Measuring resistance using non-ideal devices

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1 Abstract

It tends to be pretty difficult and obviously not convenient to perform measurements using voltage and current sensors whose impedance is comparable with the impedance of resistor itself.
That's why it makes sense to apply optimization methods to improve accuracy.
So, we've constructed all the possible schemes and proved, that there aren't any other reasonable schemes can be constructed using the given set of instruments.
Finally, we've extracted all the data from those schemes.

2 The Plan

The task is to measure the impedance of the given resistor as precise as possible.
To achieve this aim, we decided to perform these operations:

- Compute the resistance and its dispersion individually for each scheme
- Deduct which scheme's results stand out from the center (μ), for example, the threshold value of remoteness from the center might be equal to $2 \cdot \sigma$
- Dropout the data outliers (outstanding measurements)
- For those verified measurements we use them to construct an equation system.
- Optimize the function which is $\sum_{i=0}^{i<n_{equations}} \left(EQ_{i_{left}} - EQ_{i_{right}}\right)^2$ where $EQ_{i_{left}}$ and $EQ_{i_{right}}$ are left and right sides of i^{th} equation respectively
- As the result, we'll find the resultant resistance.
  Obviously, the target function won't be equal to zero because we have 8 equations and only one variable, but the accuracy of the result will be significantly better than it is for all the initial results.
- Thus, the final result should be definitely found this way.

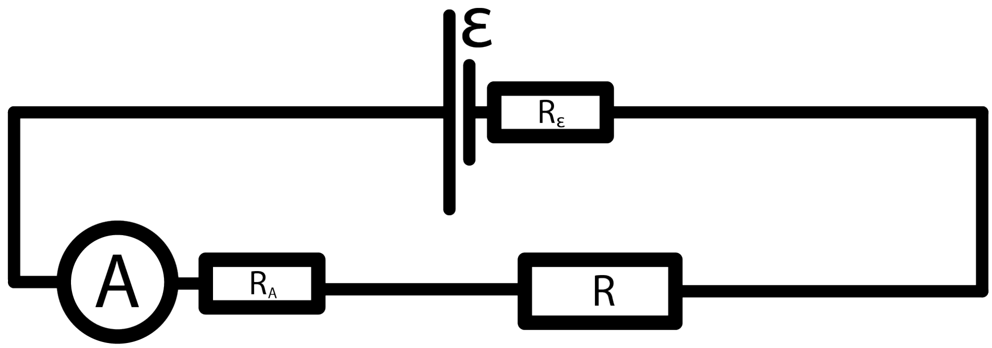
3 Schemes and Equations

There are only 8 ***reasonable*** schemes can be assembled using these items:

- Exactly one battery
- Exactly one resistor to measure
- Not more than one amperemeter
- Not more than one voltmeter

Here are the schemes:

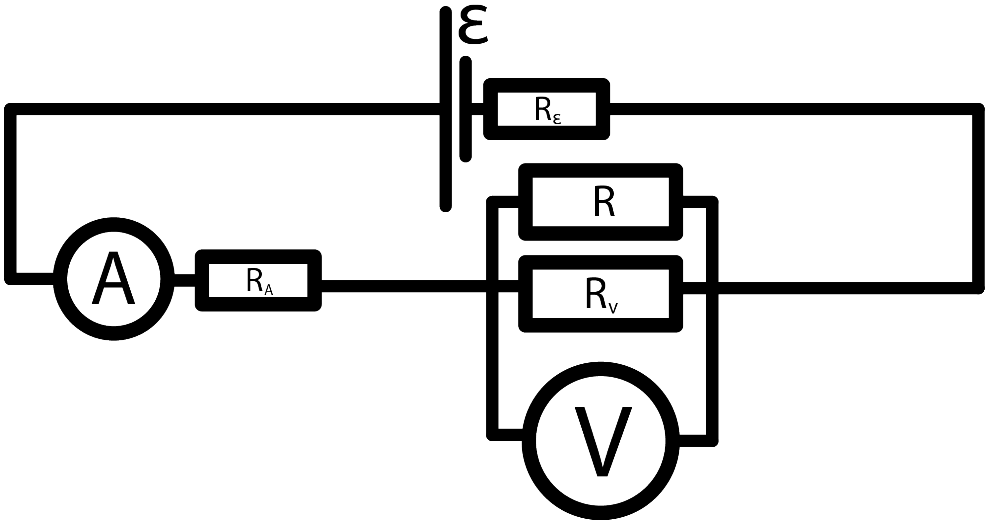
Scheme 1



$$A_1 = \frac{\varepsilon}{R_\varepsilon + R_A + R}$$

$$R = \frac{\varepsilon}{A_1} - R_\varepsilon - R_A \tag{1}$$

Scheme 2



$$A_2 = \frac{\varepsilon}{R_\varepsilon + R_A + \frac{1}{\frac{1}{R} + \frac{1}{R_V}}}$$

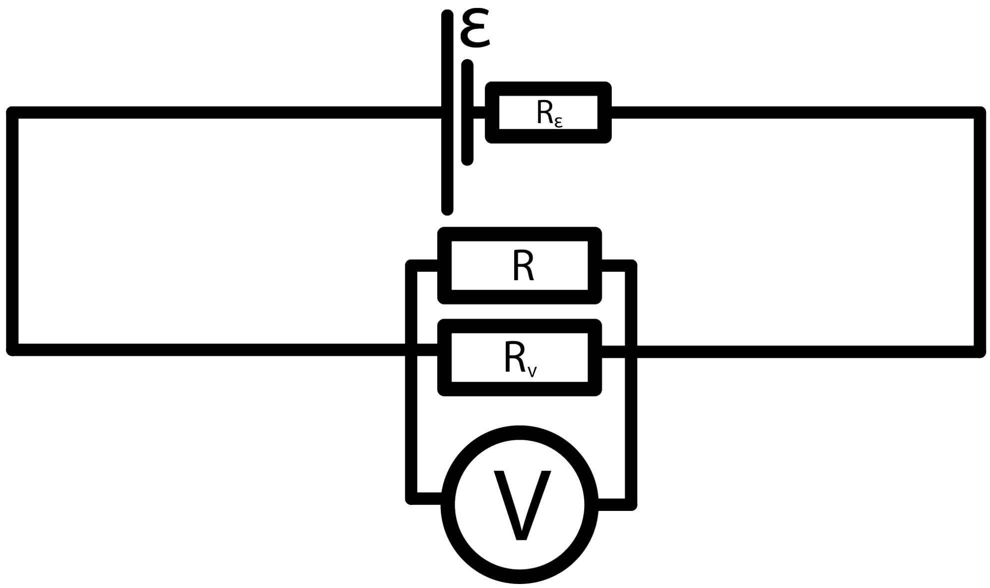
$$V_2 = A_2 \cdot \frac{1}{\frac{1}{R} + \frac{1}{R_V}}$$

