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INTRODUCTION

Over the last several years, there has been a significant increase in the importance of the global credit markets to institutional investors. This has been driven by a number of different factors, including the lower supply of government debt and the desire to enhance long-term risk-adjusted returns. All of this growth has coincided with the rapid expansion of the credit derivatives market and has heightened the demand for a better understanding of what drives credit spreads and how these dynamics then affect their nearest derivative proxy: credit default swaps.

This growth of the credit derivatives market has also created a demand for a better understanding of the new products that have emerged. In particular, default correlation products such as default baskets are gradually being seen as a new asset class to sit alongside single-name credit products.

The first article in this publication examines the drivers of credit spreads. The authors break these out into a number of fundamental factors, including default risk, re-investment risk, macroeconomic effects, risk-aversion, and liquidity, as well as technical factors such as off-equilibrium supply-demand pressures. This analysis leads to the conclusion that **the current very high levels of credit spreads are driven by historically high levels of risk premia rather than historically high levels of risks**. The authors envision substantial spread tightening once the overhang of the war, recession, and risk aversion is removed.

The second article attempts to describe how investors may begin to take advantage of the opportunities presented by the credit derivatives market to trade the spread between the cash and credit default swap market. The authors set out the various factors that cause the cash and credit default swap market to dislocate as well as the implications of the various ways one can implement a default swap basis trade. Using a model, they examine the theoretical relationship between the carry of the basis trade and any gain or loss in the event of default. A number of scenarios are presented that demonstrate the dynamics of the default swap basis and how investors can exploit this as a relative value opportunity.

For many investors, it is unclear what, if any, advantages basket default swaps present over single-name credit derivatives. In response, the authors of the third article introduce two metrics to analyze baskets. The first is the **excess spread**, defined as the difference between the spread paid by a default basket and the breakeven spread computed from historical default studies. The second is the **spread coverage ratio**, which is simply the ratio of these two quantities rather than the difference. It represents the number of times the spread is compensating the issuer for the pure default risk of holding the product. The authors find that first-to-default baskets are an ideal way to leverage excess spread. For the spread paid, they present a lower expected loss for single-name assets paying a similar spread. For investors who wish to maximise the spread coverage ratio, second-to-default are better than single-name credit derivatives, especially for pools of credits with a low default correlation.

Within the world of quantitative credit, one of the hottest problems of the moment is determining the best way to model correlated defaults. Such a model is essential for valuing and risk-managing the family of portfolio credit products that includes first-to-default baskets and both cash flow and synthetic CDOs. This question is examined in detail by the authors of the last article. Many of the models currently used in the market adopt a multi-variate normal distribution for asset returns in order to generate correlated defaults. In their article, the authors demonstrate that such an assumption significantly underestimates the likelihood of extreme events. They then propose an alternative model that allows for extreme movements and discuss how its parameters can be calibrated to agree with empirical data. The effect on the pricing of first and second-to-default baskets is presented and discussed.

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SPREADING THE PRAISE AND THE BLAME

We analyze the fundamental and technical catalysts behind credit spread movements and discuss the outlook following the terrorist attacks in the US.

Introduction

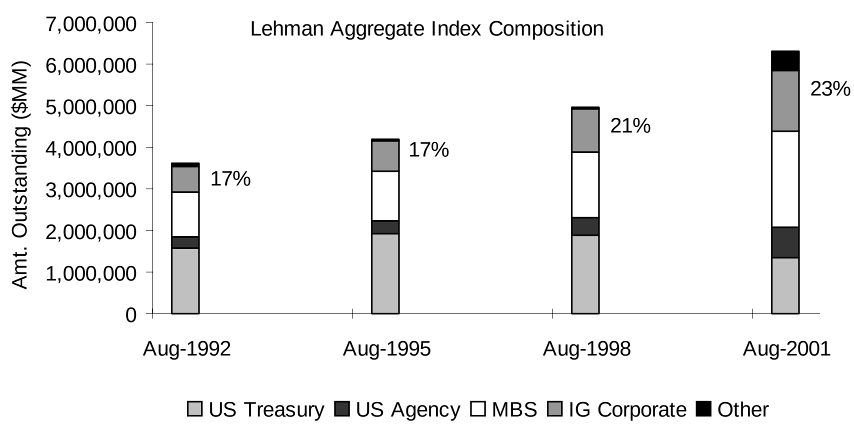
The global credit markets have become increasingly important in the asset allocation framework for institutional investors over the last five years. The U.S. has the best-developed capital markets for investment-grade and high-yield corporate bonds, but both Europe and Asia are developing rapidly. Figure 1 shows the composition of the Lehman Brothers U.S. Aggregate Index by amount outstanding in major asset classes. Corporate bonds have significantly increased in importance since 1995, mostly at the expense of U.S. Treasury bonds, which have declined recently while the government retired its debt.

Investors outside the U.S. are rapidly expanding their usage of credit product in order to enhance long-term risk-adjusted returns. Additionally, origination in the convertible securities market has grown substantially over the last 18 months. The market for structured credit product has doubled in each of the last three years and now exceeds $1 trillion. And with global interest rates steadily ratcheting lower and banks tightening their lending standards, the importance of the public credit markets has never been greater for the world's major corporations and sovereigns.

Consequently, a thoughtful interpretation of credit spread behavior (as opposed to swap spreads, which are no longer a consistent credit proxy) has significant meaning for debt investors, borrowers and equity market participants alike.

We would like to thank our colleagues Ivan Gruhl and Stephen Mandl for their contributions to this article.

Figure 1. **Lehman Brothers U.S. Aggregate Index Composition by Amount Outstanding**



What Drives Credit Spreads?

Our goal in this article is to analyze some of the relationships that govern the behavior of credit spreads, concentrating on the U.S. investment-grade corporate market. There is a similarity between current market conditions – following the September 11 terrorist attacks in the US – and those during the Gulf War. As a result, we have extended our analysis to the decade since the last recession. Since 1990, we have seen several important cycles in the behavior of the credit spreads.

Like any other asset class, credit product offers investors potential incremental return for taking on particular types of risk. The single parameter that describes the return side of this relationship is the credit spread and its behavior (tightening or widening).

The extra yield offered by credit product compensates the investor for a variety of risks. What are the particular risks that are associated with credit instruments?

- **Credit risk:** The risk that the investor will not receive the full amount of the debt repayment as promised due to default of the issuing company or non-corporate entity.

- **Reinvestment risk:** Unless the investors intend to hold the bonds until final maturity and unless they are insensitive to the mark-to-market of their holdings, such an investor bears the risk of spread level changes, even if the asset specific default risk is constant.

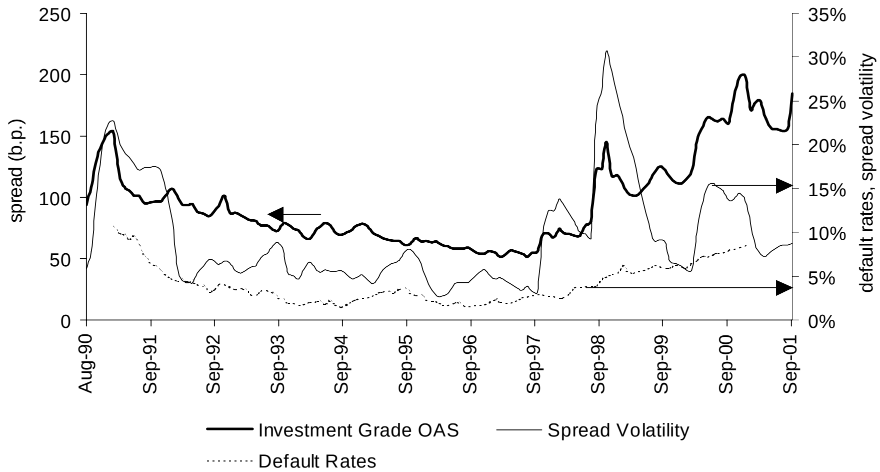
- **Technical, or supply/demand risk:** The supply/demand pressures can be significant in the short and intermediate terms, as they distort the risk and return patterns associated with normal equilibrium state of the market (especially during crisis periods).

- **Macroeconomic risk, liquidity risk, and risk aversion:** As the economy cycles through expansions and recessions and weathers cataclysmic events such as terrorist attacks or financial market disruptions, the credit markets follow in a closely related manner. The importance of macro risk is in changing levels of risk premia that investors demand for taking on asset-specific risks. The liquidity premium, while not identical, is closely related to risk aversion and reflects the added premium for holding securities other than cash or highly liquid treasuries.

The Credit Risk Factor

In terms of default risk, the U.S. credit markets in general, and the high-yield market in particular (which bears the vast majority of default risk inside of a one-year time horizon), have gone through a full cycle during the last 10 years. Figure 2 shows the rolling 12-month default experience in the U.S. high yield markets. Default rates have more than doubled since the mid 1990s.

Figure 2. **U.S. Corporate Spreads versus Default Rates and Spread Volatility**



Moreover, recovery values have deteriorated substantially, leading to a markedly higher default loss risk, which almost tripled during the same period. Because of the "new economy" nature of many recent bankruptcies, there is much less that can be salvaged by distressed investors.

The realized default rates are, by construction, lagging variables. We have shown them on a 6-month forward-looking basis; i.e., each data point refers to the realized numbers over the period covering six months prior to six months after. Figure 2 shows a remarkable accuracy with which the credit markets discount the future behavior of the credit risk factor—note the synchronicity of the pattern of spreads with future realization of default losses.

The Reinvestment (Volatility) Risk Factor

On the same Figure 2, we show the behavior of corporate spreads versus spread volatility.[1] We can clearly see four major volatility cycles: the first in the early 1990s during the U.S. recession and Gulf War, the second in 1997 after the Asian crisis, the third following the Russian crisis and hedge fund debacle in 1998, and the fourth unfolding during early 2000 with the burst of the technology bubble. Interestingly, realized spread volatility has not reached 1998 levels yet, helping to explain the excellent performance of credit markets in the first half of 2001.

In all probability, at present we are witnessing the beginning of another volatility cycle, with the U.S. economy slipping into recession and the prospect of a prolonged military conflict compounding domestic problems. This has contributed to recent spread widening and will put a floor on any recovery until the reinvestment risk subsides to acceptable levels.

1 We measure spread volatility by an exponentially weighted moving estimate with a 6-month half-life.

The Supply-Demand Factor

Given its size and liquidity, the Lehman Brothers Credit Index is a good representation of the credit marketplace. Figure 3 shows investment-grade corporate bond supply and demand due to international investor buying, as well as reinvestment of coupon and redemption flows from the Credit Index. We also show secondary trading volumes, as measured by Lehman Brothers' High-Grade cash corporate bond trading desk. The net demand for credit product, together with trading volumes and spread volatility, can be combined into a market impact variable, as explained in the Appendix.

A potentially large component of the demand for credit products is due to asset reallocation in balanced portfolios. However, we believe that this component can be ignored in our simplified framework because it is characterized by a substantially longer typical time scale. Large institutional investors, such as pension funds, normally make asset allocation decisions annually. Others, such as balanced mutual funds may adjust their allocations semi-annually or quarterly. Of course, large market dislocations like the 1998 credit crunch or the current situation do cause a revision in allocation decisions. Thus, we contend that during most of the time (except major crises), the effect of asset reallocations on monthly spread changes is relatively small.

