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Explaining the Lehman Brothers Option Adjusted Spread of a Corporate Bond

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The option adjusted spread (OAS) is a measure of the credit risk in a callable (or putable) corporate bond and has been used by investors for years. We explain what the OAS is and how it is related to the Z-spread. We present the model used at Lehman Brothers to calculate OAS and associated risk measures, e.g. option adjusted duration and convexity. LehmanLive users can access the model through the Corporate Bond Calculator (keyword: ccalc). The OAS and the risk measures for all bonds in the Lehman Brothers Corporate and High Yield Indices are also reported in POINT, where they are used for various analyses[1].

1. INTRODUCTION

This article has been written:

- as a response to numerous inquiries about how the OAS of a corporate bond is calculated at Lehman Brothers,
- to explain recent changes to our OAS model, and
- to explain the limitations of OAS as a credit spread measure and suggest a better one.

In the rest of this introductory section we give an overview of what an OAS is and how it is related to a Z-spread.

We value a fixed income security by discounting cash flow

The value of a fixed income security is usually thought of as the sum of its discounted payments. Usually we discount payments scheduled for different dates with different interest rates, the so-called the zero-coupon interest rates. For example, we will discount a payment scheduled to be made five years from today with the 5-year zero-coupon interest rate, whereas a payment scheduled to be made ten years from today will be discounted with the 10-year zero-coupon interest rate.

The Treasury and LIBOR curves are the two discount curves used most often

To value a particular fixed income security we must first decide which curve of zero-coupon interest rates to use for the discounting. The two curves most often used are the government (or Treasury) and LIBOR curves. Axel and Vankudre (1997) explain how to find the curve of zero-coupon Treasury interest rates, and Zhou (2002) explains how to find the curve of zero-coupon LIBOR interest rates.

The Z-spread of a Treasury security

If we value a Treasury security by discounting its payments with the zero-coupon Treasury rates, we usually get a value very close to the observed market price of the bond. However, it is generally not possible to find a curve of zero-coupon Treasury rates so that for any Treasury security the observed market price exactly matches the value found by discounting the payments with those zero-coupon Treasury rates. To quantify how different the discounted value and the market price are for a particular security, we can ask how much we need to change (or shift) the zero-coupon rates for the discounted value to equal the market price. The required shift to the zero-coupon interest rates is called the Z-spread of the security. The market convention is to use a parallel shift where all zero-coupon rates are

[1] *See Joneja, Dynkin et al. (2005).*

increased (or decreased) by the same (absolute) amount. For a Treasury security, the Z-spread is a relative value measure where a positive Z-spread indicates that the security is cheap and a negative Z-spread indicates that the security is rich compared with other Treasury securities.

The Z-spread to Treasuries for corporate bonds

We can calculate a Z-spread for a corporate bond exactly as we do for a Treasury security by simply treating the corporate bond as if it were a Treasury security with given fixed payments (that is, ignoring the fact that the corporate bond may default). We thus calculate the Z-spread to Treasuries for the corporate bond. Because of the credit risk, the Z-spread to Treasuries for a corporate bond will be positive. This reflects the fact that the corporate bond is worth less than a Treasury bond with the same maturity and coupon. The higher the credit risk the higher the Z-spread to Treasuries.

Z-spread to LIBOR is a better credit risk measure for corporate bonds

For a corporate bond it is often more meaningful to calculate a Z-spread to LIBOR rather than to Treasuries. It is often argued that LIBOR rates are close to (credit) risk-free and that it is the liquidity and "safe haven" benefits of Treasuries that cause Treasury rates to be lower than LIBOR rates. Since most corporate bonds do not have those benefits to the same extent as Treasuries, a Z-spread to LIBOR is likely to reflect more accurately the credit risk in a corporate bond. A Z-spread to LIBOR is calculated the same way as a Z-spread to Treasuries, that is by determining how much the curve of zero-coupon LIBOR rates must be shifted to ensure that when we sum the value of the bond's discounted payments we get the observed market price of the bond. For a corporate bond, the market convention is simply to call the Z-spread to LIBOR *the* Z-spread without emphasising that it is a spread to LIBOR.

The Z-spread is not appropriate for callable bonds

For bonds with embedded options, such as callable bonds, the Z-spread is often not meaningful. This is because it is usually not appropriate to value a callable bond simply by discounting its scheduled payments. To value a callable bond properly we need to use a model that explicitly takes into account volatility in interest rates so that the risk of the bond being called can be taken into account. We therefore use a stochastic term structure model.

Using a stochastic term structure model to calculate OAS of a callable bond

A stochastic term structure model takes as input a curve of zero-coupon interest rates and some parameters determining the volatility of these interest rates. From these inputs the model effectively generates a large number of possible scenarios for future interest rates[2]. A security is valued by first discounting the cash flow/payments of the security in each scenario separately using the interest rates of that scenario (taking into account when the option is exercised in that scenario[3]) and then averaging over all the scenarios. We can also think of this as the model generating a zero-coupon interest rate curve in each scenario which is used for discounting the cash flow in that scenario. The idea behind the OAS is now easy to explain and is exactly the same as the idea behind the Z-spread: The OAS is simply the constant (absolute) shift to the zero-coupon interest rates in all scenarios that is required to ensure that the model value (the average value over all scenarios) of the bond equals the market price.

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[2] *This can be done by simulation (the standard for calculating OAS of a mortgage backed security) or by using other numerical techniques such as trees/lattices (the standard for government and corporate bonds).*

[3] *This is difficult to do in a simulation model for American/Bermudan exercise, which is why a tree/lattice approach is usually used for such securities. The idea and concept of OAS, however, is independent of whether it is calculated in a simulation-based model or a tree/lattice.*

For bonds without embedded options, the OAS is exactly the same as the Z-spread (when adjusting for daycount and compounding conventions; see below). This is because: (1) without embedded options the bond has the same cash flow in all scenarios; and (2) the average over all scenarios of the discount factors to a given maturity equals the discount factor to that maturity calculated from the observed zero-coupon interest rate to that maturity (this is a condition we always ensure is satisfied for a stochastic term structure model).

For several years Lehman Brothers has used a Black and Karasinsky (1991) stochastic term structure model for corporate bond OAS computations. Recently we have examined that particular model choice and its implementation and made some changes. The most significant change is that we have incorporated a shift to the lognormal Black-Karasinsky dynamics to better fit the skew observed in the volatilities implied from prices of interest rate swaptions with different strikes (see Zhou (2003) for an explanation of the volatility skew).

Outline of remaining sections

The rest of this article is divided into five sections. In section 2 we explain how the Z-spread is computed and its relationship to a CDS spread. In section 3 we discuss the special considerations that must be made for bonds with embedded options and give a more detailed explanation of what the OAS is and the assumptions on which it is based. In section 4 we explain the stochastic term structure model that we use for OAS calculations. In section 5 we show numerical results from our volatility calibration and explain the effects of the recent model change on OAS for individual bonds. Section 6 gives a brief discussion of the issues we face if we want to improve upon the OAS and calculate a credit spread that better captures default risk. There is a summary in section 7.

2. THE Z-SPREAD: DISCOUNTING CASH FLOW

In the introduction we explained what the Z-spread is. In this section we repeat that description, providing details and formulas.

Consider a bond that has N>0 remaining payments to be paid at times t_i, i=1, …, N. Let C_1 denote the size of the payment (in dollars) scheduled for time t_i. Finally, let r(t) denote the continuously compounded zero-coupon rate for discounting to time t.

The discounted value of the of the bond payments is:

(2.1) $\exp(-r(t_1)t_1)C_1 + \ldots + \exp(-r(t_N)t_N)C_N$

The payment times, t_1, …, t_N, are measured in years by taking the actual number of days from the valuation date to the payment date and dividing by 365.25.

The value in (2.1) will generally be different from the market price of the bond. The continuously compounded Z-spread is the constant z that solves the equation:

(2.2) $\text{Market Price} = \exp(-(r(t_1)+z)t_1)C_1 + \ldots + \exp(-(r(t_N)+z)t_N)C_N$

where Market Price is the full (dirty) price of the bond that includes accrued interest[4]. Notice that two conventions have been used: continuous compounding and actual/365.25 daycount.

The market standard for quoting a Z-spread is to use the conventions of the bond (semi-annual compounding (in US) with 30/360 daycount). With simple compounding, equation (2.2) becomes:

(2.3) $$\text{Market Price} = \frac{C_1}{\left(1+\Delta(R(T_1)+Z)\right)^{T_1/\Delta}} + \ldots + \frac{C_N}{\left(1+\Delta(R(T_N)+Z)\right)^{T_N/\Delta}}$$

[4] *A future settlement date is taken into account by discounting the full (dirty) price from the settlement date to the valuation date. This discounting is done on the LIBOR curve.*

where $\Delta = 0.5$ for semi-annual compounding, $T_1, \ldots, T_N$ are the payment times measured in years using the 30/360 bond daycount convention, and the interest rates $R(T_1), \ldots, R(T_N)$ are related to the continuously compounded rates by:

$$\exp\left(r(t_i)t_i\right) = \left(1 + \Delta R(T_i)\right)^{T_i/\Delta} \quad (2.4)$$

The Z-spread is a more detailed measure than a standard yield spread

The Z-spread is only one of many different credit spreads used for corporate bonds. The simplest credit spread is the quoted yield spread, which is used by traders of investment-grade corporate bonds to quote prices. The quoted yield spread is simply the standard yield of the bond minus the yield of a benchmark Treasury security with a similar maturity. The advantage of quoting a spread rather than the price is that the quote will not have to be continuously updated as Treasury yields change.

It is important to recognize the limitations of the yield spread and the additional detail provided by the Z-spread. The yield spread should always be seen as simply a quoting convention and should usually be avoided for analysis.

The main advantage of the Z-spread is that it uses the full information contained in the curve of zero-coupon interest rates. In the Z-spread calculation, cash flows are discounted with a different interest rate depending on the timing of the cash flow. In the yield spread, all cash flows are discounted with the same rate. This means that the yield spread is not detailed enough to properly compare two bonds with different coupons even if the maturities (or durations) are similar. The more the interest rate curve deviates from a flat curve, the more important it becomes to use the Z-spread instead of the yield spread. See O'Kane and Sen (2004) for details (and for explanations of other types of credit spreads).

Comparing the Z-spread with a CDS spread

It is not uncommon to see comparisons between the Z-spread of a bond and a credit default swap (CDS) spread. It is important to be cautious about such a comparison and be aware of the differences between these two types of credit spreads.

The first obvious differences between a Z-spread and a CDS spread are the daycount and compounding conventions. A CDS spread is based on quarterly compounding[5] with actual/360 daycount, whereas the Z-spread is based on semi-annual compounding (in US) with 30/360 daycount.

The most important difference is that the CDS spread has a transactional interpretation: An investor is paid a running premium equal to the CDS spread for taking the risk of losing one minus the recovery rate if default occurs. The Z-spread, by contrast, is not a spread/premium that can be locked in today for taking certain risks. The Z-spread is the excess return that can be earned from buying the bond and holding it to maturity, assuming the issuer does not default and that coupons can be reinvested at forward LIBOR rates plus the Z-spread.

CDS are usually valued and analyzed in a hazard rate model which takes as inputs a curve of CDS spreads, a LIBOR curve, and a recovery rate. Based on these inputs, the hazard rate model calibrates a curve of survival probabilities using a standard method explained in O'Kane and Turnbull (2003). Based on the survival probabilities, the zero-coupon interest rates and the recovery rate, it is easy to price default contingent contracts such as CDS. We can also use the model to price a bond and ask the following question: if we were to price a bond using survival probabilities calibrated to a flat CDS spread curve, what spread would be needed to match the observed bond price? We could call the answer a *bond-implied CDS*

[5] *Payments are semi-annual rather than quarterly in some emerging market CDS.*

spread[6]. This type of bond spread is directly comparable to a CDS spread and we prefer it to the Z-spread.

A bond implied CDS spread based on a flat CDS curve implies (approximately) a constant hazard rate (the hazard rate is the instantaneous probability of default; see O'Kane and Turnbull, 2003). With a constant hazard rate we have another approximate relationship:

(2.5) CDS Spread ≈ Hazard Rate · (1 – Recovery Rate)

This means that if we use a recovery rate of zero when we calculate the bond-implied CDS spread, then the spread will approximately equal the hazard rate. This is significant because the continuously compounded Z-spread is equal to the constant hazard rate necessary to match the market price of the bond if we assume a recovery rate of zero[7]. This is clear from (2.2) because $\exp(-(r(t)+z)t) = \exp(-r(t)t)\exp(-zt)$ and $\exp(-zt)$ is the survival probability to time t, when the hazard rate is constant at z. In words: in (2.2) all bond payments are multiplied by the survival probability and discounted on the LIBOR curve. Clearly the sum of these discounted expected payments only equals the bond value if the recovery rate is zero (if the recovery rate is greater than zero there will be additional terms in (2.2)).

It is often useful to have this zero-recovery CDS spread interpretation in mind when looking at a Z-spread. Consider a premium bond of an issuer with a given CDS spread. Suppose the recovery rate of the issuer is 40%. What happens to the value of the bond if the recovery rate drops to 0% but the CDS spread is unchanged? In this situation default probabilities must have decreased and this will increase the value of the bond because it will increase the expected time over which the bond's high coupon will be paid. This means that if the bond value stays unchanged after the recovery rate drops to 0%, then the CDS spread must increase. In other words, for a premium bond the 0%-recovery bond-implied CDS spread (which approximates the Z-spread) is higher than the 40%-recovery bond-implied CDS spread. The reverse is the case for a discount bond. The effect is larger for high-spread long-maturity bonds. The effect means that even if the Z-spread of a discount bond is lower than the same-maturity CDS spread, the bond may still be cheap compared with the CDS.

Consider the following extreme example: On January 23, 2006 a hypothetical 6% bond maturing on January 1, 2036 was trading at $70. The bond's 0%-recovery CDS spread is 382bp whereas the 40% recovery CDS spread is 515bp. This is clearly a difference that must be taken into account. As a less extreme example, consider a 6% bond maturing on 1 January 2016 and trading at $80 on January 23, 2006. The bond's 0%-recovery CDS spread is 415bp whereas its 40%-recovery CDS spread is 464bp.

3. BONDS WITH EMBEDDED OPTIONS

The OAS (or option adjusted spread) should be thought of as a Z-spread that has been adjusted for any option embedded in the bond (for a bond without an embedded option, the OAS is equal to the continuously compounded Z-spread). Bonds with an embedded option are usually callable, so to simplify the terminology we focus on those, although the principles may be applied to a bond with another type of embedded option (e.g., a putable bond).

Separating a callable bond into the bond stripped of the option and a short position in a call option on the stripped bond

It is useful to compare a callable bond to portfolio with positions in two hypothetical securities. One is the identical bond stripped of its embedded call option. We call this hypothetical

[6] *Another more sophisticated bond-implied CDS spread would be to ask how far we need to shift the CDS curve to match the observed bond price. The bond-implied CDS spread could then be reported as the spread of a CDS with the same maturity as the bond plus the shift. For a third type of bond implied CDS spread see Mashal, Naldi and Wang (2005).*

[7] *and assume no recovery of coupon in default.*

security the *stripped bond*. The other security is a call option on the stripped bond with the same call schedule as the callable bond. We want to compare a long position in the callable bond with a long position in the stripped bond and a short position in the call option.

Suppose we buy the callable bond, sell the stripped bond and buy the call option. This portfolio is hedged (has a zero net cash flow) in all states of the world (i.e., in all possible scenarios for interest rates, default, etc.). First, we see that the cash flow from the callable bond matches the cash flow to the stripped bond until both bonds mature or the callable bond is called, whichever comes first. Second, by exercising the call option when/if the callable bond is called, we can ensure that we are hedged in this situation also: The proceeds from the callable bond match the exercise price of the call option and the stripped bond bought as a result of the exercise of the call option can be used to close the short position that was initially established in the stripped bond. The argument shows that we can hedge a long position in a callable bond by selling the stripped bond and buying the call option on the stripped bond. This implies that the callable bond should never cost less than the (theoretical) value of the stripped bond minus the (theoretical) value of the call option. That is:

(3.1) Market Price of Callable Bond ≥ Value of Stripped Bond – Value of Call Option

Can we use the above argument to establish a hedge for a short position in a callable bond? Unfortunately not. Suppose we sell the callable bond, buy the stripped bond and sell the call option. As long as the call option is not exercised we have no problem: the cash flow from the stripped bond will cover the cash flow required for the short position in the callable bond. However, if the call option is exercised, we would need to call the callable bond, which can be done only by the issuer. If we could be sure that the issuer would call the callable bond exactly when it is optimal to exercise the call option on the stripped bond, then the hedge would work. However, there are good reasons why this is not the case. For example, an issuer must consider the cost of refinancing before calling (i.e., prepaying) a bond. Usually an issuer would need to issue a new bond, which can be costly. The refinancing cost effectively raises the call price for the issuer and thus reduces the value of the embedded call option. This explains why the callable bond should be worth more than the (theoretical) value of the stripped bond, minus the (theoretical) value of the call option on the stripped bond.

The OAS of a callable bond is the Z-spread of the stripped bond when properly adjusting for the value of the call option

In our OAS calculation we disregard the possible refinancing cost to the issuer and assume that the issuer will call the bond exactly when it is optimal for an outside investor to exercise the call option on the stripped bond. With this assumption the value of the callable bond becomes equal to the value of the stripped bond minus the value of the call option. That is:

(3.2) Market Price of Callable Bond = Value of Stripped Bond – Value of Call Option

If we knew the value of the call option we could subtract it from the market price of the callable bond to arrive at a market-implied value of the stripped bond. Based on this we could calculate a (continuously compounded) Z-spread of the stripped bond and report it as the OAS of the callable bond. This is essentially the approach we use to calculate the OAS except that we need to use a model to value the call option.

