ME 552 Final

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```
clear all
```

Constants

```
sigma = 5.67037e-8;%[W/m^2/K^4] Stephan-Boltzmann constant
h = .3; %Height of cylinder [m]
r = 0.08; %Radius of cylinder [m]
T1 = 1300; %Temperature at surface 1, the bottom of the cylinder [K]
T2 = 400; %Temperature at surface 2, the top of the cylinder [K]
Ts = 1000; %Temperature of each surface area other than 1 and 2 [K]
Ns = [2, 4:4:20]; %Vector containing the number of surfaces used in the enclosure analysis.
% Each of these will be a separate instance of the analysis.
A = pi*r^2; %Area of the top and bottom disks
eps = 0.8; %Epsilon, the emissivity of the gray-diffuse surfaces assumption
```

Configuration Factors

Create a 5x1 cell array for the final configuration factor matrices.

```
F = cell(length(Ns),1); %Cell array for final configuration factors
```

Also, create a cell array for intermediate view factor calculations given by:

$$F_{1-dk} = F_{dk-1} = \frac{1}{2}(X_k - \sqrt{X_k^2 - 4})$$

Where:

$$X_k = 2 + \frac{1}{R_k^2}, \quad R_k = \frac{r}{k\Delta h}$$

```
F_1_dk = cell(length(Ns),1); %Cell array for intermediate configuration factor calculations
```

For pre-allocation, fill F with NaN arrays of size $N = N_s + 2$ and F_1_dk with NaN vectors of length N_s for $N_s = 4, 8, 12, 16, 20$:

```
for n = 1:length(Ns)

    N = Ns(n) + 2; %Top and bottom surface must be added to get total number of surfaces

    F{n} = NaN(N); %NxN matrix of view factors (N^2 total)
    F_1_dk{n} = NaN(Ns(n),1); %Ns number of intermediate disks - View factors
                              % from bottom disk to that disk

end
```

Compute F_{1-2} and F_{2-1}:

$$F_{1-2} = F_{2-1} = \frac{1}{2}(X - \sqrt{X^2 - 4})$$

Where:

$$X = 2 + \frac{1}{R^2}, \quad R = \frac{r}{h}$$

```
R = r/h;
X = 2 + 1/R^2;
F12 = .5*(X - sqrt(X^2 - 4));
```

Compute the configuration factors for each value of N_s:

```
for n = 1:length(Ns) %For each Ns value, compute all the configuration
                     % factors and fill the relevant cell array elements

    N = Ns(n) + 2; %Total number of surfaces for this Ns value

    %Compute top bottom and bottom top view factors
    F{n}(1,2) = F12; %View factor for 1-2 computed above is placed in the relevant position
    F{n}(2,1) = F12; %The equivalent factor for 2-1 is placed as well

    %Bottom and top self-view factors are zero
    F{n}(1,1) = 0;
    F{n}(2,2) = 0;

    %Compute intermediate view factors for dk = 1:(Ns - 1)
```

```
    dh = h/Ns(n); %Delta h for each disk
    Rk = r/dh./(1:(Ns(n)-1)); %Create a vector of Rk values from 1 to Ns-1
    Xk = 2 + 1./Rk.^2; %Create a vecotor of Xk values from 1 to Ns-1
    F_1_dk{n}(1:(Ns(n)-1)) = .5*(Xk - sqrt(Xk.^2 - 4));%Compute the intermediate
                                                     % disk view factors

    %Intermediate view factor for dk = Ns is F_1_2
    F_1_dk{n}(end) = F12;

    %F_1_3 and F_2_N are not given by inner loop 1 below and must be specified here
    F{n}(1,3) = 1 - F_1_dk{n}(1);
    F{n}(2,N) = F{n}(1,3);

    %% Inner Loop 1
    %Get view factors from surfaces 1 and 2 to surfaces 3 to Ns.
    for k = 1:(Ns(n)-1)

        %From summation and by symmetry:
        F{n}(1,k+3) = F_1_dk{n}(k) - F_1_dk{n}(k+1);
        F{n}(2,N-k) = F{n}(1,k+3);
    end

    %%Inner Loop 2
    %Use reciprocity to get the view factors from surfaces 3 to N to
    %surfaces 1 and 2. Also compute the view factor for each of these surfaces to itself.
    %Each of the subdivisions can see itself and via symmetry, this view
    %factor is shared by all
    for k = 3:N

        %From reciprocity:
        Ak = 2*pi*r*dh; % kth area
        F{n}(k,1) = A/Ak*F{n}(1,k);
        F{n}(k,2) = A/Ak*F{n}(2,k);

        %Each sees itself the same amount:
        F{n}(k,k) = 1 - 2*F{n}(3,1);
    end


    %%Inner Loop 3 contains two indices and consequently two for loops
    for j = 1:(Ns(n)-1)
        for k = 3:(N-j)

        %For two sides adjacent to one another, the view factor is the same and given by:
        F{n}(k, k+j) =  F{n}(j+2, 1) - F{n}(j+3, 1);
        F{n}(k+j, k) = F{n}(k, k+j);

        end
    end


end
```

Compute the Heat Fluxes

Create 5x1 cell arrays for the blackbody and diffuse-gray heat fluxes:

