The Relational Algebra

Database Management Systems

Handout 6

Chapter Outline

- Example Database Application (COMPANY)
- Relational Algebra
 - Unary Relational Operations
 - Relational Algebra Operations From Set Theory
 - Binary Relational Operations
 - Additional Relational Operations
 - Examples of Queries in Relational Algebra
- Relational Calculus
 - Tuple Relational Calculus

Database State for COMPANY

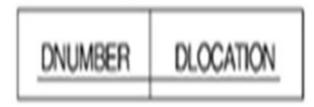
EMPLOYEE

FNAME MINIT LNAME SSN BDATE ADDRESS SEX SALARY SUPERSSN

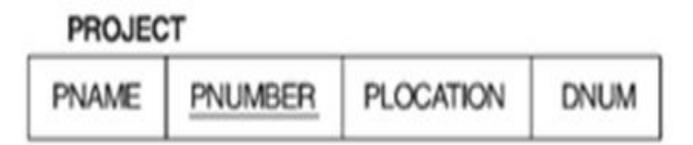
DEPARTMENT

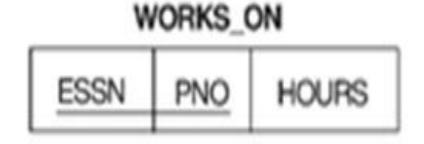


DEPT_LOCATIONS



Database State for COMPANY





DEPENDENT

ESSN DEPENDENT_NAME	SEX	BDATE	RELATIONSHIP
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Database State for COMPANY

EMPLOYEE	FNAME	MINIT	LNAME	<u>SSN</u>	BDATE	ADDRESS	SEX	SALARY	SUPERSSN	DNO
	John		Smith	123456789	1965-01-09	731 Fondren, Houston, TX	М	30000	333445555	5
	Franklin		Wong	333445555	1955-12-08	638 Voss, Houston, TX	М	40000	888665555	5
	Alicia		Zelaya	999887777	1968-01-19	3321 Casile, Spring, TX	ч.	25000	987654321	4
	Jennifer		Wallace	987654321	1941-06-20	291 Berry, Bellaire, TX	F	43000	888665555	4
	Ramesh		Narayan	666884444	1962-09-15	975 Fire Oak, Humble, TX	М	38000	333445555	5
	Joyce		English	453453453	1972-07-31	5631 Rice, Houston, TX	L L	25000	333445555	5
	Ahmad		Jabbar	987987987	1969-03-29	980 Dailas, Houston, TX	М	25000	987654321	4
	James		Borg	88866555	1937-11-10	450 Stone, Houston, TX	М	55000	null	1

Relational Algebra

- The basic set of operations for the relational model is known as the relational algebra. These operations enable a user to specify basic retrieval requests.
- The result of a retrieval is a new relation, which may have been formed from one or more relations. The **algebra operations** thus produce new relations, which can be further manipulated using operations of the same algebra.
- A sequence of relational algebra operations forms a **relational algebra expression**, whose result will also be a relation that represents the result of a database query (or retrieval request).

Unary Relational Operations – Select (σ)

SELECT operation is used to select a *subset* of the tuples from a relation that satisfy a **selection condition**.

Example: To select the EMPLOYEE tuples whose department number is four or those whose salary is greater than \$30,000 the following notation is used:

$$\sigma_{DNO = 4 OR SALARY > 30,000}$$
 (EMPLOYEE)

- Select operation is denoted by $\sigma_{\text{selection condition}}(R)$
- <selection condition> is a Boolean expression specified on the attributes of relation R

