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Beyond CADR: Searching for Value in the CDO Market[1]

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We identify some important limitations of traditional CADR analysis, and describe a simple, bottom-up approach that can be calibrated in different ways to compute 1) fair values of CDO tranches, 2) informative IRR distributions, 3) implied CADRs. We argue that these tools can be used to identify relative value opportunities in the CDO market.

1. INTRODUCTION

After experiencing a vigorous expansion in the second half of the 1990s, the CDO market has recently been challenged by a tumultuous credit environment. Although there is some evidence that a well-diversified CDO portfolio would have outperformed corporates of comparable quality over the years, CDO investors have quickly come to realize that a disciplined approach to asset selection is now a necessary condition for market survival.[2] Greater attention is being paid to the choice of collateral types and portfolio managers, and an ever increasing interest in quantitative methods has pushed researchers to challenge established analytical frameworks.

A traditional analysis of a CDO structure starts with a top-down approach. Using a constant annual default rate (CADR) assumption for the collateral, the proceeds from surviving assets are distributed to the liability side of the deal according to the contractual waterfall, and standard performance measures for the different tranches are derived accordingly. In this article, we identify the major limitations of this type of analysis and illustrate an alternative, bottom-up approach that can be used for valuation, as well as for the computation of informative risk-reward measures.

We start in section 2 with a discussion of the implicit options embedded in CDO tranches of different seniority. Section 3 explains why a traditional CADR analysis cannot correctly evaluate these options, while Section 4 identifies the sources of default uncertainty. The core of our argument is developed in Section 5, where we describe a flexible framework that can be used to produce some powerful tools for the identification of relative value opportunities in the CDO market. Section 6 concludes with a remark about the robustness of our stylized examples.

2. IDENTIFYING THE IMPLICIT OPTIONS IN CDO INVESTMENTS

In this section, we suggest an option-theoretic interpretation of the pay-off structure of CDO investments. Our goal is to highlight the importance of modeling default uncertainty. Suppose the liability side of a simple 1-year CDO is sold off in two zero-coupon tranches:

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[1] *This article is the result of many conversations with our colleagues Arthur Berd, Dominc O'Kane, Lutz Schloegl, Gaurav Tejwani and Stuart Turnbull. We would like to thank Dev Joneja, Claude Laberge, Vasant Naik and Minh Trinh for comments and suggestions, and Victor Dan for his valuable help.*

[2] *A case for investing in CDOs can also be made on the basis of the risk diversification that this asset class can bring into an aggregate portfolio. See Ganapati and Tejwani (2002) elsewhere in this issue.*

- Junior (first 20% loss),
- Senior (top 80%).

If we define S as the value of the collateral at the end of the year, the final payoff for the senior tranche (ST) is given by:

$$ST = \begin{cases} 80\ ;\ if \quad S \geq 80 \\ S;\quad if \quad S < 80 \end{cases}$$

We can write this in the form

$$ST = 80 - \begin{cases} 0;\ if \quad S \geq 80 \\ 80 - S\ ;\ if \quad S < 80 \end{cases} = SafeAsset \quad - PutOption$$

The senior tranche holder is implicitly short a put option, which is written on the value of the collateral. The initial value of the senior tranche must be equal to the present value of the safe pay-off minus the present value of the put option. We know from option theory that the value of a put depends on the distribution of the underlying asset. In particular, we expect the option value to increase with the variance of the underlying. Given that the senior tranche is short a put option, we expect the value of the senior tranche to be a decreasing function of the variance of S and to have a negative gamma.

For the junior tranche, its final payoff JT is given by

$$JT = \begin{cases} S - 80\ ;\ if \quad S \geq 80 \\ 0\ ;\ if \quad S < 80 \end{cases} = CallOption$$

The value of the call option – and, hence, the value of the junior tranche – increases as we increase the variance of the underlying asset. A more volatile value of the collateral makes the junior tranche a more valuable investment: the junior investor will greatly profit from very low default realizations, while she will have little (and possibly nothing) to lose from an additional default in case the default rate turns out to be very high.

Repeating this analysis with a richer debt structure will show that the holder of the most senior tranche is short an option, the equity holder is long an option, and mezzanine investors implicitly hold portfolios of options with both long and short positions. However, for the senior-most tranche to achieve a high rating, its implicit short option must be far out of the money, whereas the net position of mezzanine investors is generally short volatility. We can therefore conclude that, in general, an increase in the volatility of the underlying asset redistributes value from mezzanine to equity holders, leaving senior investors unaffected.

This simple example shows that a meaningful analysis of CDO tranches calls for the explicit modeling of the probability distribution of collateral defaults, which in turn determines the distribution of the terminal value of the collateral portfolio. Yet the most popular methodology for CDO analysis is based on the assumption of a deterministic collateral default rate.

3. THE LIMITATIONS OF CADR ANALYSIS

CDO tranches are often analyzed in the context of CADR scenarios. "Stress-tests" are sometimes carried out by front-loading defaults, while more favorable contingencies are generated by back-loading defaults at the end of the contractual period.

The CADR internal rate of return (IRR) has become a popular measure of reward. The *x percent* CADR IRR for a given tranche is obtained by defaulting the collateral at *x%* per year, projecting the corresponding payments to the tranche according to the waterfall, and computing the rate of return that makes the present value of the projected payments equal to the current price of the tranche.

The main problem with CADR analysis lies in the assumption that the collateral default rate is deterministic; i.e., a given percentage of collateral is assumed to default every period with certainty. This produces two major limitations.

- Valuation

  It follows from the previous section that CADR analysis is unsuitable for valuation: the implicit options embedded in CDO investments cannot be correctly evaluated by assuming a deterministic default rate. What we really have in mind when we think of, say, a "3% default environment" is a scenario in which the collateral credits have 3% probability of default. This translates into a 3% CADR only if the collateral portfolio consists of an infinite number of independent names: only in this highly theoretical case will the collateral pool default at 3% per year with certainty, as CADR assumes.[3] But for a finite portfolio of dependent credits, a 3% CADR represents an incorrect analytical translation of the idea of a "3% default environment", since it neglects the most basic ingredient of portfolio credit risk: default uncertainty. When we appropriately model the dependence structure of the collateral pool, we obtain a probability distribution for the collateral default loss, which in turn allows us to correctly evaluate the pay-offs of CDO tranches.

- IRR Computation

  Once we specify a tranche price, CADR analysis produces one value for its IRR, corresponding to the selected collateral default experience. In fact, the IRR is indeed a random variable, and we should generate its whole distribution in order to examine, for example, its median and tail behavior, or the probabilities of specific realizations. Comparing CADR IRRs for different tranches does not provide a valid relative value assessment: tranche A may have a higher IRR than tranche B at, say, 3% CADR, but its IRR may have, for example, a higher probability of being negative. In order to compute a valid IRR distribution, we need to generate a meaningful collateral default distribution. Once again, we need to model the collateral pool from the bottom, properly accounting for the dependence structure of the reference credits.

In the remainder of this article, we show that these two issues can be solved by explicitly modeling the joint default process of the collateral names under two different probability distributions.

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[3] *This follows immediately from the law of large numbers.*

4. THE DETERMINANTS OF COLLATERAL DEFAULT UNCERTAINTY

A rigorous analysis of a CDO tranche crucially depends on the probability distribution of defaults. In this section, we turn our attention to the determinants of this distribution, which will play a central role in the bottom-up approach developed in Section 5.

Consider again a simple, equally weighted collateral pool with n defaultable assets. Each asset promises to pay $1 a year from now if default does not occur, and zero if default occurs. In this simple example, the value of the collateral portfolio at the end of the year is equal to the number of survivors S.

For the j^{th} name, at the end of the year, we have either

- default, with probability p_j , or
- no default, with probability $1-p_j$.

The expected value of the portfolio at the end of the year is given by

$$E[S] = \sum_{j=1}^{n} 1 - p_j ,$$

while its variance can be written as

$$Var(S) = \sum_{i=1}^{n} \sum_{j=1}^{n} \sigma_{i,j} ,$$

where $\sigma_{i,j}$ is the covariance between the *ith* and the *jth* default events. It is instructive to separate the marginal default probabilities from the dependence structure. Separating variance and covariance terms, and standardizing covariances to correlations, we get

$$Var(S) = \sum_{j=1}^{n} \sigma_j^2 + \sum_{i=1}^{n} \sum_{\substack{j=1 \\ i \neq j}}^{n} \rho_{i,j} \sigma_i \sigma_j ,$$

where

$$\sigma_i = \sqrt{p_i (1 - p_i)}$$

is the default volatility for the i^{th} name and

$$\rho_{i,j} = \frac{\sigma_{i,j}}{\sigma_i \sigma_j}$$

is the default correlation between the i^{th} and the j^{th} name. This expression shows that for given marginal default probabilities p_i and p_j, the variance of the final portfolio value increases linearly with each default correlation $\rho_{i,j}$.

Since the present value of the different tranches, as we noticed in the previous section, depends on the volatility of the terminal portfolio value, we can now see how changes in default correlations affect the distribution of value across the seniority spectrum. An increase in default correlation(s) will generally redistribute value from the mezzanine investor, who is implicitly short a put option written on *S*, to the equity investor, who is implicitly long a call on the same underlying.

5. RELATIVE VALUE ANALYSIS FOR THE CDO INVESTOR

In this section, we model a simple CDO, evaluate its tranches and generate their IRR distributions. Central to the whole analysis is the concept of modeling dependent defaults. We argue that model prices and IRR distributions can be meaningfully used to find value in the CDO market and show that they can be produced by the same model calibrated in two different ways. Finally, we define the concept of implied CADR and use it to transpose some of our results into a familiar rate space.

5.1 A Simple Structure

Consider a 3-year deal with a collateral pool of 50 credits, each with $1 million notional, 40% recovery rate, and currently trading at a 350 bp flat spread curve. The risk-free (LIBOR) curve is flat at 5%. The liability side is tranched into $30 million senior, $15 million mezzanine, and $5 million equity. The senior tranche carries a promised coupon of LIBOR + 50 bp, while the mezzanine is issued at LIBOR + 200 bp.

We model a highly stylized waterfall, since a more realistic contract would increase the complexity of the calculations without adding anything to the intuition. Coupon payments are semi-annual. At each coupon date, promised spreads are paid to senior and mezzanine investors (in order of seniority) out of yielding (non-defaulted) collateral assets. Excess spread, if any, rewards the equity holders. Every default is recovered immediately, and the recovered amount is reinvested at the risk-free rate. At maturity, the surviving collateral and the capitalized recoveries are used to pay back the principals in order of seniority.

5.2 Relative Value I: Valuation

In order to compute the fair value of a given tranche, we simulate the joint default process of the collateral names under the risk-neutral measure. On each simulated path, we calculate the discounted cash flows accruing to the tranche according to the specified waterfall. Then, we compute the fair tranche value as the average of these discounted cash flows across paths. The results presented in this section use 50,000 simulated paths.

We assume that the risk-neutral default intensity for every name in the collateral is characterized by a non-stochastic piecewise-flat hazard rate function, which we recover from the given spread curves. The joint distribution of default times is then obtained by joining the marginal distributions with a *t* copula. For the analysis in this section, we use flat pairwise correlations of 20% and 12 degrees of freedom. The latter produces the same amount of tail-dependence that we found, for example, in the DJIA basket. [4, 5]

Once we have the joint distribution of default times, it is straightforward to simulate paths of correlated defaults and compute the theoretical prices. Figure 1 shows the model prices for

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[4] *For more details on this time-to-default simulation, see, for example, Mashal and Naldi (2002).*

[5] *There is a lively debate in the literature on the necessity of generalizing the Gaussian copula to model correlated defaults. Our choice of a* t*-copula for CDO valuation follows from recent results reported by Mashal and Zeevi (2002). Using a non-parametric approach to control for the marginals, they employ pseudo-likelihood ratio tests to show that as we increase the dimension of the basket, the assumption of Gaussian dependence becomes "easier" to reject.*

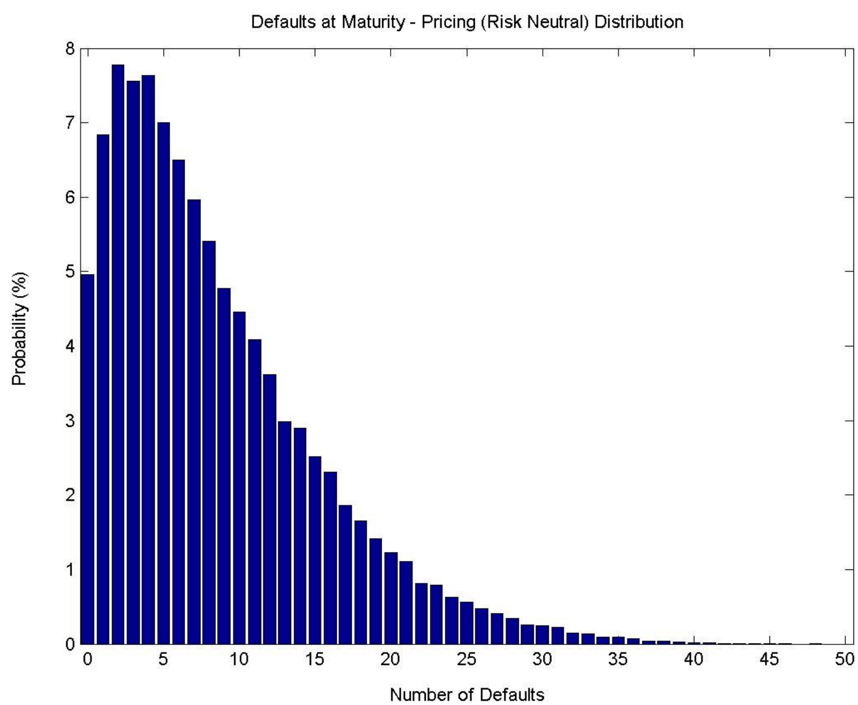
the three tranches of our stylised CDO. Comparing model and market prices can be an extremely useful tool for the identification of relative value opportunities.

Figure 1. Model Prices

	Equity	Mezzanine L+2%=7%	Senior L+.5%=5.5%
Size ($MM)	5	15	30
Price (%)	99.06	100.93	101.37

This bottom-up approach models the collateral credits directly, inferring their risk-neutral default probabilities from observable spread curves and estimating their dependence structure using time-series of observable proxies for their asset returns. A risk-neutral distribution for the collateral default rate is then implicitly generated within the time-to-default simulation: the important point is that this pricing distribution is now strictly dependent on a number of market observables that determine both the marginals and the copula of the simulation engine. Figure 2 shows the pricing distribution of defaults generated for the valuation of our CDO.[6]

Figure 2. Risk Neutral Default Distribution



6 *Our assumptions that the 50 names in the collateral portfolio trade at 350 bp spreads and have 40% recovery imply a risk-neutral cumulative probability of default over the 3-year horizon of approximately 17%, which explains why the expectation of the distribution plotted in Figure 2 is approximately equal to 50*17%=8.5.*

We can also divide the fair (model) price of a tranche into two separate components related to

1. expected discounted cash-flow, and
2. default risk premium.

The first component can be recovered by simulating the joint default process under "objective probabilities" – i.e. probabilities that reflect expected default frequencies and that are *not* shifted to account for the market price of default risk – and calculating the expected discounted cash flow of the tranche.[7] Given that we are implicitly working under the assumption that the dependence structure of the collateral names is not affected by the change of probability measure, the only difference between the "pricing" simulation and the "real-world" simulation lies in the specification of marginal default probabilities. While pricing requires us to back out market-implied default probabilities from observable spreads, the computation of expected cash flows requires the specification of a vector of objective default probabilities. The difference between the expected discounted cash flow and the model price of the tranche represents the "discount" received by the investor for tolerating default risk.

These two components of the theoretical price partition the determinants of tranche value within the model. The first component is *only* related to collateral quality (default probabilities) and diversification (number of reference credits and default correlations). The second component is *only* related to risk premium, and, unlike the first one, it would be identically equal to zero in a world of risk-neutral investors. Putting it all together, we can now represent the market price of a CDO tranche as

Market Price = Expected Cash-Flow - Risk Premium + Rich/Cheap Signal

This partition can be extremely powerful for comparing tranches across different deals. Beyond providing a rich/cheap signal, it allows one to recognize different sources of value within the model, providing the investor with a better understanding of what he is paying for.

5.3 Relative Value II: IRR Distributions

For given prices, we are generally interested in computing and comparing measures of reward from alternative CDO investments. Using the model prices in Figure 1, Figure 3 shows the traditional CADR IRRs for the tranches of our stylized CDO.

The senior tranche yields the risk-free rate for all reported CADRs when priced at its fair value, since it is basically free of default risk (remember that LIBOR is assumed flat at 5%). Both the senior and the mezzanine tranches yield less than their coupons (5.50% and 7.00%, respectively) even at CADRs that do not affect any of their promised payments. This is because their fair prices are above par, which, in turn, is a consequence of the information contained in market-implied (marginal and joint) default probabilities.

The popularity of CADR IRRs relies more on their simplicity than on their usefulness. Suppose that we want to compare the performance of two alternative mezzanine investments in a "3% default environment." Most likely, a 3% CADR scenario will simply tell us that the IRRs of the two tranches are equal to their promised yields, thus giving us a rather useless piece of information. The reason for this deficiency, as we noticed earlier, lies in the intrinsic lack of default randomness that characterizes a CADR analysis.

[7] *Of course, one can either specify subjective estimates of default probabilities or use one of the tools available on the market to back them out from market information.*

Figure 3. CADR IRRs

	Equity	Mezzanine L+2%=7%	Senior L+.5%=5.5%
Size ($MM)	5	15	30
Price (%)	99.06	100.93	101.37

CADR	IRR	IRR	IRR
0%	31.51%	6.65%	5.00%
1%	26.09%	6.65%	5.00%
2%	20.06%	6.65%	5.00%
3%	13.22%	6.65%	5.00%
4%	5.19%	6.65%	5.00%
5%	-4.72%	6.65%	5.00%
6%	-18.25%	6.65%	5.00%
7%	-25.50%	5.51%	5.00%
8%	-29.39%	3.96%	5.00%
9%	-33.60%	2.38%	5.00%
10%	-38.20%	0.77%	5.00%
11%	-43.33%	-0.86%	5.00%
12%	-49.19%	-2.54%	5.00%

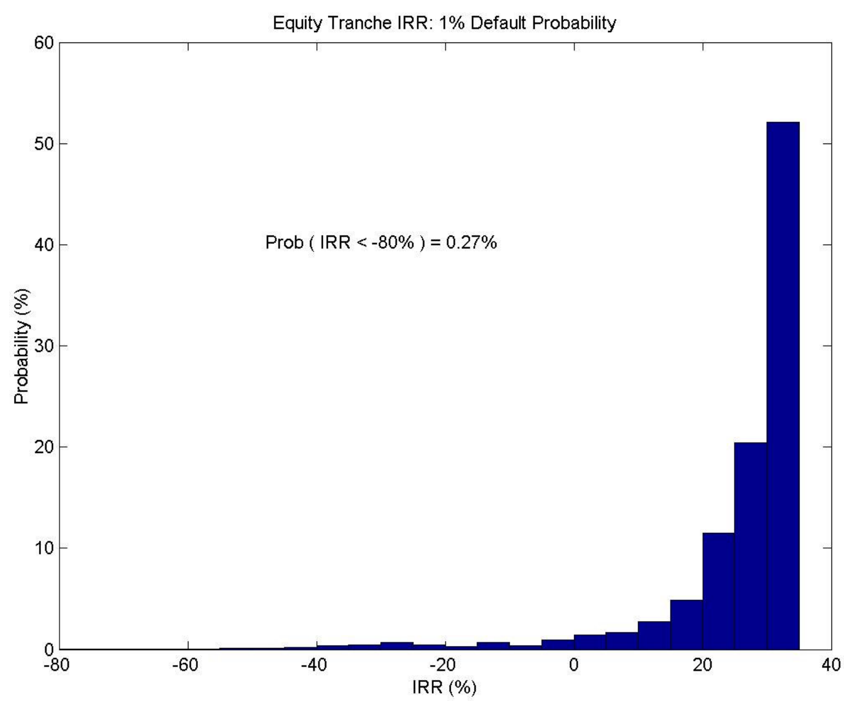
The valuation framework described in the previous section suggests a natural way to bring default uncertainty into the analysis and compute informative IRR distributions for a 3% default environment: all we have to do is simulate default times using default probabilities equal to 3% for each collateral name and compute an IRR for each tranche along every simulated path.

