Beneath the Surface: Unearthing Within-Person Variability and Mean Relations with Bayesian Mixed Models

Donald R. Williams

30 May 2019

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- Twitter thread about "measurement error"
  - Mixed-effects models (hierarchical shrinkage)
- In Rast lab, we primarily study "noise" (but from a different perspective)
  - Behavioral (in)stability or (in)consistency (not actually "noise")
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Donald R. Williams
University of California, Davis

Jeffrey N. Rouder
University of California, Irvine

Philippe Rast
University of California, Davis

Introduction

- Repeated measurement designs are common to the social-behavioral sciences
- Modeling these kinds of data requires techniques that are able to partition and account for different sources of variation:
  - Experimental effects
  - Time-varying predictors
  - Type of stimulus (i.e., items in memory research)
  - etc.
- These sources of variance can also provide valuable insight into psychological processes:
  - Temporal changes
  - Cognitive interference
  - Learning trajectories

Individual differences !

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- A variety of techniques have been proposed to account for and model individual variability
- Hierarchical mixed-effects models:
  - Average effect across individuals
  - Person-specific estimates (i.e., random effects)
  - Shrinkage can improve accuracy by smoothing the individual estimates
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  - The focus has mostly remained on the mean structure

$$\mathbf{Y} \sim N(\mu, \sigma^2)$$
$$\mu_i = \alpha + \beta x_i, \quad i = 1, ..., n.$$

  - e.g., How response times are affected by an experimental manipulation
  - Or more generally, the effect of any independent variable on the outcome of interest
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Herein lies the **untapped potential**:

Rather than viewing homogeneous variance as an assumption to satisfy, or within-person variance as "noise" or indicative of measurement "error," we can seek to explain it just like the dependent variable

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- In economics, there is a tradition of modeling heteroskedasticity (i.e., volatility)
- Variability is sometimes studied in the social-behavioral sciences, where it is referred to as *intraindividual* variability (IIV)
  - Thought to convey systematic information–e.g., behavioral consistency
  - Not regarded as reflecting mere measurement "error"
  - Inconsistency in cognitive abilities proposed as an indicator of Alzheimer's disease
  - Perhaps a predictor of death (stability and longevity)

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Woodrow (1932)

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In other words, there is likely individual variation in within-person variability (i.e., stability)

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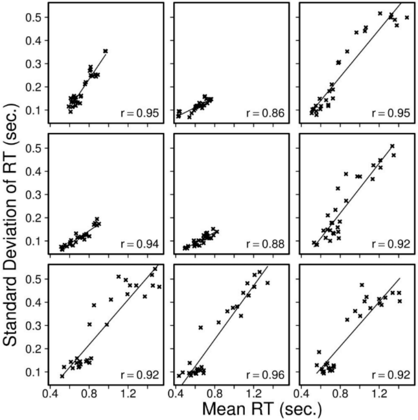

Introduction

- Mean–variance relation
  - Luce (1986) and Wagenmakers and Brown (2007) proposed a lawful relationship
- No work looking at **within-person** variance and mean relations

Aims

Paper Aims

1. To present a formal, Bayesian modeling framework, that allows for testing and visualizing what has been previously stipulated about the mean–variance relation. But, in this case, **within-person variability** and mean relations

2. More generally, to demonstrate within-person variance can provide important insights into psychological processes
   - An alternative perspective to recent papers about individual differences and measurement "error"

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Outline

This presentation is organized as follows:

- Illustrate how mixed-models can be used to investigate mean–variance relations
- Apply the model to well-known cognitive interference tasks.
  - Highlight advantages of the proposed methodology
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Illustrative Data

- Stroop task:
  - 121 participants
  - Two conditions (congruent vs. incongruent)
    - Congruent: 1, 22, or 333
    - Incongruent: 2, 11, or 222
    - About 45 trials for each condition

Random Intercepts Model

For the ith person and jth trial, the mean structure is defined as

$$y_{ij} = \beta_0 + u_{0i} + \varepsilon_{ij},$$

where β_0 is the fixed effect and u_{0i} the individual deviations.

- Note that:
  - β_0 is the **average** reaction time
  - For the first subject ($i = 1$), $\beta_0 + u_{01}$ is their average response time

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We then model the variance–i.e.,

$$\sigma^2_{\varepsilon_{ij}} = \exp[\eta_0 + u_{1i}].$$

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Check-in

Thus far, we have provided a hierarchical formulation to estimate individual means and standard deviations.

This work focuses on the relations between the mean and within-person variability.

Therefore, we need to estimate the random effects correlation–i.e., the relation between the individual means and standard deviations.

Random Intercepts Model

Assume the random effects are drawn from the same multivariate normal distribution–i.e.,

$$\begin{bmatrix} u_{0i} \\ u_{1i} \end{bmatrix} \sim N\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma_0^2 & \rho\sigma_0^2\sigma_1^2 \\ \rho\sigma_0^2\sigma_1^2 & \sigma_1^2 \end{bmatrix} \right).$$