To value the call option we use a model intended for valuing options on Treasury securities or interest rate swaps. The model takes as inputs a curve of zero-coupon interest rates and a set of parameters determining the volatility of these interest rates. Suppose we pass to the model the LIBOR zero-coupon interest rates and volatility parameters calibrated to appropriate swaption prices with the purpose of using the model to price a Bermudan swaption (say with swap maturity date equal to the bond maturity date). We know that the swap underlying the Bermudan swaption is priced correctly in the model when we input the

LIBOR rates. However, to value the call option we need to make an adjustment to the model to ensure that it correctly prices the underlying stripped bond. The adjustment is to shift all interest rates in all scenarios generated internally in the model by the OAS of the stripped bond (see a more detailed explanation in the next subsection). However, this OAS is unknown to us as it depends on the value of the call option. Putting these pieces together, we see that the OAS has to solve an equation:

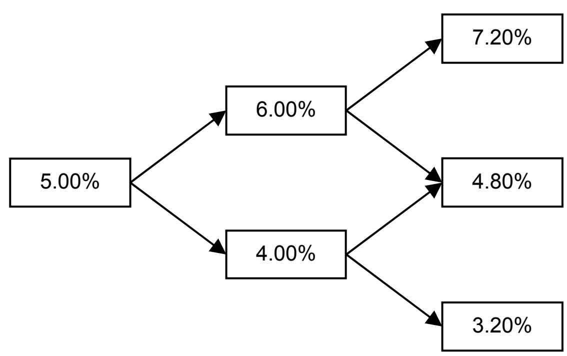
$$\text{Market Price} = \exp(-(r(t_1)+\text{OAS})t_1)C_1 + \ldots + \exp(-(r(t_N)+\text{OAS})t_N)C_N \tag{3.3}$$
$$- \text{OptionValue}(\{r(t)\}_{t>0}, \text{Volatility Parameters, OAS})$$

where Market Price is the observed price of the callable bond, the first terms on the right-hand side of the equation sum to the value of the stripped bond, and the last term is the value of the call option. The option value is written as a function of the zero-coupon interest rates $\{r(t)\}_{t>0}$, the volatility parameters and the OAS. Note that the OAS appears for the valuation of both the stripped bond and the call option. The OAS is found by numerically solving equation (3.3).

Incorporating OAS into the option valuation model

We value the option using a model that has been implemented in a recombining tree (lattice) for the short rate. The short rate is the continuously compounded interest rate earned over one period in the lattice. This is called a short rate model and is shown in Fig. 3.1.

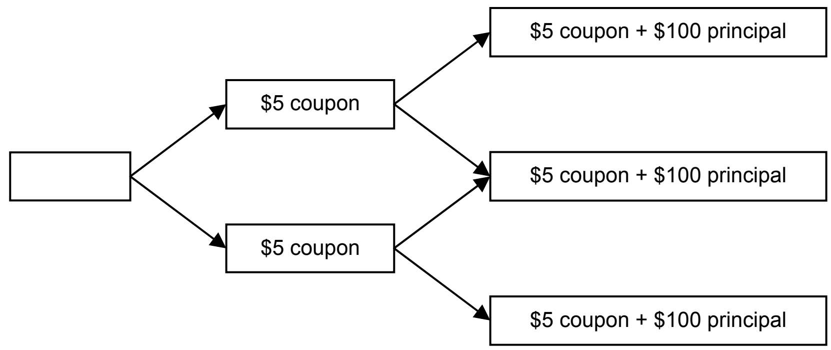
Figure 3.1. Example of a short-rate model. For simplicity, we assume that all branching probabilities are 0.5 and the period length is 1 year



Source: Lehman Brothers.

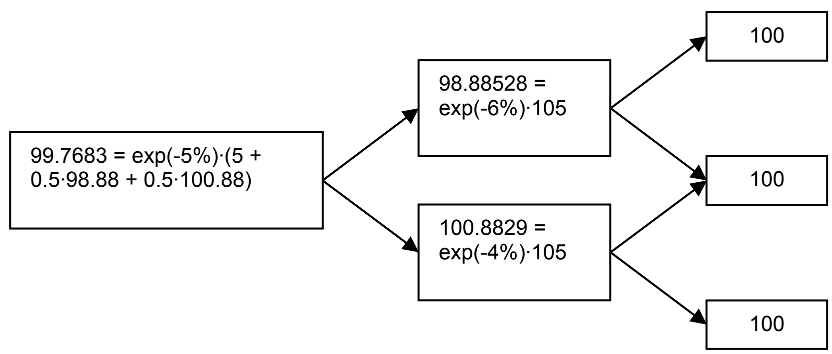
To value a security in this type of model we determine the security's cash flow in each state (i.e., node) and discount backwards in the tree. For example, if we value the bullet bond with the cash flow shown in Figure 3.2 (think of this as the stripped bond underlying the call option we want to value), we get the values shown in Figure 3.3.

Figure 3.2. Cash flow of a bullet bond to values in the short rate model in Fig. 3.1



Source: Lehman Brothers.

Figure 3.3. Values of a bullet bond (ex coupon) from Figure 3.2 in the short rate model in Figure 3.1

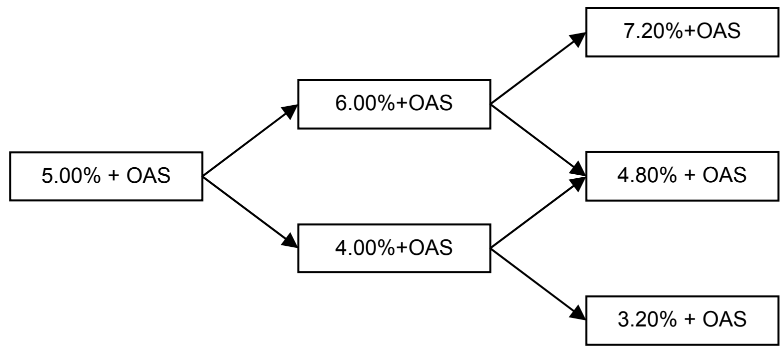


Source: Lehman Brothers.

The value of a security in a given state is the expected value of the security in the next period (including cash flow/coupon) discounted with the short rate in the state. This procedure is also used to price a call option on the bullet bond. Suppose the exercise price of the call option is 100. The option is then worthless at time 2 (end of 2nd period) and in the 6% rate state at time 1. In the 4% rate state at time 1, the option should be exercised because it is worthless at time 2 and worth 100.8829 – 100 = 0.8892 if exercised. At time 0 the option is worth exp(-5%)·0.5·0.8829 = 0.4199 if it is not exercised and has a negative value if it is exercised. This means that the option should not be exercised at time 0 and is worth 0.4199.

As explained above, when we value the call option we need to make sure that the underlying bond is priced correctly. For example, if the value of the bond is 98.8 and not 99.8 as in Figure 3.3, we need to adjust the interest rates in the tree with the OAS of the bond. This adjustment is done, as illustrated in Figure 3.4, by increasing the continuously compounded short rate in all states by the OAS.

Figure 3.4. The short rate model in Figure 3.1 has been shifted by the OAS



Source: Lehman Brothers.

If we choose an OAS = 50bp we get a bullet bond price of 98.7997. With this OAS the call option is worth only 0.1797. This means that if the callable bond (the bullet bond minus the call option) was trading at 98.80-0.18 = 98.62, the callable bond would have a 50bp OAS.

Before proceeding to the next section and explaining which type of stochastic term structure model we use, we show why the OAS and Z-spread of a bullet bond are equal when we use continuous compounding. To calculate the Z-spread we need zero-coupon interest rates to time 1 and 2. The zero-coupon rate to time 1 is clearly 5%. The zero-coupon rate to time 2, denoted R, must satisfy the equation:

$$\exp(-R\cdot 2) = \exp(-5\%)\cdot(0.5\cdot\exp(-6\%) + 0.5\cdot\exp(-4\%)) \tag{3.4}$$

where the right-hand side of the equation is the value calculated in the tree of a zero-coupon bond paying \$1 at time 2. Using this equation and letting Z denote the Z-spread we can write the value of the bond as:

$$\begin{aligned} &\exp(-(5\%+Z))\cdot 5 + \exp(-(R+Z)\cdot 2)\cdot 105 \\ &= \exp(-(5\%+Z))\cdot 5 + \exp(-5\%)\cdot(0.5\cdot\exp(-6\%) + 0.5\cdot\exp(-4\%))\cdot\exp(-Z\cdot 2)\cdot 105 \\ &= \exp(-(5\%+Z))\cdot(5 + 0.5\cdot\exp(-(6\%+Z))\cdot 105 + 0.5\cdot\exp(-(4\%+Z))\cdot 105) \end{aligned} \tag{3.5}$$

but this is also the value of the bond calculated in the tree when rates have been shifted by an OAS = Z. This shows that the OAS and Z-spread are the same. The two assumptions that make this calculation work are: (1) bond payments for a given time are independent of the state; and (2) with continuous compounding the OAS adjustment becomes a multiplicative factor on the discount factors (i.e. $\exp(-(R+Z)\cdot T) = \exp(-R\cdot T)\cdot \exp(-Z\cdot T)$).

4. THE STOCHASTIC TERM STRUCTURE MODEL

In this section we explain which stochastic term structure model we use to calculate OAS. We use a short rate model as explained in the previous section. There is extensive academic as well as practitioner literature describing the theory and uses of short rate models. Hull (2000) gives an introduction to stochastic term structure models, including short rate models. In this section we assume a familiarity with the models at the level of Hull (2000).

The Black-Karasinsky model

For several years Lehman Brothers has used a Black-Karasinsky (BK) model for corporate bond OAS computations. In the BK model the short rate is lognormally distributed. More precisely, the short rate r(t) is given by:

(4.1) $r(t) = \alpha(t)\exp(x(t))$

(4.2) $dx(t) = -\kappa(t)x(t)dt + \sigma(t)dW(t)$

where $\alpha(t)$, $\kappa(t)$ and $\sigma(t)$ are deterministic functions of time and $W(t)$ is a Brownian motion. $\kappa(t)$, $t \geq 0$, are referred to as the mean reversion speed parameters. $\sigma(t)$, $t \geq 0$, are the volatility parameters[8].

Suppose we already know the mean reversion speed and volatility parameters and that they are constant, we can then implement the model in a trinomial lattice as described in Hull (2000) in the section on the Hull and White model (starting at page 580). Although this type of trinomial lattice implementation works well, at Lehman Brothers we use a more sophisticated implementation that allows arbitrary time steps in the lattice and time-dependent volatility and mean reversion speed.

Calibrating the model

As explained in Hull (2000), given the mean reversion speed and volatility parameters $\kappa(t)$ and $\sigma(t)$, $t \geq 0$, the remaining parameters $\alpha(t)$, $t \geq 0$, are determined to ensure that the zero-coupon interest rates calculated in the model match those supplied to the model. This is called calibrating to the term structure (of interest rates).

We calibrate the volatility parameters to a set of at-the-money interest rate swaption prices. Our calibration routine uses numerical optimization to minimize a weighted sum of squared differences between model and market swaption prices. At each step in the optimization iteration, we calibrate to the LIBOR term structure.

There are two different approaches to the volatility calibration. The approach we do *not* use is to jointly calibrate the volatility and mean reversion speed parameters to best fit the entire matrix of at-the-money swaption prices, that is prices of X into Y swaptions where X ranges from 1 month to 30 years and Y ranges from 6 months to 30 years. We do not use this approach because the fit is not good.

Our approach is to use different calibrations for different maturity bonds. Suppose we want to calculate the OAS of a 10-year bond. We will then calibrate to prices of T_n into $T-T_n$ swaptions where T = 10 years and T_n = 1 month, 3 months, 6 months, 1 year, … 9 years, that is swaptions where the swap maturity date is 10 years from the calibration date[9]. With this limited set of swaptions, we can do an exact calibration. The calibration could also be done iteratively by first calibrating $\sigma_0 = \sigma(t)$, $0 \leq t < T_0$, to a T_0 into $T-T_0$ swaption, then taking σ_0 as given and calibrating $\sigma_1 = \sigma(t)$, $T_0 \leq t < T_1$, to a T_1 into $T-T_1$ swaption, and so forth. We do the calibration for bond maturities of 1, 2, 3, …, 10, 15, 20, 25, and 30 years. For a bond with maturity in between two calibration maturities, say a 9.5-year bond, we linearly interpolate the calibrated volatility parameters (in this case between the 9- and 10-year parameters).

We exogenously fix the mean reversion speed and calibrate only the volatility parameters. Currently we use $\kappa(t) = 0$ between 0 and 5 years and $\kappa(t) = 0.03$ for t greater than 5 years. The mean reversion speed is difficult to estimate empirically. The estimation is complicated by the fact the mean reversion speed under the risk-neutral probability measure (the measure used to price securities) is different from the mean reversion speed estimated directly from a time series of the short rate and by the poor empirical performance of a one-factor model. We use the stability of the calibrated $\sigma(t)$, $t \geq 0$, as a criterion for evaluating a particular choice of $\kappa(t)$, $t \geq 0$, but this is done only on an ad hoc basis.

[8] *The formulation in (4.1) and (4.2) is equivalent to $r(t) = \exp(z(t))$, $dz(t) = k(t)(\mu(t) - z(t))dt + \sigma(t)dW(t)$ where $\mu(t) = \alpha'(t)/(k\alpha(t)) + \log(\alpha(t))$.*

[9] *For the swaptions with 1- and 3-month expirations, we actually use 1-month into 10-year and 3-month into 10-year swaptions.*

Shifting the Black-Karasinsky model to better fit the Black volatility skew

The volatility skew in interest rate swaptions describes the fact that the implied Black volatility of an interest rate swaption usually increases with decreases in the strike. The simple explanation is that the Black volatility is a *proportional* volatility and this volatility tends to be higher when the interest rate is low than when it is high. For example, suppose the Black volatility for 1-year into 1-year swaption is 20%. A 1-year into 1-year forward rate of 5% then implies a standard deviation of roughly 1%. On the other hand, if the forward rate was 2.5%, the 20% volatility would imply a standard deviation of 0.5%. The skew can be thought of as simply reflecting the belief that when a rate decreases, say from 5% to 2.5%, then the standard deviation of the rate does not decrease enough to keep the volatility constant. In the example, this means that if the forward rate is 5% and the implied volatility of a 5% strike swaption is 20%, then the implied Black volatility of a swaption with a lower strike should be higher than 20%. Zhou (2003) explains this phenomenon in significantly more detail.

The BK model gives rise to swaption prices with implied Black volatilities that are almost constant as a function of the strike. This is because in the BK model, the short rate is lognormally distributed which roughly means that its standard deviation is proportional to its level.

Another model choice is to replace (4.1) with:

(4.3) $r(t) = \alpha(t) + x(t)$

The model then becomes the Hull-White model where the short rate is normally distributed, which roughly means that its standard deviation is independent of its level. The Hull-White model gives rise to a more pronounced volatility skew than is usually observed.

Our approach is to replace (4.1) with

(4.4) $\acute{r}(t) = \alpha(t)\exp(x(t))$

(4.5) $r(t) = \acute{r}(t) - \gamma$

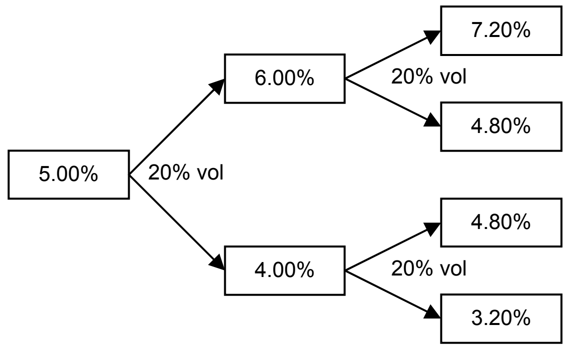
where $\gamma \geq 0$ is a fixed parameter that we call the shift. We call the model the shifted BK model. Notice that with $\gamma = 0\%$, we have $\acute{r}(t) = r(t)$ and get the BK model itself.

The parameters $\alpha(t)$, $t \geq 0$, are still calibrated to ensure that the model matches the zero-coupon interest rates. On average, loosely speaking, the level of the short rate $r(t)$ cannot be affected by the shift. Second, since the mean of $x(t)$ is zero, $\exp(x(t))$ fluctuates around 1. These two observations may be reconciled only if the αs make up for the effect of the shift. This means that the αs must be higher with a shift than with no shift and that $\acute{r}(t)$ will evolve at a level that, loosely speaking, has been shifted up by γ relative to the interest rate in the BK model.

The volatility parameters $\sigma(t)$, $t \geq 0$, determine the volatility of $\acute{r}(t)$ as for a BK model where the short rate is increased by γ. This means that the standard deviation of $\acute{r}(t)$ will be larger than the standard deviation of the short rate, $r(t)$, in the BK model if we use the same σs. We therefore need to use lower σs in the shifted model.

To better understand the effects of the shift, consider the following example. Suppose the short rate starts at 5% and has a one-period simple volatility of 20%. If we model this in a binomial tree we get the results illustrated in Figure 4.1.

Figure 4.1. Simplified binominal tree for the short rate in a lognormal model with simple 20% volatility per period[10]

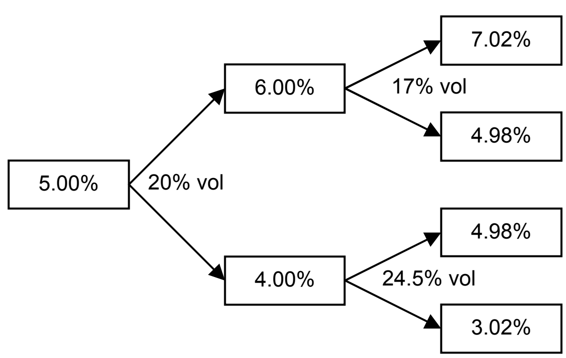


Source: Lehman Brothers.

After one period, the short rate either goes up to 1.2·5% = 6% or down to 0.8·5% = 4%. In the second period, if the rate increased in the first period, it either increases to 1.2·6% = 7.2% or decreases to 0.8·6% = 4.8%, whereas if the rate decreased the first period, it either increases to 1.2·4% = 4.8% or decreases to 0.8·4% = 3.2%. In the second period, the volatility in the up state is 20% = 7.2% / 6% – 1 = – (4.8% / 6% – 1). In the down state, the volatility is also 20% = 4.8% / 4% – 1 = 1 – 3.2% / 4%.[10]

Now consider the model with a 45% shift (Figure 4.2). Suppose we use a volatility parameter of 2%. This means that after one period the short rate either increases to 1.02·(5% + 45%) – 45% = 6% or decreases to 0.98·(5% + 45%) – 45% = 4%. We see that this is the same 20% volatility that we got in Figure 4.1 without a shift. In the second period, if the rate increases to 6%, it either continues up to 1.02·(6% + 45%) – 45% = 7.02% or falls to 0.98·(6% + 45%) – 45% = 4.98%. This implies a volatility of 17% = 7.02% / 6% – 1 = – (4.98% / 6% – 1) which is below the 20% in the non-shifted model. On the other hand, if the rate decreases in the first period, it will either rise to 1.02·(4% + 45%) – 45% = 4.98% or decrease to 0.98·(4% + 45%) – 45% = 3.02% in the second period. So if the rate decreases in the first period, the volatility over the second period is 24.5% = 4.98% / 4% – 1 = – (3.02% / 4% – 1) which is higher than the 17% volatility faced if the rate increases in the first period.

