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Tuning Correlation and Tail Risk to the Market Prices of Liquid Tranches

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The advent of the trading of standardized synthetic CDO tranches has brought greater transparency to the market participants, as well as the possibility to calibrate proprietary models. In this paper, we study two popular parametric models of correlated defaults, namely the Gaussian and the t-copula models. We describe the phenomenon of "correlation smile" in the Gaussian case, and investigate whether a t-copula model can improve upon the Gaussian specification in terms of fitting the observed mid-quotes of US Trac-X Series II tranches.

1. INTRODUCTION

The increased availability of relatively liquid prices for synthetic CDO tranches referencing standardized portfolios such as Trac-X and CDX is now providing researchers with the opportunity to compare their pricing models with the market outcome. Identifying models that are able to fit observable prices is more than an academically interesting exercise: it responds to the growing demand of practitioners and regulators to somehow "anchor" the valuation of a large notional amount of illiquid, customized exposures to the aggregate opinion of the marketplace, thereby increasing transparency and promoting the growth of these products even further.

Latent variable models with a low-dimensional factor structure have gained a lot of popularity among dealers, since they can be solved quasi-analytically and provide precise valuations of plain-vanilla multi-name instruments while allowing for fast computation of the necessary risk sensitivities[1]. In addition, closed-form asymptotic formulae are available for dealing with large portfolios, making these models very useful for portfolio risk calculations. In particular, the Gaussian copula and *t*-copula assumptions for the joint distribution of the default-triggering latent variables – usually interpreted as asset returns – have now become standard references in the credit derivatives pricing literature. In this article, we compare these two parametric pricing models in terms of their ability to explain observable market quotes for tranches referencing the US Trac-X Series II portfolio.

We start our investigation in section 2 by describing our fitting procedure to the prices of Trac-X tranches. For illustration, we use the traditional Gaussian model, where the dependence among the default-triggering latent variables is fully specified by their linear correlations. Due to its simplicity and ease of interpretability, the Gaussian model has been widely used among market participants, who often make it operational by estimating the necessary correlation coefficients from observable equity return series. A simple inspection, however, shows that the Gaussian model used in conjunction with historical equity correlations does not reproduce observed tranche prices across the capital structure of the Trac-X portfolio. We discuss alternative methods of modifying the correlation parameters in order to improve the fit of the Gaussian model to the observed tranche prices. In particular, we compare the use of a flat correlation value across the portfolio with a constant proportional shift of the historical equity correlations.

In section 3, we fit a standard *t*-copula model to the market data to investigate whether the introduction of a fat-tailed dependence structure for the latent variables improves our ability to explain the observed prices of Trac-X NA II tranches. Essentially, the *t*-copula model frees up an extra parameter controlling the likelihood of joint extreme realizations of the latent variables. Unfortunately, the pricing effects of an increase in the probability of extreme events are too similar to the pricing effects of a general increase in linear correlations for this

[1] *See Gregory and Laurent (2003) and, for the Student-t case, Andersen et al (2003).*

model to improve upon the Gaussian framework in terms of its ability to reproduce a cross-section of market prices.

More specifically, the fitting exercise reveals an undesirable feature of a one-factor *t*-copula model, namely the fact that the idiosyncratic components of the latent variables, although uncorrelated, exhibit tail dependence, both among each other and with the common factor. This specification implicitly produces a relation between the model-implied prices of junior and senior tranches which is not found in the observed market quotes. The consequence is that the *t*-copula does not improve upon the Gaussian model in terms of fitting observable market quotes across the Trac-X capital structure.

We perform our analysis on the standardized tranched US Trac-X Series II. Trac-X is a tradable portfolio CDS product, consisting of 100 US investment grade names with an average spread at 54bp as of January 7, 2004. It was launched in March 2003 with a maturity in March 2009, and quotes are available on the following tranches: 0% to 3%, 3% to 7%, 7% to 10%, 10% to 15% and 15% to 30%. Figure 1 shows a sample quote published on January 7, 2004. Note that while the mezzanine and senior tranches are quoted in terms of a single running spread (in bp) paid quarterly on the outstanding notional of the tranche, the equity tranche quote appears as a spread plus an upfront payment, expressed as a percentage of the tranche notional.

Figure 1. US Trac-X Series II tranche quotes as of January 7, 2004

	Upfront payment, %			Running spread, bp		
Tranche	**Bid**	**Ask**	**Mid**	**Bid**	**Ask**	**Mid**
0 – 3%	36	40.5	38.25	500	500	500
3 – 7%				290	340	315
7 - 10%				90	115	102.5
10 - 15%				44	60	52
15 – 30%				8	15	11.5

Source: Bloomberg.

2. GAUSSIAN COPULA

At the core of any CDO tranche pricing model is a mechanism for generating dependent defaults. Latent variable models describe default as an event generated by a latent variable – generally interpreted as asset return – falling below a pre-specified threshold, which is in turn calibrated to observable CDS spreads of the reference name. The dependence among the default times of different names is then naturally determined by the dependence structure (a.k.a. the *copula*) of the latent variables.

One of the most popular latent variable models combines a Gaussian copula with a one-factor correlation framework (see, for example, O'Kane et al. (2003)). The return of asset *i*, *X(i)*, is driven by a common market factor *M*, and an idiosyncratic variable *E(i)*:

$$X(i) = \beta(i) \cdot M + \sqrt{1 - \beta(i)^2} \cdot E(i). \tag{1}$$

Here the variables *M, E(i), i=1,2…,N* are taken to be independent standard normals, so that the asset returns *X(i)* are jointly normal with pairwise correlations given by:

$$Corr(X(i), X(j)) = \beta(i) \cdot \beta(j).$$

Within the one-factor Gaussian framework, the dependence structure is fully specified by a vector of betas, each of which can be interpreted as the correlation of an asset with the market. Conditionally on the realizations of the market factor *M*, the probability that the i^{th} credit defaults is now given by

$$P\left(X\left(i\right) < D\left(i\right) \mid M\right) = N\left(\frac{D\left(i\right) - \beta\left(i\right)M}{\sqrt{1-\beta\left(i\right)^2}}\right),$$

where *D(i)* represents the default threshold for a given time horizon calibrated to the i^{th} name's credit curve. Conditional on the market factor, the credits are independent. If we also discretize the losses on default so that the loss of the i^{th} credit is a multiple w_i of a basic loss unit, then the conditional loss distribution is a generalized binomial distribution. We can construct the conditional loss distribution recursively. If we denote by π_k^i the conditional probability that the subportfolio consisting of the first *i* credits loses *k* units, then we have the following recursion

$$\pi_k^i = \pi_k^{i-1}\left(1 - P\left[X(i) < D(i) \mid M\right]\right) + \pi_{k-w_i}^{i-1} P\left[X(i) < D(i) \mid M\right].$$

Successively adding credits in this way allows us to build up a complete loss distribution conditional on the realization of the market factor. Integrating over the market factor gives us the unconditional distribution. Once we have these loss distributions, we can compute the expected percentage loss of the tranche we are trying to price at every horizon date. These expected losses can be interpreted as "default" probabilities, and we can price a tranche using exactly the same analytics as in a single-name default swap. For a tranche with attachment point K_1 and width K_2 - K_1, the tranche "survival probability" for any given point in time is

$$Q^{tr}(K_1, K_2) = 1 - \frac{E\left[L\left(K_1, K_2\right)\right]}{K_2 - K_1},$$

and the two legs of the portfolio swap can be priced using

Protection Leg PV $= \left(K_2 - K_1\right)\sum_{i=1}^{K} B(0,t_i)(Q^{tr}(K_1, K_2; t_{i-1}) - Q^{tr}(K_1, K_2; t_i))$

Premium Leg PV $= s^{tr}\left(K_2 - K_1\right)\sum_{j=1}^{n} \Delta_j Q^{tr}(K_1, K_2; T_j) B(0, T_j)$,

where *B(0,t)* is the risk-free discount factor to time *t*, s^{tr} is the coupon paid on the tranche, Δ_j are accrual factors and T_j are the payment dates. Finally, for a seller of protection, the mark-to-market of an open contract is equal to

Tranche MTM = Premium Leg PV - Protection Leg PV.

Similar pricing approaches are widely used in the industry (see, for example, Andresen et al. (2003) and Gregory and Laurent (2003)).

In summary, we have seen that, in a one-factor Gaussian framework, specifying the correlation structure is sufficient to determine the price of any tranche of the underlying portfolio. This also means that, given the quoted market spreads on the tranches, we can use the model to back out implied correlations. Of course, the number of pairwise correlations in a 100-name portfolio is substantially larger than the number of tranches for which spreads are quoted; therefore, some additional structure needs to be imposed, and a reasonable fitting procedure has to be defined.

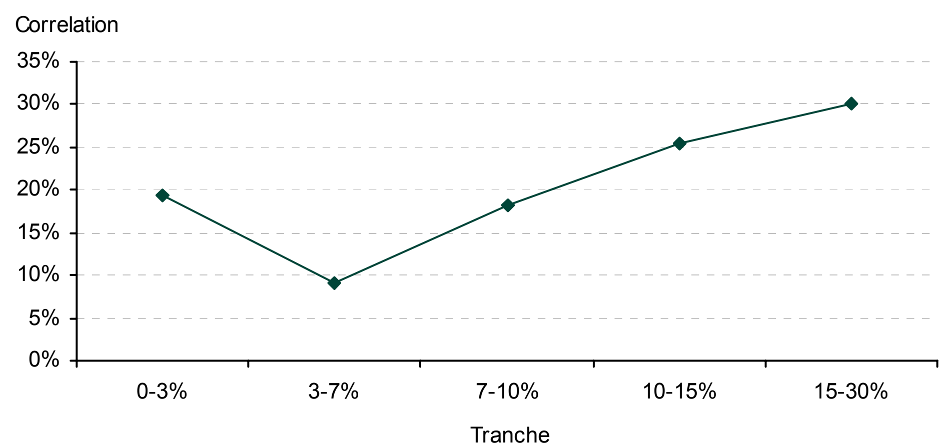
Fitting tranche prices to a Gaussian model calls for certain adjustments in the correlation structure. One approach is to find a flat correlation (across all names) which provides the "best fit across tranches"; an alternative procedure is to use a proportional shift of pairwise correlations estimated from equity return series. In both cases, the "best fit across tranches" is achieved by choosing parameter values with the goal of minimizing the total deviation of the model prices from the corresponding market prices. A convenient measure of this deviation is given by the (equally weighted) sum of the absolute values of the tranche MTMs produced by the model using the quoted mid-market spreads[2]. This is the measure of total pricing error that we will use throughout the rest of the paper for optimisation and illustration purposes.

Flat correlation and the "correlation smile"

Fitting a flat correlation structure is appealing because of its intuitive simplicity. The corresponding betas are the same for all issuers and are equal to the square root of the correlation value. Solving for the beta given a quoted tranche spread establishes a very desirable "one price, one correlation" relationship, which is convenient for developing an understanding of the subject, and quickly communicating general levels of the market. It is also worth noting that a flat structure does not require any estimate of the true correlation values, and is therefore not exposed to estimation error.

It turns out that the correlation values implied by the quoted tranche spreads are different across the capital structure: the correlation implied by the equity and super senior tranches is typically higher than the correlation implied by the prices of the mezzanine tranches. This produces a phenomenon known as the "correlation smile". We plot an illustration of this phenomenon in Figure 2: clearly, the implied flat correlation for various tranches is not the same. This shape seems to be generic, as it is inherited by other standardised tranches as well (see Isla et al. (2003)).

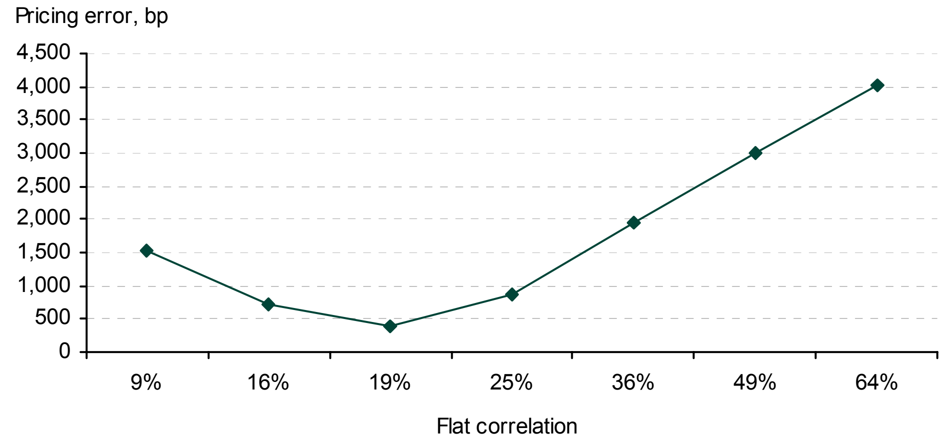
Figure 2. "Correlation smile" for US Trac-X II tranches



In Figure 3 we show the errors induced by various values of flat correlation for the tranche spreads quoted in Figure 1, according to the metric defined above. A closer look reveals an optimal flat correlation of approximately 19.48%, corresponding to an optimal implied beta of just over 44%.

2 *We assume a fixed investment of x dollars in each of the quoted tranches, and express the pricing error for each tranche in basis points. The sum of absolute pricing errors is therefore also expressed in basis points.*

Figure 3. Fitting US Trac-X II tranches with a Gaussian model and a flat correlation



One drawback of this approach is that it does not recognize the heterogeneity of the betas, and this has implications when computing spread sensitivities. Moreover, a flat implied correlation implicitly ignores the relation between correlations and spreads in the underlying pool of credits, and this is likely to introduce an artificial "skew". The effect of, say, low-beta names on the expected loss of, say, an equity tranche crucially depends on whether these names have relatively high or relatively low spreads. A similar argument holds for senior tranches as well. In light of these observations, a fit based on a non-flat correlation structure, which correctly accounts for the relation between spreads and correlations, may be preferable.

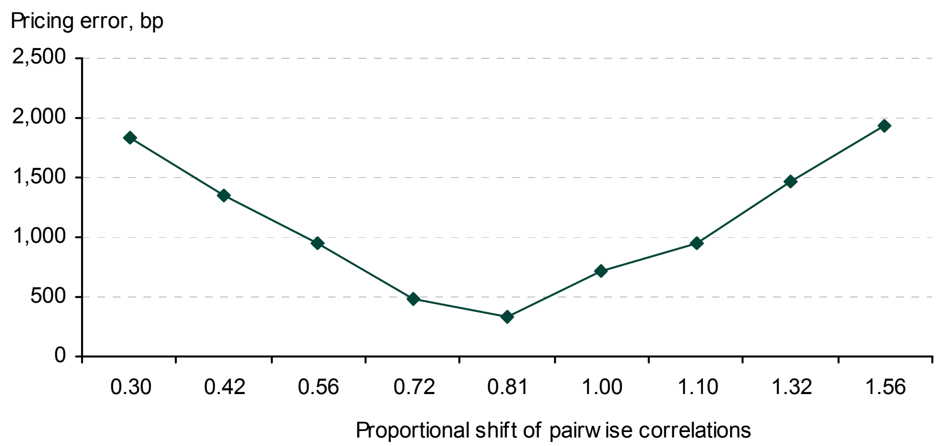
Flat proportional shift of estimated correlations

Shifting estimated correlations by the same percentage provides a relative improvement over fitting with a flat correlation value, in that it permits to take into account the heterogeneity of the betas of the underlying names. At the same time, this method still has the convenient feature of fitting just one number – i.e. the common percentage shift of each correlation estimate – so that the bulk of the procedure shown in the previous section can be used with only minor changes. Note that the corresponding multiplier for betas is also flat and equal to the square root of the correlation multiplier.

On the other hand, this method requires the knowledge of the unperturbed correlation matrix, which is usually estimated from equity returns and will therefore contain estimation error. Moreover, the set of possible multipliers is limited by the fact that the shifted correlations must still be less than unity; this constraint, however, is generally not binding.

In Figure 4, we plot the total pricing error (in the sense of the metric defined at the beginning of section 2) as a function of the flat correlation multiplier. The optimal value for the correlation multiplier turns out to be approximately 0.8057; equivalently, the optimal beta multiplier is equal to $\sqrt{0.8057} = 0.8976$. This means that, in the context of the Gaussian model, the minimum sum of absolute pricing errors is obtained by decreasing each correlation of 19.5% of its value, which is equivalent to decreasing each beta by 10.3%.

Figure 4. Fitting US Trac-X II tranches with a Gaussian model and a flat percentage shift of historical correlations



Comparing pricing errors within the Gaussian framework

Figure 5 compares the pricing errors for the five different tranches and the sum of their absolute values

1. using estimated correlations;
2. fitting a flat correlation value;
3. fitting a flat proportional shift of estimated correlations.

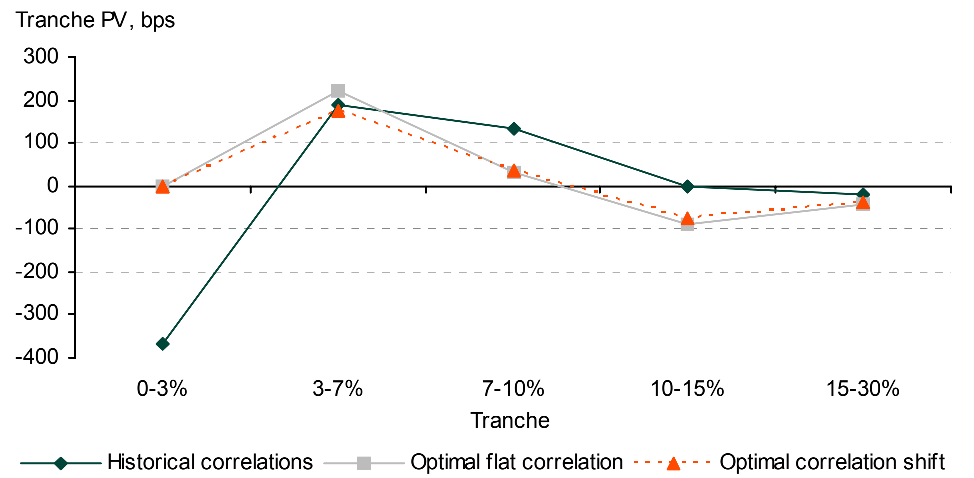
Figure 6 offers a graphical representation of the same comparison.

It is interesting to note that the minimal total pricing errors produced by the two fitting methodologies (flat correlation value and flat proportional shift of estimated correlations) are not significantly different. Moreover, when compared with the prices obtained by simply inputting estimated correlations, optimisation shifts the bulk of the error from the equity tranche to the senior mezzanine tranches, slightly improving the fit for the junior mezzanine tranches. The error in repricing the senior tranches is actually worse for both “optimised” correlations than for the unperturbed correlation matrix. In summary, it seems impossible, within the Gaussian framework, to price correctly *both* the equity and mezzanine tranches. This is not totally surprising, as it is in this part of the capital structure that market technicals have the greatest effect.

Figure 5. Fitting US Trac-X II tranches with a Gaussian model: absolute pricing errors

	Flat Implied Beta	Beta Shift	0-3%	3-7%	7-10%	10-15%	15-30%	Sum of Abs Errors
Historical	-	-	-366.9	189.6	134.3	-0.2	-18.8	709.8
Flat	44.1%	-	-0.0	219.7	29.0	-91.6	-41.5	381.9
% Shift of Historical	-	-10.3%	-0.0	176.0	37.7	-75.7	-37.7	327.0

Figure 6. Fitting US Trac-X II tranches with a Gaussian model: absolute pricing errors



3. MODELLING TAIL RISK WITH A *T*-COPULA

In the *t*-copula model, we can view the latent variables $Y(i),\ i=1,2,\ldots,n$ as being generated by a random "mixing" of the Gaussian model described in (1)

$$Y(i) = X(i)\sqrt{v/G} = \beta(i)\cdot M\cdot\sqrt{v/G} + \sqrt{1-\beta(i)^2}\cdot E(i)\cdot\sqrt{v/G}, \tag{2}$$

where the mixing variable G is distributed as a chi-square with v degrees of freedom, and is independent of M and $E(i)$, $i=1,2,\ldots,n$. The rest of the notation follows from the previous section. For the purpose of calibration, it is important to notice that, similar to the Gaussian model, we have

$$Corr(Y(i),Y(j)) = Corr(X(i),X(j)) = \beta(i)\cdot\beta(j).$$

Conditionally on the realizations of the market factor M and the mixing variable G, the probability that the i^{th} name defaults is now given by

$$P(Y(i) < D(i) \mid M, G) = N\left(\frac{D(i)\sqrt{G/v} - \beta(i)M}{\sqrt{1-\beta(i)^2}}\right).$$

Similarly to the procedure explained in the previous section, the unconditional loss distribution can be obtained by numerically integrating over the market factor M and the mixing variable G.[3] Repeating this procedure for a number of different horizons, we can produce a sequence of loss distributions that can be used to price Trac-X loss tranches.

Compared with the Gaussian framework, the *t*-copula has an extra parameter, v, which essentially controls the likelihood that the default-triggering latent variables experience joint extreme realizations. It is well known that the *t*-copula nests the Gaussian as a limiting case, converging to it as v grows to infinity.

[3] *Following the terminology of Frey and McNeil (2001), the t-copula model has a two-dimensional conditional independence structure, and thus requires a larger computational effort than its Gaussian counterpart, which has a one-dimensional conditional independence structure.*

The hypothesis that tranche investors price extreme event risk seems quite plausible, and is the main driver behind various recent attempts to depart from the Gaussian model. Different investors, however, are likely to focus on different types of extreme realizations. In particular, an investor taking on equity or junior mezzanine risk will be concerned with firm-specific tail events, while a senior investor will dread a fat-tailed market factor. Unfortunately, the *t*-copula does not distinguish between systematic and idiosyncratic extreme event risk.

