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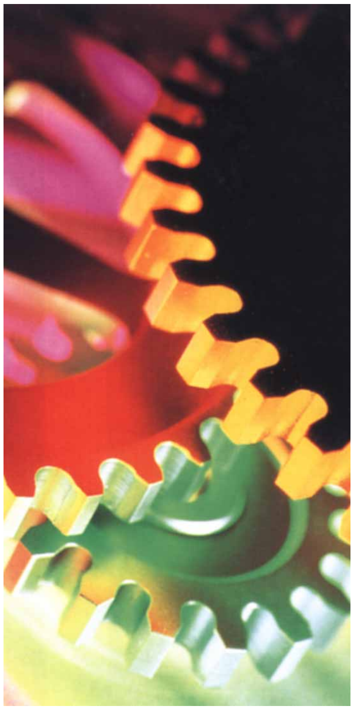

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Dynamic ESPRI: Equity as Spread Indicators with Macro Regimes

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The standard ESPRI model tends to behave differently in bullish and bearish credit market environments. We present and test an enhanced ESPRI score system, Dynamic ESPRI, which uses the new ESPRI Bull and Bear strategies to relate model signals to the broader market. We demonstrate the improved robustness and performance of the new scores, and give a detailed analysis of the macro properties of the model. The last section describes the new ESPRI Power Tool on LehmanLive which provides daily recommendations from the model.[1]

INTRODUCTION

The global credit markets of the past few years have experienced a dramatic cycle, from the high volatility and record default rates of 2001 and 2002, to the recovery and stabilization of 2003 and early 2004. The emphasis of portfolio management has naturally shifted in tandem with this changing environment, from the avoidance of fallen angels and credit blow-ups, to the search for yield and outperformance opportunities.

Throughout this period, bottom-up analysis has been critical to achieving both portfolio management objectives. Using information in both spreads and equity returns, Lehman Brothers' ESPRI model (Equity returns as SPRead Indicators) was found to be a readily applicable asset selection tool for navigating the complex credit markets, acting both as a powerful screening system and as a complement to fundamental analysis.

Following ESPRI over these two quite distinct credit market environments has yielded some valuable insights into the performance properties and characteristics of the model. ESPRI performed strongly through the highly volatile period, both in terms of correctly flagging many of the large credit deteriorations and also as an outperformance indicator. However, although the model continued to do reasonably well in the more stable or bullish credit market conditions, performance was weaker and clear differences in the ESPRI portfolio properties emerged. For instance, the high volatility/potential blow-up candidates tended to outperform in this period as a result of the downward-in-quality trade and reduction in risk-aversion.

With this deeper understanding of ESPRI portfolio behavior, it is now possible to enhance considerably the performance and robustness of the ESPRI methodology. In this report we first review the standard ESPRI model and document the updated results for both the investment grade and high yield universe; we then analyze further the relationship between the model and the macro environment and introduce the two main constituents of the Dynamic ESPRI system, ESPRI Bull and ESPRI Bear; we explain how we capture the macro regime with a dynamic allocation between ESPRI Bull and ESPRI Bear; and finally review the back-testing results for the new system. In the appendices, we review results for the euro market and describe the new ESPRI Power Tool available on LehmanLive which incorporates all the enhancements to the model.

[1] *We thank our colleagues Srivaths Balakrishnan, Manish Mehta, Vasant Naik, Marco Naldi and Peili Wang for their help in developing this project.*

ESPRI AND ESPRI–HY

The original ESPRI model for investment grade credit was launched in summer 2001 as the culmination of an extensive empirical study based on Lehman Brothers US Corporate Bond Index data. ESPRI – High Yield was launched in early 2004, extending the capabilities to the sub-investment grade arena. Both models have been shown to be robust across currencies, ratings and time horizons.

ESPRI captures information in the credit spreads and equity returns of a broad universe of issuers in order to identify potential out- and underperformers. The main insight of ESPRI is that it is possible to use information in equity returns and spread levels to predict credit performance. We have found empirically that strong (weak) credit returns tend to follow strong (weak) equity performance, everything else being equal. Equity performance is indicative of changes in future expected cashflows and future risk premium which are informative for credit. Equity returns play the role of a momentum indicator, whereas spread level is an indicator of value. By combining the momentum and value indicators, ESPRI provides superior insights for the credit selection process.

Review: The ESPRI System

The ESPRI credit selection system operates broadly consistently across the high grade and high yield universes and across currencies. The system can broadly be summarized as a multistage ranking process:

1. The base universe is the Lehman Brothers investment grade or high yield index for the currency.
2. The bonds are divided into six broad rating categories (designated ESPRI rating categories): AAs, As and BBBs for investment grade; Crossovers (XOVER), High Yield (HY) and Distressed (DISTR) for high yield.
3. The bonds of a rating category are further divided into three categories of duration: long (top 33%), medium (mid 33%) and short (bottom 33%). This step is omitted for high yield bonds and non-USD bonds because the universes are smaller.
4. The bonds are then ranked by their current option adjusted spread (OAS) level into three buckets: wide spread (top 33%), medium (mid 33%) and tight spread (bottom 33%).
5. Finally, within a spread bucket, the bonds are further divided by their issuer's past three months'[2] equity return into three further buckets: high (top 20%), medium (mid 60%) and low (bottom 20%).

This ranking process results in nine groups, or portfolios, of bonds within each duration cross rating cross currency universe, as shown in Figure 1. Classifying the bonds in this way allows us to determine the typical characteristics and properties of the bonds in each category. These characteristics are based both on economic theory and on the results of our rigorous empirical study and back-testing.

[2] *Some variations of the model use other equity return periods, depending on the investment horizon required.*

Figure 1. ESPRI Portfolio Codes and Basic Characteristics

	Past 3 Months' Equity Return of Issuer		
OAS Level	**H**igh (Top 20%)	**M**edium (Mid 60%)	**L**ow (Btm 20%)
High (Top 33%)	**HH** *Potential Outperformers*	**HM** *Neutral*	**HL** *High Volatility / Possible Blow-ups*
Medium (Mid 33%)	**MH** *Potential Outperformers*	**MM** *Neutral*	**ML** *Potential Underperformers*
Low (Btm 33%)	**LH** *Defensive / Hedge Against Shocks*	**LM** *Neutral*	**LL** *Potential Underperformers*

The full details of ESPRI for investment grade can be found in the article "Introducing ESPRI: A Credit Selection Model Using Equity Returns as Spread Indicators" in *Quantitative Credit Research Quarterly* 2002-Q1. For ESPRI HY, please refer to the publication *Introducing ESPRI – High Yield* (12 March 2004). Both of these articles also report detailed back-testing results of the standard ESPRI system.

ESPRI Performance Summary

Figures 2 and 3 show summary back-testing results for each of the ESPRI rating categories for US dollars in both high yield and investment grade (results for euros can be found in the appendix). In the interests of space, we review the performance of just the four corner ESPRI portfolios; HH, LH, HL and LL, as defined in Figure 1.

The tables report two performance numbers for each category. The first is the average monthly excess return over the respective currency/rating universe in bp per month. Note that the average excess return is monthly, assuming a three-month holding period. The second number, in italics, is the annualized information ratio. This number gives an indication of risk-adjusted return, or return per unit of volatility. The annualized information ratio is calculated as the average annualized excess return divided by the annualized volatility of the returns of this portfolio. If the information ratio is 0.5 or above, we have a fair risk-return trade-off and the strategy is of interest. If it is below 0.5 (ie, a ratio of -0.5 to 0.5), then there is either too much volatility, or the return is too low to be of interest.

Figure 2. US Dollar High Grade ESPRI Summary Results[3] – Three-Month Investment Horizon

	Performance: Jan 1990 to Apr 2004				Performance: Jan 2002 to Apr 2004			
ESPRI Rating	**HH**	**LH**	**HL**	**LL**	**HH**	**LH**	**HL**	**LL**
AA	**10.7**	**-7.7**	**7.6**	**-8.6**	**14.7**	**-8.3**	**9.5**	**-12.8**
	1.3	*-1.0*	*0.8*	*-1.3*	*2.2*	*-1.2*	*1.0*	*-2.1*
A	**15.2**	**-6.4**	**-10.9**	**-9.2**	**23.3**	**-6.6**	**-42.9**	**-14.1**
	1.7	*-0.8*	*-0.3*	*-1.3*	*1.6*	*-0.5*	*-0.5*	*-1.2*
BBB	**31.0**	**-0.4**	**-41.5**	**-12.1**	**59.1**	**2.9**	**-79.4**	**-10.1**
	1.5	*0.0*	*-0.6*	*-0.6*	*2.4*	*0.1*	*-0.6*	*-0.3*

[3] *For the US dollar high grade table, the reported numbers have been averaged across the three duration categories to give more concise results.*

Figure 3. US Dollar High Yield ESPRI Summary Results – Three-Month Investment Horizon

	Performance: Jan 1990 to Apr 2004				Performance: Jan 2002 to Apr 2004			
ESPRI Rating	**HH**	**LH**	**HL**	**LL**	**HH**	**LH**	**HL**	**LL**
XOVER	**63.9**	**-3.7**	**-83.0**	**-27.2**	**87.8**	**0.6**	**-151.6**	**-23.7**
	1.9	*-0.1*	*-0.7*	*-0.7*	*2.6*	*0.0*	*-0.8*	*-0.6*
HY	**82.2**	**13.5**	**-141.1**	**-15.7**	**128.4**	**4.5**	**-41.0**	**-37.7**
	1.5	*0.3*	*-1.0*	*-0.4*	*2.8*	*0.1*	*-0.3*	*-1.1*
DISTR	**198.6**	**59.3**	**-80.0**	**-76.6**	**464.6**	**72.4**	**-177.2**	**-148.7**
	0.9	*0.6*	*-0.4*	*-0.8*	*3.4*	*0.9*	*-0.7*	*-1.3*

In Figures 2 and 3, we see a consistent pattern across the four portfolios. The HH portfolios, which include the bonds with high spreads and high equity returns, outperform their peers in all rating categories. For AAs, the outperformance is 11bp over the AA universe, while for distressed the outperformance is 199bp in the full sample (January 1990 to April 2004). In the more recent sample we find a similar pattern, with 15bp for AAs and 465bp for distressed. The robustness of the performance can be seen through the information ratios which are all above 0.9 and can be as high as 3.4 for distressed credit since Jan 2002.

The HL portfolios, which typically include the high volatility bonds and potential credit blow-ups, underperform on average for all rating categories except for AA. The full and recent sample excess returns are quite consistent and are largely negative for lower credit quality. Since 2002, the most negative numbers are for distressed and crossover issuers; in the longer sample since January 1990, the HL bucket had the lowest excess return.

The LL portfolios, which typically consist of bonds that seem too rich relative to the equity performance of the issuer, deliver negative excess returns across all rating categories, from -9bp for AA to -80bp for distressed credit in the full sample. Performance numbers in the recent sample also confirm the underperformance of the LL portfolios. Interestingly, the magnitude of the returns of the recent sample is comparable to the returns for the HL portfolio in the most recent sample.

The last set of portfolios, the LH "defensive performers" do not systematically outperform their peers. These portfolios are intended as hedging vehicles, and perform strongly when the rest of the market underperforms (flight-to-quality effect), thus providing useful diversification and hedging, even though on average they could underperform.

The overall performance of ESPRI portfolios is thus consistent with the theory in Figure 1 apart from AA-rated bonds and the LH portfolio. The more recent period is also broadly consistent with the full sample results. In terms of performance by rating categories, only HH and LL perform as expected for AAs, mostly because default risk is less of a concern for AA, while for lower quality ratings, LH and HL work better.

As a mechanical investment strategy, ESPRI gives satisfactory results. However, it should be possible to improve on ESPRI performance by conditioning on the macroeconomic environment. In the following section, we relate ESPRI performance to macro credit conditions.

RELATING ESPRI PERFORMANCE TO MACRO CREDIT CONDITIONS

Although the documented behavior of the nine ESPRI portfolios broadly continues to hold, we find the performance profiles of the portfolios differs significantly depending on the state of the broader credit market environment. In particular, we find differing characteristics in:

- Bullish credit market conditions with relatively low volatility/general risk aversion; and,
- Weak/bearish credit market conditions with relatively high volatility/general risk aversion.

The year 2003 was a good example of a bullish credit market, whereas 2002 was an example of a bearish market. Figure 4 summarizes the broad qualitative characteristics of the key four corner portfolios, in each of these regimes as well as in normal market conditions.

Figure 4. Performance Characteristics of the Four Corner ESPRI Portfolios

Regime	HH High Spread / High Eq Rtn	LH Low Spread / High Eq Rtn	HL High Spread / Low Eq Rtn	LL Low Spread / Low Eq Rtn
Normal Market Conditions	**Moderate Outperformers**	**Defensive, Marginal Outperformers**	**Volatile, Marginal Underperformers**	**Moderate Underperformers**
	Relatively cheap names, strong equity momentum.	*On average, flat performance, but useful hedge against unexpected market shocks.*	*Unpredictable performance/ high leverage/ risky names. Potential credit blow-ups.*	*Overpriced names with poor equity momentum.*
High Volatility, High Spreads	**Moderate Outperformers**	**Defensive Moderate Outperformers**	**Highly Volatile, Significant Underperformers**	**Marginal Underperformers**
	Highly volatile, generally strong outperformance, combine with LH to hedge market shocks.	*Higher quality names with good equity momentum - outperform significantly with flight to quality*	*Riskiest, most volatile names. High chance of credit blow-ups falling here.*	*Generally marginal underperformance, inversion with strong flight to quality.*
Low Volatility, Low Spreads	**Significant Outperformers**	**Steady, But Marginal Underperformers**	**Unpredictable, Potentially Significant Outperformers**	**Moderate Underperformers**
	Performance driven by high appetite for risk/demand for yield plus strong supporting equity momentum.	*Equity momentum effect minimal, higher quality names with little room for spread tightening.*	*High spread leads to outperformance but riskier names and still potential for credit blow-ups.*	*Underperformance driven by shift to riskier names plus lagging equity indicates worsening fundamentals.*

We see from Figure 4 that the HH and LL portfolios consistently perform as expected through the different regimes. The HH portfolio for instance is a consistently moderate outperformer except in a low volatility, low spread regime in which it outperforms significantly. The other two portfolios, LH and HL, can switch from outperformance to underperformance and vice versa when we move into a low volatility, low spread regime. We examine more closely the performance of these portfolios by constructing two new portfolios, ESPRI Bull and ESPRI Bear.

INTRODUCING ESPRI BULL AND BEAR INVESTMENT STRATEGIES

To help understand and exploit these macro properties, we can combine the four different ESPRI portfolios and define two new long-short (zero-investment) strategies with appealing properties, ESPRI Bull and ESPRI Bear:

Figure 5. Description of ESPRI Bull and Bear

ESPRI Bull	ESPRI Bear
• Definition: **Long: HH; Short: LL**	• Definition: **Long: LH; Short: HL**
• ESPRI Bull is long bonds with HIGH equity returns and short bonds with LOW equity returns.	• ESPRI Bear is long bonds with HIGH equity returns and short bonds with LOW equity returns.
• ESPRI Bull is long bonds with HIGH spread and short bonds with LOW spread.	• ESPRI Bear is long bonds with LOW spread and short bonds with HIGH spread.
• ESPRI Bull should perform particularly well when the credit market is stable or performing well.	• ESPRI Bear will perform particularly well in volatile or distressed periods, benefiting from the flight to quality effect.

Figures 6 and 7 show the smoothed (12-month rolling average) time series performance profiles of these two strategies for the A-rated and Crossover categories as examples. In each case, the difference in performance profiles between ESPRI Bull and ESPRI Bear is striking. Periods in which Bear outperforms Bull are shaded in grey. These periods evidently coincide with the three major periods of high market volatility and risk aversion, namely the 1990-91 recession (for Crossovers), the 1998 Russian crisis and the 2000-2002 bear market. ESPRI Bear performs well in a high volatility environment and can work as a put option against blow-ups or market meltdowns. The performance of ESPRI Bear is intuitive: it is long volatility by going short the highly volatile HL names (high spread and equity volatility which tends to coincide with poor equity returns) and long the LH names (low spread volatility and equity volatility). Any market spread volatility will impact the issuers being shorted more than the issuers on the long side.

Also striking is the general consistency of the ESPRI Bull strategy, showing relatively few periods of negative excess returns. ESPRI Bull therefore seems a good candidate for a core strategy in all market conditions.

Other rating and currency combinations reveal similar patterns of regime-dependent performance.

Figure 6. US Dollar A-Rated ESPRI Bull and Bear 12-Month Rolling Average Excess Return over Treasuries, March 1991-March 2004

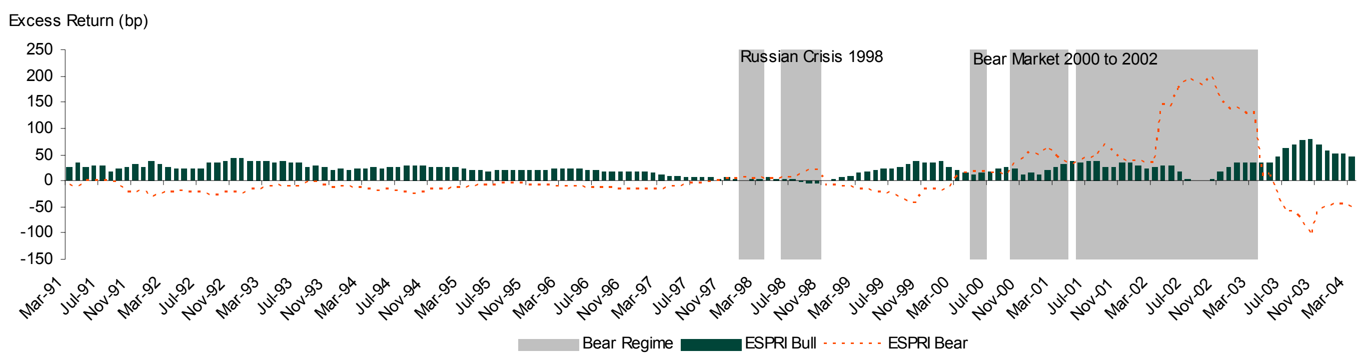
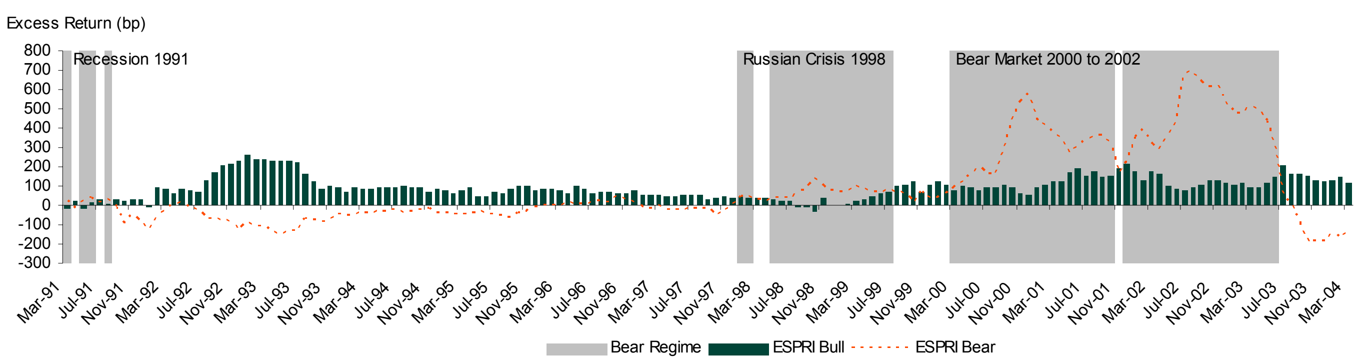


Figure 7. US Dollar Crossovers ESPRI Bull and Bear 12 Month Rolling Average Excess Return over Treasuries, March 1991-March 2004



Both the ESPRI Bull and the ESPRI Bear strategies perform reasonably well on average across the full sample period. However, these charts make it clear that each of the strategies performs well in certain periods and less well in others. The correlation between the returns of ESPRI Bull and ESPRI Bear is typically between -40% and -60%, therefore some combination of the two strategies would lead to significant diversification and potential reduced volatility.

The ESPRI Aggregate Strategy

The most obvious weightings to choose would be equal weights of 50% on each of ESPRI Bull and ESPRI Bear. This combination is what was termed the ESPRI Aggregate strategy in the original ESPRI model outputs. It corresponds to a no-view macro strategy in which we put equal weights on the Bear and Bull scenarios. This static allocation does benefit from the diversification effect and has strong results on average. It also provides some robustness since the allocation does not rely on any optimization or calculation.

However, the charts above clearly indicate that a better solution would be to choose dynamic weights between these strategies that depend on the macro regime. We should keep in mind that there will always be a trade-off between the simplicity/robustness and sophistication/data sensitivity of the modeling. In the next sections, we show how to capture the macro regime and introduce the weightings between ESPRI Bull and ESPRI Bear.

CAPTURING THE MACRO REGIME

In this section, we examine how to systematically assess how much weight to put on each strategy at each point in time to make best use of the relationship between the ESPRI portfolio performance and the macro regime.

In order to calculate these weights, we need some way to determine which regime we are in, or likely to be in over the next few months. To do this, we introduce two state variables which we believe give an indication of the relevant regime:

1. The three-month change in the credit index (either high grade or high yield) OAS level. Widening spreads generally indicate a worsening credit environment, while tightening spreads should indicate an improving trend.
2. The three-month change in the level of the VIX[4] index. A rising VIX generally indicates increasing broad market volatility and risk aversion. A falling VIX should indicate decreasing risk aversion.

Here we use three-month changes to give a compromise between a noisy shorter length change and a possibly stale longer length change. Three-month changes are also consistent with the three-month equity returns used in ESPRI.

We would expect ESPRI Bull to perform well when the credit market is stable or improving (Index OAS falling) and volatility is dropping (VIX falling), therefore we expect to find a negative correlation between ESPRI Bull returns and the two state variables. Conversely, widening spreads and increasing volatility ought to be good for ESPRI Bear performance, hence we expect a positive correlation between ESPRI Bear returns and the state variables.

Figure 8 reports contemporaneous correlations between the state variables and ESPRI Bull and Bear excess returns. As expected, we find strong negative correlations (typically -20% to -50%) for ESPRI Bull and strong positive correlations (typically +20% to +40%) for ESPRI Bear. This suggests that our state variables are indeed capturing the regimes as hoped.

