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INTRODUCTION

The first half of 2002 will probably go down in financial history as being one of the most challenging credit environments that investors have faced in a long time. The market witnessed extreme volatility in credit spreads of particular names and sectors and a stunning descent of some investment grade credits. This has sharply increased the focus on analytical tools and research studies that can help in the measurement and management of risk in credit portfolios. Consequently, in this issue of our *Quantitative Credit Research Quarterly*, we present a number of articles addressing important questions relating to the risk management of cash bond portfolios as well as of portfolios of structured credit products.

In the first article, we present the framework underlying our new risk model for the U.S. Investment Grade Credit Universe. This continues a tradition that Lehman Brothers has established in providing investors with state-of-the-art tools for managing fixed income portfolios. For over 10 years now, our flagship tool for portfolio management has been the Lehman Brothers multi-factor U.S. risk model. We are now in the process of introducing a new generation of risk models for the U.S. as well as the Euro markets. While the new framework retains the flexibility and intuitive appeal of the current methodology, we introduce a number of important innovations. These include a more accurate way of splitting bond returns, an improved method of measuring interest rate and volatility risk and a better specification for spread and non-systematic risk. Also, robust statistical techniques are used for parameter estimation which are likely to produce more reliable results when faced with data containing extreme observations such as we have experienced in the last few months. A beta version of the new risk model covering the Treasury, Agency and Credit Indices is currently available on our portfolio analytics system, POINT, and is being successfully used by a number of portfolio managers.

The second article addresses the risk management issues faced by investors in structured credit products. Here, we introduce a methodology for measuring and managing obligor and common factor risk in the mezzanine and equity tranches of collateralized debt obligations (CDOs). Though CDOs form a relatively young asset class, recent years have seen an enormous growth in their issuance as investors look for longer-term diversified investments in credit markets. While CDOs are typically buy-and-hold investments, investors still need to monitor their risk exposures to particular names and sectors on a continuous basis. Such monitoring is necessary, for example, if investors want to supplement their CDO investments with credit protection against particular names or if they want to trade their investments in the secondary markets. This article presents a comprehensive methodology that is generally consistent with the active management of cash credit portfolios. Thus, investors who combine cash and CDO positions in their portfolios will find this approach especially useful.

While the first two articles address issues arising in the management of *market* risk inherent in credit investments, the third article of this issue provides a model for analyzing *counterparty* risk for buyers of default protection through default swaps. In these contracts, the protection buyer is hedged against default of the underlying credit only if the counterparty does not default first. This article proposes a simple method for quantifying the risk of counterparty default. The methodology is applicable also at a portfolio level, where it provides a conservative estimate of risk per counterparty.

Risk management of credit portfolios is not a matter of just coming up with new analytical frameworks for measuring the magnitude of and the exposure to a variety of risk factors. The process of risk management also benefits from the advent of new derivative instruments that allow investors to transfer unwanted risk exposures to others who are less averse to such risks. An example of such an instrument that should be of great interest to global credit investors is presented in the fourth article of this issue, which explains how global investors can use cross-currency asset swaps (in particular an innovative structure called *the perfect asset swap)* to remove currency risk from global credit investments. By removing unwanted currency risk, such instruments are likely to ease the task of capitalizing on relative value opportunities provided by differential spreads at which the same credit trades in different currencies. Moreover, the advantage of the diversification afforded by global credit investing is only likely to be fully harnessed once the instruments proposed here come into regular use.

The last article moves beyond risk management issues. Here we address the question of how the investor decides if, at a given time, there is enough compensation for risk-bearing in particular credit sectors. In this article, we present an econometric methodology useful for estimating the ex-ante Sharpe ratios or prices of risk (defined as the expected excess return on an asset class per unit of volatility of excess returns) for different segments of the credit market. We call this model, *PRISM* (the *P*rice of *Ris*k *M*odel). In our model, the mean excess return and its volatility change over time, volatility is mean-reverting and estimated from observed bond price fluctuations, and the mean excess return is related to a set of predictive economic variables. Estimates of a price of risk measure from the model can be useful in such decisions as whether to overweight or underweight stocks versus bonds, Treasuries versus corporate bonds, or BBB-rated corporate bonds versus A-rated bonds. They could also be useful in determining the relative weighting of various industry sectors. We show by means of some examples how one can apply the *PRISM* framework for active investment strategies involving market timing and sector selection.

The New Lehman Brothers Credit Risk Model

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We describe the new Lehman Brothers Risk Model as it applies to the U.S. High-Grade Credit universe. The discussion focuses on our choices for splitting total return and modeling its stochastic components. We explain how to use this framework to estimate the Tracking Error Volatility of a given portfolio vs. a pre-specified benchmark, and offer some out-of-sample evidence on the performance of our estimator. Finally, we illustrate some of the Risk Model output available on POINT, our new portfolio analytic system.

1. INTRODUCTION

During the past decade, the Lehman Brothers Multi-Factor Risk Model (for the latest update, see Dynkin, Hyman and Wu [1999]) has served as an effective risk management tool for an increasing number of portfolio managers. The common practice of constraining asset allocation choices with the assignment of predetermined benchmarks has created a need for quantitative measurements of relative risk. As a result, the risk model's coverage of a variety of asset classes and its ability to measure the deviation risk between a portfolio and a pre-assigned benchmark have made it a unique tool for managers who need to track the return of a highly diversified index.

These days, credit managers are paying more attention to the importance of an adequate level of portfolio diversification. The current spread volatility and significant tail-risk are making the management of benchmarked positions harder than ever. A unique feature of our model is its ability to decompose systematic and non-systematic deviation risk, thus allowing for a quantitative evaluation of the benefits that a given position may derive from an increased level of diversification. We believe the utilization of these capabilities, combined with the employment of other quantitative tools, will become increasingly important for credit fund managers, since the complexity and volatility of the underlying credit marketplace are likely to remain elevated.

This article describes the framework underlying the new Lehman Brothers Risk Model for the U.S. investment grade credit universe. The model is calibrated using historical observations from the IG Credit Index, and it can be applied to portfolios that include any modeled issue in our database. The new U.S. Risk Model, covering the Treasury, Agency, and Credit indices is currently available on Lehman Brothers' portfolio system POINT. We are extending the model to MBS, ABS, and CMBS in order to complete the coverage of the Lehman Brothers U.S. Aggregate Index.

Relative to the previous version of the model, we have introduced a new way of splitting bond returns, a different way of measuring interest rate and volatility risk, and different specifications for systematic and non-systematic spread risk. Moreover, robust econometric techniques have been introduced for parameter estimation. What we have retained is

[1] Several people contributed to the development and the implementation of the U.S. Risk Model, including Albert Desclee, Lev Dynkin, Hank Haligowski, Jay Hyman, Dick Kazarian, Vasant Naik, and Nancy Roth. We would like to thank Arthur Berd, Ivan Gruhl, Mark Howard, Arthur Tetyevsky, Minh Trinh and Stuart Turnbull for their comments and suggestions.

the flexible and computationally convenient framework of a linear factor model and the general calibration methodology based on cross-sectional regression analysis.

Beyond providing the user with an intuitive measure of statistical distance between a given portfolio and a pre-assigned benchmark (tracking error volatility--from now on, TEV), the model lends itself to a number of applications such as:

- Understanding detailed factor exposures for a given portfolio;
- Comparing risks associated with different views;
- Identifying trades to achieve a reduction in deviation risk;
- Creating index proxies using a relatively low number of assets;
- Decomposing risk across asset classes, allowing for risk budgeting at different levels; and
- Decomposing systematic and non-systematic risk.

In the remainder of this article, we will describe the model (section 2), test its out-of-sample performance (section 3), challenge its robustness with small-portfolio examples (section 4), and illustrate some of the reports currently available to the user (section 5).

2. THE MODEL

Our methodology relies on the itemization of a security's total return into well-defined components. The model quantifies the statistical behavior of each of these sources of return. The new Credit Risk Model improves upon the existing framework in a number of ways:

- We have redefined benchmark (Treasury) risk in an OAS-based setting. Callable securities are handled in the same way as non-callable securities. We have eliminated the use of static measures that ignore embedded options (such as "yield to worst" or "ZV duration"). We allow for non-parallel changes in the benchmark Treasury curve by using 6 key rates loaded by the corresponding key-rate durations and use option-adjusted convexity as an additional risk loading.
- We incorporate volatility risk in an OAS-based framework for all volatility-sensitive securities. This risk has taken on increasing importance as the cash and derivative markets have become increasingly integrated.
- We have significantly improved our approach to handling systematic spread risk. We explain spread return by means of an industry-rating cross effect based on a new partition of the credit universe. The spread factor realizations are estimated using a robust statistical procedure that minimizes the impact of outliers. These estimated factors are easily interpretable and explain a significant fraction of each security's spread return.

2.1. Return Splitting

From a modeling perspective, our final goal is to come up with a linear factor representation for the total return of every modeled corporate bond in our database. As a first step, we employ a proprietary pricing model to perform a historical decomposition of every monthly total return in the IG Credit Index.

We first write total return as the sum of coupon return R_coup and full-price return R_fp:

$$R\_tot = R\_coup + R\_fp$$

A pricing model can be interpreted as a function P that relates current time t, benchmark curve YC_t (Treasury par curve), implied volatility surface Vol_t, and option-adjusted spread OAS_t to the full price of the bond fp_t:

$$fp_t = P(t, YC_t, Vol_t, OAS_t).$$

We now define time return *R_time* as the portion of full-price return due solely to the passage of time:

$$R\_time_{t+1} = P(t+1, YC_t, Vol_t, OAS_t) / P(t, YC_t, Vol_t, OAS_t) - 1,$$

Analogously, we define benchmark return *R_bmk* as that portion of full-price return due solely to the movement of the benchmark curve,

$$R\_bmk_{t+1} = P(t, YC_{t+1}, Vol_t, OAS_t) / P(t, YC_t, Vol_t, OAS_t) - 1,$$

and volatility return *R_vol* as

$$R\_vol_{t+1} = P(t, YC_t, Vol_{t+1}, OAS_t) / P(t, YC_t, Vol_t, OAS_t) - 1.$$

Lastly, we approximate spread return as the residual component of this decomposition

$$R\_fp - R\_time - R\_bmk - R\_vol = R\_spread.$$

This is an approximation, because *R_spread* not only contains the portion of return due to the movement in the bond's *OAS*, but also includes all cross-effects.

Notice that *R_time* takes on a negative value over months in which a coupon is paid out, since the full-price jumps down accordingly. To obtain a smooth measure of the deterministic component of return, we define carry return a

$$R\_carry = R\_coup + R\_time.$$

Using the previous definitions, we can now represent total return as

$$R\_tot = R\_carry + R\_bmk + R\_vol + R\_spread.$$

This is the basic return decomposition that we are going to use for modeling total return risk. Since *R_carry* is known in advance, we now need to specify a linear factor representation for the remaining three stochastic components. In the next sections, we index the *ith* bond with *i* while we omit the time subscript.

2.2. Benchmark Risk

The benchmark return for a bond is defined as that portion of total return that results solely from a change in the Treasury curve over the month. It is computed by re-pricing the bond using the end-of-month yield curve, holding all other variables (time, volatility, *OAS*) fixed. The curve movement is measured using the Lehman Brothers Treasury curve, which is fitted to the prices of "off-the-run" Treasury securities.

An extremely precise approximation of *R_bmk* is given by a linear combination of 6 key-rate durations (*KRD*) multiplied by their respective par yield changes (Δy), plus the impact of a convexity factor that mimics the second-order movement of the curve:

$$R\_bmk_i \cong \sum_j KRD_{i,j} * \Delta y_j + OAC_i * \left(\overline{\Delta y}\right)^2.$$

Here, *OAC* represents the option-adjusted convexity of the bond, and $\left(\overline{\Delta y}\right)^2$ is the squared average key-rate change, which proxies for the second-order movement of the curve. Extensive empirical analysis has shown that benchmark return is best approximated by using the following six maturities on the fitted par curve as key rates: 6-month, 2-year, 5-year, 10-year, 20-year, and 30-year.

2.3. Volatility Risk

The volatility return for a bond with embedded options is that portion of total return that results solely from a change in the set of implied volatilities over the month. It is, of course, equal to zero for all bonds without embedded options.

We explain volatility return using one volatility factor, which approximates the parallel movement of the implied volatility surface. The time series of the factor realizations is extracted by running cross-sectional regressions of volatility returns on vegas for all bonds with embedded options, i.e.,

$$R\_vol_i \cong \frac{vega_i}{fp_i} * F^{vol}$$

2.4. Spread Risk

The spread return is the portion of total return remaining after time, benchmark, and volatility returns have been removed. It can be approximated by the product of the security's spread duration and its *OAS* change (ΔOAS) over the period:

$$R\_spread_i \cong -oasd_i * \Delta OAS_i$$

To specify a linear factor model for ΔOAS, we first partition the IG Credit Index into 27 cells formed by intersecting 9 industries and 3 quality groups. Each of these 27 cells has its own generic spread factor. The industry partition is chosen so that a minimum number of bonds is retained in each of the cells for every month in the sample. This ensures that the estimated factor realizations are not at any time dominated by the idiosyncratic behavior of a small number of bonds, a feature which is especially important in the current market environment. To minimize the resulting loss of information, the aggregation of certain industries relies on the merger of highly correlated groups. Figure 1 describes the cells composition and assigns them short acronyms used in some of the reports.

Figure 1. **Industry—Rating Partition**

	AAA/AA	**A**	**BBB**
Banking and Brokerage	BAN1	BAN2	BAN3
Basic Industries	BAS1	BAS2	BAS3
Consumer Cyclicals	CCY1	CCY2	CCY3
Communication and Technology	COM1	COM2	COM3
Energy and Transportation	ENE1	ENE2	ENE3
Financial Companies and Insurance	FIN1	FIN2	FIN3
Consumer Non-Cyclicals	NCY1	NCY2	NCY3
Non-Corporate	NON1	NON2	NON3
Utilities	UTI1	UTI2	UTI3

Beyond cell-specific factors, we employ a slope factor for the generic credit spread curve and an *OAS* factor, on which bonds with relatively high and relatively low *OAS* load with opposite signs ("relatively" refers to the distance from the median *OAS* of all cell peers). The *OAS* factor is meant to capture liquidity risk, since liquidity is a significant determinant of the *OAS* distribution within a given cell.

Finally, we include an additive effect for non-U.S. bonds. Specifying a set of factors for different geographical areas assumed to be homogeneous in terms of spread risk would require us to change this definition as the riskiness of different countries in the same area changes over time. Therefore, we use three non-U.S. factors corresponding to the three rating groups defined above (*AAA/AA*, *A*, *BBB*). This way, a systematic change in the creditworthiness of a given country will automatically reassign its issuers to a different non-U.S. factor.

In summary, we model the spread return of bond *i* belonging to cell c as

$$R\_spread_{i,c} = -oasd_i * [\, F_c + (tomat_i - \overline{tomat_c}) * F_{twist} + (OAS_i - \overline{OAS_c}) * F_{OAS} + F_{c,nonUS}\,] + e_i \,,$$

where , F_c =1,2,…,27 represent cell-specific factors, F_{twist} is the slope factor, F_{OAS} is the *OAS* factor, $F_{c,\, nonUS}$ denotes three non-U.S. factors (since $F_{p,\, nonUS} = F_{q,\, nonUS}$ whenever cells p and q have the same rating), *tomat* is time-to-maturity, $\overline{x}_c$ indicates the median value of variable x in cell c, and e_i represents the idiosyncratic (asset-specific) component of spread return.

We estimate the time series of the 27+1+1+3=32 factor realizations by running cross-sectional regressions, one for each month in the panel. To control for outliers and pricing errors, we employ a robust estimator designed optimally to reduce their influence on the estimated factor series. [2]

Notice that the estimated series for F_c , c =1,2,…,27 can be interpreted as the series of spread changes of generic U.S. bonds with median time to maturity and median *OAS* within their cell.

[2] We employ an *M*-estimator based on iterative generalized least squares. See Wilcox (97) for more details.

2.5. Idiosyncratic Risk

The factors described in the previous section capture the systematic portion of spread returns. The error terms e_i, on the other hand, are a source of asset-specific risk. The covariance structure of the systematic factors is going to be sufficient to describe the risk of highly diversified portfolios, since diversification is going to drive to (almost) zero the variance of the portfolio of the asset-specific components. For portfolios with a moderate number of bonds, however, idiosyncratic risk can represent a significant portion of the total TEV.

Using the panel of residuals from the factor regressions, we estimate a simple model to describe the dependence of asset-specific variance on bond characteristics. To explain the idiosyncratic variance of a bond, we use its rating, its age (the percentage of its original maturity remaining, as a proxy of liquidity), and its vega (which is going to be non-zero if and only if the bond has embedded optionalities).

2.6. Putting It All Together

In matrix notation, we can think of the new Credit Risk Model for the vector of total return *R_tot* as:

$$R\_tot_{t+1} = R\_carry_{t+1} + L_t F_{t+1} + e_{t+1},$$

where L_t denotes the matrix of all loadings (key-rate durations, vegas, spread durations, etc.) at time t and F_{t+1} is the vector of factor realizations over the period $(t,t+1)$.

Using the estimated factor series, we can compute the factor covariance Σ, and with the fitted values from the asset-specific risk model, we can construct the idiosyncratic covariance matrix Ω. The latter has mostly zeroes off the main diagonal, except for cells corresponding to pairs of bonds issued by the same ticker, in which we use the conservative assumption of unit correlation. We now have:

$$Cov_t\left(R\_tot_{t+1}\right) = L_t \Sigma L_t' + \Omega.$$

If we define the vectors

$$\begin{aligned} &\theta_P = Portfolio \quad , \theta_B = Benchmark \\ &\theta = \theta_P - \theta_B , \end{aligned}$$

then we can write the corresponding tracking error volatility as

$$\begin{aligned} TEV_t(\theta) &= \sqrt{\theta' Cov_t\left(R\_tot_{t+1}\right)\theta} \\ &= \sqrt{\theta' L_t \Sigma L_t' \theta + \theta' \Omega \theta} \\ &= \sqrt{STEV_t^2(\theta) + ITEV_t^2(\theta)} \end{aligned},$$

where the last equality highlights the decomposition into a systematic (STEV) and an idiosyncratic (ITEV) component.

The new Credit Risk Model also reports an expected outperformance. The expected total return of each bond is computed as the carry return plus a convexity term. Using vector notation, we can write the expected outperformance for the difference portfolio θ as

$$E_t(\theta' R\_tot_{t+1}) = \theta' R\_carry_{t+1} + \theta' OAC_t * E_t\left(\overline{\Delta y}\right)^2.$$

All other factors have a time-series average that is statistically insignificant and is, therefore, constrained to zero. This estimate of expected return relies on the observation that the risk premium of a given bond will be reflected in its price and, therefore, in its carry return.

In the next section, we offer two tests for the out-of-sample performance of the model. For a given portfolio-benchmark pair (i.e., for a given θ), they are both based on the standardized realized outperformance (STR) defined as

$$STR_{t+1}(\theta) = \frac{\theta' R\_tot_{t+1} - E_t(\theta' R\_tot_{t+1})}{TEV_t(\theta)}.$$

3. TESTING THE MODEL PERFORMANCE

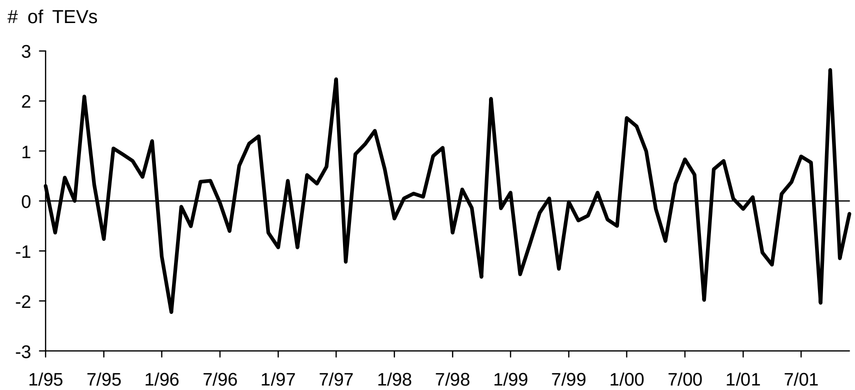
Evaluating the out-of-sample performance of a volatility estimate (such as TEV) is not an easy task because of an inconvenient property of volatility—its ex-post unobservability. Volatility is a parameter of a distribution, and, as such, it will never be "realized." Expressions such as "realized volatility" refer to a particular estimator (usually sample volatility) and not to volatility itself, which will never be known. When the predicted variable is not ex-post observable, straightforward performance measures such as "mean square prediction error" are ineffective: it is impossible to compute the distance between a predicted value and a realized value when the latter is unknown. It is, therefore, necessary to adopt different evaluation methods. In this section, we propose two simple and intuitive tests for the evaluation of the TEV statistic produced by the new Credit Risk Model.

3.1. Unit Variance Test

Our first test builds on the following intuition: dividing each realization of a process by its estimated standard deviation will produce a series of unit-variance realizations, if the estimated volatilities are correct. The chi-square test that we discuss in this section is simply a formalization of this idea.

Consider holding a market-value-weighted portfolio of all issues in the IG Credit Index with option-adjusted duration between 3 and 5 years when benchmarked to the whole IG Credit Index. Now imagine doing the same thing each month from January 1995 to December 2001 (84 periods). At the beginning of each month, we estimate TEV using the Credit Risk Model calibrated to the history available up to that point. At the end of each month, we observe the realized outperformance of

Figure 2. **Standardized Outperformance**
Portfolio = 3- to 5-yr OAD Credit Index, Benchmark = Credit Index



the portfolio. Figure 2 depicts the series of standardized realized outperformances (STR). Its estimated standard deviation is 0.97, with a 95% confidence interval (0.84,1.15), showing that we cannot reject the null of unit variance at any reasonable significance level. The graph also shows that the absolute value of the realized outperformance has exceeded one TEV 29% of the time and two TEVs 7% of the time, a behavior compatible with a distribution with slightly fatter tails than the normal. Experimenting with a variety of portfolios and benchmarks, we have found that the frequency of hits through the one-TEV band ranges between 25% and 35%. Hits in the tails become more frequent as portfolio diversification decreases, as one would expect.

3.2. Runs Test

A risk model allows the user to make probabilistic statements about uncertain events, and, in particular, it provides a framework for computing interval forecasts. Value at risk, the most widely used risk measure in the finance industry, is simply the bound of a one-sided interval forecast.

As we saw above, the risk model's conditional estimate of TEV is based on the time-varying loadings L_t and the factor covariance matrix Σ. Since the latter is estimated as the full sample covariance of the factor series, we should check whether the estimated TEV fails to capture a possible persistency in volatility.

Christoffersen (1998) has proposed a non-parametric methodology for evaluating conditional interval forecasts. Suppose that at time t, we form a forecasting interval (D_t, U_t) with probability coverage p for the realization of variable x over the next period. If we denote with H_t the information known at time t, we have

$$\Pr\left(x_{t+1} \in \left(D_t, U_t\right) \mid H_t\right) = p.$$

This simply means that, conditional on what is known at time t, the probability that the realization in period $(t, t+1)$ falls in the interval (D_t, U_t) is equal to p. Thus, p is usually called the "conditional coverage" of the interval. Next, define a sequence of random indicators $\{I_t\}_t$ as

$$I_{t+1} = \begin{cases} 1, x_{t+1} \in (D_t, U_t) \\ 0, x_{t+1} \notin (D_t, U_t) \end{cases},$$

which says that the indicator I_{t+1} takes a unit value if the realization x_{t+1} falls in the interval (a "hit") and zero if it falls outside.

The main result that we are going to use is that the sequence of interval forecasts has correct conditional coverage if and only if the hit sequence $\{I_t\}_t$ is an i.i.d. Bernoulli (p) sequence. To see this, set $\{I_t, I_{t-1}, I_{t-2}, \ldots\} = H_t$ and observe that correct conditional coverage implies that

$$p = \Pr(I_{t+1} = 1 \mid I_t, I_{t-1}, I_{t-2}, \ldots) \text{ and}$$

$$p = E[E[I_{t+1} \mid I_t, I_{t-1}, I_{t-2}, \ldots]] = E[I_{t+1}] = \Pr(I_{t+1} = 1),$$

i.e., the hit sequence is serially independent. That it is also identically distributed follows from observing that each I_t is simply a Bernoulli(p) random variable. The reverse implication is trivial.

A widely used test for the null hypothesis that a sequence of indicators is i.i.d. is the so-called runs test. A "run" is a subsequence of adjacent zeroes or ones in the sequence $\{I_t\}_t$. For example, the sequence {1110000} contains two runs, while the sequence {1100100} contains four runs. We can test the null hypothesis that $\{I_t\}_t$ is an i.i.d. sequence using the statistic

$$Z = \frac{R - 2np(1-p)}{2(np(1-p)(1-3p(1-p)))^{1/2}}.$$

Here, R denotes the number of runs in the sequence, n is the number of elements in the sequence, and p is the probability of a hit (the interval coverage). Under the null hypothesis that $\{I_t\}_t$ is an i.i.d. sequence, Z is asymptotically standard normal.[3] The test is usually made operational by substituting p with its maximum likelihood estimate $\hat{p}$.

