LEHMAN BROTHERS

Fixed Income
Quantitative Credit Research

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Quarterly

Volume 2006-Q2

To analyse the risk implications of CDO tranches in the context of a wider credit portfolio, we need a tractable way to generate the P&L distribution of the mark-to-market of these instruments. We present a hybrid approach toward this problem by combining a simulation of rating migrations and credit spreads with an analytical pricing engine. This engine is an extension of the well-known Large Homogeneous Portfolio (LHP) limit originally due to Vasicek. It partitions the portfolio underlying the tranche into different cells and applies the LHP technique in each of them, hence the name Large Homogeneous Cell (LHC). The model can be applied to both CDO tranches and large portfolios of bonds, which enables a concise risk-return modelling for many fixed income portfolios.

In this article we introduce a quantitative framework to analyze and decompose CDS spreads. In this framework, credit spreads are explained by the expected default loss and a risk premium, which is further decomposed into premia arising from the risks of default, liquidity, spread volatility, and turns in the business cycle. We describe the results of the spread decomposition of all issuers by rating. We also present applications of the framework such as spread decomposition at the issuer level, estimation of risk premia in tranche markets, and the construction of optimized portfolios under some default loss constraints.

We are introducing a new tool on LehmanLive to calculate the credit default swap (CDS) basis of a corporate bond. The basis is defined as the CDS spread to the bond maturity minus a bond-implied CDS spread. For a bond without embedded options, we calculate the bond-implied CDS spread using the standard model for valuing a CDS. For a bond with embedded options, we use a model with stochastic interest rates and credit spreads. Unlike the standard option adjusted spread (OAS) and the Z-spread, the bond-implied CDS spread is, as its name suggests, directly comparable with a CDS spread. The difference between the Z-spread/OAS and the bond-implied CDS spread can be significant especially for high yield bonds. We recommend using the bond-implied CDS spread not only when comparing a bond with a CDS but also when comparing two bonds.

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Large Homogeneous Cells: A Framework for Modelling Risk in a Credit Portfolio with CDO Tranches[1]

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To analyse the risk implications of CDO tranches in the context of a wider credit portfolio, we need a tractable way to generate the P&L distribution of the mark-to-market of these instruments. We present a hybrid approach toward this problem by combining a simulation of rating migrations and credit spreads with an analytical pricing engine. This engine is an extension of the well-known Large Homogeneous Portfolio (LHP) limit originally due to Vasicek. It partitions the portfolio underlying the tranche into different cells and applies the LHP technique in each of them, hence the name Large Homogeneous Cell (LHC). In this paper, we outline the mechanics of the model and derive analytical formulas for tranche re-pricing. The model can be applied to both CDO tranches and large portfolios of bonds, which enables a concise risk-return modelling for many fixed income portfolios. As an empirical illustration, we analyse the risk profile of a mixed portfolio of corporate bonds and CDO tranches.

1. INTRODUCTION

CDO tranches are an important asset class attractive to many institutional investors. One of the reasons for this attractiveness is the flexibility of the CDO structure: by tranching a portfolio, the risk-return profile of each tranche can effectively be decoupled from the selection of the underlying credits. A given portfolio where the credits have been selected on the basis of fundamental credit analysis can be transformed to satisfy the risk appetites of many different investors. CDO tranches, particularly those based on bespoke portfolios, are often illiquid instruments, and hence, are held till maturity. Therefore, portfolio analysis of CDO tranches is usually conducted on a buy-and-hold basis.

Nevertheless, the mark-to-market risk implications of holding a CDO tranche are important, especially when considered in the context of a wider fixed income portfolio. For example, adding protection on a CDO equity or mezzanine tranche can substantially reduce the portfolio Value-at-Risk, as well as the sensitivity to market spread moves. Stated differently, CDO tranches can be effectively used for risk management purposes. The increased liquidity of standardized tranches such as iTraxx and CDX makes the diversification motive relevant also for active trading portfolios.

An assessment of the mark-to-market risk in credit portfolios is important for financial institutions, because many of them are required to report this risk to their regulatory authorities. With bespoke financial derivatives, such as CDO tranches, mark-to-market risk evaluation and modelling becomes a challenging task.

In this paper we present a framework for efficiently modelling the dynamic behaviour of CDO tranches. Our approach is based on a Monte Carlo simulation of rating migrations as well as the credit spreads of the portfolio. For each such realization, the portfolio is re-priced using the Large Homogeneous Cell (LHC) methodology. The portfolio is considered to consist of several cells, and each cell is treated asymptotically by applying the large homogeneous limit to each of these. This extension of the conventional Large Homogeneous Portfolio (LHP) approach introduces a degree of heterogeneity into the pricing. A natural example considered in this paper is the case when portfolio cells correspond to different

[1] *We would like to thank Vasant Naik and Minh Trinh for their comments and contribution.*

credit ratings. The framework, however, can also accommodate a diversity of sector and/or country specific cells. The combination of Monte Carlo simulation and analytical pricing engine allows us to generate the tranche P&L distribution at a fixed time horizon. A generic portfolio of corporate bonds is used to illustrate the risk evaluation in a portfolio with synthetic CDO tranches. We also emphasize the importance of credit event risk, such as rating downgrades and defaults, for synthetic CDO tranches and credit portfolios overall.

The remainder of the paper is organized as follows. Section 2 provides an outline of the model, emphasizing the simulation of spreads and rating transitions, as well as the mechanics of the LHC methodology. Then we illustrate our approach by analyzing the P&L distribution of some tranches as well as demonstrating how the risk characteristics of a generic corporate bond portfolio can be improved by hedging with synthetic CDO tranches. Section 4 concludes.

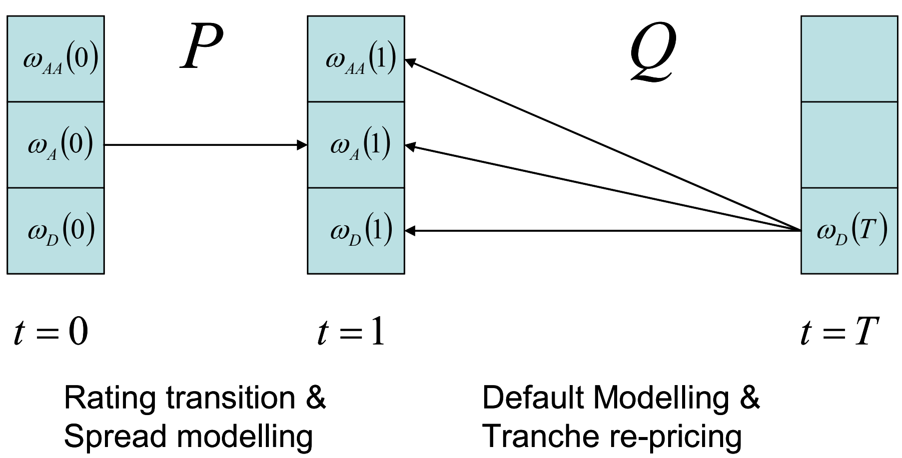
2. MODEL OUTLINE

Framework

The main objective of our framework is to give an investor a sense of the mark-to-market P&L distribution of a CDO tranche at a fixed time horizon. For linear or quasi-linear instruments, such as bonds, equities, or CDS this can be achieved by directly modelling returns or spreads. In the case of a non-linear derivative instrument, however, modelling returns is not straightforward. One way to proceed is to use an analytical pricing model jointly with a simulation technique applied to the behaviour of the underlying.

We take this approach to analyse the distribution of CDO tranche P&L. The whole modelling procedure can be divided into two steps. First, we simulate rating transitions and credit spreads in each of the underlying rating buckets to a fixed time horizon. Default is taken into account as one of the rating buckets. Then, we use the large homogenous cell (LHC) model to re-price outstanding tranches adjusted for defaults that occurred in the portfolio. It is important to note that the simulation stage uses statistical estimates of default probabilities and rating transitions, whereas at the pricing stage we need to use implied, i.e. risk adjusted default probabilities. Figure 1 depicts this two-step procedure, with the historical probability measure "P", and the risk neutral measure "Q".

Figure 1. Time outline of the LHC credit migration model



Source: Lehman Brothers

The semi-analytic tranche pricing provided by LHC is crucial for computational speed. At the same time, the model allows a higher degree of flexibility compared with the classic large homogenous portfolio (LHP) model. In particular, LHC splits the whole credit portfolio into several cells, assuming the number of credits in each cell is substantially large. While defaults in the whole portfolio are still driven by a single systematic factor, characteristic properties such as default probabilities, default correlation, and recovery rates can be different for different cells. In this paper we interpret portfolio cells as credit rating buckets. This, however, is not the only option. In particular, it is possible to use sector, country of origin, or other characteristics to split the underlying portfolio into large homogenous cells.

Rating migrations and distribution of rating cell spreads

We assume that each rating cell in the portfolio contains a large number of credits and that properties of credits in the same rating cell are similar.

In our approach, the risk of each credit stems from two components. On the one hand, there are systemic spread movements, which affect credits in each rating bucket equally. For example, these can be attributed to changes in investors' risk preferences. On the other hand, there are firm-specific changes to credit quality. These are captured by rating transitions.

In particular, suppose that the issuer universe can be described by K rating cells. We assume that the spread levels of individual credits in a given rating cell are the same. The current spread level for each rating cell is known. Assume that the time horizon is fixed at 1 year. We model spreads at the end of the year for each rating cell by assuming that these spreads are log-normally distributed with a mean equal to the current spread level. The spread changes in different cells are correlated, so that the joint dynamics of spreads can be described by the vector equation[2]

$$\ln \vec{s}_1 = \ln \vec{s}_0 - \frac{1}{2}\operatorname{diag}\left(\hat{\Omega}\right) + \hat{\Omega}^{1/2}\vec{W}_1, \tag{1}$$

where $\vec{s}_0$ is the vector of initial rating cell spreads, $\vec{s}_1$ is the vector of rating cell spreads at the end of the year, $\hat{\Omega}$ is the covariance matrix of relative spread changes, and $\vec{W}_1$ is a K-dimensional vector of independent standard normal random variables. A portfolio of credits is represented by a weighting distribution across cells. We denote these weights by $\vec{\omega}_0$. The spread evolution of each individual credit is described by the rating migration and the rating spread processes of different rating cells given in equation (1).

We model credit migrations by a Markov chain process. For the transition matrix, we use a matrix estimated by rating agencies such as Moody's or Standard and Poor's from historical migration frequencies. We allow rating transitions to be correlated with the rating spread changes. Indeed, as the market spreads widen, company downgrades become more likely. Inspired by the CDO tranche pricing literature, we use a one-factor Gaussian copula to model joint rating migrations. Effectively, this is a CreditMetrics-type of approach. Consider a K-rating cell model with transition probability matrix

$$\hat{P} = \begin{bmatrix} p_{11} & p_{12} & \cdots & p_{1K} \\ p_{21} & p_{22} & \cdots & p_{2K} \\ \vdots & \vdots & \ddots & \vdots \\ p_{K1} & p_{K2} & \cdots & p_{KK} \end{bmatrix},$$

[2] *Our model of spread behavior is very simplistic as spread modeling is outside the scope of this paper. There are no constraints, however, for the choice of the model describing spread dynamics. For example, one could model the whole term structure of credit spreads, see Duffie and Singleton (1999).*

where p_{js} is the transition probability from rating cell j to rating cell s. The migration of credit j is driven by the realization of a latent variable A_j. The latent variables are jointly normal, with a one-factor correlation structure, given by equation (2).

$$A_j = \beta_j Z + \sqrt{1-\beta_j^2}\,\varepsilon_j\,, \tag{2}$$

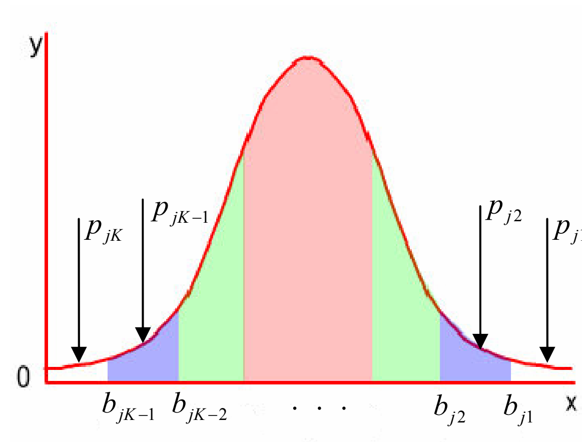
Here Z is the systematic factor common to all credits in the underlying portfolio, ε_j is the idiosyncratic factor independent of the systematic factor, and β_j is the cell-specific exposure to the systematic factor. We allow β_j to vary for different cells. The factors Z and ε_j are assumed to be independent standard normal random variables. The systematic factor Z has a negative correlation with cell spreads to capture the positive relationship between spread widening and downgrades.

The rating transition of a credit is conditional on the latent variable A_j crossing specific thresholds. These are calibrated to the migration probabilities. Specifically, for a credit belonging to rating cell j, the transition thresholds $b_{j1}, b_{j2}, \ldots b_{jK-1}$ are defined by the following system of recursive equations

$$\begin{cases} P\{A_j > b_{j1}\} = 1 - \Phi(b_{j1}) = p_{j1}, \\ \ldots \\ P\{b_{js-1} \ge A_j > b_{js}\} = \Phi(b_{js-1}) - \Phi(b_{js}) = p_{js}, \\ \ldots \\ P\{b_{jK-1} \ge A_j\} = \Phi(b_{jK-1}) = p_{jK}. \end{cases}$$

Figure 2 illustrates the idea. The probability for the latent variable to be between two given transition thresholds equals the corresponding transition probability.

Figure 2. Cell transition probabilities and latent variable thresholds



Source: Lehman Brothers.

Conditional on a realization of the systematic factor Z, transitions are independent. Therefore, using the LHC assumption we can calculate the percentage of credits initially in cell j that migrates to cell s. Conditional on the realization of the systematic factor, this percentage is equal to the conditional probability that the value of the latent variable for a single issuer in cell j is between thresholds b_{js} and b_{js-1}.

$$
\begin{cases}
P\{A_j > b_{j1} \mid Z\} = 1 - \Phi\left(\dfrac{b_{j1} - \beta_j Z}{\sqrt{1-\beta_j^2}}\right) = p_{j1}(Z), \\
\ldots \\
P\{b_{js-1} \geq A_j > b_{js} \mid Z\} = \Phi\left(\dfrac{b_{js-1} - \beta_j Z}{\sqrt{1-\beta_j^2}}\right) - \Phi\left(\dfrac{b_{js} - \beta_j Z}{\sqrt{1-\beta_j^2}}\right) = p_{js}(Z), \\
\ldots \\
P\{b_{jK-1} \geq A_j \mid Z\} = \Phi\left(\dfrac{b_{jK-1} - \beta_j Z}{\sqrt{1-\beta_j^2}}\right) = p_{jK}(Z).
\end{cases}
$$

For example, the conditional percentage of credits in rating cell s by the end of the first period is

$$\omega_{1s}(Z) = \sum_{j=1}^{K} p_{js}(Z)\omega_{0j}\ , \tag{3}$$

where ω_{0j} is the initial percentage of credits in rating cell j and $\omega_{1s}(Z)$ is the percentage of credits in cell s at the end of the year . The latter quantity depends on the realization of the systematic factor. In vector notation equation (3) can be rewritten as:

$$\vec{\omega}_1(Z) = \hat{P}^T(Z)\vec{\omega}_0\ .$$

The last element of the vector $\vec{\omega}_1$ gives the defaulted part of the original portfolio. The loss of the credit portfolio at the end of the year due to these defaults is given by

$$L_1(Z) = \sum_{j=1}^{K} p_{jK}(Z)\omega_{1j}(1 - R_j), \tag{4}$$

where R_j is the recovery rate specific to the rating cell j .

As a result, by simulating joint realizations of the rating cell spreads and the systematic factor for the rating migration, we obtain the joint distribution of rating cell weights and spreads. From this information we can re-price the asset portfolio as well as CDO tranches, using the LHC framework.