[4] *The VIX Index is an index of implied volatility of options on the S&P 500 stock index.*

Figure 9 reports lagged correlations between the state variables and the following month's ESPRI Bull and Bear excess returns. The same pattern persists of negative and positive correlations for Bull and Bear respectively. The magnitude of correlations is obviously weaker, but it demonstrates a reasonable level of predictability – or persistence – in the regime.

Figure 8. Correlations of State Variables with Same Month ESPRI Bull and Bear Excess Returns

	Correlation of ESPRI Bull Returns With:		Correlation of ESPRI Bear Returns With:	
ESPRI Rating Category	**3 Mth Chg In Index OAS**	**3 Mth Chg in VIX**	**3 Mth Chg In Index OAS**	**3 Mth Chg in VIX**
AA	-31.9%	-29.2%	43.9%	33.1%
A	-54.1%	-37.3%	35.0%	28.5%
BBB	-44.8%	-27.7%	43.5%	32.5%
XOVER	-19.0%	-26.4%	45.2%	37.4%
HY	-44.7%	-40.0%	40.5%	30.9%
DISTR	-26.5%	-7.0%	21.5%	18.7%

Figure 9. Correlations of State Variables with NEXT Month ESPRI Bull and Bear Excess Returns

	Correlation of ESPRI Bull Returns With:		Correlation of ESPRI Bear Returns With:	
ESPRI Rating Category	**3 Mth Chg In Index OAS**	**3 Mth Chg in VIX**	**3 Mth Chg In Index OAS**	**3 Mth Chg in VIX**
AA	-5.8%	-14.2%	7.2%	20.3%
A	-23.1%	-29.4%	5.4%	10.4%
BBB	-17.0%	-19.7%	15.5%	19.6%
XOVER	6.5%	-1.9%	18.6%	23.1%
HY	-21.3%	-18.0%	12.9%	17.4%
DISTR	-21.9%	-24.4%	14.5%	21.8%

DYNAMIC ALLOCATION BETWEEN BULL AND BEAR

The Bull and Bear strategies can be viewed as two assets whose historical returns and volatilities are known at each point in time. The question of weighting the strategies then becomes a simple asset allocation problem with two assets.

The classical mean-variance approach is a clear candidate for solving this. However, we also want our allocation to depend on the forecast of the macro credit regime in the following period. In particular:

- The allocation to ESPRI Bull should be higher when we forecast improving credit markets and decreasing risk aversion.
- The allocation to ESPRI Bull should be lower when we forecast worsening credit markets and increasing risk aversion.

Dynamic portfolio optimization methods can be used to compute the optimal allocation or the optimal weight, W, at each point in time for each rating and currency combination[5]. Our Dynamic ESPRI strategy now becomes:

Dynamic ESPRI = W x ESPRI Bull + (1 – W) x ESPRI Bear

[5] *In fact, different weights are calculated also for different investment horizons. We focus here only on the three-month horizon.*

Testing the Dynamic ESPRI Strategy

The new strategy is readily back-testable, allowing a direct comparison of performance with both the Bull and Bear unconditional strategies individually as well as with the ESPRI Aggregate static allocation strategy.

Figures 10 and 11 report the out-of-sample monthly excess returns over Treasuries and information ratios for these strategies for both the US dollar and euro universes.

Figure 10. US Dollar Comparison of Out-of-Sample Strategies

		AA	A	BBB	XOVER	HY	DIST
ESPRI Bull	**Avg Return**	**18.7**	**23.9**	**41.8**	**98.3**	**78.8**	**256.0**
	Inf Ratio	*1.9*	*2.0*	*1.5*	*1.7*	*1.3*	*1.5*
ESPRI Bear	**Avg Return**	**-13.8**	**7.9**	**44.1**	**91.7**	**194.7**	**226.8**
	Inf Ratio	*-1.2*	*0.2*	*0.5*	*0.6*	*1.1*	*0.8*
ESPRI Aggregate	**Avg Return**	**2.5**	**15.9**	**43.0**	**95.0**	**136.7**	**241.4**
	Inf Ratio	*0.4*	*0.8*	*1.3*	*1.4*	*1.7*	*1.7*
Dynamic ESPRI	**Avg Return**	**17.3**	**25.8**	**41.1**	**113.0**	**130.4**	**351.2**
	Inf Ratio	*1.8*	*2.3*	*1.9*	*2.1*	*2.4*	*2.0*

Figure 11. Euro Comparison of Out-of-Sample Strategies

		AA	A	BBB
ESPRI Bull	**Avg Return**	**20.0**	**26.3**	**55.3**
	Inf Ratio	*3.0*	*2.1*	*1.4*
ESPRI Bear	**Avg Return**	**-9.4**	**5.2**	**125.0**
	Inf Ratio	*-1.3*	*0.1*	*0.7*
ESPRI Aggregate	**Avg Return**	**5.3**	**15.8**	**90.2**
	Inf Ratio	*1.6*	*0.8*	*1.1*
Dynamic ESPRI	**Avg Return**	**9.3**	**24.1**	**59.4**
	Inf Ratio	*2.1*	*1.7*	*1.2*

It is interesting to compare the ESPRI Aggregate and the Dynamic ESPRI results. In the US sample, looking either at average returns or information ratios we see a clear improvement from the naïve static weighting of the aggregate strategy to the new dynamic method. The most dramatic improvements are for the higher ratings, particularly AA and A. Here, the dynamic weighting assigned to ESPRI Bull tends to be high (between 80% and 100%) in most time periods. In contrast, the lower rating categories have more balanced dynamic weights on ESPRI Bull (typically between 40% and 70%), resulting in the dynamic strategy being closer to the aggregate strategy.

In the euro sample, the results are more mixed. Although the same pattern persists of Dynamic ESPRI improving on ESPRI Aggregate, we find that ESPRI Bull on its own was the strongest performer under the average return and information ratio metrics. This suggests an unconditional optimal weight of 100% on ESPRI Bull. However, ESPRI Bear in euros suffered extreme volatility between 2000 and 2002 (due largely to the spate of fallen angels whose effect is amplified in euros because of the smaller universe). This adversely affected the information ratio, perhaps unfairly because of the short sample period, and going forward we expect that some weight on ESPRI Bear could still be optimal, in line with the US results.

How Does the Allocation to ESPRI Bull Vary?

The allocation to ESPRI Bull changes both with time and across ratings and currencies. Generally we find the weight drops as we move down the ratings spectrum. AA-rated ESPRI Bull is typically allocated close to 100% of the weight, while BBB Bull receives between 50% and 80%.

We can generate a time series of out-of-sample weights on ESPRI Bull for each rating class. Figures 12 and 13 show how these weights change over time.

Figure 12. Allocation to ESPRI Bull for A-rated bonds

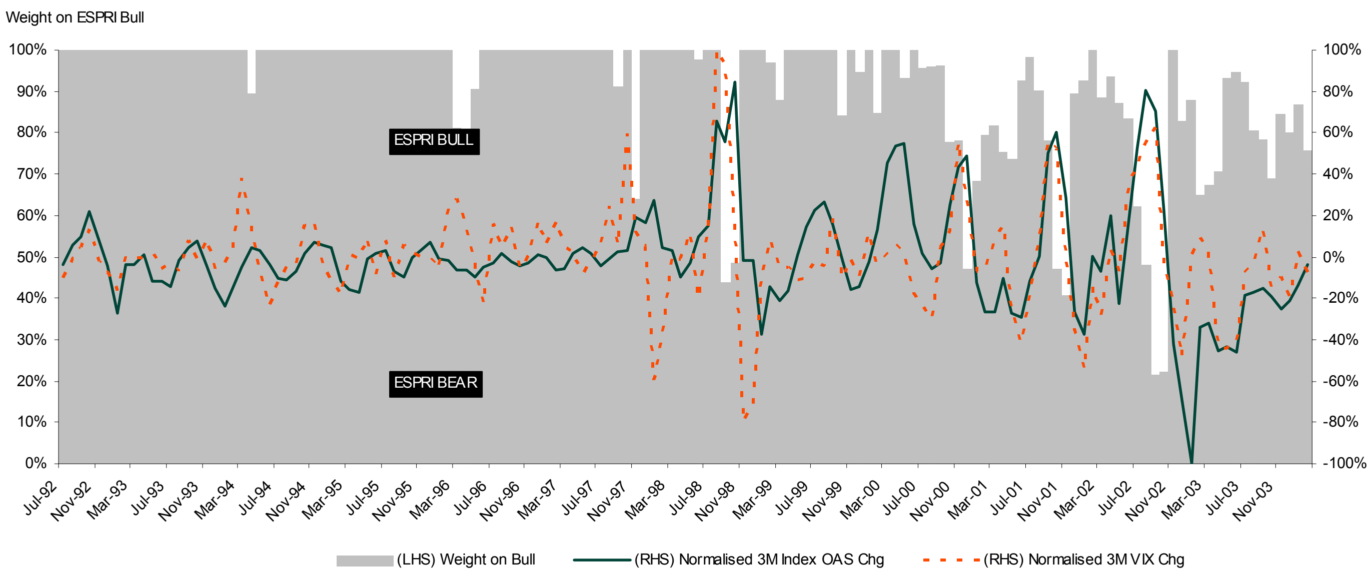
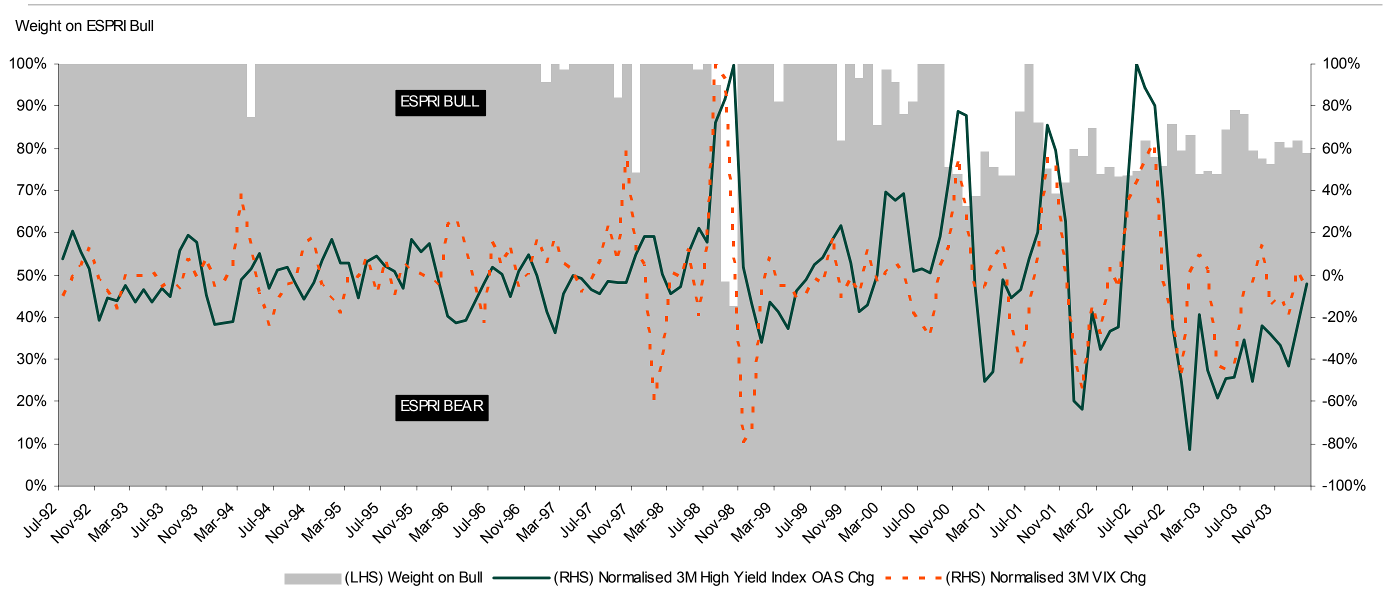


Figure 13. Allocation to ESPRI Bull for Crossover bonds



NEW ESPRI BULL AND BEAR SCORE SYSTEM

The ESPRI Bull and Bear strategies introduced above, and their relationship with the macro environment, give us a deeper understanding of the model's performance. These results have direct implications for the use of ESPRI signals, and, therefore, for the ESPRI score system which aggregates bond-level output to the issuer or sector level. In this section we describe a very natural extension of the ESPRI score system to incorporate these new ideas.

What are ESPRI Scores?

The original ESPRI score system is a simple method of summarizing the ESPRI signals of the bonds of an issuer or sector into a single number between 0 and 10, with 0 indicating the weakest aggregate signal and 10 the strongest. It avoids digging into bond-level signals when what is needed is issuer-level information. The full system is described in the article *Keeping the Score in the Credit Market: A Methodology Using ESPRI* (20 January 2003).

Under the original system, an issuer or sector received the highest score (10) if its bonds fell predominantly in **either** the **LH** or **HH** corners. Conversely, an issuer or sector received the lowest score (0) in **either** the **HL** or **LL** corners. The score did not distinguish between the regimes. We have documented that **LH** is a strong signal and **HL** a weak signal *only in weaker/bearish credit market conditions*. An optimal score system, therefore, must take into account some measure of current or forecast market conditions.

Introducing Bull and Bear Scores

In order to implement this new information we now define two new scores for an issuer or sector:

- **The ESPRI Bull Score** is a numerical value between 0 (weakest signal) and 10 (strongest signal) indicating the performance in stable / bullish credit market conditions.
- **The ESPRI Bear Score** is a numerical value between 0 (weakest signal) and 10 (strongest signal) indicating the performance in weak/volatile/bearish credit market conditions.

These new scores are constructed in a very similar manner to the old scores. The difference is in the weighting scheme assigned to the nine ESPRI portfolios (see Figure 1).

Figure 14. New and Old ESPRI Score Weighting Schemes Comparison

		Old Score System			New Score System					
		ESPRI Score			ESPRI Bull Score			ESPRI Bear Score		
		Past Equity Return			*Past Equity Return*			*Past Equity Return*		
		High	Medium	Low	High	Medium	Low	High	Medium	Low
OAS Level	High	10.0	5.0	0.0	10.0	7.5	5.0	5.0	2.5	0.0
	Med	7.5	5.0	2.5	7.5	5.0	2.5	7.5	5.0	2.5
	Low	10.0	5.0	0.0	5.0	2.5	0.0	10.0	7.5	5.0

The New ESPRI Score

Given the Bull and Bear scores for an issuer or sector we then define the new ESPRI score to be a weighted average of these:

- **The New ESPRI Score** = W x ESPRI Bull Score + (1-W) x ESPRI Bear Score

The weight W is simply the weight allocated to the ESPRI Bull strategy that was calculated in the previous section.

Testing the New Scores

Our final section looks at testing this new score system directly, and compares performance with the old system. We run a series of simple tests across the extended ESPRI universe and sample period.

Scores are calculated under the new and old systems each month. We then report the market-weighted average excess return over the rating/currency universe in the following month of the issuers with highest scores (7.5 and above) and lowest scores (2.5 and below). We also report the information ratio which can be interpreted as before (Figure 15).

Figure 15. Performance of New ESPRI Scores versus Old (Jan 1991 to Apr 2004)

		New Score System		Old Score System	
ESPRI Rating Category		**High Scores (>=7.5)**	**Low Scores (<=2.5)**	**High Scores (>=7.5)**	**Low Scores (<=2.5)**
AA	*Avg Exc Rtn*	5.1	-6.5	1.8	-1.8
	Inf Ratio	**0.5**	**-1.0**	**0.4**	**-0.3**
A	*Avg Exc Rtn*	15.0	-10.2	9.3	-14.3
	Inf Ratio	**1.6**	**-1.0**	**1.2**	**-0.6**
BBB	*Avg Exc Rtn*	33.8	-10.0	21.7	-34.6
	Inf Ratio	**1.9**	**-0.5**	**1.8**	**-1.7**
XOVER	*Avg Exc Rtn*	55.2	-30.4	42.1	-57.6
	Inf Ratio	**1.4**	**-0.8**	**1.3**	**-1.0**
HY	*Avg Exc Rtn*	75.3	-18.7	55.6	-70.6
	Inf Ratio	**2.1**	**-0.8**	**2.1**	**-1.8**
DISTR	*Avg Exc Rtn*	28.4	-183.2	124.8	-139.7
	Inf Ratio	**0.3**	**-1.3**	**1.7**	**-1.4**

The new ESPRI score system performs better than the old score system on the upside for all rating categories except distressed bonds. On the downside, the new score is improving for A and AA-rated bonds but not for BBB-rated and high-yield bonds. One reason for this is that ESPRI Bear tends to be muted in normal market conditions, potentially leading to the new scores missing some credit blow-ups that may have been caught by the old system. This indicates that ESPRI Bear should perhaps be used separately for potential blow-up detection. Nevertheless, the performance of the new score system is quite satisfactory with reasonable information ratios.

Perhaps more relevant, Figure 16 shows the same table but for the recent sample period of January 2003 onwards. Here, we see a dramatic improvement realized by the new score system versus the old, with robust returns and information ratios in every ESPRI rating category.

Figure 16. Performance of New ESPRI Scores versus Old (Jan 2003 to Apr 2004)

		New Score System		Old Score System	
ESPRI Rating Category		*High Scores (>=7.5)*	*Low Scores (<=2.5)*	*High Scores (>=7.5)*	*Low Scores (<=2.5)*
AA	*Avg Exc Rtn*	8.5	-9.0	0.1	0.7
	Inf Ratio	**1.4**	**-1.5**	**0.0**	**0.1**
A	*Avg Exc Rtn*	15.4	-17.6	7.8	-11.6
	Inf Ratio	**1.8**	**-2.3**	**1.1**	**-1.1**
BBB	*Avg Exc Rtn*	41.6	-37.8	15.0	-11.3
	Inf Ratio	**2.6**	**-3.8**	**1.4**	**-1.3**
XOVER	*Avg Exc Rtn*	32.9	-67.1	-7.7	-26.6
	Inf Ratio	**1.4**	**-3.2**	**-0.6**	**-1.3**
HY	*Avg Exc Rtn*	104.4	-45.9	40.6	-21.2
	Inf Ratio	**4.8**	**-2.8**	**3.2**	**-2.9**
DISTR	*Avg Exc Rtn*	190.6	-57.5	63.3	-77.3
	Inf Ratio	**3.2**	**-1.7**	**1.1**	**-1.7**

USING THE NEW SCORES

The new ESPRI scores are values between 0 (worst signal) and 10 (best signal), and are assigned within a currency across rating categories to every issuer and sector in the ESPRI system. Although the interpretation of the new scores is similar to the old ones, there are a few points to bear in mind:

- ESPRI scores should be compared between issuers (or sectors) of the same rating and currency for best results.
- The same issuer will have possibly different ESPRI scores for each ESPRI rating category in which the issuer has bonds.
- The new scores are less clustered around certain values such as 0, 5 and 10. It is not unlikely that in a given month there may be very few (or no) scores of 10, for example. Instead we should look to the highest scores in the rating category for the strongest signals.
- Finally the new ESPRI scores take into account a model-driven forecast of the macro credit conditions. This adds a new dynamic to the scores, in that a score can change over a month even if the underlying ESPRI bond rankings do not.

Implementing Market Views

One useful feature of the Bull and Bear score system is that ESPRI users have the flexibility to allocate their own weightings to the scores depending on their view of the market over the coming period. For example, if a user believes the model-driven weighting is too optimistic, he can reduce the weighting on ESPRI Bull accordingly. Likewise, a model weighting indicating worsening/bearish credit conditions can be moderated by using a high allocation to ESPRI Bull.

The separate Bull and Bear scores are available in many ESPRI reports and also directly via the ESPRI Power Tool (see Appendix 3).

Other Applications

The Bull and Bear scores are also useful individually for certain applications. Some examples include:

- **Portfolio Screening Using the Bear Score:** Where avoidance of high volatility names or protection from a worsening credit market is of particular interest, the ESPRI Bear scores can be used as indicators. Low Bear scores indicate potentially high volatility and underperformance in poor credit conditions. This may be applicable for screening CDOs, for example.
- **Portfolio Screening Using the Bull Score:** When risk aversion is low and return maximization is required, the ESPRI Bull score can be used as a screening tool.

CONCLUSIONS

We have reviewed the standard ESPRI system and documented the updated results for both the investment grade and high yield universes. We have explained the performance characteristics of ESPRI portfolios and considered the use of the broader market/macro environment as conditioning variables. The ability to account for the macro environment is an interesting improvement to the ESPRI model. To understand the relationship better, we introduced the ESPRI Bull and ESPRI Bear portfolios, which exhibit clearly different risk-return characteristics under different macro regimes. ESPRI Bear, for instance, performs best when the market is very volatile, spread levels are high and credit blow-ups might occur. ESPRI Bull performs steadily across the regimes. We have explained how we capture the macro regime with a dynamic allocation between ESPRI Bull and ESPRI Bear and then reviewed the back-testing results. Going forward, investors should be able to use the enhanced ESPRI model to improve their security selection procedure taking account of the macro environment.

REFERENCES

Naik, V., M. Trinh, and G. Rennison, “Introducing Lehman Brothers ESPRI – A Credit Selection Model Using Equity Returns As Spread Indicators”, *Quantitative Credit Research Quarterly* Volume 1 2002.

Rennison, G., M. Howard, and C. Kemp, *Introducing ESPRI – High Yield*. March 2004.

APPENDICES

Appendix 1: Extended ESPRI Universe

The universe of bonds on which the model operates has been significantly increased both historically and going forward. The extension includes three changes:

1. The main addition is the extension to cover the high yield universe in both US dollars and euros.
2. A new system for dealing with private/unlisted issuers has been implemented. Depending on the rating and currency, between 5% and 25% of the bonds in the ESPRI universes are issued by private companies. In order to include them in the ESPRI output, we have developed a proxy equity return system based on the weighted average equity returns of a group of listed companies carefully chosen to have similar characteristics to

the private company. While possibly not as informative as a true equity return, we believe these equity proxy returns nevertheless provide a useful signal.

3. Finally, our sample periods for US dollar high grade and high yield have been extended back from May 1994 to January 1990, significantly increasing the length of time series and statistical power of the results.

Appendix 2: ESPRI Performance Summary for Euros

Figure 17 shows a summary of the back-testing results for each of the ESPRI rating categories for euros.

