$$f(x) = e^{-x^2}$$

$$Y(\omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-x^2} e^{i\omega x} dx = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-x^2+i\omega x} dx = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-(x+\frac{i\omega}{2})^2+(\frac{\omega}{2})^2} dx$$

Definimos $x^2 + y^2 = r^2$

$$I^2 = \int_0^{\infty} \int_0^{2\pi} re^{-r^2} dr d\theta = -\pi e^{-r^2}\Big|_0^{\infty} = \pi \qquad \longrightarrow \qquad \therefore I = \pi$$

Regresando a la transformada original:

$$\frac{\sqrt{\pi}}{\sqrt{2\pi}} e^{-(\omega/2)^2} \qquad \longrightarrow \qquad Y(\omega) = \frac{1}{\sqrt{2}} e^{-\frac{\omega^2}{4}}$$