```
q_bb = cell(length(Ns),1);
q_gd = cell(length(Ns),1);
```

Fill them with NaNs of the appropriate size:

```
for n = 1:length(Ns)

    N = Ns(n) + 2; %Top and bottom surface must be added to get total number of surfaces

    q_bb{n} = NaN(N,1); %Nx1 vector of heat fluxes to solve for
    q_gd{n} = NaN(N,1); %Nx1 vector of heat fluxes to solve for

end
```

Black Surfaces

The heat fluxes for the blackbody assumption are given by:

$$q_k = \sigma T_k^4 - \sum_{j=1}^{N} F_{k-j} \sigma T_j^4$$

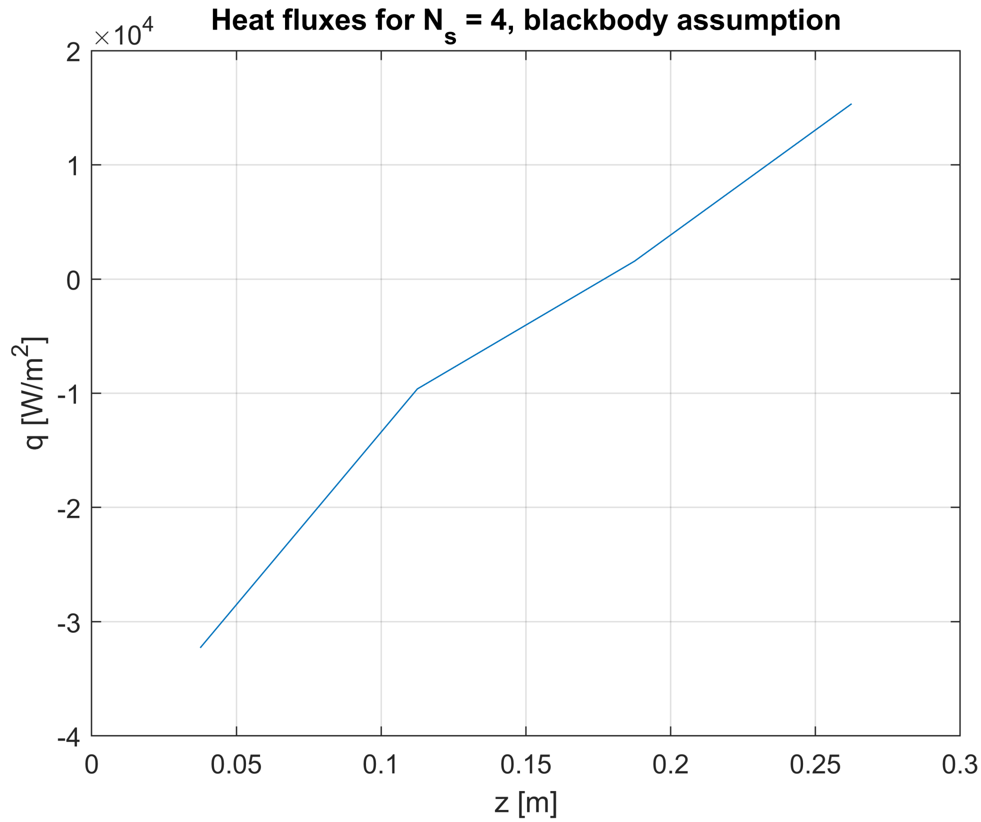
```
for n = 1:length(Ns) %For each Ns value, compute all the blackbody heat fluxes
                     % and fill the relevant cell array elements

    T = [T1; T2; Ts*ones(Ns(n),1)]; %Vector of temperatures of each of the surfaces

    q_bb{n} = sigma*T.^4 - F{n}*sigma*T.^4;