Figure 17. Euro High Grade ESPRI Summary Results – Three-Month Investment Horizon

	Performance: Jan 1999 to Apr 2004				Performance: Jan 2002 to Apr 2004			
ESPRI Rating	**HH**	**LH**	**HL**	**LL**	**HH**	**LH**	**HL**	**LL**
AA	**9.7**	**-5.0**	**4.7**	**-4.9**	**15.6**	**-5.6**	**4.3**	**-7.1**
	2.1	*-1.8*	*0.6*	*-1.7*	*4.0*	*-2.3*	*0.7*	*-2.8*
A	**13.1**	**-2.0**	**-11.2**	**-6.1**	**13.2**	**-6.0**	**15.2**	**-12.4**
	1.8	*-0.2*	*-0.4*	*-0.8*	*1.4*	*-0.6*	*0.5*	*-1.3*
BBB	**38.4**	**4.3**	**-92.6**	**-1.7**	**48.8**	**7.4**	**-120.8**	**-5.5**
	1.5	*0.2*	*-0.7*	*-0.1*	*1.3*	*0.3*	*-0.8*	*-0.3*

Appendix 3: New ESPRI Power Tool

To bring together all the enhancements made to the ESPRI system over the past few months, we have recently launched a fully revised ESPRI Power Tool on LehmanLive. The new Power Tool is now the main platform for delivery and presentation of ESPRI data, reports and analysis.

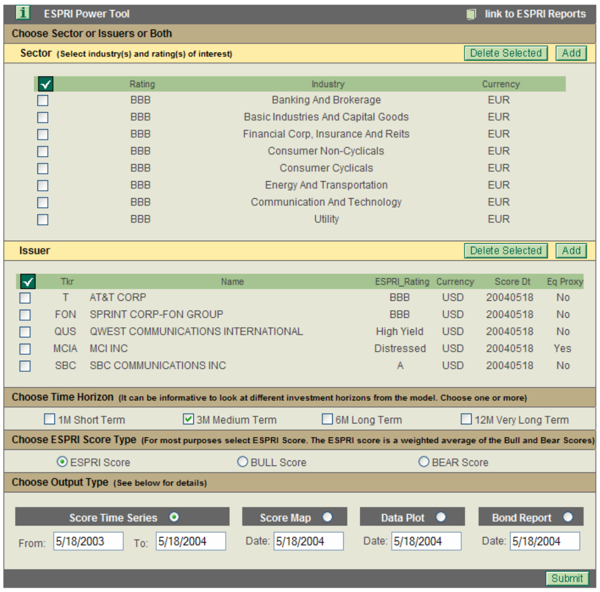
Features of the new tool include:

- Access to the daily updated ESPRI results for bonds, issuers and sectors across the ESPRI universe.
- A wide variety of display/output options, including new Issuer and Sector Score Maps, with results downloadable to Excel.
- Historical time series of ESPRI data.
- The new ESPRI Bull and Bear score system.
- Extended universes covering the full Lehman Brothers High Grade and High Yield Indices (using proxy equity returns).

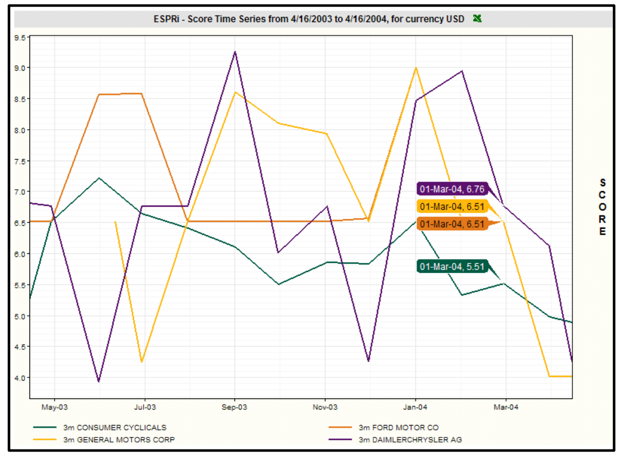
Figure 18 shows some screenshots from the tool.

Figure 18. New ESPRI Power Tool on LehmanLive

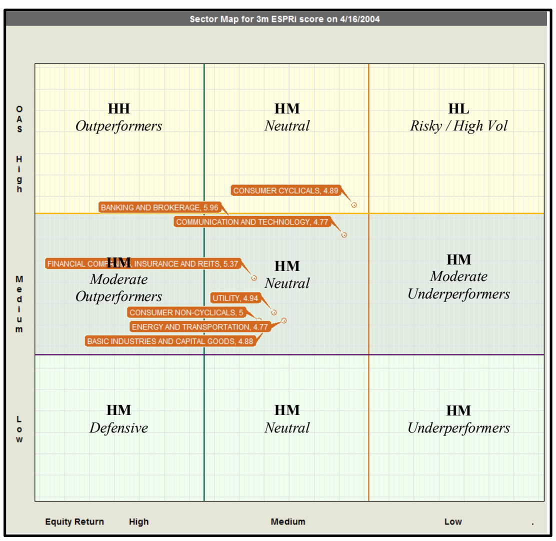
Main Power Tool Interface



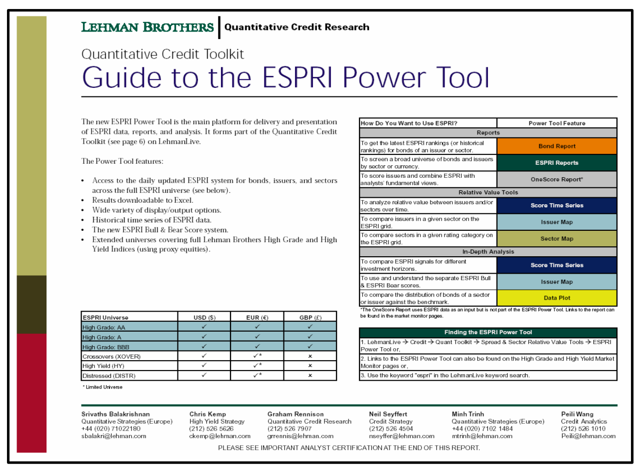
Plot Time Series of ESPRI Scores



Plot Sectors and Issuers on the ESPRI Score Map



Comprehensive Guide to the Power Tool



The Power Tool can be found under the *Quant Credit Toolkit* (under Fixed Income – Credit). Alternatively use the LehmanLive keyword "ESPRI" in the find box at the top of the screen, or follow the links from the *High Grade*, *Structured Credit* or *High Yield* market monitor pages.

For further information on using the many features of the Power Tool, see the "Guide to the ESPRI Power Tool" also available on LehmanLive.

Sources for all charts and tables: Lehman Brothers.

LH+: A Fast Analytical Model for CDO Hedging and Risk Management

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We present a closed-form model for the fast calculation of spread hedges for synthetic loss tranches. We set out the derivation of the model, explaining the main assumptions, that the portfolio is large and homogeneous. Using examples we compare this model with a slower but exact model, and find that the size of the error is small for realistic CDOs. We therefore recommend this model to those looking for a fast, simple and suitably accurate risk tool for CDO tranches.

1. INTRODUCTION

The past several years have seen the market for CDO tranches, in particular synthetics, grow massively. For investors and dealers, effectively monitoring and hedging the risk of such tranches is a top priority, and in many cases the key hurdle for continued investment in the asset class.

Current approaches to valuation and risk management of CDO tranches tend to suffer from complexity and numerical instability. Semi-analytical methods, such as those using Fast Fourier Transform techniques, require significant implementation effort. Monte Carlo approaches are slow, especially for CDOs with 100+ assets in the collateral, and suffer from numerical instability, especially for the calculation of sensitivities. Recent techniques have been developed to address some of these problems (Laurent and Gregory (2003), Andersen, Sidenius and Basu (2003)), but only to a limited extent. We therefore believe that there is room for a simple and fast model for CDO tranche risk, provided it can closely match the results from more powerful models.

Conceptually, the risks of a CDO can be split into two main categories which we shall call idiosyncratic and market. Each CDO tranche reflects some combination of the two. For example, an investor in an equity tranche is particularly vulnerable to idiosyncratic risk since each default in the collateral results in a principal loss. The equity investor is therefore concerned with monitoring the credit quality of the individual assets in the collateral, attempting to anticipate and possibly hedge individual defaults. At the other end of the capital structure is the senior investor, whose level of subordination is usually such that any principal losses would only be caused by a market-wide deterioration of the credit quality of the collateral. The senior investor is therefore concerned about monitoring market risk. Any CDO risk model should be able to measure the sensitivity to idiosyncratic risk and to market risk.

We propose a model that attempts to address all of these needs. It extends the asymptotic Large Homogeneous Portfolio (LHP) approach pioneered by Vasicek (1987). The main idea is to single out the credit for which we want to compute a particular sensitivity, and to then treat the rest of the portfolio asymptotically. In effect, we are studying a large homogeneous portfolio plus one asset, hence the name LH+. This model lets us address both idiosyncratic and market-wide risks in a transparent and tractable manner.

Idiosyncratic risk measures that are of particular interest are the spread sensitivities of a tranche to the individual issuer spreads. This is relevant to correlation traders hedging their spread risk and to risk managers wishing to quantify their exposure to a particular credit in the underlying collateral. We give a simple formula in closed form for computing these. Extensions to higher order risk measures are straightforward. Also of interest is the value-on-default (VOD), which is the impact of a particular credit's default on the value of a tranche.

Market-wide quantities of interest are the sensitivity to average spread levels as well as market-wide correlation.

2. MODEL SETUP

We develop the model within the copula approach to default dependence, initially introduced by Li (2000) and further extended by Frey and Nyfeler (2001), Mashal and Naldi (2002), Rogge and Schönbucher (2003) to name just a few. While the following framework is generic, here we focus on the Gaussian copula because of its analytical tractability which has led to its widespread adoption as the market standard for correlation products. The fact that it is rooted in Merton's (1974) firm value model has justified its calibration via the use of equity return correlations.

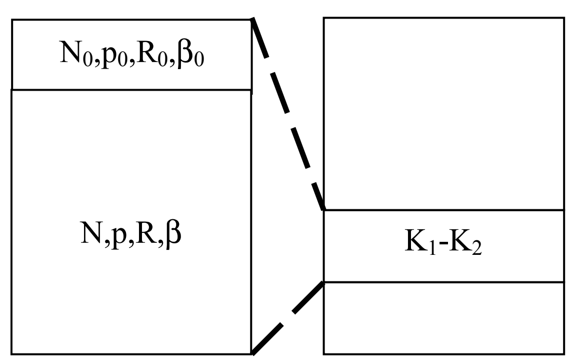
The asymptotic framework developed by Vasicek (1987) to analyze large credit portfolios has proven to be simple but powerful. In the following, we build on Vasicek's approach in the context of the one-factor Gaussian copula, although our framework could be adapted to alternative copula specifications using the results of eq, Schönbucher (2002) for Archimedean copulas or O'Kane and Schloegl (2003) for the Student-t copula.

We consider a portfolio which consists of a homogeneous part with total notional N, average default probability p, and recovery rate R. One way to formulate the Gaussian copula model is in terms of $N(0,1)$ latent variables, where each asset defaults if its return falls below the default threshold C. Denoting the standard normal distribution function by Φ, the threshold is given by:

$$C = \Phi^{-1}\left(p\right) \quad (1)$$

The market factor return is denoted by Z, the correlation of the homogeneous part of the portfolio with Z is denoted by β; this is the proxy for the market-wide level of correlation. To simplify the algebra, we restrict ourselves to the case $0 < \beta < 1$, which is also the most plausible one.

Figure 1. We split the collateral portfolio into two sections consisting of one asset which is modeled individually and a homogeneous sub-portfolio of asymptotically many assets



Source: Lehman Brothers.

In the limit of a large homogeneous portfolio, we approximate the loss sustained on the homogeneous part of the portfolio by the probability of default conditional on the market factor return Z times the loss on default *(1-R)*. Intuitively, what we are doing here is replacing the conditional default distribution, which due to the conditional independence of the credits is a binomial tending to a Poisson, with a delta function at the mean of the distribution. This approximate distribution is asymptotically correct: in the limit, as the number of assets in the portfolio becomes very large, the law of large numbers guarantees that the true conditional loss distribution tends to this form. The unconditional distribution of losses is therefore being generated by integrating over the market factor. Since it is not obvious *a priori* how good the approximation is for typical collateral sizes of synthetic CDOs, we address the issue of accuracy later in this paper.

More specifically, assuming the returns distribution for each asset in the portfolio as:

$$Z_i = \beta_i Z + \sqrt{1-\beta_i^2}\,\varepsilon_i \,,$$

where Z and all ε_i are independent standard normal random variables and all $\beta_i = \beta$, we can write the conditional probability of asset i defaulting as:

$$\pi(Z) = \Phi\left(\frac{C-\beta Z}{\sqrt{1-\beta^2}}\right).$$

The amount of loss is $(1-R)N$, and we can therefore write the loss on the homogeneous part of the portfolio as:

$$L^{\text{hom}} = (1-R)N\pi(Z) \qquad (2)$$

In addition, there is a single asset with characteristics N_0, R_0, p_0, which defaults if its asset return Z_0 falls below the implied threshold $C_0 = \Phi^{-1}(p_0)$. The idiosyncratic part of the asset return is given by ε_0, so that:

$$Z_0 = \beta_0 Z + \sqrt{1-\beta_0^2}\,\varepsilon_0 \,, \qquad (3)$$

and the default probability of the individual asset conditional on the market factor is given by:

$$\pi_0(Z) = \Phi\left(\frac{C_0-\beta_0 Z}{\sqrt{1-\beta_0^2}}\right). \qquad (4)$$

The total loss L of the portfolio is therefore given by:

$$L = \begin{cases} (1-R_0)N_0 + L^{\text{hom}}, & \text{with probability} \quad \pi_0(Z) \\ L^{\text{hom}}, & \text{with probability} \quad 1-\pi_0(Z) \end{cases} \qquad (5)$$

3. PORTFOLIO LOSS DISTRIBUTION

We first show how to derive the portfolio loss distribution. To do this we need to compute the probability of losses exceeding a given level K; we denote this by $P[L \geq K]$. To examine the loss density of the portfolio we first partition the market factor Z. The intuition is clear, since the market factor is the systemic driver of default across the portfolio. For high values of the market factor, the loss on the portfolio should be small. For low values of the market factor, the loss on the underlying portfolio is large. More specifically, for any given level K, it is possible to find two values $A(K) < B(K)$ defined by the following:

$$A = \frac{1}{\beta}\left[C - \sqrt{1-\beta^2}\,\Phi^{-1}\left(\frac{K}{(1-R)N}\right)\right],$$

$$B = \frac{1}{\beta}\left[C - \sqrt{1-\beta^2}\,\Phi^{-1}\left(\frac{K-(1-R_0)N_0}{(1-R)N}\right)\right]$$

It is easy to see that if $Z < A$, the loss on the homogeneous part is sufficiently large to cross the level K regardless of whether the additional asset defaulted or not. If $A \le Z < B$, then K is exceeded only if the single asset also defaults. Finally, if $Z \ge B$, the loss on the homogeneous part is too small, and crossing the level K is impossible. Mathematically, the above means:

$$P[L \ge K \mid Z] = \mathbf{1}_{\{Z \le A\}} + \pi_0(Z)\mathbf{1}_{\{A < Z \le B\}} \qquad (6)$$

Recall that $\pi_0(Z) = P[\,L_0 > 0 \mid Z] = P[\,Z_0 \le C_0 \mid Z]$. Integrating over the market factor lets us write the unconditional probability in terms of the bivariate normal distribution:

$$P[L \ge K] = \Phi(A) + \Phi_{2,\beta_0}(C_0, B) - \Phi_{2,\beta_0}(C_0, A) \qquad (7)$$

Here eg, $\Phi_{2,\beta_0}(C_0, A)$ denotes the CDF of the bivariate normal distribution with correlation coefficient β_0 evaluated at C_0 and A[1].

Thus we are able to compute the probability of the portfolio loss exceeding any threshold K, and from this we can generate the whole loss distribution. As there is an ample amount of reliable accurate implementations of the bivariate normal distribution function (see Genz (2002)), the formula (7) is essentially a closed form solution.

4. TRANCHE LOSSES AND DELTAS

A standard synthetic CDO tranche is described by strikes K_1 and K_2. Its notional at inception is given by $N^{tr}=K_2 - K_1$, and the tranche loss as a function of the portfolio loss is given by:

$$L^{tr} = [L-K_1]^+ - [L-K_2]^+ . \qquad (8)$$

A very important quantity for pricing or assessing the risk of a tranche is the tranche "default probability", which is the expected percentage tranche loss:

$$P^{tr} = \frac{E\left[L^{tr}\right]}{N^{tr}}$$

Using these tranche "default probabilities" for different time horizons, one can price a static pool tranche in much the same manner as a single-name default swap, see eg, Laurent and Gregory (2003) or O'Kane *et al*. (2003).

To obtain P^{tr}, we effectively need to compute the expectation of a call option payoff on the portfolio loss L. If we first condition on the realization of the market factor, then the calculation will involve multiplying the probabilities of the form (7) by the loss amount and integrating over the market variable. The resulting formula is somewhat more complicated than (7) for the loss distribution and contains not only bivariate, but also trivariate normal distribution functions:

$$\begin{aligned} E\left[(L-K)^+\right] = {} & K\left(\Phi_{2,\beta_0}(C_0, A) - \Phi(A)\right) + \left[(1-R_0)N_0 - K\right]\Phi_{2,\beta_0}(C_0, B) \\ & + (1-R)N\left[\Phi_{2,\beta}(C, A) + \Phi_{3,\Sigma}(C_0, C, B) - \Phi_{3,\Sigma}(C_0, C, A)\right] \end{aligned} \qquad (9)$$

[1] *Note that, technically, the formula for A breaks down for K=0 and K=(1-R)N and for B, when K=(1-R_0)N_0 and K=(1-R)N + (1-R_0)N_0, and therefore so do all the dependent formulas. However, these cases are easy to deal with separately, by letting A and B be infinite.*

We give the derivation in the Appendix, where it is also shown that the covariance matrix Σ is given by:

$$\Sigma = \begin{pmatrix} 1 & \beta\beta_0 & \beta_0 \\ \beta\beta_0 & 1 & \beta \\ \beta_0 & \beta & 1 \end{pmatrix}$$

The trivariate normal distribution is also fairly straightforward to implement numerically, either directly by integration of the bivariate or following the methods of Genz (2002); therefore, the formula (9) is still very appealing from the analytical point of view.

The real utility of the formula becomes apparent when we calculate the sensitivity of the tranche default probability to the spread of the single asset. If T denotes the time horizon at which we are considering the tranche loss, the risk-neutral T maturity default probability p_0 is approximately related to the default swap spread s_0 via:

$$p_0 \approx 1 - \exp\left(-\frac{s_0 T}{1 - R_0}\right) \qquad (10)$$

Since the market thresholds A and B do not depend on p_0, the only terms which depend on s_0 are those in which $C_0 = \Phi^{-1}(p_0)$ appears explicitly. The sensitivity of the tranche default probability is given by:

$$\frac{\partial P^{tr}}{\partial s_0} = \frac{1}{N^{tr}} \frac{\partial C_0}{\partial s_0} \left\{ \frac{\partial}{\partial C_0} E\left[(L - K_1)^+\right] - \frac{\partial}{\partial C_0} E\left[(L - K_2)^+\right] \right\} \qquad (11)$$

The derivatives in the right-hand side can easily be evaluated given the formulas for the default threshold and call-option payoffs. Since $C_0 = \Phi^{-1}(p_0)$, we have:

$$\frac{\partial C_0}{\partial s_0} = \frac{T(1 - p_0)}{1 - R_0} \sqrt{2\pi}\, e^{\frac{C_0^2}{2}}$$

The derivative of the call option payoff is worked out in the Appendix, where it is shown, in particular, that the derivative of the trivariate normal distribution function can be written in terms of the bivariate. The resulting formula for the derivative of the tranche "default probability" reads:

$$
\begin{aligned}
\frac{\partial P^{tr}}{\partial s_0}(T) = \frac{T}{N^{tr}}\frac{1-p_0}{1-R_0}\Bigg\{ & K_1\Phi\left(\frac{A(K_1)-\beta_0 C_0}{\sqrt{1-\beta_0^2}}\right) \\
& + \left[(1-R_0)N_0 - K_1\right]\Phi\left(\frac{B(K_1)-\beta_0 C_0}{\sqrt{1-\beta_0^2}}\right) \\
& + (1-R)N\Bigg[\Phi_{2,\rho}\left(\frac{C-\beta\beta_0 C_0}{\sqrt{1-(\beta\beta_0)^2}}, \frac{B(K_1)-\beta_0 C_0}{\sqrt{1-\beta_0^2}}\right) \\
& - \Phi_{2,\rho}\left(\frac{C-\beta\beta_0 C_0}{\sqrt{1-(\beta\beta_0)^2}}, \frac{A(K_1)-\beta_0 C_0}{\sqrt{1-\beta_0^2}}\right)\Bigg] \\
& - K_2\Phi\left(\frac{A(K_2)-\beta_0 C_0}{\sqrt{1-\beta_0^2}}\right) \\
& - \left[(1-R_0)N_0 - K_2\right]\Phi\left(\frac{B(K_2)-\beta_0 C_0}{\sqrt{1-\beta_0^2}}\right) \\
& - (1-R)N\Bigg[\Phi_{2,\rho}\left(\frac{C-\beta\beta_0 C_0}{\sqrt{1-(\beta\beta_0)^2}}, \frac{B(K_2)-\beta_0 C_0}{\sqrt{1-\beta_0^2}}\right) \\
& - \Phi_{2,\rho}\left(\frac{C-\beta\beta_0 C_0}{\sqrt{1-(\beta\beta_0)^2}}, \frac{A(K_2)-\beta_0 C_0}{\sqrt{1-\beta_0^2}}\right)\Bigg]\Bigg\}
\end{aligned} \tag{12}
$$

Completing this exercise for a set of dates, we can thus obtain a term structure of sensitivities of tranche survival probabilities to spread movements, much as is normally done for the probabilities themselves.

The derived expressions prove to be invaluable in the calculation of the risk measures known as tranche deltas. Correlation traders hedge their exposure to synthetic CDO tranches by selling protection on individual names in the collateral. The amount of protection to be sold in one name (ie, the notional of the corresponding CDS position), which makes the total position (tranche + hedge) neutral with respect to spread movements, is the definition of tranche delta. This boils down to the following:

$$\text{Tranche } \Delta(s_0) = \left(\text{Tranche PV } (s_0 + 1\text{bp}) - \text{Tranche PV } (s_0)\right)(10000 / \text{PV01}),$$

where "PV01" is the risky PV01 of a credit default swap for the name for which the delta is calculated.

