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Emerging Markets in the Lehman Brothers Global Risk Model[1]

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We describe the new Lehman Brothers Emerging Markets Risk Model and consider how it can help portfolio managers make more informed decisions. Our results suggest that emerging market bonds tend to move together more closely than other asset classes, such as high yield. Moreover, we found a weaker relationship between emerging markets spreads and other risk factors, such as interest rates or credit. This suggests that emerging markets may bring important diversification benefits. We show how this analysis is made simple through POINT, our portfolio analytics and modeling system.

1. INTRODUCTION

The Lehman Brothers Risk Model allows portfolio managers to quantify the expected volatility of performance deviation ("tracking error volatility") between a portfolio and a benchmark and find optimal transactions to reflect specific views. These tools are increasingly important because the need to manage global portfolios, monitor adequate levels of diversification, and search for investment opportunities outside the core has increased the complexity of fixed-income portfolios. It is in this context that Lehman Brothers has expanded its Global Risk Model to include emerging market debt.

For the past decade, the Lehman Brothers Multi-Factor Risk Models have been a valuable tool for fixed income money managers. Our models, delivered through POINT (Portfolio and Index Tool), cover a wide variety of currencies, products and asset classes in a single unified framework: the 23 currencies of the Lehman Brothers Global Aggregate Index, asset classes such as Treasuries, agencies, corporate investment grade and high yield, MBS, ABS, CMBS, CMOs and inflation-linked securities. We also cover derivative products, such as bond futures, interest rate and currency derivatives, and CDS. A detailed practical guide to our risk models and reports can be found in Joneja/Dynkin *et al*. (2005).

In this paper we begin with a brief review of emerging market bonds, the value the risk model can add to more traditional approaches, and a general description of the overall structure of the Lehman Brothers Risk Models. Section 2 describes the emerging markets risk model. We illustrate the model implementation through our portfolio analytical platform, POINT, in section 3. Some readers may prefer to jump directly to the example in the last section before focusing on the details of the risk model laid out in section 2.

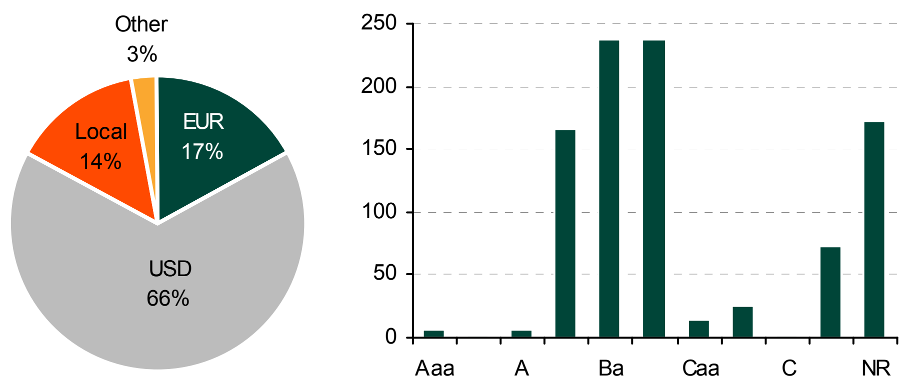
1.1. Emerging markets historical overview

The history of emerging market debt after the Second World War may be described in three distinct steps. Until the early 1990s, the market was small and the majority of the developing countries tended to borrow directly from banks or other international institutions. During the 1990s, the number of countries accessing the international public bond markets increased sharply, as the effects of the emerging markets debt crisis of the 1980s faded. The last years of the 1990s brought a new development into this market: the rise of non-sovereign emerging market debt. As countries develop, their private sectors explode and the dynamics of the market are no longer driven solely by macro/sovereign factors. In fact, recent turmoil episodes like the 2001 Argentina default or the 2002 Brazilian election seem to have had a much smaller contagious effect than previous episodes (e.g., Mondino (2005)).

[1] *I would like to thank A. Cevdet Aydemir, Dev Joneja, Joe Kogan, Bruce Phelps, Jeremy Rosten and Huarong Tang for their helpful comments and suggestions. Gary Wang and Anna Avrouchtchenko contributed to the development and implementation of the risk model in Lehman POINT.*

As of December 2004, the universe of emerging market bonds (described by Lehman Brothers as those bonds with sovereign ratings of Baa3 or below) includes 945 bonds, with a market value of $530 billion on $550 billion outstanding. Figure 1 presents some summary statistics regarding this cross section. About 60% of the issues are government related. Two-thirds of the bonds are USD denominated, 17% are EUR denominated and 14% are local issues from Indonesia, Mexico and the Philippines. Of the bonds, 19% are rated investment grade, 50% are non-distressed high yield (Ba/B), and the other 5% are distressed. 8% of the bonds are in default status while the remaining 18% are non-rated. It is important to note that this profile may change significantly across time. In particular, average ratings in December 2004 are better than historical averages.

Figure 1. Composition of the Lehman Brothers Emerging Market Index: Currency of denomination (left) and index rating (right).



Source: LehmanLive and POINT.

1.2. Risk in emerging markets: Traditional approaches and the Lehman Brothers risk model

The major sources of concern regarding emerging market debt relate to the local economic conditions. These are driven by local variables such as current accounts, inflation, economic growth, public sector spending and debt. However, under the current world market integration, local government and economic agents have only partial control over these indicators. Variables such as global growth, inflation or the general price of commodities play a significant role in determining the state of some of these economies.

Emerging market debt issued in major world currencies (such as USD or EUR) is also very sensitive to the economic environment in these major economic blocks. For instance, the level of interest rates, the business cycle and inflation in the US have a direct impact on the cost of borrowing for emerging market countries. That is why actions from the US Federal Reserve Board are closely followed by these markets.

It is extremely difficult to find an economic model that can integrate these different macro variables to explain the movements we observe in the markets. As a result, emerging market portfolio managers tend to rely on a set of portfolio statistics to measure the characteristics of their portfolios and their sensitivity to different economic variables. They monitor exposure to the yield curve (e.g., the EUR yield curve for EUR-denominated bonds). They also consider the different countries' "betas" – the relative volatility of a country's spread against a benchmark. The "betas" are used to capture a country's sensitivity to general news regarding growth, inflation or commodity prices. Portfolio managers may also look to other characteristics of their portfolios, such as their exposure to different points on the spread curve or their concentration in a specific name/country.

Analysis along these lines allows managers to formulate an educated view regarding such questions as:

- Will I outperform the benchmark if USD interest rates go up?
- If the index spread widens, what is the expected change in Mexican spreads?
- What happens if I extend the average maturity of my Russian bonds?

However, there are many other related and important questions that may be more difficult to address, such as:

- What is my risk exposure to a flattening of the USD curve?
- Is my portfolio riskier under that scenario? What is the risk impact of the flattening on my spreads?
- How much diversification do I get from adding an extra country to my portfolio? And what about expanding to a core plus strategy?
- I think my portfolio is too volatile: Where should I begin to rebalance? Where do I get the most value?
- What are the major active positions in my portfolio? Is it Asia or Venezuela? Or is it my imbalance on the 10-year point of the EUR yield curve? How should I rank them?
- How much risk am I taking for being long maturity in Brazil? How well is that hedged by my short maturity exposure to Mexico?
- What is the extra volatility associated with default events that come from my exposure to distressed bonds?
- Is my security selection diversified in terms of risk?

To answer this second set of questions, we need to translate these “directional” views into numbers. The more complex the portfolio, the harder this task is. The Lehman Brothers Global Risk Model aims to quantify all the different risk exposures of the portfolio and the benchmark in an integrated way, by netting out the effect of possible correlations between all these different effects. The result is a risk report that details and quantifies the characteristics and imbalances of the portfolio.

We formally detail the model and highlight some of the data used in section 2. However, in what follows we analyze two specific situations to illustrate what the risk model may add to more traditional approaches. Note that the examples use only a limited set of the information used in the global risk model and are constructed for illustration purposes only.

Example 1

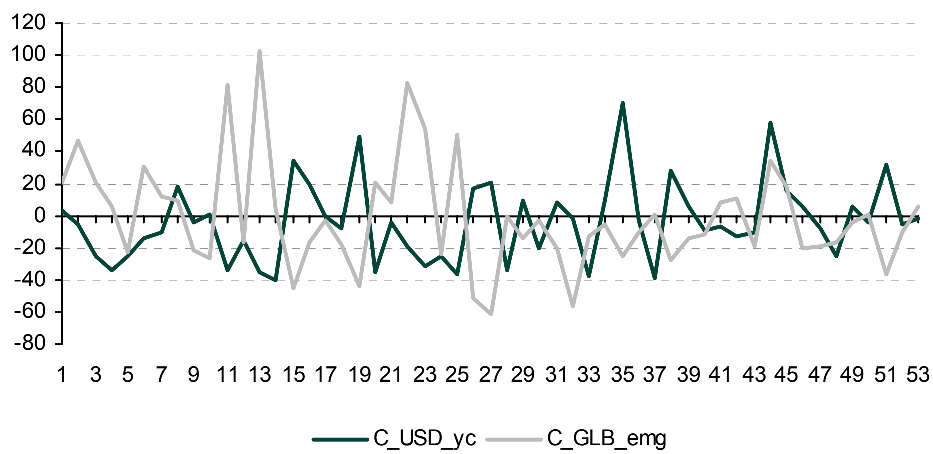
Suppose we have a generic emerging market portfolio that is short in duration and spread duration. Our working assumption is that the USD yield curve is set to rise, and we want to take full advantage of this view. We are short duration, so if interest rates do go up, we will outperform the benchmark. Suppose also that given the current market conditions, we expect spreads to tighten when interest rates go up. So our short spread duration may cancel some of our gains.

How can we quantify the extent of the trade-off? The risk model provides valuable guidance. Figure 2 shows data used by the risk model, namely the monthly changes in Treasury yields and in the OAS of the USD emerging markets index, from September 2000 to January 2005.

Specifically, we see a negative correlation (-0.44) between the two series. It confirms our reasoning that we may not be able to fully capitalize on our view about interest rates. What should we do? Ideally, we want to keep our duration exposure but increase our spread exposure. We explore two alternatives, both of which can be evaluated using our risk model.

The first alternative is to use cash bonds to rebalance the portfolio: buying bonds with small exposure to the Treasury curve but with larger exposure to changes in spreads. But there are probably too many bonds that fulfill this requirement. In this process, the risk model enables investors to fully quantify the net effect of a particular set of bonds on the portfolio's exposure to interest rates, emerging market spreads and many other risk factors.

Figure 2. Changes in the USD yield curve and on the OAS of the USD Lehman Brothers Emerging Market Index (bps/month)



Source: POINT.

A second alternative is to buy credit default swaps (CDS) written against emerging market or similar kind of debt. These instruments have low sensitivity to the interest rate curve and larger positive exposures to spread movements. The CDS allow us therefore to achieve our goals. But how can we quantify this alternative? Will we achieve better results by buying CDS from Brazilian or Mexican debt or an index of emerging market CDS? Should we buy the 3-, 5- or 7-year CDS? What extra risk/diversification will the portfolio have if we buy instead a CDX based on US credit names (like the CDX.NA.IG.4-V1 5Y)? To answer these questions, we need to know the correlation between cash bonds and CDS/CDX spreads from emerging markets, between these and yield curve movements, and so on. Also, because the correlation between emerging market cash bonds and their respective CDS spread is not perfect, we need to quantify the additional sources of risk we may be adding to the portfolio (e.g., basis risk).

How do the risk characteristics of our original portfolio compare with the new one? Is it more volatile? Was it successful in reducing our spread imbalances? Am I "buying" with the CDS/CDX some exposures I do not like or for which I have no specific views? The risk model quantifies these different issues into a unified set of *easily comparable* numbers. It details the exposures to different kinds of risks, providing a solid description of the risk of the current portfolio as well as guidance on potential portfolio changes.

In the case under consideration, the risk model would tell investors that buying the US credit-based CDX will have almost no effect on changing the correlation between spreads and Treasury yields. The old portfolio has a correlation of -0.44 against -0.38 for the "hedged" portfolio[2]. Moreover, we end up with exposure to US credit spreads, something that our initial portfolio did not have. Investors can use the risk model to evaluate other scenarios.

* * * * *

Example 2

Portfolio managers traditionally use a country's beta as a major indicator of its risk. "Beta" is defined as the sensitivity of a country's change in spreads to changes in spreads for the emerging market index. The idea is that countries with higher betas are riskier. However, betas are a very limited risk measure. Two countries may have similar betas but nevertheless present different risk characteristics. To illustrate this point, consider the cases of Turkey, Brazil and Venezuela. They have similar betas against the emerging market index, about 1.50, but are they close risk substitutes[3]? To analyze this, we look at the standard deviation of the changes in spreads on the country portfolio against the USD emerging markets index (the "Spread" Tracking Error Volatility). Figure presents some results.

Figure 3. Betas and Tracking Error Volatility (TEV) for specific countries

Country	Beta	TEV(bp/month)
Brazil	1.58	82
Venezuela	1.54	60
Turkey	1.54	84

Source: POINT.

Although the TEVs for Brazil and Turkey are close, the one for Venezuela is significantly smaller. Betas capture only the spread co-movement between the country and the benchmark. TEV, on the other hand, captures both the movements associated with the benchmark (the beta) and movements away from the benchmark (non-systematic movements). From the figure, one can infer that, historically, non-systematic volatility in Venezuela is significantly smaller than that in the other two countries.

All three countries tend to move by the same amount when the index moves. But both Turkey and Brazil have additional spread movements that are not explained by the index at all. This represents extra risk that is captured by the risk model but is neglected by the country-beta approach.

This example shows some of the limitations of the traditional approach to risk. The risk model addresses these limitations by looking explicitly at both variances and covariances of the different risk factors the portfolio is exposed to.

* * * * *

Experienced portfolio managers are able to incorporate all this information into the risk analysis of their portfolios. However, as more alternatives are considered, it becomes increasingly difficult to quantify the effect of these combined risks on the portfolio. Our risk model provides investors with a tool to do that.

In the remaining part of the paper, we introduce readers to the risk model. The model is designed to be intuitive and simple. However, familiarity with the model and its output comes with time. The best way to build it is for investors to work in POINT with a portfolio they are familiar with.

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[2] *The Global Risk Model will not display any particular correlations, but the results are based on those correlations.*

[3] *These betas are constructed using the risk model country factors. These factors proxy monthly changes in OAS since 1997 and rely on a specific treatment of outliers. The estimation of specific betas will vary based on the time span of the data used and on the frequency of estimation (monthly, weekly, daily, etc).*

1.3. Structure of the Lehman Brothers High Yield Credit Risk Model

The Lehman Brothers Global Risk Model is a unified framework for risk analysis: all product-specific models are developed within a consistent structure. In particular, the new Emerging Markets Risk Model has a structure close to the one for high yield bonds. Therefore, we begin with a brief description of the high yield model. See Chang (2003) for a more complete description of the model.

The motivation behind the model is that high yield bonds have two major sources of credit risk: (i) *market spread risk* that reflects investors' assessments of liquidity risk and changes in *future* default risk of the bond; and (ii) *default risk* that captures the risk of outright default over the short term. To capture these two kinds of risk, we describe the monthly return from bond *i* as:

$$R_{it} = (1 - I_{it}) R_{it}^{Market} + I_{it} R_{it}^{Default} \tag{1}$$

Here I is an indicative random variable for a default event – it has a value of 1 if the issuer *i* defaults during period *t*, and 0 otherwise. If there is no default during the period, the return for the bond is driven by market factors - R_{it}^{Market}. However, if the firm does default, the return is driven instead by the default process - $R_{it}^{Default}$[4].

1.3.1. Market Risk

In the model, market return is decomposed as follows:

$$R_{it}^{Market} = R_{it}^{Carry} + R_{it}^{YC} + R_{it}^{SS} + R_{it}^{VOL} + R_{it}^{Spread}, \tag{2}$$

where the first component is known in advance (e.g., coupon payments), and the remaining four are not (e.g., how much will spreads change). Return from carry captures coupon accrual and the return due to the passage of time. The yield curve (*YC*) return measures returns associated with changes only in the respective Treasury curve. Swap spread (*SS*) returns capture returns due to changes in swap spreads, and the volatility return captures the return that is due to embedded optionality, if any. Finally, the spread return captures the return due to changes in spreads over swap rates. For instance, for non-distressed high yield bonds, spread return is modeled as:

$$R_{Spread,t}^{i} = OASD_{t-1}^{i}\left[F_t^B + (TTM_{t-1}^{i} - TTM_{t-1}^{B})F_t^{TTM} + (OAS_{t-1}^{i} - OAS_{t-1}^{B})F_t^{OAS} + \varepsilon_{i,t}\right]$$

where F_t^B, F_t^{TTM} and F_t^{OAS} represent the risk factors, and the superscript *B* stands for the bucket to which the bond belongs. In the case of high yield non-distressed bonds, we have 11 buckets, based solely on industry.

This decomposition delivers a model where bond market returns are explained by two kinds of risk: the systematic risk – part of the return explained by the yield curve, volatility, swap spreads and spread factors; and the idiosyncratic risk – the portion of return not explained by the systematic factors (the $\varepsilon_{i,t}$ term above). As we show later, the role of these two kinds of risk varies widely across portfolios and across asset classes.

[4] *From a continuous time perspective, equation (1) states that the price of a credit can be viewed as the combination of a diffusion process* R_{it}^{Market} *and a jump process* I_{it}.

1.3.2. Default risk

The return given default is modeled separately as:

$$R_{it}^{Default} = (recovery_{i,t} - price_{i,t-1}) / price_{i,t-1}$$

Once portfolios are considered, a major force driving the volatility of this return is the correlation of default across all names in the portfolio.

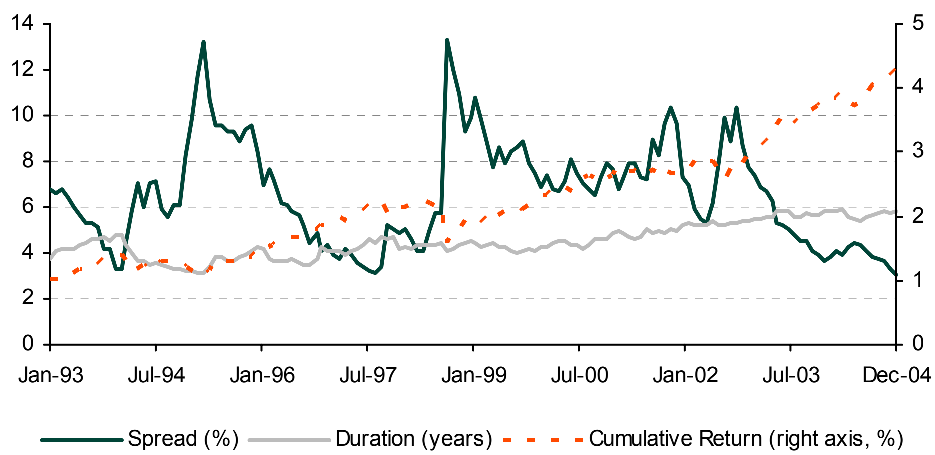
Bond returns – and credit risk – in our model are therefore explained by three types of risk: systematic, idiosyncratic and default risk. We use this rich specification to model the risk of emerging market bonds. In what follows, we detail the specifics of this asset class.

2. THE EMERGING MARKETS RISK MODEL

2.1. Market return

We begin with a very broad description of the dynamics of the USD Lehman Brothers emerging market index[5]. Figure 4 reports the spread (OAS), durations and cumulative total return (right axis) from this index. One can see that the index has returned on average about 1% per month over the past 12 years. This means that an investment of $100 in 1993 would be valued at over $400 by the end of 2004. The bulk of that return comes from the past two years, when spreads to Treasuries decreased from 10% to just above 2%. This high return is associated with the relatively high risk of this asset class measured, for instance, by the high volatility of the spreads. Finally, note that the duration of the index increased from about four years in 1993 to six years in 2004. This seems to suggest an increase in issuance and/or in the maturity of the debt issued.