Figure 4.2. Simplified binominal tree for the short rate in a shifted lognormal model with 45% shift and simple 2% lognormal volatility per period[10]



Source: Lehman Brothers.

[10] *We are using a simple volatility measure for illustration only. A binomial tree is more commonly constructed by multiplying the current level with exp(σΔ) in the upstate and exp(-σΔ) in the downstate, where Δ is the period length in years and σ is the continuously compounded volatility. In fact, the BK model can not be implemented in a binomial tree but requires a lattice with at least three branches outgoing from each node (see our discussion on implementation).*

Comments about our model choices

The BK (and the shifted BK) model is a one-factor model with one state variable. At Lehman Brothers we have more sophisticated models with several state variables and/or factors[11]. Adding state variables significantly increases the computation time to the point where three-four state variables is the maximum number practically feasible.

In a one-factor model, changes in interest rates of all maturities are perfectly correlated and the shape of the curve cannot change without the short rate, and thus the level of the curve, changing. This is an unrealistic feature that is contradicted by the data. Several empirical studies have shown that at least three factors are required to properly describe the dynamics of the interest rate curve (see Dai and Singleton (2003) and their references). Another potential problem with one-factor models is that the time-dependent parameters tend to be less constant than in multifactor models (the $\sigma(t)$, $t \geq 0$, calibrated on a given day are often decreasing or hump-shaped, and σs change more from one day to the next).

There are two main reasons why we have decided to use a one-factor/one-state variable model (at least for the time being):

- Simplicity: the model is much easier to explain and implement (and thus replicate).
- The impact of explicitly modelling default is greater than the impact of improving the interest rate modelling (especially for higher spread issuers). We model default by adding the hazard rate (the instantaneous probability of default) as a stochastic factor. This is more easily done for a one-state variable interest rate model. (See section 6 for more on this extension of the model.)

Another important modelling decision is our calibration procedure. As explained above, we perform 14 different calibrations, each using swaptions with the same underlying swap maturity date. This is because calculating an OAS is a pricing exercise, and when pricing it is important that the model is calibrated to the swaptions most similar to the option we are pricing. The option embedded in a callable bond is usually an American call option on a bullet bond which (when forgetting about default risk) is closest to an American option on a swap with the same maturity date as the bond. With our calibration choice we are calibrating to the set of European swaptions most relevant for this American swaption. In contrast, a global calibration to the entire matrix of swaption prices is unlikely to match the prices of these most relevant swaptions.

Another important calibration consideration regards the shift that must be used to best fit the skew. Here we focus on below-ATM swaptions (strike below the forward rate) with swap maturity dates about 5-30 years out and option maturity dates 1-10 years out because these are the swaptions most relevant for the typical callable bond.

5. NUMERICAL RESULTS

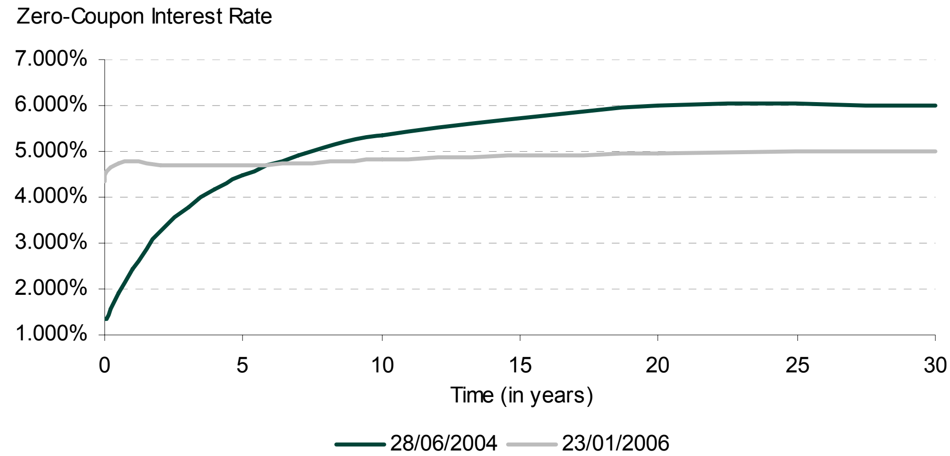
In this section we show differences in the volatility calibration for the BK and the shifted BK model on two days: January 23, 2006, when the curve was almost flat and June 28, 2004, when the curve was steeply upward-sloping. We also discuss the differences in OAS in the two models.

Volatility calibration

Figure 5.1 shows the fitted zero-coupon LIBOR curves on the two dates. The differences between the two dates are very clear.

[11] The numbers of factors is the number of Brownian motions driving the interest rates. The number of state variables is the number of dimensions (in addition to the time dimension) required to represent the model in a recombining tree.

Figure 5.1. Zero-coupon USD LIBOR interest rates on June 28, 2004 and January 23, 2006



Source: Lehman Brothers.

Figure 5.2 shows implied Black volatilities for a subset of the ATM swaptions to which we calibrate the volatility parameters. Notice that the volatilities for short-term swaptions on short-term swaps (e.g., for 3-month into 1-year) are much higher on June 28, 2004 than on January 23, 2006 whereas the volatilities for long-term swaptions on long-term swaps are lower.

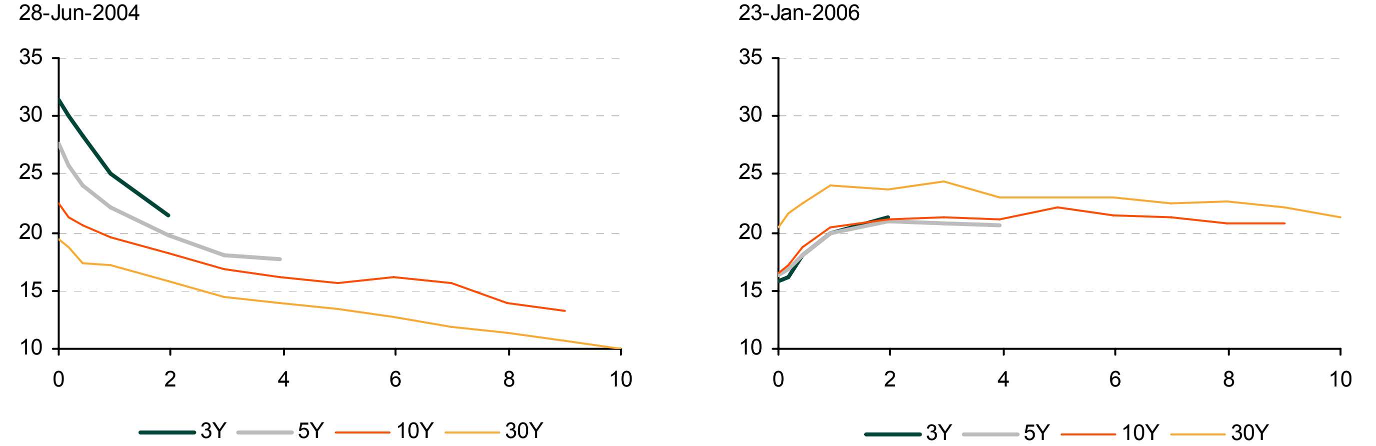
Figure 5.2. Implied Black volatilities (%) for at-the-money swaptions on 28 June 2004 and 23 January 2006 for USD

28-Jun-2004	Option Maturity						23-Jan-2006	Option Maturity					
Swap Term	3M	1Y	2Y	5Y	10Y	30Y	Swap Term	3M	1Y	2Y	5Y	10Y	30Y
1Y	37.0	30.2	24.8	18.7	13.6	9.5	1Y	13.5	17.7	20.2	20.4	18.0	12.7
2Y	33.8	27.5	23.2	18.0	13.1	9.4	2Y	15.8	18.7	20.1	20.1	17.7	12.8
3Y	30.5	25.4	22.0	17.3	12.8	9.3	3Y	16.2	18.8	19.9	19.8	17.4	12.8
5Y	26.1	22.4	20.0	16.2	12.1	9.2	5Y	16.7	18.8	19.6	19.2	16.7	12.8
10Y	20.1	18.4	17.1	14.3	10.9	8.5	10Y	16.1	18.1	18.6	17.9	15.7	12.4
30Y	14.2	13.4	12.7	10.9	9.0	6.1	30Y	15.3	16.4	16.4	15.3	13.6	11.8

Source: Lehman Brothers.

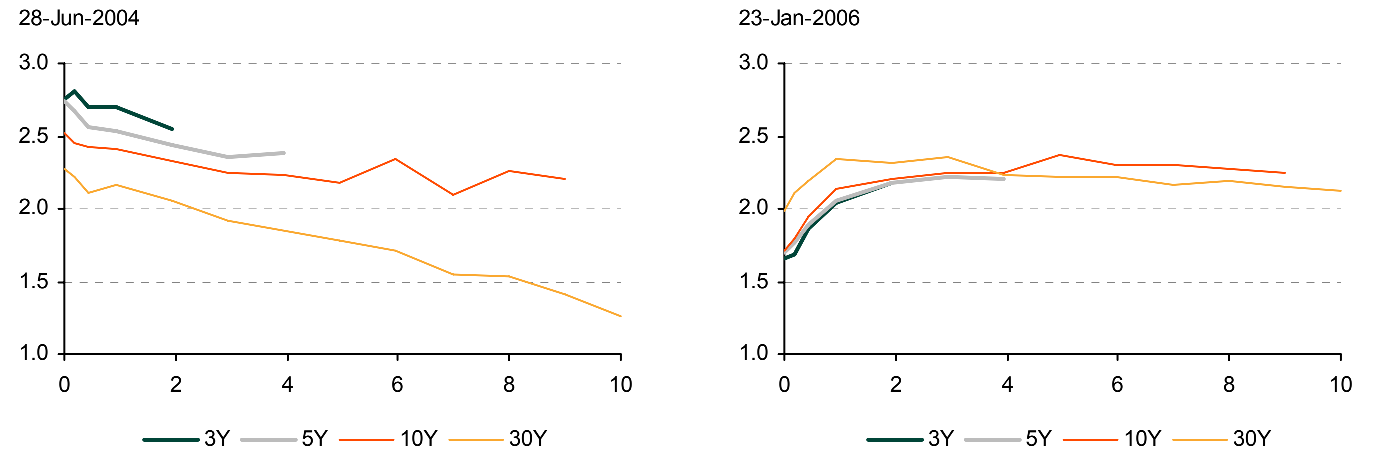
Figures 5.3 and 5.4 show a subset of the calibrated volatility parameters for the two dates using the two different models. In Figure 5.3 we see that on June 28, 2004 the BK volatility parameter curve for the shortest bond maturity (3Y) is the highest, whereas the curve for the longest bond maturity (30Y) is the lowest, and we see that this relationship has switched on January 23, 2006. This observation can also be made for the shifted BK model in Figure 5.4. The volatility parameter curves have very similar shapes in the two models and both capture the market information about the volatility changes seen in Figure 5.2. We find it interesting that the relative change in the calibrated volatility parameters between the two dates is smaller in the 40% shifted model.

Figure 5.3. Calibrated volatility parameters (%) for four different bond maturities in the BK model



Source: Lehman Brothers.

Figure 5.4. Calibrated volatility parameters (%) for four different bond maturities in BK model with 40% shift



Source: Lehman Brothers.

Effect of volatility skew parameter (shift) on OAS

In this subsection we examine the differences in the OAS between the two models. Our bond universe is a combination of the Lehman Brothers US Corporate and High Yield indices. The comparison is done using data from January 23, 2006.[12]

At the aggregate level there is very little change in OAS. However, for individual bonds some differences can be found. Out of the 4,812 bonds in our universe 1,215 are callable and 68 are putable. For 1,095 bonds there is a difference in the OAS, but only 388 of the bonds have a relative difference in OAS greater than 2% (the maximum OAS among those is 306bp). There are 78 bonds with an OAS difference greater than 10%, but among those the OAS is greater than 40bp for only 12 bonds and greater than 20bp for only 22 bonds. The maximum absolute OAS difference among all bonds is 31bp, but several of the bonds with a large absolute OAS difference have a negative OAS (this could be the result of a stale price

[12] *Thanks to Peili Wang for calculating these numbers.*

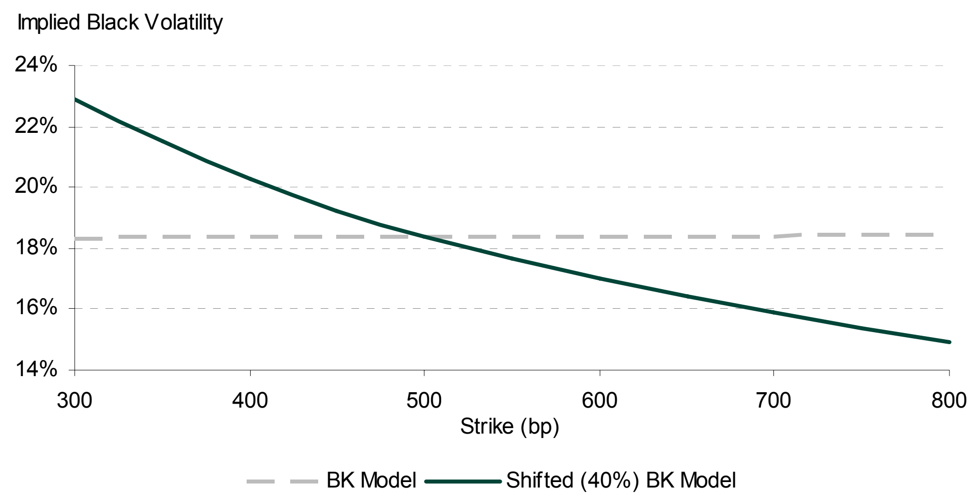
or maybe the bond has just been called[13]). Most bonds with a negative OAS in the BK model have a higher OAS in the shifted model.

The two main observations regarding the effect of the model change on the OAS of an individual bond are:

1. Bonds where the call option is in the money tend to have *higher* OAS in the shifted model.
2. Bonds where the call option is far out of the money tend to have *lower* OAS in the shifted model.

The extent to which the option is in or out of the money affects the difference in the OAS in the two models. In the shifted model, high (rate) strike swaptions will have lower Black volatility, whereas the low (rate) strike swaptions will have a higher Black volatility (Figure 5.5). Since a high rate strike swaption corresponds to an option on a high-coupon bond, in-the-money call options will tend to have a *higher* OAS in the shifted BK model. This is because a lower Black volatility means a lower option value which gives rise to a lower value of the stripped bond (to keep the value of the callable bond constant, see equation (3.2)). A lower value of the stripped bond means a higher OAS. Similarly, an out-of-the-money call option will tend to have a *lower* OAS in the shifted model.

Figure 5.5. Implied Black volatilities for a 5-year into 5-year receiver swaption assuming a flat interest rate curve at 5% and constant volatility parameters



Source: Lehman Brothers.

6. ADJUSTING FOR DEFAULT RISK

In the last subsection of section 2 we discussed how the Z-spread can be interpreted as an approximate 0%-recovery bond-implied CDS spread calculated in a hazard rate model. We can improve upon the Z-spread by calculating a bond-implied CDS spread assuming a more realistic recovery rate.

It is important to use a realistic recovery rate not only when we compare a bond with a CDS but also when we calculate the duration (interest rate sensitivity) of a bond. Consider the following extreme example. On 23 January 2006 a 6% bond maturing on January 1, 2036 was trading at \$70. The bond's 0%-recovery CDS spread is 382 and the 40%-recovery CDS

[13] *There is often a delay from the announcement of a call (and its reflection in the bond price) to the registration of the call in our bond database.*

spread is 515. If LIBOR rates are increased by 100bp, we assume recovery = 0%, and hold 0%-recovery CDS spread constant, then the price drops $7.1. If we assume recovery = 40% and hold the 40%-recovery CDS spread constant then the price only drops $4.8.

If we want to assume a recovery rate greater than 0% and directly model default, the natural next question is how to deal with a callable bond. Just as going from Z-spread to OAS means going from deterministic to stochastic interest rates, going from the bond-implied CDS spread of a non-callable bond to that of a callable bond means going from deterministic interest rates and hazard rates to *stochastic interest rates and hazard rates*. It is outside the scope of this paper to fully describe this type of two-factor model and at this point we just want to comment on the approach (a later publication will discuss the approach in detail).

There are two fundamentally different ways to implement a stochastic interest/hazard rate model. One possibility is to use Monte Carlo techniques. The advantages of this approach are that we can use a more realistic multifactor interest rate model and we can more easily incorporate a non-zero correlation between interest rates and hazard rates. The main disadvantage is that dealing with an American (or Bermudan) exercise is complicated significantly. Monte Carlo models also take longer to run to achieve the same level of accuracy as lattice-based models, which is the second type of approach.

We have implemented a stochastic interest/hazard rate model in a two-factor lattice. The main drawback of our lattice-based approach is that only one-factor is used to model interest rates which is less realistic (see section 4) and that incorporating a non-zero correlation is more difficult. The advantages of our model are that it calculates quickly, gives accurate risk measures (Greeks), and easily deals with American and Bermudan exercise.

In addition to the recovery rate (and potentially a CDS curve that will be shifted – see the discussion of bond-implied CDS spread in section 2, particularly footnote 6), a stochastic interest rate/hazard rate model needs a hazard rate volatility and an interest rate/hazard rate correlation. These two parameters are difficult to estimate and are highly dependent on the issuer of the bond. We think, in general, that if nothing else is known, the correlation should be set to zero or slightly negative. There are some market data on default swaptions that can be used to get a handle on the hazard rate volatility. For more discussion of this issue, including an account of how to build a stochastic hazard rate model (not including a stochastic interest rate), see Pedersen and Sen (2004).