It is clear that deltas are immediately related to sensitivities of tranche value with respect to spread movements. Recall that tranche PVs can be defined in terms of premium and protection leg:

$$\text{Protection leg PV} = N^{tr}\sum_{i=1}^{K} B(0,t_i)(P(t_i) - P(t_{i-1})),$$

$$\text{Premium leg PV} = s^{tr}\ N^{tr} \sum_{j=1}^{n} \Delta_j (1 - P(T_j)) B(0, T_j) \text{ , and}$$

Tranche PV = Premium leg PV – Protection leg PV.

In the above, $B(0,t)$ is the discount factor to time t; s^{tr} is the contractual spread on the tranche paid on the dates $T_1, \ldots, T_n$; and Δ_j is the accrual factor for the period between T_{j-1} and T_j. With the approximate relationship:

$$\text{Tranche PV } (s_0 + 1 \text{ bp}) - \text{Tranche PV } (s_0) \approx \frac{\partial \text{ Tranche PV}}{\partial s_0} \text{ (1 bp)}$$

we can write the tranche delta in terms of the derivative of tranche default probabilities using the following expressions:

$$\frac{\partial}{\partial s_0} \text{Protection leg PV} = N^{tr} \sum_{i=1}^{K} B(0, t_i) \left(\frac{\partial P}{\partial s_0}(t_i) - \frac{\partial P}{\partial s_0}(t_{i-1}) \right),$$

$$\frac{\partial}{\partial s_0} \text{Premium leg PV} = s^{tr}\ N^{tr} \sum_{j=1}^{n} \Delta_j \left(1 - \frac{\partial P}{\partial s_0}(T_j) \right) B(0, T_j)$$

and using formula (12) for each time horizon.

We have thus obtained an algorithm to analytically calculate deltas of synthetic CDO tranches. This is a considerable advantage over normal practice, since to determine the size of the offsetting position in CDS, hedgers usually have to manually bump the individual credit curves and re-run their models to calculate the change in the position. With an analytical formula like (12), it is possible to calculate this measure directly, saving computation time and effort.

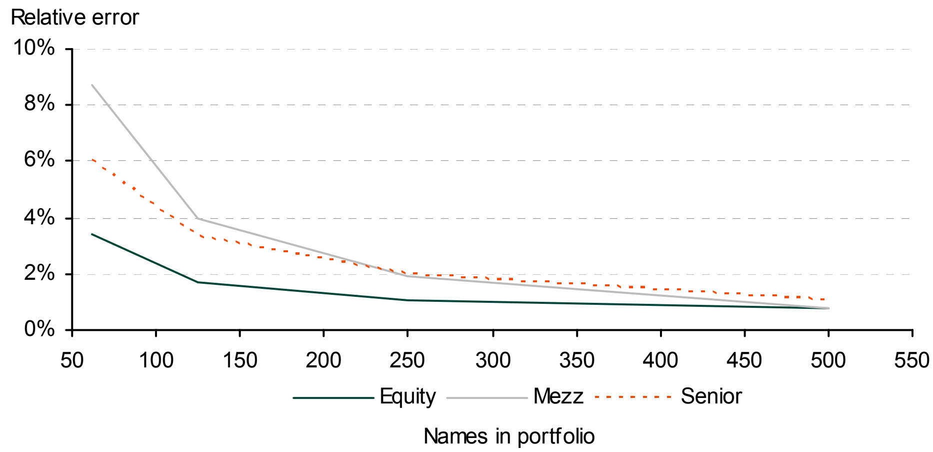
5. PERFORMANCE OF THE MODEL

We illustrate the use of our approximation formula given in equation (12). We take the following portfolio as an example. It consists of equally weighted issuers with an average 5-year default swap spread of 100 basis points and recovery rate 50%; the average pairwise correlation between issuers in the portfolio is about 20%.

We consider three tranches: an equity tranche comprising 0% to 5% of the capital structure, a mezzanine tranche from 5% to 10%, and a senior tranche with 10% subordination level, and use formula (11) to compute tranche deltas for the individual names. We compare the approximation formula to a full implementation of the Gaussian copula model.

Starting with a simple flat structure, where all the spreads, recoveries and correlations are equal to the corresponding average values, we vary the number of assets in the collateral. We plot the relative error in the deltas in Figure 2 and observe, as expected, that this error goes down as the portfolio grows, since the approximation becomes more exact. For a typical CDO collateral size of 100 to 150 assets, we get an accuracy of approximately 5% (2% for the equity tranche deltas).

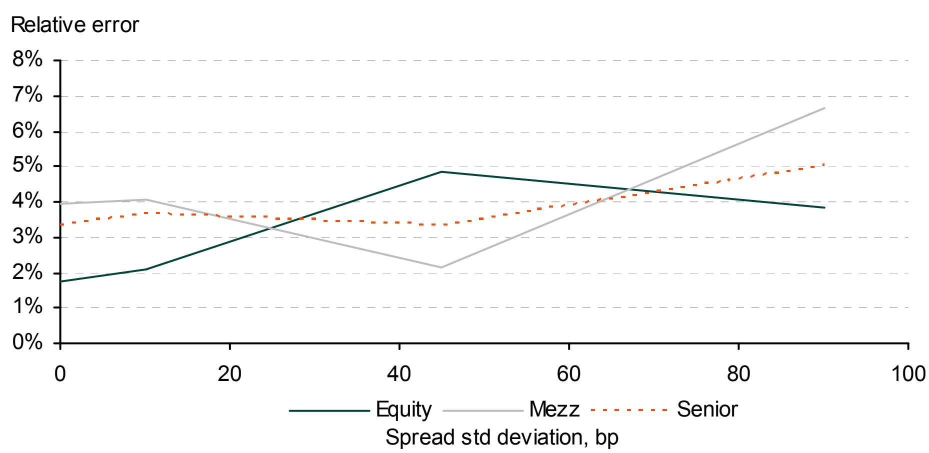
Figure 2. LH+ versus Exact: relative error as a function of portfolio size



Source: Lehman Brothers.

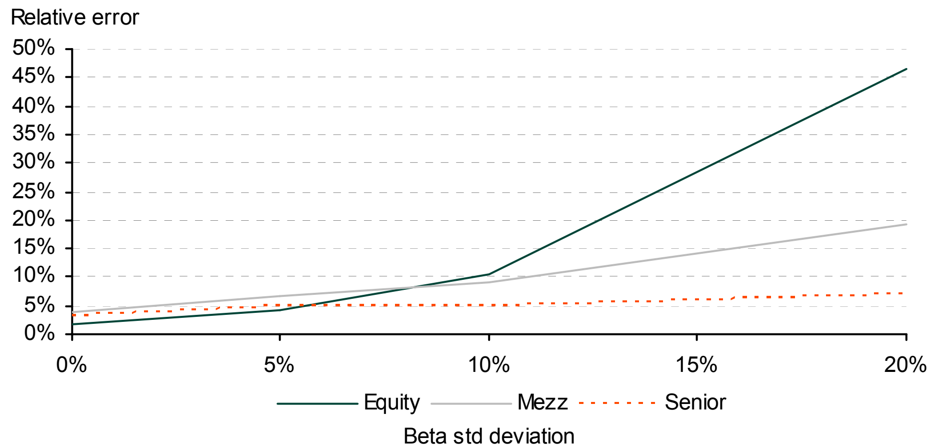
Real-life CDO collaterals are not homogeneous. To investigate the effect that this has on the accuracy of the approximation, we introduce some variability into the description. For the next three figures we investigate synthetic tranches written on a portfolio of 125 names with the same average characteristics as specified above. The plots in Figure 3 correspond to different spread distributions. The first data point is the flat configuration described above; the second data point was generated by applying the formulas to a portfolio of credits with normally distributed spreads with mean 100bp and standard deviation of 10% (ie, 10bp in absolute terms). The last two data points represent a skewed spread distribution with standard deviations of 45bp and 90bp (45% and 90% relative); the latter distribution is longer and has fatter tails. We observe that the relative error with respect to the full implementation can grow to as much as 7%, but most of the time is within the original 5%. In this calculation we keep the recoveries and correlations fixed, as in the flat configuration, to isolate the effect of spread variability.

Figure 3. LH+ versus Exact: relative error as spreads vary



Source: Lehman Brothers.

Figure 4. LH+ versus Exact: relative error as betas vary

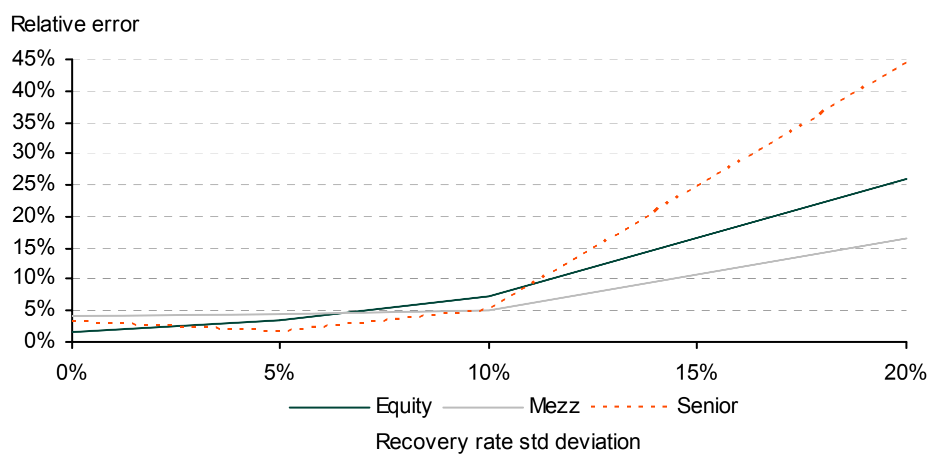


Source: Lehman Brothers.

Next, we consider names with diverse correlations. We consider two different distributions of the correlation parameter with increasing standard deviation, but the same mean as in the flat case. We observe in Figure 4 that the effect of correlation is more pronounced: even a standard deviation of 10% of the mean can increase the error to as much as 10%.

Finally, we vary recovery rates according to the same pattern in Figure 5. The effect on the error is less pronounced than for correlations, but is still capable of pushing it towards 10%.

Figure 5. LH+ versus Exact: relative error as recovery rates vary



Source: Lehman Brothers.

We can conclude that even though the error of the LH+ approximation can indeed become large for heterogeneous portfolios, an accuracy of 5−10% is usually attained for an average CDO − and often is substantially smaller. The quality of the approximation is closely related to how much of an error we are making when passing to the large portfolio limit for the part of the portfolio that we are treating as homogeneous.

Consequently, the approximation will be less efficient if this remainder of the portfolio contains large issuer concentrations or is very heterogeneous in terms of credit quality or correlation – or indeed when the portfolio of reference credits is not large enough.

6. CONCLUSIONS

The framework we have presented allows the calculation of sensitivity parameters for CDO tranches in closed form. Although we have concentrated on the spread sensitivity, it is possible to develop similar formulae to analyze, eg, correlation sensitivities and other greeks. The approximation formula is very easy to implement and typically gives sensitivities with a relative error of less than 5%. We note that it is fairly straightforward to improve the accuracy of this approach by fitting the higher moments of the conditional loss distribution. It is also straightforward to use this approach to examine other first and higher order sensitivities.

Conceptually, the LH+ framework is also very useful for analyzing CDO tranches, as it focuses on the interplay between idiosyncratic and market-wide risks, which are key in understanding the behavior of tranches at different levels of the capital structure.

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APPENDIX

Call option payoff on the loss distribution

We can write the call option payoff in the form:

$$E\left[(L-K)^{+}\right]=E\left[(L-K)\mathbf{1}_{\{L\geq K\}}\right]=E\left[L\mathbf{1}_{\{L\geq K\}}\right]-K\,P\left[L\geq K\right]. \qquad \text{(A1)}$$

The second term is given by (7), so we focus on the first term. Again, we first compute the expectation conditional on the market factor Z and then integrate. We have:

$$\begin{aligned}E\left[L\mathbf{1}_{\{L\geq K\}}\mid Z\right]=&\,E\left[L\mathbf{1}_{\{L\geq K\}}\mid Z\right]\mathbf{1}_{\{Z\leq A\}}+E\left[L\mathbf{1}_{\{L\geq K\}}\mid Z\right]\mathbf{1}_{\{A<Z\leq B\}}\\&+E\left[L\mathbf{1}_{\{L\geq K\}}\mid Z\right]\mathbf{1}_{\{Z>B\}}.\end{aligned} \qquad \text{(A2)}$$

We look at each of the three terms individually, keeping in mind the definition of thresholds A and B. In particular, if $Z\leq A$, then $L\leq K$ regardless of the realization of Z_0. Equation (6) then implies:

$$E\left[L\mathbf{1}_{\{L\geq K\}}\mathbf{1}_{\{Z\leq A\}}\mid Z\right]=\left[\pi_0(Z)(1-R_0)N_0+(1-R)N\,\Phi\left(\frac{C-\beta Z}{\sqrt{1-\beta^2}}\right)\right]\mathbf{1}_{\{Z\leq A\}}.$$

If $A<Z\leq B$, then the loss exceeds K only if the individual asset defaults, which has the probability of $\pi_0(Z)$. Once again we use equation (6) to write:

$$E\left[L\mathbf{1}_{\{L\geq K\}}\mathbf{1}_{\{A<Z\leq B\}}\mid Z\right]=\pi_0(Z)\left[(1-R_0)N_0+(1-R)N\,\Phi\left(\frac{C-\beta Z}{\sqrt{1-\beta^2}}\right)\right]\mathbf{1}_{\{A<Z\leq B\}}.$$

Finally, if Z > B, then there is no possibility that the loss exceeds K, so the third term in (A2) vanishes.

To compute the unconditional expectation $E[L\mathbf{1}_{\{L\geq K\}}]$ we need to integrate each of the terms in the two preceding equations with respect to the market factor Z. This involves evaluating expressions of the form:

$$E\left[\pi_0(Z)\mathbf{1}_{\{Z\leq z\}}\right] \qquad \text{and} \qquad E\left[\pi_0(Z)\,\Phi\left(\frac{C-\beta Z}{\sqrt{1-\beta^2}}\right)\mathbf{1}_{\{Z\leq z\}}\right]$$

for $z=A, B$. We observe that:

$$E\left[\pi_0(Z)\mathbf{1}_{\{Z\leq z\}}\right]=E\left[P\{Z_0\leq C_0\mid Z\}\mathbf{1}_{\{Z\leq z\}}\right]=P\{Z_0\leq C_0, Z\leq z\}=\Phi_{2,\beta_0}(C_0,z), \qquad \text{(A3)}$$

reducing the first of the above expressions to a bivariate normal CDF. For the second expression, we define random variables:

$$X=\beta_0 Z+\sqrt{1-\beta_0^2}\,\varepsilon_0, \quad \text{and} \quad Y=\beta Z+\sqrt{1-\beta^2}\,\varepsilon,$$

where Z, ε and ε_0 are independent standard normals. It follows that (X,Y,Z) is jointly normal with the covariance matrix given by:

$$\Sigma = \begin{pmatrix} 1 & \beta\beta_0 & \beta_0 \\ \beta\beta_0 & 1 & \beta \\ \beta_0 & \beta & 1 \end{pmatrix}.$$

At the same time, it is easy to observe that:

$$E\left[\pi_0(Z)\,\Phi\left(\frac{C-\beta Z}{\sqrt{1-\beta^2}}\right)\mathbf{1}_{\{Z\le z\}}\right] = P\{X \le C_0, Y \le C, Z \le z\} = \Phi_{3,\Sigma}(C_0, C, z), \tag{A4}$$

ie, reduced to a CDF of a trivariate normal.

We now can use equations (A3) and (A4) to write:

$$\begin{aligned} E\left[L\mathbf{1}_{\{L\ge K\}}\right] &= (1-R_0)N_0\,\Phi_{2,\beta_0}(C_0, B) + (1-R)N\,\Phi_{2,\beta}(C, A) \\ &\quad + (1-R)N\left(\Phi_{3,\Sigma}(C_0, C, B) - \Phi_{3,\Sigma}(C_0, C, A)\right). \end{aligned}$$

Combining the above with (7) and (A1) we indeed retrieve (9).

Derivative of the tranche "default probability"

To obtain the expression (12) for the derivative of the tranche default probability, we need to compute the derivative of the call option payoff on the loss distribution, given by (9). From the discussion following equation (11), it suffices to compute the derivative with respect to the default threshold C_0 and use the chain rule. Thus the problem boils down to differentiating bi- and trivariate normal CDFs with respect to their first arguments. We do this in the general case.

For independent Gaussian X and ε', define:

$$Y = \nu Z + \sqrt{1-\nu^2}\,\varepsilon'.$$

Then:

$$\begin{aligned} \Phi_{2,\nu}(x, y) &= P\{X \le x, Y \le y\} = P\left\{X \le x, \varepsilon' \le \frac{y-\nu X}{\sqrt{1-\nu^2}}\right\} \\ &= \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{x} e^{-\frac{u^2}{2}}\,\Phi\left(\frac{y-\nu u}{\sqrt{1-\nu^2}}\right)du, \end{aligned}$$

and therefore:

$$\frac{\partial}{\partial x}\Phi_{2,\nu}(x, y) = \frac{1}{\sqrt{2\pi}}e^{-\frac{x^2}{2}}\,\Phi\left(\frac{y-\nu x}{\sqrt{1-\nu^2}}\right).$$

For the trivariate normal with a covariance matrix Ω, Genz (2002) quotes the following formula:

$$\Phi_{3,\Omega}(x, y, z) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{x} e^{-\frac{u^2}{2}}\,\Phi_{2,\rho}\left(\frac{y-\omega_{12}u}{\sqrt{1-\omega_{12}^2}}, \frac{z-\omega_{13}u}{\sqrt{1-\omega_{13}^2}}\right)du,$$

with the correlation coefficient given by:

$$\rho = \frac{\omega_{23} - \omega_{12}\omega_{13}}{\sqrt{1-\omega_{12}^2}\sqrt{1-\omega_{13}^2}} .$$

We conclude that:

$$\frac{\partial}{\partial x}\Phi_{3,\Omega}(x,y,z) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} \Phi_{2,\rho}\left(\frac{y-\omega_{12}\, x}{\sqrt{1-\omega_{12}^2}}, \frac{z-\omega_{13}\, x}{\sqrt{1-\omega_{13}^2}}\right).$$

Identifying in the above $\nu = \beta_0$, $\omega_{12} = \beta_0\beta$, $\omega_{13} = \beta_0$ and $\omega_{23} = \beta$, we can explicitly compute derivatives of all terms in (9) and apply the obtained expression to derive (12).

Synthetic CDOs of CDOs: Squaring the Delta-Hedged Equity Trade

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Tight spreads in the credit markets have forced investors to turn to innovative structures in the search for yield. One such structure is the synthetic CDO of CDO tranches, also known as CDO^2. In this article, we introduce this structure, present a framework for valuation, and highlight the risk-return profile of a delta-hedged equity super tranche referencing a portfolio of mezzanine CDO tranches.

1. INTRODUCTION

In the past few years, dynamically hedged synthetic CDO tranches have had an impact on the credit derivatives market which is difficult to overstate. They have increased the liquidity and changed the dynamics of the default swap market via the "synthetic bid" for credit, and taken the CDO concept beyond the realm of structured finance into the derivatives arena. Nevertheless, synthetic CDOs are susceptible to arbitrage spreads just like their cashflow counterparts. Given the relentless spread tightening since the end of 2002, it is perhaps not surprising that it has become more difficult to obtain the yields investors had become used to via standard synthetic tranches.

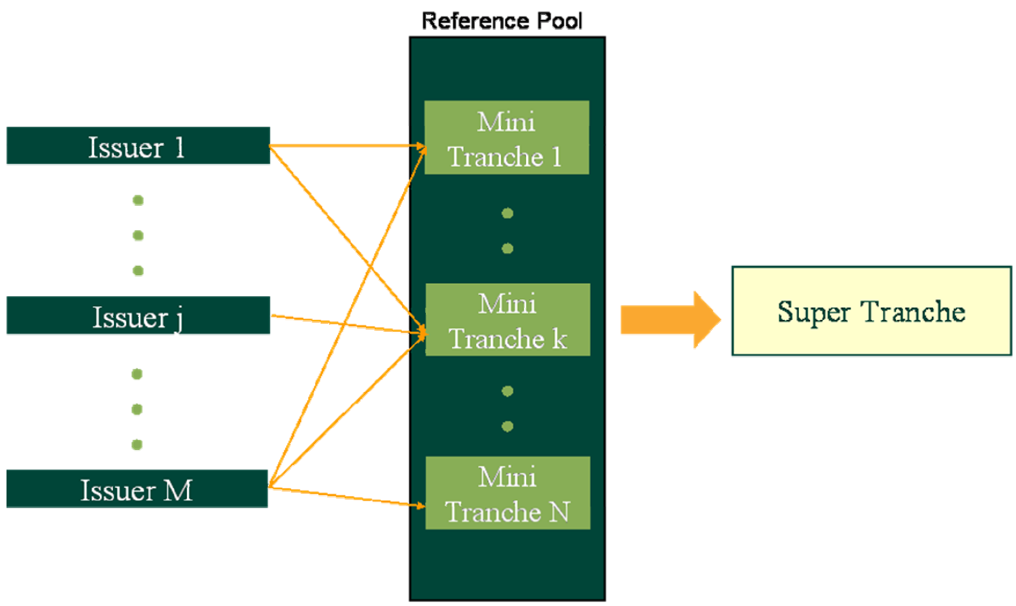
The CDO^2 concept addresses this difficulty by providing an additional layer to the capital structure. In a CDO^2, a portfolio of synthetic CDO tranches is itself tranched into so-called super tranches. This introduces quite a few new variables into the structuring equation. Not only is the composition of the underlying portfolio of individual tranches to be determined, but their joint characteristics can be tailored by varying the degree of overlap between the reference credits in the individual pools. Moreover, extra flexibility is provided to the structure by the ability to choose the level of subordination and the width of the super tranche. These additional degrees of freedom make it possible to further fine tune the risk-return profile of a loss tranche, eg, achieving high leverage while controlling the exposure to idiosyncratic default risk.

CDOs of CDOs have been used for some time in the cashflow world. However, the terms of the purely synthetic CDO^2 we are discussing here are somewhat different, so that it is worth clarifying the structure and the notation in section 2. In section 3, we extend the semi-analytical techniques for synthetic tranche valuation to the CDO^2 case, and in section 4 we illustrate the flexibility of super tranches by means of an application to a delta-hedged equity trade.

