Polytechnic University of the Philippines
Department of Computer Engineering

Digital Signal Processing

CMPE 30244

MP3
Frequency Response and Passive Filters using Multisim

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OBJECTIVES

- To reinforce the concepts behind filter circuits and frequency response
- To reinforce the idea of a phasor
  - To understand and use phasor circuit analysis
- To reinforce the procedure of deriving a transfer function
- To graphically demonstrate the effects of different passive component configurations on different ranges of frequency

MATERIALS

- The lab assignment (this document)
- Your lab parts
- Printouts (required) of the below documents:
  - Pre-lab analyses
  - Multisim screenshots e-mailed to course e-mail
- Graph paper.

INTRODUCTION

In this experiment we will analytically determine and measure the frequency response of networks containing resistors, ac sources, and energy storage elements (inductors and capacitors).

Given an input sinusoidal voltage, we will analyze the circuit using the frequency-domain method to determine the phasor of output voltage in the ac steady state. The response function is defined as the ratio of the output and input voltage phasors. It is a function of the input frequency and the values of the circuit elements (resistors, inductors, capacitors).

We start with examples of a few filter circuits to illustrate the concept.

RC Low-Pass Filter:

Consider the series combination in Fig 1 of the resistor R and the capacitor C, connected to an input signal represented by ac voltage source of frequency ω.

$$v_{in}(t) = V_s \cos(\omega t + \theta_i) \quad (1)$$

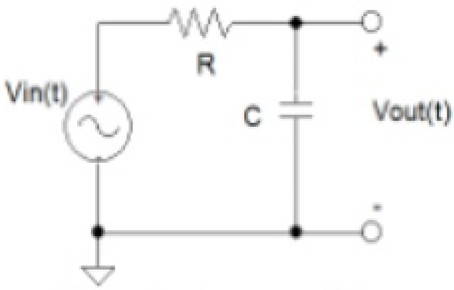

Figure 1 – Low-pass filter.

Suppose we are interested in monitoring the voltage across the capacitor. We designate this voltage as the output voltage. We know that it will be a sinusoid of frequency ω. Thus,

$$v_{out}(t) = V_o \cos(\omega t + \theta_o) \quad (2)$$

We will now determine expressions for the amplitude V_o and the phase angle θ_o. First we convert the network to frequency domain, as shown in Fig. 2.

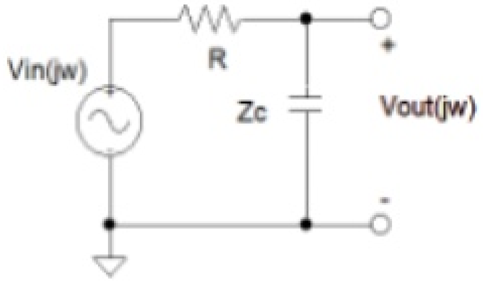

Figure 2 – Low-pass filter in frequency domain.

In the above circuit, the voltage source is represented by its phasor and the resistor and capacitor by their impedances. We wish to evaluate the phasor $\mathbf{V_{out}}$ for the output sinusoid. Since the three elements are in series, the voltage divider formula can be used and we obtain

$$\mathbf{V_{out}} = [\, Z_c / (Z_c + R) \,]\, \mathbf{V_{in}} \, , \tag{3}$$

where $\mathbf{V_{in}}$ is the phasor of the input voltage. It is given by

$$\mathbf{V_{in}} = V_s e^{j\theta_1} \tag{4}$$

$$Z_c = 1/j\omega C \tag{5}$$

The **transfer function** is defined as the output divided by the input. The **frequency response**, **H(jw)**, can be found by manipulation of equation (3),

$$\mathbf{H(j\omega)} = \mathbf{V_{out}} / \mathbf{V_{in}} = 1/(1 + j\omega RC) \tag{6}$$

The product RC has units of the inverse of angular frequency. We define (7) as a characteristic frequency of the network and write the frequency response as (8).

$$\omega_o = 1/RC \tag{7}$$

$$\mathbf{H(j\omega)} = 1/(1 + j\omega/\omega_o) \tag{8}$$

In other words, we are measuring frequency in units of ω_o (rad/s).

The sinusoid corresponding to the output voltage can be written as

$$v_{out}(t) = \mathrm{Re}\{\mathbf{V_{out}}\, e^{j\omega t}\} = \mathrm{Re}\{\mathbf{H(j\omega)Vin}\, e^{j\omega t}\} = \mathrm{Re}\{V_s e^{j\theta_1} e^{j\omega t}/(1+j\omega/\omega_o)\} \tag{9}$$

$$v_{out}(t) = \{V_s / [1+(\omega/\omega_o)^2]\}\cos(\,\omega t + \theta_I - \tan^{-1}(\omega/\omega_o)\,) \tag{10}$$

Returning to the frequency response, $\mathbf{H(j\omega)}$ is a complex number. It has a magnitude and phase. Both depend on the frequency, R and C. Thus,

$$\mathbf{H(j\omega)} = H \exp(j\theta_H) \tag{11}$$

The magnitude (absolute value) of **H** is a measure of the ratio of the amplitudes of the output and input voltages. It is given by:

$$H = |\mathbf{H(j\omega)}| = V_o / V_s = 1/[1+(\omega/\omega_o)^2]^{1/2} \qquad (12)$$

On the other hand, the phase angle of **H** measures the difference in the output and input phase angles. It is given by:

$$\theta_O - \theta_I = \theta_H = -\tan^{-1}(\omega/\omega_o) \qquad (13)$$

The frequency dependence of the magnitude H is plotted in Fig. 3. Note that the x-axis is unitless, the normalized frequency of ω/ω_o.