end

%Check answer against answer key for Ns = 2
A_mat1= diag([A, A, 2*pi*r*h/Ns(1)*ones(1,Ns(1))]);
Q_Ns_2 = A_mat1*q_bb{1}
```

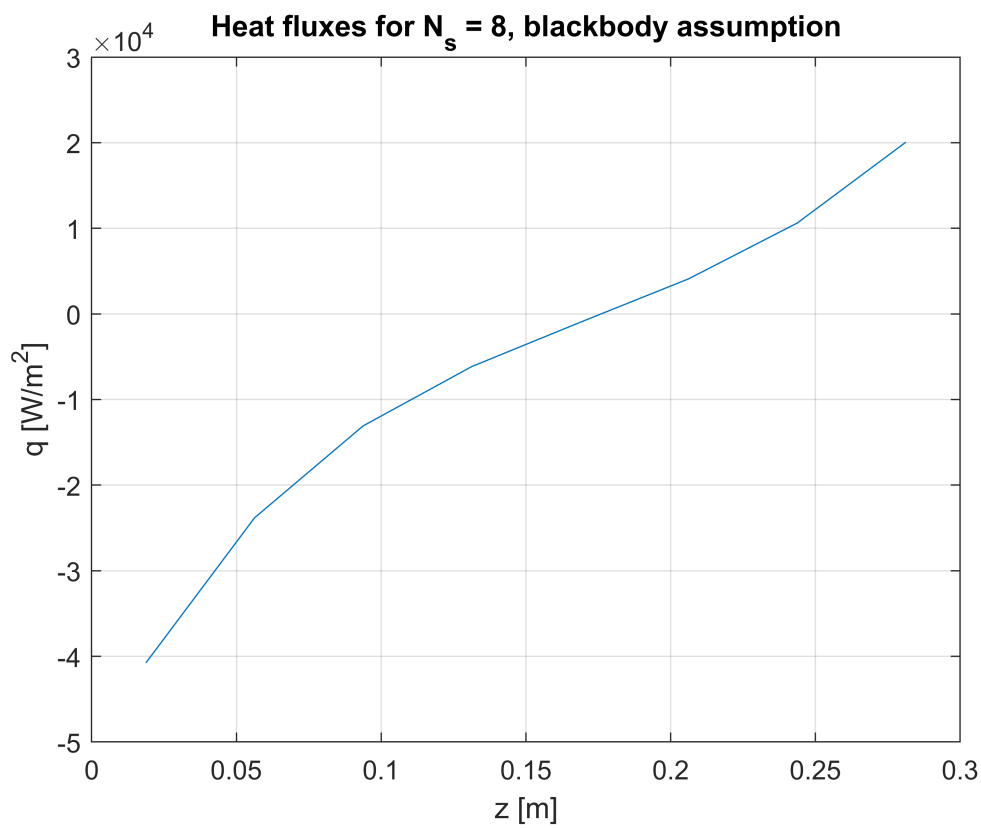
```
Q_Ns_2 = 4×1
10^3 ×