Figure 3. **U.S. Credit Index Supply, Estimated Demand and Lehman Brothers' Trading Volumes**

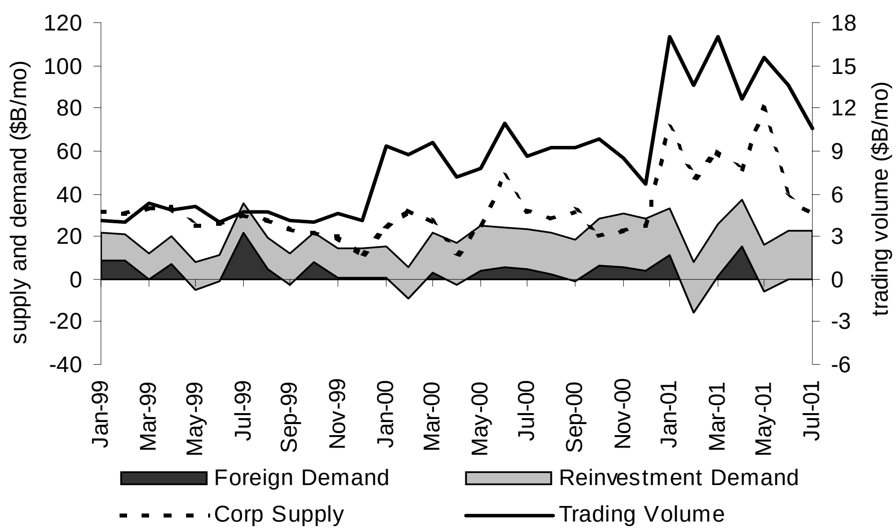


Figure 4. **Market Impact of Net Demand**

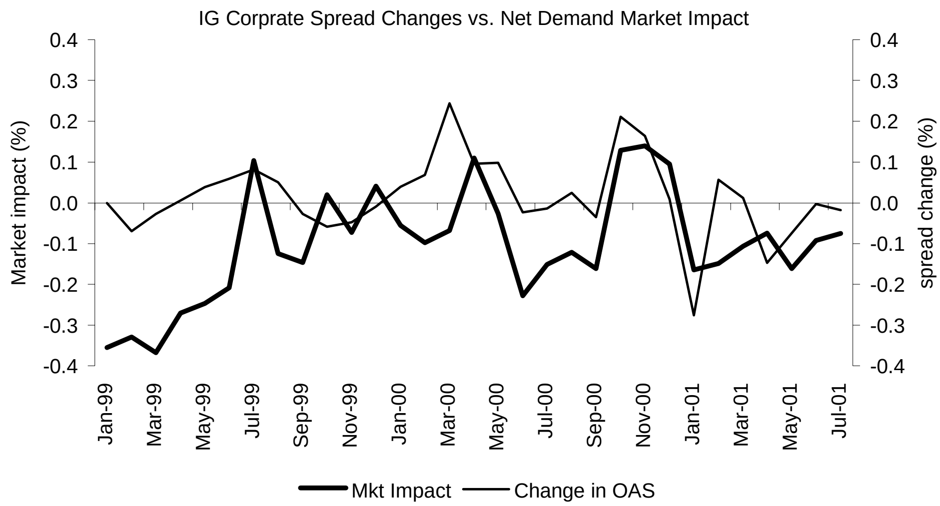
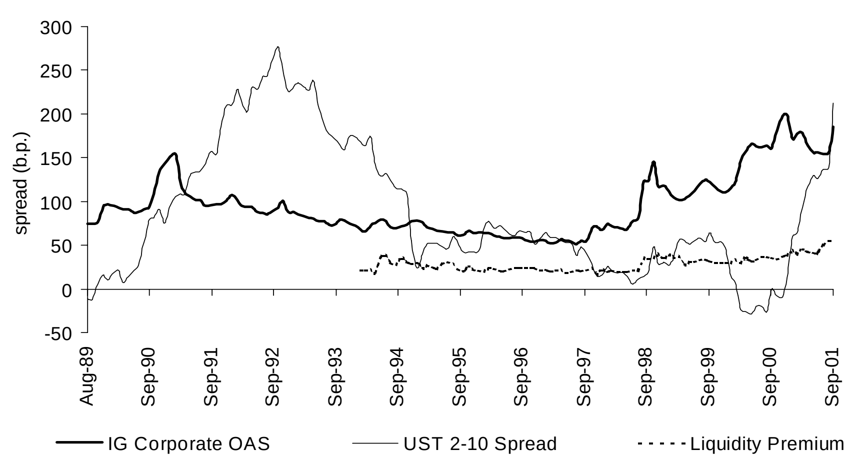


Figure 4 shows the projected impact of the net demand for credit products for the U.S. investment-grade corporate sector versus the month-over-month changes in the average OAS of the index. It is important to note that a net positive supply does not always lead to observable widening of credit spreads because it is the issuer that bears the cost of the market impact by pricing bonds at wider spreads. This phenomenon leads to new issue rallies that are not accounted for in the Aggregate Index because the bonds are included in the index only from the month following issuance. On the other hand, in the case of unbalanced positive demand, there are no other counterparties except investors, and, therefore, the market impact is not shielded.

Figure 5. **Lehman Corporate Index OAS vs. UST Slope and Liquidity Premium**



The Macro Factor

The U.S. Treasury curve slope has been a good proxy for the macro factor during normally functioning markets, with the inverted curve in early 2000 signaling a future recession risk.[1] As the Fed started reducing interest rates, the curve reverted to its normal upward sloping shape, reflecting the market's expectations of an economic recovery. As Figure 5 shows, investment-grade credit spreads usually widen when the Treasury curve is inverted and tighten when it subsequently steepens.

We also show the Lehman Brothers' LIBOR Credit Liquidity Premium on the same chart. It has been on the increase since the 1998 Russian crisis and currently stands at 50 bp, more than twice its value in 1997. These extra 25 basis points could potentially contribute to spread rallies when the more immediate political and economic risks subside.

Compared with the previous recession and war period of 1991, we observe a promising similarity in the behavior of the U.S. Treasury curve slope. We can also see that the positive slope of the curve is still almost 100 bp below the highs reached in late 1992, when the economic recovery was complete and the new expansion had started. It is likely to take both additional Fed rate cuts and signs of expected recovery (reflected in higher 10-year yields) to get to those levels.

It seems plausible that not only the value of the US Treasury slope, but also the amount of time during which this value is negative or very low may be important. Indeed, a hypothetical "deep recession" scenario when the curve stays inverted for a long time might develop if the Fed easing were not aggressive enough in pushing short rates below long rates that are depressed by prospects of the slowing economy. On the other hand, a hypothetical "soft landing" scenario could develop when curve inversion happens as a short-term phenomenon due to very active Fed tightening followed by a quick subsequent easing without affecting long term interest rates.

To quantify these possibilities we introduce the *integral macro factor*. Let us consider the long term average of the US Treasury 2-10 spread as its indicative equilibrium value. In Figure 6, we show the difference between the current value of the slope and its long-term average, shading it in dark gray when the slope is less than the average and light gray when it is greater than the average. The integral macro factor is defined as the cumulative shaded area between the spot value of the slope and its long-term average (with proper sign). This definition captures the intuitive notion outlined above that the longer the slope stays inverted (or not steep enough), the greater the Fed must lower rates (as manifested by a large positive slope) to reverse the impact of a slowing economy.

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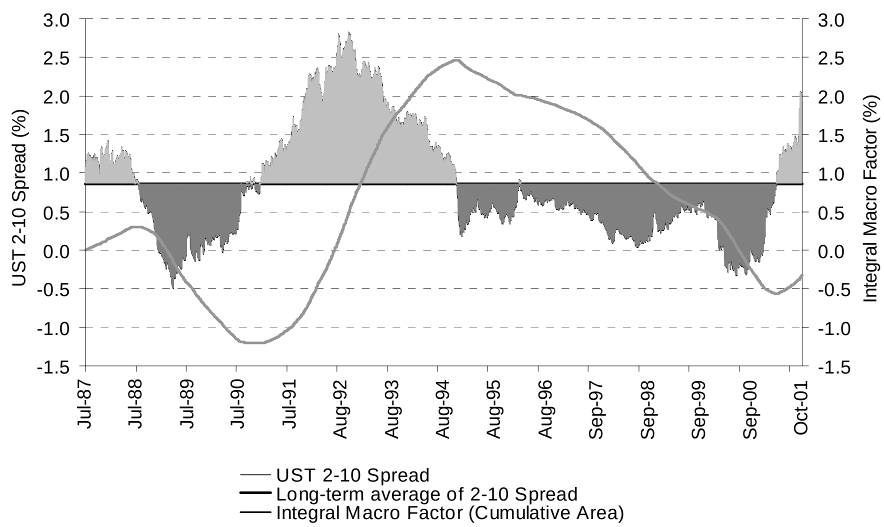
1 We proxy the Treasury curve slope by the 2s-10s, i.e. the difference between the yields of the 10-year and 2-year Treasury bonds.

Figure 6. **Defining the Integral Macro Factor**



The integral macro factor becomes negative during the recession of 1991 when the slope dips below the average, then recovers to the positive territory when the curve steepens in 1992, and then gradually decreases in value as the US Treasury curve increasingly flattens in recent years, finally resulting in negative slope and negative integral macro factor in the second half of 2000. Interestingly enough, the depth of the recent dip in the integral macro factor is considerably less than what happened in the prior recession. While the US economy has certainly missed the "soft landing" scenario, Figure 6 holds out hope that the "deep recession" scenario may not be very probable either. As Lehman Brothers' economists predict, we are likely to see a relatively shallow recession. Of course, the uncertainties still abound after the terrorist attacks of September 11, and all these indications must be taken with a grain of salt.

What to Expect in the Next Year?

As we have discussed above, credit spread movements have become more difficult to predict over the last few years as the web of inter-connecting influences has grown more complex. However, we believe that a thorough understanding of the various fundamental and technical determinants of spread activity will allow market participants to navigate today's challenging markets.

One important question on credit market participants' minds is whether the current levels of spreads are the new norm or if a return to the levels of pre-1998 is feasible. A comparison of the current situation with the 1990-91 era has merit due to the similarity of economic and political conditions. However, the unstable nature of the current capital markets renders our predictive vision limited.

It certainly appears that current spread levels are incongruously high compared with those of 1990-91. Indeed, default rates have not yet reached prior levels and may remain under those levels even if the recession materializes. Volatility risk is noticeably lower and is unlikely to reach the extremes of 1998 credit crunch. The supply/demand equation is quickly moving into net positive supply over the short term, with many companies boosting their financial leverage. Finally, the Fed is accommodating aggressively, and we will certainly see a much stronger fiscal and monetary stimulus than was previously expected.

Thus, much of the current level of spreads is due not as much to the historically high levels of risk, but to historically high levels of risk aversion and risk premia. Therefore, when the overhang of the recession and the international hostilities is removed, one should expect credit spreads to narrow substantially. The critical issue for investors, as always, is to recognize and benefit from the beginning of this process.

Appendix A. Computing Supply/Demand Impact on Spreads

In this Appendix, we outline a simple toy model that estimates the market impact in terms of spread widening or tightening induced by the supply/demand pressures. There are many dimensions to this problem. In its full complexity, the determination of the market impact is among the toughest challenges in economics and finance, because almost all standard assumptions, such as equilibrium, market efficiency, frictionless trading, symmetrical information among the market participants, etc. need to be explicitly abandoned.

In the academic literature, many have focused on modeling the market microstructure in either game-theoretic or asymmetric information markets setting. Among the industry practitioners, a more popular choice is the models, which focus on simulations of market clearing mechanisms and liquidation-equivalent prices of instruments. This approach has been particularly well studied in case of equity markets, where the best know example is Barra Market Impact model.

We, on the other hand, will approach the problem in a simplified manner. Our definition of the market impact will be limited to the determination of the fair spread premium or discount, which a hypothetical dealer would impose in order to “work the order” representing the aggregate net supply or demand. We will assume that such dealer has a good sense of the average trading volumes in the market, and does not wish to take any particular view on the direction of the market. Moreover, we will assume that there is no tradeable information content in the off-equilibrium net demand. This is clearly an oversimplification, but we believe that when considering a model for the market aggregate net demand all the client-specific information can be ignored.

Our hypothetical dealer’s main guiding rule is to have appropriate risk budget to cover his potential shortfall while making the trades. Thus, our model must depend on the net demand for credit product, the trading volume and spread volatility as the primary observable variables. Assume that in a given month the net demand for credit assets is ND, the level of the market activity is proportional to the secondary market trading volume V, and that the spread volatility is σ. Positive net demand will cause spread tightening, while negative net demand (i.e. positive net supply) will presumably cause spread widening. Everything else being equal, what is the fair spread differential that an investor should give up for entering into such market?

The risk due to off-equilibrium net demand is that by the time it takes to complete the trading, spreads will have moved and the average spread of purchased bonds will be tighter than when trading began. It will take approximately $T = |ND|/V$ time to digest the demand.

