**In addition** to understanding very well HW4, here are some additional problems:

A

Show that when $\left|\frac{\rho}{\beta}\right| < \frac{1}{2}$, we have the following simplified expressions. Plot each part separately to understand the fast and slow portions of a transient!

$\ell = MGT = \Lambda$

$$n(t) = n_0 \left[ \frac{\beta}{\beta - \rho} e^{\frac{\rho}{\beta - \rho}\lambda t} - \frac{\rho}{\beta - \rho} e^{-\left(\frac{\beta - \rho}{\ell}\right) t} \right]$$

$$C(t) = C_0 \left[ e^{\frac{\rho}{\beta - \rho}\lambda t} + \frac{\rho \ell \lambda}{(\beta - \rho)^2} e^{-\left(\frac{\beta - \rho}{\ell}\right) t} \right]$$

B

**(20 min)**

A reactor is initially critical and has been running at steady state for some time. At time t=0, a control rod is instantaneously inserted. The time evolution of the neutron density is measured. Neglect feedback effects. You may assume a single group of delayed neutron precursors.

- **a)** Derive an approximate expression for the long-term neutron density evolution as a function of time.
- **b)** Using your answer in a), explain how one may infer the dollar amount of reactivity inserted from the measurements. $\frac{\rho}{\beta}$
- **c)** On the same graph, provide a sketch for both the approximate expression you have derived in a) and the exact solution (you do not need to derive an expression for the exact solution)
- **d)** Integrate the approximate expression for the neutron density between the drop time and infinity and explain how one may infer the precursors decay constant from the measurements.

C

Statement: "Each solution of the PRKEs contains $I + 1$ exponentials."

1. [**2 pts**] What does PRKE stand for? What is "$I$" in the above statement?
2. [**5 pts**] What can you say about the time constants (we have been calling them $s_j$) in the PRKE solutions? How many are positive, how many negative, etc.?
3. [**3 pts**] In addition to the $I + 1$ exponentials, we sometimes need to add yet another function to fully describe the solutions. Why?
4. [**5 pts**] What form(s) can that extra function take? Explain.

D

Consider a low-power subcritical reactor with an external neutron source present. Assume that the time dependence of neutron population is adequately described by the PRKE with one group of precursors and that feedback effects can be neglected. At time 0, a control rod is withdrawn, adding an amount $\delta$ of reactivity in the core. At time T, another control rod is withdrawn, adding again an amount $\delta$ of reactivity in the core.

**(15 min)** Estimate the power levels at the following times (you must provide mathematical expressions as well as numerical values and state your assumptions):

- At t=0+, i.e., right after the first control rod withdrawal
- At t=T-, i.e., just before the second control rod withdrawal
- At t=T+, i.e., right after the second control rod withdrawal
- At t=2T

Hint: One may assume that time T is large compared to the reactor period.

**(5 min)** Sketch as accurately as possible the power level as a function of time, emphasizing the four instants of the previous question.

PRKE data:

| | |
|---|---|
| Mean generation time | $10^{-3}$ sec |
| precursors decay constant | 0.1 $sec^{-1}$ |
| initial reactivity | -3000 pcm |
| Fraction of delayed neutrons | 600 pcm |
| External neutron source strength | Such that the initial power is 1 W |
| amount $\delta$ of reactivity | 1000 pcm |

E

[25 min] Consider a critical reactor operating in steady state at very low power. At $t$=0, a small amount of reactivity is introduced by rapidly inserting a control rod. At $t$=$T$, the control rod is rapidly returned to its original position.

[20 min] Using the 1-group data below, derive expressions for the power level and the delayed-neutron precursor concentration for $0 \leq t \leq T$ and for $t > T$.

| $\rho$ | $\beta$ | $\Lambda$ | $\lambda$ | T | initial power |
|---|---|---|---|---|---|
| -100 pcm | 600 pcm | 1E-3 s | 0.1 $s^{-1}$ | 2 s | 10 W |

[5 min] Plot the power level and the total delayed-neutron precursor concentration as functions of time from $t$=0 to $t$>$T$. For maximum credit, the shapes (curvature, slope, etc.) of the plots in different time intervals must be correct (if needed, provide additional sketches for times around t=0 and t=T).

F

A nuclear reactor is operating in steady state with a source present.

1. **[2 pts]** Is the reactor critical, subcritical, or supercritical? Explain.
2. **[5 pts]** If the multiplication factor is $k_0$, what would you need to change it to in order to make the neutron population twice as high?

G

A nuclear reactor is observed to have a neutron population that increases linearly in time.

1. **[2 pts]** Is the reactor critical, subcritical, or supercritical? Explain.
2. **[2 pts]** Is a fixed source present or not? Explain.