click on the **Search** button will return the universe of bonds that fit the parameters set in the Inputs section. Specifying values for each parameter is optional. If no limiting values are specified for a certain attribute, then no filtering is done based on that attribute. Figure 2 shows sample search settings in the Screen Inputs window. In this particular case, we ask the tool to search for bonds of tickers GM and F having a deal size between 100 and 500 million USD and are not callable.

Should the user want to save the search, the **Save** button at the bottom of Screen Inputs can be used. A small dialogue box will appear at the bottom left-hand corner of the screen. Typing the name of the screen and clicking OK will save the displayed search.

## Monitoring the Bonds and Basis

After the search is performed, bonds fitting the search criteria will be shown on the main window of the tool along with bond indicatives and analytics. The main window in Figure 2 shows the results of the search mentioned above for GM and F bonds. Column headings and order shown in this window can be set by making the **Columns** tab the active menu and checking off the boxes that correspond to the desired column headings. From there the user can change the order of the columns by clicking on set column display order at the top of the screen. Here the column headings can be dragged and dropped into the desired order.

**Figure 2. Screener Page of the LehmanLive Basis Screening Tool**




*Source: LehmanLive Basis Screening Tool.*

It is possible to sort the bonds in ascending or descending order with respect to the entries in a certain column by clicking on the column header. For example, clicking on the **Basis** on the top row sorts the bonds with respect to their basis values. Some of the columns in the bond monitor window such as price and basis have clickable values that are marked with blue. These are linked to the LehmanLive Time Series Plotter, and clicking on such a value will bring up a window that graphs the time series of the selected data field.

The tool also links to other LehmanLive calculators and bond/issuer specific information via a small "bond menu" that is displayed when pointing to the cusip of a bond. Figure 2 shows the menu displayed for bond with cusip 345370BX. This bond menu has three items as explained below.

### Bond and Issuer Analysis

Clicking on the "Bond and Issuer Analysis" in the bond menu brings up a window with four panes displaying bond- and issuer-related graphs and information (Figure 3). The Basis History pane is used to monitor the historical trend and relationship between the Bond Implied CDS and the Maturity Matched CDS levels resulting in a rich/cheap basis calculation. Through the second pane on the right, the user can view the entire term structure of the issuer CDS against the Bond Implied CDS of the issuer's debt. The Universe Rankings pane gives statistics that benchmark the particular bond's Basis, Price Basis, and CDS – Z-Spread against the rest of the return universe to view the relative value of basis trades. Finally, the Research pane has links to Lehman Brothers research articles on the specified issuer as well as any other articles focusing on that industry sector and related industries.

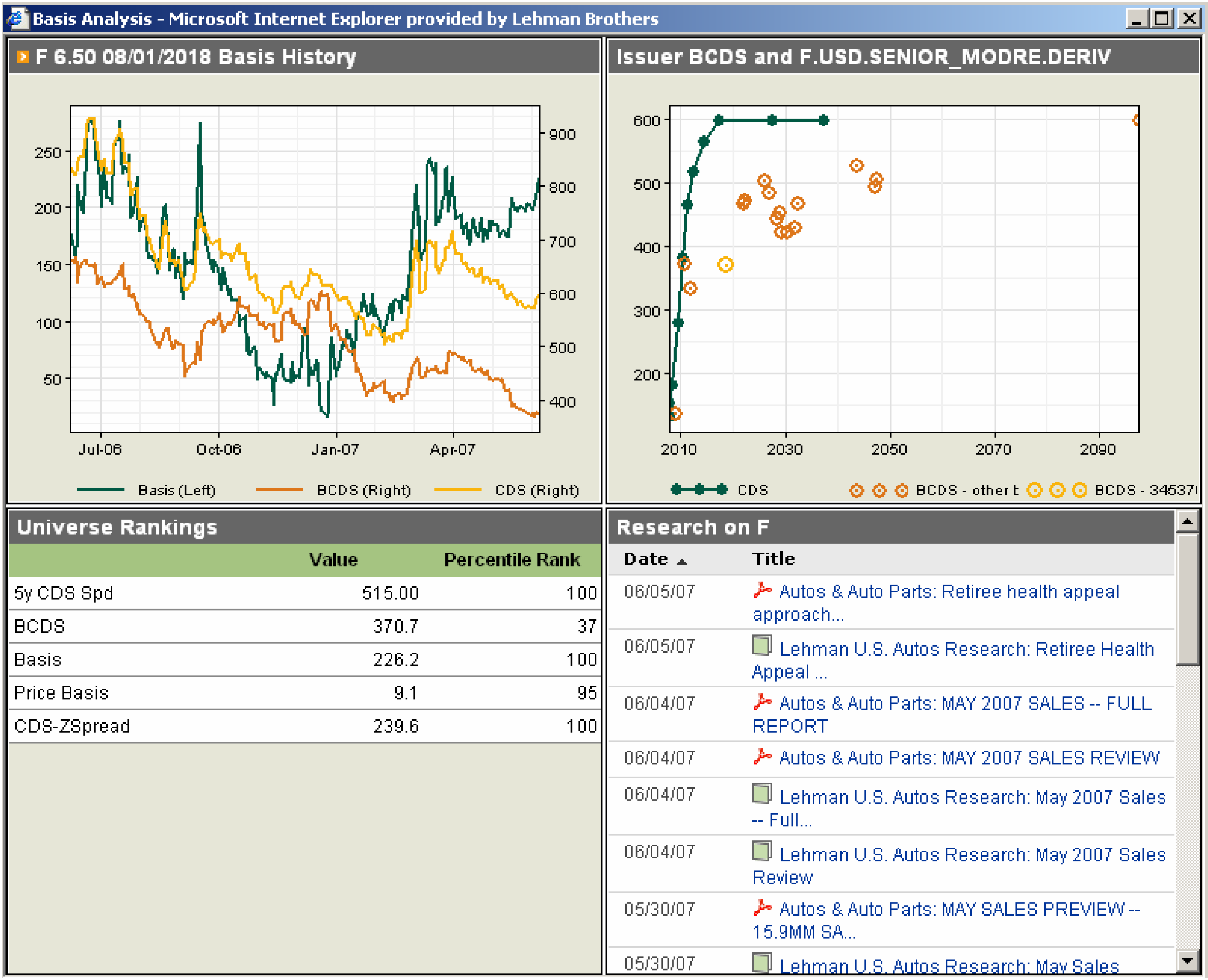
The Bond and Issuer Analysis window shown in Figure 3 corresponds to the bond with cusip 345370BX which is at the bottom of the bond list displayed in Figure 2. This is the bond with the most positive basis in that universe and therefore we may want to explore the basis trading opportunities by looking at additional information. The Basis History graph in Figure 3 shows that the basis of the bond has a decreasing trend. An investor thinking this trend will continue may put on a positive basis trade with the expectation that the basis will continue to decrease.

If a bond seems an attractive candidate for a basis trade, the user can automatically create a trade with the bond included in the trade analysis tool. This is done by selecting "Basis Trade Analysis" from the bond menu. The trade will include a CDS position and a Treasury position beside the bond position. The user can examine the characteristics of the trade and perform a what-if analysis in the trade analysis tool. We demonstrate this functionality in the following sections using the same bond as above.

It is also possible to conduct single-security analysis by selecting "Basis Calculator" from the bond menu, and loading the bond to the Basis Calculator. See Pedersen (2006) for a detailed account of Basis Calculator functionality and how it is used.

Most of the information explained above can also be accessed through the user guide to the tool. Clicking on the "User Guide" link in the upper right corner of the tool window will display a window with the user guide in it. The input and output fields in the tool are also described in the user guide. It is also possible to move the mouse pointer over a specific field and a text box will appear with an explanation of the field.

**Figure 3. Bond and Issuer Analysis Window**




*Source: LehmanLive Basis Screening Tool.*

## 4. USING THE BASIS TRADE ANALYSIS TOOL

The Basis Trade Analysis tool can be accessed using LehmanLive keyword BTA. The tool can also be opened by following the link Fixed Income->Credit->Analytic Toolkit on LehmanLive. A link to the tool is available under the “CDS and Bond Tools” section. Finally, the tool can be launched with a pre-loaded trade from the Basis Screening Tool by clicking on the bond’s cusip in the screener window.

The tool has three pages: Trade, Scenario, and My Trades. The keyword BTA links directly to the Trade page shown in Figure 4. From this page an existing trade can be loaded or a new trade can be created. Bond positions are added to a trade through the Credit Product Browser, which also supplies the most recent market prices provided by our traders. After entering all the trade positions and possibly overriding the market prices, a click on the Calculate button will calculate the credit delta, credit gamma, IR delta, IR gamma and VOD for each position and for the combined trade. Basis and bond implied CDS spread (BCDS spread) for the bond positions will also be shown.

After a trade has been selected, the Scenario page can be used to analyze how the trade will perform under a certain credit spread scenario. An unwind date and a curve of credit spreads for the underlying swap are entered. The tool then provides a valuation of the trade (i.e., a mark-to-market calculation) as of the unwind date for the scenario specified. From the mark-to-market (MTM), the realized cash flows (Carry) from the trade date to the unwind date, and the initial cost of the trade, the tool calculates the Profit/Loss (P/L) on the trade for each position and for the combined trade. Profit/Loss calculation also takes into account reinvestment returns and interest costs.

Finally, the My Trades page contains a list with links to all saved trades. We give more details on each page of the tool.

### Trade Page

The purpose of the Trade page is to:

- select an existing trade or create a new trade;
- set the market prices at which the trade is/can be executed;
- set the funding spreads used in financing the positions;
- calculate the upfront cost of CDS positions;
- calculate standard risk measures such as Credit Delta, Credit Gamma, IR Delta, IR Gamma; and
- aggregate the risk measures over all positions in the trade.

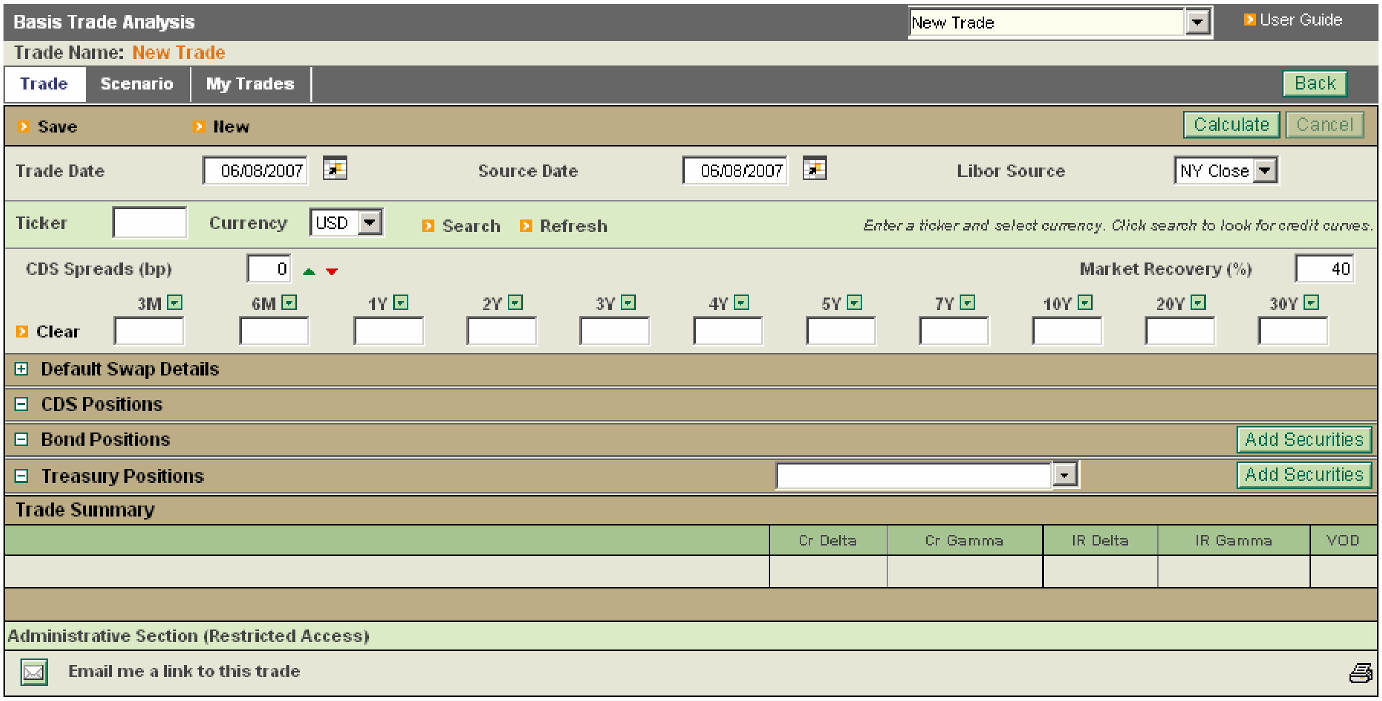
An existing trade can be loaded by selecting the My Trades page and clicking on a trade to load it into the tool, through the pull-down menu at the very top of the tool, or by clicking on a trade link distributed in an email. When a trade has been loaded, its name will appear at the top of the tool next to Trade Name.

Changes to a trade are saved when (and only when) Calculate is pressed. To make changes to an existing trade without permanently overriding it, the trade must be copied (to a different name) before the changes are made.

If an existing trade is not loaded, the Trade page will have an empty trade with the name New Trade. Any changes will be saved under that name and will be available next time the tool is entered. However, we recommend saving a trade more permanently by clicking the Save button (see Figure 4 above) and assigning a name in the pop-up box. Once a trade has been saved (with a different name from New Trade), it will appear on the list of trades and can be loaded as described above.

If the user wants to enter a new trade, the first step is to load credit spread curve data from our database of credit curves. This can be done either by entering an issuer ticker into the Ticker input box or by clicking on Search located next to the Currency input. In either case, the Credit Product Browser will be launched and choosing the appropriate credit curve in that window will load the credit spreads and recovery rate of the corresponding issuer to the tool.