The fair cost of possibility that the volatile security will have a value greater (lower) than its current level after such time is proportional to the at-the-money call (put) option price with maturity equal to trading horizon. The corresponding option premium can be written as follows:

$$\text{Premium} = \text{Price}_{\text{underlying}} \cdot \text{Price}_{\text{option}}\left(\text{Spot} = 1, \text{Strike} = 1, \sigma_{\text{price}}, T\right)$$

We then must interpret the option premium in the equation above as the potential addition (reduction) to the price of the bond at the inception of the hypothetical trade, which is necessary to cover the dealer's risk budget. For high grade fixed income bonds, the price is near par. We denote the fair premium (discount) in spread terms as Δs. It is related to price premium via usual duration relationship:

$$\frac{\text{Premium}}{\text{Price}_{\text{underlying}}} = -D \cdot \Delta \text{s}$$

Similarly, the price volatility can be re-expressed in terms of the spread volatility as follows:

$$\sigma_{\text{price}} = D \cdot \sigma_{\text{spread}}$$

For both at-the-money puts and calls, the dependence on trading horizon is well approximated by the option theta, which leads to the following simple market impact formula for short horizons:

$$\Delta s(ND, V, \sigma) \propto -\text{sign}(ND) \cdot \frac{1}{D} \cdot \Theta_{\text{ATM}}\left(\sigma_{\text{price}}, T\right) \cdot T \Big|_{T = \frac{|ND|}{V}} \approx -\text{sign}(ND) \cdot \sigma_{\text{spread}} \sqrt{\frac{|ND|}{V}}$$

The duration dependence cancels out. The fair cost is proportional to the cumulative risk during the trading horizon $\sigma\sqrt{T}$.

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TRADING THE DEFAULT SWAP BASIS

We analyse the new relative value opportunities presented by the credit derivatives market to trade the spread between the default swap and cash market.

The advent and significant growth of the credit derivatives market has presented investors with new ways to trade credit risk. In addition it has created new relative value opportunities. One such opportunity is the trading of the **default swap basis** in which investors may take a relative value view on the spread between a bond issued by some entity, and the spread demanded by a default swap contract linked to that same entity. While there is a theoretical relationship between these two spreads, there are a number of factors that may cause this relationship to break

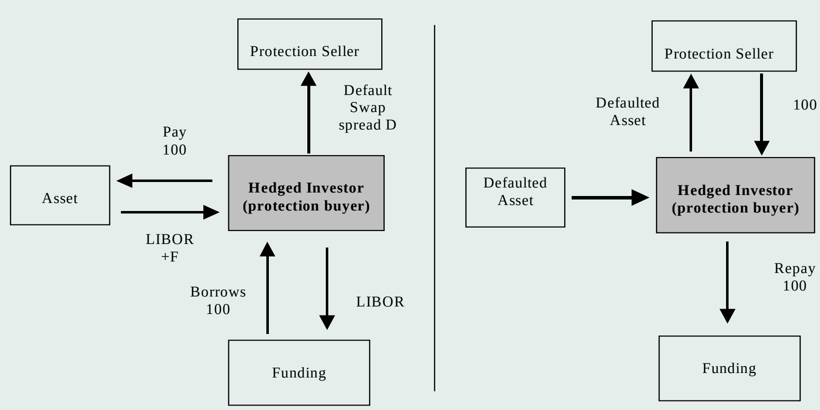
Figure 1. **The Theoretical Risk-Free Trade**

Before we explain why default swaps and cash can trade at different spread levels, we must first establish why they are related. To do this, we describe what we call the theoretical risk-free trade. This is a strategy involving a cash bond and a default swap which subject to certain assumptions can be shown to be risk-free. We consider a long basis trade, consisting of:

1) Purchase of a defaultable floating rate note **worth par** which pays a coupon of Libor plus a fixed spread *F*. We assume that the asset is funded at Libor flat.
2) Purchase of protection on this asset in the default swap market to its maturity at a default swap spread *D*.

This strategy is shown in Figure 1. In the event of default the investor delivers the defaulted asset to the protection seller in return for par. This par amount is then used to pay off the funding leg. The net strategy is therefore *credit risk free*, as the investor has no exposure to the default of the asset. The investor earns an annual spread of +*(F-D)* over Libor for assuming no credit risk. The theoretical arbitrage-free relationship requires that *D=F*.

Static hedge for a protection buyer showing a) The payments before and b) in the event of default



(a) Before default or maturity **(b) At Default**

down. In certain cases this can present clear investment opportunities to investors.

In an earlier publication[1] we set out in detail the reasons why cash and default swap spreads may differ. In what follows, our aim is to provide market participants with a framework for understanding how to analyse and implement basis trading strategies. Using a model we establish the theoretical relationship between cash and default swaps. We then discuss how one might transform a view about the default swap basis into a specific trading strategy, discussing how this theoretical relationship can break down in practice. Before doing so we define a **long basis trade** as one in which we buy the asset and buy protection. A **short basis trade** is one in which we sell the asset and sell protection. The theoretical relationship between a par floater and a default swap is shown in Figure 1.

Implementing a long basis trade

The standard way for a credit investor go long the basis is to purchase the bond on asset swap and then to buy protection. In a standard par asset swap, the investor pays par to buy a package consisting of the bond plus an interest rate swap to exchange the fixed coupon on the bond for payments of Libor plus a fixed spread known as the asset swap spread.

How precisely the credit risk of this position should be hedged depends on a number of factors, most important of which is the initial **full price** (clean price plus accrued interest) of the bond. As a default swap is a par product it can only hedge the difference between par and the recovery value of the asset. This is not a problem if the asset is initially priced at par. However, if the full price of the asset is trading away from par, a default swap does not exactly hedge the initial investment.

To see this in more detail, consider the case of an investor who purchases a fixed rate bond on asset swap, and buys protection with a default swap on the full face value of the bond to maturity. We call this a **face value hedge**. We assume that the cost of funding is Libor flat.

1 Trading the Basis: Cash vs Default Swaps Dominic O'Kane, Robert McAdie, Lehman Publication, Jue 2001.

Figure 2. **Bond on asset swap plus face value hedge showing flows for different events**

Event	Asset	Asset Swap Swap	Funding	Default Swap	Net
Initiation	-Full Price	-(100%-Full Price)	+100%	0	0
Carry	+Coupon	-Coupon+(Libor+Asset Swap Spread)	-Libor	-Default Swap Spread	+(Asset Swap Spread -Default Swap Spread)
Immediate Default	+Recovery	+(100%-Full Price)	-100%	(100%-Recovery)	+(100%-Full Price)
Default at time t	+Recovery	Swap MTM(t)	-100%	(100%-Recovery)	Swap MTM(t)
Maturity	+100%	0	-100%	0	0

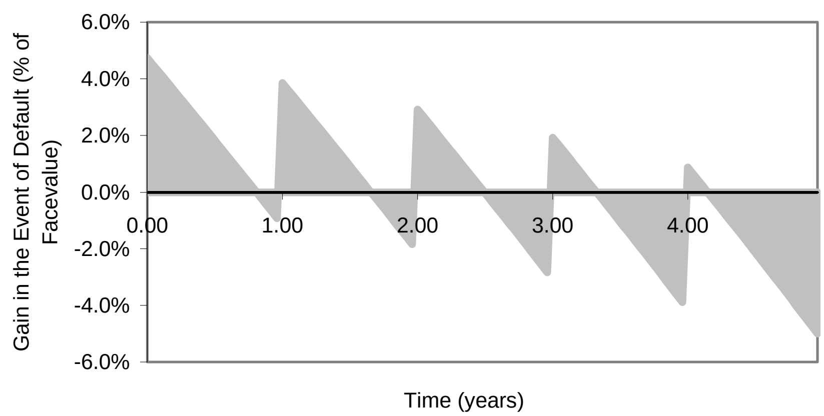
As shown in Figure 2, the annual carry for the investor in this trade is given by the difference between the asset swap spread and the default swap spread.

In the event that the asset defaults, the investor is left with a defaulted asset and an off-market swap, which must be unwound. Initially, the value of the interest rate swap equals 100%-Full Price, but this changes over time as interest rates change and as the swap rolls down the Libor curve. If the asset is a premium asset, the mark-to-market will initially be negative. If the asset is a discount asset then this swap will initially be worth a positive amount and any default will result in a positive payment from the swap. As maturity is approached, the value of the swap pulls to zero so that the net position equals par in the event of default.

Buying the asset on asset swap has the benefit that it removes most of the interest rate exposure that would be incurred by buying the fixed rate bond. It removes much of the price variation of the bond which result from movements in the Libor curve and the fixed coupon payments. This means that, on a mark-to-market basis the investor's default exposure changes less for an asset swap than for a fixed coupon bond. For example, the default exposure profile for a fixed coupon discount bond hedged on the full face value with a default swap is shown in Figure 3. To hedge this exposure, the investor would have to dynamically buy or sell default protection throughout the life of the trade. Transaction costs may make this impractical, especially given the small sizes involved. Instead, the investor may choose to hedge a slightly higher notional at the cost of losing some carry. For this reason, asset swaps are preferable.

The exception to this is bonds trading at a deep discount. These tend to trade on a price basis, where the recovery value of the asset becomes more critical and where the bond price becomes less sensitive to spread or interest rate movements. In this case investors may prefer to buy the bond outright and buy protection without entering into an asset swap.

Figure 3. **The change in gain in the event of default for a face-value hedged 5% coupon bond as the coupon payments are made and as the bond rolls down the yield curve**



An alternative hedging approach for the investor is to determine the amount of default protection in order to protect their initial investment. We call this a **market-value hedge**. It means buying protection on a notional equal to the market hedge ratio times the face value of the cash where

$$\text{Market Hedge Ratio} = \frac{\text{Full Price-Expected Recovery}}{\text{100\%-Expected Recovery}}$$

The effect of this approach is to change the carry on the position. The gain on default is initially zero provided our assumption of the expected recovery rate is correct. If default does occur and the actual recovery rate is different from the expected recovery then the gain on default is

$$\text{(1-Actual Recovery)}\left(\frac{\text{Full Price-Expected Recovery}}{\text{100\%-Expected Recovery}}\right)\text{-(Full Price-Actual Recovery)}$$

For a discount bond this means that we lose if the actual recovery is less than the expected recovery, and gain if the actual recovery is greater than the expected recovery. For a premium bond we lose if the actual recovery is greater than the expected recovery, and gain if the actual recovery is less than the expected recovery. We therefore have a recovery rate risk, which can be in our favour or against us.

A third type of hedge is to buy a notional of protection equal to the market price of the bond. This is equivalent to the market-value hedge with an expected recovery rate of zero. We therefore call this a **zero recovery market-value hedge**. The cost of protection for a discount bond is between that of a face value hedge and a market-value hedge. It guarantees that an investor can never lose more than their initial investment. However, it still leaves the investor underhedged on a mark-to-market basis as the asset accretes up to par. **We view the zero-recovery market hedge as an attractive compromise between the face value and market-value hedging strategies since it eliminates the downside recovery risk and has greater carry than a face value hedging strategy for a discount bond.**

Using a simple model of default and recovery we can establish a theoretical relationship between the default swap spread and the asset swap spread. In Figure 4(a) we have plotted the annualised carry versus the gain in the event of default for a 5-year asset with a coupon of 6%, on asset swap. This is done by varying the default probability of the issuer, and is shown for all three hedging strategies.