LHC: The Expected Loss of a CDO Tranche

In this section we outline the pricing of tranches using the semi-analytical LHC model, which translates the randomness in rating cell weights and spreads into randomness of tranche P&Ls. We assume that the reader is familiar with the way the expected losses of a tranche can be translated into the present values of premium and protection legs. Therefore, we give only the derivation of the expected loss of a tranche to a fixed time horizon in the text. Readers can find the details for the valuation of the premium and protection legs in O'Kane *et al*. (2003), p. 40.

Within each rating cell, we apply the Large Homogeneous Portfolio limit with a one-factor Gaussian copula. The correlation with the market factor is again denoted by β_j. The default threshold c_j is implied from the spread realization in cell j, so that $c_j = \Phi^{-1}(\pi_j)$, where π_j is the risk neutral default probability of cell j, which in turn is deduced from the spread realization, for example using the credit triangle

$$\pi_j(t,T) \approx 1 - \exp\left(- \frac{s_j (T-t)}{(1-R_j)} \right) \cdot \tag{5}$$

The loss in the underlying credit portfolio conditional on the realization of the systematic factor Z is

$$\ell(z) = \sum_{j=1}^{K} (1-R_j)\omega_j \Phi\left(\frac{c_j - \beta_j z}{\sqrt{1-\beta_j^2}} \right), \tag{7}$$

where ω_j is the weight of rating cell j in the portfolio and R_j is the recovery rate of credits in this cell. The portfolio loss probability distribution can be calculated semi-analytically. In particular, note that the conditional portfolio loss in (7) is a monotonically decreasing function of the systematic factor z. Therefore, there exists an inverse function $\ell^{-1}(L)$ which can be found numerically by solving the equation

$$\sum_{j=1}^{K} (1-R_j)\omega_j \Phi\left(\frac{c_j(T) - \beta_j z}{\sqrt{1-\beta_j^2}} \right) = L \cdot$$

The portfolio loss distribution is characterised by loss exceedance probabilities

$$P\{\ell(Z) \geq L\} = P\{Z \leq \ell^{-1}(L)\} = \Phi(\ell^{-1}(L)) \tag{8}$$

The expected loss of a CDO tranche is given by the equation

$$E_0\left[L^{TR}(\ell)\right] = \frac{1}{L_2 - L_1} \left\{ E\left[(\ell - L_1)^+\right] - E\left[(\ell - L_2)^+\right] \right\}, \tag{9}$$

where L_1 and L_2 are the attachment and detachment points.

It can be seen that we need to be able to calculate expectations $E_0\left[(\ell \quad - L)^+\right]$ to find an analytical formula for expected tranche loss. These calculations are, in fact, straightforward

$$E_0\left[(\ell \quad - L)^+\right] = E_0\left[\ell \quad 1_{\{\ell \quad \geq L\}}\right] - L E_0\left[1_{\{\ell \quad \geq L\}}\right]. \tag{10}$$

The second term is the portfolio loss exceedance probability

$$E_0\left[1_{\{\ell \quad \geq L\}}\right] = P\{\ell(Z) \geq L\} = \Phi(\ell^{-1}(L)). \tag{11}$$

The first term can be calculated as

$$E_0\left[\ell \quad 1_{\{\ell \quad \geq L\}}\right] = \sum_{j=1}^{K} (1-R_j)\omega_j E_0\left[\Phi\left(\frac{c_j - \beta_j Z}{\sqrt{1-\beta_j^2}} \right) 1_{\{\ell \quad \geq L\}} \right] =$$

$$= \sum_{j=1}^{K} (1-R_j)\omega_j E_0\left[1_{\{A_j \leq c_j\}} 1_{\{Z \quad \leq \ell^{-1}(L)\}}\right] = \sum_{j=1}^{K} (1-R_j)\omega_j \Phi_{2,\beta}\left(c_j, \ell^{-1}(L)\right),$$

where $\Phi_{2,\beta}(x,y)$ is the bi-variate normal cumulative distribution function with zero mean and covariance matrix

$$\Omega_\beta = \begin{bmatrix} 1 & \beta \\ \beta & 1 \end{bmatrix}.$$

As a result,

$$E_0\left[\left(\ell_T - L\right)^+\right] = \sum_{j=1}^{K}\left(1 - R_j\right)\omega_j \Phi_{2,\beta_j}\left(c_j\left(T\right), \ell_T^{-1}\left(L\right)\right) - L\Phi\left(\ell_T^{-1}\left(L\right)\right). \tag{12}$$

This expression can be readily substituted into (9) to get an analytical formula for the expected loss of a CDO tranche, which in turn can be used to obtain present values of tranche premium and protection legs.

3. EXAMPLE: MODELLING RISK IN A CREDIT PORTFOLIO

As discussed before, the risk of a credit portfolio in the LHC framework can be split into *market* and *rating migration risks*. *The market risk* is the risk of changes in credit spreads across all ratings. These changes can be caused both by fundamental reasons, such as, for example, changes in the macroeconomic environment, and by shifts in the perception of credit risk by market participants. For example, a sequence of large credit blow-ups can result in a credit spread widening across the whole market as investors become more cautious. The market risk is important on both short and long investment horizons. This risk is taken into account in the LHC framework by modelling the spread distributions of rating cells directly as described in the previous section and so we can also call it the *spread risk*. The volatility of relative spread changes is the principal parameter that determines portfolio spread risk. This risk is allocated across CDO tranches by means of the default correlation parameter. The rating *migration risk* includes rating downgrades and defaults. The relative importance of the rating migration risk increases with the investment horizon as more defaults and downgrades are likely to occur over a longer period. The allocation of the rating migration risk across synthetic CDO tranches depends on the migration correlation parameter (an equivalent of the default correlation under the physical probability measure).

Let us consider the example of a A-rated Euro credit portfolio analysed in isolation as well as in combination with synthetic CDO tranches. For this rather generic example we set the model parameters according to recent market data. We detail tranche risk modelling in the LHC framework and investigate the contributions of both market risk and rating migration risk to the total risk of the portfolio.

Risk modelling in the LHC framework consists of the joint modelling of market spreads and rating migrations. As discussed earlier, spreads and migrations can be dependent since one would expect to see a widening of market spreads as there are more downgrades in the portfolio.

As an empirical illustration of the LHC portfolio risk framework we analyse the effect of buying protection on a synthetic CDO tranche on the distribution of returns and the economic capital requirement of an A-rated corporate bond portfolio. To proxy bonds in the underlying portfolio we use an A-rated bucket of the Lehman Euro Corporate Bond Index.

The whole bond portfolio is originally concentrated in the A cell of the rating universe which we represent as consisting of five rating cells: AAA, AA, A, BAA, and HY. The implied default probabilities for these cells are deduced from Libor spreads of the corresponding rating buckets of the Lehman Euro Corporate Bond Index. Figure 3 summarizes parameters of the underlying portfolio of bonds.

Figure 3. Parameters of the underlying corporate bond portfolio

Underlying Bonds		AAA	AA	A	BAA	HY
Weights	**100%**	0%	0%	100%	0%	0%
Libor Spread	**29.7**	0.3	14.0	29.7	55.5	276.2
Recovery	**40%**	40%	40%	40%	40%	40%
Maturity	**5.9**	*	*	*	*	*
Coupon	**4.2%**	*	*	*	*	*

Source: Lehman Brothers

The average Libor spread corresponding to our A-rated investment is 29.7 bp. The average maturity of 5.9 years and coupon of 4.2 percent correspond to the average maturity and yield of the A-rated bucket in the bond index. We assume a 40% recovery rate. Libor spread levels of different rating cells correspond to the current spread levels of AAA, AA, A, BBB, and High Yield buckets of the respective Lehman Euro Bond Indices.

The joint dynamics of rating cell spreads are modelled by a vector geometric Brownian motion with a covariance matrix calibrated to the historical volatilities and correlations of bond indices for the period from Jan 1999 to Apr 2006 for AAA, AA, A, and BAA buckets (from Aug 2000 to Apr 2006 for high yield). Dependence between spreads and migrations is incorporated through the correlations between cell spreads and the systematic migration factor. We estimate it as the historical correlation between relative changes in bond spreads and returns of the DJ EURO STOXX 50. Figure 4 summarizes the simulation parameters.

Figure 4. Spread volatilities and correlations

Rating Cells	AAA	AA	A	BAA	HY
			Volatility		
	122.8%	35.6%	41.2%	47.9%	40.5%
			Correlation		
AAA	100%	45%	18%	15%	1%
AA	45%	100%	35%	43%	14%
A	18%	35%	100%	46%	45%
BAA	15%	43%	46%	100%	48%
HY	1%	14%	45%	48%	100%
Syst. Migration Factor	-2%	-6%	-23%	-32%	-36%

Source: Lehman Brothers

The correlations between spreads and the systematic migration factor show the relation between systematic downgrades in the bond portfolio and widening of the cell spread. Negative correlations imply that spreads tend to widen when the frequency of downgrades increases.

We use a transition probability matrix published by Moody’s to model individual rating migrations in the bond portfolio. The transition probability matrix is given in Figure 5.

Figure 5. Moody's historical rating migration matrix, 1920-2004

	AAA	AA	A	BAA	BA	B	CAA	D
AAA	91.73%	7.23%	0.82%	0.19%	0.03%	0.00%	0.00%	0.00%
AA	1.36%	90.53%	7.04%	0.76%	0.20%	0.04%	0.00%	0.06%
A	0.08%	3.06%	89.96%	5.89%	0.75%	0.15%	0.03%	0.07%
BAA	0.05%	0.35%	5.06%	87.26%	5.90%	0.89%	0.16%	0.32%
BA	0.01%	0.10%	0.60%	6.72%	82.37%	7.91%	0.82%	1.47%
B	0.00%	0.07%	0.23%	0.85%	7.38%	80.14%	6.47%	4.87%
CAA	0.00%	0.02%	0.04%	0.18%	1.07%	7.31%	75.30%	16.07%
D	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	100.00%

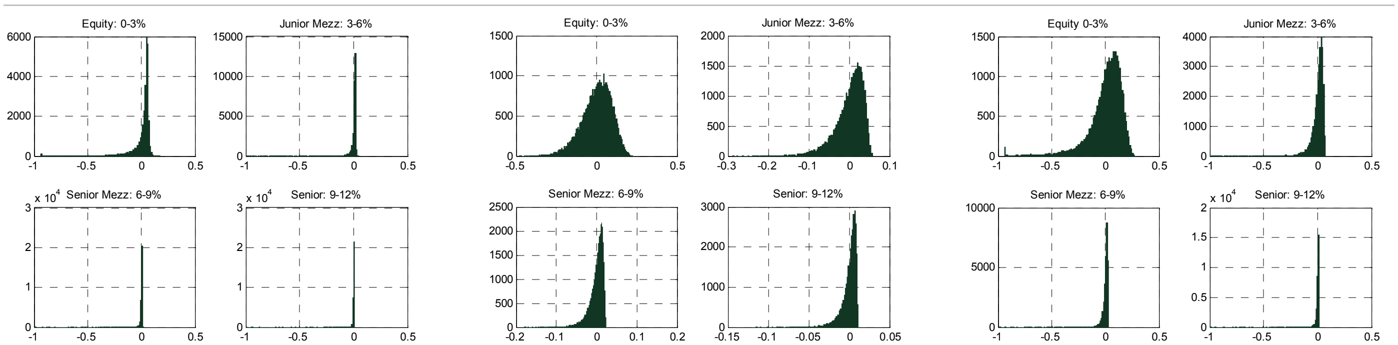
Source: Moody's

Although we assume that all high yield ratings have identical spreads, we do not consolidate them when simulating rating migration and defaults, to ensure that we maintain the Markovian property of rating transitions.

To analyse the impact of including a CDO tranche in the portfolio, we model a 5-year synthetic CDO on the underlying bond portfolio. We structure the CDO by allocating losses in the underlying notional to 0-3%, 3-6%, 6-9%, and 9-12% tranches, which we call Equity, Junior Mezzanine, Senior Mezzanine, and Senior. The default correlation is assumed to be the same for all rating cells. We calibrate the default correlation to the average correlation across German industry sectors available from the Lehman Factor Correlation Data Report[3]. The resulting default correlation is 32%.

We analyse the P&L distributions of individual tranches and highlight the contributions of the two major sources of risk: the spread risk and the migration risk. The spread risk comes from volatility of the underlying spreads; the migration risk comes from rating downgrades and defaults. To demonstrate the effect of migrations on the riskiness of tranche P&L, we consider separately three cases: isolated migration risk, isolated spread risk, and both spread and migration risks affecting tranche P&L. Figure 6 shows distributions of P&L for different tranches[4] and Figure 7 provides corresponding statistics.

Figure 6. P&L distributions of tranches: isolated migration risk (LHS), isolated spread risk (MID), spread & migration risks (RHS)



Source: Lehman Brothers

[3] *Available in the Analytic Toolkit on Lehman Live.*
[4] *Tranche P&Ls are de-meaned.*

Rating migrations significantly increase the risk associated with a tranche, especially for the equity tranche for which it makes the P&L distribution become much more skewed to the downside. Even though the default risk of an A-rated bond is relatively small (only 0.07% according to the Moody's transition matrix), the downgrade probability is substantial (5.89% to BAA and 0.93% to HY). These credit events increase the Libor spreads of downgraded bonds in a jump-like manner. As a result, protection becomes more expensive, translating into negative P&L for protection sellers.

Changes in bond spreads, when considered in isolation from rating migrations, have a less dramatic impact on tranche P&L. Figure 7 reports sample statistics of tranche P&L. As can be seen, most of the risk in the equity tranche comes from migration risk. Indeed, the equity tranche should be most sensitive to credit downgrades and defaults as it is the first to absorb losses in the underlying portfolio. For the senior tranche it is the opposite: the spread risk is 3.8% (as measured by VaR), while the migration risk is only 2.9%. Nonetheless, inclusion of the migration risk increases VaR substantially even for senior tranches. This can be explained by the positive migration correlation parameter (32%). Downgrades and defaults also increase skewness and kurtosis of the P&L distribution. Rating downgrades especially affect skewness and kurtosis of senior mezzanine and senior tranches.

Figure 7. Sample statistics of tranche P&L

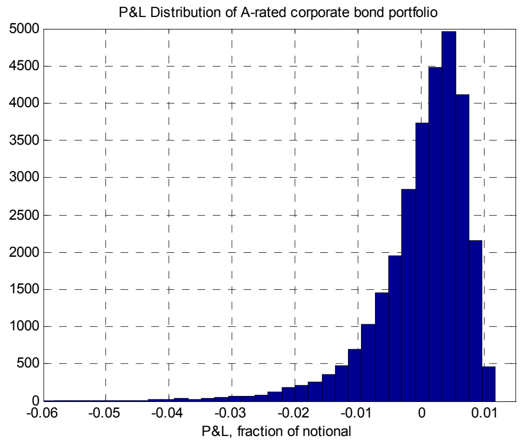
Statistics	Equity	Jr. Mezz	Sn. Mezz	Senior
Isolated Migration Risk				
Volatility	12.7%	5.5%	3.0%	1.9%
Norm. Skewness	-4	-12	-22	-36
Norm. Kurtosis	24	175	620	1635
99% VaR	64.8%	17.2%	6.4%	2.9%
Isolated Spread Risk				
Volatility	9.4%	3.7%	1.9%	1.0%
Norm. Skewness	-1	-2	-2	-3
Norm. Kurtosis	4	7	10	14
99% VaR	26.4%	12.5%	6.7%	3.8%
Migration and Spread Risks				
Volatility	17.0%	7.7%	4.4%	2.7%
Norm. Skewness	-2	-6	-11	-18
Norm. Kurtosis	10	59	189	488
99% VaR	71.5%	29.0%	13.5%	7.1%

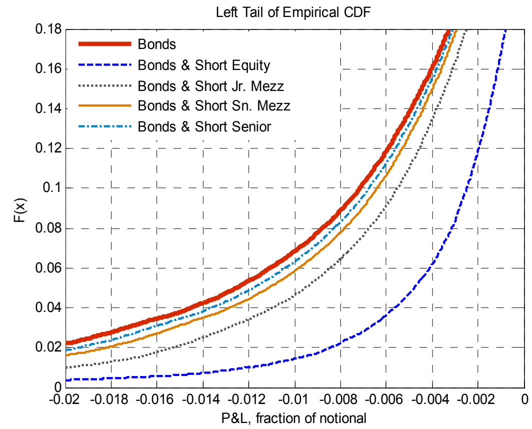
Source: Lehman Brothers

From these results, it is apparent that modelling migrations is essential for representing the riskiness of CDO tranches accurately, especially for long time horizons. The migration correlation parameter amplifies the importance of the migration risk for senior tranches.