$$\frac{1}{R} + \frac{1}{R_V} = \frac{A_2}{V_2}$$

$$\frac{1}{R} = \frac{A_2}{V_2} - \frac{1}{R_V}$$

$$R = \frac{1}{\frac{A_2}{V_2} - \frac{1}{R_V}} \tag{2}$$

Scheme 3



let R_{sum} *be* $\frac{1}{\frac{1}{R}+\frac{1}{R_V}}$

$$V_3 = \frac{\varepsilon}{R_\varepsilon + R_{sum}} \cdot R_{sum}$$

$$R_\varepsilon + R_{sum} = R_{sum} \cdot \frac{\varepsilon}{V_3}$$

$$R_\varepsilon = R_{sum} \cdot \left(\frac{\varepsilon}{V_3} - 1\right)$$

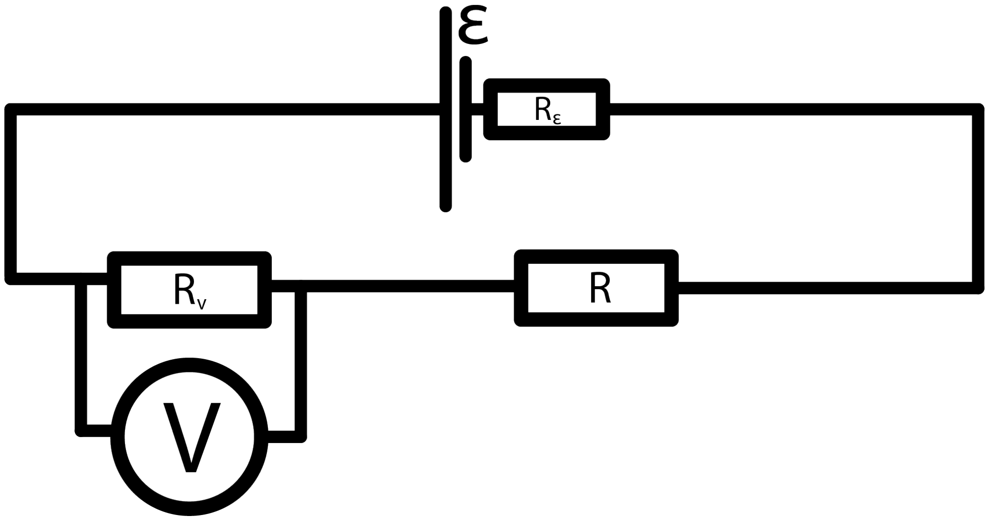
$$\frac{1}{\frac{1}{R} + \frac{1}{R_V}} = R_{sum} = \frac{R_\varepsilon}{\left(\frac{\varepsilon}{V_3} - 1\right)}$$

$$\frac{1}{R} + \frac{1}{R_V} = \frac{\left(\frac{\varepsilon}{V_3} - 1\right)}{R_\varepsilon}$$

$$\frac{1}{R} = \frac{\left(\frac{\varepsilon}{V_3} - 1\right)}{R_\varepsilon} - \frac{1}{R_V}$$

$$R = \frac{1}{\frac{\left(\frac{\varepsilon}{V_3}-1\right)}{R_\varepsilon} - \frac{1}{R_V}} \tag{3}$$