**Figure 4. Front Page of Basis Trade Analysis Tool**




*Source: LehmanLive Basis Trade Analysis.*

CDS positions may be added to the trade by clicking on the arrow icons next to the CDS maturities. For example clicking on the icon next to 2Y maturity adds a CDS position with maturity of two years and the spread equal to 2Y spread on the credit curve. The default notional is 10 million and default upfront price is zero. The user can modify the Notional, Maturity, Deal Spread, Pts Upfront and Funding Spread inputs as necessary. Certain parameters used in CDS pricing such as payment frequency and day count basis can be changed by clicking on the Default Swap Details tab.

Bond positions can be added by clicking on the Add Securities button on the right-hand side of the calculator. This will bring up a window with all available bonds of the corresponding issuer. The bond positions will be added to the trade with the most recent market prices. For the bond positions, Notional, Price and Funding Spread inputs can be modified by the user.

Finally, the user can add Treasury bond positions by clicking on the Add Securities button in the same row as the Treasury Positions field, or through the pull-down menu next to it. The pull-down menu shows on-the-run Treasury bonds only, while clicking the Add Securities button brings up a window with all available Treasury bonds. Price, Yield and Funding Spread inputs for Treasury bonds can be modified by the user.

A CDS position with positive notional is a short credit (buying protection) position, and one with a negative notional is a long credit (selling protection) position. A bond position with a positive notional means the bond has been bought and one with a negative notional means the bond has been shorted.

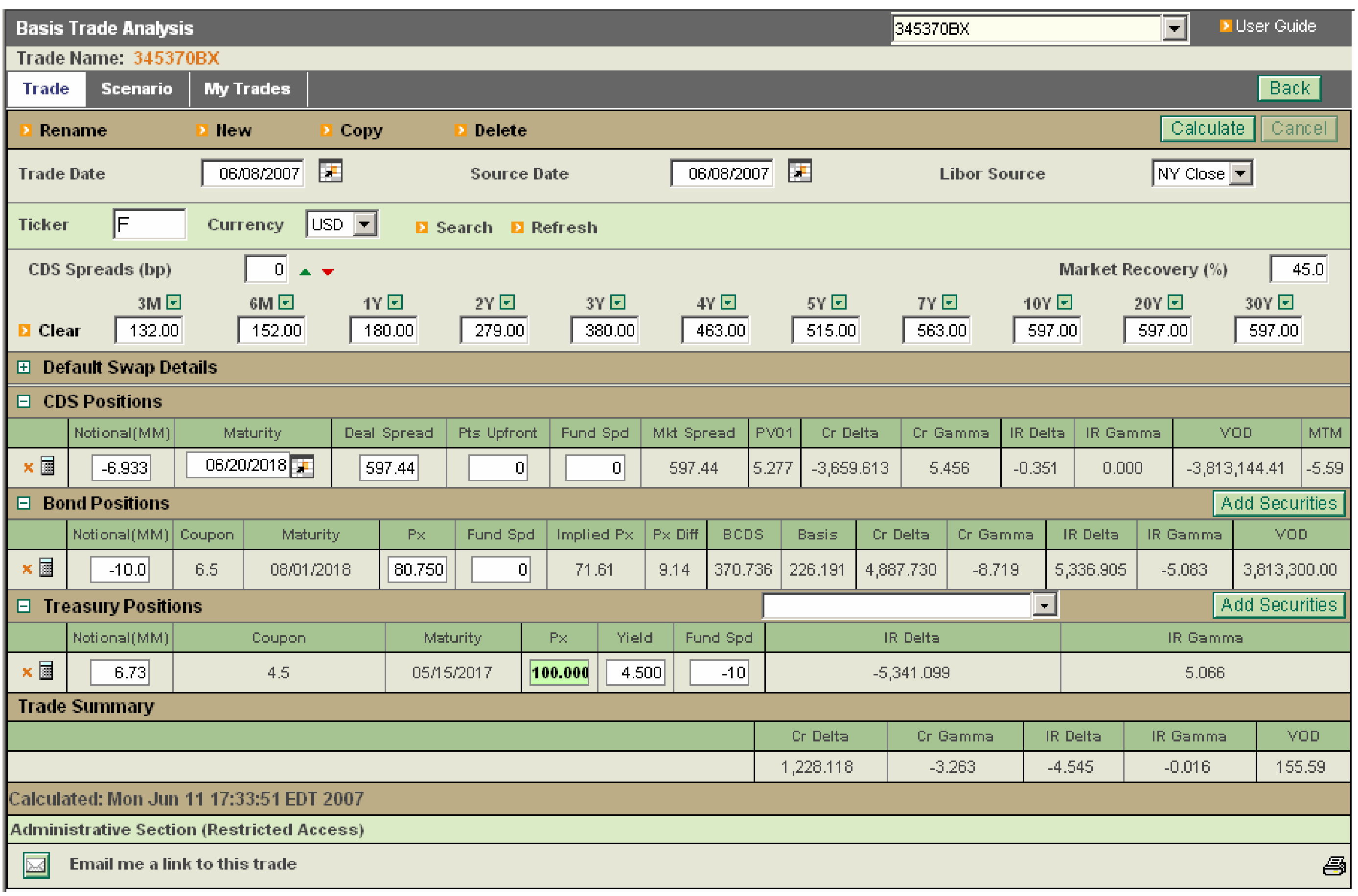
## Loading Trades using the Basis Screening Tool

Trades can also be loaded to the tool from the Screener page of the Basis Screening Tool as explained in Section 2. Hovering over the cusip of a bond on the Screener page and clicking on “Basis Trade Analysis” will launch the Trade Page with a new trade that includes the corresponding bond. The trade will also contain a CDS position and an on-the-run Treasury position both having maturities determined by the average life of the bond. The long/short direction of the bond position and the CDS position will be determined by the basis of the bond. The notional of the Treasury bond position will be selected to create a trade that is IR Delta neutral. CDS and Treasury positions are added automatically to the trade to suggest a basis trade which includes the selected bond and is hedged against interest rate moves. After the trade is loaded, the user can modify, add or delete these positions as necessary.

We revisit the example we used in the previous sections and load the bond for which the basis analysis was shown in Figure 3 to the trade analysis tool. Figure 5 shows the trade that is created by the automated process. Note that because this is a positive basis trade, both the bond notional and CDS notional are negative which means we short the bond and sell protection in CDS. The Treasury position has a positive notional that is adjusted to cancel the IR risk. This trade is automatically saved with the cusip of the bond. Clicking on calculate shows the credit and IR sensitivities of the trade as of the valuation date.

It is possible to analyze each trade position in more detail using the calculator icons to the left of each position. A click on the icon for a CDS position will bring up a new window with the Credit Default Swap Calculator (LehmanLive Keyword: CDS) and with detailed valuation output. Similarly, the calculator button next to a bond position will bring up a window with the output from the Basis Calculator (LehmanLive Keyword: BCDS), and the calculator button next to Treasury bond position will bring up a window with the output from the standard Corporate Bond Calculator (LehmanLive Keyword: CCALC).

**Figure 5. Trade Page with a Trade Loaded through Basis Screening Tool**




*Source: LehmanLive Basis Trade Analysis.*

There is a button at the bottom of the Trade page that can be used by Lehman Brothers' internal users to send a link to the trade in an email message. Clicking on the link in the email message will bring up the Trade page with the saved trade positions loaded. This facility can be used to disseminate trade ideas between clients, research, sales and trading.

### Scenario Page

The purpose of the Scenario page is to track the performance of an existing (past) trade; and to perform detailed single-scenario analysis.

On the Scenario page, we do the valuations of all the positions as of an Unwind Date that can be any date between the Trade Date and the first maturity date among all CDS and bond positions. Based on this valuation, we report quantities that show the performance of the trade such as mark-to-market, carry and P/L.

The main market input on the Scenario page is the credit spread curve on the Unwind Date. The Libor curve can be chosen either as the spot or forward Libor curve from the Source Date. When the forward curve is chosen, it is assumed that the forward rates as of the Source Date are realized on the Unwind Date.

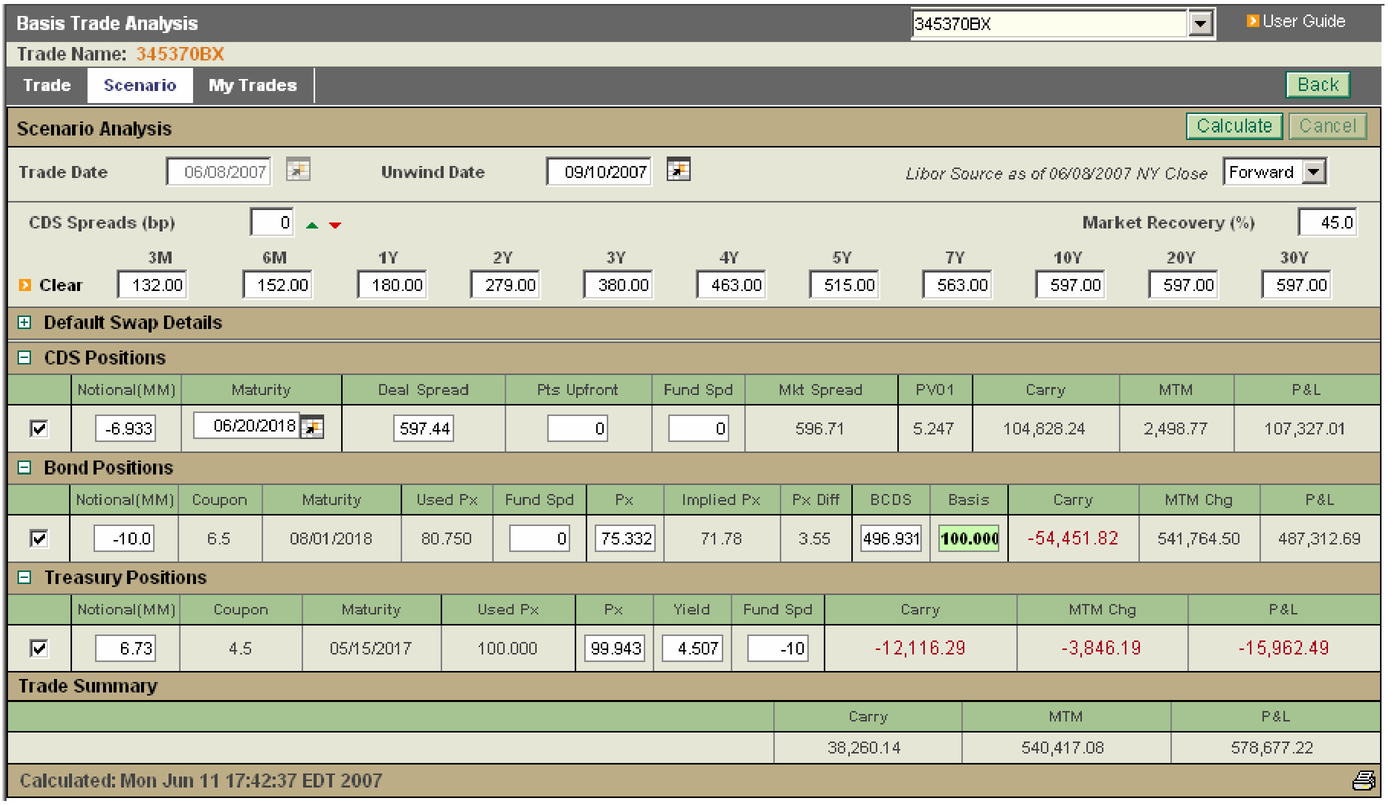
For CDS positions, mark-to-market (MTM) is the value of the position as of the Unwind Date. The Carry is the cash flow realized from the Trade Date to the Unwind Date (including reinvestment gains) minus any interest costs. We assume that any upfront payment for the CDS position is financed at the Libor rate plus the Funding Spread given for the position. Profit/Loss from the CDS positions are calculated by P/L = MTM + Carry - Upfront Cost.

When determining the value of the bond positions, the default assumption is that the basis for the bond on the Unwind Date is the same as the basis on the Trade Date. This assumption determines the price of the bond on the Unwind Date. The user can override the price, the BCDS spread or the basis to perform scenario analysis for other cases. For bond positions, mark-to-market change (MTM Chg.) gives the change in the value of the position between the Trade Date and the Unwind Date due to a change in the clean bond price. The Carry is the cash flow realized from the Trade Date to the Unwind Date minus any interest costs. We assume that the initial cost of the bond position is financed at a rate of Libor plus Funding Spread. Carry calculation also accounts for accrued coupons paid when the bond is bought and received when the bond is sold. Profit/Loss from the bond positions are calculated by P/L = MTM Chg. + Carry

The calculation of MTM Chg, Carry and P/L for Treasury bond positions is similar to calculations for corporate bond positions. The default assumption when determining the value of the Treasury bond positions is that the spread of the bond to the Libor curve on the Unwind Date is the same as the spread on the Trade Date. The user can override the price or the yield of the bond to perform scenario analysis for other cases.

Figure 6 shows the Scenario page for our example trade. Here, we look at a trade horizon of three months and would like to see the trade profit/loss if the credit curve remains the same but the basis of the bond decreases to 100bp. The tool displays the profit/loss and carry computation for each position. Under the given scenario the trade shows a profit of around USD 578,000.

