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Are Credit Markets Globally Integrated? An Initial Assessment[1]

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We present an overview of the results of an empirical study to assess the degree of globalisation in the credit markets over a period spanning just under 4 years using monthly index data of spread-changes across four major currencies (USD, EUR, JPY and GBP). We find high levels of inter-currency correlation in high-spread issuers and sectors with strong evidence of a link between correlation and spread-level as well as sector-based effects.

1. INTRODUCTION

Launched in 1999, the Lehman Brothers Global Aggregate Index provides investors with a representation of a highly diverse opportunity set, spanning currency, yield curve and credit investments. Global diversification of credit holdings ensures the index is less influenced by the idiosyncratic performance of a particular bond or issuer than traditional single currency bond indices and allows for diversity in outperformance strategies. The opportunities offered by global credit indices include sector and name selection as well as the choice of currency in which these views are implemented. These opportunities present the investor with questions of the following type: Can one assume that different bonds from the same issuer denominated in various currencies are substitutes one for the other in the sense that movements in the spread of one will be mirrored by the other? In the realm of index replication – for example with respect to the Japanese corporate market – can one use $, € or £ bonds from Japanese corporate issuers as a proxy? In risk management, does the holding of various issues from one issuer but in differing currencies of denomination lessen the issuer specific risk-exposure to that issuer or increase it? Can we talk in terms of global risk factors, or global sector effects that are common to all bonds in those sectors, appropriately defined, irrespective of their currency of denomination? It is these types of questions that we sought to address in our study.

In this article we report the results of an empirical study of spread performance in the Global Aggregate Index focusing on the major four currencies over a time history of 44 months. For identical issuer, sector or quality buckets, we observe systematic differences in spread level and volatility across currencies. We measure cross-currency correlation of average spread changes for individual issuers and attempt to explain it as a function of spread level, average rating and industry sector. Short of the conceptual framework provided by a global risk model, our results point to some simple investment conclusions for global credit investors. We find that as spread level increases, accompanied by volatility, the correlation between issuer Libor spread movements in issues denominated in different currencies rises to levels of up to 90%. In addition, our study provides evidence that correlation has increased in recent times in tandem with the general rise in spread levels.

The article is organised as follows: In Section 2 we describe the underlying data set that formed the basis of our study, the index with which the data set is to be compared and the

[1] *We would like to thank a number of people for their help throughout the study namely, Vasant Naik, Lev Dynkin, Jay Hyman, Minh Trinh, Lorenzo Isla and Robert McAdie.*

precise nature of the variables that are the focus of the study. In addition, we present some general statistics of the study universe across currencies, namely spread levels and issuer volatilities, and compare the results. Section 3 presents the issuer level results with examples, highlights and summaries of our findings together with analysis of those results. In Section 4 we state the corresponding results at the sector level and seek to establish a sector-related effect with respect to correlation. In Section 5 we consider the question of whether correlation has changed over the 44-month period of our study, and in Section 6 we suggest a number of implications of our study results for investment managers. In Section 7 we make our concluding remarks.

2. DATA AND DESCRIPTIVE STATISTICS

We defined our study universe by considering only liquid, non-collateralised bullet bonds, all members of the credit component of the Lehman Brothers Global Aggregate Index. This was achieved by setting a liquidity constraint of an equivalent in local currency terms of $500MM outstanding nominal and an average life between 3 and 10 years. The index from which we select our study universe is the Lehman Global Aggregate Index excluding treasuries and collateralised securities. Our study universe captures about 47% of this index in market value terms. Figure 1 details the various proportions of currencies in both the index and the study universe, in terms of both market value and representation of issuers.

As can be seen immediately, whilst US Dollar, Euro and Yen are well represented, Sterling is a very small proportion of both the index and our universe, reflecting the very restricted size of the £-market in comparison with the other three major currencies. Consequently, due to the thinness of the data, we were limited in the conclusions that we could draw about correlations between currency-pairs involving Sterling. To a lesser extent this is also true of Yen-denominated data where, not only are pricing issues more challenging, but in addition data is only available from Jan 2001 onwards. For the above reasons our analysis focuses primarily on the €-$ results but we state results for the other currency pairs where statistically significant.

Figure 1. Summary of Study Data As of End of August 2002

	€	£	¥	$	Other
Market Value					
% of Lehman Brothers Global Aggregate Index (ex. Treasuries and collateralised)	22	5	15	56	2
% of Study Universe	25	2	18	54	0
Study Universe as % of the Index	12	1	8	26	0
Study Universe as % of Index Currency Allocation	55	22	55	46	0
Issuers					
#Index	394	182	147	668	21
#Study Universe	269	48	106	354	0

Our analysis was carried out at three levels, issuer, sector and quality. In each case and for each currency of denomination, the study universe was divided into appropriate buckets. For example, at the issuer level all Ford €-denominated bonds formed one bucket and Ford-$

denominated bonds were in another bucket. Similarly, at the sector level, $-denominated Industrials were grouped together as were £-denominated Financials.

At each month-end for a period spanning 31/1/99 to 31/8/02, the monthly changes in Libor spreads of bonds in a given bucket are averaged to form a time-series of average change in Libor spread for that bucket. Henceforth, we refer to this average as the 'issuer spread', 'sector spread' and so on. The correlations between these time-series form the results of our study. It should be noted that all spread references in the study are with respect to the local swap curve, which was as given by LehmanLive data where available and for intermediate maturities, interpolated linearly.

Clearly, our calculations are vulnerable to inaccuracies in data. The eligibility criteria for the study universe – liquidity, the exclusion of collateralised securities and maturity of 3-10 years – were designed to reduce the likelihood of data errors. In addition, some outliers were removed manually where clearly identified.

Figure 2 presents the average spread-difference between the two currencies in each currency-pair. We have also detailed the relative numbers of issuers in that combination which, at the 95% confidence level, exhibited relative spreads of the same sign as the averages over the appropriate time-windows[2]. For example, in the case of the €-£ pair 18 out of 32 issuers issuing in both currencies (with at least 12 points of comparison for OAS) produced €-spreads higher than their £-spreads in a 95% significant manner across time. The overall average spread-difference across all tickers between €- and £-spreads was measured to be 6 bp – the average €-spread exceeding the £-spread by this margin.

Figure 2. Spread Summary Table

	€-£	€-¥	€-$	£-¥	£-$	¥-$
#Issuers tested	33	34	94	10	31	48
Ave OAS Currency 1(bp)	36	46	59	25	33	24
Ave OAS Currency 2(bp)	30	35	69	18	48	50
Ave OAS Curr1 – Ave OAS Curr2	6	10	-10	7	-15	-27
#Issuers with same sign as average at 95% conf. level	18	19	54	2	23	29

Similarly the volatilities of the changes in Libor spreads were compared between currencies and the results are summarised in Figure 3 below.

Figure 3. Comparison of Systematic Volatilities

	€-£	€-¥	€-$	£-¥	£-$	¥-$
#Issuers tested	32	33	92	10	30	48
Ave Vol (Currency 1, bp/month)	9	12	18	13	14	7
Ave Vol (Currency 2, bp/month)	14	8	19	7	13	13
Mean [Ave_1 - Ave_2] (bp)	-5	3	-2	5	1	-6
% Issuers with Vol_1-Vol_2 of same sign as mean	88	64	77	90	50	75

[2] *Note, since only those issuers with complete-pairs of data of a year or more in total were considered the number of issuers involved is very much smaller in this sample than in the universe as a whole. It should also be noted that the average issuer spread in any given currency will differ depending of the second currency in the currency pairing. This is because the spread is averaged over different time periods and issuers depending on the currency pairing.*

The above statistics indicate that, all other things being equal, of the four major currencies, US Dollar denominated issues exhibit the highest spread and that Sterling and Dollar together are at the top of the volatility range with Yen denominated currencies exhibiting the lowest spreads and volatilities.

3. ISSUER RESULTS

The methodology and results of the study are best illustrated in the following two examples.

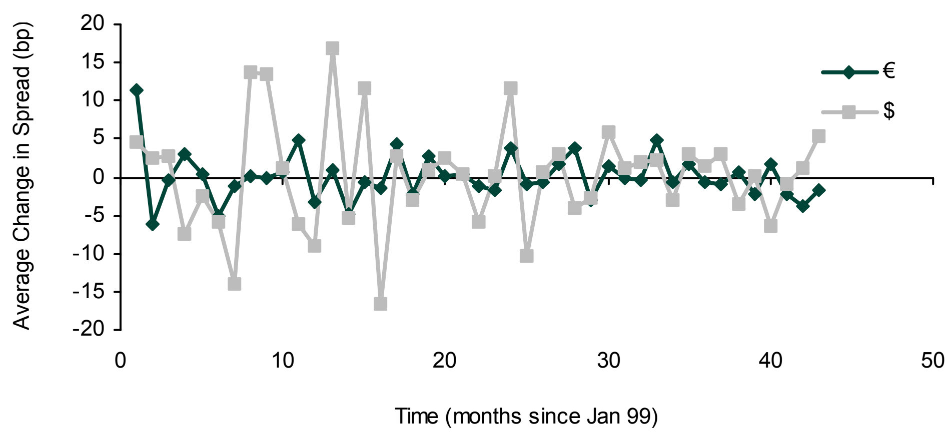
Example 1. European Investment Bank (EIB)

The EIB issued securities in all four of the major currencies during the period of study and, in particular, our universe contained a total of 22 issues denominated in Euro and 23 in US Dollar. For this issuer, we were able to form the maximum-length time series of 43 months.

The average spread over the entire 44-month period of the €-denominated bonds was measured to be approximately -6 bp, and those denominated in $ averaged approximately -13 bp. The volatilities of changes in Libor spreads was seen to be 3 and 5 bp/m respectively.

We calculated, at each month-end, the average of the changes in Libor spreads across all €-denominated EIB issues – the 'EIB(€) Issuer spread' – doing the same for the Dollar denominated EIB bonds. The two time series are charted below.

Figure 4. EIB Change in Libor Spreads Over Time in € and $

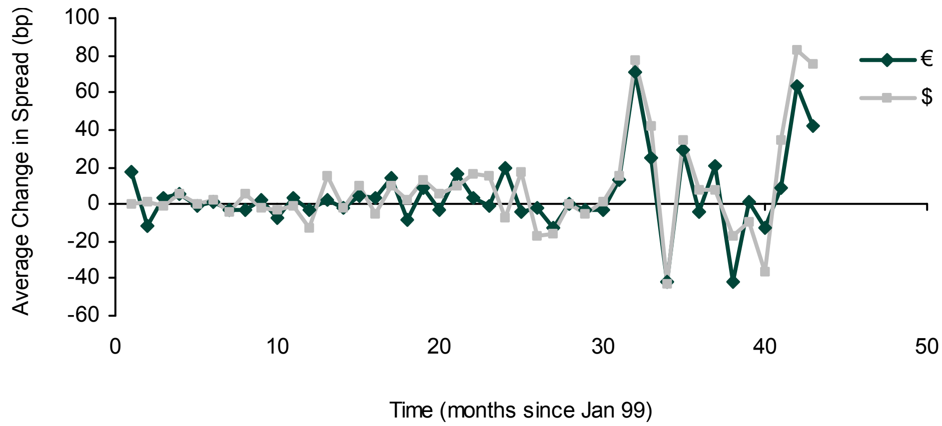


We can see that the two sets of spread behaviour are only tenuously related resulting in a low cross-currency correlation of 0.21 in Libor spread changes for EIB.

Example 2. Ford Capital

The debt of Ford, by contrast, exhibits an entirely different relationship across currencies of denomination. (It should be noted that the scales on the Ford and EIB charts are not the same.)

Figure 5. Ford Change in LIBOR Spreads Over Time in € and $



Over the 44-month period of the study, the Euro and Dollar average spread levels were far higher at 91 and 109 bp respectively reflecting its A2 average credit-rating. In addition the spread volatilities were much larger measuring 20 and 25 bp/month.

The correlation between the Euro and Dollar average changes in Libor spreads for Ford Capital was measured to be 0.86, giving a rejection of the null hypothesis (of zero correlation) at high levels of significance.

It is clear from the chart that Ford, as a credit quality, has deteriorated over the period of the study – it began with an issuer spread of approximately 30 bp, at a rating of A1 and ended with spreads averaging 350 bp with rating of A3/Baa2. The correlation can be seen to increase as the spreads and volatilities rise across the time window. The figure of 0.86 is a correlation across the whole time window and mostly reflects the highly synchronised movements over the more recent time-period during which spreads have widened.

In our empirical analysis, we calculated correlations across all currency-pairs for issuers issuing in two different currencies. In most cases the data was too scarce to draw any statistically significant conclusions. However, in a number of cases and in certain currency-pairs there was sufficient data to give meaningful results. In particular, the €-$ currency-pair, where the data is deepest, allowed a good deal of information to be gathered. Figures 6-10 summarise the results in 5 of the 6 currency-pairs (Sterling-Yen was too sparsely populated to allow such a plot.)

The correlations summarised in the figures are all based on at least 12 months of data, but each issuer need only have one bond at any given month-end in order to allow the computation of the change in issuer spread for that month.

Figure 6. €-£ Correlation of Changes in Issuer Libor-Spreads vs. OAS

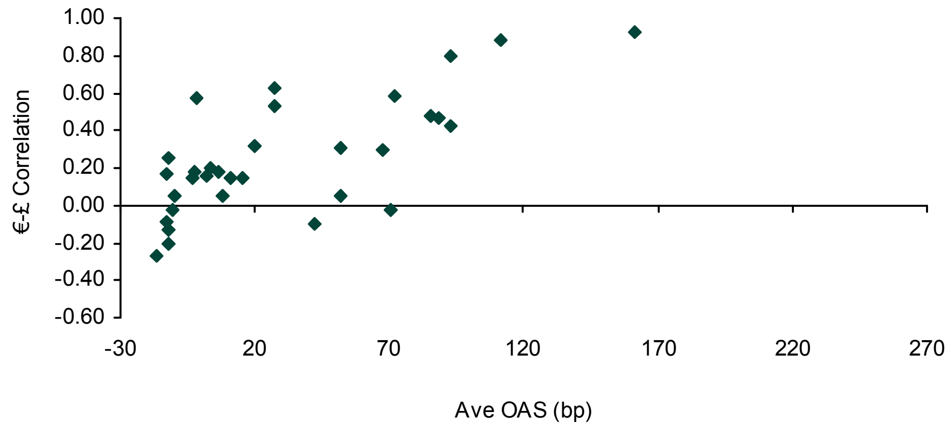


Figure 7. €-¥ Correlation of Changes in Issuer Libor-Spreads vs. OAS

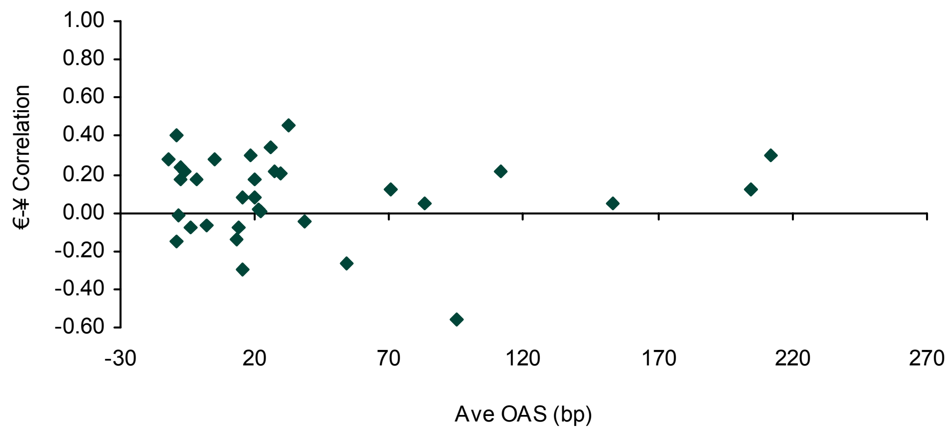


Figure 8. €-$ Correlation of Changes in Issuer Libor-Spreads vs. OAS

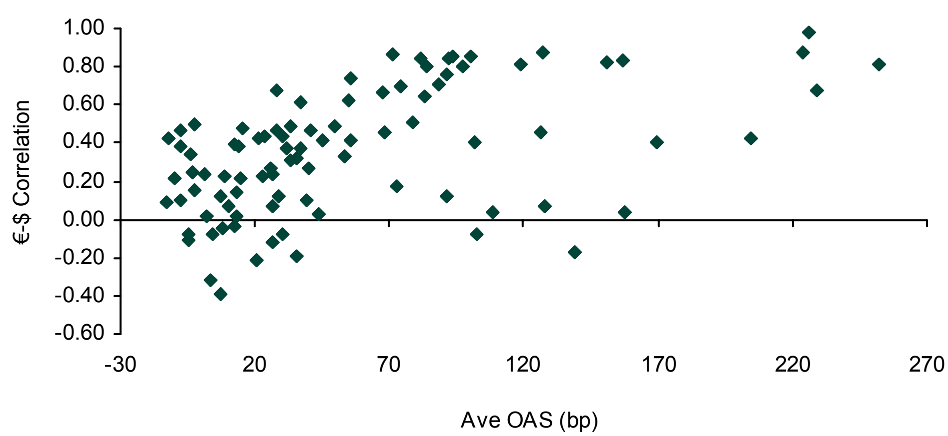


Figure 9. £-$ Correlation of Changes in Issuer Libor-Spreads vs. OAS

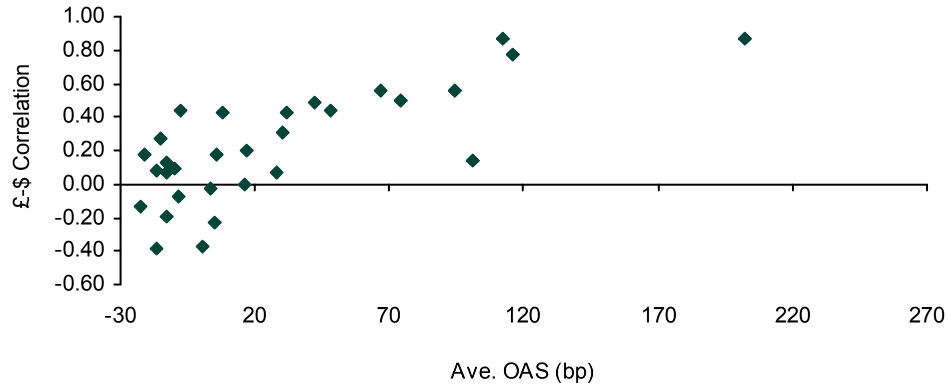


Figure 10. ¥-$ Correlation of Changes in Issuer Libor-Spreads vs. OAS

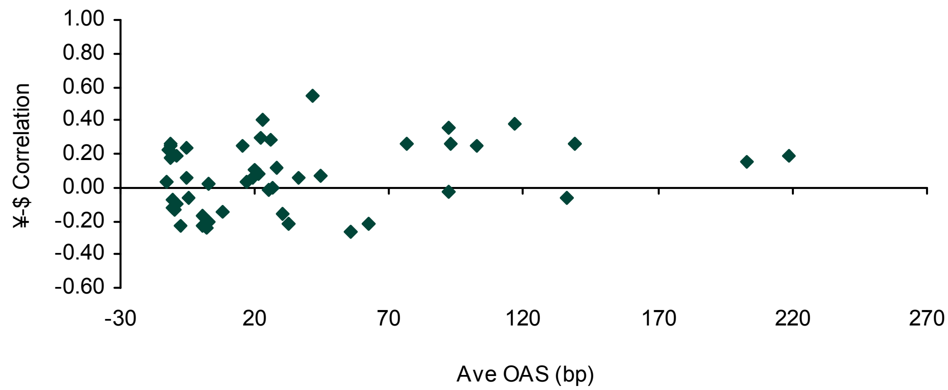
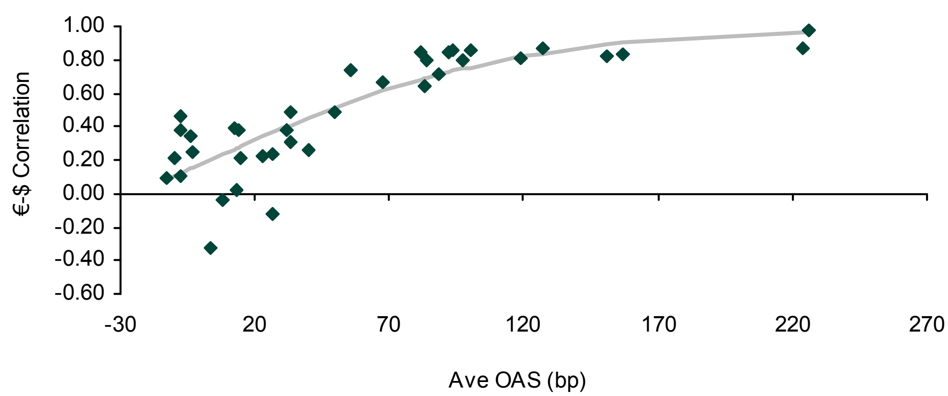


Figure 11. €-$ Correlation of Changes in Issuer Libor-Spreads vs. OAS – More Robust Data [3]



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[3] *The correlations plotted in this figure are based on only those issuers for which at least two bonds are available for any given month.*

In Figure 11, for the €-$ pair – where a good deal more data was available – we have additionally plotted all data with at least two bonds being averaged to form the issuer spread at each month-end. It is evident that, as the number of bonds making up the average from which the issuer spread is formed increases, the 'noise' of the issue-specific behaviour of any given bond is diversified away, allowing the underlying relationship between issuer spread and cross-currency correlation to be observed. As the number of bonds from which the issue spread is calculated is further increased the pattern becomes even sharper.