Scheme 4

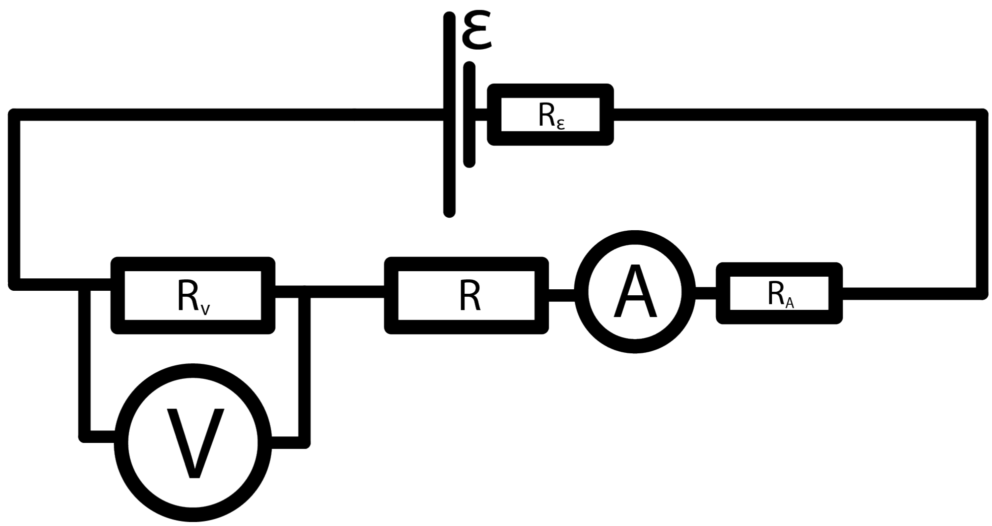


$$V_4 = \varepsilon \cdot \frac{R_V}{R_\varepsilon + R + R_V}$$

$$R_\varepsilon + R + R_V = R_V \cdot \frac{\varepsilon}{V_4}$$

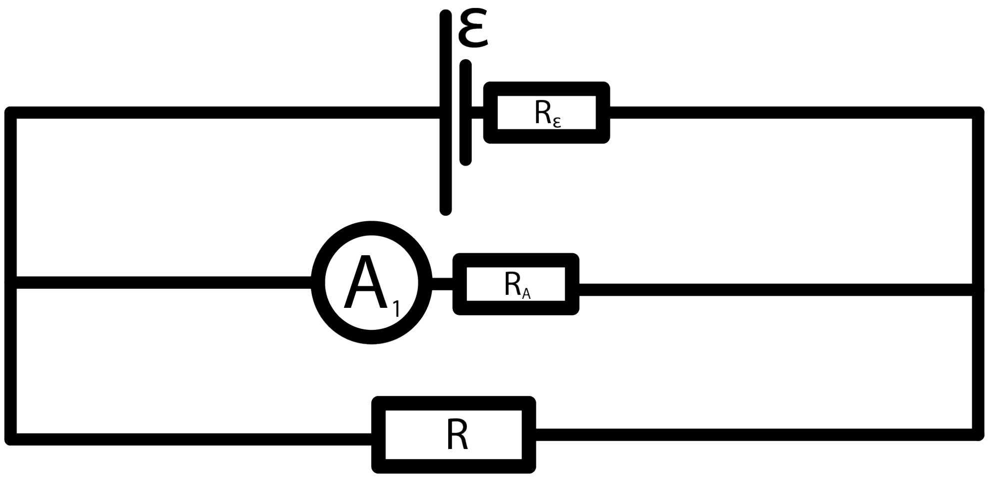
$$R = R_V \cdot \frac{\varepsilon}{V_4} - R_\varepsilon - R_V \tag{4}$$

Scheme 5



$$A_5 = \frac{\varepsilon}{R + R_\varepsilon + R_V + R_A}$$
$$R + R_\varepsilon + R_V + R_A = \frac{\varepsilon}{A_5}$$
$$R = \frac{\varepsilon}{A_5} - R_\varepsilon - R_V - R_A \quad (5)$$

Scheme 6



let R_{sum} *be* $\frac{1}{\frac{1}{R}+\frac{1}{R_A}}$

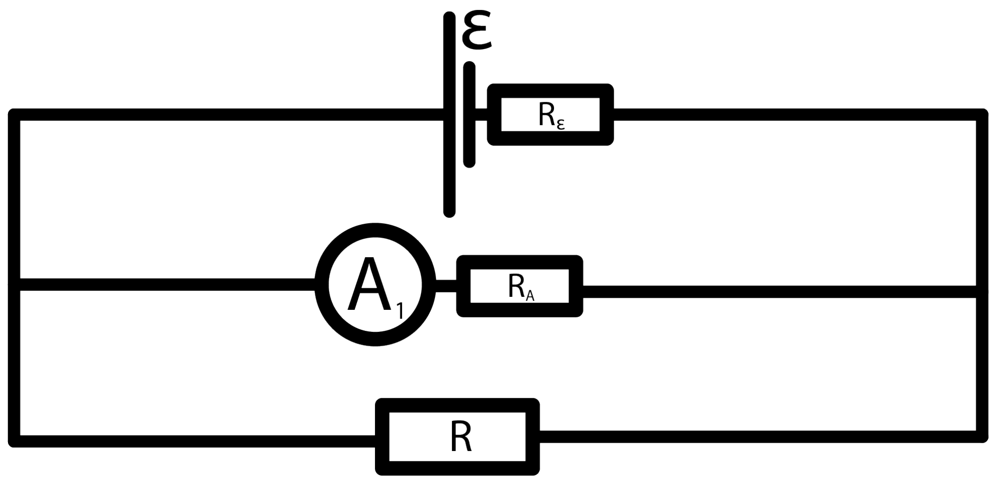
$$I_\varepsilon = \frac{\varepsilon}{R_\varepsilon + R_{sum}}$$