**Figure 6. Scenario Page in Basis Screening Tool**




*Source: LehmanLive Basis Trade Analysis.*

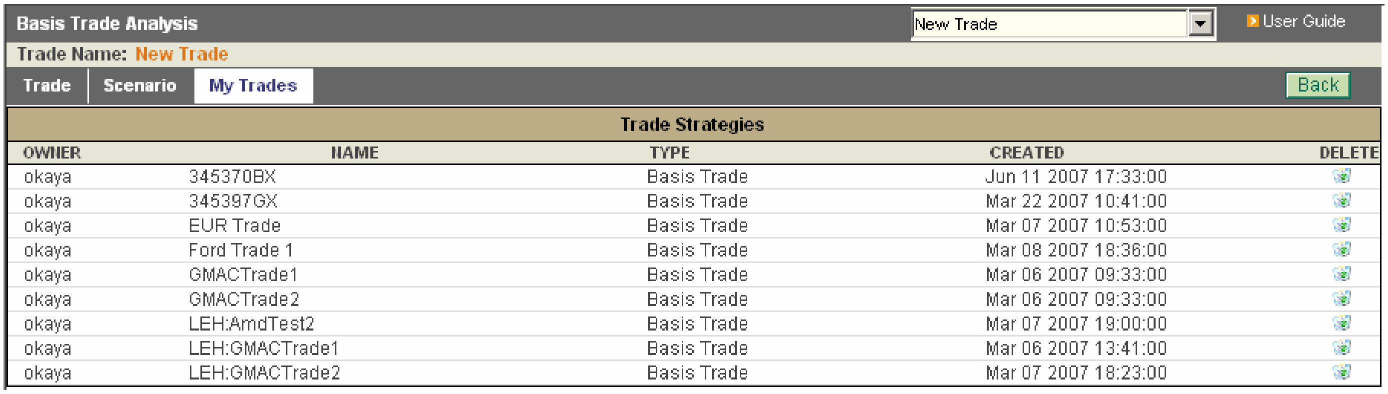
## My Trades Page

The My Trades page is shown in Figure 7. The purpose of this page is to provide easy access to all saved trades available for analysis.

Lehman Brothers researchers, salespeople and traders can send a trade as a link in an email. When a user (e.g., a client) clicks on the link, the trade will open with “LEH:” in front of the original name given to the trade by the Lehman Brothers employee. At this point, the trade has been copied and the user can modify and analyze the trade in detail. The trade will appear on the user’s My Trades page until the user deletes it. It is important to remember that any changes made to the trade after it has been opened are saved automatically when Calculate is pressed. To keep an original version, it is necessary to copy the trade to a different name before making any changes.

The user guide to the tool can be accessed by clicking on the “User Guide” link in the upper right-hand corner of the tool. The input and output fields in the tool are explained in a separate Glossary section. It is also possible to move the mouse pointer over a specific field and a text box will appear with an explanation of the field.

**Figure 7. My Trades Page**




| OWNER | NAME | TYPE | CREATED | DELETE |
|---|---|---|---|---|
| okaya | 345370BX | Basis Trade | Jun 11 2007 17:33:00 | |
| okaya | 345397GX | Basis Trade | Mar 22 2007 10:41:00 | |
| okaya | EUR Trade | Basis Trade | Mar 07 2007 10:53:00 | |
| okaya | Ford Trade 1 | Basis Trade | Mar 08 2007 18:36:00 | |
| okaya | GMACTrade1 | Basis Trade | Mar 06 2007 09:33:00 | |
| okaya | GMACTrade2 | Basis Trade | Mar 06 2007 09:33:00 | |
| okaya | LEH:AmdTest2 | Basis Trade | Mar 07 2007 19:00:00 | |
| okaya | LEH:GMACTrade1 | Basis Trade | Mar 06 2007 13:41:00 | |
| okaya | LEH:GMACTrade2 | Basis Trade | Mar 07 2007 18:23:00 | |

*Source: LehmanLive Basis Trade Analysis.*

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# The Anatomy of Credit Curve Trades Over the Economic Cycle

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*We present a quantitative analysis of the behaviour of credit steepener trades in the US, EUR and sterling markets. We find that over long periods of time, the credit steepener trade appears to generate positive alpha. Steepeners on average outperform in bullish credit market conditions. The medium to long end of lower-rated credit curves tends to flatten when market conditions turn bearish. The credit steepener trade co-moves in an intuitive fashion with the drivers of credit markets, such as corporate fundamentals and variables indicative of investor risk aversion.*

## 1. BASICS OF CREDIT STEEPENER TRADES

Macro trading strategies in credit markets have two key aspects – direction and slope. The first aspect involves view formation around the level of credit spreads in the future and is therefore 'directional' in nature. This decision could be contingent on several factors, including the stage of the economic cycle, the health of the corporate balance sheet and economic/business risks in the system. In earlier publications[1] we have documented the empirical evidence that would contribute to a quantitative approach to directional view formation. In this article, we present a quantitative analysis of credit steepener trades and document the evidence of their macro properties over different stages of the economic cycle, and their dependence on the drivers of the broad credit markets.

### Factors affecting the performance of credit curve trades

In order to understand the broad behaviour of credit curve trades, it is useful first to analyse the components of the excess returns of such trades. We choose the PV01-neutral credit 'steepener' trade (i.e. a trade that is long the front end of the credit curve and short the back end of the curve in the ratio of the PV01s) as the object of our analysis. The approximate excess returns of a 10y-2y steepener trade held for a year can be written as:

$$Steepener\ P\&L \approx -S_{10}^0 + \left( D_{10} \big/ D_2 \right) * S_2^0 + D_9 * \left( S_9^T - S_{10}^0 \right) - D_9 * \left( S_1^T - S_2^0 \right)$$

where $S_{10}^0$ and $S_2^0$ are credit spreads at the 10-year and 2-year points at the inception of the trade, and $S_9^T$ and $S_1^T$ are credit spreads at the 9-year and 1-year points at the end of the trade. $D_{10}$ is the risky PV01 at the 10-year maturity. This P&L can be rewritten as follows[2]:

$$P\&L \approx \underbrace{-S_{10} + \left( D_{10} / D_2 \right) * S_2}_{carry} + \underbrace{D_9 * \left[ \left( S_9^0 - S_{10}^0 \right) - \left( S_1^0 - S_2^0 \right) \right]}_{rolldown} + \underbrace{D_9 * \left[ \left( S_9^T - S_9^0 \right) - \left( S_1^T - S_1^0 \right) \right]}_{change\ in\ curve\ shape}$$

[1] *Naik* et al. *(2006) 'A Simple Framework to Understand the Fair Value of Credit Spreads'* Quantitative Credit Research Quarterly*, 2006-Q3.*

[2] *The above P&L expression is approximate and does not accurately account for the changes in the duration of the different legs of the trade over its life.*

The above breakdown of the steepener P&L indicates that there are three key drivers of the performance of credit curve steepener trades:

1. *Carry*: i.e. the spread pickup of the trade;
2. *Roll-down*: i.e. the component of the return arising from the upward/downward slope of the curve at the inception of the trade; and
3. *Change in the shape of the curve*: i.e. the relative change of the front end of the curve vis-à-vis the back end.

The traditional analysis of credit curve trades tends to concentrate only on the first two components of the excess returns, as they are easy to identify at the inception of the trade. However, it is the third component that is the key driver of the volatility of the excess returns of the trade. In fact, it is this component of the excess returns that embeds the investor's view on the shape of the curve over the horizon of the trade.

We present an empirical analysis of all three components of the excess returns of the steepener trade and examine its behaviour over different stages of the cycle. We find that over long periods of time, the credit steepener trade appears to generate positive alpha. Steepeners on average outperform in bullish credit market conditions. The medium to long end of lower-rated credit curves tends to flatten when market conditions turn bearish. Even in the sterling credit markets, where the average level of the credit curve has been affected by structural factors such as pension demand, we find that the macro properties of credit steepener trades do not appear to be vitiated. The credit steepener trade co-moves in an intuitive fashion with the drivers of credit markets, such as corporate fundamentals and variables indicative of investor risk aversion.

The rest of this article is organized as follows: Section 2 presents the long sample unconditional behaviour of credit steepener trades and over different stages of the economic cycle. Section 3 refines the long sample analysis by examining the properties of credit steepeners over swaps and in the EUR and sterling markets. Section 4 presents an analysis of the linkage of credit steepeners with the broad drivers of credit markets and Section 5 concludes.

## 2. BEHAVIOUR OF CREDIT STEEPENERS OVER THE ECONOMIC CYCLE

The first step in understanding the behaviour of credit curves is to analyse their behaviour in the long sample and over different parts of the economic cycle. To do this, we construct a duration-neutral credit steepener[3] trade using the Lehman Brothers US Credit Long and Intermediate Indices.

**Figure 1. Unconditional performance of credit steepeners (1973-2007)**

| | US long vs intermediate steepener | | | |
|---|---|---|---|---|
| | 1973-2007 | 1973-1990 | 1991-1998 | 1999-2007 |
| Mean excess returns (bp, monthly) | 5 | 2 | 4 | 11 |
| Median excess returns (bp, monthly) | 7 | 2 | 5 | 15 |
| Volatility (bp, monthly) | 85 | 115 | 29 | 44 |
| Annualised information ratio | 0.21 | 0.07 | 0.52 | 0.89 |

*The annualised information ratio is computed as the ratio of the mean annual excess return to the annualised volatility. Source: Lehman Brothers.*

[3] *The duration-neutral credit steepener is short one unit of the long end versus a duration-weighted long position in the intermediate end. This ensures that the trade has exposure only to the change in the difference of spreads between the long and intermediate end.*

Figure 1 presents the unconditional average excess returns of credit steepener trades (over duration-matched Treasuries) both in the long sample (1973-2007) and in sub-samples. We find that credit steepeners have positive excess returns on average, and that this finding is robust to outliers, as shown by the median monthly excess returns. It therefore appears that in the long sample, the credit curve over Treasuries offers a steepening risk premium. In order to understand the possible sources of this risk premium, we analyse the behaviour of credit steepeners over different stages of the economic cycle.

**Figure 2a. Credit steepeners over the economic cycle (1973-2007)**

| | Mean | Median | Volatility | Mean/Volatility | Median/Volatility |
|---|---|---|---|---|---|
| Expansion | 5 | 7 | 63 | 0.27 | 0.39 |
| Recession | 6 | 2 | 162 | 0.12 | 0.03 |
| Unconditional | 5 | 7 | 85 | 0.21 | 0.29 |

*The ratios of mean to volatility and median to volatility are annualized.*
*Source: Lehman Brothers, NBER.*

Figure 2a presents the average (mean and median) excess returns of the duration-neutral credit steepener trade over different parts of the economic cycle. We find that while the credit curve steepens in both expansions and recessions on average, the steepening is much stronger in expansions than recessions. To understand this better, we divide recessions and expansions into two equal halves (Figure 2b). We find that the curve steepens much more in early expansions than in late expansions and early recessions. In addition, the curve in fact flattens in late recessions.

**Figure 2b. Credit steepeners over the economic cycle (1973-2007)**

| | Mean | Median | Volatility | Mean/Volatility | Median/Volatility |
|---|---|---|---|---|---|
| 1st half expansion | 7 | 10 | 68 | 0.33 | 0.49 |
| 2nd half expansion | 3 | 4 | 57 | 0.19 | 0.26 |
| 1st half recession | 5 | 13 | 184 | 0.09 | 0.24 |
| 2nd half recession | 7 | -9 | 135 | 0.17 | -0.24 |
| Unconditional | 5 | 7 | 85 | 0.21 | 0.29 |

*The ratios of mean to volatility and median to volatility are annualized.*
*Source: Lehman Brothers, NBER.*

It is interesting to compare the performance of the credit curve steepener with that of the broad credit market and other assets such as Treasuries and equity markets (Figure 3).

**Figure 3. Asset classes over the economic cycle (1973-2007)**

| Mean Excess Returns (bp/month) | *US Credit Steepener* | US Credit BBB | US Credit Long | US Treasury Long | S&P 500 |
|---|---|---|---|---|---|
| Expansion | ***5*** | 12 | 6 | 25 | 66 |
| Recession | ***6*** | -21 | -4 | 31 | -37 |
| 1st half expansion | ***7*** | 22 | 10 | 50 | 64 |
| 2nd half expansion | ***3*** | 0 | 0 | -6 | 68 |
| Unconditional | ***5*** | 7 | 4 | 26 | 51 |

*Source: Lehman Brothers, NBER.*

The above evidence suggests that a credit steepener outperforms in periods of high economic activity (or positive growth shocks) and underperforms in periods of low economic activity (or declining/negative growth shocks). In other words, on average it appears that the steepener is a positive-beta asset, which explains its positive risk premium in the long sample. We also see that the steepener begins to underperform in the same periods as those in which the broad credit market underperforms, i.e. in late expansions and recessions. The long sample evidence therefore seems to suggest that the credit steepener is a bullish trade on the broad credit markets.

### Interpreting the long-sample evidence

The long-sample evidence suggests that the credit steepener is a bullish trade which appears consistent with the intuition that a shock to credit quality should affect the long end of the curve less than the intermediate end. This effect is based on the assumption that conditional on surviving for (say) five years, the probability of survival beyond then is not affected materially by the shock. Therefore, one must observe the intermediate end of the curve selling-off more than the long end.

However, the above intuition may not hold in all cases. A classic example is that of credit-negative events such as a leveraged buyout (LBO). The impact of an LBO on the credit curve would be determined by the market's assessment of the horizon over which the impact on credit quality (due to increased leverage) is likely to manifest. Conventional wisdom suggests that this risk is highest in the four- to six-year horizon, which implies that an LBO would cause the 5y-2y slope to steepen. By the same logic, if in a particular transaction there is an increased perception of risk further out in time, it could lead to a steepening of the longer end of the curve as well. Our analysis here does not attempt to capture such idiosyncratic effects. Rather, we focus on the properties of the steepener trade affected by the macro drivers of credit markets.

## 3. REFINING THE LONG-SAMPLE ANALYSIS OF THE CREDIT CURVE

While the analysis presented above is representative of the behaviour of the credit curve over the long sample,[4] it needs to be refined for the following reasons:

- The Lehman US Long Credit Index has an average duration of around 11 years currently while the Intermediate index has a current duration of around four. Therefore the trade analysed above need not be representative of the shorter-tenor slope trades.
- The behaviour of the credit curve over Treasuries could be different from that over the swap curve.
- The credit curve for lower-rated assets such as HY could behave differently.
- It is also important to document any differences between the credit curves in the US and those in Europe.

We address these concerns below.