Unary Relational Operations

SELECT Operation Properties

- The SELECT operation $\sigma_{\text{selection condition}}(R)$ produces a relation S that has the same schema as R
- The SELECT operation σ is **commutative**; i.e.,

$$\sigma_{\text{}(\sigma_{\text{}(R)) = \sigma_{\text{}(\sigma_{\text{}(R))$$

A cascaded SELECT operation may be applied in any order;
 i.e.,

```
\sigma_{<\text{condition}1>}(\sigma_{<\text{condition}2>}(\sigma_{<\text{condition}3>}(R))
= \sigma_{<\text{condition}2>}(\sigma_{<\text{condition}3>}(\sigma_{<\text{condition}1>}(R)))
```

 A cascaded SELECT operation may be replaced by a single selection with a conjunction of all the conditions; i.e.,

$$\sigma_{\text{condition1}}(\sigma_{\text{condition2}}(\sigma_{\text{condition3}}(R))$$
= $\sigma_{\text{condition1}} = \sigma_{\text{condition2}} = \sigma_{\text{condition3}}(R)$

Results of SELECT and PROJECT operations.

- (a) $\sigma_{\text{(DNO=4 AND SALARY>25000) OR (DNO=5 AND SALARY>30000)}}$ (EMPLOYEE).
- (b) $\pi_{LNAME, FNAME, SALARY}$ (EMPLOYEE). (c) $\pi_{SEX, SALARY}$ (EMPLOYEE)

(a)

FNAME	MINIT	LNAME	SSN	BDATE	ADDRESS	SEX	SALARY	SUPERSSN	DNO
Franklin	Т	Wong	333445555	1955-12-08	638 Voss,Houston,TX	М	40000	888665555	5
Jennifer		Wallace	987654321	1941-06-20	291 Berry,Bellaire,TX	F	43000	888665555	4
Ramesh		Narayan	666884444	1962-09-15	975 FireOak,Humble,TX	М	38000	333445555	5

(b)

LNAME	FNAME	SALARY
Smith	John	30000
Wong	Franklin	40000
Zelaya	Alicia	25000
Wallace	Jennifer	43000
Narayan	Ramesh	38000
English	Joyce	25000
Jabbar	Ahmad	25000
Borg	James	55000

(c)

SEX	SALARY
М	30000
М	40000
F	25000
F	43000
М	38000
М	25000
М	55000

Unary Relational Operations – Project (π)

Project operation selects certain *columns* from the table and discards the other columns.

Example: To list each employee's first and last name and salary, the following is used:

π_{LNAME, FNAME, SALARY} (EMPLOYEE)

General form: $\pi_{\text{<attribute list>}}(R)$,

- π (pi) is the symbol used to represent the project operation.
- <attribute list> is the desired list of attributes from the attributes of relation R.
- π removes any duplicate tuples, result is a set of tuples (hence a valid relation).

PROJECT Operation Properties

- The number of tuples in the result of projection $\pi_{< list>}$ (R)is always less or equal to the number of tuples in R.
- If the list of attributes includes a key of R, then the number of tuples is equal to the number of tuples in R.
- $\pi_{\langle list1\rangle}(\pi_{\langle list2\rangle}(R)) = \pi_{\langle list1\rangle}(R)$ as long as $\langle list2\rangle$ contains the attributes in $\langle list1\rangle$

Rename Operation

We may want to apply several relational algebra operations one after the other.

- Write the operations as a single **relational algebra expression** by nesting the operations
- We can apply one operation at a time and create **intermediate result relations**. (give names to the relations that hold the intermediate results.)

Example: To retrieve the first name, last name, and salary of all employees who work in department number 5

```
\pi_{\text{FNAME, LNAME, SALARY}}(\sigma_{\text{DNO=5}}(\text{EMPLOYEE}))
DEP5_EMPS \leftarrow \sigma_{\text{DNO=5}}(\text{EMPLOYEE})
RESULT \leftarrow \pi_{\text{FNAME, LNAME, SALARY}} (DEP5_EMPS)
```

Results of relational algebra expressions.

(a) $\pi_{\text{LNAME, FNAME, SALARY}}$ ($\sigma_{\text{DNO=5}}$ (EMPLOYEE)). (b) The same expression using intermediate relations and renaming of attributes.