This probabilistic scenario is indeed a more meaningful way to compare alternative CDO investments than traditional CADR analysis. Instead of specifying a deterministic default rate for the collateral as a whole, we express views on default probabilities for the individual credits and then generate a random number of collateral defaults by simulation. In doing so, we appropriately take into account a crucial piece of information that is ignored within the CADR framework: the amount of collateral diversification, which, in turn, depends on the number of reference assets and their default correlations.

For a given vector of default probabilities, this simulation-based analysis can be used to produce a probability distribution for the IRR of each tranche. Figures 4-6 show the IRR distributions for the equity tranche of our stylized CDO, assuming that tranches are priced at fair value and that every name in the collateral has a default probability of 1%, 3%, and 6% per year, respectively. Figures 7-9 show the same output for the mezzanine tranche. The distribution of the senior IRR is uninteresting, since this tranche practically yields LIBOR with certainty. Another way to say this is to think back to our option-theoretic interpretation, according to which the spread over LIBOR received by the senior investor is really the compensation for the option he has implicitly sold. In our example, that option is so far out of the money that it is worthless.

Notice that as far as the analysis of our CDO goes, the first two default environments (1% and 3%) are far more realistic than the last one (6%). Given our assumptions on spread curves and recovery rates, the market-implied default probabilities that we used for valuation are approximately 6% per year, and in a world of risk-averse investors, the objective default probabilities must be significantly lower than that. To confirm this intuition, most bonds now trading at similar spreads have been issued by companies with a current KMV 1-year EDF in the (0.5%, 1.5%) range.[8]

Figure 4. Equity IRR Distribution: 1% Default Probability

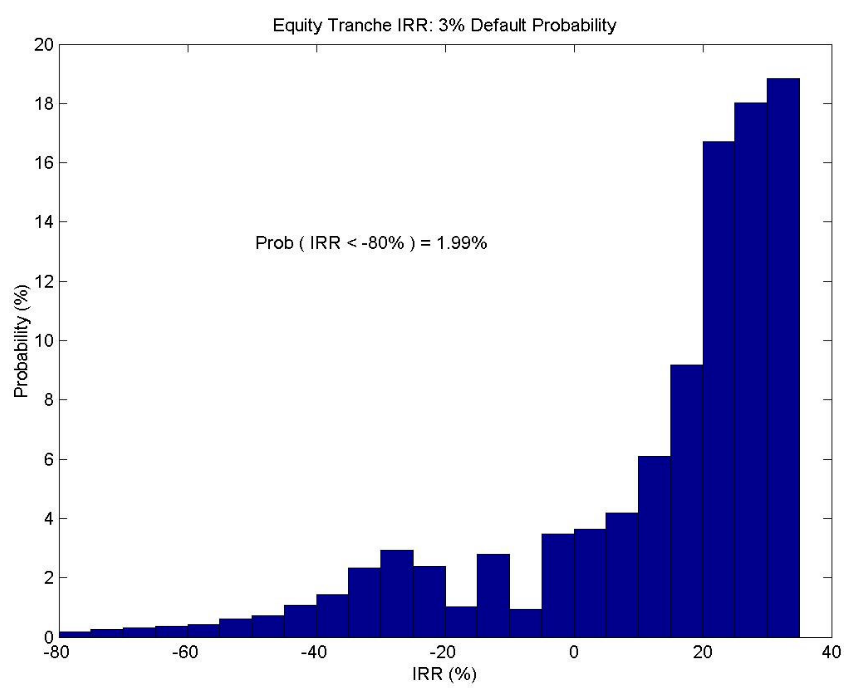


[8] *Source: CreditEdge™, KMV LLC. © 2001-2002 KMV LLC. All rights reserved. CreditEdge, Expected Default Frequency and EDF are trademarks of KMV LLC.*

It is interesting to relate the shapes of these graphs to some of the most likely default scenarios. The distribution of the equity IRR has a clear tendency to be multi-modal, i.e., to have a few local maxima. With 1% annual default probability for every collateral credit (Figure 4), the most likely equity IRR is 31.51%, which corresponds to the case where there are no defaults over the life of the contract (compare with 0% CADR IRR in Figure 3). A significant probability mass lies over the range (0%, 30%), which corresponds to scenarios where 1 or 2 of the 50 reference names default. Moving to the left of the distribution, there are also visible modes corresponding to IRRs in the (-10%, -15%) and (-30%, -25%) intervals. These correspond to scenarios where 3 and 4 names default, respectively. Notice that the distribution of probability is smooth around the modes because of the random timing of defaults.

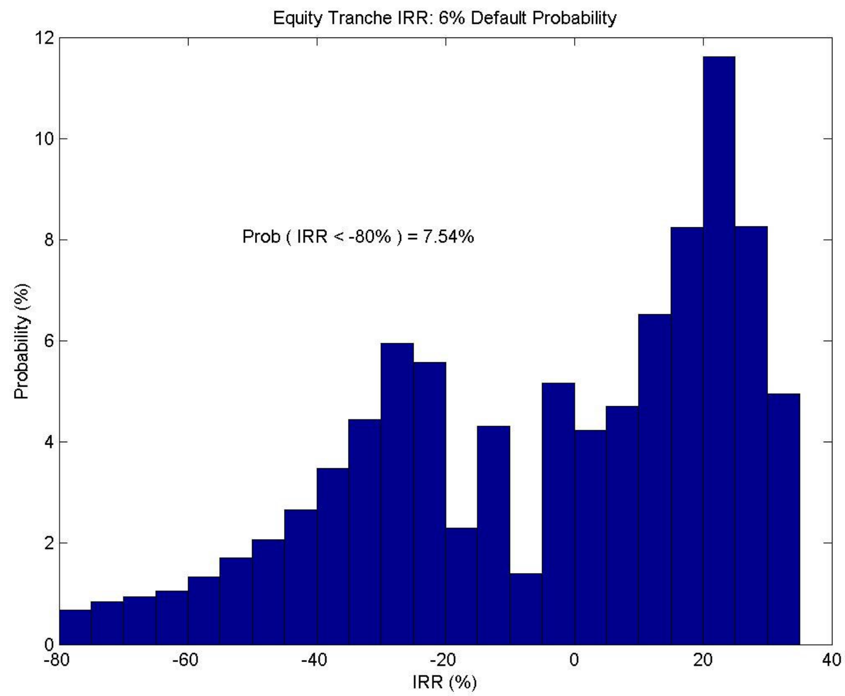
Even with marginal default probabilities as low as 1% per annum, our example shows that there is some tangible probability that a 50-name portfolio will experience 3 or 4 defaults over the next 3 years. This is a direct consequence of the fat-tailed dependence structure that a *t*-copula generates. Sellers of second-to-default protection and buyers of mezzanine risk have recently realized the importance of accounting for these events, even when dealing with investment-grade collateral.[9]

Figure 5. Equity IRR Distribution: 3% Default Probability



[9] *Elsewhere in this issue, O'Kane and Schloegl (2002) show that choosing an appropriate copula is crucial for the measurement of portfolio default risk.*

Figure 6. Equity IRR Distribution: 6% Default Probability



As we increase the annual marginal default probabilities to 3% (Figure 5), we notice a significant redistribution of probability from the no-default case to scenarios where a few defaults take place. Finally, with 6% default probabilities (Figure 6), the equity IRR distribution does not look attractive at all. The reader should keep in mind, however, that this probabilistic scenario is highly unrealistic.

Figure 7. Mezzanine IRR Distribution: 1% Default Probability

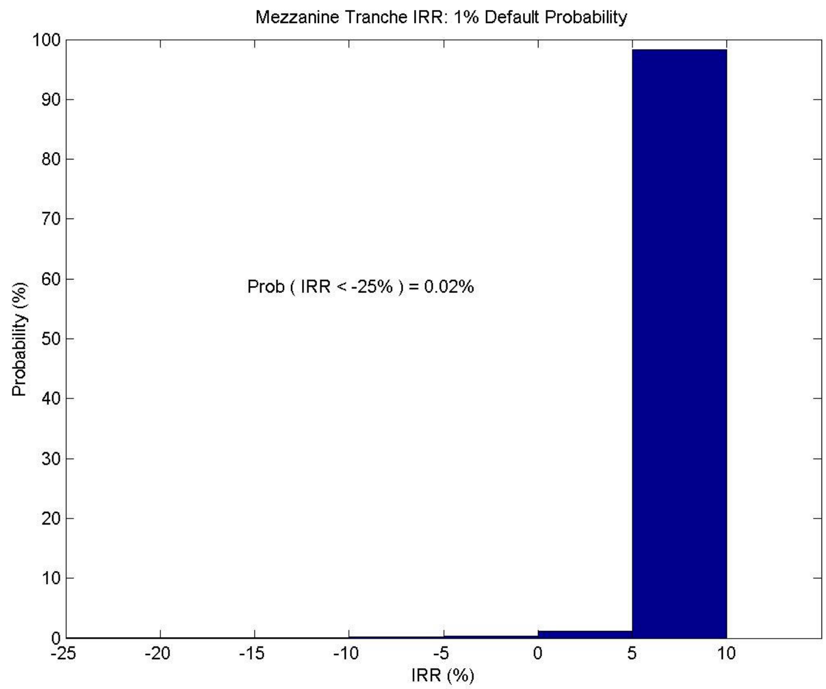


Figure 8. Mezzanine IRR Distribution: 3% Default Probability

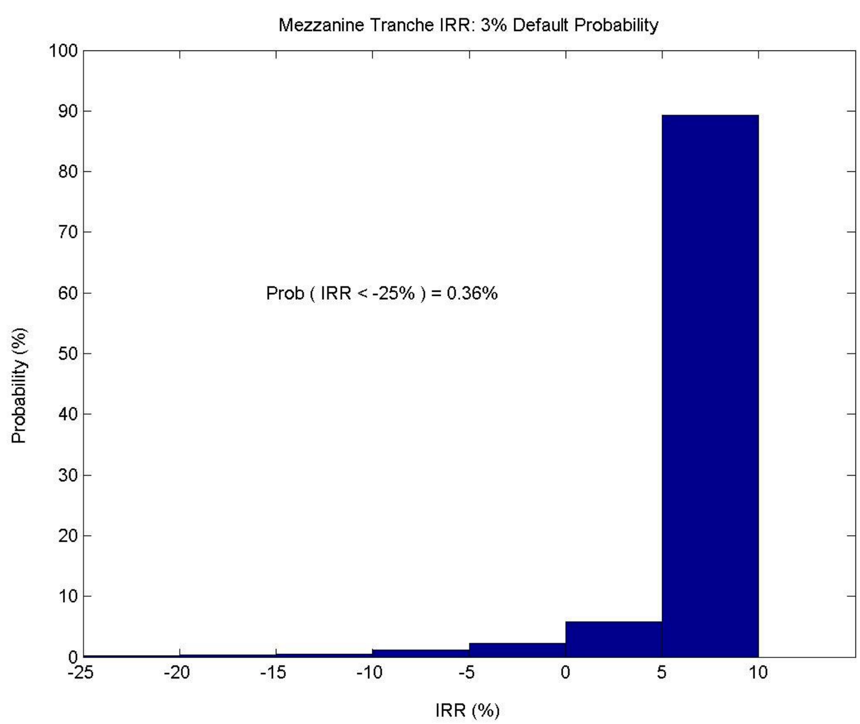
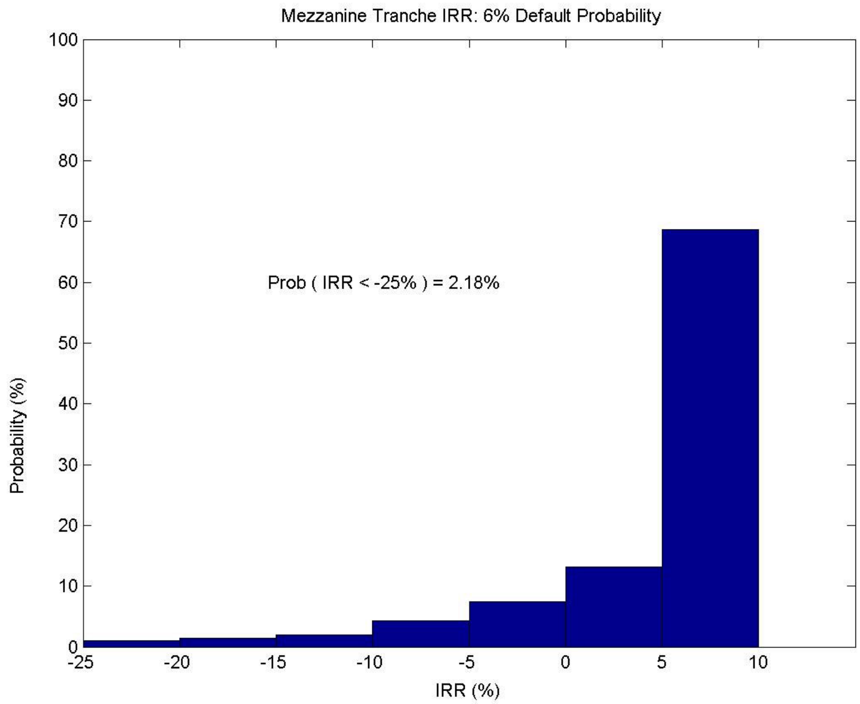


Figure 9. Mezzanine IRR Distribution: 6% Default Probability



The distribution of the mezzanine IRR (Figures 7-9) is much easier to read: most of the probability mass is on 6.65%, which is going to be the mezzanine IRR in the event that no promised payment is missed (compare with the CADR IRRs in Figure 3). As we increase the default probabilities of the collateral names, the left tail of the distribution gets thicker.

The availability of the IRR distributions makes it possible to compute statistics that cannot be obtained by means of CADR analysis alone. A few examples are given in Figure 10, where we report:

- the probability that the IRR of the equity tranche will be higher than its 2% CADR IRR (20.06%, from Figure 3),
- the probability that the IRR of the mezzanine tranche will be lower than its promised yield (6.65%),
- the probability that the IRR of the mezzanine tranche will be lower than LIBOR (5%), and
- the probability that the equity tranche will yield more than the mezzanine tranche.

Figure 10. Computing Probabilities of Specific IRR Realizations

	Default Probability of Collateral Names		
	1%	3%	6%
Event		**Probability (Event)**	
Equity IRR > 2% CADR IRR	83.84%	53.14%	24.48%
Mezz IRR < Promised IRR	2.94%	15.62%	40.28%
Mezz IRR < LIBOR	1.76%	10.73%	31.36%
Equity IRR > Mezz IRR	93.05%	71.57%	42.04%

The *x%* CADR IRR (with *x* depending on the average collateral quality) is a standard reference for CDO equity investors, and it may be interesting to know the probability that the actual yield on the investment will exceed (or fall short of) that number. For a mezzanine investor, it may be interesting to measure default risk in yield space, and compute the probability that 1) collateral defaults will be high enough to compromise the schedule of promised payments or 2) collateral defaults will be high enough for the investment to yield less than the risk-free rate. Finally, a CDO investor may be interested in probabilistic comparisons among different levels of the capital structure in order to select the amount of leverage that is most appropriate for her investment objectives.

Of course, modeling different deals will allow for comparisons across tranches backed by different collateral pools and, thus, provide a powerful tool for relative value analysis in the CDO market.

5.4 Relative Value III: Implied CADRs

What if we tried to evaluate a CDO tranche using traditional CADR analysis, i.e., defaulting the collateral at a constant expected rate? By now it should be clear that we would be facing two major issues. First, the zero-volatility assumption embedded in this analysis would not allow for a proper valuation of the implicit options discussed above. Second, a CADR used for pricing should really be a "risk-neutral" CADR; i.e., it should be higher than the expected collateral default rate in order to reflect the degree of risk aversion in the market.

Now that we have modeled the collateral pool from the bottom and recovered fair tranche prices that reflect both the value of the implicit options and the risk premium for default risk, we can always go back to the notion of CADR and ask ourselves at what constant annual default rate we should default the collateral pool in order to replicate the fair (model) price of a given tranche. Performing this exercise will allow us to compute a "model-implied CADR" for each tranche.

We can do the same exercise using observed market prices, and compute "market-implied CADRs." This allows us to transfer our rich-cheap analysis from price to CADR space: an attractive investment will have a market-implied CADR higher than the model-implied CADR, and vice versa.

Following what we did with prices, we can think of a model-implied CADR as the sum of two separate components related to

1. expected default loss, and
2. default risk premium.

Using the framework described in the previous sections, we can break down a model-implied CADR into these two components. The first component can be recovered by first calculating the expected discounted cash flow of a tranche under objective probabilities, and then computing the constant annual default rate that replicates that value. The balance to the entire model-implied CADR gives a measure of the tranche risk premium.

Once again, these two components partition the determinants of tranche value within the model. The first component is *only* related to collateral quality (default probabilities) and diversification (number of reference credits and default correlations). The second component is *only* related to risk premium. Putting it all together, we can represent the market-implied CADR of a CDO tranche as

Market-implied CADR = Expected Default Loss + Risk Premium + Rich/Cheap Signal.

The analysis of implied CADRs does not conceptually add anything to the price decomposition described earlier. It is simply a way to translate the same concepts into a space that most CDO investors have become familiar with.

6. CONCLUSION

The use of quantitative methods for the valuation and risk management of defaultable instruments has become an indispensable element in the toolbag of the credit investor. Accordingly, the quantitative analysis of CDO investments is undergoing significant changes. In this article, we have attempted to provide a link between the traditional CADR framework and a modern valuation approach: we have started with a reflection on the limitations of the former and provided some solutions within the context of the latter.

Our discussion has been illustrated with examples from a highly stylized CDO, which, admittedly, lacks many features of a realistic deal such as triggers, explicit options, and other mechanisms aimed at redistributing value among the different parts of the liability structure. This, however, does not affect in any way the validity of our conclusions. To see this, notice that all of our results were based on the representation

$$P = E[f(Y_R, \mathrm{X}_v^2)],$$

where Y_R is a vector of standard normal random variables with correlation matrix R, X^2_ν is a Chi-square with ν degrees of freedom, and f is a deterministic function. We solved this equation for both prices and yields and discussed the merits of taking the expectation under different sets of probabilities. Increasing the complexity of the contractual waterfall would only have increased the complexity of the function f, without adding anything to the conceptual framework of our discussion.

Using a bottom-up approach to default modeling, we have shown how to derive fair prices of CDO tranches, as well as informative distributions for their yields. We have argued that an appropriate utilization of these tools can significantly improve the ability of a CDO investor to recognize favorable market prices and desirable risk-reward profiles.

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CDO Equity in a Portfolio Context[1]

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CDO equity tranches offer an efficient avenue to add leveraged exposure to various asset classes. However, unlike other forms of leverage, investing in CDO equity is not just about borrowing and investing in the underlying assets.[2]. The structural aspects of CDOs make them unique investments with Sharpe ratios that are different from those of the underlying assets. In this report, we reconstruct a CDO equity index and compare it with other alternative investments in a risk-return context.