2. THE CDO^2 STRUCTURE

The fundamental inputs to a CDO^2 trade are a large pool of individual credits. The main constraint on the size of the pool is the number of credits that can be dynamically hedged due to their liquidity in the single-name default swap market. A typical pool might consist of 250-350 credits.

We denote the total number of credits in the pool by M. The credits are assigned to different so-called "mini portfolios", and we denote the total number of mini portfolios by N. It is important to note that a given credit can appear in more than one portfolio, and that the weight of a particular credit is specific to each mini portfolio.



Source: Lehman Brothers.

The percentage weight of credit j in mini portfolio k is denoted by $w_{j,k}$; these weights are constrained to be non-negative and add up to 100% within each mini portfolio, ie:

$$w_{j,k} \geq 0, \quad j = 1,2,\ldots,M, \quad k = 1,2,\ldots,N,$$
$$\sum_{j=1}^{M} w_{j,k} = 1, \quad k = 1,2,\ldots,N.$$

The matrix $\left(w_{j,k}\right)$ is the "population matrix" of the trade; it encapsulates the information about issuer concentrations and the overlap between different mini portfolios.

As the underlying risk sources of the CDO^2, we consider a tranche linked to each mini portfolio. The easiest way to describe the k^{th} mini tranche is by its percentage subordination U^k, and its percentage width V^k. Note that so far we have not fixed any absolute notional amounts yet. We denote the absolute notional of the k^{th} mini tranche by N^k, so that the total notional of the corresponding mini portfolio is N^k/V^k. This will become relevant when we describe how the individual credit losses flow through to the super tranche.

The portfolio underlying the super tranche consists of the N mini tranches, and its total notional is therefore $\sum_{k=1}^{N} N^k$. The super tranche itself is described by its percentage subordination U^{st} and percentage width V^{st}. We also refer to the portfolio of mini tranches as the super portfolio.

Let us now consider how the credit losses in the pool ultimately flow to the super tranche. Suppose that credit j defaults with a recovery rate of R. The loss to the k^{th} mini portfolio is then given by $(1-R)w_{j,k}N^k/V^k$, and the percentage notional lost is equal to $(1-R)w_{j,k}$. Once the cumulative percentage loss in any of the mini portfolios is greater than its tranche subordination, the super portfolio starts to take losses and the subordination of the super tranche is reduced. The seller of super tranche protection is obliged to make protection payments once the tranche has been eaten into, just as in a standard synthetic CDO tranche. Similarly, the contractual spread paid to the protection seller is based on the outstanding notional of the super tranche. Note that the synthetic super tranche structure is different from traditional cash CDOs of CDOs in that the mini tranches are not "physical" assets as such. They have no premium associated with them and only serve to define the subordination structure to resolve losses.

As we explain in greater detail in the next section, valuation can be performed using the concept of "tranche curve". To construct this curve, we need to define the cumulative percentage loss of the super tranche L^{st} up to a given time horizon. If the cumulative percentage loss to the k^{th} mini portfolio is L^k, then the cumulative percentage loss to the k^{th} mini tranche is:

$$L^{mt(k)} = \frac{\left[L^k - U^k\right]^+ - \left[L^k - \left(U^k + V^k\right)\right]^+}{V^k}$$

The cumulative percentage loss to the super portfolio is therefore:

$$L^{sp} = \frac{\sum_{k=1}^{N} N^k L^{mt(k)}}{\sum_{k=1}^{N} N^k},$$

and the cumulative percentage loss to the super tranche is:

$$L^{st} = \frac{\left[L^{sp} - U^{st}\right]^+ - \left[L^{sp} - \left(U^{st} + V^{st}\right)\right]^+}{V^{st}}$$

While the loss on a standard CDO tranche is a call spread on the underlying portfolio loss, the equations above show that the super tranche loss is given by compound options, effectively calls on a basket of vanilla call spreads.

3. A SIMPLE MODEL FOR VALUATION

At the core of any CDO pricing model is a mechanism for generating dependent defaults. Latent variable models describe default as an event generated by a latent variable – generally interpreted as asset return – falling below a specified threshold, which is in turn calibrated to observable CDS spreads of the reference credit. The dependence among the default times of different names is then naturally determined by the dependence structure (a.k.a. the *copula*) of the latent variables.

One of the most popular latent variable models combines a Gaussian copula with a one-factor correlation framework. The return of asset j, X_j, is driven by a common market factor Y, and an idiosyncratic variable E_j:

$$X_j = \beta_j \cdot Y + \sqrt{1-\beta_j^2} \cdot E_j.$$

Here the variables Y, E_j , $j=1,2\ldots,M$ are taken to be independent standard normal random variables, so that the asset returns X_j are jointly normal with an MxM correlation matrix given by:

$$\left(C_{j,l}\right) = \left(\beta_j \cdot \beta_l\right).$$

Within the one-factor Gaussian framework, the dependence structure is fully specified by a vector of betas, each of which can be interpreted as the correlation of an asset with the market. Given a particular realization of the market factor, the probability that the j^{th} credit defaults is now given by:

$$\pi_j(Y) = P\left[X_j < D_j \mid Y\right] = N\left(\frac{D_j - \beta_j \cdot Y}{\sqrt{1-\beta_j^2}}\right),$$

where D_j represents the default threshold for a given time horizon calibrated to the j^{th} name's credit curve.

The parsimonious one-factor structure of this model implies that, conditional on the realization of the market factor, the M individual credits are independent. When pricing a standard CDO, this conditional independence greatly facilitates the calculation of the conditional loss distribution of the tranche. In a CDO^2, however, since some of the credits may belong to several mini portfolios, the loss distributions of the mini tranches need not be conditionally independent even if the defaults of the individual credits are. The possibility of overlapping credits in the reference mini portfolios significantly complicates the task of recovering the conditional joint loss distribution of the mini tranches, which is in turn necessary to compute the conditional loss distribution of the super tranche. In the Appendix, we show how we can overcome this obstacle by means of a recursive procedure which is a multivariate extension of a well-known recursive algorithm used for standard CDOs.

Once we know how to compute the loss distribution of the super tranche for a given realization of the market factor, it is straightforward to take a probability weighted average across all possible market realizations and thus recover the unconditional loss distribution of the super tranche. Repeating the entire procedure for a grid of horizon dates, and interpreting the expected percentage loss up to time t as a cumulative default probability, we can price a tranche using exactly the same analytics as in a single-name default swap. More precisely, define the "survival probability" of the super tranche up to time t as:

$$Q^{st}(t) = 1 - E\left[L_t^{st}\right].$$

Then the two legs of the CDO^2 swap can be priced using:

$$\text{PV(Protection Leg)} = N^{st} \sum_{i=1}^{S} B(s_i)(Q^{st}(s_{i-1}) - Q^{st}(s_i)),$$

$$\text{PV(Premium Leg)} = c^{st} N^{st} \sum_{i=1}^{T} \Delta_i Q^{st}(t_i) B(t_i),$$

where c^{st} is the coupon paid on the super tranche, N^{st} is the notional of the super tranche, t_i, $i=1,2,\ldots,T$ are the coupon dates, Δ_i, $i=1,2,\ldots T$ are accrual factors, s_i, $i=1,2,\ldots,S$ discretize the timeline for the valuation of the protection leg, and $B(t)$ is the risk-free discount factor for time t.

To summarize, in the context of a one-factor Gaussian framework, we can price a super tranche if we specify:

1. the CDO^2 structure, ie,
   a. the population matrix $\left(w_{j,k}\right)$,
   b. the mini tranches triples $\left(U^k, V^k, N^k\right)$,
   c. the super tranche triple $\left(U^{st}, V^{st}, N^{st}\right)$,
2. the issuer curves of all the underlying credits (used to calibrate the thresholds D_j, $j=1,2,\ldots,M$ for each horizon date),
3. the asset correlations among all of the underlying credits (used to calibrate the market sensitivities β_j, $j=1,2,\ldots,M$).

The major advantage of the pricing approach outlined above is that the quasi-analytic valuation is convenient for the computation of precise sensitivities to the underlying parameters. As we show in the next section, these sensitivities can be highly valuable for the purpose of hedging out some of the risks and designing attractive risk-return profiles.

4. DELTA-HEDGING SYNTHETIC TRANCHES

A trade that has recently gained popularity consists of selling protection on a synthetic equity tranche and simultaneously buying protection on the individual CDS of the underlying names. Single-name protection is bought in amounts sufficient to delta-hedge the tranche position against individual spread movements, ie, to immunize the present value of the long position in the equity tranche against small changes in the underlying spread curves.

Selling delta-hedged equity protection provides the investor with a risk-return profile which generally displays positive carry, negative "Value On Default" (VOD), positive systematic convexity, and positive exposure to correlation risk. In other words, the investor earns positive carry and profits from general spread volatility through the positive convexity. In return for these features, she tolerates exposure to idiosyncratic default risk and to changes in correlations. We will come back to each of these points in greater detail below.

In this section we construct a series of stylized trades. We start by reviewing the standard delta-hedged equity exposure, and then show that similar convexity trades can in principle be constructed by hedging tranches other than the equity. However, our results indicate that by increasing the subordination of the delta-hedged tranche, the reduction in idiosyncratic default risk is necessarily accompanied by a significant loss of convexity. The main point of our discussion is then to show that, due to the increased leverage inherent in the additional capital structure layer, a delta-hedged super equity tranche of a CDO^2 retains the positive convexity while trading off idiosyncratic default risk for carry.

4.1. Delta-Hedged Equity

In a delta-hedged tranche trade, the carry is simply the difference between the compensation the investor receives for protecting the losses on the equity tranche and the premia she pays to delta-hedge. It is generally positive for a typical delta-hedged equity position.

VOD is generally defined with respect to each name in the underlying reference set, and it is equal to the change in value of the delta-hedged position in case a given name defaults instantly. Of course, VOD can be defined analogously for multiple instantaneous defaults. In order to compute VOD correctly, one has to calculate the difference between the protection payments on both sides of the trade (tranche and hedge) following the hypothesized default(s), and add the effect of the default(s) on the mark-to-market of the remaining piece of the equity tranche. VOD is generally negative for a delta-hedged equity trade, since the protection bought on each name through the CDS market, chosen to immunize the investor against spread movements, is less than the notional represented by that name in the reference portfolio. This means that, for each hypothesized default, the protection payment received on the CDS is not enough to compensate the protection payment due on the tranche.

The term “positive systematic convexity” simply refers to the fact that the delta-hedged equity position increases in value when there is a general spread widening as well as following a general spread tightening. In the first scenario, the gains from the hedges outweigh the loss on the tranche, while in the second the gain from the tranche investment outweighs the losses on the CDS shorts.

Lastly, an equity investor has positive exposure to changes in correlations. An increase in correlations increases the volatility of the loss distribution of the underlying reference portfolio, thereby decreasing the expected loss on the first-loss tranche and increasing the present value of the equity investment (changes in correlations have obviously no effect on the values of the single-name hedges). Of course, a decrease in correlations hurts the equity investor for exactly the opposite reason.

In summary, one can think of the delta-hedged equity trade as a way of gaining positive carry and positive systematic convexity in exchange for tolerating negative VOD and correlation risk. Figure 1 reports the carry, VOD and correlation exposure for a 10MM notional, delta-hedged equity investment, referencing an equally weighted homogenous portfolio of 100 names. Each name in the reference portfolio has a “beta” equal to 50% (ie, flat pairwise correlations equal to 25%), a flat CDS curve of 65bp, and a recovery rate equal to 40%. Note that we denote with VODx the change in value of the position in case x issuers instantly default[1].

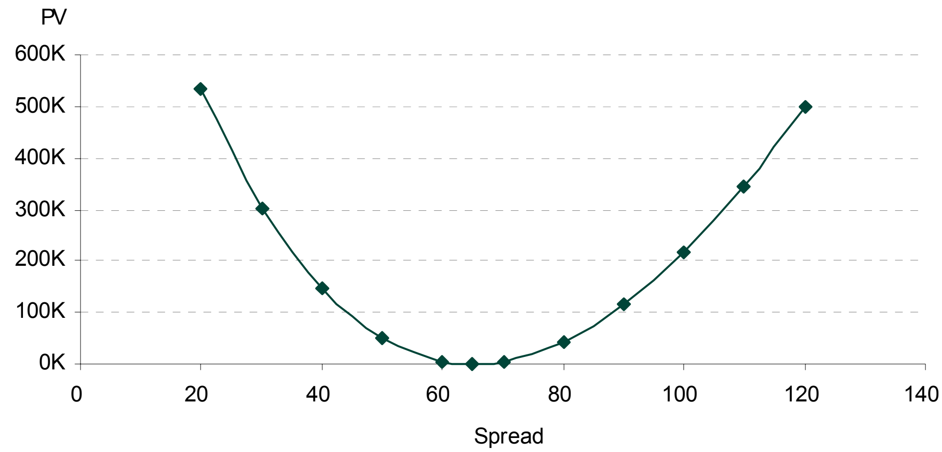
Figure 1 shows that this position offers an annual carry of about $256K. Under the modeling assumptions detailed above, a sudden default occurring immediately after the inception of the trade will cause a VOD loss of $356K, two defaults will cost the investor $672K and three defaults $940K. The position is also long correlation, with its mark-to-market gaining $53K if all betas increase by 1%. The positive systematic convexity of this position is captured in Figure 2, which shows that a generalized spread widening from 65bp to 100bp will produce a net gain of about $218,000. We stress that this is a stylized, albeit reasonably realistic trade, and that we are discussing model outputs without taking into account liquidity costs.

Figure 1. Carry, VOD, and Correlation Exposure of Delta-Hedged Equity Tranche

	Annual Carry ($)	VOD1 ($)	VOD2 ($)	VOD3 ($)	Correlation Exposure (Net $ gain for 1% increase in betas)
Equity (0-5%)	256K	356K	672K	940K	53K

1 *To compute VOD, one generally needs to specify not only the number but also the identities of the hypothesized defaulters, since their deltas depend on name-specific inputs such as credit curves and betas. When dealing with an equally weighted homogeneous portfolio, however, it is sufficient to specify the number of hypothesized defaults.*

Figure 2. Systematic Convexity of Delta-Hedged Equity Tranche



Source: Lehman Brothers.

4.2. Delta-Hedged Junior Mezzanine

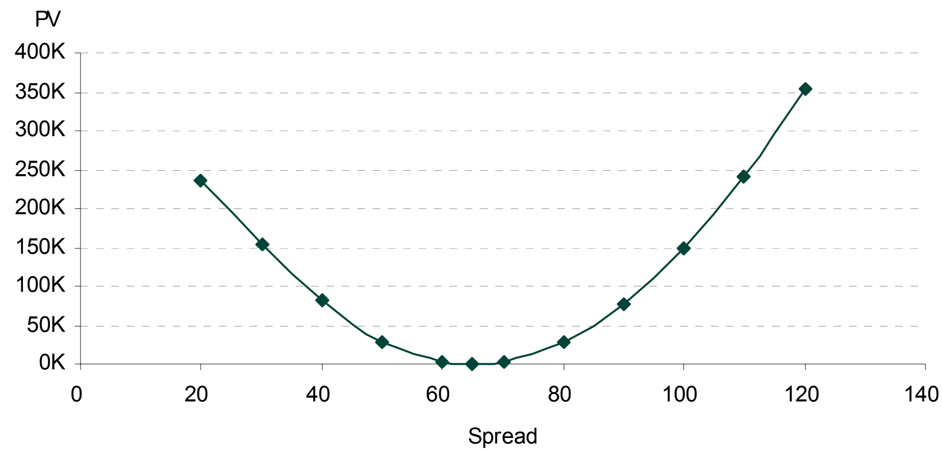
The numbers reported above show the attractiveness of the delta-hedged equity trade for investors who anticipate a systematic spread widening, possibly accompanied by an increase in correlations. However, some investors may be concerned about the relatively large VOD, and may want to look for ways to tame this idiosyncratic default risk without losing the systematic convexity of the position. This is difficult to achieve in the context of a plain synthetic CDO: as soon as we consider a delta-hedged investment in a tranche with some amount of cushion, we decrease the VOD exposure but at the same time lose the attractive feature of positive convexity. To show this point, Figures 3 and 4 replicate the information contained in Figures 1 and 2 for a 10m notional, delta-hedged investment in a 1-6% loss tranche referencing the same portfolio described above.

While we have reached the goal of trading off VOD for carry, we also significantly decreased the systematic convexity of the trade: a generalized widening of all spreads from 65bp to 100bp now delivers a mark-to-market gain of only about $149K. The correlation exposure has also decreased (to $33K gain for a 1% increase in betas), reducing the overall ability of the trade to monetize a scenario of widening spreads and increasing correlations.

Figure 3. Carry, VOD, and Correlation Exposure of Delta-Hedged Junior Mezzanine Tranche

	Annual Carry ($)	VOD1 ($)	VOD2 ($)	VOD3 ($)	Correlation Exposure (Net $ gain for 1% increase in betas)
Junior Mezz (1-6%)	31K	23K	238K	604K	33K

Figure 4. Systematic Convexity of Delta-Hedged Junior Mezzanine Tranche



Source: Lehman Brothers.

4.3. Delta-Hedged Super Equity

A delta-hedged super equity tranche of a CDO^2 referencing a portfolio of mezzanine tranches preserves a pronounced systematic convexity while efficiently trading off VOD for carry. To show this point, we first build three mini mezzanine tranches covering losses between 3% and 6%. Once again, each tranche references an equally weighted homogeneous portfolio of 100 names, each with a beta of 50%, a recovery rate of 40%, and a flat CDS curve of 65bp. The three reference portfolios overlap: each one has 40 unique names, 40 names that also belong to one of the other two portfolios, and 20 names that are common to all three portfolios.[2]

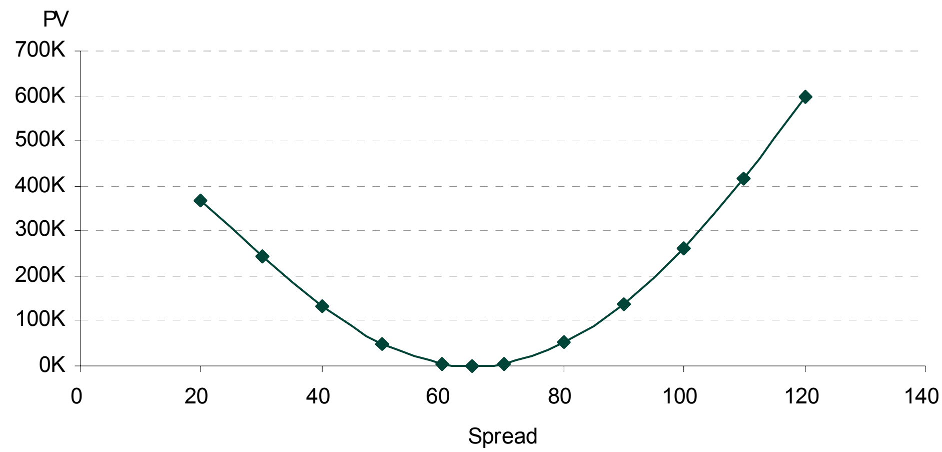
Next, we consider a 10MM notional, delta-hedged investment in a 0-5% super equity tranche referencing the three mini mezzanines described above. Figures 5 and 6 report the usual measures associated with this trade. First, notice that both the carry and the VOD profile of this trade are very close to those of the delta-hedged, 1-6% loss tranche analyzed above[3]. Most importantly, notice that this time the reduction of the idiosyncratic default risk did not come at the expense of either the correlation exposure or the systematic convexity. This "squared" delta-hedged equity investment has a positive correlation exposure of $54K for a 1% increase in betas, which is slightly higher than that of the delta-hedged 0-5% equity, and significantly higher than that of the delta-hedged 1-6% tranche. Moreover, figure 6 shows that a generalized increase in spreads from 65bp to 100bp now produces a net mark-to-market gain of approximately $261K.

Figure 5. Carry, VOD, and Correlation Exposure of Delta-Hedged Super Equity Tranche

	Annual Carry ($)	VOD1 ($)	VOD2 ($)	VOD3 ($)	Correlation Exposure (Net $ gain for 1% increase in betas)
Super Equity (0-5%) of Three Mini Mezz (3-6%)	28K	11K	246K	726K	54K

2 *This implies that the reference universe for the CDO^2 consists of 200 issuers.*
3 *Since in our CDO^2 the VOD numbers depend on whether the defaulter(s) belong(s) to one, two or all three of the reference portfolios, we conservatively compute VOD assuming that each hypothesized defaulter belongs to all three mini portfolios.*

Figure 6. Systematic Convexity of Delta-Hedged Super Equity Tranche



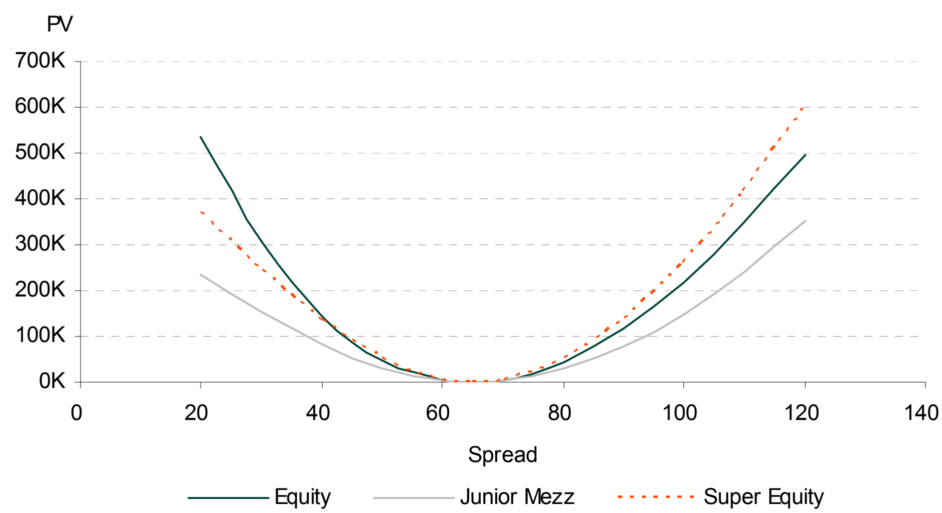
Source: Lehman Brothers.

To help the reader summarize our findings, Figure 7 compares carry, VOD and correlation exposure for the three trades we have analyzed in this section, while Figure 8 compares their systematic convexities.