(a)	FNAME	LNAME	SALARY
	John	Smith	30000
	Franklin	Wong	40000
	Ramesh	Narayan	38000
	Jayce	English	25000

(b)	TEMP	FNAME	MINIT	LNAME	SSN	BDATE	ADDRESS	SEX	SALARY	SUPERSSN	DNO
		John	В	Smith	123456789	1965-01-09	731 Fondren, Houston, TX	М	30000	333445555	5
		Franklin	T	Wong	333445555	1955-12-08	638 Voss, Houston, TX	М	40000	888665555	5
		Ramesh	K	Narayan	666384444	1962-09-15	975 Fire Oak, Humble, TX	М	38000	333445555	5
		Joyce	A	English	453453453	1972-07-31	5831 Rice,Houston,TX	F	25000	333445556	5

FIRSTNAME	LASTNAME	SALARY
John	Smith	30000
Franklin	Wong	40000
Ramesh	Narayan	38000
Jayas	English	25000

UNION Operation (∪)

The result of this operation, denoted by $R \cup S$, is a relation that includes all tuples that are either in R or in S or in both R and S. Duplicate tuples are eliminated. The two operands must be "type compatible".

Example: To retrieve the social security numbers of all employees who either work in department 5 or directly supervise an employee who works in department 5, we can use the union operation as follows:

```
DEP5_EMPS \leftarrow \sigma_{DNO=5} (EMPLOYEE)

RESULT1 \leftarrow \pi_{SSN} (DEP5_EMPS)

RESULT2(SSN) \leftarrow \pi_{SUPERSSN} (DEP5_EMPS)

RESULT \leftarrow RESULT1 \cup RESULT2
```

UNION Example

STUDENT	FN	LN
	Susan	Yao
	Ramesh	Shah
	Johnny	Kohler
	Barbara	Jones
	Amy	Ford
	Jimmy	Wang
	Emest	Gilbert

INSTRUCTOR	FNAME	LNAME
	John	Smith
	Ricardo	Browne
	Susan	Yao
	Francis	Johnson
	Ramesh	Shah

FN	LN
Susan	Yao
Ramesh	Shah
Johnny	Kohler
Barbara	Jones
Amy	Ford
Jimmy	Wang
Emest	Gilbert
John	Smith
Ricardo	Browne
Francis	Johnson

STUDENT-INSTRUCTOR

Type Compatibility

- The operand relations $R_1(A_1, A_2, ..., A_n)$ and $R_2(B_1, B_2, ..., B_n)$ must have the same number of attributes, and the domains of corresponding attributes must be compatible; that is, $dom(A_i)=dom(B_i)$ for i=1, 2, ..., n.

– The resulting relation for $R_1 \cup R_2$, $R_1 \cap R_2$, or R_1 - R_2 has the same attribute names as the *first* operand relation R1 (by convention).

Two UNION compatible relations

STUDENT	FN	LN
	Susan	Yao
	Ramesh	Shah
	Johnny	Kohler
	Barbara	Jones
	Amy	Ford
	Jimmy	Wang
	Ernest	Gilbert

INSTRUCTOR	FNAME	LNAME
	John	Smith
	Ricardo	Browne
	Susan	Yao
	Francis	Johnson
	Ramesh	Shah

INTERSECTION OPERATION

The result of this operation, denoted by $R \cap S$, is a relation that includes all tuples that are in both R and S. The two operands must be "type compatible"

Example: The result of the intersection operation (figure below) includes only those who are both students and instructors.

STUDENT ∩ INSTRUCTOR

FN	LN
Susan	Yao
Ramesh	Shah

Set Difference (or MINUS) Operation

The result of this operation, denoted by R - S, is a relation that includes all tuples that are in R but not in S. The two operands must be "type compatible".

Example: The figure shows the names of students who are not instructors, and the names of instructors who are not

students.

