# Quantitative Credit Research Quarterly

Volume 2005-Q4

### Credit Default Swaption Trade Analysis on LehmanLive......3

The Swaption Trade Analysis tool can be used to perform risk and scenario analysis on a credit default swaption trade. Risk measures, including delta, gamma, and vega, are calculated for each position and aggregated across all positions in the trade. The scenario analysis is illustrated graphically by showing the P/L of the trade under different credit spread scenarios. Users can load and modify an existing trade or create a new trade. A trade can be saved and easily tracked for future performance. Trades can be distributed as links in an email allowing easy communication of trade ideas between clients, research, sales and trading.



With the advent of new credit instruments such as CDS indices and liquid tranche products, credit-based active trading strategies have become more diverse and easier to implement. In this article, we explain how absolute return portfolios can be assembled from individual long/short trades and how selecting and allocating risk to individual trades depends on investment skill. We present a simple methodology to estimate the mark-to-market risk of liquid CDO tranches and discuss how portfolio risk models can be expanded to cover such products. We use the Lehman ORBS model for tactical risk allocation and also to describe the expected performance of funds investing across a variety of liquid credit positions.

### Base Correlation Deltas: An Empirical Investigation .......26

The main driver of price changes of a synthetic CDO tranche is changes in the spread of the underlying portfolio. Any model used to risk manage such tranches should be tested to see if it is consistent with the observed market price changes of the tranche. We show that for the standard index tranches the base correlation model spread sensitivities are consistent with empirical tranche price changes. We suggest this is a result of the widespread adoption of base correlation as a pricing and risk model for standard index tranches.



### **CONTACTS**

Quantitative Credit Research (Americas)		
Marco Naldi	212-526-1728	mnaldi@lehman.com
Prasun Baheti	212-526-9251	prbaheti@lehman.com
Bodhaditya Bhattacharya	212-526-7907	bbhattac@lehman.com
Ozgur Kaya	212-526-4296	okaya@lehman.com
Yadong Li	212-526-1235	yadli@lehman.com
Jin Liu	212-526-3716	jliu4@lehman.com
Roy Mashal	972-3-623-8675	rmashal@lehman.com
Claus M. Pedersen	212-526-7775	cmpeders@lehman.com
Minh Trinh	212-526-1712	mtrinh@lehman.com
Quantitative Credit Research (Europe)		
Lutz Schloegl	44-20-7102-2113	luschloe@lehman.com
Sebastian Hitier	44-20-7102-1410	sehitier@lehman.com
Clive Lewis	44-20-7102-2820	clewis@lehman.com
Matthew Livesey	44-20-7102-5942	mlivesey@lehman.com
Sam Morgan	44-20-7102-3359	sammorga@lehman.com
Quantitative Credit Research (Asia)		
Hui Ou-Yang	81-3-6440 1438	houyang@lehman.com
Wenjun Ruan	81-3-6440 1781	wruan@lehman.com
Quantitative Market Strategies		
Vasant Naik	44-20-7102-2813	vnaik@lehman.com
Srivaths Balakrishnan		
Albert Desclee		
Mukundan Devarajan	44-20-7102-9033	mudevara@lehman.com
Jeremy Rosten	44-20-7102-1020	jrosten@lehman.com
POINT Modeling		
Anthony Lazanas	212-526-3127	alazanas@lehman.com
Cevdet Aydemir	212-526-2097	aaydemir@lehman.com
Attilio Meucci		ameucci@lehman.com
Antonio Silva	212-526-8880	antonio.silva@lehman.com
Huarong Tang	212-526-3619	hutang@lehman.com
Credit Strategy		
David Heike Co-Head of Credit Strategy, Americas	212-526-5701	dheike@lehman.com
Ashish Shah Co-Head of Credit Strategy, Americas	212-526-9360	asshah@lehman.com
Alan Capper Head of Credit Strategy, Europe	44-20-710-24927	acapper@lehman.com
Takahiro Tazaki Head of Credit Strategy, Asia	81-3-5571-7188	ttazaki@lehman.com
Additional Contacts		
Dev Joneja Global Head of Quantitative Research	212-526-8035	djoneja@lehman.com
Michael Guarnieri Global Head of Credit Research	212-526-3919	mguarnie@lehman.com
Lev Dynkin Global Head of Quantitative Portfolio Strategies	212-526-6302	ldynkin@lehman.com
Dominic O'Kane Head of Quantitative Research, Europe	44-20-7102-2628	dokane@lehman.com
Hua He Head of Capital Markets Research, Asia	81-3-5571-7215	hhe@lehman.com

## Credit Default Swaption Trade Analysis on LehmanLive<sup>1</sup>

Claus M. Pedersen +1-212-526-7775 cmpeders@lehman.com The Swaption Trade Analysis tool can be used to perform risk and scenario analysis on a credit default swaption trade. Risk measures, including delta, gamma, and vega, are calculated for each position and aggregated across all positions in the trade. The scenario analysis is illustrated graphically by showing the P/L of the trade under different credit spread scenarios. Users can load and modify an existing trade or create a new trade. A trade can be saved and easily tracked for future performance. Trades can be distributed as links in an email allowing easy communication of trade ideas between clients, research, sales and trading. A click on a link will open the tool with the specific trade preloaded and available for analysis.

### 1. INTRODUCTION

The Swaption Trade Analysis tool (LehmanLive keyword: CDSOA) builds on our successful Credit Default Swaption Calculator (keyword: CDSO). That model has established itself as the market standard for valuation of investment grade default swaptions. All output in the new analysis tool is produced by same calculation engine as that of the calculator.

The calculator is part of our suite of single security tools created to price and calculate detailed risk measures for one security at a time. With this new analysis tool, you can price multiple positions (in CDX swaps and swaptions) at once and see risk numbers aggregated across all positions.

The ability to price multiple positions simultaneously is only one of three distinct uses of the new tool. The second use is scenario analysis of trades. The tool shows the 'hockey sticks' well-known from option textbooks. That is, the option payoff diagrams that show the P/L of a trade as a function of the price of the underlying security at the option expiry date (here as a function of the CDX spread). Our new tool allows this P/L scenario analysis for any horizon date between today and the option expiry date.

The third use of the tool is to track the performance of trades. This can be a user-defined trade, an official Fixed Income Research trade recommendation, or a trade idea/suggestion distributed from our sales force, trading desk or strategy team.

We hope that you will find the tool useful and we look forward to your feedback. We already have several extensions in mind and hope to provide similar trade analysis tools for other securities/derivatives.

This article consists of two parts: a User Guide and a Glossary. In the user guide we first give a brief overview of the tool and then explain each the tool's four pages in greater detail. In the glossary we explain all the input/output fields and terminology used throughout the tool. The user guide is also available in HTML format by clicking the *User Guide* link at the top of the tool. The explanations provided in the glossary can also be viewed by pointing the mouse to an input/output field. A text box will then appear with an explanation of the field.

The LehmanLive implementation was done by Paresh Patel, Chaitanya Penubarthi and Timorthy SooHoo. In addition, Peter Alpern, Marco Naldi, Ashish Shah and John Sissler contributed to the design of the tool.

### 2. USER GUIDE

The tool has four pages: *Trade*, *Scenario*, *Performance*, and *My Trades*. The keyword (CDSOA) links directly to the *Trade* page. From this page an existing trade can be loaded or a new trade created. Positions are added to a trade through the *Product Browser*, which also supplies the most recent market prices provided by our traders. After possibly overriding the market prices, a click on the *Calculate* button will calculate the cost, credit delta, credit gamma<sup>2</sup> and vega for each position and for the combined trade.

After a trade as been selected, the *Scenario* page can be used to analyze how the trade will perform under different credit spread scenarios. A horizon date and a range of possible spreads for the underlying CDX swap are entered. The tool then provides a valuation of the trade (i.e., a mark-to-market calculation) as of the horizon date for each scenario specified. From the mark-to-market (MTM), the realized cash flow (Carry) from the trade date to the horizon date, and the Cost of the trade, the tool calculates the Profit/Loss (P/L) on the trade for each scenario (P/L = Carry + MTM - Cost).

The *Performance* page serves two purposes: It can be used to track the performance (P/L) on an existing trade or it can be used for single scenario analysis by setting the valuation date to a future horizon date.

Finally, the My Trades page contains a list with links to all saved trades.

### 2.1. Trade Page

The purpose of the Trade page is to:

- select an existing trade or create a new trade;
- set the market prices at which the trade is / can be executed;
- calculate the upfront Cost of the trade;
- calculate standard risk measures such as Credit Delta, Credit Gamma, and Vega; and
- aggregate the cost and the risk measures over all positions in the trade.

When keyword CDSOA is first entered into the tool, the screen shown in Figure 1 appears.

Figure 1. The LehmanLive keyword CDSOA links to this screen



Source: LehmanLive Swaption Trade Analysis.

An existing trade can be loaded through the generic LehmanLive Credit Product Browser (which is accessed from the Search button at the very top of the tool<sup>3</sup>), by selecting the *My Trades* page and clicking on a trade to load it into the tool, or by clicking on a trade link distributed in an email. When a trade has been loaded, its name will appear in the top of the tool next to *Trade Name*.

When the Credit Delta and Credit Gamma have been normalized by the notional, they are sometimes loosely referred to as the spread duration and convexity, respectively.

If you know the name of the trade you can enter it in the box next to the Search button and press return (or use the pull down button). Otherwise, you press the Search button which brings up the browser from where you choose Portfolio Trades to get a list of all available trades.

Changes to a trade are saved when (and only when) *Calculate* is pressed. To make changes to an existing trade without permanently overriding it, the trade must be copied (to a different name) before the changes are made.

If an existing trade is not loaded, the *Trade* page will have an empty trade with the name *New Trade*. Any changes will be saved under that name and will be available next time the tool is entered. However, it is recommended to more permanently save a trade by clicking the *Save* button (see figure 1 above) and assigning a name in the pop up box. Once a trade as been saved (with a different name than *New Trade*), it will appear on the list of trades and can be loaded as described above.

Positions may be added to the trade by clicking the *Add Securities button*. This brings up a browser with positions/securities divided into two types: CDS Indices and Swaptions. CDS Indices include the standard CDX swaps whereas swaptions are the standard CDX swaptions currently quoted by our trader (Figure 2 shows a screenshot of the browser). Positions/securities are selected in the left side in the tool and added by clicking on *Add Securities*. The positions will be added to the trade with the most recent (mid) market prices.

Notice that when the product browser is initially opened, all CDX swaps and swaptions are listed. Typing "cdx.ig" in the search field in the very top of the browser will limit the list to only CDX.IG swaps and swaptions. To limit to series 5, say, "cdx.ig 5" can be used.