Looking back at equation (2), we can see that a joint extreme realization of the latent variables is most likely to be produced by an unusually large realization of the mixing variable G. Since this amplifies the realizations of the common factor and of the idiosyncratic components by the same magnitude, this model implicitly constrains the relation between junior and senior risk as we vary the parameter v, i.e. as we vary the likelihood that the mixing variable G – and therefore the latent variables $Y(i), i=1,2,\ldots n$ – take on extreme values. The results we report below tell us that this constraint is rejected by the market prices of US Trac-X II tranches, and that the *t*-copula model is not able to use the extra parameter available to improve upon the Gaussian fit.

As reported in the previous section, the proportional shift in betas which minimizes the sum of absolute pricing errors for the Gaussian specification is -10.3%. At this value, the sum of absolute pricing errors produced by the Gaussian model is 327 bp. When we fit a *t*-copula to the same market quotes and choose both a proportional beta shift and a number of degrees of freedom with the objective of minimizing the sum of absolute pricing errors, we get a corner solution for the parameter v that brings us back to the Gaussian model. Figure 7 illustrates the percentage decrease in betas required to minimize the sum of absolute pricing errors for different values of v. Figure 8 completes the picture by showing the minimized sum of absolute pricing errors as we vary v while optimally choosing the percentage beta shift (as indicated in Figure 7). It is easy to see that the sum of absolute pricing errors decreases monotonically with v, therefore converging to the Gaussian pricing error of 327 bp. In other words, the type of flexibility provided by the extra parameter of a *t*-copula is useless for the purpose of explaining the cross-section of US Trac-X Series II quotes as of January 7, 2004.

Figure 7. *t*-copula: optimal beta shift as a function of the degrees of freedom

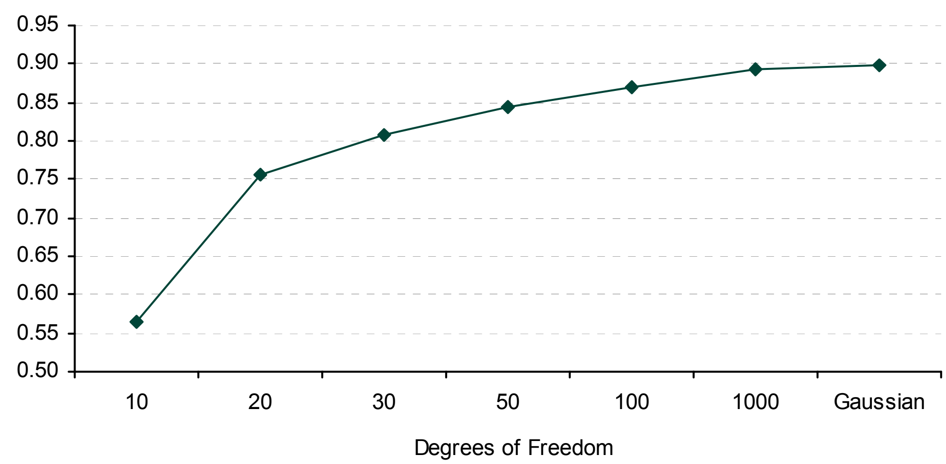
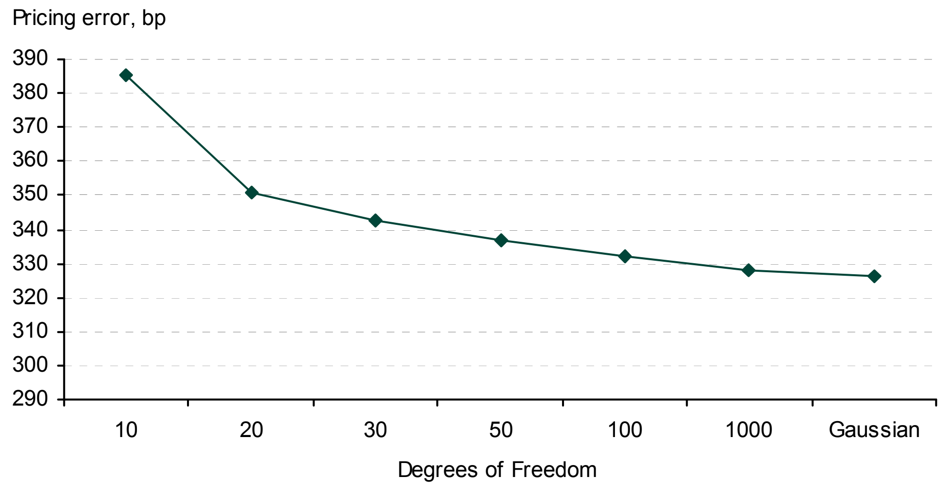


Figure 8. *t*-copula: minimal sum of absolute pricing errors as a function of the degrees of freedom



4. COMMENTS AND CONCLUSION

Latent variable models with a one-factor correlation structure have received a lot of attention from credit derivatives practitioners because of their simplicity and computational efficiency. In particular, this class of models has proved to be extremely useful for the valuation and risk management of large portfolios of synthetic CDOs.

In this article, we have provided some initial evidence regarding the ability of these models to explain the market prices of synthetic tranches referring to standardized credit portfolios. While the analysis presented here is limited to one cross-section of prices on US Trac-X Series II tranches observed on January 7, 2004, our results are qualitatively similar at different points in time. We have started our discussion by showing that the widely used Gaussian model cannot replicate the observable tranche prices across the Trac-X II capital structure when used in conjunction with either the historical equity correlations of the underlying names or a constant correlation matrix. Replicating the market prices of the different tranches requires that we use different input correlations for different seniorities. Since the correlations needed to replicate the prices of junior and senior tranches are generally higher than those needed to match the prices of mezzanine slices, this phenomenon has become known as the "correlation smile".

Simplifying a bit, we can think of at least two reasons for the existence of this "smile". First, the no-arbitrage technology implicit in the pricing methodology described above may not be appropriate for the valuation of financial instruments that are still relatively illiquid and whose payoffs are hardly replicable. If the price dynamics in the market are not driven by the absence of arbitrage, but rather by a variety of factors affecting the demand and supply of tranched risk, then the failure of the model to replicate market prices should not be surprising.

Second, even if the no-arbitrage methodology was indeed appropriate, the correct pricing of synthetic CDOs would still require the correct modelling of the dependence structure of the default-triggering variables. The Gaussian assumption is appealing for the simplicity of its parameterization, but there is some evidence that it may not appropriately describe the likelihood of joint defaults. An alternative dependence structure, which has received considerable attention in the literature, is represented by the *t*-copula. The analysis in the previous section has shown, however, that this alternative representation of the joint distribution of the latent variables does not seem to improve our ability to correctly replicate the market prices of Trac-X II tranches. Further study of alternative dependencies is therefore required if we want to uncover models that will allow credit derivatives practitioners to anchor their valuations to market observables.

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Portfolio Asset Allocation with CDOs[1]

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With the growth of the structured credit market and the improved liquidity of the credit derivatives market, synthetic CDOs have become increasingly attractive to a variety of investors. In an asset allocation framework where an investor combines CDOs with other fixed income asset classes such as government or corporate bonds, it is difficult to ignore their mark-to-market valuation. This article presents a simple framework that compares synthetic CDOs to corporate credit and treasuries, and computes the optimal allocation of CDO in a fixed-income portfolio.

1. INTRODUCTION

With the growth of the structured credit market and the improved liquidity of the credit derivatives market, investments in synthetic CDOs (i.e. CDOs backed by a pool of credit default swaps) have become increasingly attractive to institutional investors, insurance companies, banks and hedge funds. The structure of synthetic CDOs enables investors to capture the credit spread premium in excess of the default spread premium, access a diversified pool of credits, and customize their investments to their particular risk profiles.

Since CDO tranches are still a new investment class to potential buyers, managing the mark-to-market (MTM) risk of these investments and combining them optimally with other asset classes such as government and corporate bonds remains a challenge. Realistic asset allocation or statistical studies are still not feasible because of insufficient historical data on returns, even though some market information has recently been made available with the introduction of standardised tradable CDO tranches with daily bid-offer quotes.

In this article, we quantify the risk return profile of different CDO tranches (equity, mezzanine and senior tranches) for a static synthetic investment grade CDO, using simulation of forward-looking distribution of CDO returns. These returns can then be used to optimally allocate the CDO tranches with treasuries and corporate bonds.

We find that CDOs provide diversification to traditional fixed income investors seeking exposure to credit only, with potential leverage. CDOs also provide diversification to corporate credit investors in two ways. First, they provide exposure to credit risk with much less interest rate risk than corporate bonds; second, they introduce leveraged exposure to credit since CDO tranches are options on the loss in the credit collateral pool. To complete our study, we also simulate historical returns of CDO equity tranches to back-test our methodology.

The article is organized as follows: in section 2, we outline the framework and assumptions used for modelling CDOs. In section 3, we present the distribution of simulated returns for the asset class considered and their sensitivity to tranche, default and spread parameters. In section 4, we analyse CDO tranches in a portfolio context using standard mean-variance analysis and as an alternative, we use a downside risk measure through Conditional Value-At-Risk (CVAR). In section 5, we implement the same approach to produce historical returns of CDO equity investments and we conclude in section 6.

[1] *We thank Frederic Cogny for his excellent research assistance, Albert Desclee, Jay Hyman, Vasant Naik, Marco Naldi and Dominic O'Kane for their comments.*

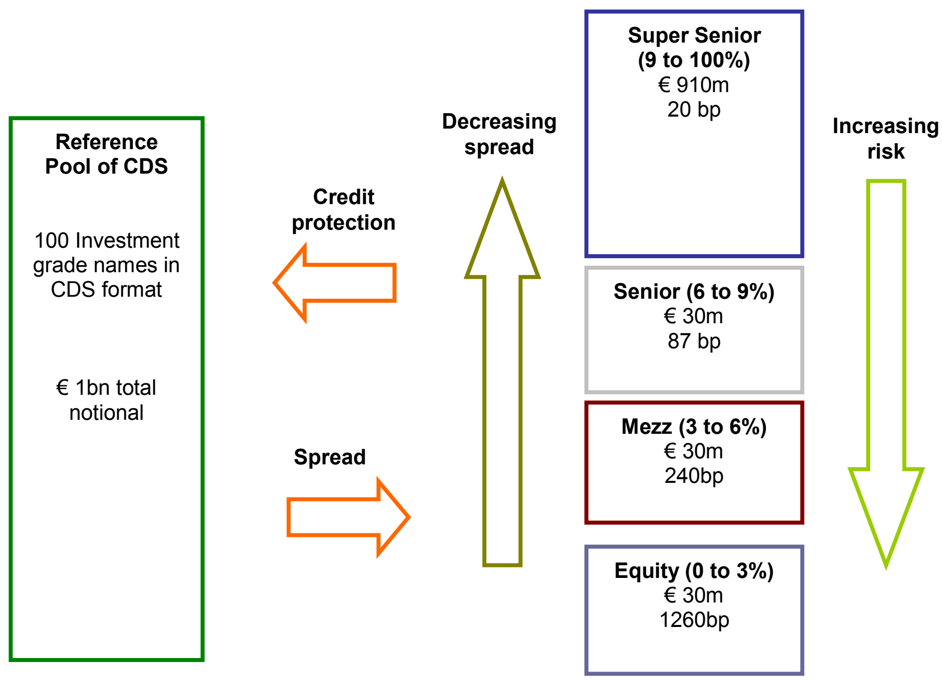
2. MODELLING OF A CDO

Synthetic CDO tranches

Synthetic CDO tranches are structured securities whose performance depends on the number of defaults in a portfolio of credit default swaps (CDS). A schematic diagram of a typical synthetic CDO with an equity, mezzanine, senior tranche and super senior tranche[2] is shown in Figure 1. Each of the tranches receives a contractual spread in exchange for absorbing losses in the portfolio. For instance, the equity tranche in this example receives a 1260bp running spread in exchange for absorbing losses from 0% to 3%.

After a default happens in the underlying portfolio, the equity tranche is written down by the loss amount and the holder makes the protection payment. The spread is paid on the surviving notional of the tranche, so the spread payment amount is also reduced at default. Once the whole equity tranche is wiped out, the mezzanine tranche starts to take principal losses and to make protection payments on default, and so on.

Figure 1. CDO structure



A representative synthetic CDO

For our modelling, we consider a representative synthetic CDO. We assume that the collateral is a pool of 100 A-rated 5-year credit default swaps with an average spread of 40bp. We assume a recovery rate of 40%. The total notional of the CDO is €1bn. The coverage ratio[3], defined as the ratio of market spread and the actuarial spread or spread explained by historical default rates, is assumed to be 3. Thus if the spread is 90bp, say, then the actuarial spread is assumed to be 30bp; the remaining 60bp is captured by the buy-and-hold investor and compensates for spread volatility, downgrade risk, liquidity risk, and systematic risk. This excess spread is the main motivation for investing in a CDO.

[2] *We do not comment here on the super senior tranche since the investor base is very narrow.*
[3] *For a discussion on spread premia see O'Kane, Schloegl and Greenberg (2003).*

The model CDO has three investable tranches: the equity tranche, the mezzanine tranche and the senior tranche. We consider a funded CDO investment, whereby the investor purchases a note paying an annual coupon. The note proceeds are invested at the Libor rate. The characteristics of the CDO are listed in Figures 2 and 3. The expected rating of each tranche is also given.

Figure 2. CDO tranche size and structure

	Tranche	Size	Notional (in €m)	Spread over LIBOR	Rating
Equity	0%-3%	3%	30	12.60%	Not Rated
Mezzanine	3%-6%	3%	30	2.40%	Baa
Senior	6%-9%	3%	30	0.87%	Aa

Figure 3. Assumptions for CDO

No. Names	100
Risk-free (Libor) rate	2.07%
Default probabilities	0.22%
Correlations	20.0%
Recovery rate	40.0%

The spreads over Libor are close to the values typically observed in the market. In the current market environment this tends to favour the equity tranche. The tranche sizes have been chosen, so that the corresponding spreads and ratings are close to the observed market values.

Valuation of tranches of the CDO

The first task is to price the different tranches of the CDO. In this paper, we use the LHP (Large Homogeneous Portfolio) approach, which is a simple yet powerful approximation for analysing synthetic CDOs. LHP is based on a one-factor Gaussian model of correlated defaults; in addition, it approximates the CDO collateral by a large homogenous portfolio of credits with the same default probability and default correlations (see Vasicek (1987)). If the actual values differ across the collateral, the averages are computed and taken as the appropriate flat values. The main advantage of this approach is that in the limit of the number of underlying names becoming large, the loss distributions of the whole collateral and all tranches can be computed in closed form, which greatly facilitates further analysis. For a realistic CDO, where the collateral consists of 100 and more names, the asymptotic formulae give very good approximations. The expression for a tranche expected loss is derived in O'Kane and Schloegl (2001) and involves bivariate normal cumulative distribution functions.

Given the expected loss on a tranche, the latter can be priced using a credit default swap analogy (see also O'Kane et al. (2003)), as follows. We define the tranche premium leg as the present value of all spread payments:

$$\text{Premium Leg PV}= E\left[\sum_{j=1}^{n} s^{tr} N^{tr}(T_j)\Delta_j B(0,T_j)\right] = s^{tr} N^{tr}(0)\sum_{j=1}^{n}\Delta_j Q^{tr}(T_j)B(0,T_j),$$

where the tranche "survival probability" is defined as one minus the expected percentage loss on the tranche: $Q^{tr}(t) = E\left[N^{tr}(t)\right]/N^{tr}(0)$, $B(0,T_j)$ is the discount factor from time 0 to T_j, $N^{tr}(T_j)$ is the notional of the tranche, s^{tr} is the spread paid to the tranche holder, contractually fixed at inception and Δ_j is the accrual factor. Since protection payments are made whenever the tranche notional is reduced, we can extend the analogy to the protection leg as well, defining

$$\text{Protection Leg PV} = N^{tr}(0)\sum_{i=1}^{K} B(0,t_i)(Q^{tr}(t_{i-1}) - Q^{tr}(t_i)) .$$

This structure is equivalent to a credit default swap for a fictitious name whose survival probabilities coincide with those of the tranche and the recovery rate is zero. We can therefore compute the mark-to-market of the tranche (from the investor's perspective) as the difference between the present value of the premium and protection legs.

Ease of implementation and fast calculations brought about by analytical formulae, combined with reasonable accuracy for most realistic portfolios, makes the LHP approach a valid technique for qualitative analysis and understanding of the performance of CDOs. More exact models are preferable for pricing and risk management purposes. For our purposes, LHP is an appropriate tool, even though in reality there are some variations in spreads, recoveries and correlations between the names, in which case the average values are taken as inputs.

The necessary model inputs are summarised in Figure 4:

Figure 4. LHP data inputs

Collateral	CDO Tranche
risk-free rate (Libor rate)	
valuation date	frequency of coupon payments
maturity date	subordination
number of assets	size of the tranche
average spread	coupon (spread over LIBOR) paid
average recovery rate	tranche notional
default correlation	

3. SIMULATING THE FORWARD-LOOKING DISTRIBUTION OF RETURNS OF CDO TRANCHES

In the previous section, we described the valuation methodology (LHP) that we use to price CDO tranches in each of the scenarios of our simulation. Each of these scenarios simulates on a six-monthly basis treasury returns and the evolution of spreads, as well as the number of defaults in the CDO collateral. Each default reduces the overall portfolio size and the tranche size of the most junior of the remaining tranches. Regarding spreads, we simulate A-rated market spreads with a mean-reverting stochastic process which we fit for European corporate credit spreads (see Appendix). The underlying credit is assumed to all be A-rated.

We compute the returns of the CDO tranches by pricing the CDO in each period. Excess returns between time t and $t+dt$ (dt is six months) are computed as follows:

- *Pricing the CDO tranches at the beginning of the period t:* we use LHP to price the CDO tranches with the average level of spread, the given tranche sizes and level of subordination.
- *Simulating correlated default times for the CDO collateral for the period t to t+dt:* we use hazard rates and a Gaussian copula to generate correlated default times (see detailed descriptions in O'Kane et al. (2003) or Li (2000)).

- *Repricing the CDO tranches at time t+dt:* we use LHP to reprice the CDO tranches with the new average level of spread, the new tranche sizes and subordination.
- *Computing CDO tranche excess returns:* we account for the spread and interest payments and accruals; the spread loss due to defaults during that period, the price at the beginning and the end of the period, and compute the CDO returns.

We simulate the CDO returns over a 5-year period, which is a typical maturity of a synthetic CDO, the 5-year CDS being usually the most liquid.

In Figure 5, we report the average annual returns of CDO tranches over the 6-month Libor-rate, as well as the volatility, skewness and (excess) kurtosis of the return distribution. The CDO tranches have been chosen to represent Aa (senior), Baa (mezzanine) and unrated (equity) securities. The Sharpe ratio is comparable across the different tranches.

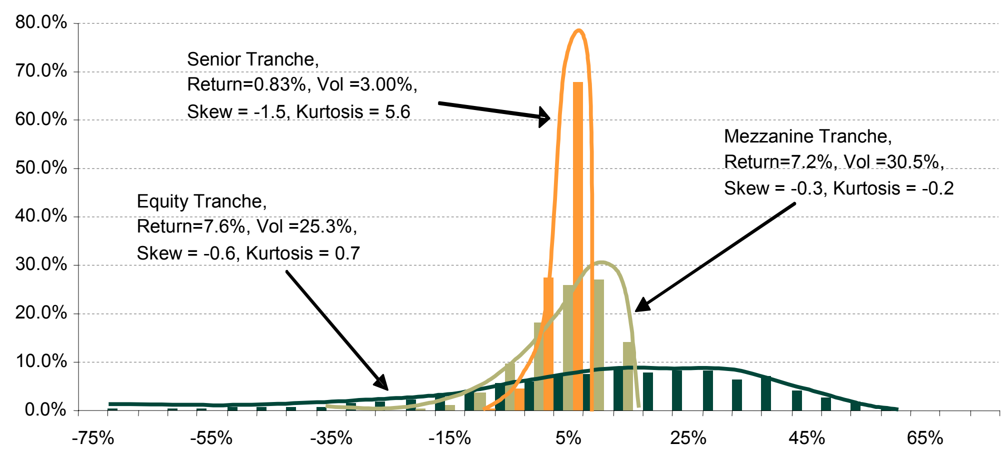
Figure 5. Average annual returns and volatility of CDO tranches (5000 simulations)

	CDO Equity	CDO Mezzanine	CDO Senior
Excess Return	8.08%	2.27%	0.85%
Volatility	23.99%	7.72%	2.99%
Sharpe Ratio	0.34	0.29	0.28
Skewness	-0.61	-2.41	-1.33
Kurtosis	2.08	18.39	3.07

The distribution of excess returns of the CDO equity tranche has small negative skewness, since unlike the more senior tranches of the CDO, the upside is less limited for the equity tranche, allowing for large positive excess returns on a mark-to-market basis. The size of the skewness will in general depend on the relative size of the equity tranche, the default correlation, default probabilities and the recovery rate.

Figure 6 shows the distribution of excess returns of all tranches for a one-year horizon. They have negative skewness and large excess kurtosis, particularly the senior and mezzanine tranches.

Figure 6. Distribution of returns of the CDO tranches



To help understand the results of the next section and, in particular, the impact of the different parameters on the distribution of CDO returns, we present the distribution of returns for different CDO tranches under different assumptions. We consider the tranche spreads fixed and we perturb various other CDO characteristics. The results are summarised in Figure 7.

