Circuit Theory

Electric circuits represent mathematical models that approximate the behavior of an actual electrical system¹.

1.1 Lumped-parameter System

Because electrical effects happen instantaneously through a system, the electric signals are assumed to affect every point of the system at the same time. The circuit has to meet certain requirements size-wise to be considered a **lumped-parameter system**: in short, if the dimension of the system (area) is $\frac{1}{10}$ th the size or smaller than the wavelength of the electric signals. Wavelength is found by dividing the speed of light (commonly referred to as c, which measures roughly $3x10^8$ meters/second) by the frequency of the electric signal, as shown by:

$$c = \lambda * f \tag{1.1}$$

$$\lambda = \frac{c}{f} = \frac{3 * 10^8}{f} \tag{1.2}$$

so, the system can be considered a lumped-parameter circuit if $\lambda \geq \frac{1}{10}$ th the area of the circuit

¹Typically the term describes both the model and the actual system

1.2 Circuit Elements

Each type of circuit element behaves in a certain and specific manner, and can be described mathematically in the circuit model. These elements are called **ideal circuit components**, and are meant to accurately represent the behavior of the components in the real world (to a degree of reasonable accuracy).

1.3 Voltage

Voltage is defined as the energy per unit charge caused by the separation of positive and negative charges. The mathematical definition of voltage is:

$$V = \frac{dw}{dq} \tag{1.3}$$

where V is the voltage in volts, w is the energy in joules and q is the charge in coulombs

Voltage is the electric force that manifests itself due to separation of charge.

1.4 Current

Current is defined as the rate at which charge flows. Mathematically, it is defined as:

$$I = \frac{dq}{dt} \tag{1.4}$$

where I is the current in amperes, q is the charge in coulombs and t id the time in seconds

Passive Sign Convention

Resistors

Resistors are electronic components that resist the flow of the current in a system, and dissipate energy from a circuit in the form of heat. Resistors are simple (non-complex) circuit elements, and are governed by a few electrical laws.

3.1 Ohm's Law

Chapter 4
Nodal Analysis

Chapter 5
Mesh Analysis

Chapter 6 Operational Amplifiers

Capacitors

Inductors

Inductors are largely related to magnetic fields, as they store energy in the induced magnetic field created when current runs through them.

8.1 Magnetic Field Contributions From Current Carrying Wires

Biot-Savart Law

From a current-carrying wire, each infinitesimally small part of wire contributes to the total magnetic field at a given point. The magnetic field is referred to as \mathbf{H} . This small contribution is given by:

$$d\overrightarrow{H} = \frac{I * d\overrightarrow{l} \times \overrightarrow{R}}{4\pi R^2} \tag{8.1}$$

where I is the current in the wire (in amps), $d\overrightarrow{l}$ is the "piece" of wire (in meters), \overrightarrow{R} is the direction vector from the "piece" of wire to the point of the magnetic field (unitless), and R is the distance from the wire "piece" to the point (in meters)

Equation 8.1.1: Partial Contribution to Magnetic Field From a Current-Carrying Wire

Therefore the total magnetic field at a given point from a currentcarrying wire is:

$$\overrightarrow{H} = \int \frac{I * d\overrightarrow{l} \times \overrightarrow{R}}{4\pi R^2}$$
 (8.2)

Equation 8.1.2: Total Magnetic Field at a Given Point from a Current-Carrying Wire

The units of \overrightarrow{H} is $\frac{amps}{meter}$, which is similar to the units of electric fields, \overrightarrow{E} , which is in $\frac{volts}{meter}$

Amperes Law

Ampere stated that if you integrated the magnetic field intensity about a closed path around a current-carrying wire, then it would equal the current enclosed by the wire, given by the formula:

$$\oint \overrightarrow{H} \cdot d\overrightarrow{l'} = I_{enclosed}$$
(8.3)

Equation 8.1.3: Amperes Law

Forces between current carrying wires also occur due to the induced magnetic fields created by the electrons moving in the wire (the current): this is called the Lorentz Force.

$$\overrightarrow{F} = q(\overrightarrow{E} + \overrightarrow{v} \times \overrightarrow{B}) \tag{8.4}$$

where \overrightarrow{F} is the force (in Newtons), q is charge (in coulombs), \overrightarrow{E} is the electric field, \overrightarrow{v} is the velocity of the charge (in meters/second, and \overrightarrow{B} is the magnetic field

Equation 8.1.4: Lorentz Force

8.2 Magnetic Field Intensity and Magnetic Flux Density

Transients, Steady-States and First Order Circuits

Chapter 10 Steady-State Sinusoidal Analysis

Chapter 11
Sinusoidal AC Power Analysis

Chapter 12 Summary