Figure 7. Carry, VOD, and Correlation Exposure of Three Delta-Hedged Loss Tranches

	Annual Carry ($)	VOD1 ($)	VOD2 ($)	VOD3 ($)	Correlation Exposure (Net $ gain for 1% increase in betas)
Equity (0-5%)	256K	356K	672K	940K	53K
Junior Mezz (1-6%)	31K	23K	238K	604K	33K
Super Equity (0-5%) of Three Mini Mezz (3-6%)	28K	11K	246K	726K	54K

Figure 8. Systematic Convexity of Three Delta-Hedged Loss Tranches



Source: Lehman Brothers.

APPENDIX: DERIVING THE LOSS DISTRIBUTION OF A SUPER TRANCHE

Our goal in this Appendix is to construct the cumulative loss distribution of a super tranche up to a given horizon. As mentioned in the main text, the possibility of overlapping credits in the reference mini portfolios significantly complicates the task of recovering the conditional joint loss distribution of the mini tranches, which is in turn necessary to compute the conditional loss distribution of the super tranche. To overcome this obstacle, we propose a recursive procedure which is a multivariate extension of a well-known recursive algorithm used for plain CDOs (see, for example, Greenberg *et al* (2004)).[4]

We first discretize losses in the event of default by associating each credit with the number of loss units that its default would produce in each of the mini portfolios: the representative element of the "loss matrix" $\left(\lambda_{j,k}\right)$ indicates the integer number of loss units in mini portfolio *k* due to the default of name *j*.

Next, we construct an *N*-dimensional hyper-cube whose k^{th} side consists of all possible loss levels for the k^{th} mini portfolio, ie, $\left(0,1,\ldots,\sum_{j=1}^{M}\lambda_{j,k}\right)$. For ease of explanation, and without loss of generality, we consider here a two-dimensional example (k=2). In this case, our hyper-cube is simply a matrix $\left(Z_{v_1,v_2}\right)$ where we can store the conditional joint distribution of the two mini portfolios; for example, we store in $Z_{3,5}$ the probability of jointly having three loss units in the first mini portfolio and five in the second. In order to obtain the correct set of joint probabilities, we first initiate each state (recursion step j=0) by setting:

$$Z_{v_1,v_2}^{0}=1,\ \text{ if }\ v_1=0 \text{ and } v_2=0,$$

$$Z_{v_1,v_2}^{0}=0 \text{ otherwise.}$$

Then, we feed the *M* credits, one at a time, through the following recursion:

$$Z_{v_1,v_2}^{j}=\left(1-\pi_j(Y)\right)\cdot Z_{v_1,v_2}^{j-1}+\pi_j(Y)\cdot Z_{\left(v_1-\lambda_{j,1}\right),\left(v_2-\lambda_{j,2}\right)}^{j-1}\quad \text{if}\quad v_1\geq\lambda_{j,1}\cap v_2\geq\lambda_{j,2}$$

$$Z_{v_1,v_2}^{j}=\left(1-\pi_j(Y)\right)\cdot Z_{v_1,v_2}^{j-1}\ \text{ otherwise,}$$

where $\pi_j(Y)$ indicates the conditional probability that issuer *j* defaults. Each credit can either survive, and every state then "keeps" its position, or default, in which case every state "moves" in the direction $[\lambda_{j,1}, \lambda_{j,2}]$. After including all the issuers, we set:

$$\left(Z_{v_1,v_2}\right)=\left(Z_{v_1,v_2}^{M}\right).$$

The matrix $\left(Z_{v_1,v_2}\right)$ now holds the joint loss distribution of the two mini portfolios conditional on the realization of the market factor. It is then straightforward to recover the conditional joint distribution of losses on the mini tranches, the conditional loss distribution of the super portfolio and the conditional loss distribution of the super tranche. We can then proceed to integrate over the market factor, construct the tranche survival curve, and price the super tranche as described in the main text.

[4] *Greenberg, A., Mashal, R., Naldi, M., Schloegl, L. (2004), "Tuning Correlation and Tail Risk to the Market Prices of Liquid Tranches", Quantitative Credit Research, March 3-12.*

Valuation of Constant Maturity Default Swaps

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A constant maturity default swap (CMDS) is a new variant of a standard credit default swap where the premium leg is periodically reset and indexed to the current market spread on a constant maturity CDS. The indexation significantly reduces mark-to-market exposure to spread widening and tightening, and instead allows investors to take exposure to changes in the steepness of the credit curve. Investors bullish about the credit fundamentals of an issuer but concerned about current market valuations are natural sellers of CMDS protection. In this article, we describe the basic mechanics of a CMDS contract and discuss in detail a number of issues concerning valuation and risk analysis.[1]

1. INTRODUCTION

Constant-maturity default swaps (CMDS) are a new class of basic credit derivatives that enable investors to tailor their exposure to specific components of credit risk. The basic CMDS contract is a variant of a standard credit default swap (CDS) where the premium leg is periodically reset and indexed to some constant-maturity tenor. For example, an investor may choose to sell protection for a 5-year period and, in return, receive a fixed fraction of the prevalent 5-year CDS spread at each premium payment date. The concept of a floating premium leg opens up several additional possibilities for structured credit products tailored to the needs of investors. These include:

- By combining the floating leg with a standard CDS protection leg, the investor creates a credit default swap with low net spread sensitivity. This combination can be used to take a shorter term view on curve steepening or flattening, or by a longer term investor bullish about the fundamentals of an issuer but bearish about current market valuations.
- By combining the floating premium leg with a standard CDS fixed premium leg, the CMDS becomes a pure synthetic contract used for hedging or taking a view on spread widening or narrowing. The investor will never need to deliver or take delivery of any bond or loan and no longer has any recovery rate risk following a credit event.
- By combining the floating premium leg with the protection leg on a standard synthetic loss tranche, a buy-and-hold investor can earn a leveraged premium by assuming the limited/tranched default risk on a diversified credit portfolio while being hedged against general portfolio spread widening.
- By swapping floating payments linked to different tenors, an investor can take a view on the shape of the credit curve while abstracting from recovery rate risk.

We discuss the mechanics, valuation and risks of a CMDS contract. The next section introduces the CMDS product and terminology. Sections 3 and 4 are devoted to valuation. Section 3 explains that the value of the floating premium leg is driven primarily by forward spreads but also that forward spreads must be adjusted by a factor dependent on spread volatility. We explain the origin of this so-called convexity adjustment. Section 4 is about valuation in a stochastic hazard rate model and the relationship between such models and well-known short-rate models for valuation of interest rate derivatives. In section 5 we provide numerical examples of the risks in CMDS. We conclude in section 6. In the Appendix we derive simple Black formulas for valuation of caps and floors on the floating premium leg.

[1] *We would like to thank Peter Alpern, Marco Naldi and Dominic O'Kane for discussions and comments.*

2. PRODUCT DESCRIPTION

Constant maturity (credit) default swaps (CMDS or CMCDS) are the core of a new product line being marketed by a number of large dealers, including Lehman Brothers. Aside from the obvious similarity to the well-established constant maturity interest rate swap products, CMDS products have been introduced primarily to address a concern among some end users about the effect of spread volatility on the mark-to-market of established credit derivatives products such as CDS and synthetic CDOs (loss tranches). This concern may be derived from a public company's obligation to mark-to-market the value of its open derivatives contracts in its financial statements − a requirement that can introduce undesired earnings volatility. Alternatively, an end user selling credit protection may be concerned about current market valuations and increasing spreads. In this section we introduce CMDS.

The Floating Premium Leg

The innovation in CMDS is the floating premium leg. A CMDS extends the features of a standard CDS by incorporating a premium leg which pays a floating coupon proportional to the observed market spread on a CDS with a fixed time to maturity.

The floating coupon usually sets in advance (at the start of the coupon period) and pays in arrears (at the end of the coupon period). After being day-count adjusted in the appropriate basis, the constant-maturity spread for the period is multiplied by a pre-determined factor called the *participation rate*. The participation rate is fixed for the life of the contract and is determined so that the contract has value zero at initiation. The constant-maturity term to which floating cashflows are linked is called the *constant maturity tenor*.

Types of CMDS Trades

Floating-for-Protection: The standard CMDS contract quoted in the market is a floating-for-protection swap where the party that is long the CMDS pays the floating premium and receives protection. The quote is given by a bid-offer for the participation rate. For example, a quote of 68%-74% indicates a seller of protection in the floating-for-protection swap will receive 68% of the reset spread, whereas a buyer of protection will have to pay 74%.

Fixed-for-Floating: The premium leg of a standard CDS can be combined with a floating premium leg to create a fixed-for-floating (or floating-for-fixed) CMDS. If this combination is quoted directly as a bid-offer on the participation rate then it is implicitly assumed that the fixed leg will pay the current market CDS spread. Alternatively the fixed leg can be quoted as a bid-offer on the fixed spread assuming a participation rate of 100% on the floating leg. For example, if the CDS spread is 60bp and the floating-for-protection participation quote is 68%-74%, then the protection-for-floating participation quote is 74%-68% and the fixed-for-floating fixed-rate quote is 81.1bp-88.2bp (60bp/74% and 60bp/68%).]

Floating-for-Floating: A floating-for-floating CMDS consists of two floating legs indexed to different constant maturity tenors (eg, 1-year and 5-year). In such instances, one of the legs is assumed to have a 100% participation rate and the trade is quoted as a bid-offer on the participation rate of the other.

In section 5 we discuss the risks and mark-to-market sensitivities of each of these types of CMDS trades. However, most of our analysis will concentrate on floating-for-protection CMDS and directly on the floating premium leg. In this article a CMDS refers to a floating-for-protection CMDS unless otherwise specified.

Caps and Floors

It is standard to incorporate a cap on the reset spread. This is to avoid circumstances in which the CDS spread becomes unobservable, for example because as the spread widens the liquidity may be significantly impaired and the market may only quote the CDS with an up-front price. The cap is typically 500-1000bp.

In this article we always assume that the cap is directly on the reset spread (before the participation rate is applied) and not on the actual payment. For example, if the cap is 1000bp and the participation rate is 80%, then the payment cap is 800bp. We believe this will become the market standard across all dealers. Although it makes no difference for valuation (since we can easily convert between a spread cap and a payment), a spread cap makes it easier to offset contracts since the caps on CMDS entered into at different participation rates will have the same strikes. For example, imagine a dealer that quotes participation rates with a 500bp cap. The dealer buys 10m floating-for-protection at 68% that can be sold at 74%. If 9.19m (68%/74%·10m) is sold, the dealer is left with 0.81m of protection that can be converted into a (credit risky) payment stream by selling standard CDS protection. If the 500bp cap had been a payment cap, the dealer would also have been left with a long and a short cap at different strikes.

Similarly, a CMDS contract may trade with a floor on the reset spread. This may be imposed to guarantee the protection seller a minimum level of spread income, or alternatively to make the contract volatility-neutral. As we explain in detail in the following sections, an uncapped floating premium leg is exposed to volatility risk. Holding the credit curve fixed, the value of the uncapped floating premium leg increases as volatility increases, that is the uncapped floating premium leg has a positive vega[2]. By embedding a cap the vega will be reduced. It is possible to choose the strike of the cap such that the initial (on the trading date) sensitivity to volatility is 0. If a low cap is desired, the vega of the uncapped floating leg will not be enough to produce a zero-vega CMDS, in which case a floor on the floating premium is needed to prevent a negative vega.

Caps and floors on CMDS trades are typically deep out of the money. For this reason, care has to be taken in their valuation, to account for the shape of the volatility smile.

Reset Mechanism

The reset mechanism is probably the biggest hurdle in making CMDS a more liquid product. In one mechanism, the party receiving the floating premium can on the reset date offer to buy protection on a CDS with a maturity equal to the constant maturity tenor and a notional equal to the CMDS notional multiplied by 0.25 (the length of the coupon period) divided by the PV01 of the CDS and multiplied by the participation rate[3]. The spread at which the protection is bid is then used as the reset spread. With this procedure the receiver of the floating spread has no incentive to report too high a spread. If the party receiving the floating premium does not provide a bid, the floating premium payer would be required to solicit quotes from a contractual list of dealers. The quotes obtained would be used to determine the reset spread, for example as the arithmetic average of the provided quotes disregarding the highest and lowest quote.

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[2] *Vega is a standard risk measure defined as the first derivative of the value of an option with respect to a volatility parameter.*

[3] *The PV01 would be the credit risky PV01 calculated on a flat CDS curve with spreads equal to the CDS spread at which the protection is being bid (using the Bloomberg CDSW calculator).*

Mark-to-Market of CMDS Trades

The mark-to-market (MTM) formula for a standard short protection CDS is

$$MTM_{CDS} = \left(S_{contract} - S_{market}\right) \cdot PV01_{market} \tag{2.1}$$

where $S_{contract}$ is the contractual spread in the CDS, S_{market} is the current market spread on a CDS with the same maturity, and $PV01_{market}$ is the value of receiving a 1bp flow on the payment dates until maturity or default.

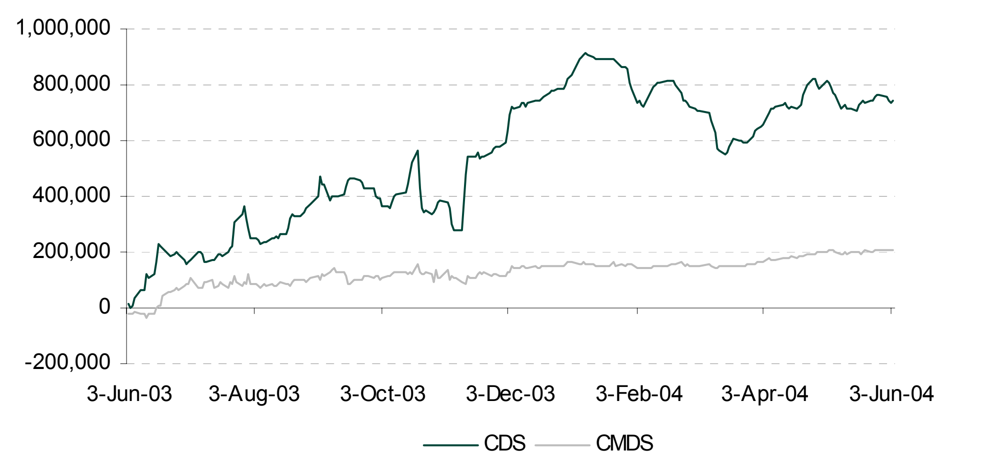
The formula for a protection-for-floating CMDS (pays protection receives floating) is

$$MTM_{CMDS} = \left(\frac{\alpha_{contract}}{\alpha_{market}} - 1\right) \cdot S_{market} \cdot PV01_{market} \tag{2.2}$$

where $\alpha_{contract}$ is the contractual participation rate and α_{market} is the current market participation rate on a CMDS with the same maturity, tenor and spread cap. The formula gives the value of the floating premium leg minus the value of the protection leg. The value of the protection leg is $PVP_{market} = S_{market} \cdot PV01_{market}$. If the participation rate is 1, then the value of the floating premium leg is $FL_{market} = PVP_{market}/\alpha_{market}$. The value of the CMDS is therefore $\alpha_{contract} \cdot FL_{market} - PVP_{market}$ which simplifies to (2.2). For this calculation to hold it is important that embedded caps and floors are on the spread itself (see above).

Figure 1 shows the MTM on a 5-year short protection CDS and a 5-year protection-for-floating CMDS with a 5-year tenor and a 1000bp spread cap. Both contracts are on Ford Motor Credit. For each trading day over the past year we calculated the participation rate for a new CMDS with the above specification. We used CDS curves from Mark-It Partners and a level of volatility close to the historical volatility over the period. We then calculated the daily MTM of the two contracts using formulas (2.1) and (2.2)[4] and assuming a notional of 10 million. Figure 1 shows the cumulative sums of the daily MTM. The positive carry is not included. With the daily rollovers assumed for the MTM, the carry would be the same on the two contracts – about 230,000 over the entire period.

Figure 1. Mark-to-Market of a CMDS and a CDS for Ford Motor Credit



Source: Mark-It Partners and Lehman Brothers.

[4] *We ignored the 1-day maturity mismatch.*

The lower volatility of the MTM of the CMDS is obvious from Figure 1. As we illustrate in section 5, unlike a CDS a CMDS is insensitive to changes in the level of the curve but sensitive to changes in the slope of the curve. Over the period, it was primarily the level of the Ford Motor Credit curve that changed and not its slope. It is consistent with the general perception of the main drivers of credit curve changes.

3. VALUATION WITH FORWARD SPREADS

The value of the floating premium leg of a CMDS contract is first and foremost determined by the level of forward spreads. We value the floating leg by valuing each individual payment. To value a single payment we find the expected payment and discount it to today's value using a credit risky discount factor. The forward spread gives a first estimate of the expected payment but it must be adjusted by a so-called *convexity adjustment*.

First Order Approximation

We can get a first order approximation of the value of the floating premium leg if we can calculate forward spreads. We use S(t;T,T') to denote the forward spread at date t on a CDS that starts at date T and ends at date T'. The forward spread is given by

$$S(t,T,T') = \frac{PVP(t;T,T')}{PV01(t;T,T')} \qquad (3.1)$$

where PVP(t,T,T') is the value of the protection leg and PV01(t,T,T') is the value of the premium leg assuming a premium of 1bp. The PV01 incorporates the value of any premium accrual to be paid by the protection buyer at default. It is well-known how to calculate the value of the two legs, and thus the forward spread, using the reduced form-approach of Jarrow and Turnbull with a piece-wise constant hazard rate of default and a constant recovery-given-default. See O'Kane and Turnbull (2003).

The value of the floating premium leg is the sum of the value of the individual payments

$$FL(T_0) = \sum_{i=1}^{n} X_i(T_0) \qquad (3.2)$$

where T_0 is the valuation date, T_1, …, T_n are the premium payment dates, and $X_i(t)$ is the value at date t of the payment scheduled for payment at date T_i.

The floating spread is reset at the last payment date preceding the scheduled payment date. The value at date T_i of the payment scheduled for date T_{i+1} is therefore

$$X_{i+1}(T_i) = PV01(T_i;T_{i+1})S(T_i;T_i+M) \qquad (3.3)$$

where M is the constant maturity tenor[5]. A natural first guess for the time T_0 value of the payment is to simply put $X_{i+1}(T_0) \approx PV01(T_0;T_i,T_{i+1})S(T_0;T_i,T_i+M)$. When there is no premium accrual on default, this simplifies to

$$X_{i+1}(T_0) \approx D(T_0,T_{i+1})\Delta(T_i,T_{i+1})S(T_0;T_i,T_i+M) \qquad (3.4)$$

where $\Delta(T_i,T_{i+1})$ is the length of the accrual period and $D(T_0;T_{i+1})$ is the credit risky discount factor at T_0 from T_0 to T_{i+1}.

In expression (3.4) we are simply discounting the forward spread by the credit risky discount factor from the valuation date to the payment date (and adjusting for the payment frequency). The interpretation is that the forward spread observed today is the expected spread to be

[5] *To simplify, we write S(t;t,T)=S(t;T) and similarly for PV01 and other security values with the same indexing.*

realized at the reset date. Indeed, the floating payment can be valued as (assuming no premium accrual on default)

$$X_{i+1}(T_0) = D(T_0, T_{i+1})\Delta(T_i, T_{i+1})E\left[S(T_i, T_i + M)\right] \quad (3.5)$$

where the expectation is under the proper risk-neutral measure. However, as we explain in detail in Appendix 1, under the proper risk neutral measure

$$S(T_0; T_i, T_i + M) \neq E\left[S(T_i; T_i + M)\right] \quad (3.6)$$

when the spread is stochastic. For reasons explained below, the difference

$$ADJ(T_0; T_i, T_i + M) = E\left[S(T_i; T_i + M)\right] - S(T_0; T_i, T_i + M) \quad (3.7)$$

is called the convexity adjustment to the forward spread.

Convexity Adjustment

We use an idealized example to show why pricing the floating leg by discounting the forward spread will lose money on average, and why we need to apply an adjustment to forward spreads in order to correctly price a CMDS trade.

Consider an investor selling 2-year protection and receiving floating spread payments linked to the 5-year CDS spread. For simplicity, we assume annual payments. We use S(i, j, k) to denote the CDS spread which resets at time i, for a contract with effective date j and maturity date k. Suppose the investor determines the participation ratio α_0 based on forward spreads, and hedges the CMDS contract with a strip of (two) forward-starting CDS.

If the issuer survives the first year, then the net carry to the investor is

$$Carry = \alpha_0 S(0,0,5) - S(0,0,1)$$

The fair value of the participation ratio at the end of the first year is α_1 = S(1,1,2)/S(1,1,6). The MTM of the CMDS trade is therefore

$$M_{CMDS} = [\alpha_0 - \alpha_1]S(1,1,6)D(1,1,2)$$

where D(i,j,k) is the risky discount factor from time j to time k as measured from time i. The MTM of the hedge can be written as

$$M_{Hedge} = [S(1,1,2) - S(0,1,2)]D(1,1,2)$$

The net MTM is therefore

$$M = [\alpha_0 S(1,1,6) - S(0,1,2)] \times D(1,1,2)$$

If forwards are realized, it can be seen that the unwind value exactly offsets the net cashflow to the investor at the end of the first year.

However, if forwards are not realized, then the MTM gain if spreads move in the investor's favor is less than the MTM loss if spreads move against the investor by the same amount. This is due to the convexity of risky discount factors. For this reason, a strategy based on the assumption that forwards are realized will, on average, lose money if forwards are not realized, ie, if spreads have non-zero volatility.

It follows that the expected spreads used to compute the value of the floating premium must be higher than the forward spreads implied by the curve. Part of the methodology for pricing CMDS therefore involves computing the correct convexity adjustment that we need to apply to forward spreads.

4. A GENERAL VALUATION FRAMEWORK

A CMDS should be valued in an integrated credit spread options framework. A stochastic hazard rate model[6] is the most obvious choice. Because of the close relationship between stochastic hazard rate models and short-rate models used for pricing interest rate derivatives, we have a number of known methods available for valuing a CMDS. We describe how to construct a lattice (or recombining tree) for the hazard rate which can be used to value the floating premium leg including any embedded cap and/or floor. As an alternative to constructing a lattice we suggest using a simpler model with quasi closed-form solutions (formulas for the survival probabilities) to find the convexity adjustment and then value any embedded cap/floor using Black formulas that we derive.

