

NUEN 301

Test 3

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1 Problem 0

"The work that I am turning in is my own. I did not work with others nor did I communicate my answers to others. An Aggie does not lie, cheat or steal, or tolerate those who do."

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2 Problem 1

$$a) W_{235} \frac{M_u}{M_{235}} + W_{238} \frac{M_u}{M_{238}} = 1$$

$$\frac{1}{M_u} = \frac{W_{235}}{M_{235}} + \frac{W_{238}}{M_{238}}$$

$$M_u = \frac{1}{\frac{0.935}{235} + \frac{0.065}{238}} = 235.19$$

$$\Sigma_a = \frac{(18.74)(1.602 \cdot 10^{23})}{235.19} \left(0.935 \frac{235.19}{235} 1.65 \cdot 10^{-24} + 0.065 \frac{235.19}{238} 0.255 \cdot 10^{-24} \right) = 0.0199 [cm^{-1}]$$

$$\Sigma_{transport} = \frac{(18.74)(1.602 \cdot 10^{23})}{235.19} \left(0.935 \frac{235.19}{235} 6.80 \cdot 10^{-24} + 0.065 \frac{235.19}{238} 6.90 \cdot 10^{-24} \right) = 0.0870 [cm^{-1}]$$

$$\Sigma_f = \frac{(18.74)(1.602 \cdot 10^{23})}{235.19} (0.935 \frac{235.19}{235} 1.40 \cdot 10^{-24} + 0.065 \frac{235.19}{238} 0.095 \cdot 10^{-24}) = 0.0168 [cm^{-1}]$$

$$b) D = \frac{1}{3\Sigma_{transport}} = \frac{1}{3(0.0870)} = 3.83$$

$$c) L = \sqrt{\frac{D}{\Sigma_a}} = \sqrt{\frac{3.83}{0.0199}} = 13.87$$

$$d) 1 = k = \frac{v\Sigma_f}{\Sigma_a + DB_g} \Rightarrow B_g = \frac{v\Sigma_f - \Sigma_a}{D} = \frac{(2.60)(0.0168) - 0.0199}{3.83} = 0.00621$$

$$e) B_g^2 = \frac{\pi}{\tilde{R}} \Rightarrow \tilde{R} = \frac{\pi}{B_g} = \frac{\pi}{0.006221} = 505.89 [cm]$$

$$f) \tilde{R} = R + 2D \Rightarrow R = \tilde{R} - 2D = 505.89 - 2(3.83) = 498.32 [cm]$$

3 Problem 2

$$1) a) 0 \leq x \leq a \quad -D_1 \frac{d^2 \phi_1}{dx^2} + \Sigma_{a,1} \phi_1(x) = S_1$$

$$\phi_1(x) = A_1 \cosh\left(\frac{x}{L_1}\right) + C_1 \sinh\left(\frac{x}{L_1}\right) + \frac{S_1}{\Sigma_{a,1}}$$

$$b) a \leq x \leq b \quad -D_2 \frac{d^2 \phi_2}{dx^2} + \Sigma_{a,2} \phi_2(x) = 0$$

$$\phi_2(x) = A_2 \cosh\left(\frac{x}{L_2}\right) + C_2 \sinh\left(\frac{x}{L_2}\right)$$

$$c) J_{x,1}(0) = -D_1 \frac{d\phi_1}{dx} \Big|_0 = 0$$

$$\phi_2(b + 2D_2) = 0$$

$$d) \phi_1(a) = \phi_2(a)$$

$$-D_1 \frac{d\phi_1}{dx} \Big|_a = -D_2 \frac{d\phi_2}{dx} \Big|_a$$

$$e) \frac{d\phi_1}{dx} = \frac{A_1}{L_1} \sinh\left(\frac{x}{L_1}\right)$$

$$\frac{d\phi_1}{dx} \Big|_0 = \frac{C_1}{L_1} \Rightarrow C_1 = 0$$

$$\phi_2(\tilde{b}) = 0 = A_2 \cosh\left(\frac{a-\tilde{b}}{L_2}\right) + C_2 \sinh\left(\frac{a-\tilde{b}}{L_2}\right)$$

$$\phi_1(a) = \phi_2(a) \Rightarrow A_1 \cosh\left(\frac{a}{L_1}\right) + \frac{S_1}{\Sigma_{a,1}} = A_2$$