We first look at the effect of default correlation after the trade was completed, by calculating the moments of the return distribution for three default correlations: high 35%, medium 25% and low 15%. As expected, equity excess returns increase with default correlation, since when correlation is high, the probability of both joint survivals and joint defaults is larger.

The equity tranche skewness and kurtosis are also affected by the correlation. A higher correlation makes the distribution more negatively skewed and with fatter tails than a normal distribution. This is due to the increase in volatility that makes the equity tranche more sensitive to larger downside risk.

We then consider the effect of tranche width and look at three different equity tranches: 0-2%, 0-3% and 0-4%[4]. When we shrink the size of the equity tranche, assuming the spread remains constant, the average equity return suffers because it is collateralized by fewer assets. Thus we see equity returns fall when the tranche size decreases. A decreasing width also makes the tranche more volatile but less skewed and with thinner tails. The latter two effects occur because when the size of the tranche becomes smaller, the higher volatility of returns increases the probability of higher returns event and thus makes the distribution less negatively skewed.

Figure 7. Moments of the CDO equity return distribution under different assumptions

		Return	Volatility	Skewness	Kurtosis
	35%	8.4%	21.2%	-1.37	5.09
Correlation	25%	8.1%	24.0%	-0.61	2.08
	15%	8.0%	27.8%	-0.11	0.40
	3%	8.1%	24.0%	-0.61	2.08
Equity size	2%	6.4%	32.3%	-0.30	0.98
	4%	9.2%	18.6%	-0.72	2.82
	30bp	8.7%	22.9%	-0.54	1.90
Spreads	40bp	8.1%	24.0%	-0.61	2.08
	50bp	7.0%	27.4%	-0.53	1.85
	25%	9.3%	22.3%	-0.35	1.76
Spread premia	33%	8.1%	24.0%	-0.61	2.08
	50%	6.1%	26.2%	-0.75	2.05

We next look at the effect of change in the spread of the assets after the trade. We perform the original analysis for an average spread of 40bp and then investigate three possible scenarios: 10bp tightening (to 30bp), neutral, and 10bp widening (to 50bp). We see that when spreads tighten, the return on the equity tranche increases, the volatility falls but the distribution has a slightly larger (negative) skewness and fatter tail. We see the reverse effect when spreads widen: the returns fall, volatility increases and the distribution is closer to Gaussian. The intuition for the kurtosis and skewness effects is that the more upside the equity tranche has, the more symmetric the distribution with thinner tails and less skewed returns, and therefore the less it resembles the returns distribution of a bond.

Finally, we examine the effect of changes in the spread premia – this is also a change in the coverage ratio. We look at the ratio of default risk over credit spread, which is the inverse of

[4] *We assume that tranche spreads are fixed and that the equity tranche is suddenly shrunk to a smaller size because of defaults in the collateral. Of course, if the CDO were to be repriced at inception with a smaller equity tranche, the corresponding spread coupon size would have to be larger to compensate for the higher risk.*

the coverage ratio. We do this by keeping the total spread constant, but varying the default spread premium between the three values of 33%, then 25% and 50%. The higher the default spread premium (and the lower the credit premium above the default premium), the lower the return (because defaults occur more often) and the higher the volatility. A lower default spread premium has, on the contrary, the benefits of a higher average return and a lower volatility. The impact on the higher order distribution moments is more limited.

Traditional credit as a CDO with a single tranche

Traditional credit can be seen as a CDO with a single tranche (0-100% tranche) with all the collateral of the same rating. We therefore define three such CDOs with Aa, A and Baa-rated underlying to give the performance of traditional credit. We simulate the average level of market spreads with correlated mean reverting stochastic processes, as for the structured credit (see Appendix) and then use the four steps described earlier in section 3 to obtain excess returns for each of these CDOs.

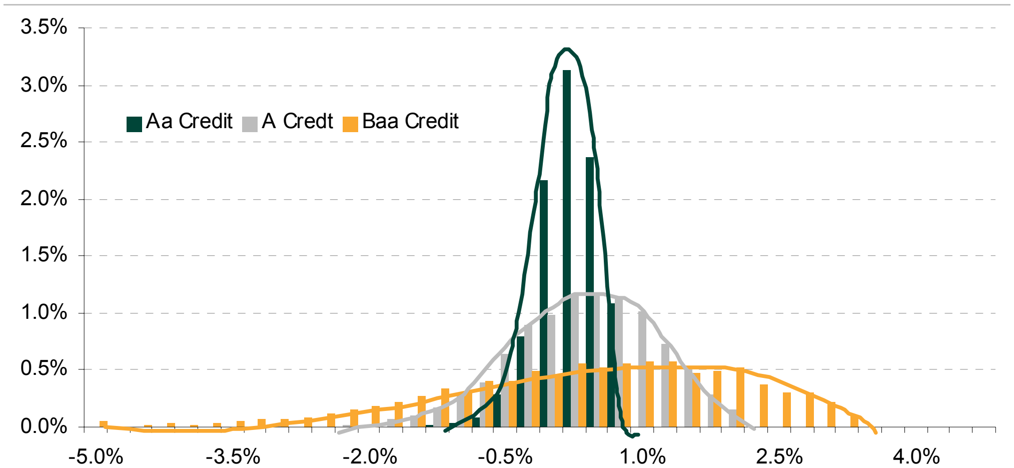
Figure 8. Average excess returns and volatility of credit bonds (5,000 simulations)

	Aa Credit	A Credit	Baa Credit
Excess Return	0.10%	0.28%	0.44%
Volatility	0.37%	0.84%	1.77%
Sharpe Ratio	0.27	0.34	0.25
Skewness	-2.29	-0.66	-0.69
Kurtosis	23.66	3.26	1.41

In Figure 8, we report the average excess returns over the 6-month Libor rate for three rating categories, as well as the volatility of these excess returns and their annualized Sharpe ratios. The Sharpe ratios range from 0.27 to 0.34 across credit quality.

For a horizon of one year, we plot the distribution of returns. Figure 9 shows the excess return distribution of credit portfolios, which has increasing skew and decreasing kurtosis as the rating is lowered.

Figure 9. Distribution of excess returns of credit portfolios



4. CDO TRANCHES IN AN ASSET ALLOCATION CONTEXT

The main input for an asset allocation analysis is the correlation matrix shown in Figure 10. This correlation matrix is implied by the joint simulation of the spreads and defaults driving the returns of the three different rated corporate credit portfolios and the structured credit portfolio. Given the low level of defaults of investment grade assets, CDO tranches and corporate credit have a relatively high correlation.

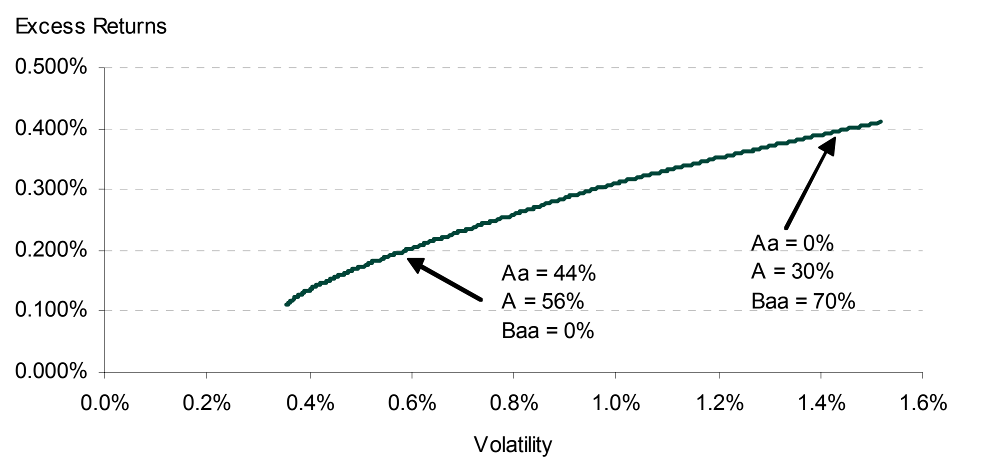
Figure 10. Correlation matrix of CDO tranche and credit

	Aa	A	Baa	CDO Equity	CDO Mezz	CDO Senior
Aa	100%	61%	64%	56%	60%	65%
A	61%	100%	82%	69%	74%	83%
Baa	64%	82%	100%	71%	77%	86%
CDO Equity	56%	69%	71%	100%	75%	75%
CDO Mezz	60%	74%	77%	75%	100%	93%
CDO Senior	65%	83%	86%	75%	93%	100%

Mean-variance and CVaR optimization

We first assume the investor wants to maximize expected returns subject to the level of risk, as measured by the standard deviation of returns, for an investment horizon of one year. Therefore the inputs needed to construct an optimal portfolio are the correlation, volatility and excess returns. These can be found in Figures 5, 8 and 10. We begin our analysis by taking a typical investor in investment grade credit (without CDOs). The resulting efficient frontier using mean-variance optimisation is given in Figure 11.

Figure 11. Typical efficient frontier for typical credit investor



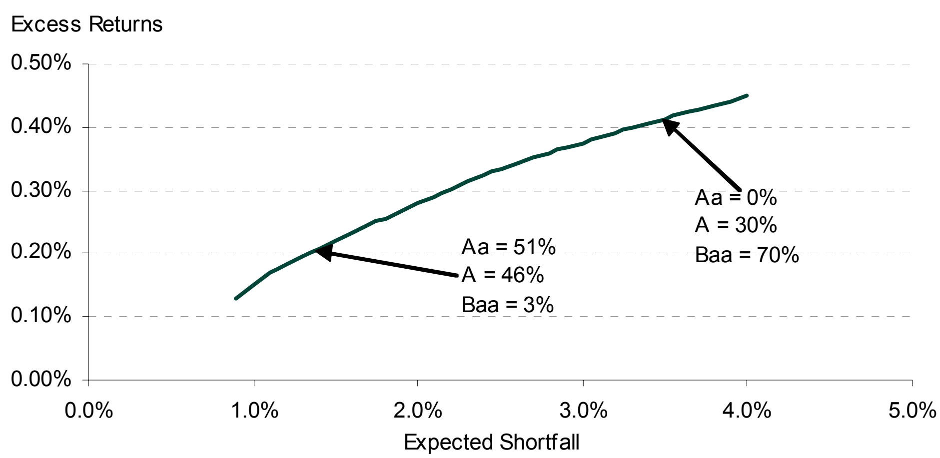
The frontier above begins in the least risky asset, Aa credit. As we move along the frontier, we move down the credit rating, through single-A credit to Baa.

Traditional mean-variance optimization of asset allocation is subject to several limitations, most notably its inability to accommodate non-Gaussian distribution of asset spreads, which characterises CDO returns (see section 3). We therefore propose to use Conditional Value-at-Risk (CVaR), also called Expected Shortfall, as the measure of risk.

The expected shortfall, or CVaR, is defined by first choosing the tail of the distribution defined by the Value-at-Risk (VaR) level (eg, a 95% VaR is the worst loss in the best 95% cases, so it defines a 5% tail, or quantile of the loss distribution). Once this level is chosen, the CVaR risk measure is further defined as the expected loss in the corresponding "tail", or quantile, conditional on being in that quantile. In this paper, we choose the 95% VaR. CVaR optimisation in this case is the maximisation of the portfolio return for a given expected loss in the 5% tail. This can be done for different levels of loss to form an efficient frontier (see Uryasev and Rockafellar (1999)).

We repeat the analysis for the traditional credit portfolio to obtain the efficient frontier, constructed by maximising returns under some expected shortfall constraint (Figure 12) and the resulting efficient frontier is very similar to that obtained using mean variance.

Figure 12. Typical Efficient Frontier for Typical Credit Investor



In Figure 13 we show the effect of including CDOs as an asset class. The CDO tranches have higher returns and volatility for an equivalent rating than standard credit, and therefore the efficient frontier extends to much higher levels of risk and return. However, for comparison, we concentrate on the part of the frontier attainable using investment grade credit only. We also show the efficient frontier that can be achieved by investing only in CDOs.

The higher levels of risk and return of the structured credit forces the asset allocation to remain almost entirely in investment grade credit for the least risky portfolio. As the level of risk is increased the efficient asset allocation uses a combination of A, Baa credit assets and the CDO equity tranche.

However, many investors are restricted to assets rated Baa and above only. Figure 14 shows how the exclusion of the equity tranche affects the efficient frontier. In this case the efficient frontier uses a combination of mezzanine CDO tranches and single-A credit to achieve portfolios with higher risk profiles.

Figure 13. Addition of structured credit as an asset class

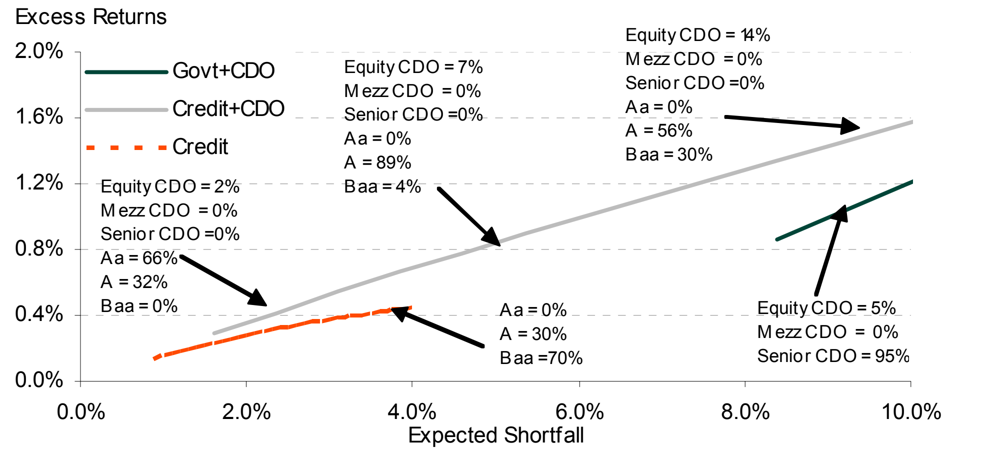
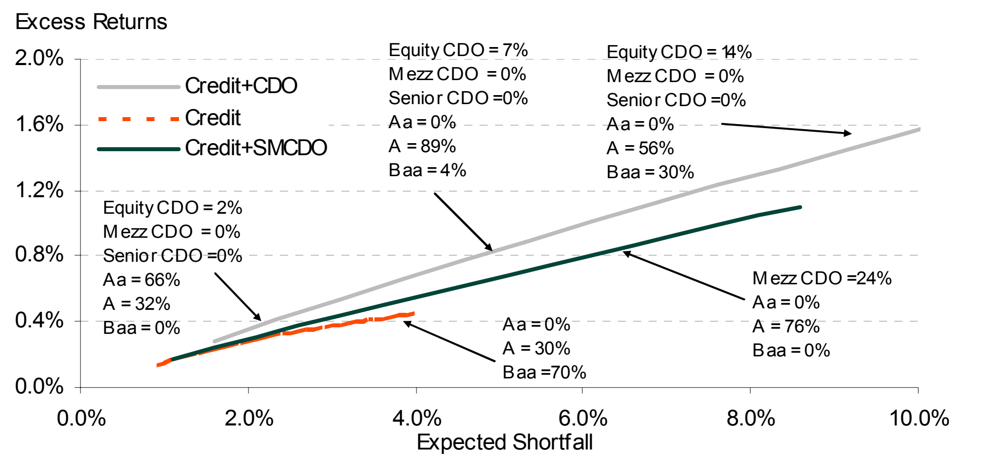


Figure 14. Addition of investment grade structured credit as an asset class



A similar result is obtained when the mezzanine tranche is excluded. In this case, the senior tranche of the CDO is included to create portfolios with higher returns.

Adding CDOs to a portfolio of corporate credit and treasury bonds

We are also interested in understanding the effect of adding CDOs to a portfolio that includes treasury bonds as well as investment grade credit.

Our modelling approach to simulate treasury returns for different maturities is explained in the Appendix: we use the Economic Factor Model (EFM) and assume that the term risk premium is such that the current (Euro) interest rate curve is expected to remain the same over the next five years. We assume there is no swap spread in order to model the credit as a spread over the treasury curve.

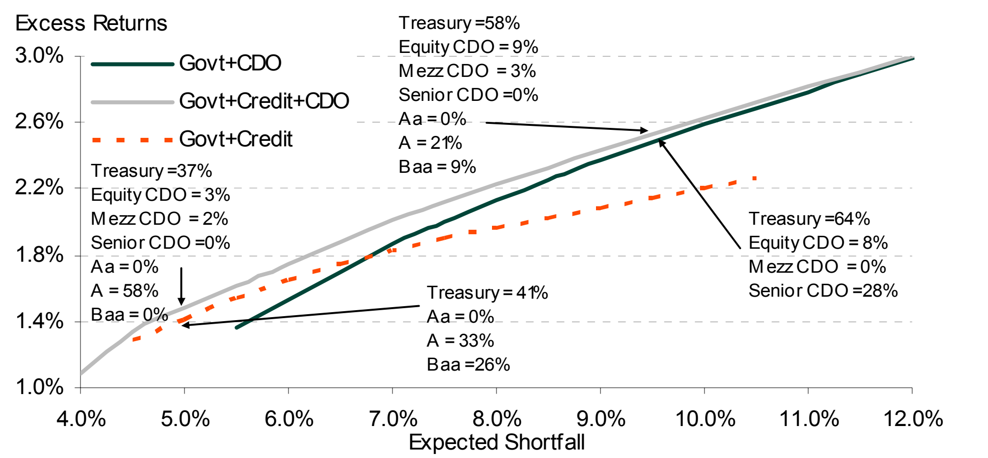
In Figure 15, we report the average excess returns over the 6-month risk-free rate of government bonds in five duration categories (which correspond to 2Y, 5Y, 10Y, 20Y and 30Y points on the curve), the volatility of these excess returns and the annualized Sharpe ratio. Among the government bonds, the long duration ones provide the highest returns but also the highest volatility. The Sharpe ratios range from 0.30 to 0.53.

Figure 15. Average excess returns and volatility of treasury bonds (5,000 simulations)

	30Y	20Y	10Y	5Y	2Y
Excess Return	2.94%	2.75%	2.24%	1.47%	0.52%
Volatility	5.53%	5.37%	4.98%	3.74%	1.80%
Sharpe Ratio	0.53	0.51	0.45	0.39	0.30
Skewness	-0.08	-0.07	-0.03	-0.01	0.00
Kurtosis	-0.06	-0.09	-0.13	-0.14	-0.16

Figure 16 shows that CDOs also play a role in a more generic fixed income portfolio. The negative correlation between credit spreads and treasury returns explains the fact that they are combined together throughout the efficient frontier. The addition of CDOs expands the efficient frontier, especially at high risk levels. At low expected loss levels standard credit tends to be preferred to senior CDOs, while for higher risk levels a combination of equity and mezzanine CDO is preferred to Baa-rated investments.

Figure 16. Asset allocation including treasuries



When we include a senior tranche to an efficient frontier made up of treasuries and credit (see Figures 17 and 18 for the effect), the main benefit is at high levels of risk. This occurs because of the high excess return of senior CDO tranches compared with equivalently rated single-name credit assets.

The inclusion of the mezzanine tranche also improves the efficient frontier for higher risk levels and the difference between the original frontier and the one including mezzanine CDO is more pronounced. Due to the lower correlation between single-A credit and mezzanine CDO than between Aa and Baa credit, mezzanine CDO is generally preferred to Baa in the asset allocation. This extra diversification lowers the risk and we therefore obtain an improved frontier.

We summarise the benefit of adding CDO tranches for two portfolios, one with low excess returns and another with high excess returns in Figures 17 and 18. The higher return level can only be reached when structured credit is included. The asset allocation obtained may be somewhat unrealistic as we allow for a large amount of curve risk to be taken (up to a duration of seven years), but is used to illustrate the improvements obtained by allowing structured credit in the investment universe.

Figure 17. Comparison of similar risk/return allocations when including CDO tranches

	Excess Return	Risk	Tsy	CDO Equity	CDO Mezz	CDO Senior	Aa	A	Baa
None	1.41%	5.00%	40.77%	0.00%	0.00%	0.00%	0.00%	33.29%	25.94%
Equity only	1.47%	5.00%	36.58%	3.02%	0.00%	0.00%	0.00%	59.82%	0.58%
Mezz only	1.43%	5.00%	39.19%	0.00%	4.58%	0.00%	0.00%	48.79%	7.44%
Senior only	1.42%	5.00%	39.84%	0.00%	0.00%	12.84%	0.00%	43.27%	4.05%
All	1.47%	5.00%	36.71%	2.58%	1.50%	0.00%	0.00%	59.21%	0.00%

Figure 18. Comparison of similar risk/return allocations when including CDO tranches

	Excess Return	Risk	Treasury	CDO Equity	CDO Mezz	CDO Senior	Aa	A	Baa
None	2.26%	10.50%	100.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
Equity only	2.48%	9.00%	58.31%	9.20%	0.00%	0.00%	0.00%	17.26%	15.23%
Mezz only	2.46%	10.00%	72.19%	0.00%	27.79%	0.00%	0.00%	0.00%	0.02%
Senior only	2.26%	10.50%	100.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
All	2.48%	9.00%	58.31%	9.20%	0.00%	0.00%	0.00%	17.26%	15.23%

5. HISTORICAL SIMULATION OF CDO RETURNS

In this section, we use the modelling approach developed above to simulate the historical performance of CDO equity. This adds an important dimension to the analysis of the asset class, showing how it would have performed in the last 14 years under very different market environments. From this exercise we can also understand the exposure of the asset class to different risk factors.