Figure 4. Lehman Brothers Emerging Market Index (USD) statistics: spread, duration and cumulative total return (right axis)



Source: LehmanLive.

We now turn to the description of the market return specification for the emerging markets risk model. The starting point is equation (2). Specifically, for emerging markets, we use six key rate durations plus a convexity term to model the return from the yield curve (again, see Chang (2003) for details). We model the return from the change in swap spreads similarly:

[5] *The analysis here is restricted to USD-denominated bonds because a longer time series is available for this currency.*

$$R_{it}^{SS} = \sum_{j=1}^{6} SSKRD_{i,j,t} \times \Delta SS_{j,t}$$

where $\Delta SS_{j,t}$ stands for the change in the jth swap spread in period t (j = 6 months, 2, 5, 10, 20 and 30 years) and:

$$SSKRD_{i,j,t} = OASD_{it} \times (KRD_{i,j,t} / \sum_{m=1}^{6} KRD_{i,m,t})$$

where *OASD* stands for option-adjusted spread duration and *KRD* for key rate duration.

In addition we set the return from volatility to zero due to the sparse information regarding embedded options in emerging market debt[6]. We are left with the spread return. Following the structure of our existing models, the first goal is to establish groups of bonds – "buckets" – with similar risk profiles. The specific groups that we end up with are based on two criteria. First, the grouping must have an intuitive appeal: for instance, most people would agree that firms in the same industry do share a set of common risk factors. In this regard, grouping bonds by industry seems to make sense. Second, the factors estimated from the model must be statistically robust. Therefore, we want to avoid groups with a small number of observations or whose characteristics vary significantly across time. In the investment grade credit models these considerations led us to define groups based on country, industry and rating. However, a different strategy is used to model distressed bonds in the high yield model: all bonds are pulled together independently of country or industry.

In emerging markets, we begin with an approach similar to that of the existing models. This procedure ensures a unified framework across all risk models. To do so, we construct a sample using data from emerging markets available from Lehman Brothers. Emerging market debt is defined as bonds from countries with sovereign ratings of Baa3 or below. The sample we use spans the period from January 1997 to December 2004 (96 months), with a total of 29,580 individual bond return observations from 46 countries and includes both EUR- and USD-denominated bonds[7].

We first group the bonds into three major geographical regions – Latin America, Europe and Asia – and three rating buckets[8]. Figure 5 presents some summary statistics for this partition.

Figure 5. Sample summary statistics by region and rating

Rating	Aaa-Baa			Ba-B			Caa-C		
Statistics	**America**	**Asia**	**Europe**	**America**	**Asia**	**Europe**	**America**	**Asia**	**Europe**
Observations	4,377	1,151	657	10,456	4,444	2,990	3,777	1,038	690
Number of Months	88	82	88	96	96	96	92	85	68
Avg. Number of Bonds per Month	50	14	7	109	46	31	41	12	10
Min. Number of Bonds per Month	20	1	1	22	23	4	1	1	1

Source: POINT.

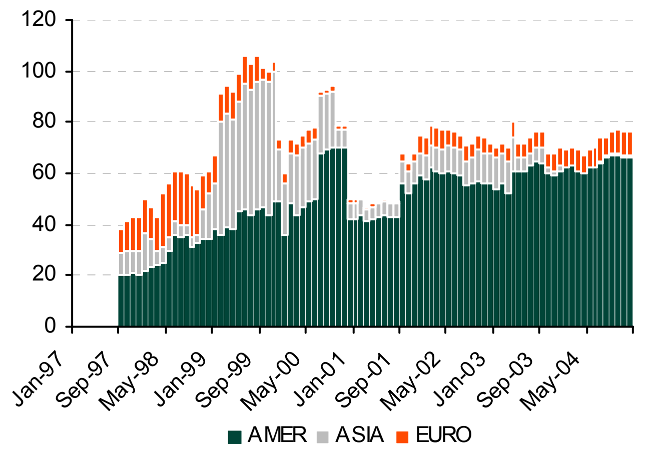
A quick analysis of the figure suggests that this partition may have some problems due to some relatively thin buckets: both investment grade (Aaa-Baa) and high yield distressed (Caa-C) for Asia and Europe. A closer look at the data is given in Figure 6. We can see that the time series profile of these buckets is not stable across time. Concerning investment grade, there are almost no data from Europe during 2000 and 2001. The same happens with Asia during the past two years. When we look into the regional profile of high yield

6 We will revisit this issue as more analytics become available for these bonds.
7 Data are available from earlier years for some Latin American countries.
8 Countries from the Middle East and Africa are included in the Asia block. Their inclusion does not materially change any of the results presented. See Appendix 1 for a full description of country/region mapping.

distressed bonds, almost the opposite happens: Europe has a relatively high number of distressed bonds after the Russian crisis in July 1998. This number begins to fade after 2000. Asia has a relatively high number of distressed bonds after the crisis in 1997. This number simply collapses after 2000.

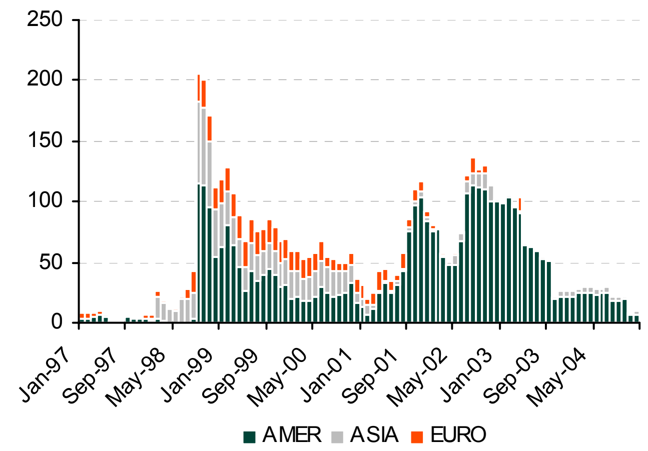
Figure 6. Number of investment grade bonds (left) and distressed bonds (right) per geographic region

Figure 6a. Investment grade bonds per geographic region



Source: POINT.

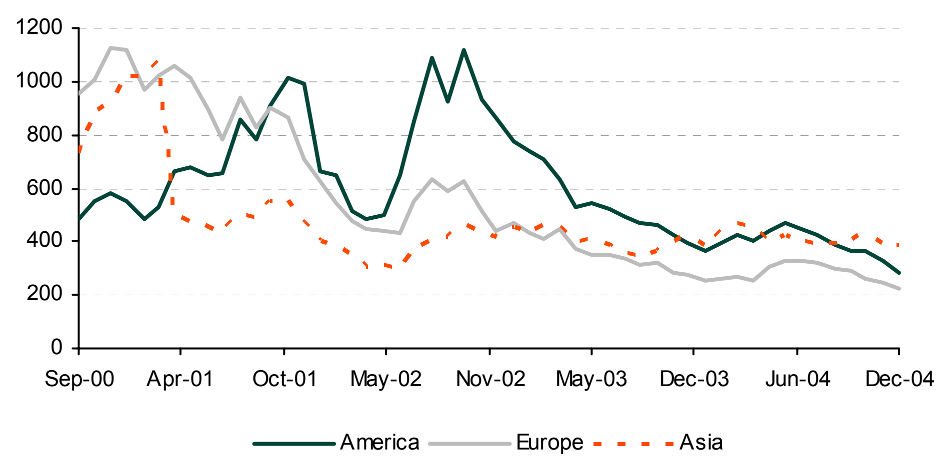
Figure 6b. Distressed bonds per geographic region



Source: POINT.

The existence of long periods when these buckets are very thinly populated poses a problem when estimating systematic risk factors. The volatility of thin buckets may be primarily driven by idiosyncratic events and we would be wrong in labeling this volatility as systematic. In other words, we cannot estimate with confidence a time series of systematic factors driving the return of these buckets. We therefore choose to merge the three investment grade buckets into a single one. We do the same for the three distressed buckets. By doing so we lose some information, but we hope to avoid any serious misrepresentation of risk.

Figure 7. Emerging markets OAS per regional block: America, Europe and Asia (bp)



Source: LehmanLive.

Regarding high yield non-distressed bonds (Ba-B), the evidence in Figure 5 suggests that we have enough data to estimate systematic factors for the three regions. But if bonds behave similarly across all three regions, there is no reason to do so. Although more information will be given later, Figure 7 sheds some light on this issue. It shows the monthly OAS for emerging market bonds denominated in USD from August 2000 to December 2004, grouped by regional block. It is apparent that although sharing some secular trends over the past four years, there are plenty of episodes when their behavior differs substantially. For instance, comparing America and Europe, one can see that their spread behavior was quite different during the first period of the sample. Also, the Argentina default of late 2001 seemed to have a strong regional effect, much bigger across Latin America than in Europe or Asia. The same is true during the events of September 2001. The figure also shows a flat OAS for Asia since mid-2002, much in contrast with both America and Europe. Finally, the OAS for European emerging market bonds has steadily decreased over these years, while the Asian OAS had an early dramatic decrease, and stayed flat almost since early 2001. In contrast, Latin American bonds have seen much more volatile behavior. This analysis supports our choice of three major regional factors – America, Asia and Europe – to describe the spread return for Ba/B-rated emerging market bonds.

In fact, we went further with this exercise, and estimated individual country factors. Portfolio managers with exposures centered on a small set of emerging market countries may find that the country-specific risk their portfolios are exposed to is being washed away by the aggregation within blocks. To avoid this dilution, we estimate individual factors for countries that are major issuers in the emerging markets[9]. Figure 8 presents the partition with the 12 buckets we use to model emerging market bonds. Later in this paper, we provide additional evidence regarding individual country factors.

In addition, we could have modeled sovereign and non-sovereign debt separately. We decided not to because we found similar volatility behavior for these two types of bonds. However, we do differentiate their default treatment (see below)[10]. Finally, some of the bonds from emerging markets are "Brady bonds". These are bonds whose principal is partially guaranteed, usually by US government bonds. The existence of these guarantees distorts the usual bond analytics. Taking that into account, we use the corrected ("stripped") analytics for these bonds, whenever needed. We also take into account their guaranteed principal when computing recovery rates.

We can now detail the model we use to estimate the return from changes in spreads for emerging market bonds. Our study suggests that there are systematic differences in spread volatility across ratings and regions. To accommodate this evidence, we estimate spread volatilities along these dimensions and attribute a common "bucket" risk factor to all bonds belonging to the same rating/region bucket (F_t^B and $F_t^{Distress}$). However, two bonds in the same bucket can still exhibit different risk profiles. We use two more risk factors to fine tune these systematic differences. One is based on the time to maturity (TTM) of the bond as compared with its bucket: even if two bonds belong to the same rating and region group, we still expect different volatilities if their time to maturity is very different. We capture this asymmetry with our "slope" risk factor. The other systematic risk factor is based on the option-adjusted spread (OAS) of the bond, again compared with the average of the bucket: we may have two bonds with the same rating, from the same country and with the same time to maturity. However, one may trade at much higher spreads – e.g., if it is less liquid – than the other. This difference in profile can be relevant for risk purposes and is fully captured by

[9] *These are also the countries for which we have enough data to perform our estimation exercise in a meaningful way.*

[10] *We differentiate their default treatment given the evidence that recovery rates for non-sovereigns are smaller. Interestingly, we did not find significant difference of behavior in the spread movements across the two sets of bonds. This may be because the recovery rates are quite small anyway for all these bonds (25% for sovereigns and 10% for non-sovereigns).*

the risk model through the “spread” risk factor. Differences in spread volatility not explained by these systematic risk factors are labeled as idiosyncratic.

More formally, spread return for non-distressed bonds is modeled as:

$$R^{i}_{Spread,t} = OASD^{i}_{t-1}\left[F^{B}_{t} + (TTM^{i}_{t-1} - TTM^{B}_{t-1})F^{TTM}_{t} + (OAS^{i}_{t-1} - OAS^{B}_{t-1})F^{OAS}_{t} + \varepsilon_{i,t}\right] \quad (3)$$

where B is one of the eleven non-distressed (Aaa-B) “buckets” described in Figure . So F^{B}_{t} is the “bucket” risk factor, F^{TTM}_{t} is the “slope” factor, and F^{OAS}_{t} the “spread” factor. The loadings to each of these factors are as described above. Finally, $\varepsilon_{i,t}$ is the idiosyncratic term.

For distressed bonds the model is similar, but returns are modeled directly, as these bonds are usually traded on price rather than spreads:

$$R^{i}_{Spread,t} = F^{Distress}_{t} + (TTM^{i}_{t-1} - TTM^{Distress}_{t-1})F^{TTM}_{t} + (\Pr ice^{i}_{t-1} - \Pr ice^{Distress}_{t-1})F^{\Pr ice}_{t} + \varepsilon_{i,t} \quad (4)$$

Factors and idiosyncratic errors are estimated monthly and their time series are used to construct both the covariance matrix and a consistent estimator for the volatility of the idiosyncratic error. Our robust estimation procedure (along the lines of the Huber M estimator) reduces the weight of the outliers and delivers monthly R-squared of 50%/60%[11]. Moreover, we use two alternative methodologies to aggregate the monthly statistics: an equal-weighted (standard) approach, where all observations are given the same weight regardless of how old they are; and a time-weighted approach, that attributes a higher weight to the more recent observations (Berd and Naldi (2002)). The latter approach uses an exponential weighting scheme with half-life defined at one year. This option is particularly useful if one believes future market volatility conditions are closer to recent volatility conditions than to historical averages.

Figure 8. Partition for emerging markets risk factors

Rating / Block	Aaa-Baa	Ba-B	Caa-C
America	EMG Investment Grade	EMG America Argentina Brazil Mexico Venezuela	EMG Distressed
Asia		EMG Asia Philippines	
Europe		EMG Europe Russia Turkey	

Source: POINT.

Figure 9 presents the standard deviation of several systematic risk factors, both from the emerging markets model (EMG) and from two other Lehman Brothers risk models: the USD

[11] *These numbers are substantially higher than those from models like the high yield model. We discuss later why this may be the case.*

investment grade (IG) and the global high yield (HY) risk models. We present the factor volatilities for both the unweighted and time-weighted methodologies. All numbers are in basis points per month. Recall from (3) and (4) that while non-distressed factors proxy for changes in OAS, the distressed factors model returns directly, so their units are distinct.

The figure shows that historically (unweighted), the factor volatility of emerging markets has been about double that of the corresponding factors from developed countries. As suggested by Figure 5, Asia seems to be an exception, with volatilities very close to those from high yield non-distressed bonds. Another interesting point to note is that this general gap is significantly reduced when we take instead the time-weighted volatilities. This expected result reflects the dramatic drop in spread volatilities from emerging markets bonds in recent years. The exception seems to be distressed bonds, where the gap did not close: although the average monthly returns (in absolute value) from high yield distressed bonds drops to 2.7% when we overweight recent observations, the one from emerging market distressed bonds is still very high, about 6.4% per month. This may still be the result of the Argentina default in late 2001.

Figure 9. Systematic risk factor volatilities (basis points/month)

Asset Class	Unweighted	Time Weighted
Investment Grade		
IG Non Corporates Baa	16.9	16.1
EMG Investment Grade	38.2	20.3
High Yield Non-Distressed		
HY non-distressed	48.4	42.2
EMG America	88.9	65.3
EMG Asia	58.1	28.9
EMG Europe	98.5	53.7
High Yield Distressed		
HY Distressed	367	270
EMG Distressed	766	637

Source: POINT.

Similarly, Figure 10 presents the idiosyncratic risk (volatility of the idiosyncratic error) for bonds from the same risk buckets. The analysis of the investment grade numbers is similar to the one made above for the systematic factors. However, the story is very different for the high yield buckets. Note that for both distressed and non-distressed high yield bonds, the idiosyncratic volatilities are smaller than those from the global high yield model.

The evidence suggests that in relative terms, emerging markets bonds tend to move together much more closely than their counterparts from more developed countries. This result is consistent with the idea that emerging markets volatility is driven by a set of macroeconomic factors, from which no individual bonds can diverge too much. This “macro” influence is only attenuated for the investment-grade bonds, possibly because investors perceive these bonds as being less correlated with their specific country’s fate: their risk is not pure sovereign risk. This perception may arise from recent default episodes, where the old across-the-board moratoria were significantly less severe. For example, during the recent defaults from Pakistan, Ukraine and Russia, payments did not stop on debt owned by foreigners (Levey and Truglia (2001)).

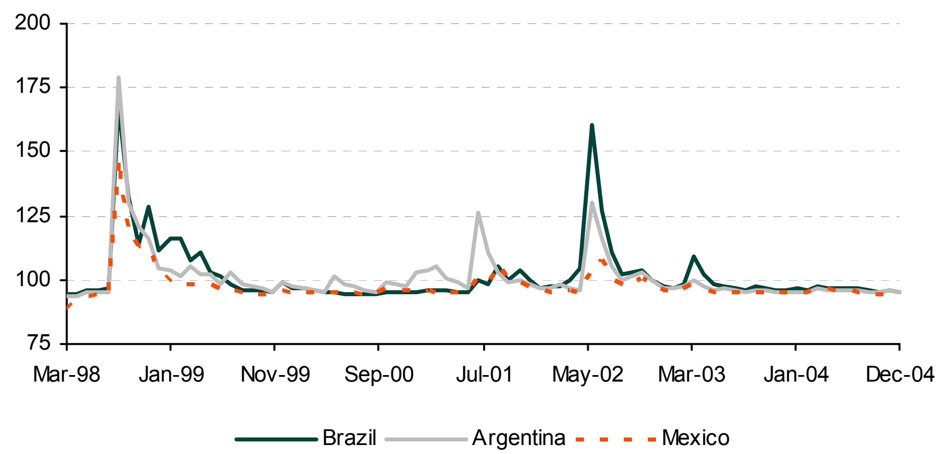
Figure 10. Idiosyncratic risk factor volatilities (basis points/month)

Asset Class	Unweighted	Time Weighted
Investment Grade		
IG Non Corporates Baa	19.2	17.0
EMG Investment Grade	38.9	26.2
HY Non-Distressed		
HY non-distressed	82.1	71.6
EMG America	59.2	45.9
EMG Asia	67.0	39.6
EMG Europe	55.4	35.8
HY Distressed		
HY Distressed	972	762
EMG Distressed	732	589

Source: POINT.

Figure 11 supports the rationale behind the use of individual country factors. Specifically, it presents the volatility in basis points of the systematic spread return of three hypothetical portfolios, each with a high yield, non-distressed, one-year OASD bond from a specific country[12]. We use the time-weighted volatilities for this example. For most of the time, the volatilities are quite similar. However, countries' risk profiles may differ substantially in the aftermath of particular shocks. For example, the volatilities increase significantly following the Russian crisis, but much less so for the Mexican portfolio. Moreover, the Argentina default of late 2001 triggered an increase in volatilities, but it was essentially a country-specific shock. On the other hand, the uncertainty around the Brazilian elections of October 2002 led to higher – but much smaller – volatilities across the border. This asymmetric behavior justifies our use of individual country factors, when possible, against the use of the more aggregated regional block factors.

Figure 11. Spread volatilities for Brazil, Argentina and Mexico (basis points/month)



Source: POINT.

[12] *We also assume that each of the bonds has average time to maturity and OAS.*

Finally, it is interesting to consider the correlation between emerging markets systematic risk factors and those outside this asset class. This may help us identify contemporaneous movements across different markets. This information can be used for portfolio hedging or diversification. These correlations are only indicative, as they can change significantly with market conditions. Figure 12 shows the (unweighted) risk model correlations[13].