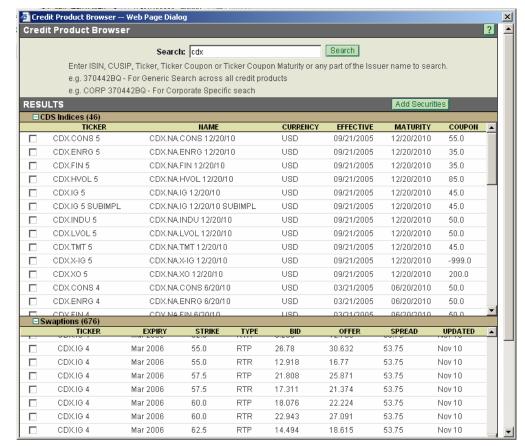


Figure 2. Browser used to add positions/securities to the trade

Source: LehmanLive Swaption Trade Analysis.

Figure 3 shows a screenshot of the Trade page where a straddle on CDX.IG has been selected. Most of the input fields are self explanatory and the Glossary below provides details. In addition, an explanation will appear when the pointer is moved to a specific text field in the tool.

A swap position with positive notional (*Amt*) is a long credit (selling protection) position whereas a negative notional is short credit (buying protection) position. A swaption position with a positive notional means that the option has been bought (buying gamma) and an upfront cost must be paid. A swaption is sold by choosing a negative notional.

An input field that may require a little more explanation is the Bid/Offer field for portfolio swap positions. The credit spread curve entered above the individual positions is supposed to be the mid quotes. The mid quotes are used to value all swaption positions. To value swaps an adjustment determined by the Bid/Offer field is made to the curve. For example, if the mid spread is 47.5bp and the Bid/Offer is 0.5bp, then portfolio swaps for which Amt (notional) is positive (these are short protection positions) will be valued using spreads of 47.50 - 0.5/2 = 47.25; that is, using a curve that has been (parallel) shifted down by half the Bid/Offer input.

Another functionality is the calculator icons to the left of each position. A click on the icon for a swaption will bring up a new window with the Credit Default Swaption Calculator and detailed valuation output. Similarly, a click on the calculator button next to a portfolio swap will bring up a window with the output from the Credit Default Swap Calculator.



Figure 3. The Trade page

Source: LehmanLive Swaption Trade Analysis.

Currently the tool is restricted to handle only trades with swaptions on the same underlying CDX swap. For example, we are currently unable to handle swaptions on both CDX.IG and CDX.HVOL. We should be able to handle such trades in a later version.

### 2.2. Scenario Page

The Scenario page calculates the profit/loss (P/L) of the trade on a user-specified Horizon Date under a number of different scenarios for the spread on the underlying portfolio swap. The realized cash flow (Carry) and the original Cost are also shown. The P/L in a particular

scenario is calculated as P/L = MTM + Carry - Cost, where MTM is the mark-to-market in that scenario. The P/L for each position and for the combined trade can be illustrated either with a *Graph View* or a *Data View*. In the *Graph View* there are various interactive features<sup>4</sup>.

Search Duser Guide Swaption Trade Analysis test1 Trade Name: test1 Performance Back Scenario Calculate Scenario Analysis 12/20/2005 11/09/2005 E Libor Source: NY Close ₹ Horizon Date: Source Date: 23.0 96.0 Spread Range: Increment: Portfolio Swaps Maturity Amt (MM) **▽** 1 - CDX.IG 5 Portfolio Swaptions Swap Mat Expiry Cost Amt (MM) Туре Price (bp) 114564.64 1 - CDX.IG 5 12/20/2010 03/20/2006 22.91 37 52.50 RTP 50.0 **▽** ■ 94328.28 2 - CDX IG 5 12/20/2010 03/20/2006 52.50 RTR 18.87 37 50.0 Trade Summary 199328.24 0 Scenario Payoff Diagram Graph View: Data View: 9001 800k 7001 500k 4001 2001 1001 40 50 90 Spread at Horizon 1 - CDX.IG 5 12/20/2010 1 - CDX.IG 5 03/20/2006 RTP 52.5 2 - CDX.IG 5 03/20/2006 RTR 52.5 Total P/L 👸 🚣 Scenario successfully completed (26.597): Thu Hov 10 14:29:49 EST 2005

Figure 4. The Scenario page

Source: LehmanLive Swaption Trade Analysis.

If the trade has swaptions that expire after the Horizon Date, an implied volatility must be provided for each swaption. Internally, the swaption is valued (and the MTM calculated) by sending a request to the LehmanLive Credit Default Swaption Calculator with the provided volatility and a flat spread curve with spreads equal to the assumed spread for the scenario for which the valuation is being done. The valuation date in the swaption calculator is set equal to the Horizon Date<sup>5</sup>.

For example, you can zoom by holding down the right mouse button while you draw a box. Unzoom by clicking the left mouse button.

The recovery rate is kept unchanged at the value provided on the Trade page. The Libor curve is set equal to the curve as of the date specified by the user in the Source Date field (the default is last business day).

### 2.3. Performance Page

The *Performance* page serves two purposes:

- to track the performance of an existing (past) trade; and
- to perform detailed single-scenario analysis.

This page looks very like the *Trade* page and also functions in a similar way. On the *Trade* page we calculate the cost and risk measures of all positions as of the *Trade Date*. On the *Performance* page we do the same calculations as of a *Valuation Date* that can be any date between the *Trade Date* and the first swaption expiry date. The main difference between the two pages is the calculation of *Carry* and *P/L* provided on the Performance page.

The mark-to-market (MTM) is the value of the position as of the valuation date. The change (Chg) shown to the right of (MTM) is the difference between the Cost calculated on the Trade page and the MTM (Chg = MTM - Cost). The Carry is the cash flow realized from the Trade Date to the Valuation Date (excluding the Cost). Finally, P/L = MTM + Carry - Cost.



Figure 5. The Performance page shows the P/L when unwinding a trade

Source: LehmanLive Swaption Trade Analysis.

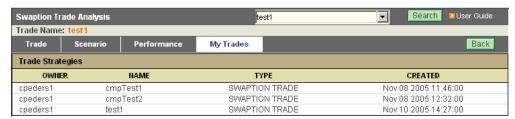
The *Bid/Offer* field explained above for the *Trade* page is also present in the *Performance* page to take into account the difference between bid and offer spreads when unwinding the trade. For example, if the mid spread is 47.5bp and the *Bid/Offer* is 0.5bp then all portfolio swap trades with positive notional (Amt) will be unwound at a spread of 47.75bp. This is because a positive notional indicates a long credit position which is unwound by buying protection at the offer spread.

Currently, bid/offer spreads on the portfolio swaption positions are taken into account only through the volatility (*Vol*). Notice that this cannot capture a bid/offer cost of unwinding the portfolio swap resulting from exercise of the swaption on the expiration date (*Expiry*). This additional functionality will be added in a later version.

### 2.1. My Trades Page

The *My Trades* page is shown in Figure 6. The purpose of this page is to provide easy access to all saved trades available for analysis.

Figure 6. The My Trades page



Source: LehmanLive Swaption Trade Analysis.

Lehman Brothers researchers, salespeople and traders can send a trade as a link in an email. When a user (e.g., a client) clicks on the link, the trade will open with "LEH:" in front of the original name given to the trade by the Lehman employee. At this point, the trade as been copied and the user can modify and analyze the trade in detail. The trade will appear on the user's *My Trades* page until the user deletes it. It is important to remember that any changes made to the trade after it has been opened are saved automatically when *Calculate* is pressed. To keep an original version, it is necessary to copy the trade to a different name before making any changes.

### 3. GLOSSARY

The explanations provided below are also available by using the mouse to point to a field. A text box will then appear with an explanation of the field.

### Amt (MM) - Swap

The notional of the position in millions. A swap position with positive notional (*Amt*) is a long credit (selling protection) position whereas a negative notional is short credit (buying protection) position.

### Amt (MM) - Swaption

The notional of the position in millions. A swaption position with a positive notional (*Amt*) means that the option has been bought (buying gamma) and an upfront cost must be paid. A swaption is sold by choosing a negative notional.

### C.Delta

The credit delta is the change in *Cost* or *MTM* of the swap or swaption caused by a 1bp (marginal) increase in the *Credit Spreads* for all maturities. When the credit delta is normalized by the notional (*Amt*) is sometimes loosely referred to as the spread duration.

### C.Gamma

The credit gamma is the change in credit delta (*C.Delta*) caused by a 1bp (marginal) increase in the *Credit Spreads* for all maturities.

### Carry

The sum of all cash payments made from the *Trade Date* to the *Valuation Date*, not including the initial *Cost*.

### Cost - Swap

The upfront cost of entering into the swap. The cost is calculated from the *Credit Spreads* and the *Coupon* using the standard CDS valuation methodology.

### **Cost - Swaption**

The (upfront) cost of the swaption.

### Coupon

Premium rate, in basis points, that determines the scheduled payments to be made by the protection buyer. For example, if the swap notional is \$10 million and the Coupon is 100bp, the premium will be approximately \$25,000 every quarter.

### **Credit Spreads (bp)**

A term structure of credit default swap (CDS) spreads in basis points. The market convention is to use a flat (constant) spread curve.

The maturity dates for the Credit Spreads are the standard CDS maturity dates (sometimes referred to as the IMM dates). For example, if the valuation date is Nov 8, 2005, the date for the 5-year point will be Dec 20, 2010.

If a flat curve is desired, only a single spread need be entered. If some spreads are unspecified, the calculator will populate the empty fields with the values that have been used to generate the results.

### **Expiry**

The date the option can be exercised.

### **Horizon Date**

The valuation date for scenario analysis. The calculations assume that the trade is unwound at this date at the volatilities specified and a scenario dependent spread. See *Spread Range* below for how the scenario dependent spreads are determined.

### Increment

Used together with *Spread Range* to determine the spread scenarios. If the *Spread Range* is 40 - 60 and the Increment is 5, the spread scenarios will be 40, 45, 50, 55, 60. A small increment leads to a many scenarios which can lead to a long computation time. Additional scenarios with spread equal to the *Strike* (for all swaptions) are always inserted.

### **LIBOR Source**

Determines together with *Source Date* the LIBOR/swap rates used. Currently only NY Close is available. The rates will be as of close of business in New York on the date specified in *Source Date*.

#### **MTM**

The mark-to-market (i.e., the market value) of the position(s) as of the specified date, assuming that the spread and volatility on that date turn out to be as specified.

### **Portfolio**

The portfolio of reference credits for the swap or swaption.

### Price (bp)

The price of the swaption in basis points. If the notional (see *Amt*) is 10 million and the *Price* is 50bp, then the *Cost* is 50,000.

### Recovery

The market recovery rate used for both swaps and swaptions. The market convention is 40%.

### **Maturity**

The maturity date of the portfolio swap.

### **Source Date**

See under LIBOR Source. If the date is left unspecified, the most recent business day is used.

### **Spread Range**

Used together with *Increment* to determine the spread scenarios. In *Spread Range* the minimum and the maximum spreads for the scenarios are set.

### **Strike**

If the option is exercised, the underlying portfolio swap will be traded at a spread equal to the specified *Strike*.

### **Swap Mat**

The maturity date of the underlying portfolio swap.

