Instructions:
Be verbose. Explain clearly your reasoning, methods, and results in your written work.

No code is necessary, but including it in your answer could result in partial credit.

Written answers are worth 10 points per question. 4 questions total – 40 possible points.

2 extra credit problems at the end is worth 10 points each. There is an additional 2 points extra credit in problem 3.

Total available points on this exam is 62.

1. Answers should be formatted as a PDF. You may convert your Python notebook, if you use one, directly to PDF.
2. Restate the question along with the question number before each answer
3. When finished, email your PDF directly to me along with your code.
4. Do not check code or answers into your repository until after the exam is completed by all.

Data for problems is generated by script.

You may use your notes and the internet for coding syntax help only. You may not work with other students – all work must be your own.

All students will be held to the Duke Community Standard

Duke's Community Standard:

Duke University is a community dedicated to scholarship, leadership, and service and to the principles of honesty, fairness, respect, and accountability. Citizens of this community commit to reflect upon and uphold these principles in all academic and non-academic endeavors, and to protect and promote a culture of integrity.
To uphold the Duke Community Standard:

1. I will not lie, cheat, or steal in my academic endeavors,
2. I will conduct myself honorably in all my endeavors; and
3. I will act if the Standard is compromised.

1. Using the data in “problem1.csv”
    a. Calculate Log Returns (2pts)
    b. Fit a normal and a Student t distribution to these series and present the fitted parameters (4pts)
    c. Which is the better fit? (3pts)

2. “problem2.csv” contains data about an European call option. Time to maturity is given in days. Assume 255 days in a year.
    a. Calculate the implied volatility (1pt)
    b. Calculate the put price (1pt)
    c. Calculate Call Delta (1pt)
    d. Calculate Call Vega (1pt)
    e. Calculate the Put Delta (1pt)
    f. Calculate the Put Vega (1pt)

    Assume you are long 1 call option and are long 1 put option.
    g. You observe a decrease in implied volatility by 5%. What is the profit or loss of your position? (3pt)
    h. Describe the 2 methods you could have used to answer part g. (1pt)

3. Using the option portfolio from Problem 2.g and the fitted distribution of returns from Problem 1.
    a. What is the profit or loss for holding this position for 100 days assuming no change in any other inputs? (2pt)
    b. Using the fitted distribution from Problem 1, what is the 5% VaR and ES of this portfolio given a 1 day holding period. (7pts)
    c. Given what you found from Problem 2 and 3 describe the risk vs expected return for this portfolio? (1pt)
    d. Why would someone enter into this position? What view are they expressing about the market? (+2 extra credit)

4. Data in “problem4.csv” is a series of returns for 3 assets. You hold a portfolio with weights of [30%, 45%, 25%].
    a. Calculate a covariance matrix from the returns using an exponential weighting factor of 0.94 (1pt)
    b. Calculate the ex-ante risk contribution of each asset as a percent of the total. (4pt)
    c. Assuming the returns are daily, the expected returns are the mean (unweighted) of the historical series, and the risk free rate is 5.25% annually (365 days), what is the daily Sharpe Ratio of this portfolio? (1pt)
    d. Calculate the maximum Sharpe Ratio portfolio weights restricting weights to be positive (4 pt)

5. Input returns in “problem5.csv” are for a portfolio. You hold 1 share of each asset. Using arithmetic returns, fit a generalized T distribution to each asset return series. Using a Gaussian Copula:
    a. Calculate ES (5%) for each asset (3pt)
    b. Calculate ES (5%) for a portfolio of Asset 1 &2 and a portfolio of Asset 3&4 (4pt)
    c. Calculate ES (5%) for a portfolio of all 4 assets. (3pt)

6. You are building a portfolio consisting of shares of a stock X and put options on that stock.
    a. Assume
        i. Initial stock price is $100
        ii. Strike price of the option is $100
        iii. Risk free rate is 0.0525%
        iv. The stock does not pay dividends.
        v. The expected return on the stock is 10%.
        vi. Implied volatility of the option equals the realized volatility of the stock which is 25% annually.
        vii. The option expires in 1 year.
        viii. You will hold the portfolio until option expiration.
        ix. You are allowed to short either the option or the stock, but no weight should be <-1
    b. Simulate 100,000 returns of both the stock and option through the holding period. Calculate the covariance between these (1 pts)
    c. Calculate the maximum Sharpe Ratio Portfolio (3 pts)
        i. (**HINT**: Because you have 2 Assets, you can quickly sweep the potential weights to find maximum values instead of using an optimizer. Weight_Stock = 1-Weight_Option)
    d. Change the Sharpe Ratio formula to use the ES (alpha=0.01) from your simulated values instead of the portfolio volatility. What weights maximize this reward/risk ratio? (3pts)
    e. Why are these values so different? (3pts)