Figure 12. Correlations across different asset classes

Risk Factors	EMG Inv. Grade	EMG America	EMG Asia	EMG Europe	EMG Distressed
A. Emerging Markets Spreads					
EMG Inv. Grade	1.000				
EMG America	0.884	1.000			
EMG Asia	0.921	0.811	1.000		
EMG Europe	0.877	0.850	0.846	1.000	
EMG Distressed	0.666	0.596	0.595	0.630	1.000
B. 5-Year Interest Rate					
USD_kr_5Y	-0.294	-0.267	-0.272	-0.265	-0.215
EUR_kr_5y	-0.140	-0.127	-0.130	-0.126	-0.102
GBP_kr_5y	-0.075	-0.069	-0.070	-0.068	-0.055
C. Other Credit Spreads					
USD Inv. Grade Credit	0.410	0.372	0.380	0.370	0.300
EUR Inv. Grade Credit	0.248	0.225	0.230	0.224	0.181
GBP Inv. Grade Credit	0.219	0.198	0.203	0.197	0.160
HY Distressed	0.392	0.356	0.364	0.354	0.287

Source: POINT.

As discussed before, Panel A shows that spreads move quite closely across the different regions. This suggests limited gains in regional diversification across emerging markets. Correlations seem to break only for the distressed group – although they are still quite large. The evidence in Panel B seems to suggest that the negative correlation between spreads and interest rates decays with rating. It is also relatively larger (in absolute value) for USD-denominated bonds. This latter relationship may be driven by the fact that the emerging market of USD-denominated bonds is larger than for other currencies (Figure 1). Still, this correlation is substantially smaller (in absolute value) than that for the other asset classes – e.g., the correlation between USD interest rates and USD credit spreads is about -0.50. Several reasons may explain this fact: the weaker relationship between the (foreign) interest rates and the state of the local economies across emerging markets. Investors may use some exposure to emerging markets to reduce the impact of interest rate changes on spreads (one needs to exercise caution though: recall the discussion in "Example 1" above). Finally, note the positive correlations between emerging market and other credit spreads, suggesting some degree of both economic and market integration/globalization. Again, this is especially true for USD credit spreads. The level of these correlations – clearly much smaller than one – seems to suggest that one can significantly reduce exposure to USD credit spread by diversifying into emerging markets. The net effect of this analysis is simplified by the risk model. There one can measure these and other trade-offs in a clear and concise way.

[13] *Weighted correlations are not too different.*

2.2. Default risk

Portfolio default volatilities in our model are driven by three factors: default probabilities, recovery rates, and correlation of default among the bonds of the portfolio. To illustrate this point, we begin with equation (1) without the market return component:

$$R_t^i \approx I_i R_{Default,t}^i$$

Although in general the recovery rate is stochastic at the beginning of the period, we treat it as deterministic, for the sake of tractability. Consequently, the standard deviation of the return due to default for a portfolio with two bonds with the same default probability *p*, return upon default *R,* portfolio weight (50%) and a correlation of default ρ is approximately:

$$stdev(\text{Re}\,turn\ due\ to\ default) \approx \sqrt{0.5p(1+\rho)}\,|R|$$

Note that the volatility is increasing in the probability of default and correlation. Moreover, the larger the loss upon default, the greater the volatility (we model *R* as having an upper bound of -10%). In what follows we analyze in more detail each of these three variables.

The default probability of a given bond is driven by the financial situation of its issuer. We assume that once default occurs for one issuer, it occurs for all its bonds. Therefore, the default probability of a bond is independent of its subordination, and all bonds from the same issuer have the same probability of default[14]. We base the default probability of a particular issuer on the rating of its senior unsecured debt[15]. Figure 13 shows the current annual default rates being used across different ratings. These numbers are updated monthly and are based on historical numbers. Again, we offer two sets of default rates: an historical average and a 12-month trailing average.

Figure 13. Annual historical default rates: long-run and 12-month trailing average (December 2004)

Average	Baa1	Baa2	Baa3	Ba1	Ba2	Ba3	B1	B2	B3	Caa-C
Long run	0.10%	0.18%	0.33%	0.60%	1.09%	1.98%	3.59%	6.50%	11.77%	21.33%
12-month trailing	0.06%	0.10%	0.19%	0.34%	0.62%	1.12%	2.03%	3.68%	6.67%	12.09%

Source: POINT.

The procedure used to attribute recovery rates to a particular bond has changed across all Lehman Brothers risk models from the one reported in Chang (2003). However, for emerging markets the changes are smaller. For the sake of completeness, we begin by describing the new general procedure. We then detail the specific assumptions used for emerging markets.

The recovery rates are based on the bond's specific subordination type. This approach replaces – for all bonds for which we calculate default TEV – the use of industry as the determinant of recovery rates. The change is based on several studies, principally Moody's model to predict loss given default detailed in LossCalc (Gupton and Stein (2002)). They find that:

1. Seniority is the main predictor of recovery rates.

[14] *For sovereign bonds this is not necessarily true. Governments can selectively default on part of their debt only. However, for simplicity, we ignore this difference.*

[15] *If the issuer does not have any senior unsecured debt, its rating is interpolated from bonds with other subordinations.*

2. Recovery rates per seniority are well predicted by their historical average and a macro indicator that captures the default "business cycle".

3. The industry effect is small and useful only if short-term industry-specific recovery rate averages are available. This is because the industry-specific recovery rates vary across business cycles.

Our approach uses the first two findings. In particular, we make recovery rates a function of seniority; we provide both a long-run historical average and a 12-month trailing average of recovery rates for senior unsecured debt. The latter series captures the essentials of the business cycle dynamics. The current values for both the long-run average and the 12-month trailing series are given in Figure 14. For all European bonds, we use 80% of these values.

We can also confirm that our 12-month trailing averages fully capture one additional important feature of the behavior of default cycle: recovery rates are usually high (low) when default rates are low (high). This correlation results in default return volatilities that can be significantly different depending on the specific average series used. For example, the default volatility for a senior unsecured B2 bond goes from 17.6% to 10.2% when one changes from using long-run averages to 12-month trailing averages.

Figure 14. Annual historical recovery rates: long-run and 12-month trailing average (December 2004)

Average	Sr. Super Secured	Sr. Secured	Sr. Unsecured	Sr. Sub.	Sub.	Jr. Sub.	Pref. Stock
Long run	71	50	31	27	27	17	9
12-month trailing	81	60	47	39	32	31	20

Source: POINT.

As mentioned previously, we treat the recovery process for emerging market bonds differently. The experience with defaults in emerging markets is significantly different from that in developed countries. The actual number of defaults is also much smaller, so we cannot model them with a partition as fine as that in Figure 14. Instead, we set recovery rates for emerging market bonds using an established fact about emerging market defaults: recovery rates for sovereign bonds tend to be higher than their corporate counterparts. In particular, we set the recovery rate to 25% for emerging market sovereign bonds and 10% for emerging market non-sovereign bonds. These numbers are conservative estimates. Their historical averages are higher, but sensitive to the small number of actual defaults.

Finally, the default volatility depends also on the default correlation. See Chang (2003) for a description of the implementation of this feature in our risk models. In short, we treat default as a systematic event: the aggregate default rate moves with the business cycle, with distressed credit conditions tending to propagate across firms.

We model default based on a model inspired by Merton (1974). Specifically, we approximate asset returns by stock returns (equity is just a call option on the company's assets). We then model the correlation of stock returns to get an estimate of the default correlation. (In particular, we use a larger cross section of individual stock returns from emerging market countries to calibrate this model to emerging market debt.) Crucial to this methodology is the use of the Student-*t* distribution to model asset returns. Empirical evidence suggests that this distribution does a much better job of explaining stock returns than the usual alternative normal distribution. This is because the *t* distribution delivers two important features: fatter tails and tail-dependence. The first implies that extreme outcomes are more likely than with the normal distribution, and the second generates more extreme co-movements. Because default itself is an extreme event, these two characteristics have a significant influence on the results of the analysis. This is particularly relevant for emerging markets. For this asset class, extreme events are even more usual, as are the propagation effects.

3. POINT IMPLEMENTATION

In this section, we examine a sample risk report for an emerging market portfolio. The report is produced through our portfolio analytical platform, POINT. POINT delivers several risk measures relating to the portfolio, benchmark and the difference between the two. A full description of the report statistics and some insights on the interpretation of the report numbers are available at the Global Risk Model Glossary in POINT.

In the example below, the portfolio is USD-denominated investment grade Brazilian bonds. The benchmark is the US non-corporate Baa credit index. It is mainly composed of Mexican bonds. We present this example purely to illustrate the level of detail in POINT risk analysis. We run the report on January 31, 2005 using the "weighted" option.

Figure 15. Portfolio/Benchmark Comparison Report (partial)

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Portfolio/Benchmark Comparison

Portfolio : us em brazil
Benchmark : US Credit Non Corporate Baa

Parameter	Portfolio	Benchmark	Difference
Market Value	5060023	72545964	
Coupon (%)	3.87	7.9	-4.03
Average Life (Yr)	6.09	10.07	-3.98
Yield to Worst (%)	6.15	5.16	0.99
ISMA Yield (%)	6.23	5.16	1.07
OAS (bps)	247	120	128
OAD (Yr)	4.38	6.22	-1.84
ISMA Duration (Yr)	4.48	6.22	-1.73
Duration to Maturity (Yr)	4.48	6.22	-1.74
Vega	-0.01	-0.0	-0.0
OA Spread Duration (Yr)	4.47	6.12	-1.65
OA Convexity (Yr^2/100)	0.23	0.71	-0.47
Total TE Volatility (bps/month)			**159.35**
Systematic Volatility (bps/month)	173.63	183.83	
Non-systematic Volatility (bps/month)	51.9	65.07	
Default Volatility (bps/month)	42.87	33.12	
Total Volatility (bps/month)	**186.23**	**197.8**	

Source: POINT.

The first report presented by POINT is the "Portfolio/Benchmark Comparison". This shows important statistics about the portfolio and benchmark, including the number of positions, currencies, market value. This report also provides a snapshot of significant analytics such as OAD, OASD, OAS. Figure 15 presents a partial view of this report. One can see that the portfolio is almost two years short duration compared with the benchmark. Such a position would suggest that the portfolio manager wants significantly less exposure to interest rate movements. The portfolio also has significantly higher spreads. At the bottom, this report also presents some summary risk measures, such as the Tracking Error Volatility (TEV) or the volatilities along the major risk sources: systematic, idiosyncratic and default risk. The TEV, that is, the expected standard deviation of the difference in returns between the portfolio and the benchmark, is 159 basis points per month. We can also see that although the duration of the portfolio is much shorter, its systematic risk is close to that of the benchmark (174bp/month vs 189bp/month). As we will see later, this is because the portfolio is exposed to the highly volatile emerging markets risk factors. The overall risk due to default or idiosyncratic events is also not too different[16].

Figure 16 presents the "Tracking Error" report. It decomposes the total TEV across several major sources of risk, namely currency, yield curve, swap spreads, volatility, investment grade spreads, high yield spreads, emerging markets spreads, idiosyncratic and default risk.

Numbers are presented for the isolated and the cumulative TEV for each of these sources. We also show the percentage of the tracking error variance that is attributed to each of these major sources. One can see that the mismatch in durations accounts for a 46bp/month TEV. The investment grade component of the TEV accounts for an isolated 101bp/month, while the emerging markets exposure mismatch is responsible for an isolated TEV of 119bp/month.

Figure 16. Tracking Error Report

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Tracking Error

Portfolio : us em brazil
Benchmark : US Credit Non Corporate Baa

Global Risk Factor	Isolated TEV (bps)	Cumulative TEV (bps)	Difference in cumulative (bps)	Percentage of tracking error variance (%)
Global				
Yield Curve	45.61	45.61	45.61	6.97
Swap Spreads	12.4	50.49	4.88	-0.0
Volatility	0.36	50.25	-0.24	-0.04
Investment-Grade Spreads	101.05	100.45	50.19	16.82
Credit and Agency Spreads	101.05	100.45	50.19	16.82
Emerging Markets Spread	118.57	126.63	26.19	39.39
Systematic risk	126.63	126.63	0.0	63.15
Idiosyncratic risk	82.5	151.13	24.5	26.8
Credit default risk	50.51	159.35	8.22	10.05
Total risk		**159.35**	**-0.0**	**100.0**

Source: POINT.

[16] *Even though the volatilities are similar, their specific characteristics may not be.*

Interestingly, note that the investment grade spreads account for 16.8% of the TE variance, while the emerging market spreads account for only 39.4%. Therefore, when all effects are taken into account, the overall effect of the non-corporate imbalances shrinks significantly. This striking result is due to covariances between these spreads and all other risk factors.

In Figure 17 we detailed this information. In particular, note that we have an imbalance on the OASD exposure of six years for the non-corporate Baa bucket and four years for emerging markets. The volatility of these factors is similar (recall that we are using weighted estimates). This means that an average monthly *uncorrelated* increase in the spread implies a change of comparable magnitude in the tracking error[17]: 97.4bp/month for the non-corporate bonds compared with -89.2bp/month for the Brazilian ones. However, in the real world risk factors are correlated. So the report also shows the correlated TE, that is respectively 43.8bp/month and -82.56bp/month. While the exposure to non-corporates is somewhat diluted by the correlation with other factors, the exposure to the Brazilian bonds is not. So at the end (see last column) the Brazilian exposure is responsible for 29% of the TE variance, while the mismatch in non-corporate spreads accounts for only 16.8%.

Figure 17. Factor Exposure – Full Detail Report (partial view)

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Factor Exposure - Full Details **1/31/2005**

Portfolio : us em brazil
Benchmark : US Credit Non Corporate Baa

Factor name	Sensitivity/Exposure	Portfolio exposure	Benchmark exposure	Net exposure	Factor volatility	TE impact of an isolated 1 std. dev. up change (bps)	TE impact of a correlated 1 std. dev. up change (bps)	Marginal contribution to TEV (bps)	Percentage of tracking error variance (%)
CREDIT IG SPREAD & VOL.									
USD Corporate volatility	Volatility Duration	0.0	0.0040	-0.0040	96.83	0.36	-24.89	15.126	-0.04
USD Non-Corporate BAA	OASD (Yr)	0.0	6.038	-6.038	16.13	97.4	43.8	-4.434	16.8
USD Corporate Spread Slope	OASD*(TTM-AvgTTM) (Yr^2)	0.0	48.794	-48.794	0.17	8.49	11.22	-0.012	0.38
USD Corporate Liquidity	OASD*(OAS-AvgOAS) (Yr*%)	0.0	2.722	-2.722	6.98	18.99	11.53	-0.505	0.86
USD Foreign Corporates BAA	OASD (Yr)	0.0	5.963	-5.963	3.29	19.63	-15.71	0.324	-1.21
EMERGING MARKETS SPREAD									
Global EMG Investment Grade	OASD (Yr)	4.467	0.079	4.388	20.33	-89.23	-82.56	10.536	29.01
Global EMG Non-Distressed Slope	OASD*(TTM-AvgTTM) (Yr^2)	3.311	-0.106	3.417	0.92	-3.15	59.73	-0.346	-0.74
Global EMG Non-Distressed Liquidity	OASD*(OAS-AvgOAS) (Yr*%)	3.412	-0.015	3.428	11.05	-37.88	-74.56	5.171	11.12

Source: POINT.

Regarding idiosyncratic risk, POINT provides two reports. In the first one, "Portfolio Issue-Specific Risk Report", we conduct issue-level risk analysis for the major holdings of the portfolio. The second, "Credit Ticker Report" (Figure 18), presents a similar analysis but with all issues from the same issuer grouped together. So if an investor has offsetting positions on two different bonds from the same issuer, the overall "issuer" risk will be small.

Interestingly, the biggest source of idiosyncratic TEV is from bonds that are not in the portfolio: Mexican sovereign bonds (62.97bp/month). This issuer is an important part of the benchmark, and the fact that we do not carry it in the portfolio creates a large imbalance. Also note that the portfolio is highly concentrated in two issuers, AMBEV and PETBRA. These two account for 58% of the portfolio. This helps explain the large idiosyncratic error we reported above.

[17] *This uncorrelated measure is an artificial construction, in the sense that we keep all other factors constant. The number is useful to give an idea of the isolated exposure to a particular factor.*

Figure 18. Credit Ticker Report (Partial)

LEHMAN BROTHERS | POINT Global Risk Model

Credit Tickers **1/31/2005**

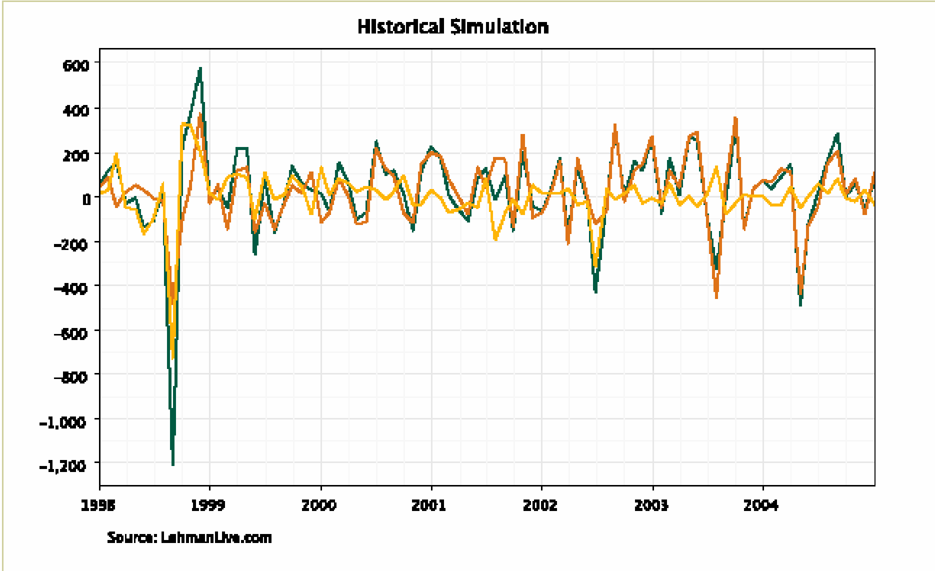
Portfolio : us em brazil
Benchmark : US Credit Non Corporate Baa

Ticker	Name	Sector	Rating	Currency	# Issues in portfolio	Portfolio weight (%)	Benchmark weight (%)	Net weight (%)	Net contribution to OASD (Yr)	Systematic TEV (bps)	Idiosyncratic TEV (bps)
MEX	UNITED MEX STATES-GLOBAL	SOVEREIGNS	BAA2	USD	0	0.0	56.21	-56.21	-3.921	116.6	62.97
AMBEV	CIA BRASILEIRA DE BEBIDAS	FOOD_AND_BEVERAGE	BAA3	USD	3	24.47	0.0	24.47	1.356	51.14	35.59
PETBRA	PETROBRAS INTL FINANCE-GLOBAL	FOREIGN_AGENCIES	BAA1	USD	5	34.01	2.37	31.63	1.049	37.76	27.53
BRADES	BANCO BRADESCO S.A.	BANKING	BAA1	USD	3	11.56	0.0	11.56	0.687	30.27	18.02
PEMEX	PEMEX FINANCE LTD	FOREIGN_AGENCIES	BAA1 BAA3	USD	0	0.0	16.93	-16.93	-0.878	26.79	14.16

Source: POINT.

Finally, POINT presents a "Historical Simulation Report" (Figure 19). The aim of this report is to help visualize the hypothetical systematic historical return of a portfolio with the same loadings. We use current loadings and past factor return realization to get that historical simulated return.