In Figure 12, below, we present more detail for the most robust results in the €-$ currency-pair, namely those of issuers having €-$ time series of length 12 or more and with, in general, at least three issues making up the average at each date in each currency. The last column, entitled 'p.value' corresponds to the p-value of the null hypothesis that the cross-currency correlations under consideration is zero. The increasing correlation down the table makes the positive relationship between spread and correlation clear.

Figure 12. Summary of Most Robust €-$ Results

			OAS					
Issuer	**Average Rating**	**Sector**	**€**	**$**	**Average**	**rho**	**#data pts.**	**p.value**
LANDWIRTSCHAFTLICHE RENTENBANK	Aaa	Agencies	3	23	13	0.02	21	0.93
I.B.R.D. (WORLD BANK)	Aaa	Supra-National	1	-17	-8	0.10	43	0.52
BANK NEDERLANDSCHE GEMEENTEN	Aaa	Finance	4	11	8	0.14	23	0.85
EUROPEAN INVESTMENT BANK	Aaa	Supra-National	-6	-13	-10	0.21	43	0.17
GENERAL ELECTRIC CAPITAL CORPN	Aaa	Finance	5	25	15	0.21	33	0.23
ABN AMRO BANK NV (CHICAGO)	Aa3	Finance	34	46	40	0.26	38	0.11
RABOBANK GROUP	Aaa	Finance	-3	30	14	0.38	23	0.07
BADEN WURTTEMBERG L-FINANCE NV	Aaa	Local Govt	5	20	12	0.40	23	0.06
KREDITANSTALT FUR WIEDERAUFBAU	Aaa	Agencies	-7	-8	-8	0.47	30	0.01
AIG SUNAMERICA GLOBAL FINANCE	Aa1	Finance	18	48	33	0.49	23	0.02
DAIMLER CHRYSLER NORTH AMERICA	A2	Industrial	65	103	84	0.80	40	0.00
FRANCE TELECOM SA	A3	Industrial	115	198	157	0.83	23	0.00
GENERAL MOTORS ACCEPT CORPN	A2	Finance	76	87	82	0.84	39	0.00
HOUSEHOLD FINANCE CORPORATION	A2	Finance	92	91	92	0.84	43	0.00
FORD CAPITAL BV	A2	Industrial	91	109	100	0.86	43	0.00
MEXICO (UNITED MEXICAN STATES)	Baa3	Sovereign	232	215	224	0.87	29	0.00

Based on the above results, we make the following observations.

Firstly, bonds denominated in Japanese Yen seem to behave in an entirely different way to their counterparts in the other three currencies – this being true, in fact, across almost all issuers. Indeed, as we will see later, Yen securities show little if any correlation with those denominated in £, $ and €, whichever way the data is bucketed.

Secondly, for the other three currencies and their pairings, it is clear that as credit quality drops and spreads increase, correlation increases in a significant fashion.

To measure the key explanatory variables, both in significance and explanatory power, we simultaneously regressed the correlations on issuer spread and issuer volatility. The results of the regression are presented in Figure 13 which shows the R^2-values, β-coefficients and t-statistics of the explanatory variables. For the issuer spread the t-statistic was found to be 5.5 and that of the volatility –1.1, clearly indicating that spread level was the major factor in explaining correlation. Furthermore the R^2 of 0.65 implies a high level of explanatory power of the Libor spread level as this is the only statistically significant variable in the regression.

Figure 13. €-$ Multi-Factor Regression Results

	Co-efficient	t-Statistic	R^2	#pts
Ave issuer spread level (bp)	0.005	5.52	0.65	37
Ave volatility of issuer spread changes (bp/month)	-0.003	-1.07		
Intercept	0.023	0.33		

3.1 Bond-by-Bond Correlations

In addition to calculating correlations of changes in issuer spreads, we further calculated all pair-wise correlations between bonds of differing currencies, and formed the average bond-by-bond correlation for that issuer in each currency-pair. So for Ford, for example, we calculated the correlation for every pair of $- and €-denominated Ford bonds and averaged the results which, in this case, gave an average correlation of 0.52. This figure is lower than that of the corresponding average *within* each currency (i.e. the average of all €-€ and $-$ correlations) since it does not benefit from the diversification of issue-specific performances.

Figure 14. Bond-by-Bond Correlations

Intra vs. Inter Currency Comparison	€-£	€-¥	€-$	£-¥	£-$	¥-$
Issuers Tested	38	37	113	10	34	57
Average Intra-Currency Correlation	0.37	0.40	0.53	0.33	0.51	0.54
Average Inter-Currency Correlation	0.24	0.07	0.29	0.03	0.14	0.14
Mean of Intra- minus Inter- Currency Correlations	0.13	0.33	0.24	0.30	0.37	0.40

The matter of the relative proportion of the issue-specific and the issuer-specific volatility is central to understanding the calculation and interpretation of the correlation results. The larger the issue-specific component in comparison with the issuer-specific one, the lower the correlation will tend to be.

We might expect that the issue-specific component would become less dominant as spread levels increased, since for bonds with high spread the larger spread movements would swamp the overall issue effects, whereas for lower spreads the issuer volatility will probably be low and the issue-specific changes in bond-spreads will be more significant. So, in the case of El Paso, a Baa rated issuer with average spread of 356 and 193 bp in € and $ respectively, we find that the issue-specific volatilities in € and $ are 0.16 and 0.25 times the issuer-specific volatility, whereas for BNG with an Aaa rating and average spreads of 4 and 11 bp the corresponding ratios are 1.65 and 1.24 respectively. Figure 14 confirms that there is such a relationship and that spread level is a highly significant factor in explaining the ratio of issue-specific volatility to that issuer-specific volatility.

Figure 15. Ratios of Issue-specific Volatility to Issuer-specific Volatility

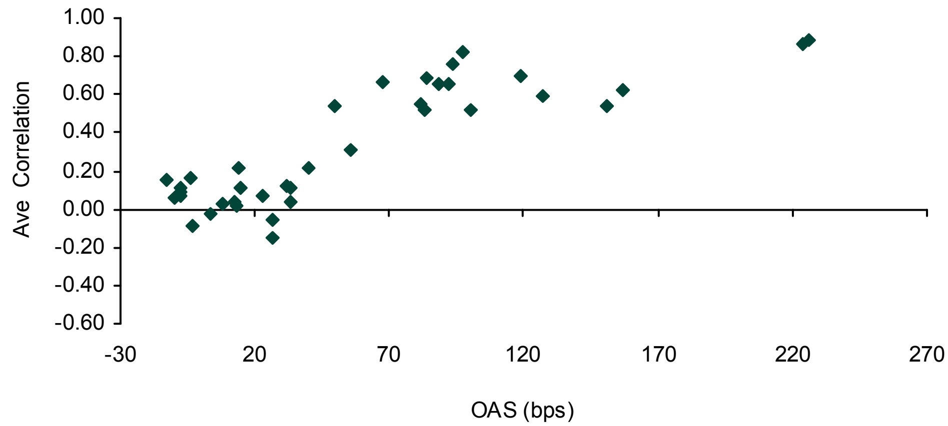
	€	$
Maximum	2.11	3.93
Median	0.59	0.54
Mean	0.66	0.62
Correlation with Libor spread	-0.39	-0.39
t-Statistic	-3.90	-4.36

However, whilst this will be a major factor for low-spread names where there are few issues outstanding at any given time, it should be expected that, as the number of bonds of a given issuer in a given currency increases, this issue-specific 'noise' will be diversified away, leaving only the issuer spread correlations, as explained above.[4] Nonetheless, the lower the issuer spread, the more issues will be necessary to accomplish this noise-reduction, given the higher proportion of bond-level spread-movement due to issue-specific effects for lower spread names. So for EIB, for example, where on average each month-end issuer spread is the mean of changes in Libor spreads from 8 bonds, we would expect the issue-specific noise to have been significantly diversified leaving, to a great extent, the issuer spread. Since the EIB correlations are low (see Figure 12 above), it is clear that there are other reasons why correlations for low-spread issuers are low aside from issue-specific factors – high issue-specific volatilities do not fully explain low correlation in low-spread securities.

The results for the average bond-by-bond correlations are presented in the chart below. The correlation between average bond-by-bond correlation and spread level is a very high 0.85, underlining the highly significant role idiosyncratic risk plays in the relative behaviour of securities. It should be noted that this is only a linear correlation while a non-linear model is required to accurately explain correlation.

[4] *This assumes that issue-specific factors are uncorrelated across bonds.*

Figure 16. Average €-$ Bond-by-Bond Correlation By Issuer



Across all currency-pairs we witnessed higher averages of bond-by-bond correlations within currencies as compared with across currencies. This points to a degree of market segmentation in the investor base, which will also be observed in the relative behaviour of issuer and sector spreads. If investors tend to focus on one currency of denomination then we would expect bonds from the same issuer to move more closely. Cross-currency movements, on the other hand, will reflect differing views of a different set of investors – for example $ and € investors – with differing risk aversions and familiarity with the issuing entities.

4. SECTOR RESULTS

We analysed sector spread in two stages. First we looked at broad sectors such as Industrials as a whole, Utilities, Sovereigns etc, then we divided them into smaller categories such as Supermarkets, Banking, Brokerage, Automotives and so on. Whilst the finer division did present problems of thinness of data, nonetheless, as for the issuer level analysis, some useful results were obtained, particularly in the €-$ currency-pair. We present here only the broad sector results in Figure 17 below.

Figure 17. Sector Correlation Results

		#Issues in Universe					Correlation Estimate					
	Quality	€	£	¥	$	Ave. OAS	€-£	€-¥	€-$	£-¥	£-$	¥-$
Supra-National	Aaa	33	27	8	76	-10	0.03	0.21	0.29	-0.07	0.12	0.13
Agencies	Aa1	138	15	219	169	-2	0.15	0.20	0.13	-0.21	0.10	-0.01
Local Govt	Aa2	121	6	95	42	7	NM	0.42	0.34	NM	NM	0.06
Finance	Aa3	293	37	249	436	36	0.28	0.11	0.61	0.08	0.53	-0.02
Sovereign	A2	35	3	32	102	43	NM	0.04	0.58	NM	NM	-0.11
Utilities	A1	54	9	98	101	71	0.27	0.04	0.87	0.05	0.20	0.06
Industrial	A3	239	26	90	534	77	0.59	-0.27	0.84	-0.24	0.73	-0.15

Once again the first observation is that Yen denominated issues showed no significant correlation in any bracket. However, in the other three currency-pairs we see highly significant correlations in the Finance and in particular the Industrial sectors, with Utilities showing very high correlation between € and $, but not so between £ and $ nor € and £.

It also seems broadly true to say that the correlation once again, as in the case of the issuer level, increases as we increase OAS or, equivalently, decrease quality rating. In order to examine sector cross-currency correlations more fully, we further dissect them into quality-sub-buckets. Figure 18 presents a summary of the basic statistics of this dissection. We have labelled 'NM' wherever the statistic was not meaningful, having too few data points to form a time-series of length at least 12 long, or too few bonds in any bucket to make up a well-diversified bucket-average.

Figure 18. Sector × Quality Correlation Results

	#Issues in Universe				Average OAS				Correlation Estimate					
	€	£	¥	$	€	£	¥	$	€-£	€-¥	€-$	£-¥	£-$	¥-$
Sovereign Aaa	7	1	9	26	0	-32	-6	-9	NM	NM	NM	NM	NM	0.24
Sovereign Aa	6	2	14	29	-12	-18	-8	-9	NM	0.54	0.22	NM	0.22	0.23
Sovereign A	14	0	7	8	28	NM	35	60	NM	0.08	0.40	NM	NM	-0.11
Sovereign Baa	20	0	9	49	122	NM	186	183	NM	0.18	0.66	NM	NM	0.05
Supra-National Aaa	33	27	8	76	-3	-19	-8	-11	0.03	0.21	0.29	-0.07	0.12	0.13
Local Govt Aaa	59	6	42	19	1	2	-1	15	NM	0.19	0.33	NM	0.02	0.01
Local Govt Aa	59	0	89	15	2	NM	11	11	NM	0.20	0.28	NM	NM	-0.02
Local Govt A	4	0	81	8	29	NM	10	22	NM	-0.18	-0.13	NM	NM	0.24
Agencies Aaa	120	14	35	148	-2	-15	-7	-8	0.10	0.20	0.09	-0.23	0.08	-0.13
Agencies Aa	26	1	192	16	4	4	6	21	NM	0.35	0.15	NM	0.29	-0.07
Utilities Aa	21	2	98	6	29	47	8	36	NM	-0.09	0.03	NM	NM	0.01
Utilities A	33	5	0	35	63	57	NM	103	0.36	NM	0.52	NM	0.11	NM
Utilities Baa	13	3	0	66	128	111	NM	187	0.40	NM	0.95	NM	0.43	NM
Finance Aaa	61	13	6	64	6	-2	6	22	0.22	0.18	0.30	0.04	0.20	-0.18
Finance Aa	157	18	43	161	22	11	17	48	0.02	0.31	0.39	0.12	0.45	0.06
Finance A	101	7	202	221	60	89	25	81	NM	0.09	0.78	NM	0.45	-0.01
Finance Baa	6	1	28	35	138	128	73	157	NM	0.07	0.70	NM	NM	0.13
Industrial Aa	44	5	46	70	27	2	16	29	0.23	-0.14	0.42	0.12	0.16	0.14
Industrial A	136	19	31	235	70	74	28	80	0.41	-0.05	0.78	-0.01	0.45	-0.14
Industrial Baa	116	12	22	298	162	155	62	161	0.62	-0.35	0.81	-0.28	0.76	-0.28

Even though we would expect the €-$ correlations to be a function of the average spread levels in the sector concerned, nonetheless, looking at the correlations purely on a spread level basis does not explain the variance between sectors of similar spread. For example, within the €-$ currency-pair, Finance-A has an average OAS across the two currencies of 71 bp similar to that of Utilities-A at 83 bp. However, despite the similarity of their spread levels, the correlations are quite different at 0.78 and 0.52 respectively.

Similarly, Finance-Aaa and Agencies-Aa have almost the same average OAS of 14 and 13 bp but their correlations are 0.30 and 0.15. Agencies-Aaa and Supra-National-Aaa have average spreads of -5 and -7 bp but correlations of 0.09 and 0.29 respectively, and Industrials Baa with an average OAS of 162 bp has an almost identical spread to Sovereign Baa (153 bp) and yet the respective correlation estimates are 0.81 and 0.66.

In general, looking through the table, it seems that there is an ordering on the sectors with respect to correlation as follows (increasing in correlation): Agencies – Sovereign/Local Govt./Supra-National – Utilities – Finance - Industrial

At the level of strict statistical significance we are less able to draw the fine distinctions above. Regressing the €-$ correlation on OAS and sectors, only Industrials gives a significant t-Statistic (at the 90% level). However, upon dividing the issuers into two sectors – Non-Corporate Credit and Corporate Credit – and performing a simultaneous regression of €-$ correlation on OAS and an indicator variable for these sectors, we found the Corporate- Non-Corporate Credit sector division to be significant at the 99% level with a t-Statistic of 2.853. Clearly sector plays a part, and a finer gradation of sectors in terms of their influence on correlation is a topic for further study.

This still leaves us, however, with a question mark over the inconsistently high correlation in Utilities BBB – and indeed in Utilities as a whole – for the €-$ currency-pair. In order to explain this we must consider the influence of rating migration on correlation. It is beyond the scope of this article to report this analysis in full, but in this case it sheds valuable light on a result which, left as it is, undermines a pattern whereby Utilities seems, as a sector, to exhibit lower correlation than Industrials and Finance for similar spreads.

In October of this year, two months after the end of the time window of this study, El Paso was downgraded to junk after a series of huge spread widening in the months leading up to this study. These spread movements dwarfed the other more modest ones of the entire Utilities sector in €-$ currency-pair. Since we would expect that events such as downgrade would carry with them very high correlations across all bonds of the issuer that is experiencing the rating migration (irrespective of the currency of denomination), the correlation of Utilities $ and Utilities € was 'artificially' boosted to figures approaching 90%.

When the market-anticipatory spread movements leading up to this downgrade[2] were expunged from the data-set, the correlation of €-$ Utilities dropped to a figure of 0.48. This confirms the pattern of lower correlation indicated in the Utilities-A bracket and indeed in the €-£ and £-$ Utilities figures which, although affected by El Paso themselves, were less so and remained at similar levels after the correction (0.24 and 0.18 respectively).[5]

[5] *We adopted the anticipatory periods described in 'Sufficient Diversification in Credit Portfolios' (Dynkin, Hyman and Konstantinovsky) in selecting those records pertaining to up-and-coming credit events.*

5. HAS CORRELATION CHANGED OVER TIME?

We divided the data also along the time-axis, considering data within not only the full time-window of 44 months but also for the most recent 20 months. This allows us to examine whether correlations have moved in a particular direction over time.

The pattern of the results, summarised in Figure 19, are unequivocal – correlations have increased through time in all of the three currency-pairs formed by £, $ and €. Assuming, under a null hypothesis of no change in correlation over time, that the conditional distributions are roughly symmetrical about the 44-month statistic, sector results are significant at the 90% level for the €-$ currency-pair.

This could be for a number of reasons:

Firstly, the last 20 months have shown particularly high volatility accompanied by a high incidence of downgrade and default which would increase volatility and spreads.

Secondly, data for this latter period was probably more robust, and so some of the previous data problems which were masking the true correlation have been eliminated.

Thirdly, markets may have matured since the launch of the Euro and monetary union. In addition, the uptake of global credit indices may have served to encourage investors to view credit markets as one global market.

Figure 19. Sector Correlations – Two Time Windows

	#Issues in Universe			Average OAS(20months/44months)			Correlation Estimate - 20months/44months		
	€	£	$	€	£	$	€-£	€-$	£-$
Supra-National	21	17	67	-1/-3	-8/-19	-10/-11	0.27/0.03	0.29/0.29	0.12/0.11
Agencies	93	11	147	7/2	-4/-13	5/-1	0.33/0.15	0.45/0.13	0.09/0.10
Local Govt	98	3	39	3/2	5/2	16/15	NM/NM	0.42/0.33	NM/NM
Finance	222	23	383	40/31	38/18	82/69	0.42/0.28	0.75/0.61	0.55/0.53
Sovereign	25	1	75	81/62	-9/-22	90/89	NM/NM	0.77/0.58	NM/NM
Utilities	51	8	101	74/61	84/68	163/146	0.32/0.27	0.97/0.87	0.31/0.20
Industrial	227	23	484	107/90	112/79	136/112	0.60/0.59	0.90/0.84	0.75/0.72

Figure 20. Quality Correlations – Two Time Windows

	#Issues in Universe				Correlation Estimate - 20months/40months					
	€	£	¥	$	€-£	€-¥	€-$	£-¥	£-$	¥-$
Aaa	197	37	99	304	0.22/0.14	0.38/0.36	0.39/0.28	-0.21/-0.19	0.10/0.11	-0.02/-0.05
Aa	226	18	474	252	0.26/0.12	0.02/0.02	0.61/0.45	0.21/0.21	0.44/0.41	-0.01/-0.00
A	240	27	489	441	0.59/0.49	-0.04/0.01	0.89/0.82	0.02/0.02	0.59/0.53	-0.15/-0.12
Baa	142	13	55	392	0.55/0.53	-0.35/-0.29	0.92/0.85	-0.42/-0.38	0.58/0.59	-0.28/-0.26

Figure 20 describes the relative correlations on a Broad-Quality level. Here, all monthly changes in Libor spreads of a given quality bracket were averaged to produce a quality spread, proxying the behaviour of quality brackets as a whole. The results indicate that there has been no significant change in correlations within quality brackets over time (the confidence intervals around lower correlations such as 0.45 are very wide and so the apparent increase in the Aa-sector from 0.45 to 0.61 is hard to interpret) and may serve to indicate the stability of the spread-correlation relationship, namely that comparable spreads are associated with similar levels of correlation irrespective of the time window employed. This merits further investigation. In addition, the high levels of correlation in the A and Baa brackets in particular indicate the existence of a genuine quality-based factor lying behind spread-movements and its global nature.