We conduct the test by producing time series of interval forecasts based on the estimated TEVs. At each time t, we set the interval forecast equal to ±1 TEV and compute the out-of-sample outperformance over $(t, t+1)$ to derive the value of the indicator I_{t+1}.

For the same portfolio and benchmark used in the previous section, the p-value for the one-sided test of the hypothesis that the hit sequence is serially independent is 15%. This shows that we cannot reject the null hypothesis at the usual 5% significance level and that the significance can be raised substantially before rejecting the null.

[3] See Campbell, Lo, and MacKinlay (1997).

Figures 3. **5-Year TRAINS Portfolio,** as of 1/15/02

Issuer	**Coupon**	**Maturity**	**Amt** (000s)	**Moody's**	**S&P**
AMERICAN ELECTRIC POWER	6.125	5/15/06	23,000	Baa1	BBB+
AOL TIME WARNER	6.125	4/15/06	50,600	Baa1	BBB+
BANK ONE CORP	6.5	2/1/06	50,600	Aa3	A
BEAR STEARNS COMPANIES I	5.7	1/15/07	50,600	A2	A
BRISTOL-MYERS SQUIBB	4.75	10/1/06	50,600	Aa2	AAA
BRITISH TELECOMMUNICATIO	7.875	12/15/05	46,000	Baa1	A-
CANADA (GOVERNMENT OF)	6.75	8/28/06	46,000	Aa1	AA+
CITIGROUP INC	5.75	5/10/06	59,800	Aa1	AA-
CONOCO FUNDING CO	5.45	10/15/06	50,600	Baa1	BBB+
FIRSTENERGY CORP	5.5	11/15/06	27,600	Baa2	BBB-
FLEETBOSTON FINL CORP	4.875	12/1/06	50,600	A1	A
FORD MOTOR CREDIT COMPANY	6.5	1/25/07	50,600	A3	BBB+
FRANCE TELECOM SA	7.7	3/1/06	32,200	Baa1	BBB+
GENERAL ELECTRIC CAPITAL	6.8	11/1/05	36,800	Aaa	AAA
GENERAL MOTORS ACCEPT CO	6.125	9/15/06	50,600	A2	BBB+
ITALY (REPUBLIC OF)	4.375	10/25/06	50,600	Aa3	AA
KRAFT FOODS INC	4.625	11/1/06	50,600	A2	A-
NATIONAL RURAL UTILITIES	6	5/15/06	36,800	A1	AA-
SPRINT CAPITAL CORP	6	1/15/07	50,600	Baa1	BBB+
TYCO INTERNATIONAL GROUP	6.375	2/15/06	50,600	Baa1	BBB
UNILEVER NV	6.875	11/1/05	50,600	A1	A+
VERIZON WIRELESS INC	5.375	12/15/06	46,000	A2	A+
WAL-MART STORES	5.45	8/1/06	46,000	Aa2	AA
WASHINGTON MUTUAL INC	5.625	1/15/07	46,000	A3	BBB+
WORLDCOM INC	8	5/15/06	46,000	Baa2	BBB

Figures 4. **10-Year TRAINS Portfolio,** as of 1/15/02

Issuer	**Coupon**	**Maturity**	**Amt** (000s)	**Moody's**	**S&P**
ALCOA ALUMINIO S A	6.5	6/1/11	48,840	A1	A+
AMERICAN TELEPHONE & TEL	6	3/15/09	39,960	A3	BBB+
AOL TIME WARNER	6.75	4/15/11	48,840	Baa1	BBB+
BANK ONE CORP	7.875	8/1/10	48,840	A1	A-
BANKAMERICA CORPN	7.4	1/15/11	35,520	Aa3	A
BELLSOUTH CORPORATION	6	10/15/11	48,840	Aa3	A+
CITIGROUP INC	7.25	10/1/10	39,960	Aa2	A+
CONAGRA INC	6.75	9/15/11	35,520	Baa1	BBB+
CREDIT SUISSE FB USA INC	6.5	1/15/12	48,840	Aa3	AA-
DAIMLERCHRYSLER NORTH AM	7.3	1/15/12	48,840	A3	BBB+
FIRSTENERGY CORP	6.45	11/15/11	26,640	Baa2	BBB-
FORD MOTOR CREDIT COMPANY	7.25	10/25/11	48,840	A3	BBB+
FRANCE TELECOM SA	8.25	3/1/11	48,840	Baa1	BBB+
GENERAL MOTORS ACCEPT CO	6.875	9/15/11	48,840	A2	BBB+
HONEYWELL INT'L	7.5	3/1/10	48,840	A2	A
INTER-AMERICAN DEVELOPME	7.375	1/15/10	35,520	Aaa	AAA
ITALY (REPUBLIC OF)	6	2/22/11	39,960	Aa3	AA
KRAFT FOODS INC	5.625	11/1/11	48,840	A2	A-
NISOURCE FINANCE CORP	7.875	11/15/10	26,640	Baa3	BBB
PROGRESS ENERGY INC	7.1	3/1/11	48,840	Baa1	BBB
QUEBEC (PROVINCE OF)	6.125	1/22/11	48,840	A1	A+
SEARS ROEBUCK ACCEPTANCE	6.75	8/15/11	48,840	A3	A-
TYCO INTERNATIONAL GROUP	6.375	10/15/11	48,840	Baa1	BBB
WAL-MART STORES	6.875	8/10/09	48,840	Aa2	AA
WORLDCOM INC	7.5	5/15/11	48,840	Baa2	BBB

Figures 5. **Long TRAINS Portfolio,** as of 2/1/02

Issuer	**Coupon**	**Maturity**	**Amt** (000s)	**Moody's**	**S&P**
ABBEY NATIONAL PLC	7.95	10/26/29	30,000	Aa3	AA-
AMERICAN TELEPHONE & TEL	6.5	3/15/29	30,000	A3	BBB+
BELLSOUTH CAP FDG CORP	7.875	2/15/30	30,000	Aa3	A+
BRITISH TELECOMMUNICATIO	8.875	12/15/30	30,000	Baa1	A-
CONOCO INC	6.95	4/15/29	30,000	Baa1	BBB+
DEVON ENERGY CORP	7.875	9/30/31	30,000	Baa2	BBB
DOW CHEM CO	7.375	11/1/29	30,000	A3	A
EL PASO ENERGY CORP MTN	7.75	1/15/32	30,000	Baa2	BBB
FIRSTENERGY CORP	7.375	11/15/31	30,000	Baa2	BBB-
FORD MTR CO DEL	7.45	7/16/31	30,000	Baa1	BBB+
FRANCE TELECOM SA	9	3/1/31	30,000	Baa1	BBB+
GENERAL MOTORS ACCEPT CO	8	11/1/31	30,000	A2	BBB+
I.B.R.D. (WORLD BANK)	7.625	1/19/23	30,000	Aaa	AAA
ITALY (REPUBLIC OF)	6.875	9/27/23	30,000	Aa3	AA
KELLOGG CO	7.45	4/1/31	30,000	Baa2	BBB
LOCKHEED MARTIN	8.5	12/1/29	30,000	Baa2	BBB
PEPSI BOTTLING GRP	7	3/1/29	30,000	A3	A-
QUEBEC PROV CDA	7.125	2/9/24	30,000	A1	A+
ROHM & HAAS CO	7.85	7/15/29	30,000	A3	A-
SPRINT CAPITAL CORP	6.9	5/1/19	30,000	Baa2	BBB+
TIME WARNER ENTMT CO L P	8.375	3/15/23	30,000	Baa1	BBB+
VERIZON GLOBAL FDG CORP	7.75	12/1/30	30,000	A1	A+
VIACOM INTERNATIONAL INC	7.875	7/30/30	30,000	A3	A-
WAL MART STORES INC	7.55	2/15/30	30,000	Aa2	AA
WORLDCOM INC	8.25	5/15/31	30,000	Baa2	BBB

In summary, our test excludes the presence of serial correlation in the hit sequence by detecting the presence of a sufficiently high number of runs in the sample. This suggests that our TEV estimator adequately captures the volatility dynamics. We obtained analogous results for a large number of test portfolios and benchmarks.

4. A ROBUSTNESS CHECK: TRACKING WITH TRAINS

On January 15, 2002, Lehman Brothers launched the 5- and 10-year TRAINS (targeted return index securities), while a longer portfolio was issued short afterward, on February 1. Each TRAINS replicates the return on a basket of 25 of the most actively traded High-Grade bonds and is constructed with the goal of tracking a specific curve segment of the overall IG Credit Index. These baskets offer investors a diversified exposure to the credit market through a single transaction. Figures 3-5 describe the portfolios on the respective closing dates.

Since inception, a couple of credits included in the TRAINS have experienced extreme idiosyncratic movements. With the benefit of hindsight, we can now perform a fairly challenging robustness check for the new Credit Risk Model.

Standing on 1/15/02 (i.e., using only information available to this date), we estimate the TEV for the 5-year TRAINS versus the 3.5- to 7-year-to-maturity portion of the Credit Index. The realized underperformance of the 5-year TRAINS over the following three-month period (ending 4/15/02) represents a 3.88 standard-deviation event.

Repeating the same exercise for the 10-year TRAINS versus the 7- to 15-year-to-maturity Credit Index , we observe a 2.76 standard-deviation underperformance. Finally, the long TRAIN realized a 1.9 standard-deviation underperformance versus the 20+year-to-maturity portion of the Credit Index.

These small-portfolio examples with extreme realizations add to the evidence presented in the previous section that the new Credit Risk Model produces reliable volatility estimates. In particular, these results witness the importance of specifying appropriate models for both the systematic and the idiosyncratic components of the return process, the latter being a crucial determinant of volatility when portfolio diversification is somewhat limited.

5. POINT REPORTS

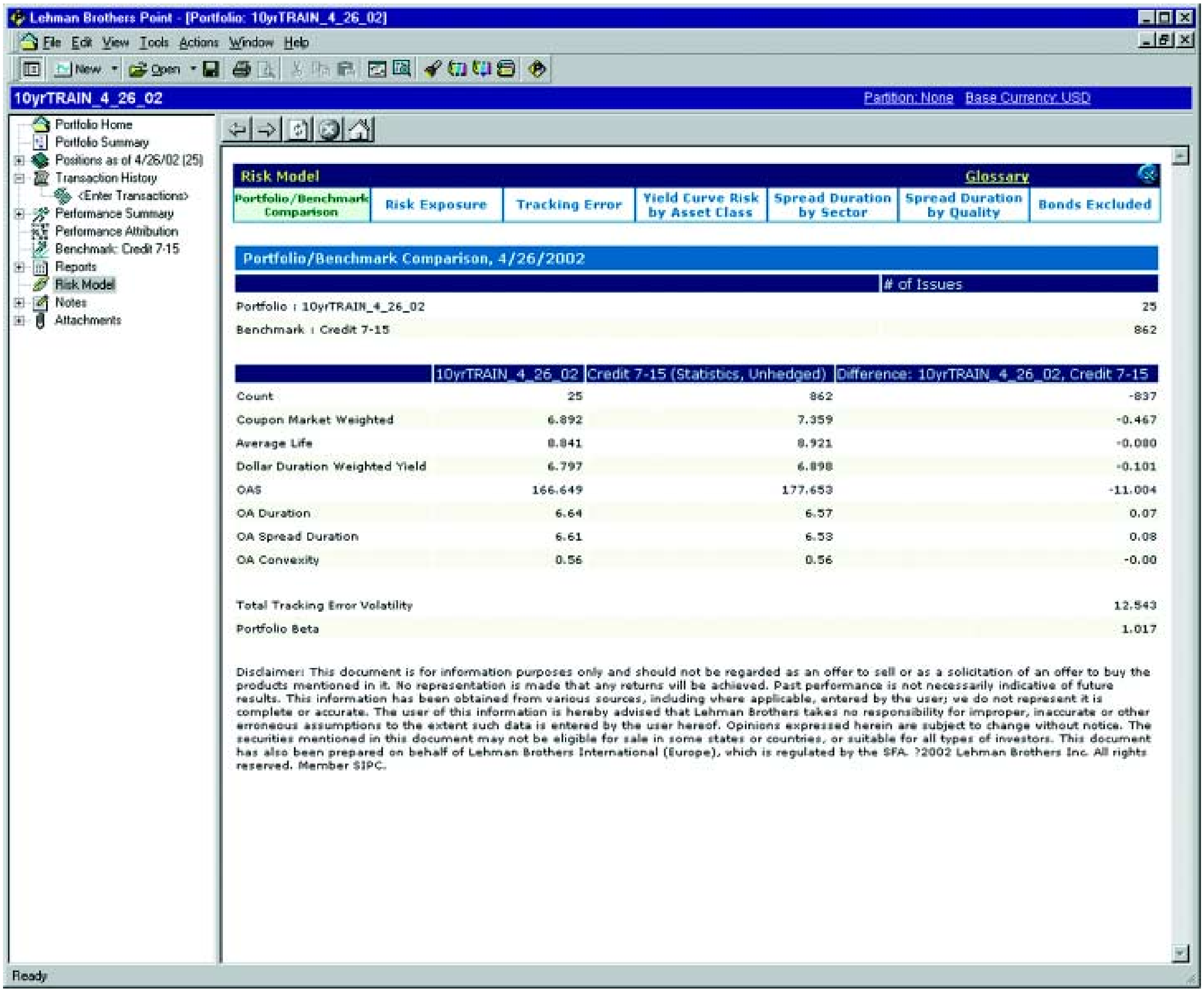
In this section, we offer a view of some of the new Credit Risk Model reports available on Lehman Brothers' portfolio system POINT.

Figure 6 refers to the 10-year TRAINS portfolio versus the 7- to 15-year-to-maturity Credit Index as of 4/26/2002. The table on the top gives summary statistics such as number of bonds included, option-adjusted spread, option-adjusted duration, etc., for the two portfolios, as well as a preliminary glance at TEV (in bp per month) and the portfolio's beta with respect to the chosen benchmark.

The bottom table reports the estimated TEV and its decomposition into different sources corresponding to different subsets of factors. This decomposition highlights both the isolated effect of a particular set of factors and its cumulative effect. Since risk factors are correlated, the cumulative decomposition depends on the order in which new sets of factors are added. It is therefore offered using two different sequences, the first focusing on adding one asset class at a time (Treasury, Agency, Credit), the second adding one type of risk at a time (benchmark risk, volatility risk, spread risk). Of course, risk measures related to asset classes other than credit are all equal to zero in the tables, since our portfolios and benchmarks both belong to the credit universe. It is interesting to notice that the idiosyncratic portion of the TEV is actually larger than the systematic part, showing again the importance of modeling idiosyncratic variance in order to capture the deviation risk of small portfolios correctly.

Figure 7 refers to the example used in section 3 (i.e., 3- to 5-year option-adjusted duration versus the whole Credit Index). This report is also produced as of 4/26/2002. This time, both the portfolio and the benchmark are highly diversified, so it is natural to expect that the idiosyncratic risk be almost eliminated, as shown by the TEV decomposition. Also, since there is a huge curve mismatch, it is not surprising that a large portion of the TEV comes from interest rate risk. It is also instructive to observe that credit spread risk, although large when isolated, does not increase the overall TEV by much when added to benchmark risk, because of the well-documented negative correlation between interest rates and credit spreads, which is captured in the factor covariance matrix.

Figure 6a. **Portfolio = 10-year TRAINS; Benchmark = 7- to 15-Year Maturity Credit Index**
Portfolio/Benchmark Summary

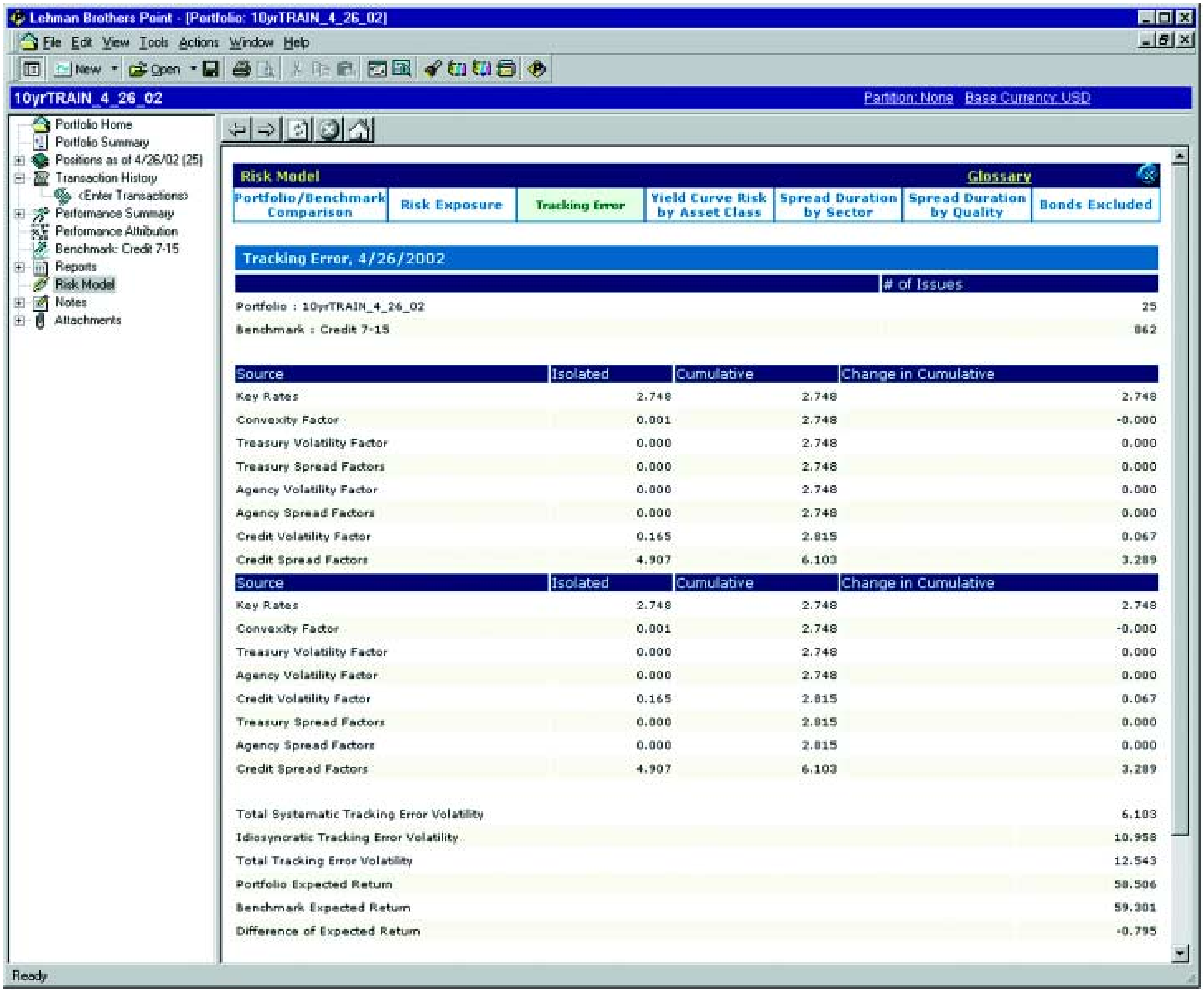


	# of Issues
Portfolio : 10yrTRAIN_4_26_02	25
Benchmark : Credit 7-15	862

	10yrTRAIN_4_26_02	Credit 7-15 (Statistics, Unhedged)	Difference: 10yrTRAIN_4_26_02, Credit 7-15
Count	25	862	-837
Coupon Market Weighted	6.892	7.359	-0.467
Average Life	8.841	8.921	-0.080
Dollar Duration Weighted Yield	6.797	6.898	-0.101
OAS	166.649	177.653	-11.004
OA Duration	6.64	6.57	0.07
OA Spread Duration	6.61	6.53	0.08
OA Convexity	0.56	0.56	-0.00

Total Tracking Error Volatility	12.543
Portfolio Beta	1.017

Figure 6b. **Portfolio = 10-year TRAINS; Benchmark = 7- to 15-Year Maturity Credit Index**
Tracking Error



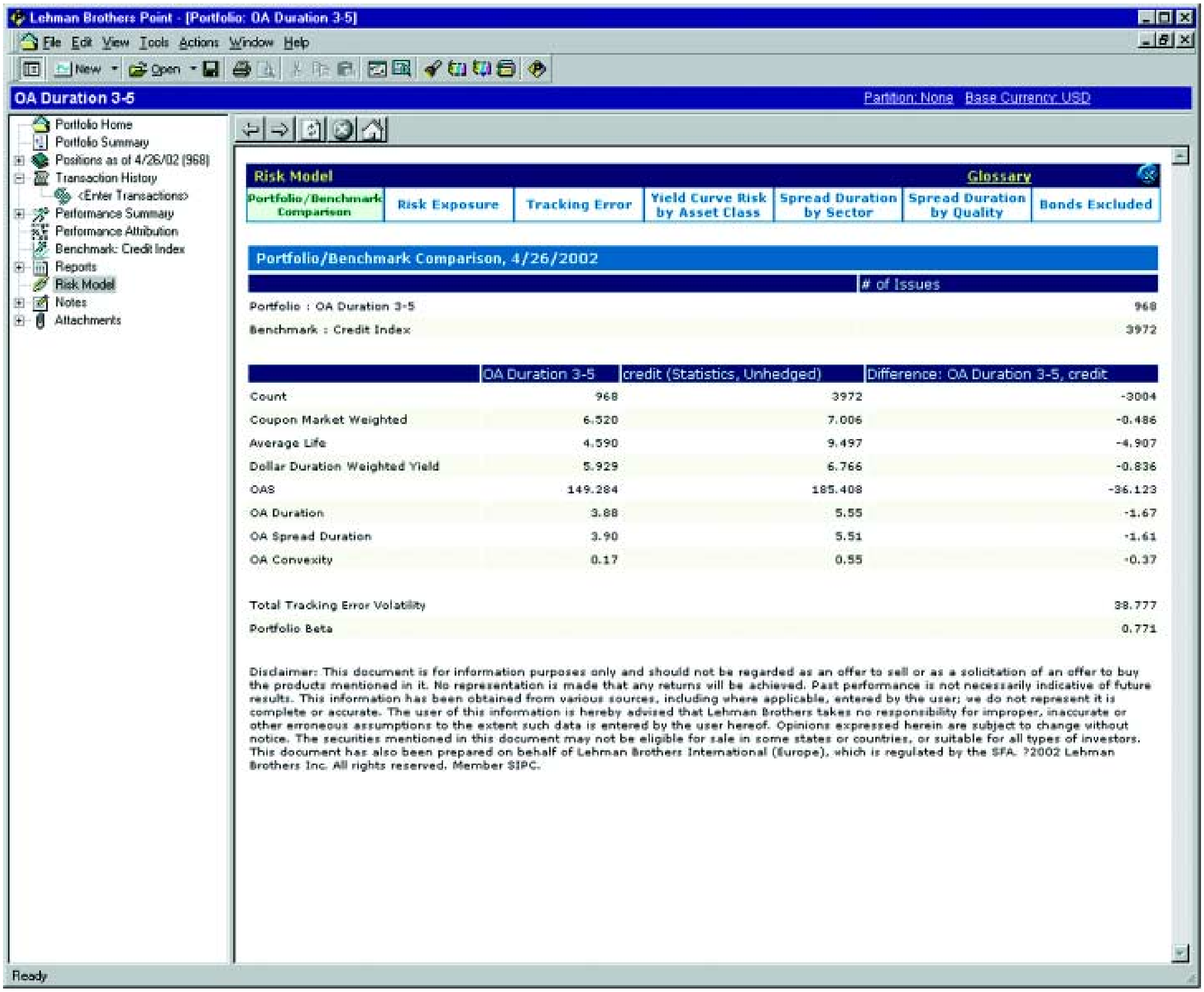
Tracking Error, 4/26/2002

	# of Issues
Portfolio : 10yrTRAIN_4_26_02	25
Benchmark : Credit 7-15	862

Source	Isolated	Cumulative	Change in Cumulative
Key Rates	2.748	2.748	2.748
Convexity Factor	0.001	2.748	-0.000
Treasury Volatility Factor	0.000	2.748	0.000
Treasury Spread Factors	0.000	2.748	0.000
Agency Volatility Factor	0.000	2.748	0.000
Agency Spread Factors	0.000	2.748	0.000
Credit Volatility Factor	0.165	2.815	0.067
Credit Spread Factors	4.907	6.103	3.289
Source	**Isolated**	**Cumulative**	**Change in Cumulative**
Key Rates	2.748	2.748	2.748
Convexity Factor	0.001	2.748	-0.000
Treasury Volatility Factor	0.000	2.748	0.000
Agency Volatility Factor	0.000	2.748	0.000
Credit Volatility Factor	0.165	2.815	0.067
Treasury Spread Factors	0.000	2.815	0.000
Agency Spread Factors	0.000	2.815	0.000
Credit Spread Factors	4.907	6.103	3.289

Total Systematic Tracking Error Volatility	6.103
Idiosyncratic Tracking Error Volatility	10.958
Total Tracking Error Volatility	12.543
Portfolio Expected Return	58.506
Benchmark Expected Return	59.301
Difference of Expected Return	-0.795

Figure 7a. **Portfolio = 3- to 5-yr OAD Credit Index; Benchmark = Credit Index**
Portfolio/Benchmark Summary