7. SUMMARY

We have explained what the Z-spread of a corporate bond is and how it is calculated. We introduced the concept of a bond-implied CDS spread and explained how the Z-spread can be interpreted as an approximate 0%-recovery bond-implied CDS spread. This was used to establish the limitations of the Z-spread for comparing a bond to a CDS. Next the OAS of a callable bond was introduced as the Z-spread of the bond stripped of its call option where the value of the stripped bond is calculated as the price of the callable bond minus the theoretical value of the call option. We explained how we use a stochastic term structure model to find the value of the call option and how to incorporate the OAS in the valuation of the call option. We also described the specific stochastic term structure model we use, its limitations, and how it has recently been changed to better fit the volatility skew observed in the interest rate swaption market. Next we presented numerical results illustrating the effect of the model change on the calibrated volatility parameters and the OAS of the bonds in the Corporate and High Yield Credit Indices. Finally, we briefly described the type of model required to calculate a bond-implied CDS spread for a callable bond.

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LEVER: A Framework for Scoring LEVeraging Event Risk[1,2]

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We introduce LEVER, a quantitative framework for measuring the relative risk of a leveraged buyout (LBO) or leveraged recapitalization in the US credit market. LEVER fits key accounting and market information into two simple risk measures: the Firm LEVER-Score and the Macro LEVER-Score. The Firm LEVER-Score seeks to capture a firm's likelihood of financial restructuring (via LBO or a leveraged recapitalization) using valuation, operation and execution variables. The Macro LEVER-Score measures the LBO-friendliness of the overall market using economic, market and financial variables. High scores on both measures suggest a higher likelihood of an LBO or leveraged recapitalization the following year. The Firm LEVER-Score showed strong predictive power in a test covering the period 1995-2005. We also discuss the LEVER PowerTool launched on LehmanLive.

1. INTRODUCTION

Investors with corporate bond portfolios are mainly exposed to default, liquidity and spread volatility risk. LBOs and leveraged recapitalizations pose an added threat because the large amount of debt raised for such transactions can lead to credit deterioration. LBO targets often fall from investment grade to BB or below. Moreover, the risk of an LBO or a leveraged recapitalization can boost spread volatility, leading to large losses in corporate bond portfolios. While actual LBOs are rare, leveraged recapitalizations occur more frequently. In the remainder of this article, the term *"LBO"* will be liberally used to refer both to an actual LBO transaction and a leveraged recapitalization, as they have similar implications for credit portfolios and are driven by similar considerations.

LBOs and leveraged recapitalizations were a newly resurgent risk in 2005. They represented the second-biggest driver of investment-grade underperformance in the past year after the US automakers. Knight Ridder was the worst-performing issuer in the Lehman Brothers US Investment Grade (IG) Index in 2005 (-1950bp excess return). Albertsons (-1275bp excess return) was at #5, followed by Kerr-McGee (-1223bp excess return), forced to recapitalize by shareholder activist Carl Icahn (Figure 1). Although our base assumption regarding leveraged transactions is that they are punitive to bondholders, we recognize that such risks can be mitigated in certain instances by tenders, exchanges and the like, particularly for issues with better covenants.

[1] *The authors would like to thank Srivaths Balakrishnan, Margery Cunningham, Hemant Dabke, Mike Guarnieri, Mary Margiotta, Eric Miller, Vasant Naik, Marco Naldi, and Brad Rogoff for their helpful comments and assistance.*

[2] *This is an updated version of our publication, LEVER: A framework for Scoring LEVeraging Event Risk, January 9, 2006, Quantitative Credit Strategies, Lehman Brothers.*

Figure 1. 10 worst-performing Lehman US IG Credit Index issuers, 2005

Source: Lehman Brothers.

Investors were largely unprepared for this risk in early 2005, with limited incremental spread priced into the LBO and leveraged recapitalization targets. While current wider spreads for LBO-associated companies suggest that the market is better prepared for 2006, we think this risk may still not be accurately priced across all firms/sectors largely because of the inherent difficulty in predicting which firms will ultimately undergo an LBO or a leveraged recapitalization.[3] This is especially true for most debt investors, who typically have limited access to the standard screening process used by financial sponsors. Moreover, equity-based analysis of LBO risk is of limited use because LBOs favor shareholders over debt-holders, causing equity prices to rise in anticipation of such events.

In this article, we propose a new framework called *LEVER* for scoring the risk of leveraged recapitalizations which is designed to narrow this informational gap. In developing such a framework, identifying the appropriate inputs (i.e. characteristics of firms that lead them to be potential LBO or leveraged recapitalization targets) is as critical as combining these inputs in a sensible functional form. To that end, we base our framework on extensive discussions with experienced practitioners in the LBO arena. The resulting model is a quantitative representation of their collective approaches.

There are two components to the *LEVER* framework. The ***Firm LEVER-Score*** identifies companies that look potentially more attractive to financial sponsors. Scores range from 0 to 10, and companies that score above 7.5 are particularly at risk. The ***Macro LEVER-Score*** identifies market environments that are more amenable to LBOs. We test our Firm *LEVER*-Score on the firms in the S&P 1500 between 1995 and 2005 and document strong evidence of its predictive power.

The rest of the article is organized as follows. In **Section 2**, we present a brief overview of the variables used in the *LEVER* framework. In **Section 3**, we explain the construction of the Macro and Firm *LEVER*-Scores. In **Section 4,** we provide some results on the historical performance of *LEVER*. In **Section 5**, we present the LEVER Powertool on *LehmanLive* which enables investors to understand and perform scenario analyses on individual issuers and sectors. We present our conclusions in **Section 6**.

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[3] *Although there have been some academic studies on this subject, including that by Opler and Titman (1993), not many are focused on forecasting LBO activity. Often, such studies are focused on the post-transaction performance of an LBO target (see for instance, Long and Ravenscraft, 1993).*

Leveraged Buyouts and Leveraged Recapitalizations: An Overview

An LBO is a transaction in which an acquirer takes a company (or a business unit of a company) private by purchasing its stock. These transactions are typically funded with 70-80% debt, resulting in a markedly weaker credit position for outstanding bondholders. The acquirer is often referred to as a financial sponsor because it provides the initial capital for the transaction, but does not intend to remain permanently invested. Post-acquisition, the acquirer uses the cash flows generated by the target firm to service the debt and provide additional returns. Effectively, the target firm finances its own acquisition. Leveraged recapitalizations operate similarly, except that the company remains public.

LBOs can be detrimental to prior debt holders because of the drastic increase in leverage. Investment-grade companies taken private through an LBO are usually rated B or BB after the transaction is complete. Furthermore, the new debt is often issued with covenants not offered to previously existing bondholders. As a result, the mere threat of an LBO is often a cause of increased spread volatility.

In identifying LBO targets, financial sponsors consider the initial equity investment required and the potential for additional financing. The investment cost is determined by the total amount of debt needed to fund the acquisition and the interest costs of raising the additional debt. Sponsors also look for an absence of restrictive covenants in the existing debt of the target firm.

Once a firm is bought out, financial sponsors can generate equity returns by improving operations, increasing incentives for the management to perform, and better utilizing free cash flows. The inherent increase in risk taking resulting from the increased leverage also creates the potential for higher returns. Finally, the financial sponsor may force corporate restructuring or asset disposals.

Even when desirable targets can be identified, the macroeconomic environment must be amenable to LBOs for a transaction to occur. Buyout firms raise capital more easily in a growing economy. Furthermore, low interest rates facilitate LBOs by reducing the cost of issuing new debt. Conversely, when the high-yield loan or debt market is experiencing numerous corporate defaults, an LBO might be expensive to finance.

2. KEY VARIABLES THAT DRIVE LBO TRANSACTIONS

2.1. Firm-level characteristics

Our model includes three broad categories of firm-level factors driving LBOs. The most attractive targets have below-market *valuations,* their *operations* generate adequate cash flows for servicing debt, and are likely to enjoy a smooth *execution* of the transaction. We discuss these factors below.

Valuation

The primary consideration for an LBO transaction is the potential for value creation.[4] Target firms usually have a large "value gap" between the current enterprise value and the potential value after restructuring, which can be measured by the following variables:

Book-to-market ratio

The book value can be used as a proxy for the disposal value of the firm's assets, or the replacement cost of the assets. A high book-to-market ratio may indicate a mispricing between the public and private equity markets. It may also indicate the potential for improving operations under new management.

Enterprise value to EBITDA[5] multiple

The enterprise value to EBITDA multiple tends to be low when the enterprise value does not fully reflect the potential earnings capacity of the firm, or when there are concerns regarding the sustainability of recent earnings levels. In particular, the relative difference in this multiple between the firm and its sector peers may reflect an attractive opportunity for an LBO acquirer. This is especially true when the target is a division or a subsidiary, which may be unfairly bundled within a larger firm with different valuation norms.

Operations

Targets must have sufficient cash flows to service the debt raised for funding an LBO transaction. Additionally, acquirers may look for firms whose operations are free from future capital commitments. The following variables can be used to capture these characteristics:

Free cash flow yield

Free cash flow yield (the ratio of free cash flow to market value) combines operational and valuation attractiveness of a potential LBO candidate. Firms with high free cash flows and low market value are attractive LBO candidates.

Growth of capital expenditure (capex)[6]

Rising capital expenditure may indicate large operational commitments that could compete for the cash flows of the firm. However, the capex growth of a firm should be compared with its sector peers, because increased capex may result from systematic factors affecting the sector rather than the commitments of an individual firm.

Execution

An attractive LBO target should also exhibit characteristics that enable the execution of a re-leveraging transaction. The following variables are indicative of these characteristics:

[4] *See Berg and Gottschalg (2004) for a survey on value creation in buyouts.*
[5] *Earnings Before Interests, Taxes, Depreciation and Amortization.*
[6] *Defined as the one-year change in capital expenditure.*

Cash-flow variability

While the size of cash flows signals operational characteristics, the stability of cash flows indicates the potential for a firm to "weather" an LBO. Stable cash flows suggest that the firm may be able to provide returns to the investor net of the debt-servicing cost. This is also a key variable that lenders consider before agreeing to finance an LBO transaction.

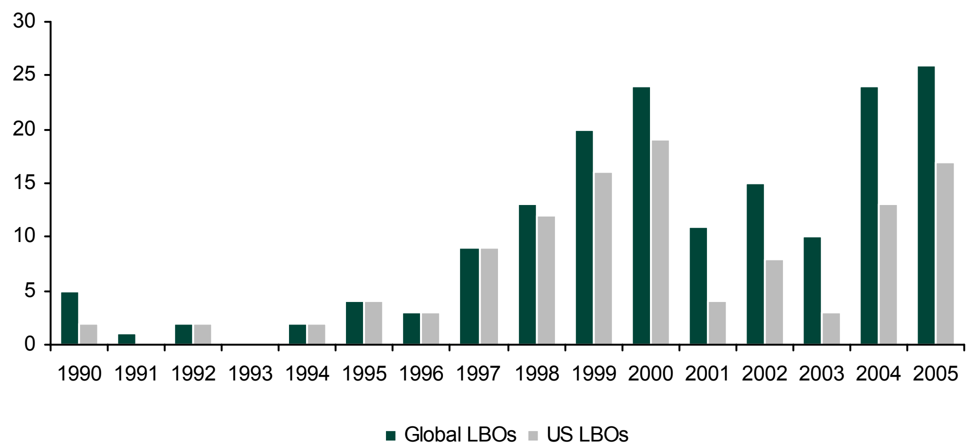
Size of the firm

A smaller firm is generally an easier LBO target as it requires a smaller amount of debt to fund the transaction. The size of the firm may be measured either by the market capitalization or the enterprise value.

2.2. Macroeconomic characteristics

We examine the overall trends in the level of LBO activity over the past 15 years to assess the correlation of LBO activity with macro drivers of leveraged transactions. In Figure 2, we show the number of large LBOs (deal size >$250m) globally and in the US between 1990 and 2005. There were two peaks in the level of LBO activity, the first in 2000 and the other in 2005. The relative drop in LBO activity in 2001 coincided with the US recession in the early part of this century.

Figure 2. LBO activity: 1990-2005[7]



Source: Lehman Brothers, SDC.

[7] *Data for 2005 are until the third quarter of the year.*

Several macro factors help explain these trends. In Figure 3, we summarize the correlation of some key variables with the number of LBO transactions.

Figure 3. Correlation of key variables and number and value of LBO transactions (1990-2005)

Category	Variable	Correlation with No. of LBOs
Economic Environment	*GDP Growth*	31%
Yield Curve	*10 year yield change*	-29%
	2y-10y slope change	-52%
Equity Markets	*S&P 500 Returns*	-12%
Macro Fundamentals	*S&P 500 Dividend Yield*	-75%
	S&P 500 Cash-flow Yield	22%
Financial risk	*Leverage of S&P 500*	-23%
	Credit spread changes	51%
Technicals	*Amount of private equity capital raised in previous year*	65%

Source: Lehman Brothers, Bloomberg, Venture Economics.

Economic growth

Overall growth (indicated by the y-o-y GDP growth) is positively correlated with the number of LBOs. Revenue and cash flow growth generally accelerate in periods of high growth, enabling more rapid debt reduction on the part of acquired companies.

Interest rates

In low rate and low equity risk premia environments, investors seek yield from alternative asset classes, such as private equity funds. This increases the amount of capital at the disposal of acquirers, hence accentuating LBO activity. Furthermore, low interest rates decrease the cost of financing new debt.

Equity market returns

A weaker equity market signals a more LBO-friendly environment since cheap target firms might be easier to find. Conversely, in periods when the equity market is strong, private equity firms must compete with strategic buyers who can use the strength of their own stock to fund acquisitions. As expected, equity market returns and the number of LBOs are negatively correlated. However, the correlation is moderate because poor equity returns are often associated with poor economic conditions that discourage LBO activity.

Cash flow and dividend yield trends

The number of LBO transactions is negatively correlated with the dividend yield of firms in the S&P 500 and positively correlated with their cash flow yields. High cash flow generation combined with a low dividend payout ratio leaves a lot of cash available for servicing new debt or paying out a special dividend.

Financial risk

Systemic financial risk, exhibited by leverage trends, shows a strong relationship with the number of LBOs. Overall leverage trends are negatively correlated with the number of LBOs because periods in which firms de-leverage are opportune for re-leveraging transactions.

Wider credit spreads also reflect systemic financial risk. However, spreads have a positive correlation with LBO trends, which likely is a consequence of the endogeneity of spreads (i.e. spreads widen in anticipation of LBO transactions). Lagged correlations between credit spreads and LBO transactions are in fact negative (approximately -13%).

LBO technicals

The level of private equity raised in the previous year has a relatively high correlation with the number of LBOs (65%). This could explain current high levels of LBO activity, as private equity firms are raising record amounts for their funds.[8]

3. SCORING LBO/RECAP RISK WITH *LEVER*

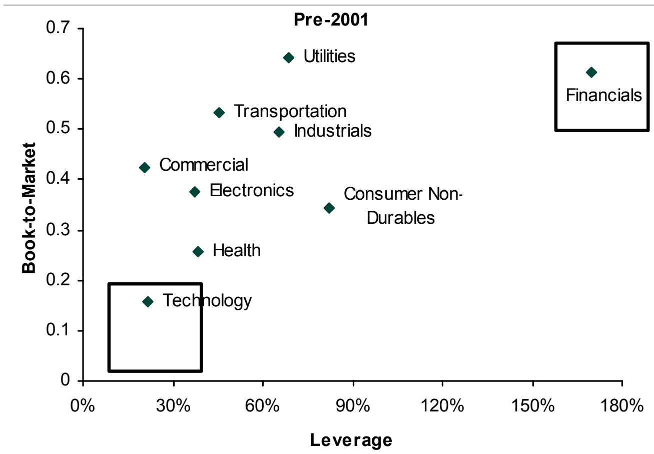
The *LEVER* framework processes the above firm-specific and market variables into two measures: the Firm *LEVER-Score* and the *Macro LEVER-Score*. The Firm *LEVER*-Score identifies particular issuers using fundamental and market information specific to each firm. The Macro *LEVER*-Score captures the overall attractiveness of the environment for LBO transactions. In this section, we discuss how these scores are calculated.

3.1. Calculating the Firm *LEVER*-Score

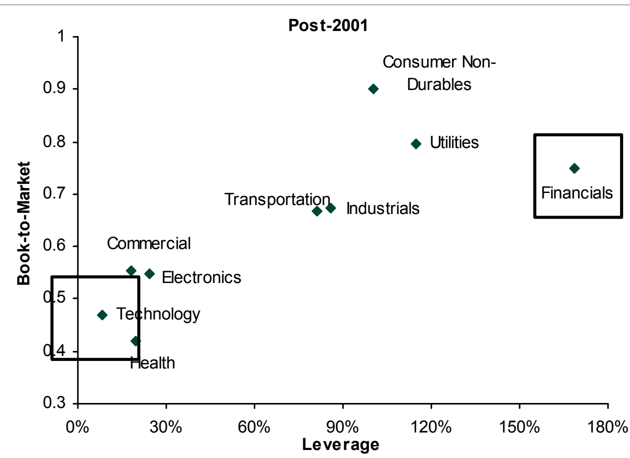
Variations across sectors

The likelihood of an LBO or leveraged recapitalization differs from sector to sector. Some sectors are fundamentally unsuited to re-leveraging operations because they are already very highly leveraged. Furthermore, growth sectors with low book-to-market ratios and less stable free cash flow are less attractive LBO candidates than value sectors, where cash flows can be used efficiently to service debt and generate returns. To analyze systematic patterns across sectors, Figure 4 charts the average leverage versus the average book-to-market ratio of different sectors.[9]

Figure 4. Sector-level LBO attractiveness



Source: Lehman Brothers.



Source: Lehman Brothers.

[8] *See for instance "Private Equity Fundraising Activity Surpassed 2004 in First Three Quarters of 2005", National Venture Capital Association, October 17, 2005.*

[9] *We define leverage as the ratio of total debt to the market value of equity.*

The Financial sector consistently displays the highest leverage, making financial firms unlikely LBO candidates. Technology firms, on the other hand, show a transformation around 2001. In the pre-2001 period (which includes the Technology boom of the late 1990s), the sector had by far the lowest book-to-market ratio, implying that it was in a "growth" phase. After 2001, however, Technology transitioned to a "value" status, exhibiting a book-to-market ratio more in line with the other sectors. Since 2001, Technology firms have become inherently more attractive for LBO transactions.