Figure 19. Historical Simulation Report (partial view)



Legend	Axis	31-Dec-97 to 31-Dec-04	Last	Minimum		Maximum		Mean	Std. Dev
	Left	Portfolio Est. Return	78.800	-1207.880	31-Aug-98	578.130	30-Nov-98	29.187	213.190
	Left	Benchmark Est. Return	112.250	-481.120	31-Aug-98	371.920	30-Nov-98	27.431	157.164
	Left	Difference	-35.650	-728.540	31-Aug-98	324.870	30-Sep-98	1.757	118.872

Source: POINT.

We can see from the graph that events such as the Russian crisis in 1998 had an extraordinary negative impact on Brazilian bonds. The general effect on the non-corporate Baa bonds is smaller, giving rise to a significant difference in performance. Other episodes, including the Brazilian elections in 2002, also caused high return differentials. This graph can be used to quantify the effect of similar stress events in the future. Note that the standard deviation of the difference between the two series (about 120bp/month) is close to the systematic TEV reported in Figure 16 above (126bp/month).

4. CONCLUSION

In this paper we describe the Lehman Brothers Emerging Markets Risk Model. The model includes three fully integrated types of risk: systematic, idiosyncratic and default risk. Systematic risk factors are based on specific regions: America, Asia and Europe. We further include country-specific risk factors when relevant, in both an economic and statistical sense.

Default correlations are modeled using a t-copula that allows for a higher level of tail dependence. The departure from the typical normal copulas is especially important in emerging markets, where tail events and credit contagion is more pronounced. Compared with similar asset classes, such as high yield, bonds from emerging markets tend to move more closely with one another, suggesting a stronger role for a macro-wide/sovereign risk factor.

Finally, we show how POINT delivers intuitive risk reports. These reports provide a detailed analysis of the major sources of risk. This exercise helps to identify and quantify the risk of the specific active exposures that managers choose for their portfolios.

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APPENDIX 1: COUNTRY – BLOCK MAPPING

In this appendix we show our mapping of specific countries to regional blocks. Note that the Asian block is in fact a residual bock, in the sense that all countries that do not fall into the America or Europe block are attributed to the Asian block. This attribution is marginal and does not affect the major results of the model. Moreover, note that this list of countries changes over time given that emerging market countries are defined as those with sovereign ratings of Baa3 or below.

Country	Block	Country	Block
Algeria	Asia	Jordan	Asia
Argentina	America	Kazakhstan	Asia
Brazil	America	Lebanon	Asia
Bulgaria	Europe	Mexico	America
Colombia	America	Morocco	Asia
Costa Rica	America	Nigeria	Asia
Croatia	Europe	Pakistan	Asia
Dominican Republic	America	Panama	America
Ecuador	America	Peru	America
Egypt	Asia	Philippines	Asia
El Salvador	America	Romania	Europe
Guatemala	America	Russia	Europe
India	Asia	Turkey	Europe
Indonesia	Asia	Ukraine	Europe
Iran	Asia	Uruguay	America
Ivory Coast	Asia	Venezuela	America
Jamaica	America	Vietnam	Asia

Base Correlation in the Large Homogeneous Portfolio with Jumps (LHPJ) Model[1]

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In this article, we show how the Large Homogeneous Portfolio with Jumps (LHPJ) model can help us understand the shape and dynamics of the base correlation skew. The key parameters of the model – asset correlation, idiosyncratic jump-to-default intensity and systematic jump-to-default intensity – influence the base correlation skew in an intuitive fashion. While the base correlation skew steepens with a higher systematic jump-to-default intensity, a higher idiosyncratic jump-to-default intensity generates a skew at the junior end of the base correlation curve. A lower value of asset correlation influences the level of the base correlation curve and creates a more pronounced skew. Using these effects, we outline the ways in which the LHPJ model can enable a more accurate interpretation of the base correlation skew and hence help form relative value views.

1. INTRODUCTION

In a previous article (Trinh, Thompson and Devarajan 2005) we presented *Asterion* – a framework that allows us to model in a simple way the presence of a "correlation smile" observed in the market for tranche products. The model renamed LHPJ, extends the framework of the Large Homogenous Portfolio (LHP) model to allow for jumps[2]. The presence of jumps creates fatter tails in the portfolio loss distribution which allow the model to capture the correlation smile commonly observed in the market. LHPJ uses three parameters: the flat asset return correlation, the market jump-to-default intensity and the idiosyncratic jump-to-default intensity.

In the past year, the market has moved from using compound correlation to using base correlation widely to quote standardised tranches. In the base correlation framework, each tranche is a portfolio of two equity tranches of a size equal to the attachment and detachment point of the tranche. A base correlation skew is the plot of equity compound correlation against the strike of each such equity tranche. Base correlation has several desirable properties: it produces unique solutions for mezzanine tranches and provides the ability to price non-standard tranches. Base correlation is also increasingly being used to price bespoke tranches (cf. O'Kane and Livesey 2004) because of such properties.

In this article, we show how LHPJ can help us to understand the shape and the dynamic of the base correlation curve. We show that the key model parameters – correlation, idiosyncratic and market jump risks – influence the base correlation skew in an intuitive manner, and that the level and the slope of the curve can be explained by the values of these parameters. While a higher systematic jump-to-default intensity steepens the base correlation skew, a higher idiosyncratic jump risk steepens the skew at the lower end of the capital structure. A lower value of asset correlation influences the level of the base correlation skew and makes it more pronounced.

The rest of the article is organized as follows. In section 2, we present a brief overview of the *LHPJ* model. In section 3, we discuss the impact of the key parameters of the model on the base correlation skew that the model generates. In sections 4 and 5 we provide the broad motivation for two potential uses of the model in conjunction with the base correlation skew – forming relative value views and arriving at interpolation-free values for the base correlation of non-standard tranches. We present our conclusions in section 6.

[1] *We thank Marco Naldi for his detailed comments and Bodhaditya Bhattacharya for his contribution to the article.*
[2] *We therefore rename it the 'Large Homogenous Portfolio with Jumps' model – or* LHPJ.

2. THE LHPJ MODEL

2.1. Basic model set-up

LHPJ builds on the LHP model[3] which assumes a large homogenous portfolio in which individual default is triggered by the asset value falling below a default barrier. In LHP, the asset value is driven by two Gaussian factors: a market factor and an idiosyncratic factor. These two factors are weighted by the asset correlation. We extend this model by allowing the possibility of idiosyncratic and systematic jumps-to-default for the collateral.

Consider a given time horizon T and a credit portfolio with m issuers. The default time τ_i of the i^{th} issuer is less than T if one of three events occurs:

1. a continuous random variable representing the asset value of the firm $A(i) = \beta(i)Z + \sqrt{1-\beta(i)^2}Z(i)$ falls below a barrier $C(i)$ where Z and $Z(i), i = 1,\ldots,m$ are i.i.d. standard normal variables; or
2. there is an idiosyncratic jump to default (modelled as the first jump time of a Poisson process $N(i,t)$ with an arrival rate $\lambda(i)$); or
3. there is a systematic jump to default (modelled as the first jump time of a Poisson process $N(t)$ with constant arrival rate λ). The difference between $N(i,t)$ and $N(t)$ is simply that when $N(t)$ jumps, all firms default simultaneously while the jump of $N(i,t)$ implies the default of the i^{th} firm alone.

All the probability distributions mentioned above and subsequently are those under the risk-neutral probability measure as we use these distributions in the pricing of tranche products.

Like the LHP model, LHPJ assumes a homogeneous portfolio. Thus, all β are assumed to be the same and β^2 is the correlation between any two assets. Also, the default barrier, and the default probabilities and recovery rates are identical across firms. We denote by N the notional amount of the portfolio and by R , the recovery rate.

3. BASE CORRELATION WITH THE LHPJ MODEL

3.1. The concept of base correlation

The standard method to quote tranches from CDX or iTraxx is generally that of "compound" correlation. The LHP pricing formulae are inverted to back up implied compound correlation numbers from the spreads of tranches. Each tranche is then assigned a compound correlation. In other words, the LHP model is used as one would use the Black-Scholes formula for options.

Recently however, another type of correlation has been proposed: "base" correlation. It is widely used in the market as it presents several advantages over compound correlation. The concept of base correlation is explained in O'Kane and Livesey (2004). In the base correlation framework, we focus only on the pricing of equity (or base) tranches. Each base tranche has a unique correlation that prices it under the LHP model. With this feature, the

[3] *See O'Kane* et al. *(2003),* Lehman Brothers Guide to Exotic Credit Derivatives *for a description of LHP. The model is due to Vasicek (1987).*

base correlation avoids the non-uniqueness of compound correlation with mezzanine tranches[4].

A value of a tranche is the difference between two adjacent base tranche values. We calculate the base correlation ρ_1 and ρ_2 for an equity tranche with width of θ_1 and a mezzanine tranche with attachment and detachment points of θ_1 and θ_2 by bootstrapping, i.e., we use information from the first tranche to solve for the second tranche. We proceed as follows:

1. We start with the equity tranche that pays a running spread S_{θ_1} and an upfront spread U_{θ_1}. We calculate the value of the correlation that sets the present value of the tranche to zero, i.e.

$$pv_{LHP}(\rho_1, \theta_1, S_{\theta_1}) = 0$$

where:

$$pv_{LHP}(\rho_1, \theta_1, S_{\theta_1}) = U_{\theta_1} + S_{\theta_1} pv_{LHP,premium}(\rho_1, \theta_1) - pv_{LHP,protection}(\rho_1, \theta_1)$$
$$= U_{\theta_1} + S_{\theta_1} \sum_{n=1}^{N} Q_{0,\theta_1}(t_n) \Delta_n Z(t_n) - \sum_{m=1}^{M} (Q_{0,\theta_1}(t_{m-1}) - Q_{0,\theta_1}(t_m)) Z(t_m)$$

Δ_n is the accrual factor between t_n and t_{n+1}

$Z(t_m)$ is the LIBOR discount factor at time t_m.

$Q_{0,\theta_1}(t_m)$ is the equity tranche survival probability. It is calculated from the tranche expected loss:

$$Q_{0,\theta_1}(t) = 1 - \frac{E_{\rho 1}^{LHP}\left[\min(L(t), \theta_1)\right]}{\theta_1}$$

The base correlation of the equity tranche is equal to its compound correlation.

2. We can now calculate the present value of a mezzanine tranche between θ_1 and θ_2 with a running spread S_{θ_1,θ_2} as the difference in present values of two adjacent equity tranches that pay the same contractual spread:

$$pv_{LHP}(\rho_1, \rho_2, \theta_1, \theta_2, S_{\theta_1,\theta_2}) = pv_{LHP}(\rho_2, \theta_2, S_{\theta_1,\theta_2}) - pv_{LHP}(\rho_1, \theta_1, S_{\theta_1,\theta_2})$$

The second base correlation ρ_2 is calculating by setting this present value to zero:

$$0 = S_{\theta_1,\theta_2} pv_{LHP,premium}(\rho_1, \theta_1) - pv_{LHP,protection}(\rho_1, \theta_1) - S_{\theta_1,\theta_2} pv_{LHP,premium}(\rho_2, \theta_2)$$
$$+ pv_{LHP,protection}(\rho_2, \theta_2)$$

In terms of survival probabilities, we can write the equation as:

$$0 = S_{\theta_1,\theta_2} \sum_{n=1}^{N} Q_{\theta_1,\theta_2}(t_n) \Delta_n Z(t_n) - \sum_{m=1}^{M} (Q_{\theta_1,\theta_2}(t_{m-1}) - Q_{\theta_1,\theta_2}(t_m)) Z(t_m)$$

The survival probability of the tranche is calculated from the expected losses of the two adjacent equity tranches:

[4] *Note that compound and base correlations are model specific and rely on LHP. However, this does not mean that the assumptions of the LHP model hold true in the market. This is analogous to the Black-Scholes framework which is used to quote equity option prices in terms of their "implied" volatilities – but equity prices do not necessarily follow a log-normal diffusion process.*

$$Q_{\theta_1,\theta_2}(t) = 1 - \frac{E_{\rho 2}^{LHP}\left[\min(L(t),\theta_2)\right] - E_{\rho 1}^{LHP}\left[\min(L(t),\theta_1)\right]}{\theta_2 - \theta_1} \tag{1}$$

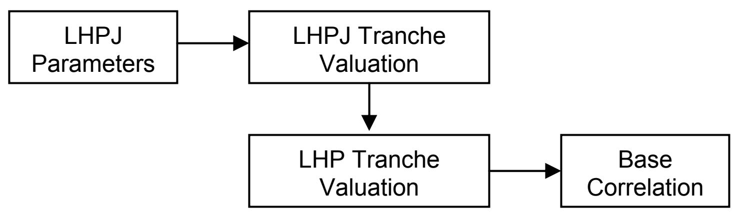
θ_1 and θ_2 are the attachment and detachment points of the tranches, and ρ_1 and ρ_2 are the base correlations corresponding to these attachment points. The equity expected losses are computed using LHP, and ρ_2 is computed assuming ρ_1 is known. It is in this sense that the base correlations are specific to the LHP model. The base correlation arrived at would be different if we were to use a model different from the LHP (for instance a heterogenous Gaussian model).

3.2. The base correlation from LHPJ

To arrive at a base correlation curve implied by the LHPJ model, we first use the key parameters of the model to arrive at breakeven tranche spreads. We then use these breakeven spreads in the LHP model to back out the corresponding base correlation skew. To continue the analogy with the options world, this is equivalent to pricing options in a model in which the price of the underlying asset follows a jump-diffusion process with some fixed parameters and backing out the implied volatility skew it generates using the Black-Scholes framework.

This process is summarized in the following flow-chart:

Figure 1. Base correlation through the LHPJ model



3.3. Effect of the parameters of the LHPJ model on the base correlation skew

In the following section we examine the effect of the three key parameters of the LHPJ model – the asset correlation, the idiosyncratic jump-to-default intensity and the market jump-to-default intensity – on the base correlation skew.

We assume values for the systematic jump intensity λ, the idiosyncratic jump intensity $\lambda(i)$, the default correlation ρ, the portfolio spread level, the recovery rate R, the interest rate r, and the maturity T of the portfolio as shown in Figure 2 below.

Figure 2. LHPJ parameter assumptions

Parameter	λ	λi	ρ	Spread	R	r	T
Value	7bp	7bp	20%	50bp	40%	4%	5 years

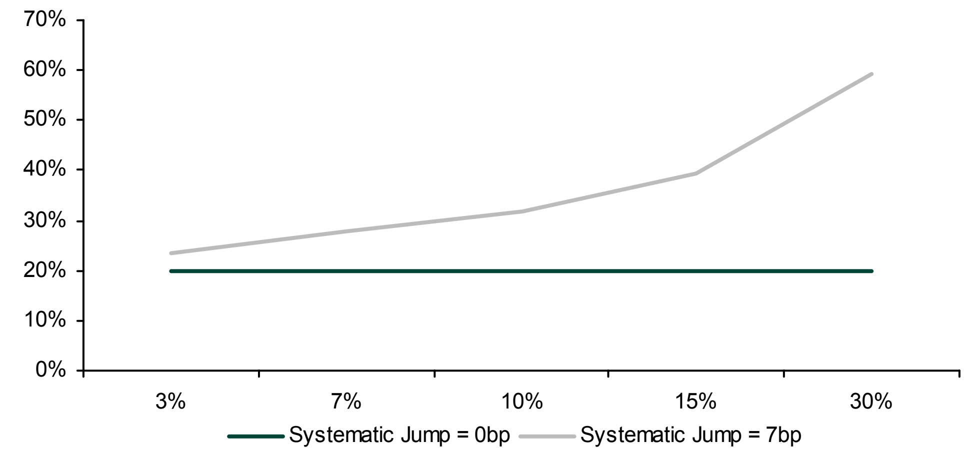
Source: Lehman Brothers.

We also use the attachment and detachment points of the standardized tranches on the DJ-CDX Investment Grade portfolio.

3.3.1. The effect of systematic jump-to-default risk on base correlation

In Figure 3, we show the impact of introducing a systematic jump-to-default risk. The flat line is the base correlation from the case where both the idiosyncratic and systematic jump-to-default intensities are set to zero, in other words the LHP case. We can see that when the systematic jump increases to 7bp, the base correlation skew becomes upward sloping.

Figure 3. Impact of increase of systematic risk on base correlation



Source: Lehman Brothers.

Figure 3 shows that when the systematic jump intensity is 7bp, the correlation implied by the junior-most equity tranche is higher than the base correlation implied when the systematic jump risk is 0bp. This can be explained as follows. With a higher level of systematic jump risk, the value of the equity tranche increases (as the equity tranche is long systematic jump-to-default risk, holding the level of collateral default probability constant – see Trinh, Thompson and Devarajan, 2005 for a detailed discussion). Therefore the base correlation of the equity tranche has to be higher with a higher level of systematic jump risk.

We also see in Figure 3 above that the introduction of systematic jump risk changes the shape of the base correlation skew to be upward sloping. The increase of the systematic jump risk changes the probability distribution of losses. It increases the survival probability of all tranches below the level of (100% – Recovery value).

The impact of this increase in the value of the base tranches on the base correlation depends on the sensitivity of their values to changes in correlation. As the size of the equity tranche becomes larger, the equity tranche increasingly becomes "in-the-money" because the detachment point is higher in the capital structure (i.e., θ is higher). Therefore, as the detachment point increases, the sensitivity of the value of the equity tranche to changes in correlation reduces[5].

Therefore, for a given increase in the value of the tranche, the base correlation needs to increase by a larger amount as the width of the equity tranche increases. In other words, the base correlation has to increase by greater amounts as we move down the capital structure (with larger equity size) to compensate for the systematic jump risk. This explains the upward sloping shape of the base correlation curve as described in Figure 3.

[5] *This is analogous to the fact that the impact of an increase in the volatility of the underlying asset on the value of a put falls as the put becomes more in-the-money.*

It is interesting to relate the base correlation curve to the level of tranche breakeven spreads. An increase in market jump risk gives a tighter spread for the equity tranche and a wider spread for the senior tranche. The base correlation has to start at a high level to reflect the tight equity spread, but then has to increase to reflect higher equity valuation at high strike levels and corresponding lower senior tranche valuation.

Figure 4. Impact of systematic jump-to-default risk on tranche breakeven spreads

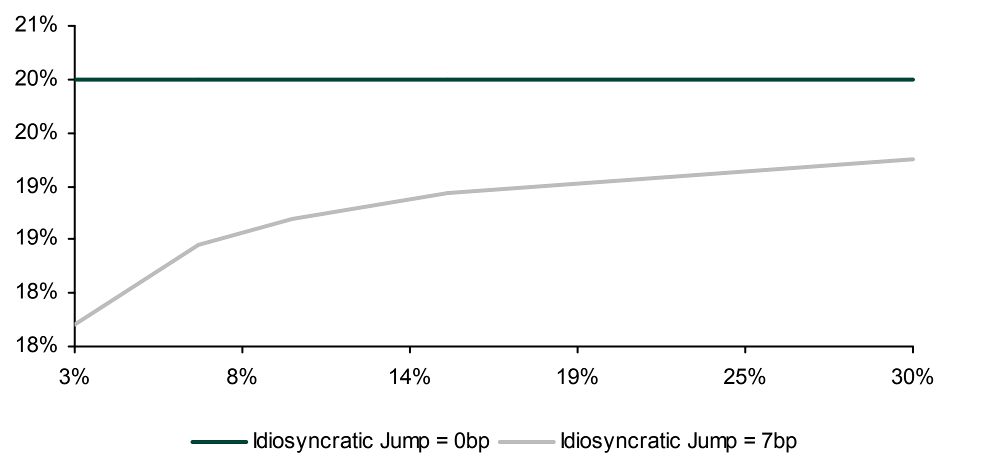
Tranche Spreads (bp)			
Attachment Point	**Detachment Point**	**Systematic Jump = 0bp**	**Systematic Jump = 7bp**
0%	3%	1859	1709
3%	7%	313	278
7%	10%	87	79
10%	15%	26	28
15%	30%	2	9

Source: Lehman Brothers

3.3.2. The effect of idiosyncratic jump-to-default risk on base correlation

The impact of the idiosyncratic jump-to-default risk can be seen in Figure 5 below. We can see that as the idiosyncratic jump-to-default intensity increases from 0bp to 7bp, the base correlation skew moves down and slopes upwards, with the largest decrease from the flat correlation (LHP) case at the junior-most equity tranche.