From the period June 1989 to December 2003, we simulate the performance of hypothetical CDO equity tranches that are constructed on a monthly basis. We mark-to-market these tranches monthly during five years (the term of the CDO) by tracking spreads and defaults of a particular cohort of investment grade corporate bonds. We therefore track 175 cohorts during 60 months, except for the cohorts starting in January 1999 or after. As we are discussing historical performance, there is no need to simulate the performance of the collateral: our approach consists in tracking the average spread and level of defaults of these cohorts. Note that despite the fact that credit default swaps did not exist for most of the time period studied, we make fairly mild assumptions in our analysis: that well diversified CDO portfolios could have been formed and that they would have traded at the same level as corporate bonds.

The steps involved in the simulation are as follows:

- *Calculating the contractual spread of the equity tranche at inception*. How much each cohort receives contractually is a key input to our analysis. We choose to use as a measure of this coupon at inception the excess spread accruing to equity investors on the month of issuance for a hypothetical CDO, ie, the difference between asset and liability spreads minus a structuring fee[5]. An alternative would be to use a flat correlation assumption with the level of spreads to calculate the coupon according to LHP. This option has two drawbacks at least. The main one is that it does not account for the fact that buyers of protection would require a higher correlation number in periods of stress

[5] *We assume a constant liability structure of 90% super senior, 3% Aaa, 1% Aa, 2.5% A and 3.5% equity tranche On the asset side, we assume 40% single-A and 60% Baa tranches from the Lehman Brothers US Corporate Credit index from June 1989 to December 1998. We assume that CDO liability spreads trade as a multiple of corporate bonds of the same rating (for the Baa category this multiple is 2.5). To calculate this multiple we look at liability prices for the last three years and assume that the relationship holds during the entire sample. Since January 1999 we assume a 60% US / 40% European portfolio with the same rating breakdown. Using a US only portfolio does not change the results much. We assume a 50bp structuring fee.*

as compensation for the highest cost of making a market in a stressed scenario; by accounting for liability spreads that move with asset spreads, we account for this effect. The alternative, modelling stochastic correlation, would be highly speculative. The numbers that we obtain are in line with the CDO equity spreads paid in the last two years in the market.

- *Pricing the CDO tranche at the beginning of the period t:* we use LHP to price the CDO equity tranche using as inputs the spread level, the tranche size and the level of subordination (percentage of losses that can happen before the tranche losses principal) available at the end of the period.
- *Calculating the average spread and average level of defaults for the CDO collateral for the period t to t+dt.* For each cohort, we calculate the rating breakdown of an investment grade portfolio with a 60% A / 40% Baa rating at inception by using Moody's data. In this way we can measure defaults and, by multiplying this vector by a vector of average spreads for the relevant rating category, we can determine both credit protection payments and average spreads for the cohort. We use Moody's Loss and Default database to calculate the trailing 12-month recovery rate for senior unsecured bonds to determine the credit protection payments, which are made at the beginning of the semester. Spread data come from the Lehman Brothers US Credit indices[6].
- *Repricing the CDO tranches at time t+dt:* we use LHP to reprice the CDO tranches with the new average level of spread, the new tranche sizes and subordination.
- *Computing CDO tranche excess returns:* returns are equal to the capital gain or loss in the period plus the coupon accruing to the investor.

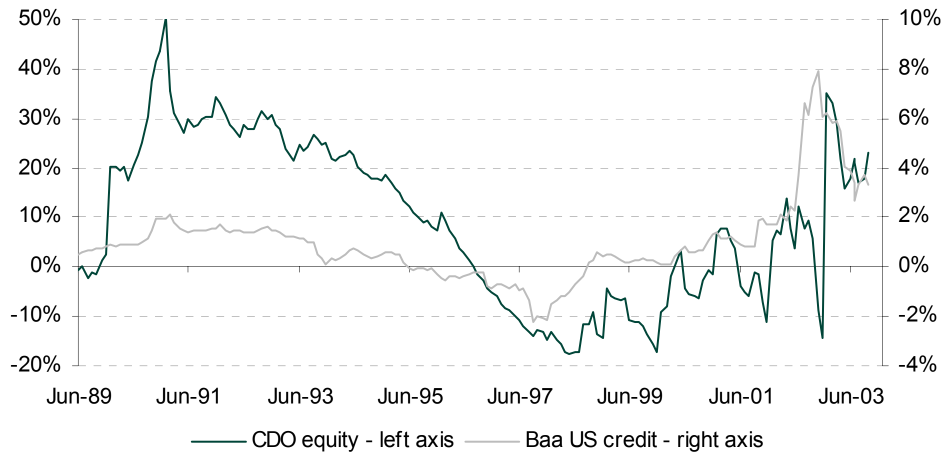
The average total monthly return in the historical returns series is 10.3%, with an annualised standard deviation of 12.3%, sensibly lower than the simulated returns that we show in section 3. The 99% monthly expected shortfall is 5.3% and the 12-month maximum drawdown is 24.3%. This highlights the fact that in the currently tight credit environment, the volatility of CDO equity investments is higher. The correlation of our panel time series with the returns of the US Credit Baa index is 42%, sensibly below the number that we find in our forward-looking simulation, which highlights the optionality embedded in CDO tranches as an important driver of diversification.

Figure 19 shows the return on an investment in each of the cohorts that we analyse using the methodology described in Ganapati and Tejwani (2002), by assuming that intermediate cash flows are invested at Libor throughout the year. All the interest proceeds are then invested in CDO equity at the end of the year. We then track the interest and principal proceeds coming from these new investments. We also show the five years cumulative return of an investment in the Lehman Brothers US Baa Credit index, which averages 51bp per month on annualised terms[7].

This data highlights the optionality embedded in equity tranches: the equity tranche is long a call option on the notional of the underlying portfolio at maturity, with a strike price equal to the difference between the notional of the underlying CDO portfolio and that of the equity tranche. In the cohorts, when default rates turned out to be low, CDO equity captures the upside. On a buy and hold basis (we show returns for the entire term of the investment), the performance of CDO equity is not much different from that of the worst credit cohorts (1997 and 1998).

[6] In our analysis, we make an assumption that favors subordinate investors: we default the exact proportion of assets defaulting in the index, which assumes a very granular portfolio.
[7] Note that for the latest cohorts, mark-to-market risk is a more important driver of the return of the investment.

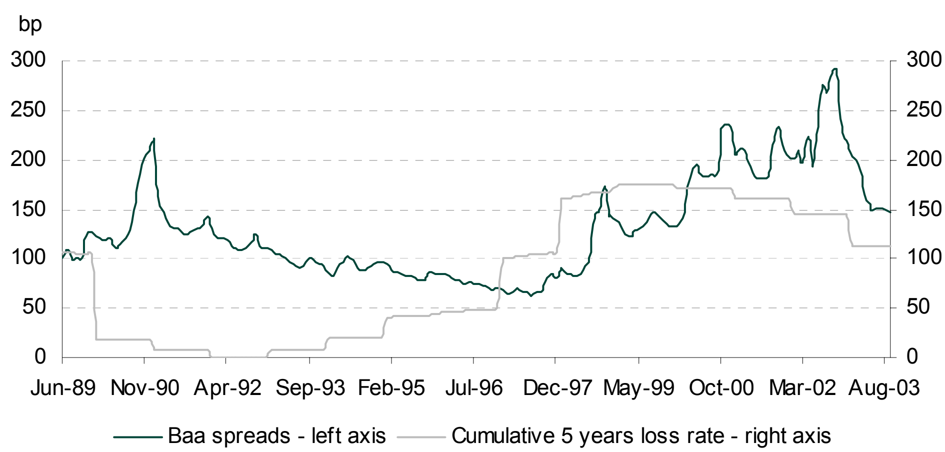
Figure 19. CDO equity returns by year of inception



Source: Lehman Brothers.

Figure 19 shows the importance of timing: unlike other asset classes the price/amount of credit risk at inception determines a big part of the performance of the transaction. This idea is explored in Figure 20, which shows for each cohort the level of Baa spreads (which are an important determinant of the coupon paid to the equity tranche) and the amount of credit losses for the remaining life of the transaction. It turns out that the weak performance of the 1997 to 2002 vintages is driven by the (ex-post) insufficient remuneration to default risk.

Figure 20. The price of credit risk and realized losses for a 5-year period



Source: Moody's Investors Services and Lehman Brothers.

We formalize this idea by estimating a panel data analysis (results available upon request) in which we estimate by GMM the parameters of a factor model which includes the Fama-French Factors, as well as a credit, liquidity risk premium, and default factors. We find that the explanatory power of the model increases sharply (from 13% to 28%) when we include dummies for the cohort, i.e. a very important determinant of the performance of CDO equity tranches is the time of inception. Apart from these dummy factors, spread changes and defaults are the most important risk factors in which CDO equity returns load.

From this analysis, we conclude that CDO equity investments outperform underlying investments in corporate credit in the following scenarios:

- For cohorts where the price of credit risk (spreads) is higher at inception than the subsequent realized default risk. One such relative value indicator is the relative implied correlation at which CDO equity trades.
- To the extent that there is predictability in the default cycle, investments coming out of the default cycle peak outperform the underlying, regardless of whether spreads tighten or not.

6. DISCUSSION AND CONCLUSION

In this report, we have quantified the risk return profile of different CDO tranches (equity, mezzanine and senior tranches) for a static synthetic investment grade CDO, using simulation of forward-looking monthly distribution of CDO returns. We have used these returns to conduct an asset allocation analysis that includes CDO tranches with treasuries and corporate bonds. We have found find that CDOs provide diversification to traditional fixed income investors by allowing investors to gain exposure to credit (delta exposure) with significant gamma exposure induced by the option-like payoff characteristic of CDO tranches. We have also accounted for the non-normal distribution of returns typically encountered in both credit and CDO investments with our CVaR framework. We have also documented results from simulated historical returns of CDO equity tranches.

There are some limitations to the analysis in this article, most notably:

- Our analysis does not model directly the spread/default process of individual assets. Instead, we model the average spread and on each period we "reset" the average market level. This should make junior tranches look safer.
- In our analysis, the mark-to-market of CDO tranches is driven by changes in their theoretical risk neutral value, whereas in reality we find that supply – demand imbalances have a large impact on the valuation of synthetic CDO tranches, as shown by the fact that market implied correlation is increasingly being used as a measure of relative value (see Isla et al. (2003)). For those who believe that supply/demand imbalances across the capital structure are responsible for the existence of a "correlation smile", modelling the stochastic aspect of the implied correlation will have a large impact in our analysis.
- Similarly LHP does not account for idiosyncratic risk either. A pure simulation approach would come at the expense of computational tractability.
- This analysis abstracts from the wide bid-ask spreads in this market: in the tranched European TRAC-X market these fluctuate between 400-500bp for the equity tranche to 5bp for an Aaa tranche.
- Our analysis abstracts from the modelling of liability benchmarks against which asset allocation risk can be measured. This modelling is a building block of our simulation and optimisation suite of models, GLASS.[8] These models also use resampling techniques to improve the optimisation and Bayesian tools to incorporate tactical asset allocation views for different asset classes, including equities and alternative investments.

[8] *Further details available upon request.*

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APPENDIX

In order simulate the behaviour of the different asset classes we must simulate:

- The term structure of interest rates. We use the EFM model described in appendix A.1.
- Credit spread movements. We use the CFM model described in appendix A.2 to simulate Aa, A and Baa spreads.
- Defaults on a basket of credit for a given spread level. We assume the spread level is given by the CFM simulated spreads for the 100 same-rated names assumed in each credit basket. We use hazard rates and a Gaussian copula to generate correlated default times (see detailed description in O'Kane et al. (2003) or Li (2000). The structured credit and single-A credit are assumed to have different underlying names. The default correlation is taken to be 20% for all the credit baskets.

We use the first of these to obtain the excess returns of different maturity treasuries and the risk-free rate of return. Credit spreads and default simulations are simulated in order to obtain excess returns for credit and structured credit. LHP is often used to evaluate structured credit, but we also use it for standard credit by assuming that traditional credit is structured credit with a single tranche.

For pricing purposes, we need risk-neutral parameters. Under risk-neutral parameters, the market is arbitrage free, i.e. the expected value of interest rates are given by the forwards and the entire spread accounts for the probability of default. Under a real world measure, interest rates are no longer expected to follow the forwards, because of term premia and the probability of default on a corporate bond is less than that implied by the spreads. Thus by simulating under the real world measure and pricing under the risk neutral measure, our different asset classes will show excess returns over the risk-free rate

A1. Simulating treasury returns

We use the Economic Factor Model (EFM) to model treasury returns, which is a term structure no-arbitrage model (see Chang and Naik 2002). Each of the factors follows a mean reverting stochastic process. We use a different form of the same model which involves the short rate (r), a slope factor (y) and a long rate (x) and we give the parametrisation of the model in Figures 21 and 22. Figure 22 gives the different mean-reversion parameters used for pricing under the risk neutral measure and simulating under the real world measure

$$\begin{array}{lll} dr = \kappa_r (R - r)\ dt + \sigma_r d\omega_r & dr = \kappa_r (x + y - r)\ dt + \sigma_r d\omega_r & \\ dR = \kappa_R (L - R)\ dt + \sigma_R d\omega_R \Rightarrow & dy = \kappa_y (\mu_y - y)\ dt + \sigma_y d\omega_y \Rightarrow & y(T) = A_T r + B_T y + C_T x + D_T \\ dL = \kappa_L (\theta - L)\ dt + \sigma_L d\omega_L & dx = \kappa_x (\mu_x - x)\ dt + \sigma_x d\omega_x & \end{array}$$

Figure 21. EFM factors

Initial factor values		Volatility parameters		Correlation parameters	
r factor	0.020	σ_r	0.006	ρ_{rx}	15%
x factor	0.070	σ_x	0.003	ρ_{ry}	40%
y factor	-0.045	σ_y	0.013	ρ_{xy}	-40%

Figure 22. Mean reversion parameters

Risk Neutral		Real World	
κ_r Q	1.208	κ_r P	1.000
κ_x Q	0.001	κ_x P	0.014
κ_y Q	0.170	κ_y P	0.076

Finally, the correlation that we assume, based on historical data, is as follows

Figure 23. Curve factors / spread correlation

Spread correlation	r	x	y
	-10%	-15%	10%

Given our real-world measure assumptions, expected excess returns for treasuries are given in Figure 24. Excess returns are close to spreads over Libor for the relevant maturity and correlations between the different maturities are close to historical levels.

Figure 24. EFM factors

	30Y	20Y	10Y	5Y	2Y
Excess Returns	2.94%	2.75%	2.24%	1.47%	0.52%
Volatility	5.53%	5.37%	4.98%	3.74%	1.80%

A2. Modelling credit returns

One important input in valuing CDO tranches is the evolution of collateral spreads (under the real world measure). In addition, corporate credit is one of the asset classes we consider in our analysis. We use a one-factor version of our CFM model to model credit (for a description of CFM, see Naik, Trinh and Rennison 2003).

We model the main CFM factor, the market factor using a square-root process. The market spread follows the following process:

$$dX_M = \alpha_M(\theta_M - X_M)dt + \sigma_M \sqrt{X_M}\, dW_M$$

The spread remains positive thanks to the square root in the volatility term. This process leads to higher spreads having higher volatility. The spread is mean reverting to the long term trend θ_M .

We also assume that each rating spread is simply the average market spread multiplied by a beta coefficient:

$$S_{Aa} = \beta_{Aa} X_M ; S_A = \beta_A X_M ; S_{Baa} = \beta_{Baa} X_M$$

The betas are decreasing with the credit rating.

We model three spread factors, for Aa, A and Baa spreads. For an annualised simulation process (spread in bp), we use the following parameters (estimated by regression on Euro data from 1999 to 2003):

Figure 25. Parameters for investment grade credit

	Aa Spread	A Spread	Baa Spread
Initial Spread	15	40	69
Alpha, α	0.912	0.912	0.912
Theta, θ	15	40	69
Beta, β	0.37	1	1.70
Sigma, σ (in bp)	2	5	9

Spread correlation is high as shown in the table below:

Figure 26. Spread correlations for investment grade credit

	Aa Spread	A Spread	Baa Spread
Aa Spread	100%	90%	89%
A Spread	90%	100%	97%
Baa Spread	89%	97%	100%

In producing the simulations, we have assumed that the starting spread level is the same level as the mean reversion level, ie, we do not have a view that spreads will widen or tighten. The credit risk premia are defined through the coverage ratio, which is set to 3.

Bond-Implied CDS Term Structures and Relative Value Measures for Basis Trading

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We introduce consistent relative value measures for CDS-Cash basis trades using the bond-implied CDS term structure derived from our implied survival rate curves. We explain why this measure is better than the traditionally used z-spread or Libor OAS and offer simplified hedging and trading strategies which take advantage of the relative value across the entire range of maturities of cash and synthetic credit markets.

1. INTRODUCTION

In a recent paper (Berd, Mashal and Wang [2003]) we introduced a new methodology for direct estimation of implied term structures of survival probabilities from credit bond prices. We have shown that this methodology is more robust than the traditional implementations of reduced-form default models (for the latter, see Litterman and Iben [1991], Jarrow and Turnbull [1995], and Duffie and Singleton [1999]). More importantly, it is more consistent with underlying market and legal practices such as debt acceleration and equal priority recovery for same-seniority bonds.

Our methodology is well suited to a direct comparison with credit derivatives, particularly credit default swaps, whose valuation is driven by the modeling of default probabilities. In this paper we introduce new relative value measures, which take advantage of the internal consistency of this pricing methodology. In particular, we define:

- Bond-Implied CDS (BCDS) term structure
- CDS-Cash curve basis
- Systematic and full bond-specific basis to CDS curve
- Risk-free-equivalent coupon (RFC) streams for credit-risky bonds

We also introduce and discuss static replication/hedging strategies of credit risk in cash bonds using forward and spot CDS. In particular, we demonstrate in detail how these strategies can be used to hedge the default risk of a credit bond with an arbitrary coupon, given an arbitrary term structure of risk-free rates. The complete hedge of credit risk in these strategies is reflected in the complementarity between the risk-free-equivalent coupon streams and the CDS hedging costs, which is formally proven in the Appendix.

2. BOND-IMPLIED CDS TERM STRUCTURE

Credit default swaps are by far the largest component of the rapidly growing credit derivatives market. They comprise as much as 70% of the market by notional volume. The outstanding notional of CDS is comparable with that of cash bonds, and their liquidity often exceeds that of the cash market for the top 200 or so names. At the same time, cash credit bonds often cover a greater range of maturities than CDS and have far more extensive historical data associated with them, providing fertile ground for research and back-testing trading strategies. One can therefore hope that an implied CDS measure derived from bond prices will be a valuable tool for consistent comparisons between the two markets across the entire range of maturities.

Deriving the Bond-Implied CDS spread term structure

The survival-based valuation approach is well suited to the CDS market. In fact it has been the market practice since its inception. By deriving the bond-implied CDS spreads within the same framework we are aiming to give investors an apples-to-apples relative value measure across the bond and CDS markets.

The pricing relationship for credit default swaps simply states that the expected present value of the premium leg is equal to the expected present value of the contingent payment (see O'Kane and Turnbull [2003]):

[1] $$\frac{S}{f} \cdot \sum_{i=1}^{N} Q(t,t_i) \cdot Z(t,t_i) = (1-R) \cdot \sum_{i=1}^{N} \left(Q(t,t_{i-1}) - Q(t,t_i)\right) \cdot Z(t,t_i)$$

where, S is the annual spread, f is the payment frequency ($f = 4$), $Q(t,t_i)$ is the issuer's survival probability between time t to time t_i, $Z(t,t_i)$ stands for the (default risk-free) discount factor from time t to time t_i, and R stands for the recovery rate. This formulation is consistent with the methodology described in Berd *et al.* (2003).

The bond-implied CDS spread term structure, hereafter denoted as BCDS term structure, is defined by substituting the survival probability term structure fitted from bond prices, $Q_{bond}(t,t_i)$, into the following equation for par CDS spreads:

[2] $$BCDS(t,t_N) = f \cdot (1-R) \cdot \frac{\sum_{i=1}^{N} \left(Q_{bond}(t,t_{i-1}) - Q_{bond}(t,t_i)\right) \cdot Z(t,t_i)}{\sum_{i=1}^{N} Q_{bond}(t,t_i) \cdot Z(t,t_i)}$$

The different payment frequencies for bonds and CDS do not represent a problem because the fitted survival probability term structure is continuous and can be evaluated at any frequency. Furthermore, the CDS contracts stipulate that at default any accrued protection premium must be paid.

The above equations approximate the settlement process of the CDS at default assuming that all defaults during a payment period are settled at the end of the period and, correspondingly, the entire period premium is paid (see O'Kane and Turnbull [2003] for a more accurate approximation). We also ignore the cheapest-to-deliver option.

Comparison with conventional spread measures

The BCDS term structure complements bond-based valuation measures defined in Berd *et al.* (2003), namely the par Libor and par Treasury spreads, and constant coupon price (CCP) term structures. The BCDS term structure is closely related to, but not equivalent to, par Libor spreads. In fact, under certain circumstances, the BCDS term structure may be significantly different from conventional measures such as a bond's z-spread and Libor spread (see "Credit Spreads Explained" by O'Kane and Sen in this issue for definitions of the conventional spread measures).