6. IMPLICATIONS FOR INVESTMENT MANAGERS

The issues raised in the study have a number of immediate implications in the realm of investment strategy.

In dealing with risk management, we have seen that cross-currency correlation can be explained by spread level and sector classification, with high spread levels implying higher correlation. This therefore necessitates an accounting of issuer-specific risk on a global basis. This is important to fund managers who apportion the management of global credit mandates to regional teams in the US, Europe and Asia for example. If each local team chooses the same high-spread names for their part of the overall portfolio, they will be increasing their overall risk exposure in comparison with a strategy of coordination whereby different high-spread names are held in differing currencies. For low-spread instruments with stable and high credit quality, typically Agency or Supra-National paper, issue-specific risk is more important, and here multi-currency holdings will add to its diversification. This also means that in higher spread assets, the benefits of cross-currency diversification may not be as large as those in low-spread sectors. In the lower spread sectors, however, it is more important to diversify the holdings among the various denominations to match the benchmark, since the $-denominated bonds are less of a proxy for the € ones, for example.

In addition to the above, our analysis indicates that when it really matters most, i.e. when risk is high, correlations are high as well. Therefore, if a portfolio manager matches the issuer level exposure of a benchmark he is probably not exposed to a great deal of idiosyncratic risk, even if at the issue and currency level allocation is quite different. When correlations are low, he may be exposed to risk but it will be of relatively small magnitude.

Concerning benchmark decisions, we note that global credit benchmarks make more sense as we cover the lower end of the credit spectrum, where the need for diversification is more acute. Our study indicates that diversification in terms of currency of denomination will lead to a lowering of issuer-specific risk in low-spread names but far less so in high-spread issuers where the cross-currency correlations are typically high. Here diversification has to be achieved through a broadening of the set of names from which the portfolio is selected rather than through cross-currency allocation techniques.

With regard to the issues of index replication, our study indicates that a strategy of representing Japanese institutional debt issued in Yen by the corresponding debt in Euro, US Dollar or Sterling has significant risks associated with it. On the other hand, a replication strategy for the JPY curve in non-domestic issuers (thereby avoiding withholding tax) based on purchasing the Yen-denominated debt of Supra-Nationals and Agencies may have merit.

The lack of correlation in high-grade debt as a whole in addition to that of the Agency and Supra-National sectors and the low volatility of such issuers with respect to the Yen swap curve would imply relatively low tracking errors with respect to the Yen curve.

Opportunities for Outperformance: Given the high bond-by-bond correlation – and consequent high average hedging efficiency – for some high-spread names, opportunities for spread-trades present themselves. For example, the debt of such an issuer has a significantly higher spread in one currency than in another and the investor takes the view that such a relationship will persist. Then, a market-value-neutral position will benefit from the positive carry in an environment of stable spread differentials while the high cross-currency correlation in spread changes will serve to mitigate the risk of the position. A risk-minimising spread trade could also be constructed taking account of the relative volatilities of spread changes in the two currencies in addition to the cross-currency correlation.

7. CONCLUSIONS

We have carried out an empirical study of the correlation between Libor spreads of credit issuers in the four major currencies, analysing the data in several dimensions. We see a highly significant link between the spread-level and the correlation in three of those four currencies – €, £ and $ – with correlation rising steadily with spread often to very significant levels. In addition, we have seen evidence of sector-related effects with Industrials, Utilities and Finance sectors showing higher correlation than an equivalent spread level in Agencies, Supra-Nationals and other Non-Corporate credit.

Intuitively, spread levels can be seen as a proxy for the risk in the underlying name – the higher the spread, the higher the implied risk. This risk could be systematic (market-wide) or idiosyncratic. The greater the systematic risk, the greater the leverage (β) to the market as a whole. If spreads are high because the market-sensitivity of an issuer's spread is high, then one also expects the cross-currency correlations of changes in the spread for the issuer to be high (as they are influenced significantly by overall market movements). At times of credit events we would expect the default-related component of risk to dominate, leading to very high correlations in those circumstances as well, such as have been witnessed in the cases of El Paso, WorldCom, Marconi and the like.

Throughout the study, however, Japanese Yen-denominated securities have shown no evidence of correlation at either the issuer or sector level. Here the reasons are less clear. Whilst it is true that the Yen data is much thinner and shorter, with no Yen issues featuring in the study universe until September 2000, nonetheless, relatively speaking the correlations in the most recent 20 months have been very low with only 1 issuer with 12 months of data showing a Yen-Dollar correlation in excess of 0.50. By way of explanation we note that the Yen credit market exhibits certain differences from its Dollar, Euro and Sterling counterparts. The presence of withholding tax, a more regional investor base and less active secondary market trading may all make the yen credit market appear segmented from the rest of the world.

The results of the study have far-reaching ramifications for investment strategy, in particular those involving issues of diversification and portfolio management. It implies that issuer-specific risk is diversified by cross-currency holdings only in so far as the spread of the issuer is low, whereas, as the credit quality of the security decreases correlations of debt in different denominations rise to significant levels of up to 80–90%. This is clearest in the €-$ currency-pair, but our results indicate that this relationship between spread levels and correlation holds for the other two pairs, namely €-£ and £-$, as well. Furthermore, we have found support for the view that in addition to spread levels, the level of cross-currency correlation is also influenced by the sector to which an issuer belongs. For the same level of spread, issuers in the Corporate Credit part of our indices demonstrates higher correlations than those in the Non-Corporate part.

We regard the results reported in this article as only the beginning of an analysis of the question of cross-currency correlation. Looking ahead, we see numerous avenues for further exploration and research. The question of whether there is a significant difference at the sector level between global and local issuers – those issuing in both currencies in question and those only in one – is a key area for study as is the comparison of downgraded issues with stable ones. Also, the question of the stability of the relationship demonstrated between spread and correlation through differing market conditions is one of great interest and importance. We look forward to presenting the results of those analyses in the near future.

Robust and Efficient Estimation of Equity Correlations[1]

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The reliable estimation of equity correlations has recently become a crucial issue for credit practitioners, who need to calibrate their default models in order to price and risk manage multi-name credit instruments. In this article, we compare the small-sample properties of two alternative correlation estimators when the observed data is generated by a multivariate distribution belonging to the elliptical family, a class of multivariate distributions that appropriately describes the joint behavior of equity returns. The results suggest that, within this class of distributions, a simple correlation estimator based on a rank statistic called Kendall's Tau is more efficient and more robust than the widely used Pearson coefficient.

1. INTRODUCTION

In our attempt to price and risk-manage multi-name credit derivatives such as default baskets and CDOs, we build models that generate defaults of the reference names. In these models, default events are often associated with some latent variables falling below a threshold level, and the correlations of the default events are implied by the dependence structure - also known as *the copula* - of these latent variables.

Consistent with the main idea behind the "structural approach" to credit modeling, the asset value of a company is the most natural candidate to play the role of the default-triggering latent variable. It seems therefore reasonable to calibrate the dependence structure of the latent variables in the model to the dependence structure of asset returns.

Unfortunately, asset values are not directly observable. Therefore, practitioners often rely on observable changes in the market value of equity to estimate the copula of the latent variables. Whether this is a reasonable approximation is a question that we try to answer elsewhere in this issue of the Quarterly.[2] In this article, we tackle the issue of estimating the linear portion – i.e. correlation – of the dependence structure of equity returns.

Generally speaking, the choice of the most appropriate correlation estimator is a function of the underlying distribution that generates the data. However, the popular Pearson coefficient (from now on, PC), defined as

$$PC(X,Y) = \frac{\sum_i (x_i - \bar{x})(y_i - \bar{y})}{\left(\sum_i (x_i - \bar{x})^2\right)^{1/2}\left(\sum_i (y_i - \bar{y})^2\right)^{1/2}},$$

is often used to estimate correlations irrespective of the distributional properties of the data. One reason for its popularity is that PC is known to have excellent statistical properties when applied to normally distributed variables (see, for example, Greene (2002)). For non-Normal

[1] *I would like to thank Roy Mashal, Dominic O'Kane, Minh Trinh and Stuart Turnbull for their comments.*

[2] *See Mashal, Naldi and Zeevi (2002), elsewhere in this issue.*

data, however, we can generally employ estimators with better small-sample properties. This is important, because there is evidence that equity returns are not normally distributed.

A large body of research has by now established that univariate equity series exhibit fatter tails than the Normal distribution.[3] More recently, the focus has shifted on the properties of the joint tails of equity returns, and strong evidence against normality has been found. Mashal and Naldi (2002) estimate the dependence structure of equity returns under the assumption of *t*-distributed marginals, while Mashal and Zeevi (2002) relax this assumption and employ a semi-parametric methodology. Both articles offer strong evidence for the rejection of the assumption that equity series have a Normal copula.

In summary, these studies suggest that equity returns follow bell-shaped multivariate distributions with marginal and joint tails that are fatter than the Normal bell. This family of distributions is often referred to as the "elliptical family", since the contour of any horizontal slice of a bell has the form of an ellipsis.

The analysis presented in this article builds on results first reported by Lindskog (2000), who shows that when the data are generated by a fat-tailed elliptical distribution, using PC to estimate true correlations can lead to large errors. He also shows that a simple estimator based on a transformation of a rank statistic called Kendall's Tau is

- more efficient (i.e. it produces estimates with lower mean square error), and
- more robust (i.e. it never produces "really bad" estimates).

Section 2 presents a simulation exercise to illustrate these two claims.

In Section 3, we extend the basic framework to compare the performance of PC and Kendall's Tau transform (from now on, KT) in the presence of shifts in the true correlation parameter. Since correlation estimates are generally "refreshed" to incorporate newly available information, Section 4 simulates a "rolling window" estimation strategy and compares the time-series properties of the rolling estimates produced by PC and KT. Section 5 briefly concludes. A formal definition of Kendall's Tau transform is offered in the Appendix.

2. EFFICIENT AND ROBUST ESTIMATES

In this section, we simulate 2,000 small samples of 200 observations each from a fat-tailed bivariate distribution. In particular, we employ an elliptical distribution with *t*-distributed marginals with 5 degrees of freedom, and a *t*-copula with 7 degrees of freedom and 50% correlation.[4] Previous statistical work has revealed that the marginal and joint tail behavior of a number of equity return series - such as AA, AXP, T, KO, EK, HON, INTC, IP, JPM, MSFT, MMM, MO, PG - is appropriately described by such a parameterization.[5]

For every sample, we estimate the correlation coefficient using two alternative estimators: PC and KT. The advantage of a simulation study is that we know the true parameters that have generated the data, and we are therefore in the position of correctly evaluating alternative efforts to estimate these parameters.

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[3] *Some of the earlier results can be found in Praetz (1972) and Blattberg and Gonedes (1974).*

[4] *Notice that this is not a multivariate t distribution, since the degrees of freedom of the marginals are different from the degrees of freedom of the copula. This implies that the fatness of the marginal tails and that of the joint tails are regulated by different parameters.*

[5] *See Mashal and Naldi (2002) for details.*

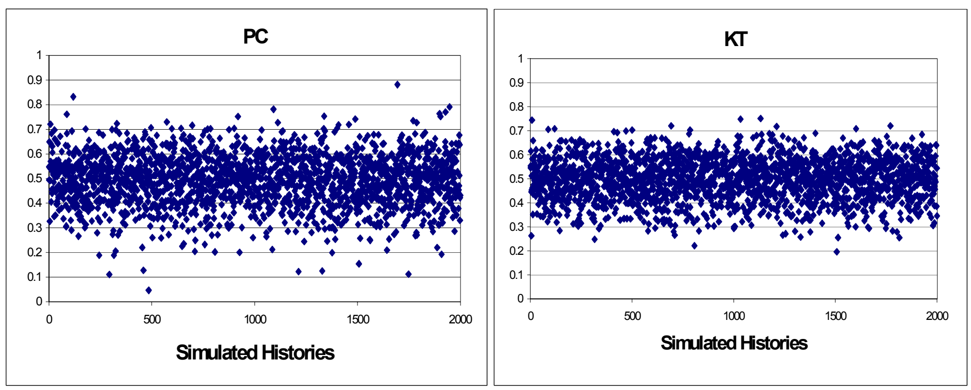
2.1 Efficiency

The Root Mean Square Error (RMSE), defined as the square root of the average across samples of the squared estimation errors, is a natural measure of efficiency of an estimator. Generally speaking, the RMSE is composed of two parts, one due to the bias and the other to the volatility (standard error) of the estimator. These measures obey the simple relation

$$RMSE^2 = Std\ Error^2 + Bias^2.$$

Our results show that both estimators are (almost) unbiased, i.e. on average they both recover the true value of the parameter they are trying to estimate. If we average our correlation estimates across all samples we obtain 49.96% for PC and 50.03% for KT, the true value being 50%. This means that both estimators will, for an average realization of history, produce a correct estimate. However, the dispersion of KT around the true value is significantly lower than the dispersion of PC, as shown in Figure 1. The RMSE is 10.17% for PC and 8.64% for KT, approximately a 15% difference.

Figure 1. Estimating Correlations with PC and KT



2.2 Robustness

What is perhaps even more important is that the distribution of PC has significantly heavier tails (i.e. a larger fraction of extreme values), allowing for a non-negligible probability of coming up with really bad estimates. As shown in Figure 1, the minimum estimate using KT was 19.6%, while the minimum estimate using PC was 4.6%. The maxima were 75.2% and 88.1%, respectively.

2.3 The Concepts of “Efficiency” and “Robustness”

Figure 1 clearly shows that there are some samples for which the correlation estimates are quite far from the true value of 50%. If we are unlucky enough to observe a sample that is not particularly representative of the true distribution, we can easily make large estimation errors.

Now imagine that each of the samples we have generated is a series of return observations for two names, and that we want to forecast their future correlation. No matter what the history behind us is saying, the future behavior of returns is still going to be regulated by the true correlation.

This is an important point, and one that is often a source of confusion. It is perfectly possible that when the true correlation is, say, 50%, the history we observe and the estimator we use will give us, say, a 35% estimate. But the most likely realized correlation we will observe over the following periods is 50%, not 35%.

It is precisely for this reason that it is important to know how badly a non-representative sample can "fool" the estimator we are using. The statistical concepts of efficiency and robustness address this issue. Efficiency tells us something about how "fooled" an estimator is going to be on average, robustness focuses on the possibility of being "fooled very badly".

Our results show that the KT estimator has lower RMSE and fewer extreme values than PC. This means that KT is going to

1. have a lower *average* deviation from the true value, and
2. have a smaller probability to *substantially* deviate from the true value.

Visually, as Figure 1 points out, KT has a more "compact" distribution around the true value.

2.4 A Remark

The previous results are by no means unique to the particular distribution we used. As noted earlier, we simulated from a distribution we found to be representative of a set of observed equity returns. However, KT retains its desirable small-sample properties for all elliptical distributions, a family that is large enough to adequately represent the universe of equity return series.

The farther away the true dependence structure is from normality, the greater the advantages of KT in terms of efficiency and robustness. If the fraction of equity volatility due to movements in fat-tailed common factors is particularly significant, the differences in RMSE and tail behavior of the two estimators can be even higher than the ones shown above.

Similarly, for a given elliptical copula, the advantages of KT increase as we reduce the number of observations. This is extremely important for those companies for which we have a limited history of equity returns.

3. REGIME SHIFTS AND TIME DECAY

It is quite common to use a weighted Pearson coefficient to estimate correlations, where the weighting scheme generally overweighs recent observations. The rationale behind this is to account for those instances when a regime shift takes place and two names experience a significant change in their pairwise correlation.

More formally, a time decay scheme is an attempt to decrease the mean square error of the estimator by decreasing its bias. Whether this attempt will be successful or not depends on whether the adopted scheme is consistent with the true data generating process. For example, if the true parameter we are trying to estimate did not in fact experience any shift, the application of time decay will end up increasing the mean square error. This is because the weighting scheme will not decrease the bias (it was already zero without decay), but will instead increase the standard error (since less information is now used in the estimation).

In this section, we show that for every weighted Pearson coefficient (WPC), we can define an effective KT estimator (EKT) that is more efficient and more robust, no matter where the regime shift took place in the sample period.

3.1 Weighted PC and Effective KT

To use WPC, we have to choose two things: the length of the data window, and the speed of the time decay. Once we have made these choices, we have in fact chosen what is usually called the number of "effective observations". For example, suppose we are using 200 weekly returns and applying a weighting scheme with a two-year (100 week) half-life.[6] This decay speed is equivalent to weighting each weekly observation 0.7% less than the previous one as we walk back in time. Another way to say this is that the decay factor is (1-0.007)=99.3%.

The number of effective observations can be computed as the sum of all weights, which in this case equals

$$\frac{1-0.993^{200}}{1-0.993}=107\,.$$

In other words, we start with 200 weekly returns, but then we weight them so that the total amount of information that we bring into the estimation is equivalent to the information contained in 107 weekly returns.

The experiment we perform in this section can be described in three simple steps:

1. Define a WPC estimator by choosing a window length N and a decay factor B, and compute the number of effective observations N_{EFF} as

$$N_{EFF} = \frac{1-B^N}{1-B}.$$

2. Define an Effective Kendall's Tau transform (EKT) estimator as a KT estimator which uses only the most recent N_{EFF} observations.

3. Compare the statistical properties of WPC and EKT for different timings of the regime shift.

3.2 Results

Once again, we simulate 2,000 small samples of 200 weekly returns from the same bivariate distribution used in the previous section. This time, however, we split the generation of each sample into two parts. In the first part, the data is generated using a correlation of 20%. In the second part, there is a regime shift and the true correlation coefficient jumps up to 50%.

What happens in reality is that we face one of these samples - the observed history - and we do not know either the current true correlation parameter or if and when a regime shift took place during the sample period.

In the next three figures we compare the WPC defined above (200-week window with a two-year half-life decay) with the associated EKT. Figure 2 shows the correlation estimates for every sample path in the case where the true correlation has jumped from 20% to 50% after 50 weeks (1 year) in the sample. Figure 3 refers to the scenario where the regime shift takes place 100 weeks (2 years) into the sample and Figure 4 deals with the case where the shift takes place 150 weeks (3 years) into the sample.