The first thing to note is that all three lines pass through the origin. This corresponds to the case when the asset prices at par in which case all three hedging strategies are equivalent and the carry is zero. The market value hedge is a vertical line since, assuming that the actual recovery rate equals the expected recovery rate, there should be zero gain in the event of an immediate default. We can see this more clearly in Figure 4(b) where we

Figure 4a. **Annualised carry versus gain in the event of an immediate default for three hedging strategies**

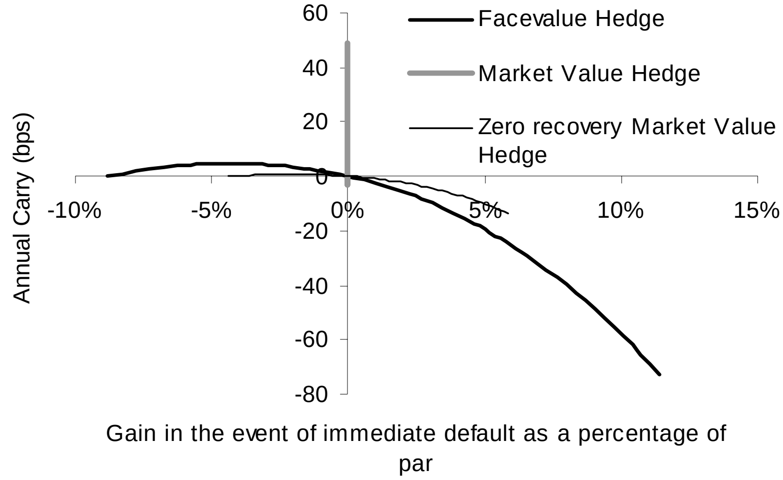
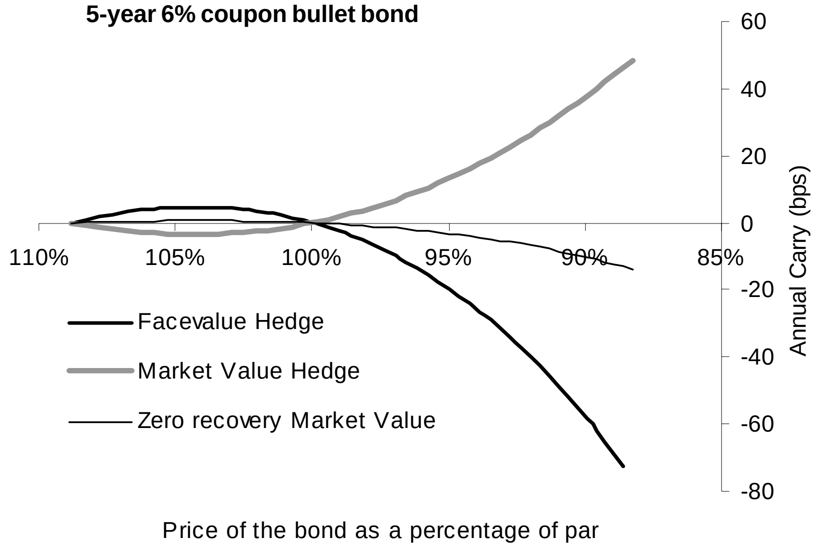


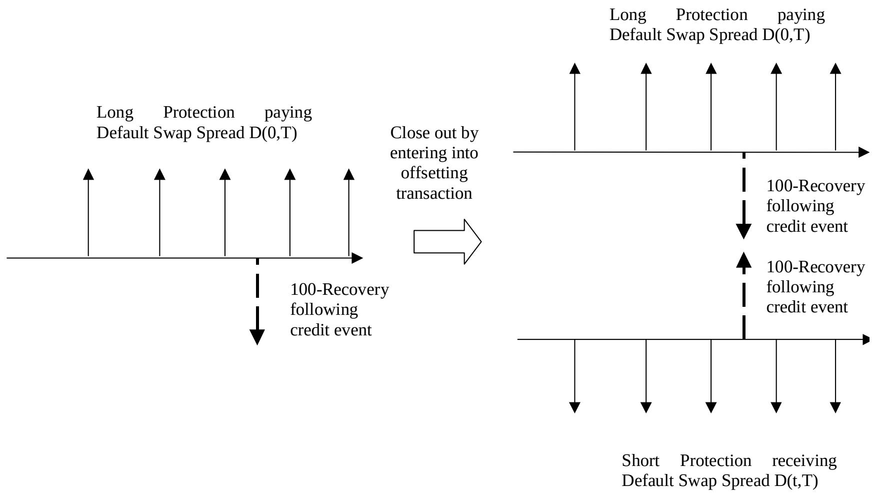
Figure 4b. **Annualised carry versus bond price. The bond chosen is a 5-year 6% coupon bullet bond**



have plotted the carry versus the bond price. Once again all three lines intersect at a price of par. The carry for the market value hedge increases as the bond price falls below par, reflecting the fact that while we are hedged for an immediate default, we are underhedged as the bond accretes to par. However we are fully hedged if the bond price falls. On the other hand, the carry becomes more negative for a face value hedge as the bond price falls since we become overhedged to an immediate default. This over-hedging reduces as the bond accretes to par.

We emphasise that Figure 4 shows the theoretical relationship between the carry and the gain on default. In practice, other factors (Figure 6) can create dislocations between the cash and default swap markets which can enable the investor to

Figure 5. **Closing out a long protection default swap position by entering into the offsetting transaction**



either increase their carry or increase the gain in the event of a default.

One way to view a **long basis** (long cash, long protection) position is as a covered put option. The option's intrinsic value is 100%-Full Price which declines if the credit improves or as the bond price pulls to par, and increases if the credit deteriorates. The negative carry on this trade is equivalent to the amortised option premium. Note that as the default swap premium terminates on default, the earlier the default, the less premium we will have paid for this option.

Default Swaps Used as a Spread Product

The default swap is essentially a spread product. This is shown most clearly in the determination of the mark-to-market of the default swap, calculated as the value of the initial trade plus the value of the offsetting transaction.

For example, an investor who has purchased protection can close out the transaction by selling protection on the same credit, to the original maturity date. Exposure to the payment at default is then hedged out. However, the remaining premium is only paid until maturity or default, whichever occurs sooner. If default happens, any remaining carry is lost. To mark the position to market, we need to know the probability of surviving to each premium payment date. Using a simple model[1], we can write the mark-to-market at time t as

$$\mathrm{MTM(t)} = \left(D(t,T) - D(0,T)\right)\sum_{i=1}^{N}\Delta_i(b)Q(i)Z(i)$$

[1] We assume independence of the interest rate process, the default process and the recovery amount. It is also assumed that recovery is paid as a fixed percentage of the facevalue of the bond. For simplicity, we have also ignored the effect of the accrued premium paid at the time of default.

Figure 6. **Reasons for the Basis**

In reality, a number of technical and market-driven factors mean that the default swap spread and the asset swap spread do not always trade at the same level, even when the bond is priced at par. We set out some of these factors:

Why default swap spreads should exceed the asset swap spread

- *Delivery Option* - the protection buyer is long an option to choose one out of a basket of deliverable assets to be delivered in the event of default
- *Risk of Technical Default* - default swaps may be triggered by events which do not constitute a full default on the corresponding cash asset. A protection seller may demand a higher spread to compensate them for this risk.
- *Profit and Loss (P&L) realisation* - unwinding a default swap by entering into the offsetting transaction means that any P&L is only realised at maturity or default whereas the P&L on a bond can be realised immediately by selling it.
- *Liquidity* in default swaps is focused at the 3 and 5 year maturities. Away from these, bid-ask spreads are wider. They also widen on a credit deterioration.
- *Demand for protection* - the difficulty in shorting a credit in the cash market makes default swaps the best alternative and so widens default swap spreads more than cash spreads.

Why the default swap spread should be less than the asset swap spread

- *Funding costs* - we believe that most market participants fund above Libor. For these investors, selling protection (which implies Libor funding) is cheaper than buying the asset and so default swap spreads narrow.
- *Counterparty credit risk* - the protection buyer is also exposed to the credit quality of the counterparty in the event of default.
- *Market Short* - the huge market for synthetic CDOs has resulted in an excess of protection sellers in the default swap market.
- *Liquidity* in default swaps is often better than that for cash at the standard 3 and 5-year maturities when comparing the same notional sizes.
- In an *asset swap*, the asset buyer is exposed to an unknown mark-to-market on the interest rate swap in the event of default. A wider asset swap spread may be demanded as compensation.

where D(t,T) is the value at time t of the default swap spread to maturity date T, D(0,T) is the initial value of the default swap to maturity date T at trade inception, Q(i) is the probability of surviving to the ith premium payment date, Z(i) is the Libor discount factor to the ith premium payment date and $\Delta_i(b)$ is the year fraction for the spread payment in the appropriate accrual basis *b*. The values of Q(i) can be extracted from a model of default and recovery which is calibrated to market default swap spreads. It is clear from the above expression for the mark-to-market that:

1) The mark-to-market of a long protection default swap position increases if the default swap spread widens. This dependence is sub-linear since Q(i) is itself a decreasing function of D(t,T). Similarly, we can show that the mark-to-market of a short protection position falls as the default swap spread increases.

2) As the maturity of the trade shortens the number of remaining default swap spread payments decreases and the mark-to-market declines. This is known as the "theta" of the position.

Taking a View on changes in the Basis

Until now, we have simply discussed the theoretical relationship between cash and default swaps. However, a number of additional factors, both technical and market-driven, can cause cash and default swap markets to dislocate. These are summarized in Figure 6. We now discuss some of the possible strategies that may be implemented to take advantage of changes in the basis caused by dynamic changes in these factors.

Investors can use the default swap basis to express a view on the delivery option (see Figure 6). This option becomes more valuable for credits as the implied probability of default increases. Consequently, the default swap spread widens faster than the bond spread. To assess the value of the option, investors should be aware as to what assets are deliverable into the contract: the type of bonds in terms of their maturity, coupon, liquidity and any non-standard features, as well as loans and other contingent liabilities[2]. If the investor's view is that the credit is on the path to default, they can play the delivery option by buying default protection and buying the cheapest-to-deliver asset, thereby maximising their gain in the event of default.

Investors can profit from a long basis position if the credit deteriorates. In this scenario, both bond and default swap spreads widen and we generally find that the bid-ask in the default swap market becomes wider than that in the cash market. As the credit becomes more distressed, the increased likelihood of default, demand for protection, and the fact that the seller of protection is short the delivery option reduces the demand to sell protection, thus driving out default swap spreads. Liquidity in default swaps reduces, leaving bonds, and in certain circumstances, loans as the only instruments with any liquidity. In this situation the basis between bonds and default widens to a maximum level and the position may be unwound at a profit.

An investor can profit from a long basis position when spreads tighten. In a spread-tightening scenario, default swap spreads lag bond spreads as the value of the delivery option is linked to the less liquid liabilities which react more slowly to the credit improvement. This is reinforced by the fact that investors expressing a positive view on the credit tend to do so in the cash market. Hence the bond price increases by more than the decline in the mark-to-market of the protection resulting in an overall gain. This will be further enhanced by the fact that the bid-ask on the bond is tighter than when the trade was entered into. At some point the default swap market will catch up with the bond market. In this case the investor can profit from a short basis position as the default swap spreads tighten by more than the cash.

If the credit quality of an issuer is not expected to change, the relationship between cash and default swaps would also not be expected to change. In this case,

2 Note that the recent ISDA supplement (May 2001) has altered the specification of the deliverable assets.

the investor should look for positive carry opportunities where the trade can be unwound at a later date at a cost lower than the accumulated carry.

Conclusion

In this article, we have explained the theoretical relationship between cash and default swaps as well as describing some of the many factors which can cause this relationship to break and how to implement default swap basis trades. We believe that an understanding of all of these issues is vital for those wishing to avail themselves of this new trading opportunity.

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LEVERAGING SPREAD PREMIA WITH DEFAULT BASKETS

An investor can use default baskets to leverage spread premia. We show that first and second-to-default baskets compare favourably to single name default swaps on a risk-return basis.

Introduction

Credit spreads reflect the additional compensation that investors demand for holding defaultable rather than default-free securities. For this reason, investors in corporate and certain emerging market government securities demand yields that are higher rather than those of US (and other) government securities. These are generally believed to be default-free. However it is well known in the credit markets that the credit spread earned can be even greater than the expected loss computed from an historical analysis of default rates for similarly rated issuers. This excess spread, known more formally as a risk-premium, is understood to be compensating investors for risk-aversion, market-to-market risk, the lower liquidity compared to government bonds, and the lack of full diversification in the investor's credit portfolio. It also incorporates the effect of other technical and structural market factors.

The best way to take advantage of the risk and liquidity premia in credit markets is through a diversified pool of credits. A large pool of well-diversified assets minimises the investor's exposure to the specific risk while enabling them to enjoy the risk premium.

Default baskets are the simplest way by which investors can gain a leveraged exposure to a pool of typically between 5 and 10 issuers. This article aims to determine how the risk premium of basket default swaps differs from that of single-name assets, and whether this provides opportunities for investors on a risk-return basis.

Default baskets have been described elsewhere in significant detail (see references). Briefly, a **first-to-default basket** is an agreement in which a protection buyer purchases protection from a protection seller against the loss made on the first asset which defaults in a specified basket of credit names. As soon as default occurs the protection buyer delivers the defaulted asset to the protection seller in return for par. In a **second-to-default basket** the payment is triggered by the second default in a basket of specified issuers. Once again the defaulted asset is delivered by the protection buyer who receives par. In both cases, the protection buyer pays for the protection in the form of a fixed regular spread which is paid until default or maturity, whichever occurs first. Clearly, third to default and higher are obvious extensions. However, the size of the baskets, typically between 5 and 10 issuers, means that 3 or more defaults have a very low probability of occurring so these are generally low yielding assets which are of limited interest to most investors. We therefore focus our analysis on first and second-to-default baskets.

