

# Numerical Comparative Dynamics: Ball Python Breeding

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## ROADMAP OF SEMINAR

1. Snake in a Vacuum
2. Tweak of Becker 1973
3. Problems
4. Approximations
  - ▶ Maximize Average Profits
  - ▶ Maximize Genes

## Snake in a Vacuum

$$\max_T \left\{ V(T) = \sum_{j=1}^T (p\Pi(j) - c)\delta^j \right\}$$

Not very close to reality.

## Tweak of Becker 1973

Supposing you start with  $I$  males and  $J$  females, and  $I + J$  is less than your total capacity.

$$\begin{aligned} \max_{\Pi} & \left\{ \sum_{t=1}^{\infty} \sum_{i \in I_t} g(m_i, f_{\Pi(i)}, t) \delta^t \right\} \\ \text{s.t.} & \quad I_t + J_t \leq C \end{aligned}$$

where  $\Pi$  maps to possible subsets of 5 or less elements of the set of females  $J_t$ , and  $C$  is the capacity.

## Problems

- ▶ Not PAM.
- ▶  $g$  may be a profit function, but there doesn't seem to be a closed form for  $\Pi$
- ▶ Brute force method is  $\mathcal{O}(IJ^4)$  in each period.

## **Approximations**

## Maximize Average Profits

$$\begin{aligned} \max_{\mathbf{X}, \mathbf{y}, \mathbf{z}} & \left\{ \sum_{i \in I} \mathbf{r}_i \mathbf{x}_i^T - 80 \mathbf{y}^T \mathbf{1}_I - 80 \mathbf{z}^T \mathbf{1}_J \right\} \\ \text{s.t.} \quad & \mathbf{X} \mathbf{1}_J \leq 5 \mathbf{1}_I \\ & \mathbf{X}^T \mathbf{1}_I \leq \mathbf{1}_J \\ & \mathbf{X} \mathbf{1}_J \leq M \mathbf{y} \\ & \mathbf{X}^T \mathbf{1}_I \leq M \mathbf{z} \\ & \mathbf{y}^T \mathbf{1}_I + \mathbf{z}^T \mathbf{1}_J \leq 15 \\ & x_{ij}, y_i, z_j \in \{0, 1\} \end{aligned}$$

## Maximize Weighted Number of Genes

This is what snake breeders actually do, and I implemented it as follows:

$$\begin{aligned}
 & \max_{\mathbf{x}, \mathbf{y}, \mathbf{z}} \left\{ \sum_{i \in I} g_i x_i^T \right\} \\
 & s.t. \quad \mathbf{X} \mathbf{1}_J \leq 5 \mathbf{1}_I \\
 & \quad \mathbf{X}^T \mathbf{1}_I \leq \mathbf{1}_J \\
 & \quad \mathbf{X} \mathbf{1}_J \leq M \mathbf{y} \\
 & \quad \mathbf{X}^T \mathbf{1}_I \leq M \mathbf{z} \\
 & \quad \mathbf{y}^T \mathbf{1}_I + \mathbf{z}^T \mathbf{1}_J \leq 15 \\
 & \quad x_{ij}, y_i, z_j \in \{0, 1\}
 \end{aligned}$$