Next, we consider possible risk-reduction strategies where the bond portfolio holder buys protection on various synthetic tranches on the portfolio assets. Figure 8 shows the P&L distribution of the bond portfolio and the tails of P&L distributions when CDO tranches are included. The right-hand-side diagram clearly demonstrates the reduction in tail risk that is achieved when protection on junior tranches is combined with the existing portfolio.

Figure 8. LHS: P&L distribution of the bond portfolio. RHS: Tail of P&L distribution of the bond portfolio with added synthetic CDO tranches





Source: Lehman Brothers.

Finally, Figure 9 quantifies the risk of the bond portfolio combined with CDO tranches by reporting sample statistics for three different cases: isolated migration risk, isolated spread risk, and combined migration and spread risks. The role of each tranche in portfolio risk reduction becomes transparent. The equity tranche reduces tail risk very significantly: migration tail risk of the bond portfolio reduces from 1.87% to 0.03% when protection on the equity tranche is added. Protections on other tranches do not eliminate risk to the same extent. In our example, spread risk cannot be fully eliminated by buying protection on an individual tranche. While the equity tranche reduces spread risk to the highest extent, buying spread risk protection in a mezzanine tranche can be more effective from a cost perspective. Again, modelling transitions is essential for capturing total risk of the portfolio as there is a substantial increase in volatility, VaR, skewness, and kurtosis when migrations are taken into account.

Figure 9. Sample statistics of the bond portfolio with synthetic CDO tranches with and without the migration risk

Statistics	Bonds	Bonds-Equity	Bonds-Jr. Mezz	Bonds-Sn. Mezz	Bonds-Senior
Isolated Migration Risk					
Volatility	0.46%	0.16%	0.31%	0.38%	0.42%
Abs. Skewness	-0.89%	-0.51%	-0.58%	-0.68%	-0.75%
Abs. Kurtosis	1.48%	1.01%	1.01%	1.10%	1.19%
99% VaR	1.87%	0.03%	1.35%	1.67%	1.78%
Isolated Spread Risk					
Volatility	0.53%	0.26%	0.42%	0.48%	0.50%
Abs. Skewness	-0.59%	-0.33%	-0.46%	-0.51%	-0.54%
Abs. Kurtosis	0.84%	0.47%	0.66%	0.74%	0.78%
99% VaR	1.72%	0.92%	1.34%	1.51%	1.60%
Migration and Spread Risks					
Volatility	0.84%	0.40%	0.62%	0.72%	0.77%
Abs. Skewness	-1.30%	-0.83%	-0.93%	-1.04%	-1.12%
Abs. Kurtosis	2.06%	1.52%	1.56%	1.66%	1.75%
99% VaR	3.01%	1.31%	2.14%	2.61%	2.81%

Source: Lehman Brothers

4. CONCLUSION

In this paper we introduced a framework which enables effective allocation and risk management in credit portfolios with synthetic CDO tranches. We call this framework the Large Homogenous Cell (LHC) model, as it relies on the assumption that credit portfolio is sufficiently large and distributed across homogeneous cells. The framework combines simulations of spreads and rating migrations with the LHC pricing engine. The specification of the simulation part is flexible and can be formulated independently from the tranche pricing model. This gives an investor a rich choice in modelling spread dynamics and migrations. The pricing part has a quasi-closed-form solution and enables portfolio heterogeneity across rating, sector, or country specific cells. This paper provides detailed description of CDO tranche pricing in the LHC framework.

As an empirical illustration to the LHC framework we considered a corporate bond portfolio protected with synthetic CDO tranches. We provide a detailed risk analysis of risk of the CDO tranches as well as of the underlying portfolio, isolating effects of migration and spread volatility. Such risk analysis can be very instrumental when allocating assets to credit derivatives within a portfolio management context.

Finally, the LHC framework can be effectively applied to asset allocation decisions in credit portfolios with CDO tranches.

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Anatomy of the Risk Premium in Credit Markets[1]

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In this article we introduce a quantitative framework to analyze and decompose CDS spreads. In this framework, credit spreads are explained by the expected default loss and a risk premium, which is further decomposed into premia arising from the risks of default, liquidity, spread volatility, and turns in the business cycle. We describe the results of the spread decomposition of all issuers by rating. We also present applications of the framework such as spread decomposition at the issuer level, estimation of risk premia in tranche markets, and the construction of optimized portfolios under some default loss constraints.

1. INTRODUCTION

In addition to compensating investors for the risk of expected default losses, credit spreads also carry premia associated with risks such as liquidity, volatility, default, and turns in the business cycle. Assessing the sources and magnitude of these risk premia is important to credit investors because it allows them to make more informed decisions on the risks to hold in their portfolios.

However, the use of measures of risk premia would differ based on whether the investor in question holds a buy-and-hold credit portfolio or is affected by mark-to-market considerations. The returns of buy-and-hold portfolios depend solely on the contractual spreads and realized defaults over the period to maturity of the instrument. Investors in this case might choose to hold bonds of issuers that pay the highest risk premium. On the other hand, mark-to-market investors would be more concerned about the risks over the horizon of the trade and would, for instance, avoid issuers that exhibit high spread volatility. As a result, they would choose issuers with the lowest exposure to volatility and the lowest premium for volatility risk.

It is therefore clear that for all investors, an analysis of the risk premia embedded in credit spreads would be interesting and instructive. In this article, we propose an econometric model to decompose credit spreads and identify risk premia therein. Our framework (that we name CSI or Credit Spread Investigator), quantifies the expected default loss per issuer and analyses the additional spread explained by a 'total risk premium'. It then identifies the main sources of this risk premium and quantifies them[2].

In this model, each source of risk that gives rise to these risk premia is proxied with systematic factors from the corporate bond, equity, volatility, and treasury markets. We argue that as long as credit returns are correlated with these systematic market factors, which cannot be diversified away, one should observe that these market factors are priced into credit spreads in some measure.

The rest of this article is organized as follows: In sections 2 and 3, we describe the different components of credit spreads and of risk premia. In section 4, we describe the model used to decompose the spreads into its various components. In section 5, we discuss the results of the decomposition. In section 6, we present some applications: issuer spread decomposition and portfolio construction and present broad conclusions in section 7.

[1] *The authors would like to thank Mukundan Devarajan, David Heike, Vasant Naik, Marco Naldi, Marc Pomper, Lutz Schloegl, and Gaurav Tejwani for their helpful comments and assistance.*

[2] *Cf. Berndt et al. (2005) for some related academic research and references therein.*

2. THE COMPONENTS OF CDS SPREADS

Fundamentally, CDS spreads (see box below) compensate for expected default loss and for the risk associated with their positions. Estimating the expected default loss accurately is critical for buy-and-hold investors. In the following, we discuss our methodology for estimating issuer-level default probabilities and the recovery risk associated with it, as well as the characteristics of total risk premium.

CDS spreads as measures of credit risk

Different possible measures of credit spreads are available for a given issuer. Here, we focus on CDS spreads instead of bond spreads because these are cleaner measures of credit risk and are less sensitive to security specific characteristics such as callability or coupon effect. Corporate bond liquidity can also vary greatly depending on the issue size and the age of the bond.

Expected Default Loss

Expected default loss depends on the probability of default and the expected recovery. To measure default risk, we can use either historical default rates such as the ones provided by Moody's or S&P or some forward-looking measures such as MKMV's (Moody's KMV) Expected Default Frequency™[3] (EDF™). The default probabilities can also be estimated by using the Merton's model and calibrating it to historical default data (Vassalou and Xing 2004, Huang and Huang 2003).

Probability of Default

For our purposes, we estimate issuer-level default probabilities using EDFs. EDFs are constructed based on information from equity markets, leverage data and historical mapping of distance-to-default to default rates. We have found that they could be usefully complemented by macro forecasts of default rates. For example, at the end of a credit cycle when equity volatility is low and there is excess liquidity in credit markets EDFs could be lower than macro-based expected default rates. For example, EDFs predict an average 5-year default probability of 1.31% for US issuers at the end of April 2006 whereas our macro-based default prediction model predicts a default probability of 2.47% (refer Appendix A for a full discussion of our default prediction model).

To adjust for this and to be on the conservative side, EDFs are scaled up whenever the average EDF is below our macro forecast. For purposes of clarity, we call these scaled EDFs by a new name – Hybrid Default Indicators (HDI). The method for scaling EDFs to get HDI is as follows:

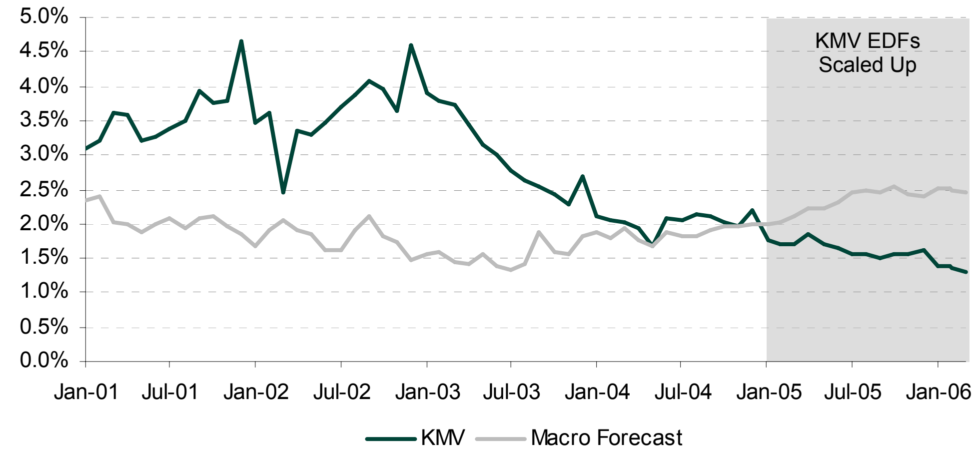
$$\mathbf{HDI_{i,t} = EDF_{i,t} * Max[1, DEF_t /(Mean\ EDF_{i,t})]}$$

DEF_t is our estimate of forward 5-year default rate for all corporates. The advantage of this methodology is that we are not only keeping the cross-sectional efficiency of the EDF, but also incorporating information from other indicators to predict the overall level of default.

As shown in Figure 1, EDFs fall below the macro forecast after January 2006 and thus are scaled up after that.

[3] *KMV®, the KMV logo™, Expected Default Frequency™, and EDF™ are trademarks of MIS Quality Management Corp. See www.creditedge.com.*

Figure 1. MKMV EDF™ and Macro Forecast of 5-year Default Rate (2001-2005)



Source: Lehman Brothers and MKMV.

In Figures 2 and 3, we report the average CDS, EDF and HDI for different ratings and sectors in IG and HY. By construction, on average, HDIs are higher than the EDFs.

Figure 2. Average 5-year EDF™, HDI and CDS (2001-2005) by rating (in bp)

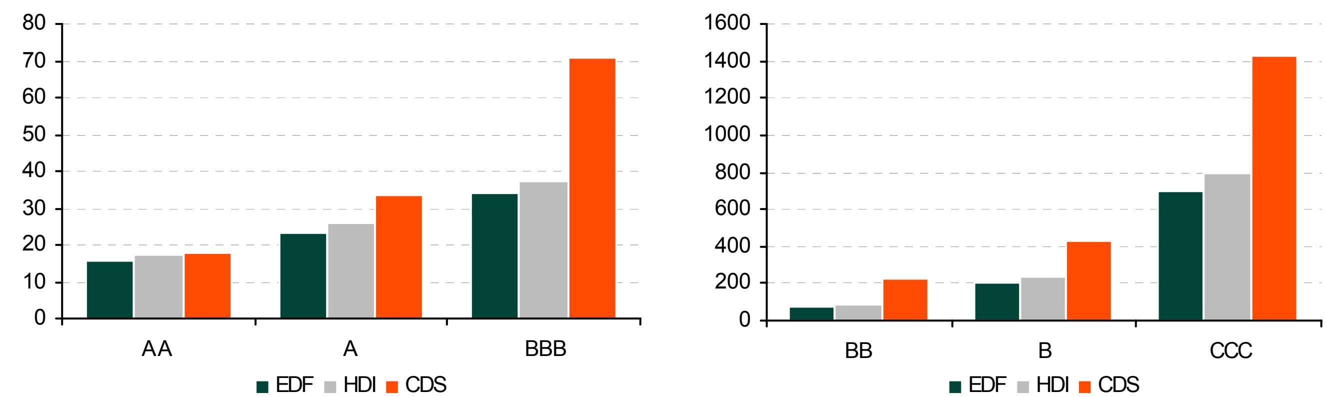
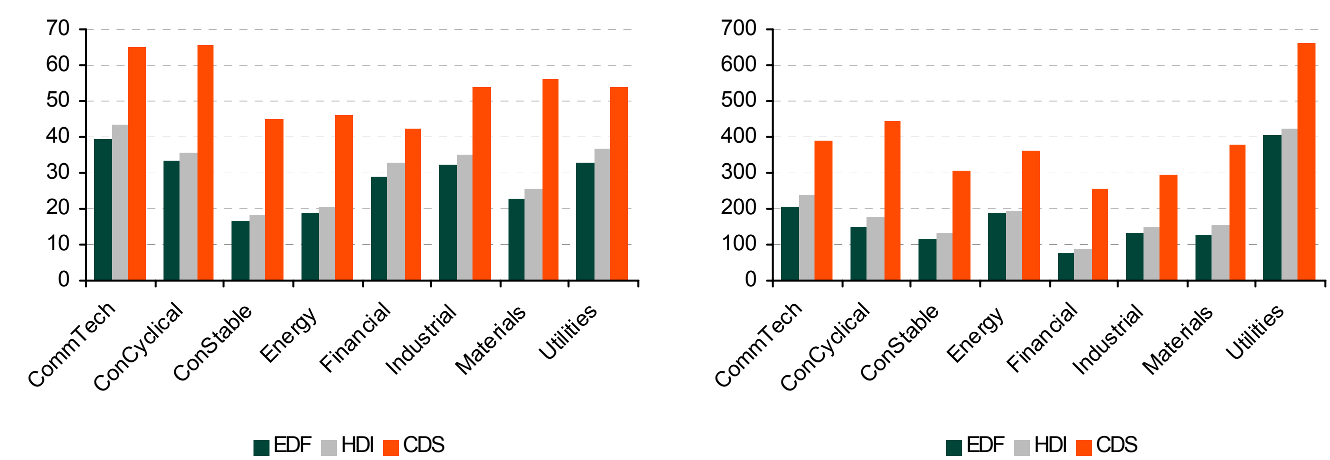


Figure 3. Average 5-year EDF™, HDI and CDS (2001-2005) by sector (in bp)



Source: Lehman Brothers and MKMV.

Recovery risk

Recovery value will change according to the macro environment (lower value in recession, higher value in expansion), geographic region, industry environment (growth, profitability) and firm specific characteristics (leverage, asset value, debt seniority, balance sheet).

