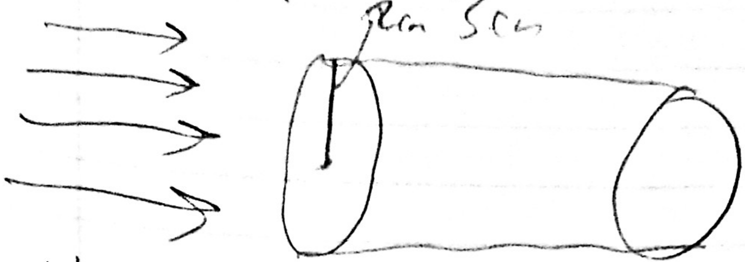
Donald Doyle
UIN: 128005953

Problem 1



$N = 10^{21}$ atoms/cm^3 $\sigma_a = 200\ b = 2\times10^{-22}\ cm^2$

$I = 10^4\ \frac{n}{cm^2 \cdot s}$

1) $LR_i = I A = 10^4 \left(\frac{n}{\cancel{cm^2} s}\right)(\pi 2^2)\ \cancel{cm^2}$

$LR_i = 1.25\times10^5\ \frac{n}{s}$

2) $AR = \sigma_a I A N \Delta x$

$AR = (2\times10^{-22})(10^4)(\pi 2^2)(10^{21})[3]$
$[\cancel{cm^2}]\ \left[\frac{n}{\cancel{cm^2} s}\right]\ [\cancel{cm^2}]\ \left[\frac{atoms}{\cancel{cm^3}}\right]\ [\cancel{cm}]$

$AR = 7.54\times10^4\ \frac{\text{reactions}}{s}$

$LR_o = LR_i - AR = 1.25\times10^5 - 7.54\times10^4$

$LR_o = 4.96\times10^4\ \frac{n}{s}$

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3) $A = 4 \text{ cm}^2 \quad N = 10^{21} \frac{\text{atoms}}{\text{cm}^3}$

$\sigma_a = 200 \text{ b} = 2 \times 10^{-22} \text{ cm}^2 \quad I = 10^4 \frac{n}{\text{cm}^2\text{-s}}$

$LR_I = IA = 10^4 \left(\frac{n}{\text{cm}^2\text{-s}}\right)(4)\left(\cancel{\text{cm}^2}\right) = 4 \times 10^4 \frac{n}{s}$

4) $AR = \sigma_a I A N \Delta x$

$A = \frac{1}{2} bh = \cancel{x}$ h is along 2 and cancels with $\frac{1}{2}$

$\Delta x = \frac{\sqrt{3}x}{2}$

$AR = \int_0^a (2 \times 10^{-22})(10^4)(10^{21})\, x \left(\frac{\sqrt{3}x}{2}\right) dx$

$AR = 1732 \int_0^a x^2 \, dx$

$AR = 1732 \left(\frac{a^3}{3}\right) \quad a = 2$

$AR = 4.62 \times 10^4$

$LR_o = LR_i - AR = 4 \times 10^4 - 4.62 \times 10^4 = -\#$

$LR_o = 0$ all neutrons are absorbed.

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Problem 2

$\rho = 10 \; g/cm^3$

1) $\Sigma = N_5 \sigma_5 + N_8 \sigma_8 + N_o \sigma_o$

$$N_{UO_2} = \frac{\rho N_a}{M_{UO_2}}$$

$$M_{UO_2} = M_U + 2M_O$$

$$\Sigma^{UO_2} = \frac{\rho_{UO_2} N_a}{M_{UO_2}} \left(w_5 \frac{M_U}{M_5} \sigma_5 + w_8 \frac{M_U}{M_8} \sigma_8 + 2\sigma_O \right)$$

$$\frac{1}{M_U} = \frac{w_5}{M_5} + \frac{w_8}{M_8}$$

$$\frac{1}{M_U} = \frac{0.5}{235} + \frac{0.5}{238}$$

$M_U = 236.5$

$M_{UO_2} = M_U + 2M_O = 268.5$

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$$\Sigma_f = \frac{(10)(6\times10^{23})}{268.5}\left[0.5\,\frac{268.5}{235}(1.2\times10^{-24}) + 0.5\,\frac{268.5}{238}(0.3\times10^{-24})\right] = 0.019\ \mathrm{cm}^{-1}$$

$$D = \frac{1}{3\Sigma_t} = \frac{1}{3(0.320)} = 1.04$$

$$L = \sqrt{\frac{D}{\Sigma_a}} = \sqrt{\frac{1.04}{0.025}} = 6.45$$

$$K = 1 = \frac{\nu\Sigma_f}{\Sigma_a + DB^2} = \frac{(3)(0.019)}{0.025 + 1.04B^2}$$

