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We use an option-pricing framework to price the extension risk embedded in some hybrid securities. We separate extension risk from default and deferral risk, two other risks common to hybrid securities, by estimating how much more a security would be worth if the issuer did not have the option to extend the maturity. This allows us to compare compensation for default and deferral risk offered by the hybrid security with that offered by selling preferred credit default swap protection. We present our modeling framework and the assumptions on which it is based. We also value the extension option for a number of hybrid securities.

We present a simple econometric framework linking the level of financial and ex-financial credit spreads to their fundamental drivers – viz. the state of the macro environment and the health of the corporate balance sheet. We illustrate the application of the framework by analysing the broad behaviour of the driving variables and conclude that investors should currently favour an underweight position in credit markets.

We extend the analysis of the behavior of corporate bond spreads to the realm of credit default swaps using a new estimation technique. The quasi-maximum likelihood approach can accommodate the stochastic nature of the relation between spread volatility and spread level. Consistent with previous results, we find support for a linear relationship between spread volatility and spread level with some evidence of non-linear effects.

The pricing of exotic portfolio products, e.g. path-dependent CDO tranches, relies on the joint probability distribution of portfolio losses at different time horizons. We discuss a range of methods to construct the joint distribution in a way that is consistent with vanilla CDO market prices. As an example, we show how our loss-linking methods provide estimates for the breakeven spreads of forward-starting CDO tranches.

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Pricing extension risk in hybrid securities

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We use an option-pricing framework to price the extension risk embedded in some hybrid securities. We separate extension risk from default and deferral risk, two other risks common to hybrid securities, by estimating how much more a security would be worth if the issuer did not have the option to extend the maturity. This allows us to compare compensation for default and deferral risk offered by the hybrid security with that offered by selling preferred credit default swap protection. We present our modeling framework and the assumptions on which it is based. We also value the extension option for a number of hybrid securities.[1]

INTRODUCTION

Hybrid securities are also referred to as hybrid capital and include not only standard preferred stocks but also other preferred and trust-preferred securities, hybrid bank capital, ECAPS and a variety of other structures[2].

In this article, we study a subset of the hybrid capital universe with the following common structure: The securities are scheduled to pay a fixed coupon until a certain date, called the step-up date, after which the coupon floats at 3-month LIBOR (or another floating rate) plus a fixed spread. The securities are callable on regular intervals on and after the step-up date. The fixed spread is often high enough that the call option is far in the money and investors often view these securities as if they mature on the step-up date. With such a perspective, investors are taking extension risk, i.e. the risk that the issuer does not call the security and the investors end up holding a longer-maturity instrument. The purpose of this article is to value the extension risk by estimating how much more a given hybrid security would be worth if it was given to mature on the step-up date.

We believe that extension risk is an important factor in some investors' decisions on whether or not to invest in hybrid capital. We present a framework that we think will give investors a better understanding of extension risk and make investors more comfortable with hybrid capital investments.

Aside from extension risk, deferral risk is another key risk that concerns investors. Deferral risk is the risk that a hybrid security stops coupon payments without the issuer defaulting on debt payments. It is different from default risk because a hybrid capital investor is often at a disadvantage relative to ordinary creditors in accelerating payment, usually has no means of recovering any value following a deferral (except selling the security) and can only hope that coupon payments will resume[3]. We do not attempt in this article to price deferral risk. However, we do take deferral risk into account in our framework. We group deferral and default events into a common class of credit events and model the probability that such an event will occur. We assume a fixed recovery rate (for the hybrid security) following a credit event.

In the next three sections we present the modeling framework we use to value the extension risk: first we describe in more detail how we incorporate default and deferral risk into the framework. Next, we discuss the cheapest-to-deliver issue and the recovery assumption. We then explain how to use an option-pricing model to price the extension

[1] Thanks to Megan Philbin, Marc Pomper, Kevin Tan and Shantanu Varma for discussions.
[2] See Pomper and Varma (2005) for an overview of the hybrid universe and a brief history of the hybrid market.
[3] Some hybrid securities have limits on how long payments may be deferred without investors getting a claim that can be collected upon.

risk. After describing the model, we use it to value the extension risk in some recently issued hybrid securities.

ACCOUNTING FOR DEFAULT AND DEFERRAL RISK

To explain how we account for default and deferral risk we use a simple example involving a special type of credit default swap (CDS) called a preferred credit default swap (PCDS). A PCDS differs from a CDS with respect to both the definition of a credit event and the set of deliverable securities[4]:

- In a standard CDS there are three types of credit events: bankruptcy, failure to make a debt payment and restructuring. In a PCDS the deferral of a preferred stock dividend or a trust preferred coupon is included as a fourth credit event.
- A PCDS references a preferred or trust preferred security. This means that after a credit event, a preferred/trust preferred security (or any security more senior in the capital structure) can be delivered to settle the PCDS. In contrast, the majority of CDS reference senior unsecured debt, although CDS referencing debt of different seniorities also exist.

Our trading desk quotes senior unsecured CDS, subordinated CDS as well as PCDS on a number of financial institutions. Figure 1 contains an example which we will use below to discuss deferral risk.

Figure 1. Senior unsecured CDS, subordinated CDS, and PCDS quotes (in bp) for 5-year USD protection on Citigroup on August 7, 2006

CDS		Sub CDS		PCDS	
Bid	Ask	Bid	Ask	Bid	Ask
9	11	13	15	29	34

Source: Lehman Brothers.

There is a simple approximate relationship between a CDS spread, a recovery rate, and the annualized (average) default probability over the life of the CDS. We refer, in this article, to this approximate relationship as the *Spread Equation*:

$$\text{Spread} = (1 - \text{Recovery}) \cdot \text{Default Probability}$$

The Spread Equation captures the fact that given an assumption about the recovery rate, we can find the market implied default probably from the spread. Or more generally, given two of the three variables in the Spread Equation we can solve for the third.

The recovery on a senior unsecured CDS is often assumed to be 40%. If we make the same assumption, we can use the mid CDS spread of 10bp (Figure 1) to back out an implied default probability of 16.7bp using the Spread Equation. The default probabilities for senior unsecured and subordinated CDS should be the same, so using the mid subordinated CDS spread of 14bp we can back out an implied subordinated recovery rate of 16%[5].

Turning to the PCDS, assume that the value of a hybrid security (deliverable to settle the PCDS) following a default event is 5% (it clearly must be lower than the 16% on the subordinated CDS). Unfortunately there is little empirical evidence on which to base the

[4] *See Shah and Kathpalia (2005) for a more complete description of PCDS and a discussion of the factors affecting the relative pricing of PCDS vs CDS.*

[5] *The standard recovery rate assumption for subordinated CDS is 20%. It is, for example, a common market practice to use a 20% recovery rate when calculating the unwind value of a subordinated CDS.*

5% assumption but it is in line with a recent study by Moody's, which uses data on the recovery rate on preferred stocks[6]. With this assumption we can ask: how much of the PCDS spread is to the result of deferral risk? A 16.7bp default probability and a 5% recovery rate imply a spread of 15.8bp. Subtracting this from the 31.5bp PCDS spread gives that 15.7bp of the spread is due to deferral risk.

We do *not* attempt to evaluate, or present a framework for evaluating, whether the 16.7bp spread compensation offered in the market for taking deferral risk on Citigroup is too high or too low. Our purpose is to value extension risk. However, if we can value extension risk in a security we can also say how much the market is paying for the other risks in the security, specifically the default and deferral risks. In particular, we would be able to compare the compensation for default and deferral risk offered by a hybrid security to that offered by selling protection in a PCDS. We need to incorporate into our model a way to account for default and deferral risk but we do not necessarily need to separate the two risks. Our approach is to model the hazard rate[7] of a combined default *or* deferral credit event and directly specify the value of the hybrid security and the extension option following such a credit event. We assume a fixed recovery rate for the hybrid security and we put the value of the extension option to zero following a credit event (see below).

By combining default and deferral events our approach to model the hybrid security is identical to the way we model other credit risky securities such as CDS and corporate bonds. In particular we can use the model presented in Pedersen (2006b). That model was specifically designed to analyze corporate bonds with embedded options and we can apply it to hybrid securities as well. One of the outputs from the model is the bond-implied CDS spread, or BCDS spread. This is a credit spread extracted from the bond price that can be compared directly with a CDS spread for the purpose of analyzing bond-CDS basis trades[8]. When we apply the model to a hybrid security we get an analogous bond-implied PCDS spread (we still call it the BCDS spread, not the BPCDS spread) that can be used to compare the hybrid security to a PCDS. If the BCDS spread is greater than the PCDS spread we would conclude that the hybrid security offers better compensation for default and deferral risk than the PCDS. Conversely, if the BCDS spread is lower than the PCDS spread then we would conclude that it is better to take default and deferral risk by selling PCDS protection than by buying the hybrid security.

IMPLICIT ASSUMPTIONS: DELIVERABILITY, RECOVERY

Our comparison of a hybrid security to a PCDS is based on a very important implicit assumption that may not always hold. We implicitly assume that the hybrid security always is the cheapest security to deliver to settle the PCDS following a default or deferral event. It is common to ignore the cheapest-to-deliver option for standard senior unsecured CDS because the price differences between senior unsecured debt following a default usually are very small. This is because bankruptcy law explicitly states that debt of the same seniority should be treated equally. For this reason it would also be innocuous to assume that preferred securities have the same recovery rate following a default. However, it is unclear whether two hybrid securities of the same seniority would have the same recovery rate following a deferral. We assume that they always have the same recovery rate and do not allow for the possibility that a hybrid with an extension option may be worth less than a similar hybrid without the extension option. This also

[6] *See Varma and Cantor (2005).*

[7] *The hazard rate is the annualized instantaneous probability of a credit event (here either a default or a deferral event). See O'Kane and Turnbull (2003) for details.*

[8] *In the next section we will explain the core elements of the BCDS spread methodology. For a more detailed description please see Pedersen (2006b).*

explains why we, as explicitly stated above, assume that the value of the extension option is zero following a credit event. The extent to which this assumption breaks down and the cheapest-to-deliver option actually plays a role when comparing a hybrid security to a PCDS determines the extent to which it is possible to price extension risk without explicitly separating default and deferral risk.

We use 10% as our benchmark recovery rate following a credit event (either a default or a deferral). This is consistent with the market practice of calculating the unwind value of a PCDS using the standard CDS valuation model with a 10% recovery rate.

However, it is difficult to determine whether 10% is the most appropriate recovery rate to use. Some market participants argue that a company will do everything to avoid a deferral and that a deferral will occur only when the company is at high risk of default. This would argue for using a low recovery rate. On the other hand, the purpose of allowing deferral is to protect creditors by giving the company a means to divert cash to make debt payments and thus prevent the company from seeking bankruptcy protection. It seems probable that if the company has substantial payments that can be deferred, deferral may indeed allow the company to avoid bankruptcy for a long period and perhaps eventually to recover. Also, in some securities, deferral is mandatory when certain financial ratio tests have been breached. If the conditions for mandatory deferral have been well designed, a deferral should give the company a chance of recovering. The recovery rate following a deferral may also be affected by whether the deferral is a deferral for payment at a later date or if the company does not ever have to pay what has been deferred. These uncertainties about deferral events all argue for using a range of recovery rates when pricing extension risk and comparing a hybrid security to a PCDS.

MODELING EXTENSION RISK

In this section we discuss how we price extension risk using an option valuation model under the simplifying assumptions for deferral risk explained earlier. With those assumptions we can use the BCDS methodology (see Pedersen (2006b)) to analyze specific hybrid securities.

Given a complete specification of a hybrid security (coupon, maturity, step-up date, step-up spread, etc.) and the following market inputs:

1) LIBOR/swap discount factors;
2) Interest rate volatility parameters;
3) PCDS spread curve;
4) Spread volatility parameter;
5) Recovery rate following a credit (default or deferral) event; and
6) Interest rate/spread correlation parameter,

we can value the hybrid security using our option valuation model. Some of the market inputs are observable. Others must be specified exogenously. For example, the LIBOR/swap discount factors are easily determined from observed rates, and interest rate volatility parameters are calibrated to interest rate swaption prices. The spread volatility, recovery rate and correlation are direct inputs[9] whereas the PCDS spread curve may or may not be observable. Whether or the not the PCDS curve is available, it only provides the starting point for the eventual valuation. The idea behind the BCDS methodology is to (parallel) shift the PCDS curve to ensure that the value of the hybrid security calculated from the model is equal to the observed market price. For example, if the

[9] *Our benchmark parameters are 40% hazard rate volatility, 10% recovery rate and zero correlation.*

original PCDS curve produces a value that is higher than the market price of the security, the PCDS curve would be shifted upwards, and vice versa. It is this shifted curve that we call the BCDS curve[10]. The level of the BCDS curve is determined by the market price of the hybrid security whereas the shape of the curve comes from the PCDS curve. Once the BCDS curve has been found we use it to quantify extension risk.

Hybrid step-ups: a bullet less a put on a floater

Conceptually, we view a long position in a hybrid security as a long position in a fixed-coupon bullet bond and a short position in an American put option on a floating rate bond that can be first exercised on the step-up date (Figure 2). The value of the put option is the price of extension risk:

Hybrid Security = Bullet Bond – Put Option on Floater

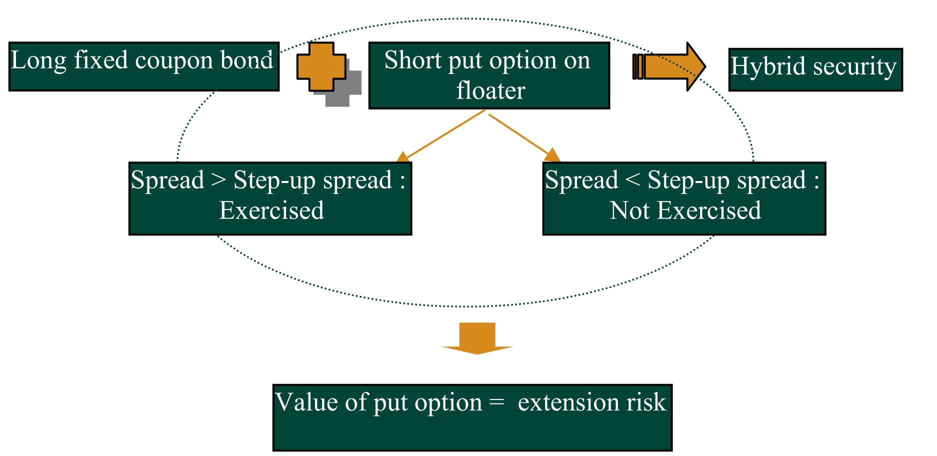
Price of Extension Risk = Value of Put Option on Floater

To value the put option, we first solve for the BCDS curve as explained above. Then we use the BCDS curve, the recovery rate, and the LIBOR curve as inputs to a standard default model to value the bullet bond component of the hybrid security. Specifically, we employ the following approach:

Price of Extension Risk = Value of Bullet Bond (using the BCDS curve) – Market Price of Hybrid Security

In other words, the market price of the hybrid security plus the price of extension risk is the value we would assign to the hybrid security if it were a given that it matured on the step-up date.

Figure 2. Hybrid securities cash flows



Source: Lehman Brothers.

We have given a conceptual explanation of how we calculate extension risk using the BCDS methodology. Now we turn to the main drivers of extension risk and give a brief introduction to the option-pricing models we use to evaluate extension risk.

Valuing the put: a payer swaption analogy for single rate structures

The majority of floating rate bonds on which we want to price a put option pay 3-month LIBOR plus a step-up spread (important exceptions are discussed below). In addition, the strike price is par ($100) for all exercise dates. Because of these simplifications, the put

[10] *See Pedersen (2006b) for more details.*

option is equivalent to an option to buy PCDS protection[11], also called a payer swaption[12], at a strike spread equal to the step-up spread over LIBOR paid by the floater. It is easier to understand this analogy by examining the position of an investor who is short a put option and long a payer swaption. When/if the put option is exercised, the investor should exercise the payer swaption. The investor will then have bought the floater at par and bought PCDS protection, which is economically equivalent to having made a deposit into a LIBOR money market account with a withdrawal at either the maturity of the floater and the PCDS (assumed to be the same maturity) or the occurrence of a credit event. This is because 1) the spread received on the floater will offset the spread paid on the PCDS and 2) if a credit event occurs the investor can settle the PCDS by delivering the floater and receive par in return. Thus, the present value of the portfolio is zero. Analogous arguments hold for an investor who is long the put option and short the payer swaption when we assume, as we also do, that there is no cheapest-to-deliver option. Thus the value of the put option is the same as the value of the payer swaption.

Value of Put Option on Floater = Value of Payer Swaption

This is an interesting observation because it can be used to illustrate which parameters are important for the price of extension risk. Specifically, the interest rate volatility and the correlation should be less important than the spread volatility. The stochastic process that we use to model credit spreads also plays a role. Our approach to modeling stochastic spreads is the same as we have used elsewhere and explained in other publications. Specifically we assume that the hazard rate (the annualized instantaneous probability of a credit event) follows a lognormal process[13] with a constant volatility. We have used this approach to model callable bonds (see Pedersen (2006b)) and credit volatility derivatives (see Pedersen and Sen (2004)).

The payer swaption analogy also explains why, as we will see in the next section, the option value is very sensitive to the forward BCDS spreads starting on exercise dates. For many of the securities the first exercise date (the step-up date) is 10 years in the future and the final maturity, if any – many hybrids are perpetual – is 30-50 years further out, so that the 10- to 60- year forward spread is important. This means the steepness (or flatness) of the curve beyond 10 years plays a role in the valuation of the option. Since it may not be possible to observe very long term spreads, we find it prudent to perform the valuation for a range of long term spreads. In addition to spread volatility and the forward BCDS spreads, the recovery rate is also important for the price of extension risk.

Valuing the put: modeling multiple reference rate structures

The analogy with a payer swaption holds only for those securities where the coupon after the step-up date is a LIBOR rate plus a fixed spread. However, other hybrid securities have more complicated floating-rate coupon structures. A common example:

Floating Coupon = min{max{3M LIBOR, 10Y CMT, 30Y CMT}+S, C}

CMT stands for Constant Maturity Treasury, S is the fixed spread and C is a cap on the coupon. In words: The coupon is set by a capped maximum of 3-month LIBOR and 10- and 30-year Treasury rates plus a fixed spread.