To simplify the analysis, we estimate a time-series regression model that relates default rates to average observed recovery rates. The regression model is

$$\text{Ln(Recovery)} = \beta * \text{Ln(Default Probability)} + \alpha$$

Here we will make the simplifying assumption that given a default probability we can extrapolate the recovery rate thanks to the estimated regression. We cap our estimates of recovery between 25% and 60% to take out the effect of outliers.

Expected Default Loss

Given our estimates of default probabilities and recovery rates, we are able to calculate the expected default loss as:

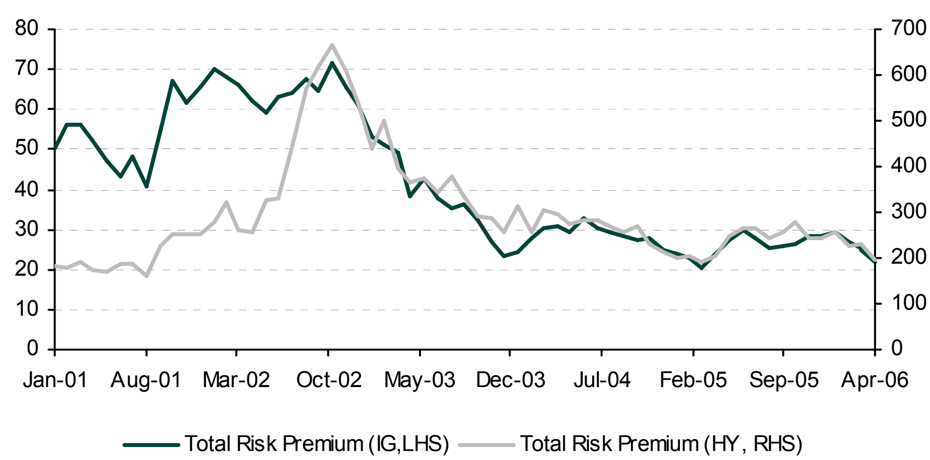
$$\text{Expected Default Loss} = \text{Default Probability} \times (1 - \text{Recovery Rate}).$$

Total Risk Premium

The total risk premium is the component of spreads not explained by expected default loss. Investors demand this premium since they are taking risks that cannot be easily diversified away. The risk premium is generally positive for credit-sensitive assets because their returns usually co-vary positively with the market[4] (Cochrane, 2001). The risk premium is also driven by the degree of risk aversion of investors. Thus more the covariation in returns and the higher the risk aversion, the higher will be the risk premium.

In Figure 4, we report the time variation of the total risk premium for the US IG and HY indexes from 2001 to 2005. We observe large variations in risk premium. The total risk premium reached a peak in October 2002 in the wake of corporate scandals and has fallen to reach current low levels.

Figure 4. Average Total Risk Premium for IG and HY (2001-2005)



Source: Lehman Brothers.

[4] *Modern asset pricing states that any financial asset that has high (low) returns when the whole market does well (poorly) is expected to have a positive (negative) risk premium because it does not provide a financial return when the investor does need it the most (when rest of her portfolio does poorly).*

Another useful way of understanding the risk-premium is to look at it on a risk-adjusted basis. The coverage ratio, defined below, gives the risk-adjusted premium.

Coverage Ratio = CDS Spreads / Expected Default Loss

A high coverage ratio is indicative of a large risk premium, adjusted for the level of risk where the risk is measured by the expected default loss. We also show the coverage ratio by rating categories (Figure 5). We expect these coverage ratios to change over time as the level of risk aversion can change in the marketplace.

3. THE COMPONENTS OF THE TOTAL RISK PREMIUM

The factors driving the total risk premium in CDS spreads are default risk, liquidity risk, volatility risk, and business cycle risk. Spreads and hence risk premium are also exposed to market technical factors that are not captured by the fundamental variables described above.

Default Risk Premium

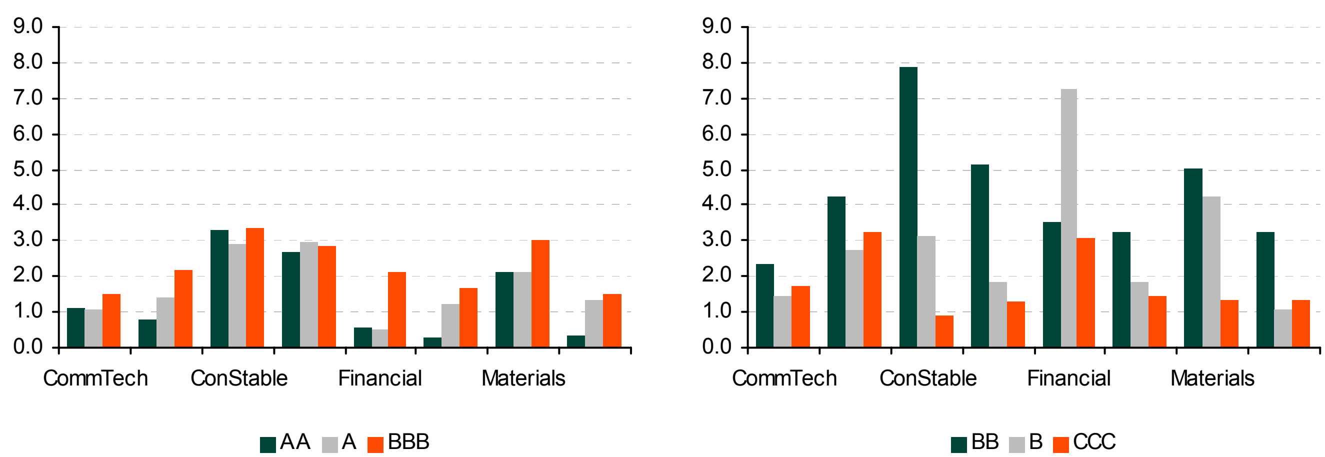
Investors demand a premium for losses beyond the expected default loss in case of a default. This is the default risk premium and we assume that it is proportional to the default probability. The default risk premium is zero either if the default probability is zero or if default risk is purely idiosyncratic and diversifiable.

Market Liquidity Risk Premium

Liquidity risk is associated with rising transaction costs which may be a result of firm or security specific reasons (small size of the firm, few investors or dealers trading securities, and lack of information about the reference entity) or macro reasons (liquidity crisis, flight to quality, and illiquid secondary markets).

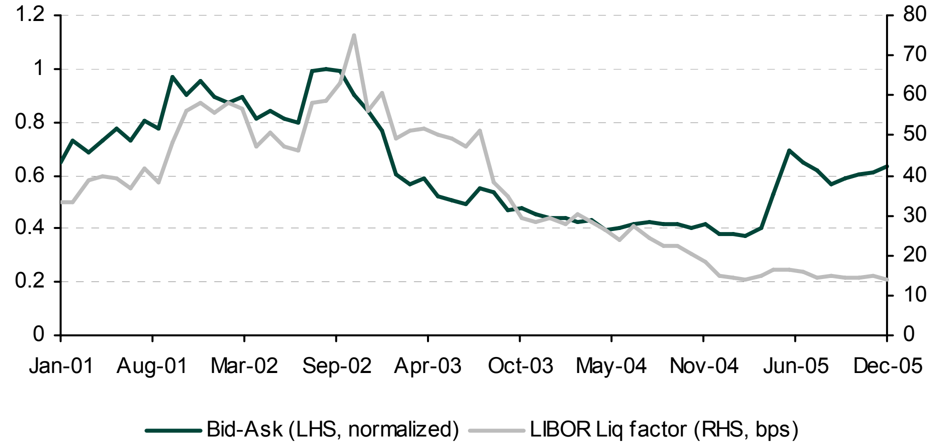
We quantify liquidity risk with a liquidity premium indicator constructed from the corporate bond market. It is the average basis point difference between the spread on the issuer's cheapest and richest bonds, adjusted by the issuer curve (Monkonnen, 2000). We also compare our indicator with the averages of bid-ask spreads in the market (Figure 6). These two indicators track each other quite well till summer 2004 when they seemed to diverge. Given the current low level of spreads, the corporate bond indicator seems to be more intuitive.

Figure 5. Coverage Ratios (2001-2005) by sector



Source: Lehman Brothers.

Figure 6. LIBOR Credit Liquidity Factor (2001-2005)



Source: Lehman Brothers.

Market Volatility Risk Premium

Short-term volatility in spreads represents mark-to-market risk for the investor. In a widening spread environment, it becomes more difficult to diversify volatility risk. To capture this volatility risk, we use VIX as a proxy. The VIX measures market expectations of short-term volatility conveyed by S&P 500 stock index option prices. An alternative is to use historical spread volatility but they wouldn't be forward looking, unlike VIX. VIX has some added advantages as it captures jump risk and some equity downside risk as well.

Business Cycle Premium

Business cycle and credit spreads are correlated with spreads tightening (widening) during expansionary (recessionary) phases. We use 5-year US treasury rates as a proxy for changes in the business cycle. Interest rates tend to be pro-cyclical: they increase when the economy expands and fall during recessions. A defensive financial asset will have a negative business cycle risk premium. Indeed, investors are willing to receive less spreads to hold these assets since they benefit from portfolio diversification when the economy does poorly.

Technicals

The part of credit spreads not explained by the default, liquidity, volatility, or business cycle risks can be a result of a host of factors, which we call 'technicals'. Technicals thus drive spreads away from their fundamental valuation. For instance, spreads can trade tighter because of excess demand or because firm-specific risk is priced. In Figure 7 we discuss several possible sources of these technicals.

Figure 7. Source of Technicals in Credit Markets

Factor	Reason
Excess Market Liquidity	Excess market liquidity due to lack of high-yielding investment opportunities can keep spreads tight
Low Volatility Environment	Investors seeking high-yielding investment opportunities might underestimate risks during low volatility periods and favor positive carry trades which keep spreads tight.
International Demand for US Assets	There is a high demand for US assets and corporate bonds from foreign investors which keeps spreads tight
Structured Credit Innovation	Structured credit innovations (CDO) have led to increasing number of market participants selling protection in order to hedge their leveraged positions. As the leverage of these positions has increased, spreads have tightened.
Idiosyncratic Risk	Event risk is a major driver of issuer specific spread movements. The technicals will also capture spread widening due to firm-specific circumstances such as financial distress.

Source: Lehman Brothers.

4. SPREAD DECOMPOSITION METHODOLOGY

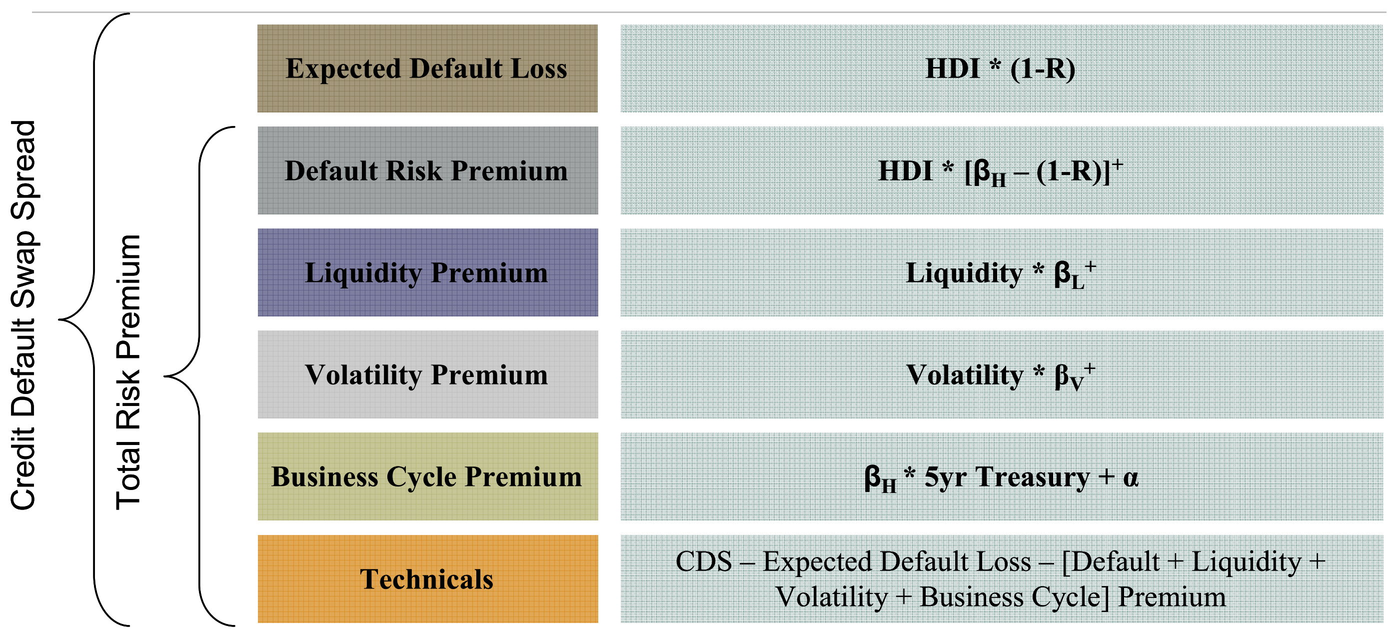
To decompose credit spreads we combine CDS and HDI information from January 2001 to December 2005. We include HY and IG US names and exclude outliers by trimming extreme observations. We decompose each CDS into the expected default loss, a default risk premium, a liquidity premium, a volatility premium and a business cycle premium.

More precisely, our CSI Model is based the following rating level panel data regression:

$$\mathbf{CDS_{i,t} = \beta_H{*}HDI_{i,t} + \beta_{V,sector}{*}Volatility_{\,t} + \beta_{L,sector}{*}Liquidity_{\,t} +}$$
$$\mathbf{\beta_{R,sector}{*}5yr\ Treasury\ Rate + \alpha + \varepsilon_{i,t}}$$

The regression is run for each rating category separately. We also impose ex-post some constraints on the regression *betas* of our model. Specifically, default, liquidity, and volatility factors should have positive *betas* as negative values do not have any natural interpretation. We choose to add the constant term to the business cycle factor because interest rate factor does not have any natural interpretation at zero value contrary to other factors.[5] Any remaining residuals are bundled into technicals. The complete specification is summarized in Figure 8.

Figure 8. Spread Decomposition: CSI Model[6]



Source: Lehman Brothers.

[5] *In other words, even when interest rates are close to zero, there are chances of an economic recession. Hence business cycle premium cannot always be zero by construction when interest rates are equal to zero.*

[6] *β^+ denotes max(β,0)*

5. SPREAD DECOMPOSITION RESULTS

The spread decomposition of US IG and HY universes are reported in Figures 9 and 10. The bold line superimposed on the decomposition chart gives the CDS spreads as observed in the market.

A negative business cycle premium, denoted as a business cycle discount, is observed if an asset outperforms in recession and underperforms in expansion. Technicals are categorized as wide or tight technicals depending on their direction.

In US IG index, technicals and business cycle premium tend to partly offset the liquidity and volatility premium. In a stressed credit environment (such as 2002), liquidity and volatility risks seem to be the main factors driving spreads wide. Interestingly, the expected default loss did not increase as much indicating that the market spread widening was not driven by fundamental default risk but rather by the market perception of risk.

For US HY, the main drivers of credit spreads in the recent period have been the business cycle premium and the default risk premium. Liquidity and volatility premium have been less significant.

Figure 9. Spread Decomposition of US IG Ex-Financials (2001-2005)

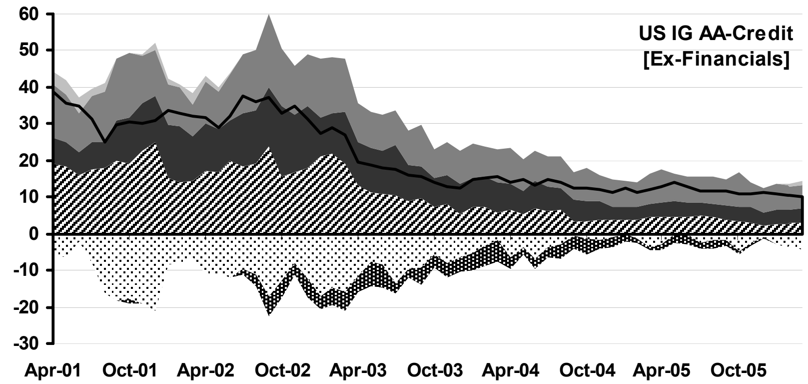
AA credit spreads are mostly driven by expected default loss, volatility premium and liquidity premium. There is a negative business cycle premium and technicals that keep spreads tight.