STUDENT-INSTRUCTOR

FN	LN
Johnny	Kohler
Barbara	Jones
Amy	Ford
Jimmy	Wang
Emest	Gilbert

• Notice that both union and intersection are *commutative* operations; that is

$$R \cup S = S \cup R$$
, and $R \cap S = S \cap R$

• Both union and intersection can be treated as n-ary operations applicable to any number of relations as both are *associative operations;* that is

$$R \cup (S \cup T) = (R \cup S) \cup T$$
, and $(R \cap S) \cap T = R \cap (S \cap T)$

• The minus operation is *not commutative*; that is, in general

$$R - S \neq S - R$$

CARTESIAN (or cross product) Operation

- Combine tuples from two relations in a combinatorial fashion. In general,
- $R(A_1, A_2, ..., A_n)$ x $S(B_1, B_2, ..., B_m)$ is a relation Q with degree n + m attributes $Q(A_1, A_2, ..., A_n, B_1, B_2, ..., B_m)$, in that order.
- The resulting relation Q has one tuple for each combination of tuples—one from R and one from S.
- Hence, if R has n_R tuples (denoted as $|R| = n_R$), and S has n_S tuples, then
 - $|R \times S|$ will have $n_R * n_S$ tuples.
- The two operands do NOT have to be "type compatible"

Example:

```
FEMALE_EMPS \leftarrow \sigma_{SEX='F'} (EMPLOYEE)
EMPNAMES \leftarrow \pi_{FNAME, LNAME, SSN} (FEMALE_EMPS)
EMP_DEPENDENTS \leftarrow EMPNAMES x DEPENDENT
```

EMPNAMES	FNAME	LNAME	SSN
	Alicia	Zelaya	999887777
	Jennifer	Wallace	987654321
	Joyce	English	453453453

DEPENDENT

	ESSN	DEPENDENT_NAME	SEX	BDATE	RELATIONSHIP
1	333445555	Alice	F	1986-04-05	DAUGHTER
	333445555	Theodore	M	1983-10-25	SON
	333445555	Joy	F	1958-05-03	SPOUSE
	987654321	Abner	M	1942-02-28	SPOUSE
l	123456789	Michael	М	1988-01-04	SON
	123456789	Alice	F	1988-12-30	DAUGHTER
Į	123456789	⊟izabeth	F	1967-05-05	SPOUSE

EMP_DEPENDENTS	FNAME	LNAME	SSN	ESSN	DEPENDENT_NAME	SEX	BDATE	* * *
	Alicia	Zelaya	999887777	333445555	Alice	F	1986-04-05	
	Alicia	Zelaya	999887777	333445555	Theodore	M	1983-10-25	
	Alicia	Zelaya	999887777	333445555	Joy	F	1958-05-03	* * *
	Alicia	Zelaya	999887777	987654321	Abner	M	1942-02-28	* * *
	Alicia	Zelaya	999887777	123456789	Michael	M	1988-01-04	* P B
	Alicia	Zelaya	999887777	123456789	Alice	F	1988-12-30	
	Alicia	Zelaya	999887777	123456789	Elizabeth	F	1967-05-05	
	Jennifer	Wallace	987654321	333445555	Alice	F	1986-04-05	* * *
	Jennifer	Wallace	987654321	333445555	Theodore	М	1983-10-25	
	Jennifer	Wallace	987654321	333445555	Joy	F	1958-05-03	
	Jennifer	Wallace	987654321	987654321	Abner	М	1942-02-28	* * *
	Jennifer	Wallace	987654321	123456789	Michael	М	1988-01-04	
	Jennifer	Wallace	987654321	123456789	Alice	F	1988-12-30	
	Jennifer	Wallace	987654321	123456789	Elizabeth	F	1967-05-05	* * *
	Joyce	English	453453453	333445555	Alice	F	1986-04-05	* * *
	Joyce	English	453453453	333445555	Theodore	М	1983-10-25	
	Joyce	English	453453453	333445555	Joy	F	1958-05-03	
	Joyce	English	453453453	987654321	Abner	M	1942-02-28	
	Joyce	English	453453453	123456789	Michael	М	1988-01-04	
	Joyce	English	453453453	123456789	Alice	F	1988-12-30	
	Joyce	English	453453453	123456789	Elizabeth	F	1967-05-05	* * *