To value a put option on this type of floater we need a model with stochastic interest rates in addition to stochastic spreads. For corporate bonds with embedded options we

11 *In this analogy we are ignoring the cheapest-to-deliver option discussed in section 2.*

12 *See Pedersen (2004) for an introduction to options on CDS.*

13 *We use the process $h_t = \alpha(t)\exp(x_t)$, $dx_t = -\kappa dt + \sigma dW_t$, where h_t is the hazard rate and W_t is a Brownian motion. κ is a constant speed of mean reversion (we put it zero) and σ is a constant hazard rate volatility. α is a function of time and calibrated to a curve of survival probabilities extracted from a spread curve and a recovery rate assumption.*

usually use a one-factor interest rate model (see Pedersen (2006a and b)). That would suffice to take into account the cap but a one-factor model will not properly capture a floating rate which is the maximum of three different maturity rates. Instead we need a multifactor model that can better model a dynamic term structure. It is often useful to think about multifactor models the following way: In a one-factor model only the level of the curve is stochastic, in a two-factor model the level and the slope are stochastic, whereas in a three-factor model the curvature is also stochastic so for example, a hump can arise stochastically. We will need a three-factor model because level, slope and curvature all matter for floating rate structures with multiple reference rate structures.

We use a proprietary model used mainly to price exotic interest rate derivatives[14]. It is a three-factor HJM type model implemented in a recombining lattice. It is (globally[15]) calibrated to the matrix of at-the-money swaption prices and also includes parameters to control the volatility skew. This model has been augmented with a one-factor lognormal hazard rate process (see Pedersen (2006b)) for which we assume a constant volatility. We put the correlation between interest and hazard rates to zero (for lack of a better choice).

NUMERICAL EXAMPLES

In this section, we provide some pricing examples. First we show how extension risk is related to the BCDS spread. Then we show how extension risk changes with different coupon step-up structures. Finally, we show the sensitivity of extension risk to such parameters as the recovery rate and spread volatility.

Extension risk of 3M LIBOR step-up hybrid securities

In Figure 3, we price nine[16] hybrid securities that pay fixed coupons until the step-up date. After the step-up date, they become floaters with step-up coupons reset every three months over 3M LIBOR (semiannually over 6M LIBOR for SCHREI 6.9%) until the final maturity. They are also callable by the issuers on coupon reset dates. However, securities issued by ILFC, ZURNVX and SWK are exceptions. They have more complex coupon step-up structures but for comparison purposes, we assume here that they have 3M LIBOR step-up coupons. The valuation date is 8/28/06, and the securities prices, the LIBOR curve and credit spread curves are from the same date.

As discussed in the previous section, given the market price of a hybrid security, we shift its PCDS curve to reprice the security. We then use the shifted (BCDS) curve to price a fixed coupon bond that matures on the step-up date. The difference of the two prices is the put option value, which we use to measure the extension risk of this hybrid security. A higher put price implies that at the step-up date, the issuer is more likely not to call the security and thereby effectively sell the floater to investors at par, and, as a result, investors will end up holding a longer-maturity security.

[14] *We thank Jinliang (Eric) Li of our interest rate modeling group for making this model available to us.*

[15] *The one-factor model used in Pedersen (2006a and b) is locally calibrated, which means we use different volatility parameters for different maturity bonds (see Pedersen (2006a)). In contrast, the three-factor model is globally calibrated to the entire swaption matrix; thus, the same volatility parameters are used for all maturity bonds.*

[16] *We use these nine hybrid securities because they are more on the run than other similar securities and are thus more liquid.*

Figure 3. Extension risk and BCDS

Issuer	Coupon	Step-Up Coupon (bp)	Step-Up Date	Final Maturity	Current Price ($)	BCDS Spread to Step-Up Date	Fwd BCDS	Fwd BCDS /Step Up	Put Price
LNC	7.000%	235.75	5/17/16	5/17/66	103.09	69	105	45%	1.53
ILFC	5.900%	155.00	12/21/10	12/21/65	99.70	19	73	47%	3.15
ILFC	6.300%	180.00	12/21/15	12/21/65	97.97	82	111	62%	2.09
ZURNVX	6.150%	175.00	12/15/10	12/15/65	98.09	35	128	73%	3.12
SCHREI	6.900%	217.80	5/25/16	5/25/66	101.62	56	163	75%	3.43
ZURNVX	6.450%	200.00	6/15/16	12/15/65	95.90	103	165	82%	3.75
JPM	6.950%	100.00	8/17/36	8/17/66	103.95	82	97	97%	0.99
WM	6.534%	148.25	3/15/11	3/15/60	97.66	67	150	101%	5.96
SWK	5.902%	140.00	12/1/10	12/1/45	93.00	78	153	109%	6.11

Source: Lehman Brothers.

Comparing put prices across different securities in Figure 3, we can see that there are 3 factors that affect the extension risk: the ratio of forward BCDS spread to the step-up coupon, the step-up date and final maturity, and the level of BCDS spread with a maturity equal to the step-up date.

First, the put price is clearly related to how close the option is to being in-the-money. Moneyness is measured by the ratio of the forward BCDS spread to the step-up coupon (Figure 3). Take LNC 7% and SCHREI 6.9% as examples. The two securities have similar step-up dates and final maturities and their BCDS spreads are 13bp apart. But LNC 7% has a forward BCDS/step-up ratio of 45%, much lower than the 75% of SCHREI 6.9%. As a result, the put price is $1.53 for LNC and $3.43 for SCHREI. All else equal, a security with a higher forward BCDS/step-up ratio has a higher extension risk.

Second, the extension risk is also related to the step-up date and final maturity. This is because the securities we consider in Figure 3 can be called by issuers at any coupon reset dates between the step-up date and final maturity; a longer period between the two dates gives issuers more options. A comparison between ILFC 5.9% and ILFC 6.3% illustrates this effect. ILFC 5.9% is further from being in-the-money than ILFC 6.3% (47% to 62% in terms of forward BCDS/step-up ratio), but it has a higher put price ($3.15 to $2.09) because of an earlier step-up date (12/21/2010 vs 12/21/2015). Approximately half of the difference in extension risk between these two securities is also explained by a lower discount factor to the step-up date for ILFC 5.9% (see below).

Third, since the put price is the expected option payoff discounted to the valuation date, the BCDS level at the step-up date also matters. A higher BCDS spread gives a deeper discount and a lower option value. A comparison between ILFC 5.9% and ZURNVX 6.15% shows this effect. The two securities have similar step-up dates and final maturities, but ZURNVX 6.15% has a lower put price ($3.12 to $3.15) although it is closer to being at the money (73% to 47% in terms of forward BCDS/step-up ratio). A look at the BCDS spreads reveals a reason: ZURNVX 6.15% is 35bp and ILFC 5.9% is 19bp.

Extension risk of max(3M LIBOR, 10Y CMT, 30Y CMT) step-up hybrid securities

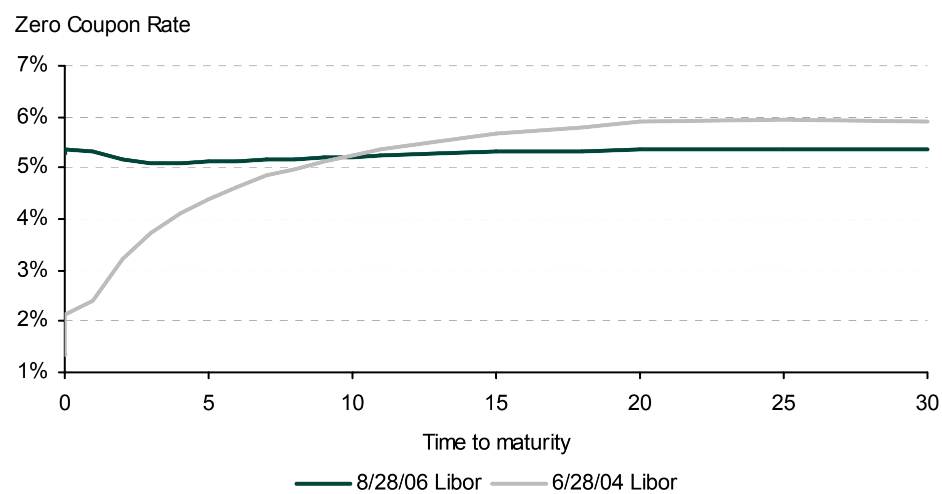
We next turn to hybrid securities that have more complex step-up structures, i.e., max{3M LIBOR, 10Y CMT, 30Y CMT} + step up with a cap.

If we look at the intrinsic value of the floaters underlying the put option, max{3M LIBOR, 10Y CMT, 30Y CMT} + step up > max{3M LIBOR, 10Y CMT} + step up > 3M LIBOR + step up. This leads to a reverse ordering of the extension risk because the put option is closer to being in-the-money when the underlying intrinsic value becomes smaller. In other words, the penalty for the issuer of not exercising the option is less severe when the step-up coupon decreases. For the same reason, a cap on top of a step-up coupon should also increase the extension risk. On the other hand, when the LIBOR curve steepens, a security with floating coupons depending on the interest rate curve shape should have a smaller extension risk because the put option is further out-of-the-money.

To illustrate the above relationships, we price the extension risk with different coupon step-up structures and interest rate curve scenarios. Figure 4 shows two LIBOR curve scenarios. The 8/28/06 curve is a flat curve, while the 6/28/04 curve is a steep curve with low front-end rates[17].

Figure 5 shows the extension risk for two hybrid securities under various coupon and LIBOR curve scenarios: ZURNVX 6.15% with the first step-up date on 12/15/2010, and SWK 5.902% on 12/1/2010[18]. Note again that the valuation date is 8/28/06. In the steep LIBOR curve scenario, we assume that the LIBOR curve is equivalent to the 6/28/04 curve and let the model take it as an input.

Figure 4. LIBOR curve scenarios



Source: Lehman Brothers.

When the step-up coupon changes from 3M LIBOR + step-up to max{3M LIBOR, 10Y CMT} + step-up, we see a decrease in extension risk under both LIBOR curve scenarios. However, we see only a very small reduction in extension risk when the step-up coupon changes from max{3M LIBOR, 10Y CMT} + step up to max{3M LIBOR, 10Y CMT, 30Y CMT} + step up, and almost no change in extension risk when a cap is imposed on the step-up coupon. Across the interest rate curve scenarios, we see a reduction in extension risk of both securities when the step-up coupons depend on the interest rate

[17] *We assume constant swap spreads to Treasury rates from the LIBOR curve.*

[18] *Figure 4 uses the three interest rate and one credit factors model discussed in the previous section. It is different from the one interest rate and one credit factor model used to produce numbers in Figure 2. One practical reason for using different models is that the computation time of the four factor model is long and we found it impractical to use it to produce BCDS spreads. We therefore used the BCDS curve produced by the one factor model as an input to the four factor model and directly valued the put option. Also note that the put values for the 3M LIBOR step-ups are slightly different in the two figures. This is due to differences between the two models.*

curve shape. For example, the extension risk of ZURNVX 6.15% would be almost one point higher if its step-up coupon was linked only to the 3M LIBOR rate. For SWK the effect is 1.4 points with the 8/28/06 curve and 1.7 points with the 6/28/04 curve. As expected we see that the difference in extension risk for the two coupon structures increase as we move to a steeper curve. However, the increase may not be as large as expected. This could be due to the securities having step-up dates that lie four years away. It may be that we have little information about the future curve shape after such a long time period, even though we start from very different initial interest rate curves.

Figure 5. Extension risk, step-up coupon structures and LIBOR curve shapes

ZURNVX 6.15	**Current LIBOR Curve**	**6/28/04 LIBOR Curve**
Coupon Structure	**Put Price($)**	**Put Price($)**
6.15%, 12/15/10	0.0	0.0
3ml + 175bp	3.2	3.0
max(3ml, 10Y CMT)+175bp	2.5	2.1
max(3ml, 10Y CMT, 30Y CMT)+175bp	2.3	2.0
max(3ml, 10Y CMT, 30Y CMT)+175bp, 13.25% cap	2.3	2.1
SWK5.902	**Current LIBOR Curve**	**6/28/04 LIBOR Curve**
Coupon Structure	**Put Price($)**	**Put Price($)**
5.902%, 12/1/10	0.0	0.0
3ml + 140bp	6.3	6.1
max(3ml, 10Y CMT)+140bp	5.3	4.6
max(3ml, 10Y CMT, 30Y CMT)+140bp	4.9	4.3
max(3ml, 10Y CMT, 30Y CMT)+140bp,		
13.25% cap	4.9	4.4

Source: Lehman Brothers.

Sensitivity analysis: how sensitive is extension risk to recovery rate and volatility?

Next we show how sensitive extension risk is to recovery rate and hazard rate volatility assumptions. In Figure 6, we use WM 6.534% as an example. The top part of the figure shows the put option values and the bottom part shows the corresponding forward BCDS spreads. An immediate observation is that starting with a given spread curve and a fixed volatility, the extension risk decreases when we increase the recovery rate. The reason is as follows: A higher recovery rate implies a higher probability of a credit event and a deeper discount from the valuation date to the step-up date. Since the put option is a knock-out option this causes a lower option value.

Figure 6. Extension risk and forward BCDS spreads of WM 6.534% under various recovery and hazard rate volatility assumptions

Put Price ($)

	Recovery					
Vol	**10%**	**20%**	**30%**	**40%**	**50%**	**60%**
30%	5.64	5.56	5.47	5.35	5.18	4.94
40%	5.94	5.85	5.74	5.60	5.42	5.15
50%	6.15	6.05	5.93	5.78	5.58	5.29
60%	6.29	6.19	6.06	5.90	5.69	5.39
70%	6.38	6.28	6.14	5.98	5.76	5.46

Forward BCDS Spread (bp)

	Recovery					
Vol	**10%**	**20%**	**30%**	**40%**	**50%**	**60%**
30%	157	159	162	167	172	181
40%	149	152	156	160	166	176
50%	144	147	151	156	162	172
60%	141	144	148	153	159	169
70%	138	141	145	151	157	168

Source: Lehman Brothers.

CONCLUSION

We have used an option-pricing framework to develop a measure of extension risk for hybrid capital securities. Specifically, we view a long position in a hybrid security as a portfolio of a long position in a fixed-coupon bullet bond and a short position in a put option on a floater. The fixed coupon bond is extended into a floater if the issuer does not call the hybrid security at the step-up date, which in our analogy corresponds to the issuer exercising the put option. We quantify the extension risk as the value of the put option and price it by combining option valuation models with a bond-implied CDS methodology.

Through numerical examples, we find that extension risk is closely related to the BCDS curve extracted from the security's market price and its PCDS curve, especially the forward BCDS spread from the step-up date to the final maturity. For securities whose step-up coupons depend on the shape of the interest rate curve, we show that the extension risk changes with different coupon structures. In general, relative to a 3M LIBOR step-up coupon structure, a max(3M LIBOR, 10Y CMT, 30Y CMT) structure increases the penalty for issuers of not calling the hybrid security at the step-up date, and reduces extension risk for investors. We have also quantified how differences in extension risk between different step-up coupon structures depend on the interest rate curve.

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A simple framework to understand the fair value of credit spreads

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We present a simple econometric framework linking the level of financial and ex-financial credit spreads to their fundamental drivers – viz. the state of the macro environment and the health of the corporate balance sheet.[1] We illustrate the application of the framework by analysing the broad behaviour of the driving variables and conclude that investors should currently favour an underweight position in credit markets.

1. INTRODUCTION

A recurring question in credit markets is to what extent credit spreads are fairly priced. Credit spreads combine default risk, loss given default and risk premium, which are in turn influenced by firm-level and macro factors. The "fair value" of credit spreads can be defined as the level of spreads that can be justified by the levels of these driving variables.

Different approaches can be used to ascertain the fair value of spreads. The fundamental approach assumes that accounting data and firm-level indicators will help to assess the fair value of credit spreads. The macro-economic approach considers macro variables such as growth and inflation, while the market approach uses market information such as equity market data to address the same question.

In designing a framework that encompasses these different approaches, we consider the period 2004-2006 when IG and HY credit spreads in the US and Europe have tightened to near historical lows. Alongside this tightening, we have also seen a strong improvement in corporate fundamentals and a largely benign macro environment.

Several factors have contributed to the tightening of credit spreads in recent years. Figure 1 presents the current level of credit spreads and their fundamental drivers, such as the level of corporate leverage and asset volatility, and equity market performance.

Figure 1. Credit spreads and key macro indicators[2]

	US		Europe	
	Aug 2006	**Hist avg[3]**	**Aug 2006**	**Hist avg**
Spreads to Treasuries (bp)				
IG Ex-Financials	107	114	59	84
IG Financials	77	108	49	46
High yield	335	483	279	674
5 year CDS indices (bp)	38	51	27	36
Leverage[4]				
Debt/market value of equity	48.5	65.1	43.7	49.5
Interest coverage (EBITDA/Interest)	6.2	5.0	11.7	7.4
Asset return volatility[5]	7.6	12.5	10.8	15.3
Equity markets (% excess return 12 month)	4.3	6.4	12.9	0.7

Source: Lehman Brothers.

[1] *The authors would like to thank Srividya J, Soumya Sarkar and Srivaths Balakrishnan for their useful insights and comments.*

[2] *Accounting data for the current period pertain to the quarter ended 31st March 2006.*

[3] *Averages are over the period 1973-2006 for the US (US High Yield from 1990) and 1999-2006 for Europe.*

[4] *US: Non-farm Non-Financial Corporates (Source: Federal Reserve Flow of Funds); Europe: Non-financial Lehman Index Issuers (Source: WorldScope, Lehman Brothers).*

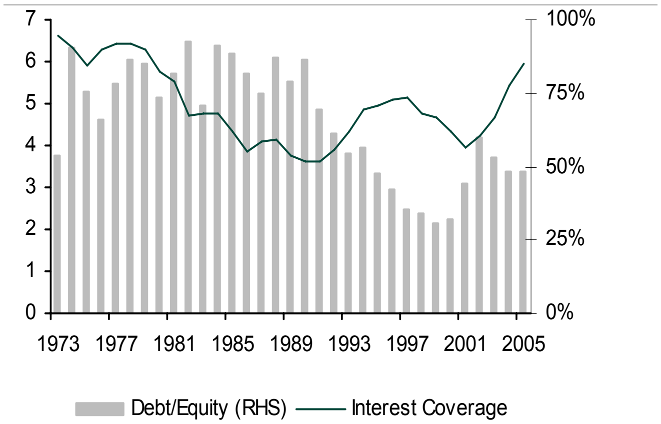
[5] *Asset return volatility is defined as the de-leveraged equity return volatility. We use the VIX Index for the US and the Implied Volatility of options on the Eurostoxx Index for Europe as our estimate of equity volatility. We then multiply this by the ratio of the value of equity to that of the total assets, to deleverage the equity volatility.*

In the past three years, global economic growth has consistently exceeded the long-term average, leading to strong earnings growth and much-improved cash flows. Corporate balance sheet health has improved sharply over the past couple of years. At the same time, reluctance to invest has resulted in a distinct rise in corporate savings across the G7. That corporates have turned net lenders has led to a contraction in their borrowing requirement and hence corporate leverage (Figure 2a).