$$B^2 = \frac{(3)(0.019) - 0.025}{1.04} = 0.031$$

$$B^2 = 0.031 = \left(\frac{\pi}{\tilde{R}}\right)^2$$

$$\tilde{R} = 17.9$$

$$R = \tilde{R} - 2D = 17.9 - 2(1.04) = 15.82\ \mathrm{cm}$$

$$M = V\rho = \tfrac{4}{3}\pi(15.82^3)(10) = 165847\ g$$

$$M = 166\ kg$$

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$$\Sigma_t = \frac{\rho_{UO_2} N_0}{M_{UO_2}} \left(w_5 \frac{M_U}{M_5} \sigma_5 + w_8 \frac{M_U}{M_8} \sigma_8 + 2\sigma_0 \right)$$

$$\Sigma_t = \frac{(10)(6\times10^{23})}{268.5} \left[0.5 \frac{268.4}{235} (7.6\times10^{-24}) + 0.5 \frac{268.4}{238} (7.8\times10^{-24}) + 2 (2.8\times10^{-24}) \right] = 0.320 \text{ cm}^{-1}$$

$$\sigma_t = \sigma_s + \sigma_a$$

$$\sigma_s = \sigma_e + \sigma_i$$

$$\sigma_a = \sigma_t - \sigma_e - \sigma_i$$

$$\sigma_{a,5} = 7.6 - 4.1 - 1.9 = 1.6$$

$$\sigma_{a,8} = 7.8 - 4.8 - 2.6 = 0.4$$

$$\sigma_{a,0} = 2.8 - 2.8 = 0$$

$$\Sigma_a = \frac{(10)(6\times10^{23})}{268.5} \left[0.5 \left(\frac{268.5}{235} 1.6\times10^{-24} \right) + 0.5 \left(\frac{268.5}{238} 0.4\times10^{-24} \right) \right] = 0.025 \text{ cm}^{-1}$$

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$\gamma = 0.5$

$M_{UO_2} = \gamma M_5 + (1-\gamma) M_8 + 2 M_O$

$M_{UO_2} = (0.5)(235) + (0.5)(238) + (2)(16) = 268.5$

$N_5 = \frac{\gamma_5 \rho N_a}{M_5} = \frac{(0.5)(10)(6\times10^{23})}{235} = 1.28\times10^{22}$

$N_8 = \frac{\gamma_8 \rho N_a}{M_8} = \frac{(0.5)(10)(6\times10^{23})}{238} = 1.26\times10^{22}$

$N_O = \frac{\gamma_O \rho N_a}{M_O} = \frac{(2)(10)(6\times10^{23})}{16} = 7.5\times10^{23}$

$\Sigma_t = N_O \sigma_O + N_8 \sigma_8 + N_5 \sigma_5$

$\Sigma_t = (7.5\times10^{23})(2.8\times10^{-24}) + (1.26\times10^{22})(7.8\times10^{-24}) +$
$(1.28\times10^{22})(7.6\times10^{-24}) = 2.30 \text{ cm}^{-1}$

$\Sigma_a = (7.5\times10^{23})(0) + (1.26\times10^{22})(0.4\times10^{-24}) +$
$(1.28\times10^{22})(1.6\times10^{-24}) = 0.0255 \text{ cm}^{-1}$

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$$\Sigma_f = (1.26\times10^{22})(0.3\times10^{-24}) + (1.28\times10^{22})(1.2\times10^{-24})$$

$$\Sigma_f = 0.019$$

$$D = \frac{1}{3\Sigma_t} = \frac{1}{3(2.3)} = 0.145$$

$$L = \sqrt{\frac{D}{\Sigma_a}} = \sqrt{\frac{0.145}{0.0255}} = 2.38$$

$$K = 1 = \frac{\nu\Sigma_f}{\Sigma_a + DB^2} = \frac{(3)(0.019)}{0.0255 + 0.145B^2}$$

$$B^2 = \frac{(3)(0.019) - 0.0255}{0.145} = 0.217$$

$$B^2 = 0.217 = \left(\frac{\pi}{\tilde{R}}\right)^2$$

$$\tilde{R} = 6.74$$

$$R = \tilde{R} - 2D = 6.74 - 2(0.145) = 6.45 \text{ cm}$$

$$M = \frac{4}{3}\pi(6.45)^2(10) = 11240 \text{ g}$$

$$M = 11.2 \text{ kg}$$

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Problem 4

$20 \times 30 \times 40$ cm $\quad D = 1 \quad \nu\Sigma_f = 0.12\ \text{cm}^{-1}$