$$-D_1 \frac{d\phi_1}{dx} \Big|_a = -D_2 \frac{d\phi_2}{dx} \Big|_a \Rightarrow -D_1 \frac{A_1}{L_1} \sinh\left(\frac{a}{L_1}\right) = -D_2 \frac{C_2}{L_2}$$

$$\begin{bmatrix} 0 & \cosh\left(\frac{a-\tilde{b}}{L_2}\right) & \sinh\left(\frac{a-\tilde{b}}{L_2}\right) \\ \cosh\left(\frac{a}{L_1}\right) & -1 & 0 \\ \frac{D_1}{L_1} \sinh\left(\frac{a}{L_1}\right) & 0 & \frac{D_2}{L_2} \end{bmatrix} \begin{bmatrix} A_1 \\ A_2 \\ C_2 \end{bmatrix} = \begin{bmatrix} 0 \\ \frac{-S_1}{\Sigma_{a,1}} \\ 0 \end{bmatrix}$$

$$L_1 = 2.236 \quad L_2 = 3.162 \quad \tilde{b} = 15$$

$$\begin{bmatrix} 0 & \cosh\left(\frac{5-15}{3.162}\right) & \sinh\left(\frac{5-15}{3.162}\right) \\ \cosh\left(\frac{5}{2.236}\right) & -1 & 0 \\ \frac{1}{2.236} \sinh\left(\frac{5}{2.236}\right) & 0 & \frac{3}{3.162} \end{bmatrix} \begin{bmatrix} A_1 \\ A_2 \\ C_2 \end{bmatrix} = \begin{bmatrix} 0 \\ \frac{-3}{0.2} \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 11.84 & -11.80 \\ 4.7314 & -1 & 0 \\ 2.068 & 0 & 0.9487 \end{bmatrix} \begin{bmatrix} A_1 \\ A_2 \\ C_2 \end{bmatrix} = \begin{bmatrix} 0 \\ -15 \\ 0 \end{bmatrix}$$

$$A_1 = -2.173 \quad A_2 = 4.720 \quad C_2 = 4.736$$

$$\text{f) } \phi_1(x) = -2.173 \cosh\left(\frac{x}{2.236}\right) + 15$$

$$\phi_2(x) = 4.72 \cosh\left(\frac{x-15}{3.873}\right) + 7.736 \sinh\left(\frac{x-15}{3.873}\right)$$

$$\text{g) } \phi_1(0) = -2.173 \cosh\left(\frac{0}{2.236}\right) + 15 = 12.83$$

$$\phi_1\left(\frac{a}{2}\right) = -2.173 \cosh\left(\frac{2.5}{2.236}\right) + 15 = 11.32$$

$$\phi_1(a) = -2.173 \cosh\left(\frac{5}{2.236}\right) + 15 = 4.72$$

$$\phi_2(9) = 4.72 \cosh\left(\frac{9-15}{3.873}\right) + 7.736 \sinh\left(\frac{9-15}{3.873}\right) = 0.967$$

2)

$$a) ARR = A \Sigma_a \int_0^a$$

4 Problem 3

$$1) a) \phi(x) = A \cosh\left(\frac{x-x_0}{L}\right) + C \sinh\left(\frac{x-x_0}{L}\right)$$

$$b) \phi(a) = 0$$

$$J(x_0) = \frac{S}{2} = \left. \frac{d\phi}{dx} \right|_{x_0} = \frac{S}{2}$$

$$c) L = 3.16$$

$$\phi(a) = A \cosh\left(\frac{x-x_0}{L}\right) + C \sinh\left(\frac{x-x_0}{L}\right) = A \cosh\left(\frac{20-7}{3.16}\right) + C \sinh\left(\frac{20-7}{3.16}\right) = 30.6A + 30.584C = 0$$

$$\left. \frac{d\phi}{dx} \right|_{x_0} = \frac{-A}{L} \sinh\left(\frac{x_0-x_0}{L}\right) + \frac{-C}{L} \cosh\left(\frac{x_0-x_0}{L}\right) = \frac{-C}{L} = \frac{S}{2} \Rightarrow \frac{-C}{3.16} = \frac{5}{2} \Rightarrow C = -7.9$$

$$30.6A + 30.584(-7.9) = 0 \Rightarrow A = 7.896$$

$$\phi(x) = -7.896 \cosh\left(\frac{x-7}{3.16}\right) + 7.9 \sinh\left(\frac{x-7}{3.16}\right)$$

2)



x	ϕ
2.5	32.81
5	14.87
7	7.90
7.5	6.24
10	3.051
12.5	1.324
15	0.603
17.5	0.229

5 Problem 4