- Here:
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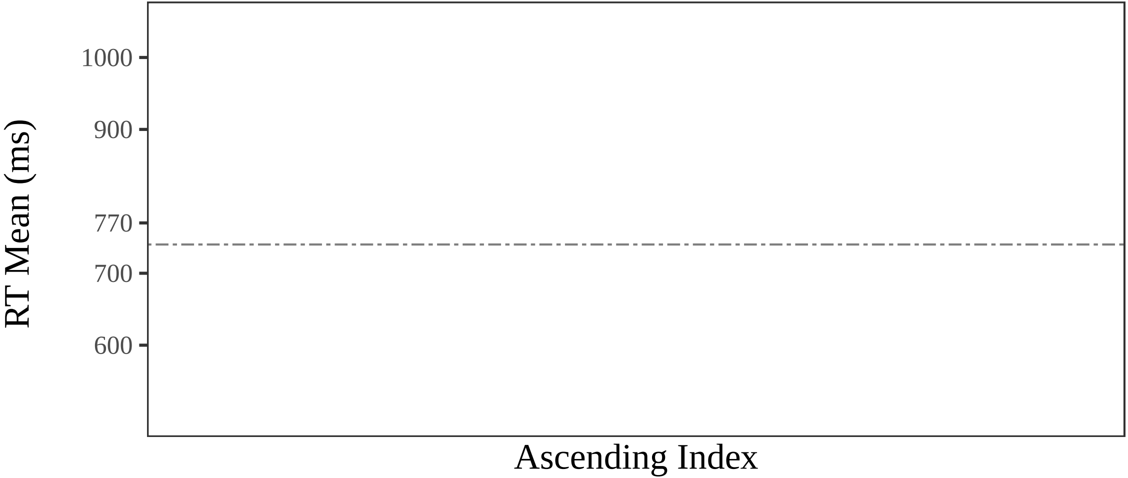

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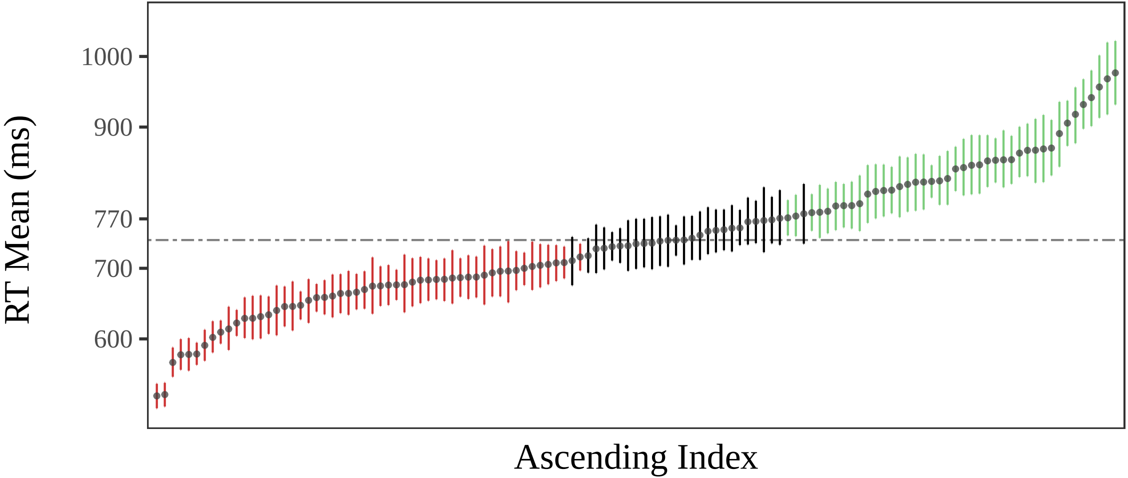

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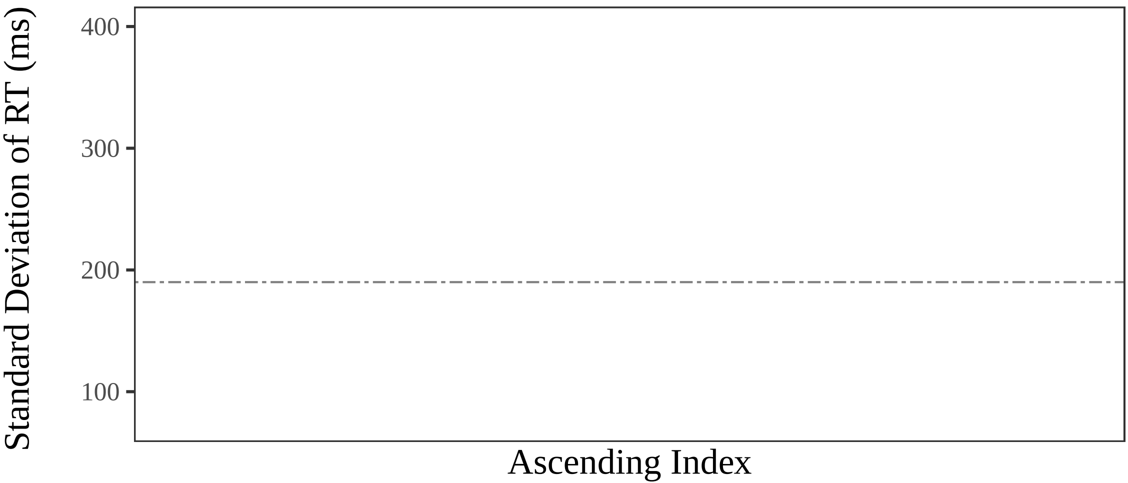

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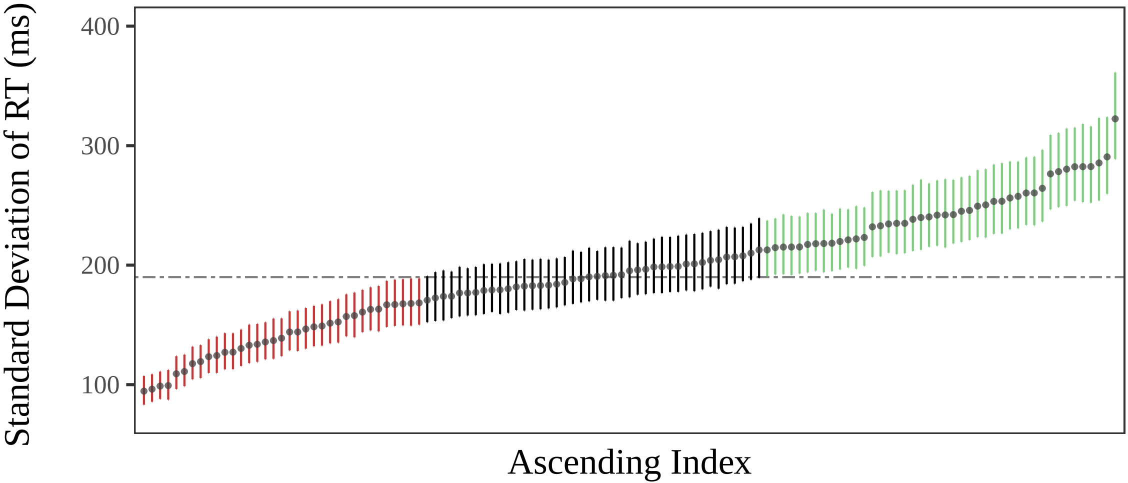