$\Sigma_a = 0.1\ \text{cm}^{-1}$

$J_0 = 1000 \frac{n}{\text{cm}^2\,\text{s}}$

1) $-10 \leq x \leq 10 \quad -15 \leq y \leq 15, \quad -20 \leq z \leq 20$

$\vec{J} = J_0 \langle x, y, z \rangle$

$\vec{J} = 1000 \left(\sin \frac{\pi x}{5} \hat{i} + \sin \frac{\pi y}{7.5} \hat{j} + \sin \frac{\pi z}{10} \hat{k} \right)$

$LR = 2\int_0^{10} dx \, 2\int_0^{15} dy \, \vec{J}$

$LR = 4\left[\left[\frac{\pi}{5} \cos\left(\frac{\pi(10)}{5}\right)\right] + \left[\frac{\pi}{7.5} \cos\left(\frac{\pi(15)}{7.4}\right)\right]\right] 1000$

$LR = 4189 \frac{n}{s}$

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Problem 4

2) $LR = 2\int_0^{10} dx \, 2\int_0^{15} dy \, 2\int_0^{20} dz \, \vec{J}$

$LR = 8\int_0^{10} dx \int_0^{15} dy \int_0^{20} dz \; 1000\left(\sin\frac{\pi x}{5}\hat{i} + \sin\frac{\pi y}{7.5}\hat{j} + \sin\frac{\pi z}{10}\hat{k}\right)$

$LR = 10891 \ \frac{n}{s}$

3) $AR = 2\int_0^{10} dx \, 2\int_0^{15} dy \, 2\int_0^{20} dz \; \Sigma_a J$

$AR = 1089 \ \frac{n}{s}$

4) $PR = 2\int_0^{10} dx \, 2\int_0^{15} dy \, 2\int_0^{20} dz \; \nu \Sigma_f \vec{J}$

$PR = 1307 \ \frac{n}{s}$

5) $K = \dfrac{\text{Production rate}}{\text{loss rate}} = \dfrac{1307}{(1089 + 10891)} = 0.11$

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6) $$k\infty = \frac{\nu \Sigma_f}{\Sigma_a} = \frac{0.12}{0.1} = 1.2$$

7) $$B^2 = \frac{\pi}{a} + \frac{\pi}{b} + \frac{\pi}{c}$$

$a = 5$
$b = 7.5$
$c = 10$

$$B^2 = \frac{\pi}{5} + \frac{\pi}{7.5} + \frac{\pi}{10} = 1.4$$

8) $$K = \frac{k\infty}{1 + L^2 B^2}$$

$$L^2 = \sqrt{\frac{D}{\Sigma_a}} = \sqrt{\frac{1}{0.1}} = 3.16$$

$$K = \frac{1.2}{1 + (1.6)(1.4)} = 0.37$$

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Problem 5

$\lambda = 0.3 \ s^{-1}$ $\Lambda = 2\times10^{-4} \ s$ $\beta = 600\times10^{-5}$

insert $\frac{\beta}{5}$ at $t=0$

1) The evolution of the neutron population in the first 2 minutes can be described by one increasing and one decreasing exponential function.

2) After the initial 2 minutes the neutron population can be described by one exponential decay function and one constant function

3) The reactor will eventually return to steady state. This state will be higher than the initial steady state due to the increase in neutron population from when the reactor was supercritical

$S = 3\times10^4 \ n/s$ $n_0 = 1000$

4) $n_0 = \frac{-S\Lambda}{\rho_{init}}$ $\rho_{init} = \frac{-S\Lambda}{n_0}$

$\rho_{init} = \frac{-(3\times10^4)(2\times10^{-4})}{1000} = -0.06$

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5) In the first 2 minutes the neutron populations can be described by one increasing exponential, one decreasing exponential and one costant function

6) After the first 2 minutes the neutron populations can be described by 2 decreasing exponentials and one costant function.

7) Eventually the reactor will reach a new steady state. This steady state will be the same as before since in both cases the steady state depends on the source term.

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8) The type of functions used to describe the neutron population would be the same as with $\$1/2$ reactivity since both are positive reactivities. So initially a increasing exponential a decreasing exponential and a constant would be used. After the reactivity is removed 2 decreasing exponentials and one constant would be used. Over the long term the neutron population would return to the same steady state as it depends on the source term again.