[6] *A weighting scheme with an x-year half-life assigns an x-year-old observation half of the weight assigned to the most recent observation.*

Figure 2. WPC vs. EKT with Correlation Shift: from 20% to 50% on Week 50

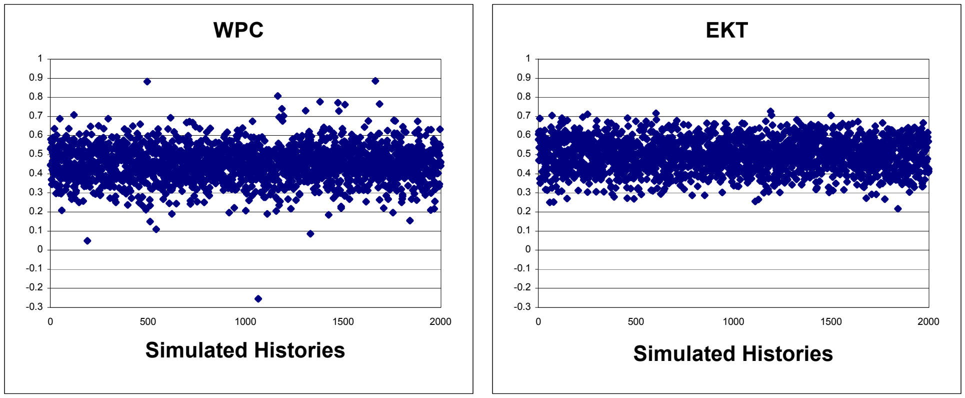


Figure 3. WPC vs. EKT with Correlation Shift: from 20% to 50% on Week 100

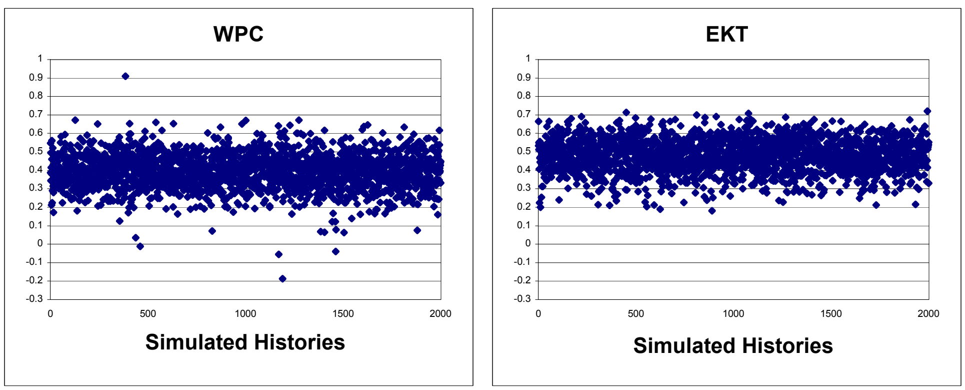
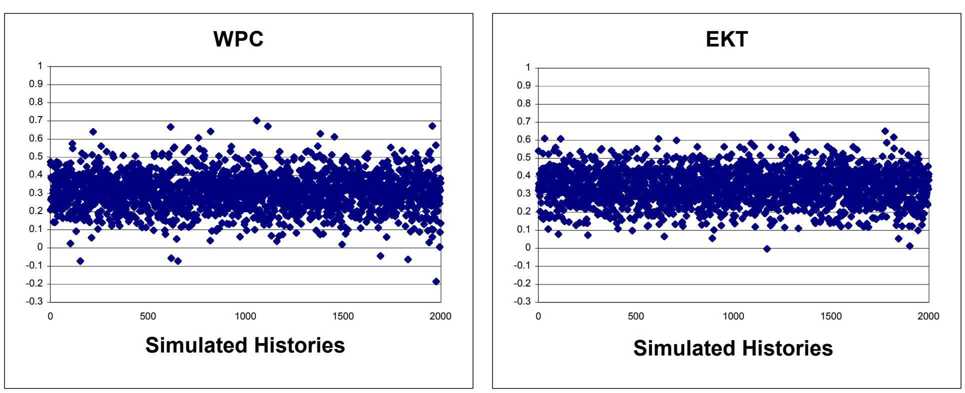


Figure 4. WPC vs. EKT with Correlation Shift: from 20% to 50% on Week 150



Notice that both estimators may now be biased, i.e. they may on average be different from the current true correlation of 50%. That is because part of the data we use to estimate the correlation was actually generated by a distribution with 20% correlation.

Figure 5 details the comparison depicted in Figures 2-4 and splits the RMSE into Bias and Standard Error for both estimators and for each of the three timings of the regime shift. These numbers show that EKT uniformly dominates WPC. EKT has lower bias and lower standard error in every regime shift scenario, and it provides a decrease in RMSE equal to 20% for an early shift, 38% for a mid-shift and 13% for a late shift.

Figure 5. Performance of WPC and EKT for Different Timings of a Correlation Shift

		EARLY SHIFT			MID SHIFT			LATE SHIFT	
	WPC	EKT	EKT/WPC	WPC	EKT	EKT/WPC	WPC	EKT	EKT/WPC
BIAS	4.89%	0.05%	1.02%	10.74%	2.11%	19.65%	18.88%	15.98%	84.64%
ST ERR	9.18%	8.27%	90.09%	9.46%	8.69%	91.86%	9.86%	9.32%	94.52%
RMSE	10.40%	8.27%	79.52%	14.31%	8.94%	62.47%	21.29%	18.49%	86.85%

In terms of robustness, Figures 2-4 show that the tails of WPC are heavier than those of EKT for every scenario. In particular, for an early regime shift, we observe WPC estimates as low as –25% when the true current correlation is 50%. The worst EKT did in this exercise was 21%.

The previous results suggest that rather than working with WPC, we can use Kendall's Tau transform and think about time-decay in terms of shortening the estimation window. This is a more natural way to overweight recent history when using a rank statistic such as KT.

4. TIME-SERIES PROPERTIES OF THE ESTIMATION ERROR

For most practical applications, correlations are periodically re-estimated in order to account for newly available data on an ongoing basis. It is therefore interesting to compare alternative estimators in terms of the time-series properties of the estimates they produce. In this section, we compare the time-series mean and volatility of the estimation errors produced by PC and KT.

4.1 Design

We simulate 1,000 paths from the same fat-tailed distribution used in Section 2. The true correlation coefficient generating the data is 50%, and each simulated path represents a history of returns for 250 periods. If we think of them as weekly returns, we are really simulating 1,000 "5-year histories".

On each path, we start by standing in week 201, and using the 200 weekly returns behind us, we estimate correlation using two different methods, PC and KT. Next, we move one week forward, stand in week 202, and using 200 observations behind us, we re-estimate correlation with both methods. And so on. At the end of each path, we have a series of 50 estimates for each method. In our "weekly" interpretation, we have rolled over a whole year, one week at a time. We repeat this procedure for each of the 1,000 histories that we have generated.

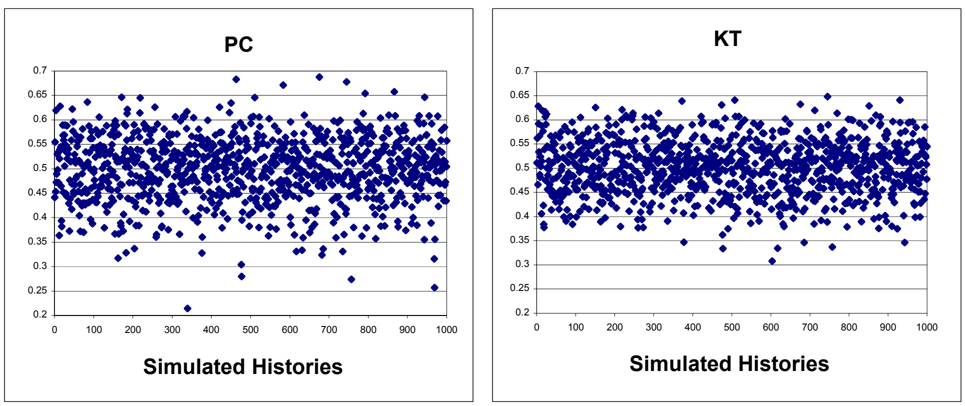
As an introduction to the analysis in the next two sub-sections, notice that if we take the time series average of our correlation estimates along every path, and then average again across all histories, we observe that both estimators are (almost) unbiased: the average across paths of all time-series averages is 49.85% for PC and 50.07% for KT. This means that both estimators will, for an average realization of history, produce a time-series of estimates that is on average correct.

This should not make us feel much better though, since in reality we are not allowed to observe many histories, but only one. Therefore, a comparison between alternative estimators should be based on their behavior across all possible histories, including the ones that are not at all representative of the true distribution.

4.2 Time-Series Means of Estimation Errors

Figure 6 shows the time series averages of the correlation estimates for each path. These graphs show that both correlation estimators, PC and KT, can be "fooled" by non-representative histories. PC, however, has more potential for doing really badly: on its worst path, it produced an average correlation estimate of 21%, a far cry from the true 50% that generated the data. The worst KT did in this exercise was an average estimate of 31%. Notice also that, not surprisingly, both estimators tend to be fooled by the same paths: the correlation between their average absolute errors across histories was 49%.

Figure 6. Time-Series Means of Correlation Estimates



A sensible way to put the two graphs of Figure 6 into comparable numbers is to compute once again the Root Mean Square Error of the two methods: it turns out to be 6.49% for PC and 5.60% for KT, approximately a 16% difference.

4.3 Time-Series Volatilities of Estimation Errors

Correlation estimates are often used as inputs for the valuation of open positions in multi-name credit derivatives. The time-series volatility of the P&L of an open position will depend on a number of factors.

- Volatility of market variables, such as spread and interest rate curves.

  The volatility of market variables should translate into P&L volatility: if it did not, we would have an inaccurate valuation.

- Volatility of the true parameters, such as correlations.

  If we have strong evidence supporting the hypothesis that a parameter has changed, we can adapt our estimation procedure to take that into account. As a simple example, consider a situation where we have reasons to believe that, at a given date in the past, there has been a regime shift that has moved the true correlations upwards. In this case, we should only use returns following that date to estimate correlations, since that is the only data we have that is representative of the new true distribution. In reality, however, it is very difficult to support specific hypothesis about changes of the true parameters. In our exercise, we have abstracted from this source of volatility: remember that in the simulation presented above, the equity returns were generated with a time-invariant correlation coefficient of 50%.

- Volatility of the estimation error.

  A volatile estimation error is a highly undesirable source of P&L volatility. It adds noise to the measurement of the value of the position, and it makes us think that our exposure is riskier than it actually is. Using a more efficient and robust estimator will sensibly reduce this source of volatility.

Figure 7. Time-Series Volatilities of Correlation Estimates

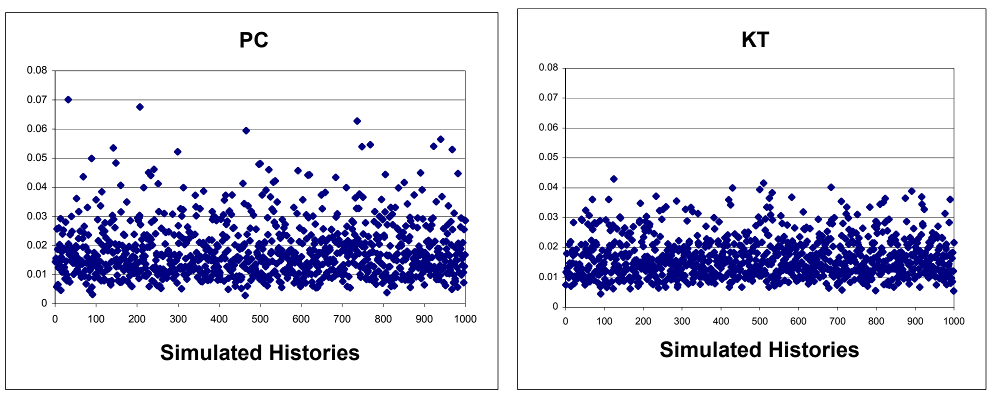


Figure 7 compares PC and KT in terms of the time-series volatility of the estimates. Notice that the true correlation is constant through time, which implies that the only source of volatility of the estimates in this exercise is estimation error.

The graphs clearly show that there is a significant fraction of histories for which PC produces an extremely volatile series of estimates, while the behavior of the KT estimator is much more uniform across paths.

The higher efficiency of KT translates into an average volatility due to estimation error that is 9% lower than that produced by PC. The greater robustness of KT can be seen in the tail behavior: for example, there is a 3.3% probability that the volatility of the correlation estimates produced by PC will be above 4%, while there is only 0.3% probability that this will happen with KT.

5. CONCLUSION

We have compared the statistical properties of two alternative correlation estimators, namely the widely used Pearson coefficient and the less-known Kendall's Tau transform. Our results show that when the data is generated by a fat-tailed multivariate distribution, Kendall's Tau transform produces estimates that have a smaller average error and a smaller probability of being dramatically different from the true value.

We have also extended the basic analysis to study two time-weighted versions of these estimators, and compare their ability to deal with a regime shift in the data generating process, i.e. a change in the value of the true correlation parameter. Once again, a modified version of Kendall's Tau transform produced more efficient and more robust estimates than the more popular weighted Pearson coefficient.

Given that joint fat tails have been shown by previous research to characterize the multivariate behavior of equity returns, our study suggests that Kendall's Tau transform can be successfully employed for the estimation of equity correlations.

6. APPENDIX: KENDALL'S TAU TRANSFORM

Kendall's Tau transform is an indirect method to obtain an estimate of linear correlation through the estimation of Kendall's Tau. The latter is a measure of dependence that is specific to the copula (i.e. the dependence structure) of the distribution, and is therefore invariant to any monotonic transformation of the marginals.

Let us first define Kendall's Tau and its estimator. For two random variables X and Y, Kendall's Tau is defined as

$$\tau(X,Y) = \Pr\{(X - X^*)(Y - Y^*) > 0\} - \Pr\{(X - X^*)(Y - Y^*) < 0\},$$

where (X^*, Y^*) is an independent copy of (X,Y).

Now imagine observing a sample of length n. Consider the $n(n-1)/2$ pairs $\{(X_i, Y_i),(X_j, Y_j)\}$, and let a denote the number of pairs such that $(X_i - X_j)(Y_i - Y_j)>0$, and b the number of pairs such that $(X_i - X_j)(Y_i - Y_j)<0$. Then define the Kendall's Tau estimator as

$$\bar{\tau}(X,Y) = \frac{a-b}{a+b}.$$

A nice property that we have exploited in this paper is that, for all elliptical copulas, we have the following simple relation between Kendall's Tau and linear correlation (see Lindskog (2000) for details):

$$\rho(X,Y) = \sin\left(\frac{\pi}{2}\tau(X,Y)\right).$$

For all simulations presented above, Kendall's Tau transform was computed as

$$\bar{\rho}(X,Y) = \sin\left(\frac{\pi}{2}\frac{a-b}{a+b}\right).$$

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The Dependence Structure of Asset Returns[1]

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The dependence structure of asset returns lies at the heart of a class of models that is widely employed for the valuation of multi-name credit derivatives. Using a statistical methodology that relies on a minimal amount of distributional assumptions, we investigate whether the popular tenet of Normal dependence between asset returns is empirically motivated. We also compare the dependence structures of asset and equity returns to provide some insight into the common practice of estimating the former using equity data. Our results provide strong evidence against the assumption of Normal dependence, and support the use of equity returns as proxies for asset returns.

1. INTRODUCTION

The valuation of default-contingent instruments calls for the modeling of default mechanisms. A well-known dichotomy in credit models distinguishes between a "structural approach," where default is triggered by the market value of the borrower's assets falling below its liabilities, and a "reduced-form approach," where the default event is directly modeled as an unexpected arrival.

Currently, a major challenge facing credit models is represented by the rapid growth of multi-name instruments, whose valuation entails modeling the joint default behavior of a set of reference names. Although both the structural and the reduced-form approaches can in principle be extended to the multivariate case, the calibration of the parameters governing the likelihood of joint defaults poses a number of problems.

If we think of defaults as generated by asset values falling below a given boundary, then the probabilities of joint defaults over a specified horizon must follow from the joint dynamics of asset values. Consistent with their descriptive approach of the default mechanism, multivariate structural models rely on the dependence of asset returns in order to generate dependent default events. In this paper, we focus on the empirical properties of the dependence structure – also known as the *copula function* – of asset returns.

Several well-known multivariate models assume a joint normal distribution for asset returns. Hull and White (2001), for example, generate default dependence by simulating correlated Brownian motions that are supposed to mimic the asset values dynamics. Similarly, two of the most commercially successful multi-name models, developed by KMV and CreditMetrics[3], rely on the joint normality of the default-triggering variables. The widespread use of the multivariate Normal distribution is certainly related to the simplicity of its dependence structure, which is fully characterized by a correlation matrix. It remains to be seen, however, whether this assumption is supported by empirical evidence.

A number of recent studies have shown that the joint behavior of *equity* returns is better described by a "fat-tailed" *t*-copula than by a Normal copula, and that correlations are

[1] *We would like to thank Stuart Turnbull for comments and suggestions.*

[2] *Assaf Zeevi is an Assistant Professor at the Columbia Business School in New York.*

[3] *A description of these models can be found in Kealhofer and Bohn (2001) and Gupton, Finger and Bhatia (1997).*

therefore not sufficient to appropriately characterize their dependence structure.[4] The first goal of this article is to apply the same kind of analysis to *asset* returns, and test the null hypothesis of Gaussian dependence versus the alternative of "joint fat tails."

Given the low liquidity of multi-name instruments, it is not yet possible to use their market prices to back-out implied values for the dependence parameters. Instead, practitioners generally estimate the copula of asset returns from historical data. From a valuation perspective, this amounts to the assumption that the dependence structure of asset returns remains unchanged when we move from the objective probability measure to the pricing (risk-neutral) distribution. Rosenberg (2001) identifies general conditions under which this equivalence holds.

Even if we are willing to rely on this invariance, we still face a major obstacle when attempting to estimate the dependence structure from historical data: asset returns are not directly observable. In fact, the use of unobservable underlying processes is one of several criticisms that the structural approach has received over the years. Given the lack of observable asset returns, it has become customary to proxy the asset dependence with equity dependence, and estimate the parameters governing the joint behavior of asset returns from equity return series.

The use of equity returns to infer the joint behavior of asset returns is often criticized on the grounds of the different leverage of assets and equity. Even those who accept it as a valid approximation for high-grade issuers, often criticize this approach when it is applied to low-grade borrowers. This is because a high-quality issuer has a relatively low probability of default, and every variation in the market value of its assets translates almost dollar-by-dollar into a variation of its market capitalization. On the other hand, high-yield borrowers are closer to the default threshold, and a variation in their asset value can potentially produce a significant variation in the market value of their debt as well. This "leverage effect" may generate significant differences in the joint dynamics of equity and asset values. The second goal of this article is to shed some light on the magnitude of the error induced by using equity data as a proxy for actual asset returns.

To provide a plausible answer to these questions, we first need to "back out" asset values from observable data. One way to estimate the market value of a company's assets is to implement a univariate structural model. Using Merton's (1974) approach, – i.e., recognizing the identity between a long equity position and the payoff of a call option written on the asset value process – one can apply standard option pricing arguments and derive two conditions that can be simultaneously solved for the asset value of the company and its volatility. This procedure is at the heart of KMV's CreditEdge,™ a popular credit tool that first computes a measure of distance-to-default and then maps it into a default probability (EDF™*)* by means of an historical analysis of default frequencies.[5] In this article, we use the asset value series generated by KMV's model to study the dependence properties of asset returns.

In Section 2, we describe a semi-parametric methodology that allows for the estimation of the dependence structure of a set of returns without imposing any parametric restriction on their marginal distributions. This section also describes a related test statistic that can be used to evaluate the statistical relevance of our point estimates. Section 3 presents the results of our empirical investigation, while Section 4 comments on our findings.

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[4] See, for example, Mashal and Naldi (2002) and Mashal and Zeevi (2002).

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2. METHODOLOGY

This section describes an estimation procedure that is used to calibrate a certain class of dependence structures to the equity and asset returns data. We first digress briefly to discuss some dependence-related concepts, and then proceed to describe the estimation methodology and an associated testing procedure.

The key ingredient in modeling and testing dependencies is the observation that any d-dimensional multivariate distribution can be specified via a set of d marginal distributions that are "knitted" together using a copula function. Alternatively, a copula function can be viewed as "distilling" the dependencies that a multivariate distribution attempts to capture, by factoring out the effect of the marginals. Copulas have many important characteristics that make them a central concept in the study of joint dependencies, see, e.g., the recent survey paper by Embrechts *et al.* (2001).

A particular copula that plays a crucial role in our study is given by the dependence structure underlying the multivariate Student t distribution. While the Gaussian distribution lies at the heart of most financial models and builds on the concept of correlation, the Student t retains the notion of correlation but adds an extra parameter into the mix, namely, the *degrees-of-freedom* (DoF). The latter plays a crucial role in modeling and explaining extreme co-movements in the underlyings.

Moreover, it is well known that the Student t distribution is very "close" to the Gaussian when the DoF is sufficiently large (say, greater than 30); thus, the Gaussian model is *nested* within the t-family. The same statement holds for the underlying dependence structures, and the DoF parameter effectively serves to distinguish the two models. This suggests how empirical studies might test whether the ubiquitous Gaussian hypothesis is valid or not. In particular, these studies would target the dependence structure rather than the distributions themselves, thus eliminating the effect of marginal returns that would "contaminate" the estimation problem in the latter case. To summarize, the t-dependence structure constitutes an important and quite plausible generalization of the Gaussian modeling paradigm, which is our main motivation for focusing on it in this study.

With this in mind, the key question that we now face is how to estimate the parameters of the dependence structure. In particular, consider a basket of d names, each following an arbitrary marginal F_i i=1,…,d, and having a joint distribution H with underlying t-dependence structure, which is denoted by

$$C(\cdot;\nu,\Sigma).$$

Here, Σ denotes the correlation matrix and ν the DoF parameter.[6] Suppose we have n observations $\{X_i\}_{i=1}^n$ on these d names, where the returns $X_i = (X_{i1},\ldots,X_{id})$ are assumed to be mutually independent and distributed according to H.

If the marginal distributions were known, then we could use the representation (1) in the Appendix to conclude that

$$U := F(X_i) \sim C(\cdot;\nu,\Sigma),$$

6 *For details on the relation between the joint distribution H and the copula C, see a version of Sklar's Theorem in the Appendix.*

where $F(X_i) := (F_1(X_{i1}),...,F_d(X_{id}))$ is the vector of marginal distributions, the symbol ":=" reads "defined as", and the symbol "~" reads "distributed according to."