Figure 5. Impact of increase of systematic risk on base correlation



Source: Lehman Brothers.

The above figure can be explained as follows. As the idiosyncratic jump-to-default intensity increases from 0bp to 7bp, the value of all equity tranches would fall because the expected losses on the equity tranche rise (because the equity tranche takes the first loss of any single-name default). Therefore, the base correlation of the junior-most equity tranche using LHP has to be lower.

However, as total portfolio losses have to be preserved, the effect discussed above reverses beyond a certain level of subordination. Since the probability of single-name default is kept constant, beyond this level the default barrier shifts down as the probability of correlated default reduces to adjust for the increase in the probability of idiosyncratic default.

With a downward shift of the barrier, the probability of the tranche being wiped out with a move of the market decreases. Therefore, for larger equity tranches, the base correlation is higher than the flat correlation to reflect the increase in value. This is also why, starting from the first equity tranche, as the size of the successive equity tranches increases, the base correlation increases.

If we relate the base correlation curve to the level of tranche breakeven spreads, we see that the base correlation curve starts at a lower level to reflect the wider equity spread and then does not increase rapidly to reflect the tighter senior spread or higher valuation of the senior tranche.

Figure 6. Impact of idiosyncratic jump-to-default risk on tranche breakeven spreads

Tranche Running Spreads (bp)			
Attachment Point	**Detachment Point**	**Idiosyncratic Jump = 0bp**	**Idiosyncratic Jump = 7bp**
0%	3%	1859	1963
3%	7%	313	292
7%	10%	87	77
10%	15%	26	23
15%	30%	2	2

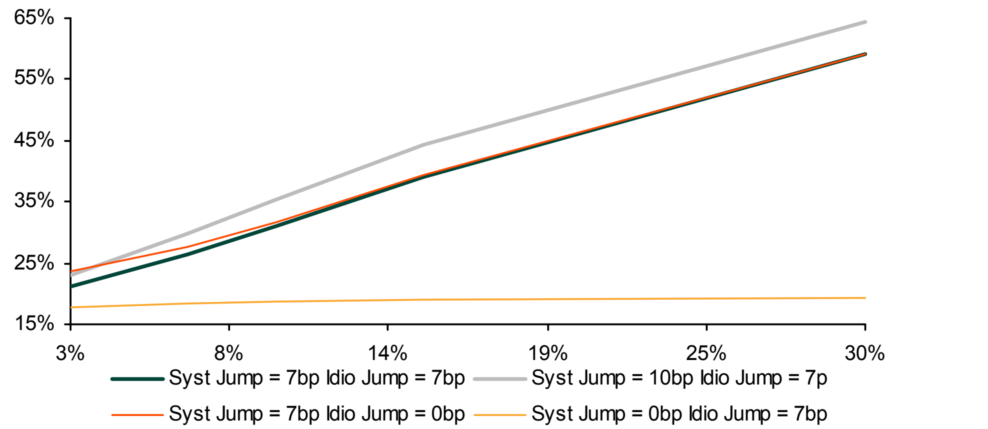
Source: Lehman Brothers.

3.3.3. The combined effect of market and idiosyncratic jump risks on base correlation

We examine the combined effect of the systematic jump risk and idiosyncratic jump risk on the base correlation skew. In Figure 7, we fix the idiosyncratic jump-to-default intensity at 7bp and increase the systematic jump-to-default intensity from 7bp to 10bp. In the case where the systematic jump-to-default intensity is 7bp, the base correlation skew lies between those when there is no systematic jump risk and no idiosyncratic jump risk. We see that as the systematic jump risk increases, the base correlation skew moves upward.

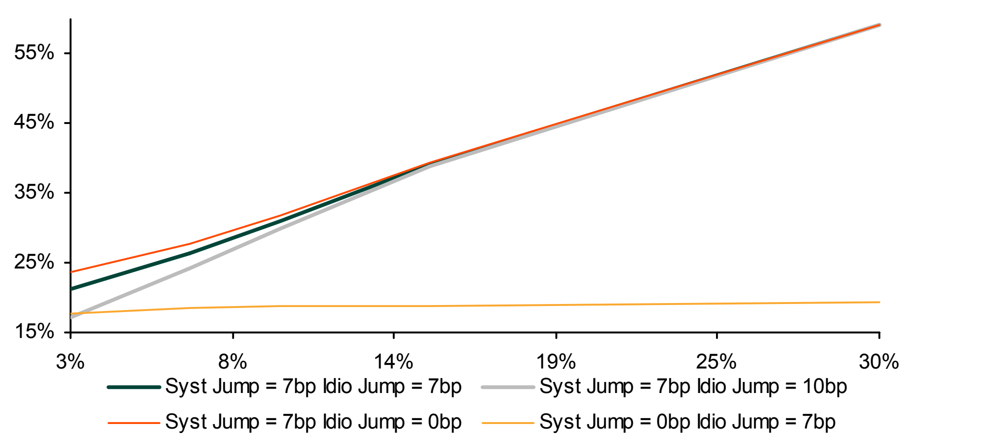
In Figure 8, we fix the systematic jump-to-default intensity at 7bp and increase the idiosyncratic jump-to-default intensity from 7bp to 10bp. We can see that the curve shifts down because the effect of idiosyncratic jump-to-default risk is to lower the overall base correlation at the junior end of the capital structure, and it does not increase the base correlation as much as the systematic jump risk on the more senior end. As a result, the whole base correlation skew moves down.

Figure 7. Impact of increased systematic jump-to-default risk in the presence of idiosyncratic jump-to-default risk on the base correlation skew



Source: Lehman Brothers.

Figure 8. Impact of increased idiosyncratic jump-to-default risk in the presence of systematic jump-to-default risk on the base correlation skew



Source: Lehman Brothers.

We can see similar effects on the breakeven spreads of tranches. In Figure 9 below the equity spread is tighter and the senior spread is wider with an increase in systematic jump risk.

Figure 9. Spread impact of increased systematic jump-to-default risk in the presence of idiosyncratic jump-to-default risk

Tranche Running Spreads (bp)			
Attachment Point	**Detachment Point**	**Sytematic Jump = 7bp**	**Sytematic Jump = 10bp**
0%	3%	1805	1737
3%	7%	256	242
7%	10%	69	67
10%	15%	25	26
15%	30%	9	11

Source: Lehman Brothers.

In Figure 10 below, we see that the spread level is wider for the equity tranche and translates into a lower level of base correlation. The higher idiosyncratic risk has little impact on senior tranche spreads and thus the base correlation is almost unchanged at these levels.

Figure 10. Spread impact of increased idiosyncratic jump-to-default risk in the presence of systematic jump-to-default risk

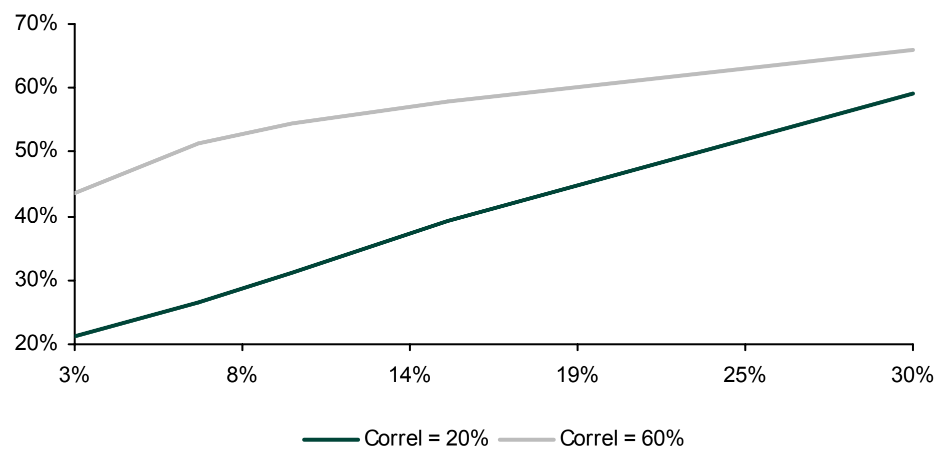
Tranche Running Spreads (bp)			
Attachment Point	**Detachment Point**	**Idiosyncratic Jump = 7bp**	**Idiosyncratic Jump = 20bp**
0%	3%	1805	1992
3%	7%	256	214
7%	10%	69	52
10%	15%	25	18
15%	30%	9	8

Source: Lehman Brothers.

3.3.4. The effect of asset correlation on the base correlation skew

In Figure 11, we hold the both the idiosyncratic and systematic jump-to-default intensities at 7bp and vary the asset correlation from 20% to 60%. We see that a higher asset correlation shifts the base correlation skew up. This is consistent with high asset correlation being similar to high systematic risk relative to idiosyncratic risk.

Figure 11. Impact of asset correlation on the base correlation skew



Source: Lehman Brothers.

In Figure 12, we see that the lower correlation causes a fall in value of the equity tranche and a transfer to the more senior tranches. The base correlation has to decrease towards the equity tranche to reflect a lower equity tranche valuation.

Figure 12. Tranche breakeven spreads with different correlation (spread=50bp; λ(i)=7bp; λ=7bp; β=√20% or √60%)

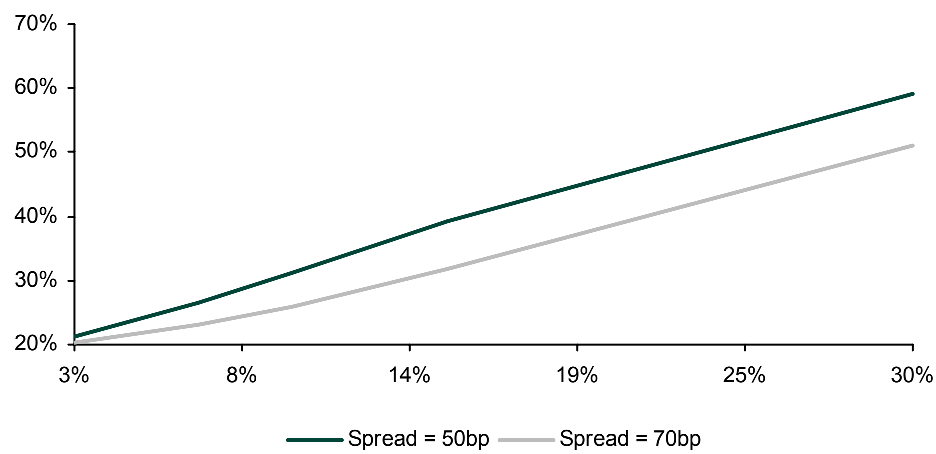
Tranche Running Spreads (bp)			
Attachment Point	**Detachment Point**	**Correlation = 20%**	**Correlation = 60%**
0%	3%	1805	849
3%	7%	256	269
7%	10%	69	163
10%	15%	25	107
15%	30%	9	49

Source: Lehman Brothers.

3.3.5. The effect of default probabilities on base correlation

In Figure 13, we hold the idiosyncratic and systematic jump-to-default intensities at 7bp and the asset correlation at 20% and vary the portfolio spread from 50bo to 70bp. We can see that a higher default probability moves the curve downward. This is because the higher default probability affects all the tranches simultaneously and alleviates the systematic jump market effect through the default barrier (which is moving up). The expected losses of the tranches are thus relatively more affected by the normal LHP market default effect than by the systematic jump effect.

Figure 13. Base correlation curve with different default probabilities (spread=50bp or 70bp; λ(i)=7bp; λ=7bp; β=√20%)



Source: Lehman Brothers

We see in Figure 14 that a higher overall level of spreads impacts all the tranches simultaneously but clearly more the junior tranches. The base correlation moves lower to reflect the lower valuation of equity tranches.

Figure 14. Tranche breakeven spreads with different default probabilities (spread=50bp or 70bp; λ(i)=7bp; λ=7bp; β=√20%)

Tranche Running Spreads (bp)			
Attachment Point	**Detachment Point**	**Spread = 50bp**	**Spread = 70bp**
0%	3%	1805	2639
3%	7%	256	488
7%	10%	69	155
10%	15%	25	56
15%	30%	9	12

Source: Lehman Brothers

4. RELATIVE VALUE WITH THE LHPJ BASE CORRELATION SKEW

4.1. Interpretation of the base correlation curve

As discussed above, the key parameters of the LHPJ model can be used to interpret the level and slope of the base correlation skew. We summarize these effects in Figure 15 below[6].

Figure 15. Impact of higher LHPJ parameters on base correlation

LHPJ Parameters	Level of Base Correlation	Slope of Base Correlation
Spreads	Lower	Flatter
Asset Correlation	Higher	Flatter
Systematic Jump Risk	Higher	Steeper
Idiosyncratic Jump Risk	Lower	Steeper (junior end)

Source: Lehman Brothers

Figure 15 presents several insights into the interpretation of the level and slope of the base correlation skew. First, the asset correlation can remain constant while the base correlation shifts according to the jump risks or the level of spreads. Second, the slope of the base correlation curve indicates the presence of tail risk: a high overall slope reflects high systematic jump risk, and steepness towards the low strike values reflects higher idiosyncratic jump risk.

The standard interpretation of the base correlation is the following: since in the LHP model the equity tranche is long correlation, any movement in the base correlation curve translates into a relative change in value of the base equity tranche; the shape of the base correlation curve indicates value transfer between the different tranches: a flattening (steepening) base correlation curve shows that the equity tranche is becoming more expensive (cheaper) relative to the senior tranche, and there is value in investing in more senior (junior) tranches[7].

The above effects show that this interpretation (steep base correlation curve = equity cheap) can sometimes be misleading. For instance, the base correlation curve can be steeper because spreads move lower without any impact on the relative valuation between the different tranches. Alternately, a higher systematic jump risk favors the equity tranche and makes the correlation curve steeper, but in such a case the equity tranche would be expensive (the spread is tight).

[6] *The level is defined as the arithmetic average of the base correlations for the different strikes. The slope is the difference of base correlation for the highest strike and the base correlation of the lowest strike.*

[7] *We use the term "value" to loosely mean a higher spread, even though the spread might just reflect a higher risk.*

4.2. Using LHPJ to interpret the base correlation skew

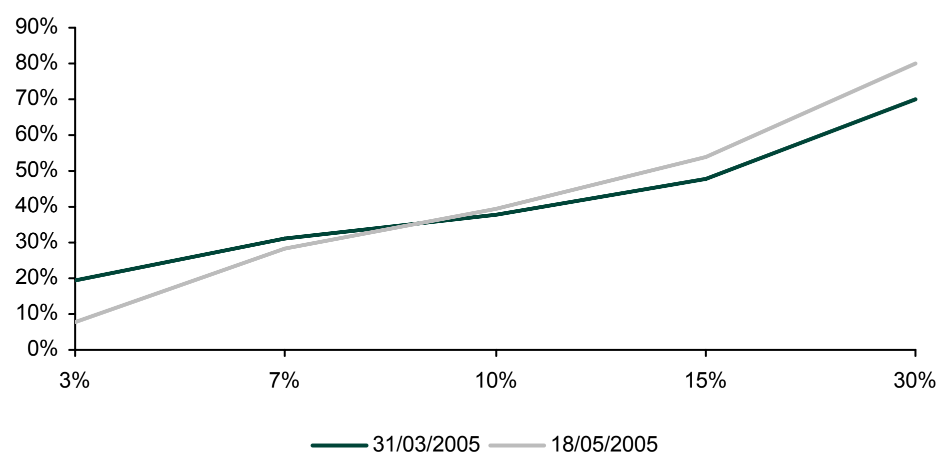
The key advantage of the LHPJ model is that it is able to replicate the shape of the base correlation skew observed in the market using three intuitive parameters. As we outlined in an earlier article (Trinh, Thompson and Devarajan 2005), these parameters can help form relative value views between tranches.

Below, we extend the discussion and outline some relative value views that can be made using the model in conjunction with the base correlation curve. This can be done in two ways:

- Due to the fact that LHPJ has relatively few parameters, not all points in the base correlation skew observed in the market would be fitted by the model exactly. A discrepancy in the fitted base correlation skew and the observed base correlation skew could indicate relative value opportunities.
- The parameters of the model can be used to interpret the level and slope of the observed base correlation skew. Such analyses could be driven by investors' views on the values of the parameters.

To illustrate, we examine the movement of CDX tranches in 2005. In May 2005, the market witnessed a sharp widening of the spread of the underlying portfolio and that of the tranches. In particular, the 0-3% (the junior-most equity) tranche widened to unprecedented wide levels of upfront spreads. This led to a significant steepening of the base correlation skew as shown in Figure 16 below.

Figure 16. Steepening of base correlation skew between March and May 2005



Source: Lehman Brothers.

The standard interpretation of this steepening would have prompted a long equity tranche trade hedged with the senior tranche – which would have been in line with the assumption of heightened idiosyncratic risk. However, this would have ignored the impact of systematic risk in the market.

In Figure 17 below, we show the actual values and the percentile ranks (vis-à-vis the history of parameter values) of the key parameters of the LHPJ model. We can see that the systematic jump intensity had increased significantly over the period, which caused the steepening of the base correlation curve. Given that the senior tranche is short systematic jump-to-default risk, a short senior trade may not have been attractive.

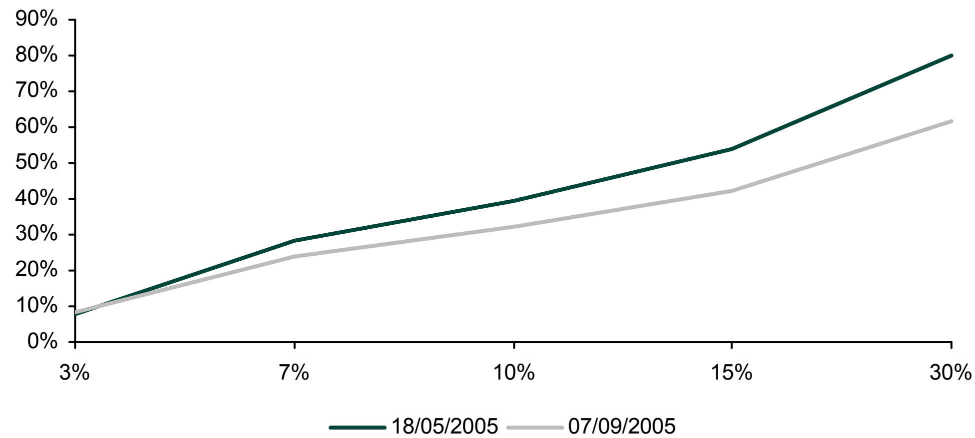
Figure 17. LHPJ parameter values – March and May 2005

	31st March 2005		18th May 2005	
	Actual (bp)	Percentile	Actual (bp)	Percentile
Systematic Risk	9	43%	22	99%
Idiosyncratic Risk	18	11%	54	99%

Source: Lehman Brothers.

Following subsidence of the perception of idiosyncratic risk since May, the spreads of both the CDX and the tranches have compressed. The base correlation skew has flattened, but with the 3% level as the pivot as shown in Figure 18 below.