Binary Relational Operations

JOIN Operation

- The sequence of Cartesian product followed by select is used quite commonly to identify and select related tuples from two relations, a special operation, called **JOIN**. It is denoted by a
- This operation is very important for any relational database with more than a single relation, because it allows us to process relationships among relations.
- The general form of a join operation on two relations $R(A_1, A_2, \ldots, A_n)$ and $S(B_1, B_2, \ldots, B_m)$ is:

$$R \bowtie_{< join \ condition>} S$$

where R and S can be any relations that result from general relational algebra expressions.

Example: Suppose that we want to retrieve the name of the manager of each department. To get the manager's name, we need to combine each DEPARTMENT tuple with the EMPLOYEE tuple whose SSN value matches the MGRSSN value in the department tuple. We do this by using the join operation.

DEPT_MGR ← **DEPARTMENT**



JOIN Operation

DEPT_MGR ← **DEPARTMENT**

MGRSSN=SSN EMPLOYEE

DEPT_MGR	DNAME	DNUMBER	MGRSSN	 FNAME	MINIT	LNAME
	Research	5	333445555	 Franklin	T	Wong
	Administration	4	987654321	 Jennifer	S	Wallace
	Headquarters	1	888665555	 James	Ε	Borg

Result of the JOIN operation DEPT_MCR ← DEPARTMENT MGRSSN=SSN EMPLOYEE

EQUIJOIN Operation

The most common use of join involves join conditions with equality comparisons only. Such a join, where the only comparison operator used is =, is called an EQUIJOIN. In the result of an EQUIJOIN we always have one or more pairs of attributes (whose names need not be identical) that have *identical values* in every tuple.

The JOIN seen in the previous example was EQUIJOIN.

NATURAL JOIN Operation

Because one of each pair of attributes with identical values is superfluous(extra), a new operation called natural join—denoted by *—was created to get rid of the second (extra) attribute in an EQUIJOIN condition.

The standard definition of natural join requires that the two join attributes, or each pair of corresponding join attributes, have the **same** name in both relations. If this is not the case, a renaming operation is applied first.

Equijoin: Example

Salior Reserves

	Sid	Sname	Rating	Age	Sid	Bid	Day
	22	Justin	7	23	22	101	2001/01/23
	31	Petter	8	21	58	103	2002/01/03
ļ	58	James	10	34			

Results of Sailor Salior. Sid = Reserves. Reserves

Sid	Sname	Rating	Age	Sid	Bid	Day
22	Justin	7	23	22	101	2001/01/23
58	James	10	34	58	103	2002/01/01

Natural Join: Example

Salior Reserves

Sid	Sname	Rating	Age
22	Justin	7	23
31	Petter	8	21
58	James	10	34

Sid	Bid	Day
22	101	2001/01/23
58	103	2002/01/01

Results of Sailor * Reserves

Sid	Sname	Rating	Age	Bid	Day
22	Justin	7	23	101	2001/01/23
58	James	10	34	103	2002/01/01

Complete Set of Relational Operations

• The set of operations including select σ , project π , union \cup , set difference -, and cartesian product X is called a complete set because any other relational algebra expression can be expressed by a combination of these five operations.

• For example:

$$\mathbf{R} \cap \mathbf{S} = (\mathbf{R} \cup \mathbf{S}) - ((\mathbf{R} - \mathbf{S}) \cup (\mathbf{S} - \mathbf{R}))$$

$$R \longrightarrow_{< \text{join condition}>} S = \sigma_{< \text{join condition}>} (R X S)$$

DIVISION Operation

Let R and S are two relations

X is a set of attributes of S, Z is a set of attributes of R

 $R(Z) \div S(X)$, where X subset Z.