### Behaviour of generic 5y-2y and 10y-5y trades over Treasuries

Using buckets of bonds that form part of the Lehman Credit indices, we construct generic spreads and excess returns at the 2-year, 5-year and 10-year points which allow us to analyse shorter-tenor slopes. We analyse two generic slopes: the 10y-5y steepener and the 5y-2y steepener over the sample 1990-2007 in USD and 1999-2007 in EUR.

In Figure 4 we present the performance of the 10y-5y and 5y-2y steepener trades in different regimes of credit market performance in USD and EUR. To do this, we first classify the history into three equally sized buckets by the performance of the USD (or EUR) BBB

[4] *The Lehman Brothers Long and Intermediate Indices are the only maturity buckets that have data over the long sample. Finer buckets are available for the sample starting 1990.*

Credit Index excess returns. We then present the mean and median excess returns of the two steepener trades in these buckets.

**Figure 4. Credit steepeners over Treasuries vs credit markets (USD: 1990-2007)**

| | | USD 10y-5y Steepener | USD 5y-2y Steepener | EUR 10y-5y Steepener | EUR 5y-2y Steepener |
|---|---|---|---|---|---|
| BBB Index outperforms | Mean (bp/quarter) | 16 | 10 | 18 | 0 |
| | Ann. IR | 0.79 | 0.64 | 1.12 | -0.04 |
| BBB Index underperforms | Mean (bp/quarter) | 9 | 31 | -31 | 26 |
| | Ann. IR | 0.41 | 1.65 | -1.04 | 2.13 |
| Unconditional | Mean (bp/quarter) | 13 | 19 | 4 | 6 |
| | Ann. IR | 0.75 | 1.25 | 0.17 | 0.45 |

*Source: Lehman Brothers.*

We find that the performance of the 10y-5y credit steepener trade over Treasuries is in line with the long-sample behaviour documented earlier. However, the 5y-2y trade seems to go in the opposite direction. In other words, we find that while the 10y-5y curve flattens in bearish credit markets, the 5y-2y curve steepens.

This observation is consistent with what a structural model like the Merton's model (Merton, 1974) would imply for the credit curve. In such a model, the impact on short- and long-term spread curves of (for instance) an increase in leverage or asset volatility would depend on the impact on the default probability conditional on surviving up to a certain period. In general the impact on conditional default probabilities at the long end is lower than that at the short end, which leads to short-end slopes steepening and long-end curves flattening.

## Credit curves over Treasuries vs over swaps

While the results of the analysis seem intuitive, it is also important to analyse the behaviour of the excess returns of credit steepeners over swaps. This is important for at least two reasons. First, it is likely that the instruments used to implement these trades would involve the standard CDS Indices, whose spreads are measured over swaps. Secondly, it is conceivable that the behaviour of credit curves over swaps is different from that over Treasuries. The credit spread curve over Treasuries comprises two components: credit spread over swaps (for instance BBB over AA) and the spread of swaps over Treasuries (i.e. AA over government yields). The dynamics of the spread curve over Treasuries would therefore be affected by both these components. Given the fact that the object of our analysis is less the behaviour of AA over government yields and more that of more risky credits, it may be useful to strip this effect out.

**Figure 5. Credit steepeners over swaps vs credit markets (USD: 1990-2007)**

| | | USD 10y-5y Steepener | USD 5y-2y Steepener | EUR 10y-5y Steepener | EUR 5y-2y Steepener |
|---|---|---|---|---|---|
| BBB Index outperforms | Mean (bp/quarter) | 14 | 13 | 16 | 6 |
| | Ann. IR | 0.70 | 0.60 | 0.97 | 0.79 |
| BBB Index underperforms | Mean (bp/quarter) | 4 | 8 | -28 | 10 |
| | Ann. IR | 0.17 | 0.33 | -1.28 | 1.20 |
| Unconditional | Mean (bp/quarter) | 7 | 11 | 2 | 5 |
| | Ann. IR | 0.36 | 0.57 | 0.09 | 0.47 |

*Source: Lehman Brothers.*

We present the excess returns of credit steepeners over duration-matched swaps in Figure 5. Comparing Figure 5 with Figure 4, we can see that the behaviour of the 10y-5y steepener over swaps appears largely consistent with that over Treasuries.

However, the behaviour of the 5y-2y USD steepener seems to be the reverse of that observed in Treasuries. This finding seems to suggest that the behaviour of the short end steepener trade may vary across different credit quality buckets. In Figure 5, by removing the effect of swap spreads we remove the effect of higher rated (AA) credits. It may therefore be valuable to disentangle the behaviour of the BBB curve and A curve vis-à-vis the AA curve.

## Impact of credit quality on the behaviour of the steepener trade

In order to analyse the behaviour of BBB and A rated vs AA rated credit curves, we compute generic USD-denominated 2-year, 5-year and 10-year spreads and excess returns over swaps. Figure 6 presents the performance of 5y-2y and 10y-5y steepeners conditioned on the performance of the BBB Credit Index.

**Figure 6. BBB and A steepeners over swaps vs credit markets (USD: 1990-2007)**

| | | USD BBB 10y-5y Steepener | USD BBB 5y-2y Steepener | USD A 10y-5y Steepener | USD A 5y-2y Steepener |
|---|---|---|---|---|---|
| BBB Index outperforms | Mean (bp/qu) | 117 | 42 | 21 | 24 |
| | Ann. IR | 1.39 | 1.40 | 0.88 | 0.98 |
| BBB Index underperforms | Mean (bp/qu) | -30 | -8 | 17 | 3 |
| | Ann. IR | -0.48 | -0.27 | 0.78 | 0.17 |
| Unconditional | Mean (bp/qu) | 48 | 16 | 14 | 13 |
| | Ann. IR | 0.64 | 0.56 | 0.67 | 0.71 |

*Source: Lehman Brothers.*

It is also interesting to move down the ratings spectrum – i.e. to analyse HY rated bonds. Figure 7 documents the evidence on generic 10y-5y and 5y-2y HY curves over swaps in USD over the period 1990-2007. We find that similar to IG curves, HY credit curves have a positive unconditional information ratio on average. We also find that the 10y-5y curve steepens along with outperforming credit markets and flattens with underperforming markets. The 5y-2y segment appears much less sensitive to the performance of the broad credit market.

**Figure 7. HY credit steepeners over swaps vs the credit markets (USD: 1990-2007)**

| | | USD HY 10y-5y Steepener | USD HY 5y-2y Steepener |
|---|---|---|---|
| BBB Index outperforms | Mean (bp/quarter) | 200 | 156 |
| | Ann. IR | 1.39 | 0.77 |
| BBB Index underperforms | Mean (bp/quarter) | -48 | 36 |
| | Ann. IR | -0.23 | 0.28 |
| Unconditional | Mean (bp/quarter) | 101 | 97 |
| | Ann. IR | 0.58 | 0.55 |

*Source: Lehman Brothers.*

## Macro properties of the sterling credit curve

Figure 8 presents the excess returns of the duration-neutral steepener trade over Treasuries in different regimes of the performance of the sterling credit market. To do this, we first classify the history into three equally sized buckets by the performance of the GBP BBB

Credit Index excess returns. We then present the mean excess returns and the annualised information ratio of the three steepener trades in these buckets.

**Figure 8. Sterling credit steepeners over Treasuries vs credit markets (1999-2007)**

| | | GBP 5y-2y | GBP 10y-5y | GBP 20y-10y |
|---|---|---|---|---|
| BBB Index outperforms | Mean (bp/quarter) | -13 | 22 | 21 |
| | Ann. IR | -0.38 | 0.45 | 0.36 |
| BBB Index underperforms | Mean (bp/quarter) | 129 | -54 | 30 |
| | Ann. IR | 1.81 | -0.56 | 0.52 |
| Unconditional | Mean (bp/quarter) | 44 | -18 | 20 |
| | Ann. IR | 0.81 | -0.26 | 0.41 |

*Source: Lehman Brothers.*

The results in Figure 8 are generally in line with those of the USD and EUR markets. We find that while the 5y-2y steepener trade outperforms markedly as the broad credit market underperforms, the 10y-5y steepener is the reverse. Such an effect would be observed in a structural model such as that of Merton (1974), wherein an increase of asset volatility or leverage would, in general, lead to a steepening of the short end of the curve and a flattening of the long end.

Based on the above line of thinking, one would expect the behaviour of the 20y-10y curve to be similar to that of the 10y-5y curve. In fact, one would expect the pro-cyclicality of the 20y-10y steepener to be even more marked than the 10y-5y steepener. In Figure 8, however, we find no perceptible difference in the performance of the 20y-10y steepener trade in bullish and bearish credit market conditions. This may be driven by the impact of dynamics of the swap spread curve, which are part of the spreads of A and BBB rated credits over government bond yields.

To strip out the impact of the dynamics of swap spreads on these results, we perform a similar analysis of the excess returns of duration-neutral credit steepener trades over swaps (Figure 9).

**Figure 9. Sterling credit steepeners over swaps vs credit markets (1999-2007)**

| | | GBP 5y-2y | GBP 10y-5y | GBP 20y-10y |
|---|---|---|---|---|
| BBB Index outperforms | Mean (bp/quarter) | -9 | 42 | 24 |
| | Ann. IR | -0.26 | 0.71 | 0.47 |
| BBB Index underperforms | Mean (bp/quarter) | 118 | -97 | 14 |
| | Ann. IR | 1.48 | -1.08 | 0.24 |
| Unconditional | Mean (bp/quarter) | 40 | -24 | 14 |
| | Ann. IR | 0.69 | -0.32 | 0.29 |

*Source: Lehman Brothers.*

On removing the effect of swap spreads, we find that the behaviour of the 20y-10y steepener is more consistent with that of the 10y-5y steepener – i.e. it exhibits stronger outperformance in bullish market conditions versus bearish market conditions.

## 4. LINKING CREDIT CURVES AND THE DRIVERS OF CREDIT MARKETS

The analysis of the contemporaneous behaviour of generic credit curves vis-à-vis that of the broad credit market suggests that the curve may also co-move with the drivers of the credit market in an intuitive fashion. In this section, we analyse the effect of some of these drivers on the performance of the credit steepener trade.

## Credit curves and corporate fundamentals

Aggregate corporate fundamentals such as leverage and asset return volatility directly impact both the level and slope of credit spread curves. Based on our analysis in the previous sections of this article, we should observe that as corporate leverage and asset volatility increase, the 10y-5y credit curve would flatten. Figure 10 presents the excess returns of the US credit 10y-5y steepener in regimes of deteriorating and improving corporate fundamentals, defined by changes in aggregate debt/equity (which is a measure of long-term leverage), changes in interest coverage (which is a measure of near-term leverage) and changes in asset return volatility.[5]

**Figure 10. 10y-5y credit steepeners vs corporate fundamentals (USD: 1990-2007)**

| | | Changes in agg debt/equity | Changes in agg interest coverage | Changes in asset return volatility |
|---|---|---|---|---|
| Improving corporate fundamentals | Mean (bp/quarter) | 13 | 24 | 16 |
| | Ann. IR | 0.92 | 1.75 | 0.87 |
| Deteriorating corporate fundamentals | Mean (bp/quarter) | 5 | 15 | 7 |
| | Ann. IR | 0.25 | 0.86 | 0.33 |
| Unconditional | Mean (bp/quarter) | 13 | 13 | 13 |
| | Ann. IR | 0.75 | 0.75 | 0.75 |

*Source: Lehman Brothers, US Federal Reserve Flow of Funds, Bloomberg.*

## Impact of risk aversion on credit curves

It is also possible that credit curves are affected by changes in investor risk aversion. As risk aversion increases, we should expect near-dated spreads (such as the 5-year) to widen more than longer-dated spreads (such as the 10-year). This effect is based on the assumption that the shocks that caused the increased risk aversion should die down over time.

In Figure 11, we present the performance of the 10y-5y steepener trade in periods of rising and falling risk aversion as evidenced by the performance of risky assets such as equity markets over cash. As risk aversion rises, investors would begin to exit risky assets in favour of less risky ones, which would lead them to underperform.

**Figure 11. 10y-5y credit steepeners vs risk aversion (USD: 1990-2007)**

| | | 10y-5y Steepener |
|---|---|---|
| Rising risk aversion | Mean (bp/quarter) | 9 |
| | Ann. IR | 0.41 |
| Falling risk aversion | Mean (bp/quarter) | 11 |
| | Ann. IR | 0.68 |
| Unconditional | Mean (bp/quarter) | 13 |
| | Ann. IR | 0.75 |

*Source: Lehman Brothers, Bloomberg.*

## Sterling credit curves and the drivers of credit markets

It is also interesting to analyse the linkage between the performance of the sterling credit steepener trade (especially at the long end) and the drivers of the credit markets. In Figure 12a we present the excess returns of the 10y-5y and 20y-10y steepener trades over swaps in

[5] *Asset return volatility is defined as the deleveraged equity return volatility.*

periods of increased and decreased investor risk aversion. In periods of heightened risk aversion (which are also periods in which the credit markets underperform), we find that the sterling credit curve flattens in both the intermediate (10y-5y) segment and the long (20y-10y) segment.

**Figure 12a. Sterling credit steepeners over swaps vs risk aversion (1999-2007)**

| | | GBP 10y-5y | GBP 20y-10y |
|---|---|---|---|
| Increased risk aversion | Mean (bp/quarter) | -90 | -7 |
| | Ann. IR | -1.04 | -0.13 |
| Decreased risk aversion | Mean (bp/quarter) | 21 | 25 |
| | Ann. IR | 0.24 | 0.41 |
| Unconditional | Mean (bp/quarter) | -24 | 14 |
| | Ann. IR | -0.32 | 0.29 |

*Source: Lehman Brothers.*

In Figure 12b, we present a similar analysis in periods of outperforming and underperforming equity markets. The results are consistent.