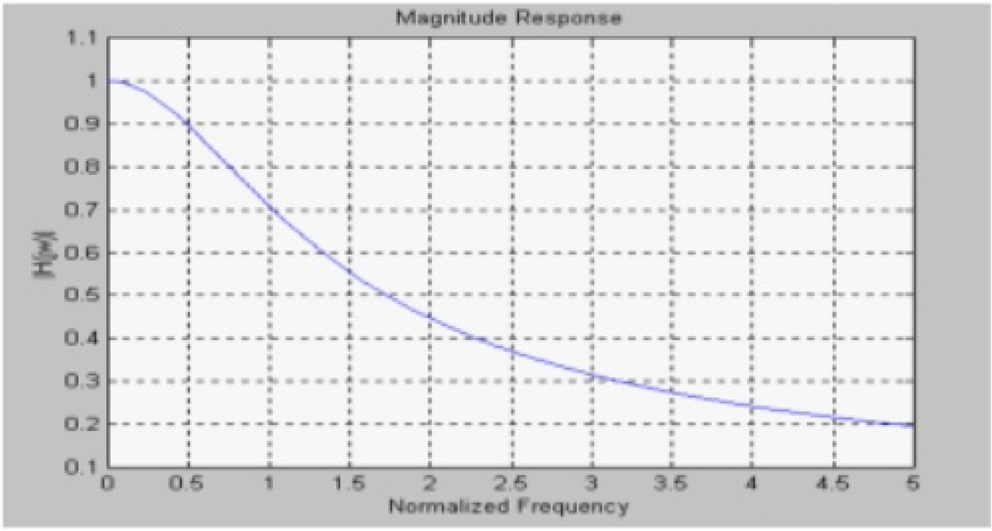

Figure 3 – Magnitude frequency response.

From Fig. 3, it is evident that for low frequencies ($\omega<<\omega_o$), H is close to one. In this frequency range, the network allows effective transmission of the input voltage. For $\omega>>\omega_o$, H becomes much less than one. This means that high frequencies do not get transmitted well by the network, but low frequencies are transmitted well. In other words, the network acts as a **low-pass filter**.

The characteristic frequency ω_o is called the **cut-off frequency**. It is defined as the frequency at which H is equal to $(1/\sqrt{2})*H_{max}$. Similarly, the frequency dependence of the phase θ_H is shown in Fig. 4. There is negligible phase shift at very low frequencies and a phase shift approaching –90° at very high frequencies.

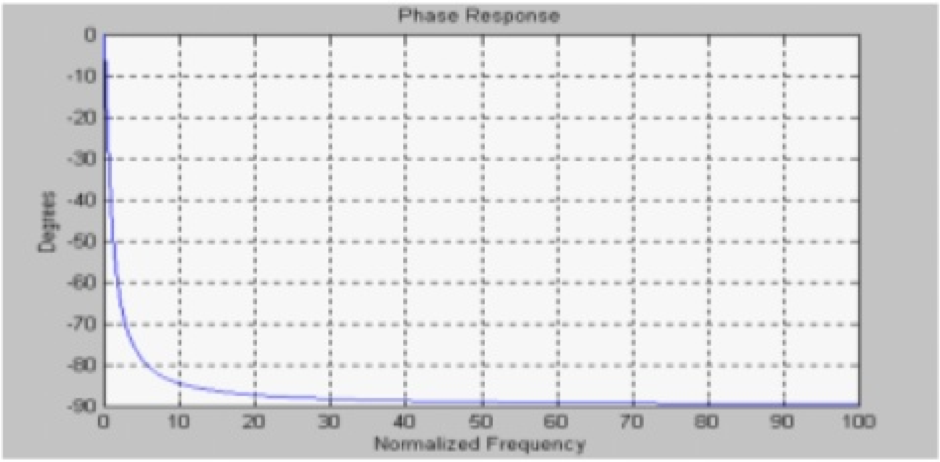

Figure 4 – Angle frequency response.

The magnitude and phase plots shown in Fig. 3 and Fig. 4 are plotted using linear scales. However, in electrical circuits, the frequency range may span several decades. For example, in audio amplifiers, the frequency range of interest is 20 Hz to 20,000 Hz. Similarly, the magnitude of the frequency response may vary over several orders of magnitude. Therefore, linear scaled plots are of little use and the frequency response is represented by **Bode Plots.**

In Bode plots, the magnitude H is plotted on the vertical axis, in units of dB, defined by the following equation:

$$H_{dB} = 20 \log H \tag{14}$$

On the horizontal axis, the frequency is represented on a log scale. On the log scale, the distance between10 and 100 rad/s is equal to that between 100 and 1000 rad/s. This is due to the fact that (log 100 – log 10) ≈ (log 1000 – log 100) = 1. The distance from 10 to 20 is 30% of the distance between 10 and 100, which can easily inferred since (log 20 – log 10) = 0.3.

Fig. 5 shows the Bode plot of the magnitude and phase of the low-pass filter of Fig. 1.

At low frequencies, the value of H_{dB} is close to 0 dB and it is represented by a straight line with zero gradient. **At the cut off frequency H_{dB} drops to – 3 dB**, and **at frequencies much larger than the cutoff frequency, the response is accurately represented by a straight line with a slope of –20 dB/decade**. If we extrapolate the two straight lines, they will intersect at the cutoff frequency. The two lines represent the **asymptotic Bode Plots**. The maximum error in asymptotic Bode plot for this case is 3 dB, occurring at the cutoff frequency.

Asymptotic Bode plots are very useful in estimating the magnitude H at any frequency fairly accurately. They are easy to sketch since only straight lines are involved. For example, if we wish to know H at a frequency 100 times larger than the cutoff frequency, we get $H_{dB} = -40$ dB, which gives $H = 0.01$, implying that the amplitude of the output voltage at this frequency is 1% of the amplitude of the input voltage.