The level of asset return volatility for the broader market and the number of defaults among investment grade credits have also declined steadily since 2002 (Figure 2b).

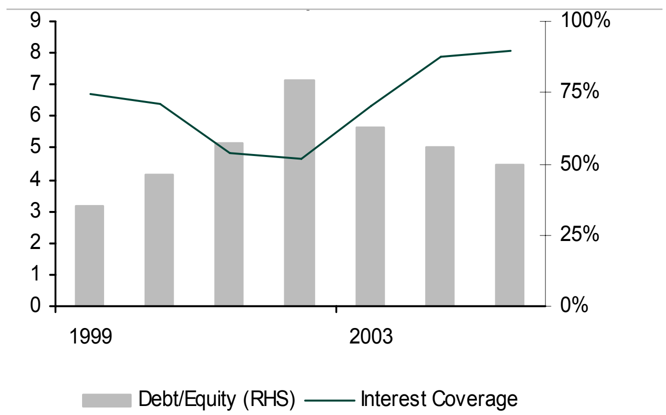
Figure 2a. Leverage measures in the US and Europe

US



Source: Lehman Brothers, Federal Reserve, WorldScope.

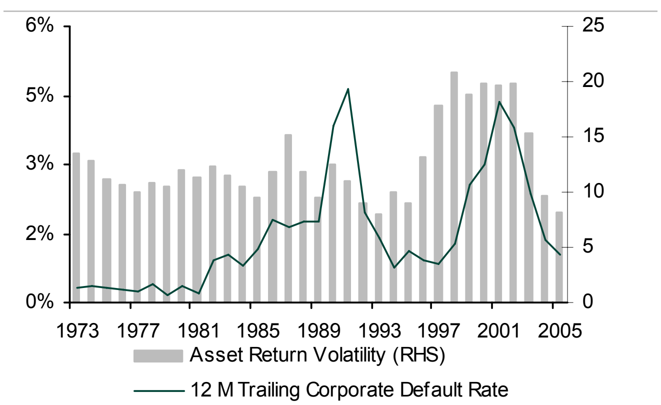
Europe



Source: Lehman Brothers, WorldScope.

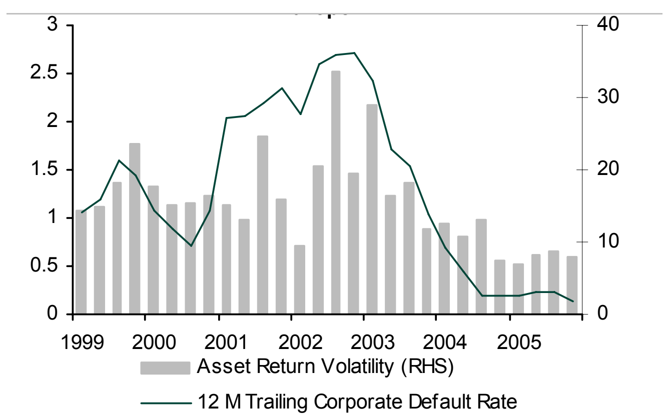
Figure 2b. Systematic risk indicators in the US and Europe

US



Source: Lehman Brothers, Moody's, Bloomberg.

Europe



Source: Lehman Brothers, Moody's, Bloomberg.

Our framework could help assess whether the tightening in corporate spreads was in line with that warranted by the improvement in fundamentals – i.e. whether there has been an over- or under-reaction in spread markets. In order to perform this analysis, we propose a simple econometric framework that links the level of credit spreads to their fundamental drivers – thereby arriving at a measure of "fair value".

This framework complements our MISTRAL model.[6] While MISTRAL directly analyses excess returns and relates them to macro factors that drive credit spreads, this framework employs a valuation approach. In other words, while MISTRAL predicts excess returns based on market variables, this framework analyses the current level of spreads vis-à-vis its fundamental drivers. Hence, this framework takes a more long-term view on the fundamental value of credit spreads.

The rest of this article is organised as follows. In Section 2 we detail the building blocks of our framework and the estimation procedure. We also present some statistical analyses of the framework that help in forming views on credit markets. In Sections 3 and 4 we analyse the current residuals and illustrate the process of understanding the future dynamics of the driving variables. We present our conclusions in Section 5.

2. A SIMPLE ECONOMETRIC FRAMEWORK FOR CREDIT SPREADS

A good econometric model should be grounded in sound theory. We know that corporate debt and equity can be regarded as contingent claims on a firm's assets.[7] More specifically, the firm's equity can be regarded as a call option on the value of its assets, while its debt can be seen as a portfolio of default-free debt and a short position in a put option on the value of its assets. Option pricing theory would therefore tell us that the "moneyness" of this claim and the volatility of the underlying assets should be important drivers of credit spreads.

We use the above insights as the building blocks of a simple econometric framework that aims to give a measure of fair value for spreads. In this framework, the drivers of credit spreads can be grouped into three broad categories:

- *Leverage measures*: The firm's leverage is indicative of the moneyness of the option on its underlying assets.
- *Risk measures*: These are indicative of the volatility of the returns of the underlying assets of the firm.
- *Macro-economic backdrop*: Macro factors could systematically lead to wider or tighter credit spreads due to a higher perception of risk.

In subsections 2.1 and 2.2 we use the above to arrive at a framework of the fair value of financial and ex-financial spreads.

2.1. Framework for the fair value of ex-financial spreads

The above building blocks have a direct interpretation in the case of ex-financial (industrial and utility) firms. We therefore use the following variables to explain the level of ex-financial credit spreads:

- **Leverage measures**
  - *Debt/Market value of equity*: This ratio is directly analogous to the "moneyness" of the option (the firm value is the underlying and the debt is the strike price). It

[6] *See Balakrishnan et al. (2005).*
[7] *See Merton (1974).*

is indicative of the firm's long-term ability to meet its debt-servicing requirements.

- *Interest coverage (EBITDA/Interest Expense)*: This ratio is indicative of the firm's short-term ability to service its debt. This is useful to capture the timing of debt repayment.

- **Risk measures**
  - *Asset return volatility*: We define the asset return volatility as the deleveraged equity return volatility. In other words:

$$\sigma_V = \frac{E}{(D+E)}\sigma_E$$

Where σ_V is the asset return volatility, σ_E is the equity return volatility, E is the market value of equity and D is the value of debt.[8] The underlying assumption is that the higher the asset volatility, the higher the probability of default and the higher the premium for the risk of defaults and increased volatility.

- **Macro backdrop**
  - We use the Chicago Fed National Activity Index (CFNAI) as an indicator of the strength of the macro-economy. A strong macro-economy should be supportive of credit markets and would hence lead to tighter credit spreads on average.

We postulate a linear relationship between the above variables and the level of credit spreads for simplicity. We also postulate that the sensitivity of US and European credit spreads to these variables is the same. However, to allow for the fact that US credit spreads could be systematically different from European spreads (due to the fact that on average European credits in the Lehman Corporate Indices could be differently rated from US credits) we introduce a dummy variable that takes a value of one for European spreads and zero of US spreads.

The resultant econometric specification is as follows:[9]

$$Spread_{i,t} = \alpha + \beta_1 * \text{Asset Return Volatility}_{i,t} + \beta_2 * \text{Leverage}_{i,t} +$$
$$\beta_3 * \text{Interest Coverage}_{i,t} + \beta_4 * \text{C}FNAI_t + \beta_5 * \text{EUR Dummy}_i + \varepsilon_t$$

Data and estimation

We apply the same econometric specification for the spreads of both investment grade and high yield ex-financial firms. To estimate the coefficients, we use US data from 1973 to 2006 for IG ex-financials and from 1990 to 2006 for HY ex-financials. European data for both IG and HY spreads are for the period 1999-2006.

To make it easier to compare the sensitivity of spreads to the driving variables, we run the regression on the (in-sample) percentile ranks of the variables (Figures 3a and 3b).

[8] *The standard Merton framework would suggest that the asset return volatility is defined as*

$\delta_E.\sigma_V = \frac{E}{V}.\sigma_E$ *where δ_E is the delta of equity with respect to the underlying asset value.*

[9] *Where i is either USD or EUR.*

Figure 3a. Regression results for IG ex-financial spreads

	Leverage	CFNAI	Asset volatility	EBITDA/Interest	EUR dummy	Constant
Beta	23.4	-17.1	56.7	-28.3	-29.4	85.4
Significance	***	***	***	***	***	***
R^2	49%					

Source: Lehman Brothers.

Figure 3b. Regression results for HY ex-financial spreads

	Leverage	CFNAI	Asset volatility	EBITDA/Interest	EUR dummy	Constant
Beta	500.7	-140.9	560.5	-189.8	188.3	34.6
Significance	***	***	***	***	***	
R^2	86%					

Source: Lehman Brothers.

2.2. Framework for the fair value of financial spreads

It is harder to define such measures as leverage and interest coverage for financial firms because they can have large off-balance sheet liabilities or liabilities matched by financial assets. We therefore use the following variables to explain the level of credit spreads, while retaining the same principles as for ex-financial spreads:

- **Default risk measures**
  - *Ratio of upgrades to total rating actions by Moody's*: This shows the overall improvement or deterioration in credit quality in the financial sector.
  - *Ratio of loan losses to total loans*: This ratio captures the total amount of defaults arising in the loan portfolios of banks. A larger proportion of loans to total loans would indicate a more risky loan book portfolio.
- **Business risk measures**
  - The level of *equity market volatility* indicates a higher level of business risk and of cash flow volatility, thus rendering debt less valuable and spreads wider.
- **Profitability measures**
  - The *net interest margin* of banks is indicative of the profitability of their core business and thus the likelihood of them servicing their debts fully and on time.
  - Long-term returns of the equity market are indicative of the profitability of the asset portfolios held by banks and insurance companies which form a significant part of the overall profitability of financial institutions.

The resultant econometric specification is as follows:

$$Spread_{i,t} = \alpha + \beta_1 * \text{EquityVol}_{i,t} + \beta_2 * \text{LoanLosses/Total Loans}_{i,t} + \beta_3 * \text{Net InterestMargin}_{i,t} + \beta_4 * \text{EquityReturns}_{i,t} + \beta_5 * \text{Moody's UG vs.DG}_{i,t} + \beta_6 * \text{EUR Dummy}_i + \varepsilon_t$$

We use data from 1986 to 2006 for US financial spreads and from 1999 to 2006 for Europe. Figure 4 presents the results of the regression.

Figure 4. Regression results for IG financial spreads

	Equity volatility	Loan losses/ Total loans	Net interest margin	Equity returns	Moody's U/(U+D)	EUR dummy	Constant
Beta	51.7	38.5	-32.0	-14.4	-15.0	-66.8	98.6
Significance	***	***	***	***	**	***	***
R^2	70%						

Source: Lehman Brothers.

2.3. Using the econometric framework to form credit views

The specification of the fair value framework in Sections 2.1 and 2.2 above assumes a cointegrating relationship between the level of spreads and the driving variables. In order to test this assumption, we perform unit root tests[10] on the residuals of the level regressions in the three sets of data (viz. ex-financial investment grade, ex-financial high yield and financials).

The residual of the regressions could have the following dynamics:

$$\varepsilon_{t+1} = \rho \varepsilon_t + \eta_{t+1}$$

Where ε_t is the residual at time t; ρ is the AR(1) coefficient and η_t is the error term. If the AR(1) coefficient ρ is significantly less than 1 (i.e. does not have a unit root), we could conclude that the levels of spreads and their driving variables above have a cointegrating relationship. Figure 5 presents the results of the Phillips-Perron test for a unit root in the residuals.

Figure 5. Unit root tests on residuals of regressions

		ρ	p-statistic	1% critical value
Ex-Financials	IG	0.78	-4.63	-3.46
	HY	0.41	-5.95	-3.44
Financials		0.72	-4.41	-3.44

Source: Lehman Brothers.

In Figure 5 above, we also present the critical values for the null hypothesis of a unit root (i.e. ρ =1) to be rejected at the 1% level. We find that in all three cases, we are able to reject a unit root at the 1% level.

Drivers of credit views based on the econometric framework

We can write the relationship between the levels of residuals as a relationship of the changes as follows:

$$\Delta \varepsilon_{t+1} = (\rho - 1)\varepsilon_t + \eta_{t+1} \qquad \text{(A)}$$

We know from the above that the residual ε_t is defined as the difference of the spread level at time t and the vector of the driving variables at time t (say, X_t) weighted by the vector of coefficients (say, B). In other words:

$$\varepsilon_t \equiv S_t - B^T X_t$$

[10] *A time series has a unit root if the regression coefficient is one in an autoregressive model (current variable regressed on its lagged value or AR(1) process).*

We can now write the relationship in (A) above as follows:

$$\Delta S_{t+1} = (\rho - 1)\varepsilon_t + B^T (X_{t+1} - X_t) + \eta_{t+1}$$

Therefore, the change in the spread level from time t to t+1 is a function of the lagged residual and the changes in the driving variables over the same period.[11]

In other words, investors can form credit views for the next period based on:

The distance of the spreads from "fair value" in the current period, and

Their views of the dynamics of the driving variables over the next period. These views could be driven by a predictive econometric specification of the driving variables – which would in turn be determined by the broad behaviour of these variables over the economic cycle.

This also means that we cannot have a simple investment strategy based on the reversion of credit spreads towards fair value without forecasting the dynamics of the driving variables. In other words, we need to either specify the dynamics of the driving variables or use a qualitative approach to form views on them – in order to determine the trade arising out of this framework.

In Sections 3 and 4 below, we illustrate such a process of view formation. While Section 3 presents an analysis of the current residuals, Section 4 analyses the behaviour of some of the driving variables in the framework over different stages of the economic cycle.

3. ANALYSING THE RESIDUALS IN THE CURRENT PERIOD

As discussed in Section 2.3 above, the first component of the credit view formation process from the framework is an analysis of the current residual of the regression. As an illustration, we present an analysis of the residuals as of the end of August 2006.

Figure 6 shows the most recent values of the fundamental drivers of spreads and the normalised residuals for investment grade ex-financial spreads.[12]

Figure 6. Fair value of IG ex-financial credit spreads as of August 2006

August 2006 Values	Spread (bp)	Asset Volatility	Leverage	CFNAI	EBITDA / Interest	Normalized Residual
US	107	7.6%	49%	0.06	6.18	1.33
EUR	59	10.8%	44%	0.06	11.74	0.39

Source: Lehman Brothers.

Normalised residuals in August 2006 are relatively small both in the US and Europe – implying that credit spreads are currently close to their fair values. US investment grade ex-financial spreads appear marginally cheaper than those in Europe.

We find a similar result for HY ex-financial and IG financials spreads. Figures 7a and 7b set out the results for the high yield ex-financial universe as of August 2006.

[11] This is often referred to as the "error-correction" representation of the cointegrating relationship.
[12] The normalised residual is defined as the ratio of the current residual to its standard deviation.

Figure 7a. Fair value of HY ex-financial credit spreads[13] as of August 2006

August 2006 Values	Spread (bp)	Asset Volatility	Leverage	CFNAI	EBITDA / Interest	Normalised Residual
US	333	11.90%	100%	0.06	3.23	-0.32
EUR	275	13.70%	74%	0.06	3.21	-0.49

Source: Lehman Brothers.

Figure 7b. Fair value of financial credit spreads as of August 2006

August 2006 Values	Spread (bp)	Equity Volatility	Moody's UG vs DG	Net Interest Margin	Loan Losses/ Total Loans	12 mth Equity Returns	Normalised Residual
US	77	18.80%	50%	3.46%	0.35%	4.29%	-0.53
EUR	49	22.40%	33%	2.71%	0.69%	12.93%	1.67

Source: Lehman Brothers.

Overall, the above analysis suggests that the case for a spread overreaction to the improvement in corporate fundamentals over the past few years seems rather weak.

Were IG ex-financial spreads at fair value earlier this year?

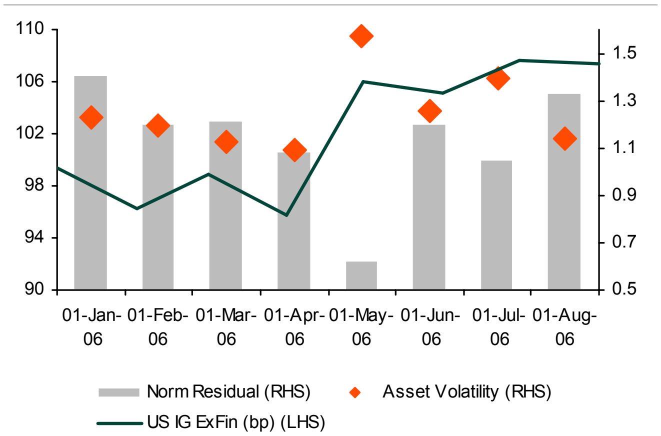
In addition to the current residual, it may also be relevant to analyse their recent history in order to take a better view on the future dynamics of spreads. This may be particularly interesting if the specification of the model of the dynamics of the residual (as in Section 2.3) includes additional lags.

Figure 8 shows that spreads were marginally cheap in early 2006 and moved much closer to their fair value following the steep rise in volatility.

Since June however, while volatility has contracted back to pre-May levels spreads have not tightened as sharply, causing them to look marginally cheap as of end-August.

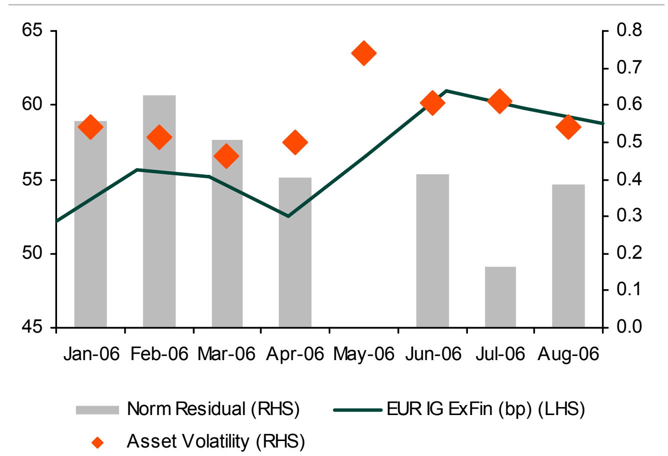
Figure 8. Recent dynamics of the fair value of IG ex-financial spreads

US[14]



Source: Lehman Brothers.

Europe[3]



Source: Lehman Brothers.

[13] *We analyse the spreads and fundamental characteristics of the HY Ex-Ford and GM universe for both the US and Europe.*

[14] *Asset return volatility has been rescaled to fit the axis.*

While the above analysis suggests that there is no case for a strong tightening or widening of credit spreads going forward due to a mean reversion of the residual – it is important to analyse the possible dynamics of the drivers going forward.