	# of Issues
Portfolio : OA Duration 3-5	968
Benchmark : Credit Index	3972

	OA Duration 3-5	credit (Statistics, Unhedged)	Difference: OA Duration 3-5, credit
Count	968	3972	-3004
Coupon Market Weighted	6.520	7.006	-0.486
Average Life	4.590	9.497	-4.907
Dollar Duration Weighted Yield	5.929	6.766	-0.836
OAS	149.284	185.408	-36.123
OA Duration	3.88	5.55	-1.67
OA Spread Duration	3.90	5.51	-1.61
OA Convexity	0.17	0.55	-0.37

Total Tracking Error Volatility	38.777
Portfolio Beta	0.771

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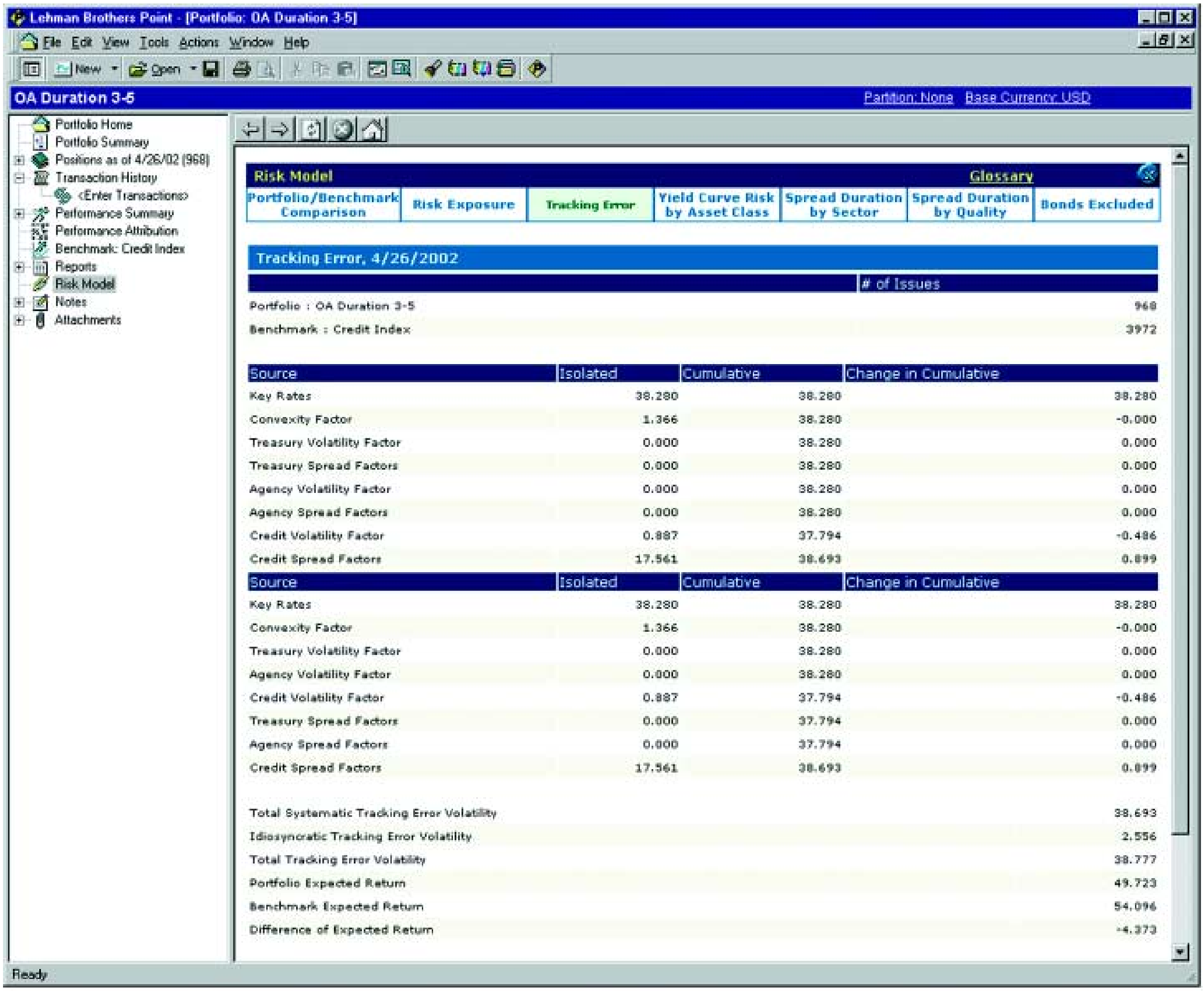
Figure 7b. **Portfolio = 3- to 5-yr OAD Credit Index; Benchmark = Credit Index**
Tracking Error



Tracking Error, 4/26/2002

	# of Issues
Portfolio : OA Duration 3-5	968
Benchmark : Credit Index	3972

Source	Isolated	Cumulative	Change in Cumulative
Key Rates	38.280	38.280	38.280
Convexity Factor	1.366	38.280	-0.000
Treasury Volatility Factor	0.000	38.280	0.000
Treasury Spread Factors	0.000	38.280	0.000
Agency Volatility Factor	0.000	38.280	0.000
Agency Spread Factors	0.000	38.280	0.000
Credit Volatility Factor	0.887	37.794	-0.486
Credit Spread Factors	17.561	38.693	0.899

Source	Isolated	Cumulative	Change in Cumulative
Key Rates	38.280	38.280	38.280
Convexity Factor	1.366	38.280	-0.000
Treasury Volatility Factor	0.000	38.280	0.000
Agency Volatility Factor	0.000	38.280	0.000
Credit Volatility Factor	0.887	37.794	-0.486
Treasury Spread Factors	0.000	37.794	0.000
Agency Spread Factors	0.000	37.794	0.000
Credit Spread Factors	17.561	38.693	0.899

Total Systematic Tracking Error Volatility	38.693
Idiosyncratic Tracking Error Volatility	2.556
Total Tracking Error Volatility	38.777
Portfolio Expected Return	49.723
Benchmark Expected Return	54.096
Difference of Expected Return	-4.373

A complete Risk Model report contains a few other tables detailing factor mismatches, decomposing interest-rate (benchmark) risk by asset class, and providing standard market structure reports by sector and quality.

6. SUMMARY

Lehman Brothers specializes in advising and working with investors who manage their holdings relative to pre-specified benchmarks. The increasingly complex nature of our fixed-income markets calls for the development of appropriate quantitative tools to evaluate and decompose the associated deviation risk.

More than a decade of experience in risk modeling and a continuous interaction with our clients have proven to be a solid basis for the development of the new Lehman Brothers U.S. Risk Model. In this article we have described its credit-related portion, emphasizing the decomposition of total return and the specification of empirical models for its stochastic components. We have also offered some hints regarding the out-of-sample testing that we use to evaluate the model's output, and a glance at some of the reports available to the user on our new portfolio system POINT.

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Managing Risk Exposures in CDO Tranches[1]

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Measuring and managing risk exposures in CDO tranches have become extremely important due to a dramatic rise in the rate of corporate defaults over the past two years. We introduce a comprehensive methodology for evaluating and managing obligor and common factor risks in CDO tranches, which is consistent with risk management practices used for cash credit portfolios and is well suited for investors who hold multiple cash and CDO positions.

1. INTRODUCTION

Collateralized debt obligations (CDOs) are a relatively young asset class, and their risk management and hedging practices are less standardized than those for cash portfolios. Though structured as buy-and-hold investments that capture liquidity premiums of underlying assets, CDOs are not immune to mark-to-market movements (see Ganapati *et.al.* [2002]). Even long-term investors are sensitive to both realized credit losses and changing credit risk expectations in the collateral portfolio that feed through to the tranche valuations. Such changes are amplified through activation of triggers that have a direct impact on cash flows. Investors increasingly realize that markets might identify problem credits before rating agencies do and would like to know the effective exposure of their tranches to the issuer in question.

The key to understanding the investor's need to manage the risks of a CDO security is the evolution with time of the collateral portfolio and market environment. Had the market evolved exactly along the "central" scenario corresponding to the expected returns of the CDO tranches, one would not need to do anything other than hold on to those notes and clip the coupons while they come. However, certain exposures that the investors were quite content with a year ago may no longer be appropriate for them, either due to an overall risk profile change or due to increased risks in those exposures. Alternatively, some exposures may become more palatable with time.

Therefore, the appropriate risk measures focus on the sensitivity of the market value of CDO tranches with respect to changes in the composition of the underlying collateral portfolio and associated credit risks. Investors in CDO tranches can apply hedging and diversification techniques that are optimal from the point of view of their risk profile and overall credit investment portfolio.

In this study, we address a set of stylized questions that investors often face:

- Seasoned CDO tranche investor: *"My credit analyst has recently become negative on ABC Inc. If the CDO collateral has a 1% exposure to this company, what is the effective exposure of my tranche? How much protection should I buy?"*

[1] We are grateful to Sunita Ganapati for guiding us through the CDO market and directing our attention to the most important issues, to Dominic O'Kane for making available a simulation model of CDO tranches that made this research possible, and to Jim Ballentine, Chris Kelly, Mark Ames, Georges Assi, James Lee, Brian Wargon, and many others for extensive comments.

- *Investor bidding on a secondary CDO:* "I think a particular distressed CDO tranche is a good value, except it has too much telecom risk for my taste. Can I effectively transform the tranche characteristics by hedging particular names?"
- *CDO fund of funds manager:* "A lot of CDO tranches in my fund have exposures to the same names. How can I monitor and manage my aggregate risk limits across different tranches?"
- *CDO manager:* "What is the impact of par creation on the future performance of the CDO?"

We would like to emphasize that we are not addressing here the subject of hedging the CDO investment as a whole. We consider instead the situation in which an investor wishes to keep his/her exposure to the asset class while managing some of the risk characteristics of the holdings. Such an approach would often be preferable to hedging or trading out of the entire investment because the transaction costs for illiquid CDO notes are likely to be very high. Indeed, this is a direct result of the fact that CDO investors (especially junior note-holders) are being generously compensated for carrying an illiquid instrument.

The purpose of this research is to develop a consistent approach for evaluating and managing the risks associated with CDO tranches. In particular, we attempt to represent the CDO investments in a framework generally consistent with active management of cash credit portfolios by identifying systematic and company-specific sources of risk and optimizing the overall portfolio exposures within either a total or excess return setting. Investors who combine multiple cash and CDO positions in credit portfolios, including managers of CDO funds of funds, should find this approach particularly useful.

The paper is organized as follows:

- In Section 2, we outline the overall framework for CDO risk profile estimation.
- In Section 3, we investigate the dependence of the risk characteristics of CDO tranches on two main parameters: the collateral portfolio composition and the underlying credit risks.
- In Section 4, we estimate the risk profile of mezzanine and equity tranches of a typical cash flow investment grade CDO.
- In Section 5, we discuss the risk management strategies on a name-by-name basis.
- In Section 6, we present a roadmap for managing of common risk exposures.
- Section 7 contains a brief summary of findings.
- Appendix A contains the description of the stylized CDO structure and modeling assumptions used in this report.
- Appendix B contains detailed definitions for effective portfolio representation of CDO tranches.

2. THE RISK MANAGEMENT FRAMEWORK

Understanding the risks associated with a given security is an important ingredient of successful investment strategies for any market. It is an even higher priority for investment in complex and unseasoned structured vehicles such as CDOs, which have not yet experienced full business cycles.

CDOs are mainly exposed to credit loss risk (a combination of default and recovery risk).[2] Default risk exposure includes both systematic and idiosyncratic components. Investors may wish to modify their exposures to either component depending on their views.

It is important to realize that risk measurement methodology and risk management objectives are intricately related. The differences between various approaches can be substantial because of the non-linear and long-term nature of the CDO investments. If investors could ignore short-term valuation risks, they would focus on the risk/return distribution over the life of the deal and optimize it according to their risk appetite using, if necessary, "static" hedges. If investors are sensitive to mark-to-market valuation risks, then a "dynamic" hedging, taking into account the current market conditions, is more appropriate.

We emphasize that the practical recommendation for a CDO note-holder who wishes to pare down his/her exposure to particular name will almost always be only a partial hedge of that name's credit risk. Indeed, from the viewpoint of an investor in a CDO equity or mezzanine note, creating a complete default hedge for a particular name will be expensive, because unrealized credit risk is shared by all tranches of the CDO. By constructing a complete external hedge, subordinated tranche holders would end up covering not only the risks of their own investments, but also those of senior note-holders, to some extent.

Even more important than the single "risk" measure is the ability to discern the influence of various driving factors. One of the main requirements for risk analysis is that it should be "actionable," both provide a measure of risk and suggest the remedy. This naturally leads to a notion of risk decomposition analysis as an important ingredient of any risk assessment. Such analysis of the CDO risk must allow one to find the risk due to individual holding in the collateral pool, as well as the risks due to market-, sector-, or industry-wide credit deterioration. Moreover, a typical CDO investor would probably invest in more than one CDO. We would like to characterize the CDO risk exposures in such a way that a direct aggregation across holdings will be possible. Before one decides whether a given credit exposure is too large, too small, or just about right, it should be considered in the context of all other risk exposures.

Decomposing the portfolio risks into contributions from particular positions or particular risk factors is a difficult task because risk is a non-linear measure. It involves complicated cross-correlations among different assets. Sometimes a relatively large position will carry surprisingly small risk contribution, and vice versa.

[2] We ignore interest rate risk, foreign exchange risk, call risk, and other non-credit risk factors with the assumption that these risks are negligible or have been appropriately hedged by the CDO collateral manager.

These difficulties are significantly amplified when one considers non-linear portfolio-based instruments, such as CDO tranches. Both the exposure profile and the risk contribution profile are important in effectively managing the risk-adjusted returns.

CDOs involve a capital structure with complex rules such as OC/IC triggers. Given a structure, the risk due to a specific name trickles down to each tranche in a non-trivial way. Intuitively, the risk profile of a CDO tranche investment is determined by the following set of factors:

- Composition of the underlying portfolio.
- Credit risk of the individual names in the underlying portfolio.
- The correlation of credit risks among the names, as well as common factor risks.
- Structure of the CDO (subordination levels, OC/IC triggers, etc.)

We concentrate on the dependence of the risk profile of CDO tranches on the first two sets of parameters. While the correlation among the credit risks is important, we consider it fixed in this investigation. We address the common factor risks in the latter part of this report.

The credit risk of individual collateral names, as measured by their respective default probabilities, can be estimated either using historical (rating agency) data; or from a fundamental structural model (such as KMV's version of the Merton model); or from a breakeven analysis of spreads, leading to so-called implied default probabilities. For managing the distribution of final outcomes at maturity, forecast or historically estimated values are more appropriate. For managing the mark-to-market risks associated with changing perceived credit risks, the implied or break-even measures are more appropriate because they also include the effect of changing risk premia in the credit market. Hence, we use spread implied default probability (also referred to as default intensity or default rate to emphasize the continuous nature of timing of default) throughout this study.

3. DEFINING CDO TRANCHE RISK EXPOSURE PROFILE

In order to clarify the dependence on the composition of the collateral portfolio and the underlying security credit risk, we consider three different model portfolios, as specified in Appendix A. We study the dependence on one parameter at a time by fixing the other. The portfolios are:

- The equal collateral weight portfolio, which has equal allocation to each name in the collateral and, thus, allows us to concentrate on the dependence of risk on the default probability of underlying credits.
- The uniform default risk portfolio, which helps clarify the dependence upon the collateral weight.
- Finally, the typical CDO portfolio, which allows us to see the complex interplay between the two dependencies.

In order to have a meaningful comparison and interpretation of results, the hypothetical portfolios chosen here are similar in structure and average characteristics to the typical portfolio analyzed later in this study. They comprise 50 names with an average weight of 2% and have a 4% equity tranche.

Due to the complex cash flow impact of changing collateral default rates or composition, the numerical estimation of the risk characteristics using a Monte Carlo simulation is the straightforward choice (see O'Kane, Schloegl [2001] for an overview). We have performed simulations on the hypothetical portfolios described above in order to illustrate the meaning of risk sensitivities and to demonstrate dependence patterns of equity and mezzanine tranches. For purposes of this analysis, we have fixed the asset correlation at 25% and recovery rates at 45%.

3.1 Default Risk Sensitivity

We define default risk sensitivity as the marginal fractional change in market value of the CDO tranche per unit change in the default rate of collateral (with a minus sign). The negative sign accounts for the fact that an increasing default rate leads to a decrease in the price of the bond and, hence, each tranche. We implicitly assume that the changes in default rate are proportional to changes in spread, with a proportionality coefficient given by the expected loss, or one minus recovery rate.[3]

Under such definition, the price of the underlying bond also changes proportionally with the change in implied default rate, with coefficient of proportionality equal to the spread duration of the bond scaled by expected loss rate. Thus, the default risk sensitivity γ_n^I of tranche I with respect to change in default rate h_n of nth underlying collateral bond is proportional to tranche price sensitivity η_n^I with respect to the same bond (see Appendix B for definition) and to that bond's spread duration SD_n and expected loss $(1 - R_n)$:

[1] $$\gamma_n^I = -\frac{\Delta PV_{tranche}^I / PV_{tranche}^I}{\Delta h_n} = \eta_n^I \cdot (1 - R_n) \cdot SD_n$$

The aggregate tranche sensitivity with respect to concurrent change in all underlying default rates is easily determined by summation:

[2] $$\gamma^I = \sum_{n=1}^{N} \gamma_n^I = \sum_{n=1}^{N} \eta_n^I \cdot (1 - R_n) \cdot SD_n$$

This is analogous to conventional portfolio spread duration calculation—the total sensitivity of the tranche value is a weighted sum of the underlying bond sensitivities. The price sensitivities act as the analogs of bond weight as applicable for each tranche. To the extent that the simultaneous increase in all default probabilities leads to a greater or smaller relative change in tranche value compared with the estimate above, it is a manifestation of the spread (or implied default rate) convexity, i.e., non-linear dependence of the tranche value on spread/default rate changes.

[3] This is the case when we use implied default rates under the condition of constant expected recovery rate.

Default Risk Sensitivity as a Function of Asset Weight

We would expect the default risk sensitivity to be higher for an asset with higher weight (all else being equal). Larger assets have greater bearing on expected returns and the value of the tranches. Exact levels of sensitivity depend on the capital structure and the cash flow waterfall. However, in a relative sense, assets with higher weight always contribute more to default risk sensitivity for all tranches. This is illustrated in the case of a uniform risk model portfolio, shown in Figure 1.

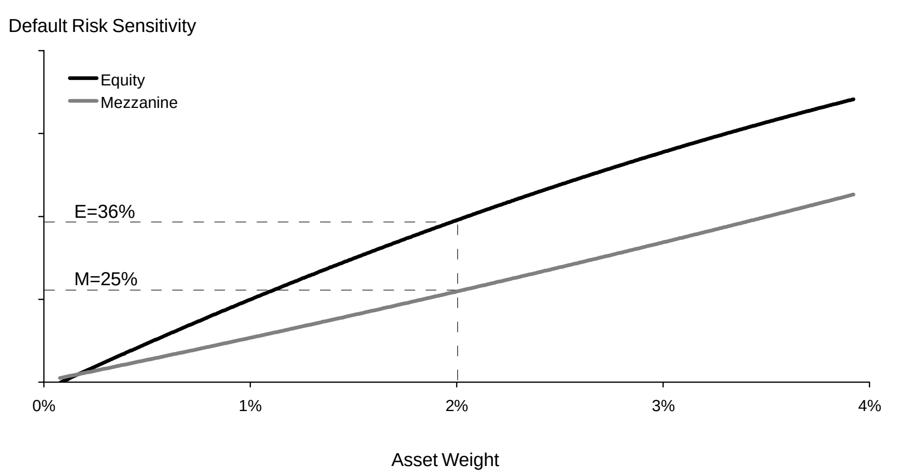
In Figure 1, we have plotted the default risk sensitivity contribution of each name in the uniform risk collateral CDO as a function of the weight that it carries in the collateral. Default risk sensitivity was calculated by inducing marginal changes in default rates of each asset, one at a time (while keeping the original coupon level), re-simulating the entire set of Monte Carlo scenarios to arrive at the new valuation of all tranches, and then measuring the fractional change in tranche value per unit of default rate increase.

For example, in Figure 1, we can see that a 10 bp increase in hazard rate of the asset with a weight of 2% leads to a decrease[4] of 25%x10 bp=2.5 bp in mezzanine value (compared to its original value) and equity loses 36%x10 bp=3.6 bp.

The relationship is almost linear, as expected. The equity tranche bears the first hit of the default and has a higher leverage, which is why the corresponding line is higher than the mezzanine. The (initial) slopes of the two curves are roughly of the order of the respective tranche leverage ratios (9x for mezzanine and 25x for equity). An interesting feature is that the equity tranche risk sensitivity exhibits a slight convexity growing less than linearly with asset weights. It is easy to understand the reason for this—while a default of the smaller asset would mostly impair the equity tranche, the default of the bigger (in weight) asset significantly affects mezzanine and higher tranches as well. Thus, part of its

[4] We plot positive numbers as default risk sensitivity has been defined with a negative sign.

Figure 1. **Default Risk Sensitivity for CDO with Uniform Default Risk Collateral**



risk is shared between the equity and mezzanine, and, therefore, the equity tranche risk sensitivity bears less than the full fraction of the asset's leveraged risk. As to the mezzanine tranche, a similar effect of risk spillover to the senior tranche is unlikely, and we see a simple linear dependence.

Default Risk Sensitivity as a Function of Asset Credit Risk

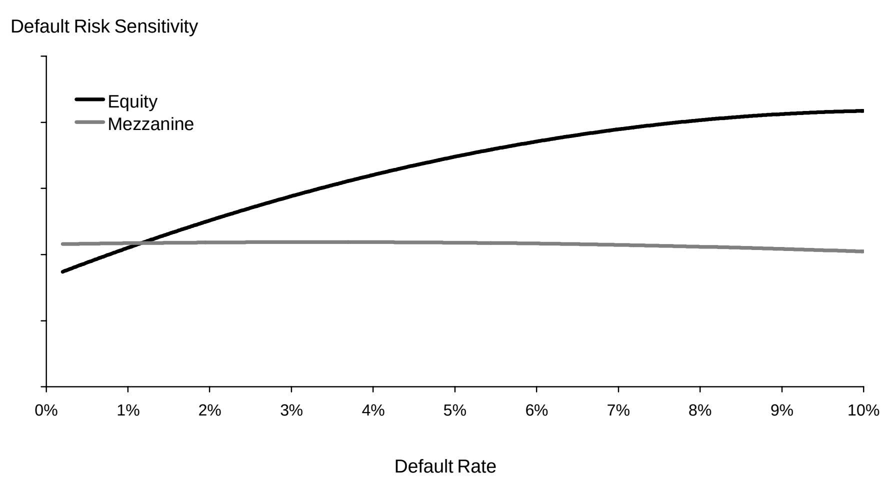
With respect to underlying asset credit risk, we know that an increase in the default probability of an asset results in an erosion of value for all tranches. The equal asset weight portfolio helps to illustrate this aspect of the CDO risk profile in Figure 2. Default risk sensitivity is plotted as a function of the (initial) default rate of assets.

In this case, the equity and mezzanine tranches exhibit significantly different dependence. The marginal increase in default risk of an asset leads to several effects on the equity tranche. First and foremost, if the bond defaults, then during the remainder of the CDO term, the equity tranche will receive less cash no matter what, simply due to lack of coupon flows from this bond. This is the main reason we see the upward sloping dependence of the equity tranche sensitivity to the underlying bond's credit risk—it simply reflects the fact that the riskier assets are also the higher-coupon assets (we assume that bonds are priced initially at par), leading to higher cash flow losses. The similar impact on the mezzanine is distant because of a cap on cash flows and the corresponding subordination in risk provided by the equity tranche.

The second effect is the higher probability that this asset will default prior to the maturity of the CDO and, therefore, will result in substantial par loss. If a bond defaults, par loss is the same, irrespective of the level of risk since we assume a constant recovery rate. The effect is on the end cash flow of the equity tranche (perhaps spilling over to mezzanine).

It is interesting to note that the dependence of equity tranche sensitivity on credit risk flattens out at higher default probabilities because the default of assets with initially high default probability during the life of the deal is already priced into the high-probability

Figure 2. **Default Risk Sensitivity for CDO with Equal Weight Collateral**



scenarios. The marginal increase of the default probability leads to change in default timing rather than a pure increase in probability of default, thus resulting in a proportionally smaller increase in the potential loss impact than in the case of the asset that is initially less likely to default. In other words, in the limit of high initial default probabilities, the default risk sensitivity approaches the plateau defined by the recovery rate (the so-called value-on-default impact).

3.2 Par Leverage Sensitivity

When a new deal is structured, the price at which an asset is purchased determines the initial over-collateralization ratio and, hence, the maximum possible capital gain. Buying cheaper bonds to fill the same amount of market value is one of the ways to "create par" and increase the OC ratios of the CDO, thereby also increasing the cash flow arbitrage.

We define par leverage sensitivity π_n^I as the marginal fractional change in a CDO tranche value $PV_{tranche}^I$ per given fractional change in the purchase price P_n of nth asset, while keeping the its market value PV_n constant.