Since the structure of the marginals is arbitrary and unknown to us, we propose to use the empirical distribution function as a surrogate, that is,

$$\hat{F}_j(\cdot) := \frac{1}{n}\sum_{i=1}^{n} I\{X_{ij} \leq \cdot\},\ j=1,\ldots,d,$$

where $I\{\cdot\}$ is the indicator function, i.e.,

$$I(A) := \begin{cases} 1, & \text{if } A \text{ is true} \\ 0, & \textit{otherwise.} \end{cases}$$

We then work with the *pseudo-sample* observations [7]

$$\hat{U}_i = \left(\hat{F}_1(X_{i1}),...,\hat{F}_d(X_{id})\right),\ i=1,\ldots,n.$$

Focusing on the t-dependence structure $C(\cdot\,;\nu,\Sigma)$ [formally given by the t-copula (2) in the Appendix], let us denote by

$$\Theta = \left\{(\nu,\Sigma) : \nu \in (2,\infty], \Sigma \in R^{d\times d} \text{ is symmetric and positive definite}\right\}$$

the feasible parameter space, and set

$$\theta := (\nu,\Sigma).$$

Then, for a given pseudo-sample $\{U_i\}_{i=1}^n$ we set the *pseudo log-likelihood* function to be

$$L_n(\theta) = \sum_{i=1}^{n} \log c\left(\hat{U}_i;\theta\right),$$

where $c(\cdot;\theta)$ is the t-copula density function associated with C (see the Appendix). Now, let

$$\hat{\theta} := \left(\hat{\nu},\hat{\Sigma}\right)$$

denote the *maximum likelihood* (ML) estimator of the DoF and correlations, i.e., the value of $\theta \in \Theta$ maximizing $L_n(\theta)$. The results reported in the next section refer to estimates obtained in this manner.

As we mentioned earlier, the DoF parameter ν controls the tendency to exhibit extreme co-movements, and also measures the extent of departure from the Gaussian dependence structure. Given its pivotal role, in the sequel we focus on the accuracy of the DoF estimates in a more detailed manner.

Specifically, we use a *likelihood-ratio* formulation to test whether empirical evidence supports or rejects a given value of ν. To begin with, we fix a value of the DoF parameter ν_0 and consider the hypotheses

[7] *This approach follows the semi-parametric estimation framework developed in a more abstract context by Genest et al. (1995).*

$$H_0 : \theta \in \Theta_0 \quad \text{vs.} \quad H_1 : \theta \in \Theta ,$$

where

$$\Theta_0 = \{ \theta \in \Theta : \nu = \nu_0 \} \subset \Theta .$$

Then, we set the *likelihood-ratio test statistic* to be

$$\Lambda_n (\hat{\nu} \mid \nu_0) = -2 \log \frac{\sup_{\theta \in \Theta_0} \prod_{i=1}^{n} c(\hat{U}_i ; \theta)}{\prod_{i=1}^{n} c(\hat{U}_i ; \hat{\theta})}$$

To determine the adequacy of each value of ν_0, we need to characterize the distribution of the statistic $\Lambda_n(\hat{\nu} \mid \nu_0)$. Since this distribution is not tractable, the standard approach is to derive the asymptotic distribution and use that as an approximation. Specifically, Mashal and Zeevi (2002) arrive at the approximation

$$\Lambda_n(\hat{\nu} \mid \nu_0) \approx (1+\gamma)\chi_1^2 ,$$

where $\gamma > 0$ is a constant that depends on the null hypothesis, χ_1^2 denotes a random variable distributed according to a Chi-squared law with one degree-of-freedom, and "≈" reads "approximately distributed as" (for large values of n).[8] Thus, we can calculate approximate p-values as a function of ν_0 as follows

$$p - \text{value} \ (\nu_0) = P\left(\chi_1^2 \geq \frac{\Lambda_n (\hat{\nu} | \nu_0)}{(1+\gamma)} \right) .$$

By letting the *null hypothesis* ν_0 vary over the parameter space, we can compute the corresponding p-values and detect the range of degrees-of-freedom that are supported (respectively, rejected) by the observed return data. Notice that *large* values of the test statistic correspond to *small* p-values, thus indicating that the hypothesized ν is not plausible on the basis of the empirical observations. This hypothesis testing formulation illustrates the "sharpness" of the estimation results in a much stronger manner than if we had just focused on the associated confidence intervals for the parameter estimates.

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[8] *A rigorous derivation and an explicit characterization of γ is given in Appendix A of Mashal and Zeevi (2002) who also validate this asymptotic numerically.*

3. EMPIRICAL EVIDENCE

In this section, we apply the methodology outlined above to study the dependence structure of asset returns and compare it with that of equity returns. For consistency, asset and equity values are both obtained from KMV's database. The reader should keep in mind, however, that equity values are observable, while asset values have been "backed out" by means of KMV's CreditEdge™ implementation of a univariate Merton model. We use daily data covering the period from 12/31/00 to 11/8/02.

In the following sub-sections, we focus our attention on two portfolios, the 30-name Dow Jones Industrial Average and a 20-name high-yield portfolio.

3.1 DJIA Portfolio

Following the semi-parametric methodology described in Section 2, we estimate the number of degrees-of-freedom (DoF) of a *t*-copula without imposing any structure on the marginal distributions. Using the test statistic introduced earlier, Figure 1 presents a sensitivity analysis for various null hypotheses of the underlying tail dependence, as captured by the DoF parameter. The two horizontal lines represent significance levels of 99% and 99.99%; a value of the test statistic falling below these lines corresponds to a value of DoF that is not rejected at the respective significance levels.

The minimal value of the test statistic is achieved at 12 DoF (ν =12) for asset returns and at 13 DoF (ν =13) for equity returns. In both cases, we can reject any value of the DoF parameter outside the range [10,16] with 99% confidence; in particular, the null assumption of a Gaussian copula (ν =∞) can be rejected with an infinitesimal probability of error.

Finally, notice that the point estimates of the asset returns' DoF lie within the non-rejected interval for the equity returns' DoF, and vice versa, indicating that the two are essentially indistinguishable from a statistical significance viewpoint. Moreover, the difference between the joint tail behavior of a 12- and a 13-DoF *t*-copula is negligible in terms of any practical application.

Figure 1. DJIA Portfolio: Asset and Equity Returns Test Statistics as Functions of Null Hypothesis for DoF

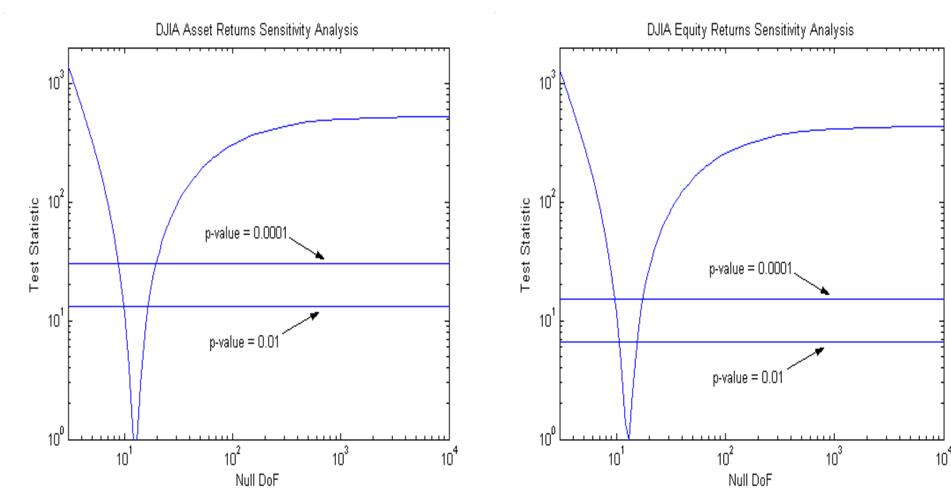

Figure 2 reports the point estimates of the DoF for asset and equity returns in the DJIA basket, as well as for three subsets consisting of the first, middle, and last 10 names (in alphabetical order). The similarities between the joint tail dependence (as measured by the DoF) of asset and equity returns are quite striking. [9]

Figure 2. Maximum Likelihood Estimates of DoF for DJIA Portfolios

Portfolio	Asset Returns DoF	Equity Returns DoF
30-Name DJIA	12	13
First 10 Names	8	9
Middle 10 Names	10	10
Last 10 Names	9	9

Next, we compare the remaining parameters that define a *t*-copula, i.e., the correlation coefficients. Using a robust estimator based on Kendall's rank statistic[10], we compute the two 30x30 correlation matrices from asset and equity returns. The maximum absolute difference (element-by-element) is 4.6%, and the mean absolute difference is 1.1%, providing further evidence of the similarity of the two dependence structures.

In summary, the empirical evidence strongly supports the widespread practice of using equity return series to estimate underlying dependencies between asset returns.

3.2 High-Yield Portfolio

In this section we investigate whether the similarities between the dependence structures of asset and equity returns persist when we restrict our attention to lower-quality, higher-leverage issuers. Figure 3 shows the constituents of a 20-name portfolio that we have randomly selected from the universe of publicly traded, high-yield companies covered by KMV.

Figure 3. High-Yield Portfolio

Names 1-5	Names 6-10	Names 11-15	Names 16-20
AES	Atlas Air Worldwide Holding Inc.	MGM Mirage	Safeway Inc.
Adaptec Inc.	Echostar Communication Corp.	Navistar International	Saks Inc.
Airgas Inc.	Gap Inc.	Nextel Communications	Service Corporation International
AK Steel Holding Inc.	Georgia-Pacific Corp.	Northwest Airlines Corp.	Solectron Corp.
Alaska Air Group Inc.	L-3 Communications Holdings Inc.	Royal Caribbean Cruises Ltd.	Sovereign Bancorp Inc.

[9] *The range of accepted DoF is very narrow in each case, exhibiting similar behavior to that displayed in Figure 1.*

[10] *See Naldi (2002), elsewhere in this issue.*

Figure 4 reports the ML estimates for the DoF of asset and equity returns for this 20-name high-yield portfolio, as well as for the four 5-name sub-portfolios shown in Figure 3. Once again, the estimated DoF for asset and equity returns are very close. When analyzing correlations, a similar behavior is also observed, specifically, the maximum absolute difference in the correlation coefficients is 6.7% and the mean absolute difference is 1.6%.

Figure 4. Maximum Likelihood Estimates of DoF for High-Yield Portfolios

Portfolio	Asset Returns DoF	Equity Returns DoF
20-Name Portfolio	15	16
Names 1-5	15	13
Names 6-10	14	12
Names 11-15	10	10
Names 16-20	13	15

4. SUMMARY

Our empirical investigation of the dependence structure of asset returns sheds some light on the two main issues that were raised in the introduction.

First, the assumption of Gaussian dependence between asset returns can be rejected with extremely high confidence in favor of an alternative "fat-tailed dependence." Multivariate structural models that employ correlated Gaussian processes for the diffusion of asset values will generally underestimate default correlations, and thus undervalue the junior risk and overvalue the mezzanine and senior risk of multi-name credit products. Fat-tailed increments of the joint value processes will produce more accurate joint default scenarios and more accurate valuations.

Second, the dependence structures of asset and equity returns appear to be strikingly similar. The KMV algorithm that produces the asset values used in our analysis is nothing else than a sophisticated way of de-leveraging the equity to get to the value of a company's assets. Therefore, the popular conjecture that the different leverage of assets and equity will necessarily create significant differences in their joint dynamics seems to be empirically unfounded, even when we analyze the value processes of low-grade issuers. Instead, our results suggest that the differences in leverage are mostly reflected in the marginal distributions of returns. From a practical point of view, these results represent good news for practitioners who only have access to equity data for the estimation of the dependence parameters of their credit models.

APPENDIX

Copula Functions and the *t*-Dependence Structure

A *copula function* is a multivariate distribution defined over $[0,1]^d$, with uniform marginals. The importance of the copula stems from the fact that it captures the dependence structure of a multivariate distribution. This can be seen more formally from the following fundamental result, known as Sklar's theorem, adapted from Theorem 1.2 of Embrechts *et al.* (2001).

Sklar's Theorem. Given a d-dimensional distribution function H with continuous marginal cumulative distributions $F_1,\ldots,F_d$, there exists a unique d-dimensional copula C: $[0,1]^d \to [0,1]$ such that

$$H(x_1,\ldots,x_d) = C(F_1(x_1),\ldots,F_d(x_d)). \tag{1}$$

As indicated in the main body, this study focuses on a natural generalization of the Gaussian dependence structure, namely the *Student t-copula*. To this end, let t_ν denote the (standard) univariate Student-t cumulative distribution function with ν degrees-of-freedom, namely,

$$t_\nu(x) = \int_{-\infty}^{x} \frac{\Gamma((\nu+1)/2)}{\Gamma(\nu/2)(\nu\pi)^{1/2}} (1 + y^2/\nu)^{-(\nu+1)/2}\, dy.$$

Then, for $u = (u_1,\ldots,u_d) \in [0,1]^d$

$$C(u_1,\ldots,u_d;\nu,\Sigma) = \int_{-\infty}^{t_\nu^{-1}(u)} \frac{\Gamma((\nu+d)/2)}{|\Sigma|^{1/2}\,\Gamma(\nu/2)(\nu\pi)^{d/2}} (1 + y^T \Sigma^{-1} y/\nu)^{-(\nu+d)/2}\, dy \tag{2}$$

is the t-copula parameterized by (ν, Σ), where Σ is the correlation matrix, and

$$t_\nu^{-1}(u) := (t_\nu^{-1}(u_1),\ldots,t_\nu^{-1}(u_d)).$$

The density of the t-copula, $c(\cdot;\nu,\Sigma)$, is obtained by differentiating the t-copula w.r.t. $u_1,\ldots,u_d$ [for details, see, e.g., Section 3.1 in Mashal and Zeevi (2002)].

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Keeping the Score in the Credit Market: A Methodology Using ESPRI

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ESPRI*, standing for Equity returns as* SPR*ead* I*ndicators, is a credit selection model that combines information in equity returns and credit spreads to find bond portfolios with superior risk-reward characteristics. This model was described in the Q1-2002 edition of* Quantitative Credit Research Quarterly*, and has found applications in portfolio management, trading, risk management and CDO structuring. We introduce an extension to the model allowing the scoring of issuers and sectors and investigate the back-testing of related strategies.*

1. INTRODUCTION

ESPRI is a credit selection model that combines information in equity returns and credit spreads to find bond portfolios with superior risk-reward characteristics. This model was first described in the Q1-2002 edition of *Quantitative Credit Research Quarterly*. Our objective was to investigate predictability in the credit market and in particular the effect of the equity market on corporate bond valuation. We have documented, using a simple strategy based on screening and sorting corporate bonds, that it was possible to out-perform our benchmark and generate good information ratios.

Since then ESPRI has found widespread applications in portfolio management, trading, risk management and CDO structuring. We introduce an extension to the model allowing the scoring of issuers and sectors. Using the ESPRI scores, we are able to select outperforming and under-performing portfolios.

In this article, we introduce the ESPRI scoring system, present some back-testing results on the score, describe the ESPRI sector and issuer maps and provide some guidelines on how to interpret the ESPRI results.

2. A REFRESHER ON ESPRI

ESPRI classifies corporate bonds into nine categories based on the equity return of the issuer and current spread level of the bond. Specifically, the bonds of a given universe are sorted into three spread buckets, H, M & L (High, Medium & Low) and then further sorted into three equity return buckets, H, M &L. The bonds are then labelled with these two letters, the first for the spread bucket, the second for the equity return bucket, as can be seen from Figure 1. Following detailed back-testing, the nine portfolios' qualitative performance characteristics are categorized as shown in Figure 1.

Figure 1. ESPRI Grid

		Equity Return		
		Top 20%	*Mid 60%*	*Btm 20%*
OAS Level	*Top 33%*	HH Potential Outperform	HM Neutral	HL Potential Underperform
	Mid 33%	MH Moderate Outperform	MM Neutral	ML Moderate Underperform
	Btm 33%	LH Defensive Outperform	LM Neutral	LL Potential Underperform

ESPRI can be adjusted for different investment horizons. Back-testing shows a symmetry between the period over which we calculate the issuer's equity return, and the subsequent horizon over which the forecasting is effective. For example, for a three month holding period, the medium term outlook version of the model, using 3 month total equity returns, is appropriate.

3. THE ESPRI SCORE

3.1 Introducing the ESPRI Score

We have recently been reporting ESPRI Scores for sectors and issuers in a variety of publications. The score is a value between 0 and 10 summarizing the signal from the ESPRI rankings of the bonds of the issuer or sector. A high score indicates the majority of amount outstanding of the issuer or sector is ranked positively (in particular HH or LH), and a low score indicates that the majority is ranked negatively (in particular, HL or LL). As with the bond-level version of the model, scores for different horizons can be computed by varying the period over which the equity return is calculated.

The ESPRI Score for an issuer or sector is computed by first calculating the percentage of total amount outstanding of bonds of the sector in the given currency falling into each of the 9 ESPRI bond portfolios. We then calculate a weighted average ESPRI ranking for the issuer or sector in the form of a co-ordinate on the ESPRI grid. The co-ordinates $(x, y) \in [0,1]^2$ are therefore:

$$\begin{aligned} x &= 0\times(HH+MH+LH)+0.5\times(HM+MM+LM)+1\times(HL+ML+LL) \\ y &= 0\times(HH+HM+HL)+0.5\times(MH+MM+ML)+1\times(LH+LM+LL), \end{aligned}$$

where, for example, HH = percentage of total amount outstanding of the issuer or sector falling into portfolio HH. Corresponding to the classifications in figure 1 we then map this co-ordinate to a score between 0 and 10 by using the weighting of ESPRI portfolios shown in Figure 2.

Figure 2. Score Weighting Scheme

OAS Level \ Equity Return	*Top 20%*	*Mid 60%*	*Btm 20%*
Top 33%	10	5	0
Mid 33%	7.5	5	2.5
Btm 33%	10	5	0

The quadratic function used is calibrated to generate the weighting for the nine ESPRI portfolios in Figure 2.

3.2 An Example Score Calculation

We take the following real example from the ESPRI August 30th run for USD Telecoms from the medium term outlook (using 3 month equity returns). Figure 3 shows the amount outstanding distribution:

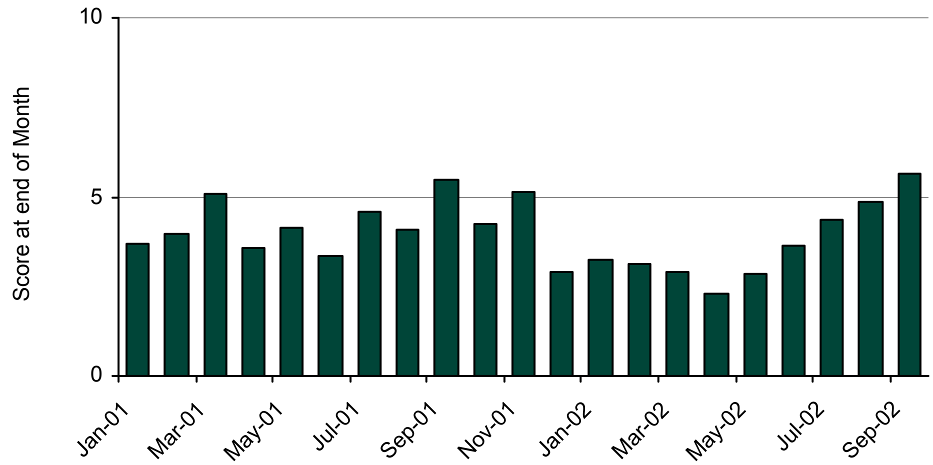
Figure 3. USD Telecom Weights 08/30/02

OAS Level \ Equity Return	*Top 20%*	*Mid 60%*	*Btm 20%*
Top 33%	17%	41%	20%
Mid 33%	6%	6%	8%
Btm 33%	0%	0%	2%

The distribution corresponds to weighted average co-ordinates of $(0.53,0.88)$, i.e. near the top of the grid slightly right of the middle, which in turn map to a score of 4.8 out of 10.

Naturally, we can then form an ESPRI Score time series from which trends may be apparent. Using the same example, the chart in figure 4 gives the medium term score series for USD Telecoms from January 2001 to September 2002.

Figure 4. ESPRI Medium Term Outlook Score Series for USD Telecoms



Note that, owing to the construction of the model, 60% of bonds are ranked as neutral (i.e. in one of the portfolios HM, MM or LM) and therefore we expect to find scores for large sectors to be clustered around 5. It is then the differences of the scores from the level 5 that are informative. In Figure 4 we can see a neutral to negative signal for telecoms throughout 2001, followed by a strongly negative period from December 2001 through the first six months of 2002 and then beginning to recover in July and August.