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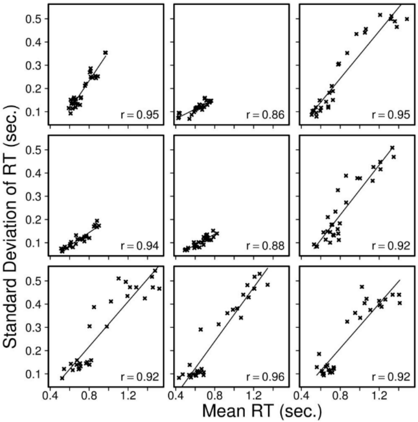

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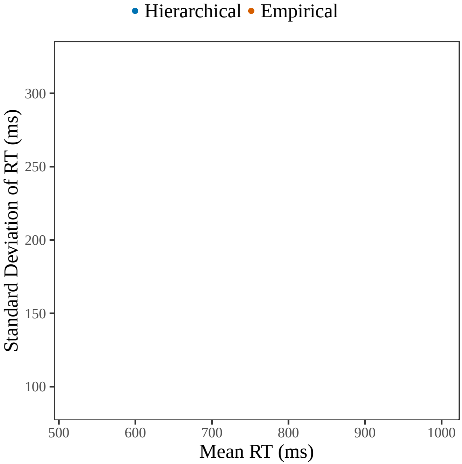

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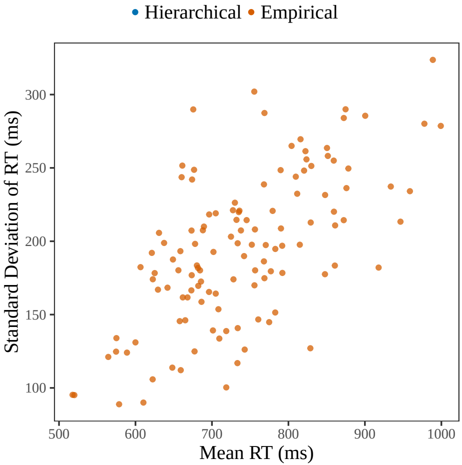

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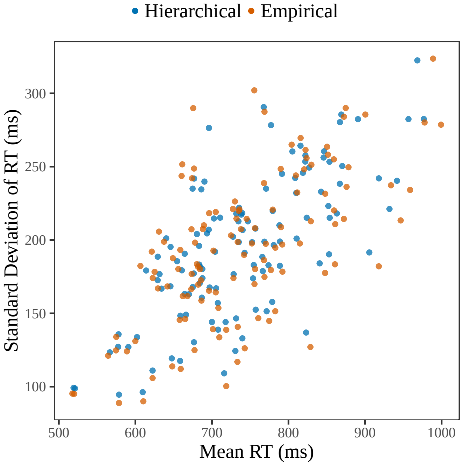

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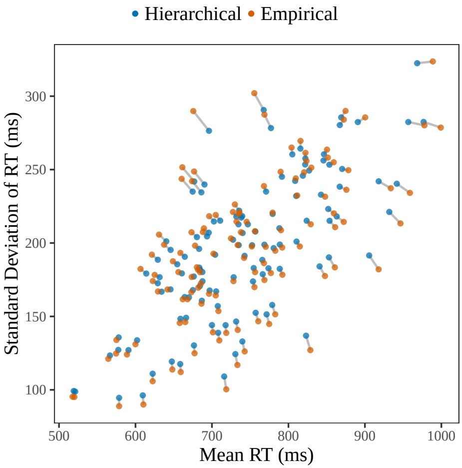

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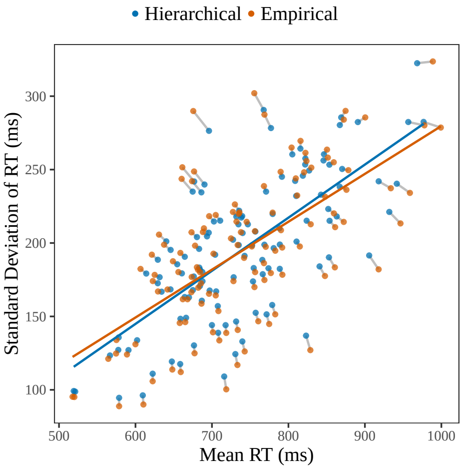

Random Intercepts Model

Check-in

This simple example illustrated several benefits of this approach for characterizing mean–variance relations:

- The hierarchical formulation reduced variability
- Nothing was lost in terms of the estimating the correlation
- The model based estimates also appeared to offer some advantages
- Concerns of overfitting were addressed

Random Slopes Model

Mean Structure:

The location sub-model of the response times for the ith person and jth trial is given as:

$$y_{ij} \sim \beta_0 + \beta_1(\text{Incongruent}_{ij}) \\ + u_{0i} + u_{1i}(\text{Incongruent}_{ij}) + \varepsilon_{ij}$$

- Note that:
  - β_0 is the *average* reaction time for the congruent condition
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  - u_{0i} denotes the random intercepts
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The covariance matrix is re-expressed as $\boldsymbol{\Sigma} = \boldsymbol{\tau}\boldsymbol{\Omega}\boldsymbol{\tau}'$–i.e,

$$\Omega = \begin{bmatrix} 1 & & & \\ \rho_{01} & 1 & & \\ \underline{\rho_{02}} & \underline{\rho_{12}} & 1 & \\ \underline{\rho_{03}} & \underline{\rho_{13}} & \rho_{23} & 1 \end{bmatrix}.$$

- $\Omega_{3,1}(\rho_{02})$: The correlation between reactions times and within-person variability for the congruent condition $cor(u_{0i}, u_{2i})$.