### **Trade Date**

The calculations are based on the assumption that all positions in the trade were entered into on this date at the prices/spreads provided.

### **Trade Name**

The name under which the trade is saved. Changes to the trade are saved when (and only when) a page is calculated, ie. when the *Calculate* button is pressed.

### **Type**

RTP = Right to Pay (or payer swaption); RTR = Right to Receive (or receiver swaption). A payer swaption gives the right to buy protection at the *Strike*. Conversely, a receiver swaption gives the right to sell protection.

### **Valuation Date**

The calculations assume that the trade is unwound at this date at the spreads and volatilities specified.

### Vega

Change in the *Cost* or *MTM* of the option caused by a 1 percentage point (marginal) increase in the implied volatility (e.g., from 50% volatility to 51% volatility).

### Vol

The volatility of the spread on the underlying portfolio swap.

### **REFERENCES**

Liu, J. and C. Pedersen (2005). "Pricing High Yield CDX Swaptions". *Quantitative Credit Research Quarterly*, 2005-Q3.

Pedersen, C. (2003). "Valuation of Portfolio Credit Default Swaptions". *Quantitative Credit Research Quarterly*, 2003-Q4.

Pedersen, C. (2004). "Introduction to Credit Default Swaptions". *Quantitative Credit Research Quarterly*, 2004-Q3/Q4.

Pedersen, C. and S. Chen (2005). "Introducing the LehmanLive Credit Default Swaption Calculator". *Quantitative Credit Research Quarterly*, 2005-Q2.

### Managing Risk Allocation to Liquid Tranches and Credit Portfolio Products<sup>1</sup>

Albert Desclée Quantitative Portfolio Strategies +44 (0) 20 7102 2474 adesclee@lehman.com

Mukundan Devarajan Quantitative Strategies +44 (0) 20 7102 9033 mudevara@lehman.com

> Jay Hyman Quantitative Portfolio Strategies +972 3 623 8745 jhyman@lehman.com

Minh Trinh Quantitative Credit Research +1 (212) 526 1712 mtrinh@lehman.com With the advent of new credit instruments such as CDS indices and liquid tranche products, credit-based active trading strategies have become more diverse and easier to implement. In this article, we explain how absolute return portfolios can be assembled from individual long/short trades and how selecting and allocating risk to individual trades depends on investment skill. We present a simple methodology to estimate the mark-to-market risk of liquid CDO tranches and discuss how portfolio risk models can be expanded to cover such products. We use the Lehman ORBS model for tactical risk allocation and also to describe the expected performance of funds investing across a variety of liquid credit positions.

### 1. INTRODUCTION

Active portfolio management relies on an efficient allocation of risk across a variety of outperformance strategies. At any time, a trading strategy can be described as a bet on which an investor is prepared to take risk in exchange for an expected performance, or alpha. Investors are faced with the challenge of optimizing the allocation between various trades. How much market risk should a portfolio exhibit given a particular return target? Should the contribution of different strategies to the portfolio volatility be equal? How does risk allocation translate into position amounts? How can we determine whether CDO-based strategies provide useful diversification opportunities? In this article, we analyse the possible answers to some of these questions.

We present a case study of an absolute-return credit portfolio for which several simple strategies are considered. Absolute-return portfolio management has several benefits. Being free from benchmark considerations, it can focus on positions supported by active views and it can use leverage to take long/short positions. In the past, the nature of credit instruments available was a constraint on exploiting such opportunities. More recently, however, absolute-return investment strategies have become easier to implement in the credit markets with the advent of such instruments as CDS Indices. With the availability of liquid CDO tranches, investors can now express their views using long-short trades designed to create exposure to particular risk factors, such as changes in default correlation.

Although this article focuses on long-short credit portfolios, it is worth noting that the analysis presented here can also be relevant for long-only credit portfolio managers. Long-only portfolios can be seen as a combination of alpha-generating strategies implemented under a "no short" constraint (and possibly others) and referenced to a benchmark index.

For clarity, it should be noted that this article deals with several distinct types of correlations. To allocate risk among various alpha-generation strategies, our risk budgeting framework considers the correlations of returns among these strategies. One particular type of strategy that is facilitated by the use of liquid CDO tranches is a view on the default correlation among the set of issuers underlying the contract (which may be measured in two different ways, as we will discuss in Section 3). When allocating risk top-down, we need to consider the correlation between such a "correlation strategy" and any other strategy in use.

This article is organized as follows. In **Section 2**, we explain with an example how absolute-return portfolios can be assembled from individual long/short trades. We show how the selection and allocation of risk to individual trades depends on directional views and investor skill. We also emphasize why understanding the risk contribution of individual trades

<sup>&</sup>lt;sup>1</sup> The authors would like to thank Bodhaditya Bhattacharya for his contribution to this article.

requires a market risk model. We use the Lehman ORBS model for tactical risk allocation and to describe expected fund performance. In **Section 3**, we continue our case study of an absolute return credit portfolio and include strategies that involve liquid CDO tranches. We examine how risk models can be expanded to cover the near term mark-to-market risk of these instruments and discuss the issues involved in building a parsimonious risk model for CDO tranches. We note that, although we refer to the Lehman Brothers Global Risk Model available in POINT, the analysis presented in this article is largely unrelated to this model and different paths may be followed when CDO products are added to the universe of securities covered by POINT.We present our conclusions in **Section 4**.

### 2. OPTIMAL RISK ALLOCATION IN ACTIVELY MANAGED PORTFOLIOS

### 2.1. Formulating active views

Actively managed portfolios aim to exploit investor views by optimally allocating risk, typically tracking error volatility with respect to a benchmark, across multiple bets. In an absolute return portfolio, the benchmark is a cash deposit and any active positions should be supported by market views, which can be formulated in a number of ways. In this article, we consider the case where investors express directional bets on individual trading relationships, without making explicit forecast as to the magnitude of potential market movement. For example, one can bet that European credit spreads will widen or that US spreads will widen relative to European ones without making a prediction on the magnitude of that widening. Such qualitative forecasts can cover a variety of risk dimensions within and outside the credit market. For practical reasons, bets are often defined with respect to trading instruments, like iTraxx, CDX, or the CDO tranches related to these CDS portfolios.

Lehman's ORBS model (Optimal Risk Budgeting with Skill) helps optimise risk allocation across individual positions. ORBS leverages on earlier work in the equity market that relates information ratio to investment skill and the breadth of the relevant trading strategies.

The ORBS model determines the optimal risk allocation to various active views using correlations between the payoffs of individual bets. These correlations can be estimated using a multi factor risk model. In addition, a risk model is necessary to incorporate investment constraints related, for example, to maximum risk exposures or leverage limits. It also helps translate the risk budget associated to active bets into position allocations to individual securities. Importantly, the risk model must cover all asset classes and instrument types covered by the alpha-generating active strategies to ensure a full representation of portfolio risk.

The Lehman multi-factor risk model available in POINT<sup>2</sup> covers traditional credit assets, including cash bonds and CDS. CDO tranches are excluded given insufficient historical market data. However, the recent advent of standardised tranches referenced to iTraxx and CDX baskets, coupled with the pricing transparency of these new instruments, allows for expansion of a risk management framework to cover strategies based on liquid credit products. We can now attempt to broaden our risk-budgeting framework, starting with a model of market risk, if only on an experimental basis, notwithstanding any subsequent modelling decisions that may be made in the flagship risk model supported by POINT.

### 2.2. Using the Lehman ORBS model

The ORBS model helps relate skill or confidence in directional views, the risk associated with individual trades and the interaction (correlation) between the various active views and

22 November 2005 14

\_

<sup>&</sup>lt;sup>2</sup> Dynkin, Joneja et al. "The Lehman Brothers Global Risk Model: A Portfolio Manager's Guide", 2005

trades. The result is an allocation between the different strategies that is optimal with respect to their risk and expected returns.<sup>3</sup>

ORBS is based on investment skill and builds on published conceptual work by Grinold and Kahn<sup>4</sup> as well as on our empirical studies of skill in fixed income portfolios<sup>5</sup>. The information ratio of an investment strategy is defined as the ratio of portfolio outperformance over excess return volatility, or active risk.

$$IR = \alpha / \sigma$$

Grinold and Kahn have shown that the information ratio of a strategy is essentially determined by two things: skill and breadth. Skill can be measured as the "information coefficient", the correlation between investment forecasts and realised market movements, while breadth refers to the number of independent decisions that the strategy implements in any particular year. The information ratio achieved by a strategy should follow the law:

$$IR \approx Skill \cdot \sqrt{Breadth}$$

Putting these together yields the following formulation of portfolio alpha:

$$\alpha = ActiveRisk \cdot IR \approx \sigma \cdot Skill \cdot \sqrt{Breadth}$$

That is, the active return of a portfolio is function of active risk, investment skill and the diversity of bets that it implements. A key implication of this is that a risk-budgeting framework can help make allocation decisions based on directional views without requiring portfolio managers to specify precise basis points forecast of market movements. Indeed, in the presence of skill, risk eventually translates into outperformance.

The above formula for alpha assumes that there is only one active strategy implemented in the portfolio, or that all active bets benefit from a uniform investment skill and all generate identical contributions to tracking error. In practice, the investment process includes different strategies supported by different sets of skills. The portfolio outperformance is the sum of contributions of individual outperformance strategies and reflects the information ratio of each active strategy as well as the risk budget allocated to it.

$$\alpha_P = \sum_i IR_i \cdot \sigma_i$$

The portfolio active risk combines the tracking errors generated by each individual strategy and is a function of the correlation structure between the payoffs of individual strategies. We derive the correlation between strategies from their exposures to individual detailed risk factors and the correlation between these factors.

$$\sigma_p^2 = T' \cdot \Omega \cdot T$$

Where T is the vector of isolated volatilities allocated to the strategies and  $\Omega$  is the correlation matrix. We can optimize the mix of active strategies in much the same way one would optimise a static asset portfolio. However, in contrast to traditional mean-variance optimization, the optimal allocation is not defined in terms of market value weights to individual assets. Rather, we find the optimal allocation of active risk to a set of individual alpha-generating strategies. Then, this allocation is translated into position amounts. So,

For further details on the ORBS model, with analysis of the interaction between security selection and macro decisions, see Desclée, Dynkin, Hyman, Konstantinovsky "Effect of Security Selection Skill on Optimal Sector Allocation", 2004 and Desclée, Dynkin, Hyman "Risk Budget Allocation to Security Selection Strategies", 2004
 Grinold, R., and R. Kahn, 1999, Active Portfolio Management, McGraw Hill

<sup>&</sup>lt;sup>5</sup> Dynkin et al. "Value of Skill in Macro Strategies for Global Fixed Income Investing", 2003

instead of making detailed forecasts on the magnitude changes in market variables, investor input is limited to identifying which trades are expected to perform.

### 2.3. A sample portfolio

Let us illustrate the working of the ORBS model with a simple sample portfolio. We assume that a credit manager has three active views:

- That iTraxx spreads will widen
- That in the European market, Financials spreads will tighten versus Cyclical spreads
- US CDX spreads will widen relative to the European iTraxx product.

