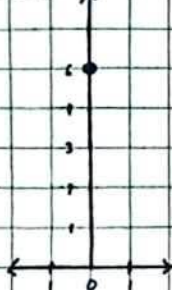


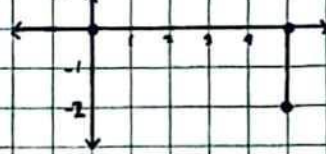
DSP: MP5 Lecture 3

3.1. Sketch each of the following special digital sequences

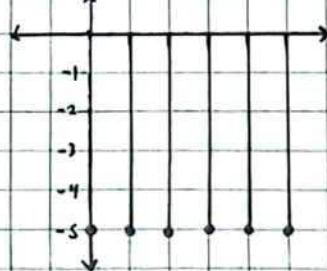
a) $5\delta(n)$



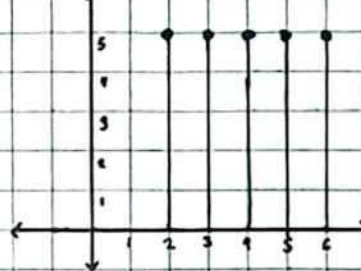
b) $-2\delta(n-5)$



c) $-5u(n)$



d) $5u(n-2)$



3.2. Calculate the first eight sample values and sketch each of the following sequences:

a) $x(n) = 0.5^n u(n)$

$0.5^0 u(0), \text{amplitude} = 1, u(n) = 0$

$0.5^1 u(1), \text{amplitude} = 0.5, u(n) = 1$

$0.5^2 u(2), \text{amplitude} = 0.25, u(n) = 2$

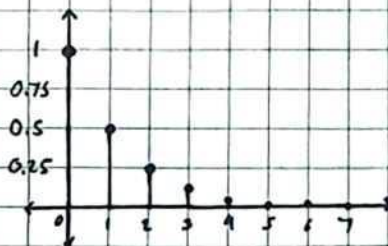
$0.5^3 u(3), \text{amplitude} = 0.125, u(n) = 3$

$0.5^4 u(4), \text{amplitude} = 0.0625, u(n) = 4$

$0.5^5 u(5), \text{amplitude} = 0.03125, u(n) = 5$

$0.5^6 u(6), \text{amplitude} = 0.015625, u(n) = 6$

$0.5^7 u(7), \text{amplitude} = 0.0078125, u(n) = 7$



b) $x(n) = 5 \sin(0.2\pi n) u(n)$

$x(0) = 5 \sin(0.2\pi(0)) u(0), \text{amp} = 0, u(n) = 0$

$x(1) = 5 \sin(0.2\pi(1)) u(1), \text{amp} = 2.94, u(n) = 1$

$x(2) = 5 \sin(0.2\pi(2)) u(2), \text{amp} = 4.75, u(n) = 2$

$x(3) = 5 \sin(0.2\pi(3)) u(3), \text{amp} = 4.76, u(n) = 3$

$x(4) = 5 \sin(0.2\pi(4)) u(4), \text{amp} = 2.94, u(n) = 4$

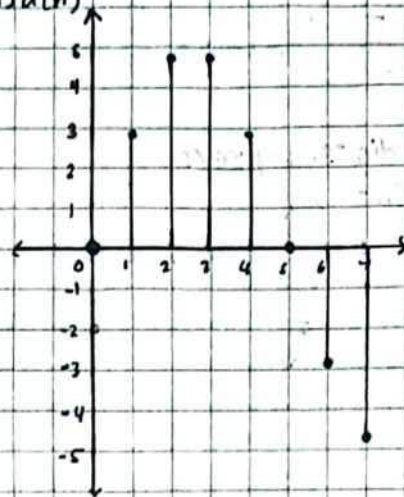
$x(5) = 5 \sin(0.2\pi(5)) u(5), \text{amp} = 0, u(n) = 5$