1. THE NEED FOR A CDO EQUITY RETURN SERIES

Asset allocation decisions are based on several parameters, an important one being the historical performance of an asset and its correlation with others. The private nature of the CDO market has made collection of historical performance data difficult. Return data, even if available, would be of limited use, as CDO equity is a relatively young asset class and, naturally, investors would like to study its performance over long periods of time covering business cycles. One proposed solution to the problem of limited CDO equity performance data is the reconstruction of returns across time using available underlying data. In this paper, we create a CDO equity return index and also discuss the risk-return benefits of adding CDO equity to an alternative investment portfolio.

1.1 We structure a hypothetical CDO every month

Historical reconstruction of a return series is not uncommon in the private equity world. Its application to CDOs presents a bigger challenge due to the sheer complexity of the problem and various parameters that need to be estimated. In our study, we assume that a CDO with identical structural features is issued every month starting in 1990, and its performance is studied over time. We first analyze the CDO equity of a typical high yield (HY) portfolio and then compare it with that of an investment grade (IG) portfolio and leveraged loan (LL) portfolio. For simplicity and generality, we take a stylized CDO that incorporates all major characteristics of a CDO, without making it unreasonably complex. We then subject it to default rates experienced for that cohort and compute CDO equity returns on each monthly cohort.

This structure is an abstract of a CDO that would have been issued under prevailing market conditions. Though the initial capital structure is fixed across cohorts, the transaction deleverages once the reinvestment period expires. The initial capital structure might change over time as the deal prepays or PIKs. Deleveraging would start after the reinvestment date even if the deal were to continue to perform well. Payment priority maintains a simple senior subordinated waterfall with a management fee. Though we model OC/IC triggers, other covenants such as WAR test and Caa bucket are not modeled, as they do not deleverage the structure.

[1] *The authors would like to thank Sridhar Bearelly, Arthur Berd, Konstantin Braun, David Deutsch, and Marco Naldi for their input.*

[2] *Adding simple leverage to an asset class does not change the Sharpe ratio of the investment.*

The capital structure for the HY deal includes the following tranches:

Figure 1. Capital structure of High Yield CDO (Base Case)

Tranche	Size
AAA	67.5%
AA	9.0%
A	5.0%
BBB	5.0%
BB	4.5%
Equity	9.0%

For the HY transaction, we assume that the asset portfolio consists of all coupon-paying bonds in the Lehman Brothers B1/B2 index with maturity between one and 12 years. The purchase price and spread earned on the assets are the average price level and spread on the "filtered" index. Purchase price and spread arbitrage together determine the probable cash flows for the transaction. However, it would be unreasonable to assume that the average price is at a steep discount or at a premium, as the manager would choose credits with due caution. We therefore apply a price cap of 100 and a floor of 85 during ramp-up. The above variables determine the structural component of a CDO's performance. The other component is the default or loss performance of the portfolio. Overall performance depends on both – the cohort effect and the performance of the manager with respect to the cohort. For our base case, we need an index of CDO returns that is independent of the manager's value addition.

1.2 An unmanaged HY CDO is our benchmark index

In order to choose a base case that is closest to "an index," we define our base case as an 8-year unmanaged CDO, issued monthly, that has the exact same defaults experience as that cohort of HY bonds rated B1/B2 at inception. For transactions that are outstanding to date, i.e., not yet matured, we assume that the default rate in the future years of that cohort would be equal to the historical average of 4.6%. This assumption of mean reversion of default rates is investigated later in this study. We also assume that the transaction has no management fee (as it is a static pool) but a small periodic fee of 10 bp that covers miscellaneous expenses such as trustee fees, etc. Interest rates are assumed to be mean reverting, and LIBOR is assumed to be 5% in the future.

The investment grade transaction involves similar assumptions, the primary difference being that the asset pool consists of the cross-over index and that the capital structure is typical of an IG transaction, with the equity tranche size being 5.5%.

We compare the performance of this benchmark HY CDO index with that of a similar IG CDO index and also study the possible value addition from active management. We also perform sensitivities to various parameters to estimate the reliability of our study. Some of the variables, such as default rates and recovery rates, are a measurement of macroeconomic condition and manager performance. Other variables, such as trigger cushions, asset spreads, asset purchase price, and liability spreads, are a function of the transaction's execution and ramp-up. A third set of variables consists of assumptions such as reinvestment rate, forward default rate, forward LIBOR, or miscellaneous parameters such as management fee, structuring fee, etc,. Wherever data are not available or a projection into the future is required, variables are set to their historical average.

1.3 We measure returns for each cohort and aggregate it over time

The return series is simply the return on each monthly cohort. The January 1995 return, for example, is the return on a CDO structured in January 1995. To compare CDO equity with other asset classes, we need the mean return on CDO equity as an asset class over the long run. Here, return has been defined as the annualized gain on investments assuming a constant reinvestment rate of 14%. The reason for using such a measure lies in the pitfalls in using the more traditional IRR method.

The calculation of IRR assumes that all annual cash flows are reinvested in another instrument with the same yield over the remaining life of the investment. This tends to overstate the actual expected return on high IRR cohorts while understating the expected return on low IRR cohorts, thus rendering returns volatile. For low-return cohorts, IRR does not provide a meaningful return measure. The IRR reinvestment assumption thus becomes overly conservative or overly aggressive depending on the cohort. CDO equity cash flows are lumpy – with higher returns in the first few periods (when cumulative defaults are low), followed by relatively low income and, finally, a possible windfall gain at maturity/call when excess principal flows in. This makes IRR a less desirable measure of return.

We use a modified return calculation:

$$Yield = \left(\frac{\sum_{i=1}^{T} C_i (1+R)^{T-i} - C_0}{C_0} \right)^{(1/T)}$$

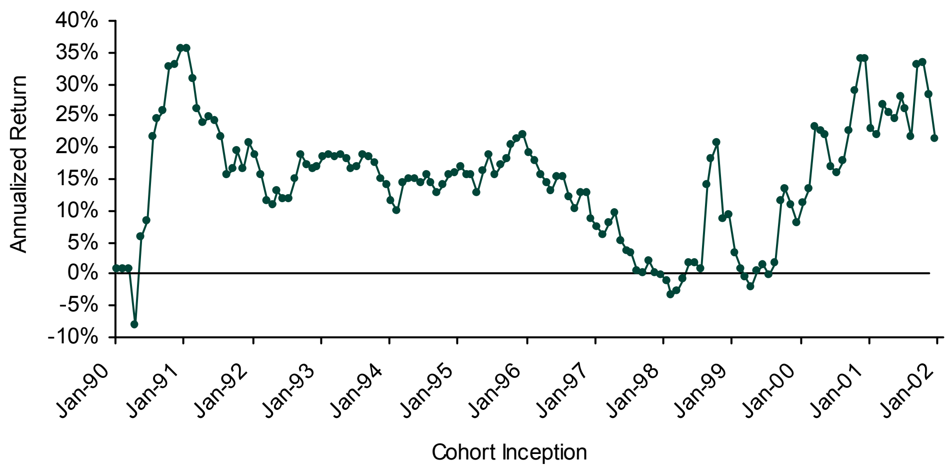
where C_i is the cash flow in the i^{th} period and R is the reinvestment rate. For this study, we have taken a reinvestment rate of 14%, which is the average total return on the S&P 500 over the past eleven years. Our assumption is that the minimum return hurdle that alternative investors would expect is that of the S&P 500. We later run sensitivities to the reinvestment rate, which is clearly an important variable. Also, high yield returns have been in the region of 9%, and CDO equity, being a leveraged position in high yield, would command a higher return.

The mean return on CDO equity as an asset class should ideally be the weighted average return on CDO equity for all transactions. The weight in this case being the par value of equity tranche and return being a number that captures the performance of the asset. For simplicity, we take the simple mean of our recreated index to be the mean return of CDO equity as an asset class. In reality, more CDOs would be issued under favorable circumstances than under adverse ones, as issuance peaks when the arbitrage levels in the market are high. Therefore, the mean return that we calculate in this study is biased downwards. A true index would reflect the change in issuance volume.

2. CDO EQUITY – THE COHORT EFFECT

Based on the reconstruction, we find that CDO equity would have returned an average of about 15% per annum over the 11 year period, despite the high default rates over the past few years (see Figure 2). We notice that CDO equity as an asset class would have returned an average of 15%-20% for one-third of all cohorts. The early 1990s would have delivered exceptional value due to a high arbitrage level helped by low default rates. The period from 1992 to 1995 represents more normalized returns when loss-adjusted spreads were attractive. Returns start to dip in 1996 as these cohorts begin to feel the impact of the high default environment that we see today. The impact was partly offset in fall 1998, when asset spreads widened dramatically and liability spreads widened less, resulting in soaring arbitrage levels. This brief rise in our equity return series is corroborated by the low downgrade activity on late-1998 cohorts despite the weak credit environment that we see today. The widening of spreads was short-lived, and returns dropped for subsequent cohorts due to lower arbitrage levels. The chart shows that different cohorts of CDO equity have wide variations in performance. **Investors should take tactical, timing-based overweight and underweight views on CDO equity investments to minimize the "cohort effect".**

Figure 2. CDO Equity Return Index (Base Case) 1990-2001 [3]



Equity returns for recent cohorts are impressive because the arbitrage level[4] has been very high. Also, the market is pricing in a very high default rate while our assumption is that default rates are mean reverting. We use a historical annual average of 4.6% for default rates in the future for all transactions outstanding today i.e. not yet matured. However, this assumption is not the prime reason, and the average return for 2001 cohorts would drop only from 26% to 23% if default rates in the future were to be 6.0% (which is approximately one half standard deviation greater than the long-term mean of 4.6%)

2.1 Over the long run, IG CDO performance is similar to that of HY CDOs

We perform an analysis on investment grade[5] (IG) CDOs, similar to the one described above,. A typical IG structure has lower excess spread but utilizes higher leverage(5.5% equity

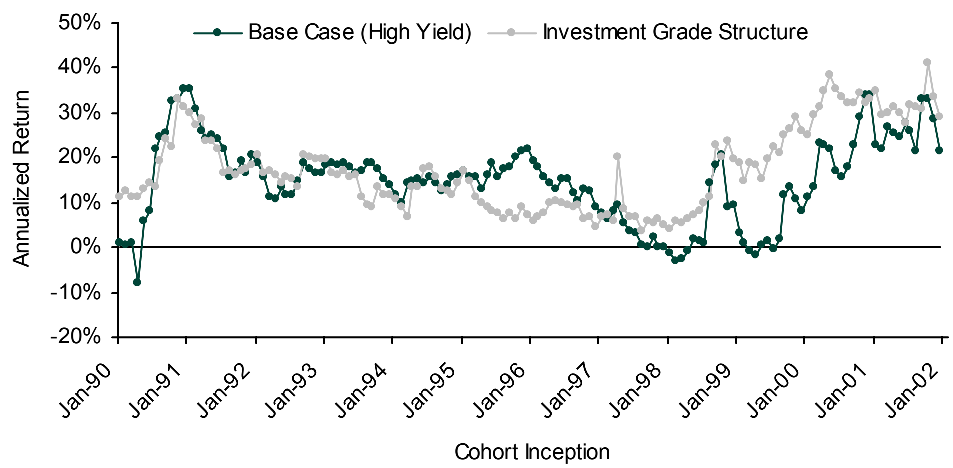
[3] *The return on a cohort is its "buy and hold" return to maturity as explained in section 1.3. We assume future default rates for cohorts not yet matured to be equal to their long term mean. Other assumptions have been stated in the text.*

[4] *The difference between the spread on assets of a portfolio of credits and the weighted average cost of liabilities issued to fund those assets minus other costs such as structuring costs, management fees, hedging costs, etc. For details, see CDO Monthly Update.*

[5] *We assume an unmanaged (low fee) IG transaction for better comparison with the HY base case.*

tranche). As can be seen from the figure 3, the long term performance of IG CDOs is not too different from that of HY CDOs (mean return for IG CDOs is 17.2% compared to 14.8% for HY while the standard deviation is similar). However, performance can differ by several % points in each cohort. The relatively poor performance in 1995 and 1996 and out-performance thereafter can be explained mostly by the arbitrage levels, which in turn reflect the tight and wide level of credit spreads during those periods respectively. One caveat is that the fallen angel effect of 2002 is not fully reflected in this return series because the increased risk of default on downgraded assets is not captured until defaults occur. We would expect this to bias down returns of recent cohorts.

Figure 3. High Yield vs. Investment Grade Collateral

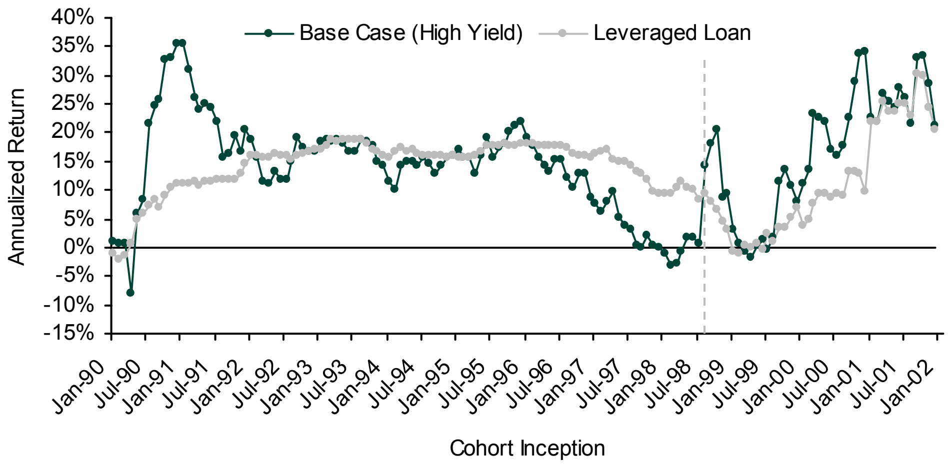


2.2 Leveraged loan CDOs provide more stable returns

A leveraged (LL) loan CDO index is tougher to construct due to lack of historical spread data for the leveraged loan market, prior to August 1998. For this period, we make simplifying assumptions that asset spreads were 350bp over LIBOR. As a result, the return series is unreliable for cohorts prior to 1998, making it difficult for us to make conclusive remarks regarding the cohort effect on LL CDOs.

Besides that we follow the same process as for reconstruction of HY CDO returns. We assume the same collateral quality as a high yield CDO. However, to reflect the fact that loan managers tend to ramp up in the new issue market and rarely have the ability to buy discounted credits, we assume that unlike a HY bond transaction, a leveraged loan CDO purchases assets at par. Though loans would have the same default experience as similar-rated bonds, they would have a higher recovery rate (60%). We assume a capital structure that would be typical of LL CDOs with an equity tranche of 7.5% and study its performance starting in 1998. The recreated LL CDO return index is shown in Figure 4.

Figure 4. High Yield vs. Leveraged Loan Collateral



LL CDOs exhibit a lower volatility compared to HY CDOs (5% vs. 10.7% post August 1998). LL CDO returns are relatively stable because of the lower volatility in arbitrage levels. (However, the apparent stability in returns prior to 1998 is partly due to the simplifying assumptions made in the absence of asset spread data). Lower arbitrage levels in the leveraged loan market, coupled with a poor credit environment, have led to low returns in late 1990s. However, the long term average return is not too dissimilar from that of HY CDOs. We would also like to point out that loan transactions are sensitive to recovery rates, and good mangers can add significant value by selling distressed assets at the right time.

3. MANAGERS ADD SIGNIFICANT VALUE TO CDO RETURNS

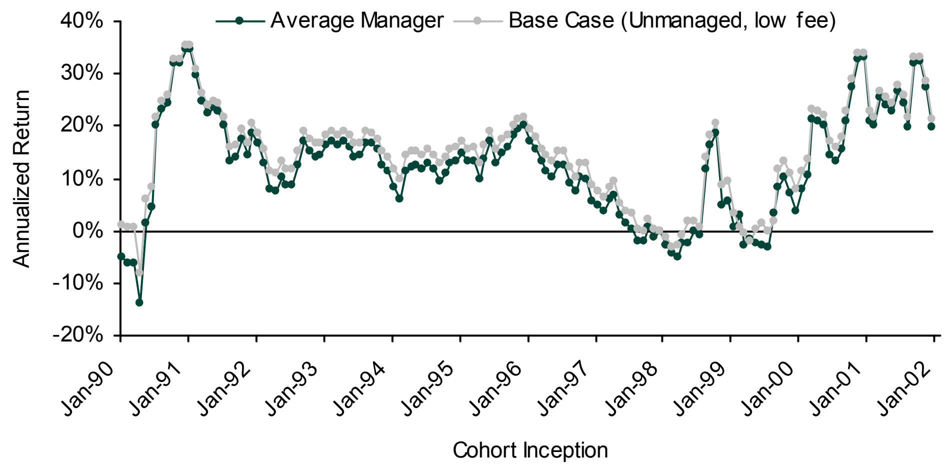
The base case we analyze in this study is an unmanaged pool of HY bonds, but a typical CDO is a managed vehicle in which the manager is expected to outperform the index. An unmanaged deal would experience defaults similar to that of the cohort (assuming that the collateral is fairly diversified). A manager, however, should be able to add value by minimizing losses and making trading gains. Also, a good manager can maximize recovery rates by choosing the right time to dispose of distressed assets (or hold on to them, as the case may be). Clearly, managers add value or investors would be better off with an unmanaged pool and low/no management fee. [6]

All else being equal, we expect managed vehicles to outperform unmanaged pools. The performance of an average manager is shown in Figure 5. The average manager here is one who does not add or take away any value but charges a management fee, hence producing returns lower than the base case. But in reality, the manager would demonstrate his or her ability to perform better than the average on several dimensions. By changing outperformance variables one at a time, we analyze a manager's potential value addition and, hence, the sensitivity of CDO return series to:

- Default rate
- Recovery rate
- Purchase price of assets
- Trading gains

[6] *We assume a management fee of 50 bp for managed transactions and only 10 bp for unmanaged.*

Figure 5. Indexed Manager vs. an Average Manager

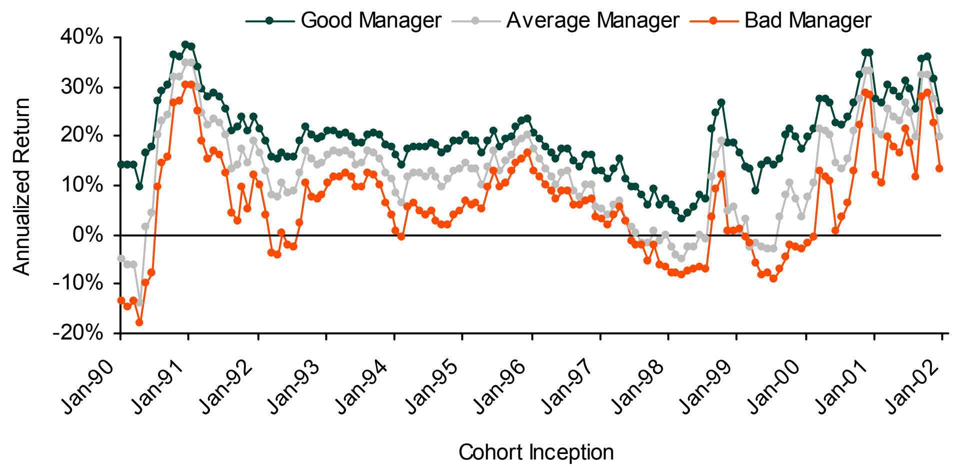


The first three factors are not entirely determined by manager's acumen. They depend, to a large extent, on the cohort being considered – or, in other words, on the macroeconomic situation that prevails during the life of the CDO. But a manager can perform better (or worse) than the market average and add (or subtract) value in the process. Default rates and recovery rates might be correlated, and it is not uncommon to find managers who outperform others by a large margin on these parameters. The third factor, i.e., purchase price of assets, seems to be less flexible at first sight (if the manager wants to maintain the same risk profile). But for illiquid assets, such as some high yield bonds, and keeping in mind how cash flow deals also need regular management (trading), purchase price is an important variable that a manager might be able to control. Trading gains, while very important determinants of CDO performance, are not possible to model.