Market participants often use the z-spread as a proxy for comparing bonds with CDS. Such analysis may be misleading because the derivation of z-spreads is based on the valuation of credit bonds with spread-based discount functions which we have shown to be incorrect in our earlier paper. This is due to the fact that credit bonds do not have fixed cashflows – they only have fixed promised cashflows, while realized cashflows may well turn out to be different from the pro-forma projections. Hence, a survival-based approach is needed to correctly model default-risky bonds.

The z-spread of a credit bond is consistent with a correct survival-based valuation framework only under the assumption of zero recovery rates. Generally speaking, such an assumption is far from observed statistics. Given that historical average recovery values are about 40% (30% during the recent credit downturn) the z-spread overestimates the losses in case of default by a significant amount. Therefore, it should not be surprising that the BCDS spread term structure can differ from the z-spread by substantial margins.

An example of this is presented in Figures 1a and 1b, which depict the relationship between the survival-based BCDS curve, the Libor spread curve and the z-spreads of individual bonds, based on spread-based discount function methodology. Figure 1a shows the results for Georgia Pacific Co. as of December 31, 2002 – ie, at a time when the bonds of the company were substantially distressed. We observe that the shape of the BCDS curve bears no resemblance to the shape of the traditionally fitted Libor spread curve or to the z-spreads of individual bonds. Figure 1b, on the other hand, shows the same set of bonds as of December 31, 2003 – when the spreads have generally tightened and the bonds no longer trade at large discounts. Here, the BCDS curve is much closer to the conventional Libor spreads.

We would like to emphasize that the methodology developed in Berd *et al.* (2003) is based on fitting the *prices* of bonds rather than their spreads, and that the deviation of the BCDS curve from bond z-spreads does not indicate a poor fit of bond values. To confirm this, we also show the constant coupon price term structures (ie, the projected prices of hypothetical bonds with 6%, 8% and 10% coupons) from the same model compared with the bond prices as of December 31, 2002. As we can see, the CCP term structures neatly envelop the scatter plot of bond prices, as indeed they should.

Figures 1a and 1b also shed light on the interesting issue regarding the "slope of the spread curve". In both cases, the Libor OAS curve which is fitted using the conventional methodology, is inverted, while the BCDS curve is not (or at least not for all maturities). One often hears that distressed bond pricing is always accompanied by an inverted spread curve. However, this is only true in the conventional, spread-discount-factor based models, and it is a consequence of the inability of these models to correctly capture the peculiar aspects of distressed bond pricing – namely the fact that such bonds trade "on price" or "to recovery".

Indeed, in case of default, all bonds of the same seniority, regardless of their maturity, should trade at the same price (equal to recovery). As the credit quality of the issuer deteriorates, the market begins pricing these bonds closer and closer to recovery scenario. As a result, bonds begin trading at similar low dollar prices across all maturities. In the conventional spread-discount-function based methodology, one can explain an $80 price of a 20-year bond with a spread of 500bp. However, to explain an $80 price of a 5-year bond, one would need to raise the spread to very large levels to achieve the required discounting effect. Thus, the inversion of the spread curve is due to the bonds trading on price. In the survival-based methodology, the low prices of bonds are explained by high default rates, which need not have an inverted term structure. Therefore, the BCDS term structure also need not be inverted in order to capture the credit risk of distressed bonds.

Figure 1a. BCDS and Libor OAS term structures and bond z-spreads, GP as of 12/31/02

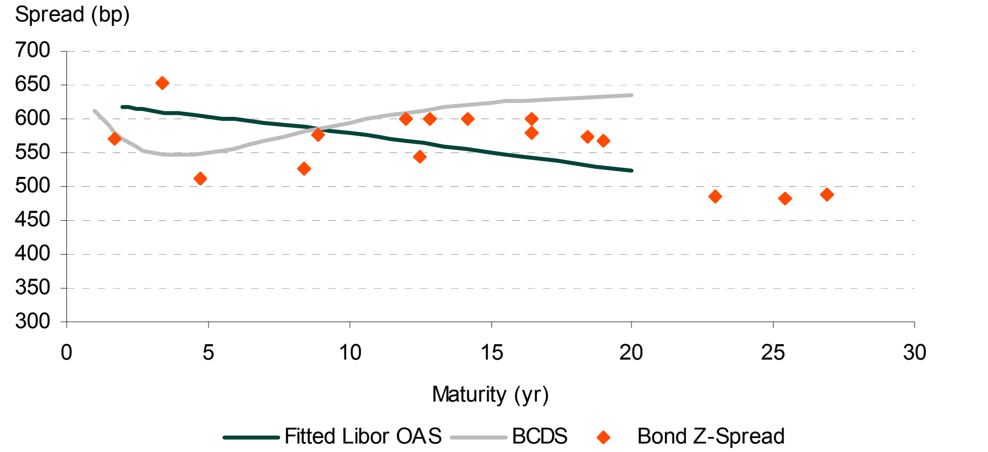


Figure 1b. BCDS and Libor OAS term structures and bond z-spreads, GP as of 12/31/03

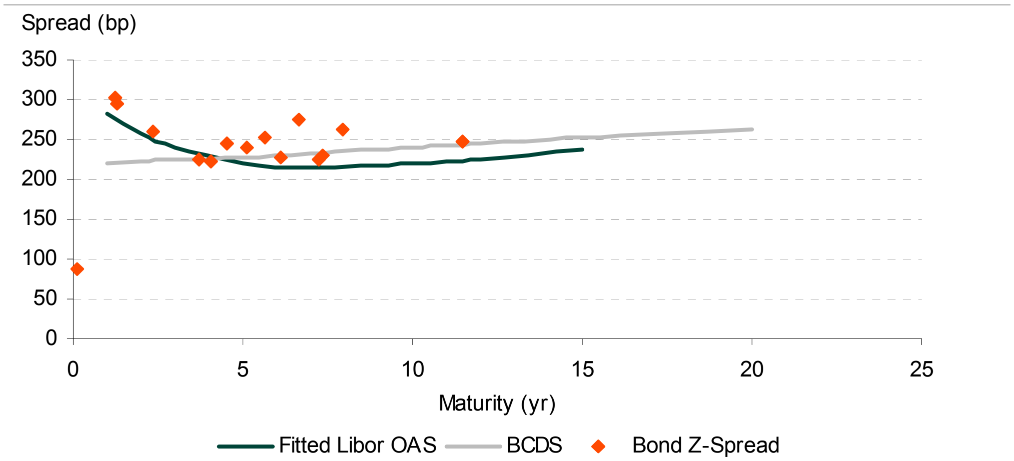
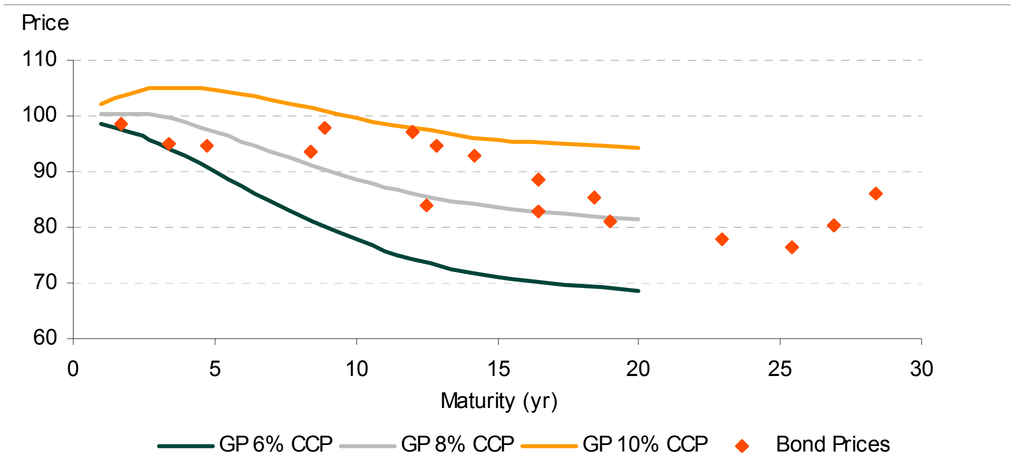


Figure 1c. CCP term structures and bond prices, GP as of 12/31/02



3. STATIC HEDGING OF BONDS WITH CDS

In the previous section we defined the BCDS term structure and explained why it is a better valuation measure than the traditional Libor OAS or z-spread measures. However, in order to be sure that the BCDS term structure corresponds to the fair value of the CDS as seen by the bond market, one should explicitly construct a hedging strategy using CDS that would completely eliminate the credit risk of any given cash bond.

Such a strategy is well known for a par bond when the underlying credit risk-free curve (which is usually assumed to coincide with the Libor swaps curve) and the hazard rates are flat. Under such assumptions, the hedging strategy turns out to be simple – buy the credit bond and buy an equal face value of CDS. The resulting combination exhibits no risk due to default. The interest rate risk of the hedged position coincides with the risk of a credit risk-free bond with a lower coupon equal to the difference of the credit-risky coupon and the CDS spread. This risk can be hedged using interest rate swaps, although residual timing risk may remain if the swap is not terminated upon default.

The situation quickly becomes more complex when the above assumptions are not valid, most notably in case of non-par bonds and/or a Libor curve that is significantly different from flat. Some of the difficulties and possible remedies have been discussed in McAdie and O'Kane (2001). There it was shown that, when hedging a given fixed coupon bond with a CDS of matching maturity, neither the face value hedge ratio nor the market value hedge ratio based on the current price of the bond is satisfactory, because they lead either to a residual mark-to-market risk during the life of the bond or to a substantial dependence of the carry cost of the hedged position on the price of the bond. McAdie and O'Kane conclude that, for a hedge using a single CDS position, an intermediate strategy called a zero-recovery market value hedge provides the best compromise.

In this section we further extend the intuition put forward in McAdie and O'Kane (2001). As it turns out, the complete hedging strategy cannot in general be accomplished by a single CDS position. Indeed, under generic conditions, a bond will have a non-trivial term structure of forward prices computed with today's risk-free discount curve and hazard rate term structure. If a bond is well hedged today, it will likely be either over- or under-hedged in the future, when its price is expected to change. After all, if the bond actually survives to maturity its price will pull to par. So the hedge at maturity should always be based on the expected price at that point, which is close to par. The question is – what should happen at prior times?

Staggered hedging strategy with forward CDS

The precise answer to this question is presented in the Appendix, where we define the notion of the *risk-free-equivalent coupon stream* (RFC) and show its complementarity with the BCDS curve. The complementarity condition defines the unique static hedging strategy based on a sequence of forward CDS contracts with notional that depends on the bond's forward price for each maturity.

A forward CDS contract provides protection during a future time period in exchange for premiums which are paid during that period but whose level is preset today. In case of a credit event prior to the starting date of that period, the forward CDS knocks out without a payment and provides no further protection. However, if a credit event occurs during the contractual future period – the CDS pays the loss amount $N_{fwd} \cdot (1-R)$. Since, based on today's calculations, the bond is expected to have a forward price $P_{fwd} \neq 100\%$ at that future date, then in order to insure the full forward price of the bond we must choose the hedge notional

[3] $$N_{fwd}(t) = \frac{P_{fwd}(t) - R}{1 - R}$$

This intuition is precisely right, as shown in the Appendix. The sequence of forward CDS hedges does, in fact, completely hedge out the credit risk of a fixed coupon bond. If we subtract from the bond's coupon the cost of hedging according to [3], the residual cashflows will coincide with the above mentioned risk-free-equivalent coupon stream. The RFC is such a pre-set sequence of term-dependent coupons which, if discounted with risk-free rate, correspond to the same term structure of forward prices as the one obtained for the credit-risky bond under consideration (see Appendix for more details).

To be more accurate, we must mention that the hedge notional shown above is only valid for the case of continuously compounded coupons and spreads. In a more realistic case of semi-annual coupon payments, one must take into account the present value of those coupons as well as the fact that the CDS market conventions stipulate a payment of the accrued premium upon default. The result is a slightly modified expression for the hedge notional applicable for the payment period ending at t_i:

[4] $$N_{fwd}(t_i) = \frac{0.5 \cdot \left(P_{fwd}(t_{i-1}) + P_{fwd}(t_i) + w \cdot C\right) - R}{1 - R}$$

The hedge depends on the average forward price during the preceding coupon period as well as the potential loss of coupon in case of default. The effective weight w depends on the assumed coupon recovery. We have found empirically that for coupon recovery of 50% the weight is also 50%, while for coupon recovery of 0% the weight is 25%.

Figure 2a illustrates this staggered forward CDS strategy in the case of a hypothetical 8% coupon 5-year bond which trades at a high initial price premium of 116.69%. The top rows of the table show the term to maturity, the spot BCDS curve and the forward BCDS curve with forward horizon equal to 0.5 years (see Berd [2003] for the relationship between spot and forward CDS spreads). The middle of the table shows how the hedge notionals are related to the forward prices of the credit bond and how they gradually decrease as the forward prices exhibit pull to par – compare the notional for a given term (column) with the forward price shown in the shaded area on the row corresponding to the same term (in this example we used 50% principal and coupon recovery in equation [4]). The next four rows demonstrate the complementarity between the hedging costs and the risk-free-equivalent coupon stream (RFC), which holds very accurately. Finally, we show that the forward prices of the resulting residual cashflow streams discounted with risk-free interest rates do indeed coincide with high accuracy with the forward prices of the original bond for all future horizons.

Staggered hedging strategy with spot CDS

One could, in principle, construct a very similar strategy using the spot instead of forward CDS. As discussed in McAdie *et al.* (2003) and Berd (2003), a forward CDS protection contract is closely related to a long-short pair trade in spot CDS, with equal notionals of the long protection position at the longer maturity and of the short protection position at the shorter maturity. Although in terms of credit protection the long-short trade is indeed equivalent to a forward CDS contract, the premium legs of these two strategies will generally differ, resulting in different forward mark-to-markets.

If we execute the hedging strategy with long-short pairs, the result becomes a staggered hedge which is nearly 100% notional for the final maturity, and which includes some additional relatively small long (or short) positions for shorter maturities depending on the forward prices of the credit bond being hedged. Each such position hedges the incremental digital price risk (with no recovery) corresponding to the next maturity interval on the hedging grid:

$$N_{pair}(t_i) = \frac{P_{fwd}(t_i) - P_{fwd}(t_{i+1})}{1-R} \quad [5]$$

Figure 2b shows an implementation of this strategy for the same hypothetical high premium bond. While the current credit risk of this bond is indeed hedged, as evidenced by close replication of spot price, this hedge does drift away from perfection with time (but remains quite close nevertheless). This is because the pair trades are not exactly equivalent to a series of forward CDS.

The coupon and price premium/discount dependence

From the discussion and examples shown above, it is clear that both the coupon level of the credit bond and the term structure of the underlying interest rates and issuer's hazard rates may substantially affect the hedging strategy with forward CDS or long-short CDS pairs. Its dependence on the underlying bond is depicted in Figures 3a, 3b and 3c.

Figure 3a shows a case of a bond with a high coupon equal to 8% and a high current price of 116.69% – the same as Figures 2a and 2b. The forward CDS hedge notional starts as high as 133% of the face value, and gradually decreases toward 100%. Despite the decrease in the hedge notional, the semi-annual cost of hedging grows gradually from 40bp to 44bp as a result of a relatively steep forward CDS curve term structure.

Figure 3b shows a case of a bond with a very low coupon equal to 3% and a current discount price equal to 94.33%. The forward CDS hedge notional starts at 89% of the face value, and gradually increases toward 100%. The semi-annual cost of hedging grows more steeply from 27bp to 43bp as both forward CDS rates and hedge notionals grow.

Figure 3c shows a case of a bond with near-par coupon equal to 4.25% and a current price of 99.93%. Despite the fact that this is a par bond, the forward bond prices and hedge notionals exhibit a non-trivial term structure, starting near 100%, then dropping to lower levels and only pulling back to par near final maturity. The semi-annual cost of hedging grows from 33bp to 43bp, which is somewhere between the high and low coupon cases.

Notes on practical implementation

The hedging strategy using a sequence of forward CDS is difficult to implement in practice. Although less precise, the strategy using spot CDS is generally easier to put to work. However, the precise hedge amounts stipulated by BCDS complementarity generally correspond to odd-lot notionals for intermediate terms, which would likely result in a loss of liquidity. As a compromise between accuracy and liquidity, we suggest a coarse-grained staggered hedge which can be constructed using a maturity grid with longer intervals. The forward price changes in these intervals will yield lumpier intermediate hedge notionals, according to [5]. The optimal hedging grid will depend on the bond coupon level and the underlying interest rates. For bonds with modest premium or discount, just one or two additional hedges can result in sufficient accuracy.

Figure 2a. Complete hedge of a premium credit bond with forward CDS, 8% coupon 5-year maturity bond

	Term	*0.00*	*0.50*	*1.00*	*1.50*	*2.00*	*2.50*	*3.00*	*3.50*	*4.00*	*4.50*	*5.00*
	Fwd BCDS	0.60%	0.60%	0.63%	0.66%	0.69%	0.72%	0.74%	0.77%	0.80%	0.83%	0.86%
	Spot BCDS	0.60%	0.60%	0.62%	0.63%	0.65%	0.66%	0.67%	0.69%	0.70%	0.71%	0.72%
Term	**Fwd Price**											
0.00	116.70%											
0.50	113.84%		1.33									
1.00	111.31%			1.27								
1.50	109.09%				1.22							
2.00	107.16%					1.18						
2.50	105.50%						1.15					
3.00	104.08%							1.12				
3.50	102.84%								1.09			
4.00	101.77%									1.07		
4.50	100.84%										1.05	
5.00	100.00%											1.03
	Hedge Notional		**1.33**	**1.27**	**1.22**	**1.18**	**1.15**	**1.12**	**1.09**	**1.07**	**1.05**	**1.03**
	Protection CF		0.40%	0.40%	0.40%	0.41%	0.41%	0.42%	0.42%	0.43%	0.43%	0.44%
	Coupon less Protection CF		3.60%	3.60%	3.60%	3.59%	3.59%	3.58%	3.58%	3.57%	3.57%	3.56%
	RCF		3.60%	3.60%	3.60%	3.60%	3.59%	3.59%	3.58%	3.57%	3.56%	3.56%
	CF Diff		*-0.01%*	*0.00%*	*0.00%*	*0.00%*	*0.00%*	*0.00%*	*0.00%*	*0.00%*	*0.00%*	*0.00%*
	Projected Fwd Price	116.69%	113.83%	111.30%	109.09%	107.16%	105.51%	104.08%	102.85%	101.77%	100.84%	100.00%
	Fwd Price	116.70%	113.84%	111.31%	109.09%	107.16%	105.50%	104.08%	102.84%	101.77%	100.84%	100.00%
	Price Diff	*-0.02%*	*-0.01%*	*-0.01%*	*0.00%*	*0.00%*	*0.00%*	*0.00%*	*0.00%*	*0.00%*	*0.00%*	*0.00%*

Figure 2b. Present value hedge of a premium credit bond with spot CDS, 8% coupon 5-year maturity bond

	Term	*0.00*	*0.50*	*1.00*	*1.50*	*2.00*	*2.50*	*3.00*	*3.50*	*4.00*	*4.50*	*5.00*
	Fwd BCDS	0.60%	0.60%	0.63%	0.66%	0.69%	0.72%	0.74%	0.77%	0.80%	0.83%	0.86%
	Spot BCDS	0.60%	0.60%	0.62%	0.63%	0.65%	0.66%	0.67%	0.69%	0.70%	0.71%	0.72%
Term	**Fwd Price**											
0.00	116.70%											
0.50	113.84%		1.33									
1.00	111.31%		-1.27	1.27								
1.50	109.09%			-1.22	1.22							
2.00	107.16%				-1.18	1.18						
2.50	105.50%					-1.15	1.15					
3.00	104.08%						-1.12	1.12				
3.50	102.84%							-1.09	1.09			
4.00	101.77%								-1.07	1.07		
4.50	100.84%									-1.05	1.05	
5.00	100.00%										-1.03	1.03
	Hedge Notional		0.05	0.05	0.04	0.04	0.03	0.03	0.02	0.02	0.02	1.03
	Protection CF		0.46%	0.45%	0.43%	0.42%	0.41%	0.40%	0.39%	0.38%	0.37%	0.37%
	Coupon less Protection CF		3.54%	3.55%	3.57%	3.58%	3.59%	3.60%	3.61%	3.62%	3.63%	3.63%
	RCF		3.60%	3.60%	3.60%	3.60%	3.59%	3.59%	3.58%	3.57%	3.56%	3.56%
	CF Diff		-0.07%	-0.05%	-0.03%	-0.01%	0.00%	0.02%	0.03%	0.05%	0.06%	0.07%
	Projected Fwd Price	116.76%	113.96%	111.48%	109.30%	107.39%	105.73%	104.29%	103.02%	101.91%	100.91%	100.00%
	Fwd Price	116.70%	113.84%	111.31%	109.09%	107.16%	105.50%	104.08%	102.84%	101.77%	100.84%	100.00%
	Price Diff	0.06%	0.13%	0.17%	0.21%	0.22%	0.22%	0.21%	0.18%	0.14%	0.08%	0.00%