We incorporate these observations into the construction and testing of our Firm *LEVER*-Score as follows:

- We completely exclude financial firms from our data set.
- We exclude observations of Technology firms prior to 2001, but retain them from 2001 onwards.

Firm-level characteristics

After filtering the universe for certain sectors as mentioned above, we analyze firm-level characteristics to identify issuers with heightened risk of an LBO or leverage recapitalization. In Figure 5 we list the variables used to compute the Firm *LEVER*-Score and our hypothesis for the relative magnitude of these variables for LBO vs non-LBO names (for instance our hypothesis is that potential LBO candidates would have a high book-to-market ratio). As discussed in Section 2, we group these variables to form three component scores: the Valuation Score, Operation Score and Execution Score (Figure 5).

Figure 5. Components of the Firm *LEVER*-Score

Valuation Score		Operation Score		Execution Score	
Book to Market	↑	Free Cash Flow Yield	↑	Firm Size	↓
EV to EBITDA	↓	Capex Growth	↓	Free Cash Flow Variability	↓

Source: Lehman Brothers.

Constructing the Firm *LEVER*-Score

To construct the Firm *LEVER*-Score, we use the cross sectional-ranking of the normalized values of the variables listed in Figure 5. We define the normalized value V_i^N of a given variable V for the i^{th} firm as:

$$V_i^N = \alpha_V \left(\frac{V_i}{V_{U_V}} - 1 \right)$$

Where:

- V_{U_V} is the market-value-weighted average of the variable V of the firms in the universe U_V. The universe U_V corresponding to the variable V includes the entire universe of firms for the variables free cash flow yield, firm size and free cash flow variability, and is limited to the sector peers of the i^{th} firm for the book-to-market ratio, EV-to-EBITDA ratio and capex growth.
- α_V is the desired direction for the given variable (e.g., the α_V corresponding to the book-to-market ratio is 1 and that corresponding to the EV-to-EBITDA multiple is -1).

Along each normalized variable V^N, the firms in the universe are ranked. The Valuation, Operation and Execution Scores for the i^{th} firm are an average of the rankings corresponding to the underlying variables. In other words, the Valuation score for firm *i* is an average of its ranking along the book-to-market ratio and EV-to-EBITDA multiple. The Firm *LEVER*-Score is then calculated as the average of all the variables underlying the Valuation, Operation and Execution scores. [10]

Examples: Toys 'R' Us and Halliburton Co.

To illustrate the process of arriving at the Firm *LEVER*-Score, let us consider two recent examples. In Figure 6a, we show the score calculation for Toys 'R' Us Inc. as of 4Q 2004.

Figure 6a. Construction of the Firm *LEVER*-Score for Toys 'R' Us Inc. (4Q 2004)

	Median of U_V [11]	Toys R Us
Firm *LEVER*-Score		9.0
Valuation Score		+++
Book-to-market	33%	97%
EV/EBITDA	9.9x	13.3x
Operation Score		+++
Capex Growth	4.5%	-36%
Cash Flow yield	4%	10%
Execution Score		++
Free cash flow variability	16%	8%
Market Value	$ 2bn	$ 4.4bn

Source: Lehman Brothers.

Toys 'R' Us Inc. had a much higher book-to-market ratio than its sector peers, making its valuation attractive for LBO investors. On the operations side, while the median capital expenditure of its sector peers rose 4.5% between 4Q 2003 and 4Q 2004, Toys 'R' Us posted a sharp fall, of 36%, in the same period. Still, it continued to generate a high level of cash flow yield, signaling strong fundamentals. In addition, the cash flows of the firm were much more stable than those of the sector. Although TOY was larger than the median value of the universe, it was not too large to deter an LBO acquirer. As a result of these characteristics, Toys 'R' Us Inc. had a high Firm *LEVER*-Score of 9.0.

In Figure 6b, we examine a case with a low score, that of Halliburton Co., as of 1Q 2005.

Figure 6b. Construction of the Firm *LEVER*-Score for Halliburton Co. (1Q 2005)

	Median of U_V	Halliburton
Firm *LEVER*-Score		0.9
Valuation Score		- -
Book-to-market	43%	20%
EV/EBITDA	9.9x	15.4x
Operation Score		- -
Capex Growth	7.3%	-14.8%
Cash Flow yield	3%	-3%
Execution Score		- - -
Free cash flow variability	21%	27%
Market Value	$ 2bn	$ 22bn

Source: Lehman Brothers.

[10] *The scores are altered so that they are uniformly distributed in the universe. This enables us to effectively examine the performance of the score, as explained in Section 4.2.*

[11] *We report the median of the universe for the cash flow yield, free cash flow variability and market value; and that of the firm's sector peers for book-to-market, EV-to-EBITDA and capex growth.*

Although Halliburton had a high EV-to-EBITDA multiple compared with its sector peers, it had a much lower book-to-market ratio. It also had worse free cash flow yields than other firms in its sector, making its operations unattractive. With respect to execution, HAL's size and its high free cash flow variability would make an LBO transaction more difficult to complete. Halliburton therefore had a low Firm *LEVER*-Score of 0.9 as of 1Q 2005.

3.2. Macro *LEVER*-Score

To construct the Macro *LEVER*-Score, we perform a lagged regression of the number of LBOs on the following macro variables (Figure 7).

Figure 7. Variables used in the construction of the Macro *LEVER*-Score

Category	Variable
Economic Environment	GDP Growth
Yield Curve	10 year yield change
	2y-10y slope change
Equity Markets	S&P 500 Returns
Macro Fundamentals	S&P 500 Dividend Yield
	S&P 500 Cash-flow Yield
Financial risk	Leverage of S&P 500
	Credit spread changes
Technicals	Amount of private equity capital raised

Source: Lehman Brothers.

We extend the Macro *LEVER*-Score to cover US and European firms.

Interpreting the Firm *LEVER*-Score with the Macro *LEVER*-Score

While the Firm *LEVER*-Score uses cross-sectional data, the Macro *LEVER*-Score is produced using time-series analysis. Interpreting them together is therefore not completely obvious. To fully utilize the information from the top-down and bottom-up analyses of the two scores, investors should consider the following:

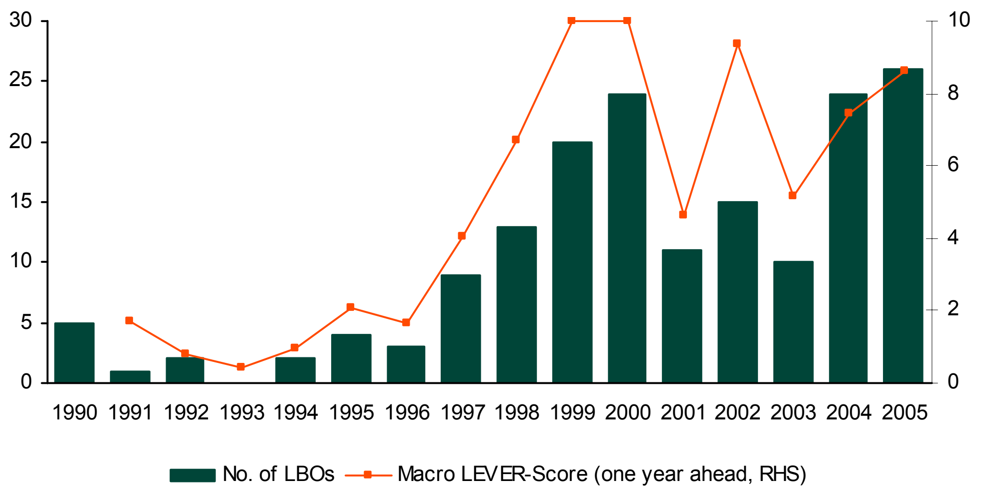
- A high Macro *LEVER*-Score indicates the possibility of a large amount of LBO activity in the year to come. This suggests that a greater weight should be placed on the Firm *LEVER*-Score when it is analyzed in conjunction with traditional credit analyses based on other indicators. Conversely, when the Macro *LEVER*-Score is low (during which periods the Firm *LEVER*-Score would flag several credits as possible LBO candidates), investors could factor in the risk of LBOs selectively.
- The Macro *LEVER*-Score could be modified into a Sector Score if the underlying regression is performed on sector-specific transactions. In such a case, using the Macro *LEVER*-Score directly with the Firm *LEVER*-Score would be informative on a cross-sectional basis for a given year. We note that this method may be difficult to implement, however, since LBO data within specific sectors may be sparse.

4. PERFORMANCE OF THE *LEVER* FRAMEWORK

4.1. Performance of the Macro *LEVER*-Score

To examine the performance of the Macro *LEVER*-Score, we need to assess if periods with high LBO activity were predicted by a high Macro *LEVER*-Score one year ahead, and vice versa. In Figure 8, we report the one-year forecasts of the Macro *LEVER*-Scores along with the annual number of LBO transactions. We can see that the profile of the Macro *LEVER*-Score one year ahead tracks the overall number of LBOs reasonably well.

Figure 8. Performance of the Macro *LEVER*-Score (one year ahead) in predicting LBO volumes[12]



Source: Lehman Brothers.

4.2. Performance of the Firm LEVER-Score

We use the concept of "performance curve" to quantify the predictive power of the Firm LEVER-Score.[13] The performance curve captures the efficacy of the score by measuring its Type I and Type II errors. In the current context, the two errors above would be defined as follows:

Type I Error: Firms that are identified as having higher risk of an LBO or leveraged recapitalization do not get acquired or undergo any leveraged recapitalization. This is generally referred to as a *false positive* error.

Type II Error: Firms that are identified as having lower risk of an LBO or leveraged recapitalization actually go on to be acquired. This is generally referred to as a *false negative* error.

Since the actual number of LBO transactions observed in the market is far fewer than the size of the universe considered, it is natural that the Type I error described above is high. Furthermore, the credit spreads of a name identified wrongly as an LBO or leveraged recapitalization candidate might still widen because of market speculation about a potential LBO, thus alleviating the negative impact of a Type I Error. It is the Type II error that is crucial to minimize from the perspective of a credit portfolio. A high Type II error level would indicate the failure of the screen to flag high-risk names in a long credit portfolio.

[12] *Data for 2005 are until the third quarter of the year.*
[13] *The performance curve is also called the "power curve" in Statistics.*

The performance curve is constructed as follows. We start with an empty rectangle and, on the horizontal axis, we mark the level of the score from right to left. Notice that, by construction, the Firm *LEVER*-Score is uniformly distributed over the entire sample; for example, exactly 10% of the firms have a score between, say, 7 and 8. On the vertical axis, we read the percentage of LBOs out of the total number of LBOs that had a score above a given level.

The first diagonal of our rectangle represents an important benchmark. A model for predicting LBO activity whose performance curve coincides with the first diagonal does not provide any improvement over the inexpensive strategy of predicting LBOs by randomly drawing company names from an urn. In fact, such a model will, on average, include 20% of the LBOs in a list of 20% of all names, 30% of the LBOs in a list of 30% of all names, and so on.

An effective model has a performance curve that lies *above* the first diagonal. The higher the power curve, the higher the model's predictive power. A perfect model will capture 100% of the LBOs by short-listing 1% of all names in the dataset. Such a model will therefore display a power curve composed of two segments, the first one overlapping with the left short side of the rectangle, the second one overlapping with the upper long side.

The share of firms that receive a high score from the model but are not LBOed measures the Type I error of the model. Symmetrically, the share of firms that receive low scores and are subsequently LBOed provides a measure of the model's Type II error. The reader can verify that these errors can be quantified using the performance curve.

A different way of looking at the performance curve is to size a "screen", i.e. a short list, in terms of an absolute number of issuers, and then observe how many LBOs the model has been able to include in that short list one quarter before the event. We perform the following back-test of the Firm *LEVER*-Score: for every quarter in the period 1995-2005, we perform a cross-sectional analysis of the names in the S&P 1500 Index and assign a score to each name as described in Section 3.

We then rank the names in descending order according to their scores, and analyze the top and bottom selections of the screen. If the screen were to perform well, the LBOs in the following quarter would predominantly fall in the top selection. In addition, only a small number of the LBOs would fall in the bottom selection.

In Figure 9 we report the number of LBOs captured one quarter ahead of the event by selecting different screen sizes over the period 1995-2005.

Figure 9. Performance of the Firm *LEVER*-Score

Total no. of LBOs	Selection	Size of selection: 100	150	200
32	Top	14	19	20
	Bottom	0	0	1

Source: Lehman Brothers.

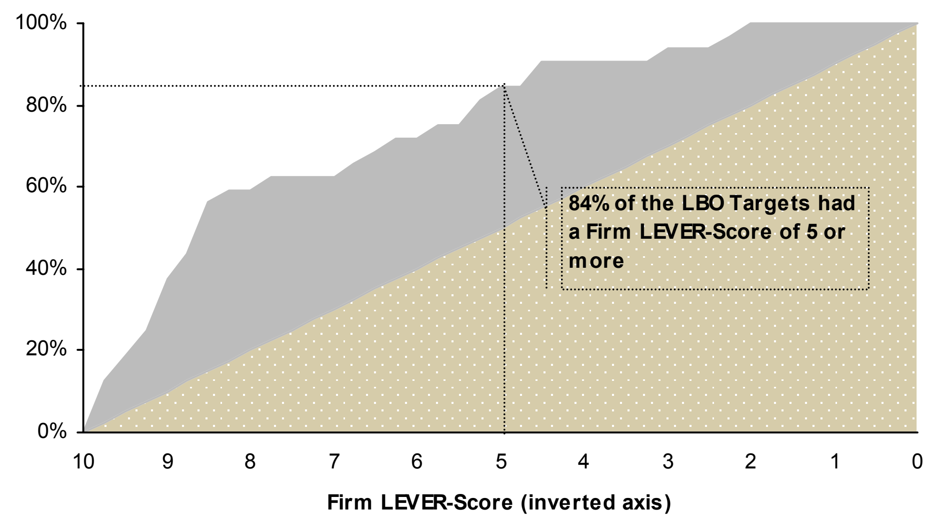
During the 1995-2005 period, there were 32 LBOs in our sample[14]. The selection of the top 100 names based on the Firm *LEVER*-Score in the previous quarter captured 14 of the names that were actually LBOed. In addition, no LBOed name was included in the list of the 100 names with the lowest Firm *LEVER*-Score. Figure 9 also shows that screening 150 names would have captured 19 LBOs, while 20 of the 32 LBOs would have been captured by short-listing 200 issuers every quarter. Only one LBO was included in a list of 200 names with the

[14] *Although we consider a universe of 1500 names, data on all the variables we consider are available for around 900 names, on average, every quarter.*

lowest Firm *LEVER*-Scores. Thus, not only is the Firm *LEVER*-Score able to capture a substantial number of LBOs one quarter ahead; the score also succeeds in minimizing the disturbing Type II error.

To analyze the profile of the scores and the degree of the Type I and II errors in further detail, we plot the performance curve of the Firm *LEVER*-Score in Figure 10a.

Figure 10a. Performance Curve of Firm *LEVER*-Score



Source: Lehman Brothers.

Figure 10a shows that the performance curve of the Firm *LEVER*-Score is well above the first diagonal. Approximately 63% of the names in our sample that were LBOed had a Firm *LEVER*-Score of 7.5 or more one quarter ahead of the transaction. This means that 63% of the LBOs were short listed in the top 25% of the names ranked according to our Firm *LEVER*-Score. In Figure 10b we report the Type I and Type II errors in our sample. Figure 10b shows that fewer than 6% of LBOed names had a score of 2.5 or less.

Figure 10b. Type I and II errors in the Firm *LEVER*-Score

	LBO Names	Non-LBO Names
Score above 7.5	63%	25%
Score below 2.5	6%	25%

Source: Lehman Brothers.

In Figure 11a below, we report the performance of the component scores. While all the component scores perform reasonably well, the Operation Score performs markedly better in terms of its Type II error. This is also evident in the performance curves shown in Figure 11b.

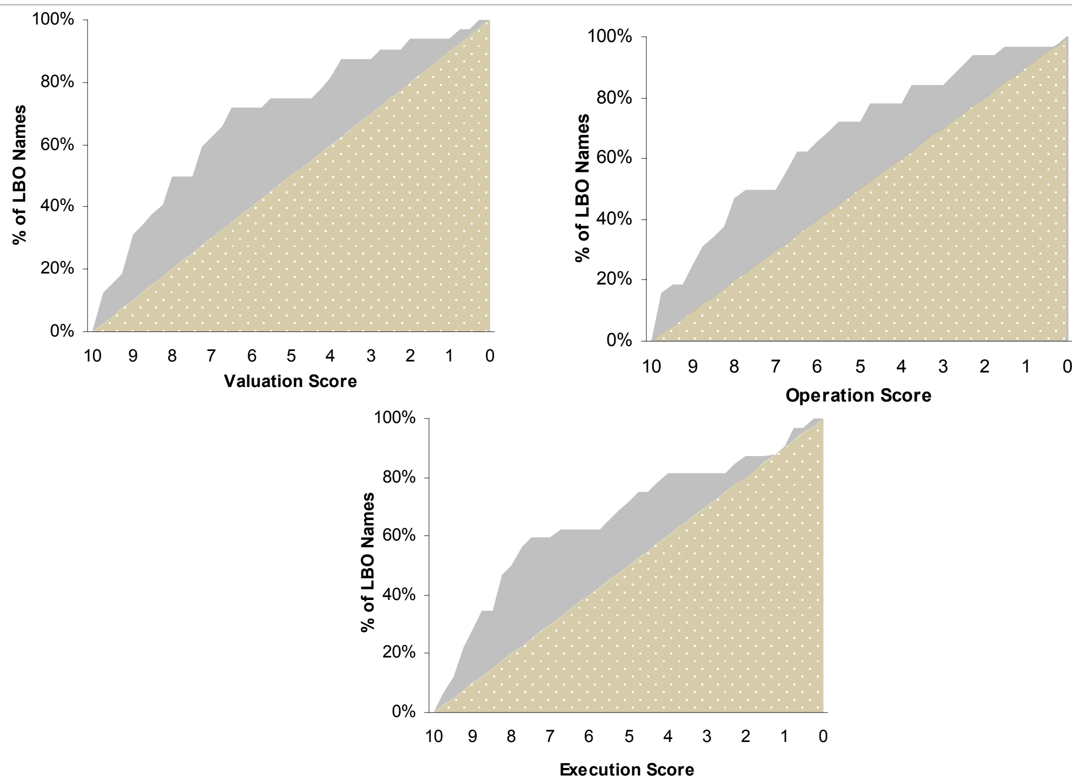
Figure 11a. Performance of the component scores[15]

% of LBO Targets	Valuation Score	Operation Score	Execution Score
Score above 7.5	50%	50%	59%
Score below 2.5	9%	9%	19%

Source: Lehman Brothers.

