

MATHCOUNTS® Problem of the Week Archive

School Makeover – September 14, 2020

Problems & Solutions

Lou has been contracted to tile classroom floors as part of the renovation of a local school. During the first five days on the job, Lou completed tiling 4, 5, 6, 4 and 8 classrooms, respectively. What is the average number of classrooms Lou tiled each day during that five-day period? Express your answer as a decimal to the nearest tenth.

To determine the average number of classrooms, we need to calculate the total number of classrooms Lou tiled over the five days and divide by five. Doing so, we find that the average number of classrooms that Lou can tile in a day is $(4 + 5 + 6 + 4 + 8) \div 5 = 27 \div 5 = 5.4$ classrooms.

Electricians Chris and Dave have been contracted to wire every classroom in the same school. Each of the classrooms requires exactly the same electrical work. Working alone, Chris can complete the wiring for one classroom in 20 hours, and it takes Dave 15 hours to do the same job by himself. After 120 hours of work, they had completed wiring $\frac{1}{2}$ of the classrooms in the school. How many classrooms are in the school?

A common measure would make Chris' and Dave's rates easier to compare. Using the least common multiple of 20 and 15, which is 60, will work. At a rate of 1 classroom every 20 hours, Chris can wire 3 classrooms in 60 hours. At a rate of 1 classroom every 15 hours, Dave can wire 4 classrooms in 60 hours. So, every 60 hours, they complete wiring a total of 7 classrooms. That means in 120 hours, they will complete wiring $(120/60) \times 7 = 2 \times 7 = 14$ classrooms. If this is $\frac{1}{2}$ of the total number of classrooms, the school must have a total of $2 \times 14 = 28$ classrooms.

In the school's main office there are four smaller offices that each require the exact same wiring. Chris and Dave can each wire an office in exactly half the time it took them each to wire a classroom. Working together, how long will it take Chris and Dave to complete wiring the four smaller offices? Express your answer as a decimal to the nearest hundredth.

If it takes Chris half the time to wire the office as the classroom, he can wire an office in $20 \div 2 = 10$ hours. Similarly, it takes Dave $15 \div 2 = 7.5$ hours to wire an office. It follows that Chris can complete $\frac{1}{10}$ of the required wiring for an office in 1 hour. Likewise, Dave can complete $\frac{1}{7.5} = \frac{2}{15}$ of the wiring for an office in 1 hour. Together, they will be able to complete $\frac{1}{x}$ of the wiring in 1 hour. That means $\frac{1}{10} + \frac{2}{15} = \frac{1}{x} \rightarrow \frac{3}{30} + \frac{4}{30} = \frac{1}{x} \rightarrow \frac{7}{30} = \frac{1}{x} \rightarrow x = \frac{30}{7}$. Since it takes $\frac{30}{7}$ hours for Chris and Dave to wire one office working together, four offices will take $4(\frac{30}{7}) = 17.14285714 \approx 17.14$ hours.