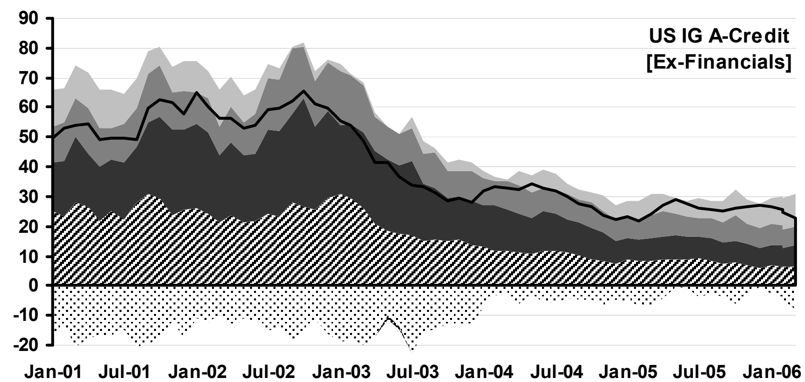
Technicals also keep spreads tight for A-rated issuers. A-rated credits remain sensitive to the business cycle but liquidity and volatility premia have dropped significantly since mid-2002.

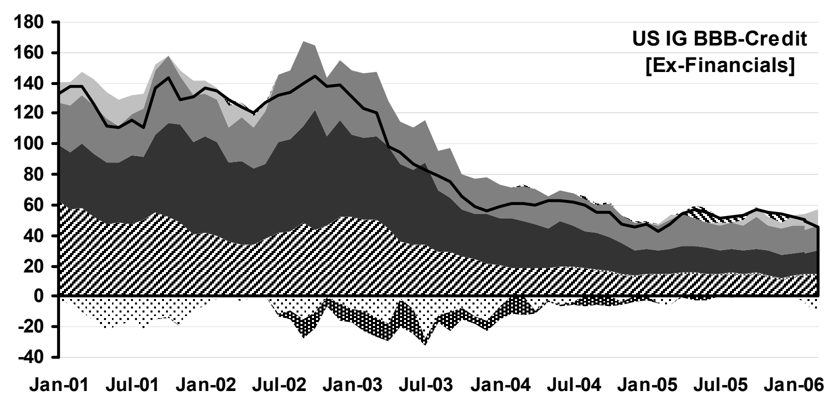
BBB spreads are driven more by fundamental factors (liquidity volatility and business cycle) than technicals compared with AA or A-rated issuers.

The effect of technicals seems to be intuitive as fewer high quality credit assets (ex-financials) are available and are thus becoming more valuable. They are also valuable in a portfolio context since they provide diversification away from equity holdings.

Liquidity premium appears to be a large part of the risk premium for all three rating categories, which is also intuitive. For instance, portfolio managers who are facing high redemption rates or margin calls might be forced to liquidate the assets in their portfolios. The cost of entering into a new trade then increases because of higher liquidity costs.







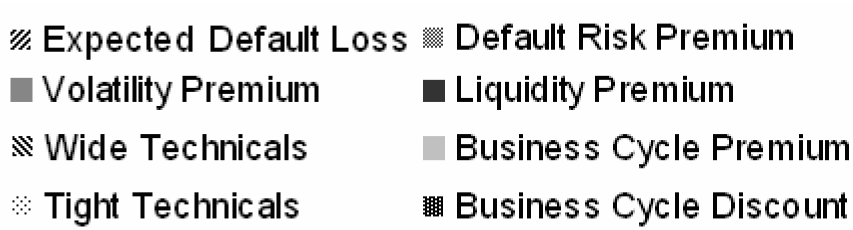


Figure 10. Spread Decomposition of US HY Ex-Financials (2001-2005)

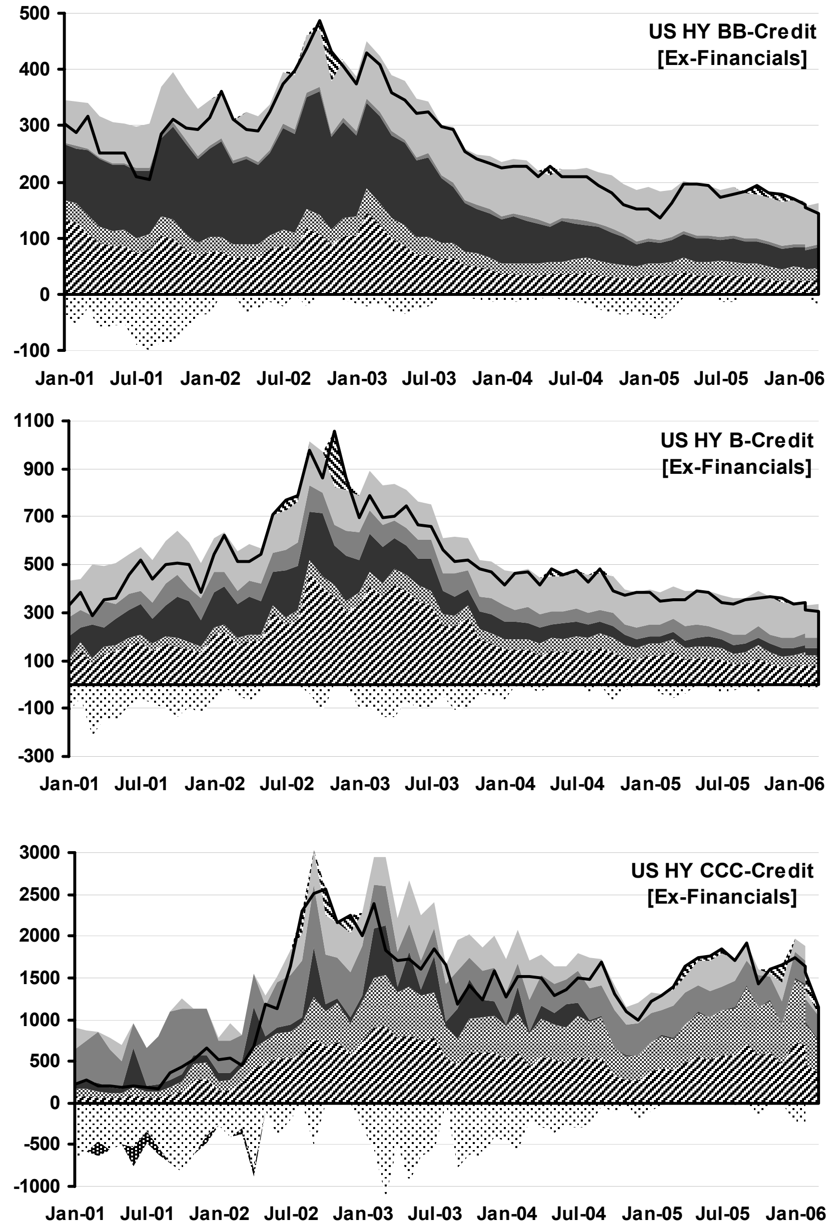
HY credit spreads are significantly more sensitive to business cycle risks than IG credits. In periods of expansion, HY assets outperform; they underperform during recessions. Business cycle premium remains high and positive across ratings for HY issuers.

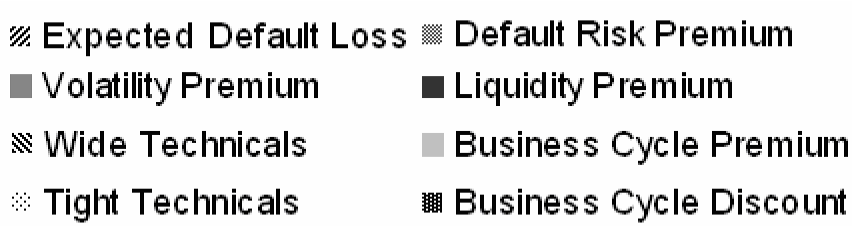
Volatility premium is in general low for BB issuers; a greater proportion of the spreads is explained by the liquidity and default premium.

Default risk seems to be priced as a systematic risk among B and BB names, which is not the case on average for IG names, probably because defaults in the HY market are more concentrated in some periods and are less diversifiable.

This is observed particularly for B issuers where default risk premium has been increasing steadily in the past two years. As a percentage of spreads, however, there is a reduction in liquidity and volatility premium.

Among CCC-rated High Yield CDS, liquidity premium is very low relative to the other risk premia. Default risk premium is the most significant component of the total risk premium, which is understandable since CCC-names are closest to default.



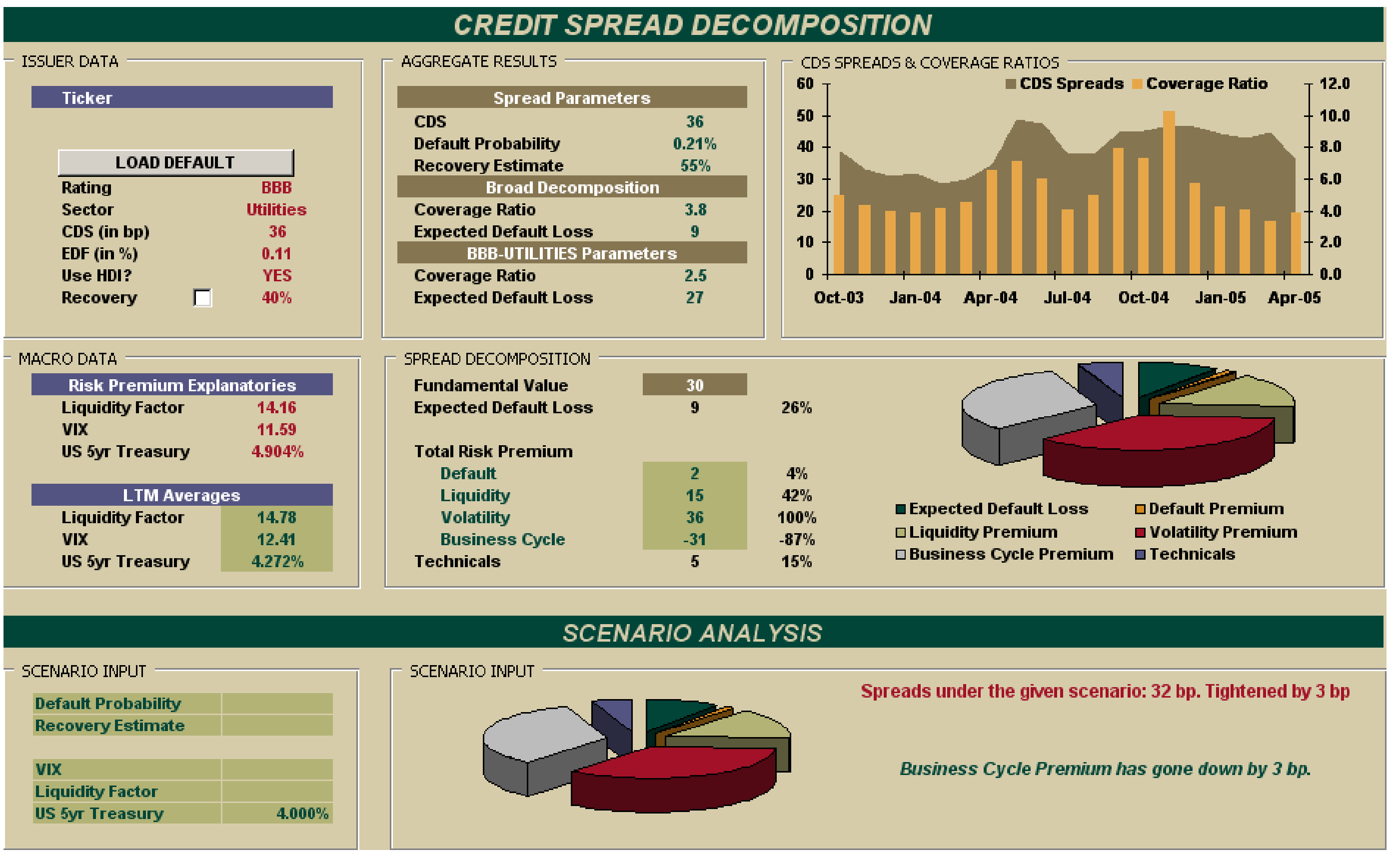


6. APPLICATIONS

Issuer Spread Decomposition & Scenario Analysis

The CSI model can be applied at the issuer level to quantify the current components of risk premium in the market. A sample scenario is presented in Figure 11. In our example, the spread is 36bp, the contribution from the expected default loss is 26% of the spread.

Figure 11. Spread decomposition: CSI Model



Source: Lehman Brothers.

A straightforward application of the spread decomposition is to conduct scenario analyses in which the user can modify the inputs and look at new outputs. The model recalculates the spreads and its constituents given the new inputs. For example, the user can ask: if the likelihood of default increases, what is going to be the impact on spread? If an issuer is downgraded, by how much the spread is going to widen? If the equity market volatility increases, how are spreads going to react? In our example, we have decreased the 5-year treasury rate to 4.0% causing spreads to tighten by 3bp.

Portfolio Optimization

An interesting application of using the decomposed spreads is to construct portfolios that are optimized for constraints such as limiting the total expected default loss. As an illustration, we constructed a long-only portfolio, which is a subset of CDX.IG.3 under the following objective and constraints.

Objective: Maximize Carry

Constraints:

- Limit on total expected default loss for the portfolio
- Limit on total volatility premium for the portfolio—the aim is to pick issuers with low exposure to spread volatility and hence lower mark-to-market risk
- Limit maximum exposure to one issuer to 5% of the total notional

The optimization was run using the spreads and rating as of September 30, 2004, i.e., just after CDX.IG.3 rolled.[1] The portfolio is held for 18 months till the end of March 2006. The total invested notional is $100 million. To benchmark our portfolio, we looked at the returns of the following portfolios

- Buy and Hold CDX.IG.3 ex-financials, ex-government
- Buy and Hold top 20 high spreads name in CDX.IG.3[2]

We summarize the performance of these sample portfolios in Figure 12

Figure 12. Performance of Sample Portfolios

Strategy	Weights	No of Positions	P&L
Buy and Hold CDX.IG.3 [*ex-financials*]	Equal Weights	96	$ 927,000
Buy and Hold top 20 high spreads issuers in CDX.IG.3	Equal Weights	20	$ 642,000
Optimized Portfolio	Optimized	21	$ 2,579,000

Source: Lehman Brothers.

The top 20 high spread had poorer returns compared to our portfolio since it included Delphi (DPH), which defaulted, and thus eroded a significant portion of the returns. Thus as a better benchmark, we compared our portfolio to a portfolio of 20 names with the maximum spread between CDS spreads and HDIs in CDX.IG.3. The results are summarized in Figure 13.

Figure 13. Performance of Sample Portfolios

Strategy	Cumulative P&L	Annualized Returns	Annual Volatility	Sharpe Ratio
Max(CDS-HDI) Portfolio	$ 2,310,000	154 bp	1.20%	1.29
Optimized Portfolio	$ 2,579,000	172 bp	1.01%	1.71

Source: Lehman Brothers.

The optimized portfolio exhibited lower volatility over the period of the test. The Sharpe ratio is thus better for this portfolio.

Similar portfolios can be created depending on the investors overall objectives. The investor can also hedge a long-only portfolio by buying protection on the Index.