[15] *Given the construction of our scores and the small number of actual LBOs as compared with the size of the sample, there would be approximately 25% of the Non-LBO names above a score of 7.5 or below a score of 2.5.*

Figure 11b. Performance curves of component scores



Source: Lehman Brothers.

4.3. Applying the LEVER framework to European firms

We apply the *LEVER* framework to the universe of names in the FTSE World Europe Index to identify firms which are at high relative risk of releveraging events.

Although all the variables considered and tested in the case of US firms are relevant in an analysis of releveraging risk in other regions such as Europe, it is important to overlay the analysis with other variables that may play an important role in certain regions. For example, LBO and takeover activity in the UK has been strongly affected by the existence of large deficits in the pension plans sponsored by potential targets. In the recent past, several takeover attempts, such as that by private equity firm Permira on the retailer W H Smith and that by Philip Green on Marks and Spencer, have been disrupted due to their pension deficits.

5. *LEVER POWERTOOL* ON LEHMANLIVE

5.1. The *LEVER Powertool* – A Brief User Guide

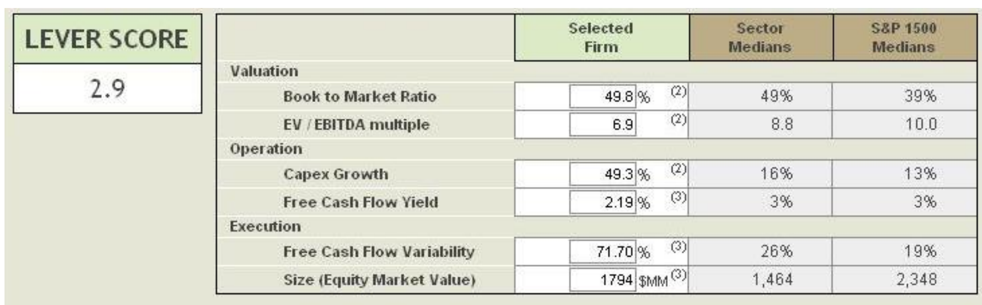
Despite the large universe of US and European names covered by the LEVER framework, there could be issuers of interest to investors. To enable investors to score names not in the S&P 1500 or the FTSE World Europe Index, the *LEVER Powertool* has been developed.

The *LEVER Powertool* [keyword LEVER in LehmanLive] is part of our suite of Quantitative Credit Analytic Tools on LehmanLive. The key features of the tool are the following:

- *Scoring individual issuers:*

  The LEVER Powertool allows users to input the variables that comprise the Firm LEVER-Score and/or the basic accounting numbers that underlie these variables (Figure 12). This enables users not only to score issuers outside the LEVER universe, but also to perform sensitivity and scenario analyses on the impact of changes in accounting variables on the attractiveness of firms to releveraging events.

Figure 12. Individual issuer scoring page



LEVER SCORE: 2.9

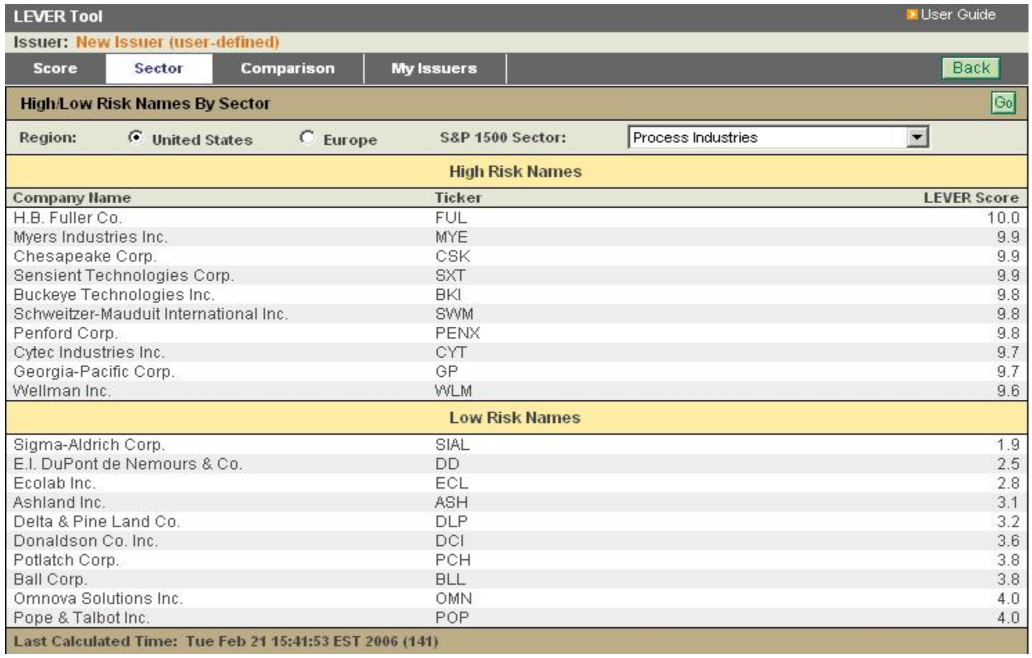
	Selected Firm		Sector Medians	S&P 1500 Medians
Valuation				
Book to Market Ratio	49.8 %	(2)	49%	39%
EV / EBITDA multiple	6.9	(2)	8.8	10.0
Operation				
Capex Growth	49.3 %	(2)	16%	13%
Free Cash Flow Yield	2.19 %	(3)	3%	3%
Execution				
Free Cash Flow Variability	71.70 %	(3)	26%	19%
Size (Equity Market Value)	1794 $MM	(3)	1,464	2,348

Source: Lehman Brothers.

- *Sector Analysis:*

  The Powertool enables users to compare issuers that appear to be at high and low relative risk of releveraging events within a given sector or sub-sector group (Figure 13). The top 10 issuers ranked according to the LEVER Score in a sector fall under the high-risk category whereas the bottom 10 names are low-risk names.

Figure 13. Sector analysis page[16]



LEVER Tool — User Guide

Issuer: New Issuer (user-defined)

Score | Sector | Comparison | My Issuers | Back

High/Low Risk Names By Sector — Go

Region: United States / Europe — S&P 1500 Sector: Process Industries

High Risk Names

Company Name	Ticker	LEVER Score
H.B. Fuller Co.	FUL	10.0
Myers Industries Inc.	MYE	9.9
Chesapeake Corp.	CSK	9.9
Sensient Technologies Corp.	SXT	9.9
Buckeye Technologies Inc.	BKI	9.8
Schweitzer-Mauduit International Inc.	SWM	9.8
Penford Corp.	PENX	9.8
Cytec Industries Inc.	CYT	9.7
Georgia-Pacific Corp.	GP	9.7
Wellman Inc.	WLM	9.6
Low Risk Names		
Sigma-Aldrich Corp.	SIAL	1.9
E.I. DuPont de Nemours & Co.	DD	2.5
Ecolab Inc.	ECL	2.8
Ashland Inc.	ASH	3.1
Delta & Pine Land Co.	DLP	3.2
Donaldson Co. Inc.	DCI	3.6
Potlatch Corp.	PCH	3.8
Ball Corp.	BLL	3.8
Omnova Solutions Inc.	OMN	4.0
Pope & Talbot Inc.	POP	4.0

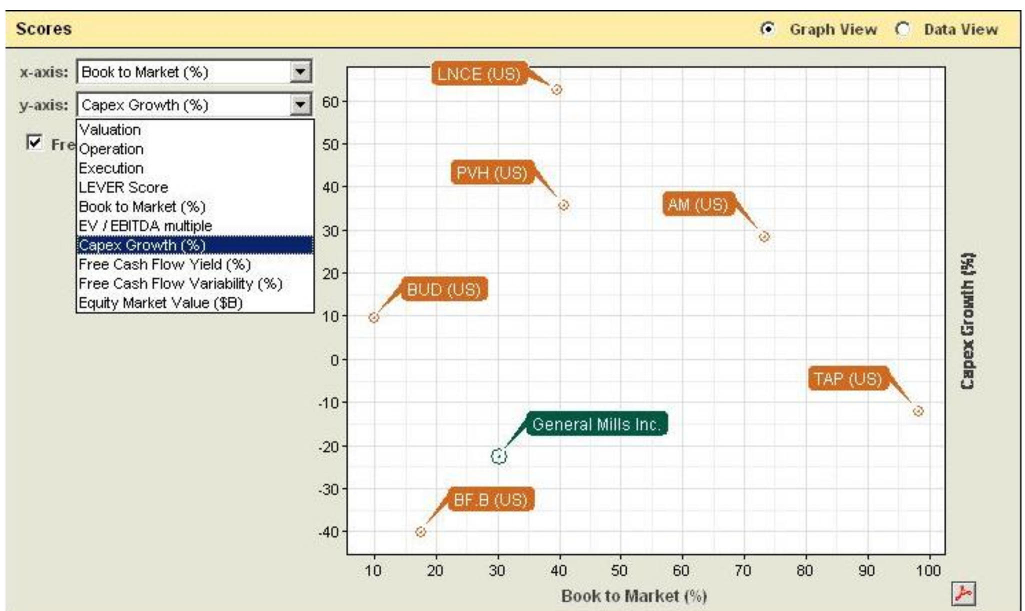
Last Calculated Time: Tue Feb 21 15:41:53 EST 2006 (141)

Source: Lehman Brothers.

- *Peer group comparison:*

  Using the "Comparison" page of the LEVER Powertool, users can compare a set of issuers defined by them along the Firm LEVER-Score or the *LEVER*-variables or any of the component scores, as chosen by them (Figure 14).

Figure 14. Peer comparison page[17]



Source: Lehman Brothers.

[16] *Please note that the list below is for illustrative purposes only. Please refer to the tool or our publications for the latest update.*

[17] *Please note that the map of LEVER variables below is for illustrative purposes only. Please refer to the tool or our publications for the latest update.*

5.2. Applications of the *LEVER Powertool*

The functionalities of the LEVER Powertool allow it to be used for a variety of purposes. In this section, we illustrate two applications that utilise the flexibility of the LEVER framework and the functionalities of the Powertool for meaningful analyses of releveraging risk. Beyond screening portfolios and scoring new issues, we suggest the following applications:

Scenario analyses

The Firm *LEVER-Score* uses the current accounting and financial information related to a particular firm and assesses its attractiveness as a target of releveraging events. The flexibility to alter the data that underlie the score using the LEVER Powertool allows the user to perform scenario analyses regarding the potential change in the level of attractiveness of a firm if its fundamental and financial profile were to change.

For instance, firm XYZ could have a Firm LEVER-Score of 6 based on its current characteristics. However, it may be interesting to analyze how its attractiveness would be altered if (other things remaining the same) its stock price fell from its current level of $12 to a lower level.

Figure 15. Illustration of scenario analysis

Price ($)	12	8.5	7.25	2.75
Book-to-Market	5%	7%	8%	21%
EV/EBITDA Multiple	7.9x	6.6x	6.2x	4.6x
Free Cash Flow Yield	5%	7%	9%	23%
Firm LEVER-Score	6.0	7.5	8.0	9.0

Source: Lehman Brothers.

Figure 15 illustrates a set of user-defined stock price scenarios and their impact on the Firm LEVER-Scores. As the stock price falls from $12 to $2.75, the book-to-market ratio increases, the EV/EBITDA multiple falls and the free cash flow yield increases, all contributing to a higher overall degree of attractiveness of the firm.

Analyzing leveraged buyout transactions

Although the Firm LEVER-Score indicates the attractiveness of an individual firm as a target of releveraging events by itself, the actual probability of the firm being a target is predicated on several factors, which include the characteristics of the acquiring firm. In addition to scoring individual targets, the LEVER Powertool can potentially be used to evaluate the leveraging capacity of the acquirer in a particular acquirer-target combination. There are of course many possible combinations and we do not provide a list of potential transactions. Moreover, using the Powertool in this way will not indicate the probability of a particular LBO combination occurring and, therefore, should be used only to complement traditional leveraged buyout transaction analysis.

To illustrate, let us consider the example of a firm, PQR, with a low Firm LEVER-Score of 4.5. However, analysis of the components of the score may show that while its Valuation characteristics are attractive (indicated by a high Valuation Score), its overall attractiveness is poor as a result of low Execution attractiveness (indicated by a low Execution Score). As such, firm PQR would appear to be an unlikely LBO target.

However, another firm with enough debt capacity and stable cash flows might be able to raise sufficient debt to fund a taking-private transaction on firm PQR, so as to capitalize on its attractive valuation.

The LEVER Powertool readily allows an analysis of a taking-private transaction on firm PQR by a range of acquirers. This can be done by scoring a hypothetical firm PQR*, which combines the characteristics of the target firm PQR and any potential acquirer, PQS. Figure 16 shows an illustration of such an analysis, where the user could input some of the characteristics of the target and others of the acquirer to evaluate the ability of the acquirer to fund the particular acquisition.

Figure 16. Illustration of the mechanics of evaluating an LBO transaction

	Valuation		Operation		Execution	
Variables	Book-to-Market	EV / EBITDA	FCF Yield	Growth of CapEx	Market Value	FCF Variability
Data from	Target	Target	Acquirer	Acquirer	Target	Acquirer

Source: Lehman Brothers.

6. CONCLUSION

In this article, we proposed a systematic approach to identifying issuers with heightened leveraged recapitalization risk and quantifying the overall conduciveness of the market for releveraging transactions. There are limitations to such a screen. First, it is liable to over-predict the risk of LBOs. Second, there may be important qualitative variables (e.g., openness of a firm's management to a buyout) that are difficult to capture in a quantitative framework. It is therefore necessary for such a screen to be complemented by a subjective analysis of the firms in the universe. Nevertheless, our historical test shows that there is value to such an approach.

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Recovery Rate Assumptions and No-Arbitrage in the Tranche Market

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At present, the standardised tranche market is liquidly traded for maturities of 5, 7 and 10 years in both the CDX IG5 and iTraxx S4 indices. Any consistent pricing model for CDO tranches must be calibrated to the market implied correlation skew. Many models have been suggested for this, fitting the market quotes with varying degrees of success, though none entirely satisfactory. In this article, we discuss a framework for detecting arbitrage in standard tranche quotes and the extent to which these are consistent with a simplistic deterministic recovery rate assumption[1].

1. INTRODUCTION

At present, the base correlation framework is the favoured method in the market for matching the standard tranche quotes. Base correlations are not derived from a consistent, arbitrage-free model. Far from being a purely theoretical consideration, this throws up important questions regarding the consistency of quotes derived from a base correlation interpolation mechanism. For example, it is entirely possible for a tranche at a certain level of the capital structure to be priced at a higher breakeven spread than one subordinate to it.

Many models have been proposed to generate portfolio loss distributions compatible with the correlation skew observed in the market, for example Andersen and Sidenius (2005), Burtschel *et al*. (2005), or Hull and White (2005), to mention just a few. So far, none of the published approaches has proved entirely satisfactory; the problem of finding a tractable model for portfolio loss distributions consistent with the observed market data has proven thorny indeed.

The complications stem from several factors. The loss distributions we deal with when pricing CDO tranches are high-dimensional objects, as they are generated by the joint default behaviour of all the credits in the underlying portfolio. Also, tranches are not European-style derivatives. Because protection payments are made as tranche losses occur, and the tranche premium is paid on a running basis, their pricing is not just a function of a loss distribution to a particular time horizon, but rather a function of the loss timings, i.e. the joint distribution of portfolio losses to multiple time horizons before the trade maturity. Finally, individual risk-neutral default probabilities and recovery rates are not directly observable, as the market trades the two risks in aggregate via default swaps.

Given the difficulties in modelling risk-neutral portfolio loss distributions in a manner consistent with the correlation skew, it is appropriate to ask whether there is an easy method to detect whether a given set of tranche quotes is compatible with any arbitrage-free model of portfolio loss. In the next section, we describe Expected Tranche Loss (ETL) functions, which provide just such a method. It is important to be precise regarding how the question is posed, particularly regarding which information we take as given. In short, a set of tranche quotes is compatible with an arbitrage-free model if we can find an increasing stochastic process L for the cumulative percentage portfolio loss, such that the expected tranche losses derived from L reproduce the given tranche quotes.

In addition to examining the quotes of traded tranches, we need to make an assumption regarding the expected portfolio loss at any point in time, as well as the range of possible realizations of the portfolio loss. The expected portfolio loss serves as the link between the

[1] *We would like to thank Sebastien Hitier for discussions and comments.*

portfolio pricing and individual credit default probabilities. The possible realisations of the loss are closely linked to recovery rate assumptions. In other words, we can determine only whether a set of tranche quotes is compatible with no-arbitrage given a certain assumption on the expected and the maximum portfolio loss. Index swap quotes give us quite a lot of confidence in our estimate of the expected portfolio loss. In contrast, the maximum portfolio loss is completely unobservable. In practice, often simplistic assumptions regarding recovery rates, driven by tractability considerations, are made. In Section 3 we test a set of tranche quotes for no-arbitrage given a deterministic recovery rate assumption. We find that mid-quotes are not necessarily compatible with a 40% recovery rate assumption.

