Midterm Problem 3-1 Hand Calculations Fourier Coefficients and Norm - Dominic Riccoboni

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FOURIER COEFFICIENTS

$$C_n = \frac{\int_0^L T_0 \sin\left(\frac{\pi x}{L}\right) \sin(\lambda_n x)\, dx}{N(\lambda_n)}$$

TRIG IDENTITY:

$$\sin(A)\sin(B) = \frac{1}{2}\left[\cos(A-B) - \cos(A+B)\right]$$

$$C_n = \frac{T_0}{2N(\lambda_n)}\left[\int_0^L \cos\left(\left(\frac{\pi}{L}-\lambda_n\right)x\right)dx - \int_0^L \cos\left(\left(\frac{\pi}{L}+\lambda_n\right)x\right)dx\right]$$

$$C_n = \frac{T_0}{2N(\lambda_n)}\left[\frac{1}{\left(\frac{\pi}{L}-\lambda_n\right)} \sin\left(\left(\frac{\pi}{L}-\lambda_n\right)x\right)\Big|_0^L - \frac{1}{\left(\frac{\pi}{L}+\lambda_n\right)} \sin\left(\left(\frac{\pi}{L}+\lambda_n\right)x\right)\Big|_0^L\right]$$

- $\sin\left(\left(\frac{\pi}{L}-\lambda_n\right)x\right)\Big|_0^L = \sin(\pi - \lambda_n L)$

$$= \sin(\lambda_n L)$$

- $\sin\left(\left(\frac{\pi}{L}+\lambda_n\right)x\right)\Big|_0^L = \sin(\pi + \lambda_n L)$

$$= \sin(-\lambda_n L)$$

$$= -\sin(\lambda_n L)$$

$$C_n = \frac{T_0}{2N(\lambda_n)}\left[\frac{\sin(\lambda_n L)}{\left(\frac{\pi}{L}-\lambda_n\right)} + \frac{\sin(\lambda_n L)}{\left(\frac{\pi}{L}+\lambda_n\right)}\right]$$

$$C_n = \frac{T_0 \sin(\lambda_n L)}{2N(\lambda_n)}\left[\frac{L^2\left(\frac{\pi}{L}+\cancel{\lambda_n}\right)+\left(\frac{\pi}{L}-\cancel{\lambda_n}\right)}{\left(\frac{\pi^2}{L^2}-\lambda_n^2\right)L^2}\right]$$

$$\boxed{C_n = \frac{T_0 \sin(\lambda_n L)\,\pi L}{2N(\lambda_n)\left[\pi^2-(\lambda_n L)^2\right]}}$$

Norm

TABLE 2-1 CASE 7

$$1/N(\lambda_n) = 2\left[\frac{\lambda_n^2 + \left(\frac{h}{k}\right)^2}{L\left(\lambda_n^2 + \frac{h^2}{k^2}\right) + \frac{h}{k}}\right]$$

$$Bi = \frac{hL}{k}$$

$$\frac{h}{k} = \frac{Bi}{L}$$

$$\Rightarrow N(\lambda_n) = \left(\frac{L}{2}\right)\left[\frac{\left(\lambda_n^2 + \left(\frac{Bi}{L}\right)^2\right) + \frac{Bi}{L}}{\lambda_n^2 + \left(\frac{Bi}{L}\right)^2}\right]\left(\frac{L^2}{L^2}\right)$$

$$\boxed{N(\lambda_n) = \left(\frac{L}{2}\right)\left[\frac{(\lambda_n L)^2 + Bi^2 + Bi}{(\lambda_n L)^2 + Bi^2}\right]}$$