Let Y = Z - X (and hence $Z = X \cup Y$); that is, let Y be the set of attributes of R that are not attributes of S.

- The result of DIVISION is a relation T(Y) that includes a tuple t if tuples t_R appear in R with $t_R[Y] = t$, and with $t_R[X] = t_s$ for every tuple t_s in S.
- For a tuple t to appear in the result T of the DIVISION, the values in t must appear in R in combination with *every* tuple in S.

 $R S R \div S$

Lecturer	Module
Brown	Compilers
Brown	Databases
Green	Prolog
Green	Databases
Lewis	Prolog
Smith	Databases

Subject
Prolog

Lecturer
Green
Lewis

R

S

 $R \div S$

Lecturer	Module
Brown	Compilers
Brown	Databases
Green	Prolog
Green	Databases
Lewis	Prolog
Smith	Databases

Subject	
Databases	
Prolog	

Lecturer
Green

R

S

 $R \div S$

Lecturer	Module
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Brown	Databases
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Green	Databases
Lewis	Prolog
Smith	Databases

Subject
Compilers
Prolog

Lecturer

Division Operation: Exercise

	sno	pno	B1	pno
	s1	p1		p2
	s1	p2	В2	
	s1	p3		pno
	s1	p4		p2
	s2	p1		p4
	s2	p2	В3	
	s3	p2		pno
	s4	p2		
	s4	p4		pl p2
				p2 p4

Find A \div B1, A \div B2, A \div B3

Retrieve the names of employees who work on all the projects that 'John Smith' works on

```
SMITH \leftarrow \sigma_{\text{FNAME='John' AND LNAME='Smith'}} (EMPLOYEE)
SMITH_PNOS \leftarrow \pi_{PNO}(WORKS_ON) ______ SMITH)
SSN_PNOS \leftarrow \pi_{ESSN,PNO} (WORKS_ON)
SSNS(SSN) ← SSN PNOS ÷ SMITH PNOS
RESULT \leftarrow \pi_{\text{FNAME. LNAME}} (SSNS * EMPLOYEE)
```

Additional Relational Operations

- Aggregate Functions and Grouping
 - Specify mathematical aggregate functions on collections of values from the database.
 - Examples: Retrieve the average salary of all employees,
 Total number of employees.
 - Common functions applied to collections of numeric values include SUM, AVERAGE, MAXIMUM, and MINIMUM. The COUNT function is used for counting tuples or values.

Additional Relational Operations (cont.)

Use of the Functional operator ${\mathcal F}$

- $\mathcal{F}_{\text{MAX (Salary)}}$ (Employee) retrieves the maximum salary value from the Employee relation
- $\mathcal{F}_{\text{MIN (Salary)}}$ (Employee) retrieves the minimum Salary value from the Employee relation
- $\mathcal{F}_{\text{SUM (Salary)}}$ (Employee) retrieves the sum of the Salary from the Employee relation
- DNO $\mathcal{F}_{COUNT\,(SSN),\,AVERAGE\,(Salary)}$ (Employee) groups employees by DNO (department number) and computes the count of employees and average salary per department.
 - [Note: count just counts the number of rows, without removing duplicates]

Aggregate Functions and Grouping-Example

Retrieve each department number, the number of employees in the department, and their average salary.

Result(DNO, NO_OF_EMPLOYEES, AVERAGE_SAL) \leftarrow (DNO $\mathcal{F}_{COUNT(SSN)}$, AVERAGE(SALARY) (EMPLOYEE))

- Q0: Retrieve the name and address of all employees who work for the department no=2.
- Q1: Retrieve the name and address of all employees who work for the 'Research' department.
- Q2:For every project located in 'Stafford', list the project number, the controlling department number, and the department manager's last name, address, and birthdate.
- Q3:Find the names of employees who work on *all the projects controlled* by department number 5.
- Q4:Make a list of project numbers for projects that involve an employee whose last name is 'Smith', either as a worker or as a manager of the department that controls the project.
- Q5:List the names of all employees with two or more dependents.
- Q6: Retrieve the names of employees who have no dependents.