More specifically, the pattern of spreads and asset return volatility in Figure 8 presents some interesting possibilities. We see that credit markets price in a higher level of volatility than that observed in the equity markets (which is the main driver of asset return volatility). This could indicate that credit markets are anticipating the arrival of a mature stage of the current economic expansion, which is marked by poorer corporate health and higher volatility. Another possibility is that at this stage of the credit cycle, there is divergence between the interests of creditors and shareholders. Company management might undertake actions favourable to the latter (higher stock price, lower equity volatility) but detrimental to the former (increased leverage and increased credit risk). We explore these hypotheses in Section 4 below.

4. FORMING VIEWS ON THE DYNAMICS OF THE DRIVING VARIABLES

The second step in forming forward-looking views on the credit markets is to analyse the dynamics of the driving variables going forward. In this section, we illustrate this process by analysing the behaviour of some of the driving variables over different stages of the economic cycle and using recent data to aid in forming views.

We present some of the key variables that investors may have to analyse in order to form views on the drivers of ex-financial credit spreads. However, the broad conclusions from the analysis below (and some of the specific variables analysed) are equally applicable to the drivers of financial spreads.

4.1. Behaviour of the drivers of ex-financial spreads over the economic cycle

In order to understand the future dynamics of the driving variables in the fair value framework, one could assess their behaviour over different stages of the economic cycle. One could then take a view on which part of the cycle we are likely find ourselves in going forward – hence taking a view on the drivers themselves.

Corporate health deteriorates as economic expansions mature

During the early part of an expansionary phase, corporate profitability improves and cash flows strengthen, leading to credit and equity markets outperforming. As the expansion matures, corporate profitability begins to slow, leading to a possible deterioration in credit quality and underperformance of equity markets. The experience of the US market as documented in Figure 9 corroborates this.

Figure 9. Corporate health over the economic cycle (June 1973-May 2006)

Average monthly excess returns[15] (bp)	Asset Volatility (%)	Debt/Equity (%)	Profit Growth (%)
Recession period	9.0	67.5	-5.4
Expansion period	7.9	54.3	3.3
1st half expansion period	7.1	55.4	5.2
2nd half of expansion period	**8.8**	**53.0**	**0.9**
Full sample	**8.1**	**57.0**	**1.9**

Source: Lehman Brothers, NBER.

[15] *The monthly returns on the relevant index are classified into Recessions and Expansions based on the NBER Classification. The returns are then averaged within each of these classifications.*

However, at such times corporate managements often prevent a fall in equity values by increasing shareholder-friendly activity, such as increased dividends and buybacks to enhance cash yields to shareholders The evidence presented in Figure 10 shows that as economic expansions mature, cash yields to shareholders increase on average.

Figure 10. Cash yields to shareholders (April 1953 – June 2006)

(Annualised yield as % of GDP)	Total	Dividends	Buy Back
Recession period	1.89	2.08	-0.19
Expansion period	2.63	2.38	0.25
1st half of expansion period	2.37	2.36	0.01
2nd half of expansion period	2.94	2.41	0.53
Full Sample	2.51	2.33	0.18

Source: Lehman Brothers, NBER, US Federal Reserve Flow of Funds.

Such cash distributions are often financed by additional debt, thereby leading to an expansion of corporate leverage. As a result, while equity markets tend to continue to outperform through the mature phase of the expansion, credit markets often underperform.

To form a view for the current period, we now have to analyse if the current expansion has begun to mature.

4.2. What do 2Q 2006 data tell us about the current expansion?

Profitability and growth prospects are strong

Corporate earnings over the first two quarters of 2006 were robust and do not point towards a significant slowdown in profitability (Figure 11).

Figure 11. Indicators of earnings health (Q4 2005 vs Q1 2006)

Indicators		Q4 2005	Q1 2006	Q2 2006
Positive to Negative Earnings Surprise Ratio	Europe	1.8	2.6	2.4
	US	2.2	4.4	3.9
Expected y-o-y Earnings Growth (%)	Europe	8.4	12.9	19.8
	US	10.7	10.1	13.8

Source: Lehman Brothers Equity Strategy.

Indications of macro-economic growth were also reasonably strong as of the first quarter. Estimates of global growth currently show only a slight slowdown in 2007 (Figure 12).

Figure 12. GDP growth – 2005 actual and estimates for 2006 and 2007

GDP Growth (%)	2005	2006	2007
Europe	1.3	2.4	2.0
US	3.2	3.4	2.9
Global	4.9	5.1	4.9

Source: Lehman Brothers, IMF World Economic Outlook September 2006.

Some signs of weakness have begun to appear

However, some signs of weakness have begun to appear. Figure 13 presents some key indicators of corporate health (pertaining to US non-farm non-financial and financial corporates) from the US Federal Reserve Flow of Funds data for Q2 2006.

Figure 13. Indicators of corporate health (Q1 vs Q2 2006)

Indicator		2004	2005	1Q 2006	2Q 2006
Profitability	y-o-y EBITDA Growth	16.2%	9.8%	7.6%	6.8%
	y-o-y Financials Profit Growth	8.1%	12.2%	6.7%	24.5%
Financing[16]	Debt/Equity	48%	49%	48%	51%
	EBITDA/Interest	5.42x	6.08x	6.17x	6.19x
	Financing Surplus/Debt	-0.9%	1.7%	0.1%	-0.8%
Investing[4]	y-o-y Capex Growth	13.6%	7.4%	9.0%	17.1%
Distribution[17]	Net Dividend/GDP	4.1%	2.6%	4.0%	4.1%
	Net Buyback/GDP	0.2%	2.3%	4.2%	3.6%

Source: Lehman Brothers, US Federal Reserve Flow of Funds.

One can see the following interesting changes:

- EBITDA growth for 1Q 2006 (revised) and 2Q 2006 are below the high levels of 2004 and 2005.
- Corporate leverage is on an uptick, possibly reflecting recent releveraging via LBOs and leveraged recapitalisations.
- Cash distribution to shareholders (in particular the level of buybacks) has increased markedly since the end of FY 2005.

The impact of monetary policy should soon begin to manifest itself

In addition to the above factors, the future dynamics of corporate profitability and hence leverage could be affected by the level of economic growth. To analyse the impact of this effect on the investor's view on the driving variables of spreads, it would be useful to examine broad monetary policy trends as well.

For instance investors could hold the view that the impact of a less accommodative monetary policy activity in the US and Europe over the past two years should begin to manifest itself in slower economic growth. In Figure 14 we present the residuals from a simple Taylor rule.[18]

---

[16] *Data pertaining to US non-farm non-financial corporates.*

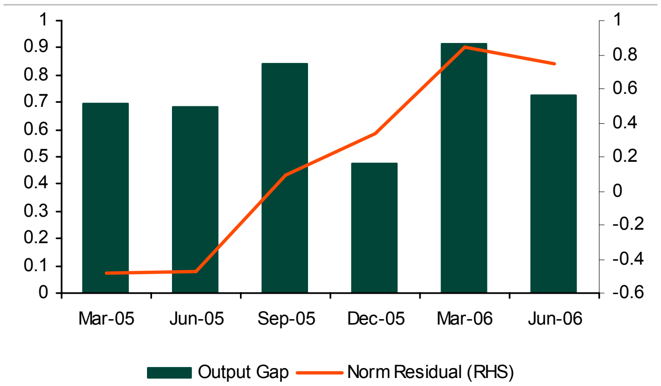
[17] *Data pertaining to US financial and non-farm non-financial corporates. Data for the first and second quarters are annualised figures.*

[18] *The Taylor rule links the short rate with the key drivers of monetary policy viz. inflation and the output gap. In the version of the rule that we use, we also factor in some inertia in monetary policy, whereby the central bank sets the next period's interest rate based on the current period's rate and a target rate that the above macro variables would imply.*
The residual in period t+1 is therefore defined as:

$$\varepsilon_{t+1} = \text{Short Rate}_{t+1} - \left\{ \rho \begin{bmatrix} \text{Neutral Rate} + \beta_{\pi}\left(\text{Inflation}_t - \text{Neutral Inflation}\right) + \\ \beta_{\pi}\left(\text{Output Gap}_t - \text{Neutral Output Gap}\right) \end{bmatrix} + (1-\rho)\text{Short Rate}_t \right\}$$

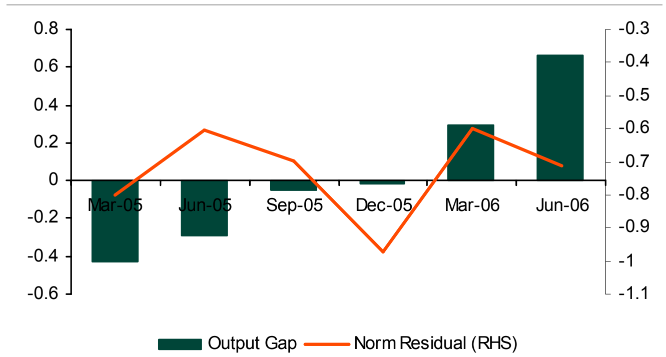
Figure 14. Taylor residuals in the US and Europe

US



Source: Lehman Brothers.

Europe

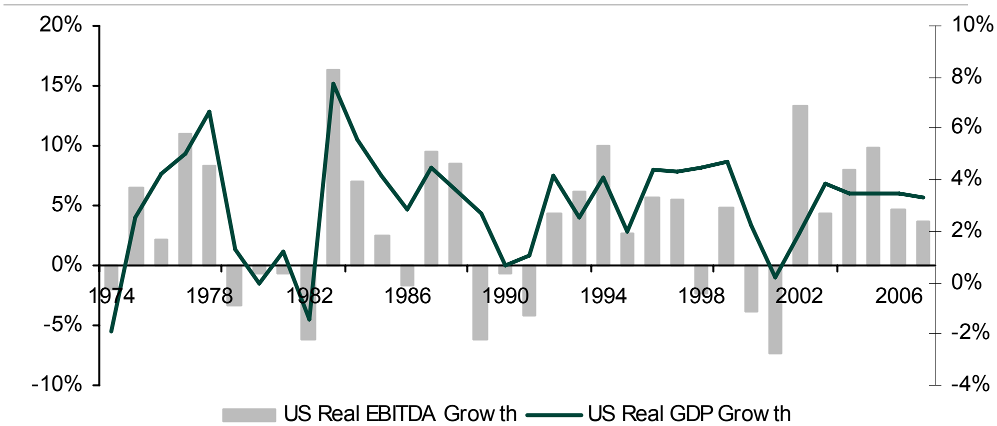


Source: Lehman Brothers.

It is clear from Figure 14 that the Taylor residual has been steadily rising in the US and is higher than its late-2005 value in Europe. This reflects the fact that monetary policy has turned less accommodative, which could have a contractionary effect on economic growth in the quarters to come.

In Figure 15 we present year-on-year real GDP growth vs real EBITDA growth (for non-farm, non-financial corporates) in the US. It is clear that monetary policy tightening could affect corporate profitability, a situation that supports an underweight position.

Figure 15. Impact of output growth on corporate profitability



Source: Lehman Brothers, US Federal Reserve Flow of Funds, Bureau of Economic Analysis.

The risk of continued releveraging activity could exacerbate credit deterioration

The recent surge in releveraging activity has begun to affect corporate leverage. Considering both the capital available for such transactions and the attractiveness of the universe in terms of releveraging targets, we believe that releveraging activity in general and LBO activity in particular are likely to continue in the foreseeable future. Such risks can be analysed using a framework such as that of LEVER which quantifies the drivers of releveraging activity from both a macro and a micro perspective.[19]

[19] *See 'Introducing LEVER: A framework for Scoring Leveraging Event Risk' Quantitative Credit Strategies January 9, 2006.*

4.3. Key insights from the current analysis

The above analysis of the long sample behaviour of the drivers of credit spreads and data pertaining to the second quarter could lead an investor to believe that we are close to entering the mature stage of the current economic expansion. The key conclusions from this analysis could be the following:

- The arrival of the mature stage of the expansion may lead to deterioration in corporate health – leading to an expansion in leverage and a reduction in interest coverage over the next few quarters.
- While asset return volatility has collapsed back to its early 2006 levels in the recent past, credit markets appear to price in a greater persistence of asset return volatility.

In other words, the above analysis may prompt an investor to have an underweight bias in credit spreads for the quarters to come.

5. CONCLUSION

We presented a simple econometric framework linking credit spreads to their fundamental drivers for both financial and ex-financial firms in the investment grade and high yield indices. Credit views based on this framework are driven by the current level of the residual from the regression, and views on the future dynamics of the driving factors.

We illustrate this process of view formation using insights from recent data and conclude that credit spreads are currently near fair value. Although there is no evidence of an overreaction in credit spreads to the improvement in corporate fundamentals in the recent past, we believe there are cyclical reasons for investors to favour an underweight bias, particularly in the US markets.

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DTS℠ (Duration Times Spread) for CDS – a new measure of spread sensitivity

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We extend the analysis of the behavior of corporate bond spreads to the realm of credit default swaps using a new estimation technique. The quasi-maximum likelihood approach can accommodate the stochastic nature of the relation between spread volatility and spread level. Consistent with previous results, we find support for a linear relationship between spread volatility and spread level with some evidence of non-linear effects.

1. INTRODUCTION[1]

Several recent studies have examined the behavior of corporate bond spreads.[2] Those studies found that the volatilities of systematic changes in spreads across a sector tend to increase linearly with the level of spreads.[3] Volatility of the non-systematic component of spread change of a particular bond or issuer was found to be proportional to its spread level as well. Furthermore, they showed that the linear relationship between spread volatility and spread level implies that excess return volatility is roughly proportional to the product of duration and spread. This new risk measure, termed "DTS"℠ (Duration Times Spread), generated better out-of-sample volatility forecasts and lower tracking error for index-replicating portfolios compared with using duration alone.

We extend the analysis of credit spread behavior beyond corporate bonds and look at credit default swaps. Establishing that the conditional volatility of spread change is proportional to the level of spread makes the DTS measure of risk exposure directly applicable to portfolios of CDS. This in turn has important applications in terms of position allocation and risk management.

A priori, we would expect all the previous results to be true for credit default swaps as, in theory, changes in their spreads and those of the underlying bonds should be closely related. In practice, however, this is not always the case. Some evidence suggests that since CDS are often more liquid than their underlying bonds, their spreads incorporate new information more quickly and exhibit higher volatility.[4] In addition, corporate bond spreads are computed relative to the Treasury curve, whereas CDS spreads represent spreads over LIBOR. Furthermore, whereas earlier studies considered data at a monthly frequency, spread changes for CDS are analyzed on a weekly basis.

Another difference between this study and previous research is the use of quasi-maximum likelihood (QML) to investigate the relation between spread volatility and spread level. This technique addresses the stochastic nature of conditional spread volatility and the fact that it is not directly observable (i.e., latent). Using QML enables us to assess the statistical validity of a pre-specified explicit functional dependence between conditional volatility and spread level, and we discuss the relative merits of such an approach compared with that employed in previous studies.

DTS is a registered service mark of Lehman Brothers.

1 *The authors would like to thank Adam Purzitsky for his advice and suggestions.*
2 *See Ben Dor et al. (2005a - 2005c).*
3 *Spread volatility in this article refers to the volatility of absolute spread changes as opposed to the volatility of log-changes in spread used in continuous time models of derivative pricing.*
4 *See for example Blanco, Brennan, and Marsh (2003).*

2. METHODOLOGY

In studying the relation between spread level and spread volatility we face a problem: volatility is ultimately unobservable. What is observed in practice are spread changes which correspond to particular realizations of the distribution of spread changes which are in turn functions of the underlying and unobservable volatility.

Sample estimates across multiple time periods can serve as a measure of the true underlying spread volatility only if the volatility is fairly stable over time. Yet, if spread volatility is related to the level of spread as was found previously, then it would fluctuate over time in response to changes in the level of spread.

Previous studies sought to address this issue by forming buckets that were populated monthly with bonds trading within a certain spread range. The time-series of average spread changes of all bonds in a given bucket was used to form an estimate of its spread volatility. While a bucket's composition may have changed over time in response to changes in spreads of the underlying bonds, its (average) spread level remained remarkably stable.[5] This allowed an analysis of the behavior of spread volatility while holding the level of spread relatively constant.

The advantage of this approach is its flexibility: no assumption is needed regarding the exact nature of the relation between spread volatility and spread level. The finding that spread volatility is linearly related to spread with a proportionality factor of about 10% was based on how the (bucket's) spread volatility reacted in response to changes in the level of spreads. However, the technique does not allow an analysis of spread volatility at the individual security or whole-sector level because of the stochastic nature of volatility as we illustrated above. It relies on buckets with a homogenous population of bonds trading at similar spreads.

We employ a different technique in this study based on "maximum likelihood". In its purest form, maximum likelihood assumes that the true distribution of the sample data is known. In our analysis, we use the normal distribution for spread changes with zero mean and volatility that is not constant over time but rather is a function of the level of spread. However, our results are not dependent on spread changes being normally distributed.[6] The idea underlying this approach is to identify the shape of the relation between spread volatility and spread level that would maximize the probability (likelihood) of the observed data (e.g., spreads changes).

We use the following specification to test the relation between spread volatility in month t and spread level at the end of the previous month:

$$\sigma_t(\Delta s) = \alpha + \beta s_{t-1} + \gamma \hat{s}_{t-1}^2 \tag{1}$$

where $\sigma_t(\Delta s)$ is the volatility of spread during period t and s_{t-1} is the beginning-of-period spread level. The last term in the specification $\hat{s}_{t-1}^2$ controls for a potential non-linear effect of changes in spread level on spread volatility.[7] The maximum likelihood procedure determines the value of the parameters α, β, and γ such that the likelihood of observing the data in the sample under the assumed (normal) distribution is maximized.

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5 *Bonds that experienced spread widening (tightening) were assigned to higher (lower) spread range buckets.*

6 *This result and its application is known as quasi (or pseudo)-maximum likelihood (QML). See Lee and Hansen (1994) who prove consistency of the quasi-maximum likelihood estimation for GARCH(1,1) specification of conditional volatility, which is methodologically similar to our specification.*

7 *The value of $\hat{s}_{t-1}^2$ is determined from an orthogonal in-sample projection of squared spreads on spread levels and the constant. See the appendix for a detailed explanation.*

For example, maximum likelihood estimates for α and γ that are not significantly different from zero would support the previous findings that spread volatility is linearly proportional to the level of spread. The proportionality factor is given by the β estimate and can be compared with the earlier result of 9-10%. If γ, the coefficient of the quadratic term (the square of the spread), is positive (and significant) this would indicate that spread volatility increases with spread in a non-linear manner. Volatility would be lower than in the linear case for tight spreads and higher for wide spreads. Alternatively, if volatility is fairly unchanged over time (which forms the basis for using spread duration as a risk measure), this would result in a significant α with the estimates of both β and γ not being significantly different from zero.