3.1 Default rate out-performance dominates returns

Default rate is one of the most important variables influencing performance. We compare an average CDO manager's performance with those of a good manager and a bad manager. For this purpose, we define an average manager as one who experiences the same default rate as the cohort. A good manager, on the other hand, avoids one in two defaults, while a bad manager has 50% higher defaults (and, thus, par loss) than the average case. The performance of such managers is reconstructed historically, similar to the base case. We compare the results in Figure 6.

Figure 6. Sensitivity to Default Rates



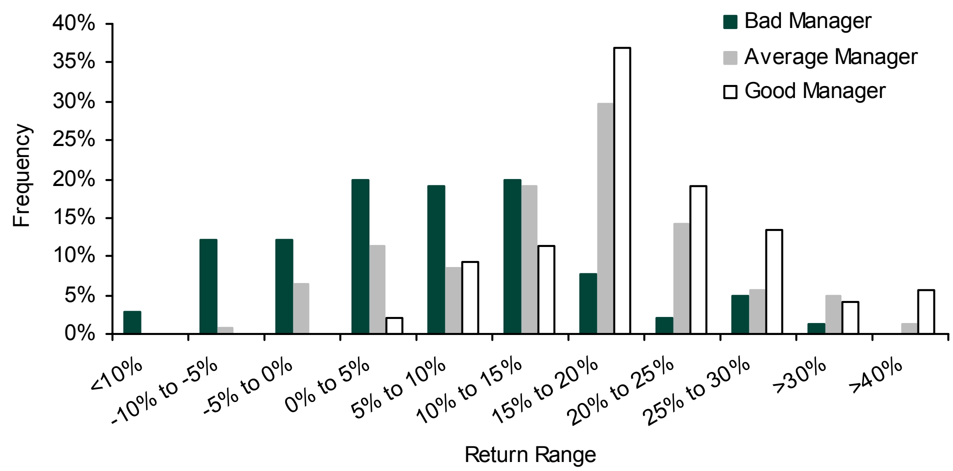
	Mean	Std Deviation	RuR[7]	Sharpe Ratio
Average Manager	12.5%	9.8%	1.28	0.62
Good Manager	19.7%	7.5%	2.63	1.77
Bad Manager	6.3%	9.9%	0.64	0.00
Base Case(unmanaged)	14.8%	9.3%	1.60	0.90

The most important way in which a good manager can enhance returns is by outperforming the index on defaults, i.e., by choosing a portfolio that experiences lower defaults than the index. **Default rate out-performance makes an enormous difference to returns, particularly on the downside**. It is worth noting that an average manager can produce positive returns for most cohorts. A good manager consistently produces strong relative and absolute performance. As shown in this example, it is only when the credit cycle reaches its lowest ebb that equity returns go negative. A bad manager, on the other hand, produces superior returns only when asset spreads are wide, average purchase price is at a discount, and the credit cycle improves. Weak CDO equity performance for bad managers gets magnified by the impact of triggers. When triggers are activated, the deal deleverages, thus increasing average funding cost and reducing returns. It is interesting to note that the return series has a distinct skew and that the skew is higher for bad managers. The skew underlines the fact that variance is not a good measure of risk for CDO equity. The bimodal nature of return probability is somewhat similar to that observed by Mashal and Naldi.[8] The implications of the skew observed here have been explained in section 4.

[7] *RuR is the Return per Unit of Risk and is defined as the mean return divided by its standard deviation. RuR is based on total return calculations while Sharpe Ratio uses an excess return measure.*

[8] *Roy Mashal and Marco Naldi, Beyond CADR - QCR Quarterly Aug2002.*

Figure 7. Return Frequency

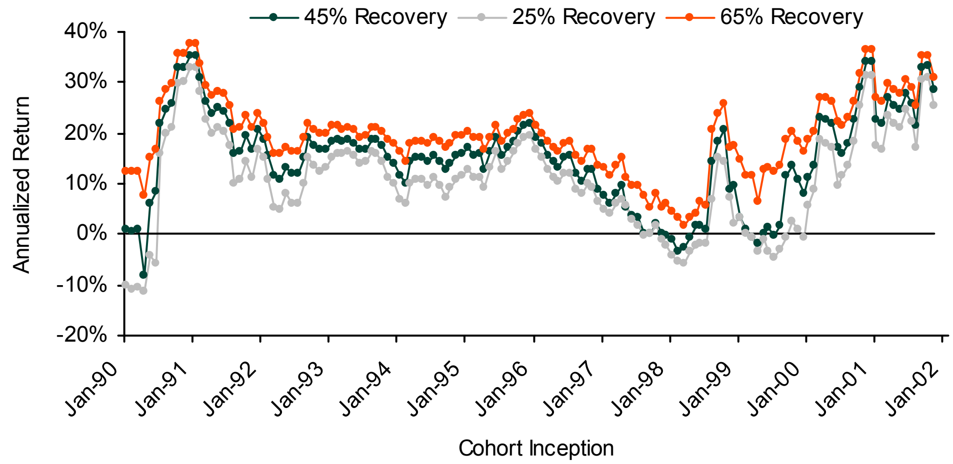


We would like to point out that we assume a flat recovery rate here. In reality, recovery rates are inversely related to default rates. This would increase both the variance and the skew that we observed using our simplistic model.

3.2 Recovery rates in isolation do not make much difference

It is the loss rate and not directly the default rate that determines CDO returns. Recovery rates by themselves are not as critical to the performance equation as default rates. As recovery rates work in tandem with default rates, poor recovery rates together with high default rates significantly hurt returns. In Figure 8, we plot reconstructed historical CDO equity returns for 25% and 65% recovery along with the 45% base case described earlier. It is evident that in times of poor credit performance such as those faced by the 1997 cohorts, recovery rates assume greater importance. The sensitivity to recovery rates is shown in Figure 8 (all other parameters set to base case). The sensitivity to recovery rates is higher in the positive direction, i.e., an increase in recovery rates has a greater positive effect on returns than a negative effect due to an equivalent decrease in recovery rates. The reason for such behavior is that for several cohorts, OC triggers get hit by a small margin and excess interest is diverted from equity to debt tranches.

Figure 8. Sensitivity to Recovery Rates

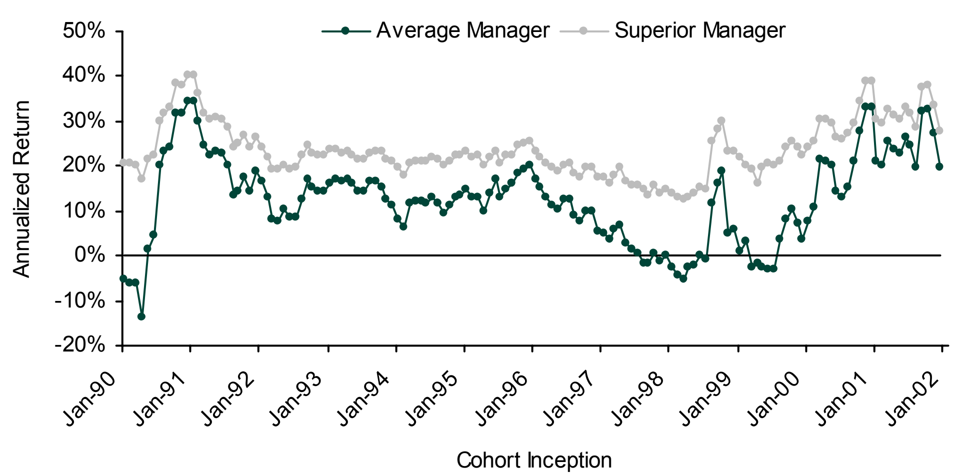


	Mean	Std Deviation	RuR	Sharpe Ratio
25% Recovery	10.8%	9.7%	1.12	0.45
45% Recovery (Base Case)	14.8%	9.3%	1.60	0.90
65% Recovery	19.5%	7.6%	2.55	1.71

Superior managers would outperform on both metrics

A superior manager would manage the portfolio in such a way that it experiences lower default rates than the cohort and also has higher recovery rates on bonds that default. Here, in Figure 9, we show the return for a manager who consistently performs well on both fronts. We assume that the manager avoids one in two defaults on the index and that the recovery rate on defaulted bonds is 65%. The average return for such a manager is in excess of 23% - about 9% more than the base case and 4% more than the good manager case shown in figure 7.

Figure 9. Average Manager vs. Superior Manager

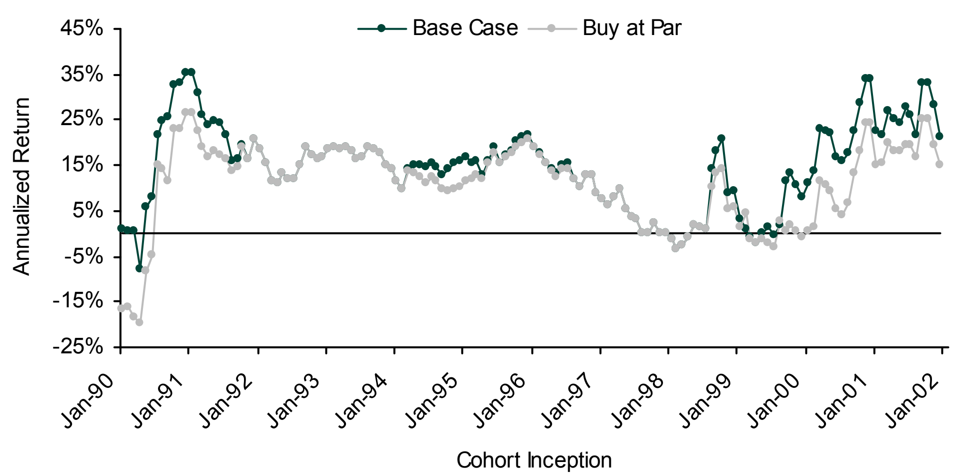


3.3 Purchase price of collateral has a great bearing on equity performance

The average purchase price of collateral has a great bearing on equity performance, almost as important an effect as default rates. Capital gains and coupon payments together determine the profitability of the portfolio. Consider a case in which asset collateral provides a spread of 400 bp and the average cost of funding is 100 bp. So the equity tranche receives 300 bp minus the impact of defaults. If the same collateral could be purchased at, say, 97 dollars, the CDO would have approximately 103 dollars of asset notional. Not only would the additional 3 dollars flow to the equity piece on maturity (minus defaults), but also, the spread offered on these 3 dollars goes directly to the equity piece. This is demonstrated by Figure 10. When the B1/B2 index average price is at a discount, capital gains contribute to CDO equity returns. For those periods in which the discount is comparatively large (1991 and 2000, for example), capital gains could account for a large fraction of return. The purchase price is determined primarily by the level of the index and is, hence, a cohort effect. For some cohorts, spread arbitrage is the dominant factor, while for other cohorts, capital gains assume greater importance.

In Figure 10, we plot the impact of purchase price. In our base case scenario, we have assumed that assets are purchased at the average level of the B1/B2 index (subject to certain constraints). If we were to assume that all assets are purchased at par (and thus exclude the impact of purchase price), we see results for a case that can be best described as a "pure spread arbitrage case." Such a purchase method also increases the volatility of returns. Also of importance (but not plotted on the graph) are the purchase at index case (i.e., no constraints on the purchase price in the form of a floor or a cap) and purchase at or below par case (no floor, but a cap on purchase price at par).

Figure 10. Sensititvity to Purchase Price



	Mean	Std Deviation	RuR	Sharpe Ratio
Base Case	14.8%	9.3%	1.60	0.90
Buy at Par	11.0%	9.0%	1.22	0.49
Buy at or below Par	14.9%	9.5%	1.57	0.89
Buy at Index	14.1%	9.6%	1.48	0.80

3.4 Trading gains

Apart from minimizing par loss by outperforming the index in terms of defaults and recoveries, the manager can add value through trading gains. Though it is not possible for us to capture trading gains in our study, they cannot be neglected. Managers can also avoid failing triggers in the short run by actively managing their portfolios. Protection offered to debt holders in the form of triggers does not usually affect the performance of the portfolio, but transfers value from equity to debt. Managers can avoid breaching a trigger by trading in and out of names.

3.5 Sensitivity to other variables

The study that we present so far involves many assumptions, and sensitivity to key variables tests robustness of the return series. Some of the sensitivities are presented in Figure 11, including those to trigger cushions, reinvestment rate, expected default rate in the future, etc. Readers would find the case of no triggers to be of special interest. It is a measure of the value that triggers divert from the equity tranche to senior tranches.

Figure 11. Sensitivity to other parameters

	Mean	Std Dev	RuR	Sharpe Ratio
Base Case	14.8%	9.3%	1.60	0.90
O/C and IC Triggers[9]				
Standard	14.8%	9.3%	1.60	0.90
Tighter triggers	14.3%	9.5%	1.51	0.83
Liberal triggers	16.5%	8.3%	1.99	1.20
No triggers	19.6%	6.1%	3.21	2.12
Structural				
Higher Leverage(7% Equity)	16.3%	9.9%	1.65	0.99
Lower Structuring Fee (-25 bp)[10]	15.0%	9.2%	1.63	0.93
Other				
Better Yield on Assets (spread 1.10 x)	16.9%	9.1%	1.85	1.14
Reinvestment Rate (12%)	13.4%	9.4%	1.43	0.75
Future default rate (6%)	14.1%	8.9%	1.58	0.86

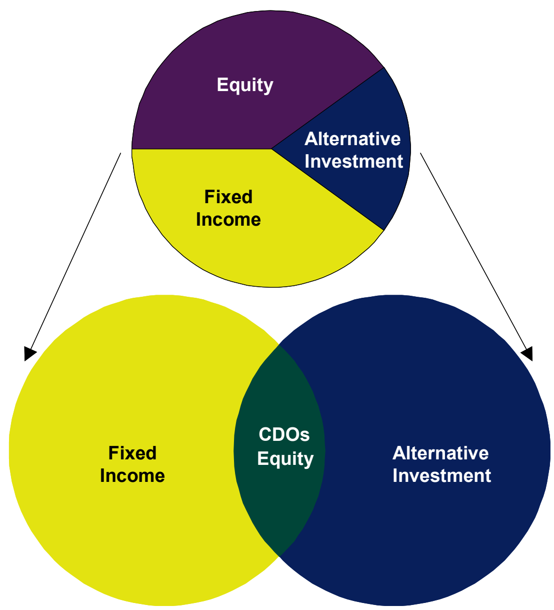
[9] *OC cushion is halved or doubled for each tranche.*

[10] *Structuring fee for base case is set to 150 bp plus 15 bp misc. legal and other fee.*

4. STRATEGIC CASE FOR CDO EQUITY

CDO equity represents a hybrid between fixed income and alternative investment strategies. It offers potential returns competitive with equity and alternative investments by leveraging a diversified portfolio of fixed income assets using unique financing with no mark to market and no recourse. But it is similar to fixed income investments, as risks/rewards are driven by credit performance rather than earnings performance. However, various characteristics of CDO equity, including its buy-and-hold nature and its higher volatility, make it more appropriate in an alternative investment context. In fact, unlike most fixed income securities, CDO equity values could be adversely affected by low interest rate environments.

Figure 12. CDO equity as a hybrid asset class



The hybrid nature of CDO equity is seen on several fronts. It is interesting to compare CDOs with other alternative investments and see how they behave as a hybrid class. CDO investments are primarily in bonds and loans – both public and private, secured and unsecured. That, in itself, would be considered more conservative than the entire spectrum of financial instruments and derivatives in which hedge funds frequently invest, or the investment in private equity and convertible debt by private equity funds. Also, CDO equity returns are front loaded, and the primary goal is income rather than growth, which makes them more like a fixed income instrument than an alternative investment. This is further supported by higher transparency through monthly portfolio disclosure. Investing in CDO equity involves a lower return volatility than most alternative investments, as the primary risk is default and related loss. Diversification is much higher than venture capital funds or private equity funds and also less volatile across time compared with most hedge funds. The comparison is summarized in Figure 13.

Figure 13. Alternative Investments – a Comparison

	CDO Equity	Fixed Income Hedge Funds	Private Equity Funds
Investments	Public and private bonds and loans, secured and unsecured	Performing and distressed public corporate, sovereign and agency bonds, derivatives	Equity and convertible debt of private companies
Investment Goals	Primary: Income Secondary: Capital Growth	Varies, usually capital growth with income secondary	Capital Growth
Targeted Returns	18 to 25% IRR 20 to 30% current yield	20 to 30% IRR, lower current yield, varies	20 to 30% IRR, low current yield
Cash Flow Return Profile	Immediate and front end loaded	Generally at managers discretion	Generally deferred, back ended
Duration of Cash Returns	3 to 6 years	Varies	5 to 15 years
Asset Diversification	Medium to high	Highly variable by timing and style	Low to medium
Primary Risks	Defaults and related loss	Interest rate & currency risk, margin calls, asset illiquidity, defaults	Execution of operating strategy, financing availability, investment valuation and terminal liquidity
Use of Leverage	Yes, term financing with no margin calls	Yes, short-term with margin calls	Yes, equity investments in highly leveraged companies
Liquidity	Limited	Quarterly or annual may have longer lock-up	Limited
Transparency	High – portfolio disclosure monthly, typically have direct access to management	Generally limited	Medium – portfolio disclosed, discussions with management rare

In an alternative asset allocation context, CDO equity can fill the "investment gap," as it offers greater current yields than most buyout or mezzanine funds. It has been observed that on average, buyout and mezzanine funds are able to invest only half of their capital commitments[11]. CDOs do not suffer an investment gap and are fully invested within a few months of closing date.

4.1 CDO equity in a mean-variance framework

One method of understanding the role of CDO equity in a strategic context is to analyze returns in a mean variance framework. Investors are concerned not only about the performance of CDO equity as an investment, but also about its performance with respect to other assets in their portfolios.

An efficient frontier is a useful tool that uses a mean variance approach in a portfolio context. The efficient frontier is a notion from Modern Portfolio Theory that considers a universe of risky investments and explores what might be an optimal portfolio based upon those possible investments. If we know the expected returns, volatilities, and correlations for all the investments in the universe, we can use that information to calculate the expected return and volatility of any portfolio that can be constructed using the instruments that compose the universe. Our equity series facilitates such an analysis.

The notion of "optimal" portfolio can be defined in one of two ways:

- For any level of market risk (volatility), consider all the portfolios that have that volatility. From among them all, select the one that has the highest expected return.

[11] *Source: Venture Economics.*

- For any expected return, consider all the portfolios that have that expected return. From among them all, select the one that has the lowest volatility.

Each definition produces a set of optimal portfolios. Actually, the two definitions are equivalent and provide the same set of results. That set of optimal portfolios is called the efficient frontier. A linear combination of two or more assets can help us obtain a smooth frontier.