$x(6) = 5 \sin(0.2\pi(6)) u(6), \text{amp} = -2.94, u(n) = 6$

$x(7) = 5 \sin(0.2\pi(7)) u(7), \text{amp} = -4.75, u(n) = 7$

Graph next page

$$x(n) = 5 \sin(0.2\pi n) u(n)$$



$$c.) x(n) = 5 \cos(0.1\pi n + 30^\circ) u(n)$$

$$x(0) = 5 \cos(0.1\pi(0) + 30^\circ) u(0), \text{ amp} = 4.33, u(n) = 0$$

$$x(1) = 5 \cos(0.1\pi(1) + 30^\circ) u(1), \text{ amp} = 3.35, u(n) = 1$$

$$x(2) = 5 \cos(0.1\pi(2) + 30^\circ) u(2), \text{ amp} = 2.03, u(n) = 2$$

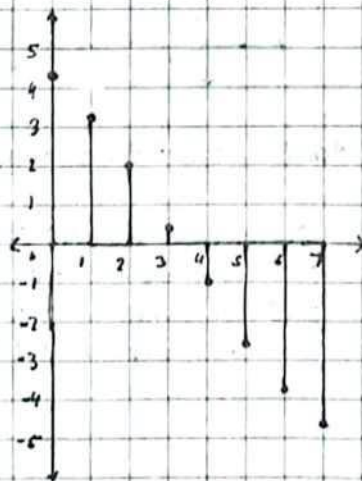
$$x(3) = 5 \cos(0.1\pi(3) + 30^\circ) u(3), \text{ amp} = 0.52, u(n) = 3$$

$$x(4) = 5 \cos(0.1\pi(4) + 30^\circ) u(4), \text{ amp} = -1.04, u(n) = 4$$

$$x(5) = 5 \cos(0.1\pi(5) + 30^\circ) u(5), \text{ amp} = -2.5, u(n) = 5$$

$$x(6) = 5 \cos(0.1\pi(6) + 30^\circ) u(6), \text{ amp} = -3.72, u(n) = 6$$

$$x(7) = 5 \cos(0.1\pi(7) + 30^\circ) u(7), \text{ amp} = -4.57, u(n) = 7$$



$$d.) x(n) = 5(0.75)^n \sin(0.1\pi n) u(n)$$

$$x(0) = 5(0.75)^0 \sin(0.1\pi(0)) u(0), \text{ amp} = 0, u(n) = 0$$

$$x(1) = 5(0.75)^1 \sin(0.1\pi(1)) u(1), \text{ amp} = 1.16, u(n) = 1$$

$$x(2) = 5(0.75)^2 \sin(0.1\pi(2)) u(2), \text{ amp} = 1.65, u(n) = 2$$

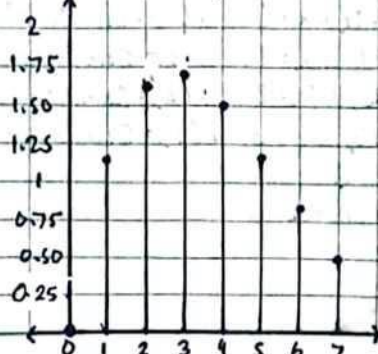
$$x(3) = 5(0.75)^3 \sin(0.1\pi(3)) u(3), \text{ amp} = 1.71, u(n) = 3$$

$$x(4) = 5(0.75)^4 \sin(0.1\pi(4)) u(4), \text{ amp} = 1.5, u(n) = 4$$

$$x(5) = 5(0.75)^5 \sin(0.1\pi(5)) u(5), \text{ amp} = 1.19, u(n) = 5$$

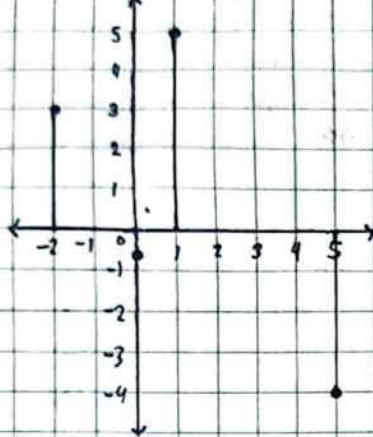
$$x(6) = 5(0.75)^6 \sin(0.1\pi(6)) u(6), \text{ amp} = 0.85, u(n) = 6$$