Figure 3a. **Hedging strategy for high coupon, premium bond**

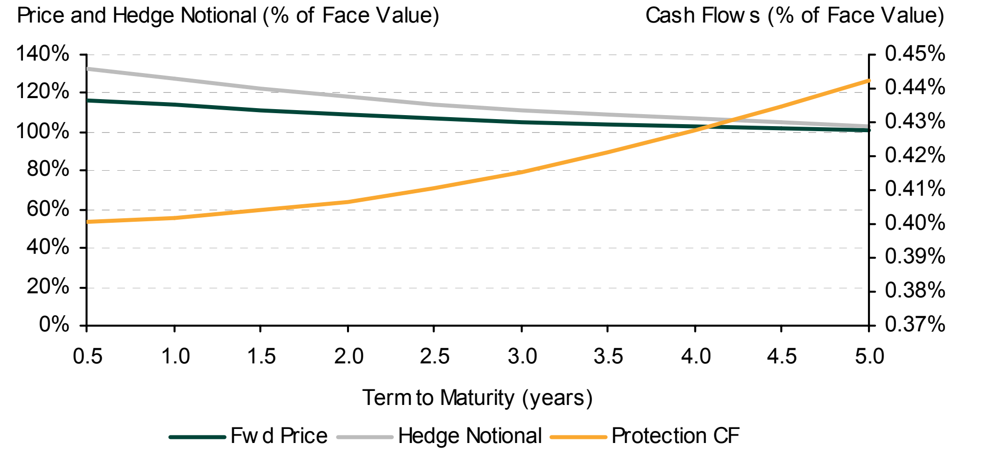


Figure 3b. **Hedging strategy for low coupon, discount bond**

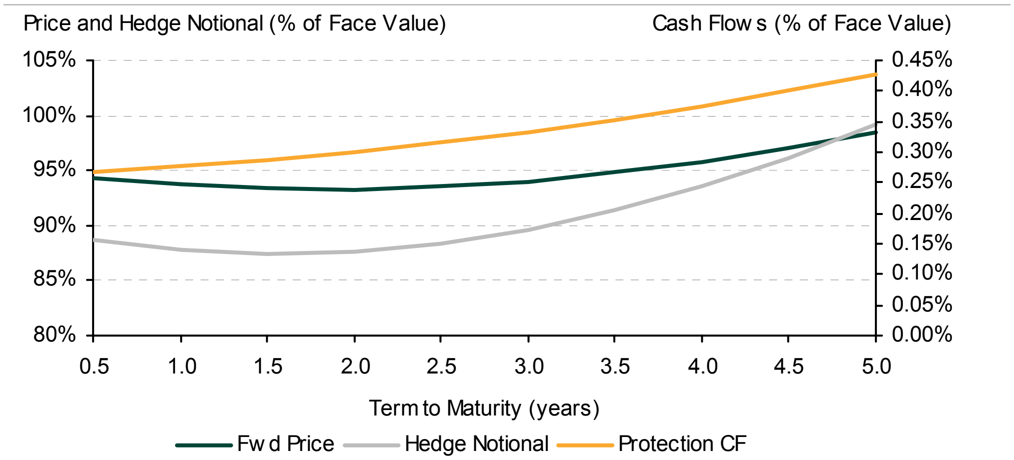
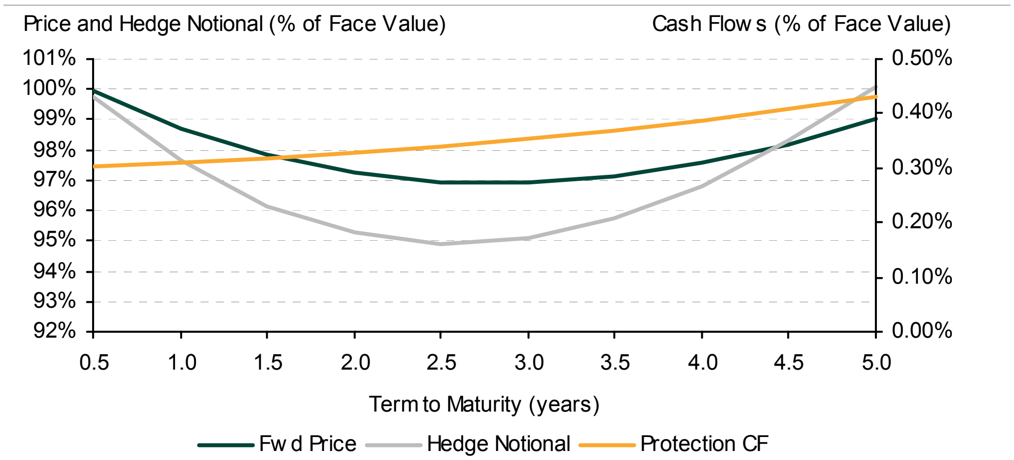


Figure 3c. **Hedging strategy for medium coupon, near-par bond**



4. RELATIVE VALUE MEASURES FOR BASIS TRADING

Although CDS and cash bonds reflect the same underlying issuer credit risk, there are important fundamental and technical reasons why the CDS and bond markets can sometimes diverge from the economic parity (see McAdie and O'Kane [2001]). Such divergences, commonly referred to as the CDS-Cash basis, are closely monitored by many credit investors. Trading the CDS-Cash basis is one of the widely used strategies for generation of excess returns using CDS. To facilitate basis trading, one must have a reliable measure of relative value between cash bonds and CDS. Here we suggest such a measure based on our approach.

The CDS-Cash curve basis

If the bonds of a given issuer were perfectly priced according to our framework, their prices would satisfy the survival-based fair value, and the BCDS term structure derived from the issuer hazard rates would correspond to a complete hedge of the credit risk as discussed in the previous section. Therefore, if the market-observed CDS term structure coincided with the BCDS term structure, then one could claim that there is no basis between the two markets since the hedged cash bond would have zero expected excess return over risk-free rates. If the market-observed CDS spreads were tighter than BCDS, one could hedge the credit bond at a cheaper price than that implied by the model, thus locking in a positive expected excess return. Vice versa, if the market CDS spreads were wider than BCDS, the hedge would have a higher cost, and the expected excess return of the hedged position would be negative.

We therefore introduce the **CDS-Cash curve basis** as a consistent relative value measure:

[6] $$\mathrm{CurveBasis}(0,T) = CDS_{market}(0,T) - BCDS(0,T)$$

This measure corrects for the biases associated with commonly used asset swap spreads and z-spreads. Since this basis measure is well defined across the entire range of maturities, one can use it not only to screen for issuers that correspond to attractive CDS-Cash trading opportunities, but also to pinpoint maturities for which such trades would be most beneficial.

Bond-specific basis and incremental relative value

The curve basis does not fully reflect the differences in the costs of hedging implied by the fair value BCDS term structure and by the market observed CDS term structure. These differences depend, as explained in the previous section, on the projected forward price term structure of cash bonds, and therefore on the coupon level of the bond under consideration.

According to the staggered hedging strategy presented in the previous section, the notional of the final maturity hedge using the spot CDS is approximately equal to the face value and therefore its contribution to the CDS-Cash basis is simply the curve basis. However, the incremental hedges for each maturity horizon are proportional to the change in the projected forward price of the bond during the short maturity span around that horizon [5]. The present value of every basis point of difference between the BCDS and the market observed CDS curve for each of these incremental hedges is given by the risky PV01 calculated in accordance with the BCDS curve. Therefore, the incremental hedging cost differential $HCD_{curve}(T)$ due to intermediate maturity curve basis is equal to:

[7]

$$HCD_{curve}(T) = -\int_0^T \mathrm{CurveBasis}(0,t) \cdot \frac{1}{1-R} \cdot \frac{\partial P_{fwd}(t,T)}{\partial t} \cdot \mathrm{RiskyPV01}(0,t) \cdot dt$$

To convert the hedging cost differential into a spread-equivalent measure, we divide it by the risky PV01 of the final maturity BCDS. This has a meaning of a weighted average curve basis in units corresponding to matching maturity CDS. We call it a **systematic bond basis**:

[8] $$\mathrm{BondBasis}_{\mathrm{systematic}}(T) = \mathrm{CurveBasis}(0,T) + \frac{HCD_{curve}(T)}{\mathrm{RiskyPV01}(0,T)}$$

Generally speaking, if the bond price premium or discount is not large, then the corrections to the systematic bond basis due to intermediate maturities will be very small, and one can use the curve basis for the final maturity as a good proxy of the systematic bond basis. However, one must note that if the matching maturity curve basis is exactly zero, then the correction term will become the main component of the systematic bond basis. Its dependence on the bond price and curve basis is non-trivial.

If the curve basis is positive at intermediate maturities, and the bond is trading at premium and is expected to gradually accrete to par, the systematic bond basis will be positive – ie, it will cost more to hedge the bond than the BCDS curve would suggest. If, on the contrary, the curve basis is positive but the bond is trading at a discount and is expected to accrete up to par, then the systematic bond basis will be negative – ie, it will cost less to hedge the bond with CDS than the bond market itself implies. This is because the staggered hedge strategy in this case actually requires selling protection at intermediate maturities, and therefore the positive curve basis will net extra benefits for the hedger. In the other cases when either the curve basis changes sign at intermediate maturities or the bond trades near par and its forward price may both increase and decrease over the future time horizon, the hedging cost differential can easily turn out to be either positive or negative – one would have to perform the full calculation to find out.

In addition to the fair value hedging cost differential embodied in the curve basis, there will also be an issue-specific pricing differential corresponding to the OAS-to-Fit (OASF) measure introduced in Berd *et. al.* (2003). This measure captures the pricing differential between the given bond and the issuer's fitted survival curve. As such, it reflects liquidity and other technical aspects of bond pricing unrelated to credit risk.

We define the **full bond-specific CDS-Cash** basis as the systematic bond basis minus an incremental amount equal to the bond's OASF:

[9] $$\mathrm{BondBasis}_{\mathrm{full}}(T) = \mathrm{BondBasis}_{\mathrm{systematic}}(T) - OASF$$

Depending on the sign and magnitude of OAS-to-Fit, a given bond may have a negative basis while the issuer curve basis is positive, and vice versa. Having said this, since the average OASF across all bonds is zero by construction (see the discussion in our previous paper) then our definition of the bond-specific basis does not add any new bias to the systematic (or curve) basis. It only serves to assist investors in picking the best candidates for execution of either positive or negative basis strategies. For example, if the investor believes that a negative curve basis will converge, then picking the cheapest bond (most positive OASF) will add an incremental expected return to the trade.

Examples of curve and bond-specific basis

Let us consider an example of curve basis in one of the most actively traded issuers, Altria Group (ticker MO). Using the data as of February 6, 2004 we can see the difference between the shapes of the BCDS and market CDS curves. A clear hump in the relative curve basis in intermediate maturities is related to significant hedging activity in the CDS market as volatility picked up in late January and early February.

If an investor's view is that the curve basis is a transient phenomenon and is expected to converge, then picking a two- to three-year maturity range would maximize the convergence potential. In this maturity range the CDS market looked cheap to cash using our relative value measures (we use the bid-side CDS marks since the bond valuation measures including BCDS term structure are also based on bid-side quotes).

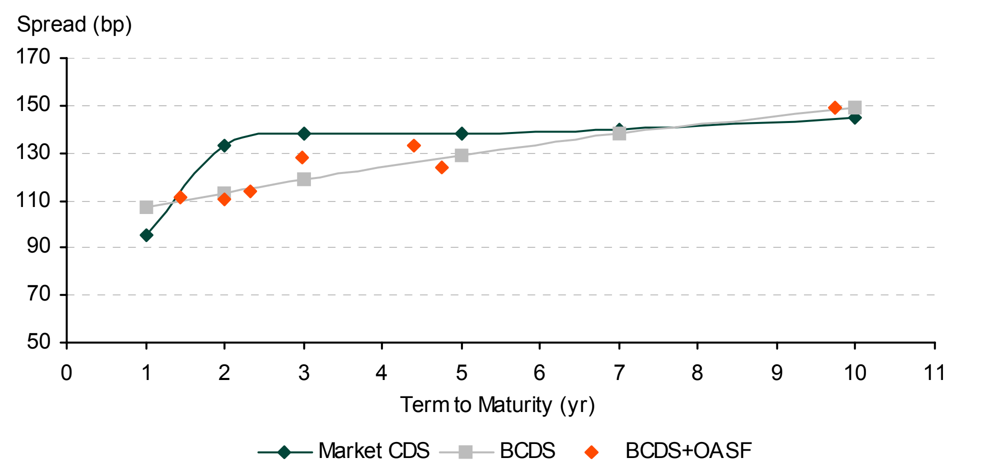
On a bond-by-bond comparison the 5-year MO 5.625 of 2008 (P=104.12) has the largest negative OAS-to-Fit and is therefore the richest cash bond among those included in this analysis. However, when considering the combined BCDS+OASF measure, the largest total basis is exhibited by the 2-year bonds MO 6.375 of 2006 (P=106.07) and MO 6.95 of 2006 (P=107.77). Note that all these bonds trade at a relatively modest premium, and therefore the systematic bond basis approximately coincides with the matching maturity curve basis.

Finally, we would like to note that comparing the BCDS and market CDS term structures may be a valuable tool not only for basis trading but for curve trading as well. For example, by looking at Figure 4, an investor may conclude that the CDS curve is too flat compared with the cash market-based BCDS curve. Depending on the investor's directional views regarding Altria, this opinion could be implemented in either bullish or bearish fashion.

A bearish curve steepener would be to sell 2-year protection and buy 7-year protection in equal notional amounts. Such a trade has an overall negative Credit01, and therefore will produce positive returns if spreads widen and/or steepen. A bullish curve steepener would be to sell 2-year protection and buy 7-year protection in amounts which make the trade Credit01-neutral. Then the overall spread moves will not affect the trade much, while the excess tightening in the front end as the spreads rally will produce positive returns.

We conclude that the newly introduced BCDS curve basis and bond-specific basis relative measures should be useful to credit investors in a variety of relative value strategies.

Figure 4. Curve and bond basis in Altria Group (as of 2/06/2004)



APPENDIX: THE COMPLEMENTARITY BETWEEN THE FORWARD CDS AND RISK-FREE-EQUIVALENT COUPON TERM STRUCTURES

Assume that the underlying risk-free discount curve $r(t)$ (usually Libor) and the issuer's hazard rate term structure $h(t)$ are given. The forward discount function for the riskless rate is given by a well known formula, where $r(s)$ is the instantaneous forward rate:

$$[10] \qquad Z(t,T) = \exp\left(-\int_t^T r(s) \cdot ds \right)$$

The forward survival probability $Q(t,T)$ stands for the probability that the issuer will survive during the time period (t,T) provided that it survived until the beginning of that period. It is related to the hazard rate by a similar relationship[1]:

$$[11] \qquad Q(t,T) = \exp\left(-\int_t^T h(s) \cdot ds \right)$$

Consider a credit-risky bond with a given coupon C and final maturity T. The projected forward price of a fixed coupon bond depends on both of these term structures as well as the level of the coupon in the following manner (for simplicity of exposition we assume continuous compounding, but very similar results hold for quarterly, or semi-annual compounding conventions as well):

$$[12] \; P(t,T) = C \cdot \int_t^T du \cdot e^{-\int_t^u (r(s)+h(s)) \cdot ds} + e^{-\int_t^T (r(s)+h(s)) \cdot ds} + R \cdot \int_t^T du \cdot h(u) \cdot e^{-\int_t^u (r(s)+h(s)) \cdot ds}$$

The first term reflects the present value of the coupon stream under the condition that the bond survived until some intermediate time t, the second term reflects the present value of the final principal payment under the condition that the bond survived until the final maturity T, the third term reflects the recovery of the fraction R of the face value if the issuer defaults at any time between the valuation time and the final maturity.

Let us define the "risk-free-equivalent coupon" stream $RFC(t,T)$ which would reproduce the same forward price term structure but only when discounted with the risk-free discount function, without any default probability. Such a coupon stream will not be constant in general and will have a non-trivial term structure, depending on both the underlying risk-free rates and, through the price of the risky bond, on the issuer hazard rates as well. The defining condition is:

$$[13] \qquad P(t,T) = \int_t^T du \cdot RFC(u,T) \cdot e^{-\int_t^u r(s) \cdot ds} + e^{-\int_t^T r(s) \cdot ds}$$

The concept of a risk-free equivalent coupon stream is necessary for consistent definition of the difference between the default-risky and risk-free bonds when the underlying interest and hazard rates have non-trivial term structures and the bonds are expected to deviate from par pricing either currently or at any time in the future.

[1] *The hazard rate and interest rate can, in general, be stochastic. In this discussion, we restrict our attention to deterministic default intensities and deterministic interest rates.*

To find the relationship between the risk-free-equivalent coupon stream $RFC(t,T)$ and the forward price $P(t,T)$ of the credit-risky bond, let us take a derivative with respect to the valuation time t of both sides of equations [12] and [13]. On one hand we get:

$$[14] \qquad \frac{\partial P(t,T)}{\partial t} = \left(r(t) + h(t)\right) \cdot P(t,T) - C - R \cdot h(t)$$

On the other hand, we get:

$$[15] \qquad \frac{\partial P(t,T)}{\partial t} = r(t) \cdot P(t,T) - RFC(t,T)$$

Since the left-hand sides are equal by construction, we can equate the right-hand sides and obtain the relationship between the risk-free-equivalent coupon stream and the forward price:

$$[16] \qquad C - RFC(t,T) = h(t) \cdot \left(P(t,T) - R\right)$$

Consider now a forward CDS contract for a short time period $(t, t+\Delta t)$. This is a contract which provides protection during the future time period in exchange for premiums which are paid during that period but whose level is preset today. In case of a credit event prior to starting date t, the forward CDS knocks out and provides no further protection during the future period. As explained in Berd (2003), the forward CDS spread is determined by:

$$[17] \qquad CDS_{fwd}(t,T) \cdot \int_t^T du \cdot e^{-\int_t^u (r(s)+h(s)) \cdot ds} = (1-R) \cdot \int_t^T du \cdot h(u) \cdot e^{-\int_t^u (r(s)+h(s)) \cdot ds}$$

It is easy to see that for a small future time interval, the forward spread is simply proportional to the hazard rate of the matching horizon:

$$[18] \qquad CDS_{fwd}(t, t+\Delta t) = (1-R) \cdot h(t)$$

Substituting this definition into equation [16], we get a complementarity condition between the risk-free-equivalent coupon streams and the forward CDS spreads:

$$[19] \qquad C - CDS_{fwd}(t) \cdot N(t,T) = RFC(t,T), \text{ where } N(t,T) = \frac{P(t,T) - R}{1-R}$$

This relationship confirms our intuition about the consistent hedging strategy for non-par credit-risky bonds which consists of a stream of forward CDS with notionals $N(t,T)$ depending on the forward price of the bond. The residual cashflows of the credit-risky bond after paying the required premiums coincide with the projected risk-free-equivalent coupon stream. Although there is still a timing risk associated with this hedging strategy, the notionals of the hedges are such that the recovered value will be equal to the correct forward price of the bond, and therefore the timing risk is unimportant when evaluating the present value of the hedged cashflows to the present time or to any future time before maturity. This is reflected in the fact that discounting these residual cashflows with riskless rates gives the correct forward prices of the bond (compare [13] and [19]).

Also note that the hedge notionals depend on the recovery rate both explicitly and implicitly, via the dependence of the implied hazard rates on the recovery rate. Therefore a credit bond hedged according to this strategy still contains recovery risk (see Berd and Kapoor [2002], and O'Kane and Turnbull [2003] for estimates of recovery dependence).

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Credit Spreads Explained

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Credit investors need a measure to determine how much they are being paid to compensate them for assuming the credit risk embedded within a security. A number of such measures exist, and are commonly known as credit spreads since they attempt to measure the return of the credit asset relative to some higher credit quality benchmark. Each has its own strengths and weaknesses. In this article, we define, describe and analyse the main credit spreads for fixed rate bonds[1], floating rate notes and the credit default swap.

1. INTRODUCTION

It is useful for credit investors to have a measure to determine how much they are being paid to compensate them for assuming the credit risk embedded within a security. This credit risk may be embedded within a bond issued by some corporate or sovereign or may be synthesized through some credit derivative. Such a measure of credit quality should enable comparison between securities issued by a company, which may differ in terms of maturity, coupon or seniority. It should also facilitate comparisons with the securities of other issuers. Ultimately it should enable the credit investor to identify relative value opportunities within the bonds of a single issuer, and across different issuers.

When we enlarge our universe of credit instruments to include not just fixed rate bonds, but also floating rate bonds, asset swaps and default swaps, it is only natural to try to define a credit risk metric which also allows a comparison across instruments. For example, we would like to know when a credit default swap is priced fairly relative to a cash bond when both are linked to the same issuer. This is especially important for determining the relative value of a default swap basis trade.

While we would like one simple credit measure, there is in fact a multiplicity of such measures. Most are called "credit spreads" since they attempt to capture the difference in credit quality by measuring the return of the credit risk security as a spread to some higher credit quality benchmark, typically either the government (assumed credit risk free) curve or the same maturity Libor swap rate (linked to the funding rate of the AA-rated commercial banking sector).

Well-known credit measures include the yield spread, the asset swap spread, the option adjusted spread (OAS), the zero volatility spread, the discount margin, the default swap spread and the hazard rate. Each is defined in its own particular way and so has its own corresponding strengths and weaknesses. However, since they play such a fundamental role in the trading, analysis and valuation of credit securities, it is essential that there exist a clear picture as to the information contained in each of these different credit spreads.

The purpose of this report is to define, explain and examine these different credit spreads. We would also like to understand the relationship between different credit spread measures. To do this we have to set up a unifying credit risk modelling framework through which we can express each of these credit-spread measures. Such a model will enable us to understand how different measures behave with changing credit quality and asset indicatives. A comparison of the various credit spread measures is left to a forthcoming Quantitative Credit Research Quarterly article.

[1] *An analysis of spread measures within the context of agency bonds has been published in Tuckman (2003).*

The remainder of this paper is organised as follows: We describe a number of credit spread measures in turn, starting with those for fixed coupon cash bonds, followed by those for floating rate bonds. We focus mainly on the spread measures quoted by Bloomberg on its YAS, YAF and ASW pages and also those used by Lehman Brothers on Lehman Live. Additionally, we define and explain the credit default swap spread.