Random Slopes Model

The covariance matrix is re-expressed as $\boldsymbol{\Sigma} = \boldsymbol{\tau}\boldsymbol{\Omega}\boldsymbol{\tau}'$–i.e,

$$\boldsymbol{\Omega} = \begin{bmatrix} 1 & & & \\ \rho_{01} & 1 & & \\ \underline{\rho_{02}} & \underline{\rho_{12}} & 1 & \\ \underline{\rho_{03}} & \underline{\rho_{13}} & \rho_{23} & 1 \end{bmatrix}.$$

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- $\boldsymbol{\Omega}_{4,1}(\rho_{03})$: The correlation between congruent responses and within-person variability that can be attributed to "Stroop effect" $cor(u_{0i}, u_{3i})$–perhaps faster (or slower) individuals in the congruent condition had relatively more (or less) stable incongruent responses

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The covariance matrix is re-expressed as $\boldsymbol{\Sigma} = \boldsymbol{\tau}\boldsymbol{\Omega}\boldsymbol{\tau}'$–i.e,

$$\boldsymbol{\Omega} = \begin{bmatrix} 1 & & & \\ \rho_{01} & 1 & & \\ \underline{\rho_{02}} & \underline{\rho_{12}} & 1 & \\ \underline{\rho_{03}} & \underline{\rho_{13}} & \rho_{23} & 1 \end{bmatrix}.$$

- $\boldsymbol{\Omega}_{3,2}(\rho_{12})$: The correlation between "Stroop effects" and stability of congruent responses $cor(u_{1i}, u_{2i})$–perhaps those with the largest slopes also had the most consistent congruent responses

Random Slopes Model

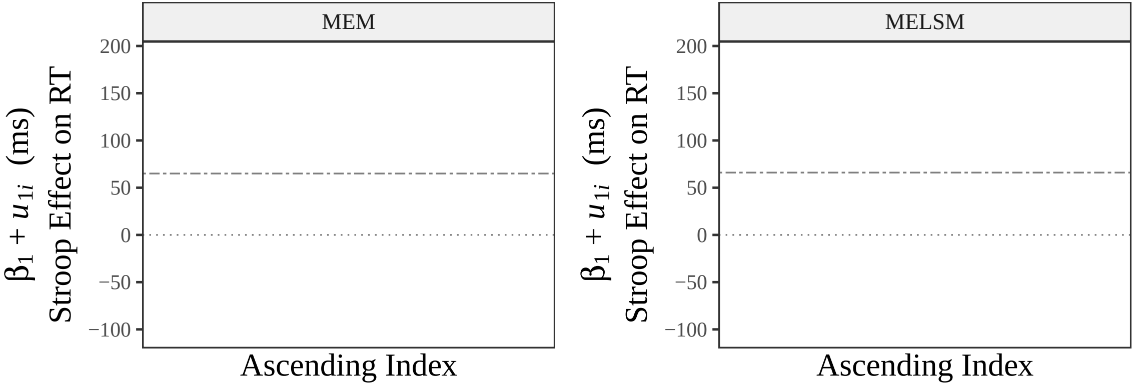
The covariance matrix is re-expressed as $\boldsymbol{\Sigma} = \boldsymbol{\tau}\boldsymbol{\Omega}\boldsymbol{\tau}'$–i.e,

$$\boldsymbol{\Omega} = \begin{bmatrix} 1 & & & \\ \rho_{01} & 1 & & \\ \underline{\rho_{02}} & \underline{\rho_{12}} & 1 & \\ \underline{\rho_{03}} & \underline{\rho_{13}} & \rho_{23} & 1 \end{bmatrix}.$$

- $\boldsymbol{\Omega}_{4,2}(\rho_{13})$: The correlation between the "Stroop effects"" on reaction times and (in)stability–perhaps those with the largest location slopes also had the most volatile incongruent responses.

Comparison

Mixed-effects model (MEM) vs. Mixed-effects **location scale** model (MELSM):



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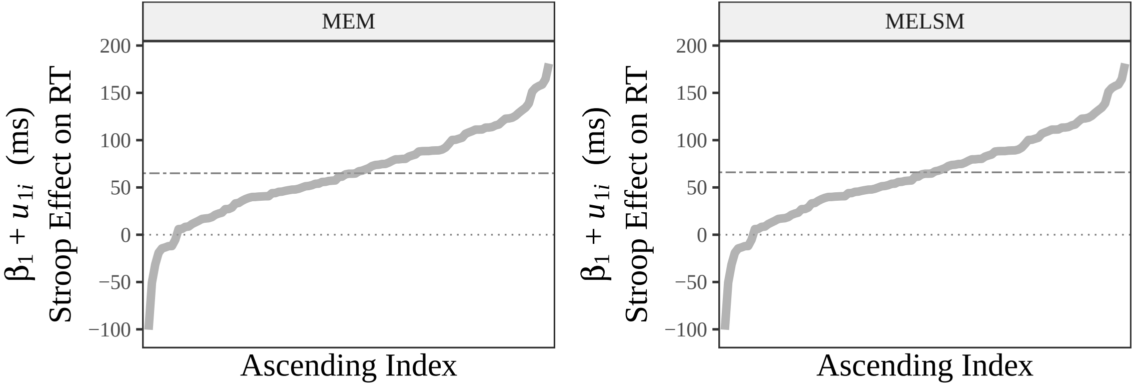