3.3 ESPRI Issuer Scores in Action

The charts in Figures 5 & 6 compare the ESPRI Issuer Score time series with the average option-adjusted spread level of bonds of the issuer falling into the respective Lehman Brothers corporate index.

Figure 5. ESPRI Medium Term Outlook Score Series for ENRON

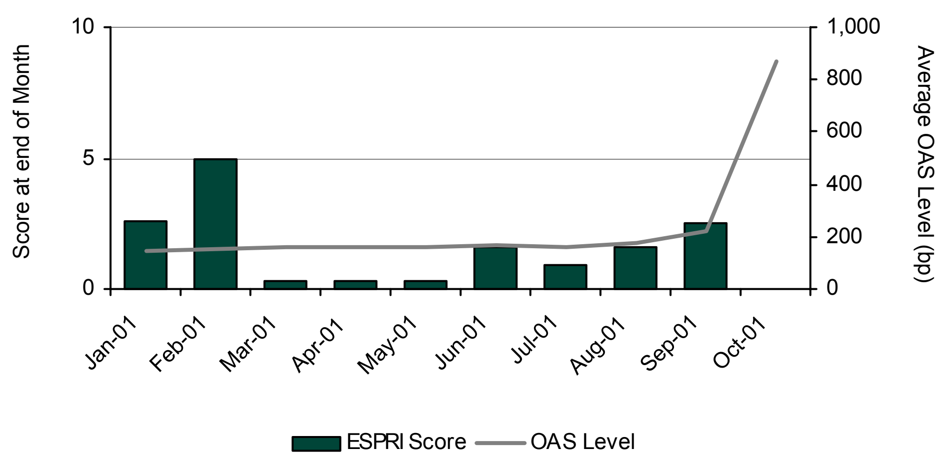


Figure 5 shows the score evolution for Enron Corp. As a score of 5 indicates a neutral outlook, scores beneath 5 can be viewed as progressively more negative. From this point of the chart clearly shows a persistent negative signal for Enron from March 2001 onwards, even though the average spread did not begin to gap out until September/October 2001.

Figure 6. ESPRI Medium Term Outlook Score Series for AT&T Corp

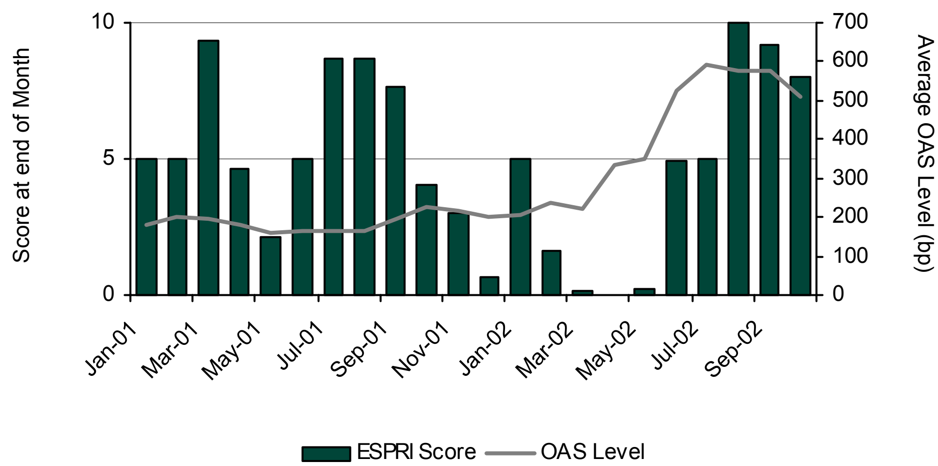


Figure 6 shows the corresponding chart for AT&T Corp. The ESPRI signal here is clearly weak from Nov 2001 onwards, and worsens further from March 2002 prior to the spread widening of approximately 300bp between April and July 2001. Interestingly we then see the score climb dramatically up through neutral to a positive 10 by August 2002. The average spread level then drops by approximately 100bp in the subsequent two months, and, on an excess return basis, the AT&T Corp bonds significantly outperform.

3.4 Testing the Score

A more rigorous testing of the ESPRI Issuer and Sector scores can be achieved in a similar back-testing framework to that used in the original bond-level ESPRI model.

Each month the bonds of our universe[1] are sorted within currency and rating groups by the score of the issuer or sector into three portfolios:

- High (Score ranks in the top 20%)
- Medium (Score ranks in the middle 60%)
- Low (Score ranks in the bottom 20%)

We then analyze the average excess returns generated by these three portfolios over a three month holding period. We use overlapping portfolios, one generated each month, employing an identical methodology to that used in our main ESPRI model back-testing described in *Quantitative Credit Research Quarterly*. The results of this are presented in figure 7. This test

[1] *Defined as bonds in the US Corporate Index with ratings between Aa and Baa for which we were able to find a related equity.*

is equivalent to buying all the bonds of a sector and analyzing results under the above bucketing.

We report first the average *monthly* excess return of each portfolio less the average excess return of the universe (in this case, the currency/rating universe) and, in italics beneath this number, the annualised information ratio (defined as the average month excess return divided by the volatility of monthly returns multiplied by the square root of twelve). The information ratio characterizes the risk-adjusted return of the strategy – i.e. the return per unit volatility. For the USD results, a value with an approximate magnitude of 0.7 indicates that the excess return is statistically significantly different from zero at the 95% level. The corresponding threshold for the Euro results is 1.0 owing to the shorter sample period.

Figure 7. Score Tests – USD Bonds May 1994 to June 2002 – Three Month Investment Horizon

BBB-Rated Bonds	Issuer SCORE	*Top 20%*	**16.7** *(2.1)*	Sector SCORE	*Top 20%*	**10.6** *(1.3)*
		Mid 60%	**1.8** *(0.5)*		*Mid 60%*	**1.5** *(0.3)*
		Btm 20%	**-24.5** *(-1.1)*		*Btm 20%*	**-12.4** *(-0.8)*
A-Rated Bonds	Issuer SCORE	*Top 20%*	**5.9** *(1.2)*	Sector SCORE	*Top 20%*	**5.9** *(1.4)*
		Mid 60%	**2.4** *(0.8)*		*Mid 60%*	**1.5** *(0.4)*
		Btm 20%	**-17.0** *(-1.1)*		*Btm 20%*	**-8.1** *(-0.9)*

In each case, bonds whose issuer or sector scores highly (in the top 20%) outperform the universe, whilst those scoring lower (in the bottow 20%) underperform. Bearing in mind these are monthly returns (assuming a three month investment horizon), the results are signifcant both on an absolute basis and a risk-adjusted basis – all of the information ratios of the top 20% and bottom 20% portfolios are above our 0.7 threshold in magnitude.

Score Maps

One limitation of the score system is the reduced information on the distribution of the bonds across the ESPRI grid system. In particular, the score gives equal weight to our HH and LH categories, despite their differing characteristics:

- The HH (High Spread, High Equity Return) portfolio is viewed as a potential out-performer: relatively cheap bonds (as indicated by the spread level) with positive signals on the fundamental side (as indicated by the strong equity return).
- The LH (Low Spread, High Equity Return) portfolio is viewed as a more defensive core holding. Historical testing has shown that bonds falling into this category do not necessarily outperform significantly on average, but possess valuable risk characteristics, in particular their hedging nature in the event of overall credit market decline and subsequent flight to quality.

The *ESPRI Sector Map* (Figure 8) is a development in the presentation of sector scores addressing this limitation. The Sector Map is best thought of as a **continuous representation of the standard ESPRI grid system** ; sectors are plotted at their weighted average position (or "center of mass") as defined by the rankings of the bonds of the sector and weighted by their par amount outstanding. Thus, a sector receiving a high score because its bonds are in the defensive LH region, will appear in the bottom left region of the map, whilst a sector concentrated in HH will appear near the top left corner. Sector Scores are shown as the contours of this map – **the lighter the shade the higher Sector Score and vice versa**. The size of the spot is proportional to the par size of the sector in the respective Lehman Brothers Corporate Index.

Figure 8.

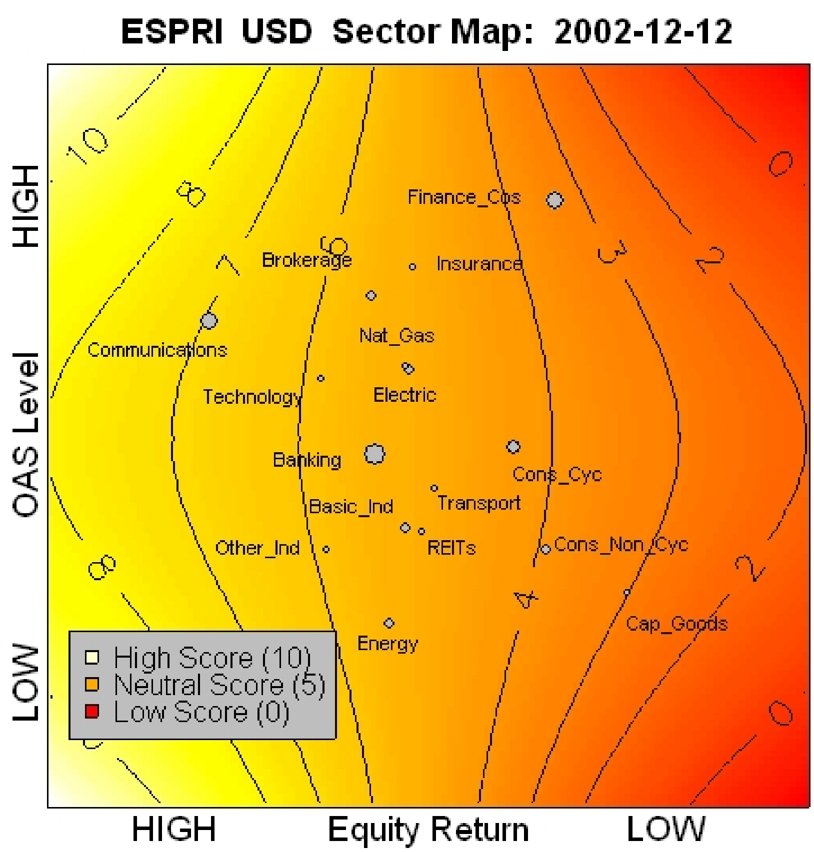


Figure 8 shows the USD ESPRI Sector map for November. Owing to the averaging effect from the score computation we find the sectors tend to be largely clustered around the centre. However, as the testing demonstrates, the scores are effective as a relative value tool as well as providing outright signals. In this map we see the communications sector featuring with the highest score of 6.8 (as can be read by looking at the score contours), whilst technology is close behind with a score of 6.0. Both sectors are plotted slightly nearer the top left corner than the bottom left, indicating the model views them as more bullish outperform recommendations (or, expressed differently, the high beta outperformers). Contrasting with this we see finance companies with a low score of 2.3, firmly located in the top right (high volatility) corner of the ESPRI grid. Sectors like transport have a weak score but fall in the middle latitudes of the map, indicating an average spread.

It is possible to draw the same maps at the issuer level for a given sector. Examples are shown in Figures 9 and 10.

Figure 9.

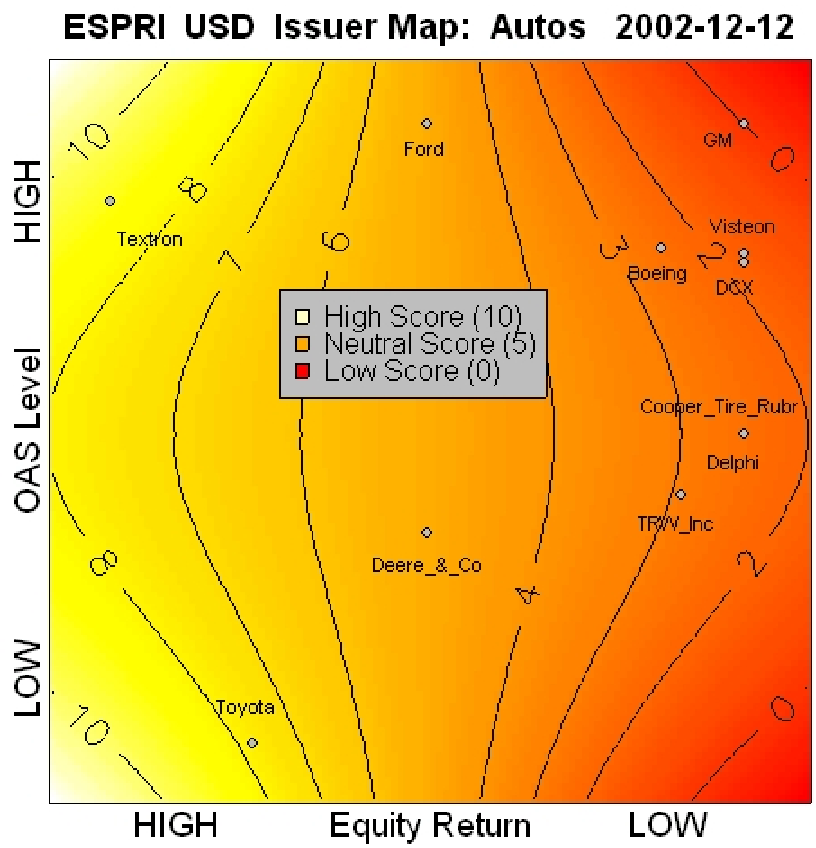
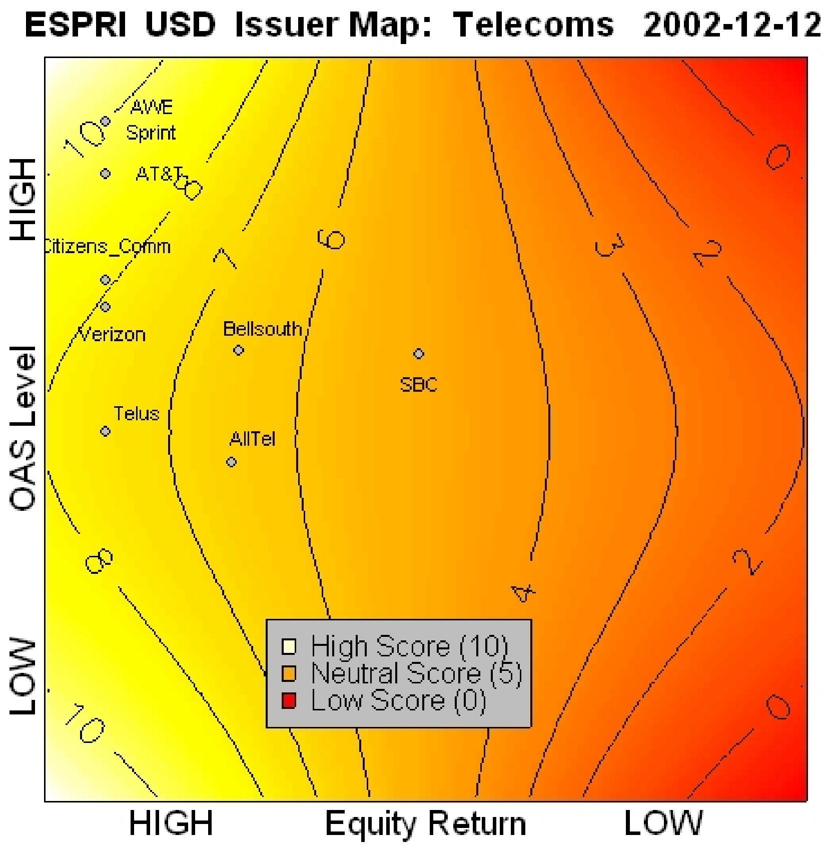


Figure 10.



4. INTERPRETING THE ESPRI RESULTS

The following are broad guidelines on using the ESPRI Scores to make investment decisions:

In general an ESPRI score of 8 and above is a positive signal (the scale is from 0 to 10) and a score of 2 and below is a rather negative signal. It is interesting to monitor the list of the issuers falling into these two categories (high and low scores).

To assess the medium-term outlook of a particular credit, one should examine the ESPRI 3-month score and its change from last month. If the score has increased, the prospect of the issuer has improved for the next three months, if the score has decreased, its prospect has worsened for the same time period.

All the ESPRI results which are published along with credit analysts' views are for a three-month horizon as fundamental credit views usually apply for an horizon of 3 to 6 months.

To assess the short-term outlook of a particular credit, one should examine the ESPRI 1-month score and its change from last month. If the score has increased, the prospect of the issuer has improved for the next month, if the score has decreased, its prospect has worsened for the same time period.

The ESPRI results which are published with a 1-month horizon, are mostly for trading purposes and for defining entry and exit points in credit investment. The one-month horizon is useful to detect turning point since it incorporates the most recent news.

The comparison of the ESPRI 3-month and ESPRI 1-month scores is interesting because it assesses the consistency of the equity momentum signal. If they both agree, the ESPRI signal is more robust. If they disagree, it might indicate that the 3-month trend is reversing and thus that the ESPRI 3-month should be interpreted with caution. This sometimes happen around earnings releases or trading updates which can significantly impact the stock price. As an example of this in action we can run a test similar to those in Figure 7. Each month we rank the bonds on their 3-month score but this time we remove bonds ranking in the top 20% having a 1-month score of less than or equal to 5. Similarly, we remove bonds bonds ranking in the bottom 20% with 1-month scores greater than or equal to 5. Though not dramatic, the effect on the returns of these portfolios is interesting. The filtered top 20% portfolios have average excess returns over the universe of 4 bp and 23 bp for A & BBB respectively, against the 6 bp and 17 bp from the unfiltered test. The downside shows a greater improvement, the filtered bottom 20% portfolios have average excess returns over the universe of –26 bp and –36 bp respectively against –17 bp and –24 bp from the unfiltered test.

5. CONCLUSION

The ESPRI model has found applications in portfolio management, trading, risk management and CDO structuring. We have introduced an extension to the model allowing the scoring of issuers and sectors and investigated the back-testing of related strategies. The ESPRI scores are effective as a relative value tool as well as providing outright signals. Along the ESPRI scores, the ESPRI sector map and ESPRI issuer map are useful graphic tool to visualize the relative value between different sectors and different issuers. The use of ESPRI is as much an art as it is a science. With practice in reading the individual bond rankings, the ESPRI issuer and sector scores and the ESPRI issuer and sector maps, the investor should be able to improve the credit selection and investment process.

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An Analytical Portfolio Credit Model with Tail Dependence

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We incorporate the effect of tail dependence into a simple one-factor portfolio credit model and present an analytical calculation of the portfolio loss distribution in the large homogeneous portfolio (LHP) limit. We examine how tail dependence changes the Value-At-Risk (VaR) of the portfolio compared to the original LHP result due to Vasicek. We show how the VaR for a finite portfolio converges to the LHP limit as a function of the number of exposures, and discuss how the incorporation of tail dependence would affect the granularity adjustment set out for bank lending portfolios in the new Basel II Capital Accord.

1. INTRODUCTION

The credit risk modelling of large portfolios is a difficult task from both a calibration and computational perspective. Even when we restrict our model to default mode – capturing the risk of multi-asset default only – the sheer amount of calibration data and the computational effort can often be sufficient to render the whole approach impractical. For example, a complete default-mode model of a 1,000 name portfolio default loss distribution requires knowledge of the default probability term structure for each asset, knowledge about each asset's recovery rate distribution and a 1,000 by 1,000 correlation matrix. Depending on the type of dependence structure used, other parameters may be required. While efficient default-mode Monte Carlo approaches do exist [Li], these are subject to simulation noise, requiring several tens of thousands of paths, especially when the quantities of interest are in the tails of the distribution.

While there has been a lot of recent effort in attempting to find semi-analytical approaches to calculating portfolio loss distributions which avoid the "noise" and computational effort required by a Monte-Carlo approach, none has managed to match the trade-off between speed and simplicity of Vasicek's large homogeneous portfolio (LHP) limit [Vasicek]. With this model, the portfolio is assumed to consist of infinitely many homogeneous assets, enabling us to derive closed-form analytical solutions. However, the model suffers from one important weakness – it ignores tail dependence, an empirically observed phenomenon [Sornette, Mashal], and an important component of portfolio credit risk and pricing [O'Kane and Schloegl, Mashal and Naldi]. In the following, we extend the LHP model to incorporate tail dependence and present analytical solutions for the computation of the loss distribution. Using simulations, we check the validity of the approach for computing portfolio tail risk measures and show how to adjust the risk measure to take the real portfolio's granularity into account.

2. THE LHP MODEL

Vasicek's LHP model is a hybrid portfolio credit model in the sense that it combines the structural and reduced form approaches [O'Kane and Schloegl 2001]. It is structural in the sense that a firm's default is triggered when the asset return of that firm falls below a specified threshold. However it is reduced-form in the sense that the level at which the threshold is set is determined by the default probability chosen for that particular credit.