[3] $$\pi_n^I = \left.\frac{\Delta PV_{tranche}^I / PV_{tranche}^I}{\Delta P_n / P_n}\right|_{PV_n = \text{const}}$$

It is worth mentioning that the definition of par leverage sensitivity does not include a negative sign because par leverage increases returns (unlike default rates, which reduce returns). For a conventional bond portfolio, the total portfolio value is independent of such a change; therefore, the par leverage sensitivity for a cash portfolio is always zero.

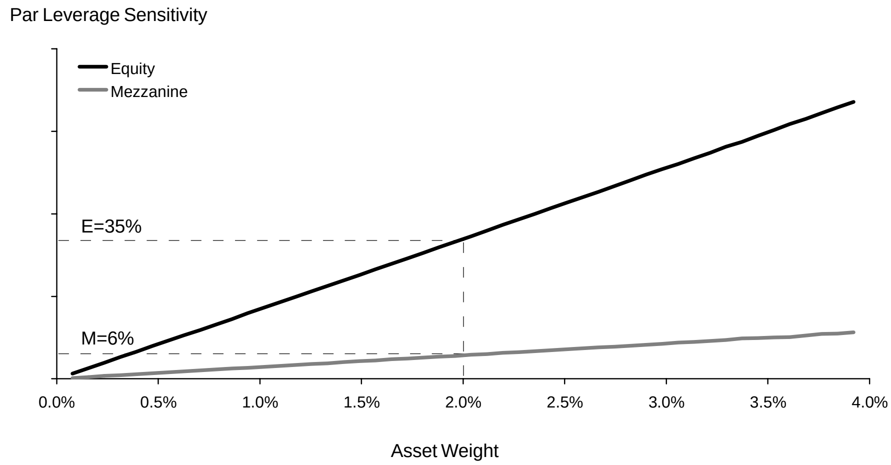
Par leverage sensitivity is one of the set of measures that can be used to analyze CDOs in a fashion comparable with conventional cash portfolios, the others being price and par sensitivity. See Appendix B for detailed definitions.

Our definition of par leverage sensitivity assumes that all other characteristics (coupon/spread and default risk) of the underlying security remained constant. Buying an asset at a cheaper price often implies that a higher credit risk might be associated with the bond. Such an effect can easily be incorporated by a linear combination of default risk sensitivity (the other partial derivative) with par leverage sensitivity. However, the higher credit risk is not the only reason for price discount—such discount may be due to technical or supply/demand factors, coupon step-up, or other bond features that are not related to creditworthiness of the borrower. Given the importance of the OC tests and recent deterioration of the over-collateralization levels in many seasoned CDOs, the "creation of par" has become an important activity on part of collateral managers. Therefore, it is especially important to gauge the investor's sensitivity to this form of increase of the portfolio leverage in the current market environment.

Par Leverage Sensitivity as a Function of Asset Weight

Let us first demonstrate the dependence of the par leverage sensitivity on underlying asset weight in the collateral. Consider the uniform default risk case (in which all assets have

Figure 3. **Par Leverage Sensitivity for CDO with Uniform Default Risk Collateral**

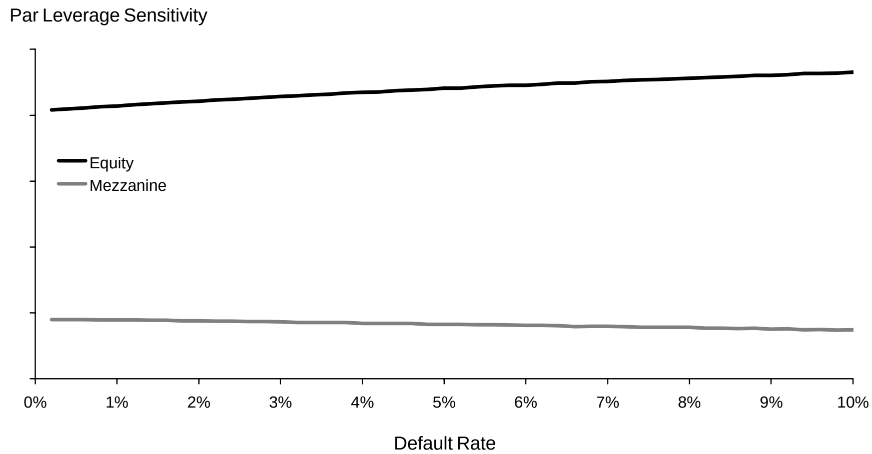


the same annual default probability but different asset weights) shown in Figure 3. For both equity and mezzanine tranches, the par leverage sensitivity is nearly linear and upward sloping, because, given our definition based on proportional change in par value, the assets with larger weight contribute greater amount of absolute OC cushion and arbitrage spread.

We pick one asset at a time, effect a 10% increase in par compared with its initial value, and observe par leverage sensitivity vis-à-vis the weight of the asset under consideration. For example, we take an asset that initially comprised 2% of the collateral weight and was priced at par and increase the face value to 2.2% of the portfolio while decreasing the price to 100/1.1 = 90.91, thereby maintaining the market value of this holding at 2% of collateral market value. We proceed to measure the impact of this change on the CDO tranches and find that the mezzanine tranche experienced 6.0%x10%=0.6% gain in PV, while the equity tranche experienced 35.0%x10%=3.5% increase in value. The equity tranche has higher par leverage sensitivity because it reaps the lion's share of excess returns due to increased cash flow arbitrage brought by over-collateralization. The mezzanine tranche also benefits from higher OC levels, but the gains are less pronounced. This is because there is no change in the maximum cash it can receive, but just an increase in the probability of receiving full coupon stream.

The level of the par leverage sensitivity values is quite easy to understand. The additional 0.2% of face value results in an eventual gain of that magnitude to equity tranche because we assume that non-defaulted bonds mature and pay out par at the maturity of CDO. This gain represents 5% of the equity tranche (recall that equity is 4% of capital structure). However, not all of it is assigned to equity tranche in the mark-to-market valuation (just as not all of the risk of default is assigned to it either). The difference between the naïve 5.0% gain and the measured 3.5% gain is roughly equal to the projected gain for mezzanine: 5.0%-3.5%=1.5% is approximately equal to the value that the extra

Figure 4. **Par Leverage Sensitivity for CDO with Equal Weight Collateral**



0.2% of par represent compared to the aggregate amount under mezzanine, i.e., 9%+4%=11% of the capital structure (0.2%/11% = 1.8%). The split between the equity and mezzanine portions of the par leverage impact is proportional to their respective leverage ratios of 25 (equity) and 9 (mezzanine).

Par Leverage Sensitivity as a Function of Asset Credit Risk

Considering the equal asset weight case, we can see the dependence of par leverage sensitivity on the default risk of the underlying assets. The results are plotted in Figure 4.

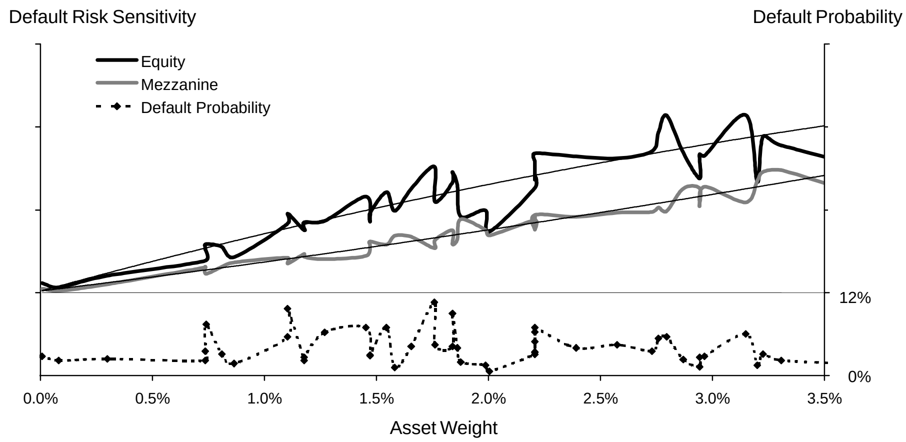
Par leverage sensitivity shows, not surprisingly, a milder dependence on the default rates than on the weight of the asset. It remains positive for all tranches for all names. However, the gains for debt tranches due to "creation of par" using names with higher credit risk (and, hence, higher spread) are lower compared with the case in which we use assets with lower credit risk. This is because over-collateralization produced using higher default rate is "riskier," while gains are capped at the level of LIBOR plus spread offered on the tranche. The equity tranche has no such cap, and the upside is more dominant than the default risk, leading to a moderately upward sloping line of par leverage sensitivity.

We notice that an asset management strategy of building par using discounted assets helps all CDO tranches as long as the asset's default risk has not changed significantly.

4. RISK CHARACTERISTICS OF TYPICAL CDO TRANCHES

The hypothetical cases considered in the previous section are important for understanding how default probabilities and asset weights determine the risk associated with individual names in the CDO collateral. In reality, however, we see a complex interplay of the two relationships. Assets with higher allocation may have low default probabilities and vice versa, thus "netting out" relative impact on risk. We look at a typical investment grade transaction to help us uncover this relationship. The collateral characteristics and the deal capital structure are defined in Appendix A.

Figure 5. **Default Risk Sensitivity for Typical CDO Equity and Mezzanine Tranches**



In Figure 5, we show the default risk sensitivity for the equity and mezzanine tranches of a typical investment grade CDO. Since we already know that the dependence on weight is far more dominant that the dependence on the initial default rate, we plot the relationship across the main dimension (asset weight) and show the other dimension (default probabilities) in a lower window of the chart. The line is jagged because we have assets with different credit risk but similar asset weight. Each point along the x-axis represents an asset in the collateral portfolio, shown in correspondence with its weight in the collateral. In reality, there are only a finite number of assets, 50, but for visual clarity, we plot a continuous curve. In the lower window, we show the annualized default probability for each of these assets (for this exposition, we are mostly concerned with the shape of the dependence distribution). The markers on this dashed curve correspond to actual collateral positions, highlighting the lumpy distribution of asset weights across the underlying portfolio.

In the upper window, we show the resulting default risk sensitivity of equity and mezzanine tranches. We see that the dependence on the collateral asset weight determines the trendlines, which are similar to those we have seen before in the case of uniform risk case study. The deviations of the annual default probability from the “uniform risk” case cause modulated dependence on the annual default probability, which is spread around the trendline and follows the shape of the annual default probability distribution curve. The scale of modulations is large, especially for the equity tranche, as can be expected from the strength of the default rate dependence in the special case of the equal weight portfolio CDO considered in the previous section.

An interesting observation is that for smaller asset weights, the modulations of the default risk sensitivity have opposite signs for equity and mezzanine tranches, while for larger asset weights, they appear to be in the same direction. The switch occurs around 2% asset weight, which is the average weight for a 50-name portfolio.

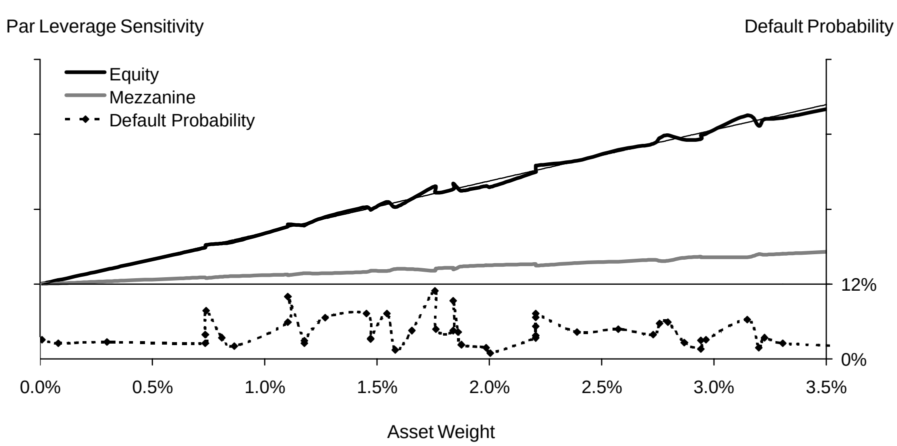
We conjecture that this reflects the complex interplay of risk apportioning among the tranches of the CDO. Namely, the marginally higher default risk for largest assets in the collateral directly affects both equity and mezzanine tranches. On the other hand, the smaller assets can affect the mezzanine only through correlated defaults with other bonds, since the equity tranche is large enough to bear a small loss both in par and in coupons. Now, remember that the equity holders essentially have a call option on correlated survival of bonds in collateral and that the writers of this option are the liability-holders of the CDO structure. The mezzanine note-holders carry most of the burden of this short call option—hence, the risk sensitivities of the mezzanine and equity are opposite in sign when the main driver of the dependence is the default correlation. Of course, this consideration is modulo the common dependence on asset weight that captures the central scenario of average default risk. However, for assets larger than a particular size, default effects spill over to the mezzanine tranche, making it more like equity (long call) than a debt or senior tranche (short call).

A similar pattern for par leverage sensitivity is shown in Figure 6. Remember that the default rate dependence for both equity and mezzanine was very mild in our investigation of an equal weight portfolio case. Therefore, it is not surprising that the scale of the modulations induced by the default rates in this typical portfolio is small, while the asset-weight driven trend line is clearly dominant.

5. MANAGING RISK EXPOSURES NAME BY NAME

At the simplest level, risk management of portfolio investments consists of controlling the exposures to common factors, as well as placing limits on specific risk exposures. This can be achieved by altering the weights of bonds in the portfolio or adding overlay trades. From the collateral manager's perspective, changing the weights in the portfolio is possible; hence, it forms the basis of risk management in practice. From an investor's

Figure 6. **Par Leverage Sensitivity for Typical CDO Equity and Mezzanine Tranches**



perspective, the CDO collateral weights are externalities that are not possible to change; thus, the overlay (hedge) portfolios are the key element of risk management practice.

Having determined the risk sensitivities of the CDO tranches, the natural question is how to hedge them if the investor's risk appetite requires putting certain limits on those exposures. As we discussed earlier in the article, two types of hedges are usually considered—the static (or "insurance") hedge and the dynamic (or mark-to-market) hedge.

By its very nature, a static hedge cannot be done for each and every name in the collateral, nor would anyone attempt such a hedge (an outright sale of the CDO investment would be a better exit strategy if exit is what the investor seeks). Indeed, the cost of such a hedge would be significantly higher than the full value of the CDO tranches under consideration, because of the following predicament—these hedges represent options (in a generic sense of this word) on a name-by-name basis, while a CDO tranche represents an option on a combination of names. The correlation of default risks is always non-negative, and an option on a combination of such risks is always cheaper than the sum of options on individual risks.

The dynamic hedging, on the other hand, takes into account the likelihood of various future scenarios, as it is based on the full re-simulation of the CDO tranche values under modified default risks of individual credits in collateral. The only situation in which a "complete default exposure" hedge may be warranted is when the probability of default is extremely high and, therefore, such a scenario becomes a central one in the valuation of not only this tranche, but also all others. Then there will be relatively little overhead in hedging the individual name risk in this manner. As the expected default risks grow, the dynamic hedges converge to the full "insurance" hedge, but the cost is less for two reasons:

1. Investors average into the hedge gradually, from relatively low risk (and hedge premium) levels until they reach high values.
2. Investors dynamically pick up or abandon the relevant names in their portfolio to hedge, avoiding the carry of premium for names that no longer represent significant risk exposure.

Of course, there remains an execution risk—since even "dynamic" hedges are likely to be revised only occasionally, there is always a chance that an unexpected large market move will leave the investor more exposed than was originally intended.

We also would like to spell out an important difference between the objectives of CDO investors and those of structurers and dealers. Dealers and structurers are not supposed to carry large credit risks on their books and have a short risk management horizon. It must not come as a surprise that they will typically hedge away as much of the residual risks on their books as possible. CDO investors, on the other hand, are being rewarded for carrying the credit risk. It is their goal to optimally choose these risks and to remain exposed to them. Hence a CDO investor will never wish to perform a wholesale or even a large-scale hedging of his/her investments. The dynamic hedging that we advocate for such investors refers to a targeted modification of the risk profile of their portfolios affecting only a handful of the exposures that have become less than optimal due to changing market conditions or risk appetite.

While some of the basic methodologies used by both types of market participants are similar, their application and outcomes are vastly different due to the substantially different objectives. In this report, we address the concerns of long-term credit investors and attempt to formulate a methodology suitable for their needs.

For a CDO investor, the question is—are there some names that are more important to hedge than the others? The answer to this question lies with both the investor's risk measurement capabilities and his forecast of the future default risks. Firstly, attention must be paid to those names that are on investor's "radar screen" for future default. If these names are held in the collateral in a significant amount, the obvious answer is to hedge the corresponding exposure. Secondly, attention must be paid to the names that significantly influence the performance of the CDO tranches regardless of whether they are on the investor's watch list. The two criteria together should be used to form the "top risk" list (e.g., top 5 or top 10 risk list) to be focused on.

Below, we list some of the instruments that an investor may use for hedging of individual name exposures:

- Default swaps. By far the most liquid and most relevant hedging instrument, especially for investment grade credits. The biggest problem with this hedge strategy is the loss of premium if the event that default doesn't materialize in the lifetime of the CDO. These hedges reduce not just the name exposure, but also the total credit exposure—something that not all investors in CDOs will necessarily want.
- Total return swaps against a properly chosen benchmark index or credit portfolio. We argue that this will, in many instances, be a better solution if the investor wishes to maintain exposure to the credit markets. Unlike in the previous case, the cost of carry can be much less for deteriorating credits that end up not defaulting until the maturity of the CDO. In particular, we would highlight the recently introduced TRAINS portfolios as a logical swapping vehicle for this strategy. Both their wide representation of the Lehman Credit Index and the available maturities of 5- and 10-year trusts make these ideal for investment grade CDO investors.
- Default baskets. A judiciously chosen default basket can be a better candidate for hedging of CDO risk exposures because of similarities in the loss distributions. After all, the baskets can be thought of as particularly concentrated CDOs. They are particularly useful for hedging correlation risks (which we did not consider in this paper).
- Bond put options (price or yield based). While this approach can theoretically work, we are not aware of sufficiently liquid markets to suggest this as a mainstream strategy.
- Short equity or long equity put options. These are generally not advised for investment grade investors, because their volatility is significantly higher than the volatility of the target investment and because their correlation with spreads is low for all but lowest-quality names. However, equity markets maybe quite relevant in cases of high yield or fallen angel names, and their use warrants a separate investigation, which is beyond the scope of this report.

6. MANAGING COMMON RISK FACTOR EXPOSURES

While controlling risk exposures name by name is a valid approach, it may not be optimal because it does not fully take into account the risk contributions of common factors and their cross-correlations. In particular, one should control the riskier exposures more tightly and may afford to loosen up the controls for less volatile factor exposures.

To illustrate this thesis, let us consider a simplified case of the single-common-factor log-normal spread/default rate assumption, which can thereafter be extended to multifactor analysis in a similar fashion.

6.1 Single (Log-Normal) Common Factor Case

While the default risk sensitivity dependence on weight and default rate in Figure 5 is illuminating, one must also remember that the larger default rates are typically associated with larger spreads and larger mark-to-market fluctuations. Recall that we are dealing with implied default rates, which are essentially proportional to the bond spreads. It is a well-known fact that the spread fluctuations are closer to a log-normal distribution (i.e., the constant proportional volatility) than to a normal one (i.e., constant absolute volatility levels), at least for the range of investment grade and crossover credits. Therefore, one can take a simplified view of a single-factor log-normal spread model and assume that a typical change in spread (and the corresponding typical change in default risk of collateral bonds) is given by a constant fraction of its initial level. In other words, the change in default probabilities of a AAA credit is lower than that of a BB credit, for example.

In Figure 7, we show a 3-dimensional graph of the marginal impact of a proportional 10% decrease in default probability of each asset on the value of the equity and mezzanine tranche. The two horizontal axes represent the asset weight in the collateral and the initial default probability for underlying bonds. The vertical axis represents the relative change in tranche value. Each peak here corresponds to one asset, with its coordinates given by its weight and initial default probability. It is evident how assets that have both large collateral weight and high default probability (the North East corner) have maximum impact on the performance of a tranche.

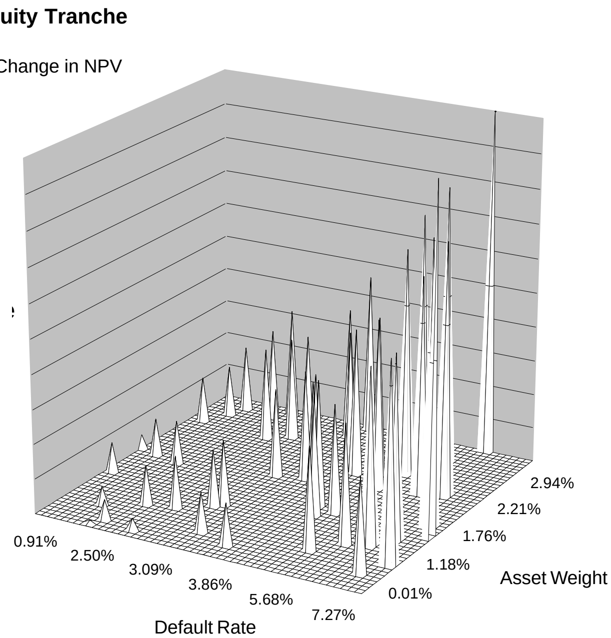
These marginal impact graphs give a clear picture of the disproportionate importance that is carried by those assets, which have both high weight and high default probability under the log-normal single-common-factor assumption. To hedge the impact of this factor, one must focus on the biggest marginal impact names.

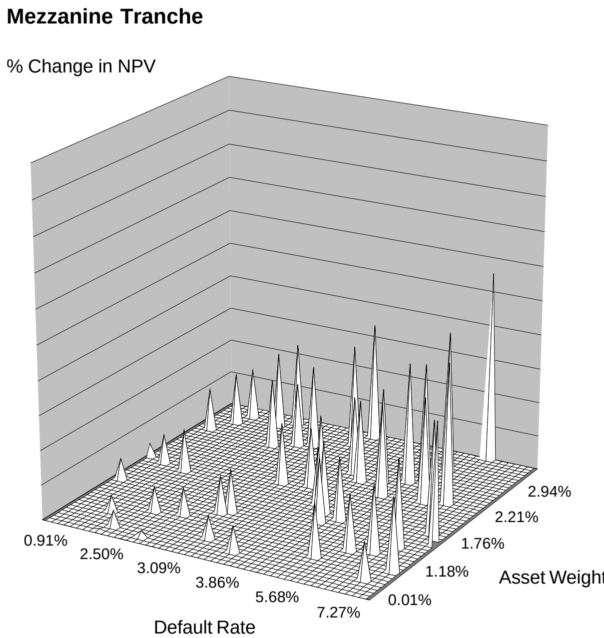
6.2 Extension to Multi-Factor Analysis

In order to generalize those results to the multi-factor world, one must go through several steps:

- Define the spread/default risk factor model. Estimate the factor time series, their correlations and volatilities.
- Define the name-by-name exposures for each factor. An example of this would be spread duration as an exposure of a bond to the common factor of industry spreads.
- Estimate the name-specific variations that are not explained by common factors.
- Define the measure of risk that is being optimized. This would depend on the investor's objective. For example, a total return investor with an intermediate holding horizon could choose the portfolio return standard deviation as the measure of risk, while an investor with a long-term horizon may alternatively choose a tail measure such as VaR or expected shortfall (see O'Kane, Schloegl [2002] for discussion).

Figure 7. **Marginal Impact of Proportional Default Risk Change**





- Define the risk decomposition procedure that unambiguously determines the contributions to risk coming from each of the common factors and from name-specific fluctuations.

Each of these steps requires a significant effort. Fortunately, not all of them need to be reinvented for CDO tranche risk management. Within the framework advocated in this paper, we consider the changes in spreads synonymous to changes in (implied) default rates under the constant expected recovery rate assumption. We take these changes as the sources of mark-to-market risk for CDO tranche investors. Therefore, we can use an existing model of credit spreads, such as the new Lehman Brothers multi-factor creit risk model (see Naldi, *et. al.* [2002]), in conjunction with our approach.

We emphasize again that the risk decomposition is an inherently portfolio concept. It is most useful in managing portfolios of cash instruments. It is applicable to CDOs insofar as each CDO note is also a portfolio product, albeit a non-linear one. Understanding the risks in a portfolio context implies understanding the correlations between various relevant parameters. In Lehman Brothers' CDO valuation model, we utilize the correlation of defaults. However, when one speaks of risk exposures, the relevant correlation is that of (implied) default rates (or equivalently spreads), which can be quite different.

Once the marginal risk decomposition methodology is defined, one can describe the CDO tranche, as well as arbitrary portfolio of CDO tranches and cash bonds, in terms of its common and name-specific exposures, on one hand, and in terms of its common and name-specific risk contributions, on the other. Together, these two profiles of the investor's portfolio provide the best information for optimal hedging and/or diversification of risk exposures. We will cover this subject in a future detailed publication.

7. SUMMARY

In this report, we have defined several risk sensitivity measures for CDO tranches and presented the results of their calculation using the Lehman Brothers Monte Carlo simulation model of tranched portfolios. A complete simulation of CDO performance and its dependence on various parameters, including characteristics of the underlying credit securities, is essential for correct measurement and management of CDO investment risk exposures.