Analogy to Interest Rate Models

In a short-rate model for the term structure of (default-free) interest rates we specify a stochastic process for the instantaneous interest rate, usually thought of as a hypothetical overnight Libor rate[7]. In a stochastic hazard rate model we simply let that process instead be the hazard rate.

If the hazard rate at a given point in time is λ, then the probability of default over a short time interval of length dt is approximately λdt. More precisely, when the hazard rate is a stochastic process the probability of surviving to T given no default at t is

$$Q(T_0,T) = E\left[\exp\left[-\int_{T_0}^{T}\lambda(s)ds\right]\right] \tag{4.1}$$

If we substitute the instantaneous interest rate, say r, for the hazard rate λ, the formula is exactly the way a default-free zero coupon bond is valued in a stochastic short-rate model. It implies that if a given process for the short rate gives rise to closed-form expressions for the price of a zero-coupon bond, then using this process for the hazard rate will produce a model with closed-form expressions for the survival probabilities.

Valuation in a Lattice

The existing literature on implementation of short-rate models is very useful when implementing a stochastic hazard rate model. For example, it is relatively easy to implement and calibrate a 1-factor lognormal model such as Black-Karasinski (BK) on a trinomial lattice with a constant hazard rate volatility.

The BK process is lognormal with mean reversion. The lognormal hazard rate gives rise to CDS spreads that are "almost" lognormal. There is little empirical evidence to suggest which distribution best models CDS spreads – it is issuer specific. However, currently we feel more comfortable with a lognormal as opposed to a more Gaussian process.

Given that we use the BK process, what level of mean reversion should be used? Mean reversion in interest rates is well established. Mean reversion in single issuer credit spreads is a more contentious issue. The fact that credit spreads can "blow up" seems to imply that mean reversion cannot be too strong. We do not think it is unreasonable to assume zero or very low mean reversion in hazard rates. We will get a better idea of the mean reversion once we begin to see prices of longer dated default swaptions. The mean reversion matters for the pricing of a CMDS.

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6 *Researchers often call this a stochastic intensity model. We generally refer to the intensity of default as the hazard rate also when the intensity is stochastic.*

7 *The short rate is hypothetical because it is more volatile than an actual observed overnight Libor deposit rate (which often stays constant for long periods of time). This is necessary because in a short-rate model the volatility of the short rate drives the movement of rates of all maturities.*

Hull (2000) describes in detail how to construct a trinomial lattice for the Hull-White (HW) model for interest rates. Modification to a BK model for hazard rates is not difficult. The steps are:

1. Choose a recovery-given-default (usually 40%) and levels for the hazard rate volatility parameter and the mean reversion speed.

2. Fit a piece-wise constant hazard rate model to a curve of CDS spreads and the chosen recovery-given-default. Produce a curve of risk-neutral survival probabilities.

3. Complete the first stage of the lattice construction as described in Hull (2000)[8]. This entails laying out the value of the hazard rate at each node in the lattice and attaching probabilities to each of the branches. Calibration is not done at this stage.[9] The hazard rate at a given node is set to $\lambda = \exp(x)$, where x is the value that would have been the short interest rate at that node in the HW model.

4. Create a new *default state* (or default node) for each time slice in the lattice. Create an extra branch from each original node to the default state on the next time slice. The probability of going to the default state is set to $p = 1-\exp(-\lambda dt) \approx \lambda dt$ where λ is the hazard rate at that node and dt is the length of the time step. For each node, the branch probabilities to the original nodes are multiplied by 1-p, where p is the default probability for that node.

5. Calibrate the model to ensure it fits the curve of survival probabilities found in step 2 above[10]. For each time slice in the lattice we shift the hazard rates at all nodes on that time slice by the same relative amount to ensure that the probability of surviving beyond the next time slice matches the given survival probability for the time slice.[11]

6. When the lattice has been calibrated, we have all the branch probabilities. We then find the one-step Libor discount factors between each pair of adjacent time slices. These are the forward discount factors from a curve fitted to current Libor/swap rates. The one-step discount factors are used together with the branch probabilities when we value a security by calculating expected discounted values at each node backwards though time.

To value the floating premium leg on a CMDS, we separately value each individual payment and any associated caplet and/or floorlet on that payment. To value the payment set at date T_i (and paid at T_{i+1}) the lattice must be constructed out to T_i+M, where M is the constant maturity tenor (eg, M = 5 years). The valuation is done by first discounting back in the lattice from T_i+M to T_i to obtain the values of $PV01(T_i,T_i+M)$, $PVP(T_i,T_i+M)$, and $PV01(T_i,T_{i+1})$ at each node on the T_i time slice. From these security values we can determine the forward spread from (3.1) and the time-T_i value of the unbounded floating payment from (3.3). Similarly, we can use equations (4.2) and (4.3) below to find the time-T_i value of a caplet and a floorlet on the floating premium scheduled for payment at T_{i+1}

$$C_{i+1}(T_i) = PV01(T_i;T_{i+1})\big(S(T_i;T_i+M) - K_C\big)^+ \tag{4.2}$$

$$F_{i+1}(T_i) = PV01(T_i;T_{i+1})\big(K_F - S(T_i;T_i+M)\big)^+ \tag{4.3}$$

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[8] *Pages 580-583.*

[9] *At this step we are constructing a lattice for the process $dx = -axdt+\sigma dW$ and setting the hazard rate to $\lambda = \exp(x)$.*

[10] *At this step we are calibrating the time-dependent mean reversion level, $\theta(t)$, to construct a lattice for $dx = (\theta(t) - ax)dt + \sigma dW$.*

[11] *The procedure is essentially the one described as stage 2 in Hull's book. First we set the initial hazard rate to $\lambda = -\log(q(dt)/dt$, where q(dt) is the probability of survival to time dt. Then we iterate over all time slices starting with the one for time dt. As we iterate forward in the lattice we keep track of the probabilities (as seen from the initial node) of reaching each node in the lattice. For a given time slice these probabilities only depend on the already fitted hazard rates at previous time slices. By combining these probabilities for a given time slice with the unadjusted hazard rates currently at nodes on that time slice and with the undetermined hazard rate shift for the time slice, we can write an equation for the probability of surviving beyond the next time slice. From the equation we find the required hazard rate shift.*

where K_C is the spread cap and K_F is the spread floor. When the values of the unbounded floating payment, the caplet and the floorlet on the T_i time slice have be found, we continue discounting in the lattice to arrive at the security values on the initial node at T_0.

Valuation with Closed-Form Expressions

As an alternative to constructing a lattice, we can use a model with closed-form solutions. There is a class of such models − called affine models − that have been studied extensively in the finance literature[12]. In an affine model the expression (4.1) for the survival probability has the form

$$Q(t;T) = \exp\left(A(t,T) + B(t,T)\lambda(t)\right) \tag{4.4}$$

where A and B are functions that either have a closed-form expression or can be easily computed using a recursive scheme. In an affine model we also have a closed-form expression for the distribution of the hazard rate at a given point in time.

The value of the floating premium payment at the reset date is a known function of the survival probabilities and can therefore be expressed directly as a function of the hazard rate at the reset date. Because the floating payment *knocks out* if default occurs before the reset date, we can value the payment by integrating over the known distribution of the hazard rate and discount using the credit risky discount factor.

If we use $X_{i+1}(T_i) = f(\lambda)$ to denote the value at the reset date of the floating payment as a function of the hazard rate λ, then we can use Taylor's approximation to write

$$X_{i+1}(T_i) \approx f(\lambda_0) + (\lambda - \lambda_0) f'(\lambda_0) + \tfrac{1}{2}(\lambda - \lambda_0)^2 f''(\lambda_0) \tag{4.5}$$

where $\lambda_0 = E[\lambda]$ denotes the expected hazard rate at the reset date. If we integrate over the hazard rate (take expectations) and discount using the credit risky discount factor we get

$$X_{i+1}(T_0) \approx D(T_0, T_i)\left[f(\lambda_0) + \tfrac{1}{2} Var(\lambda) f''(\lambda_0)\right] \tag{4.6}$$

By combining this expression with (3.5) and (3.7) we get an indication of the magnitude of the convexity correction.

When using an affine model it is important that it is fitted to the observed CDS spread curve. One model with this property that is particularly easy to implement is the one-factor Hull-White (HW) model which has a Gaussian hazard rate[13]. A Gaussian hazard rate is not our preferred distribution, but for valuing an unbounded floating payment (ie, for finding the convexity adjustment), the choice of hazard rate process is of secondary importance[14]. In fact we have obtained reasonable results using the HW model with the approximation in (4.5) and (4.6)[15].

When pricing caps and floors, the spread distribution is very important. Caps, for example, will be more valuable in a lognormal model than in the more Gaussian one-factor affine model. We therefore suggest valuing caps and floors separately, taking as given the convexity adjustments, or equivalently the values of the individual unbounded payments. In Appendix 2 we show how this is done using Black formulas that we derive.

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12 *For details about affine models see research publications by D. Duffie and K. Singleton of Stanford University. Information is available on their websites.*

13 *There are relatively simple formulas for the survival probabilities and the mean and variance of the hazard rate in the HW model (see eg, Brigo and Mercurio (2001)).*

14 *This is because the unbounded floating payment at the reset date is close to linear (no embedded options) in the reset spread.*

15 *By using the Taylor approximation we are also ignoring the fact that in the HW model there is a positive probability that the hazard rate becomes negative, which obviously is theoretically unsatisfactory. As an alternative to the HW model, closed-form solutions can be derived for the one-factor extended Cox-Ingersoll-Ross model (CIR). This model can be fitted to a spread term structure and the hazard rate cannot become negative. However, the CIR model is more complicated to implement than the HW model.*

The Hazard Rate Volatility

Aside from the initial observed CDS curve, the hazard rate volatility is an important valuation input. It should be chosen so as to match an observable default swaption price[16]. Currently, default swaption prices are not observable beyond 1-year (3- and 6-month options are the most liquid). Obviously extrapolating a hazard rate volatility ten years out can be a leap of faith.

Since the caps and floors are deep out-of-the-money, their valuation requires an opinion on a possible volatility skew or smile. It is possible to incorporate a level-dependent hazard rate volatility into the lattice construction but that is more involved. Also, we have too few market prices available to calibrate the hazard rate distribution, which must therefore be estimated from time series of credit spreads. Today the best developed credit volatility market is the portfolio swaptions market (see Pedersen (2003)) but implied portfolio swaption volatilities are not directly comparable to single-name implied volatilities.

5. NUMERICAL RESULTS

In this section we show numerical results from our valuation model. The numbers have been generated using a lognormal hazard rate model of the type described above.

Base Case Parameters

Our base case is a 5-year CMDS with the floating leg indexed to the 5-year CDS spread. The contract has four annual payments and the spread paid on the floating leg on a given date is reset on the previous payment date. The first payment is three months after the valuation date. The reference entity is hypothetical and has the CDS curve given in Figure 2. We assume that the recovery-given-default is 40% and the default-free term structure is flat at 3%.

Figure 2. Illustrative CDS curve

Term	3M	6M	1Y	2Y	3Y	4Y	5Y	6Y	7Y	8Y	9Y	10Y	20Y
Spread	30	35	40	45	50	55	60	65	70	75	80	85	90

Unless otherwise specified we always assume that the reset spread is capped at 500bp. There is no floor. The base case hazard rate volatility is assumed to be 50% and the participation rate is 70%.

Size of the Convexity Adjustment

Figure 3 shows the forward 5-year spread as well as the expected spread[17] for each payment date, computed using the lognormal model with hazard rate volatilities of 25%, 50% and 75% respectively. The figure confirms that while the convexity correction increases with volatility and time to reset, the forward spread is the main determinant of the expected spread.

[16] *In the HW model, the hazard rate volatility parameter should be chosen to match a given basis point volatility of a CDS spread. Matching a default swaption price may give misleading results as HW is not a good model for valuation of default swaptions.*

[17] *The expected spread is determined by valuing each individual payment and ensuring that relation (3.5) in section 3 is satisfied (premium accrual on default is ignored).*

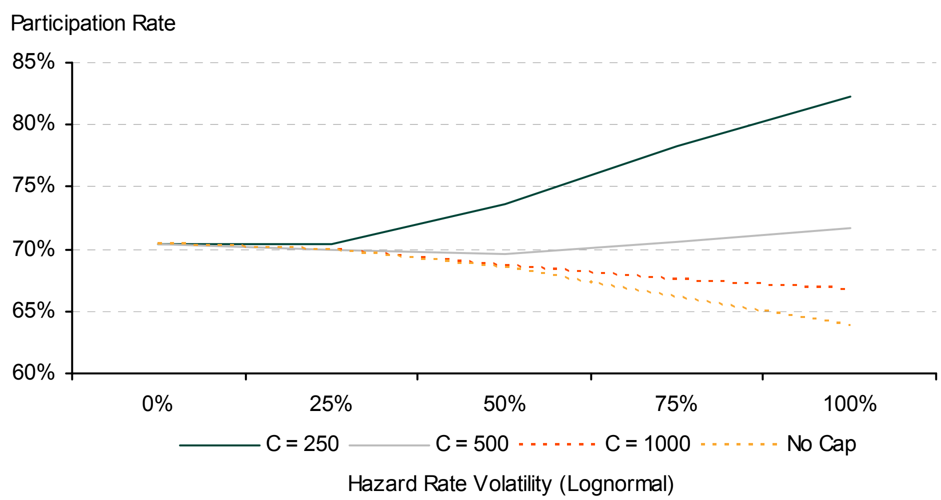
Figure 3. Convexity adjustment as a function of time to reset and hazard rate volatility. The forward spread is the main determinant of the expected reset spread

Source: Lehman Brothers.

Participation Rate

The participation rate is less than 100% if the issuer curve is upward-sloping, since spreads are expected to increase. Similarly, it exceeds 100% if the curve is downward sloping. Since the expected spread increases with volatility, the participation rate on an uncapped CMDS decreases with increasing volatility. However, if there is a low spread cap the participation rate is increasing in volatility. This is shown in Figure 4, where we plot the participation rate for a range of hazard rate volatilities and cap levels.

Figure 4. Participation rate as a function of volatility and the level of the spread cap



Source: Lehman Brothers.

We now turn to the mark-to-market (MTM) and sensitivity of CMDS trades. We examine the impact of changes to the level and slope of the issuer credit curve, and the hazard rate volatility.

Mark-To-Market Sensitivity: Effect of Curve Level

Figure 5 shows the effect of parallel shifts in the issuer credit curve on the MTM of CMDS trades, in comparison to standard CDS. MTM is presented from the perspective of the protection seller. We compare the MTM sensitivity of three trades:

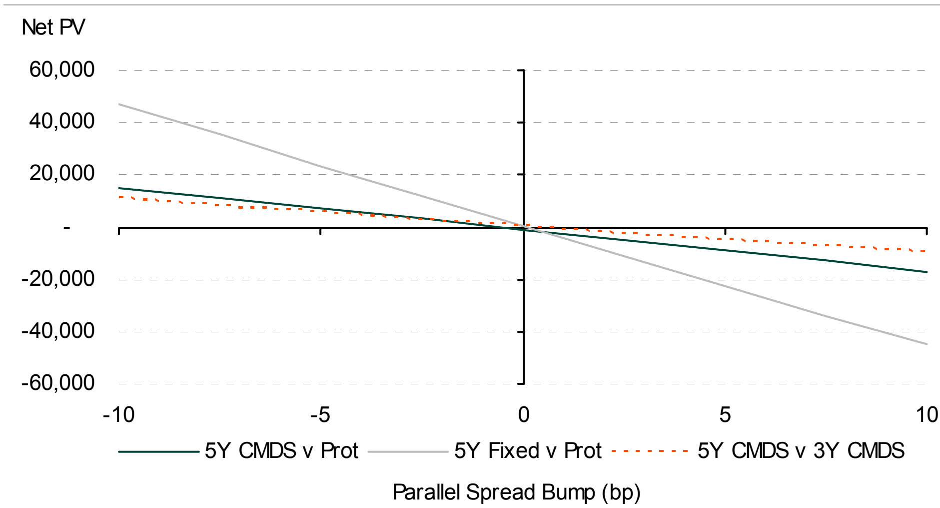
- Standard 5-year CDS at 60bp.
- Protection-for-Floating CMDS with participation ratio 70% and a constant maturity tenor of 5 years.
- Floating-for-Floating CMDS, receiving 86.50% of the 5-year constant-maturity CDS spread and paying 100% of the 3-year spread.

The CDS spread and participation rates are chosen to give all trades zero initial value. All trades are assumed to have a maturity of 5 years and an underlying notional of 10 million.

Figure 5 demonstrates that a floating-for-protection CMDS trade is less sensitive to changes in the level of the credit curve than a standard CDS. This is because the floating payments adjust according to the level of the curve, which changes with the perceived credit quality of the issuer.

Since the premium leg of a standard CDS has the same initial value as the protection leg, the MTM sensitivity of a floating-for-fixed CMDS is similar to the floating-for-protection trade shown in Figure 5. The sensitivity of MTM to parallel curve movements is least for the floating-for-floating trade.

Figure 5. MTM Sensitivity to parallel shifts in the issuer credit curve

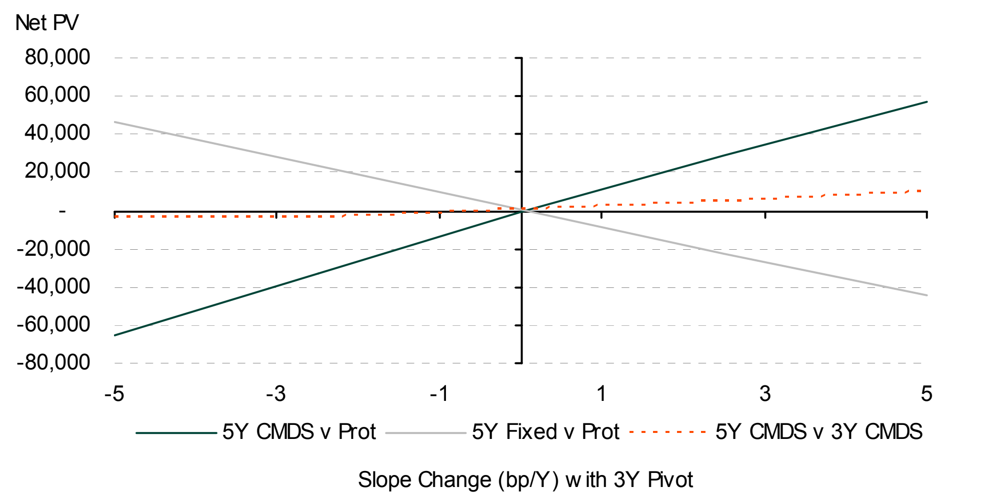


Source: Lehman Brothers.

Mark-To-Market Sensitivity: Effect of Curve Slope

Figure 6 shows the impact on MTM on the three trades for changes in the slope of the issuer credit curve. We show the case where the curve is pivoted around the 3-year point. If the pivot had been around the 5-year point, the CDS would show no sensitivity to the slope change. The MTM of the floating-for-protection CMDS is very sensitive to slope changes, since the expected reset spreads are strongly influenced by the initial shape of the curve. The impact of slope changes on the MTM of the floating-for-floating CMDS is less pronounced, as both the 3-year and 5-year constant-maturity spreads are affected by the shape of the initial curve.

Figure 6. Effect of slope changes on the MTM of CDS and CMDS



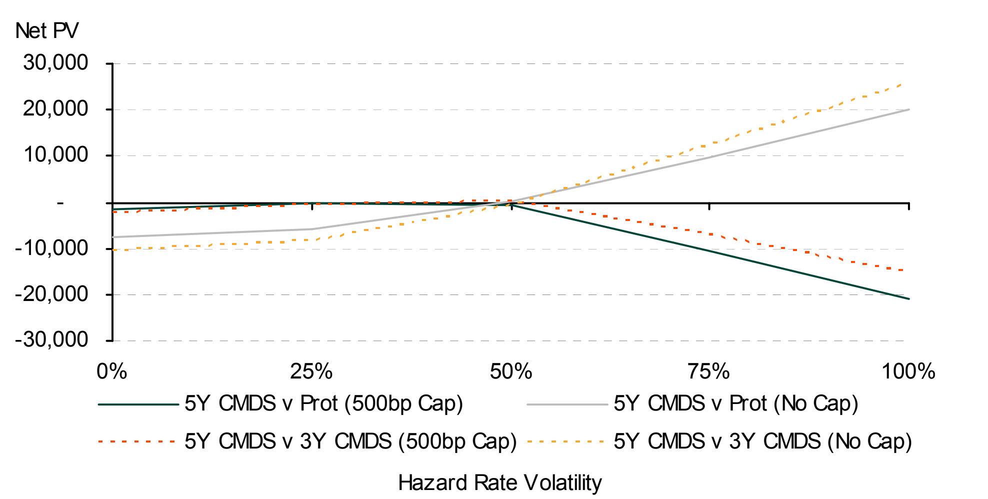
Source: Lehman Brothers.

Although Figures 5 and 6 show that the sensitivity of CMDS to slope changes is comparable to the sensitivity of CDS to level changes, we typically find that level changes occur more frequently than comparable slope changes. See section 2 for an example.

Mark-To-Market Sensitivity: Effect of Volatility

Figure 7 shows how the MTM of CMDS trades changes with volatility. We compare floating-for-protection and floating-for-floating CMDS trades with and without a spread cap. At low volatilities, the MTM of both the capped and uncapped trades increase with volatility, since this increases the expected spread while the probability of triggering the cap remains small. At high volatilities, however, we see a divergence in the MTM of capped and uncapped trades. The MTM of the uncapped trades keep increasing as the expected spreads increase with volatility. The capped trades, however, lose value as value of the caps increase with volatility and hence decrease the value of the floating premium leg.