**Figure 12b. Sterling credit steepeners over swaps vs risk aversion (1999-2007)**

| | | GBP 10y-5y | GBP 20y-10y |
|---|---|---|---|
| Outperforming equity markets | Mean (bp/quarter) | 54 | 39 |
| | Ann. IR | 0.85 | 0.62 |
| Underperforming equity markets | Mean (bp/quarter) | -112 | -23 |
| | Ann. IR | -1.30 | -0.54 |
| Unconditional | Mean (bp/quarter) | -24 | 14 |
| | Ann. IR | -0.32 | 0.29 |

*Source: Lehman Brothers.*

## 5. CONCLUSION

Based on a quantitative analysis of the behaviour of credit steepener trades over the economic cycle, we find that they have a positive alpha over long periods of time. The credit curve on average tends to steepen in bullish credit market conditions. This effect is much less pronounced in bearish markets. In fact, the long end of lower-rated credit curves tends to flatten when market conditions turn bearish.

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# Risk Attribution with Custom-defined Risk Factors[1]

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*The standard approach to risk attribution breaks down volatility in terms of its contributions from a given set of factors, such as key rates. However, practitioners often need to monitor their exposures in terms of custom factors that are combinations of the original factors, such as the PCA shift/slope/butterfly movements of the curve. We propose a generalized approach to this risk-attribution exercise.*

## 1. INTRODUCTION

Practitioners often rely on the factorization of the standard deviation to analyze the exposure of their portfolio to a given set of market risk factors (see Meucci (2005) for a comprehensive overview). In typical cases, the number of factors affecting the P&L is very large: practitioners need to organize their risk-attribution analysis according to a tree structure. Alternatively, they need the flexibility to define new factors as functions of the original ones and analyze risk in terms of these new factors. Here we propose a generalized solution to this type of problem.

In Section 2 we define the problem and its main statistics for the case where we are given a particular set of factors. In section 3 we extend the problem to arbitrary definitions of new risk factors as linear combinations of the original factors. Section 4 provides additional extensions of the general analysis and section 5 concludes.

## 2. RISK ATTRIBUTION WITH GIVEN FACTORS

Market practitioners closely monitor the risk in their portfolios. Monitoring risk means understanding the sources of risk in terms of a specified set of risk factors. This is the purpose of risk attribution. Denote by $\Pi$ the P&L of a portfolio of securities. In what follows, we consider factor models for the portfolio P&L of the following form:

$$\Pi = \sum_{m=1}^{M} L_m F_m = L'F \tag{1}$$

In this expression $F_m$ is the $m$-th risk factor and $L_m$ the corresponding portfolio loading, representing the portfolio manager's decision variable[2]. Formulation (1) is quite general. In particular, it includes as a special case standard APT-like linear pricing models. Indeed, it suffices to consider the last factor as a diversifiable idiosyncratic contribution that is independent of the remaining $M-1$ factors. Furthermore, it covers the non-linear theta-delta-gamma-vega P&L model routinely used on trading desks: in this case, the "gamma" factors are deterministic quadratic functions of the "delta" factors. Finally, it also encompasses the formulation $\Pi = \sum_{n=1}^{N} v_n R_n = V'R$, where $R$ represents the returns on a set of securities or asset classes and $V$ the market value of each of them. In this case, we can think of each security as being a specific risk factor.

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*[1] We would like to thank Anthony Lazanas, Gary Wang, Bob Durie and Diego Pontoriero for their comments.*

*[2] Indeed, $L_m$ is the weighted sum of the single-security loadings to the i-th risk factor $L_m = \sum_{n=1}^{N} v_n L_{nm}$, where $N$ is the number of securities and $v_n$ represents amount of security n in the portfolio. Note that without loss of generality, we can set the loadings to be net of a particular benchmark.*

The most widely used measure of risk in the industry is the standard deviation, also known as tracking error volatility (TEV) in the case of benchmark-driven allocations. In our case, it is a simple function of the market factors $F$ and of the allocation $L$:

$$TEV = STD(\Pi) = \sqrt{L'\Omega L}$$

where $\Omega$ denotes the covariance matrix of the risk factors. The standard deviation above has homogeneity of degree one in the loadings. The following well known results are a direct consequence of that. If we define the marginal contribution to TEV from risk factor $F_i$ as $MC_i = \partial TEV / \partial L_i$, then from Euler's homogenous function theorem:

$$TEV = \sum_{m=1}^{M} MC_m L_m$$

The total variance is not the weighted sum of the variance of the individual factors (except when the risk factors are uncorrelated – see example below). However, total volatility can now be naturally expressed as the sum of the contributions from each factor:

$$TEV = \sum_{m=1}^{M} C_m \qquad (2)$$

In this expression, the contribution from each factor is the product of the per-unit marginal contribution and the exposure of the portfolio to the factor, as represented by the respective loading:

$$C_m = MC_m L_m \qquad (3)$$

Furthermore, it is easy to check that the marginal contribution from factor *i* reads explicitly:

$$MC_m = \frac{\Omega_{m.} L}{TEV} \qquad (4)$$

where $\Omega_{m.}$ represents the *m*-th line of matrix $\Omega$[3].

### Example: Relating to POINT risk model report

In POINT[4], the risk of a particular portfolio is divided into three components: systematic (S), idiosyncratic (I) and default (D) risk. We can treat each of them as a factor $F$ in the framework described above. By construction, these three components are uncorrelated to each other. Therefore the factor covariance matrix is diagonal:

$$\Omega = \begin{bmatrix} \sigma_S^2 & 0 & 0 \\ 0 & \sigma_I^2 & 0 \\ 0 & 0 & \sigma_D^2 \end{bmatrix}$$

Given the above definition, an intuitive normalization is to set the portfolio loadings to all three factors equal to 1, such that:

$$TEV = \sigma = \sqrt{L'\Omega L} = \sqrt{\sigma_S^2 + \sigma_I^2 + \sigma_D^2}$$

In this case, the marginal contributions and the contributions are equal and read:

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[3] *More generally, we will represent $A_{i.}$ as the i-th line of matrix $A$.*

[4] *See Joneja, Dynkin et al (2005) for a detailed view of risk modeling in POINT.*

$$MC_i = C_i = \frac{\sigma_i^2}{TEV} \quad for \quad i = \{S, I, D\} \tag{5}$$

These concepts are illustrated in the following example, constructed in POINT. Consider the portfolio of the US issues in the USD investment grade credit index that have amounts outstanding larger than $1MM (as of March 27 2007). For simplicity, assume that we want to study its volatility (e.g. its benchmark is set to be cash) and that we hedge our portfolio from all sources of risk other than spreads. The following numbers are collected from the resulting POINT risk report:

**Figure 1. Details of the TEV (bp/month)**

| | |
|---|---|
| Total TEV (bp/month) | 49.6 |
| Systematic TEV | 48.4 |
| Idiosyncratic TEV | 10.4 |
| Default TEV | 3.0 |

*Source: Lehman Brothers, POINT.*

The numbers represent respectively $\sigma, \sigma_S, \sigma_I$, and $\sigma_D$. Using (5) we can now construct the contributions from these factors:

**Figure 2. Contribution to TEV from Major Components (bp/month)**

| **Contribution to TEV from:** | $C_i$ |
|---|---|
| Systematic Risk | 47.24 |
| Idiosyncratic Risk | 2.18 |
| Default Risk | 0.19 |
| Total | 49.60 |

*Source: Lehman Brothers, POINT.*

Note that the contributions sum to the total volatility of the portfolio as in (2).

## 3. RISK ATTRIBUTION WITH NEW FACTORS

Often the portfolio manager needs to measure his exposure to a set of $K \leq M$ new risk factors that are linear combinations of the original factors $F$. In formulas, we can represent the new factors in terms of a "pick" matrix $P$ with $K$ rows and $M$ columns:

$$\widetilde{F} \equiv PF \tag{6}$$

Each row of $P$ represents a linear combination of the original factors, and therefore it forms a specific "new" factor. How do we attribute risk to the new factors $\widetilde{F}$ ? In what follows we develop a general framework to think about this issue.

The new factors $\widetilde{F}$ drive the randomness in the P&L through some suitably defined factor loading vector $\widetilde{L}$. In general, if $K < M$, the new factors cannot account for all the risk in the portfolio. Therefore, there must exist some residual risk:

$$\Pi = \widetilde{L}'\widetilde{F} + \varepsilon \tag{7}$$

After defining $P$, we need to choose a suitable set of loadings $\tilde{L}$. Is there any natural choice for $\tilde{L}$ ? We propose one that satisfies two conditions. The first one is that the residual is minimized. The second is that the residual is uncorrelated with the new factors $\tilde{F}$. Note that these two conditions are satisfied once we use regression analysis. In fact, if we regress the portfolio P&L on the new factors $\tilde{F}$, the OLS estimate of $\tilde{L}$ is:

$$\tilde{L} = \left[\mathrm{cov}(\tilde{F})\right]^{-1} \mathrm{cov}(\tilde{F}, \Pi) \tag{8}$$

Or in terms of the primitives:

$$\tilde{L} = (P\Omega P')^{-1} P\Omega L \tag{9}$$

By definition these loadings are such that the residual risk $\varepsilon$ is not correlated with the new factors $\tilde{F}$. Moreover, the residual is minimal, in a least-square sense. We therefore achieved the proposed goals. With this definition of $\tilde{L}$, we can proceed in the same lines as (2) – (4). Specifically, we can define the contribution from each new factor $\tilde{F}_k$ as:

$$\tilde{C}_k = M\tilde{C}_k \tilde{L}_k \tag{10}$$

where it is easy to check that the per-unit contributions read:

$$M\tilde{C}_k = \frac{\tilde{\Omega}_{k.}\tilde{L}}{TEV} \tag{11}$$

with $\tilde{\Omega} = P\Omega P'$. In terms of the primitives the expression comes:

$$M\tilde{C}_k = P_{k.} MC \tag{12}$$

Where $MC$ is the vector of original marginal contributions with elements described in (4). Finally, note that contrary to (2), it may be the case that:

$$TEV \neq \sum_{k=1}^{K} \tilde{C}_k$$

because of the residual $\varepsilon$ in (7).

In what follows, we focus on several cases of particular interest. As we show, all of them can be analyzed using the framework outlined in (6) – (12). The cases allow us to convey some intuition on how to construct the $P$ matrix as well as how to read the corresponding results and perform risk attribution.

### Example (continued): Relating to the risk models in POINT

In Figure 1 we summarize the information regarding the systematic risk factors into a single number. However, this number comes, in our risk models, from a much richer framework[5]. If the portfolio described above in the example goes through POINT, we obtain a factor report that looks like Figure 3. In particular, note that the systematic spread volatility is explained by 34 risk factors. The contributions $C_i$ from all 34 factors sum up to the contribution of the systematic risk, 47.24 as per Figure 2. In the following sections we illustrate various ways of reorganizing this information to better suit the risk-attribution process.

[5] *See Rosten and Silva (2007) for a description of the full credit risk model available through POINT.*

**Figure 3. Details on the systematic risk factors**

| $M$ | Factor name $(F_i)$ | $L_i$ | $\sigma_{F_i}$ | $MC_i$ | $C_i$ |
|---|---|---|---|---|---|
| 1 | USD Ultra High Grade Industrials | 2.702 | 6.25 | 3.648 | 9.86 |
| 2 | USD Ultra High Grade Utilities | 0.229 | 7.96 | 3.576 | 0.82 |
| 3 | USD Ultra High Grade Financials | 3.012 | 6.74 | 4.354 | 13.11 |
| 4 | USD Ultra High Grade Non Corporate | 0.223 | 4.85 | 2.303 | 0.51 |
| 5 | USD IND Chemicals | 0.025 | 6.96 | 3.318 | 0.08 |
| 6 | USD IND Paper | 0.048 | 8.68 | 4.862 | 0.23 |
| 7 | USD IND Capital Goods | 0.04 | 10.45 | 5.311 | 0.21 |
| 8 | USD IND Div. Manufacturing | 0.01 | 9.85 | 4.954 | 0.05 |
| 9 | USD IND Auto | 0.064 | 12.09 | 6.646 | 0.43 |
| 10 | USD IND Consumer Cyclical | 0.26 | 13.95 | 6.363 | 1.65 |
| 11 | USD IND Retail | 0.142 | 8.21 | 4.355 | 0.62 |
| 12 | USD IND Cons. Non-cyclical | 0.082 | 7.62 | 3.723 | 0.31 |
| 13 | USD IND Pharmaceuticals | 0.087 | 7.97 | 3.146 | 0.27 |
| 14 | USD IND Energy | 0.161 | 8.04 | 4.172 | 0.67 |
| 15 | USD IND Technology | 0.031 | 11.27 | 6 | 0.19 |
| 16 | USD IND Media Cable | 0.189 | 13.01 | 6.58 | 1.24 |
| 17 | USD IND Media Non-cable | 0.105 | 8.87 | 5.023 | 0.53 |
| 18 | USD IND Wirelines | 0.364 | 10.95 | 5.267 | 1.92 |
| 19 | USD IND Wireless | 0.331 | 11.96 | 6.095 | 2.02 |
| 20 | USD UTI Electric | 0.136 | 11.97 | 3.525 | 0.48 |
| 21 | USD FIN Banking | 0.491 | 7.68 | 3.945 | 1.94 |
| 22 | USD FIN Brokerage | 0.515 | 12.04 | 6.433 | 3.31 |
| 23 | USD FIN Finance Companies | 0.201 | 10.09 | 5.3 | 1.06 |
| 24 | USD FIN Life and Health Insurance | 0.058 | 10.64 | 5.282 | 0.31 |
| 25 | USD FIN P&C Insurance | 0.001 | 7.89 | 4.489 | 0.01 |
| 26 | USD Non Corporate | 0.015 | 14.09 | 5.53 | 0.08 |
| 27 | USD Credit IG subordinated | 0.642 | 3.62 | 0.346 | 0.22 |
| 28 | USD IND Short Maturity | -0.639 | 1.65 | 0.178 | -0.11 |
| 29 | USD UTI Short Maturity | -0.087 | 2.23 | 0.23 | -0.02 |
| 30 | USD FIN Short Maturity | -1.016 | 2.95 | -0.114 | 0.12 |
| 31 | USD IND Long Maturity | 12.017 | 1 | 0.253 | 3.05 |
| 32 | USD UTI Long Maturity | 1.249 | 1.32 | 0.223 | 0.28 |
| 33 | USD FIN Long Maturity | 9.844 | 1.12 | 0.154 | 1.52 |
| 34 | USD NONCRP Long Maturity | 1.697 | 1.04 | 0.165 | 0.28 |

*Source: Lehman Brothers, POINT*

### Analysis with M new factors ( $K = M$ )

We start by considering the case where the investor defines a full set of $K = M$ non trivial new factors. In this case, the vector of new loadings $\tilde{L}$ is *M*-dimensional. Moreover, if the new factors are non-trivial the “pick” matrix $P$ is invertible. Using this fact and definition (9), we can write $\tilde{L}$ as:

$$\begin{aligned}&\tilde{L} = (P\Omega P')^{-1} P\Omega L = \\ &= (P\Omega P')^{-1} P\Omega P' P'^{-1} L = \\ &P'^{-1} L\end{aligned} \tag{13}$$

Therefore the new factor loadings are unequivocally defined as $\tilde{L} = P'^{-1} L$. We can rewrite (1) as:

$$\Pi = L'F = L'P^{-1}PF = \tilde{L}'\tilde{F}$$

Substituting this expression into (7), we obtain:

$$\varepsilon = 0$$

This is not surprising: with a full set of $K = M$ new factors, the P&L is fully described by the new factors. We can represent the contributions and marginal contributions as in (10) and (11). In this case the residual is driven to zero, and so, as in (2):

$$TEV = \sum_{k=1}^{K} \tilde{C}_k$$

Therefore the new factors are able to fully describe the volatility of the portfolio.