Risk Premium in Tranche Markets

Monetization of risk premium in credit markets has been a primary objective of CDOs. With our estimate of expected default loss, the total risk premium for issuers in a portfolio can easily be estimated. This information can be combined with historical correlation measures to come up with expected default losses for liquid tranches in the tranche markets. The

[1] *For estimating the components of total risk premium, we use full sample regression coefficients.*
[2] *Since spreads tightened during this period, we benchmarked our portfolio against issuers with the maximum carry.*

results of this analysis was published in 'How Large is the Risk Premium in Tranche Markets'*,* 04 April 2006, *Structured Credit Research,* Lehman Brothers.

7. CONCLUSIONS

In this article, we have used the CSI model to decompose spreads into expected default loss, liquidity risk premium, credit risk premium and technicals. The credit risk premium was decomposed into a volatility risk premium for variability of spreads, a liquidity risk premium for liquidity risk and a business cycle premium for recession risk.

In our model, factors were proxied by a liquidity indicator constructed from the corporate bond market, a volatility factor from the equity market, a business cycle factor from the treasury market and a default factor measured by the default probability of the issuer.

There can be various applications of the CSI model. The spread decomposition can be run at the issuer level to get the various risk premia. The decomposition can also be used to construct portfolios according to certain objectives like minimizing default risk. Finally, the decomposition can be used to estimate the risk premia in tranche markets.

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APPENDIX A: HYBRID DEFAULT INDICATORS

Our macroeconomic default prediction model is modeled along the lines of Moody's default prediction model introduced in 1990. Our model is based on the regression of default rate on macro factors, company specific information and market technicals. The factors are explained in Figure A1.

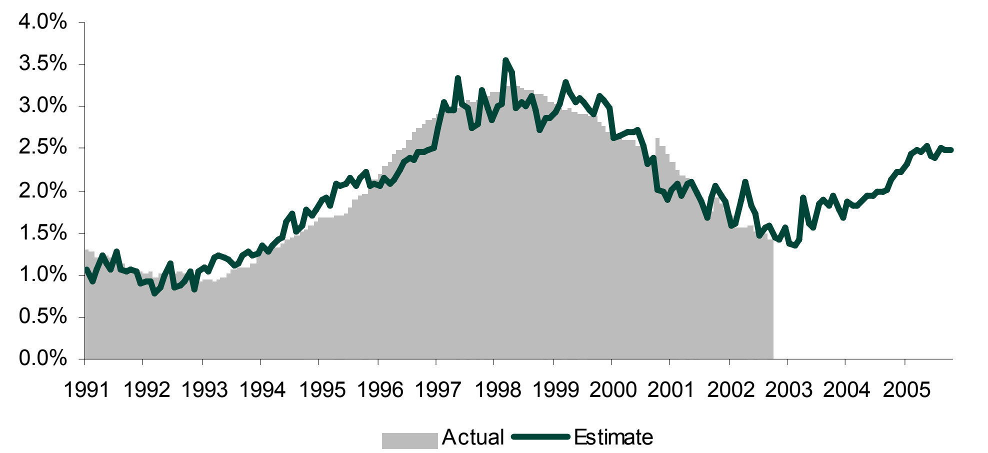
Figure A1. Variables used in Default Prediction Model

Factor	Motivation
Environment Factors	
VIX	Periods of high defaults are generally characterized by high volatility in the credit markets
2yr-10yr Slope	The shape of the yield curve is indicative of the business cycle; the curve tends to steepen in a recession and in the first stage of an economic expansion and flatten in the second stage of an economic expansion.
Issuer Specific Factors	
S&P Total Return	Profitability of firms affect future defaults
S&P1500 Dividend Yield	Defaults can increase if cash payouts increase unusually.
S&P1500 Leverage	High leverage is an indicator of potential defaults
Technicals	
1yr Lagged Private Equity	Increase in private equity raised helps finance LBO transactions
Change in 1yr Speculative Grade Default Rate	During high-default periods, speculative grade issuers are generally the first to default

Source: Lehman Brothers.

We run the regression of 5-year default rates of US All Corp on the estimators and forecast 5-year forward default rates (Figure A2).

Figure A2. Performance of our Default Prediction Model



Source: Moody's, Lehman Brothers.

Our model thus predicts an annualized 5-year default rate of 2.47% at the end of April 2006 compared with MKMV's 1.31%.

Explaining the Bond-Implied CDS Spread and the Basis of a Corporate Bond

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We are introducing a new tool on LehmanLive to calculate the credit default swap (CDS) basis of a corporate bond. The basis is defined as the CDS spread to the bond maturity minus a bond-implied CDS spread. For a bond without embedded options, we calculate the bond-implied CDS spread using the standard model for valuing a CDS. For a bond with embedded options, we use a model with stochastic interest rates and credit spreads. Unlike the standard option adjusted spread (OAS) and the Z-spread, the bond-implied CDS spread is, as its name suggests, directly comparable with a CDS spread. The difference between the Z-spread/OAS and the bond-implied CDS spread can be significant especially for high yield bonds. We recommend using the bond-implied CDS spread not only when comparing a bond with a CDS but also when comparing two bonds.[1]

1. INTRODUCTION

The basis calculator has been introduced to provide a credit spread of a bond that can be compared directly with a CDS spread and used to identify basis trading opportunities. We call this credit spread the *bond-implied CDS spread* and abbreviate it to the *BCDS* spread. It differs from other credit spreads such as the Z-spread and the OAS primarily by explicitly recognizing the recovery rate of the bond. The recovery rate does not enter into the calculation of the Z-spread nor the OAS but it is an important input for calculating the BCDS spread. Conceptually, the Z-spread and OAS are 0%-recovery BCDS spreads and can be very different from, say, a 40%-recovery BCDS spread, especially for discount bonds. In Pedersen (2006) we explained the Z-spread and OAS, how they are calculated, and why they are based on the 0% recovery rate assumption.

To explain the BCDS spread we introduce a related concept: the *CDS-implied bond price*. For standard bullet bonds, indeed for any bond without embedded options, the CDS-implied bond price is calculated using the same type of model as we used for pricing CDS. The model takes as input a curve of current market CDS spread quotes, a recovery rate assumption (often 40%) and a curve of default-free (LIBOR) discount factors, and then calibrates a curve of default probabilities. From these default probabilities, the assumed recovery rate, and the LIBOR discount factors, we can value the bond. This model-based bond value is the *CDS-implied bond price*.

Naturally, the CDS-implied bond price will be different from the bond's observed market price. The idea behind the BCDS spread is to (parallel) shift the CDS spread quotes sufficiently to ensure that they imply a bond price equal to the observed market price. If the CDS-implied bond price is higher than the market price, then the CDS quotes will be shifted up. Conversely, if the CDS-implied bond price is lower than the market price, then the CDS quotes will be shifted down. Once the shift has been found, we use the model (with default probabilities calibrated to the shifted CDS quotes) to calculate the spread of a CDS that matures on the same date as the bond. This CDS spread is the BCDS spread[2]. Finally, we calculate the spread of a CDS that matures on the bond maturity date using default

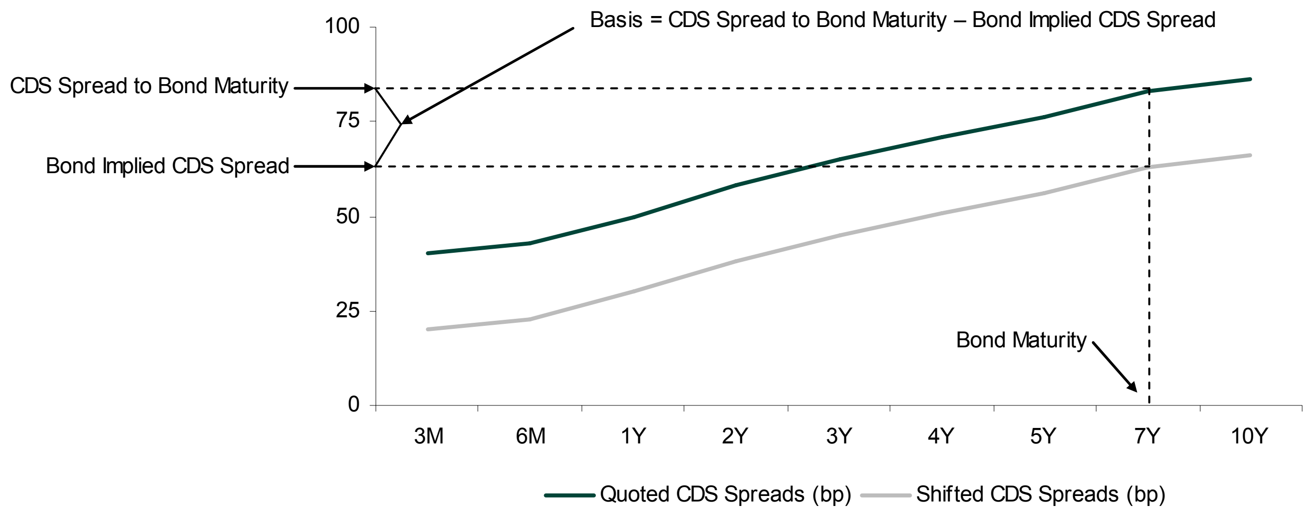
[1] *The Basis Calculator can be accessed with the LehmanLive keyword "bcds". If you enter "bcds 123456AB" and press keyword where 123456AB is the cusip of a bond, the Basis Calculator will open with that bond preloaded. Please note that the LehmanLive calculator will not be released until a few weeks after this article is published. A separate announcement will be sent when the calculator is available.*

[2] *We are only approximately parallel shifting the CDS spread quotes. More precisely, we are parallel shifting the hazard rate curve, i.e. we are increasing (or decreasing) all hazard rates by the same absolute amount, see section 2.*

probabilities calibrated to the original (non-shifted) CDS spread quotes. The basis is this CDS spread minus the BCDS spread:

$$\text{Basis} = \text{CDS Spread} - \text{Bond Implied CDS Spread}$$

Figure 1. Illustration of bond-implied CDS spread and basis



Source: Lehman Brothers.

Conceptually, the BCDS spread is no different for bonds with embedded options. However, for such bonds we run into the practical problem of choosing a good model for valuing the embedded option. We need additional parameters governing the volatility of interest rates and credit spreads. We could also incorporate a correlation between interest rates and credit spreads, but in the first release of the calculator we have chosen to set the correlation to zero. The interest rate dynamics in our model are the same as in our OAS model (see Pedersen (2006)) – we are simply extending our OAS model by adding stochastic default probabilities as a second factor.

We must also consider carefully how to report the basis for a bond with embedded options. The basis is the spread on a CDS minus the BCDS spread. However, it is not immediately clear what maturity the CDS should have. For bonds without embedded options we use the maturity of the bond, but for a callable bond, say, we should use a shorter maturity to take into account the probability that the bond is called and therefore effectively has a shorter maturity. Our solution is to first calculate the expected life of the bond taking into account when the bond may be called and when default may occur. Next we find the *equivalent CDS maturity* which is the maturity of a CDS with the same expected life as the bond. To calculate the expected life of the bond and the CDS we use the same (shifted) default probabilities used for calculating the BCDS spread. Finally we find the CDS spread to the equivalent CDS maturity using the (non-shifted) default probabilities. That CDS spread minus the BCDS spread is the basis.

In Pedersen (2006) we discuss the complications that arise when bonds have embedded options. One focus of that article was to explain how to extend the Z-spread, which is meaningful only for bonds without embedded options, to the OAS which is meaningful also for bonds with embedded options. That discussion is equally relevant for understanding the BCDS spread for bonds with embedded options.

We also encourage a careful reading of McAdie and O'Kane (2001a/b) who discuss reasons for the basis to exist and the risks associated with trading the basis (by buying the bond and buying CDS protection or shorting the bond and selling CDS protection).

The rest of this article is organized as follows: in the next section we show how the BCDS spread is calculated for bonds without embedded options, and in section 3 we explain why and how much the BCDS spread differs from the Z-spread/OAS. In section 4 we show the LehmanLive Basis Calculator (keyword: bcds). We have collected technical details on our option valuation model in section 6. The appendix contains brief explanations of the input/output in the LehmanLive Basis Calculator.

2. CALCULATING THE BCDS SPREAD

In the introduction we explained how the BCDS spread is calculated. For bonds without embedded options it is easy to calculate the BCDS spread. However, calculating the BCDS spread for bonds with embedded options is complicated since it requires an option pricing model (see section 6). In this section we focus on bonds without embedded options and complement the explanation from the introduction with a couple of formulas.

To calculate the BCDS spread we need a CDS valuation model such as the model underlying the LehmanLive CDS calculator (O'Kane and Turnbull (2003) explains this model). A CDS valuation model can calibrate a curve of survival probabilities[3] to a curve of CDS spreads (given a curve of LIBOR discount factors and a recovery rate assumption). The first step in calculating the BCDS spread is to calibrate a curve of survival probabilities to the CDS spread quotes for the same seniority as the bond (and for the same issuer). We can then use these survival probabilities to price the bond.

Consider a bond that has N>0 remaining payments to be paid at times t_i, i=1, …, N. Let C_i denote the size of the payment, in dollars (or another currency) scheduled to be made at time t_i. Let D(t) denote the LIBOR discount factor for time t, and S(t) the probability that the issuer of the bond has not defaulted before time t (the time t survival probability). Let t_0=0 be the valuation time (so $S(t_0) = 1$). Finally let R be the bond's default recovery (in dollars). Using this notation we can write the value of the bond as:

$$
\begin{aligned}
\text{Bond Value} &= D(t_1)S(t_1)C_1 + \ldots + D(t_N)S(t_N)C_N \\
&\quad + D\left(\frac{t_0 + t_1}{2}\right)\big(S(t_0) - S(t_1)\big)R + \ldots + D\left(\frac{t_{N-1} + t_N}{2}\right)\big(S(t_{N-1}) - S(t_N)\big)R
\end{aligned}
\tag{2.1}
$$

The first line gives the value of the bond assuming the recovery is zero. Each payment, say C_i, is multiplied by the survival probability, $S(t_i)$, to the time of the payment (this is the probability that the payment is made) and multiplied by the LIBOR discount factor to that time, $D(t_i)$. The second line gives an approximate value of the principal recovery. For example, $S(t_{i-1}) – S(t_i)$ is the probability that default occurs between t_{i-1} and t_i. In the formula it is assumed that if default occurs in a given time interval then it occurs halfway through and we discount the recovery payment to the midpoint of the interval (our model does not use this simple approximation for the principal recovery but an exact formula). The formula above does not include the potential recovery of coupon accrued from the last coupon date preceding the default to the time of default[4]. The LehmanLive Basis Calculator does not include coupon recovery either.

[3] *The survival probability to time t is the probability that the issuer has not defaulted by time t, that is one minus the probability that default has occurred before time t.*

[4] *An approximation of the coupon recovery can easily be included in the formula: If C_i is the coupon payment to be paid at t_i, we need only to substitute R with $(R + 0.5C_i)$ in the term multiplied by $S(t_{i-1}) – S(t_i)$.*

When we value the bond based on the survival probabilities calibrated to the CDS spread quotes, we likely get a value different from the observed price of the bond. The next step in the BCDS spread calculation is to adjust the survival probabilities until the model produces a value equal to the observed price. We introduce a constant h which can be positive or negative and adjust the survival probabilities the following way:

$$
\begin{aligned}
&S'(t_0) = 1 \\
(2.2)\quad &S'(t_i) = S'(t_{i-1})\min\left\{\frac{S(t_i)}{S(t_{i-1})}\exp\left(-h(t_i - t_{i-1})\right), 1\right\}, \, i = 1,2,\ldots,N
\end{aligned}
$$

We use the adjusted survival probabilities $S'(t_i)$, $i=1,\ldots,N$, in formula (2.2) and solve for the value of h for which the bond value is equal to the observed price. If $h = 0$ then the survival probabilities have not been adjusted. If the bond value based on the unadjusted survival probabilities is higher than the observed price, then we would need to decrease the survival probabilities and we solve to find a positive h. Conversely, if the unadjusted survival probabilities give a lower bond value than the observed price, we will find a negative h.