Figure 1. Description of CDS dataset composition

			Universe Population	
Index	**Constituents**	**Starting Date**	**Initial**	**After Exclusions**
CDX.IG	Series 1 - 6	4/17/2003	104	92[1]
ITRAXX.IG	Series 1 - 6	10/7/2003	107	99[2]
CDX.XO	Series 6	1/7/2004	35	30[3]
ITRAXX.XO	Series 5	1/7/2004	45	32[4]

1 Three issues with a substantial number of missing observations (over 30) and four issues with weekly spread changes exceeding 100bp were excluded. The Government and Technology sectors were not included due to a small number of issuers (two in each). We also exclude Arrow Electronics Inc., which became an investment grade company only in the recent past. Prior to December 2003, its CDS spreads were well above 200bp.
2 Two issues with weekly spread changes exceeding 100bp as well as one issue with time average spread level exceeding 100bp are excluded. The Government and Energy sectors are excluded due to small number of issuers (one in each). Three names with substantial numbers of missing observations were eliminated.
3 One issue in the Utilities sector is excluded. We also eliminate four names with substantial numbers of missing points (more than ten weeks).
4 Eleven names with a large number of missing observations (more than 20 weeks) as well as one name with weekly spread changes exceeding 250bp are eliminated. All credits from Utilities sector are excluded due to small number of issuers (one issuer).
Source: Lehman Brothers

3. EMPIRICAL ANALYSIS

3.1 Data

The analysis of CDS spread behavior is based on weekly data collected by Mark-It Partners. The list of individual credit default swaps is compiled from the constituents of main 5-year CDX.IG and CDX.XO for the US; and main 5-year ITRAXX.IG and ITRAXX.XO for Europe. This allows for a comparison of the results across different markets and a wide range of spread levels.

In order to accurately capture systematic spread changes, only sectors that are represented by at least eight CDS are included in the analysis[8]. In addition, several index constituents were excluded due to multiple missing observations or spread blow-ups, namely, a spread widening highly unusual for constituents of an index[9]. To ensure spread changes are not affected by insufficient liquidity, the time period analyzed varies across indices. Figure 1 displays exact details on the CDS data population.

8 The justification for using eight contracts is based on the assumption that idiosyncratic spread changes are uncorrelated with systematic spread changes but exhibit similar volatility. This assumption, backed up by our Global Risk Model, implies that we capture 95% of the systematic volatility on average.
9 In general, these extreme spread movements are associated with highly idiosyncratic events, such as corporate mismanagement or speculation about a pending LBO, and as such should not be expected to be part of the volatility anticipated by the market and reflected in a bond's spread.

In order to allow for a direct comparison of our results with those reported in previous studies, we complement the analysis with monthly spread data of corporate bonds (computed relative to the local Treasury curve). The data span the period from October 1990 to June 2006 for the US Corporate Index and from June 2000 to June 2006 for the US High Yield and Euro Corporate Indices.

Examining the returns of corporate indices and the underlying sectors reveals clear evidence of the effect of large credit events at the index level. Occurrences such as Enron and Parmalat have served to influence sector spread changes to a significant extent. In so far as these events would not be expected to be anticipated by the market, even in terms of volatility, we sought to exclude their effects. In order to do so we made use of the fact that Lehman Brothers maintains two universes of bond indices: returns and statistics.[10] The former reflects the composition of the index in the current period whereas the latter includes all the securities that would be part of the index in the subsequent period (and would then form the returns universe). The different composition of the two universes reflects issuance of new securities, downgrades, index requirements regarding minimum remaining maturity and amount outstanding, etc.

Spread changes for the returns universe reflect only spread movement of the initial index constituents (at the beginning of the period). In contrast, changes in spreads of the statistics universe also reflect changes in the index composition (due to securities entering and leaving the index at the end of the period). We analyze the time-series of spread changes of the statistics universe so as to filter the effect of individual issuers that experience extreme events (we discuss this issue further in section 3.3); we also comment on the corresponding results for the returns universe which differ substantially.

3.2 Spread volatility of credit default swaps

Panel A of Figure 2 displays volatilities and corresponding median spreads for all constituents of the CDX.IG and CDX.XO indices between July 2004 and May 2006. It clearly shows that the standard deviation of spread changes is increasing with the median level of spread and the plot for CDX.IG indicates a fairly smooth and linear relationship. While the volatility of contracts comprising the CDX.XO is not as well behaved and observations are more scattered, the same general pattern is still evident. This is consistent with Ben Dor *et al*. (2005a), which, using US high yield bonds rated Ba and B, found the linear relation extends well into spreads of 400bp.

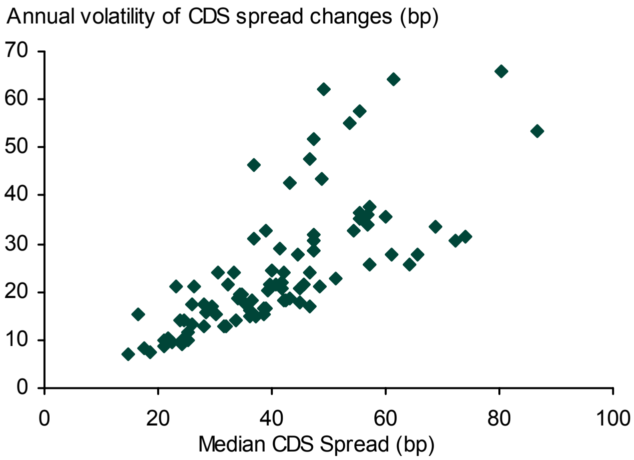
The results for European names included in the ITRAXX.IG and ITRAXX.XO are very similar (Panel B). Investment grade names are clustered along a line passing through the origin, while contracts trading at higher spreads exhibit more dispersion due to a larger idiosyncratic risk component.[11] Another phenomenon illustrated in Figure 2 is that for very low spreads, the decline in volatility seems to decelerate and converge to some "lower bound." A similar effect has been documented in Ben Dor *et al*. (2005b) for agency spreads.

[10] *The existence of these two index universes reflects the dual requirements of managers to know the return of the index if bought at the beginning of the month as well as to know the shape and structure of the up-and-coming index that will be the returns universe at the beginning of the next month*

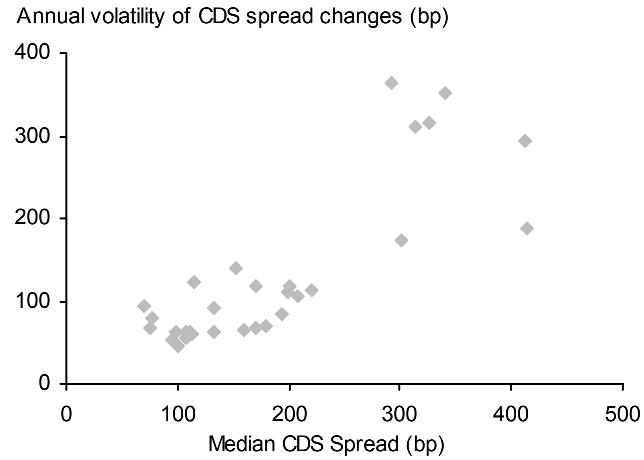
[11] *The one outlier is BAA plc, which had a low median spread of 28bp and a relatively high volatility of 41bp per year. The spreads on BAA plc widened almost 60bp after a hostile takeover bid from Ferrovia and subsequently recovered when the company agreed to include a new clause into issued debt that allowed bondholders to sell bonds at their face value in case of rating downgrade resulting from merger or acquisition.*

Figure 2. Volatility of spread changes versus median spread levels for constituents of various CDS indices

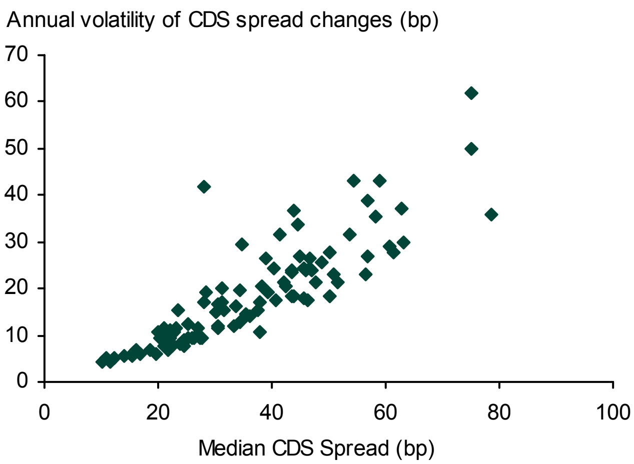
Panel A: CDX.IG Constituents



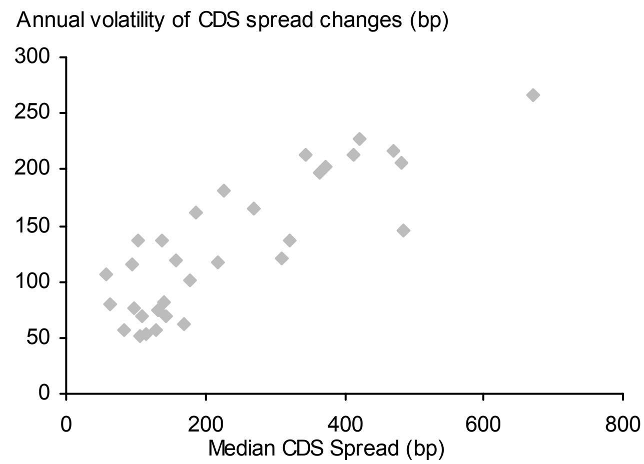
Panel A: CDX.XO Constituents



Panel B: ITRAXX.IG Constituents



Panel B: ITRAXX.XO Constituents



Note: Volatility of CDS spread changes versus median spread level; weekly observations from July 1, 2004 to June 1, 2006.
Source: Lehman Brothers.

3.2.1 Systematic volatility

To analyze the time series of systematic changes in CDS spreads we first compute the average spread change in each sector of the CDX.IG and ITRAXX.IG indices with at least eight contracts.[12] We then estimate the parameters in equation (1) separately for each sector using (quasi) maximum likelihood. For CDX.XO and ITRAXX.XO we calculate a single weekly aggregate figure since the idiosyncratic component of spread change for crossover names is relatively high therefore requiring a high degree of diversification in order to isolate systematic movements in spread.

Figure 3 reports the parameter estimates for α and β, and their associated *t*-statistics. For investment grade CDS, the estimates for the linear spread term (β) are significant across all sectors except Consumer Cyclical and Materials for CDX.IG, and Consumer Stable for ITRAXX.IG. Comparing estimates for CDX sectors with those of equivalent sectors in ITRAXX shows them to be fairly similar. In addition, most of the coefficients lie at 0.04-0.06, which is in line with the 0.09-0.10 estimated using corporate bonds data over a monthly frequency.[13] The estimates of α are always insignificant, also lending support to the previous results which found that the relation between spread volatility and spread level is best described by a linear function which intersects the origin (i.e., α is equal to zero).

Figure 3. QML estimation of the conditional relation between systematic spread volatility and spread level

Panel A:

CDX.IG	**NO CDS**	**NO OBS**	**a**	**t-stat**	**b**	**t-stat**	**c**	**t-stat**
Index*	92	161	3.7E-05	0.8	3.4%	2.9	9.5	1.73
Communications	8	161	1.5E-05	0.2	5.1%	4.5	9.0	2.28
Consumer Cyclical	17	161	6.5E-05	0.7	3.1%	1.4	31.0	2.38
Consumer Stable	11	161	4.2E-05	0.7	3.6%	2.5	18.1	2.18
Financial	19	161	-1.7E-06	0.0	6.1%	2.7	-0.2	-0.01
Industrial	14	161	-2.3E-07	0.0	4.3%	4.0	-1.4	-0.31
Materials	9	161	1.1E-04	0.9	3.2%	1.2	35.0	1.54
Utilities	8	161	-4.8E-05	-0.7	5.8%	3.8	4.8	0.65
CDX.XO								
Index	30	103	-1.2E-04	-0.3	6.6%	2.6	6.7	1.1

Panel B:

ITRAXX.IG	**NO CDS**	**NO OBS**	**a**	**t-stat**	**b**	**t-stat**	**c**	**t-stat**
Index**	99	149	1.1E-05	0.1	4.1%	2.0	22.5	1.0
Communications	14	149	1.0E-05	0.1	5.1%	2.1	26.9	1.6
Consumer Cyclical	13	149	-1.2E-04	-0.7	7.9%	2.3	36.1	1.3
Consumer Stable	14	149	1.3E-05	0.1	4.0%	1.5	34.9	1.1
Financial	21	149	-1.3E-05	-0.5	5.3%	4.1	17.7	1.0
Industrial	9	149	1.9E-05	0.2	4.3%	2.1	12.9	0.7
Utilities	14	149	2.7E-05	0.8	3.2%	3.0	10.1	1.4
ITRAXX.XO								
Index	32	103	3.4E-04	0.4	3.6%	0.9	19.6	2.6

** Excluding the energy sector. ** Excluding the Materials sector and unclassified contracts.*
Source: Lehman Brothers.

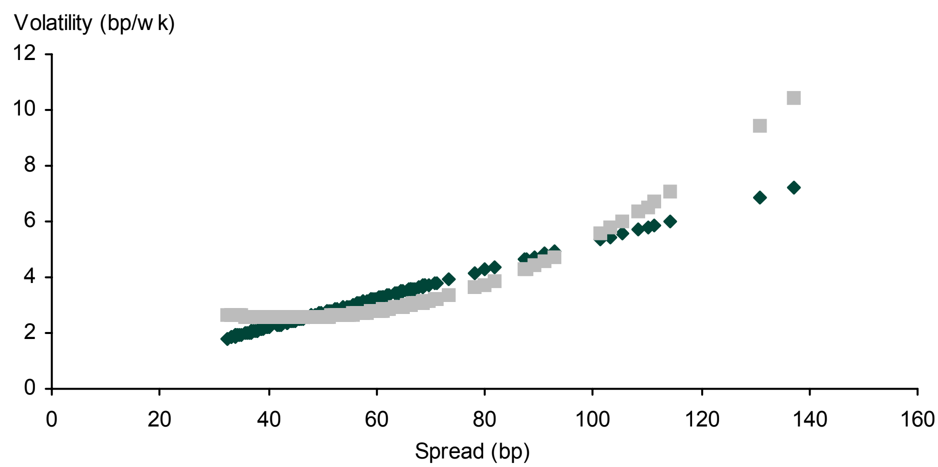
[12] *We use Markit Partners' sector classification for convenience.*
[13] *Volatilities based on weekly observations can be converted to a monthly frequency (assuming spread changes are distributed independently over time) by multiplying them by* $\sqrt{4}$.

The last two columns in the table show estimates of γ coefficient and the corresponding t-statistics which measure potential non-linear aspects of the relationship between spread level and volatility[14]. Looking at the figures reveals that such effects are evident in several sectors of the CDX.IG (Communications, Consumer Cyclical, and Consumer Stable) as well as in the aggregate CDX index at a 10% significance level. The positive values of the t-statistics imply that conditional volatility becomes less sensitive to changes in spread at low spread levels. This is consistent with Ben Dor *et al*. (2005b) who find that spread volatility is roughly constant for spreads below 20bp.[15]

Regarding crossover names, β is found to be significant for CDX.XO at a 5% level. The magnitude of the estimate (converted to a monthly frequency - 0.132) is generally higher than those for investment grade names and is consistent with previous results for high yield bonds.[16] In contrast, β is not significant for ITRAXX.XO but γ is significant at the 5% level.

The effect of the non-linear term on systematic volatility can be substantial. As an illustration, we consider predicted volatility of the CDX.IG Communications sector where both coefficients β and γ are significant. Figure 4 shows predicted volatilities with and without the non-linear term. As can be seen, the difference between predicted volatilities, especially for high spread levels, can be substantial.

Figure 4. Predicted spread volatility of CDX.IG Communications—linear versus non-linear specification



Source: Lehman Brothers.

[14] *See the Appendix for details of the exact estimation procedure.*

[15] *Ben Dor* et al. *(2005b) use Agency bond data and find that the linear relation between spread volatility and spread level holds for spreads above 20bp. For spreads below 20bp, spread volatility is roughly constant; the levels of systematic and idiosyncratic "structural" volatility are about 2.5-3.0bp and 4.5bp per month, respectively.*

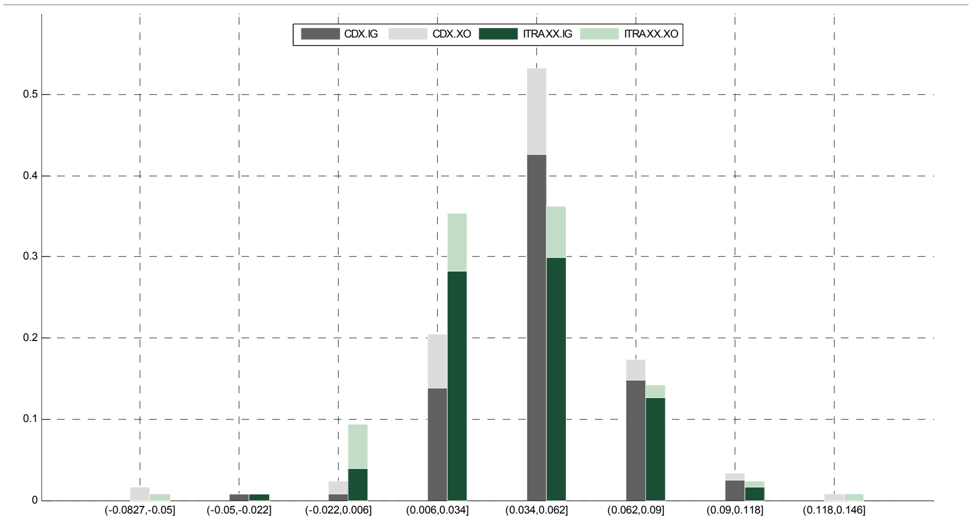
[16] *Ben Dor* et al. *(2005a) find that for HY bonds the proportionality factor is roughly 11.5%.*

3.2.2 Idiosyncratic volatility

We also analyze the idiosyncratic spread volatility of constituents of the four CDS indices. Idiosyncratic spread changes are defined by subtracting the average sector or index spread change from that of the individual bond. As before, we specify conditional volatility of idiosyncratic spread changes as a simple function of spread level (equation 1) and use maximum likelihood to estimate the parameters.

Figure 5 shows the distribution of the β estimates separately for constituents of CDX and ITRAXX. The two distributions are fairly similar with the majority of estimates falling at 0.034-0.064. Only a few estimates (three and ten for CDX and ITRAXX, respectively) have negative values.