    2.1856
   -1.2432
   -1.5799
    0.6375
```

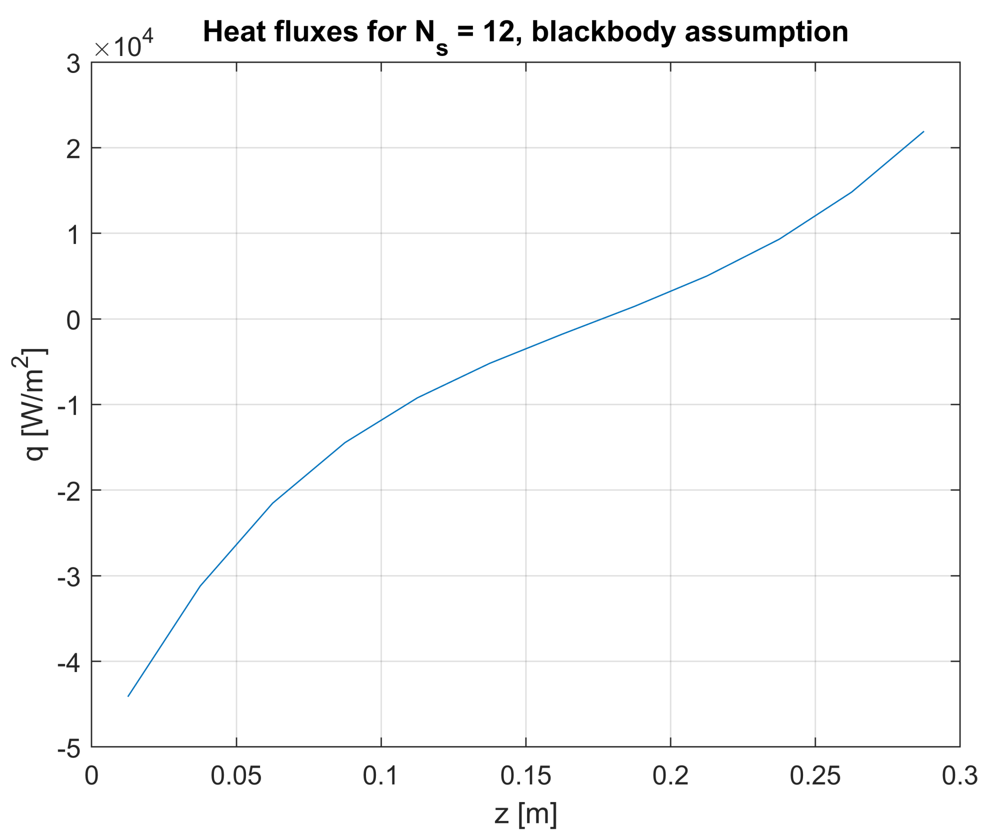
Plot the blackbody results:

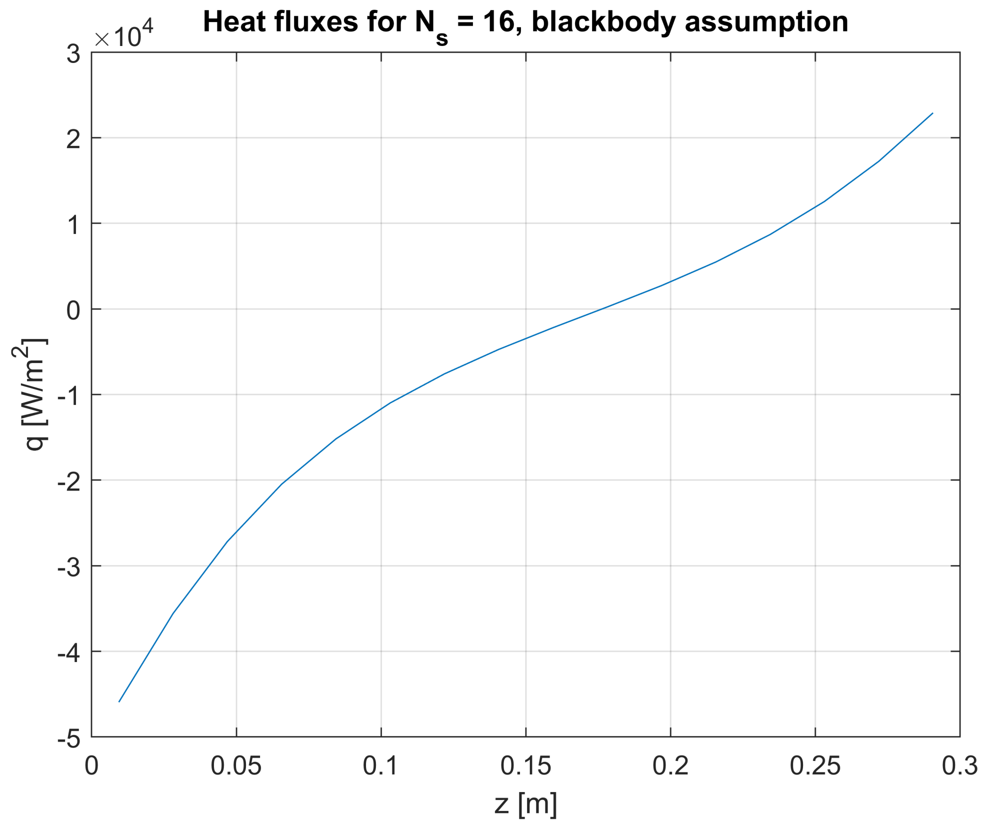
```
%Plot 1, Ns = 4
for n = 2:length(Ns)