We call the constant h for the *hazard rate shift*. This is because if we ignore the minimum function, the adjustment in (2.2) is equivalent to shifting all hazard rates by the same absolute amount[5]. The minimum is used to ensure that the shift does not result in negative hazard rates (i.e. negative default probabilities). This is particularly relevant for bonds with very steep CDS spread curves. Once the hazard rate shift has been found, we use the adjusted survival probabilities to calculate the CDS spread to the bond maturity (see O'Kane and Turnbull (2003)).

3. COMPARING CREDIT SPREADS

In the first section we gave an overview of the methodology used to calculate the BCDS spread. Our methodology ensures that the BCDS spread is based on the standard CDS conventions and that it is directly comparable with a CDS spread quote. The Z-spread and the OAS, in contrast, are not directly comparable with a CDS spread. In this section we focus on bonds without embedded options and explain four specific reasons why the Z-spread differs from the BCDS spread (in a later section we discuss how the valuation of embedded options differs in the OAS and BCDS spread calculations):

1. Par vs non-par: The effect of a non-zero recovery rate
2. Daycount convention
3. Payment frequency
4. Recovery of coupon/premium accrual in default

We also briefly discuss a fifth effect which is taken into account in the Z-spread/OAS (under the zero-recovery assumption) but which is important to understand when implementing a basis trade and which illustrates the problems of comparing a CDS spread with an asset swap spread.

5. Funded vs non-funded: The limitations of the asset swap spread

Par vs non-par: The effect of a non-zero recovery rate

[5] *The hazard rate is the instantaneous default rate. For example, if the hazard rate is 1%, then the probability of default over a short time horizon t is approximately t·1%. See O'Kane and Turnbull (2003). If the hazard rate is constant, say h, then the time t survival probability is exp(-h t).*

The recovery rate assumption is the most important cause for difference between the Z- and BCDS spreads. In this subsection we first explain the qualitative effect of the recovery rate assumption and later give numerical examples that quantify the effect.

To explain the qualitative effect we compare the return on two investments which are held to maturity with all intermediate cashflow reinvested. The investments are:

1. Buy a $100 notional bond. Reinvest all coupon payments by buying additional bonds.
2. Sell $100 notional of CDS protection (with same maturity as the bond) and invest $100 in a LIBOR money market account. Interest from the money market account and premium from the CDS is deposited into the money market account. The notional of the CDS is increased so that it always equals the balance in the money market account.

We assume that the bond's Z-spread is equal to the CDS spread. If we determine that in some situations the bond is a better investment, then we can conclude that in those situations the Z-spread should be lower than the CDS spread for the two investments to be equally attractive. Conversely, if we determine that the CDS is a better investment, then we conclude that the CDS spread should be lower than the Z-spread in a no-arbitrage world.

We compare the two investments under four simplifying assumptions:

1. The Z-spread and the CDS spread stay constant over time.
2. The (LIBOR) interest rates evolve deterministically over time as determined by the forward rates.
3. We do not consider the effects of daycount, payment frequency and coupon / premium recovery assumptions (these are discussed in later subsections). For example, assume that everything is based on continuous compounding/payment.
4. There are no bid-offer transaction costs.

Under these assumptions and assuming no default, the Z-spread is the excess return (over LIBOR) earned on the bond investment and the CDS spread is the excess return (over LIBOR) earned on the CDS investment. In other words, if there is no default the returns on the two investments are identical. This means that **the investments must be distinguished by their return in a default scenario**.

If the recovery rate is zero both investments return -100% in any scenario where default occurs before the maturity of the investments (because coupon/interest/premium is reinvested). We conclude that **the investments are equally attractive (or equally unattractive) when the recovery rate is zero.**

Now assume that the recovery rate is greater than zero, say 40%. Further assume that the bond is priced at par and stays there throughout the life of the investment. This means that the LIBOR curve must be flat and that the bond's coupon is equal to the LIBOR rate plus the Z-spread. The two investments will therefore provide the same cashflow for reinvestment and no matter when default occurs they will provide the same return. We conclude that **the two investments are equally attractive when the bond price is par and the LIBOR curve is flat**.

Suppose the bond price is below par, for example at $80, the recovery rate is 40%, and that the LIBOR curve is flat. If there is an immediate default, the bond investment will return -50% whereas the CDS investment will return -60%. In the other extreme case where default occurs immediately before maturity, the investments will provide the same return. Under the flat LIBOR curve assumption, the bond price will increase monotonically to par and the bond investment will provide a higher return than the CDS investment no matter when default occurs. Since the investments provide the same return if there is no default, we conclude that

the bond is the better investment when the bond is priced below par, the price monotonically increases to par and the LIBOR curve is flat. Using analogous arguments we conclude than **the CDS is the better investment when the bond is priced above par, the price monotonically decreases to par and the LIBOR curve is flat**.

The above case hinted at the effect of the LIBOR curve shape on the return of the two investments. Suppose that the LIBOR curve is upwards sloping but the bond price is par. In this case the cashflow from the bond investment will initially be higher than the cashflow from the CDS investment (because LIBOR rates are low initially). This will tend to favor the bond investment in default scenarios. Additionally, an initial bond price of par and an upward-sloping LIBOR curve means that the bond price must initially decrease and then later increase back to par. This means that the bond coupon reinvestment will be made into a below-par bond and therefore that the recovery on the coupon reinvestment will be greater than the recovery on the reinvested interest and premium from the CDS investment. So both the size of the initial cashflow and the recovery on the reinvested cashflow will be greater for the bond investment and we conclude that **the bond is the better investment when the LIBOR curve is upward-sloping and the bond price is par**. Conversely, **if the LIBOR curve is downward-sloping and the bond price is par then the CDS is the better investment**.

Above we explained the qualitative effects of the recovery rate being different from zero. In Figures 2a and b we quantify the effects by calculating the Z-spread and BCDS spreads based on different recovery rate assumptions for two bonds at three different dates[6]. We have used correct daycount, payment frequency and coupon/premium recovery assumptions. This means that part of the difference between Z- and BCDS spreads is due to the reasons explained in the next subsections.

Figure 2a. Comparison of Z- and BCDS spreads for VC 7% March 10, 2014

Date	Price	Z-Spread	BCDS Spread		
			0% Recovery	40% Recovery	70% Recovery
December 1, 2004	93.00	343	328	339	374
May 3, 2005	70	820	794	975	N/A
May 10, 2006	85	421	398	420	500

The bond prices are from Lehman Brothers' Index Database. The BCDS spreads are based on shifts to Lehman Brothers' indicative CDS spread curves.

Figure 2b. Comparison of Z- and BCDS spreads for XRX 7.125% June 15, 2010

Date	Price	Z-Spread	BCDS Spread		
			0% Recovery	40% Recovery	70% Recovery
December 1, 2004	107	145	141	137	130
May 3, 2005	104.75	167	161	158	150
May 10, 2006	102.5	94	91	89	85

The bond prices are from Lehman Brothers' Index Database. The BCDS spreads are based on shifts to Lehman Brothers' indicative CDS spread curves.

Notice that in all cases the 0% recovery BCDS spread is lower than the Z-spread. This difference is the combined effect of the daycount, payment frequency, and coupon/premium recovery assumptions explained in the next subsections. These effects are 2-6% of the Z-spread in the examples shown.

[6] *The discussion of the differences between Z- and CDS spreads applies to a comparison of a bond's Z- and BCDS spreads because a BCDS spread is calculated as a CDS spread.*

As explained above, increasing the recovery rate from 0% to 40% increases the BCDS spread for a discount bond and decreases the BCDS spread for a premium bond. This effect is very noticeable in the figures. The effect relative to the spread is larger, the more the bond price differs from par. For discount bonds the effect of the recovery rate assumption works in the opposite direction to the daycount, payment frequency and coupon/premium recovery effects. In some situations the two effects offset each other causing the 40% BCDS spread to be very close to the Z-spread.

When the recovery rate is increased to 70% the differences between the Z- and BCDS spreads become very large, especially for discount bonds. Interestingly, the 70% recovery assumption is not unrealistic for the VC bond (on May 10, 2006, the last date in Figure 2a, our traders' indicative recovery rate for mark-to-market of a CDS referencing VC was 65%).

Daycount convention

A CDS spread is based on the Actual/360 daycount convention. The market standard is to use the 30/360 convention for Z-spread whereas the Lehman Brothers OAS is based on Actual/365.25 daycount. This effect alone means that the CDS spread should be roughly 1.5bp lower than the Z-spread and OAS for each 100bp. For example, a 400bp CDS spread corresponds to a 406bp Z-spread/OAS when only considering the difference in daycount conventions.

Payment frequency

A CDS spread is based on quarterly payments. The market convention for Z-spread is semi-annual compounding, whereas the Lehman Brothers OAS is based on continuous compounding. One measure of the effect of the payment frequency is that a 100bp CDS with quarterly payments corresponds in value to a 100.7bp CDS with semi-annual payments and a 99.3bp CDS with continuous payments. When measured this way, the effect is roughly proportional to the spread level. The absolute size of the effect is roughly the same for the Z-spread and OAS but depends on the spread level, the level of interest rates and the coupon.[7]

Recovery of coupon/premium accrual in default

When a company defaults, for example by filling for bankruptcy protection, lenders generally have a claim on the loan's outstanding principal and the accrued interest earned between the last payment date and the date of default, although the latter may not always be recognized. Even if a bondholder's claim for accrued interest is recognized, the bond-holder will not receive the full amount but only the amount multiplied by the recovery rate. In contrast, in a standard CDS contract the protection seller will be paid the full premium accrued between the last payment date and the default event.

One way to measure the effect is by the spread difference between a standard CDS and another CDS that is identical except that following a default the protection buyer does not have to pay accrued premium. With this measure the effect is very small for low spread credits but can be significant for high spread credits. For example, with a 40% recovery rate, the effect is 0.8bp at 200bp spread, 3.3bp at 400bp spread, and 13bp at 800bp spread. This measure captures only the coupon recovery on the spread (in excess of LIBOR). The effect is larger if we compare the bond with a funded CDS (e.g. a floating rate credit linked note) with full recovery of accrued interest.

[7] *There is another effect of the payment frequency which is due to the different ways a Z-spread and CDS spread is calculated and which causes the Z- and BCDS spreads to differ if the payment frequency is the same as long as it is not continuous (even if the recovery rate is zero and the daycount convention is the same). For a 3-month frequency that effect can be 0.5%-1.5% of the spread (0.8bp at 100bp and 7bp at 500bp). This effect causes the Z-spread to be higher than the BCDS spread.*

Funded vs non-funded: The limitations of the asset swap spread

Above we compared investing in a bond with selling CDS protection and investing in a LIBOR money market account. To make the bond and CDS investments comparable we assumed that all coupons/interest/premium were reinvested. In reality, bid-offer costs renders it too costly to follow such a strategy.

Another way to make the bond and CDS investments comparable is to instead remove the funding from the bond investment in an attempt to make it comparable to selling CDS protection. We would then compare the following two investments:

1. Buy a \$100 notional bond. Borrow \$100 in a LIBOR money market account. Enter into a fixed-for-floating interest rate swap with an initial market value of \$100 minus the bond price.
2. Sell \$100 notional of CDS protection.

This type of bond investment is called buying the bond on asset swap. It entails no initial cash outlay and it pays a fixed spread, called the asset swap spread, on the notional until maturity or default, whichever arrives first. Similarly, the CDS investment pays a fixed spread (the CDS spread) until maturity or default.

We want to compare the two investments under the assumption that the asset swap spread is equal to the CDS spread. As explained in the previous paragraph, if there is no default before maturity, the two investments will provide exactly the same cashflow. This means that the two investments are distinguished by their payoff in scenarios where default occurs before maturity.

At default, the CDS investment has a payoff of -\$100·(1-R), where R is the recovery rate. This is the same as the payoff from the bond investment if the mark-to-market of the interest rate swap is zero at time of default. For example, this would be the case if the bond price is par, the LIBOR curve is flat and forward rates are realized.

If the bond price is different from par we get an effect similar to that discussed in the first subsection where we compared the Z- and BCDS spreads. For a discount bond, the initial mark-to-market of the interest rate swap is positive and it will therefore also be positive at default (under the assumption of a flat LIBOR curve with forward rates realized in the future). This means that for a discount bond, the bond investment will provide a higher payoff (less negative) than the CDS in default scenarios. The reverse is the case for a premium bond.

When the LIBOR curve is not flat there is another effect. Suppose the LIBOR curve is upwards sloping (and forward rates are realized) and the bond price is par. In this case, the initial mark-to-market of the interest rate swap is zero, but as time passes and LIBOR rates increase the mark-to-market will become positive (we are fixed rate payers in the swap). In this situation, the bond investment will provide a greater payoff in default than the CDS investment. The reverse is the case when the LIBOR curve is upwards sloping.[8]

When CDS trades on upfront

When the CDS trades on upfront it is natural to calculate a bond-implied CDS upfront price rather than a bond-implied CDS spread. In this case, the bonds usually trade at a fairly large discount and the recovery rate assumption becomes particularly important. It may be useful to determine if there is a recovery rate for which the bond-implied upfront price equals the CDS upfront price and the extent to which this recovery rate differs from the recovery rate determined from fundamental credit analysis.

[8] *In the above arguments we assumed that forward rates will be realized. This assumption can be relaxed to assuming that the time of default is independent of the LIBOR rates. Under that assumption, the arguments above will hold "on average" (where average is under the risk-neutral probability measure) which is enough for the statements made.*

Comparing bonds

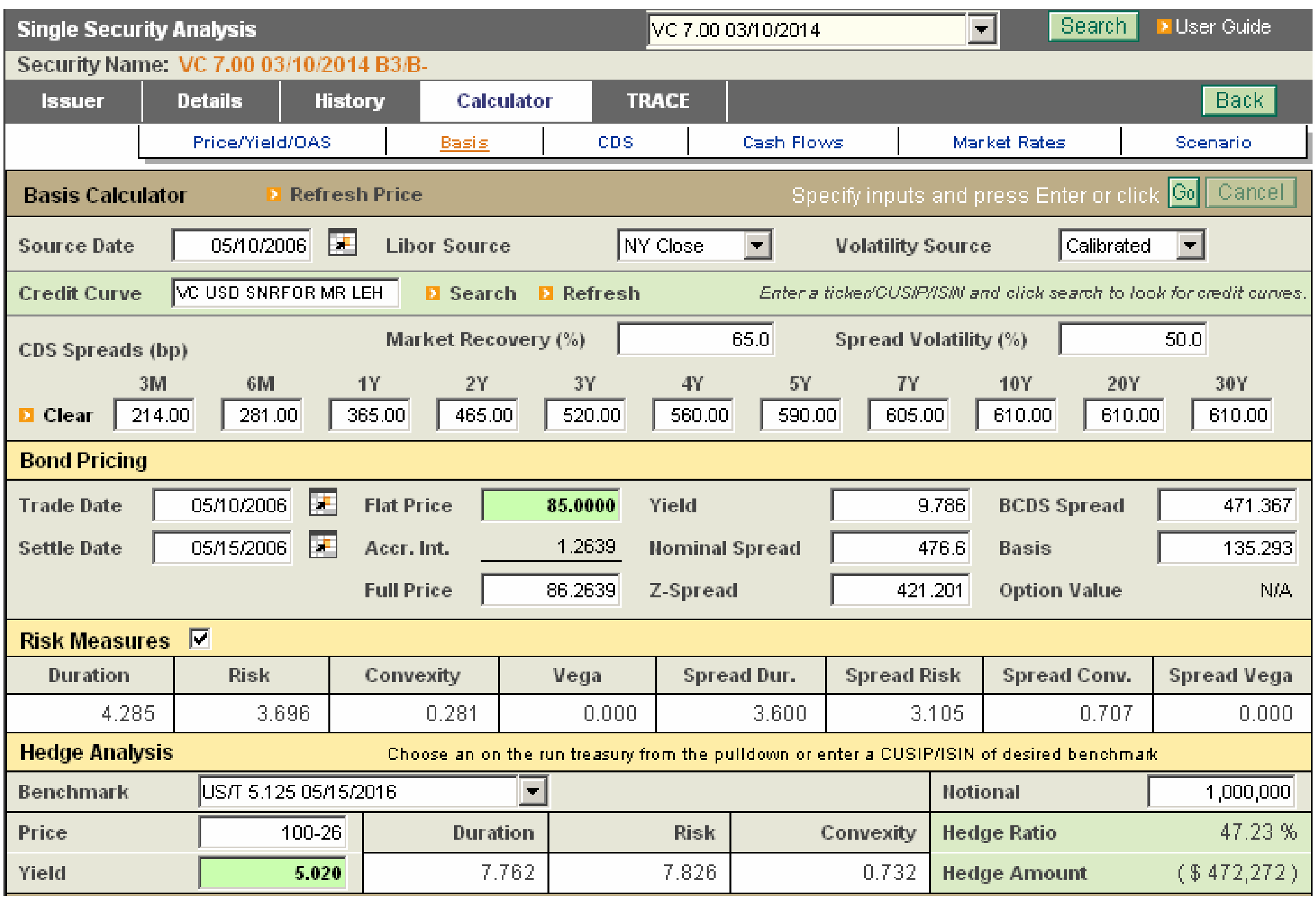
We recommend using the BCDS spread also for comparing bonds. The BCDS spread presents an improvement over the Z-spread and the OAS because of the more realistic recovery rate assumption. The daycount, payment frequency and coupon recovery assumptions, on the other hand, are not important as long as we calculate the spreads for both bonds using the same conventions.