CDO tranches, whether senior, mezzanine, or equity, continue to constitute a credit portfolio security and, as such, have many of the characteristics familiar to the managers of conventional cash corporate bond portfolios. We have attempted to highlight these similarities, as well as introduce variables that show the salient differences, such as the par leverage sensitivity of CDO tranches.

Identifying the CDO tranche risk exposures naturally leads to implementation of hedging strategies that can be employed to reduce or completely eliminate the exposures in the top risk bucket. We argued that dynamic hedging is preferable to "default insurance"-type hedging. We also discussed some of the particular hedging strategies involving single-name default swaps and total return swaps against a diversified credit portfolio such as the new Lehman TRAINS, as well as multi-name basket default swaps.

Further expanding our approach, we have outlined a way in which Lehman Brothers risk and valuation models can be synthesized to bring consistent advice to issuers, as well as investors, regarding the risk management of their CDO exposures.

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APPENDIX A. DEFINING THE PORTFOLIOS FOR CASE STUDIES

In this report, we attempt to clarify the distinction between the dependence of CDO tranche sensitivities on the underlying credit weight in the collateral portfolio and its expected default risk. The simplest way of segregating the impact of these parameters is to consider hypothetical portfolios that freeze one dimension and let us study the relationship along the second dimension. Then we can better understand the more complex dependence in the combined case, in which both parameters are allowed to vary.

We define a typical investment grade CDO capital structure. The term of the CDO is assumed to be six years. The OC test thresholds are shown in Figure 8 and are incorporated in our simulations. The IC tests are ignored in the current investigation, as their effect appears to be less important given the level of the initial arbitrage spread.

We consider three collateral portfolios—two hypothetical portfolios and a typical CDO. All three portfolios contain 50 bonds that have a 6-year maturity, coinciding with that of the CDO. All assets are assumed to be bought at par and pay a fixed coupon. The weighted average coupon (WAC) on all three portfolios is set around 8%, such that the cash flow arbitrage is the same across the three cases. The other characteristics of the portfolios are as follows:

1. Equal collateral weight portfolio. A hypothetical portfolio of 50 names that has equal amounts (2%) allocated to each name. The (implied) annual default probabilities are distributed linearly from 0.2% to 10%. The coupon on each name is chosen so that it fairly compensates for its annual default probability, i.e., the bond is priced at par.
2. Uniform default risk portfolio. A hypothetical portfolio consisting of 50 equally risky assets. However, assets are allocated different weights. We have taken weights distributed linearly so that 50 assets add up to 100%. This implies a uniform (discrete) distribution with weights ranging from 0.1% to 3.9%.
3. Typical Investment Grade Portfolio. The third portfolio closely resembles a typical investment grade CDO, with 5 A, 30 BBB and 15 BB rated bonds. The annual default probabilities range from 0.9% to 10.9%, and the weights range from 0.01% to 3.68%.

In our simulations, we assume a flat default correlation of 25% (in the context of the Merton-like asset-based model of defaults). The dependence on default correlation is beyond the scope of this paper. We evaluate 10,000 Monte Carlo scenarios for each base case and each risk sensitivity measurement. To simplify the dependence on interest rates, we take a flat LIBOR curve set at 5.5%, near the current 7-year swap level. The shape of the LIBOR curve can be important if the collateral is not fully hedged by interest rate swaps.

APPENDIX B. EFFECTIVE PORTFOLIO REPRESENTATION OF CDO TRANCHES

In the case of structured credit portfolio securities, one must recall that the value of a given CDO note is not equal to the simple sum of the bond values in the collateral portfolio. Because of this non-linearity induced by the structuring rules, one cannot simply aggregate the risk sensitivities of the underlying bonds

Figure 8. **Typical CDO Capital Structure**

Tranche	**Size**	**Spread** (over LIBOR)	**OC Tests**
A (Senior)	87%	40	107%
B (Mezzanine)	9%	220	101%
C (Equity)	4%	N/A	N/A

Instead of a simple sum, the market value of a CDO note is a nonlinear function of the set of quantities and prices of the collateral bonds and other parameters of the structure, such as the tranche weights, coupons, IC and OC tests, hedges, etc., which we summarily denote by {Struct}:

[4] $$PV^{I}_{tranche} = PV^{I}_{tranche}\left(\{q_n\},\{P_n\},\{\text{Struct}\}\right)$$

where q_n stands for the quantity of the bond in collateral portfolio (in units of face value), and P_n stands for its price. In case of a simple bond portfolio, the formula degenerates into

$PV_{port} = \sum_{n=1}^{N} q_n \cdot P_n$, as we noted above.

First, let us introduce the tranche *price sensitivity*[5] with respect to the underlying asset as the relative sensitivity of the tranche value with respect to a fractional change in the underlying bond's price (while keeping the quantity constant). This quantity is important because it coincides with the hedge ratio in case of a single-name hedging strategies.

[5] $$\eta^{I}_{n} = \left.\frac{\Delta PV^{I}_{tranche} \big/ PV^{I}_{tranche}}{\Delta P_n \big/ P_n}\right|_{q_n=\text{const}}$$

Next, let us introduce the *par sensitivity* measure as the relative sensitivity of the CDO tranche value with respect to proportional change in the underlying bond quantity (or par value):

[6] $$\theta^{I}_{n} = \left.\frac{\Delta PV^{I}_{tranche} \big/ PV^{I}_{tranche}}{\Delta q_n \big/ q_n}\right|_{P_n=\text{const}}$$

The usefulness of these two quantities becomes clear when we try to calculate them for a cash bond portfolio:

[7] $$\eta^{port}_{n} = \theta^{port}_{n} = \frac{q_n \cdot P_n}{PV_{port}} = w_n$$

In case of a cash portfolio, the price and par sensitivities are simply equal to the weight of the asset in a conventional sense. The price sensitivity generalizes the notion of the underlying security weight for a structured security tranche. We emphasize that the conventional weight measure is still meaningful for CDOs as well; however, it is related to the *collateral portfolio*. The price and par sensitivities, on the other hand, are related to a *particular tranche*.

5 This characteristic is similar to a familiar beta measure. However, beta measures are usually used with respect to common factors; therefore, we avoid denoting this quantity as tranche beta.

Finally, we introduce the *par leverage sensitivity*, i.e., relative sensitivity of the CDO tranche value with respect to proportional change in the underlying bond purchase price. We define the procedure for changing the underlying collateral portfolio as follows: we increase/decrease the price of a given asset by a certain amount while keeping the total value intact by offsetting change in quantity. For example, we reduce the price of asset A by 10% (from 100 to 100/1.1=90.91) and simultaneously increase the face value by 10% (from \$1 million to \$1.1 million) while keeping the total market value intact at \$1 million. Buying cheaper bonds to fill the same amount of market value is one of the ways to "create par" and increase the OC ratios of the CDO, thereby also increasing the cash flow arbitrage. This is why we call this quantity par leverage sensitivity. Note that for a conventional bond portfolio, the total portfolio value is independent of such a change; therefore, the par leverage sensitivity for cash portfolio is always zero.

[8] $$\pi_n^I = \left.\frac{\Delta PV_{tranche}^I / PV_{tranche}^I}{\Delta P_n / P_n}\right|_{PV_n = \mathrm{const}} = -\left.\frac{\Delta PV_{tranche}^I / PV_{tranche}^I}{\Delta q_n / q_n}\right|_{PV_n = \mathrm{const}}$$

The three sensitivities introduced above are related to each other in a simple way—the price sensitivity is equal to par sensitivity plus the par leverage sensitivity:

[9] $$\eta_n^I = \theta_n^I + \pi_n^I$$

For a cash bond portfolio, the par leverage sensitivity is zero, and both the par sensitivity and price sensitivity are equal to bond's conventional portfolio weight.

Thus, the portfolio representation of a structured product requires two independent measures — the price and par leverage sensitivities, which together generalize the single notion of the portfolio composition (weight) known for cash portfolios.

A Counterparty Risk Framework for Protection Buyers

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In a default swap contract, the protection buyer is hedged only with respect to the default of the reference credit as long as the counterparty does not default first. In this paper, we analyze this risk and provide a simple framework for quantifying it. We describe how to calibrate and implement the underlying model. The proposed framework can easily be applied at a portfolio level, where it provides a conservative estimate of risk per counterparty.

1. INTRODUCTION

Credit default swaps are bilateral *over-the-counter* derivative contracts. As such, they introduce counterparty risk between the two participants. They may, however, involve the use of collateral and margining in order to mitigate any such risk. Nevertheless, the risk underwritten by a default swap is default risk—one that has a low probability of occurrence but a significant loss if it does. The resulting large change in mark-to-market, and the associated jump in counterparty risk, may not be adequately caught by any collateralization process. The aim of this paper is to introduce a simple yet effective methodology for monitoring this risk, one that can easily be implemented and used at a portfolio level.

2. COLLATERALIZATION AND THE ISDA MASTER AGREEMENT

In practice, where there is any concern about counterparty default risk, collateral posting will generally be used. This usually involves the weaker counterparty depositing either cash or high-quality securities to the stronger counterparty. Often, the collateral agreement will be dynamic so that the amount of collateral posted changes with the mark-to-market of the position. This may even mean the stronger counterparty posting collateral to the weaker counterparty if the mark-to-market of the position is significantly in the favor of the weaker counterparty.

Collateral postings generally occur weekly or monthly. In certain circumstances, they may occur as often as daily, depending on the concerns of the credit department about the volatility of the mark-to-market and any decline in the credit quality of the counterparty.

Almost all credit derivatives are transacted within the framework of the ISDA master agreement. This provides for many eventualities and includes such features as enabling the termination of a default swap contract following the failure of a counterparty to honor the terms of another ISDA agreement. It also enables the netting across other ISDA contracts, such as interest rate swaps, which should result in some reduction of the total outstanding counterparty exposure.

In terms of seniority, all ISDA contract holders with a defaulted counterparty are ranked *pari passu*. Some may have collateral agreements and can take possession of the collateral up to the value of their loss in the event of counterparty default. Within the capital structure of the counterparty, ISDA claimants are *pari passu* with senior unsecured bond holders, who, according to Moody's default statistics, recover on average 50% of their face value in the event of default.[1] We note that recent recovery rates have been lower than the historical average.

[1] According to Moody's special comment "Default & Recovery Rates of Corporate Bond Issuers" of February 2002, the average recovery rate during the period 1982 – 2001 for senior unsecured bonds with an investment grade rating one year prior to default was 52.48%.

3. THE ASYMMETRY OF DEFAULT SWAP COUNTERPARTY RISK

The counterparty risk of a default swap depends very much on which side of the transaction one has entered into. Let us consider each in turn:

Protection Seller

For the protection seller, usually an investor, the risk can be characterized within a single scenario in which the counterparty (protection buyer) unilaterally stops paying its contractual spread due to default or other reason.

Since a default swap is unfunded, there is no loss of par to the protection seller; we ignore the case of the credit-linked note in which the investor does indeed pay par and expects par if there is no default of the reference credit. In this case, the counterparty risk is effectively the risk of the collateral in the trust or SPV which issues the credit-linked note. This should be of a high credit quality acceptable to the investor, and would ideally be uncorrelated with the reference asset of the note.

To counter this loss of spread, the investor can terminate its provision of protection under the ISDA "events of default". The investor can decide whether or not immediately to reinstate this short protection. There may be some mark-to-market gain or loss associated with this. As a percentage of the par amount, the downside risk is generally small, especially for investment grade credits. For example, a \$10 million notional 5-year default swap initially sold at a spread of 150 bp which is followed 2 years later by a narrowing of spreads to 50 bp, at which point the default swap risky PV01 is 2.5, results in a mark-to-market gain of 2.5% of the notional, i.e., \$250,000. If the counterparty defaults with zero recovery, then this is the loss to the investor.

For the protection seller, the risk is due to a narrowing of spreads. A number of factors work in the investor's favor to mitigate the counterparty risk

1) Spread narrowing, unlike spread widening, is generally a gradual process. Hence, any margining agreement should be able to collateralize most of the counterparty exposure prior to the default of the counterparty.
2) The size of the mark-to-market of a short protection position is capped by the fact that default swaps will never trade with negative spreads.
3) Spread and default correlation are generally positive. A reference credit which improves while the counterparty experiences a default is much less likely than a reference credit which deteriorates at the same time as the counterparty defaults, in which case the mark-to-market would be negative.

Protection Buyer

Compare this with a long protection position. In this case, the protection provider is essentially short the reference credit. Here we can characterize their counterparty risk through two scenarios:

1) The credit quality of the reference credit deteriorates, and the mark-to-market of the protection becomes more valuable. The asset does not default. The protection buyer attempts to monetize this gain by closing out the swap with the counterparty only to find that the counterparty defaults and is unable to make good the mark-to-

market of the long protection position. Some recovery amount may be paid on the owed mark-to-market.

In this scenario, the risk is greater than that of a protection seller since the mark-to-market gain can be significant. For example, a $10 million notional 5-year default swap initially sold at a spread of 100 bp which is followed 2 years later by a widening of spreads to 350 bp, at which point the default swap risky PV01 is 2.0, results in a mark-to-market gain of 5.0% of the notional, i.e., $500,000. If the counterparty defaults with zero recovery, then this is the loss to the investor.

We would expect that such a widening, which does not result in default, would be captured in a dynamic collateral agreement. We therefore focus on scenario 2. This is by far the most important risk:

2) In this scenario, the reference credit experiences a sudden credit event and the protection buyer is unable to receive the protection payment, as the protection seller has defaulted. The protection buyer is left with the defaulted asset and no protection. In this scenario the size of the loss is significant – equal to *(1-R)* times the face value of the protection where *R* is the recovery rate of the defaulted asset.

For the protection buyer, the risk is due to a worsening of the credit quality of the reference credit combined with an earlier default of the counterparty. The size of the potential loss is much greater than that of a short protection position. Also, this scenario is consistent with our observation that default correlation is almost always positive.

These factors make us focus on the second scenario as presenting the most counterparty risk to the protection buyer. It is only in the event of a sudden default of the reference credit that the collateral posting fails to capture the change in mark-to-market.

In Figures 1 and 2, we show examples of the two reference credit scenarios similar to those described above. Figure 1 shows indicative default swap spreads for British Airways. We see that these widened dramatically in the wake of September 11 and have been tightening gradually since. Generically, we tend to observe that credit spreads widen rapidly, whereas credit spread tightening is a much more gradual process.

Figure 2 shows the case of Enron, whose extremely rapid decline and subsequent default effectively took the default swap market by surprise. It is in a situation such as this that the protection buyer is most exposed to the credit risk of the counterparty.

4. EXPECTED LOSS OF A LONG PROTECTION POSITION

There are a number of ways to quantify the counterparty risk, which range in sophistication. Whatever measure we choose, we would at least like to be able to capture the following risk dimensions:

1) Credit quality of counterparty
2) Credit quality of reference asset
3) Recovery rate of counterparty
4) Recovery rate of reference credit

Figure 1. **Indicative Spreads for 5-Year Protection on British Airways in EUR**

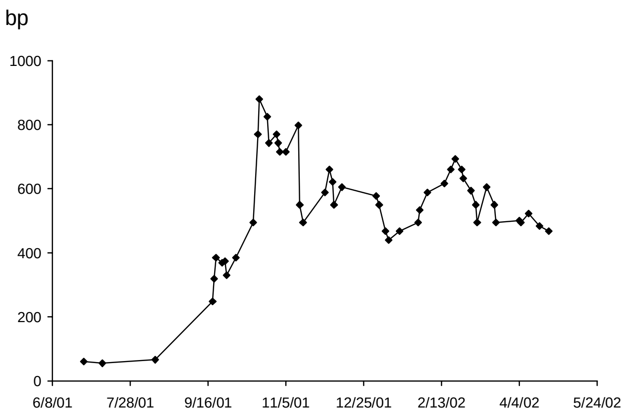
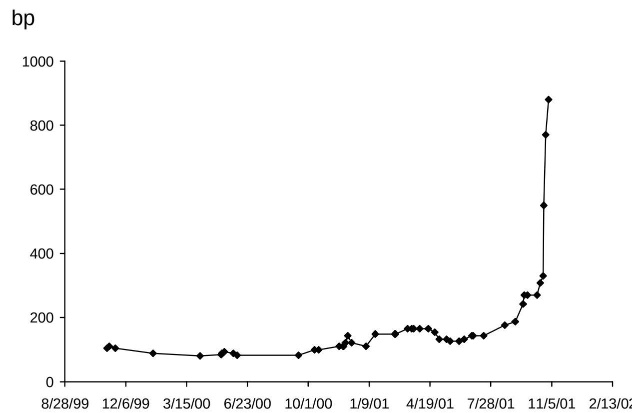


Figure 2. **Indicative Spreads for 5-Year Protection on Enron in USD**



5) Correlation between default of reference asset and default of counterparty
6) Order of default

We would also like a measure which is additive at a portfolio level. The simplest metric which captures all of these criteria is the Expected Loss (EL) defined as

$$EL = \alpha P_{A,CP}(T)(1-R_{CP})(1-R_A) \tag{1.1}$$

where the terms are

α: a factor which takes into account the ordering of the defaults—if the reference credit defaults before the counterparty, then there is no problem for the protection buyer. The derivation of α is described in section 6.

$P_{A,CP}(T)$: the joint probability of default for the reference credit and the counterparty. This is a function of the horizon date, the respective credit qualities of reference credit and counterparty, and their default correlation.

R_{CP}: the expected recovery rate of the counterparty. If this is 100%, then the expected loss is zero, as the defaulted counterparty can actually pay out all of the protection.

R_A: the expected recovery rate of the reference asset. If this is 100%, then the expected loss is zero, as there was no actual loss on the reference credit and the payoff from the default swap is zero.

T: the horizon of the trade or that to which the risk is measured

Some explanation is required on the assumptions made in the use of the expected loss measure described above. These are:

1) We have assumed that it is scenarios of the second type described in the previous section which present the main counterparty credit risk.
2) The measure is the expected loss of the long protection position in the event that the protection is not paid. This requires the counterparty to default either at or before the default of the reference credit.
3) The measure takes no account of the fact that if the counterparty is observed to default before the default of the reference credit, the protection buyer could reinstate the protection at some cost (which may be significant). In a sense, our approach assumes that the protection buyer purchases protection and then does nothing until the horizon date. As a result, this is a static risk-measure and so is conservative.
4) Equation (1.1) ignores time value by failing to discount the expected loss from the time of default. In this sense, we are quantifying the actual loss and not its present value. This also makes this risk measure conservative.

Finally, we note that expected loss is not an intuitive risk measure, which is why in a later section we convert it into the equivalent AAA-notional, i.e., the notional amount of AAA bonds which would have the same expected loss.

5. CALCULATING THE JOINT DEFAULT PROBABILITY

The term $P_{A,\,CP}(T)$ quantifies the probability that both the reference credit and the counterparty default within some horizon T. To calculate this joint probability, we need to find a way to calibrate the joint behavior of assets to market observables. One way to do this is to introduce a model in which an issuer j defaults if the return V_j of its internal firm value falls below some threshold K_j. If we assume that the distribution of asset returns is normal, then for each issuer we can find a threshold K_j so that

$$P_j(T) = P\left[V_j \le K_j\right] = N\left(V_j\right) \tag{1.2}$$

where $N(x)$ is the cumulative distribution function of the standard normal distribution. Calibrating the threshold for each issuer to the corresponding default probability means that

$$K_j = N^{-1}(P_j(T)) \tag{1.3}$$

Two issuers i and j both default before some time horizon T if both their asset returns fall below their respective thresholds. The probability of this is given by

$$P_{i,j}(T) = \Phi\left(N^{-1}\left(P_i(T)\right), N^{-1}\left(P_j(T)\right), \rho\right) \tag{1.4}$$

where $\Phi(x, y, \rho)$ is the distribution of the bivariate normal distribution,[2] and ρ is the correlation between the asset returns of issuer i and j. Note that because we are calibrating the individual issuer thresholds, we are actually making an assumption

[2] An excellent approximation is given in Hull's *Options, Futures, and other Derivatives,* Appendix 11B.

about the copula[3] function describing the dependence mechanism rather than about the marginal distributions.

The attraction of this approach is that it is possible to link the correlation of asset returns to the correlation of observable equity returns of issuer *i* and *j*. The link is based on the observation that within a Merton style framework, equity can be characterized as a call option on the value of the firm. Over a short time period, we can link a change in the value of the equity to a delta-weighted change in the firm value, where delta is the firm-value sensitivity of this call option. For different assets, it is then possible to show that, over short time periods, the correlation of the asset returns equals the correlation of equity returns.

Because we are dealing with a situation in which there are only two issuers, it is easy to write down the dependence structure explicitly. We can write the counterparty's asset return V_{CP} as

$$V_{CP} = \rho V_A + \sqrt{1-\rho^2}\, Z \tag{1.5}$$

where V_A is the asset return of the reference credit and Z is a normally distributed variable which is independent of V_A. Clearly, this construction is such that

$$Corr(V_A, V_{CP}) = \rho \tag{1.6}$$

The correlation structure of equation (1.5) is also used to determine the order of default parameter α, as we shall now describe.

6. THE ORDER OF DEFAULT

Knowing the joint default probability for two assets tells us nothing about the tendency of one asset to default before another. In many instances, this information is unnecessary. For example, in a first-to-default basket on two names, both with the same expected recovery, the present value is a function of the probability of one or the other or both of the assets defaulting within this time horizon. This depends on the joint default probability; however, the order of the default in the case of joint default is not needed.

When the issue is counterparty risk, the order of default plays an important role. This is so because a protection buyer loses only if the counterparty defaults before the reference credit. We therefore need to determine the factor which represents the fraction of all joint defaults within the time horizon in which the counterparty defaults before the reference asset. If we denote the default times of the counterparty and the reference credit by τ_{CP} and τ_A, respectively, then the adjustment factor α is given[4] by

$$\alpha = \frac{P\left[\tau_{CP} < \tau_A < T\right]}{P_{A,CP}(T)} \tag{1.7}$$

[3] A copula is a function which expresses the joint density as a function of the individual marginal distributions.
[4] Because we are dealing with continuous distributions, we don't have to worry whether the inequalities are strict or not.

Clearly

$$P\left[\tau_{CP} < \tau_A < T\right] + P\left[\tau_A < \tau_{CP} < T\right] = P_{A,CP}(T) \tag{1.8}$$

If the counterparty and reference credit are of the same credit quality, then by symmetry the two probabilities on the left-hand side of equation (1.8) are equal. This implies that

$$\alpha = \frac{1}{2} \tag{1.9}$$

meaning that if both assets default within the time horizon, there is a 50% chance that the counterparty will default first. When the counterparty and reference credit have a different credit quality, we need to use a model to calculate α. This is the subject of the next section.

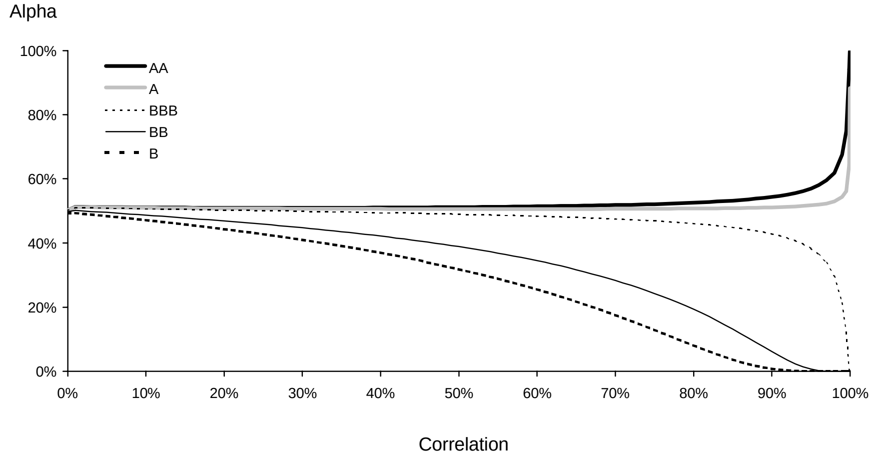
7. CALCULATING ALPHA FOR DIFFERENT CREDIT QUALITY REFERENCE CREDITS

In practice, both reference credit and counterparty will have different credit qualities. The counterparty will typically have a rating in the range of high A to AA. The reference credit will also be generally of investment-grade quality but typically further down the ratings spectrum.

Using the asset return based model described above, we have calculated and plotted the value of α for an A- rated counterparty and for reference credits of different ratings in Figure 3. The methodology is described in the appendix.

In general, we find that α does not deviate significantly from 0.5, at least for low correlations. Why this is so is related to the fact that even an asset with a low rating, such as a BBB asset, has only a 2.5% 5-year cumulative default probability. This maps to an

Figure 3. **Dependency of a on Correlation for a Counterparty of Rating A- and Reference Credits with Ratings of AA, A, BBB, BB, and B**



expected time to default of around 200 years![5] Hence the default probability is almost flat over the life of the trade. The same applies to the higher rated counterparty.

So, conditional on the fact that both assets have defaulted, the probability that the lower-rated reference asset was first is only a little bit greater than 50%. Hence, the value of α falls only slowly.

At higher correlations, both assets have a greater tendency to default together, as their asset returns are very similar. In the limit of 100% correlation, they are identical. Due to the way the asset return is mapped to the default time, this implies that the weaker asset always defaults first. If the reference credit is weaker than the counterparty, then α tends to one. Conversely, if the counterparty is the weaker credit, then α tends to one.