### Analysis with K new factors ( $K < M$ )

Now, suppose instead one is interested in computing the contributions from $K < M$ new factors. This is generally the case of interest. We can proceed using two distinct approaches. In the first, the new factors still fully describe the P&L of the report as we are able to eliminate the residual $\varepsilon$. As we show below, this is possible only with strong restrictions on the matrix $P$. In particular, the matrix is portfolio dependent.

In the second approach, we regain the control over $P$ (namely, $P$ does not depend on the portfolio), but at the expense of generating a residual. Both approaches are useful and deliver different intuitions regarding a particular portfolio or set of risk factors.

### Full risk attribution

Suppose one is interested in computing the contributions from $K < M$ exhaustive sets of the original risk factors. For simplicity, let us assume for now that these $K$ sets are also mutually exclusive. We can get this by constructing $K$ buckets $\{M_1, M_2, ..., M_k, ..., M_K\}$ that represent a non-overlapping partition of the initial set of $M$ risk factors.

We can think of each set $k$ as representing a new risk factor. It seems natural to define the contribution to risk $C_k$ from the generic $k$-th bucket as the sum of the individual contributions from each factor in the bucket:

$$C_k = \sum_{i \in M_k} C_i \tag{14}$$

That is re-write (2) as:

$$TEV = \sum_{k=1}^{K} \left( \sum_{i \in M_k} C_i \right) = \sum_{k=1}^{K} C_k \tag{15}$$

This result follows from a particular choice of the $K \times M$ matrix $P$[6]:

$$P_{ki} = \begin{cases} L_i & if \quad i \in M_k \\ 0 & otherwise \end{cases} \tag{16}$$

What is the corresponding set of loadings $\tilde{L}$ ? Note that by definition of $P$, the initial vector of loadings can be represented by:

$$L' = 1_K' P \tag{17}$$

Where $1_K$ as a $K$-dimensional vector of ones. Using (17) and (6) we can rewrite (1) as:

$$\Pi = L'F = 1_K' PF = 1_K' \tilde{F}$$

So the natural choice for $\tilde{L}$ is:

$$\tilde{L} = 1_K$$

So that:

$$\Pi = \tilde{L}' \tilde{F} \tag{18}$$

The P&L, originally driven by $M$ sources of risk, is now fully described by $K < M$ new factors (see (18)). Therefore we successfully drove the residual $\varepsilon$ to zero. In particular, because $\tilde{L}_k = 1, \forall_k$, the following is true:

$$\Pi = \sum_{k=1}^{K} \tilde{L}_k \tilde{F}_k = \sum_{k=1}^{K} \tilde{F}_k$$

Moreover:

$$M\tilde{C}_k = \tilde{C}_k$$

whereas in (14) (using (12) and (16)):

$$\tilde{C}_k = M\tilde{C}_k = P_{k.} MC = \sum_{i \in M_k} L_i MC_i = \sum_{i \in M_k} L_i MC_i = \sum_{i \in M_k} C_i$$

### Generalization

How can we set up this problem using the general framework developed in section 0? We do that by extending $P$ with a set $M - K$ residual factors. Define that extended matrix as:

$$Q = \begin{bmatrix} P \\ S \end{bmatrix} \tag{19}$$

Where $S$ is a $(M - K) \times M$ matrix of residual factors. We need only very mild conditions on $S$ to be able to reconcile the general methodology laid down in section 0 with the analysis presented before in this section. Specifically, we need to make sure that the new matrix $Q$ is invertible. If this is the case, the vector of new loadings is defined as (see (13)):

---

[6] *We can generalize this approach by redefining* $P_{ki} = \alpha_{ki} L_i$ *where* $\alpha_{ki}$ *are any numbers such that* $\sum_{k=1}^{K} \alpha_{ki} = 1$ *for all i. This condition ensures that the new factors fully capture the risk in the portfolio, as in (15).*

$$\tilde{L}_Q = Q'^{-1} L \tag{20}$$

As expected this vector has a very specific structure, given the definition of $P$ in (16). In particular (see Appendix, proof #1):

$$\tilde{L}_Q' = \begin{bmatrix} 1_K' & 0_{M-K}' \end{bmatrix}, \tag{21}$$

where $0_{M-K}$ is a $M-K$ vector of zeros. Therefore, we are able to retain all results developed previously in this section. In particular,

$$TEV = \sum_{k=1}^{K} \tilde{C}_k = \sum_{m=1}^{M} \tilde{C}_m$$

As by (10) and (21) all loadings and therefore all contributions are zero for the $M-K$ residual factors:

$$\tilde{C}_m = 0 \quad \forall_{m>K}$$

### Example (cont'd): Full risk attribution

Suppose we want to analyse the risk of our portfolio – as described in Figure 3 – using four new risk factors: industrials, utilities, financials and non-corporates. Specifically, suppose we define the new factors as:

| | |
|---|---|
| Industrials | Factors 1, 5-19, 27, 28 and 31 in Figure 3 |
| Utilities | Factors 2, 20, 29 and 32 in Figure 3 |
| Financials | Factors 3, 21-25, 30 and 33 in Figure 3 |
| Non-corporates | Factors 4, 26 and 34 in Figure 3 |

We can do that with the methodology described in this section. We start by using (16) and the loading information $L_i$ in Figure 3 to define a $4 \times 34$ matrix $P$:

$$P = \begin{bmatrix} 2.702 & 0 & 0 & 0 & 0.025 & 0.048 & \ldots & 0 \\ 0 & 0.229 & 0 & 0 & 0 & 0 & \ldots & 0 \\ 0 & 0 & 3.012 & 0 & 0 & 0 & \ldots & 0 \\ 0 & 0 & 0 & 0.223 & 0 & 0 & \ldots & 1.697 \end{bmatrix}$$

With this definition and setting $\tilde{L} = [1 \quad 1 \quad 1 \quad 1]'$, we can get $\tilde{\Omega}$ and $\tilde{C}_k$ as above. In our case:

$$C = \begin{bmatrix} 23.42 \\ 1.56 \\ 21.37 \\ 0.88 \end{bmatrix} \begin{matrix} \rightarrow & \text{Industrials} \\ \rightarrow & \text{Utilities} \\ \rightarrow & \text{Financials} \\ \rightarrow & \text{Non - corporates} \end{matrix}$$

So, we are able to attribute risk to the 4 new risk factors. Moreover, the sum of these contributions adds up to the contribution to the overall risk by the systematic component (see (15)). It is now much clearer than in Figure 3 that the bulk of risk comes from the exposure to industrials and financials. We could use the general approach to conduct this same analysis (see Appendix, Example #1).

*Partial risk attribution*

In the case above, the structure imposed on the $P$ matrix allowed us fully to describe the P&L with the new set of factors and loadings (see (18)). The new factors were constructed as a specific weighted average of the initial factors using the portfolio loadings as weights. Hence the new factors are portfolio-specific. Often we need more flexibility in defining the new factors, that is, in forming the "pick" matrix $P$. In this case, the new factors do not generally explain all the risk of the portfolio. Therefore, there is some residual risk (see (7)):

$$\Pi = \tilde{L}'\tilde{F} + \varepsilon$$

The flexibility we gain in determining a matrix $P$ that is suitable to our purposes comes with a cost, the residual risk $\varepsilon$. As discussed, we do have a natural choice for $\tilde{L}$, provided by regression analysis (see (9)):

$$\tilde{L} = (P\Omega P')^{-1} P\Omega L$$

By definition these loadings are such that the residual $\varepsilon$ is not correlated with the new factors $\tilde{F}$. Moreover, the r-square:

$$R^2 = 1 - \mathrm{var}(\varepsilon)/\mathrm{var}(\Pi)$$

attains a maximum with (9). Therefore, the regression coefficients split the risk into two separate components, namely the risk due to the new factors $\tilde{F}$, which accounts for the r-square, and the risk due to the residual $\varepsilon$:

$$\mathrm{var}(\Pi) \quad = \quad [\underbrace{\mathrm{var}(\tilde{L}'\tilde{F})}_{R^2\,\mathrm{var}(\Pi)} \quad + \quad \underbrace{\mathrm{var}(\varepsilon)}_{(1-R^2)\,\mathrm{var}(\Pi)}\ ] \tag{22}$$

Now the sum of the contributions from the new factors $\tilde{F}$ does not add up to the total volatility of the P&L (see (7)). Instead:

$$\sum_{k=1}^{K}\tilde{C}_k = \sum_{k=1}^{K} M\tilde{C}_k\tilde{L}_k = \sum_{k=1}^{K}\frac{\tilde{\Omega}_{k.}\tilde{L}}{TEV}\tilde{L}_k = \frac{\mathrm{var}(\tilde{L}'\tilde{F})}{TEV} = R^2 TEV$$

So, the (normalized) sum of the contributions from the new factors $\tilde{F}$ assumes a very intuitive interpretation. Indeed their contributions to risk sum up to the r-square of the fit:

$$\frac{\sum_{k=1}^{K}\tilde{C}_k}{TEV} = R^2 \tag{23}$$

Therefore, the risk-attribution formula (10) provides the contribution of each new factor to total risk and simultaneously provides the contribution to the r-square.

### Generalization

We now focus on how to set up this problem using again the general framework developed in section 0. The solution comes again by extending $P$ with a set $M-K$ residual factors. Once again we define the extended matrix as:

$$Q = \begin{bmatrix} P \\ S \end{bmatrix}$$

Where $S$ is a $(M-K)\times M$ matrix of residual factors. Because we want to keep the freedom to define $P$, the new set of loadings $\tilde{L}_Q = Q'^{-1}L$ will not obey (21):

$$\tilde{L}_Q' \neq \begin{bmatrix} 1'_K & 0'_{M-K} \end{bmatrix}.$$

Therefore, the contributions from the residual factors are no longer zero. Instead:

$$TEV = \sum_{k=1}^{K} \tilde{C}_k + \sum_{m=K+1}^{M} \tilde{C}_m = \tilde{C}^K + \tilde{C}^{M-K}$$

To make things worse, different choices of $S$ generally deliver different values for $\tilde{C}^{M-K}$ and $\tilde{C}^K$. This is a very undesirable result, as the residual factors have no specific interpretation. But let us remind ourselves of the principles we established on the natural choice for $\tilde{L}$. In particular, we want (i) to maximize $\tilde{C}^K$ (e.g. allow the maximum explanatory power to the factors we are interested on) and (ii) make $\tilde{C}^K$ independent of the $S$ chosen (e.g. the contributions from the factors we are interested in does not depend on the definition of the residual factors).