Figure 18. Steepening of base correlation skew in May and September 2005



Source: Lehman Brothers.

This could suggest that there has been a compression of systematic risk (which has led to the flattening) – but the idiosyncratic risk is still high (reflected by the fact that the 3% base correlation has remained more or less unchanged and that the junior end of the skew has not flattened much). This is reflected in the values of the parameters of the LHPJ model as shown in Figure 19 below. Although both idiosyncratic and systematic jump-to-default intensities have fallen, the value of the idiosyncratic jump-to-default risk is still high vis-à-vis its history.

Figure 19. LHPJ parameter values – May and September 2005

	18th May 2005		7th September 2005	
	Actual (bp)	Percentile	Actual (bp)	Percentile
Systematic Risk	22	99%	10	54%
Idiosyncratic Risk	54	99%	43	78%

Source: Lehman Brothers.

The two snapshots presented above illustrate how the parameters of the LHPJ model can be used to interpret the level and slope of the base correlation skew more accurately and therefore help form relative value views.

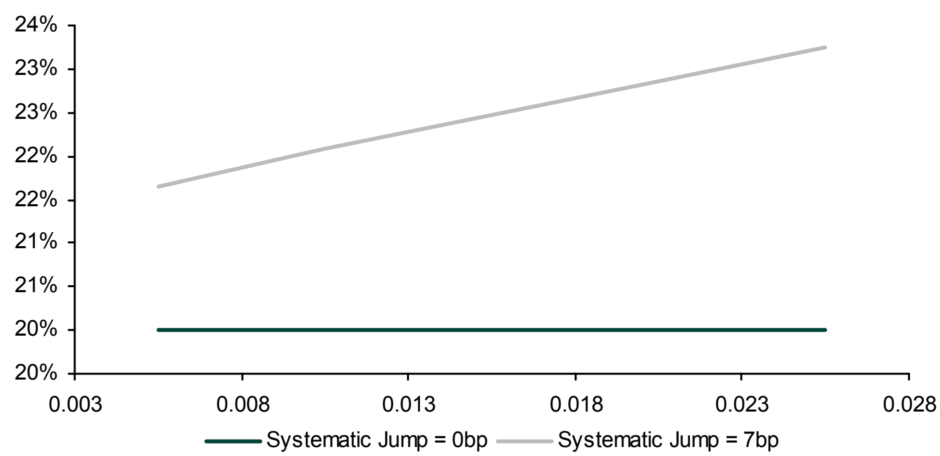
4. BASE CORRELATION FOR NON-STANDARD TRANCHES

We have mentioned that the base correlation approach is popular because it can potentially price bespoke CDO tranches. We now consider the use of LHPJ to generate such base correlation for non-standard tranches.

In general, to obtain a base correlation "skew", the LHP model is calibrated to market quotes of standard attachment points and interpolated between these standard points using methods such as cubic splines. The method used becomes important when interpolating between or extrapolating outside the quoted detachment points to price non-standard tranches. For example, if tranches are thinner than the market equity size, there is uncertainty as to the right extrapolation method. Several questions could be asked, such as whether extrapolation should be linear and whether concavity should be added.

Using a model such as LHPJ provides a solution to such problems as it helps arrive at a market implied loss distribution. This loss distribution could in theory be used for all maturities[8] and all levels of subordination. The model therefore helps generate a base correlation for all equity tranches of positive size. In other words, the model provides an alternative scheme of extrapolation which is arbitrage free across the capital structure. The advantage is that it does not rely on any interpolation of the market base correlation curve. Figure 20 shows the base correlation skew for thin tranches[9].

Figure 20. Base correlation for very thin tranches (0.5% to 2.5%)



Source: Lehman Brothers.

7. CONCLUSION

We have explained how LHPJ can help us understand the shape and the dynamic of the base correlation curve. The key parameters of the model – the asset correlation, idiosyncratic jump-to-default intensity and the systematic jump-to-default intensity – influence the base correlation in an intuitive fashion. We show that the LHPJ model can be useful in conjunction with the base correlation skew in two ways. First, the parameters of the model help us to interpret changes in the level and slope of the base correlation skew more accurately. Second, the model can be used as an interpolation-free method of arriving at base correlation values for non-standard tranches.

[8] *However, the jump-to-default intensities could have a term structure – which would provide different loss distributions for different maturities implied by market quotes across maturities.*

[9] *However, the LHPJ has few parameters and ideally should not be used as a tool for pricing tranches.*

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Pricing High Yield CDX Swaptions

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Options on investment-grade CDX are usually priced under the assumption that spread volatility is constant. In high yield CDX, some investors prefer to specify price volatility and assume that it is constant. In this note, we explain how to use the Black-Scholes formulas to price options on CDX.HY from price volatility. The approach requires that we calculate a forward price. We explain two different ways of doing so.[1]

INTRODUCTION

The choice between using a spread or price volatility model is primarily a question of deciding how best to describe the stochastic behavior of the value of the underlying CDX swap. In the price volatility model we describe below, we assume that the price volatility is constant. This is the same as saying that the standard deviation of the percentage change in the quoted price is constant,[2] or that the quoted price is lognormally distributed, which roughly corresponds to a normally distributed spread.

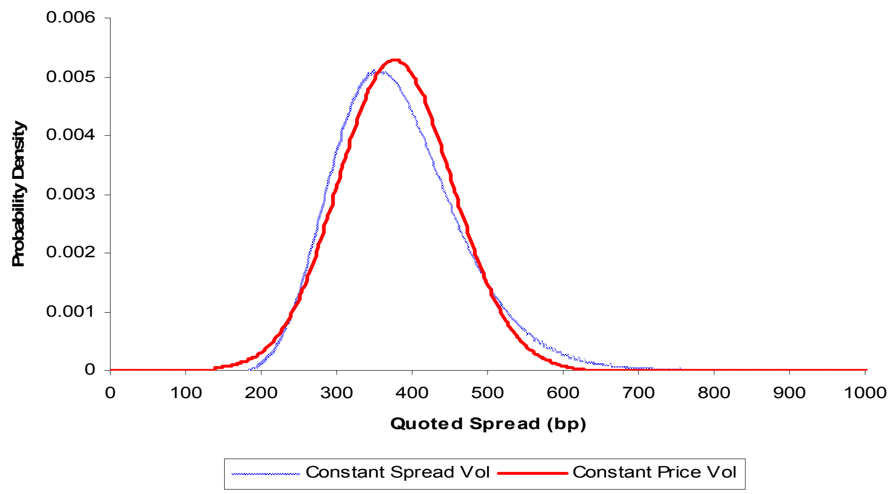
Alternatively, we could assume that the spread volatility is constant, i.e., that the spread is lognormally distributed. We have previously explained (see Pedersen (2003)) how to price CDX swaptions under that assumption. Figure 1 illustrates the two different distributional assumptions. It is clear that the constant price volatility model places more probability on low spread (i.e., high price) outcomes. In fact, in this model, there is positive probability that the spread becomes negative, although it is small for high yield. The constant spread volatility model, on the other hand, places greater probability on high spread outcomes.

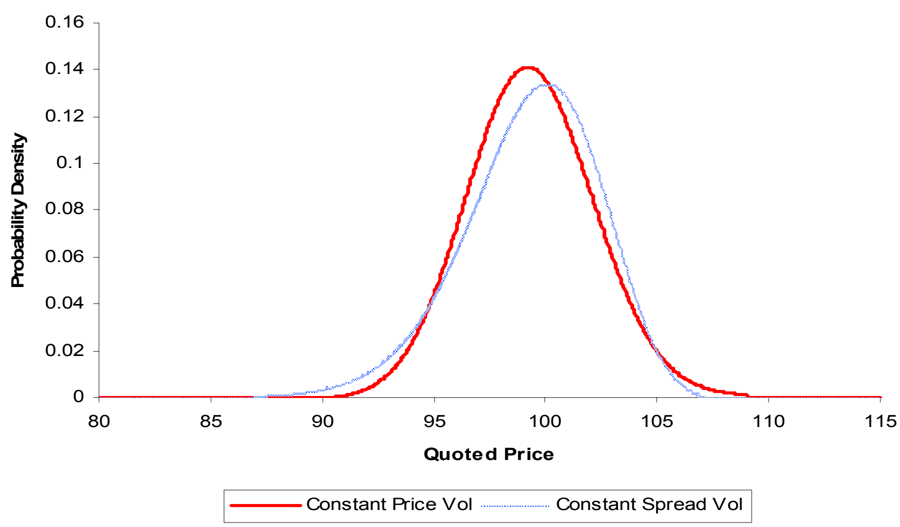
To price options on CDX assuming that the quoted price is lognormally distributed, the first step is to find the forward price associated with the option we want to value. This forward price is then plugged into a Black-Scholes formula, together with a price volatility input, to arrive at the value of the option. We first explain an easy approximate calculation of the forward price that works well in practice. We later explain a more precise calculation.

We call an option to buy protection a payer swaption, and an option to sell protection a receiver swaption. In high yield, it is common to talk about put and call options rather than payer and receiver swaptions. A put option is an option to go short credit risk and expresses a bearish view on the credit (i.e., a payer swaption), whereas a call option is an option to go long credit risk and expresses a bullish view on the credit (i.e., a receiver swaption). In Pedersen (2004), there is a detailed explanation of this terminology.

[1] *We would like to thank Jock Jones, Marco Naldi, and Jeremiah Stafford for comments and suggestions.*
[2] *The quoted price is par minus the upfront cost of buying protection in the underlying CDX contract.*

Figure 1. Distributions of quoted spread and price for 5-year CDX.HY.4 on September 20, 2005, as seen from July 15, 2005. Spread volatility is 50%; price volatility is 6.6%, which gives consistent pricing of at-the-money swaptions





Source: Lehman Brothers.

PRICING EXAMPLES

On July 15, 2005, our trader sent the Bloomberg message in Figure 2 with price quotes on September 20, 2005, expiry options on CDX.HY.4. The implied price volatilities are calculated using the model explained in this note. The implied spread volatilities are calculated using the model from Pedersen (2003).

Both the implied price and spread volatility decrease as we increase the strike price (decrease the strike spread). This implies that the market expects volatility to be higher when spreads are high than when spreads are low.

Figure 2. Market in CDX.HY.4 swaptions on July 15, 2005

7/15 10:55edit&fwd by •JEREMIAH STAFFORD, LEHMAN BROTHERS, INC 212-526-7299
originated by JEREMIAH STAFFORD of LEHMAN BROTHERS, INC NEW YORK

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Lehman HY CDX Price Strike Options 09/20/05 Expiry, Ref 100.00

Strike Price / Spd	Option Call/RTR	Option Put/RTP	Implied Vol Price / Spd	Delta Call / Put
97.00 / 442	2.67 / 2.86	0.35 / 0.54	7.69% / 51%	74% / 26%
97.50 / 428	2.28 / 2.49	0.46 / 0.66	7.59% / 51%	69% / 31%
98.00 / 414	1.87 / 2.09	0.55 / 0.77	7.19% / 49%	64% / 36%
99.00 / 387	1.19 / 1.42	0.85 / 1.09	6.71% / 47%	52% / 48%
99.50 / 373	0.87 / 1.11	1.03 / 1.27	6.33% / 45%	45% / 55%
100.00 / 360	0.56 / 0.78	1.22 / 1.44	5.68% / 41%	37% / 63%
100.50 / 347	0.34 / 0.54	1.50 / 1.70	5.32% / 39%	28% / 72%
101.00 / 334	0.15 / 0.31	1.81 / 1.97	4.7% / 35%	19% / 81%

*$25mm x $25mm with delta exchange
*European exercise,physical delivery, no knockout

Source: Lehman Brothers.

CALCULATING THE FORWARD PRICE

The first step in pricing an option is to price the underlying forward. In our context, this means calculating the forward price.

A simple approach to calculating a forward price is to forget about credit risk and use an approach similar to the way we would calculate a forward price for a Treasury bond. Later, we explain a more precise way to calculate the forward price. The approximation explained in this section is simpler to calculate and works well in practice.

Suppose P = 100 was the quoted price on 5-year CDX.HY.4 as of July 15, 2005, and we wanted to price a September 20, 2005, maturity option on that date. The coupon, or fixed rate, on 5-year CDX.HY.4 is 360bp.

Suppose we sell protection at 100 and hold this position until September 20, 2005, in an attempt to replicate a forward contract. Suppose that there is no credit risk and we know for certain that we will receive a full accrual premium of 92/360·360bp = 92bp on September 20, 2005. 92/360 is the accrual factor between June 20, 2005, and September 20, 2005 (92 days in between). The CDX contracts trade with accrued premium and the protection seller must pay 25/360·360bp = 25bp on July 15, 2005 (ignoring the 1-day settlement convention).

The convention in high yield CDX is to quote a price that is 100 minus the upfront payment. This means that, as protection sellers, we will receive on July 15, 2005:

Cash flow on July 15, 2005 = 100 – 100 – 0.25 = -0.25

where -0.25 is the accrued premium. On September 20, 2005, we will receive a full coupon payment of (see explanation above):

Cash flow on September 20, 2005 = 0.92

Now, suppose we have a money market account in which we can deposit the cash flow we receive. Suppose the LIBOR discount factor from July 15, 2005 to September 20, 2005 is 0.9935. The balance in the account after the deposit on September 20, 2005 is then:

Balance on September 20, 2005 = -0.25/ 0.9935 + 0.92 = 0.67

If we use the same convention for quoting the forward price as we use for quoting the spot price, we see that we have replicated a forward with a quoted forward price of:

Forward Price = 100 – 0.67 = 99.33

The two assumptions made in the approximation are:

1. There is no risk of a decrease in premium payments due to defaults occurring before option maturity.
2. Protection payments for defaults that occur before option maturity are assumed to be settled at default. In fact, they will be settled at option maturity.

The effect of assumption 2 is negligible, whereas assumption 1 has a small effect especially for longer-maturity options.

We can address assumption 1 using a back-of-the-envelope calculation. The price quote of 100 corresponds to a spread of 360bp. If we use a recovery rate of 40%, the 360bp spread corresponds to an annual default probability of approximately 3.60%/0.6 = 6% and, thus, a survival probability of 94%. The time to maturity in years is 0.18, so the survival probability to the maturity date is approximately $0.94^{0.18} = 0.9889$. This means that the expected premium payment on September 20, 2005, is not 0.92, but 0.9889·0.92, with the effect of increasing the forward price by 0.01.

PRICING THE OPTIONS

We can now calculate the option values by plugging the forward price into the Black-Scholes formulas. The payer swaption is a put option on the price, and the receiver swaption is a call option on the price.

$$\text{Payer Swaption} = D(0,T)\cdot\left(K\cdot N(-d_2) - \text{FwdPrice}\cdot N(-d_1)\right)$$

$$\text{Receiver Swaption} = D(0,T)\cdot\left(\text{FwdPrice}\cdot N(d_1) - K\cdot N(d_2)\right)$$

$$d_1 = \frac{\log(\text{FwdPrice}/K) + \sigma^2 T/2}{\sigma\sqrt{T}} \text{ and } d_2 = d_1 - \sigma\sqrt{T}$$

where D(0,T) is the LIBOR discount factor from today to option maturity, and K is the strike price. T is the option maturity, or more specifically, the time to maturity in years. σ is the forward price volatility. N(·) is the standard normal distribution function, and log(·) is the natural logarithm.

ALTERNATIVE CALCULATION OF THE FORWARD PRICE

In this section we explain an alternative way of calculating the forward price. The calculation illustrates the relationship between the forward price and the adjusted forward spread we use for CDX.IG swaptions (see Pedersen and Chen (2005)).

The relationship between quoted price and spread is:

$$P = 100 - \text{PV01}(S)\cdot(S - C) \tag{1}$$

P is the price and S is the spread. C is the contractual coupon, or fixed rate, in the underlying swap contract. The coupon for 5-year CDX.HY.4 is C = 360bp. PV01(S) is the value of a 1bp premium flow in the underlying swap per par ($100) notional. The valuation of PV01(S) is done using a flat credit curve with spreads equal to S. For example, on July 15, 2005, a spread of 360bp gives a PV01 of 3.72 and a price of 100 – 3.72·(3.60 – 3.60) = 100. Conversion from spread to price can be done with the LehmanLive Credit Default Swap Calculator.

A position consisting of a long payer swaption and short receiver swaption both with the same strike price (and option and swap maturities) is economically equivalent to a forward contract that guarantees that protection will be bought at option maturity and a fixed cost (determined by the strike price). For given option and swap maturities, denoted T and T_M, respectively, we define the forward price to be the strike for which this position has a value of 0.

Before we can calculate the forward price, we must find a representative credit curve that we can use with a standard model (such as the one underlying the LehmanLive CDS Calculator) to value credit risky cashflow. Because the value of a CDX swaption is relatively insensitive to the shape of the credit curve (see Pedersen and Chen (2005)), it is usually acceptable to use a flat credit curve. In this case, we simply need to convert the quoted price into a single spread using equation (1) above.

The forward consists of two legs: the protection leg and the premium leg. Payment on the premium leg begins at T and ends at T_M. It is important to recognize that the premium payments are credit risky in the CDX contract. This is because the notional is decreased each time a credit in the index portfolio defaults. We can write the value of the premium leg as:

$$\text{Premium Leg} = C \cdot PV01(T,T_M) \tag{2}$$

where C is the coupon, or fixed rate, and $PV01(T,T_M)$ is the standard knockout forward PV01 as determined from the representative credit curve discussed above. This valuation is done as if there is a single credit and premium payments stop when that credit defaults. The two arguments (T,T_M) are used to indicate that premium is accrued only between those two dates.

The protection leg in the forward is essentially the same as in the underlying swap. This is because CDX swaptions are non-knockout. This means that if an option to buy (or sell) protection is exercised, protection must be bought (or sold) on all credits, including the defaulted ones. A slight difference arises from the fact that protection for defaults that occur before time T are not paid until time T. We can write the value of the protection leg as

$$\text{Protection Leg} = FEP(0,T) + PVP(T,T_M) \tag{3}$$

FEP(0,T) is the value of front-end protection from time 0 to time T. This is the value of a contract that pays (1-R) at time T, where R is the assumed recovery rate, if default occurs before time T. $PVP(T,T_M)$ is the value of a standard knockout forward protection leg. This is the value of a contract that pays nothing if default occurs before time T, and pays (1-R) at the time of default if default occurs between times T and T_M.