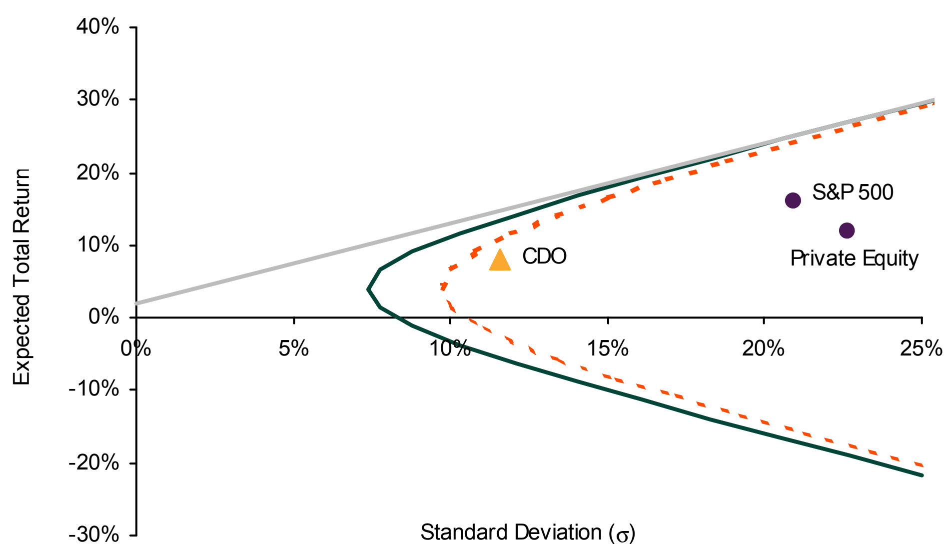
Plotting CDO equity on a risk return plane requires two inputs – a measure of returns offered by CDO equity and a measure of its volatility. The third input needed is correlation with other assets. All these inputs can be estimated from historical return data. Lack of such information implies that the best estimate we can make is by using the reconstructed historical series presented in this paper.

4.2 CDO equity expands the efficient frontier

Including CDOs in an efficient frontier comprising liquid marked to market assets is a non-trivial task. We have obtained annualized returns for the CDO index, while all other assets have marked to market returns that are significantly more volatile. The return numbers we have obtained are effectively eight-year "buy-and-hold" annualized returns that underestimate volatility and its covariance with other asset classes. For a meaningful comparison, we use eight-year buy-and-hold returns on all assets for obtaining an efficient frontier. For recent cohorts for which eight-year data are not available, we assume that the future returns and covariance matrix is equal to the historical average. Also, we use the Newey-West estimator to separate the effect of serial correlation.

In Figure 14, we plot the efficient frontier obtained for a portfolio consisting of private equity, hedge funds, venture capital funds, and the U.S. equity market.[12] The objective is to determine whether inclusion of CDO equity in the portfolio significantly expands the curve.

Figure 14. The Efficient Frontier Curve



[12] The data comprise quarterly return series for S&P 500, CSFB Tremont Hedge Fund Index and Cambridge Associates LLC indices for private equity and venture capital. Calculation based on quarterly data starting January 1994. The efficient frontier curve shown here utilizes total returns and not excess returns. CDO equity volatility might have been underestimated due to a constant reinvestment rate assumption.

We observe that CDO equity expands the efficient frontier that initially comprised other alternative investments and U.S. equity. However, as explained in the next section, a mean variance framework has its own limitations. We would not recommend using the exact asset allocation weights that such an analysis provides. However, the direction of these weights, i.e., whether an investor should go long or short a particular asset, is a useful indicator.

4.3 The pitfalls of a mean variance approach

The efficient frontier draws its own set of criticism and justifiably so. The criticism can be seen at four levels:

- The mean variance approach (as used in calculating the Sharpe Ratio or in deriving the efficient frontier) fails to capture higher moments of return distribution viz. skew and kurtosis. Variance alone is not a complete measure of risk.
- The approach uses historical mean and variance of return to be an estimate of their future values, which might not be the case.
- Performance data based on an index carry a (positive) survivorship bias. However, even if the estimated returns are unbiased, studies suggest that the frontier itself is biased outwards.
- The approach is not suitable for a portfolio containing a large number of assets due to problems of co-linearity.

Investors may prefer a positively skewed portfolio, as it is more likely to have returns exceeding the expected portfolio return. CDO equity returns might be skewed or even bimodal – which makes the standard deviation a poor measure of risk. Therefore, the efficient frontier alone does not capture the true risk-return tradeoff. It must be supported by other measures (such as skew) for an investor to make an informed decision. Nonetheless, the efficient frontier can be used as a tool for asset allocation as long as its results are interpreted with due caution.

In our study, we calculate the return to maturity for each cohort. An efficient frontier would typically require a mark-to-market return on each asset class. Such a calculation is a non-trivial task for CDO equity. Also, CDO equity investors usually hold the asset to maturity or for a fairly long time, making it tough to measure volatility in a meaningful way. A mean variable analysis that we have presented here is therefore an approximation.

5. CONCLUSION

Our reconstruction of historical equity returns and the sensitivity that we ran has established that CDO equity is a viable asset class with attractive returns over long periods of time. We recognize that we have made several assumptions to come up with this analysis; and plan to research and improve upon them in future publications. Although we have not addressed all the pitfalls of an efficient frontier approach to CDO Equity asset allocation, our analysis and the reconstructed return series makes a clear case for adding CDO equity to an alternative investment portfolio. The skewness of the return distribution, however, makes active management of underlying collateral critical to the performance equation. We have clearly established that managers can add/destroy value while managing the asset class.

Tail Dependence and Portfolio Risk

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Modelling default dependence is of prime importance when assessing portfolio credit risk. However it has recently become clear that many of the popular portfolio credit risk models fail to take into account an effect known as tail dependence which has been observed empirically. In this article we study the effect of tail dependence on portfolio measures of risk for a large and typical investment grade portfolio. This can be considerable, for example we find that the Expected Shortfall can increase by 85%.

1. INTRODUCTION

Modelling the loss distribution for a portfolio of default-risky assets is fundamental to almost all activities in today's credit markets. This applies whether one is pricing portfolio default risk in the context of multi-name credit derivatives (default baskets, synthetic CDO tranches), or risk-managing a portfolio of corporate bonds. Whereas information about the default risk of individual obligors can be elicited from the market in a relatively straightforward manner, assessing the likelihood of multiple defaults in a portfolio of issuers involves constructing a model of joint default probabilities. Consequently, one needs to be aware of how the choice of model affects the joint dependence of the assets, and how this choice affects the estimates of portfolio losses, and the corresponding risk measures. Note that in this paper we are only concerned with losses due to the actual default of assets – mark-to-market losses caused by spread movements are ignored.

A popular approach to portfolio credit modelling is to simulate joint defaults, using a copula function to describe the dependence structure. Given the individual asset default probabilities, a copula function is a distribution function which determines the probability of multiple default events occurring together. Popular commercial credit models such as CreditMetrics and KMV's Portfolio Manager both assume a Gaussian copula. In both models, correlations are calibrated via a firm-value approach to equity correlations. This choice however raises the issue of tail dependence.

In simple terms tail dependence is a measure of the extent to which a distribution captures the tendency of multiple extreme events to occur together. It has been shown by Malevergne and Sornette (2001) and also by Mashal and Zeevi (2002) that equity returns exhibit tail dependence. However, the Gaussian copula, which though attractive due to its flexibility and simplicity, does not exhibit any tail dependence, and may therefore be underestimating the tendency of extreme events to occur together.

This has led to the investigation of alternative copulas. A paper by Frey and McNeil (2001) analyzed the dependence of the quantiles of the loss distribution (i.e. the Value-at-Risk) on the choice of copula for large homogeneous portfolios, and in a recent issue of this publication, Mashal and Naldi (2002) documented the influence that the Student-t copula has on the pricing of default baskets. It is however within the context of risk-management, where we are explicitly focusing on the tail of the loss distribution, that we believe that tail-dependent copulas will have a major impact.

In this article, we consider a heterogeneous portfolio of issuers, whose composition is guided by the breakdown of the corporate sector of the Lehman Global Aggregate Index. We focus on how the choice of either Gaussian or Student-t copula influences coherent risk measures such as the Expected Shortfall.

2. JOINT DEFAULTS AND TAIL DEPENDENCE

The simplest way to look at a copula function model of default is to imagine the credit quality of each issuer to be driven by an asset returns process; the issuer defaults if the asset return falls below a certain default threshold. Consider two issuers with asset returns X and Y and default probabilities p_X and p_Y respectively. These default probabilities can be either market-implied or obtained from another source (e.g. rating agency default probabilities or KMV EDFs). Their default thresholds are C_X and C_Y, respectively, which can be calibrated to fit the individual default probabilities, i.e. we have

$$p_X = P\left[X \leq C_X(p_X)\right] \tag{2.1}$$

and

$$p_Y = P\left[Y \leq C_Y(p_Y)\right] \tag{2.2}$$

Note that we have not specified the form of probability distribution for either X or Y. The power of the copula approach is that we can separate this assumption about the form of the individual marginal distributions from the choice of dependence structure. The probability that both issuers default is therefore

$$P\left[X \leq C_X(p_X), Y \leq C_Y(p_Y)\right] \tag{2.3}$$

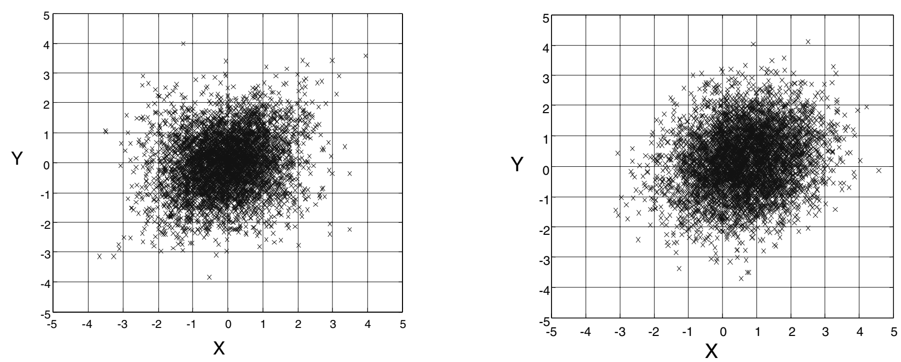
To calculate this probability we are required to model the joint distribution of X and Y. The traditional assumption for joint returns has been the multivariate Gaussian distribution, it underlies both the CreditMetrics and the KMV portfolio analytics approach. With this assumption the dependence between X and Y is completely characterized by the correlation coefficient ρ.

It is difficult to extract information about asset values directly. The most feasible way is to estimate the value of a firm's total liabilities, and to infer the asset value from the basic accounting identity that the sum of the firm's total liabilities and its shareholder equity should be equal to its total asset value. But even this approach is fraught with formidable obstacles. U.S. firms only publish their accounts at most quarterly, and in any case we would need to estimate the market values of all of the firm's debt instruments, as opposed to some accounting measure of debt.

However, it is possible to link the asset return correlation to equity return correlation. This link is based on the observation that both the CreditMetrics and the KMV model are essentially structural models of asset returns in which equity can be viewed as a call option on the assets of the firm. This is a powerful observation as it means that for publicly traded firms, there is sufficient data to make model calibration very feasible. Typically this is accomplished in conjunction with an equity factor model.

Recent empirical analysis of equity return data, e.g. by Mashal and Zeevi (2002) has shown that the multivariate Gaussian distribution can be rejected with confidence in favour of the Student-t. The Student-t distribution is a more generalized form of distribution which nests the Gaussian one. It has an additional parameter known as the *degrees of freedom n*. In Figure 1, we show the scatter plots of a Gaussian distribution and a Student-t distribution with five degrees of freedom, both with a correlation of 20%. We see that for the Student-t distribution, the tendency to observe joint extreme returns is greater – there are more points where X and Y are both large and positive and more points where X and Y are both large and negative than in the Gaussian plot. This effect is most pronounced for a small number n of degrees of freedom. As n tends to infinity, the Student-t distribution converges to the Gaussian one.

Figure 1. Scatter plot of returns with 20% correlation for Student-t distribution with 5 degrees of freedom (left), and Gaussian distribution (right)



One way to formalize the ability of a distribution to capture joint extreme events is via the concept of tail dependence. Specifically we look at the value of λ where

$$\lambda = \lim_{p_X, p_Y \to 0} P\left[X < C_X(p_X) \mid Y < C_Y(p_Y)\right] \tag{2.4}$$

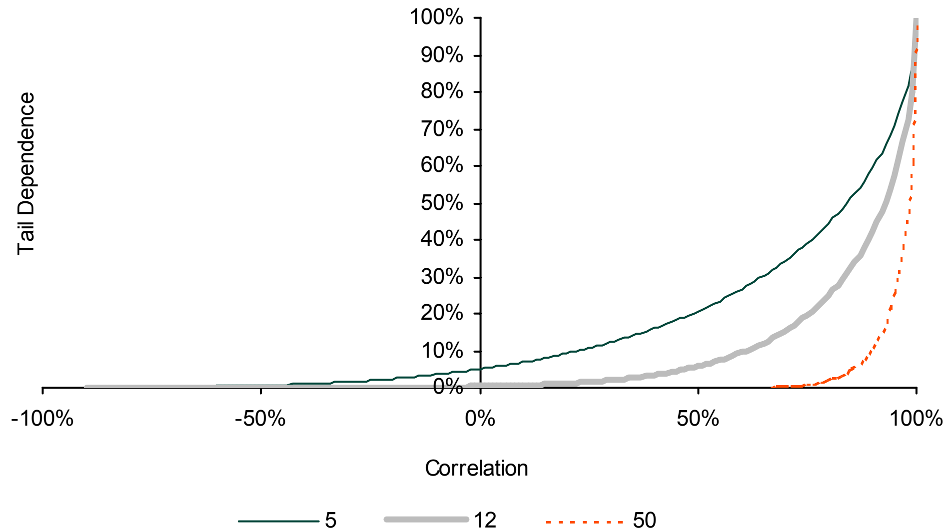
i.e. we compute the probability that X defaults conditional on Y also defaulting, in the limit where the probability of each name defaulting goes to 0%. If in this limit λ is zero, then X and Y are asymptotically independent, i.e. for rare events such as defaults, the distribution does not capture dependence between X and Y very well. For the Gaussian distribution, this limit does in fact turn out to be zero. However for the Student-t distribution, the limit can be computed[1] to be

$$\lambda = 2t_{n+1}\left(-\sqrt{\frac{(n+1)(1-\rho)}{1+\rho}}\right) \tag{2.5}$$

[1] *cf. Embrechts et al. (2002).*

where t_{n+1} is the distribution function of the Student-t distribution with n+1 degrees of freedom. Consequently, the Student-t distribution always exhibits tail dependence, even for zero or negative correlation. As shown in Figure 2, the tail dependence is greatest for small numbers of degrees of freedom and tends to zero as n goes to infinity. It is also an increasing function of correlation.

Figure 2. Tail dependence coefficient as a function of correlation for different numbers of degrees of freedom



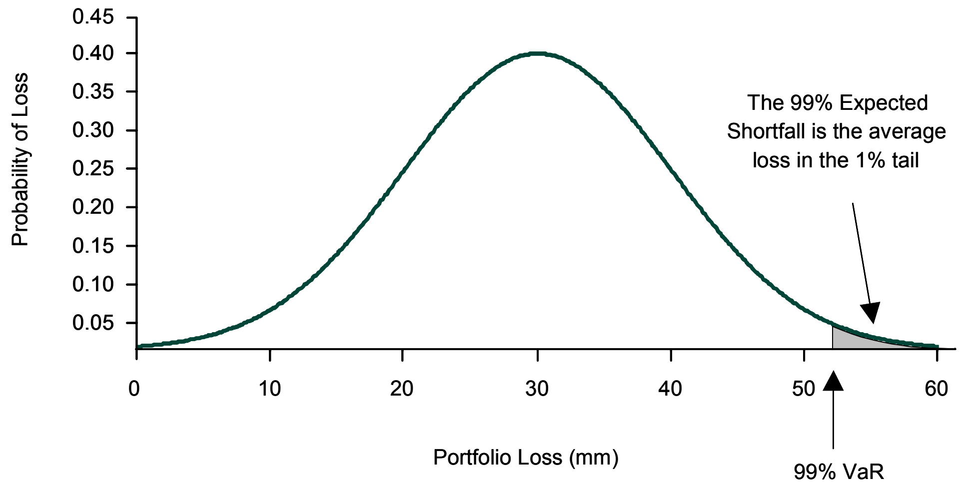
3. COHERENT RISK MEASURES FOR PORTFOLIO CREDIT

To analyze how the choice of distribution influences portfolio losses, we must choose risk metrics. In what follows we will focus on both Value-at-Risk (VaR) and Expected Shortfall (ES).

If the VaR is 23MM at a 99% level of confidence, this means that the realized portfolio loss will exceed 23MM in only 1% of all cases. However, VaR does not allow us to make an assessment of how bad things can get in this 1% tail. For example a 99% VaR would be unable to distinguish between two distributions which are identical except for their tails – in one it is a smooth Gaussian type tail which quickly declines to zero and has a total area of 1%, while in the other there is a single peak at a large loss amount which occurs with a probability equal to 1%. For liquid market instruments such as government bonds, FX rates and equities, the chances of having the latter distribution is unlikely. However, in the world of credit, the tail can definitely contain such rare but potentially large losses. This is a defect remedied when using Expected Shortfall (ES).

The Expected Shortfall is also defined in conjunction with a confidence level. For example the 99% ES computes the expected loss in the 1% tail, conditional on the loss being in that 1% tail. As such, the ES will always be greater or equal to the VaR to the same confidence level. Speaking somewhat loosely, VaR tells us where the tail of the loss distribution starts, whereas ES gives us information about the shape of the tail.

Figure 3. Value-at-Risk and Expected Shortfall are a function of the loss distribution



In addition to being able to measure the shape of the tail of the distribution, the ES has the advantage of being a so-called *coherent risk measure*, which means that it avoids certain drawbacks of VaR concerning the aggregation of risks in a portfolio[2].

4. LARGE PORTFOLIO SIMULATION STUDY

To assess the impact of the choice of Gaussian or Student-t distribution on the portfolio loss distribution, we simulate the joint defaults of an idealized portfolio of about 300 equally weighted investment grade issuers, distributed across the three sectors Industrial, Financial, and Utility, and also across the G7 countries. Here we are only concerned with losses due to default – mark to market losses due to spread movements are not taken into account. The distribution of notional amounts across these different buckets is guided by the breakdown of the corporate sector of the Lehman Global Aggregate Index, see Figure 4.

Figure 4. Portfolio composition

	US	Canada	France	Germany	Italy	Japan	UK
Finance AAA	3.72%	0.00%	0.26%	0.34%	0.02%	0.00%	0.05%
AA	8.82%	0.05%	1.76%	1.42%	0.26%	0.26%	1.98%
A	13.75%	0.03%	0.64%	0.91%	0.30%	8.45%	0.47%
BBB	1.60%	0.00%	0.03%	0.00%	0.00%	0.80%	0.12%
Industrial AAA	0.71%	0.00%	0.00%	0.00%	0.00%	0.00%	0.10%
AA	3.46%	0.07%	0.22%	0.54%	0.02%	1.71%	0.54%
A	8.66%	0.26%	0.66%	1.19%	0.00%	0.48%	1.78%
BBB	13.38%	0.58%	2.33%	1.67%	1.06%	0.47%	2.38%
Utility AAA	0.01%	0.00%	0.00%	0.00%	0.00%	0.00%	0.01%
AA	0.02%	0.00%	0.00%	0.00%	0.09%	4.02%	0.04%
A	1.61%	0.05%	0.14%	0.47%	0.12%	0.00%	1.20%
BBB	3.45%	0.01%	0.16%	0.00%	0.03%	0.00%	0.29%

[2] *For an extensive discussion of risk measures applied to portfolio credit risk, and a discussion of coherent risk measures, the interested reader is referred to O'Kane and Schloegl (2002).*

We consider the distribution of losses occurring up to a five year time horizon, using S&P's idealized default probabilities in conjunction with a recovery rate of 30%. This is consistent with average recovery rates observed in the market over the recent past.