2. CREDIT SPREAD MEASURES FOR FIXED RATE BONDS

We start by discussing the most common credit spread measures for fixed rate bonds.

THE YIELD SPREAD

The yield spread, also known as the yield-yield spread, is probably the most widely used credit spread measure used by traders of corporate bonds. Its advantage is that it is simply the difference between two yields – that of the credit bond and that of the associated treasury benchmark and so is easy to compute and sufficiently transparent that it is often used as the basis to price in the closing of a crossing trade of credit bond versus treasury bond.

Definition

The yield spread is the difference between the yield-to-maturity of the credit risky bond and the yield-to-maturity of an on-the-run treasury benchmark bond with similar but not necessarily identical maturity.

The mathematical definition of the yield-to-maturity is well known and has been discussed at length elsewhere (Fabozzi 2003). However we repeat it here for the sake of completeness. It is the constant discounting rate which, when applied to the bond's cashflows, reprices the bond. If we denote the full (including accrued) price of the defaultable bond by P^{full}, the annualised coupon by C_D, the coupon frequency by f_D, and the time to each of the cash flow payments in years by T_1, …, T_N, then the yield y_D of the defaultable bond is the solution to the following equation.

$$P^{full} = \frac{C_D / f_D}{(1+y_D/f_D)^{f_D T_1}} + \frac{C_D / f_D}{(1+y_D/f_D)^{f_D T_2}} + ... + \frac{100 + C_D / f_D}{(1+y_D/f_D)^{f_D T_N}}$$

Note that the full coupon is used for the first period, consistent with using the full price. The times to the payments are calculated using the appropriate day count convention, such as 30/360 (bond) or ACT.

A 1-dimensional root-searching algorithm is typically used to find the value of y_D which satisfies this equation.

Before we can consider the information content of the yield spread, let us consider for a moment the assumptions behind the yield to maturity measure. These are:

1. An investor who buys this asset can only achieve a return equal to the yield measure if the bond is held to maturity and if all coupons can be reinvested at the same rate as the yield. In practice, this is not possible since changes in the credit quality of the issuer may cause yields to change through time. As many investors may re-invest coupons at rates closer to LIBOR, at least temporarily, the realised return will usually be lower than the yield to maturity.

2. It assumes that the yield curve is flat which is not generally true. In practice, we would expect different reinvestment rates for different maturities. In the yield to maturity, these reinvestment rates are the same for all maturities.

To calculate the yield spread we also need to calculate the yield of the benchmark government bond y_B as above. The yield spread is then given by the following relationship:

$$\text{Yield Spread} = y_D - y_B \,.$$

Example

As an example, consider the following bond: Ford Motor Credit 7.25% 25 Oct 2011 which priced at 107.964 on the 9th February 2004, and settles on the 12th February 2004. This bond has semi-annual coupons which accrue on 30/360 (Bond) basis. With 107 days[2] of accrued interest worth 2.1549, the full price of the bond is 110.1189. A simple price-yield calculation, summarised in Table 1 below, gives a yield of 5.94%.

The benchmark for this corporate bond at issuance was the 10-yr on-the-run treasury, with 5% coupon and maturity date 15 August 2011. As the bond has rolled down the curve, the current benchmark is the 5-yr on-the-run treasury, which has 3% coupon and matures on 15 February 2009. It has a yield to maturity of 3.037%. As the yield of the Ford bond is 5.94% and that of the benchmark is 3.04%, so the yield spread is 290bp.

In this case, therefore, there is a maturity mismatch between the bond and its benchmark where the Ford bond matures almost 3 years after the benchmark. The benchmark has also changed since the bond was issued.

Table 1. Yield to maturity calculation summary

Date	Cashflow on $100 Notional	Yield	Time in Years (30/360 Basis)	Yield Discount Factor	Cashflow PV
12-Feb-04			0.008	0.9995	
25-Apr-04	3.625	5.94%	0.211	0.9877	3.5805
25-Oct-04	3.625	5.94%	0.711	0.9593	3.4773
25-Apr-05	3.625	5.94%	1.211	0.9316	3.3771
25-Oct-05	3.625	5.94%	1.711	0.9048	3.2797
25-Apr-06	3.625	5.94%	2.211	0.8787	3.1852
25-Oct-06	3.625	5.94%	2.711	0.8533	3.0934
25-Apr-07	3.625	5.94%	3.211	0.8288	3.0042
25-Oct-07	3.625	5.94%	3.711	0.8049	2.9176
25-Apr-08	3.625	5.94%	4.211	0.7817	2.8335
25-Oct-08	3.625	5.94%	4.711	0.7591	2.7519
25-Apr-09	3.625	5.94%	5.211	0.7373	2.6725
25-Oct-09	3.625	5.94%	5.711	0.7160	2.5955
25-Apr-10	3.625	5.94%	6.211	0.6954	2.5207
25-Oct-10	3.625	5.94%	6.711	0.6753	2.4480
25-Apr-11	3.625	5.94%	7.211	0.6559	2.3775
25-Oct-11	103.625	5.94%	7.711	0.6370	66.0041
Yield-Implied Full Price of the Bond:				**110.1189**	

Interpretation

We can make a number of observations about the yield spread as a credit risk measure:

- It shares all of the weaknesses of the yield to maturity measure in terms of constant reinvestment rate and hold to maturity.

[2] *The 110 calendar days between the previous coupon date (25 Oct 2003) and settlement (12 Feb 2004) correspond to 107 interest accrual days in the 30/360 basis.*

- Another disadvantage of being based on yield to maturity is that it is not a measure of return of a long defaultable bond, short treasury position.
- As a relative value measure, it can only be used with confidence to compare different bonds with the same maturity which may have different coupons.
- The benchmark security is chosen to have a maturity close to but not usually coincident with that of the defaultable bond. This mismatch means that the measure is biased if the underlying benchmark curve is sloped.
- The benchmark security can change over time, as the bond rolls down the curve. This is illustrated in the example above, where the bond switches from being benchmarked against a 10-yr on-the-run treasury security to a 5-yr on-the-run. As a result, yield spread is not a consistent measure through time.

The bottomline is that the yield and yield-spread measures are only rough measures of return. In no way do they actually measure the realised yield of holding the asset. For these reasons, the yield spread should only be used strictly as a way to express the price of a bond relative to the benchmark, rather than a measure of credit risk.

The only time when it may become useful is if the asset and Treasury are both trading at or very close to par. In this case, the yield to maturity of the defaultable bond and treasury are close to their coupon values and the yield spread is a measure of the annualised carry from buying the defaultable bond and shorting the Treasury. However this information is already known, so even then the yield spread does not add any value.

INTERPOLATED SPREAD

To overcome the issue of the maturity mismatch, it is possible to use a benchmark yield where the correct maturity yield has been interpolated off the appropriate reference curve. Rather than choose a specific reference benchmark bond, the idea is to use a reference yield curve which can be interpolated.

Definition

The Interpolated Spread or I-spread is the difference between the yield to maturity of the bond and the *linearly interpolated yield to the same maturity* on an appropriate reference curve.

The simplest way to interpolate the yield off the treasury curve is to find two treasury bonds which straddle the maturity of the defaultable bond. It is then simple to linearly interpolate the yield to maturity of these two treasury bonds to find the yield corresponding to the maturity of the credit-risky bond. If the maturities of the two government bonds are T_{G1} and T_{G2} and the yields to maturity are y_{G1} and y_{G2}, then the interpolated spread is given by

$$ISpread = y_D - \left[y_{G1} + \left(\frac{y_{G2} - y_{G1}}{T_{G2} - T_{G1}} \right) (T_D - T_{G1}) \right].$$

Other choices of reference curves include a Constant Maturity Treasury (CMT) rates curve, or the LIBOR swap rate curve. The reference curve is always specified when quoting I-spread.

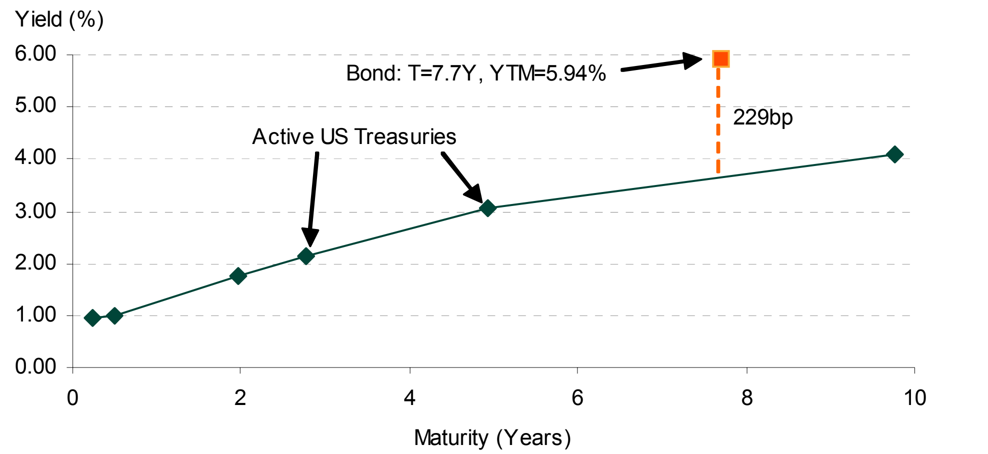
Example

Consider the following bond: Ford Motor Credit 7¼ 25 Oct 2011 which priced at 107.964 on the 9th February 2004, and settles on the 12th February 2004.

If we consider the other benchmark bonds on the US Treasury curve (shown in Figure below), we see that the two which straddle the maturity of our defaultable bond are the 3¼ Jan-09 with a yield of 3.0742% and the 4¼ Nov-13 with a yield of 4.0791%. Linearly interpolating these in maturity time gives an interpolated yield of 3.65%.

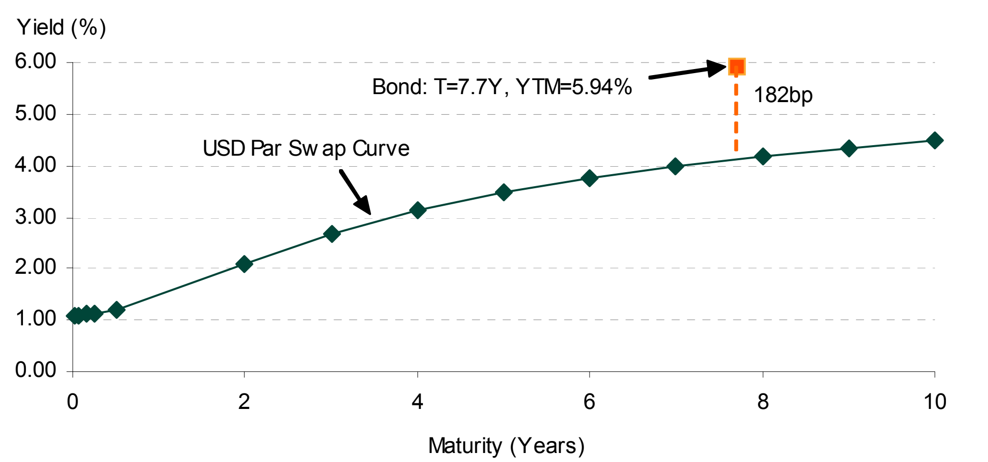
The resulting interpolated spread becomes 5.94% minus 3.65% which is 229bp. Note that this is less than the yield spread to the treasury benchmark, as the benchmark has shorter maturity than the credit bond and the treasury curve is upward-sloping.

Figure 1. I-spread against US Treasury yield curve (active)



Alternately, we may choose the LIBOR swap curve, shown in Figure 2 below, as the reference. The interpolated swap rate to this maturity is therefore a linear mixture of the 7 year swap rate which is 3.99% and the 8 year swap rate which is 4.175%. We calculate the linearly interpolated swap rate to be 4.121%. Given that defaultable bond yield is 5.94%, the interpolated spread equals the difference which is 182bp.

Figure 2. I-spread relative to US Swap Curve



Interpretation

If the reference curve is upward sloping and the benchmark has a shorter maturity then the I-spread will be less than the yield spread. If the reference curve is downward sloping and the maturity is shorter than that of the benchmark then the I-spread will be greater than the yield spread.

Viewed purely as a yield comparison, I-spread gets around the problem of mismatched maturity which affects yield spread, but it does not necessarily correspond to the yield to maturity of a traded reference bond. In addition, it inherits all the drawbacks of the yield to maturity measure, and so should be interpreted as a way to express the price of the defaultable bond relative to a curve.

I-spread does take into account the shape of the term structure of interest rates, but only in a very crude way. The option adjusted spread, which we describe next, does so in a more robust manner.

OPTION-ADJUSTED SPREAD (OAS)

The Option Adjusted Spread (OAS) was originally conceived as a measure of the amount of optionality embedded into a callable or puttable bond. However, the calculation methodology has since been borrowed by the credit markets and used for bonds which are not callable and so have no optionality. Used in this sense, the OAS becomes a convenient way to measure the credit risk embedded in a bond. For this reason, within a pure credit context, the OAS is often referred to as the zero volatility spread (ZVS) or Z-Spread. Our discussion below refers to bullet bonds only.

Definition

The Option Adjusted Spread of a bullet bond is the parallel shift to the LIBOR zero rate curve required in order that the adjusted curve reprices the bond.

If we choose discrete compounding for the OAS with a frequency equal to that of the bond f, then the OAS, denoted by Ω, satisfies the following relationship:

$$P^{full} = \frac{C}{f}\sum_{j=1}^{n}\frac{1}{\left(1+\frac{(r_{T(j)}+\Omega)}{f}\right)^{f\times T(j)}} + \frac{100}{\left(1+\frac{(r_{T(n)}+\Omega)}{f}\right)^{f\times T(n)}}.$$

where C is the annual coupon of the bond and the LIBOR zero rates are related to LIBOR discount factors Z_T as follows:

$$r_T = [(Z_T)^{-1/(f\times T)} - 1]\times f .$$

In OAS calculations, time is measured as calendar time in years, rather than being dependent on the basis of the bond, as is the case with the yield to maturity calculation. A one-dimensional root-searching algorithm is used to solve this equation.

The Lehman Brothers convention, used on LehmanLive, is to use continuous compounding for the OAS, so that the above equation becomes

$$P^{full} = \frac{C}{f}\sum_{j=1}^{n} Z_{T(j)}e^{-\Omega T(j)} + 100 Z_{T(n)}e^{-\Omega T(n)} .$$

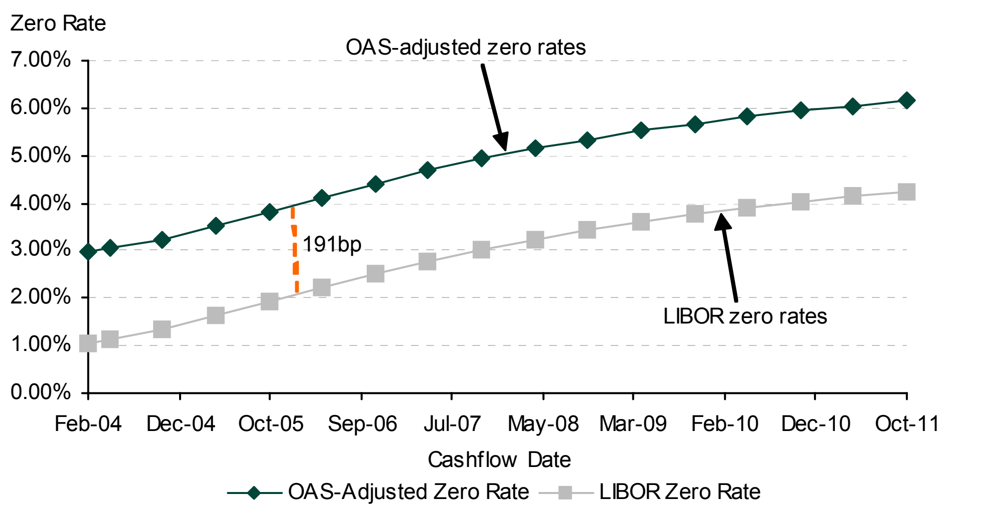
As a spread measure the OAS has a number of important differences from the yield spread. They are

1. The OAS is typically measured against LIBOR, although the reference curve is always specified and can be taken as the treasury curve as well.
2. The OAS reflects a parallel shift of the spread against LIBOR. Only the spreads are bumped rather than the whole yield. As a consequence, the OAS takes into account the shape of the term structure of LIBOR rates.
3. The OAS assumes that cashflows can be reinvested at LIBOR+OAS. As a result, future expectations about interest rates are taken into account. However there still remains reinvestment risk as it is not possible to lock in this forward rate today.

Example

Consider again the Ford Motor Credit 7.25% 25 Oct 2011 which priced at 107.964 on the 9th February 2004, and settles on the 12th February 2004. The full price of the bond, including accrued interest to the settlement date, is 110.1189. The observed US swap rates are shown in Figure 2. With semi-annual compounding, the resulting value of the OAS computed is 191bp. Figure 3 illustrates the concept of OAS as a parallel shift to the LIBOR zero rates curve.

Figure 3. Option-adjusted spread



The calculation of OAS is summarised in Table 2 below.

Table 2. Calculation of Option-Adjusted-Spread (6M-Frequency)

Date	Cashflow on $100 Notional	Calendar Time (Y)	LIBOR Discount Factor	LIBOR Zero Rate	OAS-Adjusted Zero Rate	Adjusted Discount Factor
12-Feb-04		0.008	0.9999	1.06%	2.97%	0.9998
25-Apr-04	3.625	0.208	0.9976	1.14%	3.05%	0.9937
25-Oct-04	3.625	0.708	0.9906	1.33%	3.24%	0.9775
25-Apr-05	3.625	1.207	0.9807	1.62%	3.53%	0.9587
25-Oct-05	3.625	1.707	0.9680	1.92%	3.83%	0.9374
25-Apr-06	3.625	2.206	0.9527	2.21%	4.12%	0.9140
25-Oct-06	3.625	2.707	0.9348	2.51%	4.42%	0.8885
25-Apr-07	3.625	3.206	0.9153	2.78%	4.69%	0.8620
25-Oct-07	3.625	3.707	0.8949	3.02%	4.93%	0.8349
25-Apr-08	3.625	4.206	0.8737	3.24%	5.15%	0.8076
25-Oct-08	3.625	4.707	0.8521	3.43%	5.34%	0.7804
25-Apr-09	3.625	5.206	0.8303	3.60%	5.51%	0.7534
25-Oct-09	3.625	5.707	0.8086	3.76%	5.67%	0.7269
25-Apr-10	3.625	6.206	0.7869	3.90%	5.81%	0.7009
25-Oct-10	3.625	6.707	0.7654	4.03%	5.94%	0.6755
25-Apr-11	3.625	7.206	0.7442	4.14%	6.05%	0.6508
25-Oct-11	103.625	7.707	0.7234	4.25%	6.16%	0.6267
OAS-Implied full price of the bond (OAS = 191bp):					**110.1189**	

Interpretation

The OAS is higher than the 182bp calculated for the interpolated yield spread. This difference is mainly due to the fact that the reference curve is upward sloping. The OAS is lower than the interpolated yield when the reference curve is inverted. When the reference curve is flat, both the OAS and the interpolated yield are equal, except for minor differences due to the slightly different compounding conventions. In this sense, the relationship between OAS and I-spread is similar to the relationship between zero-coupon rates and current swap rates.

The magnitude of the OAS also depends on the compounding frequency used. The relationship is analogous to discretely or continuously compounded interest rates, and is shown below in Table 3 for the Ford bond considered earlier.

Table 3. Dependence of OAS on compounding frequency

Frequency	Continuous	3M	6M	1Y
OAS (bp)	186	189	191	196

While the OAS takes into account the term structure of interest rates, it is essentially a relative value measure and should be viewed, along with yield spread and I-spread, as a way to express the price of a bond relative to a reference curve.

ASSET SWAP SPREAD (ASW)

Unlike the spread measures described so far, the asset swap spread is a traded spread rather than an artificial measure of credit spread.

Definition

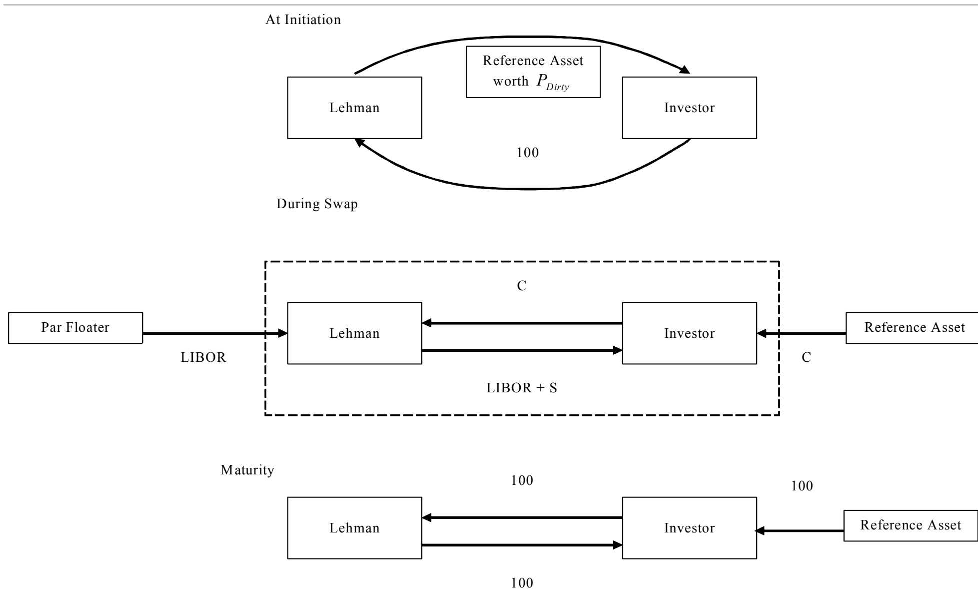
The Asset swap spread is the spread over LIBOR paid on the floating leg in a par asset swap package.