An important point to keep sight of in the following is the fact that a basket's leverage is determined by its order of protection in conjunction with the size of the underlying pool. A second-to-default basket on a pool of twenty issuers will have a larger likelihood of being triggered, and hence a much greater leverage than one on a pool of five issuers. A second-to-default basket on the larger pool behaves in a very similar fashion to a first-to-default basket.

Basket Spread Compensation

We want to compare the risk compensation an investor can receive on default baskets with that which is available via individual issuer default swaps. In practice it is difficult to disentangle the different factors contributing to default swap spreads. In addition to the pure default risk component, credit spreads are influenced by many other factors, such as liquidity, the risk of technical default, and repo considerations.

One way to obtain an indication of the risk premium inherent in spreads is to compare the market-implied default probabilities with average historical defaults. In fact, we can also follow this path in the opposite direction; i.e. construct credit curves from historical average default rates and compare the default swap spreads which would be implied by such a curve with those observed in the market.

The spread we obtain from such a "historical" curve gives us a measure of the fair compensation for the expected loss. In the language of financial theory, this is the *real-world* spread and compensates the investor for the historical average loss. The **excess spread**, defined as the (annualised) spread minus the real-world spread, even though it is a product of many factors, gives us a measure of the extra premium investors receive for holding this credit risk in default swap form.

Since higher spreads are usually associated with higher risk, what we also need is a metric that tells us about how much compensation we are receiving per unit of risk. We therefore introduce the ratio of the default spread s paid by the market to the spread $\tilde{s}$ implied from historical default rates, which we call the **spread coverage ratio**. To see why this is an informative number, note that the expected premium that will be collected over the life of a trade, and which therefore depends upon the time of default, in a default swap that pays a spread of s is given by

$$E[\text{Prem}] = s\,A \tag{1}$$

where A is the value of a risky annuity i.e. it terminates at default or maturity, and the present-value is computed with historical default rates. The break-even condition for the historical spread is given by

$$E[\text{Loss}] = \tilde{s}A \tag{2}$$

Therefore

$$\frac{E[\text{Prem}]}{\text{E}[\text{Loss}]} = \frac{s}{\tilde{s}} \tag{3}$$

The spread coverage ratio tells us how many times the expected basket premium covers the expected loss. Though both measures (excess spread and spread coverage ratio) clearly only approximate the amount of risk compensation an investor receives, they do allow us to *make a relative value comparison* of default baskets and plain vanilla default swaps.

Starting with a simple model for Basket Spreads

To gain an insight into how default baskets leverage the excess spread of the individual assets we start with a simple model. We take a basket with just two issuers A and B. The maturity of the basket is assumed to be T years, and the risk-neutral probability of A defaulting before maturity is given by P_A while the risk-neutral probability of B defaulting is given by P_B. The probability of both assets defaulting within the life of the trade is given by P_{AB}. Assuming that both assets have the same recovery rate, R, we can write the risk-neutral value of the first-to-default and second-to-default spreads as

$$S_{FTD} = (1-R)\left(P_A + P_B - P_{AB}\right) \tag{4}$$

and

$$S_{STD} = (1-R)P_{AB} \tag{5}$$

Without compromising this analysis we can, for simplicity, ignore the time value of the payment of par minus the recovery rate upon the triggering of the default - i.e. we effectively assume interest rates of zero.

We can also calculate the expected loss in the real-world measure where we denote the analogous real-world default probabilities to those defined above as $\tilde{P}_A$, $\tilde{P}_B$ and $\tilde{P}_{AB}$. Likewise we define the real-world values of the protection for first and second-to-default baskets as

$$\tilde{S}_{FTD} = (1-R)\left(\tilde{P}_A + \tilde{P}_B - \tilde{P}_{AB}\right) \tag{6}$$

and

$$\tilde{S}_{STD} = (1-R)\tilde{P}_{AB} \tag{7}$$

Hence, the first-to-default coverage ratio is given by the risk-neutral spread divided by the value of the real-world spread, i.e.

$$C_{FTD} = \frac{S_{FTD}}{\tilde{S}_{FTD}} = \frac{P_A + P_B - P_{AB}}{\tilde{P}_A + \tilde{P}_B - \tilde{P}_{AB}} \tag{8}$$

since the recovery rate dependency cancels. Likewise the second-to-default coverage ratio is given by

$$C_{STD} = \frac{S_{STD}}{\tilde{S}_{STD}} = \frac{P_{AB}}{\tilde{P}_{AB}} \tag{9}$$

To see how this depends upon default correlation, we need to specify a way of modelling correlated defaults. One choice gaining favor in the credit derivatives market is to use a Merton-style model of correlated default in which default occurs when the asset value of a firm falls below a certain threshold. In such a model the default correlation arises through the correlation of the asset values of the various issuers. The correlation structure is defined using a specific choice of copula function. A copula is a function which specifies how the joint default dependence structure depends on the marginal default distributions of the individual issuers. We can therefore write the joint default probability as a function of the individual default probabilities and the parameters of the joint distribution of the asset values, i.e.

$$P_{AB} = \Phi(P_A, P_B, \Gamma) \tag{10}$$

where $\Phi(x, y, \Gamma)$ is the cumulative density function and Γ is a vector of parameters, including the asset value correlation, which depends on the type of copula function used. In particular, choosing a normal copula implies that $\Phi(x, y, \rho)$ is the cdf of the bivariate normal distribution with correlation coefficient ρ .

Within the Merton-style framework, asset values are represented by diffusion processes, and their instantaneous correlation is the same in the real-world measure as it is in the risk-neutral measure. We can therefore write

$$\tilde{P}_{AB} = \Phi(\tilde{P}_A, \tilde{P}_B, \rho) \tag{11}$$

For reasons described above, it is generally found that the risk-neutral single-name default probabilities exceed the real-world default probabilities by an amount that consists of risk premia and other components. Let us therefore assume that the ratio of the risk-neutral to real-world default probabilities is given by *k*, where *k>1*, and that this is the same for both assets, i.e. $P_A = k\tilde{P}_A$ and $P_B = k\tilde{P}_B$. We can then write

$$C_{FTD} = \frac{k\left(\tilde{P}_A + \tilde{P}_B\right) - \Phi(k\tilde{P}_A, k\tilde{P}_B, \rho)}{\tilde{P}_A + \tilde{P}_B - \Phi(\tilde{P}_A, \tilde{P}_B, \rho)} \tag{12}$$

At zero correlation we have $\Phi(\tilde{P}_A, \tilde{P}_B, 0) = \tilde{P}_A\tilde{P}_B$ and $\Phi(k\tilde{P}_A, k\tilde{P}_B, 0) = k^2\tilde{P}_A\tilde{P}_B$. These terms are small and so, to first order, the coverage ratio for a first-to-default is approximately equal to *k*. In the limit of high correlation we have $\Phi(\tilde{P}_A, \tilde{P}_B, \rho) = \mathrm{Min}[\tilde{P}_A, \tilde{P}_B]$, and $\Phi(k\tilde{P}_A, k\tilde{P}_B, \rho) = k\,\mathrm{Min}[\tilde{P}_A, \tilde{P}_B]$ so that $C_{FTD} = k$ exactly. Hence the coverage ratio for a first-to-default is very close to *k* for all values of correlation.

For second-to-default baskets the coverage ratio is given by

$$C_{STD} = \frac{\tilde{P}_{AB}}{P_{AB}} = \frac{\Phi(k\tilde{P}_A, k\tilde{P}_B, \rho)}{\Phi(\tilde{P}_A, \tilde{P}_B, \rho)} \quad (13)$$

This has a significant dependence on the correlation between asset A and asset B. Let us take the case of zero correlation. In this case,

$$\Phi(k\tilde{P}_A, k\tilde{P}_B, 0) = k^2 \tilde{P}_A \tilde{P}_B \quad (14)$$

and

$$\Phi(\tilde{P}_A, \tilde{P}_B, 0) = \tilde{P}_A \tilde{P}_B \quad (15)$$

so that the spread coverage ratio is then given by

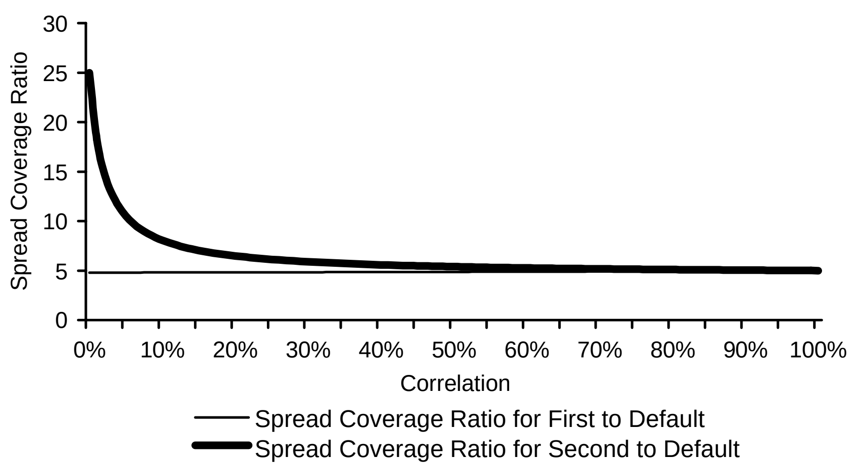
$$C_{STD}(\rho = 0) = k^2 \quad (16)$$

What is happening is that the spread coverage ratio of each asset in a second-to-default is being squared because at zero correlation the probability of two defaults is the product of the two default probabilities. This effect significantly enhances the coverage ratio of a second-to-default. In the other limit of maximum correlation, both assets will tend to default together with a probability equal to $\text{Min}[P_A, P_B]$ so that the second-to-default is effectively a first-to-default and we get

$$C_{STD}\left(\rho = \rho_{MAX}\right) = k \quad (17)$$

Between these two limits we can plot the coverage ratio as a function of the asset correlation as shown below in Figure 1.

Figure 1. **Spread coverage ratios for first and second to default baskets on A and B where k=5 and Pa=Pb=10%.**



We see clearly that the spread coverage ratio for the first to default basket is a very flat line at a value very close to 5, the chosen value for k. This agrees with the analysis above. We also see that the spread coverage ratio for the second-to-default basket begins at a value of k^2=25 for low correlation, and then falls to k=5 as the correlation increases.

For more assets we need to know about the probability of all of possible joint default events. We need to move to a proper model of correlated default that should also take into account the effect of interest rates and the term structure of credit spreads. We do this in the next section. However, despite the simplistic nature of this analysis, the results derived here apply just as much as they do to the more sophisticated model used in the next section.

A General Model of Correlated Default

Clearly the most important driver of basket spreads is the credit quality of the underlying assets. Besides this, the next most important element is the degree of dependence that is assumed between the individual issuers. What we need is a statistical model which allows us to assign probabilities to joint default events given the individual default probabilities.

An elegant and increasingly popular way of modelling the joint dependency of the default times of the issuers in a basket is via copula functions. The advantage of copula functions is that they allow us to disentangle the modelling of the dependence structure of the different default times from that of their individual distributions. For any choice of copula function, we obtain a valid model for the joint distribution of the issuer default times that allows us to evaluate a default basket. Popular copula functions include the normal and the student-t copula, they are treated in detail in the article by Mashal and Naldi, also in this issue. Once we have chosen a copula function, we can easily evaluate default baskets by generating issuer default times in a Monte Carlo simulation.

Since we know the times of defaults, we can easily take into account the present valuing of any cashflows. We can then work out the breakeven basket spread as the spread which equates the present values of the premium and protection legs, taking into account the fact that the spread leg terminates once the basket has been triggered.

Rating Based Analysis

Using a model of the type described in the previous section, we can analyse the pricing of portfolios of two or more assets, taking into account a full dependency structure as well as the effect of interest rates. In what follows, we consider collateral with a rating of A3. On average, 5-year protection on an issuer with this rating will be trading around 130 bp in the default market. For our comparison, we use the average default rates per rating as supplied by Moody's, c.f. Table 1.