    figure
```

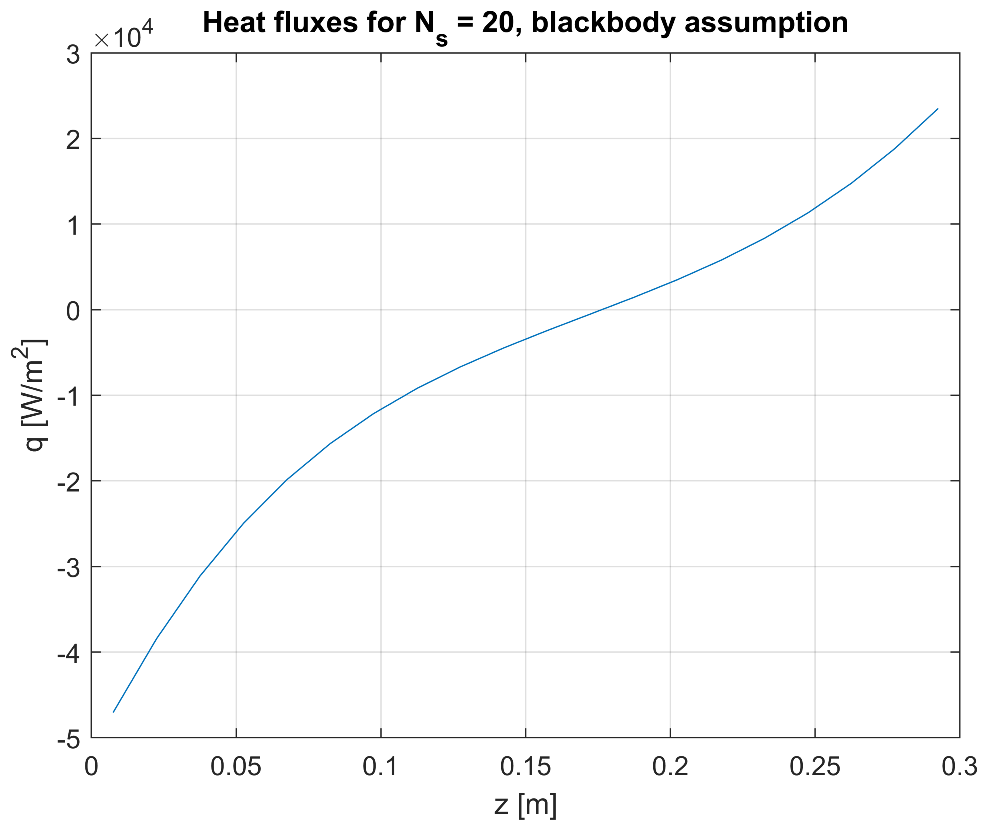
```
    z = h/Ns(n)*(.5:(Ns(n)-.5));
    plot(z, q_bb{n}(3:end))
    xlabel('z [m]')
    ylabel('q [W/m^2]')
    title(['Heat fluxes for N_s = ', num2str(Ns(n)), ', blackbody assumption'])
    grid on
end
```




Heat fluxes for N_s = 8, blackbody assumption
$\times 10^4$
q [W/m^2]
z [m]
3
2
1
0
-1
-2
-3
-4
-5
0
0.05
0.1
0.15
0.2
0.25
0.3


Heat fluxes for N_s = 12, blackbody assumption
$\times 10^4$
q [W/m^2]
z [m]
3
2
1
0
-1
-2
-3
-4
-5
0
0.05
0.1
0.15
0.2
0.25
0.3