While the first of these views is a bet on iTraxx's absolute performance, the second and third implement long-short positions in sector iTraxx baskets and iTraxx vs CDX, respectively.

How much risk should we allocate to each of these three bets? What are the optimal position amounts on each security? What is the resulting risk, in terms of portfolio return volatility, knowing that our return target is 60bp per annum? To answer these questions, we must make an assumption on investment skill. In this example, let us assume that our portfolio manager is equally talented at timing each one of these three trades and that his skill is a modest 10%. We also assume that the breadth of each one of these strategies is 12 decisions per year.

The first step in our portfolio analysis is to calculate the expected information ratio of our active bets: the product of skill and the square root of breadth is 0.35 for each one. We then estimate the correlations between bet payoffs. It may be neither possible nor desirable to estimate them from time series of CDX or iTraxx return series. Indeed, the time-series available to us for this purpose are short. Moreover, the underlying basket's composition has changed, as have the duration and spread levels of these instruments. Therefore, we would advocate mapping each position on a set of risk factors for which we have long time series and for which we can therefore estimate correlations with some robustness. Exposure to individual risk factors and factor covariances can then define the correlation between individual bet payoffs. Once we have these data, as shown in Figure 1.a., we can perform risk optimisation to determine risk allocation to each bet. We note that some correlations between individual bets can be significant as, in our sample, going long financial against cyclical appears to be directional on the bearish market view represented by our short iTraxx bet.

Our risk model can then convert the number of basis points of P&L volatility associated with individual bets into position amounts, reflecting current spread volatility levels for the relevant underlying assets. These amounts, shown in Fig. 1.b, correspond to a total portfolio market value of €100m. We put no constraints on leverage and so can have potentially large long or short positions on individual assets. A typical portfolio invested along these lines would have cash investments of €100m combined with the long-short derivative positions shown in Fig. 1.b.

The portfolio information ratio is then estimated at 46%, clearly superior to the information ratio of any component strategy evaluated on a stand-alone basis, while a large proportion of the expected return comes from the cross-market strategy (Short CDX – Long iTraxx), which seems to benefit from low correlations with the other two active views.

Figure 1.a. Example long-short portfolio – bet definitions and correlations

			Estimated correlations between strategy payoffs			
	Skill	Information Ratio	Short iTraxx	Long Financial vs. Cyclical	Long iTraxx vs. CDX	
Short iTraxx Europe 5 Yr	10%	0.35	1	0.76	0.13	
Long Financial vs. Short Cyclical	10%	0.35	0.76	1	0.14	
Long iTraxx Europe 5 Yr vs. Short CDX.NA.IG 5Yr	10%	0.35	0.13	0.14	1	

Source: Lehman Brothers.

Figure 1.b. Optimal risk allocation and position amounts for a 60bp/yr expected return

			Long position amount	Short position amount		
	Optimal Risk allocation (in bp/yr)	Alpha contribution (bp/yr)	Assuming a Portfolio Size of € 100M			
Short iTraxx Europe 5 Yr	57	20	-	2.4M (Short iTraxx)		
Long Financial vs. Short Cyclical	37	13	27.8M (Long Financial)	27.8M (Short Cyclical)		
Long iTraxx Europe 5 Yr vs. Short CDX.NA.IG 5Yr	79	27	-	73.5M (Short CDX)		
Portfolio	130	60				

Source: Lehman Brothers.

### 3. INCLUDING CDO-BASED STRATEGIES

In this section, we add an additional strategy to our set of active bets: a delta-hedged long equity/short mezzanine position in the iTraxx Europe 5 year. The same questions arise: how much risk should we allocate to each bet, including this new one, and what are the optimal position amounts? To include CDO tranches in a mark-to-market portfolio risk framework, we need to describe the risk associated with changes in default correlation since it is a new risk factor that was not needed for linear products such as single name CDS, CDX or iTraxx.

### 3.1. Return decomposition of CDO tranches

CDO tranches involve an important source of risk: changes in the default correlation that underlies their pricing. This effect is similar to the effect on option prices of changes in implied volatility. In the following, we propose attributing the mark-to-market volatility of CDO tranches with a simple linear decomposition. To empirically estimate the factors of this model however, we require data on the mark-to-market values of CDO tranches. Using the market prices for iTraxx (Europe) and CDX tranches that have been available for the last year we provide preliminary factor estimations for relevant drivers of CDO returns.

Using a standard Taylor expansion and ignoring the impact of defaults, the near term mark-to-market P&L of a CDO tranche investment can be approximated with the following relationship:

$$P \& L \approx Theta + Delta \cdot \Delta S + \frac{1}{2}.Gamma.(\Delta S)^2 + Corr01 \cdot \Delta Corr + \varepsilon$$

In the above approximation, *Theta* measures the P&L explained by the passage of time alone. While Delta measures the sensitivity to changes in the spread of the underlying portfolio, Gamma measures the sensitivity of the delta to changes of the underlying portfolio spread. Corr01 (or correlation01) measures the sensitivity to changes in default correlation<sup>6</sup>. The Corr01 can be computed using models such as the LHP (Large Homogeneous Portfolio) model.

<sup>&</sup>lt;sup>6</sup> See Schloegl and Greenberg, 2003 O'Kane and Livesey, 2004, for a justification of these risk measures.

### 3.2. Attribution of equity mark-to-market volatility

Using the P&L decomposition above, we can attribute the historical mark-to-market volatility for an investment in any CDO tranche. In the following example we use weekly data for a 14-month period (September 2004-November 2005) to attribute the mark-to-market volatility of a €10m Investment in an iTraxx 0-3% Tranche.

Figure 2 shows that relative to a €10m position, the weekly P&L volatility is 3.6%. Also, the two largest contributors to volatility are the spread (Delta) and default correlation risks. Gamma and Theta play a very limited role since we observe returns on a weekly horizon, too short for changes in these factors to significantly affect return volatility. The diversification between returns associated with Delta and default correlation also lowers the overall volatility as these two sources of returns are imperfectly correlated with each other.

€'000s/week 500 19 126 56 400 361 285 300 200 100 5 n Theta Delta Gamma Correlation Residual Diversification Tr MTM

Figure 2. Attribution of P&L volatility for a €10m position in the 0-3% tranche of iTraxx 5yr

Source: Lehman Brothers.

### 3.3. Mark-to-market risk arising from changes in default correlation

### 3.3.1. Measures of default correlation

As we also seek to include the correlation risk of tranches that are senior to the equity tranche in our risk framework, we have a choice between two types of default correlations, compound or base correlation<sup>7</sup> as the underlying risk factor. For an equity tranche, base correlation and compound correlation will be the same. Needless to say, sensitivity measures must be adapted to this choice of factors.

### Compound vs. base correlation

One standard method of quoting tranches from CDX or iTraxx is that of "compound" correlation. The LHP pricing formulae are inverted to back up implied compound correlation numbers from the spreads of tranches. Each tranche is then assigned a compound correlation. In other words, the LHP model is used as one would use the Black-Scholes formula for options. If we use changes in compound correlation, we define the P&L attributed to default correlation risk as the change of implied correlation of the tranche considered times the Correlation01 to this metric.

The other type of default correlation that has been proposed is "base" correlation, which uses only the pricing of equity (or base) tranches. Each base tranche has a unique correlation that

<sup>&</sup>lt;sup>7</sup> See O'Kane, D. and M. Livesey (2004)

0 -0.5

-1.5

prices it under the LHP model. With this feature, the base correlation avoids a problem often found in analysing mezzanine tranches: that multiple values of the compound correlation can be found to price a CDO. In this framework, the value of a tranche with an attachment point  $\theta_1$  and a detachment point  $\theta_2$  is the difference between the values of two adjacent base tranches with detachment points of  $\theta_1$  and  $\theta_2$  respectively. In other words, the value of the tranche is affected by two base correlations.

When we use base correlation as a metric, each tranche is affected by the changes in base correlations corresponding to its attachment and detachment points. For example the 3-6% tranche is affected by changes in both the 0-3% and 0-6% base correlations. This means that for any tranche senior to the equity, we attribute P&L to the changes in two correlation variables, as opposed to just one if compound correlation is used.

### 3.3.2. Default correlation risk in standardised iTraxx CDO products

In Figure 3, we plot the evolution of compound correlation and the correlation sensitivity for the 3-6% iTraxx tranches calculated with the LHP model. After March 05, we see a drop in the correlation and correlation01. It reached an extreme in May 2005, when GM spreads widened sharply and the "long equity-short mezzanine" trade underperformed substantially. Sensitivity of the 3-6% tranche to its compound correlation looks very unstable around May 2005.

12% 10%

Figure 3. Evolution of compound correlation and sensitivity to compound correlation for the 3-6% iTraxx tranche

-2 6% -2.5 4% -3 -3.5 2% -4 0% -4.5 Do 105 105 Junds Julos Julos 781. 481. 481. 481. 48 os Johns

Source: Lehman Brothers

8%

The alternative to compound correlation is the base correlation. Figure 4 shows the evolution of sensitivities to base correlations. In this case, the mezzanine 3-6% tranche is exposed to both 0-3% and 0-6% base correlation. The tranche exhibits sensitivities of opposite signs to these correlations. As Fig.4 shows, correlation sensitivities to base correlation appear more stable than those to compound correlation. We therefore choose to use base correlation factors to attribute returns in our subsequent analysis.

3-6% Compound Correl01 (RHS)

22 November 2005 19

3-6% Compound Correlation (LHS) —

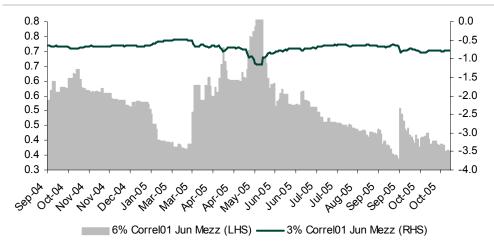


Figure 4. Evolution of sensitivity to base correlation for the 3-6% iTraxx tranche

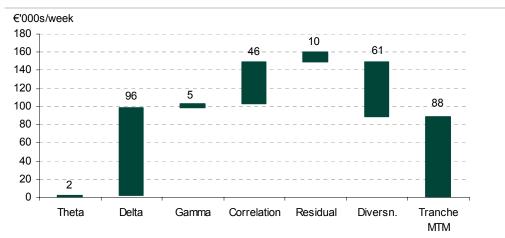
Source: Lehman Brothers.

### 3.3.3. Attribution of mark-to-market volatility of mezzanine tranche

In section 3.2, we attributed P&L volatility for the equity tranche of the iTraxx. We now extend this same analysis to mezzanine tranches and also study delta-hedged long-short positions where a long equity position is offset by a short position in a mezzanine tranche or in the underlying index (in this case, the iTraxx Europe 5 yr). It is worth mentioning that our analysis of weekly P&L covers only a short period, including the volatile month of May 2005, and may therefore not be very suitable for a forward-looking risk analysis of CDO-based trading strategies. Still, it provides useful insight and helps illustrate the workings of our portfolio management framework. Better forward-looking analysis will be possible as more market data become available over time.