We can now write the value of the forward as:

$$\text{Value of Forward} = \text{Protection Leg} - \text{Premium Leg} - (100 - K) \cdot D(0,T)$$

$$= PVP(T,T_M) + FEP(0,T) - C \cdot PV01(T,T_M) - (100 - K) \cdot D(0,T)$$

where K is the strike price and D(0,T) is the risk-free (LIBOR) discount factor from time 0 (today) to time T (option maturity). The convention in high yield CDX is that if the strike price is K, then the actual cash payment at exercise is 100 – K. If we put Value of Forward equal to 0 and solve for K, we get the expression for the forward price:

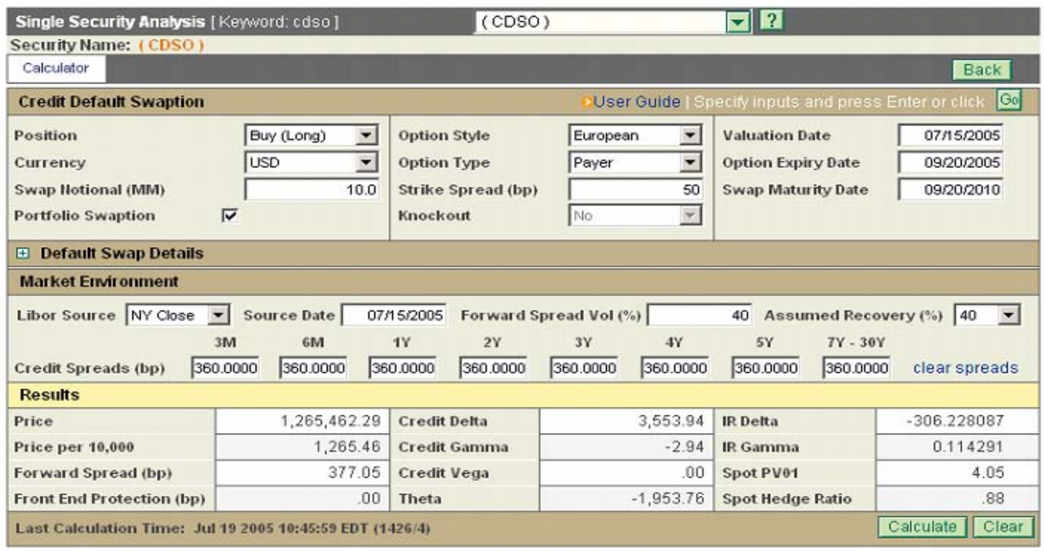
$$\text{Forward Price} = 100 - \frac{PVP(T, T_M) + FEP(0, T) - C \cdot PV01(T, T_M)}{D(0, T)} \tag{4}$$

Example

The LehmanLive Credit Default Swaption Calculator (keyword: cdso) uses a spread-based model that takes as inputs a spread strike and a spread volatility. Although we cannot currently use the calculator to price an option directly using the constant price volatility approach described above, we can use it to find the forward price, which we can then plug into the Black-Scholes formulas above to find the option values.

We did a valuation on July 15, 2005, using a flat credit spread curve of 360bp (corresponding to a quoted price of 100) with a 40% recovery rate. The option and swap maturities were set to September 20, 2005, and June 20, 2010. We assumed a coupon of 360bp.

Figure 3. Using the LehmanLive Credit Default Swaption Calculator (keyword: cdso) to calculate forward spread, front-end protection and adjusted forward spread



Source: Lehmanlive.

First, we make sure that the "Portfolio Swaption" option is not turned on, the "Knockout" field is set to "No," and we are pricing a payer swaption. We record two numbers:

Forward Spread = 360.02bp

Front-End Protection = 65.91bp

We then repeat the calculation turning on the "Portfolio Swaption" option and record the forward spread, which is an adjusted forward spread (see Pedersen and Chen (2005)).

Adjusted Forward Spread = 377.05bp

From these three numbers, we can calculate the forward price. Using the same notation as above, we have:

$$\text{Forward Spread} = \frac{\text{PVP}(T, T_M)}{\text{PV01}(T, T_M)}$$

$$\text{Adj. Forward Spread} = \frac{\text{PVP}(T, T_M) + \text{FEP}(0, T)}{\text{PV01}(T, T_M)}$$

$$\text{Front End Protection} = \text{FEP}(0, T)$$

We can now solve for $\text{PV01}(T,T_M)$:

$$\mathrm{PV01(T, T_M)} = \frac{\text{Front End Protection}}{\text{Adj. Forward Spread - Forward Spread}} = 3.87$$

Using equation (4), we get the forward price:

$$\text{Forward Price} = 100 - \frac{\mathrm{PVP(T, T_M) + FEP(0, T) - C \cdot PV01(T, T_M)}}{\mathrm{D(0, T)}}$$

$$= 100 - \frac{(\text{Adj. Forward Spread} - \mathrm{C}) \cdot \mathrm{PV01(T, T_M)}}{\mathrm{D(0, T)}}$$

$$= 100 - \frac{(3.7705 - 3.60) \cdot 3.87}{0.9935} = 99.34$$

where the discount factor D(0,T) = 0.9935 must be calculated elsewhere. Notice that the forward price is very close to the approximate forward price calculated above.

To price the swaption, we then enter the forward price and price volatility into the Black-Scholes formulas from the previous section.

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An Introduction to Recovery Products

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Recovery products allow investors to separate default risk from recovery risk. We introduce two basic contracts—recovery locks and fixed-recovery credit default swaps—and show that their prices must be related to the market price of a standard credit default swap. After discussing the calculation of the mark-to-market of a recovery lock in relation to alternative ways of documenting the trade, we illustrate a few trading applications.[1]

RECOVERY LOCKS AND FIXED-RECOVERY CDS

Standard credit default swaps (CDS) expose investors to two main risks: default risk and recovery risk. Recovery products isolate these risks and allow investors to express their views separately. Two basic recovery products are fixed-recovery CDS and recovery locks. A fixed-recovery CDS differs from a standard (market recovery) CDS in that a fixed recovery is contractually specified. In case of default, the settlement takes place in cash, and the amount exchanged is (100 – fixed recovery). A recovery lock, on the other hand, is a contingent forward contract. It does not involve either upfront or running payments, and allows the purchase or sale of underlying bonds at a predetermined price, called the recovery price, if a credit event occurs.

A recovery lock is quoted with a bid and offer recovery price, as illustrated in Figure 1. For example, suppose that the recovery quote for a certain credit is 30%/35%. Investors can sell recovery at the 30% bid and buy recovery at the 35% offer. If an investor believes the 35% offer is too low, the investor can buy recovery by trading a recovery lock that gives the investor the right (and obligation) to buy (reference) debt in a credit event at 35%.

This purchase of recovery at 35% can be documented and thought of as a single recovery lock trade. Alternatively, we can use a "recovery swap" representation and think of the trade as generated by two separate legs: a short protection position in a standard CDS and a long protection position in a 35% fixed-recovery CDS with the same spread (see Figures 2 and 3). The equality of the two spreads ensures that the running payments cancel out so that there are no net cash flows unless a credit event takes place. In a credit event, the recovery buyer takes delivery of defaulted debt and pays par as obligated through the standard CDS leg of the recovery swap. The settlement of the 35% recovery CDS leg, on the other hand, gives the recovery buyer par minus 35% in cash. Altogether, the recovery buyer pays 35% and takes delivery of the defaulted bonds, just as if the recovery lock were executed directly.

Figure 1. Recovery Lock Market Quotes on August 10, 2005


8/10 13:30 •PAUL MITROKOSTAS, LEHMAN BROTHERS, INC

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** Recovery Lock Indications **
Client buys/sells debt in Credit Event at the specified price

TKR	RECOVERY	TKR	RECOVERY
DAL	13%/16% NR	S4 99(CDX)	25%/27% NR
TPC	40%/45% NR	AW(SECURED)	64%/68% NR
CHTR	50%/55% NR	CPN *UNSEC	11%/14% NR
DPH	35%/42% MR	CPN *SEC	50%/60% NR
VC	38%/43% NR	AMT	37%/45% NR
BOW	37%/43% NR	SKS	32%/40% NR
F CO	39%/44% MR	GMAC	49%/54% MR
F_MC	49%/54% MR	GM Co	39%/44% MR

[1] *Thanks to Shu-Wie Chen, Jock Jones, Paul Mitrokostas and Jeremiah Stafford for discussions and suggestions.*

Figure 2. A recovery lock trade done at 35%

- Single trade documentation based on standard CDS with reference price set as recovery price
- No upfront or running payments
- Physical settlement upon default (defaulted debt is delivered in exchange for a cash payment)

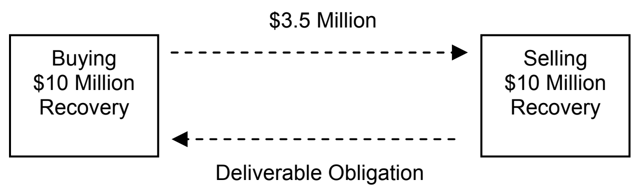
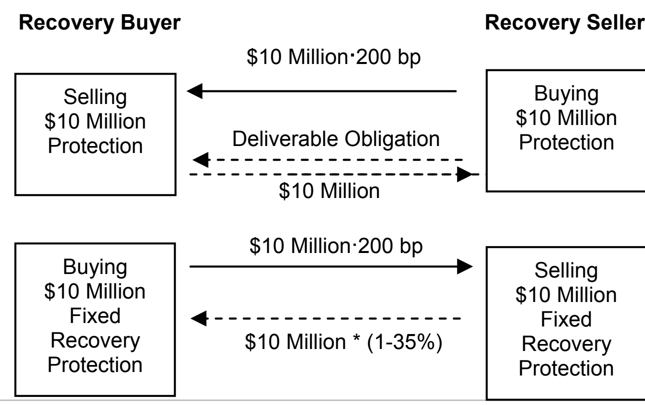


Figure 3. A recovery swap trade done at 35% when CDS is trading at 200bp

- Documented as two separate trades: a standard CDS and a fixed-recovery CDS
- The payments in the two CDS cancel out, and the combined position has no upfront or running payments
- On default: physical settlement of the standard CDS; cash settlement of the fixed-recovery CDS



Notice that the equivalence between a recovery lock and a recovery swap also holds for distressed credits, whose CDS contracts generally trade with upfront points plus a fixed running spread (usually 500bp). In this case, a 35% recovery lock can be represented by the combination of a standard CDS and a 35% fixed-recovery CDS with the same upfront payment.

REPLICABILITY AND PRICING RELATIONS

We have seen that there is an economic equivalence between a recovery lock and a position combining a standard CDS and a fixed-recovery CDS. In fact, the payoff of any of these three contracts can be perfectly replicated by a position in the other two. We now show that this payoff equivalence results in a general "no-arbitrage" relation that must hold among the market prices of these instruments.

"No-arbitrage" pricing when CDS trades with running spread only

To derive the pricing relation, we need only choose one of the three contracts and replicate its payoff with the other two. For example, let us replicate the payoff of a long position in a recovery lock trading at R_{Lock} by selling CDS protection at S and buying X-recovery CDS protection at S_X. To replicate the recovery lock, we need to make sure that the long/short position has no upfront or running payment, which requires that we buy S/S_X dollars of protection on the *X*-recovery CDS for each dollar of protection we sell on the standard CDS. If a credit event happens, the contingent net cash flow for this combined position for each dollar of notional of the standard CDS is

$$(1)\qquad (1-X)\ S\ /\ S_X - (1-R_{Realized})$$

where $R_{Realized}$ is the actual recovery realized in the credit event, i.e., the price at which reference debt trades following the credit event.

Ruling out the existence of arbitrage opportunities implies that the cash flow in (1) must be equivalent to that of the long position in a recovery lock, which, in case of a credit event, pays

$$(R_{Realized} - R_{Lock})$$

for each dollar of notional. Equating the two gives us the key pricing relation

$$(2)\qquad R_{Lock} = 1 - (1-X)\ S\ /\ S_X$$

Figures 4 and 5 illustrate the cash flow on the trades.

Figure 4. Recovery Lock

- No upfront or running payments
- Upon default, recovery buyer receives debt worth realized recovery, and pays recovery price R

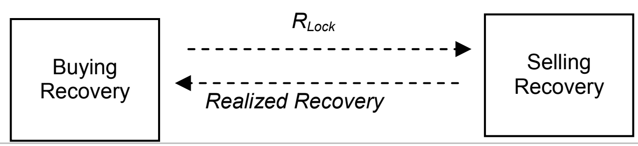
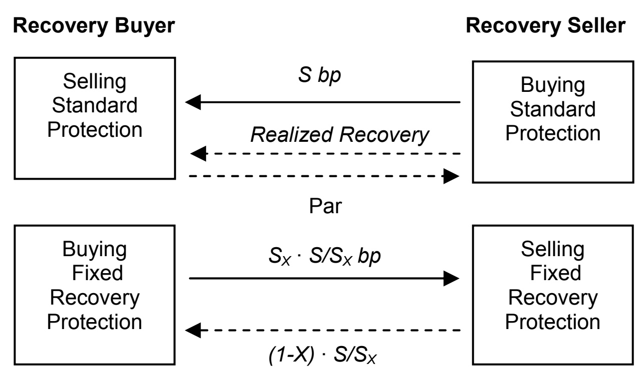


Figure 5. Combining a Standard CDS and a Fixed-Recovery CDS

- Combined position has no upfront or running payments
- Upon default, recovery buyer pays par and receives debt worth realized recovery to settle the standard CDS and receives (1-X)S/SX in cash to settle the fixed recovery CDS



To see why equation (2) *must* hold, assume for a moment that it does not. Suppose, for example, that we observe

$$R_{Lock} > 1 - (1\text{-}X)\ S\ /\ S_X.$$

Then, selling the recovery lock and entering the long/short position described above will leave investors with a zero net running payment and positive inflow in case of a credit event.

This is a gain with positive probability (the probability that the credit event is realized) and zero payoff otherwise, i.e., a pure arbitrage. The demand for entering this position must eventually drive the prices R, S, and S_X back to a point at which they satisfy the previous relation with equality. An analogous and symmetrical argument applies if we observe

$$R_{Lock} < 1 – (1\text{-}X)\ S / S_X.$$

The relation in (2) was obtained using the fact that selling protection on a standard CDS and buying protection on a fixed-recovery CDS is equivalent to buying a recovery lock. To keep this equivalence in mind, it is useful to recall that a CDS position is exposed to two types of risk—default risk and recovery risk—while a position in a fixed-recovery CDS is exposed only to default risk. The "difference" between the two, then, must contain recovery risk only. We can therefore write:

+ (rec risk + def risk) - def risk = + rec risk

CDS F.R.CDS LOCK

which is an intuitive way of representing that going long a credit with a standard CDS and shorting the credit with a fixed-recovery CDS replicates a long position in a recovery lock. Moving terms from one side to the other of the equality sign, it is now simple to see that shorting credit with a standard CDS is equivalent to shorting credit with a fixed-recovery CDS and simultaneously selling a recovery lock, or that shorting credit with a fixed-recovery CDS is equivalent to shorting credit with a standard CDS while simultaneously buying a recovery lock. We could have used any one of these replication arguments to reach the same conclusion, i.e., that the relation in (2) must hold for arbitrage opportunities to be ruled out. Conceptually, the point of this discussion is that if two out of three of these contracts are traded, the price of the third is necessarily pinned down by no-arbitrage considerations.

"No-arbitrage" pricing when CDS trades upfront plus running spread

For distressed names quoted in terms of upfront points plus a fixed running spread (500bp), we can use the same line of reasoning to derive an analogous relation. In this case, however, replication of the recovery lock requires that we use *two* fixed-recovery CDS to ensure that both the running spread and the upfront cost of the replicating portfolio are equal to zero[2].

Let P denote the upfront points of a standard CDS, P_1 the upfront points of an X_1-recovery CDS, and P_2 the upfront points of an X_2-recovery CDS. Then a \$1 long position in a recovery lock can be replicated by selling \$1 of protection on a standard CDS, buying $a=(P - P_2)/(P_1 - P_2)$ dollars of protection on the X_1-recovery CDS, and buying $b=(P_1 - P)/(P_1 - P_2)$ dollars of protection on the X_2-recovery CDS. To avoid arbitrage opportunities, prices must obey the equation:

$$R = 1 – a\cdot(1\text{-}X_1) – b\cdot(1\text{-}X_2) \quad (3)$$

Revisiting the recovery swap representation

The no-arbitrage relations we have just derived shed more light on a point we introduced in the first section, where we observed that a 35% recovery lock can equivalently be thought of as a recovery swap, i.e., a position combining a standard CDS trading at S and a 35% fixed-recovery CDS with the same premium. Equation (2) shows that, in fact, $X=R=35\%$ and $S=S_{35\%}$ was only one of the infinitely many recovery swap representations at our disposal: for example, we might have used a zero-recovery CDS with a premium of $S_{35\%} = S / (1\text{-}35\%)$ or a 10%-recovery CDS with a premium of $S_{10\%} = S*(1\text{-}10\%) / (1\text{-}35\%)$. An analogous

[2] *Unless, of course, we use a fixed recovery CDS with recovery equal to the recovery price (in the recovery lock) since in this case, as we saw in the first section, the recovery lock is simply a short protection position in a standard CDS and a long protection position in the fixed recovery CDS (both with the same notional).*

observation holds for names traded on an upfront-plus-running basis: Equation (3) shows that, in this case, we could have represented the recovery trade using any one of an infinite number of pairs of fixed-recovery CDS contracts with different upfront points.

Even if market participants resolve this indeterminacy by agreeing always to set the fixed recovery equal to the recovery price, the advantages of documenting a recovery lock as a single trade become evident when one considers the calculation of its mark-to-market in the context of the existing market conditions. We turn to this issue in the next section.

MARK-TO-MARKET OF A RECOVERY LOCK

Computing the mark-to-market of a recovery lock requires that we calculate the upfront payment necessary to close an existing contract at current market levels. Suppose we have bought one dollar of June 20, 2010, recovery at 45% and that recovery now trades at 50%. To compute the mark-to-market of the open position, we need to calculate the present value of a payment of 5 cents at the time of a credit event, in case a credit event occurs before June 20, 2010. This, in turn, requires that we compute today's market-implied probability that a credit event will take place before this time horizon. All of this can be done using a standard credit default swap calculator such as the LehmanLive Credit Default Swap Calculator, although a couple of important remarks on the actual procedure are necessary.

A standard credit default swap calculator extracts a term structure of market-implied default probabilities from a CDS spread curve and a recovery rate assumption. We will call this recovery rate the *market recovery*. The default probabilities thus obtained, together with risk-free (LIBOR) discount factors, are then used to value an existing CDS contract, which is described by an upfront payment (if any), a running spread (paid until default occurs or the contract matures), and a contingent payment to be made at the time of the credit event. This contingent payment is usually specified using the notion of *contractual recovery*, so that the actual payment is set equal to par minus this recovery rate. The important point here is the conceptual difference between the market recovery (used to extract default probabilities) and the contractual recovery (used to define the CDS payoff that we want to price).

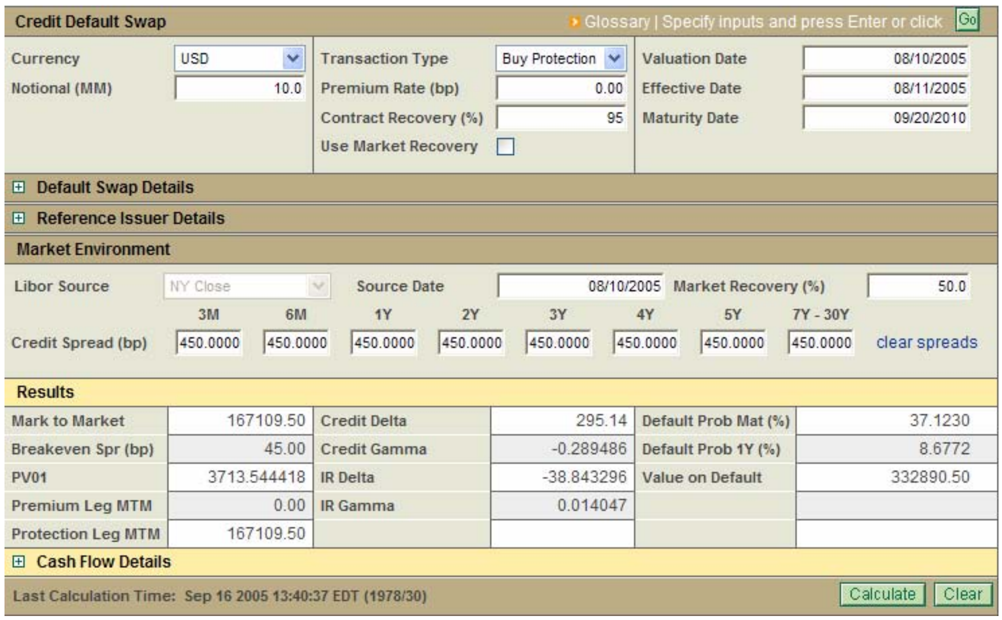
When calculating the mark-to-market of a standard CDS, market participants usually set both the market recovery and the contractual recovery equal to 40% (except in special circumstances in which they agree to use a different number). Since, until recently, no market-implied information about recovery has been available, one can think of the 40% convention as a market-wide agreement on the expected recovery of the assets deliverable into the CDS contract, which also justifies the practice of using the same number for both the market recovery and the contractual recovery.