In the original model, it was assumed that the portfolio consists of N equally weighted exposures where the asset returns of an issuer i are given by

$$\xi_i = \beta Z + \sqrt{1-\beta^2}\varepsilon_i \qquad (1)$$

where $Z, \varepsilon_1, \ldots, \varepsilon_N$ are all independent, and $Z, \varepsilon_1, \ldots, \varepsilon_N \sim N(0,1)$. The factor Z can be interpreted as the systemic market factor seen by all asset returns, while the ε_i represents the idiosyncratic contribution to the return of asset i, and β is a measure of its correlation with the systemic factor. The pair-wise correlation between any two names i and j is simply equal to β^2.

Within the framework an asset defaults if the asset return falls below some threshold D. Since the asset returns are normally distributed with mean zero and standard deviation of one, the probability of default is given by $\Phi(D)$, where Φ denotes the cumulative distribution function of the standard normal distribution. The probability of default, PD, can be calibrated by choosing a value for the threshold given by

$$D = \Phi^{-1}(PD)$$

Firms with a positive pair-wise asset return correlation tend to default together – the model exhibits the desired default correlation. The correlation of the asset returns is typically calibrated using equity correlation, the rationale being that as equity can be shown to be a call option on the assets of the firm struck at the face value of the debt [Merton 1974], the correlation of asset returns is, in the continuous time limit, equal to the correlation of equity returns. Using finite time intervals, this becomes an approximation. Nevertheless, empirical evidence presented by Mashal *et al*. elsewhere in this issue shows that the dependence structures of asset and equity returns are actually very similar.

The assumption of a normal distribution for asset returns means that Vasicek's model implicitly assumes the dependence structure of a multivariate Gaussian copula, this is the same dependence structure which underlies the CreditMetrics™ model. However, recent empirical studies of the joint behaviour of equity returns have suggested that the Gaussian copula does not accurately capture the true joint dynamics. Specifically, it underestimates the likelihood of significant positive and negative co-movements in the equity returns. The consequence is that the Gaussian copula also ignores the likelihood of large negative co-movements in the asset return processes and so underestimates the likelihood of joint default.

This tendency for large co-movements in equity returns is captured by a measure known as tail dependence. A formal definition is given e.g. in [O'Kane and Schloegl 2002b]. In the context of portfolio credit, it is the probability that one credit defaults conditional on another credit also defaulting, in the limit that both default probabilities go to zero. For a Gaussian copula this quantity is always equal to zero for all values of correlation below 100%. However, since empirical studies find a non-zero value for the tail dependence of equity returns, we need to consider alternative copulas which possess this property. Perhaps the simplest way to do this is to move to the Student-t copula. This is a generalized version of the Gaussian copula which introduces a new parameter, ν, known as the number of degrees of freedom (DoF). In the limit of large ν, the Student-t copula asymptotically approaches the Gaussian copula.

In the following, we compute the analytical form of the LHP loss distribution under the assumption of a Student-t copula asset value distribution. We then use this to compare the Value-At-Risk with and without tail dependence. We verify the accuracy of the LHP calculation using a linear extrapolation of results for finite sized portfolios. After this we consider the effect of tail dependence on the Basel II granularity adjustment.

3. THE LHP WITH TAIL DEPENDENCE

In order to introduce the tail dependence described above we must change the distribution for the asset returns. We emphasize that, because we are fitting the default threshold to the exogenous issuer default probabilities, it is the dependence structure of the asset returns which is important rather than the marginal distributions. We define the asset returns to be distributed according to a multivariate Student-t distribution with ν degrees of freedom and retain the one-factor correlation structure. This gives

$$\xi_i = \frac{\beta Z + \sqrt{1-\beta^2}\varepsilon_i}{\sqrt{W/\nu}} = \frac{\beta Z}{\sqrt{W/\nu}} + \frac{\sqrt{1-\beta^2}\varepsilon_i}{\sqrt{W/\nu}} \qquad (2)$$

where W is an additional independent random variable following a chi-square distribution with ν degrees of freedom. This is the simplest possible way to introduce tail dependence via a Student-t copula function. Note that this is no longer a "factor model" in the sense that the asset return is not composed of two independent and random factors. As both the market and idiosyncratic terms "see" the same value of W, they are no longer independent.

Default of issuer i occurs if the asset return falls below some threshold D. This can be rewritten as the condition

$$\varepsilon_i < \frac{D\sqrt{W/\nu} - \beta Z}{\sqrt{1-\beta^2}} \qquad (3)$$

Defining G as the cumulative density function of the Student-t distribution with ν degrees of freedom, to calibrate the model to the individual asset default probabilities we need to set

$$D = G^{-1}(PD) \quad (4)$$

where PD is the default probability of the homogeneous assets in the portfolio.

We can interpret this model as a *mixing model* with

$$\eta = D\sqrt{W/\nu} - \beta Z \qquad (5)$$

as *mixing variable*, cf. [Frey and McNeil] for more on the general relationship between mixing and copula models. The importance of this is that, conditional on η, the asset returns ξ_i are independent, and the conditional default probability is given by

$$P\left[\xi_i \le D \middle| \eta\right] = \Phi\left(\frac{\eta}{\sqrt{1-\beta^2}}\right) \quad (6)$$

where, as before, Φ is the cumulative distribution function of the standard normal distribution. By the law of large numbers, we can show that conditional on a specific realization of η, in the limit of N tending to infinity, the loss fraction on the portfolio equals

$$L(\eta)_{N\to\infty} = (1-R)\Phi\left(\frac{\eta}{\sqrt{1-\beta^2}}\right) \qquad (7)$$

where R is the fraction of face value recovered at default. Denoting the cumulative distribution function of η by F, we have

$$P[L \le \theta] = F[h^{-1}(\theta)] \quad (8)$$

where

$$h(x) = (1-R)\Phi\left(\frac{x}{\sqrt{1-\beta^2}}\right) \qquad (9)$$

The importance of these formulas lies in the fact that we effectively know the loss distribution if we can compute the cumulative distribution function of the mixing variable η. Similarly, the density of the loss distribution immediately follows from that of η by differentiating equation (8).

We briefly sketch how to compute F. Note that $W \sim \chi^2(\nu)$ and that the chi-square distribution is a special case of the gamma distribution. Using the incomplete gamma function, we can write the tail probabilities of W as

$$P\left[W \ge t\right] = \Gamma\left(\frac{\nu}{2}, \frac{t}{2}\right) \quad (10)$$

Conditioning first on the value of Z, we calculate

$$F(t) = P[\eta \le t] = \Phi\left(\frac{t}{\beta}\right) + \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{-t/\beta} \Gamma\left(\frac{\nu}{2}, \frac{\nu(t+\beta u)^2}{2D^2}\right) e^{-u^2/2} du \qquad (11)$$

Despite the fact that the integral on the right-hand side of equation (11) looks rather daunting, it is in fact very amenable to numerical treatment. Incomplete gamma functions are well-known and efficient implementation algorithms exist in the form of series and continued fraction representations. The integral can easily be computed using numerical integration – the trapezium rule suffices. Using similar techniques as in the derivation of equation (11), we can obtain a closed-form representation for the density as a sum over incomplete gamma functions. The interested reader is referred to the more technical paper [Schloegl 2002].

4. EFFECT OF TAIL DEPENDENCE ON TWO PORTFOLIOS

We test our analytical approach against a Monte Carlo simulation, considering two large homogeneous portfolios consisting of corporate issuers rated AA and BBB, respectively. We use the "idealized" default probabilities that Standard and Poor's uses for the rating of CDO tranches. Over a five year time horizon, these are 0.76% for AA rated corporates, and 2.5% for BBB rated ones. We consider 12 degrees of freedom to be reasonable, based on empirical studies, cf. the article by Mashal *et al*. elsewhere in this Quarterly. We have investigated three scenarios for the linear correlation between issuers: 5%, 20% and 50%. In each case we simulate the loss distribution of a homogeneous portfolio consisting of 5,000 credits using 100,000 sample paths, and overlay the (appropriately scaled) analytical density and the loss histogram derived from the simulation. For simplicity we assume a recovery rate of zero, as, for a homogeneous portfolio, this will just linearly rescale the VaR.

Figure 1. Analytic density versus Monte Carlo loss histogram for AA portfolio with 12 DoF and 20% issuer correlation

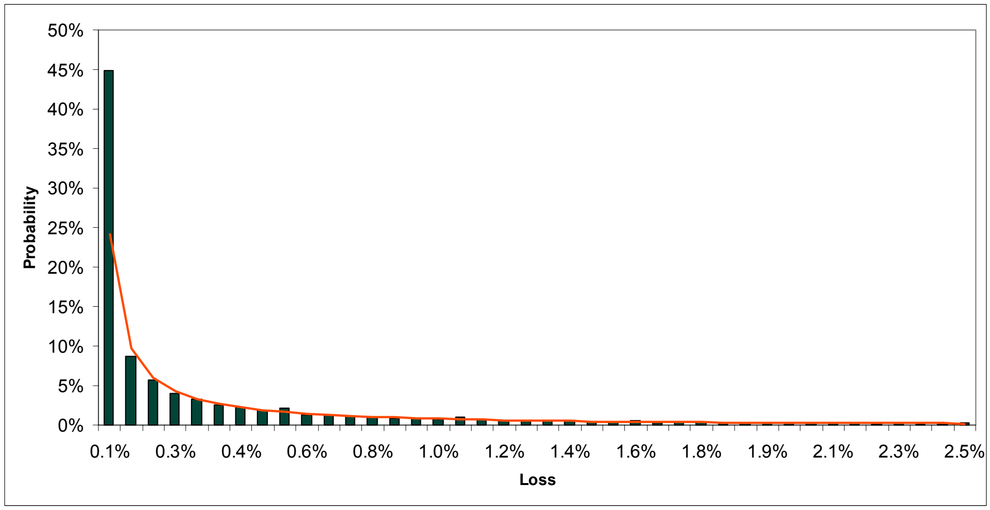
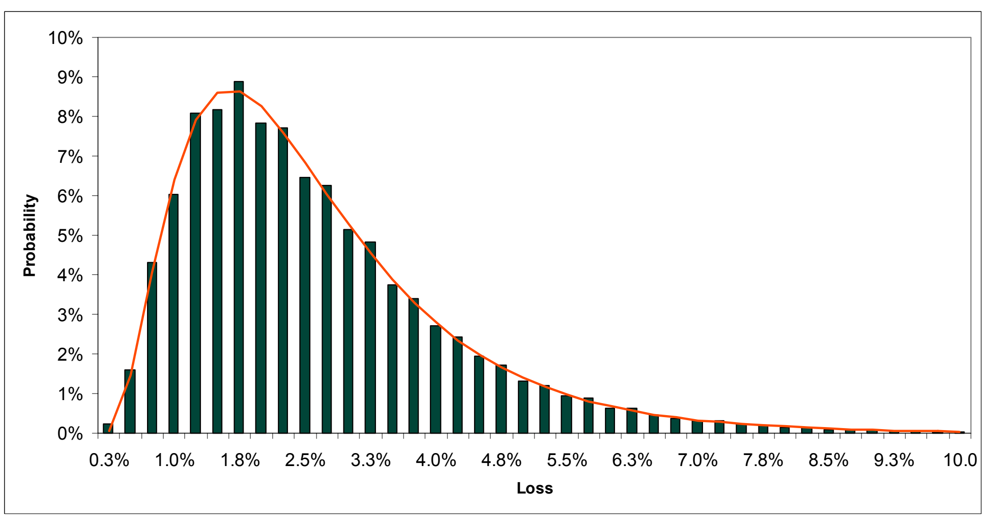


Figure 1 shows the fit of the analytic density for the AA portfolio with an issuer correlation of 20%. The fit is very good, the discrepancy for very small losses is to be expected because the density is very steep in this region. For a large number of degrees of freedom, we get results that are very similar to the Gaussian case. To illustrate this, Figure 2 shows the fit for the case of the BBB portfolio with 150 DoF and an issuer correlation of 5%.

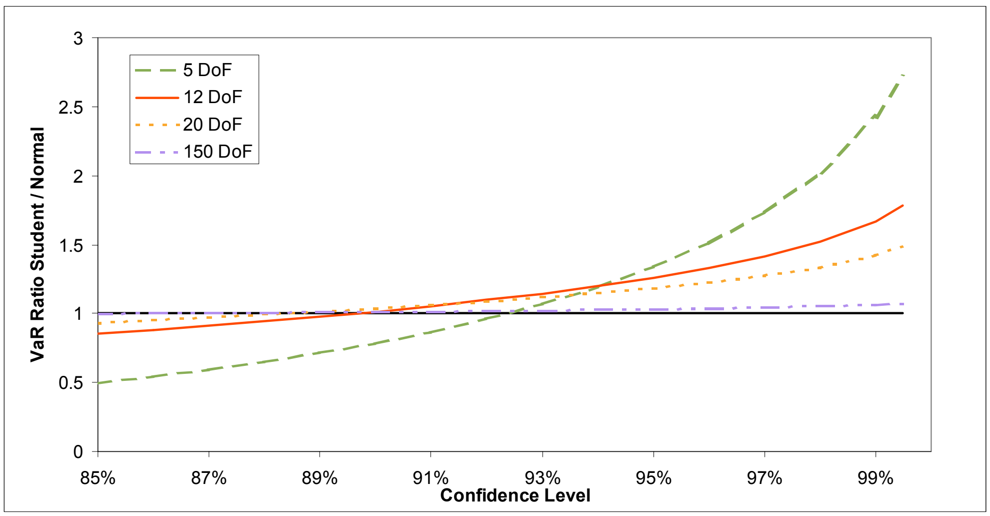
Figure 2. Analytic density versus Monte Carlo loss histogram for BBB portfolio with 150 DoF and 5% issuer correlation



5. LARGE HOMOGENEOUS PORTFOLIO VALUE-AT-RISK WITH TAIL DEPENDENCE

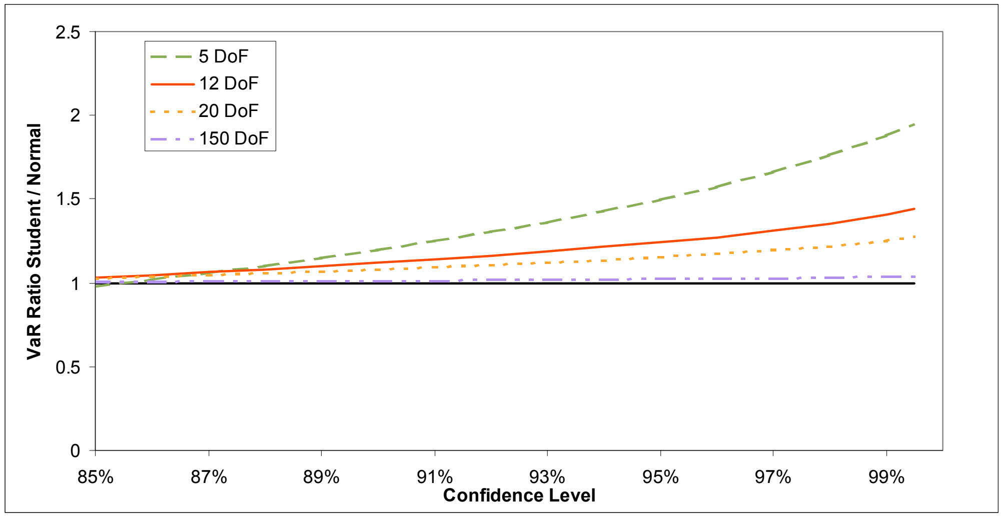
One of the main advantages of our analytical results for the LHP is that we can efficiently compute tail measures such as Value-At-Risk. All we have to do is perform a one-dimensional root search over the cdf of the loss distribution. In the following, we compare the VaR for each of the two portfolios using different values for the number of degrees of freedom, and compare this to the Gaussian case.

Figure 3. VaR ratio for AA portfolio with 20% issuer correlation



For the case of an AA portfolio with an issuer correlation of 20%, we obtain the VaR ratios shown in Figure 3. For a confidence level of 99.5%, 12 degrees of freedom gives a ratio of 1.784. The ratio decreases with increasing issuer correlation, and also when we increase the issuer default probability. Intuitively and speaking very loosely, larger losses become more likely, and the tail dependence does not manifest itself as much. For a very high issuer correlation of 50% we obtain a ratio of 1.280, whereas a low issuer correlation of 5% gives a ratio of 3.163. Note that in the extreme case of 5 degrees of freedom and an issuer correlation of 5% the ratio goes up to 5.697! In Figure 4, we show the results for the BBB portfolio with an issuer correlation of 20%. In the 12 DoF base case, the VaR ratio for the 99.5% confidence level case has gone down to 1.444.

Figure 4. VaR ratio for BBB portfolio with 20% issuer correlation



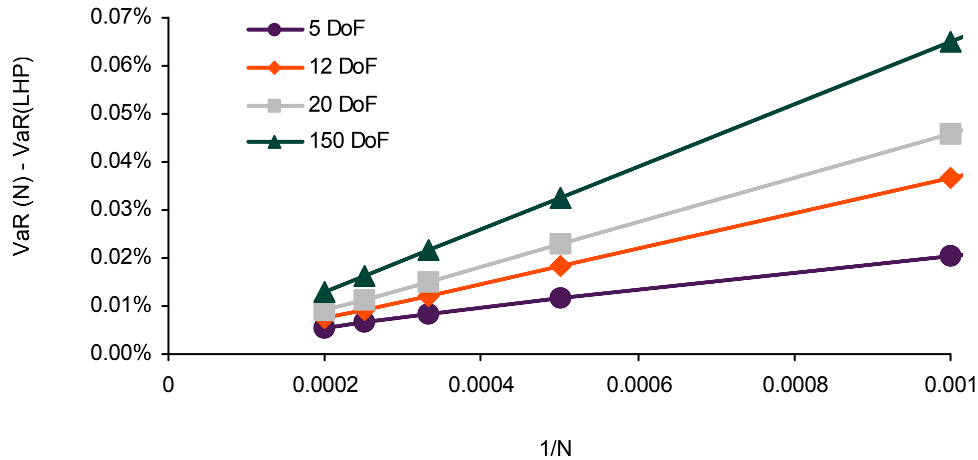
6. ASYMPTOTIC CALCULATION OF VAR FOR LARGE PORTFOLIOS

Recent work [Gordy] has shown that for a portfolio credit model with just one source of systematic risk, the difference between the Value-At-Risk of an infinitely finely grained portfolio ($N \to \infty$) and the VaR of the finite homogeneous portfolio scales as *1/N* to first order[1]. This result should also apply to a model which incorporates tail dependence. It should enable us to compute the asymptotic VaR by plotting the VaR as a linear function of *1/N* and extrapolating to infinite *N*. Since we assume a fixed loss amount (zero recovery) in the event of a default, our finite *N* loss distribution is discrete. Care must be taken to avoid "edge" effects, caused by the movement of the discrete peaks of the loss distribution as *N* increases, which can cause the portfolio VaR to be discontinuous. This can be overcome by using a simple linear interpolation scheme to compute the VaR.

To compute the VaR we can either resort to Monte-Carlo simulation or full analytic calculation of the portfolio loss distribution. While a Monte-Carlo approach is easily implemented, and importance sampling techniques can be used, it is nonetheless difficult to get accurate and stable results when looking in the small 0.5% percentile tail of the loss distribution to compute the 99.5% VaR. A better alternative is to compute the full portfolio loss distribution by numerically integrating over the mixing variable η, defined in (5). Since the portfolio loss distribution conditional on the mixing variable is the simple binomial, we can simply compute the conditional asset probability of default (6) and the resulting binomial distribution. We repeat this for each value of η and then, using the mixing density, we integrate to get the full portfolio loss distribution.

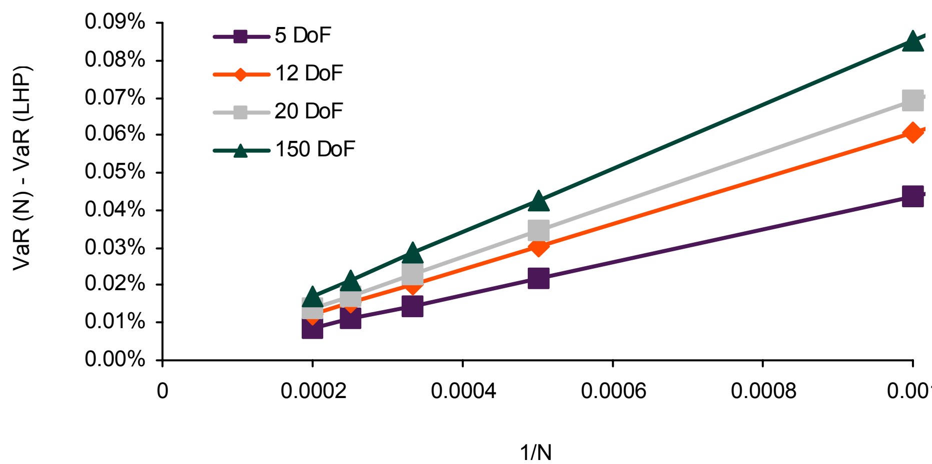
[1] *This result requires that the portfolio is statistically homogeneous and that the Value At Risk is continuous, arbitrarily differentiable and increasing.*

Figure 5. Convergence of AA-rated portfolio VaR to the LHP limit as a function of 1/N where N is the number of homogeneous assets in the portfolio for N=1000, 2000, 3000, 4000 and 5000. This is shown for 4 values of the degrees of freedom (DoF) parameter.