$$x(7) = 5(0.75)^7 \sin(0.1\pi(7)) u(7), \text{ amp} = 0.54, u(n) = 7$$

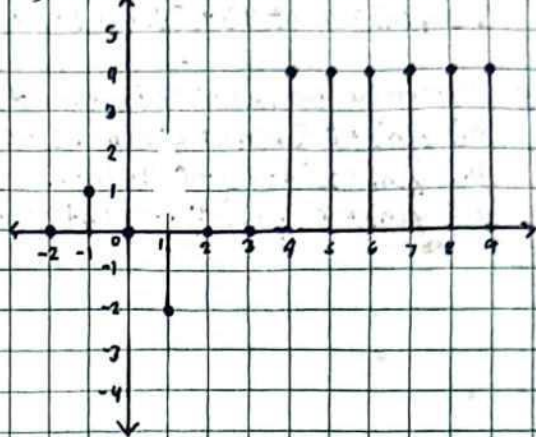


3.3. Sketch the following sequences:

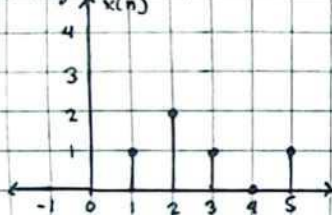
a.) $x[n] = 3\delta[n+2] - 0.5\delta[n] + 5\delta[n-1] - 4\delta[n-5]$



b.) $x[n] = \delta[n+1] - 2\delta[n-1] + 5\delta[n-4]$

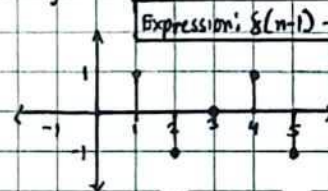
3.4. Given the digital signals $x[n]$ in Figures 3.24 and 3.25, write an expression for each digital signal using the unit-impulse and its shifted sequences.

a.) Figure 3.24



Expression: $\delta[n-1] + 2\delta[n-2] + \delta[n-3] + 0\delta[n-4] + \delta[n-5]$

b.) Figure 3.25



Expression: $\delta[n-1] - \delta[n-2] + 0\delta[n-3] + \delta[n-4] - \delta[n-5]$

3.5. Assuming that a DS processor with a sampling time interval of 0.01 second converts each of the following analog signals to $x(t)$ to the digital signal $x[n]$, determine the digital sequences for each of the following analog signals.

