Midterm Problem 3-9 Hand Calculations Fourier Coefficients - Dominic Riccoboni

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$$C_n = \frac{\frac{g_0 a}{\kappa}\int_0^a \left[\left(\frac{x}{a}\right)^2 - 1\right]\cos(\lambda_n x)\,dx}{\cosh(\lambda_n b)} \qquad n = 0, 1, 2 \ldots$$

$$= \frac{\frac{g_0 a}{\kappa}\left[\frac{1}{a^2}\int_0^a x^2\cos(\lambda_n x)\,dx - \int_0^a \cos(\lambda_n x)\,dx\right]}{\cosh(\lambda_n b)}$$

(1) (2)

(1) $\int_0^a x^2 \cos(\lambda_n x)\,dx = I$

$u = x^2$ $dv = \cos(\lambda_n x)\,dx$

$du = 2x\,dx$ $v = \frac{1}{\lambda_n}\sin(\lambda_n x)$

$$I = \frac{1}{\lambda_n}\left[x^2 \sin(\lambda_n x)\Big|_0^a - \int_0^a 2x \sin(\lambda_n x)\,dx\right]$$

$u = 2x$ $dv = \sin(\lambda_n x)\,dx$

$du = 2\,dx$ $v = -\frac{1}{\lambda_n}\cos(\lambda_n x)$

$$I = \frac{1}{\lambda_n}\left[x^2 \sin(\lambda_n x)\Big|_0^a - \frac{1}{\lambda_n}\left[-2x\cos(\lambda_n x)\Big|_0^a + 2\int_0^a \cos(\lambda_n x)\,dx\right]\right]$$

$$I = \frac{1}{\lambda_n}\left[x^2 \sin(\lambda_n x)\Big|_0^a - \frac{1}{\lambda_n}\left[-2x\cos(\lambda_n x)\Big|_0^a + \frac{2}{\lambda_n}\sin(\lambda_n x)\Big|_0^a\right]\right]$$

- $x^2 \sin(\lambda_n x)\Big|_0^a = a^2 \sin\left(\frac{(2n+1)\pi}{2}\right)$

$$\sin\left(\left(n+\tfrac{1}{2}\right)\pi\right) = (-1)^n$$

$$= a^2 (-1)^n$$

- $-2x\cos(\lambda_n x)\Big|_0^a = -2a\cos\left(\frac{(2n+1)\pi}{2}\right) - 0$

$$\cos\left(\frac{(2n+1)\pi}{2}\right) = 0$$

$$= 0$$

- $\frac{2}{\lambda_n}\sin(\lambda_n x)\Big|_0^a = \frac{2}{\lambda_n}(-1)^n$

$$\Rightarrow \quad I = \frac{1}{\lambda_n}\left[a^2(-1)^n - \frac{1}{\lambda_n}\left[\frac{2}{\lambda_n}(-1)^n\right]\right]$$

$$I = \left[\frac{a^2}{\lambda_n} - \frac{2}{\lambda_n^3}\right](-1)^n$$

(2) $$\int_0^a \cos(\lambda_n x)\,dx = \frac{1}{\lambda_n}\sin(\lambda_n x)\Big|_0^a$$

$$= \frac{1}{\lambda_n}(-1)^n$$

$$C_n = \frac{\frac{g_0 a}{K}\left[\frac{1}{a^2}\left[\frac{a^2}{\lambda_n} - \frac{2}{\lambda_n^3}\right](-1)^n - \frac{1}{\lambda_n}(-1)^n\right]}{\cosh(\lambda_n b)}$$

$$= \frac{\frac{g_0 a}{K}\left[\left[\cancel{\frac{1}{\lambda_n}} - \frac{2}{a^2\lambda_n^3} - \cancel{\frac{1}{\lambda_n}}\right](-1)^n\right]}{\cosh(\lambda_n b)} \qquad , n = 0, 1, 2 \ldots$$

$$C_n = \frac{-2g_0(-1)^n}{a\lambda_n^3 K \cosh(\lambda_n b)}, \quad n = 0, 1, 2 \ldots$$

$$C_n = \frac{2g_0(-1)^{n+1}}{a\lambda_n^3 K \cosh(\lambda_n b)}, \quad n = 0, 1, 2 \ldots$$

WITH $\lambda_n = \frac{(2n+1)\pi}{2a}$

OR

$$C_n = \frac{2g_0(-1)^n}{a\lambda_n^3 K \cosh(\lambda_n b)}, \quad n = 1, 2, 3 \ldots$$

WITH $\lambda_n = \frac{(2n-1)\pi}{2a}$