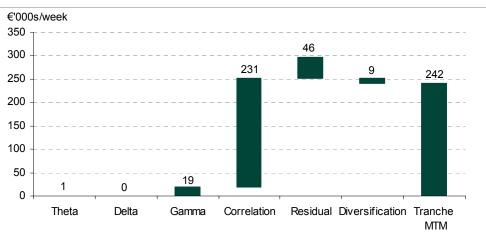
In Figures 5-7 we show the attribution of P&L volatility of an outright long position in the iTraxx Europe 5 year mezzanine tranche, and that of a long position in the equity tranche delta hedged (at a weekly interval) using the mezzanine tranche. A reassuring feature of our empirical volatility attribution is that the residual terms are relatively small. This shows that the simple linear model that we have postulated for return attribution is effective enough to describe the near-term volatility of CDO tranches and may be seen as an appropriate extension of traditional factor-based credit risk models to handle the risk contribution of exposures to default correlation risk. This unexplained residual risk should, however, be accounted for by a tranche-specific idiosyncratic risk. Our risk-budgeting model can be expanded to reflect risk contribution that is not associated with any incremental return.

Figure 5. Attribution of P&L volatility for a €10m position in the 3-6% tranche of iTraxx 5yr



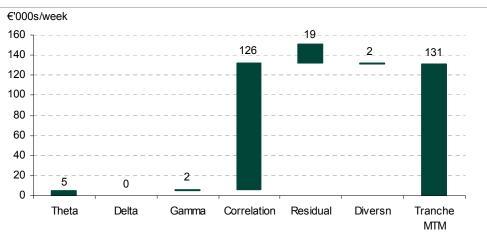
Source: Lehman Brothers.

Figure 6. Attribution of P&L volatility for a delta-hedged long equity-short mezzanine position in iTraxx 5yr for a €10m notional



Source: Lehman Brothers.

Figure 7. Attribution of P&L volatility for a delta-hedged long equity in iTraxx 5yr for a €10m notional



Source: Lehman Brothers.

Expanding our risk model to including default correlation as a set of explicit risk factors raises two additional questions:

- How many factors do we need to properly represent the risk of changes in default correlation in portfolios that invest in various tranches of the capital structure? Clearly, it is not desirable to have as many factors as there are traded tranches in the market. Parsimony is therefore a desirable feature of risk models.
- Are changes in default correlation correlated with other credit risk factors already covered by our risk model? We must answer this question to assess the directionality of correlation-based trades on other active bets included in actively managed portfolios.

### 3.4. A mark-to-market risk model with default correlation risk

### 3.4.1. How many risk factors do we need to represent default correlation risk?

The above analysis shows that each additional tranche included in the portfolio would add at least one extra correlation risk factor to our risk modelling framework. A portfolio with five tranches would seem therefore to require six correlation factors.

To see whether these correlation risk factors can be explained parsimoniously, we perform a Principal Components Analysis on weekly changes in base correlation. Figure 8 shows that the first two components explain 98% of the variation in the base correlation factors.

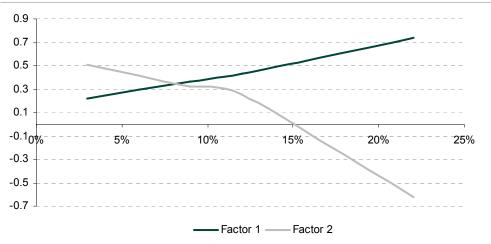
Figure 8. Variance explained by principal components

No. of PCs	% of Variance			
1	92%			
2	98%			
3	100%			
4	100%			
5	100%			

Source: Lehman Brothers.

To interpret the above principal components, we could look at the loadings of these components on the underlying factors. Figure 9 below shows that the first two factors could be interpreted as the level and slope of the base correlation skew.

Figure 9. First and second principal components (level and slope)



Source: Lehman Brothers.

Reducing changes in the base correlation skew to parallel shift only explains 78% of these changes, while a two factor (Level and Slope) model explains 95% of the variance of the entire skew. Clearly, correlation risk can be described with some parsimony at least as we analyse various tranches of the same underlying portfolio. It is, however, beyond the scope of this article to investigate the commonality of correlation risk across a diversity of underlying portfolios (like iTraxx and CDX) or even across products of different tenors.

### 3.4.2. Directionality of default correlation risk

Understanding whether and by how much the default correlation factors may be directional on other market parameters is essential to measure the interaction between individual bets involving CDO products (or not).

We investigate whether changes in base correlation are directional on spread changes in the broad iTraxx Europe index or its individual sub-sectors and use these data to expand our risk model framework, populating the cross terms between spread and default correlation factors. In Figure 10, we provide the results of this analysis performed on weekly data. We note that including May 2005 in our analysis causes all correlations to be significantly negative as spread widened at the same time as default correlation decreased. Excluding a few weeks of the month of May yields very different results. Correlation and spread risk appear much less correlated: the correlation of the changes in the spread of the underlying index with that of the implied correlation of the equity tranche is for instance 0% compared with -51% for the full sample. In a low volatility regime, correlation risk factors may therefore be less sensitive to changes of market risk.

Figure 10. Correlation between weekly changes in base correlation and in market spread levels

	Factor	3%	6%	9%	12%	22%
Market	5 year	-51%	-36%	-31%	-28%	-22%
	10y - 5y	-28%	-25%	-21%	-19%	-19%
	HiVol - Index	-45%	-34%	-29%	-26%	-18%
	TMT	-39%	-22%	-19%	-16%	-12%
Sector	Industrials	-46%	-30%	-23%	-19%	-11%
	Autos	-54%	-44%	-36%	-32%	-22%
	Sub - Sr. Fin	-40%	-29%	-27%	-24%	-14%
	Senior	-47%	-30%	-26%	-22%	-16%
	Sub	-46%	-31%	-28%	-25%	-15%

Source: Lehman Brothers.

### 3.5. Including CDOs in a portfolio risk framework

As we expanded the framework of our risk model to cover liquid CDO tranches from the already broad coverage provided by our multi-factor risk model (as implemented in the Lehman Brothers POINT application), we now have a rich set of risk factors that can accommodate a diversity of active bets. In Figure 11, we display the main categories of factors that we retain in our pro-forma model and that support the sample portfolios presented in this article. We describe the risk of any active positions on portfolio tools like iTraxx or CDX, or tranches thereof, as sets of exposures to individual risk factors in all categories of Figure 11. Since these factors are correlated with each other, factor mapping also defines correlations between individual bets as well as the P&L volatility associated with each bet in isolation. Equipped with the ability to describe individual position as well as

portfolio mark-to-market risk across new dimensions, we can now continue our portfolio example, with some additional CDO-based strategies.

Figure 11. Risk factors in pro-forma risk model:

Category	Market	Factor
Systematic risk factors	Cash and CDS	Change in risk aversion (high vs low spread premium)
		Change in spread curve slope
		Systematic change in spread across a sector / quality cell
		Senior vs. subordinated debt
		US vs. Non-US issuers
	CDS	Changes CDS/Cash basis
	Liquid CDO	Changes in CDO base correlation (Level and Slope)
Non-systematic risk	All securities	Security/issuer-specific return

Source: Lehman Brothers.

Let us now continue the portfolio optimisation example presented in section 2.3 with an additional active view, for instance, that the equity CDO tranche of 5-yr iTraxx will outperform the junior mezzanine tranche on a delta hedged basis.

As before, we assume that our fund manager is equally skilled across all active strategies: overall direction of iTraxx spreads, cyclical vs. financial, US CDX vs. European iTraxx and delta-neutral equity vs. mezzanine tranche of iTraxx CDO. The portfolio objective is to minimise P&L volatility with a target outperformance of 60 bp/yr.

Figure 12.a. reviews the four bets considered while Figure 12.b. displays the optimal risk allocation as well as the corresponding position amounts.

Figure 12.a. Example Long-Short Portfolio – Bet definitions and correlations

			Estimated correlations between strategy payoffs				
	Skill	Information Ratio	Short iTraxx	Long Financial vs. Cyclical	Long iTraxx vs. CDX	Delta-hedged long equity short junior mezzanine of iTraxx 5yr CDO	
Short iTraxx Europe 5 Yr	10%	0.35	1	0.76	0.13	0.06	
Long Financial vs. Short Cyclical	10%	0.35	0.76	1	0.14	0.02	
Long iTraxx Europe 5 Yr vs. Short CDX.NA.IG 5Yr	10%	0.35	0.13	0.14	1	0.13	
Delta-hedged long equity short junior mezzanine of iTraxx 5yr CDO	10%	0.35	0.06	0.02	0.13	1	

Source: Lehman Brothers.

Figure 12.b. Optimal risk allocation and position amounts for a 60bp/yr expected return

	Optimal Risk	Alpha	Long position amount	Short position amount	
	allocation (inbp/yr)	contribution — (bp/yr)	Assuming a Portfolio Size of € 100M		
Short iTraxx Europe 5 Yr	43	15	3.5M (Long iTraxx)	-	
Long Financial vs. Short Cyclical	31	10	23.7M (Long Financial)	23.7M (Short Cyclical)	
Long iTraxx Europe 5 Yr vs. Short CDX.NA.IG 5Yr	65	23	-	60.7M (Short CDX)	
Delta-hedged long equity short junior mezzanine of iTraxx 5yr CDO	33	12	3.0 (Long Equity of iTraxx 5yr CDO)	11.2M (Short 3-6% tranche of iTraxx CDO)	
Portfolio	117	60			

Source: Lehman Brothers.

For an unchanged return target, the portfolio risk has now been reduced to 117bp, which warrants an expected information ratio of 51%, larger than the 46% obtained when our portfolio could use only the first three bets. The CDO strategy, being only slightly correlated with the other views in our portfolio, gets a significant risk allocation and, as a result, would be expected to deliver 12bp of the 60bp/yr of expected return. Interestingly, the addition of an extra strategy to our portfolio allows for a lower net leverage, which we define in this case as the sum of all long and short positions (unadjusted for the deltas of the CDO positions). This is clearly a benefit from a more balanced allocation of risk across various positions, which warrants less aggressive position-taking to achieve our return target.

### 4. CONCLUSION

We have discussed how risk budgeting based on skill helps allocate risk optimally to individual tactical bets in liquid credit portfolio products. The ORBS risk budgeting model that we have used here takes as input a set of favoured strategies and a measure of investment skill at timing each one of them. The advantage is that these strategies can be expressed as directional as well as long-short bets on individual securities. So, instead of having to make precise numerical forecasts on a diversity of market parameters, the fund manager can focus on the monitoring and timing of a set of trading relationships, with calls expressed qualitatively and scaled according to investment skill.