Q1:

```
\begin{aligned} \text{DEPT2} \leftarrow \sigma_{\text{(Dname='Research')}} & \text{(DEPARTMENT)} \\ \text{EMPS} \leftarrow & \text{(DEPT2} & \longrightarrow & \text{DNUMBER=DNO} \\ & \text{RESULT} \leftarrow \pi_{\text{(FNAME, LNAME, ADDRESS)}} & \text{(EMPS)} \end{aligned}
```

Q2:

```
\begin{array}{l} \text{STAFFORD\_PROJS} \leftarrow \sigma_{\text{(PLOCATION='Stafford')}} \text{ (PROJECT)} \\ \text{CONTR\_DEPT} \leftarrow \text{(STAFFORD\_PROJS} \qquad \text{DNUM=DNUMBER} \\ \text{DEPARTMENT)} \\ \text{PROJ\_DEPT\_MGR} \leftarrow \text{(CONTR\_DEPT} \qquad \text{MGRSSN=SSN} \text{ EMPLOYEE)} \\ \text{RESULT} \leftarrow \pi_{\text{(PNUMBER, DNUM, LNAME, ADDRESS, BDATE)}} \text{(PROJ\_DEPT\_MGR)} \end{array}
```

```
Q3:
DEPT5_PROJS(PNO) \leftarrow \pi_{PNUMBER}(\sigma_{DNUM=5}(PROJECT))
EMP_PRJO(SSN, PNO) \leftarrow \pi_{ESSN, PNO} (WORKS_ON)
RESULT_EMP_SSNS ← EMP_PRJO ÷ DEPT5_PROJS
RESULT \leftarrow \pi_{\text{LNAME, FNAME}} (RESULT_EMP_SSNS * EMPLOYEE)
Q4:
SMITHS(ESSN) \leftarrow \pi_{SSN}(\sigma_{LNAME='Smith'}(EMPLOYEE))
SMITH_WORKER_PROJ \leftarrow \pi_{PNO}(WORKS_ON * SMITHS)
MGRS \leftarrow \pi_{\text{LNAME, DNUMBER}} (EMPLOYEE \longrightarrow_{\text{SSN=MGRSSN}} DEPARTMENT)
SMITH_MANAGED_DEPTS (DNUM) \leftarrow \pi_{\text{DNUMBER}}(\sigma_{\text{LNAME= 'Smith'}}(\text{MGRS}))
SMITH_MGR_PROJS(PNO) \leftarrow \pi_{PNUMBER}(SMITH_MANAGED_DEPTS *
PROJECT)
```

RESULT \leftarrow (SMITH WORKER PROJS \cup SMITH MGR PROJS)

Q5:

```
T1(SSN, NO_OF_DEPTS) \leftarrow_{\text{ESSN}} \mathcal{F}_{\text{COUNT (DEPENDENT_NAME)}} (DEPENDENT)

T2 \leftarrow \sigma_{\text{NO_OF_DEPTS=2}} (T1)

RESULT \leftarrow \pi_{\text{LNAME, FNAME}} (T2 * EMPLOYEE)
```

Q6:

```
ALL_EMPS \leftarrow \pi_{SSN}(EMPLOYEE)

EMPS_WITH_DEPS(SSN) \leftarrow \pi_{ESSN}(DEPENDENT)

EMPS_WITHOUT_DEPS \leftarrow (ALL_EMPS - EMPS_WITH_DEPS)

RESULT \leftarrow \pi_{LNAME, FNAME}(EMPS_WITHOUT_DEPS * EMPLOYEE)
```