Figure 5. Standard and Poor's five year cumulative default probabilities

AAA	AA	A	BBB
0.28%	0.76%	1.11%	2.50%

To obtain the correlation coefficients, we have used an equity factor model[3] which allows us to derive a correlation estimate for each sector and country that we have used in the portfolio composition as shown in Figure 4. This is derived from a maximum likelihood estimation of the distribution parameters governing the joint equity returns. In the Gaussian case these parameters are the correlation coefficients, in the Student-t case the correlation coefficients and the number of degrees of freedom. As the likelihood is maximized over all parameters, the estimated correlation coefficients are not the same in the Student-t case as in the Gaussian one. The maximum-likelihood estimate for the number of degrees of freedom of the multivariate Student-t distribution turns out to be 12. In Figure 6, we show the average correlations for each case. We have one bucket for each sector and country, giving 28 buckets overall. We report the average correlation between issuers both within each bucket and between buckets.

On the whole, the Student-t correlation matrix seems fairly similar to the Gaussian one, with the absolute differences ranging roughly between –3% and 3%. For small correlation coefficients, however, the relative differences can be much larger, exceeding 100% in some cases.

Figure 6. Average correlation coefficients between and within buckets for Gaussian and Student-t copula functions.

Copula Type	Gaussian	Student-t
Within Bucket	23.25%	22.83%
Between Buckets	7.98%	8.05%

To analyze the risk of the portfolio, we consider the VaR and ES at 95%, 99%, and 99.9% levels of confidence. Figure 7 shows the comparison for VaR, whereas the results for the ES are shown in Figure 8.

[3] *The equity factor model was constructed by our colleague Marco Naldi, whom we thank for providing the correlation estimates used in this article.*

Figure 7. Value-at-Risk for Gaussian and Student-t copula at various confidence levels

Confidence Limit	95%	99%	99.9%
Gaussian	2.85%	4.75%	7.83%
Student-t	4.03%	8.31%	14.95%
Difference	1.19%	3.56%	7.12%
Relative Difference	41.67%	75.00%	90.91%

In each case, we calculate the relative difference by dividing the difference between the two results by the result for the Gaussian copula. We see that the Student-t copula assumption produces significantly higher risk measures. At the widely-used 99% confidence limit, the Gaussian copula underestimates the VaR by a factor of 75%. This is due to the tail dependence generated by the Student-t distribution.

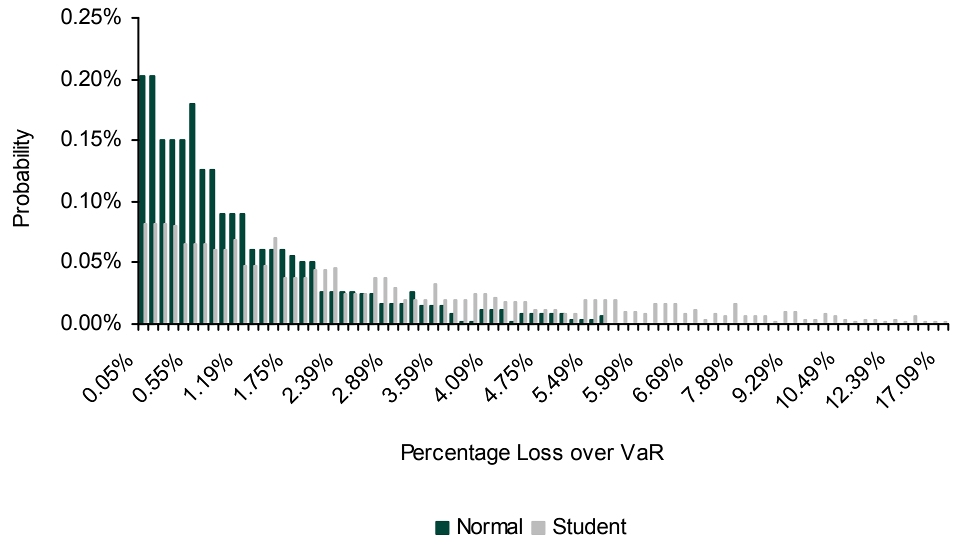
Figure 8. Expected Shortfall for Gaussian and Student-t copula at various confidence levels

Confidence Limit	95%	99%	99.9%
Gaussian	4.10%	6.06%	9.06%
Student-t	6.66%	11.22%	18.07%
Difference	2.56%	5.16%	9.00%
Relative Difference	62.58%	85.26%	99.32%

The difference is more pronounced for the ES. This is because ES actually measures the shape of the tail of the loss distribution. It is therefore better able to detect changes in the tail behavior of losses. Also, we see that for both risk measures the difference between the two distributions is more pronounced at higher confidence levels. Taking the 99% confidence limit again, we see that the Gaussian copula underestimates the value of the ES by just over 85%. This is to be expected, because the concept of tail dependence becomes more and more relevant as we go further out into the tail of the distribution.

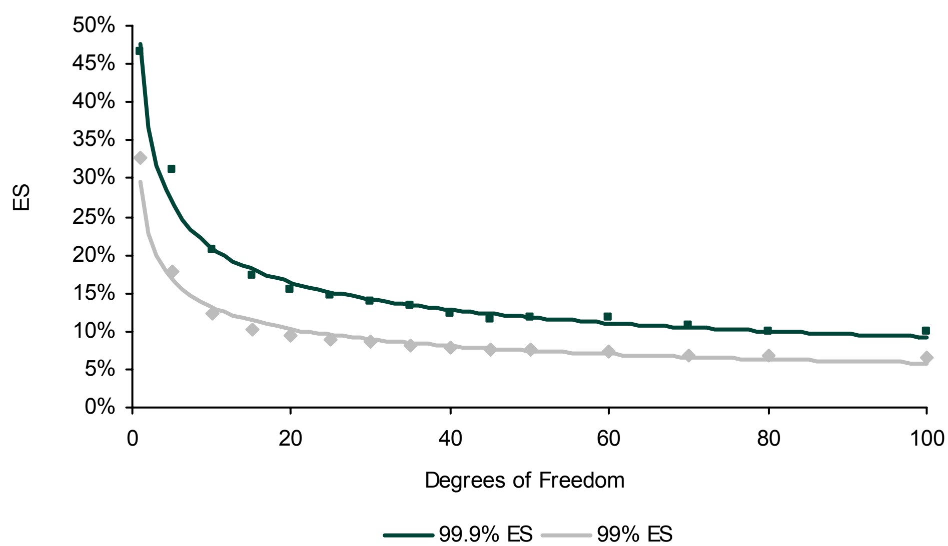
The difference in the shapes of the tails of the two distributions is shown in Figure 9. It shows histograms of the two loss distributions, where the abscissa is scaled to be the excess loss over the 99% VaR. A comparison of the two histograms immediately shows how the tail of the loss distribution computed using the Student-t copula is significantly longer than the Gaussian one.

Figure 9. Comparison of Tail of Gaussian and Student-t loss distributions



From the results of our simulation study, it is clear that the number of degrees of freedom has an important impact on the risk profile of the portfolio. Recall that the Student-t distribution converges to the Gaussian one as the number of degrees of freedom tends to infinity. To gain a better understanding of the influence of this parameter, we consider the loss distribution for various numbers of degrees of freedom. For consistency, we use the correlation matrix estimated in the Gaussian case. As the number of degrees of freedom tends to infinity, the risk parameters should converge to those obtained in the Gaussian case.

Figure 10. Expected Shortfall of portfolio as a function of the number of degrees of freedom



In Figure 10 we see that the ES of the portfolio at the 99% and the 99.9% confidence level is a monotonically decreasing function of the number of degrees of freedom. This is to be expected, as tail dependence is most pronounced for a small number of degrees of freedom. It is also interesting to note the relatively rapid convergence of the ES to its asymptotic value. The graph for the VaR statistic is qualitatively very similar, which is why we have omitted it here.

5. CONCLUSIONS

Our study has analyzed the impact of the choice of copula function on a large, but realistic investment grade credit portfolio. The effect of moving to the more realistic Student-t copula is significant, in particular if we are using the Expected Shortfall at high levels of confidence. To the commonly used 99% confidence limit, we have clearly shown that the assumption of a Gaussian copula underestimates the ES when compared to the Student-t copula by 85%. This has significant implications for risk-managers and for banks who employ a modelled approach to calculating economic and regulatory capital. We suggest that risk-managers who focus on portfolio credit risk should therefore consider amending current models to take this effect into account.

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New Estimation Options for the Lehman Brothers Risk Model: Adjusting to Recent Credit Events[1]

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We introduce an important new feature of the Lehman Brothers Risk Model. As a response to a recent, unprecedented rise in issuer-specific volatility, we are now offering the option to apply a time weighting scheme to increase the influence of recent observations on the estimate of idiosyncratic risk. An analogous choice is offered for the estimation of the systematic covariance matrix, although the two options can be exercised separately. We show that the model with time-weighted issuer-specific risk would have recently produced tracking error volatility estimates more consistent with realized observations. Finally, we introduce a marginal risk decomposition methodology that highlights the consequences of alternative estimation methods on the risk attribution profile.

1. INTRODUCTION

Most of the recent credit blow-ups stemmed from company-specific events that have drastically changed the way investors react to the dissemination of news. It is now a widely accepted opinion that the issuer-specific component of a corporate bond's return accounts for a significant fraction of its total volatility. Credit managers need to reevaluate the appropriate amount of diversification of their portfolios, and quantitative tools for credit risk management must adapt to the current environment if they are to produce realistic statistics.

In the previous issue of this *Quarterly*, Naldi, Chu, and Wang (2002) described the credit component of the new Risk Model, a tool for the risk management of benchmarked positions. The model uses a linear factor representation of bond returns to provide users with an estimate and a detailed decomposition of the statistical distance between their portfolios and a pre-assigned benchmark. This distance is known as tracking error volatility (TEV), and it is calculated as the standard deviation of the return of the active portfolio.[2] The reported TEV depends on the mismatches between the portfolio and the benchmark, on the factor covariance matrix, and on the idiosyncratic volatilities of the individual assets.

In previous versions of the Risk Model (see, for example, Dynkin, Hyman, and Wu [1999]), both the systematic covariance matrix and the issuer-specific risks were estimated assigning equal weights to a dataset of monthly observations going back to the 1980s. While the equal weighting of historical observations produces an efficient estimate of the unconditional (long-run) covariance matrix of returns, the current perception of a changed credit environment calls for an estimation methodology that is able to promptly incorporate the recent events into the estimated model parameters.

[1] *We would like to thank Hank Haligowski, Krishna Pelluru, Nancy Roth, and Gary Wang, who were responsible for the POINT implementation of the new Risk Model capabilities described in this article. We are grateful to Lev Dynkin, Vadim Konstantinovsky, Stuart Turnbull, and Dominic O'Kane for discussions and comments. AMB would also like to thank Bob Litterman, Jacob Rosengarten, Jacques Longerstaey and Peter Zangari for numerous discussions on risk decomposition methodology.*

[2] *The "active portfolio" is the difference between the portfolio held by the investor and the pre-assigned benchmark.*

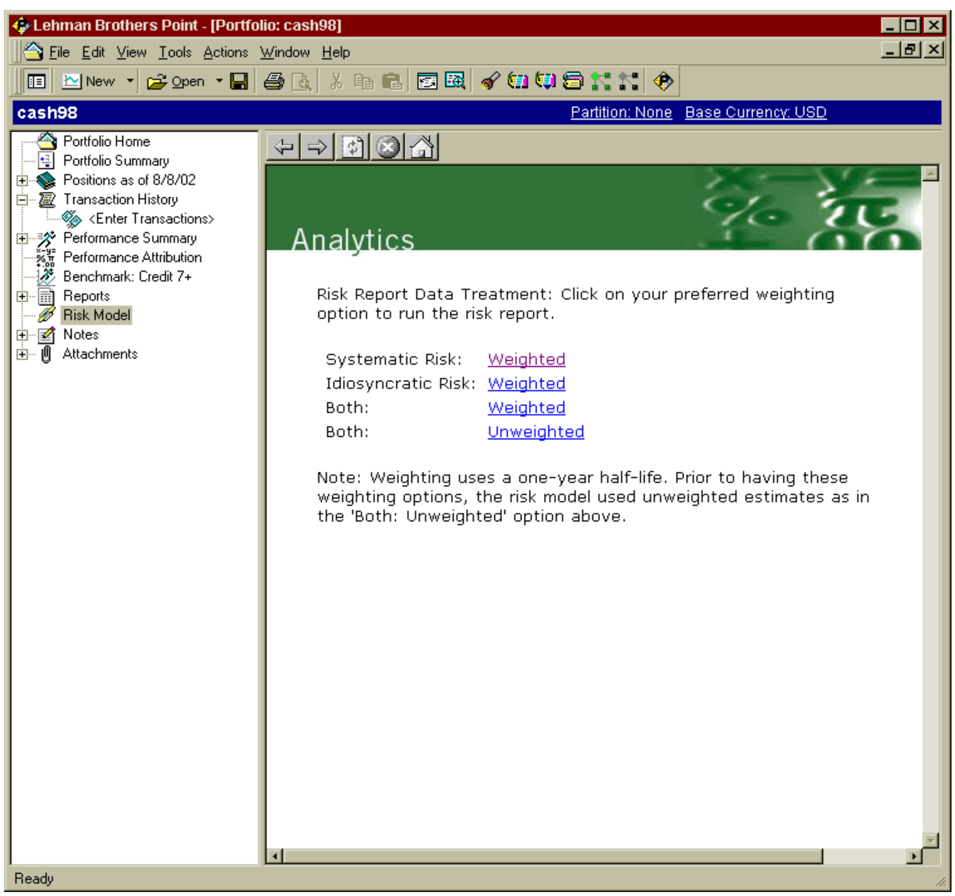
2. NEW ESTIMATION OPTIONS

In the new Risk Model available on Lehman Brothers Portfolio and Index Tool (POINT), the user now has an option to control the estimation of some crucial parameters. Figure 1 depicts the initial screen that a user will face when running a risk report. The option menu offers the possibility of applying a time-weighting scheme to the estimation of the factor covariance matrix (systematic risk) and/or the issuer-specific volatilities (idiosyncratic risk).

Consistent with the intuition that the recent credit events are largely a reflection of issuer-specific shocks, the current estimates of idiosyncratic volatilities differ dramatically when we overweight the recent company-specific components in our estimation procedure. On the other hand, the estimate of the systematic covariance matrix, as of today, is not very sensitive to the application of a weighting scheme, since the recently observed factor volatilities and correlations have not been too far from their long-run levels. Consequently, this article will focus on the effects of weighting the estimation of idiosyncratic volatilities.

An exponential time decay is used to weight the data. The speed of the time-decay is set equal to one-year half-life, which means that a one-year-old observation will receive one half of the weight received by the most recent data point. The choice of this speed has been optimized by comparing the performance of different weighting schemes in out-of-sample tests analogous to those described in Naldi et al. (2002).

Figure 1. New Estimation Options in POINT



3. WHAT DIFFERENCE DOES IT MAKE?

3.1 Adjustment Speed

The new Risk Model estimates the idiosyncratic component of a bond's spread change using a panel dataset of residuals from the systematic factor regressions. The fitted spread volatility for a credit bond depends on the industry/rating bucket that it belongs to, according to the 27-cell partition defined in Figure 2. The idiosyncratic volatility of the bond's spread return can then be obtained by multiplying this number by the bond's (option-adjusted) spread duration.

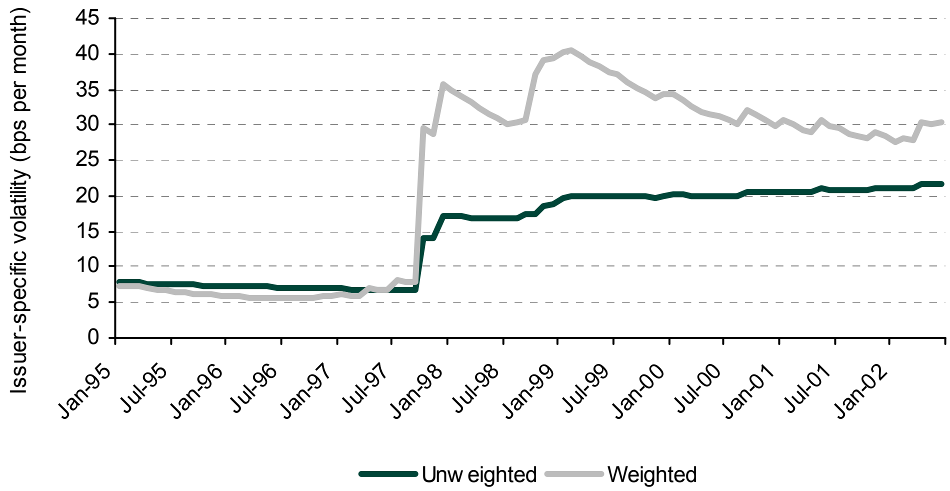
Figure 2. Industry / Rating Partition of the Credit Index for Idiosyncratic Risk Estimation

	AAA/AA	A	BBB
FINANCIALS			
Banking and Brokerage	BAN1	BAN2	BAN3
Financial Companies, Insurance and REITS	FIN1	FIN2	FIN3
INDUSTRIALS			
Basic Industries and Capital Goods	BAS1	BAS2	BAS3
Consumer Cyclicals	CCY1	CCY2	CCY3
Consumer Non-Cyclicals	NCY1	NCY2	NCY3
Communication and Technology	COM1	COM2	COM3
Energy and Transportation	ENE1	ENE2	ENE3
UTILITIES	UTI1	UTI2	UTI3
NON-CORPORATE	NON1	NON2	NON3

For some of our industry/rating cells, Figures 3-6 show the effect of time-decay on the adjustment speed of the estimates of idiosyncratic spread volatility. Every point shown in these graphs is based solely on information available at the corresponding time (marked on the x-axis), so that the dynamics depicted here highlight the speed at which new realizations are incorporated into the estimates.

The time-series of idiosyncratic spread volatility for the Non-Corporate/BBB cell (Figure 3) clearly shows the effect of the Asian crisis. Idiosyncratic volatility represents a source of diversifiable risk that is uncorrelated with the sources of systematic risk, and the Asian crisis was certainly reflected in the realizations of several systematic factors affecting Non-Corporates and foreign issuers in general. Figure 3, however, shows that the volatility of the non-systematic component of spread also increased significantly in the fall of 1997. The difference between the equally weighted estimate (jumping from 7 to 17 bp per month in the fall of 1997), and the time-weighted estimate (going up to 36 bp per month) is quite significant.

Figure 3. Idiosyncratic Spread Volatility: Non-Corporates / BBB



The Communication and Technology / BBB cell (Figure 4) shows how much faster the time-weighted volatility estimate has adjusted to idiosyncratic shocks starting from late 2000. Recently, the huge spread widenings of names such as WCOM have further increased the difference between the time-weighted and the equally weighted estimates.