8. CALIBRATING THE MODEL

Basically, three inputs are required in order to calibrate this model:

1) The default probability of the reference credit over the time horizon
2) The default probability of the counterparty over the time horizon
3) The equity correlation between counterparty and reference credit.

The first two inputs are generally taken from rating agency default statistics such as those shown in Figure 4.

[5] A 5-year cumulative default probability of 2.50% has a hazard rate approximately equal to 0.50%. The expected time to default is given by the reciprocal of the hazard rate, i.e., 1/0.50%= 200 years.

Figure 4. **S&P Idealized Cumulative Default Probabilities by Rating and Horizon**

	Horizon (Years)									
S&P Rating	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
AAA	0.02	0.06	0.12	0.19	0.28	0.39	0.52	0.66	0.82	0.99
AA+	0.02	0.07	0.14	0.24	0.36	0.50	0.66	0.84	1.03	1.25
AA	0.11	0.24	0.39	0.57	0.76	0.97	1.20	1.45	1.71	1.99
AA-	0.14	0.29	0.46	0.66	0.88	1.11	1.37	1.65	1.95	2.26
A+	0.14	0.30	0.50	0.73	0.98	1.26	1.57	1.90	2.24	2.60
A	0.14	0.32	0.54	0.81	1.11	1.45	1.81	2.20	2.61	3.04
A-	0.14	0.36	0.63	0.96	1.33	1.74	2.17	2.63	3.11	3.60
BBB+	0.22	0.53	0.91	1.35	1.84	2.37	2.92	3.49	4.07	4.66
BBB	0.22	0.64	1.18	1.81	2.50	3.21	3.94	4.67	5.38	6.08
BBB-	0.54	1.36	2.32	3.34	4.39	5.42	6.41	7.36	8.26	9.11
BB+	1.67	3.32	4.92	6.44	7.87	9.19	10.41	11.52	12.55	13.49
BB	2.77	5.26	7.50	9.49	11.25	12.82	14.20	15.42	16.50	17.47
BB-	2.79	5.67	8.38	10.83	12.97	14.83	16.44	17.82	19.01	20.04
B+	3.67	7.53	11.08	14.12	16.66	18.74	20.44	21.84	23.00	23.98
B	8.59	14.51	18.59	21.45	23.49	25.00	26.15	27.07	27.82	28.45
B-	9.56	16.63	21.56	24.96	27.32	28.99	30.21	31.14	31.88	32.50
CCC+	14.69	23.40	28.70	32.02	34.20	35.69	36.76	37.58	38.22	38.76
CCC	19.82	30.18	35.83	39.09	41.08	42.39	43.32	44.01	44.56	45.02
CCC-	46.55	53.45	57.22	59.39	60.72	61.60	62.21	62.67	63.04	63.35

The third input is the equity correlation. We use this to produce a joint default probability as described in equation (1.4) and, hence, a value for α using the methodology of the appendix. We can calculate equity correlation by doing a straightforward calculation using the two time series of equity returns. Some exponential weighting scheme may be adopted to add greater importance to recent data. Alternately, we can build a factor model based on factors including country and industry classification.

9. EQUIVALENT AAA-NOTIONAL AS A RISK MEASURE

The expected loss is not a highly intuitive measure, especially as the expected loss in this context is usually small due to the low likelihood of both reference credit and counterparty defaulting. One way to put the expected loss number in context is to determine the equivalent position in a AAA-rated security which would create the same expected loss as the counterparty risk due to the position. The smaller this notional is, the less exposed the protection buyer is to losing its default protection just when it is needed. We can therefore write the equivalent AAA notional as:

$$N_{AAA} = \frac{EL}{P_{AAA}(T)(1-R)}$$

where $P_{AAA}(T)$ is the cumulative default probability of a AAA asset to horizon T. As the 5-year AAA cumulative default probability equals 0.28%, assuming a 50% recovery, the AAA 5-year expected loss would be 0.14%. In Figure 5, we show examples where all maturities are assumed to be 5 years and the notional of the reference credit and default swap protection is \$10 million. Correlations have been estimated using a factor model.[6]

For each example, we show the computed value of α, the expected loss and the risk equivalent AAA notional. Additionally, we show the risk equivalent AAA notional one obtains using the approximate value of 0.5 for α and the relative difference between the two values.

[6] We would like to acknowledge Marco Naldi, who constructed our proprietary equity factor model which provided the equity correlations used in this analysis.

Figure 5. **Example Positions for Counterparties and Reference Credits of Different Credit Qualities with the Expected Loss Expressed as the Equivalent AAA-Rated Notional for a \$10 Million Protection Notional**

Counterparty	Reference Credit	Correlation	Alpha	Exact EL In Dollars	Equivalent Risk AAA Notional	Equivalent Risk Notional with α = 0.5	Relative Difference
US AA Bank	UK AA Financial	12.30%	50.00%	166	116,894	116,894	0.00%
US AA Bank	US BBB Energy	14.20%	49.05%	524	369,033	376,216	1.95%
US AA Bank	JPY BB Industrial	2.95%	48.63%	1,181	832,024	855,532	2.83%
US AA Bank	DEM B Telecom	4.10%	46.79%	2,393	1,685,918	1,801,709	6.87%
US A Bank	UK AA Financial	12.30%	50.55%	237	167,003	165,200	-1.08%
US A Bank	US BBB Energy	14.20%	49.33%	746	525,568	532,719	1.36%
US A Bank	JPY BB Industrial	2.95%	48.68%	1,726	1,216,326	1,249,225	2.70%
US A Bank	DEM B Telecom	4.10%	46.88%	3,500	2,465,612	2,629,653	6.65%

We can clearly see that the effects of correlation and credit quality are as expected. For example, we see that reference credits with a lower rating produce a higher equivalent AAA notional than those with a higher rating. The risk measure is also higher when correlations are higher and when the counterparty credit quality is lower.

We see the computed value of alpha is quite close to 0.5 in most cases. Only if the rating of the reference credit goes down to B do we see a larger deviation. Note that alpha decreases if the credit quality of the reference asset is lower than that of the counterparty. This means that using an approximate value of 0.5 gives a conservative estimate of the expected loss.

Overall, we note that the size of the equivalent risk measure is actually quite low, reflecting the low probability of joint default. For example, in the case of a $10 million position in a U.S. BBB energy company with a U.S. AA rated counterparty, the 5-year expected loss is $524. This equates to an additional AAA position with a $369,000 notional. Even in the case of a $10 million B-rated telecom exposure with a U.S. A-rated bank as counterparty, the counterparty risk corresponds to a 5-year expected loss of $3,500. In terms of equivalent AAA exposure, this corresponds to adding a $2.47 million AAA position to the portfolio. And we remind the reader that our risk measure is conservative for the reasons described in Section 4.

10. CONCLUSIONS

In this paper, we have presented a simple framework for measuring the counterparty risk for protection buyers. We have argued that the significant risk scenario for investors is sudden default, and we believe that our framework captures this and does so in a conservative way. The risk metric used is the expected loss, which depends on all of the important risk factors at a single trade level. At the same time, the framework is simple enough to be easily implemented and has the advantage that expected loss is additive and so can easily be extended to a portfolio. The calibration of the model is relatively straightforward, depending on market observables such as equity correlation and rating agency cumulative default probabilities.

To go beyond such a framework would require a portfolio credit model which would capture not only the default correlation risk between the reference credit and counterparty but also that between different counterparties and different reference credits. The incorporation of spread dynamics would also add to the ability to capture a broader range of scenarios. This framework would be a considerably greater undertaking, probably requiring a simulation-based approach. For this reason, we would recommend our simpler framework as a significant first step in the managing of default swap counterparty risk.

APPENDIX A. COMPUTING ALPHA

We show how to derive the adjustment factor α in the case that the counterparty and the reference credit are of different credit qualities. We denote the default time of issuer j by τ_j and assume that it is exponentially distributed with a constant hazard rate of λ_j. The default time is related to the issuer's asset return V_j via

$$1-\exp\left(-\lambda_j \tau_j\right) = N\left(V_j\right) \tag{1.10}$$

This specification matches well with our economic interpretation; large positive asset returns (which are good for the issuer) are associated with large default times. From equation (1.10) it follows that $\tau_{CP} \leq \tau_A$ if and only if

$$N\left(-V_{CP}\right) \geq N\left(-V_A\right)^{\frac{\lambda_{CP}}{\lambda_A}} \tag{1.11}$$

Using the correlation structure described in equation (1.5), we see that this is, in turn, equivalent to

$$Z \leq \frac{-\rho V_A - N^{-1}\left(N\left(-V_A\right)^{\frac{\lambda_{CP}}{\lambda_A}}\right)}{\sqrt{1-\rho^2}} \tag{1.12}$$

Conditional on the reference credit's asset return, the probability that the counterparty will default before the reference credit is given by

$$P\left[\tau_{CP} \leq \tau_A \mid V_A\right] = N\left(\frac{-\rho V_A - N^{-1}\left(N\left(-V_A\right)^{\frac{\lambda_{CP}}{\lambda_A}}\right)}{\sqrt{1-\rho^2}}\right) \tag{1.13}$$

For the computation, it is more convenient to condition on the uniform random variable $U = N(-V_A)$. The conditional default probability is then

$$\mathrm{P}\left[\tau_{CP} \leq \tau_A \middle| U\right] = N\left[\frac{1}{\sqrt{1-\rho^2}}\left\{\rho N^{-1}(U) - N^{-1}\left(U^{\frac{\lambda_{CP}}{\lambda_A}}\right)\right\}\right] \tag{1.14}$$

The default time τ_A is before the horizon T if and only if

$$U \geq \exp\left(-\lambda_A T\right) =: U_0 \tag{1.15}$$

Therefore, the unconditional probability is given by

$$\mathrm{P}\left[\tau_{CP} \leq \tau_A\right] = \int_{U_0}^{1} N\left[\frac{1}{\sqrt{1-\rho^2}}\left\{\rho N^{-1}(u) - N^{-1}\left(u^{\frac{\lambda_{CP}}{\lambda_A}}\right)\right\}\right] du \tag{1.16}$$

This one-dimensional integration can be performed numerically.

Cross-Currency Credit Explained

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By accessing foreign credit markets, credit investors can add diversity to their portfolio and take advantage of relative value opportunities. In this paper, we explain how investors can do this using the cross-currency asset swap. For investors concerned about the associated default contingent risk, we introduce and analyze the perfect asset swap structure. We also describe how currency effects play a role in the credit default swap market in terms of pricing and currency risk.

1. INTRODUCTION

Most credit investors focus solely on assets denominated in their base currency. This restricts their flexibility in two important ways. First, it limits their ability to capitalize on the possibility of improved diversification gained by accessing credit names that never issue in the investor's base currency. Second, it prevents them from taking advantage of relative value opportunities, particularly when the foreign-denominated debt trades cheap to debt of the same issuer issued in the investors' base currency. This is an effect which is particularly prevalent at the moment, when we are seeing the dollar credit spreads of some issuers trade wide to European levels. In this paper, we answer the question: how can we enable the credit investor to access these cross currency opportunities without adding significantly to their currency risk?

We begin with a discussion of the basic mechanism that enables investors to convert foreign credit assets into domestic assets: the cross-currency asset swap. The cross-currency asset swap removes the investor's interest rate and currency risk as long as the asset never defaults. If the asset does default, there is a sizeable currency and interest rate risk. While this default contingent interest rate and currency risk may be an acceptable risk for high grade credits, since the likelihood of default is low, it may not be for crossover or high-yield names.

For this reason, we introduce the concept of the perfect asset swap. This is a special structure in which the investors give up some of their spread for the asset swap seller (Lehman Brothers) to assume the default contingent currency and interest rate risk. This leaves the credit investors with exactly what they want: a pure credit play.

We then describe the credit default swap and introduce a simple pricing model which shows that credit default swap spreads linked to the same reference credit should not change significantly from currency to currency. We explain that this observation can break down in certain cases.

Figure 1. **Mid-market Basis Swaps for the Major Currencies versus the Dollar; Values Are Shown by Swap Maturity and Are Quoted in Basis Points, Which Are Added on to the Non-dollar Leg of the Swap**

Maturity	EUR	JPY	GBP
1 Year	+1.75	-1.65	-1.25
3 Year	-0.875	-5.00	-2.125
5 Year	-1.875	-8.00	-2.50
10 Year	-2.125	-14.00	-5.75

Source: ICAP.

Box 1: Cross-Currency Swaps

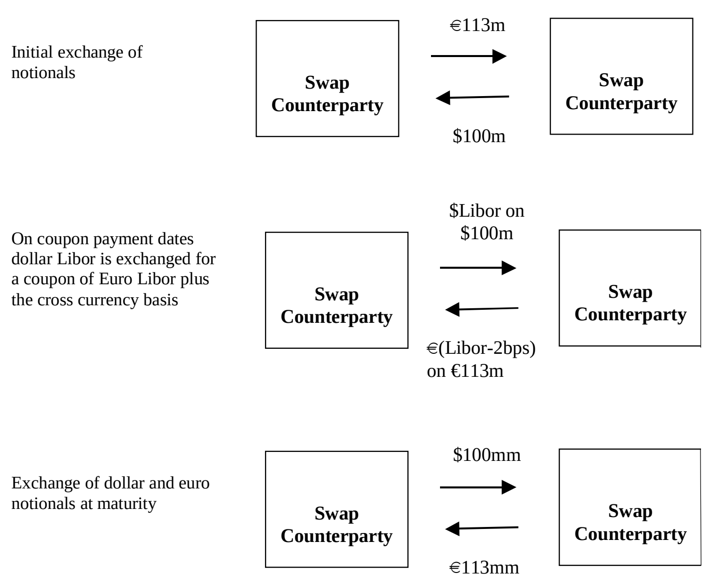
Cross-currency swaps are a component of the cross-currency asset swap. In a standard cross currency swap, there is an initial exchange of notionals in the respective currencies. Following this, each party exchanges with the other payments of Libor in one currency for payments of Libor in the other currency. Each leg of the swap is made according to the frequency and accrual conventions of the corresponding currency. The swap can be terminated early simply by re-exchanging the initial notionals.

One other feature of the cross-currency swap is the basis swap. This is an adjustment to one of the Libor legs of the swap—convention dictates that it should be the non-dollar leg—which mainly accounts for the demand to pay or receive the combination of the two floating legs involved in the swap. Figure 1 shows recent market quoted basis swap levels for a number of major currencies against U.S. dollars.

The most important factor which drives the basis swap is companies' issuing bonds outside their domestic market and swapping their future coupon liabilities into their domestic currency. In the particular case of yen Libor, there is a second factor-the fact that a significant number of the Libor panel are domestic Japanese banks who, due to their credit quality, incur a higher borrowing rate means that the fixing rate is increased. This is especially so in the setting of Tokyo yen Libor (Tibor) where all of the banks on the panel are Japanese. Since European and U.S. banks can usually borrow yen at lower rates, the basis swap becomes large and negative.

Figure 2 shows an example cross-currency swap of euros and dollars on a $100 million notional done at an initial exchange rate of 0.885 dollars per euro. Note that in this example, we have used a basis swap adjustment of -2 bp and this is added to the euro leg.

Figure 2. **Mechanics of a Cross-Currency Swap**



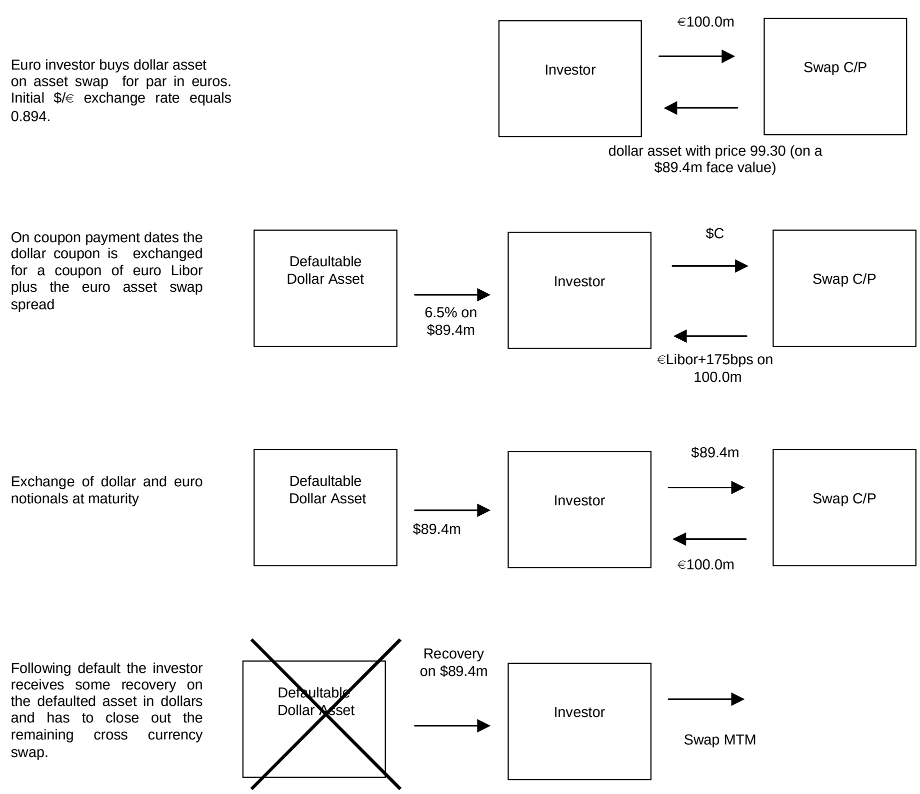
2. CROSS-CURRENCY ASSET SWAPS

Asset swaps are the traditional mechanism by which credit investors transform fixed rate bonds into synthetic par floaters. This has the benefit that it substantially reduces the interest rate risk, converting the bond into almost a pure credit play.[1] When the asset to be swapped is denominated in a currency different from that of the investor, it is possible to create a cross-currency asset swap structure to remove almost all of the interest rate and currency risk.

Figure 3 sets out the structure of a typical cross-currency asset swap in which a euro-based investor takes on the credit exposure of a dollar asset. The spot FX rate used is 0.894 in units of dollars per euro. The euro-based investor pays par in euros to buy face value $89.4 million of the dollar asset, which has a 6.5% coupon and a full price of 99.30. At the same time, the investor enters into a cross-currency swap in which the spread payment on the euro leg of the swap is 175 bp. An introduction to the standard cross-currency swap is presented in Box 1.

[1] See *Introduction to Asset Swaps*, Lehman Brothers Publications, Dominic O'Kane, January 2000.

Figure 3. **The Mechanics of a Cross-Currency Asset Swap in Which a Euro-Based Investor Gains an Exposure to a Dollar Asset**



If there is no default during the life of the asset swap, the swap closes out at maturity with an exchange of notionals, with the investor paying dollars and receiving euros on the initial notional.

However, if the asset does default, the investor loses the future coupons and principal of the asset, just receiving some recovery amount which is paid in dollars on the dollar face value. The cross currency swap is not contingent, meaning that the payments on the swap contract are unaffected by any default of the asset. The investor is therefore obliged either to continue the swap or to pay the replacement value of the remaining cash flows to the swap counterparty. This value can be positive or negative—the investor can make a gain or loss—depending on the direction of movements in interest rates and the FX rate since the trade was initiated. The risk associated with this is discussed in Section 4.

We show in Appendix A that it is possible to formulate a relationship between the cross-currency asset swap spread $ASW_€$ and that of the dollar asset swap spread $ASW_\$$. This is given by

$$ASW_€ = \frac{PV01_\$}{PV01_€} ASW_\$ + B_{€/\$} \quad (1)$$

where the PV01 terms are defined in Appendix A, and $B_{€/\$}$ is the euro-dollar basis swap. It is clear from the definition of the PV01s that if two currencies have similar levels of interest rates, they will have similar PV01s. We have listed estimates of the 5-year PV01 in Figure 4 for the major currencies.

For example, we see that the 5-year dollar PV01 is currently around 4.45, while the equivalent euro PV01 is about 4.43. So the PV01 ratio for swapping a dollar asset is about 1.005, which equates to a spread difference of 1 bp on a dollar asset swap spread of 200 bp—not significant when compared with the basis swap or bid-offer spread.

For Japanese yen, the difference can be more significant. The 5-year yen PV01 is around 4.95 so that the PV01 ratio is about 0.898. Swapping a dollar asset to yen would therefore result in a nominal spread about 10% lower than the dollar asset swap spread. This spread will be further reduced by the effect of the dollar-yen basis swap.

Figure 4. **Estimated Current PV01s for Different Currencies for a 5-Year Maturity and Semiannual Cash Flows.**

Currency	5 Year PV01
USD	4.45
EUR	4.43
JPY	4.95

3. CROSS-CURRENCY RELATIVE VALUE

When swapped into the investor's base currency using equation (1), the spread levels for bonds issued by a company may trade at different levels to those of bonds issued directly by the same company in the investor's base currency. Issuers take advantage of this relative value in order to reduce their cost of financing by issuing in currencies in which their bonds trade at narrower spreads. At the same time, this can provide relative value opportunities for credit investors who can use cross-currency asset swaps to source the bonds of an issuer denominated in other currencies in which they may trade cheap on an asset swap basis.

These spread dislocations between markets can be substantial and can persist for long periods of time. We list some of the main reasons for these dislocations below:

1) Investors in different currencies may have different degrees of risk aversion with respect to credit risk and so demand a different level of risk premia. The high number of recent U.S. defaults and worries about U.S. accounting standards are generally believed to have increased risk premia for U.S. corporate credit risk. This effect may also be reinforced by the higher U.S. spread volatility.
2) Assuming a similar level of demand, a market in which there is a lack of supply of credit risk will generally trade tight to one where there is a greater supply. For example, the significantly greater size of the U.S. credit market compared with the European market tends to drive European spreads inside U.S. spreads.
3) Different markets may be structured differently in terms of regulations such as tax, accounting treatments, and investment guidelines. For example, European pension funds are required to hold a higher proportion of their assets in bonds, including corporate bonds, than U.S. pension funds. This added demand tends to exaggerate the effect described in 2).
4) A risk-premium should be required by investors due to the default-contingent mark-to-market risk. However, this risk is symmetric.

These dislocations persist due to the fact that a large proportion of the investor base, in particular funds, is unable to enter into swap agreements because of the nature of their investment guidelines. Over time, we expect the growth of the credit derivatives market to increase the breadth of market participants who can do these sorts of trades. For the moment, the investor base for such opportunities consists mainly of banks, hedge funds, insurance companies, and corporates.

As an example of such a relative value effect, we have investigated the levels at which we can swap U.S. autos into euros. Specifically, we have looked at Ford Motor Credit which issues in both euros and dollars. This enables us to compare where spreads of the swapped dollar bonds trade relative to the euro bonds. To take two examples, consider the Ford Motor Credit bonds shown in Figure 5. When swapped to euros, the dollar-denominated bond trades at a spread of 61 bp over the comparable euro issue.

This presents a significant relative value opportunity for the investor who can implement the trade. However, before doing so, the investor should be aware that there is a default contingent cross-currency credit risk. For good-quality names, the risk is small; however, for medium-quality to crossover and high-yield names, the risk should be incorporated into any trading decision. A detailed description of this risk and the perfect asset swap structure which removes it follows.

Figure 5. **Two Bonds Issued by FMC, One in Dollars and the Other in Euros, Which Trade at Very Different Spread Levels**

Issuer	**Denomination**	**Coupon**	**Maturity**	**Asset Swap Spread** (bp)
Ford Motor Credit	USD	6.875%	Feb 2006	183 (swapped to euro)
Ford Motor Credit	EUR	5.625%	June 2006	122

4. DEFAULT CONTINGENT RISK OF A CROSS-CURRENCY ASSET SWAP

The basic cross-currency asset swap has a default contingent interest rate and currency risk—if the asset defaults, the investor is left exposed to:

1) the mark-to-market of the cross-currency swap, which can be positive or negative depending on how interest rates and FX rates have moved since trade initiation, and
2) the recovery paid on the foreign asset, which may have a different value from that paid on the same face value of domestic asset.

We can write the payoff formulas for the default contingent risks. To do this, consider a cross-currency swap in which we receive euro Libor plus a spread *S* and pay a fixed semi-annual coupon *C*. The trade is entered into at a spot FX rate of *X*, defined in terms of the number of euros per dollar.

At any time *t*, the close-out value of the swap is given by calculating the cost of the investor purchasing the remainder of the cross-currency swap which pays the dollar fixed coupon and receives Libor plus the asset swap spread in euros on the *original notionals*. Ignoring the accrued interest on both legs, in euros, the resulting mark-to-market for the investor is

$$\left[\sum_{i=i(t)}^{M} \Delta_i^{€}(L_i^{€}+S)Z_i^{€}+Z_M^{€}\right]N_{€}-\frac{X_{€/\$}(t)}{X_{€/\$}(0)}\left[\frac{C}{2}\sum_{i=i(t)}^{M} Z_i^{\$}+Z_M^{\$}\right]N_{€}$$

where

i(t) is the first coupon period after default

$X_{€/\$}(t)$ is the spot exchange rate in units of euros per dollar at time *t*

C is the annualised coupon flow on the swap fixed leg which matches that on the bond

$\Delta_i^{€}$ is the euro swap accrual factor for period *i*

$Z_{€}$(i) and $Z_{\$}$(i) represent the euro and dollar Libor discount factors to payment date *i*

M is the number of coupon payment dates on the swap

$N_{€}$ represent the euro swap notional

$L_{€}$(i) and $L_{\$}$(i) represent the euro and dollar Libor rates paid at date *i*.