a.) $x(t) = e^{-50t}u(t)$

$$x[n] = x(nT)$$

$$= e^{-50(0.01n)}u(nT)$$

$$x[n] = e^{-0.5n}u[n]$$

b.) $x(t) = 5\sin(20\pi t)u(t)$

$$x[n] = x(nT)$$

$$= 5\sin(20\pi(0.01n))u(nT)$$

$$x[n] = 5\sin(0.2\pi n)u[n]$$

c.) $x(t) = 10\cos(40\pi t + 30^\circ)u(t)$

$$x[n] = x(nT)$$

$$= 10\cos(40\pi(0.01n) + 30^\circ)u(nT)$$

$$x[n] = 10\cos(0.4\pi n + 30^\circ)u[n]$$

d.) $x(t) = 10e^{-100t}\sin(15\pi t)u(t)$

$$x[n] = x(nT)$$

$$= 10e^{-100(0.01n)}\sin(15\pi(0.01n))u(nT)$$

$$x[n] = 10e^{-n}\sin(0.15\pi n)u[n]$$

3.6. Determine which of the following is a linear system.

a.) $y[n] = 5x[n] + 2x^2[n]$ - non linear

b.) $y[n] = x[n-1] + 4x[n]$ - linear

c.) $y[n] = 4x^3[n-1] - 2x[n]$ - non linear

3.7. Given the following linear systems, find which one is time invariant

a.) $y[n] = -5x[n-10]$ - time invariant

b.) $y[n] = 4x[n^2]$ - not time invariant

3.8. Determine which of the following linear systems is causal.

a.) $y(n) = 0.5x(n) + 100x(n-2) - 20x(n-10)$ - causal

b.) $y(n) = x(n+4) + 0.5x(n) - 2x(n-2)$ - not causal

3.9. Determine the causality for each of the following linear systems.

a.) $y(n) = 0.5x(n) + 20x(n-2) - 0.1y(n-1)$ - causal

b.) $y(n) = x(n+2) - 0.4y(n-1)$ - not causal

c.) $y(n) = x(n-1) + 0.5y(n+2)$ - not causal

3.10. Find the unit-impulse response for each of the following linear systems.

a.) $y(n] = 0.5x(n) - 0.5x(n-2)$; for $n \geq 0$, $x(-2) = 0$, $x(-1) = 0$

$h(n) = 0.5\delta(n) - 0.5\delta(n-2)$ [$n \geq 0$]

b.) $y(n) = 0.75y(n-1) + x(n)$; for $n \geq 0$, $y(-1) = 0$

$h(n) = 0.75h(n-1) + \delta(n)$ [$n \geq 0$, $h(-1) = 0$]

Mathematical induction:

$h(-1) = 0$, at $n=0$: $h(0) = 0.75h(-1) + \delta(0) \rightarrow h(0) = 0.75h(-1) + \delta(0)$

$h(0) = 0.75(0) + 1 \rightarrow h(0) = 1$

at $n=1$: $h(1) = 0.75h(0) + \delta(1) \rightarrow 0.75h(0) + \delta(1) \rightarrow 0.75(1) + 0 \rightarrow h(1) = 0.75$

at $n=2$: $h(2) = 0.75h(1) + \delta(2) \rightarrow 0.75h(1) + \delta(2) \rightarrow 0.75(0.75) + 0 \rightarrow h(2) = 0.5625$

Therefore: $h(n) = 0.75^n$ [$n \geq 0$]

c.) $y(n) = -0.8y(n-1) + x(n-1)$; for $n \geq 0$, $x(-1) = 0$, $y(-1) = 0$

$h(n) = -0.8h(n-1) + \delta(n-1)$ [$n \geq 0$, $\delta(-1) = 0$, $h(-1) = 0$]

at $n=0$: $h(0) = -0.8h(-1) + \delta(-1) \rightarrow -0.8h(-1) + \delta(-1) \rightarrow -0.8(0) + 0 \rightarrow h(0) = 0$

at $n=1$: $h(1) = -0.8h(0) + \delta(0) \rightarrow -0.8h(0) + \delta(0) \rightarrow -0.8(0) + 1 \rightarrow h(1) = 1$

at $n=2$: $h(2) = -0.8h(1) + \delta(1) \rightarrow -0.8h(1) + \delta(1) \rightarrow -0.8(1) + 0 \rightarrow h(2) = -0.8$

at $n=3$: $h(3) = -0.8h(2) + \delta(2) \rightarrow -0.8h(2) + \delta(2) \rightarrow -0.8(-0.8) + 0 \rightarrow 0.64$

Therefore:

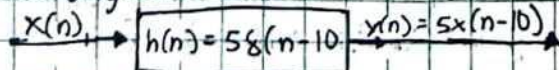
$$h(n) = \begin{cases} 0, & n=0 \\ (-0.8)^{n-1}, & n \geq 1 \end{cases}$$