Figure 4. Idiosyncratic Spread Volatility: Communication and Technology / BBB

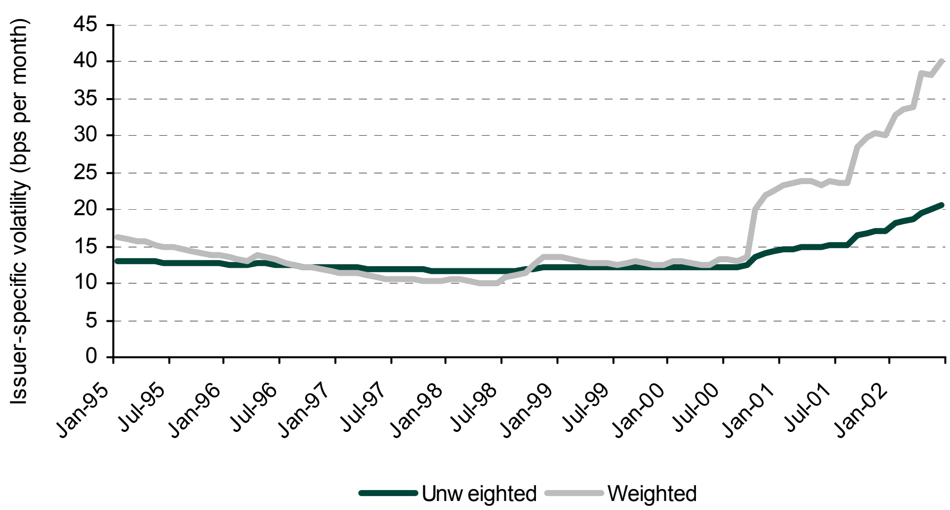
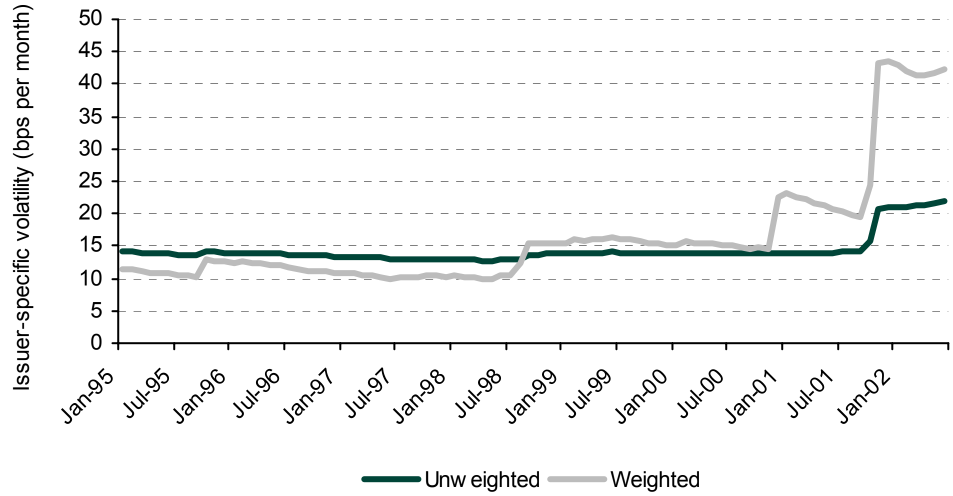
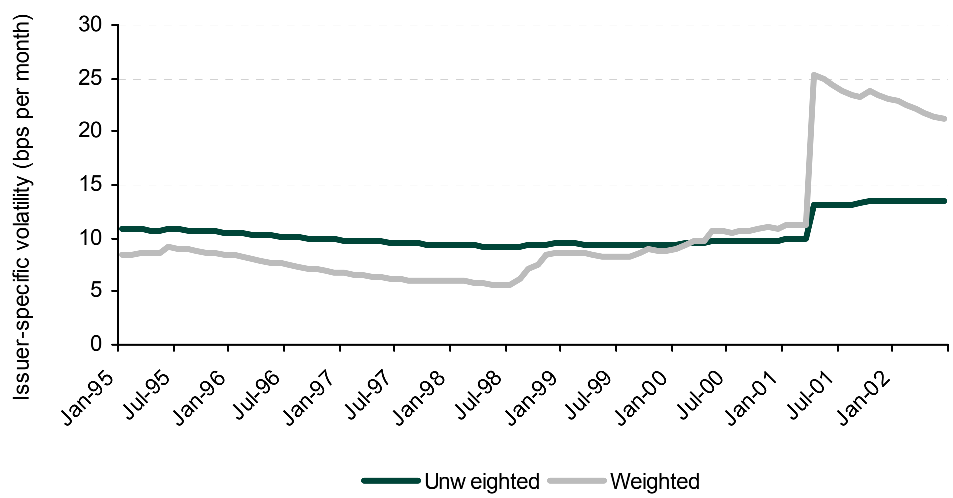


Figure 5. Idiosyncratic Spread Volatility: Utilities / BBB



The Utility/BBB cell (Figure 5) shows two sharp increases in the recent history of the estimated idiosyncratic volatility. The first, at the end of 2000, is due to the California crisis and is largely undetected by the equally weighted estimator. The second, in the fall of 2001, is due to Enron's blow-up. As a last example, the Financial Companies / A cell (Figure 6) had some large idiosyncratic movement when the equity valuation of the new economy fell abruptly in the spring of 2001.

Figure 6. Idiosyncratic Spread Volatility: Financial Companies, Insurance, REITS / BBB



3.2 Out-of-Sample Performance

Figure 7 illustrates the results of an out-of-sample comparison between the previous version of the Lehman Brothers Risk Model (delivered in PC Product) and the new Risk Model now available in POINT. The example covers the 18-month period Jan 01-Jun 02 and uses an actual portfolio of a credit fund manager benchmarked against the Lehman Brothers 7+ HG Credit Index.

At the beginning of each month, we calculated TEVs using only information available at that point in time. Then, we observed the realized outperformance over the following month: Figure 7 presents the ratio between the two numbers, i.e., the number of "realized standard deviations". The bottom rows summarize the percentage of times that the absolute values of these ratios have exceeded 1, 2 and 3, respectively.

Comparing the first two columns of Figure 7 highlights the difference in performance between the old and the new risk model. Even if the number of observations is too small to draw statistically significant conclusions, this comparison shows that over this troubled period, the TEVs produced by the new model show a higher consistency with the realized excess returns. Looking at the performance of the old model, TEVs seem to be underestimated: the true distribution of excess returns should have extremely thick tails for these realizations to exceed 3 standard deviations 28% of the times.

The difference between the first two columns of Figure 7 is not related to the application of time decay, since we are comparing two equally weighted estimators. It is rather related to the finer partition of the credit universe employed by the new risk model. While the model in PC Product employs an additive effect of industry and rating for modeling systematic risk and does not use any industry classification as a determinant of idiosyncratic risk, the new model in POINT employs a cross (multiplicative) effect of industry and rating for both purposes. This turned out to be a major improvement over the past few months, when both idiosyncratic risk and realized factor volatilities changed asymmetrically across different industry groups.

Comparing the second and third columns of Figure 7 shows the improvements that a time-weighted estimation of idiosyncratic risk can offer in similar market conditions. In particular, the weighted TEVs are consistent with a more "normal" shape of the central portion of the excess return distribution, since they allow the realized outperformance to cross the one-standard-deviation band only 38% of the times.

We suggest taking these conclusions quite cautiously for a number of reasons. First, even if we are interested in analyzing a short history because of the particular events that took place in the credit markets over the past few months, we should recognize that no statistically compelling evidence can be produced on the basis of 18 data points. Second, the comparisons above were based on a specific example, i.e., on a particular portfolio/benchmark pair. Although we highlighted results that we obtained in a variety of different exercises, the magnitudes of these results differ dramatically as a function of the selected portfolios.

Finally, readers should notice that overweighting recent observations incorporates recent events faster into the estimates by construction and does not necessarily make the time-weighted statistics better estimators. For example, in the summer of 1998, the application of an aggressive time-decay scheme for the estimation of the systematic covariance matrix would have virtually eliminated the information contained in the 1990-1991 factor realizations while overweighting the period of relative calm that preceded the storm. This would have certainly set the stage for large (and generally undesired) surprises.

Figure 7. Comparison of Out-of-sample Performance

	Number of TEVs		
	Old Model (PCProduct) Equally Weighted	**New Model (POINT) Equally Weighted**	**New Model (POINT) Weighted Idiosyncratic Risk**
Jan-01	3.20	2.21	2.18
Feb-01	-1.73	-1.00	-0.99
Mar-01	0.36	0.58	0.56
Apr-01	-0.68	-0.22	-0.21
May-01	-2.18	-1.21	-1.09
Jun-01	0.23	0.48	0.44
Jul-01	2.29	1.56	1.41
Aug-01	-0.64	-0.24	-0.21
Sep-01	1.83	1.04	0.94
Oct-01	-2.44	-1.39	-1.28
Nov-01	4.28	2.63	2.30
Dec-01	0.36	0.03	0.02
Jan-02	1.43	1.10	0.96
Feb-02	0.29	0.26	0.22
Mar-02	-3.25	-1.10	-0.94
Apr-02	4.71	2.80	2.40
May-02	-0.13	-0.12	-0.09
Jun-02	-13.78	-5.48	-4.42
	outside 1 TEV= 61%	**outside 1 TEV= 61%**	**outside 1 TEV= 38%**
	outside 2 TEV= 44%	**outside 2 TEV= 22%**	**outside 2 TEV= 22%**
	outside 3 TEV= 28%	**outside 3 TEV= 6%**	**outside 3 TEV= 6%**

Wise users of the new Risk Model will always keep in mind that the ability to react faster to market innovations must be traded-off with the potential for large surprises that a time weighting scheme inherently produces. This is why our model offers an *option* between alternative estimation methods: after all, this choice is really an expression of a view that a professional manager may want to control. This said, what we intended to show in this section is that in recent months, the TEVs obtained by weighting the idiosyncratic risk estimates would have been more consistent with realized residual returns.

4. RISK ATTRIBUTION PROFILE: THE EFFECT OF TIME-DECAY

The new estimation options highlighted above bring to the forefront the issue of decomposing tracking error volatility into its various components. Separating the relative contributions of issuer-specific and common factors is now of particular interest, since we are offering the choice of applying different weighting schemes on either source of risk.

Decomposing risk is not a trivial task. In Appendix A, we offer a brief introduction to a risk decomposition methodology that we employ in this section to analyze a sample portfolio and compare the weighted and unweighted versions of the new Risk Model.

Similarly to the analysis in the previous section, we focus on the impact of weighting the estimation of issuer-specific risk. We have already seen that switching to a fast-decaying estimate can significantly increase the level of forecasted idiosyncratic TEV. The following example will show that the impact of adopting a time-weighting scheme on the portfolio risk decomposition analysis is at least as significant.

We have taken as a sample a realistic investment grade portfolio consisting of 125 bonds, benchmarked against the Lehman Credit Index. Some basic characteristics of the portfolio are shown in Figure 8.

Figure 8. Sample Investment Grade Portfolio vs. Credit Index

	Portfolio	Benchmark	Active (Diff)
# of Bonds	125	3907	3907
Market Value (MM)	282.91	1,921.05	
OAS	256.95	170.80	86.15
OA Duration	5.22	5.54	(0.32)
OA Spread Duration	5.19	5.46	(0.28)

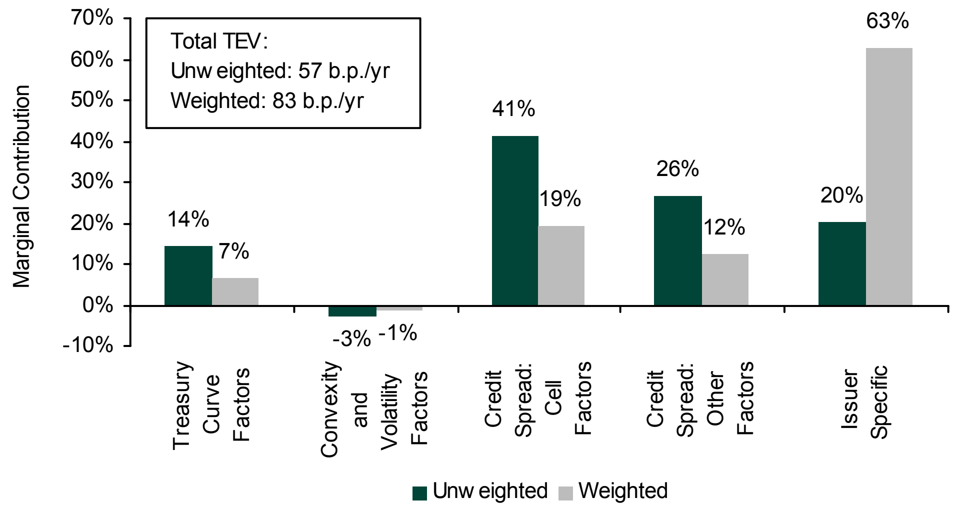
The TEV of the portfolio in the unweighted risk model turns out to be 57 bp per annum. In contrast, the TEV with a time-weighted idiosyncratic risk estimate jumps to 83 bp per annum. This alone makes it clear that the impact of weighting idiosyncratic risk is large.

The analysis of the marginal risk contributions reveals even bigger changes. Figure 9 shows a side-by-side comparison of the risk attribution profile in the unweighted and weighted risk model view of this portfolio. As we can see, the vast majority of the risk sensitivity in the context of the time-weighted risk model comes from idiosyncratic risk, while in the equally weighted case, the more dominant contribution came from systematic spread factors. We remind the reader that the marginal risk decomposition is not a decomposition of the entire TEV, but the sensitivity of TEV to small changes in the portfolio exposures (see Appendix A). Since the increase of the tracking error volatility was caused by a change in the level of the idiosyncratic risk, then the amplified contribution from this source of risk to marginal decomposition is as expected.

Let us now take an even more detailed look into the aggregated marginal risk contributions. As we explain in the Appendix, the marginal risk contributions are additive, so they can easily be sliced and diced into various buckets that a portfolio manager may like to define. One natural partition is done by the same 27 industry / rating cells that are used in the risk model to define both the systematic risk factors and the industry factors and the determinants of idiosyncratic risk (Figure 2).

Notice that in what follows, the risk contribution from a given cell is *not* the contribution from the risk factor synonymous with that cell's name, but the aggregate risk contribution of all securities that belong to that cell. For each individual asset, this contribution will include treasury risk, volatility risk, spread risk, and idiosyncratic risk. While quantitative researchers and risk managers may still prefer a view of risk based on the model's factors, portfolio managers may want to think about risk in terms of "buckets". One of the advantages of this risk decomposition methodology is that it can easily satisfy both preferences.

Figure 9. Impact of Time Weighting on Risk Attribution Profile



Using the same portfolio described earlier, Figure 10 reports the marginal contributions of the systematic and idiosyncratic components to total TEV, and shows how this decomposition changes as we move from an equally weighted to a time-weighted estimation of idiosyncratic risk.

The top 3 overweight and top 3 underweight positions are highlighted in gray. Of course, as we emphasize in the Appendix, the top risk contributors are not necessarily the buckets with the largest weight deviation from the benchmark. In both the weighted and the unweighted versions of the model, the top 5 risk contributors are highlighted in bold.

The major jump in risk when we switch to a time-weighted estimation of issuer-specific risk turns out to be driven by the Communications and Technology / BBB cell. While this group displays the largest overweight position, it did not even show up in the top 5 risk contributors of the unweighted risk model. In contrast, it shows up as the single largest risk contributor in the time-weighted model, with more than 33% of risk sensitivity. Part of that contribution is due to the demotion of the Non-corporate AA(A) sector from the first place in terms of marginal risk contribution with 23% to a distant second with only 11% of risk sensitivity.[3]

[3] *In this sector, the risk is due to the highest underweight with respect to the index – i.e., it is not the risk of the securities that portfolio managers own but the risk of the securities that they don't own.*

Figure 10. Sample Investment Grade Portfolio vs. Credit Index

		Marginal Contribution to Risk: Unweighted			Marginal Contribution to Risk: Weighted Spec. Risk		
Sector	**Active Weight**	**Total TEV**	**Issuer Specific**	**Common Factor Risk**	**Total TEV**	**Issuer Specific**	**Common Factor Risk**
Banking and Brockerage: AA(A)	-5.63%	**11.54%**	**0.98%**	**10.57%**	**5.91%**	**0.93%**	**4.98%**
Banking and Brockerage: A	0.06%	-1.23%	1.51%	-2.74%	1.32%	2.61%	-1.29%
Banking and Brockerage: BBB	0.48%	-1.49%	0.23%	-1.72%	-0.42%	0.39%	-0.81%
Finance Companies, Insurance and REITs: AA(A)	-2.93%	**8.19%**	**0.20%**	**7.98%**	3.94%	0.18%	3.76%
Finance Companies, Insurance and REITs: A	7.60%	-3.82%	1.65%	-5.47%	1.11%	3.68%	-2.58%
Finance Companies, Insurance and REITs: BBB	0.20%	1.56%	0.18%	1.38%	1.05%	0.40%	0.65%
Basic Industries and Capital Goods: AA(A)	-0.35%	0.93%	0.00%	0.93%	0.44%	0.00%	0.44%
Basic Industries and Capital Goods: A	-2.11%	5.58%	0.19%	5.39%	2.74%	0.20%	2.54%
Basic Industries and Capital Goods: BBB	-2.45%	6.44%	0.15%	6.29%	3.38%	0.42%	2.96%
Consumer Cyclicals: AA(A)	-0.88%	2.16%	0.03%	2.13%	1.02%	0.01%	1.00%
Consumer Cyclicals: A	-0.70%	4.37%	0.24%	4.13%	2.24%	0.29%	1.94%
Consumer Cyclicals: BBB	2.90%	1.73%	0.66%	1.07%	2.47%	1.97%	0.50%
Consumer Non-cyclicals: AA(A)	-1.61%	5.07%	0.02%	5.05%	2.40%	0.02%	2.38%
Consumer Non-cyclicals: A	-0.48%	**8.48%**	**0.27%**	**8.21%**	**4.04%**	**0.17%**	**3.87%**
Consumer Non-cyclicals: BBB	2.32%	-6.54%	1.42%	-7.97%	-1.63%	2.12%	-3.75%
Communications and Technology: AA(A)	-1.39%	4.64%	0.04%	4.60%	2.29%	0.12%	2.17%
Communications and Technology: A	2.18%	-0.66%	0.70%	-1.35%	**4.49%**	**5.13%**	**-0.64%**
Communications and Technology: BBB	16.08%	4.54%	8.33%	-3.78%	**33.18%**	**34.96%**	**-1.78%**
Energy and Transportation: AA(A)	-0.91%	2.60%	0.00%	2.60%	1.23%	0.00%	1.22%
Energy and Transportation: A	-0.75%	3.39%	0.09%	3.30%	1.65%	0.10%	1.56%
Energy and Transportation: BBB	0.97%	5.09%	1.40%	3.69%	2.92%	1.19%	1.74%
Utilities: AA(A)	-0.18%	0.48%	0.00%	0.48%	0.23%	0.00%	0.23%
Utilities: A	-0.33%	3.40%	0.16%	3.24%	1.77%	0.24%	1.53%
Utilities: BBB	2.50%	-4.83%	1.28%	-6.11%	3.50%	6.38%	-2.88%
Non-corporate credit: AA(A)	-9.58%	**23.47%**	**0.30%**	**23.17%**	**11.11%**	**0.19%**	**10.92%**
Non-corporate credit: A	-2.16%	**7.72%**	**0.14%**	**7.58%**	3.66%	0.09%	3.57%
Non-corporate credit: BBB	-2.88%	7.16%	0.29%	6.87%	3.97%	0.73%	3.24%

Focusing on the next level of detail, we can see that the vast majority of the 33% of risk contribution coming from the Communications and Technology / BBB cell is really due to idiosyncratic risk. We know from the previous section (Figure 3) that overweighting recent observations greatly increases the estimate of idiosyncratic risk for this group. It is therefore not surprising that the adoption of a time-weighted estimator had such a dramatic impact on this cell's risk ranking.

We can drill down even deeper and show the biggest risk contributors in terms of issuer positions. This is done in Figures 11 and 12 for the unweighted and time-weighted models, respectively. Similarly to the risk attribution profile in Figure 10, the issuer risk attribution profile is also done as an aggregation of the marginal risk contributions of individual securities. Just as before, we see not only large changes in the risk contributions of the most risky issuer positions, but even a change in the rankings of the top risk positions.