This mark-to-market is in the investor's favour if X decreases, i.e., if the euro strengthens against the dollar, since the cost of purchasing the remainder of the dollar leg of the swap falls.

To quantify this risk more precisely, we have used a model of interest rates and FX rates. We have plotted in Figure 6 the distribution of the forward swap mark-to-market on an example on-market 5-year euro-dollar swap in which the asset swap buyer is paying a fixed coupon of 6.5% on the dollar notional and receiving euro Libor plus a spread of 175 bp. The interest rate and FX volatilities used were calibrated to the respective volatility markets.

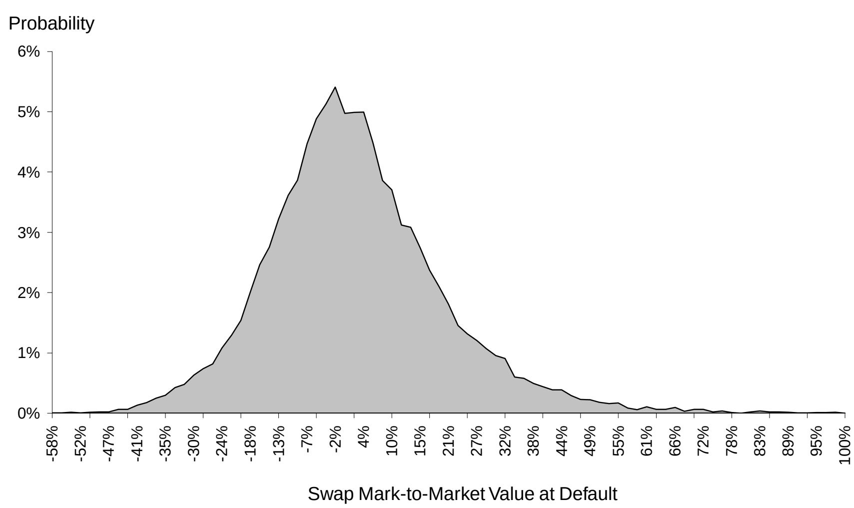
We calculated a 95% confidence limit Value-at-Risk (VaR) of about -24%. This is significant, meaning that in 5% of defaults, the loss on the swap market to market could be greater than 24% of the swap notional. It means that the default contingent swap currency risk is as important as the uncertainty in the recovery rate and, hence, of first- rather than second-order importance.

Any loss on the swap mark-to-market should be offset somewhat by the liquidation value of the defaulted dollar asset. This has a dollar value equal to $RN_{\$}$. Assuming an FX rate of $X(t)$ at default, the value in euros is given by

$$\frac{X_{€/\$}(t)}{X_{€/\$}(0)} RN_{€}$$

where R is the percentage recovery rate. If the asset had been euro denominated with a face value of $N_{€}$, then the investor would have received the same recovery rate R on the euro notional, i.e., $RN_{€}$ at default.

Figure 6. **The Simulated Distribution of the Forward Mark-to-Market at Default as a Percentage of Face Value for a 5-Year Euro-Dollar Swap in which the Investor Pays a Fixed Dollar Coupon of 6.5% and Receives Euro Libor plus 175 bp**



Box 2: The Issuer Recovery Rate Is the Same for Different Currencies

Consider a U.S.-based company which issues debt in both euros and dollars. Let us suppose that the issuer issued \$600 million face value of dollar debt and €400 million face value of euro debt at a time when the exchange rate was at parity. Both debt issues are *pari passu*. Two years later, the issuer is experiencing problems and files for bankruptcy. At that point, the value of the company assets equals \$200 million. Shareholders receive nothing, so the remaining assets are split between debt holders. Also, at that point in time, the exchange rate is 1.1 euros per dollar.

In dollar terms, the total claim is for \$600 mn + €400 mn × 1/1.1 = \$964 mn. Given that there is only \$200 mn of assets, each 1 dollar claim can only receive 20.7547 cents. Hence, holders of dollar-denominated bonds receive \$124.5283 mn, corresponding to a recovery rate of 20.7547%. Holders of euro-denominated receive \$75.4717 mn = €83.0189 mn, also corresponding to a recovery rate of 20.7547%. So both groups of bondholders receive the same effective recovery rate in their respective currencies.

See Box 2 for a discussion of why the recovery rates should be the same. Compared with the recovery of a euro asset, the gain is given by

$$\text{Recovery Gain to the investor on default} = R\left(\frac{X_{€/\$}(t)}{X_{€/\$}(0)} - 1\right) N_{€}$$

which can clearly be positive or negative, depending on whether the euro has weakened or strengthened against the dollar since trade initiation.

In Figure 7 we have plotted the forward value of this "gain" for a dollar asset which pays a 50% recovery rate on the dollar notional. We have used the forward FX rate. It shows that the "forward recovery gain" is positive and, therefore, is in-the-money to the investor for the first 3½-4 years. After that, it goes out-of-the-money. This is due to the fact that dollar interest rates are below euro interest rates until the 5 year point so that the FX forwards first rise and then fall. This is shown clearly in Figure 8, where we see that there is a crossover point between 3½-4 years forward for the dollar and euro zero rates.

It must be emphasized that this is just the forward value. In practice, interest and FX rates do not necessarily follow their forward values so that the investor's realised gain could be positive or negative. Taken together, the swap MTM risk plus the payment of the recovery amount gives a net value of

$$\left[\sum_{i=i(t)}^{M} \Delta_i^{€}(L_i^{€} + S) Z_i^{€} + Z_M^{€}\right] N_{€} - \frac{X_{€/\$}(t)}{X_{€/\$}(0)} \left[\frac{C}{2} \sum_{i=i(t)}^{M} Z_i^{\$} + Z_M^{\$} - R\right] N_{€}$$

so that the size of the second term which embodies the FX risk is reduced.

Figure 7. **This Shows How the Forward "Recovery Gain" to the Investor Changes with How Many Years Forward the Default Is. The Forward Is in the Money for the first 3.5-4.0 Years and Then Goes out of the Money**

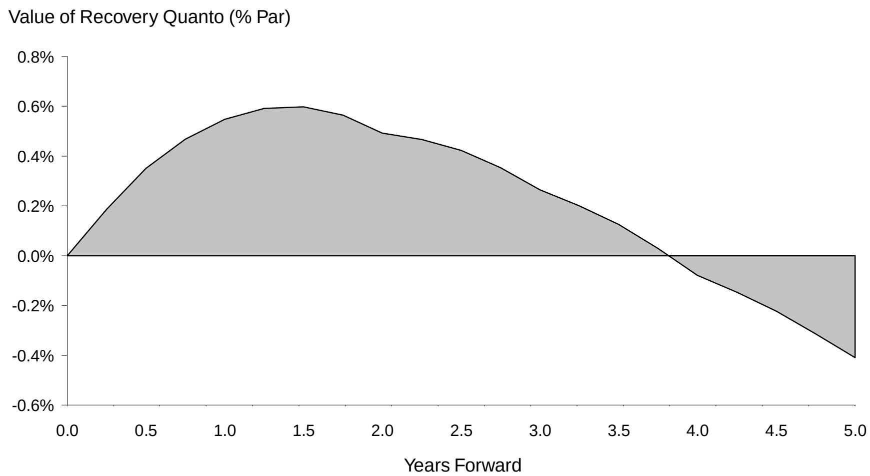
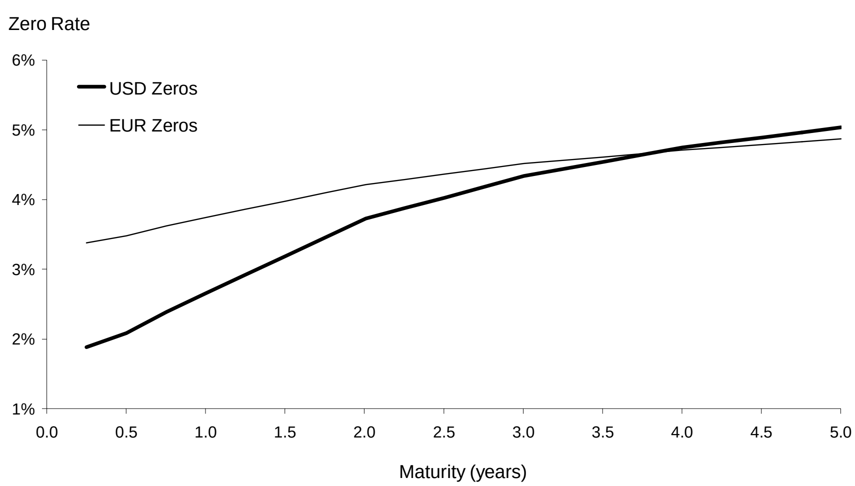


Figure 8. **Zero Rates for USD and EUR Showing That There Is a Crossover between 3½ and 4 Years. This Accounts for the Shape of the Forward "Gain" Curve Shown in Figure**



Risk-averse investors who wish to buy medium to low credit quality foreign-denominated assets on asset swap should consider perfect asset swaps. These are structured in an identical manner to a standard cross-currency asset swap except for the fact that in return for giving up some of the asset swap spread, the investor has no default contingent currency or interest rate risk.

5. PERFECT ASSET SWAPS

If there is no default, the structure of the perfect asset swap is equivalent to the standard cross-currency asset swap. It is only in the event of default that they differ. Two variations of the perfect asset swap exist:

1) Lehman Brothers takes on the default contingent swap mark-to-market risk
2) Lehman Brothers takes on both the default contingent swap mark-to-market risk and guarantees (quantoes) the recovery rate of the defaulted asset in the investor's base currency.

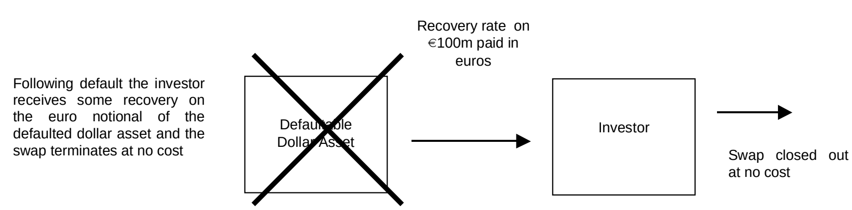
Both structures have featured widely in the CDO market where they have been used to immunize mixed-currency high-yield bond portfolios against currency risk in order to allow the structure to qualify for the desired rating from the rating agency.

The true perfect asset swap is 2) as it removes the swap risk and makes the recovery payment equivalent to that of a domestic asset. Strictly speaking, if the perfect asset swap is supposed to replicate a single currency asset swap, it should give the same swap mark-to-market as the equivalent single currency interest rate swap. However, that is an unnecessary complication; indeed, by removing this default contingent interest rate risk, this perfect asset swap is a purer credit play than even the single currency asset swap. The default event is shown graphically in Figure 9 for the same trade as shown above in Figure 3.

The cost of removing this default contingent swap mark-to-market risk and paying the recovery rate in the investor's domestic currency depends upon:

- The volatility of the FX rate.
- The credit quality of the reference credit.
- The expected recovery rate of the defaulted asset.
- The details of the swap: fixed vs. floating, maturity, spread level, etc.
- The shape of the Libor curves in both currencies.
- The volatility of interest rates in both currencies.
- The correlations between rates, FX, and the default probability.
- The cost of hedging.

Figure 9. **Perfect Asset Swap: in the Event of Default, the Investor Receives the Equivalent Recovery Value of a Domestic Asset and the Swap Closes out at No Cost**



Any pricing would require a model which captured all of these effects. The cost can be amortised over the life of the trade as a reduction in the asset swap spread paid. It is interesting to note that the cost may not be substantial, given that the two risks—the swap mark-to-market and the quantoing of the recovery rate—are actually offsetting. As we have seen in Figure 7, the shape of the currency forwards may mean that on a forward basis the quantoing of the recovery rate works in the investor's favour.

6. CURRENCY AND CREDIT DEFAULT SWAPS

Default swaps are simple bilateral contracts in which one party (the protection seller) agrees to provide protection to the protection buyer on a specified face value amount in the event that a reference credit defaults[2]. The mechanics are shown in Figure 10. Most default swap contracts are quoted in the domestic currency of the reference credit. According to a recent survey of the credit default swap market,[3] the distribution by outstanding notional of these was 44% U.S. names and 40% European names, with Asia and EMG accounting for 16%.

However, there is no reason why protection cannot be bought or sold in a currency different from that of the issuer of the reference credit. This enables investors to take the credit risk of a foreign issuer while getting paid spread in their base currency. For example, a euro investor can sell protection on Ford Motor Credit and receive the spread in euros. In the event of default, an investor who sells protection and has specified physical settlement will be delivered a face value of bonds which can be denominated in any of the G7 currencies. The amount of face value of bonds is determined by converting the face value of protection into the equivalent face value of foreign-denominated bonds at the FX rate at the time of delivery. Economically, the value of physical protection should be the same irrespective of the currency of the delivered asset(s).

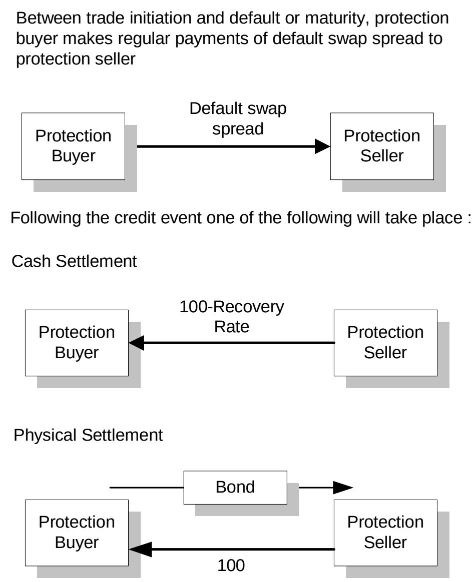
In Appendix B, we develop a very simple model for pricing default swaps which makes several assumptions including independence of the FX rate and credit risk and interest rates. We find that the theoretical default swap spread does not change noticeably when moving from one currency to another.

For example, assuming a risk-neutral default probability of 2%, a recovery rate of 50%, and a flat Libor zero rate of 1%, similar to Japanese yen rates, we calculate the fair-value 5-year default swap spread to be 100.7 bp. Using the same default probability and recovery rate but a different Libor zero rate of 4% gives a theoretical default swap spread of 101.7 bp. The 1 bp difference is very small, well within the bid-ask spread. This shows that within this simple modelling framework, moving from one currency to another should have a negligible effect on the theoretical default swap spread.

[2] Or more formally, when the reference credit experiences a *credit event*. For a complete description of default swaps and other credit derivatives, see the Lehman publication *Credit Derivatives Explained*, Dominic O'Kane, March 2001.

[3] *Risk Magazine*, February 2002

Figure 10. **Mechanics of a Default Swap. In the Event of Default, the Physical Settlement Option Involves the Delivery of a Face Value Amount of G7 Assets Equal to the Face Value of the Protection at the Prevailing FX Rate**



This effect is widely observed in the market, where euro and U.S. default swaps trade at almost exactly the same spread. Any larger differences which do occur will more likely be due to liquidity effects linked to the cash market and differences in supply and demand across currencies.

The main thing this model ignores is correlations. While the correlation between default and FX is generally believed to be small, in certain cases this may not be true. For example, the credit quality of a Japanese company which is a significant exporter will be closely related to the yen exchange rate. As a result, default may be more likely when the yen is strong. Incorporating this effect in the model would make the difference in the default swap spreads larger.

7. CONCLUSION

For investors who can enter into asset swaps and perfect asset swaps, the advantages include the ability to access new credits and increase diversification, and the ability to take advantage of relative value mispricings which exist across currencies.

Credit default swaps can be traded in a wide variety of currencies. Using a simple model, we have shown that the theoretical spread should not change significantly across currencies, and this is borne out by what we observe in the market.

APPENDIX A. CALCULATING THE CROSS CURRENCY ASSET SWAP SPREAD

Let us consider the economics of a cross-currency asset swap in order to determine the breakeven asset swap spread. In the following, we consider a euro-based investor who wishes to purchase a dollar asset on asset swap. Assume that:

1) The dollar asset has a price at initiation of P expressed as a fraction of the dollar face value and pays a semi-annual dollar coupon C.
2) We assume that counterparty credit quality is AA-rated banking so that contractual flows may be discounted at Libor flat.

The fair-value of the euro asset swap spread $ASW_€$ is given by setting the net present value equal to zero. The fair-value euro asset swap spread can be found by solving:

$$\text{upfront value} + \text{PV of swap floating leg - PV of swap fixed leg} = 0$$

which can be calculated in either currency. Note that the left hand side of the equation is the value of the asset swap to the buyer. Writing this in euros, we have

$$\begin{aligned}
&\left(N_\$ P\, X_{€/\$} - N_€\right) \\
&+N_€\left(\sum_{i=1}^{M}\Delta_i^€(L_€(i)+ASW_€)Z_€(i)+Z_€(M)\right) \\
&-X_{€/\$}N_\$\left(\frac{C}{2}\sum_{i=1}^{M}Z_\$(i)+Z_\$(M)\right) \\
&=0
\end{aligned}$$

where

$X_{€/\$}$ is the spot exchange rate in units of euros per dollar

C is the annualised coupon flow on the swap fixed leg which matches that on the bond

Δ_i is the euro swap accrual factor for period i

$Z_€$(i) and $Z_\$$(i) represent the euro and dollar Libor discount factors to payment date i

M is the number of coupon payment dates on the swap

$N_€$ and $N_\$$ represent the euro and dollar swap notionals

At initiation, the notionals are set equal in value so that $N_€ = N_\$ X_{€/\$}$. We also use the fact that a Libor floater discounted at Libor prices to par. We can then simplify the previous equation to get

$$(P-1)+\left(1+ASW_€\sum_{i=1}^{M}\Delta_i^€ Z_€(i)\right)-\left(\frac{C}{2}\sum_{i=1}^{M}Z_\$(i)+Z_\$(M)\right)=0$$

Simplifying further, we can then solve for $ASW_€$ which gives

$$ASW_{€} = \frac{\left(\frac{C}{2}\sum_{i=1}^{M} Z_{\$}(i) + Z_{\$}(M)\right) - P}{\sum_{i=1}^{M} \Delta_i^{€} Z_{€}(i)} = \frac{P^{LIBOR} - P}{PV01_{€}}$$

where

$$P^{LIBOR} = \frac{C}{2}\sum_{i=1}^{M} Z_{\$}(i) + Z_{\$}(M)$$

is the present value of the bond *discounted at Libor* and the euro PV01 is given by

$$PV01_{€} = \sum_{i=1}^{M} \Delta_i^{€} Z_{€}(i)$$

Compare the euro asset swap spread to the U.S. dollar asset swap spread for the bond given by[4]

$$ASW_{\$} = \frac{P^{LIBOR} - P}{PV01_{\$}}$$

Clearly we can write

$$ASW_{€} = \frac{PV01_{\$}}{PV01_{€}} ASW_{\$} \, .$$

where we define the dollar PV01 as

$$PV01_{\$} = \sum_{i=1}^{M} \Delta_i^{\$} Z_{\$}(i)$$

However, this ignores the fact that the investor will also receive the basis swap on the non-dollar Libor leg of the swap. We denote this as $B_{\$/€}$ bp (added by convention onto the non-U.S. dollar floating leg of the cross currency swap). Hence the actual spread paid over euro LIBOR will be:

$$\boxed{ASW_{€} = \frac{PV01_{\$}}{PV01_{€}} ASW_{\$} + B_{\$/€}}$$

[4] See *Introduction to Asset Swaps*, Lehman Publication, January 2000.

APPENDIX B. PRICING FOREIGN CURRENCY DEFAULT SWAPS

Default swaps can be priced either by replication arguments using par floaters or by using a modelling framework. Here we use the latter approach, where we make the following assumptions:

1) Default is modelled within a reduced form framework where $\lambda(t)$ denotes the **deterministic** hazard rate such that the default time t is defined by.

$$\Pr(\tau \leq t + dt \mid \tau > t) = \lambda(t)\, dt$$

   This approach has been explained in detail in another Lehman publication.[5]

2) The reference credit can default at any time and pays protection immediately.
3) We assume that the default process is the same for the domestic and foreign default swaps—there is cross-default.
4) The recovery rate R is paid as a fixed percentage of the protected face value in the same currency of the spread premium.

The fair-value default swap spread for a credit name is given by solving for the breakeven spread which equates the present value of the protection leg with that of the premium leg. Note that the premium leg terminates at default and for simplicity we have assumed no accrued premium. The default swap spread is given by

$$CDS = \frac{(1-R)\int_0^{T(M)} Z(t)\lambda(t)\exp\left(-\int_0^t \lambda(s)ds\right)dt}{\sum_{i=1}^{M} Z(i)\Delta_i \exp\left(-\int_0^{T(i)} \lambda(s)ds\right)}$$

where $Z(t)$ is the domestic Libor discount factor to time t in the domestic measure. Assuming a flat hazard rate $\lambda(t)=\lambda$ and a flat Libor zero rate r, this simplifies to give

$$CDS = \frac{(1-R)\lambda\left(1-\exp\left(-(r+\lambda)T(M)\right)\right)}{(r+\lambda)\sum_{i=1}^{M}\Delta_i \exp\left(-(r+\lambda)T(i)\right)}.$$

Within this simple model, the only thing that changes when going from one currency to another is the Libor curve used. More sophisticated approaches would take into account all of the stochastic dynamics, including the correlations.

[5] *Modelling Credit: Theory and Practice*, Dominic O'Kane and Lutz Schloegl, March 2001

A View through the *PRISM*: A *P*rice of *Ris*k *M*odel for Credit Sectors[1]

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A key variable in portfolio decision-making is the price of risk (the ex-ante Sharpe ratio) of an asset class. This is defined as the expected excess return per unit of risk, and it quantifies the risk-return trade-off for the investor. Estimates of this price-of-risk measure can be useful in decisions such as whether to overweight or underweight stocks versus bonds, Treasuries versus corporate bonds, or BBB-rated corporate bonds versus A-rated bonds. They are also useful in determining the relative weighting of various industry sectors. In this article, we present a methodology (referred to as PRISM) to estimate the price of risk for various credit sectors. For a given asset class or sector, our interest is in identifying episodes in which this price is high. If the price is high, investing in the asset provides a higher reward for undertaking a given level of risk. In our model,

- *The mean excess return and its volatility change over time,*
- *Volatility is mean reverting and estimated from observed bond price fluctuations, and*
- *The mean excess return is related to a set of predictive economic variables.*

We apply the PRISM framework to an active investment strategy of market timing and sector selection. Our framework can incorporate the effect of many variables such as spread movements, equity market movements, and other macroeconomic variables on mean excess returns.

1. INTRODUCTION

Investors need rigorously defined macro indicators to make asset and sector allocations on a timely basis. Many decisions in portfolio management involve choosing the relative weights of different asset classes or different categories within an asset class. A critical input in these decisions is the price of risk, or the ex-ante Sharpe ratio, defined as the expected excess return per unit of risk (where the excess returns are measured as returns above those on a risk-free asset). This is a key variable in financial decision making because it quantifies the trade-off between risk and return for the investor. Measures of the price of risk can be useful in macro-level asset allocation decisions such as whether to overweight or underweight stock versus bonds, Treasuries versus corporate bonds, BBB-rated corporate bonds versus A-rated bonds, or the auto sector versus the telecom sector. In this article, we present a methodology (referred to as *PRISM*) that can be used to estimate the prices of risk of various sectors of the credit market. This is a first step toward a broader goal of building quantitative tools for sector selection and asset allocation calls in the management of global credit portfolios. For a given asset class or sector, our interest is in identifying episodes in which this price is high. The times in which this price is high are also the times in which investing in the asset provides a higher reward for undertaking a given level of risk. Our model incorporates the effect of changes in macro-economic and financial market conditions on mean excess returns.

[1] We would like to thank Albert Desclée, Mark Howard, Jay Hyman, Marco Naldi and Stuart Turnbull for their valuable comments and suggestions.

This framework is flexible enough to incorporate the effect of spread movements, equity market movements, and other macroeconomic variables on mean excess returns. In addition, we capture the observed patterns of changes in the volatility of credit market returns. Our model is intended to help in top-down determination of credit portfolio strategy and is complementary to our asset selection model, ESPRI, described in Naik, Rennison, and Trinh (2002).

The evidence that the price of risk in asset market changes over time has been extensively documented for equities. See, for example, Brandt and Kang (2002) and the references mentioned therein. Indeed, the methodology used here is the same as the one used by Brandt and Kang in their estimation of the price of risk in U.S. equity markets. The evidence from equity markets has significant relevance for the corporate bond markets. Naldi (2002), for example, documents that the same fundamental factors that explain the equity risk premium are also useful in explaining the excess returns of various sectors of the U.S. credit markets. Moreover, the macroeconomic factors responsible for the variation of the price of risk in equity markets are also likely to affect the price of risk in credit markets.