Gray-diffuse Surfaces

The heat fluxes for the gray-diffuse assumption are given by:

$$[q] = [A]^{-1}[B][E_b]$$

Where:

$$A_{kj} = \frac{\delta_{kj}}{\epsilon_j} - F_{k-j}\left(\frac{1-\epsilon_j}{\epsilon_j}\right)$$

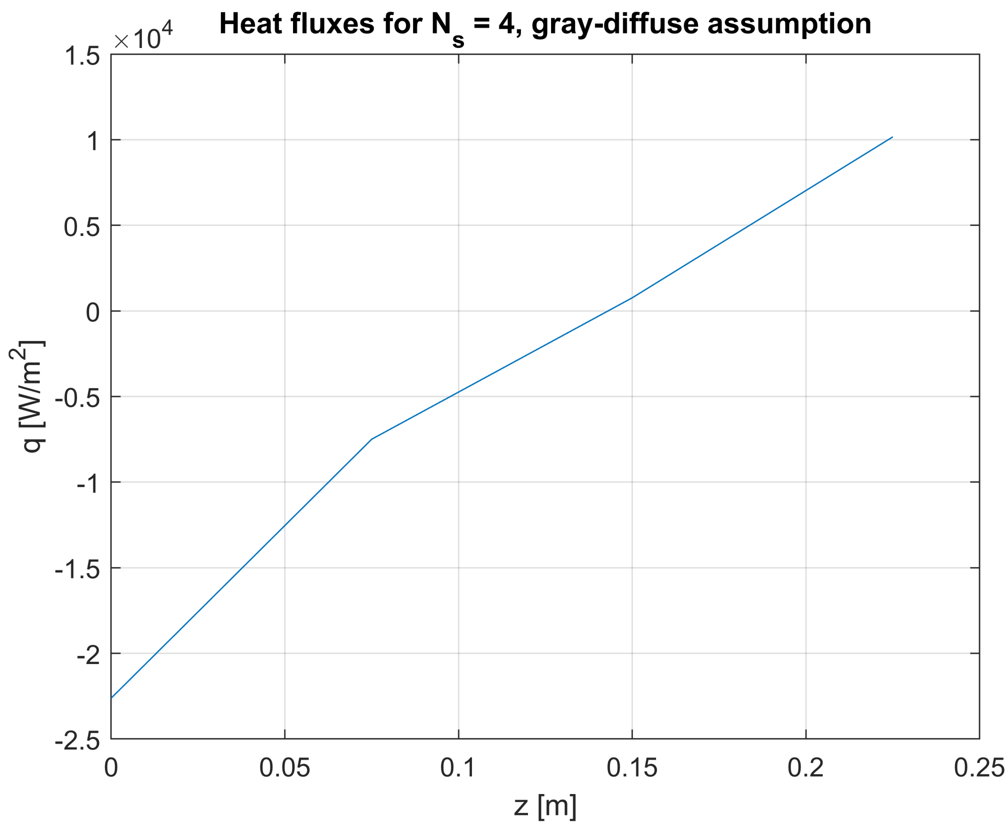
and:

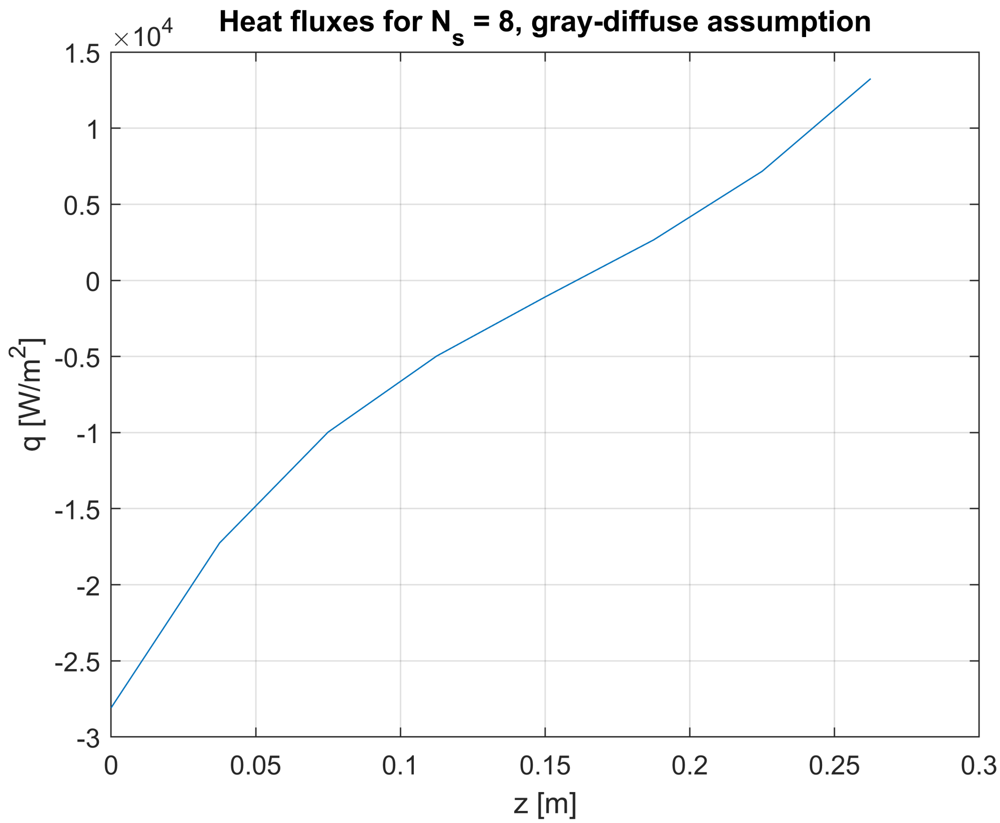
$$B_{kj} = \delta_{kj} - F_{k-j}$$

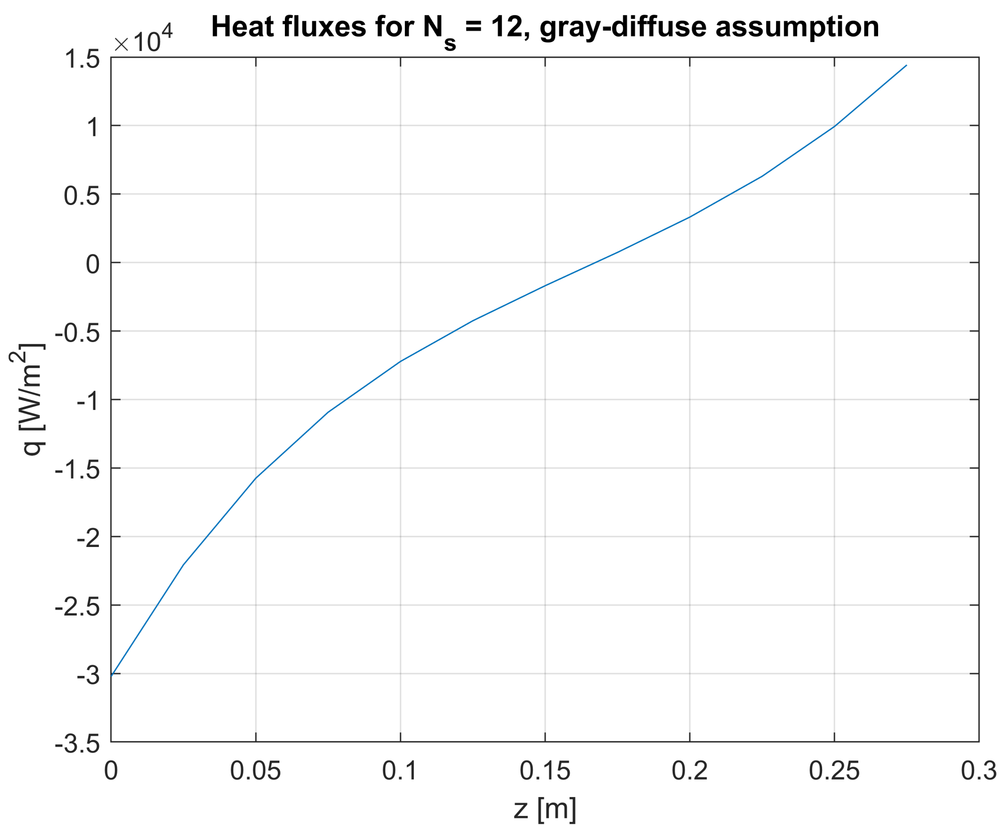
```
for n = 1:length(Ns) %For each Ns value, compute all the blackbody heat fluxes and
                     % fill the relevant cell array elements