Based on a credit curve constructed from these historical default rates, and assuming an average recovery rate of 45%, the 5-year default swap spread for an

A3 rated issuer would be 4 bp. In other words, for A3 rated credits, the default swap market is on average demanding an excess spread of 130-4=126 bp and a spread coverage ratio of 130/4=32.5. It is important to note that both of these numbers must be seen as a function of the issuer's credit quality. The spread coverage ratio in particular declines markedly as we go down the rating scale.

We consider a default basket consisting of five A3 rated issuers. The average default swap spread equals 130bp. Using a model for the issuer dependence such as we have described above, we can calculate the first and second-to-default spreads as a function of the issuer correlation. This is assumed to be the same for all issuer pairs. The basket spreads using market inputs are shown in Figure 2, while Figure 3 shows the basket spreads obtained if we use issuer spreads based on historical default rates.

In both cases, we note the typical correlation dependence of the basket spreads. The first-to-default (FTD) spread is decreasing in the issuer-issuer correlation - as correlation increases, joint events become more likely as does the probability of all of the assets surviving. On the other hand, the second-to-default (STD) spread is only mildly dependent on it, starting at a low spread at low correlations and increasing with correlation as joint defaults become more likely. In our comparison of baskets with individual assets, we use a correlation of 60%, which we regard as being conservative.

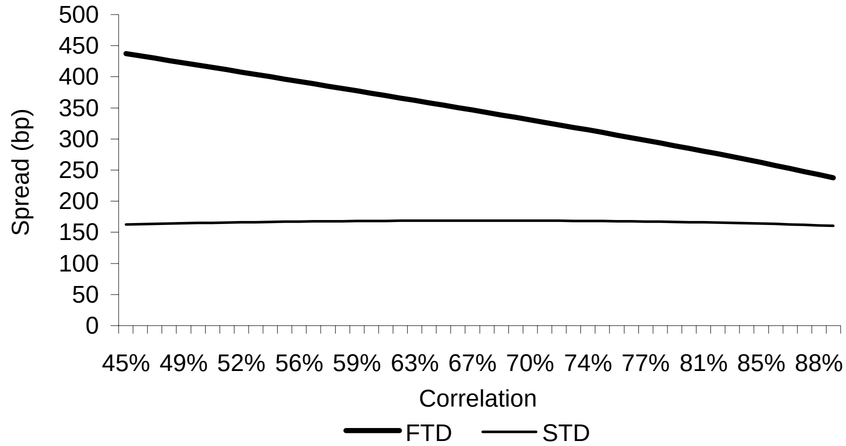
Using an issuer correlation of 60%, the FTD spread based on market spreads is 375 bp, whereas the spread based on historical default rates is 15 bp. This gives an excess spread of 375-15=360 bp and a ratio of 375/12=25. To compare this to a single-name default swap, we must look to an asset that pays a spread of 375 bp. We estimate that an issuer paying this amount of spread should be rated around

Table 1. **Average cumulative default rates by rating from 1 to 10 years for the period 1983 – 2000**

Rating	1	2	3	4	5	6	7	8	9	10
Aaa	0.00%	0.00%	0.00%	0.06%	0.18%	0.25%	0.34%	0.43%	0.43%	0.43%
Aa1	0.00%	0.00%	0.00%	0.21%	0.21%	0.35%	0.35%	0.35%	0.35%	0.35%
Aa2	0.00%	0.00%	0.06%	0.18%	0.41%	0.49%	0.59%	0.71%	0.85%	1.01%
Aa3	0.06%	0.09%	0.17%	0.26%	0.37%	0.49%	0.49%	0.49%	0.49%	0.49%
A1	0.00%	0.03%	0.30%	0.47%	0.59%	0.73%	0.79%	0.86%	0.86%	0.96%
A2	0.00%	0.02%	0.16%	0.41%	0.62%	0.84%	0.99%	1.35%	1.63%	1.71%
A3	0.00%	0.12%	0.22%	0.30%	0.35%	0.47%	0.68%	0.77%	0.97%	1.09%
Baa1	0.07%	0.30%	0.53%	0.86%	1.19%	1.43%	1.82%	2.05%	2.20%	2.20%
Baa2	0.06%	0.29%	0.61%	1.22%	1.89%	2.54%	2.93%	3.17%	3.46%	3.81%
Baa3	0.39%	1.05%	1.62%	2.47%	3.15%	4.09%	4.99%	5.95%	6.54%	7.03%
Ba1	0.64%	2.10%	3.81%	6.15%	8.12%	10.09%	11.43%	12.75%	13.35%	14.08%
Ba2	0.54%	2.44%	4.95%	7.32%	9.27%	10.88%	12.59%	13.60%	14.27%	14.71%
Ba3	2.47%	6.82%	11.68%	16.18%	20.63%	24.74%	28.39%	32.28%	35.83%	38.22%
B1	3.48%	9.71%	15.59%	20.56%	25.62%	30.78%	36.15%	40.30%	44.16%	48.01%
B2	6.23%	13.70%	20.03%	24.63%	28.24%	31.14%	32.73%	34.33%	35.03%	35.90%
B3	11.88%	20.18%	26.71%	31.95%	36.68%	39.89%	42.81%	46.80%	51.42%	53.53%
Caa1-C	18.85%	28.29%	34.51%	40.23%	43.42%	46.48%	46.48%	49.73%	53.92%	59.04%

Source: Moody's Investor Services

Figure 2. **Basket spreads for five A3 issuers with market spread inputs**



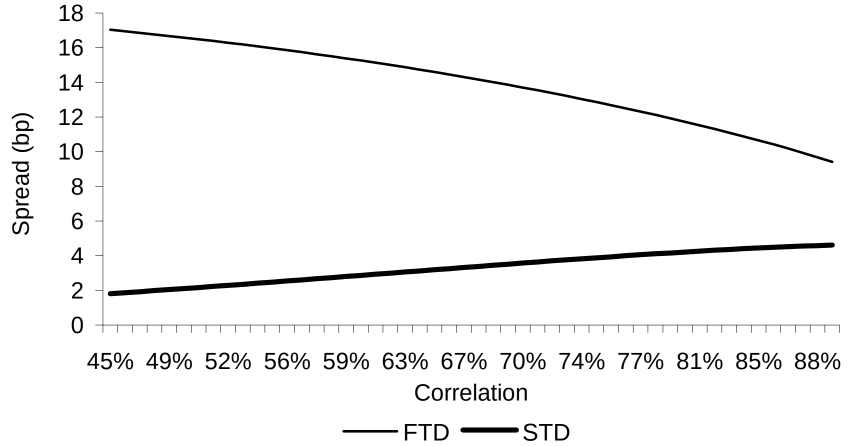
Baa3. Using historical default statistics, such an issuer provides an expected loss of 35bp. The excess spread for the single-name is therefore 375-35=340 bp, and the spread coverage ratio is only 375/35=11. **Therefore, in order to earn the same spread as from the first-to-default basket on investment grade names, the investor has to take exposure to a sub-investment grade asset, and furthermore, this asset offers a significantly lower spread coverage ratio as well as less excess spread.**

According to the data in Table 1, an issuer with the same real-world spread of 15 bp as the basket would be rated Baa1. On average, default protection on this type of issuer trades around 200 bp, giving an excess spread of 200-15=185 bp and a spread coverage ratio of 200/15=13.3. We see that the basket dominates the single asset in both metrics.

Let us now turn to the second-to-default basket. Again using an issuer correlation of 60%, the market STD spread is 168 bp, while the STD spread based on historical default rates is merely 3bp. This gives an excess spread of 168-3=165 bp and a spread coverage ratio of 168/3=56. The STD basket clearly benefits from the diversification in the issuer pool. It pays a slightly higher spread than the individual issuers in the pool, while offering a greater coverage ratio. The size difference between the coverage ratio of the second-to-default basket and an individual asset must be taken with caution, because the real-world spreads are so low. However, the second-to-default basket position has the additional safety cushion that at least two assets have to default before there is a loss.

In other words, FTD baskets are suited to investors trying to maximize the earned spread for a given level of coverage ratio, while STD baskets are suited to investors looking to earn an investment grade spread while maximizing their coverage ratio.

Figure 3. **Basket spreads for five A3 issuers with spread inputs based on historical default rates**



As the basket spread is a function of the spreads of the issuers in the pool, the obvious way to increase the spreads that can be earned is to have assets of lower credit quality in the pool. Alternatively, we can avoid exposure to sub-investment grade assets by increasing the leverage of the basket positions. This is achieved by increasing the size of the issuer pool that the basket payoffs reference. We therefore consider a pool of ten A3 rated issuers. The basket spreads are shown in Figure 6 and Figure 7, while the spread ratios are shown in Figure 8. Clearly, the larger basket means that the spreads are significantly higher than in the five-issuer case. As before, we examine the 60% issuer correlation case. The FTD spread using market inputs is 531 bp, while the spread based on historical default rates is 26 bp. This gives an excess spread of 531-26=505 bp and a spread coverage ratio of 531/26=20. The FTD position on a basket of ten A3 issuers pays a

Figure 4. **Ratio of basket spreads for five A3 issuers**

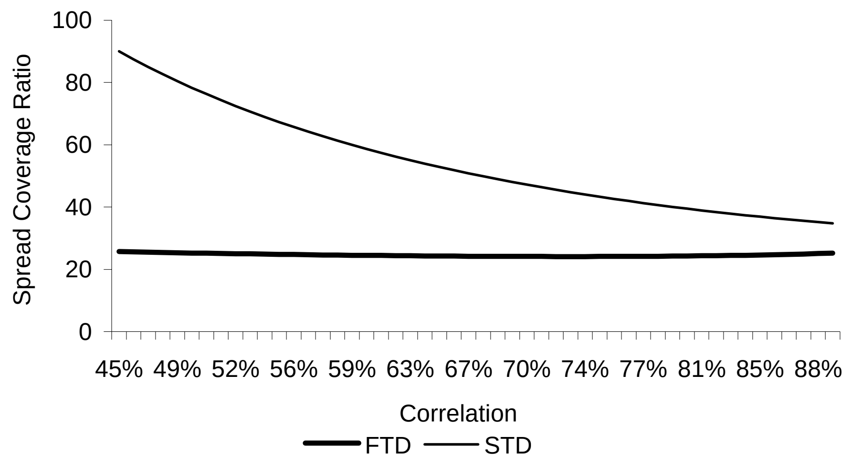
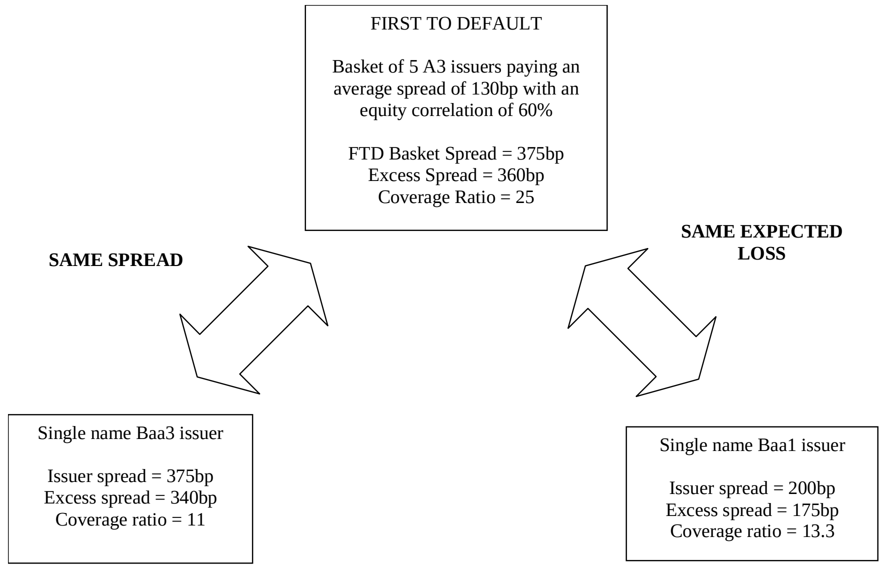


Figure 5. **A comparison of a first-to-default basket with an asset which pays the same spread and one which has the same expected loss**



high-yield type spread while still offering a better spread coverage ratio than the Baa3 asset we examined earlier.

The STD spreads are 295 bp using market inputs and 7 bp based on historical default rates, giving an excess spread of 295-7=288 bp and a spread coverage ratio of 295/7=42. We see that once again, for the spread it pays, the STD basket position maximizes the coverage ratio.