In general, the price of risk can fluctuate if the expected excess return or the volatility of excess returns are time-varying. The expected excess return of an asset, in turn, can change because of time-varying correlation of pay-offs with aggregate wealth or consumption or with some other systematic factors. The price of risk also changes if the volatility of an asset changes over time. There is considerable empirical evidence of such time-variation in volatility from equity and currency markets. In particular, this evidence suggests the existence of the phenomenon of volatility clustering, which leads to large shocks being followed by other large shocks. There is also evidence of mean reversion in the volatility process. Finally, it has also been documented that volatility changes over the different phases of the business cycle.

The changing price of risk can also be seen as related to changing risk aversion. If risk aversion increases, the reward to taking an incremental unit of risk has to entice a risk-averse agent to invest. Therefore, an episode of high prices of risk can also result from increased risk aversion. It seems plausible that risk aversion is time-varying. Several explanations are usually mentioned to explain the time variation of risk aversion (see, for example, Campbell (2002) or Cochrane (2001, Chapter 21)). For instance as wealth increases, an investor might be more willing to take risk than when his level of wealth is low (the marginal loss of one dollar is less painful) implying a lower price of risk. The effect of wealth shocks on aggregate risk aversion may be exaggerated if investors have different levels of risk aversion, and changes in wealth are unequally distributed among agents. Also, as shown by the literature on habit formation, if agents get accustomed to a certain level of consumption, then they would grow more risk averse during recessions and favor less risky assets, resulting in high prices of risk. This is because when income levels are low, the only way to meet one's target level of consumption may be by investing in less risky assets. Finally, as suggested by the behavioral finance literature, time-varying prices of risk could also be a manifestation of excessive optimism during a market boom or too much pessimism during a market bust.

Our work is related to the analysis of Naldi (2002). Naldi's work develops an innovative approach to estimate the expected excess returns in various credit sectors using the economic restriction that in arbitrage-free financial markets, risk premia are determined by co-variation of asset returns with systematic risk factors. His approach involves first estimating the betas of various credit sector returns with respect to a judiciously chosen set of systematic risk factors and then cross-sectionally estimating the price of risk corresponding to each of the systematic risk factors. Our approach focuses more on the time-series (as opposed to cross-sectional) properties of excess returns, and we aim to uncover a predictable component in the dynamics of excess returns. Moreover, we focus both on expected excess returns and on their volatility (as our interest is in expected excess return per unit of volatility). We aim to explore how the two approaches can be combined fruitfully in future work.

This article is organized as follows. In section 2, we present the model that we use for the dynamics of excess returns for a credit sector. Section 3 describes the estimation methodology and provides the estimates of the parameters of the model of excess returns. In section 4, these parameter estimates are used to compute the estimates of price of risk (or PRISM scores, as we call them) for U.S. and European A, BBB, and high-yield sectors. In section 5 and 6, we present the results of hypothetical market timing and sector selection strategies based on estimated prices of risk. Section 7 provides concluding comments.

2. MODELING THE PRICE OF RISK

Modeling the price of risk requires modeling the mean of excess returns and their volatility. The observed time series of returns of various asset classes tend to share a number of features. Returns tend to be leptokurtic and skewed, and volatility tends to cluster. Squared returns are typically heavily serially correlated, whereas the return process itself does not usually show strong serial correlation. Our model of the excess returns in credit sectors is designed to capture these effects.

It is possible to identify two separate approaches that have been developed to describe statistically the behavior of returns: an “observation-driven” approach (GARCH modeling), and a “stochastic volatility” approach. The “observation-driven” approach typically works with one stochastic process driving returns and the volatility of returns. A simple ARCH model, for example, implies an auto-regressive process for squared returns. Alternatively, rather than positing that the same source of randomness drives both returns and return volatility, it seems intuitively appealing to require that volatility is determined by a separate and latent stochastic process. This captures the idea that the true level of volatility is unobservable and may be affected by a number of different variables. This produces the stochastic volatility (SV) model, which has been used extensively in continuous-time finance models.

Both of these approaches produce volatility clustering and leptokurtosis. Both approaches produce insufficient kurtosis for highly leptokurtic series such as exchange rate returns, but the SV approach seems to be slightly better in this respect.

In this article, we use a stochastic volatility (SV) model to estimate the volatility process. The expected excess returns are related to a set of predictive economic variables. In symbols, the model that we estimate is the following:

$$
\begin{aligned}
y(t+1) &= \mu(t) + \sigma(t)\varepsilon(t+1) \\
\mu(t) &= \gamma^{T} X(t) \\
\sigma(t) &= \sigma^{*} \exp\left(\frac{\theta(t)}{2}\right) \qquad ,
\end{aligned}
$$

where $y(t)$ is the series of observed monthly excess returns on a given credit sector and $\theta(t)$ is the unobservable normalized log variance of the excess return. The excess returns are computed over duration-matched Treasury securities. The mean excess return is denoted by $\mu(t)$. This is assumed to be a linear function of $X(t)$, which is a vector of explanatory variables that determine the mean excess return at date t. The vector γ denotes the vector of coefficients on the explanatory variables determining the mean excess return (and γ^T denotes the transpose of the vector gamma). Moreover, $\sigma(t)$ is the (stochastic) volatility of the excess returns series whose dynamics are specified through that of $\theta(t)$. This is assumed to be a mean-reverting process of the following form:

$$\theta(t+1) = \varphi\theta(t) + \eta(t+1)$$

The innovations to excess returns and the log variance process are denoted $\varepsilon(t)$ and $\eta(t)$, respectively. We assume that these innovations are independently and normally distributed over time with mean zero and variance 1 and σ_η, respectively. In addition, we assume that $\varepsilon(t)$ and $\eta(t)$ have no contemporaneous correlation.

Given the above dynamics of excess returns, the price of risk can be defined as:

$$S(t) = \frac{\gamma^T X(t)}{E_t\left(\sigma^* \exp\left(\frac{\theta(t)}{2}\right)\right)},$$

where $E_t(.)$ denotes the expectation conditional on the history of observed excess returns.

3. PARAMETER ESTIMATION FOR PRISM

While observation-driven models tend to be fairly straightforward to estimate, the latency implicit in SV models makes estimation more complicated. Proposed methods include Quasi-Maximum Likelihood and Generalized Method of Moments, but these methods are likely to be subject to higher estimation error than techniques based on the exact likelihood function or on the exact density of the data. Various efficient methods—in particular, Bayesian estimation via Markov-Chain Monte Carlo—have also been utilized. We follow Brandt and Kang (2002) in adopting a conceptually simpler and efficient approach, which revolves around approximating the model with a simple form, calculating the likelihood of the approximate model, and numerically correcting for the approximation error through a Monte Carlo simulation, as suggested by Durbin and Koopman (1997).

The method we use allows us to estimate the parameters and calculate the mean expected excess return over the next period, given our information up until the current period, and the volatility over the next period. We use these to calculate the price-of-risk measure for various sectors. It should be noted that this price of risk measure is only an estimate, first because it is based on estimated parameters and second because the underlying model used to estimate it may itself not be the true model. As a result, our estimated prices of risk inherit the estimation error of the parameters, as well the risk that the underlying model may be wrongly specified. To distinguish this measure from the true ex-ante Sharpe ratio (which one would compute by using the true parameters of the true model), we refer to our estimated prices of risk as PRISM scores rather than Sharpe ratios. Our hope is that the underlying model and the estimation procedure are accurate enough to capture broadly the variations in the true price of risk.

3.1. Parameter Estimates from U.S. Data

We first apply the framework to the excess returns of the U.S. Credit A, BBB and High Yield indices. Using this data set, we compute two sets of parameter estimates, the first using the first 100 observations and the second using the full sample from August 1988 to March 2002. Figure 1 shows the set of explanatory variables used in the equation for expected excess returns. We are also interested in applying the model to the European markets. However, the short history of excess returns in Europe necessitates the use of the longer U.S. history for parameter identification.

Figures 2 and 3 provide the parameter estimates from the two cases mentioned above.

Figure 1. **Explanatory Variables in the Equation of Expected Excess Returns for U.S. Credit Sectors**

Sectors	**Explanatory variables**
Lehman U.S.-Aggregate Credit A, BBB and High Yield indices	Excess return on S&P 500 over 3-month t-bill in the previous period Change in U.S. treasury yield curve slope (10 year–6 months) Lagged excess return

Figure 2. **Parameter Estimates Based on U.S. Data,** August 1988-November 1996

Parameter Estimates	**A**	**BBB**	**HY**
φ	**0.74**	**0.81**	**0.93**
σ	**0.69**	**0.62**	**0.24**
σ^2	**0.04**	**0.11**	**1.75**
Constant	**0.06**	0.08	0.17
SPX	0.02	0.02	**0.14**
Δslope, 6-mo. to 10-yr.	**0.34**	0.16	0.42
Lagged Excess Return	0.16	**0.21**	0.19

Numbers in bold indicate statistical significance at 95% level.

Figure 3. **Parameter Estimates Based on U.S. Data,** August 1988-March 2002

Parameter Estimates	**A**	**BBB**	**HY**
φ	**0.84**	**0.85**	**0.99**
σ	**0.79**	**0.71**	**0.30**
σ^2	**0.08**	**0.18**	**3.17**
Constant	**0.06**	**0.09**	0.13
SPX	**0.01**	0.02	**0.13**
Δslope, 6-mo. to 10-yr.	**0.36**	**0.30**	0.65
Lagged Excess Return	0.07	0.15	0.10

Numbers in bold indicate statistical significance at 95% level.

Interestingly, we see from these estimates that:

- The effect of positive equity market moves is generally positive for credit excess returns. Quite intuitively, this effect is very significant for the high yield market.
- The effect of the change in slope is positive; a steepening yield curve is beneficial for spreads.
- There seems to be a mild persistence in excess returns or momentum effect. Positive returns are followed by positive returns and conversely, as seen in positive point estimates of the coefficient on lagged excess returns.
- As expected, the base-line volatility σ^* is higher for high yield than for BBB and A
- There is evidence of mean reversion in volatility (estimates of φ are less than one) for the A and BBB indices. Volatility is more persistent for the high yield market than for the BBB and A markets (φ is lower).

3.2. Estimates from Prices of Risk for the U.S. Market

Below, we show the estimates of prices of risk that we obtain using the parameter values mentioned above. In order to ensure that we do not use forward-looking information in the calculation of the estimated prices of risk, we use the parameter estimates obtained using the data from August 1988 to November 1996. The results of the estimation are presented in the following graphs for the U.S. credit A, BBB, and high-yield indices. As mentioned before, we denote our estimated prices of risk as PRISM scores. One would expect that when this score is positive and high for a particular sector, the true Sharpe ratio for investing in the sector during the following month indicates an attractive risk-reward trade-off.

The graphs of estimated prices of risk for various rating sectors illustrate their time variation during our sample period. It is seen that the liquidity crisis of fall 1998 is followed by low and negative prices of risk for all credit sectors. The bull market of 1999 is associated with generally positive prices of risk that start to turn negative in 2000 as the tech bubble bursts. Following the liquidity injection of the Fed, the prices of risk turn positive again until the September 2001 attack. This effect is reversed, however, in the following months.

Figure 4. **PRISM Score of U.S. Credit A Sector**

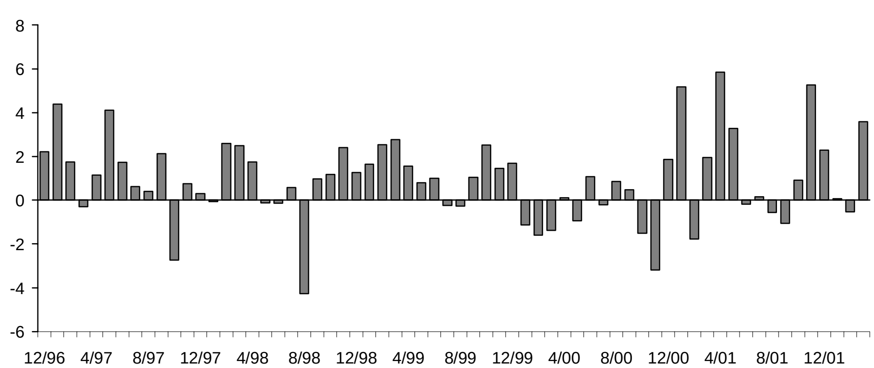


Figure 5. **PRISM Score for U.S. Credit BBB Sector**

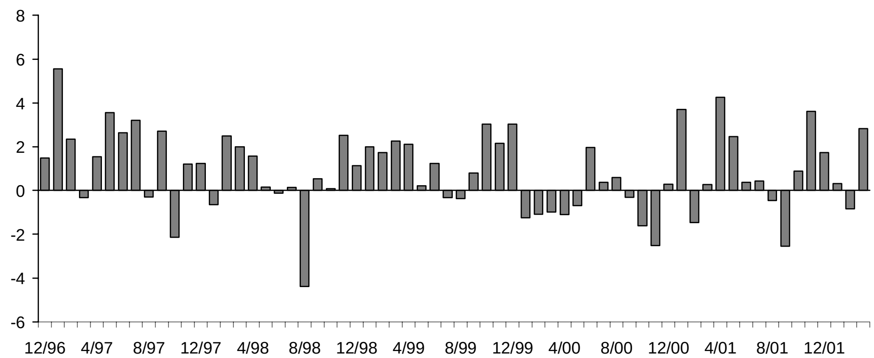
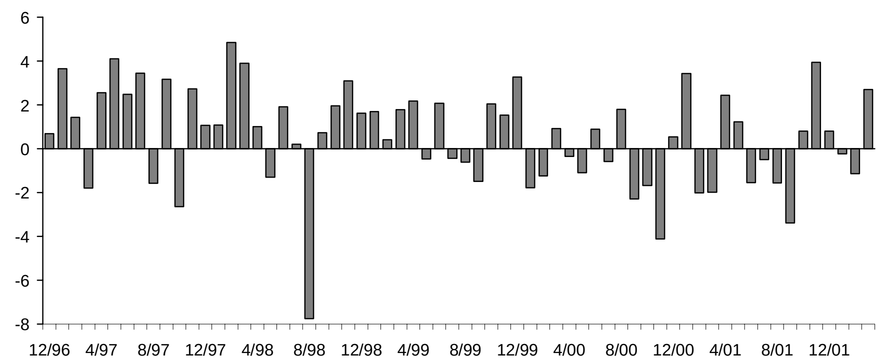


Figure 6. **PRISM Score for U.S. Credit HY Sector**



3.3. Estimates of the Prices of Risk for the European Market

We apply the same framework to the European credit A, BBB, and high yield indices. As shown in Figure 7, the explanatory variables for the expected excess returns in the euro market are taken to be the SPX excess return, the change in the euro yield curve, and the lagged excess return. However, due to the short time series for euro, we use the parameter estimates of the U.S. markets (using the data from August 1988 to November 1996). These are combined with the explanatory variables mentioned above to compute the euro PRISM score. It should be noted that for our euro estimates, one needs to focus on relative magnitudes (or changes over time) rather than the absolute magnitudes, because the underlying parameters estimates are coming from the U.S. market, where the long-term average excess returns may be different from those of the euro market.

In Figures 8, 9, and 10, we report the estimated prices of risk for the euro market. The pattern in these numbers seems broadly similar to that in U.S. estimates. We see that our estimates tend to be positive in 1999, turn negative during most of 2000 and back to positive in 2001, consistent with the U.S. experience reported above. The first half of 2001 tends to display better prices of risk, but, understandably, the reaction to events of September 2001 is negative. However, this effect is reversed relatively quickly.

4. USING PRISM FOR ACTIVE INVESTMENT STRATEGY AND MARKET TIMING

In this section, we report the results of a back-testing exercise aimed at examining whether our estimated prices of risk can be useful in active sector-selection decisions. Our back-testing is an out-of-sample exercise and implements the following idea. With our model, we can construct a series of the estimated prices of risk (PRISM score), let us say for the A index, and use the score as an investment signal. If the PRISM score is high

Figure 7. **Explanatory variables in the Equation of Expected Excess Returns for Euro Credit Sectors**

Sectors	**Explanatory variables**
Lehman Euro-Aggregate Credit A, BBB and High Yield Indices	Excess return on S&P 500 over 3-month t-bill in the previous period Change in Euro government yield curve slope (10 year–6 months) Lagged excess return

Figure 8. **PRISM Score for Euro Credit A Sector**

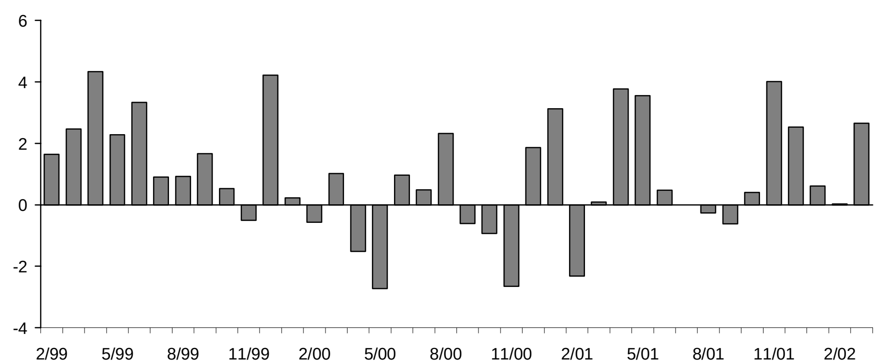


Figure 9. **PRISM Score for Euro Credit BBB Sector**

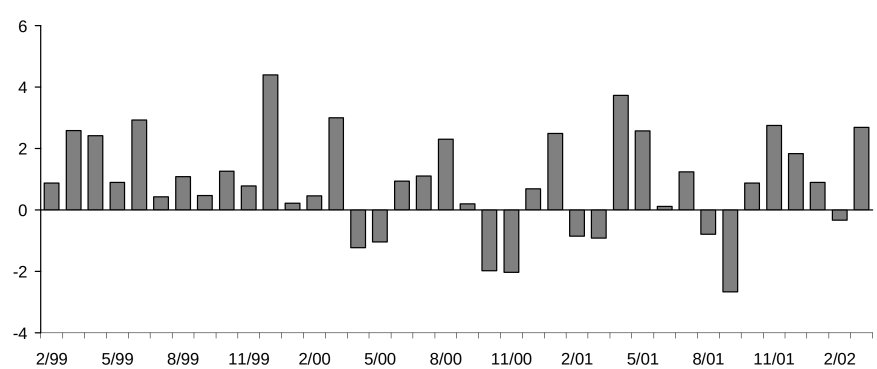
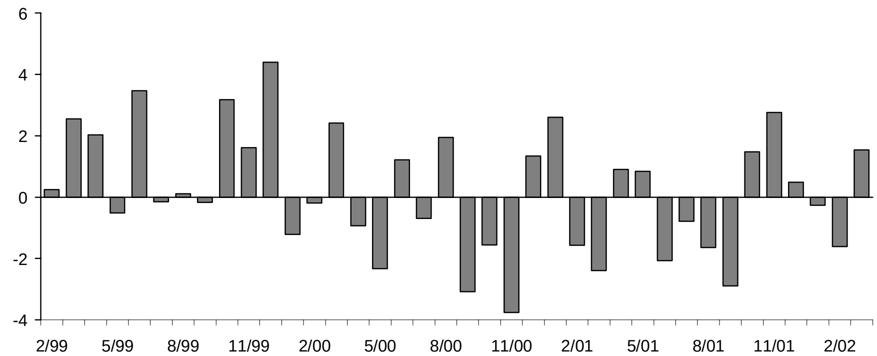


Figure 10. **PRISM Score for Euro Credit HY Sector**



(above a given threshold X), we invest in the A index, if it is below X, we invest in cash. We then calculate the average realized excess return from this strategy and its volatility. If the ratio of average realized returns to the volatility of excess returns is positive and high, then we take it as evidence suggesting that the estimated prices of risk are useful in sector selection. In this section, we report the results of the back-testing of this strategy using the Lehman Brothers U.S. and Euro-Aggregate indices

4.1. Results of the Back-Testing Exercise for the U.S. and Euro Markets

First, as a benchmark, we note some statistics from passive investing in credit indices. The passive investment strategy of buying and holding the U.S. A Credit Index would have generated an annualized (realized) Sharpe ratio of -0.15 over our testing period, December 1996 to March 2002. The average excess return is -3 basis points per month, and the monthly volatility is 70 basis points per month. The corresponding numbers for the U.S. Credit BBB Index and the U.S. High Yield Index are -0.23 and -0.38 (for the realized Sharpe ratio), -6 basis points and -28 basis points (for monthly average excess returns) and 92 basis points and 253 basis points (for monthly volatility).

Figure 11 provides the realized Sharpe ratios of following the active PRISM-based threshold strategy. There is clear improvement in the realized Sharpe ratios. In an active investment strategy—for instance, with a threshold of 1 (i.e., invest if PRISM score>1, do not invest if PRISM score<1)—the annualized Sharpe ratio is 0.96, the average excess return is 12 basis points per month, and the monthly volatility is 42 basis points. The investment strategy has a superior Sharpe ratio thanks to much lower volatility and a higher excess return. The patterns for the other two sectors are similar.

Figure 12 provides the same statistics for the euro market. Again, an improvement over the passive buy-and-hold strategy is seen here.

5. PRISM FOR SECTOR SELECTION

We have seen that the PRISM indicator could be useful for market timing. We could also implement a sector selection strategy based on the price of risk differential between sectors.

Figure 11. **Realized Sharpe Ratios of Passive and PRISM-Based Active Strategies for Various U.S. Credit Sectors,** 12/96-3/02

		Active strategy—Threshold Invest if Last Month PRISM Score > X		
Index	**Passive Strategy**	**X=0**	**X=1**	**X=2**
U.S. Credit A	-0.15	0.22	0.96	0.31
U.S. Credit BBB	-0.23	0.34	0.81	0.52
U.S. Credit HY	-0.38	0.51	0.74	0.84

Figure 12. **Realized Sharpe Ratios of Passive and PRISM-Based Active Strategies for Various Euro Credit Sectors,** 2/99-3/02

		Active strategy—Threshold Invest if Last Month PRISM Score > X		
Index	**Passive Strategy**	**X=0**	**X=1**	**X=2**
Euro Credit A	0.30	1.44	0.82	1.02
Euro Credit BBB	-0.09	0.49	0.73	0.64
Euro Credit HY	-0.62	0.29	0.60	0.63

We can illustrate this application with the choice between US Credit BBB and US Credit High Yield, and US Credit A and US Credit BBB. Consider the following strategy. If the PRISM score of US Credit BBB is higher than the score of US Credit High Yield plus a constant λ ($\lambda>0$), we invest in the US Credit BBB index for the next month. We do the reverse if the ordering of PRISM score is reversed. We include the coefficient λ which represents a confidence margin to take into account the uncertainty of the estimation procedure of the PRISM score. If λ is 0, we switch between US Credit BBB to US Credit High Yield if the score of US Credit BBB is higher than US Credit High Yield. If λ is larger than 0, the switch will occur only if the score of US Credit BBB is higher (by a margin of λ) than the score of US Credit High Yield.

Figure 13 reports the realized Sharpe ratios of the above trading strategy from December 1996 to March 2002 using different value of λ.*

As reported in Figure 14, we have similar results for the choice between U.S. Credit A and U.S. Credit BBB, but the confidence margin λ has to be higher for the trading strategy to work. This is probably due to the close behavior of U.S. Credit BBB and U.S. Credit A indices in term of risk characteristics and the less differentiating role of the SPX index compared with the U.S. Credit BBB, U.S. Credit High Yield case, in which we saw that the SPX was a strong driver for the U.S. Credit High Yield PRISM score.

Other trading strategies could also be implemented, such as industrials versus financials, autos versus telecoms, credit versus ABS, and credit versus agencies, based on the PRISM score. The challenge is to come up with the right explanatory variables to find a reasonably good estimate of the PRISM score.

*The returns calculations do not include bid-offer spreads and other transaction costs.

Figure 13. **Realized Sharpe Ratios of Sector Switching Strategy Based on PRISM Scores,** Trading between U.S. Credit BBB and U.S. Credit High Yield, December 1996-March 2002

λ	**Annualized Sharpe ratio**
0	0.69
0.5	0.72
0.75	0.73

Figure 14. **Realized Sharpe Ratios of Sector Switching Strategy Based on PRISM Scores,**Trading between U.S. Credit A and U.S. Credit BBB, December 1996-March 2002

λ	**Annualized Sharpe ratio**
0	-0.25
0.5	0.17
1	0.43

6. CONCLUSION

We have introduced the PRISM score, an indicator than can be used by investors who need rigorously defined macro indicators to make asset and sector allocations on a timely basis. The PRISM score for a credit sector is an estimate of the ex-ante Sharpe ratio, or the price of risk for that sector, and is defined as the expected excess return per unit of risk. For a given asset class or sector, our model helps in the identification of episodes in which this indicator is high. If the price is high, investing in the asset provides a higher award for undertaking a given level of risk. We show that the strategy that follows the PRISM score would have been reasonably successful for the U.S. and Euro credit markets (Credit A, BBB, and High Yield). The applications of our methodology include market timing decisions, as well as sector selection and rotation. The PRISM framework can incorporate the effect of many variables—such as spread movements, equity market movements, and other macroeconomic variables—on mean excess returns. In our examples, we have used the SPX, the change in the yield curve, and the lagged index excess returns. We plan to expand the PRISM framework to other sectors and asset classes and explore the optimal investment horizon for PRISM strategies.

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