2. EQUIVALENT ETL CONDITIONS TO NO-ARBITRAGE

In this section we formalise our analysis of no-arbitrage in the tranche market. If the cumulative percentage loss to a portfolio up to a time horizon t is denoted by L_t, the loss to a tranche with strikes K_1 and K_2 is given by:

$$L_t^{tr} = \frac{1}{K_2 - K_1}\left(\min\left(L_t, K_2\right) - \min\left(L_t, K_1\right)\right) \tag{1}$$

It is well known that the equations relevant for tranche pricing (i.e. the protection and premium legs of a tranche swap) are linear in the expected tranche loss. From equation (1) we see that the pricing of a tranche is given by linear combinations of expressions of the form:

$$H\left(t, K\right) = E\left[\min\left(L_t, K\right)\right] \tag{2}$$

For ease of description, we call H an Expected Tranche Loss (ETL) function. It describes the expected loss on an equity tranche with detachment point K. To match a set of tranche quotes, we need values $H\left(t_i, K_j\right)$ for the quoted strikes $K_1 < \ldots < K_n$ and discretisation dates $t_1 < \ldots < t_m$, we call this an ETL skeleton. The maximum loss to the portfolio $L_{\max}$ must be incorporated by setting $K_n = L_{\max}$. The expected portfolio loss enters via:

$$H\left(t_i, L_{\max}\right) = E\left[L_{t_i}\right] \tag{3}$$

The ETL skeleton completely determines the pricing of the quoted tranches. Given an assumption regarding $L_{\max}$ and the value of $E\left[L_{t_i}\right]$ for each discretisation date, a set of tranche quotes is compatible with an arbitrage-free model, if the quotes can be derived from an ETL skeleton which can be represented in the form of equation (2) with a monotonically increasing stochastic process L where the distribution of L_t is concentrated on $\left[0, L_{\max}\right]$.

This definition of no-arbitrage ensures that the actual quotes given in the market are supported by a proper model for the evolution of the aggregate portfolio loss. In particular, it ensures that tranche spreads will always behave sensibly across time and strike, i.e. tranche spreads will always be decreasing as we move up the capital structure, and the value of protection will always increase over time. Note, however, that this definition does not demand that we can always decompose the portfolio loss as the sum of individual credit losses. The definition we have given is the minimum requirement for an arbitrage-free model of the aggregate portfolio loss. The granularity of the underlying portfolio is not explicitly taken into account. The riskiness of the individual credits enters only via the expected portfolio loss.

Because we want to treat the ETL function as a primitive object, we need to characterise exactly when H can be represented in the form of equation (2). Note that this involves both the strike and the time dimension. In the strike dimension, Proposition 2 in the Appendix gives conditions under which a function $h:[0,+\infty[\to[0,+\infty[$ can be represented as the expected tranche loss of a loss distribution L via:

$$h(K)=E\left[\min(L,K)\right] \tag{4}$$

Essentially, h must be continuous, monotonically increasing and concave, while satisfying some other obvious boundary conditions. The concavity of h in the strike dimension ensures that h is in fact right-differentiable, and that we obtain a well defined cumulative distribution function F by setting:

$$F(k)=1-h'(k+) \tag{5}$$

If we did not care about time consistency, we could use the time slices $H(t,.)$ of the ETL function to generate a consistent loss distribution across strikes for any time horizon t. Equation (5) is in effect a method to calculate the market-implied risk-neutral loss distribution from expected tranche losses to a given time horizon. The technically simpler case where the loss distribution is given via a density is described in O'Kane and Schloegl (2005).

We now turn to the time dimension. Certainly a necessary condition for the different loss distributions to be aggregated to an increasing process is that the distribution functions are decreasing:

$$F_{t_1}(K)=P\left[L_{t_1}\le K\right]\ge P\left[L_{t_2}\le K\right]=F_{t_2}(K) \tag{6}$$

From equation (5), we see that this implies that the right-derivatives $k\mapsto H'(t,k+)$ must be increasing over time. This necessary condition is actually sufficient. We can explicitly construct an increasing process L such that L_t has the prescribed distribution by choosing a uniform random variable U and defining:

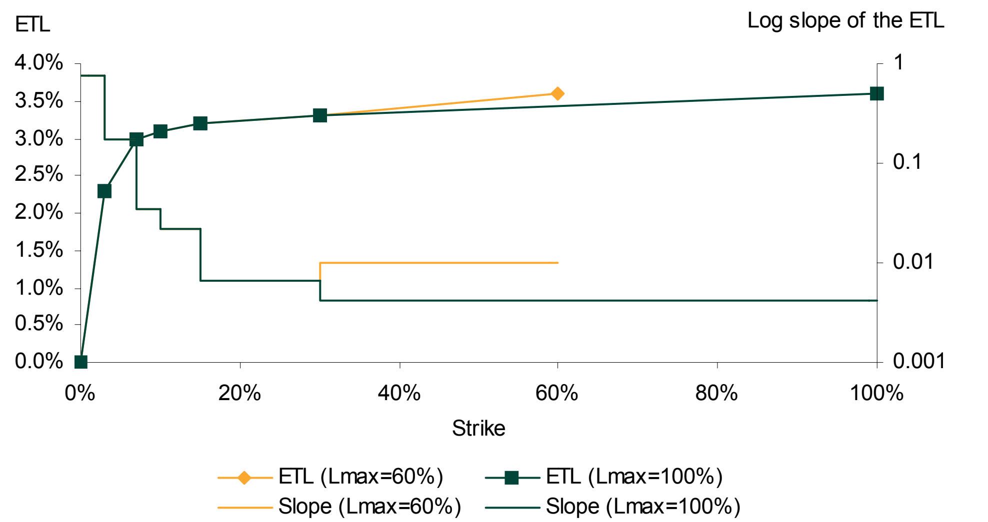
$$L_t=\inf\left\{k\in\left[0,L_{\max}\right]:1-H'(t,k+)\ge U\right\} \tag{7}$$

This is what is known as the Skorokhod construction for random variables, e.g., see Williams (1991). Equation (7) immediately makes it clear that L_t is increasing over time. To summarise, a function H is a valid ETL function, essentially if, and only if, it is monotonically increasing, concave and its right-hand derivative is increasing over time. Note that the random variables in the construction of equation (7) are co-monotonic. In other words, their time dependence is the strongest possible. This will not necessarily produce realistic forward loss distributions. However, equation (7) proves the existence result we are after in this context.

While this is an interesting theoretical result, the important point is that this is useful for detecting arbitrage in tranche quotes. When we apply our result to the ETL skeleton, we notice that all our monotonicity and concavity constraints become linear. Also recall that the pricing equations (inequalities if we are just concerned with staying inside the bid/offer) are also linear in the ETL skeleton. Hence, a set of tranche quotes is arbitrage-free if and only if a set of linear equality / inequality constraints is feasible. This is something we can easily test numerically using linear programming methods. Alternatively, we can use quadratic optimisation to find an ETL skeleton that is arbitrage-free, fits a given set of tranche quotes, and minimises the distance to a target skeleton; for example, one given by an analytical model. In the course of this, the optimiser will immediately tell us whether the optimisation problem is numerically feasible or not.

One degree of freedom in finding an ETL skeleton is the value we choose for the maximum portfolio loss, L_{max}. If we assume a fixed recovery rate of 40% for all names in the portfolio, we have L_{max}=60%. However other values are equally possible with the only constraint being that it lies between 0% and 100%. The higher the value that we choose for L_{max} the more flexibility we give the ETL skeleton to fit the quotes while remaining arbitrage-free. To illustrate the effect that L_{max} can have, Figure 1 shows two ETL skeletons that fit the market quotes, one using a value of L_{max}=100% and the other using L_{max}=60%. For convenience it also shows the log of the slopes of the ETL skeletons. The ETL skeleton is concave and arbitrage-free for L_{max}=100% but for L_{max}=60% the ETL skeleton is no longer concave and some arbitrage is introduced, as can be seen by the fact that the slope is no longer decreasing. It is clear that the higher the value of L_{max} the easier it is to keep an ETL skeleton arbitrage-free while trying to match a set of market quotes.

Figure 1. ETL Skeleton which is arbitrage-free for L_{max}=100% but not for L_{max} =60%



Source: Lehman Brothers.

3. EMPIRICAL ANALYSIS AND CONCLUSIONS

To test whether standard tranche quotes can be fitted with a model that uses a deterministic recovery rate, we have tried to find ETL skeletons that are arbitrage-free and match the market prices, using a quadratic minimisation algorithm to either find a solution or tell us that there is no solution possible. We have run this analysis for the on-the-run CDX and iTraxx standard tranches for dates over the past year.

The results show that the mid-market tranche quotes for CDX IG5 and IG4 frequently cannot be fitted using an arbitrage-free model with a fixed recovery rate of 40%; however, such a model can get to within the bid/offer spread most of the time. The iTraxx S4 portfolio can be fitted to within the bid/offer, using a fixed recovery rate of 40%, for all of the dates tested. However, it can be fitted to mid-market quotes on only half of the dates. This analysis suggests that if we want to find a model that consistently matches the market, we must take stochastic recovery rates into account.

We have analysed some biweekly data for CDX IG5, IG4 and iTraxx S4. In each case we fit tranche quotes for 5-, 7- and 10-year time horizons. We show the highest possible value of 1-L_{max} that can fit the market quotes, both to mid and within the bid/offer, or we show 40% if this allows us to fit the market quotes in an arbitrage-free way. When 40% is sufficient to fit the tranche quotes it means that there is a proper model using a fixed recovery rate of 40% that will match the market quotes.[2] The most striking results are obtained for CDX IG5. CDX IG4 also shows some deviation from 40%, but less pronounced (Figure 3).

Figure 2. Highest value of $1-L_{\max}$ able to fit CDX IG5 tranche quotes

	Bid/Offer	Mid
15-Dec-05	35%	24%
01-Dec-05	40%	34%
15-Nov-05	40%	38%
01-Nov-05	40%	35%
14-Oct-05	40%	39%
03-Oct-05	40%	29%

Source: Lehman Brothers.

Figure 3. Highest value of $1-L_{\max}$ able to fit CDX IG4 tranche quotes

	Bid/Offer	Mid
15-Sep-05	40%	40%
01-Sep-05	40%	40%
15-Aug-05	40%	40%
01-Aug-05	35%	30%
15-Jul-05	40%	40%
01-Jul-05	40%	30%
15-Jun-05	40%	38%
01-Jun-05	40%	40%
16-May-05	40%	34%
02-May-05	40%	40%
15-Apr-05	40%	40%
01-Apr-05	40%	40%

Source: Lehman Brothers.

Figure 4. Highest value of $1-L_{\max}$ able to fit iTraxx S4 tranche quotes

	Bid/Offer	Mid
15-Dec-05	40%	40%
01-Dec-05	40%	39%
15-Nov-05	40%	39%
01-Nov-05	40%	40%
14-Oct-05	40%	40%
03-Oct-05	40%	39%

Source: Lehman Brothers.

The tables show that it is not always possible to fit the market quotes using an arbitrage-free model with a fixed recovery rate of 40%. When trying to fit mid-market quotes, this problem becomes more noticeable. This means that there is no arbitrage-free model that uses a fixed recovery rate of 40% that would have been able to consistently match the market tranche quotes over the period tested.

[2] *Lmax is where we assume that the maximum loss on the portfolio can occur. For a model with a fixed recovery rate of 40%, the maximum loss on the portfolio, Lmax is 60%.*

We obtained the most extreme results for CDX IG5 on 15 December 2005. To test the robustness of this data point, and to see how sensitive these results are to changes in the input parameters, we have run the 15 December 2005 tranche market analysis with perturbed parameters. This gives us a sense of how far off fitting the market we are when using a fixed recovery of 40%.

We found that it is possible to fit to within the bid/offer using a fixed recovery rate of 40% if any of the following adjustments are made:

- Decrease the 5-, 7- and 10-year CDX IG5 index spreads by 0.5bp.
- Increase the 5-, 7- and 10-year 15-30% tranche spreads by 1bp.
- Widen the bid/offer range by a factor of 1.5.

It is possible to fit to the mid-market quotes, using a fixed recovery rate of 40% if any of the following adjustments are made:

- Decrease the 5-, 7- and 10-year CDX IG5 index spreads by 2bp.
- Increase the 5-, 7- and 10-year 15-30% tranche spreads by 4bp.

So, for CDX IG5 on 15 December 2005, to fit to mid-market requires a substantial perturbation to the market if we want to use a fixed recovery rate of 40%. Given the value of the equity tranche and the index level, the market is not putting as much risk into the senior traded tranche (15-30% strikes) as a model with a 40% fixed recovery would require. Where has this risk gone? The market appears to be placing more risk in the currently not very actively traded super-senior tranche. Intuitively, this is consistent with a risk premium being required for “meltdown” scenarios with low recovery rates. As tranche trading becomes more and more active, we expect more and more risk to be redistributed in and out of the “Black Hole” above the highest standard strike.

The huge efforts that have been expended to date in trying to fit the correlation skew have concentrated on modelling joint default probabilities, often taking recovery rates as given. Recovery swap trading, which is becoming more active in the US for distressed credits, should provide more visibility regarding expected risk-neutral recovery rates. While we would argue that our empirical results present valid reasons for the inclusion of stochastic recovery rates into portfolio credit models, it is important not to lose sight of certain caveats that make these results somewhat tentative:

Default probabilities are not directly observable; the expected portfolio loss is a function of how we extract default probabilities from individual credit spreads and how we take into account the basis adjustment between the individual credit curves and the portfolio swap. Correspondingly, there are data synchronicity issues between tranche quotes, the underlying portfolio swap and single-name credit quotes.

There are liquidity questions to be considered as well: e.g., tranche quotes can be easily skewed and “mid” is not necessarily exactly in the middle between “bid” and “offer”. On the single-name side, even in the standard portfolios not all credits are equally liquid.

Finally, note that our sample sizes are relatively small, so some caution is needed regarding the robustness of our empirical findings.

APPENDIX CHARACTERISATION OF ETL FUNCTIONS

This appendix gives mathematically precise statements of the characterisation of ETL functions. Proposition 1 collects the various properties of ETL functions, while Proposition 3 shows that these are sufficient to characterise a function as an ETL function.

Proposition 1: Let L be a non-negative random variable with values in $\left[0, L_{\max}\right]$. Define H via $H\left(k\right) = E\left[\min\left(k, L\right)\right]$. The function H is continuous, monotonically increasing and concave. We have $H\left(0\right) = 0$ and $H\left(k\right) \le k$. Finally, H is constant on $\left[L_{\max}, +\infty\right[$.

The properties of H are easily verified. The continuity of H follows from the dominated convergence theorem.

Lemma 2: Let L be a non-negative random variable with cdf F. Then for any $k \in \left[0, +\infty\right[$, we have:

$$\int_0^k 1 - F\left(t\right) dt = E\left[\min\left(k, L\right)\right] \tag{8}$$

Proof: We use Fubini's theorem:

$$\begin{aligned} \int_0^k F\left(t\right) dt &= \int_0^k P\left[L \le t\right] dt = \int_\Omega \int 1_{[0,k] \cap \left[L(\omega), +\infty\right[}\left(t\right) dt P\left(d\omega\right) \\ &= \int_\Omega \left(k - L\left(\omega\right)\right) 1_{\left\{L(\omega) \le k\right\}} P\left(d\omega\right) = kP\left[L \le k\right] - \int_{\left\{L \le k\right\}} L dP = k - E\left[\min\left(k, L\right)\right] \end{aligned} \tag{9}$$

Equation (8) follows immediately.

Proposition 3: Let $H : \left[0, +\infty\right[\to \left[0, +\infty\right[$ be a monotonically increasing, continuous, concave function with $H\left(0\right) = 0$ and $H\left(k\right) \le k$. Furthermore, suppose there is an $L_{\max} > 0$ so that H is constant on $\left[L_{\max}, +\infty\right[$. Then, there exists a probability measure μ on $\left[0, +\infty\right[$ with $\mu\left(\left[0, L_{\max}\right]\right) = 1$ such that for all $k \in \left[0, +\infty\right[$

$$H\left(k\right) = \int \min\left(k, x\right) \mu\left(dx\right) \tag{10}$$

Proof: Using the concavity of H, it is possible to show that H is right-differentiable, and that the right-hand side derivative is right-continuous and monotonically decreasing. Also, because $H\left(k\right) \le k$, we have $0 \le H'\left(k+\right) \le 1$. Because H is constant on $\left[L_{\max}, +\infty\right[$, we have $H'\left(k+\right) = 0$ for $k \ge L_{\max}$. Hence, the function F defined by $F\left(k\right) = 1 - H'\left(k+\right)$ is a well defined cdf, and the corresponding probability measure μ is concentrated on $\left[0, L_{\max}\right]$. From the previous Lemma it follows that:

$$\int \min\left(k, x\right) \mu\left(dx\right) = \int_0^k 1 - F\left(t\right) dt = \int_0^k H'\left(t+\right) dt \tag{11}$$

The function H is absolutely continuous, because for $a < b$ we have:

$$0 \le H(b) - H(a) \le (b-a)H'(a+) \le b-a \tag{12}$$

The Fundamental Theorem of Calculus now tells us that *H* is in fact differentiable λ^1-almost everywhere, H' is integrable and in particular:

$$H(k) = \int_0^k H'(t+)\,dt = \int \min(k,x)\,\mu(dx) \tag{13}$$

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Step It Up or Start It Forward: Fast Pricing of Reset Tranches

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We introduce a simple algorithm for the fast valuation of "reset tranches", a class of default-path-dependent structures that includes forward-starting contracts and subordination/leverage step-ups. After showing how to price these instruments quasi-analytically, we offer a few examples to illustrate how reset tranches can be used by buy-and-hold investors for yield enhancement and for expressing views on the timing of defaults.

1. INTRODUCTION

Since the introduction of standardized CDS portfolios (CDX in North America and iTraxx in Europe and Asia), credit markets have changed considerably. In the past two years, these products have experienced a rapid increase in traded notionals, and the extremely tight bid-ask spreads they offer today make them a very useful instrument for a constantly increasing range of market participants to take directional views and hedge open positions.

Since inception, standardized CDS portfolios have also become the underlying reference for other derivative instruments, most notably standardized tranches and standardized options. As liquidity in the portfolio swap market has expanded along the whole term structure, portfolio swaptions and tranches have started trading across a richer set of strikes and maturities.

More recently, a general spread compression across the whole quality spectrum, as well as significant changes in the rating agencies' methods, have decreased the attractiveness of the levels offered by plain-vanilla tranches. In these market conditions, one way for dealers to provide investors with a yield pick-up is to expand their offering of structures with exotic features. However single-tranche originators need reliable pricing models for managing their risk through dynamic hedging, and even the introduction of an apparently simple feature such as a forward start introduces an element of path-dependency that significantly complicates the task of valuation.