Figure 6. **Basket spreads for ten A3 issuers with market spread inputs**

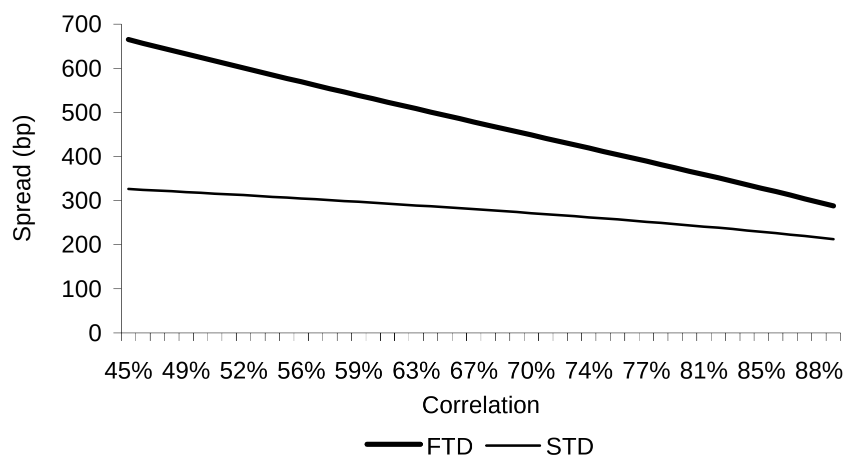
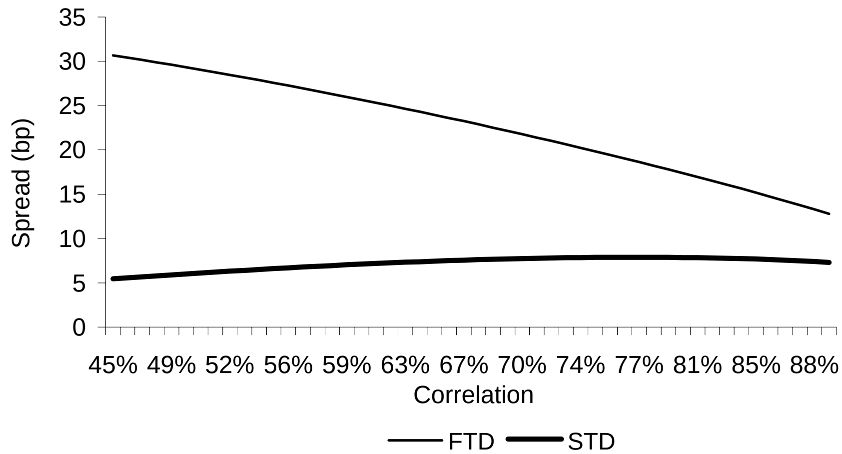


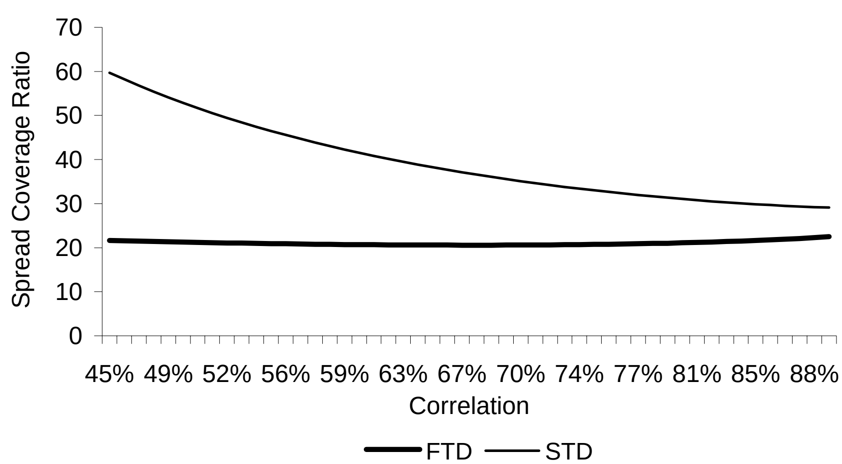
Figure 7. **Basket spreads for ten A3 issuers with spread inputs based on historical default rates**



Conclusions

Baskets provide a third way to leverage credit risk premia - the other two ways being to drop down the credit spectrum and so increase the likelihood of default, or to drop down the capital structure and so to risk losing more in the event of a default. First-to-default baskets leverage credit risk premia by exposing the investor to the first default in a pool of credits. While this has the effect of increasing the likelihood of a default event, it does so by exposing the investor to liquid issuers which are typically investment grade quality and which are well-known to the credit investor. Indeed the customisable nature of baskets means that an investor has the ability to select exactly which credit names are used. It also allows the investor to express a view about the default correlation between the assets in the pool.

Figure 8. **Spread coverage ratio of basket spreads for ten A3 issuers**



In this piece we have shown that default baskets, be they first or second-to-default, provide new ways for investors to leverage the credit risk premia embedded within credit spreads. Whether first or second-to-default should be chosen depends very much on the investor's risk appetite. Those seeking high spreads should find first-to-default baskets superior to single name assets since they leverage the excess spread more than single-name assets with the same spread. For more risk-averse investors, second-to-default baskets maximise the ratio of spread paid per unit of expected loss when compared to single name assets.

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[3] Basket Default Swaps and the Credit Cycle, Marco Naldi, Quantitative Credit Research Quarterly, June 30, 2001

[4] Extreme Events and the Valuation of Multi-name Credit Derivatives, Marco Naldi and Roy Mashal, This issue of the Quantitative Credit Research Quarterly

EXTREME EVENTS AND THE VALUATION OF DEFAULT BASKETS

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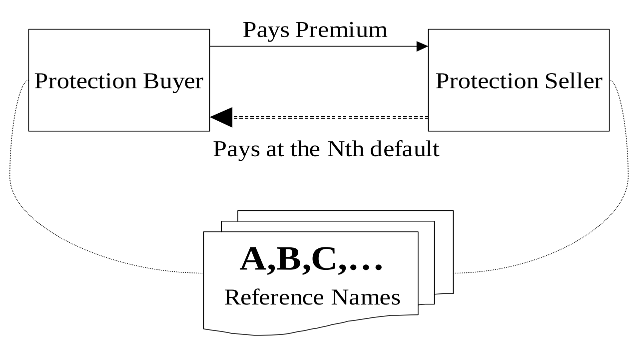
Introduction

Specifying an appropriate model for dependent defaults is the core problem in the valuation of multi-name credit derivatives. Different dependence structures produce different default distributions, which in turn affect the valuation of products such as CDO tranches and basket default swaps.

Several credit models rely on Merton's (1974) idea that a firm defaults when its asset value drops below its liabilities. A straightforward extension of this framework to the multivariate case implies that the dependence structure of defaults is determined by the assumed joint distribution of asset returns. Many existing portfolio models assume multivariate normality as the underlying joint distribution of asset returns, even if the normal distribution is clearly not compatible with the extreme joint realizations that we observe in the data. The market turmoil spurred by the tragic events of September 11 is reminding us once more that extreme joint movements do happen more often than the normal distribution would predict.

In this article we introduce POPSTAR (POrtfolio Pricing Simulator with TAil Relations), a simulation-based pricing model which accounts for extreme events by means of a more realistic distributional assumption. After describing the properties of its simulation engine, we discuss how to make the model operational, with particular reference to the estimation of the parameters that determine the likelihood of extreme events. Finally, we show the pricing impact of taking extreme events into consideration. For the purpose of this discussion, we focus our attention on basket default swaps, since the valuation of these contracts is particularly sensitive to the dependence structure of defaults.

Figure 1. ***Nth*-to-Default Basket Swap**



1 For more on *Nth*-to-default swaps and other credit derivatives, see O'Kane and Schloegl (2001).

In an *Nth*-to-default basket swap, two counterparties agree on a maturity and a set of reference assets, and enter into a contract whereby the protection seller periodically receives a premium (also called "basket spread") from the protection buyer. In exchange, the protection seller stands ready to pay the protection buyer par minus recovery of the *Nth* referenced defaulter in the event that the *Nth* default occurs before the agreed-upon maturity. First- and second-to-default swaps are the most popular orders of protection.[1]

Default Correlation and the Distribution of Asset Returns

Default correlation measures the tendency of two credits to default jointly within a specified horizon. Formally, it is defined as the correlation between two binary random variables that indicate defaults, i.e.

$$\rho_D = \frac{p_{AB} - p_A p_B}{\sqrt{p_A(1-p_A)}\sqrt{p_B(1-p_B)}} \tag{1}$$

where p_A and p_B are the marginal default probabilities for credits *A* and *B*, and p_{AB} is the joint default probability. Of course, p_A , p_B and p_{AB} all refer to a specific horizon. Notice that default correlation increases linearly with the joint probability of default and is equal to zero if and only if the two default events are independent.

Default correlations are the fundamental drivers in the valuation of multi-name credit derivatives. Unfortunately, the scarcity of default data makes joint default probabilities, and thus default correlations, very hard to estimate directly. As a result, researchers have developed alternative methods to calibrate the frequency of joint defaults within their valuation models.

One way to simulate correlated defaults relies on the use of copula functions. Generally speaking, copulas are used to link marginal and joint distribution functions. Starting with an assumption for the marginal distributions of default times, a copula function can be employed to obtain their joint distribution. This can then be used as the probability law underlying a very efficient time-to-default simulation. This procedure is extremely useful for the valuation of multi-issuer credit derivatives, since we can extract (risk-neutral) marginal default probabilities from liquid single-name products, and then value multi-name contracts by simulating correlated default times.

To illustrate this point, consider two credits *A* and *B*, whose default times T_A and T_B are exponentially distributed with hazard rates h_A and h_B. A joint distribution that correlates T_A and T_B while respecting their marginals can be obtained by means of a bivariate **normal copula**

$$F_N(x,y) = P(T_A < x,\, T_B < y) = \Phi_2(\Phi^{-1}(E_{h_A}(x)),\Phi^{-1}(E_{h_B}(y)),r), \tag{2}$$

where $\Phi_2(\cdot, \cdot, r)$ denotes a bivariate standard normal distribution with correlation r, $\Phi(\cdot)$ is a univariate standard normal distribution, and $E_h(\cdot)$ is an exponential distribution with hazard rate h. Taking limits, it is straightforward to verify that this joint distribution is perfectly legitimate in that it respects the exponential marginals that we started with.

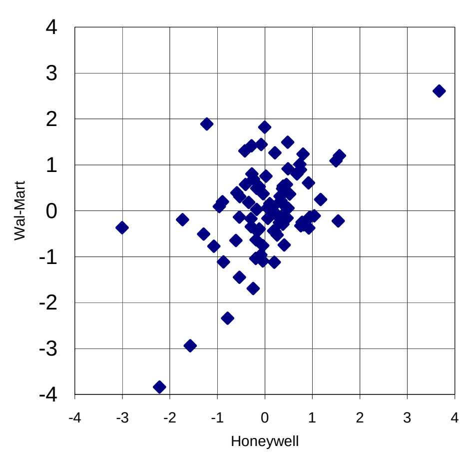
Li (2000) and Nyfeler (2000) show that several existing portfolio models generate a dependent structure of defaults that is in fact equivalent to the one produced by a normal copula. Notice that the framework described by equation (2) can be reinterpreted as a variation of Merton's model where:

1) default is caused by a firm's normally distributed asset return falling below the threshold $\Phi^{-1}(E_h(\cdot))$, and
2) the parameter r represents the correlation between the jointly normal asset returns of the obligors.

This structural interpretation is useful for two reasons. First, it simplifies calibration by relating the copula parameter r to the correlation between asset returns. This offers a clear advantage since reasonable proxies for asset correlations can be computed from observable equity data. Second, it reveals that the choice of a particular copula for survival times may be related to an implicit distributional assumption for asset returns.

The assumption of normality of asset returns is certainly not innocuous, since a multivariate normal distribution does not allow for extreme joint events to happen with the frequency that the data suggest. Figure 2 shows a bivariate scatterplot of standardized equity returns using 7 years of monthly data (August 94 - July 01). To the extent that equity returns proxy for asset returns, this figure highlights the major problem with the normality assumption. According to the normal distribution, the

Figure 2. **Standardized Monthly Returns**



most extreme joint realizations in this plot have a likelihood of happening of the order of one in a hundred thousand; yet, we observe them in a sample of 84 points. The ability of a multivariate distribution to accommodate joint extreme events can be related to the concept of "tail dependence". Formally, for two random variables X and Y with marginal distributions F_X and F_Y, (lower) tail dependence is defined as:

$$\lambda = \lim_{u \to 0^+} P\left(Y < F_Y^{-1}(u) \middle| X < F_X^{-1}(u)\right). \tag{3}$$

In words, tail dependence measures the probability that Y will have a realization in the tail of its distribution given that X has had a realization in the tail of its own. The problem with the multivariate normal distribution is that λ is identically equal to zero. Another manifestation of the same problem can be seen in the thinness of the tails of a multivariate normal density, which implies that there is very little probability mass on extreme joint events.