3.11. For each of the following linear systems, find the unit-impulse response, and draw the block diagram.

a.) $y(n) = 5x(n-10)$

$h(n) = 5\delta(n-10)$

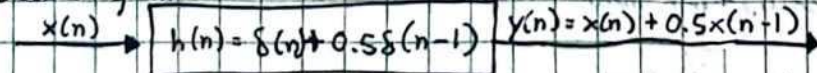
Block diagram:



b.) $y(n) = x(n) + 0.5x(n-1)$

$h(n) = \delta(n) + 0.5\delta(n-1)$

Block diagram:



3.12. Determine the stability for the following linear system.

$$y(n) = 0.5x(n) + 100x(n-2) - 20x(n+10)$$

Since the coefficients 0.5, 100, and -20 are bounded and finite, the system is **stable**.

3.13. Determine the stability for each of the following linear systems.

a.) $y(n) = \sum_{k=0}^{\infty} 0.75^k x(n-k)$

$$y(n) = \frac{1}{1-0.75} = 4 \quad \text{since the system converges, the system is BIBO stable.}$$

b.) $y(n) = \sum_{k=0}^{\infty} 2^k x(n-k)$

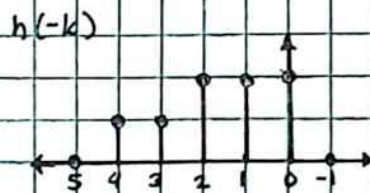
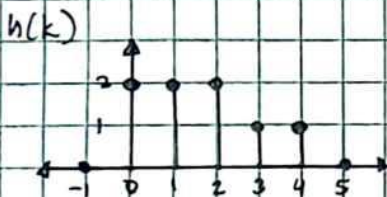
$$y(n) = \frac{1}{1-2} = -1 \quad \text{since the system diverges, the system is BIBO unstable.}$$

3.14. Given the sequence:

$$h(k) = \begin{cases} 2, & k=0, 1, 2 \\ 1, & k=3, 4 \\ 0 & \text{elsewhere} \end{cases}$$

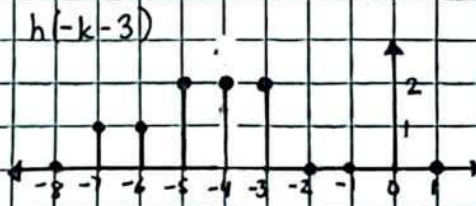
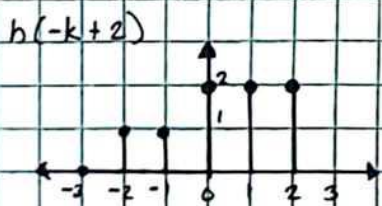
where k is the time index or sample number,

a.) sketch the sequence $h(k)$ and the reverse sequence $h(-k)$:



$$h(-k) = \begin{cases} 2, & k=0, -1, -2 \\ 1, & k=-3, -4 \\ 0 & \text{elsewhere} \end{cases}$$

b.) sketch the shifted sequence $h(-k+2)$ and $h(-k-3)$



$$h(-k+2) = \begin{cases} 2, & k=0+2, 1+2, -2+2 \\ 1, & k=-3+2, -4+2 \\ 0 & \text{elsewhere} \end{cases}$$

$$h(-k-3) = \begin{cases} 2, & k=0-3, -1-3, -2-3 \\ 1, & k=-3-3, -4-3 \\ 0 & \text{elsewhere} \end{cases}$$

$$\downarrow$$

$$h(k+2) = \begin{cases} 2, & k=-2, -1, 0 \\ 1, & k=-3, -4 \\ 0 & \text{elsewhere} \end{cases}$$

$$\downarrow$$

$$h(k-3) = \begin{cases} 2, & k=-3, -4, -5 \\ 1, & k=-6, -7 \\ 0 & \text{elsewhere} \end{cases}$$