We have shown that a multi-factor model of market risk is essential to estimate the correlations between strategy payoffs and to translate optimal risk allocation into actual position amounts. As we consider strategies that invest in liquid CDO products, we must ensure that these can be described in an augmented risk model, expanded to cover the risk of changes in default correlation. With the availability of standardised tranches, correlation factors can be estimated and included in our risk budgeting framework to account for a diversity of strategies involving long or short positions in CDO products. Our analysis should be taken with caution given the short history of market data. However, it helps set the order of magnitude for the risk and reward to be expected from trading liquid CDO tranches.

### **REFERENCES**

Grinold, R., and R. Kahn, 1999, Active Portfolio Management, McGraw Hill.

Schloegl, L. and A. Greenberg, 2003, Understanding Deltas of Synthetic CDO Tranches, *Quantitative Credit Research Quarterly*.

O'Kane, D. and M. Livesey, 2004, Base Correlation Explained, *Quantitative Credit Research Quarterly*.

Dynkin, L., Joneja, D. et al., 2005, *The Lehman Brothers Global Risk Model: A Portfolio Manager's Guide*.

Dynkin, L. et al., 2003, Value of Skill in Macro Strategies for Global Fixed Income Investing.

Desclée, A., Dynkin, L., Hyman, J. and V. Konstantinovsky, 2004, "Effect of Security Selection Skill on Optimal Sector Allocation", *Global Relative Value*, August 9, 2004.

Desclée, A., Dynkin, L. and J. Hyman, 2004, "Risk Budget Allocation to Security Selection Strategies", *Global Relative Value*, September 13, 2004.

### Base Correlation Deltas: An Empirical Investigation

Sam Morgan +44 20 7102 3359 samuel.morgan@lehman.com

> Dominic O'Kane +44 20 7102 2628 dokane@lehman.com

The main driver of price changes of a synthetic CDO tranche is changes in the spread of the underlying portfolio. Any model used to risk manage such tranches should be tested to see if it is consistent with the observed market price changes of the tranche. We show that for the standard index tranches the base correlation model spread sensitivities are consistent with empirical tranche price changes. We suggest this is a result of the widespread adoption of base correlation as a pricing and risk model for standard index tranches. <sup>1</sup>

### 1. INTRODUCTION

A standard index tranche is a synthetic CDO tranche in which the underlying reference portfolio is one of the standard CDS indices, eg, CDX or iTraxx. The primary driver of the price variation of a standard index tranche is changes in the spread of the credits in the underlying portfolio, as characterised by the spread of the corresponding index swap. A better understanding of the dynamics of this price variation is therefore of key importance to:

- 1. risk managers attempting to better understand and explain the price variation of a portfolio containing standard index tranches;
- 2. hedge funds engaged in gamma trading of standard index tranches; and
- 3. dealers attempting to hedge the spread risk of their correlation books which include standard index CDO tranches. This may also be relevant to other types of synthetic tranche on these correlation books as their pricing model may be calibrated to the pricing of standard index tranches.

Any attempt to hedge the risk of standard index tranches requires the use of some modelling approach. Within the standard index market, there has been a widespread adoption of the base correlation model as the method for quoting, pricing and risk-managing the standard index tranches. For a description of base correlation, see O'Kane and Livesey (2004). This model has also become the basis for calculating the delta-amount of the underlying index which is usually transacted with the standard index tranche in order to produce a spread-neutral package.

The aim of the analysis presented here is to use the base correlation model to calculate the sensitivity, or delta, of the standard index tranche price to changes in the average spread of the underlying portfolio index, and to see if this is consistent with the price changes seen in empirical data.

### 2. BASE CORRELATION IN THE LHP MODEL

Base correlation has become the market standard language for quoting the prices of standard index tranches in terms of correlation. For many market participants, it has also become the basis for pricing and risk-managing standard index tranches and bespoke tranches. A comprehensive description and comparative analysis of the base correlation approach can be found in O'Kane and Livesey (2004). We provide a brief description here.

We would like to thank Sebastien Hitier, Matthew Livesey, Marco Naldi and Lutz Schloegl for discussions and comments.

A correlation measure is required for quoting the value of synthetic CDO tranches because it can be shown that the risk of a tranche is driven to a large degree by the default correlation of the assets in the reference portfolio. If we use a Gaussian copula model (see O'Kane, Naldi et al. (2003) for a description) to capture this effect, we find that the correlation implied by the observed market prices of standard index tranches at various levels of the capital structure is not the same for all tranches on a given portfolio index.

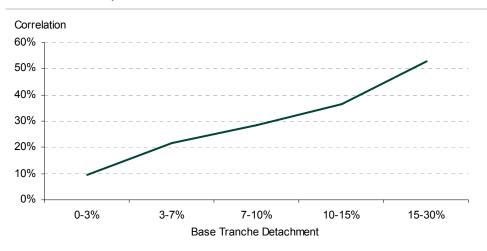


Figure 1. The base correlation skew for the CDX Series 4 5Y Index on the 1<sup>st</sup> September 2005

Source: Lehman Brothers

The base correlation idea is that any mezzanine tranche can be decomposed into a long and short position in two "base" tranches of different widths or "detachments". A base tranche is simply an equity tranche, i.e., one with zero subordination. A different implied correlation is then assigned to each base tranche. If we plot the base correlation implied by market tranche prices as a function of the detachment of the base tranche, we observe a skew. See Figure 1 for the 5 year maturity tranches of the CDX IG4 index. The base correlation skew is analogous to the volatility skew observed in option markets.

There are different ways to implement the base correlation approach. The analysis used here has been performed using the base correlation model in its Large Homogeneous Portfolio (LHP) implementation. The LHP model approximates the portfolio underlying the CDO tranche by treating it as a large collection of identical assets all with the same spread and recovery rate. This model has the advantage of being semi –analytical and so it is computationally fast. It is also simple to use as it requires a small set of input data. We justify its use on the grounds that for most investment grade portfolios, it is a good approximation to more detailed approaches which model the spread and recovery rate of individual assets in the portfolio. It is also a version of the model that is used quite widely.

The aim of the following analysis is to use historical data to determine whether the price sensitivity of a standard index tranche to changes in the spread of the underlying index is consistent with that implied by the base correlation model. Note that we are focusing on systemic price moves, i.e., those driven by a common widening in the credit spread of all of the names in the reference portfolio, rather than idiosyncratic effects.

### 3. METHODOLOGY

For the empirical analysis, we used a time series of daily tranche spreads for the standard tranches of both the CDX and iTraxx indices. We used data from both the iTraxx Series 3

and the CDX IG Series 4 indices for 5y and 10y maturity. Our time series of daily spreads and upfronts ran from 21 March 2005 until 20 September 2005. Over this period, these were the most actively traded series of these indices, i.e., they were the "on-the-run" indices.

Figure 2. Running spreads and upfronts for the liquid tranches and the index swap for 5-year maturity trades on the CDX IG Series 4 index

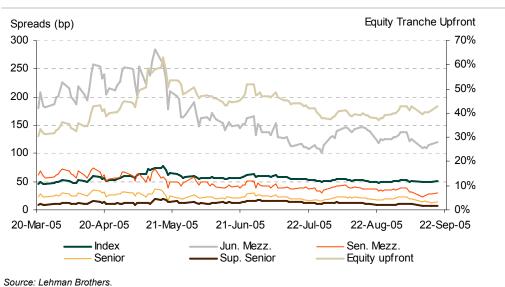
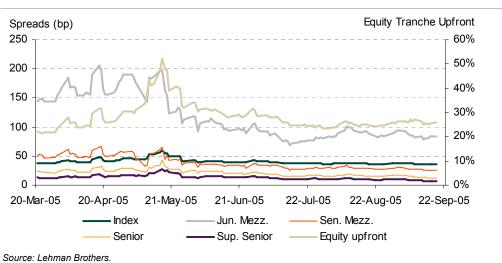


Figure 3. Running spreads and upfronts for the liquid tranches and the index swap for 5-year maturity trades on the iTraxx Series 3 index



The time series of spreads are shown in Figures 2 and 3 for 5-year maturity trades. Note that these data cover the May 2005 period in which the market experienced an increase in equity upfronts and tightening in junior mezzanine spreads following the downgrading of GM and Ford. Although neither GM nor Ford appears in the iTraxx portfolio, these effects can clearly be seen in both the iTraxx and CDX time series, indicating that the consequences of their downgrading were felt across the CDO market.

We summarise the methodology used to obtain tranche deltas as follows:

On day t,

- Enter into a tranche position at zero cost (or paying the upfront for an equity tranche) at the market spread (or upfront price).
- Enter into a position in the underlying portfolio index swap at the market spread.
- Calculate the base correlation curve implied by the market.
- Calculate the delta implied by the base correlation model.

On the next business day t+1,

- Calculate the base correlation curve implied by the market.
- Calculate the MTM of the tranche position using the new base correlation curve.
- Calculate the MTM of the index swap at the new index swap spread.
- If the index MTM is non-zero, divide the tranche MTM by the index MTM to get the implied empirical delta.

Repeat for each date until we reach the end of the time series.

This procedure was repeated for both the CDX and iTraxx tranches at both the 5- and 10-year maturity. We describe the technical details in the following sections.

### 3.1. Calculating the base correlation deltas

The base correlation delta was calculated by bumping the spread of the underlying portfolio index by 1bp and then calculating the change in PV using a base correlation LHP model. The base correlation curve was represented by the two correlations associated with the two strikes of a tranche, and these were held fixed during the spread bump.

The tranche PV change was divided by the change in the PV of the index obtained from a 1bp bump in the index spread. This gave the base correlation delta, defined as

$$\Delta_{Tr}^{BC}(t, \mathbf{S}_{I}) = \frac{PV_{Tr}^{BC}(t, \mathbf{S}_{I} + 1) - PV_{Tr}^{BC}(t, \mathbf{S}_{I})}{PV_{I}^{BC}(t, \mathbf{S}_{I} + 1) - PV_{I}^{BC}(t, \mathbf{S}_{I})}$$

To apply the base correlation model within an LHP model, we took the average spread of the reference portfolio to be the same as that of the portfolio index swap and fixed the recovery rate at 40%.