    N = Ns(n) + 2;
    T = [T1; T2; Ts*ones(Ns(n),1)]; %Vector of temperatures of each of the surfaces

    A_mat = zeros(N);
    B_mat = A_mat;
    for k = 1:N
        for l = 1:N
            %Kronecker Delta implemented here
            if k == l

                A_mat(k,l) = 1/eps - F{n}(k,l)*(1 - eps)/eps; %Compute the coefficient
                                                              % matrix for the heat fluxes
                B_mat(k,l) = 1 - F{n}(k,l); %Compute the coefficient matrix for the
                                            % blackbody hemispherical-total emissivities
            else

                A_mat(k,l) = - F{n}(k,l)*(1 - eps)/eps; %Compute the coefficient
                                                        % matrix for the heat fluxes
                B_mat(k,l) = - F{n}(k,l); %Compute the coefficient matrix for
                                          % the blackbody hemispherical-total emissivities

            end


        end
    end

    q_gd{n} = A_mat\B_mat*sigma*T.^4; %Compute the heat fluxes for this value of N_s

end
```

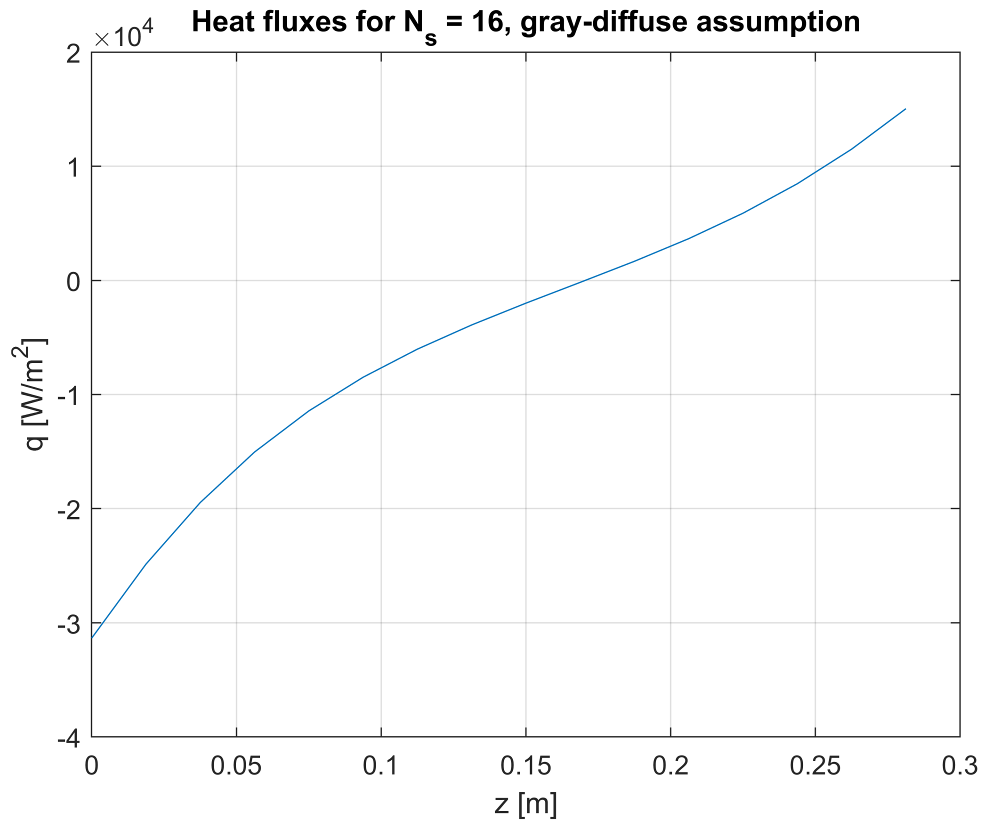
Plot the gray-diffuse results:

```
%Plot 1, Ns = 4
for n = 2:length(Ns)