Figure 5. Distributions of linear coefficients for idiosyncratic spread changes across constituents of CDX IG/XO and ITRAXX IG/XO



Source: Lehman Brothers.

Figure 6 reports the results of a "pooled estimation": the time-series of idiosyncratic spread changes of individual contracts are combined within a sector (for investment grade names) or the entire index (for crossover names) and the parameters in equation (1) are estimated as before. The result is a single set of estimates of the parameters that are representative of the relation between idiosyncratic spread volatility and spread level in each sector (or aggregate index for crossover contracts).

Overall, the results in Figure 6 lend further support to the linear specification between spread volatility and spread level we test. The estimates for the linear term are always significant at the 1% significance level. In addition, the magnitudes of the estimates are quite similar and consistent with the estimates in Ben Dor *et al*. (2005a).

The figures in the last column indicate that a non-linear effect is detected in a single sector (Utilities of ITRAXX.IG), similar to the case of systematic spread volatility (Figure 3).

Figure 6. QML estimation of idiosyncratic conditional volatility of spread changes across CDX.IG constituents

Panel A: Idiosyncratic Spread Changes in CDX

CDX.IG	NO CDS	NO OBS	a	t-stat	b	t-stat	c	t-stat
Index*	92	14,652	3.1E-05	2.0	5.1%	16.8	0.9	1.1
Communications	8	1,288	9.4E-05	2.4	3.8%	6.9	2.3	1.6
Consumer Cyclical	17	2,737	3.6E-05	1.6	4.3%	7.1	0.7	0.4
Consumer Stable	11	1,771	-3.6E-06	-0.2	6.2%	11.9	0.6	0.5
Financial	6	3,059	5.4E-05	1.7	5.1%	8.8	1.2	0.9
Industrial	19	2,254	4.5E-05	2.7	3.5%	8.2	0.6	0.7
Materials	14	1,449	6.6E-05	2.5	3.4%	5.6	0.8	0.4
Utilities	8	1,288	2.9E-05	0.6	4.0%	4.1	1.7	0.8
CDX.XO								
Index	30	3,058	3.6E-04	4.0	4.8%	10.9	0.3	1.019

Panel B: Idiosyncratic Spread Changes in ITRAXX

ITRAXX.IG	NO CDS	NO OBS	a	t-stat	b	t-stat	c	t-stat
Index**	99	14,759	2.6E-06	0.2	4.7%	17.5	0.5	0.5
Communications	14	2,128	-8.9E-06	-0.4	4.4%	10.0	1.4	1.2
Consumer Cyclical	13	1,976	-5.3E-05	-1.9	5.9%	9.8	0.0	0.0
Consumer Stable	14	2,128	-1.0E-05	-0.6	5.2%	11.1	1.1	0.8
Financial	21	3,192	9.9E-06	1.0	3.6%	9.0	2.5	1.3
Industrial	9	1,368	4.8E-05	1.3	3.4%	6.5	3.7	0.6
Utilities	14	2,128	1.1E-05	1.0	2.9%	8.2	2.4	2.6
ITRAXX.XO								
Index	32	3,230	7.7E-04	5.7	2.4%	5.4	0.0	0.2

** Excluding the energy sector. ** Excluding the Materials sector and unclassified contracts.*
Source: Lehman Brothers.

3.3 Spread volatility of corporate bonds

We complete our study with an analysis of systematic spread volatility using monthly bond data. Unlike earlier studies that relied on individual bond data, our analysis is based on (aggregated) spread changes of several Lehman Brothers indices (US and Euro Corporate Indices and US High Yield Index). We report results for three major sectors (Financials, Industrials, and Utilities) as well as for various underlying sub-sectors. To ensure proper measurement of systematic spread changes, we examine only sectors with at least ten issuers for investment grade bonds and 20 issuers for high yield bonds (monthly).

Figure 7 presents the results for the US Corporate Index. The table reports four main statistics: estimates of beta, gamma, and the associated *t*-statistics. Many investors believe that credit markets changed fundamentally in 1998 following the "Russian Crisis" and the downfall of LTCM. To ensure that our results are not an artifact of a specific time-period, we report results for the entire sample period and separately for two sub-periods: October 1990–June 1998 and October 1998–January 2006.[17]

The estimates of β for the broader Financials (12.7%) and Industrials (12.5%) sectors are in agreement with our earlier results of 10% despite the different statistical techniques and sample periods. The estimates are also significant for all sub-sectors. Comparing the results for the two sub-periods reveals that the estimates are often larger in the second period consistent with a permanent increase in spread volatility. This is in contrast with the findings of Ben Dor *et al*. (2005b) that the proportionality factor is fairly stable over time.[18]

[17] *The results for the entire sample reflect the effect of July–September 1998, which are excluded from the sub-periods.*
[18] *Unlike in our study, Ben Dor* et al. *(2005b) examine the stability of the proportionality factor year-over-year without controlling for sector classification.*

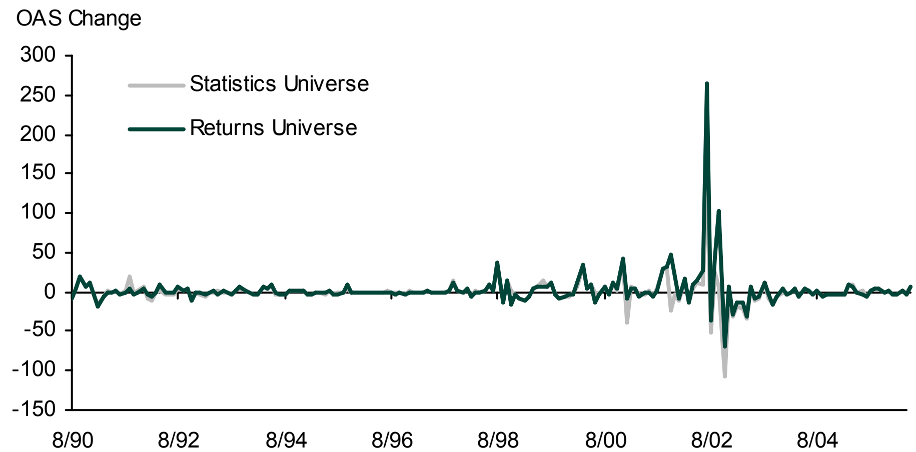
Figure 7. Estimation of systematic conditional volatility of spread changes for the US Corporate Index

	Avg. No. Bonds	October 1990 - June 2006				October 1990- June 1998				October 1998 - June 2006			
		b	t-stat	c	t-stat	b	t-stat	c	t-stat	b	t-stat	c	t-stat
Financials	886	**12.7%**	**5.4**	1.6	0.3	**9.5%**	**4.1**	6.0	1.2	**13.9%**	**5.6**	2.5	0.5
Banking	359	**12.6%**	**4.7**	1.8	0.5	**11.8%**	**4.9**	3.0	0.7	**11.8%**	**3.4**	5.5	0.6
Brokerage	105	**15.2%**	**2.6**	2.7	0.5	**16.1%**	**3.4**	1.4	0.2	**12.6%**	**3.6**	5.4	0.6
Finance Comp'	277	**16.8%**	**7.5**	0.4	0.2	**11.5%**	**4.6**	7.6	0.9	**19.3%**	**7.0**	0.4	0.2
Insurance	54	**16.2%**	**3.4**	-0.2	0.0	NA	NA	NA	NA	**16.2%**	**3.4**	-0.2	0.0
REITS	41	**5.4%**	**3.0**	2.3	0.7	NA	NA	NA	NA	**5.4%**	**3.0**	2.3	0.7
Industrials	1166	**12.5%**	**5.2**	0.2	0.0	**10.3%**	**2.7**	9.2	1.0	**11.8%**	**3.7**	2.5	0.4
Basic Industrials	144	**8.8%**	**5.8**	-1.5	-0.3	**8.0%**	**2.8**	4.8	0.6	**5.9%**	**2.2**	7.8	1.6
Captial Goods	146	**11.5%**	**5.8**	1.5	0.3	**11.3%**	**4.4**	4.8	0.9	**9.9%**	**3.0**	4.6	0.6
Communications	124	**18.8%**	**4.7**	1.0	0.3	NA	NA	NA	NA	**18.8%**	**4.7**	1.0	0.3
Consumer Cyclicals	216	**20.1%**	**5.4**	0.9	0.2	**15.5%**	**3.4**	5.0	0.9	17.2%	1.8	16.3	0.9
Energy	137	**10.9%**	**2.1**	-3.3	-0.3	**13.5%**	**2.2**	7.3	0.4	6.8%	1.7	1.4	0.2
Consumer NonCyc	228	**10.5%**	**2.7**	-1.5	-0.2	**11.4%**	**2.0**	17.4	0.5	**8.3%**	**2.3**	1.4	0.2
Technology	49	**17.2%**	**3.0**	0.2	0.0	**16.8%**	**3.6**	7.9	1.0	**17.8%**	**3.4**	0.1	0.0
Transportion	99	**10.2%**	**4.2**	-0.3	-0.1	**8.8%**	**3.5**	3.3	0.4	**15.5%**	**3.8**	-2.3	-0.7
Utilities	359	**13.6%**	**3.7**	1.0	0.3	**8.3%**	**2.3**	9.9	0.5	**16.3%**	**3.2**	1.1	0.4
Electric	229	**15.4%**	**2.9**	0.9	0.2	**11.8%**	**2.9**	7.4	0.2	**17.8%**	**2.4**	0.7	0.2
Natural Gas	80	**13.5%**	**5.5**	1.8	0.7	**5.4%**	**3.4**	1.2	0.2	**17.3%**	**4.9**	2.8	0.9

Source: Lehman Brothers.

The reported results are for the statistics universe with monthly re-balancing of the index. As explained in section 3.1, we focus on the statistics universe because it excludes, to a large extent, extreme spread changes that result from defaults or downgrades of the underlying issuers. Such bonds are excluded from the universe in the beginning of the month following the event. The degree to which the results are affected by extreme issuer-specific events can be seen if we perform a similar analysis in the returns universe. Consider, for example, the Electrical sub-sector of the US IG bonds. Enron and NRG (an Illinois power company) are both constituent issuers in this sub-sector. In the case of Enron, the company's bonds experienced extreme spread widening over the course of a few days in November 2001, trading at spreads in excess of 1000bp and were subsequently excluded from the index at the end of that month. Similarly, bonds issued by NRG suffered huge losses during July 2002 and left the index at the next month-end. These two companies have a large effect on returns of the Electrical sector in the returns universe. Figure 8 plots the time-series of spread changes of the two universes for the Electrical sector. It illustrates clearly that issuer-specific events are less pronounced in spread changes of the returns universe. If we re-estimate the parameters for the Electrical sector using the returns universe rather than the statistics universe, as in Figure 7, the estimate of β rises substantially from 15.4% to 19.3% and is less in line with estimates for the Financials and Industrials sectors.

Figure 8. Comparison of spread changes of the US IG Electric sector for the returns and statistics universes



Source: Lehman Brothers.

Figure 9. Estimation of systematic conditional volatility of spread changes for the US High Yield and Euro Corporate Indices

Panel A: USD-HY

	Avg. Number of Bonds	June 2000 - June 2006			
		b	t-stat	c	t-stat
Financials	27	**14.1%**	**2.1**	-2.5	-0.9
Industrials	418	**10.3%**	**3.2**	0.8	0.5
Capital Goods	41	**16.6%**	**2.7**	-0.2	-0.1
Communications	54	**18.3%**	**2.8**	0.1	0.1
Consumer Cyclicals	126	**8.7%**	**2.1**	1.2	0.6
Consumer Non-Cyclical	64	**7.2%**	**2.2**	-0.3	-0.1

Panel B: EUR-IG

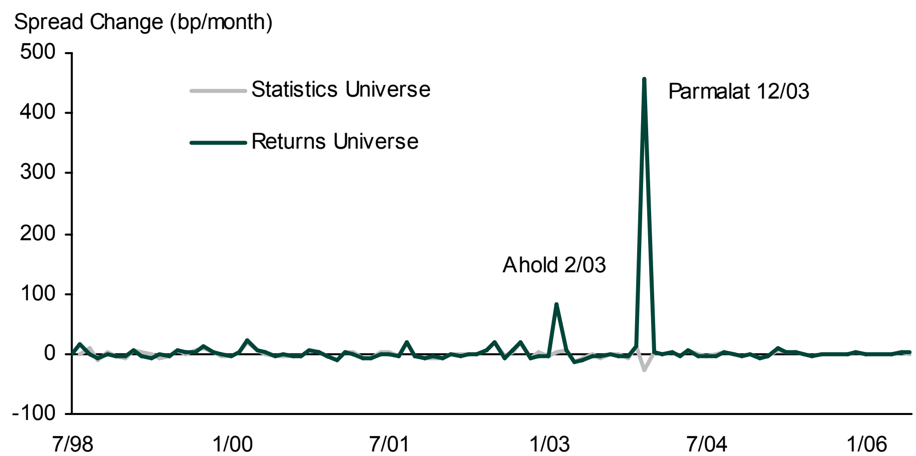
	Avg. Number of Bonds	June 2000 - June 2006			
		b	t-stat	c	t-stat
Financials	515	**12.8%**	**6.0**	5.4	0.5
Banking	385	**6.7%**	**2.7**	15.1	0.5
Finance Companies	56	**23.6%**	**5.4**	0.1	0.0
Insurance	34	14.4%	1.6	-5.5	-0.2
Industrials	353	11.4%	1.4	-2.3	-0.1
Basic Industrials	29	**14.2%**	**3.5**	4.3	0.4
Capital Goods	38	**20.0%**	**4.3**	1.0	0.1
Communications	91	**18.5%**	**2.1**	0.5	0.0
Consumer Cyclicals	74	25.1%	1.5	44.9	0.7
Consumer Non-Cyclical	62	10.2%	1.4	-3.1	-0.1
Utilities	78	**9.8%**	**2.2**	26.1	1.8
Electric	58	**6.2%**	**2.5**	**24.0**	**2.0**

Source: Lehman Brothers.

Comparing parameters estimated based on the returns and statistics universes for the US High Yield and Euro Corporate Indices further highlights the importance of accounting for issuer-specific events. For example, the result for the Communications sector of the US High Yield Index (Figure 9 – Panel A) supports the proportionality of spread changes with a significant t-statistic. However, when we re-estimate the coefficients using the returns universe, the t-statistic value drops to 0.5. Similarly, the Communications, Utilities, and Electric sectors of the Euro Corporate Index (Panel B) have t-statistics of 2.1, 2.2, and 2.5, respectively. Using the returns universe the estimated parameters have less significance with t-statistics of 1.8, 0.5, and 0.2, respectively.

Perhaps the most dramatic illustration is in the case of Consumer Non-Cyclical, where the estimate changes from 10.2% with a t-statistic of 1.4 to -98.5% (significant at the 5% confidence level with a t-statistic of -2.5). The reason for the “flip” in the coefficient estimate is illustrated in Figure 10: spread widening in the sector in February 2003 and December 2003, caused by the Ahold and Parmalat events respectively. In the statistics universe, however, both Ahold and Parmalat were dropped from the index after the blow-ups. As a result, there is no significant change in spreads.

Figure 10. Spread changes of Consumer Non-Cyclical Euro IG Index for the returns and the statistics index universes



Source: Lehman Brothers.

CONCLUSION

Previous studies regarding the relationship between spread volatility and spread level at a monthly level have focused on cash-bond spreads. These studies documented a strong linear relationship across a large spectrum of spreads from highly rated corporate bonds to high yield credit. We examine whether similar results hold for weekly variations in individual credit default swaps spreads using a maximum likelihood estimation techniques.

Our analysis indicates a strong relationship between spread level and spread volatility with high spread sectors and indices exhibiting correspondingly higher volatilities. Significance is high for the linear term in almost all cases both at the index and sector level in European and US. markets. This is generally not the case for second-order terms in the spread level, although instances of significant systematic second-order effects were in keeping with previous findings, namely that linearity of spread volatility in spread begins to break down as spreads drop to low levels. In addition to the significance of spread for spread volatility, the magnitude of the effect, as measured by the coefficients we find for the spread term in the expression for volatility, are broadly speaking in agreement with those found in previous studies. There, a slope of approximately 10% per month was found indicating an increase of one basis point of volatility for every 10bp in spread. Weekly estimates would be expected to be up to half the level of monthly figures and this is indeed the general level of the estimates provided by our analysis. We find these relationships to hold for both investment grade and crossover contracts. High yield CDS data are insufficient to make any statements for this asset class.

In addition to estimating coefficients for systematic spread movement, we also studied idiosyncratic volatilities. Here we also found results consistent with the hypothesis of a linear relationship with spread. Estimated levels also fitted previously evidenced figures for cash bonds.

Finally, we apply the maximum likelihood analysis to cash bond spreads at the index and sector levels for investment grade and high yield securities. Again, we are able to confirm previous findings in terms of both significance and magnitude of the relationship between spread volatility and initial spread level. It is also worth noting that the coefficients for high-yield bonds are comparable in magnitude to those in investment grade as was found to be the case in previous studies.

Our findings have several important implications: in the context of risk modeling, spread volatility should be expressed in terms of spread level, thereby enabling a dynamic updating of risk of securities on a real-time basis as their spreads move in the market. Explicitly incorporating spread-based factors would likely eliminate the need for using rating categories, allowing greater sector coverage as sector-quality based spread risk factors can be replaced by purely sector-based ones. Furthermore, there are reasons to believe that volatilities of “DTS-risk factors” may be more stable over time, with some of the volatility of basis-point changes being factored out in the spread level.

Finally, the results of our analysis indicate that, even for relatively short horizons, hedging of and measuring exposures to credit risk will be enhanced by the addition of exposures that explicitly take into account current spread levels as well as spread durations.

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APPENDIX

A. Quasi-maximum likelihood estimation

Maximum likelihood estimation begins with an assumption about the true distribution of the sample data. This probability density function is assumed to be known up to the value of certain parameters which need to be estimated from an available sample of observations. An optimal specification is sought for the distribution of those variables by searching for parameters that maximize an objective function – the likelihood. The MLE procedure chooses parameters in such a way that maximizes the likelihood of drawing the sample under consideration from the calibrated distribution. This likelihood optimization is carried out by means of maximization, with respect to the parameters, of the so-called (log-) likelihood function of the observed data under the parameterized probability density function.