### 3.2. Calculating the empirical tranche deltas

Mathematically, we define the systemic delta of a tranche as the notional amount of the reference portfolio index swap required to immunise the mark-to-market of the tranche to small changes in the spread of the index. In practice, we assume a small parallel shift in the spread curve of 1 basis point, so we define the delta as:

$$\Delta_{Tr}(t, \mathbf{S}_I) = \frac{PV_{Tr}(t, \mathbf{S}_I + 1) - PV_{Tr}(t, \mathbf{S}_I)}{PV_I(t, \mathbf{S}_I + 1) - PV_I(t, \mathbf{S}_I)},$$

where PV<sub>I</sub> denotes the PV of the index. At time  $t_0$  we can obtain a delta-hedged position by simultaneously entering a tranche trade and an offsetting position in the underlying portfolio with a notional  $\Delta_{Tr}(t_0, \mathbf{S}_I)$ . The delta is chosen so that the PV of the delta-hedged position does not change if the index spread moves by a small amount:

$$PV_{\Delta}(t_0,t) = PV_{Tr}(t_0,t) - \Delta_{Tr}(t_0,\mathbf{S}_I)PV_I(t_0,t) = 0.$$

To estimate the *empirical* delta from historical values of the tranche and index spreads, we assume that the ratio of the MTM changes each day are primarily the result of spread changes. More formally, we can then write:

$$PV_{Tr}(t+1,S_I(t+1)) \approx PV_{Tr}(t,S_I(t)) + \frac{\partial PV_{Tr}(t,S_I(t))}{\partial S_I(t)}(\Delta S_I),$$

$$PV_{I}(t+1,S_{I}(t+1)) \approx PV_{I}(t,S_{I}(t)) + \frac{\partial PV_{I}(t,S_{I}(t))}{\partial S_{I}(t)}(\Delta S_{I}).$$

We can therefore approximate the historical tranche deltas as follows:

$$\Delta_{Hist}(t, S_I) \approx \frac{PV_{Tr}(t+1, S_I(t+1)) - PV_{Tr}(t, S_I)}{PV_I(t+1, S_I(t+1)) - PV_I(t, S_I)}.$$

Provided the PV change of the index is non-zero, this can be calculated by taking the daily change in the MTM of a tranche divided by the change in the value of a unit of the underlying reference index, both assumed to have been entered into yesterday.

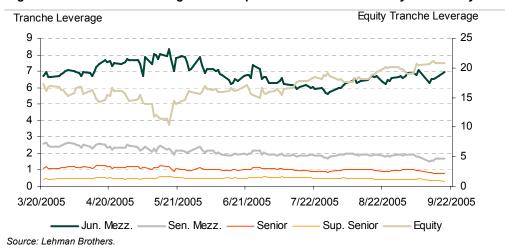
The empirical delta defined above includes the effects of all drivers of tranche value. In particular, it includes the effects of market perceptions of correlation as well as all liquidity issues and other market technicals. As such, an accurate *a priori* prediction of this quantity is practically impossible; our analysis is designed to examine how well the much simpler and analytically tractable base correlation delta, obtained by consideration of spread changes only, is able to capture the full behaviour of the historical evolution.

### 4. RESULTS

### 4.1. Tranche leverage time series

We start our analysis of the relation between the changes in value of the tranches and the index by presenting the time series for the tranche deltas predicted by the base correlation LHP model. These are shown in Figures 4 and 5 for all the liquid tranches of both the CDX IG4 and iTraxx S3 indices for five year maturity trades.

Figure 4. The model leverage for the liquid tranches of CDX IG4 5-year maturity



Figures 4 and 5 clearly show the effect of the GM and Ford downgrade in mid-May, with a drop in the equity tranche leverage and an increase in the junior mezzanine tranche leverage. The effect on the more senior tranches was rather small. Since May, the trend in both indices has been for the equity leverage to increase and the junior mezzanine leverage to decrease, along with a small decrease in the leverage of the senior mezzanine tranche.

Equity Tranche Leverage Tranche Leverage 10 30 25 8 20 6 15 10 2 5 0 n 3/20/2005 4/20/2005 5/21/2005 6/21/2005 7/22/2005 8/22/2005 9/22/2005 Sen. Mezz. -Sup. Senior Jun. Mezz. Senior

Figure 5. The model leverage for the liquid tranches of iTraxx S3 5-year maturity

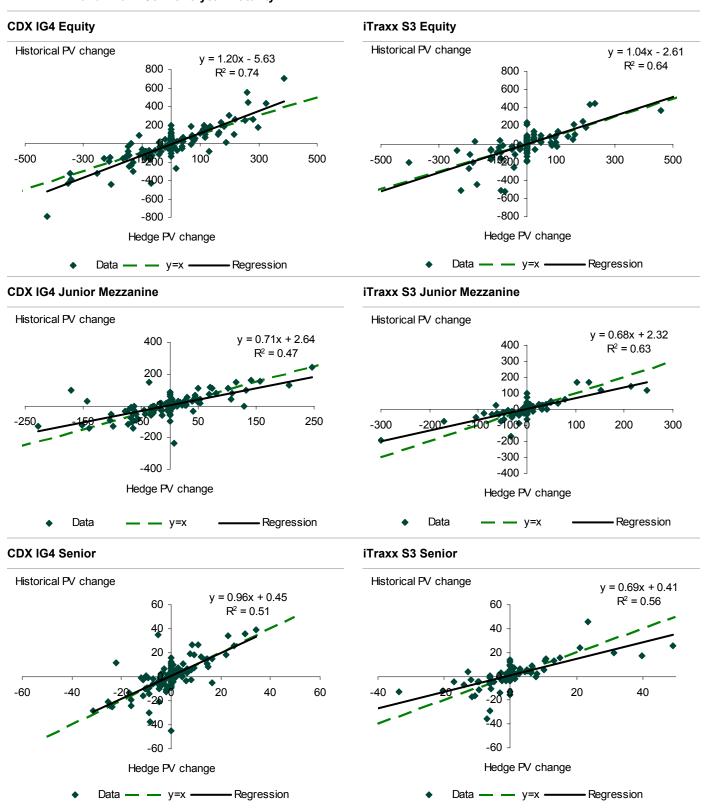
Source: Lehman Brothers.

### 4.2. Regression analysis

We can compare the base correlation model deltas shown above with those observed in the market using a regression analysis. A selection of the results are shown in Figure 6, where we plot the historical PV changes for 5-year maturity trades in equity, junior mezzanine and senior tranches of CDX IG4 and iTraxx S3 against the historical PV changes of the base-correlation-implied hedge.

If spread changes and the sensitivity implied by the base correlation model perfectly explained the tranche PV moves, each regression would have an R<sup>2</sup> and slope of 1. As can be seen from the figure, there is a significant scatter in the points but a straight line fit is clearly appropriate and the gradient in all cases is reasonably close to the estimate of 1. We have also included the line with unit slope, which looks a reasonable fit to the data in most cases.

Figure 6. Regression of historical PV changes against hedge PV changes for some of the liquid tranches of CDX IG4 and iTraxx S3 with 5-year maturity



Note: The hedge PV is the observed PV change of the index multiplied by the model leverage. Also shown are the equation of the best fit linear regression and the line with unit slope. The units are thousands of USDs or EURs on 10m notional trades.

Source: Lehman Brothers.

We obtained similar results for the senior mezzanine and super senior tranches and for tranches with a maturity of 10 years.

If we assume that the tranche deltas are approximately constant over our time-series, we can compare the time average of the historical delta against the model predictions. We obtain the time average for the empirical tranche deltas by regressing the tranche PV changes against the index PV changes (note that this is a different regression from those illustrated in Figure 6. The x-axis is now the PV change of a single unit of the index and is not leveraged by the model implied delta). For the long-time model average we took a simple arithmetic mean of the time-series shown in Figures 4 and 5.

The results of this analysis are summarised in Figure 7, where we compare the historical leverage with the time average model leverage for all liquid tranches, including trades with a 10-year maturity. The standard errors shown come from the regression fits for the historical deltas and the standard deviation of the time-series divided by the square root of the number of data points for the model deltas. These results generally show good agreement between the average observed delta and that predicted by the model, within the uncertainty implied by the standard errors in most cases. We conclude that a simple leveraging of the index provides a reasonable leading order explanation of average tranche PV movements.

Figure 7. Historical and base correlation model time-averaged leverage for the liquid tranches of CDX IG4 and iTraxx S3 for 21 March-20 September 2005

	Average D	Standard Errors		
Tranche	Historical Mod		Historical	Model
CDX IG4 (5Yr)				
Equity	18.62	16.75	0.88	0.19
Junior Mezzanine	5.25	6.82	0.49	0.05
Senior Mezzanine	1.94	2.09	0.18	0.02
Senior	1.06	1.05	0.09	0.01
Super Senior	0.39	0.49	0.04	0.01
CDX IG4 (10Yr)				
Equity	5.40	5.73	0.36	0.06
Junior Mezzanine	10.35	9.25	0.77	0.07
Senior Mezzanine	2.76	4.66	0.51	0.04
Senior	1.55	2.26	0.21	0.02
Super Senior	0.99	0.95	0.07	0.01
ITRAXX S3 (5Yr)				
Equity	18.74	19.92	1.21	0.20
Junior Mezzanine	4.60	6.10	0.33	0.07
Senior Mezzanine	1.80	2.17	0.16	0.02
Senior	0.90	1.20	0.07	0.01
Super Senior	0.56	0.67	0.04	0.01
ITRAXX S3 (10Yr)				
Equity	8.85	8.06	0.42	0.09
Junior Mezzanine	9.94	9.50	0.69	0.08
Senior Mezzanine	3.91	4.26	0.38	0.04
Senior	2.33	2.28	0.16	0.02
Super Senior	1.11	1.27	0.09	0.01

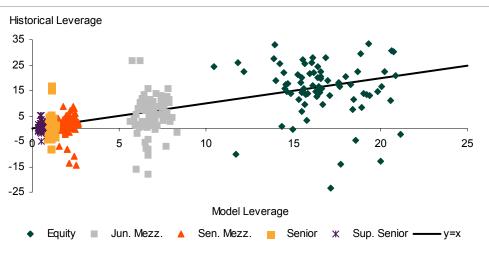
Source: Lehman Brothers.

### 4.3. Historical and model daily deltas

The above regression analysis assumes that the tranche deltas are approximately constant in time, but we can also compare the historical and model deltas on a daily basis. The results are shown in Figures 8 and 9, where we plot the full historical leverage observed in the market for the five liquid tranches of the CDX and iTraxx indices against the leverage expected from the LHP model.

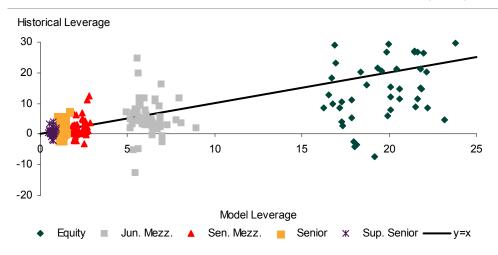
The historical leverage on the y-axis is calculated as the total MTM change between two neighbouring days divided by the total MTM change in the underlying index for the same period. We exclude from the analysis pairs of days for which there was no change in the index MTM. The model leverage on the x-axis is the MTM of the tranche expected for a parallel shift in the index spread curve equal to the observed change in the 5-year spread, with all other variables held constant.

Figure 8. Historical leverage (ratio of tranche to index MTM) vs model leverage for the liquid tranches of the CDX IG4 index with a maturity of 5 years



Source: Lehman Brothers.

Figure 9. Historical leverage (ratio of tranche to index MTM) vs model leverage for the liquid tranches of the iTraxx Series 3 index with a maturity of 5 years



Source: Lehman Brothers.

Clustering of points around the line y = x in Figures 8 and 9 corresponds to the situation where changes in value of the tranche are determined by the base correlation model, assuming that any MTM changes are solely the result of changes in the spread of the index portfolio.