4. USING THE LEHMAN LIVE BASIS CALCULATOR

The easiest way to access the Basis Calculator is to use the keyword field in the upper right-hand corner of the LehmanLive screen and enter "bcds" followed by the CUSIP or ISIN of the bond. The Basis Calculator has been linked to our existing LehmanLive corporate bond analytics and is available on a page next to the OAS calculator.

Figure 3 shows a screenshot of the calculator. The calculator has a user guide that explains all the input/output fields (see appendix) and briefly summarizes the modeling assumptions. The explanations are also available in text boxes that appear when pointing to a field. With the explanations the calculator should be straightforward to use and in this section we highlight only a few elements.

Figure 3. Screenshot of the LehmanLive Basis Calculator



Source: LehmanLive.

Input

The interest rate (LIBOR) curve and volatility assumptions are specified in the first row of the calculator. The curve can be either close-of-business on the *Source Date* or Live (if the source date is today). There are two alternative interest rate volatility assumptions: *Calibrated* or *Single Vol*. If *Calibrated* is chosen, interest rates are assumed to follow the exact same dynamics as in our corporate bond OAS model and the volatility parameters from yesterday's end-of-day calibration of the OAS model will be used. If *Single Vol* is chosen, interest rates are assumed to follow a standard Black-Karasinsky process with a constant short rate volatility specified directly by the user. For details on the interest rate dynamics and the OAS model see Pedersen (2006).

When a bond is loaded into the calculator, the *Credit Curve* input field in the second row should have a code that identifies a particular CDS curve. The spreads and recovery rate for that curve are automatically loaded and displayed but can be overridden by the user. The CDS curve as of the *Source Date* is loaded unless the curve is not available on that date. The text message on the second row shows which curve was loaded, if any (some bonds may not be linked to any CDS curve). Users can search for and load a different credit curve by using the *Search* button in the second row.

The user must also specify a *Spread Volatility* or use the 50% default value. Although we have labeled the input spread volatility, technically it is the hazard rate volatility that is specified[9]. The spread and hazard rate volatilities are closely related and it is often useful to think of the hazard rate volatility as simply the volatility of a very short term, say 3-month, CDS spread. In our model the hazard rate is lognormally distributed and driven by a single random factor. We also assume that interest rates and hazard rates are independent, which is why there is no field to input a correlation. We have limited the model in this way because the correlation is difficult to estimate and zero correlation is a natural and not unreasonable focal point (we are considering a later release of the calculator where the user will have the ability to directly specify the correlation).

Output

There are three sections with output: Bond Pricing, Risk Measures and Hedge Analysis.

The Bond Pricing section converts between six ways of specifying the bond price. Based on user input of either: (1) bond price, (2) yield, (3) spread to benchmark (nominal spread), (4) Z-spread, (5) bond-implied CDS spread (BCDS spread), or (6) basis, the remaining five variables are calculated[10]. The value of an embedded option, if any, is also calculated. The CDS-implied bond price can be calculated by entering a basis = 0. The CDS-implied bond price complements the bond-implied CDS spread as a different measure of the richness/cheapness of the bond relative to a CDS.

Interest rate and spread duration, convexity and vega are calculated in the Risk Measures section (see the appendix for definitions). These calculations take significantly longer than the calculations performed in the Bond Pricing section and can therefore be turned off in the check box.

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[9] *The hazard rate is the instantaneous default rate. For example, if the hazard rate is 1%, then the probability of default over a short time horizon t is approximately t·1%. See O'Kane and Turnbull (2003).*

[10] *There is one limitation: For a bond with an embedded option the user cannot input the BCDS spread nor the basis, except that a basis of 0 can be entered. The first step in calculating bond price from the BCDS spread is to determine the shift that must be applied to the CDS curve. This is easily done if we know the maturity of the BCDS spread (or if the CDS curve is flat) but is more involved if the bond has an embedded option because in that case the maturity of the BCDS spread depends on the BCDS spread itself. Although it is easy to solve this problem numerically, due to technical issues in the production integration of our CDS curve and option pricing models, the performance of that calculation is poor and we have decided not to include it in the first release of the calculator.*

The last section with Hedge Analysis is identical to the hedge analysis section in the OAS calculator. In this section the user chooses a benchmark Treasury bond and the calculator returns the hedge ratio by dividing the interest rate risk (duration times price) of the corporate bond by that of the Treasury.

Another interesting feature is a link to the CDS calculator on a separate page next to the Basis page. Clicking on the CDS tab will open the CDS calculator with the BCDS spread curve preloaded. From the BCDS curve, the bond's BCDS spread can be calculated.

5. OPTION VALUATION MODEL

This section contains a brief technical summary of the model we use for bonds with embedded options. See Pedersen (2006) for a low tech conceptual discussion of how we deal with embedded options when calculating a credit spread.

We use a two-factor model for the instantaneous interest rate (the short rate) and the default intensity (the hazard rate). This is the natural extension of our OAS model to include default risk. We believe it is important to use a model that is calibrated to the interest rate volatility market but also incorporates spread volatility. Our model is the simplest way to achieve both of these goals. The vast majority of bonds with embedded options are standard callable bonds and most of these are high yield bonds. It is not uncommon to use price volatility models to price options on high yield bonds and such models are often particularly appropriate for distressed bonds. However, we easily ruled out using a price volatility model because for bonds where such a model may be appropriate any embedded call option is far out of the money and irrelevant for the BCDS spread. The bonds for which the option valuation is most important have a lower spread and for those both interest rate and spread volatility are important.

The model is implemented in a two-dimensional lattice using a numerical method that is beyond this paper to explain. The two processes:

(6.1) $dx_1(t) = -\kappa_1(t)x_1(t) + \sigma_1(t)dW_1(t)$

(6.2) $dx_2(t) = -\kappa_2(t)x_2(t) + \sigma_2(t)dW_2(t)$

are modeled directly in the lattice. κ_1, κ_2, σ_1 and σ_2 are deterministic functions of time and W_1 and W_2 are correlated Brownian motions with correlation parameter ρ. The short rate, r(t), and the intensity, h(t), are given by:

(6.3) $r(t) = \alpha_1(t)\exp(x_1(t)) - \gamma$

(6.4) $h(t) = \alpha_2(t)\exp(x_2(t))$

where $\alpha_1(t)$ and $\alpha_2(t)$ are deterministic functions of time. The model also assumes a constant recovery rate, R.

Several parameters are preset in the basis calculator. We assume no mean reversion in the hazard rate ($\kappa_2(t) = 0$), constant hazard rate volatility ($\sigma_2(t) = \sigma_2$) and zero short rate / hazard rate correlation ($\rho = 0$). The shift, γ, and the interest rate mean reversion, $\kappa_1(t)$, are fixed (currently at $\gamma = 0.4$, $\kappa_1(t) = 0$ for $t<5$ and $\kappa_1(t) = 0.03$ for $t\geq5$). The user directly specifies the hazard rate volatility, σ_2, and the recovery rate, R. The interest rate volatility parameters, $\sigma_1(t)$, are calibrated daily to end-of-day interest rate swaption prices as explained below. The remaining parameters ($\alpha_1(t)$ and $\alpha_2(t)$) are calibrated live to the specified LIBOR curve, CDS spreads and recovery rate.

We calibrate 14 different piece-wise constant $\sigma_1(t)$'s based on different sets of at-the-money interest rate swaption prices. Each calibration is indexed by a maturity in years. We calibrate $\sigma_1(t;T)$, T=1,2,3,4,5,6,7,8,9,19,15,20,25,30, to the set of u-into-v swaptions where u = 1 month, 3 months, 6 months, 1 year, 2 years, 3 years, …, T-1 years and v = T years - u (rounded to nearest half-year). Thus, $\sigma_1(t;3)$ is calibrated to 1M-into-3Y, 3M-into-3Y, 6M-into-30M, 1Y-into-2Y and 2Y-into-1Y swaptions. To calculate the bond-implied CDS spread of a T maturity bond we use $\sigma_1(t) = \sigma_1(t;T)$ if T is in the above list. Otherwise, if T>30 we use $\sigma_1(t) = \sigma_1(t;30)$. If T<30 we linearly interpolate between the two maturities in the above list that surrounds T. For example, we use $\sigma_1(t) = 0.5 \cdot \sigma_1(t;9) + 0.5 \cdot \sigma_1(t;10)$ for a bond with T = 9.5 years to maturity.

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APPENDIX: EXPLANATION OF THE TERMS IN THE BASIS CALCULATOR

This section contains a list of the input/output fields in the calculator together with an explanation of their use. The explanation of a particular input/output field is also available in a text box that appears when holding the mouse pointer over the text next to the field.

Source Date

Market prices and calibrated parameters for end-of-day on this date are loaded, if available.

Libor Source

If the Source Date is today, live LIBOR rates can be chosen.

Volatility Source

There are two alternative interest rate volatility assumptions: Calibrated or Single Vol. If *Calibrated* is chosen, interest rates are assumed to follow the exact same dynamics as in our corporate bond OAS model. If *Source Date* is today, then the volatility parameters from yesterday's end-of-day calibration of the OAS model will be used. If *Single Vol* is chosen, interest rates are assumed to follow standard Black-Karasinsky dynamics with a constant short rate volatility specified directly by the user.

Credit Curve

Identifier for the credit curve linked to the bond. A click on *Refresh* will load the Market Recovery and CDS spreads on the Source Date for the chosen credit curve, if available. A click on the *Search* button will allow the user to search for a different credit curve.

Market Recovery

The recovery rate for the specified credit curve as of the Source Date.

Spread Volatility

This is the hazard rate volatility which is close to the volatility of a very short term, say 3-month, CDS spread.

CDS Spreads (bp)

The CDS spreads for the specified credit curve as of the Source Date. These are spreads to the standard CDS maturity dates (20 March, June, September and December) following the single-name CDS convention.

Trade Date

This is the valuation date. All market prices and input parameters are assumed to be valid as of this date.

Settle Date

Bond coupon starts to accrue to the buyer of the bond on this date. The settlement date is used to calculate the accrued interest that the buyer of the bond must pay to the seller in addition to the flat/quoted price. The settlement date is also the date on which the cash payment for the bond must be available in the seller's bank account.

Flat Price / Accrued Interest / Full Price

Flat price is the clean or quoted price of the bond. Accrued interest is the interest accrued on the bond from the previous coupon payment date to the settlement date. The full price, or dirty price, is the flat price plus the accrued interest.

Yield

This is the standard yield-to-maturity of the bond calculated from Full Price. The yield-to-maturity assumes the same daycount (usually 30/360) and compounding conventions as the bond. For a zero-coupon bond annual compounding is used.

Nominal Spread

The yield of the bond minus the yield of the benchmark bond. The benchmark bond is the Treasury bond selected in the *Hedge Analysis* section.

Z-Spread

The Z-spread of the bond calculated using the same daycount and compounding conventions as the bond. For details see the article "Explaining the Lehman Brothers Option Adjusted Spread of a Corporate Bond" in *Quantitative Credit Research Quarterly*, 2006 Q1.

BCDS Spread

The bond-implied CDS spread (also called the BCDS spread). This spread has been calculated from the BCDS spread curve of the bond which is the CDS curve shifted so that the model value of the bond is equal to the bond price specified in the calculator. The BCDS spread curve can be viewed by clicking the CDS tab to the right of the Basis tab.

Basis

The basis is a CDS spread minus the bond-implied CDS spread. For bonds without embedded options, the CDS spread is the spread to the bond maturity based in the credit curve specified. For bonds with embedded options, the maturity of the CDS is chosen so that the CDS has the same expected life as the bond.

Option Value

The value of the bond's embedded option. If the bond does not have an embedded option the field shows "N/A". The Option Value is calculated as the value of the bond without the embedded option minus the Full Price of the bond. The bond without the embedded option is valued using the CDS spreads implied from the bond with the embedded option.

Duration

This is the interest rate duration of the bond. It gives the percentage increase in the bond price per 100bp decrease in interest rates. The duration is calculated using the bond values arising when the swap rates are parallel shifted up 25bp and down 25bp. The BCDS spread curve is kept unchanged when the swap rates are shifted.

Risk

Risk = Duration · Full Price / 100

This is the dollar value reduction in the price of the bond per 100 basis point (1%) increase in LIBOR rates.

Convexity

This is the interest rate convexity of the bond. Together, the duration and convexity estimate the percentage change in the bond price following a parallel shift of the swap rate curve as:

$$\frac{\Delta P}{P} \approx -\text{Duration} \cdot \frac{\Delta Y}{100} + \frac{1}{2}\text{Convexity} \cdot \left(\frac{\Delta Y}{100}\right)^2$$

where ΔY is the shift in percentage points to the swap rates, i.e. a 25bp shift is $\Delta Y = 0.25$.

The convexity is calculated from the bond values when the swap rates are shifted up 25bp and shifted down 25bp using also the bond price (no shift).

Vega

This is the interest rate vega. It gives the change in the value of the bond when the interest rate swaption Black volatilities are shifted up by 100bp (keeping interest rates and BCDS spread unchanged). If the Single Vol assumption is used, the Vega gives the sensitivity of the bond price to a 100bp increase in the single vol input.

Spread Dur.

The spread duration gives the percentage increase in the bond price per 100bp decrease in the BCDS spread. The spread duration is calculated using the bond values arising when the BCDS curve is parallel shifted up 5bp and down 5bp. The swap rates are kept unchanged when the BCDS curve is shifted.

Spread Risk

Spread Risk = Spread Duration · Full Price / 100

This is the dollar value reduction in the price of the bond per 100 basis point (1%) increase in the BCDS spread.

Spread Conv.

Together, the Spread Duration and Spread Convexity estimate the percentage change in the bond price following a parallel shift of the BCDS curve as:

$$\frac{\Delta P}{P} \approx -\text{SpreadDuration} \cdot \frac{\Delta Y}{100} + \frac{1}{2}\text{SpreadConvexity} \cdot \left(\frac{\Delta Y}{100}\right)^2$$

where ΔY is the shift in percentage points to the BCDS spread, i.e. a 5bp shift is $\Delta Y = 0.05$.

The spread convexity is calculated from the bond values when the BCDS curve is shifted up 5bp and down 5bp using also the Full Bond Price (no shift).

Spread Vega

The Spread Vega gives the change in the value of the bond when the Spread Vol is increased by 100bp.

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