However, if we want to use a CDS calculator to compute the mark-to-market of a recovery lock, we need to do things a bit differently. Specifically, the market recovery must be set equal to the current market price of the recovery lock. Once the term structure of default probabilities has been computed, we can calculate the mark-to-market of the recovery lock as the mark-to-market of a CDS that pays no upfront and no running premium and has a contractual recovery equal to one minus the difference between the recovery lock's contractual recovery and the current recovery price. This corresponds to 1 - (50% - 45%) = 95% in the example above, which allows us to compute the present value of a contingent payment of 5 cents in case a credit event takes place.

Figure 6 shows how to use the LehmanLive Credit Default Swap Calculator to compute the mark-to-market of the recovery lock discussed above. To summarize:

- In the *Credit Default Swap* section, set the *Premium Rate* to 0bp and the *Contractual Recovery* to 1 – (market recovery – contractual recovery) = 95%.[3]
- In the *Market Environment* section, specify the CDS spreads and the *Market Recovery* rate as 50%. The market standard is to use a flat spread curve.
- The *Mark to Market* in the *Results* section shows the unwind value of the recovery lock.

Figure 6. Valuation of a recovery lock



Source: LehmanLive – follow the links: Fixed Income -> Credit -> Quant Toolkit -> Single Issuer Valuation and Risk Tools -> Credit Default Swap Calculator.

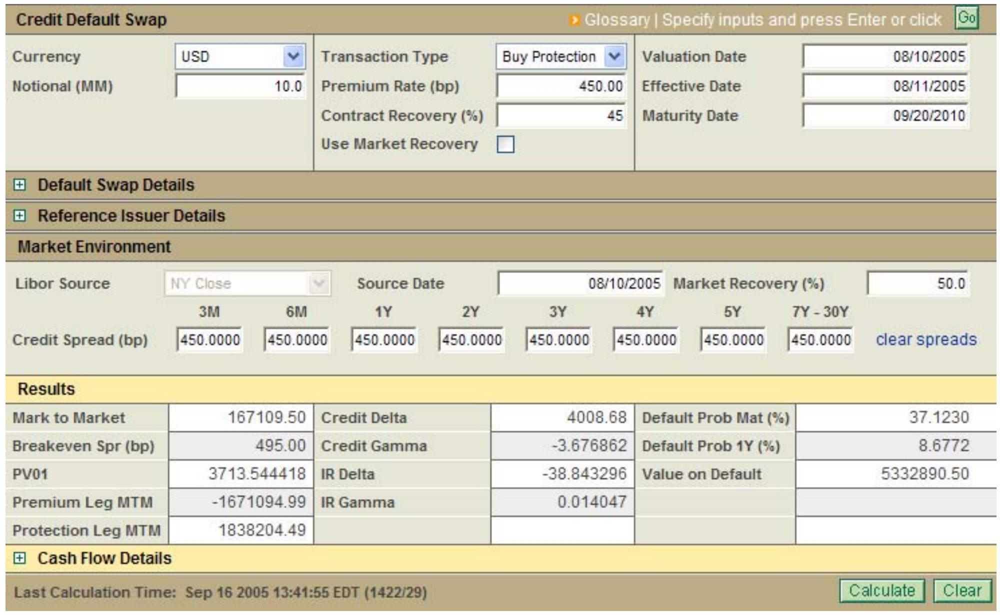
In practice, a recovery lock is often valued using a flat curve. In this case, we can also value the recovery lock by directly specifying the *Contractual Recovery* (as the contractual recovery rate in the recovery lock) and setting the *Premium Rate* equal to the CDS spread entered in the *Market Environment* section. This is due to the equivalence between a recovery lock and a recovery swap, and to the fact that by setting the *Premium Rate* equal to the market CDS spread, we ensure that the mark-to-market of the standard CDS leg of the recovery swap is equal to zero. The calculation is illustrated in Figure 7.

The *Mark to Market* is exactly the same in the two valuations in Figures 6 and 7 but the risk numbers (eg *Credit Delta*) are different. Figure 6 presents the correct risk numbers for a recovery lock trade whereas Figure 7 presents the correct risk for a fixed recovery CDS trade. To get the correct risk for a recovery lock trade through the recovery swap representation, Figure 7 must be supplemented with the valuation of a standard CDS as illustrated in Figure 8. When the risk numbers in Figures 7 and 8 are added we arrive at the correct risk seen in Figure 6.

The most important risk number in a recovery lock trade is the recovery delta (the sensitivity of the *Mark to Market* to changes in the *Market Recovery*). Recovery delta can be calculated manually by changing the *Market Recovery* input and recalculating the *Mark to Market*.

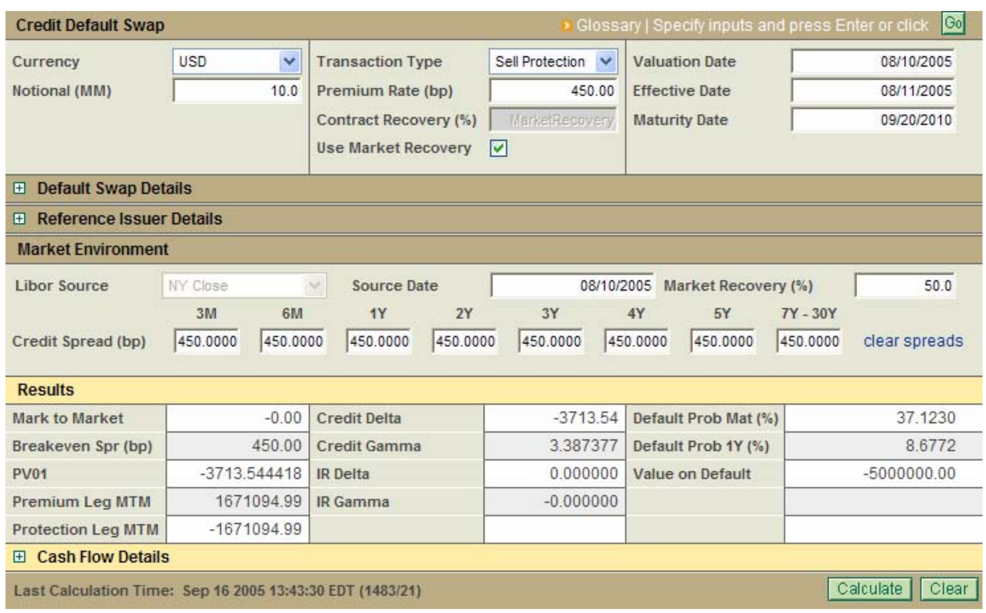
[3] *Because the calculator requires the contractual recovery rate to be less than 100%, we must enter 100% – (contractual recovery – market recovery) when market recovery is less than contractual recovery and multiply the calculated market value by -1.*

Figure 7. Valuation of a recovery lock using a recovery swap representation



Source: LehmanLive – follow the links: Fixed Income -> Credit -> Quant Toolkit -> Single Issuer Valuation and Risk Tools -> Credit Default Swap Calculator.

Figure 8. Valuation of a standard CDS. When risk numbers (such as *Credit Delta*) from this valuation are added to the risk numbers in Figure 7 we get the risk numbers seen in Figure 6



Source: LehmanLive – follow the links: Fixed Income -> Credit -> Quant Toolkit -> Single Issuer Valuation and Risk Tools -> Credit Default Swap Calculator.

The advantage of a single-trade documentation

We have seen that there is an economic equivalence between a recovery lock and a recovery swap (i.e., a position combining a standard CDS and a fixed-recovery CDS), and we have observed that this gives rise to alternative ways of documenting a recovery lock trade. An immediate consequence of the equivalence is the fact that the unwind value of a recovery swap should be the same as the unwind value of the equivalent recovery lock. However, this will be true only if we mark to market the standard CDS leg of the recovery swap using the quoted recovery price, rather than the 40% market convention.

The convention of using a fixed across-the-board recovery rate for unwinding all standard CDS is bound to disappear once recovery becomes liquidly traded for a wide range of reference assets and maturities. However, today's reality is a recovery market that is still in its infancy, and this is likely to generate a dichotomy of market practices during the transition period. For this reason, using a recovery swap documentation would require some of the CDS contracts to be marked to market at a 40% recovery and others (the ones used for representing recovery locks) at the market-implied recovery. In order to avoid this undesirable source of confusion, we feel that the single-trade documentation is preferable, in that it allows us to accommodate the existing—although probably temporary—segmentation between the CDS and the recovery markets.

Distressed credits: When CDS trades on upfront

To calculate the mark-to-market of a recovery lock on a name whose CDS trades on upfront, we need to convert the quoted upfront price of a standard CDS into an equivalent running spread. Once we have the equivalent running spread, we can calculate the mark-to-market as explained above.

We need a recovery assumption to convert from upfront price to running spread, and the only consistent choice is the observed price of a recovery lock. Once again, this presents us with the problem of the mark-to-market of a recovery swap being different from the mark-to-market of the equivalent recovery lock when the CDS leg of the recovery swap is valued using a different recovery.

CDX RECOVERY LOCKS AND FIXED RECOVERY CDX

Recently, the market has started trading CDX recovery locks and fixed recovery CDX referencing the standard IG and HY portfolios. A recovery quote of 25%, for example, means that if one of the credits in the portfolio defaults, the recovery buyer will take delivery of debt issued by the defaulted credit and pay 25% of the notional. Similarly, a zero-recovery CDX means that if one of the credits defaults, the protection buyer will receive the total notional of the defaulted credit. Just as a portfolio swap is economically equivalent to a portfolio of CDS that all have the same contractual spread, a CDX recovery lock is economically equivalent to a portfolio of single-name recovery locks that all have the same recovery price, and a fixed recovery CDX is economically equivalent to a portfolio of single-name fixed recovery CDS that all have the same fixed recovery.

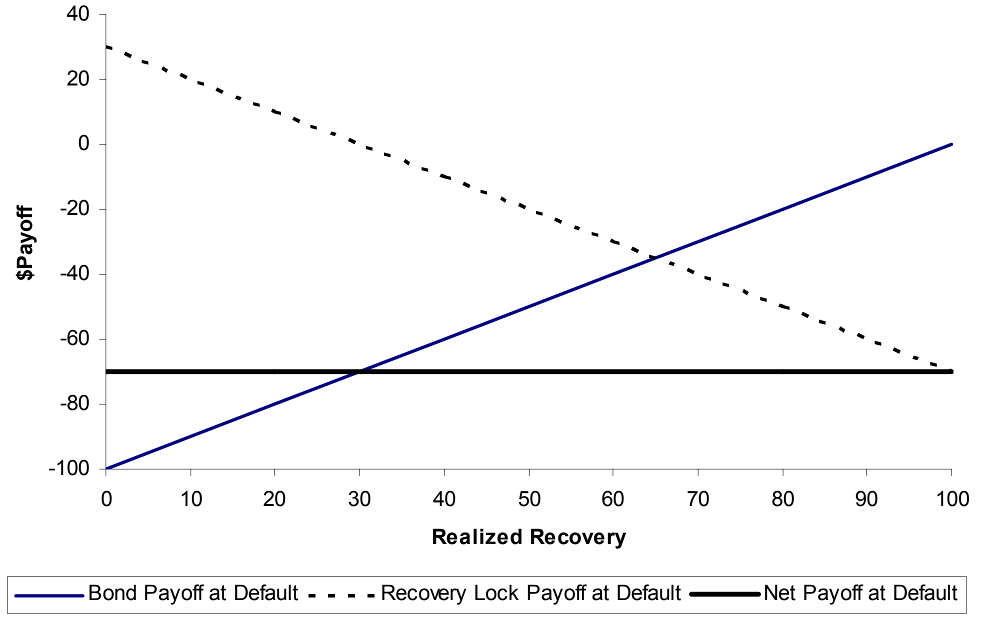
TRADING APPLICATIONS

In this section, we show through examples some possible uses of recovery products. The notional amount of all examples is assumed to be $100.

Risk reduction at default

Assume an investor owns a cash bond at par. The maximum loss on default is the total notional if the bond recovers at zero. To reduce this risk, the investor can sell a recovery at a certain reference price. This way, the investor keeps the coupon carry because the recovery lock is a forward contract. If a default occurs, the loss is the total notional minus the reference price. Assuming that the reference price of the recovery lock is 30%, Figure 9 plots the payoff at default of different positions as a function of the realized recovery of the bond.

Figure 9. Risk reduction at default with R_{Lock} = 30%

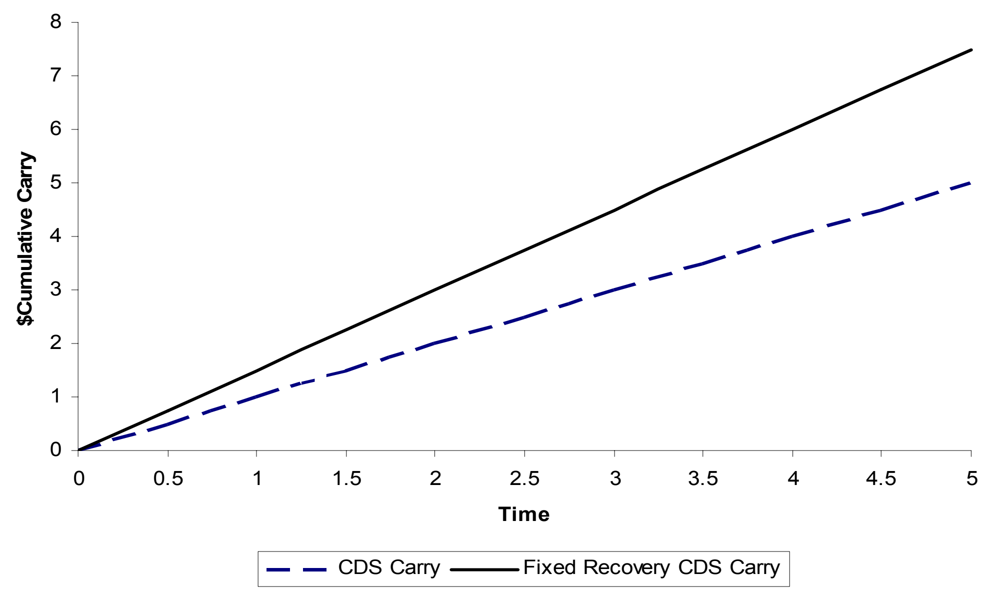


Source: Lehman Brothers.

Yield pickup

Assume an investor wants to earn more carry than she can get when selling protection in a standard CDS. This can be done by selling protection in a 0% recovery CDS. Assuming the standard CDS spread is 100bp and the 0% fixed recovery CDS spread is 150bp, Figure 10 plots the cumulative carry against the default time. The tradeoff for earning the extra carry is that if there is a credit event, the loss will be 100% of the notional instead of (100% - realized recovery) of the notional.

Figure 10. Yield pickup



Source: Lehman Brothers.

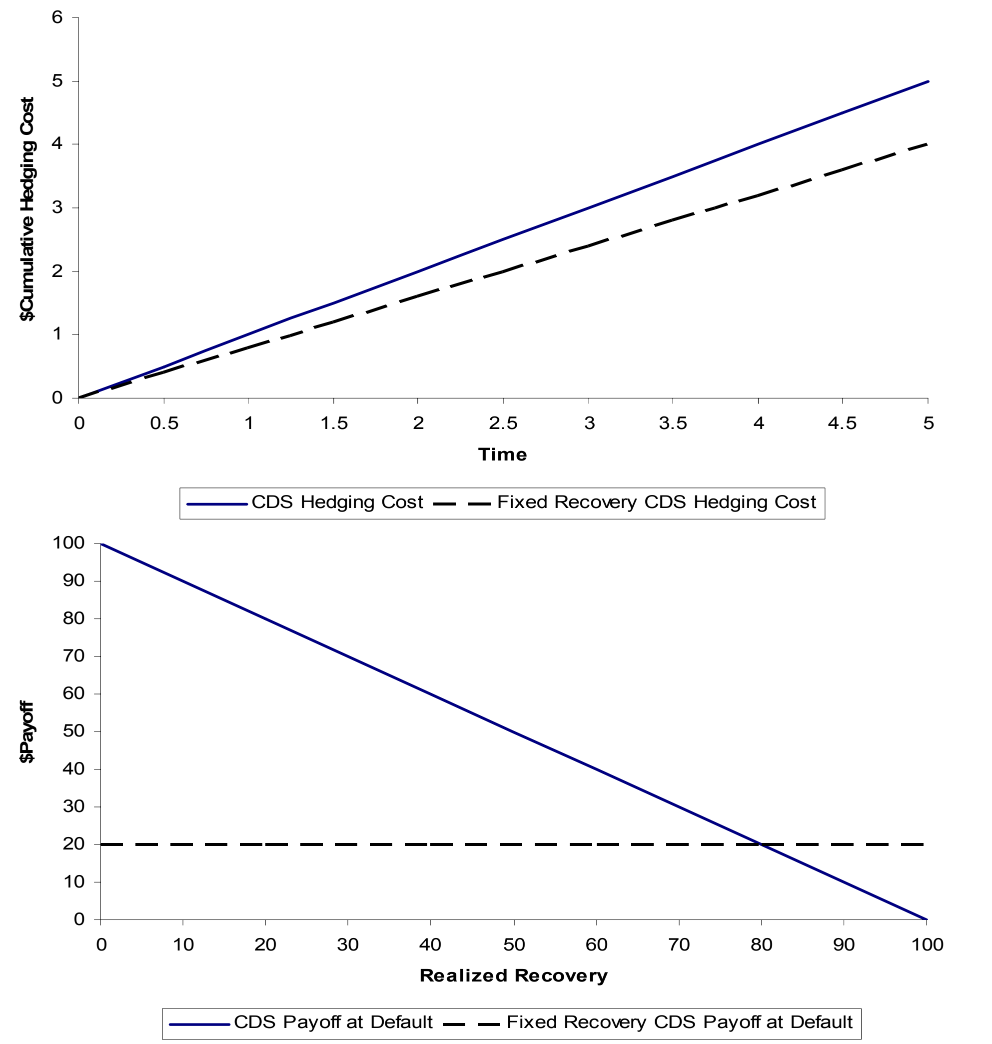
Distressed debt acquisition

Assume an investor wants to acquire a significant portion of the debt of a distressed credit when the issuer files for bankruptcy. To lock in the cost of acquisition, the investor can buy recovery on the credit and thereby guarantee that bonds can be acquired at a pre-specified price at the time of default. Note that upon default, any bond that is pari passu with the obligation referenced in the recovery lock can be delivered.

Loan hedging

Assume an investor owns a senior secured loan that will drop in value if the borrower defaults. To hedge against the default risk, the investor can buy protection using a standard CDS. Standard CDS reference senior unsecured debt, and the investor must pay for the additional risk of the senior unsecured debt compared with the senior secured loan. If the investor believes the recovery of the loan in default is 80%, the investor can buy 80% fixed recovery protection and reduce the hedging cost compared with a standard CDS. Figure 11 plots the hedging cost against default time and the CDS payoffs at default, assuming that the 80% recovery spread is 20bp lower than the CDS spread.

Figure 11. Hedging Costs and Payoffs: CDS vs. 80% Recovery CDS



Source: Lehman Brothers.