Specifically, let $f(y \mid x,\theta)$ denote the probability density function of a random variable y conditional on a random variable x and a set of parameters $\vec{\theta}$. In its application to our study, y represents the spread change, x the level of spread and $\vec{\theta}$ the set of parameters defining the dependence of spread change volatility on spread level. Given a sample of independent observations $y_1 \dots y_n$ we can write their joint probability density function $L(\vec{\theta} \mid \vec{x}, \vec{y})$, in the following way:

$$L(\vec{\theta} \mid \vec{x}, \vec{y}) \equiv f(y_1 \dots y_n \mid x_1 \dots x_n, \vec{\theta}) = \prod_{i=1}^{n} f(y_i \mid x_i, \vec{\theta}). \tag{A1}$$

This joint density function, when defined as a function of the unknown parameter vector $\vec{\theta}$, is called the likelihood function, where $\vec{y}$ and $\vec{x}$ indicate the collection of observations in the sample. The logarithm of the likelihood function is called the log-likelihood function:

$$\ln L(\vec{\theta} \mid \vec{x}, \vec{y}) = \sum_{i=1}^{n} \ln f(y_i \mid x_i, \vec{\theta}). \tag{A2}$$

Parameter estimates $\hat{\theta}$ can be obtained by maximizing the log-likelihood function with respect to the parameter set:

$$\hat{\theta} = \arg\max_{\vec{\theta}} \left[\ln L(\vec{\theta} \mid \vec{x}, \vec{y})\right]. \tag{A3}$$

It can be shown that the estimates obtained by MLE procedure are asymptotically consistent and efficient. Namely, that as the number of sample observations increases the MLE parameter estimates converge to their true values and that the asymptotic variance of MLE estimates is the smallest possible in the class of consistent estimates.

One shortcoming of the MLE procedure is that it assumes a particular form for the probability density function of the sample observations. It is usually the case that this distribution is not known *a priori*. Nevertheless, even assuming normal probability distribution for the data will, under certain conditions, lead to consistent results.[19, 20]

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[19] *The formal conditions under which consistency is assured are given, for example, in Huber (1967) or White (1982).*
[20] *The reason for this phenomenon is that estimators in (3) can be alternatively interpreted as extremum estimators with the resulting parameters converging in probability to some values, which can be interpreted as pseudo-true parameters. These parameters approximate the imposed model using the normal log-likelihood as a criteria function. These pseudo-true parameters often turn out to coincide with the true parameters of the model.*

B. Estimation methodology

In our analysis, we assume that the conditional volatility of spread change is a function of spread level, i.e.,

$$\sigma_t = \alpha + \beta s_{t-1} + \gamma \hat{P} s_{t-1}^2 , \qquad \text{(A4)}$$

in which $\hat{P} s_{t-1}^2$ is the in-sample projection of s^2 to the linear space orthogonal to $L(1, s)$. In terms of an OLS regression, $\hat{P} s_{t-1}^2$ is the residual term from regressing s_t^2 on s_t and a constant. The idea is that the estimate of the parameter γ would represent the potential non-linear relation between conditional variance of spread changes and the level of spread. The reason for using the orthogonal projection of s_t^2 rather than simply s_t^2 is that the latter is highly correlated with the level of spread. Introducing such a high level of multi-collinearity into the model would severely reduce the significance level of the estimates of $\hat{\alpha}$ and $\hat{\beta}$. As a result, the linear coefficient becomes non-informative as we cannot see whether the linear model *per se* explains the data well. To avoid this effect, we would like to split the contribution of the quadratic term into linear and non-linear components. Eventually, we would include only the non-linear component of the quadratic term into the specification leaving out the linear component. To achieve this we use the in-sample projection of the quadratic term into the orthogonal space. The result is that estimates $\hat{\alpha}$ and $\hat{\beta}$ do not depend on the inclusion of the non-linear term. This construct is purely artificial and, in fact, does not affect the conclusion regarding the quadratic term in the specification of the model. Whether we include the quadratic term or its orthogonal projection does not affect parameter estimate $\hat{\gamma}$ and its t-statistic.

Using the Gaussian conditional probability density function we can write the log-likelihood function in the form:

$$\ln L(\alpha, \beta, \gamma \mid \Delta\vec{s}, \vec{s}) = -\frac{T}{2}\ln(2\pi) - \sum_{t=1}^{T} \ln \sigma_t - \frac{1}{2}\sum_{t=1}^{T} \frac{\Delta s_t^2}{\sigma_t^2} . \qquad \text{(A5)}$$

The parameter estimates can be obtained by maximizing this likelihood function of the observed data with respect to parameters α, β, and γ.

We estimate the parameters in two steps hoping to achieve a better robustness. First, we estimate parameters of the linear specification only. Then, we fix the linear coefficients and re-run the estimation with the non-linear term included. This procedure is justified since inclusion of the orthogonal projection, as discussed above, does not change the previously estimated linear coefficients $\hat{\alpha}$ and $\hat{\beta}$.

Joint distributions of portfolio losses and exotic portfolio products

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The pricing of exotic portfolio products, e.g. path-dependent CDO tranches, relies on the joint probability distribution of portfolio losses at different time horizons. We discuss a range of methods to construct the joint distribution in a way that is consistent with vanilla CDO market prices. As an example, we show how our loss-linking methods provide estimates for the breakeven spreads of forward-starting CDO tranches.

PRICING EXOTIC PORTFOLIO PRODUCTS[1]

With the establishment of a liquid index tranche market, implied default correlation is increasingly perceived as a market observable. The current market standard for pricing vanilla CDOs is the one-factor Gaussian copula model in conjunction with base correlation. With some interpolation assumptions, the base correlation model can be used to back out portfolio loss distributions at different horizons (Figure 1). Using mapping procedures, we can also imply loss distributions for bespoke CDO tranches and CDO^{2}s.

However, as the credit correlation market evolves, more exotic structures are gaining in popularity. Among these, we find instruments where value depends on the distribution of incremental losses (e.g. forward-starting CDOs) and fully path-dependent structures such as reserve account CDOs. To price these, knowledge of the implied loss distributions to various horizons is insufficient. Instead, we need to know the joint distributions of losses at different maturities. Therefore, we must join the marginal loss distributions in a consistent and arbitrage-free way. The vanilla tranche market tells us nothing about how this should be done.

In this article we discuss the constraints we face when building a joint loss distribution from marginal loss distributions at different maturities and describe various methods to link loss distributions consistently. We also compare the different methods, in particular looking at the different breakeven spreads they give for forward-starting CDOs.

Modelling Framework

The basic modeling problem can be described as follows: Given discrete loss levels K_i and marginal loss distributions at time t_1 and t_2,

$$p_i^{(1)} := P[L(t_1) = K_i], \quad p_j^{(2)} := P[L(t_2) = K_j],$$

we want to construct a joint distribution

$$P_{ij} := P[L(t_1) = K_i \cap L(t_2) = K_j].$$

The matrix P_{ij} is constrained by the fact that the rows and columns must add up to the respective marginal distributions. Also, as the loss process is non-decreasing, P_{ij} is upper-triangular; we will refer to this as the monotonicity constraint:

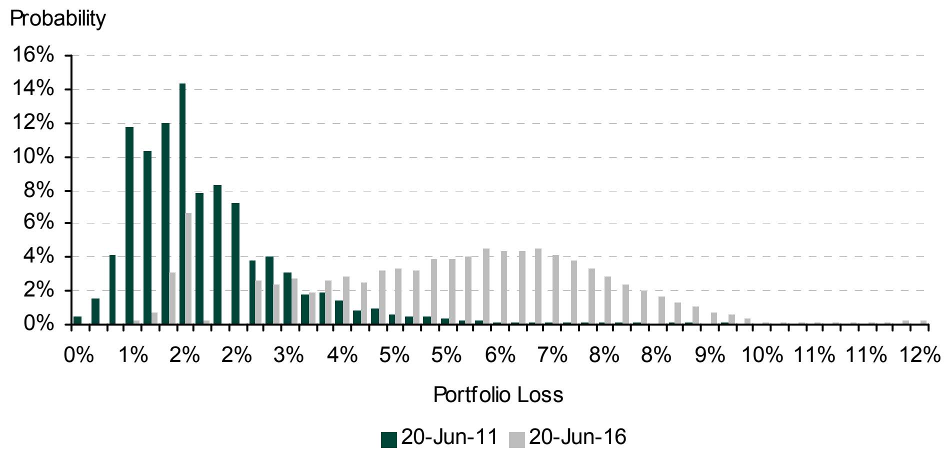
$$\begin{gathered} \sum_j P_{ij} = p_i^{(1)}, \quad \sum_i P_{ij} = p_j^{(2)} \\ P_{ij} = 0, \quad \forall\, i > j \end{gathered} \tag{1}$$

In addition to these constraints we must ensure that $0 \le P_{ij} \le 1$.

[1] *The authors would like to thank Matt Livesey for his collaboration on part of this work.*

If there is no overlap between the loss distributions at t_1 and t_2, i.e., if the maximum loss at t_1 is less than or equal to the minimum loss at t_2, monotonicity does not impose an additional constraint. In particular, it is possible to have independence between the losses at t_1 and t_2 in this case. However, with growing overlap between the two marginal distributions, the constraints from the monotonicity condition become increasingly severe. From a pricing perspective, severe constraints are attractive as they reduce the pricing uncertainty: observations of the marginal distributions allow us to learn more about the joint distribution than would otherwise be possible.

Figure 1. Marginal Loss Distributions Implied by Base Correlation



Applies to the particular reference portfolio used in this article, as of 30 May 2006.
Source: Lehman Brothers.

JOINING LOSS DISTRIBUTIONS ACROSS DIFFERENT MATURITIES

The constraints in equation (1) do not uniquely determine the joint loss distribution. In general, an infinite number of joint loss distributions are consistent with a given pair of marginal distributions. We can therefore follow a number of different approaches to link the marginals into a joint distribution. These approaches fall into two broad categories. On the one hand, we can attempt to calibrate a skew-consistent, arbitrage-free model of the portfolio loss to the vanilla CDO market and back out the implied joint distribution. This is currently a very active area of research, and many different models have been proposed, with varying degrees of success. At the end of this section we show results from a simple implementation of Schönbucher's forward loss rate model [1].

On the other hand, we can take a more model-independent view by using the marginals as inputs and imposing a dependence structure exogenously. Although the first approach clearly has more potential in terms of developing a unified framework for pricing portfolio exotics, the second allows us to focus on the effect of the copula joining the distributions across time. It will also prove useful as a way of obtaining reference points for breakeven spreads. We discuss several methods in this category next.

The Comonotonic Method

The comonotonic method is designed to introduce the maximum positive dependence between the losses at two time horizons consistent with a given set of marginal distributions. The method is developed from the well-known fact that, given a cumulative loss distribution at some time t,

$$F_t(K_i) := P\left[L(t) \le K_i\right],$$

we can sample from this distribution by generating a uniform random variate U and solving for the value of K_i such that

$$F_t(K_{i-1}) < U \le F_t(K_i)\ .$$

This approach can be extended to generate a path for the cumulative loss through time by using the same value of U to sample from the loss distributions at each time horizon. If the marginal loss distributions are arbitrage free, meaning that they satisfy

$$F_{t_1}(K_i) \ge F_{t_2}(K_i), \forall K_i, \forall t_1 < t_2\,,$$

the resulting loss path is non-decreasing. In the continuous case, this approach generates losses K_{t_1} and K_{t_2} given by

$$K_{t_1} = F_{t_1}^{-1}(U),\ K_{t_2} = F_{t_2}^{-1}(U) = F_{t_2}^{-1}\left(F_{t_1}\left(K_{t_1}\right)\right).$$

Hence, the loss at time t_2 is uniquely determined by the loss at t_1 and this method generates the maximum positive dependence between the two losses. In the discrete case, conditioning on a particular loss at t_1 is equivalent to conditioning on a range of values for U and there is a corresponding range of values for the loss at t_2. Integrating over U we obtain the distribution for the loss at t_2 conditional on the loss at t_1.

While the comonotonic method is easy to implement, it implies rather unrealistic dynamics for the underlying loss process as the loss at any given time completely determines the loss at all future maturities up to discretisation effects. However, since the comonotonic method displays the maximum positive dependence, it at least represents a useful benchmark for pricing path-dependent exotics.

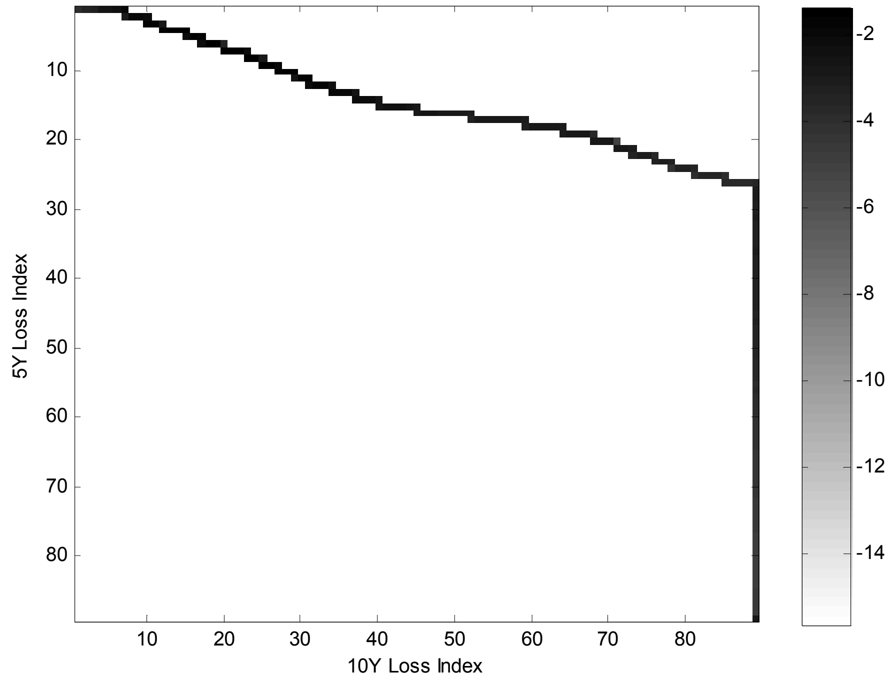
To visualize joint distributions, we use so-called heat maps, for which the joint distribution is plotted in a two-dimensional chart with different shades of grey representing the magnitude of the probability at different points in the two-dimensional loss grid. We use a base-10 logarithmic scale with dark shades indicating a high probability. The losses at t_1 and t_2 are represented along the vertical and horizontal axes, respectively.

A heat map for the comontonic method is shown in Figure 2. The marginal distributions used in the construction of the joint distribution apply to a generic bespoke portfolio[2] and market data as of 30 May 2006.

With the comonotonic method, the joint distribution takes a very simple form as the entire probability mass is concentrated along a curve in the upper right half of the loss grid. This reflects the fact that the loss at t_2 is almost always determined by the loss at t_1.

[2] *The reference portfolio used throughout this article is based on a pool of 160 equally weighted European names with an average spread of 63 basis points and a standard deviation of 34 basis points for a maturity of 10 years.*

Figure 2. Joint Loss Distribution for the Comonotonic Method



Losses at 5 years and 10 years are represented along the vertical and horizontal axes respectively. Dark shades represent high probability.
Source: Lehman Brothers.

Independent Increments

In a first attempt to build a more realistic joint loss distribution, we try to introduce independent increments, i.e., we assume that the incremental loss $\Delta L(t_1,t_2) := L(t_2) - L(t_1)$ between t_1 and t_2 is independent of the loss at t_1. From a modelling perspective, this is particularly attractive as it means that, observing losses in the near future, we do not learn anything about losses occurring later. However, as we will see, the constraints (1) imposed on the joint distribution along with the requirement $0 \le P_{ij} \le 1$ usually rule out independent increments.

It is clear that independent increments are not always possible because they are usually inconsistent with the existence of a maximum loss in the portfolio. As a simple example, consider the case in which there is a non-zero probability of obtaining the maximum loss at t_1. The incremental loss distribution conditional on having the maximum loss at t_1 must be delta-distributed at zero loss. But if the loss increment is independent of the initial loss, then this same distribution applies to all losses at time t_1 and we conclude that the two marginal distributions are identical, which of course is not generally the case.

Nonetheless, there are situations where an independent loss increment is approximately possible. It is instructive to consider this approach to see how it fails. To construct a joint distribution using independent increments, we have to find probabilities

$$q_n := P\left[\Delta L(t_1, t_2) = K_n\right], \qquad (2)$$

such that we recover the loss distribution at t_2 by carrying out the convolution of the loss distribution at t_1 with the incremental loss distribution given by the values of q_n:

$$p_n^{(2)} = \sum_k p_k^{(1)} q_{n-k} \ .$$

Based on this relation, we can easily determine the values q_n via a bootstrapping procedure. However, it is important to note that the numbers obtained in this way are neither guaranteed to take values in the interval [0,1] nor do they necessarily sum to unity. Hence, the bootstrap can easily lead to inconsistent results. We found that for short initial maturities t_1, the independent increment method works fairly well. For longer initial maturities, the bootstrap procedure becomes unstable and typically leads to values of q_n outside the allowed range of [0,1] after a few steps. This can be explained by the fact that the bootstrapping equation can be written in the form

$$\left(q_n - q_{n-1}\right) = \frac{1}{p_0^{(1)}}\left(\left(p_n^{(2)} - p_{n-1}^{(2)}\right) - \sum_{k=0}^{n-2} q_k \left(p_{n-k}^{(1)} - p_{n-k-1}^{(1)}\right) - q_{n-1} p_1^{(1)}\right),$$

which means that any increments in the marginals are magnified by a factor $(p_0^{(1)})^{-1}$ For long initial maturities, the probability of a zero loss approaches zero, rendering the bootstrap unstable.

