Algebra

Algebra at its core is made up of **terms** and **expressions**.

These are **terms:**

- **Constant:** Value that doesn't change i.e. 6, 70, -4, $\frac{4}{5}$
- **Variable:** A value that can change i.e. X, Y, ɑ, ■
- **Coefficient:** The constant multiplying a variable i.e. The 6 in 6x, $\frac{4}{5}$ in $\frac{4x}{5}$

Expressions are made of terms and operators. **Algebraic expressions** are expressions that use variables. **Polynomials**(poly meaning many) are expressions where every variable has a non-negative whole number as a power. For example, each of these is both an **algebraic expression** and a **polynomial**:

- $8a^2$
- $4x - 2$
- $-23y + 9z$
- $2x^2 - 3x - 4$

The **degree** of a polynomial refers to the highest power of a variable in the polynomial. The important degrees are:

- **Linear**(degree=1): $4x - 2$
- **Quadratic**(degree=2): $2x^2 - 3x - 4$
- **Cubic**(degree=3): $x^3 - 6x^2 + 11x - 6$
- **Quartic**(degree=4): $3x^4 - 2x^3 + x^2 + 8$

The **roots** of a polynomial are the values of the variable, when the polynomial = 0 in an equation, for example:

- For $x^2 - x - 6 = 0$, $x = -2$ & $x = 3$.

Basic ideas:

- Like terms with like terms.
    - Addition and subtraction between terms with the same variable to the same power i.e. $4x^2 + 6x^2 = 10x^2$, whereas $4x^2 + 6x$ and $4x^2 + 6y^2$ cannot be simplified.
- Anything done to the one side of an equation is also done to the other. Consider $8x - 3 = 9$. We have like terms that can be put together and simplified, namely -3 and 9. To do this, we can add 3 to both sides, following the laws of equality.
    - $8x - 3 = 9$.
    - $8x - 3 + 3 = 9 + 3$.
    - $8x = 12$.

  Now to get x on its own, we can use the same principle. 8x means x multiplied by 8, so we must simply divide both sides by 8 to get our answer.

- $x = 12/8$.
- $x = 4/3$.

- All terms can be multiplied/divided together.
  - When multiplying terms, all of the coefficients and variables are being multiplied together. For example $(4x^2)(6y^2)$, is really $(4)(x^2)(6)(y^2)$. The only multiplication we can do is $(4)(6)=24$, therefore we get $24x^2y^2$ as our result.
  - When dividing, the easiest method is to write one expression over the other, and divide whatever we can into both the bottom and the top. For example, $12x^4/8y^3$ becomes $4x^4/3y^3$, by dividing 2 into the top and the bottom.
  - When multiplying terms with the same variable, we add their powers together. For example $(4x^2)(6x^2)$, we can simplify $(4)(6)$ to 24, and also $(x^2)(x^2)$, to x^{2+2}, to x^4. This gives us $24x^4$. Similarly, when dividing we subtract the powers, for example $12x^4/6x^3 = 2x$.

Expanding brackets:

- Multiplication is **distributive**. This means that when you multiply a term by an expression in brackets i.e. $a(b + c)$, you can get the same answer by multiplying the first term by all terms in the brackets separately. So:
  - $a(b + c) \rightarrow a(b) + a(c)$, or
  - $3x(4x^2 - 6x) \rightarrow 3x(4x^2) + 3x(-6x)$
- When multiplying an expression with another expression, we apply this concept multiple times. For example:
  - $(4x^2 + 6x)(3x - 4) \rightarrow 4x^2(3x - 4) + 6x(3x - 4)$
  - $4x^2(3x - 4) + 6x(3x - 4) \rightarrow 4x^2(3x) + 4x^2(-4) + 6x(3x) + 6x(-4)$
  - $12x^3 - 16x^2 + 18x - 24x$
  - $12x^3 - 16x^2 - 6x$

Factorising:

- Factorising is the opposite of expanding brackets in many ways, where we take a polynomial and revert it to its **factors**.
- Finding the **Highest Common Factor**. When possible, simply removing the HCF is an option. For example:
  - $4x + 10 \rightarrow 2(x + 5)$
  - $5x^2 + x \rightarrow x(5x + 1)$
  - $3xy + 6x^2 \rightarrow 3x(y + 2x)$
- When factorising quadratic expressions, we can make use of the **quadratic formula**, gives us the roots of quadratic expressions:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

- The factors of quadratic expressions are generally of the form $(ax + b)(cx + d)$. For example:
  - $(x + 2)(x - 3) \rightarrow x^2 - x - 6$.
  - $(2x + 1)(x + 7) \rightarrow 2x^2 + 15x + 7$

- Therefore, when trying to solve an equation which equals 0, we know one of these factors must be equal to 0. Therefore by equating the factors to 0, we find that:
    - $(x + 2)(x - 3) \rightarrow x^2 - x - 6$.
    - $x + 2 = 0 \rightarrow x = -2$
    - $x - 3 = 0 \rightarrow x = 3$
    - For $x^2 - x - 6 = 0$, $x = -2$ & $x = 3$.

  We can thus apply this method in reverse, using the quadratic formula to find the roots.
- The **difference of two squares**. When an expression is in the form $(ax)^2 - (bx)^2$, we know its factors are $ax + bx$, and $ax - bx$. For example:
    - $4x^2 - 9 \rightarrow (2x)^2 - (3)^2 \rightarrow (2x + 3)(2x - 3)$
    - $16x^4 - 1 \rightarrow (4x^2)^2 - (1)^2 \rightarrow (4x^2 + 1)(4x^2 - 1)$

      $(4x^2 - 1) \rightarrow (2x)^2 - (1)^2 \rightarrow (2x + 1)(2x - 1)$

      $(4x^2 + 1) \rightarrow (2x)^2 - (-1)^2 \rightarrow (2x - 1)(2x + 1)$

Solving equations:

Solving equations can then be explained as finding the factors, and equating them to zero.

There are three main ways of solving quadratic equations. The most reliable method is to use the quadratic formula

- Solving equations with the quadratic formula:

  $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ where $ax^2 + bx + c = 0$.