The results of the analysis are shown in Figure 5 for the AA-rated portfolio and Figure 6 for the BBB portfolio. Both show that the VaR is a monotonic increasing function of 1/N and the lines appear straight, supporting our conjecture[2] that the result of Gordy applies in the Student-t copula case.

Figure 6. Convergence of BBB-rated portfolio VaR to the LHP limit as a function of 1/N where N is the number of homogeneous assets in the portfolio. We show N=1000, 2000, 3000, 4000, 5000. This is shown for 4 values of the degrees of freedom (DoF) parameter.



[2] *We make this conjecture on the basis of the observation that the proof by Gordy [Gordy] does not condition on the form of the distribution for the iid random variables which drive the model.*

We can use this linear relationship to verify the LHP result for the 99.5% confidence VaR. We performed the linear extrapolation using the N=4,000 and N=5,000 results. In this case the slope β is given by

$$\beta = \frac{VaR(N = 4,000) - VaR(N = 5,000)}{1/4,000 - 1/5,000}$$

so that

$$VaR(N \to \infty) = VaR(N = 5,000) - \frac{\beta}{5,000}$$

In Figure 7 we compare the extrapolated result with that computed via the LHP for different values of the degrees of freedom parameter and for both the AA and BBB portfolios. It is clear that the quality of the agreement is excellent with most results correct to within a VaR difference of 0.01%. This is a powerful demonstration of the correctness of the LHP calculation described earlier.

Figure 7. Comparison of Analytical Extrapolation results for the 99.5% VaR to the values computed from the LHP calculation for different portfolios and different values of the degrees of freedom (DoF) parameter

	99.5% VaR for AA Portfolio		99.5% VaR for BBB Portfolio	
DoF	**Extrapolated**	**LHP Calculation**	**Extrapolated**	**LHP Calculation**
5	20.89%	20.87%	35.67%	35.66%
12	13.72%	13.72%	26.44%	26.45%
20	11.42%	11.42%	23.38%	23.38%
150	8.21%	8.21%	19.03%	19.03%

One striking feature of this "finite-size" analysis is how little the portfolio VaR changes as the number of assets drops from infinity to a more realistic number. In the table below we show the computed VaR for different portfolio sizes for the BBB-rated portfolio. At a value of DoF equal to 150, going from the LHP VaR to the 100 name VaR results in a relative change of less than 4.5%. This is even more apparent at lower DoF. At 5 DoF, the change represents only 1.2% of an admittedly larger VaR. This is an encouraging result as it suggests that the LHP limit gives an approximation for the VaR of a finite portfolio which may be sufficiently accurate to be acceptable for "quick and dirty" risk-management purposes.

Figure 8. The 99.5% Value-At-Risk for a BBB-rated portfolio as a function of the number of exposures and the number of degrees of freedom

Number of Exposures	5 DoF	12 DoF	20 DoF	150 DoF
100	36.098%	27.038%	24.054%	19.873%
Infinite	35.661%	26.446%	23.381%	19.032%
% Change	1.21%	2.25%	2.89%	4.40%

Note that we have not tested the accuracy of the LHP for inhomogeneous portfolios, but expect that, provided the portfolios are large, and the asset default probability and β are replaced by their portfolio weighted averages, the accuracy of the LHP result should be good enough to get into the risk measurement "ball-park".

7. TAIL DEPENDENCY IN THE BASEL II CAPITAL ACCORD

Vasicek's LHP analysis has played a significant role in the development of a model-based approach to quantify the risk of bank lending portfolios within the new Basel II Capital Accord [Basel]. Due to come into effect in late 2006, this framework treats portfolio credit default risk in two stages.

First, an asset specific charge based on a 99.5% Value-At-Risk is computed for each exposure in the portfolio [Basel paragraph 431]. This is only possible within a modelling framework in which the portfolio is infinitely granular and there is only once source of systemic risk. The reason is that in this infinitely granular limit, all idiosyncratic credit risk has been diversified away and each asset only contributes to the portfolio risk through its exposure to the systemic risk factor. This charge is calculated using the LHP limit. The idea is that for a homogeneous portfolio of N exposures, the total portfolio loss conditional on the market factor is given by

$$L(Z) = \frac{1}{N}\sum_{n=1}^{N} p_i(Z)$$

where $p_i(Z)$ is the probability of default of asset i conditional on market factor Z.

In the limit of large N the portfolio loss is equal to its conditional mean and the 99.5% portfolio loss occurs at the value of Z_q such that $\Pr\{Z > Z_q\} = q$, where q=0.995. As a result we can write the portfolio VaR as a linear sum of the individual asset contributions

$$VaR_q = \frac{1}{N}\sum_{n=1}^{N} p_i(Z_q)$$

The second step attempts to overcome the infinite granularity assumptions of the first since this will tend to understate the true portfolio risk. This effect is accounted for using a so-called *granularity adjustment* (GA), defined as

$$GA(N) = VaR(N) - VaR(LHP) = \beta / N$$

While the current proposal switches to the CreditRisk+ model for the purposes of this adjustment, the effect is not significantly different from that computed using Vasicek's model.

However the Basel approach to capturing portfolio credit risk ignores the effect of tail dependency. To see what difference this would make, we take the realistic case of 12 DoF, a correlation of 20% as proposed by Basel II, and we look at a portfolio of assets with a range of default probabilities (PD). In this case the asset specific charge can be calculated by inverting the cumulative density function of the LHP loss distribution to find the loss corresponding to a 99.5% confidence limit. The results are shown in Figure 9.

Figure 9. Effect of tail dependence on the asset specific charge of an infinitely granular portfolio, calculated as the 99.5% VaR for the range of asset default probabilities (PD) used in Basel II

	99.5% Value-At-Risk				
PD	5 DoF	12 DoF	20 DoF	150 DoF	Normal
0.10%	4.83%	3.38%	2.72%	1.69%	1.51%
0.50%	16.53%	10.61%	8.69%	6.01%	5.57%
1.00%	23.97%	16.11%	13.58%	10.04%	9.46%
2.50%	35.66%	26.45%	23.38%	19.03%	18.32%
6.00%	48.59%	40.25%	37.40%	33.30%	32.62%
15.00%	64.19%	59.40%	57.79%	55.51%	55.14%

Clearly the asset specific charge increases as the number of degrees of freedom falls, as has already been discussed above. In the tail dependent case, the granularity adjustment is simply the difference between the LHP VaR and the finite portfolio VaR. It can therefore be derived from numbers computed above. We therefore show the value of the granularity adjustment for a 100, 200, 500 and 1,000 name portfolio for different degrees of freedom in Figure 10.

Figure 10. Granularity Adjustment for a homogeneous portfolio with PD=2.5%, correlation 20% for different portfolio sizes and different degrees of freedom

Number of Exposures	5 DoF	12 DoF	20 DoF	150 DoF
100	0.43%	0.60%	0.68%	0.84%
200	0.22%	0.30%	0.35%	0.42%
500	0.09%	0.12%	0.14%	0.17%
1,000	0.04%	0.06%	0.07%	0.09%

While portfolios with higher tail dependence exhibit a higher Value-At-Risk, it is clear from the results in Figure 10 that the size of the granularity adjustment for portfolios with a higher degree of tail dependence (lower DoF) is less than for portfolios with no tail dependence. For example, a 200 name portfolio with a DoF 150 has a granularity adjustment of 0.42%. Compare this to a 200 name portfolio with a realistic DoF of 12, we find that the granularity adjustment falls to 0.30%. This implies that as portfolios with tail dependence already have extended loss tails, making the portfolio more lumpy does not extend the loss tail as much as it does for a portfolio with no tail dependence.

8. CONCLUSIONS

We have extended Vasicek's large homogeneous portfolio approximation to incorporate tail dependency and presented the analytical formula for the cdf and the density of the portfolio loss as a function of the cdf and the density of the mixing variable distribution.

The calculation of the LHP with tail dependence was checked by comparing the computed loss distribution with that generated by Monte Carlo simulation of a large homogeneous portfolio. In addition, analytical simulation of the loss distribution was presented which supports the conjecture that Gordy's scaling result for the VaR also applies in the tail dependent case. We used it to verify the convergence of the VaR to its asymptotic LHP value. We found excellent agreement in both cases.

We can conclude that the effect of tail dependence is significant, especially for the Value-At-Risk of the portfolio at high confidence limits – when we are determining the starting point of low percentile loss tails. For realistic parameters we find that incorporating tail dependence can easily increase the VaR by a factor of two.

Finally, we showed that the implications of tail dependence for the Basel II portfolio credit risk charge are that it will significantly increase the value of the VaR. At the same time, the size of the granularity adjustment will decrease, meaning that the portfolio's risk is less sensitive to size effects.

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Learning Credit: Credit Linked Notes[1]

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This paper provides a simple explanation of credit linked notes and their uses.

1. INTRODUCTION

A Credit Linked Note (CLN) is the combination of a credit default swap and a funded underlying asset, which is usually either a highly rated bond (generally Aaa or its equivalent) or a corporate medium term note (MTN). Credit linked notes are usually issued by special purpose vehicles, corporations or trusts. In this discussion we will use the generic term Special Purpose Vehicle (SPV) to cover these possibilities.

The investor buying a CLN makes an upfront payment to the SPV. The deposited proceeds are invested in the *underlying assets*. Simultaneously, the SPV offers protection on a credit, called the *reference entity*, in the credit default swap market. The SPV provides the investor with a coupon, composed of the coupon on the underlying assets plus the premium on the default swap, and returns par at maturity provided there has not been a credit event on either the reference entity or the underlying assets. If there is a credit event, the investor can lose some or all of the remaining coupon payments and principal. Fundamentally, a CLN offers credit exposure in a structure designed to resemble a synthetic corporate bond. It is a funded asset and appears on the investor's balance sheet.

2. DEFAULT OF THE REFERENCE ENTITY

The investor in a CLN is exposed to the credit risk of both the reference entity and the underlying assets. If a credit event[2] occurs with respect to the reference entity, the SPV will terminate. The subsequent transactions will depend on whether the CLN specified cash or physical settlement. In a physical settlement, the investor will receive a principal amount of bonds issued by the reference entity equal to the notional amount of the CLN plus or minus the mark to market on the underlying assets. The protection buyer will take possession of the underlying assets. In a cash settlement, the SPV will liquidate the underlying assets and the investor will receive a cash amount equal to the following:

MV(B) * NV(C) +/- MTM

where:

MV(B) = Market Value of bonds issued by the reference entity expressed as a percentage of par

NV(C) = Notional Value of the CLN in dollars

MTM = Mark to market on the underlying asset in dollars

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1 *I thank Rene Canezin, Roy Mashal, Paul Mitrokostas, Marco Naldi, Dominic O'Kane, Lutz Schloegl and Michael Weaver for helpful discussion. Responsibility for all errors remains with the author.*

2 *We will define what constitutes a credit event in the next section.*

The notional amount of the reference entity that the protection buyer delivers depends on the value of the underlying assets. The determination of the size of the notional of the reference entity that the protection buyer delivers is predicated on the understanding that the claim of the protection buyer comes before that of the CLN.

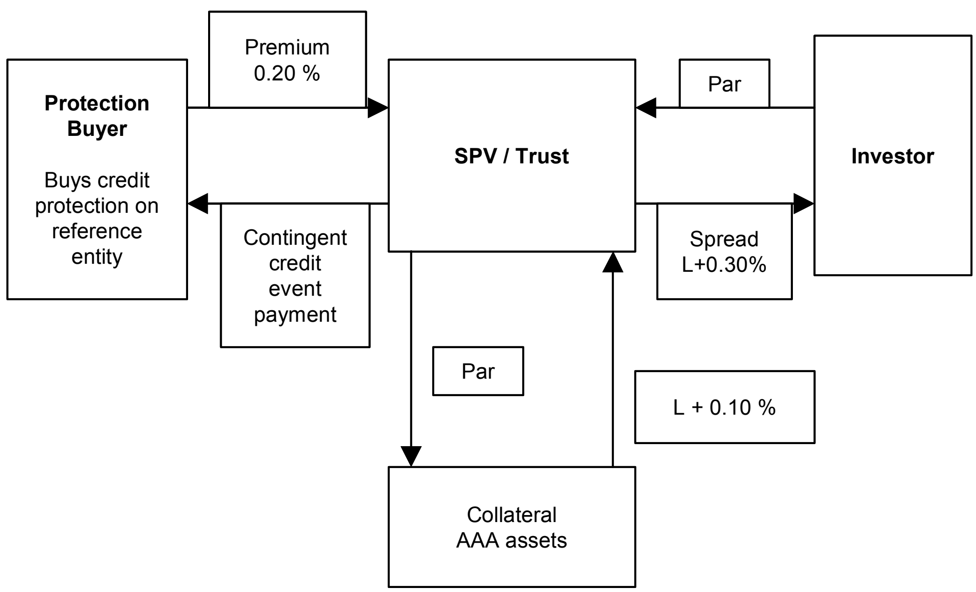
A simple numerical example is given in Appendix A.

Default By the Underlying Assets

If the underlying assets default, the investor is potentially exposed to losses regardless of the performance of the reference entity. In the event the underlying assets default, the SPV will terminate and the underlying assets will be liquidated. The investor will receive the proceeds from the liquidation plus or minus any mark to market due on the termination of the default swap.

The cash flows for the CLN are shown in Figure 1.

Figure 1. Credit Linked Note



3. CREDIT EVENTS

We need to define the credit events that trigger payment under the credit default swap. These are the standard ISDA credit events that are prevalent in the credit default swap market[3].

Default

Failure of the reference entity to make principal or interest payments when due, taking into account any applicable grace period to prevent accidental triggering due to an administrative error.

3 *Credit default swaps were described in the last issue of the Quarterly, by Turnbull (2002). See also O'Kane (2001), who described CLNs, SPVs and repackaging vehicles.*

Bankruptcy

Reference entity becomes insolvent or is unable to pay its debt (not relevant for sovereign issuers).

Restructuring/Modified Restructuring

Restructuring changes in debt obligations of the reference entity: under current ISDA standards, the restructuring must be both a restructuring of the financial conditions of the obligation and it must be associated with credit deterioration of the reference entity. Thus, changes such as a restructuring of loan covenants or a renegotiation of more favorable terms will not trigger a credit event.

Additionally, modified restructuring restricts the range of deliverable obligations in a restructuring event which must have multiple lenders (at least three) and at least two thirds of the lenders must agree to restructuring. If the protection buyer triggers the contract, the delivered instruments must have a maturity date not later than the remaining maturity of the credit default swap contract or 30 months from the date of restructuring.

4. DETERMINING THE SPREAD TO THE CLN INVESTOR

The investor is exposed to the credit risk of the reference entity and the underlying assets. The spread paid to the investor will be the sum of the spread of the underlying assets plus the premium paid by the protection buyer on the default swap. The dependence between the reference entity and the underlying assets will also in theory affect the pricing. This effect is usually small and is often ignored.

5. REASONS FOR USING CLNS

The primary benefits of investing in a CLN can be placed into three categories: economics, investment restrictions and structural flexibility. The most frequently cited reason that investors participate in the CLN is the possibility of picking up an enhanced yield relative to other securities issued by the reference entity. Additionally, a CLN provides a funded credit derivative opportunity to investors who may not be authorized to invest directly into credit derivatives or may not have the appropriate ISDA documentation. Finally, a CLN provides customized interest payment, maturity structures and credit features that may not be available in the cash market.

It is also possible to create a CLN where a protection buyer supplies a new issue MTN into which the default swap is directly imbedded. In this case there is no need to set up an SPV and the costs associated with creating a structured vehicle can be reduced or eliminated.

6. COUNTERPARTY RISK

Along with the credit risk of the underlying asset and the reference entity, buyers of credit linked notes are also subjected to a small amount of counterparty risk to the protection buyer. The risk is that the buyer of protection will fail to make the premium payments to the trust on the dates that they are due. It is possible to eliminate this counterparty risk if the protection buyer makes the entire premium payment on the default swap on the day that the transaction settles, thereby eliminating any ongoing reliance on the protection buyer to make payments to the trust. However, in this case if there is a credit event in respect of the reference entity, the notional amount paid by the protection buyer to the SPV is reduced by the amount of the premium covering the time from the date of the credit event to the maturity date, given the fact that the SPV will terminate on the credit event date, not the maturity date as originally thought. This repayment is referred to as the *Clawback Amount*.

From the perspective of the protection buyer in the default swap, CLNs can be used to overcome counterparty risk concerns traditionally associated with unfunded credit default swaps. The protection buyer in a CLN faces a bankruptcy remote SPV, not another company. The SPV is fully collateralized in the form of the underlying assets. Thus, should a credit event occur with respect to the reference entity, the protection buyer can claim the underlying assets as payment under the default swap instead of having to rely on the investor to make the payment.

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APPENDIX A

There are two mark-to-market scenarios for the underlying assets in a CLN. First, when the market value of the underlying assets is less than the notional value of the CLN; and, second, when the market value of the underlying assets is greater than or equal to the notional value of the CLN. We will use a simple numerical example to illustrate the two cases.

Case One: Underlying Asset Value Is Less Than the Notional Value of the CLN

We make the following assumptions about the relevant values.

Underlying Asset	
Principal of the underlying asset	1,000
Market value of the underlying asset	930
Mark to Market	70
Reference Entity	
Notional value	1,000
Value in default	450

Ignoring for a moment, the price decline on the underlying assets, to settle the default swap, the protection buyer delivers either 450 cash or securities issued by the reference entity with a market value of 450 and is entitled to receive cash or securities with a market value equal to the CLN notional value of 1,000. However, in this example, the value of the underlying asset is only 930, implying that if the protection buyer receives the underlying assets as compensation for the default protection, there will be a shortfall of 70 (= 1,000 – 930). The only claim of the protection buyer to collect the 1,000 is to the SPV. The claim does not extend through to the investor. Thus, the only way for the SPV to make the protection buyer whole is to require the protection buyer to deliver only 380 in cash or securities with a market value of 380 and take the underlying assets at the value of 930.

If the transaction is cash settled, the protection buyer simply reduces the amount of cash that is delivered to the SPV. However, if the transaction is physically settled, the investor needs to adjust the principal amount of securities delivered in to insure that the market value of the delivered securities is 380. In this example, for each unit of notional, the market value of the securities issued by the reference entity following the credit event is 0.45. The SPV has a shortfall of 70, so it wants to reduce the principal amount of securities that the protection buyer must deliver by an amount X, where X is defined by

$$70 = X * 0.45$$

implying that $X = 70 / 0.45$. The protection buyer delivers to the SPV a notional amount of the reference entity equal to[4]

$$1,000 - X = 1,000 - 70 / 0.45 = 844.44$$

4 In a work sheet this expression will be expressed as [Notional Amount of the reference entity minus (Underlying Asset Mark-to-Market divided by the Reference Entity Adjustment)], where Underlying Asset Mark-to-Market is defined as the Notional Amount of the Reference Entity minus the market value of the Underlying Asset (as defined by the Sales Procedures).

In dollar terms this is equal to

844.44 * 0.45 = 380

Clearly the lower the value of the underlying asset and/or the reference entity, the less the protection buyer will be required to deliver to the SPV.

<u>Case Two: Underlying Asset Value Is Greater Than or Equal to the Notional Value of the Reference Entity</u>

In this case we assume the underlying asset value is given by

Underlying Asset

Principal of the underlying asset	1,000
Market value of the underlying asset	1,050

All other information is assumed to remain unchanged. In this case the SPV can pay the protection buyer the full notional amount of the reference entity and have 50 remaining. The investor will receive 450 in cash or 450 market value of securities issued by the reference entity and will also receive the remaining 50 for a total of 500.

Publications — L. Pindyck, A. DiTizio, B. Davenport, W. Lee, D. Kramer, R. Madison, A. Acevedo, K.Kim, C. Rial, J. Batstone, K. Banham

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