The Profiled FC Technique

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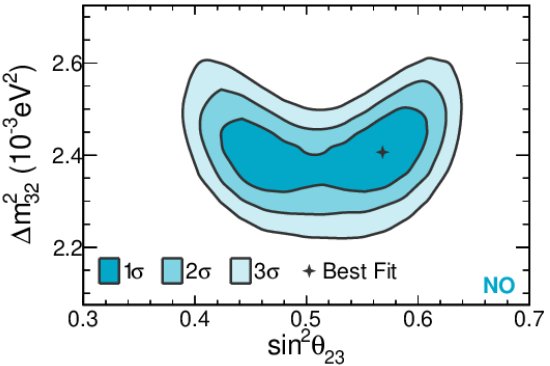
May 22, 2023

Purpose of the profiled FC technique

- "Profiled Feldman-Cousins" is the name given to a technique for constructing frequentist confidence intervals (CIs) for parameters in physical models that accounts for:
  - non-Gaussian statistics (typically due to small data samples for Poisson processes)
  - confidence intervals that are near to physical boundaries (e.g. Δm^2 must be non-negative)
  - imperfect knowledge of effects of systematic sources of uncertainty ("nuisance parameters")
- This is the technique used by NOvA, and to be used by DUNE, in their analysis of neutrino mixing parameters

Example analysis results



- This is a sample result from a NOvA publication.
- $\sin^2\theta_{23}$ and Δm^2_{32} are two of the 6 neutrino "mixing parameters" being measured.
- NOvA generally doesn't like using the term "measured", they prefer "extracted".
- Andrew prefers "estimated".

- NO means this assumes the "normal ordering" of neutrino masses (as opposed to "inverted ordering"); these are different choices of the physics model, and yield different predictions and thus different parameter measurement results. IO would give a different set of contours.

Some data sizes

- Data: the observed energy spectra of ν_e and ν_μ interactions. This is about 100 bins, maybe plus bin edges if unequal bin sizes are used.
- Theory: the data-driven prediction of what could be observed, given values of neutrino mixing parameters.
- The probability of fluctuation is a closed-form (but complicated) formula.
- There is a prediction of the idealized neutrino flux from simulation. This can be done beforehand because it is independent of the mixing and nuisance parameters.
- There is also a measurement of the observed neutrino spectrum in the near detector.
- For the different NOvA analyses, there were 50–100 nuisance parameters. More nuisance parameters were included in later analyses.

More data used to determine predictions

- Observed near-detector energy spectra.
- "Unfolding" matrices (100×100 elements, several such matrices), parameterizing the experiment's energy resolution.
- Gaussian mean and width (at 1σ) for most nuisance parameters. For a few, a histogram is used rather than a Gaussian distribution; the histograms are used to choose random numbers.
- Gaussian $\Delta\chi^2$ surfaces resulting from fits to the data. These are 2d histograms; the NOvA analysis used 40×40 bins. DUNE may use more, but not vastly more.

There is not a large amount of data needed for the calculations.

The high-level steps of the profiled FC technique

1. Generate many FC pseudoexperiments at one location in the 2D parameter space.
2. Fit of each pseudoexperiment (function minimization in about 150 parameters), to find the minimum λ for each PE.
3. Order the generated fit results to extract the value of λ that provides the desired coverage percentage (which we quote as $n\sigma$ rather than as the percentage).
4. Performing steps 1–3 at many locations over a 2D parameter space being constrained to determine the shapes of the contours.

What is profiling?

- This is the common frequentist prescription for handling nuisance parameters.
- It entails “averaging out” the effect of the nuisance parameters.
- It comes into play when we’re fitting the PEs (a few slides from now…)

Generate PEs

- For a given set of parameters (θ_1, θ_2) we will generate many (thousands to several millions, depending on which contour this pair (θ_1, θ_2) is near)
- To generate a single pseudoexperiment, we:
  - Generate random values for each of the nuisance parameters, according to their distributions (mostly independent Gaussians, a few are draws from specified histograms).
  - Using this set of values of (θ_1, θ_2) and the nuisance parameters, calculate the Poisson expectation value of the observed counts in each energy bin.
  - Using that set of Poisson means, generate Poisson random variates for the PEs counts in each bin.
- Each PE is a vector $\vec{d}$ of N integers, generated from N independent Poisson distributions, where N is the number of energy bins in the spectrum.
- Generating a single PE is not time-consuming.

Fit each PE

- For each PE, find the maximum likelihood (equivalently minimum negative log likelihood) by functional minimization in the approximately 150 dimensional parameter space.
- We actually don't care about the resulting location of this minimum; we care about the value of the function at the minimum.
- We'll use the index i to specify which PE at a specific point in the (θ_1, θ_2) space we are fitting.
- We'll call the value of the function at the minimum for PE i as λ_i.
- When run on Haswell nodes on Cori, the 3-neutrino analysis fits took 10 ± 6 minutes each. This time distribution had a long high-end tail, because some fits required multiple re-starts to find a good solution.
- A more robust fitter, parallelized, and able to handle ~150 fit parameters, is one of the biggest deliverables.
- This might be a place where automatic differentiation would be of value.

Find the $n\sigma$ value for point in parameter space

- At each point in (θ_1, θ_2) space, we use the many λ_i values resulting from the fits to the many PEs to map out the distribution of λ for that location in the 2D parameter space.
- We need to generate enough PEs to determine the value of the contour we are looking for at that location in the 2D parameters space.
- How do we know when we have generated enough λ_i values to have a good-enough estimate of the value of the $\Delta\chi^2$ surface at this point? We want to generate enough points to get a good answer, but we do not want to spend time generating more than necessary.

Search the 2-d model parameter space

If we view all the previous stuff as a function that returns a value at some point in the 2D parameter space, then this is the process of searching that parameter space to find:

- the global peak (there may also be local peaks)
- the 1-, 2-, 3- and 4-σ contours, corresponding to the specified probabilities

n	prob
1	0.6826895
2	0.9544997
3	0.9973002
4	0.9999367