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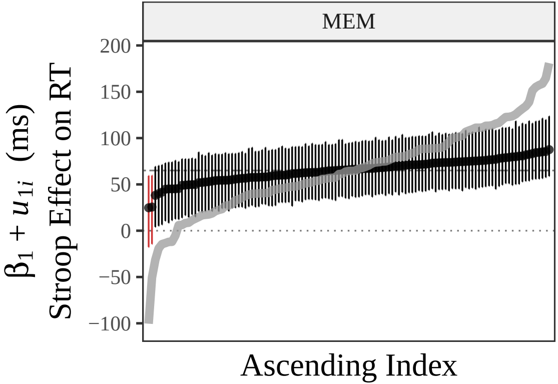

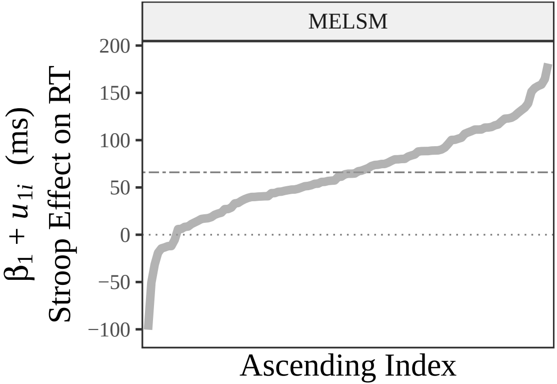

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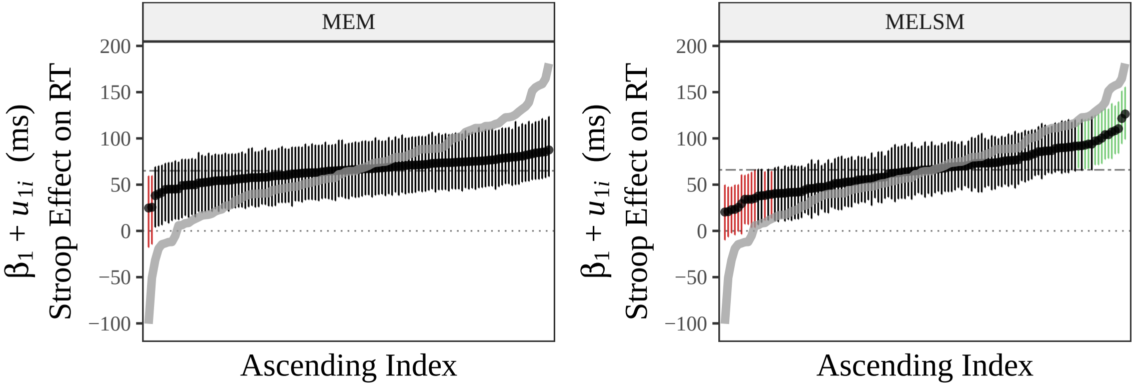
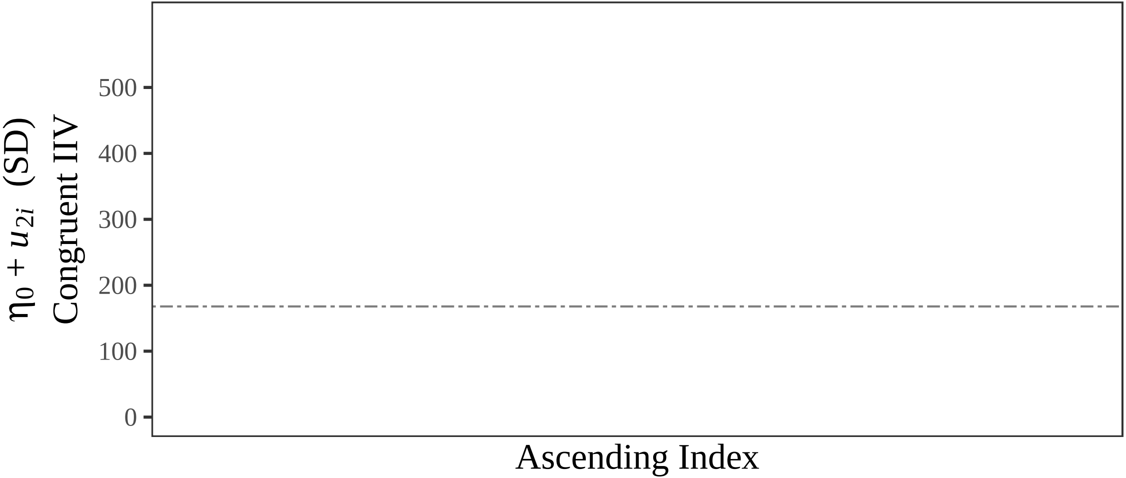


Within-Person Variability

$\eta_0 + u_{2i}$ (stability of congruent responses):