Dispersion of the points away from this line corresponds to the role of all other drivers of tranche value. The most important of these are correlation effects, effects of time changes (theta), second-order effects (e.g., gamma) and the effect of non-parallel moves in the index spread curve. The significance of the model leverage is that it is the primary quantity used to delta-hedge tranche positions in practice, and hence it is of interest to compare this with the actual moves observed in the market.

Figures 8 and 9 both show that the average changes observed in the market follow the model predictions rather well. However, there is a large dispersion in values around the hedge value, especially for the equity tranche (some additional points occur off the scale of the graphs shown).

We can also see that negative deltas have been observed on a number of occasions for each tranche. These correspond to cases where the market perceives there to be increased risk in a tranche but decreased risk in the underlying collateral or the converse. This situation can only be explained by a large change in the implied base correlation used to price the tranches.

### 4.4. Effect of changes in the implied correlation on the mark-to-market

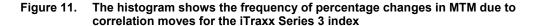
The Base Correlation LHP model obtains an implied correlation by calibration to the observed tranche spreads. All changes in the actual MTM that cannot be attributed to index spread moves or changes in the time to maturity (theta) are therefore assigned to changes in the implied correlation. Correlation changes thus represent an amalgam of real investor perception of asset correlation and model error. It is therefore of interest to see how large correlation effects are compared with the overall MTM changes.

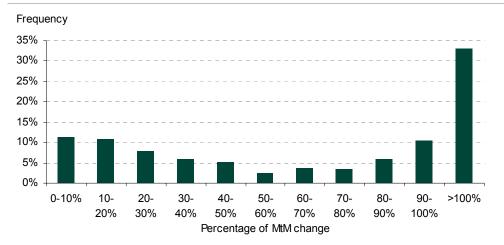
In Figures 10 and 11, we plot histograms of the percentage of the total historical tranche PV moves attributed to changes in implied correlation, averaged over all 5 year maturity tranches and over time for the CDX and iTraxx indices. We compute this by calculating the correlation sensitivities in the model and multiplying these by the observed changes in implied correlation from one day to the next. This gives the change in PV from the implied correlation. We then take the ratio of this to the total observed PV change and take the absolute value.

Frequency 25% 20% 15% 10% 5% 0% 0-10% 60-70-80-30-40-50-90->100% 10-20-80% 90% 20% 30% 40% 50% 60% 70% 100% Percentage of MtM change

Figure 10. The histogram shows the frequency of percentage changes in MTM due to correlation moves for the CDX IG4 index

Source: Lehman Brothers.





Source: Lehman Brothers.

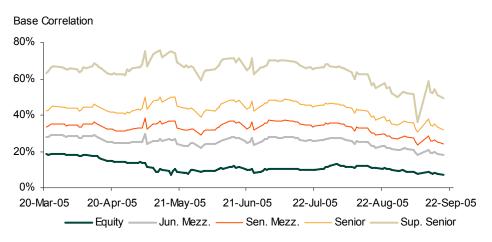
These figures show that the effects of correlation changes fall broadly into two regimes. For a substantial fraction of the time correlation changes have only a small effect on the PV changes of tranches, which are mostly explained by index spread moves. For the CDX index, for example, in about 40% of cases the correlation accounts for less than a third of the total MTM. About 20% of the time, however, the PV changes resulting from correlation are as large, or larger than the total PV change. For our time series, the effect of correlation changes on the iTraxx index is somewhat greater than on the CDX index. We see similar results for the 10 year maturity tranches of both indices.

These results show that on a significant fraction of days, delta-hedging with the index will fail substantially and we would expect relatively large moves in the overall P&L. This is no surprise since the data used include the events of May 2005, when there were significant moves in base correlation. It suggests that correlation traders must hedge their books not only against spread movements but also against changes in the prices of the liquid tranches due to changes in base correlation.

### 4.5. Base correlation

We now consider the base correlation observed in the market for the liquid CDX tranches. A time-series of the base correlation for the whole period under investigation is shown in Figure 12. We can see that the overall trend during this period has been a general decrease in the implied base correlations, especially since the end of June. As base tranches are long correlation, this represents a general perception of increased risk in these products.

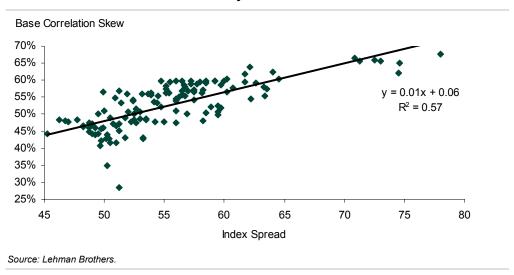
Figure 12. Time-series of base correlations for the 5-year maturity tranches of the CDX IG4 index for 21 March-20 September 2005



Source: Lehman Brothers.

We can use these data to analyse the cross-dynamics between the base correlation skew and the spread of the underlying index. We characterise the base correlation skew as the difference between the maximum and minimum base correlation values. We plot this against the spread of the underlying index in Figure 13.

Figure 13. Base correlation skew plotted against the 5y index spread for the CDX IG4 index. The skew is defined as the difference between the maximum and minimum values on the daily base correlation curve



This figure shows that the base correlation skew increases with the level of the index spread, with an approximate 1% increase in the skew for each basis point increase in spread of the index. However, the R<sup>2</sup> of the effect is only 0.57, so index spread changes by themselves do not provide a complete explanation for changes in the correlation skew.

In addition, we do not observe a significant relationship between skew and index spread for 10-year maturities or for iTraxx tranches. We conclude that any dependence of base correlation skew on the index spread is small.

### 5. CONCLUSION

We have analysed the historical behaviour of the liquid tranches of the CDX IG4 and iTraxx S3 portfolios, and compared these with the predictions of the LHP approximation to the Gaussian copula model.

We find that, on average, the deltas observed in the market are consistent with those predicted by the LHP version of the base correlation model. One reason why this may be the case is that many of the dealers in the market are using a base correlation model for pricing. As a result, on a day when the only change in the market is in the reference index, a dealer using base correlation will simply recalculate the new breakeven spreads using the model and send these out. The change in the value of a tranche when marked to market will therefore be well-explained by the base correlation delta. It is only on days when base correlations change that the base correlation delta will not explain the entire move in tranche spreads. This explanation is consistent with the effects we observe in the historical data.

As expected, we have shown that the largest factor influencing the change in the PV of tranches is a change in the index spread for the corresponding maturity. On average, changes in tranche value due to changes in base correlation usually have less than half the effect of changes in the index spread. However, roughly 20-30% of the time, correlation moves can dominate the observed PV, causing marked changes in P&L for delta-hedged tranche trades. Hedging a correlation book therefore requires careful attention to both spread risk, using the underlying index as a hedge, and correlation risk, using the standard tranches as hedges.

### **REFERENCES**

Mashal, Naldi and Tejwani (2004), *The Implications of Implied Correlation*, Lehman Brothers Fixed Income Research.

O'Kane, Naldi, Ganapati, Berd, Pedersen, Schloegl, and Mashal (2003), "The Lehman Brothers Guide to Exotic Credit Derivatives", *Risk*, October.

O'Kane and Schloegl (2001), *Modelling Credit: Theory and Practice*, Lehman Brothers Fixed Income Research.

O'Kane and Livesey (2004), *Base Correlation Explained*, Lehman Brothers Fixed Income Research.

Schloegl and Greenberg (2003), *Understanding Deltas of Synthetic CDO Tranches*, Lehman Brothers Fixed Income Research.

To the extent that any of the views expressed in this research report are based on the firm's quantitative research model, Lehman Brothers hereby certify (1) that the views expressed in this research report accurately reflect the firm's quantitative research model and (2) that no part of the firm's compensation was, is or will be directly or indirectly related to the specific recommendations of views expressed in this report.

The views expressed in this report accurately reflect the personal views of Jay Hyman, Albert Desclee, Claus Pedersen, Cong Minh Trinh and Dominic O'Kane, the primary analysts responsible for this report, about the subject securities or issuers referred to herein, and no part of such analysts' compensation was, is or will be directly or indirectly related to the specific recommendations or views expressed herein.

Any reports referenced herein published after 14 April 2003 have been certified in accordance with Regulation AC. To obtain copies of these reports and their certifications, please contact Valerie Monchi (vmonchi@lehman.com; 212-526-3173) or Larry Pindyck (lpindyck@lehman.com; 212-526-6268).

Lehman Brothers Inc. and any affiliate may have a position in the instruments or the Company discussed in this report. The Firm's interests may conflict with the interests of an investor in those instruments.

The research analysts responsible for preparing this report receive compensation based upon various factors, including, among other things, the quality of their work, firm revenues, including trading, competitive factors and client feedback.

Lehman Brothers is acting as a financial advisor to Clayton, Dubilier & Rice and the investor group with respect to their acquisition of Hertz from Ford Motor Company.

This material has been prepared and/or issued by Lehman Brothers Inc., member SIPC, and/or one of its affiliates ("Lehman Brothers") and has been approved by Lehman Brothers International (Europe), authorised and regulated by the Financial Services Authority, in connection with its distribution in the European Economic Area. This material is distributed in Japan by Lehman Brothers Japan Inc., and in Hong Kong by Lehman Brothers Asia Limited. This material is distributed in Australia by Lehman Brothers Australia Pty Limited, and in Singapore by Lehman Brothers Inc., Singapore Branch ("LBIS"). Where this material is distributed by LBIS, please note that it is intended for general circulation only and the recommendations contained herein do not take into account the specific investment objectives, financial situation or particular needs of any particular person. An investor should consult his Lehman Brothers' representative regarding the suitability of the product and take into account his specific investment objectives, financial situation or particular needs before he makes a commitment to purchase the investment product. This material is distributed in Korea by Lehman Brothers International (Europe) Seoul Branch. This document is for information purposes only and it should not be regarded as an offer to sell or as a solicitation of an offer to buy the securities or other instruments mentioned in it. No part of this document may be reproduced in any manner without the written permission of Lehman Brothers. We do not represent that this information, including any third party information, is accurate or complete and it should not be relied upon as such. It is provided with the understanding that Lehman Brothers is not acting in a fiduciary capacity. Opinions expressed herein reflect the opinion of Lehman Brothers and are subject to change without notice. The products mentioned in this document may not be eligible for sale in some states or countries, and they may not be suitable for all types of investors. If an investor has any doubts about product suitability, he should consult his Lehman Brothers representative. The value of and the income produced by products may fluctuate, so that an investor may get back less than he invested. Value and income may be adversely affected by exchange rates, interest rates, or other factors. Past performance is not necessarily indicative of future results. If a product is income producing, part of the capital invested may be used to pay that income. Lehman Brothers may, from time to time, perform investment banking or other services for, or solicit investment banking or other business from any company mentioned in this document. © 2005 Lehman Brothers. All rights reserved. Additional information is available on request. Please contact a Lehman Brothers entity in your home jurisdiction.