In a par asset swap package, a credit investor combines a fixed rate asset with a fixed-floating interest rate swap in order to remove the interest rate risk of the fixed rate asset. The mechanics of an asset swap spread are shown in Figure 4 below. There are two components to the package

1. At initiation the investor pays par, and in return, receives the bond which is worth its full price.
2. The investor simultaneously enters into an interest rate swap, paying fixed, where the fixed leg cashflows are identical in size and timing to the coupon schedule of the bond. On the floating side of the swap, the investor receives a fixed spread over LIBOR – the asset swap spread. The floating leg of the swap is specified with its own frequency, basis and settlement conventions.

If the asset in the asset swap package defaults, the interest rate swap continues or can be closed out at market and the associated unwind cost is taken by the asset swap buyer. The asset swap buyer also loses the remaining coupons and principal payment of the bond, recovering just some percentage of the face value.

Figure 4. Mechanics of a Par Asset Swap



More details on the mechanics of asset swaps, including pricing considerations and variations on the theme of par asset swaps, can be found in *Credit Derivatives Explained*

(March 2001), and Tuckman (2003), where we show that the par asset swap spread is given by the formula:

$$A = \frac{P^{LIBOR} - P^{full}}{PV01}$$

Equation 1

Here, P^{LIBOR} is the value of the bond's cashflows discounted at LIBOR, P^{full} is the market price of the bond, and $PV01$ is the LIBOR discounted present value of a 1bp coupon stream, paid according to the frequency, basis and stub conventions of the floating leg of the interest rate swap.

Example

To illustrate the calculation of asset swap spread, consider the Ford Motor Credit 6.75% 15 Nov 2006 which priced at 105.594 on the 12th February 2004, and settles on the 17th February 2004. The full price, including accrued interest, is 107.3193. Assume that the floating leg of the swap pays quarterly and is computed using the ACT360 basis. We calculate the asset swap spread to be 214bp. Cashflows from this par asset swap spread are shown in Table 4 below.

The first two columns of Table 4 show the cashflow schedule of the bond. LIBOR discount factors are shown in the third column. Using these, we have:

$$P^{LIBOR} = 113.0877$$

$$P^{full} = 107.3193$$

The floating leg of the swap has 3M frequency and accrues interest on ACT360 basis. These accrual factors for each period are shown in column 4. Note that the first cashflow on the floating leg is adjusted for the stub period to the first cashflow date. The PV01 of the floating leg is to:

$$PV01 = 2.7017$$

Using these figures, the Asset Swap Spread works out to 214bp.

Table 4. Cashflows in an Asset Swap Spread

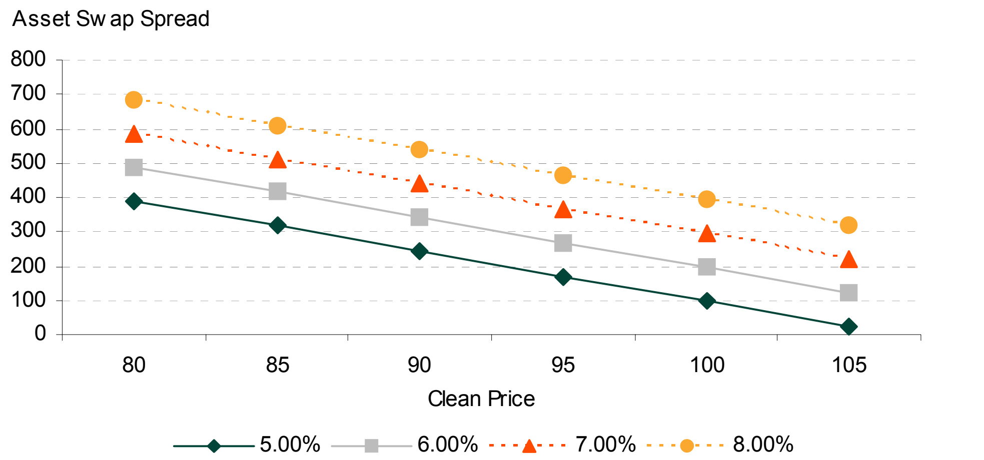
Date	Cashflow on $100 Notional	LIBOR Discount Factor	Accrual Factor (ACT360)	Forward LIBOR	ASW Fixed Leg	ASW Floating Leg
17-Feb-04		1.0000				
15-May-04	3.375	0.9971	0.244	1.19%	-3.375	0.813
15-Aug-04		0.9939	0.256	1.26%		0.869
15-Nov-04	3.375	0.9899	0.256	1.59%	-3.375	0.951
15-Feb-05		0.9852	0.256	1.85%		1.018
15-May-05	3.375	0.9800	0.247	2.15%	-3.375	1.060
15-Aug-05		0.9740	0.256	2.43%		1.166
15-Nov-05	3.375	0.9674	0.256	2.66%	-3.375	1.227
15-Feb-06		0.9602	0.256	2.93%		1.295
15-May-06	3.375	0.9524	0.247	3.32%	-3.375	1.350
15-Aug-06		0.9436	0.256	3.62%		1.471
15-Nov-06	103.375	0.9344	0.256	3.86%	-3.375	1.533

Interpretation

One thing to understand about the asset swap spread is that if the asset in the asset swap defaults immediately after initiation, the investor, who has paid 100 for the asset swap, is left with an asset which can be sold for a recovery price R, and an interest rate swap worth 100-P where P is the full price of the bond at initiation. The loss to the investor is -100-R-(100-P) = P-R, the difference between the full price of the bond and the recovery price of the defaulted bond. If the price of the asset is par, i.e. P = 100, then the loss on immediate default is 100-R, similar as we shall see to a default swap.

The point here is that if we hold the credit quality of the asset constant and increase its price, by, say, considering another bond of the same issuer with the same maturity but with a higher coupon, the loss on default is greater and the asset swap spread should increase.

Figure 5. Dependence of asset swap spread on bond coupon



What is more relevant is what happens if we keep the details of the bond constant but allow the credit quality to change. Suppose we fix the LIBOR curve and coupon and the price of the bond changes. As any change in the bond price can only be due to a change in the perceived credit quality of the issuer, as the bond price falls this can only be because the implied credit risk of the issuer is increasing and vice versa. This is clear from Equation 1. An increase in the bond price results in a fall in the asset swap spread. This supports the idea of the asset swap spread as a measure of credit quality.

Table 5 below summarises the various credit spread measures for fixed rate bonds.

Table 5. Summary of credit spread measures for fixed rate bonds

Spread Measure	Summary	Comments
Yield Spread or Yield-Yield Spread	Difference between YTM of the bond and YTM of the benchmark treasury bond.	Assumes reinvestment at same rate as the yield, and assumes the bond is held to maturity. Can be biased as maturities may not be the same and the benchmark bond changes over time.
I-Spread	Difference between YTM of the bond and corresponding rate for the same maturity on a benchmark curve (swaps or treasuries).	Reference curve rates are linearly interpolated. Gets around the maturity mismatch problem of yield spread, but suffers drawbacks from being based on the yield to maturity measure.
OAS or Z-Spread	Parallel shift to treasury or LIBOR zero rates required to reprice the bond.	Relative value measure for the bond against a reference curve. A rough measure of credit quality. Expect a difference in the computed OAS based on compounding frequency: Bloomberg uses discrete compounding, while Lehman uses continuous.
Asset Swap Spread or Gross Spread	Investor pays par and receives LIBOR+ASW instead of paying full price and receiving fixed coupons.	This is a tradable spread – not a spread "measure" – it corresponds to a real cashflow. A better measure of compensation for assuming credit risk as the cashflows are real and the interest rate exposure is residual.
CDS Spread[3]	Compensation for expected loss due to a credit event. A "real" spread.	Cleanest measure of credit risk. Similar to OAS if recovery rates are zero, but a pricing rather than a yield measure. Better than ASW since the contract terminates following a credit event (no residual interest rate swap MTM).

[3] *The CDS spread is defined in a later section but included here for completeness.*

3. CREDIT SPREAD MEASURES FOR FLOATING RATE NOTES

We now turn to Floating Rate Notes (FRNs). In contrast to fixed rate bonds, there are fewer commonly quoted measures of spread for FRNs. These are the quoted margin which determines contractual cashflows, discount margin which is similar to yield-to-maturity, and zero-discount margin which is similar to OAS. We now describe each in turn.

QUOTED MARGIN (QM)

The quoted margin is not strictly a spread measure; it is simply the spread over the LIBOR index paid by a floating rate note.

Definition

The quoted margin is the fixed, contractual spread over the floating rate index, usually LIBOR, paid by a floating rate note.

Once issued, the quoted margin of the bond is contractually fixed. In certain cases, defined within the bond prospectus, it may step up or down. It is therefore not a dynamic measure of ongoing credit quality. At most it only reflects the credit quality of the issuer *on the issue date of the bond* since this was the spread over LIBOR at which the bond could be issued at par

Example

To illustrate credit spread measures for FRN's, we consider a Ford Motor Credit bond with maturity 6 January 2006. This bond pays quarterly coupons indexed to the 3M-Euribor with accrued interest computed on an ACT360 basis. It is currently trading at a clean price of 101.09.

The quoted margin for this bond is 175bp. The floating rate is set at the start of each period, so that the coupon is known 3 months in advance. In this case, the size of the coupon due on 6 April 2004 is 3.87% (annualised), which means that the floating rate was fixed at 2.12% on 6 January 2004.

DISCOUNT MARGIN (DM)

Unlike the quoted margin, the discount margin is a dynamic spread measure which reflects the ongoing perceived credit quality of the note issuer. It is a simple measure of spread which assumes that the underlying reference curve is flat.

Definition

The discount margin is the fixed add-on to the current LIBOR rate that is required to reprice the bond.

Discount margin measures yield relative to the current LIBOR level and does not take into account the term structure of interest rates. In this sense, it is analogous to the YTM of a fixed rate bond.

The expected cashflows of an FRN are usually based on forward LIBOR rates. In the discount margin calculation, however, the assumption is that all future realised LIBOR rates will be equal to the current LIBOR rate. Cashflows are therefore projected as the current LIBOR plus quoted margin (except for the first cashflow, which is known for sure). Discount factors are also based on the current level of LIBOR, adjusted by a margin. The size of the margin is chosen to reprice the FRN, in which case it is called the discount margin.

The discount margin δ satisfies the following relationship:

$$P^{full} = \frac{L_{fix} + q}{1 + \Delta_1 (L_{stub} + \delta)} + \sum_{j=2}^{n} Z_\delta(T_j)\Delta_j (L + q) + 100 Z_\delta(T_n)$$

where

$$Z_\delta(T_j) = \frac{Z_\delta(T_{j-1})}{1 + \Delta_j (L + \delta)} ; Z_\delta(T_1) = \frac{1}{1 + \Delta_1 (L_{stub} + \delta)}$$

and we have used the following notation:

P^{full}	=	Full price of the FRN
q	=	Quoted margin on the FRN
L_{fix}	=	Known LIBOR rate for the current coupon period
L_{stub}	=	LIBOR between the valuation date and the next coupon date
L	=	Current LIBOR for the term of the FRN coupons (e.g. 3M)
$\Delta_1,\ldots,\Delta_n$	=	Coupon accrual periods in the appropriate basis (e.g. ACT360)
$T_1,\ldots,T_n$	=	Cashflow dates for the FRN

This calculation assumes that all future LIBOR cash flows are equal to the previous fixing. As a result, no account is taken of the shape of the LIBOR forward curve as in the par floater calculation.

Example

The concept of discount margin is best illustrated using an example. Consider the same bond as before, i.e. Ford €+175bp 6 Jan 2006. The previous Euribor fixing, which together with the quoted margin determines the cashflow at the next coupon date, is 2.12%. The stub Euribor to the next coupon date, used to determine the discount factor, is 2.057%.

The bond pays quarterly coupons. The current level of 3M-Euribor is 2.064%. For all cashflows beyond the next coupon date to maturity, the discount margin calculation assumes that the realised Euribor rate is equal to 2.064%. This is also used in discounting.

The calculation of discount margin is illustrated in Table 6. Since the Euribor fixing for the current period is known, we can compute accrued interest. The full price of the FRN is 101.50. The discount margin comes out to 116.3bp.

As with most spread calculations for fixed rate bonds, we typically need to use a 1-dimensional root-searching algorithm to solve for the discount margin.

Table 6. Discount margin calculation

Date	Accrual Factor (ACT360)	Projected LIBOR	LIBOR + Discount Margin	Adjusted Discount Factor	Projected Cashflows
06-Jan-04				1.0000	
06-Apr-04	0.253	2.057%	3.220%	0.9952	0.978
06-Jul-04	0.253	2.064%	3.227%	0.9871	0.964
06-Oct-04	0.256	2.064%	3.227%	0.9791	0.975
06-Jan-05	0.256	2.064%	3.227%	0.9711	0.975
06-Apr-05	0.250	2.064%	3.227%	0.9633	0.954
06-Jul-05	0.253	2.064%	3.227%	0.9555	0.964
06-Oct-05	0.256	2.064%	3.227%	0.9477	0.975
06-Jan-06	0.256	2.064%	3.227%	0.9399	100.975
Implied full price from Discount Margin (116.3bp):				**101.498**	

Interpretation

Discount margin is similar to the yield-to-maturity measure for fixed rate bonds. It expresses the price of an FRN relative to the current LIBOR level, and as such does not take into account the shape of the yield curve.

ZERO DISCOUNT MARGIN

The zero discount margin is the analogous spread measure to the zero volatility spread used for fixed rate bonds.

Definition

The Zero Discount Margin (Z-DM) is the parallel shift to the forward LIBOR curve that is required to reprice the FRN.

Forward LIBOR rates are used to project cashflows, and adjusted by the Z-DM to calculate discount rates. Z-DM therefore takes into account the term structure of interest rates. The calculation of Z-DM is similar to that for discount margin, except that forward LIBOR rates are used rather than current rates. It is determined using the equation:

$$P^{full} = \frac{L_{fix} + q}{1 + \Delta_1 (L_{stub} + \zeta)} + \sum_{j=2}^{n} Z_\zeta (T_j) \Delta_j (L(T_{j-1}, T_j) + q) + 100 Z_\zeta (T_n)$$

where

$$Z_\zeta (T_j) = \frac{Z_\zeta (T_{j-1})}{1 + \Delta_j (L(T_{j-1}, T_j) + \zeta)} ; Z_\zeta (T_1) = \frac{1}{1 + \Delta_1 (L_{stub} + \zeta)} .$$

Here, $L(T_{j-1}, T_j)$ is the forward LIBOR rate between the two cashflow dates T_{j-1} and T_j, ζ is the Z-DM, and the rest of the notation is the same as for the discount margin calculation.

Example

Table 7 below illustrates the calculation of the zero discount margin for the bond in the previous example. The calculated Z-DM is 116.2bp, which is close to the discount margin since the FRN has short maturity. In general for upward-sloping yield curves, the Z-DM is less than the discount margin.

Table 7. Calculation of Z-DM

Date	Accrual Factor (ACT360)	Projected LIBOR	LIBOR + Zero-DM	Adjusted Discount Factor	Projected Cashflows
06-Jan-04				1.0000	
06-Apr-04	0.253	2.057%	3.219%	0.9952	0.978
06-Jul-04	0.253	2.077%	3.239%	0.9871	0.967
06-Oct-04	0.256	2.107%	3.269%	0.9789	0.986
06-Jan-05	0.256	2.191%	3.353%	0.9706	1.007
06-Apr-05	0.250	2.416%	3.578%	0.9620	1.042
06-Jul-05	0.253	2.678%	3.839%	0.9528	1.119
06-Oct-05	0.256	2.829%	3.991%	0.9431	1.170
06-Jan-06	0.256	2.976%	4.137%	0.9333	101.208
Implied full price from Z-DM (116.2bp):					**101.498**

Interpretation

The Z-DM is similar to a par floater spread; in fact they are numerically equivalent when the FRN is priced at par which is certainly the case when the bond is issued. At other prices the two measures differ since, as we have seen, they use a different method for implying the value of the future coupons. Also by convention, the fixed spread over LIBOR paid by a floating rate note is also called the quoted margin. If $\zeta = q$ and we are on a coupon refix date, the price of the bond equals par.

Both DM and Z-DM are yield measures, and as such should be viewed as ways to express the price of an FRN relative to some curve. Only the quoted margin is a real cashflow measure.

4. CREDIT DEFAULT SWAP SPREAD

The CDS spread is the contractual premium paid to a protection seller in a credit default swap contract. As such it measures the compensation to an investor for taking on the risk of losing par minus the expected recovery rate of a bond if a credit event occurs before the maturity of the CDS contract. It is termed a "spread" even though it does not explicitly reference any interest rate curve. However, implicitly, the reference curve is Libor.

Definition

The CDS spread is the contractual spread which determines the cashflows paid on the premium leg of a credit default swap.

It is the spread which makes the expected present value of the protection and premium legs the same. The valuation of CDS is described fully in the references and reviewed in the next section, but we can consider a simple example to explain the basic concepts.

Example

Suppose an investor sells protection on $10mm notional to a 5-year horizon on a credit risky issuer at a default swap spread of 200bp. The investor is paid for protection in the form of fixed quarterly instalments of approximately $50,000. The payments stop if the issuer defaults prior to maturity, in which case the value of protection delivered by the seller is par minus the recovery rate. Assuming a recovery rate of 40%, the investor would lose $6mm.

It has been shown that the valuation of a CDS position requires a model. The reader is referred to O'Kane and Turnbull (2003) for a more detailed discussion.

Interpretation

The CDS spread is arguably the best measure of credit risk for several reasons. First, a CDS contract is almost a pure credit play, with low interest rate risk. Second, it corresponds to a realisable stream of cashflows, which compensates an investor for a loss of par minus the recovery rate of the issuer following a credit event. All cashflows cease and the contract is settled following a credit event. Third, an investor can trade CDS to a number of fixed terms, so we should be able to observe a term structure. Finally, the CDS market is relatively liquid, so that CDS spreads accurately reflect the market price of credit risk.

5. CONCLUSIONS

In this article, we have tried to explain the precise definition and significance of the plethora of market terms used to express the credit risk embedded in a bond. There is an important distinction between measures of yield and tradable measures of spread. The former should be viewed as convenient ways to express the price of a bond or FRN relative to some benchmark instrument (bond, rate or curve). The latter can be translated into physical cashflows. There remains the important issue of how these spreads compare with each other, particularly in regard to their relative magnitudes and sensitivities to changes in the credit quality of the underlying bond. This analysis is left to a forthcoming paper in the Quantitative Credit Research Quarterly series.

6. REFERENCES

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Introducing the Lehman Brothers Basket Rating Tool

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In recent years N^{th}-to-default baskets have proven to be a popular tool for a variety of different investors. The risk/return profile of baskets can vary considerably and may be attractive for hedging as well as for portfolio yield enhancement. Due to the significant market growth, the rating agencies have developed new models that are used to rate investments in these instruments. Lehman Brothers has developed a proprietary Basket Rating Tool which replicates the main assumptions used by the rating agencies with respect to the stochastic properties of default events and recovery rates. The Basket Rating Tool can therefore be used to estimate the expected rating of a given basket and is now available to our clients via LehmanLive.

N^{TH}-TO-DEFAULT BASKETS

In an N^{th}-to-default basket, two counterparties agree on a maturity and a set of reference assets (usually between 3 and 10), and enter into a contract whereby the protection seller periodically receives a premium (also called "basket spread") from the protection buyer. In exchange, the protection seller stands ready to pay the protection buyer par minus recovery of the N^{th} referenced defaulter in the event that the N^{th} default occurs before the agreed-upon maturity. First- and second-to-default swaps are the most popular orders of protection.

Basket exposures can be created by using either one of two basic contractual forms, namely a default swap or a credit linked note (CLN). In the swap format, there is no initial exchange of money but rather a bilateral agreement to exchange premium and protection payments. In the CLN format, the protection seller collateralizes the full basket notional. The collateral is usually invested in a note issued by the basket arranger, in a note issued by a third party, or in Libor-yielding assets. To estimate the expected loss to the basket investor, it is therefore important to consider these cases separately and, for each alternative format, model the joint default behavior of all the relevant defaultable entities.

MAIN FEATURES OF THE BASKET RATING MODEL

Lehman Brothers' Basket Rating Tool replicates the main assumptions used by the rating agencies with respect to the stochastic properties of default events and recovery rates. The main features of the underlying model are:

1. The basket rating is based on the computation of the expected loss to the investor (protection seller).
2. Losses are computed by means of a stepwise simulation of latent variables that may trigger the default of the reference names, the basket arranger, as well as the note issuer.
3. The simulation is based on a Gaussian copula parameterized by correlations estimated by country and industry.
4. Default probabilities are based on ratings.
5. Recoveries are stochastic and correlated with defaults.

The model produces the following outputs:

1. Estimated basket rating.
2. Expected loss to the investor.
3. Relative standard error of the simulation.
4. Expected recovery given that the basket is triggered.
5. Probability of triggering the basket.

Below is a screenshot of the user interface on LehmanLive. Interactive pop-up windows are provided to simplify the choice of the reference portfolio and collateral issuer.