In this article we introduce a simple algorithm for the fast pricing of "reset tranches", a class of default-path-dependent structures that includes forward-starting contracts and subordination/leverage step-ups. The remainder of the paper is organized as follows:

- **Section 2** reviews the basics of synthetic tranche pricing in the context of a model with conditionally independent defaults.
- **Section 3** derives a new algorithm for the quasi-analytical valuation of reset tranches.
- **Section 4** presents a few specific examples that show how reset tranches can be used for yield enhancement and for expressing views on the timing of defaults.

2. PRICING SYNTHETIC TRANCHES: A BRIEF REVIEW

At the core of any CDO pricing model is a mechanism for generating dependent defaults. If a simple factor structure is used to join their marginal distributions, the default times of the underlying credits are independent conditionally on the realization of the common factor(s). This conditional independence of defaults is very useful because it allows one to use quasi-analytical algorithms to compute the term structure of expected tranche losses, which is the fundamental ingredient for the valuation of a synthetic CDO.

Because of their analytical tractability, conditionally independent models have become a standard in the synthetic CDO market. In particular, the one-factor Gaussian-copula model has played a dominant role since the early days of single-tranche trading.

2.1. The Gaussian-copula model

In the one-factor Gaussian-copula framework, the dependence of the default times is Gaussian, and is therefore completely specified by their correlations. In this model, given a particular realization of a normally distributed common factor *Y*, the probability that the j^{th} credit defaults by time *t* is equal to:

$$\pi_{j,t}(Y) = N\left(\frac{D_{j,t} - \beta_j \cdot Y}{\sqrt{1-\beta_j^2}}\right),$$
$$j = 1,2,\ldots M ,$$

where *N(.)* denotes the standard Gaussian distribution function, the vector $\{\beta_j\}$ determines the correlations of the default times, $\{D_{j,t}\}$ are free parameters chosen to satisfy

$$p_{j,t} = \int_Y \pi_{j,t}(Y)\, dN(Y) \quad ,$$

and $p_{j,t}$ is the (unconditional) probability that name *j* defaults by time *t.* Importantly, for the CDO model to price the underlying CDS correctly, $p_{j,t}$ must be backed out from the term structure of observable CDS spreads.

Given a realization of the Gaussian factor *Y*, the *M* individual credits are independent, and a simple recursive procedure (see Andersen *et al* (2003)) can then be employed to recover the conditional loss distribution of the underlying portfolio, as well as the loss distribution of any particular tranche of interest. Once we know how to compute the loss distribution of a tranche for a given realization of the common factor, it is straightforward to take a probability-weighted average across all possible realizations of *Y* and thus recover the unconditional loss distribution of the tranche.

Repeating this procedure for a grid of horizon dates and interpreting the expected percentage loss up to time *t* as a "cumulative default probability", we can price the tranche using exactly the same analytics that we would use for pricing a CDS. More precisely, we can define the "tranche curve" as the term structure of expected surviving percentage notionals of the tranche, i.e:

$$Q(t) = 1 - E\left[\frac{[L_t - U]^+ - [L_t - (U+V)]^+}{V}\right],$$

where L_t is the number of loss units experienced by the reference portfolio by time *t*, *U* is the number of loss units that the tranche can withstand (attachment), and *V* is the number of loss units protected by the tranche investor. Then the two legs of the swap can be priced using:

$$\text{Premium} = cN\sum_{i=1}^{T} \Delta_i Q(t_i) B(t_i) ,$$

$$\text{Protection} = N\sum_{i=1}^{T} B(t_i)(Q(t_{i-1}) - Q(t_i)),$$

where c is the annual coupon paid on the tranche, N is the notional of the tranche, t_i, $i=1,2,\ldots,T$ are the coupon dates, Δ_i, $i=1,2,\ldots T$ are accrual factors, and $B(t)$ is the risk-free discount factor for time t. Notice that, for ease of notation, we have used the coupon dates t_i, $i=1,2,\ldots,T$ to discretize the timeline for the valuation of the protection leg.

2.2. Fitting to liquid tranche prices

Before the emergence of standard index tranches, most market participants used the Gaussian copula model described above for the valuation and risk management of their synthetic CDO tranches. Since no market information about correlation was available, it was customary to calibrate the "betas" to some historical estimate of equity or spread correlations.

With the development of a relatively liquid market for tranches of standardized portfolios such as CDX, it became clear that the Gaussian copula model parameterized by historical correlations was unable to replicate the observable tranche prices. Suddenly, pricing a tranche on the CDX portfolio with non-standard strikes and/or non-standard maturity had become a complicated task; pricing more exotic payoff structures seemed even harder.

Essentially, two different paths have been explored in the attempt to develop valuation frameworks that are able to calibrate to observable CDX tranche prices and extrapolate to non-standard strikes/maturities:

1. Base Correlation Approach: The idea here is to keep using the Gaussian-copula model to map observable (equity) tranche prices into implied (base) correlations, and then apply *ad hoc* interpolation methods to reconstruct a market-implied portfolio loss distribution and price non-standard tranches (see O'Kane and Livesay (2005) for a detailed description of this approach).
2. Non-Gaussian default times: A second approach is to modify the dependence structure of default times and explore non-Gaussian copulas.

The main limit of the base correlation approach is that it is specifically built around the payoff structure of plain-vanilla tranche swaps and cannot give any guidance with respect to the valuation and risk management of more exotic contracts.

On the other hand, coming up with non-Gaussian models that can calibrate to the available term structure of CDX tranches and that are fast enough to perform on-demand calibration, valuation and risk, has probably proved to be more difficult than researchers initially expected. Nevertheless, this approach has given useful results. The "random factor loading" model presented in Andersen *et al* (2005), the "perfect copula" approach of Hull and White (2005), and the time-homogeneous specification suggested by Skarke (2005) are all examples of non-Gaussian models that can capture the "correlation skew" exhibited by market prices. Moreover, these models share the convenient property of conditionally independent defaults, which means that the same fast recursive procedure mentioned in the previous section is available for pricing plain-vanilla tranche swaps, and therefore for fast calibration to observable CDX tranche prices.

But what about more exotic payoffs, e.g. tranches with such path-dependent features as forward starts or subordination resets? While a proper model of defaults, unlike a base correlation approach, allows us to deal with these types of contracts, one may think that a simulation of the (market-implied) joint distribution of default times is the only way to handle the path-dependent features of these contracts.

In the remainder of this article we describe how to expand the set of multi-name instruments for which we can derive quasi-analytical pricing in the context of a model with conditionally independent defaults.

3. FAST PRICING OF RESET TRANCHES

A reset tranche is defined as a path-dependent tranche whose attachment and width are reset *at a predetermined time (the reset date) as predetermined functions of the random amount of losses incurred by the reference portfolio up to that time*. It will soon become clear that forward-starting tranches and tranches whose attachment point resets at a predetermined future date both belong to this class.

3.1. Pricing a reset tranche

Let t_s denote the reset date, λ_j, $j=1,2,...M$, the number of loss units produced by the default of the j^{th} name, $\lambda = \sum \lambda_j$ the maximum number of loss units that the portfolio can suffer, $p(\omega)$ the probability today that the reference portfolio incurs exactly ω loss units by the reset date t_s.

A reset tranche can be defined by the vector

$$\{t_T, t_s, U, V, U(\omega), V(\omega)\},$$

where $U(\omega) \geq \omega$ is the attachment point of the tranche (in loss units) after the reset date, and $V(\omega)$ is the number of loss units protected by the tranche investor after the reset date. We can price the two legs of this swap as:

$$\text{Premium} \quad = cN \sum_{\omega=0}^{\lambda} p(\omega) \sum_{i=1}^{T} \Delta_i Q(t_i;\omega) B(t_i),$$

$$\text{Protection} \quad = N \sum_{\omega=0}^{\lambda} p(\omega) \sum_{i=1}^{T} B(t_i) \left(Q(t_{i-1};\omega) - Q(t_i;\omega) \right),$$

where we have defined the *conditional* tranche curve $Q(t;\omega)$, $t_0 \leq t \leq t_T$, as:

$$Q(t;\omega) = T(t,\omega) \cdot \left(1 - E\left[\frac{[L_t - U(t;\omega)]^+ - [L_t - (U(t;\omega) + V(t;\omega))]^+}{V(t;\omega)} \mid L_{t_s} = \omega \right] \right),$$

$$T(t,\omega) = 1 - 1_{\{t > t_s\}} \frac{[\omega - U]^+ - [\omega - (U+V)]^+}{V},$$

$$U(t;\omega) = \begin{cases} U, t \leq t_s \\ U(\omega), t > t_s \end{cases},$$

$$V(t;\omega) = \begin{cases} V, t \leq t_s \\ V(\omega), t > t_s \end{cases}.$$

In words, the conditional tranche curve $Q(t;\omega)$ represents the (risk-neutral) expected percentage surviving notional of the tranche at time t, conditional on the event that the reference portfolio experiences a cumulative loss of ω units up to the reset date.

Equally, we can write down the valuation in terms of the *unconditional* tranche curve:

$$Q(t) = \sum_{\omega=0}^{\lambda} p(\omega) \cdot Q(t;\omega),$$

and thus obtain the familiar equations:

$$\text{Premium} \quad = cN\sum_{i=1}^{T}\Delta_i Q(t_i)B(t_i)$$

$$\text{Protection} \quad = N\sum_{i=1}^{T} B(t_i)(Q(t_{i-1}) - Q(t_i))$$

However, while the unconditional tranche curve for $t_0 \le t \le t_s$ reduces to the standard tranche curve defined in section 2:

$$Q(t) = \sum_{\omega=0}^{\lambda} p(\omega)\cdot Q(t;\omega) = 1 - \sum_{\omega=0}^{\lambda} p(\omega) E\left[\frac{[L_t - U]^+ - [L_t - (U+V)]^+}{V} \mid L_{t_s} = \omega\right] =$$

$$= 1 - E\left[\frac{[L_t - U]^+ - [L_t - (U+V)]^+}{V}\right],$$

the unconditional tranche curve for $t_s < t \le t_T$

$$Q(t) = \sum_{\omega=0}^{\lambda} p(\omega)\cdot Q(t;\omega) =$$

$$= \sum_{\omega=0}^{\lambda} p(\omega)\cdot T(t,\omega)\cdot\left(1 - E\left[\frac{[L_t - U(\omega)]^+ - [L_t - (U(\omega)+V(\omega))]^+}{V(\omega)} \mid L_{t_s} = \omega\right]\right)$$

incorporates the added complexity of the path-dependent valuation.

3.2. Deriving the conditional tranche curve

Our discussion so far leaves open the problem of constructing the conditional tranche curve. From the previous discussion, it should be clear that to achieve this goal we need to be able to compute conditional expectations of the form $E\left[f(L_{t_u},\omega) \mid L_{t_s} = \omega\right]$ for some function f, and this, in turn, requires that we know the joint distribution of the cumulative losses on the reference portfolio at two different horizons. In this section we present a two-dimensional recursive algorithm for computing this joint distribution; the methodology is conceptually similar to the one introduced by Baheti *et al* (2005) for pricing "squared" products.

As anticipated, we assume that the underlying default model exhibits the property of conditional independence. We exploit this by conditioning our procedure on a particular realization of a common factor Y. We first discretize losses in the event of default by associating each credit with the number of loss units that its default would produce: we indicate with λ_j the integer number of loss units that would result from the default of name j. Next, we construct a square matrix (Z_{v_1,v_2}) whose sides consist of all possible loss levels for the reference portfolio, i.e. $(0,1,...,\lambda)$. In this matrix we will store the joint probabilities that the reference portfolio incurs v_1 loss units up to time t_s and v_2 loss units up to time t_u, with $t_u \ge t_s$. By definition of cumulative loss, the matrix must be upper triangular, i.e.:

$$Z_{v_1,v_2} = 0 \text{ if } v_2 < v_1.$$

For the non-trivial elements where $v_2 \ge v_1$, we set up the following recursion. We first initiate each state (recursion step j=0) by setting:

$$Z^0_{v_1,v_2} = 1, \text{ if } \quad v_1 = 0 \text{ and } v_2 = 0,$$

$$Z^0_{v_1,v_2} = 0 \text{ otherwise.}$$

We preserve the notation adopted during our description of the Gaussian-copula model and denote with $\pi_{j,t}(Y)$ the probability that name *j* defaults by time *t*, conditional on the market factor taking value *Y*. Now we feed one credit at a time into the recursion and update each element according to:

1. If $v_1 \geq \lambda_j$:

$$Z^j_{v_1,v_2} = \left(1-\pi_{j,u}(Y)\right)\cdot Z^{j-1}_{v_1,v_2} + \pi_{j,s}(Y)\cdot Z^{j-1}_{(v_1-\lambda_j),(v_2-\lambda_j)} + (\pi_{j,u}(Y)-\pi_{j,s}(Y))\cdot Z^{j-1}_{(v_1),(v_2-\lambda_j)}$$

2. If $v_2 < \lambda_j$:

$$Z^j_{v_1,v_2} = \left(1-\pi_{j,u}(Y)\right)\cdot Z^{j-1}_{v_1,v_2}$$

3. If $v_1 < \lambda_j \leq v_2$:

$$Z^j_{v_1,v_2} = \left(1-\pi_{j,u}(Y)\right)\cdot Z^{j-1}_{v_1,v_2} + (\pi_{j,u}(Y)-\pi_{j,s}(Y))\cdot Z^{j-1}_{(v_1),(v_2-\lambda_j)}$$

After including all the issuers, we set:

$$\left(Z_{v_1,v_2}\right) = \left(Z^M_{v_1,v_2}\right).$$

The matrix $\left(Z_{v_1,v_2}\right)$ now holds the joint loss distribution of the reference portfolio at the two horizon dates t_s and t_u, conditional on the realization of the market factor *Y*, and we can numerically integrate over the common factor to recover the unconditional joint loss distribution. Using the joint distribution of losses at different horizons, it is then straightforward, for any function *f(.)*, to compute conditional expectations of the form $E\left[f\left(L_{t_u},\omega\right) \mid L_{t_s} = \omega\right]$, which is how we construct the conditional tranche curve.

4. APPLICATIONS

In this section we present a few examples of reset tranches that can be handled using the methodology outlined above.

Generally speaking, these structures offer buy-and-hold investors additional flexibility to express views on the timing of defaults and fine-tune the risk-return profile of their exposure. For example, an investor concerned about short-term defaults can start out with a reassuring level of subordination and moderate leverage, and decrease the subordination and/or lever up the tranche if no defaults happen until the reset date. This controls for the exposure to defaults in the short term while providing extra yield to the investor. Similarly, an investor uncertain about the performance of the underlying credits in the long term can agree to enhance the subordination or de-lever the tranche at the reset date, either unconditionally or conditionally on portfolio losses having reached a pre-specified threshold by the reset date.

Next we present some specific trades that can be priced using the simple algorithm described above. The reader should note that a much wider range of contractual agreements fall within our definition of reset tranches. For notational convenience, and without loss of generality, we will assume throughout these examples that one loss unit corresponds exactly to 1% of the initial portfolio notional.

Example 1: Subordination Reset

A 5-year, 5-8% mezzanine tranche whose subordination resets after two years to 5% of the initial portfolio notional can be priced by specifying:

$$\left\{\begin{array}{l} t_T = 5 \text{ years} \\ t_s = 2 \text{ years} \\ U = 5, \\ V = 3, \\ U(\omega) = 5 + \omega, \\ V(\omega) = \left(3 - (\omega - 5)^+\right)^+ \end{array}\right\}.$$

Example 2: Leverage Reset

A 5-year, 5-8% tranche that is (possibly) de-levered at the reset date by restoring the width to the initial level of 3% can be priced by specifying:

$$\left\{\begin{array}{l} t_T = 5 \text{ years} \\ t_s = 2 \text{ years} \\ U = 5, \\ V = 3, \\ U(\omega) = \max(5, \omega) \\ V(\omega) = 3 \end{array}\right\}.$$

Example 3: Conditional Subordination Reset

An investor may want to reset the subordination only in states where the original cushion has been eroded by portfolio losses. Consider again an initial 5-year, 5-8% tranche. One may choose to reset the subordination to the initial 5% only if the portfolio losses have reached at least 2% by the reset date:

$$\left\{\begin{array}{l} t_T = 5 \text{ years} \\ t_s = 2 \text{ years} \\ U = 5, \\ V = 3, \\ U(\omega) = \begin{cases} 5 + \omega, & \text{if } \omega \geq 2 \\ 5, \text{ if } \omega < 2 \end{cases} \\ V(\omega) = \left(3 - (\omega - 5)^+\right)^+ \end{array}\right\}.$$

Example 4: Conditional Leverage Reset

An investor may want to lever up the investment, but only in scenarios where the default environment has been particularly benign up to the reset date. Starting again with a 5-year, 5-8% tranche, one may want to reduce the tranche width to 2% if the reference portfolio has had no losses after the first two years:

$$\left\{\begin{array}{l} t_T = 5 \text{ years} \\ t_s = 2 \text{ years} \\ U = 5, \\ V = 3, \\ U(\omega) = \max(5, \omega) \\ V(\omega) = \begin{cases} \left(3 - (\omega - 5)^+\right)^+, \text{if } \omega > 0 \\ 2, \text{if } \omega = 0 \end{cases} \end{array}\right\}$$

Example 5: Forward Start

A forward-starting tranche can be seen as a particular case of a reset tranche. A 5-year, 5-8% mezzanine tranche starting two years from now can be priced using a conditional curve that is constant at 1 until the start date and is defined as:

$$Q(t;\omega) = \left(1 - E\left[\frac{\left[L_t - (5+\omega)\right]^+ - \left[L_t - (5+\omega+3)\right]^+}{3} \mid L_{t_s} = \omega\right]\right)$$

for $t > 2$.

SUMMARY

In this article we have presented a simple methodology for pricing a class of default-path-dependent tranches quasi-analytically. The proposed methodology is general in the sense that it can be easily applied to any model with conditionally independent defaults. We have then argued that the set of structures that can be handled by this approach provide investors with opportunities to enhance their yields and choose from a rich menu of risk-reward profiles that can be made contingent upon the realization of defaults.

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