    figure
    z = h/Ns(n)*(.5:(Ns(n)-.5));
    plot(z, q_gd{n}(3:end))
    xlabel('z [m]')
    ylabel('q [W/m^2]')
    title(['Heat fluxes for N_s = ', num2str(Ns(n)), ', gray-diffuse assumption'])
    grid on
end
```

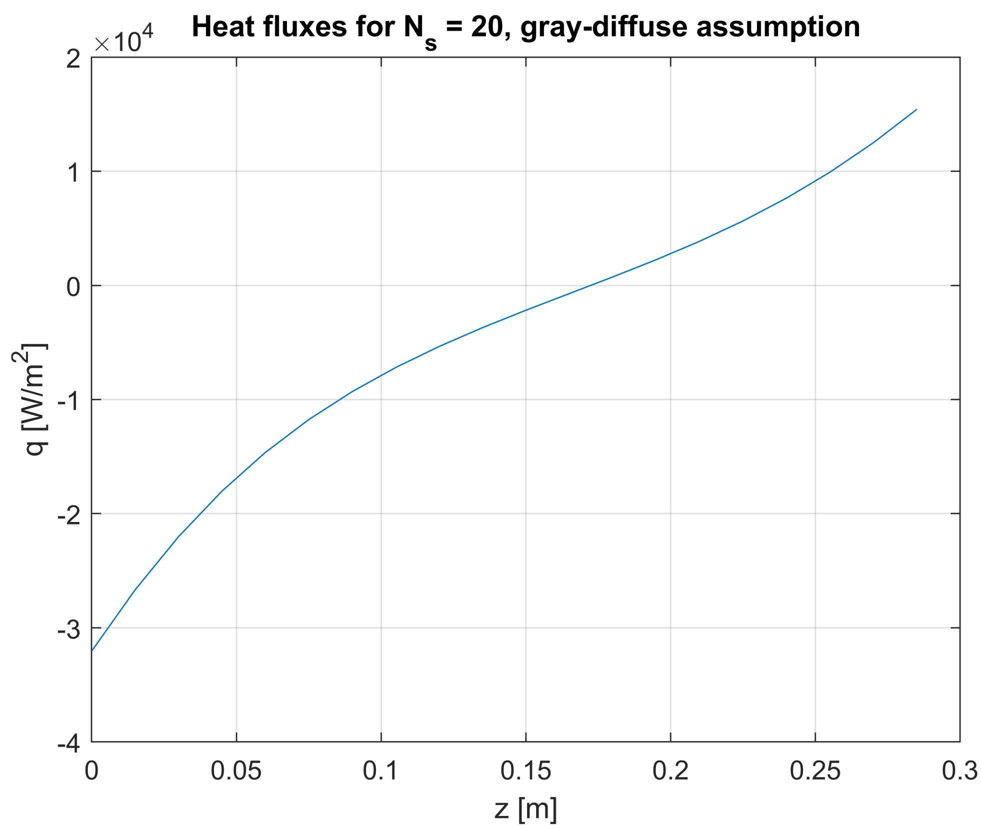



Heat fluxes for N_s = 8, gray-diffuse assumption
×10^4
1.5
1
0.5
0
-0.5
-1
-1.5
-2
-2.5
-3
q [W/m^2]
0
0.05
0.1
0.15
0.2
0.25
0.3
z [m]


Heat fluxes for N_s = 12, gray-diffuse assumption
×10^4
1.5
1
0.5
0
-0.5
-1
-1.5
-2
-2.5
-3
-3.5
q [W/m^2]
0
0.05
0.1
0.15
0.2
0.25
0.3
z [m]



Heat fluxes for N_s = 16, gray-diffuse assumption
×10^4
q [W/m^2]
z [m]

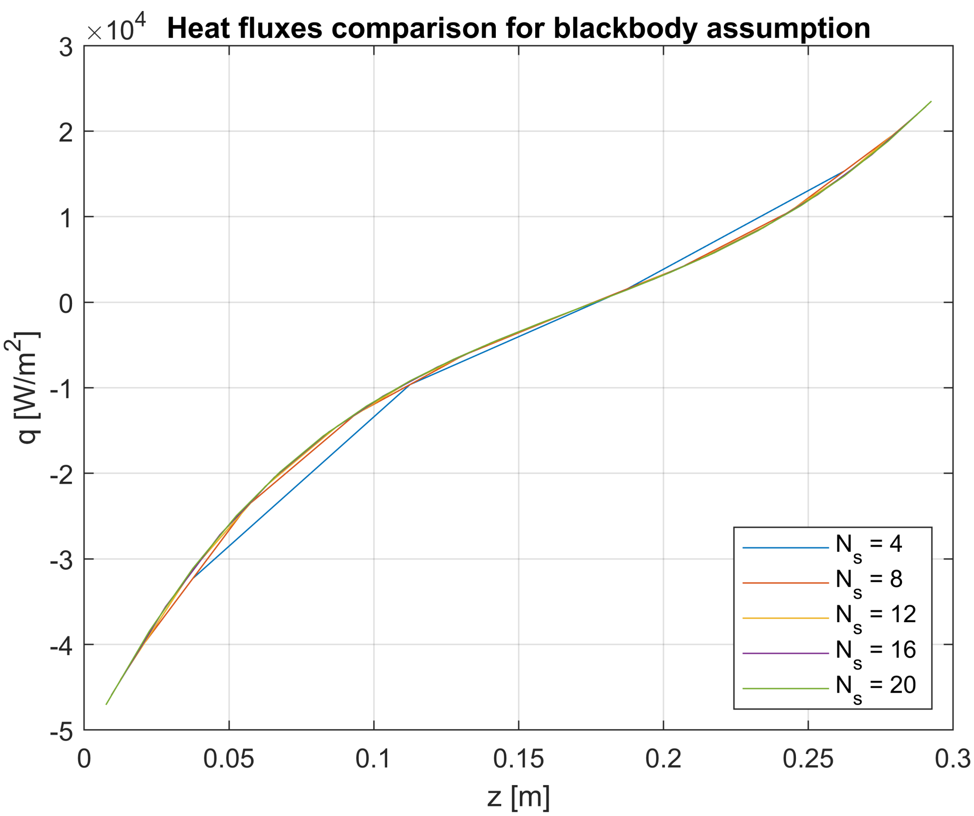


Heat fluxes for N_s = 20, gray-diffuse assumption
×10^4
q [W/m^2]
z [m]

Discussion

As N_s increases, the resolution of the curve of q vs z increases and it appears smoother. The heat flux estimate also decreases as N_s increases. The sparser curves appear to be a linear interpolation of the denser curve at $N_s = 20$ implying that there may be some "true" curve at $N_s = \infty$. Perhaps there exists some analytical solution involving differential area elements and integration.

```
figure
    z2 = h/Ns(2)*(.5:(Ns(2)-.5));
    z3 = h/Ns(3)*(.5:(Ns(3)-.5));
    z4 = h/Ns(4)*(.5:(Ns(4)-.5));
    z5 = h/Ns(5)*(.5:(Ns(5)-.5));
    z6 = h/Ns(6)*(.5:(Ns(6)-.5));
    plot(z2, q_bb{2}(3:end),z3, q_bb{3}(3:end), z4, ...
        q_bb{4}(3:end), z5, q_bb{5}(3:end), z6, q_bb{6}(3:end))
    xlabel('z [m]')
    ylabel('q [W/m^2]')
    title('Heat fluxes comparison for blackbody assumption')
    grid on
    legend('N_s = 4', 'N_s = 8', 'N_s = 12','N_s = 16', 'N_s = 20', 'Location', 'southeast')
```



For the gray-diffuse surfaces with emissivity 0.8, the magnitude of the heat fluxes required to maintain the prescribed temperatures are less than those of the black-body surfaces. The curve is steaper everywhere for the blackbody case and the extremes at the bottom and top surfaces are greater.

```
figure
z = h/Ns(end)*(.5:(Ns(end)-.5));
plot(z, q_bb{end}(3:end),z, q_gd{end}(3:end) )
xlabel('z [m]')
ylabel('q [W/m^2]')
title(['Heat fluxes comparison for N_s = ', num2str(Ns(n))])
legend('Blackbody', 'Gray-diffuse')
grid on
```