Don Radcliffe
SEFS 502 Assignment 2: Matrix Algebra
Jon Bakker
January 18th, 2022

1. Matrix algebra (16 points). Use the following objects:

a) Premultiply M by VT. Solve by hand, showing your work. (2 points)

$V^{T} \%\cdot\% M =$

$\begin{bmatrix} 1 & 7 \end{bmatrix} \cdot \begin{bmatrix} 5 & 2 & 7 \\ 10 & 1 & 8 \end{bmatrix} =$

$\begin{bmatrix} (1\cdot5)+(7\cdot10) & (1\cdot2)+(7\cdot1) & (1\cdot7)+(7\cdot8) \end{bmatrix} = \begin{bmatrix} 5+70 & 2+7 & 7+56 \end{bmatrix} = \begin{bmatrix} 75 & 9 & 63 \end{bmatrix}$

For each of the following equations, determine by hand (i.e., not using R) whether it will work mathematically, and the dimensions of the solution. Then, do the calculation in R and compare the R output with what the correct answer should be. (1 point each)

b) V %*% M

Dimensions of $V = 2\times\underline{1}$
Dimensions of $M = \underline{2}\times3$

Will not work, inside dimensions don't match (1 and 2)

In R → Error, which is correct

c) vector %*% M

Vector	M.
$2\times\underline{1}$	$\underline{2}\times3$

Will not work

In R → [75, 9, 63], which shouldn't be calculated

d) M %*% vector

M	vector
$2\times\underline{3}$	$\underline{2}\times1$

Will not work

In R → will not work, which is correct

e) M %*% M

$$\underset{2\times \underline{3}}{M} \cdot \underset{\underline{2}\times 3}{M}$$

Will not work

In R → error, which is collect

f) M %*% t(M)

$$\underset{2\times \underline{3}}{M} \quad \underset{\underline{3}\times 2}{M^t}$$

Will work, inner value match

Resulting matrix 2x2 (outer values)

In R → 2x2 matrix $\begin{bmatrix} 78 & 108 \\ 108 & 165 \end{bmatrix}$

g) t(M) %*% M

$$\underset{3\times \underline{2}}{M^t} \quad \underset{\underline{2}\times 3}{M}$$

Will work, produce a 3x3 matrix (outer values)

In R → 3x3 matrix $\begin{bmatrix} 125 & 20 & 115 \\ 20 & 5 & 22 \\ 115 & 22 & 113 \end{bmatrix}$

h) V %*% V

$$\underset{2\times \underline{1}}{V} \quad \underset{\underline{2}\times 1}{V}$$

Will not work

In R → Error, which is collect

i) vector %*% vector

$$\underset{2\times \underline{1}}{\text{vector}} \quad \underset{\underline{2}\times 1}{\text{vector}}$$

Will not work

In R → [50], so it incollectly transposed the first vector

j) V %*% vector

$$\underset{2\times \underline{1}}{V} \quad \underset{\underline{2}\times 1}{\text{vector}}$$

Will not work

In R → $\begin{bmatrix} 1 & 7 \\ 7 & 49 \end{bmatrix}$, so it incollectly transposed vector

k) vector %*% V

vector V will not work

2x1 2x1

In R → [50], so it incorrectly transposed vector

l) What is the difference between vector and V? (2 points)

vector is considered a vector by R, which means:

- R displays it as a row vector
- R will transpose it to make an operation work, even if erroneously, as we saw in the example above

V is considered a matrix, so R will retain its original dimensions when attempting matrix algebra.

m) What is the difference between premultiplication and postmultiplication? How is the dimensionality of the solution affected by the order of the terms? (2 points)

Premultiplication of X by Y is Y%*%X
Postmultiplication of X by Y is X%*%Y

This affects dimensionality/feasibility if the matrices aren't square, because outside values of a matrix multiplication problem affect dimensionality, inside values affect feasibility, and the order of multiplication affects where values fall.

2. Euclidean distances (7 points). Assume that the matrix M above contains data for 3 variables on 2 plots.

• Calculate, by hand, the Euclidean distance between these two plots and show your work. Verify your calculation using R. (2 points)

$$ED = \sqrt{(5-10)^2 + (2-1)^2 + (7-8)^2}$$
$$= \sqrt{5^2 + 1^2 + 1^2}$$
$$= \sqrt{25+1+1}$$
$$= \sqrt{27} = 5.20$$

verified in R

• Then, relativize M by variable maxima and recalculate the Euclidean distance. How has it changed? (2 points)

M relativized by max =

$$\begin{bmatrix} 0.5 & 0.2 & 0.7 \\ 1.0 & 0.1 & 0.8 \end{bmatrix}$$

$$ED = \sqrt{(0.5-1.0) + (0.2-0.1)^2 + (0.7-0.8)^2}$$
$$= \sqrt{0.5^2 + 0.1^2 + 0.1^2}$$
$$= \sqrt{0.25+0.1+0.1}$$
$$= \sqrt{0.27}$$
$$= 0.52$$

verified in R

It changed by a decimal place

• Under what conditions would the Euclidean distance between two plots be at its minimum? (1 point)

If both plots had the exact same values across their rows, the Euclidean distance would be 0.

• Is there a maximum value for the Euclidean distance and, if so, under what conditions would it be reached? (2 points)

No, if we assume theoretical limits of infinity for our variables, such as species abundances. In practice Euclidean distance is maximized when one plot has 0 species and the other has the maximum abundance of species in the dataset.

3. Bray-Curtis distances (7 points). The Bray-Curtis distance measure is one of the most commonly used in community ecology. The intent of this question is to help you understand how it works. Here are two equivalent formulations for it:

Assume that the matrix M above contains abundances of 3 species on each of 2 plots.

• Calculate, by hand, the Bray-Curtis distance between these two plots and show your work. Verify your calculation using R. (2 points)

$$M = \begin{bmatrix} 5 & 2 & 7 \\ 10 & 1 & 8 \end{bmatrix}$$

$$\text{Bray} = 1 - (2(5+1+7)/14+19)$$
$$= 1 - 26/33$$
$$= 1 - 0.7878$$
$$= 0.21$$

verified in R

• Then, relativize M by species maxima and recalculate the Bray-Curtis distance. How has it changed? (2 points)

M relativized by max =

$$\begin{bmatrix} 0.5 & 0.2 & 0.7 \\ 1.0 & 0.1 & 0.8 \end{bmatrix}$$

$$\text{Bray} = 1 - (2(0.5+0.1+0.7)/0.14+0.19)$$
$$= 1 - 0.26/0.33$$
$$= 1 - 0.7878$$
$$= 0.21$$

Verified in R

• Under what conditions would the Bray-Curtis distance between two plots be at its minimum? (1 point)

When the plots share no species in common

Ex: $\begin{bmatrix} 2 & 0 & 10 \\ 0 & 4 & 0 \end{bmatrix}$

• Is there a maximum value for the Bray-Curtis distance and, if so, under what conditions would it be reached? (2 points)

The maximum value for Bray-Curtis is 1, and it would be maximized when both plots have the same abundance values for all the same species

Ex: $\begin{bmatrix} 2 & 4 & 10 \\ 2 & 4 & 10 \end{bmatrix}$