$$U_R = U_{R_A} = \varepsilon - I_\varepsilon = \varepsilon - \frac{\varepsilon}{R_\varepsilon + R_{sum}} = \varepsilon \cdot \left(1 - \frac{1}{R_\varepsilon + R_{sum}}\right)$$

$$U_{R_A} = R_A \cdot I_{R_A} = R_A \cdot A_6$$

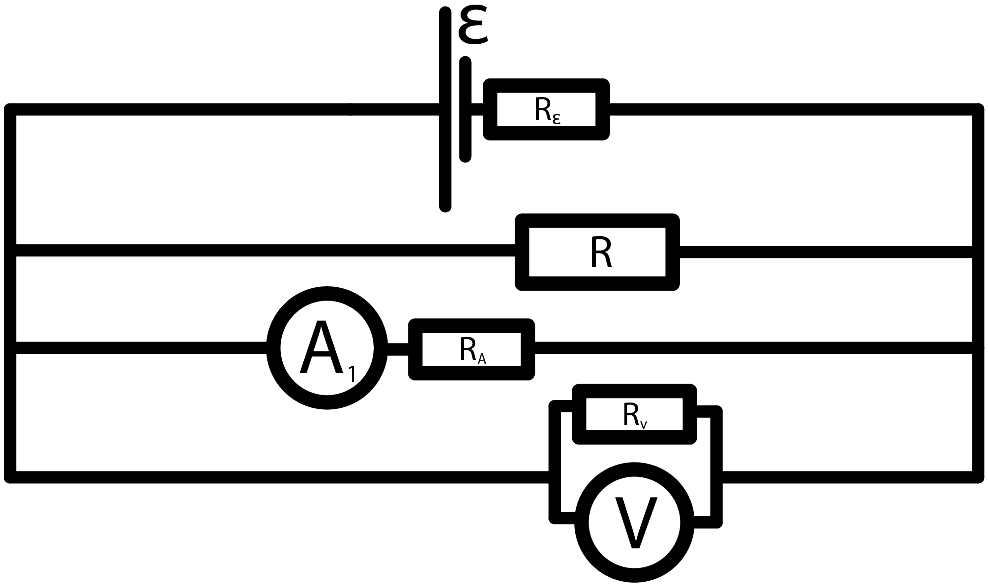
$$R = ...TOOLONGTOCOUNT \tag{6}$$

Scheme 7



There are too much computations required to count this scheme which dramatically decreases the precision, so, I didn't solve this equation.

Scheme 8



There are too much computations required to count this scheme which dramatically decreases the precision, so, I didn't solve this equation.

4 Measurement Results

Here they are:

$$Scheme1: \; A_1 = (0.98 \pm 0.03) \times \frac{15.8}{2000} Amperes$$

$$Scheme2: \; V_2 = (1.5 \pm 0.1) \times \frac{6.2}{6} Volts$$

$$A_2 = (1.10 \pm 0.03) \times \frac{15.8}{2000} Amperes$$

$$Scheme3: \; V_3 = (2.0 \pm 0.1) \times \frac{6.2}{6} Volts$$

$$Scheme4: \; V_4 = (2.6 \pm 0.1) \times \frac{6.2}{6} Volts$$

$$Scheme5: \; V_5 = (2.2 \pm 0.1) \times \frac{6.2}{6} Volts$$

$$A_5 = (0.50 \pm 0.03) \times \frac{15.8}{2000} Amperes$$

$$Scheme6: \; A_6 = (1.30 \pm 0.03) \times \frac{15.8}{2000} Amperes$$

$$Scheme7: \; V_7 = (3.0 \pm 0.1) \times \frac{6.2}{6} Volts$$

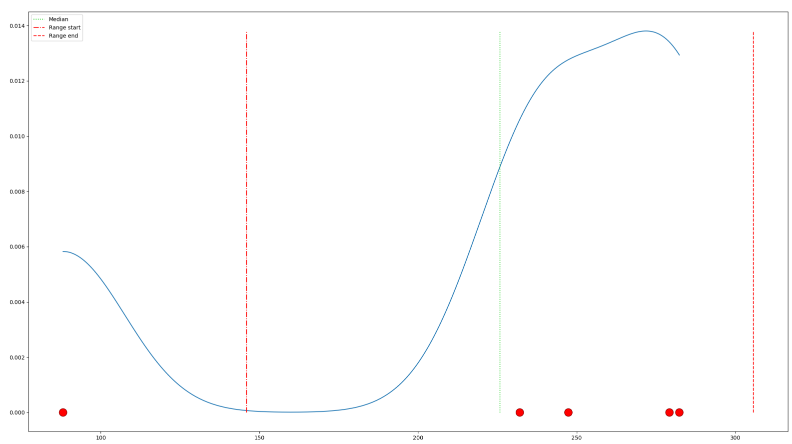
$$A_7 = (0.40 \pm 0.03) \times \frac{15.8}{2000} Amperes$$

$$Scheme8: \; V_8 = (1.0 \pm 0.1) \times \frac{6.2}{6} Volts$$

$$A_8 = (1.20 \pm 0.03) \times \frac{15.8}{2000} Amperes$$

5 Looking for Data Outliers

As mentioned above, there's always a non-zero probability, that some measurements or equation system solvings might be incorrect.
So, it makes sense to filter them: the ones, that are out of $n \cdot \varsigma$ from the center to fix that.
This graph shows density of R distribution and its dependence on the resistance itself. Here we clearly see, that one of the points doesn't fit to the gap: there's probably a mistake there.



(Actually, the main purpose of building this graph is to explain the machine, which points are OK to make automation possible)

6 Initial Dispersions

We've tried to perform the measurements more than once, but soon I realized, that it's quite a stupid idea. The random dispersion is negligibly small compared to the measurement dispersion.
As mentioned above (The Plan), the accuracy of the result will be significantly better than it is for all the initial results.
To estimate the dispersions of "R"s, I'll use that library from
`https://github.com/sitandr/dispersion_counter`

7 Optimizing the Target Function

When we've got rid of measurements with mistakes, we can find the optimal solution taking all the Firstly, lets define that target function. It is weighted sum of sides of some equations to optimize. It's important for them to have the same order of magnitude to avoid unnecessary high valuability of some of the equations. So, lets formulate them all in terms of voltage:

$$Scheme1: \ \varepsilon = A_1 \cdot (R_\varepsilon + R_A + R)$$

$$Scheme2: \ V_2 = \frac{A_2}{\frac{1}{R} + \frac{1}{R_V}}$$

$$Scheme3: \ V_3 = \frac{\varepsilon}{R_\varepsilon + R_{sum}} \cdot R_{sum} = \frac{\varepsilon}{R_\varepsilon + \frac{1}{\frac{1}{R}+\frac{1}{R_V}}} \cdot \frac{1}{\frac{1}{R} + \frac{1}{R_V}}$$

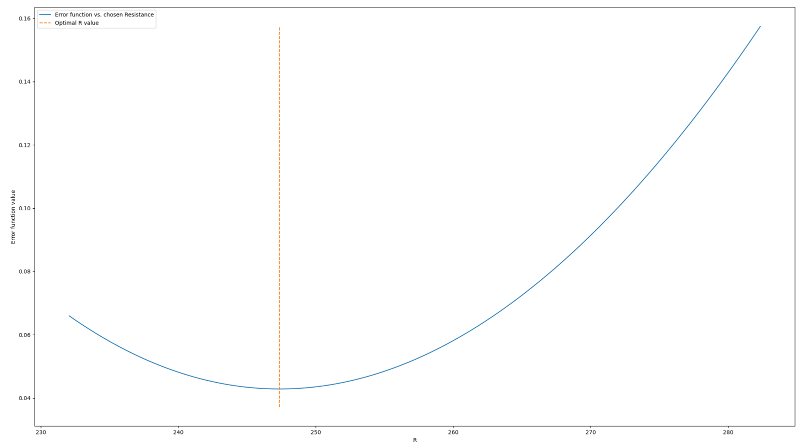
$$Scheme4: \ V_4 = \varepsilon \cdot \frac{R_V}{R_\varepsilon + R + R_V}$$