The Maximum Entropy Method

The failure of independent increments to provide a reliable method to join marginal distributions can be stated as follows: Given a pair of marginal loss distributions, the non-decreasing property of the loss process may require a dependence structure that allows us to gain some information about the incremental loss $\Delta L(t_1, t_2)$ by observing the initial loss $L(t_1)$. As we cannot eliminate this information gain from our model in every case, we will try to minimize it instead. This leads to the maximum entropy method, which is guaranteed to produce a valid joint distribution from any pair of marginal distributions. Entropy is a concept from information theory and statistical physics that is used as a measure of disorder. For a bivariate distribution with probabilities P_{ij}, it is defined as

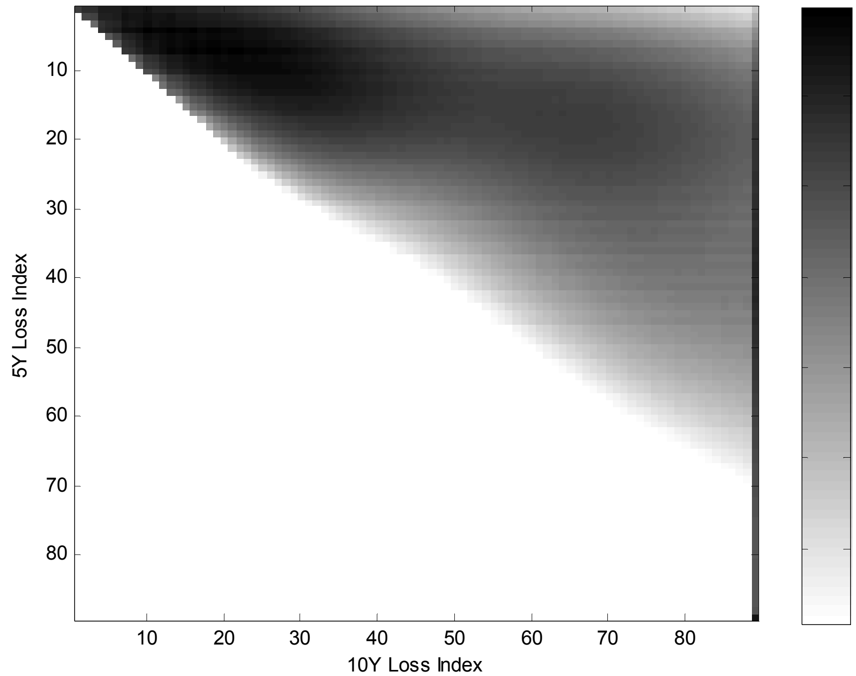
$$H = -\sum_{ij} P_{ij} \ln P_{ij} ,$$

where the sum is over all states with non-zero probability. Equivalently, we may include zero-probability states if we employ the limit $\lim_{p \to 0} p \ln(p) = 0.$

Maximizing the entropy H, minimizes the amount of information we inject into the joint distribution over and above that which is already contained in the marginal distributions and the no-arbitrage requirement that losses cannot decrease over time. One can prove that in the case of zero overlap between losses at t_1 and t_2, independence gives the joint distribution with maximum entropy. In the case of non-zero overlap, we construct the joint distribution via a numerical optimization, solving for the P_{ij} subject to the constraints (1) and using the entropy of the joint distribution as the objective function.

The resulting joint distribution is much more dispersed than for the comonotonic method, and looking at the heat map one can immediately see that there is relatively little dependence between losses at t_1 and t_2. For example, there is a significant probability of having a small initial loss at t_1 and yet a large loss at t_2.

Figure 3. Joint Loss Distribution for the Maximum Entropy Method



Losses at 5 years and 10 years are represented along the vertical and horizontal axes respectively. Dark shades represent high probability.
Source: Lehman Brothers.

The Markov Forward Rate Model

Another approach to linking losses through time is the Markov forward loss transition rate model described by Schonbucher [1]. The model is built on the concept of a forward loss transition rate $a_{ij}(t)$ which gives the instantaneous rate at which losses flow from K_i to K_j. The no-arbitrage condition gives

$$a_{ij} = 0, \forall i > j\,; \quad a_{ij} \geq 0, \forall i < j\,.$$

The so-called generator matrix A with elements a_{ij} is therefore upper triangular, with only positive elements above the diagonal. The diagonal elements a_{ii} describe the rate at which losses flow out of the level K_i and we also have the consistency condition (coming from the conservation of probability) that

$$a_i := -a_{ii} = \sum_{j \neq i} a_{ij}\,.$$

Thus the diagonal elements of the generator matrix are negative.

We present results from a simplified version of the full model in which we restrict ourselves to one-step transitions so that a_{ij} is only non-zero for j = i, and j = i + 1. The generator matrix A is therefore bi-diagonal and has the form

$$A = \begin{bmatrix} -a_0 & a_0 & 0 & \cdots & 0 \\ 0 & -a_1 & a_1 & \cdots & 0 \\ 0 & 0 & -a_2 & \ddots & 0 \\ \vdots & \vdots & \vdots & \ddots & a_{n-1} \\ 0 & 0 & 0 & 0 & -a_n \end{bmatrix},$$

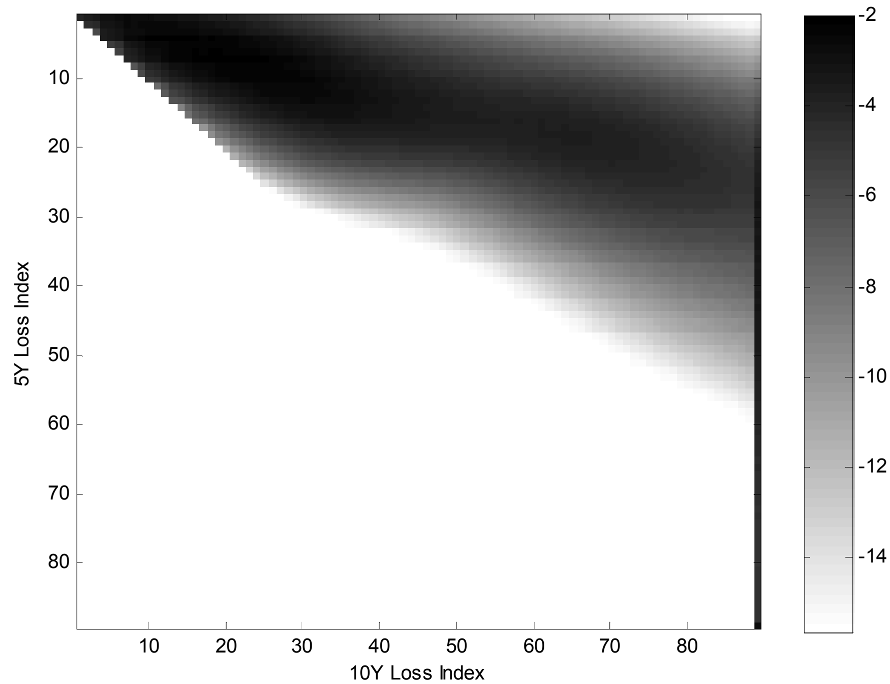
where the last diagonal element, a_n, is set to zero because of the conservation of probability.

In this approach the discrete portfolio loss distribution obeys the differential equation

$$P_i(t) := P(L_t = K_i), \quad \frac{dP_i(t)}{dt} = -a_i P_i(t) + a_{i-1} P_{i-1}(t).$$

If we assume that the transition rates a_i are piecewise constant in time, the solution of this equation is straightforward and we can easily obtain the loss distribution at time t_2 conditional on losses at an earlier time t_1.

Figure 4. Joint Loss Distribution for the Markov Forward Rate Method



Losses at 5 years and 10 years are represented along the vertical and horizontal axes respectively. Dark shades represent high probability.
Source: Lehman Brothers.

The piecewise constant generator matrices A are obtained by calibration to the loss distributions implied from the vanilla tranche market. The advantage of the bi-diagonal structure of the generator is that a single transition rate controls the loss flowing off each discrete loss level. This allows us to obtain a unique solution that matches the marginal distributions. Once we have the generator matrices, we can obtain the distribution of the loss at time t_2 conditional on the loss at t_1 by setting the loss distribution at t_1 equal to the delta-distribution at the loss in question and evolving this to time t_2. Repeating this procedure for all possible losses at t_1 gives us the full conditional distribution of the losses at the two time horizons and multiplying by the known marginal distribution at t_1 we obtain the joint distribution.

COMPARISON OF THE DIFFERENT METHODS

Given the range of loss-linking methods at our disposal, we would like to describe the dependence introduced by the different methods in a way that is suitable for comparison.

A number of different measures of dependence will be used in the analysis. The data we present in this section are based on a bespoke portfolio, with market inputs as of 30 May 2006. In terms of maturities, we are concerned with the period between 20-Jun-11 (referred to as 5y) and 20-Jun-16 (10y).

Correlation Between Future Losses

First, the correlation between losses at t_1 and t_2 provides a broad and intuitively accessible measure of dependence. We have calculated both linear correlation and Spearman's rank correlation for losses at 5y and the loss increment between 5y and subsequent maturities, using

$$q_n = \sum_{k=0}^{N-n} P_{k,k+n}$$

to convert the joint distribution to the distribution of loss increments (cf. equation (2) for the definition of q_n), where K_N is the maximum portfolio loss.

Of the three methods we tested, maximum entropy gives the lowest correlation, although the value, at 38.6% for a final maturity at 10y, is still significantly above zero. This is a reflection of the fact that the Independent Increments method is not possible for the 5y to 10y period. If it were possible to have independent increments, the corresponding joint distribution would coincide with the solution obtained from maximum entropy and it would obviously have a correlation of 0%.

Figure 5. Correlation of Loss at 20 June 2011 and the Loss Increment to Later Dates

	Linear Correlation			Spearman's Rank Correlation		
	20-Jun-12	20-Jun-14	20-Jun-16	20-Jun-12	20-Jun-14	20-Jun-16
Comonotonic	74.3%	71.0%	67.9%	84.4%	95.9%	97.6%
Markov	55.0%	57.0%	52.0%	32.8%	45.9%	55.4%
Max Entropy	40.1%	45.9%	40.9%	18.7%	29.5%	38.6%

Source: Lehman Brothers.

The comonotonic method results in a very high positive correlation as expected from the way it is constructed. The Markov forward rate model gives correlations in between the other two methods but is slightly closer to maximum entropy.

As a function of the second maturity, rank correlation is increasing for all three methods. However, there is no clear trend in terms of linear correlation. Since the loss distributions are highly non-Gaussian, rank correlation is the preferred correlation measure because it is purely a function of the copula joining the losses and is independent of the marginals.

Entropy of the Joint Distribution

In terms of the entropy, the order of the three methods is reversed. Obviously, maximum entropy gives the highest entropy. Of the other two methods, the Markov forward rates model has higher entropy than the comonotonic method. The latter has a low entropy because it is highly structured in the sense that there is very little 'randomness' in the joint distribution: knowing the loss at t_1 determines the loss at t_2 and vice versa, up to discretisation effects. While the Markov forward rate model again falls in between the other two methods, it is much closer to maximum entropy according to this measure. In Figure 6 we list the entropy for all three methods. As a reference, the entropy of the marginal distributions is also included.

Figure 6. Entropy of Joint Distributions for the Loss at 20-Jun-11 and Later

Date	Marginals	Comonotonic	Markov	Max Entropy
20-Jun-11	2.848	2.848	2.848	2.848
20-Jun-12	3.084	3.510	4.799	4.971
20-Jun-13	3.241	3.657	5.210	5.406
20-Jun-14	3.418	3.759	5.521	5.721
20-Jun-15	3.598	3.919	5.767	5.967
20-Jun-16	3.739	3.965	5.950	6.148

Source: Lehman Brothers.

For the three loss-linking methods we discuss in this article, correlation and entropy form an inverse relation. However, this is not always the case. Entropy should therefore not be seen as an inverse proxy for correlation. It is a measure of disorder and accordingly it is possible to construct highly structured joint distributions which have low correlation as well as low entropy. Note also that distributions with negative correlation exist, at least for some marginal distributions, whereas entropy is always positive.

As a function of the second maturity, the entropy is strictly increasing for all three methods. This is because the marginal distribution at 5 years contains progressively less information about the loss distribution at the later date.

Loss Increment Distributions

It is instructive to look at the distribution of loss increments more closely. The expected loss increment is model-independent as it depends only on the marginal distributions at t_1 and t_2,

$$E[\Delta L(t_1,t_2)] = E[L(t_2)] - E[L(t_1)] .$$

In our case, the expected loss increment for the 5y to 10y period is 4.15% of the portfolio notional. While the expected loss is not model-dependent, the standard deviation of the loss increment distribution does depend on how we link the losses. The distribution is widest for maximum entropy and narrowest for the comonotonic method. This has implications for pricing forward-starting CDOs, as we will see in the next section.

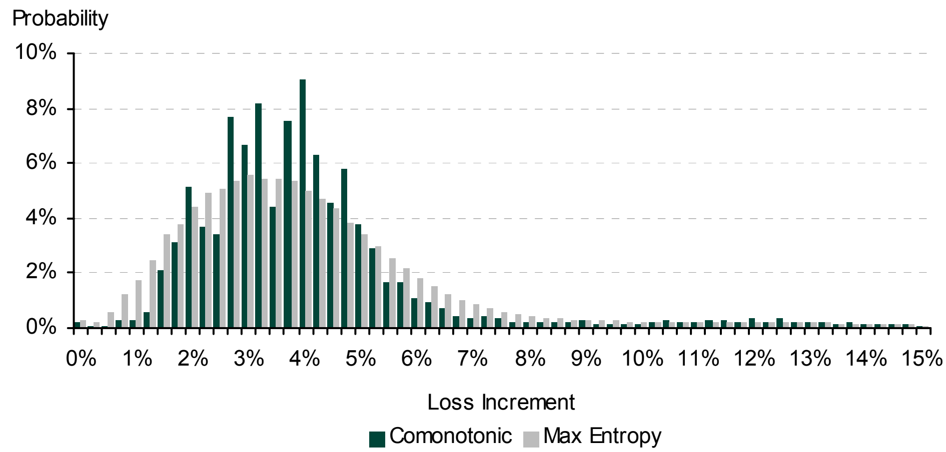
Figure 7. Properties of Loss Increment Distributions for 5y to 10y

	Comonotonic	Markov	Max Entropy
Expected Loss Increment	4.15%	4.15%	4.15%
Standard Deviations	2.32%	2.51%	2.66%

Source: Lehman Brothers.

The full loss increment distributions for the comonotonic and maximum entropy methods can be found in Figure 8.

Figure 8. Loss Increment Distribution for the Period from 5y to 10y



Applies to the reference portfolio as of 30 May 2006.
Source: Lehman Brothers.

FORWARD STARTING CDOS

Forward Starting CDOs (referred to henceforth as FDOs) provide a simple application of our modelling framework. In an FDO, the investor sells protection on a CDO tranche starting at a future date, t_1 (the protection start date) and extending to a final maturity T. However, FDOs come in two flavours: If the contract specifies that losses occurring before t_1 lead to a reduction of the tranche subordination, we can replicate the FDO by selling protection on a CDO tranche with maturity t_2 and buying protection on a CDO tranche with maturity t_1. Hence, the price is uniquely determined by the marginal distributions. Here, we are concerned with FDOs of the second kind, i.e. FDOs for which the subordination of the tranche is increased to offset any losses between the value date and the protection start date. The pricing of the latter depends only on the distribution of the loss increments $\Delta L(t_1, t_2)$ with t_2 in $[t_1, T]$. This can be seen as follows:

At any time t, the tranche loss is

$$L_{Tr}(t) = \left(L(t) - K_A(t)\right)^+ - \left(L(t) - K_D(t)\right)^+,$$

where $K_A(t)$ and $K_D(t)$ are the time-dependent attachment and detachment points of the tranche in question. For a refreshing tranche the strikes are

$$K_A(t) = K_A + L(t_1), \quad K_D(t) = K_D + L(t_1),$$

for $t \geq t_1$ and constant K_A and K_D. It follows that the tranche loss at any time $t_2 \geq t_1$ can be written as

$$L_{Tr}(t_2) = \left(\Delta L(t_1, t_2) - K_A\right)^+ - \left(\Delta L(t_1, t_2) - K_D\right)^+,$$

which depends only on the loss increment. As the expected loss increment is model-independent, the main determinant for the pricing of FDO tranches is the width of the loss increment distribution. As the maximum entropy method, with a standard deviation of 2.66%, gives the widest distribution it implies high risk for the upper end of the capital structure and low risk for the lower end, relative to the other two methods. Similarly, the comonotonic method, which leads to a narrow loss increment distribution, implies relatively high risk at the low end and low risk at the upper end of the capital structure. The different widths of the loss increment distribution are reflected in the breakeven spreads that we calculated for a series of thin FDO tranches with a protection start date at 5y and a final maturity of 10y.

Figure 9. Breakeven Spreads for a 5y into 5y Forward-Starting CDO

Tranche	Comonotonic	Markov	Max Entropy
0-1%	**1500**	1443	*1413*
1-2%	**1113**	1059	*1034*
2-3%	**790**	749	*727*
3-4%	**501**	489	*477*
4-5%	*250*	282	**288**
5-6%	*118*	153	**166**
6-7%	*75*	92	**102**
7-8%	*59*	66	**71**
8-9%	*50*	53	**56**
9-10%	*42*	44	**45**

Source: Lehman Brothers.

Note that in Figure 9 we have set the highest breakeven spread for a given tranche in bold and the lowest spread in italics. There is an inflection point as the order of breakeven spreads flips when going from the 3-4% to the 4-5% tranche. This coincides with the expected loss increment at 4.15%.

As a simple illustration of the effects of a narrow loss increment distribution, consider a delta-distributed loss increment. In this limiting case, a thin tranche below the expected loss increment is certain to be wiped out while a thin tranche above this point is certain to survive. A narrow loss increment distribution therefore implies high risk for junior tranches and low risk for senior tranches, consistent with our results above.

As the comonotonic method is based on maximum dependence, we regard the breakeven spread it predicts as providing a valid bound in practical circumstances. Similarly, maximum entropy, giving in some sense minimum dependency, could be seen as an approximate bound in the other direction. However, two cautionary notes are in order. First, the maximum entropy method gives a much weaker bound than the comonotonic method, as can be seen from the following argument: By its nature, the comonotonic method introduces high positive correlation between losses at different maturities, while maximum entropy leads to a small positive correlation. However, it is in fact possible to construct joint distributions with negative correlation. These would typically give a wider loss increment distribution than maximum entropy as the probability for very large loss increments increases. The maximum entropy method should therefore be viewed as producing the joint distribution that is the closest to the independence case while respecting the no-arbitrage constraints.

Also, we should note that even the spreads predicted by the comonotonic method do not represent a strict bound as it is likely that we can construct joint distributions which concentrate incremental losses more heavily in any particular region of the capital structure. However, we believe that these joint distributions are not likely to represent the loss distribution in realistic situations.

CONCLUSIONS

We have discussed a range of ways to value exotic correlation products where the value depends on the joint distribution of the portfolio loss at two or more time horizons. The Markov forward rate model is a skew-consistent arbitrage-free model which implies a certain joint distribution once calibrated to vanilla CDO prices. The comonotonic and maximum entropy methods impose a dependency structure exogenously. While the comonotonic method gives the maximum dependence between future losses, the maximum entropy method is designed to give the minimum dependence.

As an application of our modelling framework, we have priced forward-starting CDO tranches using the different loss-linking methods. The comonotonic and maximum entropy methods can be regarded as providing reference points for the breakeven spreads. While the comonotonic method provides an upper bound for equity tranches and a lower bound for senior tranches in realistic cases, the maximum entropy method gives low spreads for equity tranches and high spreads for senior tranches. For a thin tranche, the crossover occurs when the strike equals the expected loss increment over the life of the trade.

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[1] Philipp J. Schönbucher (ETH Zürich): "Portfolio Losses and the Term Structure of Loss Transition Rates: A new methodology for the pricing of portfolio credit derivatives", November 2005.

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