As we derived above, the first goal is accomplished by defining the loadings over the $K$ factors of interest as in (9). The second is achieved by a judicious choice of $S$ – a choice that imposes only a minor restriction in the selection of residual factors[7]. It can be expressed as an orthogonality constraint:

$$S = null(P\Omega)' \tag{24}$$

Where $null(A)$ represents the null space of matrix $A$. To see how these conditions helps us, let's start by defining:

$$\tilde{L}_Q' = \begin{bmatrix} \tilde{L}_K' & \tilde{L}'_{M-K} \end{bmatrix}$$

$$\tilde{F}_Q = QF = \begin{bmatrix} PF \\ SF \end{bmatrix} = \begin{bmatrix} \tilde{F} \\ \tilde{F}_{M-K} \end{bmatrix}$$

Note that the first set of loadings and factors ($\tilde{L}_K$ and $\tilde{F}$) are the ones we do care about. The others are residual factors and, as shown below, have no particular interpretation. With these definitions we can represent the variance of the P&L of the portfolio as:

$$\begin{aligned} \mathrm{var}(\Pi) &= \mathrm{var}(\tilde{L}_Q' \tilde{F}_Q) = \\ &= \mathrm{var}(\tilde{L}'_K \tilde{F}) + \mathrm{var}(\tilde{L}'_{M-K} \tilde{F}_{M-K}) + \mathrm{cov}(\tilde{L}'_K \tilde{F}, \tilde{L}'_{M-K} \tilde{F}_{M-K}) \end{aligned}$$

We show in the appendix (proof #2) that under (24), $\mathrm{cov}(\tilde{L}'_K \tilde{F}, \tilde{L}'_{M-K} \tilde{F}_{M-K}) = 0$, therefore:

$$\mathrm{var}(\Pi) = \mathrm{var}(\tilde{L}'_K \tilde{F}) + \mathrm{var}(\tilde{L}'_{M-K} \tilde{F}_{M-K})$$

Moreover, we show in the appendix (proof #3) that $\tilde{L}_K = \tilde{L}$, so:

$$\mathrm{var}(\Pi) = \mathrm{var}(\tilde{L}' \tilde{F}) + \mathrm{var}(\tilde{L}'_{M-K} \tilde{F}_{M-K})$$

---

[7] *This restriction is not important as we do not give any particular interpretation to the residual factors. Their only role is to help us generalize the results from the several cases we discuss in this paper into a single framework.*

Using (22), we get that:

$$\operatorname{var}(\varepsilon) = \operatorname{var}(\tilde{L}'_{M-K}\tilde{F}_{M-K})$$

So, the residual factors are the residual and are orthogonal to the factors of interest $\tilde{F}$ . With (24) in place, all the analysis in (22)-(23) follows, namely:

$$\tilde{C}^{K} = \sum_{k=1}^{K}\tilde{C}_{k} = \tilde{L}\tilde{\Omega}\tilde{L} / TEV = R^2 TEV \tag{25}$$

$$\begin{aligned}\tilde{C}^{M-K} &= \sum_{m=K+1}^{M}\tilde{C}_{m} = \tilde{L}_{M-K} S\Omega S' \tilde{L}_{M-K} / TEV = \\ &= \operatorname{var}(\varepsilon)/TEV = (1-R^2)TEV\end{aligned} \tag{26}$$

### Example (cont'd): Partial risk attribution

The partition imposed in (16) is rather restrictive. In particular, note that by using the portfolio loadings, the risk attribution is portfolio-specific. We may want instead to define generic factors that are not specific to a particular portfolio. For instance, suppose we want to aggregate the risk factors in Figure 3 by factor-type: Ultra-High-Grade (1 to 4), DTS Industry factors (5 to 26) and maturity (28 to 34). In particular, note that we are ignoring factor 27, e.g., we want it to be added to the residual. To that end, we can define generically $P$ as:

$$P = \begin{bmatrix} 1 & 1 & 1 & 1 & 0 & 0 & \dots & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & \dots & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \dots & 1 \end{bmatrix} \tag{27}$$

Because we are not using any portfolio-specific information to define these new factors, we can use this set of new factors for any portfolio to be analyzed. However, by defining the matrix $P$ this way, we are creating a residual into the risk attribution analysis. In our example, the procedure developed in this section with $P$ defined as above delivers:

$$C = \begin{bmatrix} 16.74 \\ 20.63 \\ 1.30 \\ 8.58 \end{bmatrix} \begin{matrix} \rightarrow & \text{Ultra High Grade} \\ \rightarrow & \text{DTS factors} \\ \rightarrow & \text{Maturity} \\ \rightarrow & \text{Residual} \end{matrix} \tag{28}$$

And so the $R^2$ in our case is around 80%. Why is this number not close to 100%? Note that in (27) we are basically defining a new ultra high grade factor that is an equal sum of all the four ultra high grade factors. Our portfolio has almost no exposure to two of them (utilities and non-corporates, see Figure 3), so the equal sum will not be a faithful description of this particular portfolio. The same is true for the maturity factor. As one can see in Figure 3, the loadings to the short and long factors are very different. We may be imposing too much structure by joining them together under one single new factor "maturity" (more on this later on). This particular example shows us the trade-off we referred to above. To be able to define a generic partition that can be applied to different portfolios, we may lose some ability to attribute risk in a particular portfolio.

Note that we can use the extended matrix $Q$ to achieve the same goal (see appendix, example #2).

## 4. EXTENSIONS

### Defining the residual in terms of the original factors $F$

One question regards the interpretation of the risk due to the residual $\varepsilon$ . If the choice of the user-defined new factors $\tilde{F}$ is not the most suitable to describe the exposures of the portfolio, the residual can be large. In this situation, the portfolio manager needs to determine the nature of the residual, typically in terms of the original factors. Again, we can fully express the residual as a combination of the original factors:

$$\begin{aligned}\varepsilon &= \Pi - \tilde{L}'\tilde{F} = L'F - \tilde{L}'PF = \\ &= (L' - \tilde{L}'P)F = L^{\varepsilon}F = \\ &= \sum_{m=1}^{M} L_m^{\varepsilon} F_m\end{aligned}$$

where:

$$L^{\varepsilon} \equiv L - P'\tilde{L}$$

The risk of the portfolio splits into two distinct components as in (22): a new-factor component and a residual component. The risk-attribution process further splits the new-factor component into separate terms due to each of the new factors:

$$\mathrm{var}(\Pi) \quad = \quad [\underbrace{\sum_{k=1}^{K} \tilde{C}_k}_{R^2\,\mathrm{var}(\Pi)} \quad + \quad \underbrace{\sum_{m=1}^{M} C_m^{\varepsilon}}_{(1-R^2)\,\mathrm{var}(\Pi)}]$$

To compute in practice the contributions

$$\tilde{C}_k = M\tilde{C}_k \tilde{L}_k$$

$$C_m^{\varepsilon} = MC_m^{\varepsilon} L_m^{\varepsilon}$$

With $M\tilde{C}_k$ defined as in (11) and $MC_m^{\varepsilon} = \Omega_{m.} L^{\varepsilon} / TEV$

### Example (cont'd): Checking the residual

If we calculate $C^{\varepsilon}$ on the example we have been following, we can understand better where the residual 8.58 (see (28)) is coming from. In other words, we can understand in what dimensions the new factors mis-represent the old ones. Taking $P$ as in (27), the following are the major elements in $C^{\varepsilon}$ :

$$\begin{bmatrix} C_{30}^{\varepsilon} \\ C_{31}^{\varepsilon} \\ C_{33}^{\varepsilon} \end{bmatrix} = \begin{bmatrix} 1.25 \\ 3.13 \\ 2.33 \end{bmatrix}$$

Two conclusions follow: first, note that these three factors represent the bulk of the residual (6.71 out of the 8.58). If we could correct this situation, the residual would be drastically reduced. Second, all these three original factors are summarized by our new factor "maturity". This suggests that we only need to change the definition of this factor. A closer look at Figure 3 highlights the problem. The nature of the loadings on the short and long factors (#28 to #34) is quite different. By pooling them into a single new factor, we are blurring their individual contribution. With this diagnosis, we can proceed in various directions. For instance, we could separate the factors into a short and a long maturity. In this case, we would have 4 new factors, instead of the current 3. Suppose we want to keep the number of new factors to 3. Then, given the evidence in Figure 3, it seems more promising to drop the short factors from the new factor "maturity". This is because the extreme of the loadings is coming from the long maturity factors. If we do this, by redefining the third row of $P$ in (27) accordingly (zero, except for factors 31 to 34), the contributions from the three factors plus residual come as:

$$C = \begin{bmatrix} 19.77 \\ 19.85 \\ 4.28 \\ 3.34 \end{bmatrix} \begin{matrix} \rightarrow & \text{Ultra High Grade} \\ \rightarrow & \text{DTS factors} \\ \rightarrow & \text{LONG Maturity} \\ \rightarrow & \text{Residual} \end{matrix} \tag{29}$$

Comparing with (28), we see that this redefinition contributed substantially to reduce the residual, now at 3.34. The $R^2$ of the current set of factors is now at 91%, a substantial increase from the previous 80%.

### Adding idiosyncratic risk to the analysis

As referred above, the formulation in (1) is quite general. One specific practical application is to define the last "factor" as idiosyncratic:

$$\Pi = \sum_{i=1}^{M} L_i F_i = \sum_{i=1}^{M-1} L_i F_i + \nu$$

Here $\nu \equiv L_M F_M$ is independent of any of the other factors $\{F_1, F_2, ..., F_{M-1}\}$. In this case, all the previous analysis goes through, but we need to re-interpret some of the results.

Define $I_M$ as the $M \times M$ identity matrix. Moreover, let the "pick" matrix $P$ be a truncation of $I_M$. In particular let $P$ be the first $M-1$ rows of $I_M$, $P^T = I_{1,2,...,M-1;1,2,...,M}$. Finally, define the (truncated (T)) loadings' vector as $L^T = L_{1,2,...,M-1}$ and $F^T = P^T F$, then:

$$\text{var}(\Pi) = \text{var}(L^T{}' F^T) + \text{var}(\nu)$$

Note that as previously:

$$\frac{\sum_{k=1}^{M-1} C_k^T}{TEV} = R^2$$

Where $R^2$ has the standard goodness-of-fit interpretation. If we define a new set of factors and loadings as in (6) and (9) with respect to the truncated series of factors $F^T$ and loadings $L^T$, then:

$$\begin{aligned} \text{var}(\Pi) &= \sigma_{SYS}^2 + \sigma_{IDIO}^2 = \text{var}(\tilde{L}^T{}' \tilde{F}^T) + \text{var}(\varepsilon) + \text{var}(\nu) = \\ &= \text{var}(\tilde{L}^T{}' \tilde{F}^T) + \text{var}(\omega) \end{aligned}$$

Where $\omega = \varepsilon + \nu$. Then in the spirit of (22), if we define $r^2 = \text{var}(\tilde{L}^T{}' \tilde{F}^T) / \text{var}(\Pi)$:

$$\mathrm{var}(\Pi) \quad = \quad [\underbrace{\mathrm{var}(\tilde{L}^T{}'\tilde{F}^T)}_{r^2\,\mathrm{var}(\Pi)} \quad + \quad \underbrace{\mathrm{var}(\omega)}_{(1-r^2)\,\mathrm{var}(\Pi)}\ ]$$

and the normalized sum of factor contributions is now:

$$\frac{\sum_{k=1}^{K} \tilde{C}_k^T}{TEV} = r^2$$

So the r-square should now be interpretable against $\mathrm{var}(\Pi)$, not $\mathrm{var}(L^T{}'F^T)$ as before.

### Example (cont'd): Adding idiosyncratic risk to the analysis

Recall that up to now we focus on the contribution to systematic risk only. To extend the approach to the full volatility of our portfolio, we need to add the volatility that is coming from idiosyncratic and default risk (see Figure 1 and Figure 2). We can do this using the definitions of $R^2$ and $r^2$. In particular, note that:

$$R^2 = \frac{\sum_{k=1}^{3} \tilde{C}_k}{std(L^T F^T)} = \frac{43.9}{48.4} = 0.91$$

$$r^2 = \frac{\sum_{k=1}^{3} \tilde{C}_k}{std(\Pi)} = \frac{43.9}{49.6} = 0.89$$

Where we use the numbers from (29) (the sum of the contributions from the 3 new factors) and Figure 1.

## 5. CONCLUSIONS

We propose a general framework to attribute risk to arbitrary user-defined combinations of factors in a given market. In this approach the portfolio P&L is represented as a function of the user-defined factors plus a residual. The loadings on the factors are defined as the standard regression coefficients. This way, the residual is minimal and uncorrelated with the user-defined factors. As intuition suggests, the sum of the contributions to total risk from the user-defined factors represents the r-square of the regression of the factors on the portfolio P&L. We also show that the residual can be broken down into contributions from the original factors allowing us to characterize the part of the portfolio risk (as described by the original factors) that the new user-defined factors fail to capture.

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## APPENDIX

### PROOF #1

Starting with (20):

$$\tilde{L} = Q'^{-1} L$$

We can rewrite it as:

$$Q'\tilde{L} = L$$

Using the definition of $Q$ given by (19):

$$\begin{bmatrix} P' & S' \end{bmatrix}\tilde{L} = L$$

We also know from (17) that:

$$L = P'1_K$$

If we define $\tilde{L}' = \begin{bmatrix} \tilde{L}'_K & \tilde{L}'_{M-K} \end{bmatrix}$, then:

$$P'\tilde{L}'_K + S'\tilde{L}'_{M-K} = P'1_K$$

or

$$P'(\tilde{L}'_K - 1_K) + S'\tilde{L}'_{M-K} = 0$$

We can use again (19) to get:

$$Q'\begin{bmatrix} \tilde{L}'_K - 1_K \\ \tilde{L}'_{M-K} \end{bmatrix} = 0$$

If $Q$ is invertible then:

$$\begin{bmatrix} \tilde{L}'_K - 1 \\ \tilde{L}'_{M-K} \end{bmatrix} = 0 \Leftrightarrow \begin{bmatrix} \tilde{L}'_K \\ \tilde{L}'_{M-K} \end{bmatrix} = \begin{bmatrix} 1_K \\ 0_{M-K} \end{bmatrix}$$

Or

$$\tilde{L}' = \begin{bmatrix} 1_K & 0_{M-K} \end{bmatrix}$$

As in (21). ■

### PROOF #2

Start by defining:

$$\tilde{L}_Q' = \begin{bmatrix} \tilde{L}_K' & \tilde{L}'_{M-K} \end{bmatrix}$$

$$\tilde{F}_Q = QF = \begin{bmatrix} PF \\ SF \end{bmatrix} = \begin{bmatrix} \tilde{F} \\ \tilde{F}_{M-K} \end{bmatrix}$$

Therefore:

$$\begin{aligned} TEV^2 &= \mathrm{var}(\tilde{L}_Q'\tilde{F}_Q) = \mathrm{var}(\tilde{L}_K\tilde{F}) + \mathrm{var}(\tilde{L}'_{M-K}\tilde{F}_{M-K}) + \\ &+ 2\,\mathrm{cov}(\tilde{L}_K\tilde{F}, \tilde{L}'_{M-K}\tilde{F}_{M-K}) \end{aligned}$$