$$Scheme5: \ \varepsilon = A_5 \cdot (R + R_\varepsilon + R_V + R_A)$$

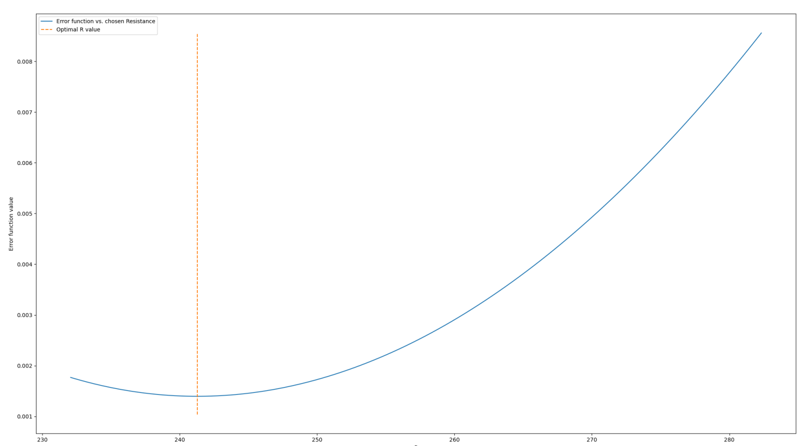
So, the final error function looks like this:

$$(\varepsilon - A_1 \cdot (R_\varepsilon + R_A + R))^2 + (V_2 - \frac{A_2}{\frac{1}{R} + \frac{1}{R_V}})^2 + (V_3 - \frac{\varepsilon}{R_\varepsilon + \frac{1}{\frac{1}{R}+\frac{1}{R_V}}} \cdot \frac{1}{\frac{1}{R} + \frac{1}{R_V}})^2$$
$$+ (V_4 - \varepsilon \cdot \frac{R_V}{R_\varepsilon + R + R_V})^2 + (\varepsilon - A_5 \cdot (R + R_\varepsilon + R_V + R_A))^2$$

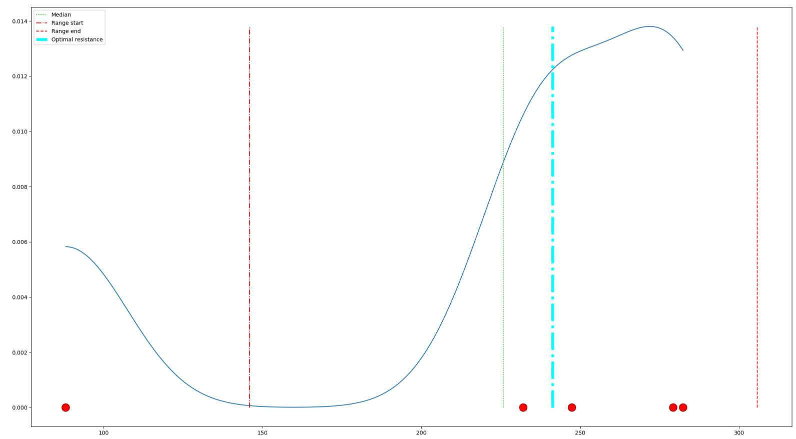
As the result, we have this error function values:



But it definitely makes sense to use inversed resultant resistance dispersions as weights in error functions.



We get a slightly different result.
Let's view this value plotted on density graph:


Median
Range start
Range end
Optimal resistance
0.014
0.012
0.010
0.008
0.006
0.004
0.002
0.000
100
150
200
250
300

8 Total Dispersions

The resultant dispersion should be probably smaller than all the usual ones. This approximation seems to be quite reasonable:

$$dispersion_{res} = \frac{dispersion_{average}}{\frac{N}{2}} \approx 17.26 \approx 17(Ohm)$$

9 The Answer

$$R = 241 \pm 17 Ohm$$

10 Conclusion

The result looks pretty reasonable!

All the code is available in my github repository: `https://github.com/donRumata03/Experiments/tree/master/MeasuringResistance`