POPSTAR allows for "fat-tailed" returns by means of a non-normal assumption. It does so by joining the marginal distributions of default times through a copula function that introduces extreme events while retaining the structural interpretation described above. The latter point is important for practical purposes because it permits relating it to the copula parameters to the distribution of asset returns.

More precisely, POPSTAR obtains the joint distribution of default times by joining the exponential marginals $E_h(\cdot)$ with the ***t*-copula**

$$F_t(x, y) = P(T_A < x,\ T_B < y) = t_{2,v}(t_v^{-1}(E_h(x)), t_v^{-1}(E_h(y)), r), \tag{4}$$

where $t_{2,v}(\cdot, \cdot, r)$ is a bivariate standard t distribution with v degrees of freedom and correlation r , and $t_v(\cdot)$ is a univariate standard t distribution with v degrees of freedom.

One can immediately see that the structural interpretation of default is maintained. In fact, the choice of a t-copula for default times may be viewed as an implicit assumption that asset returns follow a **multivariate *t* distribution**. Since two jointly $t_{2,v}$ variables are marginally t_v distributed, one can immediately interpret $t_v^{-1}(E_h(\cdot))$ as a default threshold and $t_{2,v}(\cdot, \cdot, r)$ as the joint distribution of asset returns.

Not all copula functions lend themselves to a structural interpretation. Lindskog (2000) and Schonbucher and Schubert (2001), among others, study different forms of dependence such as Clayton and Gumbel copulas. These are perfectly legitimate ways to join the marginal distributions of default times and introduce tail dependencies. However, from a practical point of view, it is not clear where to look for reasonable estimates of the copula parameters.

There is an impressive amount of literature documenting the non-normality of equity returns (see for example Blattberg and Gonedes (1974)). When modeling the joint distribution of equity returns, a multivariate t fits the data much better than a multivariate normal. We will formally show this in the next section. Compared with a normal, a t distribution has an additional parameter, the number of degrees of freedom. As this number goes to infinity, a t distribution tends to a normal distribution; i.e., it displays no tail-dependence. But for a finite number of degrees of freedom, a t distribution allows for extreme joint realizations.

Figure 3 shows the values of tail dependence λ for a bivariate t distribution as a function of the correlation coefficient ρ and the degrees of freedom ν. The last row refers to the normal case $(\nu=\infty)$. To understand the meaning of these numbers, consider the case where the distribution is characterized by 10 degrees of freedom and 30% correlation. In this scenario, knowing that the return of the first name has had an extremely negative realization leaves us with a 3.31% probability that the return of the second name will also have an extremely negative realization. This is very different from the prediction of a jointly normal distribution. In the last section of this article, we will quantify the pricing impact of this difference.

Figure 3. **Tail Dependence (λ)**

ν / ρ	**-0.5**	**0**	**0.3**	**0.5**	**0.9**	**1**
3	0.0257	0.1161	0.2161	0.3125	0.6701	1
10	0.0001	0.0068	**0.0331**	0.0819	0.4627	1
15	3e-6	0.0010	0.0097	0.0346	0.3724	1
20	9e-8	0.0002	0.0029	0.0151	0.3051	1
∞	0	0	0	0	0	1

The relation between the tail dependence of asset returns and the dependence of default events is depicted in figure 4. Using a 5-year horizon and two credits with constant hazard rates of 1%, this graph compares a normal copula and a t-copula with 3 and 10 degrees of freedom. Tail dependence increases default correlation for any value of asset correlation. In particular, notice that even when asset returns are uncorrelated (i.e. linearly independent), tail dependence can produce a significant amount of default correlation.

Estimation and Testing

To simulate default times from the joint distribution specified in equation (4), we need to estimate its parameters. Using time series of equity returns to proxy for asset returns, we can estimate these parameters by maximum likelihood. Moreover, since a multivariate t distribution tends to a normal distribution as the number of degrees of freedom increases (i.e. $F_t \to F_N$ as $\nu \to \infty$), the normality

Figure 4. **Correlation**

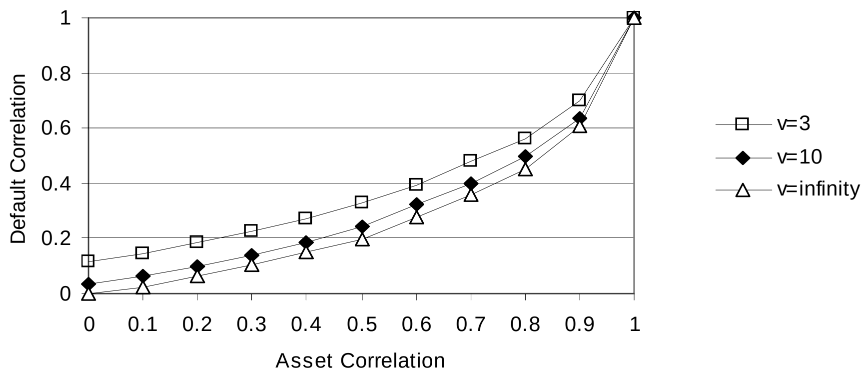


Figure 5. **Basket Examples**

Basket	**Reference Names**
#1	Fleet Boston, AT&T, IBM, Eastman Kodak.
#2	Honeywell, BellSouth, Wal-Mart, First Data
#3	Union of the names in baskets 1 and 2

assumption is actually nested in our framework as a special case. Therefore, we can carry out likelihood ratio tests for the null hypothesis that returns are jointly normal. For the analysis in this section and the next, we consider the baskets described in figure 5.

Using 7 years of monthly data (Aug 94 – Jul 01) and assuming serial independence of returns, figures 6-8 plot the likelihood of the multivariate return processes for these three baskets as a function of the degrees of freedom. The maximum likelihood estimates are shown to be 9, 7 and 9, respectively, which signal the presence of a significant amount of tail fatness in the data. To confirm the inadequacy of the normal distribution, the same figures also report the p-values of the likelihood ratio tests for the null hypothesis of normality $(\nu = \infty)$. They suggest that we can reject normality with an infinitesimal probability of making a mistake. This result is by no means specific to the chosen examples: we have obtained analogous results with dozens of different portfolios.

Extreme Events and Valuation

Taking extreme events into account has significant consequences on the valuation of multi-name credit derivatives. Other things equal, simulating defaults by means of a fat-tailed copula increases the probability of joint defaults and, therefore, default correlations as defined in equation (1).

Figure 6. **Basket #1 Log-Likelihood Function**

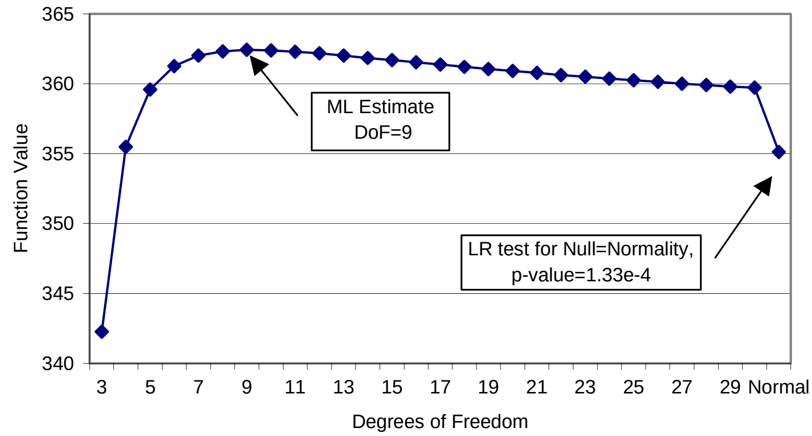


Figure 7. **Basket #2 Log-Likelihood Function**

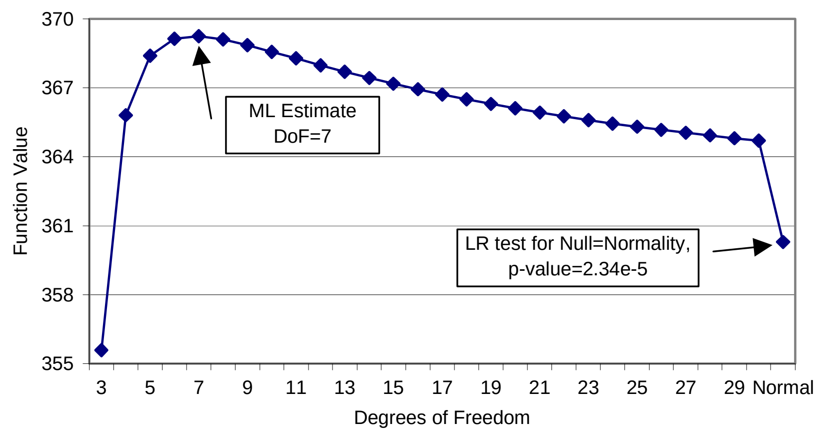


Figure 8. **Basket #3 Log-Likelihood Function**

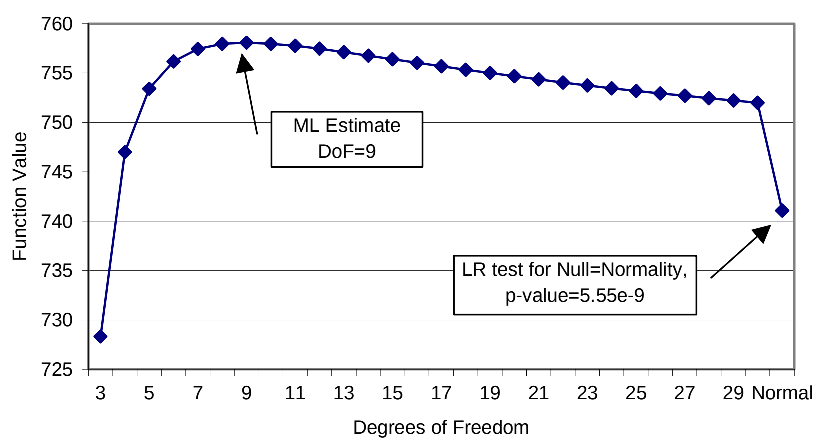


Figure 9 reports pricing results for first- and second-to-default swaps on the baskets described in figure 5. All deals have a maturity equal to 3 years and have been priced using information (LIBOR curve, market-implied hazard rates, estimated parameters of *t* copula) available as of the end of July 2001. Premia are in basis point and percentage standard errors are reported in brackets.

Figure 9. **Valuation of 3-year Default Baskets (as of the end of July 2001): Normality vs. POPSTAR**

Basket		Premium (StdErr)	
		1st-to-default	2nd-to-default
	Normal	236 (0.28%)	29 (0.75%)
#1	POPSTAR	224 (0.26%)	36 (0.67%)
	Diff	-5%	24%
	Normal	140 (0.34%)	12 (1.15%)
#2	POPSTAR	129 (0.35%)	19 (0.92%)
	Diff	-8%	58%
	Normal	350 (0.29%)	64 (0.71%)
#3	POPSTAR	316 (0.31%)	76 (0.66%)
	Diff	-10%	19%

Recall from the previous section that accounting for extreme events increases default correlations. As we showed in a previous issue of this *Quarterly* (see Naldi (2001)), the sign of the relation between basket premia and default correlations depends on the order of the basket. The value of first-to-default protection is always monotonically decreasing in default correlations. Therefore, when we allow for joint extreme events, first-to-default protection gets cheaper. The value of second-to-default protection is not necessarily monotonic in default correlations. Rather, it generally increases up to a certain point, then it becomes decreasing. The location of this turning point depends on all other parameters and, in particular, on the number of names in the basket. With a low number of names, second-to-default protection is generally increasing in default correlations over most of the domain. Intuitively, with only a handful of names in the portfolio, the only way that two of them can default within 3 years is if they have a significant tendency to default together. That explains the results we observe in figure 9 regarding second-to-default valuation.

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