Our goal is to make the two set of factors uncorrelated, so that we can interpret the second set as the OLS residual. To that end, we need that:

$$\mathrm{cov}(\tilde{F}_K, \tilde{F}_{M-K}) = 0 \tag{A.1}$$

or

$$\mathrm{cov}(PF, SF) = P\,\mathrm{cov}(F, F)S' = P\Omega S' = 0$$

By definition of the null space of a matrix, this happens if:

$$S = null(P\Omega)'$$

### PROOF #3

Let's start with the definition given by (8):

$$\tilde{L} = (\tilde{F}\tilde{F}')^{-1}\tilde{F}\Pi$$

We can rewrite it as:

$$(\tilde{F}\tilde{F}')\tilde{L} = \tilde{F}\Pi$$

If we use (A.1), we can re-write the above expression as:

$$\begin{bmatrix} \tilde{F}_K'\tilde{F}_K & 0 \\ 0 & \tilde{F}_{M-K}'\tilde{F}_{M-K} \end{bmatrix} \begin{bmatrix} \tilde{L}_K \\ \tilde{L}_{M-K} \end{bmatrix} = \begin{bmatrix} \tilde{F}_K\Pi \\ \tilde{F}_{M-K}\Pi \end{bmatrix}$$

And in particular:

$$\left(\tilde{F}_K'\tilde{F}_K\right)\tilde{L}_K = \tilde{F}_K\Pi$$

Or:

$$\widetilde{L}_K = \left(\widetilde{F}_K{}'\widetilde{F}_K\right)^{-1}\widetilde{F}_K\Pi = \left(P\Omega P'\right)^{-1}P\Omega L$$

From (9) we can therefore establish that:

$$\widetilde{L}_K = \widetilde{L}$$

**EXAMPLE** #1

In particular, we can define $Q$ as:

$$Q = \begin{bmatrix} 2.702 & 0 & 0 & 0 & 0.025 & 0.048 & \ldots & 0 \\ 0 & 0.229 & 0 & 0 & 0 & 0 & \ldots & 0 \\ 0 & 0 & 3.012 & 0 & 0 & 0 & \ldots & 0 \\ 0 & 0 & 0 & 0.223 & 0 & 0 & \ldots & 1.697 \\ - & - & - & - & - & - & - & - \\ & & & & S_{30,34} & & & \end{bmatrix}$$

Where $S_{30,34}$ is any $30 \times 34$ matrix so that $Q$ is invertible. In this case, $\widetilde{L} = Q'^{-1} L = \begin{bmatrix} 1'_4 & 0'_{30} \end{bmatrix}$ and:

$$C = \begin{bmatrix} 23.42 & 1.56 & 21.37 & 0.88 & 0 & \ldots & 0 \end{bmatrix}'$$

**EXAMPLE** #2

In particular, let's define $\widetilde{L}_Q = Q'^{-1} L$ and $\widetilde{F}_Q = QF$, where:

$$Q = \begin{bmatrix} P \\ null(P\Omega)' \end{bmatrix}$$

and $P$ is defined as in (27). In this case, the vector of contributions $C$ has a dimension of 34. In particular, the vector comes as:

$$C_Q = \begin{bmatrix} 16.74 & 20.63 & 1.30 & -0.11 & 0.18 & \ldots & 0.15 \end{bmatrix}'$$

As expected from (25) and (26), the first 3 elements of $C_Q$ corresponds to the first 3 of $C$ in (28). The other 31 elements of $C_Q$ sum up to 8.58, the fourth element of $C$ in (28). Remember these last 31 factors are artificially created. They do not convey individually any specific intuition or information. What is relevant is to understand that, as a whole, these residual factors do represent $\varepsilon$.

# A Note on Pricing LCDX Swaptions

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*With the launch of LCDX investors have a new way to gain exposure to the loan market. We also expect demand for LCDX swaptions, which allow investors to leverage their view on the loan market and gain exposure to loan volatility. LCDX differs from other CDX products by incorporating a cancellation feature. We explain how this impacts the pricing of swaptions.*

## 1. INTRODUCTION

An LCDX swap is economically equivalent to an equally weighted portfolio of 100 single-name loan CDS (LCDS) with the same coupon. An LCDX swap trades on a price basis exactly as a CDX.HY swap: If a swap is quoted at price P, the protection buyer must pay the protection seller (100-P) points upfront. The upfront payment is in addition to the running coupon which is paid quarterly on 20 March, June, September and December. See Laberge *et al.* (2007) for an overview of the LCDX index, constituents and swap mechanics.

The difference between LCDX and CDX swaps lies in the difference between CDS and LCDS. LCDS has a cancellation clause which causes LCDS to terminate (without any settlement payment) if the reference entity repays all deliverable obligations without issuing new ones (usually only syndicated secured first lien debt would be deliverable for an LCDS). This is in contrast to standard CDS which do not terminate even if the reference entity has no outstanding debt. If one of the LCDX reference entities experiences a cancellation event, that entity will be removed from the LCDX portfolio, existing swaps will no longer reference that entity, and swap notionals will be reduced accordingly. In this sense, a cancellation event is similar to a credit event with a 100% recovery.

We look at the pricing of LCDX swaptions which are options to buy or sell protection through an LCDX swap at a future date at a fixed strike price. Swaptions on high yield and high grade CDX are traded liquidly, and we expect that an options market will also develop for LCDX as swap trading volume continues to increase. In the following sections, we analyze the implications of cancellation for the valuation of LCDX swaptions and explain the relationship to the model used for CDX.HY swaptions.

## 2. VALUATION OF A FORWARD STARTING LCDX SWAP

Before thinking about the pricing of swaptions, we need to be able to value a forward contract. One can create a synthetic forward by simultaneously buying a payer swaption and selling a receiver swaption with the same maturity and strike. Any option-pricing model should generate prices that are in line with the forward price, otherwise there would be an arbitrage. In other words, the following put-call parity for the value of a payer swaption, a receiver swaption and a forward must be satisfied:

$$Payer(K,T) - Receiver(K,T) = Forward(K,T) \tag{1}$$

$K$ is the strike price and $T$ is the option maturity. Forward(K,T) is the value of a non-knockout forward starting swap for buying protection at time $T$ at a price $K$.

### Ignoring cancellation

In deriving the value of the forward, we initially ignore the cancellation feature. The effect from cancellation will be incorporated later. Our derivation is similar to the one found in Liu and Pedersen (2005).

If the current swap price is $P$, then a protection buyer must pay (100-$P$) upfront in addition to paying a running coupon at a rate denoted C. In exchange, the protection buyer receives protection payments if there are defaults before the swap terminates. This means that the swap price should satisfy the following equation:

$$100\text{-}P = \text{ProtectionLeg}(0, T_M) - \text{PremiumLeg}(0, T_M) \tag{2}$$

where time 0 is today's date, $T_M$ is the swap maturity date and the notation on the right-hand side shows that we are evaluating the present value of protection and premium legs from today to swap maturity.

Next we would like to value a non-knockout forward with maturity $T$ to buy protection at price $K$. For the forward, the premium payments will not begin until $T$, but the protection buyer will still receive protection payments for defaults occuring between today and $T$ as the contract is non-knockout. At time $T$, the protection buyer will pay an amount determined by the set price of the contract, $K$. The present value of such a contract is:

$$Forward(K,T) = \text{ProtectionLeg}(0, T_M) - \text{PremiumLeg}(T, T_M) - (100\text{-}K) \cdot \text{D}(0, T) \tag{3}$$

where D(0, $T$) is the Libor discount factor to time $T$. Different from the spot contract value in equation (2), the premium leg in the forward contract value covers the interval from time $T$ to swap maturity. We can add and subtract the value of the premium leg from today to time $T$ in the right-hand side of equation (3) in order to obtain the relationship between the value of the forward and the current swap price:

$$Forward(K,T) = (100\text{-}P) + \text{Premium Leg}(0, T) - (100\text{-}K) \cdot \text{D}(0, T) \tag{4}$$

Here, we are making a small approximation by assuming that the protection leg values in equation (2) and equation (3) are the same. In reality, for a forward contract, the default payment for a default occurring between today and time $T$ will be made at time $T$. This will cause the protection leg value in the forward value to be slightly lower, but we omit this small difference for simplicity.

We still need the value of the premium leg in equation (4) to obtain the value of the forward. This value will depend on the probabilities of default between time 0 and time $T$, so we need to fit a default probability curve. We fit to a flat credit curve with spreads equal to (an estimate of) the spread on the portfolio to maturity T (option maturity). Using this fitted credit curve, we can calculate the PV01 to time T and write the value of the premium leg as:

$$\text{Premium Leg}(0, T) = \text{PV01}(0,T) \cdot C$$

Plugging this value into equation (4), we obtain:

$$Forward(K,T) = (100\text{-}P) + \text{PV01}(0,T) \cdot C - (100\text{-}K) \cdot \text{D}(0, T) \tag{5}$$

We can use this equation to obtain the value of the forward, but as we noted before, we ignored the cancellation feature in the derivation. Therefore, although equation (5) can be used to obtain the value of a CDX.HY forward swap, we need to make an adjustment for cancellation to obtain the value of an LCDX forward.

### Incorporating cancellation

The first term in equation (5) depends only on the current price of the index and is not affected by cancellation. However, the second term involving the coupon, and the third term involving the forward strike price $K$ will be affected. If a name in the LCDX portfolio is removed because of cancellation, then the trade notional will be reduced by 1% of the original notional and the coupon will accrue on that reduced notional. The amount of payment $K$ will also be based on the new notional. In that sense, cancellation causes part of the trade to be knocked out. This means that although the forward contract is non-knockout with respect to default, it is knockout with respect to cancellation. We need to take this into account by modifying the payments. If we have an estimate of the proportion of issuers in the portfolio for which a cancellation event will occur by time $T$, then we can modify the equation above to obtain the value of the forward as:

$$\text{Forward}^{\text{LCDX}}(K,T) = (100\text{-}P)+(1\text{-}P^{\text{CANCEL}}(T))\ [\ \text{PV01}(0,T) \cdot C - (100\text{-}K) \cdot D(0, T)\ ] \qquad (6)$$

where $P^{\text{CANCEL}}(T)$ denotes the proportion of LCDS that is expected to be cancelled by time $T$. We can equivalently view this as a cancellation probability for the portfolio by time $T$. Note that an approximation is used above for the coupon term. We use the cancellation proportion until time $T$ when adjusting the value of this term even though the notional that applies to the coupon value will be reduced at the time of cancellation[1]. Another important assumption used here is that cancellation is independent of default, which may not be realistic. It can be argued that cancellation is most likely to occur when the credit quality of the firm increases (see Leberge *et al.* (2007)) which would imply that cancellation and default are negatively correlated. From equation (6) we see that this assumption will only affect the coupon part and will not be significant when the time horizon is short.

Equation (6) gives us a way of valuing a forward if we have an estimate of the cancellation proportion $P^{\text{CANCEL}}(T)$ which may be obtained from historical estimates or perhaps from calibrated cancellation probabilities for the individual issuers. In the second case, a valuation model is needed to bootstrap the cancellation probability curve of an issuer by taking into account the spread between the regular bond CDS curve and the LCDS curve of an issuer.

The effect of cancellation adjustment is relatively small for typical swaption maturities of three to six months. In order to take advantage of the price discrepancy in the forward prices, one would need to create synthetic forward contracts using swaptions. Dealer prices are quoted with a bid/ask spread, i.e. the price to buy a swaption is not the same as the price to sell the same swaption. A long forward contract would be created by buying the payer swaption at the ask price and selling the receiver swaption at the bid price. Conversely, a short forward contract would be created by buying the receiver at the ask price and selling the payer at the bid price. Thus, the bid/ask spread on the swaptions naturally creates a bid/ask spread on the forward contracts. The difference in the forward prices starts to become important when it is higher than this bid/ask spread. Even if we take a conservative estimate of 5% cancellation probability per year, the cancellation probability for a six-month forward contract will be around 2.5%. The exact value of adjustment will depend on the coupon and the forward strike price $K$, but for typical values, the difference is inside the bid/ask spread we observe in the swaption market.

## 3. PRICING THE SWAPTIONS

Even though the size of the adjustment to the forward value due to cancellation is relatively small, we may still want to obtain swaption prices that are in line with the adjusted forward price. The market standard for pricing portfolio swaptions is a lognormal spread model such as in Pedersen (2003). The main input in the model is the value of the forward which is converted into an adjusted forward spread that is used in pricing the swaptions. We can follow the above reasoning and adjust the value of the forward for cancellation but otherwise follow the same steps as in Pedersen (2003). This way, we obtain swaption prices that are consistent with the forward contract value and satisfy the put-call parity.

[1] *If we disregard premium recovery on default and cancellation, the PV01 is a sum of terms $D(t_i)S(t_i)\Delta(t_{i-1},t_i)$ where $D(t_i)$ is the Libor discount factor, $S(t_i)$ is the survival (non-default) probability, and $\Delta(t_{i-1},t_i)$ is the daycount fraction. A more accurate adjustment of the PV01 to incorporate cancellation would be to multiply each term by the non-cancellation probability to time $t_i$, $1\text{-}P^{CANCEL}(t_i)$, as opposed to the probability to time T (as done in equation 6). We could also argue that the LCDX spread used to determine the default probabilities used to calculate the PV01 would be lower if LCDX did not have the cancellation clause. In other words, by using a spread that incorporates cancellation, that PV01 automatically has some adjustment for calculation even without the specific adjustment we suggest.*

Another model that is sometimes used for quoting implied price volatility assumes that the quoted forward price is lognormally distributed. This model is explained in Liu and Pedersen (2005). It is necessary to determine the forward price to price swaptions using the price model. This is the value of *K* in equation (6) that makes the initial forward value zero. Thus, the forward price is given by:

$$\text{Forward Price} = 100 - \frac{100 - \mathrm{P}}{(1 - \mathrm{P}^{\mathrm{CANCEL}}(\mathrm{T})) \cdot \mathrm{D}(0,\mathrm{T})} - \frac{\mathrm{PV01}(0,\mathrm{T}) \cdot \mathrm{C}}{\mathrm{D}(0,\mathrm{T})} \tag{7}$$

The swaption prices are calculated by plugging the forward price into Black's option pricing formula. An adjustment for cancellation is needed on the discount factor used in discounting the option payoffs.

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