Figure 11. Top 5 Most Risky Issuer Positions: Unweighted Risk Model

		Marginal Contribution to Risk		
Issuer	**Active Weight**	**Total TEV**	**Issuer Specific**	**Common Factor**
SPRINT CAPITAL CORP	2.10%	5.89%	1.10%	-8.61%
GENERAL ELEC CAP CORP MTN	-1.68%	5.14%	0.20%	6.23%
CITIZENS COMMUNICATIONS	2.05%	4.56%	0.92%	-8.02%
ITALY (REPUBLIC OF)	-1.29%	4.13%	0.10%	4.15%
AT&T WIRELESS GROUP	1.16%	3.66%	0.36%	-5.08%

Figure 12. Top 5 Most Risky Issuer Positions: Time-Weighted Risk Model

		Marginal Contribution to Risk		
Issuer	**Active Weight**	**Total TEV**	**Issuer Specific**	**Common Factor**
DEUTSCHE TELEKOM INTL FIN	1.70%	7.18%	8.27%	-5.18%
SPRINT CAPITAL CORP	2.10%	6.87%	4.61%	-4.06%
CITIZENS COMMUNICATIONS	2.05%	5.66%	3.94%	-3.78%
KONINKLIJKE KPN NV	1.92%	3.22%	5.84%	-4.53%
AT&T WIRELESS GROUP	1.16%	3.09%	1.54%	-2.39%

A comparison between Figures 11 and 12 shows that Deutsche Telecom, which does not even show up in the top 5 when using the unweighted risk model, unseats Sprint from 1st to 2nd place when we adopt a time-weighted estimator. Also, GE and Italy drop out of the list of the top 5 riskiest positions. Of course, these rank reversals depend on the relative changes in the estimates of idiosyncratic risk that take place when we apply our time-weighting scheme.

5. CONCLUSIONS

We have described some estimation options now implemented in the new Lehman Brothers' Risk Model, which is delivered to clients via our Portfolio and Index Tool (POINT). Focusing on the model's credit component, we have shown that over the past several months, the adoption of a time-weighted estimation methodology for issuer-specific risk would have allowed for a faster incorporation of observed credit events into the model and would have produced tracking error volatility estimates more consistent with subsequent excess return realizations.

We have also offered a brief outline of an intuitive risk decomposition methodology, and we have used it to show that switching between unweighted and time-weighted estimates produces significant changes in the portfolio's risk attribution profile. The understanding of the risk contributions is as important as the accurate prediction of the level of risk. Indeed, asset managers will be able to reduce or increase their portfolio's tracking error volatility dynamically by adjusting the largest risk concentrations, as highlighted in the risk attribution profile.

In persistently volatile markets, such as the ones we have witnessed in the past couple of years, asset managers may be inclined to use the time-weighted version of the model in order to capture the most recent dynamics of the markets. However, it is worth noting that over the entire credit cycle, there are periods during which an excessive focus on the recent past would have produced very low risk estimates, setting the stage for unexpectedly large deviations. Our recommendation is to keep in mind both the long-term and the dynamic view of risk. The choice among the two is ultimately a portfolio manager's call. In this paper, we have attempted to highlight the tradeoffs that they face in such a choice and to describe the expanded toolkit that Lehman Brothers now provides for the analysis of portfolio risk.

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APPENDIX A. RISK DECOMPOSITION ANALYSIS

In this Appendix, we briefly outline the risk decomposition analysis methodology in a multi-factor model setting.[4] We will give a more detailed exposition of this subject in a forthcoming publication.

We take a limited definition of risk measure as the standard deviation of the portfolio returns. We also assume that all portfolios are benchmarked against some benchmark. For cases in which we are interested in relative risk (i.e., tracking error volatility), the benchmark is the appropriate index portfolio. For cases in which we are interested in total return risk (i.e., total return volatility), the benchmark is simply the cash (riskless) asset. Consequently, the "active portfolio" security weight is the difference between the weight of that security in the asset manager's holdings and its weight in the benchmark (if any). Importantly, the "active portfolio" encompasses not only the securities held by the asset manager but also any security that is a component of the benchmark portfolio.

A.1. Marginal Risk Decomposition Methodology

Risk decomposition and attribution is the key part of any risk management system. It allows the investment managers and traders to focus on the most critical positions in their portfolios and reveals the magnitude of both intended and unintended bets that they are taking. However, questions such as "*what portion of the risk in my portfolio comes from exposure to asset A*" or "*what portion of the risk comes from exposure to factor F*" do not have an unambiguous answer. The risk contributed by the asset A depends on all other assets in the active portfolio. We denote the question above as the *aggregate risk decomposition problem*.

[4] *For a general introduction to multi-factor models and their use for portfolio management, see the book by Grinold and Kahn (1999).*

Instead of finding a consistent answer to the *aggregate risk decomposition problem*, which is not possible, one must change the problem itself. A question that can be unambiguously answered is "*what is the sensitivity of my portfolio's risk to exposure to asset A or factor F*", or, in other words, "*how much change in risk can I anticipate if the exposure to asset A or factor F is slightly changed*". We denote such a question as *marginal risk decomposition problem*.[5]

This problem can be solved because by considering small changes instead of the totality of the risk, we essentially linearize the initial problem. And decomposing a linear combination is always unambiguous. Moreover, even in practical applications, the marginal approach is more relevant than the aggregate one. Indeed, a typical portfolio manager will change positions in small increments, and, therefore, the impact of such small changes is what is relevant. For example, when a portfolio manager wishes to decrease a fund's tracking error volatility from 200 bp to 180 bp, he really cares about where the 20 bp of the change will come from, not where the entire 200 bp of initial risk came from.

First of all, let us clarify that all the risk analysis that is considered in this paper is single-period in nature, namely it is an ex-ante analysis of distribution of returns 1-month in future. Because single-period returns are additive, they can be sliced and diced into components in arbitrary ways without losing the consistency of portfolio view.

Let us illustrate the definition and main properties of risk decomposition analysis by considering a sub-division of the (active) portfolio into two components, a given sub-portfolio with weight w_{sub} and the complementary portfolio that holds the rest of securities with weight w_{rest}. The question that we would like to answer is what is the marginal risk contribution from our pre-defined sub-portfolio. The consistent way to define the change in portfolio's exposure with respect to the given subset (e.g., a sub-portfolio) without making assumptions about how all other assets are changed is to assume that the particular position under consideration is leveraged up or down by borrowing (lending) in a riskless cash security. Assuming that we leveraged our sub-portfolio by α percent in this fashion, the return of the active portfolio R_{port} can be represented as a weighted sum of the return of the sub-portfolio R_{sub}, the return of the complementary portfolio R_{rest} and the return of the riskless cash asset $R_{riskless}$:

[1] $$R_{port} = w_{sub} \cdot R_{sub} + w_{rest} \cdot R_{rest} + \alpha \cdot w_{sub} \cdot \left(R_{sub} - R_{riskless}\right)$$

The portfolio risk, as a function of the degree of leverage in chosen sub-portfolio, is:

[2] $$\begin{aligned}\sigma^2_{port}(\alpha) \;&=\; (1+\alpha)^2 \cdot w^2_{sub} \cdot \mathrm{var}(R_{sub}) + w^2_{rest} \cdot \mathrm{var}(R_{rest}) \\ &+\; 2 \cdot (1+\alpha) \cdot w_{sub} \cdot w_{rest} \cdot \mathrm{cov}(R_{sub}, R_{rest})\end{aligned}$$

[5] *R. Litterman (1996) has introduced and popularised this view of risk decomposition analysis under the term of HotSpots™. Our risk attribution profiles are consistent with HotSpots™ view of a given portfolio.*

One can easily derive from this expression for the sub-portfolio's marginal contribution to risk as the fractional sensitivity of portfolio risk with respect to α, which is the fractional change in sub-portfolio's weight in the total portfolio:

$$MCR_{sub} = \frac{1}{\sigma_{port}(\alpha)} \cdot \frac{\Delta\sigma_{port}(\alpha)}{\Delta\alpha} = \frac{\mathrm{cov}(w_{sub} \cdot R_{sub}, R_{port})}{\mathrm{var}(R_{port})} \qquad [3]$$

This definition of marginal contribution to risk has a number of desirable qualities:

- **Consistency:** Marginal contributions are measured in percentage terms and are directly comparable across assets and factors.

Indeed, we have not restricted our choice of sub-portfolio in any way during the derivation – it can be a single asset, or even a specific "overlay" that mimics the exposure of the portfolio to a particular risk factor.

- **Completeness:** Marginal contributions take cross-covariances into account and sum up to 100%.

Indeed, if we decided to slice and dice our portfolio into many sub-portfolios, the total sum of their marginal contributions will have the same denominator, while in the numerator, we will have the covariance of sum of all sub-sets with the total, which indeed is the variance of the total portfolio; hence, the sum will be equal to 1.

A.2. Attributing Risk in a Multi-Factor Model Setting

In a multi-factor model, each asset in turn is represented as a mini-portfolio of common return factors, which are the same across all assets, and a specific return factor, which is the same only for assets from the same issuer. Therefore, our simple definition of marginal risk decomposition can be easily carried through not only to the level of granularity corresponding to a single asset but even deeper, to a level of granularity corresponding to a single factor "within" a particular asset under consideration.

Most interestingly, we can separate the contributions from asset-specific (idiosyncratic) risk and common factor risk. Then, since the marginal risk decomposition is additive by construction, we can add up the contributions from idiosyncratic risk across all assets to arrive at the original question that we posed – what is the relative importance of the common factor and specific risk in a given portfolio and how this breakdown has changed when switching from old to the new weighting scheme in the risk model?

Rewriting the main equation [3] for a single asset, we obtain **asset level risk contribution**:

$$MCR(Asset_i) = \frac{\mathrm{cov}(Asset_i, Portfolio)}{\sigma^2_{Total}} \qquad [4]$$

Note that the above notation includes the appropriate (active) weight of the *i*-th asset.

Furthermore, one can introduce **asset level specific risk contributions** and **asset level factor contributions to risk**, which should be interpreted as the portion of the asset-level marginal contribution that appears due to the change in the factor exposures driven by the changing of the weights of this particular asset:

[5] $$MCR(Asset_i) = MCR(Asset_i | Specific) + \sum_{k=1}^{K} MCR(Asset_i | Factor_k)$$

The summation of asset level specific risk contributions across all assets belonging to a given issuer gives the **issuer specific risk contribution**:

[6] $$MCR(Issuer_I | Specific) = \sum_{i=1}^{N_I} MCR(Asset_i | Specific)$$

Adding up the asset level factor contributions or specific risk contributions separately across all securities within a given bucket (such as an industry/rating cell) gives the **aggregated marginal contributions to risk** from common factor and idiosyncratic risk, respectively:

[7] $$MCR(Bucket | Factor_k) = \sum_{i=1}^{\#\text{ in bucket}} MCR(Asset_i | Factor_k)$$

[8] $$MCR(Bucket | Specific) = \sum_{i=1}^{\#\text{ in bucket}} MCR(Asset_i | Specific)$$

If we sum up all of the asset level factor contributions in entire active portfolio, we obtain the **total factor level marginal contributions to risk**:

[9] $$MCR(Factor_k) = \sum_{i=1}^{N} MCR(Asset_i | Factor_k)$$

On the other hand, summing up the asset level specific risk contributions across the entire portfolio gives the **total specific risk contribution**:

[10] $$MCR(Specific) = \sum_{i=1}^{N} MCR(Asset_i | Specific)$$

The aggregated factor marginal contributions and asset specific marginal contributions sum up to unity (i.e., provide the full breakdown of the risk):

[11] $$MCR(Specific) + \sum_{k=1}^{K} MCR(Factor_k) = \frac{\sigma^2_{Specific}}{\sigma^2_{Total}} + \frac{\sigma^2_{Factor}}{\sigma^2_{Total}} = 1$$

The definitions above form the basis of the risk analysis, results of which are presented in the main text of this paper.

Learning Credit: Introduction to Credit Default Swaps[1]

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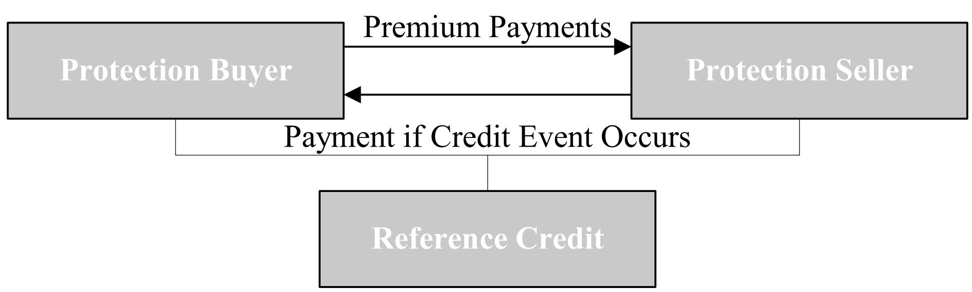
This paper provides a simple explanation of credit default swaps and their uses.

1. INTRODUCTION

A Credit Default Swap (CDS) is an agreement between two parties to exchange credit risk on a reference asset issued by a specified *reference entity*. The reference entity is typically a corporate, bank, or sovereign issuer. A CDS allows you to buy or sell protection on a specific class of instruments, typically bonds or loans on a particular third party (the *reference name*). One party, the *protection buyer*, wants protection from the consequences of a defined credit event and agrees to make periodic payments to the protection seller. The second party to the contract, the *protection seller*, agrees to pay the protection buyer a payment if the defined credit event occurs. Figure 1 illustrates the basic framework. The protection buyer is effectively short the credit of the reference name, while the protection seller is effectively long the credit.

Figure 1. Credit Default Swap



The British Bankers Association estimates the size of the global credit derivative market to be USD 586 billion at the end of 1999 and to be over USD 1000 billion in 2002. About half of global derivatives are traded in London. The size of the London market was estimated to be USD 272 billion in 1999 and over USD 500 billion in 2002.

2. SETTLEMENT MECHANICS

Physical Settlement: The contract usually specifies a class of securities that are ranked *pari passu*[2]. Any of the assets may be delivered in place of the reference asset. The protection buyer delivers one of the eligible securities to the protection seller and receives the notional in

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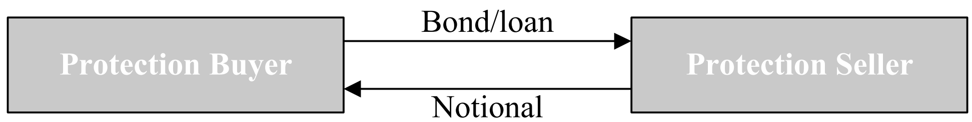
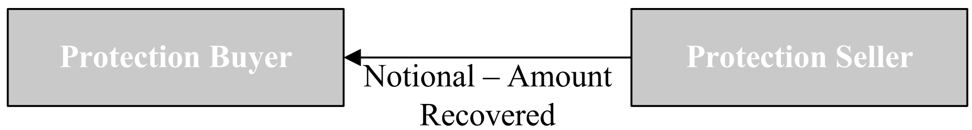
[1] *I thank Arthur Berd and Dominic O'Kane for many helpful comments.*

[2] *The Latin term* pari passu *means of equal standing.*

cash from the protection seller. This implies that the protection buyer is effectively long a "cheapest to deliver" option.



Cash Settlement: The protection seller pays amount equal to notional less the recovery on the obligation. The protection buyer does not deliver anything. Cash settlement implies that the protection buyer does not have a "cheapest to deliver" option.



Physical settlement is the market standard[3].

3. SUMMARY OF ISDA DEFINED CREDIT EVENTS[4]

We need to define the credit event that trigger payment by the protection seller.

Default: Failure to Pay

Failure of the reference entity to make due payments, taking into account some grace period to prevent accidental triggering due to administration error.

Bankruptcy

Reference entity becomes insolvent or is unable to pay its debt (not relevant for sovereign issuers).

Restructuring / Modified Restructuring[5]:

Restructuring changes in debt obligations of the reference creditor but excluding those that are not associated with credit deterioration, such as renegotiation of more favorable terms.

Modified restructuring is to specify that a loan to qualify for restructuring it must have multiple lenders (at least three) and at least two thirds of the lenders must agree to restructuring. The protection buyer must deliver instruments maturing not later than 30

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[3] *The protection buyer has a 30 day period from a defined credit event to deliver a notice of intended settlement. Physical delivery must take place within five days of settlement.*

[4] *IDSA has recently dropped Obligation Acceleration and Repudiation/Moratorium. Obligations Acceleration become due and payable earlier than they would have been due to default or similar condition. Repudiation/Moratorium: A reference entity or government authority rejects or challenges the validity of the obligations*

[5] *In the U. S. A. default swaps trade with Modified Restructuring, while in Europe they can trade with or without Modified Restructuring. In Europe contracts trade mainly with standard restructuring.*

months from the date of restructuring. In Europe, the standard restructuring definition is still used.

4. DIGITAL CREDIT DEFAULT SWAP

Digital credit default swaps are similar to credit default swaps in structure. However, there is an important difference. If a credit event occurs, the protection seller pays the full notional amount of the contract to the protection buyer. Digitals are only a small part of the CDS market.

5. DETERMINING THE CREDIT DEFAULT SWAP PREMIUM

The protection buyer purchases at par a floating rate note issued by the reference name. The coupon is LIBOR + F. This is financed by issuing at par, a floating rate bond with coupon of LIBOR + B. The protection buyer also purchases a credit default swap. The initial outflow is zero. The protection buyer agrees to pay periodic premium payments of D. The total outflow of funds is zero.

An approximate non-arbitrage relationship[6] is

$$D = F - B$$

which provides a rough and ready ball park estimate of the default premium. The *default swap basis* is defined as

$$\text{Default Swap Basis} = \text{Credit Default Spread (D)} - \text{Par Floater Spread (F)}$$

Credit default swaps can be used for relative value plays of trading the basis[7].

6. COUNTERPARTY RISK

A CDS is a bilateral contract and consequently either party to the contract might default. The protection buyer is more exposed to the consequences of non-performance due to counterparty risk than the protection seller.

7. EXAMPLES

7.1 Buying Credit Protection

There are many situations where there is demand to purchase protection against the consequences of credit events. Credit default swaps provide such protection. The party that buys credit insurance is referred to as buying (credit) protection.

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6 *If default occurs, the payment from the protection seller compensates the protection buyer. However, there is still an interest rate financing cost to cover and the financing must also be terminated, implying a negative cash flow. Consequently the usual argument used to justify the relation D = F – B is invalid.*

7 *O'Kane and McAdie (May, 2001) provide a detailed discussion of the basis spread.*

Credit default swaps can facilitate the controlling of credit exposure in long term contracts that often occur in many different sectors of the economy such as leasing, the property office rental market and in the utility sector.

A manufacture enters into a two year contract to supply goods to a nation wide retailer. The manufacture incurs up-front costs to fulfill the contract. If the retailer goes into default over this period this will adversely affect the profitability of the contract. The manufacture enters into a two year credit default swap. If a credit event occurs, the payoff from the CDS will help to offset the loss on the contract.

Two recent examples where buying credit protection would have helped firms exposed to credit risk are the bankruptcies of WorldCom and Winstar. Regional phone companies like Verizon and SBC Communications are owned more than $200 million by WorldCom. The bankruptcy of Winstart cost the "vendor financier" Lucent Technologies more than $600 million.

A bank makes a five year loan to an obligor. To reduce its credit exposure to the obligor, the bank buys credit protection, a five year credit default swap, with the loan being the reference asset . In the contract the bank can specify the seniority of the reference asset. The bank buys credit protection and it also has confidentiality. A CDS allows the bank to control its exposure without damaging its lending relationship with the obligor. The sale of a loan requires disclosure, while the buying of credit protection requires no disclosure.

CDS can be written on sovereign names. This can provide a hedge against political events and for export credit guarantees.

These examples illustrate the case of buying credit insurance. In each case there must be parties willing to sell credit insurance.

7.2 Selling Credit Protection

A party that sells credit insurance is referred to as selling credit protection. Selling credit protection is effectively equivalent to being long a credit issued by the reference name. Asset managers by selling credit protection can gain exposure to credits that may not be readily available in the cash market.

To increase the yield and diversify the risk of its loan portfolio, a bank can sell credit protection. If the reference name does not default, then the bank enhances its return by collecting the premium payments. The willingness of a bank to sell protection depends on its funding rate. If a bank's funding rate is above LIBOR, the economics of the transaction tend to favor selling credit protection.

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