Figure 7. Effect of increasing volatility

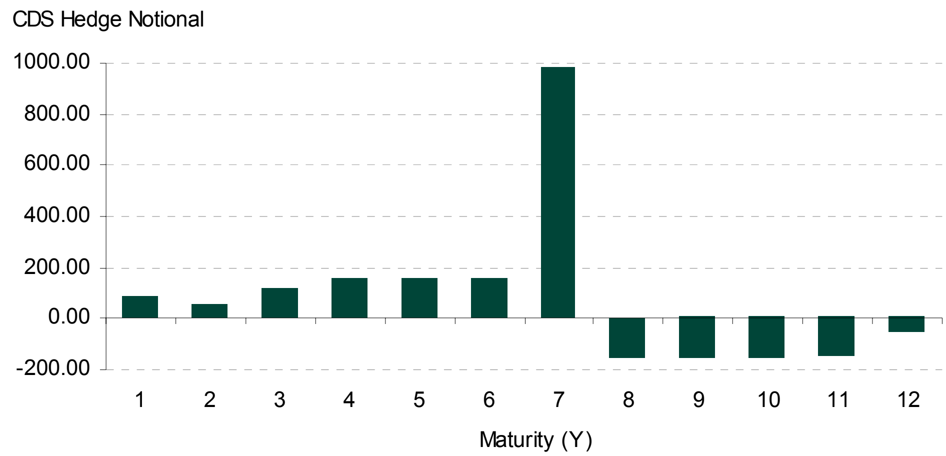


Source: Lehman Brothers.

Delta Hedging

Figure 8 shows the notional amounts of standard CDS contracts of various maturities required to delta hedge the protection-for-floating CMDS trade described earlier. The PV sensitivity of the CMDS is computed for each marked point on the issuer curve (ie, we compute the change in PV for a 1bp bump to the 3M, 6M, 1Y, …, 10Y points singly) and converted into a CDS notional by dividing by the credit risky PV01 to that maturity.

Figure 8. Delta-hedging a CMDS with standard CDS



Source: Lehman Brothers.

From Figure 8, it is apparent that, to the first order, a CMDS contract is hedged using a strip of forward-starting CDS, which are synthesized by buying protection at the short maturities (less than 5Y) and selling corresponding protection at long maturities (more than 5Y). The difference between maturities of long-short pairs is 5 years, the constant maturity tenor. The large exposure at the 5-year point is due to the protection leg.

6. SUMMARY

Constant maturity default swaps are among a new line of credit derivative products designed to reduce mark-to-market volatility while allowing investors to gain exposure to the steepness of the credit curve.

We have described the mechanics of CMDS trades, outlined different approaches to valuation, and examined the risks of the trades. We have demonstrated that the shape of the credit curve is the first-order determinant of the expected reset spread, while spread volatility mainly influences the valuation of caps or floors embedded in the trade. We have also shown how to price the floating premium leg within a lognormal hazard rate model, and discussed how to approximate its value using closed-form expressions. Through numerical examples we have illustrated the size of the convexity adjustment that must be applied to forward spreads and the mark-to-market exposure of CMDS to curve and volatility changes.

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8. APPENDICES

Appendix 1: A Technical Explanation for the Convexity Adjustment

From finance theory we know that for any given security, say B, that makes no payment between today's date, say T_0, and a given date T, there is a probability distribution such that for any other security, say A, that also does not make a payment between T_0 and T, the ratio of today's values of the securities is equal to the expectation of the ratio of the security values at T[18]. We can write this as

$$\frac{A(T_0)}{B(T_0)} = E^B\left[\frac{A(T)}{B(T)}\right] \tag{A.1}$$

In a concrete valuation problem, A is the derivative we are valuing, B is called the numeraire for the valuation, and the probability distribution is called a valuation measure for the numeraire B. Depending on the specification of the underlying model there may be several valuation measures for a given numeraire, but generally we restrict the model so that for each numeraire there is only one valuation measure indicated by the superscript B on the expectation operator in the above equation.

Using the notation from section 2, in our valuation problem A = X_{i+1} (X_{i+1} is the value of the unbounded floating premium scheduled for payment at T_{i+1}). If we ignore premium accrual on default, use T = T_{i+1}, and choose as numeraire the credit risky zero-coupon bond that matures at T_{i+1}, then the valuation equation (A.1) becomes[19]

$$X_{i+1}(T_0) = D(T_0;T_{i+1})\Delta(T_i,T_{i+1})E^{D_{i+1}}\left[S(T_i;T_i+M)\right]$$

The convexity adjustment is needed because

$$S(T_0;T_i,T_i+M) \neq E^{D_{i+1}}\left[S(T_i;T_i+M)\right]$$

In fact, there is another numeraire, namely B = PV01(·;T_i,T_i+M), such that $S(T_0;T_i,T_i+M) = E^B[S(T_i;T_i+M)]$. As shown in (3.1), the forward spread is a ratio of two security values. If we

[18] *The result is valid even if B_T can be 0 as long as A_T = 0 when B_T = 0. The states where B_T = 0 are simply ignored in that case, and the distribution will be such that the probability that B_T = 0 is 0. This is important when B is a security that has value 0 if default occurs before T. In this case, A should be a security that knocks out, ie, a security that also has value 0 if default occurs.*

[19] *Using that $B(T_0) = D(T_0;T_{i+1})$ and $B(T_{i+1})$ = 1 when there is no default by T_{i+1}.*

let A be the forward starting protection leg and B the forward starting PV01, then from (A.1) we have the relationship[20]:

$$S(T_0;T_i,T_i+M)=\frac{PVP(T_0;T_i,T_i+M)}{PV01(T_0;T_i,T_i+M)}$$
$$=E^{PV01(\cdot;T_i,T_i+M)}\left[\frac{PVP(T_i;T_i+M)}{PV01(T_i;T_i+M)}\right]=E^{PV01(\cdot;T_i,T_i+M)}\left[S(T_i;T_i+M)\right]$$

We can use this result together with equation (3.3) to write

$$X_{i+1}(T_0)=PV01(T_0;T_i,T_i+M)E^{PV01(\cdot;T_i,T_i+M)}\left[\frac{PV01(T_i;T_{i+1})}{PV01(T_i;T_i+M)}S(T_i;T_i+M)\right] \quad (A.2)$$

We see that if the PV01 ratio was constant, $X_{i+1}(T_0) \approx PV01(T_0;T_i,T_{i+1})S(T_0;T_i,T_i+M)$ would be exact and there would be no convexity adjustment. However, since

$$\frac{PV01(T_i;T_{i+1})}{PV01(T_i;T_i+M)}=\left(1+\frac{PV01(T_i;T_{i+1},T_i+M)}{PV01(T_i;T_{i+1})}\right)^{-1}$$

the PV01 ratio is generally an increasing function of the spread and the value of the floating payment will be higher than the (credit risky) discounted forward spread.

In an affine model we can express the PV01 ratio in (A.2) as a function of the hazard rate and invert the hazard rate to CDS spread function. This allows us to write the expression under the expectation operator in (A.2) as a function only of the spread $S(T_i,T_i+M)$. We can then numerically integrate this function over a given distribution of the spread. Obviously the approach is not consistent in the sense that the PV01 ratio as a function of the spread is based on a different stochastic hazard rate model than the model used to determine the distribution of the spread. Still, the approach can give us an idea of the importance of the distribution of the spread for the value of the floating payment.

Appendix 2: Black Formulas for Caps and Floors

We derive Black-type formulas to value caps and floors on a CMDS. The presentation is focused on a cap but a formula for a floor can be derived using exactly the same steps. We use the notation defined in section 3.

A cap is a portfolio of caplets, so to value a cap we only need to know how to value caplets. Consider the caplet on the payment to be made at T_{i+1}. The value of the caplet at T_i is

$$C_{i+1}(T_i)=PV01(T_i;T_{i+1})\left(S(T_i;T_i+M)-K\right)^+ \quad (A.3)$$

where K is the cap strike.

We can value the caplet by using $PV01(\cdot;T_i,T_{i+1})$ as numeraire (see Appendix 1). From equation (3.3) it is clear that this also is the natural numeraire to use for valuation of the unbounded payment. If we know the value of the unbounded payment we can infer the expected spread under the valuation measure associated with $PV01(\cdot;T_i,T_{i+1})$. Specifically, we have

$$E^{PV01(\cdot;T_i,T_{i+1})}\left[S(T_i;T_i+M)\right]=\frac{X_{i+1}(T_0)}{PV01(T_0;T_i;T_{i+1})}$$

where $X_{i+1}(T_i)$ is the value of the unbounded payment at date T_i.

[20] *This relationship is also used when valuing a default swaption with Black formulas (see the "Modelling Credit Options" section in O'Kane* et al. *(2003)).*

To arrive at a Black-type formula we assume that $S(T_i;T_i+M)$ is lognormally distributed with constant volatility σ. We can then repeat the steps from the derivation of the Black-Scholes formula to arrive at

$$
\begin{aligned}
C_{i+1}(T_0) &= PV01(T_0;T_i;T_{i+1})E^{PV01(\cdot;T_i;T_{i+1})}\left[\left(S(T_i;T_1+M)-K\right)^+\right] \\
&= PV01(T_0;T_i;T_{i+1})\left(E^{PV01(\cdot;T_i;T_{i+1})}\left[S(T_i;T_1+M)\right]N(d_1)-KN(d_2)\right) \\
&= X_{i+1}(T_0)N(d_1)-PV01(T_0;T_i;T_{i+1})KN(d_2)
\end{aligned}
$$

where

$$d_1 = \frac{\log\left(X_{i+1}(T_0)/\left(K\cdot PV01(T_0;T_i,T_{i+1})\right)\right)+\sigma^2\Delta(T_0,T_i)/2}{\sigma\sqrt{\Delta(T_0,T_i)}} \text{ and}$$

$$d_2 = d_1 - \sigma\sqrt{\Delta(T_0,T_i)}$$

and $\Delta(T_0,T_i)$ is the time to maturity (in years) of the caplet.

Similarly, the value of the floorlet is

$$F_{i+1}(T_0) = PV01(T_0;T_i;T_{i+1})KN(-d_2) - X_{i+1}(T_i)N(-d_1)$$

where K is the floor level, or strike, and d_1 and d_2 are the same as above.

The Implications of Implied Correlation

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Implied correlation is increasingly used for relative value considerations when comparing alternative investments in synthetic CDO tranches. Here we show that, by neglecting the heterogeneity of the underlying portfolio, implied correlation may lead to misleading relative value assessments. We argue that a modified implied correlation measure, which we call the "implied correlation bump", may be more appropriate for the relative value analysis of alternative tranched investments.

INTRODUCTION

Synthetic collateralized debt obligations (CDOs) are instruments whose payouts are linked to the performance of a portfolio of synthetic credit exposures. This market has experienced continuous innovation over the past few years. While in the early days synthetic CDOs were mainly used by banks for capital relief, most of the issuance is now generated by the dealer community in the form of one-off, bespoke tranches referencing investment grade credit default swaps. These structures allow for a high degree of customization, and can therefore be used to tailor specific exposures to the risk preferences of a variety of different credit investors. This customization has spurred significant growth in the credit derivatives business, as reported last year in a *Risk* survey (February 2003, page 20).

In 2003, default swaps on the TRAC-X and CDX portfolios were introduced, and tranches linked to these reference sets also started to be actively quoted. This portfolio standardization has allowed for the creation of a more liquid and transparent market for tranched risk, and the recent merger of the two portfolio products is likely to drive this process even further. The new availability of relatively liquid market levels has led to the quotation of tranche prices in terms of "implied correlation" – a practice that is clearly reminiscent of the use of implied volatility in the options markets. Here, we explain what implied correlation is and why this measure alone may be insufficient for comparing two alternative investments.

Synthetic CDO market participants often face investment decisions such as the following: "I can sell protection on one of two equity tranches – which one offers better value?"

Investment A

- *0%-3% (first loss) tranche.*
- 100 name reference pool with avg spread of 60bp and avg historical correlation of 25%
- Investor can sell protection at a spread of 17.4% corresponding to **an implied correlation of 21.5%.**

Investment B

- *0%-3% (first loss) tranche.*
- 115 name reference pool with avg spread of 85bp and avg historical correlation of 26.5%
- Investor can sell protection at a spread of 20.6% corresponding **to an implied correlation of 28.9%.**

Investment A has a lower implied correlation than investment B. Also, investment A is being quoted at an implied correlation that is lower than the average historical correlation on the reference pool, while investment B has an implied correlation higher than the average on its portfolio. What conclusions, if any, can I draw from these observations? Is it fair to say that investment A is cheap to historical correlation while investment B is rich?

WHAT IS IMPLIED CORRELATION?

Increasingly, market participants are quoting the implied correlation rather than the spread or the price of a CDO tranche. The implied correlation of a tranche is the uniform asset correlation[1] number that makes the fair or theoretical value of a tranche equal to its market quote. Currently, the most common models used to price synthetic CDOs are variants of the one-factor Gaussian copula model. In this model, the correlation of default times is determined by the correlation of asset returns, so tranche values are directly related, though in a complex way, to the assumed asset correlations.[2] In the above example (investment A), plugging a uniform correlation of 21.5% into our pricing model (thus ignoring the actual correlation structure) would produce a fair spread on the equity tranche equal to the market quote of 17.4%.

This is somewhat analogous to the equity derivatives market, where quoting implied volatility is equivalent to stating the price, since all other variables are known. Given the model and all the assumptions that go into it, quoting the spread on the tranche is equivalent to stating its implied correlation. Although it is an elegant way of representing a tranche price, there are two key differences in the analogy drawn with the equity market, and investors should be aware of them:

- Unlike the equity options market, where the Black-Scholes model has gained universal acceptance, the models utilized in the CDO market vary across market players and keep evolving over time. Consequently, the implied parameters, which are model-dependent, will be different as well.
- While volatility is a single parameter in equity derivatives models, a typical reference portfolio of a synthetic CDO tranche generally has thousands of pairwise correlation parameters – for example, a 100-name portfolio involves 4,950 parameters in its correlation matrix. Fitting a flat correlation structure is appealing because of its intuitive simplicity, but expressing a complex relationship in one number can often be inaccurate as it does not reflect the heterogeneity of a portfolio. We illustrate this point below with an example and show why investors should be cautious in interpreting the results.

We next use a set of hypothetical investments to demonstrate how tranches on portfolios with identical average characteristics may have significantly different fair prices and, therefore, significantly different fair implied correlations.

EXAMPLE 1: PRICING A FIRST LOSS (0-3%) SYNTHETIC CDO TRANCHE

In this example, we construct four hypothetical reference portfolios, each with the same average characteristics but different relationships among the reference assets within the portfolio. Each portfolio consists of 100 names with an average spread of 60 basis points and an average observed asset correlation of 25% with other assets.[3] The key characteristics of the portfolios are the following:

- Homogenous or base case – flat (constant) spreads (60bp) and uniform correlation (25%).
- Portfolio 1 – flat (constant) spreads, but variable asset correlations.

---

[1] *The cashflows to a tranche are determined by the default realization in the reference pool. Most models utilize asset correlation (or its proxy - equity correlation) as a means of generating correlated defaults.*

[2] *See D. O'Kane, M. Naldi, S. Ganapati, A.M. Berd, C. Pedersen, L. Schloegl and R. Mashal (2003),* The Lehman Brothers Guide to Exotic Credit Derivatives*, Risk, October.*

[3] *As shown in Figure 1, the average spread of the investment-grade CDX portfolio on 22 April 2004 was 59bp, while the average historical correlation was close to 25%. The hypothetical portfolios we choose here are therefore similar to the CDX portfolio in terms of their average characteristics.*

- Portfolio 2 – spreads positively related to asset correlations.
- Portfolio 3 – spreads negatively related to asset correlations.

Next, we compare the prices of the four equity (0-3%) tranches[4] using a typical one-factor Gaussian copula model. The results of our analysis are shown in Figure 1.

Figure 1. Comparing the prices of four equity (0-3%) tranches

	Portfolio Description				Equity Tranche	
	Spreads (bp)	Correlation	Avg. Spread (bp)	Avg. observed Correlation	Fair Spread	Implied Correlation
Base Case	Flat 60	Flat 25%	60	25%	16.11%	25.00%
Portfolio 1	Flat 60	Variable[5]	60	25%	16.09%	25.03%
Portfolio 2	30 to 90	Higher correlation amongst higher spread names	60	25%	15.02%	28.88%
Portfolio 3	30 to 90	Higher correlation amongst lower spread names	60	25%	17.43%	21.49%
CDX.IG.NA 4/22/04	Varies		59	25.1%	16.77%	24.28%

Figure 1 shows the fair spread of the tranche using the observed pairwise correlations of the reference pool, as well as the implied correlation that can be backed out from the fair spread. For example, an investor who sells protection on the first loss piece of portfolio 2 should receive a spread of 15.02% as a fair compensation for his risk exposure. Although the average observed correlation on the underlying portfolio is 25%, the implied correlation is 28.88%; that is, we can substitute the actual correlation structure with a flat number (28.88%) and achieve the same price (fair value spread) on the equity tranche.

The example clearly shows how fair implied correlations of the tranches referencing the four hypothetical portfolios are different, even though the average characteristics of the reference pools are the same. For example, investors considering the first loss piece from portfolio 3 should receive a fair implied correlation of 21.49% compared with 28.88% for portfolio 2 – a difference of 7.39%! This demonstrates our first premise – that a portfolio with *uniform* pairwise correlations of X (25% in this case) does not necessarily have the same risk profile as a portfolio with an *average* pairwise correlation of X (25%). If the above hypothetical tranches traded at their fair values, investors should be indifferent in their choice. However, an investor choosing on the basis of a comparison between implied correlation and average historical correlations would be mistakenly attracted to portfolio 3.

A careful look at the characteristics of the reference pools and their heterogeneity helps explain the previous results. Because of the positive relation between spreads and correlation of the underlying names, the loss distribution of portfolio 2 is more volatile than that of the other three portfolios. This means portfolio 2 has a greater probability of realizing a very low number of defaults as well as a higher likelihood of realizing a very large number of defaults, with average default realizations being less likely. Equity investors benefit from the extra probability of very benign default scenarios, while they are not particularly hurt by the extra

[4] A similar analysis can be performed using mezzanine or senior tranches, but the relatively high sensitivity of the equity tranche makes it a better candidate for illustrative purposes

[5] In the context of a one-factor model, correlations are generated by the product of the sensitivities to the common market factor, generally called the "betas". More formally, we produce a heterogeneous correlation matrix by taking the cross product of a vector of betas with its transpose, and varying the elements of this vector from 25% to 75%. The average correlation is thus 50% x 50% = 25%

probability of a large number of defaults. Consequently, the fair spread to be paid to the equity investor on portfolio 2 is lower than the fair spread to be paid to the equity holders of the other three portfolios. A symmetric argument explains why the fair spread to be paid to the equity investor on portfolio 3 is higher than the other three fair spreads.

The above examples have shown how changing the heterogeneity of a portfolio, even while maintaining a similar average profile, changes the theoretical value of a tranche. Therefore, in the real world, comparing the implied correlations of two tranches referencing different portfolios carries even lesser meaning. Let us now revisit the investment decision we posed at the beginning of this note, and compare investments A and B – two equity tranches referencing different portfolios.

EXAMPLE 2: COMPARING INVESTMENTS A AND B

Investment A has been used as portfolio 3 in the earlier example, and given our assumption that it is being shown at its fair value spread, the present value or mark-to-market on the tranche is equal to zero on day one. Investment B, on the other hand, references a higher spread portfolio. As shown in Figure 2, at a market quote of 20.6% (which corresponds to an implied correlation of 28.9%), and using historical pairwise correlations to price the tranche, the investment is worth, on day one, $94,613 for a $10 million notional. Clearly, investment B would be a better choice here. Yet, a naive comparison between historical and implied correlations would suggest that investment A is a better value.

Figure 2. Comparing two alternative investments

	Investment A (0-3% Tranche) $10m Notional	Investment B (0-3% Tranche) $10m Notional
Reference Pool		
Number of Names	100	115
Average Spread	60bp	85bp
Average Correlation	25%	27%
Market Quote		
Spread	17.4%	20.6%
Implied Correlation	21.4%	28.9%
Present Value	$0	$94,613

AN ALTERNATIVE: THE "IMPLIED CORRELATION BUMP"

One of the main reasons for which the notion of implied correlation has gained so much ground in the market is the simple fact that it is just one number, making it easy to quote. But, as we have just seen, implied correlation is a poor measure for relative-value analysis because it neglects the heterogeneity of a portfolio. As an alternative, we suggest using an "implied correlation bump", ie, a single number that multiplies all historical pairwise correlations in order to re-price a tranche.

Revisiting the comparison between investment A and investment B, there is evidently no need to bump the correlations for investment A since we assumed the equity piece trades at fair value. The implied correlation bump for investment B, on the other hand, is 92.5%, that is, all the elements in the historical pairwise correlation matrix would have to be scaled down by 7.5% to price the tranche at its current market (spread) quote. We can therefore say that the tranche is "cheap to historical correlation" – an observation that cannot be made by looking at implied correlation alone. This bump in correlation is worth $94,613 in terms of present value, as discussed earlier.

The implied correlation bump has the advantage of respecting the specific diversification of the portfolio, while retaining the convenient feature of fitting just one number. The main drawback, however, is that contrary to implied correlation, it is based on a historical estimate of the correlation matrix. Consequently, the implied correlation bump is likely to be more useful for relative-value analysis than for quoting the price of a tranche.

SUMMARY AND CONCLUSION

The increased liquidity of standardized bespoke tranches has brought more transparency to the market, as well as the ability to better calibrate proprietary models. One of the recent trends in the synthetic market is the quotation of tranche prices by means of their implied correlations. However, implied correlations are increasingly being used for other purposes such as relative-value analysis. In this article, we have highlighted some of the potential drawbacks of using implied correlations for evaluating the relative attractiveness of alternative tranched investments. In particular, we have shown that using a flat implied correlation number does not properly account for the cross-sectional variability of pairwise correlations between individual credits, and this can lead to misleading investment choices. Consequently, we do not recommend drawing conclusions about the relative attractiveness of two tranches based on the comparison of their implied correlations, especially if the two tranches refer to different portfolios. As an alternative, we have suggested looking at the "implied correlation bump", a measure that in our opinion can be more meaningfully employed to detect relative value across tranches.

It must be pointed out that our previous examples were all based on the popular one-factor Gaussian framework. Using this model, the correlations implied by the observable prices of junior and senior tranches are generally higher than those needed to match the prices of mezzanine slices – a phenomenon known as the "correlation smile". As recently observed by Duffie (2004)[6], the inability of the model to explain observable market prices across the capital structure using the same correlation matrix raises doubts about the appropriateness of the underlying distributional assumptions.

Looking further into the future, we believe that identifying models that are able to fit observable prices is more than an academically interesting exercise: it responds to practitioners' and regulators' growing demand to somehow "anchor" the valuation of a large notional amount of illiquid, customized exposures to the aggregate opinion of the market-place, thereby increasing transparency and promoting the growth of these products even further.

[6] *See D Duffie (2004), Time to adapt copula methods for modelling credit risk correlation,* Risk *April 2004, page 77.*