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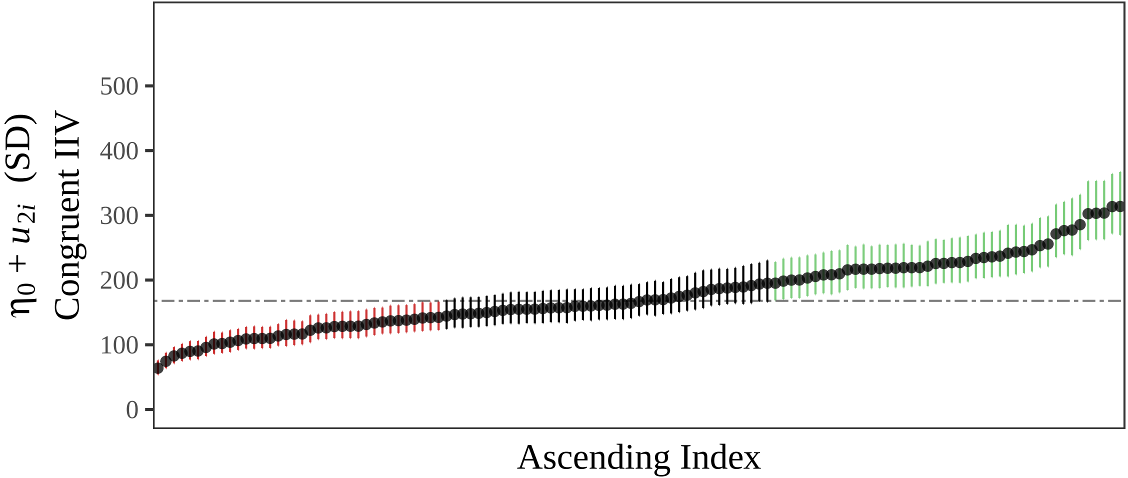

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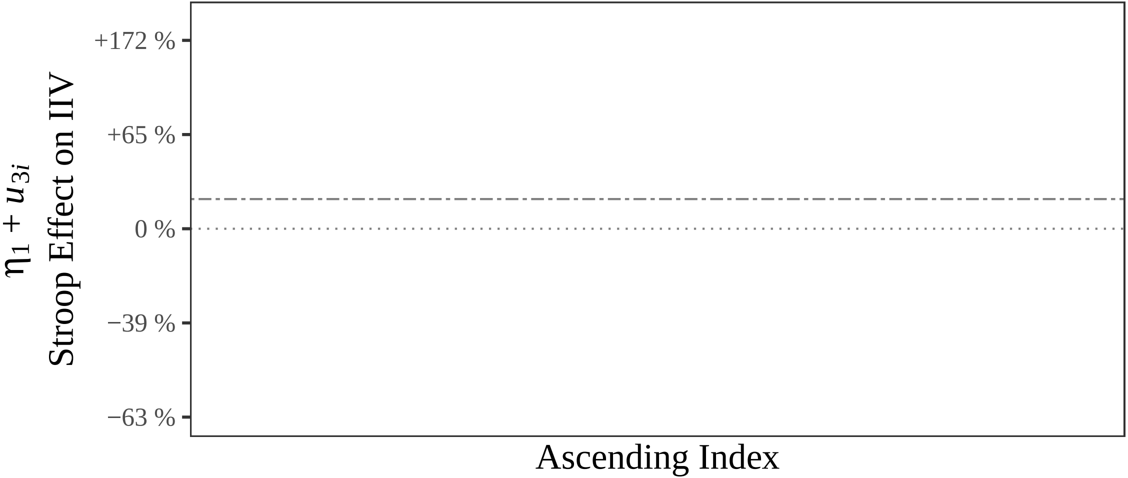
$\eta_1 + u_{3i}$ ("Stroop Effect" on within-person variability):



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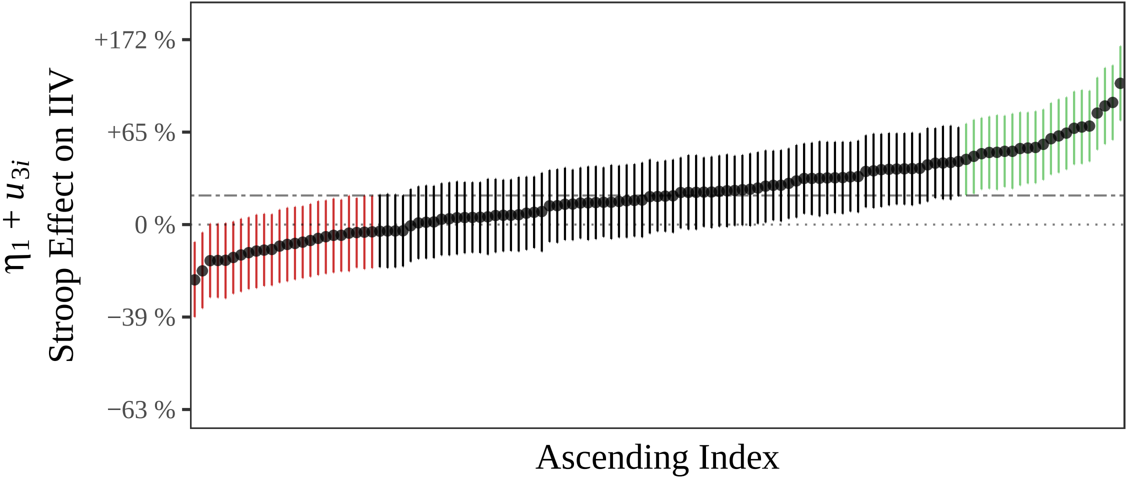
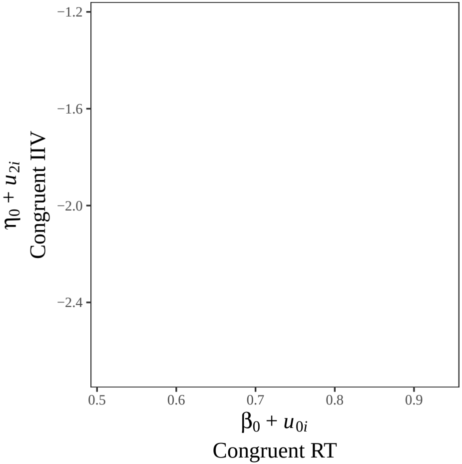


Within-Person Variability and Mean Relations

$\Omega_{3,1}(\rho_{02})$:



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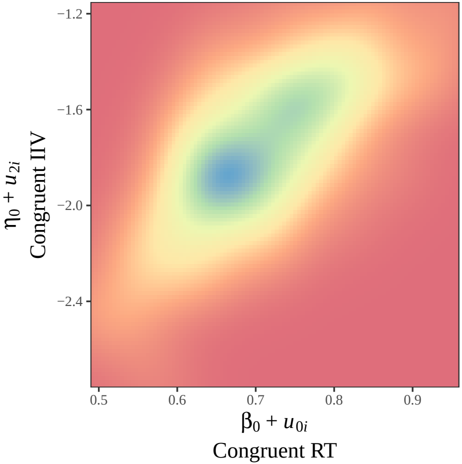

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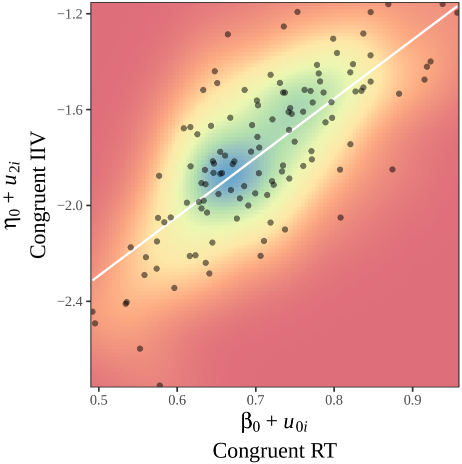

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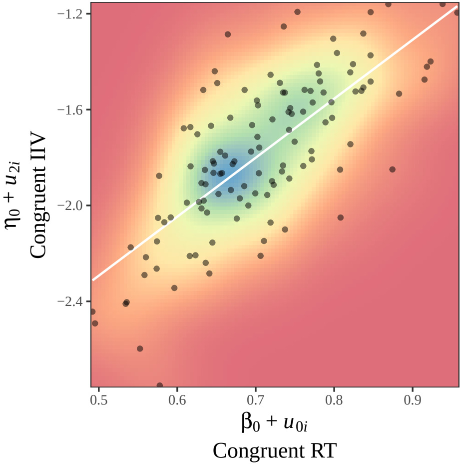

$$\mathcal{H}_0 : \rho_{ij} = 0$$
$$\mathcal{H}_1 : \rho_{ij} > 0$$
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Parameter	M	$p(\mathcal{H}_0 \mid \mathbf{Y})$	$p(\mathcal{H}_1 \mid \mathbf{Y})$	$p(\mathcal{H}_2 \mid \mathbf{Y})$
$cor(u_{0i}, u_{2i})$	0.68	0.00	1.00	0.00

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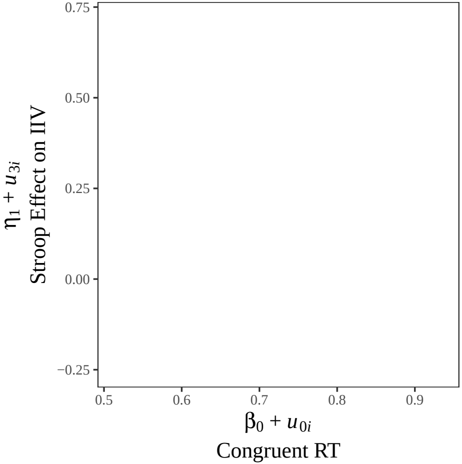

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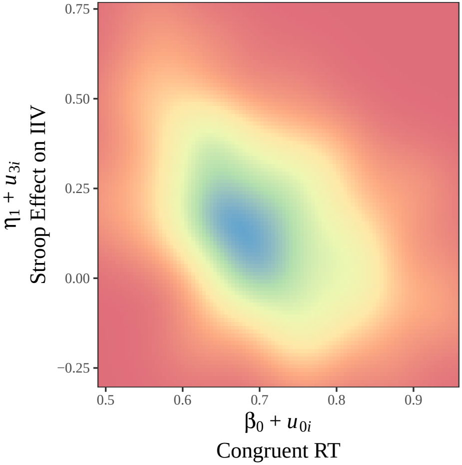

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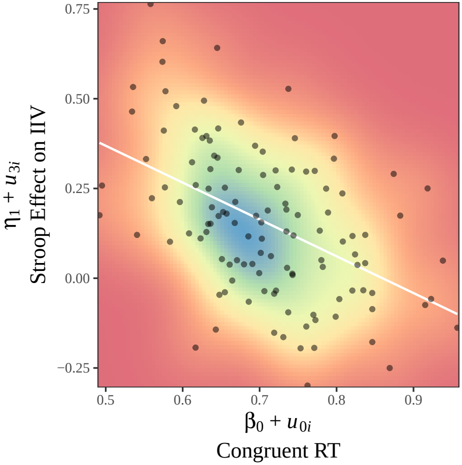

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$cor(u_{0i}, u_{3i})$	-0.40	0.00	0.00	1.00

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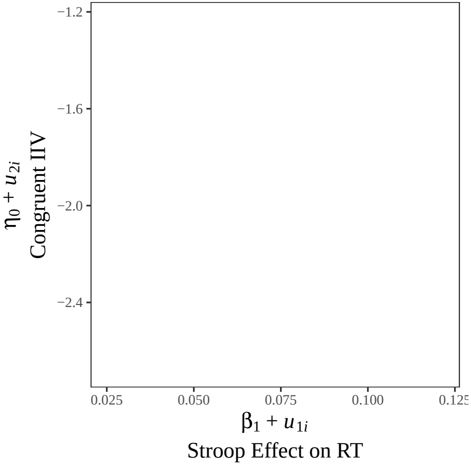
$\Omega_{3,2}(\rho_{12})$:



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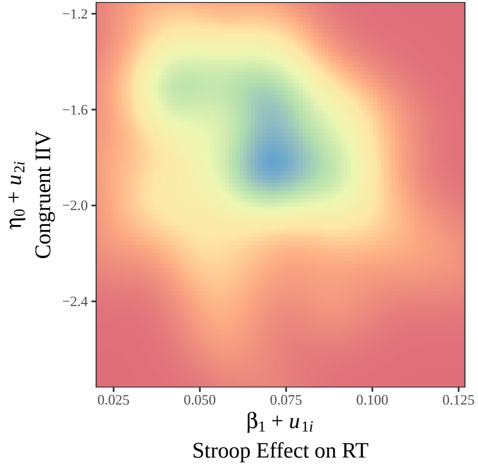

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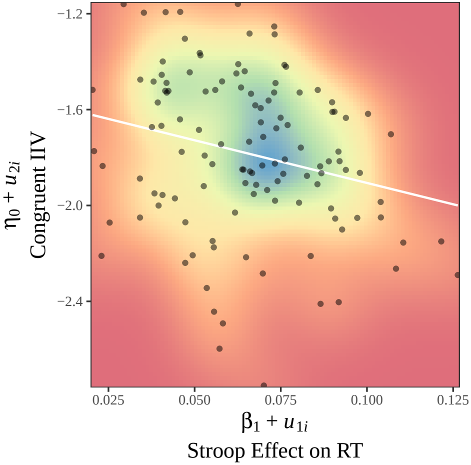

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$cor(u_{1i}, u_{2i})$	-0.19	0.42	0.05	0.53

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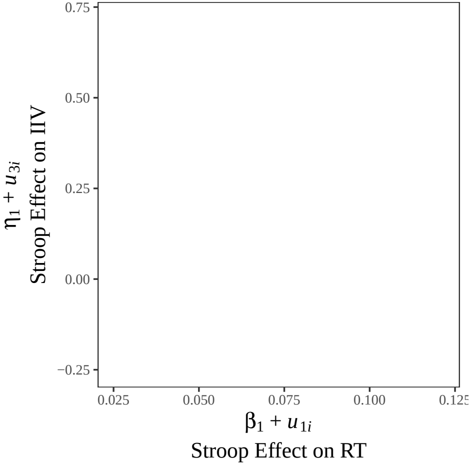

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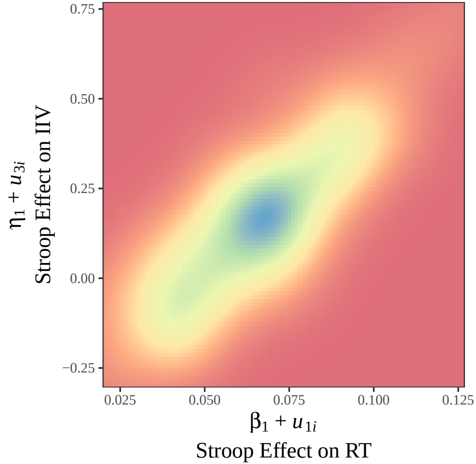

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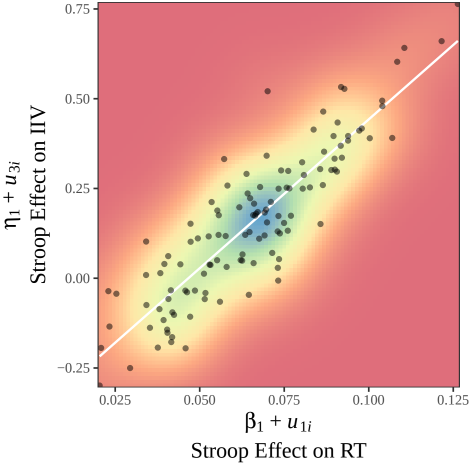

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$cor(u_{1i}, u_{3i})$	0.80	0.00	1.00	0.00

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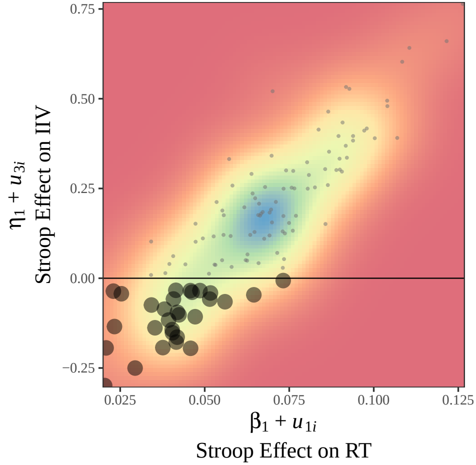

Some people became slower and more consistent–i.e., a higher mean yet lower variance

This contradicts the lawful relationship at the individual level !

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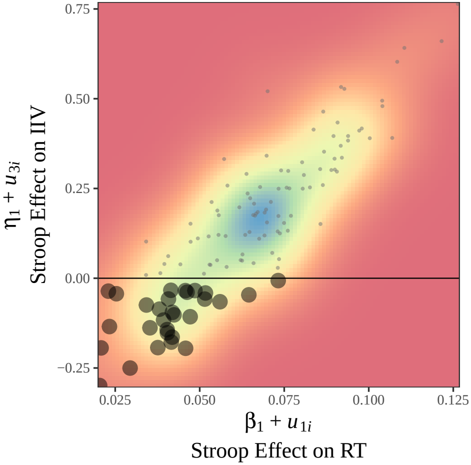

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Substantive:

- An intricate web of within-person variability and mean relations
- Individual differences in within-person variability
  - Individual differences in "noise" or measurement "error" ?
  - or, **Individual differences in response time (in)stability !**

Methodological:

- Mixed-models assume each person (or) group is equally consistent (same sigma)
- Shrinkage is determined by the ratio of between-individual to total variance
  - Thus, in the case of varying within-person variability, many estimates will be penalized too much
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